

ABSTRACT

Title of dissertation: ESSAYS ON SOVEREIGN DEBT:
 DEFAULT, SETTLEMENT, AND
 REPAYMENT HISTORY

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This dissertation consists of two essays and studies topics on sovereign debt. The first essay analyzes, using duration models, the factors that affect the length of time a country in default is excluded from the international credit markets. It shows that disclosure of information by the debtor will lead to a reduction in the time of the negotiation; that is, an increase in the settlement rate. This disclosure of information can be in the form of previous default or the credible announcement of fiscal reforms. Engaging in debt reduction or rescheduling under concessionary rates hurts the debtor's probability of reaching an agreement and settling the debt. The second essay proposes a dynamic model of sovereign debt that combines default, settlement, and repayment history. The model addresses two questions: 1) how level of debt and income profile affect the length of time a country in default is excluded from the international credit market, and 2) how repayment history shapes the credit limit and interest rate lenders offer, and the borrower's incentive to default. In the model, borrowers weigh the benefits from defaulting against the penalties associated

with it, namely a lengthy exclusion from the market and a potential deterioration in the credit terms. The settlement is modeled as a random process conditioned on the portion of the defaulted debt borrowers agree to repay. The model incorporates repayment history into the information set used by lenders. Quantitative analysis shows that the model can replicate some key stylized facts of sovereign debt: 1) settlement offers depend positively on a debtor's current income level and negatively on its level of debt; 2) the debt-to-GDP ratio that new borrowers or serial defaulters can support is well below the ratio that proven debtors can safely manage; 3) the probability of default is larger for debtors with recent payment difficulties; 4) once a country defaults, it takes many years of flawless repayment and low levels of debt for that country to gain continuous access to international credit markets at low cost; this process is slow and backsliding into default is difficult to avoid.

ESSAYS ON SOVEREIGN DEBT:
DEFAULT, SETTLEMENT, AND REPAYMENT HISTORY

by

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Chapter 1

Default and Settlement: Payment Resumption and Disclosure of Information

1.1 Introduction

Most of the sovereign debt literature assumes that after defaulting, a borrower is punished with exclusion from the international credit markets for an infinite period of time. The evidence suggests that this assumption is flawed: not only are sovereigns often able to borrow again soon after the settlements of their defaulted debts, but more importantly, credit markets are eager to lend to them. There is plenty of historical evidence to support this assertion.

During the British financial frenzy of 1824-1825 and even before all the Latin American Republics achieved their independence, their governments were able to issue £17,000,000 out of the £25,000,000 of government securities sold during these years in London. Even the nonexistent “Kingdom of Poyais” was able to successfully market £200,000 worth of bonds.¹ After the European commercial and banking crisis of 1825-1826 spread to Latin America, Peru suspended payments and shortly after followed Gran Colombia and Chile. By mid 1828 all the Latin American nations (Argentina, Mexico, Central America) had defaulted, with the exception of Brazil. Customs revenues, which were the main income for national treasuries, had

¹See Marichal (1989).

decreased sharply after the decline in transatlantic trade as a consequence of the 1825 crash in Europe. Therefore, unable to cope with debt service, all defaulting governments refused to resume payments. After the default episodes, most Latin American Republics witnessed decades of internal and external conflicts. The first country to resume payments was Chile in 1842. The Chilean government proposed to resume payments on the debt and capitalize the arrears of interest since 1826 with a new 3% bond. After that, most countries followed a similar scheme: Peru in 1849, Ecuador in 1856, Argentina in 1857, Venezuela in 1859 and Colombia in 1861. Guatemala in 1856 and Honduras in 1867 were able to capitalize arrears of interest in new 5% bonds. Costa Rica paid off the principal at 85% of par value in 1844 and so did Nicaragua in 1872 and El Salvador (90% par) in 1860. The case of Mexico is very complex and ultimately led to a full-scale military invasion by French, British, and Spanish forces. The economic recovery that allowed the Latin American nations to resume payments, accompanied by a new expansive phase of the international economy, led to a new loan boom. The most impressive case is Honduras. After 40 years in default, from 1828 to 1867, Honduras was able to issue £1,000,000 in 1867, the same year of the settlement, in London and Paris in order to finance railways. Colombia, 2 years after the settlement, raised £200,000 in 1863 for public works. Peru, after 24 year in default, raised £400,000 in London to finance railways by 1853, four years after it settled its debt. Argentina, by 1865, after 31 years in default, raised £2,500,000 after 9 years of the settlement.

The same story has been repeating itself from then on. Suter (1992) characterizes these successions as global debt cycles and identifies them as having three

consecutive phases: 1) the expansion of international loans, a period during which capital shifts from developed economies to emerging economies (foreign investment); 2) the outbreak of a payment crisis; and 3) renegotiation and debt settlement. Suter and Stamm (1992) identify four periods of wide-spread debt-servicing incapacity (phase 2 and 3) since 1800: from the late 1820s to the early 1840s, from the mid-1870s to the early 1880s, from the early 1930s to the late 1940s, and from the early 1980 to the mid-1990s. These periods have been studied and documented, among others, by Marichal (1989) for Latin American countries between 1823-1930, Lindert and Morton (1989) from the early 19th century, Eichengreen and Portes (1989) and Jorgensen and Sachs (1989) for the interwar years. Although it is relatively easy to identify the outbreak of the repayment crises, it is much harder to pin down the time they start receding. This is because the duration of the third phase identified by Suter —renegotiation and debt settlement— varies greatly from country to country and within countries. Suter (1992) affirms that there are marked differences between the settlement processes before and after World War II (WWII), mainly that debt crises were managed more quickly after. Suter explains that the long duration of defaults before WWII, particularly the ones that occurred during the period 1821-1870, was due to the low level of institutionalization of bondholders; that is, the lack of permanent bondholders committees. He also wonders about the comparatively quick settlement of bonds in default during the period 1871-1925 and states that is more difficult to explain. He attributes the reduction in the duration of the settlement/rescheduling process post WWII to changes in the actor structure on the creditors' side: individual bondholders were displaced by international organizations

(IMF, World Bank, regional development banks), and large bank syndicates which in his view should have helped facilitate the settlement/rescheduling process due to the reduction in the number of actors.

Contrary to Suter's interpretation, Mitchener and Weidenmier (2004) state that the structural change occurred around 1904 after president Theodore Roosevelt outlined United States interventionist policy in Latin America aimed to protect the interest of European investors in the region, and in this way avoid any military expansion of Europeans using the defense of economic interests as an excuse. What came to be known as the Roosevelt Corollary to the Monroe Doctrine was intended to assert U.S. dominance in the region (Central America and Caribbean countries). Direct intervention of the United States in the Dominican Republic in 1905, and the submission on 55% of custom revenues by the Dominican government sent a powerful message to countries under the United States' sphere of influence and led countries in Central America and the Caribbean to settle long-standing defaulted debts. Mitchener and Weidenmier (2004) conclude that this was a regional phenomenon characterized by positive abnormal returns that affected only countries under the influence of the United States² but that did not take place in other developing countries.

Jorgensen and Sachs (1989) tangentially go over the duration of the settlement

²Mitchener and Weidenmier (2004) found that countries like Brazil and Argentina, that were considered at the time outside the sphere of economic interest of the United States, did not experienced abnormal returns as did not British Guiana, located in the Caribbean but unlikely to be the target of U.S. gunboats.

process and find that in the interwar period the longer a debtor held out for more favorable renegotiation terms, the better it fared in the conditions of the settlement.

Although some empirical studies have hinted on factors that may have affected the duration of the settlement process during past debt crises, the literature has not directly addressed the question of what determines the length of the negotiation process after a default occurs. This paper is intended to empirically answer some questions regarding the length of the settlement process of defaulted debts and relies on econometric models of duration to do so. These models analyze the length of time spent in a given state (default in the case of interest) before transition to another state (good standing with the international credit market), and allow us to estimate the probability of one of such transitions taking place at any point in time. Duration models and the hazard function estimation would permit us to determine the conditional probability that a country excluded from the international credit markets due to a default at time t is still excluded at $t+1$. Since the main interest of the paper is studying the duration of the event (exclusion from international credit markets after a default occurs and while a settlement takes place) and the probability that the event will end “in the next period” given that it has lasted as long as it has, duration models seem to be more appropriate than logit or probit models which would estimate the probability of a country being in default (or in good standing with the international credit markets) at a particular point in time. Given that most countries have experienced multiple default episodes, the models used account for the possibility of dependence of the default durations within a country. In order to correct for possible correlations between default durations

within countries, I used shared frailty models and variance correction models.

After analyzing the debt cycles from the 1820's onwards, and using the hints proposed by the previous literature, it seems reasonable to suggest that the duration of the negotiation process; that is, the period of exclusion from the international credit markets after a default occurs, depends on the characteristics of the settlement (the lower the debtor's demands the shorter the length of the negotiation), the type of debt (bonds vs. bank loans), the conditions of the international capital markets (if interest rates are too low creditors will be more willing to offer loans to risky countries, if there is a lending boom the market sentiment toward emerging markets will be favorable), the debtor credit history, etc. It also seems reasonable to believe that "fundamentals" are important since they determine access to the credit markets in the first place.

Studying the effect of different variables on the hazard functions, I claim that the Mitchener and Weidenmier's argument and the experience acquired by debtors and creditor dealing with previous settlement processes, rather than a change in the structure of the creditors' side (Suter's thesis), may help explain the reduction in the duration of defaults. I found, contrary to common wisdom, that bank loans are not settled faster than bonds and that disclosure of information in the form of previous default (history of default) or the announcement of credible fiscal reforms are mayor factors in the reduction of the length of the negotiation. The variable that affect the most the duration of a default, nevertheless, seems to be the portion of the defaulted debt that borrowers are willing or able to pay back; that is, the settlement conditions.

The remainder of the paper is organized as follows. Section II discusses the different models considered and the methodology used. Section III discusses the data used for the analysis. Section IV presents the most important results. Section V concludes and discusses possible extensions.

1.2 Model Specification

This section discusses the models and its assumptions. It is worth mentioning that the empirical problem was approached using reduced form models. The reason for this choice was to let the functional form of the hazard function be explained by the data instead of constraining its form by theoretical considerations. When using a structural approach, the choice element in each transition (default-settlement) is emphasized and the countries are assumed to occupy the state that they prefer given the opportunity set they face. Although this sounds appealing, choosing the wrong theoretical model for the structural approach would lead to deceptive or contradictory conclusions. A reduced form approach stresses the chance aspect of movement between states (the desirability of a different state and the opportunities open to the economic agents vary in a stochastic way over time) but allows for the results to be ample enough as to shed light about the type of structural model that can reproduce these empirical results. Using the right theoretical model to describe the settlement process should be essential when deciding, for example, on an international restructuring mechanism for defaulted debt (either Collective Action Clauses -CAC, or Sovereign Debt Restructuring Mechanism -SDRM, for

instance). Based on the empirical results and the work by Kennan and Wilson (1989) on strategic bargaining models, I will be able to identify the theoretical model that best describes the behavior of borrowers and lenders when negotiating a settlement agreement after a default takes place.

Before presenting the models to be employed, it proves useful to introduce the following notation conventions. Let $T \geq 0$ denote the default duration, the length of time measured in years at which a country leaves the state of default and can potentially gain access to the international credit market. It is, rather than calendar time, a measure of time on country-specific clocks that are set to zero whenever the country enters in default.³ T has some distribution in the population, and the population is considered homogeneous with respect to a set of time-invariant regressors or covariates, Z , that allow for measured, and potentially for unmeasured, systematic differences between countries and that affect the distribution of T .

The *cumulative distribution function* of the duration, conditional on Z , is defined as

$$F(t; Z) = P(T < t, Z), \quad t \geq 0 \tag{1.1}$$

which specifies the probability that the random duration of the default T , conditioned on the covariates Z , is less than some value t .

The *survival function*, the probability of “surviving” past time t ; that is, the probability that the random duration of the default T , conditioned on the covariates

³This assumption can be modified depending on the model.

Z , will equal or exceed t , is defined as

$$S(t; Z) \equiv 1 - P(T < t, Z) = P(T \geq t, Z) \quad (1.2)$$

In the case of interest, the survival function is the probability that a country in default stays in that state past time t .

The *density function* of T is defined as

$$f(t; Z) = \frac{dF(t; Z)}{dt} \quad (1.3)$$

The *hazard function* for T , the probability of leaving the state of default in the interval $[t, t + h)$ given that the country is in default at time t , is defined as

$$\lambda(t; Z) = \lim_{h \rightarrow 0} \frac{P(t \leq T < t + h \mid T \geq t, Z)}{h} \quad (1.4)$$

The hazard function is the instantaneous settlement rate per unit of time; the rate at which defaults will be settled when the length of the default is t , given that the country default has lasted until t .

Other definition that will prove useful is the *cumulative or integrated hazard function*:

$$\Lambda(t; Z) = \int_0^t \lambda(s; Z) ds = -\ln S(t; Z) \quad (1.5)$$

Although this function does not have a convenient interpretation, it is key for specification checks. Under any distribution of T , the transformation $\Lambda(t; Z)$ becomes a constant hazard by modifying the time scale; that is, the probability of a settlement does not depend on how long the country has been in default in the new time scale. $\Lambda(t; Z)$, independently of the distribution of T , is unit exponentially

distributed.⁴

Duration models differ in the assumptions they make about how the vector of explanatory variables Z (covariates) affects the hazard function. *Proportional Hazard* (PH) models assume separability; the baseline hazard function $\lambda_0(t)$ depends on t but not on Z and it is common to every subject.⁵ Differences in the subjects' characteristics are incorporated by a non-negative function of covariates that does not depend on t ; this function scales the baseline function common to every subject.⁶

$$\lambda(t; Z) = \lambda_0(t) \exp(Z' \beta) \quad (1.6)$$

In *Accelerated Failure time* (AFT) models, the function $\psi(Z, \beta^*)$ of the covariates scales time itself, $t\psi(Z, \beta^*)$, instead of a baseline function. A covariate can accelerate ($\psi(Z, \beta^*) > 1$) or decelerate survival time ($\psi(Z, \beta^*) < 1$), where:

$$\psi(Z, \beta^*) \equiv \exp(-Z' \beta^*) \quad (1.7)$$

$$\lambda(t, Z) = \lambda_0[t \exp(-Z' \beta^*)] \exp(-Z' \beta^*) \quad (1.8)$$

The interpretation of the parameters reported in each case (β and β^*) differs considerably. An AFT coefficient “relates proportionate changes in *survival time* to a unit change in a given regressor, with all other characteristics held fixed” while a

⁴The survival function of the new variable $X = \Lambda(t)$ is defined as $S_X(x) = P(X \geq x)$. Using the change of variable, $S_X(x) = P[\Lambda(t) \geq x] = P[t \geq \Lambda^{-1}(x)] = S_X[\Lambda^{-1}(x)] = \exp[-\Lambda[\Lambda^{-1}(x)]] = \exp(-x)$.

⁵The assumption that every subject faces the same baseline hazard $\lambda_0(t)$ can be relaxed in favor of different baseline hazards for different groups but constraining the coefficients β to be the same.

⁶When dealing with time-varying covariates the treatment is more complicated.

PH coefficient “relates a one unit change in a regressor to a proportionate change in the *hazard rate*, not survival time.⁷”

Although the underlying transition process from a default state to a good standing state is continuous in nature, the data is not provided in that form. Survival times have been grouped into discrete intervals of time (number of years in state of default). Survival analysis techniques allow for the use of discrete and continuous time models, which include non-parametric and parametric models. Parametric models use parametric distributions (Exponential, Weibull, Log-logistic, Gompertz, Lognormal, and Generalized Gamma),⁸ which impose very different behavior structures to their respective hazard functions. It is uncertain which of the distributions best describes the process of interests: the duration of the settlement process after a sovereign debtor defaults and the shape of the settlement rate (hazard function).

The models proposed attempt to analyze the duration of the settlement process, and how that duration depends on certain exogenous variables. The variable of interest is the length of time that elapses from the time of the default, either until an agreement between debtor and creditors is reached or until the measurement is taken (censored observation), which may precede termination and corresponds to the last observation date. The end of the default; that is, the time of the settlement, is considered as a proxy for the time the debtor country can reenter the credit markets.

There are some potential problems that can arise when using duration models.

⁷See Jenkins (2004).

⁸These are the distributions available in Stata for survival analysis.

Let assume that the observation interval is $[0, b]$ and that the sample consists of countries that enter into default at some point during that interval. One of these problems is due to the fact that the default events might start at different calendar dates. The other problem is that some countries might still be in state of default when the observation interval is closed⁹ (right censoring).

Let the triplet (a_i, t_i^*, Z_i) be a random draw of the population. Where, a_i is the time at which country i enters the state of default (starting time), where $a_i \in [0, b]$ and $[0, b]$ is the observation period.

t_i^* is the length of time in the state of default

Z_i is the vector of observed covariates

It is assumed that t_i^* , the random duration of a default, has a continuous conditional density $f(t; Z_i, \theta)$, where $t \geq 0$ and θ is the vector of unknown parameters of the distribution. The observed duration t_i , nevertheless, is the smaller of t_i^* and c_i and is defined as $t_i = \min(t_i^*, c_i)$ where c_i is the censoring time for country i measured from the time the default took place in the country-specific clocks (right censoring). Since there is a fixed calendar date at which the sample is truncated (December 2004), the censoring time differs by country because different countries default on different calendar dates.

It has been noticed in the literature that either because of global economic recessions, which undermines the ability of foreign debtors to service their debt, or

⁹The many other problems will be discussed later.

because of a free rider problem,¹⁰ countries tend to default in block. Suter (1992) suggests four principal debt crisis periods: late 1820s, the 1870s, the 1930s, and 1980s, which correspond with stagnation phases of the world economy. He also identifies three minor debt crises during 1840s, 1890s and during and after World War II.

Under these circumstances, it will be expected that the times at which the countries entered the state of default (a_i s) be correlated among them. The default duration, however, does not seem to depend on when the country enters the state of default, or when the censoring occurs (c_i s). It is assumed that conditional on the covariates, the duration in the state of default is independent of the starting point a_i , and the censoring time, c_i :

$$D(t_i^*; Z_i, a_i, c_i) = D(t_i^*; Z_i) \quad (1.9)$$

In order to guarantee that the assumption holds, a dummy variable could be introduced in Z_i to identify different debt cycles. This dummy variable, however, should not be significantly different from zero under assumption (1.9). The sample will consist then on random draws (d_i, t_i, Z_i) of the population, where d_i is defined as a censoring indicator such that $d_i = 0$ if the observation is censored and $d_i = 1$ otherwise.

Other potential problem in the case of interest is the existence of multiple “failures” or default-settlement episodes by country. Since defaults can become a way of

¹⁰Lindert and Morton (1989) notice that borrower countries will not be particularly punished by cutting repayments and demanding partial write-downs of debt, as long as they do so collectively. The only countries punished are the ones defaulting in isolation (p. 231-232).

life for many countries, it is not difficult to find countries that have experienced multiple default episodes, countries that Reinhart, Rogoff and Savastano (2003) have called serial defaulters. Multiple spells for each country in the sample gives rise to possible dependencies across spells for the same country. For the k^{th} ($k = 1, \dots, K_i$) settlement, let T_{ki} be the settlement time or default duration of the i^{th} country ($i = 1, \dots, n$). A model that intends to analyze these multiple default experiences (multiple failure, in the survival analysis jargon) must account for the possibility of dependence of the default durations (failure times, $T_{1i}, T_{2i}, \dots, T_{K_i}$) within a country. Although the covariates $Z_{ki}(t)$ are assumed to be constant within each default episode they are allowed to vary over episodes (k's). Traditional survival models assume the independence of default durations among observations, assumption that might be violated in this case because of the existence of multiple default episodes per country. Although the models proposed allow for intra-country correlation, observations corresponding to different countries are assumed to be independent from each other.

A simplified way to deal with this dependence problem is to analyze the time from first default to first settlement and ignore additional default-settlement episodes, but doing so implies wasting information. Dealing with the dependencies between default durations, however, can be done in two different ways.

First, using shared frailty models or mixture models. In this case the associations between default durations within a country are modeled as a random effect term, an unobserved covariate common to default events experienced by a country, which creates the dependence (and may account for unobserved heterogeneity

among countries). Thus, conditional on those country specific quantities (multiplicative term ν_i), the default durations are independent and identically distributed within countries although the distributions they are drawn from are allowed to differ from country to country. The hazard function of shared frailty models is described as:

for Proportional Hazard Models

$$\lambda_{ik}(t; Z_{ik}(t), \nu_i) = \nu_i \lambda_0(t) \exp(Z'_{ik}(t) \beta_0) \quad (1.10)$$

for Accelerated Failure Time Models

$$\lambda_{ik}(t; Z_{ik}(t), \nu_i) = \nu_i \lambda_0[t \exp(Z'_{ik}(t) \beta_0^*)] \exp(Z'_{ik}(t) \beta_0^*) \quad (1.11)$$

Where the hazard function now involves a country-specific, time invariant quantity ν_i , which is assumed to be a realization of a random variable V , where $\nu_i \sim G(\theta)$, θ are the parameters of the distribution G , and $E(\nu_i) = 1$.

Second, we can deal with the dependencies between default durations using variance correction models. In this case the dependencies between default durations are not included explicitly in the models, and in order to account for default duration correlations within a country the covariance matrix of the estimators are adjusted. The models, within the second group, to be considered in the paper are those proposed by Andersen and Gill (1982); Wei, Lin and Weissfeld (1989); and Prentice, Williams, and Peterson (1981). These models do not account for possible heterogeneity among countries, they assume all the durations are identically distributed but allow for possible dependencies within countries. I will refer to these models as “ordered events” models.

Andersen and Gill (1982) (AG) consider a counting process model where the event analyzed is recurrent, but not more than one event may happen at a given time; the model can be regarded as a special case of the Cox proportional hazard model where the underlying hazard function is identical for every recurrent event, is unaffected by earlier events, and its modeling is continued beyond the subject's first failure. The risk set at time t for event k is all subjects under observation at time t . The hazard function is defined as:

$$\lambda_{ik}(t) = Y_{ik}(t)\lambda_0(t) \exp(Z'_{ik}(t)\beta_0) \quad (1.12)$$

Where Y_{ik} takes the values $\{0, 1\}$ and indicates ($Y_{ik} = 1$) when the i^{th} individual is under observation (N_{ik} counts observed events in the life of individual i^{th} and jumps only when $Y_{ik} = 1$).

Prentice, Williams, and Peterson (1981) (PWP) propose handling the dependence between event times by stratifying the hazard function using the previous number of failures $N(t) = \{n(u) : u \leq t\}$, where $n(u)$ is the number of failures on a study subject prior to time u . In their model, the risk set at time t for recurrence k is composed by all the subjects under observation at time t that have experienced event $k - 1$. They considered two approaches to measure time to event: from entry time and from previous event.¹¹ The hazard functions in each case, are defined as, from entry time, PWP(1):

$$\lambda(t | N_i(t), Z_{ik}(t)) = \lambda_0(t) \exp(Z'_{ik}(t)\beta_0) \quad (1.13)$$

¹¹When time to event is measured from entry time, risk begins accumulating since the time of the first default occurrence and it's not set to zero after a settlement takes place. When time to event is measured from previous event, the risk is set to zero after a settlement (failure) occurs.

from previous event, PWP(2):

$$\lambda(t | N_i(t), Z_{ik}(t)) = \lambda_0(t - t_{n(t)}) \exp(Z'_{ik}(t)\beta_0) \quad (1.14)$$

Wei, Lin and Weissfeld (1989) (WLW) allow the underlying hazard functions to vary among different recurrences (each event has its own hazard function, $\lambda_{k0}(t)$). The risk set at time t for recurrence k is composed by all the subjects under observation at time t that have not experienced event k . Since in this case the coefficients $\hat{\beta}_k$ for each recurrence k can be estimated, they show that the “average” effect of the covariate can be approximated using a joint normal distribution (where the covariance matrix can be estimated). The hazard function for this model is defined as:

$$\lambda_{ik}(t) = \lambda_{k0}(t) \exp(Z'_{ik}(t)\beta_k) \quad (1.15)$$

In all the models, a country that has defaulted more than once is considered to have experienced repeated occurrences of the same type of event (ordered events in the latter group), and once in default the “failure” occurs when it exits from a default state to a good standing state.¹²

In addition to analyzing the data using the proposed Cox proportional hazard models, the parametric distributions will be used in order to shed lights about the shape of the settlement rate (hazard rate), and about how default history affects that settlement rate.

The analyses were carried out conditioned by a set of fixed covariates (within

¹²More complex models can be considered, for example, multi-state transitions (default, good-standing without new credit, good-standing with new credit).

default episode) instead of time-varying covariates. In the case of fixed covariates, it is assumed that they remain constant from the time of the default until the time of the settlement (or until the last observation was taken), which is a strong assumption, especially in the case of economic data constantly monitored by the credit markets. The values associated with the covariates correspond with the date of the default to avoid endogeneity problems.¹³

1.3 Data Description

The data used to estimate the models was taken from Standard and Poor's, the World Bank's Global Development Finance Database, the World Bank's World Development Indicators, and the IMF's International Financial Statistics. The analysis was carried out for a sample of 92 default episodes from 55 different countries (an average of 1.58 defaults per country, an a maximum of 4 defaults per country) from 1962 to 2004, a period that comprises lending expansions and collapses. The variables used are described below:

1.3.1 Dependent Variable (Spell duration):

Defined as the time elapsed between entering the state of default and settling the defaulted debt. The definition of default used is taken from Standard and Poor's: "the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt

¹³In the case of time-varying covariate models, the endogeneity problems will also be present but those problems can be solved using instrumental variables.

issue.¹⁴” The episodes of default were dated using Beers and Bathia (1999), and later actualizations by Beers and Chambers (2003 and 2004). Standard and Poor’s dataset includes default episodes that date back to the 1820s, the type of debt in default (either bank debt or bond debt), and whether the debt is rated or not by Standard and Poor’s rating services.

1.3.2 Exogenous Variables (Covariates):

Depending on the type of information creditors can extract from the variables, these will either hurt or improve the probability of a debtor settling the defaulted debt and then leaving the state of default.

GDP per capita growth (annual %): a higher GDP per capita growth a priori indicates an improvement in the state of the economy, which might facilitate the accumulation of resources needed to repay the debt. The coefficient should be positive meaning that an increase in the growth rate will increase the settlement rate.

¹⁴“Debt is considered in default in any of the following circumstances: i) For local and foreign currency bonds, notes, and bills, when either scheduled debt service is not paid on the due date, or an exchange offer of new debt contains terms less favorable than the original issue. ii) For Central Bank currency, when notes are converted into new currency of less than equivalent face value; and iii) For bank loans, when either scheduled debt service is not paid on the due date, or a rescheduling of principal and/or interest is agreed to by creditors at less favorable terms than the original loan. Such rescheduling agreements covering short- and long-term bank debt are considered defaults even where, for legal or regulatory reasons, creditors deem forced rollover of principal to be voluntary.”

