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5 February 2007

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Re.: *The Nature of Time*

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The Nature of Time

Edited by T. GOLD

WITH THE ASSISTANCE OF

D. L. SCHUMACHER

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THE NATURE OF TIME *is the report of a meeting held at Cornell University on the thirtieth and thirty-first of May and the first of June, 1963. The meeting was convened by Professor H. Bondi and Professor T. Gold, and it was to a large extent supported by a contract and grant from the United States Air Force Office of Scientific Research (No. AFOSR AF49 [638]–1527).*

Preface

The scientists who participated in the meeting recorded in this book were chosen not only for their eminence but also for their willingness to discuss a subject that quite clearly is not well understood. By limiting the gathering to twenty-two people, Professor Bondi and I hoped to promote informal discussion among them and to relieve their embarrassment at having to voice opinions about the nature of time.

It is an embarrassment for a scientist who has concerned himself with the basic nature of the physical laws to have to admit that the coordinate system in which the laws are imbedded is itself quite mysterious. Lack of understanding is not the only difficulty; many other areas of physical science are not well understood. But in this case the problem is so fundamental that no thoughtful scientist can claim to have given it no consideration. Most believe that they have gained some basic understanding and are then distressed to find a divergence from the views of their colleagues. Introspective understanding of the flow of time is basic to all our physics, and yet it is not clear how this idea of time is derived or what status it ought to have in the description of the physical world.

The invitation to the participants contained the statement that the subjects we intended to discuss would include the following:

1. Cosmic time and the notion of large-scale change; thermodynamic or entropic time in cosmology; advanced and retarded potentials; suitability of the notion of progressing time for the discussion of cosmology and small-scale physics.

2. Irreversible processes in general; entropy in thermodynamics and information theory.

3. Microphysics—irreversibility in quantum physics; theory of measurement; PCT theorem and related matters.

Most of these subjects are in fact included in the conversations as reported here. But the reader must not expect any very systematic treatment or course of instruction. Instead he will find, in addition to a number of prepared papers, a lot of conversation that is sometimes deep, sometimes flippant, and sometimes, I am afraid, incomprehensible.

There were severe doubts whether the proceedings of the conference should be published. The informality of the meeting might make for difficult reading; the absence of an audience allowed the participants a degree of freedom of expression not generally acceptable in scientific writing. The impression that a spoken remark makes on a small group of colleagues can be judged well on the spot—but it may not be at all the same as that which it will make on the reader. Still, at the end of the conference I urged publication, and all but one of the participants were agreeable. I urged publication because it seemed to me that students of the subject should be informed not only of the aspects that are understood and presented here, but also of the profound uncertainties, the basic divergences of opinion that exist in so fundamental a discussion. Most young persons beginning work in a field believe it to be fairly systematic and well understood, and as they learn more are disappointed at the muddled thinking, the ignorance, and the uncertainty among experts. This disillusionment is an essential part of the learning process. It is this that usually gives the student the courage to enter the fray himself. If this report fails to have any other useful results, I am sure it will make amply clear, through the degree of understanding that has been achieved, that the subject is not forbidding.

One participant, hereafter referred to as "Mr. X," was against publication. To him the informality of the conversation had been such an essential part of the conference that he felt our promised privacy was being violated. He would have prepared his remarks quite differently for publication. Really, of course, he was more consistent than the rest of us, who pretended to ignore the tape recorder when it should have looked to us like a large audience listening to our deliberations. Still, Mr. X did not insist on forbidding the publication. His contributions have, however, not had the benefit of his corrections and may be on occasions incorrect through mistakes in transcription. The reader is therefore requested not to quote Mr. X.

I hope that the process of transcribing and editing and the work of the participants in checking their contributions have led to a text that in the main represents what was said. I am sure, though, that some errors and discrepancies will remain, and for these I offer my apologies.

The difficult work of transcribing from the tapes was carried out by Mrs. A. Echandi and the first round of editing by Mr. B. Ulrich. The main tasks of editing, corresponding with the participants, and preparing the final draft were carried out by Mr. D. L. Schumacher. These three have my thanks for what proved to be an arduous task.

But chiefly I must thank the participants, for they made this book through their efforts in coming to the meeting, contributing papers and discussion, correcting transcripts, and above all giving us the benefit of their understanding of many aspects of this vexing subject.

