

ABSTRACT

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Classic decision theories assume that gamble payoff and probability are independent in subjective evaluation. This study argues that when a simple gamble is paired with a sure outcome, the independence assumption is violated. Important aspects of this study, unlike most others in the literature, were (1) no parametric assumptions were made in evaluating independence, and (2) participants were paid according their performance in an attempt to motivate to seek maximum gain and minimum loss. Experiment 1 used a new procedure, relays, to test independence. Then three dependent models, which used functional expressions, were proposed and evaluated in Experiments 1 and 2 to pinpoint a two-way interaction pattern in which the contrast along one attribute of a gamble, either payoff or probability, influences the evaluation of the other. The results showed that the contrast between a gamble payoff and a sure outcome is more likely to affect the evaluation of probability than the other way around. This phenomenon and the accompanying uncommon value and weighting

functions may be explained by the task by which participants were encouraged to gain much and lose less by contingent final payment.

DEPENDENT EVALUATION OF PAYOFF AND PROBABILITY IN CHOICE

By

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Dedication

Special thanks are due to Dr. Thomas S. Wallsten, my advisor, for extensive discussion of theories and models analyzed in the dissertation.

Table of Contents

Dedication	ii
Table of Contents	iii
List of Tables	iv
List of Figures	v
Chapter 1: Introduction	1
Dependent Evaluation of Payoff and Probability in Choice	1
Assumption of Independence between Payoff and Probability	2
Testing Independence between Subjective Payoff and Probability Weight	6
Chapter 2: The Experiments	11
Experiment 1	11
The Contrast within Payoff or Probability	17
Models Associated with Dependence	19
Maximum Likelihood Estimators and Model Comparisons	23
Experiment 2	29
Chapter 3: Conclusions and General Discussion	35
Appendices	61
Appendix A	61
Appendix B	62
Appendix C	63
Appendix D	65
Footnotes	66
References	67

List of Tables

Table 1. The Results of Minimum Window of 41 Participants in Experiment 1	39
Table 2. Pair-Wise Model Comparisons between Higher Ranked and Nested Models in Experiment 1.....	40
Table 3. The Median (and Inter-Quartile) Values of Parameter Estimates in Four Models in Experiment 1.....	41
Table 4. Pair-Wise Model Comparisons between Higher Ranked and Nested Models in Gain Domain in Experiment 2.....	42
Table 5. Pair-Wise Model Comparisons between Higher Ranked and Nested Models in Loss Domain in Experiment 2.....	43
Table 6. The Median (and Inter-Quartile) Values of Parameter Estimates in Four Models in the Gain Domain in Experiment 2.....	44
Table 7. The Median (and Inter-Quartile) Values of Parameter Estimates in Four Models in the Loss Domain in Experiment 2.....	45
Table 8. The Median (and Inter-Quartile) Values of Parameter Estimates for sub-group 2 in the Gain Domain in Experiment 2.....	46
Table 9. The Median (and Inter-Quartile) Values of Parameter Estimates for sub-group 3 in the Gain Domain in Experiment 2.....	47

List of Figures

Figure 1. An example of monotonic (left panel) and non-monotonic (right panel) value functions. The square points came from one relay and the triangles from another. The circled pairs of points differ in their monotonicity relation.	48
Figure 2. Display of an experimental trial.	49
Figure 3. The flowchart of the dynamic procedure of relays. For relay i , each iteration includes a gamble with a fixed probability and a payoff obtained from CE of the previous gamble within that relay (except for the first gamble, which has a payoff 10,000). The initial sure outcome of the new gamble is a random number between 0 and the gamble payoff. Then paired sure outcome is adjusted up and down through PEST in the next several trials until the certainty equivalent of the gamble is obtained. There are six iterations in one relay (except for the relay with p of 5%).	50
Figure 4. Illustration of minimum window size. Data points A and B represent minimized maximal inverse relationship. Weak monotonicity is satisfied only when the two data points move to the center of the rectangle, indicated by the arrows. Strong monotonicity is achieved for any two data points if either moves further beyond the center. Thus, $\frac{1}{2}$ of the full height and width of the rectangle are the two dimensions of the minimum window).	51
Figure 5. The structure of models with highest ordered model on the top and lowest at the bottom.	52
Figure 6. The empirical distribution of the m.l.e. of parameter α in Experiment 1. P represents for prospect theory; I for interaction model; PW for probability-weighted mode and VW for value-weighted model.	53
Figure 7. The empirical distribution of the m.l.e. of paramete β . P represents for prospect theory; I for interaction model; PW for probability-weighted mode and VW for value-weighted model.	54
Figure 8. The subjective value $v(x)$ is adjusted by the power $(1 + \gamma(1 - p))$ in the interaction and probability-weighted models. Higher values of γ increase the subjective feeling of the payoff. (The curves are based on $\alpha = 1.0743$ and fixed $1 - p = 0.95$).	55
Figure 9. The subjective value of payoff is adjusted by the probability p of the gamble (Take $\gamma = -0.11725$ as one example, The lines from low to high represent for the subjective payoff values adjusted by probability p equals 0, 0.2, 0.4, 0.6 0.8 and 1, respectively).	56

Figure 10. The probability weight is adjusted by the power $(1 + \delta \frac{s}{x})$. Higher δ tends to decrease the evaluation of the probability (Take $\beta = 1.4717$ and ratio $\frac{s}{x} = 0.1$ as one example)..... 57

Figure 11. The weighting function adjusted by the ratio of the sure outcome to the gamble payoff. (Take $\beta = 1.4717$ and $\delta = 2.197$ as one example, the lines from left to right represent the probability weight adjusted by ratios of 0.05, 0.1, 0.3, 0.6, 0.9, 1, respectively.)..... 58

Figure 12. The empirical distribution of the m.l.e.s of α in the gain domain. P represents the prospect theory estimates; I the interaction model estimates; PW the probability-weighted model estimates and VW the value-weighted model estimates..... 59

Figure 13. The empirical distribution of the m.l.e.s of β in the gain domain. P represents the prospect theory estimates; I the interaction model estimates; PW the probability-weighted model estimates and VW the value-weighted model estimates..... 60

Chapter 1: Introduction

Dependent Evaluation of Payoff and Probability in Choice

Classic decision-making theories, such as expected utility theory and prospect theory assume that evaluations of multiple alternatives are independent, that is, any effect produced by either payoff or probability does not influence the subjective value of the other. Take one gamble as an example: According to expected utility theory, the expected utility of the gamble $(x_1, p_1; x_2, p_2)$, which means to gain x_1 with probability p_1 or to gain x_2 with probability $p_2 = 1 - p_1$, expressed by $EU(g) = v(x_1)p_1 + v(x_2)p_2$, where $v(x)$ is a real-valued utility function. According to prospect theory, the expected utility is evaluated by $EU(g) = v(x_1)w(p_1) + v(x_2)w(p_2)$, where $w(p)$ is the subjective weight accorded to probability p . It is not necessary in prospect theory that $w(p_1) + w(p_2) = 1$ when $p_1 + p_2 = 1$. Even though the two theories take the probability of each payoff differently (and differ in certain other ways), both agree that the subjective magnitude of payoffs x_i are always unique and will not be affected by the probabilities of occurrence. Similarly, the subjective impact of probabilities is unique as well and is independent of the payoff they modify¹.

