

ABSTRACT

Title of Dissertation: THE EFFECTS OF SELF-SIMILAR
TRAFFIC ON THE PERFORMANCE OF
PLAYTHROUGH RING NETWORKS

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PLAYTHROUGH ring performance is studied with self-similar traffic patterns. Measurements made by others on local networks connected to the Internet have shown that TCP traffic is self-similar. Self-similar traffic can be generated using heavy-tailed distributions. In particular, others have shown that the Weibull distribution provides a good fit for TCP connection interarrival times. The Weibull distribution, with specific parameters measured in real networks, is used to simulate the operation of the PLAYTHROUGH ring under self-similar traffic.

Simulation results reveal that the mean waiting time performance of the PLAYTHROUGH ring under self-similar traffic is markedly worse than that of hitherto assumed traffic patterns using the exponentially distributed interarrival

times and geometrically distributed message lengths. Furthermore, it appears that, in general, mean waiting times are significantly greater for PLAYTHROUGH under exponential interarrival times and Weibull-distributed message lengths than in the case when message interarrival times and message lengths are assumed to be Weibull-distributed and geometrically distributed, respectively.

An analytical model is derived for various PLAYTHROUGH ring performance metrics under the assumption of exponential interarrivals and Weibull-distributed message lengths, including the moments of the number of minipackets, control frame round trip time, transmission time, service time, blocking duration, and waiting time. When Weibull interarrival times are assumed, finding an analytical model for waiting times is a seemingly intractable problem because the Laplace transform of the Weibull distribution does not have a closed form. However, it is shown that, under heavy loads, mean waiting times under the assumption of exponentially distributed interarrival times and geometrically distributed message lengths are, in general, a lower bound on mean waiting times under the assumption of Weibull interarrivals and geometrically distributed message lengths. Moreover, under heavy loads, mean waiting times under the assumption of exponentially distributed interarrival times and Weibull message lengths are, in general, upper bounds on mean waiting times under the assumption of Weibull interarrivals and geometrically distributed message lengths.

This work provides the first analytical approximation that predicts the performance of PLAYTHROUGH ring under self-similar traffic. In fact, no prior analytical model exists for any ring network under self-similar traffic, including TOKEN ring.

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OF PLAYTHROUGH RING NETWORKS

by

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DEDICATION

To my parents, Lucien and Marcelline Wantou-Siantou, for their unfaltering
love and their indefatigable support.

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CHAPTER 1

INTRODUCTION

1.1 Motivation and Overview of Ring Networks

The ever increasing and widespread use of the Internet, the World Wide Web, and other distributed computer systems for real-time computations and multimedia applications requires high speed and reliable data transmission.

One way of achieving fast data transmission rates is through the use of optical fibers. Optical fibers have several advantages over copper media. In particular, security, reliability, and performance are all enhanced with the use of optical fibers. They have low attenuation and as such can be used for communications over a long distance. In addition, optical fibers have a high bandwidth. Optical fibers do not emit electrical signals and, therefore, cannot be tapped, unlike copper media, which can be tapped easily, allowing access to the data that is being transmitted in the medium. Moreover, fiber is immune to electrical interference from radio frequency interference and electromagnetic interference. All these features make optical fiber a very attractive medium for data communications and computer networks. However, low loss optical signal splitting is difficult to perform effectively, so most practical optical links involve point-to-point connections. This prevents an easy fiber optic implementation for bus network configurations. On the other hand, ring networks are very suitable for fiber optics communications. Stations are arranged in a circle. Each station

receives data from the station on its left, and transmits data to the station on its right. In ring networks, the links between stations are point-to-point links and they can take advantage of the high throughput, the efficiency and the high performance of optical fibers. Because of their ability to take advantage of the superior performance provided by optical fibers, the study of ring networks is important and of great interest.

Several types of rings networks have been proposed, including token rings, slotted rings, register insertion rings, and circuit-switched rings.

The Token ring (*e.g.*, Farmer and Newhall Loop [FN69], IEEE Token ring [Ins85], Fiber Distributed Data Interface (FDDI) [Ros86]) uses a token that is passed from station to station. A station must await the token before it can transmit its message. When a station is finished transferring its message, it releases the token to the next station. Packets are circulated around the ring until they reach their destination, using stations between their source and their destination as repeaters. After reaching their destination, packets must travel back to their source and be removed. Consequently, only one station may transmit at a time on token ring.

On slotted rings (*e.g.*, Pierce Loop [Pie72], Distributed Computing System (DCS) [FFH⁺73], Cambridge Ring [WW79]), the ring is divided into an integer number of fixed-size segments called slots. Each slot is marked as either containing data or empty. The slots are circulated perpetually around the ring. When a station wishes to transfer a packet, it fragments the packet into minipackets that can fit into slots and waits for the passing of an empty slot by the station. Upon seeing an empty slot, the station inserts a minipacket into the empty slot. At its arrival at the destination, the minipacket is copied into the destination buffer. On slotted rings, either source removal or destination

removal of transferred packets can be used. But typically packets are removed by the source.

Register (or buffer) insertion rings (*e.g.*, DLCN [Liu78], [LB77], [BLP77], MetaRing [CO93]) provide concurrent transfer of multiple variable length packets. Two shift registers are used: an insertion register for packets sourced at this station, and a variable length delay buffer (register) to temporarily store incoming message bits while the station inserts its own data packet onto the ring.

Circuit-switched rings (*e.g.*, Jafari's New Loop [JLS80], Leventis New Loop [LPK82], Parallelring [QLL92], Pipeline ring [WY94], PLAYTHROUGH ring [WS79], [WS80a], [Sil86a], [GSSW88], [SGP92], [CS95]) use virtual or dedicated circuits for packet transfer. When a source station wishes to transmit to a destination station, a virtual circuit is established between the two stations. All links and stations in between the source and the destination station are dedicated for the duration of the packet transfer, after which time the circuit is destroyed. On circuit-switched rings, multiple packets can be transferred simultaneously over contiguous, non overlapping ring segments. The ability for multiple packets to be transmitted simultaneously on circuit-switched rings is called spatial reuse and represents an important advantage over simple token rings. As a result of spatial reuse, throughput is increased on circuit-switched rings. With a physical layer based on fiber optics, circuit-switched rings can achieve very high throughput and be a very attractive solution for high speed and multimedia applications.

The PLAYTHROUGH protocol is a circuit-switched medium access control protocol that has been specified [Ste91], [SS92] to operate with the physical layer specification of the commercially available fiber implementation of the

FDDI token ring network. Chai [CS94], [CS95], [Cha95] completed a performance analysis of the PLAYTHROUGH ring carrying multimedia traffic and demonstrated the PLAYTHROUGH ring's suitability for modern network applications. The PLAYTHROUGH ring therefore deserves further research.

The following is a brief description of the PLAYTHROUGH ring. A control frame, comprising a leading *FLAG* character, a variable number of control messages, and a trailing *GO* character, circulates perpetually around the ring. *START* and *STOP* control messages are inserted into the control frame, just ahead of the *GO* character, for the initiation and destruction of virtual circuits using a form of fixed-length register insertion. While packet data is removed from the ring by the packet's destination station, control messages circulate once around the entire ring and are removed by their source stations.

1.2 Objective

The performance of the PLAYTHROUGH ring has been studied extensively [Sil85], [Sil86b], [SW87], [SG88a], [SG88b], [YSG89], [GS89], [Gha89] [CS95], [CS94], [Cha95], [Hen98]. However, all prior studies have used Markovian assumptions for traffic patterns, whereby the interarrival times between packets follow an exponential distribution and packet lengths are exponentially distributed or geometrically distributed. However, it has been shown that Markovian assumptions are not consistent with self-similar traffic. Self-similar traffic displays high variability at all time scales, which is not observed in Markovian-type traffic [LTWW94], [PF95]. Heavy-tailed distributions such as the Weibull, the Pareto, and the Lognormal distribution can be used to model self-similar traffic because of their high variability over many time scales. Anja Feldmann [Fel00] showed through measurements on real

network traffic that TCP traffic connection interarrivals can best be modelled with the Weibull distribution. Deng [Deng96] also independently showed that the Weibull distribution gives a good fit for Web connection interarrivals. It has also been shown that the reliable transfer of files whose sizes are drawn from heavy-tailed distributions generates self-similar traffic [PKC00], [PKC96a]. Our goal in this research is to study the performance of the PLAYTHROUGH ring under self-similar traffic. We use discrete event simulations and we find analytical models for performance metrics of the PLAYTHROUGH that will ease the study of the performance of PLAYTHROUGH ring.

1.3 Contribution

This research makes original contributions in several areas. First, the performance of PLAYTHROUGH rings under self-similar traffic is simulated and compared with the performance of PLAYTHROUGH rings under Markovian assumptions. This provides understanding as to how self-similar traffic affects the performance of PLAYTHROUGH ring and it reveals the shortcomings of prior models. Second, analytical models are developed for various performance metrics of the PLAYTHROUGH ring under self-similar traffic. Simulations of the performance of the PLAYTHROUGH ring under self-similar traffic requires an extensive amount of time. Analytical models provide a much faster way to study the performance of PLAYTHROUGH ring under self-similar traffic. This is especially important in the context of self-similar traffic because the high variability of self-similar traffic implies that simulations take a long time to reach steady state. An analytical model is provided for various performance metrics of the PLAYTHROUGH ring, including mean waiting times, when it operates under exponential interarrival

times and Weibull message lengths. Finding an analytical model for waiting times on the PLAYTHROUGH ring when it operates under Weibull message interarrival times and geometric message lengths is seemingly intractable because the Laplace transform of the Weibull distribution does not have a closed form. Nonetheless, we provide heavy-loads upper and lower bounds for message waiting times when PLAYTHROUGH ring operates under Weibull interarrival times and geometric metric lengths.

1.4 Organization of the Dissertation

In Chapter 2, we give an overview of the PLAYTHROUGH ring and its operation. We subsequently discuss the topic of self-similar traffic. We describe self-similar traffic and present evidence and causes for this type of traffic from the literature. In addition, we discuss work done on the characterization of self-similar traffic. In particular, we describe the work reported by Anja Feldmann [Fel00] on the characterization of TCP connection interarrivals. Feldmann's work shows that TCP connection arrivals are self-similar and can best be modelled using the Weibull distribution.

In Chapter 3, based on the results of Feldmann, we present simulation results for message mean waiting times when the PLAYTHROUGH ring operates with TCP-like message traffic, whereby the message interarrival process is self-similar and can be modelled using the Weibull distribution. We complement those results with simulations of the PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths, motivated by the work of Paxson, Kim, and Crovella showing that the reliable transfer of files whose sizes are drawn from a heavy-tailed distribution causes self-similar traffic [PKC96a], [PKC96b].

In Chapter 4, we give analytical models for many performance metrics of PLAYTHROUGH ring under exponential interarrival times and Weibull distributed message lengths. In particular, we find a tight upper bound for the mean number of round trips of the control frame. Approximations for the second and third moments of the number of round trips of the control frame are also found. Subsequently, we give an analytical model for the moments of transmission time and the moments of service time. An elaborate model for blocking duration is presented, and mean waiting time is derived.

In Chapter 5, we show that, at heavy load, mean waiting time for messages in PLAYTHROUGH rings under Weibull message interarrival times and geometric message lengths has as upper bound mean waiting time for PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths. We also show that mean waiting time for messages in PLAYTHROUGH ring under Weibull message interarrival times and geometric message lengths has as lower bound mean waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths.

CHAPTER 2

LITERATURE REVIEW

2.1 PLAYTHROUGH Ring

A PLAYTHROUGH ring allows stations to transfer messages concurrently. Multiple messages can be transmitted simultaneously over contiguous but nonoverlapping ring segments. A station participates in a PLAYTHROUGH ring through its ring interface unit (RIU). There are three classes of messages in a PLAYTHROUGH ring having varying priorities. From the lowest to highest priority these classes are data messages, the synchronizing token (called GO), and update control messages. Update control messages include *START/STOP* messages and acknowledgement messages. Data messages are user data to be sent at a particular node. The token and update control messages are used to implement the PLAYTHROUGH protocol. A message with a higher priority can preempt, i.e. insert itself ahead of, a message with a lower priority.

At any given time, the status of a station in a PLAYTHROUGH ring can either be a source, a destination, a bridge (i.e. a repeater) or an idle station. Messages travel from a source to a destination and stations between a source and a destination are called bridges. An idle station is neither a source nor a destination nor a bridge. In a ring made of N stations, stations are numbered consecutively from 0 to $N-1$, and a station's position number is called its address. Each station's RIU maintains two registers, a status register and a

range register. The status register is used to keep the status of a station, either source, destination, bridge, or idle. A given station keeps another station's address in its range register if all stations between the two are idle; in other words, if the links between the two stations are free. A station can send a data message to any station within the range specified by its range register. At ring start up, all stations are idle and each station's range register includes all other stations by having its own address in its range register.

A control frame bracketed by a leading *FLAG* and a trailing *GO* symbol is circulated perpetually around the ring, and each station receives the *FLAG* character at regular time intervals. When a station has a message to transmit, it first checks its range register to ensure that the destination of the message is in its range. If it is, the station waits for the *GO* symbol. When *GO* arrives, the station inserts an update control message in front of it. The update control message includes the station's own address, (the source of the message), the destination station's address, and a *START* command to initiate a virtual circuit between the source and the destination. The *GO* symbol preceded by the control message continues to circulate around the ring until it reaches the destination station, updating status and range registers of the stations on its path and establishing a virtual circuit between the source and the destination. At the destination, the *START* control message is changed into an acknowledgement message and continues to travel around the ring back to the source, updating registers on its path. When the acknowledged *START* control message arrives back at the source, the source removes the control message and continues sending the data message characters to the destination right after *GO* departs the station. The destination station is in charge of removing the data message from the ring when it receives it. After all characters of the data

message have been transmitted, the source node waits for the return of *GO*. When it arrives, the source station transmits a *STOP* control message to inform all downstream stations that it is releasing the virtual path dedicated to the data transfer. When the *STOP* control message arrives at the destination, it is changed into an acknowledgement message and sent back to source, updating registers on its path. Upon receipt of the *STOP* control message, the source removes it from the ring.

In our study of the PLAYTHROUGH ring, several performance parameters are of interest to us, including data message waiting time, service time, and transmission time, and blocking duration. The waiting time or queuing time is measured from the time a message arrives at a particular station's RIU until the time its RIU receives the *GO* character and inserts a *START* message to begin the transmission of the message. The transmission time is measured from the insertion the *START* message until the time the last bit of the data message is sent. The service time is measured from the insertion of the *START* message until the return of the *STOP* message to the source for removal.

2.2 Self-similar Traffic

A self-similar phenomenon displays structural similarities across a wide range of time scales. Objects that possess the self-similar property are sometimes referred to as fractals. In the context of networking, self-similarity refers to a process that displays high variability (or burstiness) at all time scales. The work of Leland, Taqqu, Willinger, and Wilson [LTWW94] on the self-similar nature of network traffic spurred an explosion of work on this phenomenon. In particular, work has been done in the area of measurement-based traffic modeling and in the area of physical modeling. In the area of

measurement-based traffic modeling, traffic traces from physical networks have been collected and analyzed to detect, identify, and quantify traffic characteristics. They have shown that scale-invariant burstiness or self-similarity is a widespread phenomenon found in local-area networks, wide-area networks, IP and ATM protocol stacks, copper and fiber optical communications. Leland, Taqqu, Willinger, and Wilson [LTWW94] showed self-similarity in a LAN environment (Ethernet); PAXON and Floyd [PF94], [PF95] demonstrated self-similar burstiness in Wide Area Network Internet Protocol (IP) traffic. Crovella and Bestavos [CB96] showed self-similarity in World Wide Web (WWW) traffic. Feldmann [Fel00] showed that TCP connection arrival processes are self-similar. In addition, Feldmann [Fel00] showed that distributions with heavy-tails give a better fit for connection interarrival times than Markovian distributions. Prior to the explosion of work on traffic self-similarity, Markovian assumptions were the paradigm for studying the performance of voice and data traffic in communications systems. One of the reasons why Markovian models have been cherished in the communications and the performance analysis community is that they have the advantage of tractability and are sometimes amenable to exact analysis. Indeed, queueing theory involving Markovian inputs is a very mature area. Closed-form solutions exist for many queueing problems involving Markovian distributions. Many local area networks and wide area networks have been studied extensively and modeled using Markovian assumptions. The problem with these models is that they are not consistent with self-similar traffic.

In the area of physical modeling, attempts have been made at explaining the physical causes of self-similarity in network traffic based on network mechanisms and empirically established properties of distributed systems. For

example, one cause of self-similarity has been attributed to the reliable transfer of files drawn from heavy-tailed distributions [PKC96a], [PKC96b]. This causal relationship is important because there is strong empirical evidence based on file system measurements that UNIX file systems sizes are heavy-tailed.

2.2.1 Self-similarity and Burstiness

Self-similar traffic is bursty across a wide range of time scales. The traffic measure in question can be packet interarrival times, throughput, etc. The burstiness of self-similar traffic over many time scales implies high variability over many time scales. Examinations have shown that Internet traffic is highly variable over a wide range of time scales [PF94], [PF95]. The variability over wide time scales means that bursts do not average out over long enough time scales. This is in sharp contrast with what is observed in the case of Markovian traffic, where bursts average out over long enough time scales. Figure 2.1, taken from [WP98], compares the self-similar behavior of Internet traffic arrivals to that of voice (Poisson) traffic arrivals. It shows a comparison between Markovian traffic and self-similar traffic. The fact that bursts do not average out implies that, unlike voice networks, the Internet cannot be engineered to reduce ill-effects such as packet loss below any desired threshold, since bursts occur at all time scales.

2.3 Characteristics of TCP Connection Arrivals and Its Impact

2.3.1 Packet Arrival Process

The observation of self-similar traffic at the packet level dates as far back as the work of Leland et al. [LTWW94], in which they showed that on a local-area network (LAN) the packet arrival process shows self-similar behavior. Paxson and Floyd [PF95] subsequently showed self-similarity at the packet level in

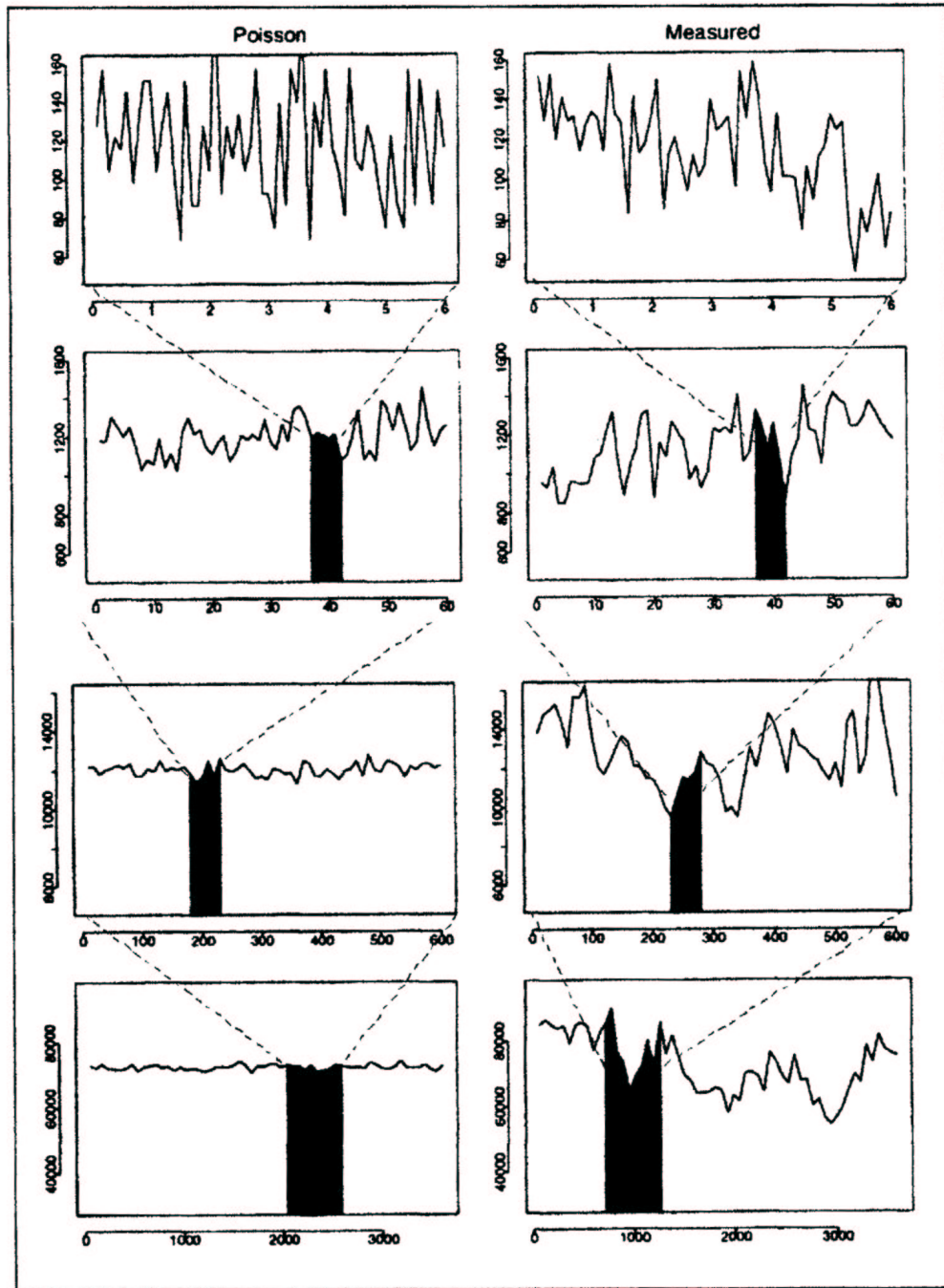


Figure 2.1: Self-similarity of Internet Traffic (Measured) vs Poisson Traffic (Ordinary Telephone Traffic).

wide-area networks. Willinger et al. [WTSW95] explain self-similarity at the packet level observed on Ethernet traffic as the result of a superposition of many on/off sources, where the lengths of the on and off periods are drawn from heavy-tailed distributions.

2.3.2 Connection Arrival Process

Although packets are the basic unit of the Internet, most operations involve more than one packet and user experience is usually based on a set of packets, rather than a single packet [Fel00]. Network operations such as error control and congestion control are done on a set of packets. In fact, operating on single packets can lead to unstable solutions [Fel00]. For this reason, studying networks from the point of view of a set of packets is important and of great interest in the networking community. At the edge of the internet, TCP provides the abstraction of a connection, which represents a set of packets related to the same message (example: HTTP connection) or user session (example: telnet). For applications running on top of TCP, resources are typically allocated on a per-connection basis, or on the time scale of a connection. For example, at the beginning of a TCP communication between a sender and a receiver, the receiver sets the window size, which is the maximum amount data the client can send per transmission. The receiver does so based on the availability of its resources (buffer and CPU) and the round trip time in the medium, which depends on the bandwidth. The window size for the connection is adjusted dynamically by the receiver, depending on the availability of its resources and the medium bandwidth usage. Anja Feldmann [Fel00] did an extensive study of the characteristics of TCP connection interarrivals. The study revealed that the TCP connection interarrival process is self-similar

(bursty). In addition, the study revealed that heavy-tailed distributions such as the Pareto, the lognormal and the Weibull distribution provide a better fit for the TCP connection interarrival process than does the exponential distribution.

2.3.3 Impacts of the Burstiness of TCP Connections

The burstiness of TCP connection interarrivals impacts the signaling, routing and resource allocation for web servers. Signaling deals with connection admission and rejection. Connection interarrival burstiness requires a higher performance from a router's CPU to perform algorithms necessary for connection admission or rejection. Dynamic routing, which is a goal in the Internet, may not be possible at the connection level if connection arrivals are too bursty. Web servers that allocate resources on a per-connection basis may not perform well if connection interarrivals are too bursty.

2.4 Network Traces Used By Feldmann to Characterize TCP Connection Intervals

Feldmann's study [Fel00] of TCP connection interarrival times was based on traces collected on three different Ethernet segments at Carnegie Mellon University (CMU), ATT Bell Laboratories, and ATT Labs-Research.

The first set of data, collected on an Ethernet segment of the School of Computer Science at CMU, is one of 18 Ethernet segments. The second traffic data set was collected on an Ethernet segment at ATT Bell Laboratories. The third traffic data set was collected on the same segment shortly after the split of ATT Bell Laboratories into ATT Labs-Research and Lucent Bell Laboratories. The second traffic data set was collected using a workstation connected to the internal network of ATT Bell Laboratories; whereas, the third traffic data set was collected using a workstation connected to an Ethernet segment outside

ATT's firewall. For this reason, the second trace is referred to as the internal trace, whereas the third trace is referred to as the external trace.

2.5 Connection Arrival Process Self-Similarity

Prior to the work of Feldmann, Paxson and Floyd [PF95] had pointed out that arrival processes of FTP DATA and SMTP connection arrivals are not consistent with Poisson processes. Feldmann [Fel00] showed that TCP connection interarrival times, in general, are self-similar. He attributes this in part to the fact that the World Wide Web is now the dominant application on the Internet. According to Feldmann, a user is likely to download several Web pages during a Web session, since each web page can consist of several embedded images [BC98]. This results in self-similar traffic. Figure 2.2, taken from [Fel00], depicts the number of connection arrivals over time for HTTP traffic on three different time scales for the external ATT trace. The plot shows that the burstiness of the connection arrivals does not decrease as the time resolution is increased, which indicates self-similarity for the arrival process of HTTP connection arrivals.

2.6 Characterization of Connection Interarrival Times

Feldmann modeled the cumulative distribution functions of the CMU, the external ATT, and the internal ATT traces using the exponential distribution and three heavy-tailed distributions: the Weibull distribution, the lognormal distribution, and the Pareto distribution. Feldmann considered all interarrival times, application specific interarrival times, application specific interarrival times over a shorter time periods, and interarrival times for specific sources. In all cases, the Weibull distribution fit the data better than did all other

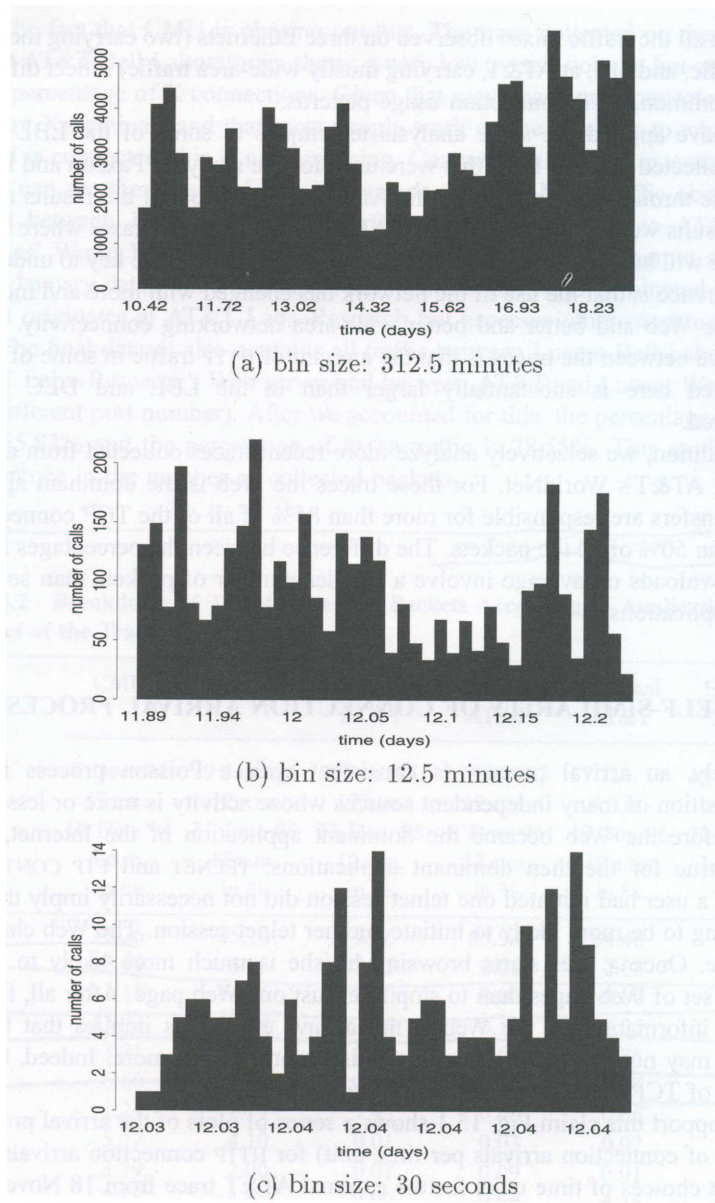
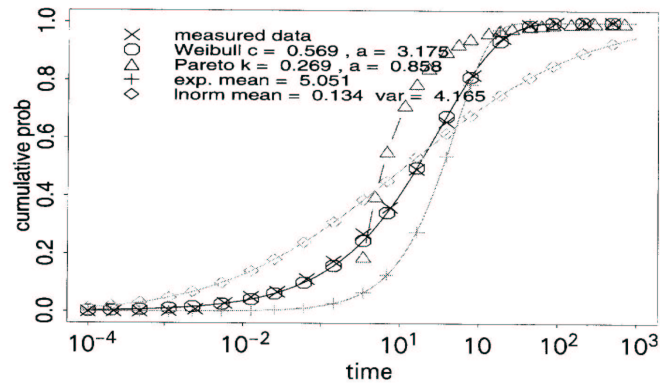
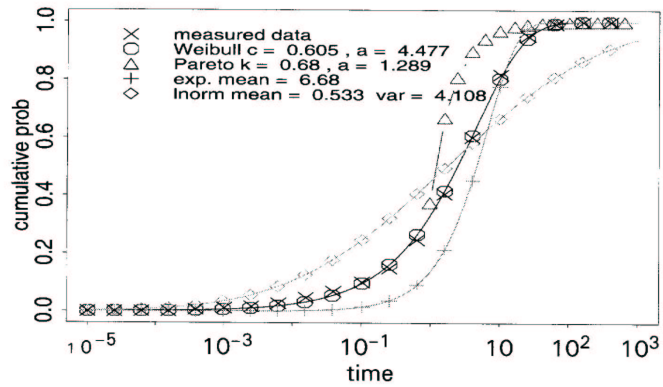


Figure 2.2: Pictorial indication of self-similarity: number of connection arrivals over time for HTTP traffic on three different time scales [Fel00].



(a) 29–30 June 95 CMU



(b) 18 Nov.–8 Dec. 95 ext. AT&T

Figure 2.3: Empirical and fitted cumulative interarrival time distributions of one CMU and one external ATT dataset [Fel00].

distributions. Figure 2.3, taken from [Fel00], shows the empirical and the fitted cumulative interarrival time distributions of one CMU and one external ATT dataset for all interarrival times. The figure shows that Weibull distribution gives a better fit than all other distributions. Feldmann also showed that the Weibull distribution gave a good fit for specific applications, including http, telnet, and smtp. In particular, the Weibull distribution provides an excellent fit for http connection interarrival times. Feldmann also shows that the Weibull

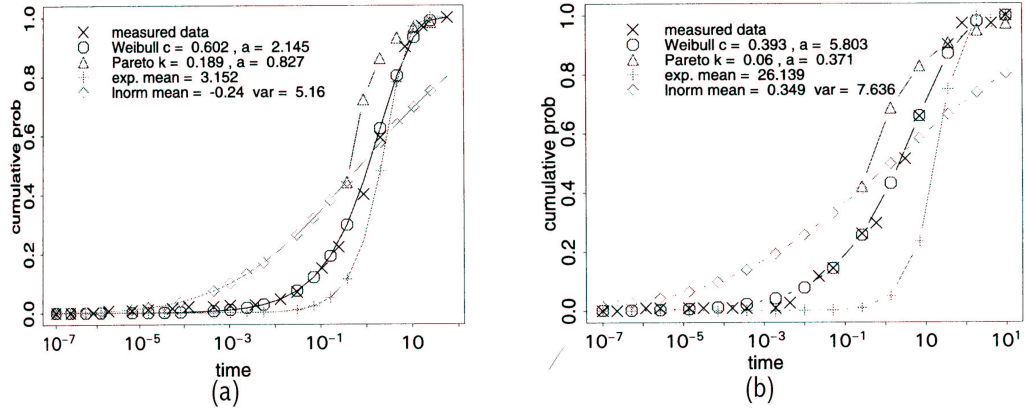


Figure 2.4: The empirical and fitted cumulative distributions of TCP traffic from 2 different sources [Fel00].

distribution provides a good fit for all time periods that are long enough to allow sufficient observation. Figure 2.4, taken from [Fel00], shows that the Weibull distribution also provides a superior fit if traffic from specific sources is aggregated.

2.7 Heavy-tailed Distribution

2.7.1 Definitions

Let $F(x)$ be the cumulative distribution function (cdf) of a random variable X and let $F^c(X) = 1 - F(X)$ be its complementary cdf (ccdf). The cdf $F(X)$ is said to heavy-tailed (also known as long-tailed or fat-tailed) if the ccdf $F^c(X)$ decays more slowly than exponentially, i.e., if:

$$e^{at} F^c(x) \rightarrow \infty \quad \text{as} \quad x \rightarrow \infty \quad \text{for all} \quad a > 0 \quad (2.1)$$

An interpretation of the definition of heavy-tailed distributions is that the probability of large events is nonnegligible.

The cdf $F(X)$ is said to short-tailed if the ccdf $F^c(X)$ decays exponentially, i.e., if there exists some $a > 0$ such that:

$$e^{at} F^c(x) \rightarrow 0 \quad \text{as} \quad x \rightarrow \infty \quad (2.2)$$

Power-tailed distributions are a subclass of heavy-tailed distributions. The cdf $F(X)$ is said to be power-tailed if its cumulative density function $F(x)$ satisfies the equation :

$$F^c(x) \sim \alpha x^{-\beta} \quad \text{as} \quad x \rightarrow \infty, \quad (2.3)$$

where α and β are positive constants and the notation “ $f(x) \sim g(x)$ ” means that $f(x)/g(x) \rightarrow 1$ as $x \rightarrow \infty$.

An example of a power-tailed distribution is the Pareto distribution, whose ccdf is defined as:

$$F^c(x) = (1 + bx)^{-a} \quad \text{where} \quad a, b > 0 \quad (2.4)$$

Heavy-tailed distributions have high or even infinite variance and therefore show extreme variability over all time scales. More intuitively, heavy-tailed distributions show a wide range of values, including very large ones, even if almost all values are small. For this reason, heavy-tailed distributions are appropriate for the modeling of self-similar processes.

Some studies [AW96], [CB96], [Ir193], [PF94] have shown that distributions with heavy-tails such as the Weibull, the Pareto and the lognormal distribution yield better models for file size in file systems. Other studies [PKC96a], [PKC96b], [PKC97], [PKC00] have shown that the TCP transfer of files drawn from heavy-tailed distributions, such as the Weibull and the Pareto distribution, causes self-similarity in network traffic. And, as mentioned before, heavy-tailed distributions, and the Weibull distribution in particular, have been shown to provide good fit for TCP connection interarrivals [Fel00].

2.7.2 The Weibull Distribution

In our study of PLAYTHROUGH ring we will use the Weibull distribution extensively, based on work performed by Feldmann [Fel00] that shows that

Weibull distribution provides a good fit for TCP connection arrival times. The Weibull distribution is a 2-parameter distribution whose cumulative distribution function $F(x)$ is given by:

$$F(x) = \begin{cases} 1 - e^{-(\frac{x}{a})^c}, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (2.5)$$

where a is a positive number called the scale parameter and c is a positive number called the shape parameter of the Weibull distribution. When $c < 1$ the Weibull distribution is heavy-tailed, but not power-tailed. As the value of c decreases the probability of longer, as well as shorter, values increases, which results in an increase of burstiness [Fel00].

The probability density function of the Weibull distribution is given by:

$$f(x) = \begin{cases} c \frac{1}{a} \left(\frac{x}{a}\right)^{c-1} e^{-(\frac{x}{a})^c}, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (2.6)$$

or

$$f(x) = \begin{cases} c \frac{x^{c-1}}{a^c} e^{-(\frac{x}{a})^c}, & x \geq 0 \\ 0, & x < 0. \end{cases} \quad (2.7)$$

And its mean and second moment are given by:

$$E[X] = \int_{-\infty}^{+\infty} x f(x) dx = \int_0^{+\infty} c \frac{x^c}{a^c} e^{-(\frac{x}{a})^c} dx = a \Gamma\left(1 + \frac{1}{c}\right) \quad (2.8)$$

and

$$E[X^2] = \int_{-\infty}^{+\infty} x^2 f(x) dx = \int_0^{+\infty} c \frac{x^{c+1}}{a^c} e^{-(\frac{x}{a})^c} dx = a^2 \Gamma\left(1 + \frac{2}{c}\right), \quad (2.9)$$

where, Γ refers to the gamma function. It is defined by:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \quad (2.10)$$

Generally, the n^{th} moment of the Weibull distribution is given by:

$$E[X^n] = a^n \Gamma\left(1 + \frac{n}{c}\right) \quad (2.11)$$

2.8 Notation

2.8.1 Operators

Throughout this dissertation, the operator \oplus designates addition *modulo* N , where N is the number of stations on the ring. Let us consider station i on a PLAYTHROUGH ring, where $i \in \{0, \dots, N\}$. The station distance j downstream from station i is station $i \oplus j$. The operator \oplus is commutative, that is $i \oplus j = j \oplus i$. The operator \ominus designates subtraction *modulo* N . The operators \oplus and \ominus are additive inverses, that is, $x \oplus y \ominus y \equiv x$ for all integers x and y .

The unitary factorial operator $!$ is defined recursively as:

$$n! = \begin{cases} 1 & n = 0, 1 \\ n[(n-1)!] & n = 2, 3, 4, \dots, \end{cases} \quad (2.12)$$

where n is a non-negative integer.

The binomial coefficient, denoted $\binom{n}{k}$, where n and k are two nonnegative integers, $k \leq n$, is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!} \quad (2.13)$$

2.8.2 Stochastic Notation

Let \mathcal{E} be an event and let X be a continuous random variable. Throughout this dissertation, the probability of the event \mathcal{E} is denoted $Pr[\mathcal{E}]$. The probability

distribution function of X , denoted $F_X(x)$ is given by:

$$F_X(x) = Pr[X \leq x] \quad (2.14)$$

The probability density function of X , denoted $f_X(x)$, is given by:

$$f_X(x) = \frac{dF_X(x)}{dx}. \quad (2.15)$$

For any function $g(X)$ of the random variable X , the expected value of $g(X)$, denoted $E[g(X)]$, is given by:

$$E[g(X)] = \int_{x=-\infty}^{\infty} g(x)f_X(x)dx. \quad (2.16)$$

In particular, the n^{th} moment of the random variable X , denoted $E[X^n]$ is the expected value of the function $g(X) = X^n$ and is given by:

$$E[X^n] = \int_{x=-\infty}^{\infty} x^n f_X(x)dx. \quad (2.17)$$

The first moment, also called mean or expected value, of the random variable X is denoted $E[X]$ and is given by:

$$E[X] = \int_{x=-\infty}^{\infty} x f_X(x)dx \quad (2.18)$$

The variance of the random variable X , denoted σ^2 , is given by:

$$\sigma^2 = E[X^2] - E[X]^2 \quad (2.19)$$

The Laplace-Stieltjes transform of the density $f_X(x)$, which is often also referred to as the Laplace-Stieltjes transform of the random variable X , is denoted $X^*(s)$ and given by:

$$\begin{aligned} X^*(s) &= E[e^{-sX}] \\ &= \int_{x=-\infty}^{\infty} e^{-sx} f_X(x)dx; \end{aligned} \quad (2.20)$$

Abbreviation	Meaning
LST	Laplace-Stieltjes transform
pdf	probability density function
PDF	probability distribution function
PGF	probability generating function
pmf	probability mass function
<i>r.v.</i>	random variable

Table 2.1: Abbreviations.

We designate by $X^{*(n)}(s)$ the n^{th} derivative with respect to s of $X^*(s)$.

Let Y be a discrete random variable with probability mass function $g_k = Pr [Y = k]$. The probability generating function of Y , denoted $\hat{Y}(z)$, is

$$\hat{Y}(z) = \sum_{k=-\infty}^{\infty} g_k z^k. \quad (2.21)$$

We designate by $\hat{Y}^{(n)}(z)$ the n^{th} derivative with respect to z of the probability generating function $\hat{Y}(z)$.

2.8.3 Abbreviations

Table 2.1 lists abbreviations used in this dissertation and their meanings.

2.9 Conclusions

In this chapter, an overview of PLAYTHROUGH ring operation was given. The phenomenon of self-similar traffic as it relates to network traffic was discussed. Evidence and causes of self-similar traffic were presented. The work of Anja Feldmann on the characterization of TCP connection arrivals was discussed. It revealed that the arrival process for TCP traffic is self-similar. In addition, Feldmann showed that the Weibull distribution with specific parameters provides a best fit for TCP connection arrivals. We also brought to light the work of Paxson, Kim, and Crovella, which reveals that the reliable transfer of files whose sizes are drawn from a heavy-tailed distribution causes self-similar

traffic. Heavy-tailed distributions, which include the Weibull distribution, were defined and discussed. Particular attention was given to the Weibull distribution.

In Chapter 3, we will provide a detailed description of the operation of the PLAYTHROUGH ring. Motivated by the work of Feldmann, we will give simulation results for the PLAYTHROUGH ring when it operates under Weibull message interarrival times and geometric message lengths. We will compare those results to the results obtained under previously used assumptions, that is exponential message interarrivals and geometric message lengths. In light of the causal relationship between heavy-tailed file size and self-similarity, we will also provide simulation results for PLAYTHROUGH ring when it operates under exponential message interarrivals and Weibull message lengths. We will compare those results to results obtained using exponential message interarrivals and geometric message lengths.

CHAPTER 3

SIMULATIONS OF THE PERFORMANCE OF PLAYTHROUGH RINGS UNDER SELF-SIMILAR TRAFFIC

3.1 Ring Operation

Figure 3.1 shows a simplex PLAYTHROUGH ring network. It consists of N stations in a ring configuration. Each station consists of a host, or attached component, and a ring interface unit (RIU). Figure 3.2 depicts a ring interface unit, RIU. To identify the consecutive stations on the ring, a unique one-character ring-relative station address in the range $[0 \dots N - 1]$ is assigned to each station at ring initialization. A station i 's RIU possesses a transmitter, denoted Tx_i , and a receiver, denoted Rx_i . Station i 's transmitter, Tx_i , feeds into station $i \oplus 1$'s receiver, $Rx_{i \oplus 1}$, where the operator \oplus refers to addition *modulo* N . Transmitters and receivers are therefore connected in a point-to-point fashion, making the ring suitable for a fiber optics physical layer.

We refer to station $i \oplus 1$ as the downstream neighbor of station i ; similarly, we refer to station i as the upstream neighbor of station $i \oplus 1$. Given two stations with addresses i and j , *modulo* N , where $j > i$, the distance between i and j is $j - i$, *modulo* N . It is the number of “hops” that must be made when going from station i to station j in the direction of transmission. For example, the distance between station i and station $i \oplus j$ on a simplex ring is j . We denote the path from station i to station $i \oplus j$ as $i \rightarrow i \oplus j$. It is the set of RIUs

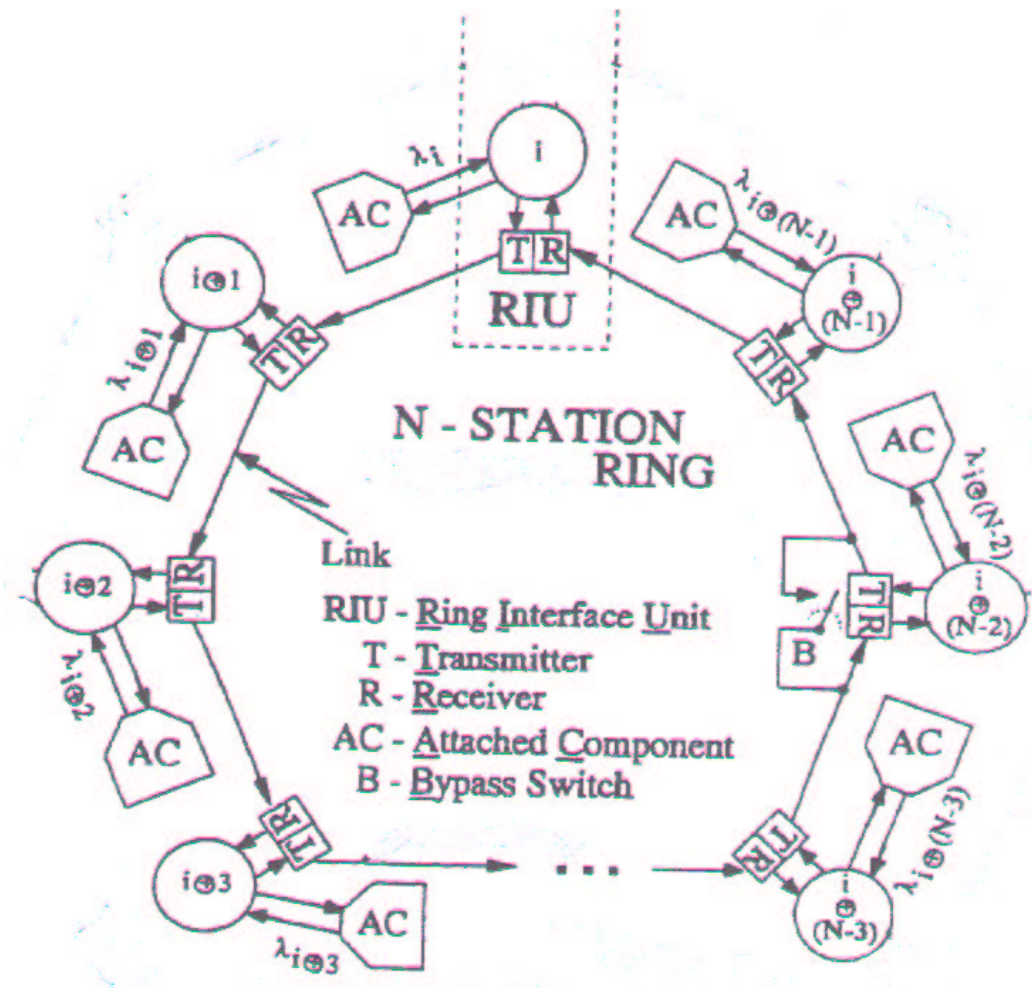


Figure 3.1: An N station simplex ring.

in between station i and station $i \oplus j$.

Traffic passing through the ring experiences several delays. As characters pass through the RIU of station i , they are delayed by a nominal latency of k_R character times by the station i 's RIU receive buffer. The link from station i 's upstream neighbor to station i adds an additional propagation delay of k_{T_i} character times. The total nominal latency for station i is denoted τ_i . It includes the upstream link propagation delay k_{T_i} and the nominal latency k_R .

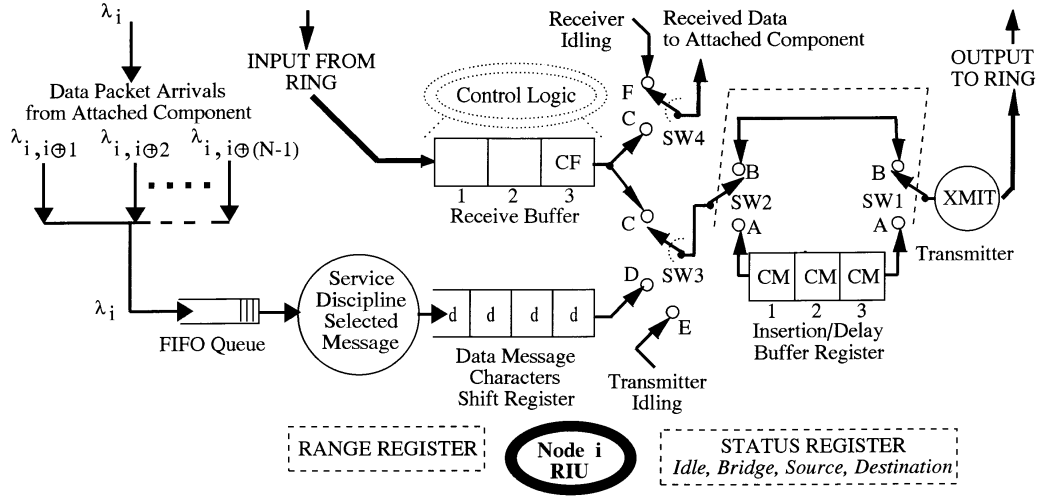


Figure 3.2: A PLAYTHROUGH Ring Interface Unit.

It is given by:

$$\tau_i = k_{T_i} + k_R. \quad (3.1)$$

The total nominal ring latency τ is obtained by summing all the N stations nominal latencies on the ring. It is the nominal time required to make a round trip across the ring and it is given by:

$$\tau = \sum_{i=0}^{N-1} \tau_i. \quad (3.2)$$

The transmission of a message from one station to another includes M message characters and h header characters. The M message characters are supplied by the source host to be delivered to the destination host; the h header characters are a constant number of overhead characters used for message integrity checks, network routing, or other protocol-related function. In previous works on PLAYTHROUGH ring, the term packet was used to refer to the logical unit of data transmitted from one host to another, consisting of M message characters and h header characters. We prefer not to use the term packet here because in this work, we think of the transmission of a message as

generally corresponding to a connection (for example a TCP connection) between the source and the destination. Messages sourced by station i are buffered at the queue within station i 's RIU until they may be transmitted across the ring to the destination station. The queue in each station RIU is assumed to be infinitely long. Messages are served from the RIU's queue in First-In, First-Out (FIFO) order. The message at the head of the queue is termed the head of the line (HOL) message.

A virtual circuit is constructed along the path $i \rightarrow i \oplus j$ in order for the transfer of a message and its header to take place. The transmitters along the path $i \rightarrow i \oplus j$ are dedicated to this virtual circuit during the transmission of the message and its header. When the transfer of the message and its header is completed, the virtual circuit is destroyed and the transmitters along the path $i \rightarrow i \oplus j$ become available for another message transfer.

The PLAYTHROUGH protocol uses destination removal. This means that the destination RIU that receives the message and its header removes them from the ring. This is in contrast to source removal used in IEEE Token ring, where the source RIU that sends data also removes the data from the ring. Source removal of data implies that all transmitters on the ring must participate in the transmission of every data message. This limits the ring to the service of at most one message at a time. With destination removal, only the transmitters on the transferred data path must be dedicated to the data transmission. This leaves the other transmitters on the ring available for additional data transfers, if no two simultaneous data transfers require the use of the same transmitters. The ability of PLAYTHROUGH ring to support simultaneous data transfers on different paths on the ring is known as spatial reuse.

The transmission of data on PLAYTHROUGH ring is accomplished through

the transmission of control a frame. At any given time on the ring, exactly one control frame and possibly several data frames are present, distributed in delay buffers and along transmission links. The control frame circulates continuously as it is passed from station to station.

The control frame uses control messages of fixed size k_D characters to establish and destroy virtual circuits on the ring, and to control the global state of the ring. It is delimited at its start by a unique character denoted *FLAG*, and at its end by a unique character denoted *GO*. Control messages are inserted into the control frame by an RIU through a register insertion mechanism employing the RIU's insertion-delay buffer (*id_buffer*), and are removed by the same RIU using a source removal protocol.

Control messages are inserted immediately before the *GO* delimiter at the end of the control frame, following any control messages already present in the frame. When the control message returns to its sending RIU it immediately follows the initial *FLAG* delimiter because any control messages present in the frame when the control message was inserted are already removed. To send a control message, an RIU fills its *id_buffer* with the desired message, waits for *GO* to arrive, and switches the *id_buffer* into the transmission circuit, inserting the control message into the control frame. Upon returning exactly $\tau + k_D$ character times later, the control message will be completely contained within the *id_buffer* buffer where it originated. The RIU then switches the *id_buffer* out of the transmission circuit, removing the message from the ring. Each station can send at most one control message per control frame, as each station has only one *id_buffer*. Hence, at most N control messages are present in any given control frame. The size of a control frame varies with the number of control messages it contains. An empty control frame has k_G characters of

delimiters and physical layer overhead bits. Each control message adds k_D characters to the size of the control frame, resulting in a maximum size of $k_G + Nk_D$ characters for the control frame.

Delimiting the data frame at its start is the end of the control frame (marked by *GO*). Each data frame consists of exactly one minipacket, a unit of data of fixed size $m = \tau - k_G$ characters. The data frame may contain up to m characters of message data and message overhead, padded at the end with *IDLE* characters, if necessary, to completely fill the minipacket. When transmitting data, the RIU automatically fragments the data message and its overhead into minipackets of fixed size m characters. The final minipacket is filled with *IDLE* characters if the total size of the message and its overhead is not a multiple of m . Hence, the transfer of every data message results in the transmission of an integral number of minipackets of size m .

At any given time, the global state of the ring is maintained in the RIU's internal registers. Each station's RIU updates the global state of the ring through the passing of control messages. Each control message transmitted on the ring passes through every RIU since it is removed by its source. This allows each RIU to update its internal registers. Many control messages have been specified for PLAYTHROUGH ring, including control messages used to support packet transmission with multiple levels of priority, control messages used for broadcast transmissions to multiple simultaneous destinations, and control messages for management functions at ring startup. In this dissertation, however, we restrict our attention to the *START* and the *STOP* control messages. Detailed specifications for all control messages are provided by DelCoco [Del88]. Every control message consists of three characters: a command character specifying the type of control message and containing

control and status bits; a one-character source station address that indicates the RIU sending the control message; and a one-character destination address that indicates the destination RIU for the control message. Both the *START* and the *STOP* control message contain an acknowledgment bit, which is transmitted as zero by the source station and changed to one by the destination station.

When station i wants to send a message to station $i \oplus j$, the *START* control message sent by station i to station $i \oplus j$ establishes a virtual circuit for the path $i \rightarrow i \oplus j$. All stations between i and $i \oplus j$ become engaged as repeaters, if the outbound distance j is two or greater. A station engaged as repeater simply repeats verbatim all data frames to its downstream neighbor, and processes control frames normally. When the destination station $i \oplus j$ receives the *START* control message, it sets the acknowledgment bit in the command character and gets ready to receive minipackets from source station i . Station $i \oplus j$ removes each received minipacket from the ring. Unless station $i \oplus j$ is concurrently transmitting its own minipackets, it replaces each minipacket with a minipacket filled with *IDLE* characters.

Station i continues to fill data frames with consecutive m -character pieces of data as long as its current HOL message contains more data to be transmitted. After completing the transmission of the minipacket containing the last characters of the HOL data message, the source station sends a *STOP* control message in the control frame, which marks the end of its data transfer.

Upon receiving the *STOP* control message, the repeating stations are released from the virtual circuit. The acknowledgment bit of the *STOP* command character is set by the destination station. In addition, the destination station forwards the received data to its host and updates its internal registers.

The internal registers of each station's RIU include a status register and a range register. The status register of a station's RIU keeps the status of the station, which can be a source, a bridge, a destination, both a source and destination, or idle. A station is a source if it is sending a message to another station on the ring. A station is a bridge if it is in the path of another station's message transfer, in which case it is simply repeating the data in transmission. A station is a destination if it is the receiving station of a message sent by another station on the ring. Finally, a station is idle if it is neither a source, nor a destination, nor a bridge. A station's RIU range register records the farthest downstream station to which it can transmit a message. Station $i \oplus j$, $j \geq 1$, is in station i 's range register if there is a free path from station i to station $i \oplus j$ and station $i \oplus j$ is a source. In the event that no station on the ring is currently transmitting, station i 's range register contains the ring-relative address of station i itself. Let us suppose that station i wants to send a message to station $i \oplus j$. This is only possible if station i 's status register indicates that it is idle and if station $i \oplus j$ is an upstream neighbor to the station contained in station i 's range register. Station i waits for the passing of the control frame preceded by the leading *FLAG* character and followed by the *GO* character in its receive register. Station i inserts a *START* control message immediately ahead of *GO*. The *START* control message contains the address of source station i and the address of destination station $i \oplus j$. The *START* control message preceded by the leading *FLAG* character and followed by *GO* continues to travel around the ring, causing all the stations on the ring to update their status and range registers. Stations on the path of station i 's message transfer become bridges. Station $i \oplus j$ becomes a destination and, of course, station i is a source. Other stations on the ring can insert a *START* control message on the ring after

station i 's *START* control message and before *GO*. *START* control messages are removed by their source stations in the order in which they were inserted. When the destination station receives the *START* control message it sets the acknowledgment bit in *START* control message. The *START* control message continues on its way around the ring to its source station, where it is removed. The source station i can now begin the transmission its message. After station i has transmitted the last character of its message, it waits for *GO* and inserts a *STOP* control message immediately ahead of *GO* which it sends to the destination station $i \oplus j$. The *STOP* message contains both the source and the destination address. The destination station, upon receiving the *STOP* message, sets the acknowledgment bit and the *STOP* message makes its way back to the source station. As the *STOP* message passes through the other stations, they update their status and range registers.

The PLAYTHROUGH ring uses single-character ring-relative source and destination addresses ranging from zero to one less than the number of stations. Because the station addresses are consecutive, they allow for proper computation of station to station distances and they keep address lengths short, which makes control message transmission efficient. In addition to ring-relative station addresses chosen at ring initialization, a permanent absolute address is used, which provides a unique identity for each station. This absolute address typically consists of six characters. The host provides the source and destination absolute addresses to the RIU and the RIU converts these absolute addresses into ring-relative addresses. When two PLAYTHROUGH rings are connected together via a bridge, the source and destination addresses are included in the h header characters. Messages destined to another ring are sent to the bridge's ring-relative address. The bridge determines the ring-relative

address of the destination station on the other ring using the absolute addresses in the header and forwards the message to its destination.

The PLAYTHROUGH ring was designed to allow the use of FDDI's physical layer in the specification of the PLAYTHROUGH media access control layer protocol given by DelCoco [Del88]. Accordingly, the *FLAG* character is defined as FDDI's unique JK symbol. The GO character consists of a leading T symbol, a nominal number of IDLE bits for elastic buffer adjustment, and a trailing T symbol. The nominal size of an empty control frame with no IDLE bits is $k_G = 2$ characters. This is assumed throughout this dissertation for simulations and analytical results. Minipackets are padded with FDDI IDLE symbols if they are empty or partially full. Each station transmits data at the same nominal bit rate.

In addition to the simplex ring configuration described thus far, the PLAYTHROUGH ring supports a counter-rotating double ring version. As shown in Figure 3.3, a double PLAYTHROUGH ring comprises an inner ring and an outer ring. Each of the N RIU's contains a pair of transmitters and receivers, one set connected to the inner ring and one set connected to the outer ring. Transmitters and receivers are connected differently on the inner and outer ring. On the inner ring, transmitters and receivers are connected in the same way as for simplex rings, that is the transmitter of station i is connected to the receiver of station $i \oplus 1$. On the outer ring, transmitters and receivers are connected in the opposite direction, that is RIU i 's transmitter is connected to RIU $i \ominus j$'s receiver, where the operator \ominus designates subtraction modulo N , the number of stations. As a result, transmission on the outer ring occurs in the direction opposite transmissions on the inner ring, in the direction of decreasing host number. The distance between two stations can be measured along either

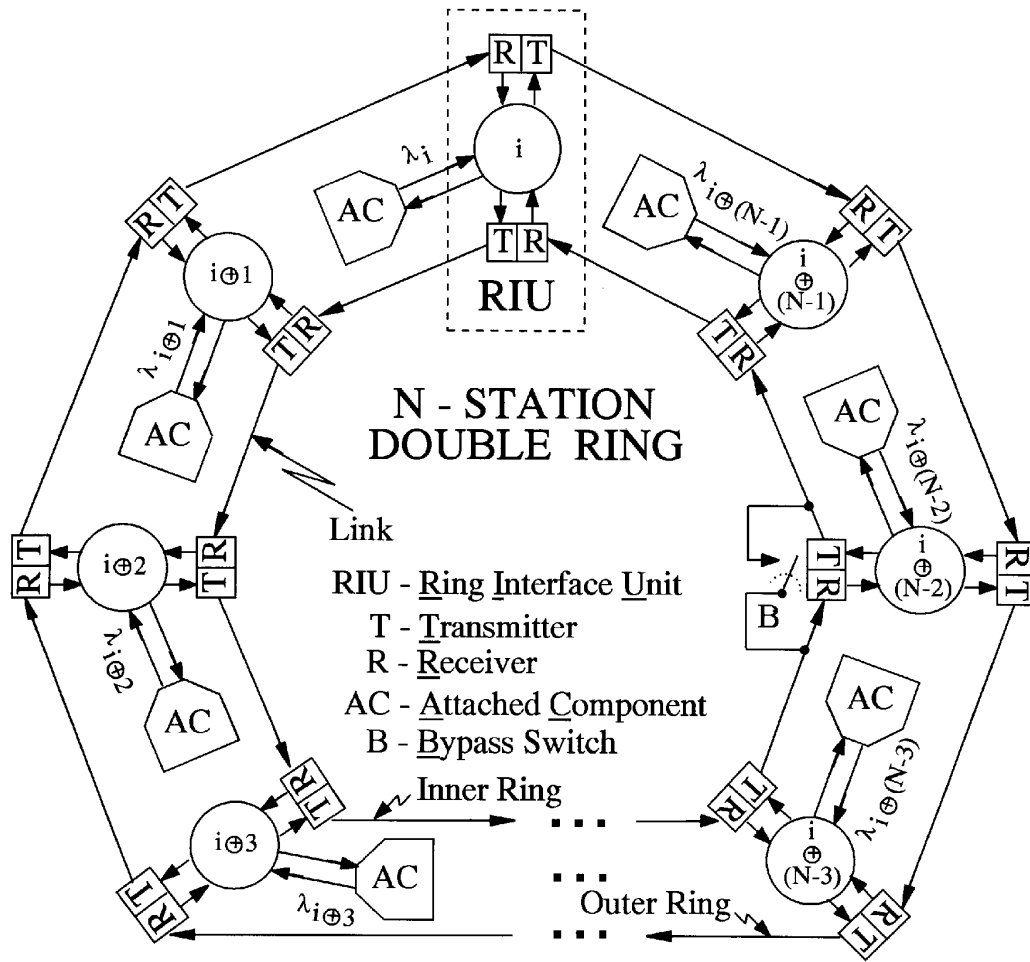


Figure 3.3: An N station double ring.

the inner or the outer ring. When measured along the inner ring, station $i \oplus j$ is a distance j away from station i . When measured along the outer ring, station $i \oplus j$ is a distance $N - j$ away from station i . Messages arriving at host i for transmission to host $i \oplus j$ can be routed along either the inner or outer ring. The double PLAYTHROUGH ring makes use of a routing protocol to determine which ring will carry the message. Shortest distance to destination routing is usually assumed in double ring models. In shortest distance to destination, a message sourced by station i and destined to station $i \oplus j$ will be placed into the inner ring queue if $j < N/2$, and into the outer ring queue if

$j > N/2$. In the event that the number of stations on the ring is even, station $i \oplus N/2$ is equidistant from station i when measured along either the inner ring or the outer ring. In this case, messages of outbound distance $N/2$ are placed randomly with equal probability into the queue for either ring.

The double PLAYTHROUGH ring considerably improves the reliability of the ring. The loss of a link or RIU on one of the rings can be detected, and this information coupled with simple additional switching circuits at each RIU can be used to isolate the failed station or bypass the failed link by reconfiguring the ring to maintain communications on what appears to be a single ring, providing graceful degradation in service.

In addition to improving the reliability of single PLAYTHROUGH ring, double PLAYTHROUGH ring improves the throughput of simple PLAYTHROUGH ring. On a single ring with uniform and symmetric traffic, the average outbound distance traveled by a message is $N/2$. Thus, two message transmissions can proceed concurrently on average with spatial reuse. On a double ring with uniform and symmetric traffic, a message transfer on the inner ring or outer ring has a maximum outbound distance of $N/2$, and travels an average distance of roughly $N/4$. Therefore, with spatial reuse, four messages may be transferred concurrently on average per ring, with a total of eight messages on the entire ring. This ability of double ring PLAYTHROUGH ring to quadruple the throughput of a simplex ring with only twice the hardware cost of the simplex ring makes double PLAYTHROUGH ring very attractive.

3.2 Traffic Assumptions

In this dissertation, all data lengths such as packet lengths and token sizes are in characters, and all time units are in character times. We designate by $\mathcal{C}_{i,i\oplus j}$

the class of traffic sourced by station i with outbound distance j , $0 \leq i \leq N - 1$ and $1 \leq j \leq N - 1$. Each of the N stations is allowed to transmit to any of the $N - 1$ remaining stations, yielding a maximum of $N(N - 1)$ classes of traffic on a PLAYTHROUGH ring. Packets belonging to the different classes of traffic $\mathcal{C}_{i,i \oplus j}$, $0 \leq i \leq N - 1$, $1 \leq j \leq N - 1$, are assumed to arrive at each station according to arrival rate $\lambda_{i,i \oplus j}$ and are mutually independent.

Each message of class $\mathcal{C}_{i,i \oplus j}$ is made up of a constant header of length $h_{i,i \oplus j} \geq 0$ and a variable-size payload of length $M_{i,i \oplus j}$.

Unless otherwise stated, we assume uniform and symmetric traffic (UST). With this type of traffic, messages belonging to all the traffic classes $\mathcal{C}_{i,i \oplus j}$, $0 \leq i \leq N - 1$, $1 \leq j \leq N - 1$, have identical statistical characteristics and each station transmits the same amount of traffic to every other station.

We define certain message events on the PLAYTHROUGH ring as follows. We denote by $\mathcal{T}_{i,i \oplus j}$ the event that station i transmits a message to station $i \oplus j$, and by $\mathcal{T}_i = \bigcup_{j=1}^{N-1} \mathcal{T}_{i,i \oplus j}$ the event that station i transmits a packet of arbitrary outbound distance. We denote by \mathcal{R}_i the event that station i 's transmitter is engaged as a repeater, and let us designate by $\mathcal{R}_{i \oplus j, i}$ the event that station $i \oplus j$ sources packets that must be repeated by station i 's transmitter; let $\mathcal{I}_{i \oplus j, i}$ be the event that station $i \oplus j$ sources packets that are independent of station i 's transmitter (*i.e.*, are not repeated by station i). Traffic sourced by station $i \oplus j$ (event $\mathcal{T}_{i \oplus j}$) can be divided into traffic requiring the use of station i as a repeater (event $\mathcal{R}_{i \oplus j, i}$) and traffic independent of station i (event $\mathcal{I}_{i \oplus j, i}$). Therefore, $\mathcal{T}_{i \oplus j} = \mathcal{R}_{i \oplus j, i} \cup \mathcal{I}_{i \oplus j, i}$. In addition, traffic that uses transmitter $Tx_{i \oplus j}$ can be divided into two disjoint sets: traffic repeated by station $i \oplus j$ (event $\mathcal{R}_{i \oplus j} = \bigcup_{k=2}^{N-1} \mathcal{R}_{i \oplus j \oplus k, i \oplus j}$), and traffic originating at station $i \oplus j$ (event $\mathcal{T}_{i \oplus j}$).

3.3 Simulations of the Performance of PLAYTHROUGH Under Weibull Connection Interarrivals and Geometric Message Lengths

In this section, we present the simulation results for PLAYTHROUGH rings using the assumptions stated below. The purpose of these simulations is to gain insight into the performance of PLAYTHROUGH rings under self-similar connection interarrivals, to later explain them, and to validate analytical results. The queueing discipline assumed is First-in, First-out (FIFO). In order to gain understanding of how the PLAYTHROUGH ring performs under Weibull interarrivals and geometric message lengths, an earlier C++ simulation model for PLAYTHROUGH [Hen98] was modified. Sample means and moments for waiting time, message service time, message transmission time, and other parameters were collected on a simulated sample of 1,000,000 messages transferred on the ring for each nominal arrival rate applied to the ring.

3.3.1 Traffic Characteristics

Messages of each class $\mathcal{C}_{i,i\oplus j}$ are assumed to arrive at station i 's queue according to a Weibull process with scale parameter $a_{i,i\oplus j}$ and shape parameter $c_{i,i\oplus j}$. In addition, the arrival processes for different message classes are assumed to be independent. The probability distribution function of message interarrival times for messages of class $\mathcal{C}_{i,i\oplus j}$ is given by:

$$F(t) = 1 - e^{-\left(\frac{t}{a_{i,i\oplus j}}\right)^{c_{i,i\oplus j}}}, \quad (3.3)$$

The arrival rate for messages of class $\mathcal{C}_{i,i\oplus j}$ is the inverse of the mean interarrival time for messages of class $\mathcal{C}_{i,i\oplus j}$, which is the mean of the message interarrival time distribution and is given by:

$$\lambda_{i,i\oplus j} = \frac{1}{a_{i,i\oplus j} \Gamma(1 + \frac{1}{c_{i,i\oplus j}})}. \quad (3.4)$$

The aggregate message arrival rate to station i 's queue is denoted λ_i and comprises all the $N - 1$ classes of traffic $\{\mathcal{C}_{i,i\oplus 1}, \dots, \mathcal{C}_{i,i\oplus(N-1)}\}$ that arrive at station i . We show that the confluence of independent Weibull arrival processes results in a Weibull process.

Let $X_1, X_2 \dots X_k$ be K independent Weibull distributed interarrival times r.v.'s with the same scale parameter a_0 and the same shape parameter c_0 . The aggregation of the K independent random variables results in the random variable $Y = \min(X_1, X_2 \dots X_k)$, where \min designates the minimum among the K random variables. We have:

$$\begin{aligned} P[Y > y] &= P[X_1 > y \dots X_k > y] \\ &= P[X_1 > y] \dots P[X_k > y] \\ &= e^{-(\frac{y}{a_0})^{c_0}} \dots e^{-(\frac{y}{a_0})^{c_0}} \\ &= e^{-K(\frac{y}{a_0})^{c_0}} \end{aligned} \quad (3.5)$$

As a result, we have:

$$P[Y \leq y] = 1 - e^{-K(\frac{y}{a_0})^{c_0}} \quad (3.6)$$

Therefore, the aggregation of K independent Weibull distributed interarrival time streams with scale parameter a_0 and shape parameter c_0 results in a Weibull distributed interarrival time stream with new scale parameter $\frac{a_0}{c_0^{1/K}}$ and shape parameter c_0 .

Because we assume uniform and symmetric traffic (UST), we let:

$$a_{i,i\oplus j} = a, \quad \text{and} \quad (3.7)$$

$$c_{i,i\oplus j} = c, \quad (3.8)$$

for all $i, j \in \{0, \dots, N-1\}$

Hence, the arrival rate of the aggregate stream sourced by station i is given by:

$$\lambda_i = \sum_{j=1}^{N-1} \lambda_{i,i\oplus j} = \frac{\sqrt[c]{N-1}}{a\Gamma(1 + \frac{1}{c})} \quad (3.9)$$

and the overall arrival rate on the ring is given by:

$$\lambda = \sum_{i=0}^{N-1} \lambda_i = N\lambda_i = \frac{\sqrt[c]{N-1}\sqrt[c]{N}}{a\Gamma(1 + \frac{1}{c})} \quad (3.10)$$

Each message of class $\mathcal{C}_{i,i\oplus j}$ comprises a constant-size header of discrete length $h \geq 0$ and a variable-size payload of length $M_{i,i\oplus j}$, a discrete random variable satisfying $M_{i,i\oplus j} \geq 1$. $M_{i,i\oplus j}$ is assumed to be geometrically distributed with parameter $\beta_{i,i\oplus j}$ and probability density function

$$Pr [M_{i,i\oplus j} = k] = (1 - \beta_{i,i\oplus j})^{k-1} \beta_{i,i\oplus j}; \quad k = 1, 2, 3, \dots \quad (3.11)$$

We assume uniform symmetric traffic (UST), which means that each station transmits an equal amount of traffic to every other station, and all message lengths are identically distributed. Under these circumstance, $\lambda_i = \frac{\lambda}{\sqrt[c]{N-1}}$, $\lambda_{i,i\oplus j} = \frac{\lambda_i}{\sqrt[c]{N-1}}$, and $\beta_{i,i\oplus j} = \beta$.

3.3.2 Simulations and Comparison With Prior Assumptions

Plots in this section show of the average waiting time of messages in the PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths. The average waiting time of messages in the PLAYTHROUGH ring

under exponential interarrival times and geometric message lengths is also included for comparison purposes.

Two sets of plots are presented. The first set of plots includes Figures 3.4–3.18, in which the value of the shape parameter of the Weibull interarrival times distribution is $c = 0.6$. The second set of plots includes Figures 3.19–3.32, in which the value of the shape parameter of the Weibull interarrival times distribution is $c = 0.4$.

For each data point shown on the plots, a Weibull distribution with scale parameter a and shape parameter c is applied to the ring, resulting in an overall ring arrival rate λ given by:

$$\lambda = \frac{1}{a\Gamma\left(1 + \frac{1}{c}\right)} \quad (3.12)$$

The arrival rate λ_i of messages arriving at station i is given by $\lambda_i = \frac{\lambda}{\sqrt[N]{N}}$ and the arrival rate for messages of class $\mathcal{C}_{i,i\oplus j}$ is given by $\lambda_{i,i\oplus j} = \frac{\lambda_i}{\sqrt[N-1]{N-1}}$. Message lengths for each data point are geometrically and identically distributed with average common mean length equal to $E[M]$.

The average waiting time $E[W]$ is plotted versus the measured offered load $\lambda E[M]$, which is the result of the measured arrival rate λ times the measured average message length $E[M]$. The measured arrival rate λ is the mean number of messages that arrive to the ring per character time.

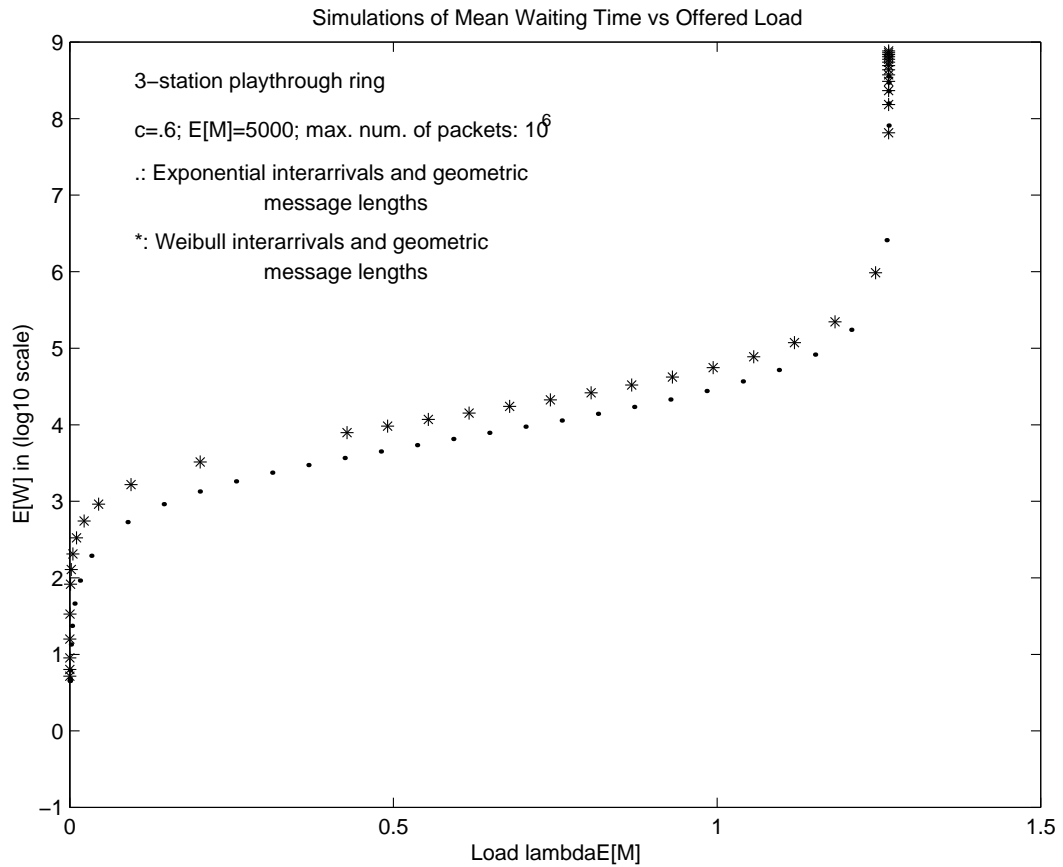


Figure 3.4: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 3$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.4 shows the simulated average waiting time on an $N = 3$ -station ring, with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.265$.

