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Design Optimization of Spur Gear Sets

by

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ABSTRACT

Design optimization problems are typically characterized by a large number of inequality constraints, many of them satisfied as equalities at the optimum. To take advantage of this fact, we have used an augmented Lagrangian approach together with monotonicity analysis to optimize the design of spur gear sets. Two examples are included; namely, minimizing the pinion diameter of a gear set and minimizing the weight of a gear reducer.

INTRODUCTION

The importance of identifying the critical requirements for a design is well recognized by engineer designers. In design optimization context this corresponds to determining the active constraints at the optimal solution. An active constraint is an inequality constraint which is satisfied in the form of equality at the optimum. The method of monotonicity analysis originally proposed by Wilde (1) and developed by Papalambros (2) aims at establishing rules for constraint activity identification. A summary of the developments in this area is given by Papalambros and Li (3). Zhou (4) and Azarm (5) developed algorithms for nonlinear constrained optimization which utilized local monotonicy information. In this paper we present an algorithm for constrained nonlinear optimization problems which is based on the local monotonicity information together with an augmented Lagrangian method. It may be applied to a problem which is formulated in the following form:

Minimize $f(x)$

$$\text{subject to:} \quad \begin{array}{ll} g_j(x) \le 0 & j = 1,\ldots,m \\ g_j(x) = 0 & j = (m+1),\ldots,p \end{array} \qquad (1)$$

where f and g_j are (scalar) objective and constraint functions, and x is a n-vector of design variables. The algorithm is similar to the one proposed by Zhou (4) coupled with the local monotonicity information proposed by Azarm (5). A short description of the algorithm is presented in the next section. Subsequent sections present two examples involving gear design. It is appropriate to note that the program should be considered as still under development.

OUTLINE OF THE ALGORITHM

Here we focus on the basic operation of the algorithm which is based on the local monotonicity information together with an augmented Lagrangian method. We use the local monotonicity information (5) to the problem of Eq. (1) to identify the active constraints. The active inequalities together with equalities are used to form a penalty function which is then minimized using the augmented Lagrangian method of the Powell-Hestenes (6,7). The solution to this penalty function is used to estimate a new set of active constraints and thus a new penalty function is formed and minimized. This operation is repeated until an optimal solution to the problem of Eq. (1) is obtained.
We can now summarize the basic steps of the algorithm. Consider an initial point $x^{(o)}$ and set k=1 to begin:
Step 1: Find partial derivatives of the objective and constraint functions.

Step 2: Use local monotonicity (5) to identify the active constraints.

Step 3: Find the minimum of the following penalty function using the Fletcher and Reeves method (8):

$$P(x,\lambda^{(t)}) = f(x) - R\sum_{j=1}^{J} \{[g_j(x)+\lambda_j^{(t)}]^2 - [\lambda_j^{(t)}]^2\} \quad (2)$$

where R is a constant penalty term, $\lambda^{(t)}$ is the multiplier estimate, and J is the total number of active inequalities and equalities. The multipliers at (t+1)st stage of Step 3 are formed according to the following rule:

$$\lambda_j^{(t+1)} = g_j(x)^{(t)} + \lambda_j^{(t)} \qquad j = 1,\ldots,J \quad (3)$$

Step 4: Check the termination criteria, if satisfied then stop; otherwise set k = k+1 and go to Step 1.

EXAMPLES

In this section we present two examples of the design optimization of spur gear sets where the algorithm was implemented. In the first example which has two design variables, the objective is to minimize the pinion diameter. In the second example which has seven design variables, the objective is to minimize the weight of a gear reducer.

Example 1

The model for this example expands the work of Carroll and Johnson (9) by including the AGMA safety factor considering the overload and load distribution factor. In addition, face width and standard pitch considerations are taken into account. The design variables for this example are the pitch (P) and the number of pinion teeth (T_1). The objective is to minimize the pinion diameter, i.e. T_1/P, and the constraints for this example are:
Interference (g_1) - In order to have correct tooth action, it is necessary for the point of contact of two mating teeth to lie on the involute profile. Interference is said to occur when one of the teeth having a larger addendum comes in contact with non-involute portion of the mating tooth. For no interference, the maximum allowable addendum circle radius ($R_{a(max)}$) is (10):

$$R_{a(max)} = [R_2^2\cos^2\phi + (R_1\sin\phi + R_2\sin\phi)^2]^{1/2} \quad (4)$$

Using standard addendum for no interference, we have:

$$\frac{2R_1}{T_1} < R_{a(max)} - R_2 \quad (5)$$

So, this constraint is:

$$K_1 - T_1 < 0 \ ; \text{ where } \quad K_1 = 2/[(G^2 + \sin^2\phi(2G+1))^{1/2} - G]. \quad (6)$$

