

ABSTRACT

Title of Document: INVESTIGATING TEACHERS' EXPERIENCES WITH THE HISTORY OF LOGARITHMS: A COLLECTION OF FIVE CASE STUDIES

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This study investigated five secondary mathematics teachers' efforts to study and use the history of a specific topic. A professional development experience, constructed to reflect the features of effective professional development identified by Garet, Porter, Desimone, Birman, and Yoon (2001) and Smith (2001), was designed to engage teachers in the study of the historical development of logarithms. Modifications of activities found in the *Exponentials and Logarithms* module (Anderson, Berg, Sebrell, & Smith, 2004), as well as various print and electronic resources, were used during the professional development.

Two primary research questions guided the study. First, the study addressed how teachers with different background knowledge and experiences responded to the professional development. Second, the study investigated how teachers' background variables and experience with the professional development influenced the teachers'

personal mathematical knowledge and instructional practice. Exploratory case study methodology was used to describe the experiences of five participants; four teaching in a public high school and one teaching in a private day school. Data sources used in the case study included teacher background, attitudes, and content knowledge instruments; participant observation during all professional development sessions and classroom instruction (during a unit on logarithms); and semi-structured interviews.

The study found that engagement during the professional development sessions was stronger on the part of participants who reported high participation in previous professional learning activities and who were able to consider alternatives for dealing with the barriers to incorporating the history of logarithms. Similarly, the extent to which participants incorporated the history of logarithms during their instruction was directly related to the extent of their engagement during the professional development. Lastly, the two teachers with the strongest professional development engagement and implementation of the history of logarithms exhibited the most improvement in content knowledge.

The study conveyed important information for what Barbin (2000) indicated is essential for qualitatively analyzing “the changes that can occur when history has a place in the teaching of mathematics” (p. 66).

INVESTIGATING TEACHERS' EXPERIENCES WITH
THE HISTORY OF LOGARITHMS:
A COLLECTION OF FIVE CASE STUDIES

By

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In their classic, “Closer to Fine,” the Indigo Girls observed,

There's more than one answer to these questions
Pointing me in a crooked line
The less I seek my source for some definitive
The closer I am to fine.

This refrain spoke volumes to me back then, and although my questions were much less defined, the journey over the past several years has been made much sweeter by the many individuals who have aptly guided me along “crooked lines.”

I have had the honor and pleasure of working with the most amazing doctoral committee. I know that no collection of words can appropriately express how I feel about the following individuals. I want to publicly thank my advisor during my entire Maryland experience and the chair of my committee, Dr. Anna Graeber. I thank you for your sustaining guidance and wisdom. You have been a constant source of human and scholarly comfort to me over the past four years and I am a better mathematics educator for it.

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have learned volumes from you, Dr. Fey, and what I treasure the most is that the key exists in asking the right question and often it is the question which sustains us.

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My ability to contribute to mathematics education in the small way that this work will set forth in the following pages would not be possible without the willingness and contributions of Mandy, Sue, Ted, Shirley, and Mary. Thank you for allowing me to share my passion with you and for letting me into your professional lives and your classrooms. I hope that we will all meet again in the future and that history – in more ways than one – will be the vehicle.

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Finally, I realize now that I never needed “to seek my source” to guide me to my next opportunity. Instead, I take great comfort in knowing that each of these individuals composed the source responsible for bringing me just that much “closer to fine.”

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Chapter 1 Introduction

My interest in studying the use of the history of logarithms originates from my personal experience with the history of mathematics and began in 1989 when I took a graduate-level history of mathematics course. In the course, Dr. David Stout chose to teach with an emphasis on the mathematics, specifically, working with mathematical topics as would have been done in a given century with the tools and concepts known to the mathematicians of the time. I found the course frustrating because I consistently tried to use my modern view of mathematics to solve the problems. This was often difficult because most of the problems did not share a context from modern times and often the solution processes that I tried to employ were over-qualified, so to speak. At the end of the semester, I was convinced that my frustrations with “doing” mathematics from an historical perspective were parallel in many ways to the frustrations of the students I had taught. I briefly held the thought that I would tap into this valuable resource when I returned to teaching.

Unfortunately, in my next teaching position I was faced with many of the same challenges secondary teachers confront: developing new curricula, creating meaningful tasks for students, learning the procedures of a new school, developing a “teacher personality,” and balancing academics with extra-curricular responsibilities. My good intentions of using history of mathematics in my teaching were quickly forgotten, save for an occasional anecdote about Pythagoras or Archimedes. Although the ideas from my history of mathematics class had the potential for profound impact on how I considered teaching various mathematical topics, I was lacking experience in two major aspects of using history of mathematics with students: the ability to focus on the historical specifics

of topics I was teaching and access to materials to use with secondary students. Over time, my interest in the history of mathematics has deepened and provided me with the motivation to examine other teachers' experiences with using the history of mathematics for my dissertation research.

The purpose of this study was to explore teachers' efforts to study and use the history of mathematics. The topic of logarithms is often situated between seemingly unrelated topics, such as conic sections and sequences and series. The topic of logarithms was chosen in part because often this placement is viewed as arbitrary to both teachers and students. Examining logarithms from an historical perspective aids in transitioning between problematically-placed topics. The history of logarithms depends upon mathematical topics which students typically encounter prior to logarithms, such as exponents; and sets the stage for later topics, such as sequences. Additionally, the topic of logarithms possesses a history rich in examples providing students with experience in moving from the concrete to the abstract (Liu, 2003).

The focus on classroom teachers was influenced by Sfard's call for continued investigations with a "prevalent focus on the teacher and teacher practice" (2005, p. 397). The research was composed of two critical components. First, a professional development experience was designed and implemented to engage teachers in the historical development of logarithms. During the implementation of the professional development, each teacher's participation was documented for the purpose of describing their engagement. Second, each teacher's instruction during the unit on logarithms was observed to determine how he or she ultimately incorporated the history of logarithms. To investigate the claim made by scholars that the use of the history of mathematics is

truly a beneficial teaching tool, I planned to engage with and study five teachers attempting to use the tool of history in actual secondary classrooms. The study investigated how the participants labored to understand historical materials and perspectives and their subsequent use in classrooms already burdened by the constraints created by high-stakes assessments, time, and school contexts.

Background

Russ (1991) observed that many mathematics teachers may not have reason or occasion to use the history of their subject, “and taking it up may seem a daunting additional burden of doubtful value” (p. 7). The consideration of the value and importance of using the history of mathematics has occurred over numerous decades, although mostly through the endeavors of a small collection of individuals. In the United Kingdom, encouragement for the inclusion of historical aspects of mathematical topics has appeared in documents for that country’s National Curriculum – off and on – for almost 100 years. Fauvel (1991) cited several excerpts:

From 1919: That portraits of the great mathematicians should be hung in the...classrooms, and that reference to their lives and investigations should be frequently made by the teacher in his lessons, some explanation being given of the effect of mathematical discoveries on the progress of civilization.

From 1958: The teacher who knows little of the history of Mathematics is apt to teach techniques in isolation, unrelated either to the problems and ideas which generated them or to the further developments which grew out of them.

And, from 1982: The mathematics teacher has the task...of helping each pupil to develop so far as is possible his appreciation and enjoyment of mathematics itself and his realization of the role which it has played and will continue to play both in the development of science and technology and of our civilization. (p. 3)

Fauvel also noted, however, that beginning in the early 1990s, “the historical perspective [was] less noticeable...than in any official document about mathematics education for a century” (p. 3).

The same “history” is mirrored in mathematics education documents in the United States. In the opening chapter of the thirty-first yearbook of the National Council of Teachers of Mathematics (1969), *Historical Topics for the Mathematics Classroom*, Phillip Jones articulated the struggle in using history in the mathematics classroom:

Teaching so that students understand the “whys,” teaching for meaning and understanding, teaching so that children see and appreciate the nature, role, and fascination of mathematics, teaching so that students know that men are still creating mathematics and that they too may have the thrill of discovery and invention – *these are objectives eternally challenging, ever elusive* [italics added]. (p. 1)

This elusiveness may be in part because, as the editors of the yearbook themselves admitted, their goal was to “emphasize the mathematical content of the material and to leave the method of bringing it into the individual classroom in the hands of the person most qualified to make this decision – the teacher” (Baumgart, Deal, Vogeli, & Hallerberg, 1969, pp. x – xi).

Identifying scholars who testify to the importance of incorporating the history of mathematics into the teaching of topics is not a difficult task (Borasi, 1987; Brown, 2001; Fauvel, 1991; Katz, 1997; Ransom, 1991; Siu, 1997). The stance of such research, however, has been largely focused on the benefits realized by students, including pre-service teacher education students (Fleener, Reeder, Young, & Reynolds, 2002) and other undergraduate students (Lit, Siu, & Wong, 2001). Although mathematicians and mathematics historians attest to the power of using history in teaching mathematics, there exists the overwhelming problem of how to enable secondary mathematics teachers to

use the history of mathematics in meaningful ways in their teaching. As Jones (1969) claimed, “the history of mathematics will not function as a teaching tool unless the users (1) see significant purposes to be achieved by its introduction and (2) plan thoughtfully for its use to achieve these purposes” (p. 5). Indeed, there are numerous examples (Fauvel & van Maanen, 2000; Jones, 1969; Liu, 2003; Siu, 1997; Tzanakis & Thomaidis, 2000) of how teachers might “plan thoughtfully” for using the history of mathematics with students. Among those most often cited are:

- (1) use of brief historical anecdotes;
- (2) use of historical problems (in conjunction with some prior discussion);
- (3) assigning reports or papers (mathematical or biographical); and
- (4) reading and interpreting original documents.

Regardless of the choice of classroom activity or task, a common caveat is that unless the teacher has significant training or course work in the history of mathematics, the inclusion of mathematics history requires some form of intervention.

Although recommendations for the use of the history of mathematics persist, there is a paucity of research which has been conducted to investigate its use by secondary classroom teachers in the United States. There are, however, several studies which have been conducted in other countries that qualitatively analyze “the changes that can occur when history has a place in the teaching of mathematics” (Barbin, 2000, p. 66). Several of the articles archived by Barbin focus on the presentation of “case studies by [French] teachers of work in their own classrooms” (p. 66). Other studies conducted in Hong Kong (Lit, Siu, & Wong, 2001) have focused on teachers’ perceptions of the teaching and

learning experienced when using the history of mathematics with secondary students in that country.

Rationale

There are several reasons to incorporate the use of the history of mathematics in the secondary classroom. Perhaps the most influential consideration for incorporating the use of history of mathematics in teaching is the impact of such a practice on the development of the mathematical disposition of students. In addition to the more obvious effect of the use of history on students' attitudes towards mathematics (McBride & Rollins, 1977; Marshall, 2000), the use of historical materials can offer a significant portrait of "what it means to be a mathematician, or, more broadly, a research scientist" (Furinghetti, 1993, p. 33). For example, biographies help to unveil "the mysterious...nature of the discipline than targeted explanations of the subject" (Furinghetti, p. 33).

What does this mean for the classroom teacher and his or her students? By rethinking school mathematics as naturally containing a historical element, other goals of instruction can be realized. The reading and writing activities often associated with the inclusion of the history of mathematics become a natural activity in the mathematics classroom, not a contrived one. For students who generally "hate" mathematics or find it uninteresting, the use of historical examples can be of significant help. Fischbein (1987) noted that, "the fact that great scientists and mathematicians have absolutely believed in ideas that later on became obsolete, may by itself be very encouraging to students" (p. 39). Using the history of mathematics enables students to connect with the aesthetic as

well as human side of mathematics. Furinghetti (1993) referred to the following concerning ethnomathematical studies:

Since the means used for transmitting mathematics are the natural language together with icons, symbols, figures, the differences between the various languages affect this transmission less than they do in the case of other subjects. This feature emphasizes the universal character of mathematics. (p. 34)

It is naïve to infer that mathematics can then be magically and universally learned by all students simply because the history of mathematics is used as a learning tool. Many believe a student's mathematical disposition is strengthened when students become more familiar with the developmental nature of mathematics. Gulikers and Blom (2001) contended that, "effective learning requires that each learner has to retrace the main steps in the historical evolution on the studied subject" (p. 225). This effect is sometimes referred to as the historical-genetic-principle. When the principle is transferred to the realm of learning mathematics, it is interpreted as the notion that "the development of the mathematical understanding of an individual follows the historical developments of mathematical ideas" (p. 225). Thus, Gulikers and Blom claimed that, "the...teaching and learning mathematics along the line of its historical development" (p. 226) was a necessity. They caution, however, not to take the statement literally throughout the curriculum, citing the example that, "no one has ever suggested that a child should be kept away from the concept of zero until he has completed the study of Greek geometry in which the concept does not occur" (p. 226).

Using the history of mathematics also strengthens students' mathematical dispositions in another important way. Historical approaches make the circuitous routes leading to important discoveries and refinements of mathematical ideas explicit, enabling students to take comfort in the notion that they are not alone in experiencing

mathematical difficulties, misunderstandings, and mistakes (Borasi, 1987; Fauvel, 1991; Fischbein, 1987; Gulikers & Blom, 2001). As such, the use of the history of mathematics allows for learning mathematics in a non-linear fashion. Modern textbooks expertly disguise the non-linear manner in which much of mathematics is developed. At the same time, such textbooks lull students into believing that subsequent mathematical topics follow smoothly from all that they previously know, essentially stripping the human considerations from the development of mathematics. Brown (2001), in his criticism of this very characteristic of textbooks, observed that textbooks are poor candidates for a humanistic orientation towards mathematics. The fact that traditional texts are ordered with such supreme attention to logical development and contain weak “story lines” detracts from students’ abilities to wonder or ask why certain topics appear or how the topics fit into the grand scheme of things (p. 203).

Mathematics is a purely human construction (the question of mathematics as discovery or as invention will not be debated here). Thus, a second reason for using ideas from the history of mathematics or incorporating the historical development of particular mathematical topics in teaching mathematics helps to fortify “the role of mathematics in society, shows the development of mathematics as a human activity, helps to create a lively classroom atmosphere, and helps to increase students’ interest for learning” (Gulikers & Blom, 2001, p. 230). Russ (1991) characterized the history of mathematics in this way:

It enables pupils to gain a more accurate picture of mathematics and their own role in the learning process. And it ensures that a more human face is put on a subject which for too long has been regarded as cast in concrete, impenetrable and rather frightening. (p. 7)

An important aspect of the use of the history of mathematics is relating the actual stories of the men and women behind the mathematical contributions. The National Council of Teachers of Mathematics (NCTM) listed “learning to value mathematics” as the first goal for students in the *Curriculum and Evaluation Standards for School Mathematics* (1989). The goal specified that:

Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities... It is the intent of this goal – learning to value mathematics – to focus attention on the need for student awareness of the interaction between mathematics and the historical situations from which it has developed and the impact that interaction has on our culture and lives. (pp. 5 – 6)

The most common use of historical materials, incorporating biographical information and historical vignettes, is often an essential element for addressing the “learning to value mathematics” goal. Students’ awareness of mathematics as a human activity is further supplemented by including students in the investigation of which mathematicians and under what contexts particular mathematical developments occurred (Noddings, 1993). Most importantly, by linking mathematical developments to the history of the world and the cultures of the people involved, the history of mathematics has the power to remind students that “mathematics did not just happen; it was encouraged – or discouraged – by the zeitgeist – the nature of the prevailing culture – and the political, social, and economic conditions under which mathematicians had to work” (Lightner, 2001, p. 780).

A third reason to use of the history of mathematics in teaching is the potential for students to learn from “errors, alternative conceptions, change of perspective, [and]

revision of implicit assumptions” (Tzanakis & Arcavi, 2000, p. 219). Brown (2001)

observed that when we engage students in mathematical problem solving, we are asking much more of them than to arrive at a solution. Instead, we want students:

- to consider what might have been in other people’s minds when they were forming their mathematical ideas;
- to react to the intellectual and emotional dimensions of their own experiences in relation to the mathematical concepts they are studying;
- to appreciate that ideas take on a variety of forms, elegance, and interest;
- to use what many people would dismiss as errors for the purpose of expanding the kinds of questions and analyses they might consider; and
- to use their mathematical experiences as stepping-stones to consider important related issues in their own lives and in their culture. (p. 103)

If we invite students to consider problem solving along these dimensions, we empower them to “profit from an interpretation of errors as the motivation and means for exploration in mathematics” (Borasi, 1987, p. 2). As an example, when discussing the different ‘proofs’ of the Parallel Postulate (Euclid’s Fifth Postulate) students are able to consider alternatives to Euclidean plane geometry. Discussing the acceptance or rejection of the postulate places students in the powerful role of problem poser, where problem posing is identified as an essential component of humanistic mathematics education (Brown, 1996; 2001).

One cannot consider the benefit of examining errors and alternatives that arise from including a historical perspective in mathematics teaching without considering the problems behind the errors committed or alternative conceptions or perspectives created. Fried (2001) noted that, “historical problems serve to motivate, illustrate, or enlighten classroom topics” (p. 393), even if a mathematics curriculum is not structured according to historical development. Using authentic problems from the history of mathematics provides material for students to actively engage in classroom discourse. In addition,

when students examine the advance from the ancient to the modern, they “become aware that methods are changing and they can see that improvements in formats have made it easier to learn mathematics” (Gulikers & Blom, 2001, p. 227).

Significance

If the use of the history of mathematics is so widely considered by mathematicians, mathematics historians, and mathematics educators to be beneficial to secondary students, why is its use problematic and not widespread as of yet? As Gulikers and Blom (2001) outlined, obstacles to using the history of mathematics include teachers not possessing “enough historical expertise,” teachers’ lack of “access to the right materials,” and the “lack of time” available to teachers to pursue either expertise or materials (pp. 230 – 231). Some research exists that describes the affective benefits for using the history of mathematics with K – 12 students (Marshall, 2000; Furinghetti, 1997). Other research discusses the impact on university students’ experiences with the history of mathematics (McBride & Rollins, 1977), including how pre-service teachers’ mathematical knowledge is impacted (Fleener et al., 2002). Research that examines “how well...received the new way of teaching” (Lit, Siu, & Wong, 2001, p. 17) which incorporates the history of mathematics is difficult to find and is more typically investigated in countries other than the United States. This study is significant in that it will contribute to an ignored segment of research investigating the use of the history of mathematics with secondary students in the United States: how teachers are influenced by the study and use of the history of mathematics.

Research addressing how teachers learn about the history of mathematics and in turn decide how to incorporate what they study in their teaching is practically non-

existent within the United States. Identifying what aspects of history of mathematics teachers use or do not use in their teaching, and under what conditions (influenced by attitudes, beliefs, or school contextual features) will provide valuable information to guide continuing efforts for advocating the inclusion of historical topics and perspectives in K – 12 teaching.

Purpose of the Study

The purpose of this study is to describe secondary mathematics teachers' study of the historical development of logarithms, as well as the implementation of the history of logarithms in their teaching. The study included two components that contributed to the description of the teachers' experiences. The first component was composed of professional development sessions designed to introduce teachers to materials that can be used when teaching logarithms from an historical perspective. The second component focused on the teachers' instructional practices and use of history during a unit on logarithms. I chose to narrow the mathematical topic to logarithms so that analysis of the data would not be further complicated by the participants' dispositions of some mathematical topics over others. Also, I anticipated that in choosing the topic of logarithms, the use of historical activities would not be competing with other "favorite" activities that teacher would use for motivating the topic. By providing materials on the historical development of logarithms for discussion and teacher use during the professional development sessions, I heeded the suggestion of Bruckheimer and Arcavi (2000) and provided teachers with "more extensive knowledge" (p. 135) to aid in a more effective use of history (in this case, history of logarithms) in the classroom.

The study focused on the teachers' experiences in two important ways. First, investigating in what ways and at what level teachers chose to participate in the professional development sessions that focused on the historical development of logarithms provided valuable information on the teachers' conception of the use of the history of mathematics within their given contexts. The second facet of teachers' experiences that I examined was how and what the teachers ultimately decided to use from the history of logarithms with students. The intent of this study was to explore and describe how teachers study, plan for, and use the history of logarithms in a Precalculus-type course. Most importantly, I wanted to capture in the teachers' own words the benefits they perceived from as well as the affordances and obstacles to including the history of logarithms. The perceived benefits and articulated affordances obstacles by actual teachers are presented as case studies to generate conversation about how secondary teachers can both consider and incorporate the history of mathematics (logarithms) with students after engaging in their own study of a topic from an historical perspective.

Theoretical Perspective

There are two complementary theoretical perspectives, social constructivism and humanistic mathematics, which influenced the design of this study. Social constructivism is the predominant theoretical lens through which I viewed two major aspects of the study:

- the teachers' engagement during the professional development sessions, and their subsequent interaction with the materials during instruction and
- the revision (and in some instances, creation) of materials for both student and teacher use for the purpose of encouraging both teacher and student engagement with them.

Social Constructivism

Vygotsky's social constructivism recognized the importance of the social origins of constructs and that they are "learned through interaction with others" (Oxford, 1997, p. 43). The use of "interaction with others" was a critical consideration in the design and implementation of the professional development sessions. During the sessions, the dynamic process of learning (Forman, 2003) was modeled with teachers as they engaged in the study of a topic that represents a rich example of how mathematics exists as a product of human interaction.

The identification and revision of materials for incorporating the historical development of logarithms was also motivated by a social constructivist lens. The original collection of *Exponentials and Logarithms* module activities constructed by Anderson, Berg, Sebrell, and Smith (2004) contained rich and meaningful mathematical tasks for students to complete. The form of many of the exercises and prompts implied that the student would interact with the materials individually or in isolation from discussion with others. Consequently, I changed much of the formatting to include additional historical comments, as well as encouragement for students to consult a partner and prompts for introduction, pre-work, exploration, conclusion, and wrap-up portions of the lessons. As the lesson installments (modified for this study) progress, students are encouraged to question and discuss their results with each other and to reconnect with the historical importance of the invention of logarithms.

The lesson installments (Appendix B) used in this study were revised to incorporate aspects of a social constructivist perspective in the classroom outlined by

Bauersfeld (1995). The lesson installments encourage teachers to develop the social and mathematical culture of the classroom in the following ways:

- Self-organized problem solving and small-group work on “new” tasks;
- Continued engagement in the process of constructing, as well as reflection on completed tasks, consideration and discussion of alternative thinking;
- Assigned work related to the results of the lesson, such as written tests, homework, and “debugging procedures”; and
- Recognition of the teacher’s role of an expert and as one who can move the culture of mathematics in the classroom forward. (pp. 157 – 158)

Using the history of mathematics, or specifically, the historical development of logarithms, provides a context in which students are able to participate in what the NCTM calls “experiences related to the cultural, historical, and scientific evolution of mathematics” (1989, p. 6). As a result of engaging teachers during the professional development sessions using a social constructivist perspective, teachers can in turn create an environment for their students to participate in what Dewey (1938, as cited in Cole & Wertsch, 1996) called “a world of persons and things which is in large measure what it is because of what has been done and transmitted from previous human activities” (The Primacy of Cultural Mediation section, para. 1). In other words, in addition to the social and individual constructions highlighted by Vygotsky and Piaget that guide a child’s mathematical development, we must consider the third component Cole and Wertsch include in the process of construction of knowledge: “the accumulated products of prior generations, **culture**, the medium within which the two active parties to development interact” (emphasis in original, Introduction section, para. 5). Whitehead (1929) also emphasized “the relationships among ideas over traditional content focus” (as cited in Fleener et al., 2002, p. 74). Specifically, Whitehead’s observed that students should be guided in their intellectual growth by providing them with “living thoughts” and

discouraging them away from “inert ideas” (p. v). The same observation holds true when “students” is replaced with “teachers” (as mathematical learners in their own right) in the previous sentence.

Humanistic Mathematics

The second perspective, humanistic mathematics, is intimately related to a social constructivist perspective when considering the use of the history of mathematics when teaching logarithms. Placing the study of logarithms within a historical context provides a reason to study them beyond the usual practice of a definition and a few properties. Indeed, logarithms were developed out of a need of the scientific culture of the times (sixteenth and seventeenth centuries). Mathematicians and astronomers continually struggled with charting the heavens and trying to learn and formulate theories about what lies beyond this planet. Unlike many other mathematical inventions throughout history, Napier’s logarithms were embraced enthusiastically and were quickly accepted across Europe (Calinger, 1999). There was such a need for the discovery of logarithms that many observed as Laplace did, that logarithms “shorten[ed] the labors [and] doubled the life of the astronomer” (as cited in Eves, 1990, p. 312). When teachers allow students to engage in the calculation work of mathematicians from long ago to arrive at the values to which they have frequent access (i.e., logarithmic values stored in scientific and graphing calculators), they can “adapt their own constructs toward an effective orientation for their actions,” as Bauersfeld (1995, p. 156) implied.

Teachers may be able to incorporate affective ways in which students are able to experience the usefulness of studying logarithms. By doing so, they provide students with the opportunity to appreciate the human side of the mathematical developments that

continue to shape our global society. In this sense, the *humanization of mathematics* context has been overlooked too often in the teaching of school mathematics.

The use of the history of mathematics creates opportunities to discuss the origins of mathematical discoveries and advancements and that people, as opposed to textbooks, are the vehicles for such knowledge. A humanistic perspective suggests that mathematics teaching and learning at the secondary level is impacted by the following principles:

- Appreciating the role of intuition in creating and understanding concepts.
- Appreciating the human dimensions that motivate discovery.
- Understanding of the value judgments implied in the growth of any discipline.
- Using teaching and learning formats that help wean students from a view of knowledge as certain or something that is to-be-received.
- Providing opportunities for students to think like a mathematician, including a chance to work on tasks of low definition, to generate new problems, and to participate in controversy over mathematical issues. (Brown, 1996, p. 1301)

Instead of relying on broad descriptions of mathematical activities given in the *Principles and Standards for School Mathematics* (NCTM, 2000), we should employ a humanistic perspective to redefine mathematical activity so that students experience mathematics as a healthy and human process that does not always have a correct test answer as the goal. Pimm (1983) observed that, “the beauty of the study of the history of mathematics is that it can give a sense of place...from which to learn mathematics, rather than merely acquiring a set of disembodied concepts” (p. 14). Most importantly from a humanistic perspective, “history can convey the notion of a culturally-based and culturally-bound mathematics which is open-ended and changing, a challenge to a more prevalent view of mathematics as a static list of accumulated truths” (Pimm, p. 14). Indeed, philosophers of mathematics of the 20th and 21st centuries have challenged the paradigm of absolute truth that often plagues mathematics teaching and learning in grades

K through 12. In response to the *Principles and Standards for School Mathematics* document's call for students to "acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the classroom" (NCTM, 2000, p. 52), humanism enables a person to create and develop "personal knowledge of mathematics rather than... 'absolute' knowledge" (Pimm, 1983, p. 14). Thus, while embracing the history of either a problem or the mathematics within a problem, we create a space in the classroom to debunk dangerous myths with respect to problem solving. History enables students to see that axioms "were not handed down in a Mt. Sinai sort of way" (Brown, 1996, p. 1293) nor were important mathematical results "created in a way that involved no labor pains" (Brown, p. 1293).

Research Questions

I used several research questions to frame the study of teachers' use of the history of logarithms. The primary research questions were:

1. How do teachers with different background knowledge and experiences respond to professional development focused on understanding the history of mathematics?
2. How do background variables and professional development experiences with history of mathematics combine to influence teachers' personal mathematical knowledge and instructional practice?

I also formulated secondary research questions to investigate the overarching issues within each of the primary research questions, including:

1. How do teachers engage in professional development sessions about the history of logarithms?

2. How do teachers implement the materials and methodological and pedagogical ideas discussed during those sessions when teaching logarithms in a Precalculus-type course?
3. What do teachers identify as benefits when using the history of logarithms during their teaching of a unit on logarithms?
4. What obstacles and affordances do teachers identify when using the history of logarithms? How does the teacher deal with the obstacles and affordances?

Definitions, Assumptions, and Limitations

The following definitions, descriptions, assumptions, and limitations are included to clarify understanding with respect to this study.

Definitions

Logarithm. From Greek word, *logarithmos*. The term, *logarithm*, is composed of the two words meaning ratio and number, or “the number that counts the ratios.” In most traditional textbooks, *logarithm* is often defined in terms of functions: If $a^y = x$, then $\log_a x = y$, and y is the logarithm base a of x . In either case, the logarithm is a real number value, and exists when $a > 0$ and $a \neq 1$. Another definition of logarithm which does not rely as heavily on a functional relationship is:

The value of an exponent x that will make b^x , where $b > 0$, equal to a positive number. (Washington, 1969, p. 274)

Logarithmic function. Euler’s (1748) definition of a logarithmic function is the function which is the inverse of an exponential function: If $a^x = y$ is an exponential function with base $a > 0$ and not equal to 1, then $a^y = x$ is the inverse exponential or logarithmic function given as $\log_a x = y$.

Two particle argument. In 1619, Napier defined or described *logarithm* as:

The logarithm of a given sine is that number which has increased arithmetically with the same velocity throughout as that with which radius began to decrease geometrically, and in the same time as radius has decreased to the given sine. (Text from Calinger, 1995, p. 285)

In his two particle argument to support his definition of *logarithm*, Napier used a kinematic model to describe the movement of two particles. One particle moved along a line segment of a given fixed distance and the other moved along a parallel line (or ray) of indefinite length. An interpretation of the two particle argument, as presented to participants in Lesson Installment 2, is provided in Appendix B.

Prosthaphaeresis. The term *prosthaphaeresis* is a combination of the Greek words for addition, *prosth* and subtraction, *aphaeresis*. The term describes the process in which potentially complex products are calculated using only sums and differences. A common prosthaphaeretic (trigonometric) formula is: $\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$

Affordance. With regard to the fourth secondary research question (*What obstacles and affordances do teachers identify when using the history of logarithms?*), I have understood *affordance* to mean the characteristics or features of a teacher's particular situation – both personal and instructional – which assisted them in incorporating the history of mathematics in the classroom.

Assumptions

I made two primary assumptions in the construction of this study. First, I conjectured that using the history of mathematics with students is difficult if a teacher is not somewhat of an expert. With this in mind, I believed the results of the study would be more informative if the historical component were limited to a single topic. It was assumed that the descriptive case study would entail a richer and more informative

account if the focus on the history of one mathematical topic (logarithms) was utilized. Also, a goal of the study was to provide not only materials, but professional development and ongoing support to teachers for the purposes of studying the history of logarithms and incorporating the history of the topic with students. By restricting the topic, I was able to provide both focus and support for the participants.

Second, the study was conducted with the expectation that teachers would use the historical materials, resources, and knowledge in ways which they personally determined. The participants were not required to use any of the lesson installments and supporting resources. Instead, the participants were informed that the case study would focus on describing all of their experiences, including what they personally determined to use (or not use) during their instruction.

Limitations

The study was limited along three dimensions. Conditions resulting from both teacher circumstance and beliefs and school context contributed to the professional development sessions being considered less-than-ideal according to features identified in the literature as essential for effective professional development (Garet et al., 2001; Smith, 2001). An example of one such school contextual feature was that one of the school sites (Mulberry High School) had limited days available for teacher professional learning. Consequently, several professional development sessions needed to occur in one day as opposed to occurring over several days. These conditions impacted teacher engagement in professional development activities.

Next, the topic choice of logarithms may have constituted a limitation. Each participant volunteered primarily because they believed the topic of logarithms was

problematic for students with respect to placement of the topic in a Precalculus-type course. However, it is possible that the participants viewed logarithms as too difficult of a topic for which to consider an historical perspective.

Lastly, the fact that the topic of logarithms was pre-selected by the researcher may have inhibited teacher engagement during the study. Logarithms appear at the end of the Precalculus curriculum and teachers may have begun to experience an increased perception of lack of time (as an obstacle), given the number of topics left to cover and the amount of time left in the school year.

Chapter 2 Review of the Literature

The volume, *History in Mathematics Education: The ICMI Study* (Fauvel & van Maanen, 2000) begins with:

When the English mathematician Henry Briggs learned in 1616 of the invention of logarithms by John Napier, he determined to travel the four hundred miles north to Edinburgh to meet the discoverer and talk to him in person. (p. xi)

It is mildly amusing that this text, which I have used extensively since beginning my journey to investigate how teachers use the history of mathematics in their teaching, begins with the very topic that has captured my interest for so long. Although Fauvel and van Maanen claimed that, “the meeting of Briggs and Napier is one of the great tales in the history of mathematics” (p. xi), I propose that many teachers are not aware of this particular “great tale.”

In this review of pertinent literature, I first dedicate a major portion to presenting recent work concerning the use of the history of mathematics, particularly within the secondary school setting. Next, I include specifics regarding the history of logarithms (“the great tale”) and the historical argument that provides the backbone of the curricular materials used with teachers during the professional development sessions. Lastly, literature pertaining to other theoretical and practical issues that arise when working with teachers is also reviewed.

History of Mathematics in Teaching

In a special issue of *Mathematics in School* on the history of mathematics, Paul Ernst (1998) observed that, “an historical approach [in mathematics] can help to improve perceptions of mathematics and attitudes to it, by making it interesting, alive and part of human history and culture” (p. 25). The heart of my dissertation research was focused on

the work of secondary teachers and their engagement with and use of the history of mathematics. Unfortunately, there is a paucity of research that examines the experience of in-service teachers' use of the history of mathematics. Consequently, this section will focus on what many scholars have to say about the use of the history of mathematics in classrooms, often from the perspective of the presumed impact of its use on students.

During the past quarter-century, and especially during the last 10 years, the use of the history of mathematics in the teaching of mathematics has received a great deal of attention. There is a growing body of literature that point to this particular interest. For example, Bruckheimer and Arcavi (2000) claimed, "we make far too little use of the history of mathematics in our everyday teaching at all levels" (p. 135). In this introductory statement they made for a text by Popp in 1975, Bruckheimer and Arcavi proposed a method for incorporating history at all levels. They noted that before the history of mathematics can be used effectively, there must be a more extensive knowledge of elementary facts (on the part of teachers). Only then can "the next step of using and integrating history into the general process of teaching and learning mathematics" (p. 135) be taken.

In another document, *History in Mathematics Education: The ICMI Study*, Barbin (2000) indicated that in order to investigate the impact of using history in the mathematics classroom, two kinds of materials must be studied. First, we should collect experiences of teachers who use history, including their aims, steps, problems that arise in teaching, and the advantages and disadvantages they report. Second, we should collect questionnaires and conduct interviews of teachers and students about their study of mathematics (p. 90).

More recently, a topic study group from the International Congress on Mathematical Education (ICME – 10), which focused on the role of the history of mathematics, established the reasons for integrating the history of mathematics in order to enhance learning. Since the publishing of the International Commission on Mathematical Instruction (ICMI) report in 2000, the study group’s aim has been to provide a forum for participants to share their teaching ideas and classroom experience in connection with the history of mathematics. The orientation for Topic Study Group [TSG] 17 (the role of the history of mathematics in mathematics education) for ICME – 10 was given as:

History of mathematics, besides its intrinsic value, is just one of the many means which may help (some) students to learn better and/or some teachers to teach better. Likewise, mathematics is important but is not the sole subject worth studying. It is the harmony of mathematics with other intellectual and cultural pursuits that makes the subject meaningful and worthwhile. In this wider context history of mathematics has a yet more important role to play in providing a fuller education of the community. (TSG 17, 2004, Aims and Focus section)

An outcome of the Topic Study Group 17 sessions of ICME – 10 was the identification of two needs with respect to introducing a historical dimension in mathematics education. The *Report of the Topic Study Group 17* (Siu & Tzanakis, 2004) identified two points:

- (i) There is a need to construct and develop appropriate relevant ***didactical material*** which can either be used directly in the classroom or constitute a resource for mathematics teachers. The material should aim to motivate and guide the teacher to improve the teaching approach or understand better students’ difficulties or their idiosyncratic ways in learning mathematics.
- (ii) There is a need to enrich ***teachers’ education*** at all levels in this direction, both by introducing courses in...the history of mathematics and its relation to other disciplines, and by letting them become acquainted with historically inspired material that can be, or has been used in the classroom. (emphasis in original, Summary section)

Use of the History of Mathematics: Reasons and Strategies

The ICMI study also investigated how the position of a variety of societies concerned with mathematics education differed (Fasanelli, 2000). The position of the National Council of Teachers of Mathematics (NCTM), however, is usually the position that is quoted with regard to the teaching standards and curriculum recommendations. The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) stated that, "students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics" (p. 6). In addition, the document urged that the intent of the first goal, learning to value mathematics, is "to focus attention on the need for student awareness of the interaction between mathematics and the historical situations from which it has developed and the impact that interaction has on our culture and lives" (p. 6). Equivalent recommendations concerning the role of the history of mathematics are found in the NCTM's *Principles and Standards for School Mathematics* (2000), with the objective of students developing an appreciation of mathematics as "being one of the greatest cultural and intellectual achievements of humankind" (p. 4). Unfortunately, there has not been overwhelming evidence that this objective is being achieved (Lui, 2003, p. 418). However, the *Principles and Standards for School Mathematics* provided the recommendation that, "students develop a much richer understanding of mathematics and its applications when they can view the same phenomena from multiple perspectives" (p. 289). One such perspective is the historical perspective.

The inclusion of curricular materials that present students with historical contexts and problems helps teachers to create learning opportunities for students to engage in "the

development of the mathematical understanding" that "follows the historical developments of mathematical ideas" (Gulikers & Blom, 2001, p. 225). The work of Gulikers and Blom, although situated within a geometric context, provided an argument that seeks to answer the question: Why use history of mathematics in teaching and learning? Their argument included:

- Students derive comfort from realizing that they are not the only ones with problems so that they get less discouraged by misunderstandings and mistakes.
- History of mathematics helps students to learn in a non-linear way. The development of mathematical ideas proceeded not as smoothly as modern textbooks mostly suggest.
- History of mathematics helps students to acquire a balance between 'rigor' and 'imagination.'
- Since history of mathematics can help develop a multicultural approach in the classroom, it may help teachers work with multi-ethnic classes and can help develop tolerance and respect among fellow students.
- The use of history of mathematics provides opportunities for cross-curricular work between mathematics and other disciplines.
- History of mathematics helps to explain the role of mathematics in society, shows the development of mathematics as a human activity, helps to create a lively classroom atmosphere, and helps to increase students' interest for learning. (pp. 227 – 230)

Liu (2003) echoed many of the aspects outlined by Gulikers and Blom. In addition to revisiting Fauvel's (1991) "list of fifteen reasons for including the history of mathematics in the mathematics curriculum," Liu highlighted five reasons of his own:

- History can help increase motivation and helps develop a positive attitude toward learning.
- Past obstacles in the development of mathematics can help explain what today's students find difficult.
- Historical problems can help develop students' mathematical thinking.
- History reveals the humanistic facets of mathematical knowledge.
- History gives teachers a guide for teaching. (p. 416)

Other scholars (Borasi, 1987; Brown, 2001) have observed similar reasons for using the history of mathematics in teaching. Furinghetti (2004) examined several studies that appeared in academic journals in a four-year time period and noted two themes to “clarify the place that history may take in the classroom” (p. 1). First, history is used “for reflecting on the nature of mathematics as a socio-cultural process” and second, that history is used to “construct mathematical objects” (p. 1). Furinghetti discussed Tymoczko’s work with regard to attempting to define the concept of humanistic mathematics. To integrate a humanistic view of the socio-culture process found in pure mathematics, Furinghetti recalled Tymoczko’s answer to the question, “What made mathematics one of the humanities?” (p.3). Tymoczko answered:

Certainly not the mere fact that humans did it? Humans do science too... Pure mathematics is ultimately humanistic mathematics, one of the humanities because it is an intellectual discipline with a human perspective and a history that matters. (as cited in Furinghetti, 2004, pp. 3 – 4)

In her review of the literature, Furinghetti noticed that secondary teachers and university instructors alike have used anecdotes, stories, and vignettes in order to humanize mathematics. The justification offered by the educators for the use of such activities was the reliance on “the affective factors which intervene in [the] teaching/learning process” (p. 4).

Man-Keung Siu has also spent a great deal of his career examining the use of history of mathematics in mathematics teaching. Siu (1997) recognized that the use of history is not a panacea for improving mathematics test scores. He believes, however, that the history of mathematics “can make learning mathematics a meaningful and lively experience, so that (hopefully) learning will come easier and will go deeper” (p. 154). Siu

offered four strategies for using the history of mathematics in the undergraduate classroom. The “ABCD’S” as Siu called them included:

- A is for Anecdotes;
- B is for Broad Outline;
- C is for Content; and
- D is for Development of Mathematical Ideas. (p. 144)

With respect to the impact of the use of the history of mathematics on teachers, he stated that the “evolutionary aspect of mathematics can make a teacher more patient, less dogmatic, more humane, less pedantic” (p. 154).

In research conducted with colleagues to investigate the effectiveness of the use of the history of mathematics, Lit, Siu, & Wong (2001) noted that, “mathematics becomes a part of the learner’s assets if we can let the learner appreciate ‘mathematics-in-the-making’ and not just ‘mathematics-as-an-end-product’” (p. 18). With this interpretation of mathematics as a human construction, Lit et al. echoed Swetz’s (1994) observation that “by incorporating some history into teaching mathematics, teaching can lessen its stultifying mystique” (Lit et al., p. 18). Although the research was focused mainly on how effective the use of history was with students, a small portion of the study investigated the teacher’s experience with the history of mathematics in a three-week study of the Pythagorean Theorem. The teacher noted that the use of historical materials was successful and “his sole concern [was] the provision of good teaching material for teachers to use in the classroom” (Lit et al., p. 25). This teacher’s concern is reminiscent of the suggestions offered by Fauvel (1991) a decade earlier:

It is important to move beyond this stage, to take it as read, for the moment, that using history is a good thing...and to show *how* it might be incorporated into

some classroom activities, *how* it might make the teaching of various specific things easier, *how* the extra work which may be needed at first has a long-term payoff in improving the attainment of objectives within the mathematics syllabus, and so forth. (italics in original, p. 4)

Recent efforts have focused on the important work of “*how*” suggested by Fauvel. The *Historical Modules for the Teaching and Learning of Mathematics* (Katz & Michalowicz, 2004) were “designed to demonstrate the use of the history of mathematics in the teaching of mathematics” (Katz, 2005, p. 20). The materials within each of eleven modules “can fit many different types of objectives and can be used in variety of mathematics classes” (Katz, p. 20) and offer a significant contribution for access to rigorous and meaningful materials appropriate for secondary mathematics students.

The Historical Development of Logarithms

Most of the literature reviewed above discussed the benefits of using the history of mathematics in the secondary classroom in general. Several resources also describe the specific use of the history of mathematics when teaching logarithms. Interestingly, Fauvel (1995) provided the anecdote that when he was discussing with a friend on how to teach logarithms, the friend responded, “Whatever for? Surely no one needs to learn about those any more, now that we have calculators and computers” (p. 39). Many teachers, when approached about the possibility of looking at the teaching of logarithms using a historical context, may express the same opinion. Fauvel continued to offer a counter-argument to his friend by observing that, “logarithms are a good and accessible example of something fundamentally changing its conceptual role within mathematics” (p. 45). To clarify, examining the historical development of logarithms with students by exploring arithmetic and geometric progressions allows students “a more deeply rooted understanding of what is going on” (Fauvel, p. 42). When logarithms are first studied in

school mathematics, typically late in a second (or advanced) algebra course (grades 10 or 11), the use of the historical development of logarithms is an appropriate curricular enhancement to serve such a purpose. Subsequently, historical problems and contexts can be used to accentuate and revisit the use of logarithms in trigonometry (other uses for working with astronomical numbers) and in calculus (areas under hyperbolas).

A form of enlightenment results when teachers resist the temptation to simply present a definition and several properties, along with exercises to practice them.

Primarily, this enlightenment results from “showing pupils how concepts have developed” (Fauvel, 1991, p. 4). Using the history of mathematics in general and the history of logarithms in particular also brings to light many of the benefits Fauvel (1991) highlighted, including:

- Students can experience the human side of mathematics;
- Comparing historical with modern establishes value of modern techniques;
- Provides opportunities for investigations; and
- Past obstacles to development may assist students with what they find difficult. (p. 4)

Katz (1995; 1997) provided a succinct argument for examining the development of logarithms from an historical perspective. To provide an appropriate context for secondary teachers to include the historical development of logarithms as an adaptation of Napier’s original argument, Katz observed that Napier developed logarithms “for use in the extensive plane and spherical trigonometrical calculations necessary for astronomy” (Katz, 1995, p. 49). Although the motivation for the invention of logarithms is significant, Katz noted that in general, students today often know very little about astronomy and the magnitude of both the numbers and the calculations involving them that was necessary to advance the science of astronomy. Astronomical advances have

remained critical throughout civilization, however, and Katz (1997) indicated that, “it is well for us to introduce it whenever possible” (p. 63). In addition, the inclusion of the invention of logarithms and the development of tables for easing astronomical calculations would complement the use of a humanistic perspective when teaching mathematics. When teachers use historical problems with students to exemplify the human effort and struggle of mathematicians “students are pedagogically enlightened when they realize that such problems are not created in a vacuum and...that mathematicians make mistakes too” (Liu, 2003, p. 419). Reducing contributions such as John Napier’s invention of the logarithm to the presentation of a mere definition and a few properties simply perpetuates the view that the mathematical study students participate in is pulled from thin air and is not the result of cultural and intellectual achievements.

The Professional Development of Teachers

The professional development sessions conducted for this study were designed to serve as a vehicle to translate each teacher’s study of the history of logarithms into their instructional practice. Although the professional development component of this study was not a large-scale or long-term effort, I examined literature addressing the following aspects of professional development:

- Characteristics of effective professional development;
- Opportunities for professional development and professional learning; and
- Contextual features affecting the professional learning of teachers.

Characteristics of Effective Professional Development

Shaha, Lewis, O’Donnell, and Brown (2004) stated that, “the purpose of professional development is to help teachers become better teachers” (p. 1). Although I

did not study the effectiveness of the professional development component of this study with respect to increased student learning, it was important to identify the various features of effective professional development with Shaha et al.'s purpose in mind.

Hawley and Valli (1999) identified similar characteristics associated with professional development activities and noted that, "professional development is more likely to result in substantive and lasting changes in the knowledge, skills, and behaviors of educators that strengthen student learning when it includes these characteristics" (p. 137). Professional development should:

- Include a content focus on what students are to learn and how to address different problems students may have in learning the material.
- Include analyses of the differences in actual student performance, as well as goals and standards for student learning.
- Involve teachers in identifying what they need to learn and developing the learning experiences in which they will be involved.
- Be school-based and built into the day-to-day work of teaching.
- Be organized around collaborative problem solving.
- Be continuous and ongoing and include follow-up and support for further learning.
- Incorporate evaluation of multiple sources of information.
- Provide opportunities to understand theory underlying the knowledge and skills being learned.
- Be connected to a comprehensive change process focused on improving student learning.

Still other scholars have discussed principles for designing effective professional development. For example, Wilson and Berne (1999) cite numerous examples of sets of characteristics from the literature that are necessary for meaningful professional development. Many of the characteristics identified are similar to those already mentioned. As Wilson and Berne observed, however, there is "consistency across such lists [of characteristics]" (p. 176). They also noted that Putnam and Borko (1997, as cited

in Wilson & Berne) encapsulated the essentials of professional development into the following “mantras” or “truisms”:

1. Teachers should be treated as active learners who construct their own understanding.
2. Teachers should be empowered and treated as professionals.
3. Teacher education must be situated in classroom practice.
4. Teacher educators should treat teachers as they expect teachers to treat students. (p. 176)

Others, however, have brought attention to the difficulties associated with designing and providing high-quality professional development.

For example, Fullan (1993), recognized that whereas schools are called upon to engage in reforming their practices to meet the needs of students, we also have the problem that,

the way teachers are trained, the way schools are organized, the way the educational hierarchy operates, and the way political decision makers treat educators results in a system that is more likely to obtain the status quo. (p. 12)

If we are to have any hope in addressing this dilemma, we must begin to pay attention to the claim made by Smyth (1995). Smyth stated that, “if we wish to both understand and influence the way in which teachers develop professionally, then we need to be prepared to canvass possibilities that might lie outside our current range of vision” (p. 69). He further urged that the stakeholders involved in the professional development of teachers must move beyond what Hargreaves (1982) called the “culture of individualism” (p. 166).

Richardson (2003) claimed that the cultural norm of American individualism appears to work against the use of research-based professional development which seeks to bring teachers together around a problem and that requires their joint effort. Many teachers express their individual work as:

This is my space, and I am responsible for it. It is mine. It reflects me. I am the teacher here. This classroom is unique and is therefore unlike any other classroom because of my uniqueness and my particular group of students. (p. 402)

Consequently, this individualist view underlies many teachers' doubts about the feasibility of implementing what professional development programs espouse.

Richardson maintained, however, that research-based professional development can call for mutual participation and collaboration, while allowing participants to “maintain their sense of autonomy, expertise, and individual efficacy” (2003, p. 402). “The adage, “that won’t work in my classroom” is a derivative of the view that each teacher’s classroom is unique and thus application of reform practices will not work there because the unique features of a particular teacher’s classroom are supposedly not known to others. Along these lines, it is important to “understand this insular way of life and its consequences” (Richardson, p. 402) in order to work effectively with teachers in a change process.

Richardson (2003) also attempted to quantify how teachers, who may be considered resistant to change in the context of professional development, approach change. Teachers change their practice frequently and a common scenario includes experimenting with new activities. Richardson observed,

When these new activities engage the students, do not violate the teacher’s particular need for control, match the teacher’s beliefs about teaching and learning, and help the teachers respond to system-determined demands for such outcomes as high test scores, they are deemed to work. (p. 403)

Thus, when a teacher experiences the trajectory afforded by a successful “experiment,” the change becomes part of the teacher’s new practice. Richardson claimed then, that the “first step as professional developers is to try to operate within this naturalistic sense of

teacher change” (p. 403). In order to do this, Richardson proposed that the approach professional developers should employ is the inquiry approach, which entails:

- (1) Determine ways in which teachers make their decision to change, provide input, and help when they do so.
- (2) Help teachers see the usefulness of a collective approach to some change-related decisions and actions. (p. 403)

Garet, Porter, Desimone, Birman, and Yoon (2001) identified three structural features and three core features of effective professional development practices by examining the literature and analyzing survey data collected from the mathematics and science professional development experiences of teachers. The three structural features identified were: form (reform-minded activity or a traditional workshop/conference), duration (number of hours and over what time period), and participation (i.e., groups of teachers from the same school, department, or grade level). The three core features they identified were: content focus (activity focusing on an in-depth study of one topic), active learning (teachers actively involved in a meaningful analysis of teaching and learning), and coherence (continuation of professional development efforts, linked to teacher goals and state standards or assessments). Garet et al. noted that,

[s]ome studies conducted over the past decade suggest that professional development experiences that share all or most of these characteristics can have a substantial, positive influence on teachers’ classroom practice and student achievement. (p. 917)

The implications Garet et al. (2001) derived from analyzing the results of their national sample both support and extend the essential features of effective professional development previously identified. Although Garet and his colleagues found that the type of professional development activity (i.e., conference, workshop) had a less profound impact on teacher outcomes, the results did indicate that, “sustained and intensive

professional development is more likely to have an impact, as reported by teachers, than is shorter professional development” (p. 935). Further, professional development incorporating a focus on academic subject matter (core feature of content), providing teachers with “hands-on” work (core feature of active learning), and which is situated in the daily work of teachers (core feature of coherence), was “more likely to produce enhanced knowledge and skills” (p. 935).

With regard to the core features of active learning and coherence identified by Garet et al. (2001), Smith (2001), who focused on the professional development of mathematics teachers, also indicated that in order to “transform teachers’ knowledge, beliefs, and habits of practice...the professional development of teachers should be situated in practice” (p. 7). The central feature of practice-based professional development for mathematics teachers is that the approach is “centered in the critical activities of the profession” (Smith, p. 8). The approach is further characterized by using the “cycle of teachers’ work” (p. 8) to design professional learning activities. The cycle includes planning for instruction, enacting the plan developed, and reflecting on the impact of teaching and learning on students. To assist this approach focused on active learning and situated within the daily work of teachers, Smith discussed a variety of tools which can be incorporated into teachers’ professional learning. Smith outlined three forms of practice which should be considered in a practice-based professional development approach, including:

- The exploration of mathematical tasks;
- Opportunities to analyze and critique episodes of teaching; and
- Examining students’ work. (pp. 10 – 13)

Smith (2001) also echoed the core feature of content focus when considering the design of meaningful professional development experiences for teachers. Specifically, Smith urged that,

Decisions [of what teachers learn] should be based on an assessment of what teachers need or want to learn and what knowledge, skills, and experiences they bring to the enterprise. With this information in hand, professional developers can select appropriate materials and create professional learning tasks that build on teachers' prior knowledge and have the potential to foster the intended learning. (p. 40)

Others (Fullan, 1993; Loucks-Horsley, Hewson, Love, & Stiles, 1998) also emphasized the importance of expanding the professional and content knowledge of teachers via professional development efforts. Fullan discussed the importance of teachers' mastery as a critical element in the change process in education. He noted that simply acquiring skills and knowledge is not enough and that, "beyond exposure to new ideas, we have to know where they fit, and we have to become skilled in them, not just like them" (p. 14).

Lappan's (1997) work is particularly informative with respect to viewing professional learning as a means "to help teachers build a sense of success and satisfaction from more demanding mathematics goals and the complex teaching such goals require" (p. 211). She continued by suggesting that "new kinds of working relationships and conditions in which teachers can function and grow as professionals are needed" (p. 211). An examination of one such condition is teacher decision making. Lappan, using the evolving vision of NCTM's *Professional Standards for Teaching Mathematics*, discussed several factors of mathematics teaching resulting from the decisions teachers make. Among these are:

- The selection, adaptation, or creation of worthwhile mathematical tasks;
- The use of discourse along several dimensions to aid in the building of a community of learners;

- The on-going examination of the environment, including the emotional climate of the classroom; and
- Commitment to engaging in reflection and self-analysis to search for evidence of how the teacher's decisions are supporting or failing to support student learning. (pp. 213 – 217)

In addition to the factors influencing teacher decision-making, Lappan (1997) also identified several demands that are endemic to the work of reform-oriented classroom teachers. Demands such as teaching to reach all students, challenging teacher-as-mathematical-authority paradigm, monitoring the work of students in groups, and teaching mathematics through bigger problems are examples of “huge hurdle[s] for teachers and for those who work to support teachers” (p. 218).

Opportunities for Professional Development

Judith Warren Little (1990) observed that there are essentially two main opportunities for secondary teachers to participate in professional development opportunities: formal and informal (and which occur during the salaried workday). Within the genre of formal professional development activities, teachers can participate in university coursework, district-provided or sponsored activities, or activities sought out by the secondary teacher in the larger professional community (such as teachers' involvement with subject-area professional associations). Little noted a decline in teachers choosing university coursework as a professional development alternative. A possible reason for the decline was that, “the extrinsic incentives for university study reside nearly exclusively in the salary schedule” (Little, p. 206) and as teachers approach the top of the salary ladder, “major incentive to participate in coursework is lost” (p. 206).

District-provided or sponsored professional development activities are often problematic and ineffective because “although teachers attend staff-development events together, they are only incidentally in one another’s presence; rarely is collective participation in such an event part of a collective engagement in professional growth (Little, 1990, p. 207). Consequently, professional development activities that are sponsored or provided by the district often fall into the “one-size-fits-all” trap and tend to lack many of the structural and core features identified by Garet et al. (2001) and fall seriously short of the recommendations provided in Hawley and Valli’s (1999) work. Scribner (1999) also identified the necessity for “school leaders to initiate changes that place professional development at the core of teacher work to ingrain the value of continuous professional learning throughout teachers’ careers” (p. 261).

Examining informal opportunities for professional growth are most often considered within the context of the salaried workday. Little (1990) discussed such informal obligations and opportunities “by examining the nature and extent of teachers’ workload, by examining the distribution of out-of-classroom time and responsibilities, and by examining how teachers are organized (or not) to benefit from one another’s expertise” (p. 210). Still, situating professional learning within teachers’ workplaces is difficult because of the obstacles present in many school contexts.

Contextual Features Affecting the Professional Learning of Teachers

In addition to considering the characteristics of effective professional development and the types of learning opportunities available, it is also necessary to consider the contextual features that promote or hinder such opportunities. Scribner (1999) observed that, “existing research does little to clarify why professionals engage in

learning activities” (p. 246). In an attempt to understand why teachers engage in professional learning, Scribner identified several context factors that influence teacher learning, including those related to school-level features and district-level features.

Among school-level features, the teachers in the study conducted by Scribner (1999) reported that the leadership styles impacted how professional learning opportunities manifested in their schools. In the schools studied, “the approaches and philosophies toward professional development reflected the inherent tension between organizational and individual learning goals” (Scribner, p. 253). In addition to each school’s leadership playing a central role in determining the balance between “organizational imperatives and faculty learning needs” (Scribner, p. 253), school administrators also influenced the allocation of resources that were applied to professional learning activities within schools. The resources most often cited by teachers were appropriate workspace, time, and appropriate funds (either to bring in outside professional developers or to attend workshops or other opportunities).

Scribner (1999) also reported that, “strong faculty norms shaped teacher attitudes and expectations for professional development” (p. 255). Many of the faculty norms were shaped as a result of the “hectic pace of high school teaching and such stressors as maintaining safe environments for students and staff” (p. 256). Scribner found that the factors present in the daily life of the teachers he studied created a structure within schools that impacted teacher learning. In similar fashion to the culture of individualism which Richardson (2003) discussed, Scribner noted that,

Isolation from peers also has an insidious effect on teacher learning by creating invisible walls between teachers and diminishing the valuable role that activities such as collaboration can have in their practice. (pp. 255 – 256)

In addition to school-level factors, the teachers Scribner (1999) studied reported that district policy reforms and district professional development priorities influenced their work context (p. 256). For example, the implementation of accountability testing and other district-wide initiatives created associated learning opportunities for teachers so that they would become better equipped to participate in district activities. Scribner observed, however, that:

Despite its reform agenda, the prevailing attitude among teachers across schools remains that the district's impact on their professional development was minimal because district-sponsored activities did not address critical issues. (p. 257)

Loucks-Horsley, Hewson, Love, and Stiles (1998) also highlighted several contextual factors which influence professional development. They discussed nine factors and corresponding critical issues that should be considered when designing professional development for mathematics and science teachers. All of the factors which were emphasized in Scribner's work were also identified as critical factors by Loucks-Horsley et al., including organizational culture and structures, leadership factors, availability of resources, and practices related to instruction, assessment, and the learning environment (p. 174). Loucks-Horsley and her colleague also observed that students must be considered in the design of professional development, since they ultimate benefit from teachers' participation. Likewise, teachers represent an important contextual factor in the design of professional development. Loucks-Horsley et al. observed, "no factor is more important to consider than the teachers themselves" (p. 176).

Borko and Putnam (1996) observed that, "contextual factors may work against learning to teach in new ways....the working conditions in most public schools are often not conducive to promoting teacher reflection and learning" (p. 700). Yet, as discussed

previously (Fullan, 1993; Garet et al., 2001; Hawley & Valli, 1999; Smith, 2001), such reflection is vital for effective professional learning. Borko and Putnam also emphasized the urgency for examining the professional learning of teachers to include topics beyond complying with district-imposed mandates and meeting certification requirements. They observed that, “if we truly expect teachers to learn to teach in new ways...we must begin to view schools as places for teachers, as well as their students, to learn” (p. 702).

The contextual features at the levels of district, school, department and classroom contribute to not only the view of what professional learning is, but how or even whether it occurs. One such contextual feature is that of time. Little (1990) observed that, “even under the best circumstances, opportunities for professional development must compete for time and the press of daily work” (p. 210). Teachers’ views of the importance of continued professional development are shaped by the school or district’s vision or plan for the professional learning of its teachers. The expectations that schools and districts hold for the professional development of teachers vary widely. Examples of teacher complaints with respect to lack of vision speak to the dilapidated professional culture within schools:

- There has been no program for professional development during the years that I have been here. The administration does not understand the process of development or what is involved in personal, professional, or institutional change.
- In-service programs have been a waste of time.
- Our school...does nothing really for development ‘in house’ except for an occasional ‘pep talk’ by an outside speaker. (Beall, 1999, p. 72)

Each of the preceding observations focuses further attention to the importance of the impact of teacher learning and creating contexts for such learning to happen. Borko and Putnam claimed that, “if teachers are to be successful in creating classroom learning

environments in which subject matter and learners are treated in new ways, they need to experience such learning environments themselves” (p. 703).

Many of the recommendations suggested by Driscoll and Lord (1990) are rooted in the professional learning of teachers. Driscoll and Lord addressed, specifically for mathematics educators, the issues of changing roles and responsibilities in the classroom, in the profession, and in the broader community. In order for teachers to actively engage learners, for example, “teachers must alter the ways in which they perceive the field of mathematics and the processes of teaching and learning” (p. 238). This implies that teachers must often be placed in the role of learner so that their professional practice is able to change along with new observations of effective practice to promote meaningful student learning. Correspondingly, the professional disposition of a teacher provides a context for their conception of professional development.

A teacher’s ideological view of teaching is another significant contextual feature which influences their view of professional learning. Speer (2005) noted that,

[b]eliefs appear to be, in essence, factors shaping teachers’ decisions about what knowledge is relevant, what teaching routines are appropriate, what goals should be accomplished, and what the important features are of the social context of the classroom. (p. 365)

Grundy (1987) outlined three orientations or “interests” in education, the technical, practical, and emancipatory, which also shape teachers’ professional learning and decisions for instruction. Grundy described the technical cognitive interest with respect to curriculum claimed,

The objectives model of curriculum design is informed by a technical cognitive interest. This means that implicit within objectives models of curriculum, such as Tyler’s (1949), is an interest in controlling pupil learning so that, at the end of the teaching process, the product will conform to the *eidōs* (that is, the intentions or ideas) expressed by the original objectives. (emphasis in original, p. 12)

The practical interest, Grundy (1987) argued, aids in generating subjective knowledge. Instead of curriculum being driven and maintained by objectives, the practical interest is fundamentally concerned with “understanding the environment through interaction based upon a consensual interpretation of meaning” (p. 14). Reminiscent of Vygotsky’s work, curriculum design (and consequently, the instructional process through which it is delivered) “is regarded as a process through which pupil and teacher interact in order to make meaning of the world” (Grundy, p. 15). Grundy further characterized the practical interest as one which depends more on teacher judgment, as opposed to teacher direction (p. 15).

The final orientation described by Grundy (1987), the emancipatory cognitive interest, is defined as, “fundamental interest in emancipation and empowerment to engage in autonomous action arising out of authentic, critical insights into the social construction of human society” (p. 19). This interest, fundamentally informed by the work of Habermas and Freire, seeks to address several levels of freedom. For example, when an emancipatory curriculum appears in practice, it “involve[s] the participants in the educational encounter” and it “entails a reciprocal relationship between self-reflection and action (p. 19).