T. Gold

Cornell University
March 1967

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MR. X [Richard Feynman]

THE NATURE OF TIME

Introduction

T. Gold and H. Bondi

Gold

I wish first to welcome my distinguished colleagues; we are very honored that you have all responded to this invitation.

The organization of this meeting is minimal. We wanted to make it a discussion meeting around the table, and we have therefore tried to avoid the distractions that would arise from a larger group and from holding the meeting in a bigger room with a large audience. I had to resist strong pressures in this respect. Many other people wanted to join the meeting, and I am sure that some of them would have made interesting contributions. Nevertheless, the meeting would have become much more formal. I felt that there was a strong case for maintaining a kind of informality such that we can speak our minds and have a free give-and-take in an argument, rather than be forced to make speeches.

The problem that we are here to discuss, the problem of time, is a very strange one. Nearly everyone who works in physics has come to the conclusion that an understanding of the nature of time is basic. Each physicist has developed a point of view about it. There are great divergences in these points of view which have not been well aired; the literature is disappointingly limited in this subject. I think we shall discover that there is no agreement regarding even the most basic matters. It amazes me that a concept like time can have such a profound tradition in the physical sciences in spite of the fact that it is regarded in such widely different ways.

Is the lack of agreement due perhaps to the fact that a sensible outlook has not yet been developed? We seem to derive the notion of a flow of time in the first place from introspection. We then use the introspective notion to classify observations in the physical world. But does

VI. Infinite Red-Shifts in General Relativity

C. MISNER

I would like to talk about how people get out of touch with each other. One example of this situation occurs in the horizons that appear in cosmology. Two observers who were able to talk to each other head off in different directions and eventually there is no longer a possibility of communication between them. Another question is whether it is possible for the universe genuinely to fission. If so, we would get the sort of behavior depicted in Figure VI-1, where the universe we started

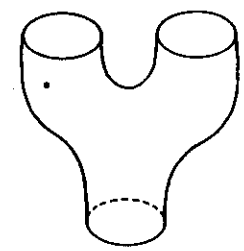

FIGURE VI-1

with evolves into two. But I shall ignore these cosmological questions and talk about a local situation in which two observers can get out of touch with each other. This situation occurs in the Oppenheimer-Snyder problem of continuing stellar collapse.[1] The problem has become interesting recently from a number of points of view, and we shall try to review it here. In particular, there is a geometrical picture of this Oppenheimer-Snyder solution, given by D. L. Beckedorff,[2] which I will summarize.

How can violent stellar collapse come about in the real world? When stars evolve and get cold, the radiation pressure is no longer able to support them, and they begin to collapse. Eventually, the star might become merely a Fermi sea, whose pressure prevents collapse. As the star crushes down, there is less space, but the electrons still need the same number of quantum states. These fermions then fill up more of the momentum space, since they have less configuration space available to them. The work it takes for them to fill up momentum space determines the pressure that resists further collapse. As we fill a state with momentum p, we have to supply an energy $E=p^2/2m$, and this determines the force law. Eventually, as things crush down, the electrons become relativistic and $E=pc$. The difference between the square power of p and the linear power of p in the nonrelativistic and relativistic cases is a crucial one. It turns out that when we work with the relativistic force law, there is no equilibrium. Either the system will expand and continue expanding as long as this relativistic relation holds, or else it will contract and continue contracting as long as this formula holds. This depends entirely on the total mass of the system; the critical mass is given, for an electron gas, by the Chandrasekhar limit,[3] which says that there cannot be equilibrium of a white-dwarf-type star of degenerate electron gas having total mass greater than about 1.5 solar masses.

What happens after the star collapses? If the electrons are crushed onto most of the protons, a degenerate neutron gas results. But what happens when the neutrons become relativistic? The critical mass in this case is smaller; it is on the order of .65 solar masses.[4] Above that there is no equilibrium supported by a neutron gas. Still, equilibrium can be found for other possibilities, such as those in which the star breaks up into smaller pieces, or in which angular momentum prevents the collapse. Both these possibilites are very real; both shedding of mass and spinning are known to be effects in stars. If we ignore these, it is because we cannot calculate them. My own feeling is that there is not an equilibrium in these circumstances. We should look forward to a continuing collapse.