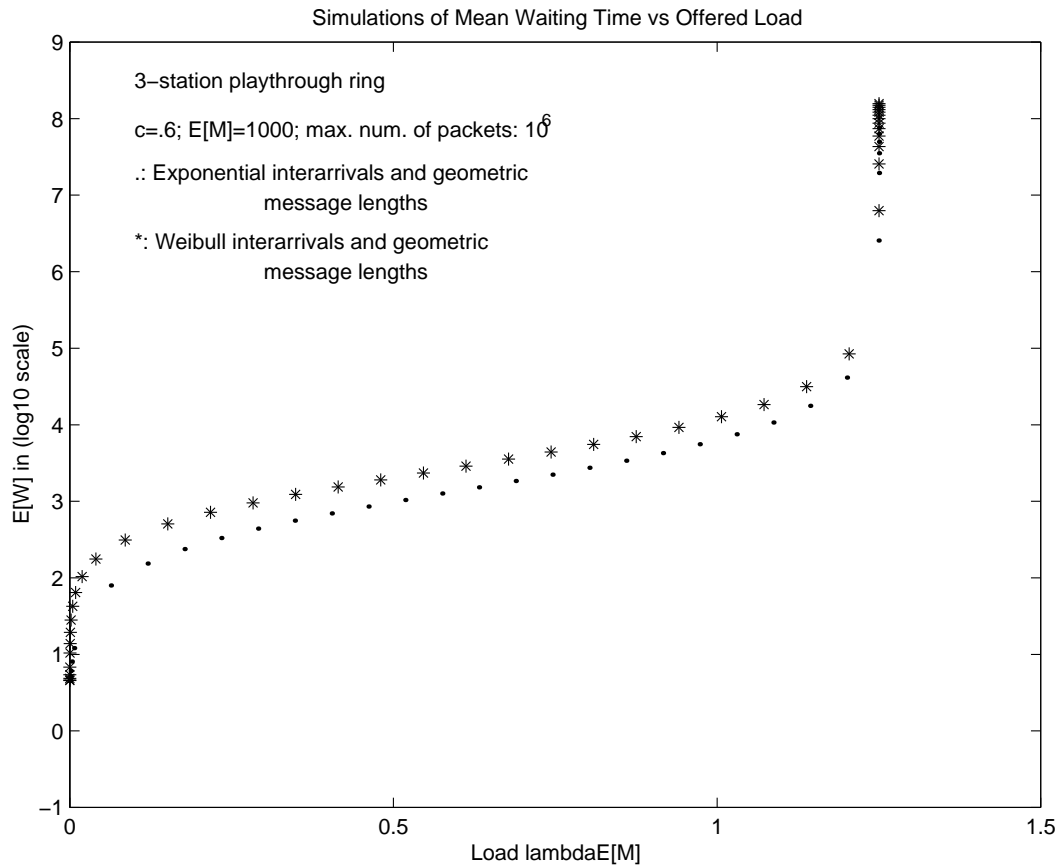


Figure 3.5: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 3$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.5 shows the simulated average waiting time on an $N = 3$ -station ring, with 1000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.250$.

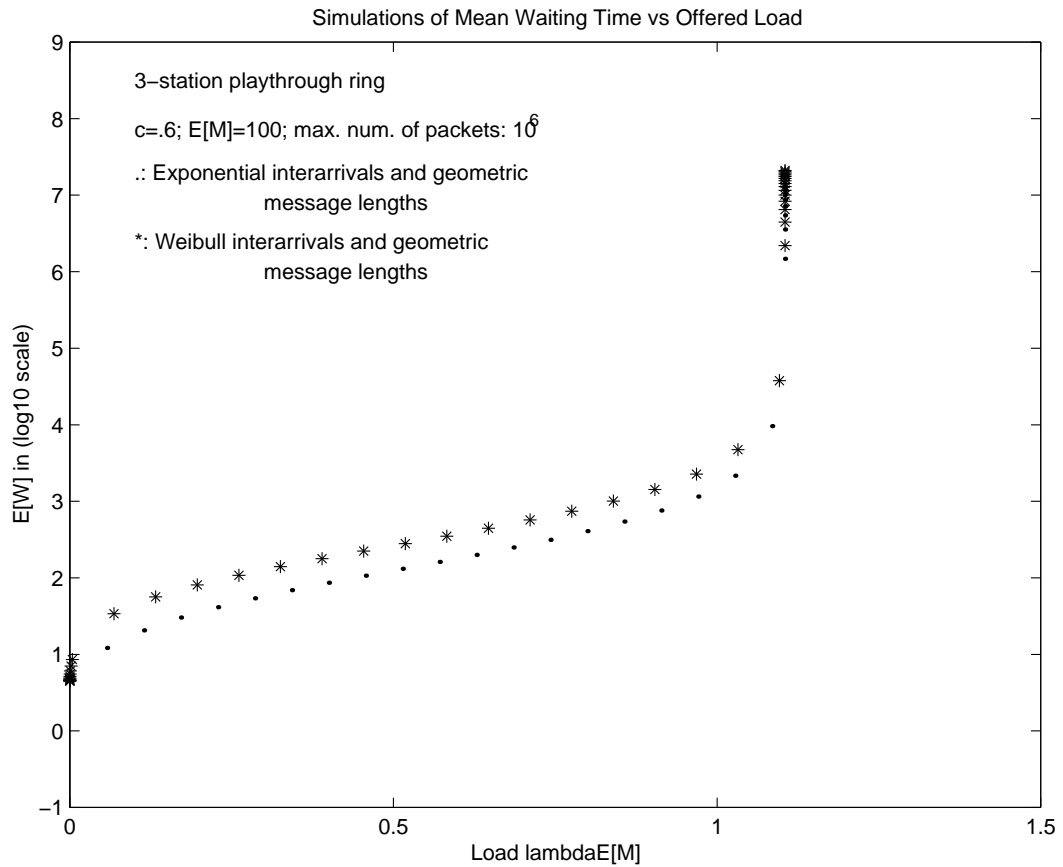


Figure 3.6: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 3$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 100$ characters.

Figure 3.6 shows the simulated average waiting time on an $N = 3$ -station ring, with 100 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.104$.

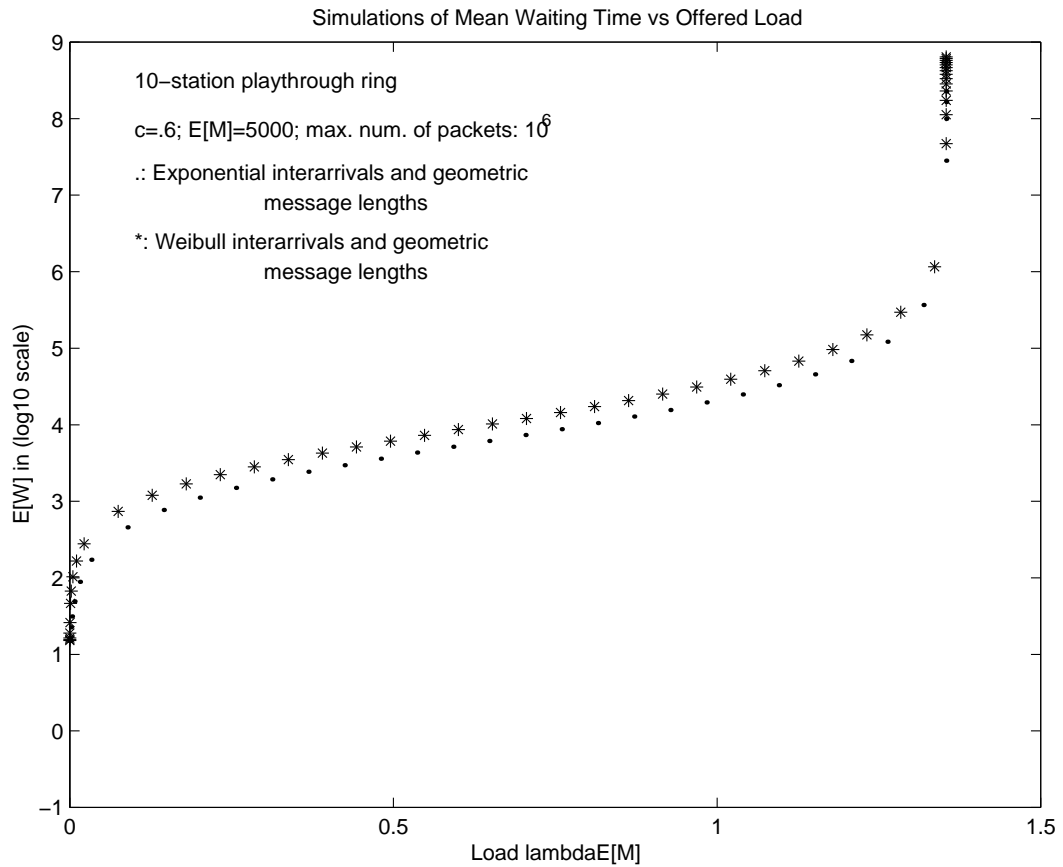


Figure 3.7: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 10$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.7 shows the simulated average waiting time on an $N = 10$ -station ring, with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.354$.

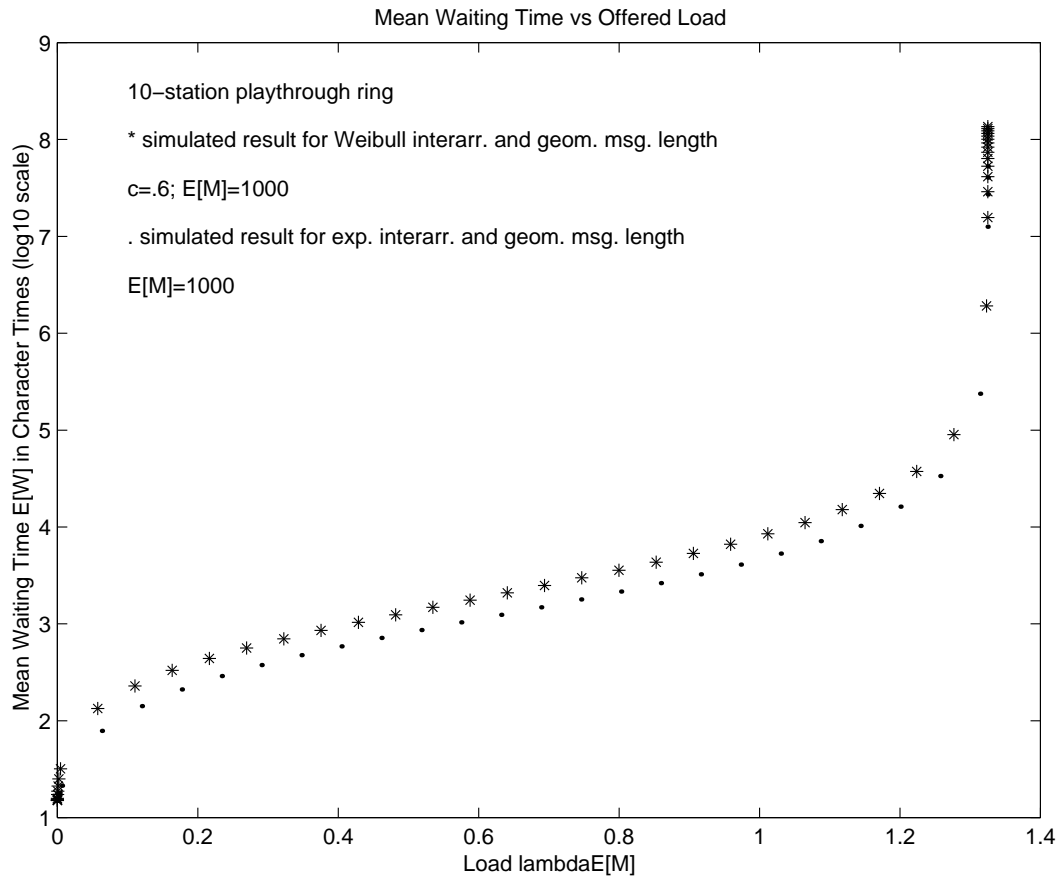


Figure 3.8: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 10$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.8 shows the simulated average waiting time on an $N = 10$ -station ring, with 1000 character mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.325$.

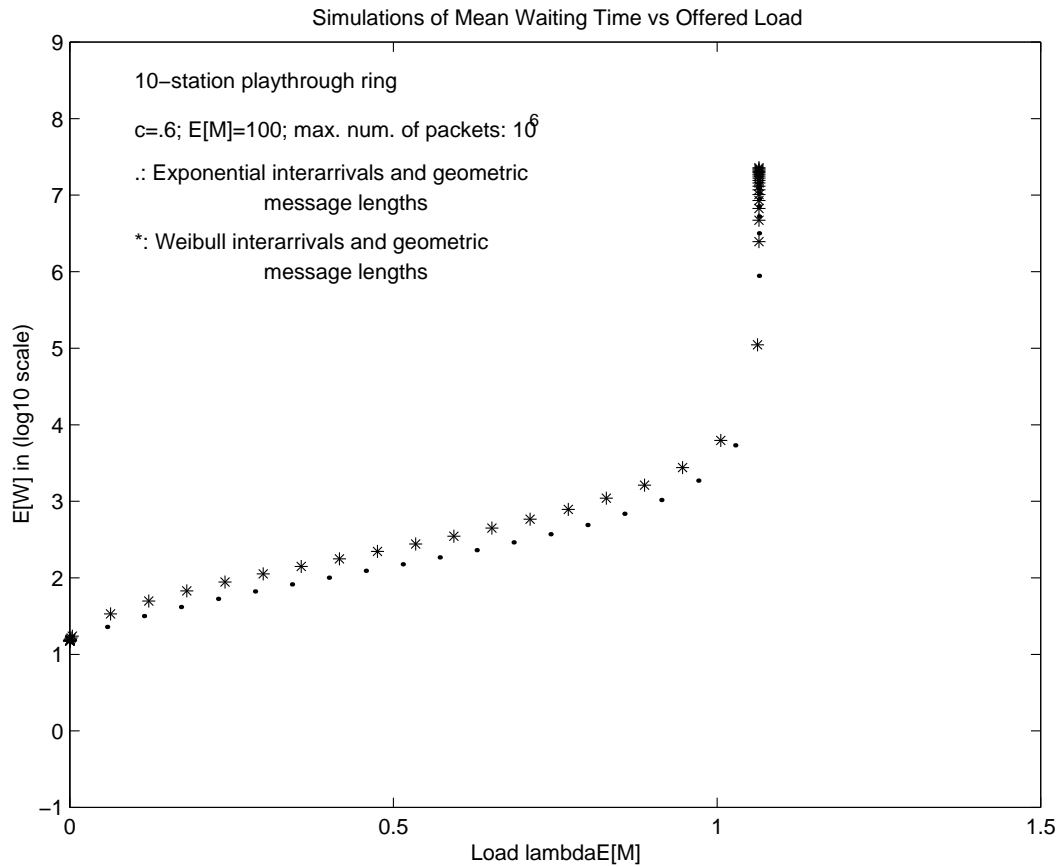


Figure 3.9: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 10$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 100$ characters.

Figure 3.9 shows the simulated average waiting time on an $N = 10$ -station ring, with 100 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both case is reached at $\lambda E[M] = 1.064$.

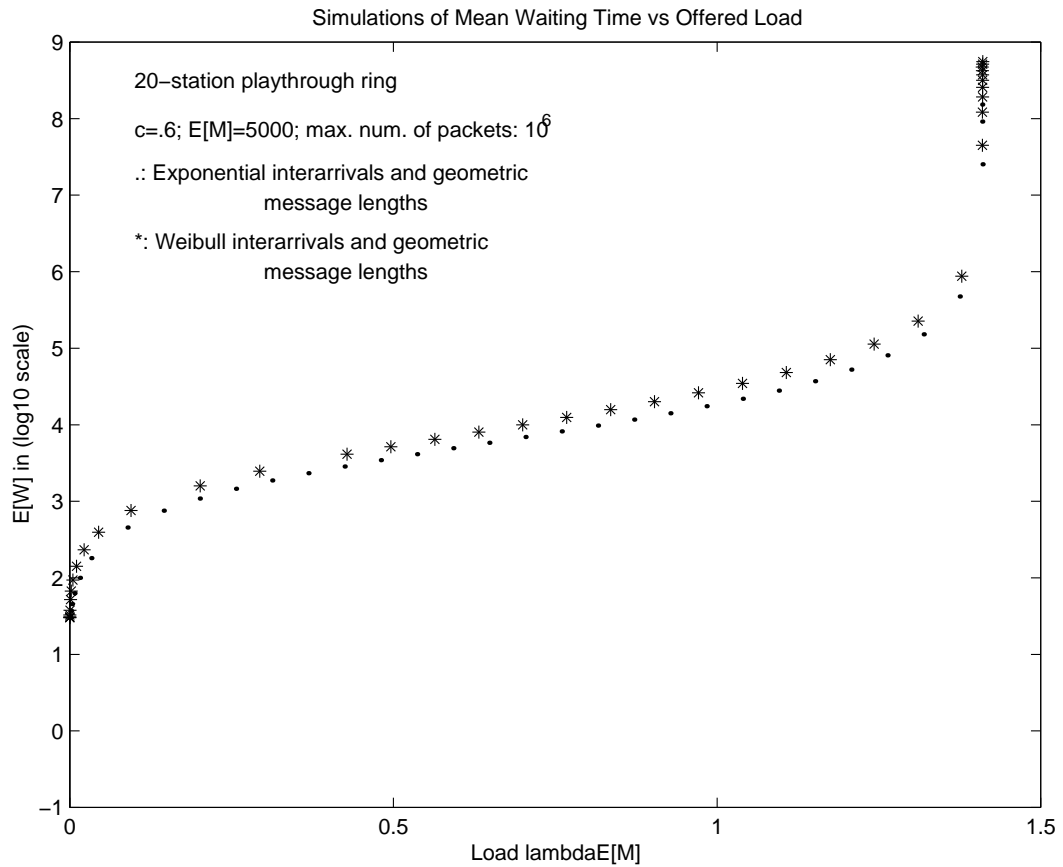


Figure 3.10: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 20$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.10 shows the simulated average waiting time on an $N = 20$ -station ring, with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.410$.

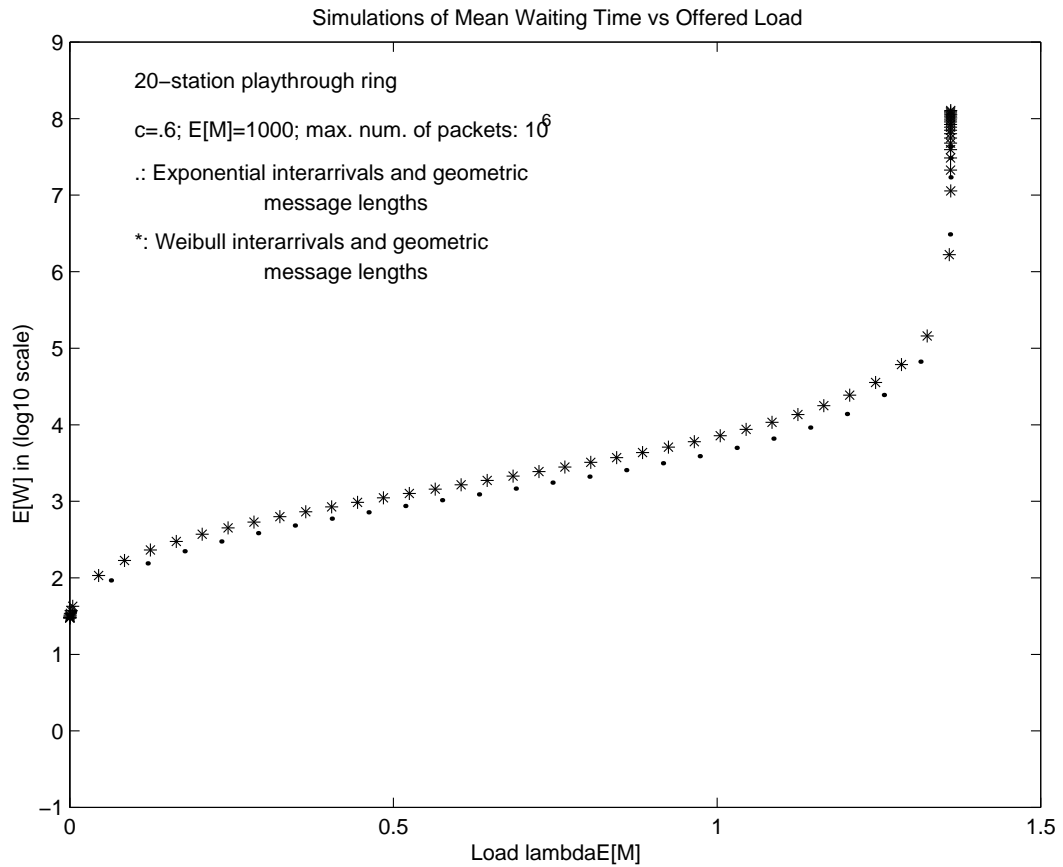


Figure 3.11: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 20$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.11 shows the simulated average waiting time on an $N = 20$ -station ring, with 1000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both case is reached at $\lambda E[M] = 1.360$.

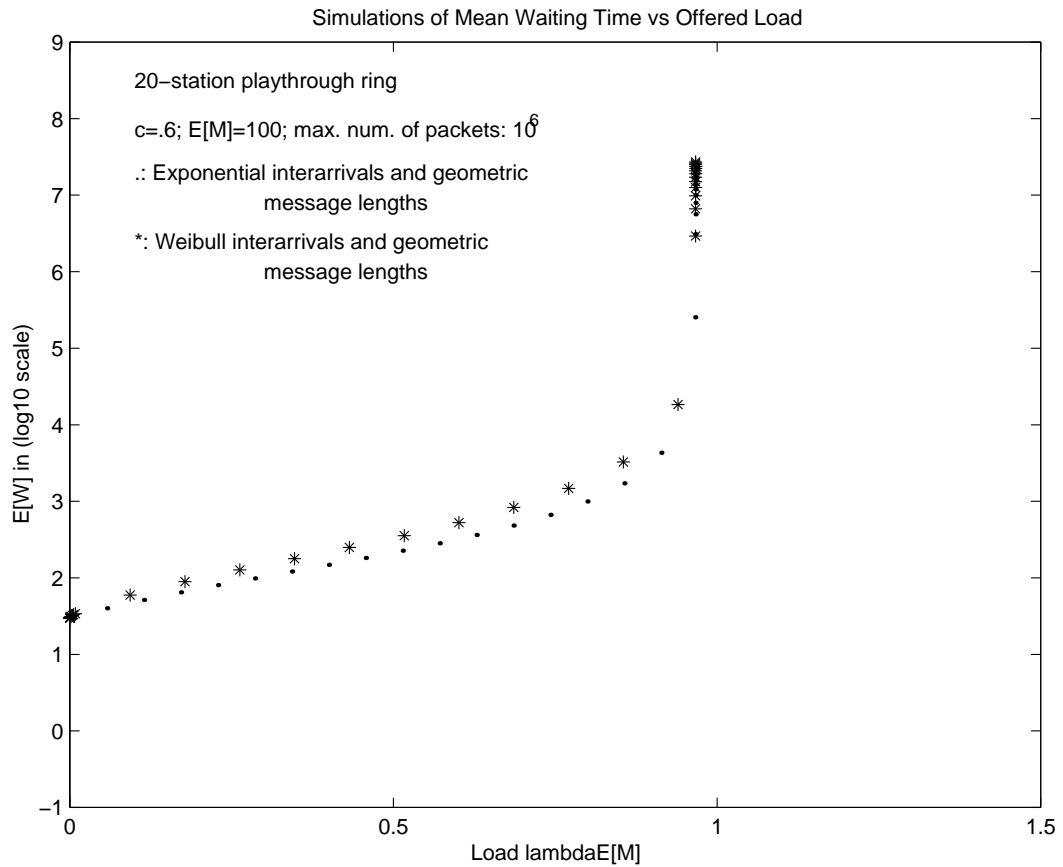


Figure 3.12: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 20$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 100$ characters.

Figure 3.12 shows the simulated average waiting time on an $N = 20$ -station ring, with 100 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 0.96$.

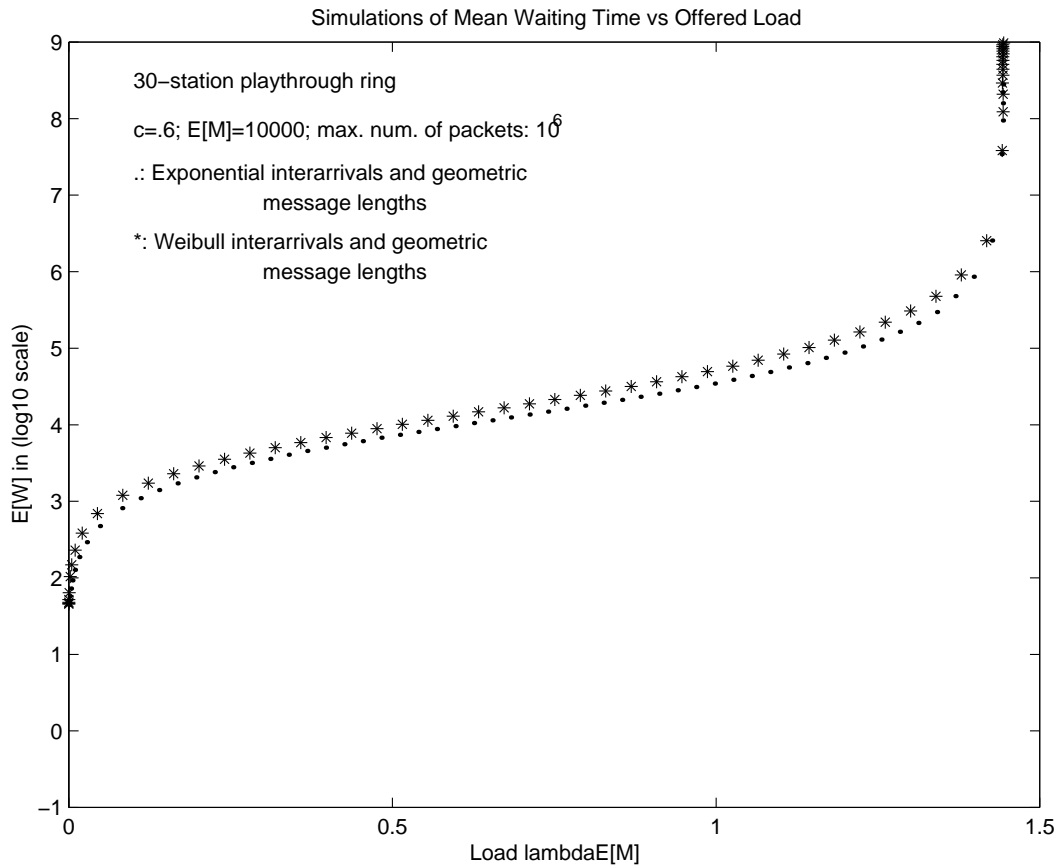


Figure 3.13: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 30$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 10000$ characters.

Figure 3.13 shows the simulated average waiting time on an $N = 30$ -station ring, with 10000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.44$.

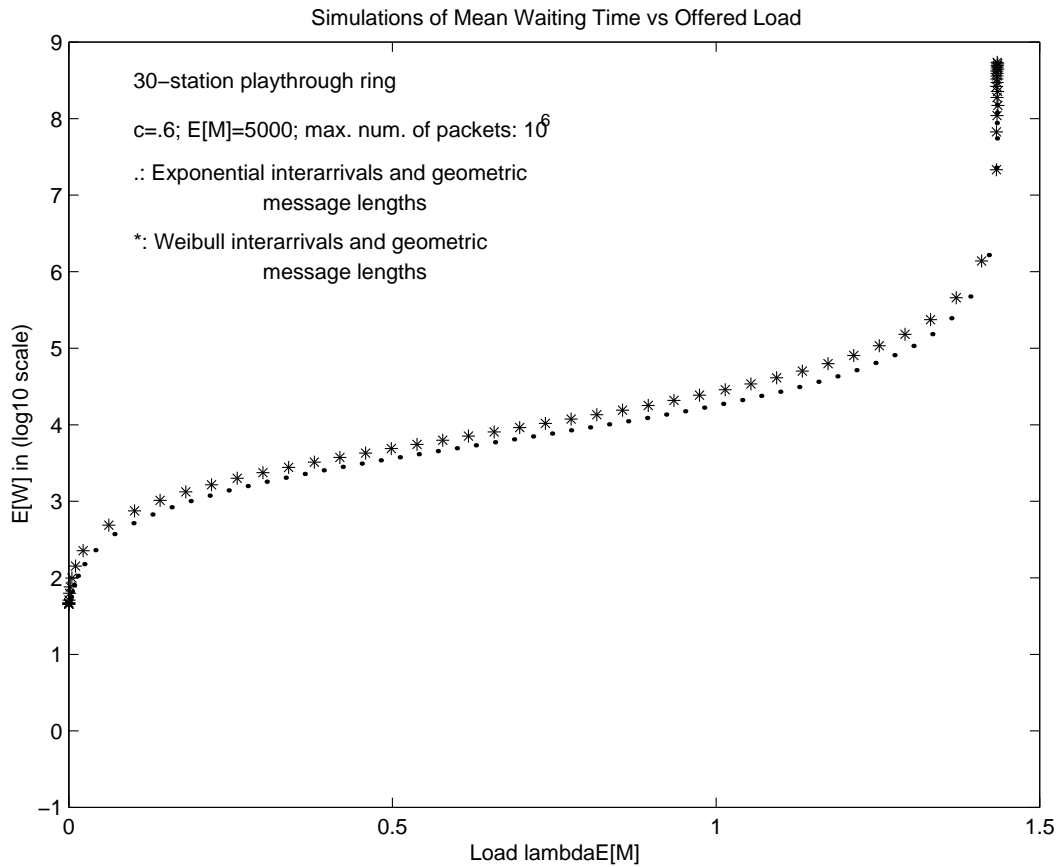


Figure 3.14: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 30$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.14 shows the simulated average waiting time on an $N = 30$ -station ring, with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.43$.

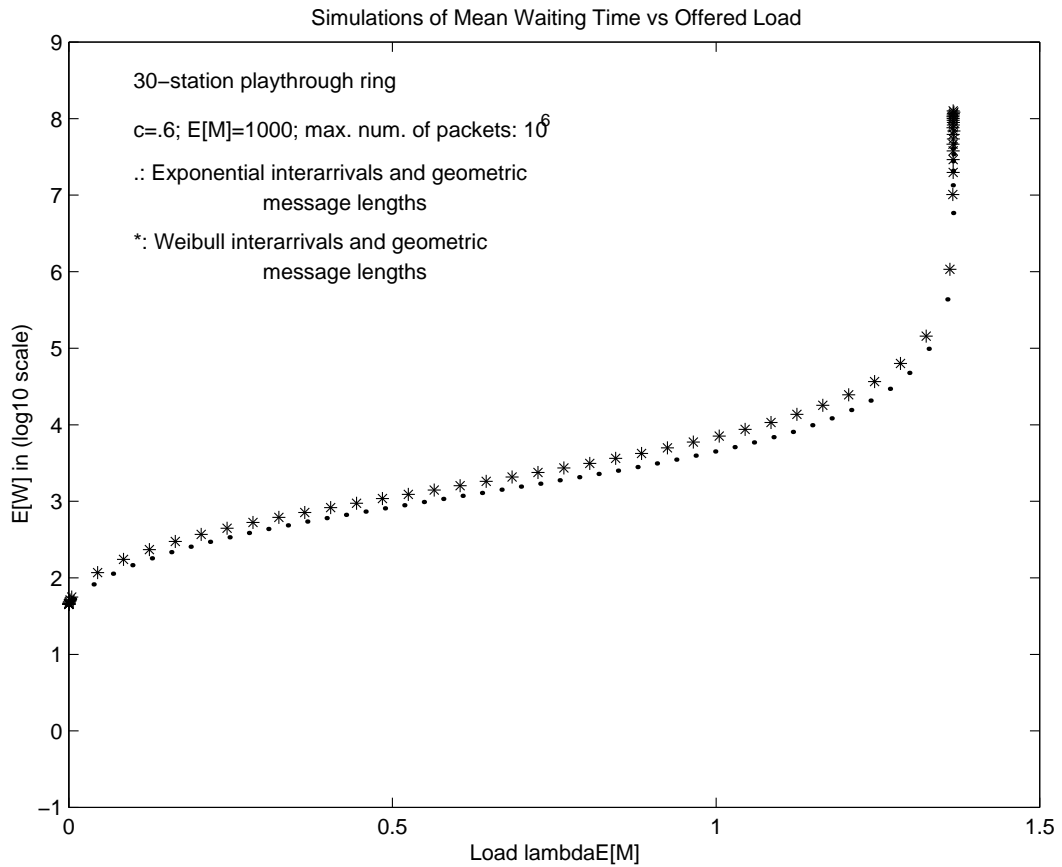


Figure 3.15: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 30$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.15 shows the simulated average waiting time on an $N = 30$ -station ring, with 1000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.36$.

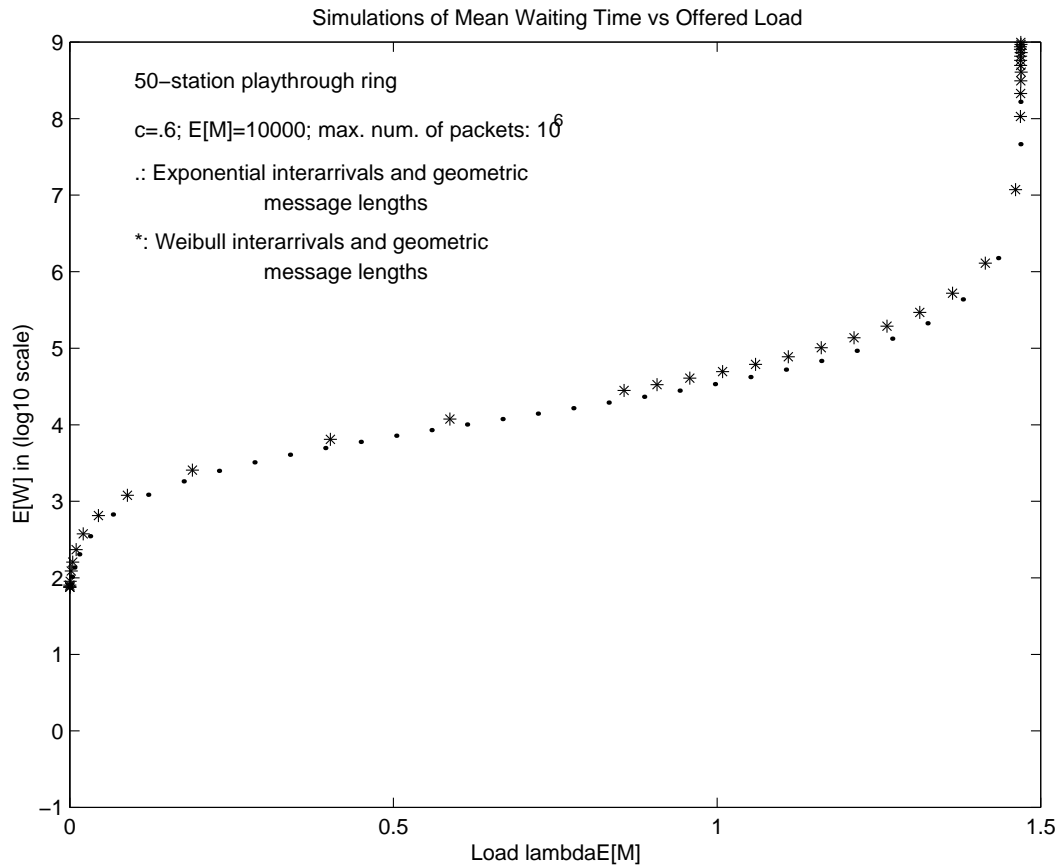


Figure 3.16: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 50$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 10000$ characters.

Figure 3.16 shows the simulated average waiting time on an $N = 50$ station ring with 10000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.469$.

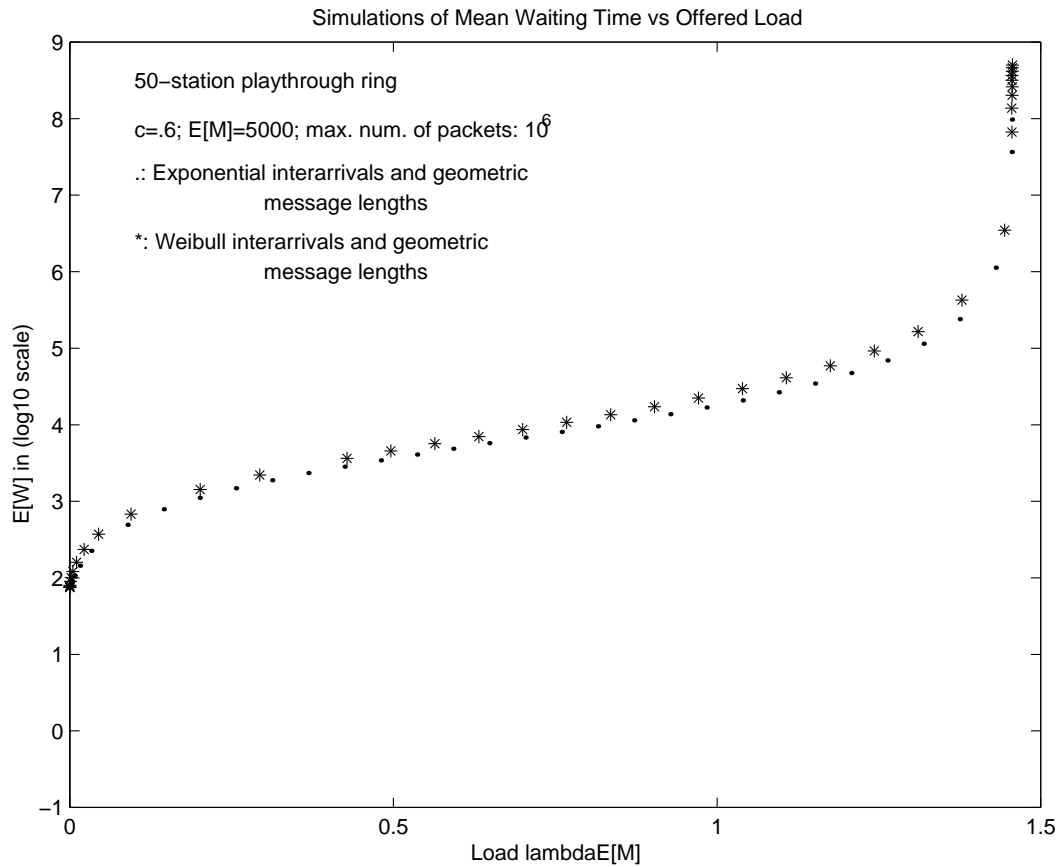


Figure 3.17: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 50$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.17 shows the simulated average waiting time on an $N = 50$ station ring with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.456$.

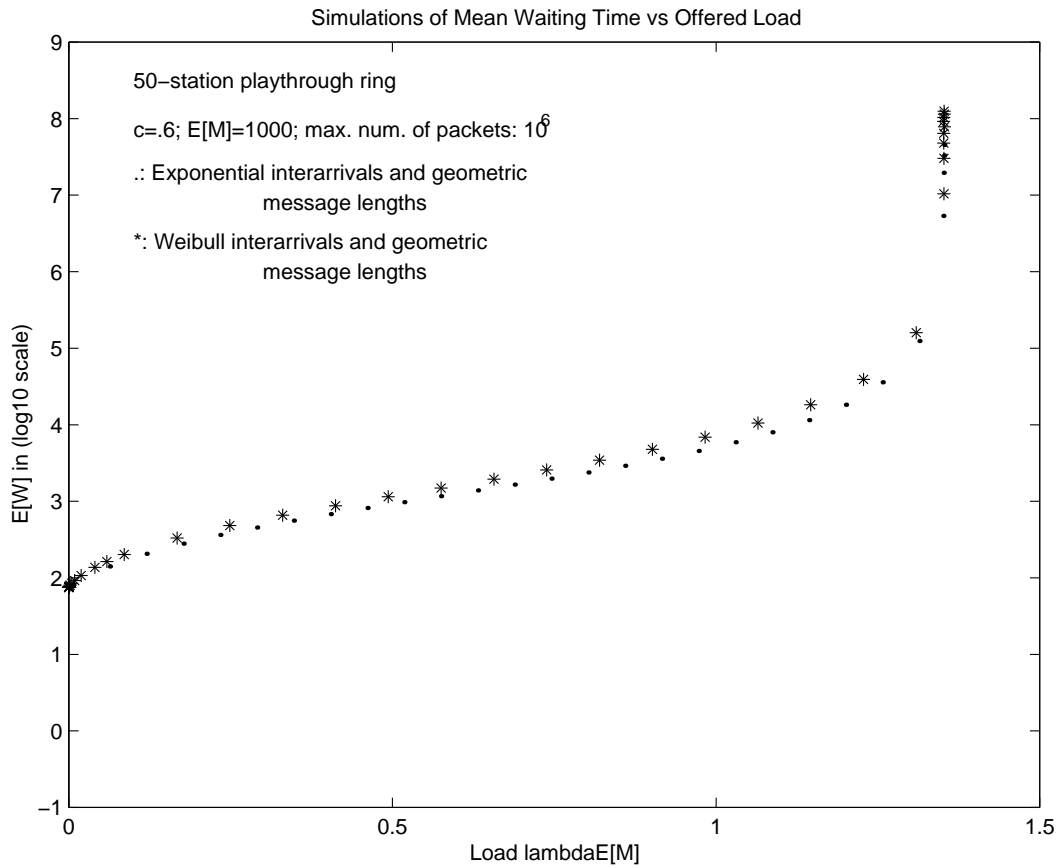


Figure 3.18: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.6$ and exponential interarrival times. $N = 50$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.18 shows the simulated average waiting time on an $N = 50$ station ring with 1000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is higher than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.352$.

The greater the number of stations, the more time is required to obtain simulation results because of the fact that the number of events to be simulated is increased. For practical reasons we did not simulate PLAYTHROUGH ring performance for $N > 50$. In addition, we believe simulating PLAYTHROUGH ring for $3 \leq N \leq 50$ gives us a wide enough range of stations to be able to determine trends. For $N = 10$, and $N = 20$, we used the same average message lengths as were used for $N = 3$, that is 100, 1000, and 5000 characters. However, 100 character is in fact a small average of message length for an $N = 20$ -stations PLAYTHROUGH ring. Indeed, for $N = 20$, and $E[M] = 100$, the number of control frame round trips needed to transfer an average-size message would be approximately $\frac{100}{3*20} \approx 1.7$. This would render PLAYTHROUGH ring inefficient for average transfers and result in a waste of bandwidth, given that at least two overhead round trips are required to transfer the payload on PLAYTHROUGH ring. We nonetheless simulate the performance of PLAYTHROUGH ring for $N = 20$, and $E[M] = 100$ to be able to compare it with the performance of PLAYTHROUGH ring for $N = 3$, $E[M] = 100$ and $N = 10$, $E[M] = 100$ on a consistent basis. For $N = 30$ and $N = 50$, we use $E[M] = 1000$, $E[M] = 5000$, and $E[M] = 10000$. These are adequate average message lengths for this number of stations. For example, an average message length of 1000 characters on a 30-station PLAYTHROUGH ring results in an average number of control frame round trips approximately equal to $\frac{1000}{3*30} \approx 11.1$. An average message length of 1000 characters on a 50-station PLAYTHROUGH ring results in an average number of control frame round trips approximately equal to $\frac{1000}{3*50} \approx 6.7$.

From Figures 3.6–3.18 we can make several observations. First, the average message waiting time on PLAYTHROUGH ring under Weibull interarrival times and geometrically distributed message lengths appears to always be

greater than average message waiting time on PLAYTHROUGH ring under exponential interarrival times and geometrically distributed message lengths.

Second, waiting times on PLAYTHROUGH ring saturate at approximately the same offered load under the assumptions of either Weibull interarrival times or exponential interarrival times, both with geometrically distributed message lengths.

We now consider the effect of the shape parameter on average waiting time under the assumption of Weibull interarrival times. In Figures 3.6–3.18, the Weibull distribution shape parameter was chosen to be 0.6. In Figures 3.21–3.32, we choose $c = 0.4$

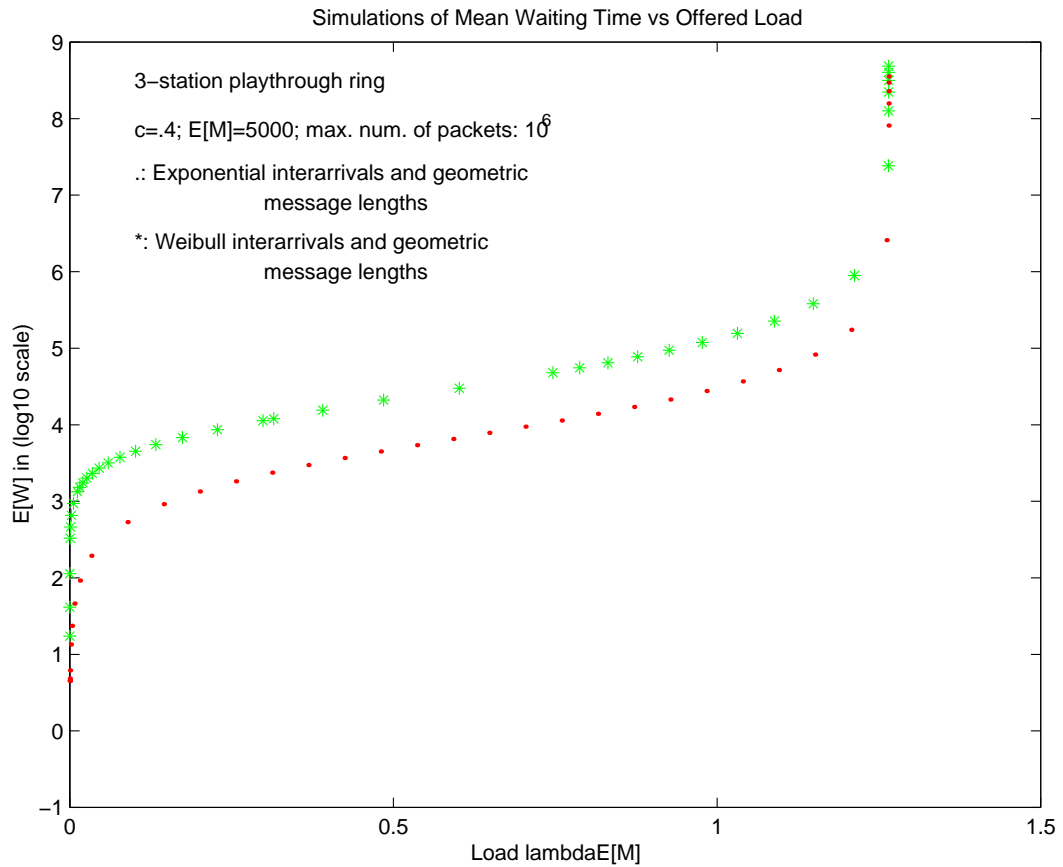


Figure 3.19: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 3$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.19 shows the simulated average waiting time on an $N = 3$ -station ring, with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.265$.

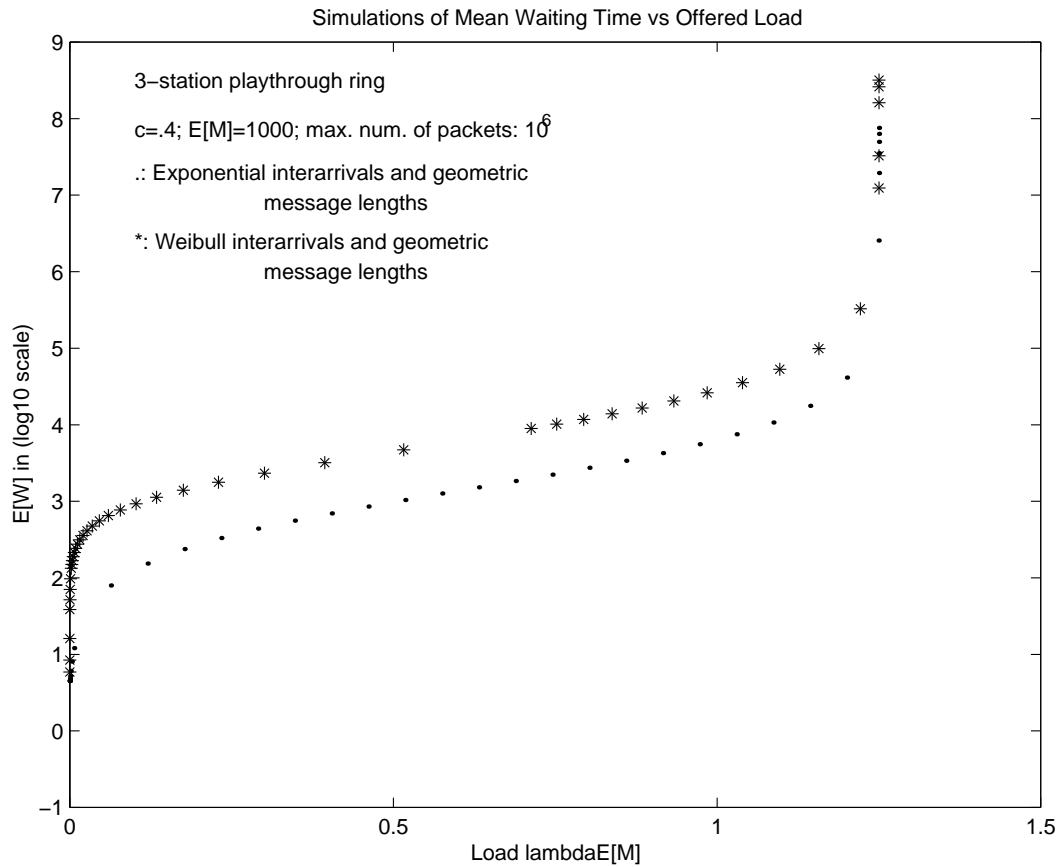


Figure 3.20: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 3$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.20 shows the simulated average waiting time on an $N = 3$ -station ring, with 1000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.25$.

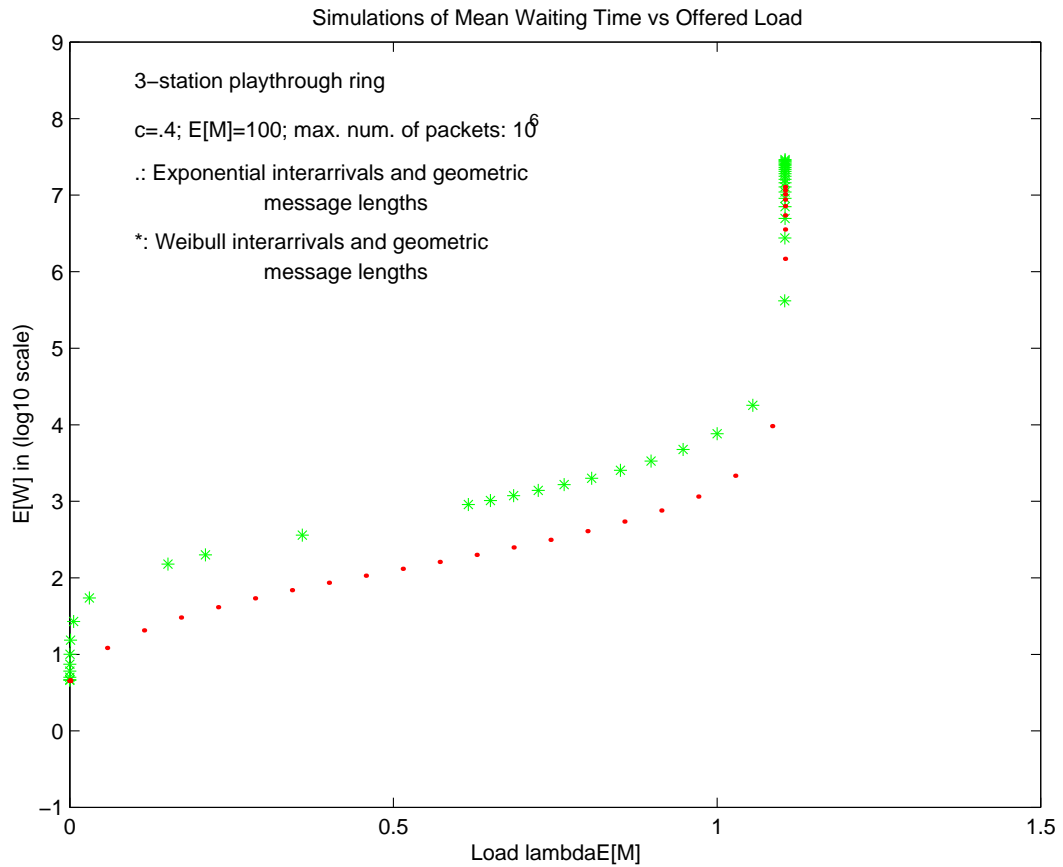


Figure 3.21: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 3$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 100$ characters.

Figure 3.21 shows the simulated average waiting time on an $N = 3$ -station ring, with 100 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached at $\lambda E[M] = 1.104$.

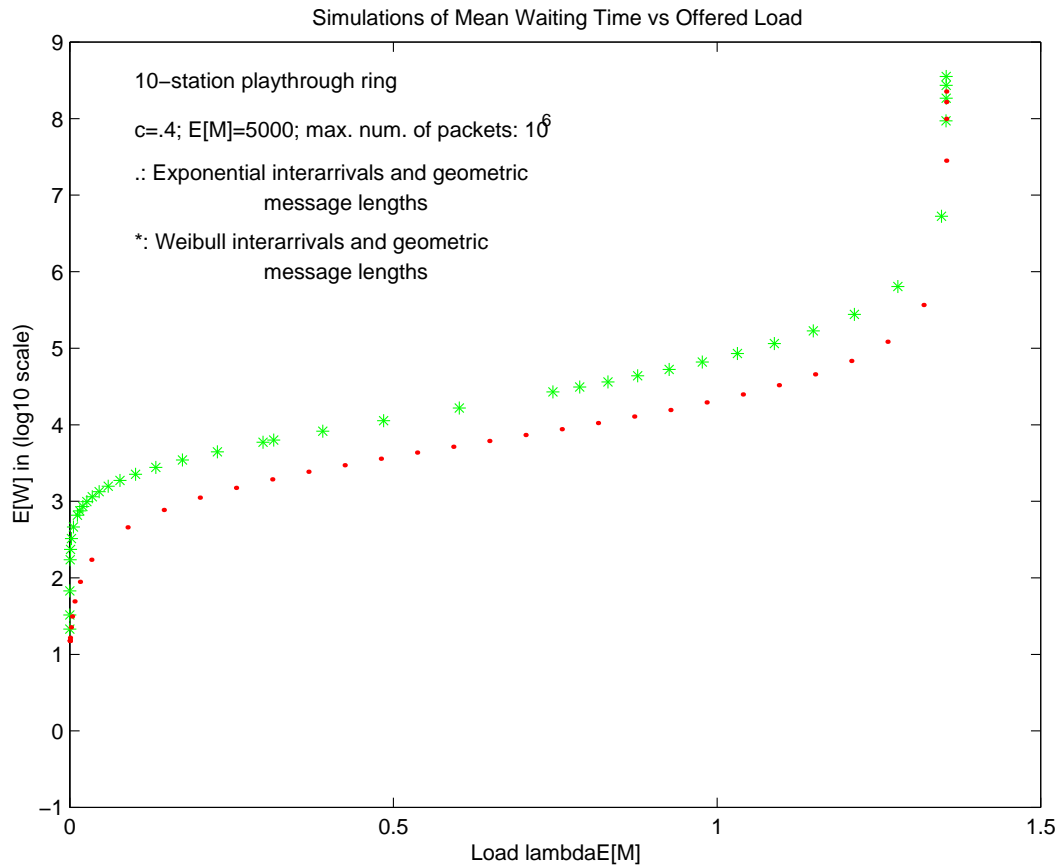


Figure 3.22: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 10$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.22 shows the simulated average waiting time on an $N = 10$ -station ring, with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.35$.

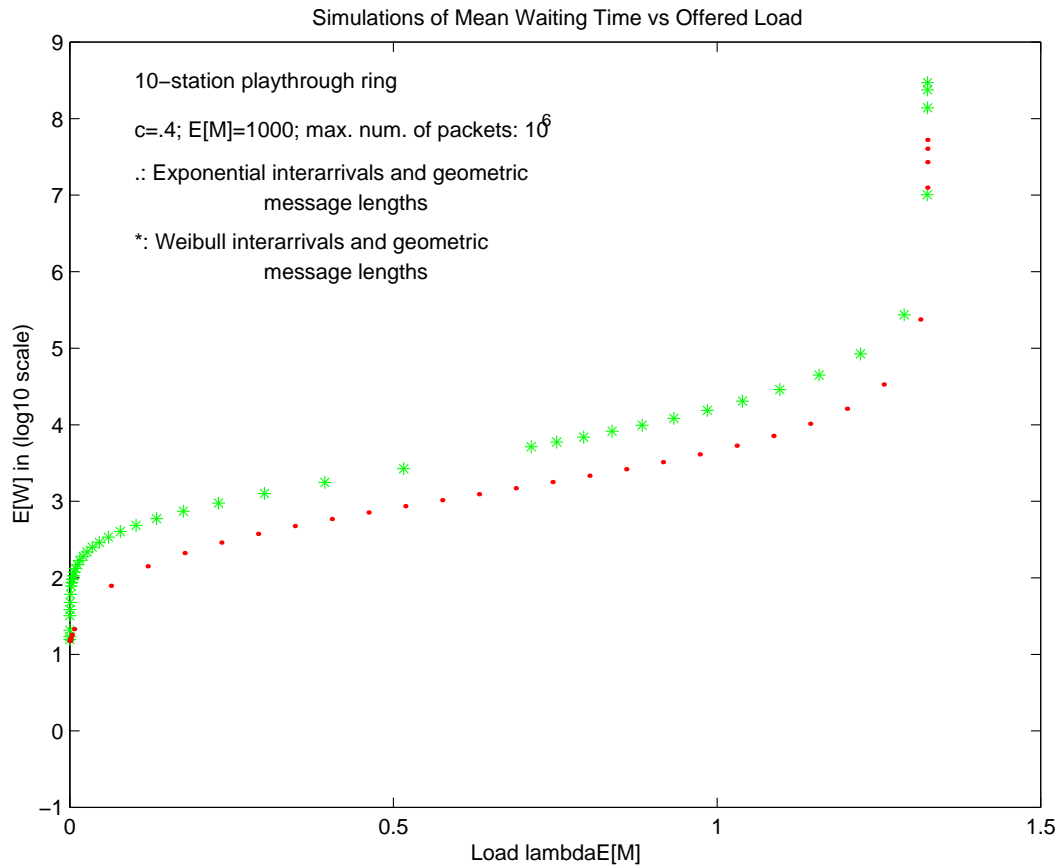


Figure 3.23: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 10$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.23 shows the simulated average waiting time on an $N = 10$ -station ring, with 1000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.32$.

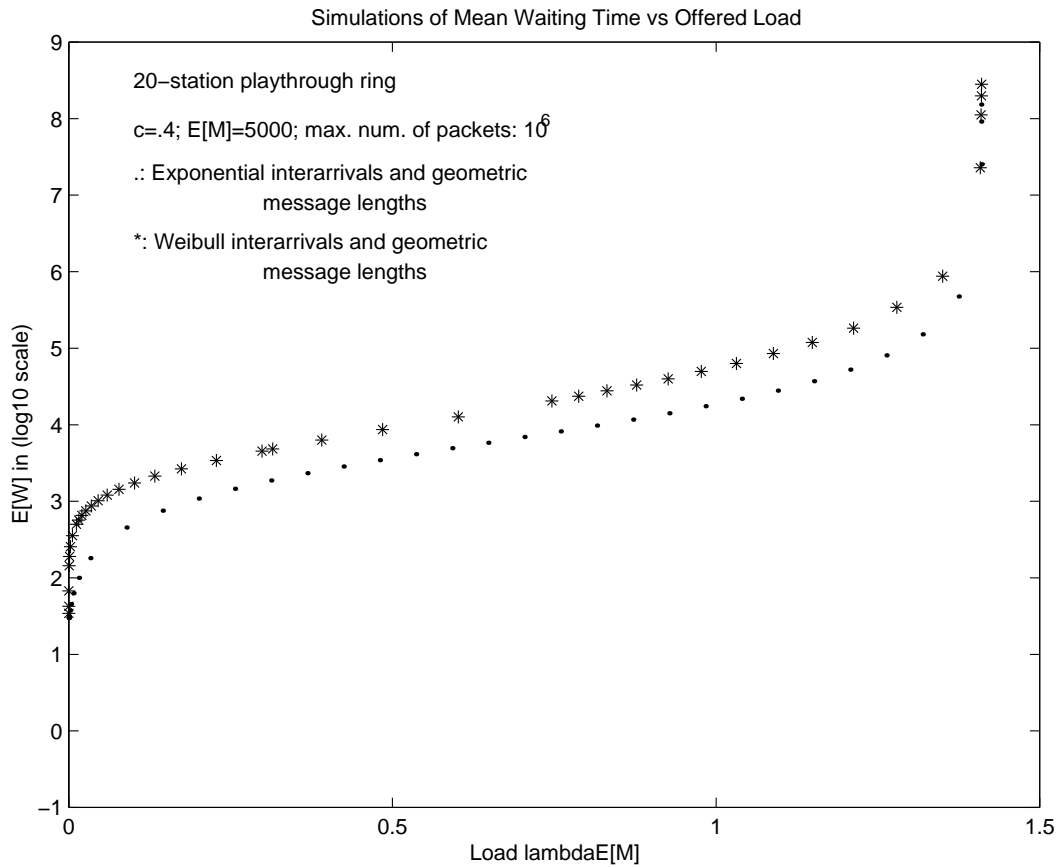


Figure 3.24: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 20$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.24 shows the simulated average waiting time on an $N = 20$ -station ring, with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.40$.

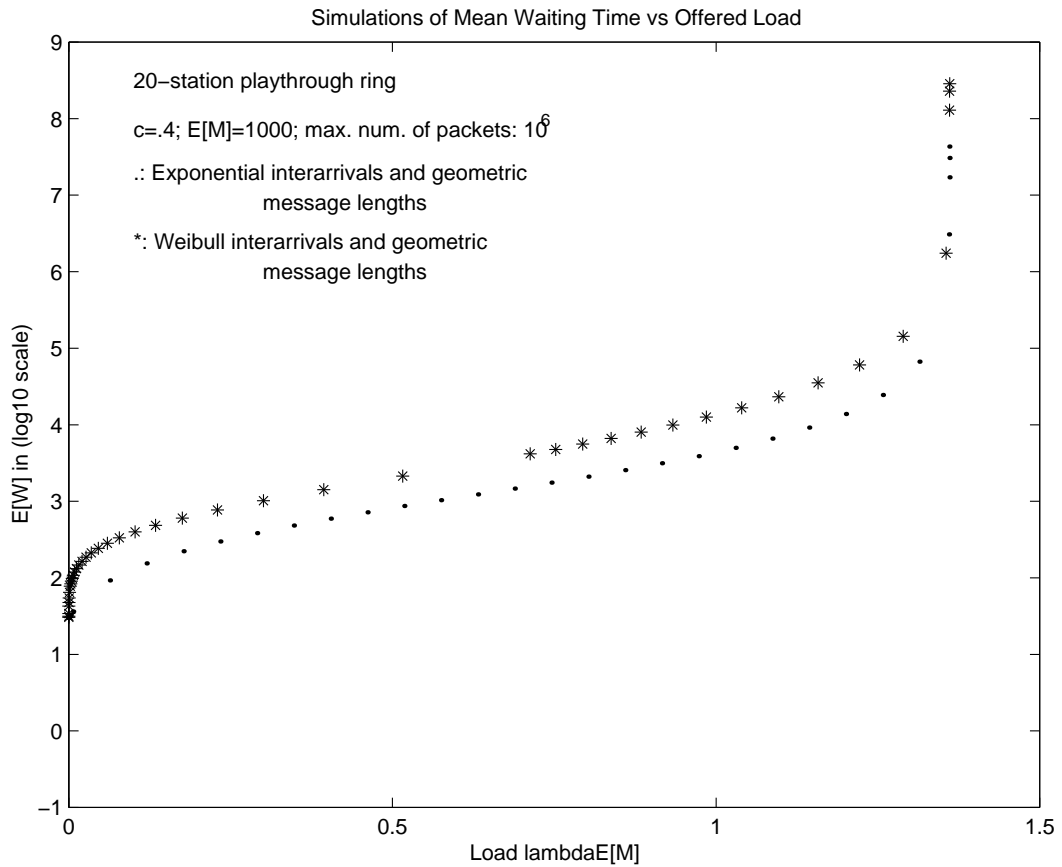


Figure 3.25: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 20$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.25 shows the simulated average waiting time on an $N = 20$ -station ring, with 1000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.36$.

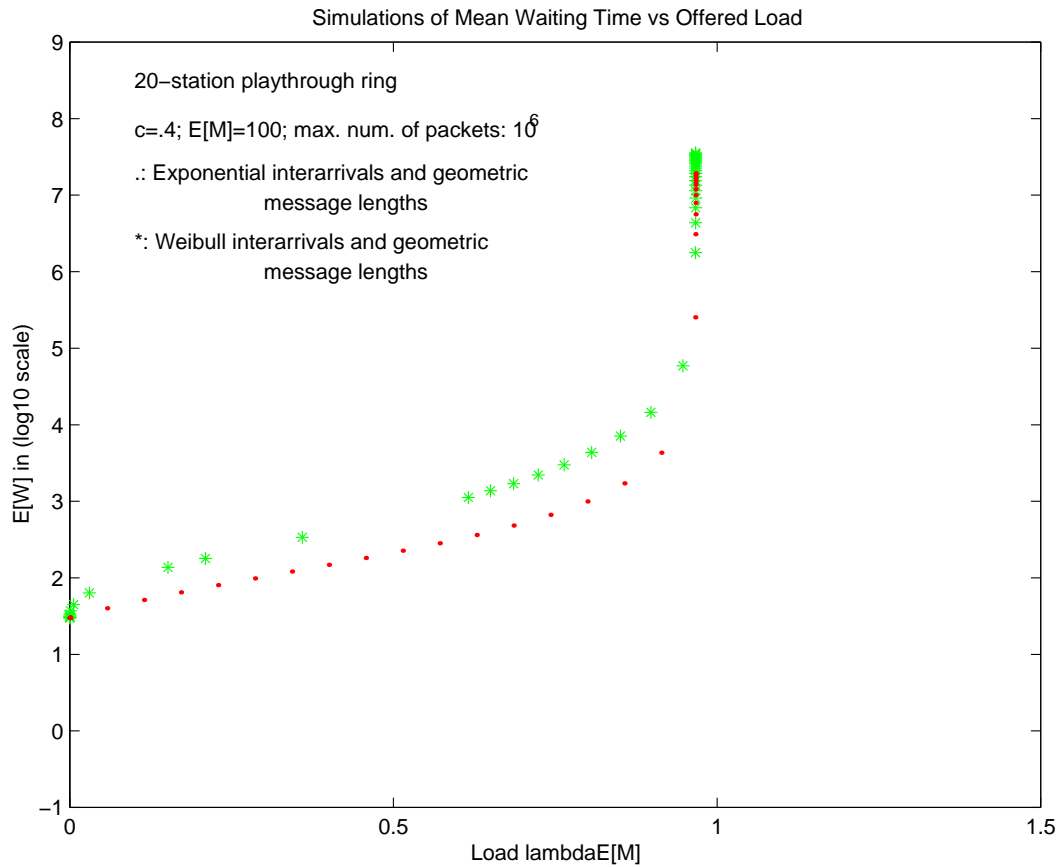


Figure 3.26: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 20$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 100$ characters.

Figure 3.26 shows the simulated average waiting time on an $N = 20$ -station ring, with 100 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 0.96$.

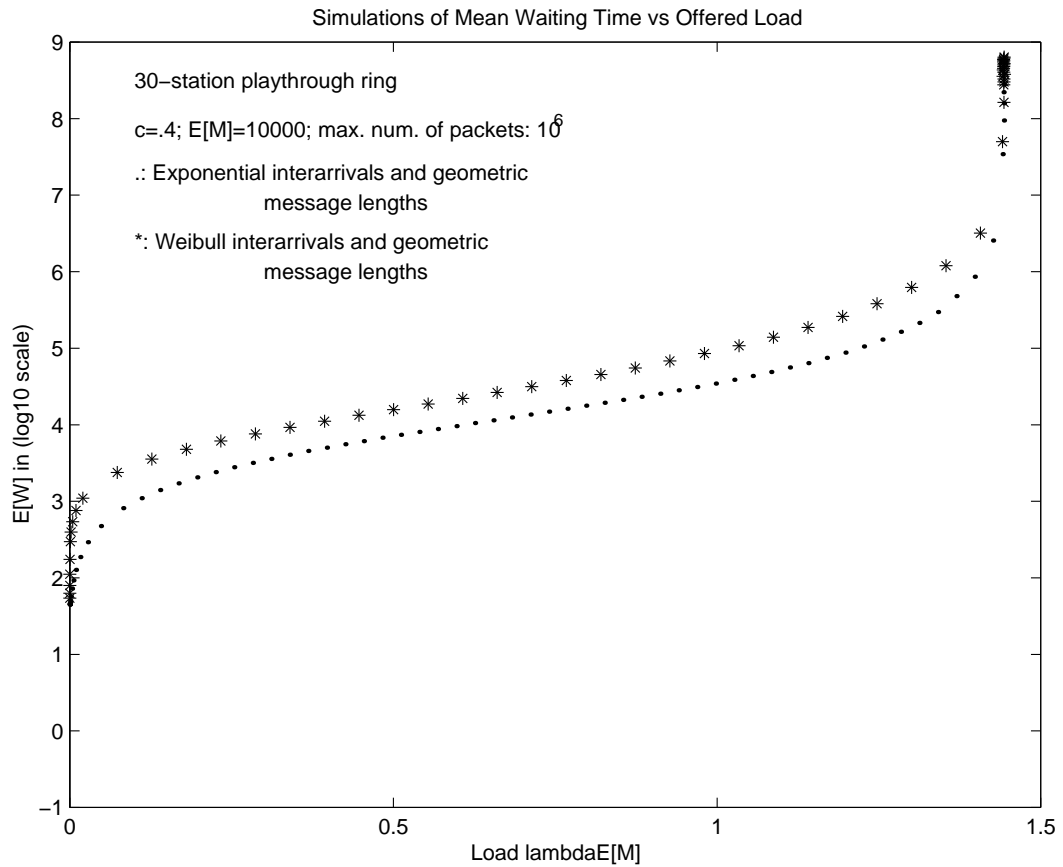


Figure 3.27: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 30$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 10000$ characters.

Figure 3.27 shows the simulated average waiting time on an $N = 30$ -station ring, with 10000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.44$.

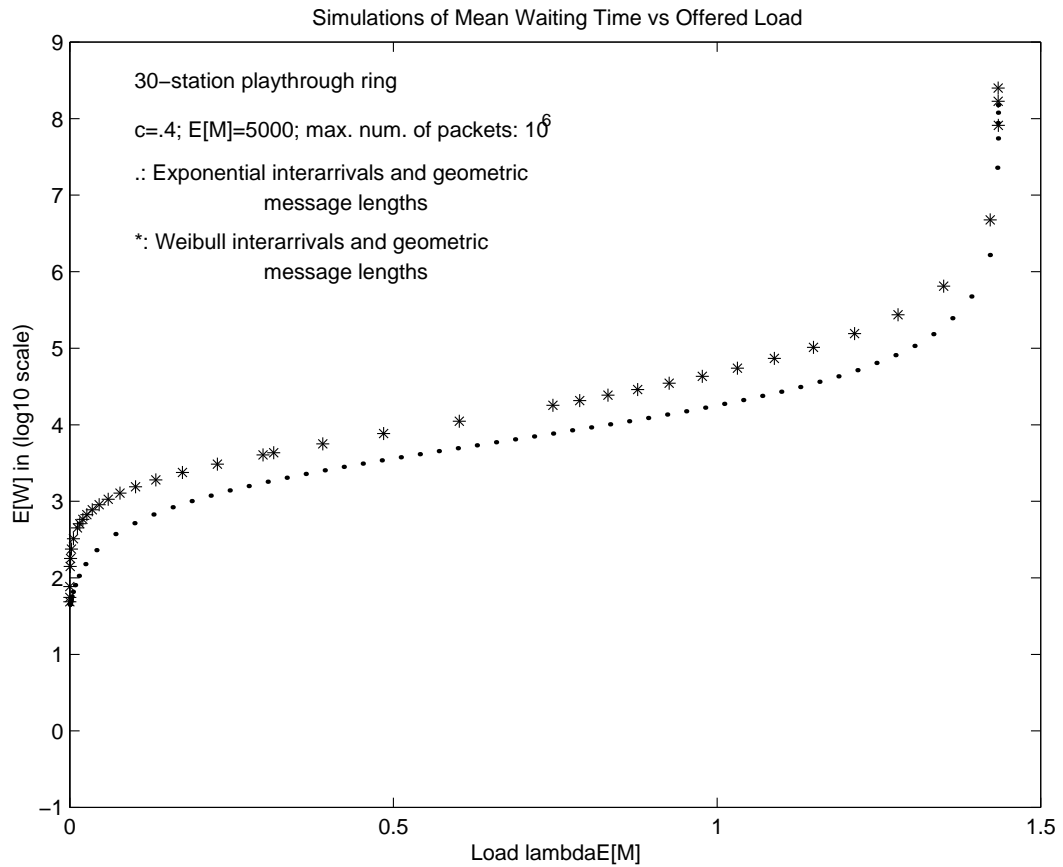


Figure 3.28: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 30$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.28 shows the simulated average waiting time on an $N = 30$ -station ring, with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.43$.

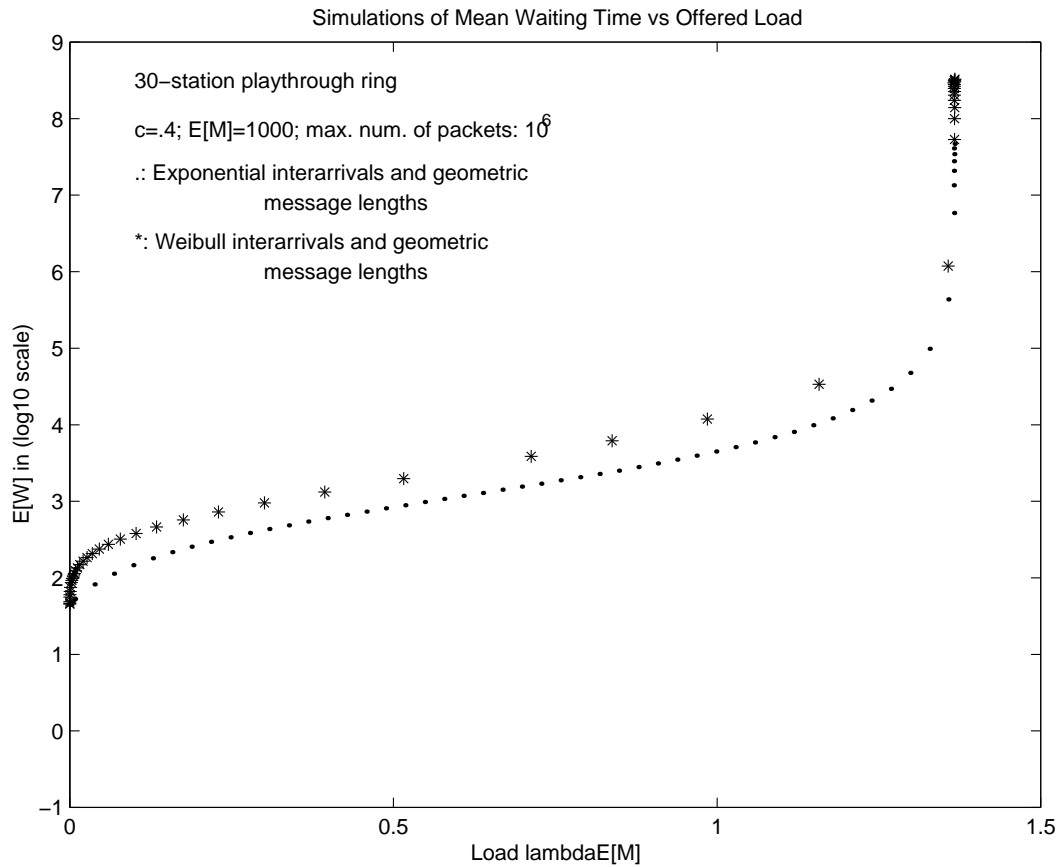


Figure 3.29: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 30$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.29 shows the simulated average waiting time on an $N = 30$ -station ring, with 1000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached at $\lambda E[M] = 1.36$.

