ABSTRACT

Title of Document: OPTIMIZATION MODELS FOR COMPARING CONVENTIONAL BUS, DIAL-A-RIDE AND TAXI SYSTEMS IN RURAL AREAS

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This thesis formulates, analyzes, optimizes and compares total cost models for conventional, taxi and dial-a-ride systems in rural areas. The models are developed for a rural region with a town at its center. By considering characteristics of rural areas and rural passengers, the models minimize total cost by optimizing key decision variables, i.e. headway and the number of taxis in a system.

The analysis of these models aims to identify thresholds of demand where different systems are preferable and explore the effects of various operating conditions on cost and optimized decision variables for each transportation system. The results of this thesis show that in general the taxi system has the lowest total cost per trip, but the dial-a-ride and conventional bus systems have the lowest user and operator cost, respectively. This analysis gives policymakers in rural regions

guidelines for developing efficient public transportation systems given various circumstances.

OPTIMIZATION MODELS FOR COMPARING CONVENTIONAL BUS, DIAL-A-RIDE AND TAXI SYSTEMS IN RURAL AREAS

By

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Thesis submitted to the Faculty of the Graduate School of the University of Maryland, College Park, in partial fulfillment of the requirements for the degree of Master of Science
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Professor Kelly Clifton
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Dedication

To my mom and dad without which this would not be possible….

Mom, thanks for everything and then some. I love you.

Dad, I am who I am because of who you were. I miss you.

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List of Abbreviations

Symbol	Definition	Units
A	Total service area	$miles^2$
A_D	Service area served by a dial-a-ride collection tour	$miles^2$
A_o	Service area outside of ¼ of a mile of bus stop	$miles^2$
A_T	Total area served by bus stop	$miles^2$
A_w	Service area within ¼ of a mile of bus stop	$miles^2$
B	Bus operating cost ($/vehicle- hour)	$/vehicle- hour
C	Total cost	$/hour
c	Average cost	$/trip or $/ passenger-mi
C_d	Total schedule delay cost	$/hour
c_d	Average schedule delay cost	$/trip or $/ passenger-mi
c_F	Circuity factor	
C_o	Total operator cost	$/hour
c_o	Average operator cost	$/trip or $/ passenger-mi
C_u	Total user cost	$/hour
c_u	Average user cost	$/trip or $/ passenger-mi
C_v	Total in-vehicle cost	$/hour
c_v	Average in-vehicle cost	$/trip or $/ passenger-mi
C_w	Total wait cost	$/hour
c_w	Average wait cost	$/trip or $/ passenger-mi
C_x	Total access cost	$/hour
c_x	Average access cost	$/trip or $/ passenger-mi
d	Average user access distance	miles
D	Equivalent average system round trip distance	miles
D_c	Distance of one collection tour	miles
d_T	Average total distance traveled by passenger	miles
h	Headway	hours/vehicle
H^*	Minimum of h^* and h_{cap}	hours/vehicle
h^*	Optimized headway	hours/vehicle
h_{cap}	Headway constrained by capacity of vehicle	hours/vehicle

Symbol	Definition	Units
K	Constant for Stein's Formula	
k	Number of taxis in system	
k_c	Proportionality constant	
L	Length of service area	miles
l_{bus}	Load factor for bus	
l_{car}	Load factor for car	
n	Number of bus stops in the system	
N	Number of passengers in one collection tour	
O	System operating cost	$/vehicle-hour
$p(0)$	Probability of having zero requests for taxi service in system	
P_o	Percentage of users beyond ¼ mile of stop	
P_w	Percentage of users within ¼ mile of stop	
q	Demand density in both directions	trips/square miles/hr
Q	Demand in both directions	trips/hr
s	Distance between stops	miles
S	Dial-A-Ride operating cost	$/vehicle-hour
T	Taxi operating cost	$/vehicle-hour
u	Average number of passengers per pick-up point	
v	Average service speed	miles/hour
v_{bus}	Average bus speed	miles/hour
v_{car}	Average vehicle speed	miles/hour
v_d	Value of schedule delay	$/passenger-hour
v_t	Value of time	$/passenger-hour
v_{walk}	Average walk speed	miles/hour
w	Average wait time	hours
W	Width of service area	miles
w_{max}	Maximum wait time passengers are willing to wait	hours
x_{bus}	Seat capacity of bus	seats/vehicle
x_{car}	Seat capacity of car	seats/vehicle
λ	Average number of requests for taxi service per system hour	
μ	Average number of passengers served per hour (v/d_T)	passengers/hour

Chapter 1: Introduction

1.1 Problem Statement

According to the Status of Rural Public Transportation Report, about 91 million people or 36 percent of the United States population live in rural areas (CTAA 2000). The U.S. Census Bureau describes rural as a "territory, population and housing units not classified as urban (U.S. Census Bureau 2005)." Rural or non-metropolitan counties are located outside the boundaries of metropolitan areas and have no cities with more than 50,000 residents. By contrast urbanized areas are census block groups or groups having a population density of at least 1,000 people per square miles of land. Additionally, urban areas have a minimum residential population of 50,000 people. Rural areas often comprise open country and settlements with fewer than 2,500 residents, while urban areas are more densely settled (USDA 2003). Table one shows average statistics for rural single-counties in the U.S.

Table 1.1 Mean Statistics for Rural Single Counties in the United States

Mean Density	23 people/mi^2
Mean Service Area	2329 mi^2
Mean Population	52573 people
Percentage of population without a car	0.077

Source: CTAA 2000

Rural areas account for 83 percent of the United States' land, 21 percent of its population, 18 percent of its jobs and 14 percent of its earnings. Compared to urban areas, rural areas contain greater percentages of males, whites, elderly, persons in poverty, households with incomes below the national median, homeowners and car owners (Burkhardt 1999).

Transportation issues in rural areas are so significant because 32 percent of all rural residents are classified as transit dependent (CTAA 2000). One in 14 households in the rural United States has no vehicle. Of this population, 45 percent of the rural elderly and 57 percent of the rural poor have no car. Thirty-eight percent of all rural residents live in counties with no public transit service. Additionally, intercity and interstate bus, train, and air services to rural areas have greatly diminished and many areas have no taxi service (Burkhardt 1999).

Captive riders of rural transportation systems include those disadvantaged by age, disabilities or income with no alternative to using public transportation systems. This is a major issue because rural communities with a high proportion of residents without cars are also characterized by high poverty rates (USDA 2005). Furthermore, over 90 percent of individuals on public assistance do not have a car. These potential users of rural transit must be provided efficient ways of traveling to work, school, doctor's appointments, etc. Many of these users are recipients of government programs such as Medicaid and Welfare to Work.

Over the years, the U.S. federal, state and local governments have tried to address the unmet transportation needs of rural United States citizens. In 1998 there were about 1,600 rural transportation systems using over 10,000 vehicles (FHWA and FTA 2001). Most of these systems receive some type of government funding. The federal government's Surface Transportation Assistance Act of 1978 began the trend of ongoing federal funding for rural transit. In 1999, the U.S. Department of Transportation's Transportation Equity Act for the 21st Century Congress authorized a $134 million ceiling for this program which is now called Section 5311 (Burkhardt 1999). In the current transportation bill, SAFETY-LU, over $2 billion has been designated for rural transportation. However, there is still great potential for improving and optimizing transportation services in rural areas and these improvements can have a great impact on the culture of rural America. A study by Burkhardt submits that rural transit services can generate economic benefit by providing a means for resident to get to jobs and enabling rural community residents to live independently (Burkhardt 1999).

The policy implications of rural public transportation are obvious and interests in improving the system are justified. Several papers have examined rural public transportation systems qualitatively and analysis has been extensive for system optimization in urban systems; however, methodologies for quantitatively developing rural systems have not been adequately studied. Therefore, this thesis attempts to develop theoretical models that can be practically applied in efforts to improve the rural public transportation system in the United States.

1.2 Research Objectives

The goal of this thesis is to formulate, optimize, analyze and compare models for conventional bus, taxi and dial-a-ride systems in rural areas. The first objective is to analyze in detail the interactions between various parameters that characterize each transportation system. A model is formulated for each system that optimizes decision variables such as headway and number of vehicles in a system. The second objective is to identify thresholds of demand beyond which different systems are preferable. The third objective is to explore the effects of various operating conditions on optimized decision variables.

1.3 Organization

This thesis is organized into six chapters. The second chapter reviews current literature on rural and urban public transportation systems. First the chapter considers literature devoted to exploring types of public transportation systems and the characteristics and needs of rural transportation systems. Then the chapter examines literature that theoretically and empirically considers urban public transportation models by optimizing total cost functions.

Chapter 3 proposes a basic model formulation for rural public transportation systems. The chapter develops optimization models to fairly compare three alternative systems at their best. These models ensure that the most effective design choices are used for each of the modes. The systems studied are conventional bus, dial-a-ride and taxi.

The next two chapters present analyses of the optimization models. In Chapter 4, a system evaluation is performed. Values are determined for the decision variables and minimized cost for each type of system given a set of input parameters. Threshold and sensitivity analyses are performed in the fifth chapter. The level of demand for which each type of system is most appropriate is defined and the effect of the input parameters on the model will be determined.

Lastly, chapter 6 summarizes the findings of this thesis and makes recommendations on a series of policy issues applicable to the practical use of these models. Additionally, recommendations for further research are discussed.

Chapter 2: Literature Review

Chapter 2 summarizes previous research related to this thesis. The review considers three categories of literature. The first section highlights literature that deals with various types of transit. Then the review examines the needs and characteristics of rural transportation systems. The third section reviews literature that deals with transit models used to optimize total cost.

2.1 Types of Transit

The most conventional type of transit is fixed-route bus systems. These systems have fixed schedules and fixed routes and require substantial demand densities to be economically viable (Vuchic 1991). Systems that deviate from this convention are considered paratransit. These systems are adaptable in their scheduling and/or routing. As a consequence of this characteristic, these systems have been deemed more appropriate for rural or low-density service areas (Chang and Schonfeld 1991a, Gray 1992). Paratransit has the potential to provide attractive, high quality alternatives to the auto. Because paratransit is service-oriented, it has the potential to lead to net savings in vehicle miles traveled and reductions in time-related congestion costs, energy consumption and vehicle emissions. Also, paratransit ensures mobility and access opportunity for those individuals who are unable to use auto or conventional transit (Gray and Hoel 992).

Taxis and dial-a-ride services are considered paratransit. Taxis systems offer service at taxi stands, telephone calls or cruising streets. They provide individualized, but labor-intensive and costly service (Vuchic 1981). Among various types of paratransit, taxis are also associated with higher social cost with respect to congestion, noise and air pollution (Vuchic 1981). Dial-a-ride is considered most suitable for areas where demand is dispersed with poor street networks (Vuchic 1981). Service by conventional transit would be inappropriate and taxi service would be too costly in these networks. Dial-a-ride provides cheap door to door service by allowing ridesharing. Types of routing include many to many and one to many (or vice versa). This type of paratransit also offers variability in vehicle size. These vehicles can be as small as taxis, vans, or small to medium size buses. Dial-a-ride can easily be adapted to serve disabled users.

2.2 Characteristics of Rural or Low-Density Transportation Systems

Several articles were found that examine the qualitative characteristics of the rural public transportation systems or demand in rural or low-density areas.

Rural America and rural transportation are given a close examination, in a report released by the Federal Highway Administration (FHWA) and Federal Transit Administration (FTA) (2001). The United States Department of Transportation defines rural in two ways. For highway functional classification, it is defined as anything outside of an area with a population of 5,000. For planning purposes, rural is considered areas outside of metropolitan areas 50,000 or greater in population. However, in the report rural is considered to be "non-metropolitan areas outside the limits of any incorporated or unincorporated city, town or village." The report further categorizes rural into three distinctive types. The first type is called basic rural. This includes dispersed counties or regions with few or no major population centers of 5,000 or more. These areas are described as having stable or declining populations, agricultural and natural resource based economies and farm-to-market localized transportation patterns. Transportation issues include reduced funding for maintenance and preservation of the road and bridge system and high expense to service the small public transit dependent segment of the population. The second category is called developed rural. These are area with dispersed counties or regions with one or more population center(s) of 5,000 or more. Its characteristics include stable or growing populations, more diverse transportation and economies with mixed industrial and service based cities and rural areas that are agriculture and natural resource based. Developed rural areas face transportation issues such as maintaining a regional system that enables access to regional service centers, farm-to-market or ranch-to-market transportation, funding for capacity improvements and providing public transportation options. The third type is called urban boundary rural. This consists of counties or regions that border metropolitan areas and are highly developed. In these areas economic and population growth and transportation are connected to the urban center. Transportation issues include balancing economic development and preserving rural character, maintaining roads and bridges amidst traffic growth, funding capacity improvements and providing adequate public transportation.

This report further characterizes the rural transportation system as decentralized because roads are funded and maintained by all levels of the government. The rural public transit system is primarily funded by state and federal governments, but locally operated. Rural transportation is essential for connecting people to jobs, family and healthcare, as well as contributing to regional economic growth by connecting business to customers, goods to markets and tourists to destinations. However, the article highlights four geographic challenges of rural areas. These challenges include long distances between population centers, steep grades and mountain passes, more dramatic weather events and effects on road conditions and a dispersed system with high unit costs for service delivery, operations and maintenance.

Radow and Winters highlight the major differences between urban and rural transit systems. Rural transportation providers must operate in large geographic areas with low population densities and serve rural residents that generally have lower incomes than urban residents. Additionally, these rural providers are challenged to operate demand-response or subscription services to largely transit-dependent groups such as the elderly, youth, low-income and people with disabilities.

In a 1978 Institute of Transportation Engineers (ITE) Technical Information Report that examined the public transportation needs and demands of rural citizens, the typical rural county is described as having five distinctive features. The characteristics are as follows: a geographically scattered population, users' desire travel to a limited number of destinations usually in a nearby town or county seat, trip lengths are longer than those provided by urban transit systems, potential population densities are insufficient to support conventional fixed-route services and rural networks are not highly connective. Furthermore, the report suggests that rural public transportation systems must carefully balance need of users and the travel demand of the system. In these communities, there is a certain amount of fixed travel that is deemed necessary to provide adequate standard of living; thus, this type of travel is not affected by price of travel. In rural areas, the travel demand is still used to estimate vehicle requirements, vehicle utilization and operating costs, but is highly correlated to the user's income level and extent of travel occurring relative to a precise set of environmental circumstances. Additionally, travel impedance must include price, difficulty associated with arranging the trip, waiting time and scheduling compatibility (ITE 1978).

In an overview of problems in rural passenger transportation, Burkhardt submits that several factors influence the number of persons that can be expected to ride a given rural system. First, he cites a relation with the monthly number of bus-kilometers served by the system. As more service is provided, there should be an increase in ridership. Secondly, availability of service, expressed as frequency or reservation time, impact the demand of the system. Next, as the population of the system increases, the number of riders increases. In contrast, ridership decreases as trip distances and cost increase (Burkhardt 1978).

2.3 Models for Rural Public Transportation Systems

Few articles look at methods of mathematically modeling service characteristics of systems for rural or low-density areas. Specifically, most models on rural public transportation discuss estimation procedures for determining demand in rural areas. Articles were also found that formulate models for the total cost per vehicle mile for rural transportation systems and that perform cost-effectiveness comparisons between fixed and flexible routes systems in low-density areas. Unfortunately, no articles were found that optimized the total cost of a system as function of service characteristics for rural or low-density areas public transportation systems.

Neumann and Byrne (1978) developed a Poisson model for ridership on rural public transportation routes. The model assumes a loose loop rural transit route that leaves a central city and returns to the city picking up people with no drop-offs. The model estimates the probability associated with different groups within the total ridership of a particular route. For example, the model estimates can be used to find the probability of different ages, gender or socioeconomic groups using the system. To estimate the probability, a maximum likelihood estimator is derived. This estimator is the same as that used in cross-classification trip generation. The advantages of the model include that it is a disaggregate model such that the users are disaggregated into socioeconomic groups and usage relationships are developed for each group. Secondly, the model can be used to determine the likelihood that demand would exceed capacity of various sizes of buses since it produces probabilities of attaining a given number of riders.

Burkhardt and Lago (1976) also developed a model for predicting demand for rural transit systems. First the authors explain that the range of possible trip rates in rural areas is very large. The possible range of trips for the total area population is on the order of 0.01 to 3.0 trips per person per year with a variation of 300 percent. However, most of the variation is probably between 0.10 and 0.70 trip per person per year for a total population of the area. The model was based on three assumptions. First, the model assumes that transportation systems currently being operated are representative of the systems appropriate for rural areas and various conditions. The second assumption was that counties are the appropriate geographical unit to focus projections of aggregate demand, but routes can be used to estimate the distribution of demand. Thirdly, the patterns of travel behavior vary for definite and discoverable reasons such as characteristics of transportation services available and characteristics of the traveler and service area. Using regression analysis, several models were developed to estimate demand for fixed-route and demand-responsive systems on macro and micro patronage levels.

Ceglowski et al. (1978) examine rural transportation cost. The model was developed while trying to determine potential future demand for the Urban Mass Transit Association (UMTA) capital and operating assistance program in rural areas. The model calculates a standardized cost per vehicle mile for a specified vehicle type given cost and operating characteristics information. The total system cost are the summation of the following major cost categories: costs dependent on vehicle miles, costs dependent on vehicle hours, costs dependent on number of vehicles, capital costs and overhead costs. Two of the major outcomes of this study are that economies of scale are not obvious in rural transportation operations and in rural transit operations, the bulk of the total system costs are directly attributable to driver's wages, overhead costs and vehicle capital costs.

A theoretical analysis by Ward (1975) compares conventional and subscription bus services for low density urban areas. In this approach, Ward develops supply models for each type of service and determines ranges of relative efficiency. The fixed route feeder system is described in terms of vehicle productivity (passenger trips per vehicle hour) and service ratio (ratio of average passenger travel time to auto travel time). The flexible route feeder service is analyzed on the basis of variation of productivity and service ratio with size of service sectors. The study finds that flexible routes have a lower sensitivity of cost to level of service provided than fixed route buses. The flexible route bus offers better service at the same or higher level of productivity at all demand levels below 100 passengers per square miles per hour. Additionally, even when unit operating cost are assumed to be 50 percent higher than fixed route bus, flexible route bus can provide as good or better service for the same cost. Fixed route feeder systems only become competitive when providing a very low level of service.