In the current study, I argue that payoffs and probabilities are not evaluated independently in choice situations in which a simple gamble is paired with a sure outcome and one of them is chosen according to preference. In other words, the contrast between two options along one attribute, either payoff or probability, will

affect the subjective value of the other in the gamble. Even more, the strength of this cross-attribute influence is goal oriented. Thus, the payoff contrast affects the subjective value or weight of the gamble probability if the payment is contingent on performance.

This paper is arranged as follows: In first section, I introduce the independence assumption between payoff and probability in decision making and discuss its wide influence. The second section develops a novel method, the relay, to test whether subjective values of payoff and probability weight are independent in choice. The third section presents two experiments that test whether payoff and probability are evaluated independently. Three dependent models are proposed as alternatives to the independent model, prospect theory. Experiment 1 uses the relay system developed in the preceding section. Experiment 2 uses a different design to test conclusions derived from applying the models to Experiment 1. The final section discusses the results and conclusions.

Assumption of Independence between Payoff and Probability

As a descriptive decision model, prospect theory follows the normative models of rational choice by assuming that payoff and probability are evaluated independently in valuing a gamble. Both prospect theory and expected utility theory take the parsimonious formula $EU(g) = \sum_i v(x_i)w(p_i)$ to calculate the expected utility of the gamble regardless of other potential options (In expected utility theory, $w(p) = p$.) This formula suggests two different types of independence. One is the independence between options: the evaluation of one gamble is not influenced by the

presence of other options. This means that decision makers independently evaluate the options in hand, and any similarity or contrast between options does not influence the evaluation of either option. This type of independence implies that for any set of options, a, b, c , and d , $p(a, b) > p(c, b)$ if and only if $p(a, d) > p(c, d)$, where $p(x, y)$ reads “the probability that x is preferred to y ”. This independence is proved to be equivalent to strong stochastic transitivity (Tversky & Russo, 1969). The second type of independence states that probability and payoff independently contribute to the evaluation of a gamble without any interaction between two attributes. Any change in one attribute, either payoff or probability, does not change the subjective magnitude of the other. Note that first type of independence could be the premise of the second type if dependence of attributes is caused by the contrast among options.

The fundamental assumption of independence between the subjective payoff $v(x)$ and the probability weight $w(p)$ is widely accepted. Researchers in psychology and economics interested in the functional form of payoff and weight assume independence of $v(x)$ and $w(p)$. The most widely acceptable form for the value function, $v(x)$, where x is the change of wealth in prospect theory, is that it is concave above the reference point (which is 0) and convex below it. That is, $v''(x) < 0$, for $x > 0$ and $v''(x) > 0$, for $x < 0$ (Galanter & Pliner, 1974; Fishburn & Kochenberger, 1979; Kahneman & Tversky 1979). And the most widely accepted form of the weighting function, $w(p)$, is that humans overweight lower probabilities ($w(p) > p$ for low p) and underweight higher probabilities ($w(p) < p$ for high p) (Tversky & Kahneman, 1992; Camerer & Ho, 1994; Tversky & Fox, 1995; Wu &

Gonzalez, 1996; Prelec, 1998; Wu & Gonzalez, 1999). Earlier studies along this line usually assumed independent functional forms of value and weight before fitting the data (Tversky et.al., 1992; Camerer et.al., 1994; Tversky et.al, 1995), while later studies avoided prior assumptions of functional forms. For instance, Wu and Gonzalez (1996, 1999) used a non-parametric procedure to estimate values and weights. If any specific functional form fits the nonparametric estimates well, it is chosen for parametric estimation. But their nonparametric method still depends implicitly on the assumption of independence. Gonzalez & Wu (1999) developed an algorithm by treating levels of $v(x)$ and $w(p)$ as parameters and then used an alternating least square method to estimate them.. More recently, Abdellaoui (2000) elicited value and weighting functions with a parameter-free method. He first constructed a sequence of outcomes equally spaced in subjective value using the “trade-off” method (see Wakker & Deneffe, 1996), and then used the sequence of outcomes to obtain a sequence of probabilities equally spaced in terms of probability weighting. This procedure is like conjoint measurement, which establishes the required intervals on variable A to compensate a certain difference on interval B and then establish the equal intervals on A in a similar fashion. However, just as with conjoint measurement, Abdellaoui’s parameter-free method required the independence assumption. This assumption is strong and it matters when the dependence is real.

This fundamental assumption in decision theories has been studied. Tversky (1967) reported the strict additivity, which implies independence between the payoff and probability, However, in that study, participants were asked to write out the

gamble's selling price (indifference value) which ultimately was determined not only by the gamble itself, but also by the buyer's (the experimenter's) distribution of buying prices. In that sense, the selling price is not an indifference price because it is affected by the buying price and possibly by the participant's perception of its probability distribution. The second issue that may affect the conclusion of independence is that two-factor analyses of variance were conducted after logarithmic transformations of risky as well as riskless bids, with significance levels set at 0.1. The decision about interactions, thus, is a matter of setting the alpha-level of the test. A stricter criterion results on interactions for more persons. In addition, studies by Slovic (1966) and Irwin (1953) found that the value (payoff) or the desirability of the event influenced its subjective probability.

More recently, other researchers investigated the dependence between options. The stochastic difference model (Gonzalez-Vallejo, 2002) pointed out that the comparison occurs at the level of attributes rather than options. Operating at the level of attributes, the function sums proportions of the difference between two options relative to the larger value. The contrast weighting model (Mellers, Chang, Birnbaum and Ordóñez, 1992) states that the weight of an attribute depends on the difference (similarity) of the two alternatives on that dimension. Later, some studies looked at the influence of dimension contrasts on the other dimensions. For example, decision field theory (Busemeyer and Townsend, 1993; Roe, Busemeyer and Townsend, 2001) focused on relative attractiveness between options. Attribute values are compared across options and weighted differences are summed across attributes to produce momentary valence for each option. Mellers and Biagini (1994) investigated

interactions between payoff and probability, which indicates the violation of the second type of independence. They proposed a second version of contrast weighting theory, which states that the judged strength of preference for simple gamble a over b is expressed as $S(a,b) = J[u(x_a)^{\alpha(p)} s(p_a)^{\beta(x)} - u(x_b)^{\alpha(p)} s(p_b)^{\beta(x)}]$, where $u(x_a)$ and $u(x_b)$ are the subjective payoffs associated with the amounts to win; $s(p_a)$ and $s(p_b)$ are the probability weights of winning. The subjective payoffs are weighted by $\alpha(p)$, the probability contrast; the subjective probabilities are weighted by $\beta(x)$, the payoff contrast.