Money and quasi money (M2) as % of GDP: inverse of the velocity of money. Assuming that money can be used as a store of value, a higher ratio indicates people are willing to hold money because they are confident the economy is stable (low inflation, sound banking sector, etc.). A large ratio might also indicate international creditor the possibility that part of those resources could finance local currency debt, which may reduce the financial burden on them. In both cases the coefficient should be positive.

General government final consumption expenditure(annual % growth): annual percentage growth of government current expenditures for purchase of goods and services, including compensation of employees. A high growth rate reveals information about an improvement in the state of the economy so much so that the government has enough resources to increase its expenditures. Under these conditions, any attempt by the government to get any concession by the creditor should be unsuccessful. Creditors might be willing to wait in order to get a better offer from the government. An increase in the growth rate should reduce the settlement rate, so the coefficient should be negative.

Gross fixed capital formation as % of GDP: Investment in infrastructure both public and private sectors. The sign of this coefficient is ambiguous. Creditors may read this piece of information in two different ways: first, if the private sector is large compared to the public sector, and increase in the variable indicates the economy is improving so the accumulation of resources would facilitate the repayment of the defaulted debt (positive coefficient). Second, if the public

sector is larger than the private one, an increase in the variable indicates that resources are deviated from debt servicing to government investment, under these circumstances creditors might be unwilling to grant any concession and instead they might want to wait in order to get a higher offer from the government decreasing the settlement rate, in this case the coefficient should be negative.

Paris Club: This categorical variable was constructed using data from debt renegotiations under Paris Club terms. The Paris Club defines itself as “an informal group of official creditors whose role is to find coordinated and sustainable solutions to the payment difficulties experienced by debtor nations. Paris Club creditors agree to reschedule debts due to them. Rescheduling is a means of providing a country with debt relief through a postponement and, in the case of concessional rescheduling, a reduction in debt service obligations.¹⁵” Since 1965, and until January 2006, 81 debtor countries have agreed to negotiate under the Paris Club terms. Under different terms, debt reduction (cancellation) ranges from 0% up to 90% of the debt treated. When no debt reduction is agreed upon, reschedule may offer temporary relief through deferral of payments. The values assigned to the categorical variable are as follow:

1. Country has not negotiated the default episode under Paris Club terms.
2. Country negotiated under “Classic” terms, the standard treatment where debt is rescheduled at market interest rates without cancellation (debt

¹⁵<http://www.clubdeparis.org/en/presentation/presentation.php?BATCH=B01WP01>

relief).

3. Country has negotiated under “Houston” terms; that is, debt is rescheduled at concessionary rates, repayment periods are lengthened, grace periods are conceded and debt swaps are allowed. Under these terms there is no nominal debt cancelation.
4. Country has negotiated under “Toronto”, “London”, or “Naples” terms. These terms allow cancelation of 33.3% (Toronto) of non “Official Development Assistant” (ODA) credits throughout either principal or interest reduction. That percentage was raised to 50% (London) in 1991 and later, in 1999, to up to 67% (Naples). The terms also lengthen the repayment period and include a grace period. This group also include countries that negotiated under “Lyon” or “Cologne” terms. These terms are the most generous of all, contemplate the cancelation of up to 90% the outstanding non ODA credits (initially 80% under Lyon). The terms allow the lengthening of the repayment period and include a grace period.

Although this categorical variable is ordinal in nature, the particular numbers assigned to each category are arbitrary. For this reason the categorical variable was then converted into three indicator variables in order to capture the relative magnitude of belonging to each group. The omitted group is the one formed by countries that negotiated under the Lyon-Cologne terms and under Toronto-London-Naples terms, so all comparisons are made with respect to this group. Given the definition of the variables, and assuming worse terms

for the creditors imply larger negotiation processes, it is expected a positive sign in the coefficients of all the indicator variables, these coefficients are expected to be greater than one (all the settlement rates are expected to be greater than the one of the omitted group).

Default History: This variable indicates the ordering of default episodes experienced by each country as required by Prentice, Williams, and Peterson; and Wei, Lin and Weissfeld.

Type of Debt: This dummy variable indicates the type of debt in default, either bank loans (0) or bond debt (1). The coefficient should be negative since bonds should be settled in a longer time than bank loans because the number of actors is larger.

The expected signs of the parameters discussed above correspond to the parameters in PH models. Coefficients estimated with AFT models are expected to have the opposite signs.

1.3.3 Comparability of Treatment

Although the Paris Club treats just official debt —debt owed by or guaranteed by national governments to bilateral lending agencies (multilateral debt)— the “comparability of treatment” principle adopted by the Club is aimed to “ensure a balanced treatment among all external creditors of the debtor country.” Non-multilateral creditors —mainly private creditors in the form of banks, bondholders, and suppliers; and other official bilateral creditors not members of the Paris Club

— reschedule on terms comparable to the ones agreed by the Paris Club. Multilateral debt is considered to have seniority over other obligations and its treatment is excluded except for “Heavily Indebted Poor Countries” under the HIPC initiative. The idea behind the “comparability of treatment” is that Paris Club’s leniency should not pay for private creditors’ better terms.

Making a direct comparison between the terms agreed by the Paris Club and other creditors can be very complex —given the broad range of instruments and their different characteristics (maturity, denomination, etc.)— but the experience suggests that other non-Paris Club official bilateral creditors often reschedule under very similar terms. The same is true for commercial banks grouped in the “London Club.” Since most of the sovereign debt issued from 1956, year of the Paris Club creation, until the mid 1990s was in the form of bank loans, debt rescheduling and reduction involved the participation of the London Club that generally cooperated in the application of comparable treatment. The exclusion of bond debt from the negotiations until the mid 1990s, because it was practically non-existent, gave the false idea of its seniority over bank loans. The large stock of bond debt created as a result of the Brady plan (after the late 1980s and early 1990s debt crisis) changed the composition of the stock of debt increasing bonds in detriment of bank loans and boosting the importance of bond markets as a source of finance for emerging economies. The idea of bond seniority was dismissed in the late 1990s after bondholders were required by the Paris Club and sovereign debtors to “take a haircut” too; that is, share the burden of the loss. Comparability of treatment, which can be supervised by the Paris Club since it requires debtors to share information on

the outcome of negotiations with other creditors, has been present in the cases of Pakistan, Ukraine, and Ecuador.

1.4 Results

The empirical models shown in this section are the ones whose covariates, from the range of the potentially relevant models, were individually and jointly significant, had the expected signs and best fit the data. Results are presented for a variety of models that were grouped into i) Shared Frailty Models (models with potentially unobserved heterogeneity), ii) Models without Unobserved Heterogeneity, and iii) Ordered Events Models. In addition to the models, I present some non-parametric tests for the equality of the survivor function across groups, and discuss the strategic bargaining models best suited to describe the default-settlement experiences of sovereign debtors.

1.4.1 Shared Frailty Models

These models were estimated using non-parametric and parametric distributions. The distributions used for the parameter ν were the Standard Gamma and the Inverse Gaussian, but the results under both distributions are not very different from each other. The results presented in table (1.1) are the ones corresponding with the Standard Gamma distribution.

Even though different distributions impose very different behavior structures to their respective hazard functions, the coefficients estimated under the different

distributions are fairly similar.

Table 1.1: Shared Frailty Models

	Cox	Exponential	Weibull	Gompertz	Lognormal	Log-Logistic
Covariates	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)
Constant		-2.1976 0.000	-3.7360 (0.000)	-3.1047 (0.000)	1.8311 (0.000)	1.8199 (0.000)
Capital/GDP	-0.0621 (0.010)	-0.0523 (0.009)	-0.0721 (0.019)	-0.0595 (0.022)	0.0405 (0.011)	0.0446 (0.002)
M2/GDP	0.0155 (0.045)	0.0121 (0.070)	0.0193 (0.044)	0.0166 (0.045)	-0.0096 (0.060)	-0.0102 (0.033)
Govt. Expend. growth	-0.0337 (0.003)	-0.0271 (0.003)	-0.0277 (0.056)	-0.0308 (0.016)	0.0146 (0.059)	0.0135 (0.075)
GDP per capita growth	0.1336 (0.000)	0.1028 (0.000)	0.1752 (0.000)	0.1387 (0.000)	-0.0981 (0.000)	-0.0964 (0.000)
Paris1 (No Paris Club)	2.1492 (0.000)	1.5880 (0.000)	2.6061 (0.000)	2.2372 (0.000)	-1.3956 (0.000)	-1.4364 (0.000)
Paris2 (Classic)	2.2982 (0.000)	1.6360 (0.000)	2.8837 (0.000)	2.4571 (0.000)	-1.6828 (0.000)	-1.7296 (0.000)
Paris3 (Houston)	1.7109 (0.001)	1.3993 (0.002)	1.6206 (0.011)	1.6884 (0.003)	-1.0779 (0.001)	-1.1002 (0.002)
	θ	$\ln(\theta)$	$\ln(p)$ $\ln(\theta)$	γ $\ln(\theta)$	$\ln(\sigma)$ $\ln(\theta)$	$\ln(\gamma)$ $\ln(\theta)$
Parameters of the distribution	0.2206	-15.1154 (0.985)	0.5982 (0.000) -0.1116 (0.787)	0.1241 (0.005) -0.7676 (0.238)	-0.3241 (0.008) -1.0674 (0.163)	-0.8395 (0.000) -1.3083 (0.138)
$\chi^2(df)$ (p-value)						
Likelihood-ratio test ($\theta=0$)	0.62 (0.216)	0.00 (1.000)	7.88 (0.002)	2.71 (0.050)	2.42 (0.060)	1.74 (0.094)

The parameter θ is an estimate of the variance of the frailties. Neither the Cox nor the Exponential model shows that this variance is statistically different from zero (Likelihood-ratio test). All the other models, nevertheless, indicate that there is heterogeneity among countries that cannot be explained by the covariates.

The results obtained from the Cox model, which does not impose any particular form to the hazard function, indicate that the settlement rate (hazard function) increases up to a maximum and then declines. Given the structural forms imposed by parametric models, is then no surprise that the ones that best fit the data are

the ones that allow for this type of shape; that is, the log-normal and log-logistic models. Although the parameter p estimated under the Weibull model is larger than one ($p = 1.8$), which indicates a positive duration dependence (settlement rate increases with time), when integrating over the unobservable heterogeneity ν_i , the estimated population hazard decreases after a certain point, yielding a population hazard very similar to the one estimated under the log-normal model. It is worth noticing that the variance $theta$ of the frailty is smaller for the log-normal (0.34) and log-logistic (0.27) models compared with the estimated value of $theta$ for the Weibull model (0.89); that is, heterogeneity is larger under the latter model, which is necessary to “bend” the population hazard. The change in the slope of the population hazard is due to the fact that countries with higher values of ν leave the state of default sooner than countries with lower values. Due to this selectivity bias, the effect of the covariates on the population hazard will diminish with analysis time, in favor of the frailty effect. For this reason the proportional hazard assumption does not generally hold for the population hazard. The proportionality of hazards assumption made by the Cox model, nevertheless, was tested and it is satisfied at a 10% significance level.¹⁶

Diagnostic Test Based on the Integrated Hazard Function

In order to determine the distribution that better fits the data, I used graphical tools based on the integrated hazard function. Under the correct specification, the

¹⁶Tests based on Schoenfeld residuals support the proportionality of hazards assumption; tests based on re-estimation do not.

generalized (Cox-Snell) residuals should follow the unit exponential distribution, and when plotting the estimated cumulative hazard function against the generalized residuals the relationship should be a 45° line. A deviation of the cumulative hazard from the 45° line indicates a misspecification of the model. These tests were carried out for all the models, with (Gamma and Inverse Gaussian) and without unobserved heterogeneity, for which the integrated hazard expression is available.

None of the models with unobserved heterogeneity, neither using the gamma nor the inverse gaussian distribution, seem to be well specified, indicating that frailty models are not the appropriate choice (See figures 1.7-1.18). The plot of the cumulative hazard function against the generalized residuals that best approximate a 45° line is precisely the one corresponding with the exponential model; for this model the variance of the frailty is not statistically different from zero. The parametric models that best fit the data are the lognormal and loglogistic without unobserved heterogeneity.

These results are particularly relevant for explaining the form of the hazard function. The data is best fitted with models that allow the settlement rate to increase to a maximum and then decline indicating that negotiations dwindle pass certain point. That point is reached in both the lognormal and loglogistic models in the fourth year after a default takes place (See figures 1.1-1.2).

1.4.2 Models without Unobserved Heterogeneity

The graphical tools for detecting misspecification indicate that shared frailty models are not a correct choice for the case of interest, and instead point to models

without unobserved heterogeneity, potentially indicating that default durations are all drawn from the same distribution and that the differences that arise among countries can be explained through the observed covariates. Since frailty models were used to model default duration correlation within countries, and the measure of the correlation is the variance θ of the frailties, the results indicate the lack of statistical evidence for the correlation within countries. Even though the tests point to misspecification when using frailty models, the values of the parameters estimated using frailty models do not change dramatically when estimated using models without unobserved heterogeneity as shown in table (1.2).

Table 1.2: Models without Unobserved Heterogeneity

	Cox	Exponential	Weibull	Gompertz	Lognormal	Log-Logistic
Covariates	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)
Constant		-2.1976 (0.000)	-2.8680 (0.000)	-2.6441 (0.000)	1.8774 (0.000)	1.8675 (0.000)
Capital/GDP	-0.0597 (0.004)	-0.0523 (0.009)	-0.0675 (0.002)	-0.0572 (0.005)	0.0450 (0.003)	0.0471 (0.001)
M2/GDP	0.0141 (0.046)	0.0121 (0.070)	0.0154 (0.026)	0.0145 (0.038)	-0.0104 (0.045)	-0.0107 (0.029)
Govt. Expend. growth	-0.0352 (0.000)	-0.0271 (0.003)	-0.0335 (0.000)	-0.0335 (0.001)	0.0193 (0.012)	0.0172 (0.026)
GDP per capita growth	0.1248 (0.000)	0.1028 (0.000)	0.1288 (0.000)	0.1227 (0.000)	-0.0990 (0.000)	-0.0964 (0.000)
Paris1 (No Paris Club)	1.9852 (0.000)	1.5880 (0.000)	1.9979 (0.000)	1.8767 (0.000)	-1.4722 (0.000)	-1.5122 (0.000)
Paris2 (Classic)	2.0504 (0.000)	1.6360 (0.000)	1.9927 (0.000)	1.9291 (0.000)	-1.6531 (0.000)	-1.7066 (0.000)
Paris3 (Houston)	1.7127 (0.000)	1.3993 (0.002)	1.7052 (0.011)	1.6539 (0.000)	-1.2245 (0.000)	-1.2280 (0.001)
			ln(p)	gamma	ln(sigma)	ln(gamma)
Parameters of the distribution			0.2716 (0.002)	0.0541 (0.029)	-0.1774 (0.027)	-0.7091 (0.000)

The parameters estimated for the PH and AFT models have opposite signs as expected; the interpretation of the results is as follows:

- The initial three indicator variables created to distinguish among the four different groups under which the debt is assumed to be negotiated (Non Paris Club, Classic, Houston, Toronto-London-Naples and Lyon-Cologne), confirm the expected results. Since all the groups are compared with the Toronto-London-Naples and Lyon-Cologne group (omitted indicator variable), the coefficient of the indicators shown are all positive and greater than one.
- Renegotiating defaulted debt under more generous terms to the debtors reduces the settlement rate. Restructuring defaulted debt under market conditions (Classic) takes less time to negotiate than restructuring under concessionary rates (Houston). The settlement rate for countries with a cancellation level of 67% or 90% (Toronto-London-Naples and Lyon-Cologne) is the lowest, indicating that for these countries it takes the longest time to negotiate their defaulted debt.
- Countries that settled their defaulted debts without participating in the Paris Club did so comparatively in more time than countries that negotiated under the Classic terms, but in less time than countries receiving some kind of debt relief by their creditors (either through concessionary interest rates or debt cancellation). The principle of conditionality that guides the Paris Club's operations requires that "debt treatments are applied only for countries that need a rescheduling and that implement reforms to resolve their payment difficulties. In practice conditionality is provided by the existence of an appropriate program supported by the IMF, which demonstrates the need for debt

relief.” Debtor countries are “expected to provide a precise description of their economic and financial situation.” The disclosure of information and the commitment by the debtor country to implement reforms under Classic terms might explain the differences in the settlement rates between Non-Paris Club countries and countries that negotiate under market conditions. Uncertainty reduction increases the settlement rate.

- The characteristic that weights the most when determining the settlement rate of the defaulted debt is how the country approaches the negotiations rather than the state of its economy. For instance, agreeing to disclose information and commit to abide under an IMF program (Classic) vs. not doing so (Non Paris Club) increases the settlement rate as much as an increase of 0.47% in the annual growth rate of GDP per capita, or as much as a reduction of 1.77% in the growth rate of government consumption expenditure.¹⁷
- As indicated above, an increase in the GDP per capita growth rate increases the settlement rate as expected.
- The ratio M2/GDP (inverse of velocity of money) has a positive effect on the settlement rate of the defaulted debt. A high ratio can indicate the willingness of the people to hold money as store of value indicating their confidence in the economy. It also shows that part of those resources might be available to finance local currency denominated debt reducing the burden on international creditors.

¹⁷Figures estimated using the coefficients of the Cox model without unobserved heterogeneity.

- There is a negative relationship between the government consumption expenditure growth rate and the settlement rate of the defaulted debt. An expansion of the government consumption expenditure reveals information about the possible lack of commitment to honor its defaulted debt. Creditors are willing to wait under these circumstances in order to get a better deal.
- Capital investment/GDP ratio (Gross fixed capital formation as % of GDP) has a negative effect on the settlement rate. A high ratio might reveal information about possible future growth of the economy. Creditors might be willing to wait in order to be able to recover a larger portion of the defaulted debt. The sign of this coefficient could be indicator that serial defaulter countries have a large public sector when compared with the private sector of the economy. Was this not the case, the coefficient should not be significantly different from zero.

1.4.3 Ordered Events Models

Models presented in this subsection correspond to Cox proportional hazard models proposed by Andersen and Gill (AG); Prentice, Williams, and Peterson (PWP); and Wei, Lin and Weissfeld (WLW).

Post estimation tests¹⁸ suggest that potentially omitted variables have little or no explanatory power (the results of these tests are not shown). However, not all the specifications satisfy the global tests¹⁹ for the proportionality of hazards (tests

¹⁸Under the assumption that $Z\beta_z$ is the correct specification, the coefficient of the squared linear predictor is insignificant, that is, $\beta_2 = 0$, where $LRH = \beta_1(Z\hat{\beta}_z) + \beta_2(Z\hat{\beta}_z)^2$.

¹⁹Under the proportional hazard assumption, the interaction of the covariates with analysis

Table 1.3: Ordered Events Models

	AG	PWP(1)	PWP(2)	WLW
Covariates	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)
Capital/GDP	-0.0599 (0.003)	-0.0552 (0.009)	-0.0435 (0.015)	-0.0860 (0.003)
M2/GDP	0.0122 (0.069)	0.0125 (0.013)	0.0135 (0.007)	0.0121 (0.221)
Govt. Expend. growth	-0.0302 (0.000)	-0.0377 (0.000)	-0.0357 (0.000)	-0.0422 (0.000)
GDP per capita growth	0.1254 (0.000)	0.1024 (0.000)	0.1114 (0.000)	0.1246 (0.000)
Paris1 (No Paris Club)	1.7766 (0.000)	2.1165 (0.000)	1.7039 (0.001)	2.6950 (0.000)
Paris2 (Classic)	1.8073 (0.000)	1.9548 (0.000)	1.9130 (0.000)	1.7826 (0.000)
Paris3 (Houston)	1.6623 (0.000)	1.6209 (0.000)	1.4903 (0.002)	2.3584 (0.000)
$\chi^2(df)$ (p-value)				
PH Test (Schoenfeld Res.)	1.14 (0.9923)	1.16 (0.9918)	9.51 (0.2182)	5.10 (0.6478)
PH Test ($\beta_2 = 0$)	0.49 (0.4834)	0.72 (0.3953)	8.31 (0.0039)	2.79 (0.0950)

based on Schoenfeld residuals, and on re-estimation are shown in table 1.3), hence, no conclusions can be drawn from these specifications (PWP(2) and WLW) under the Cox proportional hazard. An in-depth analysis of a variable-by-variable case indeed shows that the latter models failed the proportionality hazard assumption test for some variables; that is, the effect of these variables changes with time.

As in the case of frailty models, I used the integrated hazard function approach as an informal way to detect misspecification on the models used. The cumulative time should not be different from zero; if the model is correctly specified that interaction is already parameterized in the model. If $Z\beta_z$ is the correct specification, then $\beta_2 = 0$, where $LRH = \beta_1(Z\hat{\beta}_z) + \beta_2(Z\hat{\beta}_z)t$. The test can also be performed for each covariate, $LRH = \beta_1(Z\hat{\beta}_z) + \beta_2(Z_1t)$.

hazard function plotted against the generalized (Cox-Snell) residuals suggests that the model that best fit the data is the AG model. The results of all sets of tests combined (potential omitted variables, proportionality of hazard functions, and misspecification) point to the AG model as the one that best describes the case of interest.

The parameters estimated using the AG model have the same sign and roughly the same values as the parameters estimated using the models without unobserved heterogeneity (Cox PH Model).²⁰ The interpretation of the coefficients estimated under the latter models can then be extended to these coefficients.

Since all these models consider the order of the default-settlement episodes, it is possible to analyze how the default history influences the settlement rate of the default in progress. Using these same specifications for parametric models I estimate the median settlement time for countries that default for the first, second, third time and so on. It turns out, that the indicators that uniquely identify the strata (default order) are statistically significant up to the third default; from then on the settlement rates are not significantly different from each other. Moreover, countries that default for the first time settle their debts in a median time of about 8.67 years (or an average of 10.46 years), the second default is settled in a median time of 4.02 years (or an average of 8.6 years) and the third in a median time of 2.52 years (or an average of 5.31 years).²¹

²⁰Lognormal and loglogistic models are AFT models and their coefficients have opposite sign to PH models.

²¹From a Weibull model under specification proposed by PWP(1). Similar results are found under different parametric model and different specifications.

Once the debtor country has defaulted, one of the few variables it can control—given the state of nature—is its decision to enter or not into a “Paris Club” type of agreement with its creditors. In the negotiation process, which final goal is the settlement of the defaulted debt, the borrower can decide whether to repay in full, if it has the ability to do so, and leave the “punishment” period as soon as he settles; or the borrower can engage in a negotiation in order to reduce the outstanding debt and remain excluded from the international credit markets for a longer period. From the empirical model, and as shown in figures 3 and 4, the higher the portion of debt repaid, the shorter the period of exclusion of the debtor country from the international credit markets; that is, the higher the probability of regaining access to the international credit markets in the next period (settlement rate). A debtor country that has not engaged in any type of debt reduction program is excluded, on average, 11.05 years from the international credit markets; if in addition that country commits itself to implement reforms, sending a costly signal to the credit markets through an agreement with the IMF, it will be excluded, on average, 8.98 years from the credit markets, 2.1 years less than the one that does not do so. On the other hand, a debtor country that belongs to the Paris Club and that might have received a reduction of up to 90% of its debt will be excluded, on average, 21.7 years. A country that has only rescheduled its debt but that has not received any nominal reduction (renegotiated under concessionary rates) is excluded, on average, 17.09 years.²²

These cases have different implications for both the borrower and the lender:

²²ibid.

the longer the period of exclusion the higher the cost of default for the borrower and the less often debtor countries will be able to default. Since creditors are able to recover just a portion of the defaulted debt in a period of time longer than that originally stated in the contract—and they incorporate this information when establishing the credit ceiling and the interest rate they will charge for each loan—the lower the portion of repayment, the more expensive the loan and the lower the credit ceiling.

1.4.4 Non Parametric test for equality of survival functions

Using non parametric tests (Log-rank, Wilcoxon, Tarone-Ware, Peto-Peto-Prentice) one can determine the equality of the overall settlement functions by comparing, at each settlement time, the expected vs. the observed number of settlements for each group, and then combining these comparisons over all observed settlement times. The tests differ on how they weight each of these individual comparisons that occur at each settlement time to form the overall test statistic.²³ Although the tests have the advantage that we do not need to know anything about the shape of the survival curve or the distribution of the survival times, the literature suggests that the Wilcoxon test is more powerful when the data is drawn from a log-normal or log-logistic distribution because it places more weight on early failures (settlements). The null hypothesis is that there is no difference in the probability of a settlement at any point in time across groups.

²³Log rank test weights all differences equally, Wilcoxon and Tarone-Ware tests give more weight to differences that occur at time points where there is the most data.

The results of these tests are only valid under the assumption that the duration of default episodes are all drawn from the same distribution, as the results of shared frailty models seem to indicate (no correlation within countries). These tests were carried out using the complete dataset that includes all defaults experienced since the 1820s.

Bank loans vs. bonds

The results related with the type of debt in default —bank loans vs. bonds— are striking and, a priori, unexpected. As hinted by Suter (1992) in explaining the reduction in the duration of defaults after WWII when debt shifted from bonds to bank loans, common wisdom suggests that because bondholders are numerous, scattered, and potentially anonymous, bonds are very hard to settle/reschedule. Until recently, there was even the belief that bonds were senior to bank loans and that they carried hardly any risk of default. Contrary to bonds, bank loans are in the hands of few investors (large bank syndicates) and this should, in theory, facilitate the negotiation process because the number of actors is smaller. A dummy variable indicating the type of debt in default should, following this reasoning, be significantly different from zero. Bank loans, however, are registered on the banks' balance sheets at face value and are not mark-to-market as are bonds. Building enough capital such that banks are able to write down the losses after a debt crisis requires time and this may explain why there is no difference between bank loans and bonds regarding the duration of the settlement process. As stated by Rieffel (2003, page 3) when describing the 1980s debt crisis: “The outstanding loans of the world's biggest banks

to developing countries were a multiple of the capital on the banks' balance sheets. Writing down these loans at the onset of the crisis to reflect their current market value would have reduced bank capital to the point of risking serious disruption in the international financial system and a global economic depression. The financial authorities in the major creditor countries adopted a strategy to buy time until the banks were able to accumulate enough capital to absorb these losses.”

The log-rank test for equality of survival functions for syndicated loans (bond=0) and bonds (bond=1) does support the hypothesis that both groups have the same survival functions.

Table 1.4: Test for Equality of Survival Functions: Bank Loans vs. Bonds

	$\chi^2(1)$	$Pr > \chi^2$
Log-rank Test	0.00	0.9594
Wilcoxon Test	0.06	0.8069
Tarone-Ware Test	0.02	0.8966
Peto-Peto-Prentice Test	0.02	0.8907

Default Order

The log-rank tests for equality of the survival functions across groups confirm the results found with the ordered event models: the experience gained by debtors and creditors in negotiating the settlement conditions reduces the duration of defaults up to the third event; that is, history of default matters until the third default is reached. After the third default, experience does not help debtors and creditors reduce the time of the negotiations. Pairwise comparisons confirm that there is no difference between any pair of defaults after the third event occurs.