Although each of the cognitive orientations Grundy discussed consider curriculum a social construction, the technical interest is still pervasive in K – 12 education. In their work with pre-service teachers, Fleener et al. (2002) identified “impact of context of choice” (p. 81) as a context feature that has significant influence on teachers’ “abilities to approach their own thinking about mathematics teaching and learning from hermeneutical or critical perspectives” (p. 81). Even with the existence of

recommendations from NCTM (1989; 2000) and the National Research Council (1990) that encourage either implicitly or explicitly to approach the teaching of mathematics from either hermeneutical or emancipatory perspectives, the resiliency of the technical orientation in mathematics teaching remains – and is quite possibly exacerbated – “in the era of *No Child Left Behind*” (Fleener et al., 2002, p. 81).

Chapter 3 Methodology

This study describes five teachers' engagement in learning about the history of logarithms and their use of that history in a Precalculus-type high school course. Description, as defined by Strauss and Corbin (1998), "draws on ordinary vocabulary to convey ideas about things, people, and places" (p. 16). Basic description, however, also "involves purpose and audiences and the selective eye of the viewer" (p. 17). In order to address the research questions of this study for the purpose of understanding and broadening the use of the history of mathematics in secondary mathematics classrooms, I examined the intended and actual classroom practices of five high school teachers during and after their participation in a professional development program (described in Chapter 4) that focused on the historical development of logarithms.

Guiding Principles of the Study

The six conceptual responsibilities of qualitative case researchers guided my commitment to this research. Stake (2000) outlined six responsibilities as follows:

1. Bounding the case, conceptualizing the object of the study;
2. Selecting phenomena, themes, or issues...to emphasize;
3. Seeking patterns of data to develop the issues;
4. Triangulating key observations and bases for interpretation;
5. Selecting alternative interpretations to pursue; and
6. Developing assertions or generalizations about the case. (p. 448)

Stake also noted that, "the more the researcher has intrinsic interest in the case, the more the focus of the study will be on the case's uniqueness, particular context, issues and story" (p. 448). I intended to study each teacher's particular engagement with the history of logarithms, both during and after professional development sessions focused on the topic, as well as their instructional practice related to the topic of logarithms.

Consequently, it is the “story” of uniqueness, context, and issues related to the teachers’ experiences that is the focus of this study.

The case studies compiled for this study document five secondary mathematics teachers’ experiences with the history of logarithms. Stake (2000) described case study in depth. For example, he stated that, “case study is not a methodological choice but a choice of what is to be studied” (p. 435). He pointed out that although we may study a case using any variety of methods, what we truly concentrate on, “at least for the time being, [is]...the case” (p. 435). Stake’s definition of instrumental case study is of particular interest. He defined a case study as instrumental if “a particular case is examined mainly to provide insight into an issue or to redraw a generalization” (p. 437). Here, the essential feature of providing “insight into an issue” is of most concern. The participants’ experiences with the history of logarithms were investigated in depth to characterize the benefits and obstacles that teachers encounter when using the history of mathematics with students.

The general characteristics of qualitative design (Janesick, 2000, pp. 385 – 386) were also important for the methodological choices considered when constructing the study. Table 1 lists several features of qualitative design, along with a description of their application to this study.

Table 1
Qualitative Design Characteristics

| Characteristic | Application to the study |
|-----------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|
| Construction is holistic; not constructed to prove something or to control people | Each component of the study aided in constructing a description of teachers’ experiences with the history of logarithms. |
| Looks at relationships within systems or cultures | Teacher and school characteristics were considered in the interpretation of data, as well as for reflections, conclusions, and |

| Characteristic | Application to the study |
|----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | recommendations. |
| Concerned with the personal, face-to-face, and immediate | Participant observation retained my stance as one of “along with” the teacher participants. |
| Focused on understanding, not necessarily making predictions about social settings | With very little research of this same type, understanding for the purpose of making future recommendations or suggesting other research activities was essential. |
| Demands that researcher stay in the setting over time | Two research sites and two different course syllabi created the need to interact with study participants over time. |
| Demands time in analysis equal to the time in the field | The time in analysis actually exceeded time in the field. |
| Model development sometimes necessary | An outcome of the study resulted in describing the characteristics of the schools and teachers who were most amenable for incorporating the history of logarithms. |
| Requires the researcher to become the research instrument | As a participant in each professional development and instructional session, I was able to sharpen my observation skills throughout the study. My participation during each phase of the research also enabled me to develop relationships with each of the participants that ultimately assisted in interviews and group discussions. |
| Incorporates informed consent and is responsive to ethical concerns | The goals and activities of the research study and the consent form were explained to each of the participants. Other concerns such as anonymity and confidentiality were discussed. |
| The design incorporates room for description of the researcher’s role, own biases, and ideological preference | I revealed my own experiences with using the history of mathematics with students and teachers and how they biased my stance in the study. |
| Requires the construction of an authentic and compelling narrative of what occurred in the study and the various stories of the participants | A significant goal of the research was to construct an “authentic and compelling narrative” so that the study may serve as a guide for other research and activities that allow for the use of history of mathematics. |
| Requires ongoing analysis of the data | This was presented as a significant characteristic of the study. |

Sampling

It was necessary to use “purposeful sampling” for this study. Bogdan and Biklen (2003) described the method of purposeful sampling as choosing “particular subjects to include because they are believed to facilitate the expansion of the developing theory” (p. 65). Stake (2000) observed that, “instrumental and collective casework regularly requires researchers to choose their cases” (p. 446). In this sense, it was “required” that I choose participants who would be teaching logarithms during the 2004 – 2005 school year. In order to describe teachers’ experiences with studying and using the history of logarithms with students, it was imperative to identify teachers willing to participate on a variety of levels. In particular, I needed teachers to volunteer their time for the professional development component of the study. This component included the expectation that they would continue to research, study, and engage with the materials on their own time between and beyond the professional development sessions. Additionally, the teachers needed to at the very least consider the use of the historical materials and knowledge possible when teaching logarithms.

Beyond the actual work associated with the lesson installments on the historical development of logarithms, the teacher participants also gave of their time in additional smaller increments for completing surveys, interviews, and follow up contact. In addition to explicit commitments to participating in the research, there were implicit considerations in the purposeful sampling. The most important implicit consideration was that each teacher would teach a unit on logarithms during the study duration. Mary Long is an exception to this consideration as she was not currently teaching a Precalculus-type course. Mary volunteered to participate for reasons different from the other four

participants. The study of her experience revealed meaningful information and consequently, her unique case remains included in the collection.

When I initially tried to acquire access to teachers willing to participate in the study, I was most concerned with being able to identify public school teachers. The motivation for this was due in part to a desire to research, interpret, and suggest recommendations for the use of the history of mathematics in a more “typical” classroom environment. When Mandy Wilson (High Acres School) volunteered, however, I decided examining teachers’ experiences with the history of logarithms in classrooms which are structurally and philosophically different from each other would allow for additional and potentially powerful interpretations of the data.

Setting Descriptions

Two schools served as research sites for the study.

Mulberry High School

Mulberry High School (all names used are pseudonyms), is one of three high schools in Peterson County Public Schools. Peterson County is a rural school district in the southeastern United States. The selection of this school was motivated first by the inability to identify teachers in my local area willing to participate in professional development focused on content that did not appear explicitly on county-provided unit assessments (N. Gaines, personal communication, September 10, 2003). Secondly, Mulberry’s mathematics department chair, a personal friend and professional colleague, volunteered the Trigonometry and Advanced Algebra teachers at her school to participate in the study. (The course, Trigonometry and Advanced Algebra, will be referred to as Trigonometry for the remainder of this text.) In Sue’s words, she was confident that she

was not the only teacher in her department who struggled to present logarithms in such a way to make the topic more appealing to students. Additionally, Sue claimed that she and her peers struggled to present the topic of logarithms in a more meaningful way for students within a traditional Precalculus-type course (S. Moe, personal communication, August 29, 2004).

After this initial conversation with Sue, I agreed to send her a written description about what the study would entail. She agreed to share the information with the principal and interested teachers in her department. Ultimately, Sue obtained permission from the principal, which allowed me to work with interested Mulberry mathematics teachers. Additionally, Sue notified me that the Trigonometry and Advanced Algebra teachers (Sue, Ted, and Shirley) met to discuss the written description of the study and that they all agreed to participate.

According to the information provided by the state, Peterson County is a rural district and the 11th fastest growing school district in the United States. The projected growth of the district calls for an additional 1200 students each year. The district, which serves approximately 21,000 students, is described as 80% White, 15% Black, with the remaining 5% a combination of Hispanic, Asian American, and other ethnicities. The average total Scholastic Aptitude Test (SAT) score for the district in 2003 – 2004 was 964, which was 53 points below the national average and 17 points below the state average. However, 96% of all eleventh grade students passed the state high school mathematics graduation exam. (Notably, 94% of Black students and 97% of White students passed.) Graduation rates for the district include an overall rate of 69.5%, with 69.7% of Black students and 69.5% of White students graduating in 2004. The

percentage of Peterson County students eligible for the Free and Reduced Meals (FARMS) program is 24%. Twelve percent of students received services for disabilities and only one percent were limited English proficient.

Demographics are similar for Mulberry High School. For the academic year 2003 – 2004 Mulberry's student population was 75% White, 21% Black, and 4% from other ethnic groups. The school reported a student enrollment of 1,998 on October 5, 2004. The average total SAT score in 2003 – 2004 was 941 and 97% of all eleventh grade students passed the state high school mathematics graduation exam. (Notably, 97% of Black students and 98% of White students passed.) The graduation rate for the school was 69.4%, with 77.6% of Black students and 66.8% of White students graduating in 2004. Nineteen percent of students at Mulberry High School are eligible for the FARMS program, 11% of students received services for disabilities and 1% were limited English proficient.

High Acres School

High Acres School is a private (day) school near a large suburban area in the mid-Atlantic United States. In October 2004, the school's Precalculus teacher attended the presentation of my research interest at the Eastern Regional Conference of the National Council of Teachers of Mathematics. The teacher, Mandy Wilson, attended the session specifically because she was interested in a humanities approach to presenting mathematical topics in her Precalculus classes. Mandy shared that her school's philosophy provided the freedom and flexibility to explore alternative curricular approaches, even with respect to the mathematics curriculum. The school's website claimed that the mission of High Acres School is to "provide a rigorous, liberal arts

education” (retrieved February 13, 2005). In addition, the school believed “that a challenging, content-based curriculum trains the intellect while fostering self-discipline, independence, creativity, and curiosity.” Since the description of my study appeared to fit with both her plan for the Precalculus students and the philosophy of High Acres, Mandy approached me at the end of the session and inquired whether she could participate in the study. I responded with an enthusiastic “yes”!

At the time of the study, demographic information for High Acres School was only sporadically available. The 2000 – 2001 Narrative Report issued by the school’s advisory board reported an Upper School (grades 9 – 12) enrollment of 144 students. Minorities comprised 7% of the student population and 18% of students received financial aid to attend High Acres. The average SAT score of High Acres juniors and seniors in 2002 was 1189, with an average verbal score of 614 and an average mathematics score of 575.

A key difference between the two settings is that High Acres was not burdened by state- or federally-mandated assessments. Significant differences also existed between the two school sites with respect to parent involvement on curricular issues.

Participants

Each individual participant (‘unit of analysis’ sounds so sterile) in the case study is defined as a Trigonometry (Mulberry) or Precalculus (High Acres) teacher within the two different research sites. Three participants (Shirley Corson, Ted Jones, and Sue Moe) taught trigonometry at Mulberry High School. Shirley and Ted each taught two trigonometry classes and Sue taught one such class. Each class met for approximately 55 minutes each day, Monday through Friday. In this course, students covered traditional

trigonometry course material during the first semester of the school year (August through December). Second semester topics included higher-order equations, conic sections, exponential and logarithmic functions, and sequences and series.

In addition to the three trigonometry teachers, one Mulberry High School participant volunteered when she heard about the study from the department chair, Sue Moe. Via Sue, Mary Long requested to participate because she was interested in learning more about the history of mathematics in general and the history of logarithms in particular. Mary taught Mulberry's Advanced Placement Calculus class and was interested in the study because she covered logarithmic differentiation, which required her students to know and understand the three properties for simplifying logarithms (S. Moe, personal communication, October 1, 2004). It is important to note that Mary Long has not taught a unit on logarithms (as might be found in a Precalculus-type course) in many years. Mulberry High School had only one Advanced Placement Calculus class, which included a student enrollment of ten students during the 2004 – 2005 academic year.

The final participant, Mandy Wilson, taught three Precalculus classes at High Acres School and was the only person who taught the course at her school. The class schedule included all classes meeting for 45 minutes on Mondays and three classes, each meeting in 80-minute blocks, on Tuesday through Friday. Consequently, Mandy's Period 1 class met Mondays, Tuesdays, and Thursdays and Periods 2 and 4 met Mondays, Wednesdays, and Fridays. The Precalculus course at High Acres included a variety of mathematical topics necessary for success in studying calculus, including solving equations, complex numbers, sequences and series, exponential and logarithmic

functions, limits, and trigonometric functions. Trigonometric functions were the last major topic covered in the course.

Data Collection

For each school, logarithms were taught as a second semester topic within each year-long course (Trigonometry or Precalculus). To ensure sufficient time for the professional development component of the study, the initial data collection phase began in November 2004 for Mandy Wilson at High Acres School and December 2004 for Mulberry High School teachers. The unit covering logarithms began in mid-February 2005 for Mandy Wilson and late March 2005 for Mulberry High School teachers. Each teacher expected to spend approximately three weeks on the unit that included the study of logarithms.

Chronology of Study Events

The various activities necessary to conduct the study occurred according to the following schedule:

| <u>Time Interval</u> | <u>Event</u> |
|--------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| May 2004 through November 2004 | Identified cases |
| | Scheduled professional development sessions with teachers |
| | Developed instruments: background, attitudes, and content knowledge (Appendix C) |
| | Acquired (University) Institutional Review Board approval |
| | Mailed materials for first half of the history of logarithms lesson installments (Appendix B) to Mulberry High School teachers in November 2004 |

November 2004 through January 2005

Administered pre-assessments to High Acres School participant; Mulberry High School participants

Conducted professional development sessions; audio taped professional development sessions; take field notes

Administered post-assessments (at the end of each site's professional development sessions)

Reviewed professional development data and began initial interpretations

Updated classroom materials based upon participant feedback

Created on-line reflection community for participant interaction between professional development phase and instructional phase (potential data sources: discussion board postings, chat transcripts, notes).

January 2005 through April 2005

Transcribed audiotapes from professional development sessions in preparation for observations of instruction during logarithms unit

February 2005 – April 2005

Observed participants' instruction during logarithms unit; audio taped sessions; took field notes

Collected artifacts (i.e., handouts, student work samples)

Began initial data analysis from classroom observations: coded instructional practice data

Constructed interview questions (Appendix D) from observations of classroom practice and teachers' additional engagement (since the professional development sessions) with the history of logarithms

| | |
|------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | Conducted informal participant interviews during and semi-structured participant interviews after logarithms unit |
| | Administered post-assessments (after each teachers' instruction of logarithms) |
| April 2005 – June 2005 | Conducted additional review of the data: identified consistency of professional development engagement themes (after searching for disconfirming evidence of original themes) |
| June 2005 – October 2005 | Articulated tentative conclusions (organized by research questions); formulated structure for case descriptions; continued to review data and returned to participants for additional validation data |
| October 2005 – February 2006 | Dissertation writing |
| January 2006 – February 2006 | Conducted member check |

Data Collection Techniques

Data were collected from a variety of sources, including: background, attitudes, and content knowledge instruments; observations; field notes; analytic memos; interviews; artifacts; and interactions from an on-line professional development community.

Instruments

Data from three instruments (Appendix C) were used to describe some of the participant background variables necessary to address the research questions. The first instrument requested demographic information, including years of experience, certification route, mathematics content preparation, and previous professional development activities. This background survey, adapted from one used by the National

Assessment of Educational Progress (2003), provided data used to construct a professional profile of each participant.

The second instrument was in the form of a content knowledge assessment. The assessment was focused on logarithms only and was composed of questions found in the traditional high school curriculum as well as questions taken from the history of logarithms lesson installments. Changes in content knowledge assessment performance were used to describe the participants' experiences when studying and using the history of logarithms.

Finally, the third instrument was an attitudes assessment (adapted from Marshall, 2000), which measured attitudes regarding the history of mathematics and attitudes, beliefs, and teaching practices regarding the teaching of logarithms. Changes in attitudes were considered in conjunction with each teacher's level of participation during the professional development sessions and their decision to incorporate materials from the history of logarithms.

Both the content test and attitudes survey were re-administered after the professional development sessions and at the end of each teacher's instructional unit which contained the topic of logarithms.

Observations

I used participant observation during two key phases of the study. Participant observation is described as the mode of observation where the "researcher enters the world of the people he or she plans to study, gets to know them and earns their trust, and systematically keeps a detailed written record of what is heard and observed" (Bogdan & Biklen, 2003, p. 2). Since I delivered and participated in the professional development

sessions focused on the historical development of logarithms, I audio taped each session so that I could participate more fully. The transcriptions of the taped sessions provided me with the “detailed written record” suggested by Bogdan and Biklen. In addition, I took field notes to document the essential happenings of each session. (The predominant records, however, were the transcriptions of the audio taped sessions.) When necessary, I captured chalk board and white board images using a digital camera.

The data collected during the professional development component of the study (transcripts and field notes) were used to capture the participants’ comments, difficulties, and suggestions related to the potential use of the materials. Review of these data was subsequently used to construct open-ended interview questions. Observation data and interview responses from the professional development component of the study were then compared to instructional observation data.

I also used participant observation during each teacher’s classroom instruction about the unit on logarithms. Class meetings involving instruction on logarithms was audio taped to capture each participant’s teaching practice. In addition, I took extensive field notes during each class period of instruction to assist my efforts during the data analysis. On the few occasions that the observation of a class session was not possible, I solicited the individual teacher participant’s help in audio taping their instruction and I followed up with any questions about the particular class session.

Field Notes

Field notes served as my systematic written record. As Bogdan and Biklen (2003) defined, field notes served as a written account of what I heard, saw, experienced, and thought (p. 111) during the study of the teachers’ experiences with the history of

logarithms. In my field journals, I recorded essential elements of instruction (including time spent on the various elements of instruction: i.e., homework review, administrative tasks, and teaching new content), classroom discussion, and assignments during the logarithms unit. I also recorded instances of when participants incorporated historical information or materials that were (and were not) discussed during professional development sessions and how the participants responded to student reaction (individually or whole-class) regarding the historical development of logarithms. Instances of participants requesting my involvement in classroom activities related to the history of logarithms were also noted.

Memos

As needed, I used memos to reflect and expand on the descriptions captured in my field notes. Bogdan and Biklen (2003) described memos as “the additional “think pieces” about the progress of the research” (p. 114). The reflections covered such methodological and status concerns as analysis, ethical dilemmas and conflicts, and clarification, and aided in the construction of interview questions and site and participant descriptions.

Interviews

There was moderate geographical distance between the two research sites and myself. Consequently, I used open-ended interviews that were “focused around particular topics” or “guided by some general questions” (Bogdan & Biklen, 2003, p. 95). When necessary, I used e-mail correspondence and informal conversations to follow up between observations and on-site interviews. I also conducted semi-structured interviews after all instruction related to logarithms was completed at each research site. Teacher-specific interview guides were constructed based upon at least two reviews of the data collected

during the professional development sessions and the observed instruction. I developed questions attempting to link each teacher's level of engagement with the materials discussed during the professional development component of the study with the actual use of the history of logarithms with students. In addition, the interview guides constituted another source of data for determining themes related to each participant's experience with the teaching logarithms using an historical perspective or historical materials. The interview guides were constructed prior to each post-instruction interview, and I utilized a core set of common questions to ask of each participant. I treated the post-instruction interview with the intent of treating each participant as an expert with respect to their own experiences (Bogdan & Biklen, p. 99).

In this research, I endeavored not only to describe how a teacher's knowledge of and affinity toward the history of logarithms manifests in their teaching of the topic, but I sought to capture the entire spectrum of their experiences with using the history of mathematics as a teaching tool in their own words, from within and in spite of the context of actual high school classrooms.

Artifacts

Merriam-Webster (1993) defined an artifact as "something created by humans usually for a practical purpose" (p. 65). During this study, teachers occasionally created documents, for both their own use and student use, necessary to expand their instructional practice. I collected various types of artifacts to support observational data resulting from the professional development sessions and classroom instruction. The artifacts of interest were those which teachers created for use with their students that were directly related to the study of the history of logarithms. Artifacts included handouts and assignments used

with students, copies of historical text, student work samples, and digital photographs of board work.

'Tapped In' On-line Community

An online community was established on the educational website *Tappedin.org* for the purpose of establishing an ongoing professional development environment for the participants. *Tapped In* was created to respond to the need for professional development providers to

Expand face-to-face programs to include online activities and content that engage teachers anytime, anywhere. The growing recognition that no single organization can satisfy teachers' ongoing professional development needs requires that educators and providers form communities to share strategies, resources, and support. (Tapped In, About section, para. 2)

Providing a place for the case study participants to go for support, sharing, and identification of additional resources was planned as a necessary component with respect to the assumed unfamiliarity teachers will have with the history of logarithms. The online resource allowed me to provide additional support to teachers as they reflected on their study of the historical development of logarithms and as they considered the use of the history of logarithms with their students. Transcripts from the *Tapped In* website served as additional data to document teachers' proposed use or experience with using the history of logarithms in the teaching of logarithms.

Data Analysis

Data collection, interpretation, and analysis constituted an ongoing process. Strauss and Corbin (1998) offered a structure of "techniques and procedures to those...who want to do qualitative analysis but who do not wish to build theory" (p. x). They further stated that:

Building theory is not the only goal of doing research. High-level description and what [is called] *conceptual ordering* also are important to the generation of knowledge and can make a valuable contribution to a discipline. (p. x)

Analysis of Professional Development Data

I relied upon several aspects of grounded theory in the data analysis.

Microanalysis, as defined by Strauss and Corbin (1998), is the “detailed line-by-line analysis necessary at the beginning of a study to generate initial categories” (p. 57).

Using a process of “careful...examination and interpretation” (p. 58), I examined all of the transcripts from the professional development sessions “line-by-line” for the purpose of identifying themes of each teacher’s engagement. I began with reviewing the transcripts from Mandy Wilson’s three professional development sessions. By using the tool of open coding from microanalysis, I determined a collection of descriptive codes to describe Mandy’s participation during the professional development sessions. Open coding is defined as the process that a researcher uses to identify “potential themes by pulling together real examples from the text” (Ryan & Bernard, 2000, p. 783). One way to perform open coding is to conduct a “close examination of data, phrase by phrase and sometimes word by word” (Strauss & Corbin, 1998, p. 119). Subsequent to this practice of close examination, I identified six themes describing Mandy’s participation during the professional development sessions. The resulting themes are described in Chapter 4.

After I analyzed Mandy’s professional development data using this collection of participation themes, I applied the same framework to the participation data for the Mulberry High School teachers. In an effort to search for disconfirming evidence of the themes already identified, I next examined the transcripts of the two Mulberry professional development sessions. Creswell and Miller (2000) described the search for

disconfirming evidence as, “the process where investigators first establish the preliminary themes or categories in a study and then search through the data for evidence that is consistent with or disconfirms these themes” (p. 127). I was aware of the caution Creswell and Miller offered regarding the inability of some researchers to identify disconfirming evidence because they are overtly engaged with seeking confirmation. However, each utterance from the Mulberry professional development session was determined to fit within one of the original six themes identified. In addition, each of the Mulberry teachers’ engagement during the professional development sessions could be characterized using a subset of the six themes identified for Mandy. After each teacher’s significant contributions during the professional development sessions were classified using either primary or secondary themes, I next identified their level of engagement. Each participant’s level of engagement was determined from both the number of primary and secondary themes exhibited and the extent to which each theme was exhibited during the teachers’ professional development participation. The discussion of the resultant themes and engagement classifications also appears in Chapter 4.

To aid in the identification and articulation of important themes I used *in vivo* coding as an examination tool. In this sense, *in vivo* coding is a process that enables identification (or confirmation) of potential themes “taken from the words of [the participants] themselves” (Strauss & Corbin, 1998, p. 105). Plans for the use of the history of logarithms articulated by the participants during the professional development sessions, along with their actual classroom practice contributed to characterizing each participant’s experience. *In vivo* coding allowed me to use the participants’ voices in the

case study description, which was fueled by the identification of themes surrounding the inclusion of the history of mathematics in teaching practice.

Analysis of Instructional Practice Data

The instructional practice data collected from each participant's instruction during a unit on exponential and logarithmic functions was examined for two purposes. First, I was interested in collecting data on how each teacher incorporated the historical development of logarithms. When a participant did use the history of logarithms with students, I examined their practice against what they indicated they would do during the professional development sessions ("pedagogical decisions" engagement theme). When the outcome (actual instructional practice) differed from initial plans (professional development engagement), I created interview questions. The interview questions were used to understand the differences.

The second purpose of examining instructional practice data was to collect evidence of the participants' beliefs about their role as teacher and about the role of students. I formulated interpretations of these beliefs for each participant for whom I collected instructional data and also asked about their beliefs during semi-structured interviews. The interview responses were used in conjunction with the instructional observation data to characterize differences in practice among the cases.

Cross-case Analysis

After open coding was used to successfully identify a core set of engagement themes, I employed constant comparison techniques to examine the influence of background variables and the engagement during the professional development sessions

focused on the history of logarithms on each participants' instructional practice.

LeCompte and Preissle (1993) defined constant comparison to include:

The discovery of relationships, or hypothesis generation, [beginning] with the analysis of initial observations, undergoes continuous refinement throughout the data collection and analysis process, and continuously feeds back into the process of category coding. As events are constantly compared with previous events, new typological dimensions as well as new relationships may be discovered. (p. 256)

With respect to the professional development data, then, I endeavored to use several aspects of constant comparison (Charmaz, 2000, p. 515). I first compared the different participant's experiences during the professional development sessions to confirm the six engagement themes originally identified. I also revisited the data and my characterization for each participant at several points during the study. For example, after initially I identified each participant's themes and level of engagement (after the professional development sessions), I returned to the data and characterization for each during instruction, after instruction, and while writing the first draft of each teacher's case description. Lastly, I continued to evaluate specific examples identified for each engagement theme for fidelity with the theme originally identified.

The Issue of Credibility: Validity, Generalizability, and Reliability

My views on the issues of credibility, with respect to qualitative case study research, are aligned with those of Janesick (2000). In her response to the issue of credibility in qualitative research, Janesick (2000) suggested replacing validity, generalizability, and reliability with referents more befitting of the qualitative paradigm. She noted that, "qualitative research has to do with description and explanation and whether or not the explanation fits the description. In other words, is the explanation

credible?” (p. 393). What exists at the core of this very argument, however, is the fact that a qualitative researcher’s interpretation is just that: an interpretation. Since there is no single “correct” interpretation of an event or collection of events that define a case, it is not realistic to discuss validity in case research in traditional terms (Janesick, p. 393). In addition, a traditional definition of generalizability does not fit with the purpose of this study. What seemed most critical for the case study research I proposed was to question the “meaning and interpretation [of] individual cases” (Janesick, p. 394). This aspect of the study would be limited by what Janesick identified as “old notions of generalizability” (p. 394). Lastly, discussing reliability and replicability also becomes meaningless within the examination of a case. Janesick observed, “the value of the case study is its uniqueness” (p. 394). With this case study, I anticipated conducting a “carefully done, rigorous [study] that uncovers the meanings of events in individuals’ lives” (p. 394).

Bogdan and Biklen (2003) differentiated the reliability of a qualitative research study with that of a quantitative research study when they observed:

Among certain research approaches, the expectation exists that there will be consistency in results of observations made by different researchers or by the same researcher over time. Qualitative researchers do not exactly share this expectation. (p. 35)

They further observed that, “qualitative researchers tend to view reliability as a fit between what they record as data and what actually occurs in the setting under study, rather than the literal consistency across different observations [by different researchers]” (p. 36). This conception is much like the ‘uniqueness of a case’ idea put forth by Janesick.

The need to recognize the importance of the uniqueness of this case study does not diminish the significance of the research itself. It is important to contribute to the almost non-existent research investigating secondary teachers' use of the history of mathematics within the United States in the ways outlined by Barbin (2000). The data collected will support the aims, steps, problems, advantages, and disadvantages teachers experience when using history (Barbin, p. 90). Equally necessary, however, is to determine why teachers choose not to use the history of mathematics after participating in an intervention that attempts to provide tools and background for its use with respect to a specific topic. The data will provide the media to construct a narrative account of each participant's experiences that will inform scholars seeking existence proofs or counter-examples of secondary teachers considering the benefits and advantages of using the history of mathematics with students. In addition, the research may provide ideas for increasing the capacity to use curricular interventions dependent upon historical contexts. Thus, my analysis based upon the data collected from multiple data sources of *these* teachers with *their* particular experiences, in *these* settings, at *this* particular time, and with *this* particular mathematical topic will be as reliable as my interaction with each of these elements. To crystallize my descriptions, interpretations, and explanations of the participants' experiences with the history of logarithms, I used several validity procedures.

Creswell and Miller (2000) identified nine types of validity procedures (p. 126). Although they endeavored to situate appropriate procedures within difference research paradigms, I utilized procedures which would aid in describing each unique case. Of the nine procedures they identified, I used triangulation, disconfirming evidence, researcher

reflexivity, member checking, and prolonged engagement in the field. The use of searching for disconfirming evidence and my participation with the teachers over time at each research site has already been discussed. My “assumptions, beliefs, and biases” related to the use of the history of mathematics are revealed in Appendix A.

Creswell and Miller (2000) defined triangulation as, “a validity procedure where researchers search for convergence among multiple and different sources of information to form themes or categories in a study” (p. 126). It was important to use transcripts, field notes, artifacts, and interview data to fully describe each participant’s instructional practice when teaching logarithms in a Precalculus-type course. For example, several participants noted during the professional development sessions that they would use a particular lesson installment in their instruction. Classroom observation data provided evidence of the actual use of the lesson installment, although in a slightly different manner than discussed during the professional development. Subsequent interviews were used to clarify the differences between the professional development data and the classroom observation data. Lastly, artifacts (in the form of handouts or classroom activities used with students) were collected from the participants to provide further crystallization established by professional development session, classroom instruction, and interview data.

Creswell (1994) outlined several aspects “that lend internal validity to a study” (p. 158). In addition to triangulation procedures and indicating how the researcher and participants were involved with each aspect of the research, he also noted that member checks enable the researcher to “take categories or themes back to the informants and ask whether the conclusions are accurate” (p. 158). I provided each participant their

completed case account and requested that they read the account and provide feedback along several dimensions (Appendix E). By doing so, I was able to “take data and interpretations back to the participants in the study so that they [could] confirm the credibility of the information and the narrative account” (Creswell & Miller, 2000, p. 127). Four participants, Mandy, Sue, Ted, and Shirley, responded during the member check phase. A discussion of each participant’s feedback appears in Appendix E.

Chapter 4

The Professional Development Experience: Theoretical Design, Planning, and Implementation

Teachers often find it difficult to incorporate the history of mathematics into their teaching, especially if they have not had the occasion to study the history of mathematics. With this reality in mind, a priority of this study was designing a professional development component to use with the teacher participants. The professional development was designed to possess features common to effective professional development programs.

Theoretical Design of the Professional Development Sessions

A collection of structural and core features outlined by Garet, Porter, Desimone, Birman, and Yoon (2001) were used as guidelines for the professional development I designed for this study. When the six features identified by Garet et al. were compared with features of effective professional development proposed by other scholars, each could be classified within either the three structural features or the three core features found in the Garet et al. work. Consequently, the professional development sessions were organized with the features of form, duration, participation, content focus, active learning and coherence in mind.

Structural Features

There are three structural features commonly found in effective professional development efforts: form, duration, and participation.

Form

As recommended by Garet et al. (2001), I wanted to develop a sequence of sessions that would be considered seminar-like, where the responsibility of presentation

and knowledge authority was shared among the participants. With respect to this feature, my role was best described as participant observer because of my dual role comprised of participating as a student of the historical development of logarithms (i.e., learning from others' perspectives) as well as observing how the teacher participants engaged in and shared knowledge about the historical development of logarithms. As I considered the overall design of the sessions, I wanted to be sure to allow for time between the sessions for the teachers to work with the materials and resources outside of our structured professional development sessions. In this way, the *form* of the professional development I envisioned defied many traditional structures of professional development programs wherein teachers attend a workshop, meeting, or conference for a limited amount of time with little or no intervening time provided for participant reflection or individual and group study.

Duration

I originally wanted to work with teachers over eight 60- or 90-minute sessions. The goal of the first session would be to provide an overview of the seven lesson installments. In addition, I would include a brief introduction to the electronic and print resources we would be using during our study of the historical development of logarithms. Each week thereafter would be spent discussing the mathematical and pedagogical ideas within each of the seven lesson installments. Between each of the 60- to 90-minute sessions teachers would have the opportunity to participate in online discussions via a *Tapped In* electronic learning community. Regardless of each teacher's decision to participate online however, I planned for teachers to review the lesson installments – much like they would expect of their students – prior to the next session.

Consequently, the *duration* of the professional development would span several weeks, with the expectation that the teachers would struggle with the new perspective of teaching logarithms in between and during our sessions together.

Participation

The final structural feature I considered was the participation dynamic. When I began my search for a research site, I focused on contacting mathematics department chairs and mathematics resource teachers so that I could discuss the possibility of working with either an entire mathematics department or a subset of the department responsible for teaching courses in which logarithms were part of the curriculum. My intent was to work with a group of teachers who would not only share course concerns, similar student populations, and curricular issues but who would also be able to rely upon each other as resources outside of the professional development sessions. With this collaborative ideal in mind, the *participation* dynamic was a significant consideration while formulating the professional development session structure.

Core Features

Garet et al. (2001) identified three core features necessary for successful professional development of teachers. The professional development activity should possess a content focus, include opportunities for active learning, and aid in the creation of a coherent program of teacher learning.

Content Focus

Limiting the content focus of the professional development sessions to the study of the historical development of logarithms was an essential factor in not only the design of the professional development but also for the construction of the study's research

questions. The *content focus* of the history of logarithms was intended to provide a concentrated examination of how the historical perspective of a particular mathematical topic can be studied and utilized as a teaching tool in classroom practice. It is often considered an optimal practice for teacher participants to share in the decision-making process surrounding the choice of the content covered during a professional development program. For this study, however, I anticipated that many teachers would not possess enough background knowledge related to the historical development of many secondary mathematics topics to aid in choosing the content focus.

Active learning

Essential to the creation of the study's professional development component was the need to include opportunities for teachers to analyze teaching and learning when incorporating an historical perspective to teaching a particular mathematical topic. The sessions were devised to engage teachers in the materials so that they could consider their use with students on two different levels. First, while reflecting upon their own learning and experiences with the lesson installments and resource materials, teachers would be able to discuss their perceptions of students' experiences and potential for learning if the students were to engage in the same materials. And, secondly, by bringing the teachers together as a group to study the historical development of logarithms we would be able to discuss particular ways in which the historical development could be used as a tool while teaching logarithms in a Precalculus-type course. As part of the on-going nature of the professional development sessions, the teachers would also be encouraged to utilize the online professional development community (*Tapped In*) to continue conversations about

the potential ways in which the teachers envisioned incorporating the history of logarithms in their teaching.

Coherence

Often, the core feature of coherence combines the ideas of continuing professional communication among teachers, incorporating the experiences of teachers, and striving for alignment among state standards and teacher goals. Each of the structural and core features of effective professional development lend themselves to creating a coherent program in which teachers would be able to simultaneously expand their professional experience and consider opportunities for student learning. Two of the aforementioned ideas were difficult to identify for the two research sites while planning the sequence of professional development sessions for this study. Evidence of professional communication among teachers (particularly within the departments at each research site) and alignment of professional development goals with state standards were often not easily identifiable. Consequently, providing opportunities for these to occur remained an impetus for using the core feature of *coherence* to bring together each of the structural and core features guiding the design of the professional development sessions.

Planning and Living the Experience: The Historical Development of Logarithms

There were several characteristics related to teaching and experiences with the history of logarithms which the five participants did not share, as subsequent data analysis will reveal. As such, it is important to describe the professional development sessions that I conducted at each site and my efforts to provide essentially the same experience for each. Prior to beginning their participation in the study, each teacher

indicated that they had no experience with the historical development of logarithms – a characteristic that the participants shared.

The Professional Development Sessions

The school and teacher characteristics of the two school research sites necessitated that the actual professional development sessions differed from the original plan for the professional development sessions. The format of the professional development sessions also differed between the two sites.

High Acres School

Mandy Wilson was the only participant at High Acres School. We spoke at length after a presentation outlining my proposal for the *Investigating Teachers' Experiences with the History of Logarithms* study at a regional National Council of Teachers of Mathematics meeting. Within four days of meeting we established e-mail contact to arrange a schedule of times for me to visit her school. During a single visit (November 4, 2004) I was able to complete all of the pre-content tasks: providing an overview of the research study, explaining and obtaining her signature on the consent form, collecting Mandy's responses on the three instruments (background, attitudes, and content knowledge), and describing the contents of the resource binder.

In addition to providing Mandy with the necessary background for a study of her experiences with the history of logarithms, I also received valuable information during the initial visit to High Acres. Before we began, Mandy took me on a tour of the upper school, describing the school's educational philosophy and introducing me to several administrators along the way. Even within the early stages of our collaboration, Mandy exhibited that she would be actively sharing in the exchange of information during our

association together – a theme that would be prevalent throughout her participation. At one stop during our tour, Mandy alerted my attention to a history of mathematics timeline displayed outside of the upper school dean’s office. The timeline, Mandy explained, was completed by her Precalculus classes during the first six weeks of school. The students studied the mathematical developments of the Dark and Middle Ages, concentrating on the various contributions of non-western cultures.

Scheduling the professional development sessions with Mandy Wilson was an easier task than for the Mulberry High School site, given that the school was located closer to my home. This is not to say, however, that scheduling – in particular, adhering to the schedule – was without problems. Beginning on November 4, 2004, Mandy and I agreed to meet weekly for the remainder of the fall semester. Due to several last minute conflicts coupled with a long-term illness (Mandy’s), weekly sessions did not always occur. In addition, completion of the professional development sessions focused on Lesson Installments 5, 6, and 7 in the same manner as Installments 1 through 4 was never possible. The professional development sessions with Mandy are outlined in Table 2.

Table 2
High Acres School: Professional Development Sessions

| Date (Day) | Time | Activity |
|-------------------|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| November 4, 2004 | 11:30 – 3:00 PM | Research study overview; consent forms; survey and pre-assessments |
| November 15, 2004 | 2:15 PM – 3:45 PM | Discussed Lesson Installments 1 and 2; overview of Lesson Installment 3 |
| November 22, 2004 | 2:15 PM – 4:00 PM | Discussed Lesson Installment 3; overview of Lesson Installments 4 and 5 |
| November 29, 2004 | 2:15 PM – 3:45 PM | Discussed Lesson Installment 4 (Mandy had not completed study of Installment 5); overview of Lesson Installments 6 and 7; discussed additional slide rule activity not originally included in professional development materials |

Although the High Acres content sessions took place over a longer period of time when compared to Mulberry High School, we were only able to complete an examination of the first four lesson installments due to Mandy's protracted illness lasting from November 30, 2004 until January 5, 2005. Mandy and I worked together over a series of three content sessions. In addition to discussing various resource materials included in the resource binder and others which both Mandy and I later identified, we also reviewed Lesson Installments 1 through 4 in detail and Lesson Installments 5 through 7 briefly. In the time between each content session, Mandy was consistent in her efforts to work through the lesson installments in detail. She was also motivated to share reflections of her work with the historical development of logarithms and her anticipated use of the history of logarithms with students.

During the first content session, Mandy and I discussed the usability of the resources provided in the resource binder. Mandy also offered several other resources for review that she came across during her internet search to complete Installment 1. The remainder of the session was spent on interacting with Mandy about her experience with researching the people, motivation, and mathematics behind the development of logarithms (Lesson Installment 1) and presenting Napier's two particle argument, which defined logarithm using the correspondence between two sequences (Lesson Installment 2). We ended the session with two activities. First, we had a brief discussion about Mandy's impressions for how to use the history of logarithms with her students and her initial plans for the ordering of topics in the spring semester to best accommodate the use of materials. Second, I provided Mandy with a copy of Lesson Installment 3, which we briefly reviewed in order to make plans for the second content session.

At nine pages in length, Lesson Installment 3 is the longest installment out of the seven. Consequently, we spent most of the second content session on this installment. Our interactions were quite varied during this professional development session. Mandy and I shared our mathematical work, as well as critical feedback related to directions to students and typographical errors found within the lesson pages. Additionally, careful examination of the Installment 3 content enabled us to reflect on how a presentation of Napier's two particle argument may impact the use of the installment with students. Mandy also continued to reflect on how she believed her students would react to her use of an historical approach to teaching logarithms. At the conclusion of the second content session we agreed to investigate Lesson Installments 4 and 5 during our next meeting together.

In what would be our final content session together, Mandy and I discussed a variety of topics during the third professional development sessions. Although a lengthy discussion about Lesson Installment 4 was the central activity of the session, we also spent time discussing:

- the inclusion of a slide rule activity;
- the implications of what it means to really understand the concept of a logarithm;
- the practicality of using original documents to introduce and explore a mathematical topic; and
- the content of Installments 5, 6, and 7.

Mandy was unable to complete her study of Installment 5 in time for our third session, so we agreed to cover our investigation of the associated material during our next meeting. As stated earlier, Mandy became ill (she eventually had to have oral surgery and missed a great deal of school) so we were not able to continue formal review of the last three installments.

Mulberry High School

It was necessary to conduct the professional development sessions with the Mulberry High School participants under less-than-ideal circumstances due to the long distance between myself and the school. The four participants agreed to dedicate three half-days during the fall semester examination week and one-half of their professional learning day at the beginning of the spring semester to the study of the history of logarithms. In an effort to encourage their study of the history of logarithms outside of the formal sessions scheduled to take place as a group, I compiled a set of materials in a resource binder for each of the teachers and then mailed the binders to Sue Moe (mathematics department chair, a participant in the study) two weeks prior to my arrival. Sue distributed the binders to the other three teacher participants. The following materials were included in the binders:

- Letter of introduction for participation in the research study;
- Overview of the research study;
- Tentative timeline of professional development activities;
- Lesson Installments 1, 2, and 3;
- Various teacher resources, including a list of useful websites and portraits of mathematicians involved in the development of logarithms.

The letter of introduction alerted the teacher participants to the schedule of activities for the week of December 13, 2004. In an effort to be sensitive to the participants' needs and schedules for the week, which was the last week of the fall semester, the schedule was tentative and based upon their input communicated via Sue Moe. After arriving on campus, I met with the teacher participants to discuss the study in detail. I wanted to ensure that the participants understood the two components of the study and using both the written overview and the consent form, I outlined the major features of each

component. I also wanted to allow sufficient time for questions; however, there were none.

After the initial meeting with the participants, I modified the schedule to allow them time to work on their concluding activities of the semester, in particular, grading student exams, entering grades, calling parents, and attending meetings. This resulted in a significant loss of time for any professional development activities on December 16.

Table 3 outlines the schedule that was established after the initial meeting with teachers on December 14, 2004.

Table 3
Mulberry High School: Professional Development Sessions – Part I

| Date (Day) | Time | Activity |
|-------------------|--------------------|--------------------------------------------------------------------------|
| December 14, 2004 | 3:30 PM – 4:30 PM | Research study overview; consent forms |
| December 15, 2004 | 1:00 PM – 4:00 PM | Questions about the particulars of the study; survey and pre-assessments |
| December 17, 2004 | 9:00 AM – 12:00 PM | Lesson Installments 1 – 3 |

I decided to use the first two days (December 14 – 15) as a way to get to know the teacher participants since it was necessary to delay discussing the content related to the historical development of logarithms. I used before school, during lunch (all Mulberry mathematics teacher shared the same lunch period), and after school to become acquainted with each participant. For the most part, this was very easy for several reasons. First, Sue Moe, my close personal friend and professional colleague, is the mathematics department chair and the other participants made frequent visits to her room for various purposes. Each time they visited Sue’s classroom, I made a point to engage in conversation with them. Second, two of the participants, Ted Jones and Shirley Corson, each have classrooms adjacent to Sue’s classroom. This close proximity made it

convenient for me to interact with Ted and Shirley often and as result, I was able to feel less like a stranger in the building. Third, Mary Long, the final Mulberry participant, had become very close to Sue during the school year. I was immediately accepted by Mary under the ideal that ‘any friend of Sue is a friend of mine.’ Mary, Sue, and another mathematics teacher, Carrie Morgan, often congregated in Sue’s classroom at the end of the school day to work and socialize. The after school get-togethers, often lasting as long as three hours, helped me to gain access to the teachers in valuable, socially-oriented ways.

In addition to building rapport with the teacher participants via social conversation and classroom visits, I also made a concerted effort to introduce the teachers to the study in such a way that we only concentrated on two activities for each of the introductory sessions. On Tuesday (December 14), I explained the project overview, which was included in the teachers’ binders, from a very personal standpoint. I invited teachers to think about their questions concerning the research itself and I indicated that I would answer any that they had at the beginning of the next session. In addition, I explained the consent forms, allowed the teachers ample time to read the form, and requested their signatures as a final step for agreeing to participate in the project. Each teacher also chose a pseudonym and I provided each with an identification number to be used on all instruments.

None of the teacher participants presented me with any questions or concerns about the study when we met as a group on Wednesday (December 15). With the exception of Mary Long, each teacher participant completed the three instruments collecting background, attitudes, and content knowledge information. Mary needed

additional time to complete Page 3 of the background survey in order to respond completely to Question 8.

Another factor was instrumental in the decision to delay the major content portion of the professional development sessions until Friday. Sue Moe indicated that the other participants may not have reviewed all of the lesson installments prior to my arrival, and as such, the later I scheduled our discussion of them the more significant the teachers' participation would be (personal communication, 12/13/04). This would remain a consistent theme of the Mulberry High School teachers with respect to the professional development aspect of the study. Although eager to participate in the study and genuinely interested in the history of logarithms, the expectations and lack of effort towards continued professional learning of Mulberry teachers presented a barrier for each teachers' participation in the actual professional development sessions. As a result, the teachers were more comfortable with the idea of professional development as a mode of information gathering, i.e., the level of participation they were most comfortable with was to attend, listen, and possibly be able to take away with them materials ready for classroom use. Consequently, I used this final opportunity to encourage the participants to "at least try to do something within each of the three installments by Friday" (Field Notes Journal, December 14, 2004) in hopes that the session on Friday (December 17) would enable me to "capture their engagement of the material" (Field Notes Journal, December 13, 2004).

In anticipation of the Mulberry participants not familiarizing themselves with the content of the first three installments, I devised an alternative plan for engaging the teachers during our time together. I was most concerned with avoiding the lecture-

delivery method for the professional development sessions, intending instead to establish a more collaborative environment for which to work with the teacher participants.

During the first professional development session, a variety of activities took place. First, since all but one of the teachers had in fact spent time on Lesson Installment 1 (timeline activity), we were able to discuss a variety of issues related to not only the key mathematicians and motivational forces behind the development of logarithms, but also the pedagogy and reality of using such an activity with students.

Next, I shared Napier's two particle argument that accompanied his definition for logarithm. Teachers were given a written introduction to the argument with Lesson Installment 2 in the resource binder of materials sent in November 2004. This segment of the professional development was the only lecture segment planned and it enabled me to provide the background necessary for future lesson installments. The content of Lesson Installment 2 also gave me the opportunity to establish the necessary context for many of the historical documents and supplementary resources that I provided to the participants. As the lecture moved forward from my explanation of initial information, however, the participants assisted in constructing the two sequences necessary to arrive at Napier's definition.

The content portion of this session concluded with an examination of Lesson Installment 3. Most of the discussion related to Lesson Installment 3 focused on the format of the lesson installment. For example, teachers commented on its length, the abundance of text, and the focus on the use of sequences, which Mulberry curriculum would not cover until after exponential and logarithmic functions. None of the Mulberry participants asked questions about particular aspects of the lesson installment. The

discussion about Napier's idea of connecting two sequences to eventually define logarithm did promote discussion about the translation of the word, *logarithm*, as well as what motivation prompted Napier to relate an arithmetic sequence and a geometric sequence to describe his invention of logarithm. In our discussion about these ideas, we returned to the content of Lesson Installment 1.

Finally, we discussed the teachers' perceptions of the materials and potential use of the materials with students. To encourage the participants' on-going examination of the historical development of logarithms, I explained the purpose of the *Tapped In* online community as well as registration procedures for the site. In addition, the teachers were given the remaining lesson installments (Appendix B) and several resources to assist them in their study of logarithms. The resources included:

- Excerpts from Napier's *Constructio* (1619), including parts of his table of logarithms;
- Excerpts from W. R. MacDonald's translation (1966 reprint) of Napier's *Constructio*;
- Several articles from *From Five Fingers to Infinity* (Swetz, 1994); and
- An updated list of websites.

The second professional development session with the Mulberry High School participants was designed to include a sharing session among the participants (including me). I believed the teachers would need time to discuss the history of logarithms with each other during the second content session since the semester break (December 18, 2004 – January 2, 2005) would have prevented them from meeting as a group. This presumption was based on the idea that the teachers would have worked on the new materials (Lesson Installments 4 – 7) individually, with the understanding that on January 3, 2005 we would come together again and operate much as we did during the December 17, 2004 session. In reality, however, only one teacher (Sue Moe) made a sustained effort

to work on the materials before the second content session, and another (Ted Jones) had skimmed through parts of Installments 4 and 5.

I decided to deal with this scenario by asking the participants how they would like to proceed. They unanimously agreed that they were in fact interested in the content of the lesson installments and that they wanted to work through them together in the session with my guidance. We continued through the second session, alternating between Sue and Ted serving as the “expert” on the content of the lesson installments that they had spent the most time on. When either Sue or Ted was unable to successfully address the mathematical ideas, I stepped in to assist. Lastly, as we discussed the historical development of logarithms mathematically we continually addressed pedagogical issues related to the use of the history of logarithms while teaching logarithms and school and student characteristics that may present obstacles. We spent the most time on the computational aspects of Lesson Installments 4 and 5 and worked through the progression of exercises in Lesson Installment 6. Lesson Installment 7 received only a cursory treatment.

Describing the Experience

Analyzing the data from the five formal professional development sessions (two for Mulberry High School; three for High Acres School) enabled me to identify six themes of teacher engagement. The six themes provided a framework from which to construct a description of how each teacher participated during the professional development component of the study. Each teacher’s engagement was not determined by all six themes. Instead, some teachers only exhibited a subset of the six.

Determining the Six Themes

I holistically examined the data from all of the professional development sessions at the conclusion of the professional development component of the study (January 2005). Mandy's participation data were rich with examples of engagement which I determined could be classified in six reoccurring themes. Table 4 displays the six themes, divided into primary and secondary features of Mandy's professional development participation, and their descriptions. Themes were identified as primary if they were easily categorized as one theme and were prevalent in Mandy's participation. A theme was identified as secondary if they were ancillary to a primary theme and not easily coded (i.e., potentially overlapped depending upon interpretation and context). A primary theme was also less prevalent in Mandy's overall participation.

Table 4
Identification and Description of Participation Themes

| Theme | Level | Description |
|---------------------------------------|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Collaboration | Primary | Form of contribution during the discussion of content, resources, and lesson installments; described according to overall tone of contributions |
| Anticipation of student engagement | Primary | Reflection on aspects of student use and interaction with the content, resources, and lesson installments, including articulation of particular student difficulties, benefits, and enjoyment |
| Pedagogical decisions | Primary | Articulated plans for use of the content, resources, and lesson installments during instruction |
| Commitment to learning | Secondary | Evidence of ongoing study of the history of logarithms materials (taking place outside of the actual professional development sessions) |
| Critical reflection | Secondary | Reflection about the quality and content of the actual lesson installments and related resources, both for teacher study and student use |
| Self-identification of knowledge gaps | Secondary | Identification of further study needed to expand understanding and use of the historical development of logarithms |

After identifying the six themes relevant to Mandy’s professional development engagement, I examined the professional development data from the Mulberry High School sessions to search for disconfirming evidence of the six themes. What occurred, instead, was that each of the Mulberry teachers’ engagement could be classified using a subset of the original six themes determined from Mandy’s engagement. In addition to determining which engagement themes applied to each of the participants, I also determined their level of engagement along a continuum based upon the number of themes exhibited in their professional development participation and the intensity to which each was exhibited. Table 5 displays the primary and secondary participation themes for each teacher and the corresponding level of their engagement during the professional development sessions.

Table 5
Professional Development Participation Themes and Level of Engagement

| Participant | Participation themes | Level of engagement |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
| Mandy | Collaboration (“active”) Anticipation of student engagement Pedagogical decisions Commitment to learning Critical reflection Self-identification of gaps in knowledge | Eager |
| Sue | Collaboration (“facilitative”) Anticipation of student engagement Pedagogical decisions | Moderate |
| Ted | Collaboration (“assertive”) Anticipation of student engagement (single utterance only) Pedagogical decisions (single utterance only) Critical reflection | Moderate |
| Shirley | Collaboration (“mathematical”) Anticipation of student engagement (negative case) | Limited |
| Mary | Collaboration (“biographical”) Anticipation of student engagement Commitment to learning (negative case) | Limited |

Summary

In Chapters 5 through 7, I present three types of data. The first type of data, based upon the results of the three instruments (background, attitudes, and content knowledge), will contribute to the background profile for each teacher participant. The background data will aid in establishing each participant's

- previous professional experiences (teaching assignment and professional development);
- attitudes about the use of the history of mathematics (in general);
- previous instructional practices with respect to teaching the topic of logarithms; and
- personal mathematical knowledge of logarithms, including performance on items with an historical context.

Next, data from the professional development component of the study are reported. The data will be presented in order to describe the teachers' engagement during the professional development sessions, which focused on the study of the history of logarithms. The professional development engagement data will be organized using a framework of three primary themes and three secondary themes of engagement, previously described. The types of themes, and the degree to which they were identified, will be used to identify a teacher's engagement as eager, moderate, or limited.

Lastly, data collected during each participant's instruction during the unit or chapter in which logarithms occurred are reported. In a similar fashion to the classification of professional development engagement, the instructional practice data are described using the extent to which each participant incorporated the historical development of logarithms. Each participant's implementation of the history of

logarithms matched the characterization of their professional development engagement as eager, moderate, or limited.

Chapter 5
An Eager Study and Use of the History of Logarithms:
The Case of Mandy Wilson

This chapter describes the case of Mandy Wilson by presenting data organized across several dimensions. In order to address the research questions outlined in Chapter 1, the following sub-sections describe

- Mandy’s professional background;
- her engagement during the professional development sessions designed to examine the historical development of logarithms;
- her prior instructional practice related to logarithms and her beliefs about her role as a teacher and the role of her students as learners; and
- her implementation of the historical content with students in her three Precalculus classes.

Lastly, I summarize Mandy’s experience with the history of logarithms, including a description of the benefits, obstacles, and affordances related to Mandy’s use of the historical development of logarithms.

Professional Background

Mandy was the most experienced teacher of the five participants, having taught mathematics for 37 years. Her teaching trajectory followed a slightly non-traditional path. She first taught secondary mathematics for ten years and then taught for 20 years in a community college system. For the last seven years, Mandy was teaching both secondary mathematics during the day and community college mathematics in the evening.

Mandy’s professional preparation was unique. Mandy earned a bachelor’s degree in political science (with a statistics and economics emphasis), with a minor in mathematics. Consequently, at the undergraduate level, Mandy’s mathematics preparation appeared less substantial than the other four participants. She reported taking

only Calculus I, II, and III and Introductory Statistics and Statistical Analysis at the undergraduate level. As was the case with the other participants, Mandy had never taken a formal history of mathematics course. She did, however, report previous experiences with personal reading about the history of mathematics. Mandy also earned a master's degree in mathematics education and reported a minor in mathematics and special emphases in statistics and educational research. She also reported completing additional graduate course work in curriculum and instruction, with an emphasis in mathematics education. Mandy possessed a regular teaching certification, which was obtained after completing her undergraduate degree.

Mandy actively pursued professional development experiences during the two years prior to this study. Of the 12 potential professional development activities, Mandy reported participating in seven, including

- School, county/district, or state-provided programs, workshops, training sessions, or institutes;
- Conference or professional association meetings;
- Observational visit of mathematics instruction to another school;
- Committee focusing on mathematics curriculum, instruction, or student assessment in mathematics;
- Individual or collaborative research;
- Independent reading on a regular basis; and
- Consultation with a mathematics specialist. (Background Survey, 11/04/04)

Attitudes and Knowledge

Mandy's responses to the items on the first part of the Attitudes Instrument Pre-assessment (see Table 6) indicated that she possessed a strong inclination toward the use of the history of mathematics in both the teaching and learning of mathematics. On the Likert-type scale used, Mandy either "strongly agreed" or "moderately agreed" with each of the eight items on Part I of the instrument.

Table 6

Attitudes Instrument (Part I) Pre-Assessment Results: Mandy Wilson

| Survey item | Pre response |
|---------------------------------------------------------------------------------------------------------------------------------------------------|--------------|
| 1. Understanding the history of mathematics is an important part of understanding mathematics. | 6 |
| 2. Including history enriches the teaching and learning of mathematics. | 6 |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | 5 |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | 5 |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | 5 |
| 6. Using historical materials in my mathematics classes has been an integral part of my instruction in: Precalculus/Trigonometry. Calculus. | 6 6 |
| 7. Prospective mathematics teachers should be required to study the history of mathematics. | 6 |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | 6 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

Mandy's extended responses on Part II of the Attitudes Instrument provided insight into several of the items in Part I. In addition to her reported practice of incorporating the history of mathematics into Precalculus and Calculus instruction (Part I, Item 6), Mandy also indicated several reasons for why it was important for her students to experience the history of mathematics. Mandy referenced (1) the importance of the human element contributing to the development of mathematical ideas; (2) the important role mathematics played in the development of the human thought process; (3) the use of quality materials (which she equated to use of primary documents) that appeal to students less interested in mathematics; and (4) the challenge that using historical problems poses for students. Most of Mandy's views expressed on the Attitudes Instrument Pretest were framed by her interest in using the history of mathematics (M. Wilson, personal communication, 11/04/04). Her existing practice related to the use of the history of

mathematics included the use of primary documents and the historical problems posed within them.

Of the eight content items related to the study of logarithms, four were purely historical in orientation and were taken directly from the historical development of logarithms lesson installments. Three were traditional in nature, and asked the participants to either evaluate a logarithmic expression or solve a logarithmic equation. A final item, asking participants to define *logarithm*, was considered a “swing” item, as it could be answered using either the traditional or historical definition. Mandy’s performance on the logarithms content knowledge pre-assessment was similar to that of the other participants. Mandy accurately completed four out of the eight items. She correctly answered all three traditional items and one of the historical items. The items, their classification (historical or traditional), and the results of Mandy’s efforts on the content knowledge pre-assessment are presented in Table 7.

Table 7
Content Knowledge Pre-Assessment Results: Mandy Wilson

| Item | Historical/Traditional | Response given |
|---------------------------------------------------------------------------------------|---------------------------------------------|-----------------------------------------------------------------------------|
| 1. Define <i>logarithm</i> . | Either, depending upon participant response | Defined <i>logarithmic function</i> instead of <i>logarithm</i> (incorrect) |
| 2. Describe the basic idea or motivation for the invention of logarithms. | Historical | Described in terms of calculations necessary for astronomical measurements |
| 3. Construct the values for $\log_{10}2$ and $\log_{10}3$ without using a calculator. | Historical | Attempted using approximation method (incorrect) |
| 4. Let $u = b^n$ and $v = b^m$. Verify $L(u) - L(v) = L\left(\frac{u}{v}\right)$. | Historical | Proof attempted (incorrect) |
| 5. Evaluate: $\log_{32}16$. | Traditional | Evaluated successfully by converting to exponential form |
| 6. Evaluate: | Traditional | Evaluated successfully by |

| Item | Historical/Traditional | Response given |
|-------------------------------------------------------------------------------------|------------------------|----------------------------------------|
| $\log_{\frac{1}{3}} 81$. | | converting to exponential form |
| 7. Calculate the product of 8409.5 and 951.49 using the method of prosthaphaeresis. | Historical | No mathematical response |
| 8. Solve for x : $2\log 3 + \log x = \log 45$. | Traditional | Solved correctly using a unique method |

Although her stance indicated that she was eager to incorporate the history of logarithms in her practice, Mandy’s responses to several of the items signaled that she was also in a position to strengthen her historical knowledge of the topic. First, Mandy was unable to define *logarithm* in Item 1. In her attempt to correctly describe what a *logarithm* was Mandy instead defined *logarithmic function*. Second, Mandy pursued the construction of the values for $\log_{10} 2$ and $\log_{10} 3$ (Item 3) by examining the graph of a logarithmic function and describing how she could show a geometric approximation. Her attempt, however, again depended upon the use of a logarithmic function. Item 3 was intended as a historically-situated item, and consequently was meant to be completed using means other than functional analysis. In the third item of note, Item 7, Mandy circled “prosthaphaeresis” in the item and simply responded that she had “never used [the] term” (Content Knowledge Assessment, 11/04/04).

Professional Development Engagement

As we approached the end of our third content session, Mandy and I proceeded to wrap up our time together as we usually did. I provided her with the next set of lesson installments and we outlined the tasks that we would complete before the next session. And, as was becoming habit, Mandy would offer some kernel of knowledge or practical

anecdote. This late November session was not unlike the others, as Mandy shared the following exchange that evinced her eager participation in this study.

Mandy: Now, we've had a very interesting discussion here at the school because every administrator here comes from the social sciences. And of course, the kids are preparing for history day and they use all the original documents and everything for their AP classes and all this. So, I was in there right now with the dean of curriculum. She said, "What are you doing" – I was working on this [the history of logarithms installment materials]. I said, "Well, we're going to do this experiment with kids using original works" – [and the administrator said] "you can't do that in math!"

Kathy: She did not say that to you!

Mandy: She just said it! She just looked at me and said, "Guess you could." And I said, "If you could use it in government classes; if you can use it in history classes, why not?" And she said, "I never thought of that much. But you know me and math." She's the most math-phobic person I've ever seen. But I said – while we were having this little conversation – had you had a different approach to the math yourself, being a very research/social science-oriented person, there may have been times when your interest would have been peaked and you would have learned things from learning them rather than memorizing formulas. (Mandy's emphasis, Professional Development Session 3, 11/29/04)

This sentiment epitomized Mandy's view of how the history of a subject – including the examination and use of original documents within the discipline – can serve as a learning tool for the subject. The strength of a student's interest in a subject and the fact that the process of learning is just as important as a product of that learning (i.e., a formula for later use) was a motivating factor for Mandy's study of the history of logarithms. Such a belief also sustained Mandy as she eagerly engaged in the professional development component of the study.