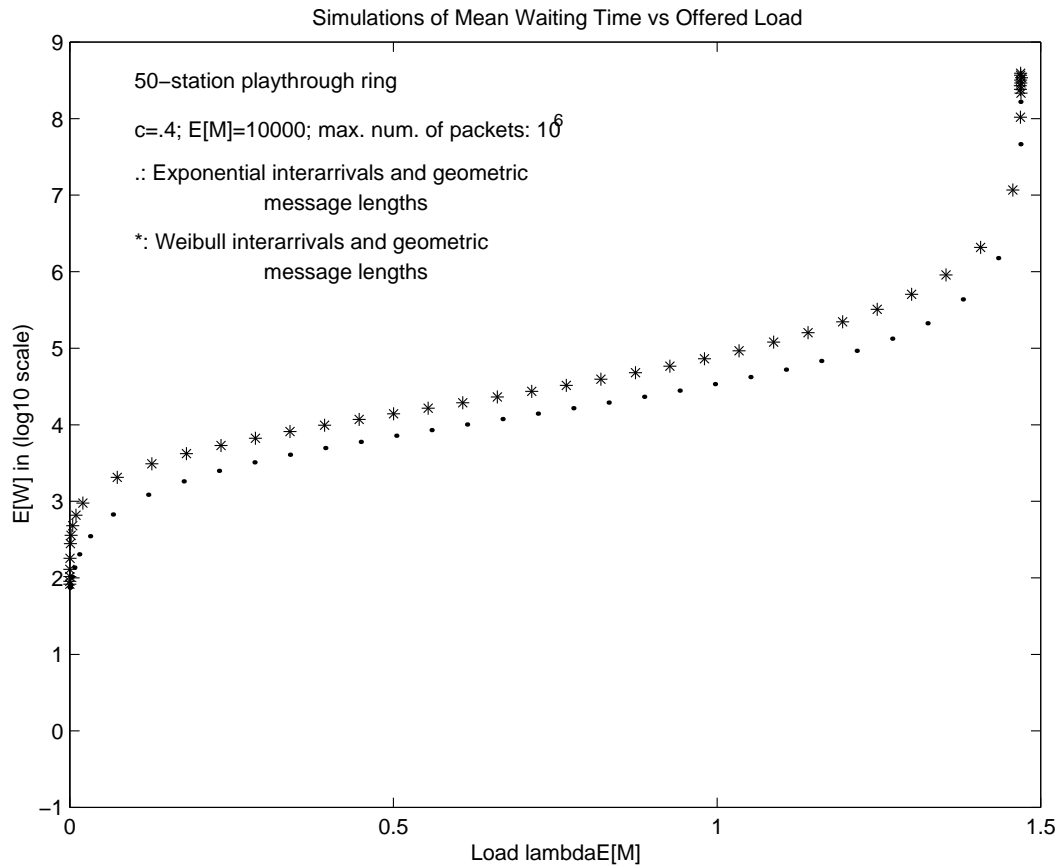


Figure 3.30: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 50$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 10000$ characters.

Figure 3.30 shows the simulated average waiting time on an $N = 50$ -station ring, with 10000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.46$.

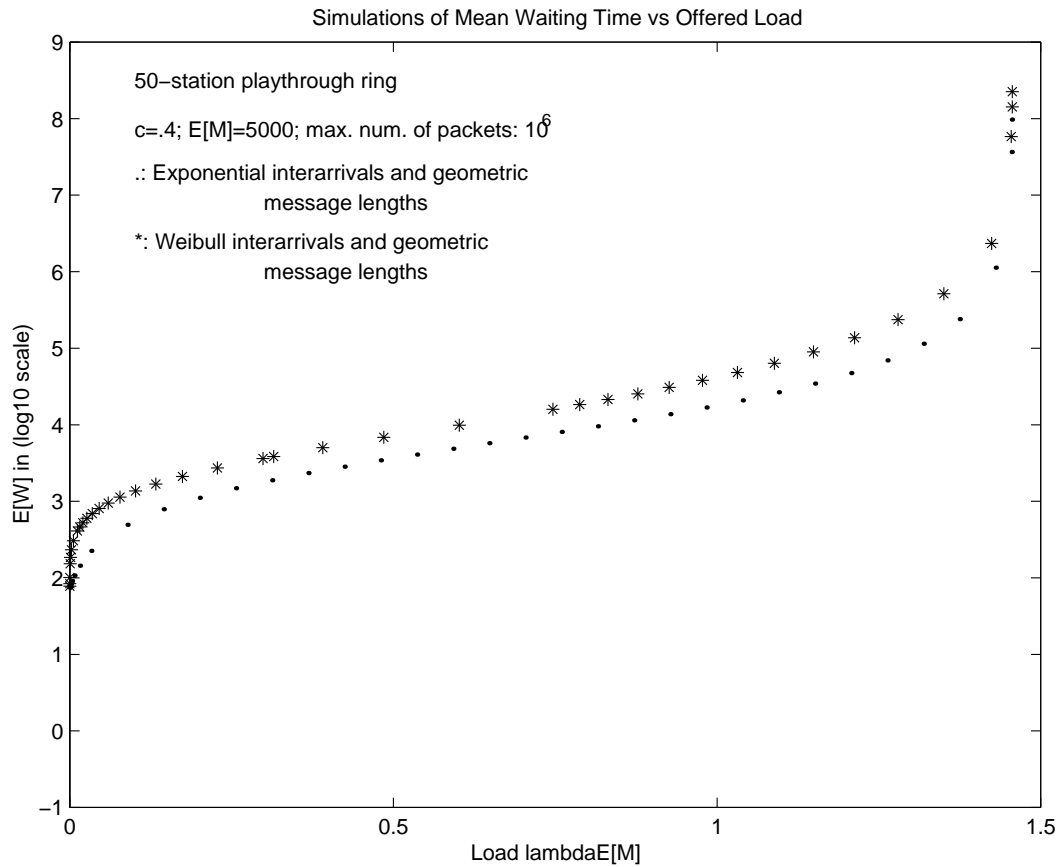


Figure 3.31: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 50$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 5000$ characters.

Figure 3.31 shows the simulated average waiting time on an $N = 50$ -station ring, with 5000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.45$.

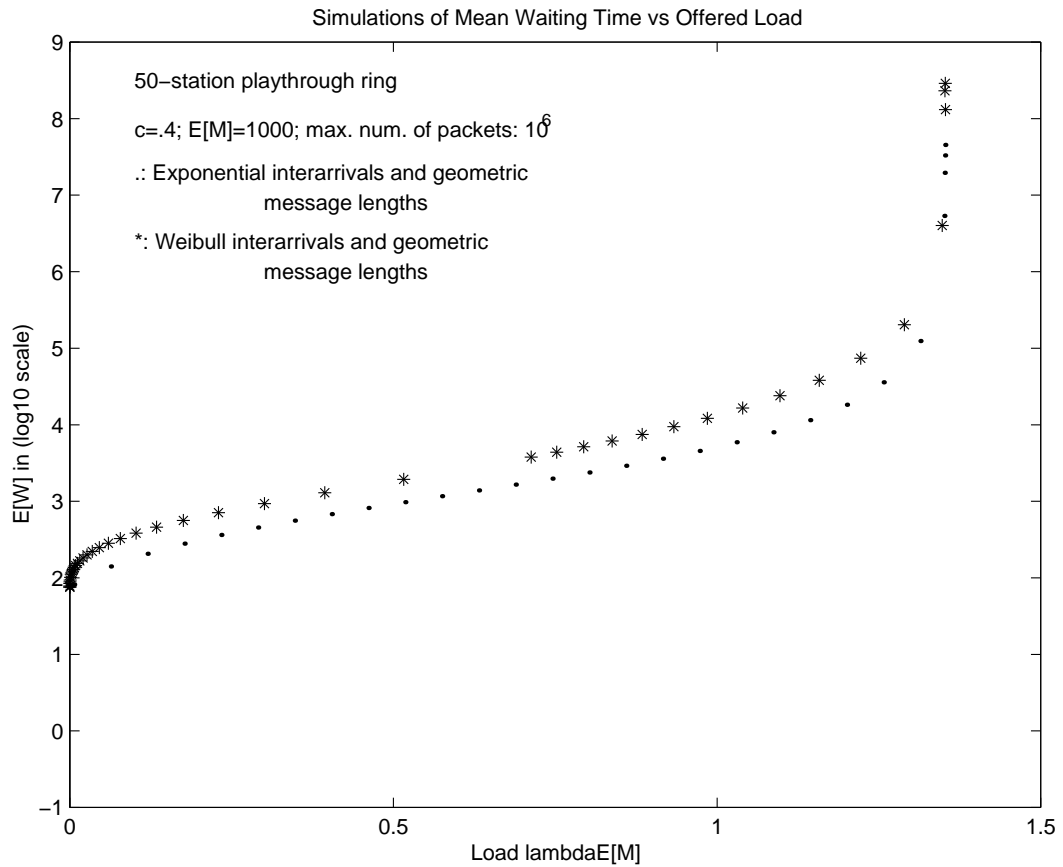


Figure 3.32: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for Weibull interarrival times with parameter $c = 0.4$ and exponential interarrival times. $N = 50$ station, simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and geometrically distributed message lengths with mean $E[M] = 1000$ characters.

Figure 3.32 shows the simulated average waiting time on an $N = 50$ -station ring, with 1000 characters mean message lengths. The average waiting time for PLAYTHROUGH under Weibull message interarrival times and geometrically distributed message lengths is greater than the average waiting time under exponential message interarrival times and geometrically distributed message lengths. Saturation in both cases is reached approximately at $\lambda E[M] = 1.35$.

By inspecting plots from Figures 3.4–3.18 and those from Figures 3.19–3.32 we can make the following observations.

Average waiting times in Figures 3.19–3.32 are greater than corresponding waiting times in Figures 3.4–3.18. The lower value of the shape parameter $c = 0.4$ used in Figures 3.4–3.18 has the effect of worsening the performance of PLAYTHROUGH ring than the shape parameter of $c = 0.6$ used in Figures 3.4–3.18. A lower value of the shape parameter c results in increased variability of message interarrival times. The plots therefore suggest that increased variability of message interarrival times worsens the performance of PLAYTHROUGH Ring.

The saturation point of PLAYTHROUGH Ring under Weibull message interarrival times is the same as the saturation point of PLAYTHROUGH Ring under exponential message interarrival times.

3.4 Simulations of the Performance of PLAYTHROUGH Under Exponential Connection Intervals and Weibull Message Lengths

In this section, simulations of the performance of PLAYTHROUGH ring under exponential message interarrival times and Weibull message lengths are presented. We assume that messages are served by PLAYTHROUGH ring with a FIFO queueing discipline. To be able to simulate the performance of PLAYTHROUGH ring under exponential message interarrival times and Weibull message lengths, an earlier simulation model written by Henry [Hen98] was modified. Sample means and moments for waiting times and other performance metrics were collected on a simulated sample of one million messages transferred on the ring for each nominal arrival rate applied to the ring.

For each measured parameter Y , let n denote the number of samples, $E[Y]$

denote the sample mean, and $SD[Y]$ denote the sample standard deviation. Approximate 95% confidence intervals were calculated (and are plotted in Chapter 4) as

$$E[Y] \pm 1.96(SD[Y]/\sqrt{n}) \text{ [Kob78]}, \quad (3.13)$$

which form the upper (+) and lower (-) confidence limits, respectively. In our simulations, $n=1,000,000$.

3.4.1 Traffic Characteristics

Messages of each class $\mathcal{C}_{i,i\oplus j}$ are assumed to arrive at station i 's queue according to a Poisson arrival process with rate $\lambda_{i,i\oplus j}$ arrivals per character time. The arrival processes of different message classes are assumed to be independent. Hence, the probability $p_{i,i\oplus j}(k, t)$ that k new messages of outbound distance j arrive at station i 's queue during a time interval of duration t character times is given by

$$p_{i,i\oplus j}(k, t) = \frac{(\lambda_{i,i\oplus j}t)^k}{k!} e^{-\lambda_{i,i\oplus j}t}. \quad (3.14)$$

Let λ_i be the aggregate message arrival rate to station i 's queue. The aggregate message arrival process to station i 's queue comprises the sum of all the $N - 1$ independent classes of traffic $\{\mathcal{C}_{i,i\oplus 1}, \dots, \mathcal{C}_{i,i\oplus(N-1)}\}$ which arrive at station i . Since the $N - 1$ different Poisson arrival processes are independent, the aggregation of these arrivals is a Poisson arrival stream [Kle75] with rate equal to the aggregate arrival rate λ_i .

$$\lambda_i = \sum_{j=1}^{N-1} \lambda_{i,i\oplus j}. \quad (3.15)$$

Let $p_i(k, t)$ be the probability that k new messages of any outbound distance arrive at station i 's queue during a time interval of duration t character times.

We have:

$$p_i(k, t) = \frac{(\lambda_i t)^k}{k!} e^{-\lambda_i t}. \quad (3.16)$$

The aggregate arrival rate to the entire ring, denoted λ , is the sum of the arrival rates for each of the N stations.

$$\lambda = \sum_{i=0}^{N-1} \lambda_i = \sum_{i=0}^{N-1} \sum_{j=1}^{N-1} \lambda_{i,i\oplus j}. \quad (3.17)$$

The fraction $f_{i,i\oplus j}$ of messages of outbound distance j sourced by station i is given by:

$$f_{i,i\oplus j} = \frac{\lambda_{i,i\oplus j}}{\lambda_i}. \quad (3.18)$$

Each message of class $\mathcal{C}_{i,i\oplus j}$ is made up of a constant-size header of discrete length $h \geq 0$ and a variable-size payload of length $M_{i,i\oplus j}$, a *r.v.* which satisfies $M_{i,i\oplus j} \geq 1$. $M_{i,i\oplus j}$ is assumed to be Weibull distributed with parameters $a_{i,i\oplus j}$ and $c_{i,i\oplus j}$. We have:

$$Pr [M_{i,i\oplus j} \leq x] = 1 - e^{-\left(\frac{x}{a_{i,i\oplus j}}\right)^{c_{i,i\oplus j}}} \quad (3.19)$$

We assume uniform and symmetric traffic (UST), which means that each station transmits an equal amount of traffic to every other station, and all message lengths are identically distributed. Under these circumstance, $\lambda_i = \frac{\lambda}{N}$, $\lambda_{i,i\oplus j} = \frac{\lambda_i}{N-1}$, $a_{i,i\oplus j} = a$, $c_{i,i\oplus j} = c$.

3.4.2 Simulations and Comparison With Prior Assumptions

Plots in this section show of the average waiting time of messages in the PLAYTHROUGH ring under the assumption of exponential interarrival times and Weibull message lengths. The average waiting time of messages in the PLAYTHROUGH ring under the assumption of exponential interarrival times and geometric message lengths is also included for comparison purposes.

Two sets of plots are presented. The first set of plots includes Figures 3.33–3.47, in which the value of the shape parameter of the Weibull message lengths distribution is $c = 0.6$. The second set of plots includes Figures 3.48–3.62, in which the value of the shape parameter of the Weibull message lengths distribution is $c = 0.4$.

For each data point shown on the plots, an exponential interarrival times distribution and a Weibull message length distribution with scale parameter a and shape parameter c are applied to the ring, resulting in an overall ring arrival rate λ and an average message length $E[M]$ given by:

$$E[M] = \frac{1}{a\Gamma\left(1 + \frac{1}{c}\right)} \quad (3.20)$$

The arrival rate λ_i of messages arriving at station i is given by $\lambda_i = \frac{\lambda}{N}$ and the arrival rate for messages of class $\mathcal{C}_{i,i\oplus j}$ is given by $\lambda_{i,i\oplus j} = \frac{\lambda_i}{N-1}$.

The average measured waiting time $E[W]$ is plotted versus the measured offered load $\lambda E[M]$, which is the result of the measured arrival rate λ times the measured average message length $E[M]$. The measured arrival rate λ is the number of messages that arrive to the ring per character time.

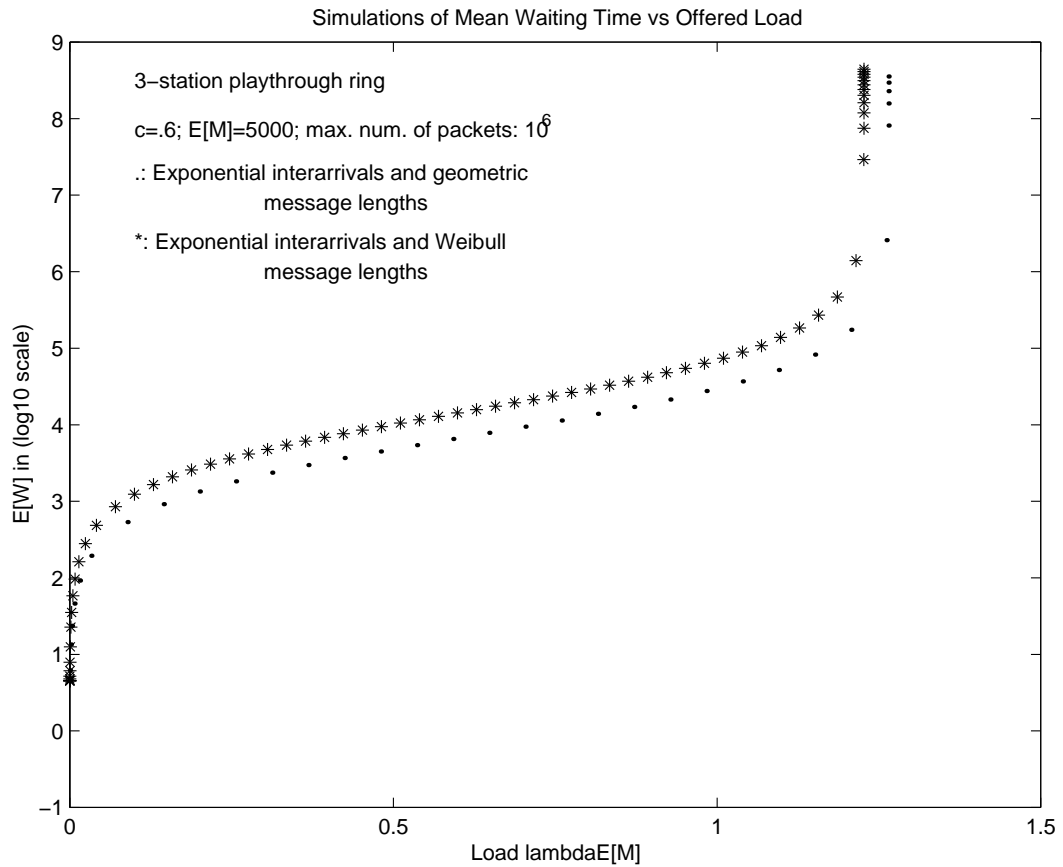


Figure 3.33: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 5000$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.33 shows the simulated average waiting time on an $N = 3$ -station ring, with 5000 characters mean message lengths. In Figure 3.33, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 100% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.226) than it does under geometrically distributed message lengths (at 1.265).

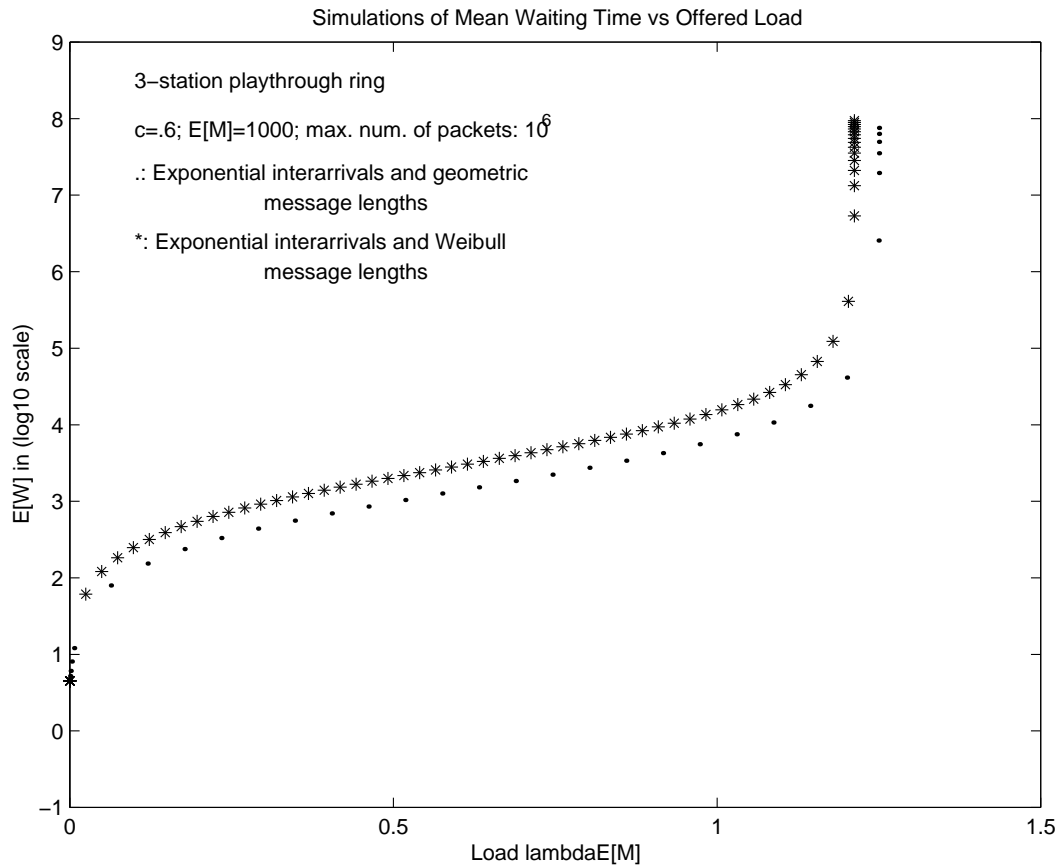


Figure 3.34: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 1000$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.34 shows the simulated average waiting time on an $N = 3$ -station ring, with 1000 characters mean message lengths. In Figure 3.34, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 100% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.212) than it does under geometrically distributed message lengths (1.250).

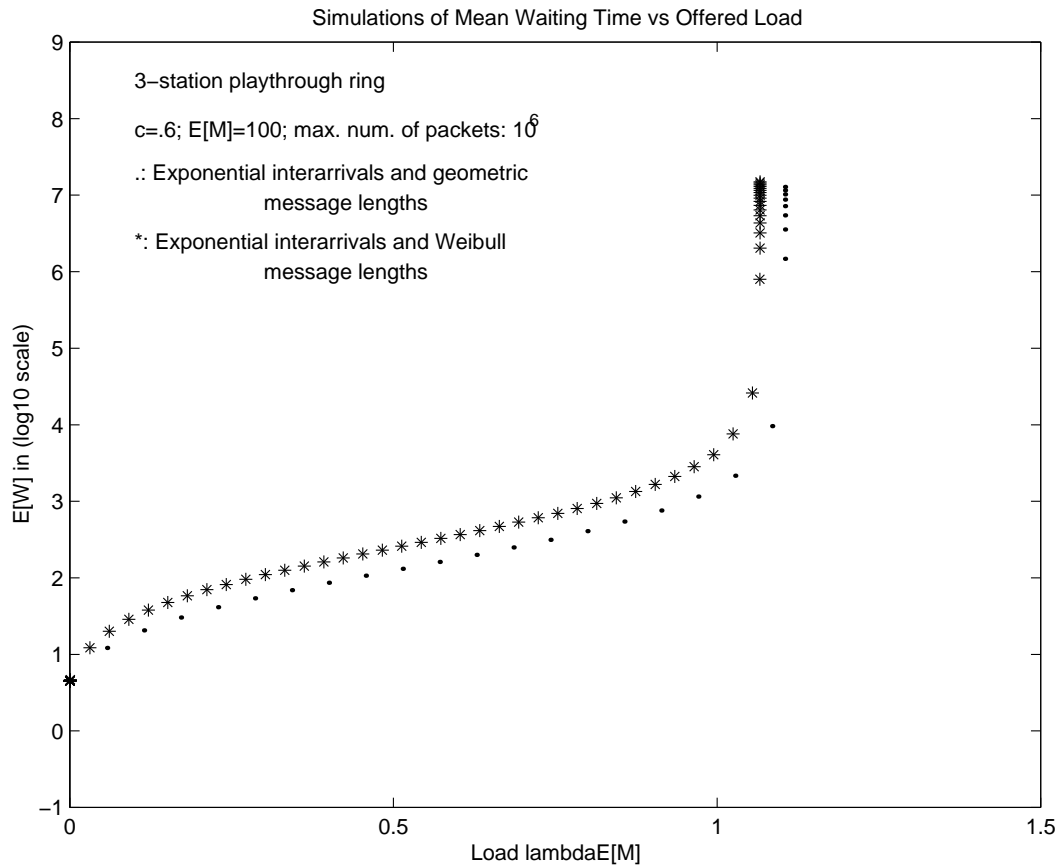


Figure 3.35: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 100$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.35 shows the simulated average waiting time on an $N = 3$ -station ring, with 100 characters mean message lengths. In Figure 3.35, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 60% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.066) than it does under geometrically distributed message lengths (1.104).

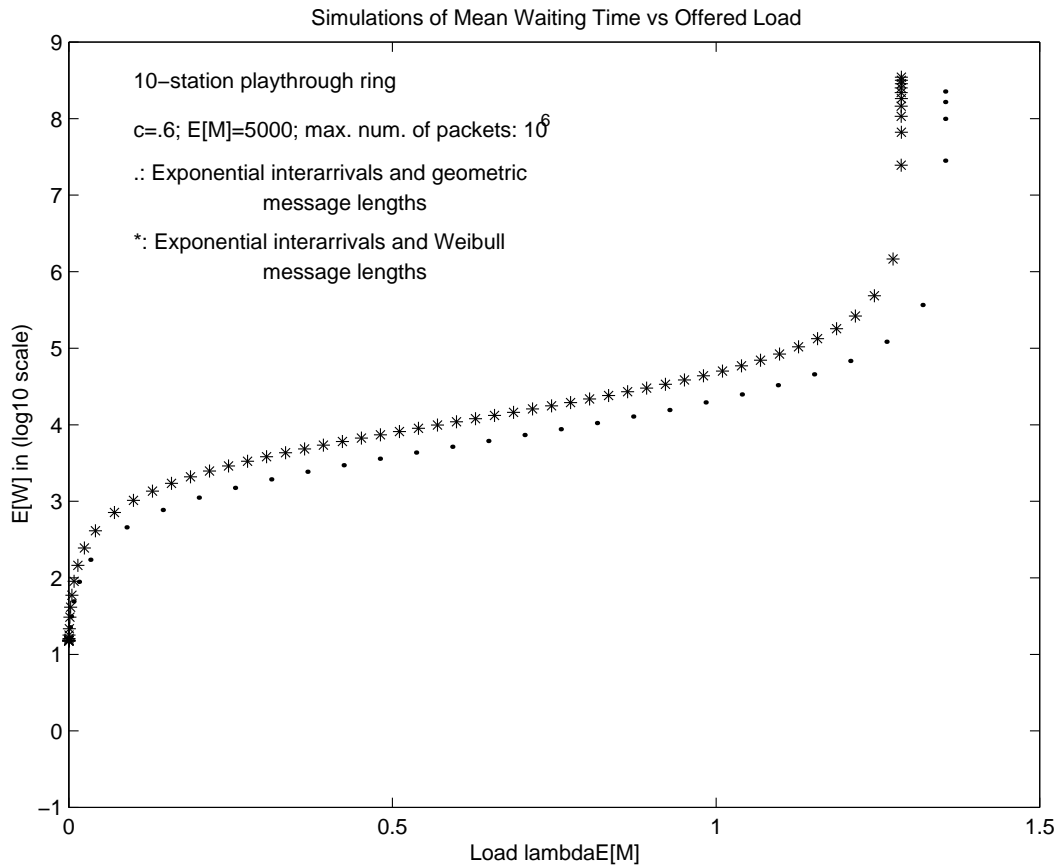


Figure 3.36: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 5000$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.36 shows the simulated average waiting time on an $N = 10$ -station ring, with 5000 characters mean message lengths. In Figure 3.36, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 125% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.286) than it does under geometrically distributed message lengths (at 1.35).

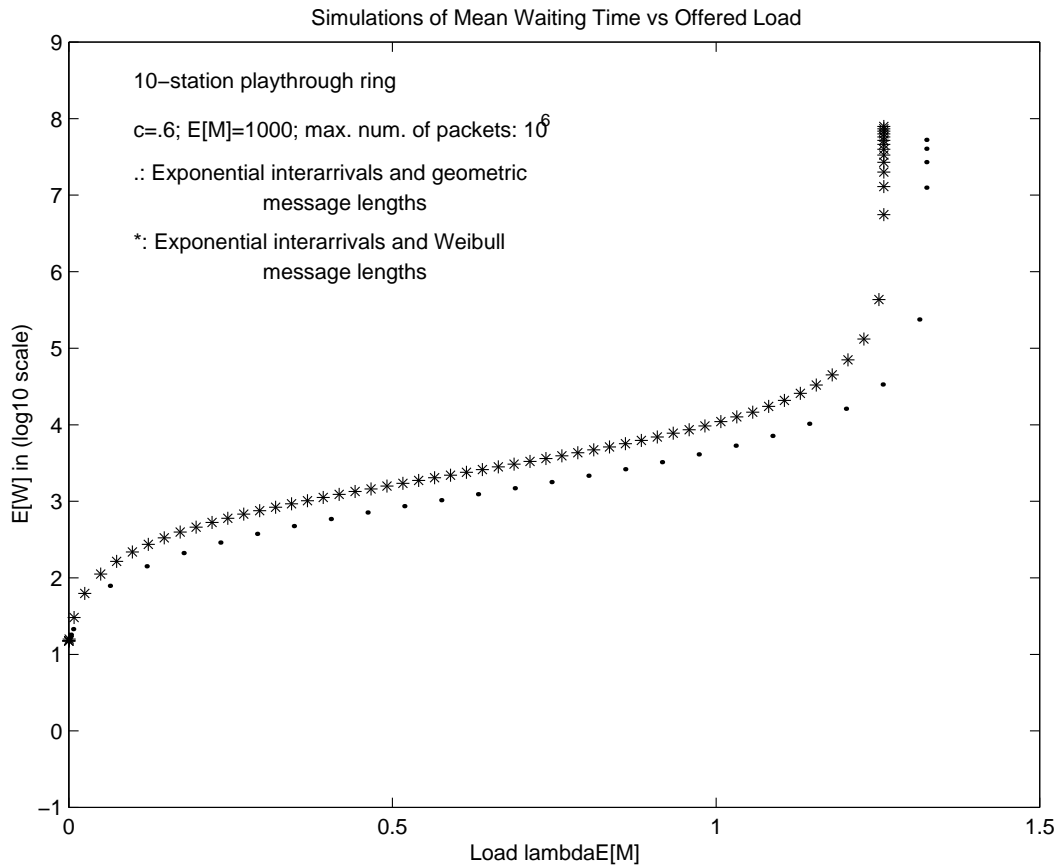


Figure 3.37: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 1000$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.37 shows the simulated average waiting time on an $N = 10$ -station ring, with 1000 characters mean message lengths. In Figure 3.37, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 100% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.25) than it does under geometrically distributed message lengths (at 1.32).

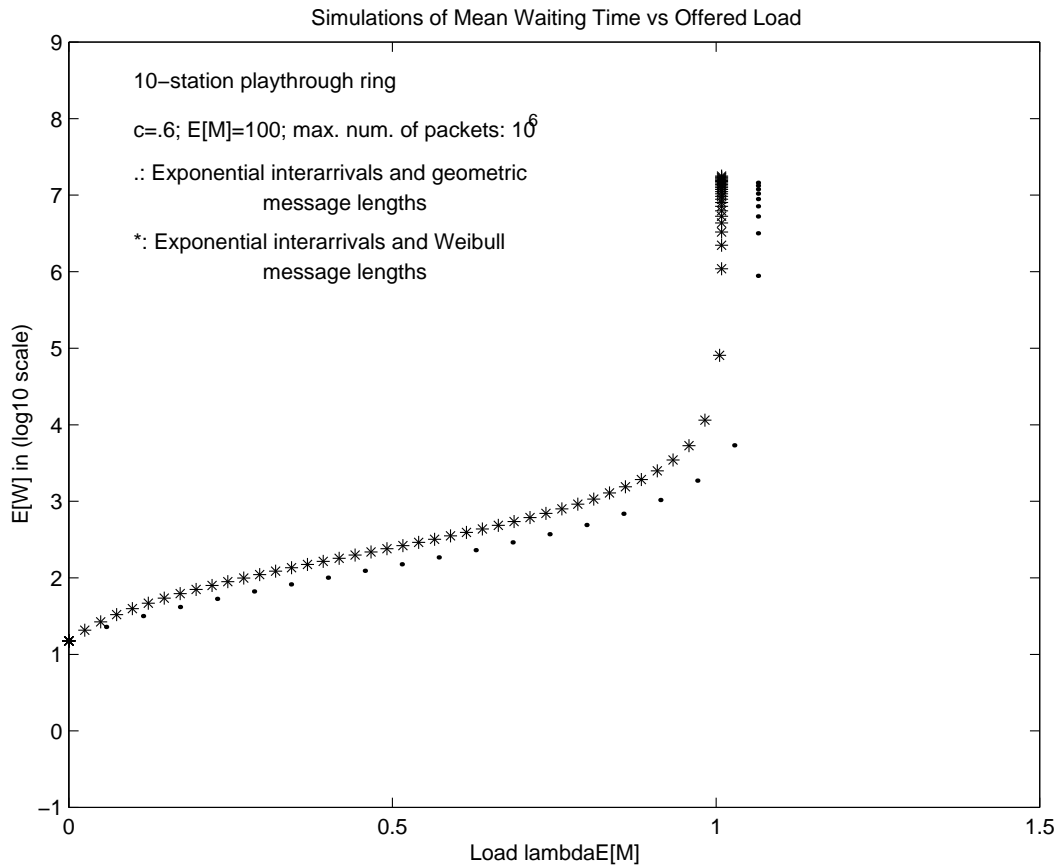


Figure 3.38: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 100$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.38 shows the simulated average waiting time on an $N = 10$ -station ring, with 100 characters mean message lengths. In Figure 3.38, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 80% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.008) than it does under geometrically distributed message lengths (at 1.0646).

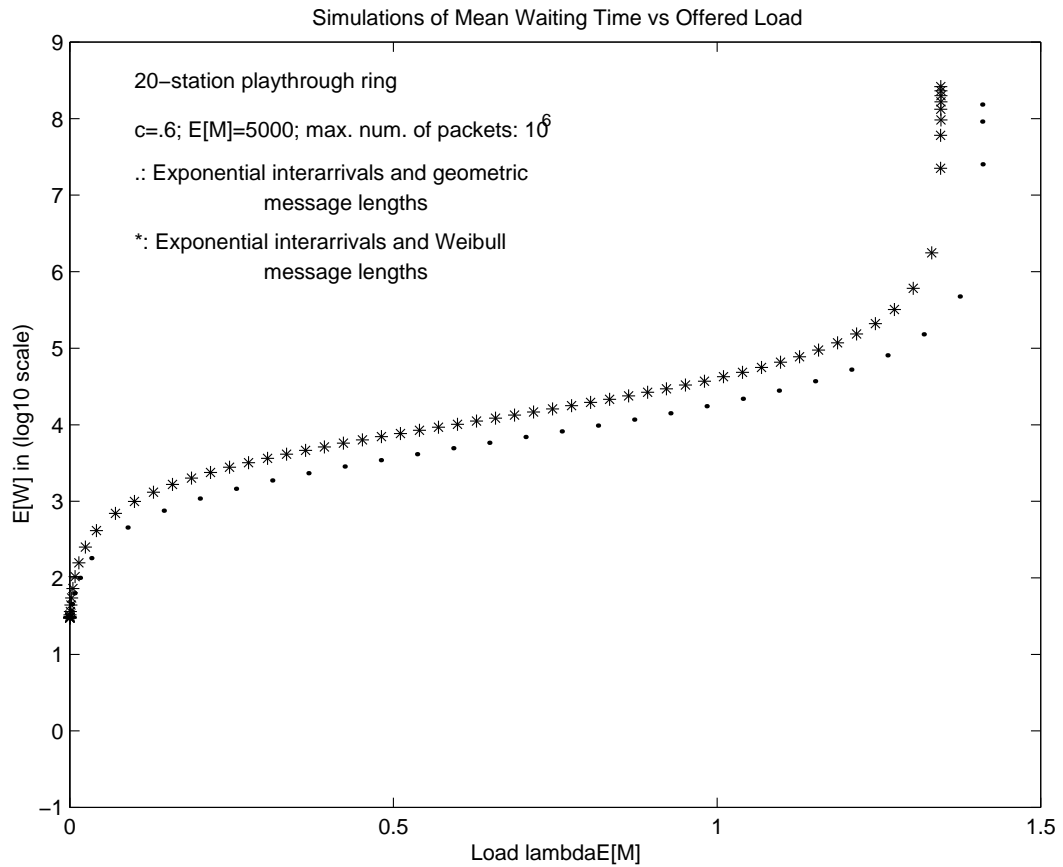


Figure 3.39: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 5000$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.39 shows the simulated average waiting time on an $N = 20$ -station ring, with 5000 characters mean message lengths. In Figure 3.39, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 150% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.345) than it does under geometrically distributed message lengths (at 1.40).

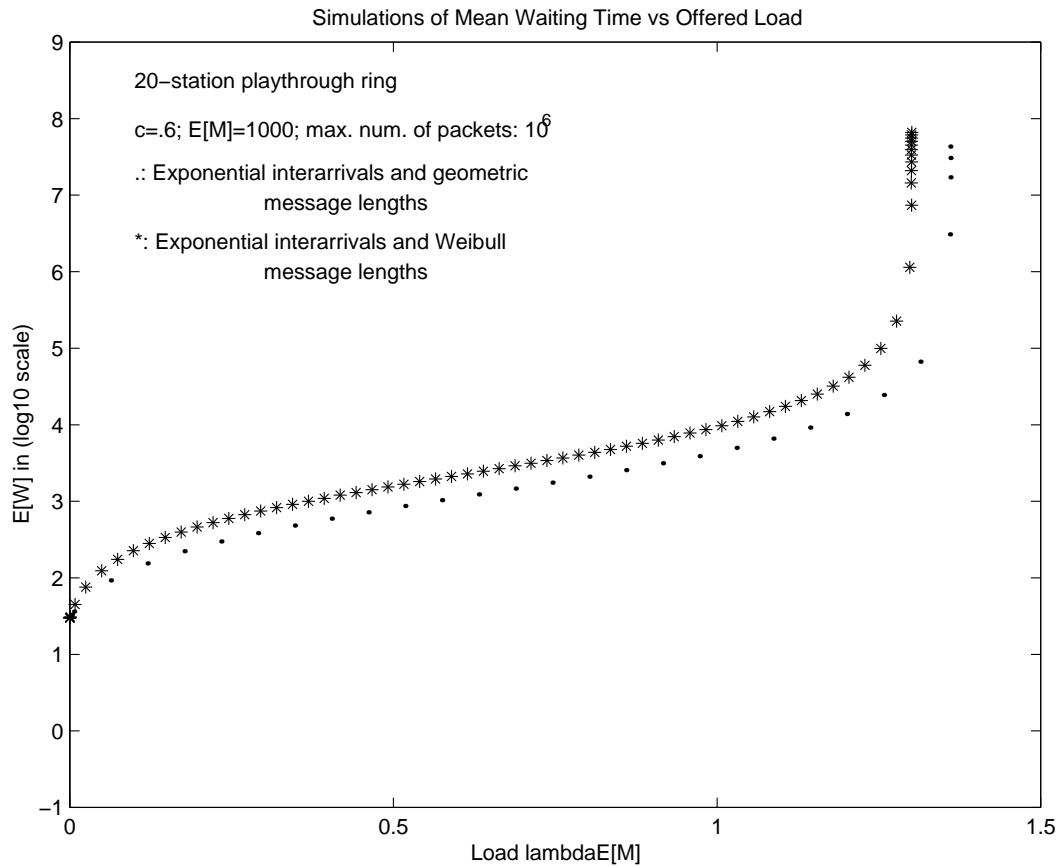


Figure 3.40: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 1000$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.40 shows the simulated average waiting time on an $N = 20$ -station ring, with 1000 characters mean message lengths. In Figure 3.40, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 100% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.30) than it does under geometrically distributed message lengths (at 1.35).

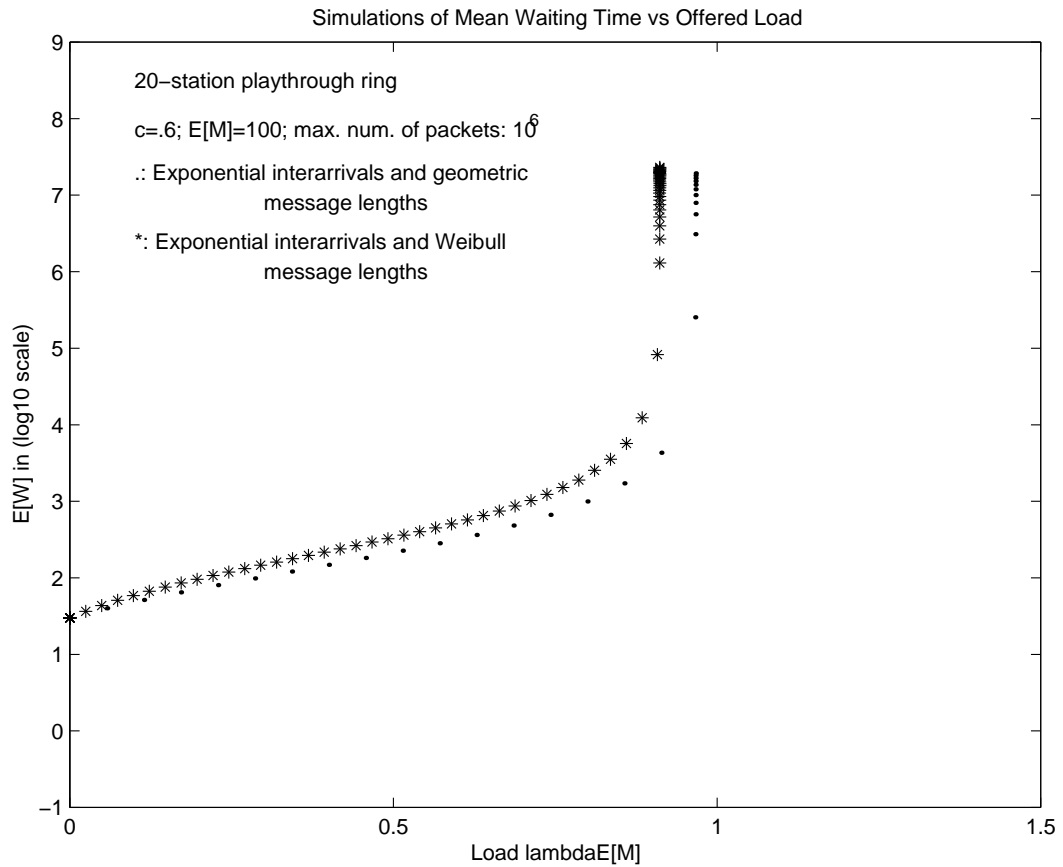


Figure 3.41: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 100$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.41 shows the simulated average waiting time on an $N = 20$ -station ring, with 100 characters mean message lengths. In Figure 3.41, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 60% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 0.911) than it does under geometrically distributed message lengths (at 0.966).

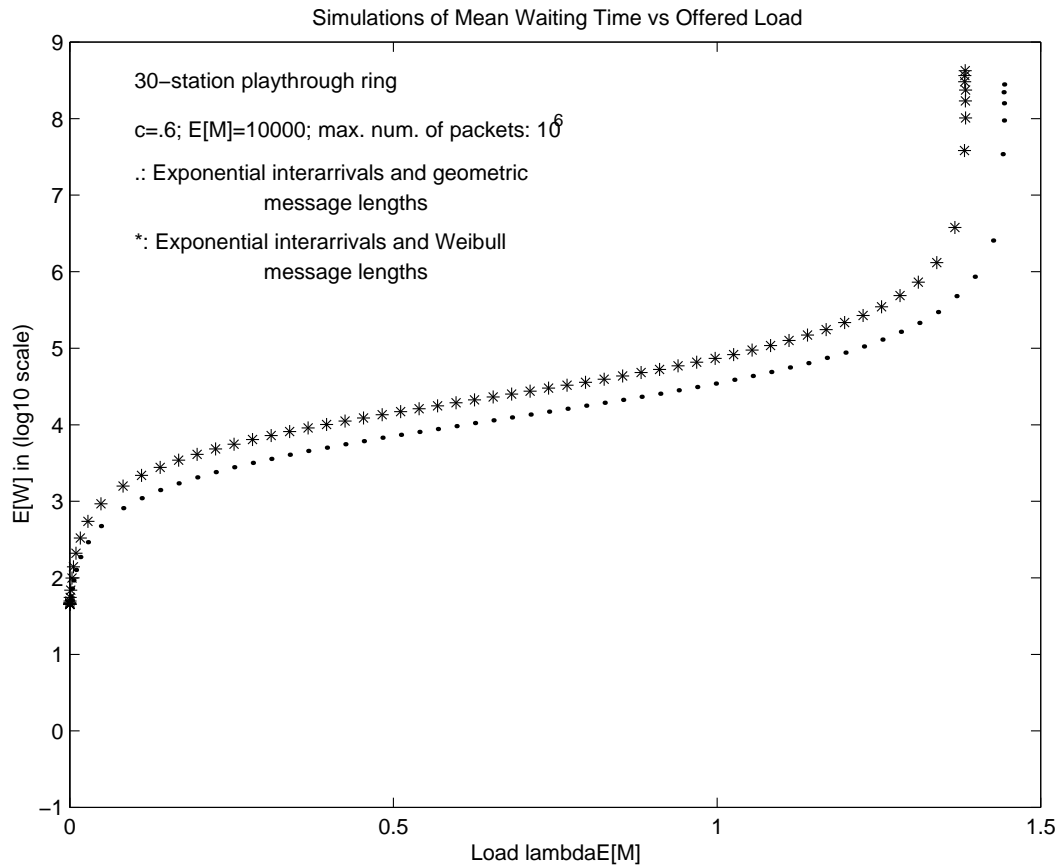


Figure 3.42: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 10000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.42 shows the simulated average waiting time on an $N = 30$ -station ring, with 10000 characters mean message lengths. In Figure 3.42, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 100% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.38) than it does under geometrically distributed message lengths (at 1.44).

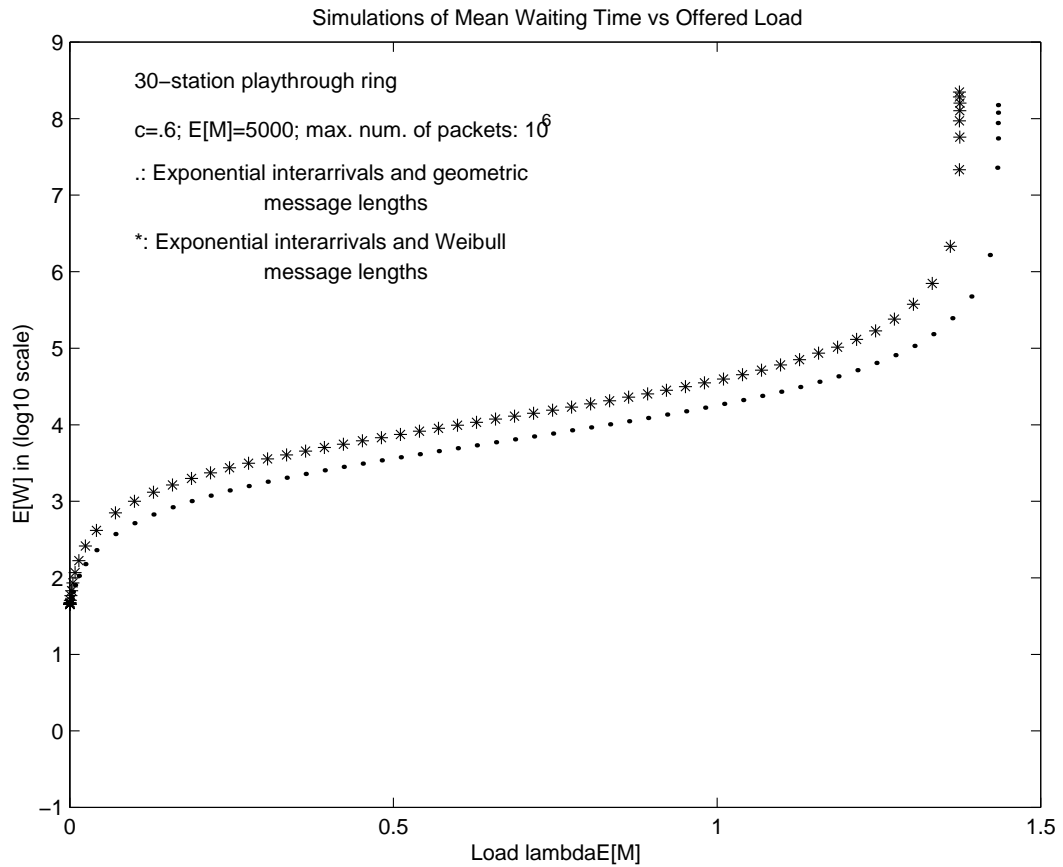


Figure 3.43: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 5000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.43 shows the simulated average waiting time on an $N = 30$ -station ring, with 5000 characters mean message lengths. In Figure 3.43, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 100% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.37) than it does under geometrically distributed message lengths (at 1.43).

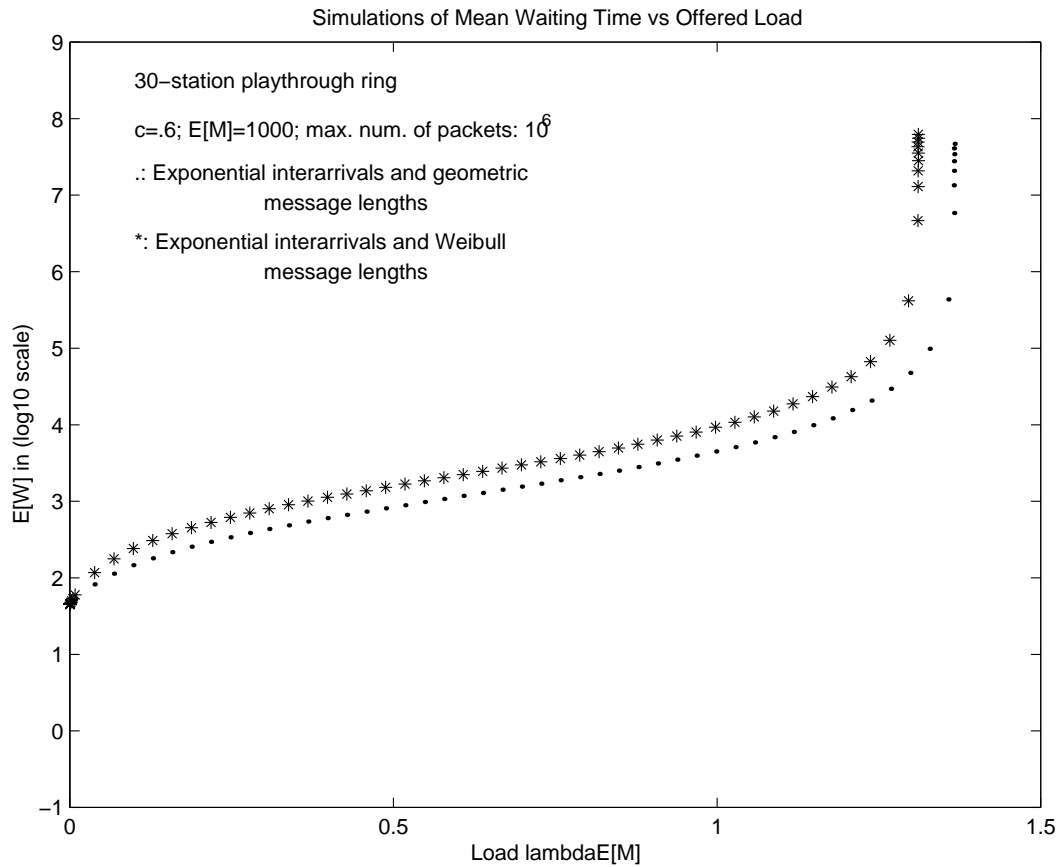


Figure 3.44: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 1000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.44 shows the simulated average waiting time on an $N = 30$ -station ring, with 1000 characters mean message lengths. In Figure 3.44, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 135% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.31) than it does under geometrically distributed message lengths (at 1.36).

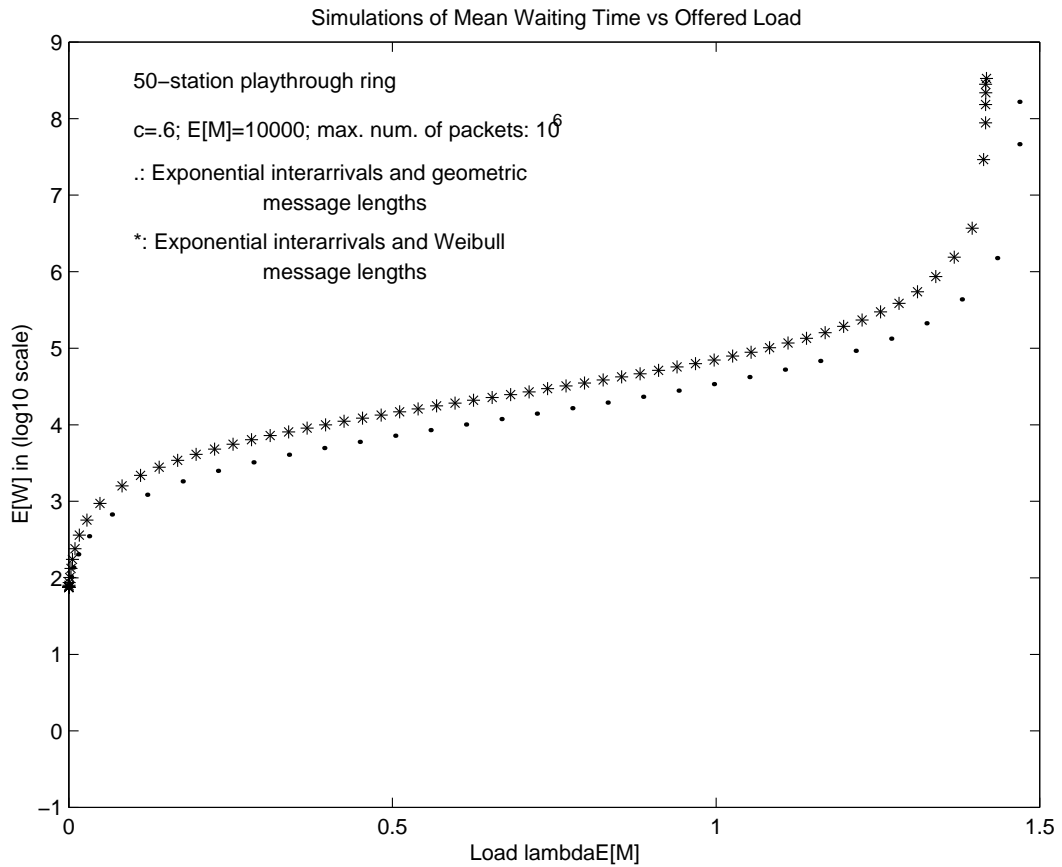


Figure 3.45: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 10000$ characters, $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.45 shows the simulated average waiting time on an $N = 50$ -station ring, with 10000 characters mean message lengths. In Figure 3.45, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 130% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.41) than it does under geometrically distributed message lengths (at 1.46).

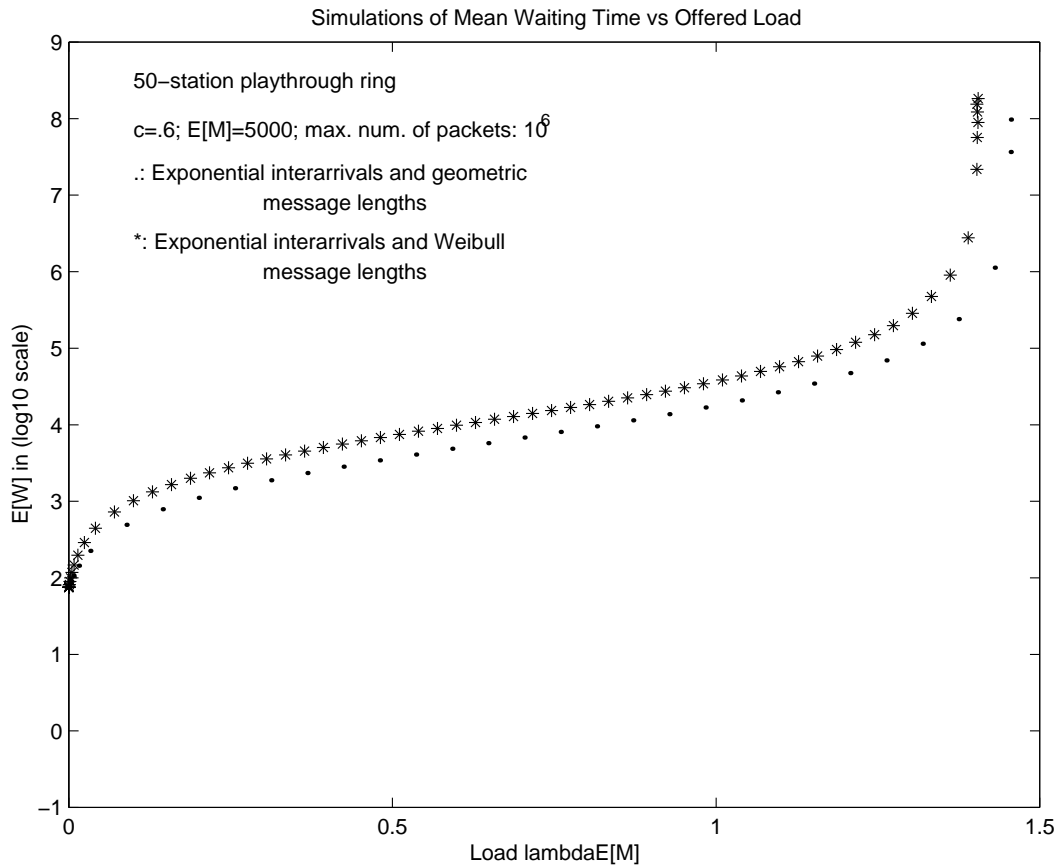


Figure 3.46: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 5000$ characters, $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.46 shows the simulated average waiting time on an $N = 50$ -station ring, with 5000 characters mean message lengths. In Figure 3.46, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 150% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.40) than it does under geometrically distributed message lengths (at 1.45).

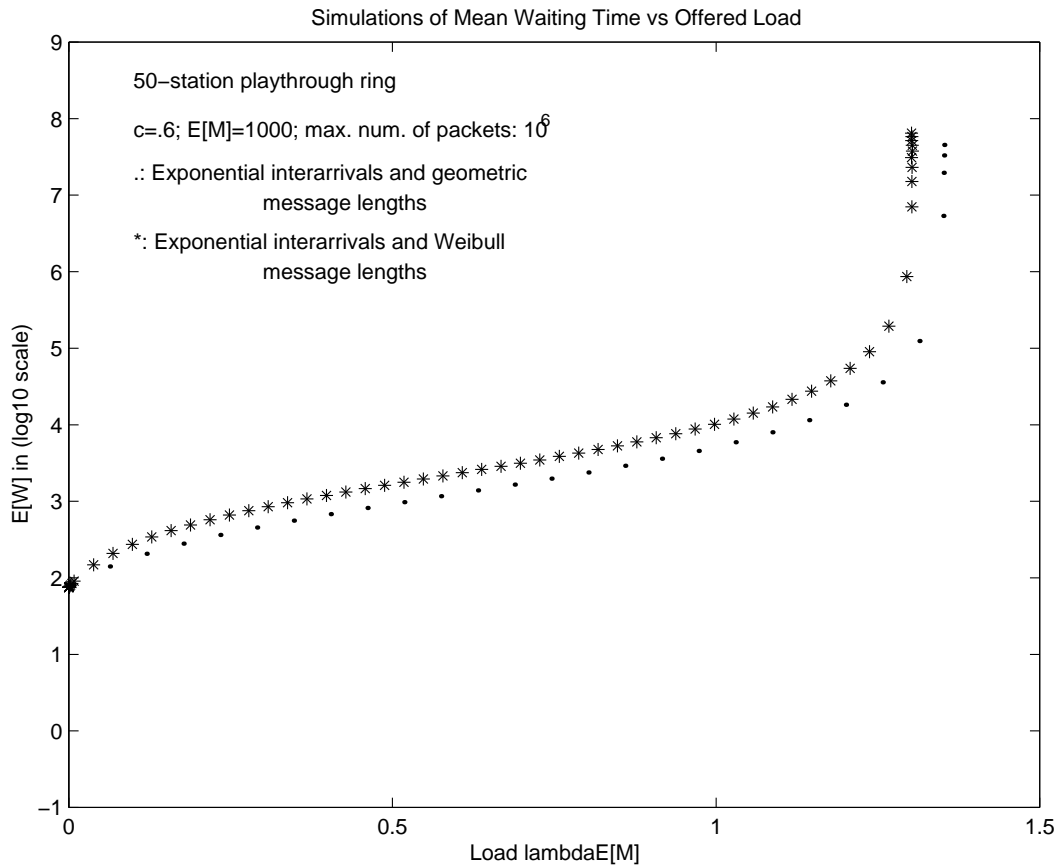


Figure 3.47: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 1000$ characters, $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.47 shows the simulated average waiting time on an $N = 50$ -station ring, with 1000 characters mean message lengths. In Figure 3.46, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 80% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.302) than it does under geometrically distributed message lengths (at 1.35).

From Figures Figure 3.33–Figure 3.47 we can make several observations.

First, the average message waiting times on PLAYTHROUGH ring under exponentially distributed interarrival times and Weibull distributed message lengths are generally greater than the average message waiting times on PLAYTHROUGH ring under exponentially distributed interarrival times and geometrically distributed message lengths.

Second, PLAYTHROUGH ring generally saturates earlier under exponentially distributed interarrival times and Weibull distributed message lengths than it does under exponentially distributed interarrival times and geometrically distributed message lengths.

We now consider the effect of a smaller shape parameter c on waiting times in PLAYTHROUGH ring under the assumption of exponentially distributed message interarrival times and Weibull distributed message lengths. In Figure 3.33–Figure 3.47, the shape parameter of the Weibull distribution was chosen to be $c = 0.6$. In Figure 3.48–Figure 3.62, it is chosen to be $c = 0.4$.

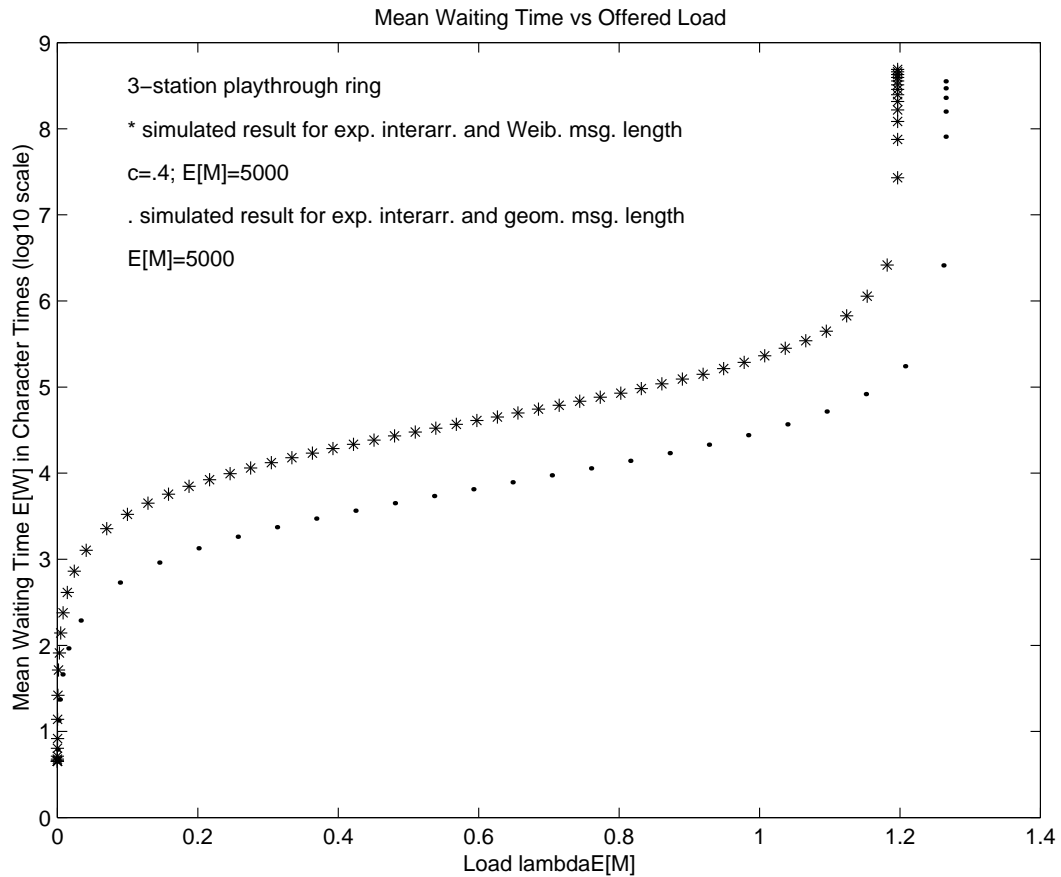


Figure 3.48: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 5000$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.48 shows the simulated average waiting time on an $N = 3$ -station ring, with 5000 characters mean message lengths. In Figure 3.48, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 530% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.196) than it does under geometrically distributed message lengths (at 1.265).

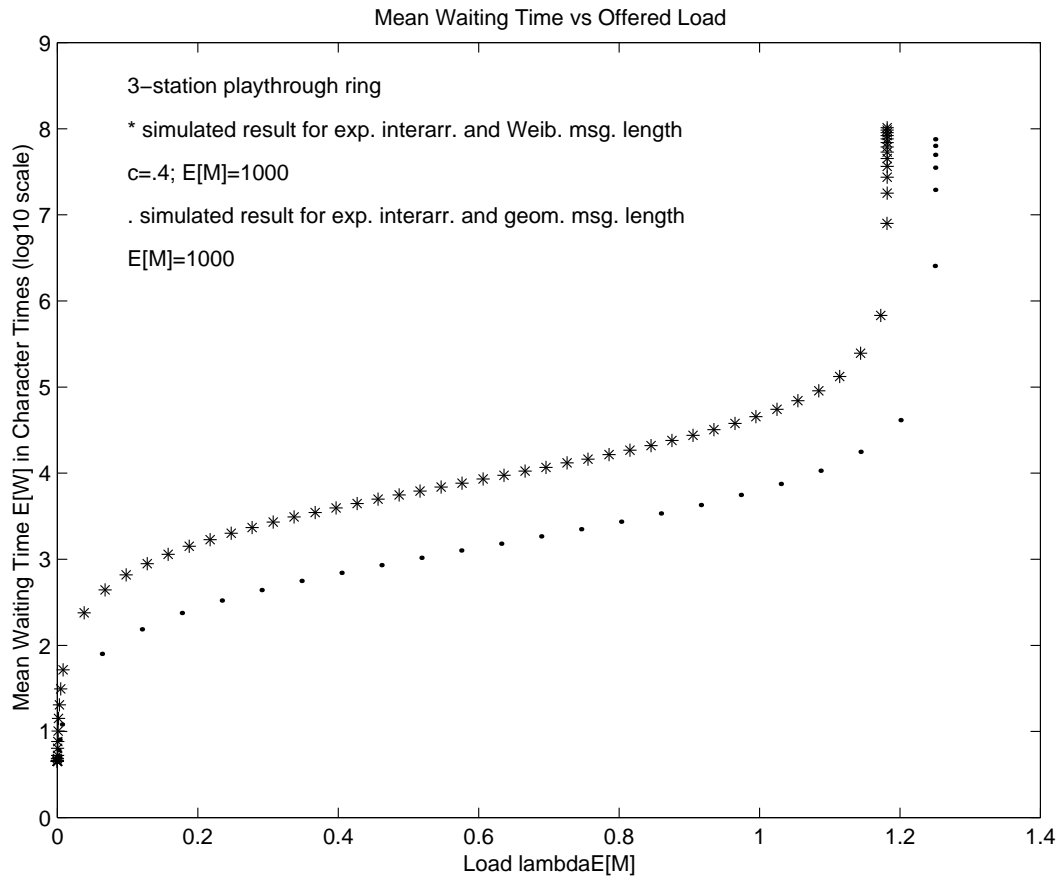


Figure 3.49: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 1000$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.49 shows the simulated average waiting time on an $N = 3$ -station ring, with 1000 characters mean message lengths. In Figure 3.49, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 530% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.181) than it does under geometrically distributed message lengths (1.250).

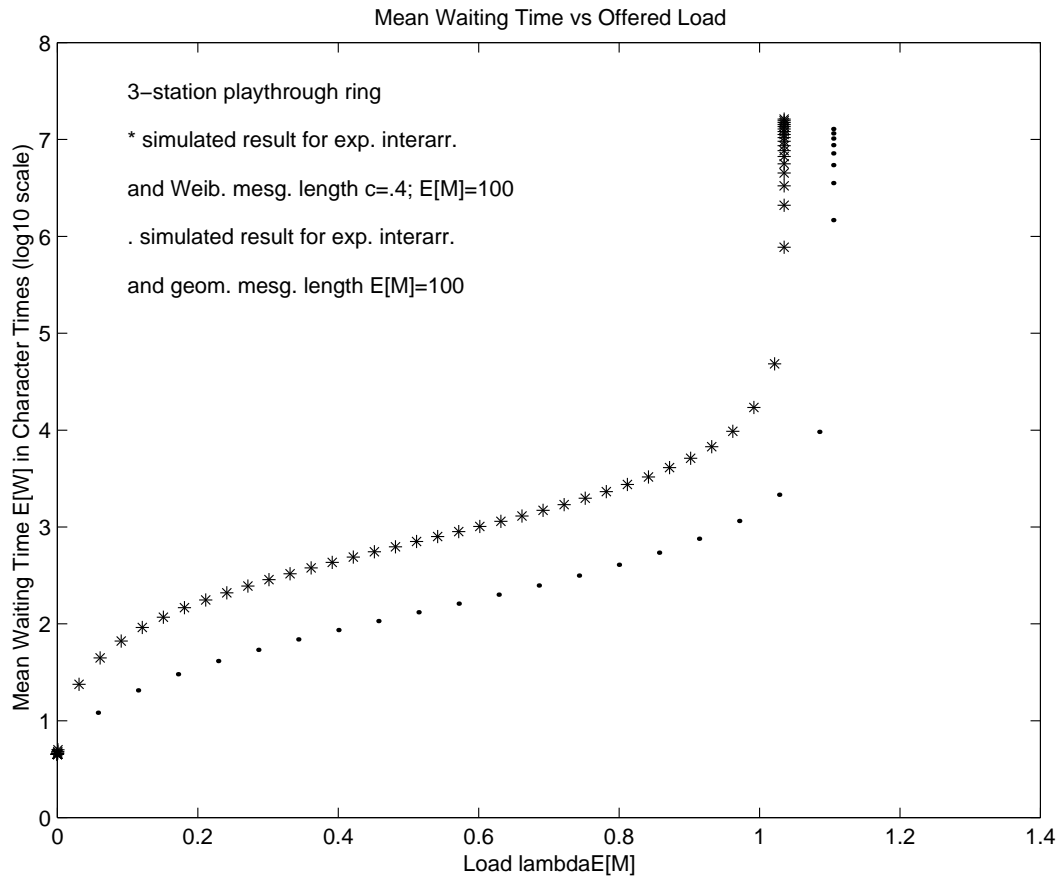


Figure 3.50: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 100$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.50 shows the simulated average waiting time on an $N = 3$ -station ring, with 100 characters mean message lengths. In Figure 3.50, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 400% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.03) than it does under geometrically distributed message lengths (1.104).

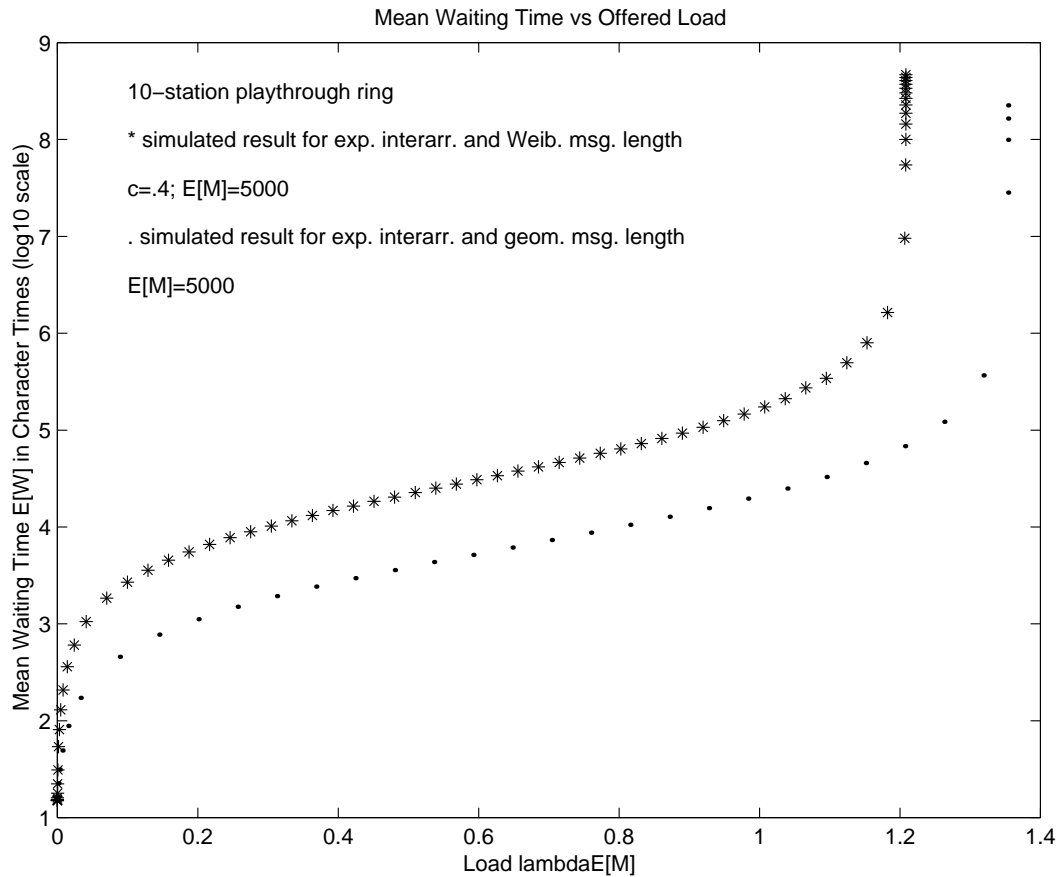


Figure 3.51: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 5000$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.51 shows the simulated average waiting time on an $N = 10$ -station ring, with 5000 characters mean message lengths. In Figure 3.51, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 400% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.20) than it does under geometrically distributed message lengths (at 1.35).

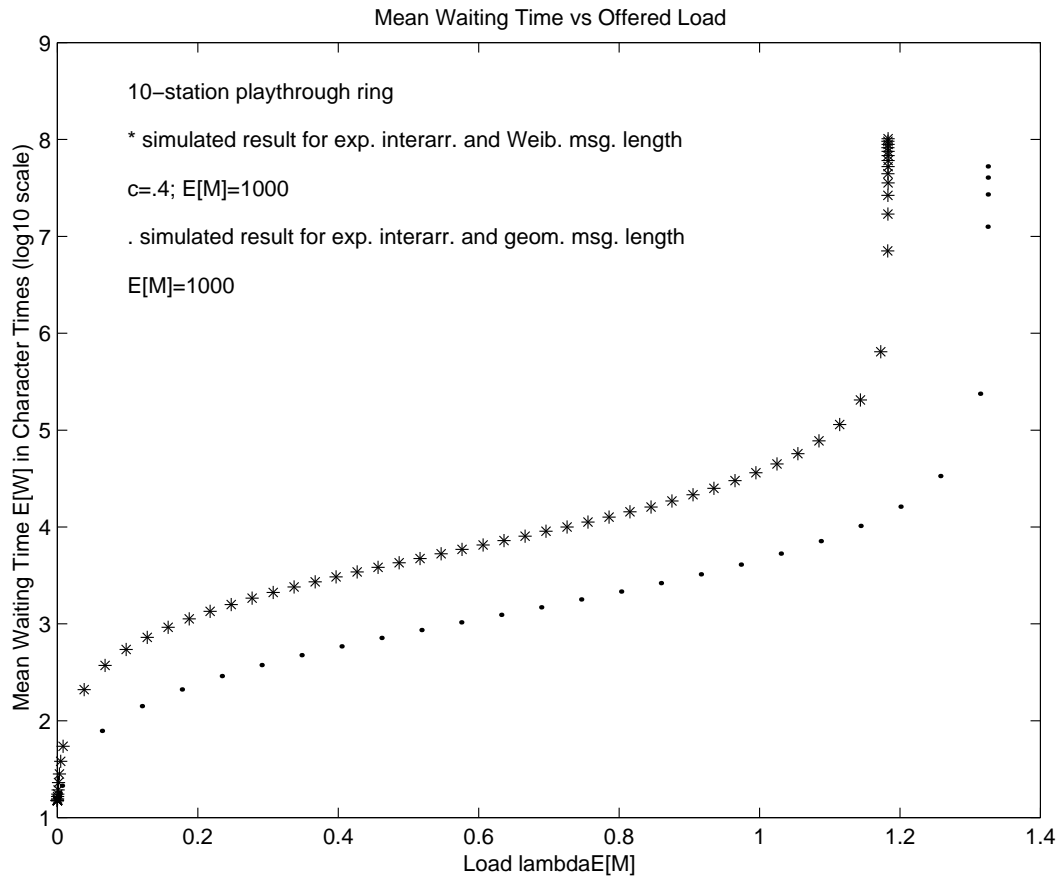


Figure 3.52: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 1000$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.52 shows the simulated average waiting time on an $N = 10$ -station ring, with 1000 characters mean message lengths. In Figure 3.52, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 300% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.18) than it does under geometrically distributed message lengths (at 1.32).