In the current study, I argue that probability weight and subjective payoff are dependent when a simple gamble is paired with other options in choice, in particular when the participants' payoffs depend on their performance. The experiments focus on the simplest version of choice paradigms, in which decision makers choose between a simple gamble and a sure outcome. That is, given a pair of choices that include one sure outcome, s , and one simple gamble $(x, p; 0)$, which means gaining x with probability p otherwise nothing, participants are asked to choose which one they prefer. With this paradigm, let us first look at whether independence holds between payoff and probability in choice.

Testing Independence between Subjective Payoff and Probability Weight

Prospect theory provides a framework for testing the independence between subjective payoff and probability weight in choice. Two important assumptions in prospect theory are independence of subjective payoff and probability weight, and that value and weighting functions are monotonically increasing. Independence can

be inferred from the basic seminal formula $U(x_1, p_1; \dots; x_i, p_i) = \sum_i v(x_i)w(p_i)$, which implies that any change of payoff or subjective value will not influence the probability weight and vice versa. The second condition, monotonically increasing value and weighting functions, indicates that $v(x_1) > v(x_2)$ if and only if $x_1 > x_2$ and $w(p_1) > w(p_2)$ if and only if $p_1 > p_2$. This property can be inferred from the functional forms used in prospect theory (Tversky et. al., 1992).

In prospect theory, independence and monotonicity exist simultaneously. I test independence following the logic of reductio ad absurdum. In detail, the testing starts from the premise of independence between subjective payoff and probability weight. Coupled with prospect theory independence implies certain relationships between variables. The aim is to find a solution that makes these relationships meet the condition of monotonicity. If no solution exists, the independence assumption is rejected.

One novel method, which I call relays, is developed for evaluating independence between subjective payoff $v(x)$ and probability weight $w(p)$. The advantage of using relays is that the relationship between x and p can be examined without assuming any functional forms for $v(x)$ and $w(p)$. A relay can be constructed by starting from a simple gamble $(x, p; 0)$ denoted by g_1 . An individual can always find a certainty equivalent c_1 such that the expected utilities of g_1 and c_1 are equal, thus, $EU(g_1) = EU(c_1)$. Substitute c_1 for x in gamble g_1 . The new gamble $(c_1, p; 0)$, denoted by g_2 , can also yield a certainty equivalent c_2 . The relationship between g_2 and its certainty equivalent c_2 is expressed by $EU(g_2) = EU(c_2)$. Again, substitute

c_2 for c_1 in gamble g_2 to generate a new gamble $g_3 \sim (c_2, p; 0)$, which has its own certainty equivalent c_3 . The relay continues to yield iterations until the certainty equivalent of the last gamble is too small to be meaningful. Note that probability p in one relay is identical cross iterations.

According to prospect theory, the equation $EU(g_1) = EU(c_1)$ can be expanded into $v(x)w(p) = v(c_1)$, $EU(g_2) = EU(c_2)$ into $v(c_1)w(p) = v(c_2)$, and so on. Then convert the equations into

$$\frac{v(c_1)}{v(x)} = w(p), \frac{v(c_2)}{v(c_1)} = w(p), \quad (\text{Equation 1})$$

Connect the two equations by $w(p)$, yielding $\frac{v(c_1)}{v(x)} = \frac{v(c_2)}{v(c_1)} = w(p)$. A more general relation among n iterations within one relay can be expressed by

$$\frac{v(c_1)}{v(x)} = \frac{v(c_2)}{v(c_1)} = \dots = \frac{v(c_n)}{v(c_{n-1})} = w(p) = r, \quad (\text{Equation 2})$$

where $w(p)$ is an unknown constant, and I take it as one parameter, denoted by r .

The gambles in one relay keep the same probability p but vary their payoffs. Eight different probabilities can be used to construct 8 different relays with different values p and r , denoted by p_1, p_2, \dots, p_8 and r_1, r_2, \dots, r_8 , respectively. Each relay provides several gambles that follow the relationship in Equation 2. Using the subscript i to denote relays, where $i = 1, 2, \dots, 8$, Equation 2 can be rewritten to

$$\frac{v(c_{i,1})}{v(x)} = \frac{v(c_{i,2})}{v(c_{i,1})} = \dots = \frac{v(c_{i,n})}{v(c_{i,n-1})} = w(p_i) = r_i. \quad (\text{Equation 3})$$

If the relays all start from a gamble with payoff x and probability p_i . $v(x)$ is an unknown constant. Divide $v(c_{i,j})$ by $v(x)$ in order to normalize $v(x)$ to 1. Then the relations in Equation 3 can be converted into $v'(c_{i,1}) = r_i$, $v'(c_{i,2}) = v'(c_{i,1}) \times r_i = r_i^2$... $v'(c_{i,n}) = v'(c_{i,n-1}) \times r_i = r_i^n$, where $v'(c_{i,j})$ is the normalized value of $v(c_{i,j})$. Keep in mind that all $v'(c_{i,j})$ can be expressed by r_i^j via parameters r_i . Recall that the value function $v(c_{i,j})$ is an increasing function of $c_{i,j}$. So, too, is the normalized value of it: $v'(c_{k,l}) > v'(c_{g,h})$ if and only if $c_{k,l} > c_{g,h}$.² In the experiment, $c_{i,j}$ is observable. Plotting $v'(c_{i,j})$ against $c_{i,j}$ is equivalent to plotting r_i^j against $c_{i,j}$. According to prospect theory, the data points are supposed to meet the requirement of monotonicity. Now, the task is to find a solution for the parameters r_i that satisfies the monotonicity of $v'(c_{i,j})$, and also ensures that $v(x)$ is a monotonically increasing function of x . Note that r_i is the subjective weight of p_i , that is $r_i = w(p_i)$. The values of r_i are subject to the limit of monotonicity of the weighting function regardless of its functional form. If there is at least one set of solutions for the r_i s, then it may be concluded that the assumption of independence between subjective payoff and probability weight is not violated. However, if no solution can be found, then the independence assumption must fail because it cannot co-exist with the condition of monotonicity of value and weighting functions.

Take two relays, as one example, to explain how the parameter search works. Data points from two relays are plotted into one graph (see Figure 1). The data marked by squares are from one relay with probability p_1 and data marked by

triangles are from the other relay with probability p_2 . The slashed lines link normalized subjective payoffs to corresponding values of $c_{i,j}$, which are observable. The left panel in Figure 1 demonstrates perfect monotonicity in which normalized subjective payoffs monotonically increase along with the objective value x . The right panel displays a non-perfect situation in which monotonicity is not met. To compare the points in the circle on each panel, the point represented by the square is off the monotonous track unless the parameter r_2' (in the right panel of Figure 1) is adjusted to r_2 (in the left panel of Figure 1). When it is impossible to find an appropriate value for parameter r_1 or r_2 , then independence fails under the constraint of monotonicity. Experiment 1 collected data for eight relays to examine the independence assumption.