It is worth mentioning that first defaults range in year of occurrence from 1824

to 2003, second defaults from 1827 to 2002, thirds from 1837 to 2002, fourths from 1854 to 2002, and fifths from 1866 to 2001.

Table 1.5: Test for Equality of Survival Functions: Order of the Default Events (1st, 2nd, 3rd and higher)

	$\chi^2(2)$	$Pr > \chi^2$
Log-rank Test	10.71	0.0047
Wilcoxon Test	10.30	0.0058
Tarone-Ware Test	10.62	0.0049
Peto-Peto-Prentice Test	10.18	0.0061

Pre vs. Post World War periods

Taking advantage of the conclusions we can draw regarding the lack of correlation within countries, I tested Mitchener and Weidenmier’s claim about the influence of the Roosevelt Corollary on the credit market. There are several other reasons for comparing the pre vs. the post World War periods (both, WWI and WWII):

- The Roosevelt Corollary²⁴ to the Monroe Doctrine²⁵ outlined in 1904, deterred

²⁴President Theodore Roosevelt stated on December 6th, 1904 in his annual message that "Any country whose people conduct themselves well can count upon our hearty friendship. If a nation shows that it knows how to act with reasonable efficiency and decency in social and political matters, if it keeps order and pays its obligations, it need fear no interference from the United States. Chronic wrongdoing, or an impotence which results in a general loosening of the ties of civilized society, may in America, as elsewhere, ultimately require intervention by some civilized nation, and in the Western Hemisphere the adherence of the United States to the Monroe Doctrine may force the United States, however reluctantly, in flagrant cases of such wrongdoing or impotence, to the exercise of an international police power We would interfere with them only in the last resort, and then only if it became evident that their inability or unwillingness to do justice at home and abroad had violated the rights of the United States or had invited foreign aggression to the detriment of the entire body of American nations."

²⁵The Monroe Doctrine in December 1823 declared the Western Hemisphere an area of particular interest to the United States which would oppose any new attempt at colonization by the European powers.

European powers, especially after WWI, from intervening in Central, South America and the Caribbean reducing gunboat diplomacy incidents. There is empirical evidence that indicates that the Roosevelt Corollary modified investors' and creditor's behavior regarding defaults (Mitchener and Weidenmier (2004)).

- World War I marked a dramatic shift regarding creditors' location. Previous to WWI, sovereign debts were issued primarily on the London Exchange (France, Germany and the Netherlands played a secondary role). After WWI underwriting of sovereign securities moved to the United States.
- After WWII, multilateral institutions were created and official creditor programs were introduced.

In order to test for equality of the survival functions among the periods, a new variable *century* was created, which registers default episodes that occurred previously to WWI (1914), during the beginning of WWI (1914) and the end of WWII (1945), and after the end of WWII (1945). The different tests for equality of the survival functions cannot clearly reject the null hypothesis that the survival functions of the three groups are the same (See table(1.6)).

Table 1.6: Test for Equality of Survival Functions: Pre, Post and Interwar Periods

	$\chi^2(2)$	$Pr > \chi^2$
Log-rank Test	4.23	0.1209
Wilcoxon Test	5.11	0.0776
Tarone-Ware Test	4.37	0.1127
Peto-Peto-Prentice Test	4.53	0.1036

In order to determine the point where the structural change took place, I conducted pairwise tests comparing two consecutive groups. When the *century* variable is defined for default episodes that occurred before (*century*=0) and after (*century*=1) the beginning of WWI (1914), dropping observations that occurred after WWII (1945), the null hypothesis that the survival functions of the two groups is the same is accepted at a 5% significance level; that is, the empirical evidence suggests that there is no difference between settlement experiences that occurred before WWI and in the interwar period (1914-1945).

Table 1.7: Test for Equality of Survival Functions: Pre, and Post WWI

	$\chi^2(1)$	$Pr > \chi^2$
Log-rank Test	2.72	0.0994
Wilcoxon Test	0.36	0.5501
Tarone-Ware Test	1.03	0.3105
Peto-Peto-Prentice Test	0.37	0.5436

If the *century* variable is defined for default episodes that occurred before (*century*=0) and after (*century*=1) the end of WWII (1945), dropping defaults that occurred before WWI (1914), the null hypothesis that the survival function of the two groups is the same cannot be rejected at a reasonable significance level.

Table 1.8: Test for Equality of Survival Functions: Pre, and Post WWII

	$\chi^2(1)$	$Pr > \chi^2$
Log-rank Test	0.01	0.9172
Wilcoxon Test	1.68	0.1946
Tarone-Ware Test	0.60	0.4370
Peto-Peto-Prentice Test	1.06	0.3026

There is no empirical evidence that suggest differences in the survival function between the interwar period (1914-1945) and post WWII or between WWI and the

interwar period. However, if the sample is split before and after 1904, year of the announcement of the Roosevelt Corollary, the tests point to a large difference in the settlement experiences of both groups. A stratified test allows for a deeper analysis regarding the possible causes for these differences: after creating a new variable LA that registers those countries that might have been affected by the Roosevelt Corollary (Latin American countries), and performing the test for both subgroups (Latin American and Non Latin American countries), it is clear that the differences in the default experiences before and after 1904 are driven by Latin American countries. The Latin American countries after 1904 experienced a reduction in the period of exclusion, which indicates that the probability of leaving the state of default increased after the announcement of the Roosevelt Corollary. This behavior is not observed in non Latin American countries. The causes for the differences in the default experiences seems to rest in the announcement of the Roosevelt Corollary rather than the shift in the creditors' location from Britain to the United States, or the creation of multilateral institutions after WWII. The latter results confirm Mitchener and Weidenmier's.

Table 1.9: Test for Equality of Survival Functions: Roosevelt Corollary (1904)

	All Countries		LAC Countries		Non LAC Countries	
	$\chi^2(1)$	$Pr > \chi^2$	$\chi^2(1)$	$Pr > \chi^2$	$\chi^2(1)$	$Pr > \chi^2$
Log-rank Test	8.66	0.0033	6.15	0.0132	2.54	0.1113
Wilcoxon Test	6.22	0.0126	4.76	0.0291	1.57	0.2103
Tarone-Ware Test	7.26	0.0071	5.51	0.0189	1.84	0.1754
Peto-Peto-Prentice Test	6.27	0.0123	5.01	0.0252	1.53	0.2166

It is important to recall, at this point, the wide range in the distribution of years of occurrence of the first, second, third, and fourth defaults. Because these events occurred before as well as after 1904, the experience gained by the parties in the negotiation is not the only factor driving the results.

1.4.5 Strategic Bargaining Models

Following Kennan and Wilson (1989), the results indicate that the appropriate strategic bargaining models to describe the settlement process after a sovereign debt default are screening or signalling models rather than attrition models. Attrition models (“winner takes all”) predict that the bargaining process will continue as long as the privately known cost of holding out, which increases with time, is less than the expected gain, which depends on the other party capitulating. The perceived probability of the other party conceding (quit rate) declines with the passage of time, so the settlement rate, which is the sum of both players quit rates is a decreasing function of time, that is, default duration. Because the winner takes all, the settlement conditions (Paris club terms of agreement) are uncorrelated with default durations. As pointed out by the empirical results, there is a correlation between settlement conditions and default durations, which cannot be explained by war-of-attrition models. These models are also unable to replicate the functional form of the settlement rate found in the empirical model. Even when one allows for heterogeneity, which might lead to an inverse U-shape settlement rate for the whole population, as found in the results, it is necessary for some individuals to have settlement rates with positive duration dependence.

Screening models (optimal discrimination by one of the parties), on the other hand, predict a declining correlation between the negotiated settlement conditions (percentage of debt reschedule, level of debt cancelation, grace period, interest rate) and default duration, and often forecast a flexible settlement rate. These models characterize impatience using a discount factor, instead of linear delay costs as in attrition models, and assume the existence of information asymmetries. Offers are made periodically at discrete intervals. The shape of the settlement rate will depend on whether just one or both parties are able to make offers, on the magnitude of the discount factors with respect to one another, and the distribution of the party type.

Signalling models focus on costly delays as a credible signal that the offer cannot be improved. The parties choose the timing of each offer such that delay has a signalling role and must be sufficient to prevent profitable mimicking behavior by another type; that is, must satisfy incentive-compatibility conditions such that the parties select their optimal behavior depending on their types. As in the case of screening models, the settlement rate is potentially variable and might allow for the correlation between settlement conditions and default duration to exist. Given the anecdotal evidence, these theoretical models seem to be the better suited to describe the negotiation process after a sovereign default.

1.5 Concluding Remarks

The use of survival analysis techniques for the study of the duration of a country's exclusion from international credit markets, as a consequence of a default,

supports the idea that the more information lenders have regarding the debtor country, the shorter the settlement process. That information can come from experience gained in previous defaults, or from credible announcements of fiscal reforms. In order for these announcements to be credible, they need to be costly: in our case the commitment to implement these fiscal reforms under the sponsorship of an IMF program, which tends to be highly unpopular among the constituency but guarantees disclosure of information to the creditors. Experience gained by debtors and creditors in dealing with a previous default seem to be a factor in reducing the length of the negotiation of future episodes. The credit history of a country, however, is statistically significant in explaining the duration of the settlement process up to a certain point; after that point is reached (the third default) history does not matter any more. The reduction in the negotiation period, which I attribute in part to the experience gained by debtors and creditors in dealing with defaults, was explained by Suter (1992) arguing that the marked difference between the settlement process before and after WWII, was driven by the creation of bondholders committees and the change in the actors' structure of the creditor's side (bondholders were replaced by bank syndicates). Although the conclusions seem to be in part the same, that the experience gained in past events matter, a further look reveals the differences: my results indicate that it does not matter if the debt is issued in form of bonds or in form of bank loans.

The benefits that common wisdom attribute to negotiating bank loans, namely that it is easier because the debt is held in the hands of very few investors (bank syndicates), are lost because the seemingly faster bank negotiators need to accu-

multate capital. Building capital is necessary for banks to write down the loans in distress, to reflect their true value, without jeopardizing their operations. If banks were able to register their loans at their market value, marking-to-market them as investors do with their holdings of bonds, the myth that banks negotiate faster than bondholders would be probably valid. But whatever time is gained against bonds because the number of agents is low, is lost because the way banks register loans in their balance sheet.

The difference documented by Suter (1902) in the settlement process before and after WWII is driven then, in part by the experience gained by creditors in dealing with debtors (creation of bondholders committees), but mainly by the change endured by Latin American countries under the sphere of influence of the United States after the announcement of the Roosevelt Corollary of the Monroe Doctrine. The difference, as suggested by the results, is driven neither by the shift in the creditors' location from Britain to the United States, nor by the creation of multilateral institutions after WWII, but by the announcement of the Roosevelt Corollary. The thesis of the effect of President Theodore Roosevelt on the credit market, first proposed by Mitchener and Weidenmier (2004), is confirmed by the results of the log-rank tests.

The results also show that when a portion of the defaulted debt is forgiven, the debtor country is kept from reentering the international credit markets for a longer period. Even when a portion of the debt is rescheduled, providing the debtor country with a temporary relief in the form of a grace period or concessionary interest rates, lenders keep the country out of the credit market longer than a country that

has not rescheduled its debt. It seems reasonable to believe that countries that engage in these kinds of deals (debt reduction) are the ones categorized as “Heavily Indebted Poor Countries” (HIPC), and that these are the countries with the worse economic indicators. This reasoning should lead us to conclude that fundamentals are key to understanding the settlement process and that the better the economic conditions the shorter the period of exclusion. The results, however, show that there are few economic indicators that significantly affect the settlement process. The variables that consistently explain the length of the period of exclusion are the dummy variables for debt reduction (different Paris Club indicators).

The parametric models that best fit the data (log-normal and log-logistic), and the correlation between the settlement conditions and the duration of the negotiation, shed lights about the strategic bargaining models best suited to describe the experience of sovereign debtors in default: signalling models where the parties use the timing of their announcements, and consequently the delays, as signals that their offer cannot be improved.

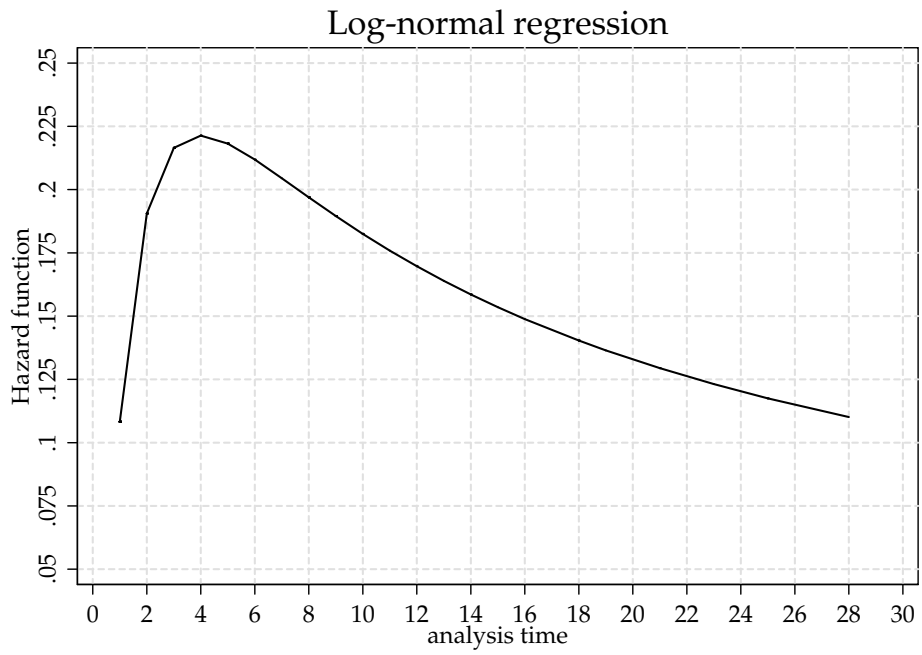


Figure 1.1: Settlement Rate under LogNormal Model without Unobserved Heterogeneity

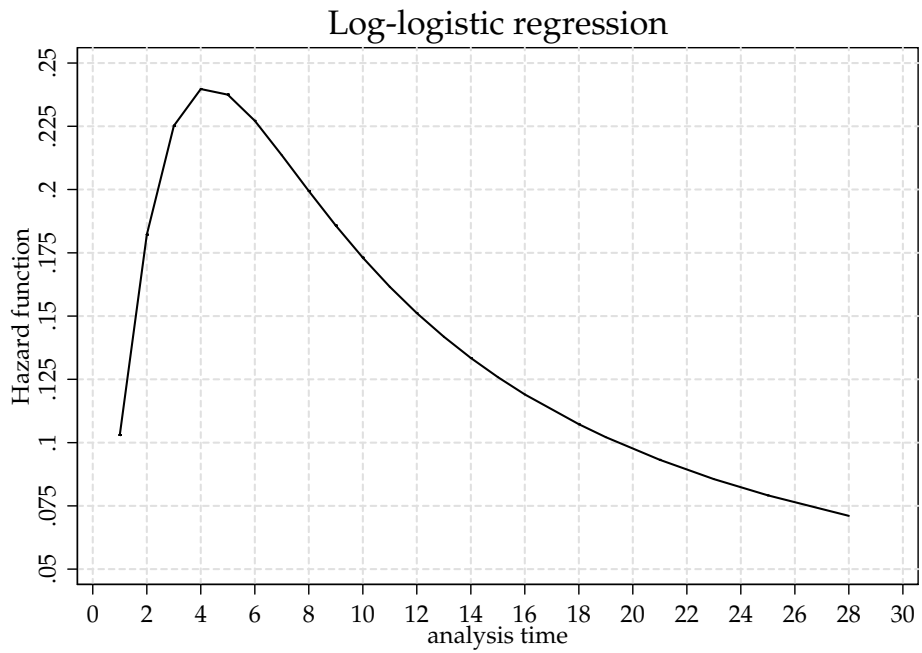


Figure 1.2: Settlement Rate under LogLogistic Model without Unobserved Heterogeneity

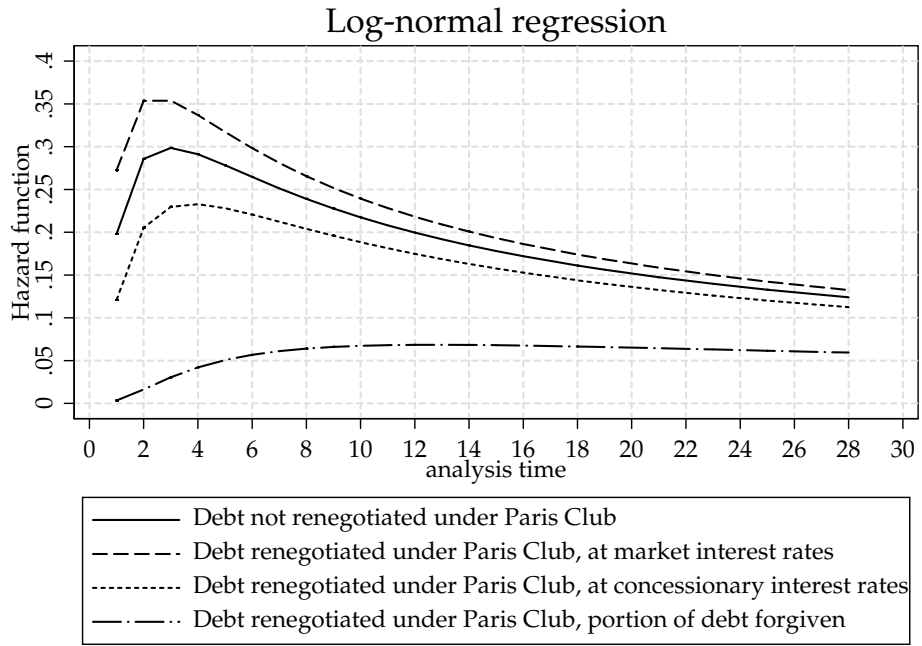


Figure 1.3: Settlement Rate for the LogNormal Model without Unobserved Heterogeneity at different values of the indicator “Paris Club”

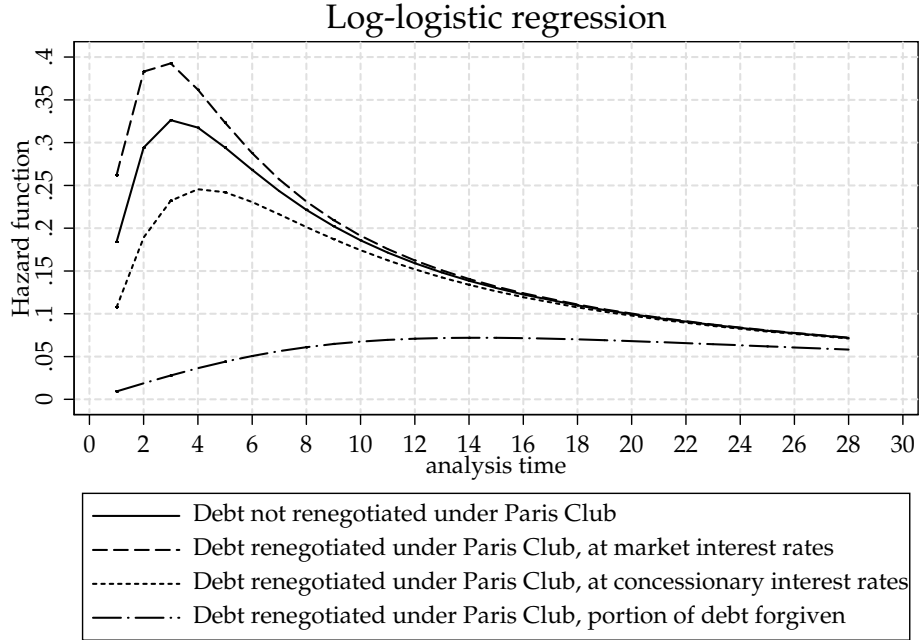


Figure 1.4: Settlement Rate for the LogLogistic Model without Unobserved Heterogeneity at different values of the indicator “Paris Club”

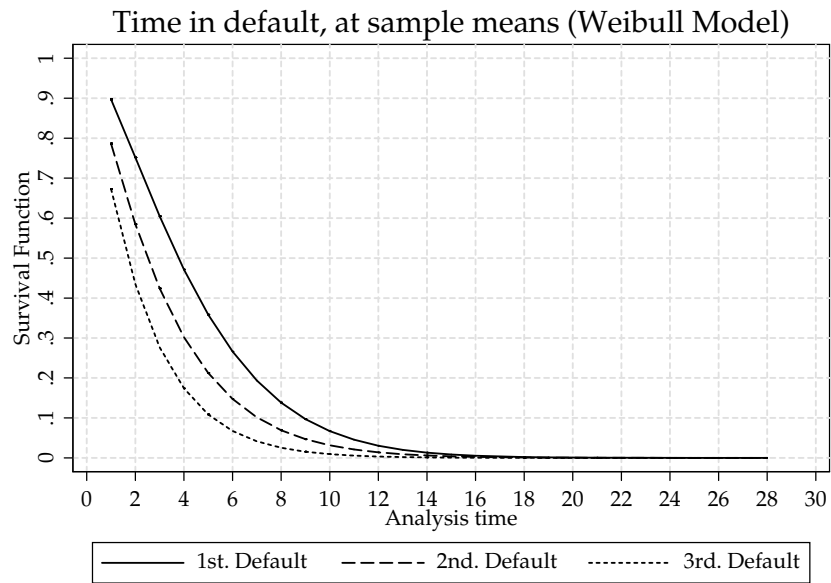


Figure 1.5: Probability of Staying in State of Default @ Pclub2=1 (Weibull Model, under PWP specification)

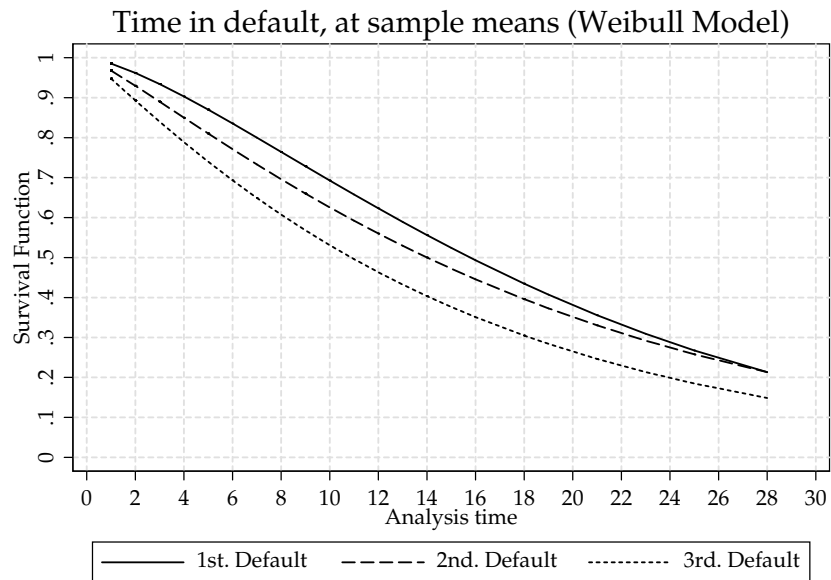


Figure 1.6: Probability of Staying in State of Default @ Pclub4=1 (Omitted Variable) (Weibull Model, under PWP specification)

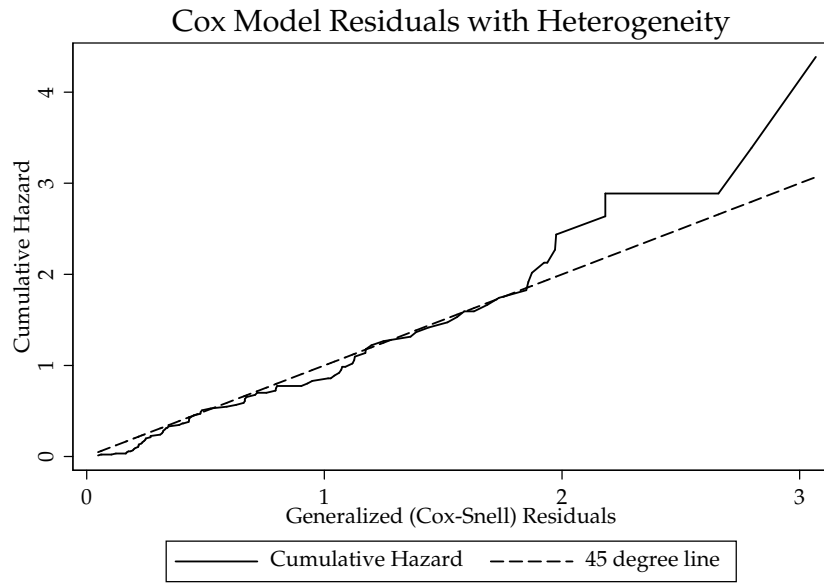


Figure 1.7: Cumulative Hazard Function of Cox-Snell Residual, Cox Frailty Model

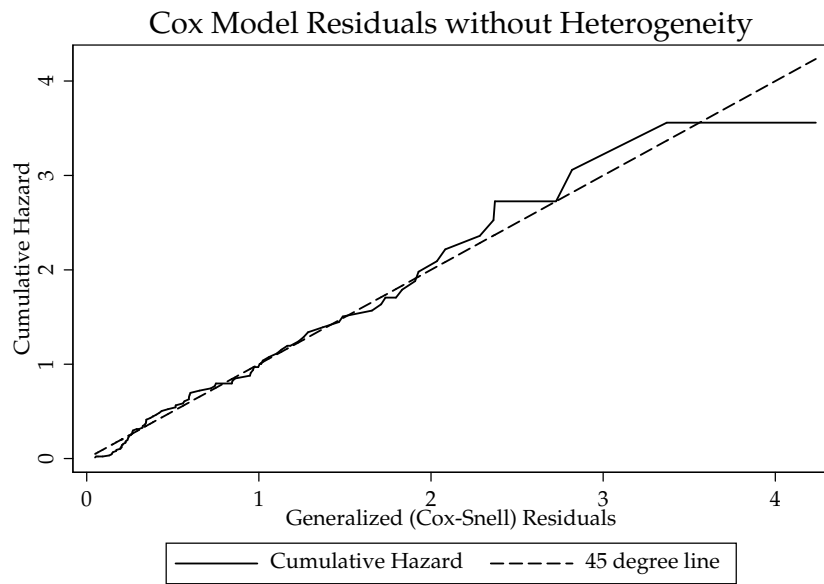


Figure 1.8: Cumulative Hazard Function of Cox-Snell Residual, Cox Model without Unobserved Heterogeneity