We have been talking in terms of the degenerate-electron-gas equation of state, and the degenerate-neutron-gas equation of state. Wheeler has discussed also a detailed equation of state,[5] and Ambartsumian[6] has discussed the problem of composition of superdense stars. After the collapse, neutron levels are built up, and it becomes cheaper energetically to produce mesons, and so on, rather than to fill higher energy neutron levels. Therefore we have to ask about elementary particles before we can determine the equation of state. However, we can also

try to do the problem in the complete limit. Suppose we consider an incompressible fluid. By "incompressible fluid" is meant the following: we look for equilibrium situations, and the pressure is to be whatever is required to permit them. The density, which is the constant ρ occurring in the formula for $T_{\mu\nu}$, is specified beforehand.

We are then considering the standard and well-known problem of the interior Schwarzschild solution.[7] We still have a limit in this case; if there is too much mass in a given region, then we cannot find an equilibrium solution.[8] I will do the work in detail later, but the result is that the density which we assume in such a case has to satisfy

$$\rho \leq \left(\frac{M_\odot}{M}\right)^2 \times 1.4 \times 10^{16} \frac{\text{gm}}{\text{cm}^3}, \tag{1}$$

where M is the mass of the object and $M_\odot$ is the mass of the sun. We find an equilibrium solution of the Einstein equation for a mass M of incompressible fluid only when this condition is satisfied. At the center we have infinite pressure; in the $T_{\mu\nu}$ this implies an infinite curvature scalar of some kind. The separation of positive and negative energy states goes to zero at the center.

All the limits found by the particular equations of state ultimately fall below those given by the inequality (1) above. We may draw some conclusions from this inequality. If there is a big enough mass, the collapse can begin at low densities. So these elementary-particle spectra, nuclear-force laws, hard cores, and so forth are irrelevant to this question, in principle. In particular, let us put in a mass of $10^8\ M_\odot$ which just cancels the factor of 10^{16}. A body of $10^8\ M_\odot$ gathered together in one sphere cannot attain a zero-temperature equilibrium unless the density is less than that of water. So a very ordinary density would come about in this case.

Morrison

You must say something about the localization of the density, because the density of real matter is not really anything like this. It is on the order of either 10^{18} or zero.

Misner

Yes. Here we have assumed constant density.

MORRISON

Grainy matter is somehow singular. What will we see at short distances inside this object?

MISNER

If we put in a very small mass, such as the mass of the proton, then the density limit is very high; the density of the proton does not exceed it.

BERGMANN

This problem is more relevant to the collapse of a galaxy; for a galaxy, 1.4 gm/cm^3 is quite a hefty density. It is about 10^{30} times the normal density.

MISNER

I am not claiming that these are normal densities. I am just claiming that we do not have to worry about quantum effects. This is not nuclear matter. The answer to this problem does not in principle rely on the elementary-particle spectrum, unless we want to discuss collapse of relatively small objects of less than one solar mass. And for values of the order of one solar mass, densities somewhat higher than that of nuclear matter arise. So this case is on the borderline as regards our knowing or not knowing the equation of state.

The main thing that concerns us is the nonequilibrium solution when collapse begins. Only an ideal case is known—the case in which there is spherical symmetry and zero pressure; this is a free-falling collapse.

The question of hard nuclear cores has arisen in connection with the "borderline" case of collapse of stars having about one solar mass. S. Weinberg[9] has in this connection considered world models instead of stars, which is really the same question. He was concerned with oscillating universes, and how dense they can get. If we want the model to bounce rather than collapse, we must have attractive nuclear cores. The introduction of hard nuclear cores does not make the model bounce. Repulsive forces make the model collapse faster, because repulsive forces correspond to positive potential energies. Positive potential energies produce more energy and more gravitational attraction. When the limit has been passed, the hard core makes the object contract even faster. What we need therefore is a short-range, deep potential for attraction, if the object, whether world model or star, is to bounce.

Now we consider the state which can be treated as an exact solution in our idealization. We obtain it by putting together several solutions which we are familiar with. The Friedman world model turns out to describe properly the geometry corresponding to the state of pressure-free collapsing dust inside the star. To get a model representing the whole metric for the collapsing star, we match the Friedman metric onto the exterior Schwarzschild solution, which we follow in Kruskal's representation.