Chapter 2: The Experiments

Experiment 1

Experiment 1 was designed to test the independence of subjective payoff and probability weight. The test details are presented first, and then a set of dependent models is proposed and compared with prospect theory based on the data from Experiment 1.

Participants:

Fifty-five participants, 23 male and 32 female, all undergraduate students from the University of Maryland at College Park, participated in the experiment to earn credit for completing psychology research. Among them, 41 participants completed the entire experiment and their data were used in all analyses. The other 14 participants took a pilot experiment that yielded incomplete relays. Their data were usable in subsequent model comparisons, but not in the initial analyses.

Stimuli and Procedures:

Participants viewed the stimuli displayed on the computer screen. During each trial, a pair of options, one simple gamble and one sure outcome were presented (see Figure 2) and participants clicked on the option they preferred. Sure outcomes and simple gambles were used in this study to simplify the choice task and to avoid causing confusion. Gambles with two or more branches are usually more difficult to process, and the complicated stimuli and procedure prevent individuals from making accurate decisions.

As mentioned in the previous sections, eight probabilities yielded eight relays. The probabilities under study were 5%, 20%, 30%, 40%, 50%, 70%, 90% and 95%.

The payoff of the first gamble in each relay started from $x=10,000$. The sure outcome was an integer randomly generated between 0 and x , the payoff of the gamble. The method of *parameter estimation by sequential testing* (PEST) (Bostic, Herrnstein and Luce, 1990) was adopted to derive the certainty equivalents of gambles. For details of this method, please see the description in Appendix B. Recall that certainty equivalents in one iteration of a relay were used as the payoffs of gambles in the next iteration within that relay. Thus, the payoff of the second gamble in each relay was determined only after the certainty equivalent of the first gamble in that relay was obtained, and so forth. Six iterations were derived for each of the eight probabilities (relays) except for 5% since certainty equivalents in the relay with probability 5% always ended up very small. The sure outcome and payoff of the gamble are not meaningful if their values are lower than 1. So, for relays with probability of 5%, 5 or fewer iterations were conducted for each person. Figure 3 illustrates the procedure for each relay. Trials from different relays show up alternatively.

Before the formal experiment, participants received training and tried a few trials until they understood and felt comfortable with the experiment. The participants were informed beforehand that their goal is to accumulate as many points as possible and that their performance would determine how much extra money they could earn in addition to the academic credit. The participants understood they should choose their favored options in order to earn points, and computers played gambles once they were chosen. The results were not disclosed until the end of the experiment. Gambles from the eight relays were displayed on the screen in random order. For the first two

rounds, gambles irrelevant to the study were interspersed between relay gambles simply as distraction. Refer to Appendix A for instructions.

Results and Discussion

Analyses were conducted at an individual level. A constraint minimization procedure (*sequential quadratic programming* (SQP)) searched for parameter estimates r_1, r_2, \dots, r_8 , where $r_1 < r_2 < \dots < r_8$, that minimized the sum of inverse relations, $\sum_{i,j} s_{i,j}$, to zero, where $s_{i,j}$ is defined as follows: for any two point values c_i , and c_j

$$s_{i,j} = \begin{cases} v'(c_i) - v'(c_j) & \text{if } c_i < c_j \text{ and } v'(c_i) > v'(c_j) \\ 0 & \text{otherwise} \end{cases}$$

Note that standardized value function $v'(c)$ is monotonically increasing if and only if the value function $v(c)$ is monotonically increasing. If there is at least one set of solutions, r_i , I can conclude both (1) subjective value and probability weight are monotonic functions, and (2) the two functions are independent of each other.

The results of the optimization (refer to Appendix B for the details of the optimization procedure) showed that none of the 41 participants had a single set of solutions. Thus, two primary assumptions necessary for prospect theory can not hold simultaneously. Either (1) payoff and probability do not combine independently, (2) the weighting and value functions are not monotonically increasing, or both.

However, monotonicity of value and weighting functions is more straightforward, and it is supported by the study (Galanter et.al., 1974) on the shape of the value function. So, when two assumptions can not hold simultaneously, I question the validity of the independence assumption.

Assuming that independence actually holds, violation of it may occur just due to stochastic variability. One way to take variability into account is to relax the requirement of perfect accuracy in evaluating independence. Remember that data points from eight relays were plotted on one graph of $v'(x)$ versus x . To reduce precision, I thinned the data points at two levels. The first level was to thin the adjacent data points so that no two were closer than 50 along the x -axis. The procedure started with the point corresponding to the lowest x value; the adjacent point was removed if it was less than 50 points away from the first point. This procedure continued through the highest x . The second level was to thin adjacent points no closer than 100 along the x -axis.

We searched for solutions again after thinning the data. The results showed that at first level, 8 out of 41 participants had at least one set of solutions; at the second level, 17 out of 41 had solutions. However, thinning the data reduced the number of data points to a very low level. On average, 65% and 49% of original data points remained at the first and the second levels, respectively. And the results after thinning the data were not very good in terms of the proportion of participants who had solutions, 19.5% on the first and 41.5% on the second level. Hence, although some individuals had solutions after thinning the data, the solutions at these levels are not quite valid because thinning data not only eliminates random error due to the cognitive fluctuation but also eliminates systematic error due to canceling too many data points. To be conservative, I measured the minimum degree of errors that yield results consistent with independence.

Minimal window size. To what extent does the tolerance of error or fluctuation guarantee the independence? To answer this question, I defined a two-dimensional (height, width) window center for each pair of inverted points in the plot of $v'(x)$ versus x , (see Figure 4). The height was defined by $h = v'(x_1) - v'(x_2)$ and the width as $w = x_2 - x_1$, where $x_1 < x_2$ and $h > 0$. The optimization procedure searched for r_i that minimized $\sum h$. I denoted the solution by h^* , where $h^* = \max(h)$. The magnitude of h^* must be interpreted relative to w^* , the width of that window yielding a solution center at (h^*, w^*) . I choose to define the the minimum window size, instead $(0.5h^*, 0.5w^*)$, as those values that indicate the minimum movement required of the points to yield weak monotonicity. Any further movement yields strong monotonicity. Moving the data points within this two-dimensional window would guarantee that all the reverse relationships can convert to follow strong monotonicity, except the maximal one represented by data points A and B in Figure 4. The minimal window size is the minimal movement required by data points A and B to follow weak monotonicity. The larger the window the greater tolerance the data requires in restoring monotonicity. Independence holds only when monotonicity exists. However, to my knowledge, no previous studies show how large a window is acceptable for accommodating the error or noise from behavioral data. The minimum window size was estimated at an individual level. Note the width of the window is along the scale of payoff, x , and the height of the window was the normalized subjective value of payoff, $v'(x)$. Their scale units are different, so the absolute value of window size is not meaningful. Instead, the relative size against the full scale is

reported in Table 1. The maximum value of payoff, x , in the current study was 10,000. The maximum value of scaled subjective value of payoff, $v'(x)$, was 1.