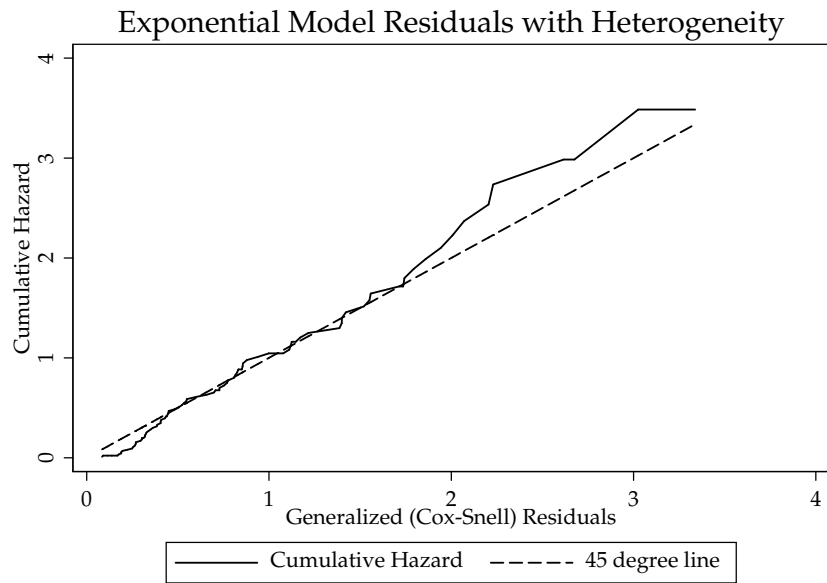


Figure 1.9: Cumulative Hazard Function of Cox-Snell Residual, Exponential Frailty Model

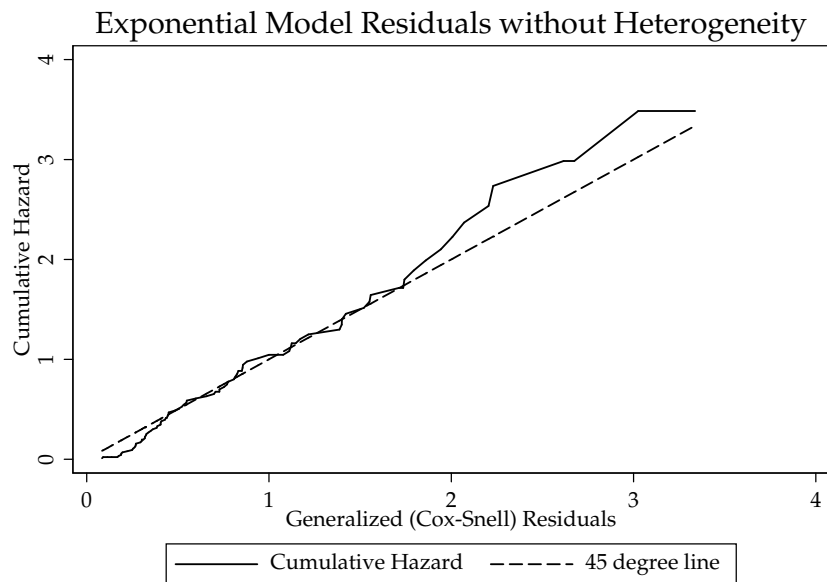


Figure 1.10: Cumulative Hazard Function of Cox-Snell Residual, Exponential Model without Unobserved Heterogeneity

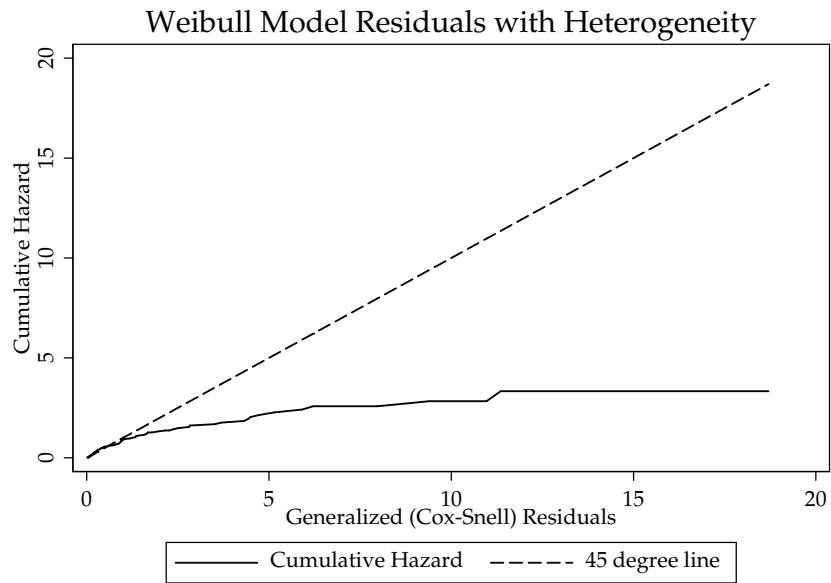


Figure 1.11: Cumulative Hazard Function of Cox-Snell Residual, Weibull Frailty Model

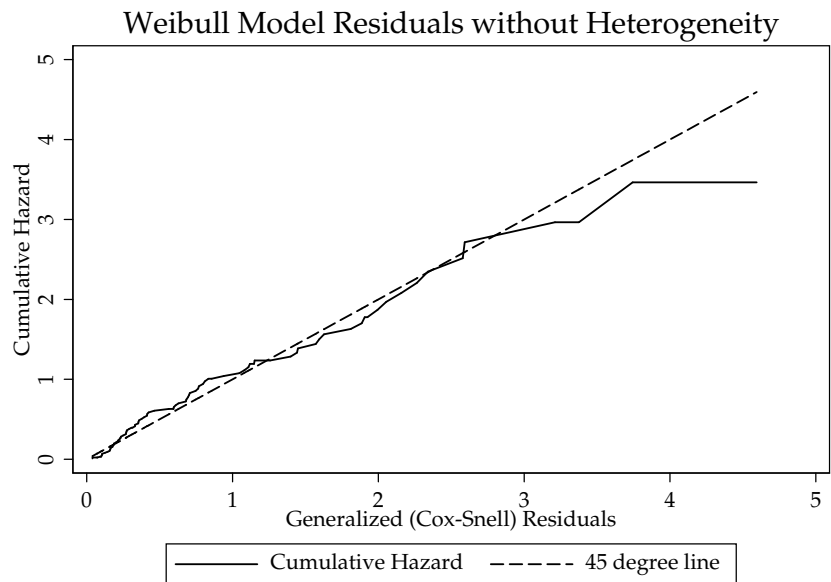


Figure 1.12: Cumulative Hazard Function of Cox-Snell Residual, Weibull Model without Unobserved Heterogeneity

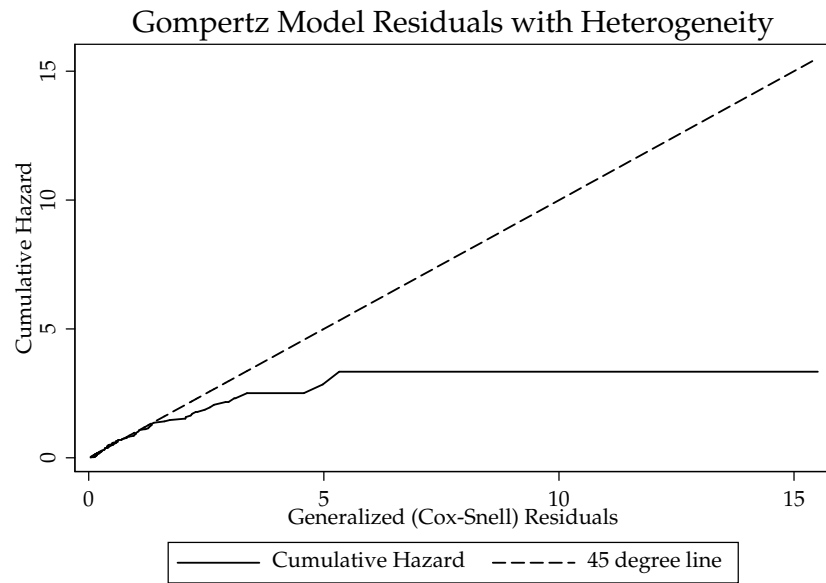


Figure 1.13: Cumulative Hazard Function of Cox-Snell Residual, Gompertz Frailty Model

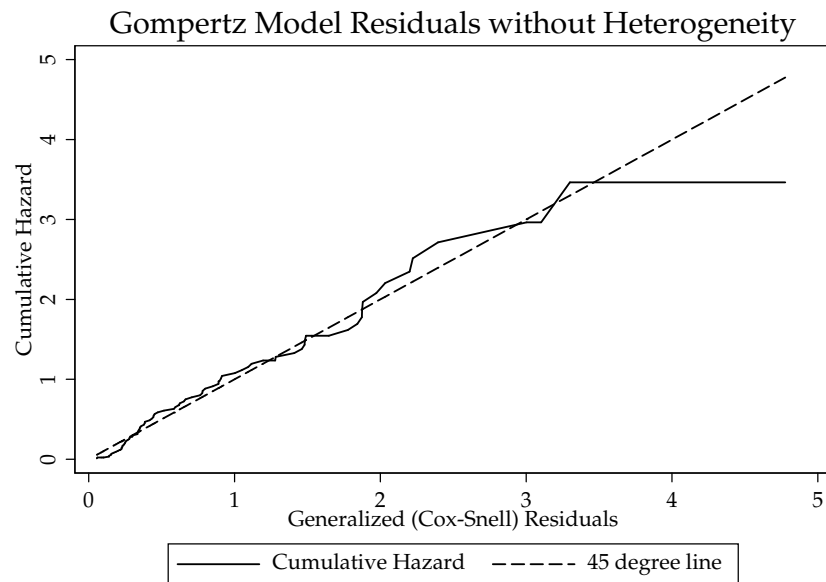


Figure 1.14: Cumulative Hazard Function of Cox-Snell Residual, Gompertz Model without Unobserved Heterogeneity

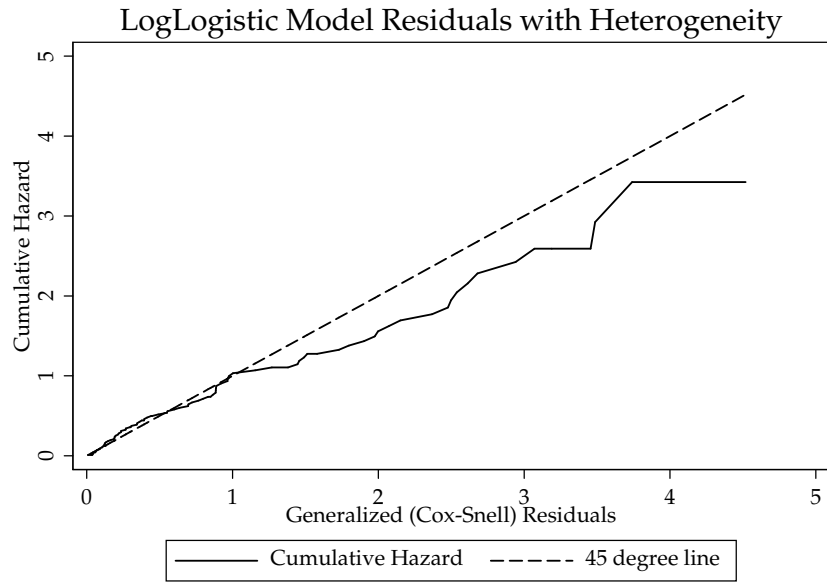


Figure 1.15: Cumulative Hazard Function of Cox-Snell Residual, LogLogistic Frailty Model

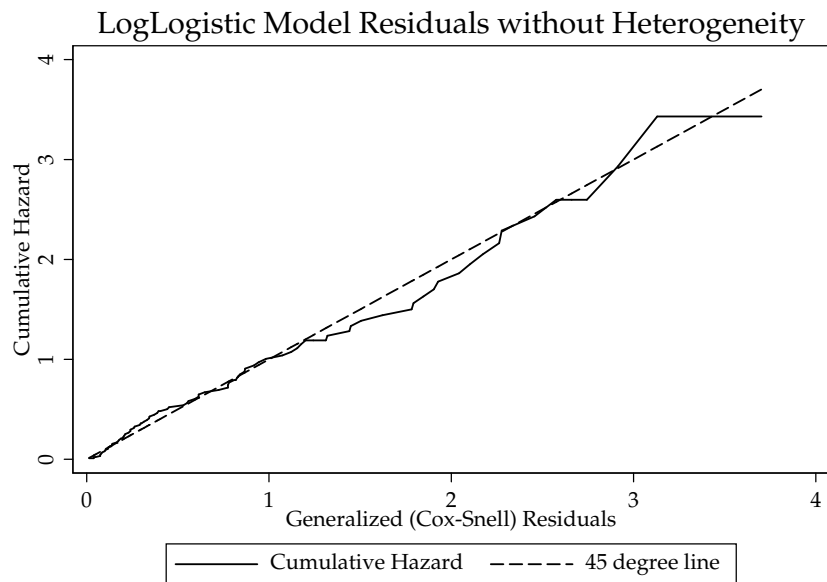


Figure 1.16: Cumulative Hazard Function of Cox-Snell Residual, LogLogistic Model without Unobserved Heterogeneity

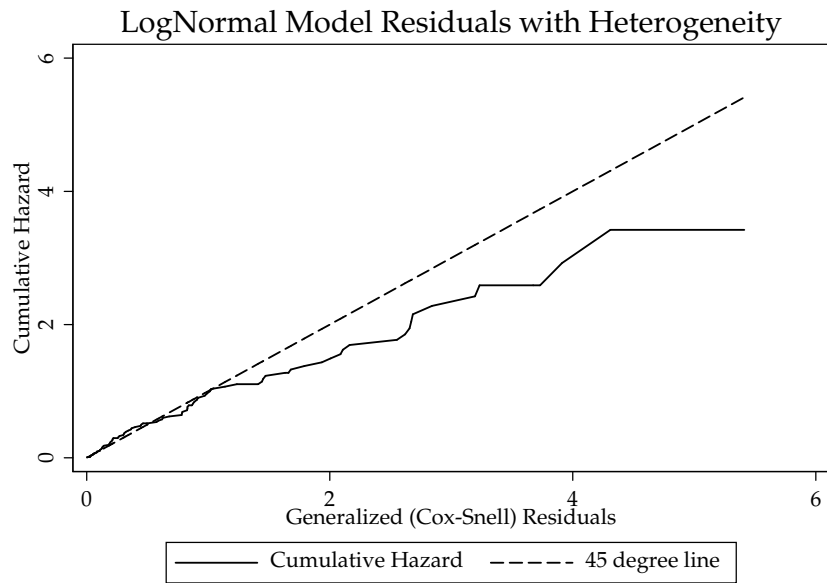


Figure 1.17: Cumulative Hazard Function of Cox-Snell Residual, LogNormal Frailty Model

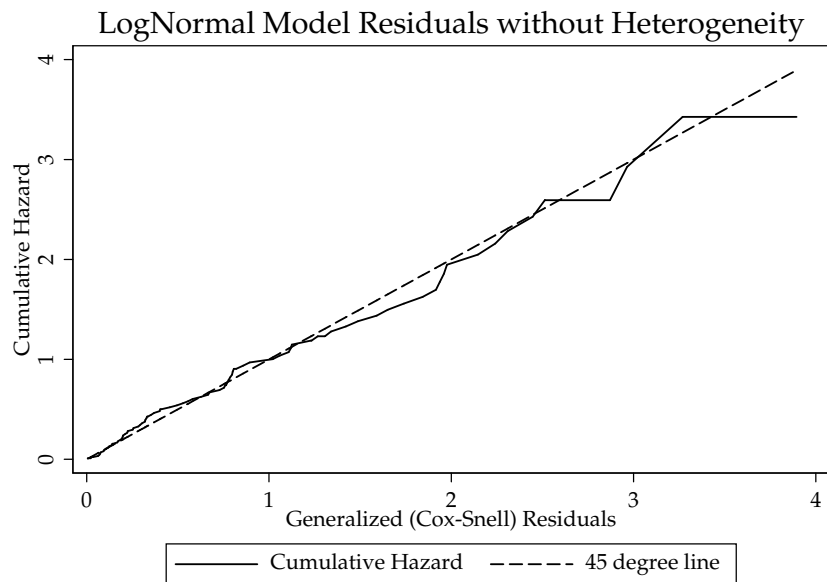


Figure 1.18: Cumulative Hazard Function of Cox-Snell Residual, LogNormal Model without Unobserved Heterogeneity

Chapter 2

Default, Settlement, and Repayment History: A Unified Model of Sovereign Debt

2.1 Introduction

Lenders and credit rating agencies often classify debtor countries into different categories depending on their ability to access international credit markets. There are countries with virtually unlimited access, others that rarely are able to tap the market, and a middle group of countries characterized by having sporadic access to the international credit market. A number of studies, including Reinhart, Rogoff and Savastano (2003), Tomz (2001), Özler (1993) and Cruces (2003), have documented this fact. These classifications go hand-in-hand with several stylized facts about sovereign debt: i) unfamiliar borrowers are charged the highest risk premium, and this premium declines over time provided that the government services its debt. I argue that as a result of the differences in the cost of servicing the debt, ii) the debt-to-GDP ratio that new borrowers (or serial defaulters) can support is well below the ratio that proven debtors can safely manage; and that the decline in the risk premium over time gives rise to the fact that iii) rating revisions are strongly serially correlated and more volatile for middle-rated countries where credit surprises are more likely to occur. The frequently interrupted access of many emerging economies to international credit markets has been partially blamed on their histories of serial default. It is well documented that iv) countries that default are unable to gain

access to new credit unless they work out a settlement with their creditors; and that v) the length of time countries in default are shunned in international credit markets depends on the attractiveness of the settlement offer to creditors, and consequently on the duration of the negotiation process. The data also suggest that vi) once a country defaults, it will take many years of perfect repayment and low levels of debt for that country to gain continuous access to international credit markets at low cost; and that vii) this process is exceedingly slow, and backsliding into default is very difficult to avoid. This paper deals with countries in the middle group and focuses on explaining the reasons why their access to international credit markets is irregular, and on rationalizing some stylized facts previously documented in the literature (ii,iii,vi, and vii).

The paper addresses two related issues: 1) assuming that the probability of a settlement depends on the conditions of the offer (i.e., the portion of the defaulted debt to be repaid by the borrower) and provided that debtors and lenders know that probability, how the level of debt and income affect the length of time a country is excluded from the international credit market after a default; 2) assuming lenders charge a risk premium that depends on the number of periods the borrower has been active in the international credit market, how repayment history affects the menus of debt contracts lenders are willing to offer and the borrower's incentives to default. The paper argues that repayment history affects a borrower country in the same way it affects an individual borrower; namely, the credit market charges higher borrowing cost and imposes tighter credit limits upon new borrowers or individuals

that have recently filed for bankruptcy.¹

I tackle these two issues using a dynamic, stochastic, general equilibrium model in which both default and the settlement conditions are endogenously determined, and in which repayment history affects access to the credit market and the cost of the debt. The borrowing cost of the one-period bonds negotiated by risk-neutral lenders and risk-averse borrowers, as well as the debt ceiling, are determined endogenously and depend not only on “fundamentals”, characterized by the borrower’s income profile, but also on lenders’ expectations regarding a potential settlement process after a default and on the score that lenders assign to borrowers based on their repayment history. This score depends on the number of periods the borrower has been active in the international credit market; the number of periods may be zero either because the borrower is new or because it has recently gained access to the market after a settlement. Lenders’ expectations about the time the negotiation process will take and about the portion of the defaulted debt that the debtor will be willing to pay affect the menu of debt contracts lenders are prepared to offer and the incentives borrowers have to default. Provided the state of the economy is the same, only the government’s repayment history affects the cost of debt.

The model assumes that although lenders know the borrower’s income profile,

¹The model is particularly relevant to individual borrowers’ default decision in light of the new Bankruptcy Abuse Prevention and Consumer Protection Act of 2005. The new act forces individuals who earn more than their state’s median income to file for bankruptcy under Chapter 13 rather than under Chapter 7. Under Chapter 7 the borrower’s debt is discharged and all non-exempt assets are seized by his creditors. Under Chapter 13, debt is not discharged or assets relinquished, but debtors are required to submit to a repayment plan that involves partial reduction of the debt.

they cannot verify the current state of the economy. After a default, the existence of private information translates into inefficient delays in the negotiation and this inefficiency is captured in the stochastic nature of the settlement process. The paper, however, does not model the negotiation process, but simply takes for granted the outcome of the negotiation. It assumes, as shown in Chapter 1, that after a default, the probability that lenders will accept a settlement offer depends positively on the portion of the defaulted debt borrowers are willing to repay. Costly delays are a credible signal that the parties cannot negotiate to improve the offer. The model also assumes that lenders charge an additional risk premium to new borrowers as well as to debtors that are able to settle the debt after a default. This risk premium declines asymptotically over time provided that the borrower faithfully services its debt. The role of this assumption is to capture differences in the costs of defaulting for countries with identical income profiles but different repayment histories.

Given the nature of the model, and in order to illustrate the two main contributions of the paper, I present the numerical solution in two sections: one that deals with the settlement process without accounting for repayment history, and a second section that introduces repayment history into the model. The results show that the model is not only able to replicate the frequency of default observed in several countries, but is also able to replicate periods of exclusion from the credit markets after a default comparable with the ones observed in the data. Based on these results, it is clear that default episodes are not rare events and serial defaults are not extraordinary.

The contribution of this paper to the existing literature is a dynamic bor-

rowing model that combines default, settlement, and repayment history. The main differences with the existing literature are twofold. First, the model allows for the possibility of restructuring the stock of debt after a default occurs.² The model takes into account the fact that the negotiation is a lengthy process, and that the government cannot access the international credit market until the bad debt is settled.³ In order to reflect the inefficient delays in reaching a mutually beneficial agreement due to the existence of private information, the paper models the length of exclusion from the credit market as a random process that depends on the borrower's choice of settlement offer. The model conditions the outcome of the settlement not only on the country's level of indebtedness, but also on its current income level. The cost of settling the defaulted debt accounted in the model helps explain why the previous literature based on the seminal work of Eaton and Gersovitz (1981) —with exogenous reentry to the credit market and no settlement cost because the defaulted debt is forgiven— was unable to generate large levels of indebtedness: the endogenous debt limit is closely related to the cost of defaulting. Second, the model explores the phenomenon documented by Tomz (2001): lenders use a learning approach to update their perception about the likelihood of a default in response to the borrower's payment history. Although Tomz's theory about lender behavior is recent,

²Arellano (2003), Aguiar and Gopinath (2004), Chatterjee, Corbae, Nakajima, and Rios Rull (2002) develop dynamic borrowing models with endogenous default. Although they allow for the defaulting country —consumer in the case of Chatterjee et al.— to reenter the financial markets, ex-post renegotiation and debt rescheduling is ruled out a priori. Their models assume the defaulting agent is redeemed with a known probability and that all past debt is forgiven.

³Yue (2005) models the renegotiation process and assumes the country is excluded from the financial market for one period after a default.

the consequences of this behavior have been amply documented in the literature. The introduction of repayment history helps to answer questions such as: why some countries become serial defaulters, why countries with identical income profiles face different borrowing costs, why serial defaulters are unable to handle debt-to-GDP ratios that developed economies handle without any trouble, and why debt crises are preceded by loan booms and followed by credit slumps.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the theoretical model. Section 4 discusses the numerical solution and presents the results. Section 5 concludes with a discussion of the implications of the model and comments on possible extensions.

2.2 Related Literature

Economists have modeled default episodes as an equilibrium phenomenon in which creditors agree to offer debt contracts even in cases when default episodes are possible outcomes (Eaton and Gersovitz (1981), Grossman and Van Huyck (1988), Kletzer and Wright (2000)). In some models, all creditors need to know is the income distribution of the prospective borrower in order to extend a menu of debt contracts. The only decision for a risk-neutral lender in this position is what interest rate to charge. After a default episode has taken place, however, the lender faces a very different problem. Since there is no way to verify the financial position of the debtor country, the lender needs to decide whether to agree to the restructuring deal offered by the borrower (which by definition is disadvantageous compared with the initial contract) or wait in hopes of getting a better offer and thereby incurring costs

associated with delay. The borrower might be offering the new deal in good faith because it cannot repay in full, but it also might be trying to wear the lender down. The cost borne by the borrower associated with delays in the negotiation process is exclusion from international credit markets until a deal is reached.⁴ After analyzing default episodes occurring since 1820, it is striking that almost no country has been able to access international credit markets while in state of default (Tomz (2001)). Creditors seem to cooperate in excluding countries in default from the credit market as a way to protect their own reputation and guarantee their future profitability. Wright (2002) reckons that the existence of syndicated loans and multiple sovereign debtors create incentives for creditors to collude in punishing a country in default.

Although historical evidence suggests regularities in the timing of borrowing and the patterns of default (debt cycles), it is hard to identify a pattern of debt rescheduling (the period of exclusion from the international credit market greatly varies from country to country, and within countries). This randomness reflects the inefficiencies induced by strategic behavior (debtor-creditor bargaining problems, coordination failures, etc.) because the benefits from reaching a restructuring agreement are private information to the debtor and its creditors (Haldane, Pealver, Saporta and Shin (2003)). The literature has paid little attention to why some countries are able to regain access to credit markets sooner than others and what affects the length of the period of exclusion. Further, how past performance affects

⁴Countries do not need to repay the defaulted debt in full in order to regain access to the credit market; they simply need to express their commitment to do so under the new conditions reached during the negotiation process.

new debt contracts has received little attention.

Cruces (2003) documents some stylized facts about credit ratings. Using Institutional Investors Ratings (IIR) data, he finds that the ratings of middle-rated countries are more volatile than the ratings of very high or very low rated countries. For the latter two groups, credit surprises are relatively rare. He refers to this phenomenon as volatility clustering. Cruces also shows that rating revisions are strongly serially correlated; he estimates that the probability of two consecutive revisions being in the same direction is larger than two thirds. However, he notes that adjustments are asymmetric. Countries rated in the 20-70 range⁵ have maximum downwards adjustments about four times as large as their maximum credit upgrades.

These results are consistent with the findings of Reinhart, Rogoff and Savastano (2003). In their paper, they define three debtors' clubs. Club A consists of advanced economies with virtually continuous access to capital markets, no default history and high IIRs (above 67.7). These are countries that never surprise their creditors. Club C is formed by countries with low IIRs (below 24.2), and very infrequent access to capital markets that spend most of the time in a state of default and rely heavily on grants and official loans as their only source of external financing. Club B, the intermediate region, is formed by a large variety of countries, some of which have experienced recent default episodes and some that have never experienced one. This group includes countries that might graduate to Club A soon, and

⁵Institutional Investors ratings range from 0 to 100, with 100 corresponding to the highest quality credit risk.

countries on the verge of a default. The latter group's access to capital markets is not guaranteed and its safe debt-to-GDP threshold is lower than that of advanced economies. Reinhart et al. show that debt intolerance⁶ is a long lived phenomenon and that although countries can eventually outgrow debt intolerance, the process tends to be exceedingly slow; it requires many decades of impeccable repayment and low levels of debt to graduate from Club B to Club A. Further, backsliding is extremely difficult to avoid. Countries in Club B can easily fall into Club C (consistent with the asymmetric adjustments in credit ratings found by Cruces).

Gelos, Sahay and Sandleris (2003) find that economies that fail to access credit markets usually have lower GDPs, suffer higher political instability, have worse perceived policies and institutions, and are more vulnerable to external shocks than economies that have access.