Mandy exhibited three primary modes of participation (as described in Chapter 4) during the time spent together in the professional development sessions. Her engagement

during these sessions can be characterized via personas of an active collaborator, a teacher anticipating student engagement, and a pedagogical decision maker. Several secondary themes were also identified which describe Mandy's engagement during the study of the history of logarithms. There was ample, although often overlapping evidence, for each of these secondary themes related to Mandy's experiences:

- commitment to continuing her learning outside our scheduled meeting times;
- ability to critically reflect on the materials and resources that we discussed; and
- desire to continue her study of related topics to address "holes" in her current understanding of the history of logarithms.

In the remainder of this section, I will describe Mandy's engagement during the professional development sessions, which were designed to mirror a potential program of study of the historical development of logarithms to be used with students. Elaboration of the three primary and three secondary themes, using description and supporting data, are used to characterize her experiences during this phase of the research.

Mandy as an Active Collaborator

Merriam-Webster (1993) defined *active* as "characterized by action rather than by contemplation or speculation" (p. 12) and *collaborate* as "to work jointly with others or together especially in an intellectual endeavor" (p. 224). In her case, Mandy's engagement "personality" during and outside of the professional development sessions was most appropriately described as an active collaborator. She was a dynamic and attentive participant during the times that we met to discuss the history of logarithms. Although there were times in which I needed to dominate our conversation to present essential information, Mandy consistently offered her work on lesson installments for examination, completed mathematical arguments that were previously unknown to her,

and shared mathematical and pedagogical ideas. At the end of our second content session on the history of logarithms, I recorded the following observations in my field notes journal:

For Mandy Wilson it is important to her to actually do the lesson installments. What is unique about this participant is that she really goes deep into the work to do the mathematics. It's not just about the "fluff" of the history. What's more, she is constantly thinking about what her students are doing or will be doing and how things fit together for them. She is always willing to share these comments! Mandy is also committed to looking for experiences that connect and bring students to engage with the mathematics. She always shares something with me in such a way that I learn from the experiences that we're sharing. I intend to tell her about this important benefit (for me) of this work when we meet on 11.29.04. (Emphasis in original, Field Journal, 11/22/04)

These observations were helpful in organizing observations of Mandy's actions as we continued to work jointly to understand the historical development of logarithms and how her intellectual endeavors would manifest in her classroom practice.

Sharing resources. One of Mandy's most consistent practices was her desire to complete each of the lesson installments as they were presented to her. In doing so, she was able to provide critical feedback on the lessons, a practice which I elaborate on later as a secondary theme. Mandy's commitment to remaining an active participant during the professional development component of the research motivated her to approach the content of each lesson installment as if she were one of her students. Whenever she worked on a lesson and discovered something of particular interest, she shared the discovery with me. In particular, Mandy frequently used the resources that I suggested and searched for others on her own. With respect to the research she conducted to complete Installment 1 (Appendix B), Mandy came upon a web site called *Scotland's People*, and she shared her experience with me:

Mandy: As we're going through here [on Installment 1]...I was able to get most of these people. A few of them, I was surprised with.

Kathy: Surprised because you weren't able to find them or surprised because of the information that they offered?

Mandy: Information. First of all, this one on Scotland's people [a website], there are many errors in this. You might want to check that. I "Googled" to this one, *Scotland's People* – a history of Scotland's famous people.

Kathy: I've only been to that site really briefly.

Mandy: I found at least two errors on the first page! (Mandy's emphasis, Professional Development Session 1, 11/15/04)

Engaging with difficult content. In addition to sharing resources (and providing corresponding critiques) identified during her review of Lesson Installment 1, Mandy was also actively engaged during a segment of the first content session in which I presented the mathematical background of Napier's two particle argument. In this argument, Napier described his concept of a logarithm. Presenting the argument of particles moving along parallel line segments to teachers is considered by some to lead to challenging problems (J. van Maanen, personal communication, 9/1/04), not the least of which occurring in the form of accepting Napier's original description of logarithm or struggling to understand how the historical ideas relate to our "modern" logarithm. In the experience I shared with Mandy, however, she was able to follow the initial structure of the argument easily. After I translated Napier's text into initial numerical conditions for the purpose of calculating distances traveled by the two particles, I continued to complete the next set of necessary calculations (see also Appendix B).

Kathy: And if he's traveling at the same velocity as he started, if this is one unit and that's the same unit of time – that's two, so: boring! I don't have anything else to do with that one. This one's a little

more interesting. So, in order to calculate what's going on, I know AZ ; I now need to know –

Mandy: BZ .

Kathy: What is BZ ? So, BZ is a little easy. It's all of this guy [points to the entire segment AZ] – minus one. But, just in case I need something else later, I'm going to do a factoring trick on this. If I factor out that ten to the seventh because I want to be explicit what I'm left with –

Mandy: One minus ten to the negative seventh.

Kathy: Yes! So there's the –

Mandy: That's where that [the value one minus ten to the negative seventh as seen earlier in the table example that we did] comes from.

Kathy: Now, here's the second part... somewhere else in the *Constructio Napier* is telling us that the velocity of this point [point B] is proportional to the remaining distance. So, BZ ... which is a distance is actually the same as the velocity at B .

Mandy: Right! (Professional Development Session 1, 11/15/04)

Mandy continued assisting with the reconstruction of Napier's two particle argument during this particular collaborative exchange, until finally completing the pattern to identify the two sequences necessary to arrive at Napier's definition of a logarithm. Without her active collaboration, this segment of the session would have fixed me in the role of information giver and Mandy in the passive role of information receiver.

Sharing unique methods. A final exemplar of Mandy's collaborative effort with regard to the lesson installments is found in the session in which we discuss the construction of the logarithm (base 10) values for the numbers 1 to 13 (Appendix B). I worked with the estimation of the logarithm of these values on several occasions but I had not previously considered the technique which Mandy shared with me:

- Mandy: I got some really, really crude ones [estimates of logarithm values]. And then I started going back over here with the seven. I'm curious of how you did seven.
- Kathy: I used a numerical relationship that we already know the logs for: log of 2 and log of 100, because 49 was really close to 50.
- Mandy: And I did it the other way. I had done it that way the first time then started playing around with, suppose we just try looking at some exponential powers here.
- Kathy: Ah! Now I did not think about that!
- Mandy: Which was to try to see you know, how close could we get to it [the actual value of log 7]?
- Kathy: So seven to the 3.9 is really close to 2000 [reading from Mandy's work]?
- Mandy: Yes. And so we came very, very close [to the actual value for the logarithm of 7]. (Professional Development Session 3, 11/29/04)

We continued discussing Mandy's technique for calculating the logarithm of 5, 11, and 13. Our conversation about student involvement in this activity continued and Mandy observed,

No, it's not just about logarithms anymore. It's just developing that number sense. I would love to have had these kids that I have in calculus right now go through this kind of an exercise because it's hard for them to think about where they can approximate. They ask, how do I go about *doing* it? (Mandy's emphasis, Professional Development Session 3, 11/29/04)

Although Mandy's method for estimating the logarithmic values in Lesson Installment 4 was unique, her method was calculator-dependent and circumvented the original intent of the lesson. However, it was important to Mandy to consider opportunities for her students to make connections along the entire trajectory of their mathematical development. Thus, Mandy modified the directions for the lesson

installment and completed the activity in a way that provided a compromise between the historical intent of the lesson and the needs of her students.

Mandy as an Anticipator of Student Engagement

Mandy expressed interest in the history of logarithms on two different levels. On one level, she expressed interest and sometimes surprise when learning about mathematicians and mathematical ideas for the first time. Manifestations of this level of interest are more powerful with regard to Mandy's instructional practice and are discussed later in this chapter. On the second level, however, Mandy articulated anticipation of her students' level of engagement after examining and completing particular activities for herself. In this section, I highlight three examples of this facet of Mandy's professional development experience.

Napier's invention. I was appreciative of Mandy's enthusiasm for the presentation of Napier's two particle argument (Appendix B), for two reasons. First, her engagement with the development of the definition of logarithm, as described by Napier and interpreted by me, provided evidence that I had made an appropriate decision to include Lesson Installment 2 in the history of logarithms materials. Second, her enthusiasm indicated that she also considered the presentation appropriate for her Precalculus students.

My presentation of Napier's two particle argument consisted of two parts. In the first part, I wanted to model the use of the history of mathematics in the same way that topics were often (historically) developed: moving from a concrete example to an abstract idea. Consequently, in the first part of the presentation Mandy and I worked on an example of multi-digit multiplication the "old-fashioned way." By this I mean that we

used the process associated with astronomical calculations of the early 17th century, which combined prosthaphaeresis and tables of sine values. After we calculated one product, I related the result to tables of values which appeared in Napier's tables from 1616. I wanted the example to hang in the air for a moment so that Mandy could digest the algorithm, the terminology, and the use of the tables. When I asked, "Any questions so far?" Mandy replied, "No! But it's fascinating! I can see some of [my students] really jumping off on that one!" (Professional Development Session 1, 11/15/04).

I looked forward to the second part of the two particle argument presentation – composed of both abstractions and calculations – to go as well. After we persevered through three iterations of calculations resulting from the movement of particles along the parallel segments, we were able to apply Napier's definition of logarithm through an analysis of our calculations. Mandy commented that she had researched several college mathematics books and could not locate a presentation of logarithms that reflected any significant adherence to the historical definition. I asked Mandy if she thought that presenting the two particle argument leading to the definition of logarithm would be appealing for her Precalculus students, particularly those also enrolled in a physics course. She replied,

I think this would be very appealing for that particular audience; to see the reality of where these two things fit together. I think it would be very appealing to these kids. Also, it would appeal to the physics instructor because he has a very strong math background. I think it would be kind of interesting to share this with him. (Professional Development Session 1, 11/15/04)

Calculations are like puzzles. Our review of Lesson Installment 4, *Calculation of Logarithms Using the Method of Napier and Briggs*, occurred at an opportune moment during the professional development component. Mandy and I reviewed Installment 4 after we spent a significant amount of time working together on the construction of Napier's two particle argument (Lesson Installment 2) and the development of logarithms using sequences (Lesson Installment 3). We welcomed a discussion about a task that was slightly less formal and taxing. The moment I turned on the recording equipment Mandy proclaimed, "I think the kids are going to thoroughly enjoy this!" (Professional Development Session 3, 11/29/04). When I probed further, Mandy stated that:

It's like working a puzzle. It's a jig-saw kind of arrangement. I know the personality of some of my students and they're going to get into it and go and they're going to have a delightful time with it. (Professional Development Session 3, 11/29/04)

In the name of religion. High Acres School was previously affiliated with a particular religious denomination. The school divided interests in the early 1990s and a new, nondenominational High Acres School opened at its current location in 1996. On several occasions during my visits as part of the High Acres School family, however, various ideas of how religion was possibly related to the development of logarithms were raised. One such instance occurred during the discussion Mandy and I had about the mathematicians and motivations behind the development of logarithms. As we examined the relationship of trigonometry and the contributions of ancient cultures to the development of logarithms, we also discussed less secular reasons motivating their development.

Kathy: If you look at any Greek survey of mathematics at the time, one of the big three motivations for the mathematics was charting the heavens.

Mandy: And this connection – which I think is very interesting – the religious connection; the fact that so much of the astronomy was based on astrology also.

Kathy: That goes back to their whole idea – of people and not folklore – but the telling of the lives of people which they link to gifts from heaven.

Mandy: Yes; this was such a large part of their philosophy. (Professional Development Session 1, 11/15/04)

Our meanings of my original use of “heavens” converged by the end of the discussion. Although Mandy did not explicitly verbalize an anticipation of interest on behalf of her students, our numerous informal conversations about the underlying religious dedication of much of the High Acres School staff and student body prompted me to hold out this example as one of implicit anticipation.

Mandy actively anticipated her students’ interest and engagement with the history of logarithms through expressions of her own interest when learning about a familiar mathematical topic from an historical perspective. The impact of affective factors such as interest is evident in the numerous examples of Mandy’s articulation of pedagogical decisions related to introducing her students to the historical development of logarithms.

Mandy as Pedagogical Decision Maker

Mandy appeared to make three types of pedagogical decisions during our discussions about the use of the materials. These decisions were focused upon:

- Modifications of her existing traditional practice when teaching logarithms;
- Considerations for using and offering improvements on the existing lesson installments to better fit the needs of her students; and
- Links to bring together the mathematical concepts included in studying the history of logarithms with topics previously presented.

Modifications of existing practice. Mandy highlighted several instances in which she was willing to alter her existing traditional practice for teaching logarithms. A particular mathematical theme of the historical development of logarithms is the correspondence between an arithmetic sequence and a geometric sequence. During our initial discussions on the feasibility of including all or part of Lesson Installment 3 (Appendix B), which focused on the idea of the correspondence between two sequences, Mandy shared that, “I don’t think at this point in time my students have enough experience with geometric and arithmetic sequences and series. I think they need a little bit more along that line, looking for that” (Professional Development Session 1, 11/15/04). Mandy was essentially saying that prior to her instruction on logarithms her students would not be familiar enough with different types of sequences to be able to appreciate the intended importance of Lesson Installment 3. Consequently, Mandy initially believed she would need to cover the topic of sequences in its entirety prior to beginning the topic of logarithms. Later in our conversation, however, Mandy began to consider a different order of mathematical topics. Specifically, she considered incorporating sequences along with her instruction on logarithms, stating that it would be a “big impetus now to look at it [sequences and series]” earlier in the spring (Professional Development Session 1, 11/15/04).

The next time we met, we began the session by discussing Mandy’s experience with Lesson Installment 3. She expressed her plans more affirmatively and stated, “This sort of sequence concept and everything, I think it’s going to be a good tie in [with the topic her students will be studying just prior to logarithms]. So what I would have done with fractals will really lead them into this” (Professional Development Session 2,

11/22/04). Mandy's consideration for the new order of topics prior to her unit on logarithms was confirmed during the third professional development session:

Kathy: At one point you had said that the sequences and series [within the various lesson installments] might make you want to readjust [your schedule of topics], but then you realized that, 'yeah, they would have had enough study of sequences.'

Mandy: Yes, they've had enough in the sequences and series.

Kathy: So then the February dates are still kind of firm [for the introduction in the history of logarithms]?

Mandy: Yes; no problem with that at all. (Professional Development Session 3, 11/29/04)

The second modification of Mandy's practice that we discussed dealt with the possible placement of the historical materials within the unit on logarithms and exponentials. While we worked through Lesson Installment 3, I raised the question of placement to Mandy.

Kathy: Thinking about the placement, because one of the versions in the curriculum plan (Appendix F) was either do this prior to even opening the textbook during this unit or –

Mandy: Oh! I'll want to do this prior! Because I went back last night and got a couple of other precalculus books – other than the one we use – and looked at their presentation and the more I looked at it, the more I hate it! (Professional Development Session 2, 11/22/04)

In this passage, when Mandy mentioned her displeasure with the presentation of logarithms in the traditional Precalculus textbooks with which she was familiar, she was referring to a presentation of the topic of logarithms which adheres to the following order:

- Define and discuss exponential functions; practice graphing exponential equations;
- Establish (or typically, review) the properties of exponents;

- Define and discuss logarithmic functions as inverses of exponential functions; practice graphing logarithmic functions;
- Establish and practice the properties of logarithms, with special emphasis on the “log of a power” property;
- Using the “log of a power” property, practice taking the logarithm of both sides of an equation for the purpose of solving exponential logarithmic equations;
- Use each of the above skills and concepts in real-world applications.

In this bulleted list of topics and tasks, the concept of logarithm is not explicitly covered. Instead, most traditional treatments of the topic of logarithms include only an examination of *logarithmic functions*, with very little time spent on what a *logarithm* itself is. When Mandy proposed to “do this prior” I understood her to mean that she would first use the installments with her students and then proceed to complete the unit with the topics and tasks that were not included in the historical materials. The decision to first focus on what a logarithm is and then present exponential and logarithmic functions is important in that it utilizes an instructional approach that examines logarithms in the reverse order from how they are traditionally studied. Mandy’s plan to incorporate materials that focus on the historical development of logarithms as an introduction to the unit represents a significant change in her practice.

Mandy’s decision to incorporate the construction and use of slide rules with her students represents a final example of her intent to change her instructional practice while teaching the topic of logarithms. We discussed another way to present the change of base formula for calculating logarithms as well as the construction of tables of logarithms over the centuries and Mandy then focused on the use of slide rules.

- Mandy: The other thing, too, is I definitely would like to bring in a collection of slide rules from home and let them actually play around with them.
- Kathy: This is jumping ahead a little bit, but there's an activity which would lead in to the use of slide rules. Lesson Installment 4 talks about Briggs's methodology for approximating the log of just thirteen numbers. In the original module, there is a lesson on 'how to make your own slide rule' – it's just done for the thirteen numbers that they calculate in Lesson Installment 4. So I don't know if before they played around if you wanted to –
- Mandy: Oh, to have them make it!
- Kathy: It's pretty easy, but if you already have a collection that they could use, it's the same idea.
- Mandy: Yeah, but I think making them would be even better! (Professional Development Session 2, 11/22/04)

We revisited Mandy's idea of using and making slide rules in our third content session. I brought Mandy a copy of the lesson from the original *Exponentials and Logarithms Module* (Anderson et al., 2004), which provided students with directions for the construction of a slide rule. In addition to the physical construction of the slide rule, the activity also utilizes the calculations of logarithmic values from Lesson Installment 4. As I reviewed the activity with Mandy, we discussed the various mathematical considerations it contained, including the emphasis of Oughtred's work related to the invention of the slide rule, the manifestation of the logarithmic scale, and the actual use of a slide rule. We practiced using the slide rule that Mandy and I constructed by first calculating the product of two and three (Appendix G).

- Kathy: Okay, so if you want to multiply two times three, what you do is line up the first number [on the second scale] with the "one" of the first scale and then you look for the three on the first scale. Then, two times three would be six, since the six on the second scale is now lined up with the three on the first scale.

Mandy: Oh, I don't think this is too simple! Let's take three times two and a half. So you've got three times two point five and it's seven.

Kathy: But three times two and a half...it should be seven and a half.

Mandy: Seven and a half, so yeah, I think you're going to get close to seven and a half. (Professional Development Session 3, 11/29/04)

Mandy's plan for her students to construct and learn to use a slide rule that utilized the three properties of logarithms (the logarithm of a product, quotient, and power) is significant for two reasons. First, Mandy considered the use of a more tactile strategy to help students attain a more conceptual understanding of the three logarithmic properties. Mandy stated that she could see that the slide rule activity would help her students to avoid making some of the typical errors related to the application of the three properties, such as calculating the logarithm of a product to be the product of the two logarithms (Professional Development Session 3, 11/29/04). Second, Mandy considered the exposure to slide rules to be a worthwhile activity. The importance of slide rules is often only trivially mentioned during instruction about logarithms. With modern calculating devices such as computers and scientific and graphing calculators, slide rules are clunky, outdated, and often (depending upon the age of the mathematics teacher) difficult to figure out. Mandy believed that her own knowledge would be enhanced by (re)learning how to use a slide rule and that her students' experience with the history of logarithms would be strengthened by including an examination of the construction, use, and application of the device.

Usage and improvements of lessons. Throughout our discussions during the professional development sessions, Mandy shared several instances of her plans for using the existing lesson installments. Her plans for using the lesson installments varied across a broad spectrum, ranging from order of usage to experimenting with the content of the lesson installments with an independent study student. Table 8 outlines several examples of how Mandy planned to use the lesson installments.

Table 8

Decisions Related to Lesson Installment Usage: Mandy Wilson

| Description of practice | Evidence of plans or decisions |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Mandy described her vision for how students could investigate the content of Installment 1, the installment which asks students to investigate the particular people and influences involved in the development of logarithms. | Mandy stated that she wanted the activity to remain “very open-ended with this particular group of students” and that she planned for them to “come back and hold a very good discussion” over each student’s research of historical figures (mathematicians, scientists) involved in the development of logarithms (Professional Development Session 1, 11/15/04). |
| Lesson Installment 3 was quite long and covered a broad range of mathematical topics and techniques related to the development of logarithms. In order to consider possible improvements on the lesson as well as potential student difficulties and engagement with the material, Mandy planned to have an independent study student work on Lesson Installments 1, 2, and 3. | During a discussion concerning the directions associated with the proofs found in Part II of Lesson Installment 3, Mandy shared that she would experiment with the lesson by having the student who was taking Precalculus via independent study work through the material first. She revealed that she would “try this whole thing through with her first, to see how it works out” (Professional Development Session 2, 11/22/04). |
| In many instances, the student directions that were given within the lesson installments became a topic of discussion related to Mandy’s considerations for classroom use. In both the <i>Introduction/Transition</i> and <i>Exploration</i> section of Lesson Installment 3, Part III, students are asked to complete sequences using two equivalent notations. Mandy and I talked about the usefulness of the alternative notations in the context of the | Although I had concerns about the potential of student confusion resulting from the directions and the redundancy of two of the questions, Mandy once again thought that she would “leave the directions open-ended and see which way they’re looking at it” (Professional Development Session 2, 11/22/04). |

| Description of practice | Evidence of plans or decisions |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| students' study of logarithms. | |
| The <i>Recall</i> section of Lesson Installment 4 ends with asking students to investigate an approximation of 75 using only powers of 2 and 3. This example was designed to get students thinking about one way for approximating the logarithmic values explored in Installment 4. | “And I think I’m going back to 75 on the previous page... I think I would definitely do this one on the board” (Professional Development Session 3, 11/29/04). |

Closely related to Mandy’s decisions for how to use the individual lesson installments and the content contained within them are the connections Mandy identified as bringing together the mathematical concepts included in studying the history of logarithms with topics previously presented to her students.

Connections for students. Mandy’s motivation for asking to be included in the study originated from her broad interest in the history of mathematics. She asked her students to spend time during the first six weeks of the 2004 – 2005 academic year exploring the historical development of the number system. Students summarized their study with the construction of a time line (M. Wilson, personal communication, 11/04/04). With her underlying interest in the history of mathematics as a tool to situate Precalculus topics for her students within the broader context of human development and learning, Mandy viewed the historical materials and lesson installments covering the historical development of logarithms as a way to make explicit connections to other topics. Mandy provided a specific example of this type of connection when she reflected about presenting a more contemporary historical topic of fractals. In her example, Mandy planned to motivate a study of the history of logarithms by covering the history of the development of fractals (a unit which would occur just prior to logarithms). When we discussed Mandy’s second semester curricular plans, she noted that her students’ current

(November – December 2004) work of following up the study of complex numbers with an exposure to fractals would fit nicely with the historical approach to logarithms.

Mandy: Actually, in planning what I'm doing right now, we just finished complex numbers today. It's a good stopping point before break. We're going to do fractals right afterwards. And this sort of – the sequence concept and everything – I think it's going to be a good tie in. So what I would have done with fractals will really lead them into this.

Kathy: Do you get to the point on fractals where they work with non-rational dimensionality? Is that where you end up?

Mandy: I'm not sure... it's a better group than I've ever had before. I'd like to get there, so we'll see. They're doing Internet research over the holiday on fractals. I think it would be nice for them to see something very, very new within the history of mathematics and contemporary along with their historical position of it. (Mandy's emphasis, Professional Development Session 2, 11/22/04)

Mandy's ability to consider appropriate links between the curriculum she was currently providing for her students and the opportunities that the inclusion of the historical development of logarithms would present for them was a significant feature in her practice.

A powerful example of Mandy's desire to strive for providing a coherent Precalculus course for her students is found in her concern for weaving an earlier exposure to closure notation together with the study of history of logarithms. In Lesson Installment 3, directions guide students to summarize their findings after manipulating terms of a sequence using multiplication and division. Mandy found it helpful to connect the investigation of the early motivation for and the development of logarithms in Installment 3 with the work her students had previously completed using formal mathematical notation. We discussed an alternative to the directions found in Installment 3 to better meet the needs of her students based upon this previous work:

Mandy: I had them talking earlier about closure. So they used the notation of just the idea of closure coming in through here. Would you accept that as notation for them?

Kathy: Yes; in fact, when I was writing up some answers to something later on, I was using mathematical notation, like ‘is an element of’, or positive integers, ‘ Z^+ ’, for that. And I don't think we get the opportunity to use that kind of notation with high school students, because it's left out.

Mandy: Yes! I think it's great to be exposed to it at least.

Kathy: This is a great way to practice that. So yes, that would be fine. I know somewhere early on, I said, ‘be sure to use complete sentences in your response’. But, if the point of writing like, ‘the reason why the operation is addition in S_1 is because...’ – if that is not necessarily something that needs to be practiced –

Mandy: I think this group does not need to practice writing for communication on that level.

Kathy: Then that can be a direction that is taken out, I think. I think maybe something instead of ‘please use complete sentences,’ use ‘make your mathematical idea clear.’

Mandy: Yes. And let them see how the closure notation looks in this instance. (Professional Development Session 2, 11/22/04)

At the end of this discussion, Mandy adjusted the directions for Problem 3 of Installment 3 to read “In summary, use closure notation” (Professional Development Session 2, 11/22/04). Whereas the intent of the original directions was to provide opportunities for students to summarize in writing what the results of the manipulations of the sequence terms yielded, Mandy felt that the lesson could be altered to better meet the needs of her students by connecting to their prior experience with closure notation. In this way, Mandy’s students would be able to practice an important aspect of mathematical communication during their initial study of the historical development of logarithms.

A final example of Mandy’s ability to think about the historical development of logarithms in such a way that she simultaneously considered connections across topics for her students is embedded in our discussion of Lesson Installment 4 (Appendix B). Our investigation of Installment 4’s content allowed us to engage in several tangent conversations related to how students might benefit from an activity that asks them to calculate several logarithmic values. Although the activity is valuable for its cultural-historical perspective which signals the importance of how such values came about in the first place, Mandy found the inclusion of Installment 4 important for other reasons. After we discussed Mandy’s plans to incorporate the additional slide rule activity, we contemplated the value of using a physical model to assist with students’ conceptual understanding of logarithm:

Kathy: And so this physical model is helpful because of the fact that it appeals to the tactile side of the brain. It helped my students a little bit. I’m not saying that it eliminated the trouble of students grasping the logarithmic properties. Your students might not even experience errors with using the properties.

Mandy: No, I can see some of them that will.

Kathy: It goes back to that idea – that’s not actually a logarithm thing – it’s a definition of function thing.

Mandy: It’s really the concept of it!

Kathy: Yeah, it’s conceptual, because they see “log of the quantity x plus y ” and, unfortunately they think of the distributive property. It’s not a number; it’s a concept, like you just said.

Mandy: But I think this would help. We [her calculus class] just did [she writes the following expression down on a sheet of paper]:

$$f(x) = \frac{\left[(3x^2 + 5x)^3 (8x^3 - 2x + 3)^{\frac{1}{2}} \right]^3}{(4x^3 - 3x^2 + 2x)^3}$$

Kathy: Factoring these kinds of things?

Mandy: No! What *is* it? And it's something that I heard years ago that I think is just really, really neat. A biblical statement, when God talked to Adam and he said, "You name the animals. If you name the animals, you have control over them." (Mandy's emphasis, Professional Development Session 3, 11/29/04)

For the next several minutes, Mandy and I worked on "naming the animal." In this case, the "animal" was a rational expression composed of many terms. During this segment of the professional development session, Mandy shared with me how she considered using the history of logarithms with her students in order to increase their conceptual, and hopefully long-term, understanding of what a logarithm is. This example of identifying and providing connections for her students was a longitudinal consideration. Instead of identifying an outcome of the use of the history of logarithms that students would benefit from in the short-term (during their Precalculus course), Mandy was thinking forward to when the same students would encounter logarithms in a calculus course. Specifically, if students gain a more conceptual understanding of what a logarithm is – instead of relying upon memorization of properties which are often incorrect – then the realization that the application of the properties assists in tasks such as logarithmic differentiation becomes a long-term benefit.

Mandy anticipated that connecting her "naming the animal" technique with the lesson installments that focus on the historical development of logarithms would result in improved and sustained understanding for her students. She summarized this observation by saying,

Then what I would have them do is go through and tell me how you're going to write it in logarithmic form. And then I show them how much easier it is to do the logarithmic differentiation. (Professional Development Session 3, 11/29/04)

Significant Secondary Features

After I identified the three primary features characterizing Mandy's engagement during the three professional development content sessions, I realized that three secondary features surfaced as reinforcements for quantifying Mandy's practice as an active collaborator, anticipator of student engagement, and pedagogical decision maker. Most of the evidence used to identify the three secondary features of Mandy's engagement during the professional development sessions overlapped with examples previously set forth in the examination of the primary features. The next three subsections highlight Mandy's commitment to continue her learning outside our scheduled meeting times, her ability to critically reflect on the materials and resources that we discussed, and her desire to continue her study of related topics to address "holes" in her current understanding of the history of logarithms.

Commitment to Learning

Teacher interest in the history of logarithms and a commitment to actively participate were key assumptions that motivated my work during the professional development component of this research. Although the initial design of the lesson installments was completed before meeting with the teachers, I continually probed the teachers for their improvement ideas, surprises and challenges that they encountered during their completion of the lesson installments, and additional content or pedagogical needs. Upon reflection of her interest in the activities presented in the lesson installments, it is worthwhile to comment on Mandy's commitment to learning about the history of logarithms, which extended beyond our series of sessions together.

The importance of historical figures. Previously in this chapter I provided an example of Mandy’s eagerness to share resources, specifically materials that she encountered while conducting background research on the historical figures and mathematical motivation for the development of logarithms discussed in Lesson Installment 1. Mandy commented that several individuals captured her attention during her research including Briggs, Glaisher, Stifel, Bürgi, Chuquet, and Ibn Yunus. Her commitment to know more about these individuals with respect to their contribution to the historical development logarithms is evident in the questions she asked. Table 9 contains examples of questions and comments raised by Mandy during the first professional development session. The first session occurred eleven days after Mandy had received her resource binder, which included the first two lesson installments among other items. In the intervening time period she utilized the internet to investigate the mathematicians and motivations behind the development of logarithms.

Table 9
Significance of Historical Figures: Mandy Wilson

| Mathematician (historical figure) | Sample question or comment (Evidence) |
|-----------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Briggs | “I was very surprised at Briggs. That’s one mathematician I had <u>never</u> encountered before at all!” (Mandy’s emphasis, 11/15/04) |
| Stifel | “And then there was Stifel. Is he attributed to having invented logarithms also?” (11/15/04) |
| Chuquet | “I’ve never, never come across him before at all. Very, very interesting. And it was interesting because x to the zero power or any number to the zero power – where did that originate?” (11/15/04) |
| Ibn Yunus | “Well, I was interested in Ibn Yunus.” (11/15/04) |

The final comment regarding Ibn Yunus encouraged our conversation to take an interesting detour. Mandy stated on numerous occasions that her interest in the history of logarithms was precipitated by her long-held view that the human contribution factor was a necessary facet for teaching mathematics. Mandy's commitment to study for herself and subsequently provide a cultural-historical view for her students underlies the following exchange:

Mandy: Well, I was interested in Ibn Yunus because I'm working on some other material. Have you read a book called *Aristotle's Children*?

Kathy: I haven't read that one yet.

Mandy: It's well worth looking into. He goes into some of the very early Islamic cultures, for the connections going through here...more from the logic and philosophy. I had no idea his [Ibn Yunus] observations were as sophisticated as they were for that time frame.

Kathy: I also think what is very strong about his [Ibn Yunus] inclusion is we forget, especially if you're very Anglo/European-American, you forget that so much of mathematics is being done during the Middle and Dark Ages.

Mandy: I think you would enjoy reading this book because it talks about how it was developed in the East and then it ceased, and where that transition ceased. (Mandy's emphasis, Professional Development Session 1, 11/15/04)

Mandy had a desire to connect her current study of the history of logarithms with other directions of study that she pursued. Investigating the contribution of all cultures to the development of mathematical thought was a self-proclaimed and consistent theme of Mandy's practice. Her investigations were enhanced by the opportunities to study the historical figures instrumental in the development of logarithms.

Revisiting the content. I provided Mandy a brief description of Napier's two particle argument in her resource binder at our first meeting in November 2004. Napier's description of logarithm was the focus of Lesson Installment 2 and we planned to explore the argument during our first professional development session. After developing the argument which culminated in Napier's definition of logarithm, I promised Mandy a written version of the development at our next session and she indicated that she would try to reconstruct the argument for herself. Before I could provide her with the written copy, however, Mandy broached the idea of where to include Lesson Installment 2 in her unit on logarithms.

Mandy: I started looking at where am I going to go from there [logarithmic notation] or what I am going to do with that. And I went back on over here.

Kathy: To Napier's argument?

Mandy: I do think that on Napier's argument, on going with that, I think we need to go at least one more iteration.

Kathy: I go to the fourth power, actually [in the written version].

Mandy: Do you? Okay, and just part of it to recall that they're working with the factoring and coming up through with it. I would definitely say go on to one more iteration with it. (Professional Development Session 2, 11/22/04)

At this point in our professional relationship, I was not surprised at Mandy's intent to work on not only the lesson installments that we would be covering in any one session, but the material from installments already covered. Mandy viewed herself as a learner and as such, needed to return to previous content as she reflected on the placement, purpose, and outcome of subsequent material.

Perception of her role as a learner. Mandy was completely comfortable participating during the professional development sessions as both a collaborator and a learner. During the third session, Mandy admitted that she was unable to complete Lesson Installment 5 before meeting together. I asked her if she wanted to work through the installment together, to which she replied, “No. I’m going to work on it, because I really want to dig in to that” (Professional Development Session 3, 11/29/04).

There are several instances from the three professional development sessions in which Mandy offered other information about her efforts related to the seven lesson installments. I inferred from Mandy’s verbal clues that she consistently worked through the problems and activities contained in the individual lesson installments. Table 10 outlines several examples from our conversations that prompted me to quantify a consistent commitment to learning about the history of logarithms beyond the formal professional development sessions as a feature of Mandy’s engagement. It was important for Mandy to “dig in to” the material on her own rather than waiting to receive the information passively during the professional development sessions.

Table 10
Evidence of a Sustained Commitment to Learning: Mandy Wilson

| Lesson installment number (and title) | Example description | Evidence |
|-----------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2 (Napier’s Description of a Logarithm: The Two Particle Argument) | Mandy and I discussed the integration of the contributions of Napier and Briggs related to the descriptions provided in Napier’s <i>Descriptio</i> . | After a summarization of Briggs’s recommendations to Napier on how to fix his table of logarithms and related definitions Mandy replied, “Fix that up; get this rule in there. Yeah, I remember reading that part [from the online Wright translation of the <i>Descriptio</i>]” (Professional Development Session 1, 11/15/04). |
| 3 | After working on Lesson | “When I went back last night and got |

| Lesson installment number (and title) | Example description | Evidence |
|---------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (The Development of Logarithms Using Sequences) | Installment 3, Mandy considered the placement of it relative to her traditional instruction while teaching about logarithms. This prompted her investigation of how various textbooks have presented the development of logarithms. | a couple of other precalculus books, other than the one we use, and looked at their presentation and the more I looked at it, the more I hate it!" (Professional Development Session 2, 11/22/04). |
| 4 (Calculation of Logarithms Using the Method of Napier and Briggs) | Mandy identified the student appeal of the structure of Installment 4's content and discussed (relative to two different examples) her engagement with the calculations required. | In an effort to explain her pages of calculations from the previous week, Mandy stated: "I was literally just sketching these things [the calculations required to approximate the logarithm of 1 through 13] really, <i>really</i> quickly." Additionally, while we discussed how to calculate the logarithm of 13, Mandy referred back to her calculations for log 3: "I lost my threes. I've got them here somewhere." (Mandy's emphasis, Professional Development Session 3, 11/29/04) |

Critical Reflection

Many of the instances selected to help describe Mandy's commitment to learning could also be referenced in relation to her ability to critically reflect on the content, materials, and perspectives related to the historical development of logarithms. Merriam-Webster defined *critical* as "exercising or involving careful judgment or judicious evaluation" (1993, p. 275). I used this definition as a lens to characterize many of Mandy's utterances regarding the quality and application of the lesson installments.

Observations of quality and suggestions. Much of the evidence pointing to Mandy's desire to critique the lesson installments and associated resources was based in her desire to improve upon the materials. Since she was fully aware that another group of teachers (and, hopefully students) would be using the lesson installments in the very near future, I considered her comments to be extremely helpful. Additionally, when Mandy offered suggestions for improvements on question stems or activity directions she did so with the intent to not only inform me for future use of the materials, but also for her future use of them with her students.

One such example of Mandy's reflection on the quality of resources appeared in the narrative related to the sharing of resources, within the characterization of Mandy as an active collaborator. In that example, Mandy observed that the website, *Scotland's People*, contained at least two errors. This was not one of the original websites offered in the resource binder, rather one that Mandy found during her own Internet search. Mandy did not only identify actual problems with resources, however. She also deliberated on the potential problems that such resources could cause. In the following excerpt, Mandy questioned one source's claim that Michael Stifel also invented logarithms, even though her initial research informed her that Briggs and Napier originally developed logarithms.

Mandy: And then there was Stifel. Is he attributed to having invented them also?

Kathy: No, not the logarithms. He did extensive work with exponential notation and he did some work with linking arithmetic and geometric sequences. I found several errors with regard to him [on the internet]. That since he is linked to logarithms –

Mandy: Yes, it says, “Stifel’s research was arithmetic and algebraic. He invented logarithms independently from Napier.” (Professional Development Session 1, 11/15/04)

Mandy's dedication to considering each resource and each lesson activity in this critical manner is an important piece of describing her practice during the professional development sessions. Later in this chapter I discuss the connection between Mandy's professional development experiences while studying the historical development of logarithms and the experiences she provided her students. While describing these connections, Mandy's engagement with the historical material at the one level (during the professional development sessions) impacts the experiences she provided for her students.

Additional critiques related to lesson installment quality focused on suggestions for improvements to enhance student use of the materials. For example, Mandy and I discussed the feasibility of asking students to provide three cases for Problem 2 of Lesson Installment 3.

Kathy: I picked three cases. Do you think that three is reasonable?

Mandy: I think that's a very reasonable number. This is a question I wanted to know, though. When you were talking about notation – how formal of notation?

Kathy: I think at this level, the notation is basically whatever you want to use. It's not meant to be highly formal at this point. (Professional Development Session 2, 11/22/04)

This exchange continued with a discussion about the use of closure notation (*Connections for Students* within the *Mandy as Pedagogical Decision Maker* section), however Mandy was concerned about understanding the directions as they are and how they may be improved.

We continued to concentrate on directions to students in the lesson installments when Mandy asked about Part II of Lesson Installment 3. The directions for Exercise 10 of Lesson Installment 3 are:

Exploration: Work with a partner on each of the following. You should record several cases for each of the three properties you are confirming in Exercise 9. You will only need to verify each property once in Exercise 10. With your partner, decide *before* beginning Exercise 10 if you are going to work independently first and then compare results; or if you want to work together to decide on how to verify each.

[Exercise] 10. To verify your conclusions in Exercise 9, use the function notation below. Let u and v be terms of $S4$. Use algebraic manipulation to show the following three properties hold.

As we compared our individual work for Exercise 10, Mandy indicated that she wanted to see how I had completed the proofs for each of the three properties of logarithms.

After she considered the mathematics involved in completing the proofs, she decided that she wanted to alter the directions so that they better reflected the students' experiences with proofs:

Kathy: Would you imagine changing the directions to give them guidance on that, for this particular one?

Mandy: Let me think about that. I might. What I might do, Kathy, is I might go through and say, let's do one numerically. Now, go through and do a formal proof on it because these students haven't done proofs since geometry.

Kathy: So it's been two years?

Mandy: It's been two years for many of them; two or more years. And see, many of them took geometry in eighth grade. So there was not a whole lot of proof at that stage of the game. I'm wondering if maybe doing one of them and actually demonstrate for them.

Kathy: And maybe prompt them, if you do this numerically, or even for the generic case of their base b notation. But give numerical values for the exponents and then ask them, what would be a good thing

to call “u” in general? (Professional Development Session 2, 11/22/04)

Although Mandy consistently exhibited her ability as a pedagogical decision maker when she considered the best way to use the lesson installments with her students, she was able to do so by exercising careful judgment and by suggesting critical improvements. While summarizing her suggestions for Exercises 9 and 10 of Lesson Installment 3 Mandy offered that, “we need some kind of transition in here [between Exercises 9 and 10]” (Professional Development Session 2, 11/22/04). Such a transition also served to improve the flow of the activity. The new flow included Mandy’s suggestion of requiring students to numerically investigate one property, followed by her demonstration proof of the property and students proving the remaining two properties.

A final example of Mandy’s willingness to be open and constructive in her critical feedback is evident in her opinion of the question stems accompanying Lesson Installment 4. After students (or, in this case, Mandy) are asked to calculate the logarithm of the numbers 1 through 10, they are prompted to answer several questions about the numerical relationships that they encountered. Sample questions included:

Which of the above calculations yield exact results?

Which of the above calculations yield approximate results?

How do you know?

Was it more difficult to calculate the logarithm of some values than others?

Why?

In response to whether to include the questions, Mandy observed that, “the ‘why’ part, that’s the most important part” (Professional Development Session 3, 11/29/04).

Mandy's comment provided a validation of sorts for the construction of the lesson installment. The version of Lesson Installment 4 used for this research reflects significant changes when compared to how it appeared in the Anderson et al. (2004) *Exponentials and Logarithms Module*. Specifically, the edited version of the lesson contains less guidance in the calculation of logarithms activity and an increased cognitive demand in the accompanying questions. Mandy's brief observation about the importance of the questions related to the student investigation was not only helpful, but also enabled us to continue our conversation about the underlying mathematics contained within the rest of the Lesson Installment 4 questions.

Throughout the time we spent together, Mandy and I examined the historical development of logarithms, the lesson installments and associated resources, and how these professional development experiences would inform her practice with students. On several occasions, Mandy was openly candid about her need for additional knowledge or assistance to complete the lesson installments.

Self-identification of Knowledge Gaps

On two occasions, Mandy expressed that she was not as knowledgeable as she wanted to be with regard to topics related to the historical development of logarithms. In both instances, Mandy commented that she desired to learn more about the topic and I spent time discussing relevant background information that might be helpful to her as she began her investigation.

Astronomy and trigonometry. The first entry of Table 8 referred to Mandy's plan for using Lesson Installment 1 with her students. In a discussion about her plans, Mandy reminded me of the first activity that she did with her Precalculus students at the

beginning of the school year, which involved historical research on the development of the various number systems.

Kathy: Did you by chance get to look at any of the links with the astronomers [while working on Lesson Installment 1]?

Mandy: Just a little bit. I got to Tycho Brahe and that was about it. I know very little about astronomy [laughs] and so I do need to do a lot more work on that topic.

Kathy: Do you mind if I stepped in and said a little bit about that?

Mandy: Oh, no!

Kathy: Developing naturally into logarithms, I think, the motivation was in fact a need for the improvement of astronomical calculations. Just to share, I brought these little photographs of Tycho Brahe's property in Sweden. (Professional Development Session 1, 11/15/04)

I continued (for too long, perhaps) to share with Mandy what I had learned about Brahe's life in Denmark (now Sweden) in an effort to put into perspective the calculations being performed by Kepler, Brahe, and Napier. The astronomical calculations in the 16th and 17th centuries required the trigonometric sine of angles and these sine values were based on lengths of half-chords of central angles of a circle with a very large radius. Consequently, Mandy directed me to the next gap in her knowledge.

Kathy: Did you have any other questions about things that you found?

Mandy: No, but you know, I want to go back. I want to go back more into the trigonometric functions. I think that's just research I need to do myself.

Kathy: I found some [Internet] links on Hipparchus and Ptolemy – they created the first tables of sines. But even when you find it on the Internet, it's a little difficult to follow because everything's done in sexagesimal notation. But at least you can get the background. Also, Aryabhata, from 900ish, there's a really great website that I found. It gives a brief overview, but it tells you all the nuts and

bolts of the trigonometry and the tables, so I can send you that link as a place to start. (Professional Development Session 1, 11/15/04)

Mandy's willingness to articulate that the mathematical story leading to the development of logarithms was for her incomplete allowed us to interact and collaborate during the professional development component of this research. By doing so, we were able to construct the story of logarithms for Mandy and reconstruct and revisit the story of logarithms for me.

Instructional Practice

Mandy used the history of logarithms content from the professional development sessions to reconceptualize and restructure her instruction related to logarithms. Of the four lesson installments that were the focus of our professional development sessions, Mandy chose to use all four with her students. Mandy presented her students with experiences about the historical development of logarithms prior to a traditional treatment of the topic. Unlike traditional instructional practice, Mandy opted to present logarithms before exponentials. She also focused on the idea of a logarithm as the value of an exponent x that will make b^x (where b is a positive real number) equal to a positive number, as opposed to a strict treatment of logarithmic functions. In addition, Mandy made several pedagogical decisions related to incorporating the lesson installments, including modifying Lesson Installment 1 to better fit an activity completed early in the school year and enhancing Lesson Installment 2 by requiring students to read a translation of an original document (Napier's *Descriptio*, 1614).

The next four sub-sections discuss Mandy's self-reported instructional practice relative to logarithms which she utilized prior to this investigation, as well as her beliefs

about her role as the teacher, her beliefs about the student's role, and her consideration of school factors influencing instruction.

Existing Instructional Practice

Item 6 of Part III of the Attitudes Instrument (pre-assessment only) asked the participants to *outline how you usually approach the teaching of logarithms* (Appendix C). Mandy's approach to teaching logarithms originated with an examination of exponential functions, followed by the use of geometric transformations; evaluating expressions using logarithmic and exponential conversions; modeling applications involving logarithmic and exponential functions, and finally, solving logarithmic equations (Attitudes Instrument Pretest, 11/04/04). For the most part, Mandy's prior instructional practice utilized a functions-based approach.

Beliefs about the Role of the Teacher

Mandy believed her role as a mathematics teacher to be one in which she served as

[a] resource for the students so that they can become self-learners. I do a very didactic approach to teaching and I keep asking them questions. And I will not stop asking them questions until they start looking for and finding the answers. I don't see my role as just imparting knowledge. (Interview, 4/20/05)

Mandy also noted that "a very minor part of [her] teaching" (Mandy's emphasis, Interview, 4/20/05) was delivering content via lectures. She observed that on occasion, it was more efficient and sometimes necessary to deliver a lecture. Mandy further articulated her teaching philosophy to include helping students to "learn how to learn" (Interview, 4/20/05). She summarized her philosophy with the notion that,

What you're going to be learning 25 years from now hasn't been invented yet. So you've got to learn how to learn. My role is to give you confidence so that you

can learn; to try to give you techniques so that you can learn efficiently; to learn how to use resources; and to have a joy of learning. (Interview, 4/20/05)

Beliefs about the Role of the Student

Mandy's view of her role as teacher was parallel to her description of what she considered a student's role to be. Mandy acknowledged that,

I want the student to take ownership of his learning. The worst thing to me is to have a student regurgitate knowledge as facts. So I want a student to approach a problem thoughtfully...thoroughly...with great precision of language, both written and spoken. (Interview, 4/20/05)

A concentration on a "great precision of language, both written and spoken" was consistent with High Acres focus on a rigorous, liberal arts education and Mandy's interest in a humanities approach to presenting mathematical topics. In addition, an emphasis on the use of written and spoken language will be highlighted in a subsequent section describing Mandy's implementation of the historical development of logarithms.

I prompted Mandy further on her desire for students to take ownership of their learning. In response to whether she had any expectations for students which would go beyond the boundaries of learning and participating in class, Mandy observed,

This environment is very flexible. For example, I do think that by the time they get into the eleventh and twelfth grade, or the few tenth graders who have gotten into [Precalculus], they need to take responsibility of my giving them long-term assignments and saying, okay, this is what we're covering over the next three weeks. Here are your test dates; I'm not going to say the problem has to be turned in tomorrow or the next day. You've got many other requirements and responsibilities – you learn to balance it. As an example, seniors are working on their thesis right now. So we're balancing; we're letting go of a few things. (Interview, 4/20/05)

Here, Mandy revealed that not only do students need to learn how to balance academic responsibilities, but the school as a community participates in the process of "balancing." To prevent interpreting the "letting go of a few things" to be synonymous with loosening

the rigor of the content, Mandy clarified with, “we have some very gifted students here and I won’t accept mediocre work from gifted students. They’re going to be prodded until they work up to their ability” (Interview, 4/20/05). Additionally, High Acres teachers revisited student learning goals and the activities designed to address them after they conferred with each other about lengthy and demanding projects.

Influence of School Features

Mandy indicated several school factors which influenced her daily work in general and her instruction focused on the history of logarithms.

Assessment. Mandy identified High Acres School as a school environment that was “very flexible.” Without the pressures of high-stakes testing which public school teachers experience Mandy was able to incorporate the historical development of logarithms during the unit on exponential and logarithmic functions. Although the school was not required to operate under state or local assessment and accountability systems, many students arrived at High Acres with recent high-stakes assessment experience. On rare occasion, students new to the High Acres community associated learning new mathematics content with identifying the correct answer at the end of a problem. Mandy reflected on her experience of incorporating the history of logarithms using the lens of assessment.

Students were surprised and pleased when I said that there would be no formal evaluation of what they had done in that unit. Instead, it was an experience of learning for the sake of learning without testing. It was nice for them to see that they’re not tested over everything that they have to learn and I think it’s very much an impetus for them. (Interview, 4/20/05)

Awkward scheduling. Mandy identified that the High Acres daily instructional schedule coupled with weather events of the second semester hindered some of her

efforts at incorporating the historical development of logarithms. At High Acres, each class met for 45 minutes on Monday. For the remainder of the week, each class met for 80 minute blocks, with odd-numbered period classes meeting on Tuesdays and Thursdays and even-numbered period classes meeting on Wednesdays and Fridays. Disregarding snow events that forced school to be closed, Mandy felt that “with this type of student, the timing was really rough, with the one short block, long block, long block” (Interview, 4/20/05). When weather events forced school to be closed (inevitable during the region’s winters), however, the loss of an 80-minute class impacted the flow of topics. Thus, when Mandy considered what she would do differently, she responded that she “may have condensed some of the topics a little bit” when incorporating the historical development of logarithms (Interview, 4/20/05), especially in light of lost class time.

Multi-tasking. Traditional secondary mathematics instruction is comprised of teachers covering content one unit at a time. For example, exponential and logarithmic functions may appear in Chapter 11 in a particular textbook and sequences and series in Chapter 12. During Mandy’s instruction, however, it was typical for her to cover multiple topics simultaneously. When I asked Mandy whether her choice to cover the historical development of logarithms while also providing her students opportunities to examine arithmetic, geometric, and other recursive sequences was out of necessity (i.e., to save time) or typical of her practice, Mandy observed,

That’s typical and I often cover multiple ideas at the same time. I think one of the criticisms that I have of education is that math has been so much in a vacuum, without topics being related to each other. So any way that you can put two of them together is a good thing. (Interview, 4/20/05)

Chronology of Instruction

Mandy taught three Precalculus classes in 2004 – 2005. The chronology of instruction related to logarithms appears in Table 11. Mandy’s instruction on the topic of logarithms occurred from February 14, 2005 until March 25, 2005. Data were collected until March 8, 2005, with the exception of the week of February 21, 2005 (High Acres’s winter break) and February 28 and March 1, 2005 (classes cancelled due to snow). Regrettably, I was unable to collect data beyond that date. Mandy’s schedule of topics included an historical treatment of logarithms (2/14/05 – 3/10/05), followed by a traditional survey of exponential functions, logarithmic functions, converting between exponential and logarithmic notation, solving exponential and logarithmic equations, and applications of exponential and logarithmic functions (3/15/05 – 3/18/05; 3/21 – 3/25/05).

Table 11

Instructional Schedule: Mandy Wilson

| Date | Class activities: Period 1 | Class activities: Period 2 and 4 |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2/14/05 | Discussion of research assigned the previous week: what is the history of logarithms? | <u>Period 2:</u> Same as Period 1 <u>Period 4:</u> Began similar to Periods 1 and 2, but of the three students in the class, only one had begun the assignment. Mandy assigned students to study hall to complete their research. |
| 2/15/05 | I. Review of homework: arithmetic and geometric sequences. II. Review of history of logarithms research III. Students began examination of the 1616 English translation of Napier’s <i>Mirifici logarithmorum canonis description</i> | Classes not scheduled to meet |
| 2/16/05 | Class not scheduled to meet | <u>Period 2:</u> Same as period 1, 2/15/05 <u>Period 4:</u> Same as Period 1, 2/15/05 |

| Date | Class activities: Period 1 | Class activities: Period 2 and 4 |
|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2/17/05 | <p>I. Students responded to and posted responses to three reflective questions</p> <p>II. Class examined the first ten terms of the sequence</p> $a_n = \left(1 - \frac{1}{10}\right)^n.$ <p>III. Mandy presented Napier's two particle argument to the class</p> | Classes not scheduled to meet |
| 2/18/05 | Class not scheduled to meet | <p><u>Period 2:</u> Same as Period 1, 2/17/05</p> <p><u>Period 4:</u> Students shared research on proving/disproving Lord Moulton's statement; continued with content similar to Period 2</p> |
| 3/02/05 | Class not scheduled to meet | <p><u>Period 2:</u> I. Students shared content of their research papers II. Students worked on first three pages of Lesson Installment 3</p> <p><u>Period 4:</u> Same as Period 2</p> |
| 3/03/05 | Same as Period 2, 3/02/05 | Classes not scheduled to meet |
| 3/04/05 | Class not scheduled to meet | Classes completed Lesson Installment 3 |
| 3/07/05 | Completed Part II of Lesson Installment 3 | <p><u>Period 2:</u> Began work on Lesson Installment 4</p> <p><u>Period 4:</u> Students shared content of their research papers; began work on Lesson Installment 4</p> |
| 3/08/05 | Completed Part III of Lesson Installment 3 and Lesson Installment 4 | Classes not scheduled to meet |
| 3/09/05 | Class not scheduled to meet | <p><u>Period 2:</u> Class constructed and used a slide rule using the estimated values from Lesson Installment 4</p> <p><u>Period 4:</u> Same as Period 2</p> |
| 3/10/05 | Same as Period 2, 3/09/05 | Classes not scheduled to meet |
| 3/11/05 & 3/14/05 | Test: Sequences and Series | Test: Sequences and Series |
| 3/15/05 – | Covered traditional topics from | Covered traditional topics from |

| Date | Class activities: Period 1 | Class activities: Period 2 and 4 |
|----------------------------------|-------------------------------------------------------------------|----------------------------------------------------------------|
| 3/18/05; 3/21/05 – 3/23/05 | textbook, Chapter 10: Exponential and Logarithmic Functions | textbook, Chapter 10: Exponential and Logarithmic Functions |
| 3/24/05 – 3/25/05 | Test: Exponential and Logarithmic Functions | Test: Exponential and Logarithmic Functions |

Incorporating the History of Logarithms

Mandy implemented the history of logarithms materials by incorporating the following activities:

- Research on the “human element” of the historical development of logarithms, including (1) informal student research on the history of logarithms; (2) class discussions about the who, when, where, why, and what (five journalistic questions) of the historical development of logarithms; and (3) student research papers on some aspect of the historical development of logarithms (modification of Lesson Installment 1)
- Original document reading: 1616 English translation of Napier’s *Descriptio* (enhancement of Lesson Installment 2)
- Two particle argument presentation (Lesson Installment 2)
- Development of logarithms using sequences (Lesson Installment 3)
- Calculation of logarithmic values (Lesson Installment 4)
- Construction and use of a slide rule (an addition to the original history of logarithms lesson installments)

Researching the “Human Element”

Mandy indicated on her pretest response to Item 1 of Part II of the attitudes instrument that she believed researching mathematicians “helps students understand the human element of mathematics” and that investigating “a mathematician’s process of invention helps students understand the balance between creativity and proof” (Attitudes Instrument Pretest, 11/04/04). She also claimed during the first professional development session that she wanted to incorporate Lesson Installment 1 in an open-ended manner and

that she planned “to come back and hold a very good discussion” (11/15/04) about each student’s understanding of contributions to the historical development of logarithms.

Mandy did in fact incorporate the content of Lesson Installment 1 in an open-ended manner. Instead of simply assigning students to construct a timeline, Mandy began by asking her students to research the history of logarithms over the weekend prior to beginning the unit. Then, students were asked to share the information they found by responding to the “journalistic questions” (Observation, 2/14/05) of who, where, when, why, and what. Mandy initially requested responses to address the first four of these Mandy explained to her students that they would address the “what” later in the week.

For three days and in the three different class periods, Mandy probed students for historical information that would lay the foundation for the research papers students were required to complete over winter break. While students shared their initial research, Mandy called attention to the human element contributing to the development of logarithms, as well as the experiences of her students. For example, when a student in the first period class described Napier’s work, *The Construction of the Wonderful Canon of Logarithms* (1619), Mandy used the students’ interest in their religious faith to question them about “the meaning of the term *canon* in that context” (Observation, 2/14/05). In the second period class, Mandy asked students what work they did with logarithms in their Algebra II class and one student replied, “I didn’t like logarithms because we didn’t really talk about what they were” (Observation 2/14/05). Mandy addressed this statement by questioning students about the applications of logarithms in other disciplines (i.e., physics, chemistry) and used the opportunity to focus on the “why” pertaining to the development of logarithms. The classes discussed the various efforts that aided in the

development of logarithms, including the lengthy undertaking of the creation of tables of logarithmic values by Napier, Briggs, and Vlacq. The critical question that Mandy posed to her students was, “If you have a calculator that gives you logarithmic values, why do you need to study logarithms” (Observation, 2/14/05)? The students’ collective response was, “You still need to know where they came from. Maybe if you know what they are, that may be helpful in their use” (Observation, 2/14/05).

To provide a meaningful research experience for the students, Mandy asked the students to choose a topic for their research paper. The only guideline given was that the topic must address an aspect of the historical development of logarithms. The range of student topic choices for the first two class periods is given in Table 12.

Table 12
Student Research Paper Topic Choices

| Period 1 topic choices | Period 2 topic choices |
|-------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| People contributing to the development of logarithms | The historical contributions of mathematicians from the Near East and China |
| Events contributing to the development of logarithms | The time/eras during which logarithms were developed |
| An historical perspective from the standpoint of time | Why logarithms were developed/invented |
| How logarithms are used, specifically their application in the sciences | People contributing to the development of logarithms |
| Why logarithms were developed/invented | The Napier family history |
| Applications of logarithms in engineering and business | The locations where the work on the development of logarithms took place |
| Connection between mathematicians, philosophy, and religion | |

The research paper assignment for the fourth period class, which consisted of only three students, was provided by Mandy. This was a pedagogical decision made by Mandy after she discovered that the fourth period students had not completed the initial Internet research assignment. For this class, Mandy developed the topic from a conversation that

she and I had concerning the validity of the statement made by Lord Moulton in 1914 concerning the development of logarithms.

The invention of logarithms came on the world as a 'bolt from the blue.' No previous work had led up to it; nothing had foreshadowed it or heralded its arrival. It stands isolated, breaking in upon human thought abruptly, without borrowing from the work of other intellects or following known lines of mathematical thought. (as cited in Hawkins, 1982, pg. 159)

Mandy decided that she would provide the topic of the research paper given that the fourth period class was smaller (only three students) and their original research efforts were weaker. The three students were asked to construct and present an argument to prove or disprove Lord Moulton's statement.

Mandy used a portion of the next class day (2/15/05 for Period 1; 2/16/05 for Periods 2 and 4) to review what she considered important historical information from the students' research. In each of the classes, Mandy placed the list of mathematicians, which also appeared in Lesson Installment 1 on the board. The list contained 13 names:

| | |
|-----------|--------|
| Brahe | Kepler |
| Briggs | Napier |
| Bürgi | Stifel |
| Chuquet | Viète |
| Craig | Vlacq |
| Euler | Werner |
| Ibn Yunus | |

Mandy indicated which mathematicians had already been mentioned in class discussions and added Aristotle, Bernoulli, Copernicus, Galileo, Newton, and Wallis to the list at the request of various students. In addition to referring students back to their extended "Great 100s List," Mandy also urged the students to note the emerging connections between the mathematicians and scientists contributing to the development of logarithms (i.e., the work of Kepler and Brahe; Napier, Briggs, and Bürgi).

Mandy's primary emphasis on the "human element of mathematics" was evident in her concentration on the mathematicians contributing to the development of logarithms. Mandy connected the students' research on the history of logarithms with the persons from history comprising the extended "Great 100s List" with which students were familiar from other courses at High Acres. In addition, when students asked about the background of Ibn Yunus, Bürgi, and Vlacq, Mandy used her knowledge of individual students' background, asked about their particular heritage, and encouraged them to think about the answers to their own questions.

Mandy remained true to her intent to use Lesson Installment 1 in a "very open-ended" fashion. She decided to let students choose either the topic of their research paper or the position they would argue with respect to the Lord Moulton's claim. Two related decisions also highlighted the open-ended nature of Mandy's implementation of Lesson Installment 1. First, Mandy did not suggest particular websites for students to reference during their research. I asked Mandy why she chose to do this and she initially stated that steering students in the direction of a particular website, such as the MacTutor History of Mathematics archive, would in fact be helpful. However, she never directed students toward the use of particular texts or Internet resources. When students shared information which Mandy considered suspect, she probed students about the credibility of the website (Observation, 2/14/05; 2/18/05; 3/2/05), often asking students to provide validation of website information.

A second open-ended feature of Mandy's implementation of Lesson Installment 1 was allowing student freedom in choosing the focus for their research paper. Mandy allowed students to focus on a particular aspect of the historical development of

logarithms. This pedagogical decision enabled students to choose a feature, person, or connection which was meaningful to them personally. For example, one student, a descendent of John Napier, decided to research the Napier family history. Another student was intrigued by Napier's religious works and developed a metaphor to characterize his work in mathematics. She wrote:

In order for a great mind to become great, in order for that one great idea to be formed inside the head, there must be background. There must be events that led up to that one moment which made everything come together. For a light bulb to come on, it must be fed by electricity. That electricity must travel through wires which are channeled to hold it. If John Napier's mathematical ideas were his light bulb, then religion and his religious beliefs made up the electricity which turned it on. (Student Paper, Period 1, 2/28/05).

Mandy reported that although she wanted to return to the students' research and construct the timeline of the development of logarithms. However, she was unable to do this at the end of the semester because of lack of time.

Original Document Reading

Mandy incorporated two activities which were not part of the original seven lesson installments. The first was an exercise in reading. After each class discussed the general content of the intended research papers, Mandy presented students with a 1616 English translation of Napier's *Mirifici logarithmorum canonis descriptio* (or, the *Descriptio*). Mandy transitioned from reviewing the historical figures involved in the development of logarithms into reading the 29 translated pages. Mandy did not discuss including this activity during the professional development sessions, although the website containing the 1616 translation was provided in the resource binder. When I asked Mandy why she decided to use an original document with the students she replied that she "wanted to expose them to the language" (Interview, 2/16/05). Indeed, Mandy asked

a variety of questions while students read Napier's definitions for "surd quantities" and "logarithmes" and corollaries for "proportionall lines" and "whole sines" (Wright, 1616), including:

- What language is it in?
- Can you read any of it?
- Did you have difficulty with the reading?
- What does the word "canon" mean? (Observation, 2/15/05; 2/16/05)

Mandy recognized that the primitive English translation, along with the technical mathematical terms and geometric arguments would be difficult for her students. She valued the role that Napier's *Descriptio* played in preparing her students for Napier's two particle argument. In addition to gauging the students' experience with reading an original document in their mathematics class, Mandy also guided their encounter with the document in order to highlight essential features necessary for the two particle argument. Mandy focused students' attention the importance of the value 10^7 (10,000,000) and the relationship of distance, velocity, and time. After a quick look through the *Descriptio*, a student declared, "What does that have to do with anything? We're talking about logarithms" (Observation, 2/16/05). Mandy replied that she would develop the definitions for them (in the form of Napier's two particle argument) during the next class.

