ABSTRACT

Title of Dissertation: HETEROGENEITY AND INPUT REALLOCATION

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In this dissertation, we analyze some patterns of aggregate job reallocation that are significantly determined by the coexistence of heterogeneous businesses in any industry. First, we argue that the interaction of non-strictly convex adjustment costs and learning about true efficiency can explain the significant growth of survivors in a cohort of entering firms. Using Portuguese data we find evidence that survivors are the main source of growth in the cohort's average size, and that their contribution varies across sectors. By simulation, we show that we need adjustment costs to match this evidence with a selection model of industry dynamics. In a calibration of the model, we find that proportional costs and the fixed exit cost are key parameters in matching the evidence, and that firms in manufacturing learn relatively less initially about their efficiency, and are subject to much larger adjustment costs than firms in services.

Second, we analyze how does structural heterogeneity across classes of firms affects the cyclical behavior of aggregate job flows. We find that types of firms whose optimal

employment is relatively more determined by aggregate shocks than by idiosyncratic shocks influence the dynamics of aggregate job flows by more than they affect average aggregate flows. In Portuguese data, we conclude that large and old firms tend to affect aggregate dynamics by more than their already large employment shares would suggest. This tends to make job reallocation less procyclical than otherwise, and affects aggregate behavior in some sectors.

Finally, as a background for the empirical analysis that is used in this dissertation, we analyze basic facts about the business cycle and gross job flows in Portugal from 1986 to 2000. We conclude that gross job flows are large and react in predictable ways to the business cycle and that patterns of job reallocation vary widely across sectors and firm's age and size.

HETEROGENEITY AND INPUT REALLOCATION

by

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To my parents

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Chapter 1

Introduction

Since the mid 1980s the analysis of large micro panel datasets on business units has changed our understanding of the extent to which establishments and firms reoptimize their input and output decisions in face of idiosyncratic and aggregate shocks to technology and to input and output markets. The lesson that we have now learned from this effort is that net aggregate changes hide an enormous amount of gross additions and losses to inputs by individual firms. This suggests that firms are subject to a wide range of idiosyncratic effects which leads to the coexistence of firms that significantly add jobs and others that significantly reduce jobs, while in the aggregate we find more smooth net changes. Because this process of job and worker reallocation among businesses occurs in tightly defined sectors, it fundamentally reflects the need to reoptimize the use of expensive resources by firms that suffer different fates in the market. Efficient firms tend to expand and new firms enter the market leading to the creation of jobs. Inefficient firms tend to contract and exit, leading to the destruction of jobs. Simultaneously, for a given set of jobs, there is a large frequency of changes in the workers attached to them, as firms and workers attempt a better match between requirements and qualifications.

The patterns of this process of reallocation vary widely across classes of businesses units, defined in terms of age, size, ownership type and other characteristics. Small and young businesses reallocate more than large and old business; net growth rates are larger among small and young businesses; young and small businesses tend to reallocate more in good times, whereas old and large businesses tend to reallocate more in bad times. The composition of sectors in terms of classes of businesses also varies significantly, with manufacturing being dominated by large and old firms, while services is dominated by small and young firms.

An important property of the process of input reallocation is that it displays substantial lumpiness and intermittence at the micro-level, in contrast with the smooth and sluggish adjustment observed at the aggregate level. This dynamic behavior is usually rationalized by considering costs to adjustment in input markets. The macro-level evidence on partial adjustment suggests convex costs which generate the sluggish adjustment to shocks. However, we need some form of non-strictly convex adjustment costs in order to generate the micro-level evidence on non-smooth adjustment. This suggests that idiosyncratic shocks and non-strictly convex adjustment costs are able to explain to explain the evidence both at the micro and macro level. Even if we are interested in aggregate responses, the micro-level information can be very useful in telling us how aggregate activity might change in response to macroeconomic shocks that affect the distribution of firms.

This dissertation builds on the evidence on heterogeneity in labor reallocation, and attempts to make some theoretical and empirical contributions. We use *Quadros de Pessoal*, an administrative database with information on all firms, except for public administration, with paid employees in the Portuguese economy. The database spans the period from 1985 to 2000, and contains information on firms, establishments and workers Since we are building macroeconomic patterns from micro-level adjustment, our models use linear and nonconvex adjustment costs, and build aggregate patterns from the intermittent and lumpy micro level adjustment that these costs imply.

The dissertation is structured in three main chapters (2, 3, and 4) and we now describe some contributions in each of them. The second chapter advances an explanation for the recent empirical evidence that emphasizes post entry growth of survivors, as opposed to exit of inefficient and small firms, as the main source of growth over time in the average size of a cohort of entering firms. We suggest that the interaction of adjustment costs with learning by entering firms about their true efficiency gives them incentives to start small and adjust upwards as they learn they are efficient. We consider linear and nonconvex adjustment costs, i.e., proportional and fixed costs, and conclude that for many plausible configurations of adjustment costs firms will start small and grow rapidly after entry. Initial uncertainty about true profitability makes entering firms prudent since they want to avoid incurring superfluous costs on jobs that prove to be excessive ex post. Even though there is less pruning of inefficient firms, surviving firms will grow faster and therefore the survivors' contribution to growth in the cohort's average size will increase.

We start by analyzing the 1988 cohort of entering firms in the Portuguese economy and decompose the change in the cohort's average size into a survivor component and a selection component. We conclude that survivors have the highest contribution to changes in the cohort's average size. However, manufacturing and services are at opposite ends: initial selection is stronger and the survivor's component is much smaller in services than in manufacturing. We then provide simulations for a finite learning horizon version of the model, with positive dispersion in entry size, and conclude that adjustment costs are needed to account for a high survivors' contribution. Finally, a calibration of the model to the overall economy and the manufacturing and services cohorts suggests that proportional costs and the fixed exit cost are key parameters in matching the evidence on firm dynamics. Firms in manufacturing learn relatively less initially about their efficiency, and are subject to much larger adjustment costs than firms in services.

Therefore, this chapter looks at heterogeneity along the age dimension, which is implied by a learning-about-efficiency mechanism. We conclude that growth in the average size of a cohort of entering firms is not merely the result of sample bias due to a selection mechanism that censors the lower end of the size distribution, but instead that surviving firms do tend to grow substantially, and a pure learning with selection model cannot fully account for this evidence.

The third chapter analyzes to what extent does the composition in terms of heterogeneous classes of firms matter for aggregate job flows dynamics. The starting point is that young and small firms, on the one hand, and old and large firms, on the other hand, display quite different dynamics of gross job flows. Previous studies have identified an effect of the age and size distribution on the cyclical behavior of reallocation, besides the simple trend effect as a simple (S,s) model of employment adjustment would indicate. Based on this, we reformulate an (S,s) model by considering two types of firms that are structurally different, and obtain analytical expressions for gross job flows statistics. We conclude that classes of firms whose optimal employment tends to be relatively more affected by aggregate shocks than by idiosyncratic shocks will influence aggregate job flow dynamics by more than their sample weight, as is the case for their influence in average aggregate job flows.

We then analyze with Portuguese data which types of firms are relatively more sensitive to aggregate shocks, and find that large and old firms are more sensitive than young and small firms. Therefore, old and large firms determine aggregate job flow dynamics by more than what their already large employment shares would indicate. And because large and old firms have countercyclical reallocation whereas small and young firms have procyclical reallocation, this tends to make aggregate job flows less procyclical than otherwise. By looking at one-digit sectoral data, we find that the particular behavior of old and large firms in each sector determines the aggregate dynamics of gross job flows in that sector. Therefore, institutional, technological, and competitive restrictions determine the composition of each sector in terms of classes of firms, and this decisively influences the aggregate cyclical dynamics of job reallocation in the sector.

The fourth chapter provides basic facts concerning macroeconomic performance and job reallocation for Portugal. We present statistics on gross job flows for the overall economy, and describe how they respond to the business cycle. We analyze the evolution of sectoral gross job flows and point out possible structural changes occurring in some sectors. We also provide some evidence on gross job flow dynamics by age and size classes. We conclude that patterns are significantly different across sectors and age and size classes.

We conclude in chapter 5 by outlining the main contributions of this dissertation, and stressing some questions that remain open and that deserve to be analyzed in future work.

Chapter 2

Firm Dynamics with Infrequent Adjustment and Learning

2.1 Introduction

In recent years there has been a renewed interest in explaining patterns of firm dynamics. New longitudinal datasets have confirmed heterogeneities between firms of different size and age. Small and young (surviving) firms tend to grow faster and have higher failure rates than large and old firms, so that job destruction due to plant exit and job creation due to the scaling-up of firm size decrease with age.[1] Entering plants tend to be small, but survivors grow rapidly after entry.[2] These patterns differ significantly across countries and sectors, suggesting that technological differences are important, but that country specific factors also matter.[3]

The two broad types of explanations for these facts are theories based on selection of inefficient and small firms, and theories based on financing constraints. Selection

[1] See Dunne, Roberts and Samuelson (1989a, 1989b).

[2] See Mata and Portugal (1994) and Cabral and Mata (2003).

[3] See Bartelsman, Scarpetta and Schivardi (2005). In particular, the US displays a size distribution with much more dispersion, a smaller relative entry size, and hazard rates that decline less steeply with age.

theories stress the tendency for firms that accumulate bad realizations of productivity to exit the market. This implies a composition bias towards larger more efficient firms as smaller, inefficient, slow-growing firms gradually exit the industry. Meanwhile, financing constraints theories argue that some imperfection in financial markets causes young firms to have limited access to credit, forcing them to enter at a suboptimally small scale. As firms get older and survive, they establish creditworthiness and build up internal resources that enable them to expand to their optimal size.

Cabral and Mata (2003) provide some evidence on the empirical validity of these theories for the Portuguese manufacturing sector. They find that the shift of a cohort's size distribution to the right is mostly due to growth of surviving firms rather than exit of small firms. The authors suggest that a model with financial constraints is better able to match these facts. This chapter tries to interpret this and other cross-sector evidence from a different perspective. At the core of our theory is the interaction of learning with adjustment costs. Unlike existing selection theories, our model generates plausible cohort dynamics even in the absence of any pruning of inefficient firms (e.g. when exit is not allowed), although allowing for exit amplifies the effects of learning and adjustment costs on the contribution of survivors to growth in the cohort's average size.

Our contribution is twofold. We contribute to the empirical literature by introducing a decomposition of a cohort's change in average size that allows a quick assessment of the match between particular theories and data. We apply this decomposition to the 1988 cohort of entrants in the Portuguese economy, using the *Quadros de Pessoal* dataset. For the overall economy, we conclude that growth of survivors is the main force behind the change in average firm size. However, there are significant cross-sector differences in this measure. In particular, initial exit rates are smaller and the survivors' contribution to changes in size are much higher in manufacturing than in services. We also find significant inaction and lumpiness in labor adjustments, in varying degrees across sectors. This suggests the presence of non-strictly convex costs in labor adjustment that vary by sector, possibly due to technological heterogeneities.

We contribute to the theoretical literature by using this evidence to motivate the introduction of linear and nonconvex adjustment costs into a model of Bayesian learning about efficiency. Our model builds on Jovanovic (1982) by adding both proportional and fixed costs that potentially apply not only to regular labor adjustment, but also at entry and exit. We argue that under most forms of adjustment costs, firms will have a tendency to start smaller and grow faster. We prove this analytically in a simplified model in which there is no exit of firms. This shows that non-strictly convex costs can generate firm growth without selection. When firms are allowed to exit, selection intensifies the effects of adjustment costs on firm growth, while costs to adjustment reduce exit rates. Therefore, adjustment costs increase the contribution of surviving firms to growth in the cohort's average size. All that is needed for firm growth under linear and nonconvex costs is the existence of a learning environment that generates a stochastic process for perceived efficiency with both persistence and decreasing uncertainty in age. For example, firm growth would occur in our model even if exit was random with a constant probability for all firms, whereas this would not be true in a pure selection model.

The intuition for why firms grow faster and display smaller exit rates under most forms of fixed and proportional costs is that initial uncertainty about true profitability makes entering firms prudent; that is, they enter small and "wait and see" since they want to avoid incurring superfluous entering/hiring costs and firing/shutdown costs on jobs that prove to be excessive *ex post*. This implies that surviving firms will grow faster, even though adjustment costs imply that there are fewer firms exiting the market and therefore less pruning of inefficient firms. The assumption that entering firms face a Bayesian learning problem concerning their efficiency is standard in selection theories and has been advanced as an explanation for the high rates of exit, job creation and job destruction among young firms. The initial literature on adjustment costs used a convex specification in an attempt to explain the sluggishness in input responses to aggregate shocks. However, the assumption that costs of adjustment are linear and/or nonconvex is now standard in dynamic factor demand models, following a number of studies since the late 1980s that have documented lumpiness and inaction in adjustment at the micro level. Since strictly convex costs imply smooth adjustments over time, whereas linear and nonconvex costs imply immediate adjustment when it occurs, allowing for strictly convex costs instead of non-strictly convex costs would make our argument stronger. In the case of hiring/entering costs, firms would not adjust immediately to their optimal size, after changes in their perceived productivity, but would do so gradually. For firing/exiting costs, firms experiencing large negative shocks would want to adjust downwards in various steps, a scenario that makes firms start smaller to attenuate its effects. Therefore, our decision to assume only linear and nonconvex costs is conservative, and permits a simplification of the methods employed to measure the effects of adjustment costs.

To assess our model quantitatively, we calibrate and simulate a finite learning horizon version with positive dispersion in entry size, using recursive expressions for all relevant densities and moments. We conclude that linear and nonconvex costs can account for the high empirical contribution of survivors to changes in a cohort's average size. In particular, the key elements needed to match the evidence on firm dynamics are the proportional costs and the fixed exit cost. The proportional costs enable us to match the high value and flatness over age of the survivor component, while the fixed exit cost enables us to reduce initial selection. A calibration to the manufacturing and services cohorts in the Portuguese data also suggests that firms in manufacturing learn relatively less initially about their efficiency, and are subject to much larger setup and adjustment costs than firms in services.

The main implication of this chapter for economic policy is that substantial growth of survivors does not necessarily mean that some imperfection in capital markets causes entering firms to be inefficiently undersized. On the contrary, small entry size might be an optimal response by firms as they try to save on adjustment costs, given the uncertainty about their long-run profitability. Therefore, any government intervention in capital markets or incentives for larger entry size might be a waste of resources. To the extent possible, these resources should alternatively be applied in helping potential entrants better predict their post-entry efficiency. However, there is an intrinsic random element in the creation of efficient firms, with respect to which the government can do little. Taking into account the cross country evidence, another implication of this chapter is that a particular set of institutional configurations that potentially can be mapped into a particular structure of adjustment costs may explain why firms enter bigger and experience smaller post-entry growth in some European countries than in the US.

A short review of the literature follows. Representative papers of selection theories are Jovanovic (1982) and Ericson and Pakes (1995). In Jovanovic (1982), *ex ante* identical firms learn about their *ex post* average efficiency by observing their performance. Because accumulation of market experience makes firms more reliant on their Bayesian posterior estimate, the threshold productivity below which firms exit is increasing in age. Therefore, inefficient firms gradually exit from the market while efficient firms remain in the industry and adjust their employment level in accordance with perceived productivity. The model predicts increasing average size among surviving firms of a given cohort, and a decreasing failure rate with age. In Ericson and Pakes (1995) the profitability of a firm is determined by the stochastic outcomes of investment projects realized by the firm and its rivals, and by the competitive environment in which firms interact. Therefore, firms must spend resources in order to improve their relative position in the industry, and selection is partly an endogenous process.

With respect to financing constraints theories, two important contributions are those of Cooley and Quadrini (2001) and Albuquerque and Hopenhayn (2004). In Cooley and Quadrini (2001) productivity shocks are persistent, while there is a transaction cost on equity and a default cost on debt. This financial friction implies that equity and debt are not perfect substitutes, so that size depends positively on the amount of equity. Assuming that firms observe their efficiency before entering the industry, new firms will tend to be of high productivity and will borrow more intensively. This implies that they will face higher volatility in their performance. The model is able to generate both the negative dependence on age (due to financial frictions) and the negative dependence on size (due to persistence of shocks) of growth, volatility of growth, job creation, job destruction and exit. In Albuquerque and Hopenhayn (2004), limited liability of borrowers and limited enforcement of debt contracts imply that it is optimal for lenders to introduce credit constraints, which can only be loosened as the firm gets older and more profitable. This credit constraint implies higher growth among young and smaller firms.

To our knowledge, this is the first work that suggests adjustment costs as an explanation for differences in firm dynamics by age. The paper by Cabral (1995) is nearest to this chapter. In his model, firms must pay a proportional sunk cost to increase their production capacity. He argues that, in a model with Bayesian learning, a proportional capacity cost would make small entering firms grow faster than large entering firms. The reason is that small entrants are those whose initial profitability signals were not good, so their exit probabilities are higher, and therefore they choose to invest more gradually. Unlike our model, Cabral's model depends on the existence of selection. Also, by analyzing a size-growth relationship, his model is not able to explain why some large entering firms also grow substantially.

Previous studies on linear and nonconvex adjustment costs have concluded that their effects on average labor demand and the firm size distribution are negligible.

Bentolila and Bertola (1990) study the effects of linear firing and hiring costs on average labor demand and the dynamics of labor adjustment. They conclude that high firing costs do not have a significant effect on average labor demand, but make adjustments more sluggish. Hopenhayn and Rogerson (1993) analyze the effects of a tax on job destruction on employment, productivity and welfare. They conclude that such a tax would significantly reduce employment, productivity and welfare, but would have insignificant effects on the size distribution of firms.

The chapter is organized as follows. In section 2.2, we present evidence of firm dynamics for a cohort of entering firms. In section 2.3, we build the general model, obtain optimality conditions, and provide heuristic arguments explaining the effects of adjustment costs. In section 2.4, we analytically prove the main results in a simplified version of the model. In section 2.5, we calibrate a finite learning horizon version and quantify the contribution of adjustment costs to firm dynamics. Section 2.6 concludes with plans for future research. All proofs are left for an appendix.

2.2 Firm Dynamics in a Cohort of Entering Firms

There is a well established literature on the identification and explanation of differences in behavior between young and old firms. In this section, we analyze firm dynamics in a cohort of entering firms. We use *Quadros de Pessoal*, a database containing information on all Portuguese firms with paid employees. This dataset originates from a mandatory annual survey run by the Ministry of Employment, which collects information about the firm, its establishments and its workers. All economic sectors except public administration are included. The panel we have access to covers the period 1985-2000. Information refers to March through 1993, and to October since the reformulation of the survey in 1994. On average the dataset contains 250,000 firms, 300,000 establishments, and 2,500,000 workers in each year.

The literature on firm dynamics typically finds that young firms grow faster than old firms. Using kernel density estimates of the firm size distribution in a cohort of entrants, Cabral and Mata (2003) argue graphically that the cohort's evolution is mostly due to growth of survivors rather than exit of small firms. Their analysis points to the need for a measure of the contribution of survivors versus nonsurvivors to the growth in a given cohort's average size. To accomplish this, we propose a decomposition of the cohort's cumulative growth that will later allow an assessment of the empirical relevance of adjustment costs. We consider the following decomposition:

$$\frac{1}{N(S_\tau)}\sum_{i\in S_\tau} l_{i,\tau} - \frac{1}{N(S_0)}\sum_{i\in S_0} l_{i,0} = \underbrace{\frac{1}{N(S_\tau)}\sum_{i\in S_\tau} l_{i,\tau} - \frac{1}{N(S_\tau)}\sum_{i\in S_\tau} l_{i,0}}_{\text{Survivor Component}} + \underbrace{\frac{N(D^\tau)}{N(S_0)}\left(\frac{1}{N(S_\tau)}\sum_{i\in S_\tau} l_{i,0} - \frac{1}{N(D^\tau)}\sum_{i\in D^\tau} l_{i,0}\right)}_{\text{Selection Component}}$$

where τ is the firm's age, $l_{i,\tau} = \ln(L_{i,\tau})$ is log-employment at firm i in period τ, S_τ is the set of age-τ surviving firms, D^τ is the set of age-τ non-surviving firms, so that $\{S_\tau, D^\tau\}$ is a partition of S_0, and $N(X)$ is the number of firms in set X.[4]

In general, the growth in a cohort's average size can originate from significant growth of survivors or from smaller initial size of nonsurvivors. Any theory of firm dynamics should consider both these sources of growth. Our measure enables us to see if a particular theory can explain the key source of growth in a cohort's average size. The survivor component compares the current average size of period τ survivors with their initial average size, so that it measures how much survivors have grown. The selection component compares the average initial size of period τ nonsurvivors with the average initial size of period τ survivors, so that it measures how relatively small nonsurvivors were initially.

[4] Throughout the paper we will assume that firms enter in some generic period 0. Therefore, τ will represent both the firm's age and the period (after entry) we are analyzing.

We can obtain a similar decomposition for employment-weighted moments. The weighted decomposition contains information about the entire distribution of employment, not just its cross-sectional mean, and is affected both by within- and between-firm growth. Therefore, the weighted decomposition would be more relevant for assessing a richer model that considers the reallocation of employment shares between firms within the cohort. In the results that follow we focus on the unweighted decomposition because it analyzes within-firm growth, which in our model is the most relevant statistic to assess the effect of adjustment costs on the incentives for firms to grow.[5]

We can also produce a decomposition based on the cohort's annual growth instead of the cohort's cumulative growth. However, the annual version of the above decomposition is more sensitive to two aspects that would complicate the analysis in the paper. First, the annual survivor component is significantly affected by the business cycle, especially after the first few years of life. To control for this, we would need to somehow remove the cyclical part of the survivor component. Second, as the age of the cohort increases, the annual survivor component becomes increasingly sensitive to downsizing and exit by some survivors that become technologically outdated and consequently relatively less efficient. To fully consider this aspect of the data would force us to introduce additional parameters into the model that we present in section 2.3. Therefore, we believe that by employing a decomposition based on the cohort's cumulative growth we avoid having to adjust the analysis for these two aspects, and

[5] For the weighted decomposition, the cumulative change would be $\sum_{i\in S_\tau}\omega_{i,\tau}^{S_\tau}l_{i.\tau}-\sum_{i\in S_0}\omega_{i,0}^{S_0}l_{i,0}$, where $\omega_{i,\tau}^{X}$ is the weight of firm i in period τ in set X, with $\omega_{i,\tau}^{X}=L_{i,\tau}/\sum_{i\in X}L_{i,\tau}$ The weighted survivor component can be further decomposed as

$$\sum_{i\in S_\tau}\omega_{i,\tau}^{S_\tau}l_{i,\tau}-\sum_{i\in S_\tau}\omega_{i,0}^{S_0}l_{i,0}=\sum_{i\in S_\tau}\omega_{i,0}^{S_\tau}\left(l_{i,\tau}-l_{i,0}\right)+\sum_{i\in S_\tau}\left(\omega_{i,\tau}^{S_\tau}-\omega_{i,0}^{S_\tau}\right)l_{i,0}+\sum_{i\in S_\tau}\left(\omega_{i,\tau}^{S_\tau}-\omega_{i,0}^{S_\tau}\right)\left(l_{i,\tau}-l_{i,0}\right).$$

The first term is a within-firm component, measuring average growth weighted by initial size; the second term is a between-firm component, measuring the contribution of changes in employment shares; and the third is a cross component. For the unweighted decomposition, the last two terms are zero, since in this case $\omega_{i,\tau}^{X}=N\left(X\right)^{-1}$.

Table 2.1: 1988 Firm Cohort: All Sectors

Year	*Unweighted*				*Weighted*			
	CumEx	*AvEmp*	*CGrEmp*	*SurComp*	*CumEx*	*AvEmp*	*CGrEmp*	*SurComp*
1988		1.11				2.58		
1989	17.2	1.29	17.3	69.5	14.5	2.80	21.3	106.0
1990	27.7	1.38	27.1	70.4	22.5	2.99	40.9	87.9
1991	34.7	1.45	33.2	69.7	28.5	3.03	45.0	87.5
1992	39.3	1.47	35.9	69.3	33.1	3.11	52.9	89.9
1993	44.1	1.48	36.1	68.9	38.9	3.10	51.4	95.3
1994	49.6	1.48	37.0	69.2	44.8	3.18	59.3	94.0
1995	52.6	1.49	37.2	68.9	47.7	3.22	64.1	93.5
1996	55.9	1.50	38.4	67.3	50.7	3.29	70.8	91.3
1997	57.5	1.52	40.6	68.6	52.0	3.42	83.2	90.3
1998	59.6	1.53	41.3	68.5	54.2	3.40	81.4	88.4
1999	61.6	1.55	43.5	69.0	56.0	3.61	102.3	88.9

Notes: *CumEx* is the cumulative exit rate (in %); *AvEmp* is the mean of log-employment among survivors; *CGrEmp* is the cumulative growth rate (in %) of mean log-employment among survivors; *SurComp* is the survivor component (in %) associated with the cumulative change in *AvEmp*.

instead focus on how intense is survivor's growth while learning-about-efficiency effects are significant.

In table 2.1, we present the evolution of exit rates and the share of firm growth due to the survivor component in the 1988 cohort of entering firms for the overall economy.[6] In 1988 there were $22,810$ entering firms. The exit rate is very high initially but tends to decrease as firms get older.[7] However, ten years after entry

[6]We identify entering firms in year t as those firms that have not been in the database before t. Given the high incidence of temporarily missing firms, we select the 1988 entering cohort, using 1985 and 1986 to detect false entries. Similarly, we identify exiting firms in the τ-th period (after entry in 1988) as those firms that are present in the database in period $\tau - 1$, but do not reappear in any of the following periods. Therefore, we display results only up to 1999, using 2000 to detect false exits. This procedure eliminates most false entries and false exits.

[7]To avoid inconsistent sets of firms at period τ and period 0, we adopt the following procedure concerning temporarily missing firms. In measuring exit and growth between period 0 and period τ, we consider only the set of nonmissing firms in period τ. Across all years, this procedure excludes a maximum of 11.7% of all entering firms. We do not exclude all firms which ever had temporarily missing values during their lifetime, because that would eliminate too many firms (about 1/3 of the entering cohort). Therefore, the exit rate among all firms, including temporarily missing firms, is slightly smaller than that reported in table 2.1: 15.6% in 1989, 40% in 1993, and 59.9% in 1999. We also analyzed the effect of excluding temporarily missing survivors only until 1993. In this case, we exclude around 1/5 of all entering firms and the implied exit rates are 19.9% in 1989 and 46.5% in 1993, but the survivor component becomes slightly higher. The same would occur if we had used the 1991 cohort instead. Finally, in 1994 there is a higher than normal exit rate because the survey moved from March to October in this year. A corrected annual cumulative exit rate for 1994 would

Table 2.2: 1988 Firm Cohort: Summary Characteristics by Sector

Sector	*EmpSh*	*CumEx*			*AvEmp*	*CGrEmp*			*SurComp*	*SurCompw*
	88	*89*	*92*	*99*	*88*	*89*	*92*	*99*	*89-99*	*89-99*
All	100.0	17.2	39.3	61.6	1.11	17.3	35.9	43.5	69.0	92.1
Manu	41.8	15.7	39.2	61.7	1.58	19.7	40.7	46.1	83.3	113.9
Serv	20.1	18.9	41.0	60.9	0.99	12.7	32.2	40.6	61.6	77.8
Reta	11.1	16.3	37.6	60.8	0.80	15.2	32.5	43.0	69.7	92.1
Cons	10.5	16.1	37.9	59.5	1.32	17.9	32.7	36.9	67.1	88.2
Whol	6.7	14.3	34.6	62.5	1.16	24.2	47.1	60.6	79.2	78.6

Notes: *EmpSh* is the employment share of the sector in the cohort (in %); *CumEx* is the cumulative exit rate (in %); *AvEmp* is the mean of log-employment among survivors; *CGrEmp* is the cumulative growth rate (in %) of mean log-employment among survivors; *SurComp* and *SurCompw* are the survivor component (in %) associated with the cumulative change in *AvEmp*, based on unweighted and employment weighted moments, respectively.

only 40% of the initial entrants remain active. There is significant growth in the cohort's average size, which is mostly due to the growth of survivors rather than to the exit of small inefficient firms. For the unweighted decomposition, survivors' growth contributes around 69% to the growth in the cohort's average size. The employment-weighted results show that larger firms have smaller exit rates, and therefore average employment increases more intensely. This, and the fact that high growth firms increase their weight over time, explains the larger shares for the survivor component.[8]

Table 2.2 presents similar evidence on cohort dynamics at roughly the one-digit sectoral level.[9] We include the employment shares of each sector in the 1988 cohort of entering firms, which are close to shares in the overall economy. Even though manufacturing has a much higher employment share than services, the number of entering firms in services surpasses that of manufacturing (6074 and 4834, respectively). All sectors display a cumulative exit rate around 61% by 1999. However, initial differences in exit rates are more significant, with manufacturing and wholesale trade displaying

be around 47.8%.

[8] A similar exercise for labor productivity revealed that survivors account for about 90% of the change in the cohort's unweighted average productivity.

[9] In order to obtain equivalent one-digit SIC87 sectors, we use the following correspondence in terms of CAE Rev. 1 codes : construction($= 5$), manufacturing($= 3$), wholesale trade($= 6.1$), retail trade($= 6.2$), and services($= 6.3 + 8.3.2 + 8.3.3 + 9.2 + 9.3 + 9.4 + 9.5$). We omit agriculture, finance, mining, and transportation and public utilities because of the small entering cohorts.

the smallest values, and services displaying the highest value. In terms of initial size, manufacturing has the largest entrants, and services and retail trade the smallest. Although manufacturing and wholesale trade have the largest entrants, they exhibit more growth in average employment and a larger contribution of survivors to that growth than services and retail trade.

We perform two robustness checks on the previous findings. First, we redo our calculations using establishments rather than firms as the unit of analysis. For the 1988 establishment cohort, we obtain similar results, although exit rates and the survivor component are higher than in the case of firms. Second, we examine an alternative cohort to make sure our results are not driven by business cycle conditions. The Portuguese economy experienced an expansion between 1986 and 1991, a recession between 1992 and 1994, and another weaker expansion between 1995 and 2000. The growth rates of real GDP were 6.4% in 1989, 1.1% in 1992, and 4.3% in 1995, so that the 1991 cohort did not face as favorable a macroeconomic environment as the 1988 cohort. However, the results for the 1991 cohort are, in all dimensions, very similar to those presented above. The results for the 1994 cohort are also very similar, but with slightly smaller values for the survivor component in the first few years after entry.[10]

Two symptoms of linear and nonconvex costs are inaction and lumpiness in labor adjustment. Lumpiness is typically taken as evidence of fixed adjustment costs, while inaction is associated with both proportional and fixed adjustment costs. In table 2.3, we provide evidence on both of these phenomena in the 1988 cohort of entering firms. We use the unweighted and the employment-weighted distributions of the adjusted growth rate conditional on survival. Following Davis and Haltiwanger (1992), the

[10] Reflecting our previous argument about the greater cyclical sensitivity of the decomposition based on the cohort's annual growth rate, we observe a substantial reduction in the annual survivor component associated with the 1988 and 1991 cohorts during the 1992-1994 recession. However, a similar pattern does not occur with the 1994 cohort. This is one of the reasons why we choose a decomposition based on cumulative growth rates. Note also that the annual selection component is not as sensitive to the business cycle as the annual survivor component.

Table 2.3: 1988 Firm Cohort: Characteristics of Labor Adjustment by Sector

Sector	*Unweighted*						*Weighted*					
	89			*93*			*89*			*93*		
	N30	*NA*	*P30*	*N30*	*NA*	*P30*	*N30*	*NA*	*P30*	*N30*	*NA*	*P30*
All	7.9	43.0	13.7	13.7	45.3	17.1	18.0	19.1	27.3	30.3	21.0	30.6
Manu	10.8	31.5	20.7	20.9	33.4	24.6	20.2	11.6	33.3	38.6	15.6	33.9
Serv	7.3	47.7	11.8	11.3	50.3	15.0	19.5	25.6	22.4	22.5	24.9	31.0
Reta	4.7	55.5	9.1	8.2	55.8	12.7	9.8	40.5	17.1	16.4	36.3	27.5
Cons	10.5	31.4	13.9	17.5	34.3	17.1	15.5	14.0	22.2	31.8	16.9	25.0
Whol	8.1	35.8	17.5	12.6	42.1	21.5	14.0	21.3	31.0	23.9	24.5	28.0

Notes: *N30* is the fraction of firms with an adjusted growth rate of employment, conditional on survival, in the interval (-30%,0%); *NA* is the fraction of firms that do not adjust employment, conditional on survival; *P30* is the fraction of firms with an adjusted growth rate of employment, conditional on survival, in the interval (0%,30%). All values in %.

adjusted growth rate in period τ is defined as $100 \times (L_\tau - L_{\tau-1}) / \tilde{L}_{\tau-1}$, where $\tilde{L}_{\tau-1} = \frac{1}{2}(L_\tau + L_{\tau-1})$. This $\tilde{L}_{\tau-1}$ is also used to define employment weights. The table shows that the incidence of inaction is very high, increases with age, and is higher in sectors with smaller firms like services or retail trade. This may be due to technology-induced differences in adjustment costs, or due to indivisibilities in jobs, or due to a smaller impact of fixed adjustment costs for larger firms. An additional fact is the left skewness of the 1989 distributions, showing that survivors tend to grow initially, especially in manufacturing and wholesale trade. This skewness is less evident in 1993 (especially in the weighted data), suggesting that adjustment patterns are different in initial years.

This evidence on inaction and lumpiness justifies our assumption of linear and nonconvex adjustment costs in the model that we present next.

2.3 A Model of Learning with Linear and Nonconvex Costs

In this section, we introduce linear and nonconvex adjustment costs into a model of Bayesian learning about efficiency. We derive conditions for optimal employment over time and present heuristic findings about the effects of adjustment costs on the path of employment. Our model is similar to Jovanovic (1982), although we change the way the idiosyncratic shock is specified and add adjustment costs.