2.4 Transit Models for Optimizing Total Cost

Many studies have been conducted to investigate different aspects of urban or high density transit and optimize the total cost function. Unfortunately, none have optimized public transportation systems for rural areas. However, some studies focused on modeling different aspects of paratransit systems, which are popular forms of public transportation in rural environments.

Chang and Schonfeld (1991a) used analytical models to compare average trip cost between feeder services of a fixed route conventional bus and a flexible bus route subscription system. Optimized results are found for vehicle size, route spacing, headway and service zone areas. The study then finds favorable situations for the operation of temporally integrated systems and formulates and analyzes mathematical models of total system costs for an integrated system. Specifically, the study submits that in temporally integrated systems fixed-route systems should be provided during higher-demand periods and flexible-route services are provided during lower-demand periods. In numerical results optimal vehicle size in integrated systems were a compromise between the optimal vehicle sizes in pure fixed-route and pure flexible-route services. Additionally, the average system cost per trip for integrated systems can be lower than either pure system. It is hypothesized that the benefits of temporal integration increase as the relative duration of low-demand periods increases.

Chang and Schonfeld (1991b) compared average trip cost between conventional and subscription bus feeder services with a uniformly distributed, fixed demand. This comparative analysis optimized vehicle size and service zone size to minimize the objective function of total system cost which included operator and user costs. The results of this study showed that the average cost functions are quite similar in magnitude for the two services, but there are significant differences in the operator and user costs. Operator cost for the subscription bus is significantly higher, while the user cost is much lower than the conventional bus service. Additionally, sensitivity analyses found that subscription services are favored for cases with smaller service areas, higher express speeds, high values of access and wait time and lower values of in-vehicle time.

2.3 Summary

This literature review concludes that there is virtually no literature that focuses on mathematically modeling public transportation systems for rural areas. This thesis will expand the earlier works on urban public transportation to systems that exist in characteristically rural areas, i.e. having low-densities and street networks that are not dense and rectangular.

Chapter 3: Model Formulation

In this chapter, the conventional bus, dial-a-ride and taxi systems are designed on a framework developed for rural areas. Additionally, the models have been formulated to optimize the objective function of total system cost. The table on page xii defines the notation for variables that are used in this thesis.

3.1 Total Cost Objective Function

This analysis optimizes functions for cost of the total system. The total system cost formulated here is the sum of the operator cost C_o and the user cost C_u. It should be noted that all cost formulations consider roundtrip travel. The operator cost incorporates cost such as the capital, maintenance and fuel costs for the system vehicle as well as driver wages. This cost is found by multiplying the total round-trip distance of the system's route D and the vehicle operating cost O and then dividing by the product of the vehicle speed v and headway h. The generic form of operator costs given is given in dollars per hour as:

$$C_o = \frac{2DO}{vh} \tag{3.1}$$

The user cost is a sum of user access cost, wait cost, schedule delay cost and in-vehicle cost. The following expressions represent these expressions which are all given in dollars per hour. The access cost is the cost incurred by the user to travel to and from a system pick-up point to his or her destination. It is modeled as the distance d from to the user's origin to the bus stop and then once the user has arrived at the destination bus stop from that point to the user's actual destination in each direction. The distance is multiplied by the demand density q, the value of time v_t and the area A and divided by the speed v.

$$C_x = \frac{2 \times 2dqv_tA}{v} \tag{3.2}$$

The wait cost refers to the cost incurred by the user while waiting for service once a particular pick-up has been chosen for departure. For instance, this cost accounts for differences in passenger arrival times to a bus stop and how service varies with scheduled pick-up times. The expression of wait cost in Equation 3.3 is determined by multiplying the average wait time w, the demand density q, the area A and value of time v_t for each direction of the trip. The expression is multiplied by two to account for the wait time in both directions of travel.

$$C_w = 2wqAv_t \tag{3.3}$$

Schedule delay is defined as the difference between the desired and nearest available or actual departure time. Thus, individuals who travel earlier or later than they would like to travel incur schedule delays (de Palma and Lindsey 2001). The cost associated with schedule delay is expressed as the product of half of the headway h, the demand density q, the area A and the value of time v_t for each direction of the trip.

$$C_d = 2\left(\frac{h}{2}\right)qAv_d = hqAv_d \tag{3.4}$$

The in-vehicle costs accounts for the cost incurred by the user during the time he or she is on a system vehicle and traveling to a destination. The expression for the in-vehicle cost is shown in Equation 3.5. It is the product of the average distance traveled by users on the system vehicle or half of the system's route, i.e. D/2, the demand density q, area A, and the value of time v_t for each direction of the trip divided by the speed the vehicle.

$$C_v = \frac{2DqAv_t}{2v} = \frac{DqAv_t}{v} \tag{3.5}$$

Thus, the objective function of the analysis becomes (Chang and Schonfeld 1991a and b, Spasovic and Schonfeld 1993):

$$C = C_o + C_u = C_o + C_x + C_w + C_d + C_v \tag{3.6}$$

The final analysis will compare average cost c in dollars/round trip and dollars/ passenger mile.

3.2 Assumptions for Transportation Systems

To formulate and compare models, all transportation systems are based on the same basic service area. The following simplifying assumptions are made about the service area:

1. As shown in Figure 3.1, the service area is rectangular (L × W) and served by two major perpendicular roads that intersect in the town of the rural area's center. However, this model could be applied to any radial or diametrical network system. Existing features of the region must be taken into account such as land use and geographic constraints, topography and environmental factors and the existing transportation network when designing the system (Vuchic 2005).

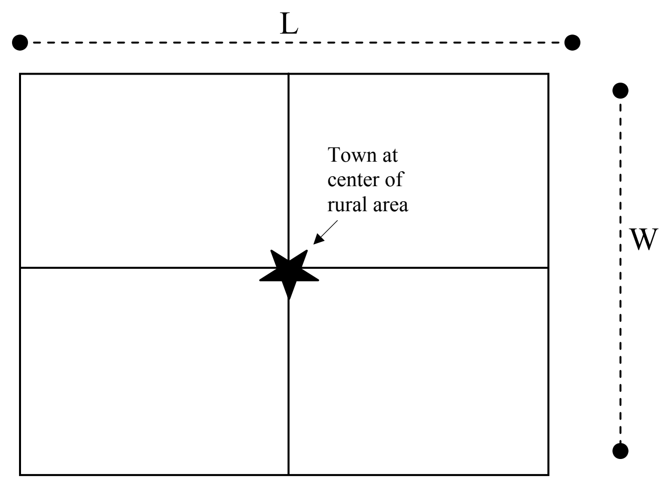

Figure 3.1 Service Area for the Rural Transportation Systems

2. A circuity factor, c_F of 1 is assumed in this thesis with respect to rectilinear space. It should be noted that although they are not depicted in the figure, local roads are present. These roads make-up a rectangular, grid street network. However, to account for the differences in the street network such as lack of connectivity and road density as well as geographic factors such as mountains, lakes and reservations, circuity factors can be used. Ballou et al. (2002) define circuity factors as multipliers used to approximate actual travel distances from straight-line distances. It is found as a ratio of actual travel distance to calculated distance. For straight line distances, the circuity factor should be equal to or greater than one because travel distances can not be shorter than the straight-line distance. For example, Ballou et al. found straight-line circuity factors for the United States to be 1.79, 1.20 and 1.21 for Alaska, east of the Mississippi River and west of the Mississippi River, respectively. Additionally, Ballou et al. submit that cities that are connected directly with high-level roads have circuity factors approaching one, while

cities on a low-density network with little direct connection and/or significant obstacles will have a circuity factor approaching 2 or even higher. In this thesis, circuity factors are used to adjust rectilinear distances. Thus, rectilinear circuity factors can be less than one.

3. The town, or center of the rural community, serves as the major trip attractor. Several places of user interest such as the market, courthouse, doctors' offices and schools are ideally located here.

Several assumptions are made about the demand in the service area used in this formulation.

1. We assume that a demand is uniformly distributed over the service area.
2. Demand is assumed to be fixed, i.e. perfectly inelastic with respect to price and service quality.
3. The demand in this research studies the round-trip travel of passengers. Thus, q represents the combined two-way demand density.

Assumptions for the routing and operational characteristics of each system are defined below.

3.2.1 Conventional Bus System

Figure 3.2 shows the service area used to formulate the model for the conventional bus service. The following assumptions are made in formulating the model.

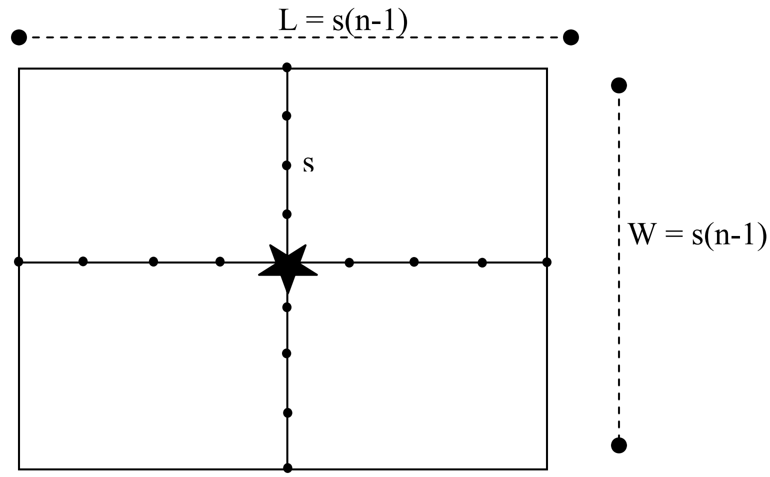

Figure 3.2 Service Area for the Conventional Bus System

1. In this system, each bus only travels along the two major roadways of the rural area. The buses pick up passengers at n stops along the roadways that are separated by equal distances s, where s is the length or width divided by the number of proposed stops minus one, i.e. s = L/(n-1) or W/(n-1).
2. Each bus travels the length or width of the service area at an average speed of v_{bus}.
3. All vehicles in the system are assumed to be equal in capacity.
4. The wait time is assumed to be half the headway h for headways up to 30 minutes. Long headways are usually classified as longer than 10 minutes (Vuchic 2005). The industry standard, however, accepts 15 to 20 minutes as the maximum time passengers are willing to wait for services (Tan 2004). Studies show that people who use transit services with long headways tend to use timetable schedules to adjust their arrivals to scheduled departure times. Passengers want to arrive only a few minutes before the bus or train is

scheduled to arrive. Thus, the average wait time becomes somewhat shorter for random passenger arrivals and remains approximately constant for long headways. Figure 3.3 shows the function used in this model for wait time.

5. In this study, average schedule delay is modeled as half the headway. Due to the low densities found in rural areas, headways for the conventional bus service are classified as long headways. To account for the inconvenience of longer headways, we consider schedule delay costs for the user. These delays are unavoidable with public transportation because vehicles do not depart continuously. Figure 3.3 shows the schedule delay function used in this formulation.

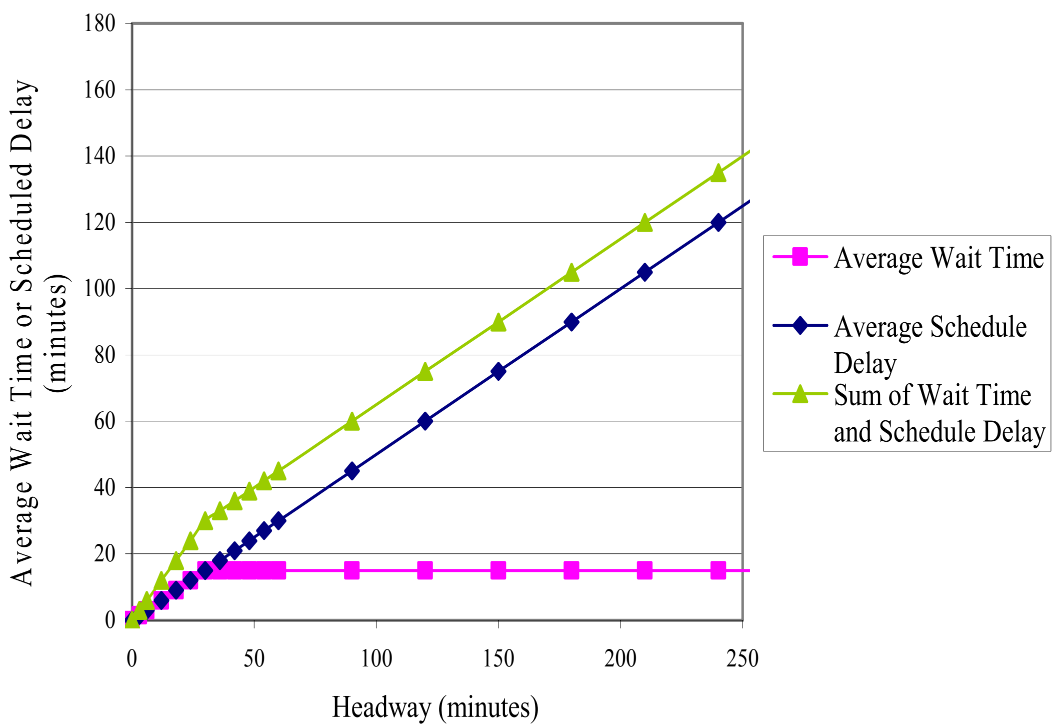

Figure 3.3 Wait Time and Schedule Delay Functions

The following equations have been derived for each component of the total cost of the conventional bus system. The form of operator cost defined in Equation 3.1 is used to formulate the operator cost for the conventional bus system. The operating cost O is replaced for the conventional bus operating cost B. The total distance traveled by the vehicle D is equal to the route length for the conventional bus system. The route length is equal to the stop spacing s multiplied by the number of bus stops *n* subtracted by one, i.e. (n-1)s. The hourly operator cost for the conventional bus system is given in dollars per hour as:

$$\mathrm{C_o} = \frac{2\mathrm{B}}{\mathrm{hv_{bus}}}\left[(n-1)\mathrm{sc_F}\right] \tag{3.7}$$

The hourly user wait cost can be formulated in dollars per hour as shown in equation 3.8. Notice that unlike the form in Equation 3.3, the wait time has been defined as half the headway for this system.

$$\mathrm{C_w} = 2\mathrm{qAw}v_\mathrm{t} = 2\mathrm{qA}\left(\frac{\mathrm{h}}{2}\right)v_\mathrm{t} = \mathrm{qAh}v_\mathrm{t} \;\; (\text{for h} \le 30 \text{ minutes}) \tag{3.8}$$

The hourly user schedule delay cost is shown in equation 3.9 in dollars per hour.

$$\mathrm{C_d} = 2\mathrm{qA}\left(\frac{\mathrm{h}}{2}\right)v_\mathrm{d} = \mathrm{qAh}v_\mathrm{d} \quad (\text{for } 0 \le \mathrm{h} \le \infty) \tag{3.9}$$

The hourly user in-vehicle cost can be formulated in dollars per hour as

$$\mathrm{C_v} = \frac{2\mathrm{qA}(n-1)\mathrm{sc_F}v_\mathrm{t}}{2\mathrm{v_{bus}}} = \frac{\mathrm{qA}(n-1)\mathrm{sc_F}v_\mathrm{t}}{\mathrm{v_{bus}}}. \tag{3.10}$$

The average in-vehicle distance is assumed to be half of the route length, (n-1)s/2.

It is often assumed that the maximum distance users are willing to walk is 0.25 miles (Kocur and Hendrickson 1982). In this model, since service trip origin is uniformly distributed over the area, we assume that those users within 0.25 miles walk and all others use a vehicle to arrive at the bus stop. Thus, we define the variables P_w and P_o for the percentages of users who are walking and using a vehicle to get to the bus stop, respectively. The following equations are used to determine these percentages, where A_T is the total served by the bus stop, A_w defines the service area within ¼ mile of the bus stop and A_o accounts for the remainder of the service area:

$$P_w = \frac{A_w}{A_T} \tag{3.11}$$

$$P_o = \frac{A_o}{A_T} \tag{3.12}$$

We assume that on average users within 0.25 miles of a bus stop travel an average distance of $k_c\sqrt{A_w}$, where k_c is the proportionality constant for determining average travel distances. We model the area as a four-sided diamond along the major roadway with right-angle travel and the bus stop located at the center of the area. Thus, from Odoni and Larson (1981), k_c is equal to 0.471. Users outside of the 0.25 mile boundary travel an average distance of s/4 between stops and (n-1)s/2 to the roadway. As mentioned in section 3.1, the user is modeled as traveling the same access distance at both ends of the trip. Thus, the access cost can be modeled as the following equation.

$$C_x = 2 \times 2qAc_F v_t \left[\left(\frac{s}{4} + \frac{(n-1)s}{2} \right) \frac{P_O}{v_{car}} + \left(0.471\sqrt{A_w} \right) \frac{P_w}{v_{walk}} \right]$$

$$C_x = qAc_F v_t \left[\left(s + 2(n-1)s \right) \frac{P_O}{v_{car}} + \left(0.471\sqrt{A_w} \right) \frac{P_w}{v_{walk}} \right] \tag{3.13}$$

From these equations, it follows that the total system cost in dollars per hour can be expressed as:

$$C=\frac{2B}{hv_{bus}}\left[(n-1)s\right]c_F+qAhv_t+qAhv_d+\frac{qA(n-1)sc_F v_t}{v_{bus}}+qAc_F v_t\left[\left(s+2(n-1)s\right)\frac{P_O}{v_{car}}+\left(0.471\sqrt{A_w}\right)\frac{P_w}{v_{walk}}\right]$$

(3.14)

The average cost given in dollars per trip is given by dividing by the total trip demand 2qA in the following equation.

$$c=\frac{2B}{qAhv_{bus}}\left[(n-1)s\right]c_F+\frac{h}{2}v_t+\frac{h}{2}v_d+\frac{(n-1)sc_F v_t}{v_{bus}}+c_F v_t\left[(s+2(n\text{-}1)s)\frac{P_0}{v_{car}}+(0.471\sqrt{A_w})\frac{P_w}{v_{walk}}\right]$$

(3.15)

The average cost in dollars per passenger mile can be found by dividing the average cost per trip by the total average distance traveled by passengers per trip. For the conventional bus system, the average distance traveled per passenger is shown in equation 3.16. The average distance per passenger is the sum of the user's average in-vehicle distance and the average access distance.

$$d_T=\left[2\times\left(\frac{(n-1)s}{2}\right)+4\times\left(\left(\frac{s}{4}+\frac{(n-1)s}{2}\right)P_O+\left(0.471\sqrt{A_w}\right)P_w\right)\right]c_F$$

$$d_T=\left[(n-1)s+\left[s+2(n-1)s\right]P_O+1.884P_w\sqrt{A_w}\right]c_F \qquad (3.16)$$

The average total cost is then minimized by setting its derivative with respect to the headway equal to zero and solving for optimized headway. The following equations show this analysis:

$$\frac{dc}{dh}=0=-\frac{4B}{qAh^2v_{bus}}\left[(n-1)s\right]c_F+v_t+v_d \qquad (3.17)$$

$$h^* = \sqrt{\frac{4B(n-1)sc_F}{qA(v_t + v_d)v_{bus}}} \tag{3.18}$$

It should be noted that the optimized headway may not be feasible. The optimized headway is constrained by the capacity of the vehicle. To compensate for this, the headway used will be the minimum of optimized headway, h*, and the headway required by the capacity of the system, h_{cap} as shown in equation 3.19.

$$H^* = \min(h^*, h_{cap}) \tag{3.19}$$

h_{cap} is defined as x*l*/qA where x is the seat capacity of the vehicle and *l* is the load factor of the vehicle (Chang and Schonfeld 1991b).