In order to incorporate a discussion about Napier's two particle argument to introduce students to the historical development of logarithms, Mandy believed that her students also needed to flexibly understand the basic concepts of sequences. Mandy required her students to investigate the basic concepts of sequences while they simultaneously worked on their historical research assignments. The two particle argument would provide a connection between these two topics and Lesson Installment 3 would provide a way for students to explore both in depth.

The Two Particle Argument

Mandy transitioned into the two particle argument by directing her students to “look at logarithms the way Napier did them” (Observation, 2/17/05). She asked students to find the first ten terms of the sequence $a_n = \left(1 - \frac{1}{10}\right)^n$, and to examine “the relative magnitude of the values” (Observation 2/18/05). Mandy initially asked students to examine this decreasing sequence because it was similar to other sequences that they encountered. The sequence given by a_n was similar to the sequence which resulted from Napier’s two particle argument, or $L_n = 10^7 \left(1 - \frac{1}{10^7}\right)^n$.

Mandy did not ultimately require students to obtain the generalized result (L_n) upon examining four successive iterations from Napier’s definition (Appendix B). She did, however, expect students to connect their research on logarithms, the original document reading, and their ability to simplify numerical expressions through factoring and properties of exponents. In addition to exposing students to reading the *Descriptio* prior to the presentation of the two particle argument, Mandy was careful to include a study of arithmetic and geometric sequences. During the first professional development session, Mandy noted that the use of Lesson Installment 3 would motivate her to cover sequences and series before logarithms. After the second session, Mandy compromised by planning to modify the order of the mathematical content in the Precalculus classes for the second semester so that sequences and series were covered simultaneously with the topic of logarithms. In this way, Mandy incorporated (1) the “human element” of the historical development of logarithms (the inclusion of an actual work of Napier’s); (2) the

mathematical rigor of the invention (presenting Napier's two particle argument); and (3) the content students needed to be successful in studying the history of logarithms and subsequent topics.

In keeping with her philosophy that she served as a "resource for the students so that they can become self-learners," Mandy addressed several student questions raised during the two particle argument lecture in essentially the same manner. In one instance, a student inquired about the value 10^7 , including its importance highlighted by Mandy in a previous discussion, as well as its appearance in the two particle argument. Responding to the whole class, Mandy urged,

That was something you were supposed to have figured out with your research! Where did that 10^7 come from? That's something that if you had done the appropriate research, you would have come across. Why did he [Napier] use 10^7 ? (Observation, 2/18/05)

In another instance, Mandy asked students to respond to three questions after the two particle argument presentation:

What have you learned about the history of logarithms?
What is interesting to you, with respect to the history of logarithms?
What questions do you have about the history of logarithms? (Observation, 2/18/05)

The questions prompted one student to ask, "So what exactly is a logarithm? Is that a logarithm [referring to the resulting expression from the two particle argument]" (emphasis by student, Observation, 2/18/05)? Mandy again pushed for her students to use the resources before them and replied, "I don't know! My question is, what have you learned, what has surprised you the most, and what do you want to know? (Observation, 2/18/05).

When Mandy concluded the two particle argument with the Period 4 class she again used a student question to encourage further student research.

Mandy: Can you understand this iterative process? How many times does he [Napier] have to do it?

Student: Did he do it for every degree (of angle measure), or something?

Mandy: That's part of the research you're supposed to be working on. Very good question! (Observation, 2/18/05)

Prompting students to locate an error in Napier's work in anticipation of a traditional treatment of logarithms is another example of Mandy's role as a resource to promote self-learning. Directing the students' attention to both the *Descriptio* text and the two particle argument, Mandy observed,

Napier had a really good idea, but as [a student] said, while he built on ideas, he did not perfect it. He has an error in his thinking and it will take somebody else to think about it. You've got the clues; the clues are sitting right there. What kind of correction did he have to make to his thinking? You're probably not going to think of it right now. You're probably going to find it as you do a little bit more of your research. (Observation, 2/18/05)

In this passage, Mandy referred to Napier's definition of the logarithm of 10,000,000 to be zero ($\log 10000000 = 0$). Briggs collaborated with Napier to define the logarithm of 1 to be zero ($\log 1 = 0$), a fact that the class would use in Lesson Installment 4.

Development of Logarithms Using Sequences

Mandy introduced students to Lesson Installment 3 by recalling the use of sequences from the two particle argument. She noted that the loss of class time (snow event) prevented using another original document referenced in the lesson installment.

Ideally, I would like to have had you read an excerpt from Mark Napier's work, but we don't have *quite* enough time to go through and do as much reading as I'd like to. One thing I think that this project needs is a little bit more time than we're able to allot to it. It would be nice if we had the freedom to do that. But it says

over here that we read and discussed the original argument of “Napier’s great invention” – which we did before break. (Observation, 3/02/05)

Mandy asked students to consider “from a time perspective, what was coming into play just about this same time frame” (Observation, 3/02/05)? Student responses included the introduction of scientific notation, the use of decimal notation, and the fact that mathematical notation was just beginning to be standardized across cultures and languages.

Students investigated Lesson Installment 3 in pairs over approximately two class periods (80 minutes in length). Although her students were academically able to handle the lesson installment as one long investigation, Mandy chose to provide them with the lesson pages one part at a time. However, Mandy used the lesson in the way it was designed, with several modifications (see Table 13).

Table 13
Lesson Installment 3 Modifications: Mandy Wilson

| Lesson part | Modification |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| I | Mandy added an additional table at the end of the lesson, organized as <i>S1</i> and <i>S2</i> , but using powers of 3. Students were instructed to answer the same questions for the new sequence and compare the results. |
| II | Problem 10 originally asked for students to <i>Verify your conclusions in Exercise 9, using the function notation below. Let u and v be terms of $S4$. Use algebraic manipulation to show the following three properties hold.</i> Mandy did not emphasize the algebraic verification and instead asked students to verify by confirming using three numerical examples. |
| II | Mandy directed the students to do a “critical reading” of the introduction to Part II and to complete the table provided. Mandy asked for the careful reading so that students could “find the two important facts” – one of which was missing from the original directions. In the <i>Pre-work</i> section, the restriction that b is greater than zero should have also included $b \neq 1$. |
| III | Problem 14 asked students to <i>Practice the new “log” notation to associate the corresponding terms of sequences $S5$ and $S6$ to find each of the following</i> [eight exercises were given]. In addition to determining the logarithmic values requested, Mandy also asked students to convert the given logarithmic expressions to the corresponding exponential form. |

Calculation of Logarithmic Values

Unlike Lesson Installment 3, Mandy introduced students to Lesson Installment 4 differently in the three classes. In Periods 1 and 2, Mandy did not use the launching example given on the first page of the installment:

Use of estimation and number sense abilities for a particular purpose. For example, approximate the value of 75 using only powers of 2 and 3. (Or, thought of another way: construct a product *close* to 75 that uses only powers of 2 and 3.)

Instead, Mandy first reviewed the three properties of logarithms and powers of ten:

..., $10^3 = 1000$; $10^2 = 100$; $10^1 = 10$; $10^0 = 1$; $10^{-1} = 0.1$; $10^{-2} = 0.01$; $10^{-3} = 0.001$,

She then directed students' attention to the progression of values and asked, "How can I get some pretty close values to these logs? Suppose I want the log of 2? How am I going to get that" (Observation 3/07/05, Period 2)? After eliciting several student responses in the Period 1 class, Mandy realized that students struggled with the goal of the activity. Students failed to recognize that they needed to calculate approximations of logarithmic values using previously determined values. They also expressed difficulty with how to determine which mathematical relationship would eventually yield the value desired. At this point, I assisted (at Mandy's prompting) the class with formulating a technique to determine an approximation for \log_2 , which was essentially an explanation of the example provided in the chart on Page 2 of the lesson installment.

When Mandy introduced Lesson Installment 3 to the Period 1 students, the directions for and goal of the task were more direct and students completed the example to (approximate the value of $\log_{10} 2$) easily. For clarity, Mandy discussed an additional example.

Mandy: So what we're going to do is play a guessing game to determine approximations. How am I going to look at log of 3? And I can look at anything that I want to.

Student: This is kind of fun, though!

Mandy: Yeah, it's kind of a fun thing. It's what I would call 'mental gymnastics' – playing a game with the mind. (Observation, 3/08/05)

In Period 1, Mandy's implementation of Lesson Installment 4 began to take on the spirit that Mandy expressed during the third professional development session. At that time, Mandy observed that the students would enjoy the approximation and calculation task and that it would be "like working a puzzle."

The students continued to work with their partner to determine approximations of the values for $\log 3$ through $\log 10$, and $\log 11$ and $\log 13$. Each pair of students generated approximate values using a slightly different technique, including one pair who developed a method very similar to Mandy's (Professional Development Session 3, 11/29/04). For example, instead of using a numerical relationship that involved 7 as a whole-number factor and depended only on known logarithmic values (e.g., $\log_{10} 10^3 = 3$) or previously approximated values (e.g., $\log_{10} 2 \approx 0.3$), the student used a calculator to determine that $7^{3.55} \approx 10^3$. This relationship then allowed him to determine an approximate value for the logarithm (base 10) of 7. Mandy questioned the student's partner about the calculation method used.

Mandy: What did your calculation look like?

Student 1: I don't know if I can explain it [the calculation method] very well.

Mandy: It is a matter of creativity for which ones [logarithmic values] you pick up and how close you get. (Observation, 3/07/05).

In Periods 1 and 2, Mandy did not emphasize the intent of the lesson installment as given in the *Introduction*.

Keep in mind, however, that we will be finding approximations for only a few common logarithms and will not have nearly the accuracy that Briggs found when he was developing his logarithm tables! (And all of his work without a calculator, no less!) (Lesson Installment 4)

Mandy did, however, ask the students to compare the approximate logarithmic values which they calculated with the approximate logarithmic values (to the nearest ten thousandth) given by their calculators. She concluded the Lesson Installment 4 experience by encouraging students to answer the six accompanying questions.

Students in the fourth period class calculated logarithms in Lesson Installment 4 in a similar manner as the students in Periods 1 and 2. The primary difference in how the installment was used was in Mandy's initial launch of the activity. Again, the final comment in the *Recall* section of Lesson Installment 4 asked students to

Approximate the value of 75 using only powers of 2 and 3. (Or, thought of another way: construct a product *close* to 75 that uses only powers of 2 and 3.)

The goal of this sample task was for students to consider approximating the value of the logarithm of 75 using both the logarithm of smaller numbers and the properties of logarithms. In addition, by selecting a number which could not be factored into powers of 2 and 3, students were prompted to think about some of the processes which may have been required of Briggs and others as they built tables of logarithms.

Mandy's implementation of this initial example was somewhat different, however. After a review of the three properties of logarithms, Mandy asked students, "How can I express 75 in terms of logs? You have your tables of 2s and 3s there" (Observation, 3/07/05). Mandy attempted to introduce students to a strategy for creating

numerical relationships which would enable them to approximate logarithmic values. Instead of first requiring students to think about how to express a given value using factors which may not lead to the exact value, Mandy modified the task to prompt students to think about how to find the logarithm of the given value (75, in this case). She also modified the allowable factors, changing from the factors 2 and 3, to the factors 3 and 5. This initial exploration was essentially the same as the calculations students were asked to do on Page 2 of the lesson installment. In this manifestation of the launch to the activity, it was necessary for students to use their calculators to determine log of 5 and log of 3. Thus, students calculated $\log_{10} 75$ as:

$$\begin{aligned}\log_{10} 75 &= \log(3 \cdot 5^2) \\ \log_{10} 75 &= \log_{10} 3 + 2 \log_{10} 5 \\ \log_{10} 75 &\approx 0.4771 + 2(0.6989) \\ \log_{10} 75 &\approx 1.8710. \\ &(\text{Observation, 3/07/05})\end{aligned}$$

In addition to modifying the launching example, Mandy also proceeded differently with the remainder of the lesson installment with the students in Period 4. Instead of completing just one example with the whole class, Mandy and the class completed three examples together, approximating the values of $\log 2$, $\log 3$, and $\log 4$. In addition, Mandy interjected her technique of using non-integer powers (using the calculator) to arrive at the closest possible value. Again, although a unique method, the continued concentration on calculator usage (raising integers to non-integer exponents) to arrive at the best approximation shifted the focus of the lesson installment away from its historical focus.

Slide Rules and Properties of Logarithms

From the beginning of the study of logarithms, Mandy tempted students' curiosity by placing slide rules, including circular slide rules, in front of them at strategic moments. She encouraged students to try to figure out what information the slide rules provided and how one might use them. When students were asked to respond to the three questions about what they had learned, what interested them, and what questions they still have with respect to the history of logarithms, two students commented on the various instruments laid before them.

I would like to know why we have flight computers out on the desks.
I want to know how to work these nifty aviation things. (Artifact, 2/17/05)

These student comments, combined with her desire to link the historical with the practical, motivated Mandy to include a lesson not included in the original seven designed for this study.

Mandy motivated the incorporation of constructing a slide rule by asking students to first think about the logarithmic values calculated in Lesson Installment 4.

Mandy: The first thing on Wednesday morning, we're going to use these values to make our own slide rules.

Student: So how long is the slide rule going to be? It would be pretty long.

Mandy: Well, let's think about that. How would you go about constructing the slide rule? (Observation, 3/07/05)

The students ascertained that they needed to place the values just calculated on the slide rule. However, representing the scale needed was troublesome for them. Students were unsure about how to deal with the increasing density of the logarithms of the numbers from 1 through 13.

Mandy addressed the density issue by asking students to compensate for it when constructing their slide rule. Using two sheets of notebook paper for the sliding parts,

Mandy described how to construct a slide rule:

1. Fold each sheet of notebook paper into quarters along the longer edge. The paper now resembles a ruler.
2. Orient one “ruler” so the lines of the notebook paper are oriented vertically and the original top margin of the notebook paper is on the left. Along the bottom half of the “ruler,” label the first marking “0” (zero).
3. Continued labeling every other mark (lines of the notebook paper) on the bottom half of the “ruler” by tenths until the end of the paper: from 0.1 until about 1.5.
4. On the same scale near the folded edge, label the values for log 1 through log 15. You will need to use your calculator to obtain the approximate the values of log 14 and log 15, since these were not calculated in Lesson Installment 4. Since every other mark was used in step 2, you will have a little more room to record your values for the logarithms as they become more dense.
5. Next, line your second scale (“ruler”) up with your first scale and place just the logarithmic values for 1 through 15 along the folded edge. (Activity adapted from Anderson et al., 2004; Audiotaped Class Session, 3/08/05)

After constructing the slide rules, the class discussed how to use the slide rule by practicing the operations of multiplication, division, and raising a number to a power.

Mandy included this activity for her students to combine the use of the approximated logarithmic values, the properties of logarithms, and their interest in the development of early calculation devices.

Summary

In this study, I addressed two primary research questions directed at understanding the potential and challenges for incorporating effective historical approaches to mathematics teaching.

How do teachers with different background knowledge and experiences respond to professional development focused on understanding the history of mathematics?

How do background variables and professional development experiences with the history of mathematics combine to influence teachers' personal mathematical knowledge and instructional practices?

This sub-section summarizes Mandy's professional background, change in attitudes toward and knowledge of history of mathematics, engagement during the professional development sessions, the influence of beliefs about teacher and student roles and school factors, implementation of an historical approach in teaching, and perceptions of benefits, barriers, and affordances associated with using the historical approach.

Professional Background

Mandy was a veteran teacher with 37 years of classroom experience. For the past several years, she taught high school mathematics at High Acres School during the day and college mathematics at night at a nearby community college. She expressed a strong commitment to participating in professional development and personal learning experiences and endeavored to use such experiences to not only enhance her teaching ability, but the learning opportunities of her students as well. Mandy reported that she did not participate in any formal history of mathematics courses or training. Nevertheless, she did express a deep interest in the history of mathematics and how it could be used to motivate and supplement the content of the courses that she taught.

Attitudes

Mandy's attitudes towards the history of mathematics did not change significantly during this research, given her already strong inclination for the use of the history of mathematics in teaching. On Part I of the Attitudes Instrument Post-assessment, Mandy strongly agreed with all of the statements except for one part of Item 6. Mandy

moderately agreed with the statement, *Using historical materials in my mathematics classes has been an integral part of my instruction in Statistics*. Mandy stated that she would “love to see what I could do with statistics, because I do use a historical approach when we go into probability. But not for statistical testing and I would like to do that” (Interview, 4/20/05). Her responses to items in Part II of the Attitudes Instrument Post-assessment was similar to those she articulated in the pre-assessment. The primary difference between her pre- and post-assessment responses was the level of detail in response completions. An example is given in Table 14.

Table 14
Attitudes Instrument Comparison: Sample Response

| Item stem | Pre response (11/04/04) | Post response (4/19/05) |
|-----------------------------------------------------------------------------------|---------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>I would consider incorporating historical problems in the curriculum as...</i> | ...most likely because students enjoy this type of challenge. | ...possible because these are motivating and challenging, especially for gifted students. Time is a limiting factor in AP classes, unfortunately. |

Content Knowledge

Prior to the professional development component of this study, Mandy indicated that she was unaware of the historical development of logarithms. Mandy’s eagerness for teaching logarithms using an historical approach was evident in her commitment to study them and her personal study, combined with her professional development participation and classroom instruction impacted Mandy’s content knowledge related to the history of logarithms (see Table 15). On the Content Knowledge Pre-Assessment, Mandy answered Items 2, 5, 6, and 8 correctly. On the post-assessment, however, Mandy answered all but Item 4 correctly. Although Mandy had successfully completed each of the purely traditional items on the pretest, two notable improvements occurred on the post-

assessment. Mandy solved the equation in Item 8 on the pretest by first substituting approximate values for $\log 3$ and $\log 45$. Although her solution method was correct, it yielded an approximate value for x of 4.996. On the post-assessment, however, Mandy recognized that by first simplifying the equation she could use the property of logarithms, if $\log_b x = \log_b y$ then $x = y$. From this step, Mandy obtained the exact value of x by solving the resultant equation $9x = 45$.

The second notable improvement is Mandy's correct definition of *logarithm* in Item 1. On the pre-assessment, Mandy defined *logarithmic function* instead of *logarithm*. On the post-assessment, Mandy correctly defined logarithm and differentiated between *logarithm* and *logarithmic function*.

Table 15
Content Knowledge Post-Assessment Results: Mandy Wilson

| Item | Historical/Traditional | Response given |
|---------------------------------------------------------------------------------------|------------------------|-----------------------------------------------------------------------------------------------------------------------|
| 1. Define <i>logarithm</i> . | Traditional | Correctly defined <i>logarithm</i> and described a <i>logarithmic function</i> |
| *2. Describe the basic idea or motivation for the invention of logarithms. | Historical | Described in terms of calculations necessary for astronomical measurements |
| 3. Construct the values for $\log_{10}2$ and $\log_{10}3$ without using a calculator. | Historical | Approximated values using techniques discussed during professional development sessions and instruction with students |
| 4. Let $u = b^n$ and $v = b^m$. Verify $L(u) - L(v) = L\left(\frac{u}{v}\right)$. | Historical | Proof attempted (incorrect) |
| *5. Evaluate: $\log_{32}16$. | Traditional | Evaluated successfully by converting to exponential form |
| *6. Evaluate: $\log_{\frac{1}{3}}81$. | Traditional | Evaluated successfully by converting to exponential form |
| 7. Calculate the product of 8409.5 and 951.49 using the | Historical | Calculated correctly using the identity |

| Item | Historical/Traditional | Response given |
|----------------------------------------------------|------------------------|---------------------------------------------------------------------------------------------|
| method of prosthaphaeresis. | | $\cos ab = \frac{1}{2}[\cos(a + b) + \cos(a - b)].$ Also refined estimate by interpolation. |
| *8. Solve for x : $2\log 3 + \log x = \log 45$. | Traditional | Solved correctly using properties of logarithms |

Note. Items with an (*) were answered correctly on the Content Knowledge Pre-Assessment.

The only item answered incorrectly on the post-assessment, Item 4, involved content which Mandy did not cover as written in the lesson installments. Item 4 is taken from Part II of Lesson Installment 3. When she used the installment with students, Mandy did not require students to prove the properties of logarithms using the “ L ”-function notation. Instead, students were asked to verify that the properties held by using three confirming examples.

Professional Development Engagement

During the professional development sessions, Mandy eagerly participated as both a teacher thinking about her future practice and as a student learning about new ideas related to concepts she previously understood pertaining to logarithmic functions. During the professional development sessions, Mandy was active in collaborating, anticipating student engagement, and articulating decisions for how and when to use the history of logarithms materials. She consistently reflected on the quality of the materials and resources and identified aspects of her own learning needing improvement. Her commitment to learning about the historical development of logarithms continued beyond the formally scheduled sessions.

Due to illness, Mandy and I were only able to review Lesson Installments 1 through 4 together during the professional development sessions. We utilized the

additional resources throughout our examination of the historical development of logarithms. As a result of her dedication to sustained professional learning and her experiences during the professional development sessions, Mandy implemented each of the Lesson Installments 1 through 4 during her instruction. She also incorporated two additional activities that were directly related to the content of two existing lesson installments.

Influence of Beliefs and School Factors

Mandy did reform her typical instruction of exponential and logarithmic functions to include the historical development of logarithms due to several factors. Her stance regarding continued professional learning and the professional development experience in this study contributed to the viability of her reformed instruction. Mandy's vision of her role and the role and responsibilities of her students, along with the contextual features of High Acres School, also contributed. The overarching theme of Mandy's experience was that the use of the history of logarithms served as a lens to describe the development of the mathematical idea of *logarithm*, from both a mathematical and humanistic perspective. The philosophy of High Acres School held providing "a rigorous, liberal arts education" (High Acres website, *Mission and Philosophy* section) as its primary goal. Further, they believed that "a challenging, content-based curriculum trains the intellect while fostering self-discipline, independence, creativity, and curiosity" (High Acres website, *Mission and Philosophy* section). The challenging and content-based curriculum is presented with an emphasis on effective and disciplined communication.

The nature of Mandy's teaching context at High Acres supported her desire to teach logarithmic functions in such a way that students would experience the challenges

outlined in the school's philosophy. As a private school, High Acres was not burdened by high-stakes accountability testing, restrictive curricula, or large class sizes. In addition, Mandy's teaching philosophy and her beliefs about the student's role in the learning process were aligned quite well with the school's educational philosophy.

Mandy considered herself to be a "resource for the students so that they can become self-learners" (Interview, 4/20/05). Throughout her implementation of the history of logarithms materials, she consistently adhered to her desire to keep asking the students questions "until they start looking for and finding the answers" (Interview, 4/20/05). The most notable manifestation of her philosophy of teaching was found when examining Mandy's responses to her students' questions throughout the study of the historical development of logarithms. In most instances, Mandy would turn the student question into an opportunity to encourage them to pursue the knowledge for themselves; to conduct additional research in response to their own queries.

In addition to using the history of logarithms materials with students as a way to enhance her role of serving as a resource, Mandy maintained a desire for her students to "approach problems thoughtfully, thoroughly, [and] with great precision of language, both written and spoken" (Interview, 4/20/05). To assist her students in "learning how to use resources and to have a joy of learning" (Interview, 4/20/05), Mandy provided students with an initial research task that required them to cast a wide net to capture any information related to the development of logarithms. She did not always address the subtle inconsistencies in the claims students offered when sharing their research. She did, however, create a didactic classroom environment which enabled students to share and respond to each others' work.

Incorporating the History of Logarithms

The combined effect of her professional development participation, beliefs about teaching and students, and the contextual features of High Acres School led Mandy to use six historical activities with her students. The six activities, including the first four lesson installments, enabled Mandy to appeal to the students' sense of using information as a resource for their learning. For example, she used the individual research experiences and their experience of reading an original document to set the context for presenting Napier's definition of *logarithm*. Mandy viewed the implementation of the lesson installments focused on the historical development of logarithms as a way in which she could provide resources to her students. After her instruction of logarithms, Mandy stated that she would use quality materials from the history of mathematics with students if they were available because her students "were very interested in the historical development of ideas" and that incorporating the history of logarithms materials "worked very well with both motivated students and those who have not previously been too excited about math" (Attitudes Instrument, 4/19/05).

Benefits of Using the History of Logarithms

A closer examination of Mandy's implementation of the historical development of logarithms provides insight into the benefits which resulted and the obstacles which existed. Mandy claimed that incorporating the historical development of logarithms enabled her students "to attach personal and contextual meaning to the development of mathematical thought" (Attitudes Instrument, 4/19/05). Prior to taking Precalculus, students experienced logarithms as a purely procedural topic. One student lamented, "I didn't like logarithms because we didn't really talk about what they were" (Observation,

2/14/05). When Mandy reflected upon the experience of incorporating the history of logarithms she noted that students relented on the reluctance they initially exhibited and that they benefited from “having to do some thinking about the topic; it’s not just a calculator approach to the world” (Interview, 4/20/05).

Mandy also observed that the added benefit of incorporating the history of logarithms enabled her to make connections to previously studied topics as well as those which typically appeared after logarithmic functions in a Precalculus curriculum. Mandy anticipated that advantages to studying *logarithms* before *logarithmic functions* were significant enough to alter her usual instructional practice (see Table 16).

Table 16
Modifications in Instructional Practice: Mandy Wilson

| Previous instructional practice | New instructional practice |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Exponential and Logarithmic Functions: <ol style="list-style-type: none"> Definition and examination of exponential functions Geometric transformations (i.e., a logarithmic function is the inverse of an exponential function) Evaluating expressions using logarithmic and exponential conversions Modeling applications involving logarithmic and exponential functions Solving logarithmic equations | Historical development and definition of <i>logarithm</i> : <ol style="list-style-type: none"> Background research and discussion[†] Completion of research papers[†] Original document reading (translation of Napier’s <i>Descriptio</i>) Napier’s two particle argument leading to the definition of <i>logarithm</i> Developing properties of logarithms (Lesson Installment 3) Approximating logarithmic values (Lesson Installment 4) Constructing and using a slide rule |
| Sequences and Series: <ol style="list-style-type: none"> Calculating terms given a rule for a sequence and determining a rule given terms in a sequence Study of arithmetic sequences Study of geometric sequences Study of arithmetic and geometric series | Traditional study of exponential and logarithmic functions: <ol style="list-style-type: none"> Calculator exploration of exponential functions and their graphs; graphic transformations; and applications Logarithmic functions and their graphs; graphic transformations Properties of logarithms Solving exponential and |

| Previous instructional practice | New instructional practice |
|---------------------------------|-----------------------------------------------------|
| | logarithmic functions e) Applications and models |

Sequences and series were simultaneously studied (with an emphasis on arithmetic and geometric sequences) in preparation for the remainder of the historical work.

The opportunity for her students to study multiple mathematical ideas simultaneously enabled Mandy to model the notion that topics do not develop in isolation.

Incorporating the historical development of logarithms also facilitated Mandy’s ability to make connections across topics and subjects. During the class periods devoted to more research-oriented activities, Mandy often called upon students’ use of their “Great 100s” list, which was integrated across subject areas at High Acres School. Mandy also planned to “introduce and revisit the historical approach using prosthaphaeresis later in trigonometry [the next unit in Precalculus]” (Interview, 4/20/05).

The Obstacle of Time

Mandy recognized that she was “fortunate enough to have the flexibility to incorporate this unit in its entirety – this may not be possible in another teaching environment” (Interview, 4/20/05). While it is true that Mandy taught in a school environment which was more conducive to incorporating alternative teaching approaches, she still experienced a common obstacle. In some instances, Mandy lacked adequate time to fully integrate the history of logarithms as she intended. She indicated, for example, that,

We still plan to create the timeline. We will work on this as the class project toward the end of the semester. This will be a very good “review” activity during a time when there are many interruptions. (Interview, 4/20/05)

The “many interruptions” at the end of the school year – Field Day, senior papers, class fieldtrips – prevented Mandy from revisiting the conclusion of the timeline activity.

Mandy also indicated that time was an obstacle to incorporating historical problems in the curriculum in general. She noted that such uses of the history of mathematics “are motivating and challenging especially for gifted students. Time is a limiting factor in Advanced Placement classes, unfortunately” (Attitudes Instrument, 4/19/05).

Affordances

The mission of High Acres School to present students with a rigorous, liberal arts education, while “fostering self-discipline, independence, creativity, and curiosity,” afforded Mandy the flexibility to incorporate the history of logarithms while using a cognitive practice focused on presenting mathematical topics in concrete terms before an abstract treatment of them. Typical instruction related to logarithms omits an emphasis on what logarithms are. Instead, exponential functions are typically presented first, progressing through a study of their definition, features, graphs, calculations, and solving equations which include expressions for which the exponent contains a variable term (abstract to concrete). Finally, logarithmic functions are presented as inverses of exponential functions, and the same progression ensues. The flexibility within Mandy’s teaching context enabled her to begin the unit with an investigation of the historical development of logarithms. Mandy was able to provide students with an examination of what logarithms were and how their invention led to functions which depend upon them.

Moving from concrete to abstract while incorporating the historical development of logarithms also permitted Mandy to address less able students’ attitudes towards difficult topics. Students who struggled with the concept of logarithmic functions during Algebra II began to question further study of logarithms, including their historical development. One student complained, “how is this ever going to help” (Observation,

3/04/05)? Mandy replied that, “sometimes, intellectually, it’s helpful to see how things have developed” (Observation, 3/04/05). At the conclusion of the unit, Mandy shared that this particular student, “who showed the little bit of reluctance did a flip-flop in attitude” (Interview, 4/20/05), remarked that the unit on logarithms “was the most fascinating unit they had ever done” (Interview, 4/20/05).

Chapter 6 A Moderate Attempt to Study and Use the History of Logarithms: The Cases of Sue Moe and Ted Jones

In this chapter I present the cases of Sue Moe and Ted Jones, teachers whose participation during the professional development sessions I characterized as moderate. This characterization follows the definition of moderate addressed in Chapter 4. Sue and Ted also selected specific materials from the study of the historical development of logarithms to use in their teaching and their use of the history of logarithms is characterized as moderate as well.

The chapter is organized by case, and addresses the research questions outlined in Chapter 1. Each case study begins with a description of the participant's professional education and teaching experience. Accompanying the description for each, I also discuss their

- previous experiences with the history of mathematics;
- expressed attitudes and beliefs related to the role of the history of mathematics in teaching; and
- evoked knowledge about the topic of logarithms, both in traditional and historical contexts.

Next, I describe their participation during the professional development sessions, which focused on the historical development of logarithms. I then examine how Sue and Ted incorporated the historical content and materials into their instruction. Lastly, I summarize each case, including identification of the obstacles, benefits, and affordances related to using the historical development of logarithms.

The Case of Sue Moe

Sue was the youngest of the five participants and has taught for four years. All of Sue's teaching experience has taken place at Mulberry High School. She has been the chair of the mathematics department for two years and was the Algebra I lead teacher in 2004 – 2005.

Professional Background

Sue's completed a traditional teacher preparation program at an accredited institution in the southern United States. She majored in mathematics with a special emphasis on mathematics education and she is currently pursuing a master's degree in mathematics education. Sue's undergraduate mathematics preparation is quite strong. She reported taking four semesters of calculus, differential equations, linear algebra, modern geometry, abstract algebra, and introductory statistics as an undergraduate. Sue reported no prior experience with the history of mathematics beyond reading occasional biographies of mathematicians.

During the two years prior to this study, Sue indicated that she actively pursued a variety of professional development experiences, including

- Mathematics content courses (at the college level);
- School, county, or state-provided programs, workshops, training sessions, or institutes;
- Conference or professional association meetings;
- Formal mentoring or peer observation and coaching;
- Committee focusing on mathematics curriculum, instruction, or student assessment in mathematics;
- Regularly scheduled discussion or study group;
- Independent reading on a regular basis;
- Co-teaching/team teaching; and
- Consultation with a mathematics specialist. (Background Survey, 12/15/04)

Attitudes and Knowledge

Sue moderately or strongly agreed with statements in Part I of the Attitudes Instrument Pre-assessment which focused on using the history of mathematics in teaching (Table 17). Her response of “strongly disagree” to Item 6 indicated that she did not currently use the historical materials in her teaching.

Table 17
Attitudes Instrument (Part I) Pre-Assessment Results: Sue Moe

| Survey item | Pretest response |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------|
| 1. Understanding the history of mathematics is an important part of understanding mathematics. | 5 |
| 2. Including history enriches the teaching and learning of mathematics. | 6 |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | 6 |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | 5 |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | 5 |
| 6. Using historical materials in my mathematics classes has been an integral part of my instruction in: Algebra I or II. Geometry. Precalculus/Trigonometry. Calculus. | 1 1 1 1 |
| 7. Prospective mathematics teachers should be required to study the history of mathematics. | 6 |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | 6 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

To clarify her response to Item 1, Sue indicated that she moderately agreed with the statement for teachers, but from the perspective of lower level students, she slightly disagreed that understanding the history of mathematics is important for understanding mathematics (Attitudes Instrument, 12/15/04).

Consideration for what the use of the history of mathematics would enable Sue to do with students was prevalent in three of the four completions to the items on Part II of the Attitudes Instrument. Sue reported that,

- researching mathematicians contributes a lot to a mathematics course because it helps the students make connections and realize how long math has been around, and if they do experience the history of mathematics long enough they will see how math has evolved;
- mathematics teachers should require some history work in their classes since it helps answer “Who ever came up with this?” and “Why is this important?”; and
- she would often use available materials from the history of mathematics with students because the history of mathematics is interesting to her and her students say they want a break from lecture all of the time. (Attitudes Instrument, 12/15/04)

Sue correctly answered the three traditionally-oriented questions on the Content Knowledge Instrument Pre-assessment. The evaluation methods that Sue employed to answer Items 5 and 6 indicate that she is adept at evaluating simple logarithmic expressions by using equivalent exponential expressions and applying the property of equality for exponents (if $a^x = a^y$, then $x = y$) to solve. In Item 8, Sue applied the properties of logarithms correctly in order to solve the given logarithmic equation. Sue attempted a fourth item, Item 3, by representing $\log_{10} 2$ in exponential form. She left the four remaining items blank, except for placing a question mark (“?”) in the blank space provided.

Professional Development Engagement

The professional development experience with the Mulberry High School participants often felt less intense than the sessions I worked with Mandy Wilson at High Acres School. Whereas Mandy always approached each discussion with passion and enthusiasm, the Mulberry High School teachers were often passive. After our discussions

about Lesson Installments 1, 2, and 3, I shared one hope that I held for the Mulberry teachers as participants in this research. My hope was that after the teachers reflected on our work together, they would consider that a study of the historical development of logarithms would provide them – and potentially their students – a truly different way of thinking about logarithms.

Sue was the most vocal and candid of the four Mulberry teachers and after reflecting on the discussions of the first three lesson installments she observed that,

The only thing that I really wonder about these – I really like these nine pages [of Lesson Installment 3]. They were great and all, but I *know* logarithms (her emphasis) and it was really interesting to see the connections after I did it. So I'm really wondering where I would or when I would insert this. Whether I would teach logarithms the way I always do and then go back and say, "Okay, let's look at it this way" – so that they have a basic understanding about what we are talking about [historically speaking]. (Professional Development Session 1, 12/17/04)

Although Sue referred to a specific lesson installment, the sentiment was prevalent through this entire professional development experience. Sue grappled with many of the issues teachers typically face with they consider revising their instructional practice. Her willingness to publicly verbalize her struggles with many of these issues may be attributed to her number of years teaching (completing her fourth year of teaching) or her ability to take risks in her role as a school leader (completing her second year as mathematics department chair). Regardless, I characterized Sue's engagement during the professional development sessions (two formal and two informal online discussions in her case) as moderate. In the case of Sue, her participation during the formal sessions and her personal study outside of the sessions is best described as moderate because of her struggle to integrate her new knowledge of the historical

development of logarithms, her considerations of how to bring this knowledge to her students, and the obstacles she perceived she would encounter in the classroom setting.

When Sue's participation during the professional development sessions is viewed from the framework of the six engagement themes discussed in Chapter 4, only the three primary themes prevail. Sue's engagement during the professional development sessions includes acting as a facilitative collaborator, anticipating student engagement, and considering pedagogical decisions she would make with respect to using the history of logarithms with students. An absence of specific examples of the three secondary themes of (1) commitment to continued learning; (2) ability to critically reflect on the materials and resources; and (3) identification of gaps in historical knowledge, accentuated Sue's overall characterization of "moderate engagement." Sue's overall engagement lacks the three secondary themes because a considerable proportion of her effort focused on the anticipation of student use of the history of logarithms materials and the associated decisions to use the materials in her practice.

The following sub-sections describe the salient features of Sue's professional development experience using examples from the two professional development content sessions that took place at Mulberry High School. I first provide transcript passages that illuminate Sue's facilitative collaborator persona. In many ways, Sue's involvement during the professional development sessions was directly related to the beliefs she held as mathematics department chair. Second, I will use Sue's own words to outline several examples of her ongoing struggle to foresee how students would interact with the history of logarithms materials and content. Sue often discussed her anticipation of student engagement from a negative perspective. These considerations were often in conflict with

the third theme of Sue's professional development engagement, in which she outlined different ways to incorporate the study of the history of logarithms with her students.

Sue as a Facilitative Collaborator

Sue's overall participation during the professional development sessions is described as moderate because her efforts were often constrained due to the departmental duties for which she was responsible. Sue identified several school and district responsibilities associated with her department chair position, including budget, teacher evaluation, and various meetings. With respect to the school-based responsibilities, Sue reported:

This takes more time than I have. I spend probably two to three hours a week in meetings (more at the beginning of school and parent conference week). I spend five hours a week on paperwork, purchasing, observing, sending memos and creating reports. I do not have the time to look for grants and extra things for our teachers. (Interview, 10/21/05)

Also, planning for and conducting professional development that occurs at the beginning or end of a semester is problematic given the many activities that distract teachers away from personal and professional learning opportunities. Tasks such as giving and grading exams, calculating grades, contacting parents, departmental and school faculty meetings, and planning for future instruction all impose great demands on teachers' time. In Sue's case, she faced each of the above tasks as well as other time-consuming duties required of Mulberry High School department chairs: school leadership meetings, textbook inventory and distribution, supply distribution, and student schedule repairs.

The combination of the instructional and planning tasks to which Sue attended and the departmental tasks required of her often caused her to engage with the professional development activities only as time permitted. Sue confided in me several

times that she just did not have time to address the content of the lesson installments, but that she would “make time” (Field Journal, 12/13/04; 1/01/05). During the formal professional development sessions, Sue would often need to leave the room to report to the front office to meet with an administrator, counselor, teacher, or parent. Each interruption prevented Sue from fully engaging in the discussions taking place for two reasons. First, and most obvious, if Sue was not present, she could not participate. Second, even upon rejoining the group, it often took Sue a few moments to refocus her concentration on the discussion taking place.

Sue admitted that what she did not like about her role as department chair was “the pressure to lead 16 people in the right direction” (Interview, 10/21/05). Consequently, I believe Sue felt somewhat responsible for the other teachers’ participation during our study of the history of logarithms. Her collaborative efforts were often facilitative in nature, enabling the session to be conducted in an easy and unimpeded manner. When I arrived at Mulberry High School for the December professional development session, Sue indicated that other than Mary Long, who was very excited about the timeline activity (Lesson Installment 1), Ted Jones and Shirley Corson may not have completed anything in time for our first session together. Sue also added that she would not have time to look at Lesson Installment 1 until the next morning (S. Moe, personal communication, 12/13/04). Additionally, two days before the second professional development session, Sue confessed that she had just begun working on the “homework” (Lesson Installments 4 – 7) that day. Mary called her the same afternoon and stated that she had not begun to review the lessons either. Sue was fairly certain that

neither Ted nor Shirley “would have done any either” (S. Moe, personal communication, 1/01/05).

After becoming aware of the level of the participants’ “extracurricular” engagement with the history of logarithms materials, I interpreted Sue’s efforts during the more mathematically challenging discussions as an extension of her leadership role within the mathematics department. Sue may have felt it was her responsibility to “keep the conversation going” because she was aware of her own and her colleagues’ work ethic relative to their study of the history of logarithms. Table 18 outlines three episodes from the discussion of Lesson Installment 6 highlighting Sue’s collaborative efforts during the two Mulberry High School professional development sessions.

Table 18

Examples of Facilitative Collaboration Efforts: Sue Moe

| Description of effort | Episode evidence |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Only one teacher (Sue) worked through Lesson Installment 6 prior to the second professional development session, and a second teacher (Ted) “got lost” beginning with Exercise 3 of Lesson Installment 6. Consequently, the group decided to work through on the lesson installment together.</p> <p>Sue came to the board to record the work of the group.</p> | <p>Ted: I have a question on [Installment] 6. I got lost on number three, four, and –</p> <p>Kathy: Did any you just want to go through all of [Installment] 6?</p> <p>Mary: Yeah! I’m up for that!</p> <p>Sue: Do you want me to write?</p> <p>Kathy: Yeah, do you mind? (Professional Development Session 2, 1/03/05)</p> |
| <p>While discussing Exercise 2 of Lesson Installment 6, Mary questioned how we obtained a particular value for the common ratio in the increasing geometric sequence. Unlike several other questions which were directed at me, Sue authoritatively provided the explanation Mary needed.</p> | <p>Kathy: But what’s say an expression for the b sub zero term?</p> <p>Sue: [Writes an expression for this term on the board.]</p> <p>Mary: Where did you get that?</p> <p>Sue: Dividing any one of the terms by its pervious term. So you take like ten, zero, zero, zero, four hundred (10,000,400) and</p> |

| Description of effort | Episode evidence |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | divide by ten, zero, zero, zero, three hundred (10,000,300). And it will work for all of them. So that's the common ratio. (Professional Development Session 2, 1/03/05) |
| As we continued working with the algebraic manipulation necessary to complete Lesson Installment 6, we arrived at Exercise 3, the problem at which Ted began experiencing problems. Sue, who had completed Lesson Installment 6 the night before (personal communication, 01/02/05), stepped in to navigate toward the expression necessary for Exercise 3. | Ted: So it's one plus, one over ten to the fifth $\left(1 + \frac{1}{10^5}\right)$. Kathy: Right. So that's using exponents as well. And then the exponent of all of that? Ted: I put n minus one. Kathy: [considering this] Does that give us the first term, though? Sue: Don't you have to have the zero, to get zero? Kathy: Yeah, because the first term is ten to the seventh. And if I put n minus one, then that would not get me there. Ted: Okay, the first term is ten to the seventh. Sue: So this [pointing to the exponent in question] has to be a zero. (Professional Development Session 2, 1/03/05) |

The nature of Sue's collaborative efforts during the formal professional development sessions was not the strongest dimension of her engagement. As Sue struggled to understand the content and considered how a different perspective for teaching logarithms would fit with her students' needs and abilities, she openly discussed the anticipation of her students' engagement with the material.

Sue as an Anticipator of Student Engagement

Sue's verbal plans for using particular lesson installments and her anticipation of how students would interact with the material during class were an integral part of the

professional development discussions at Mulberry High School. Most of the anticipatory episodes highlighted in this sub-section were negatively situated by Sue, such as anticipating student difficulty with the mathematics or student disengagement from the material. In two instances, however, Sue considered how the mathematical content of the lesson installments would positively affect student engagement.

Appreciation of cultures and computing. Sue felt that her students held an incomplete world view when it came to certain cultures. At the end of our discussion about Lesson Installment 1, she observed that,

Most students don't attribute anything to the Middle East at this point other than terrorism and I don't think they realize that some of the mathematics came from Iran and Iraq. But I think that might actually be something that they can learn from, because you hear comments that they make – especially from things that they see on *Channel One*. (Professional Development Session 1, 12/17/04)

Sue offered that if anything, when students studied the history of mathematics, it would help them to “have better cultural relations because they might see that a lot of great mathematicians came out of the Middle East” (Professional Development Session 1, 12/17/04).

After the Mulberry teachers discussed Lesson Installment 3, I distributed a set of resources related to the first three lesson installments. Two of the resources I presented to the teachers were excerpts from a microfiche copy of Napier's *Constructio* (1619) and a copy of a table from a Gravelaar's 1899 translation of Napier's *Descriptio* (1614). In response to a question about the origin of the word *logarithm* raised by both Sue and Mary, I demonstrated a use of the tables by multiplying two very large numbers: 9,999,974 and 9,999,967. The example allowed me to recall portions of the two particle argument to emphasize the meaning of *logarithm*. Sue commented that such a problem

could be used to launch the study of logarithms and that “it could also lead the way into the appreciation for the people who [later] programmed the calculators” (Professional Development Session 1, 12/17/04).

In the two preceding examples, Sue articulated how her students might benefit from examining contributions of different cultures to the development of logarithms. Several other examples, however, established Sue’s concern with students’ engagement with the mathematical content of history of logarithms materials.

Connections for students. On many occasions, Sue would raise questions about some facet of the historical development of logarithms well after she processed the related information. After presenting the two particle argument, I asked the teachers about their experience with anything similar to Napier’s development of the definition of *logarithm*. Ted responded with the same question that I had presented to the teachers at the beginning of the session: “What motivated him to use this method?” (Professional Development Session 1, 12/17/04). I knew that I wanted to probe teachers on this very topic individually at the end of the research, so I left the question “hanging” for them to contemplate. At this point, Sue appeared to need some form of summary or closure to the two particle argument. She asked, “Is this what it all really means – the definition of logarithm?” (Professional Development Session 1, 12/17/04). We decided as a group to take a short break, but Sue pressed on with her question while the others remained.

Sue: See where it says, “the logarithms of a given sine,” that number...maybe I’m just thinking about it the wrong way. But does that have to do with sine, cosine, and tangent?

Kathy: Were you here when we discussed the half-chord thing?

Sue: Yes, I saw that and how you talked about that before. But, what I am seeing them [the students] asking me is, ‘What does that have to do with

where we say the sine of 60 is one-half or whatever?’ I don’t understand that. (Professional Development Session 1, 12/17/04)

The lack of apparent connection between the trigonometric sine her students would have already studied and “the logarithms of a given sine” was a legitimate concern for Sue. I did not probe her further at this point in the professional development session concerning her plans to incorporate the two particle argument (Lesson Installment 2) in her instruction. The fact that Sue was able to raise such a concern indicated that she was considering the use of the material as well as student engagement with the content.

Student difficulty and disengagement. As Sue anticipated her students’ engagement with the history of logarithms material, she was overwhelmingly influenced by the difficulties she perceived her students having. In one instance, related to the order of topics in the curriculum, Sue believed her students would be successful in handling the potential difficulty. Lesson Installment 3 used the relationship of two sequences to bridge the historical idea of logarithms with the more modern concept. At Mulberry High School, however, sequences (Chapter 12 in the text) were covered after logarithms (Chapter 11 in the text). Before we began a detailed discussion of Lesson Installment 3, I asked the teachers to share with the group their general impressions of both the mathematical content and the potential use of the material with students. Sue and Mary commented on the placement of Lesson Installment 3.

Sue: Yeah, these were pretty – [sequences] S1 and S2 – are pretty basic on the first page. They can’t answer what kind of sequence it is because of the way we teach it. They won’t know it until we teach Chapter 12 and logarithms are in Chapter 11. But they can see that it increases by one every time. They will be able to see that and they will be able to see that if you divide one term by the one before it, you get the same number every time. So they will essentially know what it means later.

Mary: But they don’t even really have to know that.

Sue: That's what I'm saying. They won't know how but they will realize it later when you come back to Chapter 12. (Professional Development Session 1, 12/17/04)

Although this initially sounded like Sue's observation of a potential student difficulty, ("they can't answer what kind of sequence it is"), Sue went on to identify how students would be able to recognize patterns and common operations without having formally studied sequences before working on Lesson Installment 3. In two other examples, however, Sue observed that the difficulty presented in the content of the lesson installment would outweigh student persistence or mathematical ability.

The first example is related to Sue's observation given at the opening of the section on professional development. In that particular passage, Sue articulated her overarching concern of how to incorporate the history of a topic after having previously taught the topic in a purely traditional, textbook-influenced manner. Intrigued by her comment, I probed Sue a bit further.

Kathy: You were saying that maybe one use could this like a summary thing – almost like a review. Because wouldn't they have already had exposure to some of the important ideas?

Sue: Yes, but I don't know that they would even...explore it. Even though you don't have to know anything about the topic, it's easier to take this number and raise it to this power, or something. But even I got stuck on one of them and I couldn't figure out why it didn't work. But then that intrigues me, so then I went and worked it because I thought I saw a pattern. I was actually doing the wrong pattern because I couldn't get the number to work out right. But I see the kids not doing that, because, you know, they have no...*drive* (Sue's emphasis).

Kathy: So you have a characteristic trait of mathematical persistence and maybe that is something that students in general don't possess and so maybe they wouldn't stick with it?

Sue: They're going to look at this and ask, 'Why are there so many words on it?' (Professional Development Session 1, 12/17/04)

Sue originally shared her considerations for how to insert material from Lesson Installment 3 – the lesson we were discussing in this passage – because she felt it was worthwhile for her students. However, she eventually contemplated a potential lack of student engagement. Sue predicted a lack of student engagement with the content of Lesson Installment 3 because it contained “so many words.” She also noted, however, that her students lacked a type of persistence she felt students needed to stay engaged in a sequence of activities such as those in the installment.

In a second example, Sue cited student difficulty with a proof exercise as a potential deterrent of student engagement. In Problem 4 of Lesson Installment 6, students are asked to provide reasons for each transformation of the expression for the n th term of b_n from Problem 3 into an alternative form. During the professional development session, the teachers worked through the transformation together. We discussed how students might express reasons for each step in the transformation appearing in Problem 4. The participants began to discuss how one step, changing a decimal to a fraction, might be problematic for students because as Sue and the others indicated, they would find any use of fractions difficult. Further, Mary, Ted, and Shirley each reported that they did not believe students would be able to describe what was called for in each step of the transformation. Sue, however, offered:

Sue: Yes. I think mine would be okay because when we do identities, I give them the problems that are all worked out and they have to fill in the blanks and show what changed from one step to the next and they've gotten kind of used to doing that.

Kathy: Even though those were problems with trig identities?

Sue: Yes, because what I teach them to do is: look! What is the one thing that has changed from this step to this one? And they'd say,

‘well, we went from a decimal to a fraction.’ They would see that, automatically. (Professional Development Session 2, 1/03/05)

In this particular exchange, Sue progressed from an initial reaction that students would balk at the idea of having to interpret an algebraic transformation toward the stance that her students would in fact rise to the challenge of thinking about such a problem. Sue supported her opinion shift by noting that her students were previously given opportunities to consider problems like this during their study of trigonometric identities. It is also important to note that Sue was the only Mulberry teacher who was eventually able to consider positive student engagement with respect to a mathematical task like Problem 4 of Lesson Installment 6.

Outlining examples of how Sue considered her students’ engagement with the history of logarithms materials also provides the backdrop for describing her contributions about the pedagogical decisions she offered during the professional development sessions.

Sue as Pedagogical Decision Maker

During the course of the two professional development sessions at Mulberry High School, I identified several examples of how Sue envisioned using the history of logarithms materials with students. A significant proportion of these were accommodations Sue hypothetically constructed to account for obstacles which she felt would otherwise prohibit the use of history of logarithms resources and lesson installments. The types of difficulties for which Sue offered potential instructional decisions included:

- lack of previous class assignments or activities possessing an historical context;

- the order of required content to allow for the inclusion of the history of mathematical topics;
- inadequate instructional time to allow for incorporating the history of logarithms materials; and
- perceived limited student ability.

Difficulty of context. Sue recognized that without previous experience in researching and summarizing historical information, her students would experience frustration and difficulty with Lesson Installment 1. Although Sue did not completely construct a timeline during her study of how the individual mathematicians and the overarching mathematical ideas contributed to the development of logarithms, she did sufficient research and observed:

I did the same chronological thing that Mary did but the part that I didn't do, but that I think I will do, is not like a time line. I got confused when I was reading the history, because somebody was studying somebody else's work, who was born at a different time. And then some of them were only two years apart but they read something from 15 years earlier. So it was very confusing. I was going to draw a thing like on a time line, but include their birth and their death so that you can see the span of when they lived and who overlapped who and where they overlapped for *that* part (Sue's emphasis), but not for the content – for just like when they lived. (Professional Development Session 1, 12/17/04)

Sue raised an important issue regarding the dissemination of information during the critical beginning of the Scientific Revolution. Sue's observation that several individuals were studying (and possibly finding out that they were simultaneously working on the same mathematical ideas) one another's work is an aspect of studying the history of mathematics that is often untapped. Sue recognized, however, that the context of such an exercise may cause confusion for mathematics students who have not previously used an historical perspective when studying mathematical topics. Sue

continued to consider an alternative to Lesson Installment 1 that would fit with her students' needs and experiences.

I have thought about taking them to the computer lab, in a more structured kind of assignment, and giving them the timeline by assigning a specific person to each student. That way, they only have one person to concentrate on. Maybe put them in pairs and let two of them research one person at the same time. That way they can just get an idea of the time frame we're talking about and then they're not really overwhelmed with it because it was very...time consuming, was it not?

I just thought, because sometimes we have "odd" days, like testing days, that there might be a day to do something different with them. (Professional Development Session 2, 1/03/05)

Finally, Sue refined her pedagogical decisions regarding student research even further when she engaged me in an online conversation via our *Tapped In* community. In March 2005, Sue e-mailed me and asked if we could "meet" online to discuss incorporating some of the lesson installments in her impending chapter (Chapter 11) on logarithms. We met in our *Tapped In* chat room on two different occasions during Mulberry's spring break (Mary joined us as well). Sue and I had begun to formulate an additional assignment to use prior to giving students Lesson Installment 1, which would include an Internet web quest component. Together we decided that the additional guided assignment would allow the students to rely on websites that contained reliable and accurate information, and which would not be blocked by the school district's firewall software. While becoming familiar with these websites (Appendix H), students would be prepared to investigate the information required to complete Lesson Installment 1. During our second online meeting, Sue discussed her tentative plans.

Sue: I have not worked on the questions at all, but I have been thinking about what I will be doing.

Kathy: I only did a few, so I can keep working on them and then we can merge.

Sue: I was thinking on Monday [after spring break] about just going through some neat history things, like showing them how to use the slide rule and tell them about why we rationalize and maybe some other random facts I can find. Then I could begin the talk about logs.

Kathy: Do you want to divide the work [for the webquest]? I could do the top half of the one-pager and you can start with "Discussed logarithms in 1614..." down to the bottom (ending with Euler).

Sue: Okay. (*Tapped In* Session 2 Transcript, 3/17/05)

Sue spent a considerable amount of time on modifying the original idea of Lesson Installment 1 to introduce students to the key mathematicians behind the development of logarithms into a series of activities to use with her students. As Sue indicated in the excerpt from January 2005, the week after spring break would in fact be filled with “odd” days because of the administration of the state high school graduation test. This particular week would also coincide with the beginning of the chapter which focused on exponents and logarithms. Sue planned to use the confluence of these circumstances to provide the opportunity for her students’ first exposure to the history of mathematics. Sue planned to develop and use activities that would be meaningful to her students and that would provide a different experience than they had previously had with respect to the study of logarithms.

Difficulty of the order of topics. Sue highlighted considerations for the existing order of the presentation of logarithms on two occasions during the professional development sessions. In the first example, Sue offered a mostly general observation about the difficulty of including content other than what appears in the textbook for the Trigonometry course. Prior to presenting Napier’s two particle argument, I asked the teachers about their experience with the length of half-chords (an ancient application) related to the trigonometric sine values (a modern application). Only Ted was familiar

with the notion of half-chords. After his contribution of relating the lengths of half-chords with the table of sine values students are typically required to memorize in a trigonometry course, Sue inquired about the placement of historical information within traditional course content. With respect to the difficulties of doing this, Sue observed that, “You would have to teach things totally out of order from the way the textbook is set up, just to incorporate anything other than what’s in the text” (Professional Development Session 1, 12/17/04).

A second instance of Sue’s struggle with both the order and intensity of using actual lesson installments occurred at the end of the first professional development session. After we discussed Lesson Installment 3 and the accompanying resources that I compiled for them, several of the Mulberry teachers described other uses for the installments as an alternative to considering them replacement activities (“supplanting versus supplementing”). This idea was particularly applicable to Lesson Installment 3 because the teachers viewed the length and the fact that it contained “so many words” intimidating to students.

Sue: That’s what they always skip, the word problems. I’m just saying, I don’t know when it would be better to say, okay, do this page this day, then this, and this...

Kathy: That’s maybe an option. I don’t think you could just say, ‘Okay, now we’re going to do this, because I have this really cool activity.’ I think there’s something about a classroom culture that they have to be used to.

Sue: It’s just going to mean that you’re going to have to take the trig [meaning the textbook used for both the Trigonometry and Advanced Algebra courses] book and just do this for all the chapters and match.

Kathy: Well, I don’t think it’s necessary for all topics; maybe just for key topics. But you would need to start early on. Maybe instead of just doing the unit circle, you could do one sample from ancient Babylonia, where you do actually talk about a half-chord. Then after you spend a lot of time on the

unit circle, you can keep coming back to half-chords and the origin of the trigonometric functions. (Professional Development Session 1, 12/17/04)

And, as Sue continued to think about the specific content of Lesson Installment 3, she added,

The pure computational problems are going to mean a lot when we get to Chapter 12 to do sequences and series. So it's going to be easy to say, 'Remember when...?' (Professional Development Session 1, 12/17/04)

Although Sue grappled with the reality that incorporating history of mathematics material of any kind in her teaching would require thinking about the curriculum in new ways – ways which could also be quite time consuming – she was still willing to participate in the struggle. Her desire to consider alternatives to the original presentation of the lesson installments in ways that would best meet the needs of her students, at the same time allowing for the inclusion of alternative perspectives to introduce mathematical topics was indicative of Sue's pedagogical decision making practice.

The obstacle of time. Whereas Sue articulated the difficulty of rethinking the order of topics for her Trigonometry course in order to include the historical development of logarithms, making time for the actual use of the materials was an explicitly discussed obstacle. In several of Sue's proposed plans for lesson installment usage, she made the point that time was a precious commodity. Sue was amenable to the idea of actively including the use of the history of logarithms with her students, yet she tempered her plans with the following.

Sue: Most of this is, I would say, an obstacle. I can tell you one thing that I am *quite* sure of as of today that I will do, will be using the tables to multiply the numbers – Installment 5 (Sue's emphasis). Quite sure about that one. If nothing else than to say, this is why – this is why all the stuff came, is because they needed to multiply together – just so they will be thankful they have calculators today. This is the one thing that I am pretty sure I

will take the time to do. But that's just another problem [laughs]: time, It's our big problem. (Professional Development Session 2, 1/03/05)

Perception of limited student ability. In addition to identifying the lack of time required to incorporate the history of logarithms in her teaching, Sue also recognized a lack of student persistence and ability to engage in such materials. She observed,

The rest of it [the other lesson installments], I don't see the students understanding or attempting. Maybe after I go through the rules with them, depending upon what kind of questions they ask, I might be able to see about the other ones. I have thought about taking them to the computer lab, in a more structured kind of assignment, and giving them the timeline in assigned pieces. (Professional Development Session 2, 1/03/05)

As has been seen in with the timeline activity (Lesson Installment 1) and other examples (see *Student difficulty and disengagement* within *Sue As Anticipator of Student Engagement*), the duality of focusing on what students may not be able to do and altering the content that she was newly exposed to via the lesson installments presented Sue with the task of considering accommodations to address context, order, time, and student ability.

Instructional Practice

Sue translated her study of the historical development of logarithms into instructional practice by incorporating the activities that she felt were most appropriate for her students. When compared to that of the other participants, the extent of how Sue used the history of logarithms was situated between the continuum of implementation, which ranged from incorporating all of the materials studied (Mandy) to incorporating none of them (Shirley and Mary). Ted's implementation of the history of logarithms was also situated between the two extremes and his particular case is discussed in the second half of this chapter.

Sue's moderate implementation of the history of logarithms involved a balance between the use of biographical and historical information and a mathematical examination of the development of logarithms. Sue included Lesson Installments 1, 4, and 5 during her instruction of Chapter 11. Additionally, she created another activity prior to Lesson Installment 1 to introduce her students to the use of important websites and to particular knowledge she wanted to ensure they obtained by the end of the activity. Sue also included a class activity which focused on the number e . This activity was motivated by her personal interest.

The next four sub-sections discuss (1) Sue's self-reported instructional practice relative to logarithms; (2) her beliefs about her role as the teacher; (3) her beliefs about the role of the student; and (4) school factors which influence her instruction.

Existing Instructional Practice

In Item 6 of Part III of the Attitudes Instrument pre-assessment, Sue stated that her usual approach to teaching logarithms was "purely computational." She added that she "shows students the simplest way to calculate logarithms, but [does] not usually include word problems or application problems" (12/15/04). Sue did not identify a progression of topics she planned to cover in Chapter 11 as the other participants did in response to Item 6. She did, however, indicate on another occasion that she planned to cover the curriculum as indicated in the semester plan, which included all but section 11.7 in the *Advanced Mathematical Concepts* text (S. Moe, personal communication, 12/17/04).

Beliefs about the Role of the Teacher

Sue's beliefs about her role of the teacher were heavily influenced by how she believed students felt about mathematics and what they needed to be able to do by the end of any one course. With regards to her role, Sue believed,

I disseminate information. Although sad, I have given up on trying to get my students to appreciate or love math, because I have found it impossible for the most part. I just need them to learn it for the end of course test and so they know enough when they get to college. I believe I have high expectations of my students, although I adjust it every year to the level and type of students I have. (Interview, 10/21/05)

Similar to the other Mulberry High School participants, Sue viewed her role as one in which she was possessor of information that must be conveyed to students. The information that Sue provided was influenced by the need of a prescribed curriculum – specifically, material covered on a county-wide, end of course test – as opposed to the needs and interests of her students and herself.

Beliefs about the Role of the Student

Sue believed students were to obtain information from the instruction provided. She characterized this belief when she reported,

I expect my students to be sponges, and when they start leaking, I expect them to study more and ask questions! I do not believe it is my job to entertain them, so I expect them to take notes and learn the information no matter how boring they think it is or how adamant they are that they will never use it. (Interview, 10/21/05)

Sue believed that she held the mathematical authority in the classroom and that student input involved asking questions when they failed to retain (or “soak up”) information. The way in which Sue described her role and the role of her students is reminiscent of Freire's (1972) characterization of the technical interest, with the teacher

as narrator and with the aim of “turning [students] into containers...to be filled by the teacher” (as cited in Grundy, 1987, p. 101).