We assume an industry with competitive output and input markets. Current profits of a representative firm are defined by

$$\Pi(L, \theta) = pF(L)\theta - wL,$$

where $F(L)\theta$ is the production function; L is the amount of labor input; θ is a productivity shock; p is the output price; and w is the wage rate. Given the competitive environment, the firm considers both p and w as known constants.

Concerning technology we make the following assumption.

Assumption 1 *The two components of the production function satisfy:*
(a) $F : \mathbb{R}_+ \to \mathbb{R}_+$ *is* C^2, $F' > 0$, $F'' < 0$, $F(0) = 0$, $F'(0^+) = \infty$, *and* $F'(\infty) = 0$.
(b) *Letting* τ *denote the firm's age and* 0 *the period in which the firm enters, the stochastic process of* θ *is defined by*

$$\theta_\tau = \xi(\eta_\tau), \qquad \eta_\tau = \mu + \varepsilon_\tau, \qquad \mu = \mu_0 + \mu_1, \qquad \tau = 0, 1, \ldots \tag{2.1a}$$

$$\varepsilon_\tau \sim N\left(0, \sigma^2\right), \qquad \mu_0 \sim N\left(\bar{\mu}, \sigma^2_{\mu_0}\right), \qquad \mu_1 \sim N\left(0, \sigma^2_{\mu_1}\right), \tag{2.1b}$$

where μ_0, μ_1, $\{\varepsilon_\tau\}_{\tau \geq 0}$ *are mutually independent,* $\xi : \mathbb{R} \to \mathbb{R}_+$ *is* C^1, $\xi' > 0$ *and* $\xi(-\infty) = \nu_1 \geq 0$, $\xi(\infty) = \nu_2 < \infty$.

Part **(a)** basically ensures a well defined interior optimum. In some of the analyses below, we will further specialize by assuming that F is Cobb-Douglas. Meanwhile, part **(b)** establishes that in each period productivity is stochastic with a constant mean over the firm's lifetime. The long-run productivity coefficient, μ, has two components: μ_0, which is observed at entry but prior to the initial choice of employment, and μ_1, which is never directly observed by the firm. The introduction of μ_0 is essential to obtain a non-degenerate initial distribution of size in the cohort of entering firms. Intuitively, μ_0 can be thought of as indexing *ex ante* efficiency, measuring such aspects as initial research and technology choice, while μ_1 indexes *ex post* productivity, measuring how well a firm performs within its technology choice.

The introduction of μ_0 is essential to our posterior analysis of the survivors' contribution to growth in the cohort's average size. In Jovanovic (1982) there is only one component to the productivity parameter, so that the distribution of initial size is degenerate. In this case, all future nonsurvivors will have the same average initial size as all future survivors, so that our measure of the survivors' component would be 100%. By assuming $\sigma_{\mu_0} > 0$, we avoid this aspect of Jovanovic's model.

At entry the firm knows the parameters governing the stochastic process of θ, i.e., $\bar{\mu}$, $\sigma^2_{\mu_0}$, $\sigma^2_{\mu_1}$ and σ^2, and the realization of *ex ante* productivity μ_0. Over time, the firm will learn about its specific μ_1 by observing the realizations of θ_τ. To form a prediction for its productivity, in each period the firm forms a posterior estimate for μ based on past realizations of productivity, $\{\theta_s\}_{s=0}^{\tau-1}$, and on the *ex ante* efficiency parameter μ_0. Similarly to Zellner (1971), a firm with age $\tau \geq 0$ has the following Bayesian posterior distribution for μ at the beginning of period τ:

$$\mu|_{\Omega_\tau} \sim N\left(Y_\tau, Z_\tau\right), \qquad \Omega_\tau \equiv \left\{\mu_0, \{\eta_s\}_{s=0}^{\tau-1}\right\} \tag{2.2a}$$

$$Y_\tau = \frac{\tau\sigma^{-2}}{Z_\tau^{-1}}\bar{\eta}_\tau + \frac{\sigma_{\mu_1}^{-2}}{Z_\tau^{-1}}\mu_0, \qquad \bar{\eta}_\tau = \frac{1}{\tau}\sum_{s=0}^{\tau-1}\eta_s, \qquad Z_\tau = \frac{1}{\tau\sigma^{-2} + \sigma_{\mu_1}^{-2}}. \tag{2.2b}$$

In lemma 2 of appendix A.1 we show that, for purposes of predicting μ, Ω_τ can be summarized by (θ^*_τ, τ), where θ^*_τ is the prediction of the productivity coefficient at τ based on the information available at the beginning of period τ. That is, $\theta^*_\tau = E_\tau(\theta_\tau)$ where $E_\tau(\cdot) \equiv E(\cdot \mid \Omega_\tau)$ is the conditional expectation given the information set at τ.

We now lay out the timing assumptions.

Assumption 3 *A potential entering firm, at the beginning of period* 0*, takes the following actions:*

(i.a) *Fixed cost and ex ante productivity: the firm pays a fixed cost* $I > 0$*, associated with the process of initial research, after which it observes its realization of ex ante productivity,* μ_0*.*

(i.b) *Entry decision: based on the idiosyncratic realization of* μ_0*, the firm chooses whether to enter the industry or not.*

(i.c) *Initial employment and production decisions: conditional on entering the industry, the firm chooses how much labor to use and how much output to produce in period* 0*.*

A firm of age $\tau > 0$ *takes the following actions:*

(ii.a) *Update of posterior productivity: at the beginning of period* τ*, the firm updates its posterior expectation of* θ_τ*,* θ^*_τ*, based on the observation of* $\theta_{\tau-1} = \xi\left(\eta_{\tau-1}\right)$ *at the end of period* $\tau - 1$*.*

(ii.b) *Exit decision: given the new posterior productivity estimate,* θ^*_τ*, and employment from last period,* $L_{\tau-1}$*, the firm chooses whether to stay or exit the industry.*

(ii.c) *Employment and production decisions: conditional on staying, the firm chooses how much labor to use and how much output to produce in the current period. At the end of period* τ*, the firm observes the productivity realization,* θ_τ*, and the process repeats itself again until the firm decides to leave the industry.*[11]

[11] In this model we do not consider the possibility that as firms get older they might decay or

While deciding whether to stay one more period or to exit, the firm compares the expected profit in case it stays, V, with the opportunity cost of doing so, W. In a model where the capital decision is explicitly considered, this opportunity cost could be defined as the expected profit of applying the firm's capital resources in an alternative activity. Therefore, in the absence of adjustment costs we have:

$$V\left(\theta_\tau^*, \tau\right) = \max_{L_\tau} \left\{ \Pi\left(L_\tau, \theta_\tau^*\right) + \beta E_\tau \left[\max\left\{ W, V\left(\theta_{\tau+1}^*, \tau+1\right)\right\}\right]\right\} \qquad (2.3)$$

where V represents expected profits conditional on staying in period τ.

At entry, we have $\Omega_0 \equiv \mu_0$, and in equilibrium expected profits net of the fixed research cost I must compensate for the opportunity cost, i.e., $E\left[V^{EN}\left(\theta_0^*\right)\right] - I = W$. Since markets are competitive and there is no friction in the entry and exit processes, the assumption of a strictly positive initial fixed cost, $I > 0$, is essential to avoid the extreme situation where entry and exit are so high that only the highest productivity firms would enter and remain in the industry. Although I enables us to obtain a non-degenerate initial distribution of size, we do not explicitly consider a capital stock decision. The main reason for not including capital in the model is that there is no reliable capital stock variable in *Quadros de Pessoal*.

Up to this point, the only differences between our model and Jovanovic (1982) are that in the latter model the efficiency parameter implicitly affects the cost function and the initial distribution of productivity in the entering cohort is degenerate. Therefore, without adjustment costs there would be no intertemporal linkages in our model aside from the exit decision. Because V is strictly increasing in θ^*, the exit decision is characterized by an age-dependent exit threshold. For values of θ_τ^* above or equal to that threshold, the firm would stay and choose employment to maximize current period profits. For values of θ_τ^* below that threshold, the firm would leave the

become obsolete. This could be achieved by assuming exogenous probabilities for those two events. This could generate both a decrease in size of old firms (decay) and the exit of old firms (decay and obsolescence).

industry, since its expected profitability is below the opportunity cost. The increasing confidence the firm puts in θ_τ^* as it grows older implies that the exit threshold is increasing with age. This is the driving force underlying Jovanovic's result that the size distribution and the survival probability increase with age.

We now introduce linear and nonconvex costs into the model, allowing entry and exit costs to differ from regular adjustment costs. The adjustment cost function for continuing firms, $C^S : \mathbb{R}_+^2 \to \mathbb{R}_+$, is defined as

$$C^S(L_\tau, L_{\tau-1}) = \begin{cases} C^{SU} = F^{SU} + P^{SU}(L_\tau - L_{\tau-1}), & \text{if } L_\tau > L_{\tau-1}, \\ C^{SN} = 0, & \text{if } L_\tau = L_{\tau-1}, \\ C^{SD} = F^{SD} + P^{SD}(L_{\tau-1} - L_\tau), & \text{if } L_\tau < L_{\tau-1}, \end{cases}$$

where the superscript indexes ^{S}, ^{SD}, ^{SN}, and ^{SU} stand for "staying", "staying and adjusting downwards", "staying and not adjusting", and "staying and adjusting upwards". Therefore, $F^{SU} \geq 0$, $P^{SU} \geq 0$ are the fixed and proportional hiring costs, and $F^{SD} \geq 0$, $P^{SD} \geq 0$ are the fixed and proportional firing costs, respectively. The entry and exit (adjustment) cost functions, $C^{EN} : \mathbb{R}_+ \to \mathbb{R}_+$ and $C^{EX} : \mathbb{R}_+ \to \mathbb{R}_+$, respectively, are defined as

$$\begin{aligned} C^{EN}(L_0) &= F^{EN} + P^{EN} L_0, \\ C^{EX}(L_\tau) &= F^{EX} + P^{EX} L_\tau, \end{aligned}$$

where the superscript indexes ^{EN} and ^{EX} stand for "entering" and "exiting" the industry, respectively. Then, $F^{EN} \geq 0$, $P^{EN} \geq 0$ are the fixed and proportional entry costs, and $F^{EX} \geq 0$, $P^{EX} \geq 0$ are the fixed and proportional exit costs, respectively.

With entry, adjustment and exit costs, the optimization problem now becomes,

$$V^{S}\left(L_{\tau-1},\theta_{\tau}^{*},\tau\right)=\max_{L_{\tau}}\left\{\left[\Pi\left(L_{\tau},\theta_{\tau}^{*}\right)-C^{S}\left(L_{\tau},L_{\tau-1}\right)\right]+\right.$$
$$\left.\beta E_{\tau}\left[\max\left\{V^{EX}\left(L_{\tau}\right),V^{S}\left(L_{\tau},\theta_{\tau+1}^{*},\tau+1\right)\right\}\right]\right\}, \quad (2.4)$$

for all periods after entry ($\tau \geq 1$) in which the firm remains in the industry, and

$$V^{EN}\left(\theta_{0}^{*}\right)=\max_{L_{0}}\left\{\left[\Pi\left(L_{0},\theta_{0}^{*}\right)-C^{EN}\left(L_{0}\right)\right]+\beta E_{0}\left[\max\left\{V^{EX}\left(L_{0}\right),V^{S}\left(L_{0},\theta_{1}^{*},1\right)\right\}\right]\right\}, \quad (2.5)$$

for the entry period, where V^{EX}, the value of exiting, is defined as

$$V^{EX}\left(L_{\tau}\right)=W-C^{EX}\left(L_{\tau}\right).$$

Note that contrary to the case without adjustment costs, the previous period employment is a state variable for the current period optimization problem. Also, in V^{EN} and in V^{EX} the costs of entry and exit are taken into account.

In general, we can allow for asymmetry among the cost parameters in C^{S}, C^{EN}, and C^{EX}. However, asymmetries between exit and firing costs or between entry and hiring costs lead to biases in entry and exit decisions. For example, if hiring proportional costs are higher than entry proportional costs, then firms will tend to hire more workers at entry in order to save on expected future hiring costs; if firing proportional costs are smaller than exit proportional costs, then firms facing the prospect of exit will tend to decrease their employment right before exiting the industry. Therefore, in what follows we will consider the standard case to be $F^{EN}=F^{SU}=F^{H}$, $P^{EN}=P^{SU}=P^{H}$, $F^{EX}=F^{SD}=F^{F}$, and $P^{EX}=P^{SD}=P^{F}$, where the superscript indexes ^{H}, ^{F} stand for "hiring" and "firing". We will comment on the effects of asymmetries when relevant.

Because the adjustment cost function introduces a nondifferentiability and discontinuity of the objective function at the frontiers between adjustment and non-adjustment, some of the usual properties assumed for the value function do not hold in general, and some proofs of its properties must be adapted. We briefly discuss here how we tackle the problem and leave proofs for the appendix. We consider a two-step optimization procedure where the firm first chooses optimal employment in each of three possible scenarios, and then selects the scenario with the highest pay-off. More precisely,

$$V^S(\cdot) = \max\left\{V^{SD}(\cdot), V^{SN}(\cdot), V^{SU}(\cdot)\right\},$$

where V^{SD} and V^{SU} are obtained by maximizing the objective function in (2.4) over $L_\tau \leq L_{\tau-1}$ (with C^S replaced by C^{SD}) and $L_\tau \geq L_{\tau-1}$ (with C^S replaced by C^{SU}), respectively, and V^{SN} is obtained by choosing $L_\tau = L_{\tau-1}$ in (2.4).

In proposition 4 we present some properties of the value function V^S and its associated optimal exit policy function.

Proposition 4 *Let V^S be defined as in (2.4). Then:*
(a) *There exists a unique value function $V^S(L_{\tau-1}, \theta^*_\tau, \tau)$ satisfying (2.4) that is bounded, continuous in $(L_{\tau-1}, \theta^*_\tau)$, and strictly increasing in θ^*_τ.*
(b) *There exists a unique optimal exit policy function $\chi^*_\tau(L_{\tau-1}, \theta^*_\tau) = \mathbf{1}\left(\theta^*_\tau < \theta^{EX}(L_{\tau-1}, \tau)\right)$, where $\theta^{EX}(L_{\tau-1}, \tau)$ is a unique continuous function in $L_{\tau-1}$.*

Proof. See appendix A.1. ■

Because V^S is not concave, at least when fixed costs are positive, we cannot prove the usual differentiability properties of the value function. Therefore, in what follows, we implicitly assume that $V^S\left(L_\tau, \theta^*_{\tau+1}, \tau+1\right)$ is differentiable at L_τ with probability one, in terms of $F\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right)$ for all $\theta^*_\tau \in \Theta$. By part **(b)** of proposition 4 and the dominated convergence theorem, this implies that the objective functions associated with V^{SD}, V^{SN} and V^{SU} are continuously differentiable in L, so that marginal conditions can be applied to find interior optima. This assumption also implies that

$V^S(L_{\tau-1}, \theta_\tau^*, \tau)$ is differentiable at $L_{\tau-1}$ with probability one.[12] In general, the optimal employment will not be characterized by a simple target and threshold policy due to the presence of fixed adjustment costs.[13]

We have the following proposition concerning the optimal employment policy.

Proposition 6 *For any period $\tau > 0$, if the firm adjusts upwards, optimal employment satisfies*

$$[pF'(L_\tau^*)\theta_\tau^* - w] + \sum_{s=1}^{\infty} E_\tau \beta^s \left\{ \tilde{\chi}_{\tau+s}^* \left(-P^{EX}\right) + \hat{\chi}_{\tau+s}^* \left[pF'\left(L_{\tau+s}^*\right)\theta_{\tau+s}^* - w\right] \right\} = P^{SU}, \tag{2.6}$$

and we must have $V^{SU} > V^{SN}$, whereas if the firm adjusts downwards optimal employment satisfies

$$[pF'(L_\tau^*)\theta_\tau^* - w] + \sum_{s=1}^{\infty} E_\tau \beta^s \left\{ \tilde{\chi}_{\tau+s}^* \left(-P^{EX}\right) + \hat{\chi}_{\tau+s}^* \left[pF'\left(L_{\tau+s}^*\right)\theta_{\tau+s}^* - w\right] \right\} = -P^{SD}, \tag{2.7}$$

and we must have $V^{SD} > V^{SN}$.

In period 0, the firm enters the industry if $E\left(V^{EN}(\theta_0^*)\right) - I \geq W$, in which case optimal employment satisfies

$$[pF'(L_0^*)\theta_0^* - w] + \sum_{s=1}^{\infty} E_0 \beta^s \left\{ \tilde{\chi}_s^* \left(-P^{EX}\right) + \hat{\chi}_s^* \left[pF'(L_s^*)\theta_s^* - w\right] \right\} = P^{EN}. \tag{2.8}$$

$L_{\tau+s}^*$ is the optimal employment in period $\tau+s$, and $\tilde{\chi}_{\tau+s}^*$, $\hat{\chi}_{\tau+s}^*$ are functions of the optimal exit decision, $\chi_{\tau+j}^*$, in periods $\tau+1$ to $\tau+s$, such that $\tilde{\chi}_{\tau+s}^*$ equals one when the firm has remained in the industry until period $\tau+s-1$, but decides to exit in

[12] In proposition 5 of appendix A.1, we prove that this property holds both in a model with a finite lifetime horizon and in a model with infinite-lived firms that face a finite learning horizon (as in sections 2.4 and 2.5).

[13] A simple policy is composed of two thresholds and two targets, i.e., (t, T, U, u), where if $L < t$, then $L' = T$, while if $L > u$, then $L' = U$, and $L' = L$ otherwise. In the case with fixed costs, even in the simpler problems of Chen and Simchi-Levi (2003) and Ye and Duenyas (2006), it is still unknown what property of the value function ensures that the optimal employment policy will be a simple policy. Some concept similar to that of (K_1, K_2)-concavity might be useful here.

period $\tau+s$, and $\hat{\chi}^*_{\tau+s}$ equals one when the firm is still in the industry in period $\tau+s$.

Proof. See appendix A.1. ■

Equations (2.6), (2.7) and (2.8) are marginal conditions, similar to the smooth pasting conditions in the (S,s) model literature, and they state that if the firm adjusts then the marginal adjustment cost must equal the expected present discounted value of the marginal revenue product for all future periods in which the firm is still in the industry, minus the increase in the exit cost when the firm decides to exit. This is the discrete-time analog of the continuous-time result present in Nickell (1986) and Bentolila and Bertola (1990), adjusted for the fact that now we also have an exit decision in each period. The firm will not adjust if the marginal cost of adjustment exceeds its marginal benefit for the first unit of adjustment. The conditions involving V^{SD}, V^{SN}, V^{SU}, and V^{EN} are similar to the value matching conditions in the (S,s) model literature, and they state that adjustment will only occur when its pay-off is higher than non-adjustment. When there are fixed adjustment costs (contrary to Nickell and Bentolila and Bertola), the fact that the marginal benefit of adjustment exceeds the marginal cost of adjustment at the current employment level $(L_{\tau-1})$ does not imply that it is optimal for the firm to adjust, because the total benefit from adjustment must also exceed the total cost of adjustment (including the fixed cost). Therefore, proportional costs imply inaction whereas fixed costs imply both inaction and lumpiness in the employment decisions of the firm.

In equilibrium, the entry condition will hold with equality, since if $E\left(V^{EN}\right)-I>W$ new firms will enter the industry, causing a decrease in price p until equality is restored.

Even though the results in proposition 6 do not enable us to solve the model analytically, and therefore prove formally the effects of adjustment costs in this general model, the following corollary of proposition 6 will allow us to make qualitative heuristic statements.

Corollary 7 *For any period $\tau \geq 0$, the marginal benefit of one additional unit of labor, that is, the LHS of expressions (2.6), (2.7), and (2.8), can be recursively represented as*

$$MB\left(L_{\tau-1}, \theta_\tau^*, \tau\right) = \left(pF'\left(L_\tau^*\right)\theta_\tau^* - w\right) + \beta E_\tau\left[\chi_{\tau+1}^*\left(-P^{EX}\right) + \left(1-\chi_{\tau+1}^*\right) MB\left(L_\tau^*, \theta_{\tau+1}^*, \tau+1\right)\right] \quad (2.9)$$

where $L_\tau^* = L_\tau^*\left(L_{\tau-1}, \theta_\tau^*\right)$, $\chi_{\tau-1}^* = \chi_{\tau-1}^*\left(L_{\tau-1}, \theta_\tau^*\right)$ are the optimal employment and exit decisions.

Proof. See appendix A.1. ■

As we have seen above, when there are no costs to adjustment, optimal employment is determined solely to maximize current period profits. Therefore, firms' growth is essentially a by-product of a selection mechanism: those firms that are inefficient, and therefore small, exit, while those firms that are efficient survive and grow. There is an additional source of positive growth when the frictionless employment decision rule is convex in θ^*. Because of Jensen's inequality and because θ_τ^* is a Martingale, surviving firms will grow over time: $E_\tau\left[L^*\left(\theta_{\tau+1}^*\right)\right] > L^*\left[E_\tau\left(\theta_{\tau+1}^*\right)\right] = L^*\left(\theta_\tau^*\right)$. However, L^* will not be convex in θ^* for general $F(L)$.[14]

We now analyze each cost in turn, assuming symmetry both between hiring and entry costs and between firing and exit costs. In the heuristic arguments that follow, we use the property that MB_τ is weakly increasing in θ_τ^*, and that L_τ^* is locally weakly increasing in θ_τ^*. We present in figure 2.1 the case where there is a proportional hiring

[14] In general, we have

$$\bar{L}''\left(\theta^*\right) = \left(\frac{F'''\left(\bar{L}\right)F'\left(\bar{L}\right) - 2F''\left(\bar{L}\right)^2}{F''\left(\bar{L}\right)^2\theta^*}\right)\bar{L}'\left(\theta^*\right),\ F''\left(\bar{L}\right) < 0,\ L'\left(\theta^*\right) > 0,$$

whose sign depends on $F'''\left(\bar{L}\right)$. Therefore, if decreasing returns to labor do not decrease too fast, that is, $F'''\left(\bar{L}\right) < 2F''\left(\bar{L}\right)^2/F'\left(\bar{L}\right)$, then we will have $\bar{L}''\left(\theta^*\right) < 0$. When $F(L) = \ln(L)$, then $\bar{L}''\left(\theta^*\right) = 0$, and when $F(L) = L^\alpha$, $\alpha \in (0,1)$, then $\bar{L}''\left(\theta^*\right) > 0$.

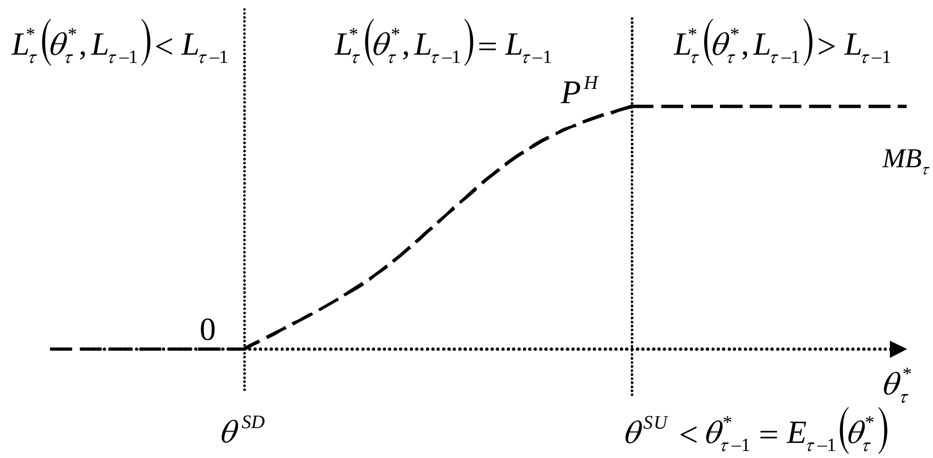

Figure 2.1: Proportional Hiring/Entry Cost

cost, $P^H > 0$, and all other costs are zero. This figure assumes a given $L_{\tau-1}$. For that specific value of L_{t-1}, θ^{SU} and θ^{SD} are the frontiers between non-adjustment and upward and downward adjustment, respectively. Therefore, if $\theta_\tau^* \in \left[\theta^{SD}, \theta^{SU}\right]$ there will be no adjustment. In this case there is inaction but no lumpiness in employment, and the marginal benefit of an additional unit of labor (represented by the dashed line) is contained in the interval $\left[0, P^H\right]$. To make the argument clearer, we consider a firm whose sequence of productivity draws is such that in every period it has a perceived productivity equal to the unconditional mean of θ^*, even though the firm's uncertainty over next period θ^* decreases with age.

Case 1: Proportional Hiring Cost: $P^H > 0$ *(all other costs are zero)*

Because the firm starts at the hiring margin, we must have $MB_0 = P^H$ at entry, and $MB_\tau \in \left[0, P^H\right]$, for all subsequent periods, $\tau = 1, 2, \ldots$, with the two extremes of the interval representing firing and hiring of workers, respectively. Consider first a situation where exit is not allowed.[15] Then (2.9) would become

$$MB\left(L_{\tau-1}, \theta_\tau^*, \tau\right) = \left(pF'\left(L_\tau^*\right)\theta_\tau^* - w\right) + \beta E_\tau MB\left(L_\tau^*, \theta_{\tau+1}^*, \tau+1\right)$$

[15] This could be obtained by assuming a sufficiently large fixed cost of exit, F^{EX}.

For the entry period, we have $MB(\theta_0^*) = P_H$, which implies that the firm will start smaller when $P^H > 0$ than when $P^H = 0$.[16] Since $MB(\theta_1^*) \in [0, P^H]$, $E_0 MB_1 < P^H$ and thus we must have $pF'(L_0) - w > 0$, for all $\beta \in (0,1)$, if $P^H > 0$. In the following period, firms will adjust upwards as frequently with $P^H > 0$ as when $P^H = 0$, because they start at the hiring margin and $E_0\theta_1^* = \theta_0^*$, even though they might have smaller magnitudes of adjustment due to the hiring cost.[17] The proportional hiring cost implies that firms will adjust downwards only if $\theta_1^* < \theta_1^{SD}$, so that there is a region of inaction when $P^H > 0$ that is not present when $P^H = 0$. That is, firms hire fewer workers initially because the resulting smaller probability of having to fire them, and therefore wasting the initial hiring cost, compensates for the expected decrease in profits this period. Consequently, in period 1 more firms will hire than fire, and this tendency towards growth in young firms will persist for several periods.

The Bayesian learning mechanism implies both persistence and a reduction in variance with age in the Markov process associated with θ_τ^*. The effect of persistence, that is, the fact that $E\left(\theta_{\tau+1}^* \mid \theta_\tau^*, \tau\right) = \theta_\tau^*$, was analyzed in the previous paragraph. The reduced uncertainty in the posterior estimate of productivity will be reflected in a smaller inaction region; that is, θ^{SU} will decrease. This causes an increase in $E_\tau MB_{\tau+1}$ for those firms already at the hiring margin, which must be balanced by an increase in L_τ^* for the right hand side of (2.9) to remain equal to P^H. As firms become more certain about their true productivity they are more willing to adjust to their long run optimal size. Because most firms are at the hiring margin, this will cause a further increase in average size.

Consider now the possibility of exit. In this case, the uncertainty reduction as the firm ages implies a decrease in the exit probability, and a further increase in the future-periods component of MB in (2.9). Consequently, L_τ^* needs to increase

[16] When there is no exit and no fixed costs, we can prove that V^S is concave (and continuously differentiable) in L, so that L_0 must decrease for MB_0 to increase.

[17] In fact, firms should adjust upwards more frequently, since the inaction region becomes smaller with the reduction in uncertainty over next period productivity as firms get older.

further in order to compensate.[18] On the other hand, the smaller exit probability makes selection less intense as a cohort ages, which tends to make growth in average firm size smaller. This occurs because there is less pruning of inefficient slow-growing firms. Therefore, we will have less growth due to selection and more growth due to survivors, so that survivors' contribution to average firm growth should increase when exit is allowed.[19]

Case 2: Proportional Firing Cost: $P^F > 0$ *(all other costs are zero)*

In this case we have $MB_0 = 0$, $MB_\tau \in \left[-P^F, 0\right]$, $\tau = 1, 2, \ldots$. Assume first that exit is not allowed. The intuition is the same as in case 1. In comparison with $P^F = 0$, when $P^F > 0$ firms start smaller and subsequently hire more frequently than they fire. As firms age, the reduction in variance of θ^* causes an increase in $E_\tau MB_{\tau+1}$, which must be compensated by an increase in L_τ^* for firms at the hiring margin. When exit is possible, those effects become more intense, since the exit probability will decrease as firms age.

Case 3: Fixed Hiring Cost: $F^H > 0$ *(all other costs are zero)*

Contrary to above, now $MB_0 = 0$ at entry and $MB_\tau \geq 0$, $\tau = 1, 2, \ldots$, with equality whenever the firm adjusts in either direction. When $F^H > 0$, the firm will only hire if the total benefit of hiring exceeds the fixed cost. This implies that even if MB_τ is positive, the firm might not hire in future periods because the total benefit does not compensate for F^H. Thus, hiring fixed costs cause an increase in equilibrium price and entry size, and a reduced frequency of hiring in the following periods.[20] Firms realize that they will be less willing to adjust upwards next period because of

[18] This effect is similar to that of Cabral (1995).

[19] There is an additional effect of adjustment costs that is due to the increase in price. This change in equilibrium price affects the level of average size, but has a second order impact on the growth rate of average size.

[20] This is related to the point made by Jovanovic and Stolyarov (2000), in the sense that a fixed capacity formation cost and proportional adjustment costs for labor would be a good explanation for both the initial high excess capacity of firms and the reduction in excess capacity with age, as firms increase the relative use of labor.

the fixed cost. Therefore, reduced profits from overhiring at entry are compensated by smaller expected future hiring costs and higher profits in case of non-adjustment. Only firms that experience a sufficiently high increase in productivity will increase employment in the future, and they will do so by the same magnitude as in the case with no fixed cost. Meanwhile, all firms that receive a bad signal will reduce their size. Fixed hiring costs will thus create an incentive for firms to start large and to grow slowly thereafter.

Contrary to previous cases, reduced uncertainty in θ^* causes a decrease in $E_\tau MB_{\tau+1}$ since it produces a decrease in θ^{SU}, while entering firms are located at θ^{SD}. Therefore, L^*_τ will tend to decrease with age as firms realize that indeed they have overhired. However, the decrease in the probability of exit will have an effect similar to that in the previous cases, causing an increase in the future-periods component of MB_τ, and creating an incentive for growth as firms survive.

Case 4: Fixed Firing Cost: $F^F > 0$ (all other costs are zero)

Here we have $MB_0 = 0$ at entry and $MB_\tau \leq 0$, $\tau = 1, 2, \ldots$, with equality whenever the firm adjusts. Assume first there is no exit. Firms will be more willing to adjust upwards than downwards. Therefore, a firing fixed cost will cause an increase in equilibrium price, a decrease in entry size, and a reduced frequency of firing in the following periods. The reduced uncertainty in θ^* as firms age causes an increase in θ^{SD}, and an increase in $E_\tau MB_{\tau+1}$ since entering firms are located at θ^{SU}. Therefore, there will be a tendency for employment to increase, as firms realize a reduction in the probability of future overemployment. Consider now the effect of exit. Once again, a reduction in the probability of exit will cause an increase in the future-periods component of MB_τ, so that optimal current employment must increase to maintain equality in (2.9).

The heuristic intuition we have given above analyzes each cost in isolation. When we have more than one of those costs simultaneously, with possible asymmetries between hiring and firing, the results will be a mix of the effects presented. Note that proportional hiring and proportional firing costs reinforce each other, whereas fixed hiring and fixed firing costs counteract each other. If fixed costs are more important for capital and proportional costs are more important for labor, we could explain why firms tend to have initial excess capacity, why capital adjustments tend to be lumpier and more intermittent than labor adjustments, and why firm employment size tends to grow with age.

Our heuristic analysis suggests that most forms of (labor) adjustment costs create incentives for growth. In the end, our assessment of the relevance of adjustment costs will depend on how well a pure selection model can fit the empirical evidence, and on how well adjustment costs can improve the fit. Before we move into a quantitative assessment, we present analytical results for a simple version of the general model.

2.4 Model with One Period Learning Horizon and No Exit

In this section, we analyze a model where firms learn their true efficiency after the first period of life. The introduction of adjustment costs implies an additional expected operating cost for entering firms. Therefore, the equilibrium price must increase to generate higher expected future profits that compensate for the costs incurred while adjusting to optimal size. These higher profits translate into less selection, so that more inefficient firms will be able to survive. In fact, this is optimal from a social point of view, since there will be some saving in unrecoverable costs.

In order to investigate the implications of firms' lifetime horizon, we assume that firms live for $\bar{T}$ periods, $\bar{T} \in \{2, \ldots, \infty\}$. We also assume that no exit is allowed prior to age $\bar{T}$. Therefore, we eliminate any selection and focus only on the incentives for survivors to grow. We know that adjustment costs decrease the amount of selection in the industry, so that if adjustment costs create incentives for survivors to grow faster, then we would have a higher relative contribution of survivors to growth in the cohort's average size even if we allowed exit. We examine the impact of adjustment costs on the log growth rate of employment rather than the standard growth rate, in order to attenuate the effect of Jensen's inequality on firm growth.[21]

To formulate the problem, we use the fact that once the firm learns its true efficiency in period 2, it will adjust once and for all to its long run employment level, and not adjust in any of the following periods. This result is formalized in proposition 11 below. Therefore, assuming that upon exit at age $\bar{T}$ the firm receives its opportunity cost net of exit costs, the optimization problem in period 2 is

$$V^S(L_1, \theta_2^*) = \max_{L_2} \left\{ \delta\left(\bar{T}\right) \Pi(L_2, \theta_2^*) - C^S(L_2, L_1) + \beta^{\bar{T}-1} V^{EX}(L_2) \right\} \tag{2.10}$$

where $\delta\left(\bar{T}\right) \equiv \sum_{s=0}^{\bar{T}-2} \beta^s = \left(1 - \beta^{\bar{T}-1}\right) / (1-\beta)$, and C^S and V^{EX} are defined as above. In period 1, we then have

$$V^{EN}(\theta_1^*) = \max_{L_1} \left\{ \Pi(L_1, \theta_1^*) - C^{EN}(L_1) + \beta E_1 \left[V^S(L_1, \theta_2^*) \right] \right\}.$$

Finally, the equilibrium price is determined by the condition that potential entrants break even, i.e., $E_0\left[V^{EN}(\theta_1^*)\right] - I = W$.