3.2.2 Dial-A-Ride System

The service area used to model the dial-a-ride system is shown in Figure 3.4. The following assumptions are made in formulating the model.

1. In this model the four quadrants defined by the two perpendicular major roads are service zones A_1, A_2, A_3 and A_4 for the dial-a-ride system. All dial-a-ride routes are tours starting and ending at the town center.
2. The users are collected at their doorsteps (i.e. access distance is neglected) through a tour of stops.
3. It is also assumed that dial-a-ride vehicles operate on preset schedules with variable routing designed to minimize the tour distance, D_c. Under this assumption, all users along a particular route are traveling between their respective service zone and the town center. Once in town, passengers have

the option to transfer to other vehicles to travel to other service zones if desired.

4. As in the conventional bus system, average wait time is assumed to equal half of the headway for headways up to 30 minutes. Otherwise, the wait time is capped at 15 minutes.
5. Dial-a-ride vehicles can be shared among routes.
6. The schedule delay is modeled as half of the headway as in the model for the conventional bus system.

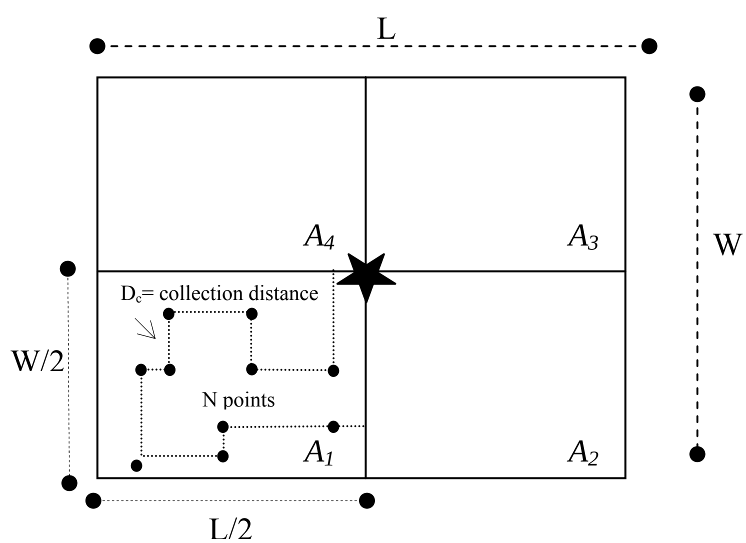

Figure 3.4 Total Service Area for the Dial-A-Ride System

The length of the tour, D_c can be estimated using Stein's formula (Stein 1978b). Stein's formula is shown in Equation 3.20.

$$D_c = K\sqrt{NA} \quad \text{(Stein 1978b)} \tag{3.20}$$

where K is a constant, N is the number of points in the collection tour and A is the area in which the tour takes place. Based on the optimized traveling salesman tour problem, Stein's equation assumes that the number of points, N is randomly, independently and uniformly distributed over an area A (Odoni and Larson 1981). It has been shown that small values of N may be adequate in providing good approximations to the expected length of the optimal traveling salesman tour if the area is "fairly compact and fairly convex." Odoni and Larson define an area as "fairly compact and fairly convex" if one dimension is not much greater than the other dimension and major barriers or boundaries indentations do not exist in the area. Using a set of simulation experiments, the value of K has been found to be approximately equal to 0.765 (Odoni and Larson 1981). In this model, we assume that N is determined endogenously and equal to the hourly demand of service multiplied by the headway and divided by the number of passengers per pick-up, i.e. N = 2qAh/u where u is the average number of passengers per pick-up point (Chang and Schonfeld 1991b).

The following equations have been derived for each component of the total cost of the dial-a-ride system. The operator cost only includes the total distance traveled by the vehicle accounts for only the collection tour since all zones are adjacent to the town. However, the model accounts for a tour at both ends of the trip in each direction. The hourly operator cost is formulated in dollars per hour as:

$$C_o = \frac{4S}{hv_{bus}}(K\sqrt{NA_D})c_F = \frac{4S}{hv_{bus}}\left(K\sqrt{\frac{2qhA_D{}^2}{u}}\right)c_F = \frac{4S}{h^{1/2}v_{bus}}\left(K\sqrt{\frac{2qA_D{}^2}{u}}\right)c_F \quad (3.21)$$

where S is the operating cost for dial-a-ride systems and A_D is the service area served by the collection tour.

Similarly to the wait cost for the conventional bus system, the users wait cost is formulated as in dollars per hour in equation 3.22.

$$C_w = 2qA_D w v_t = 2qA_D\left(\frac{h}{2}\right)v_t = qA_D h v_t \text{ (for } h \le 30 \text{ minutes)} \qquad (3.22)$$

The user schedule delay is shown in the following equation in dollars per hour.

$$C_d = 2qA_D\left(\frac{h}{2}\right)v_d = qA_D h v_d \qquad (\text{for } 0 \le h \le \infty) \qquad (3.23)$$

The user in-vehicle cost models the average in-vehicle distance as half of the tour distance. It can be expressed as:

$$C_v = \frac{2qA_D v_t}{v_{bus}}\left(\frac{K}{2}\sqrt{NA_D{}^2}\right)c_F = \frac{2qAv_t}{v_{bus}}\left(\frac{K}{2}\sqrt{\frac{2qA_D{}^2 h}{u}}\right)c_F = \frac{qA_D v_t K h^{1/2} c_F}{v_{bus}}\sqrt{\frac{2qA_D{}^2}{u}} \qquad (3.24)$$

Again, the access cost is assumed to be zero, i.e. C_x=0, because the system provides door to door service.

The total system cost for the dial-a-ride system is formulated in dollars per hour as:

$$C = \frac{4S}{h^{1/2}v_{bus}}\left(K\sqrt{\frac{2qA_D{}^2}{u}}\right)c_F + qA_D h v_t + qA_D h v_d + \frac{qA_D v_t K h^{1/2}}{v_{bus}}\sqrt{\frac{2qA_D{}^2}{u}} \qquad (3.25)$$

The average cost in dollars per trip is calculated by dividing C by the trip demand 2qA, as shown in Equation 3.26.

$$c = \frac{2S}{qA_D h^{1/2} v_{bus}}\left(K\sqrt{\frac{2qA_D{}^2}{u}}\right)c_F + \frac{h}{2}v_t + \frac{h}{2}v_d + \frac{v_t K h^{1/2}}{2v_{bus}}\sqrt{\frac{2qA_D{}^2}{u}} \qquad (3.26)$$

As in the model for a conventional bus system, the average cost in dollars per passenger mile can be found by dividing the average cost per trip by the total average distance traveled by passengers per trip. For the dial-a-ride system, the average distance traveled per passenger is the average collection tour length. It is can be expressed as:

$$dT = \frac{2K\sqrt{NA_D}}{2}c_F = K\sqrt{\frac{2qA_D{}^2h}{u}}c_F \quad (3.27)$$

The average cost in dollars per trip is then minimized by setting its derivative with respect to the headway equal to zero. We will numerically solve for the optimized value of headway. Equation 3.28 is the formula to be used in this analysis.

$$\frac{dc}{dh} = 0 = -\frac{SK}{qA_D h^{3/2} v_{bus}}\sqrt{\frac{2qA_D h^2}{u}} + \frac{v_t}{2} + \frac{v_d}{2} + \frac{v_t K}{4h^{1/2} v_{bus}}\sqrt{\frac{2qA_D h^2}{u}} \quad (3.28)$$

As in the formulation for the conventional bus system, we must again consider the constraints of the vehicle's capacity. Thus, the headway used in the model is the minimum value of the optimized headway, h* and the headway required by the capacity of the system, h_{cap}.

3.2.3 Taxi System

For the taxi system, we make the following assumptions in developing a total cost function. The service area is depicted in Figure 3.1.

1. There is a central base for the taxi located in the town of the rural area. Users must call the taxi stand to request services. Once a request is received, a taxi is immediately notified and dispatched to the location of the pick-up from its present location or of the drop-off point of the current passenger.
2. Queuing theory is used to model the taxi system. The supply and demand for taxis are intertwined through relations for taxi availability and taxi utilization (Manski and Wright 1976). However, as in Manski and Wright, we focus on

taxi availability (1976). The wait time is based on a probability function that accounts for queuing in the system due to a back-up of service requests and/or traffic. This model assumes a queuing process where all the taxis in the system are servers in parallel with identical exponentially distributed service times of μ. The passenger calls for service are Poisson distributed with a mean arrival rate of λ and form a single queue that has a service order of first in- first out with unlimited queuing capacity. In other words, these assumptions imply that passenger calls are uncoordinated and random. Additionally, we assume that μ is equal to the time needed to travel the distance of the average trip, i.e. v/d_T. The number of requests for service per unit hour, λ is equal to the demand, Q.

3. We assume door-to-door service; thus, the cost of access, C_x is zero.
4. All vehicles are identical with respect to size and cost. Also, there is an infinitely elastic supply of taxi drivers.

The total cost of a taxi system has been derived in dollars per hour. The operator cost is derived by estimating the average distance traveled by the taxi. The average distance between any two random points in a uniformly distributed area can be estimated. Thus, for this system, the average distance for one segment of the service is estimated as the (L + W)/3 (Larson and Odoni 1981). The taxi distance includes the travel distance from the taxi stand or drop-off point of the last passenger to the pick-up location of the current call and then from the pick-up location to the drop-off location.

At first glance the origin-destination assumptions for the taxi system seem different than the other rural transportation systems. The taxis in the system are not required to pass through the town to respond to request for pick-up; however, the conventional bus and dial-a-ride systems provide a many to one service to the town. There are three ways that this model compensates for this inconsistency. First, the average taxi distance overestimates the actual distance that the taxi will travel to provide service. For instance, as the number of taxis in the system increases, the model does not consider that the taxi closest to the request for service will respond. The model assumes that all taxis will travel the same distance. Secondly, the user of the conventional bus and dial-a-ride systems can make connections in town to travel to areas in the rural region. Thus, users of all three systems have access to the entire service area. Lastly, costs per system mile can be used to compare the three systems. This type of analysis takes into account that the taxi system is less restricted in the routes that it may travel.

The operator cost considering both directions of the passenger's trip can be formulated as follows:

$$C_o = Tk \quad (3.29)$$

where T is the operating cost for the taxi system, v_{car} is the speed of the car and l_{car} is the load factor for the car.

The wait cost is modeled using queuing theory as described in assumption 2. This cost is given in the following equation in dollars per hour.

$$C_w = 2qAwv_t \quad \text{(for w} \leq \text{15 minutes)} \qquad (3.30)$$

The average wait time is calculated using Equation 3.31. The average number of requests for taxi service per system hour λ is defined in Equation 3.32, where u is the average number of passengers per pick-up point. The average number of passengers served per hour μ is equal to the speed of the taxi divided by the average distance for a taxi driver to complete a call, i.e. distance traveled to arrive at the pick-up point of the passenger and drop-off at destination. The average distance is defined in Equation 3.33. k is the number of vehicles in the system having a service rate of μ and $p(0)$ is the probability of having zero requests for service in the system.

$$\mathrm{w} = \frac{\mu\left(\lambda/\mu\right)^{\mathrm{k}}}{(\mathrm{k}-1)!(\mathrm{k}\mu-\lambda)^2}\,p(0) \tag{3.31}$$

$$\lambda = \frac{2\mathrm{qA}}{\mathrm{u}} \tag{3.32}$$

$$\mathrm{d_T} = 2\left(\frac{\mathrm{L}}{3}+\frac{\mathrm{W}}{3}\right)\mathrm{c_F} \tag{3.33}$$

$$\text{where } p(0) = \frac{1}{\left[\sum_{n=0}^{\mathrm{k}-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^{n}\right]+\frac{1}{\mathrm{k}!}\left(\frac{\lambda}{\mu}\right)^{\mathrm{k}}\frac{\mathrm{k}\mu}{\mathrm{k}\mu-\lambda}} \tag{3.34}$$

Numerical analysis will be used to optimize the wait time based on the number of taxis in the system, k. The schedule delay cost is shown in Equation 3.35.

$$\mathrm{C_d} = 2\mathrm{qA}\frac{\mathrm{w}}{2}v_{\mathrm{d}} = \mathrm{qAw}v_{\mathrm{d}} \quad (\text{for } 0 \le \mathrm{w} \le \infty) \tag{3.35}$$

The in-vehicle cost uses the average distance traveled by the passenger. As in the formulation for operator cost, the average distance of one segment of the roundtrip is estimated as (L + W)/3 for the user in both directions. It is formulated in Equation 3.36.

$$C_v = \frac{2qAv_t}{v_{car}}\left(\frac{L}{3}+\frac{W}{3}\right)c_F = \frac{2qAv_t}{3v_{car}}(L+W)c_F \tag{3.36}$$

Thus, the total cost function for the taxi system can be expressed in dollars per hour as:

$$C = Tk + 2qAwv_t + 2qAwv_d + \frac{2qAv_t}{3v_{car}}(L+W)c_F \tag{3.37}$$

The average cost given in dollars per trip is found by dividing the total system cost by the total demand 2qA. It is expressed as:

$$c = \frac{Tk}{2qA} + wv_t + wv_d + \frac{v_t}{3v_{car}}(L+W)c_F \tag{3.38}$$

The average cost given in dollars per passenger mile can be formulated by dividing the cost per trip by the average distance traveled by users per trip. The average distance was defined in Equation 3.33. The average distance for the taxi system is equal to the average distance used in formulating the operator and in-vehicle cost.

Chapter 4: System Evaluation

To analyze the effects of various input parameters on the minimum total cost for the rural public transportation systems, Chapter 4 presents the system evaluation results using the optimization models developed in Chapter 3. Section 4.1 describes input parameter values, while section 4.2 evaluates each system individually.

4.1 Input Parameter Values

This section presents input parameters for the system evaluation. The inputs shown in Table 4.1 were used to calculate the optimized minimum cost for each type of system. For example, the service areas from Figures 3.1, 3.2 and 3.4 are defined to have rectangular area with a length and width of 48 miles and total area of 2304 square miles in this study. The operating cost for the bus is set at $80/vehicle-hour, while the dial-a-ride system's cost is $60/vehicle-hour and $30/vehicle-hour for the taxi system. The value of time cost for passengers is $12/passenger-hour and the value of schedule delay is $5/passenger-hour. The walking, bus/dial-a-ride and car/taxi speeds are 2.5mph, 20mph and 40mph, respectively. The demand is uniformly distributed and studied for demands of less than 5 trips per hour. The inputs for cost and speed are from previous studies (Chang and Schonfeld 1991 a and b). The costs have been adjusted to reflect inflation.

Table 4.1 Definitions and Baseline Values for System Evaluation

Symbol	Definition	Units	Baseline Value
A	Total service area	miles^2	2304
A_D	Total area served by dial-a-ride services	miles^2	576
A_o	Service area outside of ¼ of a mile of bus stop	miles^2	2302.625
A_T	Total area served by bus stop	miles^2	
A_w	Service area within ¼ of a mile of bus stop	miles^2	1.375
B	Bus operating cost	$/vehicle- hour	80
C	Total cost	$/hour	
c	Average cost	$/trip or $/passenger-mi	
c_F	Circuity factor		1
C_o	Total operator cost	$/hour	
c_o	Average operator cost	$/trip or $/ passenger-mi	
C_d	Total schedule delay cost	$/hour	
c_d	Average schedule delay cost	$/trip or $/ passenger-mi	
C_u	Total user cost	$/hour	
c_u	Average user cost	$/trip or $/ passenger-mi	
C_v	Total in-vehicle cost	$/hour	
c_v	Average in-vehicle cost	$/trip or $/ passenger-mi	
C_w	Total wait cost	$/hour	
c_w	Average wait cost	$/trip or $/ passenger-mi	
C_x	Total access cost	$/hour	
c_x	Average access cost	$/trip or $/ passenger-mi	
d	Average user access distance	miles	
D	Equivalent average system round trip distance	miles	
D_t	Distance of one collection tour	miles	
d_T	Average total distance traveled by passenger	miles	
h	Headway	hours/vehicle	
H^*	Minimum of h^* and h_{cap}	hours/vehicle	
h^*	Optimized headway	hours/vehicle	

Symbol	Definition	Units	Baseline Value
h_{cap}	Headway constrained by capacity of vehicle	hours/vehicle	
K	Constant for Stein's Formula		0.765
k	Number of taxis in system		
k_c	Proportionality constant		0.471
L	Length of service area	miles	48
l_{bus}	Load factor for bus		1.0
l_{car}	Load factor for car		1.5
n	Number of bus stops in the system		7
N	Number of passengers in one collection tour		
O	System operating cost	$/vehicle-hour	
$p(0)$	Probability of having zero requests for taxi service in system		
P_o	Percentage of users beyond ¼ mile of stop		0.9994032
P_w	Percentage of users within ¼ mile of stop		0.0005968
q	Demand density in both directions	trips/square miles/hr	
Q	Demand in both directions	trips/hr	
s	Distance between stops	miles	8
S	Dial-A-Ride operating cost	$/vehicle-hour	60
T	Taxi operating cost	$/vehicle-hour	30
u	Average number of passengers per pick-up point		1
v	Average service speed	miles/hour	
v_{bus}	Average bus speed	miles/hour	20
v_{car}	Average vehicle speed	miles/hour	40
v_d	Value of schedule delay	$/passenger-hour	5
v_t	Value of time	$/passenger-hour	12
v_{walk}	Average walk speed	miles/hour	2.5
w	Average wait time	hours	
W	Width of service area	miles	48
w_{max}	Maximum wait time passengers are willing to wait	hours	0.25
x_{bus}	Seat capacity of bus	seats/vehicle	16
x_{car}	Seat capacity of car	seats/vehicle	
λ	Average number of requests for taxi service per system-hour		
μ	Average number of passengers served per hour (v/dT)	passengers/hour	0.625

4.2 Evaluations

Using the input values shown in Table 4.1, we evaluated the models developed in Chapter 3 using two basic steps. First the optimized values were determined for the decision variables used for each system. For the conventional bus and dial-a-ride systems, headway was optimized to minimize total cost. The number of vehicles was optimized in the taxi system to minimize total cost. The second step involved using the determined values for the decision variables to calculate the minimized cost. Each cost is divided into several components. Besides determining the total cost, operator, user, wait, schedule delay, access and in-vehicle cost are evaluated. Additionally, each system was evaluated in terms of total cost and average cost.