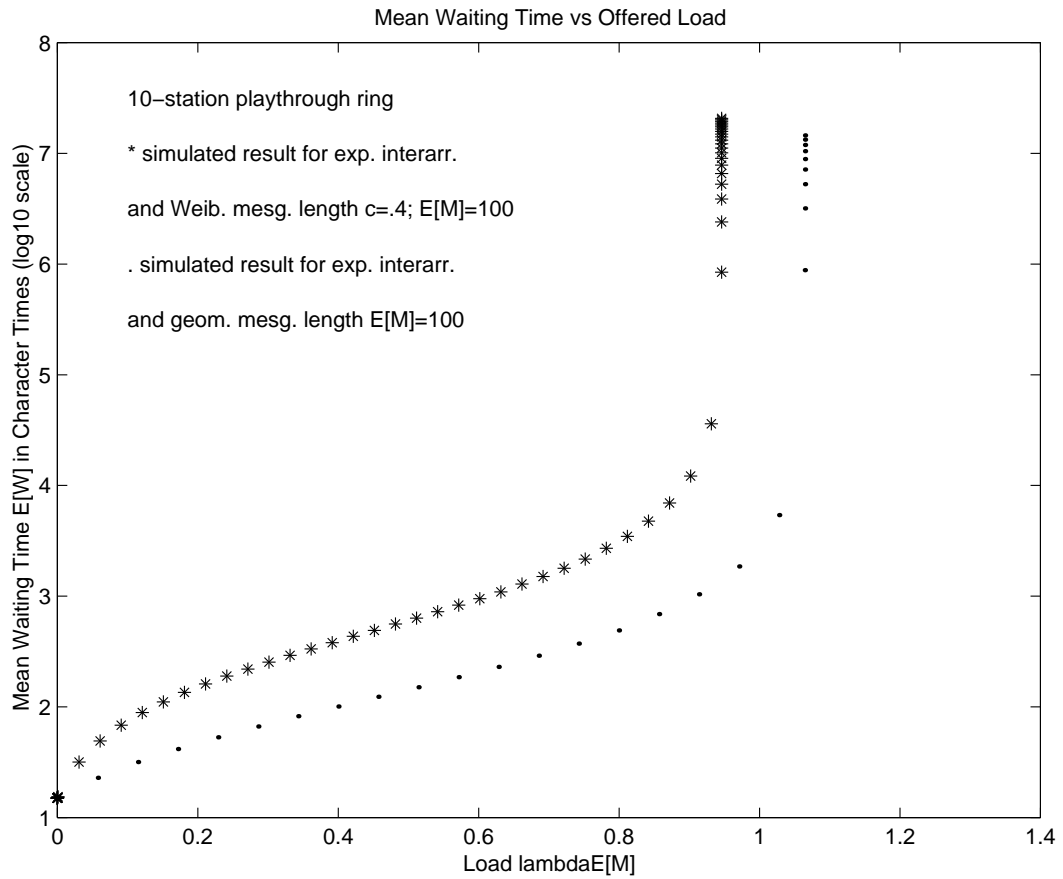


Figure 3.53: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 100$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.53 shows the simulated average waiting time on an $N = 10$ -station ring, with 100 characters mean message lengths. In Figure 3.53, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 400% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 0.94) than it does under geometrically distributed message lengths (at 1.0646).

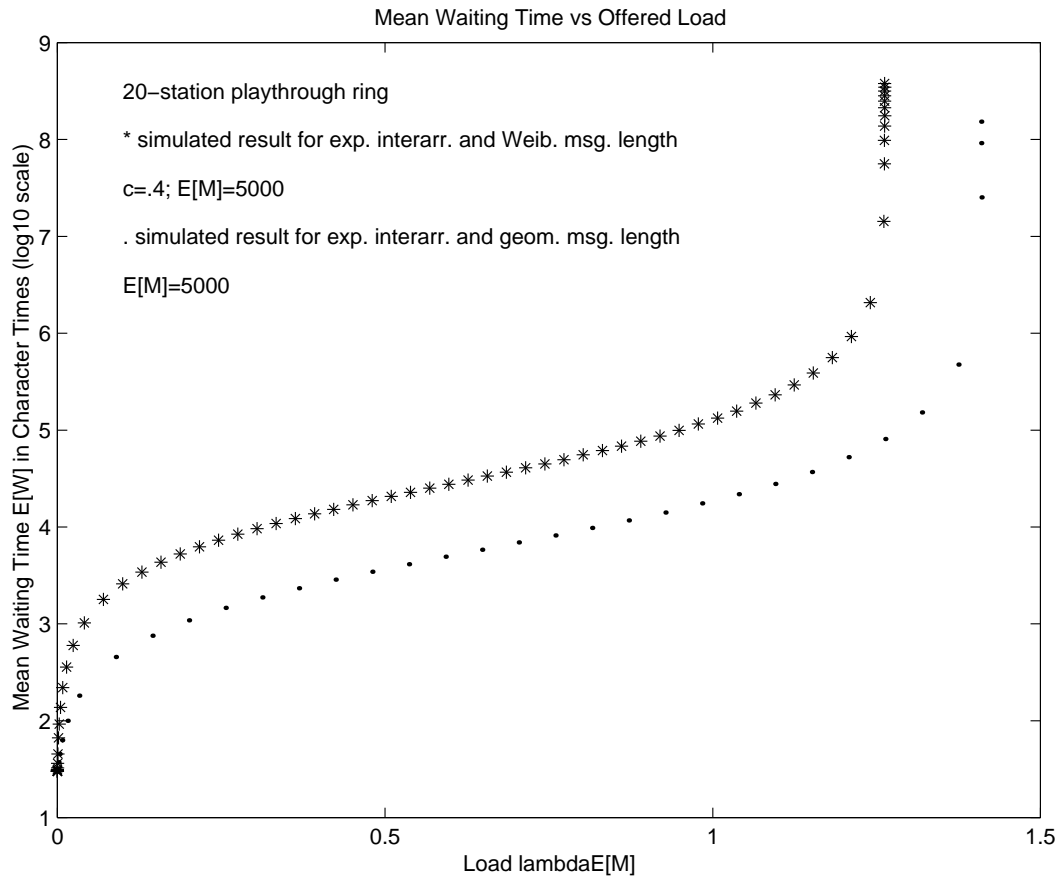


Figure 3.54: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 5000$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.54 shows the simulated average waiting time on an $N = 20$ -station ring, with 5000 characters mean message lengths. In Figure 3.54, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 200% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.26) than it does under geometrically distributed message lengths (at 1.40).

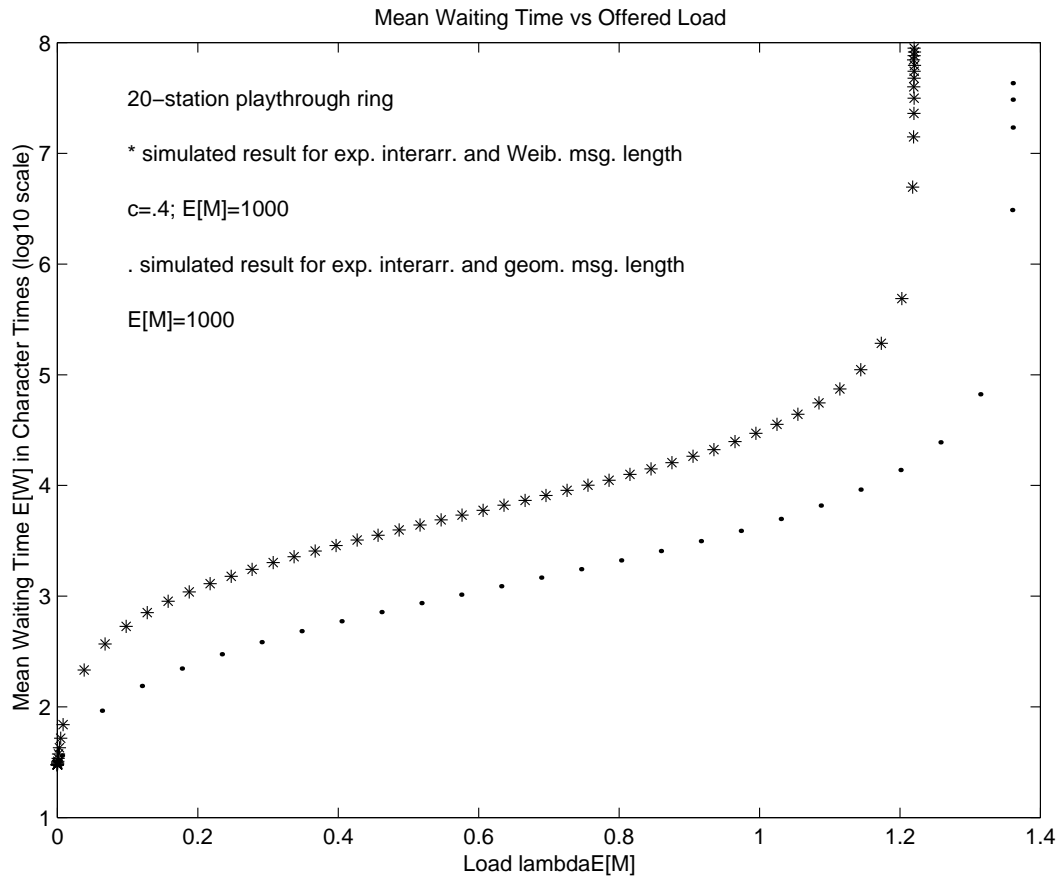


Figure 3.55: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 1000$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.55 shows the simulated average waiting time on an $N = 20$ -station ring, with 1000 characters mean message lengths. In Figure 3.55, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 150% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.21) than it does under geometrically distributed message lengths (at 1.35).

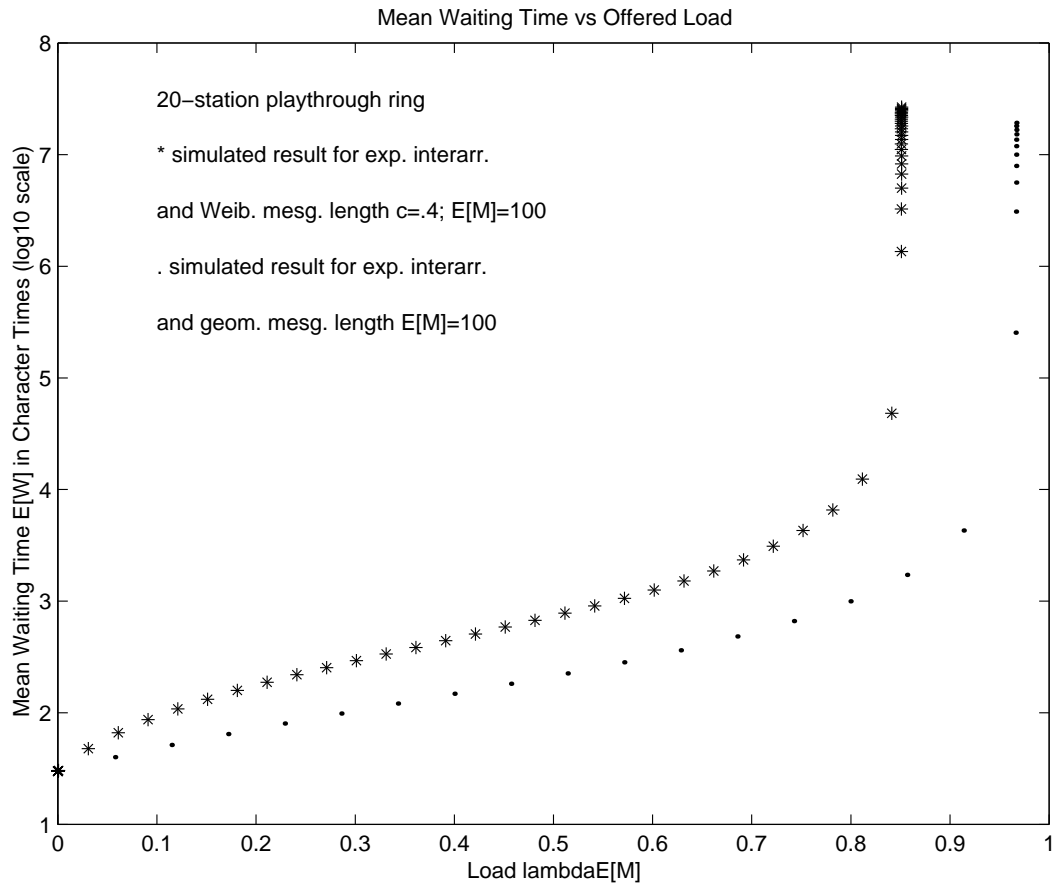


Figure 3.56: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 100$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.56 shows the simulated average waiting time on an $N = 20$ -station ring, with 100 characters mean message lengths. In Figure 3.56, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 150% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 0.85) than it does under geometrically distributed message lengths (at 0.966).

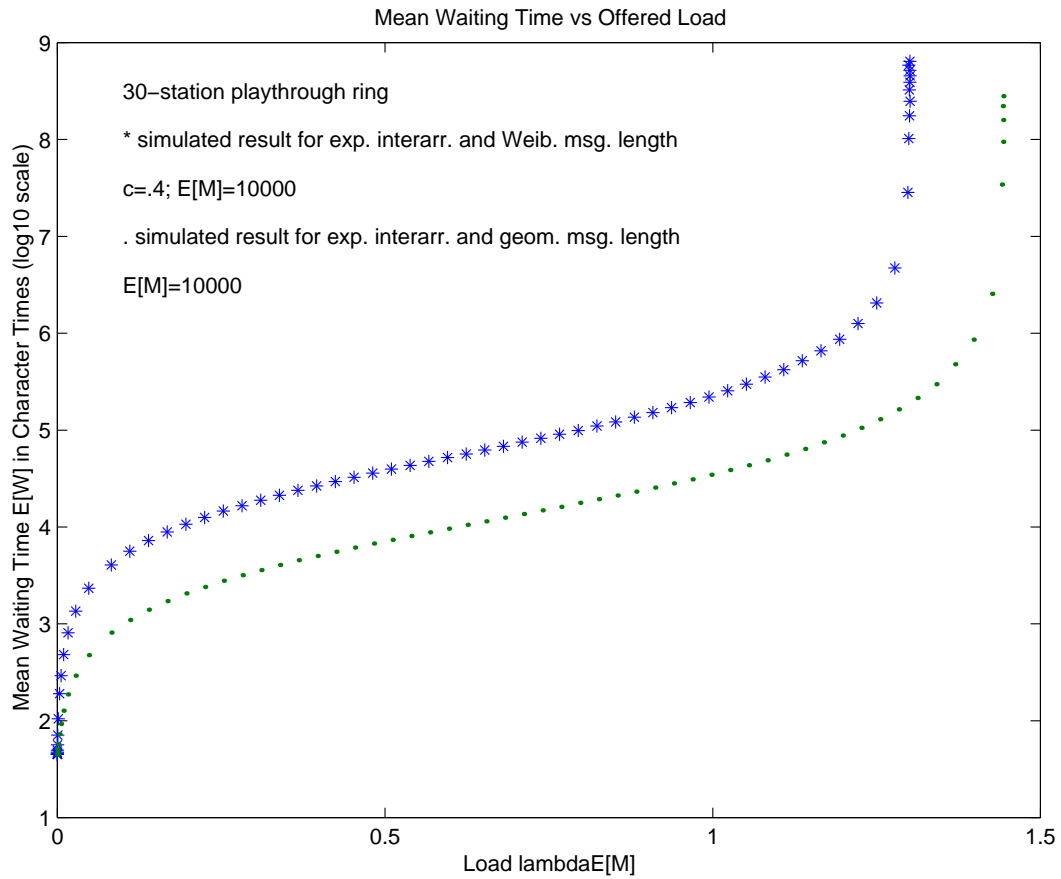


Figure 3.57: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 10000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.57 shows the simulated average waiting time on an $N = 30$ -station ring, with 10000 characters mean message lengths. In Figure 3.57, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 130% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.29) than it does under geometrically distributed message lengths (at 1.44).

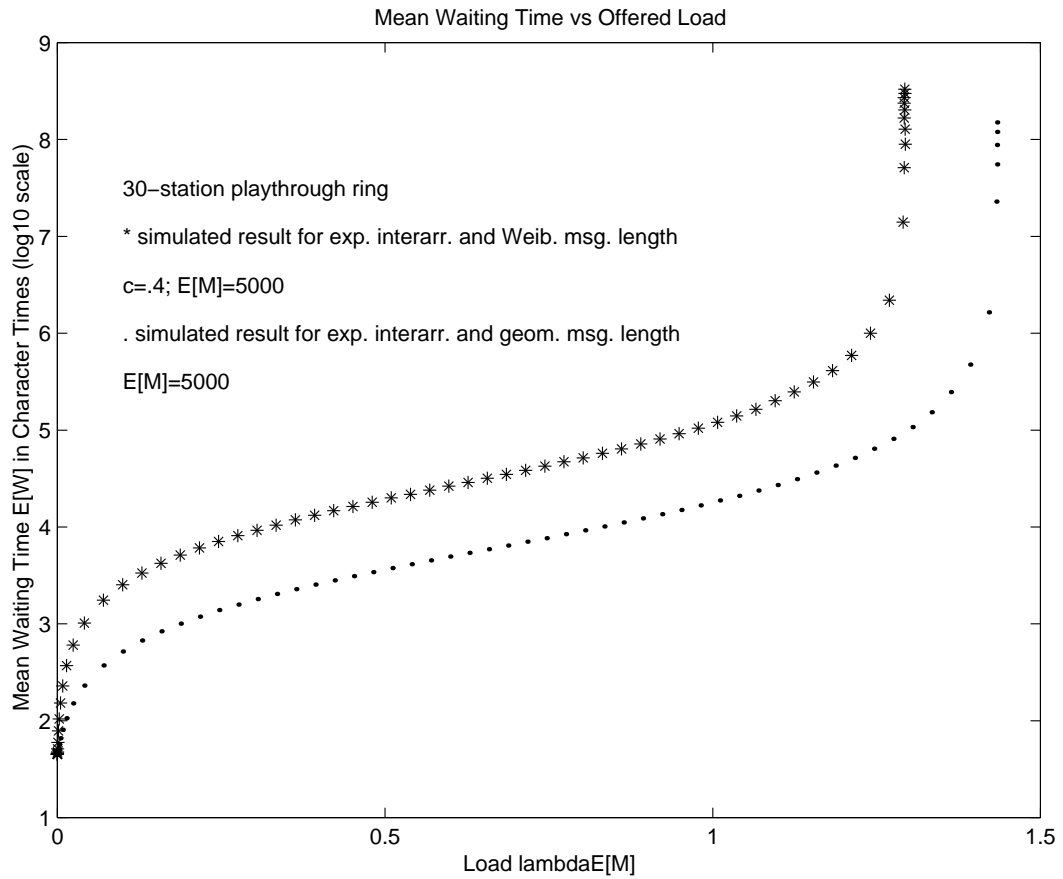


Figure 3.58: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 5000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.58 shows the simulated average waiting time on an $N = 30$ -station ring, with 10000 characters mean message lengths. In Figure 3.58, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 200% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.29) than it does under geometrically distributed message lengths (at 1.44).

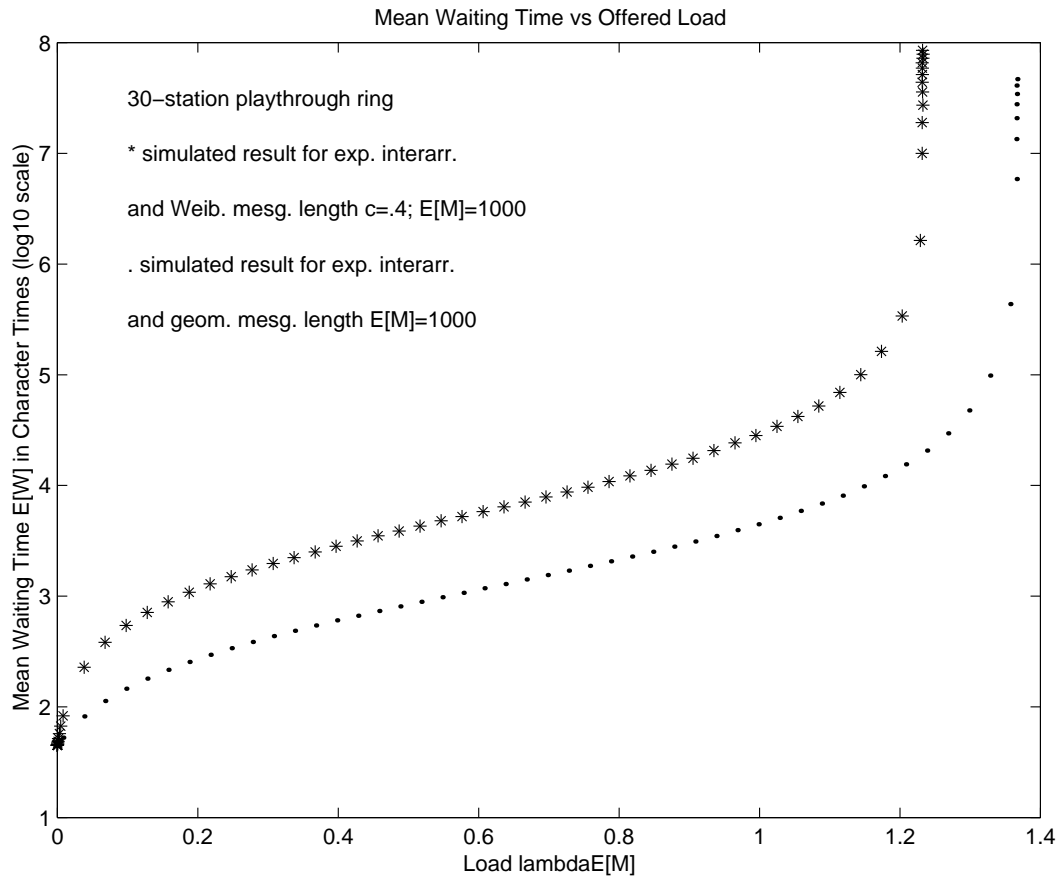


Figure 3.59: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 1000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.59 shows the simulated average waiting time on an $N = 30$ -station ring, with 1000 characters mean message lengths. In Figure 3.59, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 200% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.23) than it does under geometrically distributed message lengths (at 1.36).

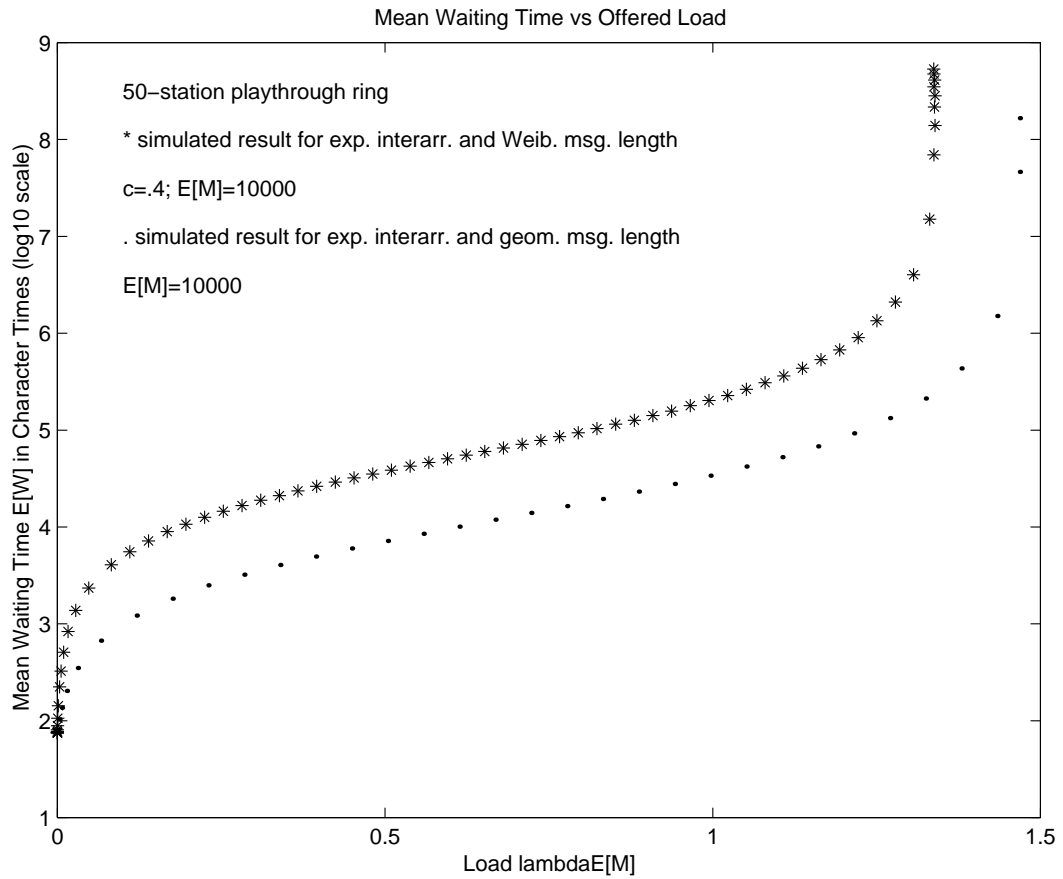


Figure 3.60: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 10000$ characters, $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.60 shows the simulated average waiting time on an $N = 30$ -station ring, with 10000 characters mean message lengths. In Figure 3.60, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 100% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.33) than it does under geometrically distributed message lengths (at 1.44).

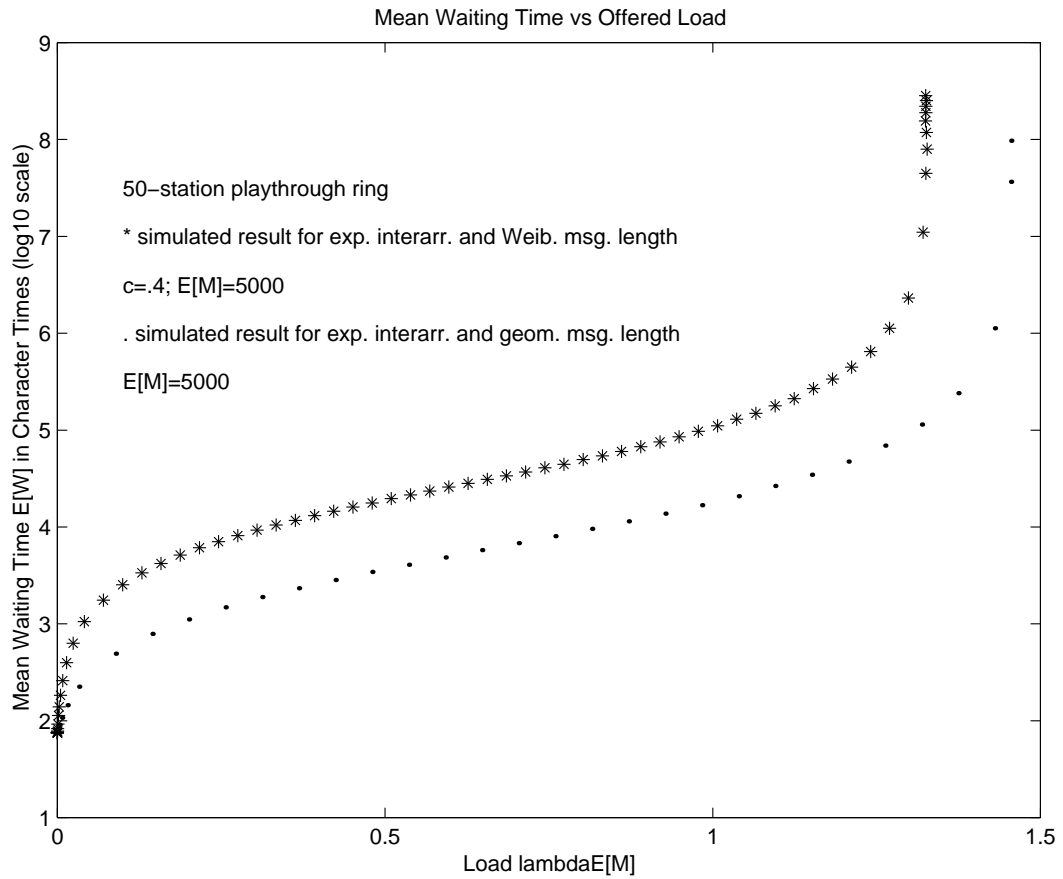


Figure 3.61: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 5000$ characters, $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.61 shows the simulated average waiting time on an $N = 50$ -station ring, with 5000 characters mean message lengths. In Figure 3.61, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 150% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.32) than it does under geometrically distributed message lengths (at 1.45).

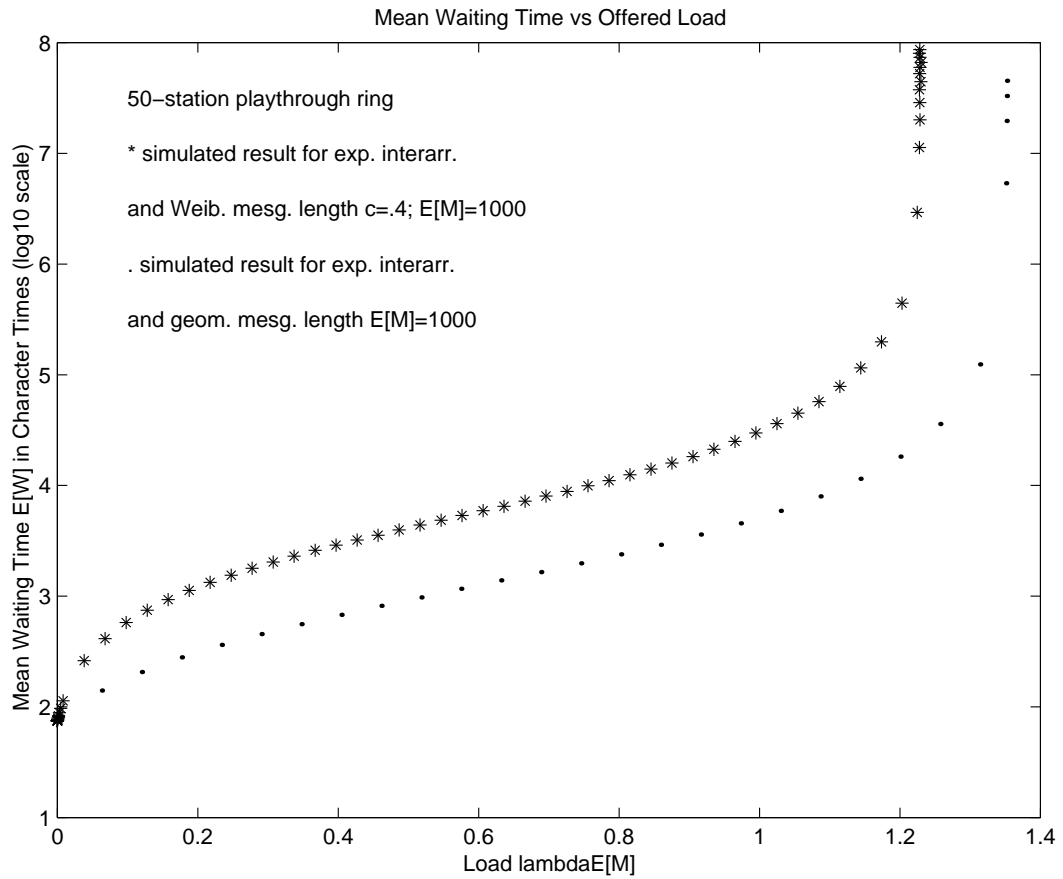


Figure 3.62: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 1000$ characters, $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

Figure 3.62 shows the simulated average waiting time on an $N = 50$ -station ring, with 1000 characters mean message lengths. In Figure 3.62, the average waiting times under Weibull distributed message lengths are greater than the average waiting times under geometrically distributed message lengths (by about 80% at medium range). Furthermore, PLAYTHROUGH ring saturates earlier under a Weibull message length distribution (at 1.22) than it does under geometrically distributed message lengths (at 1.35).

From Figures 3.33–3.47 and Figures 3.48–3.62 we can make the following observations.

Average waiting times in Figures 3.48–3.62 are greater than corresponding average waiting times in Figures 3.33–3.47. The smaller value of the shape parameter $c = 0.4$ used in Figures 3.48–3.62 has the effect of worsening the performance of PLAYTHROUGH ring than when the shape parameter is $s = 0.6$ as used in Figures 3.33–3.47. A lower value of the shape parameter c results in increased variability of message lengths. The plots suggest that increased variability of message lengths worsens the performance of PLAYTHROUGH Ring, using mean waiting times versus load as the performance measure.

Under Weibull distributed message lengths, PLAYTHROUGH ring generally saturates at a lesser value of average load than it does under geometrically distributed message lengths.

3.5 Conclusions

In this section, simulation results for PLAYTHROUGH Ring under Weibull message interarrival times, on the one hand, and under Weibull message lengths, on the other hand, were presented. Simulation results for PLAYTHROUGH Ring under Weibull interarrival times were presented for values of the shape parameter of the Weibull interarrival times distribution equal to 0.6 and 0.4. Those values correspond to values measured on actual networks by Anja Feldmann [Fel00]. The simulation results reveal that the average waiting time performance of PLAYTHROUGH ring under Weibull message interarrival times is worse than the performance of PLAYTHROUGH ring under the hitherto assumed exponential message arrival times. The

simulation results also showed that the saturation point is approximately the same for PLAYTHROUGH ring under exponential message interarrival times and under Weibull message interarrival times.

Consistent with simulations of PLAYTHROUGH Ring under Weibull interarrival times, shape parameters of values 0.6 and 0.4 were used to simulate PLAYTHROUGH Ring under Weibull distributed message lengths. Simulation results showed that mean average waiting times on PLAYTHROUGH Ring under Weibull message lengths are generally greater than waiting times under exponential message lengths. PLAYTHROUGH ring generally saturates at a lesser value of average load under Weibull distributed message lengths than it does under geometrically distributed message lengths.

Although simulations are a good tool to study PLAYTHROUGH Ring under self-similar traffic, they require a long time to perform. Analytical models would provide a much faster way to study and obtain the performance parameters of PLAYTHROUGH Ring.

Obtaining an analytical model of PLAYTHROUGH Ring under Weibull interarrival times appears to be an intractable problem because the Laplace of the Weibull distribution does not have a closed form expression. Solutions have been proposed for obtaining the Laplace transform of the Weibull distribution [Fel96], [Fel98]. But those solutions are numerical and only approximative.

Predicting mean waiting times for PLAYTHROUGH ring under the assumptions of Weibull message lengths and Poisson arrivals appears to be more amenable to an analytical solution. In the next chapter, we obtain an analytical model for PLAYTHROUGH Ring under exponential interarrivals and Weibull message lengths. Its predictions are in agreement with simulation

results over a wide range of number of stations and average message lengths.

CHAPTER 4

ANALYTICAL MODELING OF THE PERFORMANCE OF PLAYTHROUGH RINGS UNDER SELF-SIMILAR TRAFFIC

4.1 Minipacket Statistics

Let us consider an arbitrary message comprising M characters and h header characters, where M is a discrete positive random variable (r.v) and h is a fixed discrete nonnegative constant. In order to transmit the $M+h$ characters, an integral number n_G of minipackets of size m characters is required. Each round trip conveys at most $m = \tau - k_G$ characters such that

$$n_G = \left\lceil \frac{M+h}{m} \right\rceil. \quad (4.1)$$

We assume $h \leq m$ and we seek the probability mass function

$$\begin{aligned} Pr [n_G = k] &= Pr [k-1 < n_G \leq k] \\ &= Pr \left[k-1 < \frac{M+h}{m} \leq k \right] \\ &= Pr [m(k-1) < M+h \leq mk] \\ &= Pr [m(k-1) - h < M \leq mk - h]. \end{aligned} \quad (4.2)$$

In addition, we seek the moments of n_G . In particular, the first three

moments of n_G are given by:

$$E[n_G] = \sum_{k=0}^{\infty} k Pr [n_G = k], \quad (4.3)$$

$$E[n_G^2] = \sum_{k=0}^{\infty} k^2 Pr [n_G = k], \quad (4.4)$$

$$E[n_G^3] = \sum_{k=0}^{\infty} k^3 Pr [n_G = k]. \quad (4.5)$$

The cumulative distribution function of Weibull distributed messages of length y is given by

$$F_y(Y) = \begin{cases} 1 - e^{-(\frac{y}{a})^c}, & y > 0 \\ 0, & y \leq 0. \end{cases} \quad (4.6)$$

Thus,

$$\begin{aligned} Pr [n_G = 1] &= Pr \left[0 < \frac{M+h}{m} \leq 1 \right] \\ &= Pr [0 < M+h \leq m] \\ &= Pr [-h < M \leq m-h] \\ &= Pr [0 \leq M \leq m-h] \\ &= (1 - e^{-(\frac{m-h}{a})^c}), \end{aligned} \quad (4.7)$$

and

$$\begin{aligned} Pr [n_G = k] &= Pr \left[(k-1) < \frac{M+h}{m} \leq k \right] \\ &= Pr [(k-1)m-h < M \leq km-h] \\ &= (1 - e^{-(\frac{km-h}{a})^c}) - (1 - e^{-(\frac{(k-1)m-h}{a})^c}) \\ &= e^{-(\frac{(k-1)m-h}{a})^c} - e^{-(\frac{km-h}{a})^c} \\ &\approx e^{-(\frac{(k-1)m}{a})^c} - e^{-(\frac{km}{a})^c}, \quad \text{assuming that } h \ll \tau. \end{aligned} \quad (4.8)$$

We seek the probability generating function of n_G in order to calculate its

moments.

$$\begin{aligned}
\widehat{n}_G(z) &= \sum_{k=1}^{\infty} z^k Pr [n_G = k] \\
&= \sum_{k=2}^{\infty} z^k Pr [n_G = k] + z Pr [n_G = 1] \\
&\approx \sum_{k=2}^{\infty} z^k (e^{-\left(\frac{(k-1)m}{a}\right)^c} - e^{-\left(\frac{km}{a}\right)^c}) + z(1 - e^{-\left(\frac{m-h}{a}\right)^c}). \tag{4.9}
\end{aligned}$$

The first moment of n_G is given by

$$\begin{aligned}
E[n_G] &= \widehat{n}_G'(1) \\
&= \sum_{k=2}^{\infty} k(e^{-\left(\frac{(k-1)m}{a}\right)^c} - e^{-\left(\frac{km}{a}\right)^c}) + (1 - e^{-\left(\frac{m-h}{a}\right)^c}) \\
&= \sum_{k=2}^{\infty} k(e^{-\left(\frac{(k-1)m}{a}\right)^c}) - \sum_{k=2}^{\infty} k(e^{-\left(\frac{km}{a}\right)^c}) + (1 - e^{-\left(\frac{m-h}{a}\right)^c}) \\
&= \sum_{K=1}^{\infty} (K+1)(e^{-\left(\frac{Km}{a}\right)^c}) - \sum_{k=2}^{\infty} k(e^{-\left(\frac{km}{a}\right)^c}) + (1 - e^{-\left(\frac{m-h}{a}\right)^c}), \quad \text{where } K=k+1 \\
&= 2e^{-\left(\frac{m}{a}\right)^c} + \sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c} + (1 - e^{-\left(\frac{m-h}{a}\right)^c}) \\
&\approx 1 + e^{-\left(\frac{m}{a}\right)^c} + \sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c}, \quad \text{assuming that } h \ll \tau. \tag{4.10}
\end{aligned}$$

Let us prove that $\sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c}$ converges.

Let:

$$a_K = e^{-\left(\frac{Km}{a}\right)^c}; \tag{4.11}$$

then

$$a_{K+1} = e^{-\left(\frac{(K+1)m}{a}\right)^c}. \tag{4.12}$$

Let:

$$r = \frac{a_{K+1}}{a_K}. \tag{4.13}$$

We have

$$\begin{aligned}
r &= \frac{e^{-\left(\frac{K+1}{a}\right)^c m}}{e^{-\left(\frac{K}{a}\right)^c m}} \\
&= \frac{\left[e^{-(K+1)^c}\right]^{\frac{m}{a^c}}}{\left[e^{-K^c}\right]^{\frac{m}{a^c}}} \\
&= \left[\frac{e^{-(K+1)^c}}{e^{-K^c}}\right]^{\frac{m}{a^c}}.
\end{aligned} \tag{4.14}$$

Because the shape parameter c is greater than 0, the following inequality holds

$$\frac{e^{-(K+1)^c}}{e^{-K^c}} < 1. \tag{4.15}$$

Thus we have $r < 1$ and $\sum_{K=1}^{\infty} e^{-\left(\frac{K}{a}\right)^c m}$ converges, according to the ratio test for positive series [EG94a].

The infinite summation $\sum_{K=1}^{\infty} e^{-\left(\frac{K}{a}\right)^c m}$ is not practical for computations and we would like to find an upper bound for it so that we can determine a worst case for the number of minipackets. To that end, first we prove that

$$\sum_{K=1}^{\infty} e^{-\left(\frac{K}{a}\right)^c m} \leq \int_0^{\infty} e^{-\left(\frac{x}{a}\right)^c m} dx, \tag{4.16}$$

and

$$\int_0^{\infty} e^{-\left(\frac{x}{a}\right)^c m} dx \leq \int_0^{\infty} c \left(\frac{m^{c-1} x^c}{a^{c-1}}\right) e^{-\left(\frac{x}{a}\right)^c m} dx. \tag{4.17}$$

We begin by proving that Equation 4.16 is true. Let us consider the partition $P = [0, 1), [1, 2), [2, 3), \dots, [n, n + 1), \dots$ of the interval $[0, \infty)$ and the function $f(x) = e^{-\left(\frac{x}{a}\right)^c m}$. For every interval $[K, K+1)$ of the partition P , we have $e^{-\left(\frac{K+1}{a}\right)^c m} < e^{-\left(\frac{i}{a}\right)^c m}$ for $i \in [K, K + 1)$. Therefore, $e^{-\left(\frac{K+1}{a}\right)^c m}$ is the minimum value of $f(x)$ in $[K, K + 1)$.

The lower Riemann sum of the function $f(x)$ associated with the partition P is defined in [EG94b] as: $L_f(P) = \sum_{k=1}^{\infty} m_k \Delta_k$, where m_k is the minimum

value of $f(x)$ in $[k, k + 1)$ and $\Delta_k = (k + 1) - k = 1$. Thus the sum $\sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c}$ is the lower sum $L_f(P)$ of the function $f(x) = e^{-\left(\frac{xm}{a}\right)^c}$ associated with the partition P . Therefore, we have [EG94b]:

$$L_f(P) = \sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c} \leq \int_0^{\infty} e^{-\left(\frac{xm}{a}\right)^c} dx \quad (4.18)$$

Thus, we have proven that Equation 4.16 is true.

We now turn our attention to Equation 4.17 and prove its validity. We have:

$$\int_{0^+}^{\infty} c \left(\frac{m^{c-1} x^c}{a^{c-1}} \right) e^{-\left(\frac{xm}{a}\right)^c} dx = \int_{0^+}^{\infty} xc \left(\frac{m^{c-1} x^{c-1}}{a^{c-1}} \right) e^{-\left(\frac{xm}{a}\right)^c} dx. \quad (4.19)$$

Let $u(x) = x$, $v(x) = 1 - e^{-\left(\frac{xm}{a}\right)^c}$. We have: $u'(x) = 1$ and $v'(x) = c \left(\frac{m^{c-1} x^{c-1}}{a^{c-1}} \right) e^{-\left(\frac{xm}{a}\right)^c}$. Using integration by parts, we have:

$$\int_{0^+}^{\infty} xc \left(\frac{m^{c-1} x^{c-1}}{a^{c-1}} \right) e^{-\left(\frac{xm}{a}\right)^c} dx = \int_{0^+}^{\infty} u(x)v'(x)dx = [u(x)v(x)]_{0^+}^{\infty} - \int_{0^+}^{\infty} u'(x)v(x)dx. \quad (4.20)$$

From Equation 4.19 and Equation 4.20 we have:

$$\begin{aligned} \int_{0^+}^{\infty} c \left(\frac{m^{c-1} x^c}{a^{c-1}} \right) e^{-\left(\frac{xm}{a}\right)^c} dx &= [x(1 - e^{-\left(\frac{xm}{a}\right)^c})]_{0^+}^{\infty} - \int_{0^+}^{\infty} (1 - e^{-\left(\frac{xm}{a}\right)^c}) dx \\ &= [x]_{0^+}^{\infty} + [-xe^{-\left(\frac{xm}{a}\right)^c}]_{0^+}^{\infty} - [x]_{0^+}^{\infty} + \int_{0^+}^{\infty} e^{-\left(\frac{xm}{a}\right)^c} dx \\ &= -\lim_{x \rightarrow \infty} (xe^{-\left(\frac{xm}{a}\right)^c}) + \lim_{x \rightarrow 0^+} (xe^{-\left(\frac{xm}{a}\right)^c}) + \int_{0^+}^{\infty} e^{-\left(\frac{xm}{a}\right)^c} dx. \end{aligned} \quad (4.21)$$

We have:

$$\begin{aligned} \lim_{x \rightarrow \infty} xe^{-\left(\frac{xm}{a}\right)^c} &= \lim_{x \rightarrow \infty} \frac{x}{e^{\left(\frac{xm}{a}\right)^c}} \\ &= \lim_{x \rightarrow \infty} \frac{x}{\sum_{n=0}^{\infty} \frac{\left(\frac{m}{a}\right)^{cn} x^{cn}}{n!}} \\ &= 0. \end{aligned} \quad (4.22)$$

Thus, Equation 4.21 becomes:

$$\int_{0^+}^{\infty} c \left(\frac{m^{c-1} x^c}{a^{c-1}} \right) e^{-\left(\frac{xm}{a}\right)^c} dx = \lim_{x \rightarrow 0^+} (xe^{-\left(\frac{xm}{a}\right)^c}) + \int_{0^+}^{\infty} e^{-\left(\frac{xm}{a}\right)^c} dx. \quad (4.23)$$

We have $\lim_{x \rightarrow 0^+} (xe^{-(\frac{xm}{a})^c}) \geq 0$. Therefore,

$$\int_{0^+}^{\infty} e^{-(\frac{xm}{a})^c} dx \leq \int_{0^+}^{\infty} e^{-(\frac{xm}{a})^c} dx + \lim_{x \rightarrow 0^+} (xe^{-(\frac{xm}{a})^c}) \quad (4.24)$$

and

$$\int_{0^+}^{\infty} e^{-(\frac{xm}{a})^c} dx \leq \int_{0^+}^{\infty} c \left(\frac{m^{c-1} x^c}{a^{c-1}} \right) e^{-(\frac{xm}{a})^c} dx. \quad (4.25)$$

Hence, we have proven that Equation 4.17 is true. As a result, we have,

$$\begin{aligned} \sum_{K=1}^{\infty} e^{-(\frac{Km}{a})^c} &\leq \int_0^{\infty} xc \left(\frac{mx}{a} \right)^{c-1} e^{-(\frac{mx}{a})^c} dx \\ &= \frac{a\Gamma(1 + \frac{1}{c})}{m} \\ &= \frac{E[M]}{m} \\ &\leq E[M]. \end{aligned} \quad (4.26)$$

So the following is true:

$$\begin{aligned} E[n_G] &= 2e^{-(\frac{m}{a})^c} + \sum_{K=1}^{\infty} e^{-(\frac{Km}{a})^c} + (1 - e^{-(\frac{m-H}{a})^c}) \\ &\leq 2e^{-(\frac{m}{a})^c} + E[M] + (1 - e^{-(\frac{m-H}{a})^c}) \\ &= 2e^{-(\frac{m}{a})^c} + a\Gamma(1 + 1/c) + (1 - e^{-(\frac{m-H}{a})^c}) \\ &= \text{Upperbound..} \end{aligned} \quad (4.27)$$

In Figure 4.1, Figure 4.2, and Figure 4.3, an approximation of $E[n_G]$, the mean number of minipackets, evaluated using Equation 4.10 with K terms in the summation $\sum_{K=1}^{\infty} e^{-(\frac{Km}{a})^c}$ is plotted versus $E[M] = a\Gamma(1 + 1/c)$, the mean message length, for various values of K and for various numbers of stations. Figure 4.1 reveals that the upper bound found in Equation 4.27 is not a tight upper bound. Given that $E[n_G]$ converges, we seek a tighter bound for $E[n_G]$.

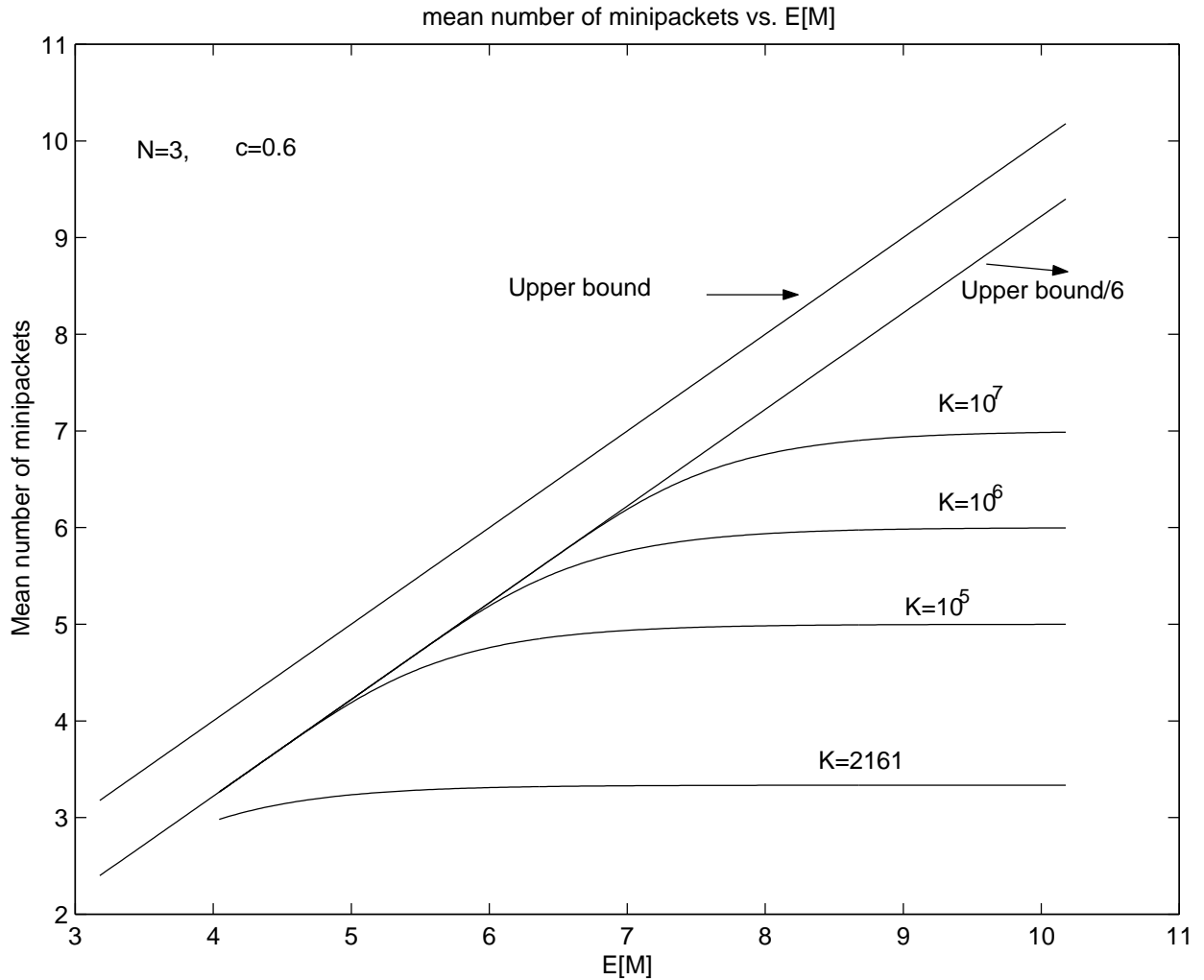


Figure 4.1: Mean number of minipackets vs. mean message length $E[M]$ for $N = 3$ station PLAYTHROUGH ring, uniform and symmetric traffic (UST), and Weibull distributed message length.

The number of minipackets required to send a message of length $E[M]$ appears to always be bounded tightly by

$$\frac{E[M]}{3(N-1)}. \tag{4.28}$$

Intuitively, this can be explained by the fact that the control frame goes through $N - 1$ buffers of length $k_R = 3$ before GO returns to the source station.

We can show analytically that $\frac{E[M]}{3(N-1)}$ is a tighter upper bound on the

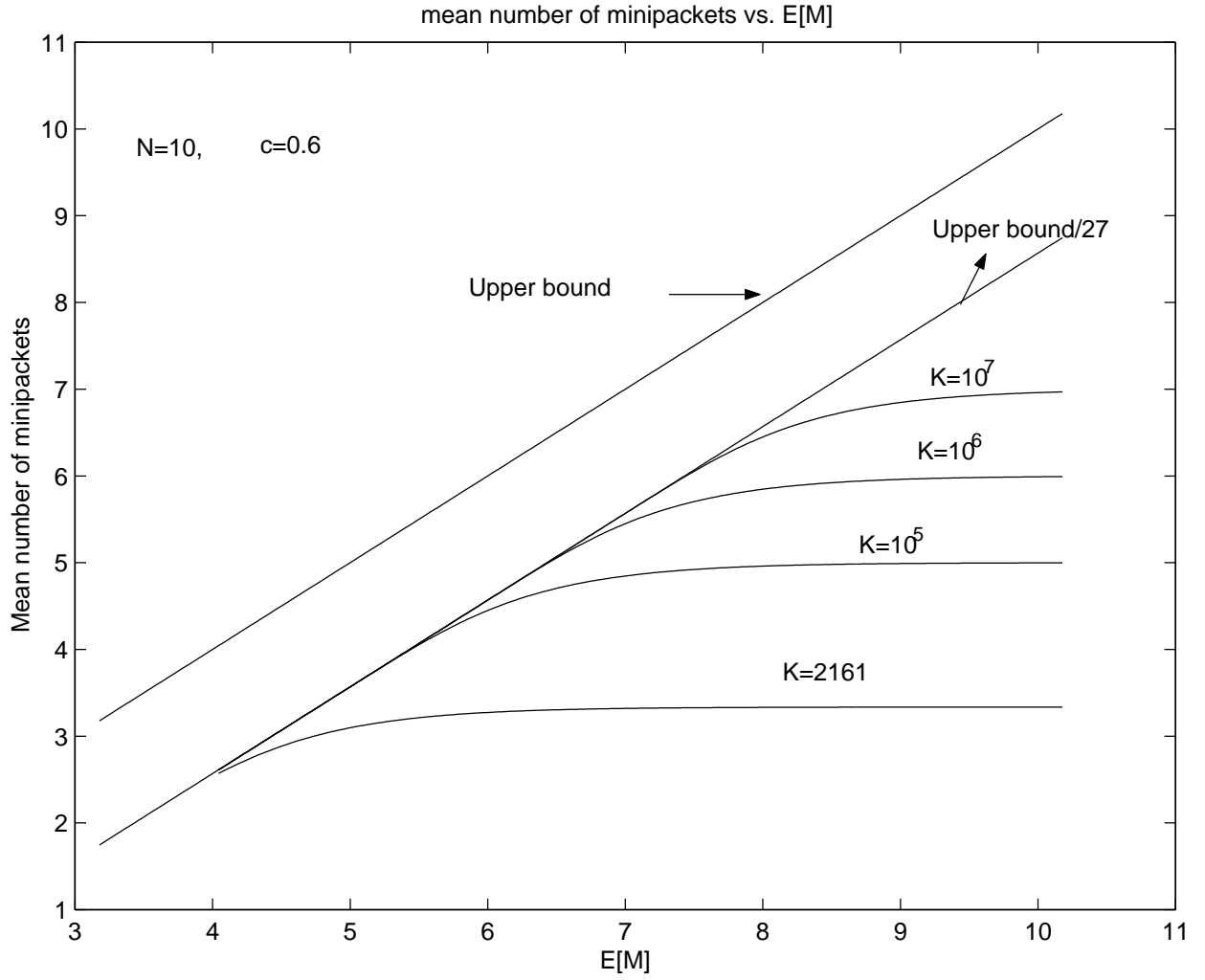


Figure 4.2: Mean number of minipackets vs. mean message length $E[M]$ for $N = 10$ station PLAYTHROUGH ring, uniform and symmetric traffic (UST), and Weibull distributed message length.

expression for $E[n_G]$ from Equation 4.10 than *Upperbound*. We must show to that:

$$E[n_G] \leq \frac{E[M]}{3(N-1)} \leq \textit{Upperbound}. \quad (4.29)$$

We have:

$$\frac{E[M]}{3(N-1)} \leq E[M], \quad (4.30)$$

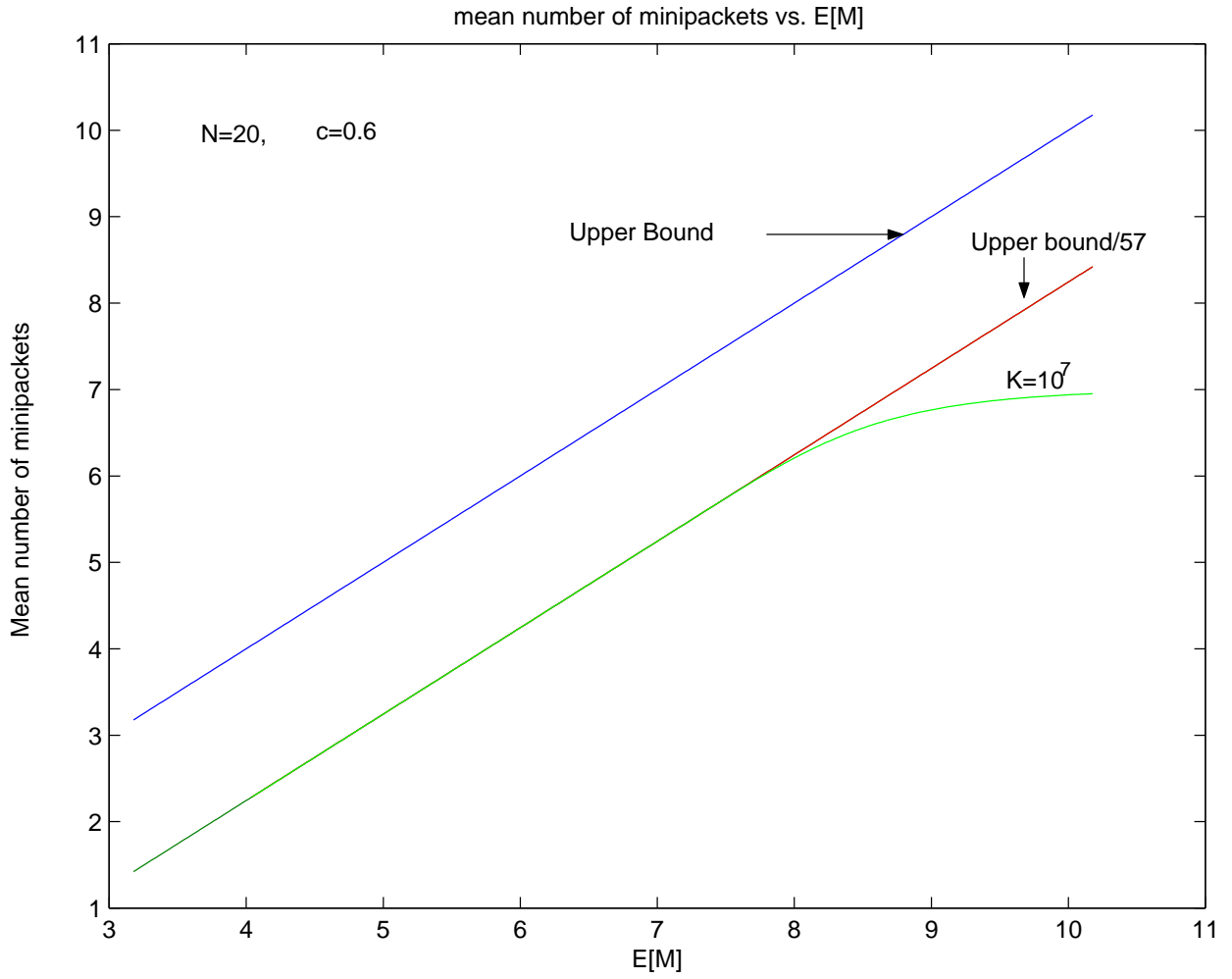


Figure 4.3: Mean number of minipackets vs. mean message length $E[M]$ for $N = 20$ station PLAYTHROUGH ring, uniform and symmetric traffic (UST), and Weibull distributed message length.

Therefore,

$$\frac{E[M]}{3(N-1)} \leq 2e^{-\left(\frac{m}{\alpha}\right)^c} + E[M] + (1 - e^{-\left(\frac{m-H}{\alpha}\right)^c}). \quad (4.31)$$

The expression on the right side of Equation 4.31 is *Upperbound*. So we have,

$$\frac{E[M]}{3(N-1)} \leq \textit{Upperbound}. \quad (4.32)$$

We now prove that $E[n_G] \leq \frac{E[M]}{3(N-1)}$. We have:

$$E[n_G] = 2e^{-\left(\frac{m}{a}\right)^c} + \sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c} + (1 - e^{-\left(\frac{m-h}{a}\right)^c}). \quad (4.33)$$

Since $h < m$, we have $e^{-\left(\frac{m-h}{a}\right)^c} > e^{-\left(\frac{m}{a}\right)^c}$. Therefore,

$$E[n_G] < e^{-\left(\frac{m}{a}\right)^c} + 1 + \sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c}. \quad (4.34)$$

We have,

$$1 + \sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c} \leq 1 + \int_0^{\infty} xc\left(\frac{x}{a}\right)^{c-1} e^{-\left(\frac{xm}{a}\right)^c} dx = 1 + \frac{E[M]}{m} = 1 + \frac{E[M]}{3N-2}. \quad (4.35)$$

Hence we have:

$$\begin{aligned} E[n_G] &\leq 1 + e^{-\left(\frac{m}{a}\right)^c} + \frac{E[M]}{3N-2} \\ E[n_G] &\leq \frac{3N-2 + (3N-2)e^{-\left(\frac{m}{a}\right)^c} + E[M]}{3N-2}. \end{aligned} \quad (4.36)$$

In the typical case where $M \gg m = 3N-2$, we have:

$$\frac{3N-2 + (3N-2)e^{-\left(\frac{m}{a}\right)^c} + E[M]}{3N-2} \approx \frac{E[M]}{3N-2}, \quad (4.37)$$

and

$$E[n_G] \leq \frac{E[M]}{3N-3}. \quad (4.38)$$

This upper bound would hold for any value of c since the mean of n_G depends only on mean message length and on the data frame size m . Similarly, we will calculate the second moment of the number of minipackets required to send a message of size M and we will find an upper bound for that second moment. The second moment of n_G can be obtained from the probability generating function of n_G as follows:

$$E[n_G^2] = \widehat{n}_G''(1) + \widehat{n}_G'(1) \quad (4.39)$$

We have:

$$\begin{aligned}
\widehat{n}_G''(1) &= \sum_{k=2}^{\infty} (k^2 - k) \left[e^{-\left(\frac{(k-1)m}{a}\right)^c} - e^{-\left(\frac{km}{a}\right)^c} \right] \\
&= \sum_{k=2}^{\infty} \left[k^2 e^{-\left(\frac{(k-1)m}{a}\right)^c} - k^2 e^{-\left(\frac{km}{a}\right)^c} - k e^{-\left(\frac{(k-1)m}{a}\right)^c} + k e^{-\left(\frac{km}{a}\right)^c} \right] \\
&= \sum_{K=1}^{\infty} (K+1)^2 e^{-\left(\frac{Km}{a}\right)^c} - \sum_{k=2}^{\infty} k^2 e^{-\left(\frac{km}{a}\right)^c} - \sum_{K=1}^{\infty} (K+1) e^{-\left(\frac{Km}{a}\right)^c} \\
&\quad + \sum_{k=2}^{\infty} k e^{-\left(\frac{km}{a}\right)^c}, \quad \text{where } K = k + 1 \\
&= \sum_{K=1}^{\infty} K^2 e^{-\left(\frac{Km}{a}\right)^c} + \sum_{K=1}^{\infty} 2K e^{-\left(\frac{Km}{a}\right)^c} + \sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c} \\
&\quad - \sum_{k=2}^{\infty} k^2 e^{-\left(\frac{km}{a}\right)^c} - \sum_{K=1}^{\infty} (K+1) e^{-\left(\frac{Km}{a}\right)^c} + \sum_{k=2}^{\infty} k e^{-\left(\frac{km}{a}\right)^c} \\
&= e^{-\left(\frac{m}{a}\right)^c} + \sum_{K=1}^{\infty} K e^{-\left(\frac{Km}{a}\right)^c} + \sum_{k=2}^{\infty} k e^{-\left(\frac{km}{a}\right)^c} \\
&= 2e^{-\left(\frac{m}{a}\right)^c} + 2 \sum_{k=2}^{\infty} k e^{-\left(\frac{km}{a}\right)^c}. \tag{4.40}
\end{aligned}$$

Thus,

$$\begin{aligned}
E[n_G^2] &= \widehat{n}_G''(1) + \widehat{n}_G'(1) \\
&= 2e^{-\left(\frac{m}{a}\right)^c} + 2 \sum_{k=2}^{\infty} k e^{-\left(\frac{km}{a}\right)^c} + 2e^{-\left(\frac{m}{a}\right)^c} + \sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c} + (1 - e^{-\left(\frac{m-H}{a}\right)^c}) \\
&= 4e^{-\left(\frac{m}{a}\right)^c} + 2 \sum_{k=2}^{\infty} k e^{-\left(\frac{km}{a}\right)^c} + \sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c} + (1 - e^{-\left(\frac{m-H}{a}\right)^c}). \tag{4.41}
\end{aligned}$$

In Figure 4.4, an approximation of the second moment of the number of minipackets required to send Weibull distributed messages of mean length $E[M]$ and shape parameter $c = 0.6$ on a 15-station PLAYTHROUGH ring is plotted using K terms in the summations $2 \sum_{k=2}^{\infty} k e^{-\left(\frac{km}{a}\right)^c}$ and $\sum_{K=1}^{\infty} e^{-\left(\frac{Km}{a}\right)^c}$ in Equation 4.41. It appears to always be bounded tightly by

$$4 \left(\frac{E[M]}{3(N-1)} \right)^2 \tag{4.42}$$

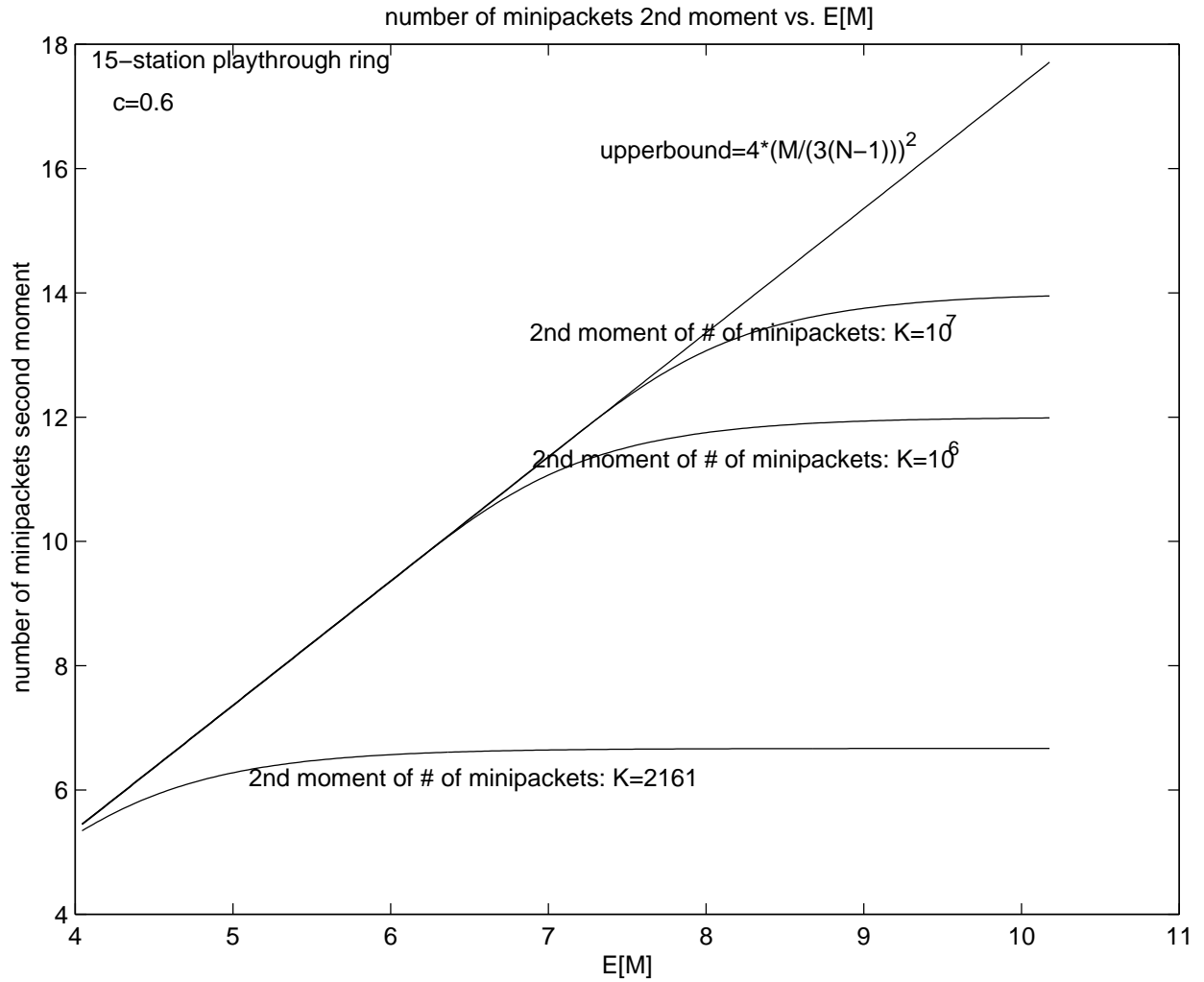


Figure 4.4: Second moment of minipackets vs. mean message length $E[M]$ for $N = 15$ station PLAYTHROUGH ring, uniform and symmetric traffic (UST), and Weibull distributed message length.

We give an analytical justification for the factor 4 that appears in the upper bound formulation in the case where $c = 0.6$. In addition, we give a generalized formula for the upper bound on the second moment of the number of round trips that applies for any value of c . The first and second moments of a Weibull

distributed random variable X are given by:

$$E[X] = a\Gamma\left(1 + \frac{1}{c}\right), \quad \text{and} \quad (4.43)$$

$$E[X^2] = a^2\Gamma\left(1 + \frac{2}{c}\right), \quad (4.44)$$

respectively.

We have

$$E[M] = a\Gamma\left(1 + \frac{1}{c}\right), \quad (4.45)$$

$$E[M^2] = a^2\Gamma\left(1 + \frac{2}{c}\right), \quad (4.46)$$

and

$$\begin{aligned} E[n_G^2] &= E\left[\left[\frac{M^2}{m^2}\right]\right] \\ &\approx \frac{E[M^2]}{m^2}. \end{aligned} \quad (4.47)$$

Thus,

$$\begin{aligned} \frac{E[n_G^2]}{E[n_G]^2} &\approx \frac{\frac{E[M^2]}{m^2}}{\frac{(E[M])^2}{m^2}} \\ &= \frac{E[M^2]}{(E[M])^2} \\ &= \frac{\frac{a^2\Gamma(1+\frac{2}{c})}{m^2}}{\frac{a^2\Gamma(1+\frac{1}{c})^2}{m^2}} \\ &= \frac{\Gamma(1 + \frac{2}{c})}{\Gamma(1 + \frac{1}{c})^2}. \end{aligned} \quad (4.48)$$

In the specific case where $c = 0.6$ the expression

$$\frac{\Gamma(1 + \frac{2}{c})}{\Gamma(1 + \frac{1}{c})^2} \quad (4.49)$$

evaluates to

$$\frac{\Gamma(1 + \frac{2}{.6})}{\Gamma(1 + \frac{1}{.6})^2} \approx 4. \quad (4.50)$$

Mess. length Distribution	Ratio	
	$\frac{E[n_G^2]}{E[n_G]^2}$	$\frac{E[n_G^3]}{E[n_G]^3}$
<i>geometric</i>	2	6
<i>weibull, c = 1</i>	2	6
<i>weibull, c = 0.6</i>	4.0908	35.2322
<i>weibull, c=0.4</i>	10.865	382.354

Table 4.1: Ratio $\frac{E[n_G^i]}{E[n_G]^i}$ for various values of the Weibull parameter c .

and we have

$$\frac{E[n_G^2]}{E[n_G]^2} \approx 4. \quad (4.51)$$

Using a similar argument, we can obtain an upper bound on the third moment of the number of minipackets, n_G

$$\begin{aligned} \frac{E[n_G^3]}{E[n_G]^3} &\approx \frac{\frac{E[M^3]}{m^3}}{\frac{(E[M])^3}{m^3}} \\ &= \frac{E[M^3]}{(E[M])^3} \\ &= \frac{\frac{a^3 \Gamma(1 + \frac{3}{c})}{m^3}}{\frac{a^3 \Gamma(1 + \frac{1}{c})^3}{m^3}} \\ &= \frac{\Gamma(1 + \frac{3}{c})}{\Gamma(1 + \frac{1}{c})^3}. \end{aligned} \quad (4.52)$$

In the case where $c = 0.6$ the expression

$$\frac{\Gamma(1 + \frac{3}{c})}{\Gamma(1 + \frac{1}{c})^3} \quad (4.53)$$

evaluates to

$$\frac{\Gamma(1 + \frac{3}{.6})}{\Gamma(1 + \frac{1}{.6})^3} \approx 35.2322, \quad (4.54)$$

and we have

$$\frac{E[n_G^3]}{E[n_G]^3} \approx 35.2322. \quad (4.55)$$

Table 4.1 contains the ratio $\frac{E[n_G^i]}{E[n_G]^i}$ for $i = 2, 3$. It is clear that the ratio $\frac{E[n_G^i]}{E[n_G]^i}$ is considerably greater for Weibull distributed message lengths than for

geometrically distributed message lengths. For Weibull distributed message lengths, the ratio $\frac{E[n_G^i]}{E[n_G]^i}$ grows quite rapidly as i increases. For example, for $c = 0.6$, $\frac{E[n_G^4]}{E[n_G]^4} = 506.106$ and $\frac{E[n_G^5]}{E[n_G]^5} = 10743.3$; for $c = 0.4$, $\frac{E[n_G^4]}{E[n_G]^4} = 29748$ and $\frac{E[n_G^5]}{E[n_G]^5} = 4.21942e + 06$.

4.2 Basic Transmission and Service Time

A message sourced by station i and destined to station $i \oplus j$ requires $n_{G_{i,i \oplus j}}$ minipackets of size m to be transmitted to its destination. In the absence of control message interruptions, the time the message spends in transmission is denoted $\check{T}_{i,i \oplus j}$ and the time the message spends in service is denoted $\check{S}_{i,i \oplus j}$. The basic transmission time is defined as the amount of time from the time a start message is inserted into the control message to initiate message transmission to the time the last character of the message is transmitted. The basic service time includes the transmission time plus the time for the STOP message to be transmitted and for GO to return. Recalling that k_D is the length of a control message in character times ($k_D = 3$), we have :

$$\check{T}_{i,i \oplus j} = n_{G_{i,i \oplus j}} \tau + k_D \quad (4.56)$$

$$\check{S}_{i,i \oplus j} = \check{T}_{i,i \oplus j} + \tau + k_D. \quad (4.57)$$

The moments of the basic transmission time are given by:

$$\begin{aligned} E \left[\check{T}_{i,i \oplus j}^n \right] &= E \left[(n_{G_{i,i \oplus j}} \tau + k_D)^n \right] \\ &= E \left[\sum_{k=0}^n \binom{n}{k} (n_{G_{i,i \oplus j}} \tau)^{n-k} k_D^k \right] \\ &= \sum_{k=0}^n \binom{n}{k} \tau^{n-k} E \left[n_{G_{i,i \oplus j}}^{n-k} \right] k_D^k. \end{aligned} \quad (4.58)$$

The moments of the basic service time are given by:

$$\begin{aligned}
E \left[\check{S}_{i,i\oplus j}^n \right] &= E \left[(\check{T}_{i,i\oplus j} + \tau + k_D)^n \right] \\
&= E \left[(n_{G_{i,i\oplus j}} \tau + \tau + 2k_D)^n \right] \\
&= E \left[\sum_{k=0}^n \binom{n}{k} (n_{G_{i,i\oplus j}} \tau)^{n-k} (\tau + 2k_D)^k \right] \\
&= \sum_{k=0}^n \binom{n}{k} \tau^{n-k} E \left[n_{G_{i,i\oplus j}}^{n-k} \right] (\tau + 2k_D)^k. \tag{4.59}
\end{aligned}$$

The product $\lambda_{i,i\oplus j} E \left[\check{T}_{i,i\oplus j} \right]$ is the fraction of time the ring spends transmitting messages sourced by station i and destined to station $i \oplus j$. It is denoted $\check{\rho}_{T_{i,i\oplus j}}$. We write:

$$\check{\rho}_{T_{i,i\oplus j}} = \lambda_{i,i\oplus j} E \left[\check{T}_{i,i\oplus j} \right] \tag{4.60}$$

The total traffic intensity sourced by station i is denote given by:

$$\check{\rho}_{T_i} = \sum_{j=1}^{N-1} \check{\rho}_{T_{i,i\oplus j}}. \tag{4.61}$$

The sum of all traffic intensities on the ring is denoted $\check{\rho}_T$ and is given by

$$\check{\rho}_T = \sum_{i=0}^{N-1} \check{\rho}_{T_i}. \tag{4.62}$$

If we designate by $\check{\rho}_{T_{i\oplus k,i}}^I$ the total traffic intensity sourced by station $i \oplus k$ that is independent of station i , we have:

$$\check{\rho}_{T_{i\oplus k,i}}^I = \sum_{j=1}^{N-k} \check{\rho}_{T_{i\oplus k,i\oplus k\oplus j}}; \tag{4.63}$$

If we designate by $\check{\rho}_{T_{i\oplus k,i}}^R$ the total traffic intensity sourced by station $i \oplus k$ that uses station i as a repeater, we have:

$$\check{\rho}_{T_{i\oplus k,i}}^R = \check{\rho}_{T_{i\oplus k}} - \check{\rho}_{T_{i\oplus k,i}}^I. \quad (4.64)$$

4.3 Control Message Interruption Rate

In the absence of control message interruptions, the control frame circulates around the ring with round trip duration τ . Each control message of size k_D adds k_D character times to the duration of the control frame round trip time. Because the transmission of every message generates two control messages, a *START* and a *STOP* control message, the total delay of the control frame due to the transmission of a message is $2k_D$ character times.