Face Width (g_2) - Quite often gear sets which have face width greater than five times the circular pitch, have nonuniform load distribution (11). Therefore, this constraint is:

$$F < \frac{5\pi}{P} \qquad \text{or} \qquad T_1 - K_2 < 0 \ ; \text{ where } \quad K_2 = 5\pi/\lambda. \quad (7)$$

Bending Failure (g_3) - The original model for bending failure developed by Lewis is given as (11):

$$\sigma_B = \frac{WP}{Fy} \quad (8)$$

where y is the Lewis form factor. Then $\sigma_B = S_B/N_G$, where $N_G = K_o K_m n$ is used to compute the safety factor. Based on the AGMA recommendation (11) in order to guard against failure we use $n > 2$. Therefore, this constraint is:

$$K_3 P^3 - T_1^2 < 0 \ ; \text{ where } \quad K_3 = (792000\ H_p K_o K_m)/(\pi N \lambda y S_B). \quad (9)$$

Pitting Failure (g_4) - Pitting failure occurs when excessive compressive stresses are applied on gear tooth. The Hertz contact stress for a spur gear is given as (11):

$$\sigma_H^2 = W[(1/\rho_1) + (1/\rho_2)]C_p^2/(F\cos\phi) \quad (10)$$

where $C_p = [(1/\pi)/((1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2)]^{1/2}$. Since σ_H and W are not linearly related, then the permissible transmitted load (W_p) is:

$$W_P = N_G W \tag{11}$$

where $N_G = K_O K_m n$ is used to compute the safety factor. Based on the AGMA recommendation, $n \geq 2$ is selected. Note that W_P is derived from Eq. (14) by changing σ_H to S_H and W to W_P. Therefore, this constraint is:

$$K_4 P^3 - T_B T_1^{\ 3} \leq 0 \tag{12}$$

where $$K_4 = (792000\ H_P E K_O K_m \sin\phi)/((1-\nu^2)\pi^2 S_H^{\ 2} \lambda N \cos^2\phi) \tag{13}$$

$$T_B = \theta_B[\sin\phi - \theta_B \cos\phi/(1+G)] \tag{14}$$

$$\theta_B = [(1+2/T_1)^2 - \cos^2\phi]^{1/2}/\cos\phi - 2\pi/T_1 \tag{15}$$

Score Failure (g_5) - Once pitting starts and gear is kept in operation, the surface of gear teeth will be worn away; i.e., scoring failure. Scoring failure may even happen before pitting due to the lack of lubrication. Similar constraint as g_4 can be derived for scoring failure in terms of θ_C:

$$K_5 P^3 - T_C T_1^{\ 3} \leq 0 \tag{16}$$

where $$K_5 = K_4 \tag{17}$$

$$T_C = \theta_C[\sin\phi - \theta_C \cos\phi/(1+G)] \tag{18}$$

$$\theta_C = [(1+G)\sin\phi - ((G+2/T_1)^2 - G^2\cos^2\phi)^{1/2}]/\cos\phi \tag{19}$$

Minimum Pitch (g_6) - This is a lower bound on pitch; $1 - P \leq 0$

Example 1: Solution

In (9) an optimal design was given for an example with G = 5; ϕ = 20 deg; HP = 20; λ = 0.25 S_H = 200 ksi; S_B = 60 ksi; E = 30×10^6 psi; ν = 0.25, k_v = 1, N = 1260.5 rpm. The optimal design was given as d = 2.25 (T_1 = 36; P = 16). We solved this problem by applying the algorithm to the newly developed model which accounts for AGMA safety factor including overload (k_o = 1.7) and load distribution factor (k_m = 1.5) with the face width and standard pitch considerations. The result was d = (T_1 = 36, P = 9) for which constraints g_3 and g_5 are active. The difference between our result and the result in (9) is due to the introduction of more constraints here. The optimal solution of (9) would be infeasible considering this model.

Example 2

In this example, the optimal design of a gear reducer for a given application is considered. This example was modeled by Golinski (12) and solved by several optimization schemes such as an adaptive optimization approach (13) utilizing five deterministic and stochastic algorithms and a heuristic combinatorial approach by Lee (14). The solutions obtained for all of the techniques are given in the cited references. However all of the reported optimized designs are infeasible for the given problem parameters. Here we start by simplifying the problem using monotonicity analysis. We then apply the algorithm to the simplified model. We also applied GRG2 (15-16) to the problem. The results of all computer runs are in agreement with each other, and moreover, the obtained solution is feasible. Here only the final model is presented. The reader may consult the cited references for further information on the original model development. The design variables for the example are:

x_1 = gear face width ; x_2 = teeth module ; x_3 = number of teeth of pinion ;

x_4 = distance between bearings of shaft 1 ; x_5 = distance between bearings of shaft 2 ;

x_6 = diameter of shaft 1 ; x_7 = diameter of shaft 2.