Summary. All behavioral data are subject to random error, which is why I conducted complementary analyses. Instead of concluding immediately that independence between payoff and probability was violated in choice, I looked for the tolerance of independence. After thinning the data, only 65% (at 50-point thinning) and 49% (at 100-point thinning) of the original data points remained and only 19.5% and 41.5% of the participants achieved independence at two different levels, respectively. These results hardly lead to the conclusion that independence holds. Look at the alternative window method: the width of the window demonstrated a great inverse between data, from 78 to 4938, and the distribution of this inversion was approximately uniformly distributed. Compared with the width, the height of the window is less intuitive because the normalized subjective value of payoff is not an absolute number with a natural unit. Even though the relative height was very small, from 0.001% to 0.94% of the scale, the majority of participants do not share the small window height of 0.001% of the original scale because the distribution of height was an approximately uniform. Even more so, the window height ought to be offset by the larger width. The window size was regarded as small only when both height and width were small.

Considering that neither the results from thinning data nor the results from minimal window size favored the independence assumption, I am reluctant to accept the independence. To the contrary, I prefer to place doubt on the argument of independence between payoff and probability in choice.

Despite noise in the data, the results strongly suggest that payoff and probability do not combined independently. To pinpoint the nature of the possible dependence, I proposed three stochastic models carrying different versions of dependence. The dependence could be a one-direction influence or a two-way interaction between two attributes, payoff and probability. The contrast within each attribute was used to specify the nature of the dependence. Each model was compared to prospect theory, one representative of the family of independence models.

The Contrast within Payoff or Probability

The idea that contrasts within attributes regulate preference can be found in previous studies. The stochastic difference model (Gonzalez-Vallejo, 2002) argues that subjects are sensitive to attribute differences and the advantage/disadvantage of a certain option is a linear combination of attribute differences. For example, given two options x and y , denoted by $x \sim (a, p)$ and $y \sim (b, q)$, where a and b are values from dimension A, p and q are from dimension P, the decision maker prefers x over y if and only if the combination of attribute differences,

$$d = \frac{\max(|a|, |b|) - \min(|a|, |b|)}{\max(|a|, |b|)} - \frac{\max(|p|, |q|) - \min(|p|, |q|)}{\max(|p|, |q|)} > \delta + \varepsilon,$$

where $a > b$ and $p < q$, δ is the threshold and ε is the random error. Dimension A can represent payoffs and dimension B can represent probabilities of gambles. The stochastic difference model emphasizes that comparisons between options begin at the attribute level rather than by evaluating each option independently before making a comparison.

The contrast-weighting model (Mellers, Chang, Birnbaum and Ordóñez, 1992) takes the difference of two options within each attribute as the weight of that attribute. Weights are monotonic with differences along dimensions. Later, Mellers and Biagini (1994) proposed another version of a contrast weighting model in which the similarity of levels along one dimension enhances the weight of the other dimension. The authors asserted that this version, in addition to accounting for similarity effects and violations of strong stochastic transitivity, better captures the intuition that similarity along one dimension enhances differences on another dimension. To present this new version, let us to see one example: Consider two simple gambles a and b , in which gamble a has probability p_a to earn x_a , otherwise nothing; and gamble b has probability p_b to earn x_b , otherwise nothing . A very general expression for the judged strength of preference for gamble a over b , denoted by our notation is

$$T(a, b) = f[v(x_a)^{j(p)} w(p_a)^{k(x)} - v(x_b)^{j(p)} w(p_b)^{k(x)}],$$

where $v(\cdot)$ is the subjective value of the payoff and is weighted by $j(p)$, the probability contrast; $w(\cdot)$ is the probability weight and is weighted by $k(x)$, the value contrast of the gambles. Mellers and Biagini declared that the second version of the contrast weighting model seemed to better capture the intuition of similarity and fit the data better.

The current study stands with the second version of the contrast-weighting model because it specifies the contrast effect across dimensions by raising the contrast on one dimension to a power dependent on the evaluation of the other, while other theories mentioned above mainly look at the contrast effects with dimensions. To

explore the possible dependence cross the dimensions (payoff and probability), I first acknowledge that the attribute differences between two options are important and then further expect that the difference along one dimension influences the weights of the other dimension. There are three possible ways to qualify the cross-dimension influence. Two possibilities are that either payoff or probability influences the subjective evaluation of the other dimension, but not both. The third possibility is the two-way interaction that probability weight is affected by the payoff contrast and simultaneously the subjective payoff is influenced by the probability contrast. The goal now is to determine whether dependent models perform better than independent models, and if the answer is yes, which possibility of cross-dimension influence works the best. For this purpose, define an attribute contrast as the ratio of the absolute difference in attribute values of the two options relative to the greater of the two values. I believe that this relative difference better embodies the contrast between two values since relative values as it eliminates concern regarding different units across attributes.

Models Associated with Dependence

The pairs of stimuli in Experiment 1 included one simple gamble $g \sim (x, p; 0)$ and one sure outcome s . The utility of a simple gamble is expressed most generally by $u(g) = v(x)^{j(p)} w(p)^{k(x,s)}$. The weights $j(p)$ and $k(x,s)$ convey the influence from the contrast between the simple gamble and the sure outcome along the attributes of probability and payoff, respectively. Dependent models assume that contrasts apply to the utility of the simple gamble but leave the utility of the sure outcome unchanged, which is expressed by the original $v(s)$. In other words, the presence of the sure

outcome affects the relative evaluation of the gamble but not conversely. Intuitively, that is acceptable because the sure outcome is one element without uncertainty or risk, so relative evaluation is less necessary for sure outcomes than for gambles.

The weight $j(p)$ is a function of the contrast along the attribute of probability. The relative contrast along the attribute of probability is

$$\frac{\max(p,1) - \min(p,1)}{\max(p,1)} = 1 - p.$$

Weighting this contrast by γ , the exponent $j(p)$ can

be expressed as:

$$j(p) = 1 + \gamma(1 - p), \quad (\text{Equation 4})$$

where the parameter γ describes the magnitude of the influence of the probability and $-1 < \gamma < +\infty$. The lower bound of γ is no less than -1, such that the $v(x)^{j(p)}$ keeps the evaluation of the payoff monotonically increasing. The sign of the parameter γ reflects the direction of the change of the subjective payoff. If $p=1$, then the gamble becomes a sure outcome and $j(1) = 1$, which means that the utility of the sure outcome x is original $v(x)$. This is consistent with the assumption that contrasts only influence the evaluation of gambles not sure outcomes. If $p = 0$, $j(p) = 1 + \gamma$. However, when $p = 0$, there is no gamble, as the outcome is a sure 0.

The weight $k(x,s)$ is a function of the contrast along the attribute of payoff. Gambles in the experiment always have payoffs greater than the paired sure outcome,

so $\frac{\max(x,s) - \min(x,s)}{\max(x,s)} = \frac{x-s}{x} = 1 - \frac{s}{x}$. The constant 1 can be ignored for simplicity

because ratio $\frac{s}{x}$ is the key factor adjusting the probability weight. The exponent

$k(x, s)$ is expressed by:

$$k(x, s) = 1 + \delta \frac{s}{x} \quad (\text{Equation 5})$$

where δ represents the magnitude of influence of the contrast along the attribute of payoff and $-\infty < \delta < +\infty$. The higher $k(x, s)$, the lower the probability weight is since the original $w(p)$ is less than 1 when $p < 1$.