Let us designate by $\nu_{i,i\oplus j}$ the rate at which messages of class $\mathcal{C}_{i,i\oplus j}$ are interrupted by control messages. We are interested in investigating how the the control message interruption rate is affected by the transmission of Weibull distributed messages. Previous works done on control message interruption rate assumed that message lengths were geometrically distributed or exponentially distributed.

The simplest and earliest attempt to approximate the control message interruption rate was done by Ghafir [Gha89]. According to Ghafir's model, the control message interruption rate is given by:

$$\nu_{i,i\oplus j} = \sum_{k=j}^{N-1} \sum_{\ell=1}^{N-k} 2\lambda_{i\oplus k,i\oplus k\oplus \ell}. \quad (4.65)$$

Ghafir's model of control message interruption rate would be unaffected by heavy-tailed distributed message lengths because the control message interruption rate depends exclusively on arrival rate of messages whose path does not overlap with the path of the message of class $\mathcal{C}_{i,i\oplus j}$ in transmission.

Ghafir's model was found to give a low approximation by Chai [Cha95], who improved Ghafir's model by including the quantity $1 - 2\lambda k_D$ in the

denominator. Chai's model gave a better estimate of control message interruption rate. Chai's model would not be affected by heavy-tailed distributed message lengths either, given that it depends only on the arrival rate of messages whose path does not overlap with the path of the message of class $\mathcal{C}_{i,i\oplus j}$ in transmission and on the aggregate arrival rate on the ring.

Henry [Hen98] improved on both previous models by using a conditional argument, which increased the control message interruption rate. Henry's derivation of his model for control frame round trip time was based on the following reasoning. The transmission of a message of class $\mathcal{C}_{i,i\oplus j}$ is considered. Messages that interrupt the transmission of the message of class $\mathcal{C}_{i,i\oplus j}$ are messages whose transmission does not overlap with the transmission of a message of class $\mathcal{C}_{i,i\oplus j}$. Such messages are of class $\mathcal{C}_{i\oplus k,i\oplus k\oplus \ell}$ for $j \leq k \leq N-1$ and $1 \leq \ell \leq N-k$. The transmission of each of those messages generates two control messages, a *START* control message and a *STOP* control message, that delay the transmission of the message of class $\mathcal{C}_{i,i\oplus j}$. Let us designate by $\mathcal{T}_{i,i\oplus j}$ the event that station i transmits a message to station $i \oplus j$. If events \mathcal{A} and \mathcal{B} are mutually independent, the following is true.

$$Pr[\mathcal{A} | \overline{\mathcal{B}}] = \frac{Pr[\mathcal{A}]}{1 - Pr[\mathcal{B}]} \quad (4.66)$$

We designate by $\mathcal{C}_{i\oplus k,i\oplus k\oplus \ell}$ for $j \leq k \leq N-1$ and $1 \leq \ell \leq N-k$ a class of messages whose transmission does not overlap with the transmission of the messages of class $\mathcal{C}_{i,i\oplus j}$, and by $\mathcal{C}_{i\oplus g,i\oplus g\oplus h}$ a class of messages that overlap the transmission of the message of class $\mathcal{C}_{i,i\oplus j}$. Because event $\mathcal{T}_{i,i\oplus j}$ implies event $\overline{\mathcal{T}_{i\oplus g,i\oplus g\oplus h}}$, the following approximation is made.

$$\begin{aligned} Pr[\mathcal{T}_{i\oplus k,i\oplus k\oplus \ell} | \mathcal{T}_{i,i\oplus j}] &\approx Pr[\mathcal{T}_{i\oplus k,i\oplus k\oplus \ell} | \overline{\mathcal{T}_{i\oplus g,i\oplus g\oplus h}}] \\ &= \frac{Pr[\mathcal{T}_{i\oplus k,i\oplus k\oplus \ell}]}{1 - Pr[\mathcal{T}_{i\oplus g,i\oplus g\oplus h}]}; \end{aligned} \quad (4.67)$$

If we designate by $F(i, j, k)$ the total traffic intensity of class $\mathcal{C}_{i \oplus g, i \oplus g \oplus h}$ that overlaps with the transmission of the message of class $\mathcal{C}_{i, i \oplus j}$, we have:

$$Pr[\mathcal{T}_{i \oplus k, i \oplus k \oplus \ell} \mid \mathcal{T}_{i, i \oplus j}] = \frac{Pr[\mathcal{T}_{i \oplus k, i \oplus k \oplus \ell}]}{1 - F(i, j, k)} \quad (4.68)$$

So,

$$\nu_{i, i \oplus j} = \sum_{k=j}^{N-1} \sum_{\ell=1}^{N-k} \frac{2\lambda_{i \oplus k, i \oplus k \oplus \ell}}{1 - F(i, j, k)}. \quad (4.69)$$

The total traffic intensity of class $\mathcal{C}_{i \oplus g, i \oplus g \oplus h}$ is given by:

$$F(i, j, k) = \sum_{g=1}^{j-1} \check{\rho}_{T_{i \oplus g, i \oplus k}}^R + \sum_{g=j}^k \check{\rho}_{T_{i \oplus g, i}}^R + \sum_{g=k+2}^{N-1} \check{\rho}_{T_{i \oplus g, i \oplus k}}^R. \quad (4.70)$$

Therefore,

$$\nu_{i, i \oplus j} = \sum_{k=j}^{N-1} \sum_{\ell=1}^{N-k} \frac{2\lambda_{i \oplus k, i \oplus k \oplus \ell}}{1 - (\sum_{g=1}^{j-1} \check{\rho}_{T_{i \oplus g, i \oplus k}}^R + \sum_{g=j}^k \check{\rho}_{T_{i \oplus g, i}}^R + \sum_{g=k+2}^{N-1} \check{\rho}_{T_{i \oplus g, i \oplus k}}^R)}. \quad (4.71)$$

It is clear from the above derivation and the resulting equation for control message interruption rate that the control message interruption rate obtained by Henry is not affected by the heavy-tailed distributed message lengths.

4.4 Control Frame Round Trip Time

As stated earlier, the insertion of control messages adds to the duration of the control frame round trip time. In the absence of control message interruptions, the duration of the control frame round trip time is simply τ . Each control message of size k_D characters adds k_D character times to the duration of the control frame round trip time. Let S_{GO} designate the duration of the control frame round trip time for a control frame containing η control messages. S_{GO} is given by:

$$S_{GO} = \tau + \eta k_D, \quad (4.72)$$

where k_D is the duration of a control message. The first three moments of the control frame round trip time were found by Henry [Hen98] and they are given by:

$$E[S_{GO}] = \frac{\tau}{1 - \nu k_D}, \quad (4.73)$$

$$E[S_{GO}^2] = \left(\frac{\tau}{1 - \nu k_D} \right) \left(\frac{\tau + \tau \nu k_D + \nu k_D^2}{1 - \nu^2 k_D^2} \right), \quad (4.74)$$

and

$$\begin{aligned} E[S_{GO}^3] &= E[S_{GO}^2] \left(\frac{3\nu^2 k_D^3 + 3\tau \nu^2 k_D^2}{1 - \nu^3 k_D^3} \right) \\ &+ E[S_{GO}] \left(\frac{\nu k_D (k_D^2 + 3\tau k_D + 3\tau^2)}{1 - \nu^3 k_D^3} \right) + \left(\frac{\tau^3}{1 - \nu^3 k_D^3} \right). \end{aligned} \quad (4.75)$$

These three moments were derived assuming exponentially distributed message interarrival times and are independent of the message lengths distribution.

4.5 Message Transmission and Service Time

In the presence of control message interruptions, transmission and service times are increased. The transmission time in the presence of control message interruptions, denoted $T_{i,i\oplus j}$, is found by replacing τ , the empty control frame round trip time duration, by S_{GO} in the expression for basic transmission time. The service time, denoted $S_{i,i\oplus j}$, is equal to the transmission time plus the time for the *STOP* message to be transmitted and for *GO* to return. We have:

$$T_{i,i\oplus j} = n_{G_{i,i\oplus j}} S_{GO_{i,i\oplus j}} + k_D, \quad (4.76)$$

and

$$S_{i,i\oplus j} = T_{i,i\oplus j} + \tau + k_D. \quad (4.77)$$

Messages sourced by station i and destined to station $i \oplus j$ arrive at station i 's queue with arrival rate $\lambda_{i,i\oplus j}$ messages per character time and have an average transmission time equal to $E[T_{i,i\oplus j}]$ character times. The product $\lambda_{i,i\oplus j}E[T_{i,i\oplus j}]$ is then the fraction of time the ring spends transmitting messages of class $\mathcal{C}_{i,i\oplus j}$, and is known as transmission traffic intensity, denoted $\rho_{T_{i,i\oplus j}}$, and is given by:

$$\rho_{T_{i,i\oplus j}} = \lambda_{i,i\oplus j}E[T_{i,i\oplus j}]. \quad (4.78)$$

The total transmission traffic sourced by station i is denoted ρ_{T_i} and is given by:

$$\rho_{T_i} = \sum_{j=1}^{N-1} \rho_{T_{i,i\oplus j}} = \sum_{j=1}^{N-1} \lambda_{i,i\oplus j}E[T_{i,i\oplus j}]. \quad (4.79)$$

The fraction of time the ring spends serving messages of class $\mathcal{C}_{i,i\oplus j}$, known as traffic intensity, is denoted $\rho_{i,i\oplus j}$ and is given by:

$$\rho_{i,i\oplus j} = \lambda_{i,i\oplus j}E[S_{i,i\oplus j}]. \quad (4.80)$$

The total traffic intensity ρ_i sourced by station i is given by:

$$\rho_i = \sum_{j=1}^{N-1} \rho_{i,i\oplus j} = \sum_{j=1}^{N-1} \lambda_{i,i\oplus j}E[S_{i,i\oplus j}] \quad (4.81)$$

Finally, the total traffic intensity ρ on the ring is:

$$\rho = \sum_{i=0}^{N-1} \rho_i. \quad (4.82)$$

4.6 Blocking Duration

A message of class $\mathcal{C}_{i,i\oplus j}$ at the head of the line (*HOL*) of station i 's queue and destined to station $i \oplus j$ will experience blocking if there is another message that is using the path $i \rightarrow i \oplus j$. Let us suppose that the message using the path $i \rightarrow i \oplus j$ is of class $\mathcal{C}_{i\oplus g,i\oplus g\oplus h}$. We denote by $BL_{i,i\oplus j}$ the duration of

blocking experienced by the message sourced by station i of outbound distance j . The use of the path $i \rightarrow i \oplus j$ while the message of class $\mathcal{C}_{i,i \oplus j}$ is at the head of the line of station i 's queue constitutes a blocking event whose duration is the residual time of service of the message of class $\mathcal{C}_{i \oplus g, i \oplus g \oplus h}$ in transmission. We denote by $R_{S_{i \oplus g, i \oplus g \oplus h}}$ the residual service time of a message of class $\mathcal{C}_{i \oplus g, i \oplus g \oplus h}$. Messages that block the message of class $\mathcal{C}_{i,i \oplus j}$ are messages whose transmission overlaps the path $i \rightarrow i \oplus j$. Such messages can be divided into four regions as shown in Figure 4.5: Messages sourced by a station in the path $0 \rightarrow i \ominus 1$ that use station i as a repeater (region (1)), messages whose source and destination are in the path $i \rightarrow i \oplus j$ (region (2)), messages whose source is in the path $i \rightarrow i \oplus j$ and that use station $i \oplus j$ as a repeater (region (3)), and finally messages whose source is in the path $i \oplus j \rightarrow N - 1$ and that use station i as a repeater (region (4)).

We recall that $\rho_{i \oplus g, i \oplus g \oplus h}$ denotes the traffic intensity of messages of class $\mathcal{C}_{i \oplus g, i \oplus g \oplus h}$. It is the probability that a message of class $\mathcal{C}_{i \oplus g, i \oplus g \oplus h}$ is being transmitted. Traffic in region (1) occurs with probability:

$$\rho_{bl}^{(1)} = \sum_{g=N \ominus i}^{N \ominus 1} \sum_{h=N \ominus g \oplus 1}^{N \ominus 1} \rho_{i \oplus g, i \oplus g \oplus h}. \quad (4.83)$$

Traffic in region (2) occurs with probability:

$$\rho_{bl}^{(2)} = \sum_{g=1}^{j \ominus 1} \sum_{h=1}^{N \ominus g \oplus j} \rho_{i \oplus g, i \oplus g \oplus h}. \quad (4.84)$$

Traffic in region (3) occurs with probability:

$$\rho_{bl}^{(3)} = \sum_{g=1}^{j \ominus 1} \sum_{h=N \ominus g \oplus j \oplus 1}^{N \ominus 1} \rho_{i \oplus g, i \oplus g \oplus h}. \quad (4.85)$$

Traffic in region (4) occurs with probability:

$$\rho_{bl}^{(4)} = \sum_{g=j}^{N \ominus i \oplus 1} \sum_{h=N \ominus g \oplus 1}^{N \ominus 1} \rho_{i \oplus g, i \oplus g \oplus h}. \quad (4.86)$$

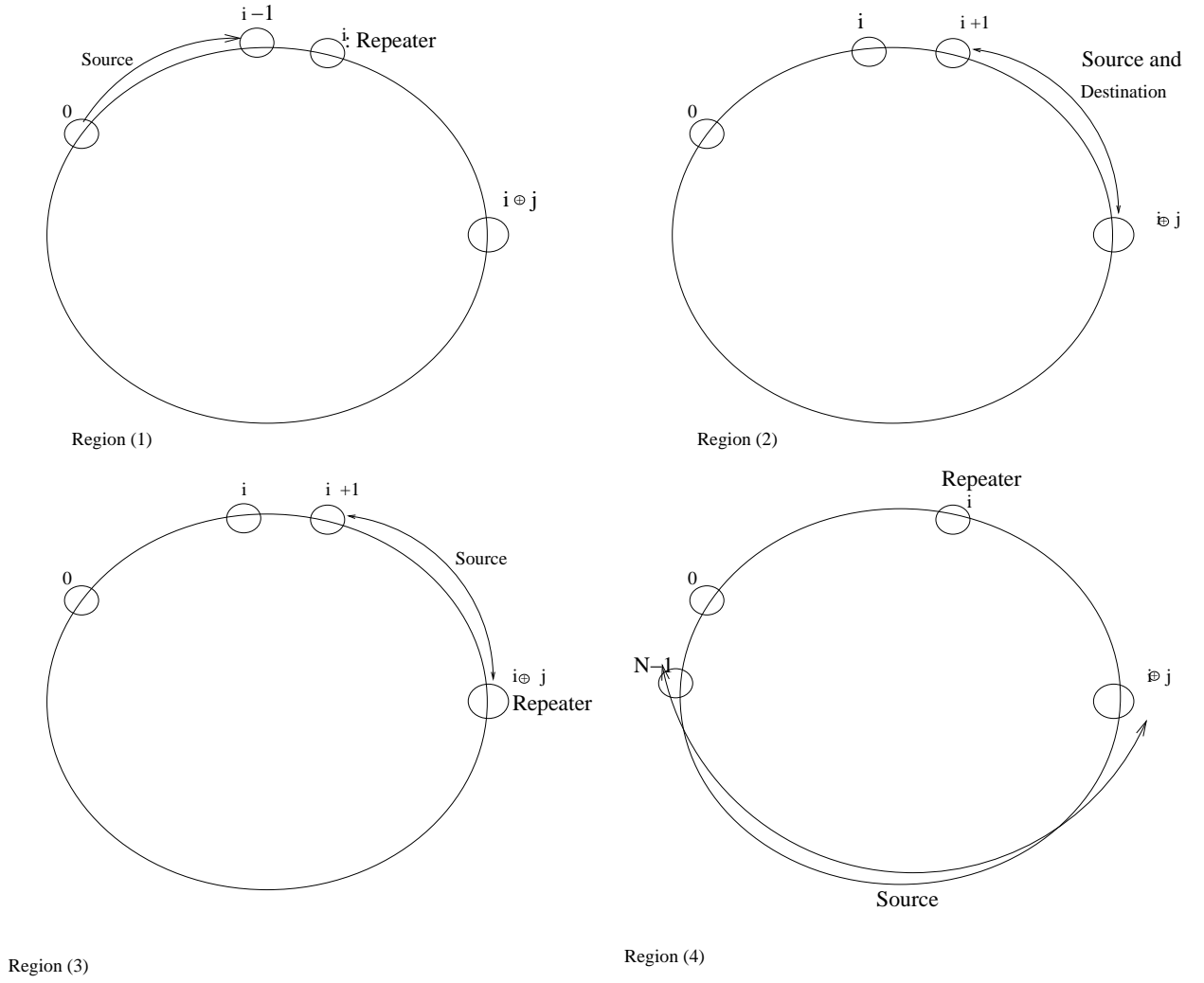


Figure 4.5: Regions of messages that overlap the path $i \rightarrow i \oplus j$.

The total traffic intensity ρ_{bl}^T for messages that block the message of class $\mathcal{C}_{i, i \oplus j}$ is:

$$\begin{aligned}
 \rho_{bl, i, i \oplus j}^T &= \rho_{bl}^{(1)} + \rho_{bl}^{(2)} + \rho_{bl}^{(3)} + \rho_{bl}^{(4)} \\
 &= \sum_{g=N \ominus i}^{N \ominus 1} \sum_{h=N \ominus g \oplus 1}^{N \ominus 1} \rho_{i \oplus g, i \oplus g \oplus h} + \sum_{g=1}^{j \ominus 1} \sum_{h=1}^{N \ominus g \oplus j} \rho_{i \oplus g, i \oplus g \oplus h} \\
 &\quad + \sum_{g=1}^{j \ominus 1} \sum_{h=N \ominus g \oplus j \oplus 1}^{N \ominus 1} \rho_{i \oplus g, i \oplus g \oplus h} + \sum_{g=j}^{N \ominus i \ominus 1} \sum_{h=N \ominus g \oplus 1}^{N \ominus 1} \rho_{i \oplus g, i \oplus g \oplus h}.
 \end{aligned}$$

The probability that a message of class $\mathcal{C}_{i \oplus g, i \oplus g \oplus h}$ blocks the path $i \rightarrow i \oplus j$

given that the path $i \rightarrow i \oplus j$ is blocked is $\frac{\rho_{i \oplus g, i \oplus g \oplus h}}{\rho_{bl}^T i, i \oplus j}$. The blocking duration of the message of class $\mathcal{C}_{i, i \oplus j}$ is obtained by averaging over all possible blocking events and we have:

$$\begin{aligned}
BL_{i, i \oplus j} &= \sum_{g=N \ominus i}^{N \ominus 1} \sum_{h=N \ominus g \oplus 1}^{N \ominus 1} \frac{\rho_{i \oplus g, i \oplus g \oplus h}}{\rho_{bl}^T i, i \oplus j} R_{S i \oplus g, i \oplus g \oplus h} \\
&+ \sum_{g=1}^{j \ominus 1} \sum_{h=1}^{N \ominus g \oplus j} \frac{\rho_{i \oplus g, i \oplus g \oplus h}}{\rho_{bl}^T i, i \oplus j} R_{S i \oplus g, i \oplus g \oplus h} \\
&+ \sum_{g=1}^{j \ominus 1} \sum_{h=N \ominus g \oplus j \oplus 1}^{N \ominus 1} \frac{\rho_{i \oplus g, i \oplus g \oplus h}}{\rho_{bl}^T i, i \oplus j} R_{S i \oplus g, i \oplus g \oplus h} \\
&+ \sum_{g=j}^{N \ominus i \oplus 1} \sum_{h=N \ominus g \oplus 1}^{N \ominus 1} \frac{\rho_{i \oplus g, i \oplus g \oplus h}}{\rho_{bl}^T i, i \oplus j} R_{S i \oplus g, i \oplus g \oplus h}. \tag{4.87}
\end{aligned}$$

The Laplace-Stieltjes Transform of $BL_{i, i \oplus j}$ is given by:

$$BL_{i, i \oplus j}^*(s) = E[e^{-s BL_{i, i \oplus j}}]. \tag{4.88}$$

The first three derivatives of the Laplace-Stieltjes transform of $BL_{i, i \oplus j}(s)$ are given by:

$$BL_{i, i \oplus j}^{*'}(s) = -E[BL_{i, i \oplus j} e^{-s BL_{i, i \oplus j}}] \tag{4.89}$$

$$BL_{i, i \oplus j}^{*''}(s) = E[BL_{i, i \oplus j}^2 e^{-s BL_{i, i \oplus j}}] \tag{4.90}$$

$$BL_{i, i \oplus j}^{*'''}(s) = -E[BL_{i, i \oplus j}^3 e^{-s BL_{i, i \oplus j}}], \tag{4.91}$$

and the first three moments of the Laplace-Stieltjes transform of $BL_{i, i \oplus j}(s)$ are given by:

$$E[BL_{i, i \oplus j}] = -BL_{i, i \oplus j}'(0) \tag{4.92}$$

$$E[BL_{i, i \oplus j}^2] = BL_{i, i \oplus j}''(0) \tag{4.93}$$

$$E[BL_{i, i \oplus j}^3] = -BL_{i, i \oplus j}'''(0). \tag{4.94}$$

Because of the linearity property of the Laplace-Stieltjes transform [Kle75], we have:

$$\begin{aligned}
BL_{i,i\oplus j}(s) &= \sum_{g=N\ominus i}^{N\ominus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} R_{S i\oplus g,i\oplus g\oplus h}(s) \\
&+ \sum_{g=1}^{j\ominus 1} \sum_{h=1}^{N\ominus g\oplus j} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} R_{S i\oplus g,i\oplus g\oplus h}(s) \\
&+ \sum_{g=1}^{j\ominus 1} \sum_{h=N\ominus g\oplus j\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} R_{S i\oplus g,i\oplus g\oplus h}(s) \\
&+ \sum_{g=j}^{N\ominus i\oplus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} R_{S i\oplus g,i\oplus g\oplus h}(s). \tag{4.95}
\end{aligned}$$

Thus, the mean of the blocking duration is given by:

$$\begin{aligned}
E[BL_{i,i\oplus j}] &= -BL_{i,i\oplus j}^{*'}(0) = - \sum_{g=N\ominus i}^{N\ominus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} R_{S i\oplus g,i\oplus g\oplus h}^{*'}(0) \\
&- \sum_{g=1}^{j\ominus 1} \sum_{h=1}^{N\ominus g\oplus j} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} R_{S i\oplus g,i\oplus g\oplus h}^{*'}(0) \\
&- \sum_{g=1}^{j\ominus 1} \sum_{h=N\ominus g\oplus j\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} R_{S i\oplus g,i\oplus g\oplus h}^{*'}(0) \\
&- \sum_{g=j}^{N\ominus i\oplus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} R_{S i\oplus g,i\oplus g\oplus h}^{*'}(0). \tag{4.96}
\end{aligned}$$

So,

$$\begin{aligned}
E[BL_{i,i\oplus j}] &= \sum_{g=N\ominus i}^{N\ominus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} E[R_{S i\oplus g,i\oplus g\oplus h}] \\
&+ \sum_{g=1}^{j\ominus 1} \sum_{h=1}^{N\ominus g\oplus j} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} E[R_{S i\oplus g,i\oplus g\oplus h}] \\
&+ \sum_{g=1}^{j\ominus 1} \sum_{h=N\ominus g\oplus j\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} E[R_{S i\oplus g,i\oplus g\oplus h}] \\
&+ \sum_{g=j}^{N\ominus i\oplus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bli,i\oplus j}^T} E[R_{S i\oplus g,i\oplus g\oplus h}]. \tag{4.97}
\end{aligned}$$

Likewise, we have:

$$\begin{aligned}
E[BL_{i,i\oplus j}^2] = BL_{i,i\oplus j}^{*''}(0) &= \sum_{g=N\ominus i}^{N\ominus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bl,i\oplus j}^T} R_S^{*''}{}_{i\oplus g,i\oplus g\oplus h}(0) \\
&+ \sum_{g=1}^{j\ominus 1} \sum_{h=1}^{N\ominus g\oplus j} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bl,i\oplus j}^T} R_S^{*''}{}_{i\oplus g,i\oplus g\oplus h}(0) \\
&+ \sum_{g=1}^{j\ominus 1} \sum_{h=N\ominus g\oplus j\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bl,i\oplus j}^T} R_S^{*''}{}_{i\oplus g,i\oplus g\oplus h}(0) \\
&+ \sum_{g=j}^{N\ominus i\ominus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bl,i\oplus j}^T} R_S^{*''}{}_{i\oplus g,i\oplus g\oplus h}(0).
\end{aligned} \tag{4.98}$$

Thus,

$$\begin{aligned}
E[BL_{i,i\oplus j}^2] &= \sum_{g=N\ominus i\ominus 1}^{N\ominus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bl,i\oplus j}^T} E[R_{S_{i\oplus g,i\oplus g\oplus h}}^2] \\
&+ \sum_{g=1}^{j\ominus 1} \sum_{h=1}^{N\ominus g\oplus j} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bl,i\oplus j}^T} E[R_{S_{i\oplus g,i\oplus g\oplus h}}^2] \\
&+ \sum_{g=1}^{j\ominus 1} \sum_{h=N\ominus g\oplus j\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bl,i\oplus j}^T} E[R_{S_{i\oplus g,i\oplus g\oplus h}}^2] \\
&+ \sum_{g=j}^{N\ominus i\ominus 1} \sum_{h=N\ominus g\oplus 1}^{N\ominus 1} \frac{\rho_{i\oplus g,i\oplus g\oplus h}}{\rho_{bl,i\oplus j}^T} E[R_{S_{i\oplus g,i\oplus g\oplus h}}^2].
\end{aligned} \tag{4.99}$$

In Equation 4.97 and Equation 4.99, $R_{S_{i\oplus g,i\oplus g\oplus h}}$ is the residual service time and its n th moment is given by:

$$E[R_{S_{i\oplus g,i\oplus g\oplus h}}^n] = \frac{E[R_{S_{i\oplus g,i\oplus g\oplus h}}^{n+1}]}{(n+1)E[R_{S_{i\oplus g,i\oplus g\oplus h}}]}. \tag{4.100}$$

Thus the first and second moments of the residual service time of a message of class $\mathcal{C}_{i\oplus g,i\oplus g\oplus h}$ and of service time duration $S_{i\oplus g,i\oplus g\oplus h}$ are given by:

$$E[R_{S_{i\oplus g,i\oplus g\oplus h}}] = \frac{E[S_{i\oplus g,i\oplus g\oplus h}^2]}{2E[S_{i\oplus g,i\oplus g\oplus h}]} \tag{4.101}$$

and

$$E [R_{S^{i \oplus g, i \oplus g \oplus h}}^2] = \frac{E [S_{i \oplus g, i \oplus g \oplus h}^3]}{3E [S_{i \oplus g, i \oplus g \oplus h}]}. \quad (4.102)$$

Figures 4.6–4.26 show the analytical results for blocking duration plotted versus the load $\lambda E [M]$. The first set of plots includes Figures 4.6–4.17, where the shape parameter c is assumed to be equal 0.6. The second set of plots includes Figures 4.18–4.27, where the shape parameter c is assumed to be equal 0.4. For comparison purposes, mean blocking duration under the assumption of exponential message interarrival times and geometric message lengths is also included.

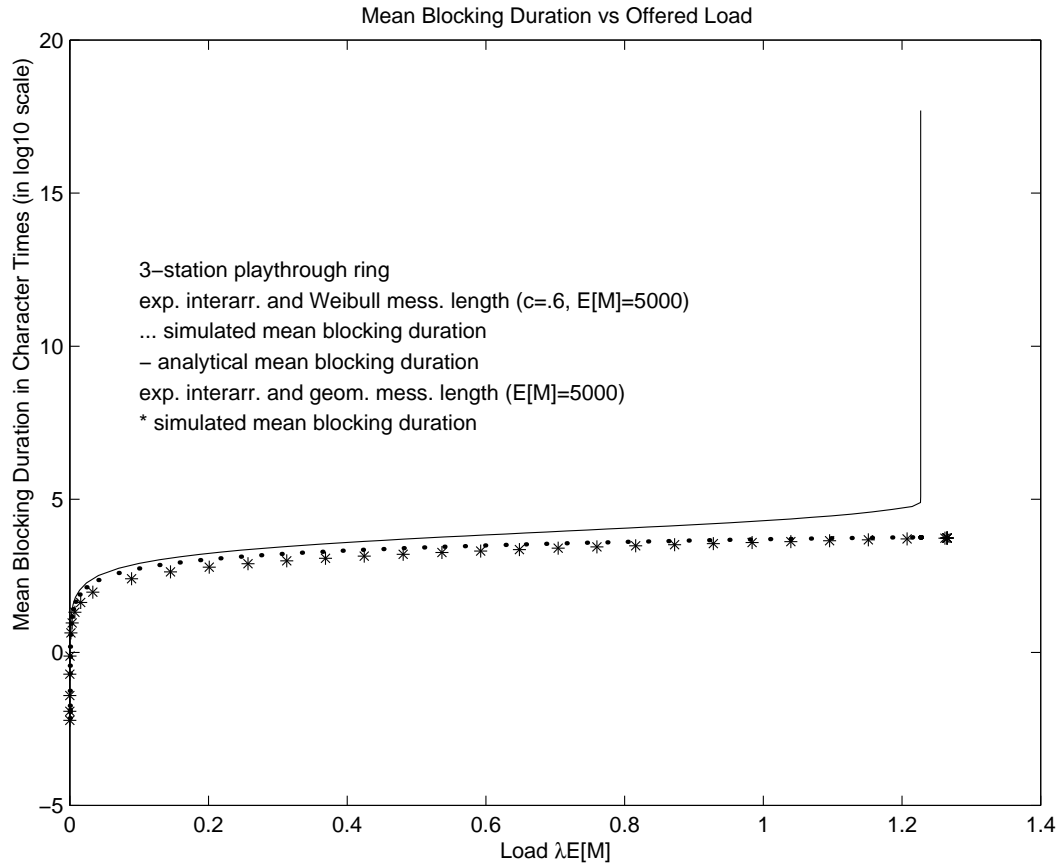


Figure 4.6: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 5000$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

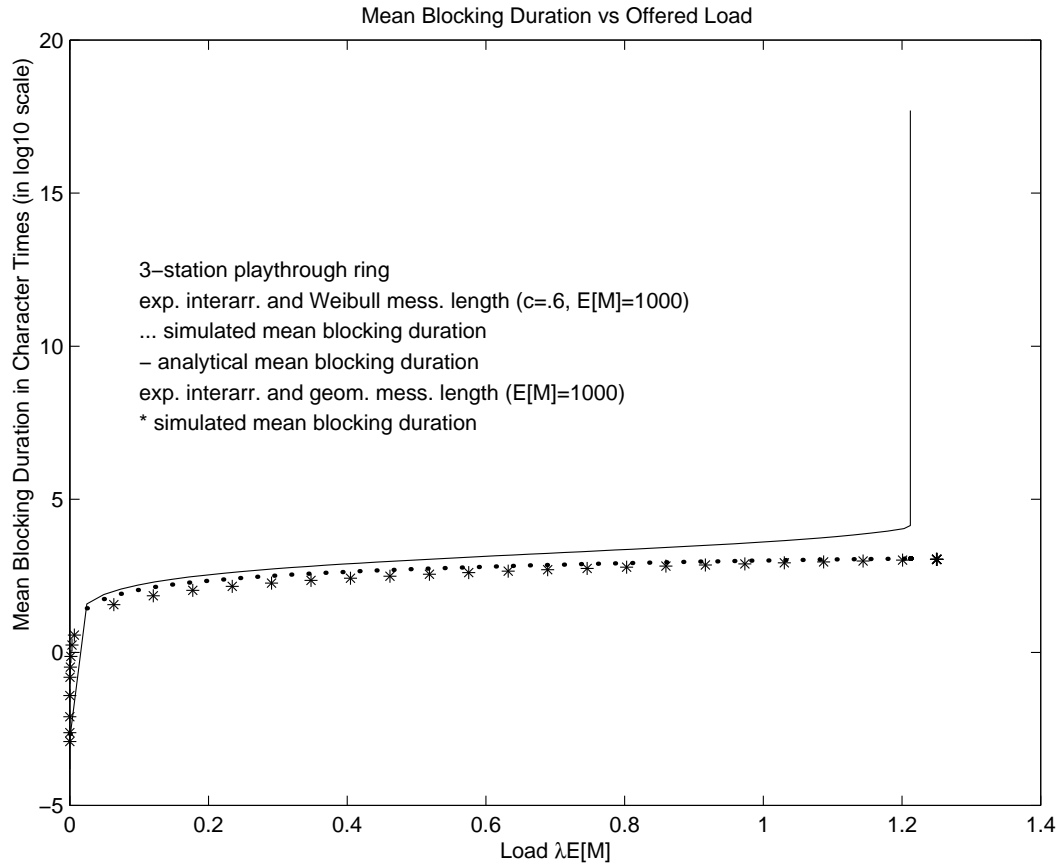


Figure 4.7: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 1000$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

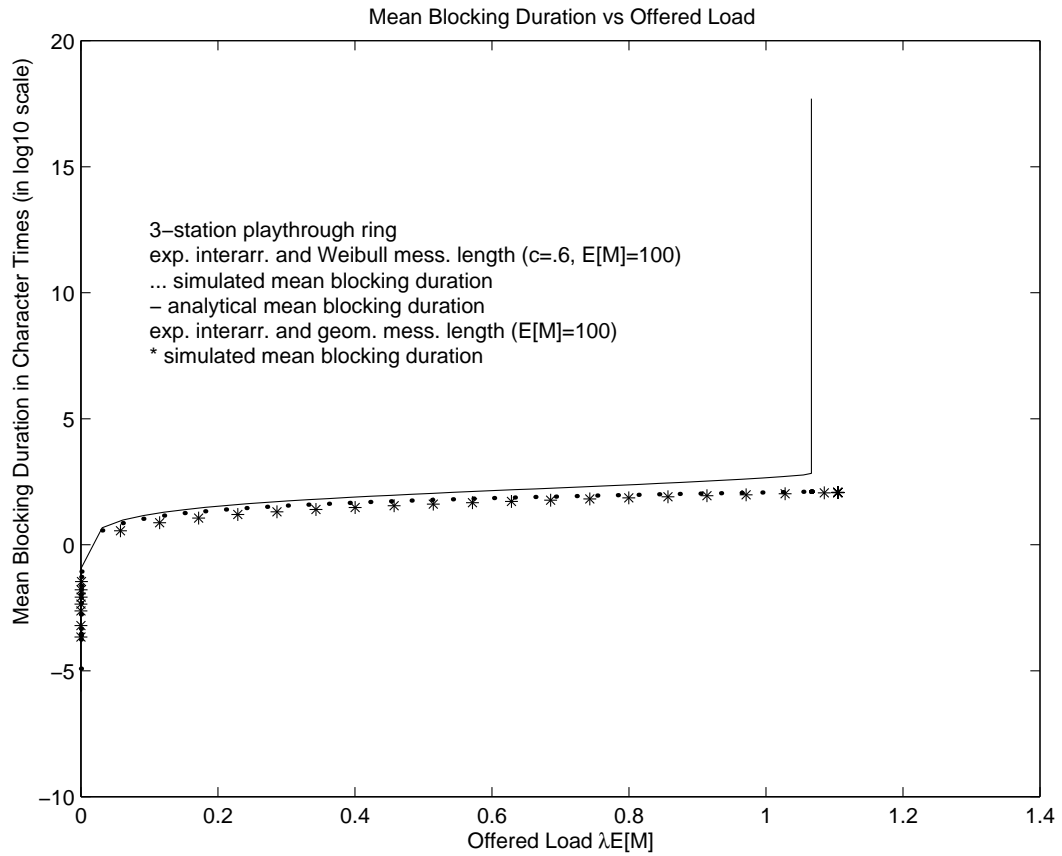


Figure 4.8: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 100$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

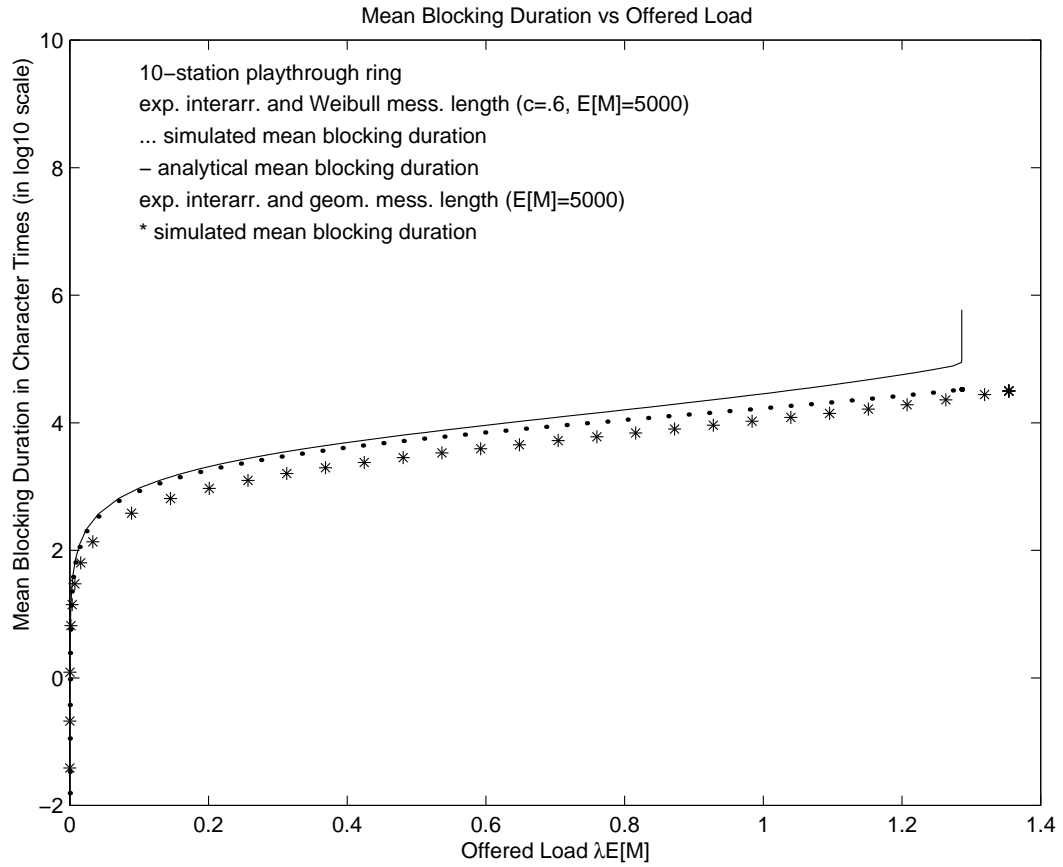


Figure 4.9: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 5000$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

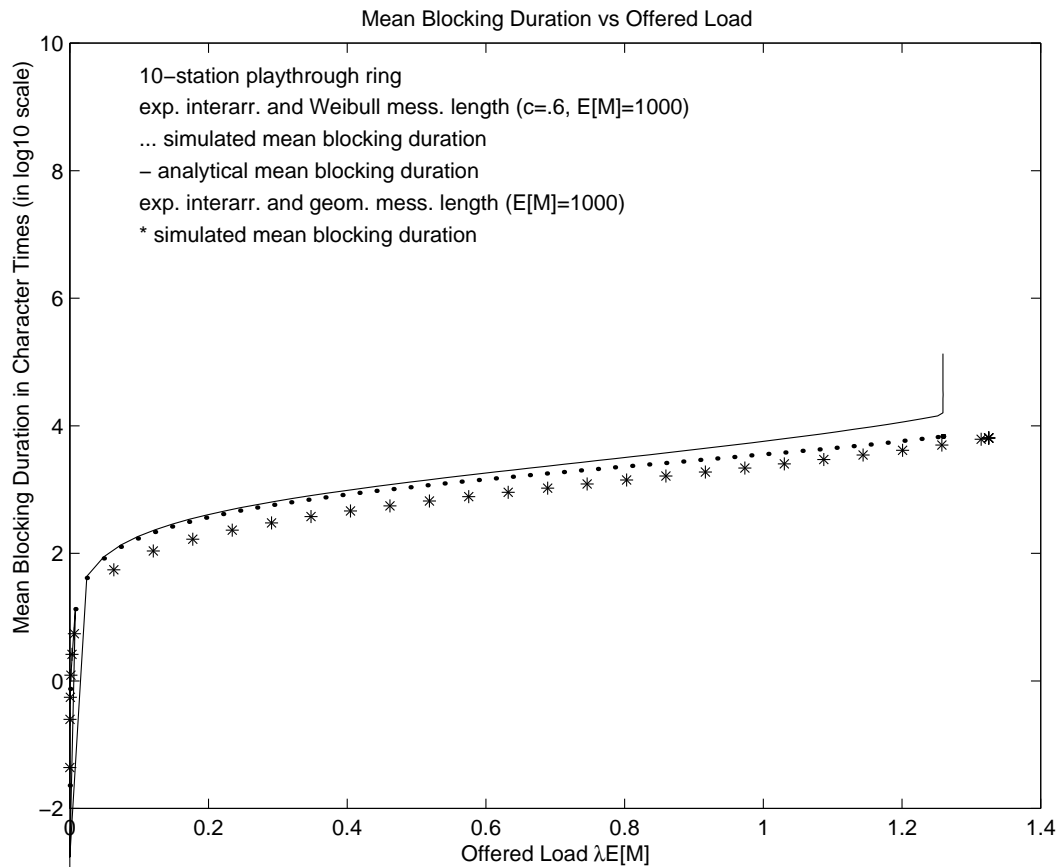


Figure 4.10: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 1000$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

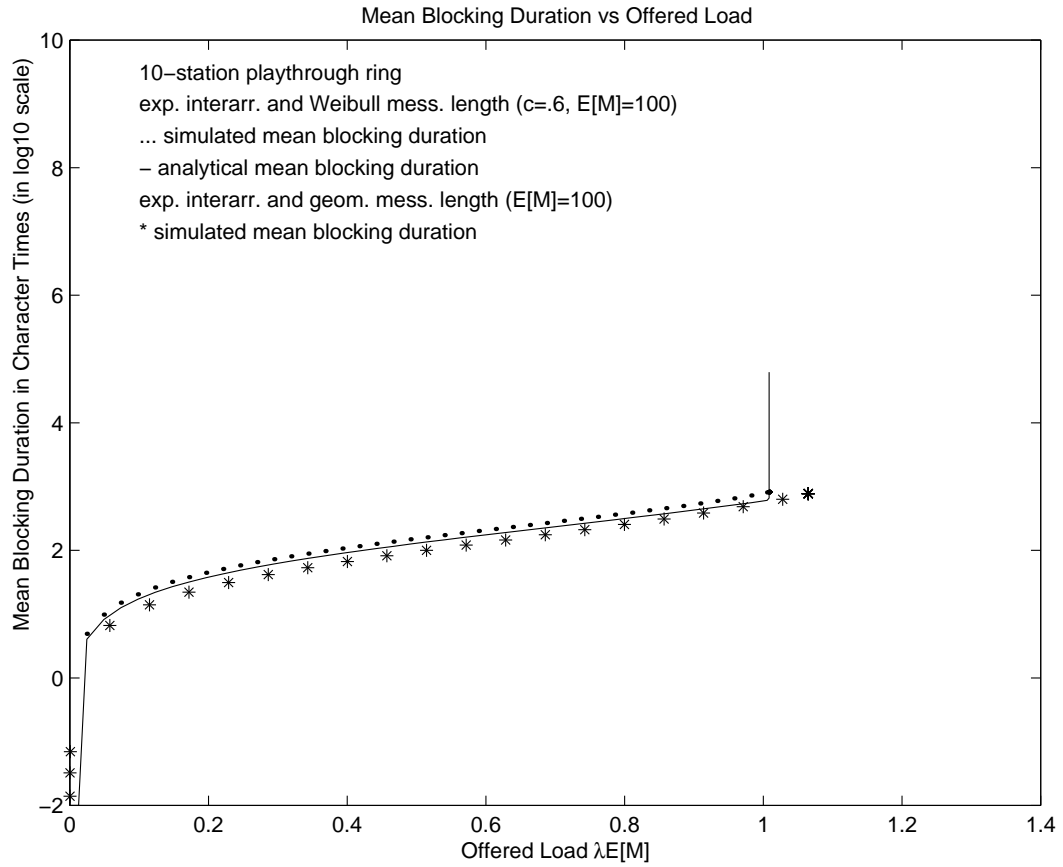


Figure 4.11: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 100$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

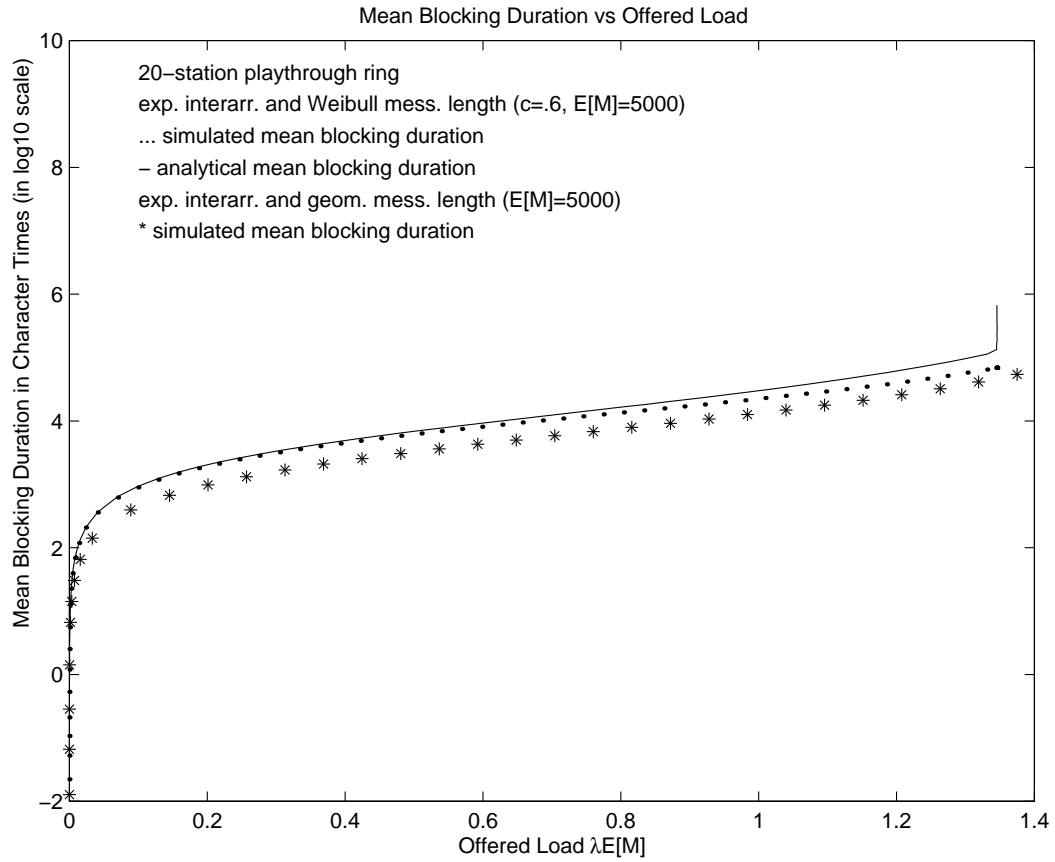


Figure 4.12: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 5000$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

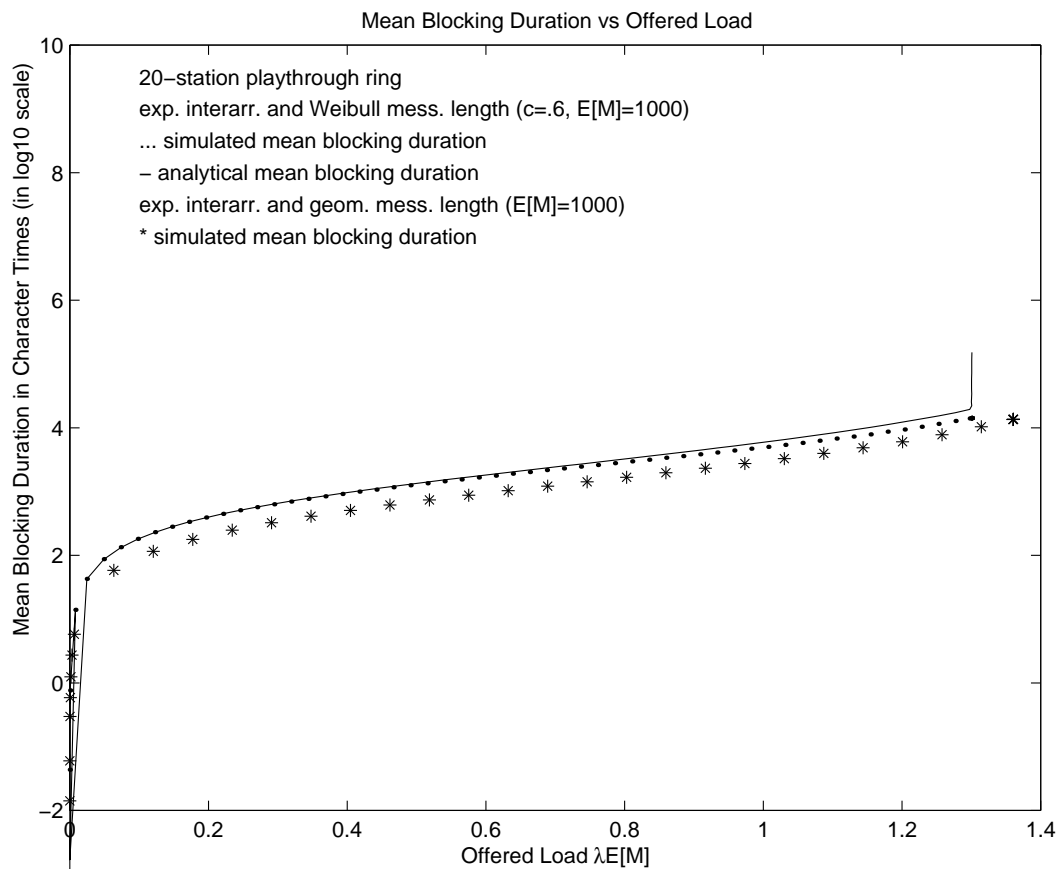


Figure 4.13: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 1000$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

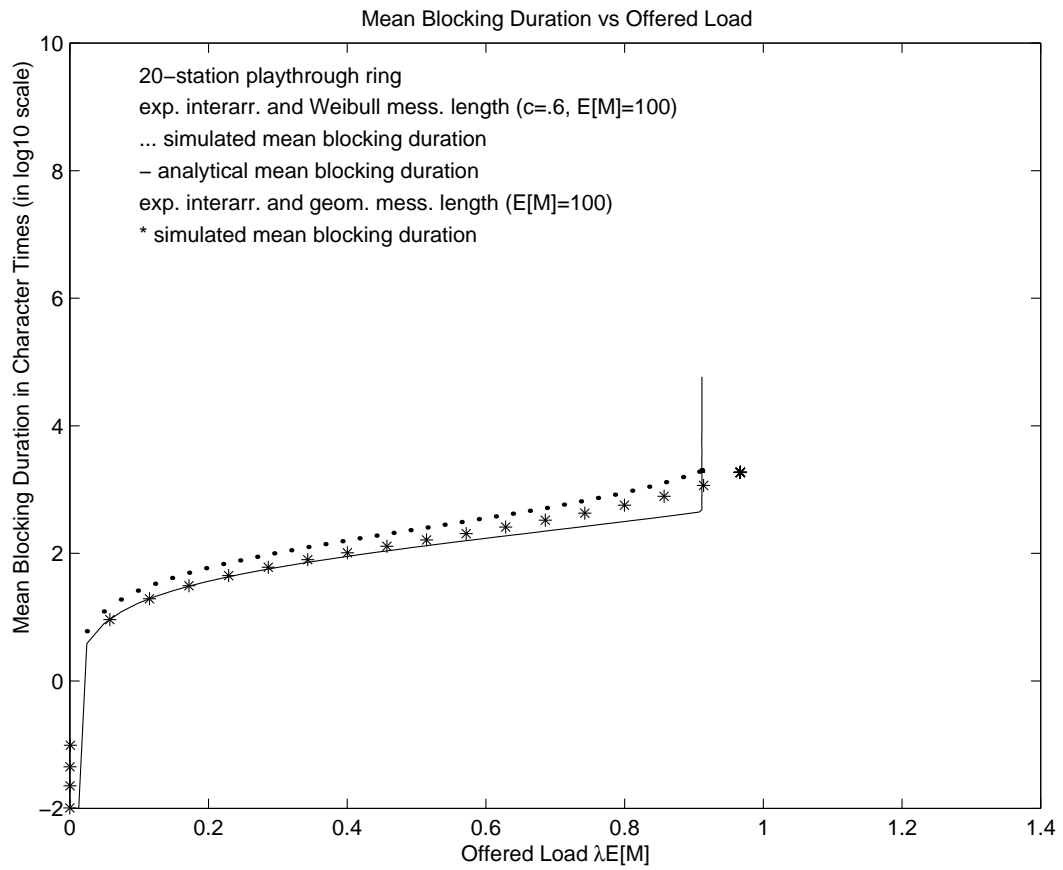


Figure 4.14: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 100$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

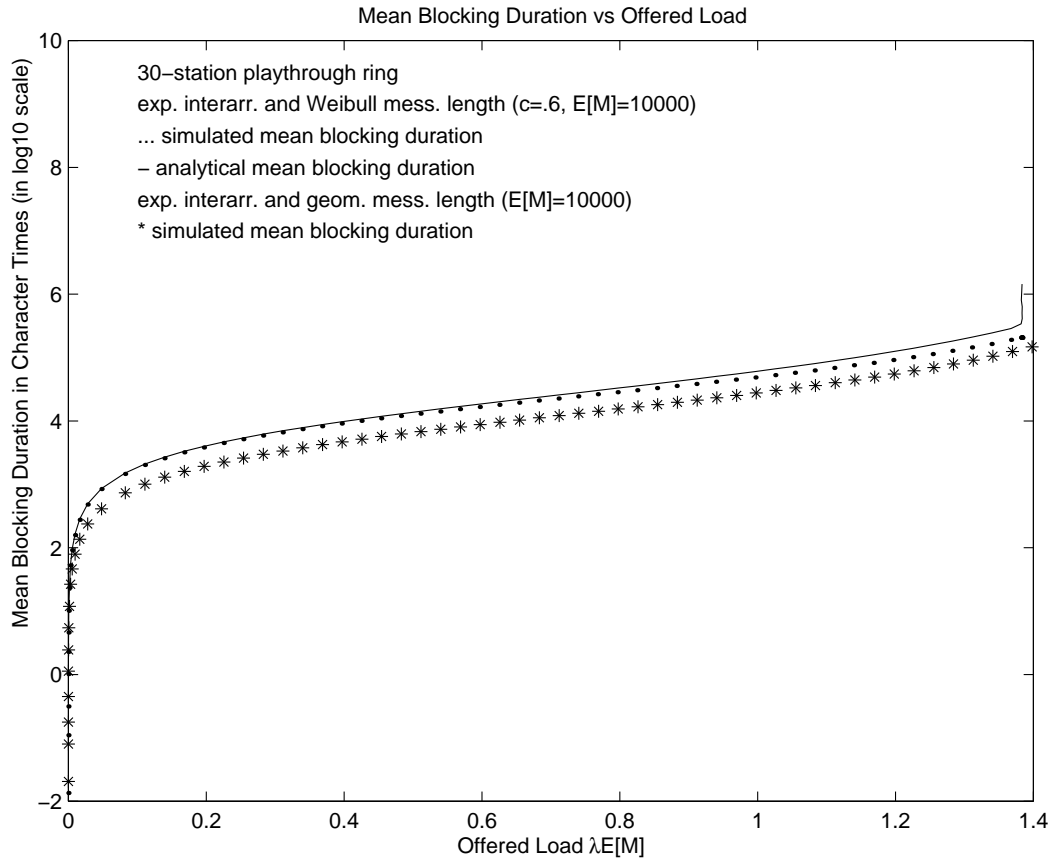


Figure 4.15: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 10000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

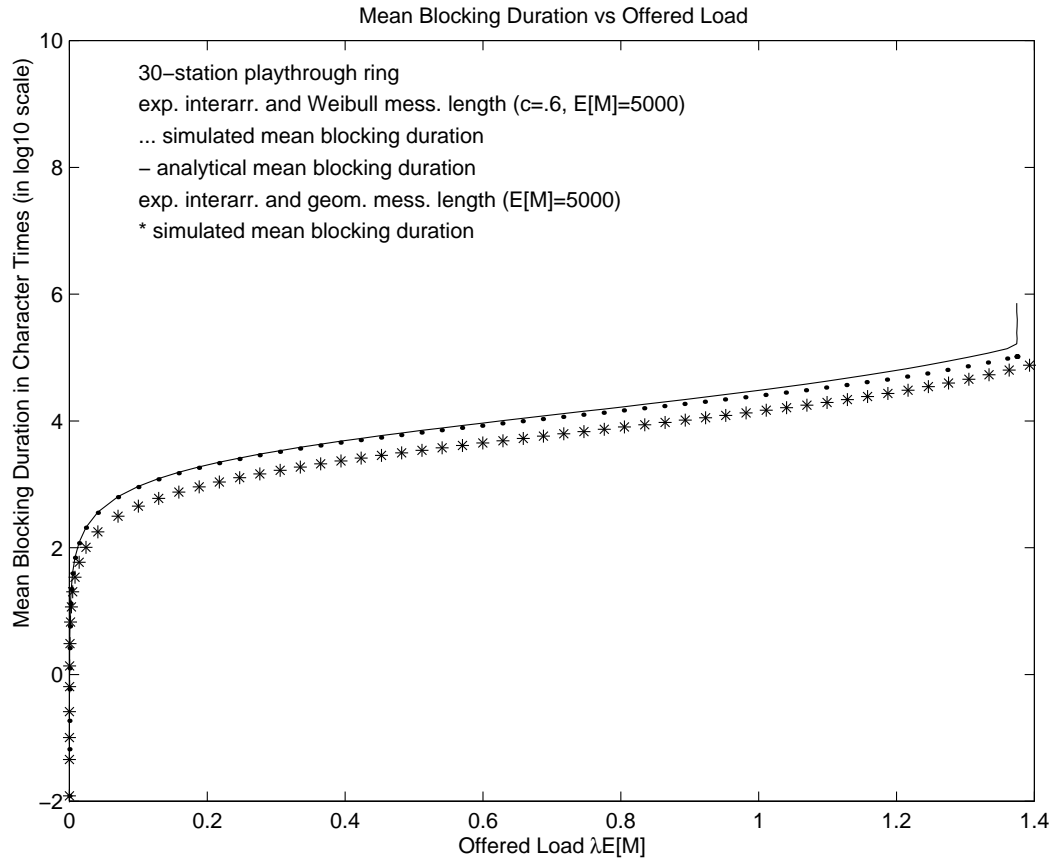


Figure 4.16: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 5000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

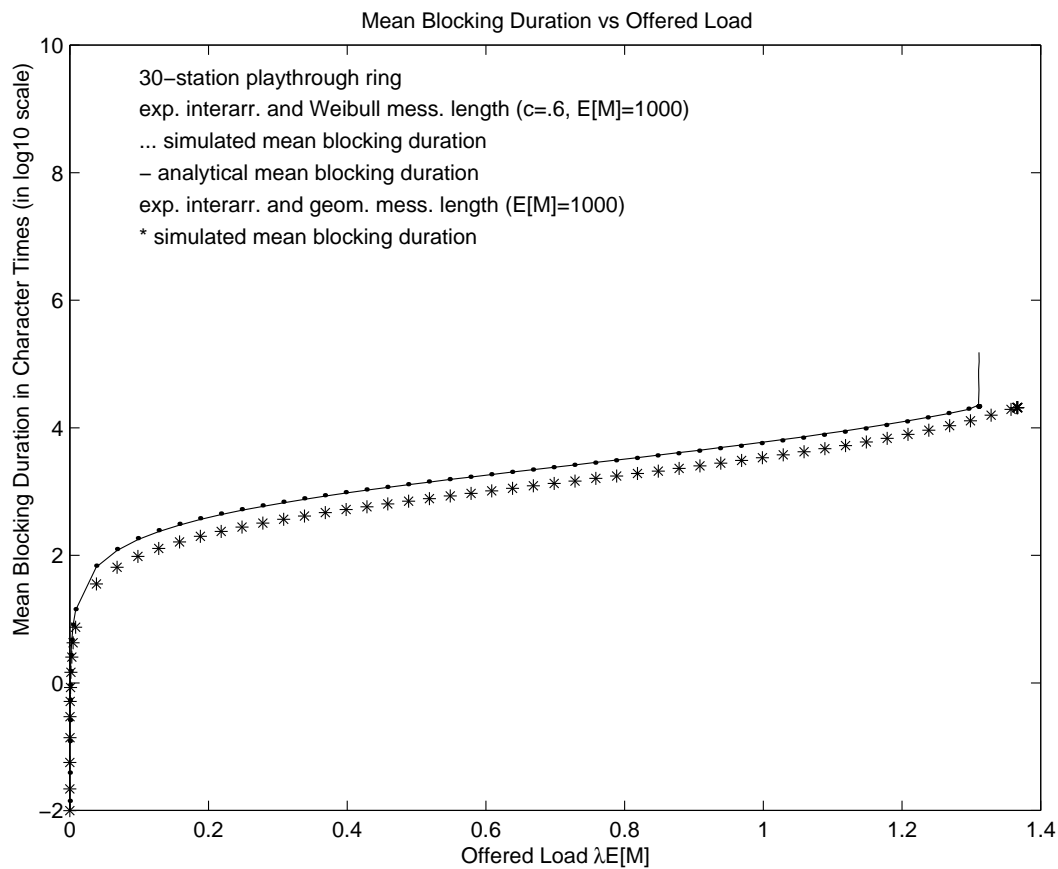


Figure 4.17: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.6$; mean msg. length $E[M] = 1000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

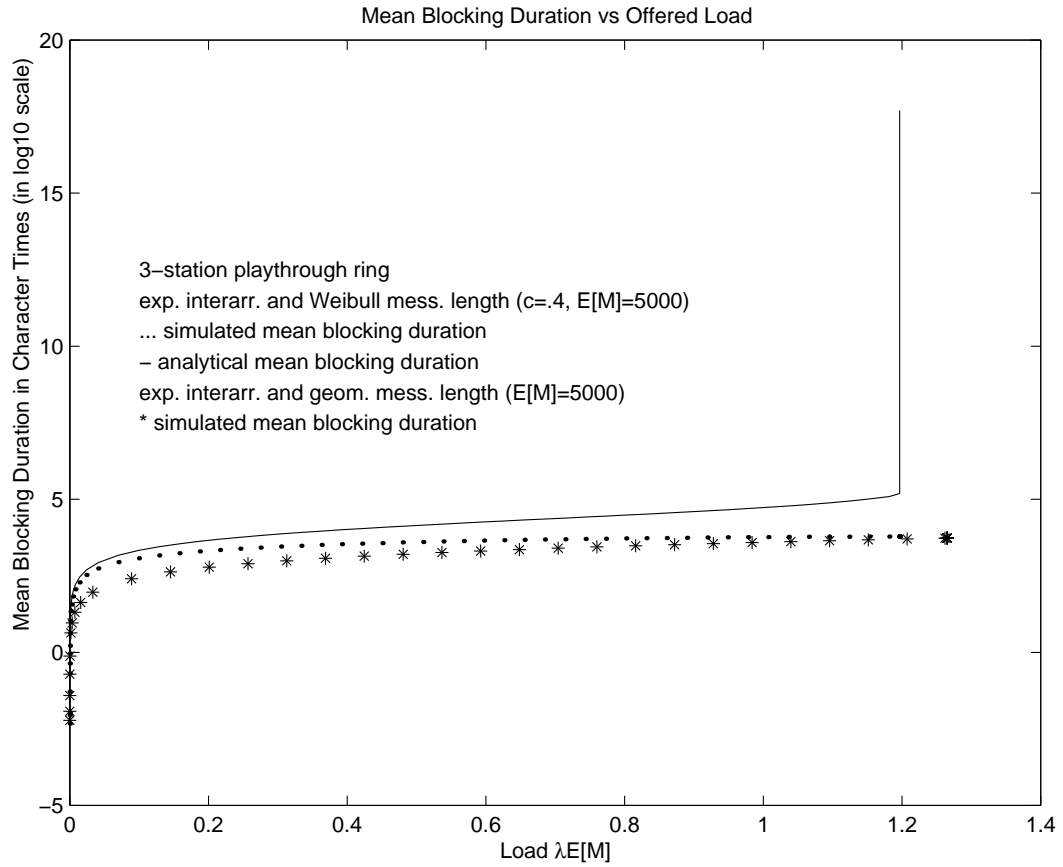


Figure 4.18: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 5000$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

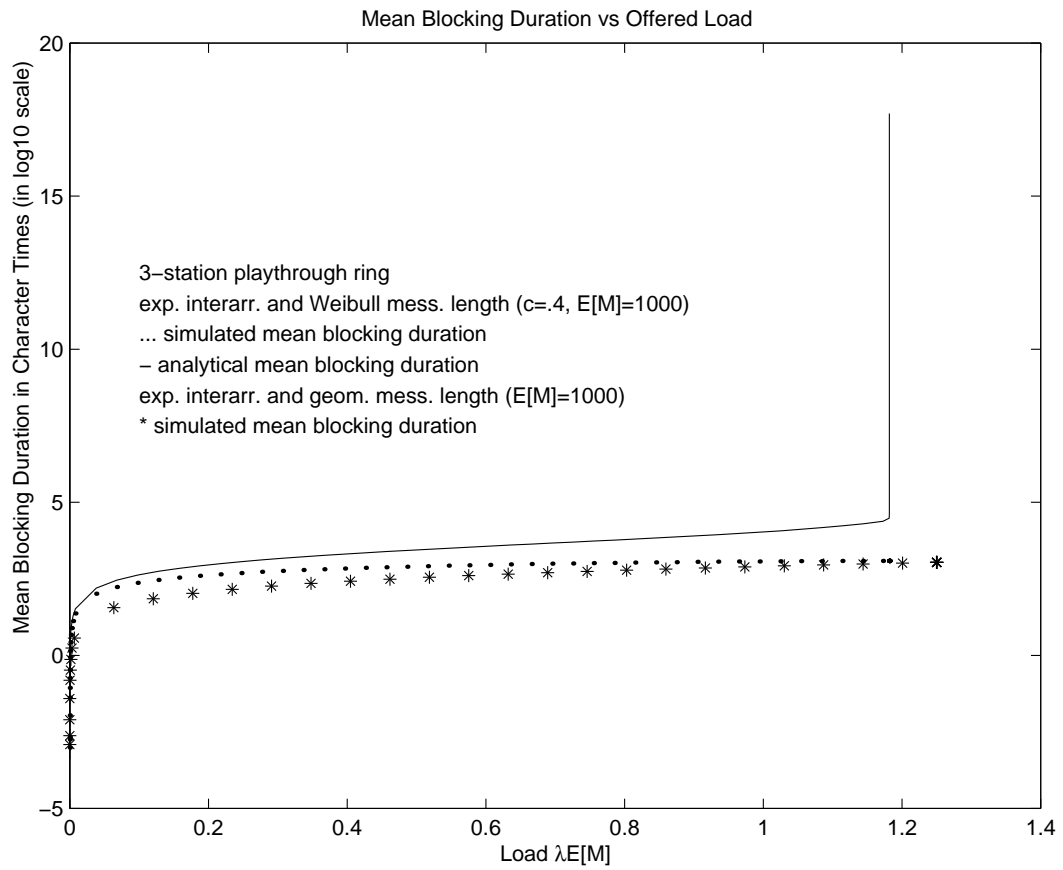


Figure 4.19: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 1000$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

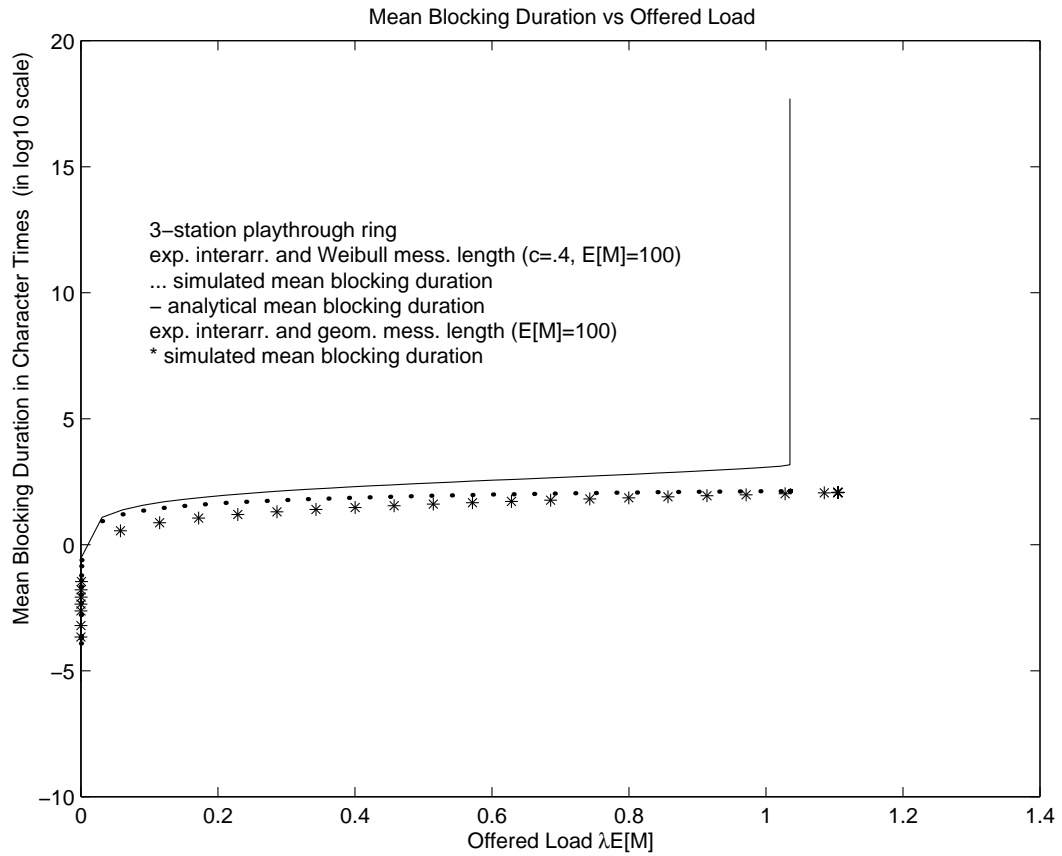


Figure 4.20: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 100$ characters, $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

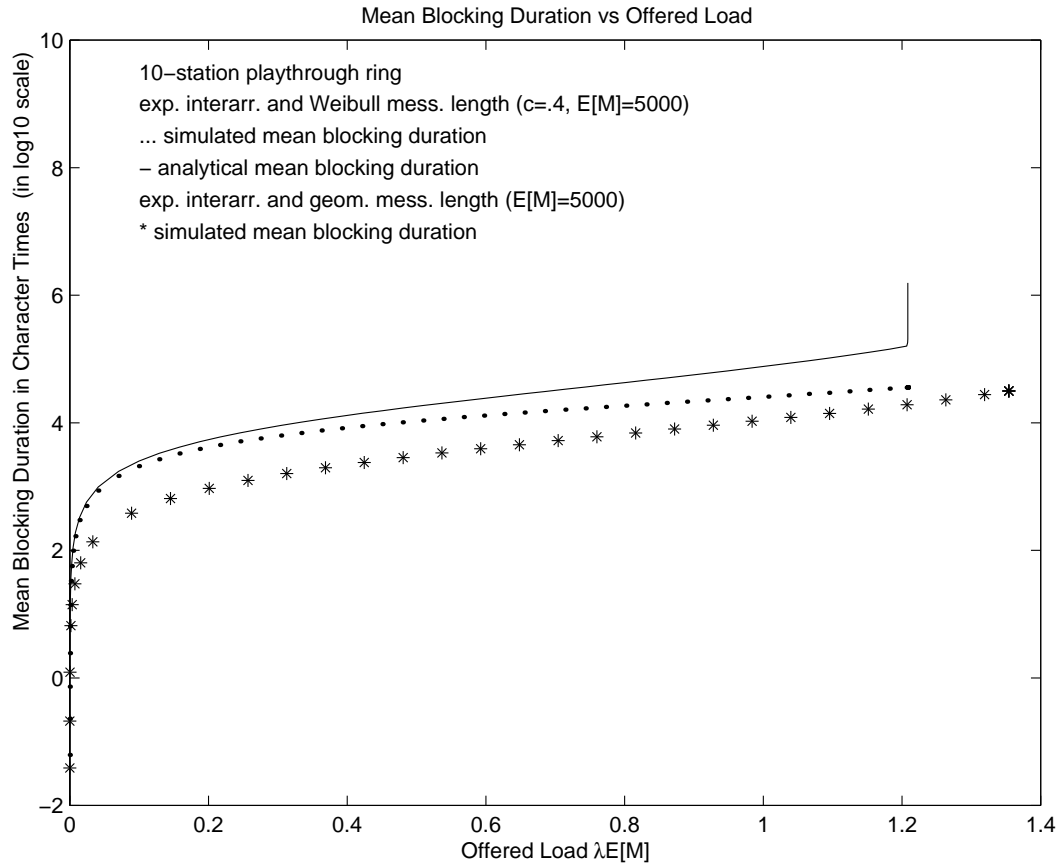


Figure 4.21: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 5000$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

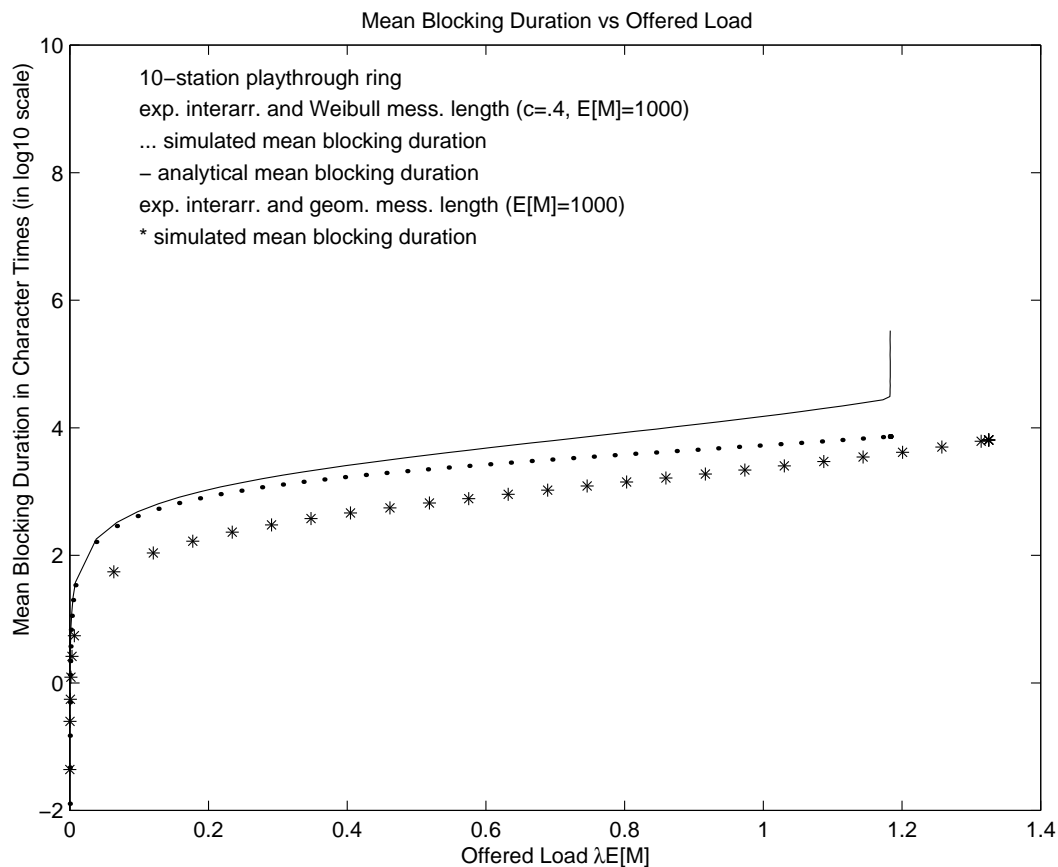


Figure 4.22: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 1000$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