Influence of School Features

Sue’s instructional practice was influenced by the existing Trigonometry curriculum and accountability testing, her perception of student ability and interest, and the time available to incorporate an alternative teaching perspective.

Existing curriculum and testing. Sue’s instructional practice was heavily influenced by the curriculum plan established by the school district and approved by the mathematics department. In addition, her Trigonometry instruction was guided by the textbook used in the course. During the professional development session in which we discussed Lesson Installment 3, Sue noted, “they can’t answer what kind of sequence it is because of the way we teach it. They won’t know it until we teach Chapter 12 and logarithms are in Chapter 11” (Professional Development Session 1, 12/17/04). Thus, maintaining a particular order of topics, completing the prescribed curriculum for successful completion of the end of course test, and providing students ample practice with the content they would encounter on the course assessment each influenced Sue’s instructional practice. To achieve her curricular goals during a typical class in which she did not incorporate material from the history of logarithms, Sue would spend approximately 40 minutes reviewing examples or assigned homework problems or asking students to work problems on the board.

Student ability and interest. Sue’s judgment of student ability and interest also influenced her instructional practice. For example, Sue recognized that the overall ability

of her students changed every year (Interview, 10/21/05). When asked about using all of the lesson installments with students, Sue observed,

I don't know if I would ever be able to show the two particle argument. I think it is well above my students. I used to have many more students who were also in physics. It may have worked with them. (Interview, 4/15/05)

The ability to capture student interest was a reality that Sue struggled with, even though she stated that it was not her "job to entertain them." Sue indicated that several students equated her efforts to incorporate the history of logarithms with "giving them busy work" (Interview, 10/21/05). She shared that "it is still upsetting that that is the impression they have of you when all you are trying to do is hook their interest in something" (Interview, 10/21/05).

In addition, Sue anticipated resistance from her students with regard to class activities which strayed from the typical practice of observing examples worked by the teacher followed by students practicing similar problems. Sue reported that, "if I had been doing these types of lessons all year, it would have been easier to use the lessons without meeting so much resistance" (Interview, 4/15/05).

Lastly, Sue doubted her students' mathematical persistence with materials which did not ask for traditional textbook-like problem solving. When contemplating a particular lesson installment, Sue admitted,

I don't know that they would even explore it. I thought I saw a pattern, but I was actually doing the wrong pattern because I couldn't get the number to work out right. But I see the kids not doing that, because, you know, they have no...*drive*. (Sue's emphasis, Professional Development Session 1, 12/17/04)

Time. As was the case with most of the other participants, Sue lacked adequate time to accomplish some of the tasks associated with teaching. She considered incorporating historical problems in the Trigonometry course as possible and she planned

to look into resources for ideas when she had more time (Attitudes Instrument, 12/15/04). Sue experienced additional burdens on her time which prevented her from planning instruction using multiple resources and perspectives due to her mathematics department chair responsibilities. It was considerably easier however, for Sue to incorporate the history of logarithms into her instruction because of the availability of the lesson installments and associated resource materials. While preparing for Chapter 11, Sue stated,

I looked over all of the lesson installments to see which ones I would use with my classes and when. I looked up some information on the number e to talk about where it came from. (Interview, 4/15/05)

As she considered the use of the lesson installments during her instruction of Chapter 11, Sue expressed concern for the time commitment necessary to incorporate the development of logarithms. Sue looked for opportunities to include activities which she initially felt would be time consuming. For example, Sue found creative ways to use class time which would have been impacted by school interruptions in order to provide students with “a different approach to Chapter 11” (Observation, 3/21/05). On other occasions, however, Sue planned to use additional lesson installments in subsequent chapters and was unable to do so because of lack of time (S. Moe, personal communication, 10/24/05).

Chronology of Instruction

Sue taught one Trigonometry class in 2004 – 2005 and her instruction of Chapter 11, which covered exponential and logarithmic functions, occurred from March 21, 2005 until April 18, 2005. Observation data of Sue’s instruction were collected until April 15,

2005. No data were collected during the week of April 4, 2005 due to Mulberry's spring break. Sue's Period 3 Trigonometry class was 55 minutes in length.

Incorporating the History of Logarithms

Sue incorporated the following lesson installments and activities:

- A history of logarithms webquest which guided students to specific information and websites for future use (Class days: 3/21/05 – 3/22/05);
- Construction of a history of logarithms timeline (Lesson Installment 1), assigned to students in pairs (Class day: 3/23/05);
- Class lecture on the development of the number e and vignette activity (Class days: 3/30/05 – 3/31/05);
- Lesson Installment 4: Calculation of logarithmic values (Class day: 4/14/05); and
- Lesson Installment 5: Examination of astronomical calculations using the method of prosthaphaeresis (Class day: 4/15/05).

History of Logarithms Webquest

Sue began Chapter 11 by alerting students to the fact that they were going to take a “different approach” with the chapter (Observation, 3/21/05). Juniors at Mulberry High School were just finishing the first day of the high school graduation test as they began the third period class on March 21, 2005. Sue viewed these test days as the “odd” days she identified during the second professional development session. Even though most juniors were finished by the time third period began, Sue anticipated that she should “plan to do something different” (Professional Development Session 2, 1/03/05) on the days impacted by the graduation test administration (March 21 – March 24, 2005).

Sue asked students several questions in preparation for the research assignment, including:

How many of you are good at history?
What was happening in the 1600s?

Was it good or bad to be moving away from religion and toward reason?
(Observation, 3/21/05)

After a brief orientation to the assignment, Sue escorted her students to the computer lab to begin the webquest assignment.

McCoy (2004) defined a webquest as “an activity in which students utilize World Wide Web resources to obtain information that is then used in a group project” (What is a Webquest? section). The history of logarithms webquest (Appendix H), designed by Sue and I, contained a series of 14 questions. Sue planned for the questions to focus on particular aspects of the development of logarithms in the form of “a more structured kind of assignment” (Professional Development Session 2, 1/03/05). The webquest also provided students with key websites that they would utilize for the subsequent timeline project. Because of the length of the webquest, the poor condition of the computer lab, and the impact of graduation testing, Sue allowed two days for the activity.

At the end of the first day of working on the webquest, Sue asked whether I overheard what the students said as they worked on the webquest. Sue noted that the students wished they could do more of “this type of work.” Sue was not clear, however, whether they were referring to the historical nature of the assignment or the fact that it was Internet-based. Sue observed, “We’ll see what they come up with when we talk about it on Tuesday” (Field Notes Journal, 3/21/05).

Students completed the webquest assignment at the end of the second class day. Sue did not collect the students’ work; however, she did review the responses in preparation for the next activity, a timeline for the development of logarithms. Sue indicated prior to incorporating the webquest activity that she would feel more comfortable discussing the historical background with my assistance.

The thing I am most worried about with all of this is the questions that are going to come up from the kids and I am not really knowledgeable about most of it. That is why I was asking you if you could participate in the discussions. (*Tapped In Session 2 Transcript, 3/17/05*).

Sue chose to review student responses as a whole class and students actively offered responses to the 14 questions.

History of Logarithms Timeline

The webquest experience enabled students to obtain information and resources to complete a timeline for the development of logarithms, a project Sue assigned to students to complete in pairs. Sue introduced the activity by asking each student pair to read Lesson Installment 1. Sue emphasized that it was up to the students to determine the format of their timelines and what information to include.

You need to decide how you're going to do this tonight because I want it tomorrow in order. I know right now, a lot of your things are out of order, so if you're doing it on paper, you need to find a way to get that together tonight, because I want it finished tomorrow. (*Observation, 3/23/05*)

Students chose various formats for the presentation of their timelines, including electronic (using Microsoft PowerPoint), poster, and written formats. The majority of student pairs chose a handwritten format, which essentially gave a listing of dates, people, and mathematical milestones relevant to the development of logarithms. For the most part, students did not indicate the sources used to construct the timelines. Sue did not evaluate the quality or accuracy of the timelines. When I asked Sue whether she would incorporate or refer back to the timeline activity, she responded,

It comes up when we are discussing things in class. It is helpful that they know some of that because that prompts them to ask more questions. I will allow them to write about something they included in their timeline on their test. (*Interview, 4/15/05*)

In addition to adding an historical dimension to class discussions, Sue also believed the webquest and timeline activities “allowed the students to tell me what they have learned in their history classes about what else was happening in the same time period” (Interview, 4/15/05). Sue did not implement the timeline activity as she originally intended. During the second professional development session, Sue considered requiring students to research one mathematician in pairs to allow for a more significant study of one individual contributing to the development of logarithms. Instead of students focusing on particular mathematicians during the study of exponential and logarithmic functions, Sue referred to the students’ research experience in subsequent class sessions and focused mainly on Napier and Euler.

The Number e

The Mulberry Trigonometry text, *Advanced Mathematical Concepts* (Holliday, Cuevas, Carter, McClure, & Marks, 2001), used Section 11.3 to bridge the study of exponential and logarithmic functions by discussing the number e (2.71828...); in particular, its occurrence in formulas for various applications. Sue chose to supplement the text’s introduction by including an historical examination of the number e . The supplementary material consisted of two parts. First, Sue provided the first page of Vignette 52 from *Agnesi to Zeno* (Smith, 1996) for students to read about Euler’s contributions related to the number e . On the second day, Sue used the MacTutor History of Mathematics archive to create a timeline for discussion “about the number e and where it came from” (Observation, 3/31/05).

The students were given a few minutes at the beginning of class to copy the timeline from the board and Sue discussed the influence of the contributions of various

mathematicians leading to the development and use of the number e . Sue spent 15 minutes briefly reviewing the contributions of each of the following in the development of e :

- 1618 – Napier
- 1624 – Briggs
- 1647 – Saint-Vincent
- 1661 – Huygens
- 1668 – Mercator
- 1683 – Bernoulli (Jacob)
- 1690 – Correspondence between Huygens and Leibniz
- 1731 – Correspondence between Euler and Goldbach
- 1748 – Euler

Sue concluded her review by developing the value e from Euler's famous sum,

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

Sue incorporated the work of Euler on several occasions. First, of the five questions Sue created for the history of logarithms webquest, one focused on Euler's work: *Who was Euler and what is he most known for?* Second, when I asked Sue if she had conducted any other research related to the history of logarithms materials since the professional development sessions, she reported that she "looked up some information on the number e to talk about where it came from" (Interview, 4/15/05). Lastly, the final question on the quiz covering Sections 11.1 through 11.5 included the question, "Name two contributions Euler made to mathematics" (Artifact, 4/15/05).

Euler's contributions to the development of logarithmic functions were highlighted in Lesson Installment 1 and Lesson Installment 7. Sue decision to extrapolate more detailed information about Euler suggests that Sue had a personal interest in this mathematical topic motivated by the development of logarithms.

Calculation of Logarithmic Values

Sue reported that what her students previously enjoyed most about studying logarithms was that they represented “fairly easy computational problems” (Attitudes Instrument, 12/15/04). Sue later stated that she felt using Lesson Installment 4 with students would be more effective after they reviewed the essential content in Chapter 11, i.e., the properties of logarithms (S. Moe, personal communication, 4/11/05).

Consequently, after completing Sections 11.4 and 11.5, which focused on logarithmic functions and the properties of common logarithms, Sue presented Lesson Installment 4 to her students.

Hopefully you all recognize the properties on the first page. Let’s look at the chart on the second page. You and your partner are going to be approximating some common logarithms. Please know that we are only talking about base 10. (Observation, 4/14/05)

Sue implemented Lesson Installment 4 in the same way as she did Lesson Installment 1, by requesting that students work in pairs and to begin by reading the installment to ensure they understood the task. Sue continued by explaining the example provided, which called for approximating the value of the logarithm (base 10) of 2. In her explanation of the calculation of $\log 2$, Sue directed her students to

Find two numbers that are pretty close to each other. One thousand twenty-four and 1000 – those are pretty close. But you’ve got to use a [power] of ten. (Observation, 4/14/05)

This direction, to use powers of ten only to construct a relationship, was what Sue understood to be the lesson’s task. It is likely that Sue was trying to simplify the task for her students since they had just completed a study of common logarithms which use a base of 10. Regardless, however, as students needed help thinking about different power relationships, Sue relented on the direction of using only powers of ten, as long as

students created (1) a relationship of two numbers which were approximately equal; (2) which could be expressed as powers; and (3) which utilized the base of the logarithm desired. For example, constructing the relationship

$$3^7 \approx 2^8$$
$$2187 \approx 2048,$$

would have been acceptable for approximating the value of $\log_{10} 3$.

Students worked in pairs for the remainder of the class period on April 14, 2005 (30 minutes). During the class period, both Sue and I circulated among the students to answer questions and observe student work. At the end of the class, Sue observed, “See, that went well” (Observation, 4/14/05).

The next day, students continued working with their partner for the first ten minutes of class. During that time, students calculated approximations, determined actual logarithmic values, and responded to the accompanying questions in Lesson Installment 4. Sue then began a review of responses to questions 2 through 7. In the brief review (ten minutes), Sue focused on the response to question 5, “Why are some of your results better approximations than others?” Instead of soliciting a student response, Sue summarized by asking, “Some people got very, very close on every single one of them. Some got maybe eight of them very, very close and then two of them were nowhere near [the true value]. Why is this?” (Observation, 4/15/05)? Several students conjectured what was causing the approximated values to differ from the calculator values. Sue eventually told students, “When you use a calculation that you already know, it makes it a little bit closer than if you did not use that and you just went with a pure approximation” (Observation, 4/15/05). Sue misrepresented the reason behind the varying accuracy of approximated logarithmic values. The brief amount of time that Sue allotted for

reviewing the more conceptual aspects of this lesson did not enable her to sufficiently examine the features contributing to the accuracy of the approximated values.

Lack of time also impacted the incorporation of Lesson Installment 4 in another way. Question 7 asked students to *Estimate log 11 and log 13 using an appropriate power relationship for each*. Sue was unable to review question 7 and noted, “You can do number 7, but for the sake of time, we won’t do number 7” (Observation, 4/15/05).

The Method of Prosthaphaeresis

Although Sue identified insufficient time for a full treatment of Lesson Installment 4, she did so to ensure adequate time to incorporate Lesson Installment 5. During the professional development sessions Sue commented that,

Most of this is, I would say, an obstacle. I can tell you one thing that I am *quite* sure of as of today that I will do, will be using the tables to multiply the numbers – Installment 5 (Sue’s emphasis). Quite sure about that one. (Professional Development Session 2, 1/03/05)

Sue used the remainder of class on April 15, 2005 to introduce students to the method of prosthaphaeresis, a key process in the development of the logarithmic properties they had previously used in Sections 11.5 of their text, as well as in Lesson Installment 4.

Implementation of Lesson Installment 5 began with a student requesting to read the *Introduction*. After briefly discussing the historical background, Sue guided students through determining the product of 275.6 and 9848. In addition to determining the product of these two numbers by using the sum of two quantities, Sue was also able to review trigonometric identities students encountered the previous semester. Several students inquired about the process resulting in approximate values, as opposed the exact ones obtained on modern calculators.

- St1: You know how they did this back in the day? When did they get to the point when they actually started getting the real exact values?
- Sue: Well, the longer people worked on it, you know how we were reading on the internet? Some people worked on it more after the other people were dead. They read about it; they heard about it; they thought it was interesting and they had been working on it to. So they just added to it. Eventually it just became more and more accurate. And they did it by hand!
- St2: When they did it by hand, what was the number that they used inside the cosine? [Here, the student is referring to the values used in $\cos(x + y)$ or $\cos(x - y)$ from the identity $2\cos x \cos y = \cos(x + y) + \cos(x - y)$.]
- Sue: They calculated it. They had tables for all of it. They had big tables like this for *everything* (Sue's emphasis).
- St2: So one person sat down and actually did this?
- Sue: It was more than one person. A lot of people worked on it. (Observation, 4/15/05)

Sue was quite certain that she would include this lesson installment in her instruction. She was able to make connections to previous course material (using and proving trigonometric identities) and to the historical research she required students to do at the beginning of the chapter. Sue observed,

With the timeline activity, it was interesting to see them think things are neat that they did not know, or for them to see the connections between when and why something was discovered and how we use it or need it today. With the other activities (e.g., Lesson Installments 4 and 5), I think the only benefit they got out of it is that they have an appreciation for the calculator. (Interview, 10/21/05)

Summary

Sue was moderately engaged during the professional development sessions in an attempt to study, understand, and consider the use of the historical development of logarithms in her teaching. Her contributions during the sessions indicated that she did not complete every lesson installment. She spent the time studying the history of

logarithms judiciously for the purpose of planning the best use of the materials with her students. For the most part, Sue's actual implementation paralleled her plans for incorporating the historical development of logarithms. The following sub-sections summarize the case of Sue, including her

- professional background;
- change in attitudes towards the history of mathematics;
- content knowledge of logarithms;
- engagement during the professional development sessions focused on the history of logarithms;
- beliefs about the roles of teacher and students and influential school features; and
- instructional practice, including the use of the history of logarithms.

The summary ends with an examination of the obstacles, affordances, and benefits related to Sue's experience of incorporating the historical development of logarithms.

Professional Background

Sue possessed a bachelor's degree in mathematics and completed an approved teacher certification program before beginning her teaching career at Mulberry High School. The 2004 – 2005 school year was Sue's fourth year of teaching and her second year as mathematics department chair. She was currently pursuing a master's degree in mathematics education. Sue's participation in professional development activities was strong when compared with her Mulberry colleagues. Sue reported no previous formal experience with the history of mathematics; however, she expressed interest in studying the historical development of logarithms with hopes of strengthening instruction of a topic which often left her and her students frustrated.

Attitudes

Sue’s attitudes towards the use of history of mathematics in teaching remained strong throughout her participation in this study. Table 19 shows Sue’s responses to Part I of the Attitudes Instrument for each administration.

Table 19
Attitudes Instrument (Part I) Results: Sue Moe

| Survey item | Pre response (12/15/04) | First post response (1/03/05) | Second post response (4/15/05) |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|-------------------------------|--------------------------------------|
| 1. Understanding the history of mathematics is an important part of understanding mathematics. | 5 | 6 | 6 |
| 2. Including history enriches the teaching and learning of mathematics. | 6 | 6 | 5 |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | 6 | 6 | 6 |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | 5 | 5 | 5 |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | 5 | 6 | 5 |
| 6. Using historical materials in my mathematics classes has been an integral part of my instruction in: Algebra I or II. Geometry. Precalculus/Trigonometry. Calculus. | 1 1 1 1 | 1 1 1 1 | 1 no response 4 no response |
| 7. Prospective mathematics teachers should be required to study the history of mathematics. | 6 | 6 | 6 |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | 6 | 6 | 6 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

In addition to Sue’s responses indicating her favorable attitudes toward using the history of mathematics in teaching, they also revealed that her attitudes were consistent over time.

Two changes in Sue’s attitudes are notable. First, prior to the professional development sessions and just after, Sue strongly agreed that including history enriches the teaching and learning of mathematics. After her experience with using the history of logarithms, however, Sue moderately agreed with the statement. Her response may indicate that she held a less-idealized view of the enrichment experience – from her perspective and that of her students – after trying to incorporate the history of a topic in teaching. The second modification, found in Sue’s response to the Precalculus/Trigonometry category in Item 6, reflects her actual practice of incorporating elements of the historical development of logarithms.

Two of the four items of Part II of the Attitudes Instrument support features of Sue’s implementation of the historical development of logarithms. The two main features of Sue’s practice included requiring students to

- Research the mathematicians central to developing logarithms (webquest and Lesson Installment 1) and
- Work on historically-situated problems related to the historical development of logarithms (Lesson Installments 4 and 5).

Pre- and post-responses to Items 1 and 4 of Part II illustrate the impact of Sue’s instructional experience related to the four class activities (see Table 20).

Table 20
Attitudes Towards Particular Historical Activities (Part II): Sue Moe

| Item stem and choice | Pre-assessment completion (12/15/04) | Post-assessment completion (4/16/05) |
|-----------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Researching a mathematician contributes... | <u>a lot</u> to a mathematics course because it helps the students make connections and realize how long math has been around, and if they do it long enough they will see how math has evolved. | <u>a little</u> ... because it offers them the chance to see what the original uses of some things were and how it fits together with other things. |
| 4. I would consider incorporating | <u>possible</u> because it is something I will look into when I have the | <u>most likely</u> because I find it interesting and it sometimes |

| Item stem and choice | Pre-assessment completion (12/15/04) | Post-assessment completion (4/16/05) |
|---------------------------------------------|--------------------------------------------------------------|----------------------------------------------------|
| historical problems in the curriculum as... | time, because no resources are available to me at this time. | prompts the students to ask interesting questions. |

Sue added “it would be ‘a lot’ if the kids cared” to her post-assessment completion for Item 1. On different occasions during and after her use of the webquest and Lesson Installment 1, Sue commented about the success of including such research activities with students. When I asked Sue what she did when students did not seem interested in the historical information, she said,

I keep going anyway. Hopefully, even if they never admit it, they will think about how they are glad they did not have to go through what others did without technology and such. I try to tell them they would not have anything they have without the history of mathematics and that they should at least be grateful if nothing else. (Interview, 4/15/05)

At the end of the study, Sue reported that incorporating historical problems would be a likely part of her practice (Item 4). Both reasons Sue provided were teacher-centered, related to her interest in the material and potential students questions.

Content Knowledge

Sue’s performance on the content knowledge post-assessments was identical. She improved from answering only the three purely traditional items correctly on the pretest to answering three traditional and three historically-oriented items correctly on the two post-tests. As with two of the other Mulberry participants, Sue was unable to successfully define *logarithm*. The results of the content knowledge post-assessments are given in Table 21.

Table 21

Content Knowledge Instrument Results: Sue Moe

| Item | Historical/ Traditional | First post response (1/03/05) | Second post response (4/16/05) |
|---------------------------------------------------------------------------------------|----------------------------|-------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|
| 1. Define <i>logarithm</i> . | Traditional | No response | No response |
| 2. Describe the basic idea or motivation for the invention of logarithms. | Historical | Correctly described basic idea in terms of calculations necessary for astronomical measurements | Same as 1/03/05 response |
| 3. Construct the values for $\log_{10}2$ and $\log_{10}3$ without using a calculator. | Historical | Approximated each value correctly | Approximated each value correctly (power relationship for $\log_{10}2$ different) |
| 4. Let $u = b^n$ and $v = b^m$. Verify $L(u) - L(v) = L\left(\frac{u}{v}\right)$. | Historical | Attempt only | No response |
| *5. Evaluate: $\log_{32}16$. | Traditional | Evaluated successfully by converting to exponential form and solving | Evaluated successfully by converting to exponential form and solving |
| *6. Evaluate: $\log_{\frac{1}{3}}81$. | Traditional | Evaluated successfully by converting to exponential form and solving | Evaluated successfully by converting to exponential form and solving |
| 7. Calculate the product of 8409.5 and 951.49 using the method of prosthaphaeresis. | Historical | Approximated value successfully using prosthaphaeresis | Approximated value successfully using prosthaphaeresis |
| *8. Solve for x : $2\log 3 + \log x = \log 45$. | Traditional | Solved correctly using properties of logarithms | Solved correctly using properties of logarithms (combined two steps for more direct solution) |

Note. Items with an (*) were answered correctly on the Content Knowledge Pretest.

Professional Development Engagement

Sue viewed the study of the historical development of logarithms as providing her with “a deeper understanding of logarithms” (Interview, 4/15/05). Sue was moderately engaged in the study of the historical development of logarithms during the professional development sessions. This characterization was a result of observing Sue’s collaborative efforts during discussion of the lesson installment content, her perception of student engagement with the material, and her plans for using the historical materials with students.

Sue’s consideration for how to employ an historical approach to teaching logarithms was also prevalent in her planning activities outside of the professional development sessions. Sue focused her study of the lesson installments and resources on activities in which she was interested and which she believed her students were able to handle. She created additional classroom activities for students when the lesson installment content did not meet her needs.

Influence of Beliefs and School Features: Obstacles to Including History

Sue held beliefs about her role as teacher and the role of students which were similar to Grundy’s (1987) characterization of the technical orientation to instruction. Sue believed her main responsibility was to disseminate information and that students should take in the information and ask questions to eliminate gaps in the transfer of knowledge. Yet her technical view of teaching did not negatively impact Sue’s desire to seek alternative methods of disseminating information to students. Sue recognized that her decision to modify her instructional practice to accommodate an historical approach impacted her students’ comfort zone with the already established roles. She observed, “If

I had been doing these types of lessons all year, it would have been easier to use the lessons without meeting so much resistance” (Interview, 4/15/05).

Three types of school-related factors (obstacles) also influenced Sue’s instructional practice: (1) the pressures of a prescribed curriculum and accountability assessments (end of course test); (2) perceived student ability and interest; and (3) adequate time for planning and instruction. The impact of multiple obstacles often influenced Sue’s decisions related to her instructional practice. For example, Sue discussed the feasibility of using Lesson Installment 3 during Chapter 12 (sequences and series) since the computational background of logarithms required the concept of sequences. Sue did not consider changing the order of topics in the course (prescribed curriculum), however she did not have adequate time during Chapter 12 to include an activity which would require certain student abilities and interest to make the connection between the topics.

Incorporating History of Logarithms

Sue incorporated five distinct activities into her instruction. The five activities included two that required students to conduct Internet research about the significant mathematicians and motivations behind the development of logarithms (webquest and Lesson Installment 1); one enrichment and discussion episode (history of the number e); and two that were focused on a mathematical process (Lesson Installments 4 and 5).

Sue’s implementation of each was purposeful. For example, Sue was certain she did not want students to begin their experience with the history of mathematics being overwhelmed by an Internet search requiring them to reconstruct the development of a topic. To make the timeline activity more meaningful, Sue helped to design a webquest

which would introduce students to key aspects of the history of logarithms, as well as the electronic resources they would encounter. Sue's discussion about the history of e enabled students to connect their previous research on the history of logarithms with the description of e given in their textbook (Holliday et al., 2001):

The number e in the formula is not a variable. It is a special irrational number. This number is the sum of the infinite series show below.

$$e = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \dots + \frac{1}{1 \cdot 2 \cdot 3 \cdot \dots \cdot n} + \dots \quad (\text{p. 712})$$

Lastly, Sue used Lesson Installments 4 and 5 after completing traditional exercises with the properties of logarithms and calculations with common logarithms. In this way, Sue illuminated students' experience with calculations and applications by integrating two activities which accentuated the evolution of the historical development with modern calculating devices. Sue noted that, "Even if they never admit it, they will think about how they are glad they did not have to go through what others did without technology and such" (Interview, 4/15/05).

Benefits of Using the History of Logarithms

Sue identified three benefits of incorporating the historical development of logarithms. As previously noted, Sue believed she benefited from studying and using the history of logarithms. Sue also believed that by introducing her students to the history of this topic she was able to eliminate the frustration many of her students experienced when studying logarithms. Lastly, Sue observed that using the history of mathematics enabled students to incorporate content from other courses. She noted,

For myself, I have a deeper understanding of logarithms, which I would assume helps me to explain it better to my students. All of the years I have taught trigonometry, this is the one section that students always ask me, 'what is this for?' or 'who came up with this?'. That did not happen this year because I already provided the information to them about why it was developed. It also allowed the

students to tell me what they have learned in their history classes, about what else was happening in the same time period. (Interview, 4/15/05)

Affordances of Using the History of Logarithms

Sue identified several aspects of teaching at Mulberry High School which would obstruct her efforts to incorporate an historical approach to teaching mathematics. The curriculum, assessments, textbook, student interest and ability, and time available to study, develop, and implement new ideas all impacted Sue's instructional practice. Sue admitted that time, the Trigonometry curriculum, and student experience were most influential. When I asked Sue what it would take with respect to school or student factors or the lesson installments themselves to use the content from all seven, she said,

The timing is important. I can't use sequences and series until after Chapter 12. If I had been doing these types of lessons all year, it would have been easier to use the lessons without meeting so much resistance. I would have to "train" my students to understand why we do history activities all year long so that their complaining would not bother me so much. (Interview, 4/15/05)

Despite Sue's recognition that including the history of mathematics is difficult, she was able to incorporate a moderate number of historical activities. Sue's interest in the history of mathematics enabled her to experiment with an historical approach to logarithms. In addition, Sue appeared to be challenging the characterization of her role in the classroom. Except for the timeline and lecture on the number e , Sue did not simply disseminate historical information. Students engaged in the research activities and the mathematical work of the lesson installments and for the most part, Sue made time for each to be completed and discussed.

The Case of Ted Jones

In this section of the chapter, I present the case of Ted Jones. Similar to the presentation of the cases of Mandy and Sue, the following sub-sections describe

- Ted's professional background;
- his engagement during the professional development sessions designed to examine the historical development of logarithms;
- his prior instructional practice related to logarithms and his beliefs about his role as a teacher and the role of his students as learners and;
- his implementation of the historical content with students in his two Trigonometry classes.

The chapter ends with a brief summary of Ted's experience with the history of logarithms, including identification of the obstacles, benefits, and affordances related to Ted's use of the historical development of logarithms.

Professional Background

Ted has taught secondary mathematics for 16 years, the last two of which at Mulberry High School. Prior to Mulberry, Ted taught in three other locations (in two different states), including a private, Christian school.

Ted's academic and teacher education preparation included a wide variety of experiences. After serving in the military, Ted pursued a Bachelor of Science degree in Bible studies. With the aim to teach mathematics, Ted also earned a bachelor's and master's degree at another institution, each with an emphasis in mathematics education. Ted's undergraduate mathematics content preparation is similar to that of Mandy's, which included relatively few upper level mathematics courses. Ted reported taking only College Algebra, Calculus I and II for his undergraduate degree and Statistics, Statistical Analysis, and Linear Algebra at the graduate level. In addition to earning a master of arts

in teaching, Ted completed 18 additional hours in mathematics and mathematics education coursework and earned state certification for teaching secondary mathematics. Ted reported that he had not taken a formal course in the history of mathematics but that he experienced a small exposure to history during professional conferences. With respect to using the history of mathematics in teaching, Ted reported that he “read through a classic textbook on the history of mathematics and developed and used a mini-lesson and overview” (Background Instrument, 12/15/04). He also noted that, “the Math Tutor (sic) website at Saint Andrews is a key site [he has] used for about five years” (Background Instrument, 12/15/04).

Ted’s experience with a variety of professional development activities during the two years prior to his participation in this study was less pronounced than the average professional development experience of the five participants. Ted claimed experience with four distinct activities (out of 12 potential activities listed), including taking a mathematics content or methods college course; attending county-provided workshops and training sessions; regular, independent reading for professional purposes; and consulting with a mathematics specialist. Ted also participated in leadership activities within the mathematics department by serving as the lead teacher for the Analysis course.

Attitudes and Knowledge

Table 22 displays Ted’s responses to Part I of the Attitudes Instrument Pre-assessment. For Part I, a Likert-type scale was used, with responses ranging from “strongly disagree” (corresponding to a score of 1) to “strongly agree” (corresponding to a score of 6). Ted agreed with each item on Part I, although he only slightly agreed with a majority of the items.

Table 22

Attitudes Instrument (Part I) Pre-Assessment Results: Ted Jones

| Survey item | Pretest response |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|
| 1. Understanding the history of mathematics is an important part of understanding mathematics. | 4 |
| 2. Including history enriches the teaching and learning of mathematics. | 5 |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | 5 |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | 4 |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | 4 |
| 6. Using historical materials in my mathematics classes has been an integral part of my instruction in: Algebra I or II. Geometry. Precalculus/Trigonometry. Calculus. | 4 4 4 No response |
| 7. Prospective mathematics teachers should be required to study the history of mathematics. | 6 |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | 6 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

When a subset of the eight items of Part I are categorized by purpose – either using history for understanding and learning mathematics or using history in the teaching of the subject – Ted’s responses were oriented with respect to selective aspects of using the history of mathematics in teaching. Ted moderately agreed that including history enriches teaching and learning (Item 2) and that including biographies of mathematicians makes mathematics classes more enjoyable (Item 3). Ted slightly agreed with items directed toward the connection between understanding history and understanding mathematics (Item 1) or knowledge of the development of a topic related to learning the topic (Item 4).

Examining the completions to the four items of Part II of the Attitudes Instrument provided additional insight into Ted's selectivity in using the history of mathematics. Ted indicated that researching mathematicians contributes a little to "giving a human face to math" (Attitudes Instrument, 12/15/04). In addition to considering the use of the history of mathematics as a way to incorporate biographical information, Ted also noted that requiring some history work and sometimes using the history of mathematics with students, given access to quality materials, would "provide another dimension to the work for students" and "enhance the instruction" (Attitudes Instrument, 12/15/04). When I questioned Ted further, he suggested that talking about the people behind the mathematics – only briefly – would in fact provide this additional dimension and instructional enhancement (Interview, 3/28/05).

There were several inconsistencies in Ted's responses to Item 6 of Part I of the Attitudes Instrument. First, Ted indicated that he slightly agreed that using historical materials had been an integral part of his instruction in geometry, a course he did not teach. Next, for a course he did teach, Mathematical Analysis, Ted did not indicate whether he used historical materials. Lastly, Ted reported that he moderately agreed with the statement, *Including history enriches the teaching and learning of mathematics* (Item 2, Attitudes Instrument, 12/15/04). Thus, Ted's views regarding use of the history of mathematics and his reported practice were somewhat contradictory.

Ted did well on the Content Knowledge Pre-assessment compared with the other participants' performance. He answered five of the eight questions correctly, four of which were traditionally-oriented (see Table 23).

Table 23

Content Knowledge Pre-Assessment Results: Ted Jones

| Item | Historical/Traditional | Response given |
|---------------------------------------------------------------------------------------|---------------------------------------------|--------------------------------------------------------------------------------------------------|
| 1. Define <i>logarithm</i> . | Either, depending upon participant response | Correctly defined <i>logarithm</i> in a traditional sense |
| 2. Describe the basic idea or motivation for the invention of logarithms. | Historical | Correctly described in terms of making computations easier (including astronomical measurements) |
| 3. Construct the values for $\log_{10}2$ and $\log_{10}3$ without using a calculator. | Historical | Attempted by simply converting from logarithmic to exponential form (incorrect) |
| 4. Let $u = b^n$ and $v = b^m$. Verify $L(u) - L(v) = L\left(\frac{u}{v}\right)$. | Historical | Proof attempted (incorrect) |
| 5. Evaluate: $\log_{32}16$. | Traditional | Evaluated successfully by converting to exponential form and solving |
| 6. Evaluate: $\log_{\frac{1}{3}}81$. | Traditional | Evaluated successfully by converting to exponential form and solving |
| 7. Calculate the product of 8409.5 and 951.49 using the method of prosthaphaeresis. | Historical | Attempted; converted multiplication to the sum of two logarithmic values (incorrect) |
| 8. Solve for x : $2\log 3 + \log x = \log 45$. | Traditional | Solved correctly using properties of logarithms |

Ted was the only participant who successfully defined *logarithm* in Item 1 and did so without confusing *logarithm* with *logarithmic function*. Additionally, Ted included an attempt at an historical translation of the two components of the word *logarithm*.

Ted's response to Item 2, although considered correct, also contained a minor error. He identified the basic motivation for the invention of logarithms as:

In the application and study of mathematics, especially astronomy, the work with large and small numbers along with involved computations necessitated an easier way to compute. (Content Knowledge Pre-assessment, 12/15/04)

For the astronomical calculations in the 16th and 17th centuries, computations involved large numbers only.

Professional Development Engagement

Ted's engagement during the professional development sessions was considered moderate and due in large part to the selectivity of his participation. Using *select* to mean "judicious or restrictive in choice" (Merriam-Webster, 1993, p. 1058) assisted in characterizing Ted's contributions during the professional development sessions. Other levels of Ted's interaction, including questions that he asked and plans for student use of the history of logarithms that he articulated, were selective in nature as well. In some instances, Ted was limited in his choice of contributions because they appeared to be dependent upon how he engaged with the history of logarithms materials on his own. In other instances, Ted's careful choices were driven by his personal knowledge base and interests. Regardless of the individual underlying reasons for such choices, how and when Ted chose to participate during the professional development sessions was heavily influenced by his selectivity.

One particular theme of Ted's selectivity is represented by the following observation. In this excerpt, taken from our discussion regarding Lesson Installment 1, Ted expressed how the "human side" of using the history of mathematics with students is helpful, yet must be used with caution.

And another thing I would do is go to the [Mac] Tutor web site and just print a poster picture of these guys – just to be with them not too much. If you do too much with the students, you start losing them. If you just do a few things and do a real good job, you can always add to these guys later. So it puts a face on it. It puts a face to what you are talking about. And I do think it is helpful to talk about the historical background. (Professional Development Session 1, 12/17/04)

Thus, Ted advocated a selective use of the history of logarithms: sharing the personal and mathematical contributions of the mathematicians to the development of logarithms. However, he also advocated the selective use of how much of this information to include: “to be with them not too much.” In another sense, Ted’s affinity for discussing the human contributions behind the development of logarithms during the professional development sessions was another form of selectivity.

Ted as an Assertive Collaborator

Ted’s collaborative contributions during the professional development sessions were assertive in nature. Ted was willing to boldly and confidently share (Merriam-Webster, 1993, p. 69) techniques and insights related to the study of the history of logarithms during the professional development sessions. Often assertiveness carries negative connotations, as it is often characterized by forceful or aggressive influence (Merriam-Webster, p. 69). This was not true in Ted’s case. Instead, he often contributed in such a way that he exuded confidence about the topic of which he spoke, even in instances of misunderstanding directions or erring mathematically. There are four particular episodes which occurred during the two professional development sessions which exemplify Ted’s participation as an assertive collaborator. The corresponding data describe examples of Ted’s ability to:

- highlight the religious influence of the time period during which logarithms were developed;
- explain the concept of half-chords to the other teacher participants;
- discuss a classroom activity that engages students during initial exposure to the properties of logarithms; and
- share his own mathematical thinking during a discussion about calculating approximations for logarithmic values.

The influence of religion. While I was getting to know Ted, he revealed to me that he had majored in Bible studies, taught in a parochial school, and was very active with the youth program at his church. His personal interest and commitment to religious study and service was evident on at least one occasion. Ted actively participated during our discussion about the background research that the participants conducted for Lesson Installment 1. It was clear from his contributions that Ted was very interested in the combination of the humanistic and non-secular influences on the mathematical ideas motivating the development of logarithms. To this end Ted observed,

All of these things are related to what's happening in Europe; to what's happened with the Protestant Reformation. That's a big thing at that time. All of these guys seem to be into astronomy, it seems like. And all of these things go together. And these guys are in separate places, trying to solve problems or to make the computation easier. Or they're trying to investigate some area of science. Some guy comes up with something and it is published and it spreads around to different parts of Europe. So the thing Napier did that was really, really helpful was his...his ivory rods. And he makes a statement that they were being sold to foreign countries. Well that stuff happened all the time during that time period. And there were some really good, interesting people and figures in history that really set the stage for what we study in math and science today. (Ted's emphasis, Professional Development Session 1, 12/17/04)

Although Ted did not specify "what happened with the Protestant Reformation" with regard to the development of logarithms, he was able to assert his view of the link between the scientific advancements of the time (i.e., by Kepler and Napier) and the need to break away from the restrictive edicts of the Catholic church concerning the positional role of the Earth in the universe. The other Mulberry teachers appeared to accept Ted's contribution in such a way as to view Ted as an authority on the topic. Only one participant, Mary Long, interjected with the comment that astrology was also something that "these guys seem[ed] to be into" during the time of Napier and his contemporaries (Professional Development Session 1, 12/17/04).

Understanding half-chords. An essential element to following and understanding Napier’s two particle argument requires the understanding of the relationship between half-chords of a circle and the trigonometric sine of an angle. Prior to presenting the two particle argument to the Mulberry teachers, I reviewed the mathematical background knowledge necessary for the two particle argument. After I highlighted the reason why Napier identified his values as *the logarithm of a sine* as opposed to just *logarithm*, I pointed out that, “at this time, and hundreds of years previous, that when they talked about sines, they were talking about half-chords of half a central angle of a circle” (Professional Development Session 1, 12/17/04). I provided a diagram similar to Figure 1 on the board to illustrate the construction of the sine of the half-chord of an angle.

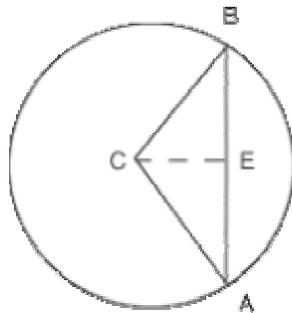


Figure 1. Segment BE (\overline{BE}) is a half-chord corresponding to the central angle BCA.

After some discussion about the ordering of topics in a Trigonometry course to accommodate the inclusion of an historical perspective, Ted contemplated my diagram (Figure 1) further and offered,

You know there is a really good illustration to explain the nature of sine. And all it is, is just a unit circle with a line going this way [uses hand motions] and then every 10 degrees just go around the circle and then just draw at the top with one color marker, the half-chords. Then on the bottom, the other [half-chords]. It is essentially the same thing there [points to diagram on the board]. But for a student, it is very easy for them. If you want something to tell them in five seconds what sine is, here it is right here. (Professional Development Session 1, 12/17/04)

I initially misunderstood the type of illustration Ted was describing because I had in my mind another, similar graphical representation of the sine function. Ted and I realized our miscommunication and he was motivated to address the misunderstanding.

Ted: Are you talking about what I'm talking about?

Kathy: I think so. What you are talking about?

Ted: Well, I'll show you. Like this, every ten degrees you put a little dot. What you want to tell the students, you want them to understand. Now you go, you just take this marker and see this is 90 [degrees] right here and this is 45 and that's 30. All you're doing is just taking the height of this little thing right here.

Kathy: Oh, I was thinking of something else. I'm glad that you wrote that then, because I was thinking of the tangent-secant model.

Ted: And here's 180. Anyone can see that 180 has got to be zero because there is no height there. [He really means the sine of 180 is zero.]

Ted: Then all you've got to do is just take these red guys (half-chords above the axis in the sine curve; in the upper half of the circle), put them over here [continues drawing, see Figure 2]. And take the blue guys (half-chords below the axis in the sine curve; in the lower half of the circle) and put them over here.

But for a student, you've got to give them something to hang onto... And it's easy to go from that to what [you have with the half-chords].
(Professional Development Session 1, 12/17/04)

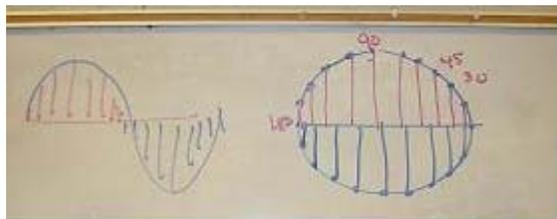


Figure 2. Ted's rendition of showing half-chords graphically.

Here, Ted communicated a particular confidence and command of both the topic and the pedagogical considerations. Consequently, his collaborative stance during the exchange is characterized as assertive rather than solely facilitative.

In the beginning. Ted attempted to portray his current instruction as already including aspects from the history of mathematics. In this way, his contributions during the professional development activities were offered as examples of what could work when accentuating the historical development of logarithms. In one particular instance, however, Ted offered commentary on how his traditional introduction to the basic properties of logarithms typically included historically-influenced experiences for students.

One way I have just taught the properties and the other one is were I let the students pick two to three digit numbers and then we, sort of, I guide them to discovering – actually I’m guiding them but they’re discovering – and when they see that that little key on their calculator, “log,” really does enable them to add the power of the two of those guys together and it’s the power of the product, they buy that. I want as many of those experiences that I can for my students because sometimes we are conveying that the culture of mathematics has always been established. And you [not really me, specifically] do that too much. All of the students say, “You just want to fill my brain up with a bunch of other brain information from other people’s brains, you know.” And so to try to make it real to them and at the same time give them what they really need, they need to understand the properties. That’s a really quick way to do that. So I found that to be more or less helpful, plus they remember the way we did it. (Professional Development Session 1, 12/17/04)

Here, Ted outlined a guided-discovery activity that he used to introduce the properties of logarithms, which was purely mathematical (computational) in nature. He pointed out that this experience would “convey that the culture of mathematics” has *not* always been established. However, this is contradictory. Although Ted felt it was important to convey that not all mathematics appears from out of nowhere in an organized, established form (i.e., school mathematics versus mathematics as a discipline), his example has the potential to convey this very idea to the students who participate. Ted asserted a strong position on the importance of how we represent the human contribution

to the development of mathematics, yet the example he shared is in effect purely computational and devoid of historical or human intervention.

Computation as an end. Perhaps the most assertive example of Ted's professional development participation occurred during his recollection of the completion of Lesson Installment 4. Although the explicit directions in this installment failed to mention that a calculator is not to be used for the calculations in Problem 1, they are implied. For example, the introduction to the activity reads,

Napier and Briggs each spent many years doing lengthy computations to determine tables of logarithms. Since Napier did not use base 10 for his logarithms, but Briggs did, this activity will introduce you to Briggs' approach. Keep in mind, however, that we will be finding approximations for only a few common logarithms and will not have nearly the accuracy that Briggs found when he was developing his logarithm tables! (And all of his work without a calculator, no less!)

Thus, the intent of the activity was to provide an experience with Briggs-like calculations without a calculator, with the main purpose to examine the use of the properties of logarithms established in Lesson Installment 3. A secondary purpose of the lesson was to promote an appreciation of the development of logarithmic values. With respect to this activity, Ted was forthcoming in the methodology he developed to complete Lesson Installment 4. Interestingly, however, the following excerpt exhibits a similar lapse in connecting the history of logarithms to the actual mathematical work, as was found in the previous example on the introduction to logarithmic properties. In this instance, Ted viewed Problem 1 as a purely computational problem for the purpose of obtaining the best value for the logarithm of each given number. Ted's quest for the closest logarithmic value essentially stripped the activity of its connection to the work of

Briggs (done without the aid of a calculator) and how Lesson Installment 4 fit within the greater scheme of the proposed history of logarithms materials.

Kathy: So, we didn't talk about [Installment] 4 at all, other than Ted giving it a try. Do you think you have the concept of the table and everything?

Ted: Yeah, I think I pretty much got [Installment] 4. The thing about [Installment] 4 was...I studied your example for a while and then I figured out, that I could put it in my little calculator, without cheating, obviously. I put in here on my little table on my graphing calculator, and on the next one [function equation] I did three to the x [3^x]. And then I went to a table and I said I'm looking for a value on the right that's close to one. Either nine point something really high or one point something. So the whole purpose of this is try to find the log of three, without using the log key on your calculator. That's what I understood the point was.

So, I did three to the nineteenth. Now how did I get three to the nineteenth? I didn't just take my calculator and go, three to the ten, three to the eleven, three to the twelfth. I just put it in a little – my equation editor three to the x . I went to my table and I just started running down here by integers and on nineteen, I got 1.16, which was the closest I got to a "1." How did I know to do one, because in your example up there, you had the log of two. And then you had two to the tenth is one zero two four [1024]. And that's close to a thousand. So I kind of took that as my cue and I said, I'm looking for something close to...one [he actually means a power of 10].

So, what I did was three to the nineteenth is actually equal to one point one six, times ten to the ninth [$3^{19} = 1.16 \times 10^9$]. So, what I did is in that little box, I said, three to the nineteenth is approximately ten to the ninth. And then what I did is I took the log of both. I just said, the log of three to the nineteenth is approximately the log of ten to the ninth (Ted's emphasis).

Kathy: So you got the log of three is equal to nine divided by nineteen?

Ted: Right. And then I did that for all of them. I just tried to find a number that was pretty close [to a power of 10].

Kathy: So you did not, for instance, on log of six, take your log of two value and add it to your log of three value?

Ted: No, I just did them all just that way. (Professional Development Session 2, 1/03/05)

Several aspects of this particular excerpt aid in characterizing Ted's collaboration as assertive. Ted's confidence in his ability, in spite of making mathematical errors (e.g., stating that 3^{19} is equal to 1.16 times 10^9 , as opposed to approximately equal) is emphasized by the fact that no one challenged his techniques or explanations – including myself. During the explanation of the methods which Ted used to complete Problem 1 of Lesson Installment 4, none of the other teacher participants raised the issue with Ted concerning his lack of adherence to the directions of the activity or the lack of historical context within his methods. In short, Ted was viewed as the authority on the completion of this problem. Although Ted's interpretation of the problem solution was unique and efficient, it was heavily dependent upon technology (using the table feature of a graphing calculator to determine values of 3^x close to a power of ten, for example) and was fairly disconnected from the intended purpose of the activity.

On several occasions during the professional development sessions, Ted selectively engaged with the content of the history of logarithms materials. He offered observations resulting from his personal knowledge of the relationship between religion and scientific thought and he was confident in sharing his knowledge about how to consider the relationship between half-chords and modern trigonometric sine values. Although Ted's assertiveness did not always reflect accuracy, he often contributed to the discussion with authority and went unchallenged by the other participants.

Ted as Pedagogical Decision Maker

Ted's articulation of how he planned to incorporate the historical development of logarithms was also selective and was centered on his observation that it is "helpful to talk about the historical background" (Professional Development Session 1, 12/17/04).

History as biography. With respect to his desire to “put a face on” the mathematics he was teaching, Ted specified the extent of how he would incorporate historical and biographical information in his instruction:

Ted: If I was going to use this in a class, I would take just some dates – just a very short number of dates.

Kathy: Like by year?

Ted: For example, give the date and then Michael Stiffel.

Kathy: Sty-fel [corrects pronunciation].

Ted: Stifel. Invented in 1544, then Napier wrote that book, then this fellow Briggs, he read it and then he met him. Then we have his little rods going on there. And then in 1628 we have that production of that book by...

Kathy: Vlacq?

Ted: Yes, then that added to Briggs’ work of logs from 1 to 2,000 – 20, 000 – and so forth. And then [the] tables were printed in London. So that’s one thing there I would do. (Professional Development Session 1, 12/17/04)

I probed all of the Mulberry participants further about other options to the timeline activity of Lesson Installment 1 by asking, “Could you keep something like almost keeping a running timeline that you could access on different occasions?” Ted only replied that, “Yeah, that might work” (Professional Development Session 1, 12/17/04). I did not pursue additional questioning about Ted’s ideas on options for incorporating biographical information into a unit on logarithms. On the basis of the scope of his other contributions, however, I determined that if he already had an idea about pedagogical plans that he would indeed share them. If Ted chose not to expend effort on considering uses of the historical materials, it was either due to his selective engagement (i.e., not substantially considering the materials outside of the two

professional development sessions) or because he was not committed to student use of the materials or ideas taken from the installments.

Possibilities only. If Ted did not explicitly offer concrete examples for how to incorporate the history of logarithms in his instruction (i.e., presenting biographical information), then he was only willing to submit to the possibility of offering an idea from any lesson installment while teaching. At one point during the first professional development session I tried to engage the teachers in thinking about what alternatives to the existing format of Lesson Installment 3 they would consider using with students. I probed for this information because Sue had raised the issue that students would “ask why [there are] so many words on it” and that they would struggle with the installment in its present form. I continued to press for information.

Kathy: Is there anything from Installment 3 that any of you would consider using? Are there any ideas from it that you would incorporate into your teaching? Perhaps not necessarily an exact problem off of the lesson installment, but just an *idea* of something that you’ve come across?

Ted: Probably more that way. (Professional Development Session 2, 1/03/05)

When I questioned Ted further, he stated that he would possibly introduce students to overarching ideas from the historical development of logarithms. He stated that he knew what his students could handle and would be mindful of that before he would distract them for too long with any one historical activity, such as the nine-page Lesson Installment 3.

Potential instructional plans. One outcome of Ted’s moderate study of the history of logarithms outside of the formal sessions was his confusion about the inclusion of lesson installments within the traditional curriculum plan for teaching logarithms. With limited exposure to the mathematics and the historical context of the different lesson

installments, Ted was unable to initially describe an appropriate sequence of instructional events for the purpose of including activities related to the historical development of logarithms.

Ted: Well for me, I would do it toward the beginning, because in that way you're not using any of the rules and I would just ask them questions: ten to the log of five is what, and have them figure out the log of five. And the rest of the students would go, well the log of zero's one – the log of one is ten – so what they're going to do is they're going to try to take pattern of one point two – point two, point three, point four, point five, you know? And so they'll get closer. If you make it a contest for these kids, they will really work. And then I'll say here's another way that we can do that.

Kathy: So you don't think that they need the properties? They would know to use the power rule [for logarithms]?

Ted: Oh, they need the power rule, yeah, that's true. They have to know the properties, for sure. So I'd do that right after the properties. (Professional Development Session 2, 1/03/05)

Prior to the beginning of the second professional development session, Ted shared with me that he usually arrived at school between 6:30 and 7:30 in the morning. On the morning of January 3 he arrived as usual and began to work on the four lesson installments that I had provided the participants in December 2004. As a result, Ted was only able to hastily review the content within Lesson Installments 4, 5, and the first two problems of Lesson Installment 6. As a result, Ted described one potential placement of Lesson Installment 4. However, without sufficient time to process the prior knowledge that students needed in say a traditional treatment of logarithms, Ted was unable to correctly outline the placement of Lesson Installment 4. He eventually corrected himself when I questioned him further.

It is also noteworthy to mention that Ted's use of basic knowledge related to logarithms was also somewhat flawed. Within his description for using Lesson

Installment 4, Ted made errors such as identifying the logarithm of zero as one ($\log_{10} 0 = 1$) and the logarithm of one as ten ($\log_{10} 1 = 10$). These errors may also be attributed to a limited engagement with the history of logarithms and that prior to the morning of January 3, Ted may not have had occasion to work with logarithms since teaching them during the previous spring semester.

Ted's discussion of when and how he could use Lesson Installment 4 was not without pedagogical merit. As with his earlier emphasis on bringing about the "human side" of the historical development of logarithms, it was important for Ted to consider how to motivate students in an endeavor to use Installment 4. Ted noted that if you "make it kind of contest for these kids, they will really work" (Professional Development Session 2, 1/03/05). Thus, a theme of Ted's pedagogical considerations was to frame any instructional event within an activity that he felt students could handle.

Ted as an Anticipator of Student Engagement

Ted's ability to anticipate student engagement with the history of logarithms materials is the least represented in his professional development data. Ted viewed student interaction with the history of logarithms in terms of content and duration. He believed that if you did "too much" with students with regard to including historical and biographical information during instruction that you would lose students (specifically, their attention). Ted did not share why he felt that students would only respond to a limited exposure of content from the historical development of logarithms. His beliefs of limited student ability and attention span reappear during the second professional development session.

Kathy: So, decimals to fractions? That might be something that you would accept? Sue actually gave the numbers: change this number to this form. Can you think of anything else?

Sue: I can think of what all they're [the students] are going to think.

Mary: Fractions! Blech!

Kathy: They don't know how to describe what they did? They can't verbalize it at all?

Ted: No. (Professional Development Session 2, 1/03/05)

This excerpt is part of a longer passage used in the discussion of Sue's anticipation of student engagement (*Student difficulty and disengagement*). Whereas Sue was eventually able to contemplate positive results from her students' engagement with a mathematical task like Problem 4 of Lesson Installment 3, Ted was not. His abrupt answer of "no" to the questions about student ability to describe each transformation of Problem 4 is indicative of his selective engagement during the professional development sessions. Ted was either unwilling or unable to discuss the alternatives to his initial beliefs, which were for the most part negatively oriented. As in his example of students' inability to engage in "too much" biographical and historical information, students would also be ill-equipped to navigate through the difficult algebraic manipulations of Lesson Installment 6.

Significant Secondary Feature: Critical Reflection

Ted was the only Mulberry teacher for which a significant secondary feature was prevalent – in a positive sense – in his professional development participation. In contrast to the case of Mandy, whose experience exhibited all three secondary features of (1) commitment to continued learning; (2) ability to critically reflect on the materials and

resources; and (3) identification of gaps in historical knowledge, only the critical reflection piece was exhibited in Ted's participation.

There are three noteworthy examples of Ted's capacity to critically reflect on the history of logarithms materials. Each instance also supports the identification of Ted's assertive stance while participating in the professional development sessions. In each of the three descriptions below, Ted offered feedback for the sole purpose of contributing to a more meaningful use of the history of logarithms materials.

Importance of accurate resources. Another aspect of Ted's assertive engagement was the sharing of his experience with the Mac Tutor History of Mathematics archive website. On different occasions participants requested my opinion of helpful resources in addition to the resources already compiled to accompany the seven lesson installments. While discussing Lesson Installment 1, Sue asked how to determine if the information found was reliable and appropriate to use for students. Ted provided his critical analysis of this particular website:

I think that the St. Andrews website is a good site because it relies upon a lot of citations from the actual sources. How many times in the election season did I hear something and everybody got an e-mail. And I thought, I don't think that is true and I found out it wasn't true. You've got to find some good sources. That's a good point. Some things you never will know for sure. (Professional Development Session 1, 12/17/04)

Quality control and suggestions. Ted's suggestions for improvement of two of the lesson installments were similar to Mandy's critique of the lesson installments and associated resources. The two improvements were offered during the second professional development session, which occurred on the same morning in which Ted had completed the lesson installments for himself. Both appraisals were helpful with respect to improving the lesson installments for student and teacher use alike. I can only conjecture

about the overall impact of Ted’s critical analysis. If the professional development component was structured as a series of seven sessions as opposed to two, I believe Ted’s critical analysis would have been more frequent and the impact of his suggestions more pervasive. A summary of Ted’s suggestions is found in Table 24.

Table 24
Suggestions for Improving Lesson Installments: Ted Jones

| Lesson installment number (and title) | Description of Ted’s analysis | Outcome |
|-------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5 (Astronomical Calculations Made “Easy”: Ancient Influences on the Development of Logarithms) | Before the second professional development session began, Ted approached the group with a concern that he had about the identities on Page 1 of Lesson Installment 5. He claimed that three of the identities [$2\sin x \sin y = \sin(x + y) + \sin(x - y)$; $2\cos x \sin y = \sin(x + y) - \sin(x - y)$; and $2\sin x \sin y = \cos(x - y) - \cos(x + y)$] appearing on Page 1 were incorrect. | As a group, we proved each of the identities on Page 1 and determined that one of them, $2\sin x \sin y = \sin(x + y) + \sin(x - y)$, was indeed incorrect. We changed the identity to its correct form, $2\sin x \cos y = \sin(x + y) + \sin(x - y)$ on the lesson installment. (Professional Development Session 2, 1/03/05) |
| 6 (A Glimpse Into the Future: Translating Napier’s Method To Include the Natural Base e) | While discussing Problem 7 of Lesson Installment 6, I raised the issue that the directions felt a bit “clunky.” I had modified Installment 6 quite a bit from its original form in Anderson et al. (2005) and was unhappy with the results for certain directions. Ted observed, “I didn’t know what you were asking on it.” Eventually, however, he offered a reasonable alternative to the directions I originally wrote. | My version of Problem 7’s directions: Using the rules of exponents, determine what number needs to be substituted into k below so that it is of the same form as the final expression of Problem 4. Ted’s suggestion included, “I wouldn’t say, ‘use the rules of exponents;’ I would just say, ‘determine what number needs to be substituted in for k so that it looks like the final result in problem four’ (Professional Development Session 2, 1/03/05). |

Instructional Practice

Ted selectively incorporated only “human interest” aspects from the history of logarithms during his instruction of the unit on logarithms. Of the seven lesson installments included for participant study and use with students, Ted incorporated only brief elements from Lesson Installment 1. The next three sub-sections discuss Ted’s self-reported instructional practice relative to logarithms prior to this investigation, as well as his beliefs about his role as the teacher and about role of a student, and the identification of school factors influencing Ted’s instruction.

Existing Instructional Practice

On the Attitudes Instrument Pre-assessment, Ted outlined his usual approach to teaching logarithms as

I start with working problems related to exponential equations; then make the transition to log equations, talk about applications, discover the properties and then work to solve log equations. (12/15/04)

I assumed that Ted’s description of “working problems related to” exponential and logarithmic equations would also include graphing of both types of functions. It was interesting that Ted stated he would “talk about applications” rather than require students to solve application problems. The outline of Ted’s planned instruction mirrored the range and order of topics in the course textbook, *Advanced Mathematical Concepts* (Holliday et al., 2001), and appear in Table 25.

Beliefs about the Role of the Teacher and Student

Ted believed that his role was to “encourage the student to take their education seriously and to encourage them to get a good math background” (Interview, 4/14/05). Ted claimed that he was able to help students accomplish each of these behaviors by

“teaching the course [he was] assigned to teach” (Interview, 4/14/05). When I questioned Ted more deeply about his teaching philosophy, he stated that,

My philosophy really has a lot to do with knowing your role. That’s key. Knowing your role and responsibilities. The teacher’s role is basically to teach. Since the teacher is the adult, and if the teacher’s doing their job, they know what the child needs. And it’s the job of the teacher to motivate; encourage; convince the student to want to do the things they need to do. (Interview, 4/14/05)

The demarcation of roles was also prevalent when Ted shared his beliefs about the role of the student.

The student’s role is basically to engage in learning activities, whatever they are. Listening, or reading, or talking, or discussing; following some established guidelines – that sort of thing. A student should go to your class; they should be respectful to you because you’re an adult; they should respect your knowledge; and they should do what you need them to do. (Interview, 4/14/05)

Thus, Ted’s beliefs focused on the idea that teachers have a plan for students to follow. If necessary, teachers should provide the additional motivation for students to do what the teacher needs them to do. The authority in the classroom belongs to the teacher and students follow a set of prescribed tasks and behaviors.

Influence of School Features

Ted highlighted three school features which influenced his instructional practice at Mulberry High School and articulated their combined impact when he observed

As a public school teacher, I am expected to adhere to the objectives from the State’s Comprehensive Curriculum (SCC). So, time is an issue because I have a responsibility to the SCC. Beyond that, interest on the part of the students is necessary. In their mindset, students want to be working problems. They basically want more of this. Students need to give their time and attention and there must be an interest level on the part of the student. (Interview, 3/28/05)

Thus, the features of time (having a sufficient amount to cover the curriculum), the prescribed curriculum (covering what is necessary for a given course as indicated by the

state), and student interest (being strong enough) were considered when planning instructional activities for students.

Chronology of Instruction

Ted taught two Trigonometry classes in 2004 – 2005. His instruction on the topic of logarithms occurred between March 25, 2005 and April 20, 2005, with no instruction during the week of April 4, 2005 due to Mulberry’s spring break. Ted’s instruction followed the same order that he indicated on the Attitudes Instrument pre-assessment and was essentially the same for each of the two class periods. Period 4 was 58 minutes long (which included time for three rolling lunch periods); Period 5 was 55 minutes in length. In brief, Ted covered the material of Sections 11.1 through 11.6 as they appeared in the text (see Table 25).