4.2.1 Evaluation of Conventional Bus System

Figures 4.1 through 4.3 show the cost of the conventional bus system. Several trends are evident in the plots for the conventional bus system. First, for the total cost of the system, in $/hr, all cost increase with demand. Although the operator cost is the most expensive component of the total cost per hour, none of the component costs seem to dominate the model. In Figures 4.2 and 4.3, we see that the average operator cost decreases non-linearly as demand increases. This shows that as demand increases, the marginal cost per trip or per mile decreases for the operator. In other words, the fixed cost associated with conventional bus system is spread across more trips.

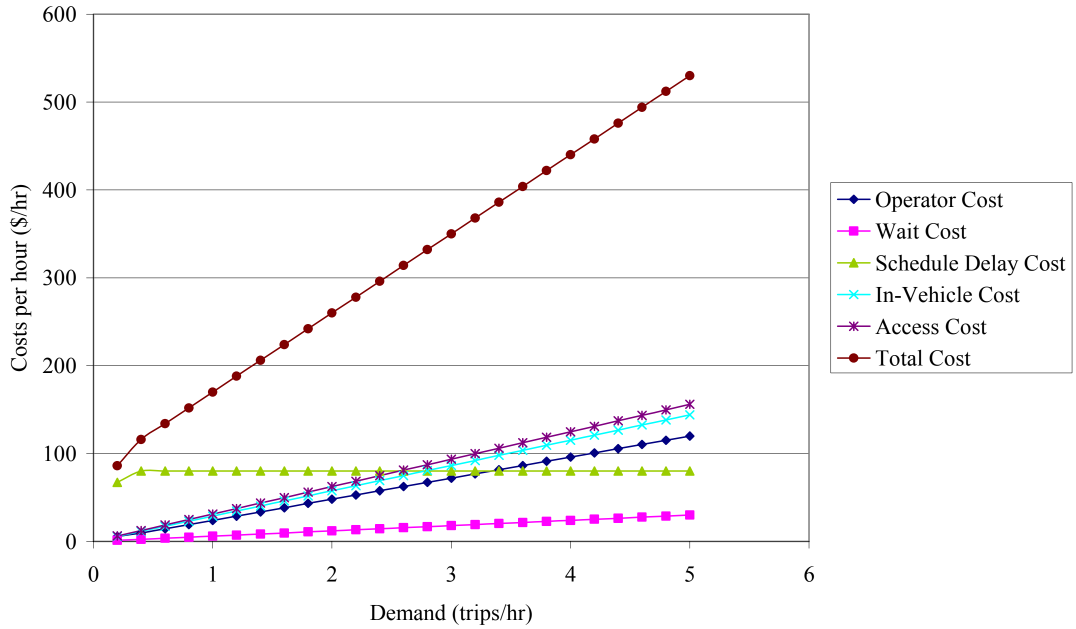

Figure 4.1 Costs for the Conventional Bus System in $/hr

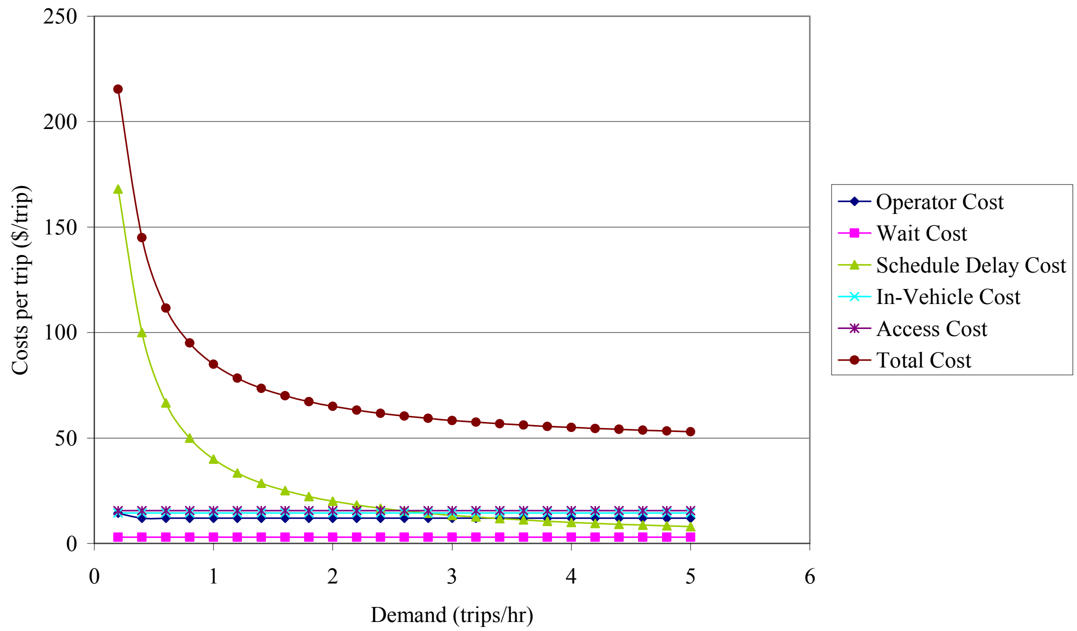

Figure 4.2 Average Costs for the Conventional Bus System in $/trip

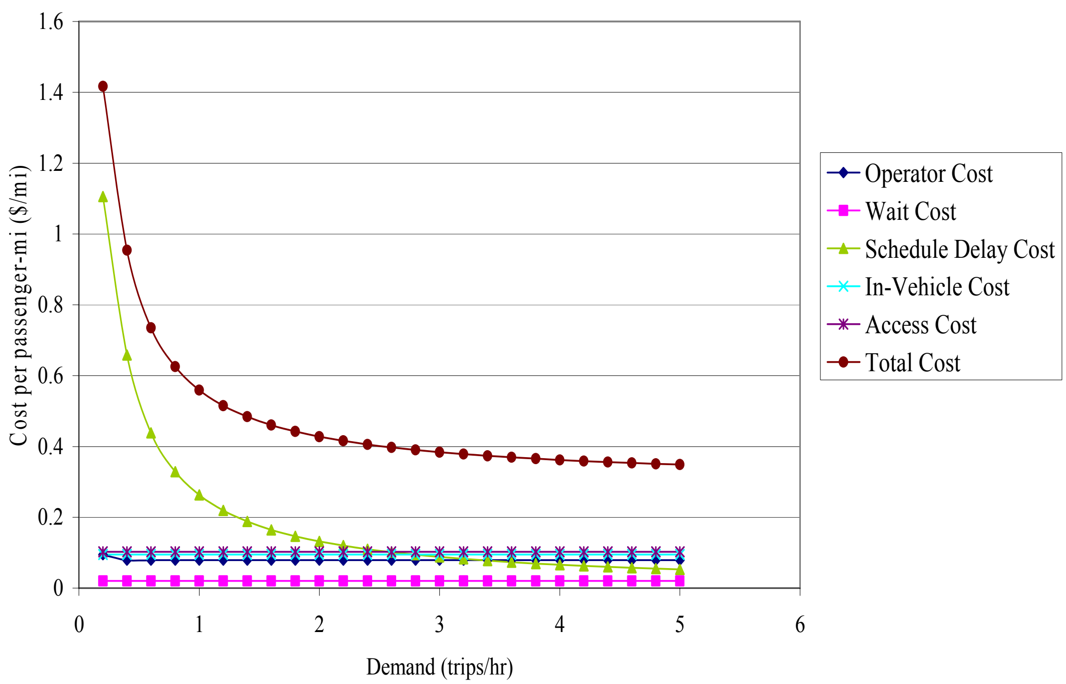

Figure 4.3 Average Costs for the Conventional Bus System in $/mi

4.2.2 Evaluation of Dial-A-Ride System

This section discusses the results of the model evaluation for the dial-a-ride system. To optimize the headway for the dial-a-ride system a numerical analysis was performed. For example, for a demand of 0.5 trips/hour, the optimized headway was determined to be 5.04 hours/vehicle. In Figures 4.4 through 4.6 the cost of the dial-a-ride system is considered. We can see that in Figure 4.4 the total cost of the dial-a-ride increases with demand. However, unlike for the conventional bus system, it is evident in all three figures for the dial-a-ride costs that the operator cost is a larger fraction of the total cost. This is expected because of the more personalized service provided by dial-a-ride services. This type of service leads to higher cost for the operator and low cost for the user. As shown in the figures, the user has no access cost expenses due to the door to door service offered by dial-a-ride. In Figure 4.5, there is a slight increase in dial-a-ride user costs as demand increases. This is due to a slight increase in the in-vehicle cost that occurs as demand increases, since tours get longer; however, this levels off as demand increases and headway is limited by vehicle size. User schedule delay decreases as demand increases. This occurs because as demand increases, headway increases and vehicles arrive more frequently. Otherwise, in Figures 4.5 and 4.6, user cost is approximately constant regardless of the demand.

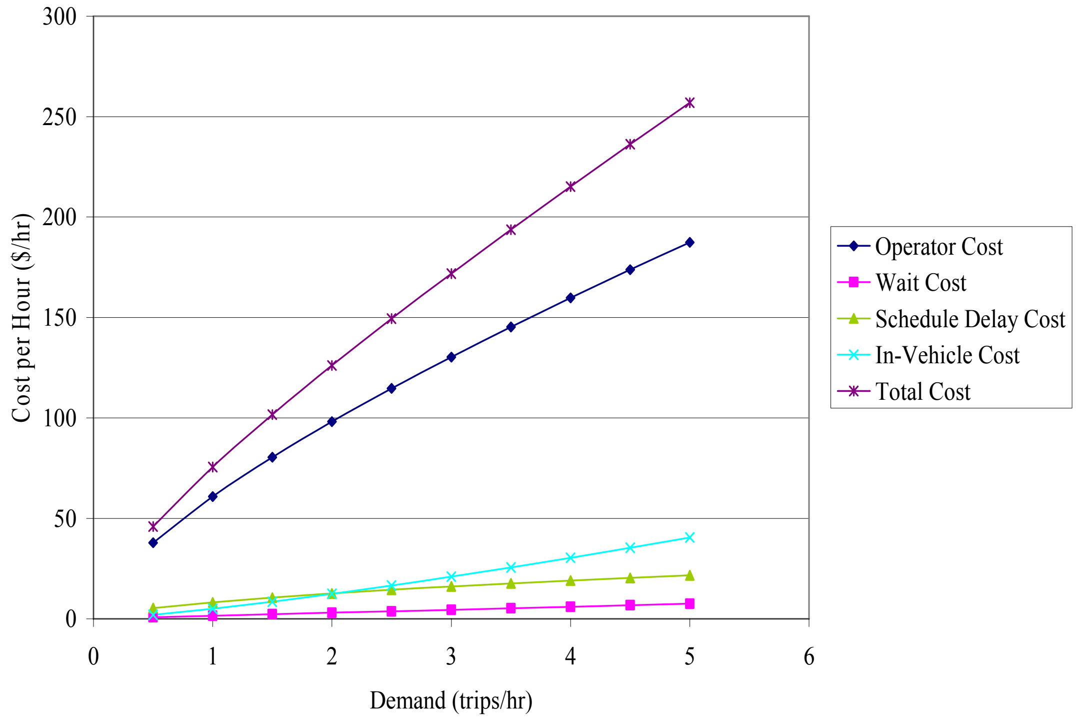

Figure 4.4 Costs for the Dial-A-Ride System in $/hr

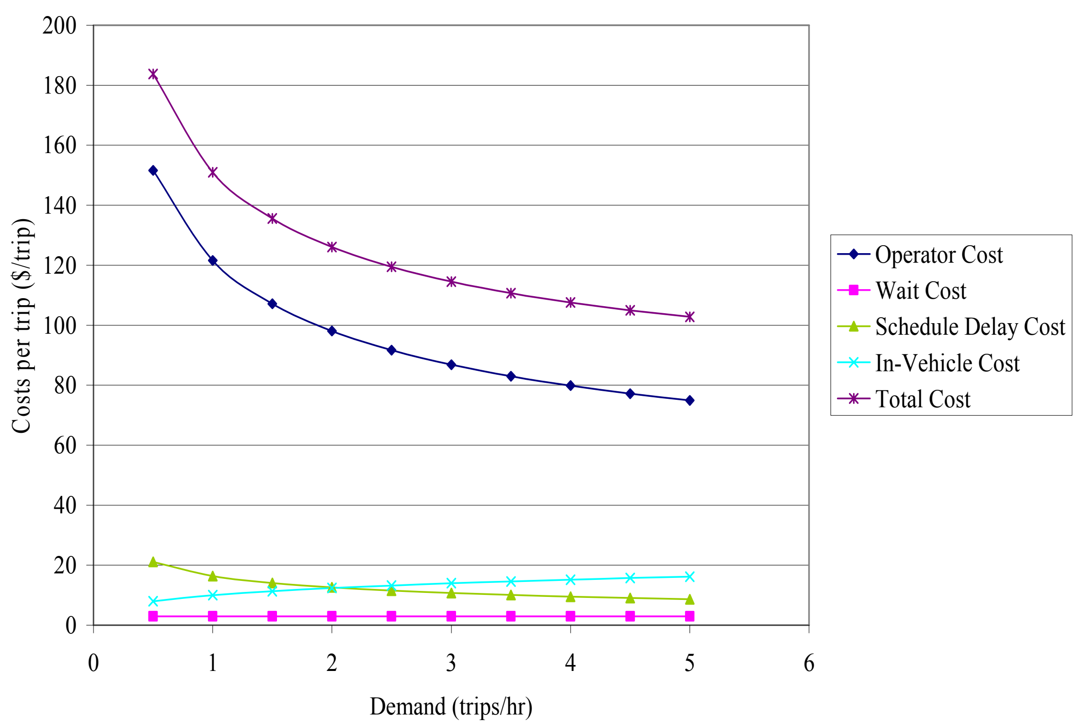

Figure 4.5 Average Costs for the Dial-A-Ride System in $/trip

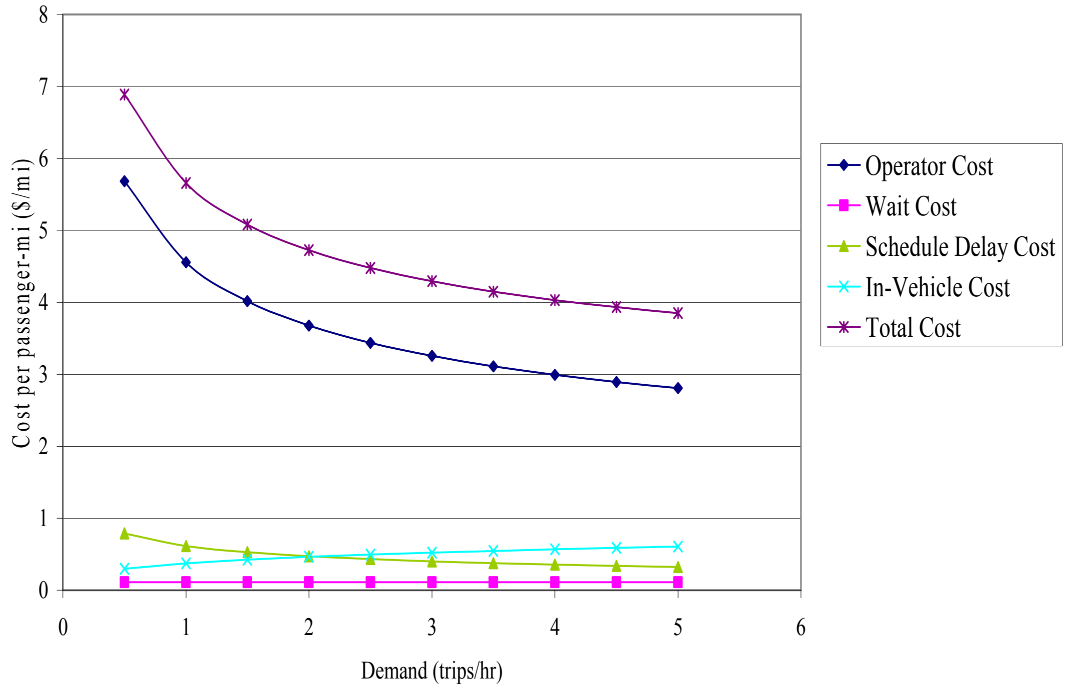

Figure 4.6 Average Costs for the Dial-A-Ride System in $/mi

4.2.3 Evaluation of Taxi System

A numerical analysis was also performed for the taxi system, using queuing theory to determine the users' wait time. Table 4.2 shows an example of the numerical analysis performed for the evaluation of this system. For example in the table, where Q is 1 trip/hr, the wait time for minimum cost is 2.84 hours with three taxis serving the area. Note that with either one or two taxis in the system, the equation for wait time outputs a negative value because $\lambda > k\mu$. This indicates that this value is infeasible. Additionally, these analyses show how the wait time decreases for additional taxis in the system. For example, for a Q of 1 trip/hour, the wait time would be 0.04 hours with five taxis in the system, but the total cost increases as a result.

Table 4.2 Example of the Numerical Analysis for the Taxi System*

Q = λ	**k**	**w**	**Operator Cost**	**User Wait Cost**	**Total**
0.5	1	-25.6	30	-425.6	-395.6
	2	0.636025	60	18.78012	78.78012
	3	0.088069	90	11.09718	101.0972
	4	0.01378	120	9.834267	129.8343
	5	0.002047	150	9.634804	159.6348
	6	0.000278	180	9.604721	189.6047
Q = λ	**k**	**w**	**Operator Cost**	**User Wait Cost**	**Total**
1	1	-3.01176	15	-41.6	-26.6
	2	-13.2129	30	-215.019	-185.019
	3	0.939214	45	20.29607	65.29607
	4	0.17858	60	12.63586	72.63586
	5	0.042013	75	10.31421	85.31421
	6	0.009935	90	9.768897	99.7689

*The shaded cell denotes the wait time for a system optimized for lowest cost.