The metric for the *interior Schwarzschild solution* is[10]

$$ds^2 = R^2(dX^2 + \sin^2 X\, d\Omega^2) - \tfrac{3}{2}(\cos X_0 - \tfrac{1}{3}\cos X)\, dt^2$$
$$\text{where} \quad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2. \qquad (2)$$

This model is spherically symmetric, so we use $d\Omega^2$ as the metric on the sphere; we do not need to write out those angles explicitly all the time. The space part of the interior Schwarzschild solution is a three-dimensional sphere, and X is the third angle in addition to the usual two, and $\sin^2 X$ is the radius of a two-dimensional sphere of constant X. Then R is the radius of the three-dimensional sphere, because the interior of the Schwarzschild model looks like a section of a sphere within limiting angle X_0; we use the metric (2) only out to a certain maximum radius, determined by X_0, (Figure VI-2). This section of the three-dimensional

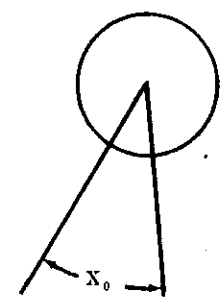

FIGURE VI-2

sphere filled with matter is to be joined to the exterior Schwarzschild solution. Because of the factor $\frac{1}{3}$ in the metric, we need $\cos X_0 > \frac{1}{3}$ in order to preserve the correct signature, so we can get only a moderate size angle X. The limit density which we wrote down is based on this fact, so if the density exceeds the limit, g_{00} possibly goes to zero at the center. The pressure, which is given by a formula which involves the same thing in the denominator, goes to infinity.

We can picture this whole Schwarzschild solution by drawing the curvature of the solution imbedded somehow in another space. Instead of having the exterior Schwarzschild solution go on towards its singularity, we stop it and fill it in with a section of the three-dimensional sphere where the matter lies (Figure VI-3).

The *Friedman solution* has essentially the same geometry as the three-

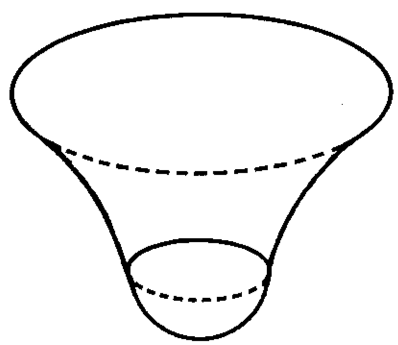

FIGURE VI-3

dimensional sphere, but this solution uses all of the sphere. This world model is filled with dust at zero pressure and uniform density; it differs locally from the Schwarzschild interior not in the instantaneous picture of the geometry, but in the dynamical picture. The Friedman universe is a collection of dust that expands and then collapses. So the Friedman universe has the same sort of spatial metric as the Schwarzschild interior, but with a time-dependent radius $R(t)$, again with three angles representing the three-dimensional sphere, $d\Sigma$.

$$\begin{aligned} ds^2 &= R^2(t)\, d\Sigma^2 - dt^2 \\ d\Sigma^2 &= dX^2 + \sin^2 X\, d\Omega^2. \end{aligned} \tag{3}$$

If we choose proper time for t we can get $g_{00}=1$, or at least a spatial constant for g_{00}, which otherwise could or could not be time-dependent. So $R^2 d\Sigma^2$ represents spatial sections, or three-dimensional spheres. The geometry is the same as before. Again we have the same angle X, but now it has the full range of values, and the radius of this sphere expands and collapses according to a certain law, which is a cycloid curve if there is no pressure.

The next familiar solution to consider is the *exterior Schwarzschild solution* in the Kruskal representation. With this we can know what is happening at the singularity. We need to know this because in stellar collapse, the matter follows geodesics and in finite proper time falls right into the Schwarzschild singularity at $r=2m$.

The following is the Schwarzschild metric in standard coordinates.

$$ds^2 = -\frac{dt^2}{\dfrac{2m}{t} - 1} + \left(\frac{2m}{t} - 1\right) dz^2 + t^2\, d\Omega^2 \tag{4}$$

I merely changed the names: r is usually written where we write t; t is usually written where we write z. What does the metric look like for $r<2m$? This form is appropriate for the case in which we are going towards what is normally called $r=0$. I simply point out that for these small values of Schwarzschild's r, which we call t, the signature is such that the r (our t) has a time interpretation. What is the sort of geometrical picture here? The space sections are the terms in dz^2 and $d\Omega^2$, and we see that as t goes to zero, the coefficient of dz^2 becomes positively infinite. So the z dimensions are stretching in the z direction as t goes to zero. The cross sections, which are spheres, are collapsing. So the geometrical picture is that of a cylinder, with z measured along it, and with cross sections being ordinary two-dimensional spheres. It is just like a piece of rubber tubing being pulled out as the time coordinate changes. It has cylindrical symmetry, because z does not appear anywhere else. The object stretches in one dimension and collapses in two. The infinite curvature arises when cross sections have zero radius.