Table 25
Instructional Schedule: Ted Jones

| Section: Topic | Dates covered |
|-------------------------------|-----------------------|
| 11.1: Real Exponents | 3/25/2005; 3/28/2005 |
| 11.2: Exponential Functions | 3/29/2005 – 3/30/2005 |
| 11.3: The Number e | 3/31/2005 |
| 11.4: Logarithmic Functions | 4/01/2005; 4/11/2005 |
| 11.5: Common Logarithms | 4/12/2005 – 4/13/2005 |
| 11.6: Natural Logarithms | 4/14/2005 – 4/15/2005 |
| Chapter 11 Review and testing | 4/18/2005 – 4/20/2005 |

Incorporating the History of Logarithms

Ted’s philosophy about engaging students in the historical development of logarithms centered on the idea of using only key, brief components. On three of the four occasions that he brought an aspect of the history of logarithms into his teaching, Ted’s predominant focus was the relationship between the inventions of logarithms and the slide rule.

Introduction to Modern Calculation Devices

To transition between chapters which appeared to have nothing in common, Chapter 10 (conic sections) and Chapter 11 (exponential and logarithmic functions), Ted decided to redirect students' attention to a satellite they discussed in Chapter 10. Students then read the opening example in Chapter 11 which discussed the use of astronomically large numbers in the context of a NASA application. Finally, Ted introduced students to the connection between astronomy of the sixteenth century, the invention of slide rules, and logarithms.

On a timeline poster he had created for introducing Chapter 11, Ted highlighted inventions leading up to the development of the first modern calculator. He emphasized the role of the slide rule in making calculations with very small and very large numbers possible by showing students an actual slide rule. To develop an interest in logarithms, Ted prompted,

What we want to do is take a little time and we want to think about where all of this stuff came from. Where did the idea of a slide rule come from? From logarithms, right? Someone had to think about it, right? That someone has a name. (Observation, 3/25/05)

Ted had prepared an overhead copy of one panel of the timeline and a student read from the overhead slide, which focused on John Napier and the infamous "Napier's bones." To emphasize the contribution of John Napier, Ted displayed a picture of him and a set of his calculating rods. Finally, Ted ended the introduction to Chapter 11 by displaying another overhead slide which indicated professions using logarithms and by telling students, "I want you to know logarithms by the end of the chapter" (Observation, 3/25/05).

When I asked Ted why he decided to use the poster to introduce students to the history of computing, Ted replied

All of what's on the banner shows the progression towards the modern development of computers and the origins of all that development is because of logarithms. The banner helps students realize the "whole picture" of the use of logarithms. (Interview, 3/28/05)

The examples which follow highlight Ted's focus on the invention of the slide rule as being central to the development of the "whole picture" of the use of logarithms.

Continued Connections to the Slide Rule

Ted continued to emphasize the importance of the slide rule invention during two additional class sessions, one just prior to spring break and the second just after. In the first, during instruction on Section 11.4, Ted asked students to find the values of $\log_{10}7$, $\log_{10}25$, and $\log_{10}175$ on the calculator. Ted then asked students to determine a relationship between the three logarithmic values. Rounding the calculator values created difficulty for some students to recognize the relationship. However, many were able to recognize that while the product of 7 and 25 yielded 175; the sum of the logarithms of 7 and 25 (approximately 0.8451 and 0.1.398, respectively) was equal to the logarithm of 175 (approximately 2.2243). At the end of this exercise, Ted explained to the students,

Some guy figured out – it was John Napier and some other folks – they figured out this way to be able to take numbers that are big that you want to multiply and put the numbers on these little rods [of the slide rule]. And you could slide these rods back and forth a certain way and you could actually multiply the numbers by adding the logarithms of the numbers, just like we're doing. And that's how the slide rule works. (Audiotape, 4/01/05)

After spring break, while still covering Section 11.4, Ted approached each of three properties of logarithms by asking students to calculate a number of quantities. For the "log of a product" property, for example, Ted asked students to:

Pick two 2-digit numbers and find their product;
Find the logarithm of each factor and the product;
Look at each of the calculations to see if a pattern emerges. (Observation, 4/11/05)

After the class agreed that the sum of the two individual logarithm values equaled the logarithm of the product, Ted asked,

If you have to multiply a two-digit by a two-digit, or a three-digit by a three-digit, or a *four*-digit by a *four*-digit without the calculator. I'm talking without the calculator, okay (Ted's emphasis)? What would be easier, multiplying them or just adding them? Now, before we had the calculator, people would use this little instrument right here [showed students a slide rule]. They took a number, and they found the log of the number. Then they found the log of the other number; they added them together in their head; then they found out ten to that power would be what number? And they figured that out using this little tool right here. (Observation, 4/11/05).

Historical Vignette: John Napier

Ted distributed a two-sided handout to students for the warm up activity at the beginning of the second class day covering Section 11.5. The handout included two vignettes from *Agnesi to Zeno* (Smith, 1996), which was given to each of the participants for their participation in this research. On one side, "M.C. Escher: Artist and Geometer (Vignette 90)"; on the other, "Napier Invents Logarithms (Vignette 39)." Ted included only the first page from each vignette, which entailed a reading exercise. After spending 13 minutes reading about and discussing the life and work of M.C. Escher, Ted turned the class's attention to the life and work of John Napier. Ted highlighted several brief tidbits of information about Napier's life, including who he was; where he lived; when he lived; what he invented; why he invented logarithms; and what he wrote. At the end of the five-minute summary, Ted concluded, "I wanted to highlight Mr. Napier for you" (Observation, 4/13/05).

Summary

Ted Jones represented a case of moderate engagement during the professional development sessions focused on the historical development of logarithms. His efforts to incorporate the history of logarithms were also moderate, when compared to the two extremes of implementation encountered during the study.

Professional Background

Ted came to teaching as a second career after serving in the military. He obtained an initial degree in Bible studies and he possessed a weak, undergraduate mathematics content preparation. Ted possessed undergraduate and graduate degrees in mathematics education and regular teaching certification. At the time of the research, Ted had taught for 16 years, spending the last two years at Mulberry High School.

Attitudes

The results of the three Attitudes Instrument administrations (pre-assessment and two post-assessments) are shown in Table 26.

Table 26
Attitudes Instrument (Part I) Results: Ted Jones

| Survey item | Pre response (12/15/04) | First post response (1/03/05) | Second post response (4/15/05) |
|-----------------------------------------------------------------------------------------------------------|-------------------------|-------------------------------|--------------------------------|
| 1. Understanding the history of mathematics is an important part of understanding mathematics. | 4 | 5 | 6 |
| 2. Including history enriches the teaching and learning of mathematics. | 5 | 6 | 5 |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | 5 | 4 | 5 |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | 4 | 4 | 5 |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | 4 | 5 | 5 |
| 6. Using historical materials in my mathematics | | | |

| Survey item | Pre response (12/15/04) | First post response (1/03/05) | Second post response (4/15/05) |
|-------------------------------------------------------------------------------------------------|-------------------------|-------------------------------|--------------------------------|
| classes has been an integral part of my instruction in: Algebra I or II. | 4 | 5 | 6 |
| Geometry. | 4 | 5 | 6 |
| Precalculus/Trigonometry. | 4 | 5 | 6 |
| Calculus. | No response | 5 | 6 |
| 7. Prospective mathematics teachers should be required to study the history of mathematics. | 6 | 6 | 5 |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | 6 | 6 | 6 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

Three significant observations can be made about Ted's attitudes toward the use of the history of mathematics. First, at the conclusion of the research (i.e., after the professional development sessions and instruction on logarithms), Ted moderately or strongly agreed with each of the eight statements on Part I of the Attitudes Instrument related to the broad use of the history of mathematics in teaching and learning mathematics. Next, although his responses to Items 2 and 3 fluctuated slightly after the professional development sessions, Ted still moderately agreed that including history enhances the teaching and learning of mathematics and that the use of biographies of mathematicians make mathematics classes more enjoyable. Ted's agreement with these items was apparent in the justifications he provided for including any history during his instruction. Lastly, Ted's response to Item 6 over the three administrations was inconsistent with what was observed during his instruction. Ted reported that he strongly agreed that the use of historical materials was an integral part of his instruction in four courses, two of which he did not teach. I did not observe instruction in Ted's Algebra II

classes. However, observation of his Trigonometry classes indicated that the use of historical materials were not an integral part of his instruction.

Content Knowledge

On the first post-assessment administration of the Content Knowledge Instrument, Ted improved upon his pre-assessment performance by completing an additional item, Item 7, correctly. Thus, Ted correctly answered six out of the eight items; four were traditionally oriented and two were historically oriented. On the second post-assessment administration, however, Ted answered only four items correctly, and was the only participant to perform less proficiently on the final post-test than on the pre-test. Table 27 outlines Ted's content knowledge performance on three administrations of the instrument.

Table 27
Content Knowledge Instrument Results: Ted Jones

| Item | Historical/ Traditional | First post response (1/07/05) | Second post response (4/15/05) |
|---------------------------------------------------------------------------------------|----------------------------|-------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| *1. Define <i>logarithm</i> . | Traditional | Correctly defined <i>logarithm</i> in a traditional sense | Correctly defined <i>logarithm</i> in a traditional sense |
| *2. Describe the basic idea or motivation for the invention of logarithms. | Historical | Correctly described basic idea in terms of calculations necessary for astronomical measurements | Incomplete response which referenced ease of calculations, but no specifics were given |
| 3. Construct the values for $\log_{10}2$ and $\log_{10}3$ without using a calculator. | Historical | Converted from logarithmic to exponential form and attempted to solve for unknown exponents | Wrote each expression in exponential form; no approximate values obtained |
| 4. Let $u = b^n$ and $v = b^m$. Verify $L(u) - L(v) = L\left(\frac{u}{v}\right)$. | Historical | Proof attempted | Proof attempted |
| *5. Evaluate: | Traditional | Evaluated | Evaluated |

| Item | Historical/ Traditional | First post response (1/07/05) | Second post response (4/15/05) |
|-------------------------------------------------------------------------------------|----------------------------|----------------------------------------------------------------------|----------------------------------------------------------------------|
| $\log_{32}16.$ | | successfully by converting to exponential form and solving | successfully by converting to exponential form and solving |
| *6. Evaluate: $\log_{\frac{1}{3}}81.$ | Traditional | Evaluated successfully by converting to exponential form and solving | Evaluated successfully by converting to exponential form and solving |
| 7. Calculate the product of 8409.5 and 951.49 using the method of prosthaphaeresis. | Historical | Approximated value successfully using prosthaphaeresis | Reverted to same (incorrect) method used on the pretest |
| *8. Solve for x : $2\log 3 + \log x = \log 45.$ | Traditional | Solved correctly using properties of logarithms | Solved correctly using properties of logarithms |

Note. Items with an (*) were answered correctly on the Content Knowledge Pretest.

Professional Development Engagement

Ted's engagement during the professional development sessions and between professional development sessions was characterized as moderate for several reasons. Ted exhibited all three of the primary participation themes, although his contributions focused on anticipating student engagement with the history of logarithms and proposed pedagogical decisions related to the use of the materials were minimal. During the professional development sessions much of Ted's interaction was selective in nature. Many of the contributions Ted offered were focused on the human interest aspect afforded by the use of the history of mathematics in teaching, such as anecdotes about mathematicians or the religious and historical influences of the 16th and 17th centuries related to the development of logarithms. Ted participated in two lesson installment discussions which were more mathematically intensive. However, when he did so, the ideas he offered contained mathematical errors which other participants did not articulate.

Perhaps the most selective aspect of Ted's professional development participation was his anticipation of student engagement with the history of logarithms. Ted's beliefs about limited student attention and ability negatively impacted his ability to plan for the use of the historical development of logarithms. Ted suggested that students would find some of the mathematical content too difficult (e.g., Lesson Installment 6) and he did not discuss potential student engagement with the history of logarithms content. When Ted discussed how he might incorporate the historical development of logarithms into his teaching, his ideas always centered on brief, biographical insertions.

Influence of Beliefs and School Features: Obstacles to Including History

Ted's beliefs about his responsibilities as the teacher and those of students influenced his stance during the professional development and his instructional practice. It was Ted's primary belief that his role was to teach the course he was assigned to teach. The students' role was to engage in the learning activities that Ted provided. The corresponding roles clashed when viewed from the influence of school-specific features, however. For example, time available for instruction – which was impacted by the prescribed curriculum Ted was assigned to teach – influenced his decisions in conflicting ways. When I asked Ted whether he would use historical materials with his students he stated that,

It would have to be in a format where you can do a one-page deal. So what I would do is I would have to make a one-page deal out of something. Really, that's my problem. My problem is I don't have time to do that, honestly. I think it makes for a better class, but I'm trying to teach them like it's some kind of enrichment class. I forget all the time, I'm supposed to be teaching them Trigonometry and going over the problems and teaching them new stuff. And I do that, but to me, I have (Ted's emphasis) to do this other stuff [meaning the "enrichment"]. (Interview, 3/28/05)

Here, Ted's emphasis that such materials needed to be in a one-page format was related to his inclusion of the history of logarithms only during the warm-up segment of his lessons. A few weeks later, however, Ted admitted,

Because the curriculum is laid out, my instructional decisions start with making sure that I teach the minimum and not doing a lot of extra stuff. So any decision I make will be regarding the curriculum that's already set out for me to do.
(Interview, 4/14/05)

Ted noted that he often had only 30 minutes to prepare for any given lesson and that he did not plan ahead. Instead, he planned each class just prior to teaching it because "that's just the reality of it here [at Mulberry High School]" (Interview, 3/28/05). The lack of available time and the demands of a prescribed curriculum, combined with Ted's perception of student interest, influenced his study of the historical development of logarithms and plans for its use in his instructional. In addition, lack of time, curriculum demands, and waning student interest were obstacles in implementing the history of logarithms.

Incorporating the History of Logarithms

Early in the first professional development session, Ted stated that,

If you do too much with the students, you start losing them. If you just do a few things and do a real good job, you can always add to these guys later. So it [including historical information] puts a face on it. (Professional Development Session 1, 12/17/04)

Several months later while teaching the topics of exponential and logarithmic functions, Ted maintained that since he "knew his own students" (Interview, 3/28/05), he was only able to use brief, key information from the history of logarithms. Of the four class occasions which Ted included historical information about the development of logarithms, three lasted less than five minutes each and one took approximately 15

minutes of class time. The focus of each was on the invention or use of the slide rule. John Napier was mentioned on two of the four occasions, as were his invention, “Napier’s bones.” Similar to his professional development experience, Ted also committed errors while incorporating historical comments in his lectures. For example, Ted incorrectly identified “Napier’s bones” as the rods which were slid back and forth within a slide rule to calculate products of numbers.

Benefits of Using the History of Logarithms

Ted believed that it was “important for students to see how logarithms were developed by building on someone else’s work from before” (Interview, 3/28/05). However, Ted did not incorporate complete information about the history of logarithms. He did not focus on their development based upon the work of others, such as Napier’s original definition, Ibn Yunus’s prosthaphaeretic processes, or Archimedes’s or others’ work with arithmetic and geometric sequences. Instead, he focused on the invention of the slide rule and how those calculation devices used logarithms

In an effort to understand Ted’s identification of the benefit of seeing a mathematical topic develop from the work of others, I asked him why he chose not to incorporate Lesson Installment 4, which used concepts similar to those in the textbook exercises he required of students. He replied,

If I was going to use this, I would like to give it as sort of as an enrichment to some folks and then report back from those guys what they did, to the class. Because then it’s not really my deal, it’s kind of their deal. But I feel if I give that handout to them and said, “I want you all to spend time working on this,” I don’t think many of them would do it. The reason why is they would say, “What does that got to do with what we’re doing? I don’t see that in the book.” (Interview, 4/14/05)

Affordance of Using the History of Logarithms

I failed to explicitly ask Ted what afforded him to incorporate the history of logarithms in his instruction. On several occasions, however, Ted noted that his ability to know his students enabled him to incorporate tidbits about the history of logarithms. Ted held the belief that this knowledge of students enabled him to capture the interest of students so that they could really come to “know logarithms” by the end of instruction. (The outcome of students “knowing logarithms” resulting from including brief, anecdotal comments about John Napier and slide rules appears inconsistent, however.) Ted regarded historical content that would interest his students in the necessary curriculum as a positive intervention and that “it makes math meaningful. It makes the study of math and the learning of math and the teaching of math more meaningful to the student” (Interview, 3/28/05). Further, Ted dealt with both the interpreted affordance of knowing his students well enough and with the obstacles of time, student interest, and student ability by using the history of logarithms only in the form of brief anecdotes.

Chapter 7
Limited Engagement with the History of Logarithms:
The Cases of Shirley Corson and Mary Long

This chapter presents participant background information and data necessary to tell the story of Shirley Corson and Mary Long and their experiences with the history of logarithms. In each of their cases, Shirley and Mary struggled to participate in the study of the history of logarithms, during both formal professional development sessions and personal study. Due to their struggle to participate, their explicit engagement during the professional development component is characterized as limited according to the engagement framework outlined in Chapter 4. In addition, Shirley and Mary did not incorporate the historical development of logarithms into their classroom practice, albeit for very different reasons.

In this chapter, I address the research questions outlined in Chapter 1. The chapter is organized by case. Each case study begins with a description of features of their professional education and teaching experience. Related to these features, I also discuss

- their previous experiences with the history of mathematics;
- the attitudes and beliefs that they expressed relative to the role of the history of mathematics in teaching; and
- the knowledge they possess about the topic, both in traditional and historical contexts.

Next, I describe the participation during the professional development sessions (and personal study), which focused on the historical development of logarithms. I then examine the implementation of the professional development content, which for both Shirley and Mary was an examination of non-implementation. Lastly, I summarize each case.

The Case of Shirley Corson

In order to address the research questions outlined in Chapter 1, the following sections

- describe Shirley's professional background variables;
- describe her engagement during the professional development sessions designed to examine the historical development of logarithms;
- describe her practice during instruction about logarithms; and
- describe the obstacles and affordances Shirley identified for using the historical development of logarithms.

Lastly, I summarize the changes in Shirley's attitudes and content knowledge, as well as identify the features influencing her professional development engagement and her instructional practice.

Professional Background

Of the five participants, Shirley was the second-most experienced teacher, with 28 years of teaching experience. Shirley taught in the midwestern United States for 10 years and 18 years in the southeast. Although 2004-2005 was her first year at Mulberry High School, Shirley taught for 17 years in an adjacent county. Twenty-seven of Shirley's 28 years of teaching were at the secondary level.

Much of Shirley's professional preparation mirrors that of the other participants. She holds a bachelor's degree in mathematics education, a master of arts in teaching mathematics, and regular teaching certification. Only three of the five participants have earned a masters degree; the other two are currently pursuing a master's in mathematics education. Her mathematical preparation includes a predictable array of courses. As was the case with the other four participants, Shirley has never taken a course in the history of

mathematics. For the prompt, *Describe any previous experience with the history of mathematics*, Shirley simply responded, “N/A” (Background Survey, 12/15/04).

Shirley’s experience and variety with professional development activities during the last two years is significantly different from that of the other participants. On her Background Survey, Shirley reported that the only professional development activity (out of 12 potential activities) that she participated in during the last two years was mentoring (i.e., a student teacher or novice teacher). Shirley also noted that prior to 2003, the only staff development courses that she took were required for certification and not necessarily mathematics related.

Attitudes and Knowledge

Table 28 displays Shirley’s responses to Part I of the Attitudes Instrument Pre-assessment. A Likert-type scale was used, ranging from strongly disagree (corresponding to a score of 1) to strongly agree (corresponding to a score of 6).

Table 28
Attitudes Instrument (Part I) Pre-Assessment Results: Shirley Corson

| Survey item | Pretest response |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------|
| 1. Understanding the history of mathematics is an important part of understanding mathematics. | 2 |
| 2. Including history enriches the teaching and learning of mathematics. | 5 |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | 4 |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | 4 |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | 4 |
| 6. Using historical materials in my mathematics classes has been an integral part of my instruction in: Algebra I or II. Geometry. Precalculus/Trigonometry. Calculus. | 1 No response 1 No response |

| Survey item | Pretest response |
|-------------------------------------------------------------------------------------------------|------------------|
| 7. Prospective mathematics teachers should be required to study the history of mathematics. | 4 |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | 4 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

Several patterns emerge when examining Shirley’s responses to these items. First, when Items 1 through 5 are categorized by purpose, either using history for learning mathematics or using history in the teaching of the subject, Shirley’s responses indicate that she initially viewed knowing the development of a topic as somewhat helpful in learning the particular topic. To a lesser degree, she felt that understanding the historical development of mathematics was essential for understanding mathematics. With regard to the second purpose, using history in the teaching of the subject, Shirley agreed that incorporating the history of mathematics would add particular enriching dimensions to teaching mathematics, such as adding enjoyment and contributing to the quality of instruction.

Second, in many ways, Shirley’s views about the instructional worthiness of including the history of mathematics were inconsistent with her actual (reported) instructional practice. Shirley’s completion of Item 2 of Part II provides additional insight into her consideration of the requirement that students work with the history of mathematics. Shirley completed Item 2 by stating that teachers “should require no history work in their mathematics classes” (Attitudes Instrument, 12/15/04). She added that the use of history “is not part of the state-mandated performance standards” (Attitudes Instrument, 12/15/04). Shirley also reported that for the two courses she was currently

teaching, Algebra I and Trigonometry, that she strongly disagreed that using historical materials were an integral part of her instruction.

The third pattern connects Shirley’s response to Item 8 of Part I with her reported professional development participation on the background instrument. Shirley slightly agreed with the statement, *As a mathematics teacher, it is important for me to continue my own learning of mathematics*. The fact that Shirley was participating in professional development which used the historical development of logarithms as content could be evidence of her self-reported importance for continued learning. However, upon examination of Shirley’s reported professional development activities during the last two years, the validity of this particular statement weakened. In that time, Shirley reported that she had not (1) taken any mathematics courses; (2) participated in state, school, district, or state professional development workshops; (3) attended any professional mathematics or mathematics education conferences; or (4) conducted any research or independent reading for the continuation of her own learning of mathematics. Shirley’s infrequent commitment to the study of mathematics will re-emerge with respect to her participation during the professional development component of this research. The theme of limited participation will also reverberate within her instructional practice.

Shirley answered only three of the eight questions on the Content Knowledge Pre-assessment. Table 29 identifies the content items, the classification of items as either historical (taken from a lesson installment) or traditional, and a description of Shirley’s response to the item.

Table 29
Content Knowledge Pre-Assessment Results: Shirley Corson

| Item | Historical/Traditional | Response given |
|------------------------------|------------------------|----------------|
| 1. Define <i>logarithm</i> . | Either, depending upon | None |

| Item | Historical/Traditional | Response given |
|----------------------------------------------------------------------------------------|------------------------|----------------------------------------------------------------------|
| | participant response | |
| 2. Describe the basic idea or motivation for the invention of logarithms. | Historical | None |
| 3. Construct the values for $\log_{10}2$ and $\log_{10}3$ without using a calculator. | Historical | None |
| 4. Let $u = b^n$ and $v = b^m$. Verify $L(u) - L(v) = L\left(\frac{u}{v}\right)$. | Historical | None |
| 5. Evaluate: $\log_{32}16$. | Traditional | Evaluated successfully by converting to exponential form and solving |
| 6. Evaluate: $\log_{\frac{1}{3}}81$. | Traditional | Evaluated successfully by converting to exponential form and solving |
| 7. Calculate the product of 8409.5 and 951.49 using the method of prosthaphaeresis. | Historical | None |
| 8. Solve for x : $2\log 3 + \log x = \log 45$. | Traditional | Solved correctly using properties of logarithms |

Unlike the other four participants, Shirley did not attempt any of the problems derived from the history of logarithms lesson installments (Problems 2, 3, 4, and 7) or the definition of logarithm (Problem 1). The first question, which asked participants to define *logarithm*, was considered either historical or traditional depending upon the participant's response. It is unclear why Shirley left this item blank, especially in light of the fact that she teaches an Algebra II class that covers the topic.

Professional Development Engagement

Shirley participated very little during the professional development sessions and represented a case of limited engagement. Only two of the primary themes identified in Chapter 4 were prevalent in Shirley's professional development experience, although

each was superficially manifested when compared with a case of either eager or moderate engagement.

Since Shirley did not contribute in any substantial way during the first professional development session to provide insight into her collaborative participation, there are essentially no data representing Shirley's participation during that session. In an analytic memo written after the session, I observed that Shirley

was the most resistant and did not respond positively to anything discussed on She did, however, take notes on Installment 2 (the two particle argument) and Sue later relayed to me that she "perked up" during this particular discussion because of the algebraic manipulation utilized. (12/17/04)

Thus, during the first professional development session, I was unable to determine in what ways Shirley anticipated student engagement with the history of logarithms materials or what decisions she contemplated for the use of the materials. Additionally, because she did not reveal anything about her own personal study of the lesson installments and accompanying resources, there is no evidence to describe Shirley's commitment to learning, critical reflection, or identification of gaps in knowledge about the historical development of logarithms. Her participation during the second professional development session, however, provided a small number of occasions to examine Shirley's professional development engagement.

Shirley as a Mathematical Collaborator

The second professional development session at Mulberry High School motivated Shirley to collaborate with her peers during a group examination of Lesson Installment 6. During the same conversation, she also concurred with the group about their perception of student ability and engagement with the materials.

Lesson Installment 6 was difficult for each of the Mulberry teachers. There was variation due to the participants' lack of desire to work on the collection of lesson installments (4, 5, 6, and 7) over the semester break and difficulty with the mathematical content itself. Shirley never revealed that she engaged with the materials outside of the formal professional development sessions. After Ted admitted that he "got lost" beginning with Problem 3 the group decided to work through Lesson Installment 6 together.

Throughout our examination of the lesson installment – which was perhaps the most difficult content-wise because of notation and algebraic manipulation – Shirley remained focused on the mathematics. She questioned notation (i.e., that b_0 is equivalent to 10^7 in Problem 1), struggled for equivalent function notation, and helped with troubleshooting on the graphing calculators to compute the quantity, $\left(1 - \frac{1}{10^7}\right)^{10^7}$.

Shirley's interest in the mathematics of Lesson Installment 6 motivated her to share in the completion of the installment – a level of engagement which she exhibited at no other time during the professional development.

Shirley's View of Student Engagement

Albeit to a different degree, Mandy, Sue, Ted, and Mary were able to anticipate how their students might engage with the historical materials and resources related to the development of logarithms. For the Mulberry teachers, the anticipation of student engagement was often negatively framed. Shirley's single utterance of considering student engagement with the history of logarithms took place during one of the same exchanges referenced in each of the other three Mulberry cases. The discussion was precipitated by the directions in Problem 4 of Lesson Installment 6:

Suppose we wish to manipulate the expression for the n th term of b_n from Problem 3. In the steps below, describe the reason for each transformation.

Initially, each Mulberry teacher commented that students would not be able to complete such a task because of the level of algebraic manipulation required, specifically with respect to providing justification for each step in the transformation. When I asked, “They don’t know; they can’t verbalize it at all?” (Professional Development Session 2, 1/03/05), both Ted and Shirley responded, “no.” Although Shirley’s contribution to this discussion was quite brief, it is significant in that it was the only time during the professional development sessions in which she offered her view about student engagement with a lesson installment.

Instructional Practice

The subsequent data describe Shirley’s instructional practice related to the topic of logarithms. In contrast to the instructional choices made by Mandy, Sue, and Ted, several factors contributed to Shirley’s decision to forgo incorporating the historical development of logarithms during her instruction of Chapter 11. These features include Shirley’s existing practice, her beliefs about her role as the teacher and the role of the students, and the interplay of particular school features.

Existing Instructional Practice

Item 6 of Part III of the Attitudes Instrument Pre-assessment asked the participants to *Outline how you usually approach the teaching of logarithms*. Shirley’s approach to teaching logarithms included:

Review rules for exponents.

Go over examples and show why the rules are such.

Define logarithms in terms of exponential expressions. (Attitudes Instrument, 12/15/04).

Shirley’s outline of her approach to teaching logarithms is an accurate depiction of the instruction she carried out in March 2005. Chapter 11 of the Mulberry High School Trigonometry course textbook presents exponential and logarithmic functions in a traditional manner. The seven sections of Chapter 11 in *Advanced Mathematical Concepts* (Holliday et al., 2001) are shown in Table 30.

Table 30
Chapter 11 Textbook Topics Covered: Mulberry High School

| Section | Topic |
|---------|---------------------------------------------------------------------|
| 11.1 | Real Exponents |
| 11.2 | Exponential Functions |
| 11.3 | The Number e |
| 11.4 | Logarithmic Functions |
| 11.5 | Common Logarithms |
| 11.6 | Natural Logarithms |
| 11.6B | Natural Logarithms and Area (Graphing Calculator Exploration. |
| 11.7 | Modeling Real-World Data with Exponential and Logarithmic Functions |

Shirley followed the curriculum of the text closely, omitting only Sections 11.6B and 11.7. When I first arrived at Mulberry High School to conduct classroom observations, Shirley had already begun Chapter 11 with her two Trigonometry classes.

Table 31 outlines Shirley’s instructional schedule for Chapter 11.

Table 31
Chapter 11 Instructional Schedule: Shirley Corson

| Date | Section and topics | Comments |
|--------------------|-----------------------------------------------------|---------------------------------------------------------------------------------------------------|
| Prior to 3/21/2005 | 11.1: Real Exponents 11.2: Exponential Functions | Instruction occurred prior to data collection |
| 3/21/2005 | 11.2 continued | Emphasis of instruction was on graphing exponential functions and applications (word problems) |
| 3/22/2005 | 11.3: The Number e | Emphasis on applications, including exponential growth and decay and compound interest |
| 3/23/2005 | 11.4: Logarithmic Functions | Review of exponents and defining a logarithmic function as the inverse of an exponential function |
| 3/24/2005 | 11.4 continued | Solving logarithmic equations. Review of Sections 11.1 – 11.4 for quiz on March 25, |

| Date | Section and topics | Comments |
|-----------|--------------------------|----------------------------------------------------------------------------------------|
| | | 2005. |
| 3/25/2005 | 11.1 – 11.4 | Quiz (no data collection) |
| 3/28/2005 | 11.5: Common Logarithms | Evaluating expressions, including change of base; solving logarithmic equations |
| 3/29/2005 | 11.6: Natural Logarithms | Same as 3/28/3005, with base e |
| 3/30/2005 | 11.1 – 11.6 | In-class review assignment; take home portion of the Chapter 11 test given to students |
| 3/31/2005 | 11.1 – 11.6 | Review for Chapter 11 test |
| 4/01/2005 | 11.1 – 11.6 | Chapter 11 test (no data collection) |

On each of the first four days I observed Shirley's instruction, Mulberry High School was administering the state high school graduate test. On each of these class days and during each of Shirley's Trigonometry classes, an average of 12 out of 24 students were taking the graduation test in another location of the school. Still, Shirley continued with her instruction since she had already communicated with students that it was their responsibility to keep up with the material on their own during the four days that they would be absent from class. Additionally, several students were absent from class on March 29, 2005 (five from Period 1 and 10 from Period 2) due to a senior field trip. Each class day (Table 32), Shirley's instructional methods remained approximately the same.

Table 32

Daily Instructional Plan: Shirley Corson

| Description of activity | Dominant voice | Approximate time |
|--------------------------------------------------------------------|--------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| Review of homework answers (read out loud) and/or warm-up activity | Shirley | 5 – 10 minutes |
| Responding to homework questions by working problems for students | Shirley (with initial student request of problem and some student input) | 0 – 30 minutes (if a quiz or test was just taken, new material began immediately) |
| Presentation of new material | Shirley | 15 – 30 minutes |

| Description of activity | Dominant voice | Approximate time |
|----------------------------------|----------------|------------------|
| Students begin working exercises | None | 5 minutes |

Beliefs about the Role of the Teacher

Investigating Shirley’s beliefs about her role as the teacher provided valuable information about her decision not to incorporate any materials, resources, or information about the historical development of logarithms during her instruction about logarithms.

Teacher as conveyor-of-information. Shirley believed it was her job to provide mathematical information to students because they would not obtain it through any other resource. Shirley described her role as a mathematics teacher in the following terms:

I feel like – and I know that’s not the politically correct way to view things anymore – but...sort of that conveyer of information. Because I know that kids do not read textbooks. If you leave it to them to “discover,” that’s not going to happen [laughs]. I don’t want to give that impression that I can tell you everything you [the student] need. But [I can be] more of a...I guess a guide. (Interview, 4/14/05)

When I prompted Shirley further about why she felt that her instructional approach was not a “politically correct way to view things anymore,” she discussed her view in terms of her experience with educational approaches of other teachers:

I think anymore most of the educational philosophies are that...it’s a student-centered classroom – that it should be. And I don’t think that always works, because I don’t think the students necessarily know where to focus. And maybe that’s my interpretation of that and it’s wrong, too. I do know that a few years back there was – and it scares me now because she is head of teacher education at one of the local colleges – and she taught at [Shirley’s previous school]. She had a geometry class where a lot of my Algebra I kids went. There was some class she was taking at the time where the discovery method was the key. And so they [the geometry students] were doing all of these little “activities.” Now can you “formulate” some of the geometric philosophies or formulas or whatever. And, oh my gosh, those kids hated it (Shirley’s emphasis). They would come back to me on a regular basis saying, ‘that lady is crazy’ [laughs]. (Interview, 4/14/05)

And you know, they wanted to know what they needed to know. These were kids at the time that came out of an Honors Algebra I class. So they were serious

students and they're like, 'I need to know certain things; tell me what I need to know.' And she wasn't doing that and it just drove them crazy. That's a kind of a take on it from a good student's standpoint, so if you have the less studious kids, how would they react to that kind of thing? That scares me. (Interview, 4/14/05)

Shirley believed that as a teacher she not only possessed what her students needed to know, but she knew the most efficient manner in which to provide the information to them. Shirley indicated that one feature of her instruction, although not "popular either," was for students to be given "lots of practice." Shirley observed, "I try to relate it to band and sports. You don't get good unless you practice" (Interview, 4/14/05).

Much of Shirley's classroom practice was representative of her view of teacher as a conveyor-of-information. Of the 640 minutes of observed classroom instruction (omitting the two days on which Shirley administered tests), Shirley dominated 580 minutes with delivery of instruction, answering homework questions, and reviewing for tests. Although a small fraction of this time was devoted to answering homework questions raised by students, Shirley controlled virtually all but the asking of the original question of the classroom talk. Thus, even when a student asked a question motivated by the difficulty they experienced on a homework problem, Shirley felt that hers was the most reliable mathematical authority in the classroom and her instructional stance was representative of her need to tell her students "what they needed to know" (Interview, 4/14/05).

Beliefs about the Role of the Student

In many ways, Shirley's instructional practice is reminiscent of Grundy's description of the technical interest. Grundy (1987) stated that "the technical interest is an interest in control" (p. 47). Grundy further observed that when the technical interest informs a teacher's knowledge and work, that:

An important aspect of their endeavours would be gaining control over their teaching situation so that they can produce what they set out to produce....Thus, the knowledge generated is 'skill knowledge': knowledge of how to act in certain situations to improve the outcome of the act of teaching. (p. 47)

This aspect of the technical interest is apparent in Shirley's view of the student's role in the classroom.

Student as absorber-of-knowledge. When I asked Shirley for her description of the role of the student in her classroom, she claimed that:

[T]he best description, quick description is for the student to be like a sponge. Absorb (Shirley's emphasis) what I'm telling you but then take it, internalize it, make it meaningful to you. Sometimes the kids surprise me. Even in my Algebra I classes. We were talking about finding the zeroes of a polynomial the other day and I had not made that connection, but someone said, 'So those are your x -intercepts.' I'm going, yeah, exactly. But it's like they had kept that knowledge from a previous chapter and now it's like, 'oh, well, that was the same thing.' So it's nice when they see those links and I don't have to point them out (Shirley's emphasis). Every once in a while they will do that on their own. (Interview, 4/14/05)

Shirley's characterization of students as sponges and her own declaration that she desired to tell students what they need to know seemed at odds with her desire for students to "take it [the mathematical content], internalize it, and make it meaningful." Shirley's instruction was similar to Freire's characterization of the technical interest, with the teacher as narrator and with the aim of "turning [students] into containers...to be filled by the teacher" (as cited in Grundy, 1987, p. 101).

Another manifestation of Shirley's belief in students as absorbers-of-knowledge is found in her choice to continue with Chapter 11 regardless of the absence of over half of her students. On March 21 – 24, 2005, Mulberry High School administered their state high school graduation test to all 11th grade students. The testing took most of the morning (Periods 1, 2, and 3) for each of the four days. Shirley's Trigonometry classes

(Periods 1 and 2) were impacted on each of the four days. Although Shirley was farther along in Chapter 11 than either Sue or Ted, she chose not to supplement the exponential and logarithmic functions content found in Chapter 11 in other ways. Instead, Shirley continued with her planned instruction and expected the students to pick up the material on their own or to seek after-school help.

Shirley conducted ‘class as usual’ for ten of the sixteen class periods (see Table 31) for which I observed or collected data, even though a large proportion of the students were absent. Shirley’s decision to conduct class supports her expectations of her students to absorb mathematical knowledge like a sponge – even when they were not present to absorb the knowledge she was dispensing. In the end, however, Shirley offered an analysis of her students’ performance on the Chapter 11 test:

Shirley: I thought they understood it [the material from Chapter 11] better. I think what I thought and what was the reality was different, from questions that I got when some of them came in for help right before the test. Because I thought they understood the mechanics of, this is your relationship with your base and exponent and this is how it works and they could do the process. But as far as understanding, I think that was weak. I think they had more concerns there and I *think* the process; the mechanics of it, I think most of them have it (Shirley’s emphasis).

Kathy: Mechanics meaning like taking log of both sides [of an equation], or changing forms, those kind of things?

Shirley: Yeah, they can go through and do that.

Kathy: But as far as articulating “logarithm” from say some other mathematical topic, that that would be something that they’re weak in?

Shirley: I think that’s it; they’re very weak. (Interview, 4/14/05)

Two items from the Attitudes Instrument also revealed pivotal beliefs which Shirley held about her students’ engagement with mathematics.

Engagement and accountability. The first item of note (Item 4 of Part III) asked: *What would you say your students enjoy the least about logarithms?* Shirley responded, “My students probably enjoy the least being held accountable for having prior knowledge of rules of exponents” (Attitudes Instrument, 12/15/04). Although Shirley reported that her students disliked being held responsible for the laws of exponents which she viewed as essential to the study of exponential and logarithmic functions, two outcomes of Shirley’s practice indicated that she did not hold them accountable for recalling or using this prior knowledge.

First, students were asked to review exponents in Section 11.1 of their text as a preview of applying laws of exponents in calculations, solving exponential equations, and connecting the laws of exponents to the construction of the properties of logarithms. In addition to the review of exponents in Section 11.1, however, Shirley also gave the laws of exponents to students as they began study of Section 11.4, which covered logarithmic functions. In this sense, Shirley did not hold the students’ command of a basic topic in high regard. Upon starting the lesson for Section 11.4, Shirley oriented her students to the topic of logarithms in the following manner:

In talking about logarithmic functions, you need to make sure that you are comfortable with your relationships with exponents. And going back and reviewing some rules of exponents that we talked about and you’ll need to know this. We’ve already talked about it this year; we reviewed it in the first lesson last week; we’re going to come back to it. But you’ll need to know this on the quiz on Friday. (Observation, 3/23/05).

Shirley then proceeded to display five rules of exponents for her students.

A second outcome of Shirley’s instructional practice which contradicted her views about student accountability occurred when she first dictated the four properties of logarithms. As she displayed the properties on the overhead, Shirley did not situate each

of them with respect to their relationship to exponents. With the exception of explicitly noting the similarity of the relationship between subtraction and division found between the two properties of

$$\frac{a^m}{a^n} = a^{m-n} \quad \text{and} \quad \log_b \frac{m}{n} = \log_b m - \log_b n ,$$

Shirley did not engage students in determining the properties of logarithms for themselves to, as she said, “make sure that [they were] comfortable with [their] relationships with exponents.” Instead, Shirley rewrote the properties during her lecture just as they appeared on the handout she provided the students at the beginning of class. Thus, Shirley indicated that she believed her students disliked being held responsible for their prior knowledge of exponents yet her practice did not indicate that she was in fact holding them accountable beyond the tasks that would appear on the chapter quiz and test.

Student ability. The second item (Item 4 of Part II) asked participants to make a choice and provide a brief completion for the statement:

I would consider incorporating historical problems in the curriculum as (circle one) possible, most likely, improbable because...

In all three instances, one pre-assessment and two post-assessments, Shirley circled *improbable* and completed the statement with:

...few of my students have more than a basic working knowledge of trig

(12/15/04);

...of time constraints (1/03/05); and

...the students I have do not seem to be able to (4/14/05).

In her response on both the pre-assessment and the final post-assessment, Shirley indicated that incorporating historical problems into the curriculum would be dependent upon student ability – or the lack of it. Shirley was the only participant to indicate that this aspect of including the history of mathematics would depend upon student ability, and would in fact prohibit her use of such a curricular innovation.

Student apathy and attention. After the completion of Chapter 11, I asked Shirley the following about her use of instructional time:

I noticed that you spent varying lengths of time on the homework questions-slash-new problems of the day because I know that the first four days the students were testing and sometimes there were fewer people. Taking all that time together, anywhere from 10 to say 30 minutes and recalling your observation of ‘they know the mechanics but do they really understand it’, under what conditions would you consider using historical activities? Such as either something that we did or something that you know about or would have put together on your own, to replace [traditional instruction] or to balance that with problem review?
(Interview, 4/14/05)

She responded in terms of student attention span and their lack of persistence, observing that,

I found that it’s almost like sound bytes kind of thing. If I can’t give it to them quickly and concisely, it doesn’t matter how perfect it is. That’s something that I get aggravated about a lot. If you have a problem that has, let’s say eight steps in it; say solving an equation. They [the students] get so upset (Shirley’s emphasis). ‘It’s going to take half a page to do this problem.’ And it’s like, so what? I’ve done problems before in college that have taken pages to do. There’s not...a maximum amount of space a problem should take. They have that real short attention span and like I said, there’s a lot of stuff that I used to elaborate on. For instance, when we did the quadratic formula, I have, in the past gone through and shown them that it’s not something that someone made up. We’ve done completing the square. This is how – if you do it in general – this is what you come up with. That’s where it came from. (Interview, 4/14/05).

When I inquired further about the students’ ability to follow the derivation of the quadratic formula by completing the square, Shirley simply commented that, “I think

there are a lot of them, even though they are good note-takers, they wouldn't even write it down. They just don't care" (Interview, 4/14/05).

I followed up on Shirley's acknowledgement of the effectiveness of the use of the "sound bytes kind of thing" with respect to a particular example about the calculation of the logarithm of particular values. Shirley again acknowledged the students' short attention span.

Kathy: So if you could every once in a while drop in some historical idea, such as when you take the log of 2, that's from the calculation – remember? Ten to some power point 3 (0.3) in this case, about, equals 2. So, kind of interspersing those types of things to remind students of how the values were constructed. Would that be a potential way to use the materials? I know that it wouldn't call for the whole historical activity of constructing the values, but maybe putting in one idea here or there. Would that be one potential way to include the history of logarithms?

Shirley: That would be a way, yeah. But like I said, it seems like it has to be sort of short and sweet. (Interview, 4/14/05)

The themes of time and student ability were also evident when Shirley discussed characteristics of the Mulberry school environment.

The Influence of School Features

Shirley identified two school-specific features which influenced her classroom practice.

Time and scheduling. In addition to indicating that student ability would preclude her use of historical problems when teaching mathematics, Shirley also discussed how time would make such inclusions improbable. When asked why she chose not to include aspects of the historical development of logarithms in relation to a particular segment of her instruction, Shirley again raised the issue of time.

- Kathy: With respect to the motivation behind either logs or the exponents, such as including Euler or Napier, why did you choose not to include something like, ‘Okay, now we are going to do something just to give you a little historical tidbit’?
- Shirley: Well, that’s one of those things that at some point in time, in a perfect world, there will be that point in time. Take yesterday. By the time I was finished going over enough of the examples I wanted to do and get whatever questions they had, they had less than ten minutes left in class.
- Kathy: Yeah, I think it was seven [minutes].
- Shirley: I’m getting better at this. But let me tell you, if you’d seen me at the beginning of this school year, it was panic (Shirley’s emphasis) because I’ve been going for the last I don’t know how many years, on a block schedule. So I had 90 minutes. You know, 90 minutes is a lot different than 55. So it’s the pacing; it’s just the time limit.
- Kathy: I know that you must have some interest in it [in the history of mathematics] because of the time that you put into designing the project that you would eventually like them to do.
- Shirley: They’ve asked me, ‘are we going to do a project?’ I said, I don’t know, it depends on just how much stuff we get done now and if we can sacrifice some time. I’d like to have maybe a day or at least a good chunk of time, to at least talk about it [the history project she designed]. I just feel like it’s constantly pushing them to get everything done. (Interview, 3/24/05)

Shirley identified the obstacle of lack of time as the reason for not being able to incorporate the historical development of logarithms during her instruction. Additionally, Shirley had designed a semester project focused on researching famous mathematicians (Appendix I), yet was never able to include the project during the semester because of lack of time. Another aspect of this critical feature of time, however, is that Shirley was also dealing with adjusting to 55 minute classes after years of teaching 90 minute classes. Thus, Shirley implied that the school-imposed feature of not operating on a block

schedule influenced her curricular decisions with respect to using the history of logarithms.

Difference in students. Shirley implied that student attitude, behavior, and ability had changed greatly over the course of her teaching career and as a result, impacted what she was able to accomplish with and require of her students. This issue surfaced when I asked Shirley whether her teaching philosophy had changed over the years. She replied

Yes, because I think when I first started teaching, I didn't have "bad classes." It was first of all, it was a different community. The things that I would have thought then were major disruptions, now I think I would just ignore and just go right on because it would be no big deal. But I think *then* my focus was, let me tell you this math stuff (Shirley's emphasis). Let's just deal with this. Over the years I've had some classes that have been just horrible (her emphasis). I mean, I have some kids now, they've never passed a math class. (Interview, 4/14/05)

Although I did not prompt Shirley further about the differences between her current school community (Mulberry High School) and her previous community (Owens High School, in a neighboring county), the implication of Shirley's sentiment was that differences existed and that other pressures existed (i.e., discipline, student apathy) which prohibited her from telling her students the "math stuff" they needed to know.

Incorporating the History of Logarithms

No aspect of Shirley's instruction about logarithms (or logarithmic functions) included elements from their historical development. She did not interject anecdotal stories or biographical information, make use of historical problems, or draw upon any of the lesson installments utilized in the professional development. Shirley's apparent purposeful exclusion of the historical development of logarithms was very much aligned with her limited engagement during the professional development sessions.

Summary

Shirley represents a case of limited professional development engagement coupled with complete exclusion of the historical development of logarithms during her instruction. Shirley represents the only case of a participant who infrequently engaged in the professional development who also chose not to incorporate any content from the historical development of logarithms in her instruction.

The professional development participation data, along with Shirley's performance on the content knowledge test, suggest that more active engagement during professional development introducing teachers to unfamiliar content is necessary for teachers to entertain the inclusion of such content in their instruction. The data also imply that interest in a mathematical topic or participating in a professional development experience is not sufficient to influence modification of a teacher's practice.

After instruction related to logarithms, Shirley's attitudes about using the history of mathematics appear to have shifted favorably, for the most part, so that she at least slightly agreed with four of the first five statements Part I of the Attitudes Instrument (Table 33). Although Shirley agreed with using the history of mathematics in some forms (biographies, historical facts), she slightly disagreed with only one of the first five items of Part I: *Knowing the historical development of a key mathematical topic facilitates the learning of that topic*. By articulating this particular attitude, Shirley implied that she more firmly believed in her current practices to facilitate the teaching of mathematics.

Table 33
Attitudes Instrument (Part I) Results, Pre- and Post-Assessments: Shirley Corson

| Item | Pre response (12/15/04) | Post response 1 (1/03/05) | Post response 2 (4/14/05) |
|---------------------------------------------------|----------------------------|---------------------------------|---------------------------------|
| 1. Understanding the history of mathematics is an | 2 | 2 | 4 |

| Item | Pre response (12/15/04) | Post response 1 (1/03/05) | Post response 2 (4/14/05) |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------|---------------------------------|------------------------------------------------|
| important part of understanding mathematics. | | | |
| 2. Including history enriches the teaching and learning of mathematics. | 5 | 5 | 6 |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | 4 | 5 | 4 |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | 4 | 2 | 3 |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | 4 | 2 | 5 |
| 6. Using historical materials in my mathematics classes has been an integral part of my instruction in: Algebra I or II. Geometry. Precalculus/Trigonometry. Calculus. | 1 No response 1 No response | 1 1 1 1 | 1 No response 1 No response |
| 7. Prospective mathematics teachers should be required to study the history of mathematics. | 4 | 2 | 4 |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | 4 | 4 | 4 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

Shirley’s responses to Item 2 of Part II of the Attitudes Instrument, *Mathematics teachers should require (circle one) no, some, much history work in their mathematics classes*, evolved over time. Shirley’s initial perspective, both prior to and directly after the professional development sessions, was that such work could not be required because history is not “part of the state-mandated performance standards” (Attitudes Instrument, 12/15/04). On the final post-test, however, Shirley indicated that teachers “should require some history work” because it “gives them [students] a broader knowledge base” (Attitudes Instrument, 4/14/05).

Shirley's performance on the final Content Knowledge Post-assessment indicated that her traditional use of logarithms, for the purpose of solving equations and manipulating expressions, was firmly intact. As with the Content Knowledge Pre-assessment, Shirley made no attempt on any of the problems with an historical orientation. Of note is that Shirley again omitted a response to first item, *Define logarithm*. Shirley's treatment of the definition of *logarithm* during her instruction provided additional support of her beliefs about her role as conveyor-of-information.

What we're going to do today is we're going to talk about inverses of exponential expressions. Your first definition is your logarithm of x . You've got this on your notes. If you have x is equal to a to the y power, that's an exponential function. Then that can also be written as the log base a , of x , is equal to y . Now, when you write logarithmic expressions, please, if you have to exaggerate the size of your numbers – but make sure that you know that this is your base [points to a , in $\log_a x = y$] and that these numbers [points to x and y] are larger than the a number here, when you're writing this. (Observation, 3/23/05)

Here, Shirley stated the definition of a logarithmic function, not the definition of *logarithm*. In this respect, Shirley's flawed understanding of the concept of *logarithm*, her lack of engagement during the professional development sessions, and her lack of implementation of the historical materials appear related and suggest a need for further examination, which the data collected for this study did not enable.

The paucity of data available to describe Shirley's knowledge of logarithms is unfortunate and allows for only three broad conclusions. First, Shirley's lack of effort on the Content Knowledge tests is indicative of her overall participation in this study. Second, Shirley's ability to work with logarithmic equations and the evaluation of logarithmic expressions – to the extent that they are represented on the Content Knowledge Assessment – is consistent and thorough. And third, Shirley may be weak in her overall conceptual understanding of what a logarithm is, as evidenced by her

omission of a definition on the pre- and post-assessments and her inability to provide a meaningful definition for students. To this end, a more active participation during the professional development sessions, as well as an expanded personal study of the history of logarithms, may have contributed to both Shirley's mathematical knowledge and the demystification of the concept of logarithm for her students.

In the case of Shirley, other factors that influenced her decision to not incorporate anything from the historical development of logarithms include her beliefs about her role as teacher, her beliefs about students and their abilities, and her identification of school constraints. Shirley's identification of herself as a conveyor-of-information could be viewed as compatible with telling anecdotes or relaying the historical origin of logarithms. Ultimately, however, she chose not to do this. Her attribution of students as absorbers-of-knowledge conflicted with even the most superficial inclusion of historical knowledge due to Shirley's traditional perception of the essential mathematical knowledge necessary for solving logarithmic equations. Shirley identified several obstacles which prevented her use of the historical development of logarithms in the classroom, with lack of time and lack of student ability (including inadequate student attention span) as the predominant obstacles. Indeed, time contributed to Shirley's lack of participation related to the professional development sessions and her personal study of the history of logarithms as well. When questioned whether she had had time to review anything else from the materials discussed during the professional development sessions, Shirley simply responded, "no" (Interview, 3/24/05). It is likely that Shirley represents a case of a teacher who joins professional development activities for the sake of

professional inclusion. For example, Shirley may have felt pressure to participate because the other Trigonometry teachers did so.

The Case of Mary Long

Like Shirley, Mary experienced limited engagement with the historical development of logarithms. Mary Long was the only one of the five participants not currently teaching a Precalculus-type course. Her teaching assignment included teaching Algebra I, Geometry, and Advanced Placement Calculus. Upon hearing about the proposed professional development and associated study from Mulberry's Trigonometry teachers, however, Mary requested to attend the professional development sessions and participate in all of the related activities. Although some of the data applicable to the other participants were not collected for Mary, she is included in the study to further explore the impact of professional development designed around the specific purpose of engaging teachers in the history of a particular mathematical topic.

Professional Background

Mary, as in the case of Ted, began teaching as a second career. Mary spent sixteen years as a computer programmer and systems analyst. She earned undergraduate degrees in both computer science and mathematics and after her first career, completed all of the education course work necessary to teach secondary mathematics. After achieving certification, she taught high school mathematics for ten years in an adjacent state and one year in a neighboring district before arriving at Mulberry, where she had been for two years.

Although a career-switcher, Mary's mathematics content preparation was very similar to that of the other participants. Mary took most of the courses typically required for an undergraduate degree to teach secondary mathematics, with the exception of abstract algebra. Also, Mary is currently pursuing a master's degree in mathematics education. As was the case with the other four participants, Mary has never taken a course in the history of mathematics. For the prompt, *Describe any previous experience with the history of mathematics*, Mary simply responded, "none" (Background Survey, 12/15/04).

On the Background Survey, participants were asked to indicate which of twelve possible professional development activities they participated in during the last two years. The average number of type of activity reported was six; however, participants were not required to report the frequency for each type of professional development experience. Mary indicated that she was involved with six of the activities. Additionally, Mary noted that she served as the mathematics department's Geometry lead teacher, which entailed organization of review materials for the state end-of-course test, addressing curriculum schedules and content, and delegating the construction of commonly administered end-of-chapter tests.

Attitudes and Knowledge

In addition to the Background Survey items regarding professional development participation, the Attitudes Instrument asked participants to respond to the question (using a scale from "strongly disagree" to "strongly agree"): *As a mathematics teacher, it is important for me to continue my own learning of mathematics*. Given that Mary asked to join the other teachers specifically for the professional development, Mary's response

of “strongly agree” was not surprising. This sub-section discusses the data from Mary’s initial responses to the attitudes and content knowledge items in an effort to characterize her attitudes about the history of mathematics and knowledge of the development and use of logarithms prior to the professional development sessions. Since Mary was not currently teaching Trigonometry at Mulberry High School and the teaching of logarithms was no longer in her recent memory, she exempted herself from responding to Part III (questions focused on the participant’s approach to teaching logarithms) of the Attitudes Instrument.

Mary’s responses to Part I of the Attitudes Instrument Pre-assessment indicated that she held somewhat varied opinions about the importance and use of the history of mathematics (Table 34).

Table 34
Attitudes Instrument (Part I) Pre-Assessment Results: Mary Long

| Survey item | Pretest response |
|-----------------------------------------------------------------------------------------------------------|------------------|
| 1. Understanding the history of mathematics is an important part of understanding mathematics. | 1 |
| 2. Including history enriches the teaching and learning of mathematics. | 4 |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | 4 |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | 1 |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | 1 |
| 6. Using historical materials in my mathematics classes has been an integral part of my instruction in: | |
| Algebra I or II. | 1 |
| Geometry. | 4 |
| Precalculus/Trigonometry. | 1 |
| Calculus | 1 |
| 7. Prospective mathematics teachers should be required to study the history of mathematics. | 5 |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | 6 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

When the first five items are categorized by purpose, using history for learning mathematics or using history in the teaching of the subject, Mary's responses indicated that she initially viewed the history of mathematics as useful for enrichment purposes only. Using the categorizations, Mary responded that she was slightly comfortable with including the history of mathematics for enrichment or enjoyment (Items 2 and 3). Also, she was unable to commit to identifying the use of the history of mathematics as essential for learning or understanding mathematics (Items 1 and 4) or quality instruction (Item 5).

Mary also slightly agreed that she used historical materials as an integral part of her instruction in geometry (Item 6). When asked whether she used history of mathematics on any occasion, Mary responded

I have used some in geometry. I've done the rope stretchers demonstration. Actually when I was in [another state], I brought rope and it had knots in it and we did that. We would actually form that right triangle like they did and we talked about that and of course I would tell them how geometry started in Egypt. When the floods would come and they [the ancient Egyptians] would have to redo everything and then they built their pyramids. But yes, we would talk about things like that. (Interview, 3/25/05)

Her recollection of the rope stretchers activity, which took place more than three years ago, made it unclear whether Mary answered Item 6 in terms of her overall practice or only in terms of her most recent experience within Mulberry High School.

Mary's completion of Items 1 and 2 on Part II of the Attitudes Instrument are contradictory to her responses in Part I of the instrument. For example, in Item 1 of Part II, Mary completed the statement, *Researching a mathematician contributes nothing to a mathematics course because it* "has nothing to do with math concepts that I have to teach. As long as I teach correct concepts, it doesn't really matter where it [mathematics] came

from” (Attitudes Instrument, 12/15/04). Her comfort with using the history of mathematics for enrichment (Item 2, Part I) is obscured by the view stated in completion of Item 2 of Part II: *Mathematics teachers should require no history work in their mathematics classes since this* “is not part of the curriculum and there is not enough time” (Attitudes Instrument, 12/15/04). These two responses, coupled with her strong disagreement that quality mathematics instruction includes important facts from the history of mathematics, equates Mary’s view of quality instruction as that which highlights only the mathematical concepts included within the prescribed county curriculum.

Mary attempted six of the eight items on the Content Knowledge Pre-assessment.

Table 35 identifies the content items, the classification of items as either historical (taken from a lesson installment) or traditional, and a description of Mary’s response to the item.

Table 35

Content Knowledge Pre-Assessment Results: Mary Long

| Item | Historical/Traditional | Response given |
|---------------------------------------------------------------------------------------|---------------------------------------------|----------------------------------------------------------------------------------------------|
| 1. Define <i>logarithm</i> . | Either, depending upon participant response | Described <i>logarithmic function</i> instead of defining <i>logarithm</i> |
| 2. Describe the basic idea or motivation for the invention of <i>logarithms</i> . | Historical | Stated that she “had no idea” |
| 3. Construct the values for $\log_{10}2$ and $\log_{10}3$ without using a calculator. | Historical | Attempted by simply converting $\log_{10}2$ from logarithmic to exponential form (incorrect) |
| 4. Let $u=b^n$ and $v=b^m$. Verify $L(u) - L(v) = L\left(\frac{u}{v}\right)$. | Historical | Proof attempted (incorrect) |
| 5. Evaluate: $\log_{32}16$. | Traditional | Evaluated successfully by converting to exponential form and solving |
| 6. Evaluate: $\log_{\frac{1}{3}}81$. | Traditional | Evaluated successfully by converting to exponential form and solving |

| Item | Historical/Traditional | Response given |
|-------------------------------------------------------------------------------------|------------------------|-------------------------------------------------|
| 7. Calculate the product of 8409.5 and 951.49 using the method of prosthaphaeresis. | Historical | None |
| 8. Solve for x : $2\log 3 + \log x = \log 45$. | Traditional | Solved correctly using properties of logarithms |

Mary successfully completed each of the purely traditional items. She evaluated logarithmic expressions and solved the logarithmic equation; however, she was unable to define *logarithm* correctly. Her definition, “another way to write exponential function” (Content Knowledge Instrument, 12/15/04), indicated that she possessed a weak conceptual understanding of what a logarithm was. Mary commented on her own understanding of logarithm, “I think the whole topic is kind of like some mysterious thing. Like someone just all of a sudden – it just appeared” (Interview, 3/25/05).

Professional Development Engagement

Mary represented a different kind of volunteer to the study. Sue Moe contacted me in early October 2004 about Mary’s desire to participate in the project with the group of Trigonometry teachers. Mary inquired about the “history study” that Sue had discussed with Ted and Shirley. When Mary expressed interest in participating – even though she was not currently teaching Mulberry’s Trigonometry course or any other course that focused on logarithms – Sue told her that she would ask about the possibility of Mary joining the group. Although Mary would not represent a participant with whom I could investigate each of the research questions, I felt that she would illustrate an interesting case nonetheless. In particular, she was interested in the history of mathematics, yet she had not recently taught a high school course with logarithms as a topic of study. Consequently, Mary’s engagement during the professional development sessions, with a concentration on her study of and engagement with the content would constitute a case

with the potential to inform only a subset of the research questions. There were no data available on her actual use of the history of logarithms.

The data in this sub-section address the choice of characterizing Mary's engagement as limited during the professional development sessions. Merriam-Webster (1993) defined *limited* as "confined within limits; restricted" (p. 676). Mary's desire to study the history of logarithms, especially with a group of her peers (Mary characterized herself as a very social person), was quite strong as evidenced by her volunteering despite not teaching the target course. The strength of her desire was insufficient, however, when compared to the required action of interacting with the materials and resources during and outside of the structured professional developments sessions.

An analysis of the evidence associated with Mary's participation (and engagement) during the professional development sessions showed a sporadic manifestation of two primary themes of engagement:

- Mary as a biographical collaborator (a very specific practice); and
- Mary's focus on student engagement motivated by a desire to understand underlying reasons for the development of a mathematical idea.

A secondary analysis of the professional development data, however, also points to underlying reasons for why Mary's engagement during the sessions was restricted. Mary often expressed difficulty with the content of the lesson installments during the professional development discussions. She did so in two different ways. First, Mary experienced difficulty in following the discussion of several tasks and activities within the different lesson installments. Second, certain questions and statements offered by Mary indicated that she had not worked with the history of logarithms materials on her own outside of the formal professional development sessions. Regardless of the reason, a

limited commitment to studying the history of logarithms (a secondary theme of engagement) in preparation for the discussions planned for the professional development sessions prevented Mary from engaging in many of the same ways that Mandy, Sue, and Ted were able to do during the sessions.

Framed by the two primary themes and one secondary theme of professional development engagement, I next highlight examples from Mary's professional development experience to support my characterization of her engagement as *limited*.

Mary as a Biographical Collaborator

Mary was particularly focused on the worth of the biographical information related to the historical development of logarithms. Her collaborative efforts – even in relation to sharing ideas about student engagement and activity – were centered on the type of information gained from the activity in Lesson Installment 1. Mary expressed her opinion about the benefit and utility of such information as

I didn't even know that they [logarithms] were developed to make astronomy easier. I think that's kind of neat for the kids to know. I'm sure there are probably a lot of other things that were developed for practical reasons; that this is the real problem that they had back then and this math was developed. (Professional Development Session 1, 12/17/04)

Mary's interest in bringing information concerning not only the "why" behind the development of the mathematics, but the people behind the mathematics was prevalent throughout the first professional development session. In the first professional development session, she initiated the discussion about the completion of the timeline activity.

Mary: Well, I did mine because I started writing it and I thought about the timelines where you have this long strip of paper. They [the mathematicians suggested on Lesson Installment 1] were so intertwined

that I just decided well, I'll just go to the computer and then I can just insert them after I found them.