Figures 4.7 through 4.9 show the cost of the dial-a-ride system. Again, as observed with the previous systems, operator cost makes up the largest fraction of the total cost. In Figure 4.7, the brown line shows the minimum number of taxis needed to service the given demand. As expected, the total cost and operator cost closely follow this curve. First we observe that these cost increase at a very fast rate. This is due to the small number of passengers per trips that can be serviced with each vehicle. With the other systems, increased demand can easily be accommodated for larger demands without the need for additional vehicles in the system. Secondly, in this system the operator cost, on average, is at least double all other components of the cost. As in the dial-a-ride system, this is a result of the personalized service provided by the taxi system. The user access cost is zero due to the door-to-door service.

In Figures 4.8 and 4.9, we make two very interesting observations. First, the plots of average total cost, user wait cost and user schedule delay cost are saw-toothed. This is due to the constraint that there must be an integer number of taxis in the system. The wait cost increases until the limit for the amount of demand that can be served is reached, then the number of taxis increases. For instance, one spike corresponds to a set of wait times that can be serviced optimally with a particular number of taxis. At the start of the spike, the shorter wait time corresponds to a smaller demand. As the demand increases, so does the wait time and the user wait cost. However, the operator cost per trip decreases as demand increases within the analysis for an equal number of taxis. At the peak, the system has reached the optimal demand that can be serviced with that number of taxis. The next point corresponds to the next integer of taxis needed to optimally service that demand. Additionally, as the demand increases, the peak in the plot for operator cost occurs at a slightly lower cost. Second, the average in-vehicle cost is constant because the model for this cost is independent of demand. It assumes that a passenger's trip is independent of other users of the system and that the same average distance is traveled from all origins to destinations.

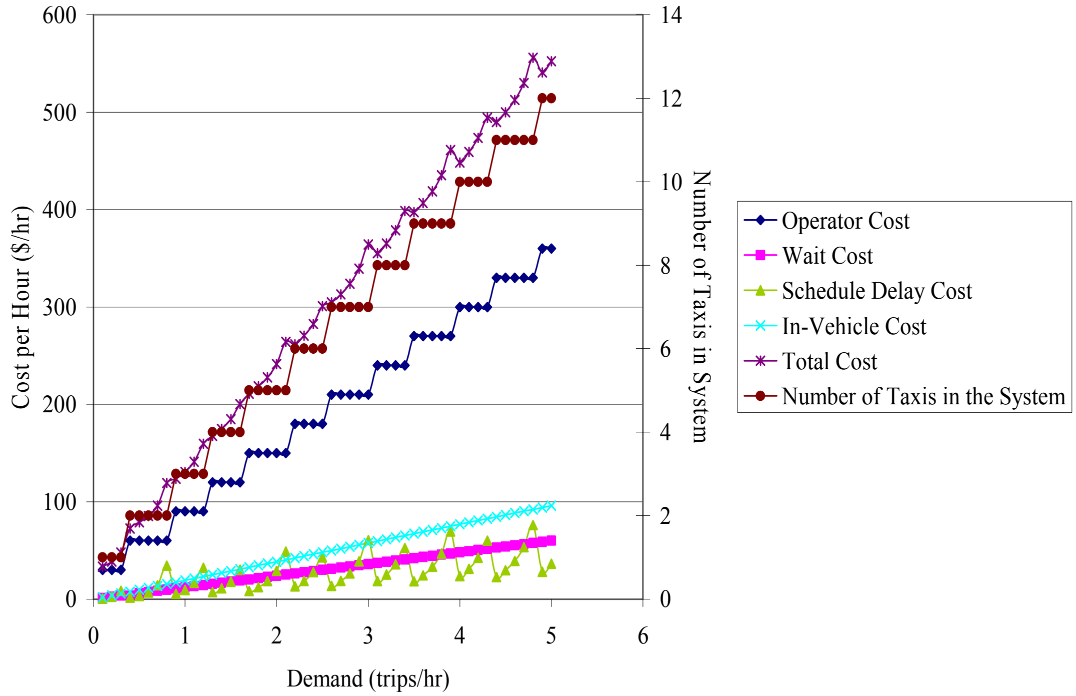

Figure 4.7 Costs for the Taxi System in $/hr

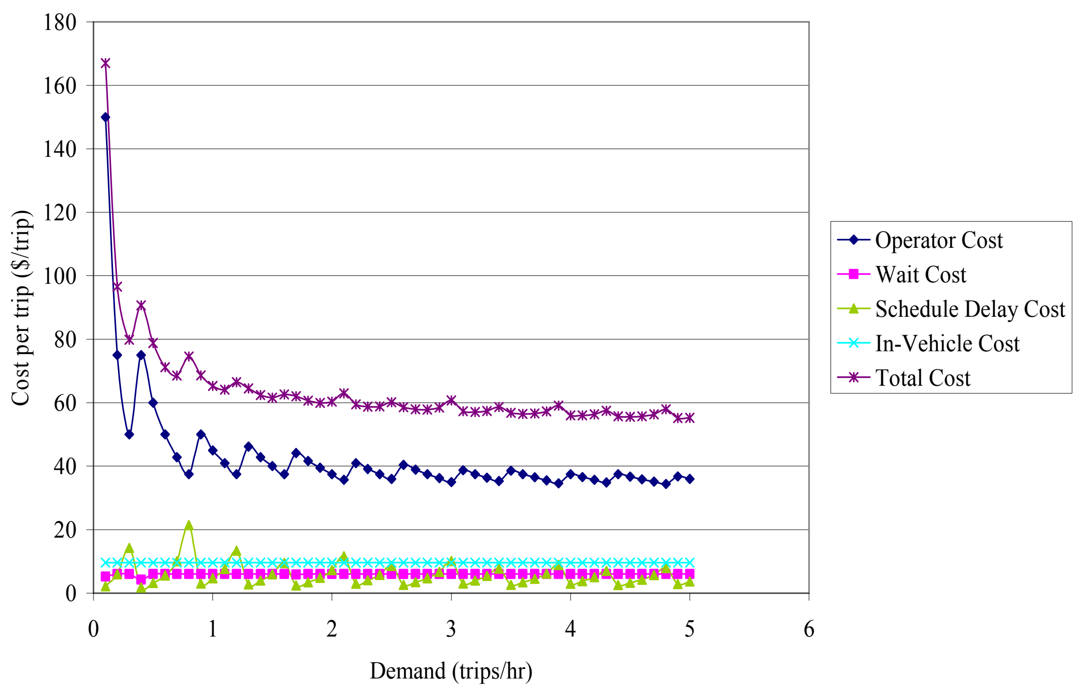

Figure 4.8 Average Costs for the Taxi System in $/trip

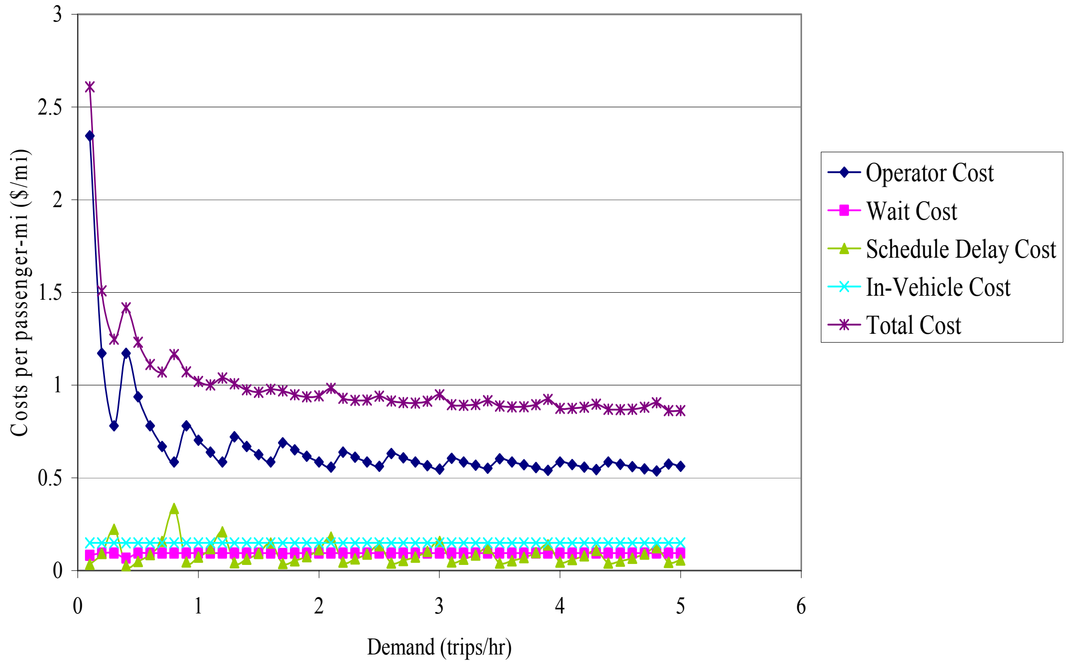

Figure 4.9 Average Costs for the Taxi System in $/mile

Chapter 5: System Analysis

This chapter compares the three rural transportation systems and analyzes how each modeled system is affected by various input parameters. In Section 5.1, a threshold analysis is used to determine which service type has the minimum total cost under given circumstances. The cost per trip is used to identify the critical demand Q, at which each type of public transportation mode, conventional bus, dial-a-ride or taxi is preferable. These analyses provide useful guidelines to policy makers for determining efficient public transportation system designs. In Section 5.2, a sensitivity analysis is performed to investigate the effects of various input parameters on the resulting optimized values for the conventional bus, dial-a-ride and taxi evaluations specified in Chapter 4. These analyses show how sensitive the conclusions of this work are to the values of input parameters.

5.1 Threshold Analysis

In this analysis, we consider different demand densities in an effort to find ranges within which each mode best serves the rural area. "Best" is defined as the system that minimizes the total system cost. In other words, we want to find the mode that is best suited for a particular demand or range of demand. Usually, determining the best public transportation system for a rural region is based on various other policy issues that each region must address independently. Often there is a balancing act between the costs of the user and those of the operator.

5.1.1 Threshold Analysis in $/trip

Figures 5.1 through 5.3 show the results of the threshold analysis. It should be noted that because the optimization process for the taxi system constrains the solution to an integer number of taxis in the system, the cost plots for the taxi has fluctuations. The models for the conventional bus and dial-a-ride systems do not constrain the models to an integer number of vehicles. The decision variable in these models is headway. These models ensure that the headway can be accommodated by the size of the vehicle, but it does not constrain the number of conventional buses or dial-a-ride vehicles to an integer. Thus, these resulting curves are a smooth.

In Figure 5.1 we see that the average cost per trip in scenario ranges from $55 to $220 per trip for all systems. However, considering overall total cost the taxi system seems to be the most appropriate service for demands less than 3 trips/hr. At this point, the conventional bus and taxi systems are equally viable. Upon closer review of the comparison of systems, we see that the user cost of the dial-a-ride systems can be as much as about $15 per trip cheaper than the other services. On the other hand, the conventional bus system is much less expensive than the other systems when considering operator cost.

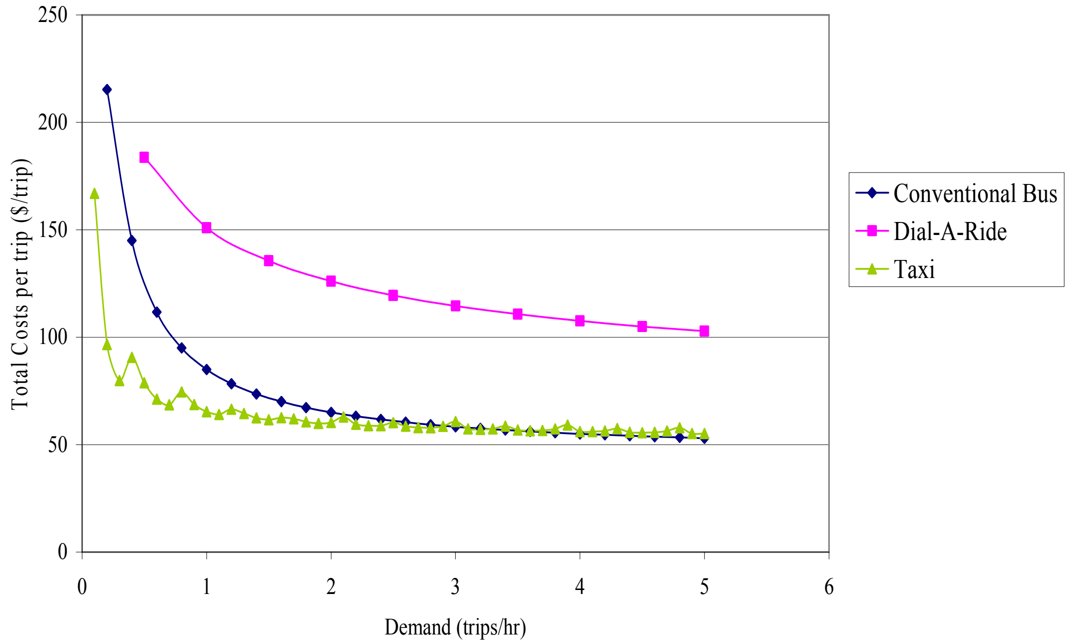

Figure 5.1 Comparison of Total Cost of Three Systems in $/trip

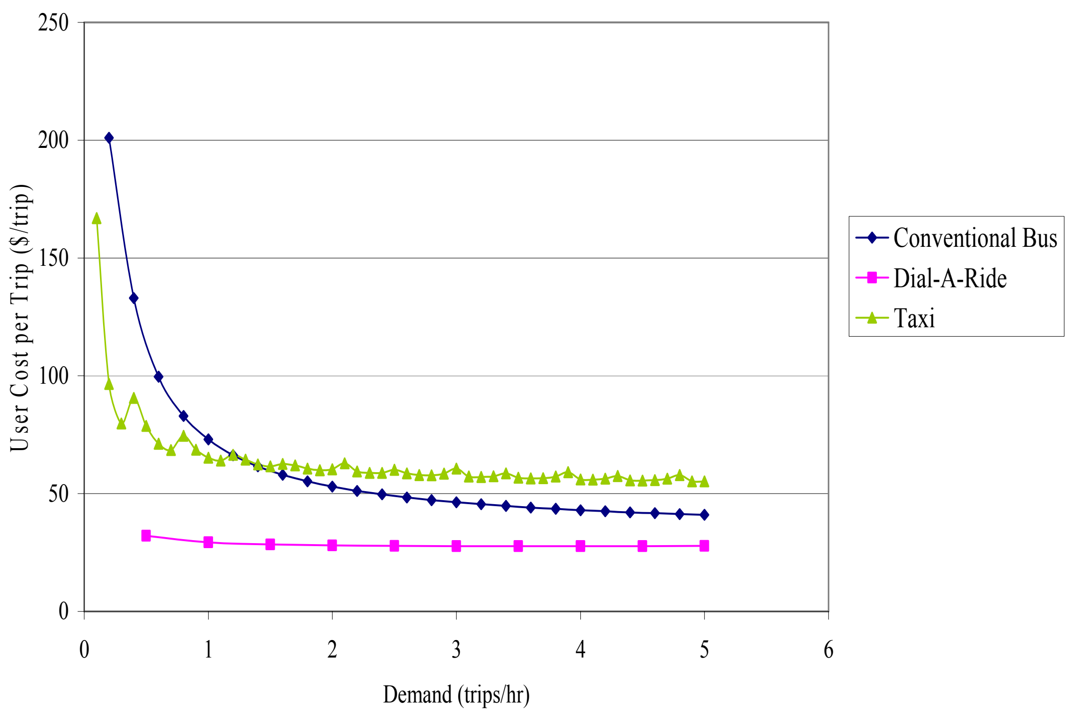

Figure 5.2 Comparison of User Cost of Three Systems in $/trip

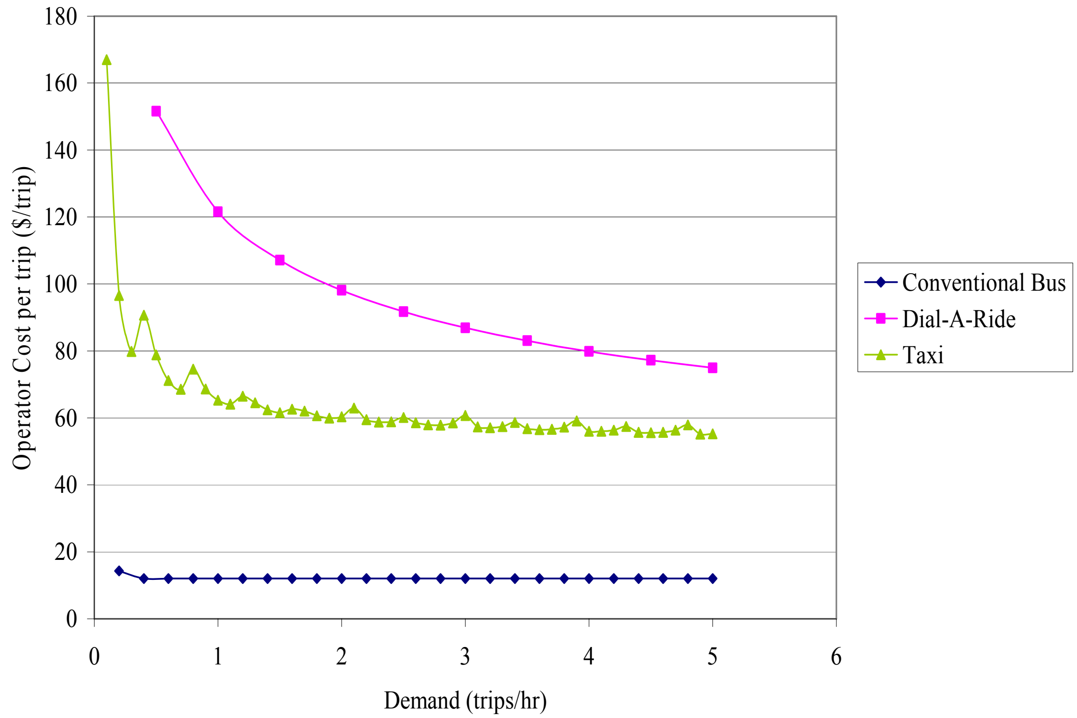

Figure 5.3 Comparison of Operator Cost of Three Systems in $/trip

5.1.2 Threshold Analysis in $/mi

Figures 5.4 through 5.6 compare the average cost of all three types of rural public transportation in $/passenger-mile. Figure 5.4 show that the conventional bus system has the least total cost when considering the average number of passenger-miles traveled by the system. This occurs because the conventional bus service does not provide door-to-door service. The conventional buses stay on major roads and do not deviate from their preset routes, while dial-a-ride and taxi systems provide more personal service. On average the conventional bus passenger's average travel distance is longer and thus, the cost per passenger-mile is less than the other systems. This is even more evident when examining operator cost in Figure 5.6. However, for user cost at lower demand densities, the taxi becomes competitive with the conventional bus system.