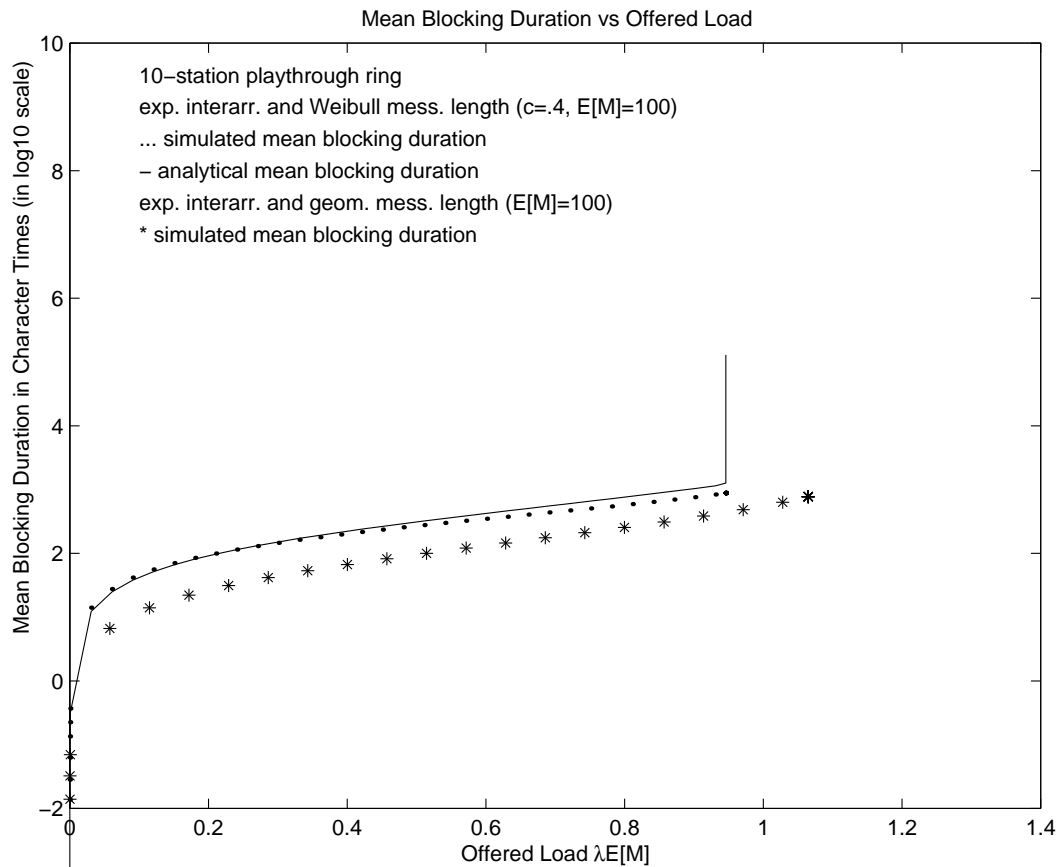


Figure 4.23: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 100$ characters, $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

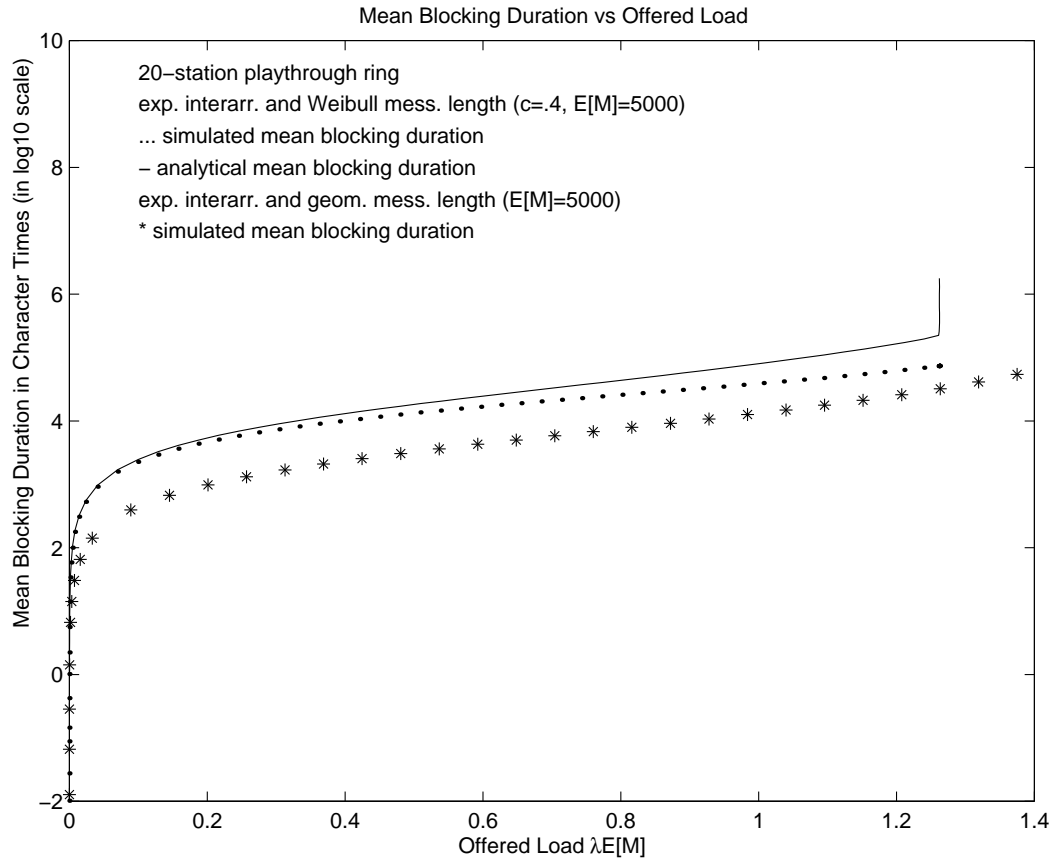


Figure 4.24: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 5000$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

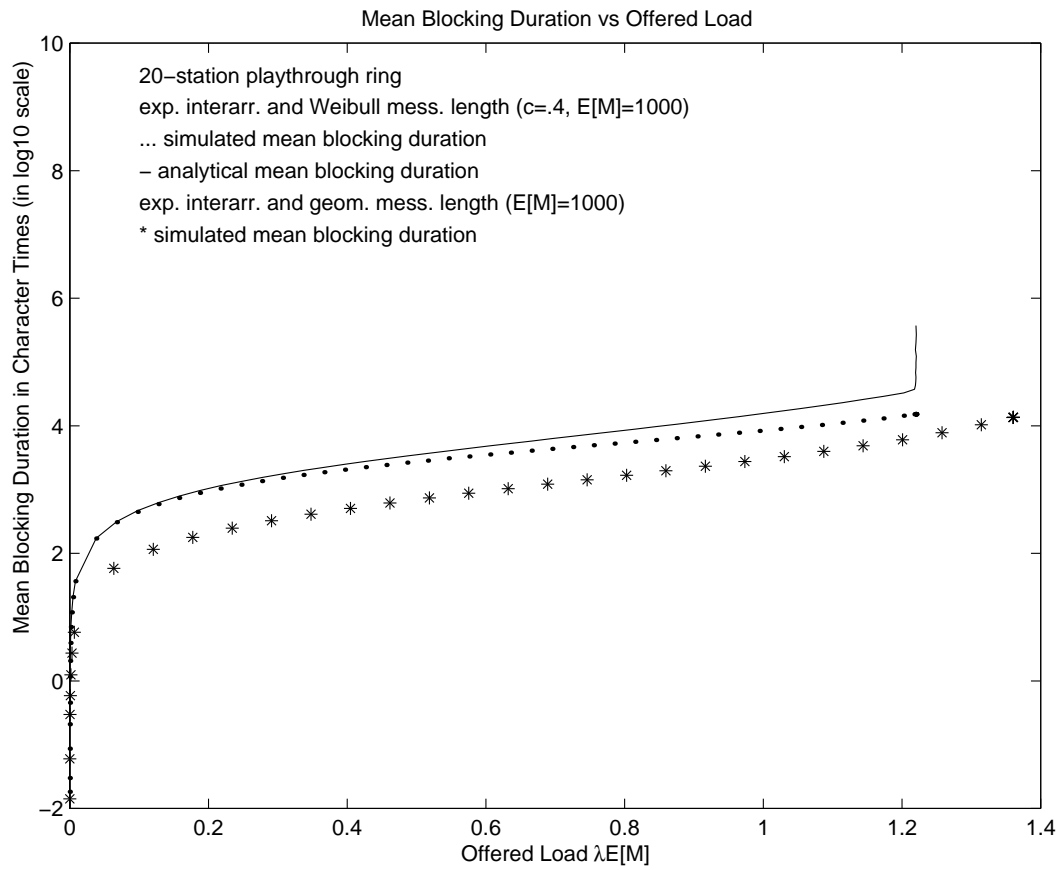


Figure 4.25: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 1000$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

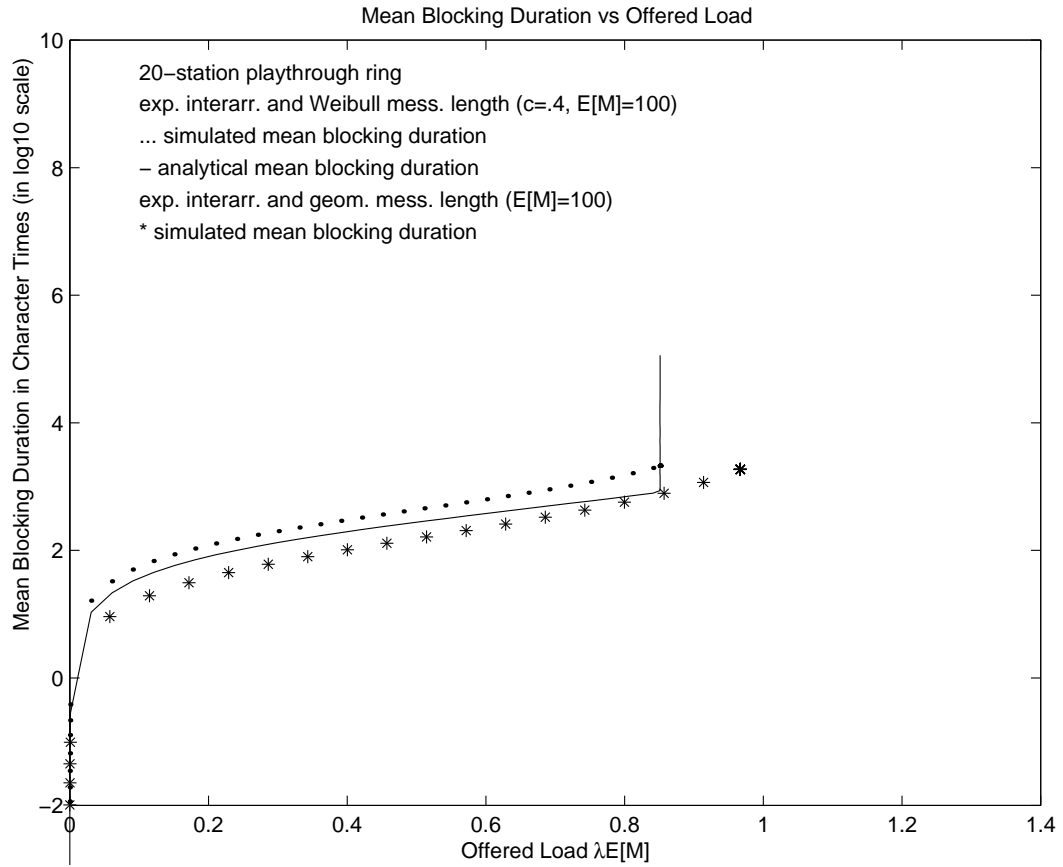


Figure 4.26: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 100$ characters, $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

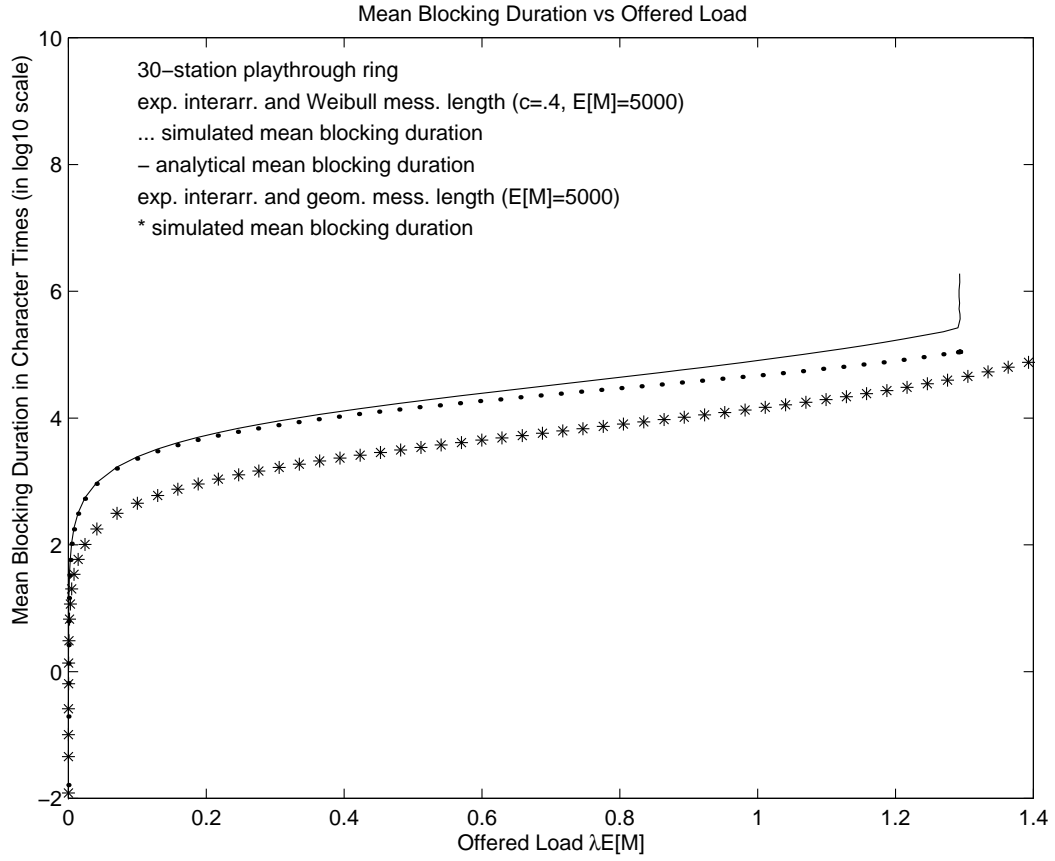


Figure 4.27: Mean blocking duration $E[BL]$ vs. offered load $\lambda E[M]$ for geometric and Weibull distributed message (msg.) lengths with parameter $c = 0.4$; mean msg. length $E[M] = 5000$ characters, $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential msg. interarrival times.

4.7 Waiting Time

Let us consider a PLAYTHROUGH ring consisting of N stations numbered $0, 1, \dots, N - 1$. Messages of outbound distance d arrive at station i according to independent Poisson arrival processes with rate $\lambda_{i,i\oplus d}$, $d \in \{1, 2, \dots, N - 1\}$ and the probability of k arrivals at station i is $p_i(k, t) = \frac{(\lambda_i t)^k}{k!} e^{-\lambda_i t}$. The total arrival rate at station i is $\lambda_i = \sum_{d=1}^{N-1} \lambda_{i,i\oplus d}$. The fraction of the total number of messages arriving at station i that have outbound distance d is $f_{i,i+d} = \frac{1}{N-1}$. Throughout this section, we designate by $S_{i,i\oplus d}$ the service time of an outbound

distance d message at station i 's HOL, and by $BL_{i,i\oplus d}$ the blocking duration experienced by station i 's HOL message of outbound distance d . Because the derivation of models in this section are made with respect to station i , we write $S_d \equiv S_{i,i\oplus d}$, $BL_d \equiv BL_{i,i\oplus d}$, and $f_d \equiv f_{i,i\oplus d}$.

We model the queue at station i using a two-state embedded Markov chain. Messages are assumed to arrive at station i using the FIFO queueing discipline.

The \mathcal{VS} state is the *Virtual Service* state, during which a head of the line (HOL) message at station i is possibly blocked and subsequently serviced. The \mathcal{G} state is the *Go* state, during which station i has no message at HOL and must wait for the arrival of GO . We recall that S_{GO} denotes the round trip time of GO . Let us designate by BL_d the blocking duration experienced by a HOL message of outbound distance d at station i and by S_d the service time of a HOL message of outbound distance d . The system remains in state \mathcal{VS} for $BL_d + S_d$ character times and in state \mathcal{G} for S_{GO} character times.

Let us designate by $\alpha_{k,d}$ the probability that k new messages arrive during a \mathcal{VS} epoch of duration $S_d + BL_d$ given a distance d HOL message, and by β_k the probability that k new messages arrive during a Go epoch of duration S_{GO} . We have:

$$\alpha_{k,d} = E [p_i(k, S_d + BL_d)]. \quad (4.103)$$

$$\beta_k = E [p_i(k, S_{GO})]. \quad (4.104)$$

The probability generating function $\hat{A}_d(z)$ of $\alpha_{k,d}$ is given by:

$$\begin{aligned}
\hat{A}_d(z) &= \sum_{k=0}^{\infty} z^k \alpha_{k,d} \\
&= \sum_{k=0}^{\infty} z^k E [p_i(k, S_d + BL_d)] \\
&= \sum_{k=0}^{\infty} z^k E \left[\frac{(\lambda_i(S_d + BL_d))^k}{k!} e^{-\lambda_i(S_d + BL_d)} \right] \\
&= E \left[e^{-\lambda_i(S_d + BL_d)} \sum_{k=0}^{\infty} \frac{(z \lambda_i(S_d + BL_d))^k}{k!} \right] \\
&= E [e^{-\lambda_i(1-z)(S_d + BL_d)}] \\
&= (S_d + BL_d)^* (\lambda_i(1 - z)); \tag{4.105}
\end{aligned}$$

where $(S_d + BL_d)^*$ is the Laplace-Stieljes transform of the *r.v.* $S_d + BL_d$ evaluated at $\lambda_i(1 - z)$. In a similar fashion the probability generating function $\hat{B}(z)$ of β_k is given by:

$$\hat{B}(z) = S_{GO}^*(\lambda_i(1 - z)). \tag{4.106}$$

Averaging the generating functions $\hat{A}_d(z)$ across outbound distance d , we obtain:

$$\hat{A}(z) = \sum_{d=1}^{N-1} f_d \hat{A}_d(z). \tag{4.107}$$

$\hat{A}_d(z)$ and $\hat{B}(z)$ have the following derivatives.

$$\begin{aligned}
\hat{A}_d'(z) &= -\lambda_i(BL_d + S_d)^{*'}(\lambda_i(1 - z)) & \hat{A}_d'(1) &= \lambda_i E [(BL_d + S_d)]. \\
\hat{A}_d''(z) &= \lambda_i^2(BL_d + S_d)^{**}(\lambda_i(1 - z)) & \hat{A}_d''(1) &= \lambda_i^2 E [(BL_d + S_d)^2]. \\
\hat{A}_d'''(z) &= -\lambda_i^3(BL_d + S_d)^{***}(\lambda_i(1 - z)) & \hat{A}_d'''(1) &= \lambda_i^3 E [(BL_d + S_d)^3]. \\
\hat{B}'(z) &= -\lambda_i S_{GO}^{*'}(\lambda_i(1 - z)) & \hat{B}'(1) &= \lambda_i E [S_{GO}]. \\
\hat{B}''(z) &= \lambda_i^2 S_{GO}^{**}(\lambda_i(1 - z)) & \hat{B}''(1) &= \lambda_i^2 E [(S_{GO})^2]. \\
\hat{B}'''(z) &= -\lambda_i^3 S_{GO}^{***}(\lambda_i(1 - z)) & \hat{B}'''(1) &= \lambda_i^3 E [(S_{GO})^3].
\end{aligned} \tag{4.108}$$

Let j_n be the number of customers in the system at the *beginning* of the n^{th} epoch and let j_{n+1} be the number of customers in the system at the *end* of the n^{th} epoch. Let $\pi_{j,d}^{\mathcal{VS}}$ designate the steady-state probability of a *Virtual Service* epoch with j customers at the beginning of the epoch, where d is the outbound distance of station i 's HOL message. Moreover, let $\pi_j^{\mathcal{G}}$ be the steady-state probability of a *Go* epoch. The *Virtual Service* epoch cannot start without a HOL message. Therefore, $\pi_{0,d}^{\mathcal{VS}} = 0$. In addition, $\pi_j^{\mathcal{G}} = 0$ for $j \geq 1$, since the presence of any HOL message must initiate a *Virtual Service* epoch. The probability density π_j of being in one of the states $\{\mathcal{G}, \mathcal{VS}\}$ with j customers in queue is:

$$\pi_j = \pi_j^{\mathcal{VS}} + \pi_j^{\mathcal{G}}. \quad (4.109)$$

Because the total probability of being in one of the valid states must be equal to one, we have:

$$\sum_{j=0}^{\infty} \pi_j = \sum_{j=0}^{\infty} (\pi_j^{\mathcal{VS}} + \pi_j^{\mathcal{G}}) = 1. \quad (4.110)$$

There are two ways a *Go* epoch can be started.

1. if no new message have arrived at the end of a *Go* epoch.
2. if the last message of outbound distance l in the queue was served during the prior *Virtual Service* epoch, and no new messages have arrived, which occurs with probability $\alpha_{0,l}$.

Thus,

$$\pi_0^{\mathcal{G}} = \sum_{\ell=1}^{N-1} (\alpha_{0,\ell} \pi_{1,\ell}^{\mathcal{VS}}) + \beta_0 \pi_0^{\mathcal{G}}. \quad (4.111)$$

A *Virtual Service* epoch that starts with $j \geq 1$ messages in queue and a HOL message of outbound distance d may be entered in one of two ways:

1. the prior epoch was a *Go* epoch (which occurs with probability π_0^G), j new arrivals have entered station i 's queue in time S_{GO} (which occurs with probability β_j), the new HOL message has outbound distance d (which occurs with probability f_d);
2. the prior epoch was a *Virtual Service* epoch with k messages in queue ($1 \leq k \leq j + 1$) and a message of outbound distance ℓ at HOL (which occurs with probability $\pi_{k,\ell}^{VS}$), the previous HOL message has departed and $j - k + 1$ new arrivals have entered station i 's queue in time $BL_\ell + S_\ell$ (which occurs with probability $\alpha_{j-k+1,\ell}$).

At the end of a *Virtual Service* or *Go* epoch with a non-zero number of customers in queue, the outbound distance of the HOL message takes the value d with probability f_d . Hence,

$$\begin{aligned}
\pi_{j,d}^{VS} &= f_d \beta_j \pi_0^G \\
&\quad + \sum_{k=1}^{j+1} f_d \sum_{\ell=1}^{N-1} (\alpha_{j-k+1,\ell} \pi_{k,\ell}^{VS}) \\
&= f_d \beta_j \pi_0^G \\
&\quad + f_d \sum_{k=1}^{j+1} \sum_{\ell=1}^{N-1} (\alpha_{j-k+1,\ell} \pi_{k,\ell}^{VS}). \tag{4.112}
\end{aligned}$$

The probability generating function for $\pi_{j,d}^{VS}$ is given by:

$$\begin{aligned}
\hat{\pi}_d^{VS}(z) &= \sum_{j=1}^{\infty} \pi_{j,d}^{VS} z^j \\
&= \sum_{j=1}^{\infty} f_d \beta_j \pi_0^G z^j \\
&\quad + \sum_{j=1}^{\infty} f_d \sum_{k=1}^{j+1} \sum_{\ell=1}^{N-1} \alpha_{j-k+1,\ell} \pi_{k,\ell}^{VS} z^j
\end{aligned}$$

Using results from Section A.3 of Appendix A, the probability generating

function for $\pi_{j,d}^{\mathcal{V}\mathcal{S}}$ is computed:

$$\begin{aligned}
\hat{\pi}_d^{\mathcal{V}\mathcal{S}}(z) &= \sum_{j=1}^{\infty} \pi_{j,d}^{\mathcal{V}\mathcal{S}} z^j \\
&= \sum_{j=1}^{\infty} f_d \beta_j \pi_0^{\mathcal{G}} z^j \\
&\quad + \sum_{j=1}^{\infty} f_d \sum_{k=1}^{j+1} \sum_{\ell=1}^{N-1} \alpha_{j-k+1,\ell} \pi_{k,\ell}^{\mathcal{V}\mathcal{S}} z^j \\
\hat{\pi}_d^{\mathcal{V}\mathcal{S}}(z) &= f_d \left[\hat{B}(z) - \beta_0 \right] \pi_0^{\mathcal{G}} \\
&\quad + f_d \sum_{\ell=1}^{N-1} \left\{ z^{-1} \hat{A}_\ell(z) \hat{\pi}_\ell^{\mathcal{V}\mathcal{S}}(z) - \alpha_{0,\ell} \pi_{1,\ell}^{\mathcal{V}\mathcal{S}} \right\} \\
&= f_d \hat{B}(z) \pi_0^{\mathcal{G}} + f_d z^{-1} \sum_{\ell=1}^{N-1} \hat{A}_\ell(z) \hat{\pi}_\ell^{\mathcal{V}\mathcal{S}}(z) \\
&\quad - f_d \left[\beta_0 \pi_0^{\mathcal{G}} + \sum_{\ell=1}^{N-1} \alpha_{0,\ell} \pi_{1,\ell}^{\mathcal{V}\mathcal{S}} \right] \\
&= f_d z^{-1} \sum_{\ell=1}^{N-1} \hat{A}_\ell(z) \hat{\pi}_\ell^{\mathcal{V}\mathcal{S}}(z) \\
&\quad + f_d \left[\hat{B}(z) - 1 \right] \pi_0^{\mathcal{G}}; \tag{4.113}
\end{aligned}$$

Let:

$$\hat{J}(z) = \sum_{\ell=1}^{N-1} \hat{A}_\ell(z) \pi_\ell^{\mathcal{V}\mathcal{S}}(z). \tag{4.114}$$

$$\hat{G}(z) = z^{-1} \hat{J}(z) + \left[\hat{B}(z) - 1 \right] \pi_0^{\mathcal{G}}. \tag{4.115}$$

We have:

$$\hat{\pi}_d^{\mathcal{V}\mathcal{S}}(z) = \hat{R}_d(z) \hat{G}(z); \tag{4.116}$$

where

$$\hat{R}_d(z) = f_d. \tag{4.117}$$

Let:

$$\hat{H}(z) = \sum_{d=1}^{N-1} \hat{A}_d(z) \hat{R}_d(z). \tag{4.118}$$

We have:

$$\begin{aligned}
\hat{J}(z) &= \sum_{\ell=1}^{N-1} \hat{A}_\ell(z) \hat{\pi}_\ell^{\mathcal{V}^S}(z) = \sum_{d=1}^{N-1} \hat{A}_d(z) \hat{\pi}_d^{\mathcal{V}^S}(z) \\
&= \sum_{d=1}^{N-1} \hat{A}_d(z) \hat{R}_d(z) \hat{G}(z) \\
&= \hat{H}(z) \hat{G}(z) \\
&= \hat{H}(z) \left\{ z^{-1} \hat{J}(z) + [\hat{B}(z) - 1] \pi_0^{\mathcal{G}} \right\}. \tag{4.119}
\end{aligned}$$

$$\hat{J}(z) [1 - z^{-1} \hat{H}(z)] = \hat{H}(z) [\hat{B}(z) - 1] \pi_0^{\mathcal{G}}. \tag{4.120}$$

$$\hat{J}(z) = \frac{\hat{H}(z) [\hat{B}(z) - 1]}{[1 - z^{-1} \hat{H}(z)]} \pi_0^{\mathcal{G}}. \tag{4.121}$$

$$\begin{aligned}
\hat{G}(z) &= z^{-1} \hat{J}(z) + [\hat{B}(z) - 1] \pi_0^{\mathcal{G}} \\
&= z^{-1} \frac{\hat{H}(z) [\hat{B}(z) - 1]}{1 - z^{-1} \hat{H}(z)} \pi_0^{\mathcal{G}} + [\hat{B}(z) - 1] \pi_0^{\mathcal{G}} \\
&= \left[\frac{z^{-1} \hat{H}(z)}{1 - z^{-1} \hat{H}(z)} + 1 \right] [\hat{B}(z) - 1] \pi_0^{\mathcal{G}} \\
&= \frac{\hat{B}(z) - 1}{1 - z^{-1} \hat{H}(z)} \pi_0^{\mathcal{G}}. \tag{4.122}
\end{aligned}$$

We now express the probability generating function for the probability density π_j found in Equation 4.109 in terms of a new auxiliary variable $\hat{L}(z)$.

$$\begin{aligned}
\hat{\pi}(z) &= \sum_{d=1}^{N-1} [\hat{\pi}_d^{\mathcal{V}^S}(z)] + \pi_0^{\mathcal{G}} \\
&= \sum_{d=1}^{N-1} [\hat{R}_d(z)] \hat{G}(z) + \pi_0^{\mathcal{G}}. \tag{4.123}
\end{aligned}$$

Let:

$$\hat{L}(z) = \sum_{d=1}^{N-1} [\hat{R}_d(z)]. \tag{4.124}$$

$$\begin{aligned}
\hat{\pi}(z) &= \hat{L}(z)\hat{G}(z) + \pi_0^g \\
&= \hat{L}(z)\frac{\hat{B}(z) - 1}{1 - z^{-1}\hat{H}(z)}\pi_0^g + \pi_0^g \\
&= \left\{ \hat{L}(z)\frac{\hat{B}(z) - 1}{1 - z^{-1}\hat{H}(z)} + 1 \right\} \pi_0^g.
\end{aligned} \tag{4.125}$$

$\hat{\pi}(z)$ is the transform of a probability density. Therefore, we must have:

$$\lim_{z \rightarrow 1} \hat{\pi}(z) = 1.$$

$$\begin{aligned}
\lim_{z \rightarrow 1} \hat{\pi}(z) &= \lim_{z \rightarrow 1} \pi_0^g \left\{ 1 + \hat{L}(z)\frac{\hat{B}(z) - 1}{1 - z^{-1}\hat{H}(z)} \right\} \\
&= \pi_0^g \left\{ 1 + \hat{L}(1) \lim_{z \rightarrow 1} \frac{\hat{B}(z) - 1}{1 - z^{-1}\hat{H}(z)} \right\}.
\end{aligned} \tag{4.126}$$

$$\lim_{z \rightarrow 1} \frac{\hat{B}(z) - 1}{1 - z^{-1}\hat{H}(z)} = \frac{1 - 1}{1 - 1} \rightarrow \frac{0}{0}. \tag{4.127}$$

Using L'Hospital's rule, we have:

$$\begin{aligned}
\lim_{z \rightarrow 1} \hat{\pi}(z) &= \lim_{z \rightarrow 1} \pi_0^g \left\{ 1 + \hat{L}(z)\frac{\hat{B}(z) - 1}{1 - z^{-1}\hat{H}(z)} \right\} \\
&= \pi_0^g \left\{ 1 + \hat{L}(1) \lim_{z \rightarrow 1} \frac{\hat{B}(z) - 1}{1 - z^{-1}\hat{H}(z)} \right\} \\
&\stackrel{\text{L'H}}{=} \pi_0^g \left\{ 1 + \hat{L}(1) \lim_{z \rightarrow 1} \frac{\hat{C}'(z)}{-z^{-1}\hat{H}'(z) + z^{-2}\hat{H}(z)} \right\} \\
&= \pi_0^g \left\{ 1 + \frac{\hat{L}(1)\hat{C}'(1)}{\hat{H}(1) - \hat{H}'(1)} \right\} = 1.
\end{aligned} \tag{4.128}$$

Hence, the steady state probability π_0^g of being in the *Go* state is

$$\pi_0^g = \frac{1 - \hat{H}'(1)}{1 - \hat{H}'(1) + \hat{L}(1)\hat{C}'(1)}; \tag{4.129}$$

where (from Appendix B)

$$\hat{H}'(1) = \sum_{d=1}^{N-1} \hat{A}_d(1)\hat{R}_d(1); \tag{4.130}$$

and

$$\hat{L}(1) = \sum_{d=1}^{N-1} \left[\hat{R}_d(1) \right]. \quad (4.131)$$

Let us designate by $\pi_{j,d}^{SE}$ the conditional probability of the event that j messages are in queue at the *end (completion)* of an epoch e , given that e was a *Virtual Service* epoch and the served message had outbound distance d . Furthermore, let $\pi_{k,d}^{SI}$ designate the conditional probability of the event that k messages are in queue at the *start (initiation)* of an epoch e , given that the epoch e is a *Virtual Service* epoch and the HOL message has outbound distance d . We have:

$$\pi_{j,d}^{SE} = \sum_{k=1}^{j+1} \pi_{k,d}^{SI} \alpha_{j-k+1,d}; \quad (4.132)$$

where $\alpha_{j-k+1,d}$ is the probability of the event that $j - k + 1$ new messages arrive in the queue during the virtual service of a message of outbound distance d . $\pi_{j,d}^{SI}$ is the steady-state probability of the event that j messages are in queue at the initiation of an epoch *and* the current epoch is a *Virtual Service* epoch *and* the HOL message has outbound distance d . Hence,

$$\pi_{k,d}^{SI} = \frac{\pi_{k,d}^{\mathcal{V}S}}{\sum_{\ell=1}^{\infty} \pi_{\ell,d}^{\mathcal{S}}} = \frac{\pi_{k,d}^{\mathcal{V}S}}{\hat{\pi}_d^{\mathcal{V}S}(1)}. \quad (4.133)$$

From Equation 4.132 and Equation 4.133, we have:

$$\pi_{j,d}^{SE} = \sum_{k=1}^{j+1} \left[\frac{\pi_{k,d}^{\mathcal{V}S}}{\hat{\pi}_d^{\mathcal{V}S}(1)} \right] \alpha_{j-k+1,d}; \quad (4.134)$$

and the probability generating function of $\pi_{j,d}^{SE}$ is given by:

$$\begin{aligned} \hat{\pi}_d^{SE}(z) &= \sum_{j=0}^{\infty} \pi_{j,d}^{SE} z^j \\ &= \sum_{j=0}^{\infty} \sum_{k=1}^{j+1} \left[\frac{\pi_{k,d}^{\mathcal{V}S}}{\hat{\pi}_d^{\mathcal{V}S}(1)} \right] \alpha_{j-k+1,d} z^j \\ &= \left[\frac{1}{\hat{\pi}_d^{\mathcal{V}S}(1)} \right] z^{-1} \hat{A}_d(z) \hat{\pi}_d^{\mathcal{V}S}(z). \end{aligned} \quad (4.135)$$

Let Q_d^{SE} be the number of messages in queue at the service completion of a message of outbound distance d . The mean or first moment of Q_d^{SE} is given by:

$$\begin{aligned} E [Q_d^{SE}] &= \sum_{j=0}^{\infty} j \pi_{j,d}^{SE} \\ &= \hat{\pi}_d^{SE'}(1). \end{aligned} \quad (4.136)$$

$$\begin{aligned} \hat{\pi}_d^{SE'}(z) &= \left[\frac{1}{\hat{\pi}_d^{\mathcal{V}S}(1)} \right] \left\{ z^{-1} \left[\hat{A}_d(z) \hat{\pi}_d^{\mathcal{V}S'}(z) + \hat{A}'_d(z) \hat{\pi}_d^{\mathcal{V}S}(z) \right] \right. \\ &\quad \left. - z^{-2} \hat{A}_d(z) \hat{\pi}_d^{\mathcal{V}S}(z) \right\}. \end{aligned} \quad (4.137)$$

$$\begin{aligned} \hat{\pi}_d^{SE'}(1) &= \left[\frac{1}{\hat{\pi}_d^{\mathcal{V}S}(1)} \right] \left\{ \hat{\pi}_d^{\mathcal{V}S'}(1) + \hat{A}'_d(1) \hat{\pi}_d^{\mathcal{V}S}(1) - \hat{\pi}_d^{\mathcal{V}S}(1) \right\} \\ &= \left[\frac{\hat{\pi}_d^{\mathcal{V}S'}(1)}{\hat{\pi}_d^{\mathcal{V}S}(1)} \right] + \hat{A}'_d(1) - 1. \end{aligned} \quad (4.138)$$

Let us designate by $S_d^{\mathcal{V}S}$ the virtual service time of a message of outbound distance d and by $W_d^{\mathcal{V}S}$ the duration between the time a message arrives in queue and the time it enters into virtual service. From Little's result [Kle75],

$$E [Q_d^{SE}] = \lambda_i (E [W_d^{\mathcal{V}S}] + E [S_d^{\mathcal{V}S}]); \quad (4.139)$$

Solving Equation 4.139 for $E[W_d^{\mathcal{V}S}]$, we have:

$$\begin{aligned} E [W_d^{\mathcal{V}S}] &= \frac{E [Q_d^{SE}] - \lambda_i E [S_d^{\mathcal{V}S}]}{\lambda_i} \\ &= \frac{1}{\lambda_i} \left[\hat{\pi}_d^{SE'}(1) - \hat{A}'_d(1) \right] \\ &= \frac{1}{\lambda_i} \left[\frac{\hat{\pi}_d^{\mathcal{V}S'}(1)}{\hat{\pi}_d^{\mathcal{V}S}(1)} + \hat{A}'_d(1) - 1 - \hat{A}'_d(1) \right] \\ &= \frac{1}{\lambda_i} \left[\frac{\hat{\pi}_d^{\mathcal{V}S'}(1)}{\hat{\pi}_d^{\mathcal{V}S}(1)} - 1 \right] \end{aligned} \quad (4.140)$$

From Equation 4.116, Equation 4.117, and Equation 4.122

$$\begin{aligned} \hat{\pi}_d^{\mathcal{V}S}(z) &= \hat{R}_d(z) \hat{G}(z) \\ &= (f_d) \left(\frac{\hat{B}(z) - 1}{1 - z^{-1} \hat{H}(z)} \right) \pi_0^{\mathcal{G}} \\ &= \pi_0^{\mathcal{G}} \left(\frac{\hat{X}(z)}{\hat{Y}(z)} \right); \end{aligned} \quad (4.141)$$

where we define the auxiliary variables $\hat{X}(z)$ and $\hat{Y}(z)$ as:

$$\hat{X}(z) = (f_d)(\hat{B}(z) - 1). \quad (4.142)$$

$$\hat{Y}(z) = 1 - z^{-1}\hat{H}(z). \quad (4.143)$$

The derivatives associated with the auxiliary variables $\hat{X}(z)$ and $\hat{Y}(z)$ can be found in Appendix B. We have: $\hat{X}(1) = \hat{Y}(1) = 0$. Therefore,

$$\hat{\pi}_d^{\mathcal{V}\mathcal{S}}(1) = \lim_{z \rightarrow 1} \pi_0^{\mathcal{S}} \frac{\hat{X}(z)}{\hat{Y}(z)} \rightarrow \frac{0}{0}, \quad (4.144)$$

and the use of L'Hospital's rule is required to find $\hat{\pi}_d^{\mathcal{V}\mathcal{S}}(1)$.

$$\begin{aligned} \hat{\pi}_d^{\mathcal{V}\mathcal{S}}(1) &= \lim_{z \rightarrow 1} \pi_0^{\mathcal{S}} \frac{\hat{X}(z)}{\hat{Y}(z)} \\ &\stackrel{\text{L'H}}{=} \lim_{z \rightarrow 1} \pi_0^{\mathcal{S}} \frac{\hat{X}'(z)}{\hat{Y}'(z)} \\ &= \pi_0^{\mathcal{S}} \frac{\hat{X}'(1)}{\hat{Y}'(1)}. \end{aligned} \quad (4.145)$$

$$\hat{\pi}_d^{\mathcal{V}\mathcal{S}'}(z) = \pi_0^{\mathcal{S}} \frac{\hat{X}'(z)\hat{Y}(z) - \hat{X}(z)\hat{Y}'(z)}{[\hat{Y}(z)]^2}. \quad (4.146)$$

We have: $\lim_{z \rightarrow 1} \hat{\pi}_d^{\mathcal{V}\mathcal{S}'}(z) \rightarrow \frac{0}{0}$. Therefore, L'Hospital's rule must be applied once again.

$$\begin{aligned} \hat{\pi}_d^{\mathcal{V}\mathcal{S}'}(1) &= \lim_{z \rightarrow 1} \pi_0^{\mathcal{S}} \frac{\hat{X}'(z)\hat{Y}(z) - \hat{X}(z)\hat{Y}'(z)}{[\hat{Y}(z)]^2} \\ &\stackrel{\text{L'H}}{=} \pi_0^{\mathcal{S}} \lim_{z \rightarrow 1} \frac{\hat{X}''(z)\hat{Y}(z) - \hat{X}(z)\hat{Y}''(z)}{2\hat{Y}(z)\hat{Y}'(z)} \\ &\stackrel{\text{L'H}}{=} \pi_0^{\mathcal{S}} \lim_{z \rightarrow 1} \frac{\hat{X}'''(z)\hat{Y}(z) + \hat{X}''(z)\hat{Y}'(z) - \hat{X}'(z)\hat{Y}''(z) - \hat{X}(z)\hat{Y}'''(z)}{2\hat{Y}(z)\hat{Y}''(z) + 2[\hat{Y}'(z)]^2} \\ &= \pi_0^{\mathcal{S}} \frac{\hat{X}''(1)\hat{Y}'(1) - \hat{X}'(1)\hat{Y}''(1)}{2[\hat{Y}'(1)]^2}. \end{aligned} \quad (4.147)$$

From Equation 4.140,

$$\begin{aligned}
\lambda_i E [W_d^{\mathcal{V}S}] &= \frac{\hat{\pi}_d^{\mathcal{V}S'}(1)}{\hat{\pi}_d^{\mathcal{V}S}(1)} - 1 \\
&= \pi_0^g \left(\frac{\hat{X}''(1)\hat{Y}'(1) - \hat{X}'(1)\hat{Y}''(1)}{2[\hat{Y}'(1)]^2} \right) \left(\pi_0^g \frac{\hat{X}'(1)}{\hat{Y}'(1)} \right)^{-1} - 1 \\
&= \left(\frac{\hat{X}''(1)\hat{Y}'(1) - \hat{X}'(1)\hat{Y}''(1)}{2\hat{X}'(1)\hat{Y}'(1)} \right) - 1 \\
&= \frac{[\hat{X}''(1) - 2\hat{X}'(1)]\hat{Y}'(1) - \hat{X}'(1)\hat{Y}''(1)}{2\hat{X}'(1)\hat{Y}'(1)} \\
&= \left(\frac{\hat{X}''(1)}{2\hat{X}'(1)} - 1 \right) - \frac{\hat{Y}''(1)}{2\hat{Y}'(1)}. \tag{4.148}
\end{aligned}$$

From Appendix B,

$$\begin{aligned}
\frac{\hat{X}''(1)}{2\hat{X}'(1)} &= \frac{f_d \{ \hat{C}''(1) \}}{2f_d \hat{C}'(1)} \\
&= \frac{\hat{C}''(1)}{2\hat{C}'(1)}; \tag{4.149}
\end{aligned}$$

and

$$\begin{aligned}
\frac{-\hat{Y}''(1)}{2\hat{Y}'(1)} &= \frac{\hat{H}''(1) - 2\hat{H}'(1) + 2}{2[1 - \hat{H}'(1)]} \\
&= \frac{\hat{H}''(1) + 2[1 - \hat{H}'(1)]}{2[1 - \hat{H}'(1)]} \\
&= \frac{\hat{H}''(1)}{2[1 - \hat{H}'(1)]} + 1. \tag{4.150}
\end{aligned}$$

We now have:

$$\begin{aligned}
\lambda_i E [W_d^{\nu S}] &= \left(\frac{\hat{X}''(1)}{2\hat{X}'(1)} - 1 \right) - \frac{\hat{Y}''(1)}{2\hat{Y}'(1)} \\
&= \left(\frac{\hat{X}''(1)}{2\hat{X}'(1)} - 1 \right) + \frac{\hat{H}''(1)}{2[1 - \hat{H}'(1)]} + 1 \\
&= \frac{\hat{B}''(1)}{2\hat{B}'(1)} + \frac{\hat{H}''(1)}{2[1 - \hat{H}'(1)]}, \tag{4.151}
\end{aligned}$$

where (from Appendix B)

$$\hat{H}'(1) = \hat{A}'(1) \tag{4.152}$$

$$\hat{H}''(1) = \hat{A}''(1). \tag{4.153}$$

We recall that:

$$\hat{A}'(1) = \lambda_i E [BL + S]$$

$$\hat{A}''(1) = \lambda_i^2 E [(BL + S)^2]$$

$$\hat{B}'(1) = \lambda_i E [S_{GO}]$$

$$\hat{B}''(1) = \lambda_i^2 E [S_{GO}^2].$$

Therefore,

$$\lambda_i E [W_d^{\nu S}] = \frac{\lambda_i E [S_{GO}]}{2E [S_{GO}^2]} + \frac{\lambda_i^2 E [(BL + S)^2]}{2[1 - \lambda_i E [BL + S]]};$$

and,

$$E [W_d^{\nu S}] = \frac{E [S_{GO}]}{2E [S_{GO}^2]} + \frac{\lambda_i E [(BL + S)^2]}{2[1 - \lambda_i E [BL + S]]}. \tag{4.154}$$

$W_d^{\nu S}$ is the duration between the time a message of outbound distance d arrives at station i 's queue and the time it possibly experiences blocking at HOL by the transmission of another message. The actual waiting time of a message of outbound distance d in station i 's queue is the duration between the time it arrives at station i 's queue and the time it goes into service after experiencing

blocking. Let W_d denote the actual waiting time of a message of outbound distance d at station i 's queue. We have:

$$W_d = W_d^{\nu S} + BL_d. \quad (4.155)$$

So,

$$E[W_d] = E[W_d^{\nu S}] + E[BL_d]. \quad (4.156)$$

Hence,

$$E[W_d] = \frac{E[S_{GO}]}{2E[S_{GO}^2]} + E[BL_d] + \frac{\lambda_i E[(BL + S)^2]}{2[1 - \lambda_i E[BL + S]]}. \quad (4.157)$$

The average waiting time $E[W]$ for a message of arbitrary outbound distance is obtained by averaging $E[W_d]$ over the outbound distance d . Therefore,

$$\begin{aligned} E[W] &= \sum_{d=1}^{N-1} f_d E[W_d] \\ &= \frac{E[S_{GO}]}{2E[S_{GO}^2]} + \sum_{d=1}^{N-1} f_d E[BL_d] + \frac{\lambda_i E[(BL + S)^2]}{2[1 - \lambda_i E[BL + S]]} \\ E[W] &= \frac{E[S_{GO}]}{2E[S_{GO}^2]} + E[BL] + \frac{\lambda_i E[(BL + S)^2]}{2[1 - \lambda_i E[BL + S]]} \end{aligned} \quad (4.158)$$

In Equation 4.158, the first term is the mean residual round trip time of GO . The second term is the mean blocking duration experienced by a message of arbitrary outbound distance at HOL. The third term is the expression of waiting time in an M/G/1 queue with arrival rate λ_i and service time $BL + S$.

Figures 4.28–4.42 show the analytical results for mean waiting time in Equation 4.158 along with simulated results plotted versus the load $\lambda E[M]$. Each simulated data point is the average $E[W_{sim}]$ of $n = 1,000,000$ simulated samples W_{sim} corresponding to a load $\lambda E[M]$ applied to the ring. The 95% confidence interval shown in Figures 4.28–4.42 is calculated as:

$$E[W_{sim}] \pm 1.96(SD[W_{sim}]/\sqrt{n}) \text{ [Kob78]}, \quad (4.159)$$

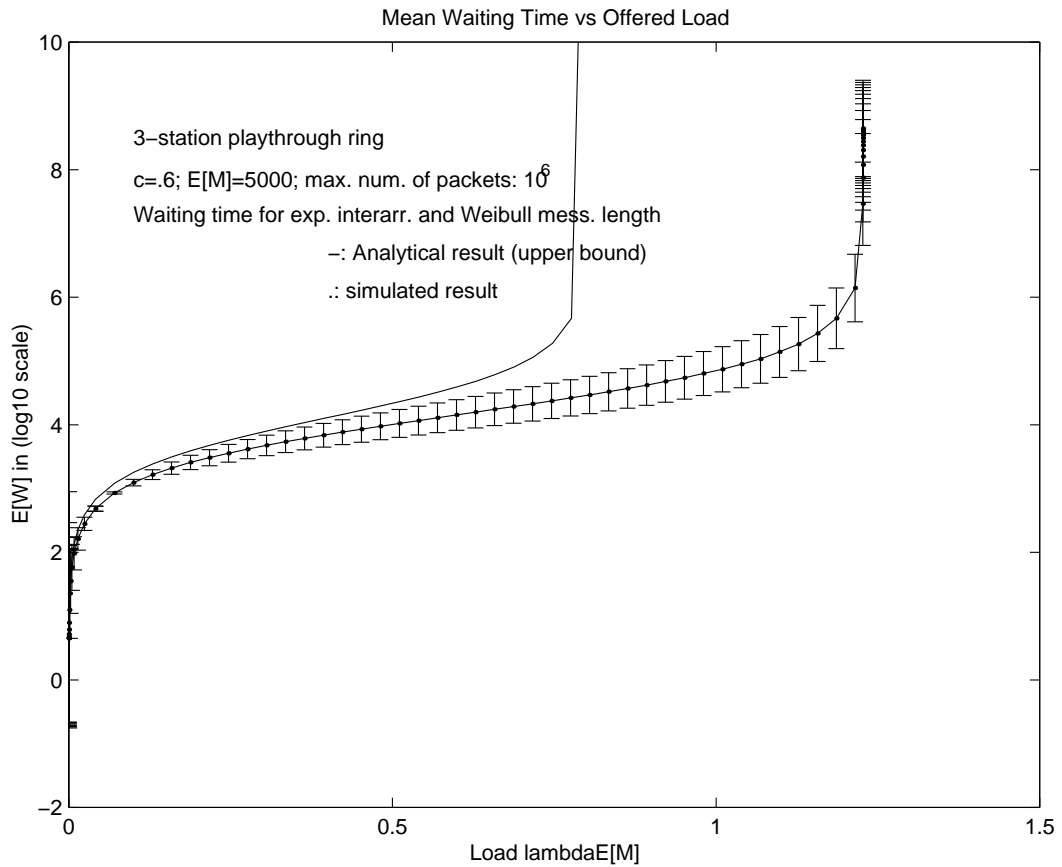


Figure 4.28: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 5000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

where $SD[W_{sim}]$ is the standard deviation of the samples W_{sim} from their mean $E[W_{sim}]$. Figure 4.28 shows the analytical and simulation results of average waiting time on a 3-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 5000. The analytical model overestimates the simulated mean waiting time. Except for very light loads, the analytical model is outside the confidence interval. The analytical model saturates at $\lambda E[M] = 0.785$, whereas the simulation results saturate at $\lambda E[M] = 1.265$.

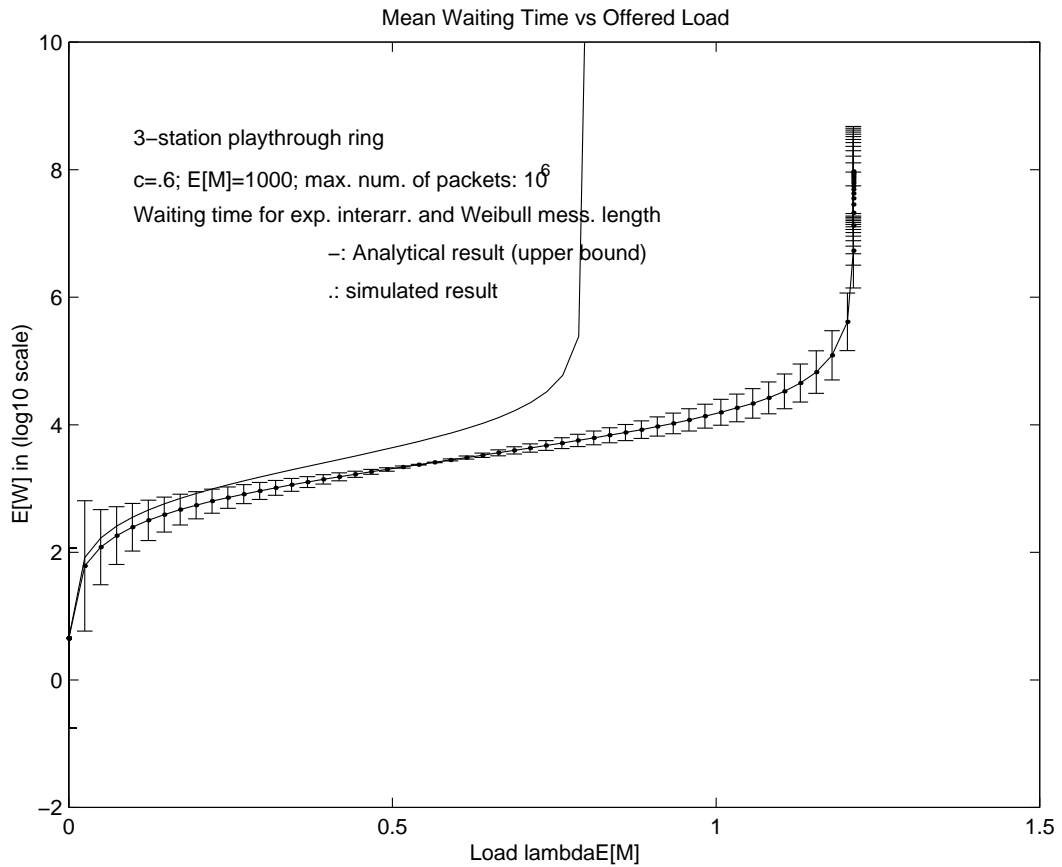


Figure 4.29: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 1000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.29 shows the analytical results of average waiting time on a 3-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 1000. The simulation results are included on the same plot. The plot shows that the analytical model is in good agreement with the simulations at light load but overpredicts the simulated waiting time for the rest of the load range. The analytical model saturates at $\lambda E[M] = 0.812$, whereas the simulation results saturate at $\lambda E[M] = 1.250$.

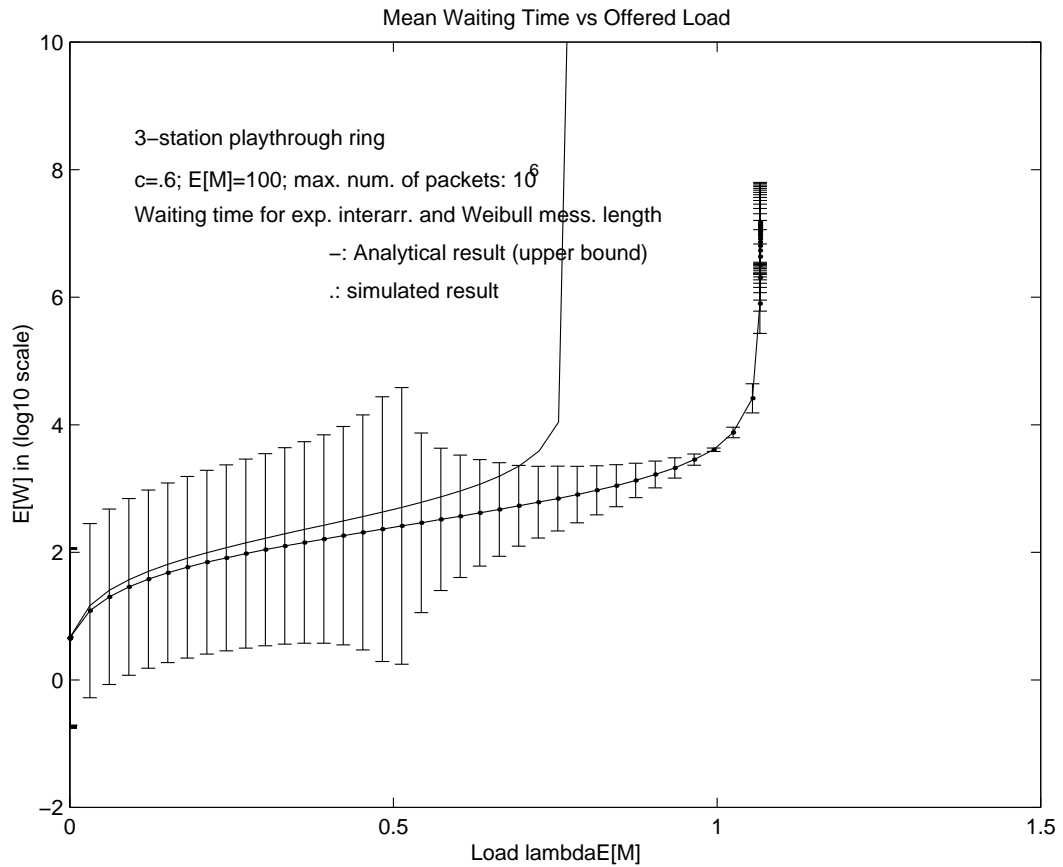


Figure 4.30: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 100$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.30 shows the analytical results of average waiting time on a 3-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 100. The simulation results are included on the same plot. The plot shows that the analytical model is in good agreement with the simulations at light and moderate loads. The analytical model saturates at $\lambda E[M] = 0.785$, whereas the simulation results saturate at $\lambda E[M] = 1.104$.

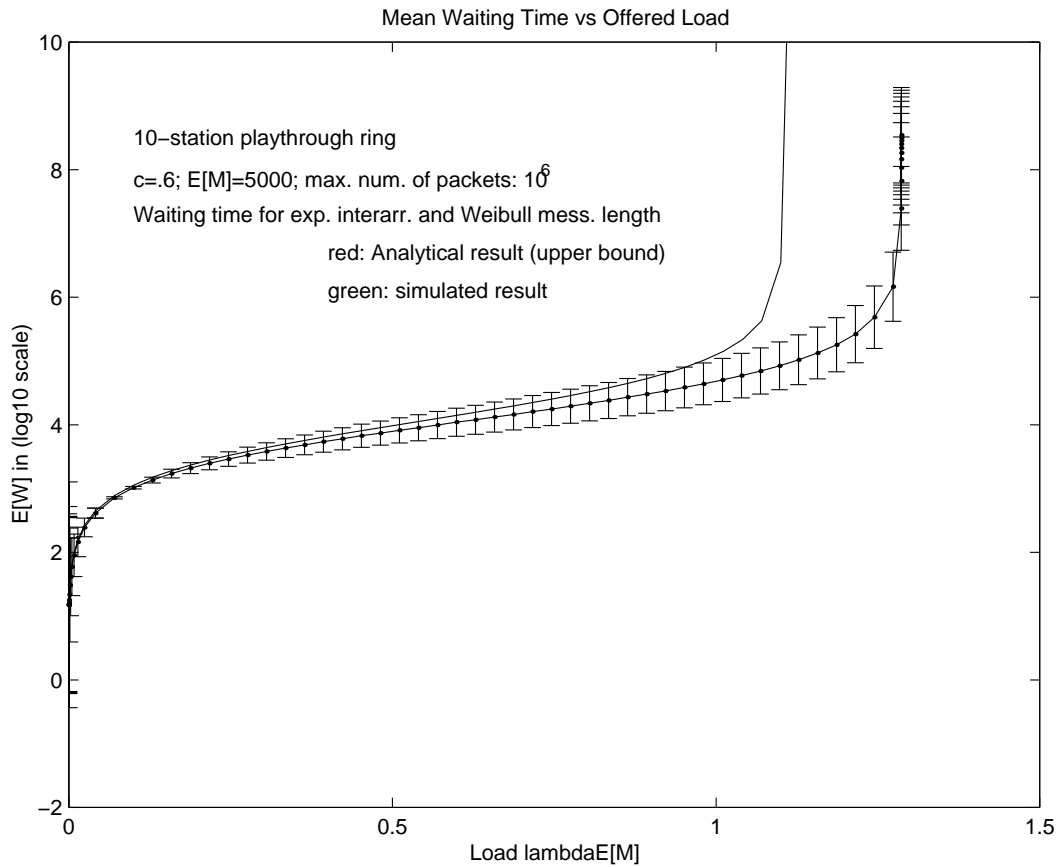


Figure 4.31: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 5000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.31 shows the analytical results of average waiting time on a 10-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 5000. The simulation results are included on the same plot. The plot shows that the analytical model is in good agreement with the simulations at light and medium loads, remaining within the confidence interval until it saturates. The analytical model saturates at $\lambda E[M] = 1.129$, whereas the simulation results saturate at $\lambda E[M] = 1.35$. The analytical waiting time gives a rather good prediction of the simulated

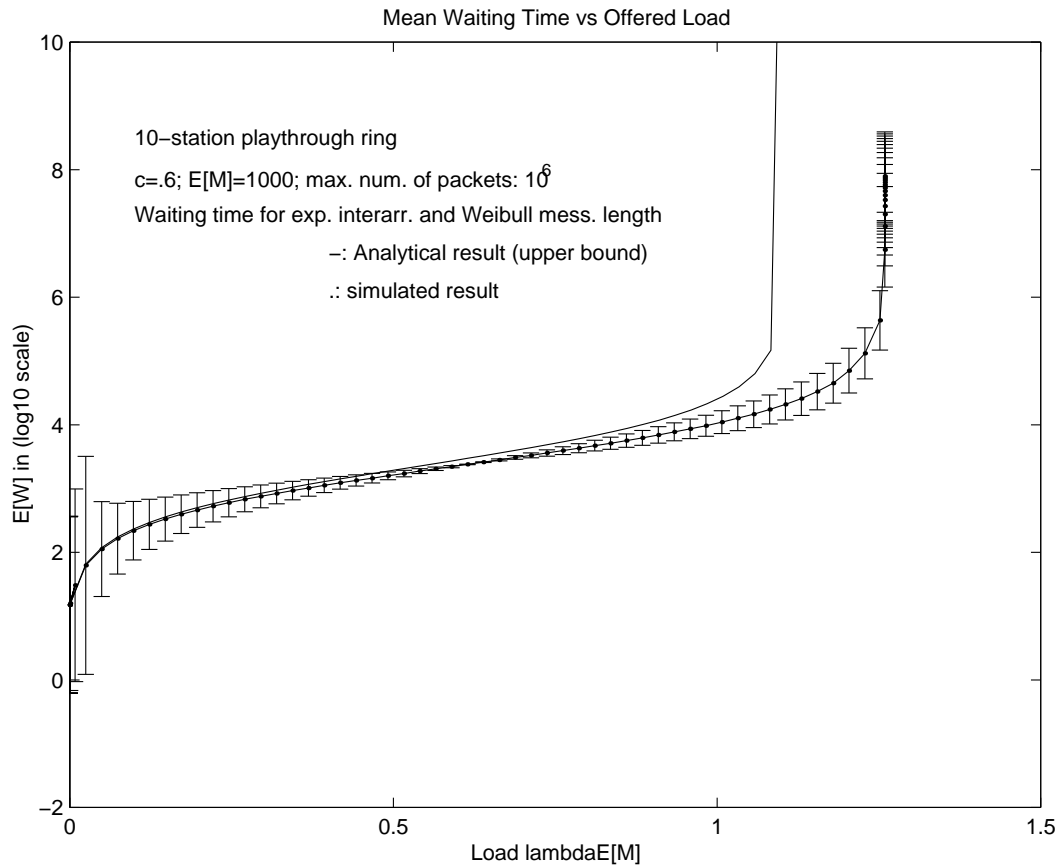


Figure 4.32: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 1000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

waiting time over a very wide range of values. Figure 4.32 shows the analytical results of average waiting time on a 10-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 1000. The simulation results are included on the same plot. The analytical model saturates at $\lambda E[M] = 1.107$, whereas the simulation results saturate at $\lambda E[M] = 1.32$. The analytical model gives a rather good prediction of the simulated waiting time at light load and appears to be close to the simulated waiting time at moderate and heavy loads. Figure 4.33 shows the

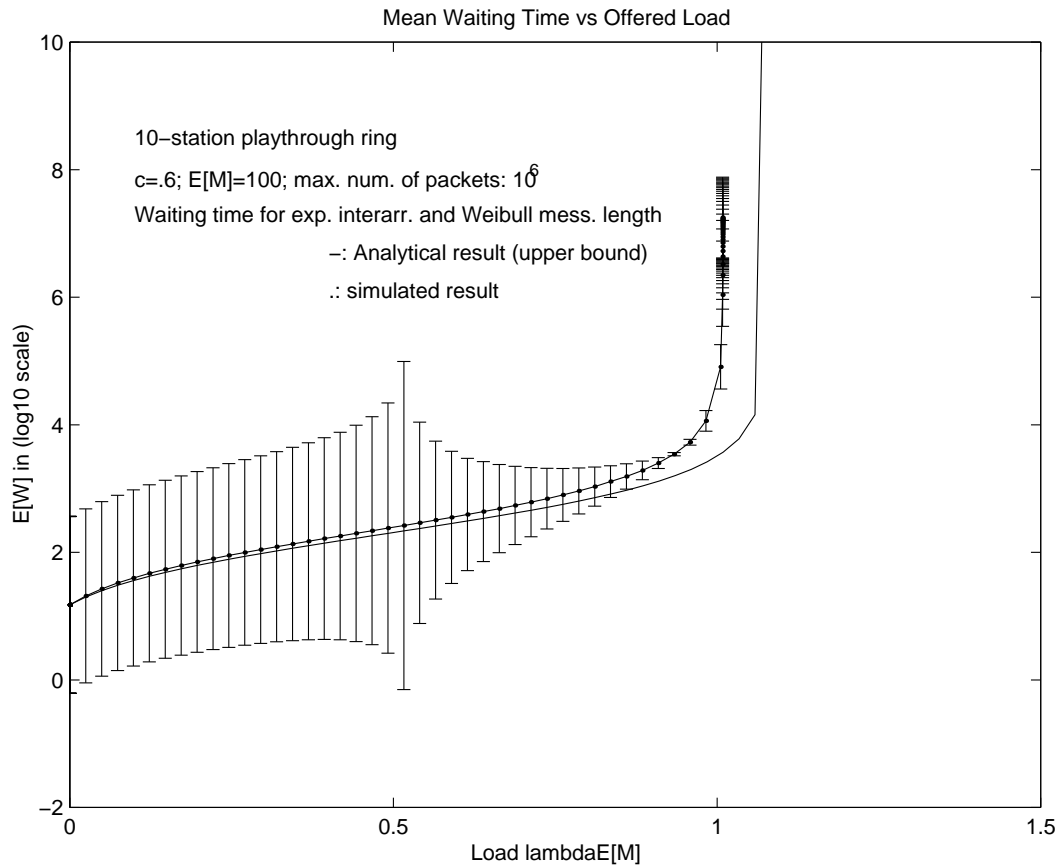


Figure 4.33: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 100$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

analytical results of average waiting time on a 10-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 100. The simulation results are included on the same plot. The analytical results saturate at $\lambda E[M] = 1.083$, whereas the simulation results saturate at $\lambda E[M] = 1.064$. The analytical waiting time saturates at slightly heavier load than the simulated waiting time. The analytical model gives a very good prediction of the simulated waiting times for light loads and moderate loads, but underpredicts the simulated results at heavy load.

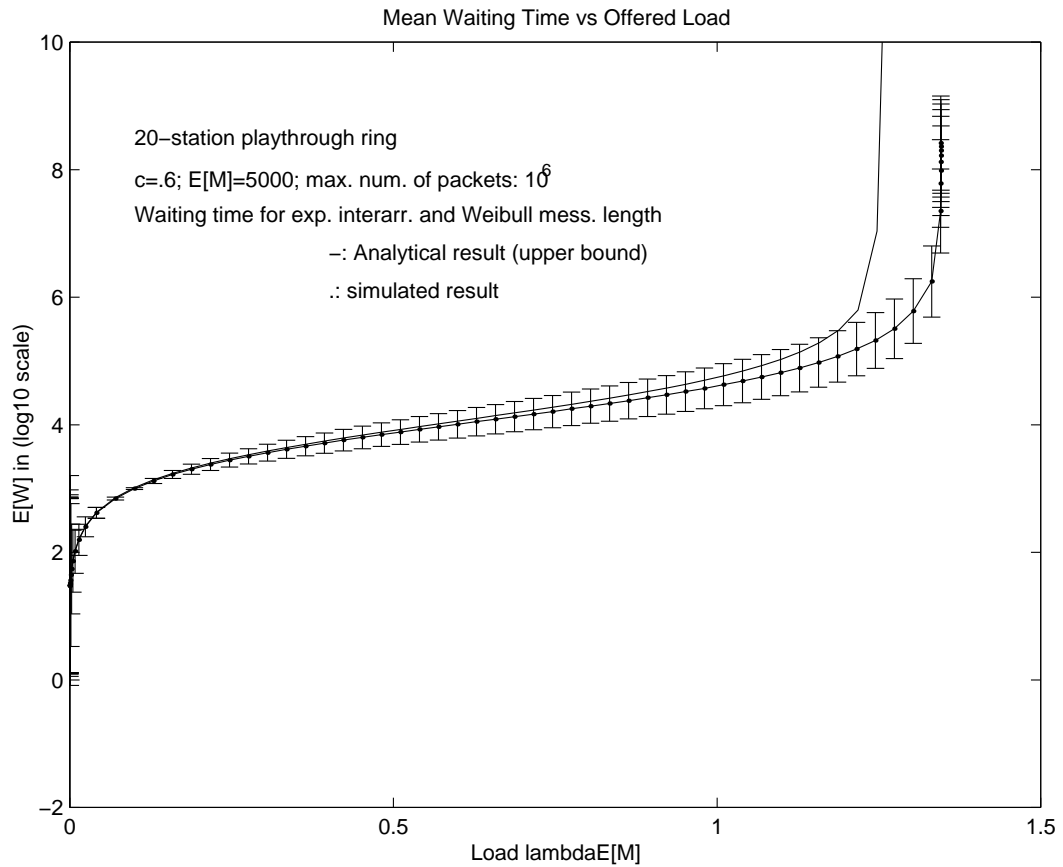


Figure 4.34: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 5000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.34 shows the analytical results of average waiting time on a 20-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 5000. The simulation results are included on the same plot. The plot shows that the analytical model is in good agreement with the simulations at light load, moderate load, and heavy load, but it saturates at lighter load than the simulated waiting time. Saturation is reached at $\lambda E[M] = 1.34$ for the simulated waiting times and at $\lambda E[M] = 1.27$ for the analytical waiting time.

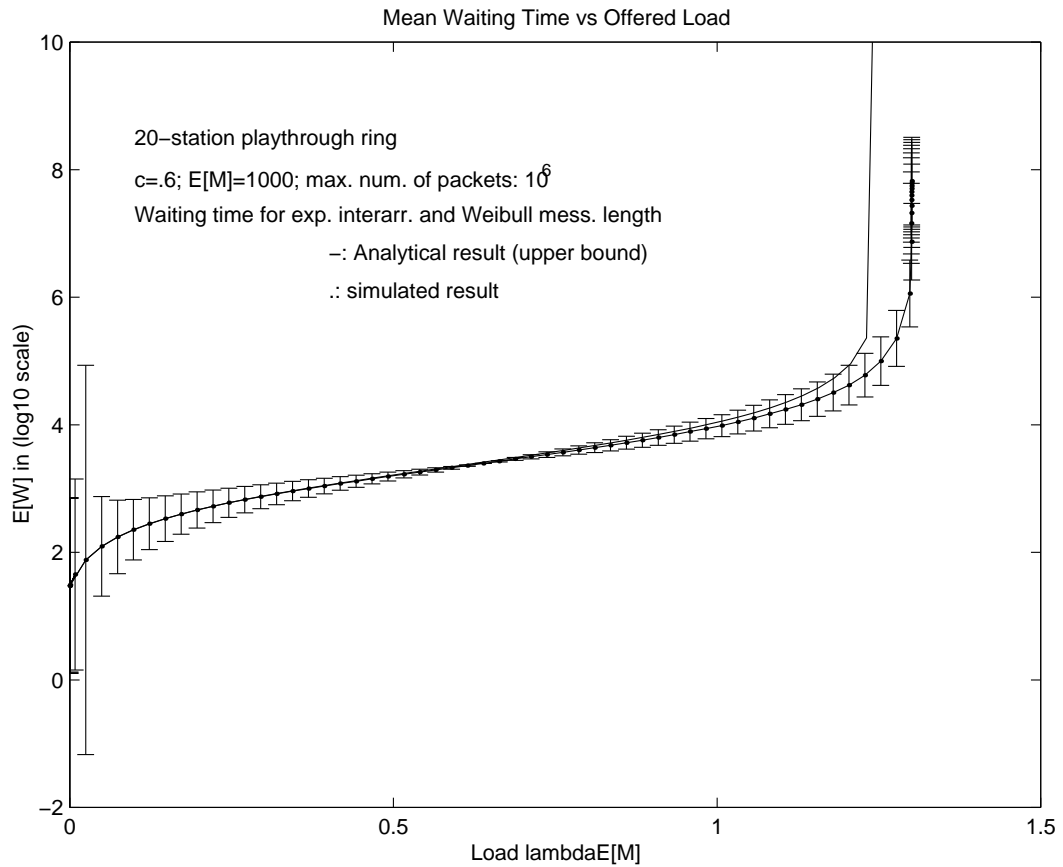


Figure 4.35: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 1000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.35 shows the analytical results of average waiting time on a 20-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 1000. The simulation results are included on the same plot. The analytical waiting time tracks the simulated waiting times very well, even at moderate load, where the confidence intervals appear to be very slim. Saturation is reached at $\lambda E[M] = 1.30$ for the simulated waiting times and at $\lambda E[M] = 1.25$ for the analytical waiting time.

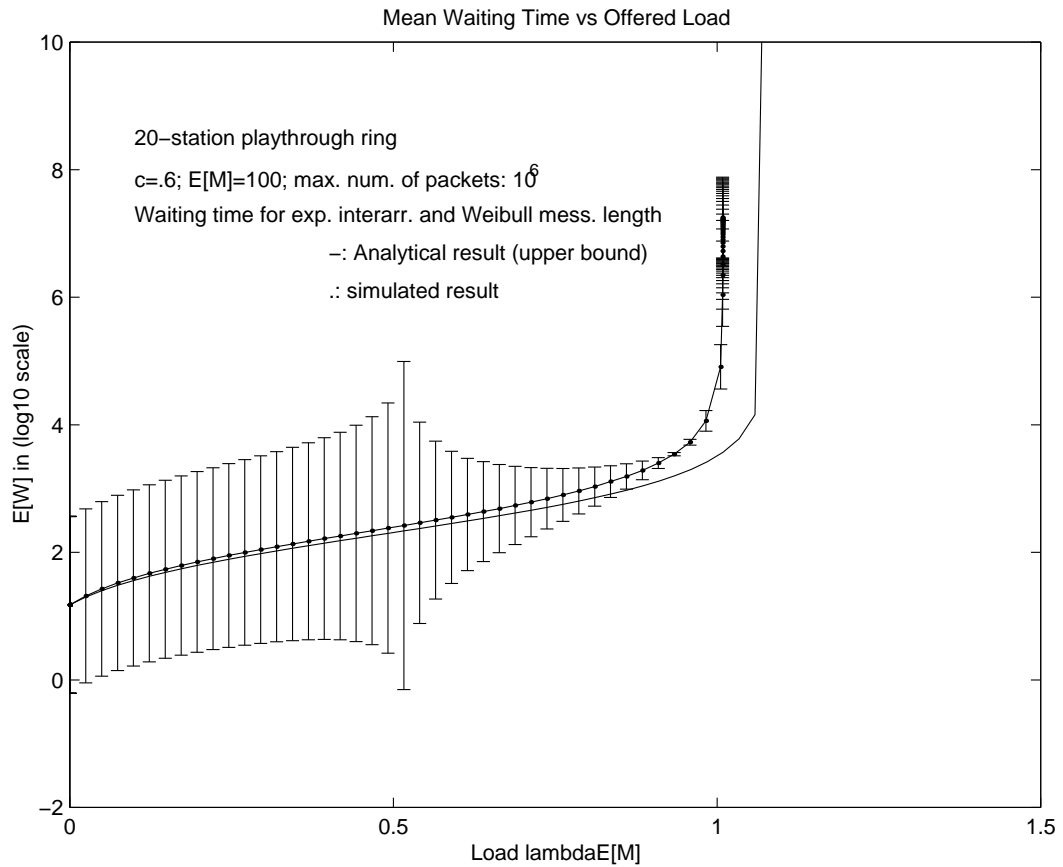


Figure 4.36: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 100$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.36 shows the analytical results of average waiting time on a 20-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 100. The simulation results are included on the same plot. Despite the fact that the analytical waiting time saturates at heavier loads than the simulated waiting times, it gives a very good prediction of the simulated waiting before saturation. Saturation is reached at $\lambda E[M] = 0.90$ for the simulated waiting times and at $\lambda E[M] = 0.93$ for the analytical waiting time.