Tomz (2001) studies credit classification empirically. After analyzing international debt data collected from the eighteenth and nineteenth centuries, he argues that creditors use Bayesian principles to update their beliefs about the likelihood of a default in response to new information about compliance with international debt contracts. Like Reinhart et al. (2003) and Cruces (2003), Tomz distinguishes among three different groups: "stalwarts" that pay their debts under virtually any conditions, "fair-weather payers" that will default under duress but are expected to pay during auspicious years, and "lemons" that will default regardless of the situation. He states that because creditors cannot directly observe the resolve and

⁶The inability of many emerging economies to handle debt levels that would seem manageable by developed economies.

competence of a foreign government, they use credit history —exogenous shocks and actual payments— to draw inferences. Tomz argues that a government alters its reputation by surprising creditors: a “stalwart” will keep its high rating only if it pays under nearly all circumstances, a “lemon” has the opportunity to enhance its low ratings by offering a settlement on defaulted debt, and a “fair-weather payer” should not suffer much loss in its credit rating by defaulting when under economic pressure, provided that it offers an acceptable settlement to creditors.

Others have also found evidence that not only does repayment history affect a country’s perceived creditworthiness, but also that settlement of bad debt is key to regaining access to credit markets after a default. The *IMF Policy Development and Review Department (2001)* finds that countries in default are not able to re-access capital markets until the uncertainty over their negotiations with creditors is resolved, and that a country’s payment history affects its subsequent market access.

Cole, Dow, and English (1995) argue that settlement of the defaulted debt is a tool to reestablish the debtor’s reputation. Their paper focuses on how countries can regain access to international credit markets once they have defaulted. Since settlements cannot be compelled, the authors claim that debtors agree to settle bad debt as a way of signaling their commitment to repaying future loans; that is, settlement can signal a change in their attitude toward complying with their promises. Their work emphasizes the importance of incomplete information and the passage of time.

While creditors may view settlement as a sign that a country that defaulted has a renewed commitment to service its debt, recent empirical evidence (Chapter

1) suggests that once a government has defaulted, engaging in debt reduction or rescheduling hurts the debtor's probability of leaving the period of exclusion. This, in turn, can increase the length of negotiations. When the debtor government decides to repay in full, the debtor-creditor bargaining problem disappears and credit markets are soon open again. There is a positive relationship between the favorableness of the terms of the settlement for the borrower and the duration of the negotiation process. However, the more favorable are the terms to the lenders, the shorter the negotiation span. Jorgensen and Sachs (1989) find similar patterns in the interwar period: the longer a debtor held out for more favorable terms, the better it fared in the conditions of the settlement. Lindert and Morton (1989) have also documented this phenomenon. They state additionally that the returns realized by creditors on foreign government bonds, although below the ex-ante returns implied by the bond issue price and repayment terms, were higher than the returns on risk-free home government bonds.⁷ In extending credit, lenders seem to weigh not only the probability of a default, but also the probability of recovering part of the defaulted debt.

Nineteenth-century reports of the Corporation of Foreign Bondholders on Paraguay and Mexico illustrate two extreme examples of how creditors perceive the defaulting government's resolve and competence, and how that affected the length of the negotiation process. After a visit to Paraguay, Mr. Granville, an envoy of the

⁷Lindert and Morton (1989) estimate a net return premium of 0.44% per year on 1,519 bonds outstanding between 1850 and approximately 1970. These bonds were issued by the governments of Argentina, Brazil, Chile, Mexico, Australia, Canada, Egypt, Japan, Russia, and Turkey.

Corporation of Foreign Bondholders, described the situation in 1880 as follows:

Such is a slight sketch of Paraguay as I found it, and it is to be hoped that some external effort will be made to assist the present enlightened Government (many of the members of which have had the advantages of an European education) to raise the country from its present poverty.

A word more with regard to the Loans. The question of the cession of the “yerbales”, or tea lands, appear to be quite out of the question, at least for the present, public opinion is the country being so much against it; but I think that before the Bondholders finally renounce the idea of receiving lands in exchange for bonds, they should enter more fully into the matter. Until the country becomes more populated, and is properly developed, there is no prospect of their claim, or even a small portion of it, being paid if this exchange is refused, for there is absolutely nothing else that the Government has to offer.

The Bondholders’ case, however, as will be seen by my report of the state of the country, is not hopeless. The remedy is in the future, and depends greatly upon proper assistance being given to the Government in its laudable endeavors to raise the country and inhabitants to the former state of prosperity in which the war of 1865-70 found them (1880).

On the other hand, the description offered by Mr. F. Bennoch, Chairman of the Council of Foreign Bondholders, regarding the situation of the defaulted Mexican debt in 1876, was completely different:

He [Mr. Bennoch] has thus given them [the bondholders] a rapid account of the formation of the gigantic debt and the resources of the country. The debt was large, but not too large to be borne with comparative ease, could they only secure an honest and patriotic administration. He had good reason for believing that the present customs dues and general taxation were amply sufficient to provide for the necessities of the State, and meet the interest on their several Loans. He was told, on most reliable authority, that barely two-thirds of the Customs Dues came into the Treasury, and that merchants would almost feel degraded were it once supposed that they honestly paid the duties. In this one source of revenue, carefully collected, they had enough to meet all their engagements, instead of year by year getting deeper into difficulty by a deficiency of nearly a million and a quarter sterling (1876).

The ultimate settlement conditions of these debts, as documented by the Annual Report of the Council of the Corporation of Foreign Bondholders (1919), are difficult to compare.⁸ It is clear from the reports, however, that Mexico was in a

⁸Paraguay: (i) Reduction of the outstanding principal of the debt by 50%, (ii) Interest on the reduced debt at 2% for five years, 3% for the next five years, and 4% thereafter, together with a sinking fund of $\frac{1}{2}$ % from June 1896 onward, (iii) An assignment of 2,177,344 acres of public lands in full payment of the arrears of interest —amounting to £1,306,217— from the default to the 1st of July, 1886

Mexico: (i) New bonds bore 1% interest for 1886, 1.5% for 1887, 2% for 1888, 2.5% for 1889, and 3% thereafter. (ii) The Government could purchase the bonds in the market at the rate of 40% of their nominal value up to December 31st, 1890; and 50% after that date. (iii) The bonds were issued to the holders of old bonds on the following principle: (a) £109 of New bonds for £100 of the 3% 1851 Arrangement, with arrears since January 1867, (b) £50 of New bonds for £100

better position than Paraguay to service its debt. Consistent with the results discussed above, it took 20 years for the bondholders to accept the Mexican settlement offer, while the bondholders of the Paraguayan debt reached an agreement with the Government after 12 years of suspension of payments.

Though anecdotal, this evidence suggests that the settlement process tended to be protracted even prior to the existence of official export credits, Paris Club reschedulings, IMF stabilization loans, and World Bank development assistance, as asserted by Eichengreen and Portes (1989). They state that although there is heterogeneity in the returns realized on foreign loans floated during the interwar period, in none of the cases they studied did the bondholders recover the total of interest arrears incurred between suspension and readjustment. They conclude that when macroeconomic conditions improved, the cost of settlement declined and the negotiation processes accelerated.

The literature on sovereign debt also notes that past behavior might be a good predictor for future actions. Lindert and Morton (1989) point out that countries that have defaulted in the past are significantly more likely to become problem debtors again. Reinhart et al. (2003) emphasize that debt intolerance is intimately linked to the pervasive phenomenon of serial default. Standard and Poor's state in their "Primer" (2004) that countries with a recent history of default —usually at

of the 3%, 1864 bond —bond issued in settlement of unpaid interest—, (c) £20 of New bonds for £100 of deferred bonds issued in 1837, (d) £20 of New bonds for £100 of unpaid Certificates of Conversion of 1851, (e) £20 of New bonds for £100 of Baring's Certificates (f) £29 of New bonds for £100 of 1834 Bonds.

The prevailing risk-free interest rate in 1885 —U.K. Consol's yield— was 3%.

the lower end of the rating scale— still carry the risk of policy reversals that can result in renewed default.

Given the asymmetry of information, Bayesian creditors should offer worse credit conditions to unknown governments than to countries that have proven their creditworthiness. Razaghian (2004) concludes that financial credibility, in the form of independent financial institutions responsible for debt repayment policies, affected yields of government securities in the U.S. antebellum period, compensating investors for the associated risk. Özler (1993) finds that countries with histories of repayment difficulties were charged higher interest rates than countries with no repayment difficulties, and that new borrowers (countries that acquired sovereignty after World War II) were charged interest rates as high as those charged to countries that experienced default episodes in the past. Recent defaults, Özler concludes, are more important in influencing the risk premium, which suggests the size of the premium fades with time.

Meanwhile, Eichengreen (1989) finds that between 1920 and 1929, Scandinavian countries, members of the British Commonwealth, small Western European countries, and small Central American republics economically or politically dependent on the United States were consistently charged with the lowest risk premia. He concludes that national reputation and political considerations, more than economic developments, influenced the pricing of foreign bonds. On the other hand, Lindert and Morton (1989) conclude that defaulting governments have seldom been punished, either with direct sanctions or with discriminatory denial of later credit, as long as they default collectively —presumably during a global recession— and as

long as they offer a settlement of at least a portion of the defaulted debt. However, as pointed out by Tomz (2001), Lindert and Morton's finding that defaulting governments are rarely penalized might be consistent with creditors' behavior. Countries with a history of default that already pay a high risk premium should not experience a further increase in the cost of the debt were they to experience a new default.

2.3 Model

This section discusses the model and its assumptions. Consider a small open economy characterized by an infinitely lived representative agent, the government, who borrows from the international credit market. The international credit market consists of competitive, risk-neutral lenders who are willing to offer credit contracts to the government up to a certain limit and at a risk premium. That premium reflects the probability of default, as perceived by the lenders. The debt ceiling is determined endogenously at the point at which it is no longer optimal for the lenders to offer credit and for the borrower to exceed that limit. The ceiling rules out the possibility of governments accumulating debt with the sole purpose of repudiating it later and living in autarky thereafter.

The government will find it optimal to default when the cost of exclusion from the international credit market is lower than the direct cost of repaying the debt. The risk premium reflects the probability that the government finds itself in those states of nature where it is optimal to default. Lenders, who are assumed to know relevant characteristics of the borrower, will impose a risk premium that increases with the size of the loan. That risk premium will also depend on the number of

periods the borrower has been active in the international credit market; this number may be zero either because the borrower is new or because it has recently gained access to the market after a settlement. The government's repayment history will influence the risk premium charged by the lenders. Although governments may change periodically, the model does not account for political risk.

A government that defaults has the opportunity of regaining access to the international credit market as long as it offers an acceptable settlement to the lenders. The settlement offer consists of the portion of the defaulted debt the government is willing to repay. The benefits from reaching a restructuring agreement are private information to the debtor and its creditors, and this lack of perfect information induces strategic behavior by both parties. In order to capture this, the probability that the settlement takes place depends on the size of the offer. A government that cannot afford or is unwilling to repay a large portion of the defaulted debt is likely to be excluded from the credit market for a long period of time. Only a long, costly period of exclusion will convince lenders that the settlement offer is, in fact, the best the debtor government can make. Although the settlement offer is determined endogenously and depends on a country's level of indebtedness and its current income level, the negotiation process is not modeled explicitly.

In determining the menu of debt contracts they offer to the borrowers, lenders take into account the number of periods the borrower has been active in the international credit market as well as the possibility of settlements in the event of a default. The menu of debt contracts depends on the lenders' expectations about the likely amount of time that will elapse during any post-default negotiation (the period of

exclusion from the international credit markets) and the likely characteristics of the settlement (the portion of the defaulted debt that the debtors agree to repay). The number of periods the borrower has been active in the international credit market evolves over time and depends on the government's decision to default or repay the debt each period. After a default, the "clock" is set to zero, and borrowers have to start all over again.

2.3.1 The Government

The government receives in each period a stochastic, non-storable⁹ endowment y_t that follows an AR(1) process:

$$y_t = \bar{y} + \rho(y_{t-1} - \bar{y}) + \vartheta_t, \quad \vartheta_t \sim N(0, \sigma^2) \quad (2.1)$$

In order to smooth consumption, the government can access international credit markets, provided that it fully repays the debt d_t (principal and interest) contracted in the previous period. Otherwise, the government is punished by intervals of exclusion from the credit market, and by disruptions to its trade or production during those intervals by the intervention of private creditors (it is assumed that the endowment is a tradable good). For the purposes of the model, it is assumed that after defaulting, and during the period of exclusion from the credit markets, the

⁹This assumption is made for simplicity. The results would not change significantly if the government were allowed to save because default occurs when the level of indebtedness is high; that is, after the government has depleted its savings. The most important consequence is an increase in the debt limit due to the buffer effect provided by the savings. Not to be confused with the Bulow and Rogoff argument: after a default occurs I still assume the government loses access to the financial markets.

government consumes a fraction α of its endowment. The fraction $(1 - \alpha)$ accounts for losses of output and trade while the government is in state of default.

The government consumption will depend, therefore, on its decision to repay:

$$c_t = y_t + b_t - d_t \quad \text{Consumption in case of repayment}$$

$$c_t = \alpha y_t \quad \text{Consumption in case of default}$$

Where,

$$d_t = R(b_{t-1}) = [1 + r(b_{t-1})]b_{t-1} \text{ is the repayment function;}$$

$r(b_{t-1})$ is the interest rate at which the previous period debt, b_{t-1} , was contracted;

b_t is the debt denominated in units of real goods contracted in the current period,

$$b_t \in B_t.$$

$B_t = B(s_t)$, and $B_t = \{0\}$ are the sets of allowed debt contracts offered by the

lenders. These contracts consist of one-period bonds such that a loan of b at

$t - 1$ (b_{t-1}) entails a payment of d at t (d_t). Loans offered by the lenders are

bounded; that is, $b_t \in [0, \bar{b}]$. The debt ceiling \bar{b} is defined as the maximum

of the inverse of the repayment function $d = R(b)$; $\bar{b} \equiv R^{-1}[\bar{d}]$ such that

$$\bar{b} \geq R^{-1}[d] \quad \forall d.$$

The sets of allowed debt contracts will depend on the government's decision to default or repay. If the government defaults, or is restructuring the defaulted debt, lenders offer an empty set $B_t = \{0\}$ and the government loses access to

the credit market. If the government is in good standing with the credit market, lenders offer sets $B_t = B(s_t)$ that depend on the repayment score s_t , which is a monotonic transformation of the number of periods the borrower has been active in the international credit market.

The government's objective function is:

$$U = E \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \quad (2.2)$$

where $\beta \in (0, 1)$, $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ and $c \geq 0$.

Once the government defaults, there is an ad hoc probability $(1 - \theta(\varepsilon))$ of regaining access to the international credit market in the next period that is conditional on the government making a settlement offer ε . This probability is the settlement rate, and ε is the portion of the defaulted debt d the government is willing to pay. There is a positive relationship between the size of the offer ε and the probability of a settlement taking place. The functional form of $\theta(\varepsilon)$ is an approximation of the empirical model derived in Chapter 1 and will be discussed later.

The value of the objective function given the decision to default is given by:

$$v^d(y_t, d_t, 0) = u(\alpha y_t) + \max_{\varepsilon \in [0,1]} \beta E \left\{ \theta(\varepsilon) v^d(y_{t+1}, d_{t+1}, 0) + (1 - \theta(\varepsilon)) v^r(y_{t+1}, d_{t+1}, s_1) \right\} \quad (2.3)$$

where

$$d_{t+1} = \begin{cases} d_t, & \text{in } v^d(y_{t+1}, d_{t+1}, 0); \\ \varepsilon d_t, & \text{in } v^r(y_{t+1}, d_{t+1}, s_1). \end{cases}$$

While in default, the government loses access to the international credit market and consumes a fraction α of its endowment. Given the endowment y_t and the defaulted debt d_t , the government chooses the settlement offer ε that maximizes its utility. Choosing a large value of ε increases the probability of the offer being accepted, but decreases consumption in the next period if the offer is, in fact, accepted. In this case, consumption next period will be given by $c_{t+1} = y_{t+1} + b_{t+1} - \varepsilon d_t$, where ε is the settlement offered by the borrower and accepted by the lenders, and d_t the debt under negotiation at t .

Were the government to repudiate its debt, making an offer each period of $\varepsilon = 0$, it would live in autarky and consume a fraction α of its endowment. The empirical evidence indicates that countries do not find eternal autarky optimal. Countries are not able to access the credit market while in default. When the probability of regaining access to the international credit market is zero, that is $\theta(0) = 1$, equation (2.3) boils down to equation (2.4) which correspond to the case discussed in Eaton and Gersovitz (1981):

$$v^d(y_t, d_t, 0) = v^d(y_t) = E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} u(\alpha y_{\tau}) \right] \quad (2.4)$$

Were creditors to forgive the debt in default allowing the delinquent government to reenter the credit market with an exogenous probability $(1 - \psi)$, the value of the objective function given the decision to default would be given by equation

(2.5) which correspond to the cases discussed in Arellano (2003), and Aguiar and Gopinath (2004). Equations (2.4) and (2.5) will be used for comparison purposes.

$$v^d(y_t, d_t, 0) = v^d(y_t) = u(\alpha y_t) + \beta E \left\{ \psi v^d(y_{t+1}) + (1 - \psi) v^r(y_{t+1}, 0, s_1) \right\} \quad (2.5)$$

When the government is in default, the “clock” that measures the number of periods the borrower has been active in the international credit market, and its transformation s_t , are set to zero, $s_t = 0$. The government, however, has the opportunity to improve its low evaluation —imagine s_t as a repayment score— by offering a settlement on the defaulted debt. The borrower repayment score s_t , although positive after the settlement $s_t = s_1$, is not necessarily good. It may take many years of faithful repayment before a lender reduces the risk premium it charges the borrower.

The dynamic programming problem given the decision to repay is:

$$v^r(y_t, d_t, s_t) = \max_{b \in B(s_t)} \left\{ u(y_t + b - d_t) + \beta E \max \left\{ v^d(y_{t+1}, d_{t+1}, 0); v^r(y_{t+1}, d_{t+1}, s_{t+1}) \right\} \right\} \quad (2.6)$$

The set of available loans, $B(s_t)$, changes each period and depends on the borrower’s repayment score s_t . The expression $\max \{ v^d(y_{t+1}, d_{t+1}, 0); v^r(y_{t+1}, d_{t+1}, s_{t+1}) \}$ indicates that the decision to default is reconsidered next period.

Default is optimal when:

$$v^d(y_t, d_t, 0) > v^r(y_t, d_t, s_t) \quad (2.7)$$

Then, the probability of default is given by:

$$\lambda(d_t, s_t) = Pr[v^d(y_t, d_t, 0) > v^r(y_t, d_t, s_t)] \quad (2.8)$$

The probability of default $\lambda(d_t, s_t)$ is defined as the cumulative distribution function of y_t , $F(y_t) \sim N(\bar{y}, \frac{\sigma^2}{1-\rho^2})$, evaluated at the point $y_t^*(d_t, s_t)$ where the value functions given the decisions to default and repay are equal:

$$v^d[y_t^*(d_t, s_t), d_t, 0] = v^r[y_t^*(d_t, s_t), d_t, s_t] \quad (2.9)$$

It is important noticing that in order to determine the probability of default of d_t , given that the government has a repayment score s_t , we only need to know the cumulative distribution function $F(y_t)$ and be able to determine the point $y_t^*(d_t, s_t)$. The probability of default $\lambda(d_t, s_t)$ is then $Pr[y_t \leq y_t^*(d_t, s_t)] = F(y_t^*(d_t, s_t))$.

2.3.2 International Lenders

International lenders are risk neutral and know the income profile of the borrower country (equation (2.1)). However, they are unable to verify the borrower's current income level y_t . This assumption implies that lenders can infer the probability of default $\lambda(d_t, s_t)$, but cannot assess the repayment capabilities of the borrower country after a default occurs. This conjecture is key to model the borrower's lengthy exclusion from the international credit market before a settlement takes place. Given that there is no reason preventing the government and its creditors from settling immediately after a default, they would be better off if they could settle under the same terms but without the protracted negotiation. The inefficiency

characterized by the delay in reaching a mutually beneficial agreement is explained in the literature on bargaining processes by informational differences. Incomplete information about some aspect critical to reaching an agreement, in this case how much the government should repay given its current income level, is an answer that the literature in economics has tried for costly disputes (exclusion from credit markets for the borrowers and uncertainty about the recovery value of the defaulted debt and potential losses for the lenders). Incomplete information plays no role when the government is allowed to reenter the market without repaying the defaulted debt (Equation (2.5)) because there is no implicit negotiation taking place.

Creditors will be willing to lend as long as the expected value from lending to a risky government is equal to the expected value from investing in a risk-free asset. If r is the return of the risk-free asset, then the optimal credit contract must satisfy:

$$b_{t-1}(1+r) = d_t[1 - \lambda(d_t, s_t)]s_{t-1} + \lambda(d_t, s_t)E_t \left[\sum_{j=0}^{\infty} \left(\prod_{i=0}^j (\theta[\varepsilon(y_{t+i}, d_t)])^{1_{(i < j)}} \right) \frac{[1 - \theta(\varepsilon(y_{t+j}, d_t))]\varepsilon(y_{t+j}, d_t)d_t}{(1+r)^{j+1}} \right] \quad (2.10)$$

On the right hand side, the first term of equation (2.10) is the expected value of the debt payment (principal and interest) weighted by the repayment score. The second term is the expected recovery value of the defaulted debt d_t , provided that the government defaults. This value takes into account whether the settlement offer is accepted or not by the lenders, and when acceptance occurs (see equation(2.11)). Under equations (2.4) and (2.5), this second term is zero because in one case exclusion is indefinite and in the other debt is forgiven by the lenders. Because the

optimal settlement offer the government can make at any point in time $\varepsilon(y_t, d_t)$ depends on that period's income level y_t , lenders take expectations over the income process y_t of the present value of the stream of possible payments based on the best offer the government can make at each state $\varepsilon(y_t, d_t)$:

$$\begin{aligned}
& E_t \left[\sum_{j=0}^{\infty} \left(\prod_{i=0}^j (\theta[\varepsilon(y_{t+i}, d_t)])^{1_{(i < j)}} \right) \frac{[1 - \theta(\varepsilon(y_{t+j}, d_t))] \varepsilon(y_{t+j}, d_t) d_t}{(1+r)^{j+1}} \right] = \quad (2.11) \\
& E_t \left[\frac{(1 - \theta(\varepsilon(y_t, d_t))) \varepsilon(y_t, d_t) d_t}{(1+r)} \right. \\
& + \frac{\theta(\varepsilon(y_t, d_t)) (1 - \theta(\varepsilon(y_{t+1}, d_t))) \varepsilon(y_{t+1}, d_t) d_t}{(1+r)^2} \\
& + \frac{\theta(\varepsilon(y_t, d_t)) \theta(\varepsilon(y_{t+1}, d_t)) (1 - \theta(\varepsilon(y_{t+2}, d_t))) \varepsilon(y_{t+2}, d_t) d_t}{(1+r)^3} \\
& + \frac{\theta(\varepsilon(y_t, d_t)) \theta(\varepsilon(y_{t+1}, d_t)) \theta(\varepsilon(y_{t+2}, d_t)) (1 - \theta(\varepsilon(y_{t+3}, d_t))) \varepsilon(y_{t+3}, d_t) d_t}{(1+r)^4} \\
& \left. + \dots \right]
\end{aligned}$$

The borrower's repayment score plays an important role in the profile of the debt contracts lenders will optimally choose to offer. Lenders update this score using repayment history; that is, the number of periods the borrower has been active in the international credit market. When the government defaults, the "clock" that registers the number of active periods as well as the repayment score are both set to zero, and each remains at zero as long as the debt is in default. When the government is in good standing with the credit market, the number of periods the borrower has been active in the international credit market increases, and lenders update the borrower's repayment score s_t accordingly.

Lenders thus use two pieces of information to set the borrower's repayment

score: the previous period repayment score s_{t-1} , and the borrower's decision to repay (R) or default (D), a_t . The scoring function $s_t = f(s_{t-1}, a_t)$ is intended to capture some stylized facts about sovereign debt: i) unfamiliar borrowers are charged the highest risk premium, ii) this premium declines over time provided that the government services its debt, iii) the premium declines asymptotically, iv) unfamiliar borrowers are charged interest rates as high as the ones charged to countries that have experienced default episodes in their recent past. Motivated by facts (i)-(iii), it is assumed that repayment score updates are asymmetric; that is, at any point in time, positive adjustments are smaller than negative ones. Moreover, positive adjustments get smaller as s_t increases. Fact (iv) is used to justify the following assumption: when a government borrows for the first time, the lenders assign an initial, minimum repayment score s_1 (entry level repayment score). After a settlement is completed, lenders will give that same minimum entry level repayment score to governments that recently experience a default episode but were able to negotiate a settlement.

The scoring function $s_t = f(s_{t-1}, a_t)$ is defined as:

$$s_t = f(s_{t-1}, a_t) = \begin{cases} g(s_{t-1}), & \text{if } a_t = R; \\ 0, & \text{if } a_t = D. \end{cases} \quad (2.12)$$

Where g is bounded $[0, 1]$, and satisfies $g' > 0$ and $g'' < 0$. Were the government to repay several periods in a row, the borrower's repayment score would increase until $s_\tau - s_{\tau-1} \approx 0$. When that point is reached $s_\tau \simeq \bar{s} = 1, \forall t \geq \tau$. As the number of periods the borrower has been active in the international credit mar-

ket increases, $g'' < 0$ guarantees that the repayment score lenders assign converges asymptotically to $\bar{s} = 1$. As a consequence, the risk premium that lenders charge converges asymptotically as long as the borrowing government faithfully repays its debt.

2.3.3 Competitive Equilibrium

The competitive equilibrium of the model will be given by:

- (a) An optimal set of loans $B(s)$ satisfying (2.10) such that the optimal borrowing policy $b(y, d, s) \in B(s)$
- (b) An optimal repayment function $d = R(b(y, d, s))$, and an optimal settlement offer $\varepsilon(y, d, 0)$
- (c) Value functions $v^r(y, d, s)$, $v^d(y, d, 0)$ and an optimal probability of default $\lambda(d, s)$ such that,

(1) The government maximizes its utility and decides optimally whether or not to default, implicitly defining the optimal probability of default given its optimal decisions $b(y, d, s)$ and $\varepsilon(y, d, 0)$; (2) The government's optimal demand for loans $b(y, d, s)$ belongs to the optimal set of loans offered by the international lenders $B(s)$; (3) The lenders, knowing the functional form of $\theta(\varepsilon(y, d, 0))$, aware of the optimal settlement offer $\varepsilon(y, d, 0)$, knowing the income distribution $F(y)$, and having established a repayment score function $s = f(s_{-1}, a)$, define the optimal set of loans $B(s)$ and the optimal repayment function d such that (2.10) is satisfied.