Following numerous studies on value and weighting functions, I adopt two general and flexible formulas for $v(x)$ and $w(p)$. In accordance with prospect theory, I set

$$v(x) = x^\alpha, \text{ where } x > 0 \text{ and } 0 < \alpha < +\infty. \quad (\text{Equation 6})$$

The one-parameter weighting function in Prelec's (2000) study provides the basic functional form for probability weight in this study:

$$w(p) = \exp[-(-\ln p)^\beta], \text{ where } 0 < p < 1 \text{ and } \beta > 0 \quad (\text{Equation 7})$$

When $0 < \beta < 1$, the probability weight is inversely S-shaped; if $\beta > 1$, the probability weight is a S-shaped function of objective probability. Obviously, when $\beta = 1$, it is a linear function. As some readers may note, Prelec defined β as $0 < \beta < 1$. I extend Prelec's original one-parameter weighting function by allowing $\beta > 1$ to reflect a variety of shapes of weighting functions in order to accommodate individual differences. I did not employ Prelec's two-parameter weighting function, $w(p) = \exp[-\lambda(-\ln p)^\beta]$, because the only relaxation of the two-parameter over the

one-parameter version is that diagonal concavity is not required. In brief, diagonal concavity says that the function is concave where there is overweighting and convex where there is underweighting. Since diagonal concavity is supposed to be a characteristic of the weighting function, there is no benefit in relaxing that requirement. In addition, the one-parameter version crosses the diagonal at $1/e$, which is stated to be very close to the empirical functions in several studies (Prelec, 2000). So varying the position of the reflection point by adding one more parameter is not necessary. In sum, the one-parameter version makes the weighting function as simple as possible, meanwhile, it keeps the essential characteristics of empirical weighting functions.

As we know, prospect theory takes the simple gamble utility as $v(x)w(p)$.

The complete version of the dependent model in this study is called interaction model, which is expressed as

$$u(x, p, s) = v(x)^{j(p)} w(p)^{k(x,s)}. \quad (\text{Equation 8})$$

This full interaction model captures a bi-directional interaction. The other two nested dependent models cover uni-directional influences. One of them specifies that the contrast along the attribute of probability affects the subjective payoff evaluation of the gamble. This model is called the probability-weighted model. The expected utility of the gamble is expressed by:

$$u(x, p) = v(x)^{j(p)} w(p). \quad (\text{Equation 9})$$

The other nested model is opposite in that the contrast along the attribute of payoff enhances probability weight. This model is called the value-weighted model, in which the expected utility of the gamble is expressed by:

$$u(x, p, s) = u(x)w(p)^{k(x,s)}. \quad (\text{Equation 10})$$

The relationship of the models is shown in Figure 5:

The parameters of the three dependent models and one independent model (prospect theory) were estimated and each dependent model was compared with the independent model by means of a likelihood ratio test.

The α -level for all tests was set as 0.05. Tests 1 to 4, as numbered in Figure 5, had 1 degrees of freedom (df). Test 5 had $df=2$. Data collected from all trials were involved in the likelihood ratio tests.

Maximum Likelihood Estimators and Model Comparisons

On each trial the strength of preference for the gamble, g , over the sure outcome, s , is defined as

$$T(g, s) = U(g) - U(s) + \varepsilon. \quad (\text{Equation 11})$$

$U(s)$ is the utility of the sure outcome, s , $U(s) = v(s) = s^\alpha$. $U(g)$ is the utility of the gamble and is contingent on the model that it serves. $U(g) = v(x)w(p)$ in prospect theory; $U(g) = v(x)^{[1+\gamma(1-p)]}w(p)$ in the probability-weighted model;

$U(g) = v(x)w(p)^{\frac{(1+\delta^s)}{x}}$ in the value-weighted model; and

$U(g) = v(x)^{[1+\gamma(1-p)]}w(p)^{\frac{(1+\delta^s)}{x}}$ in the interaction model. These four models share the same components, $v(x)$ and $w(p)$, which are defined by Equation 6 and 7,

respectively. The probability of choosing g over s is expressed by

$$P(g, s) = P[U(g) - U(s) + \varepsilon \geq 0] \quad (\text{Equation 12})$$

The item ε reflects the variability in the data. I assume that ε is i.i.d. and $\varepsilon \sim N(0, \sigma^2)$, where $\sigma = \theta \text{var}(g)$, and $\theta > 0$. $\text{Var}(g)$ is the variance of the gamble, which varies from trial to trial due to different gambles. For a simple gamble $g \sim (x, p; 0)$, $\text{var}(g)$ is calculated as $(x - EV)^2 p + (0 - EV)^2 (1 - p)$, where EV is the expected value of the gamble. The parameter θ captures the degree to which the gamble variance can affect the choice. The reason for considering the variance of the gamble is that several previous studies identified the importance of gamble variance in choice. Edwards (1962) pointed out the concept of *variance preference* and claimed that people might base their preferences among bets not only on the first moment of the distribution of outcomes, such as the mean, expected value or the subjective expected value, but also on the higher moments of the distribution, such as the variance of gambles. Decision field theory (Busemeyer & Townsend, 1993) also took gamble variance as one important component to define the momentary valence, which explains why attention varies from moment to moment. Weber, Shafir and Blais (2004) used the coefficient of variance (standard deviation divided by mean) to predict human and lower animal sensitivity to risk. All of these results suggested that gamble variance is one important factor in human decision-making. $\text{Var}(g)$ may explain why same values of $u(g) - u(s)$ can lead to different choices.

The parameters $\alpha, \beta, \gamma, \delta$ and θ were estimated separately for each individual. On each trial, participants chose either the gamble or the sure outcome. The optimization procedures (see Appendix B) maximized the joint probability of those choices. To avoid local maxima, each model was optimized 500 times per person. Each run used new initial values of parameters randomly generated within

their domains. For those parameters with at least one end of the domain as infinity, I set broad ranges in parameter estimation (Interested readers can refer to Appendix C). The broad ranges were determined by running optimizations with even broader ranges beforehand, which then provided a reasonable range for each parameter. The maximum likelihood achieved in the 500 runs was taken as the maximum likelihood of that model and the associated parameter values were regarded as m.l.e. (maximum likelihood estimators). As shown in Figure 5, the interaction model is a complete model with five parameters, $\alpha, \beta, \gamma, \delta$ and θ . The probability-weighted, value-weighted and prospect theory models are nested under the interaction model. The model with more parameters will fit the data better than nested models do, but not necessarily significantly so. I compared each model to its nested model(s), if any, by likelihood ratio test.