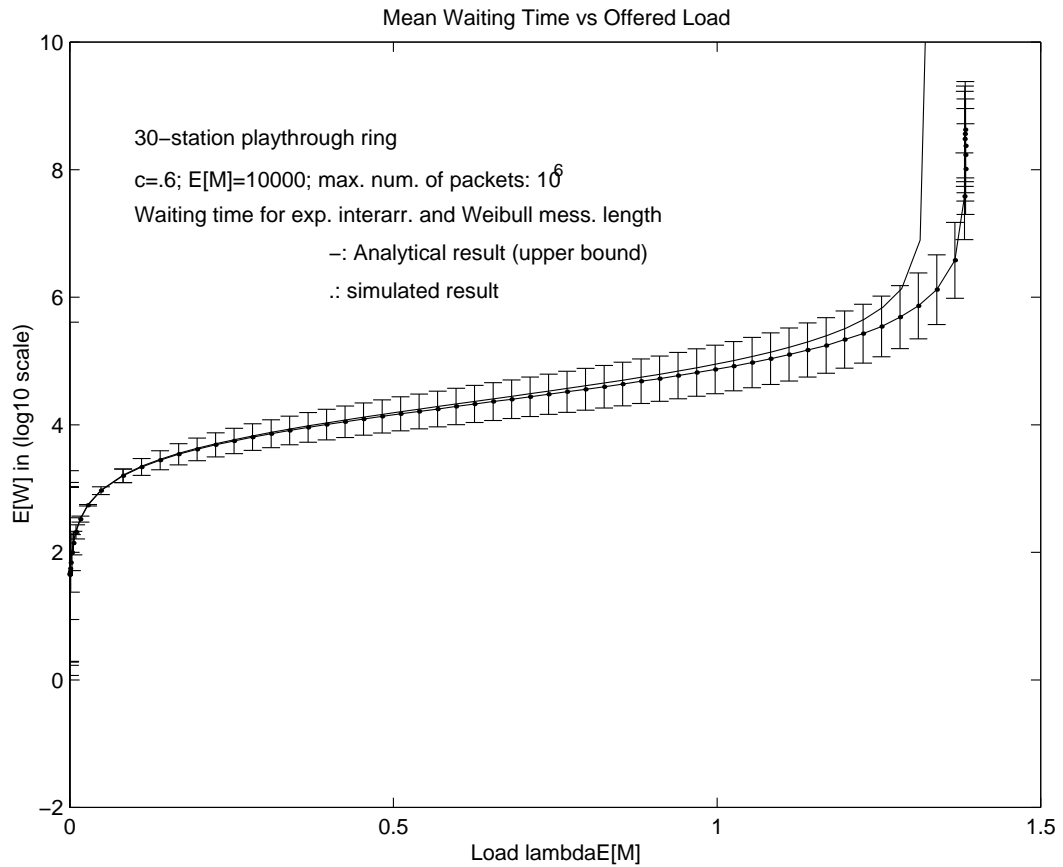


Figure 4.37: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 10000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.37 shows the analytical results of average waiting time on a 30-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 10000. The simulation results are included on the same plot. The analytical results are in very good agreement with the simulations until saturation. Saturation is reached at $\lambda E[M] = 1.38$ for the simulated waiting times and at $\lambda E[M] = 1.33$ for the analytical waiting time.

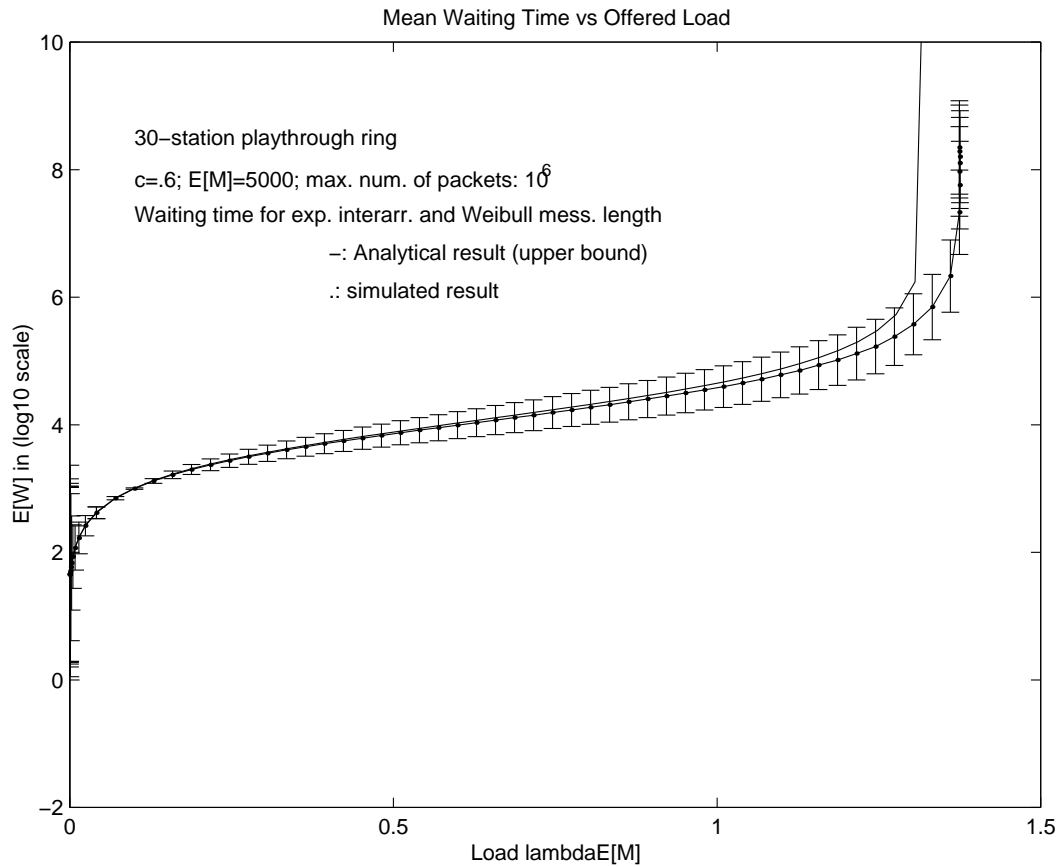


Figure 4.38: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 5000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.38 shows the analytical results of average waiting time on a 30-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 5000. The simulation results are included on the same plot. As in Figure 4.37, the analytical waiting time shows impressively good agreement with the simulations. Saturation is reached at $\lambda E[M] = 1.37$ for the simulated waiting times and at $\lambda E[M] = 1.33$ for the analytical waiting time.

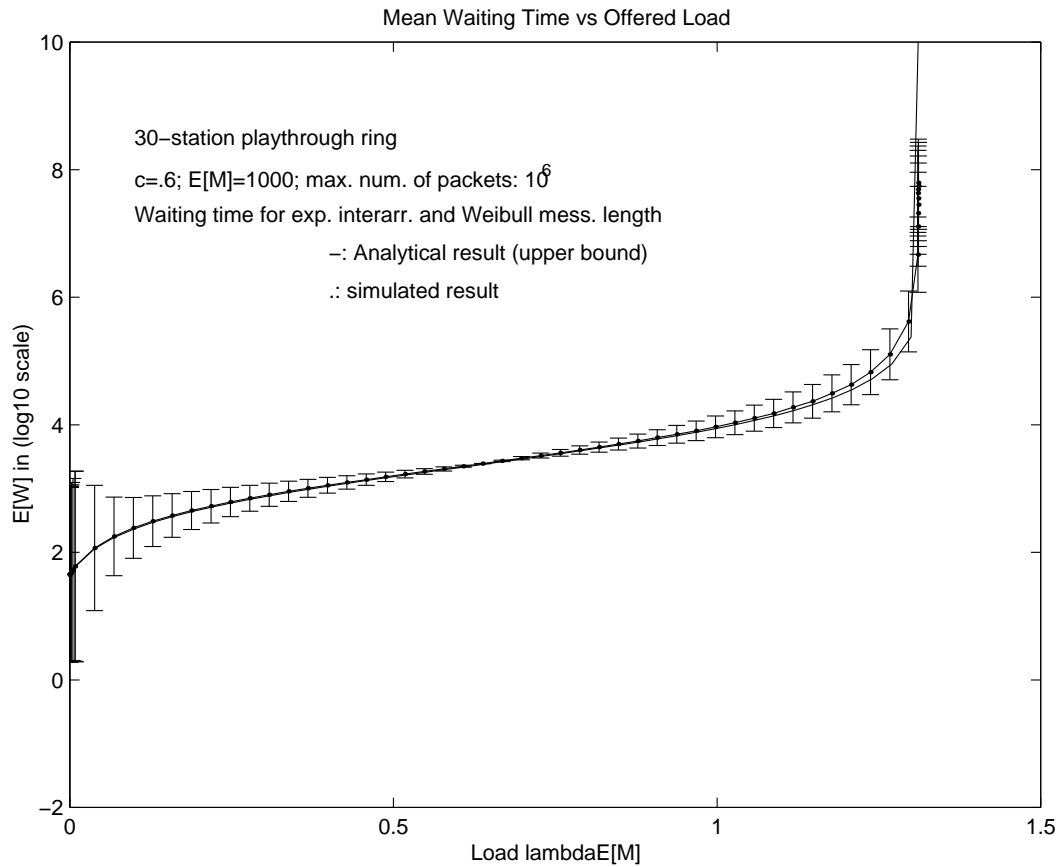


Figure 4.39: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 1000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.39 shows the analytical results of average waiting time on a 30-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 1000. The simulation results are included on the same plot. The analytical waiting time is virtually identical to the simulated mean waiting time, remaining within the confidence interval for all loads. Saturation is reached at $\lambda E[M] = 1.31$ for the simulated waiting times and at $\lambda E[M] = 1.32$ for the analytical waiting time.

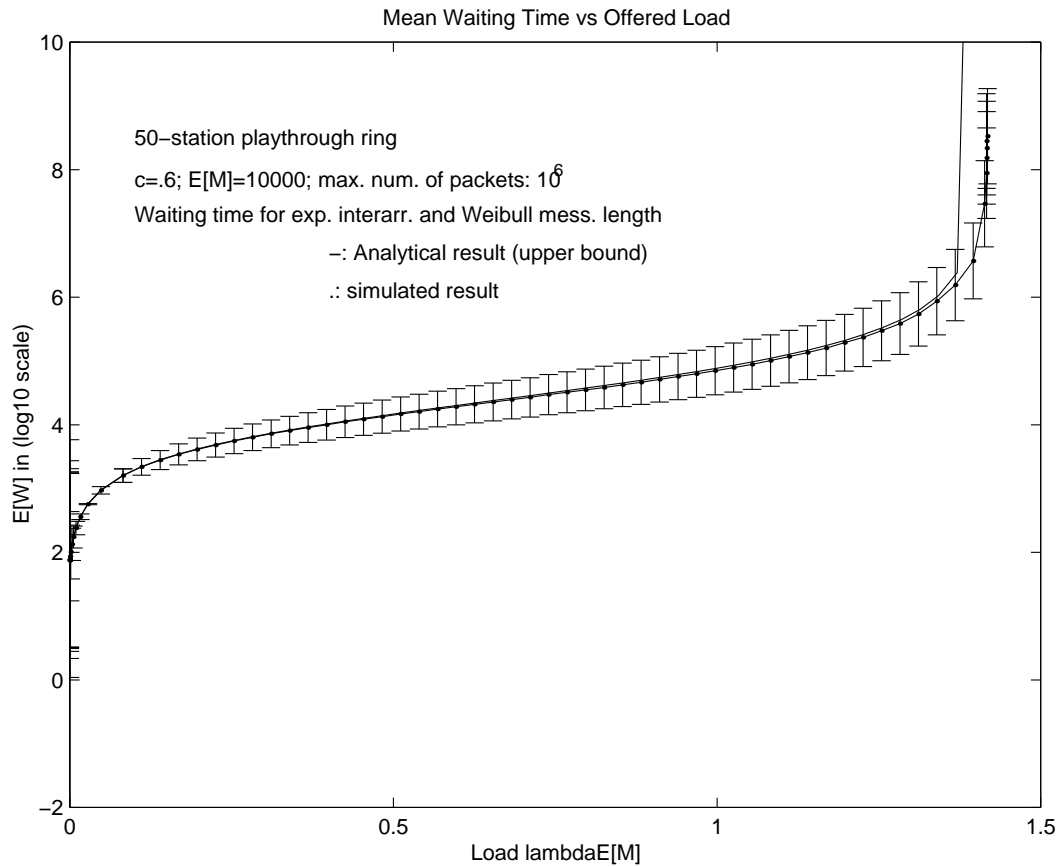


Figure 4.40: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 10000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.40 shows the analytical results of average waiting time on a 50-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 10000. The simulation results are included on the same plot. The analytical waiting time tracks the simulated waiting times very well. Saturation is reached at $\lambda E[M] = 1.41$ for the simulated waiting times and at $\lambda E[M] = 1.38$ for the analytical waiting time.

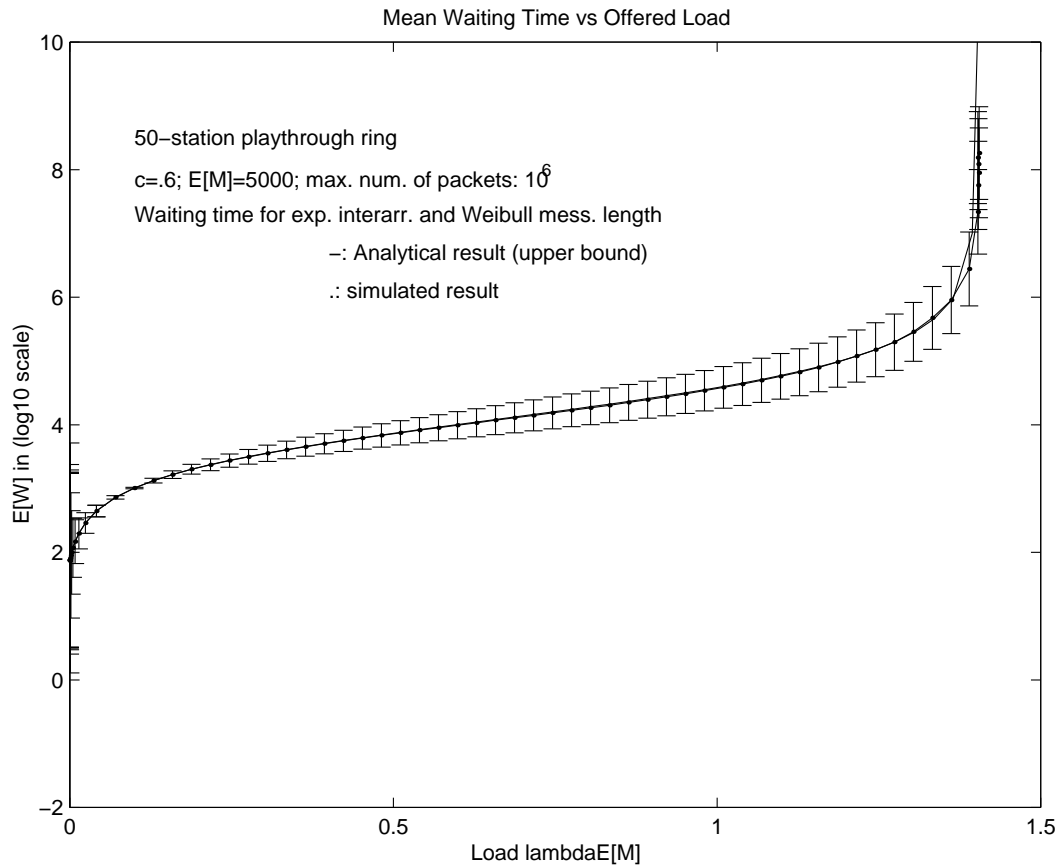


Figure 4.41: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 5000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.41 shows the analytical results of average waiting time on a 50-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 5000. The simulation results are included on the same plot. The analytical waiting time is identical to the simulated waiting times for all loads, giving a very accurate prediction. In addition, the analytical results remain within the 95% confidence interval at all values. Saturation is reached at $\lambda E[M] = 1.40$ for the simulated waiting times and at $\lambda E[M] = 1.39$ for the analytical waiting time.

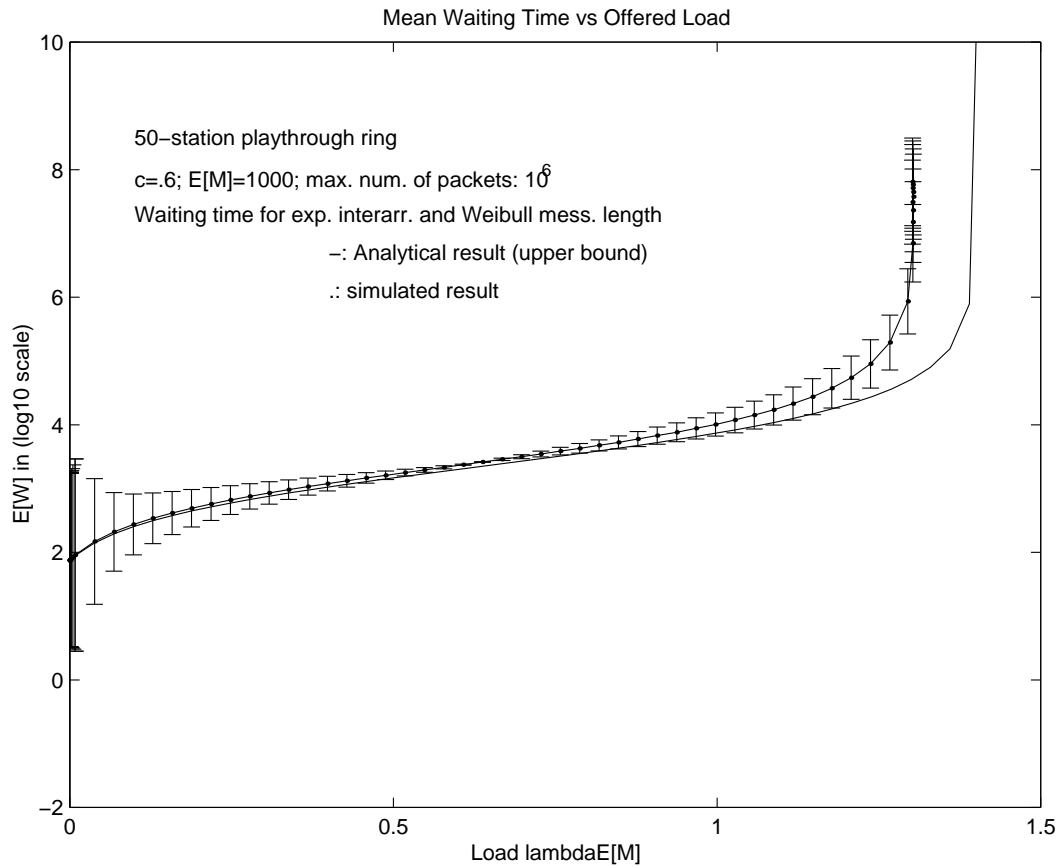


Figure 4.42: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and exponential interarrival times, Weibull distributed message lengths with parameter $c = 0.6$ and mean $E[M] = 1000$ characters. Comparison with simulated mean waiting times under exp. interarrival times and Weibull message lengths.

Figure 4.42 shows the analytical results of average waiting time on a 50-station PLAYTHROUGH ring with exponential message interarrival times and Weibull distributed message lengths with mean 1000. The simulation results are included on the same plot. The analytical waiting time slightly underpredicts the simulated waiting times at heavy loads but gives a very good prediction at light loads and moderate loads. Saturation is reached at $\lambda E[M] = 1.30$ for the simulated waiting times and at $\lambda E[M] = 1.37$ for the analytical waiting time.

From Figures 4.28–4.42, we can make the following observations. Except for the limiting case where the number of stations N is equal to 3, the analytical model for average waiting time gives a very good prediction of the simulated waiting time for a wide range of loads. The analytical prediction of the simulated results appears to improve for a large range of average message lengths as the number of stations increases.

4.8 Conclusions

In this chapter, a tight upper bound of the mean number of minipackets required to send a message on PLAYTHROUGH ring under exponentially distributed message interarrival times and Weibull distributed message lengths was found. From that upperbound, analytical expressions for higher moments of the number of minipackets necessary to transmit a message were obtained. Approximate control frame round trip time, transmission time, service time, and blocking duration were given. An analytical model for waiting time was derived. The analytical model was shown to provide very good predictions of the simulated mean waiting times for a wide range of number of stations and loads. As the number of stations increases, the analytical predictions improve for a wide range of average message lengths.

We did not provide an analytical model for waiting time under Weibull message interarrival times and geometric message lengths. This case is less amenable to analysis because the Laplace transform of the Weibull distribution does not exist. In the next chapter, we will show that at heavy load, the mean waiting time for PLAYTHROUGH ring under exponentially distributed interarrival times and Weibull message lengths is greater than the mean waiting time for PLAYTHROUGH ring under Weibull interarrival times and geometric

message lengths when identical mean message interarrival times and mean message lengths are used. In addition, we show that at heavy load, the mean waiting times for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths are greater than mean waiting times for PLAYTHROUGH ring under exponentially distributed interarrival times and geometric message lengths.

CHAPTER 5

PERFORMANCE OF PLAYTHROUGH RINGS UNDER WEIBULL INTERARRIVAL TIMES AND GEOMETRIC MESSAGE LENGTHS

5.1 The Heavy Traffic Approximation for the G/G/1 Queue.

Let us consider a queueing system with one server, an arbitrary distribution for the interarrival time random variable \tilde{t} , an arbitrary distribution for the service time \tilde{S} . The mean waiting time for this G/G/1 queueing system is given by [Kle75]:

$$E[W] = \frac{\sigma_a^2 + \sigma_b^2 + (\bar{t})^2(1 - \rho)^2}{2\bar{t}(1 - \rho)} - \frac{\bar{I}^2}{2\bar{I}}, \quad (5.1)$$

where σ_a^2 is the variance of the interarrival times distribution, σ_b^2 is the variance of the service times distribution \tilde{S} , \bar{t} is the mean interarrival time $E[\tilde{t}]$, $\rho = \frac{E[\tilde{S}]}{\bar{t}}$, \bar{I} is the mean idle time, and \bar{I}^2 is the second moment of idle time. The idle time is defined as the interval of time from the departure of a customer (in our context a message) that leaves the system until the next arrival. At heavy load, we have:

$$\frac{\bar{I}^2}{2\bar{I}} \rightarrow 0, \quad (5.2)$$

$$(\bar{t})^2(1 - \rho)^2 \approx 0. \quad (5.3)$$

As a result, we obtain the following heavy traffic approximation:

$$E[W] \approx \frac{\sigma_a^2 + \sigma_b^2}{2\bar{t}(1 - \rho)}. \quad (5.4)$$

In the rest of this chapter, we will apply the heavy traffic approximation in many instances. To be concise, we will refer to exponentially distributed interarrival times as exponential interarrival times and Weibull distributed interarrival times as Weibull interarrival times. In addition, we will refer to geometrically distributed message lengths as geometric message lengths and Weibull distributed message lengths as Weibull message lengths.

The notation exp/geom/1 will be used to refer to a single queue in which messages arrive according exponentially distributed interarrival times and have geometrically distributed message lengths. The notation exp/weib/1 will be used to refer to a single queue in which messages arrive according to exponentially distributed interarrival times and have Weibull message lengths. The notation weib/geom/1 will be used to refer to a single queue in which messages arrive according to Weibull distributed interarrival times and have geometrically distributed message lengths.

5.2 Comparison of Waiting Time for Two Queueing Systems, One with an exp/geom/1 queue, and the other with a weib/geom/1 queue.

We consider a queueing system comprising a computer station at which messages arrive according to exponential interarrival times and have geometric message lengths and another queueing system comprising a computer station at which messages arrive according to Weibull interarrival times and have geometric message lengths. We assume that the mean interarrival times and the mean service times are the same for the two queueing systems. The mean waiting time $E[W_1]$ for the first queueing system comprising message arriving

with exponential interarrival times and geometric message lengths is given by:

$$E[W_1] = \frac{\sigma_{a_1}^2 + \sigma_{b_1}^2 + (\bar{t}_1)^2(1 - \rho_1)^2}{2\bar{t}_1(1 - \rho_1)} - \frac{\bar{I}_1^2}{2\bar{I}_1}, \quad (5.5)$$

where $\sigma_{a_1}^2$ is the variance of the interarrival times distribution \tilde{t}_1 , $\sigma_{b_1}^2$ is the variance of the service time distribution \tilde{S}_1 , \bar{t}_1 is the mean interarrival time, $\rho_1 = \frac{E[\tilde{S}_1]}{\bar{t}_1}$, \bar{I}_1 is the mean idle time, and \bar{I}_1^2 is the second moment of the mean idle time.

At heavy traffic, Equation 5.5 can be approximated by:

$$E[W_1] \approx \frac{\sigma_{a_1}^2 + \sigma_{b_1}^2}{2\bar{t}_1(1 - \rho_1)}. \quad (5.6)$$

The mean waiting time $E[W_2]$ for the second queueing system comprising message arriving with Weibull interarrival times and geometric message lengths is given by:

$$E[W_2] = \frac{\sigma_{a_2}^2 + \sigma_{b_2}^2 + (\bar{t}_2)^2(1 - \rho_2)^2}{2\bar{t}_2(1 - \rho_2)} - \frac{\bar{I}_2^2}{2\bar{I}_2}, \quad (5.7)$$

where $\sigma_{a_2}^2$ is the variance of the interarrival times distribution \tilde{t}_2 , $\sigma_{b_2}^2$ is the variance of the service time distribution \tilde{S}_2 , \bar{t}_2 is the mean interarrival time, $\rho_2 = \frac{E[\tilde{S}_2]}{\bar{t}_2}$, \bar{I}_2 is the mean idle time, and \bar{I}_2^2 is the second moment of the mean idle time.

At heavy traffic, Equation 5.7 can be approximated by:

$$E[W_2] \approx \frac{\sigma_{a_2}^2 + \sigma_{b_2}^2}{2\bar{t}_2(1 - \rho_2)}. \quad (5.8)$$

We have:

$$E[W_2] - E[W_1] \approx \frac{\sigma_{a_2}^2 + \sigma_{b_2}^2}{2\bar{t}_2(1 - \rho_2)} - \frac{\sigma_{a_1}^2 + \sigma_{b_1}^2}{2\bar{t}_1(1 - \rho_1)} \quad (5.9)$$

In addition,

$$\begin{aligned}
\bar{t}_1 &= \bar{t}_2 = \bar{t}, \\
\rho_1 &= \rho_2 = \rho, \\
E[\tilde{S}_1] &= E[\tilde{S}_2], \\
\sigma_{b_1}^2 &= \sigma_{b_2}^2.
\end{aligned} \tag{5.10}$$

So,

$$E[W_2] - E[W_1] \approx \frac{\sigma_{a_2}^2 - \sigma_{a_1}^2}{2\bar{t}(1 - \rho)}. \tag{5.11}$$

Let a and c be the parameters of the Weibull interarrival times distribution of the second queueing system. We have:

$$\bar{t}_2 = a\Gamma\left(1 + \frac{1}{c}\right) = \bar{t}_1 = \bar{t}. \tag{5.12}$$

So,

$$a = \frac{\bar{t}}{\Gamma\left(1 + \frac{1}{c}\right)}. \tag{5.13}$$

and

$$\begin{aligned}
\sigma_{a_2}^2 &= E[\tilde{t}_2^2] - \bar{t}_2^2 \\
&= a^2\Gamma\left(1 + \frac{2}{c}\right) - a^2\Gamma\left(1 + \frac{1}{c}\right)^2 \\
&= \left(\frac{\bar{t}}{\Gamma\left(1 + \frac{1}{c}\right)}\right)^2 \Gamma\left(1 + \frac{2}{c}\right) - \left(\frac{\bar{t}}{\Gamma\left(1 + \frac{1}{c}\right)}\right)^2 \Gamma\left(1 + \frac{1}{c}\right)^2 \\
&= (\bar{t})^2 \left(\frac{\Gamma\left(1 + \frac{2}{c}\right)}{\Gamma\left(1 + \frac{1}{c}\right)^2} - 1\right).
\end{aligned} \tag{5.15}$$

We plot the ratio:

$$\frac{\Gamma\left(1 + \frac{2}{c}\right)}{\Gamma\left(1 + \frac{1}{c}\right)^2} \tag{5.16}$$

as a function of c for $c \leq 1$. We have:

$$\frac{\Gamma(1 + \frac{2}{1})}{\Gamma(1 + \frac{1}{1})^2} = 2 \quad (5.17)$$

and as shown in Figure 5.1, the ratio $\frac{\Gamma(1+\frac{2}{c})}{\Gamma(1+\frac{1}{c})^2}$ is a decreasing function for $0 \leq c \leq 1$. Hence, we have:

$$\frac{\Gamma(1 + \frac{2}{c})}{\Gamma(1 + \frac{1}{c})^2} \geq 2 \quad \text{for } 0 \leq c \leq 1. \quad (5.18)$$

Therefore,

$$\sigma_{a_2}^2 = (\bar{t})^2 \left(\frac{\Gamma(1 + \frac{2}{c})}{\Gamma(1 + \frac{1}{c})^2} - 1 \right) \geq (\bar{t})^2. \quad (5.19)$$

And since

$$\sigma_{a_1}^2 = (\bar{t})^2, \quad (5.20)$$

we have

$$\sigma_{a_2}^2 - \sigma_{a_1}^2 > 0. \quad (5.21)$$

So,

$$E[W_2] - E[W_1] > 0. \quad (5.22)$$

5.3 Comparison of Waiting Time for Two Queueing Systems, One With a weib/geom/1 Queue and the Other With an exp/weib/1 queue.

We now consider a queueing system comprising a computer station at which messages arrive according to Weibull interarrival times and have geometric message lengths and another queueing system comprising a computer station at which messages arrive according to exponential interarrival times and have Weibull message lengths. We assume that the mean interarrival time and the mean service time for the two queueing systems are the same. As stated before, the mean waiting time $E[W_2]$ for the first queueing system comprising messages

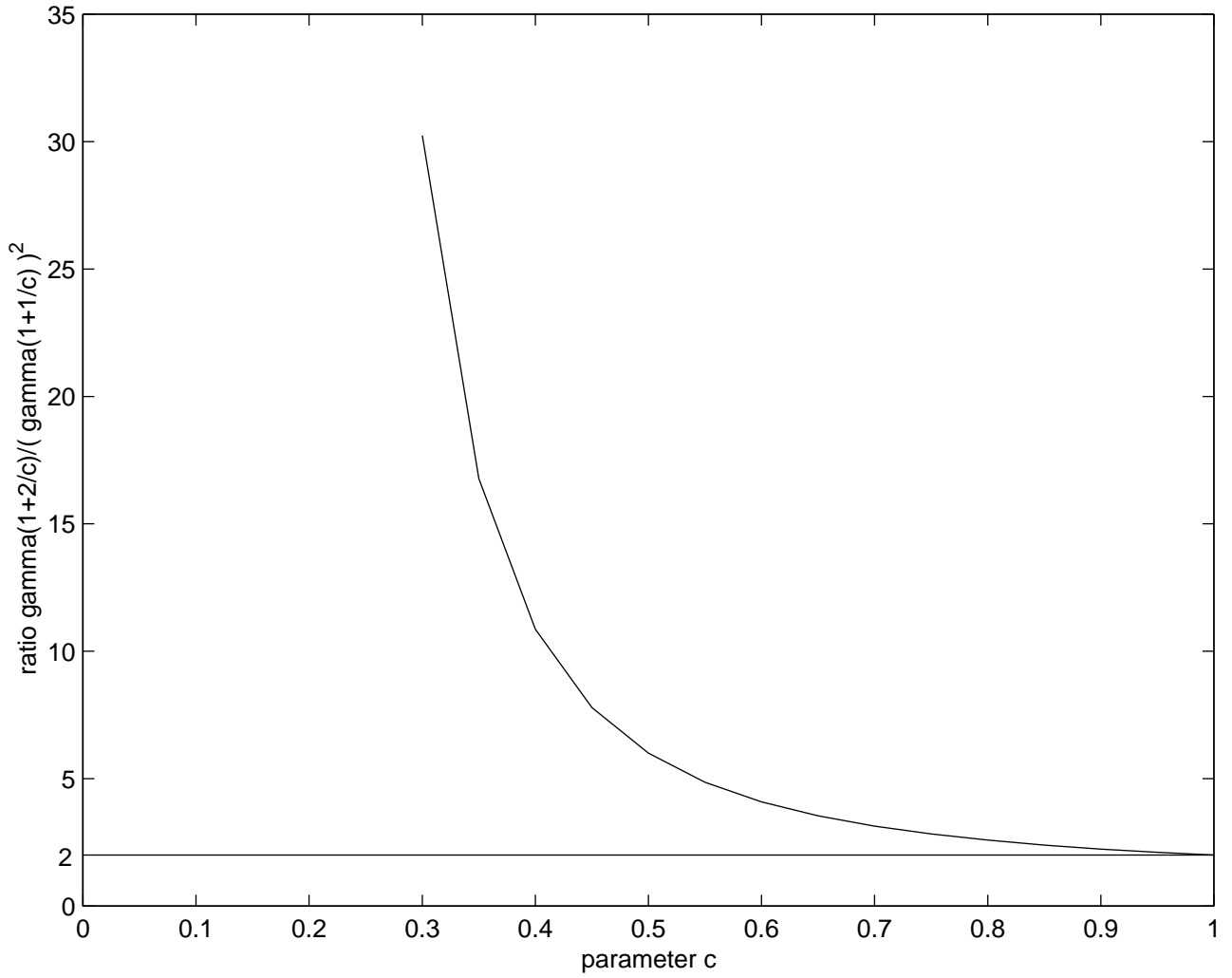


Figure 5.1: plot of the ratio from Equation 5.16 as a function of c .

arriving with Weibull interarrival times and geometric message lengths is given by:

$$E[W_2] = \frac{\sigma_{a_2}^2 + \sigma_{b_2}^2 + (\bar{t}_2)^2(1 - \rho_2)^2}{2\bar{t}_2(1 - \rho_2)} - \frac{\bar{I}_2^2}{2\bar{I}_2}. \quad (5.23)$$

At heavy traffic, Equation 5.23 can be approximated by:

$$E[W_2] \approx \frac{\sigma_{a_2}^2 + \sigma_{b_2}^2}{2\bar{t}_2(1 - \rho_2)}. \quad (5.24)$$

The mean waiting time $E[W_3]$ for the second queueing system comprising

messages arriving with exponential interarrival times and Weibull message lengths is given by:

$$E[W_3] = \frac{\sigma_{a_3}^2 + \sigma_{b_3}^2 + (\bar{t}_3)^2(1 - \rho_3)^2}{2\bar{t}_3(1 - \rho_3)} - \frac{\bar{I}_3^2}{2\bar{I}_3}, \quad (5.25)$$

where $\sigma_{a_3}^2$ is the variance of interarrival times distribution \tilde{t}_3 , $\sigma_{b_3}^2$ is the variance of the service time distribution \tilde{S}_3 , \bar{t}_3 is the mean interarrival time, $\rho_3 = \frac{E[\tilde{S}_3]}{\bar{t}_3}$, \bar{I}_3 is the mean idle time, and \bar{I}_3^2 is the second moment of the mean idle time.

At heavy traffic, Equation 5.25 can be approximated by:

$$E[W_3] \approx \frac{\sigma_{a_3}^2 + \sigma_{b_3}^2}{2\bar{t}_3(1 - \rho_3)}. \quad (5.26)$$

We have:

$$E[W_3] - E[W_2] \approx \frac{\sigma_{a_3}^2 + \sigma_{b_3}^2}{2\bar{t}_3(1 - \rho_3)} - \frac{\sigma_{a_2}^2 + \sigma_{b_2}^2}{2\bar{t}_2(1 - \rho_2)}. \quad (5.27)$$

At heavy traffic, we have:

$$\begin{aligned} E[\tilde{t}_2] &\approx E[\tilde{S}_2], \\ E[\tilde{t}_3] &\approx E[\tilde{S}_3]. \end{aligned}$$

If we assume

$$E[\tilde{S}_3] \geq E[\tilde{S}_2], \quad (5.28)$$

we have

$$\sigma_{b_3}^2 \geq \sigma_{b_2}^2, \quad (5.29)$$

because a Weibull distributed r.v. \tilde{w} having a mean greater or equal to the mean of an exponential distribution \tilde{e} has a greater variance than the exponential distribution \tilde{e} .

In addition, as a consequence of the assumption $E[\tilde{S}_3] \geq E[\tilde{S}_2]$, we have:

$$E[\tilde{t}_3] \geq E[\tilde{t}_2],$$

since $E[\tilde{t}_2] \approx E[\tilde{S}_2]$, and $E[\tilde{t}_3] \approx E[\tilde{S}_3]$. Using an argument similar to the one used to justify Equation 5.29 we have:

$$\sigma_{a_3}^2 \geq \sigma_{a_2}^2. \quad (5.30)$$

5.4 Comparison of waiting time for PLAYTHROUGH ring under two different schemes, One with an exp/geom/1 queues, and the other with a weib/geom/1 queues.

We now turn our attention to the comparison of the waiting time for PLAYTHROUGH ring under two different assumptions. We seek to compare the waiting time for a PLAYTHROUGH ring in which message interarrival times are exponentially distributed and message lengths are geometrically distributed to the waiting time for a PLAYTHROUGH ring in which message interarrival times are Weibull distributed and message lengths are geometrically distributed.

5.4.1 Waiting Time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths.

Using a line of reasoning similar to that used to obtain Equation 4.158, a model for PLAYTHROUGH ring operating under exponential interarrival times and geometric message lengths can be obtained. According to that model, the mean waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths is given by the expression:

$$\begin{aligned} E [W_{exp/geom}] &= E [R_{GO}] + E [BL] \\ &+ \frac{\lambda_i E [(S + BL)^2]}{2(1 - \lambda_i E [S + BL])}. \end{aligned} \quad (5.31)$$

In this expression, the first term is the mean residual time for GO to return to the station. The second expression is the mean blocking duration for a message at the head of the queue at a given station, the third term can be recognized as the expression for waiting time for an M/G/1 queue in which blocking is included as part of service time.

5.4.2 Waiting time for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths.

In a similar way, the waiting time for the PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths can be modeled as the sum of 3 components. The first one being the mean residual time for GO , the second being the mean blocking duration for a message at the head of the queue at a given station, the third term being the waiting time time for a G/G/1 queue in which blocking is included as part of service time. So we can write:

$$E [W_{weib/geom}] = E [R_{GO}] + E [BL] + W_{g/m/1}. \quad (5.32)$$

5.4.3 Comparison of waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths to waiting time under Weibull interarrival times and geometric message lengths.

In Equation 5.31 and Equation 5.32, mean residual round trip time for GO and mean blocking duration are the same. However, as shown previously in this chapter, at heavy traffic, waiting time for a queue with Weibull interarrival times and exponential service times is greater than waiting time for a queue with exponential interarrival times and geometric message lengths. Therefore, at heavy load, we have:

$$E [W_{weib/geom}] \geq E [W_{exp/geom}]. \quad (5.33)$$

- 5.5 Comparison of waiting time for PLAYTHROUGH ring under two different schemes, One with an weib/geom/1 queues, and the other with a exp/weib/1 queues.

We now seek to compare the waiting time for a PLAYTHROUGH ring in which message interarrival times are Weibull distributed and message lengths are geometrically distributed to the waiting time for a PLAYTHROUGH ring in which message interarrival times are exponentially distributed and message lengths are Weibull distributed.

- 5.5.1 Waiting time for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths.

As stated in the previous section, waiting time for the PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths can be written as:

$$E [W_{weib/geom}] = E [R_{GO}] + E [BL] + E[W_{g/m/1}]. \quad (5.34)$$

- 5.5.2 Waiting time for PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths.

The mean waiting time for PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths was found in Equation 4.158 and it is given by:

$$E [W_{exp/weib}] = E [R_{GO}] + E [BL] + \frac{\lambda_i E [(S + BL)^2]}{2(1 - \lambda_i E [S + BL])}. \quad (5.35)$$

- 5.5.3 Comparison of waiting time for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths to waiting time under exponential interarrival and Weibull message lengths.

At heavy load, $E[W_{g/m/1}]$ in Equation 5.32 can be written as [Kle75]:

$$E[W_{g/m/1}] \approx E[R_{GO}] + E[BL_2] + \frac{\lambda_i(\sigma_{t_2}^2 + \sigma_{S_2+BL_2}^2)}{1 - \lambda_i E[S_2 + BL_2]}. \quad (5.36)$$

If we neglect the mean residual time $E[R_{GO}]$, we can write:

$$E[W_{g/m/1}] \approx E[BL_2] + \frac{\lambda_i(\sigma_{t_2}^2 + \sigma_{S_2+BL_2}^2)}{1 - \lambda_i E[S_2 + BL_2]}. \quad (5.37)$$

The expression:

$$\frac{\lambda_i E[(S + BL)^2]}{2(1 - \lambda_i E[S + BL])} \quad (5.38)$$

in Equation 5.35 is the mean waiting time in an M/G/1 queue with arrival rate λ_i and mean service time $E[S + BL]$. At heavy load, we have [Kle75]:

$$\frac{\lambda_i E[(S + BL)^2]}{2(1 - \lambda_i E[S + BL])} \approx \frac{\lambda_i(\sigma_{t_3}^2 + \sigma_{S+BL}^2)}{1 - \lambda_i E[S + BL]}, \quad (5.39)$$

and Equation 5.35 can be rewritten as:

$$E[W_{exp/weib}] \approx E[R_{GO}] + E[BL_3] + \frac{\lambda_i(\sigma_{t_3}^2 + \sigma_{S_3+BL_3}^2)}{1 - \lambda_i E[S_3 + BL_3]}. \quad (5.40)$$

If we neglect the mean residual time $E[R_{GO}]$, we can write:

$$E[W_{exp/weib}] \approx E[BL_3] + \frac{\lambda_i(\sigma_{t_3}^2 + \sigma_{S_3+BL_3}^2)}{1 - \lambda_i E[S_3 + BL_3]}. \quad (5.41)$$

From equations Equation 5.37 and Equation 5.41, we have:

$$\begin{aligned} E[W_{exp/weib}] - E[W_{weib/geom}] &\approx E[BL_3] + \frac{\lambda_i(\sigma_{t_3}^2 + \sigma_{S_3+BL_3}^2)}{1 - \lambda_i E[S_3 + BL_3]} \\ &\quad - E[BL_2] - \frac{\lambda_i(\sigma_{t_2}^2 + \sigma_{S_2+BL_2}^2)}{1 - \lambda_i E[S_2 + BL_2]} \\ &= E[BL_3] - E[BL_2] \\ &\quad + \frac{\lambda_i(\sigma_{t_3}^2 + \sigma_{S_3+BL_3}^2)}{1 - \lambda_i E[S_3 + BL_3]} - \frac{\lambda_i(\sigma_{t_2}^2 + \sigma_{S_2+BL_2}^2)}{1 - \lambda_i E[S_2 + BL_2]}. \end{aligned} \quad (5.42)$$

We have:

$$\begin{aligned} E[\tilde{t}_2] &= E[\tilde{t}_3] = \bar{t}, \\ E[\tilde{S}_2] &= E[\tilde{S}_3] = \bar{S}. \end{aligned}$$

In addition, because we are considering heavy traffic intensities, we have:

$$E [\tilde{t}_2] = E [S_2 + BL_2], \quad (5.43)$$

$$E [\tilde{t}_3] = E [S_3 + BL_3]. \quad (5.44)$$

The blocking probabilities $E [BL_2]$ and $E [BL_3]$ are given by:

$$E [BL_2] = E [S_2], \quad (5.45)$$

and

$$E [BL_3] = \frac{E [S_3^2]}{2E [S_3]}, \quad (5.46)$$

respectively.

Because the service time \tilde{S}_3 is Weibull distributed, we have from Appendix C:

$$\frac{E [S_3^2]}{2E [S_3]} > E [S_3], \quad (5.47)$$

and since $E [S_2] = E [S_3]$, we have:

$$E [BL_3] > E [BL_2] \quad (5.48)$$

The random variables \tilde{BL}_3 of blocking duration and the random variable \tilde{S}_3 of service time are independent variables because they are generated by independent events. Likewise, the random variables \tilde{BL}_2 of blocking duration and the random variable \tilde{S}_2 of service time are independent variables. Hence, we have:

$$\sigma_{\tilde{S}_3 + \tilde{BL}_3}^2 = \sigma_{\tilde{S}_3}^2 + \sigma_{\tilde{BL}_3}^2, \quad (5.49)$$

$$\sigma_{\tilde{S}_2 + \tilde{BL}_2}^2 = \sigma_{\tilde{S}_2}^2 + \sigma_{\tilde{BL}_2}^2. \quad (5.50)$$

We have:

$$\sigma_{\tilde{S}_3}^2 \geq \sigma_{\tilde{S}_2}^2, \quad (5.51)$$

because \tilde{S}_3 is Weibull distributed, \tilde{S}_2 is exponentially distributed and $E[\tilde{S}_3] = E[\tilde{S}_2]$. We have:

$$\sigma_{BL_3}^2 = E[BL_3^2] - E[BL_3]^2 = \frac{E[\tilde{S}_3^3]}{3E[\tilde{S}_3]} - \frac{E[\tilde{S}_3^2]}{2E[\tilde{S}_3]} > E[\tilde{S}_2^2], \quad (5.52)$$

$$\sigma_{BL_2}^2 = E[BL_2^2] - E[BL_2]^2 = \frac{E[\tilde{S}_2^2]}{2E[\tilde{S}_2]} = E[\tilde{S}_2^2]. \quad (5.53)$$

Because we have

$$E[\tilde{t}_3] = E[S_3 + BL_3] > 2E[S_3],$$

$$E[\tilde{t}_2] = E[S_2 + BL_2] = 2E[S_2],$$

at heavy load, it follows that

$$E[\tilde{t}_3] > 2E[S_2] = E[S_2 + BL_2],$$

$$E[S_3 + BL_3] > E[S_2 + BL_2] = E[\tilde{t}_2],$$

since $E[S_3] > E[S_2]$ and $E[BL_3] > E[BL_2]$. Because \tilde{t}_3 and $\tilde{S}_2 + \tilde{BL}_2$ are both exponentially distributed and \tilde{t}_2 and $\tilde{S}_3 + \tilde{BL}_3$ are Weibull distributed, we have:

$$\begin{aligned} \sigma_{\tilde{t}_3}^2 &> \sigma_{\tilde{S}_2 + \tilde{BL}_2}^2, \\ \sigma_{\tilde{S}_3 + \tilde{BL}_3}^2 &> \sigma_{\tilde{t}_2}^2. \end{aligned} \quad (5.54)$$

In addition, we have:

$$1 - \lambda_i E[S_3 + BL_3] < 1 - \lambda_i E[S_2 + BL_2]. \quad (5.55)$$

Therefore, we have:

$$E[W_{exp/weib}] > E[W_{weib/geom}]. \quad (5.56)$$

Figures 5.2–5.30 show comparisons of mean waiting times for PLAYTHROUGH ring under 3 different assumptions: exponential interarrival

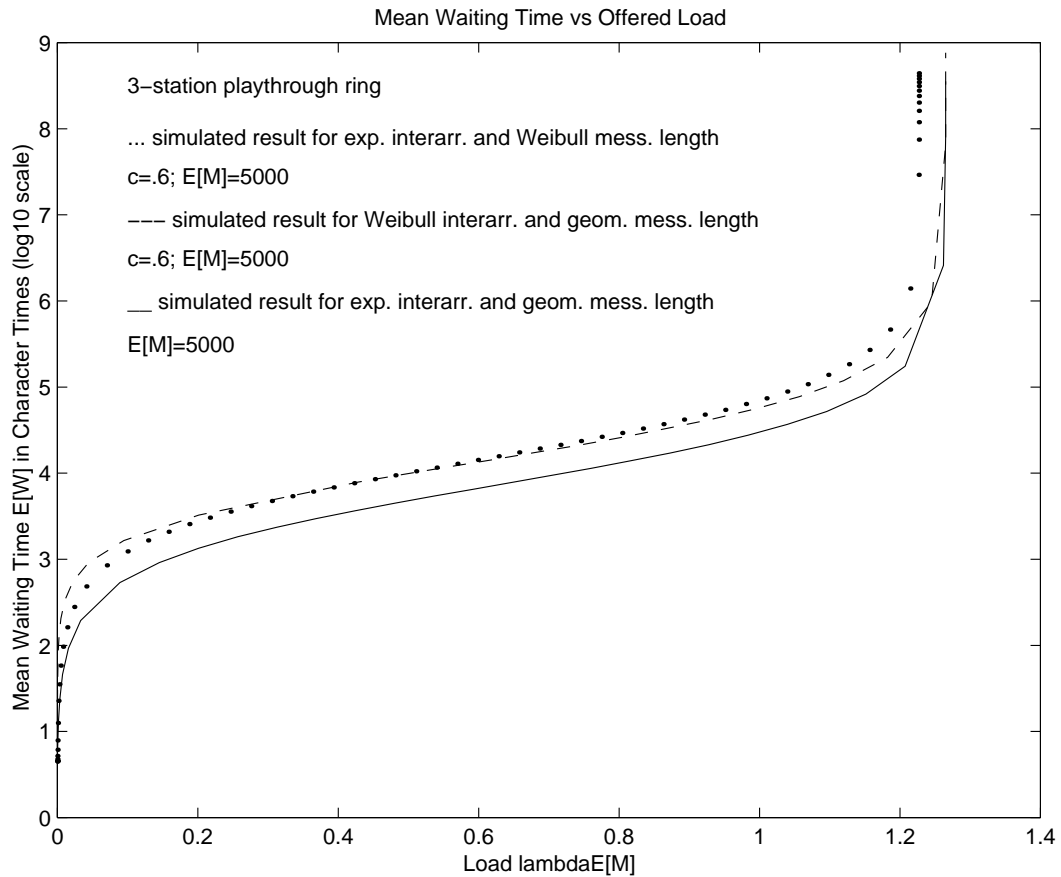


Figure 5.2: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

times and geometric message lengths, Weibull interarrival times and geometric message lengths, exponential interarrival times and Weibull message lengths.

Figure 5.2, Figure 5.3, and Figure 5.4 depict the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths for an $N = 3$ -station PLAYTHROUGH ring with average message lengths of 5000, 1000, and 100 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.6. The plots show that at heavy loads the average message waiting

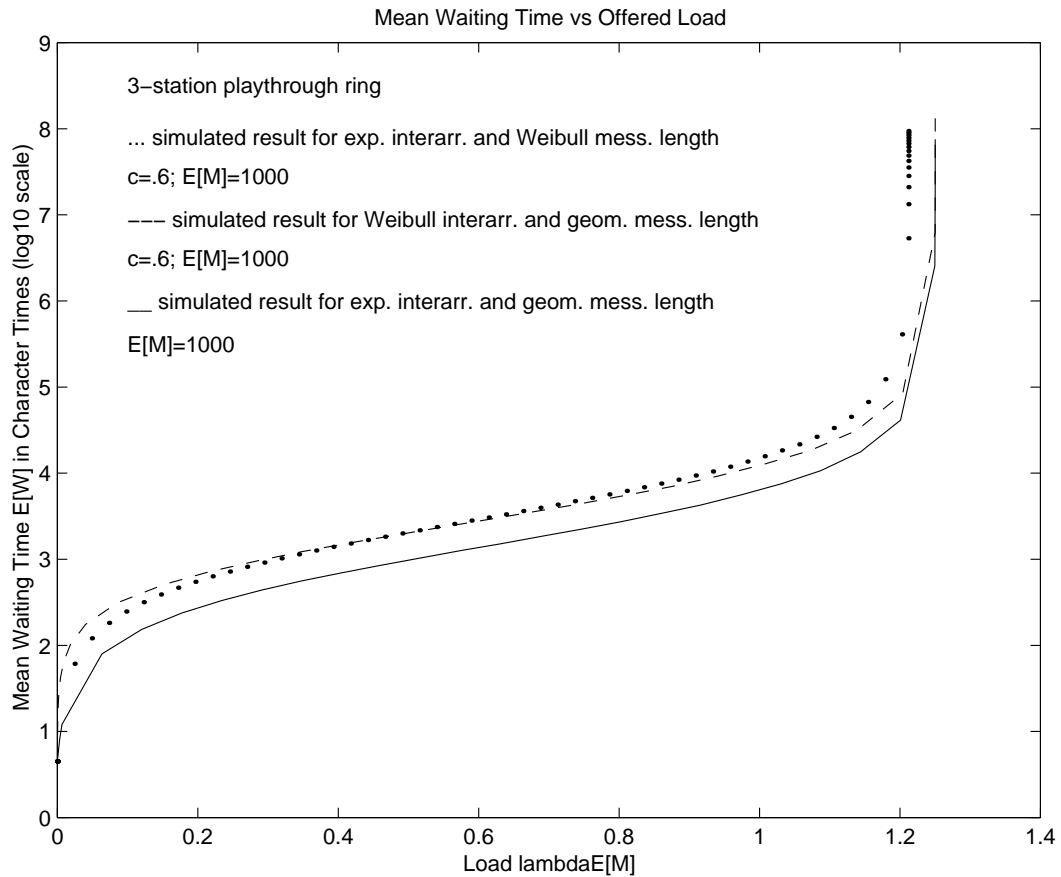


Figure 5.3: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is greater than the average waiting time for PLAYTHROUGH ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. In fact, only at light load does average waiting time for the PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths appear to be greater than average waiting times

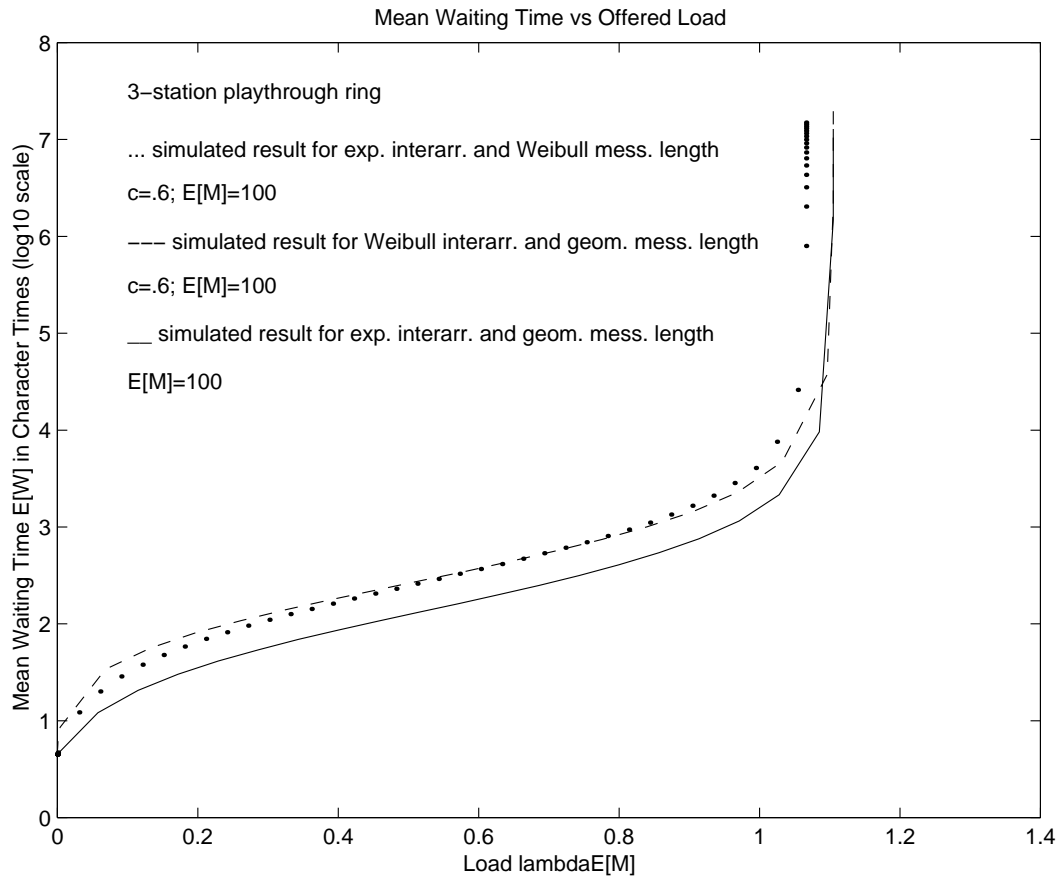


Figure 5.4: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 100$ characters. Comparison for different assumptions.

for the PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths in Figure 5.2, Figure 5.3, and Figure 5.4. There is a small segment of Figure 5.4 where, after saturation, mean waiting times for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths look as if they were greater than mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. But this is solely due to the fact that the solid line that connects simulated mean waiting times of the PLAYTHROUGH ring under exponential interarrival times and geometric message lengths is very coarse in

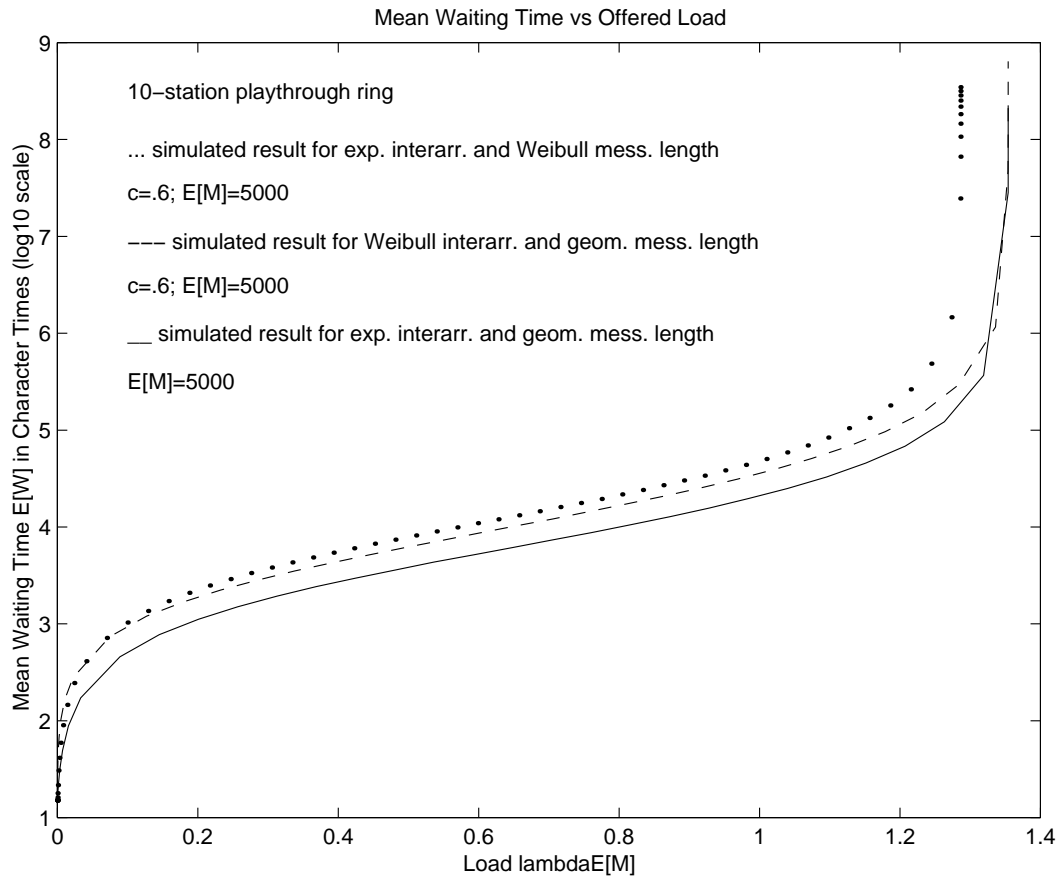


Figure 5.5: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

that segment. The actual simulated mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths are lower than the simulated mean waiting times for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths.

Figure 5.5, Figure 5.6, and Figure 5.7 show the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths for an $N = 10$ -station PLAYTHROUGH ring with average message lengths of 5000,

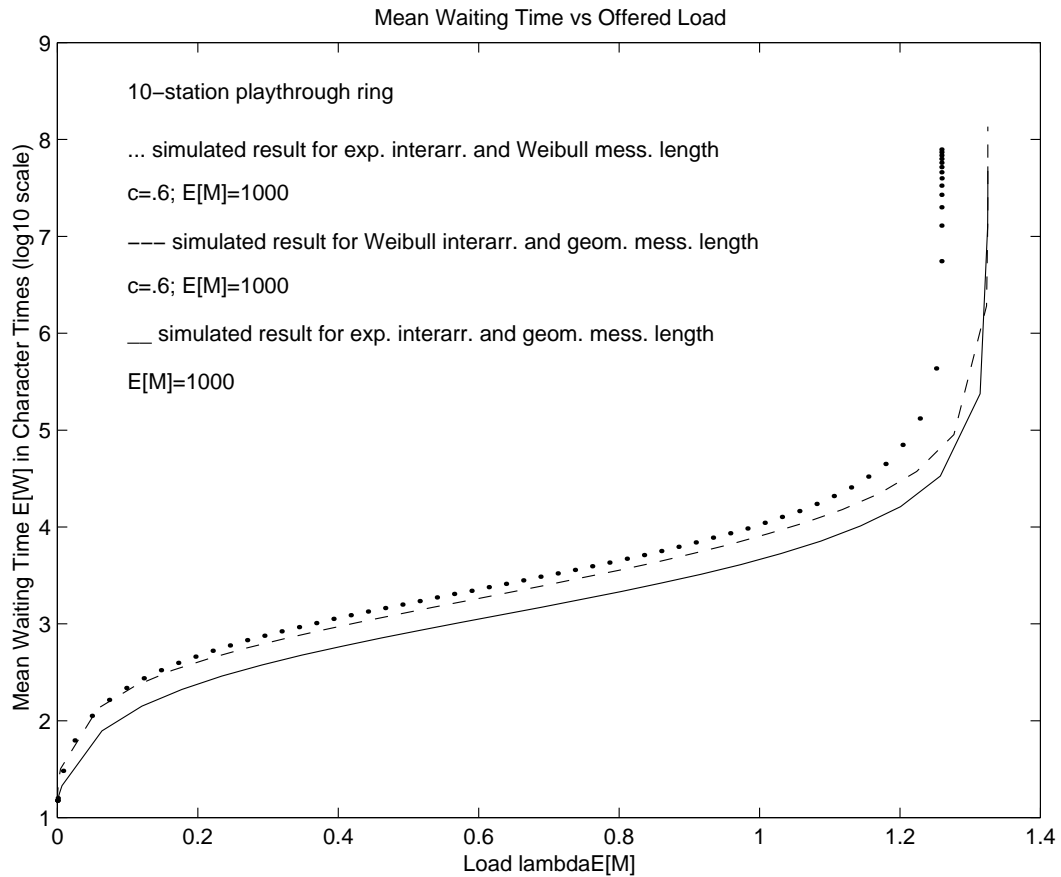


Figure 5.6: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

1000, and 100 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.6. As in Figure 5.2, Figure 5.3, and Figure 5.4, at heavy loads, the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is greater than the average waiting time for PLAYTHROUGH ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric

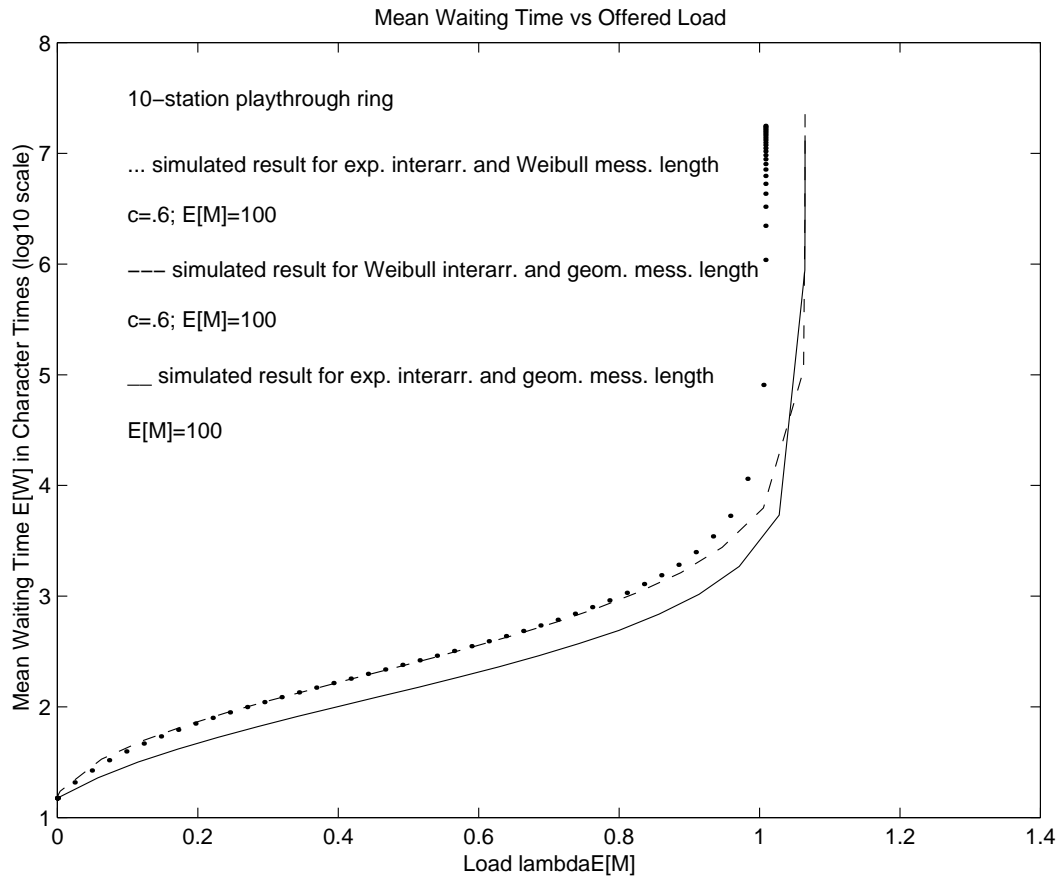


Figure 5.7: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 100$ characters. Comparison for different assumptions.

message lengths. There is a small segment of Figure 5.7 where, after saturation, mean waiting times for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths look as if they were greater than mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. This is due to the fact that the solid line that connects simulated mean waiting times of the PLAYTHROUGH ring under exponential interarrival times and geometric message lengths is very coarse in that segment. The actual simulated mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths are

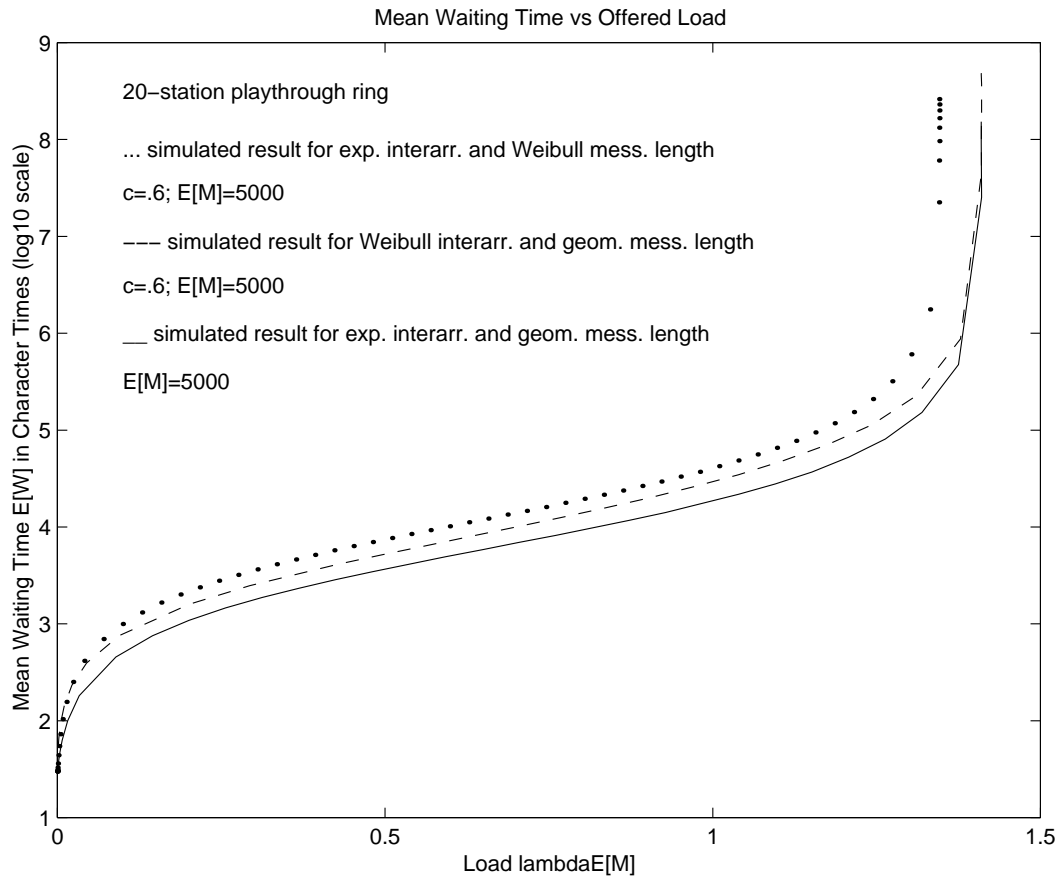


Figure 5.8: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

lower than the simulated mean waiting times for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths.

Figure 5.8, Figure 5.9, and Figure 5.10 show the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths for an $N = 20$ -station PLAYTHROUGH ring with average message lengths of 5000, 1000, and 100 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.6. At heavy loads, the average message waiting time

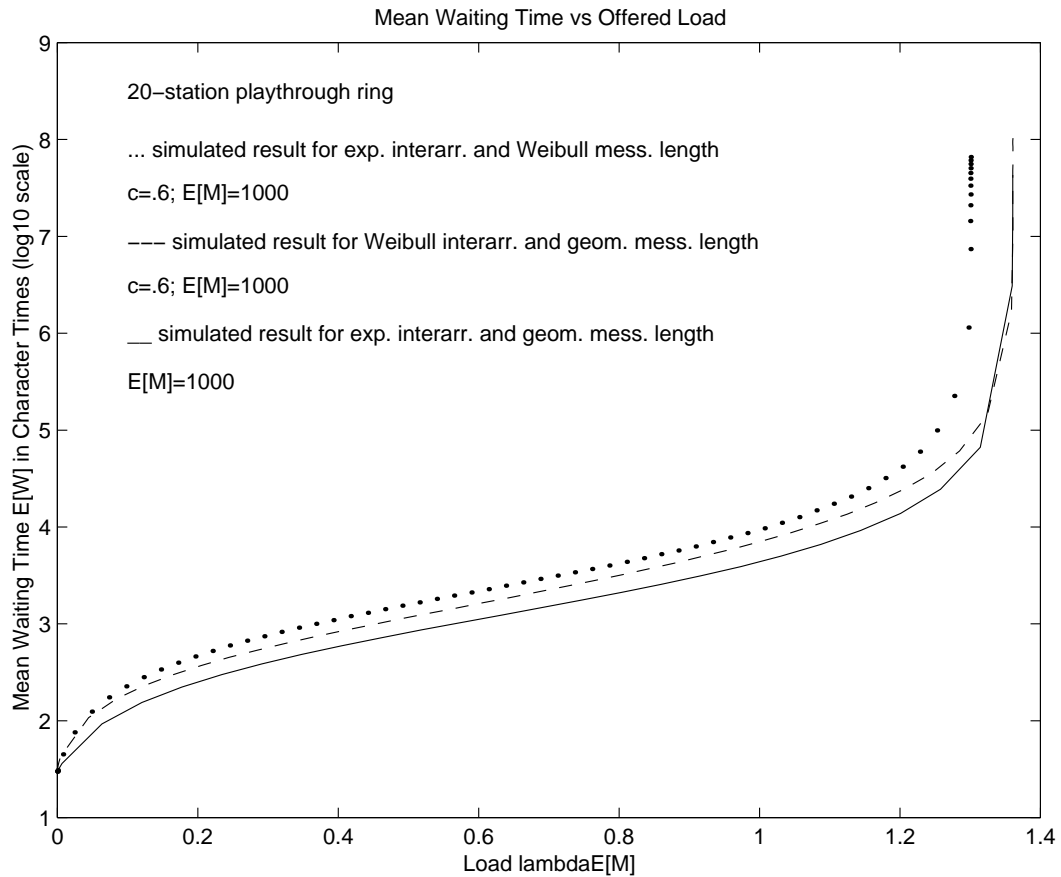


Figure 5.9: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is greater than the average waiting time for PLAYTHROUGH ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. In Figure 5.10 the average waiting times for PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths appear to be nearly equal to the average waiting times under

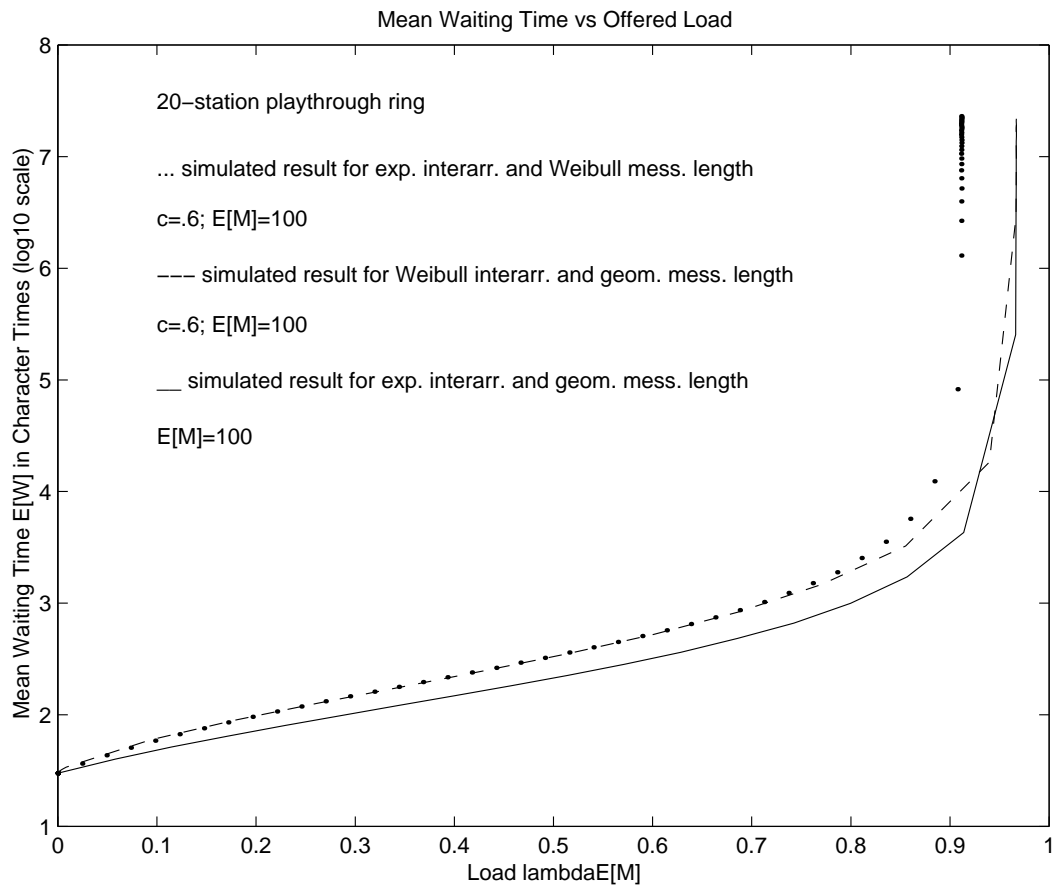


Figure 5.10: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 100$ characters. Comparison for different assumptions.

Weibull interarrival times and geometric message lengths at light and medium loads. Near saturation the mean average waiting times for PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths becomes greater than the mean average waiting times for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths.