Now, this solution ought to have something to do with the normal exterior Schwarzschild metric, which is a static situation, because if we follow the equation of a geodesic, it goes from $r>2m$ to $r<2m$. How do we put these solutions together? We introduce a single system which covers both sides of the "singularity." Kruskal[11] introduced coordinates u and v, by the following sort of transformation.

$$u^2 - v^2 = (r-1)e^r$$

$$\frac{u}{v} = \tanh t$$

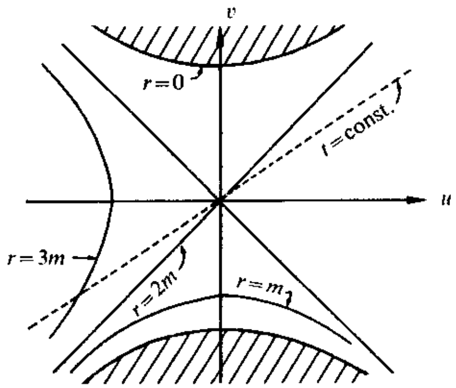

FIGURE VI-4

I do not guarantee every detail of these equations; we may need factors of $2m$ and such quantitative features. The variables r and t are now the same as the standard Schwarzschild r and t, and if we draw the u, v plane, we see that $r=2m$ corresponds to $(r-1)e^r=0$, or $u=\pm v$ (Figure VI-4). Standard surfaces, $t=$constant, the spacelike surfaces in the normal exterior Schwarzschild solution, are $u/v=$constant, so they are slanted lines through the origin of this diagram. An observer who is on a satellite orbiting around the star, or in any orbit of constant r, lies on curves $u^2-v^2=$constant, which are hyperbolas. On the other hand, r is a singular function of u and v at $r=0$, and the Schwarzschild curvature invariants become infinite there. Then $r=0$ corresponds to $u^2-v^2=-1$, which is another hyperbola. We never get into the shaded region because at $r=0$ we already have infinite curvature. The metric is roughly some function of r times du^2-dv^2, plus the other coordinates $r^2d\Omega^2$:

$$ds^2 = \frac{e^{-r}}{r}(du^2 - dv^2) + r^2\,d\Omega^2. \qquad (5)$$

The important feature of this metric is that the light cones are at 45° in the u, v plane, so we can conveniently discuss causality questions by using this diagram, so long as the light has no angular momentum and goes radially inwards or outwards.

The transformation laws are such that the metric is nonsingular throughout the entire region that is unshaded. It includes both areas $r>2m$ and $r<2m$, so we can see both of them. The part just next to the shaded region is the part which can be described as a collapsing cylinder. That is the $r<2m$ region. Below that is the familiar $r>2m$ region. The metric at $u=v$ shows no singular behavior whatever, so it gives somehow a smooth transition in the diagram between one of these behaviors and the other. I do not know how to think of that transition, but it is there.

How can we visualize the content of Figure VI-4? We should look at the geometry of this spacelike surface, $t=0$ where $v=0$. We can think of the surface as a succession of concentric spheres, each of radius r. We get some idea of how it looks by seeing how r varies with u. We start out with $r=\infty$ where $u=-\infty$; we then come to a sphere of a certain finite radius $r=2m$ at $u=0$, and we keep on going until $r=\infty$ again at $u=+\infty$. So this sequence of concentric spheres starts with large ones, comes to those of small finite size, and then comes to large ones again. This suggests Figure VI-5.