Results and Discussion

The data from all 55 participants were used to do model comparisons at the individual level. The interaction model was compared to the three other models, while the probability and value-weighted models were compared to prospect theory only. The results of the comparisons are shown in Table 2. For the majority of participants (52 out of 55; 94.6% of all participants) the interaction model was a significant improvement over prospect theory. These 52 participants also demonstrated a significant improvement of the probability-weighted and the value-weighted models over prospect theory. The results suggest that dependence models describe choice in this context better than the independence model does. To identify whether the cross-attribute influence is one-way or two-way, it is necessary to compare the interaction

model with the probability and value-weighted models. Twenty-four out of 55 participants (43.6% of participants) showed a significant difference between the interaction and probability-weighted models. Only 10 out of 55 participants (18.2% of participants) showed a significance effect between the interaction and value-weighted models. The results suggest that the value-weighted model performed almost as well as the interaction model did, because 81.8% of participants' data are described by both models with the same accuracy. The probability-weighted model is inferior to these two models because fewer participants (56.4%) showed the same accuracy as the interaction model did. Thus, contrast along the attribute of payoff strongly influenced the evaluation of probability weight, but not the other way around.

The optimization was conducted 500 times per participant per model. Each time the initial values were randomly generated by the computer within the ranges determined in advance. The median and the inter-quartiles values of the m.l.e.s accounting for the best fit of 55 participants are reported in Table 3. The empirical distribution of those m.l.e.s of α and β are plotted in Figure 6 and 7. The median of parameter α has a value close to 2 for all four models. Less than 10% of participants have values lower than 1. The result suggests that the subjective value is a convex function of value, i.e., that the increase of subjective values is faster for high than for low payoffs. The median of parameter β is greater than 1 in all models except prospect theory, where β is almost 1. For the other three models fewer than 30% of the participants had values less than 1. This result indicates that for most participants

probability weight is an S-shaped function rather than inversely S-shaped weighting function, as reported by previous studies.

Parameters α and β in Experiment 1 only describe human behavior in the gain domain, but showed different trends for $v(x)$ and $w(p)$ compared to prospect theory results from numerous previous studies (Tversky et.al. 1992; Camerer et.al, 1994; Tversky et.al, 1995; Wu et.al, 1996; Prelec, 1998; Wu et.al, 1999). Comparing Experiment 1 with other studies, one primary difference is that a clear task goal was identified in Experiment 1. Participants were encouraged to earn as many points as they could, and they acknowledged that their performance would determine the final amount of bonuses in addition to guaranteed academic credits. Also, participants' performances would determine if they could upgrade to the next experiment (not reported in this study). By entering the next level, participants had a chance to gain extra credits. On the contrary, the common instruction in previous studies usually asked participants to choose one favored option out of a pair for a number of trials. For instance, some studies on the weighting function (Wu et.al. 1996, 1999) paid participants \$5 to finish a survey, in which participants were asked to choose the favorable option. There was no real link between performance and obtaining rewards. The real pressure in the current study might cause participants to crave more points, thus over-evaluating the payoff, in particular higher possible payoffs. Meanwhile, participants underweighted lower probability, such as 5%, and overweighted higher probability, such as 95%. One possible reason for this phenomenon is that participants wanted to earn some points on each trial because of the goal. Low

probabilities, such as 5%, were regarded as essentially 0%. High probabilities were regarded as close to 100%.

The parameter γ in the interaction and probability-weighted models was negative. Recall that γ adjusts the subjective value of the payoff via formula $v(x)^{[1+\gamma(1-p)]}$. Increasing the value of γ raises the subjective payoff (Figure 8). Similarly, for fixed values of α and γ , increasing p increases the subjective evaluation of payoff x as illustrated in Figure 9. The smaller the contrast of the probability between the sure outcome and the gamble, the higher the adjusted subjective value is. Psychologically, the subjective value of the gamble payoff looms larger as the probability of obtaining this payoff increases. This result is consistent with the intuition that as the probability of earning payoff x increases, not only the overall gamble, but also the payoff of the gamble becomes more attractive.

The parameter δ , which adjusts the probability weight, was positive. The formula $w(p)^{(1+\delta\frac{s}{x})}$ indicates that probability weight decreases as δ increases (Figure 10). The adjusted probability weight curve moves to the right as the ratio s/x increases. Note that a high ratio of sure outcome versus gamble payoff causes underweight of probability. Figure 11 plots 6 adjusted probability weight curves described by $w(p)^{(1+\delta\frac{s}{x})}$, where $\beta = 1.4717$ and $\delta = 2.197$. The most left curve is modified by the ratio $\frac{s}{x} = 0.05$, and the most right by $\frac{s}{x} = 1$. Intuitively, this means that high values of sure outcomes make the payoff of gamble payoff subjectively small. When the sure outcome is small relative to the gamble payoff, the significance of the

probability looms large and reaches its maximum when the sure outcome is too small to be considered. In that case, the power raised to $w(p)$ is close to 1, and the ratio $\frac{s}{x}$ does not adjust the original probability weight at all. It also means that when the gamble is presented without a sure outcome, against which to compare it, the evaluation of the gamble probability will be $w(p)$, which is not affected by a cross-attribute contrast.

Experiment 1 was designed to implement the relay methodology, not to provide data for the subsequent post hoc modeling, thus outcome spacing was unequal and much denser at lower values. Also, the relay method caused outcomes to depend on previous responses, although not sequentially, as the 8 relays were cycled through in random order. Experiment 2 was run to correct these problems and to provide validation of the modeling results. In addition, the loss domain was involved to complement the gain domain.

Experiment 2

Possible dependence between payoff and probability was evaluated again in Experiment 2. The paradigm was similar to that of Experiment 1 except that (1) gamble outcomes were specified independently of responses, (2) gambles were randomly ordered, and (3) losses were included.

The functional forms for the gain domain are same as those in Experiment 1. In the loss domain, however, the value function is expressed by $v(x) = -\xi(-x)^\alpha$, where $x < 0$ and $\xi > 0$ (Tversky & Kahneman, 1992).

Participants:

The twenty-seven participants, 9 male and 18 female, were undergraduate students in the University of Maryland at College Park and earned credits for Psychology courses.