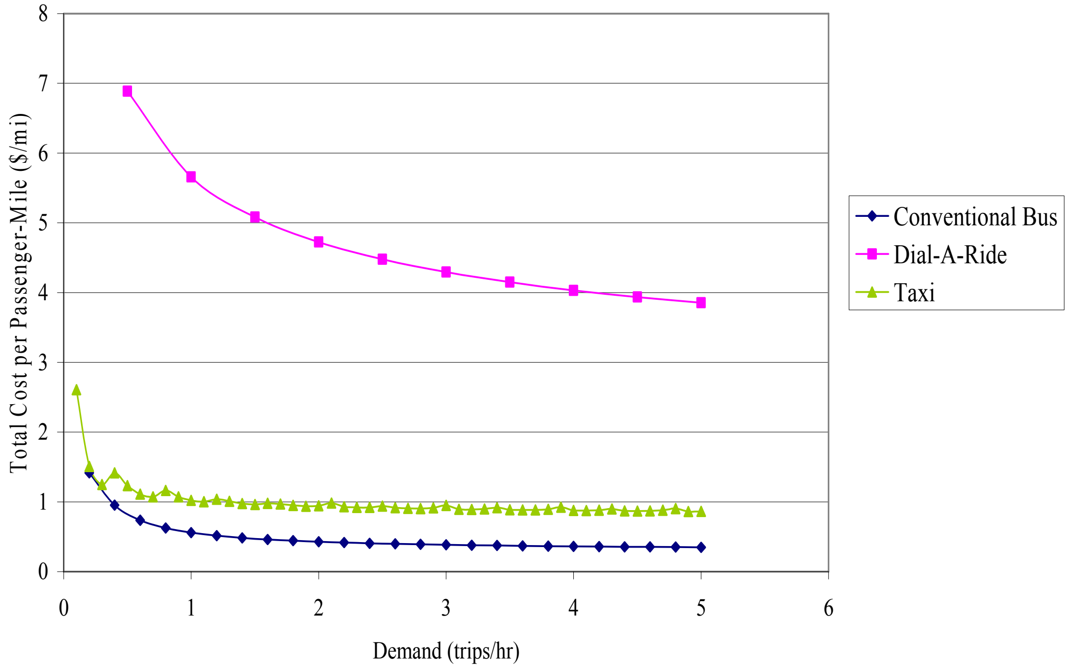

Figure 5.4 Comparison of Total Cost of Three Systems in $/mile

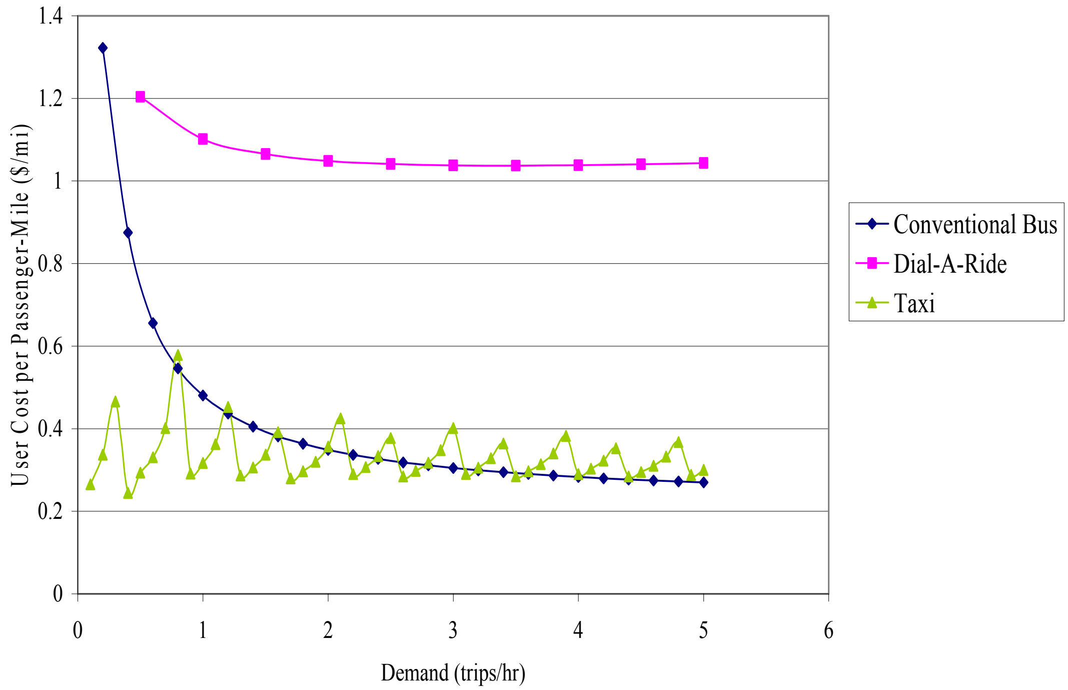

Figure 5.5 Comparison of User Cost of Three Systems in $/mile

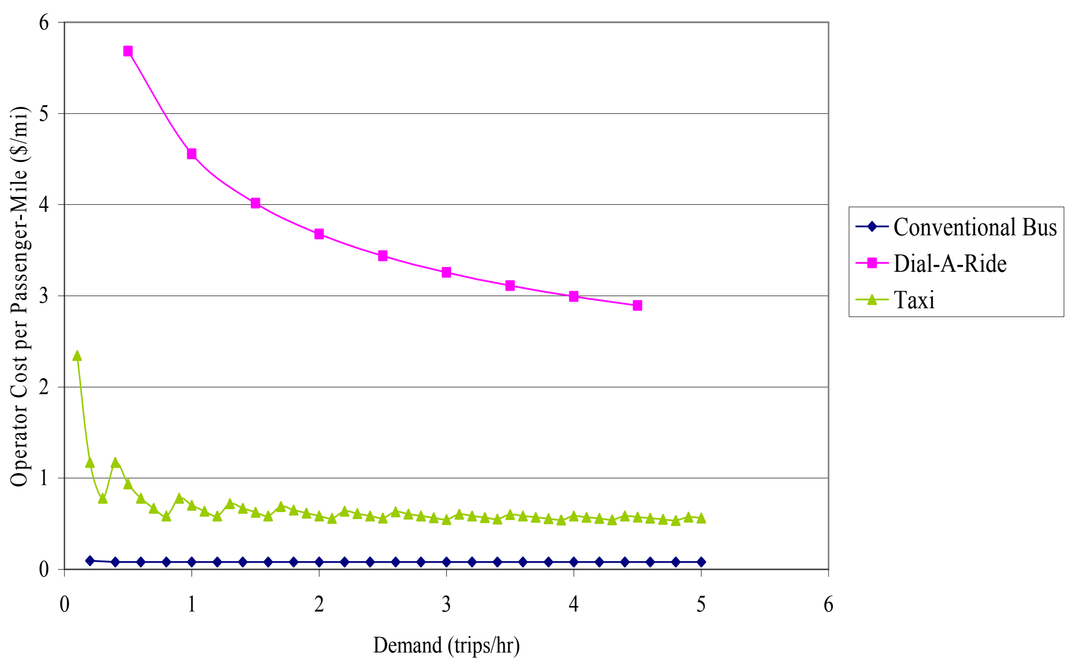

Figure 5.6 Comparison of Operator Cost of Three Systems in $/mile

5.1.3 Threshold Analysis for Demand within ¼ Mile of Bus Stops

Two assumptions for the conventional bus system model make this system (and model) unlikely to be worthwhile in real-life applications. The model assumes that conventional bus system users outside of the ¼ mile perimeter of the bus stop have access to automobiles. Access to an automobile means that users either have a car to drive, can borrow a car or can get a ride to the bus stop. Secondly, the model assumes that these users will use the vehicles to get to the bus stop instead of just driving straight to town. These are issues that are usually associated with conventional bus systems inability to ensure mobility and access opportunity to individuals in low-density areas. To better compare the conventional bus system to the other models, this section only examines the service area within ¼ mile of the bus stops for all rural public transportation systems. Thus, this entire population is efficiently served by all systems. In other words, all users can directly access all three systems by walking to the system pick-up point.

Figures 5.7 through 5.9 show the comparison of public transportation systems serving only the demand within walking distance to the bus stops. It should be noted that the costs in this analysis are extremely high due to the low demands and large distances. The analysis demonstrates the difficulty of developing cost efficient public transportation system with such low population densities and large service areas. Figure 5.7 depicts the comparison of the total cost for the three systems. The total cost of all three systems decreases non-linearly with an increasing demand. The conventional bus is has the lowest total cost. Figure 5.8 shows that within smaller areas, the dial-a-ride system has lower user costs. In Figure 5.9, the plot shows that conventional bus is the least expensive.

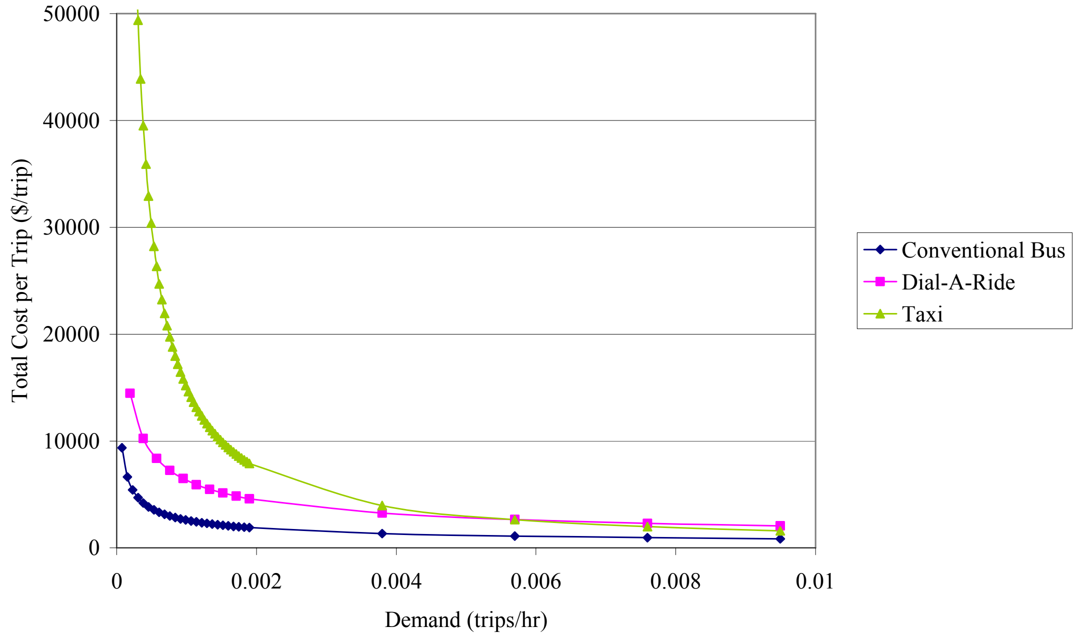

Figure 5.7 Comparison of Total Cost of Three Systems

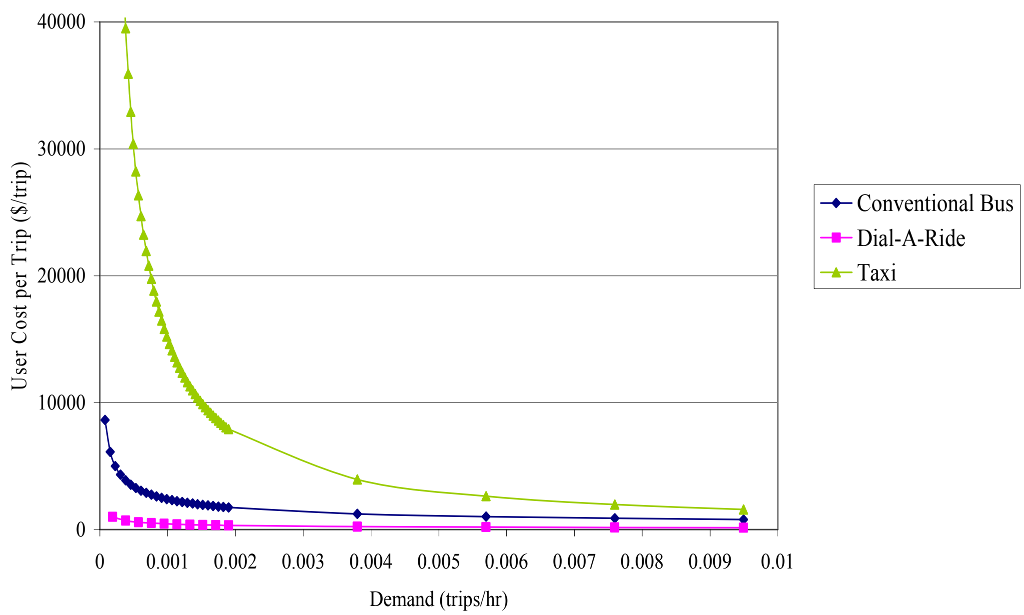

Figure 5.8 Comparison of User Cost for Three Systems

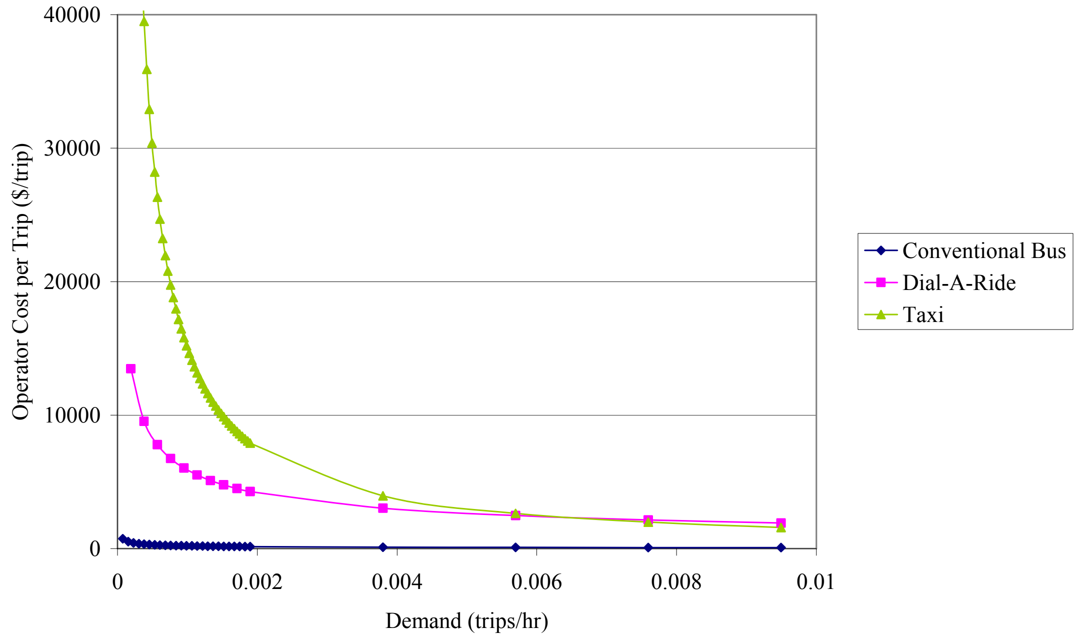

Figure 5.9 Comparison of Operator Cost for Three Systems

5.2 Sensitivity Analysis

It is important in any analysis of a model to determine how the model responds to changes in input parameters. This section determines these relations for each type of rural public transportation. The analysis examines how the service area, operator cost, value of time, speed and demand affect the total cost as well as the decision variables used to optimize each system. If the model is very sensitive to changes in a particular input parameter, that parameter should be predicted as accurately as possible and decisions should be made more cautiously. Unless otherwise stated the service area from Figure 3.1 and the baseline values from Chapter 4 are used in this analysis. The analysis assumes a demand density of 0.001 trips/square miles/hour.

5.2.1 Effects of Service Area

In Figures 5.10, 5.11 and 5.12, we examine the effects of service area on the model. In this analysis, the service areas range from 25 to 5625 square miles. We see that on average total cost initially decreases and then increases with service area. For the conventional bus and dial-a-ride systems, there is a trade-off between lower headways and higher operator cost per trip that causes this effect. For the taxi, the operator cost per trip is higher at lower demands, but at higher demands user cost increases.

Figures 5.11 and 5.12 examine how the decision variables, headway and number of taxis in the system are affected by service area. In Figure 5.11, we see that headway decreases as area increases in the conventional bus and dial-a-ride systems. The headway for both systems decreases non-linearly. The number of taxis in the system is shown to increase somewhat linearly with service area in Figure 5.12.