A common implication of proportional and fixed costs is the existence of an inaction region which is a function of previous period employment and the adjustment cost parameters. In this simple model, the inaction region can be expressed as an interval: $\Theta^{SN} = \left[\theta^{SD}, \theta^{SU}\right]$. Therefore, the average log growth rate between period 1

[21] As we saw above, Jensen's inequality implies positive expected growth, even in the absence of adjustment costs, when optimal employment is a convex function of θ^*. Because the log transformation is concave, it will counteract the convexity of the optimal employment function. For example, for a Cobb-Douglas specification of $F(\cdot)$, using a log growth rate will eliminate the effect of Jensen's inequality, since $\ln(L_\tau^*)$ becomes linear in θ_τ^*.

and period 2, conditional on θ_1^*, is defined as

$$g\left(\theta_1^*\right) = E\left[\ln\left(L_2^*\right) - \ln\left(L_1^*\right)\right] =$$
$$\int_{\nu_1}^{\theta^{SD}} \left\{\ln\left(L_2^{*SD}\right) - \ln\left(L_1^*\right)\right\} dF\left(\theta_2^* \mid \theta_1^*\right) + \int_{\theta^{SU}}^{\nu_2} \left\{\ln\left(L_2^{*SU}\right) - \ln\left(L_1^*\right)\right\} dF\left(\theta_2^* \mid \theta_1^*\right)$$

where $\Theta \equiv [\nu_1, \nu_2]$ is the support of the distribution of θ_2^*, and $\theta^{SD}\left(L_1^*\right)$ and $\theta^{SU}\left(L_1^*\right)$ are the frontiers between non-adjustment and downward and upward adjustment, respectively. Depending on the specific value of θ_1^* and the magnitude of the adjustment cost parameters, we might have $\theta^{SD}\left(L_1^*\right) = \nu_1$ and/or $\theta^{SU}\left(L_1^*\right) = \nu_2$. However, in the results that follow, we assume that θ_1^* and the adjustment cost parameters are such that both downward adjustment and upward adjustment occur with positive probability, i.e., $\theta^{SD}\left(L_1^*\right) > \nu_1$ and $\theta^{SU}\left(L_1^*\right) < \nu_2$. While analyzing the effect of each type of adjustment cost, we assume the other costs to be zero, but the sign of the partial effects would not change if we allowed other costs to be present. We also ignore the indirect effects of adjustment costs due to changes in the equilibrium price. These indirect price effects influence average firm size in both periods, but are of second order importance for the average log growth rate.[22] We first consider proportional costs and then fixed costs.

Case 1: Proportional Costs: either $P^F > 0$ or $P^H > 0$.

In this case, optimal employment in period 2 is determined by

$$L_2^*\left(L_1, \theta_2^*\right) = \begin{cases} L_2^{*SU} = F'^{-1}\left(\dfrac{w + \frac{\beta^{\bar{T}-1} P^F}{\delta(\bar{T})} + \frac{P^H}{\delta(\bar{T})}}{p\theta_2^*}\right), & \theta_2^* > \theta^{SU}\left(L_1\right) \\ L_2^{*SN} = L_1, & \theta^{SU}\left(L_1\right) \geq \theta_2^* \geq \theta^{SD}\left(L_1\right) \\ L_2^{*SD} = F'^{-1}\left(\dfrac{w + \frac{\beta^{\bar{T}-1} P^F}{\delta(\bar{T})} - \frac{P^F}{\delta(\bar{T})}}{p\theta_2^*}\right), & \theta^{SD}\left(L_1\right) > \theta_2^* \end{cases}$$

[22]In the proof of proposition 8 below, we show for proportional adjustment costs that if the production function is Cobb-Douglas then the indirect price effects cancel out.

where the frontiers of adjustment are defined as

$$\theta^{SU}(L_1) \equiv \frac{w + \frac{\beta^{\bar{T}-1}P^F}{\delta(\bar{T})} + \frac{P^H}{\delta(\bar{T})}}{pF'(L_1)}, \theta^{SD}(L_1) = \frac{w + \frac{\beta^{\bar{T}-1}P^F}{\delta(\bar{T})} - \frac{P^F}{\delta(\bar{T})}}{pF'(L_1)}.$$

Note that the numerator of θ^{SU} equals the pro-rated per-period cost of adding another worker, including the wage, the marginal hiring cost, and the discounted cost of firing the worker after period $\bar{T}$. The numerator of θ^{SD} has a similar interpretation, as the benefit of shedding a worker.

We then have the following result concerning the effects of changes in P^H and P^F on the cohort's average log growth rate of employment.

Proposition 8 [23] *Assuming that $F(L)$ is Cobb-Douglas and that $\theta^{SD}(L_1^*) > \nu_1$ and $\theta^{SU}(L_1^*) < \nu_2$:*

(a) *The marginal effect of P^H on $g(\theta_1^*)$, assuming all other costs are zero, is positive for a high enough value of $\bar{T}$.*

(b) *The marginal effect of P^F on $g(\theta_1^*)$, assuming all other costs are zero, is positive for all $\bar{T}$.*

Proof. See appendix A.1. ■

Consider first the hiring cost. In the proof, we show that an increase in P^H decreases both L_1^* and L_2^{*SU}. The impact of P^H on the growth rate depends on two opposing effects. First, while in the case of L_2^{*SU} the cost of hiring can be equally spread out over $\bar{T}-1$ periods with certainty, in the case of L_1^* it will be spread out over either $\bar{T}$ periods or one period, depending on whether the firm learns in period 2 that it has overhired. Therefore, *ex ante* a proportionately greater part of P^H is attached to period 1 in the case of L_1^*. This explains the positive effect on growth

[23] In the proof, we consider a general production function and then specialize to a Cobb-Douglas specification in order to obtain the sign of the effect. From that general setup, we can say that the form of the production function should not be determinant for these results when the elasticity of the marginal product of labor does not change much with the amount of labor used.

of P^H for $\bar{T}=\infty$. Second, the hiring cost on L_1^* can possibly be spread out over $\bar{T}$ periods, while the hiring cost on L_2^{*SU} can only be spread out over $\bar{T}-1$ periods. This explains why the effect of P^H on growth is not necessarily positive for finite $\bar{T}$. However, as $\bar{T}$ increases the first effect dominates so that P^H decreases L_1^* more than L_2^{*SU} and growth increases.[24]

With respect to P^F there is always a positive effect on growth, independently of the lifetime horizon. This occurs because an increase in P^F decreases L_1^* and increases L_2^{*SD}. This positive effect always dominates the uncertain effect due to the fact that L_2^{*SU} also decreases with P^F.

When there are both hiring and firing costs and these costs are identical ($P^H = P^F = P$), then an increase in P has a positive effect on $g(\theta_1^*)$, for sufficiently high $\bar{T}$, where the required $\bar{T}$ is lower than in item **(a)** of proposition 8.

Case 2: Fixed Costs: either $F^F>0$ or $F^H>0$.

In this scenario, optimal employment in period 2 is determined by

$$L_2^*(L_1,\theta_2^*) = \begin{cases} L_2^{*SU} = F'^{-1}\left(\frac{w}{p\theta_2^*}\right), & \theta_2^* > \theta^{SU}(L_1) \\ L_2^{*SN} = L_1, & \theta^{SU}(L_1) \geq \theta_2^* \geq \theta^{SD}(L_1) \\ L_2^{*SD} = F'^{-1}\left(\frac{w}{p\theta_2^*}\right), & \theta^{SD}(L_1) > \theta_2^* \end{cases}$$

with the frontiers of adjustment defined as

$$\begin{aligned} V^{SU}\left(L_1,\theta^{SU}\right) &= V^{SN}\left(L_1,\theta^{SU}\right), \\ V^{SD}\left(L_1,\theta^{SD}\right) &= V^{SN}\left(L_1,\theta^{SD}\right), \end{aligned}$$

where V^{SD}, V^{SN}, and V^{SU} have an analogous definition to that in the general model of the previous section.

The following proposition summarizes the effects of F^H and F^F on $g(\theta_1^*)$.

[24]In our simulations, $\bar{T}=3$ was enough to generate a positive effect on growth.

Proposition 9 *For a general production function, under the assumption that* $\theta^{SD}(L_1^*) > \nu_1$ *and* $\theta^{SU}(L_1^*) < \nu_2$*:*

(a) *The marginal effect of* F^H *on* $g(\theta_1^*)$*, assuming all other costs are zero, is negative for all* $\bar{T}$*.*

(b) *The marginal effect of* F^F *on* $g(\theta_1^*)$*, assuming all other costs are zero, is positive for all* $\bar{T}$*.*

Proof. See appendix A.1. ■

Excluding the indirect effect of prices, the intuition is clear. With a fixed hiring cost, firms know that they will be less willing to adjust upwards; that is, $\partial\theta^{SU}/\partial F^H > 0$. The consequence of this is that they hire more initially, $\partial L_1^*/\partial F^H > 0$. Because optimal employment in period 2, conditional on adjustment, is not affected directly by F^H, then we must have a smaller average growth. A similar inverse logic applies to the fixed firing cost.

In the results above we have assumed symmetry both between hiring and entry costs and between firing and exit costs. This assumption is innocuous for fixed costs, since fixed entry and exit costs do not affect the incentives for firms to grow. For proportional costs, if we analyze each in turn, hiring costs (P^{SU}) would reduce growth while entry costs (P^{EN}) would promote growth. On the other hand, firing costs (P^{SD}) would increase firm growth, while exit costs (P^{EX}) would reduce growth. As we discussed above, we avoid introducing these asymmetries because they lead to arguably artificial biases in entry, exit and growth decisions. For example, if the entry cost is smaller than the hiring cost, then firms would start larger in order to save on expected hiring costs in the following period.

2.5 Calibration Under Finite Learning Horizon

In the previous two sections, we developed heuristic arguments about the effect of adjustment costs on the cohort's average growth rate and then proved those results in a simplified version of the model. We now wish to assess the contribution of adjustment costs to explain some of the basic facts on firm dynamics found in section 2.2, both for the overall economy and for the manufacturing and services sectors in Portugal. Therefore, in this section we perform some computational and calibration experiments.

Simulation of an infinite learning horizon model is potentially a difficult task because V^S depends on the firm's age. This prevents us from using an iterative method that in each iteration provides some converging approximation to the value function or the policy function. Therefore, we follow the suggestion of Ljungqvist and Sargent (2000, p. 109) and consider an approximation where firms live forever, but learn their *ex post* true productivity component, μ_1, with certainty at some age T.[25]

In our simulations, we assume that F is Cobb-Douglas, i.e., $F(L) = L^\alpha$, $\alpha \in (0,1)$. Under this assumption, with no adjustment costs, optimal employment would be given by

$$L_\tau^* = L(\theta_\tau^*, \tau) = \begin{cases} \bar{L}(\theta_\tau^*) \equiv \left(\frac{\alpha p \theta_\tau^*}{w}\right)^{\frac{1}{1-\alpha}}, & \text{if } \theta_\tau^* \geq \theta_\tau^{EX}, \\ 0, & \text{if } \theta_\tau^* < \theta_\tau^{EX}. \end{cases} \tag{2.11}$$

Therefore, with $\alpha \in (0,1)$, optimal employment conditional on survival is a convex function of θ_τ^*, so that Jensen's inequality implies growth of employment even in the absence of selection. As in the previous section, in order to avoid any growth due to Jensen's inequality, we take logs of all variables and analyze the effects of adjustment costs on the log-growth rate.

Concerning the productivity distribution, we assume $\xi(\eta) = \exp\{\eta\}$, so that

[25] Note that this T differs from the lifetime horizon $\bar{T}$ used in section 2.4, with $\bar{T} \geq T$. In this section, we assume an infinite horizon, so that $\bar{T} = \infty$. In our simulations and calibrations below, we assume that $T = 14$ (years), and present results until year 10.

θ_τ follows a lognormal distribution.[26] This assumption is made for computational simplicity, although it is also reasonable on empirical grounds (see Aw, Chen and Roberts 2004). Besides this, the results in section 2.4 suggest that the distribution of productivity mostly affects the intensity of the effect of adjustment costs on growth of firms, but not the sign. In fact, propositions 8 and 9 are derived independently of the particular distribution of θ_τ^*. Given this assumption, we have the following proposition concerning the transition law for the θ^*s.

Proposition 10 *Let $\theta_\tau = \exp\{\eta_\tau\}$ be generated as in assumption 1. Then,*

(a) *The posterior distribution of $\theta_{\tau+j}$ $(j \geq 0)$, given the information set at time τ, $\Omega_\tau = \left\{\mu_0, \{\eta_s\}_{s=0}^{\tau-1}\right\}$ if $\tau < T$, and $\Omega_\tau = \{\mu_0, \mu_1\}$ if $\tau \geq T$, is*

$$\theta_{\tau+j} \mid_{\Omega_\tau} \sim \log N\left(Y_\tau, Z_\tau + \sigma^2\right),$$

where, for $\tau < T$, Y_τ and Z_τ are defined in (2.2), and, for $\tau \geq T$, $Y_\tau = \mu$ and $Z_\tau = 0$. Let $\theta_\tau^* = E(\theta_\tau \mid \Omega_\tau) = E(\theta_\tau \mid \theta_\tau^*, \tau)$. Then the distribution of $\theta_{\tau+j}^*$ $(j \geq 1)$ given (θ_τ^*, τ) is

$$\theta_{\tau+j}^* \mid_{(\theta_\tau^*,\tau)} \sim \log N\left(\ln(\theta_\tau^*) - \frac{1}{2}(Z_\tau - Z_{\tau+j}), Z_\tau - Z_{\tau+j}\right).$$

Also, the unconditional distribution of θ_τ^* $(\tau \geq 0)$ is

$$\theta_\tau^* \sim \log N\left(\bar{\mu} + \frac{1}{2}\left(Z_\tau + \sigma^2\right), \sigma_\mu^2 - Z_\tau\right),$$

where $\sigma_\mu^2 = \sigma_{\mu_0}^2 + \sigma_{\mu_1}^2$.

Proof. See appendix A.1. ■

Since we assume that the firm enters the industry already knowing its *ex ante* productivity component μ_0 (see assumption 1), we will get a non-degenerate distrib-

[26] This assumption makes $\nu_1 = 0$, and $\nu_2 = \infty$. Even though this violates assumption 1, it does not pose any problem in this section, since we will be using a discrete approximation to this distribution.

ution of initial sizes. This occurs because $L_0 = L_0^* (\theta_0^*)$, and θ_0^* has positive variance in the cohort's initial distribution. The next proposition analyzes the properties of the optimization problem after μ is revealed to the firm in period T.

Proposition 11 *If μ is revealed to the firm at period T, then all adjustments are made at period T, and the firm will not change its exit and employment decisions after that period. This means that*

$$V^S (\theta_T^*, L_{T-1}, T) = \max_L \left\{ \frac{1}{1-\beta} \Pi (L, \theta_T^*) - C^S (L, L_{T-1}) \right\}, \qquad (2.12)$$

$$L_s^* = L^* (\theta_T^*, L_{T-1}, T), \ s \geq T, \ \chi_T^* = \mathbf{1} \left[V^S (\theta_T^*, L_{T-1}, T) < V^{EX} (L_{T-1}) \right].$$

Proof. See appendix A.1. ■

This result enables us to simplify the computational algorithm significantly, since it implies a finite horizon dynamic programming problem. In appendix A.2, we present some details concerning the computational algorithm. Our method requires only that we compute the optimal employment and exit decisions numerically, and then include these in the recursive expressions used to obtain all relevant densities and moments in the model. This is an important simplification since our dynamic programming problem is not age independent, and we are using a finite learning horizon, so that we could not implement an iterative procedure converging to an ergodic distribution of optimal employment. In the next subsection, we present a calibration of the learning model with adjustment costs, and leave for the following subsection a sensitivity analysis.

2.5.1 Calibration with Costly Adjustment

We now calibrate our model to match statistics from the 1988 cohort of all entering firms. We first calibrate parameters related to inputs directly from the data. We then search for values of the parameters associated with the learning process to match the evolution of firm size and exit observed in the data. Finally, we search for values of the adjustment cost parameters that produce a survivor component close to that found in the data. Similarly to section 2.2, our decomposition of the change in the cohort's average size is the following

$$E\left[\ln\left(L_\tau\right) \mid S_\tau\right] - E\left[\ln\left(L_0\right) \mid S_0\right] = \underbrace{E\left[\ln\left(L_\tau\right) - \ln\left(L_0\right) \mid S_\tau\right]}_{\text{Survivor Component}} +$$
$$\underbrace{\Pr\left(D^\tau \mid S_0\right)\left\{E\left[\ln\left(L_0\right) \mid S_\tau\right] - E\left[\ln\left(L_0\right) \mid D^\tau\right]\right\}}_{\text{Selection Component}}$$

In appendix A.2, we give details on how to compute the densities associated with each of these moments.

We now explain in more detail our calibration method. The parameters α and w are calibrated with data from INE (1997) containing the *Inquérito Annual às Empresas* from 1990 to 1995. This data is reliable and covers all firms in the Portuguese economy, with sampling among firms with less than 20 workers. We measure α as the 1990-1995 average of the cost share of labor in value added, and w as the 1990-1995 average cost per worker. We can also obtain these values at the one-digit sectoral level. We deflated all nominal variables using the GDP sectoral price indices available in the updated version of *Séries Longas para a Economia Portuguesa* in Banco de Portugal (1997). The real interest rate is calibrated as the 1990-1995 average of the implicit real interest rate on public debt transactions on the secondary market of the Lisbon Stock Exchange. The data was also taken from Banco de Portugal (1997). We deflated the nominal interest rates using the December-to-December consumer price index from INE (1990–1995). The discount rate was then obtained as $\beta = \frac{1}{1+r}$, where r is the average real interest rate.

The calibration of W deserves some discussion. First, the main purpose of this parameter is to induce endogenous exit of firms in our model. In Hopenhayn (1992), the same is accomplished using a fixed per period operating cost, instead of an exit opportunity cost (Jovanovic 1982). In proposition 12 of appendix A.1, we show the equivalence between the two mechanisms, since the per period opportunity cost is just the annuity value of the exit opportunity cost. Second, because we omit capital in our model, the calibration of W should be based on the expected discounted future stream of value added minus labor costs, adjusted for the initial research cost, I. Therefore, we obtain a measure for W equal to the annuity value of the 1990-1995 average of value added minus labor costs, using the same deflators as for w. However, this is certainly an overestimate for the true value of W, since the sample is biased towards surviving firms. Third, if for a given value of W we use the parameters governing the learning process to match the exit rate in the cohort of firms, then W will indirectly determine the cohort's average size. Because of this, and because of the stated overestimation of W, we calibrate W in order to match both the exit rates and the average size of firms in the cohort.[27]

The remaining parameters are calibrated to match the evidence on cohort dynamics presented in section 2.2. First, for the given values of α, w, β we search for values of $\bar{\mu}$, σ_{μ_0}, σ_{μ_1}, σ, and W that make the model's implications for the time-series of the cross-sectional mean of log-employment conditional on survival, $E\left[\ln\left(L_\tau\right) \mid S_\tau\right]$, the time-series of annual changes in the cross-sectional standard deviation of log-employment conditional on survival, $SD\left[\ln\left(L_\tau\right) \mid S_\tau\right]$, and the time-series of the cumulative exit rate, $\Pr\left(D^\tau \mid S_0\right)$, closest to the equivalent moments in the data. Second, we search for values of the adjustment cost parameters that enable the model to simultaneously match the time-series of the survivor component, in addition to

[27] When we calibrate W as measured from the data (1373.8 for the overall economy cohort, 3317.1 for the manufacturing cohort, and 269.1 for the services cohort) we obtain qualitatively similar results for the calibration of adjustment cost parameters, and we are able to match the observed survivor component, but largely overestimate the cohort's average size. This shows that W is not determinant to explain the survivors' contribution to firm growth.

Table 2.4: Calibration to the 1988 Firm Cohort

Sector	*Overall Economy* *NAC*	*AC*	*Manufacturing* *AC*	*Services* *AC*
α	0.56	0.56	0.57	0.73
w	11.8	11.8	13.1	7.5
β	0.956	0.956	0.956	0.956
W	666.3	716.3	1500	178.5
σ_{μ_0}	0.180	0.180	0.110	0.121
σ_{μ_1}	0.210	0.218	0.212	0.146
σ	0.350	0.350	0.350	0.250
$\bar{\mu}$	3.380	3.387	3.800	2.528
I	114.3	102.9	180.9	44.6
P	0.0	0.55	1.6	0.2
F^{EX}	0.0	7.0	25.0	1
$dw(mlL)$	1.643	0.973	7.011	2.452
$dw(sdlL)$	0.4796	0.5030	1.0737	1.9898
$dw(ER)$	10658	10652	14004	13031
$dw(SuC)$	32676	4874	51994	12817

Notes: NAC refers to a calibration when adjustment costs are absent; AC refers to a calibration when adjustment costs are present; $P = P^H = P^F$; $dw(\cdot)$ is the weighted distance measure associated with average size, mlL, standard deviation of size $sdlL$, exit rate ER, and survivor component SuC. In all cases, $F^{SU} = F^{SD} = F^{EN} = 0$.

the previous moments. In doing this, we adjust the other parameters as needed, so that the goodness of fit in terms of cross-sectional size and exit rates, at the least, does not worsen significantly. Finally, p is normalized to 1 and I is obtained by the equilibrium condition $I = E\left(V_0\left(\theta_0^*\right)\right) - W$.[28]

The values for all parameters are in table 2.4, where we include calibrations for the overall economy, the manufacturing sector and the services sector cohorts. To illustrate the process of calibration, for the overall economy cohort we include the calibrated parameters both with and without adjustment costs. In all cases, we consider that $F^{SD} = F^{SU} = 0$, since these two fixed costs have effects of opposite sign on the survivor component. However, we consider a positive value for the fixed

[28] As a measure for the goodness of fit between the model generated cross-sectional moments, x, and the observed moments, y, we use a weighted quadratic form, $dw = (x-y)' \, \Sigma^{-1} (x-y)$, where Σ is obtained from the variance-covariance matrix of the cross-sectional (unconditional) distribution of θ_τ^* (see proposition 10). Therefore, a larger weight is given to the distance between the first few moments after entry.

cost of exit, even though the results are very similar if instead we consider a positive value for the fixed cost of entry. As we argue below, these fixed costs of entry and exit do not directly affect the incentives for firms to grow, but reduce firms' exit rates.

From table 2.4, we verify in our calibrated model without adjustment costs that most information is revealed *ex post* ($\sigma_{\mu_1} > \sigma_{\mu_0}$), and that there is significant noise in the learning process ($\sigma > \sigma_{\mu_0}$ $\sigma > \sigma_{\mu_1}$). For the overall economy cohort, we were able to significantly improve the fit of the survivor component with a symmetric proportional adjustment cost that amounts to 4.7% of the annual wage, and a fixed exit cost that amounts to 60% of the annual wage. This fact can be seen in figure 2.2, where we plot the data moments and the equivalent moments in the calibrated model with (AC) and without (NAC) adjustment costs.

Figure 2.2 shows that even though the model with no adjustment costs is able to generate moments on firm size and exit rates that are close to equivalent moments in the data, it cannot satisfactorily match the observed values for the survivors' contribution. That is, the model with no adjustment costs cannot explain the true source for growth in the cohort's average size, since survivors contribute much more to growth in the data than in such a model. This shortcoming occurs especially in the initial years after entry, since in the data the path of the survivors' component is almost flat, whereas in the model with no adjustment costs the survivors' component is increasing. This positive slope of the model generated survivor component reflects the fact that with no adjustment costs, learning has a larger initial impact on selection than that on growth of survivors.

In table 2.4, we also calibrate the model to the data for manufacturing and services, and in table 2.5 we present a summary of the implied moments. Firms in manufacturing initially learn relatively less about their efficiency than firms in services, and adjustment costs need to be much higher in manufacturing than in services to account for the much higher survivor component in manufacturing. This compari-

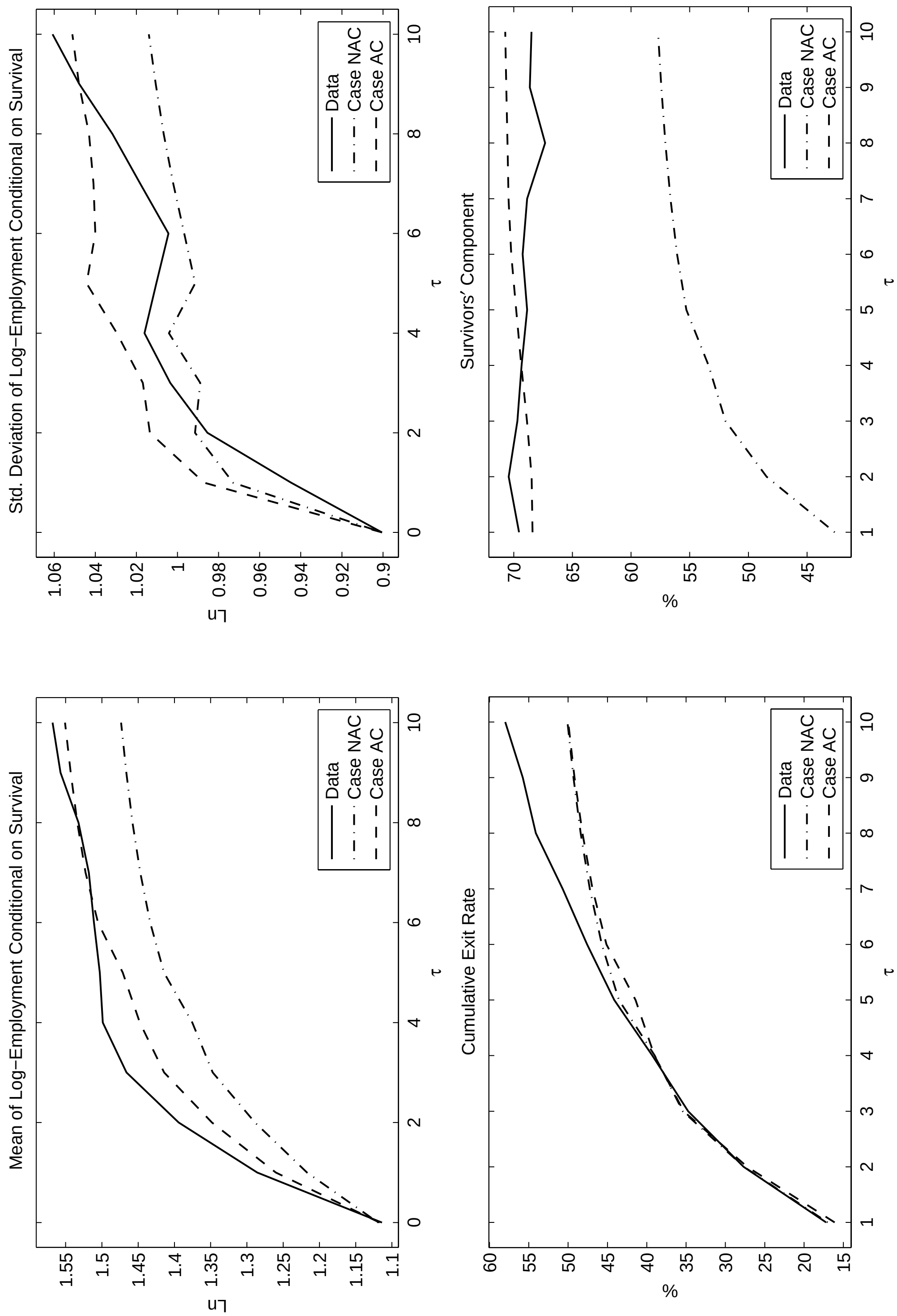

Figure 2.2: Firm Dynamics for Overall Economy Cohort

Table 2.5: Firm Dynamics in Calibration for the 1988 Firm Cohort

Sector	*AvEmp*		*CumEx*		*SurComp*	
	89	*98*	*89*	*98*	*89*	*98*
All: Data	1.11	1.57	17.2	58.0	69.5	68.5
All: AC	1.12	1.55	16.1	50.0	68.4	70.7
Manu: Data	1.58	2.25	15.7	58.3	87.8	83.5
Manu: AC	1.66	2.15	14.5	50.9	81.8	84.9
Serv: Data	0.99	1.35	18.9	57.6	60.3	60.9
Serv: AC	0.97	1.43	15.5	46.6	61.1	65.2

Notes: *AvEmp* is the mean of log-employment among survivors; *CumEx* is the cumulative exit rate (in %); *SurComp* is the survivor component (in %); *Data* refers to observed values; *AC* refers to simulated values from a calibration when adjustment costs are present.

son exercise between manufacturing and services suggests that firms in manufacturing pay a higher initial research cost (I is higher) and face a higher fixed exit cost (F^{EX} is higher), so that even though they learn less initially about their efficiency ($\sigma_{\mu_0}/\sigma_{\mu_1}$ is smaller), there is more initial selection ($1 - \Pr(S_0)$ equals 22.5% in manufacturing and 20.6% in services). Because manufacturing firms face much higher proportional adjustment costs and they know relatively less about efficiency at entry, they have higher incentives to start smaller and to gradually adjust to optimal size as they survive and their uncertainty is resolved. Simultaneously, a larger fixed exit cost implies that in manufacturing selection at entry is larger and post-entry selection is smaller. The consequence is that the contribution of survivors to growth in the cohort's average size is larger in manufacturing than in services.

2.5.2 Sensitivity Analysis

In this subsection we explain some aspects of the calibration exercise and provide a detailed sensitivity analysis to all parameters in the model. First, note that we do not attempt to match the level of the cross-sectional variance of log-employment, but only its change over time. This is because to fit the observed level of dispersion in employment, we would need to make both σ_{μ_0} and σ_{μ_1} much larger. This would enable us to match $SD\left[\ln\left(L_\tau\right) \mid S_\tau\right]$ and $\Pr\left(D^\tau \mid S_0\right)$, but would imply an excessive rate of growth in $E\left[\ln\left(L_\tau\right) \mid S_\tau\right]$. However, this failure is not a serious problem. It just means that only a fraction of the observed cohort's employment dispersion can be attributed to a Bayesian learning process about efficiency. The remaining part could be attributed to heterogeneity in the initial choice of technology. For instance, consider a model with capital and suppose that a firm chooses its initial stock of capital, K_0, based on its observation of a random variable indexing technology choice. Assume further, that after selecting K_0 the firm keeps its capital stock unchanged for its remaining life. Then, conditional on the chosen value of K_0, the problem would become

$$\begin{aligned} V\left(K_0, L_{\tau-1}, \theta_\tau^*, \tau\right) = \max_{L_\tau} \{ & \Pi\left(K_0, L_\tau, \theta_\tau^*\right) - C^S\left(K_0, L_\tau, L_{\tau-1}\right) + \\ & \beta E_\tau\left[\max\left\{V^{EX}\left(K_0, L_\tau\right), V\left(K_0, L_\tau, \theta_{\tau+1}^*, \tau+1\right)\right\}\right]\} \end{aligned}$$

If the production function has constant returns to scale, if the opportunity cost is proportional to K_0, i.e., $W = \tilde{W} K_0$, where $\tilde{W}$ is the opportunity cost per unit of capital, and if all fixed adjustment costs are also proportional to K_0, then the above problem could be equivalently stated as[29]

$$\begin{aligned} V\left(K_0, L_{\tau-1}, \theta_\tau^*, \tau\right) = K_0 \tilde{V}\left(\frac{L_{t-1}}{K_0}, \theta_\tau^*, \tau\right) = K_0 \max_{L_\tau}\left\{\Pi\left(1, \frac{L_\tau}{K_0}, \theta_\tau^*\right) - \right. \\ \left. C^S\left(1, \frac{L_\tau}{K_0}, \frac{L_{\tau-1}}{K_0}\right) + \beta E_\tau\left[\max\left\{V^{EX}\left(1, \frac{L_\tau}{K_0}\right), \tilde{V}\left(\frac{L_\tau}{K_0}, \theta_{\tau+1}^*, \tau+1\right)\right\}\right]\right\} \end{aligned}$$

Therefore, in this alternative framework, dispersion in K_0 would govern the initial dispersion in employment and only the subsequent evolution in this dispersion would

[29] If fixed adjustment costs were less than proportional to K_0, then this model would explain why larger firms tend to exhibit smaller exit rates, since for the given equilibrium price larger firms would be relatively more profitable.

depend on the Bayesian learning process. This is the reason why we attempt to match only the evolution of $SD\left[\ln\left(L_{\tau}\right) \mid S_{\tau}\right]$, but not its level. After simulating the calibrated model we can infer what fraction of the observed dispersion in the cohort's log-employment is attributable to the learning process itself.

In order to make clear the dynamic behavior of the cohort's log-employment dispersion, we rescale the model's implied initial value of $SD\left[\ln\left(L_{\tau}\right) \mid S_{\tau}\right]$ to the level found in the data. In the calibrated model without adjustment costs the value of $SD\left[\ln\left(L_{0}\right) \mid S_{0}\right]$ is 0.2833, while in the data it is 0.9005. Therefore, only little more than 30% of the observed dispersion in the cohort's log-employment is needed to account for the exit rate and growth in average size.