Kathy: So you have times down the side of the page and then you've inserted text about whatever particular contribution?

Mary: Right, because I tried to include when this person was born and when he died, and I think it would have been overall easier to see the big picture. I did find a whole lot of neat things about people. And I like genealogy. That is one of my hobbies. It is nice to know dates when they were born and died, married, how many children and all that but it is also interesting – I think you remember more (Mary's emphasis) – if you know little things about the person.

Kathy: Like anecdotal information?

Mary: Yeah, things about them like what they did, how they lived. (Professional Development Session 1, 12/17/04)

Mary concluded her contribution about Lesson Installment 1 by telling a story about the connection between Dr. John Craig and Napier and the inspiration for Shakespeare's *The Tempest*.

Mary: What it said was that this may have been or they think it may have been this trip was the trip that inspired Shakespeare to write *The Tempest*.

Kathy: Yes! I am so glad somebody came across that.

Mary: And so, I thought, wow, for some of your students who were more English- and literature-oriented, that would be interesting. (Professional Development Session 1, 12/17/04)

In the process of collaborating with the other Mulberry teachers over the content of Lesson Installment 1, Mary implied a reason for her attraction to biographical and historical information. Her hobby of genealogy generated her interest in the content of Lesson Installment 1. Mary did not engage in the same collaborative manner with any of the other lesson installments during the professional development sessions.

Mary as an Anticipator of Student Engagement

In similar ways to Sue and Ted but on a much smaller scale, Mary anticipated student engagement with the history of logarithms materials with negative undertones.

Student difficulty. As in the case of Sue Moe, Mary was influenced by the difficulties she perceived students would have. Although Mary noted that she did not get very far in her study of Lesson Installment 3, she offered the following mixed assessment of the overall idea of the lesson.

With the worksheets, I finished the first one [Lesson Installment 3] but I thought that was very interesting and I have done some stuff with Pascal's triangle and all the different patterns in that. I think kids enjoy seeing those, even though I think sometimes to detect the pattern is hard – it's difficult for some students to do – but I think that is something that is kind of enjoyable to look at; to find those patterns and see that connection. (Professional Development Session 1, 12/17/04)

Mary eventually agreed with Sue in that students would not need to have formal knowledge of different types of sequences (i.e., arithmetic and geometric) to be able to complete the activity in Lesson Installment 3.

Connections for students. Mary felt it was important to expose students to biographical and historical information. While discussing the history of logarithms materials broadly at the end of the first professional development session, Mary offered

Well, the students never think that math relates to anything. I don't know what they think. They are always asking, 'When I am going to use this?' I just think that it's kind of neat to know that a lot of this was just developed because it helped to meet some kind of a need for real people. (Professional Development Session 1, 12/17/04)

Although Mary was unable to engage in the more substantial mathematical content of the lesson installments, her contributions with regard to the importance of focusing on the “human side” of the historical development of logarithms were consistent throughout her participation.

Commitment to Learning

The professional development data provided relevant evidence of Mary's commitment to learning that may have hindered her participation during the professional development sessions. Mary experienced difficulty with the content of the lesson installments, which may have occurred because of the topic of logarithms being removed from her immediate practice. In addition, several of Mary's questions and exchanges during the professional development sessions indicated that she had not reviewed the lesson installments and resources prior to the formal professional development sessions. Consequently, Mary's lack of familiarity and limited personal study of the history of logarithms significantly limited her participation.

Basic concepts for studying sequences. During the second professional development session, we worked on Lesson Installment 6 as a group. We began the installment by calculating the common ratio for the sequence b_n . After Sue and Ted determined the ratio, we moved on to Problem 2. Mary interjected with a question, which resulted in Sue explaining the calculation of the common ratio. Mary's unfamiliarity with calculating the common ratio for a geometric sequence limited her understanding of the content required to complete Lesson Installment 6. On another occasion, Mary admitted to not working with sequences and series in a long time. Her comment however, was ill-placed because at the time, we were discussing approximation methods for calculating logarithmic values. The techniques being discussed did not require the use of sequences.

Lack of review of materials. At the end of the first professional development session, I reviewed the various handouts and resources given to the participants in preparation for our next session. (This was similar to the description provided in the letter

of introduction which I included with the resource binders sent in November 2004.) Mary responded, “Oh, we’re to do these then?” This led me to believe that she did not quite understand that an expectation of the professional development was to review, attempt, and complete as much of the material contained in the lesson installments as possible prior to the formal professional development sessions.

In addition, Mary requested to participate in two synchronous, online sessions (via *Tapped In*) with Sue and me. During the online conversations, however, Mary did not actively engage in the sessions. Her contributions were limited to questions about the resources and content that Sue raised.

Intended Changes in Instructional Practice

Mary represented a unique subset of teachers who seek to increase their knowledge for teaching through engagement with both content and instructional materials. Although Mary did not currently teach a course which contained an explicit unit on logarithms, her interest in the history of mathematics compelled her to participate in this research. As with several of the other teacher participants, however, the demands of teaching, such as available time to contribute to the study of a new topic, appeared to limit Mary’s ability to engage in the professional development component of the project.

Although Mary was not currently teaching Trigonometry at Mulberry High School, she revealed important insights about her anticipated instructional use of the history of mathematics.

Using History to Highlight the Human

Mary possessed a somewhat humanistic orientation with respect to the use of the history of logarithms with students. When considering the need to make connections for

students, Mary felt that if students could realize that “a lot of this was just developed because it helped to meet some kind of a need for real people” (Professional Development Session 1, 12/17/04), then it would strengthen students’ ability to see that mathematics relates to processes in the physical world around them. Regarding the use of the history of mathematics in teaching, Mary observed that

You can sit there and learn the facts, but if you know a little background, I think it might make it more meaningful or at least more interesting if you could tell a story about something. That’s just one thing that’s just been on my mind lately, is just how can we make this more interesting for the students? Because they just seem so uninterested. (Interview, 3/25/05)

For Mary, using the history of mathematics with students was much more about the story of the mathematics – people, anecdotes, and relationships – than it was of the mathematics or the notable historical problems. Mathematical content influenced Mary’s choices to some extent. For example, Mary had previously used not only the story of the rope stretchers of ancient Egypt with her geometry students, but she also asked them to build right triangles and relate the measures of the three sides. When an opportunity to include mathematical content related to the history of a topic in Mary’s AP Calculus class, however, her practice remained narrative.

Remember the time I e-mailed you and I said that someone asked about the number e ? I think it was related to natural logarithms, and so I tried to tell them a little bit about how it came about. (Interview, 3/25/05)

When I prompted Mary further about other attempts to use the history of mathematics in her calculus class, she said,

No, I really haven’t and I should, I guess. I’ve never taught any other calculus besides this AP Calculus and so I’ve just been really trying to concentrate on what they have to know. But there’s a lot that could be put into it with just little notes here and there. Actually, I just don’t know enough of it, I guess. I’ve just for the last two years been trying to relearn the calculus well enough to teach it. (Interview, 3/25/05)

Although Mary identified that access to materials and the time to study and plan for their use was an obstacle to her consideration of including the history of mathematics, she noted that the actual time of using history with students was not as much of a concern. Mary observed:

I think a lot of kids don't have any idea where this [mathematics] comes from. That it ever could have come from a person. There's just no reality, I think. It doesn't take long to mention the history of a topic in the classroom. It's just knowing it enough to mention it. (Interview, 3/25/05)

History of Logarithms

Mary acknowledged that she would definitely utilize the seven lesson installments should she ever teach a course which included the topic of logarithms. Although she initially admitted that she had not worked on the lesson installments beyond the professional development sessions, Mary stated,

If I ever taught the topic of logarithms, I would want to use the history of logarithms. I would definitely want to do that because I think that that's really neat. And I don't know exactly what Sue did, but the activity sounded neat. I just really like to do things different like that. (Interview, 3/25/05)

In this instance, Mary made a specific reference to the webquest activity which Sue had modified (see Chapter 6) to accompany the timeline lesson installment. This “different” activity also focused on the mathematicians who contributed to the early development of logarithms and was consistent with Mary's desire to use the history of mathematics to accentuate the human story of its development, as opposed to using historical problems to accentuate the mathematical development.

Lastly, Mary observed that including the historical development of logarithms would aid in the demystification of logarithms:

Usually when you teach logarithms, and I did back a few years ago. It's something you just start talking about. There's just no reason why it's there; it's just there. It's just 'learn this.' (Interview, 3/25/05)

I did not pursue Mary's view for the reintroduction of the properties of logarithms which occurs in Advanced Placement Calculus. If Mary's calculus students learned logarithms devoid of context or reason in Algebra II or Precalculus courses, it would seem reasonable to include some aspect of their historical development when they revisited logarithms. Mary did not, however, include anything from the history of logarithms when teaching differentiation and integration of logarithmic and exponential functions.

Access to Historical Materials

The final component of Mary's intended practice is her identification of the need for appropriate materials and resources. Mary discussed her desire to take a history of mathematics course as one of the five mathematics courses needed for her master's degree program. Sharing this prompted her to recognize that if student textbooks included more historical information than is presently encountered, teachers would "incorporate more into their lessons" and that it "would make it easier for a teacher" (Interview, 3/25/05), provided student textbooks included historically correct and appropriate material. Additionally, Mary inquired about whether there were any books available "that would be appropriate for a high school teacher" (Interview, 3/25/05) such that

If we were studying a topic and I opened the book, could I go to a chapter and find something about that topic? For instance, is there very much for an Algebra I class? (Interview, 3/25/05)

I completed the interview by providing Mary with the names and authors of various materials which would provide her significant historical information and

appropriate student activities to use in her classes. After sharing a story about Girolamo Cardano and Niccolo Fontana Tartaglia's race to derive a formula for solving cubic equations, Mary declared, "We could do that! I mean, how long did it take to tell that story?" (Interview, 3/25/05).

Mary's desire to locate quality history of mathematics materials was also echoed in her responses to items in Part II of the Attitudes Instrument post-assessments. Mary stated that if she had access to such materials, she would use them often with students because "it might make students remember concepts if they can relate to some interesting fact" (Post-assessment, 1/03/05) and because "mathematics would become more interesting and then students would learn more" (Post-assessment, 4/06/05).

Summary

Mary represented a unique case in this study. She did not teach the course targeted for this study, yet she appeared enthusiastic to learn about the history of mathematics regardless of topic. Mary's engagement during the professional development sessions was limited. Still, the case of Mary provides valuable information for addressing a subset of the study's research questions.

Mary was eager to learn about the history of logarithms. Her desire and intent were not sufficient for her to maintain a high level of engagement during the professional development component. The data suggested that Mary struggled with the content of professional development sessions, thereby limiting her preparation for and participation during the sessions. Also, Mary indicated that dedicating the time for her study of the content was a factor. In Mary's opinion, however, time was not as significant an obstacle to the use of the history of mathematics with students. Another feature of Mary's

professional development participation was her emphasis on the use of the history of mathematics for the purpose of telling the story of mathematics. The recognition of the influence of time (and the corresponding lack of it) and the importance of the biographical and anecdotal contributions of the history of mathematics were evident in Mary's responses to the Attitudes Instrument items.

Mary's general attitude towards the use of the history of mathematics in teaching changed significantly during the study. Specifically, Mary responded more favorably to all but one item in Part I of the Attitudes Instrument Post-assessment. (In Item 8, Mary continued to strongly agree for the duration of the research study.) Table 36 presents Mary's responses to Part I of the instrument across the three administrations.

Table 36
Attitudes Instrument (Part I) Results: Mary Long

| Survey item | Pre response (12/15/04) | Post response 1 (1/03/05) | Post response 2 (4/06/05) |
|-----------------------------------------------------------------------------------------------------------|-------------------------|---------------------------|---------------------------|
| 1. Understanding the history of mathematics is an important part of understanding mathematics. | 1 | 3 | No response |
| 2. Including history enriches the teaching and learning of mathematics. | 4 | 6 | 6 |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | 4 | 6 | 6 |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | 1 | 5 | 4 |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | 1 | 3 | 4 |
| 6. Using historical materials in my mathematics classes has been an integral part of my instruction in: | | | |
| Algebra I or II. | 1 | 1 | 3 |
| Geometry. | 4 | 2 | 3 |
| Precalculus/Trigonometry. | 1 | No response | 3 |
| Calculus | 1 | 2 | 3 |
| 7. Prospective mathematics teachers should | 5 | 6 | 6 |

| Survey item | Pre response (12/15/04) | Post response 1 (1/03/05) | Post response 2 (4/06/05) |
|-------------------------------------------------------------------------------------------------|----------------------------|---------------------------------|---------------------------------|
| be required to study the history of mathematics. | | | |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | 6 | 6 | 6 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

Mary’s final post-assessment responses indicate that she minimally “slightly agreed” with each of the statements in all but Item 6. Her responses for each of the courses in Item 6 are somewhat puzzling, as Mary was currently not teaching Trigonometry at Mulberry High School. Additionally, when I asked Mary about her current uses of the history of mathematics, she indicated that she previously used some history with geometry, some three years earlier in another state. She explained that except for a brief question in her calculus class about the number e , she had not incorporated any history of mathematics during the current year. Thus, her responses of “slightly disagree” for each of the four courses listed in the final post-test statement, *Using historical materials in my mathematics classes has been an integral part of my instruction*, is inconsistent with her reported practice.

The first five items of Part I from Mary’s final post-assessment responses were again examined by purpose for the use of the history of mathematics, i.e., for learning or understanding mathematics versus teaching mathematics. Mary still strongly favored the use of the history of mathematics for the purpose of enriching the story of mathematics. Her recognition of the history of mathematics contributing to learning, and understanding mathematics shifted considerably, however. Whereas Mary strongly disagreed with such items (Items 1, 4, and 5), she slightly agreed with the items on the final post-assessment.

Perhaps the most significant modification in Mary's reported attitudes about the use of the history of mathematics is evident in her responses to Items 1 and 2 of Part II of the Attitudes Instrument. Mary's pre-assessment responses and post-assessment responses to Items 1 and 2 appeared at opposite ends of the attitude spectrum. In both instances, Mary progressed from her initial responses, which were heavily influenced by existing curriculum and lack of time, toward a view that the introduction of the history of mathematics would enable students to understand the origin of mathematical ideas and the significance of the individuals behind them. Mary initially reported that researching mathematicians contributes nothing to a mathematics course because such a task "has nothing to do with math concepts that I have to teach. As long as I teach correct concepts it doesn't really matter where it came from" (Attitudes Instrument, 12/15/04). On the post-assessment, however, Mary declared that the use of such research contributes "a lot" because it makes mathematics real as opposed to mystical; the course becomes more relevant; and students can relate to the work of the mathematicians that they research (Attitudes Instrument; 1/03/05; 4/06/05).

Similarly for Item 2, Mary initially believed that mathematics teachers should not require any history work in their classes because it "is not part of the curriculum and there is not enough time" (Attitudes Instrument, 12/15/04). Summaries of her post-assessment responses to the same question reveal that Mary believed teachers should require much history work. She observed that history work in mathematics classes may help students understand where the content comes from and it aids in showing students a progression of mathematics, which is not arbitrary (Attitudes Instrument, 1/03/05; 4/06/05).

Items 3 and 4 of Part II supported Mary's perception that the use of the history of mathematics provides students with interesting and anecdotal insight into the development of mathematical ideas. With respect to Item 4, Mary once again indicated that time was a factor when considering the use of the history of mathematics. Mary stated that, "I would consider incorporating historical problems in the curriculum as possible because they would enrich the curriculum provided that I had time" (Attitudes Instrument, 1/03/05). It was unclear, however, whether Mary was referring to her own time needed to study the problems for inclusion in the curriculum, or the amount of time needed to use the historical problems with students.

Mary's content knowledge of logarithms was not significantly impacted by her participation in the professional development sessions. Mary attempted the same six of eight items on the two Content Knowledge Post-assessments and completed only one additional item correctly, Item 2. Mary's response to Item 2 on the second post-assessment (4/06/05) was much more descriptive than the first post-assessment response. On the first post-assessment, Mary simply responded "astronomy" (1/09/05) to the prompt, *Describe the basic idea or motivation for the invention of logarithms*. On the second post-assessment, however, Mary explained, "They were invented to make multiplication and division of large numbers easier for astronomers who had to make tedious calculations" (4/06/05). In addition, Mary's response to the first item on the second post-test, *Define logarithm*, was beginning to assume language more appropriate for defining *logarithm* as opposed to *logarithmic function*. She responded with "exponent" (4/06/05), and although this definition was not mathematically clear or

complete, Mary refrained from moving directly to “another way to write exponential function” (12/15/04) as she did on the Content Knowledge Pre-assessment.

Without instructional data to pinpoint specific practices which Mary was inclined to include in her teaching of logarithms, I could not address the research questions relating to implementation of the historical materials or identification of benefits or obstacles encountered when incorporating the historical development of logarithms. Instead, Mary’s case highlighted two important aspects of incorporating the history of mathematics: the more typical practice of emphasizing human contributions to the development of mathematical ideas and the need for readily accessible, high-quality, “teacher-friendly” resources for incorporating the history of mathematics. Also, time was an influential factor underlying each of these important aspects in the case of Mary. Mary recognized that including the story of mathematics during instruction would not require much time; however, the time available to locate and study appropriate resources was inadequate. Indeed, the obstacle of time limited Mary’s participation in the study of the historical development of logarithms, even when access to materials was provided.

Chapter 8 Reflections across the Cases: Conclusions, Implications, and Recommendations

This chapter contains three discussions. The first discussion presents conclusions related to the salient features of each participant's experience with the history of logarithms and which resulted from an examination of the two primary research questions:

How do teachers with different background knowledge and experiences respond to professional development focused on understanding and using the history of mathematics?

How do background variables and professional development experiences with the history of mathematics combine to influence teachers' personal mathematical knowledge and instructional practice?

The second discussion focuses on the implications and recommendations for the professional development of teachers and for using the history of mathematics in teaching. Lastly, the third discussion presents recommendations for future research and activities focused on incorporating the history of mathematics in teaching. Only the experiences of Mandy, Sue, Ted, and Shirley will be considered in Chapter 8, as Mary did not teach the content of interest during this study.

Conclusions

Mandy, Sue, Ted, and Shirley entered the teaching profession via different paths. Sue and Shirley obtained regular secondary certification in mathematics as part of an undergraduate program in mathematics. Mandy and Ted were certified in secondary mathematics, but not as part of their initial undergraduate program. The comparison of the participants' knowledge and experience is organized according to certification path.

Background Variables Summary

Two instruments, a Background Survey and Attitudes Instrument, were used to collect information about each participant's background. Results concerning certification route, years of experience, mathematics preparation, previous professional development activities, and attitudes towards the history of mathematics were examined for their potential influence on each participant's experience during this study.

Certification, experience, and content preparation. Sue and Shirley each completed a traditional secondary certification program in mathematics in conjunction with their undergraduate degree program. Sue was currently pursuing a master's degree in mathematics education, whereas Shirley earned a master of arts in teaching (MAT) in mathematics in 1979. Sue and Shirley experienced essentially the same mathematics preparation (Table 37). Sue and Shirley differed the most in regard to teaching experience; Sue in her fourth year of teaching and Shirley in her twenty-eighth.

In contrast, Mandy and Ted followed alternative certification programs; however both were fully certified. Mandy's certification was earned after her undergraduate program, which included a major in political science and a minor in mathematics. Mandy earned a master's of science degree in mathematics education. Ted served in the military for his first career, completed undergraduate programs in Bible studies and mathematics education, and a master's degree in mathematics education. Ted had an additional 18 semester hours in graduate coursework beyond his master's degree. Nine hours of this additional coursework was mathematics content taken at the undergraduate level. Although Mandy and Ted's mathematical preparation was similar in number of courses, it differed in range of content (Table 37) when compared with Sue and Shirley's. Ted and

Mandy also differed in how long they had been teaching. Ted was completing his sixteenth year; Mandy, her thirty-seventh.

Table 37

Mathematics Certification and Preparation Comparison

| Mandy Wilson | Sue Moe | Ted Jones | Shirley Corson |
|-------------------------------------------------|-------------------------|-------------------------------------------------|------------------------|
| Regular certification (alternative route) | Regular certification | Regular certification (alternative route) | Regular certification |
| College Algebra | | | College Algebra |
| Calculus I, II, III | Calculus I, II, III, IV | Calculus I, II | Calculus I, II, III |
| | Differential Equations | | Differential Equations |
| Introductory Statistics Statistical Analysis | Introductory Statistics | Introductory Statistics Statistical Analysis | |
| | Linear Algebra | Linear Algebra | Linear Algebra |
| | Abstract Algebra | | Abstract Algebra |
| | College Geometry | | College Geometry |

Prior professional development activities. In the two years prior to this study, Sue and Shirley's reported professional development experiences varied in quantity and range of experience (see Table 38). The differences in Mandy and Ted's reported professional development experiences were equally disparate (also Table 38). Sue and Mandy reported the strongest engagement with previous professional development activities. For this reason, it is not surprising that they were the most active during the professional development focused on the historical development of logarithms.

Table 38

Professional Development Experiences Related to Teaching Mathematics

| Type of activity | Mandy Wilson | Sue Moe | Ted Jones | Shirley Corson |
|-----------------------------------------------------------------|--------------|---------|-----------|----------------|
| Content or method college course | | X | X | |
| School/district/state-provided workshop, training, or institute | X | X | X | |
| Conference or professional meeting | X | X | | |

| Type of activity | Mandy Wilson | Sue Moe | Ted Jones | Shirley Corson |
|----------------------------------------------------|--------------|---------|-----------|----------------|
| Observation visit to another school | X | | | |
| Mentoring or peer observation or coaching (formal) | | X | | X |
| Committee or task force work | X | X | | |
| Regularly scheduled discussion or study group | | X | | |
| Teacher collaborative network | | | | |
| Individual or collaborative research | X | X | | |
| Independent reading on a regular basis | X | X | X | |
| Co- or team-teaching | | X | | |
| Consultation with a mathematics specialist | X | | X | |

Note. An “X” indicates that the individual reported participating in the activity.

Attitudes. Each of the participants reported favorable attitudes toward the use of the history of mathematics. Of the eight items in Part I of the Attitudes Instrument, four focused on general and particular uses of history of mathematics (Items 1, 2, 3, 5); three on the importance of learning about the history of mathematics (Items 4, 7, 8); and one item about the extent of the use of historical materials each participant employed in their teaching (see Table 39).

Table 39

Average Responses: Attitudes Instrument Assessments (Part I)

| Item | Average pre-response | Average post-response (after professional development) | Average post-response (after instruction) |
|-----------------|----------------------|--------------------------------------------------------|-------------------------------------------|
| 1 | 4.25 | 4.33 | 5.5 |
| 2 | 5.5 | 5.67 | 5.5 |
| 3 | 5 | 5 | 5.25 |
| 4 | 4.5 | 3.67 | 4.75 |
| 5 | 4.5 | 4.33 | 5 |
| 6(Algebra I/II) | 2 | 2.33 | 2.67 |
| 6(Geometry) | 2.5 | 2.33 | 6* |
| 6(Precalc/Trig) | 3 | 2.33 | 4.25 |
| 6(Calculus) | 3.5 | 2.33 | 6 [#] |
| 6(Statistics) | No response | No response | 5 [†] |

| Item | Average pre-response | Average post-response (after professional development) | Average post-response (after instruction) |
|-------------------------------|----------------------|--------------------------------------------------------|-------------------------------------------|
| 7 | 5.5 | 4.67 | 5.25 |
| 8 | 5.5 | 5.33 | 5.5 |
| Combined average, less Item 6 | 4.96 | 4.74 | 5.25 |

Note. Responses ranged from 1 (strongly disagree) to 6 (strongly agree).

*One respondent only: Ted Jones

†One respondent only: Mandy Wilson

#Average of two respondents: Ted Jones and Mandy Wilson

In general and across each of the Attitudes Instrument assessments, Mandy, Sue, Ted, and Shirley expressed favorable attitudes about the use of the history of mathematics. With the exception of Item 6, on average, each participant moderately agreed with each statement related to the uses of history of mathematics and the importance of history of mathematics in learning on the pre-assessment. Responses to Item 6 were varied, due to each participant's perception of previous and current use of the history of mathematics in teaching. The inconsistencies in some of the participant's responses to Item 6 are discussed in previous chapters. For example, it is unexplained why Ted responded "moderately agree" to using historical materials in Calculus after the professional development sessions, yet "strongly agreed" at the end of instruction. (Ted did not teach Calculus in 2004 – 2005, thus, his reported use of historical materials could not have improved.)

The slight drop in average reported agreement to the eight items on the first post-assessment (administered at the conclusion of the professional development sessions) may be attributed to participants articulating clearer beliefs about the obstacles which impact the use of history of mathematics.

Impact of Background Variables on Professional Development Engagement

Three background features appeared to impact the level of each teacher's participation during the professional development, including number of years teaching experience, recent experience with professional development activities, and identification of the obstacles present in their school context.

Teaching experience. The two teachers with the most and least number of years of teaching experience, Mandy and Sue, respectively, exhibited the strongest engagement during the professional development sessions at their school sites. Mandy eagerly participated throughout the three professional development sessions at High Acres School. Sue's engagement, although not as extensive as Mandy's, was the most enthusiastic of the Mulberry participants. I conjectured that Mandy's eager engagement may have been a factor of her desire to pursue professional learning experiences which she viewed as necessary for her effectiveness in the classroom. Her eager participation in this study (and the accompanying professional development) was the result of her desire to seek an alternative approach to teaching logarithms, which was better aligned with her school and teaching philosophy. Sue expressed the same desire to teach logarithms in a way so that students would understand their motivation and development (Interview, 4/15/05). Sue, still young in her teaching career, exhibited a strong record of pursuing professional learning to enhance her instructional practice. The quality of her participation during this study may have been impacted by her ambition to pursue professional learning opportunities early in her career.

Professional development activities. The strength of participation in professional development activities during the previous two school years was positively related to Sue

and Mandy's engagement during the history of logarithms professional development sessions. Sue participated in nine of the twelve activities listed and served in two leadership capacities at Mulberry High School (mathematics department chair and Algebra I lead teacher). Similarly, Mandy reported participating in seven of the twelve professional development activities during the two previous years. She also served as a mathematics consultant for various mathematics education endeavors. In addition to reporting similar professional development profiles, Sue and Mandy each participated in activities which neither Ted nor Shirley indicated, including:

Conference or professional meetings;

Committee or task force work; and

Individual or collaborative research.

A limitation of the professional development participation data collected for this study was that I did not ask for additional qualitative information concerning duration of activity, type of participation, or the number of each type of activity reported. Further information regarding the number of workshops, training sessions, or institutes Ted attended may have changed his participation profile considerably. With additional prompting, for example, Ted may have reported attending 15 content or pedagogy workshops in a two-year time period in comparison to Sue attending just one on classroom management.

Identification of obstacles. Lastly, the obstacles Mandy, Sue, Ted, and Shirley each perceived about their teaching context were related to the intensity of their participation during the professional development sessions. Each teacher identified barriers that could hinder the use of particular innovations in teaching. The significant

difference among the participants existed in how they looked beyond the barriers to consider the use of the history of logarithms. The teachers who also articulated positive attributes of using the history of mathematics with students during the professional development sessions evidenced greater participation during the sessions.

Mandy and Sue each identified time as an obstacle to including the history of logarithms in their teaching albeit from different perspectives. Mandy's reasons arose from the awkward class meeting times, which were sometimes affected by events beyond her control. Mandy's teaching context, however, supported her efforts to incorporate alternative teaching perspectives. At High Acres School, Mandy did not have to deal with a strictly prescribed curriculum and state accountability assessments. Additionally, the mission of High Acres School included a liberal arts focus which embraced an historical perspective. Sue, on the other hand, identified lack of time available to include the history of logarithms because of a restrictive curriculum and testing demands.

Regardless of reason, Mandy and Sue also believed that including the history of mathematics was worth doing and made the essential accommodations. Of the four participants considered in this chapter, Mandy and Sue were the most engaged during the professional development sessions focused on the study of the historical development of logarithms. Each of these two teachers exhibited a strong capacity to anticipate student engagement and to share pedagogical plans for the use of the materials. Time did not adversely affect their participation, except for lacking sufficient time to study the lesson installments as needed prior to each professional development session. Even so, Mandy and Sue were the only two participants who worked on the lesson installments in between professional development sessions. In Mandy's case, she worked on the installments

individually and then shared her questions, discoveries, and comments with me during our sessions together. Sue preferred working with others on the lesson installments. At the beginning of the professional development experience, Mary and Sue discussed the lesson installments. After Mary ceased review of the lessons, Sue requested to work on the lesson installments with me.

In addition to recognizing the barrier of lack of time, Sue also identified the obstacles of student ability and interest. Although some aspects of Sue's perception of student ability were negative (e.g., "they can't answer what kind of sequence it is"), others were positively oriented. Thus, by considering positive outcomes of using an historical approach in teaching, Sue's professional development engagement was also positively impacted.

In contrast, Ted observed that his instructional time, as well as his planning time, could not accommodate the inclusion of the history of mathematics. Ted was unable to complete or review all of the lesson installments between professional development sessions or prior to his instruction of Chapter 11. The reason he provided was lack of time. Ted's conclusion that incorporating the history of mathematics would take time that he did not have may have impacted his professional development participation in a similar fashion. For example, most of Ted's contributions during the professional development sessions focused on Lesson Installment 1 and Lesson Installment 4, the first installments to be covered during the two sessions. Ted admitted to preparing for his work day (in this case, for December 17, 2004 and January 3, 2005) for 30 minutes at the beginning of the day. Consequently, the contributions Ted offered during the professional development sessions were focused on these two lesson installments.

Ted was confident of his knowledge of how students would respond to classroom activities. He identified lack of student interest as an instructional obstacle which also limited his professional development engagement. Ted believed that, “if you do too much with the students, you start losing them” (Professional Development Session 1, 12/17/04). Ted appeared to use the professional development sessions to evaluate what to use with students and as a result, much of Ted’s engagement remained focused on what “few things” he could include to “put a face on it [the development of logarithms]” (Professional Development Session 2, 12/17/04).

Similar to Ted, Shirley noted student apathy and lack of class time as barriers to including the history of mathematics in her instruction. Shirley also noted that insufficient time prevented her from further study of the historical logarithms as she approached Chapter 11. The lack of class time exerted the strongest influence on Shirley’s instructional choices, however. During one interview, Shirley directed my attention to a display of Trigonometry projects which her students completed during the first semester. Shirley provided me a copy of the history of mathematics project which she designed after participating in the professional development sessions. She admitted that on several occasions, her students had inquired about when they would be assigned their next project. The barrier of time proved so influential in the case of Shirley that she was unable to incorporate the project she designed, even when student interest was present. The lack of time and student interest which Shirley noted numerous times may have been the overarching features which prevented her from participating in a substantial manner during the professional development sessions. Shirley’s case may have been an example of a teacher believing that they could not possibly implement such

an approach in their teaching, thus any significant effort during the professional development experience was not possible.

Impact on Knowledge and Instruction

Each teacher volunteered to participate in this study because they were interested in investigating an alternative approach to teaching logarithms in a traditional Precalculus-type course. Examination of the participants' performance on the Content Knowledge Pre-assessment revealed that each possessed strong procedural knowledge related to using properties of logarithms to evaluate expressions and solve simple logarithmic equations. The Content Knowledge test performance provided less information about the participants' conceptual understanding of what a *logarithm* is and how *logarithms* differ from *logarithmic functions*. Improved performance on the eight content knowledge items appeared related to the participant's level of engagement during the professional development sessions and the intensity of the use of the historical materials during instruction.

Impact of Professional Development on Knowledge

Mandy and Sue, the teachers who exhibited the strongest engagement during the professional development sessions and incorporated the most content focused on the historical development of logarithms, out-performed the other teachers on the Content Knowledge Post-assessment. Mandy improved her pre-assessment performance of correctly answering four out of eight items (three traditional and one historical) to correctly answering seven out of eight items on the post-assessment (four traditional and three historical). Furthermore, Mandy correctly defined *logarithm* on the post-assessment, which was an improvement of her previous performance. Sue improved her

pre-assessment performance of correctly responding to three traditionally-oriented items to correctly answering six out of the eight post-test items (three traditional and three historical). However, Sue was unable to correctly define *logarithm* on each of the three Content Knowledge assessment administrations.

Ted was moderately engaged during the professional development sessions and incorporated minor biographical and historical information on four occasions during his instruction of Chapter 11. His performance on the final Content Knowledge Post-assessment, however, was not as strong as either the pre- or first post-assessment (administered at the end of the professional development). On the final post-assessment, Ted was unable to correctly answer two historically-oriented items; one of which he answered correctly on both of the prior assessments and one which he answered correctly on the first post-assessment. However, Ted was the only Mulberry participant correctly defined *logarithm* from a traditional perspective.

Shirley's participation during the professional development sessions at Mulberry High School was very limited. Shirley also chose not to incorporate the historical development of logarithms during her instruction of Chapter 11. Her performance on the Content Knowledge Pre-assessment and final post-assessment was exactly the same. Shirley correctly answered the three traditional items and made no attempt on any of the other items.

Impact of Background Variables on Instruction

Shuell (1996) outlined five dimensions of classroom environments and teachers which "have implications for the way in which instruction is delivered and the effects that

teachers have on student learning” (p. 729). The five dimensions, which were originally proposed by Anderson in 1989, included:

- (1) The academic goals of schooling;
- (2) Perceptions of the teacher’s instructional roles;
- (3) Students’ roles in promoting their own learning;
- (4) The nature of academic tasks; and
- (5) The social environment as the context for individual learning. (as cited in Shuell, p. 729)

The background variables most influential on the participants’ instructional practice for incorporating the history of logarithms were the beliefs related to the roles of the teacher and the student in the classroom and the context of each of the schools. The contextual setting of each school (High Acres and Mulberry) was more closely related to Anderson’s dimension of academic goals.

Mandy held the belief that she served as a resource for students and that students should take ownership of their learning. These beliefs were compatible with her desire and ability to incorporate the history of mathematics in her teaching. The private school context of High Acres also afforded Mandy the flexibility and support to modify her typical instruction during the unit containing exponential and logarithmic functions. In addition, Mandy’s commitment to continue her professional learning in order to better serve as a resource for students was evident in the range of activities that she participated in during the two years prior to the study and by her eager engagement during the professional development sessions focused on studying and using the history of logarithms.

Sue, Ted, and Shirley each contended with constrictive features of the public school setting. Each teacher easily identified obstacles which would prevent or hinder incorporating an historical approach to teaching mathematical topics, such as the rigidity of curriculum, the pressure of end-of-course testing, and the time required to complete instruction as required. Also, each Mulberry participant articulated essentially the same beliefs with respect to the role of the teacher and the role of the student. For instance, Ted delineated the two roles as the teacher teaches and the student engages in the learning activities provided by the teacher. He also added that it was the teacher's job to "motivate; encourage; convince the student to want to do the things they need to do" (Interview, 4/15/05). Shirley maintained that her role as a mathematics teacher was to convey information and that the students were to absorb what she told them, but then to "take it [and] make it meaningful" (Interview, 4/14/05). In contrast to Ted's belief that who should motivate and encourage students, Shirley's beliefs concentrated on providing students with the information and practice they needed to master skills.

Sue also characterized students as sponges. Although Shirley expected students to make the information they absorbed meaningful, she did not offer any concrete examples of how that would occur other than through the act of sitting in class, listening to lectures, and completing assignments. While Sue indicated that she expected her students "to be sponges," she also expected them to "study more and ask questions" (Interview, 10/21/05) if the information they tried to absorb failed to stay with them. The subtle difference in how Sue allowed for student control over the knowledge they possessed may have enabled her to incorporate an alternative approach to providing information and learning opportunities for students.

Impact of Professional Development on Instruction

The cliché, “If you build it, they will come” resonated with me as I worked with the participants in studying the historical development of logarithms and investigating what the use of such an approach would look like in the classroom.

In order to “build” the potential for using an historical approach to teaching mathematics, it was necessary for the participants to express a concern for the problematic nature of teaching logarithms in a Precalculus-type course. Especially in the case of the Mulberry teachers, the majority of students taking the course, Trigonometry and Advanced Algebra (Mulberry’s Precalculus-type course), did not continue on to take a calculus course. Thus, students often found the topic of logarithms “difficult and useless” (S. Moe, personal communication, 12/15/04). Additionally, students expressed a dislike for the topic of logarithms because their teachers could not tell them why they were required to study them (S. Moe, personal communication, 12/15/04). Consequently, the desire on the part of the teachers to investigate an alternative approach for including the study of logarithms prompted them to volunteer to participate in this study. For the most part, each of the Mulberry teachers participated for reasons similar to those expressed by Sue. Additionally, Mandy wanted to study and implement an historical approach to logarithms because such an approach was compatible with her interests, her students’ interests, and the mission of High Acres School (M. Wilson, personal communication, 10/15/04).

In addition to addressing the interests and concerns of the participants, other influential aspects of the professional development component of this study included providing access to the historical materials and an ongoing, concentrated program of

study during which participants could engage with colleagues. The professional development sessions focused on the study and use of materials in the form of lesson installments and supporting resources. Mandy, Sue, and Ted, who used the professional development sessions to critique, discuss, and verbalize plans for the use of an historical approach to teaching logarithms, also incorporated the history of logarithms in their instruction. In the cases of Mandy and Sue, their professional development engagement included numerous instances of anticipating student engagement and discussing plans for using the historical materials and resources.

Comparing both their instructional practice with the plans they articulated during the professional development sessions revealed that Mandy and Sue each implemented an historical approach to teaching logarithms in the manner they anticipated. Lack of adequate class time served as an obstacle in each instance. Both Mandy and Sue were unable to complete their plans as desired. As an example, Sue wanted to use the content from Lesson Installment 3 during her instruction of sequences in Chapter 12 because she felt the connections linking sequences and the purely computational aspect of logarithms would enrich the content of the Trigonometry course. Sue was unable to use Lesson Installment 3 as she planned.

To a lesser degree, Ted indicated during the professional development sessions that he would only briefly incorporate historical or biographical information into his instruction. Ted did introduce his students to the history of logarithms and only using the brief exposure (using dates, people, and the nature of their work) that he anticipated.

Mandy, Sue, Ted, and Shirley participated in this study to investigate the use of an historical approach to logarithms. After participating during the professional

development sessions designed to facilitate their individual and group study of the history of logarithms, each decided whether to incorporate aspects of an historical approach. The strength of their professional development engagement ranged from eager to limited. Furthermore, this engagement was directly related to the intensity of the use of the historical development of logarithms in their instruction.

Implications, Limitations, and Recommendations

Conducting an investigation of five teachers' experiences with the history of logarithms has reminded me that working with teachers and considering ways to incorporate the history of mathematics is simultaneously challenging and rewarding. Working with Mandy, Sue, Ted, Shirley, and Mary also reminded me that the numerous forces acting on the daily life of teachers are powerful and can serve as strong indicators of how teachers will choose to incorporate different teaching approaches.

As a researcher, I learned that interest alone in a topic, approach, or professional learning endeavor is insufficient for teachers to fully participate and learn from professional development activities. In addition, teachers must commit to significant engagement in ongoing learning which lives beyond the professional development experience. Such a commitment is vital if teachers plan to incorporate innovative approaches, in light of the many obstacles which serve to derail alternative teaching perspectives.

The following discussion summarizes what I have learned from participating in and studying the experiences of five teachers as they engaged in and used (or chose not to use) the historical development of logarithms.

Implications for the Professional Development of Teachers

Professional development which aims to provide teachers with learning experiences that are both content-specific and which introduce pedagogical approaches or instructional perspectives is strengthened when the structural and core features identified by Garet et al. (2001) are considered. The professional development used in this study was designed with the intent to include the essential features of form, duration, participation, content, active learning, and coherence. After conducting two iterations of professional development and reflecting on the relationship between the professional development sessions and the participants' experiences with incorporating the history of logarithms, it was informative to revisit the six features. The following sub-sections suggest improvements based upon what actually occurred in the professional development sessions with respect to form, duration, participation, content, active learning, and coherence.

Form. Each of the professional development sessions was conducted in seminar style, in which discussion among participants was more prevalent than providing information via lecture. Presenting the two particle argument (Lesson Installment 2) was the exception to this because of the mathematical language and content used in Napier's original description. In many ways, the form of the professional development sessions was the strongest feature to which I attended in the initial design of the sessions. There are two improvements which would strengthen the seminar format, however.

The first, related to *duration* (below), is to incorporate more sessions so that during the time between individual sessions participants would have the opportunity to study and reflect on the historical material. It was necessary to cover multiple lesson

installments or resource materials during each of the professional development sessions held at High Acres School and Mulberry High School. The demands on the participants' time required me to modify the original plan of eight sessions at each site to either two (Mulberry High School) or three (High Acres School) sessions only. As a result, the amount of material covered per session, as well as the amount of material reviewed for the next session may have been too overwhelming for participants. Consequently, participants' choice of what to review before the next session and ultimately what to consider for use during instruction may have been impacted by the number of professional development sessions.

Especially in the case of the Mulberry professional development sessions, teachers may have viewed the professional development as more workshop-like based on their previous experiences. As such, they may have regarded the materials as "ready for use" in their classrooms and continued to participate during the sessions without the intention of engaging with the materials prior to a seminar discussion. This perception may have also detracted from further review the content and limited the participant's consideration of potential uses of the materials within their classroom context.

Unfortunately, suggesting that more professional development sessions (each approximately one hour in length) is also problematic. I needed to reduce the number of times to meet with the Mulberry teachers because their schedule would not allow multiple sessions. They were willing to meet for longer time periods, but for fewer sessions, either because of actual time constraints or because of their inability to engage in a long-term professional development experience. The success of professional learning programs for practicing teachers depends upon not only incorporating the structural and

core features necessary for effective experiences, but also shifts in teachers' attitudes and beliefs towards continuing their professional learning in significant ways. Such shifts require reconceptualizing the instructional day to include professional learning opportunities as part of teachers' daily lives.

The second improvement is related to the specific content of one of the lesson installments. To maintain the seminar format throughout the professional development sessions, I would present Lesson Installment 2, a review of Napier's two particle argument in the same way as the other lesson installments. Instead of providing teachers a brief description of the argument prior to presenting it, I would ask teachers to read a translation of Napier's argument (e.g., Wright, 1616) along with a mathematical interpretation of the argument. Then, during a seminar focused on Lesson Installment 2, I would ask teachers to discuss the relationship of Napier's original argument to the concept of logarithm as presented to students in traditional textbooks. Motivating a seminar discussion from this perspective may have also impact teachers' understanding of the definition of *logarithm* (as opposed to the definition of *logarithmic function*), which was flawed among several of the participants of this study.

Duration. Garet et al. (2001) discussed the wide range of contact hours reported by the professional development programs they studied. The amount of time spent on professional learning activities and the fact that they extend over time are both influential factors in effective professional development practices. I believe that for topics and perspectives (history of mathematics) which are less familiar, it is essential to design opportunities which provide sufficient time enough in time per session to allow for "in-depth study [and] interaction" (Garet et al., p. 922). Equally important, the professional

development effort must span a long enough period to provide adequate time for reflection (Garet et al., p. 922). The professional development component of this study may have impacted instructional practice differently if teachers would have experienced sustained engagement with the content and historical perspective.

Participation. Two aspects of the participation dynamic would have improved the professional development experience for teachers. First, it would have been more ideal for the sessions at High Acres School to include additional participants. Although Mandy remained actively engaged during each professional development session, research indicates the advantages of collective participation, including “the opportunity to discuss concepts, skills, and problems that arise during their professional development experiences” (Garet et al., 2001, p. 922). For example, if the same topic of focus was used (historical development of logarithms), I would include the Algebra II teachers from both schools since each indicated that the topic of logarithms was included in that course as well. The additional participation would incorporate another dimension to the discussion of the potential use of the history of logarithms.

Content focus. Garet et al. (2001) identified at least four dimensions along which content covered during professional development may vary. The dimensions include:

- Emphasis given to the subject matter that teachers are expected to teach;
- Specificity of the changes in teaching practice;
- Goals for student learning; and
- Emphasis on the ways students learn particular subject matter. (p. 923)

Garet et al., (2001) reported that, “the degree of content focus [is] a central dimension of high quality professional development” (p. 925). In this study, a greater

emphasis was placed on the first two dimensions. However, a focus on suggesting a change in teaching practice (i.e., using an historical perspective) also implied an interest in addressing the difficulties students experience in learning and retaining understanding of logarithms. In the future, professional development efforts focused on using an historical perspective in teaching should also include careful attention to how teachers perceive student mastery of particular mathematics topics. Additional emphases to this end should include review and analysis of student work.

Shirley observed that she was disappointed that students did not possess basic conceptual understanding at the end of their study of exponential and logarithmic functions. Shirley's instruction was consistently delivered in a 'lecture, examples, practice' mode, yet did not result in the results she desired. By including review and analysis of student work in the professional development experience, participants could begin to examine critical questions, such as, what is it about the 'lecture, examples, practice' method of instruction that does not work? Also, a goal of the professional development could include an investigation of alternative ways to connect with students and address content at the same time.

Active learning. In addition to including review and analysis of student work, the professional development component would have been enhanced by providing the participants the opportunity to "observe expert teachers and to be observed teaching" (Garet et al., 2001, p. 925).

The necessary aspects of incorporating an historical perspective and historical materials include belief in the worth of such an endeavor and the action required to make it happen. Each participant either strongly or moderately agreed that including history

enriches the teaching and learning of mathematics (Attitudes Instrument Post-assessment), indicating that the belief was present. Access to action during the professional development sessions was not prevalent among the Mulberry High School participants. The Mulberry participants were only able to share their prospective plans of how to incorporate the history of logarithms. By extending the professional development effort to take place throughout their instruction of Chapter 11, Sue, Ted and Shirley may have benefited from observations of each others' practices.

For example, when compared to Ted and Shirley, Sue was able to incorporate considerable material from the history of logarithms. Sue was challenged by the same curriculum, testing, student, and time constraints that Shirley and Ted were, yet she was still able to incorporate an historical perspective and complete the curriculum as prescribed. By observing Sue's practice, Shirley may have realized that there were conditions already present which would allow for the inclusion of the history of mathematics beyond just "sound bytes" (Interview, 4/14/05). Similarly, Ted may have observed that using history of mathematics with all students (as Sue did) rather than with just "some folks" (Interview, 4/14/05) was worthwhile and that many students would rise to the challenge of engaging with the history of mathematics. In addition to observing how others incorporate the history of mathematics, the participants would also have the opportunity to provide feedback on how to improve future use of historical material.

Coherence. Lastly, Garet et al. (2001) identified the importance of "the extent to which professional development activities are perceived by teachers to be a part of a coherent program of teacher learning" (p. 927). Garet et al. identified the following three dimensions as contributing to such a coherent program:

- Connections with goals and other professional learning activities;
- Alignment with state and district standards and assessments; and
- Communication with others. (pp. 927 – 928)

The lack of clear connection with these dimensions of coherence in the professional development provided in this study was the feature most in need of improvement, but which requires attention to school and district features. In the case of Mandy, High Acres School supported her professional learning both internally and externally. If Mandy needed to attend a conference, institute, or workshop because of the learning the event afforded, she was supported in doing so. She was also expected to share her knowledge with other High Acres mathematics teachers and did so. This aspect of Mandy's professional development experience was prevalent during our sessions together. In many instances, Mandy would contribute by sharing knowledge and techniques with me (e.g., "you've got to name the animal") as she considered me part of her "network...involved in change" (Garet et al., 2001, p. 928).

In the case of the Mulberry High School teachers, however, I was unable to attend to the dimension of "connect[ing] with goals and other professional learning activities." Each of the Mulberry participants possessed different goals for professional learning and the district did not provide a structured professional development plan. Instead, the only professional development activities teachers needed to participate in were those which would assist them in recertification. Thus, the professional learning culture at Mulberry was individual and isolated. The individuality was also revealed in how Sue, Ted, Shirley, and Mary addressed the historical materials between professional development sessions. Sue wanted to work on the lesson installments with others. She was unable to

do so (beyond Lesson Installment 1) because the other Mulberry participants lacked the time and commitment to do so. Ted and Shirley's lack of experience with professional development activities which emphasized connections with other activities and communicating with a network of engaged teachers may have contributed to their level of engagement in this study.

The final dimension of coherence was the most difficult to consider for improvement. Mandy was not constrained by assessments in the way that the Mulberry teachers were. She was, however, committed to creating a Precalculus course which would best serve her students. To create such a course, Mandy consulted with mathematics department faculty and considered the future academic needs of students, whether they would continue on with Applied Calculus, Advanced Placement Calculus, Advanced Placement Statistics, or Statistics. In addition, the mission of High Acres School heavily influenced Mandy's pursuit of professional development activities. The curricular program of High Acres School was focused on providing rigorous, liberal arts content and Mandy sought professional learning opportunities which would provide content focused in a similar manner.

The professional development sessions were originally designed with the content of a typical Precalculus course in mind; a course which included a study of logarithms. The content of the professional development sessions was focused on the historical development of logarithms. Although the Mulberry High School end-of-course test did not contain material from the history of logarithms, it did include problems from the study of exponential and logarithmic functions. Designing a professional development which focused on the history of a topic covered in a state, district, and school curriculum

(and assessments) was not sufficient, however. The Mulberry teachers were overwhelmed by the scope of the Precalculus curriculum and the consideration of alternative teaching practices was not a strong enough dimension to provide a coherent learning opportunity for them. In this respect, the influence of school features and obstacles to reforming instructional practice at Mulberry High School outweighed the effort to design a coherent professional development experience for teachers. In future work, it will be necessary to work with teachers on how to actively reflect upon the prescribed curriculum and alternative teaching approaches during the professional development experience.

Implications for Using the History of Mathematics

In addition to identifying the features of effective professional development, it is also appropriate to recall the recommendations Barbin (2000) discussed with respect to “pursuing an investigation on the effectiveness of history in the classroom” (p. 90).

Barbin and others observed that we should:

- Collect experiences of teachers who use history, including their aims, steps, problems that arise in teaching;
- Present the advantages and disadvantages they report; and
- Collect questionnaires and conduct interviews of teachers and students about their study of mathematics. (p. 90)

This study was focused on the experiences of teachers and thus, no interviews were conducted with students. The study was designed, however, to address each of the other recommendations proposed by Barbin. The following implications for the use of the history of mathematics result from listening to the experiences of the five teacher participants.

Using history to facilitate connections. All of the participants reported that including information about mathematicians in their instruction would provide a “human interest” dimension to teaching mathematics. Two participants, Mandy and Sue, went beyond including historical and biographical information. In order for Mandy and Sue to incorporate the history of logarithms, it was mandatory for the use of history to provide connections for students.

For Mandy, the connections were related to personal values, mathematical topics, and other academic skills. Mandy and her students connected the life and work of John Napier to Catholicism. Mandy chose to modify her instruction so that sequences and series were studied concurrently with the historical development of logarithms. In this way, students had access to the mathematics they needed to understand Napier’s original definition of logarithms. Lastly, Mandy required her students to research the history of logarithms and present an aspect of their development in a formal paper, using the same guidelines required for their composition courses.

Sue also articulated the importance of using the history of mathematics to provide connections between topics and courses. She generated interest in researching the development of a topic by drawing on students’ talents in history. Sue also strategically incorporated Lesson Installments 4 and 5 so that their placement would review the properties of logarithms (Lesson Installment 4) and trigonometric identities (Lesson Installment 5). Lastly, Sue extended her study of the historical development of logarithms to research the history of the number e in order to provide her students with a meaningful connection to relate exponential and logarithmic functions.

Including history should begin early. Sue reported that if she “had been doing these types of lesson all year, it would have been easier to use the lessons without meeting so much resistance” (Interview, 4/15/05). Understandably, Sue’s students showed some resistance to engaging in the historical activities she presented to them in late March and early April. The material represented a different form of mathematical activity than what the students were used to, which included more of a ‘lecture, work examples, practice problems’ style. Sue also noted that even though some students had difficulty with the “newness” of studying the history of mathematics, she continued with her plans to incorporate it and told them, “how they would not have anything they have without the history of mathematics and that they should at least be grateful, if nothing else” (Interview, 4/15/05).

Mandy’s experience also supported the notion that the use of the history of mathematics should begin early in a course. Mandy described the activity she used which focused on the history of the number system and which students worked on for the first six weeks of school. The activity enabled students to discuss the evolution of the number system, including an examination of the concrete operations and the abstract concepts pertaining to each. Mandy claimed that the students’ experience with this in-depth historical study made it possible for her to incorporate the historical development of logarithms.

Using history should go beyond the anecdotal. Siu (1997) and others identified the use of brief anecdotes as one way to incorporate the history of mathematics in the classroom. Incorporating anecdotal information only, as in the case of Ted, appeared to have less impact than incorporating original documents (Mandy), student research

(Mandy and Sue), or historical problems (Mandy and Sue). In one particular instance, Ted used a brief vignette about the life of John Napier prior to discussing common logarithms. At the end of class, several students asked Ted if he wanted the copy of the vignette back. Despite his answer of “no,” several students returned their copy to Ted. The students did not attach significance to the use of the vignette, as it was disconnected from the work required of the students for the rest of the period (and chapter).

Using historical materials should include teacher construction. As previously discussed, interest in a different approach to teaching logarithms was not sufficient for ensuring professional development engagement or instructional use of the history of logarithms. On several occasions, the participants intimated that the form of the lesson installments would prevent their use. For example, during our discussion of Lesson Installment 3, Sue commented that,

They’re going to look at this and ask, ‘why are there so many words on it’? Because that’s what they always skip – the word problems. I’m just saying, I don’t know when it would be better...to say, okay, do this page this day, then this, and this....(Professional Development Session 1, 12/17/04)

Sue’s concern about how to use the lesson installments was valid. Providing access to historical materials such as those found in *Historical Modules* (Katz & Michalowicz, 2004) or modified versions such as the ones used in this study is a major hurdle which needs to be addressed. However, as professional developers and creators of historical materials, we must also accommodate teachers endeavoring to use historical materials. The lesson installments provided appropriate content to launch discussions about the aspects of the historical development of logarithms. However, an alternate use of the lesson installments may be more appropriate.

One alternative to providing teachers with lessons to study and discuss would entail working with teachers to create their own lessons from studying historical content. Teacher participants would be presented with the important content in the form of a reader. After each installment in the reader, participants would then work together to construct lessons to use with students. The professional development effort would involve increased teacher engagement. In addition, the lessons created may have a greater chance of being used because teachers would create them for their particular students. As stated previously, however, lack of sufficient time to engage in this form of professional learning is a reality. Whereas I recognize how difficult it is to conduct the work I am proposing, I am unable to offer solutions.

Additional Recommendations and Future Research Direction

In this final section of the chapter, I make three recommendations for future research focused on potential efforts to incorporate the history of mathematics in both the professional learning of teachers and their classroom teaching.

History-friendly Textbooks

Marshall (2000) discussed his effort of contacting numerous publishers about the extent to which they included historical content in high school textbooks. Mandy and Sue each identified concern with the lack of historical content in the textbook used in the Precalculus (Mandy) or Trigonometry (Sue) textbook. Mandy's concerns focused on her displeasure with the decontextualized treatment of logarithms. Sue's concerns focused on the lack of support the course textbook provided when she considered the amount of work involved in taking the content of the textbook and creating opportunities for

incorporating the history of mathematics. In this regard, the Mulberry teachers' curriculum was primarily driven by the content of the *Advanced Mathematical Concepts* textbook. For example, Sue, Ted, and Shirley were unable to consider alternative teaching approaches, such as teaching sequences and series (Chapter 12) before exponentials and logarithms (Chapter 11), based upon the order of topics presented in the text.

Traditional textbook content is often developed based upon the need to address the content standards of multiple states simultaneously. To suggest additional textbook content focused on the history of mathematics runs the risk of including much of the same current treatment of historical information. Such a treatment presents historical content on a limited number of topics and at the surface only. As an alternative, I propose examining existing textbooks for the purpose of creating "historical content guides." This idea was precipitated by Mary's observation that if student textbooks included more historical information than is currently available, that teachers may be inclined to incorporate more history into their teaching. The "historical guides" would provide teachers with an outline for courses of study and would direct teachers to appropriate historical materials and approaches, such as the *Historical Modules* (Katz & Michalowicz, 2004), original sources, and other classroom materials.

One obstacle in providing teachers with access to historical materials to use in the teaching of mathematics is clarifying exactly what materials are available. An "historical guides" project would not only provide such information, but would seek to match appropriate materials with corresponding mathematical topics. The *Historical Modules* (Katz & Michalowicz, 2004) already include one level of this information, in reverse. For

example, in the *Logarithms and Exponentials Module*, each course in which an activity could be used is identified for teachers at the beginning of the module.

Topical Focus for Future Professional Development

An additional response to the call for focusing professional development efforts on content (Garet et al., 2001) is the involvement of teachers in the choice of the content focus. In this study, the focus on the historical development of logarithms was predetermined for the teachers who volunteered. A proposal for increasing teacher engagement with the history of mathematics is to allow teachers to direct the focus of the content.

An alternative to designing professional development focused a given topic would include asking teachers to identify some number of topics that they deem difficult to teach (i.e., teachers are frustrated with a traditional approach; the topic appears disconnected from a particular course) and which could be enhanced by using an historical approach. Then working with teachers, individuals (professional developers) with expertise in the history of mathematics would create a program of study of the collection of topics for use with students. In addition to becoming more expert in the history of the topics they select, teachers would also help create classroom activities, identify historical resources, and discuss curricular modifications necessary to enact an historical approach surrounding the topics chosen.

Studying Professional Relationships in Schools

In addition to creating classroom resources and learning opportunities for teachers, it is also necessary to continue to study the contexts in which teachers continue their professional learning. School environments continue to represent “intellectual

isolation” (Lortie, 1975, p. 70). Pomson (2005) suggested that teacher isolationism is either

[A]n adaptive strategy in environments where the resources required to meet instructional demands are in short supply or an ecological condition, encouraged by workplace settings where physical isolation is pervasive. (p. 787)

In either case, until the structure of a typical instructional day is transformed to facilitate the professional education of teachers to occur in what Smith (2001) terms “in practice,” researchers and professional developers should concentrate efforts on examining “how (and why) teachers’ investment in community can fluctuate among cooperation, collegiality, and collaboration” (Pomson, 2005, p. 787). Such an examination will begin to inform future professional development structures and how each form of social interaction capital (cooperation, collegiality, and collaboration) can be utilized to create opportunities for sustained and cohesive (Garet et al., 2001; Smith, 2001) professional development efforts.

Appendix A: My Personal Experiences with the History of Mathematics

In 1997 I was given the opportunity to recreate my school's History of Mathematics course, which had been reduced to a "write a research paper about your favorite mathematician" course over the years prior to my arrival. I was excited about the chance to do so and set to work to outline a viable curriculum. I knew that one course – almost ten years prior – on the history of mathematics would not provide the background that I needed to do such a course justice. I was able to create a course that had as its backbone a timeline that would guide the topics I wanted to cover. In addition to incorporating biographical information about mathematicians, I also included stories with a "human interest" quality that accentuated characteristics such as collaboration and overcoming obstacles. The remainder of my energies focused on choosing appropriate mathematical documents for my students to read and interpret and historical problems that they could discuss and (hopefully) solve. Interestingly enough, with respect to the problem solving, my students experienced the same frustration that I had experienced many years previous. They continued to want to solve problems using their modern techniques.

The first experience with teaching History of Mathematics did not end poorly, however. Eventually, as we addressed new topics, the students asked whether we were going to do mathematics like "they did in the old days" and their frustration evolved into interest and engagement. I began to feel that teaching the history of mathematics could impact my other courses in similar ways. About this time (Spring 1999), my office colleague mentioned an ad in the Mathematical Association of America *Focus* magazine asking for high school teachers to participate in a History of Mathematics Institute being

sponsored by Victor Katz and his group in Washington, DC. I applied and during the next two years (three summers) I participated in studying the history of particular mathematical topics, reviewing new materials (the *Historical Modules for the Teaching and Learning of Mathematics*) that were being developed with the secondary student in mind, and providing feedback and evaluations of the materials.

I worked most extensively with the two parts of the *Trigonometry* module, the *Functions* module, and the *Exponentials and Logarithms* modules during the three summers and two school years. I was teaching a History of Mathematics course each year, however, so I had the opportunity to use material from many of the eleven modules. During the time that I endeavored to create a History of Mathematics course that was meaningful for high school students, I found that incorporating historical activities and using an historical perspective became a powerful curriculum tool in my other courses. I believed I was experiencing what Russ (1991) called a “general and specific awareness of the historical origins of mathematics” (p. 7) and that this “second nature” was permeating and influencing my use of the history of mathematics with students.

During graduate school, I have remained interested in examining the use of history in the secondary classroom. I have, however, shifted my focus to investigating how others (teachers) experience, study, and subsequently choose to use the history of mathematics in the classroom. I chose the topic of logarithms because it is one of the topics that I have studied more in depth since becoming interested in the using the history of mathematics. In addition, the choice to focus on the historical development of logarithms resulted from logarithms proving to be problematic for my students. Their experience with logarithmic functions being defined as the inverse of exponential

functions was not too difficult to accept: merely another definition in their study of various functions. What was most difficult for them was gaining meaningful understanding about what exactly a *logarithm* is, and subsequently, how and why the mysterious properties worked. By infusing the topic of logarithms with their historical development – and relating their invention as a product of human need and invention – along with activities related to their development, I was able to provide a more successful experience for my students. A significant factor in the choice of logarithms as the topic to drive this investigation was my desire to introduce teachers to the same experience.

Appendix B: Lesson Installments

[Note: Lesson Installments 3 – 7 are adapted from Anderson et al. (2004).]

Lesson Installment 1

Who Are the Key Players in the Invention of Logarithms?

Introduction: The invention of logarithms (and later, logarithmic functions) required the ideas of several individuals collected and refined over many centuries. Although the definition and original idea of logarithms is attributed to one individual and occurred over a relatively short amount of time, it is also important to identify the contributions of others with respect to the development of logarithms.

Exploration: Using the list of names and list of “big ideas,” construct a timeline that tells the story of the invention of logarithms. Since not all material on the Internet is entirely accurate, you may want to use the web sites your teacher provides. (In either case, provide references for each item in your timeline.) In any case, any conflicting information that you locate should be noted. In addition, you may identify other mathematicians and “big ideas” to include in your timeline. As you construct your timeline, think about the following questions:

- What was the driving motivation behind the invention of logarithms? Was this surprising to you? In what ways (or why not)?
- What obstacles or challenges did the men involved in the invention of logarithms have to face?
- If logarithms had not been invented when they were, do you think it would have impacted future mathematical or scientific developments? If so, in what ways?
- If you have any prior knowledge of logarithms, did the research you conducted match that knowledge or did it raise new ideas for you? If it raised new ideas, what are they?

Individuals

Brahe
Briggs
Bürgi
Chuquet
Craig
Euler
Ibn Yunus
Kepler
Napier
Stifel
Viète
Vlacq
Werner

“Big Ideas”

Astronomy
Exponents
Prosthaphaeresis
Trigonometry

Share: After your individual research, we’ll regroup as a whole class to construct one common timeline to use as a reference during our study of logarithms.