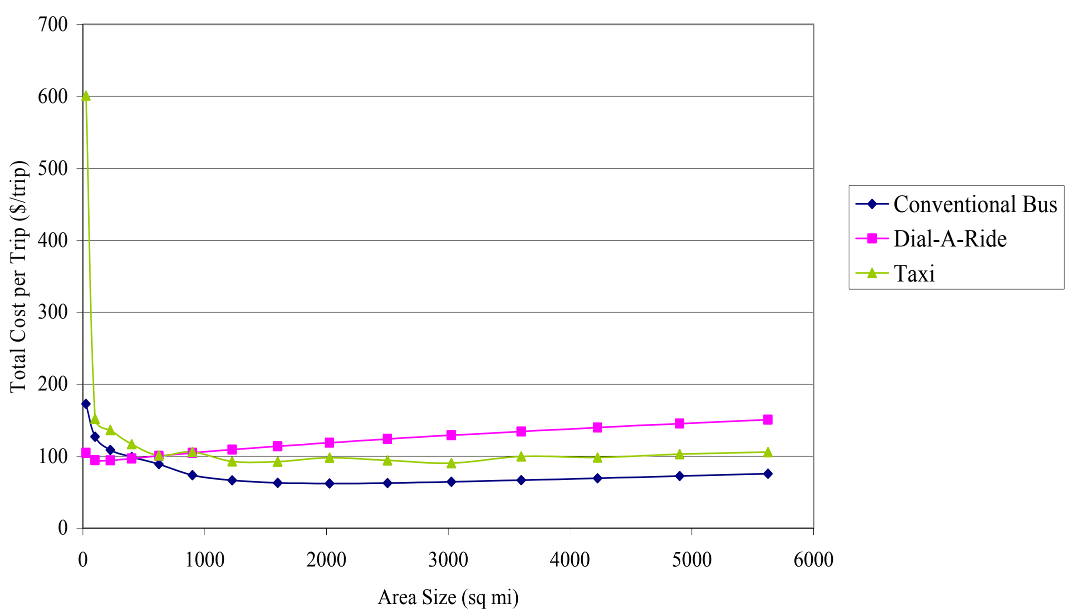

Figure 5.10 Effects of Area on Average Cost of System in $/trip

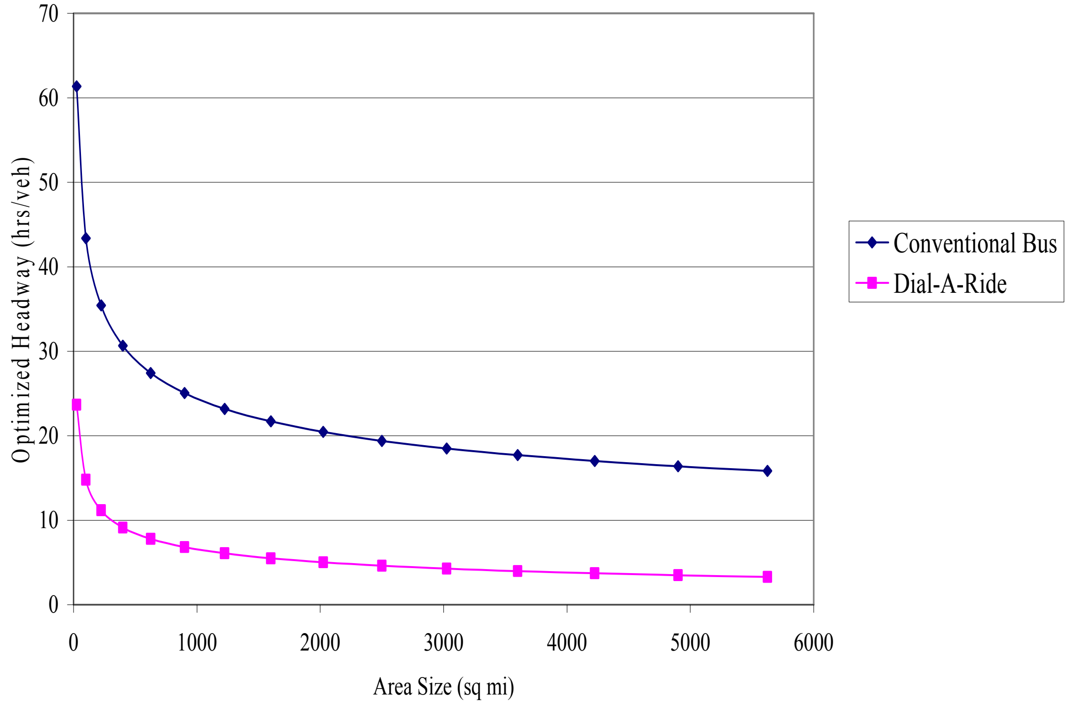

Figure 5.11 Effects of Area on Optimized Headway

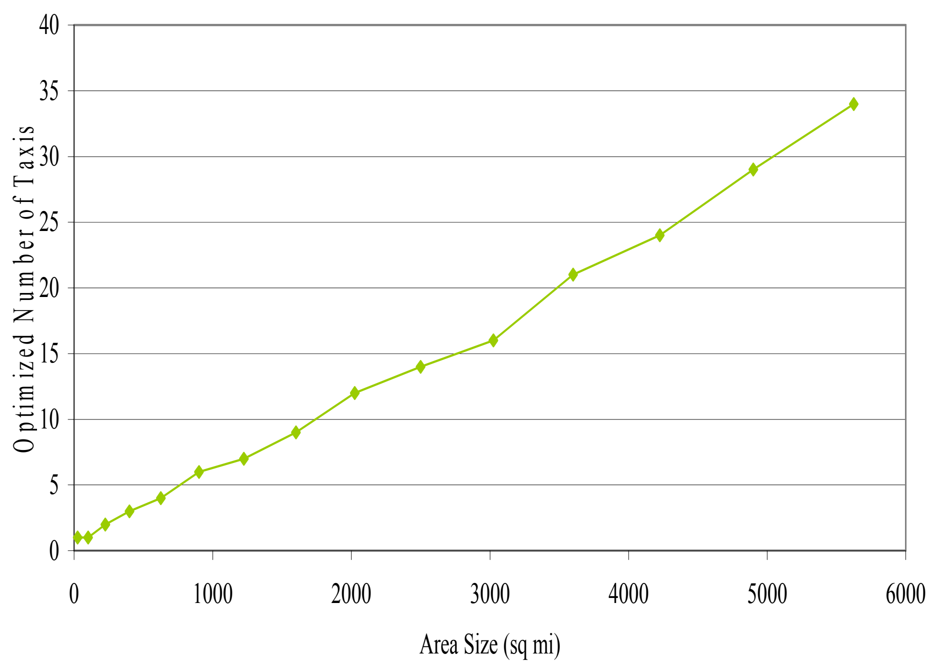

Figure 5.12 Effects of Area on Optimized Number of Taxis in the System

5.2.2 Effects of Vehicle Speed

Figures 5.13 to 5.15 show how the model responds to vehicle speed. This analysis examines speeds from 5 to 75 mi/hr. In Figure 5.13 it is shown that as speed increases, the total cost per trip decreases non-linearly for all systems. This is a result of the operator, in-vehicle and scheduled delay costs decreasing with increased vehicle speed. At about 55 miles/hour, the total costs for the conventional bus and taxi intersect. All three types of rural public transportation begin to converge at 75 mi/hr. As expected, it is shown in Figures 5.14 and 5.15 that both the headway and number of taxis in the system decrease with as vehicle speed increases. It should be noted that this analysis does not take into account the effects of vehicle speed on the operating costs.

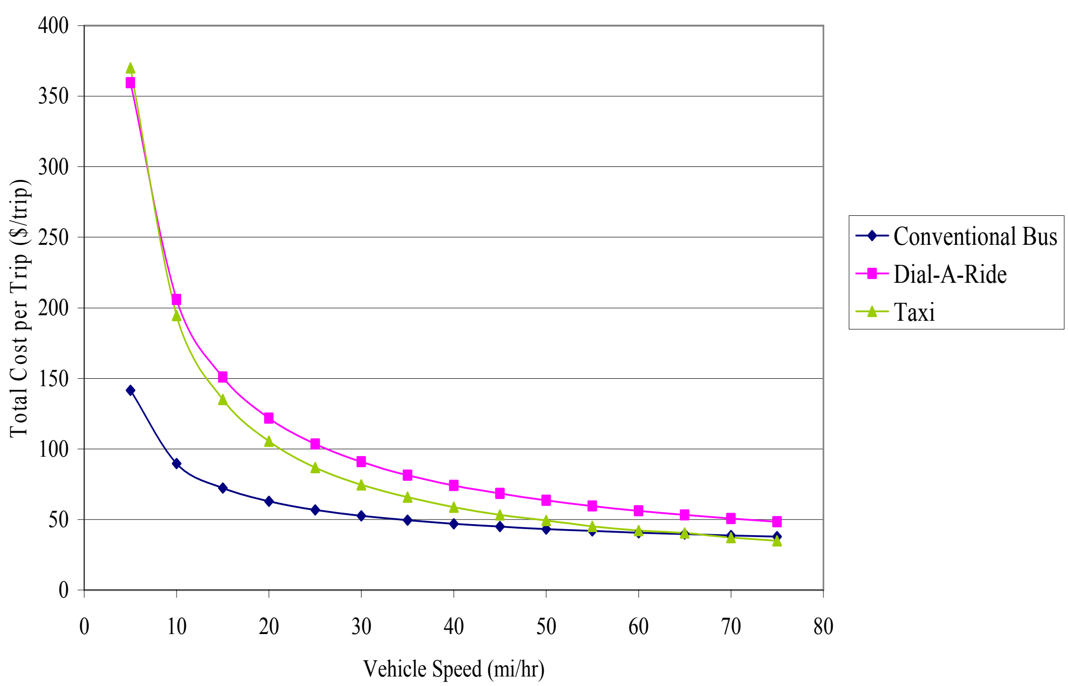

Figure 5.13 Effects of Speed on Average Total Cost of System in $/trip

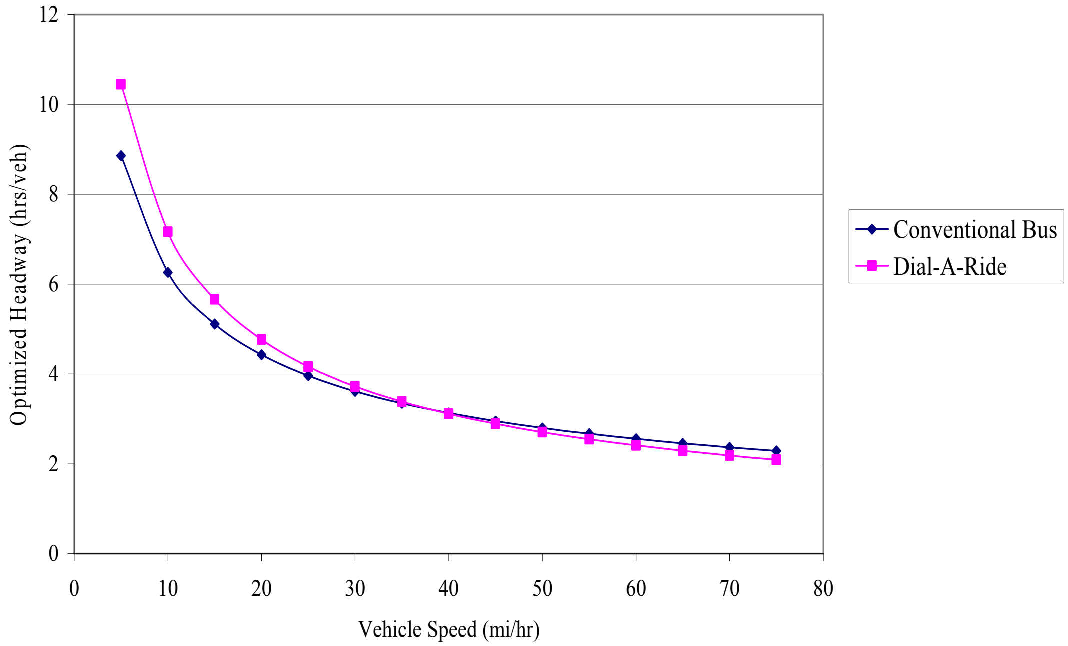

Figure 5.14 Effects of Speed on Optimized Headway

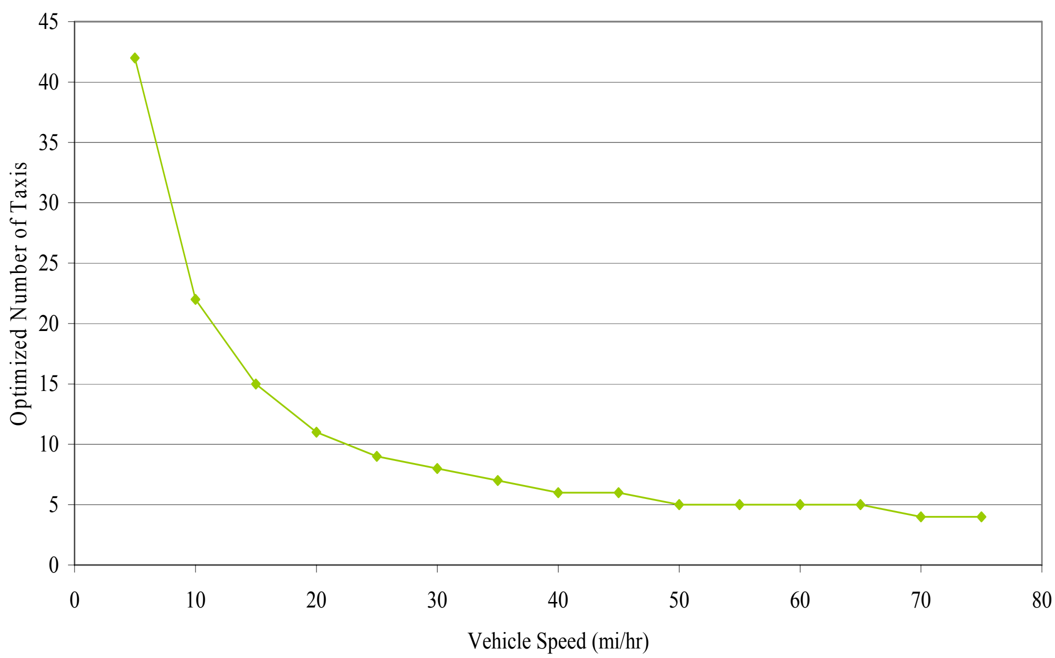

Figure 5.15 Effects of Speed on the Optimized Number of Taxis in the System

5.2.3 Effects of Operator Cost

The effects of operator cost on total cost and the decision variables are shown in Figures 5.16 through 5.18. Operator cost is studied between the values of $10 and $110 per vehicle-hour.

As anticipated, the total cost increases as operator cost increases in Figure 5.16. The total cost per trip for the dial-a-ride system is strongly affected by the operator cost. On the other hand, the total cost for the taxi system increases at a much slower rate as the operator cost increases. Furthermore, in Figure 5.17 the headways for the conventional bus and dial-a-ride models increase with an increased operator cost. In Figure 5.18 it is shown that the number of taxis in the system decrease with an increased operator cost. At an operator cost of $20/vehicle-hour, the taxi model compensates for the higher operator cost by decreasing the number of taxis in the system.