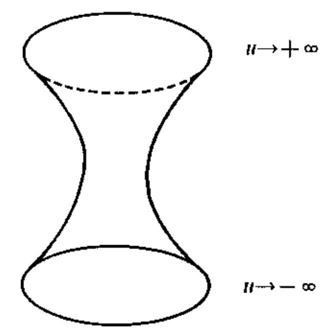

FIGURE VI-5

Now the entire metric will describe the dynamics of this. Suppose that an observer is at the neck of Figure VI-5. By looking at Figure VI-4, he interprets the $u=0$ line to be a geodesic as shown. It is a timelike geodesic, since light cones are at 45°. So an observer who starts at the neck will eventually end up at $r=0$. If we draw a spacelike surface $v=$const.>0 through him and ask for a description of this surface, we see a set of concentric spheres with $r=+\infty, \cdots 3m, 2m, m, 2m, 3m, \cdots +\infty$. We get qualitatively the same picture, Figure VI-5, as before, except that the minimum circumference does not correspond to a radius of $2m$ but to a smaller radius. Therefore what we find is that this space is essentially dynamic, and that the center is collapsing. We will tie this central collapse to the collapse of the star by throwing out some of the empty space and replacing it with an interior solution.

Figure VI-4 can also be used to visualize the causality relationships. We need to consider light rays, the radial null geodesics. Consider an "orbiting" observer at 1 who stays on curve $r>2m$, as in Figure VI-6. He can send a light signal which goes into the $r<2m$ region. If, however, this light ray hits another observer at 2, who is in the $r>2m$ region,

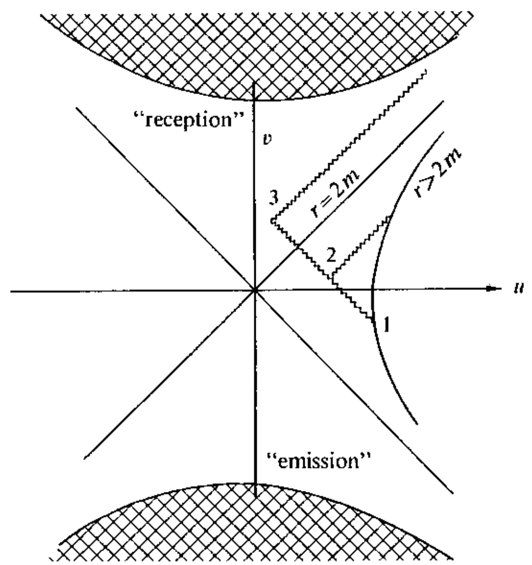

FIGURE VI-6

the first observer can get a reply. Observer 2 can send another message back which would intercept the first observer's orbit $r>2m$. So there can be communication between these two observers. Suppose however that the light ray goes on farther before it hits someone who replies, say, at some event 3. A message sent back never gets into the region $r>2m$ again! So their times have become disconnected. The phenomenon shows that there exists here an horizon of a type similar to that in cosmology; two observers who were originally in communication get out of communication. The observer following the radial geodesic cannot send a message to the observer orbiting on $r>2m$ after the former attains $r=2m$.

Of course, this solution has time symmetry. Messages from the $r<2m$ region with $v>0$ can never get out although messages can be received from outside. If we consider the lower part of the diagram, we see that an observer in the region $r<2m$ but $v<0$ can send messages out of this region, but he can never receive any from the exterior region. So we may label the regions, "emission" and "reception." For $v<0$, the region can emit but not receive messages from the outside, and for $v>0$ the $r<2m$ region can receive but not transmit communications to the outside.

One other thing that will be useful to us is to notice what happens in this diagram if we want to change the zero of time. Suppose there is an observer far out, and difficult to watch. Suppose we want to bring him back. We can do this just by changing the zero of time. Changing the zero of time will change the v/u ratio, but at constant r it keeps u^2-v^2 invariant. So a shift in the zero of time in the standard Schwarzschild coordinates corresponds to a Lorentz transformation in the u, v plane, namely a linear transformation which preserves the quadratic form u^2-v^2. To ask what happens to a given geodesic if we look at it from a different vantage point, we make such a pseudo-Lorentz transformation to obtain an equivalent geodesic differently situated in the diagram.

For the case of a collapsing star, we must take an exterior Schwarzschild solution as initial conditions. This is a section of a sphere representing uniform density of matter. If there is enough pressure, the geometry will stay static. If there is too much matter present, even infinite pressure will not prevent a collapse. But, under any circumstances, if the pressure is "turned off" the assembly will collapse. These initial conditions also represent, with changes of scale, the way the geometry looks all during the collapse. How do we represent these conditions, say, on the Kruskal diagram? At $t=0$ we accept the exterior Schwarzschild solution outside a certain radius. Inside, we erase it and match up the piece of a Friedman sphere with it. So part of the Kruskal diagram stops being meaningful at $t=0$. What will be the time development from these initial conditions? If there is no pressure, the particle that is on the surface falls on a geodesic. There are no forces, and the motion describes a geodesic for both the exterior and the interior metric. So we must throw away the shaded part of the diagram and replace it by some other solution, as in Figure VI-7.