Due to the convergence of s_t to \bar{s} , the dynamic programming problem can be separated into two parts: a finite horizon part (for $s_t < \bar{s}$), and an infinite horizon part (for $s_t = \bar{s}$). In the infinite horizon part, the borrower's repayment score does not depend on t . The future looks the same whether the government is in state (y_t, d_t, \bar{s}) at time t , or state $(y_{t+k}, d_{t+k}, \bar{s})$ at time $t+k$, provided that the states are the same ($y_t = y_{t+k}$, and $d_t = d_{t+k}$). The only variables that affect the government's view about the future are the values of y and d . The problem becomes stationary, and the optimal decision rule and corresponding value function are time invariant and are defined as:

$$v_\infty^r(y, d, \bar{s}) = \max_{b \in B(\bar{s})} \left\{ u(y + b - d) + \beta E \max \left\{ v_\infty^d(y', d', 0); v_\infty^r(y', d', \bar{s}) \right\} \right\} \quad (2.13)$$

$$b(y, d, \bar{s}) = \arg \max_{b \in B(\bar{s})} \left\{ u(y + b - d) + \beta E \max \left\{ v_\infty^d(y', d', 0); v_\infty^r(y', d', \bar{s}) \right\} \right\} \quad (2.14)$$

Once a settlement offer is accepted, or when a debtor country borrows for the first time, the lenders assign an entry level repayment score s_1 to the borrower. This entry level repayment score corresponds with the solution to v_1^r in the finite horizon case. When the government is in default, the problem is also stationary; the only variables that affect the government's view about the future are y and d , and time per se plays no role. As when the government is in good standing with the credit market and $s_t = \bar{s}$, the optimal decision rule and corresponding value function when the government is in default are time invariant and are defined as:

$$v_\infty^d(y, d, 0) = u(\alpha y) + \max_{\varepsilon \in [0,1]} \beta E \left\{ \theta(\varepsilon) v_\infty^d(y', d, 0) + (1 - \theta(\varepsilon)) v_1^r(y', \varepsilon d, s_1) \right\} \quad (2.15)$$

$$\varepsilon(y, d, 0) = \arg \max_{\varepsilon \in [0,1]} \left\{ u(\alpha y) + \beta E \left\{ \theta(\varepsilon) v_\infty^d(y', d, 0) + (1 - \theta(\varepsilon)) v_1^r(y', \varepsilon d, s_1) \right\} \right\} \quad (2.16)$$

In the finite horizon part ($T < \infty$), however, the government's decision depends not only on the state of the economy (y_t, d_t) , but also, and more importantly, on the repayment score lenders assign based on the government's repayment history. Because the debt contracts that lenders are willing to offer vary each period, and because the government's incentive to default changes each period, the dynamic programming problem is not stationary but can be solved using backward induction. The terminal period v_T^r and b_T are defined by:

$$v_T^r(y_T, d_T, s_T) = \max_{b \in B(s_T)} \left\{ u(y_T + b - d_T) + \beta E \max \left\{ v_\infty^d(y_{T+1}, d_{T+1}, 0); v_\infty^r(y_{T+1}, d_{T+1}, \bar{s}) \right\} \right\} \quad (2.17)$$

$$b_T(y_T, d_T, s_T) = \arg \max_{b \in B(s_T)} \left\{ u(y_T + b - d_T) + \beta E \max \left\{ v_\infty^d(y_{T+1}, d_{T+1}, 0); v_\infty^r(y_{T+1}, d_{T+1}, \bar{s}) \right\} \right\} \quad (2.18)$$

In periods $t = 1, 2, \dots, T - 1$, v_t^r and b_t are defined by:

$$v_t^r(y_t, d_t, s_t) = \max_{b \in B(s_t)} \left\{ u(y_t + b - d_t) + \beta E \max \left\{ v_\infty^d(y_{t+1}, d_{t+1}, 0); v_{t+1}^r(y_{t+1}, d_{t+1}, s_{t+1}) \right\} \right\} \quad (2.19)$$

$$b_t(y_t, d_t, s_t) = \arg \max_{b \in B(s_t)} \left\{ u(y_t + b - d_t) + \beta E \max \left\{ v_\infty^d(y_{t+1}, d_{t+1}, 0); v_{t+1}^r(y_{t+1}, d_{t+1}, s_{t+1}) \right\} \right\} \quad (2.20)$$

The number of periods in the finite horizon case ($T < \infty$) will depend on $f(s_{t-1}, a_t)$, and s_1 . The number of periods T is defined as the value of t such that, given the government's decision to faithfully repay from $t = 1$ to $t = T$, ($a_1 = R, a_2 = R, a_3 = R, \dots, a_T = R$), the lenders' positive update at $T + 1$ is zero. The borrower's repayment score at $T + 1$ is the same as in T , so $s_{T+1} - s_T \approx 0$.

Given the existence and uniqueness of a solution to equations (2.13) and (2.15), it is straightforward to verify that $b_t(y_t, d_t, s_t)$ solves (2.19) for $t = 1, 2, \dots, T$, where $v_{T+1}^r = v_\infty^r$. Because the state space $S = \{y, d, s\}$, actions $a = \{R, D\}$, and decision rules $b = (y, d, s)$ and $\varepsilon = (y, d, 0)$ are bounded, and because the Bellman equations (2.13) and (2.15) satisfy Blackwell's sufficient conditions for a contraction mapping (monotonicity and discounting), the existence and uniqueness of a solution is guaranteed (see Appendix A).

2.4 Numerical Solution

This section summarizes the algorithm used to solve the model using numerical methods. The choice of the parameters is discussed and justified.

2.4.1 Numerical Method

The competitive equilibrium is solved, in the infinite horizon part and the finite horizon part, using a recursive numerical solution method (value function iteration). I approximate the state space (y, d) to evenly spaced discrete grids. The points on the debt repayment grid $D = \{d_1, d_2, \dots, d_m\}$ are determined such that $d_1 = 0$ and \bar{d} satisfies $\bar{d} < d_m$. The AR(1) income process is transformed into a n -state Markov process following Hussey and Tauchen (1991).¹⁰ The Markov process is characterized by a finite state space $Y = \{y_1, y_2, \dots, y_n\}$, a $n \times n$ transition probability matrix P defined as $P_{ij} = Pr\{y_{t+1} = y_j | y_t = y_i\}$ such that $y_j, y_i \in Y$, and a $n \times 1$ stationary distribution P^∞ where $Pi^\infty = Pr(y = y_i)$. The number of nodes for D was set equal to 80, while the number of nodes for Y was set equal to 20.

The solution algorithm is summarized in the following steps:

0. Initialization: Specify the values of the parameters of the AR(1) income process ρ and σ ; effective income when in default α ; settlement rate η , μ , and ζ ; risk free interest rate r ; the coefficient of risk aversion γ ; the scoring function $s = f(s_{-1}, a)$; and the number of years until graduation T . Stipulate an initial guess for v_∞^r , v_∞^d , $B(s)$, and the probability of default $\lambda(d, s)$. Set the convergence tolerance ξ , and π .

1. Update the value function v_∞^r

$$v_\infty^r \leftarrow \max_{b \in B(\bar{s})} \left\{ u(y + b - d) + \beta P \max \left\{ v_\infty^d(y', d', 0); v_\infty^r(y', d', \bar{s}) \right\} \right\} \quad (2.21)$$

¹⁰I used Markovappr.m from Paul Fackler and Mario Miranda's toolbox.

2. Specify the terminal value function $v_{T+1}^r = v_\infty^r$ and set $t \leftarrow T$
3. Given v_{t+1}^r , compute v_t^r

$$v_t^r \leftarrow \max_{b \in B(s_t)} \left\{ u(y_t + b - d_t) + \beta P \max \left\{ v_\infty^d(y_{t+1}, d_{t+1}, 0); v_{t+1}^r(y_{t+1}, d_{t+1}, s_{t+1}) \right\} \right\} \quad (2.22)$$

4. Termination check: If $t = 1$, stop; otherwise set $t \leftarrow (t - 1)$ and return to step 3.
5. Given v_1^r , compute v_∞^d

$$v_\infty^d \leftarrow u(\alpha y) + \max_{\varepsilon \in [0,1]} \beta P \left\{ \theta(\varepsilon) v_\infty^d(y', d, 0) + (1 - \theta(\varepsilon)) v_1^r(y', \varepsilon d, s_1) \right\} \quad (2.23)$$

6. Termination check: If $\| \Delta v_\infty^r \| < \xi$, and $\| \Delta v_\infty^d \| < \xi$, set:

$$b(\bar{s}) \leftarrow \arg \max_{b \in B(\bar{s})} \left\{ u(y + b - d) + \beta E \max \left\{ v_\infty^d(y', d', 0); v_\infty^r(y', d', \bar{s}) \right\} \right\} \quad (2.24)$$

$$b(s_t) \leftarrow \arg \max_{b \in B(s_t)} \left\{ u(y_t + b - d_t) + \beta E \max \left\{ v_\infty^d(y_{t+1}, d_{t+1}, 0); v_{t+1}^r(y_{t+1}, d_{t+1}, s_{t+1}) \right\} \right\} \quad (2.25)$$

$$\varepsilon \leftarrow \arg \max_{\varepsilon \in [0,1]} \left\{ u(\alpha y) + \beta E \left\{ \theta(\varepsilon) v_\infty^d(y', d, 0) + (1 - \theta(\varepsilon)) v_1^r(y', \varepsilon d, s_1) \right\} \right\} \quad (2.26)$$

7. Given v_∞^r , v_t^r for $t \in \{1, 2, \dots, T\}$, v_∞^d , and $\varepsilon(y, d)$, compute the probability of default $\lambda(d, s)$ and the debt contracts $B(s)$.

$$\lambda(d, s) \leftarrow Pr[v^d(y, d, 0) > v^r(y, d, s)] \quad (2.27)$$

$$B(s) \leftarrow \frac{1}{(1+r)} d[1 - \lambda(d, s)]_{s-1} + \quad (2.28)$$

$$\lambda(d, s) E \left[\sum_{j=0}^{\infty} \left(\prod_{i=0}^j (\theta[\varepsilon(y_i, d)])^{1_{(i < j)}} \right) \frac{[1 - \theta(\varepsilon(y_j, d))] \varepsilon(y_j, d) d}{(1+r)^{j+1}} \right]$$

8. Termination check: If $\| \Delta \lambda(d, s) \| < \pi$, and $\| \Delta B(s) \| < \pi$ stop; otherwise return to step 1.

Although the state space is discrete, the maximization over the level of debt b is done approximating the repayment function d' at values of b different from the nodes on the grid using quadratic spline interpolation. The maximization procedure uses the Newton-Raphson method. In order to evaluate the value functions at points of d different from the grid nodes, I use piecewise linear interpolation.

The convergence tolerance ξ was set equal to 1e-05, and π to 1e-04.

2.4.2 Calibration

Income Process

The parameters in equation (2.1), $y_t = \bar{y} + \rho(y_{t-1} - \bar{y}) + \vartheta_t$ with $\vartheta_t \sim N(0, \sigma^2)$, are estimated using real GDP per capita series for different countries. The series are detrended and normalized such that $\bar{y} = 1$, so that the values of d , b and ε

can be easily compared with mean income. The variance is adjusted such that it is constant with respect to the trend.¹¹ The estimated autocorrelation parameter ρ ranges between 0.22 and 0.97, while the estimated standard deviation of the disturbances σ varies between 0.011 and 0.081. The parameters chosen for the numerical solution are $\rho = 0.72$ and $\sigma = 0.055$, which correspond to Argentina.¹² Parameters corresponding to other countries were calculated for the sensitivity analysis.

One of the main criticisms of existing models has been their inability to reproduce high levels of debt-to-GDP (or Gross National Income (GNI) as reported by the World Bank's Global Development Finance (GDF)).¹³ However, adjusting the value of the parameter α , which represents the effective portion of the income consumed while the government is in state of default, can generate realistic levels of indebtedness. The value of α is thus a major determinant of the debt ceiling. The empirical literature suggests that this value ranges from 92% to 99.01%. Sturzenegger (2004) finds an accumulated drop of 4% in output over the 4 years after a default (approximately 0.99% per year). Rose (2005) estimates that renegotiation of defaulted debt is associated with a decline in bilateral trade of approximately 8%

¹¹After performing some relevant tests (Dickey-Fuller, Durbin-Watson, Jarque-Bera), most of the countries' parameters were estimated using MLE on the model $y_t = a + bt + cy_{t-1} + e_t$, where $e_t \sim N(0, \sigma^2(a + bt + cy_{t-1})^2)$.

¹²The Hodrick and Prescott filter yields similar results for the parameters ρ and σ when using a smoothing parameter of 100 for annualized data.

¹³Argentina, for example, reported a total debt/GNI of 136% for 2003, the highest reported by the GDF database for a non-African country, excluding Guyana with 207.3% and Nicaragua with 174%. The average in 2003 for the 136 countries that report public and publicly-guaranteed debt to the World Bank Debtor Reporting System was 85.7%, the average being 65.7% during the 1970-2003 period.

a year that persists for around fifteen years. The parameter α is set equal to 99.01% as proposed by Sturzenegger.¹⁴

Settlement Rate

The settlement rate, which represents the probability that the lender will accept the settlement offer in the current period given that the government is in default at the beginning of the period, is an approximation based on the results derived in Chapter 1. The main conclusion of that work, which uses survival analysis techniques to estimate the duration of exclusion from credit markets, is that renegotiating defaulted debt under more generous terms to the debtors reduces the probability of the settlement offer being accepted by the lenders. Although the data suggest that the settlement rate, or hazard rate in the terminology of duration models, changes with time, in order to simplify the model and make the numerical solution tractable, the settlement rate $(1 - \theta(\varepsilon))$ is assumed to be constant over time but dependent on the size of the settlement offer. Thus, the settlement rate is constant provided that the settlement offer ε remains fixed. The structure of the problem guarantees, however, that the settlement offer, and consequently the settlement rate, change over time. This variation arises because the income level, which along with the level of indebtedness directly affects the settlement offer, follows an AR(1) process.

The settlement rate used in the present model is calculated using the mean time spent in default at each value of ε , as shown in Figure 2.1. The settlement

¹⁴Chatterjee et al. use 99.6%, while Arellano and Yue use, both of them, 98%.

rate is given then by $(1 - \theta(\varepsilon)) = \eta \exp(\mu - \zeta\varepsilon)$, which is constant over time. The values of η , μ , and ζ are calculated using the estimated parameters \hat{p} and $\hat{\delta}$ under the Weibull distribution¹⁵, and the mean time \hat{t} for each value of ε . Those values are set to $\eta = 0.0050$, $\mu = -1.9973$, and $\zeta = -5.3968$. There is no convenient interpretation for the parameters η , μ , and ζ .

International Lenders

The risk-free interest rate, r , is set to 2.8%, which is the average real return (1962-2004) of the 10-year U.S. Treasury bond. The average real interest rate of the 1-year Treasury bill during the same period was 1.9%.

The repayment score function $s_t = f(s_{t-1}, a_t)$ is calculated such that it will take a government approximately 23 years¹⁶ to gain full access to the credit market given that it has faithfully repaid its debt during that period; that is, $s_t = \bar{s} \approx 1$ for $t = 23$. This condition can be satisfied for values of s_1 such that the interest rate a lender charges a borrower with no repayment history is reasonable (below 100%).¹⁷ It is important to note that in the model, the credit “rating” should include the probability of default $\lambda(d, s)$, the borrower’s score s , and the recovery value of the defaulted debt. s_t can be understood as part of the rating going from $s_t = 0$, when

¹⁵The hazard function under the Weibull distribution is given by $h(t|\varepsilon) = p t^{p-1} \exp(\delta\varepsilon)$, where p and δ are the parameters of the model. The survival function, which captures the probability of staying in state of default at time t , is given by $S(t|\varepsilon) = \exp(-\exp(\delta\varepsilon)t^p)$.

¹⁶This number corresponds to the minimum number of years it takes for a country to “graduate from debt intolerance”. Following Reinhart et al. (2003), the country that graduated the fastest was Chile, whose last default occurred in 1983. Increasing the number of periods beyond this point renders the numerical solution cumbersome.

¹⁷Low values of s_1 imply that the lenders will charge extremely high interest rates.

Figure 2.1: Estimated probability $\theta(\varepsilon)$ of the settlement offer, ε , being rejected by the lenders

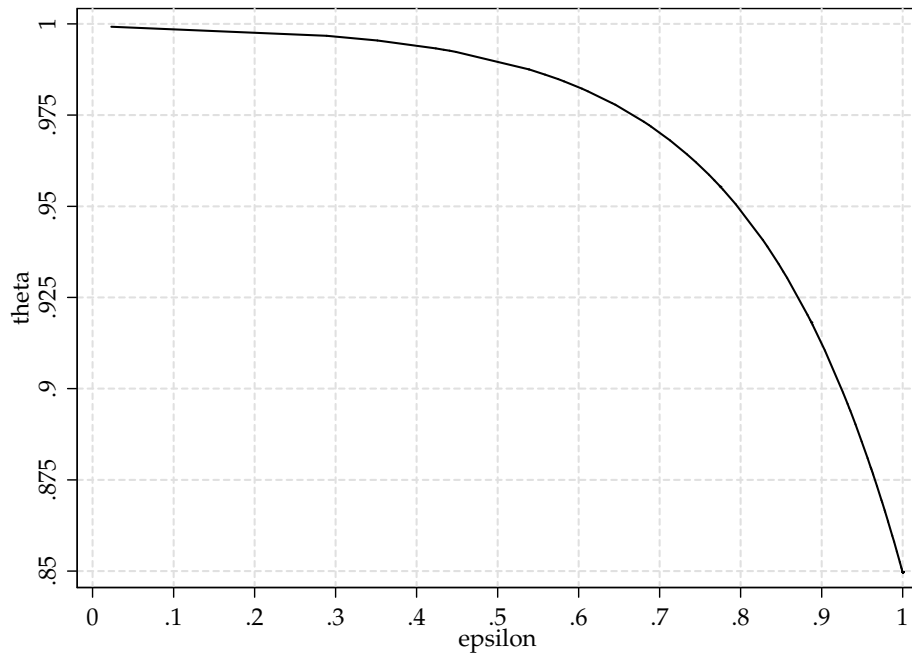
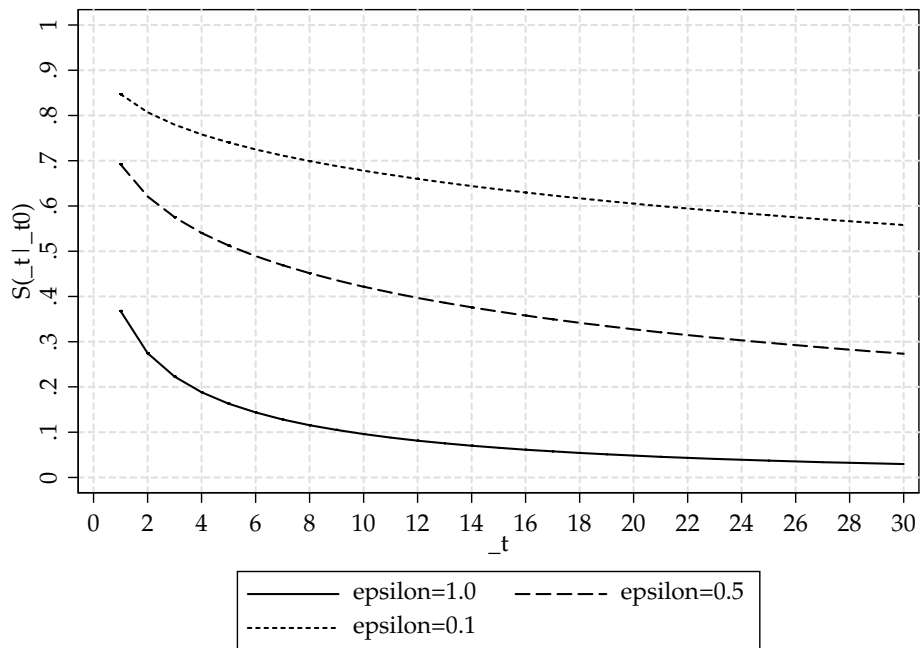


Figure 2.2: Estimated probability of staying in default given the settlement offer, ε



the government is in default, to $s_t = \bar{s} = 1$, when the government has continuous access to credit markets. The value of s_1 might reflect “market sentiment,” and the implications of different values of s_1 will be studied in the sensitivity analysis.

Other Parameters

The value of β is set such that $\beta = \exp(-r)$. The coefficient of risk aversion γ of the utility function (CRRA) is set to 5.

Table 2.1: Parameter Values

	Parameter	Value
Mean Income	\bar{y}	1
Autocorrelation Coefficient	ρ	0.72
Standard Deviation	σ	0.055
Effective Income when in Default (%)	α	0.9901
Settlement Rate Parameters	η	0.005
Settlement Rate Parameters	μ	-1.9973
Settlement Rate Parameters	ζ	-5.3968
Risk-free Interest Rate	r	0.028
Number of Years until Graduation	T	23
Discount Factor	β	$\exp(-r)$
Coefficient of Risk Aversion	γ	5

2.4.3 Results

The results are presented in two sections in order to better illustrate the two main contributions of the paper to the existing literature: (i) the introduction of settlements and the random length of exclusion from credit markets after a default, and (ii) the introduction of repayment history as a variable affecting the menu of debt contracts offered to borrowers. First, this section presents and analyzes results related to the settlement process abstracting from repayment history. For

this analysis, it is assumed that the variable s_t does not play any role, so v_1^r is substituted by v_∞^r in equations (2.16) and (2.15). In the second step, this restriction is lifted and the variable s_t reintroduced.

Settlement Process

The optimal settlement offer borrowers will propose after a default $\varepsilon(y, d)$ depends negatively on the size of the defaulted debt d . Because the derivative of the settlement rate $(1 - \theta(\varepsilon(y, d)))$ with respect to ε is positive, income y and debt d affect the settlement rate in the same way that they affect the settlement offer. The impact on $\varepsilon(y, d)$ of variations in y will depend on both the size of the defaulted debt d and the level of income y . For large values of d , ε increases monotonically in y . This is intuitive since higher income increases ability to pay (figure (2.3)). For relatively low levels of indebtedness, an increase in y will cause a decline in ε for low values of y , and an increase in ε for large values of y (figure (2.4)). This nonmonotonicity only occurs when the autocorrelation of the income process is positive. When income is low (and expected to remain low in the future ($\rho > 0$)), the government offers a high settlement, even though it is expensive to do so, because the benefit of accessing the credit market as soon as possible outweighs the cost of repaying the debt. For high levels of income, the cost of repaying the debt is relatively low, so the settlement offer is also high. For middle levels of income, however, borrowers might be willing to wait longer. If income improves, the settlement cost is smaller relative to today, and the cost of waiting is not as high as when income is low. Because the probability of default is zero for such low levels of indebtedness, though, debtors and creditors

in this case will not attempt to negotiate. These results are robust to changes in the parameters of the model.

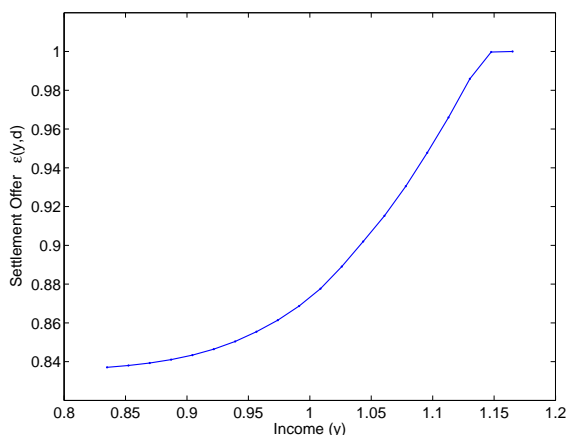


Figure 2.3: $\varepsilon(y, d)$ for high values of d

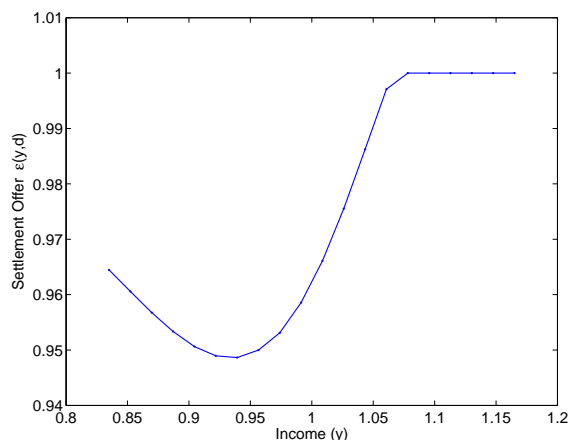


Figure 2.4: $\varepsilon(y, d)$ for low values of d

The results generated by the model closely match the empirical evidence. The analysis of Chapter 1 shows that an increase in income (GDP per capita growth) positively affects the settlement rate. That paper also shows that renegotiating under terms set by the Paris Club¹⁸ significantly affects the settlement rate. Under Paris Club terms, countries are eligible for greater leniency in terms of repayment if they suffer from low income levels and high degrees of indebtedness (measured as debt to GDP, debt to exports, and scheduled debt service over exports ratios). Countries with the lowest settlement rates are those with the highest levels of indebtedness, which makes it more difficult for them to offer an acceptable settlement

¹⁸“The Paris Club is an informal group of official creditors whose role is to find coordinated and sustainable solutions to the payment difficulties experienced by debtor nations. Paris Club creditors agree to reschedule debts due to them. Rescheduling is a means of providing a country with debt relief through a postponement and, in the case of concessional rescheduling, a reduction in debt service obligations.” Since 1965, 80 debtor countries have agreed to negotiate under the Paris Club terms.

to the creditors.¹⁹ These results are intuitive. After a default, debtor countries weigh the cost of settlement against the benefits of regaining access to the credit market. As macroeconomic conditions get better—higher levels of income given the same level of debt, or lower levels of debt given the same level of income—the cost of settlement declines relative to the benefits of smoothing consumption through credit markets, and the government can improve its settlement offer.

The results generated by the model indicate that the settlement offer $\varepsilon(y, d)$ is more sensitive to changes in debt (figure (2.5)) than to changes in the level of income (figure (2.6)), one of the reasons being that once in default, it is harder to modify the level of indebtedness than to wait for the level of income to improve.²⁰

The values of $\varepsilon(y, d)$ generated by the model range between 0.47 and 1 (Figure 2.7). These numbers seem high compared with the 35 cents on the dollar offered by Argentina in February 2005 to its aggrieved bondholders. However, Argentina's offer corresponded to 64% of the total public debt, which at that time included not only bonds, but also multilateral loans.

In no case is it optimal for the debtor to stay in default indefinitely by setting $\varepsilon = 0$. Were debtors in default denied the opportunity to settle their debt, as in

¹⁹Under “Naples terms,” creditors can reduce their debt up to 67%; under “Cologne terms,” this reduction can reach 90%. Eligibility for these terms depends on a country's track record with the Paris Club and the IMF, its level of indebtedness, and its eligibility for IDA-only financing. IDA loans are offered by the World Bank under highly concessional terms with maturities between 35 and 40 years and interest rates of 0.75%. “IDA-only” countries are not considered creditworthy enough to pay back under market conditions.