Second, the value of $\sigma_{\mu_0}/\sigma_{\mu_1}$ affects the long-run contribution of survivors, since a relatively smaller initial dispersion would make the average size of exiting firms closer to the average size of surviving firms in the entry period, and in this case most growth would be due to survivors. In the aforementioned extended model with an initial choice over K_0, if we had $\sigma_{\mu_0} = 0$ we would have a non-degenerate initial distribution of size, entirely due to the heterogeneity in K_0, but the survivors' component would still be 100% in each period. This would occur because the distribution of initial size among exiting firms would be equal to the distribution of initial size in the cohort of all entering firms. This also explains why even with heterogeneity over K_0, we would still need to assume $\sigma_{\mu_0} > 0$ in order to match the empirical facts on the importance of the survivor component.

While we could increase the long-run contribution of survivors by tinkering with the ratio $\sigma_{\mu_0}/\sigma_{\mu_1}$, without adjustment costs the model cannot match the observed flatness in the path of the survivors' component. For any choice of σ_{μ_0} and σ_{μ_1}, it will always be the case that the survivors' component will exhibit an increasing path in the absence of adjustment costs. This is because without adjustment costs the reduction in uncertainty is more important than the persistence aspect associated

Table 2.6: Sensitivity Analysis for All Economy Cohort (NAC, I)

Param	*NAC*	*Range*	*CumEx*		*SurComp*	
			89	*98*	*89*	*98*
Data			17.2	59.6	69.5	68.5
Case NAC			17.0	52.1	42.7	58.5
α	0.56	0.44–0.71	15.3–26.3	46.1–60.7	39.6–47.4	56.0–60.8
β	0.956	0.931–0.976	18.7–15.3	52.3–50.5	45.6–39.6	59.8–56.4
w	11.8	9.3–13.8	17.0	52.1	42.7	58.5
W	666.3	586.3–766.3	20.2–18.7$^{(1)}$	51.4–52.8$^{(1)}$	39.4–45.6	55.1–59.7
σ_{μ_0}	0.18	0.09-0.36	17.8-17.5$^{(1)}$	53.1–46.3	57.3–29.5	76.6–38.8
σ_{μ_1}	0.21	0.12–0.39	5.94–46.0	26.6–74.6	20.9–72.4	33.3–79.8
σ	0.35	0.26–0.53	27.8–9.8	56.7–41.9	47.1–34.3	58.1–53.3
$\bar{\mu}$	3.38	3.13-3.58	17.0	52.1	42.7	58.5
I	114.3	73.9–163.9	20.4–18.2	57.5–48.1	48.4–36.4	62.5–53.1

Notes: *CumEx* is the cumulative exit rate (in %); *SurComp* is the survivor component (in %); *NAC* refers to calibrated and simulated values when adjustment costs are absent. $^{(1)}$: does not behave monotonically due to discretization.

with the learning mechanism Since the intensity of selection is mostly dependent on the reduction in uncertainty, then its relevance for firm growth decreases as firms age. Note also that the ratio $\sigma_{\mu_0}/\sigma_{\mu_1}$affects both the exit rate and the evolution of the cross-sectional firm size dispersion. If this ratio becomes too small, post-entry exit rates become excessively high and the size dispersion increases too fast. This is the reason why in the model without adjustment costs we cannot find a value for this ratio that attains the long-run contribution of survivors found in the data, and simultaneously matches the behavior of the cumulative exit rate and the evolution of cross-sectional size dispersion. Therefore, the value we select for this ratio is disciplined by the exit rates and the evolution of firm size dispersion in the cohort.

Third, to show that proportional adjustment costs are crucial for our model to fit the evidence on the survivors' contribution to growth in the cohort's average size, we perform a sensitivity analysis with respect to each parameter in the model. We take as benchmark the calibrated model with no adjustment cost for the overall economy cohort. In table 2.6 we present the sensitivity analysis with respect to every parameter except adjustment costs, and in table 2.7 we present the sensitivity analysis

Table 2.7: Sensitivity Analysis for All Economy Cohort (NAC, II)

Param	*NAC*	*Range*	*CumEx*		*SurComp*	
			89	*98*	*89*	*98*
Data			17.2	59.6	69.5	68.5
Case NAC			17.0	52.1	42.7	58.5
P^H	0	0.0–5.0	17.0–13.9	52.1–46.5	42.7–71.8	58.5–73.7
P^F	0	0.0–5.0	17.0–13.9	52.1–46.5	42.7–71.8	58.5–73.8
F^H	0	0.0–10.0	17.0–13.9	52.1–47.2	42.7–$^{(1)}$	58.5–$^{(1)}$
F^F	0	0.0–10.0	17.0–13.9	52.1–47.1	42.7–74.1	58.5–78.3
P	0	0.0–5.0	17.0–14.7	52.1–44.7	42.7–68.3	58.5–73.5
F	0	0.0–10.0	17.0–10.9	52.1–43.1	42.7–34.8	58.5–68.9

Notes: *CumEx* is the cumulative exit rate (in %); *SurComp* is the survivor component (in %); *NAC* refers to calibrated and simulated values when adjustment costs are absent; P^H corresponds to $P^H = P^{EN} = P^{SU}$, and similarly for F^H; P^F corresponds to $P^F = P^{EX} = P^{SD}$, and similarly for F^F; P corresponds to $P = P^H = P^F$, and similarly for F. $^{(1)}$: decreases and eventually becomes negative.

with respect to the adjustment cost parameters.

From table 2.6, we see that the model without adjustment costs cannot match simultaneously the exit rate and the contribution of survivors to growth in the cohort's average size even if we allow parameters to vary one by one from their benchmark values. The only parameters that significantly affect the survivor component are σ_{μ_0} and σ_{μ_1}. However, they also affect significantly the exit rates and the evolution of firm size dispersion (not shown in the table). In particular, we can see that reducing σ_{μ_0} and increasing σ_{μ_1} increases the survivor component, while maintaining its positive slope, and increases exit rates.

From table 2.7, we conclude that no other parameters besides proportional costs and fixed firing costs enable us to match both the exit behavior and the survivor component. The main effect of these costs is to put more emphasis on individual firm growth in the initial years of life, when selection is very intense. Another result that comes out of this sensitivity analysis is that P^H and P^F produce almost identical results. This should be expected when β is very high, as is the case here, since the incentives created by proportional hiring/entry and firing/exit costs differ only in the

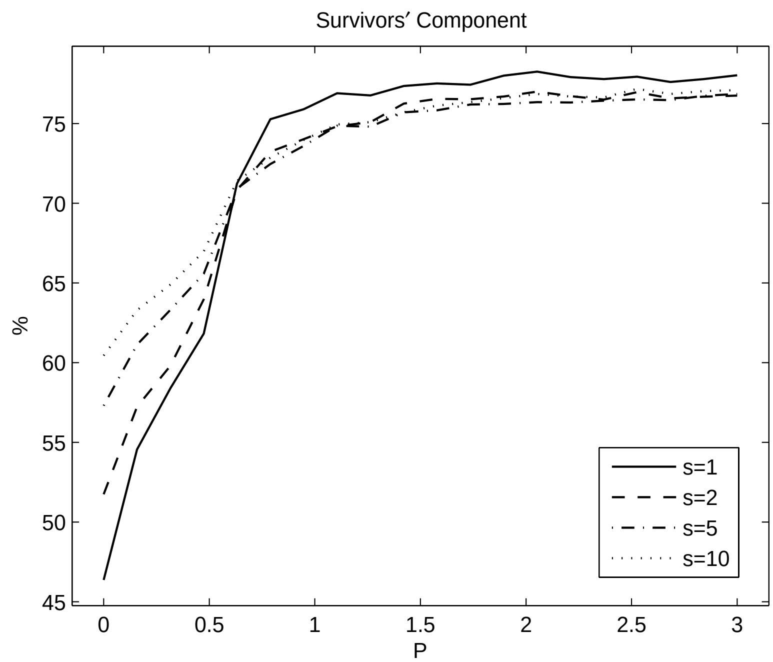

Figure 2.3: Proportional Adjustment Cost: $P = P^H = P^F$

displacement of timing by one period.[30]

The key element in replicating the evidence on the contribution of survivors is the proportional adjustment cost. To show that proportional adjustment costs are in fact the key factor, and that the fixed exit cost plays a secondary role in this respect, we present in figure 2.3 the impact of changes in $P = P^H = P^F$ on the survivor component, assuming the values in column 3 of table 2.4 for the remaining parameters, including $F^{EX} = 7$. We conclude that $F^{EX} > 0$ is not the crucial element to increase the value of the survivor component, especially in the initial post-entry periods, and that allowing for even a small value of P has a significant impact on the survivors' contribution, with a larger effect in the initial years of life.

F^{EX} is important to control both the fraction of firms that never enter the indus-

[30]This suggests that if we used data on firm dynamics to estimate adjustment cost parameters, we would find P^H and P^F to be nearly unidentifiable, since asymmetry between them only slightly affects the distribution of the employment growth rate. Therefore, our estimation would need to assume $P^H = P^F = P$.

try, $1-\Pr(S_0)$, and the amount of exit in the initial post-entry periods, $\Pr(D_\tau \mid S_{\tau-1})$. As F^{EX} increases the first fraction increases, since it is costly to actually exit the industry once the firm decides to enter, and the second probability decreases since there is a larger initial selection. Therefore, by changing F^{EX} we have more freedom to adjust $\sigma_{\mu_0}/\sigma_{\mu_1}$ to account for the long-run survivor component, without distorting much the initial exit rates. The proportional adjustment costs, besides increasing the long-run value of the survivor component, make its path flatter. This is what makes P crucial, since it puts more emphasis on firm growth, and not so much on firm exit in the first few post-entry periods.

2.6 Conclusion

In this chapter, we propose a model of Bayesian learning with linear and nonconvex adjustment costs, and show that most forms of adjustment costs will generate a bias towards firm growth. We present new evidence showing that firm size dynamics in the Portuguese economy are driven largely by growth of survivors rather than selection and pruning of small weak firms, and that there exist significant cross-sectoral differences between manufacturing and services in the contribution of survivors to average size growth. Our calibrations and simulations indicate that adjustment costs can generate plausible patterns of cohort size dynamics, and can explain differences between manufacturing and services. In particular, proportional adjustment costs and the fixed exit cost are the key parameters to explain the high contribution of survivors to growth in the cohorts' average size, and these costs are much larger in manufacturing than in services. Therefore, our theory is an alternative to financing constraints as an explanation for the significant growth of survivors, and seems to be better suited to explain cross industry differences.

This project could be extended in several ways. First, we could use the model to estimate adjustment cost parameters for particular economic sectors, namely manufacturing and services.[31] Second, we could attempt to distinguish our model empirically from theories based on financing constraints. We could also attempt to explain the evidence that, even for narrowly defined industries, there are significant differences in the firm size distribution between countries. Bartelsman *et al.* (2005, p. 26) interpret this as evidence that different institutional settings manifest themselves in different adjustment costs structures, which then have implications for firm size distributions:

> ... if certain administrative costs at entry are fixed, then the higher these costs (as in a number of European countries compared with the United States and the United Kingdom) the greater the disincentives for relatively small units to enter the market and then expand in the initial years. Likewise, post entry adjustments in employment may be hindered by tight hiring and firing restrictions and the latter are more restrictive in a number of European countries than in the United States.

Matching cross-country and cross-industry variation in the structure of adjustment costs with variation in the firm size distribution and cohort dynamics would give strong support for our hypothesis.

[31] Good references in this respect are Cooper and Haltiwanger (2005) and Cooper, Haltiwanger and Willis (2004).

Chapter 3

Does Structural Heterogeneity Matter for Job Flow Dynamics?

3.1 Introduction

The literature studying the process of labor reallocation across businesses is now large. It has emphasized both the magnitude and cyclicality of flows of job creation and job destruction, and their different patterns across heterogeneous units defined by characteristics such as size and age. It has also found the significant sectoral differences in the cyclical behavior of job reallocation (the overall flows of job creation and job destruction) and in the age and size composition of business units. In this chapter, we analyze whether the sectoral distribution of types of firms is an important element in understanding aggregate job flow dynamics and cross-sector differences in the patterns of job reallocation. We also gauge the empirical relevance of sectoral differences in firm types using firm level job flows data in the Portuguese economy.

Some of the stylized facts associated with gross reallocation activities were first laid out by Davis and Haltiwanger (1990, 1992) and Davis, Haltiwanger and Schuh (1996) with plant level information for the U.S. manufacturing sector from the Census Bureau's Longitudinal Research Database (LRD). One of their stylized facts that deserves special attention is the larger volatility of job destruction relative to job creation, meaning that job reallocation increases in recessions and decreases in expansions.[1] In their analysis of different patterns across types of firms, the authors find that the countercyclical reallocation of jobs is a phenomenon associated with older and larger plants. Davis *et al.* (1996) and Lane, Stevens and Burgess (1996) (using unemployment compensation data for the state of Maryland) also conclude that young and small firms have higher rates of job creation, job destruction and reallocation, and that young firms display larger growth rates than old firms. Along a similar line, based on the same dataset used by Lane *et al.* (1996), Burgess, Lane and Stevens (2000) emphasize the impact of the firm's lifecycle on the patterns of reallocation. They conclude that young and dying firms account for about a third of all job reallocation, and that job reallocation is more important among young firms whereas worker reallocation (the overall flows of workers over a set of jobs, including job-to-job transitions) is more important among mature firms.

This particular evidence led to the proposal of theories that could account for the level and countercyclical behavior of reallocation. Caballero (1992) explains the higher relative volatility of job destruction using an (S,s) model with asymmetric aggregate shocks, in the sense that contractions are shorter and more severe than expansions. In Caballero and Hammour (1994) and Mortensen and Pissarides (1994), job reallocation is countercyclical because job creation is time-consuming and costly, whereas job destruction is not and responds immediately to recessions. In Campbell and Fisher (2000), symmetric proportional costs of creation and destruction cause shrinking plants to be more sensitive to aggregate shocks than growing plants, so

[1] Given that job reallocation is the sum of job creation and job destruction, and that net employment growth equals their difference, countercyclical reallocation activity implies a higher cyclical variability of job destruction than job creation. This is so because job creation will tend to be procyclical and job destruction will tend to be countercyclical, and the higher volatility in job destruction will cause job destruction to be the dominating force in the volatility of job reallocation.

that job destruction becomes more volatile than job creation.

The view that reallocation is countercyclical was later questioned based on additional evidence for the U.S. and other countries. Boeri (1996) argues that the apparent countercyclical nature of job reallocation is the result of a selection bias against small and young firms in the LRD data, since only establishments with more than five workers are considered and the sample is renewed every five years. This sample bias, and the fact that the LRD data contains only information for manufacturing, causes an underestimation of gross job creation.

Foote (1998) confirms this argument in unemployment insurance based data for Michigan, where the higher volatility of job destruction in manufacturing does not hold in other sectors like services and retail trade. Foote then presents a simple model in which the cyclical properties of input reallocation are a function of the sector's trend growth rate, since the sign of trend employment growth indicates the margin on which firms are more active. In an expanding sector, firms are more active on the creation margin, especially in a favorable aggregate state, so that reallocation is procyclical. Foote tests his model empirically and finds that the model cannot fully account for the magnitude of sectoral differences between manufacturing and services in the relative volatility of job destruction and creation.

In an attempt to account for the unsatisfactory results of Foote (1998), Davis and Haltiwanger (1999) analyze if composition effects can explain cross-sector differences in the cyclical behavior of gross job flows. The authors conclude that, among four-digit manufacturing sectors, the relative volatility of job destruction is negatively affected by trend growth and positively affected by firm size and age. This suggests that the higher relative volatility of job destruction in manufacturing partly results from it being mostly composed of larger and older firms, while the opposite is true for the services sector.

This evidence suggests that different industries face different technological, demand, and legal constraints which determine the characteristics of the average firm in the sector, and the dynamics of firm entry and exit. These characteristics could then explain, in part, the behavior of gross job flows in the sector. In this paper, we investigate to what extent differences in the age and size distributions can explain differences in the cyclical properties of gross job flows across sectors.

In order to analyze the impact of age and size heterogeneities on job flows, following Bertola and Caballero (1990) and Foote (1998), we use an (S,s) descriptive model of the process of labor reallocation over the business cycle, and consider structural heterogeneity among the firms in a sector. We derive analytical expressions for job creation and job destruction and show how their dynamics depend on the structural parameters of the model. These expressions show us the parameters that might explain the marked differences between young and small firms, on the one hand, and large and old firms, on the other hand. We conclude that the distribution of types is relevant for the cyclical properties of aggregate gross job flows when (and only when) heterogeneous types display differences in their relative responsiveness to aggregate shocks versus idiosyncratic shocks.

Using Portuguese data, we find evidence that large and old firms are relatively more affected by aggregate shocks than small and young firms. This means that large and old firms affect aggregate job flow dynamics more than their employment share would suggest. Consequently, aggregate job reallocation tends to be more countercyclical than what would be expected from employment shares of each type of firm. We then analyze the cyclical properties of job reallocation in four Portuguese sectors: manufacturing, transportation, services, and retail trade. In each of the sectors, we identify characteristics of old and large firms that influence aggregate job flow dynamics.

This chapter is concerned with a topic similar to that of Campbell and Fisher (2004). Campbell and Fisher argue that the larger volatility of job reallocation among young firms relative to old firms is the result of an optimal substitution by young firms from structured jobs, with high productivity nut high adjustment costs, to unstructured jobs with lower productivity and lower adjustment costs. This substitution occurs because young firms face larger idiosyncratic shocks. Therefore, young firms benefit from increased flexibility that compensates for the decrease in productivity. This substitution, motivated by a higher idiosyncratic risk, in turn implies that young firms become more sensitive to aggregate shocks due to increased flexibility. In this chapter, instead of emphasizing standard deviations, we emphasize coefficients of variation of gross job flows. Because young firms display larger gross job flows on average, it is not surprising that the cyclical volatility of gross flows is also larger among young firms, since standard deviations are scale dependent. On the contrary, coefficients of variation are scale independent, and we show that they identify the importance of each type of firm to aggregate job flow dynamics. We also show that coefficients of variation of gross job flows are larger for old and larger firms, an indication that they are more important for aggregate cyclical behavior.

A final remark about why we should look at heterogeneity along the age and size dimensions is in Boeri (1996, p. 620):

> ... the variance of establishment-level employment changes in any industry is by and large the main component of job turnover [...]. Insofar as this heterogeneity plays an important role in the time variation of gross job flows [...] the identification of the main sources of this heterogeneity can also shed some light on the determinants of aggregate outcomes.

In section 3.2, we present a basic (S,s) model of employment adjustment in continuous-time and derive expressions for the implied job flows statistics. In section 3.3, we extend the model to consider structural heterogeneity across firms. In section 3.4, we present Portuguese evidence for the aggregate economy and for four sectors. We conclude in section 3.5.

3.2 (S,s) Model of Employment Adjustment

In this section, we analyze what a model of (S,s) adjustment predicts for the behavior of gross job flows as a function of the underlying structural parameters of the firm. By better understanding the determinants of job flows we can potentially explain the marked differences between young and old firms, and analyze whether age and size heterogeneities can account for the sectoral differences in the behavior of aggregate gross job flows. We use the same model employed by Bertola and Caballero (1990) to study the consumption of durable goods in the U.S., and later by Foote (1998) to study the cyclical volatility of gross job flows across U.S. sectors.

3.2.1 Basic Assumptions

We now briefly describe the main assumptions of the model. In appendix B.1, we present more detail about the model's assumptions and solution. To generate cyclical variability in the pattern of employment adjustment in the cross-section of firms, the model assumes that each plant's optimal employment level is driven by both an aggregate and an idiosyncratic component. Specifically, the stochastic process governing the frictionless log of employment has an aggregate drift and exhibits both aggregate and idiosyncratic uncertainty. The aggregate drift and uncertainty can be seen as sector specific, or, more generally, as involving both an economy wide and a sector specific aggregate component. For our analysis, only the net aggregate component would be relevant, since we are not interested in analyzing the co-movement across sectors in gross job flows.

In terms of notation, we use t to indicate time, and i to name the i-th firm. We assume that, in the absence of adjustment costs, the optimal level of log-employment for the firm is driven by an arithmetic Brownian motion,

$$e^*_{t,i} = a_t + \sigma_I w_{I,t,i},\ a_t = \mu t + \sigma_a w_{a,t}, \tag{3.1}$$

where $e^*_{t,i}$ is the time t frictionless log-level of employment for the i-th firm, a_t is the aggregate component of the stochastic process, $\sigma_I w_{I,t,i}$ is the idiosyncratic component, and $w_{I,t,i}$ and $w_{a,t}$ are independent Wiener processes. This implies that the growth rate of employment per unit of time has mean $\mu + \frac{1}{2}\sigma^2$, and fluctuates around the mean due to normally distributed aggregate and idiosyncratic shocks with mean zero and variances σ_a^2 and σ_I^2, respectively. Since for each firm the source of uncertainty is irrelevant, we can rewrite the above process as

$$e^*_{t,i} = \mu t + \sigma w_{t,i},\ \sigma = \left(\sigma_a^2 + \sigma_I^2\right)^{1/2}, \tag{3.2}$$

where $w_{t,i}$ is a Wiener process, and μ, σ are sector specific.[2] Note that $\sigma dw_{t,i}$ has exactly the same stochastic properties as $\sigma_I dw_{I,t,i} + \sigma_a dw_{a,t}$, so that the correlation coefficient between the shocks of different firms is σ_a^2/σ^2.[3]

Note that we could justify the structure of (3.1) and (3.2) by considering a simple model of frictionless employment determination in which the firm sets the marginal revenue product equal to the wage each period (assuming input and output markets are perfectly competitive). If the production function is Cobb-Douglas, subject to a productivity shock, that is, $F(E,\theta) = E^\alpha \theta$, and the firm observes θ before taking its decision, then optimal employment in logs would be given by,

$$e^* = (1-\alpha)^{-1}(\ln\alpha - \ln W) + (1-\alpha)^{-1}\ln\theta,$$

[2] Since the log of employment follows an arithmetic Brownian motion, the level of employment follows a geometric Brownian motion. Namely, using the algebra of Itô's calculus, if we denote the firm's employment level by $E^*_{t,i} = \exp\left(e^*_{t,i}\right)$, then

$$\frac{dE^*_{t,i}}{E^*_{t,i}} = \left(\mu + \frac{1}{2}\sigma^2\right)dt + \sigma dw_{t,i}.$$

The implication is that the mean (instantaneous) growth rate of employment is constant, and the standard deviation of employment level increments grows with the (employment) size of the firm.

[3] Of course, the actual path of $\{w_{t,i}\}$ must be chosen so that $\sigma w_{t,i} = \sigma_I w_{I,t,i} + \sigma_a w_{a,t}$, at all points in time.

where W denotes the wage. Now, if W is fixed over time, and the dynamic behavior of θ is characterized by

$$(1-\alpha)^{-1}\ln\theta_t = \mu t + y_t,\; y_t = y_{t-dt} + \varepsilon_{t,dt},\; \varepsilon_{t,dt} \sim N\left(0, \sigma^2 dt\right), \tag{3.3}$$

then log employment would follow (3.2). Note that equation (3.3) is valid whenever technology shocks follow a random walk, since we can rewrite (3.3) as

$$\ln\theta_t = \tilde{\mu} + \ln\theta_{t-dt} + \tilde{\varepsilon}_{t,dt},\; \tilde{\mu} \equiv (1-\alpha)\,\mu dt,\; \tilde{\varepsilon}_{t,dt} \sim N\left(0, (1-\alpha)^2\sigma^2 dt\right).$$

In general, the presence of firing and/or hiring costs implies that the firm will not choose to set the employment level equal to the frictionless optimal value. In the (S,s) model under consideration, the structure of those costs is as follows. Whenever firms decide to adjust their actual log-level of employment, $e_{t,i}$, they incur both a fixed and a proportional cost, so that the adjustment cost function is given by[4]

$$C(\Delta e) = \begin{cases} C_l + c_l\Delta e, & \text{if } \Delta e > 0, \\ C_u - c_u\Delta e, & \text{if } \Delta e < 0. \end{cases}$$

The existence of adjustment costs implies that, in deciding the optimal level of e, firms will balance the benefits of closely following e^* against the costs of doing so. Therefore, following Bertola and Caballero (1990), we define $z = e - e^*$ as the excess of current employment over the frictionless employment, and assume that firms choose an optimal path for z_t in order to minimize the expected present value of lost profits, due to the presence of adjustment costs, plus adjustment costs. Note that, in the absence of adjustment, z follows a Brownian motion process with drift $-\mu$ and

[4] Here we are assuming that proportional costs apply to percentage changes in employment. The alternative case, where employment follows a geometric Brownian motion and proportional costs apply to level changes, would not allow us to get an optimal policy so simple as the one for the case we are considering.

standard deviation σ.

Harrison, Sellke and Taylor (1983) have shown that the optimal policy is characterized by four parameters (L, l, u, U), such that $z \in (L, U)$, and

$$\Delta e = \begin{cases} l - L > 0, & \text{if } z = L, \\ u - U < 0, & \text{if } z = U. \end{cases}$$

This means that there is an inaction region between L and U, and that whenever z hits L employment increases by $\Delta e = l - L > 0$, and whenever z hits U employment decreases by $\Delta e = u - U < 0$. In this policy rule there is both infrequent adjustment, in that the employment gap must be high enough so that the marginal benefit of adjustment equals the marginal cost of adjusting, and lumpy adjustment, in that adjustment must be high enough so that the (discrete) benefit of adjusting equals the (discrete) costs of adjusting. The first aspect of adjustment, intermittence, is a function of the proportional and fixed adjustment costs, whereas the second aspect of adjustment, lumpiness, is a function of the fixed adjustment costs.[5] Both of these features are easily observed in the micro data on employment adjustment.[6]

In this model, there is an ergodic distribution for the firm location over the state variable z (see appendix B.1). However, the cross-sectional distribution over z will not converge in the face of aggregate shocks. This is due to the fact that in (3.1) there is an aggregate uncertainty component, $\sigma_a w_{a,t}$, which is common to all firms in the sector. This implies that when there is a positive aggregate shock, $dw_{a,t} > 0$, all firms in the sector will face the same decrease in $z_{t,i}$, so that the cross-sectional distribution over z moves in a parallel way to the left by $\sigma_a dw_{a,t}$. However, following Foote (1998), we solve for job flows statistics using the firm's ergodic distribution as an approximation

[5] Note that if $C_u = C_l = 0$, then we will have $L = l < 0$, and $u = U > 0$, so that there is no lumpy adjustment. On the other hand, if $c_u = c_l = 0$, then we will have $l = u = 0$, so that firms fully adjust.

[6] See for example Hamermesh and Pfann (1996) and Caballero, Engel and Haltiwanger (1997).

for the time varying cross-sectional distribution, even if in a simulation of the model it would be more accurate to use the second. This approximation allows us to obtain analytical expressions for the impact of the underlying structural parameters on the long-run statistical properties of job flows.

3.2.2 Job Flows Statistics

In order to obtain expressions for job flows, we use a random walk discrete-time approximation to the Brownian motion process (see appendix B.1). Similarly to Bertola and Caballero (1990) and Dixit (1991a), we consider the case of arbitrarily small changes, so that

$$a_{t+dt} = \begin{cases} a_t + da, & \text{with probability } p_a, \\ a_t - da, & \text{with probability } q_a = 1 - p_a, \end{cases}$$

where $da = \sigma_a \sqrt{dt}$, $p_a = \frac{1}{2}\left(1 + \mu \frac{dt}{da}\right)$, and dt, da represent, respectively, the duration of the discrete-time period and the length of each step taken by a in each period.

Without any adjustment, the employment gap for firm i evolves as

$$z_{t+dt,i} = \begin{cases} z_{t,i} + dz, & \text{with probability } p_x, \\ z_{t,i} - dz, & \text{with probability } q_x = 1 - p_x, \end{cases}$$

where $dz = \sigma\sqrt{dt}$, and the right expression for p_x depends on whether we are conditioning or not on the realization of the aggregate state.

$$p_x = \begin{cases} p_{z|b} = \frac{1}{2}\left(1 - \frac{da}{dz}\right), & \text{if } da_{t+dt} = da, \\ p_{z|r} = \frac{1}{2}\left(1 + \frac{da}{dz}\right), & \text{if } da_{t+dt} = -da, \\ p_z = \frac{1}{2}\left(1 - \mu\frac{dt}{dz}\right) = p_a p_{z|b} + q_a p_{z|r}, & \text{unconditionally,} \end{cases}$$

where b stands for a boom, and r stands for a recession.[7] Therefore, when there is a boom in the sector, the probability that firm i's employment gap will decrease is higher than the probability that the employment gap will increase, because most probably optimal employment will increase. Note, however, that the impact of the aggregate shock on the firm-level probability is increasing in da/dz, which in turn depends on the relative importance of aggregate and idiosyncratic shocks for firm-level volatility. If σ_a/σ is low, then da/dz is low and aggregate shocks will have a small impact on the direction of firm-level adjustment.

We now use the above stochastic process in order to derive appropriate expressions for the job creation rate, the job destruction rate and their standard deviations. We will start by deriving the discrete-time expressions, and then show that, as $dt \to 0$ and $dz \to 0$, with $dz = \sigma\sqrt{dt}$, these expressions converge to appropriate expressions in continuous-time. Foote (1998) derived correct discrete-time expressions for the job flows statistics, but without making clear the dependence of those expressions on the structural parameters. Also, he did not derive the limiting continuous-time expressions, which clarify that structural dependence.

In lemma 1 we prove that the discrete-time ergodic distribution converges in distribution to the continuous-time ergodic distribution.

Lemma 1 *Let $F_d(z; dz)$, $F_c(z)$ be the discrete-time and continuous-time ergodic distributions of z implied by the (S,s) model under consideration. Then F_d converges in distribution to F_c. That is*

$$F_d(z; dz) \to F_c(z), \text{ as } dz \downarrow 0, \text{ for all } z \in \mathbb{R}.$$

[7] Similarly to what is done in appendix B.1, it is possible to prove that this binomial random walk mimics the main features of the continuous Brownian motion process, namely, $E(\Delta a_{t+dt}) = \mu dt$, $Var(\Delta a_{t+dt}) = \sigma_a^2 dt$, $E(\Delta z_{t+\Delta t}) = -\mu dt$, $Var(\Delta z_{t+dt}) = \sigma^2 dt$. It is also true that as $dt \to 0$, the above random walk process converges, in some appropriate way, to the Brownian motion process with aggregate shocks in (3.1) and (3.2).

Proof. See appendix B.2. ■

Now, we use this property to obtain continuous-time expressions for gross job flows statistics. The results are in proposition 2.

Proposition 2 *In the (S,s) model under consideration, assuming that the ergodic distribution provides a good approximation to the cross-sectional distribution for purposes of characterizing the long-run stochastic properties of the model, the aggregate job creation rate ($E(JC)$), the aggregate job destruction rate ($E(JD)$), the aggregate variance of job destruction ($Var(JD)$), and the aggregate variance of job creation ($Var(JC)$) have the following continuous-time expressions:*

$$E(JC)_c = f_c'(L)\frac{\sigma^2}{2}(l-L)\,,\; E(JD)_c = -f_c'(U)\frac{\sigma^2}{2}(U-u)\,,$$

$$Var(JC)_c = [E(JC)_c]^2\frac{\sigma_a^2}{\sigma^2},\; Var(JD)_c = [E(JD)_c]^2\frac{\sigma_a^2}{\sigma^2}.$$

Proof. See appendix B.2. ■

Before we derive more explicit expressions for the continuous-time version of these statistics, we describe as an example how $E(JC)$ is calculated. Using the discrete-time approximation, the job creation rate is obtained by

$$E(JC)_d = \frac{f_d(L_z+dz;dz)\,q_z\,(l_z-L_z)}{dt},$$

that is, the expected job creation rate equals the per unit of time fraction of firms located at the job creation border, $\frac{f_d(L_z+dz;dz)}{dt}$, times the probability that each of those firms will receive a positive shock to employment, q_z, times the amount of (proportional) adjustment that a positive shock will generate, l_z-L_z.[8] The expression for the job destruction rate has a similar interpretation.

[8]Note that the return and trigger points are defined in terms of logs, so that l_z-L_z represents the proportional increase in employment.

In corollary 3, we present closed-form expressions for $E\left(JC\right)_c$ and $E\left(JD\right)_c$.

Corollary 3 *In the (S,s) model under consideration, assuming that the ergodic distribution provides a good approximation to the cross-sectional distribution for purposes of characterizing the long-run stochastic properties of the model, the aggregate job creation rate ($E\left(JC\right)$), the aggregate job destruction rate ($E\left(JD\right)$) have the following continuous-time expressions:*

(a) $\mu \neq 0$:

$$
\begin{aligned}
E\left(JC\right)_c &= \mu\left[1-\frac{(U-u)\left(\exp\left\{2\frac{\mu}{\sigma^2}l\right\}-\exp\left\{2\frac{\mu}{\sigma^2}L\right\}\right)}{(l-L)\left(\exp\left\{2\frac{\mu}{\sigma^2}U\right\}-\exp\left\{2\frac{\mu}{\sigma^2}u\right\}\right)}\right]^{-1}, \\
E\left(JD\right)_c &= \mu\left[\frac{(l-L)\left(\exp\left\{2\frac{\mu}{\sigma^2}U\right\}-\exp\left\{2\frac{\mu}{\sigma^2}u\right\}\right)}{(U-u)\left(\exp\left\{2\frac{\mu}{\sigma^2}l\right\}-\exp\left\{2\frac{\mu}{\sigma^2}L\right\}\right)}-1\right]^{-1}.
\end{aligned}
$$

(b) $\mu = 0$:

$$
\begin{aligned}
E\left(JC\right)_c &= \frac{\sigma^2}{(U+u-l-L)}, \\
E\left(JD\right)_c &= \frac{\sigma^2}{(U+u-l-L)}.
\end{aligned}
$$

Proof. These expressions follow from proposition 2 and the expressions for f_c in appendix B.1. ■

We now make some comments on proposition 2 and corollary 3. First, we should mention that $E\left(JC\right)_d$, $E\left(JD\right)_d$ (in the appendix) are similar to the corresponding expressions in Foote (1998), even though his statistics are not invariant to the step size assumed for the random walk approximation. Second, the continuous-time (approximate) expressions make clear the contribution of underlying parameters for aggregate gross job flows behavior. By examining the case in which $\mu = 0$, we can see that adjustment costs asymmetries have a minor effect on relative job flows. The main factors affecting the relative magnitude of $E\left(JC\right)$ and $E(JD)$ are μ and σ^2.