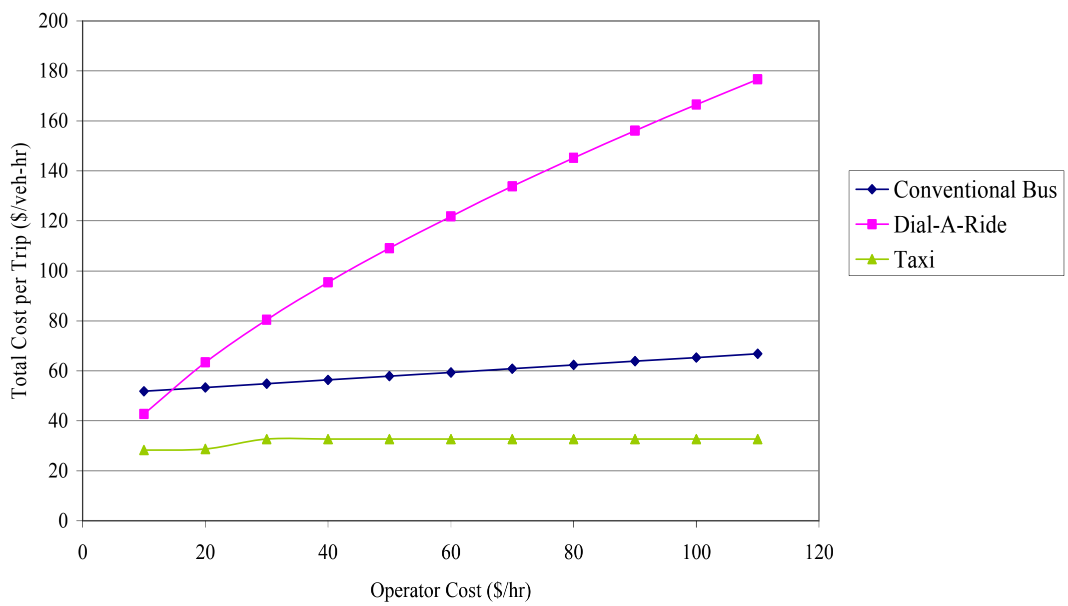

Figure 5.16 Effects of Operator Cost on Average Cost of System in $/trip

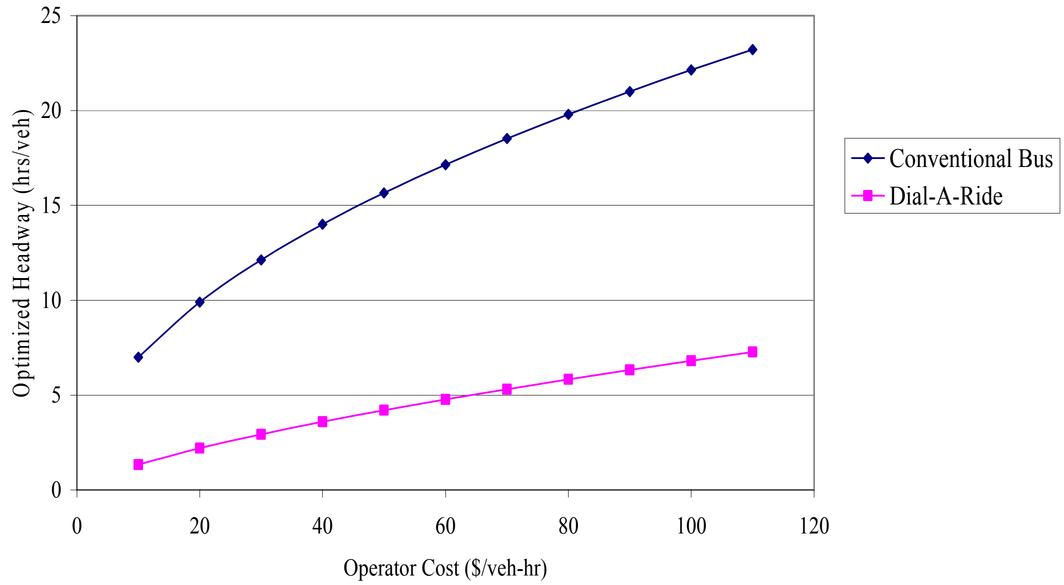

Figure 5.17 Effects of Operator Cost on Optimized Headway

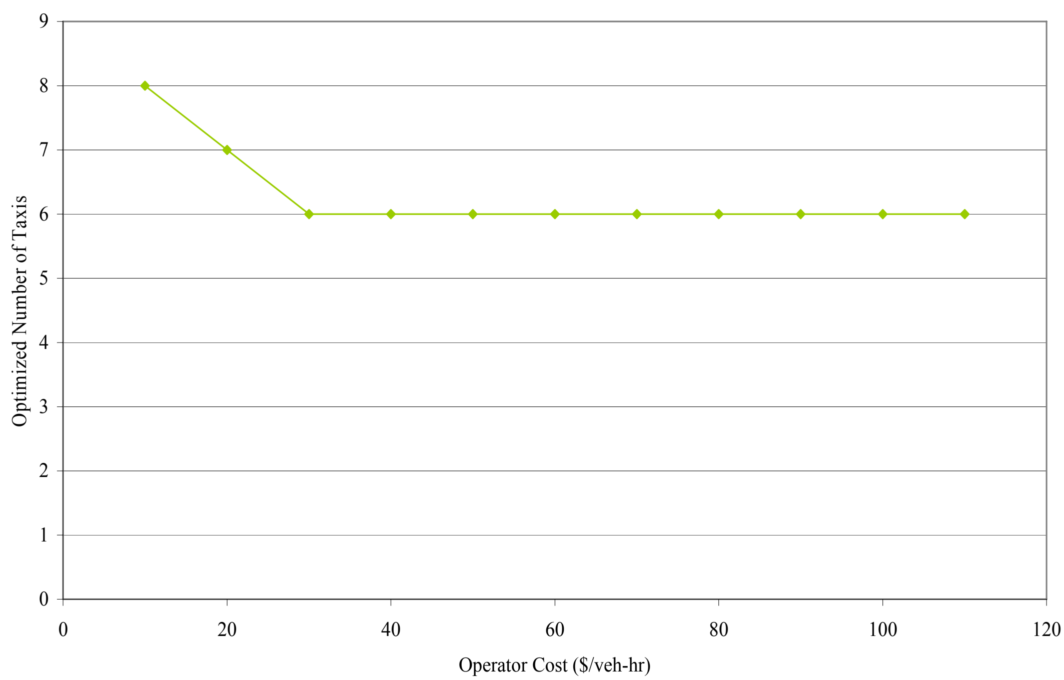

Figure 5.18 Effects of Operator Cost on Optimized Number of Taxis in the System

5.2.4 Effects of Value of Time

In Figures 5.19, 5.20 and 5.21 we examine the effects of value of time on the model. The values of time studied range from \$6 to \$30 per passenger-hour. As shown in Figure 5.19, the average total cost increases with an increased value of time. It should also be noted that at about \$6/passenger hour, the conventional bus and taxi systems are nearly equal. At this point, either alternative is viable when considering average total cost. In Figure 5.20 the optimized headway declines at a decreasing rate with an increased value of time. Figure 5.21 shows that as user's value of time increases, the optimized number of taxis in the system increases. The shift in the curve for the taxi system at \$14/passenger hour is a result of the model adjusting the number of taxis in the system to minimize total cost. At this value of time, the total cost is decreased by introducing an additional taxi into the system and decreasing the user wait time.

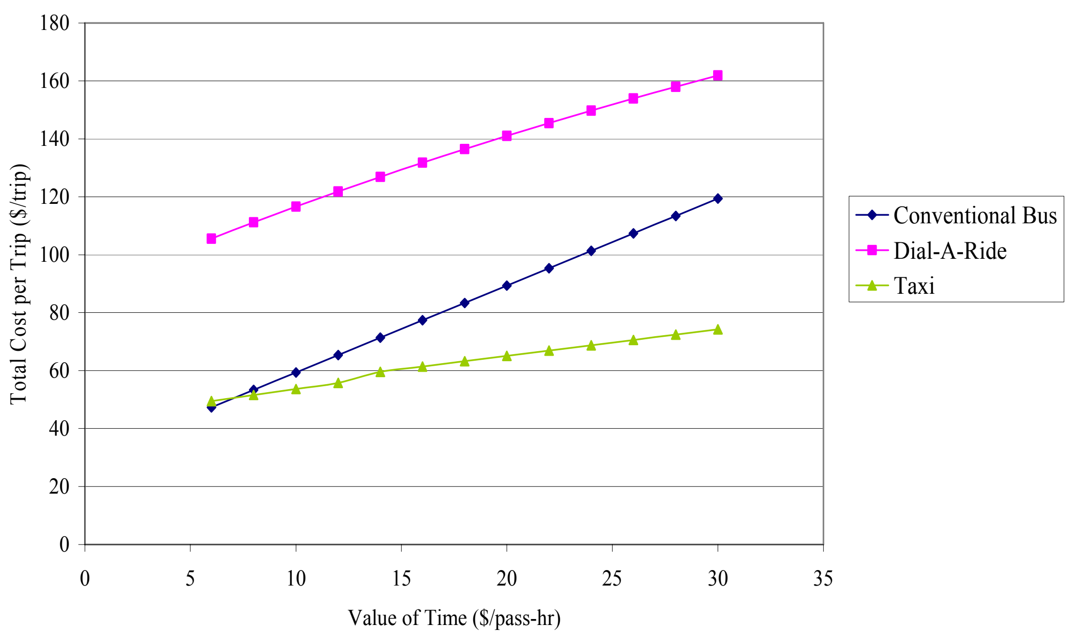

Figure 5.19 Effects of Value of Time on Average Cost of System in \$/trip

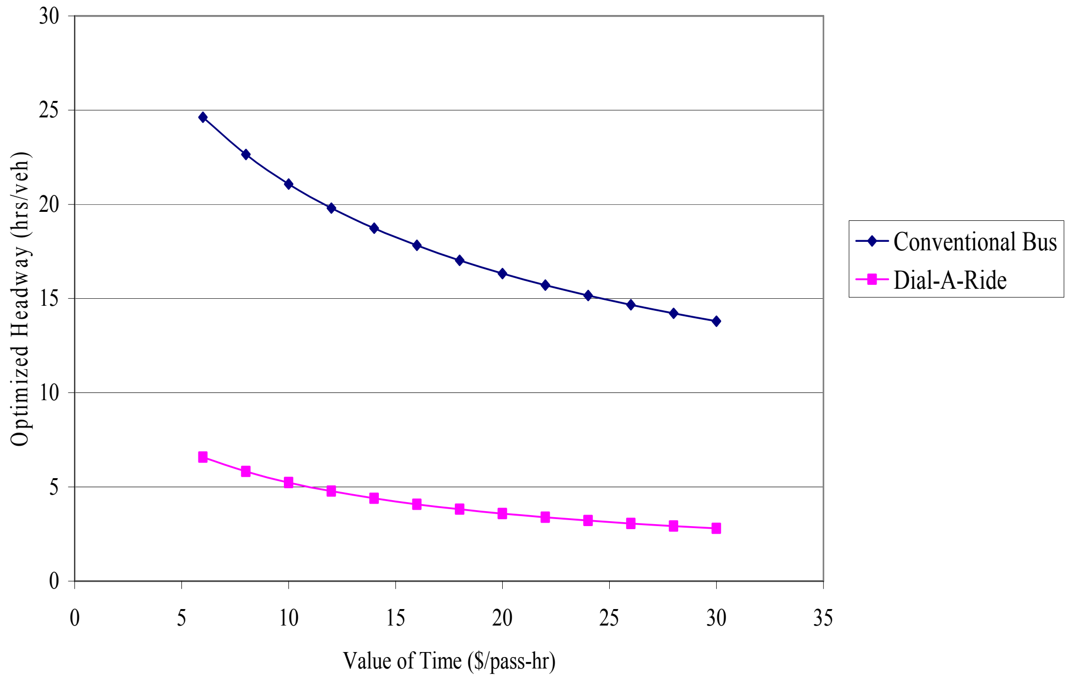

Figure 5.20 Effects of Value of Time on Optimized Headway

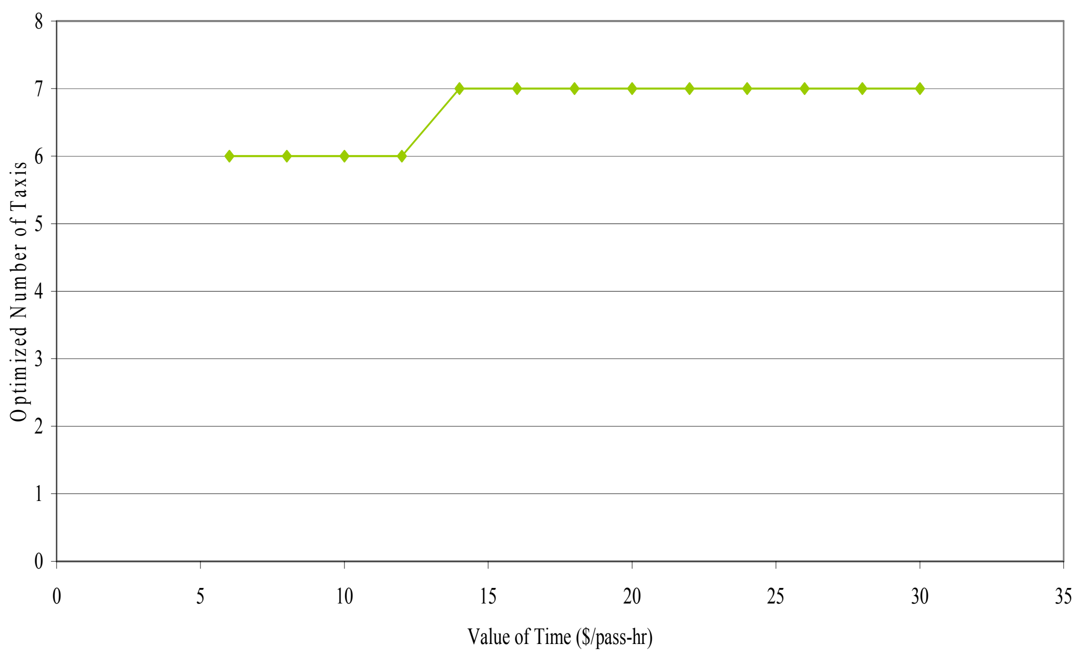

Figure 5.21 Effects of Value of Time on Optimized Number of Taxis in the System

5.2.5 Effects of Demand

In this section we explore the effects of demand on headway and the number of taxis in the system. The effect of demand on cost is thoroughly discussed in Section 5.1. In Figure 5.22, as the demand increases, the optimized headway decreases at a non-linear rate. Additionally, the dial-a-ride system has a shorter headway for densities less than 5 trips/hr. As expected, Figure 5.23 shows that the number of taxis in the system increases almost proportionally with increased demand.

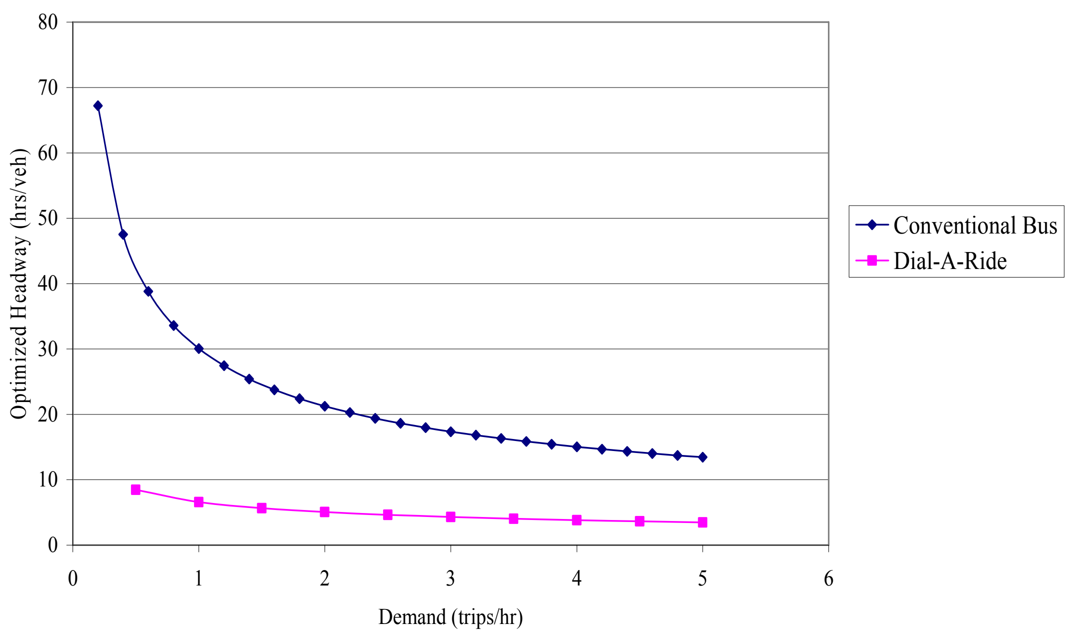

Figure 5.22 Effects of Demand on Optimized Headway

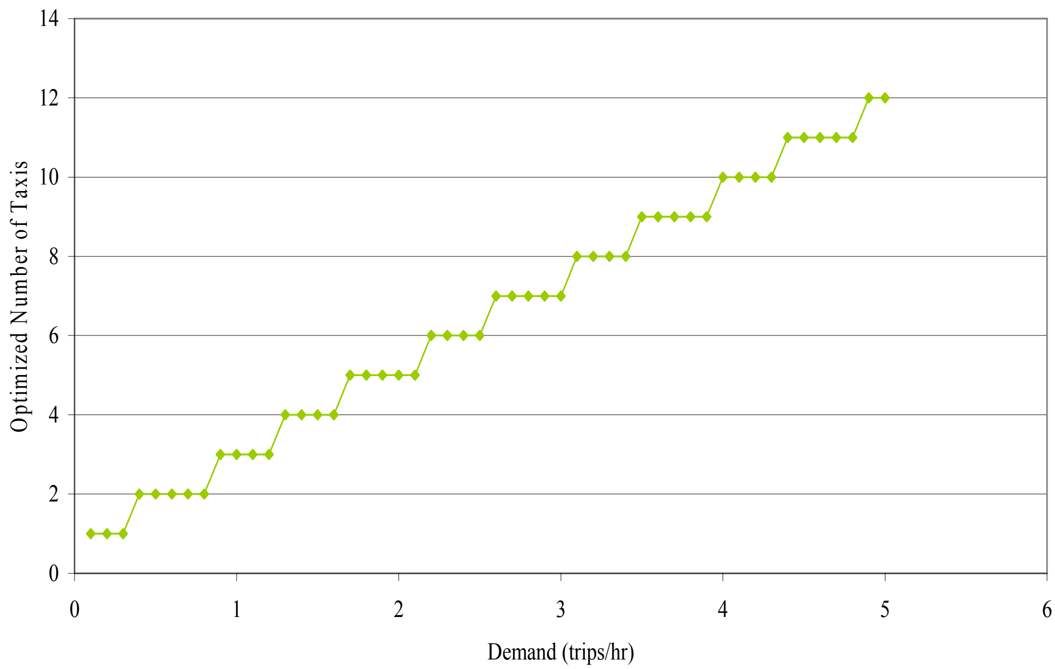

Figure 5.23 Effects of Demand on the Optimized Number of Taxis in the System

Chapter 6: Conclusions and Recommendations

6.1 Summary of Results

The primary outcomes of this research are:

1. Optimization models have been developed for conventional bus, dial-a-ride and taxi systems in rural areas in order to consistently compare these modes. The decision variables were headway for the conventional bus and dial-a-ride systems and number of taxis in the system for the taxi system.
2. Demand thresholds among the three modes have been determined for average total cost, average user cost and average operator cost for various circumstances.
3. For each mode, the effects of several input parameters, including service area, vehicle speed, operator cost and value of time on the total cost and the decision variables have been examined.

6.2 Conclusions

In this thesis, several forms of public transportation are analyzed and models developed in an effort to provide rural transportation services at minimum total system cost. Models are developed and evaluated for conventional bus, dial-a-ride and taxi transportation systems. After threshold and sensitivity analyses, several characteristics of the models are identified for each type of system. The major conclusions of this thesis are listed below.

1. The taxi system has the least expensive total cost per trip. However, when considering user cost the dial-a-ride has the lowest cost per trip. This can mostly be attributed to this system having no access cost due to the door-to-door service that it provides as well as the operator cost being dispersed amongst more passengers per route. The conventional bus system, on the other hand, has a lower operator cost. This is a result of fewer vehicles being required to operate the conventional bus system. The dial-a-ride and taxi services provide more personal service to the user; thus, they often require more vehicles and longer routes.

2. The analysis of cost per passenger-mile resulted in the conventional bus system having the least total cost per trip. On average the conventional bus passenger's average travel distance is longer and thus, the cost per passenger-mile is less than the other systems.

3. When examining a specific population for which all rural public transportation systems can provide service without the use of an automobile, the conventional bus system had the lowest cost per trip. This is due to the extremely low cost associated with the conventional bus service compared to the other modes because they offer door-to-door service.

4. The models respond differently to various input parameters. The total cost per trip increases with increased operator cost and value of time demand, but decreases with increased speed and demand. Headway increases with increased operator cost, but decreases with increased service area, speed, value of time and demand. The number of taxis in the system increases with increase service area, value of time and demand but decreases with increase speed and operator cost.

5. These models provide policymakers in rural regions some methods for evaluating the public transportation options in region. First, the models can easily be modified to consider important characteristics of their region such as area, major existing roadways, speed limits and demand. Next, guidelines for developing public transportation systems can be observed for policymakers. Lastly, the models can be used to balance operator and user costs according to budgetary needs of the agency as well as the socioeconomic needs of the users.

6.3 Recommendations for Further Research

The following suggestions are made for further research:

1. Although these models are applicable to many geographical regions, it would be interesting to consider other rural area types. For example, these models assume that there is a town that serves as an attractor for the trips. However, future research might consider rural regions that do not have a concentrated

center of activity. Instead, a region might have several small towns in which none attract the majority of the population.

2. This research assumes that passengers are uniformly distributed within a rectangular service area and over time. Future research should consider other spatial and temporal distributions of demand. For example, passenger distribution could be a function of distance from the town or major roadways that service the region. Additionally, the model could be modified in the future to analyze areas with various shapes rather than just rectangular ones.

3. To specifically deal with the characteristics of rural passengers, some studies that analyze the value of rural passengers' time and actual times that rural passengers are willing to wait for service would be beneficial to the models developed here. Moreover, it would be fascinating to compare these results with those of urban residents.

4. In analyzing a taxi system, the arrival rate should be considered as a function of exogenous factors, such as the distance from the pick-up point to the town, value of time and/or the connectivity of the street network. This could better link within the model the temporal characteristics of a region to the relation between taxi availability and the taxi utilization.

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