²⁰There exists the possibility of a country offering a settlement, reducing its indebtedness level, and soon after defaulting again, with the possibility of an additional reduction in the level of debt. These episodes were very common during the 1800s.

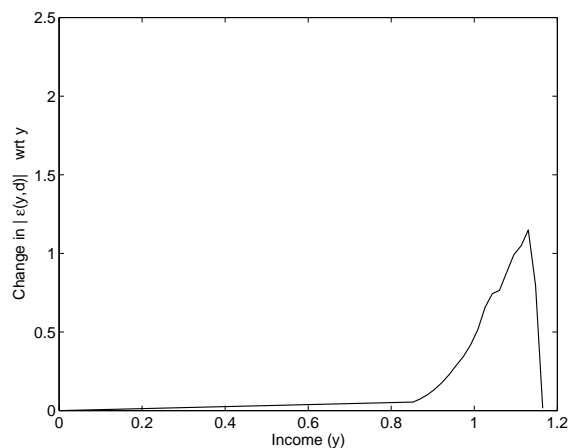
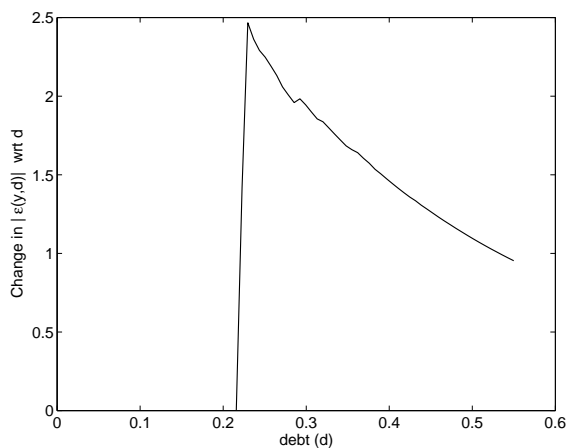


Figure 2.5: $\Delta|\varepsilon(y, d)|$ due to changes in d Figure 2.6: $\Delta|\varepsilon(y, d)|$ due to changes in y

equation (2.4), the credit ceiling would be higher, increasing from 28.4% to 43.8%.²¹ Because debtor countries can regain access to the credit market after lenders accept the settlement offer, the incentives for defaulting become larger when settlement is allowed, since the cost associated with default is a finite rather than an infinite period of exclusion. Other cost associated with default is the repayment that lenders require from delinquent debtors in order to allow them renew access to the market. The cost of repayment is larger than the zero cost associated to defaulting under equation (2.5). In this case debtors in default are forgiven their debt and allowed to reenter the market with an unconditional probability $(1 - \psi)$; incentives to default become even larger and the credit ceiling that governments can support is reduced dramatically to a mere 4.66%.²²

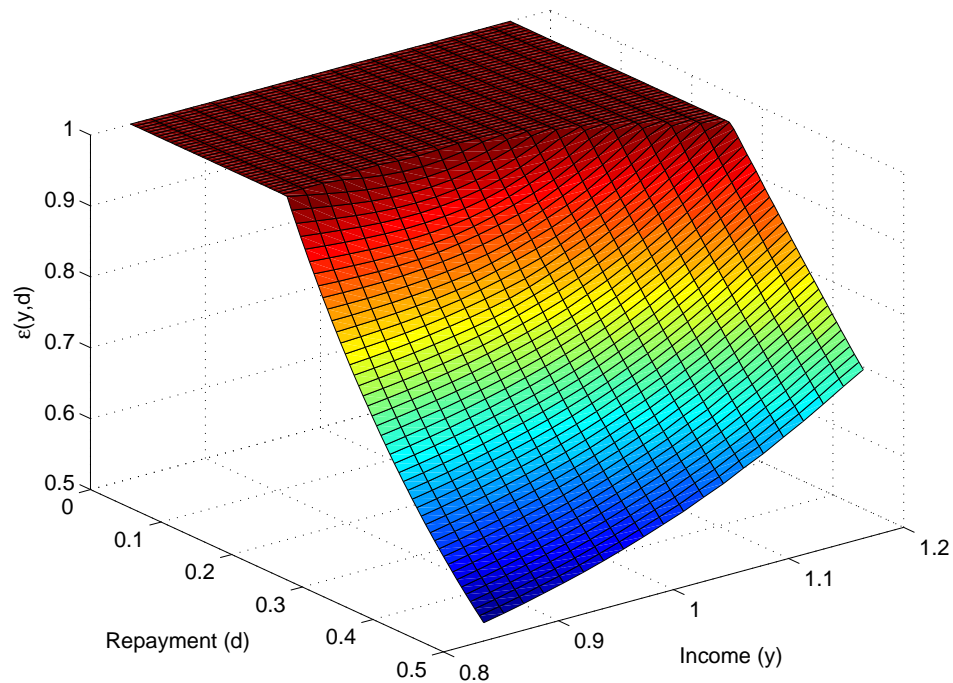
Under equation (2.4), when default carries a maximum penalty of indefinite

²¹The results reported here correspond with the debt ceiling-to-mean income ratio (\bar{b}/\bar{y}) . Recall the mean income \bar{y} was normalized to one. When evaluated at the minimum income, the debt ceiling-to-income ratio increases from 28.4% to 34.0%, and from 43.8% to 52.5%.

²²Arellano reports a debt ceiling equal to 5% of total output. The parameter $(1 - \psi)$, as in the case of Arellano, was set equal to 0.7

expulsion from the international credit markets, the highest debt-to-GDP ratio that the country could manage would imply a default probability of only 0.42%. Under equation (2.3), although the maximum debt-to-GDP ratio lenders allow is smaller, the default probability associated with this level is 1.36%. In this case, default is costly because it entails finite exclusion from the market and renew access only after a settlement is reached. Under equation (2.5), debtors have the highest incentives to default because they can reenter the market without having to repay their defaulted debt, these incentives are reflected in the large default probability of 2.94% at the debt ceiling. Reducing the cost of default dramatically increases the incentives for borrowers to fail payments: these incentives are reflected in higher probabilities of default and low debt ceilings.

Figure 2.7: Optimal Settlement Offer $\varepsilon(y, d)$



From the menu of debt contracts, b can be understood as the present value of a zero coupon bond, with principal d to be paid in the next period. As the probability of default increases, and the interest rate rises (see figure(2.9)), the present value b starts decreasing after the debt ceiling \bar{b} is reached (see figure (2.8)). It is not optimal for a debtor to choose any value b such that $R(b) = d > \bar{d}$ (see figure (2.10)) where $\bar{d} = R(\bar{b})$. The lender imposes the debt ceiling \bar{b} through an increase in the interest rate charged to borrowers, and thereby deters them from borrowing past \bar{b} .

Figure 2.8: Debt Ceiling

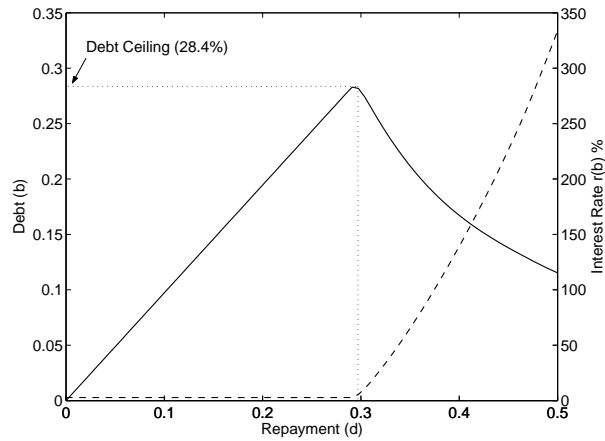


Figure 2.9: Probability of Default and Interest Rates (Detail)

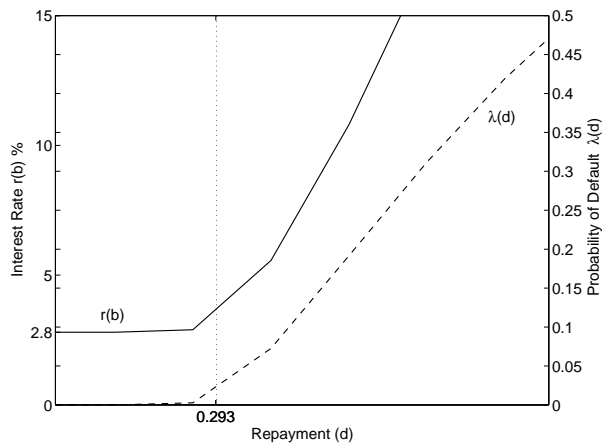
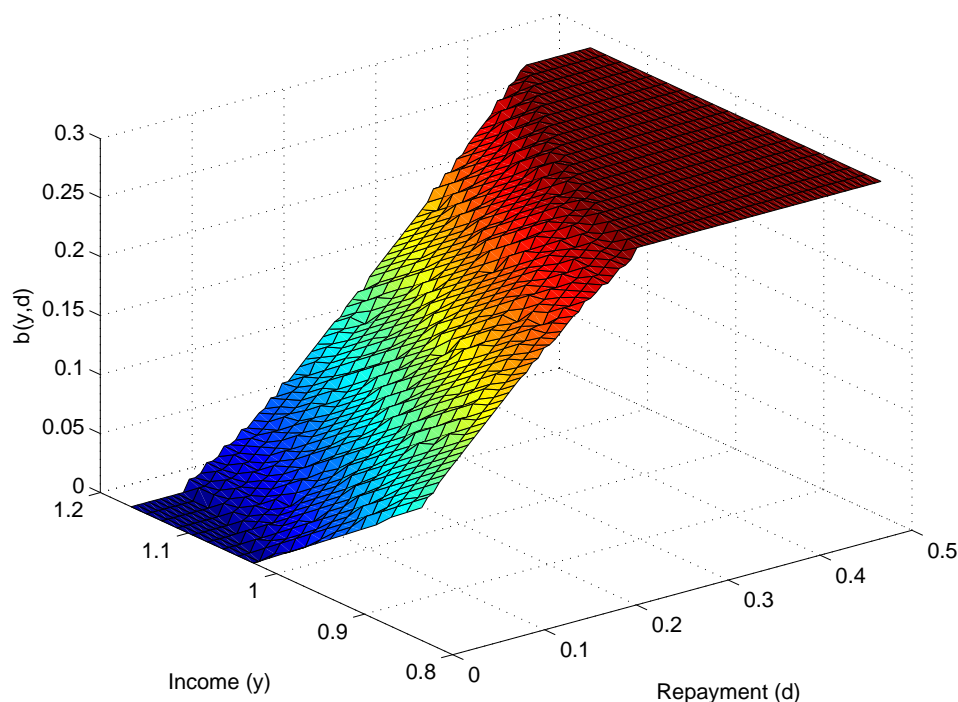


Figure 2.10: Optimal Borrowing Function $b(y, d)$



In order to evaluate the model's goodness of fit, 1000 simulations of 180 periods each were generated. The number of periods per simulation was chosen to match the number of years in the Standard and Poor's dataset, which includes default episodes since the 1820s. The length of each default episode was calculated using Beers and Bathia (1999) and Beers and Chambers (2003 and 2004). The model is capable of generating an average number of periods of exclusion (L) from the credit market (12.25 years) comparable to the average observed in the data (10.46 years).²³ It also produces a large number of default episodes. In 45.6% of the simulations, there was at least one default episode, and in 6.14% of these cases, the country experienced

²³As reported in Chapter 1 for a country that defaults for the first time. Suter and Stamm (1992) reports an average default duration of 10 years between 1821 and 1975, and Tomz and Wright report a median length of exclusion of 12 years between 1824 and 1969.

default at least three times. The model thus suggests that default episodes are not rare events, and that serial default is not extraordinary. Over a 180-year period, the maximum number of defaults generated by a simulation was 5. Such serial defaulting occurs with a probability of 0.3% (3 times in 1000 simulations). Standard and Poor's dataset indicates that Argentina, the country used for the calibration, has defaulted on its foreign debt five times in the same amount of time. The maximum number of defaults generated depends on a country's income profile, as will be shown in the sensitivity analysis. With a different income process, the model is capable of explaining cases like Costa Rica, Ecuador, or Venezuela, which each had 8 defaults, or Brazil, Colombia, Guatemala, Liberia, Mexico, Peru, and Uruguay, which each had at least 6 episodes of default. From the Standard & Poor's dataset, one can calculate that by the end of 2004, 37 out of the 98 countries (37.8%) that have defaulted on their foreign debt since the 1820s have done so on at least three occasions.

The model fails to reproduce the high volatility of the interest rate on sovereign debt observed in the data (the standard deviation of the interest rate is 0.314% in the model at its maximum vs. 3.89% in the data for Argentina while it was in good standing with the credit market).²⁴ Given the fact that the risk-free interest rate on the alternative asset is not allowed to change, this low volatility is not surprising.

While the model has difficulties replicating the volatility of interest rates, it produces degrees of volatility (σ) in consumption (c), and current account (CA) that closely match the data. The model replicates as well the correlation (ρ) between

²⁴Similar results are reported by Arellano (2003) and Yue (2005).

income and consumption, but fails to reproduce the countercyclical nature of the current account observed in the data: capital flows to emerging economies (CA deficits) in good times.²⁵ In the model, the government increases debt in order to smooth consumption so we observe surpluses in good times and deficits in bad times, which is reproduced in the positive correlation between current account and income (see table (2.2)).

Table 2.2: Long-run Business Cycle Statistics

	Data	Endogenous Settlement	Repayment History	Infinite Exclusion	Exogenous Entry
		Equation(2.3)		Equation(2.4)	Equation(2.5)
$\sigma(c)$	0.056	0.042	0.041	0.038	0.055
$\sigma(y)$	0.055	0.055	0.055	0.055	0.055
$\sigma(c)/\sigma(y)$	1.010	0.755	0.746	0.693	1.001
$\rho(y, c)$	0.947	0.901	0.861	0.853	0.995
CA/y %	-1.335	-0.074	-0.164	-0.100	-0.008
$\sigma(CA/y)$	0.034	0.033	0.030	0.032	0.007
$\rho(y, CA)$	-0.692	0.569	0.694	0.748	0.085
b/y %	45.6	12.42	15.75	19.42	4.52
\bar{b}/\bar{y} %	—	28.4	40.4	43.82	4.66
$\lambda(\bar{d})\%$	—	1.36	0.57	0.57	2.94
$E(L)(years)$	10	12.25	18.35	—	3.28

Repayment History

The results presented in this subsection correspond to the case in which repayment history plays a role in the way lenders optimally choose the menu of debt contracts offered to the borrowing government. When the variable s_t is reintroduced in the model, the results described in the previous section are still valid, although

²⁵See Kaminsky, Reinhart, and Végh (2003).

the range of values of $\varepsilon(y, d)$ changes. Because the probability of default is positive for a range of values of d larger than in the case when repayment history is left aside, the settlement offer $\varepsilon(y, d)$ always increases in y independent of the value of d , as indicated in figure (2.3). The minimum settlement offer $\underline{\varepsilon}(y, d)$, safe debt-to-GDP thresholds $\bar{b}(s_t)$,²⁶ and probabilities of default at these thresholds, depend on the law of motion of the borrower's repayment score; that is, on the function $f(s_{t-1}, a_t)$ and, in particular, on the entry level repayment score s_1 . In none of the cases analyzed and reported does the government in default prefer to be excluded from the credit market indefinitely; that is, in no case is $\varepsilon(y, d) = 0$.

As anticipated, the model incorporating repayment history can replicate the empirical results obtained by Reinhart et al. (2003). The debt-to-GDP threshold that a country can “safely” manage is closely tied to that country's repayment history. When the repayment score is low, either because of a recent default or because the country is a new borrower, lenders charge an additional premium that translates as a higher cost of servicing the debt and higher probabilities of default at the debt ceiling. As the repayment score improves over time due to consistent repayment, the safe debt-to-GDP threshold eases and the probability of defaulting becomes almost nonexistent.

Figures (2.11) and (2.12) depict the evolution of probabilities of default and debt contracts for a selection of values of s_t . The value chosen for s_1 is 0.7. The probability of default on the left corresponds to the lowest repayment score value,

²⁶This threshold is the debt ceiling $\bar{b}(s_t)$: the maximum level of debt creditors are willing to offer given the repayment score s_t assigned to the government.

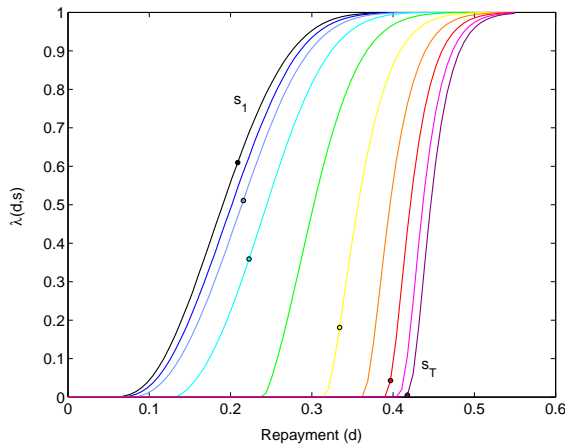


Figure 2.11: Probabilities of Default

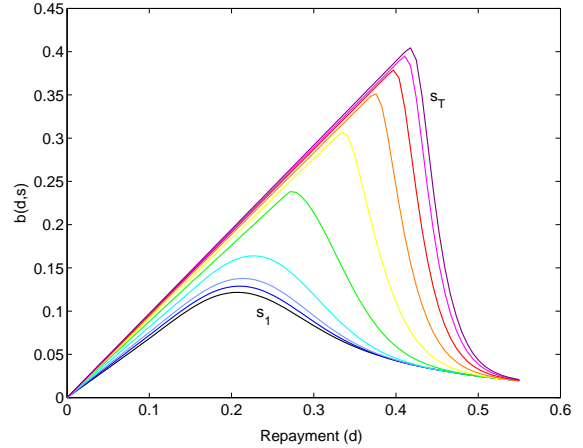


Figure 2.12: Debt Contracts

or entry level value, s_1 . The probability of default on the right corresponds to the highest repayment score value, $s_T = \bar{s} = 1$. For a given value of d , the probability of default decreases as s_t increases. The points over the curves in figure (2.11) represent the probability of default evaluated at the debt ceilings $\bar{b}(s_t)$. As the repayment score improves, the maximum probability of default decreases monotonically.

The debt contracts corresponding to the lowest value s_1 are represented in the curve at the interior of figure (2.12), which in turn has the lowest debt ceiling (see figure (2.8)). The debt contracts corresponding to the highest value $s_T = \bar{s} = 1$ are depicted in the exterior curve in figure (2.12). This curve has the highest debt ceiling. The variations in $\bar{b}(s_t)$ are small from one period to another, but relatively large when moving from s_1 to \bar{s} . Although this result might be difficult to test empirically, it can explain why some countries default at relatively low levels of debt-to-GDP while others can manage much larger ratios without experiencing any distress.²⁷ It also can explain why countries may not experience a sizable increase in

²⁷Turkey, as reported by Standard & Poor's, defaulted on bank debt in 1978. That same year,

the cost of the debt after a default: if a country was already paying a high premium because it had a low repayment score, defaulting would not change the terms of credit significantly. The difference might only be observable if a country with an excellent repayment record were to default. The probability of this occurring, however, is very low because the borrower's incentives to default in this case are weak for two reasons. First, it would be hard to settle the debt due to the country's high level of indebtedness. As a result, the borrower in this case would be excluded from the international credit market for a long period of time. Second, a country with a good record of repayment that suddenly defaulted would suffer a sharp deterioration in the terms of credit; namely, it would face sharply higher borrowing costs and a dramatic tightening in the debt ceiling.

Figure (2.13) illustrates the evolution of the debt ceilings $\bar{b}(s_t)$ over time, provided the government honors its debt. The behavior is similar for different values of s_1 .

The model also replicates the stylized facts documented by Cruces (2003). Lenders' revisions are strongly serially correlated and ratings are more volatile for middle-rated countries. In the model, the repayment score has a positive autocorrelation of 0.59. The figure reported by Cruces for the 1980-2000 period based on IRR data is 0.66. The autocorrelation coefficient of the repayment score is also positive and equal to 0.89.

the World Bank reported a total debt-to-GNI ratio for Turkey of 22.13%. Mexico, Argentina, Brazil, and Venezuela defaulted in the 1980s with debt-to-GNI ratios no larger than 55%. Japan, on the other hand, handles a debt-to-GDP ratio of 130%, the largest among OECD members.

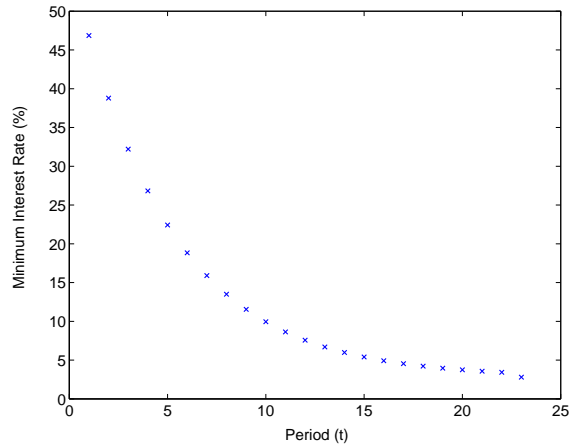
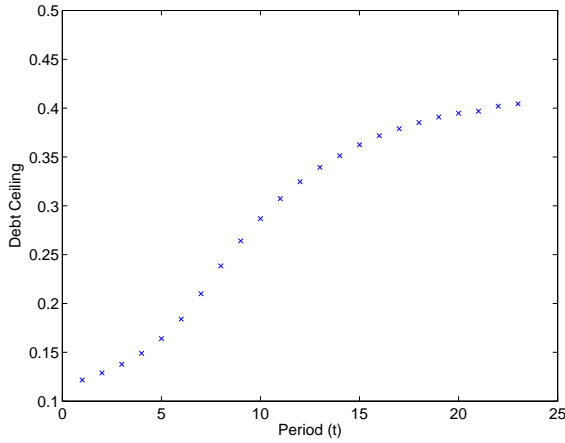


Figure 2.13: Evolution of Debt Ceilings Figure 2.14: Evolution of the Interest Rate

$$\bar{b}(s_t)$$

Rate

As indicated before, s_t should not be thought of as a measure of debt rating because it does not capture the probability of default given by the “fundamentals” $\lambda(d, s)$: the strategic decision to default because the government is better off doing so. The debt rating should include the probability of default $\lambda(d, s)$, the repayment score s , and the expected value of the recovered defaulted debt (expression (2.29) derived from equation (2.10)). Measured in this way, the debt rating exhibits “volatility clustering”. The variety of values that the “rating” can take is larger when the repayment score is in the middle range than when repayment score is low ($s_t \leq 0.7$) or high ($s_t \geq 0.9$). The rating volatility diminishes at the extremes because credit surprises are less likely to occur for countries in those regions.

The debt rating is determined by the probability of default and the expected value of the recovered defaulted debt at each level of the repayment score s_t . These two forces act in opposite directions. When the repayment score is at its lowest value (s_1), the likelihood of a default at the debt ceiling reaches the maximum.

However, because the level of indebtedness is low in this case (the debt ceiling is at its minimum at s_1), the recovery value of the defaulted debt is high. When the repayment score is in the high range of values ($s_t \geq 0.9$), the probability of default is at its minimum, but the recovery value of the defaulted debt is low because the level of indebtedness is at its maximum. When the repayment score is in the middle range, however, neither of the two opposing forces dominates the other, and the range of values that the “rating” takes broadens (figure (2.15)).

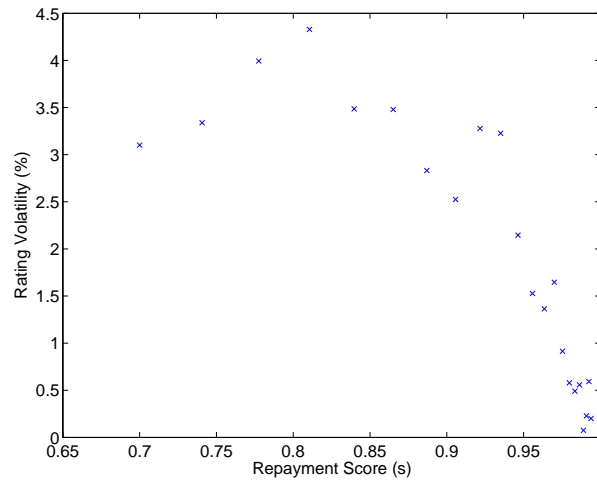
$$\frac{b_{t-1}(1+r)}{d_t} \equiv \left[[1 - \lambda(d_t, s_t)]s_{t-1} + \lambda(d_t, s_t)E_t \left[\sum_{j=0}^{\infty} \left(\prod_{i=0}^j (\theta[\varepsilon(y_{t+i}, d_t)])^{1_{(i < j)}} \right) \frac{[1 - \theta(\varepsilon(y_{t+j}, d_t))]\varepsilon(y_{t+j}, d_t)d_t}{(1+r)^{j+1}} \right] \right] \quad (2.29)$$

Figure (2.14) shows the evolution of the risk-free interest rate charged to the country, depending on the number of periods the country has been active in the international credit market. As Tomz (2001) and Özler (1993) show empirically, the premium declines over time provided that the government services its debt; that is, the premium declines as a country’s repayment score improves. In the model, however, this result is a direct consequence of the form assumed for $f(s_{t-1}, a_t)$.

Repayment history also helps to explain the large variation observed in the interest rates borrowers pay for their debt.²⁸ When issued in 1993, the Argentinean Par bonds offered an average yield to maturity (YTM) of 12.95%. After the 2001 default, the Argentinean debt was negotiated in the secondary market at a maximum

²⁸The standard deviation of the interest rates is calculated using the prices of the debt in the secondary market.

Figure 2.15: Volatility Clustering



yield of 2,945% (October 2002). The standard deviation of the YTM of Argentinean Par bonds, including post default data (1993-2002), is 65.56% (the figure is calculated using annualized data). The standard deviation of the interest rate generated by the model incorporating repayment history is 42.59%.

2.4.4 Sensitivity Analysis

In order to understand better the forces that drive the behavior of creditors and debtors alike, the sensitivity analysis is carried out in two different sections. The first section analyzes the impact of changes in the parameters on the settlement process as well as on the frequency of default. This analysis abstracts from the repayment history (s_t plays no role). The second section analyzes the influences of the repayment history on the credit market.

Settlement Process

This section discusses the sensitivity of the results to changes in the underlying parameters of the model that abstracts from repayment history. The results are most sensitive to the value of α , the percentage of income a country receives while in default that accounts for losses of output and trade.

Allowing for the settlement of the defaulted debt largely modifies the incentives of both borrowers and lenders. Under the assumption that defaulted debt is forgone, and that countries lose access to the international credit market indefinitely after a default (equation (2.4)), lenders relax credit to borrowers. Under these circumstances, the benefit of ensuring continual access to credit markets and smoothing consumption outweighs the cost of repaying the debt during a recession, and as a result, defaults rarely occur. When settlement is a possibility, the chances of the lenders recovering a large percentage of the defaulted debt increase, but the cost of defaulting for the borrowers declines. Relative to the case in which settlement is not an option, lenders become less enthusiastic about extending credit because the cost of defaulting is reduced; the debtor can delay the payment until it is optimal to do so; that is, a debtor can postpone payment until economic conditions improve.