If $\mu > 0$, then $E(JC) > E(JD)$, and the difference will be higher the smaller is σ^2. This is because a positive trend growth rate causes firms to be bunched at job creation border (a high value for $f_c'(L)$, and a low value for $-f_c'(U)$), and the lower is σ^2 the higher will be this tendency, since there is less dispersion in employment movements. Because the standard deviations of job flows are a linear function of their means, these two factors are also the main source of asymmetries between the volatility of job creation and the volatility of job destruction. This is precisely the main insight of Foote (1998).

In fact, the continuous-time version of the main result in Foote follows from proposition 2:

$$\frac{std\left(JD\right)_c}{std\left(JC\right)_c} = \frac{E\left(JD\right)_c}{E\left(JC\right)_c},$$

that is, the relative volatility of job destruction is just a linear function of the ratio of the means of job destruction and job creation. We can see that, in continuous-time, there is no need to find a proxy for $\ln\left(q_z/p_z\right)$, as Foote did in the implementation of his regression, and which caused an increase in the uncertainty associated with the estimation of the parameter associated with $\ln\left(E\left(JD\right)/E\left(JC\right)\right)$.

A third comment is that, in continuous-time, the coefficient of variation for job creation or for job destruction depends only on the weight of aggregate uncertainty in total uncertainty:[9]

$$\frac{std\left(JC\right)_c}{E\left(JC\right)_c} = \frac{std\left(JD\right)_c}{E\left(JD\right)_c} = \frac{\sigma_a}{\sigma}.$$

We can also prove, using corollary 3, that the net growth rate of employment is just equal to the trend growth rate, that is

$$E\left(NET\right) = E\left(JC\right) - E\left(JD\right) = \mu.$$

[9] Note that these expressions explain the result in table 1 of Caballero (1992). In fact, unless we abandon the assumption of symmetric shocks, as he does in table 2, the coefficient of variation will be identical for JC and JD.

Therefore, in this model the net growth rate and the coefficient of variation for job flows are sufficient to reveal important deep parameters characterizing the shocks facing firms.

Before moving to the next section we should just mention that this model can be realistically calibrated. We give a brief example. First note the dependence of the parameters on the units of time ($[T]$), cost ($[\$]$) and length of adjustment ($[L]$) (see Dixit 1991b): σ^2 and σ_a^2 are in $[L]^2 [T]^{-1}$, μ is in $[L][T]^{-1}$, C_l and C_u are in $[\$]$, c_l and c_u are in $[\$][L]^{-1}$, b is in $[\$][L]^{-2}[T]^{-1}$, and ρ is in $[T]^{-1}$. Given that z is in $[L]$, it can easily be proven that (L, l, u, U) are in $[L]$ and that f_c is in $[L]^{-1}$. We consider $[T]$ to be one year, $[\$]$ to be dollars, and $[L]$ to be units of percentage. Then, consider the following calibration:

$$\rho = 0.02,\ \mu = -3,\ \sigma^2 = 100,\ \sigma_a^2 = 9,\ b = 100,\ C_u = C_l = 1000,\ c_u = c_l = 100. \quad (3.4)$$

We can interpret these values as follows: the annual discount factor is 0.98; the trend growth rate of frictionless employment is -3% *per year*; the volatility of optimal employment corresponds to a standard deviation of 10% *per year*; the contribution of aggregate shocks to the uncertainty a firm faces is 30%; a gap between actual and optimal employment of 10% for one year costs the firm $\$5,000$ of lost profits in that year; the fixed cost of adjustment is symmetric and amounts to $\$1,000$; and the proportional cost of adjustment is symmetric and amounts to \$100 for each percentage point variation in the number of workers.

With this calibration we would get $L = -12.3597$, $l = -3.1014$, $u = 0.79212$, $U = 10.874$. This means that, for example, the firm would only decide to hire new workers if its employment level fell below the target by roughly 12%, and that such a firm would increase the work force by roughly 9%. The job flows statistics implied by this example would be $E(JC) = 2.4$, $E(JD) = 5.4$, $std(JC) = 0.72$, $std(JD) = 1.62$,

$std(JD)/std(JC) = 2.3$. Therefore, this example would be consistent with Foote's argument that a negative trend growth rate causes firms to be concentrated near the job destruction margin, causing a higher relative volatility of JD.

3.3 (S,s) Model with Structural Heterogeneity

We now extend the previous model to the case where there is heterogeneity among the firms in a given sector. To simplify the analysis we consider the simplest case where there are two types of firms, with p being the proportion of employment accounted for by firms of type 1. We have in mind both the distinction of young *versus* old firms and of small *versus* large firms. In the first case, the appropriate approach would be to consider that the trend growth rate and the uncertainty vary smoothly with the age of the firm. However, this would complicate the solution of the model, by making the trigger and return points age-dependent. This model could only be solved numerically. To avoid these complications, we consider that the discount rate is high enough so that young and old firms can be considered to be facing different structural parameters μ, σ_a, and σ_I (and, eventually, C_l, C_u, c_l, and c_u). Given this, in proposition 4, we show that aggregate statistics are just a weighted average of type specific statistics.[10] Then, in corollary 5, we derive the general expression for the correlation coefficient of job reallocation and net job growth.

Proposition 4 *In the (S,s) model under consideration, when there are two types of firms in a given sector, with p the fraction of sector employment accounted for by firms of type 1, the aggregate job creation rate ($E(JC)$), the aggregate job destruction rate ($E(JD)$), the standard deviation of aggregate job destruction ($std(JD)$), and the standard deviation of aggregate job creation ($std(JC)$) are weighted averages of the*

[10] Note that even though the standard deviations are also a weighted average of the type specific standard deviations, the same does not occur with the variance, since $Cov(JD_1, JD_2) \neq 0$.

type-specific corresponding magnitudes, that is:

$$E(JC) = E(JC)_1 p + E(JC)_2 (1-p),$$
$$E(JD) = E(JD)_1 p + E(JD)_2 (1-p),$$

$$std(JD) = std(JD)_1 p + std(JD)_2 (1-p),$$
$$std(JC) = std(JC)_1 p + std(JC)_2 (1-p).$$

Proof. See appendix B.2. ■

Corollary 5 *In the (S,s) model under consideration, when there are two types of firms in a given sector, with p the fraction of sector employment accounted for by firms of type 1, the cyclical behavior of job gross flows satisfies:*

$$\frac{std(JD)_c}{std(JC)_c} = \frac{E(JD)_{c,1}\left(\frac{\sigma_a}{\sigma}\right)_1 p + E(JD)_{c,2}\left(\frac{\sigma_a}{\sigma}\right)_2 (1-p)}{E(JC)_{c,1}\left(\frac{\sigma_a}{\sigma}\right)_1 p + E(JC)_{c,2}\left(\frac{\sigma_a}{\sigma}\right)_2 (1-p)}. \tag{3.5}$$

$$\begin{aligned} Cov(Rea, Net)_c = & \left[E(Rea)_{c,1}\left(\frac{\sigma_a}{\sigma}\right)_1 p + E(Rea)_{c,2}\left(\frac{\sigma_a}{\sigma}\right)_2 (1-p)\right] \times \\ & \left[E(Net)_{c,1}\left(\frac{\sigma_a}{\sigma}\right)_1 p + E(Net)_{c,2}\left(\frac{\sigma_a}{\sigma}\right)_2 (1-p)\right] \end{aligned} \tag{3.6}$$

Proof. See appendix B.2. ■

Some comments on these results follow. First, from proposition 4, the mean and standard deviation of aggregate gross job flows are employment-weighted averages of the respective measures in each type of firm. Therefore, industries characterized by larger shares of young and smaller firms will be characterized by larger rates of job reallocation and by higher standard deviations of gross job creation and destruction. Second, from corollary 5, $\ln\left(\frac{std(JD)}{std(JC)}\right)$ is no longer a linear function of $\ln\left(\frac{E(JD)}{E(JC)}\right)$, when $\left(\frac{\sigma_a}{\sigma}\right)_1 \neq \left(\frac{\sigma_a}{\sigma}\right)_2$, that is, when the two types of firms have different degrees of relative sensitivity to aggregate shocks. In particular, firm types that are relatively more sensitive to aggregate uncertainty have disproportionate impact on both the relative volatility of job destruction and creation and on the covariance between job reallocation and net job creation. This latter covariance is composed of two parts, one associated with Rea the other with Net, and in each component firms more sensitive to aggregate shocks have weight disproportionate to its employment share. In the next section, we analyze empirically how different firm types affect the two components of $Cov\,(Rea, Net)$, which we use as a summary measure of the cyclical behavior of job flows.

3.4 Structural Heterogeneity and Job Flow Dynamics

In this section, we analyze the influence of different age and size classes of firms on aggregate gross job flows dynamics. We use *Quadros de Pessoal*, a database containing information on all Portuguese firms with paid employees. This is an administrative dataset which collects information about the firm, its establishments and its workers. All economic sectors except public administration are included. The panel we have access to covers the period 1985-2000. Information refers to March through 1993, and to October since the reformulation of the survey in 1994. On average the dataset contains 250,000 firms, 300,000 establishments, and 2,500,000 workers in each year. In the following, we consider the period 1987-1999, in order to minimize false entries and exits.

In table 3.1, we present some properties of firm job flows for the overall economy and for four one-digit sectors, manufacturing (*Manu*), transportation (*Tran*), services

Table 3.1: Firms Job Flows: 1987-1999

Sector	*EmpSh*	*JC*	*JD*	*Net*	*NetC*	$\frac{SD(JD)}{SD(JC)}$	$C(Net, Rea)$	$C(JD, JC)$
All		13.0	10.4	2.6	1.0	0.93	0.09	−0.59
Manu	40.3	10.2	9.9	0.2	−0.1	0.98	0.03	−0.68
Tran	7.5	7.5	7.3	0.2	−1.5	1.43	−0.55	0.84
Serv	18.0	17.2	10.8	6.4	3.0	0.57	0.56	−0.51
Reta	9.8	16.6	11.1	5.6	3.0	0.57	0.54	−0.38

Notes: *EmpSh* is the employment share (in %); *JC* and *JD* are the mean gross job creation and job destruction rates (in %); *Net* and *NetC* are the net job creation rate (in %) among all firms and among continuing firms, respectively; $SD(JD)$ and $SD(JC)$ are the standard deviations of *JD* and *JC*; $C(Net, Rea)$ is the correlation between *Net* and the rate of reallocation $(JC + JD)$; $C(JD, JC)$ is the correlation between *JD* and *JC*.

(*Serv*) and retail trade (*Reta*).[11] In general, gross job flows are quite large in all sectors, but net job creation and the cyclical behavior of job flows vary significantly across sectors. Consistent with Foote (1998), sectors with larger net job creation (*Serv* and *Reta*) exhibit a larger relative volatility of job creation and more procyclical reallocation.[12]

In table 3.2, we provide information on cross-sector differences in the age and size composition of firms, and on the cyclical properties of gross job flows across age and size classes. The dynamics of gross job flows over the business cycle are quite distinct across firms of different age and size. Young firms, and to a lesser extent small firms, grow faster and exhibit a larger relative volatility of job creation. More importantly, for young and small firms, fluctuations in gross job flows tend to be less determined by aggregate shocks, and more by idiosyncratic shocks as evidenced by their relatively low coefficient of variation for job flows. This is consistent with a theory of learning and growth, where firms in the first few years of life are learning and

[11] In order to obtain equivalent one-digit SIC87 sectors, we use the following correspondence in terms of CAE Rev. 1 codes : manufacturing$(= 3)$, transportation and public utilities$(= 7 + 4)$, services$(= 6.3 + 8.3.2 + 8.3.3 + 9.2 + 9.3 + 9.4 + 9.5)$, and retail trade$(= 6.2)$.

[12] The positive value for the correlation between *JC* and *JD* for *Tran* can only be explained by the dominance of old and large firms in this sector, most of them controlled by the state, at the least for the largest part of the period under consideration. Therefore, we should be careful when analyzing the results for this sector.

Table 3.2: Sectoral Heterogeneity

Class	$\frac{SD(JD)}{SD(JC)}$ *All*	*NetC* *All*	$CV(JC)$ *All*	$CV(JD)$ *All*	*EmpSh* *All*	*Manu*	*Tran*	*Serv*	*Reta*
Size									
$[1-3]$	0.42	1.0	0.11	0.07	7.6	2.6	3.0	13.3	21.5
$[4-9]$	0.35	2.1	0.15	0.08	13.7	7.6	4.7	20.5	29.5
$[10-24]$	0.37	2.3	0.17	0.09	14.7	13.0	5.7	17.2	17.3
$[25-99]$	0.62	1.9	0.16	0.12	22.5	28.1	10.1	19.4	15.7
$[100-\cdot]$	1.19	-0.2	0.19	0.23	41.5	48.7	76.5	29.6	16.0
Age									
$[0-3]$	0.38	9.2	0.10	0.13	8.0	6.0	2.7	10.8	11.3
$[4-9]$	0.62	4.0	0.17	0.08	15.7	13.7	5.5	19.2	20.2
$[10-24]$	0.68	1.3	0.13	0.07	25.0	24.9	8.5	29.7	31.8
$[25-\cdot]$	1.68	-1.4	0.20	0.20	51.3	55.4	83.3	40.3	36.7

Notes: $SD(JC)$ and $SD(JD)$ are the standard deviations of job creation (JC) and job destruction (JD); *NetC* is the net job creation rate (in %) among continuing firms; $CV(JC)$ and $CV(JD)$ are the coefficients of variation of JC and JD; *EmpSh* is the employment share of each class (in %).

adjusting accordingly to their ex-post optimal scale. This suggests that the influence of old and large firms on the cyclical variation of aggregate gross job flows is larger than their influence on average aggregate gross job flows, which is already large given their employment shares. Table 3.2 also shows that, because of technological and institutional restrictions, the size and age distributions are quite different across the four sectors. In particular, *Tran* and *Manu* have a larger proportion of old and large firms, whereas *Reta* and *Serv* have a larger proportion of young and small firms. We now analyze the impact of these differences on the cyclical properties of job flows in each sector.

In order to apply the theoretical results in sections 3.2 and 3.3 in the empirical analysis, we need to recognize that our measure of the relative sensitivity to aggregate and idiosyncratic shocks depends on whether it is obtained from JC or from JD. Therefore, we adopt the following strategy. We rewrite the two components of the ratio of volatilities in (3.5) as a weighted sum of average JC and JD across all classes of firms. Let n be the number of types of firms ($i = 1, \ldots, n$). Then, we compute the

Table 3.3: Overall Economy: Heterogeneity and Job Flow Dynamics

Class	Net	JD	JC	$CV(JD)$	$CV(JC)$	p	w_{jd}	w_{jc}	$\frac{SD(JD)}{SD(JC)}$
Size				*Relative Volatility:* $(0.93, 0.73)$					
$[1-3]$	9.6	18.3	28.0	0.07	0.11	7.6	3.6	4.9	0.42
$[4-9]$	6.9	13.3	20.2	0.08	0.15	13.7	7.2	12.0	0.35
$[10-24]$	4.3	11.6	15.9	0.09	0.17	14.7	8.4	14.7	0.37
$[25-99]$	2.1	9.8	11.9	0.12	0.16	22.5	18.1	21.2	0.62
$[100-\cdot]$	−0.1	7.8	7.7	0.23	0.19	41.5	62.6	47.1	1.19
All	2.6	10.4	13.0	0.15	0.17	100.0	100.0	100.0	0.93
Age				*Relative Volatility:* $(0.83, 0.78)$					
$[0-3]$	9.2	6.3	15.4	0.14	0.12	8.0	7.3	5.9	0.48
$[4-9]$	4.0	7.6	11.6	0.11	0.17	15.7	10.8	15.3	0.43
$[10-24]$	1.3	6.8	8.1	0.09	0.13	25.0	14.5	19.4	0.58
$[25-\cdot]$	−1.4	6.6	5.2	0.21	0.20	51.3	67.4	59.5	1.34
All	1.0	6.6	7.6	0.16	0.17	100.0	100.0	100.0	0.83

Notes: For size classes values refer to all firms, whereas for age classes values refer to continuing firms. JD, JC, and Net are the mean values of job destruction, job creation, and net job creation (in %); $CV(JD)$ and $CV(JC)$ is the coefficient of variation of JD and JC (see (3.7c) and (3.7d)); p is the employment share; w_{jd} and w_{jc} are the weighted shares in JD and JC dynamics (see (3.7b)); $SD(JD)$ and $SD(JC)$ is the standard deviation of JD and JC; in (z, y) z is the observed relative volatility of job destruction, and y is a version of z corrected for sectoral heterogeneity.

two terms in (3.5) as follows

$$std\left(X\right) = CV\left(X\right) \sum\nolimits_{i=1}^{n} E\left(X\right)_i w_{X,i},\ X = JD, JC \tag{3.7a}$$

$$w_{X,i} = \frac{CV\left(X\right)_i}{CV\left(X\right)} p_i,\ X = JD, JC \tag{3.7b}$$

$$CV\left(X\right)_i = \frac{std\left(X\right)_i}{E\left(X\right)_i},,\ X = JD, JC \tag{3.7c}$$

$$CV\left(X\right) = \sum\nolimits_{i=1}^{n} CV\left(X\right)_i p_i,\ X = JD, JC, \tag{3.7d}$$

where p_i $(\sum_{i=1}^{n} p_i = 1)$ is the employment share of each class.

In tables 3.3 to 3.7, we analyze how each type of firm contributes to the relative volatility of job destruction and job creation, both for the overall economy and for the four sectors. For the overall economy, we see that young and small firms have larger net job creation rates (column 2), and have a smaller relative sensitivity to aggregate shocks (columns 5 and 6). This implies that in the determination of aggregate job flow dynamics old and large firms have a larger weight than what would be expected from its (already large) employment share (compare column 7 with columns 8 and 9). Because young and small firms tend to have procyclical reallocation and large and old firms tend to exhibit countercyclical reallocation (see column 10), then the larger sensitivity of large and old firms to aggregate shocks implies that aggregate reallocation activity is more countercyclical than what would be if firms were equally sensitive to aggregate shocks. In rows 2 and 9 we show that if instead of using the weights $w_{JC,i}$ and $w_{JD,i}$ we use the employment share of each type of firm, p_i, then the ratio of volatilities of job destruction and job creation decreases from 0.93 to 0.73 (size classes) and from 0.83 to 0.78 (age classes).[13]

The same analysis can be done at the one-digit sectoral level. To avoid repetition, we stress the main cross-sector differences, and how they are determined by specific heterogeneities in each sector. The results for *Manu* are in table 3.4. Relative to other sectors, *Manu* displays small differences in the relative sensitivity to aggregate shocks in large/old firms versus small/young firms. Therefore, the employment share of each class is close to its respective weight in job flow dynamics. However, there is an interesting asymmetry for large manufacturing firms. Because they are more active on the destruction margin, we have the interesting result $w_{JD} > p > w_{JC}$, which implies that, for size classes, reallocation would be slightly more countercyclical if all types were equally sensitive to aggregate shocks. For age classes, results are similar to those of the overall economy.

Table 3.5 contains the results for *Tran*. This sector is dominated by old and large firms with much larger relative sensitivities to aggregate shocks than young and small

[13] Note that for age classes we consider only job flows among continuing firms. We do this because the job creation rate for the class $[0-3]$ years is very large, a consequence of the job creation due to births. When all firms are considered, we obtain a completely different result: if all firms were equally sensitive to aggregate shocks, aggregate job flows would be extremely procyclical. This fact shows that even though job creation and job destruction rates are very large for the class $[0-3]$, their impact on aggregate job reallocation is limited because these flows are not very sensitive to aggregate conditions.

Table 3.4: Manufacturing: Heterogeneity and Job Flow Dynamics

Class	*Net*	*JD*	*JC*	$CV(JD)$	$CV(JC)$	p	w_{jd}	w_{jc}	$\frac{SD(JD)}{SD(JC)}$
Size				*Relative Volatility:* $(0.98, 1.01)$					
$[1-3]$	9.1	18.6	27.7	0.07	0.15	2.6	1.0	2.0	0.31
$[4-9]$	6.9	14.1	21.0	0.07	0.22	7.6	3.2	8.5	0.23
$[10-24]$	3.6	12.0	15.6	0.11	0.26	13.0	8.3	17.6	0.33
$[25-99]$	0.8	9.7	10.5	0.16	0.27	28.1	26.0	38.6	0.56
$[100-\cdot]$	-2.1	8.2	6.1	0.22	0.13	48.7	61.5	33.3	2.23
Manu	0.2	9.9	10.2	0.18	0.20	100.0	100.0	100.0	0.98
Age				*Relative Volatility:* $(1.21, 1.02)$					
$[0-3]$	11.5	4.9	16.4	0.20	0.16	6.0	6.1	5.0	0.38
$[4-9]$	4.2	6.2	10.3	0.15	0.22	13.7	11.0	16.0	0.42
$[10-24]$	1.0	5.7	6.7	0.15	0.19	24.9	19.8	25.6	0.67
$[25-\cdot]$	-2.8	6.8	4.1	0.22	0.18	55.4	63.0	53.4	2.03
Manu	-0.1	6.3	6.2	0.19	0.19	100.0	100.0	100.0	1.21

Notes: See table 3.3.

Table 3.5: Transportation: Heterogeneity and Job Flow Dynamics

Class	*Net*	*JD*	*JC*	$CV(JD)$	$CV(JC)$	p	w_{jd}	w_{jc}	$\frac{SD(JD)}{SD(JC)}$
Size				*Relative Volatility:* $(1.43, 0.87)$					
$[1-3]$	13.7	16.0	29.7	0.08	0.10	3.0	0.3	0.4	0.44
$[4-9]$	10.6	12.1	22.7	0.12	0.10	4.7	0.8	0.7	0.64
$[10-24]$	6.2	10.7	16.8	0.20	0.13	5.7	1.5	1.1	1.00
$[25-99]$	3.2	8.3	11.6	0.35	0.20	10.1	4.9	3.0	1.27
$[100-\cdot]$	-1.7	6.4	4.6	0.88	0.84	76.5	92.4	94.8	1.44
Tran	0.2	7.3	7.5	0.73	0.68	100.0	100.0	100.0	1.43
Age				*Relative Volatility:* $(1.89, 1.51)$					
$[0-3]$	13.7	4.9	18.6	0.21	0.14	2.7	0.9	0.8	0.40
$[4-9]$	6.8	7.0	13.8	0.25	0.19	5.5	2.2	2.2	0.69
$[10-24]$	4.4	6.5	10.9	0.25	0.21	8.5	3.3	3.7	0.72
$[25-\cdot]$	-3.3	5.5	2.2	0.72	0.53	83.3	93.6	93.3	3.35
Tran	-1.5	5.5	4.0	0.64	0.47	100.0	100.0	100.0	1.89

Notes: See table 3.3.

Table 3.6: Services: Heterogeneity and Job Flow Dynamics

Class	Net	JD	JC	$CV(JD)$	$CV(JC)$	p	w_{jd}	w_{jc}	$\frac{SD(JD)}{SD(JC)}$
Size				*Relative*	*Volatility:*	$(0.57, 0.61)$			
$[1-3]$	9.7	18.6	28.3	0.06	0.07	13.3	5.8	6.3	0.58
$[4-9]$	6.9	12.7	19.6	0.09	0.10	20.5	12.6	13.6	0.58
$[10-24]$	5.4	10.0	15.5	0.12	0.09	17.2	14.3	10.3	0.87
$[25-99]$	6.1	8.4	14.5	0.10	0.12	19.4	13.4	14.6	0.51
$[100-\cdot]$	5.4	7.5	12.9	0.27	0.29	29.6	53.8	55.3	0.54
Serv	6.4	10.8	17.2	0.15	0.16	100.0	100.0	100.0	0.57
Age				*Relative*	*Volatility:*	$(0.57, 0.62)$			
$[0-3]$	7.3	6.7	14.0	0.11	0.10	10.8	7.0	6.1	0.50
$[4-9]$	4.1	8.0	12.1	0.11	0.18	19.2	12.7	18.7	0.41
$[10-24]$	2.5	6.8	9.3	0.11	0.17	29.7	19.4	28.4	0.45
$[25-\cdot]$	1.9	6.0	7.9	0.25	0.21	40.3	60.8	46.8	0.90
Serv	3.0	6.6	9.6	0.17	0.18	100.0	100.0	100.0	0.57

Notes: See table 3.3.

firms.[14] This is the sector where the difference in the sensitivity to aggregate shocks between young/small firms and old/large firms is more pronounced. Therefore, job reallocation is much more countercyclical than would be expected from the employment share of each class of firms.

Finally, tables 3.6 and 3.7 display the results for *Serv* and *Reta*. We combine comments for these two sectors because their patterns are very similar. In both *Serv* and *Reta*, large and old firms are relatively sensitive to aggregate shocks. However, large and (to a lesser extent) old firms tend to exhibit high positive net job creation rates. This implies that in these two sectors large and old firms have relatively low values for the relative volatility of job destruction (except old firms in *Serv*). Then, we have two competing effects: the smaller weight of young and small firms causes them to contribute less to procyclical reallocation, but the larger weight of old and large firms, which display strongly procyclical reallocation, causes them to contribute more to procyclical reallocation. We conclude that procyclical reallocation activity in these two sectors is associated with all classes of firms, so that a large relative

[14] As mentioned above, this might reflect the fact in *Tran* we find some large state owned firms.

Table 3.7: Retail Trade: Heterogeneity and Job Flow Dynamics

Class	Net	JD	JC	$CV(JD)$	$CV(JC)$	p	w_{jd}	w_{jc}	$\frac{SD(JD)}{SD(JC)}$
Size				*Relative Volatility:* $(0.57, 0.71)$					
$[1-3]$	7.8	16.5	24.4	0.07	0.13	21.5	10.8	19.1	0.36
$[4-9]$	4.4	10.7	15.1	0.06	0.12	29.5	13.2	23.9	0.36
$[10-24]$	3.4	9.4	12.9	0.07	0.10	17.3	9.1	11.4	0.54
$[25-99]$	3.4	8.1	11.5	0.18	0.13	15.7	20.2	14.0	0.94
$[100-\cdot]$	8.9	8.3	17.2	0.41	0.30	16.0	46.7	31.5	0.66
Reta	5.6	11.1	16.6	0.14	0.15	100.0	100.0	100.0	0.57
Age				*Relative Volatility:* $(0.36, 0.53)$					
$[0-3]$	7.2	5.3	12.5	0.12	0.12	11.3	10.2	8.2	0.42
$[4-9]$	3.7	6.9	10.6	0.10	0.14	20.2	15.6	17.0	0.47
$[10-24]$	1.4	6.5	7.9	0.10	0.12	31.8	22.9	22.0	0.68
$[25-\cdot]$	2.9	6.2	9.1	0.19	0.24	36.7	51.2	52.9	0.52
Reta	3.0	6.2	9.2	0.13	0.17	100.0	100.0	100.0	0.36

Notes: See table 3.3.

sensitivity to aggregate shocks in old and large firms does not significantly change the dynamics of the job reallocation in the sector.

To put the above sectoral analysis into context, we should mention that during the period 1986 to 2000, the sectoral employment shares evolved as follows: *Manu* decreased from 45.8% to 32.7%; *Tran* decreased from 9.2% to 6.6%; *Serv* increased from 14.0% to 24.2%; and *Reta* increased from 7.9% to 11.5%. *Manu* was subject to a large structural change mainly due to international competition. Because this process should be somehow independent of business cycles, the job reallocation of old and large firms was less cyclically sensitive than in other circumstances. In *Serv*, and especially in *Reta*, the opposite occurred with the expansion of existing and the creation of new industries. The scale and the first-mover advantages seem to have been important factors for success in these sectors, such as telecommunications and the big retail segment.

3.5 Conclusion

In this chapter, we analyze to what extent structural heterogeneity among firms can affect the dynamic behavior of aggregate job flows. We use an (S,s) model of employment adjustment, and derive continuous-time expressions for gross flows statistics. We extend the model by considering two types of firms and derive the impact of each type on the cyclical behavior of job reallocation. We then analyze cyclical properties of job reallocation in the Portuguese economy, considering the overall economy, and the manufacturing, transportation, services, and retail trade sectors. In general, the impact of old and large firms on aggregate job flow dynamics is larger than what should be expected from their employment share. This occurs because old and large firms tend to be affected relatively more by aggregate shocks than young and small firms. This higher relative sensitivity to aggregate shocks tends to decrease the procyclicality of job reallocation in all sectors (except in retail trade and services for specific reasons). However, we find a larger effect in transportation than in other sectors. In manufacturing the structural differences between the two types of firms are smaller than in other sectors. In services and retail trade, old and large firms have large procyclical job reallocation rates, contrary to what occurs in other sectors.

The detailed analysis in this chapter enables us to realize that aggregate job flow dynamics are substantially affected by sector-specific technological and institutional factors that determine how the age and size distribution of firms evolves. Notwithstanding this, the analysis makes it clear that old and large firms are more important to the cyclical behavior of job flows than their (already large) employment shares would indicate.

Chapter 4

Gross Job Flows in Portugal

4.1 Introduction

In this chapter, we study regularities of gross job flows for the Portuguese economy. We use *Quadros de Pessoal* (*QP*), a Portuguese longitudinal employer-employee matched with annual data covering the period 1985-2000.[1] The main findings are consistent with those of other studies: high rates of gross job flows; wide disparity of results across different sectors; industries with positive trend growth rates tend to exhibit procyclical reallocation; different age and size classes of firms display quite different patterns of gross job flows. We contribute to the literature by providing a detailed analysis of gross job flows in Portugal by economic sector and age and size classes. Previous studies, such as Blanchard and Portugal (2001), only contained information for manufacturing and no analysis of heterogeneity across age and size classes.

[1] The structure of this database is described in appendix C.1.

4.2 Reallocation of Jobs in Portugal

4.2.1 Macroeconomic Performance

We start with basic institutional and macroeconomic facts about the Portuguese economy. During the period 1985-2000, Portugal went through a modernization process in infrastructures and market regulations. Having joined the European Union in 1986, jointly with Spain, Portugal benefited, to a great extent for free, from an enormous amount of funds to invest in infrastructure. Simultaneously, until the mid 1990s, Portugal had to adopt reforms to enhance competition and liberalize financial markets, in the process leading to the creation of an economic union in Europe. Additionally, in the late 1980s and early 1990s there was a significant amount of privatizations, especially of big public utilities. In this restructuring process, some traditional manufacturing sectors, like textiles, suffered considerably while new opportunities emerged, especially in the services sector.

The macroeconomic performance during this period is summarized in figure 4.1, where we plot the real growth rate of GDP, the unemployment rate, and the net job growth rate (Net). This picture shows that Net matches quite closely real GDP growth, and that the unemployment rate responds (countercyclically) with a lag of about one year. In terms of business cycles, the late 1980s is a period of high growth with a declining unemployment rate. This expansion is then followed by a recession, which starts in 1990 and hits the bottom in 1993. However, the ensuing mild recovery was not so successful in terms of net job creation.

4.2.2 Gross Job Flows by Sector

With this business cycle information as background, we now move to the analysis of gross job flows. We use *Quadros de Pessoal*, an administrative database on all Portuguese firms with paid employees. We describe in more detail this database in

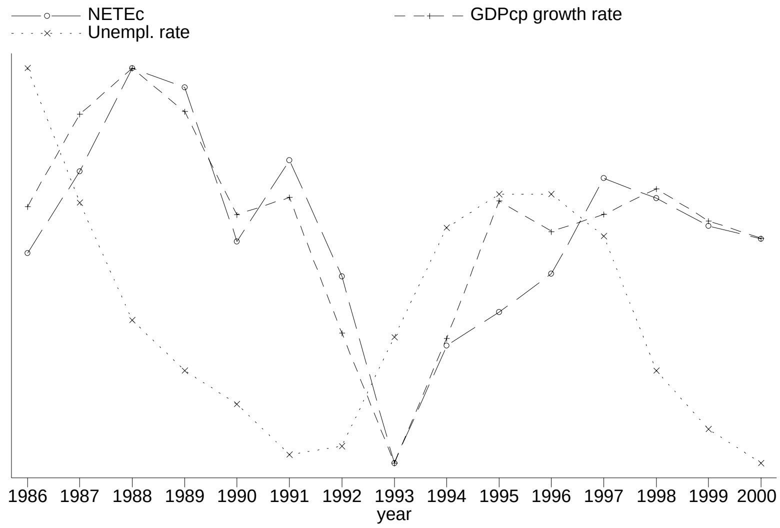

Figure 4.1: Portuguese Macroeconomic Performance

appendix C.1, but it is the same database used in Blanchard and Portugal (2001). We also discuss methodological questions associated with the measurement of gross job flows in appendix C.2. We have considered gross job flows for firms, for establishments and for establishments with at least five employees (the U.S. standard in the LRD), which account for about 90% of all jobs. The greatest difference across the three data categorizations occurs between the statistics for firms and for establishments, especially in terms of levels, but not so much in terms of covariation.