Lesson Installment 2
Napier's Two Particle Argument
[Teacher Pages Only]

Installment 2 focuses on John Napier's original explanation of his invention of logarithms in which he employed a kinematic model (Calinger, 1999, p. 488). Using this model, Napier makes an argument that is the essence of comment 26 of *Mirifici logarithmorum canonis constructio* (The construction of the wonderful canon of logarithms, 1619):

The logarithm of a given sine is that number which has increased arithmetically with the same velocity throughout as that with which radius began to decrease geometrically, and in the same time as radius has decreased to the given sine. (Calinger, 1995, p. 285)

Thus, the goal of Installment 2 is for the teacher to present one version of Napier's geometric explanation of how he defined *logarithm*. The explanation is somewhat difficult, however if students are enrolled in a precalculus course, and have had some experience with physics or a physical science course, the explanation is engaging, quite beautiful, and not addressed in traditional mathematics textbook chapters on exponential and logarithmic functions. To understand the necessity of the use of sequences in Installment 3, a sample explanation found in Installment 2 follows.

Napier's Two Particle Explanation

In his kinematic model, Napier described the movement of two particles, one (point P) moving along a line segment of fixed distance (AZ) and another (point Q) moving along a line (or ray) of indefinite length ($A'Z'$). Also, Napier defined the line segment and the ray to be parallel to each other:

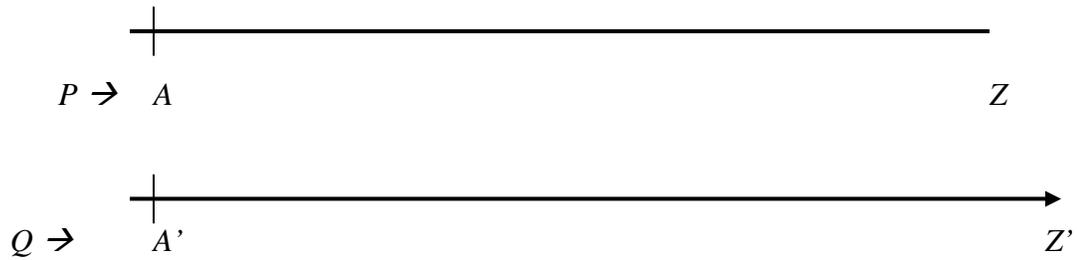


Figure 1. Two Particle Model

To define the movement of points P and Q , Napier established three rules. First, the points P and Q begin movement along their paths with the same initial velocity. Second, point Q keeps this velocity along its path. And lastly, point P 's velocity slows down in such a way that its velocity is proportional to the distance it has left to travel along AZ . Napier also defined the initial length of segment AZ to be equivalent to 10^7 units, since this was the value of the radius of the circles used to construct his tables of sines. By defining such a length, however, this meant that the initial velocities of points P and Q were also 10^7 , as well as point Q 's constant velocity.

Using the initial conditions Napier established, we can begin to describe subsequent movement along the segment and the ray, which will in turn provide the pair of sequences alluded to in comment 26 of Napier's *Constructio*. First, we can consider P and Q moving along their respective paths to the next position:

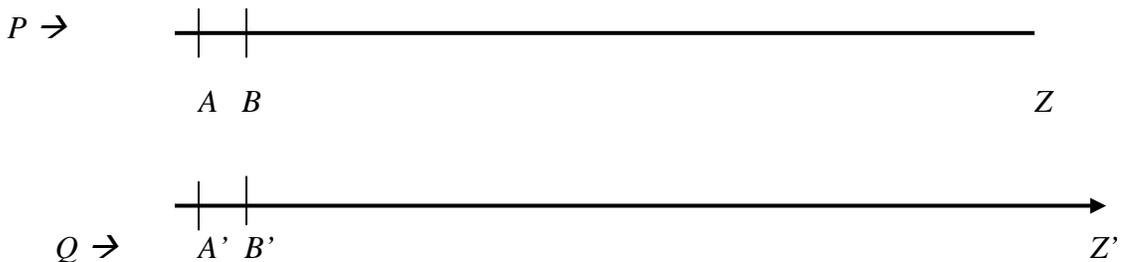


Figure 2. First Movement

Particle P 's velocity is diminishing at each point (Figure 2, at point B) in such a way that the velocity is "proportional to the distance remaining in the line's terminus point of Z " (Calinger, 1999, p. 488). A series of calculations will help to create the necessary sequences. (Units are omitted for convenience.)

First, we can calculate the increment of time used for each movement:

The initial velocity (v_1) of Q is 10^7 and the distance (d_1) $A'B'$ is defined to be 1 (Scott, 1969, p. 130). Thus, the time it takes to travel from A' to B' is found by: $t_1 = d_1/v_1$ or 10^{-7} .

Since this is a relatively short increment of time, the distance from A to B is also very close to one (1). If these initial calculations are used, along with the same interval of time (10^{-7}), then the geometric sequence corresponding to the remaining distance left to travel along segment AZ is found as follows:

$$BZ = 10^7 - 1 \text{ or } 10^7(1 - 10^{-7})$$

$BC = (\text{velocity at } B) \times (\text{time})$ [since the velocity at each point is proportional to the remaining distance along AZ]

$$BC = (10^7(1 - 10^{-7}))(10^{-7})$$

$$BC = (1 - 10^{-7})$$

Now, CZ will equal $AZ - AB - BC$, or $10^7 - 1 - (1 - 10^{-7})$. Simplifying, CZ equals $10^7(1 - 10^{-7})^2$.

Continuing this process yields the following geometric sequence corresponding to the remaining distance for particle P to travel along AZ :

$10^7(1 - 10^{-7})^0, 10^7(1 - 10^{-7})^1, 10^7(1 - 10^{-7})^2, 10^7(1 - 10^{-7})^3, \dots$ corresponds to AZ, BZ, CZ, DZ, \dots

The arithmetic sequence corresponding to how far Q has traveled on $A'Z'$, however is increasing, or $A'A'$ (Q has not moved yet), $A'B'$, $A'C'$, $A'D'$, ... is given by:

0, 1, 2, 3,

Finally, and numerically, Napier described his logarithms as the common ratio of the two sequences of numbers (Calinger, 1999, p. 487). Thus, in the example given in Figure 3, Y is the logarithm of X , or the logarithm of $10^7(1 - 10^{-7})^3$ would equal 3. In other notation, we would have:

$$A'D' = \log(DZ).$$

References

Calinger, R. (1995). *Classics of mathematics*. Englewood Cliffs, NJ: Prentice Hall.

Calinger, R. (1999). *A contextual history of mathematics*. Upper Saddle River, NJ: Prentice Hall.

Scott, J. F. (1969). *A history of mathematics*. London: Taylor & Francis.

Lesson Installment 3
The Development Of Logarithms Using Sequences
Part I: Relationships Between Two Sequences

Introduction/Transition:

In Lesson II, we read and discussed the original argument of “Napier’s great invention.” A descendent of John Napier, Mark Napier, provided a description of logarithms in his 1834 *Memoirs*. In that work he used sequences of base 10 in order “to adopt the series most easy to multiply into such a progression.” For example, Mark Napier provided the following:

| | | | | | | | |
|----|-----|------|-------|--------|---------|----------|-----------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1, | 10, | 100, | 1000, | 10000, | 100000, | 1000000, | 10000000. |

He noted (1834, p. 437):

It is a short-hand *exemplification* of the most convenient system of Logarithms; the ciphers stand in place of the *arithmetical* progression, 1, 2, 3, &c. as adapted to the *geometrical* progression, 1, 10, 1000, 1000, &c. and the whole is based upon the denary scale in use.

And, Mark Napier eventually provided an example of how to use the two progressions:

Now, 1000 multiplied by 10000 must give 10000000; for the numbers above the *factors* are 3 and 4, which added, give 7, which number points to the product sought, 10000000.

To investigate logarithms from the perspective of sequences, we’ll use Mark Napier’s idea using a base of 2.

Consider the two given sequences:

| | | | | | | | | | | | | | | | |
|-----------|---|---|---|----|----|----|-----|-----|-----|------|------|------|------|--|--|
| S1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | | |
| S2 | 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 | 1024 | 2048 | 4096 | 8192 | | |

Pre-work: What kind of a sequence is $S1$? (How do you know?) What kind of a sequence is $S2$? (How do you know?)

Exploration: Work with a partner on the following exercises. When appropriate, please use complete sentences for your responses. You may want to begin to think symbolically in order to organize your observations for Exercise 3.

1. Rewrite the numbers in sequence $S2$ as powers of a single number. How are the two sequences, $S1$ and $S2$, related?

2. a. Note that when two terms of $S2$ are multiplied (i.e., 64 and 128), their product, $(64)(128) = 8192$, is another term of $S2$. Pick a few pairs of numbers from $S2$ of your own choosing to multiply. Is the product always a term of $S2$ (for the examples you chose)? Do you think this is always true? Why or why not? [*Note:* You may have to extend the terms in each sequence to verify your observations.]

b. Note that when you multiplied two terms from $S2$ that there was a corresponding operation that describes what happened to the terms in $S1$. What is this operation? Check that this holds for the other pairs you chose in Exercise 2a.

3. In a brief paragraph, summarize the results of your observations thus far. If you are able, give a justification for your results using your knowledge of exponents.

You have just examined the relationship between two sequences when the product of two numbers in one of the sequences (S_2) is desired. Let's investigate what relationships hold when other operations are used...

4. Now take any two terms in S_2 and divide the smaller value into the larger. Record at least three cases. In each case, identify the corresponding numbers in S_1 of the two terms you chose, as well as the term (again, in S_1) corresponding to the quotient.

5. After trying several pairs of terms in S_2 , describe (in words) how you can divide any two terms in S_2 *without* using a calculator or doing any computations by hand.

6. Let's try one more operation: take any term in S_2 and raise it to a positive integer power. Record at least three cases. With each calculation, observe the corresponding numbers in S_1 of the terms you choose, as well as the term in S_1 of the resulting power calculation.

7. After examining several cases in Exercise 6, describe (in words) how you can raise any term in S_2 to a positive integer power *without* using a calculator or doing any computations by hand.

Wrap-up (For Class Discussion):

8. From the previous exercises, you have investigated how to multiply, divide, and raise to powers by *relating terms in sequences S_1 and S_2* in certain ways. Assume that you did not have access to a calculator or computer device. Describe how you would use these relationships to help you perform complex computations. Make your explanation as clear as possible, and feel free to draw upon your prior knowledge!

Lesson Installment 3
The Development Of Logarithms Using Sequences
Part II: Generalizing the Results

Introduction/Transition:

Following the path of all great mathematicians in history, we are now going to take the results from our specific examples in the previous handout, and work on making generalizations about the results of the operations within the two sequences $S1$ and $S2$. Recall from Lesson II that John Napier explained his results using a geometric argument involving two particles moving continuously along two parallel lines over time. Today, we will try to generalize (numerically) the association of a geometric and an arithmetic progression (sequence).

Pre-work: First, let's look at the two following sequences where $b > 0$ and $b \neq 1$ (complete the table):

| | | | | | | | | | | | | | | |
|-----------|-------|-------|-------|-------|---|---|---|---|---|--|--|--|--|-----|
| S3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | | | | | n |
| S4 | b^1 | b^2 | b^3 | b^4 | | | | | | | | | | |

We begin by defining a function, $L(x)$, which has as its domain the elements of $S4$. The “value” of the function $L(x)$ is its corresponding term in $S3$. For example:

$$L(b^4) = 4.$$

In general, what would be the “value” of $L(b^n)$? _____, (for n any positive integer).

Exploration: Work with a partner on each of the following. You should record several cases for each of the three properties you are confirming in Exercise 9. You will only need to verify each property once in Exercise 10. With your partner, decide *before* beginning Exercise 10 if you are going to work independently first and then compare results; or if you want to work together to decide on how to verify each.

9. Review your results from the Part I handout. Do you think the three results hold here? If so, use the *general terms* of $S3$ and $S4$ to check if the properties hold.

Product:

Quotient:

9, *continued.*

Power:

10. To verify your conclusions in Exercise 9, use the function notation below. Let u and v be terms of $S\mathcal{A}$. Use algebraic manipulation to show the following three properties hold:

i. $L(u) + L(v) = L(u \cdot v)$

ii. $L(u) - L(v) = L(u/v)$

iii. $L(u^k) = kL(u)$

Lesson Installment 3
The Development Of Logarithms Using Sequences
Part III: Extending the Results to Reach Napier's Invention

Introduction/Transition:

At the time of John Napier, it was perfectly acceptable to work with rational and negative numbers within the context of the calculations Napier was performing. Given this information, what is one way that we could extend each sequence for the purpose of verifying our properties to include these types of numbers?

Complete the table using your idea:

| | | | | | | | | | | | | | | | |
|-----------|-----|--|--|--|--|----|---|-------|-------|-------|-------|-------|--|--|-----|
| S3 | ... | | | | | -1 | 0 | 1 | 2 | 3 | 4 | 5 | | | ... |
| S4 | ... | | | | | | | b^1 | b^2 | b^3 | b^4 | b^5 | | | ... |

Exploration: With your partner, investigate the following:

11. Using the sequences above, investigate the validity of the general results found in Exercise 10 on page 5. (You may wish to do this using a few cases of pairs of terms from sequences *S3* and *S4*.)

12. To review notation from the study of exponents, what is an alternative notation to use for the first five terms of *S4* above? (Complete the table again using the second notation.)

| | | | | | | | | | | | | | | | |
|-----------|-----|--|--|--|--|----|---|-------|-------|-------|-------|-------|--|--|-----|
| S3 | ... | | | | | -1 | 0 | 1 | 2 | 3 | 4 | 5 | | | ... |
| S4 | ... | | | | | | | b^1 | b^2 | b^3 | b^4 | b^5 | | | ... |

Now let's return to our original sequences, $S1$ and $S2$:

| | | | | | | | | | | | | | |
|-----------|---|---|---|----|----|----|-----|-----|-----|------|------|------|------|
| S1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| S2 | 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 | 1024 | 2048 | 4096 | 8192 |

John Napier called the terms of the sequence $S1$ the “logarithm” of the corresponding terms of the sequence $S2$. To review, the word “logarithm” is a translation of the Greek word *logarithmos*, which comes from the Greek words meaning ratio and number; it therefore means “the number that counts the ratios.” In the context of his original geometric explanation, Napier called his logarithms “equidistant comparisons of proportional numbers.”

We can now determine exactly what “ratios” we are counting when we determine the logarithm of a number given our modern examination of sequences. As an example, Napier would have called **7** the logarithm of 128. In this case, **7** indicates how many ratios (as in the common ratio of the geometric sequence $S2$ is 2) are required for the term 128 to be found. (*Important historical note:* Again, Napier did not use base $b = 2$. In fact, he did not articulate the concept of base at all, but the idea is similar.) We now define the function developed in Part II as a logarithmic function for base $b = 2$.

If we replace our $L(x)$ notation from the last part of Installment 3 with the new notation “ \log_2 ” (read: “the logarithm to the base 2” since our sequence $S2$ uses base 2) we obtain the following results:

| Exponential Notation (from $S1$ and $S2$) | Previous Notation | New Notation |
|-----------------------------------------------|-----------------------|--------------------------|
| $1 = 2^0$ | $L(2^0) = \mathbf{0}$ | $\log_2(1) = \mathbf{0}$ |
| $2 = 2^1$ | $L(2^1) = \mathbf{1}$ | $\log_2(2) = \mathbf{1}$ |
| $4 = 2^2$ | $L(2^2) = \mathbf{2}$ | $\log_2(4) = \mathbf{2}$ |
| $8 = 2^3$ | $L(2^3) = \mathbf{3}$ | $\log_2(8) = \mathbf{3}$ |
| | | |
| | | |
| | | |
| | | |

← Complete a few more lines of the table to practice the new notation.

A little aside #1: Use the pattern of the bolded numbers in the table on the previous page to describe the relationship between the exponent in the exponential notation and the logarithm in either of the other two notations.

A little aside #2: Practice the use of the word “logarithm.” For example, how many ratios are required (i.e., what is the logarithm) for the term 8 to be found in $S2$?

13. Complete the table for sequences $S5$ and $S6$:

| | | | | | | | | | | | | | | | |
|-----------|----|----|----|----|----|----|---------------|---|---|---|---|---|---|---|---|
| S5 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| S6 | | | | | | | $\frac{1}{3}$ | 1 | 3 | 9 | | | | | |

What is the common ratio (or base) for sequence $S6$?

14. Practice the new “log” notation to associate the corresponding terms of sequences $S5$ and $S6$ to find each of the following:

a. $\log_3(1) = \underline{\hspace{2cm}}$ b. $\log_3\left(\frac{1}{243}\right) = \underline{\hspace{2cm}}$ c. $\log_3(81) = \underline{\hspace{2cm}}$

d. $\log_3(27) = \underline{\hspace{2cm}}$ e. $\log_3(3) = \underline{\hspace{2cm}}$ f. $\log_3\left(\frac{1}{9}\right) = \underline{\hspace{2cm}}$

g. $\log_3(2187) = \underline{\hspace{2cm}}$ h. $\log_3\left(\frac{1}{3}\right) = \underline{\hspace{2cm}}$

Create two other examples for your partner to calculate:

j.

k.

Conclusion/Connection:

In general, you will see the following notation in modern textbooks:

$$\log_b(a) = c \quad \text{if and only if} \quad b^c = a,$$

where b is a positive real number ($b \neq 1$), a is a positive real number, and c is any real number.

15. Based on your study of using pairs of sequences, describe the relationship between logarithms and exponents.

Wrap-up:

What questions do you still have lingering about the development of logarithms, the notation, or their basic calculations or definition?

Rewrite the three properties from Exercise 10 (Part II) using the “ \log_b ” notation:

Lesson Installment 4
Calculation Of Logarithms Using The Method Of Napier And Briggs
(But without the “log” button of your calculator!)

Introduction:

Napier and Briggs each spent many years doing lengthy computations to determine tables of logarithms. Since Napier did not use base 10 for his logarithms, but Briggs did, this activity will introduce you to Briggs’ approach. Keep in mind, however, that we will be finding approximations for only a few common logarithms and will not have nearly the accuracy that Briggs found when he was developing his logarithm tables! (And all of his work without a calculator, no less!)

Recall:

Using whatever variable names you wish, you should be familiar with the following to work with the computations in this activity:

- Definition: $\log_b x = y$ if and only if $b^y = x$.
- Three properties of logarithms:
 - $\log_b(uv) = \log_b u + \log_b v$
 - $\log_b\left(\frac{u}{v}\right) = \log_b u - \log_b v$
 - $\log_b(u^n) = n \log_b u$
- Use of estimation and number sense abilities for a particular purpose. For example, approximate the value of 75 using only powers of 2 and 3. (Or, thought of another way: construct a product *close* to 75 that uses only powers of 2 and 3.)

1. Work with a partner to approximate the common logarithms (logarithms base 10) for the positive integers 1 through 10.

Table:

| <i>Value</i> | <i>Logarithm</i> | <i>Sample Relationship</i> (for use in calculation) | <i>Calculation of</i> <i>Logarithm</i> (to four places) | <i>Calculator</i> <i>Value</i> (Exercise 4) |
|--------------|------------------|---------------------------------------------------------------------------|--------------------------------------------------------------------|---------------------------------------------------|
| 1 | log 1 | | | |
| 2 | log 2 | <i>Example:</i> $2^{10} \approx 10^3$ $1024 \approx 1000$ —————→ | $\log(2^{10}) \approx \log(10^3)$ (Now complete on your own...) | |
| 3 | log 3 | | | |
| 4 | log 4 | | | |
| 5 | log 5 | | | |
| 6 | log 6 | | | |
| 7 | log 7 | | | |
| 8 | log 8 | | | |
| 9 | log 9 | | | |
| 10 | log 10 | | | |

Questions:

2. Which of the above calculations yield exact results? Which of the above calculations yield approximate results? How do you know?

3. Was it more difficult to calculate the logarithm of some values than others? Why?

4. For each of the logarithm values you calculated, use your calculator to find the same values. Why are the calculator values and the values you calculated not the same?

5. Why are some of your results *better* approximations than others? Provide an example in your explanation. Can you improve upon any of your results? If so, recalculate and update your table.

6. If your estimate of $\log 2$ is too small and your estimate of $\log 5$ is too large, explain why this occurred.

7. Estimate $\log 11$ and $\log 13$ using an appropriate power relationship for each.

Lesson Installment 5
Astronomical Calculations Made “Easy”:
Ancient Influences On The Development Of Logarithms

Introduction:

As with the vast majority of new advances in mathematics, the concept of logarithms did not appear totally without earlier suggestive techniques. There are widely considered to be two major influences on the development of logarithms, one from trigonometry and with ancient roots, and one a more algebraic influence much closer to Napier’s own time.

The first influence was a trigonometric technique that changed products into sums, first discussed in the writings of the Islamic mathematician Ibn Yunus (c. 1000 A.D.). Translated into modern terms, Ibn Yunus’s methods states that:

$$2\cos x \cos y = \cos(x + y) + \cos(x - y).$$

This was called the method of *prosthaphaeresis*, a Greek word meaning ‘addition and subtraction.’ Francois Viète, a French mathematician (1540 – 1603), extended this idea to produce (again in modern notation):

$$2\sin x \cos y = \sin(x + y) + \sin(x - y)$$

$$2\cos x \sin y = \sin(x + y) - \sin(x - y)$$

$$2\sin x \sin y = \cos(x - y) - \cos(x + y).$$

Equivalent formulas involving quotients were developed using secant and cosecant functions. These formulas were widely used by astronomers to aid in their computations, since the formulas transformed multiplications into additions and subtractions. Such a process not only shortened the labor associated with such astronomical computations, but it also lessened the number of errors made. While such labors might seem excessive today with the advent of modern technology, by the time of Napier, mathematicians had developed highly accurate trigonometric tables to a very large number of decimal places. These highly accurate tables made the process of *prosthaphaeresis* considerably more effective than it might seem at first glance.

With respect to the influences on the formalization of logarithms, we also know that a friend of Napier’s, a doctor named John Craig, met the Danish astronomer Tycho Brahe when Craig accompanied the Scottish King James IV to Denmark to meet the king’s bride. Because of storms during the trip, the wedding party was forced to land near Brahe’s observatory at Uraniborg and Brahe showed Craig some of the methods he had been using, including *prosthaphaeresis*. Upon Craig’s return to Scotland, he discussed his exposure to Brahe’s methods with Napier.

The second influence is attributed to Michael Stifel, who was broadly known in Europe in Napier’s time. In his book *Arithmetica integra* he clearly displayed an arithmetic series (a list of terms obtained by adding a constant) with a geometric series (a list of terms obtained by multiplying by a constant), as you have seen in an earlier exploration. It seems clear that Stifel realized the computational advantages to such a list, and it is highly probable that Napier was aware of this text.

Exploration:

Since we have already examined (in Installment 3) how Stifel's work with corresponding arithmetic and geometric sequences influenced Napier, let's investigate a sample calculation of an earlier influence, *prosthaphaeresis*.

1. Using *prosthaphaeresis*, calculate the product of _____ and _____ using only the formulas and tables provided. Outline the strategies you used and be sure to include the level of accuracy that is assumed (and why).
2. Now, determine the answer by actual multiplication and determine the percent accuracy of the answer found using *prosthaphaeresis*.
3. Use a different formula than the one you chose for Problem 1 and compare this value with the value of the product you have already found.
4. Although using calculators today is obviously easier, comment on the advantages and challenges of using *prosthaphaeresis* in earlier centuries (11th – 17th).
5. Write a brief reflection about the development of logarithms. Include in your reflection observations about:
 - the motivation for their development;
 - the use of previous mathematics that aided in their development; and
 - the human aspect of their development.

Lesson Installment 6
A Glimpse Into The Future:
Translating Napier’s Method To Include The Natural Base e

Revisiting Previous Lessons:

Recall that the reason Napier developed logarithms was so that computations with very large or very small numbers could be done “easily.” This is seen in our modern study of logarithms when we find the rules for logarithms as given in the textbook and that we discussed in class. To recap, logarithms have the ability to change multiplication into addition, division into subtraction, and exponentiation into multiplication (by a constant).

From previous explorations, we know that $\log_2 4 = 2$, $\log_2 8 = 3$, $\log_2 16 = 4$, etc. However, to consider logarithms to be of true practical use, we need to be able to calculate the logarithm of *all* numbers between 4 and 8 and between 8 and 16, and so on. The goal of this activity is to find a way (still firmly entrenched in our historical investigation) to “fill in the gaps” between the numbers 4, 8, and 16.

Napier recognized this problem and he chose a series that was “dense.” We saw that there are *very* small gaps between successive terms of his geometric series when we examined the calculations associated with Napier’s original explanation. Napier believed that his calculations should be accurate to seven decimal places (the accuracy goal for the early 17th century). Consequently, he chose 10^7 as the first term of his sequence. He took as his common ratio a number that is extremely close to 1: $(1 - 10^{-7})$ or .9999999. Napier then constructed several intricate tables that eventually became the basis for complex calculations. Here is a sample of a piece of one of his tables:

| <i>Exponent</i> | <i>Napier’s Exponential Form</i> | <i>Value</i> |
|-----------------|----------------------------------|--------------|
| 0 | $10^7(1 - 10^{-7})^0$ | 10,000,000 |
| 1 | $10^7(1 - 10^{-7})^1$ | 9,999,999 |
| 2 | $10^7(1 - 10^{-7})^2$ | 9,999,998 |
| 3 | $10^7(1 - 10^{-7})^3$ | 9,999,997 |
| ... | ... | ... |
| 100 | $10^7(1 - 10^{-7})^{100}$ | 9,999,900 |

You can see that the numbers in the right column are very “dense.” That is, they are very close to each other (unlike the sequences we have previously used), which is exactly what Napier wanted.

Guided Exploration:

Our goal for this exploration is to examine Napier’s choice of sequences in light of values that we may have reason to use in our modern study. Before we begin, let’s revisit the idea of identifying relationships between arithmetic and geometric sequences.

Consider the two sequences:

| | | | | | | |
|-------|------------|------------|------------|------------|------------|-----|
| a_n | 0 | 10 | 20 | 30 | 40 | ... |
| b_n | 10,000,000 | 10,000,100 | 10,000,200 | 10,000,300 | 10,000,400 | ... |

1. What is the common ratio for b_n ?
2. Write each term of b_n using exponents:
3. (Let $n = 0$ for the first term given.) What is an expression for the n th term of a_n ? What is an expression for the n th term of b_n ?
4. Suppose we wish to manipulate the expression for the n th term of b_n from Problem 3. In the steps below, describe the reason for each transformation:

$$b_n = \underline{\hspace{2cm}} \quad \text{From Problem 3}$$

$$10^7 \left(1 + \frac{1}{10^5}\right)^n \quad \underline{\hspace{2cm}}$$

$$10^7 \left(1 + \frac{1}{10^5}\right)^{\frac{10n}{10}} \quad \underline{\hspace{2cm}}$$

$$b_n = 10^7 \left(1 + \frac{1}{10^5}\right)^{\frac{a_n}{10}} \quad \underline{\hspace{2cm}}$$

5. Calculate: $\left(1 + \frac{1}{10^5}\right)^{10^5}$. (Give as many digits as your calculator displays.)

6. Does the number that you found in Problem 5 look familiar? It should! What is the value (to the same number of decimal places that you used for Problem 5) that your calculator stores for the actual value of _____ (fill in the “name” of the value here!)?

Now we’re really going to test our algebraic manipulation powers! (No pun intended.) What we seek is an equation for the expression of the relationship between the sequences (see the end of Problem 4) that will eventually enable us to use our modern

“ $\log_b a$ ” notation. It may be difficult to see now, but the result will be worth it! (Then, you’ll try the same technique using a different (more “Napierian”) sequence for b_n .)

7. Determine what number needs to be substituted into k below so that the exponential expression looks like the final result of Problem 4:

$$b_n = 10^7 \left[\left(1 + \frac{1}{10^5} \right)^{10^5} \right]^{\frac{a_n}{k}}$$

8. Now, simplify the expression for b_n , using the name of the approximate value of $\left(1 + \frac{1}{10^5} \right)^{10^5}$ (from Problem 6):

9. Another substitution! Letting $x = \frac{b_n}{10^7}$ and $y = \frac{a_n}{10^6}$, rewrite the result in Problem 8:

10. Last one! Using the definition of a logarithmic function, rewrite the equation in Problem 9:

Whew! You have just completed the derivation for the natural logarithmic function using the relationship between an increasing arithmetic and increasing geometric sequence. When Napier constructed his argument for his invention of logarithms, he used a *decreasing* geometric sequence. If the number e “naturally” arose in the instance of an increasing geometric sequence, what will happen when we examine a decreasing geometric sequence?

On Your Own:

Now we want to follow a similar investigation to Problems 1 – 10, only using the same common ratio that Napier used $[(1 - 10^{-7})]$ when constructing his tables of logarithms.

11. Fill in the missing values in the table (and additional interim terms if you desire) for the corresponding sequences, using a common ratio of $(1 - 10^{-7})$ for b_n .

| | | | | | | | | | |
|-------|----------|----|----|----|----|--|--|--|-------|
| a_n | 0 | 10 | 20 | 30 | 40 | | | | $10n$ |
| b_n | 10000000 | | | | | | | | |

12. Find the general expressions for a_n and b_n .

13. Now, find a relationship between a_n and b_n , as we did in Problem 4.

14. *The finale!* Rewrite the equation in Problem 13 to take on a form similar to the equation found in Problem 9. Then, use this equation to write an equation with the “log” notation. Explain all substitutions that you make.

Postscript:

The beauty of the invention of logarithms aside, if they were to be used to simplify computations, they needed to be constructed in such a way that the terms in the geometric sequence were close together, or “dense.” By choosing such a sequence, with ratios very close to 1, the numbers e and $1/e$ naturally arose. Napier did *not* see this. Later mathematicians (we’ll meet one of them in the next installment) discovered this interesting fact. The discovery was to be so impressive that they decided to denote a special logarithm with e as its base. Thus, the natural logarithm was born, forever changing the history of mathematics.

Lesson Installment 7
Summarizing Logarithms: What Did Euler Have To Say About Them?

Introduction:

1. Before you read Leonhard Euler's description of the logarithmic function (don't worry, it has been translated from the original Latin!), which he published in 1748, familiarize yourself with Euler's life and why he would have been motivated to describe exponentials and logarithms in his mathematical endeavors.

2. Read the excerpt of Euler's *Introductio in analysin infinitorum* (1748; paragraphs 102 – 105, 107; attached).

Understanding...

Paragraph 102:

3. What is the definition of the LOGARITHM of y that Euler gives?

4. Is this the same definition that you have become familiar with in this course? Explain why you think so.

5. In the paragraph, what is " a "? What properties must a have?

6. What does the last sentence say about the domain of the logarithmic function?

Paragraph 103:

7. Explain why Euler says that, no matter the base, $\log 1 = 0$ (or, in our notation, $\log_a 1 = 0$).

8. Explain why " $\log a = 1, \log a^2 = 2, \log a^3 = 3, \log a^4 = 4, \text{ etc.}$ "

9. Explain why $\frac{1}{a}, \frac{1}{a^2}, \dots$ are positive numbers less than 1 and why their logarithm is negative.

Paragraph 104:

10. What does Euler say the product rule, the quotient rule, and the power rule for logarithms are?

11. Outline the explanation Euler uses for the product rule.

12. Outline a similar explanation for the quotient rule.

13. How can these rules be used to “find the logarithms of many numbers from a knowledge of the logarithms of a few”? Where have you experienced this before?

Paragraph 105:

14. What does the first sentence mean? Give an example of a logarithm of a number that is rational.

15. What is a surd? (You may have to reference a dictionary.)

16. Why, according to Euler, is the logarithmic function a transcendental function?

Paragraph 107:

17. Give the change of base rule that you learned in the unit on logarithms in this course.

18. Translate this passage into your own words: “If the base of one system is a and that of the other is b , if also the number n has logarithm p in the first system and logarithm q in the second, then $a^p = n$ and $b^q = n$. Therefore $a^p = b^q$, so that $a = b^{q/p}$ and the value of p/q is constant, no matter the value of n may be.”

19. Show that Euler is describing how to change base 10 logarithms into base 2 logarithms in his example.

Summary:

20. Compare and contrast each mathematician’s (Napier and Euler) work with logarithms.

Introduction to Logarithms
From
Introductio in analysin infinitorum

Taken from Introductio in analysin infinitorum (1748) by Leonhard Euler

Chapter VI. On Exponentials and Logarithms

102. Just as, given a number a , for any value of z , we can find the value of y [= a^z], so, in turn, given a positive value for y , we would like to give a value for z , such that $a^z = y$. This value of z , insofar as it is viewed as a function of y , it is called the LOGARITHM of y . The discussion about logarithms supposes that there is some fixed constant to be substituted for a , and this number is the base for the logarithm. Having assumed this base, we say the logarithm of y is the exponent in the power a^z such that $a^z = y$. It has been customary to designate the logarithm of y by the symbol $\log y$. If $a^z = y$, then $z = \log y$. From this we understand that the base of the logarithms, although it depends on our choice, still it should be a number greater than 1. Furthermore, it is only of positive numbers that we can represent the logarithm with a real number.

103. Whatever logarithmic base we choose we always have $\log 1 = 0$, since in the equation $a^z = y$, which corresponds to $z = \log y$, when we let $y = 1$ we have $z = 0$. From this it follows that the logarithm of a number greater than 1 will be positive, depending on the base a . Thus $\log a = 1$, $\log a^2 = 2$, $\log a^3 = 3$, $\log a^4 = 4$, etc. and, after the fact, we know what base has been chosen, that is the number whose logarithm is equal to 1 is the logarithmic base. The logarithm of a positive number less than 1 will be negative. Notice that $\log 1/a = -1$, $\log 1/a^2 = -2$, $\log 1/a^3 = -3$, etc., but the logarithms of negative numbers will not be real, but complex, as we have already noted.

104. In like manner if $\log y = z$, then $\log y^2 = 2z$, $\log y^3 = 3z$, etc., and in general $\log y^n = nz$ or $\log y^n = n \log y$, since $z = \log y$. It follows that the logarithm of any power of y is equal to the product of the exponent and the logarithm of y . For example $\log \sqrt{y} = \frac{1}{2}(z) = \frac{1}{2}(\log y)$, $\log 1/\sqrt{y} = \log y^{-1/2} = -\frac{1}{2}(\log y)$, and so forth. It follows that if we know the logarithms of any number, we can find the logarithms of any power of that number. If we already know the logarithms of two numbers, for example $\log y = z$ and $\log v = x$, since $y = a^z$ and $v = a^x$, it follows that $\log vy = x + z = \log v + \log y$. Hence, the logarithm of the product of two numbers is equal to the sum of the logarithms of the factors. In like manner $\log (y/v) = z - x = \log y - \log v$, that is, the logarithm of a quotient is equal to the logarithm of the numerator diminished by the logarithm of the denominator. These rules can be used to find the logarithms of many numbers from a knowledge of the logarithms of a few.

105. From what we have seen, it follows that the logarithm of a number will not be a rational number unless the given number is a power of the base a . That is, unless the number b is a [rational] power of the base a , the logarithm of b cannot be expressed as a rational number. In case b is a power of the base a , then the logarithm of b cannot be an irrational number. If, indeed, $\log b = \sqrt[n]{n}$, then $a^{\sqrt[n]{n}} = b$, but this is impossible if both a and b are rational. It is especially desirable to know the logarithms of rational numbers, since from these it is possible to find the logarithms of fractions and also surds. Since the logarithms of numbers which are not the powers of the base are neither rational nor irrational, it is with justice that they are called transcendental quantities. For this reason, logarithms are said to be transcendental. [Note: For Euler, an “irrational” number was what we would call an “algebraic” number, namely, a real number which is the solution to a polynomial equation. Real numbers which were not solutions of polynomial equations were called “transcendental”.]

107. There are as many different systems of logarithms as there are different numbers which can be taken as the base a . It follows that there are an infinite number of systems of logarithms. Given two different systems of logarithms, there is a constant which relates the logarithms of the same number. If the base of one system is a and that of the other is b , if also the number n has logarithm p in the first system and logarithm q in the second, then $a^p = n$ and $b^q = n$. Therefore $a^p = b^q$, so that $a = b^{q/p}$ and the value of p/q is constant, no matter what the value of n may be. If the logarithms of all numbers have been computed in one system, then it is an easy task, by means of this golden rule for logarithms, to find the logarithms in any other system. For example, we have logarithms for the base 10. From these we can find the logarithms with any other base, for instance the base 2. We look for the logarithm of a number n for base 2, which will be q , while the logarithm with base 10 of the same number n will be p . Since for base 10, $\log 2 = 0.3010300$ and for base 2, $\log 2 = 1$, then $p/q = 0.3010300/1$ and $q = p/0.3010300 = 3.3219277p$. If every common logarithm is multiplied by 3.3219277 then we will have produced a table of logarithms for base 2.

Appendix C: Survey and Instruments

Background Survey

1. Gender: Female Male
2. Which of the following best describes you? Circle one or more.
 - A White
 - B Hispanic or Latino
 - C Black or African American
 - D Asian
 - E American Indian or Alaska Native
 - F Native Hawaiian or other Pacific Islander

For the next two questions, include any full-time teaching assignments, part-time teaching assignments, and long-term substitute teaching assignments, but not student teaching.

3. Counting this year, how many years have you worked as an elementary or secondary teacher?
4. Counting this year, how many years have you taught mathematics in grades 6 through 12?
5. What type of teaching certificate do you hold?
 - A Regular or standard state certificate or advanced professional certificate in secondary mathematics
 - B Probationary certificate in secondary mathematics
 - C Provisional or other type of certificate given to persons who are still participating in what the state calls an “alternative certification program”
 - D Temporary certificate for teaching mathematics
 - E Emergency certificate or waiver to teach mathematics
 - F No certificate in teaching mathematics

6. Did you have a major, minor, or special emphasis in any of the following subjects as part of your **undergraduate** coursework? Circle one response per line.

| | | | |
|--------------------------|-------------------------|---------------------------------------------|-----------|
| | Yes, a major | Yes, a minor or special emphasis | No |
| a. Mathematics education | A | B | C |

| | | | |
|----------------|-------------------------|---------------------------------------------|-----------|
| | Yes, a major | Yes, a minor or special emphasis | No |
| b. Mathematics | A | B | C |

| | | | |
|--------------------------------------------------------------------------|-------------------------|---------------------------------------------|-----------|
| | Yes, a major | Yes, a minor or special emphasis | No |
| c. Other mathematics-related subject such as statistics or physics | A | B | C |

| | | | |
|-------------------------------------------------|-------------------------|---------------------------------------------|-----------|
| | Yes, a major | Yes, a minor or special emphasis | No |
| d. Education (including secondary education) | A | B | C |

7. Did you have a major, minor, or special emphasis in any of the following subjects as part of your **graduate** coursework? Circle one response per line.

| | | | |
|--------------------------|-------------------------|---------------------------------------------|-----------|
| | Yes, a major | Yes, a minor or special emphasis | No |
| a. Mathematics education | A | B | C |

| | | | |
|----------------|-------------------------|---------------------------------------------|-----------|
| | Yes, a major | Yes, a minor or special emphasis | No |
| b. Mathematics | A | B | C |

| | | | |
|--------------------------------------------------------------------------|-------------------------|---------------------------------------------|-----------|
| | Yes, a major | Yes, a minor or special emphasis | No |
| c. Other mathematics-related subject such as statistics or physics | A | B | C |

| | | | |
|-------------------------------------------------|-------------------------|---------------------------------------------|-----------|
| | Yes, a major | Yes, a minor or special emphasis | No |
| d. Education (including secondary education) | A | B | C |

8. Please indicate which courses you took in your **undergraduate** program (place an “X” on the blank adjacent to the course):

- | | |
|---------------------------------|----------------------------------|
| a. College Algebra _____ | b. Introductory Statistics _____ |
| c. Calculus I _____ | d. Calculus II _____ |
| e. Calculus III _____ | f. Calculus IV _____ |
| g. Statistical Analysis _____ | h. Linear Algebra _____ |
| i. Differential Equations _____ | j. Abstract Algebra _____ |
| k. College Geometry _____ | l. History of Mathematics _____ |

9. **During the last two years**, did you participate in or lead any of the following professional development activities related specifically to the teaching of mathematics? Circle one response per line.

| | Yes | No |
|------------------------------------------------------------------------------------------------------------------|------------|-----------|
| a. Mathematics content or methods college course taken after your first certification | A | B |
| b. School, county/district, or state-provided programs, workshops, training sessions or institutes | A | B |
| c. Conference or professional association meeting | A | B |
| d. Observational visit of mathematics instruction to another school | A | B |
| e. Mentoring and/or peer observation and coaching as part of a formal arrangement | A | B |
| f. Committee or task force focusing on mathematics curriculum, instruction, or student assessment in mathematics | A | B |
| g. Regularly scheduled discussion or study group | A | B |
| h. Teacher collaborative or network (such as one organized by an outside agency or over the Internet) | A | B |
| i. Individual or collaborative research | A | B |

9, *continued*. **During the last two years**, did you participate in or lead any of the following professional development activities related specifically to the teaching of mathematics? Circle one response per line.

| | Yes | No |
|-------------------------------------------------------------------------------------------------------|------------|-----------|
| j. Independent reading on a regular basis (for example, educational journals, books, or the Internet) | A | B |
| k. Co-teaching/team teaching in mathematics | A | B |
| l. Consultation with a mathematics specialist | A | B |

10. Do you have special leadership responsibilities for mathematics or mathematics education at your school (for example, responsibilities as a mentor teacher, lead teacher, resource specialist, department chair, or master teacher)?

Yes No

If Yes, please describe:

11. Describe any previous experience with the history of mathematics (for example, courses taken, workshop participation, or personal reading).

Attitudes Assessment

Using History In Teaching Mathematics Attitudes Survey – Revised (Survey adapted from G. L. Marshall Dissertation (2000))

PART I:

Please rate the extent to which you agree or disagree with each statement by circling the appropriate letter. Use the following code.

SA = strongly agree MA = moderately agree LA = slightly agree
LD = slightly disagree MD = moderately disagree SD = strongly disagree

| <i>Statement</i> | <i>Response</i> |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Understanding the history of mathematics is an important part of understanding mathematics. | SA MA LA LD MD SD |
| 2. Including history enriches the teaching and learning of mathematics. | SA MA LA LD MD SD |
| 3. Biographies of relevant mathematicians make mathematics classes more enjoyable. | SA MA LA LD MD SD |
| 4. Knowing the historical development of a key mathematical topic facilitates the learning of that topic. | SA MA LA LD MD SD |
| 5. Quality mathematics instruction includes major facts from the history of mathematics. | SA MA LA LD MD SD |
| 6. Using historical materials in my mathematics classes has been an integral part of my instruction in: Algebra I or II. Geometry. Precalculus/Trigonometry. Calculus. | SA MA LA LD MD SD SA MA LA LD MD SD |
| 7. Prospective mathematics teachers should be required to study the history of mathematics. | SA MA LA LD MD SD |
| 8. As a mathematics teacher, it is important for me to continue my own learning of mathematics. | SA MA LA LD MD SD |

Using History In Teaching Mathematics Attitudes Survey – Revised
(Survey adapted from G. L. Marshall Dissertation (2000))

PART II:

Please circle an underlined response choice and provide a short answer that completes the following statements. Your honest opinion is desired.

1. Researching a mathematician contributes

(circle one) nothing, a little, a lot

to a mathematics course because it

2. Mathematics teachers should require

(circle one) no, some, much

history work in their mathematics classes since this

3. If I had access to quality materials from the history of mathematics, I would

(circle one) not, sometimes, often

use them with students because _____

4. I would consider incorporating historical problems in the curriculum as

(circle one) possible, most likely, improbable

because _____

Content Knowledge Assessment

Please answer each of the following questions.

1. Define *logarithm*.

2. Describe the basic idea or motivation for the invention of logarithms.

3. Construct the values for $\log_{10}2$ and $\log_{10}3$ without using a calculator.

4. Let $u = b^n$ and $v = b^m$. Verify $L(u) - L(v) = L\left(\frac{u}{v}\right)$.

5. Evaluate: $\log_{32}16$.

6. Evaluate: $\log_{\frac{1}{3}}(81)$.

7. Calculate the product of 8409.5 and 951.49 using the method of prosthaphaeresis.

8. Solve for x : $2\log 3 + \log x = \log 45$.

Appendix D: Sample Interview Protocol

- 1) You have essentially used Lesson Installments 1 through 4 with your classes to introduce the study of logarithms.
 - (a) Did you use the Installments as you originally envisioned?
 - (b) What did you move on to next? (In the study of logarithms – history-related or not; and in trigonometry) Did you use Installment 5 with the classes after covering trigonometric identities? If so, how did it go?
 - (c) If you were to teach logarithms again, what would you do differently?
- 2) You incorporated the use of an original document (the *Descriptio*) as a preface to Installment 2. Could you talk a little about your decision to do that and how that decision was influenced by your perception of what the students need?
- 3) Could you describe your overall impression of what the presentation of logarithms from their historical perspective meant for your students?
- 4) With respect to your own students (possibly considering each class period separately), what was more influential on your decisions of what to include and how: the human aspect behind the development of logarithms or the mathematical evolution?
- 5) Early in the week of February 14th, you stated that you would have the students come back after they completed their papers and create a ‘timeline of the development of logarithms’ to be displayed in the classroom. Is this something you still plan on doing? If so/not, talk a little about the plan/why not.
- 6) Is your classroom environment one such that the students are not “tied” to the textbook? In other words, are they able to work for an extended period of time without needing to depend on their book? (Or is this a constant struggle, regardless of topic?)
- 7) How would you describe what you consider your role to be as a mathematics teacher?
- 8) How would you describe what you consider a student’s role to be in your class? Or, what expectations do you have of students in your classes?
- 9) Could you briefly describe your teaching philosophy? (If this is different from what you consider your role to be in the classroom.)

Appendix E: Member Checks

Letter Mailed to Participants

January 9, 2006

Dear <Participant Name>:

Happy New Year! I am finally finished with a draft that is complete enough to send out for comment! I am asking each participant in the study to read their chapter and to return it to me with their comments. Please feel free to write on the document. I am enclosing a completed label and postage for returning it to me. I researched the mailing options, and if you could please return it via **Priority Mail Flat Rate Envelope (USPS)**, which is "\$4.05, regardless of weight or destination" – I would greatly appreciate it. I am enclosing cash for the mailing cost.

I thought it would be helpful to outline a few guidelines for providing feedback. First, to guide you in the reading, your chapter or section was written to address the following research questions:

Primary research questions:

How do teachers with different background knowledge and experiences respond to professional development focused on understanding the history of mathematics?

How do background variables and professional development experiences with history of mathematics combine to influence teachers' personal mathematical knowledge and instructional practice?

Secondary research questions:

- 1. How do teachers engage in professional development sessions about the history of logarithms?*
- 2. How do teachers implement the materials and methodological and pedagogical ideas discussed during those sessions when teaching logarithms in a Precalculus-type course?*
- 3. What do teachers identify as benefits when using the history of logarithms during their teaching of a unit on logarithms?*
- 4. What obstacles and affordances do teachers identify when using the history of logarithms? How does the teacher deal with the obstacles and affordances?*

Second, please comment/correct/reinterpret anything that I have presented which you feel is a misrepresentation of what you said, did, wrote, or believed. For example, if I stated, "Jane's traditional instruction focused on teaching that all logarithmic expressions must be written in green ink," and you know for a fact that that is not what you either told me

or did, then I welcome your corrections. If you also want to provide a summary statement, I will be happy to include that as well.

Third, as publishing criteria for the University of Maryland are quite rigid and the dissertation must be “approved” with regard to these criteria, it is not necessary for you to provide any feedback that is technical. Feedback on spacing, dangling section headings, margins, and so forth is not needed. Also, it is not necessary to provide any grammatical editing. That’s partially what my committee is for, and is ultimately my responsibility. (But, if you’re like me, it is often difficult to review something without making corrections of that sort. It’s the teacher in us, I think.)

Related to editing issues, you will notice in the text of your chapter that I have made reference to appendices which I am not providing. For the most part, those appendices contain the various instruments, the seven lesson installments, and large transcript excerpts. Also, I am aware that each table is labeled with a **bold** number. That is as a reminder to myself, so that once the entire document is compiled, I’ll remember to go back and number all of the tables consecutively.

Lastly, you may find it helpful to know *why* I am including the opportunity for you to provide comments. Member checking (“sharing draft study findings with the participants, to inquire whether their viewpoints were faithfully interpreted, whether there are gross errors of fact, and whether the account makes sense to participants with different perspectives”) is a technique that allows the individual participants to serve as another type of data source. Since my work is wholly qualitative, I believe this is an essential piece to triangulating my data so that I can tell the best possible story. All comments that you send will be compiled (unless you provide a comprehensive statement that you want me to include) into an appendix at the end of the dissertation. In order to include them in time for my committee review of the dissertation before I defend my work, I need to receive your comments by **February 3, 2006**. If I do not receive corrections/comments/statements from you by that time, I will assume that you believe the representation that I have constructed is accurate.

Thank you again for making this dissertation a reality. It has been personally and academically rewarding to spend all these months with you (in person or in spirit while I was writing!). If you have any questions, please do not hesitate to e-mail me.

With deep appreciation,

Kathy

Comments: Mandy Wilson

Mandy returned her chapter and a collection of other documents in response for my request for comment. She included a letter, her exponential and logarithmic functions unit test, and her spring semester exam.

The letter from Mandy included responses to several questions I added to the letter I included with Mary's chapter. She confirmed the order of topics that she covered during the time that I spent in her classroom, including sequences and series; the historical development of logarithms; and a traditional survey of exponential and logarithmic functions. Her letter closed with an invitation to get together after I "have finished with this mammoth task."

It is interesting to note that Mandy's semester exam included three questions (out of 63) about the history of logarithms. They were:

John Napier can take sole credit for the invention of logarithms.

Henry Briggs was a German mathematician who created or invented exponential notation.

Logarithms were created to assist with the calculation of large numbers related to chemistry.

In addition to the letter, Mandy highlighted three passages in her chapter and corrected one typographical error. Two of the passages were given as corrections and one was an observation. First, I incorrectly identified Mandy's additional graduate work as focusing on gifted education. Instead, the work focused on mathematics education. Second, when I described Mandy's use of the historical development of the number system (which she incorporated at the beginning of 2004 – 2005), the reference implied that that is all she did for the first six weeks of school. Mandy noted that, "not all of that was devoted to historical" work, and I incorporated the change she suggested. Lastly, Mandy provided an observation related to my discussion of her use of Napier's two particle argument. She said, "I attempted [this] with college students this fall – BUST! You need academically and intellectually engaged students!!" I found it interesting that Mandy's chose "engaged" rather than "able" in her description. I also interpreted her comment to mean that the engagement of the student (the academically and intellectually engaged students which Mandy encountered at High Acres School) is another affordance for using the history of mathematics.

Comments: Sue Moe

I received a phone call from Sue on January 17, 2006. Sue called to let me know that she had received her chapter section and decided to begin looking at it. Once she started reading however, she found it was "very easy to read, so [she] just kept reading" (S. Moe, personal communication, 1/17/06). When I asked her about other feedback as

outlined in the letter I had sent, she told me that she found two typographical errors, but that was all. Also, she told me that it was a little strange reading about her but that once she got used to that, the reading went quite smoothly.

I next asked Sue if the chapter accurately represented her practice, in the professional development sessions, on the instruments, and in her instruction. She responded, “Yes, that’s pretty much what happened” (S. Moe, personal communication, 1/17/06).

Sue identified the two typographical errors, which I recorded so that I could fix them when I got off the phone.

Comments: Ted Jones

I received Ted’s chapter section on January 26, 2006. As with the other participants, I included questions particular his case along with the letter accompanying the manuscript. In addition to requesting Ted’s comments on the interpretations and overall representation of his case, I also asked Ted specifically about the dates corresponding to the end of his instruction in Chapter 11 (I was not present for the end of Ted’s instruction). Ted did not respond to the question I had about his instructional schedule (I included a note to Ted on page 25). He did include one comment about the presentation of his responses to Part I of the Attitudes Instrument Pre-Assessment (page 3 of text). In response to the identification of inconsistencies in his response to Item 6, Ted noted:

I am not sure why I responded to Geometry and not Calculus. I have taught Geometry in the past. My only guess is yes, I believed it was important to include the teaching [of history] when I taught it in the past.

I am unsure whether Ted continued reading after page 3, since the inconsistency remained when I discussed Ted’s responses to Item 6 in his case summary. For example, on the final post-assessment, Ted responded that he strongly agreed to incorporating historical materials in teaching Calculus (compared to slightly agreeing on the pre-assessment), yet he did not teach Calculus in 2004 – 2005.

Comments: Shirley Corson

Shirley did not return her manuscript with comment to me. Instead, she spoke with Sue about reading her chapter section and only identified two typographical errors. I indicated that I was least worried about typographical errors and more worried about each participant’s comment on the interpretations and overall representation of their case. Sue relayed the information to Shirley and I did not receive any other communication from Shirley.

Comments: Mary Long

I did not receive any communication from Mary. My letter indicated that, “If I do not receive corrections/comments/statements from you by [February 3, 2006], I will assume that you believe the representation that I have constructed is accurate.” I have assumed that since the four week window was sufficient time for the other four participants to respond, that Mary did not disagree with the presentation of her case.

Appendix F: Sample Curriculum Plan
Mandy Wilson

Curriculum Outline: Teaching Logarithms from an Historical Perspective

| Lesson Title | Lesson Placement | Lesson Objective(s) | Activity Description |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>I. Identifying the key players in the invention of logarithms (Biographical and background information of key mathematicians; their contributions; and the historical context of their work)</p> | <p>Before the mathematical study of logarithms begins, i.e., after a test covering laws of exponents and/or exponential functions (Time: One class period*)</p> <p style="text-align: center;"><i>*All times based upon 45-minute class periods.</i></p> | <p>To provide students with experiences in the cultural, historical, and scientific evolution of the mathematical topic of logarithms</p> | <p>Students will construct the story of the invention of logarithms using resources provided in the classroom (including several internet websites and print materials). Students will work in small groups to complete a timeline summarizing the people, accomplishments, and places involved in the invention of logarithms. To summarize, the students will come together as a large group to agree upon a common timeline that relies on information that can be verified in original sources and respected historical resources. <i>Materials Needed:</i> Two copies of logarithm timeline for students to complete during small group work and whole-class discussion; world map; portrait posters of key mathematicians</p> |
| <p>II. Napier's description of a logarithm: The two particles argument (Knowledge of the sine function is necessary for this</p> | <p>First lesson of introduction to logarithms (Time: 1 class period)</p> | <p>To establish John Napier's original argument for the development of logarithms.</p> | <p>The teacher will present a modified version of the original development of logarithms, for which Napier provided a geometrical explanation involving the movement of two particles along</p> |

| Lesson Title | Lesson Placement | Lesson Objective(s) | Activity Description |
|------------------------------------------------|-------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| activity) | | | <p>parallel lines. The primary motivation of the lesson is to engage students in the authentic historical development of logarithms, which is significantly different from the traditional context it is usually presented.</p> <p><i>Materials Needed:</i> Excerpts from primary sources</p> |
| III. Development of logarithms using sequences | Second lesson of the introduction to logarithms (Time: 2 class periods) | Students will explore the ideas of the historical invention of the logarithm in modified form by associating the terms of a geometric sequence with those of an arithmetic sequence. | <p>This lesson contains three parts:</p> <ol style="list-style-type: none"> <li data-bbox="1154 821 1443 1507">I. Exercises requesting students to recall and reflect on their knowledge of sequences. A link is established between this prior knowledge and how to perform multiplication and division of large numbers. <li data-bbox="1154 1514 1443 1873">II. Part I is extended by asking students to summarize their results in terms of a function, L. Students are asked to |

| Lesson Title | Lesson Placement | Lesson Objective(s) | Activity Description |
|----------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | | | <p>prove their results algebraically.</p> <p>III. The results from Parts I and II are used to establish a more formal symbolic representation that results in the modern “$\log_b a$” notation.</p> <p><i>Materials Needed:</i> Lesson handouts</p> |
| <p>IV. Calculation of logarithms using the method of Napier and Briggs</p> | <p>Used as a reinforcement of the definition of a logarithmic function and the three laws of logarithms. If used as a supplement to traditional textbook instruction, should appear after properties of logarithms are taught. (Time: 1 class period)</p> | <p>Students will investigate, using small integer values, one way to establish logarithmic values. The relationship between the work of Napier and Briggs is emphasized in this activity.</p> | <p>Students are asked to calculate the base 10 logarithms for the integers 1 through 11, using the laws of logarithms they developed in the previous lesson. Students are also required to apply properties of exponents and factorization. Multiple solution paths are possible, and students are asked to articulate number relationships.</p> <p><i>Materials Needed:</i> Handout; calculator with common (base 10) logarithmic values</p> |
| <p>V. Astronomical calculations made “easy”: Ancient influences on the development of logarithms</p> | <p>After traditional textbook practice of applying the definition and laws of logarithms</p> | <p>To examine the method of <i>prosthaphaeresis</i> and its potential influence on the development and calculation of</p> | <p>Students will use the example of multiplying two large numbers (8409.5 and 951.49) to experience the method of <i>prosthaphaeresis</i> and its potential contribution to</p> |

| Lesson Title | Lesson Placement | Lesson Objective(s) | Activity Description |
|------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | (Time: 1 class period) | logarithms. | <p>the development of logarithms (specifically, tables). Students will also work with other formulas developed in the 16th century. The exercise highlights the potential influences on Napier’s calculation of logarithms, including the work of Viète, Ibn Yunus, Craig, and Brahe. Students will need skills working with exponential notation and inverse trigonometric functions.</p> <p><i>Materials Needed:</i> Handout, calculator with trigonometric values</p> |
| <p>VI. A glimpse into the future: Translating Napier’s method to include the natural base e</p> | <p>After working with natural logarithms in a traditional text (Time: 1 class period, with potential homework for reading the historical background and completing the second part of the investigation)</p> | <p>To investigate the relationship between Napier’s development of logarithms and the modern use of natural logarithms.</p> | <p>Extensive background information is provided to guide students toward questioning how to establish the logarithms of numbers “filling in the gaps” between the values of the geometric sequence given in Lesson III. Students then work with two <i>new</i> sequences based upon Napier’s first term choice of 10^7 and common ratio of $(1 - 10^{-7})$. The analysis the students engage in enables them to derive the value of e (and $\frac{1}{e}$) from Napier’s work.</p> <p><i>Materials Needed:</i> Handout, calculator (for complex exponential</p> |

| Lesson Title | Lesson Placement | Lesson Objective(s) | Activity Description |
|-------------------------------------------------------------|-----------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VII. Summarizing logarithms: What did Euler say about them? | End of unit on logarithms, possibly in place of review exercises (Time: 1 class period) | Students will read and analyze the excerpt from Euler's <i>Introductio in analysin infinitorum</i> (1748) for the purpose of investigating the evolution of logarithmic properties and definitions from the time of Napier (c. 1614) to the time of Euler (c. 1748). | calculations) Students will read five paragraphs from the <i>Introductio</i> . A series of questions follow that guide the students through an interpretation of the excerpt. Students are asked to make comparisons to their modern definitions as well as to the work of Napier. <i>Materials Needed:</i> Excerpt, Handout |

Appendix G: Slide Rule Calculation

Example:

To find 2 multiplied by 3 (2×3), put the “1” from scale 1 below the “3” on Scale 2 and locate the 2 on Scale 1. The answer for (2×3), will be on Scale 2, above the “2.”

Graphic:



Appendix H: WebQuest Created by Sue Moe

History of Logarithms WebQuest

1. What contribution did Ibn Yunus make to the eventual development of logarithms?

<http://www.hps.cam.ac.uk/starry/tychomaths.html>

How long ago did Ibn Yunus work on his trigonometric contributions?

2. Exponents are believed to be the precursors to the use of logarithms. Locate on this web site: <http://www.veling.nl/anne/templars/operation.html> the mathematicians who worked with began to use:

positive integers as exponents: _____

negative integers as exponents: _____

and fractional exponents: _____

3. What formula (or formulas) linked together the following mathematicians: Brahe, Ursus, Clavius, Bürgi, and Werner?

<http://www.hps.cam.ac.uk/starry/tychomaths.html>

4. What is the definition of *prosthaphaeresis*?

5. How does Johannes Kepler's work related to logarithms link the work of Brahe and John Napier? <http://www.mathpages.com/rr/s8-01/8-01.htm>

6. What is the name of the Scottish doctor who suggested the mathematical technique of *prosthaphaeresis* to John Napier?

7. What was the professional connection between Kepler and Bürgi?

8. How did John Napier define *logarithm* in 1614?
http://www.johnnapier.com/table_of_logarithms_005.htm

9. How does your textbook define *logarithm*? (This is a trick question!!!)

10. What are Napier's bones for?

11. Who was Euler and what is he most known for?
<http://www-history.mcs.st-and.ac.uk/history/index.html>

12. What is the link between Dr. John Craig and the development of logarithms?
<http://www.mathpages.com/rr/s8-01/8-01.htm>

13. Briggs was responsible for finding the logarithm of all natural numbers from _____ to _____. Why is that significant?
<http://www-history.mcs.st-and.ac.uk/history/index.html>

14. Describe the significance of Bürgi and Vlacq.
<http://www-history.mcs.st-and.ac.uk/history/index.html>

Appendix I: History of Mathematics Project Created by Shirley Corson

**Advanced Algebra and Trig
Spring Semester Project Info
“Mathematicians”**

- http://www-history.mcs.st-andrews.ac.uk/Indexes/Full_Alph.html
- Poster size 24” x 18”
- Picture of person
- Birthplace and date
- Date of death
- Country
- Major contributions to mathematics
- Quotations
- “Illustrations” of contributions, e.g., curves, graphs, formulas

- | | |
|----------------------------|---------------------------|
| 1. Archimedes of Syracuse | 17. Fermat, Pierre de |
| 2. Aristotle | 18. Fourier, Joseph |
| 3. Bacon, Roger | 19. Galois, Evariste |
| 4. Banneker, Benjamin | 20. Gauss, Carl Friedrich |
| 5. Bernoulli, Daniel | 21. Groot, Johannes de |
| 6. Bruno, Giuseppe | 22. Mobius, August |
| 7. Cantor, Georg | 23. Mandelbrot, Benoit |
| 8. Copernicus, Nicolaus | 24. Napier, John |
| 9. da Vinci, Leonardo | 25. Nash, John |
| 10. Descartes, Rene | 26. Newton, Sir Isaac |
| 11. Einstein, Albert | 27. Nightingale, Florence |
| 12. Eratosthenes of Cyrene | 28. Pascal, Etienne |
| 13. Euclid of Alexandria | 29. Polya, George |
| 14. Euler, Leonard | 30. Pythagoras of Samos |
| 15. Faraday, Michael | 31. Schoenberg, Isaac |
| 16. Fibonacci, Leonardo | 32. Venn, John |

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