We know what the exterior behavior looks like. We know that just before the radial geodesics cross $r=2m$, we can see the surface of the collapsing star. It emits light which we can see for a while, but eventually any light it emits no longer escapes. Even if there is some finite temperature at the surface of the star, but not enough pressure to interfere with the collapse, radiation cannot get out after the star has collapsed past $r=2m$. This represents an infinite red-shift, or infinite Doppler shift. Actually, we have a combination of gravitational red-shift and Doppler

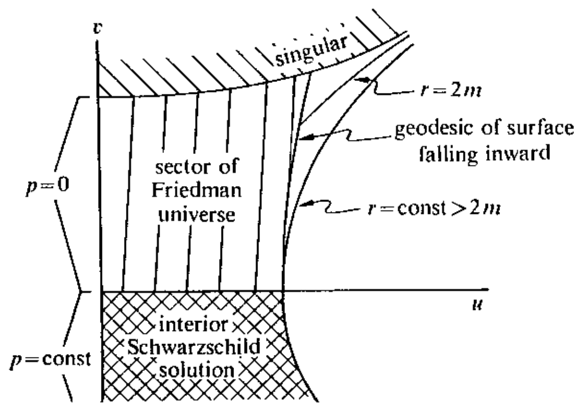

FIGURE VI-7

shift in this case. It is perhaps not meaningful to try to separate the two effects.

For the external observer, the time is constant along radial lines of this diagram. As the radial line swings up toward $u=v$, we have $t\to\infty$. So it takes an infinite length of time for the particle to disappear. An observer at constant r reaches $t\to\infty$ before noting the disappearance of the particle falling in. The observer outside sees it getting redder and redder.

Now, the method of making sure that we have the whole solution under control is to match a solution for the exterior, represented in Schwarzschild or Kruskal coordinates, to an interior solution. We know that the borderline representing the falling particles must be a geodesic. So we know what border we require in that part of the diagram. We now put a border on the Friedman model, a border which must be, as we know from the previous discussions of the Friedman universe, the geodesics at constant angle X. So we choose some fixed constant angle X; the particle sitting there follows the geodesic as the radius of the whole model changes. So a geodesic bounding-surface is made by taking a part of the whole model. We also know the geometry for this object. The question now arises whether or not they match.

The first matching condition is that the circumferences of the two surfaces be the same as a function of time. We see that the formula for the circumference of the surface in the Schwarzschild solution is given by the cycloid formula for r and t, where t is the proper time. We find just the same thing for the other solution. We readily verify that they both have the same law of collapse. But this is only matching the metric on the surfaces, so to speak, and we know that for a second-order differential equation to match, both metric and first derivative must match. How do we match the first derivative? Oppenheimer and Snyder of course represented the whole thing in one coordinate system and matched the first derivatives. This is awkward because we then have to definite strange coordinate systems in order to be able to see the whole picture. A way to avoid that technique is through matching what is called the "second fundamental form." This form is an intrinsic geometric characterization of the surface which has the same information in it as the first derivative, but which is independent of coordinate system. We can compute it independently from inside and outside using the appropriate coordinates. We can check whether the two pieces fit together by checking intrinsic geometrical properties of the interface. One of the properties is the metric on the interface and the other is the way the metric is embedded in its neighboring geometry, and this property is measured by the second fundamental form. Beckedorff simply computed the second fundamental form from outside and from inside and verified that they agreed. So we know that these two solutions do in fact match together. Anyhow, it is best to think of all the relationships from this Kruskal diagram. I like the diagram because I can see how it matches up the standard exterior Schwarzschild region to the interior regions. We cannot use this diagram if we use the other approach.

The total solution gives us just the picture we described at first, but tells us that the radius of the interface, the circumference of the inner sphere, follows the law of collapse which we mentioned. The density is still moderate while we use the equation of state. We know that we can stop worrying about mesons, and so on, being produced until the surface of the star gets across this critical $r=2m$ surface and vanishes.

MORRISON

But now we have no problem, because if there is a meter stick half outside and half inside the interior region, then one end does not experience the forces on the other end.