In tables 4.1 and 4.2, we present job flows statistics for establishments and firms, respectively. The values for gross job flows are comparable to other international evidence, such as Davis *et al.* (1996) and Baldwin, Dunne and Haltiwanger (1998). Average gross job creation and job destruction are large, and the contribution of births and deaths to gross job flows is also large. The fact that the values for job reallocation are larger than the average values presented in Blanchard and Portugal (2001) are due to the fact that those authors consider only the Portuguese manufacturing sector. We can see that both job creation and job destruction react in predictable ways to the business cycle and with a time profile that is consistent with figure 4.1. Job

Table 4.1: Establishment Level Job Flows

Ano	*JCC*	*JC*	*JDC*	*JD*	*Net*	*Rea*	*ERea*
1987	8.1	14.1	6.3	10.6	3.5	24.6	21.2
1988	8.9	15.7	6.0	10.3	5.5	26.0	20.5
1989	9.1	16.0	7.0	10.9	5.1	26.9	21.8
1990	8.6	14.0	7.6	11.9	2.1	25.8	23.7
1991	9.0	17.0	8.2	13.3	3.7	30.3	26.7
1992	7.7	13.5	8.6	12.1	1.4	25.5	24.1
1993	6.8	12.6	10.1	14.9	−2.3	27.5	25.3
1994	5.7	12.4	7.0	12.4	0.0	24.8	24.8
1995	7.8	13.0	8.2	12.3	0.7	25.3	24.6
1996	8.1	13.2	7.7	11.7	1.5	24.9	23.5
1997	8.9	14.6	7.3	11.2	3.3	25.8	22.4
1998	9.0	14.7	7.2	11.7	2.9	26.4	23.5
1999	8.7	14.6	7.8	12.2	2.4	26.8	24.4

Notes: JC and JD are the rates of job creation and job destruction among all units; JCC and JDC are the rates of job creation and job destruction among continuing units; $Net(= JC - JD)$ is the net job creation rate; $Rea(= JC + JD)$ is the job reallocation rate; $ERea(= Rea - |Net|)$ is the excess job reallocation rate. All rates are in %.

reallocation is also mostly due to excess reallocation, and only a small part is due to trend growth rate.

In table 4.3, we analyze firm level gross job flows in some one-digit sectors. We consider manufacturing (*Manu*), services (*Serv*), retail trade (*Reta*), construction (*Cons*), wholesale trade (*Whol*), and transportation and public utilities (*Tran*).[2] In the process of calculating job flows by economic sector, we had to face a change in the sector classification system in 1995. In appendix C.2, we describe the methodology used to harmonize the two *CAE* codes.

For the overall economy, even though the average net job growth rate is significantly positive, reallocation is only slightly procyclical. This result is not entirely consistent with the prediction of Foote (1998), but it can be explained by the larger relative sensitivity to aggregate shocks by old and large firms, as was the case in the previous chapter. Now we discuss the results for the one-digit sectors. As evidence of

[2]In order to obtain equivalent one-digit SIC87 sectors, we use the following correspondence in terms of CAE Rev. 1 codes : $Manu(= 3)$, $Serv(= 6.3 + 8.3.2 + 8.3.3 + 9.2 + 9.3 + 9.4 + 9.5)$, $Reta(= 6.2)$, $Cons(= 5)$, $Whol(= 6.1)$, and $Tran(= 7 + 4)$.

Table 4.2: Firm Level Job Flows

Ano	*JCC*	*JC*	*JDC*	*JD*	*Net*	*Rea*	*ERea*
1987	6.9	12.3	5.1	8.9	3.4	21.2	17.8
1988	8.0	14.3	5.3	9.0	5.3	23.2	17.9
1989	8.4	15.2	5.6	8.7	6.5	23.9	17.4
1990	7.6	13.1	6.4	10.1	2.9	23.2	20.2
1991	7.5	13.7	7.5	11.1	2.5	24.8	22.2
1992	6.9	12.1	7.3	10.8	1.3	22.9	21.7
1993	5.9	11.2	8.9	13.1	−1.9	24.4	22.5
1994	5.2	11.3	6.8	11.1	0.2	22.4	22.1
1995	7.2	11.9	6.9	10.8	1.1	22.7	21.5
1996	7.8	12.3	6.9	10.6	1.6	22.9	21.2
1997	9.1	13.9	6.4	9.9	4.1	23.8	19.8
1998	9.1	14.4	6.5	10.6	3.8	25.0	21.2
1999	9.0	13.9	6.7	11.0	2.9	25.0	22.1

Notes: See table 4.1.

Table 4.3: Firm Level Job Flows by Sector

Sector	*ESh87*	*ESh99*	*JC*	*JD*	*Net*	*Rea*	$C_{Rea,Net}$	$C_{Nets,Net}$
All			13.0	10.4	2.6	23.5	0.09	
Manu	45.7	34.1	10.2	9.9	0.2	20.1	0.03	0.93
Serv	14.2	23.2	17.2	10.8	6.4	28.0	0.56	0.78
Reta	8.1	11.4	16.6	11.1	5.6	27.7	0.54	0.65
Cons	8.2	10.5	18.4	14.1	4.3	32.5	0.83	0.89
Whol	7.7	7.7	13.8	10.6	3.2	24.3	−0.12	0.88
Tran	9.0	6.6	7.5	7.3	0.2	14.8	−0.55	0.81

Notes: *ESh87* and *ESh99* are the employment shares (in %) of each sector in 1987 and 1999; $C_{x,y}$ is the correlation coefficient between x and y; *Nets* is the net job creation rate (in %) at the sectoral level; for all other variables see table 4.1.

the structural changes that occurred in the Portuguese economy during this period, we can see that *Manu* has suffered a large decline in its employment share, whereas *Serv* and *Reta* have registered a significant increase. This is reflected in the much larger net job creation rates for the last two sectors. In general, reallocation tends to be more procyclical in sectors with higher growth rates, suggesting that trend related reallocation increases with the absolute value of trend.

4.2.3 Gross Job Flows by Age and Size Classes

We have also obtained gross job flows by age and size classes. In order to avoid repetition, we comment on the information in tables 3.2 and 3.3 in the previous chapter. Heterogeneity across age and size classes is very significant. Young and small firms have higher rates of job creation and job destruction, accounting for a significant fraction of all job creation and all job destruction, and have larger net growth rates (both among all units and among continuing units). It follows that young and small firms tend to display procyclical reallocation, whereas old and large firms tend to have countercyclical reallocation.

4.3 Conclusion

In this chapter we have provided evidence on the macroeconomic performance of the Portuguese economy during the period form 1985 to 2000 and have analyzed the cyclical behavior of gross job flows. The evidence is similar to studies in other countries. We have also analyzed gross job flows across one-digit SIC87 sectors and across age and size classes. We observe significant differences both across sectors, especially between manufacturing and services, and across age and size classes. This evidence is also consistent with studies for other countries.

Chapter 5

Conclusions

This dissertation analyzes some heterogeneities in patterns of gross job flows across different classes of firms, defined by age and size. We now summarize the main contributions that are made to the literature on job reallocation and industry dynamics. First, we propose a measure that decomposes changes in the average size of a cohort of entering firms into growth of survivors and exit of small firms. This is a useful instrument to empirically distinguish theories that attempt to explain firm growth. Second, we propose a mechanism for growth that relies on linear and nonconvex adjustment costs and decreasing idiosyncratic uncertainty with age due to a process of learning. This gives incentives for firms to be cautious by entering small and adjusting upwards if they survive and are efficient. Third, we show that firms whose optimal employment is determined relatively more by aggregate shocks than by idiosyncratic shocks affect the dynamics of aggregate job flows by more than they affect average job flows. Fourth, we measure the impact of heterogeneity along the age and size dimensions in Portuguese data, and conclude that old and large firms tend to make reallocation less procyclical than what their employment shares would indicate. Finally, we produce an updated account of business cycle behavior of gross job flows in Portugal.

These contributions open further questions that deserve to be analyzed in future work. We discuss some of these questions for chapters 2 and 3. In chapter 2, one desirable extension is the formal estimation of the adjustment cost parameters, especially if we can do it for different sectors, such as manufacturing and services. The eventual cross-sector differences in the estimates would reinforce our argument. Another extension is to build a model with both adjustment costs and financing constraints. In this case, the firm is not interested in borrowing as much money as it can, and adjustment costs might make it more costly to default. It would also be interesting to analyze empirically the importance of these two arguments for firm growth. We could do it using balance sheet information on the weight of debt among entering firms and among continuing firms. Sectors where entering firms have the same weight of debt as existing firms should not be subject to financing constraints, whereas sectors where entering firms are less indebted than existing firms might be an indication of financing constraints. In this dissertation we have explored the cross-industry evidence on firm dynamics. However, there is also evidence of significant cross-country differences in firms dynamics, even for tightly defined industries. This is an indication that both technological and institutional factors matter for firms dynamics, eventually because they reflect themselves in different structures of adjustment costs.

In chapter 3, we have concluded that the sensitivity of firms with respect to aggregate versus idiosyncratic shocks is an important element in the heterogeneous dynamic behavior of job flows between young and small firms, on the one hand, and old and large firms, on the other hand. Therefore, we should build a model that simultaneously explains the larger volatility of gross job flows and the smaller relative sensitivity to aggregate shocks among young and small firms. We believe that a model with observable aggregate shocks, linear adjustment costs, and some element of learning about efficiency will lead to that result.

Appendix A

Appendices for Chapter 2

A.1 Appendix: Proofs

Lemma 2 $\Omega_\tau \equiv \left\{\mu_0, \{\eta_\tau\}_{\tau\geq 0}\right\}$ *can be summarized by* (θ_τ^*, τ)*, and the distribution function* $F\left(\theta_{\tau+1}^* \mid \theta_\tau^*, \tau\right)$ *is a continuous and strictly decreasing function of* θ_τ^**.*

Proof. From (2.2) we have

$$\theta_\tau^* = g\left(Y_\tau, \tau\right) = E\left(\xi\left(\eta_\tau\right) \mid Y_\tau, \tau\right) = \nu_1 + \int_{-\infty}^{\infty} \left[1 - F_\eta\left(\eta_\tau \mid Y_\tau, \tau\right)\right] d\xi\left(\eta_\tau\right),$$

where $F_\eta\left(\cdot \mid Y_\tau, \tau\right)$ is the posterior distribution of η_τ. Because $F_\eta\left(\eta_\tau \mid Y_\tau, \tau\right)$ is continuous and strictly decreasing in Y_τ, and $\xi\left(\eta_\tau\right)$ is strictly increasing in η_τ, we conclude that $g\left(Y_\tau, \tau\right)$ is continuous and strictly increasing in Y_τ (see theorem 3.4.1 in Swartz 1994). Therefore, for the purpose of predicting θ_τ, $\Omega_\tau \equiv \left\{\mu_0, \{\eta_s\}_{s=0}^{\tau-1}\right\} \equiv \{Y_\tau, \tau\} \equiv \{\theta_\tau^*, \tau\}$, since $Y_\tau = g_Y^{-1}\left(\theta_\tau^*, \tau\right)$, where g_Y^{-1} is the inverse function of g with respect to Y_τ. Using the recursion

$$Y_{\tau+1} = \frac{\sigma^{-2}}{Z_{\tau+1}^{-1}}\eta_\tau + \frac{Z_\tau^{-1}}{Z_{\tau+1}^{-1}}Y_\tau,$$

the conditional distribution of $\theta^*_{\tau+1}$ can be represented as

$$F\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right) = F_\eta\left[\frac{Z^{-1}_{\tau+1}}{\sigma^{-2}} g^{-1}_Y\left(\theta^*_{\tau+1}, \tau+1\right) - \frac{Z^{-1}_\tau}{\sigma^{-2}} g^{-1}_Y\left(\theta^*_\tau, \tau\right) \mid g^{-1}_Y\left(\theta^*_\tau, \tau\right), \tau\right],$$

since we need to integrate the density of η_τ over the domain where $g\left(Y_{\tau+1}, \tau+1\right) \leq \theta^*_{\tau+1}$. From this, we conclude that $F\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right)$ is a continuous and strictly decreasing function of θ^*_τ. Therefore, the transition function associated with $F\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right)$ is monotone and satisfies the Feller property (see pp. 376-9 in Stokey, Lucas and Prescott 1989). ■

Proof of proposition 4. We use the following notation: *(i)* $X \equiv \mathbb{R}_+ \times \Theta \times \mathbb{N}_0$ and $x \equiv (L, \theta, \tau) \in X$, where $\Theta \equiv [\nu_1, \nu_2] \subset \mathbb{R}_+$, $\nu_1 \geq 0$, $\nu_2 < \infty$; *(ii)* T is the operator associated with (2.4); *(iii)* M denotes the following operator

$$\left(MV^S\right)\left(L_\tau, \theta^*_\tau, \tau\right) = \int_{\nu_1}^{\nu_2} \max\left\{V^{EX}\left(L_\tau\right), V^S\left(L_\tau, \theta^*_{\tau+1}, \tau+1\right)\right\} dF\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right);$$

(iv) V^S_O, V^{SD}_O, and V^{SU}_O denote the objective functions associated with V^S, V^{SD}, and V^{SU}, that is, for $j = S, SD, SU$

$$V^j_O\left(L_\tau; L_{\tau-1}, \theta^*_\tau, \tau\right) = \Pi\left(L_\tau, \theta^*_\tau\right) - C^j\left(L_\tau, L_{\tau-1}\right) + \beta\left(MV^S\right)\left(L_\tau, \theta^*_\tau, \tau\right).$$

We prove the proposition in various steps.

(a.i) *Existence and Uniqueness:* This part is similar to the case without linear and nonconvex costs (see Jovanovic 1982)

(a.ii) *Continuity in* $(L_{\tau-1}, \theta^*_\tau)$: Because the objective function in (2.4), $V^S_O\left(L_\tau; L_{\tau-1}, \theta^*_\tau, \tau\right)$, is not continuous the usual argument is slightly modified. Let $C_{12}(X)$ be the space of bounded functions on X which are continuous in $(L_{\tau-1}, \theta^*_\tau)$. This is clearly a closed subset of $B(X)$, the space of bounded functions $V^S : X \to \mathbb{R}$. Since $B(X)$ with the sup norm $\left\|V^S\right\| = \sup_{x\in X}\left|V^S(x)\right|$ is a Banach space, then $C_{12}(X)$

is also a Banach space. Now consider $V^S \in C_{12}(X)$. Because $\max\left\{V^{EX}, V^S\right\}$ is also continuous and $F\left(\theta^*_{\tau-1} \mid \theta^*_\tau, \tau\right)$ satisfies the Feller property (see lemma 2), then MV^S is continuous in (L_τ, θ^*_τ) (see lemma 9.5 in Stokey *et al.* 1989). Then $V^{SN}(L_{\tau-1}, \theta^*_\tau, \tau)$ is continuous in $(L_{\tau-1}, \theta^*_\tau)$, and $V^{SN}_O(L_\tau; L_{\tau-1}, \theta^*_\tau, \tau)$ and $V^{SU}_O(L_\tau; L_{\tau-1}, \theta^*_\tau, \tau)$ are continuous in $(L_\tau; L_{\tau-1}, \theta^*_\tau)$. Therefore, applying the maximum theorem, we conclude that $V^{SD}(L_{\tau-1}, \theta^*_\tau, \tau)$ and $V^{SU}(L_{\tau-1}, \theta^*_\tau, \tau)$ are continuous in $(L_{\tau-1}, \theta^*_\tau)$. For V^{SD} the set of admissible values for employment is naturally compact. For V^{SU} we can make it compact by choosing a value for L_τ high enough, say L^{UB}, such that $L^{SU*}_\tau(L_{\tau-1}, \theta^*_\tau, \tau) \leq L^{UB}$, for all $L_{\tau-1} \leq L^{UB}$, so that all values of interest are considered. L^{UB} is finite since $F'(\infty) = 0$, and MV^S is bounded. Therefore, V^S as defined by (2.4) is continuous in $(L_{\tau-1}, \theta^*_\tau)$.

(a.iii) *Strict Monotonicity in* θ^*_τ: From lemma 2 (the transition function associated with $F\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right)$ is monotone) if $V^S\left(L_\tau, \theta^*_{\tau+1}, \tau+1\right)$ is weakly increasing in $\theta^*_{\tau+1}$, then $\left(MV^S\right)(L_\tau, \theta^*_\tau, \tau)$ is also weakly increasing in θ^*_τ. Then, because $\Pi(L_\tau, \theta^*_\tau)$ is strictly increasing in θ^*_τ (and the constraint set is not affected by θ^*_τ), $V^S(L_{\tau-1}, \theta^*_\tau, \tau)$ is strictly increasing in θ^*_τ (see corollary 3.1 in Stokey *et al.* 1989).

(b) *Exit Policy*: The exit policy is determined by the condition

$$V^{EX}(L_{\tau-1}) \equiv V^S(L_{\tau-1}, \theta^*_\tau, \tau).$$

Because, for each $L_{\tau-1}$, V^{EX} is constant and V^S is strictly increasing in θ^*_τ, then it is obvious that $\theta^{EX}(L_{\tau-1}, \tau)$ is a unique function defined by the value of $\theta^* \in [\nu_1, \nu_2]$ that satisfies the above equation, if it exists, or by ν_1, when $V^{EX}(L) < V^S(L, \nu_1, \tau)$, or by ν_2, when $V^{EX}(L) > V^S(L, \nu_2, \tau)$. Because both V^{EX} and V^S are continuous functions, then θ^{EX} is also a continuous function in L. ■

Proposition 5 *Let* $\bar{T}$ *be the maximum allowed age, so that a firm entering in period* 0 *must exit the industry at the end of period* $\bar{T}$. *Then* $\Pr\left(\theta^*_{\tau+1} \in \Theta^D_{\bar{T}}(L_\tau, \tau+1) \mid\right.$

$\theta^*_\tau, \tau) = 1$, *for all* $L_\tau \in \mathbb{R}_+$, $\tau \in \{0, \ldots, \bar{T} - 1\}$, *where*

$$\Theta^D_{\bar{T}}(L_\tau, \tau + 1) = \left\{\theta^*_{\tau+1} \in \Theta : V^S_{\bar{T}}\left(L_\tau, \theta^*_{\tau+1}, \tau + 1\right) \text{ is differentiable at } L_\tau\right\}.$$

Consequently, the objective functions associated with $V^{SD}_{\bar{T}}$ *and* $V^{SU}_{\bar{T}}$ *are continuously differentiable in* L, *and all optima are interior in the region of their definition.*[1]

Proof. We prove this by induction. In period $\bar{T}$, we have

$$V^S_{\bar{T}}\left(L_{\bar{T}-1}, \theta^*_{\bar{T}}, \bar{T}\right) = \max_{L_{\bar{T}}} \left\{\Pi\left(L_{\bar{T}}, \theta^*_{\bar{T}}\right) - C^S\left(L_{\bar{T}}, L_{\bar{T}-1}\right) + \beta V^{EX}\left(L_{\bar{T}}\right)\right\},$$

so that $V^{SD}_{\bar{T},O}$, $V^{SN}_{\bar{T}}$ and $V^{SU}_{\bar{T},O}$ are continuously differentiable functions of $L_{\bar{T}}$, $L_{\bar{T}-1}$, and $L_{\bar{T}}$, respectively. Since $V^{SD}_{\bar{T},O}\left(L; L, \theta^*, \bar{T}\right) \leq V^{SN}_{\bar{T}}\left(L, \theta^*, \bar{T}\right)$, $V^{SU}_{\bar{T},O}\left(L; L, \theta^*, \bar{T}\right) \leq V^{SN}_{\bar{T}}\left(L, \theta^*, \bar{T}\right)$, $F'\left(0^+\right) = \infty$, $F'\left(\infty\right) = 0$, and V^{EX} is bounded above, then $V^{SD}_{\bar{T},O}$ and $V^{SU}_{\bar{T},O}$ have interior optima in the regions of definition of $V^{SD}_{\bar{T}}$ and $V^{SU}_{\bar{T}}$. Therefore, those optima are independent of $L_{\bar{T}-1}$, and we must have $\partial V^{SD}_{\bar{T}}/\partial L_{\bar{T}} = -P^{SD}$, $\partial V^{SU}_{\bar{T}}/\partial L_{\bar{T}} = P^{SU}$, in the regions of their definition, and

$$\frac{\partial V^{SN}_{\bar{T}}}{\partial L_{\bar{T}-1}} = pF'\left(L_{\bar{T}-1}\right)\theta^*_{\bar{T}} - w - \beta P^{EX}.$$

We conclude that $V^S_{\bar{T}}\left(L_{\bar{T}-1}, \theta^*_{\bar{T}}, \bar{T}\right)$ is continuously differentiable at $L_{\bar{T}-1} \in \mathbb{R}_+$, with probability one (given $F\left(\cdot \mid \theta^*_{\bar{T}-1}, \bar{T} - 1\right)$ and for all $\theta_{\bar{T}-1} \in \Theta$).

Now consider a generic period $\tau \in \{1, \ldots, \bar{T} - 1\}$, and assume that $V^S_{\bar{T}}\left(L_\tau, \theta^*_{\tau+1}, \tau + 1\right)$ is continuously differentiable at $L_\tau \in \mathbb{R}_+$ with probability one. Because $\theta^{EX}\left(L_{\bar{T}-1}, \bar{T}\right)$ is a unique continuous function of L, we can apply the dominated convergence theorem to conclude that $\left(MV^S_{\bar{T}}\right)\left(L_\tau, \theta^*_\tau, \tau\right)$ is continuously differentiable at L_τ, for all $\theta^*_\tau \in \Theta$ (see theorems 3.2.16 and 3.4.3 in Swartz 1994). Consequently, the

[1] A similar result would hold for the case of infinite-lived firms that face a finite learning horizon, as in sections 2.4 and 2.5. However, in this case we would need to use proposition 11 first.

same argument used for period $\bar{T}$ can be repeated here. ■

Proof of proposition 6. For given $(L_{\tau-1}, \tau)$ we partition the state-space associated with θ^*_τ, Θ, into regions of exit, Θ^{EX}, downward adjustment, Θ^{SD}, non-adjustment, Θ^{SN}, and upward adjustment, Θ^{SU}:[2]

$$\begin{aligned}
\Theta^{EX}(L_{\tau-1}, \tau) &= \left\{\theta : V^{EX} > V^{S}\right\}, \\
\Theta^{SD}(L_{\tau-1}, \tau) &= \left\{\theta \in \Theta : V^{SD} > V^{SN},\ V^{SD} \geq V^{SU},\ V^{SD} \geq V^{EX}\right\}, \\
\Theta^{SN}(L_{\tau-1}, \tau) &= \left\{\theta \in \Theta : V^{SN} \geq V^{SD},\ V^{SN} \geq V^{SU},\ V^{SN} \geq V^{EX}\right\}, \\
\Theta^{SU}(L_{\tau-1}, \tau) &= \left\{\theta \in \Theta : V^{SU} > V^{SN},\ V^{SU} \geq V^{SD},\ V^{SU} \geq V^{EX}\right\}.
\end{aligned}$$

If it is optimal for the firm to adjust upwards, then we must solve

$$A_{SU} = \left[pF'(L^*_\tau)\,\theta^*_\tau - \left(w + P^{SU}\right)\right] + \beta \frac{\partial\left(MV^S\right)(L^*_\tau, \theta^*_\tau, \tau)}{\partial L} = 0,$$

and if it is optimal for the firm to adjust downwards, we must solve

$$A_{SD} = \left[pF'(L^*_\tau)\,\theta^*_\tau - \left(w - P^{SD}\right)\right] + \beta \frac{\partial\left(MV^S\right)(L^*_\tau, \theta^*_\tau, \tau)}{\partial L} = 0$$

Now, the derivative can be rewritten as

$$\begin{aligned}
\frac{\partial\left(MV^S\right)(L_\tau, \theta^*_\tau, \tau)}{\partial L} = &\int_{\Theta^{EX}} \frac{\partial V^{EX}(\cdot)}{\partial L_\tau} dF\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right) + \int_{\Theta^{SD}} \frac{\partial V^{SD}(\cdot)}{\partial L_\tau} dF\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right) \\
&+ \int_{\Theta^{SN}} \frac{\partial V^{SN}(\cdot)}{\partial L_\tau} dF\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right) + \int_{\Theta^{SU}} \frac{\partial V^{SU}(\cdot)}{\partial L_\tau} dF\left(\theta^*_{\tau+1} \mid \theta^*_\tau, \tau\right),
\end{aligned}$$

where some of the regions might be empty, and in separating the integrals we have taken into account the continuity of the integrand at the frontiers.

[2] In Θ^{SD} and Θ^{SU} we need to use $V^{SD} > V^{SU}$ and $V^{SU} > V^{SD}$ because V^S is not concave in L.

For each of the above derivatives we have

$$\frac{\partial V^{EX}\left(L_\tau\right)}{\partial L}=-P^{EX},$$

$$\left.\frac{\partial V^{SD}\left(L_\tau,\theta^*_{\tau+1},\tau+1\right)}{\partial L}\right|_{\theta^*_{\tau+1}\in\Theta^{SD}(L_\tau,\tau+1)}=$$

$$-P^{SD}=\left[pF'\left(L^*_{\tau+1}\right)\theta^*_{\tau+1}-w\right]+\beta\frac{\partial\left(MV^S\right)\left(L^*_{\tau+1},\theta^*_{\tau+1},\tau+1\right)}{\partial L},$$

$$\frac{\partial V^{SN}\left(L_\tau,\theta^*_{\tau+1},\tau+1\right)}{\partial L_\tau}=\left[pF'\left(L_\tau\right)\theta^*_{\tau+1}-w\right]+\beta\frac{\partial\left(MV^S\right)\left(L_\tau,\theta^*_{\tau+1},\tau+1\right)}{\partial L},$$

$$\left.\frac{\partial V^{SU}\left(L_\tau,\theta^*_{\tau+1},\tau+1\right)}{\partial L}\right|_{\theta^*_{\tau+1}\in\Theta^{SU}(L_\tau,\tau+1)}=$$

$$P^{SU}=\left[pF'\left(L^*_{\tau+1}\right)\theta^*_{\tau+1}-w\right]+\beta\frac{\partial\left(MV^S\right)\left(L^*_{\tau+1},\theta^*_{\tau+1},\tau+1\right)}{\partial L},$$

where we have used the fact that $A_{SU}=0$, when it is optimal to adjust upwards, and $A_{SD}=0$, when it is optimal to adjust downwards. Therefore, we have

$$\frac{\partial\left(MV^S\right)\left(L^*_\tau,\theta^*_\tau,\tau\right)}{\partial L}=E_\tau\left(\chi^*_{\tau+1}\left(-P^{EX}\right)+\left(1-\chi^*_{\tau+1}\right)\left\{\left[pF'\left(L^*_{\tau+1}\right)\theta^*_{\tau+1}-w\right]+\right.\right.$$
$$\left.\left.\beta\frac{\partial\left(MV^S\right)\left(L^*_{\tau+1},\theta^*_{\tau+1},\tau+1\right)}{\partial L}\right\}\right),$$

Using the law of iterated expectations, we can rewrite the above as

$$\frac{\partial\left(MV^S\right)\left(L^*_\tau,\theta^*_\tau,\tau\right)}{\partial L}=\sum_{s=1}^{\infty}E_\tau\beta^{s-1}\left\{\tilde{\chi}^*_{\tau+s}\left(-P^{EX}\right)+\hat{\chi}^*_{\tau+s}\left[pF'\left(L^*_{\tau+s}\right)\theta^*_{\tau+s}-w\right]\right\},$$

The result now follows by plugging this expression in A_{SU}, and A_{SD}. ■

Proof of corollary 7. We can rewrite the LHS of (2.6) and (2.7) as follows

$$
\begin{aligned}
MB\left(L_{\tau-1},\theta_\tau^*,\tau\right) &= \left(pF'\left(L_\tau^*\right)\theta_\tau^* - w\right) + \beta E_\tau \tilde{\chi}_{\tau+1}^*\left(-P^{EX}\right) + \\
&\quad \beta E_\tau \left\{ \hat{\chi}_{\tau+1}^*\left(pF'\left(L_{\tau+1}^*\right)\theta_{\tau+1}^* - w\right) + \sum_{s=1}^{\infty} E_{\tau+1}\beta^s\left[\tilde{\chi}_{\tau+1+s}^*\left(-P^{EX}\right) + \right.\right. \\
&\quad \left.\left. \hat{\chi}_{\tau+1+s}^*\left(pF'\left(L_{\tau+1+s}^*\right)\theta_{\tau+1+s}^* - w\right)\right]\right\}.
\end{aligned}
$$

Taking into account that $\hat{\chi}_{\tau+1}^*\tilde{\chi}_{\tau+1+s}^* = \tilde{\chi}_{\tau+1+s}^*$, $\hat{\chi}_{\tau+1}^*\hat{\chi}_{\tau+1+s}^* = \hat{\chi}_{\tau+1+s}^*$, $\hat{\chi}_{\tau+1}^* = 1 - \chi_{\tau+1}^*$, and $\tilde{\chi}_{\tau+1}^* = \chi_{\tau+1}^*$, then we get the stated result. ■

Proof of proposition 8. With proportional adjustment costs, optimal employment at entry is determined by

$$
\begin{aligned}
& pF'\left(L_1\right)\theta_1^* - \left(w + P^H\right) + \beta\left(\int_{\nu_1}^{\theta^{SD}} -P^F dF\left(\theta_2^* \mid \theta_1^*\right) + \right. \\
& \left. \int_{\theta^{SD}}^{\theta^{SU}} \left\{\delta\left(\bar{T}\right)\left[pF'\left(L_1\right)\theta_2^* - w\right] - \beta^{\bar{T}-1}P^F\right\} dF\left(\theta_2^* \mid \theta_1^*\right) + \int_{\theta^{SU}}^{\nu_2} P^H dF\left(\theta_2^* \mid \theta_1^*\right)\right) = 0
\end{aligned}
$$

(a) In the case of a proportional hiring cost, assuming $P^F = 0$, we have

$$
\begin{aligned}
L_1 = F'^{-1}\left(\frac{w}{p\theta^{SD}}\right) &= F'^{-1}\left(\frac{w + \frac{P^H}{\delta(\bar{T})}}{p\theta^{SU}}\right), \\
\frac{\partial L_1^*}{\partial P^H} &= \frac{F'\left(L_1^*\right)}{F''\left(L_1^*\right)}\left(\frac{\tilde{w}_H}{w\tilde{w}_w + P^H\tilde{w}_H} - \frac{\partial p/\partial P^H}{p}\right), \\
\tilde{w}_w = 1 + \beta\delta\left(\bar{T}\right)\left[F\left(\theta^{SU} \mid \theta_1\right) - F\left(\theta^{SD} \mid \theta_1\right)\right]&, \ \tilde{w}_H = 1 - \beta\left[1 - F\left(\theta^{SU} \mid \theta_1\right)\right]
\end{aligned}
$$

Then, considering the indirect effects of P^H through the equilibrium price, after some algebra we get

$$
\begin{aligned}
\frac{\partial g}{\partial P^H} &= \psi\left(L_1^*\right) \frac{\tilde{w}_H}{w\tilde{w}_w + P^H \tilde{w}_H} F\left(\theta^{SD} \mid \theta_1^*\right) + \\
&\int_{\theta^{SU}}^{\nu_2} \left\{ \psi\left(L_1^*\right) \frac{\tilde{w}_H}{w\tilde{w}_w + P^H \tilde{w}_H} - \psi\left(L_2^{*SU}\right) \frac{1}{w\delta\left(\bar{T}\right) + P^H} \right\} dF\left(\theta_2^* \mid \theta_1\right) + \\
&\qquad \frac{\partial p / \partial P^H}{p} \left\{ \int_{\nu_1}^{\theta^{SD}} \left[\psi\left(L_2^{*SD}\right) - \psi\left(L_1^*\right)\right] dF\left(\theta_2^* \mid \theta_1^*\right) + \right. \\
&\qquad\qquad \left. \int_{\theta^{SU}}^{\nu_2} \left[\psi\left(L_2^{*SU}\right) - \psi\left(L_1^*\right)\right] dF\left(\theta_2^* \mid \theta_1^*\right) \right\}
\end{aligned}
$$

where $\psi\left(L\right) = -F'\left(L\right) / \left(LF''\left(L\right)\right) > 0$ and $\frac{\partial p / \partial P^H}{p} > 0$. If $F\left(L\right) = AL^\alpha$, we have $\psi\left(L\right) = \left(1 - \alpha\right)^{-1}$. Therefore, the indirect effects cancel out, and the above expression simplifies to

$$
\begin{aligned}
\frac{\partial g}{\partial P^H} = \left(1 - \alpha\right)^{-1} \Bigg(& \frac{\tilde{w}_H}{w\tilde{w}_w + P^H \tilde{w}_H} \left\{ F\left(\theta^{SD} \mid \theta_1^*\right) + \left[1 - F\left(\theta^{SU} \mid \theta_1^*\right)\right] \right\} - \\
& \frac{1}{w\delta\left(\bar{T}\right) + P^H} \left[1 - F\left(\theta^{SU} \mid \theta_1^*\right)\right] \Bigg),
\end{aligned}
$$

which is positive when $\bar{T}$ is high enough so that

$$
\begin{aligned}
& w\left\{ \delta\left(\bar{T}\right) F\left(\theta^{SD} \mid \theta_1^*\right) - \beta^{\bar{T}-1} \left[1 - F\left(\theta^{SU} \mid \theta_1^*\right)\right] \right\} + \\
& \qquad P^H \left\{ 1 - \beta \left[1 - F\left(\theta^{SU} \mid \theta_1^*\right)\right] \right\} F\left(\theta^{SD} \mid \theta_1^*\right) > 0
\end{aligned}
$$

(b) In the case of a proportional firing cost, assuming $P^H = 0$, we get similarly

$$
\frac{\partial g}{\partial P^F} = -\int_{\nu_1}^{\theta^{SD}} \left\{ \psi\left(L_2^{*SD}\right) \frac{\beta^{\bar{T}-1} - 1}{w\delta\left(\bar{T}\right) + \left(\beta^{\bar{T}-1} - 1\right) P^F} - \right.
$$

$$\left. \psi\left(L_1^*\right) \frac{\beta \tilde{w}_F}{w\tilde{w}_w + \beta P^F \tilde{w}_F} \right\} dF\left(\theta_2^* \mid \theta_1\right) -$$

$$\int_{\theta^{SU}}^{\nu_2} \left\{ \psi\left(L_2^{*SU}\right) \frac{\beta^{\bar{T}-1}}{w\delta\left(\bar{T}\right) + \beta^{\bar{T}-1} P^F} - \psi\left(L_1^*\right) \frac{\beta \tilde{w}_F}{w\tilde{w}_w + \beta P^F \tilde{w}_F} \right\} dF\left(\theta_2^* \mid \theta_1\right) +$$

$$\frac{\partial p/\partial P^F}{p} \left\{ \int_{\nu_1}^{\theta^{SD}} \left[\psi\left(L_2^{*SD}\right) - \psi\left(L_1^*\right)\right] dF\left(\theta_2^* \mid \theta_1^*\right) + \right.$$

$$\left. \int_{\theta^{SU}}^{\nu_2} \left[\psi\left(L_2^{*SU}\right) - \psi\left(L_1^*\right)\right] dF\left(\theta_2^* \mid \theta_1^*\right) \right\}$$

where $\frac{\partial p/\partial P^H}{p} > 0$, and

$$\tilde{w}_w = 1 + \beta\delta\left(\bar{T}\right)\left[F\left(\theta^{SU} \mid \theta_1^*\right) - F\left(\theta^{SD} \mid \theta_1^*\right)\right]$$

$$\tilde{w}_F = F\left(\theta^{SD} \mid \theta_1\right) + \beta^{\bar{T}-1}\left[F\left(\theta^{SU} \mid \theta_1^*\right) - F\left(\theta^{SD} \mid \theta_1^*\right)\right]$$

We need $P^F < \frac{w}{1-\beta}$, under the assumption that marginal utility is always positive, since otherwise the firm would prefer to pay the worker, instead of firing him. If $F(L) = AL^\alpha$, we have $F'/\left(LF''\right) = (\alpha - 1)^{-1}$. Therefore, the indirect effects cancel out, and the above expression simplifies to

$$\frac{\partial g}{\partial P^H} = (1-\alpha)^{-1} \left(\frac{\beta \tilde{w}_F}{w\tilde{w}_w + \beta P^F \tilde{w}_F} \left\{F\left(\theta^{SD} \mid \theta_1^*\right) + \left[1 - F\left(\theta^{SU} \mid \theta_1^*\right)\right]\right\} - \right.$$

$$\left. \frac{\beta^{\bar{T}-1} - 1}{w\delta\left(\bar{T}\right) + \left(\beta^{\bar{T}-1} - 1\right) P^F} F\left(\theta^{SD} \mid \theta_1^*\right) - \frac{\beta^{\bar{T}-1}}{w\delta\left(\bar{T}\right) + \beta^{\bar{T}-1} P^F} \left[1 - F\left(\theta^{SU} \mid \theta_1^*\right)\right] \right)$$

which is positive for all $\bar{T}$. ■

Proof of proposition 9. With fixed adjustment costs, optimal employment at entry is determined by

$$pF'(L_1)\,\theta_1^* - w + \beta \int_{\theta^{SD}}^{\theta^{SU}} \delta\left(\bar{T}\right)\left[pF'(L_1)\,\theta_2^* - w\right] dF\left(\theta_2^* \mid \theta_1^*\right) = 0,$$

exactly the same as without those costs. With fixed costs we do not consider the indirect effects that operate through changes in the equilibrium price, but once again they should be of second-order magnitude.