Model Description, Simplification, and Solution

The problem is to minimize the total weight of gear wheels and transmission shafts. The constraints used are as follows:

g_1 : upper bound on the bending stress of the gear tooth due to the tangential component of the gear load.

g_2 : upper bound on the contact stress of the gear tooth.

g_3-g_4 : upper bounds on the transverse deflection of the shafts.

g_5-g_6 : upper bounds on the stresses of the shafts.

g_7-g_{23} : dimensional restrictions based on space and/or experience.

g_{24}-g_{25} : design conditions for shafts based on experience.

The nonlinear programming statement for this model is:

$$\begin{aligned}\text{minimize } f(x) = {} & 0.7854x_1x_2{}^2(3.3333x_3{}^2 + 14.9334x_3 \\ & - 43.0934)-1.508x_1(x_6{}^2+x_7{}^2)+7.477(x_6{}^3+x_7{}^3) \\ & + 0.7854(x_4x_6{}^2+x_5x_7{}^2)\end{aligned} \tag{20}$$

$$\begin{aligned}
\text{subject to: } & g_1 : 27x_1^{-1}x_2^{-2}x_3^{-1} \leq 1 \; ; \quad g_2 : 397.5x_1^{-1}x_2^{-2}x_3^{-2} \leq 1 \\
& g_3 : 1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} \leq 1 \; ; \quad g_4 : 1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} \leq 1 \\
& g_5 : A_1/B_1 \leq 1100 \\
& \qquad A_1 = [(\frac{745x_4}{x_2x_3})^2 + 16.9\text{x}10^6]^{1/2} \; ; \quad B_1 = 0.1x_6^3 \\
& g_6 : A_2/B_2 \leq 850 \\
& \qquad A_2 = [(\frac{745x_5}{x_2x_3})^2 + 147.5\text{x}10^6]^{1/2} \; ; \quad B_2 = 0.2x_7^3 \\
& g_7 : x_2x_3 \leq 40 \\
& g_8 : 5 \leq x_1/x_2 \leq 12 : g_9 \qquad g_{16}: 7.3 \leq x_4 \leq 8.3 : g_{17} \\
& g_{10}: 2.6 \leq x_1 \leq 3.6 : g_{11} \qquad g_{18}: 7.3 \leq x_5 \leq 8.3 : g_{19} \\
& g_{12}: 0.7 \leq x_2 \leq 0.8 : g_{13} \qquad g_{20}: 2.9 \leq x_6 \leq 3.9 : g_{21} \\
& g_{14}: 17 \leq x_3 \leq 28 : g_{15} \qquad g_{22}: 5.0 \leq x_7 \leq 5.5 : g_{23} \\
& g_{24}: (1.5x_6+1.9)x_4^{-1} \leq 1 \qquad g_{25}: (1.1x_7+1.9)x_5^{-1} \leq 1
\end{aligned}$$

From the monotonicities of the variables for the model the following simplifications were made (5):

(1) With respect to x_1 constraint g_8 is active and g_1 is redundant.
(2) With respect to x_6 constraint g_5 is active, g_3 and g_{20} are redundant.
(3) With respect to x_7 constraint g_6 is active, g_4 and g_{22} are redundant.

We also found that constraints g_1-g_4, g_7, g_9, g_{20} and g_{22} are redundant (5). We then assume that g_5, g_6, and g_8 are as equalities in the algorithm to solve the simplified problem. The algorithm found that the constraints g_{12}, g_{14}, g_{16}, and g_{25} are also active at the solution where x = (3.5, 0.7, 17, 7.3, 7.72, 3.35, 5.29) for which the objective function has the value 2994.

CONCLUSION

An attempt has been made to develop a methodology for solving a general nonlinear programming problem in the form of Eq. (1). In the methodology presented here, an augmented Lagrangian together with local monotonicity information were used to identify the critical requirements. The methodology was applied to two examples in gear design optimization. It is a simple methodology as it has been implemented on a small personal computer.

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NOMENCLATURE

d	diameter of pinion	S_H	surface strength
E	elastic modulus	T	number of teeth
F	face width	W	transmitted
G	gear ratio	W_P	permissible load
H_P	horsepower	y	form factor
k_o	overload factor	θ_B	pinion roll angle to lowest point of single tooth contact
k_m	load distribution factor		
k_v	dynamic factor	θ_C	pinion roll angle to lowest point of tooth contact
N	speed in rpm		
N_G	gear safety factor	λ	length to diameter ratio
n	AGMA safety factor	ν	Poisson's ratio
P	pitch	$\rho 1, \rho 2$	radii of adenda circle
R_1, R_2	pitch circle radii	σ_B	bending stress
S_B	bending strength	σ_H	Hertz contact stress
		ϕ	pressure angle

Subscripts: 1 for pinion; 2 for gear