Table (2.3) suggests that there is no clear relationship between the debt ceiling and the expected value that lenders can recover after a default. Facilitating the negotiation process by increasing the settlement rate (which can be accomplished by a reduction in the parameter μ or by an increase in η or ζ) causes an increase in the probability of default, which, as a consequence, decreases the debt ceiling

Table 2.3: Sensitivity Analysis

	Description	Value	\bar{b} %	$E(b)$ %	$\lambda(\bar{d})$ %	Max. No. of def.	$E[L]$ years	$E[\varepsilon(y, \bar{d})]$ %	$\sigma(CA)$	$\rho(CA, y)$
σ	Standard Deviation (Income)	0.090	31.02	10.31	10.68	8	12.14	91.59	0.056	0.347
		0.055	28.37	12.42	1.36	5	12.25	88.08	0.033	0.569
		0.020	26.41	15.17	0.00	0	0.00	83.09	0.013	0.872
ρ	Autocorr. Coefficient (Income)	0.92	24.27	9.20	4.13	6	15.79	87.57	0.026	0.309
		0.72	28.37	12.42	1.36	5	12.25	88.08	0.033	0.569
		0.52	32.03	14.53	3.09	5	12.68	87.04	0.042	0.677
		0.00	38.88	20.05	0.58	1	7.44	85.05	0.049	0.958
γ	Coefficient of Risk Aversion	8.0	29.93	11.76	3.10	6	11.87	90.59	0.037	0.519
		5.0	28.37	12.42	1.36	5	12.25	88.08	0.033	0.569
		2.0	26.42	12.25	1.36	6	15.10	84.96	0.033	0.523
\bar{r}	Risk-free Interest Rate	0.048	17.45	06.90	4.97	7	13.24	88.74	0.028	0.409
		0.028	28.37	12.42	1.36	5	12.25	88.08	0.033	0.569
		0.008	67.53	31.78	0.57	3	13.93	85.68	0.043	0.787
α	Effective Income in Default	0.9990	03.41	0.84	10.41	12	12.88	90.97	0.010	0.139
		0.9901	28.37	12.42	1.36	5	12.25	88.08	0.033	0.569
		0.9800	54.59	24.35	3.10	5	13.00	86.42	0.051	0.610
η	Settlement Rate Parameter	0.008	23.46	9.16	8.75	8	9.41	86.91	0.039	0.366
		0.005	28.37	12.42	1.36	5	12.25	88.08	0.033	0.569
		0.002	35.76	16.45	0.54	2	23.83	89.32	0.030	0.731
μ	Settlement Rate Parameter	3.000	36.30	16.71	0.51	2	24.49	89.41	0.030	0.733
		1.997	28.37	12.42	1.36	5	12.25	88.08	0.033	0.569
		1.000	17.79	7.18	6.42	10	5.95	86.53	0.033	0.361
ζ	Settlement Rate Parameter	6.400	19.70	8.33	3.03	7	5.77	89.04	0.032	0.437
		5.397	28.37	12.42	1.36	5	12.25	88.08	0.033	0.569
		4.400	35.16	16.12	0.34	2	25.97	85.85	0.030	0.728

lenders will impose upon debtors. An increase in the settlement rate reduces the cost of defaulting because the length of exclusion from the credit market shrinks. Governments have more incentives to default and they will do so more often.

This exercise sheds light on the policy implications of the model. Any international restructuring mechanism for defaulted debt (such as Collective Action Clauses and Sovereign Debt Restructuring Mechanism) that is intended to expedite the resolution of debt crises may bring about an increase in the cost of the debt and a reduction in the debt ceiling the international credit market can optimally support. As suggested by Alfaro and Kanczuk (2005), any proposal for a debt restructuring mechanism should account for this possibility and must be applied with caution.

In general, any reduction in costs associated with default —low output cost (high α) or high settlement rate (low μ or high η or ζ)— significantly increases the probability and frequency of defaults and decreases the debt ceiling lenders can support.

It is worth noticing that large output costs (low α), and low interest rates which correspond to high values of β ($\beta = \exp(-\bar{r})$) are able to generate large debt ceilings comparable to the ones observed in the data. An increase in the uncertainty related to the income process (higher volatility or lower autocorrelation) also increases the debt ceiling (see table (2.5)).

Table (2.4) shows some statistics corresponding to a selection of countries with different income profiles.²⁹ In this exercise, all parameters but ρ and σ are fixed to their original values. This exercise is complemented with additional sensitivity analyses for ρ and σ shown in tables (2.5), (2.6), (2.7), and (2.8).

Although the probability of default varies considerably depending on the income profile of a given country, the debt ceilings and expected settlement offers (evaluated at the debt ceilings) are less sensitive to changes in the parameters. It is important to note that the minimum settlement offer evaluated at the debt ceiling $\underline{\varepsilon}(y, \bar{d})$, as reported in tables (2.4) and (2.6), closely matches the figure reported by Suter and Stamm (1992). Suter and Stamm estimate that during the 1821-1975 period, the average percentage reduction in the face value of outstanding bonds was

²⁹The simulations are created using the same seed for the random number generator. In this way it is guaranteed that the “market conditions” are identical for each country so the results are comparable.

Table 2.4: Default Episodes for Several Countries

Country	Data			Model Simulations			
	ρ	σ	No. of Defaults	Max. No. of Defaults	$\underline{\varepsilon}(y, \bar{d})$	\bar{b}	$\lambda(\bar{d})$
Argentina	0.72	0.055	5	5	83.71%	28.37%	1.36%
Cameroon	0.82	0.058	1 [‡]	3	82.81%	26.31%	2.97%
Chile	0.87	0.039	4	6	82.63%	25.52%	1.40%
Costa Rica	0.92	0.042	9	4	82.38%	24.79%	1.46%
Malaysia	0.90	0.037	0 [‡]	3	82.47%	25.14%	1.17%
Mexico	0.81	0.033	8	2	82.85%	26.23%	0.57%
Peru	0.80	0.057	7	6	82.58%	26.57%	5.59%
Sweden	0.82	0.015	0	0	82.28%	25.98%	0.00%
Turkey	0.58	0.038	6	7	83.14%	28.75%	4.18%
United Kingdom	0.85	0.016	0 [†]	0	82.33%	25.90%	0.00%
Venezuela	0.84	0.045	9	7	82.57%	25.80%	3.12%
Zimbabwe	0.57	0.053	2 [‡]	2	84.05%	30.99%	1.36%

(§) $\underline{\varepsilon}(y, \bar{d})$ evaluated at \bar{d} . (†) Standard and Poor's does not report the United Kingdom defaulting on its sovereign debt. Eichengreen (1988) reports that in 1934, and due to the Great Depression, Great Britain defaulted on First World War debts and reparations; payments were never resumed. (‡) The number of periods per simulation was adjusted to reflect the number of years since independence —Cameroon (45), Malaysia (48), and Zimbabwe (40). Turkey was included as the Ottoman Empire before 1923 so the number of periods was set to 180. Although Zimbabwe gained independence from the United Kingdom in April 1980, it unilaterally declared independence in 1965 and its first default dates back to that year.

17.9%, which corresponds to an average repayment percentage of 82.1% of defaulted debt.

Differences in income profiles by themselves cannot explain either the observed volatility of interest rates or the fact that the debt ceiling that debtors can “safely” manage varies significantly from one country to another (given the parameters corresponding to the income profile of the countries shown in the sample). The model also cannot justify observed cross-country differences in settlements offered by debtors to their creditors when in distress. Nonetheless, the income profile can explain variations in the likelihood of default and the frequency with which they occur. If, as

proposed, repayment history plays a role in the cost of the debt, then countries that are more likely to default are expected to pay a higher premium in the long run if they are not able to escape the default trap and gain greater credibility in the eyes of lenders.

The results of table (2.4), (2.7), and (2.8) indicate that countries with volatile incomes that exhibit high levels of autocorrelation are expected to default more often than countries with more stable incomes and lower levels of autocorrelation. The ability of a given country to generate stable income might be correlated with its ability to avoid the default trap. This result is consistent with the findings of Gelos et al. (2003): economies that fail to access the markets are usually more vulnerable to external shocks than economies that do have access.

Importantly, the maximum frequency of defaults occurs in every country for the same sample path of shocks (under the same “market conditions”) even though every country has a different income profile distribution. This highlights the importance of economic shocks in triggering debt crises and in determining their precise timing.

Table 2.5: Sensitivity Analysis, Debt Ceiling

$\rho \backslash \sigma$	0.09	0.07	0.055	0.04	0.02
0.92	23.31	23.87	24.28	24.72	25.51
0.72	31.02	29.30	28.37	27.44	26.40
0.52	40.09	35.22	32.03	29.61	27.00
0.32	48.03	39.99	35.23	31.21	27.36
0.00	57.39	45.80	38.88	33.08	27.75

Table 2.6: Sensitivity Analysis, Minimum Settlement Offer

$\rho \backslash \sigma$	0.09	0.07	0.055	0.04	0.02
0.92	80.54	82.43	81.94	82.09	82.33
0.72	81.98	83.02	83.67	83.44	82.57
0.52	84.47	84.52	83.99	83.69	82.48
0.32	85.96	85.35	84.81	83.83	82.42
0.00	88.39	86.79	85.05	83.70	82.32

Table 2.7: Sensitivity Analysis, Probability of Default

$\rho \backslash \sigma$	0.09	0.07	0.055	0.04	0.02
0.92	10.78	3.13	4.13	3.53	0
0.72	10.68	5.77	1.36	0.57	0
0.52	5.73	2.91	3.09	0.58	0
0.32	5.72	3.59	0.58	0	0
0.00	2.91	0.48	0.58	0	0

Table 2.8: Sensitivity Analysis, Maximum Number of Defaults (180 years)

$\rho \backslash \sigma$	0.09	0.07	0.055	0.04	0.02
0.92	7	6	6	6	0
0.72	8	7	5	4	0
0.52	5	4	5	2	0
0.32	5	3	2	0	0
0.00	2	1	1	0	0

Repayment History

As indicated before, s_1 can be seen as reflecting “market sentiment”. Suter (1992) characterizes a global debt cycle as having three consecutive phases: 1) the expansion of international loans, a period during which capital shifts from developed economies to emerging economies (foreign investment); 2) the outbreak of a payment crisis; and 3) renegotiation and debt settlement. During the loan boom phase, optimistic lenders are willing to offer credit to almost every government, and they may assign a high score to unknown governments (s_1 tends to be high). During a

credit slump, just after the settlement process, the lenders' confidence in the market tends to be low, which may induce them to assign a lower entry level repayment score s_1 . Inflows of foreign debt are meager until a new loan boom phase begins; that is, capital flows remain depressed until s_1 increases. The model captures this fact. Debt frenzies are followed by debt crises. When the markets are calm and keep their composure, debt crises are uncommon.

Table (2.9) shows the relation between the value of s_1 , the entry level repayment score, and variables like the minimum interest rate charged, the debt ceiling $\bar{b}(s_1)$, and the maximum probability of default at the moment the government taps the market for the first time or after a settlement.

Table 2.9: Interest Rate, Settlement Offer, Debt Ceiling, and Probability of Default

s_1	Minimum interest rate @ s_1 (%)	$\underline{\varepsilon}(y, d)$ (%)	$\bar{b}(s_1)$ (%)	$\lambda(\bar{d}(s_1))$ (%)	$\bar{b}(\bar{s})$ (%)	$\lambda(\bar{d}(\bar{s}))$ (%)
0.6	71.3	21.71	10.7	58.7	40.9	0.1
0.7	46.9	26.49	12.2	55.3	40.4	0.6
0.8	28.5	34.00	14.3	49.8	39.8	0.6
0.9	14.2	44.19	17.9	39.5	38.1	1.6

The minimum interest rate and the probability of default $\lambda(\bar{d}(s_1))$ decrease as the credit market gains confidence, or as lenders assign high entry level repayment scores. Under these circumstances, the debt ceiling $\bar{b}(s_1)$ relaxes and the borrower increases its payment to the lender in the event of a default: $\underline{\varepsilon}(y, d)$ is positively related to s_1 . In the long run, however, the probability of default in the stationary equilibrium, $\lambda(\bar{d}(\bar{s}))$, increases compared with smaller values of s_1 . Default is not as costly in the long run because the stigma of default is lower — the change in s_t

smaller.

The variations in the minimum interest rate associated with changes in s_1 are dramatic. As the market tightens, the cost of borrowing becomes prohibitive and foreign capital is rendered scarce. Although countries have access to credit markets, in many states of nature they choose not to borrow due to the exorbitant cost. Such episodes have occurred over the course of history. Jenks (1937), for instance, reports the situation regarding the failed Colombian loan of 1866: “one million pounds and a half to be remitted to a country with a diminutive volume of trade. There was a floating debt to be scratched off, a road company to be financed, some warships to encourage the Colombian sense of statehood. And when deduction has been made for a year’s interest and commissions and expenses Colombia found herself in receipt of some \$560,000 cash each year until 1873 in remittance of the loan proceeds, and owing the contractors in return in interest and amortization, \$800,000 a year for twenty years.”

One of the most interesting results is the relationship between the entry level repayment score, s_1 , and the incidence of serial default. The model cannot replicate serial default for low values of s_1 (credit slump), but does well when s_1 is increased (credit boom). The reason is that the value of $\bar{b}(s_1)$ depends positively on s_1 , as shown in table (2.9). When the cost of borrowing is too large, or when s_1 is small, the value $b(y, d, s_1)$ ³⁰ a borrower chooses is close to zero. In the next period, however, the value to be repaid will be small compared with the new set of loans lenders are

³⁰ $b(y, d, s)$ can be understood as the present value of a zero coupon bond with principal d to be paid in the next period.

willing to offer given that the government repays its debt. More importantly, even though it has the opportunity to increase its debt, the government will borrow only a small amount in the next period because the mechanism to maximize its utility is through smoothing consumption across periods and across states. This behavior reduces the probability of default in subsequent periods.

However, if the government receives a low endowment in the first period and chooses to default, it needs to offer sufficiently attractive settlements $\varepsilon(y, d, 0)$ for lenders to accept and thereby make it possible for the government to access the credit market again. Lenders are more likely to accept the offer $\varepsilon(y, d, 0)$ the larger the value of y . Given the nature of the income process (AR(1)), a large value of y at t makes large values of y at $t + 1$ more likely, which implies low values of $b(y, d, s_1)$. Once the government settles the debt, it finds itself in the situation described above; a low level of indebtedness that reduces the probability of default in subsequent periods.

As s_1 increases, the values of $b(y, d, s_1)$ also increase. This permits the government to raise its level of indebtedness more rapidly without jeopardizing its objective of maximizing its utility through consumption smoothing. However, increasing indebtedness boosts the chances of default in subsequent periods, potentially leading to serial default. Although low values of s_1 are associated with a reduction in the number of defaults, they are also associated with high costs for debtors and a reduction in the amount of capital flowing to emerging economies. In this way, lenders can control the inflow of capital to emerging economies. Large inflows will invariably be associated with debt crises.

The introduction of s_t into the model also creates variation in the minimum settlement borrowers will offer to the lenders after a default. When repayment history plays no role, variations in the income profile or other parameters cannot explain the large range of settlement offers observed in the data; as shown in table (2.4), the minimum settlement offer is around 83.42%, while in the data one can find cases where the value of ε is not larger than 56% (as reported by the World Bank in the GDF database for the years when the defaulted debt was settled). When repayment history is introduced, however, the maximum level of indebtedness, and hence the minimum settlement offer, depend on s_t . The settlement offers when repayment history is incorporated into the model mimic the values observed in the data and display larger volatility.

Table 2.10: Borrowers' Expected Consumption, Lenders' Expected Net Return, and Maximum No. of Defaults per Simulation

s_1	E(c)	Net return (%)	Maximum No. of Defaults
0.6	0.9959	0.2115	3
0.7	0.9958	0.0752	3
0.8	0.9958	0.0252	4
0.9	0.9957	-0.1490	5
1.0 [§]	0.9964	-0.4900	5

(§) A value of $s_1 = 1$ corresponds with the case where repayment history does not matter.

Moreover, introducing repayment history into the model increases the net return lenders get after extending debt contracts. As table (2.10) suggests, this comes at the expense of the borrower's average consumption. The net return is calculated accounting for partial default; that is, the return includes losses due to the settlement of a defaulted debt below its par value. The net return premium

reported by Lindert and Morton (1989) based on data from ten countries is 0.44% per year on 1,519 bonds outstanding between 1850 and approximately 1970.

2.5 Concluding Remarks

This paper combines endogenous default, endogenous settlement, and repayment history into a dynamic model of sovereign debt. It was written to address two related questions: 1) how the level of indebtedness and the income profile affect the length of time a country in default is excluded from the international credit market, and 2) how repayment history impinges upon the credit limit and interest rate lenders are willing to offer, and upon the borrower's incentive to default. In order to model the defaulting country's potentially lengthy exclusion from the international credit market, the paper assumes that although lenders know the income profile of the borrowing country, they cannot verify the current income level. The existence of private information translates into inefficiencies and induces strategic behavior by borrowers and lenders during the settlement process (inefficiencies because borrowers and lenders are better off settling under the same final conditions without costly delays). Although the negotiation is not modeled explicitly, the likelihood of a settlement is assumed to be a random process that depends positively on the portion of the defaulted debt that the borrowing government is willing to repay. Based on the current state of their economy, borrowers then choose what settlement offer to make to the lenders, and can influence in this way the length of time they are excluded from the credit market. The paper also studies how lenders respond to new information in the form of repayment history, or the number of years a borrower

has been active in the international credit market. Lenders incorporate information about a country's record of repayment into the debt ceiling they impose as well as the interest rate they charge in the following period.

The results of this paper, which are obtained using numerical solution methods and by calibrating the model to replicate the income profile of emerging economies, are presented in two stages: a first stage that analyzes the settlement process, and a second stage that incorporates repayment history into the analysis. The findings of this study suggest that it is always optimal for a defaulting country to offer a positive settlement; that is, defaulting countries never choose to stay excluded from the international credit market for an indefinite period of time. The downside of this finding is that since the cost associated with a default is a finite rather than an infinite period of exclusion from the credit market, lenders reduce the debt ceiling they are willing to offer through an increase in the cost of the debt. Any reduction in the cost of default increases borrowers' incentives to fail payments and as a result lenders reduce the debt ceiling they optimally support. It is in the extreme case, when borrowers can reenter the credit market without having to repay the defaulted debt, that lenders offer the lowest debt ceiling. These findings are robust for a number of different parameterizations of the model.

This paper has very important implications for any international restructuring mechanism for defaulted debt intended to expedite the resolution of debt crises. The model reveals that reducing the settlement time, or the cost of defaulting, increases borrowers' incentives to default. In response, lenders increase the cost of the debt and reduce the debt ceiling they can optimally support. This result highlights

the need to carefully consider the implications of any proposed debt restructuring procedures.

The analysis further indicates that the best settlement offer a borrower can make to a lender after a default depends positively on the borrower's current income level and negatively on the size of its defaulted debt. These results are consistent with the empirical literature. The model matches several statistical moments of the economy used for the calibration and closely replicates the default experiences, including the number of defaults and the length of exclusion from the international credit market of several countries. Furthermore, the model shows that not only are defaults not rare events, but also that serial defaulters are not extraordinary.

When repayment history is introduced in the analysis, the model is able to replicate some stylized facts about sovereign debt: the debt-to-GDP ratio that new borrowers or serial defaulters can support is well below the ratio that proven debtors can safely manage; countries that have defaulted in the past are significantly more likely to become problem debtors again in the future; countries with identical income profiles may face different borrowing costs depending on their repayment history; and once a country defaults, it will take many years of impeccable repayment and low levels of debt for that country to gain continuous access to international credit markets at low cost. The process of regaining continuous access to credit markets at low cost is exceedingly slow, and backsliding into default is very difficult to avoid.

In the framework of the model, debt crises are preceded by an expansion in credit activity provoked by lenders' overconfidence in the ability of the debtor governments to service their debt. Conversely, tight credit markets are associated

with meager capital inflows to emerging economies. Economic shocks are crucial in explaining widespread debt crises and the timing of their occurrence.

Even though a country's income profile, which reflects the variance and autocorrelation of its income process, influences the debt-to-GDP ratio that the country can safely manage, repayment history is a more important determinant of that ratio. When creditors charge large premia to new borrowers and serial defaulters, the higher cost of servicing the debt results in larger probabilities of default. As a consequence, these countries fail more often, making it more difficult for them to escape a vicious cycle of defaulting. Only competent and cautious governments committed to servicing their countries' debt faithfully might, after decades of timely payments, become proven debtors and enjoy full access to international credit markets at a low cost. The ability to avoid the default trap seems to be associated with a country's ability to generate stable income.

The model proposed is only a first approximation to the question of why repayment history matters. A direction for further research is to use repayment history as a screening device to sort out different types of government. Under this specification, lenders would charge a premium to new borrowers or serial defaulters due to the uncertainty they face regarding the borrowers' willingness (or ability) to pay. One type of government in this framework might be characterized by impeccable repayment record and unlimited access to the international credit market, whereas another type might include countries that spend most of the time in state of default and suffer from little if any access to the market. Adding this feature would complete the circle in the sense that it would allow for the emergence of a middle group

of countries —countries that have only sporadic access to the market such as those analyzed in this paper— to be a consequence of the international credit market's inability to determine with certainty the willingness of each and every borrowing government to service its debt.

Appendix A

Appendix to Chapter 2

The contraction mapping theorem establishes that given a set of continuous functions $f(x)$, $g(x)$, $h(x)$, mapping values from a bounded closed interval into a bounded close interval, if T is a contraction mapping then there exists exactly one function satisfying $f(x) = Tf(x)$. Furthermore, Blackwell's contraction mapping theorem asserts that the operator T is a contraction if it has the properties of *Monotonicity* and *Discounting*:

Monotonicity If $f(x) \leq g(x)$ for all x , then $Tf(x) \leq Tg(x)$ for all x .

Discounting Let a be a positive constant. There exists some $\delta \in (0, 1)$ such that

$$T(f + a)(x) \leq Tf(x) + \delta a.$$

Equations 2.15 and 2.13 satisfy Blackwell's contraction mapping theorem.

Proof of Monotonicity. Because the state space $S = \{y, d\}$ is closed and bounded, u is bounded below some \bar{u} , then $V_\infty^r(y, d) \leq \frac{\bar{u}}{1-\beta} < \infty$ and $V_\infty^d(y, d) \leq \frac{\bar{u}}{1-\beta} < \infty$ are both bounded and therefore have a maxima.

Let $Q_\infty^r(y, d)$ be an arbitrary function defined as:

$$Q_\infty^r(y, d) = \max_{b \in B} \left\{ u(y + b - d) + \beta E \max \left\{ Q_\infty^d(y', d'); Q_\infty^r(y', d') \right\} \right\} \quad (\text{A.1})$$

and b_Q the optimal choice of b when $Q_\infty^r(y, d)$ is maximized.

Let $Q_\infty^d(y, d)$ be an arbitrary function defined as:

$$Q_\infty^d(y, d) = u(\alpha y) + \max_{\varepsilon \in [0,1]} \beta E \left\{ \theta(\varepsilon) Q_\infty^d(y', d) + (1 - \theta(\varepsilon)) Q_\infty^r(y', \varepsilon d) \right\} \quad (\text{A.2})$$

and ε_Q the optimal choice of ε when $Q_\infty^d(y, d)$ is maximized.

Suppose $V_\infty^r(y, d) > Q_\infty^r(y, d)$ and $V_\infty^d(y, d) > Q_\infty^d(y, d)$.

Then it must be the case that:

$$\begin{aligned} T(V_\infty^r)(y', d') &\equiv \max_{b \in B} u(y + b - d) + \beta E \max \left\{ V_\infty^d(y', d'); V_\infty^r(y', d') \right\} \quad (\text{A.3}) \\ &\geq u(y + b_Q - d) + \beta E \max \left\{ V_\infty^d(y', d'); V_\infty^r(y', d') \right\} \\ &\geq u(y + b_Q - d) + \beta E \max \left\{ Q_\infty^d(y', d'); Q_\infty^r(y', d') \right\} \\ &\equiv T(Q_\infty^r)(y', d') \end{aligned}$$

and

$$\begin{aligned} T(V_\infty^d)(y', d) &\equiv u(\alpha y) + \max_{\varepsilon \in [0,1]} \beta E \left\{ \theta(\varepsilon) V_\infty^d(y', d) + (1 - \theta(\varepsilon)) V_\infty^r(y', \varepsilon d) \right\} \quad (\text{A.4}) \\ &\geq u(\alpha y) + \beta E \left\{ \theta(\varepsilon_Q) V_\infty^d(y', d) + (1 - \theta(\varepsilon_Q)) V_\infty^r(y', \varepsilon_Q d) \right\} \\ &\geq u(\alpha y) + \beta E \left\{ \theta(\varepsilon_Q) Q_\infty^d(y', d) + (1 - \theta(\varepsilon_Q)) Q_\infty^r(y', \varepsilon_Q d) \right\} \\ &\equiv T(Q_\infty^d)(y', d) \end{aligned}$$

□

Proof of Discounting. Let b^* the optimal choice of b when $T(V_\infty^r)(y', d')$ is maximized

$$\begin{aligned} T(V_\infty^r)(y', d') + \beta a &\equiv u(y + b^* - d) + \beta E \max \left\{ V_\infty^d(y', d'); V_\infty^r(y', d') \right\} + \beta a \quad (\text{A.5}) \\ &\equiv u(y + b^* - d) + \beta E \max \left\{ V_\infty^d(y', d') + a; V_\infty^r(y', d') + a \right\} \\ &\geq u(y + b^* - d) + \beta E \max \left\{ V_\infty^d(y', d') + a; V_\infty^r(y', d') \right\} \\ &\equiv T(V_\infty^r + a)(y', d') \end{aligned}$$

Let ε^* the optimal choice of ε when $T(V_\infty^d)(y', d)$ is maximized

$$\begin{aligned}
T(V_\infty^d + a)(y', d) &\equiv u(\alpha y) + \beta E \left\{ \theta(\varepsilon^*) (V_\infty^d(y', d) + a) + (1 - \theta(\varepsilon^*)) V_\infty^r(y', \varepsilon^* d) \right\} \text{(A.6)} \\
&\equiv u(\alpha y) + \beta E \left\{ \theta(\varepsilon^*) V_\infty^d(y', d) + (1 - \theta(\varepsilon^*)) V_\infty^r(y', \varepsilon^* d) \right\} + \beta a \theta(\varepsilon^*) \\
&\leq u(\alpha y) + \beta E \left\{ \theta(\varepsilon^*) V_\infty^d(y', d) + (1 - \theta(\varepsilon^*)) V_\infty^r(y', \varepsilon^* d) \right\} + \beta a \\
&\equiv T(V_\infty^d)(y', d) + \beta a
\end{aligned}$$

□

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