X

A meter stick is too long!

MORRISON

Yes, but suppose there is an atom at the boundary $r=2m$. What happens?

X

As the atom crosses $r=2m$, it is all right locally. Only when the atom is seen from infinity could this give trouble.

MISNER

Now we want to see what happens when the curvature gets high. We have followed the star until it went out of sight. We followed it for $t=\infty$, as far as we are concerned. The collapse at $r=2m$ is still at moderate density; there are only moderate curvatures, and no quantum mechanics has to be worried about. But according to the picture we have constructed, the collapse continues by itself. Eventually the assembly attains infinite density and infinite curvature, and we will have these problems. Will they bother us? Will they change the solution? The answer is that they will not. We have a uniqueness theorem due to Stellmacher which says that if we have the initial data for such a solution which are given on a spacelike surface, then the solution is unique. It depends only on the initial data within its past light cone, which is at angle 45° in our diagram. Now, to an observer at a point where $r<2m$, the physics can depend on the high density which develops, but for an $r>2m$ region, the physics is uniquely determined by the equation of state of the material before it reaches high density. If the stars explode and emit huge amounts of radiation inside $r=2m$, none of this can ever affect what is seen outside. All radiation, all the new elementary particles or anything else that arises in this object, will be entirely contained.

Suppose, however, that we inquire how the collapsing star looks to an observer at $r>2m$ as $t\to\infty$. We use a Lorentz transformation in the u, v plane. This is equivalent to a change of the time origin of the Schwarzschild coordinates. The geodesic describing the surface of the star is changed into another geodesic. This geodesic tends asymptotically toward the light cone $v=-u$. Eventually the star collapses at the speed of light. So as $t\to\infty$, the figure looks just like the first one we drew for the u, v plane, Figure VI-4, except that the region to the left (or below) the null line $v=-u$ is to be replaced by an interior solution, or ignored.

HOYLE

Does it worry you that this disaster as seen from the inside happens in a finite proper time, in a very short proper time in fact?

MISNER

Well, it suggests to me that I would prefer not to get into this situation!

ROBINSON

An observer in the Schwarzschild solution whose coordinates r, θ, ϕ are fixed is not in a state of free fall. He is in an accelerated state, and the gravitational force on him is constant. On the other hand, suppose that there is an observer in Minkowski space who is given a constant acceleration. If that observer carries out just the same series of experiments with light signals, he encounters the same difficulty as the observer in the Schwarzschild metric does. Of course this does not dispose of the "disaster" which Misner describes, because the observer moving around the center of the object is in free fall, and will also observe the "disaster." But the general argument must be based on geodesics because otherwise time could be fragmented by artificial means.

Notes and References

Introduction (T. GOLD)

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I. Absorber Theory of Radiation (J. HOGARTH)

Note

1. See *Proc. R. Soc., Ser. A*, **267** (1962), 365.

II. Time-Symmetrical Electrodynamics and Cosmology (F. HOYLE AND J. V. NARLIKAR)

Note

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III. Cosmological Boundary Conditions for Zero Rest-Mass Fields (R. PENROSE)

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IV. Retarded Potentials and the Expansion of the Universe (D. W. SCIAMA)

Note

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VI. Infinite Red-Shifts in General Relativity (C. MISNER)

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3. S. Chandrasekhar, *Mon. Not. R. Astr. Soc.*, **91** (1931), 456, and **95** (1935), 207,

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VIII. The Strong Cosmological Principle, Indeterminacy, and the Direction of Time (D. LAYZER)

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IX. The Instability of the Future (P. MORRISON)

Note

1. A more rigorous treatment separate from the discussions presented here appears in brief in *Preludes in Theoretical Physics*, ed. A. de-Shalit *et al.* (Amsterdam: North-Holland, 1966), p. 347.

X. The Anisotropy of Time (A. GRÜNBAUM)

Notes

1. This is an amended version of the paper presented at this meeting. The original version appeared in *Monist*, **48** (1964), 219.
2. H. Weyl, *Philosophy of Mathematics and Natural Science* (Princeton: Princeton University Press, 1949), p. 116. For a critique of basic misunderstandings and irrelevant criticisms of Weyl's statement by Max Black and Milič Čapek, see A. Grünbaum, *Philosophical Problems of Space and Time* (New York: Knopf, 1963), ch. 10. Hereinafter