(a) Consider first the case of hiring costs, i.e., $F^F = 0$. Then, we get

$$pF''(L_1^*)\frac{\partial L_1^*}{\partial F^H}\left\{\theta_1^* + \beta\delta\left(\bar{T}\right)\int_{\theta^{SD}}^{\theta^{SU}} \theta_2^* dF\left(\theta_2^* \mid \theta_1^*\right)\right\} = \\ -\beta\delta\left(\bar{T}\right)\left[pF'(L_1^*)\,\theta^{SU} - w\right] f\left(\theta^{SU} \mid \theta_1^*\right)\frac{\partial \theta^{SU}}{\partial F^H}$$

$$\frac{\partial \theta^{SU}}{\partial F^H} = \frac{1 + \delta\left(\bar{T}\right)\left[pF'(L_1^*)\,\theta^{SU} - w\right]\partial L_1^*/\partial F^H}{\delta\left(\bar{T}\right) p\left[F\left(L_2^{*SU}\right) - F\left(L_1^*\right)\right]}.$$

Now, because L_1^* must be an interior local maximum, we have from local concavity

$$pF''(L_1)\left\{\theta_1^* + \beta\delta\left(\bar{T}\right)\int_{\theta^{SD}}^{\theta^{SU}} \theta_2^* dF\left(\theta_2^* \mid \theta_1^*\right)\right\} \leq \\ -\frac{\beta\delta\left(\bar{T}\right)\left[pF'(L_1^*)\,\theta^{SU} - w\right]^2 f\left(\theta^{SU} \mid \theta_1^*\right)}{p\left[F\left(L_2^{*SU}\right) - F\left(L_1^*\right)\right]},$$

where we have used

$$\frac{\partial \theta^{SU}}{\partial L_1} = \frac{pF'(L_1)\,\theta^{SU} - w}{p\left[F\left(L_2^{*SU}\right) - F\left(L_1^*\right)\right]}.$$

From this we conclude that $\partial L_1^*/\partial F^H > 0$, and $\partial \theta^{SU}/\partial F^H > 0$. Therefore, the marginal effect of F^H on $g\left(\theta_1^*\right)$, assuming all other costs are zero, is negative, i.e.,

$$\frac{\partial g}{\partial F^H} = \int_{\nu_1}^{\theta^{SD}} -\frac{\partial L_1^*/\partial F^H}{L_1^*} dF\left(\theta_2^* \mid \theta_1^*\right) + \int_{\theta^{SU}}^{\nu_2} -\frac{\partial L_1^*/\partial F^H}{L_1^*} dF\left(\theta_2^* \mid \theta_1^*\right) - \\ \frac{\partial \theta^{SU}}{\partial F^H}\left[\ln\left(L_2^{*SU}\right) - \ln\left(L_1^*\right)\right] f\left(\theta^{SU} \mid \theta_1^*\right) < 0$$

(b) For the case with firing costs, i.e., $F^H = 0$, we have similarly

$$pF''\left(L_1^*\right)\frac{\partial L_1^*}{\partial F^F}\left\{\theta_1^* + \beta\delta\left(\bar{T}\right)\int_{\theta^{SD}}^{\theta^{SU}}\theta_2^* dF\left(\theta_2^* \mid \theta_1^*\right)\right\} =$$

$$\beta\delta\left(\bar{T}\right)\left[pF'\left(L_1^*\right)\theta^{SD} - w\right] f\left(\theta^{SD} \mid \theta_1^*\right)\frac{\partial\theta^{SD}}{\partial F^F}$$

$$\frac{\partial\theta^{SD}}{\partial F^F} = \frac{1 + \delta\left(\bar{T}\right)\left[pF'\left(L_1^*\right)\theta^{SD} - w\right]\partial L_1^*/\partial F^F}{\delta\left(\bar{T}\right)p\left[F\left(L_2^{*SD}\right) - F\left(L_1^*\right)\right]}.$$

Because L_1^* must be an interior local maximum, we need

$$pF''\left(L_1\right)\left\{\theta_1^* + \beta\delta\left(\bar{T}\right)\int_{\theta^{SD}}^{\theta^{SU}}\theta_2^* dF\left(\theta_2^* \mid \theta_1^*\right)\right\} \leq$$

$$\frac{\beta\delta\left(\bar{T}\right)\left[pF'\left(L_1^*\right)\theta^{SD} - w\right]^2 f\left(\theta^{SD} \mid \theta_1^*\right)}{p\left[F\left(L_2^{*SD}\right) - F\left(L_1^*\right)\right]},$$

where we have used

$$\frac{\partial\theta^{SD}}{\partial L_1} = \frac{pF'\left(L_1\right)\theta^{SD} - w}{p\left[F\left(L_2^{*SD}\right) - F\left(L_1^*\right)\right]}.$$

From this we conclude that $\partial L_1^*/\partial F^F < 0$, and $\partial\theta^{SD}/\partial F^F < 0$. Therefore, the marginal effect of F^F on $g\left(\theta_1^*\right)$, assuming all other costs are zero, is positive, i.e.,

$$\frac{\partial g}{\partial F^F} = \int_{\nu_1}^{\theta^{SD}} -\frac{\partial L_1^*/\partial F^F}{L_1^*} dF\left(\theta_2^* \mid \theta_1^*\right) + \int_{\theta^{SU}}^{\nu_2} -\frac{\partial L_1^*/\partial F^F}{L_1^*} dF\left(\theta_2^* \mid \theta_1^*\right) +$$

$$\frac{\partial\theta^{SD}}{\partial F^F}\left[\ln\left(L_2^{*SD}\right) - \ln\left(L_1^*\right)\right] f\left(\theta^{SD} \mid \theta_1^*\right) > 0$$

■

Proof of proposition 10. The result concerning the posterior distribution of $\theta_{\tau+j}$ follows directly from

$$\ln\left(\theta_{\tau+j}\right)|_{\Omega_\tau} = \mu\,|_{\Omega_\tau} + \varepsilon_{\tau+j},\ \mu\,|_{\Omega_\tau} \sim N\left(Y_\tau, Z_\tau\right).$$

For the distribution of $\theta^*_{\tau+j}$ conditional on (θ^*_τ, τ), we use the fact that

$$\ln\left(\theta^*_{\tau+j}\right) |_{\Omega_\tau} = Y_{\tau+j} |_{\Omega_\tau} + \frac{1}{2}\left(Z_{\tau+j} + \sigma^2\right)$$

$$Y_{\tau+j} = \sigma^{-2} Z_{\tau+j} \sum_{s=\tau}^{\tau+j-1} \eta_s + \frac{Z_{\tau+j}}{Z_\tau} Y_\tau,$$

$$Z_{\tau+j} = Z_\tau - \sigma^{-2} Z_{\tau+j} Z_\tau j,$$

$$\eta_s |_{\Omega_\tau} \sim N\left(Y_\tau, Z_\tau + \sigma^2\right), \; Cov\left(\eta_s, \eta_{s'} \mid \Omega_\tau\right) = Var\left(\mu \mid \Omega_\tau\right) = Z_\tau, \; s, \; s' \geq \tau, \; s \neq s'$$

so that, in the end, we get

$$E\left[\ln\left(\theta^*_{\tau+j}\right) \mid \Omega_\tau\right] = Y_\tau + \frac{1}{2}\left(Z_{\tau+j} + \sigma^2\right),$$

$$Var\left[\ln\left(\theta^*_{\tau+j}\right) \mid \Omega_\tau\right] = Z_\tau - Z_{\tau+j}.$$

From here the result follows by noting that $\ln\left(\theta^*_\tau\right) = Y_\tau + \frac{1}{2}\left(Z_\tau + \sigma^2\right)$.

For the unconditional distribution, just note that $\ln\left(\theta^*_\tau\right)$ is a sum of normal random variables, and that

$$E\left[\ln\left(\theta^*_\tau\right)\right] = \bar{\mu} + \frac{1}{2}\left(Z_\tau + \sigma^2\right)$$

$$Var\left[\ln\left(\theta^*_\tau\right)\right] = \sigma^2_{\mu_0} + \left(Z_0 - Z_\tau\right)$$

■

Proof of proposition 11.

After period $T-1$ the optimization problem is time invariant, since there is no uncertainty concerning $E(\theta)$. Therefore, for periods s, $s \geq 0$, we have

$$V^S\left(\theta^*_T, L_{s-1}, T\right) = \max_{L_s \geq 0, \chi_s \in \{0,1\}} \Big\{\left[\Pi\left(L_s, \theta^*_T\right) - C^S\left(L_s, L_{s-1}\right)\right] + \beta\left\{\chi_s\left[W - C^{EX}\left(L_{\tau+s}\right)\right] + \left(1-\chi_s\right) V^S\left(\theta^*_T, L_s, T\right)\right\}\Big\}.$$

Consider a firm that is in the industry at time s, $s \geq 0$. We now prove that this firm will not change its employment level in period $s+1$. For this, we use the easily proven fact that it is less costly to adjust in one step than in two steps, i. e.,

$$C^S\left(L_{s+1}, L_s^*\right) + C^S\left(L_s^*, L_{s-1}\right) \geq C^S\left(L_{s+1}, L_{s-1}\right),$$

where $L_s^* = L_s\left(\theta_T^*, L_{s-1}, T\right)$. We then have

$$\begin{aligned}
& \Pi\left(L_{s+1}, \theta_T^*\right) - C^S\left(L_{s+1}, L_s^*\right) + \beta \max\left\{V^{EX}\left(L_{s+1}\right), V^S\left(\theta_T^*, L_{s+1}\right)\right\} \\
= & \Pi\left(L_{s+1}, \theta_T^*\right) - \left[C^S\left(L_{s+1}, L_s^*\right) + C^S\left(L_s^*, L_{s-1}\right)\right] + \\
& \beta \max\left\{V^{EX}\left(L_{s+1}\right), V^S\left(\theta_T^*, L_{s+1}\right)\right\} + C^S\left(L_s^*, L_{s-1}\right) \\
\leq & \Pi\left(L_{s+1}, \theta_T^*\right) - C^S\left(L_{s+1}, L_{s-1}\right) + \beta \max\left\{V^{EX}\left(L_{s+1}\right), V^S\left(\theta_T^*, L_{s+1}\right)\right\} + \\
& C^S\left(L_s^*, L_{s-1}\right) \\
\leq & V^S\left(\theta_T^*, L_{s-1}\right) + C^S\left(L_s^*, L_{s-1}\right) \\
= & \Pi\left(L_s^*, \theta_T^*\right) - C^S\left(L_s^*, L_{s-1}\right) + \beta \max\left\{V^{EX}\left(L_s^*\right), V^S\left(\theta_T^*, L_s^*\right)\right\} + C^S\left(L_s^*, L_{s-1}\right) \\
= & V^{SN}\left(\theta_T^*, L_s^*\right).
\end{aligned}$$

Therefore, at time $s+1$ it is optimal to set $L_{s+1}^* = L_s^*$.

We now give conditions under which the firm does not exit at time $s+1$ after remaining in the industry at time s, $s \geq 0$. Because the firm stays at time s, then $V^S\left(\theta_T^*, L_{s-1}^*, T\right) \geq V^{EX}\left(L_{s-1}^*\right)$. Now assume that in period $s+1$ the firm exits, so that

$$V^S\left(\theta_T^*, L_s^*, T\right) < V^{EX}\left(L_s^*\right) \Leftrightarrow \Pi\left(L_s^*, \theta_T^*\right) < (1-\beta) V^{EX}\left(L_s^*\right).$$

This then implies

$$\begin{aligned}
V^S\left(\theta_T^*, L_{s-1}^*, T\right) & < (1-\beta) V^{EX}\left(L_s^*\right) - C^S\left(L_s^*, L_{s-1}^*\right) + \beta V^{EX}\left(L_s^*\right) \\
& = V^{EX}\left(L_s^*\right) - C^S\left(L_s^*, L_{s-1}^*\right).
\end{aligned}$$

For $s \geq 1$, this is a contradiction. For $s = 0$, we can have $L_T^* \neq L_{T-1}^*$. In order to exclude that possibility for $s = 0$, we must have

$$V^{EX}\left(L_s^*\right) - C^S\left(L_s^*, L_{s-1}^*\right) \leq V^{EX}\left(L_{s-1}^*\right).$$

If $L_s^* \geq L_{s-1}^*$ this inequality holds. If $L_s^* < L_{s-1}^*$ this inequality is equivalent to

$$P^{SD} \geq P^{EX} - F^{SD} \frac{1}{L_{s-1}^* - L_s^*}.$$

Therefore, for $s = 0$ and for all L_{s-1}^*, we must have $P^{SD} \geq P^{EX}$. This means that it must be at least as expensive to fire workers while in the industry than to fire them when exiting from the industry. ■

Proposition 12 *The model of learning with adjustment costs can be equivalently formulated, such that instead of an exit opportunity cost we have a per period operating cost.*

Proof. In order to simplify on notation, we omit any adjustment costs. The formulation with adjustment costs is the following

$$V^S\left(\theta_\tau^*, \tau\right) = \max_{L_\tau} \left\{\Pi\left(L_\tau, \theta_\tau^*\right) - \beta E_\tau \max\left\{W, V^S\left(\theta_{\tau+1}^*, \tau+1\right)\right\}\right\}.$$

Now let $\tilde{V}^S(\cdot) = V^S - W$. Then, it is easily seen that we can write

$$\tilde{V}^S\left(\theta_\tau^*, \tau\right) = \max_{L_\tau} \left\{\Pi\left(L_\tau, \theta_\tau^*\right) - r + \beta E_\tau \max\left\{0, \tilde{V}^S\left(\theta_{\tau+1}^*, \tau+1\right)\right\}\right\},$$

where

$$W = \frac{r}{1-\beta}.$$

■

A.2 Appendix: Computational Algorithm

We present in various steps the algorithm we use to solve the finite learning horizon model.

(i) Discretization and transition probability matrices associated with θ^*:

We discretize θ^* based on a uniform discrete approximation to the (cross-section) distribution of $\theta_T^* = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$, which is $\log N\left(\mu + \frac{1}{2}\sigma^2, \sigma_\mu^2\right)$. We then employ the method of Tauchen (1986) to build the transition matrices associated with this discrete approximation, using Gauss-Legendre quadrature for numerical integration. We use a grid with 25 points.

(ii) Discretization of L

From the decision rules for problem (2.12) in case of hiring and in case of firing, we consider

$$\ln(L) \sim N\left(\mu_L, \sigma_L^2\right),$$
$$\mu_L = \frac{1}{1-\alpha}\left\{\bar{\mu} + \frac{1}{2}\sigma^2 + \ln\left(\frac{\alpha p}{\sqrt{\left[w + P^{SU} + \beta P^{EX}\right]\left[w - (1-\beta) P^{SD}\right]}}\right)\right\},$$
$$\sigma_L^2 = \frac{1}{(1-\alpha)^2}\sigma_\mu^2.$$

For the mean of $\ln(L)$ we assume that if a firm is at the upper end of the grid for L, then it should optimally decrease employment even at θ_{N_θ}, and that if a firm is at the lower end of the grid for L, then it should optimally increase employment and exit next period, even at θ_1. We then find the upper and lower end of the grid for L such that those decisions occur, and use the same procedure to discretize L as the one used for θ, considering 200 gridpoints.

(iii) Choice for T

We choose $T = 15$, and display results until period 10.

(iv) Updating rule for p

In the sensitivity analysis of table 2.5 while changing each parameter we determine a new equilibrium price such that $E\left(V^{EN}\right) - I = W$. The updating rule for p is based on problem (2.12) when we consider C^{EN} instead of C^S. We apply

$$\begin{aligned} p_{i+1} &= p_i + \frac{W + I - E\left(V^{EN}\right)}{\eta\left(p_i\right)} \\ \eta\left(p_i\right) &= \left(\frac{\alpha p E\left(\theta\right)}{w + \left(1-\beta\right) P^{EN}}\right)^{\frac{\alpha}{1-\alpha}} \frac{E\left(\theta\right)}{1-\beta}\beta^x \end{aligned}$$

where x is changed in each iteration in order to improve the approximation. On average, about 6 iterations are needed for convergence.

(v) Basic densities:

The first result contains recursive expressions to compute all relevant densities and moments in the model

Proposition 13 *Consider a generic period τ, and let $NE^{\tau-1}$ represent the event that a firm has survived through period $\tau - 1$, and it has not decided yet if it will remain in period τ, and let $NE_{\tau-1}$ represent the event that a firm will produce in period $\tau - 1$. That is,*

$$NE_{\tau-1} = \left\{\left(\theta^*_{\tau-1}, L_{\tau-2}\right) : \theta^*_{\tau-1} \geq \theta^{EX}_{\tau-1}\left(L_{\tau-2}\right)\right\},$$

$$\begin{aligned} NE^{\tau-1} = \Big\{\left(\theta^*_0, L_0, \theta^*_1, \ldots, L_{\tau-2}, \theta^*_{\tau-1}\right) : \theta^*_0 \geq \theta^{EX}_0, \theta^*_1 \geq \theta^{EX}_1\left(L_0\right), \ldots, \\ \theta^*_{\tau-1} \geq \theta^{EX}_{\tau-1}\left(L_{\tau-2}\right)\Big\}. \end{aligned}$$

where $NE^{\tau-1} = NE^{\tau-2} \cap NE_{\tau-1}$. *Then, for* $\tau \geq 2$

$$f\left(\theta_\tau^*, L_{\tau-1} \mid NE^{\tau-1}\right) =$$
$$\frac{\sum_{NE_{\tau-1}} f\left(\theta_\tau^* \mid \theta_{\tau-1}^*, \tau-1\right) \mathbf{1}\left[L_{\tau-1} = L_{\tau-1}^*\left(\theta_{\tau-1}^*, L_{\tau-2}\right)\right] f\left(\theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}\right)}{\sum_{NE_{\tau-1}} f\left(\theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}\right)}$$

and for $\tau = 1$

$$f\left(\theta_1^*, L_0 \mid NE^0\right) = \frac{\sum_{NE_0} f\left(\theta_1^* \mid \theta_0^*, 0\right) \mathbf{1}\left[L_0 = L_0^*\left(\theta_0^*\right)\right] f\left(\theta_0^*\right)}{\sum_{NE_0} f\left(\theta_0^*\right)}$$

Proof. Note that

$$f\left(\theta_\tau^*, L_{\tau-1} \mid NE^{\tau-1}\right) = \frac{\sum_{NE_{\tau-1}} f\left(\theta_\tau^*, L_{\tau-1}, \theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}\right)}{\sum_{NE_{\tau-1}} f\left(\theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}\right)},$$

and the integrand in the numerator can be computed as

$$f\left(\theta_\tau^*, L_{\tau-1}, \theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}\right) =$$
$$f\left(\theta_\tau^* \mid \theta_{\tau-1}^*, \tau-1\right) f\left(L_{\tau-1}, \theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}\right) =$$
$$f\left(\theta_\tau^* \mid \theta_{\tau-1}^*, \tau-1\right) f\left(L_{\tau-1} \mid \theta_{\tau-1}^*, L_{\tau-2}, NE^{\tau-2}\right) f\left(L_{\tau-1}, \theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}\right) =$$
$$f\left(\theta_\tau^* \mid \theta_{\tau-1}^*, \tau-1\right) \mathbf{1}\left(L_{\tau-1} = L_{\tau-1}^*\left(\theta_{\tau-1}^*, L_{\tau-2}\right)\right) \times f\left(\theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}\right)$$

■

(vii) Decomposition densities:

We define $S_\tau = NE^\tau = NE^{\tau-1} \cap NE_\tau$, $D_\tau = NE^{\tau-1} \cap \widetilde{NE}_\tau$, where $\tilde{A}$ is the complement of A, and $D^\tau = \cup_{s=1}^\tau D_s$. Then,

$$E\left(\ln\left(L_\tau\right) \mid S_\tau\right) = \frac{\sum_{NE_\tau} \ln\left[L_\tau^*\left(\theta_\tau^*, L_{\tau-1}\right)\right] f\left(\theta_\tau^*, L_{\tau-1} \mid NE^{\tau-1}\right)}{\sum_{NE_\tau} f\left(\theta_\tau^*, L_{\tau-1} \mid NE^{\tau-1}\right)}$$

$$E\left[\ln\left(L_0\right) \mid S_0\right] = \frac{\sum_{NE_0} \ln\left[L_0^*\left(\theta_0^*\right)\right] f\left(\theta_0^*\right)}{\sum_{NE_0} f\left(\theta_0^*\right)}$$

$$E\left[\ln\left(L_0\right) \mid S_\tau\right] = \sum_{NE_0} \ln\left[L_0^*\left(\theta_0^*\right)\right] f\left(\theta_0^* \mid NE^\tau\right)$$

where for $\tau \geq 1$

$$f\left(\theta_0^* \mid NE^\tau\right) = \frac{\left[\sum_{NE_\tau} f\left(\theta_\tau^*, L_{\tau-1} \mid NE^{\tau-1}, \theta_0^*\right)\right] f\left(\theta_0^* \mid NE^{\tau-1}\right)}{\sum_{NE_\tau} f\left(\theta_\tau^*, L_{\tau-1} \mid NE^{\tau-1}\right)}$$

and for $\tau \geq 2$

$$f\left(\theta_\tau^*, L_{\tau-1} \mid NE^{\tau-1}, \theta_0^*\right) =$$
$$\frac{\sum_{NE_{\tau-1}} f\left(\theta_\tau^* \mid \theta_{\tau-1}^*, \tau-1\right) \mathbf{1}\left[L_{\tau-1} = L_{\tau-1}^*\left(\theta_{\tau-1}^*, L_{\tau-2}\right)\right] f\left(\theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}, \theta_0^*\right)}{\sum_{NE_{\tau-1}} f\left(\theta_{\tau-1}^*, L_{\tau-2} \mid NE^{\tau-2}, \theta_0^*\right)}$$

and for $\tau = 1$

$$f\left(\theta_1^*, L_0 \mid NE^0, \theta_0^*\right) = f\left(\theta_1^* \mid \theta_0^*, 0\right) \mathbf{1}\left[L_0 = L_0^*\left(\theta_0^*\right)\right] \mathbf{1}\left(\theta_0^* \in NE^0\right)$$

Note also that

$$E\left[\ln\left(L_0\right) \mid D^\tau\right] = \frac{1}{\Pr\left(D^\tau \mid S_0\right)} \left\{E\left[\ln\left(L_0\right) \mid S_0\right] - \Pr\left(S_\tau \mid S_0\right) E\left[\ln\left(L_0\right) \mid S_\tau\right]\right\}$$

(viii) Weighted moments:

Let E_ω represent an employment weighted moment. Then, as an example, we compute the employment weighted mean of log-employment conditional on survival as

$$E_\omega\left[\ln\left(L_\tau\right) \mid NE^\tau\right] = \sum\nolimits_{NE^\tau} \frac{L_\tau^*\left(\theta_\tau^*, L_{\tau-1}\right)}{E\left[L_\tau \mid NE^\tau\right]} \ln\left(L_\tau^*\left(\theta_\tau^*, L_{\tau-1}\right)\right) f\left(\theta_\tau^*, L_{\tau-1} \mid NE^\tau\right).$$

Appendix B

Appendices for Chapter 3

B.1 Appendix: Impulse Control of Brownian Motion

In this appendix, we present some technical details underlying the results in sections 3.2 and 3.3 of chapter 3.[1] An agent wants to control some stock variable, x, so that it is as near as possible to its optimal frictionless value, x^*. The stochastic law governing the evolution of x^* is characterized by an arithmetic Brownian motion process with drift μ and standard deviation σ, that is,

$$dx_t^* = \mu dt + \sigma dw_t,$$

where w_t is a Wiener process. A Wiener process, also called a standardized Brownian motion, is a continuous-time stochastic process whose increment, dw_t, follows a normal distribution with zero mean and variance dt. Therefore, the Brownian motion process x_t^* has increments, dx_t^*, over the infinitesimal time interval, dt, which follow a normal distribution with mean μdt and variance $\sigma^2 dt$.

[1] The references are Cox and Miller (1965), Harrison *et al.* (1983), Harrison (1985), Bertola and Caballero (1990), Dixit (1991b, 1991a, 1993), and Stokey (2003).

Because it is costly to exert control over x_t, in general, it will not be optimal for the agent to make x_t coincide with x_t^* at every point in time. Instead, he will try to make x_t as close as possible to x_t^*, so that the benefits of closely following x_t^* balance the costs of doing so. In the (S,s) model under consideration there are both fixed and proportional adjustment costs, that is

$$C(dx_t) = (C_l + c_l dx_t)\,\mathbf{1}(dx_t > 0) + (C_u - c_u dx_t)\,\mathbf{1}(dx_t < 0),$$

where $\mathbf{1}(\cdot)$ stands for the indicator function.

Following Bertola and Caballero (1990), define $z_t = x_t - x_t^*$ as the excess of current stock over the frictionless optimal stock, and assume that the agent chooses an optimal path for z_t in order to minimize the expected present value of the lost profits due to the presence of adjustment costs. The stochastic process of z can be specified as follows

$$dz_t = dx_t - dx_t^* = -\mu dt + \sigma dw_t + dl_t - du_t,$$

where $\{l_\tau\}$, $\{u_\tau\}$ denote the cumulative upward and downward adjustment on z_t up to time τ.[2] Let the sets of indexes associated with moments in time where there is adjustment be $\{\upsilon \in \mathbb{N} : du_{\tau_\upsilon} > 0, \tau_\upsilon > t\}$, $\{\lambda \in \mathbb{N} : dl_{\tau_\lambda} > 0, \tau_\lambda > t\}$. If $f(z_\tau)$ denotes the instantaneous current value of lost profits due to $z_\tau \neq 0$, then the optimization problem faced by the firm can be represented by:

$$\begin{aligned} V(z) &= \min_{\{u_\tau\}_{\tau\geq t},\{l_\tau\}_{\tau\geq t}} E\left\{\int_t^\infty e^{-\rho(\tau-t)} f(z_\tau)\,d\tau + \int_t^\infty e^{-\rho(\tau-t)}(c_{u,j}du_\tau + c_{l,j}dl_\tau)\right. \\ &\quad \left. + \sum_{\upsilon=1}^\infty e^{-\rho(\tau_\upsilon - t)} C_u + \sum_{\lambda=1}^\infty e^{-\rho(\tau_\lambda - t)} C_l \mid z_t = z\right\} \qquad \text{(B.1)} \end{aligned}$$

[2] Note that, in the absence of adjustment, z follows a Brownian motion process with drift $-\mu$ and standard deviation σ.

The solution to this problem was shown by Harrison *et al.* (1983) to be characterized by four parameters (L, l, u, U), where L is the lower trigger point, l is the lower return point, U is the upper trigger point and u is the upper return point. This means that there is an inaction region between L and U, and that whenever z hits L the stock is increased by $\Delta x_t = l - L > 0$, and whenever z hits U the stock is decreased by $\Delta x_t = u - U < 0$. As shown in Stokey (2003), the value function V satisfies the Hamilton-Jacobi-Bellman equation:

$$\frac{1}{2}\sigma^2 V''(z) - \mu V'(z) - \rho V(z) + f(z) = 0. \tag{B.2}$$

For a given specification of f, the solution to this second-order ordinary differential equation is the difference of two components: the first is the present discounted value of lost profits in the absence of any control over z, and the second is the present discounted value of the reduction in lost profits, minus adjustment costs, due to control.

As shown in Dixit (1993), an analytical solution to V can be found when f is assumed to be a polynomial in z_τ. When $f(z_\tau) = \frac{b}{2}z_\tau^2$, treating (B.1) as a maximization problem, the solution to (B.2) is given by

$$V(z) = -\frac{b}{2}\left\{\frac{2\mu^2}{\rho^3} + \frac{\sigma_j^2 - 2\mu z}{\rho^2} + \frac{z^2}{\rho}\right\} + Ae^{-\alpha z} + Be^{\beta z} \tag{B.3a}$$

$$\alpha = -\frac{\mu - \sqrt{\mu^2 + 2\rho\sigma^2}}{\sigma^2}, \quad \beta = \frac{\mu + \sqrt{\mu^2 + 2\rho\sigma^2}}{\sigma^2} \tag{B.3b}$$

The two unknown constants in the solution to the differential equation and the precise values of (L, l, u, U) are found by solving the following system of equations

$$V(l) - V(L) = C_l + c_l(l - L), \; V(u) - V(U) = C_u + c_u(U - u),$$

$$V'(l) = V'(L) = c_l, \; V'(u) = V'(U) = -c_u,$$

where the two first conditions, called the value matching conditions, determine whether or not the agent adjusts, in the sense that the benefits from adjusting should equal the costs from doing so, and the last four conditions, called the smooth pasting conditions, determine the optimal adjustment size.[3] Because this is a nonlinear system of equations, in general, we can only find the solution numerically.