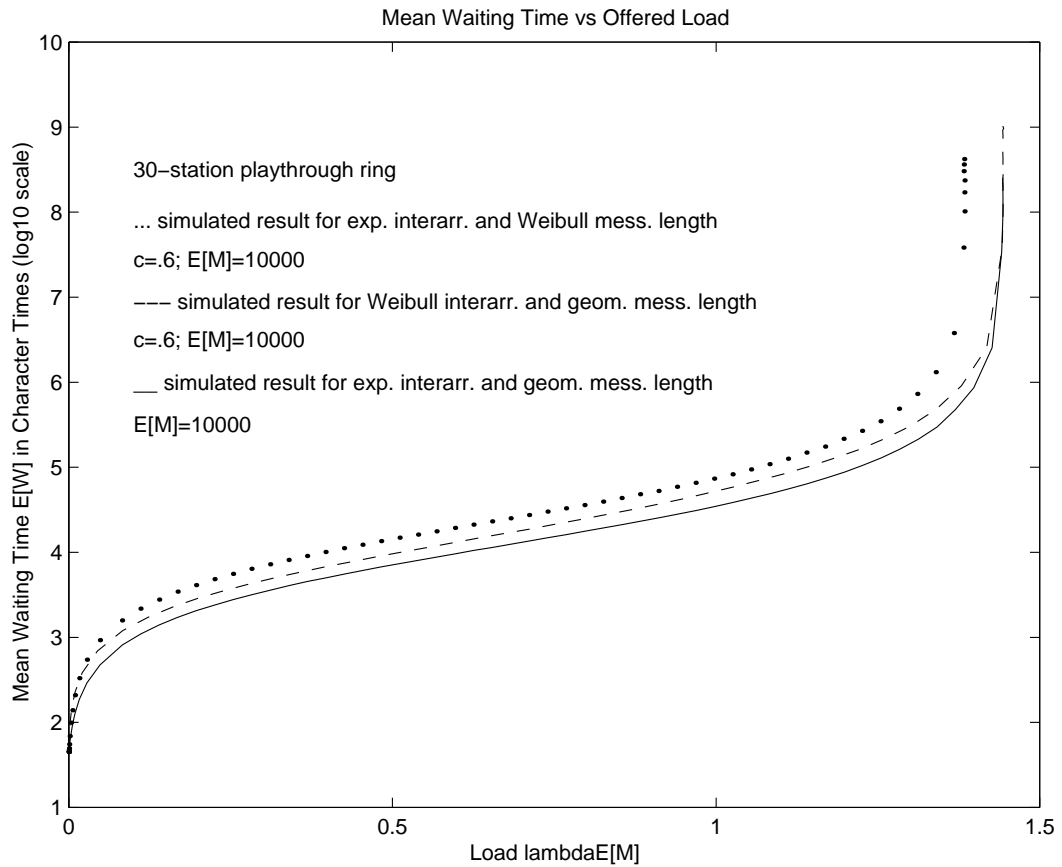


Figure 5.11: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 10000$ characters. Comparison for different assumptions.

Figure 5.11, Figure 5.12, and Figure 5.13 show the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths for an $N = 30$ -station PLAYTHROUGH ring with average message lengths of 10000, 5000, and 1000 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.6. At heavy loads, the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is greater than the average waiting time for PLAYTHROUGH

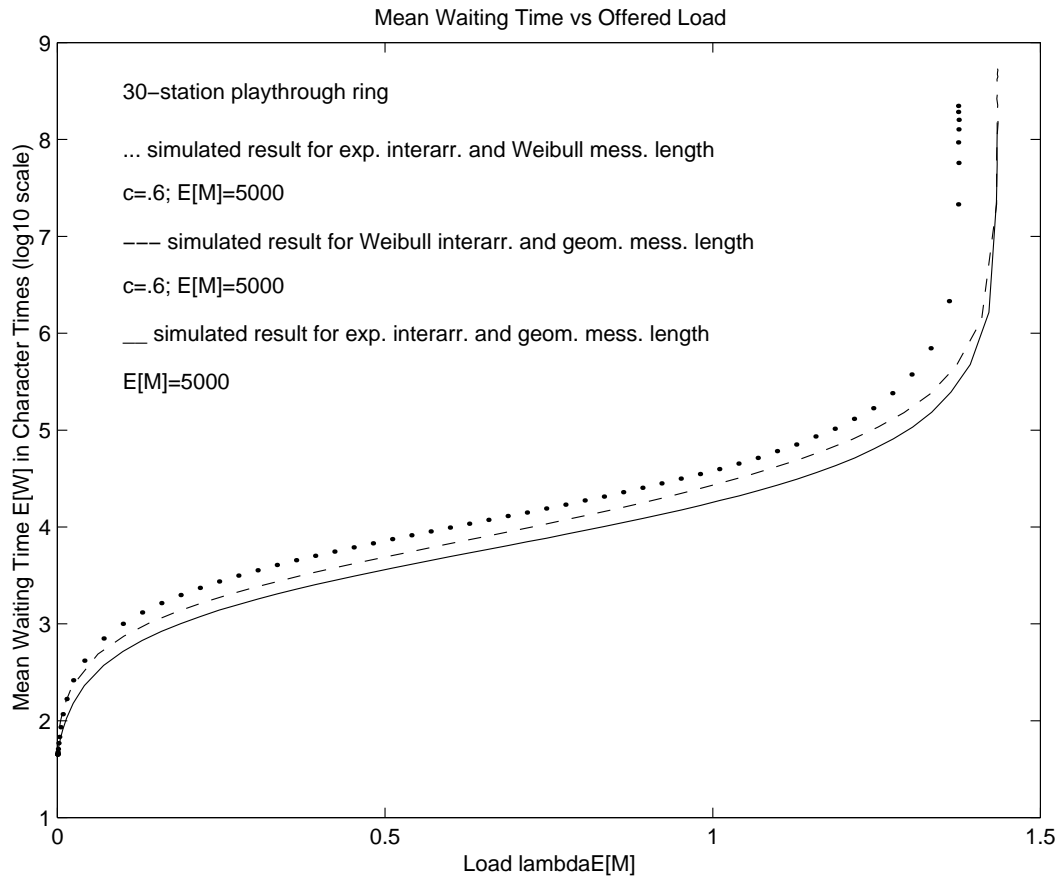


Figure 5.12: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. In fact, the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths appears to be strictly greater than the average waiting time for PLAYTHROUGH ring with Weibull interarrival times and geometric message lengths for all load values in Figure 5.11, Figure 5.12, and Figure 5.13.

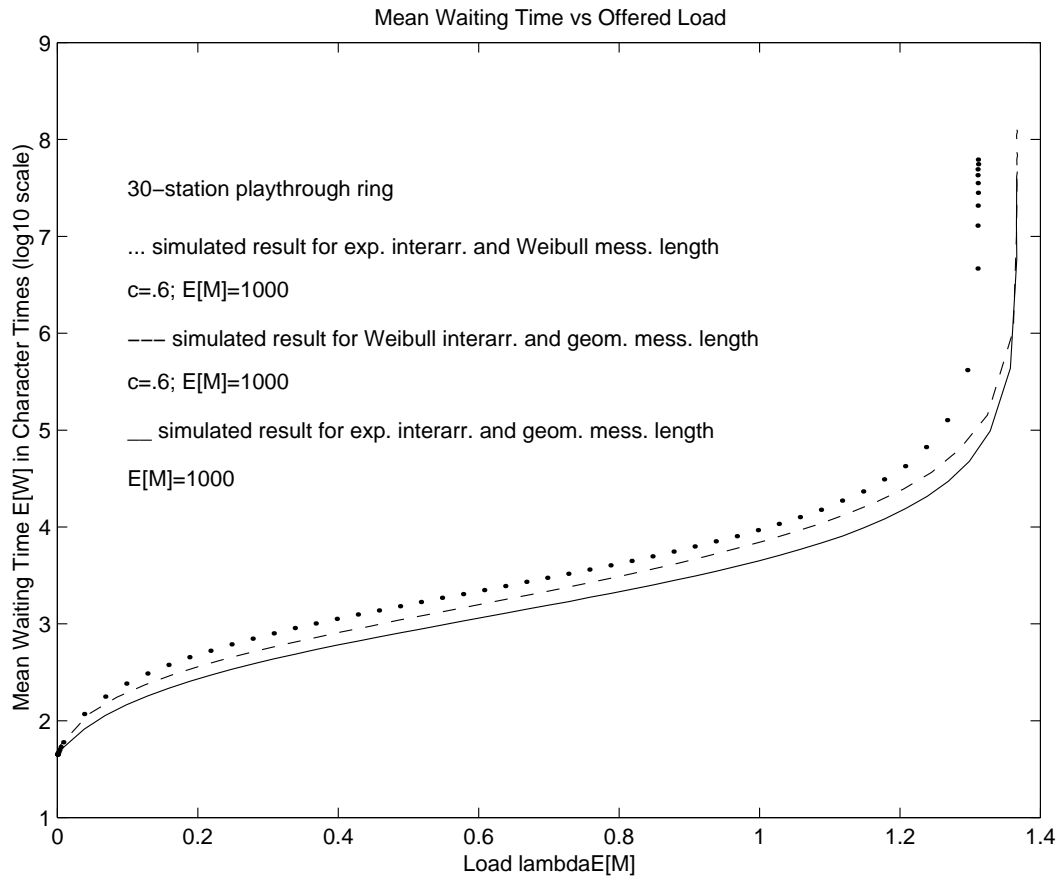


Figure 5.13: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

Moreover, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be strictly greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths for all loads in Figure 5.11, Figure 5.12, and Figure 5.13.

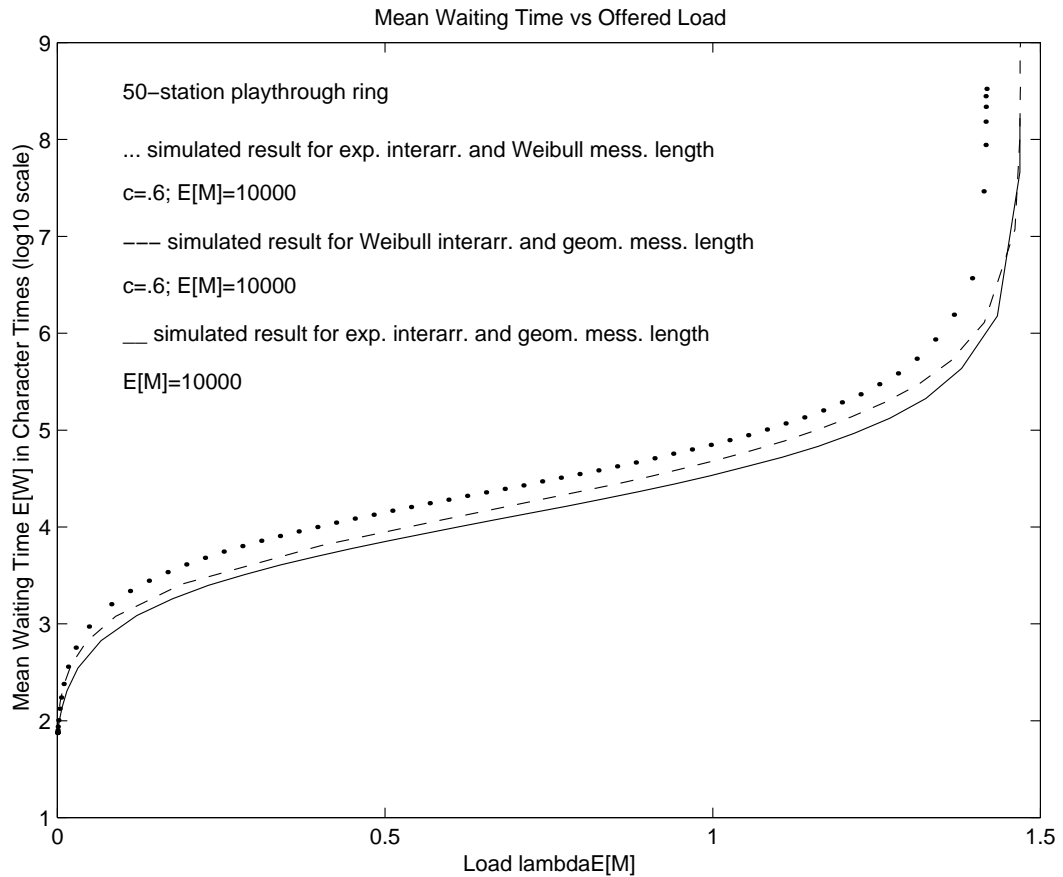


Figure 5.14: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 10000$ characters. Comparison for different assumptions.

Figure 5.14, Figure 5.15, Figure 5.16 show the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths for an $N = 50$ -station PLAYTHROUGH ring with average message lengths of 10000, 5000, and 1000 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.6. At heavy loads, the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is greater than the average waiting time for PLAYTHROUGH

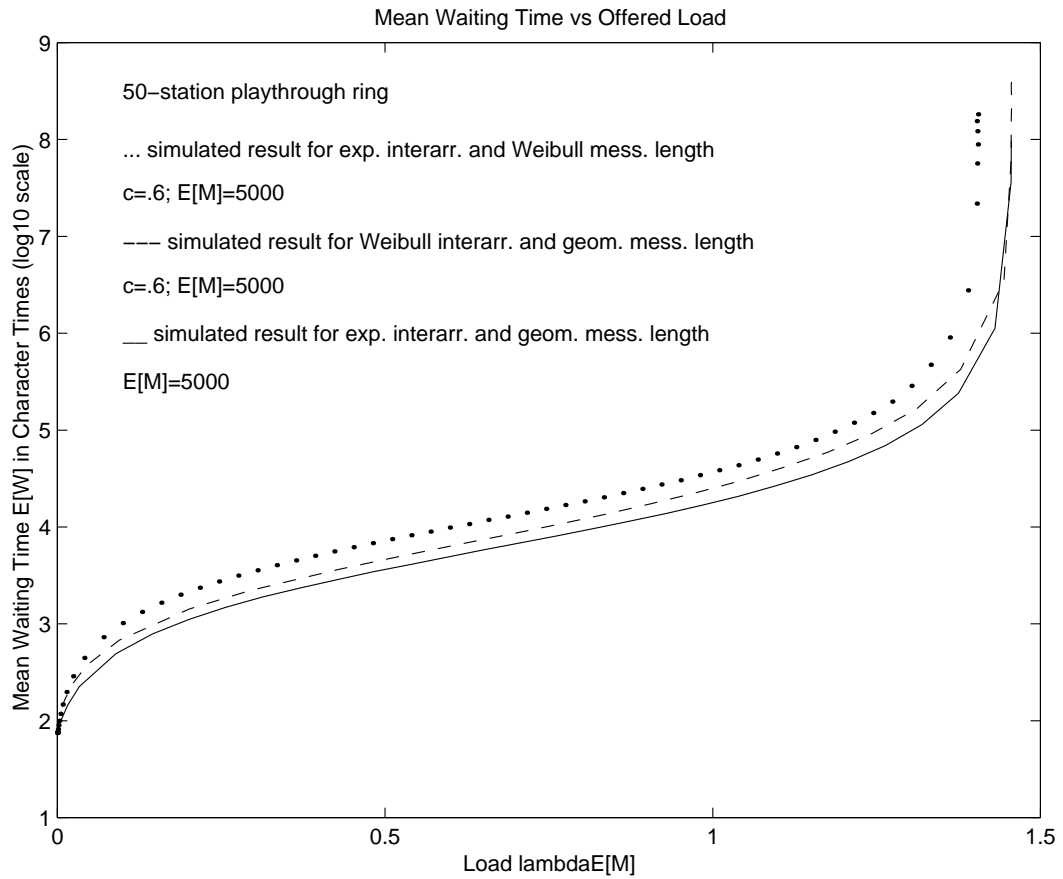


Figure 5.15: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. In fact, the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths appears to be strictly greater than the average waiting time for PLAYTHROUGH ring with Weibull interarrival times and geometric message lengths for all load values in Figure 5.14, Figure 5.15, and Figure 5.16.

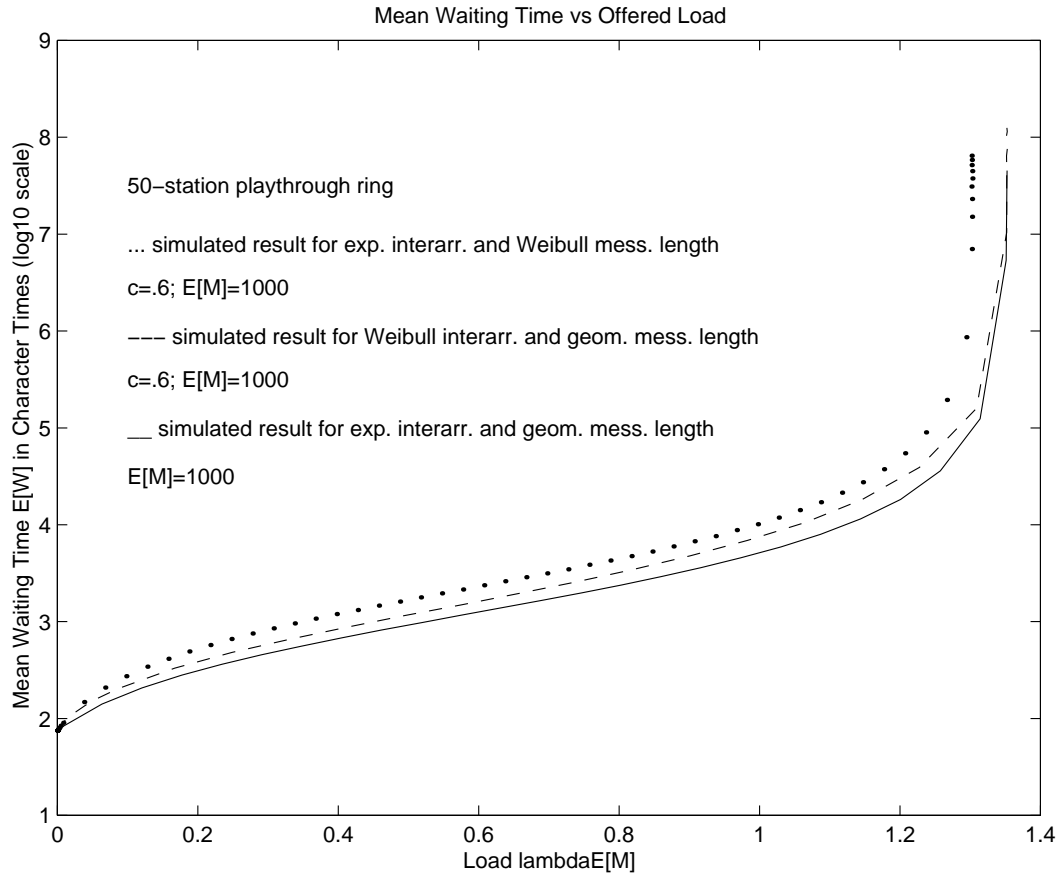


Figure 5.16: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

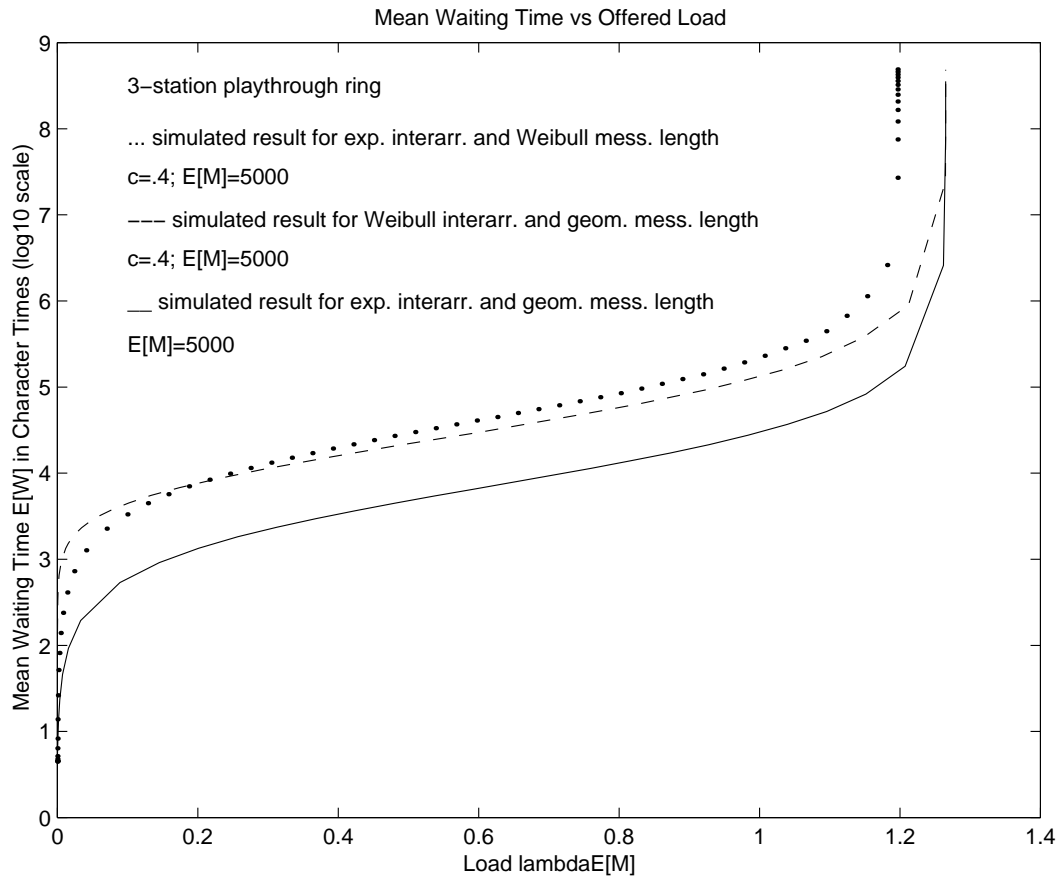


Figure 5.17: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

Figure 5.17, Figure 5.18, and Figure 5.19 depict the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths for an $N = 3$ PLAYTHROUGH ring with average message lengths of 5000, 1000, and 100 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.4. The plots show that at heavy loads the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is greater than the average waiting time for PLAYTHROUGH

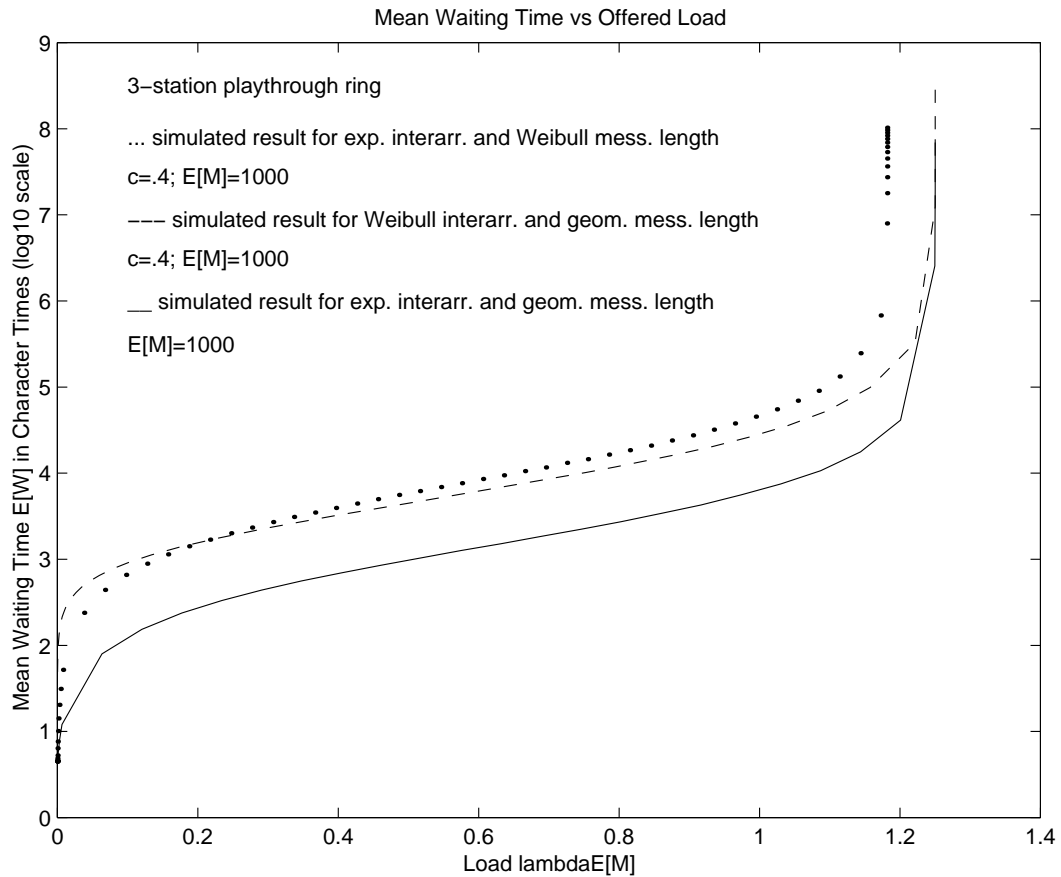


Figure 5.18: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. In fact, only at light load does average waiting time for the PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths appear to be greater than average waiting times for the PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths in Figure 5.17, Figure 5.18, and Figure 5.19.

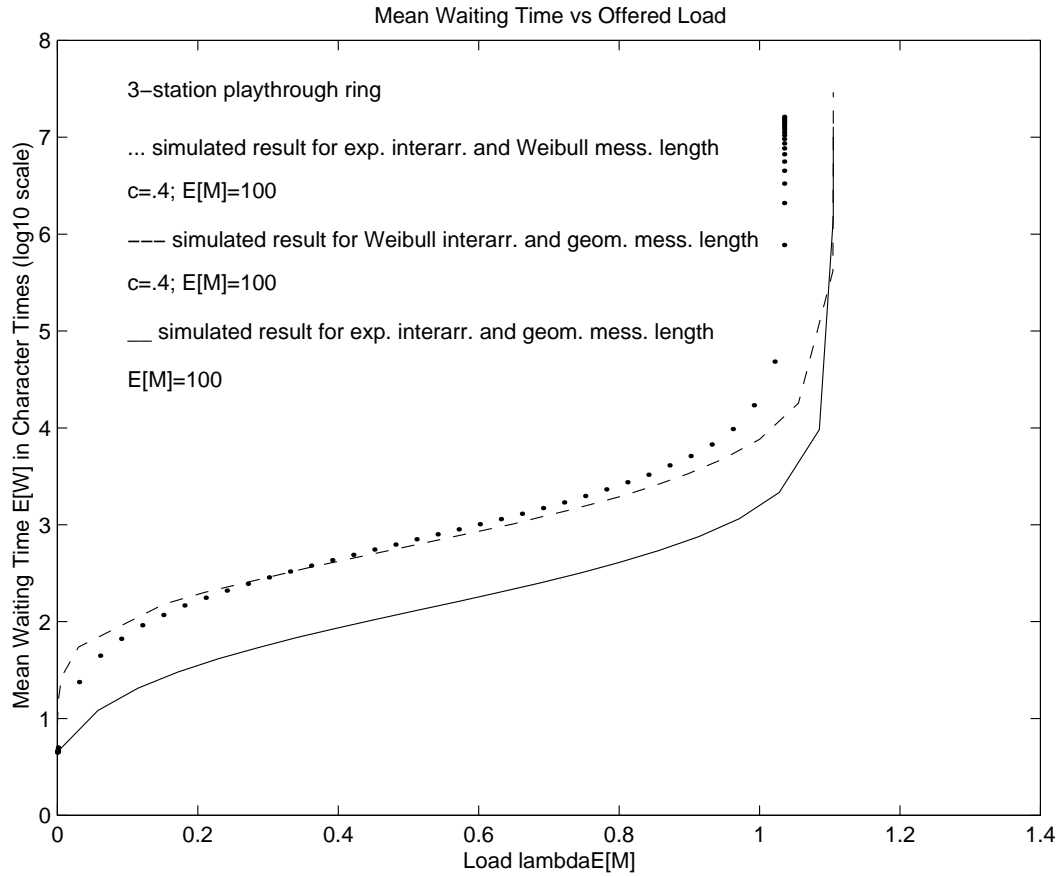


Figure 5.19: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 3$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 100$ characters. Comparison for different assumptions.

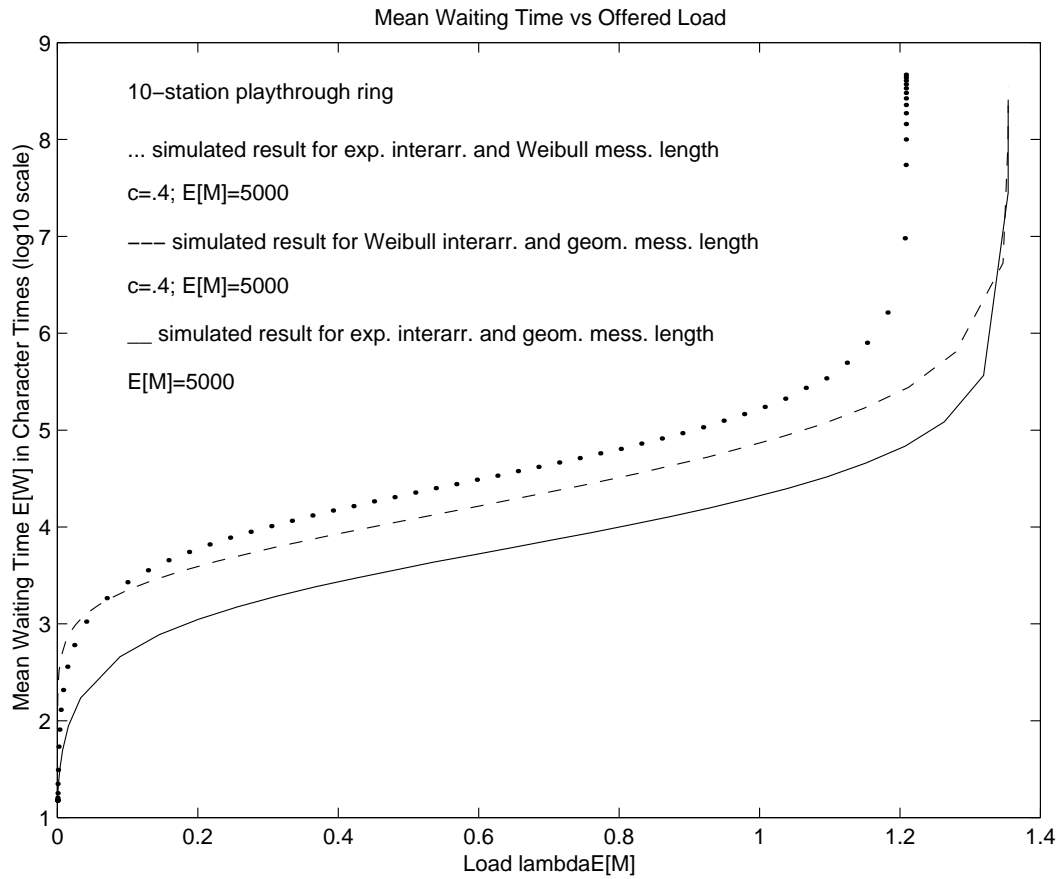


Figure 5.20: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

Figure 5.20 and Figure 5.21 show the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths for an $N = 10$ PLAYTHROUGH ring with average message lengths of 5000, 1000, and 100 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.4. The plots show that at heavy loads the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is greater than the average waiting time for PLAYTHROUGH

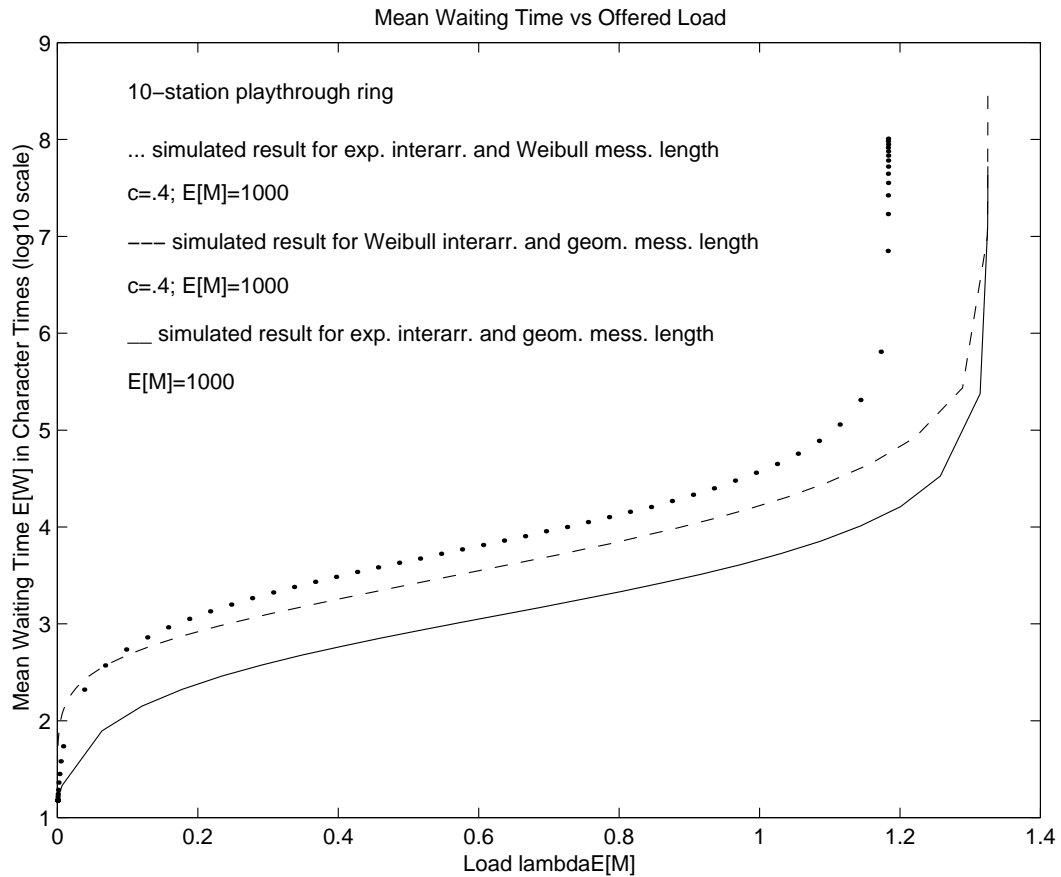


Figure 5.21: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 10$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. In fact, only at light load does average waiting time for the PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths appear to be greater than average waiting times for the PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths in Figure 5.20 and Figure 5.21.

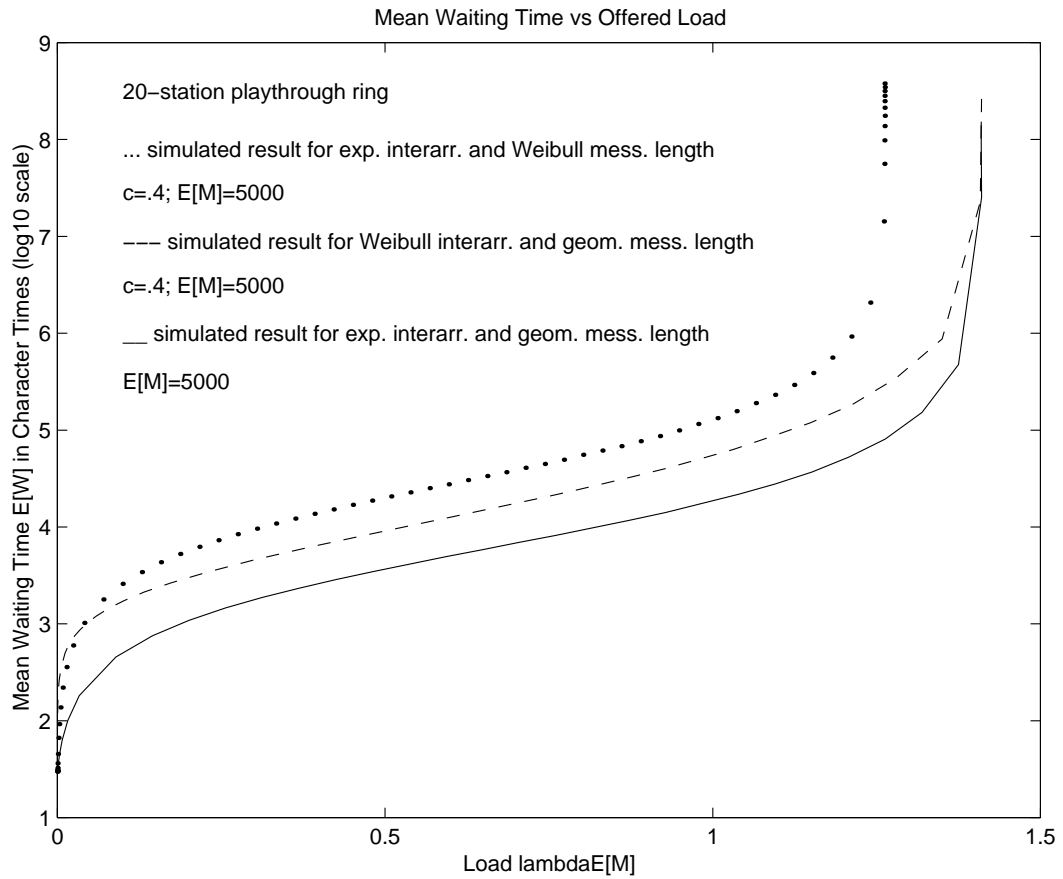


Figure 5.22: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

Figure 5.22, Figure 5.23, and Figure 5.24 show the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths for an $N = 20$ -station PLAYTHROUGH ring with average message lengths of 5000, 1000, and 100 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.4. The plots show that at heavy loads the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is greater than the average waiting time for

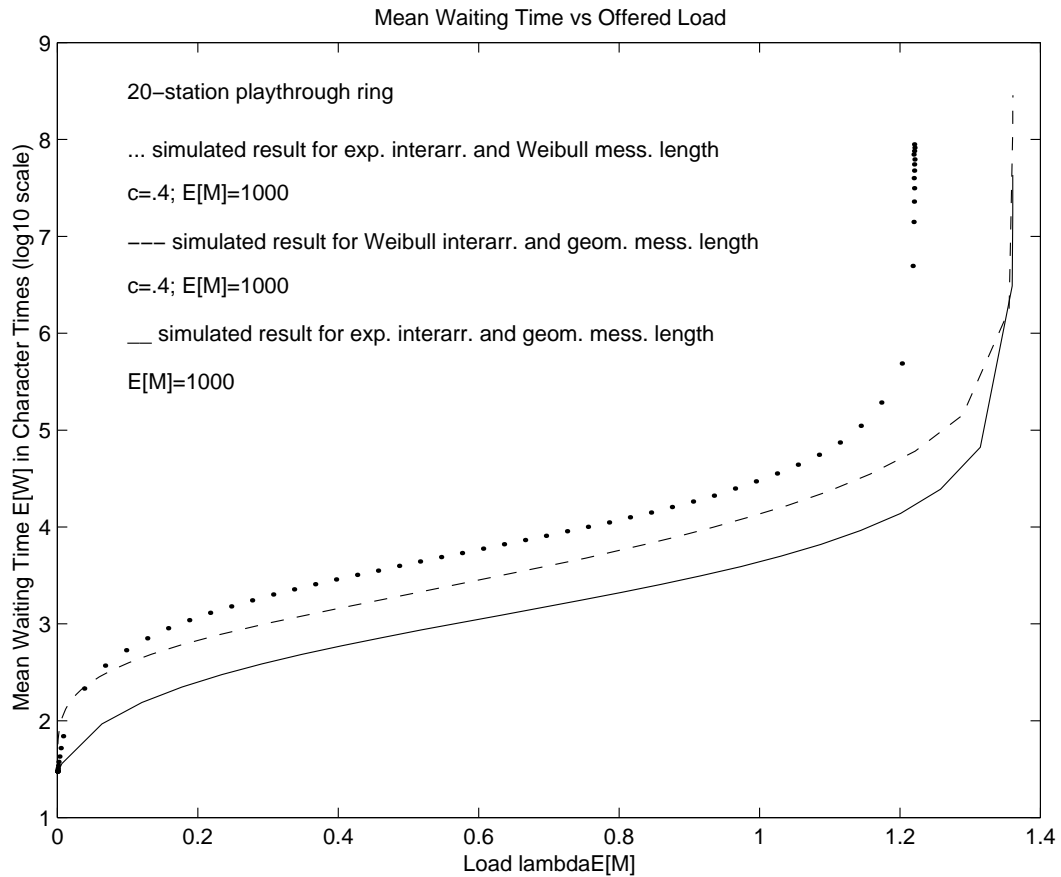


Figure 5.23: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

PLAYTHROUGH ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. In fact, only at light load does average waiting time for the PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths appear to be greater than average waiting times for the PLAYTHROUGH ring under exponential interarrival times and Weibull message

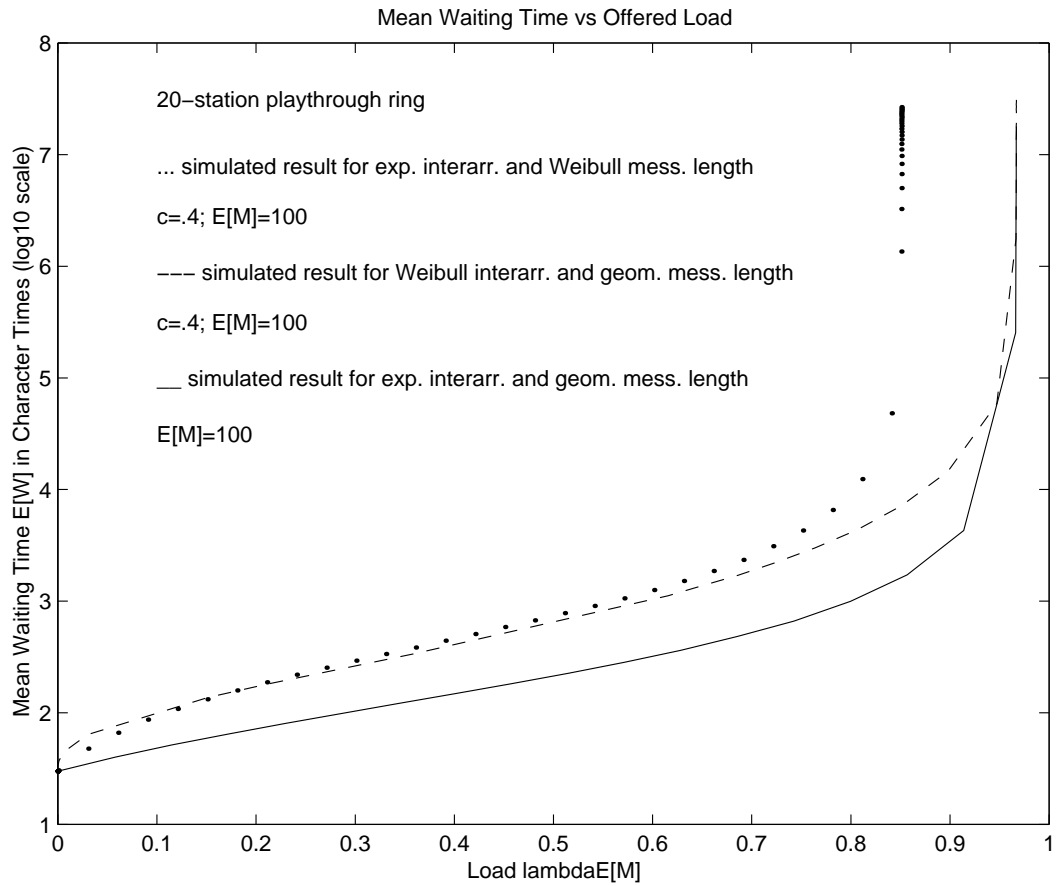


Figure 5.24: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 20$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 100$ characters. Comparison for different assumptions.

lengths in Figure 5.22, Figure 5.23, and Figure 5.24. At heavy load, the discrepancy between the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message and the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is larger than that observed in Figure 5.8, Figure 5.9, and Figure 5.10, where the shape parameter was assumed to be 0.6.

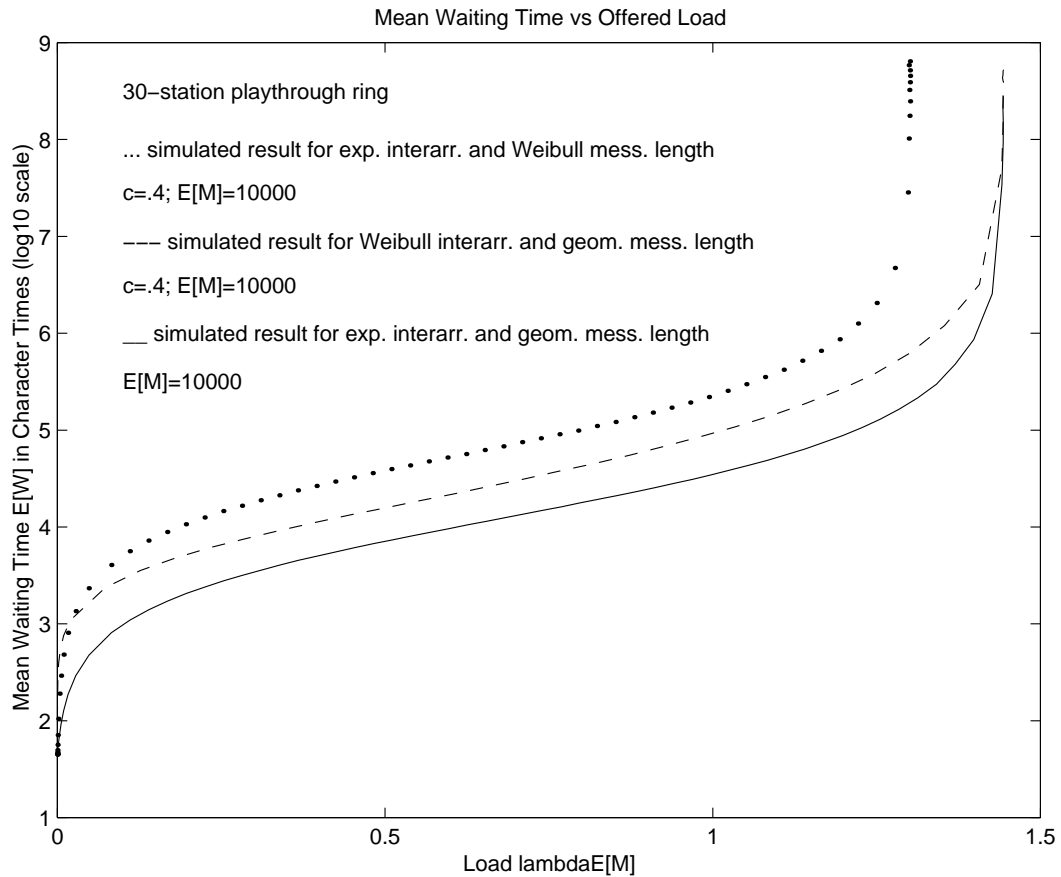


Figure 5.25: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 10000$ characters. Comparison for different assumptions.

Figure 5.25, Figure 5.26, and Figure 5.27 show the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths for an $N = 30$ -station PLAYTHROUGH ring with average message lengths of 10000, 5000, and 1000 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.4. The plots show that at heavy loads the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is greater than the average waiting time for

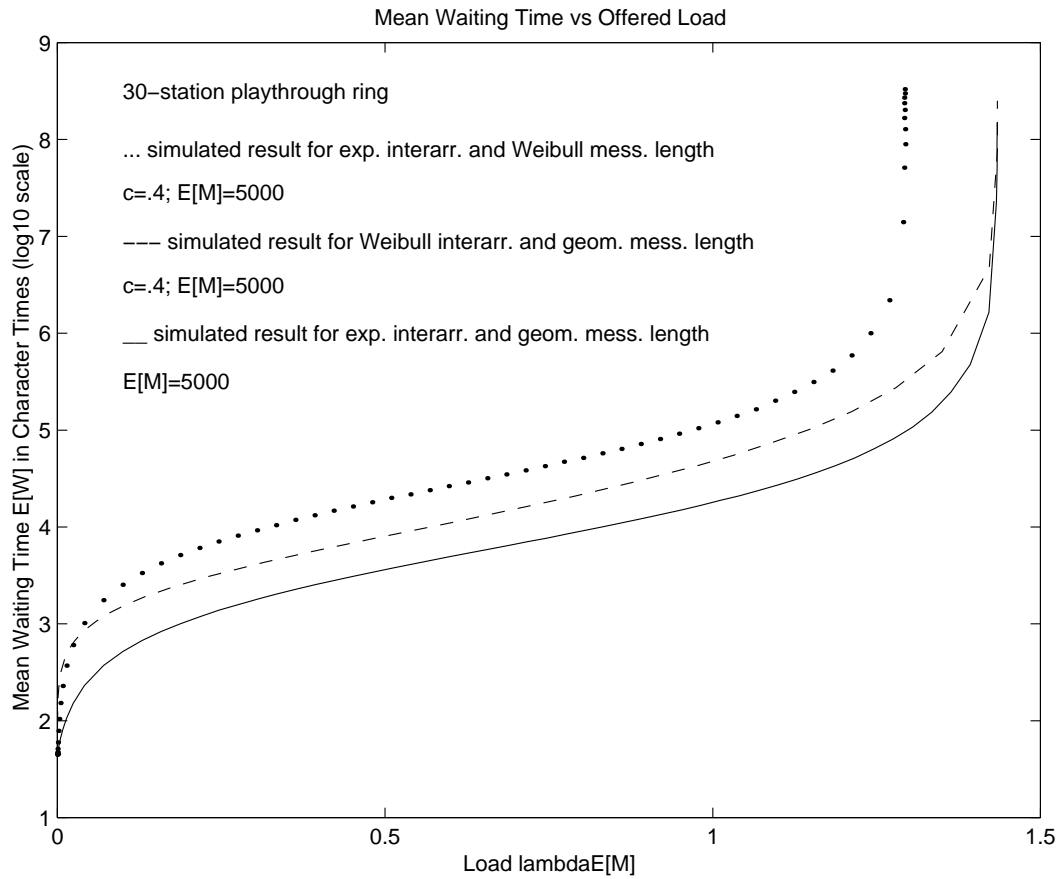


Figure 5.26: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

PLAYTHROUGH ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. At heavy load, the discrepancy between the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message and the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is larger than that

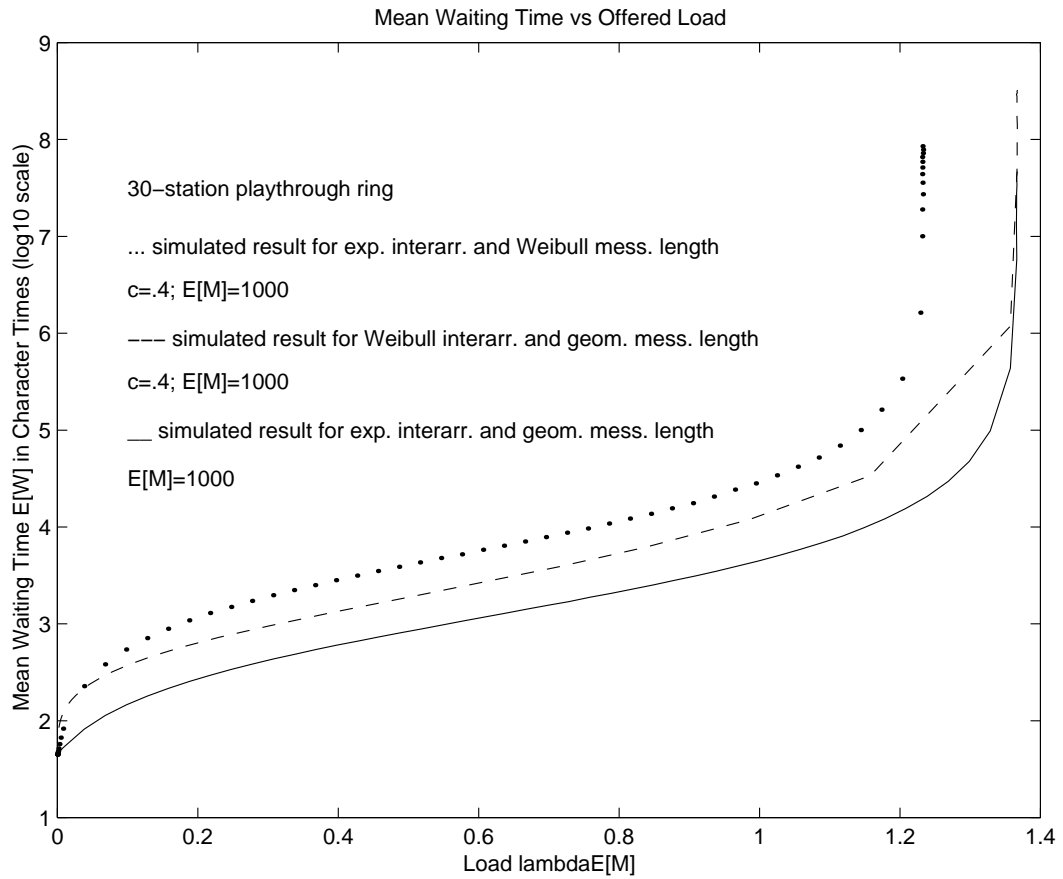


Figure 5.27: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 30$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

observed in Figure 5.11, Figure 5.12, and Figure 5.13, where the shape parameter was assumed to be 0.6.

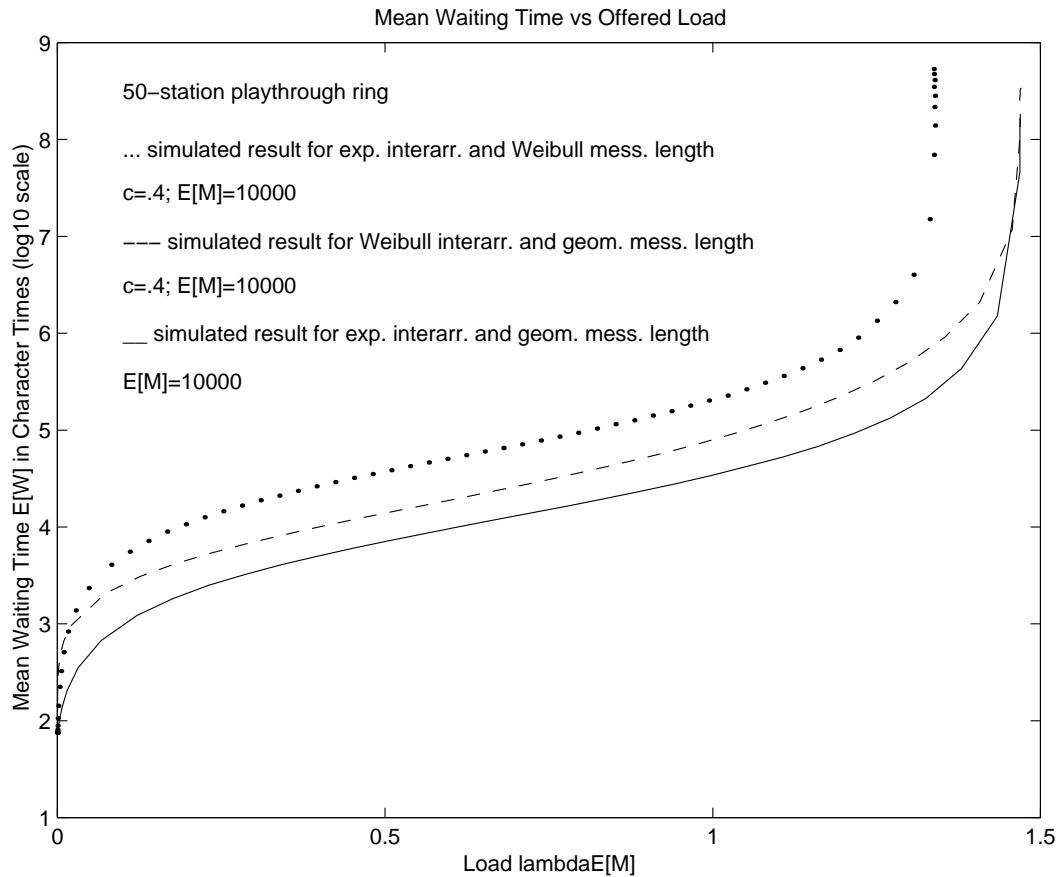


Figure 5.28: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 10000$ characters. Comparison for different assumptions.

Figure 5.25, Figure 5.26, and Figure 5.27 show the average mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, Weibull interarrival times and geometric message lengths, and exponential interarrival times and Weibull message lengths. The number of stations N on the PLAYTHROUGH ring is assumed to be 50 and the average message lengths are 10000, 5000, and 1000 characters. The shape parameter c of the Weibull distribution is assumed to be equal to 0.4. The plots show that at heavy loads the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message

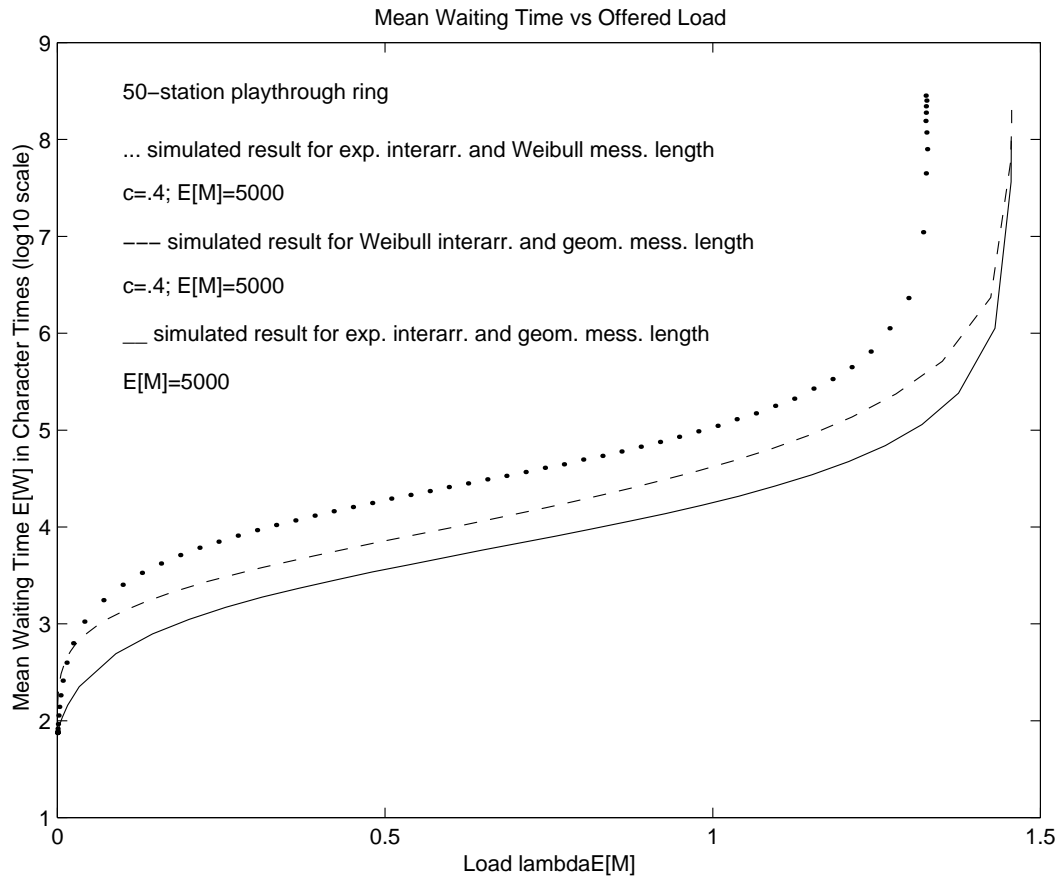


Figure 5.29: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 5000$ characters. Comparison for different assumptions.

lengths is greater than the average waiting time for PLAYTHROUGH ring with Weibull interarrival times and geometric message lengths. In addition, at heavy loads, the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message lengths appears to be greater than the average waiting time for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. At heavy load, the discrepancy between the average waiting time for PLAYTHROUGH under Weibull interarrival times and geometric message and the average message waiting time for PLAYTHROUGH ring with exponential interarrival times and Weibull message lengths is larger

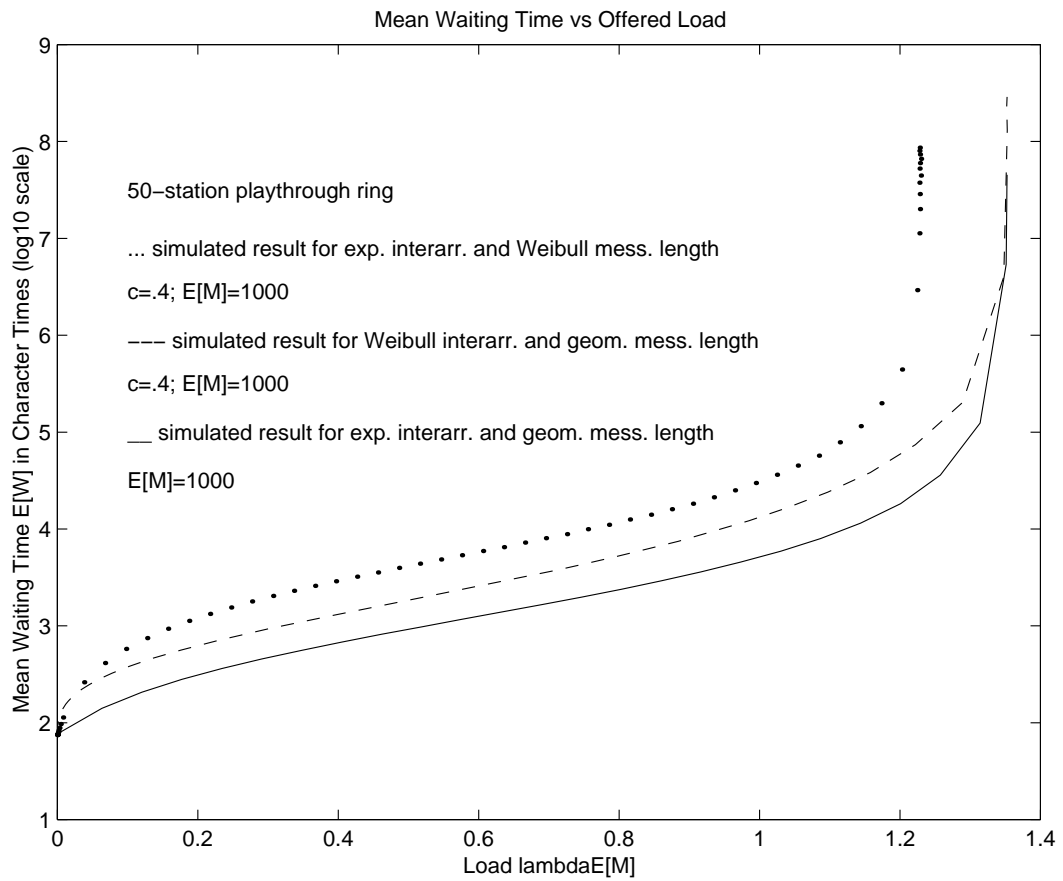


Figure 5.30: Mean message waiting time $E[W]$ vs. offered load $\lambda E[M]$ for $N = 50$ station simplex PLAYTHROUGH ring, uniform and symmetric traffic (UST), and mean message length $E[M] = 1000$ characters. Comparison for different assumptions.

than that observed in Figure 5.14, Figure 5.15, and Figure 5.16, where the shape parameter was assumed to be 0.6.

5.6 Conclusions

In this chapter, we proved analytically that at heavy loads mean waiting times for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths can be bounded. We showed that mean waiting times for PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths are upper bounds to mean waiting times for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths at heavy load, with the same mean interarrival times and the same mean message lengths. Furthermore, we showed that mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths are lower bounds to mean waiting times for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths at heavy load, with the same mean interarrival times and the same mean message lengths.

CHAPTER 6

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

6.1 Summary of Contributions

This research made contribution in many areas. The first contribution, found in Chapter 3, is the simulations of message waiting time in PLAYTHROUGH ring under self-similar traffic. The second contribution, found in Chapter 4, is a suite of analytical approximations for various parameters of PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths. No prior analytical models for PLAYTHROUGH ring under self-similar traffic have been presented. The third contribution, found in Chapter 5, is the establishment of upper and lower bounds for waiting times in PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths. Queueing problems involving Weibull interarrival times have been shown to be very difficult to solve. This work represents an original way of bounding waiting time for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths.

The first contribution made by this work is the simulation of the performance of PLAYTHROUGH ring under self-similar traffic. Self-similar traffic can be generated using heavy-tailed distributions. Feldmann made measurements on local networks which revealed that the Weibull distribution with shape parameters ranging from 0.4 to 0.6 was most suitable for modeling

TCP connection interarrival times. Park, Kim and Crovella showed that the reliable transfer of files whose sizes are drawn from heavy-tailed distributions causes self-similar traffic. Based on those works, simulation results for message waiting times in PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths were presented. In addition, simulation results for message waiting times in PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths were presented. Those simulation results show waiting times that are worse than those obtained for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths, which had always been assumed prior to this work.

The second contribution of this research is a suite of analytical approximations for various performance parameters in PLAYTHROUGH ring when it operates under exponential interarrival times and Weibull message lengths. The average number of round trips of GO required to transmit a message is approximated. A tight upper bound for the average number of round trips of GO required to transmit a message was found. This tight upper bound of the average number of round trips of GO required to transmit a message was used to approximate the average number of round trips of GO . Higher moments of the number of round trips of GO were found. An approximation for the moments of basic transmission time and basic service time were found using the moments of the number of round trips of GO . It was shown that prior models for the control frame interruption rate are applicable when message interarrival times are exponentially distributed and message lengths are Weibull distributed. An approximation was given for the control frame round trip time. Models were presented for transmission and service time. A new and elaborate model was given for the first and second moment of blocking duration. A model

for average message waiting time was derived based on the operation of PLAYTHROUGH ring.

The third contribution achieved by this research was the establishment of analytical bounds for mean message waiting time for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths at heavy loads. The fact that the Laplace transform of the Weibull distribution does not have a closed form renders the problem of finding an analytical model for waiting times in PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths highly intractable. We expect the waiting time in a queueing system to become unstable at heavy loads. Thus, finding analytical bounds at heavy loads on the mean message waiting time in PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths is a very significant result. We showed that at heavy loads, the waiting time of PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths is an upper bound to the waiting time of PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths. We also showed that waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths are lower bounds to the waiting time for PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths at heavy loads. These analytical proofs were validated by simulation results.

6.2 Suggestions for Further Research

Throughout this dissertation, we considered uniform and symmetric traffic. Uniform traffic implies that each station transmits the same amount of traffic. Symmetric traffic means that each station transmits to any other station on the

ring with the same probability. Often, a server, which transmits more traffic than other stations, may be present on the ring. This would result in asymmetric traffic. In addition, all stations on the ring may not have the same mean message lengths and the same arrival rates. The results presented in this dissertation may be extendable to nonuniform or asymmetric traffic.

In Chapter 5 we showed analytically that at heavy loads waiting times for messages in PLAYTHROUGH ring under Weibull message interarrival times and geometric message lengths are less than mean waiting times for messages in PLAYTHROUGH ring under exponential interarrival times and Weibull message lengths. We also showed analytically that, at heavy loads, waiting times for messages in PLAYTHROUGH ring under Weibull message interarrival times and geometric message lengths are greater than mean waiting times for PLAYTHROUGH ring under exponential interarrival times and geometric message lengths. It might be possible to achieve more precision as to where those results holds. In addition, it might be possible to determine the conditions under which those results hold for all loads, not just heavy loads.

Finding solutions to queueing problems involving Weibull interarrival times is a topic of ongoing research. With new findings in this area, it may be possible to find more precise analytical approximations for waiting times in PLAYTHROUGH ring under Weibull interarrival times and geometric message lengths.

APPENDIX A

STOCHASTIC IDENTITIES

Some useful stochastic identities are listed in the following sections.

A.1 Probability Identities

Let \mathcal{A} and \mathcal{B} be two mutually exclusive events, where $\mathcal{A} \cap \mathcal{B} = \emptyset$. The conditional probability that \mathcal{A} occurs given \mathcal{B} does *not* occur is

$$\begin{aligned} Pr[\mathcal{A} | \overline{\mathcal{B}}] &= \frac{Pr[\mathcal{A} \cap \overline{\mathcal{B}}]}{Pr[\overline{\mathcal{B}}]} \\ &= \frac{Pr[\mathcal{A}]}{Pr[\overline{\mathcal{B}}]} \\ &= \frac{Pr[\mathcal{A}]}{1 - Pr[\mathcal{B}]}. \end{aligned} \tag{A.1}$$

A.2 Transform Identities

We give some transform identities that were presented by Kleinrock [Kle75].

Let X be a random variable with probability distribution function $F_X(x) = Pr[X \leq x]$ and Laplace-Stieltjes transform $X^*(s)$. The moment generating property of the Laplace-Stieltjes transform is given by:

$$\begin{aligned} E[X^n] &= (-1)^n \left. \frac{d^n X^*(s)}{ds^n} \right|_{s=0} \\ &= (-1)^n X^{*(n)}(0). \end{aligned} \tag{A.2}$$

If X is a discrete random variable with probability mass function $g_k = Pr[X = k]$ and probability generating function $\hat{X}(z)$, the following

identities are true.

$$\hat{X}(1) = 1 \quad (\text{A.3})$$

$$\hat{X}'(1) = E[X] \quad (\text{A.4})$$

$$\hat{X}''(1) = E[X(X-1)] = E[X^2] - E[X] \quad (\text{A.5})$$

$$\hat{X}'''(1) = E[X(X-1)(X-2)] = E[X^3] - 3E[X^2] + 2E[X] \quad (\text{A.6})$$

$$\hat{X}^{(n)}(1) = E[X(X-1)(X-2)\cdots(X-n+1)], \quad n = 4, 5, 6, \dots \quad (\text{A.7})$$

A.3 Generating Function Identities

The following identities involving probability generating functions are derived.

$$\sum_{j=1}^{\infty} f_j z^j = \hat{f}(z) - f_0. \quad (\text{A.8})$$

$$\begin{aligned} \sum_{j=1}^{\infty} \sum_{k=1}^j f_{j-k} g_k z^j &= \sum_{k=1}^{\infty} \sum_{j=k}^{\infty} f_{j-k} g_k z^j \\ &= \left(\sum_{k=1}^{\infty} g_k z^k \right) \left(\sum_{j-k=0}^{\infty} f_{j-k} z^{j-k} \right) \\ &= \hat{f}(z) [\hat{g}(z) - g_0]. \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} \sum_{j=1}^{\infty} \sum_{k=1}^{j+1} f_{j-k+1} g_k z^j &= \sum_{k=1}^{\infty} \sum_{j=k-1}^{\infty} f_{j-k+1} g_k z^j - f_0 g_1 \\ &= z^{-1} \left(\sum_{k=1}^{\infty} g_k z^k \right) \left(\sum_{j-k+1=0}^{\infty} f_{j-k+1} z^{j-k+1} \right) - f_0 g_1 \\ &= z^{-1} \hat{f}(z) [\hat{g}(z) - g_0] - f_0 g_1. \end{aligned} \quad (\text{A.10})$$

$$\sum_{j=0}^{\infty} \sum_{k=1}^{j+1} f_{j-k+1} g_k z^j = z^{-1} \hat{f}(z) [\hat{g}(z) - g_0]. \quad (\text{A.11})$$

APPENDIX B

DERIVATIVE CALCULATIONS FOR WAITING TIME MODEL

The following are derivatives of some variables used in Chapter 4 Section 4.7.

$$\hat{R}_d(z) = f_d. \quad (\text{B.1})$$

$$\hat{R}'_d(z) = 0. \quad (\text{B.2})$$

$$\hat{R}''_d(z) = 0. \quad (\text{B.3})$$

$$\hat{R}_d(1) = f_d. \quad (\text{B.4})$$

$$\hat{R}'_d(1) = 0. \quad (\text{B.5})$$

$$\hat{R}''_d(1) = 0. \quad (\text{B.6})$$

$$\hat{H}(z) = \sum_{d=1}^{N-1} \hat{A}_d(z) \hat{R}_d(z). \quad (\text{B.7})$$

$$\hat{H}'(z) = \sum_{d=1}^{N-1} \left[\hat{A}_d(z) \hat{R}'_d(z) + \hat{A}'_d(z) \hat{R}_d(z) \right]. \quad (\text{B.8})$$

$$\hat{H}''(z) = \sum_{d=1}^{N-1} \left[\hat{A}_d(z) \hat{R}''_d(z) + 2\hat{A}'_d(z) \hat{R}'_d(z) + \hat{A}''_d(z) \hat{R}_d(z) \right]. \quad (\text{B.9})$$

$$\begin{aligned}
\hat{H}(1) &= \sum_{d=1}^{N-1} \hat{A}_d(1) \hat{R}_d(1) \\
&= \sum_{d=1}^{N-1} (1)(f_d) = 1.
\end{aligned} \tag{B.10}$$

$$\begin{aligned}
\hat{H}'(1) &= \sum_{d=1}^{N-1} \left[\hat{A}_d(1) \hat{R}'_d(1) + \hat{A}'_d(1) \hat{R}_d(1) \right] \\
&= \sum_{d=1}^{N-1} \left[(1)0 + \hat{A}'_d(1) f_d \right] \\
&= \sum_{d=1}^{N-1} f_d \hat{A}'_d(1) \\
&= \hat{A}'(1).
\end{aligned} \tag{B.11}$$

$$\begin{aligned}
\hat{H}''(1) &= \sum_{d=1}^{N-1} \left[\hat{A}_d(1) \hat{R}''_d(1) + 2\hat{A}'_d(1) \hat{R}'_d(1) + \hat{A}''_d(1) \hat{R}_d(1) \right] \\
&= \sum_{d=1}^{N-1} \left[\hat{A}_d(1)0 + 2\hat{A}'_d(1)0 + \hat{A}''_d(1) f_d \right] \\
&= \sum_{d=1}^{N-1} \left[f_d \hat{A}''_d(1) \right] \\
&= \hat{A}''(1).
\end{aligned} \tag{B.12}$$

$$\hat{L}(z) = \sum_{d=1}^{N-1} \left[\hat{R}_d(z) \right]. \tag{B.13}$$

$$\hat{L}'(z) = \sum_{d=1}^{N-1} \left[\hat{R}'_d(z) \right]. \tag{B.14}$$

$$\hat{L}''(z) = \sum_{d=1}^{N-1} \left[\hat{R}''_d(z) \right]. \tag{B.15}$$

$$\hat{L}(1) = \sum_{d=1}^{N-1} [\hat{R}_d(1)]. \quad (\text{B.16})$$

$$\begin{aligned} \hat{L}'(1) &= \sum_{d=1}^{N-1} [\hat{R}'_d(1)] \\ &= 0 \\ \hat{L}''(1) &= \sum_{d=1}^{N-1} [\hat{R}''_d(1)] \\ &= 0. \end{aligned} \quad (\text{B.17})$$

$$\hat{X}(z) = [f_d] [\hat{B}(z) - 1]. \quad (\text{B.18})$$

$$\hat{X}'(z) = [f_d] \hat{B}'(z). \quad (\text{B.19})$$

$$\hat{X}''(z) = [f_d] \hat{B}''(z). \quad (\text{B.20})$$

$$\begin{aligned} \hat{X}(1) &= [f_d] [\hat{B}(1) - 1] \\ &= [f_d] [1 - 1] = 0. \end{aligned} \quad (\text{B.21})$$

$$\hat{X}'(1) = [f_d] \hat{B}'(1) \quad (\text{B.22})$$

$$\hat{X}''(1) = [f_d] \hat{B}''(1). \quad (\text{B.23})$$

$$\hat{Y}(z) = 1 - z^{-1} \hat{H}(z). \quad (\text{B.24})$$

$$\hat{Y}'(z) = -z^{-1} \hat{H}'(z) + z^{-2} \hat{H}(z). \quad (\text{B.25})$$

$$\hat{Y}''(z) = -z^{-1} \hat{H}''(z) + 2z^{-2} \hat{H}'(z) - 2z^{-3} \hat{H}(z) \quad (\text{B.26})$$

$$\hat{Y}(1) = 1 - \hat{H}(1) = 0. \quad (\text{B.27})$$

$$\hat{Y}'(1) = \hat{H}(1) - \hat{H}'(1). \quad (\text{B.28})$$

$$\hat{Y}''(1) = -\hat{H}''(1) + 2\hat{H}'(1) - 2. \quad (\text{B.29})$$

APPENDIX C

MEAN RESIDUAL TIME OF A WEIBULL DISTRIBUTED RANDOM VARIABLE

Let X be a Weibull distributed random variable with shape parameter c and scale parameter a . The mean residual time of the random variable X is given by:

$$R_X = \frac{E[X^2]}{2E[X]} \quad (\text{C.1})$$

$$= \frac{\sigma_X^2 + E[X]^2}{2E[X]} \quad (\text{C.2})$$

$$= \frac{\sigma_X^2}{2E[X]} + \frac{E[X]^2}{2E[X]} \quad (\text{C.3})$$

$$= \frac{E[X^2] - E[X]^2}{2E[X]} + \frac{E[X]}{2} \quad (\text{C.4})$$

$$= \frac{a^2(\Gamma(1 + \frac{2}{c}) - \Gamma^2(1 + \frac{1}{c}))}{2a\Gamma(1 + \frac{1}{c})} + \frac{E[X]}{2} \quad (\text{C.5})$$

$$\geq \frac{a^2(2\Gamma^2(1 + \frac{1}{c}) - \Gamma^2(1 + \frac{1}{c}))}{2a\Gamma(1 + \frac{1}{c})} + \frac{E[X]}{2} \quad (\text{C.6})$$

$$= \frac{E[X]}{2} + \frac{E[X]}{2} \quad (\text{C.7})$$

$$= E[X], \quad (\text{C.8})$$

where we use the inequality:

$$\frac{\Gamma(1 + \frac{2}{c})}{\Gamma(1 + \frac{1}{c})^2} \geq 2 \quad \text{for } 0 \leq c \leq 1 \quad (\text{C.9})$$

from Equation 5.18. Hence, $\frac{E[X^2]}{2E[X]} \geq E[X]$.

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