Using the arguments in Bertola and Caballero (1990), by solving a system of equations, we can find the ergodic distribution for the location of the agent in the z state space. For the case where $\mu \neq 0$, we have the following continuous-time density

$$f_c(z) = \begin{cases} A_1^c + B_1^c \exp\left\{-2\frac{\mu}{\sigma^2} z\right\}, & \text{if } z \in [L, l], \\ A_2^c + B_2^c \exp\left\{-2\frac{\mu}{\sigma^2} z\right\}, & \text{if } z \in [l, u], \\ A_3^c + B_3^c \exp\left\{-2\frac{\mu}{\sigma^2} z\right\}, & \text{if } z \in [u, U], \end{cases}$$

where

$$B_3^c = \frac{\exp\left\{-2\frac{\mu}{\sigma^2}(l+u)\right\} - \exp\left\{-2\frac{\mu}{\sigma^2}(L+u)\right\}}{\begin{array}{c}\left[(U-u)\left(\exp\left\{-2\frac{\mu}{\sigma^2}(L+u+U)\right\} - \exp\left\{-2\frac{\mu}{\sigma^2}(l+u+U)\right\}\right)\right. \\ \left. +(l-L)\left(\exp\left\{-2\frac{\mu}{\sigma^2}(L+l+U)\right\} - \exp\left\{-2\frac{\mu}{\sigma^2}(L+l+u)\right\}\right)\right]\end{array}}$$

$$A_1^c = \frac{\exp\left\{-2\frac{\mu}{\sigma^2}(L+l+u)\right\} - \exp\left\{-2\frac{\mu}{\sigma^2}(L+l+U)\right\}}{\exp\left\{-2\frac{\mu}{\sigma^2}(L+u)\right\} - \exp\left\{-2\frac{\mu}{\sigma^2}(l+u)\right\}} \times B_3^c,$$

$$B_1^c = \frac{\exp\left\{-2\frac{\mu}{\sigma^2}(l+u)\right\} - \exp\left\{-2\frac{\mu}{\sigma^2}(l+U)\right\}}{\exp\left\{-2\frac{\mu}{\sigma^2}(l+u)\right\} - \exp\left\{-2\frac{\mu}{\sigma^2}(L+u)\right\}} \times B_3^c,$$

$$A_2^c = 0,$$

$$B_2^c = \frac{\exp\left\{-2\frac{\mu}{\sigma^2}u\right\} - \exp\left\{-2\frac{\mu}{\sigma^2}U\right\}}{\exp\left\{-2\frac{\mu}{\sigma^2}u\right\}} \times B_3^c,$$

$$A_3^c = -\exp\left\{-2\frac{\mu}{\sigma^2}U\right\} \times B_3^c.$$

[3] Although there are four trigger and return points and six equations, the additional two value matching conditions are needed to determine the two *a priori* unknown constants in V.

For the case where $\mu = 0$, we have the density

$$f_c(z) = \begin{cases} A_1^c + B_1^c z, & \text{if } z \in [L, l], \\ A_2^c + B_2^c z, & \text{if } z \in [l, u], \\ A_3^c + B_3^c z, & \text{if } z \in [u, U], \end{cases}$$

where

$$A_1^c = -\frac{2L}{(l-L)(U+u-l-L)},$$

$$B_1^c = \frac{2}{(l-L)(U+u-l-L)},$$

$$A_2^c = \frac{2}{U+u-l-L},$$

$$B_2^c = 0,$$

$$A_3^c = -\frac{2U}{(u-U)(U+u-l-L)},$$

$$B_3^c = \frac{2}{(u-U)(U+u-l-L)}.$$

We can view a Brownian motion process as the limit of a random walk when the time interval and the step size go to zero simultaneously at appropriate rates. This is a useful property because it enables us to characterize some properties of the Brownian motion process by taking the limit per unit of time of the corresponding property of the random walk process when the time interval goes to zero. We follow Bertola and Caballero (1990) and Dixit (1991a) to approximate the continuous-time process with a discrete-time, discrete state-space Markov chain, which in the limit converges to the above process. Namely, we consider

$$z_{t+\Delta t} = \begin{cases} z_t + \Delta z, & \text{with probability } p_z, \\ z_t - \Delta z, & \text{with probability } q_z = 1 - p_z, \end{cases}$$

where $\Delta z = \sqrt{\sigma^2 \Delta t + \mu^2 (\Delta t)^2}$, $p_z = \frac{1}{2}\left(1 - \mu \frac{\Delta t}{\Delta z}\right)$, and Δt, Δz represent, respectively, the duration of the discrete-time period and the length of each step taken by z in each period. If we are considering only arbitrarily small changes, then we would have $dz = \sigma\sqrt{dt}$, $p_z = \frac{1}{2}\left(1 - \mu \frac{dt}{dz}\right)$.[4, 5]

We now prove that this binomial random walk mimics the main features of the continuous Brownian motion process, namely,

$$E\left(\Delta z_{t+\Delta t}\right) = p_z \Delta z + (1 - p_z)(-\Delta z) = \mu \Delta t$$

$$Var\left(\Delta z_{t+\Delta t}\right) = p_z \left(\Delta z - \mu \Delta t\right)^2 + (1 - p_z)\left(-\Delta z + \mu \Delta t\right)^2 = \sigma^2 \Delta t$$

It is also true that, as $dt \to 0$, with $dz = \sigma\sqrt{dt}$, this random walk converges, in some appropriate way, to the Brownian motion process (3.1) and (3.2).[6] Similarly to the continuous-time case, for given values of (L, l, u, U) we can find the firm's ergodic distribution over z. To do that, we first make a change of scale. Consider $i \in M = \{0, 1, 2, \ldots, m\}$, and let $L_i \equiv 0$, $U_i \equiv m$, l_i, $u_i \in M$. Then we have the following relation between the values assumed by z, in the discrete-time case, and the values assumed by i

$$z(i) = i \times dz + L_z \Leftrightarrow i(z) = \frac{z - L_z}{dz}.$$

Obviously,

$$i(L_z) = L_i,\ i(l_z) = l_i,\ i(u_z) = u_i,\ i(U_z) = U_i.$$

[4] In terms of stochastic calculus, we can ignore all terms of higher order than dt. However, if we were to make a simulation of the model in discrete time, then we should also consider the terms in $(\Delta t)^2$.

[5] Note that the meaning of dz here is different from its meaning in the definition of a Brownian motion. Here it represents the step size of the binomial random walk, and therefore is a deterministic quantity, whether in the definition of the Brownian motion process it was representing the stochastic infinitesimal change in the process.

[6] See, for example, Cox and Miller (1965).

Then, for the case where $\mu \neq 0$, we have the following discrete-time density

$$f_d(z;dz) = \begin{cases} A_1^d + B_1^d \left(\frac{p_z}{q_z}\right)^{\frac{z}{dz}}, & \text{if } z \in \{L_z, \ldots, l_z\}, \\ A_2^d + B_2^d \left(\frac{p_z}{q_z}\right)^{\frac{z}{dz}}, & \text{if } z \in \{l_z, \ldots, u_z\}, \\ A_3^d + B_3^d \left(\frac{p_z}{q_z}\right)^{\frac{z}{dz}}, & \text{if } z \in \{u_z, \ldots, U_z\}, \end{cases}$$

where

$$B_3^d = \frac{\left(\frac{p_z}{q_z}\right)^{\frac{l_z+u_z}{dz}} - \left(\frac{p_z}{q_z}\right)^{\frac{L_z+u_z}{dz}}}{\left[\frac{(U_z-u_z)}{dz}\left(\left(\frac{p_z}{q_z}\right)^{\frac{L_z+u_z+U_z}{dz}} - \left(\frac{p_z}{q_z}\right)^{\frac{l_z+u_z+U_z}{dz}}\right) + \right. \\ \left. + \frac{(l_z-L_z)}{dz}\left(\left(\frac{p_z}{q_z}\right)^{\frac{L_z+l_z+U_z}{dz}} - \left(\frac{p_z}{q_z}\right)^{\frac{L_z+l_z+u_z}{dz}}\right)\right]}$$

$$A_1^d = \frac{\left(\frac{p_z}{q_z}\right)^{\frac{L_z+l_z+u_z}{dz}} - \left(\frac{p_z}{q_z}\right)^{\frac{L_z+l_z+U_z}{dz}}}{\left(\frac{p_z}{q_z}\right)^{\frac{L_z+u_z}{dz}} - \left(\frac{p_z}{q_z}\right)^{\frac{l_z+u_z}{dz}}} \times B_3^d,$$

$$B_1^d = \frac{\left(\frac{p_z}{q_z}\right)^{\frac{l_z+u_z}{dz}} - \left(\frac{p_z}{q_z}\right)^{\frac{l_z+U_z}{dz}}}{\left(\frac{p_z}{q_z}\right)^{\frac{l_z+u_z}{dz}} - \left(\frac{p_z}{q_z}\right)^{\frac{L_z+u_z}{dz}}} \times B_3^d,$$

$$A_2^d = 0,$$

$$B_2^d = \frac{\left(\frac{p_z}{q_z}\right)^{\frac{u_z}{dz}} - \left(\frac{p_z}{q_z}\right)^{\frac{U_z}{dz}}}{\left(\frac{p_z}{q_z}\right)^{\frac{u_z}{dz}}} \times B_3^d,$$

$$A_3^d = -\left(\frac{p_z}{q_z}\right)^{\frac{U_z}{dz}} \times B_3^d.$$

For the case where $\mu = 0$, we have

$$f_d(z;dz) = \begin{cases} A_1^d + B_1^d z, & \text{if } z \in \{L_z, \ldots, l_z\}, \\ A_2^d + B_2^d z, & \text{if } z \in \{l_z, \ldots, u_z\}, \\ A_3^d + B_3^d z, & \text{if } z \in \{u_z, \ldots, U_z\}, \end{cases}$$

where

$$A_1^d = -\frac{2L_z dz}{(l_z - L_z)(U_z + u_z - l_z - L_z)},$$

$$B_1^d = \frac{2dz}{(l_z - L_z)(U_z + u_z - l_z - L_z)},$$

$$A_2^d = \frac{2dz}{U_z + u_z - l_z - L_z},$$

$$B_2^d = 0,$$

$$A_3^d = -\frac{2U_z dz}{(u_z - U_z)(U_z + u_z - l_z - L_z)},$$

$$B_3^d = \frac{2dz}{(u_z - U_z)(U_z + u_z - l_z - L_z)}.$$

B.2 Appendix: Proofs

Proof of lemma 1.

We first assume that L_z, l_z, u_z, and U_z, are chosen so that

$$|L_z - L| < dz,\ |l_z - l| < dz,\ |u_z - u| < dz,\ |U_z - U| < dz.$$

This implies that discrete-time trigger and return points converge to the continuous-time analogs. Now, note that the discrete-time ergodic distribution function is defined as

$$F_d(z; dz) = \begin{cases} 0, & \text{if } z < L_z, \\ \sum_{i=0}^{\left[\frac{z-L_z}{dz}\right]} f_d(i \times dz + L_z; dz), & \text{if } L_z \leq z \leq U_z, \\ 1, & \text{if } z > U_z, \end{cases}$$

where $[x]$ represents the largest integer not higher than x. Note that F_d is continuous from the right, has the usual aspect of a ladder, and given the restriction imposed on f_d (see appendix B.1) $F_d(L_z; dz) = 0$, $F_d(U_z; dz) = 1$. With respect to the continuous-time ergodic distribution, it is defined as

$$F_c(z) = \begin{cases} 0, & \text{if } z < L, \\ \int_L^z f_c(z), & \text{if } L \le z \le U, \\ 1, & \text{if } z > U. \end{cases}$$

It is obvious that, for $z \le L$ or $z \ge U$, $\lim_{dz \to 0} F_d(z; dz) = F_c(z)$. We now prove that it is also true for $z \in [L, l]$, and the remaining cases would be similar. First note that $l_z - dz < l < l_z + dz$, $L_z - dz < L < L_z + dz$. Now, for $z \in [L, l]$, we have

$$\begin{aligned} F_z(z) &= \int_L^z \left(A_1^c + B_1^c \exp\left\{ -2\frac{\mu}{\sigma^2} z \right\} \right) dz \\ &= A_1^c (z - L) + \left(2\frac{\mu}{\sigma^2} \right)^{-1} B_1^c \left(\exp\left\{ -2\frac{\mu}{\sigma^2} L \right\} - \exp\left\{ -2\frac{\mu}{\sigma^2} z \right\} \right), \end{aligned}$$

$$\begin{aligned} F_d(z; dz) &= \sum_{i=0}^{\left[\frac{z-L_z}{dz}\right]} \left(A_1^d + B_1^d \left(\frac{p_z}{q_z} \right)^{i + \frac{L_z}{dz}} \right) = \sum_{i=1}^{\left[\frac{z-L_z}{dz}\right]} \left(A_1^d + B_1^d \left(\frac{p_z}{q_z} \right)^{i + \frac{L_z}{dz}} \right) \\ &= A_1^d \left[\frac{z - L_z}{dz} \right] + B_1^d \left(\frac{p_z}{q_z} \right)^{1 + \frac{L_z}{dz}} \frac{1 - \left(\frac{p_z}{q_z} \right)^{\left[\frac{z-L_z}{dz}\right]}}{1 - \frac{p_z}{q_z}} \\ &= \frac{A_1^d}{dz} dz \left[\frac{z - L_z}{dz} \right] + \frac{B_1^d}{dz} \left(1 - \frac{\mu}{\sigma^2} dz \right) \left(2\frac{\mu}{\sigma^2} \right)^{-1} \times \\ &\quad \left[\left(\frac{p_z}{q_z} \right)^{\frac{L_z}{dz}} - \left(\frac{p_z}{q_z} \right)^{\frac{1}{dz}\left(dz \left[\frac{z-L_z}{dz}\right] + L_z \right)} \right]. \end{aligned}$$

Given that

$$\lim_{dz \to 0} \left(\frac{p_z}{q_z} \right)^{\frac{a}{dz}} = \lim_{dz \to 0} \frac{\left(1 - \frac{\mu}{\sigma^2} dz\right)^{\frac{a}{dz}}}{\left(1 + \frac{\mu}{\sigma^2} dz\right)^{\frac{a}{dz}}} = \exp\left\{ -2\frac{\mu}{\sigma^2} a \right\},$$

and

$$\lim_{dz \to 0} dz \left[\frac{z - L_z}{dz} \right] = z - L,$$

then it is easily shown that

$$\lim_{dz\to 0}\frac{A_i^d}{dz}=A_i^c,\ \lim_{dz\to 0}\frac{B_i^d}{dz}=B_i^c,$$

and

$$\lim_{dz\to 0}F_d\left(z;dz\right)=F_c\left(z\right).$$

■

Proof of proposition 2.

First of all, note that these are quantities per unit of time. Then, for the job creation rate we have in discrete-time

$$E\left(JC\right)_d=\frac{f_d\left(L_z+dz;dz\right)q_z\left(l_z-L_z\right)}{dt}=\frac{f_d\left(L_z+dz;dz\right)\frac{\sigma^2}{2}\left(1+\frac{\mu}{\sigma^2}dz\right)\left(l_z-L_z\right)}{\left(dz\right)^2},$$

given that $dz=\sigma\sqrt{dt}$. Now, we will use the fact that

$$f_d\left(L_z+dz;dz\right)=F_d\left(L_z+dz;dz\right)=F_d\left(L_z+dz;dz\right)-2F_d\left(L_z;dz\right)+F_d\left(L_z-dz;dz\right),$$

since $F_d\left(L_z;dz\right)=F_d\left(L_z;dz\right)=0$. Then, in continuous-time, we will have

$$\begin{aligned}
E\left(JC\right)_c &= \lim_{dz\downarrow 0}E\left(JC\right)_d\\
&= \lim_{dz\downarrow 0}\frac{\lim_{dz_\downarrow 0}\left[F_d\left(L_z+dz;dz_1\right)-2F_d\left(L_z;dz_1\right)+F_d\left(L_z-dz;dz_1\right)\right]}{\left(dz\right)^2}\\
&\quad\times\frac{\sigma^2}{2}\lim_{dz\downarrow 0}\left(1+\frac{\mu}{\sigma^2}dz\right)\left(l_z-L_z\right)\\
&= \lim_{dz\downarrow 0}\frac{\lim_{dz_1\downarrow 0}\frac{F_c(L+dz)-F_c(L+dz-dz_1)}{dz_1}-\lim_{dz_1\downarrow 0}\frac{F_c(L)-F_c(L-dz_1)}{dz_1}}{dz}\times\frac{\sigma^2}{2}\left(l-L\right)\\
&= \lim_{dz\downarrow 0}\frac{f_c\left(L+dz\right)-f_c\left(L\right)}{dz}\frac{\sigma^2}{2}\left(l-L\right)\\
&= f_c'\left(L^+\right)\frac{\sigma^2}{2}\left(l-L\right).
\end{aligned}$$

For the job destruction rate we have in discrete-time

$$E(JD)_d = \frac{f_d(U_z - dz; dz)\, p_z(U_z - u_z)}{dt} = \frac{f_d(U_z - dz; dz)\, \frac{\sigma^2}{2}\left(1 - \frac{\mu}{\sigma^2}dz\right)(U_z - u_z)}{(dz)^2}.$$

For the continuous-time version, we use the fact that

$$f_d(U_z - dz; dz) = -F_d(U_z; dz) + 2F_d(U_z - dz; dz) - F_d(U_z - 2dz; dz),$$

since $F_d(U_z; dz) = F(U_z - dz; dz) = 1$. Then, in continuous-time, we will have

$$\begin{aligned}
E(JD)_c &= \lim_{dz\downarrow 0} E(JD)_d \\
&= -\lim_{dz\downarrow 0} \frac{\lim_{dz_1\downarrow 0}\left[F_d(U_z; dz_1) - 2F_d(U_z - dz; dz_1) + F_d(U_z - 2dz; dz_1)\right]}{(dz)^2} \\
&\quad \times \frac{\sigma^2}{2} \lim_{dz\downarrow 0}\left(1 - \frac{\mu}{\sigma^2}dz\right)(U_z - u_z) \\
&= -\lim_{dz\downarrow 0} \frac{\lim_{dz_1\downarrow 0} \frac{F_c(U) - F_c(U - dz_1)}{dz_1} - \lim_{dz_1\downarrow 0} \frac{F_c(U - dz) - F_c(U - dz - dz_1)}{dz_1}}{dz} \frac{\sigma^2}{2}(l - L) \\
&= -\lim_{dz\downarrow 0} \frac{f_c(U) - f_c(U - dz)}{dz} \frac{\sigma^2}{2}(l - L) \\
&= -f_c'\left(U^-\right)\frac{\sigma^2}{2}(l - L).
\end{aligned}$$

Now, for the variance of the job creation rate, we have

$$\begin{aligned}
Var(JC)_d &= p_a\left[E(JC \mid b)_d - E(JC)_d\right]^2 + q_a\left[E(JC \mid r)_d - E(JC)_d\right]^2 \\
&= \left[E(JC)_d\right]^2 \left\{ p_a \left[\frac{q_a\left(q_{z|b} - q_{z|r}\right)}{q_z}\right]^2 + q_a\left[\frac{p_a\left(q_{z|r} - q_{z|b}\right)}{q_z}\right]^2 \right\} \\
&= \left[E(JC)_d\right]^2 \frac{p_a q_a}{q_z^2} \frac{\sigma_a^2}{\sigma^2} \\
&= \left[E(JC)_d\right]^2 \frac{\left(1 + \frac{\mu}{\sigma_a\sigma}dz\right)\left(1 - \frac{\mu}{\sigma_a\sigma}dz\right)}{\left(1 + \frac{\mu}{\sigma^2}dz\right)^2} \frac{\sigma_a^2}{\sigma^2}.
\end{aligned}$$

where the second line comes from $E(JC \mid i)_d = \frac{f_d(L_z + dz) q_{z|i}(l_z - L_z)}{dt} = E(JC)_d \frac{q_{z|i}}{q_z}$,

$i = b, r$, and $q_z = p_a q_{z|b} + q_a q_{z|r}$, and the third line uses $q_{z|b} - q_{z|r} = \frac{\sigma_a}{\sigma}$. Finally, for the variance of the job destruction rate, we have

$$
\begin{aligned}
Var\left(JD\right)_d &= p_a \left[E\left(JD \mid b\right)_d - E\left(JD\right)_d\right]^2 + q_a \left[E\left(JD \mid r\right)_d - E\left(JD\right)_d\right]^2 \\
&= \left[E\left(JD\right)_d\right]^2 \left\{ p_a \left[\frac{q_a\left(p_{z|b} - p_{z|r}\right)}{p_z}\right]^2 + q_a \left[\frac{p_a\left(p_{z|r} - p_{z|b}\right)}{q_z}\right]^2 \right\} \\
&= \left[E\left(JD\right)_d\right]^2 \frac{p_a q_a}{p_z^2} \frac{\sigma_a^2}{\sigma^2} \\
&= \left[E\left(JD\right)_d\right]^2 \frac{\left(1 + \frac{\mu}{\sigma_a \sigma} dz\right)\left(1 - \frac{\mu}{\sigma_a \sigma} dz\right)}{\left(1 - \frac{\mu}{\sigma^2} dz\right)^2} \frac{\sigma_a^2}{\sigma^2}.
\end{aligned}
$$

where the second line comes from $E\left(JD \mid i\right)_d = \frac{f_d(U_z - dz) p_{z|i} (U_z - u_z)}{dt} = E\left(JD\right)_d \frac{p_{z|i}}{p_z}$, $i = b, r$, and $p_z = p_a p_{z|b} + q_a p_{z|r}$, and the third line uses $p_{z|r} - p_{z|b} = \frac{\sigma_a}{\sigma}$. The result for the continuous-time versions of the variances is now obvious. ■

Proof of proposition 4.

The first two results for the job creation rate and job destruction rate are obvious. For the standard deviations, note that, for example for the job creation case

$$
\begin{aligned}
Var\left(JC\right) &= p_a \left[E\left(JC \mid b\right) - E\left(JC\right)\right]^2 + q_a \left[E\left(JC \mid r\right) - E\left(JC\right)\right]^2 \\
&= p_a q_a \left[E\left(JC \mid b\right) - E\left(JC \mid r\right)\right]^2,
\end{aligned}
$$

since $E\left(JC\right) = p_a E\left(JC \mid b\right) + q_a E\left(JC \mid r\right)$. Therefore, because $E\left(JC\right) = pE\left(JC\right)_1 + \left(1-p\right) E\left(JC\right)_2$, we have the result that

$$
\begin{aligned}
std\left(JC\right) &= \sqrt{p_a q_a} \left\{ p \left[E\left(JC \mid b\right)_1 - E\left(JC \mid r\right)_1\right] + \right. \\
& \qquad \left. \left(1-p\right)\left[E\left(JC \mid b\right)_2 - E\left(JC \mid r\right)_2\right] \right\} \\
&= p \times std\left(JC\right)_1 + \left(1-p\right) \times std\left(JC\right)_2.
\end{aligned}
$$

■

Proof of corollary 5.

The first result follows directly from propositions 2 and 4. The second result follows from

$$Cov\left(Rea, Net\right) = Var\left(JC\right) - Var\left(JD\right)$$

and proposition 4. ■

Appendix C

Appendices for Chapter 4

C.1 Appendix: The *Quadros de Pessoal* Database

In this appendix, we discuss the methods used in the empirical exercises on job reallocation with *Quadros de Pessoal* (*QP*). *QP* is a Portuguese longitudinal database containing annual information on workers, establishments and firms, and covering the period 1985-2000. The database originates from a mandatory annual survey run by the Ministry of Employment, and it covers all economic entities, excluding public administration, with at least one worker. Information refers to March up to 1993, and to October since the reformulation of the survey in 1994. The database is composed of three linkable datasets (separated in annual files): the dataset on workers, the dataset on establishments, and the dataset on firms. The dataset on firms covers the period 1985-2000, and in each year it includes an average of 250,000 firms. The variables are the following:

(i) Firm ID: A unique identifier given to a firm when it first answers the survey. Some procedures are used to avoid giving a new identification number to an already existing firm, based mainly on the location of its main establishment.

(ii) Location: It contains information on the firm location up to the second of three administrative territorial divisions.

(iii) CAE (SIC): An industrial classification of the firm main activity equivalent to the 4-digit SIC code. During the reference period, this classification has changed from "CAE Rev. 1" to "CAE Rev. 2", starting in 1995.

(iv) Legal nature: It contains information on the firm legal status (public firm, corporation, joint stock company, partnership, etc.).

(v) Sales: The value, at current prices, of shipments by all firm's establishments.

(vi) Year of birth: The year in which the firm was created (starting only in 1995).

(vii) Common stock: The shareholders' equity minus retained earnings.

(viii) Common stock structure: A decomposition of the shareholders' common stock into public, private, and foreign owners.

(ix) Number of establishments: The number of establishments the firm currently holds (starting only in 1994).

(x) Number of employees: The number of employees in all establishments held by the firm.

Two notes of caution concerning the firms dataset are: first, for each year the *common stock* and *sales* variables are missing for around 40% and 10% of all firms, respectively; second, a significant fraction of firms change their *CAE* at some point in time, but most of these changes keep them in the same 3 or 4–digits *CAE*.

The dataset on establishments spans the period 1985-2000, and on average includes 300,000 establishments per year. This dataset includes these variables:

(i) Firm ID: The unique identifier of the firm the establishment is included in (see above).

(ii) Establishment ID: The unique establishment identifier within the firm Similar procedures to avoid duplicate entries/exits are employed here, but, apparently, are much less successful.

(iii) Location: The establishment location (see above).

(iv) CAE (SIC): The establishment industry specialization (see above).

(v) Number of employees: The number of employees at the establishment.

Some notes about this dataset are: first, only 5% of all establishments pertain to multi-establishment firms, but these make a more significant share of total employment since they belong to larger firms; second, even though the Ministry of Employment staff implements routines to avoid false entries and false exits of establishments, in the dataset there is evidence that they still occur, especially for multi-establishment firms and around 1990, precisely the year in which the workers dataset was not processed.

The dataset on workers covers the periods 1985-1989 and 1991-2000, and it includes an average of 2,500,000 workers per year. The variables are the following:

(i) Firm ID: The unique identifier of the firm the worker is employed in (see above).

(ii) Establishment ID: The unique identifier (within the firm) of the establishment the worker is attached to (see above).

(iii) Worker ID: The unique worker identifier, which is given by its social security number.

(iv) Sex: The sex of the worker.

(v) Position: It contains information on the situation of the worker at the firm (boss, non-self-employed worker, family worker, etc.).

(vi) Education: The higher educational degree attained by the worker.

(vii) Part/Full-Time: It tells whether the employee is working at full-time, at part-time, or with no defined schedule.

(viii) Date of birth: The worker's date of birth.

(ix) Date of hiring: The date when the worker was hired by the firm.

(x) Date of promotion: The date of the last promotion of the worker within the firm.

(xi) Wages: The amount of regular wages, in current prices, earned by the worker.

(xii) Tenure wages: The amount of compensation, in current prices, earned by the worker that is due to the tenure duration in the current qualification.

(xiii) Overtime wages: The amount of overtime wages, in current prices, earned by the worker.

(xiv) Grants: The amount of grants, in current prices, received by the worker, decomposed into regular and irregular grants.

(xv) Regular hours: The number of hours the employee should normally work.

(xvi) Regular hours worked: The actual number of regular hours the employee worked.

(xvii) Overtime hours worked: The number of overtime hours the employee worked.

(xviii) Profession: The worker's profession as given by a national classification of professions.

(xix) Qualification: The qualification associated with the worker's job at the firm.

(xx) Labor agreement: The specific collective labor agreement under which the workers connection to the firm is partially regulated.

(xxi) Professional category: The worker's professional category as specified in the collective labor agreement by which he is covered.

Given the highly detailed information contained in this dataset, its quality is questionable in some aspects. First, some variables, such as *wages* and *date of hiring*, are missing for about 10% of the workers in each year. Second, the other components of worker compensation besides *wages* only exist for a smaller fraction of the population of workers (at maximum 50%, and usually around 15%).

Notwithstanding these remarks, the *QP* is a valuable source of information to study macroeconomic aspects associated with the reallocation of labor over time, industrial economics topics such as the size distribution of firms, and labor economics questions such as the determinants of earnings. One shortcoming of this database is the absence of information of the firm's capital stock and materials usage, which does not enable an analysis of multifactor productivity. Even the analysis of labor productivity is subject to the problem that some industries and types of firms register a high incidence of missings

C.2 Appendix: Procedures Used in Analysis of *Quadros de Pessoal*

Definitions

We define the rates of job creation (JC) and job destruction (JD), both for continuing and entering establishments/firms as in Davis and Haltiwanger (1990). Let i stand for the i-th unit (firm or establishment), s for the sector the unit is in, and t for the time period in question. Then the job creation rate among all units, JC_{st}, among continuing units, JCC_{st}, and among births, JCB_{st}, are defined by

$$
\begin{aligned}
JC_{st} &= \sum_{i \in I_{st},\ g_{ist}>0} \frac{X_{ist}}{X_{st}} g_{ist}, \\
JCC_{st} &= \sum_{i \in IC_{st},\ g_{ist}>0} \frac{X_{ist}}{X_{st}} g_{ist}, \\
JCB_{st} &= \sum_{i \in IB_{st},\ g_{ist}>0} \frac{X_{ist}}{X_{st}} g_{ist},
\end{aligned}
$$

$$X_{ist} = \frac{1}{2}\left(L_{ist} + L_{is,t-1}\right), \qquad X_{st} = \sum_{i \in I_{st}} X_{ist} = \frac{1}{2}\left(L_{st} + L_{s,t-1}\right)$$

$$g_{ist} = \left(L_{ist} - L_{is,t-1}\right)/X_{ist}$$

where $JC_{st} = JCC_{st} + JCB_{st}$, L_{ist} is the number of employees at unit i, in sector s and period t, X_{ist} is the size of unit i at time t, defined as the average employment in periods t and $t-1$, L_{st} and X_{st} have similar definitions but at the sectoral level, I_{st} is the set of units relevant for job flows in sector s at time t, IC_{st} the set of units in sector s at time t that remained in activity from the previous period, and IB_{st} the set of units that started activity at time t in sector s.

Similarly, the job destruction rates among all units, JD_{st}, among continuing units, JDC_{st}, and among deaths, JDD_{st}, are defined by

$$
\begin{aligned}
JD_{st} &= \sum_{i \in I_{st},\ g_{ist}<0} \frac{X_{ist}}{X_{st}} g_{ist}, \\
JDC_{st} &= \sum_{i \in IC_{st},\ g_{ist}<0} \frac{X_{ist}}{X_{st}} g_{ist}, \\
JDD_{st} &= \sum_{i \in ID_{st},\ g_{ist}<0} \frac{X_{ist}}{X_{st}} g_{ist},
\end{aligned}
$$

where $JD_{st} = JDC_{st} + JDD_{st}$, and ID_{st} is the set of units that exited at time t in sector s.

Entering and Exiting Units

We select entering/exiting units at time t by requiring that $t/t-1$ was the earliest/latest period their ID showed up in the dataset (with positive employment).[1] Because there is some incidence of temporary exits, especially among establishments, we recover all units with only one missing value, and exclude all others in all years with missing values. For the recovered units, the missing value is taken to be the average of the two closest years.

However, in the case of establishments, we notice abnormal values of JCB and JDD in some years. This is especially true around 1990, precisely the year in which the workers dataset was not processed by the Ministry staff.[2] In order to reduce the amount of false creation and destruction of establishments we adopt the following rule. We exclude from period t births and deaths all establishments belonging to firms that in periods $t-2$, $t-1$, t, and $t+1$, have a maximum of simultaneously operating establishments less than the total number of establishments operating in those years.[3] With this exclusion rate, we are able to obtain smoother values for JCB and JDD.

Year 1994

In year 1994 the reference month of the survey becomes October instead of March. In order to reduce proportionally the amount of job flows, we create a new employment variable referring to March 1994. With probability 7/19 this new variable is randomly assigned the value in March 1993, and with probability 12/19 it is randomly assigned

[1] This method is equivalent to selecting entering units in year t as those units whose ID was higher than the highest ID in $t-1$.

[2] Among establishments, in year 1991 we get $JCB = 25.9\%$, and $JDD = 24.4\%$.

[3] For year 1986, the rule only includes years 1985 to 1987, and for 2000, the rule only includes years 1998 to 2000.

the value in October 1994. That is

$$npess94a = \begin{cases} npess94, & \text{if } u \leq \frac{12}{19}, \\ npess93, & \text{if } u > \frac{12}{19}. \end{cases}$$

Harmonization of *CAE* Codes

The *CAE* industry classification system was revised in 1995. Therefore, to enable the longitudinal analysis by economic sector, we adopt the following procedure. First, we reduce the amount of miscoding by converting all 6-digits CAE Rev. 1 codes into 4-digits CAE Rev. 1 codes. We do this in the *CAE* variable from 1985 to 1994, and construct the correspondence table between 6-digits CAE Rev. 2 codes and 4-digits CAE Rev. 1 codes. Second, we use the information in 1994 and 1995 to construct a probability transition matrix associated with the transformed equivalence table. Third, using the transformed equivalence table, for each 5-digits CAE Rev. 2 codes, we list all possible 4-digits CAE Rev. 1 codes. Starting in 1995, and going iteratively until 2000, we first select the correctly entered CAE Rev. 2 codes, and check if in the previous year the unit has one of the 4-digits CAE Rev. 1 codes appearing in the transformed equivalence table. If that is the case, it becomes the firm's equivalent 4-digit CAE Rev. 1 code for the current year. If that is not the case, namely for new births, then we use the transformed equivalence table to randomly select the 4-digits CAE Rev. 1 code from the set of possible codes associated with the current year 5-digits Rev. 2 code. Finally, because there is some incidence of incorrectly entered CAE codes, for those 5-digits Rev. 2 codes that are miscoded, we first convert them into 3-digits Rev. 2 codes and then apply the same procedure as above, but now using the equivalence table between 3-digits CAE Rev. 2 and 4-digits CAE Rev. 1 codes.

This method seems to be very efficient for correctly allocating units to economic sectors from 1995 to 2000, as we conclude by looking at a large number of units.

Proxies for Units' Age

Concerning the age of each unit, we have two proxies available: the *year of birth* variable from the firms dataset and the *year of hiring* from the workers dataset. With respect to the *year of birth* we only have observations from 1995 until 2000. We assume the age of the firm to be the mode across these years, and try to use all firms that enter before 1995, survive at least until 1995, and have nonmissings for the *year of birth* variable.

With respect to the *year of hiring*, we first correct and or omit the variable for erroneous entries, and proceed in two steps. First, for each firm we calculate the mode, across all years, for each worker with a valid id. Then we take the minimum across all workers to be the year of entry by the firm. For those firms that do not have any worker with a valid id, we select the minimum *year of birth* across all workers in each year, and then obtain the mode of this minimum across all years.

In the end, we use the proxy constructed from the *year of hiring* variable, since the proxy constructed from the *year of birth* refers to the firm, and not the establishment, and is only available from 1995 onwards, leaving only 6 years for analysis.

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