Stimuli and Procedures:

Like Experiment 1, Experiment 2 required participants to view the stimuli displayed on computer screens and to respond by clicking the option they prefer. The instructions are included in Appendix A. A pair of options, one simple gamble and one sure outcome, was shown on each trial. The stimuli were organized into two blocks: gain and loss. The computer randomly decided which block showed up first. Each block included 360 trials, obtained by crossing 6 payoffs, 6 probabilities and 10 sure outcomes per gamble. The probabilities were 5%, 20%, 40%, 60%, 80% and 95%, and the gamble payoffs were 100, 500, 1000, 4,000, 6,000, 10,000. For each gamble, five out of ten sure outcomes paired with it were higher than the expected value of the gamble and five were lower than it. For the five higher sure outcomes, each was higher than the gamble's expected value (EV) by $n/6$ of the difference between the gamble payoff and EV. Thus, the five higher sure outcomes equaled $xp + (x - xp)\frac{n}{6} = x[p + (1 - p)\frac{n}{6}]$, where n was 1, 2, ..., 5. The five lower sure outcomes were less than the gamble EV, obtained by subtracting $n/6$ of the difference between EV and 0 from EV. This amount could be expressed as $xp(1 - \frac{n}{6})$, where n also equaled 1, 2, ..., 5. All participants went through the same 720 trials. Pairs of stimuli were presented in a random order, and participants were asked to choose the one they preferred. Once again, their goal was to accumulate as many points as they

could in the gain domain and to minimize the loss of points in loss domain. Their performances in both domains determined whether they could get one extra credit in addition to the credit for their work. The performances were recorded and calculated by the computer, which displayed the percentages of optimal choice in both domains at the end of the experiment. The optimal choice was defined as the option with higher EV. Participants did not know this definition until the end of the experiment and only 5 out of 27 participants were rewarded by one extra credit because they had around 90% optimal choices in both domains.

Results and Discussion:

The optimization procedure, conducted at the individual level for all 27 participants, was identical to that used in Experiment 1. I will present the results of the model comparisons and values of the m.l.e.s for gain and loss domains separately in order to compare patterns in Experiments 1 and 2, and to compare patterns in the gain with those in the loss domain.

Model Comparisons. The results of model comparisons in the gain domain are presented in Table 4. As in Experiment 1, the interaction, probability-weighted and value-weighted models all performed much better than prospect theory for the majority of participants. The value-weighted model performed similarly to the interaction model, with only 8 out of 27 participants showing a significant difference between the two. However, the probability-weighted model did not perform equally as well, with 18 out of 27 comparisons showing significantly worse performance than the interaction model. In sum, the interaction model and the value-weighted model described data almost equally well while the probability-weighted model did not.

This pattern of results is the same as obtained in Experiment 1. Based on likelihood ratio tests in the gain domain, the interaction and value-weighted models performed similarly (81.8% and 70.4% of participants did not show significant differences in Experiment 1 and 2) while the probability-weighted model was worse, (43.6% and 66.7% showed significantly inferior performance in Experiment 1 and 2). But all three dependent models performed better than prospect theory did.

The results of model comparisons in the loss domain are presented in Table 5. The interaction, probability-weighted and value-weighted models performed better than prospect theory in well over 70% of the participants. Interestingly, the same pattern was found in both the gain and loss domains: value-weighted models described data better than probability-weighted model did (see Table 4 and 5). In the loss domain, the value-weighted model described the data as well as the interaction model did for 81.5% of the participants, the probability-weighted model performed significantly inferior to the interaction model for 40.7%. This suggests that participants tended to be affected by the payoff contrast in both domains. Intuitively, this result can be interpreted as showing that subjective payoff values increase as probability of obtaining payoffs increase.

Values of m.l.e.s. Each model was optimized 100 times per person. One set of random initial values within parameter domains was generated for each optimization. The best fit m.l.e.s in the gain domain showed the same patterns (Table 6) as those in Experiment 1. γ is negative and δ is positive in the gain domain. The only difference is that the parameter estimates between the two experiments. The absolute values of the medians of α , γ and θ are lower in Experiment 2 while median values

of β and δ are higher in Experiment 2. The pattern occurs in all four models, across both experiments and in Experiment 2 within both domains (Tables 6 and 7).

According to the median parameter estimates from 27 participants, α and β are greater than 1. γ is negative and δ is positive. Recall that the utility of a simple gamble, $g \sim (x, p; 0)$, is expressed by $U(g) = v(x)^{[1+\gamma(1-p)]} w(p)^{(1+\delta\frac{s}{x})}$. The value of γ is so small that the exponent $1 + \gamma(1 - p)$ almost equals 1, which means that contrasts along the attribute of probability may not influence the subjective evaluation of gamble payoffs very much. Meanwhile, the exponent $1 + \delta\frac{s}{x}$ matters, since the value of δ is substantial. This shows once again that the payoff contrast affects subjective evaluation of the probability.

Individual differences must be emphasized. As reported in the gain domains of Experiment 2, the best fit m.l.e.s of 27 participants vary tremendously (see Figures 12 and 13 as one example). I have more confidence reporting the empirical distribution than reporting an exact value of a parameter for a group of people. Essentially, for most people subjective payoff value is convex in the gain and concave in the loss domain; Probability weight is an S-shaped function of the gamble probability. Payoff contrasts have a greater influence on the evaluation of probabilities than the other way around.

Since there are great individual differences in parameter estimates, and the performance of the interaction model was significantly better than other models for some people but not for others, it is interesting to look at the parameter estimates for all participants best described by each of the models. I only focus on the model

comparisons in the gain domain in Experiment 2 because Experiment 2 was particularly designed for model comparisons and the pattern in the gain domain can give us some ideas about the circumstance under which each model performs equally well as the interaction model does. Three sub-groups of people can be identified. Participant in sub-group 1 did not show significant differences between the interaction model and prospect theory. Participants in sub-group 2 showed significant differences between prospect theory and the value-weighted model, but not between the value-weighted and interaction models. People in sub-group 3 had significant differences between the probability-weighted model and prospect theory, but not between the probability-weighted and interaction models. The descriptive statistics of the parameter estimates for sub-group 2 and 3 in the gain domain are displayed in Tables 8 and 9. The statistics for sub-group 1 is not very meaningful since there are only three persons in that group, and the interquartile range does not provide much information. Table 8 suggests one interesting change of subgroup 2 from overall: The median value of parameter δ is much higher than the estimate overall. This suggests that great value of δ at least partially accounts for the good performance of the value-weighted model. The higher the value of δ , the greater the effect of the payoff contrast on probability weight. However, Table 9 does not follow the same line. The median value of γ is similar between the sub-group 3 and overall. One reason is that the range of parameter estimate is too narrow to allow significant change when the data are broken down.

Chapter 3: Conclusions and General Discussion

In both experiments, participants chose between a simple gamble and a sure outcome. The goal was to accumulate as many points as possible in the gain domain and to minimize the loss of points in the loss domain. Under this paradigm, the results suggested that evaluations of gamble payoff and probability are dependent in the context of choice. The interaction model was most general and therefore described the data best. The results of likelihood ratio tests showed that the value-weighted model performed similarly to the interaction model and did better than the probability-weighted model, which implies that the contrast between a gamble payoff and a sure outcome adjusted the subjective evaluation of the gamble probability. This is consistent with the previous result that the desirability of the event, or payoff, affects the subjective probability (Irwin, 1953; Slovic, 1966). This adjustment explained much of the excellent performance of the interaction model over prospect theory. When comparing against a sure outcome, decision makers tended to estimate the magnitude of gamble payoffs higher when they were with high probabilities than with low probabilities. All the patterns displayed in this study were same for the gain and the loss domains, except for the shape of $v(x)$. In addition, the similarity of model comparisons in Experiment 1 and 2 showed that the results were not affected by the relay method of Experiment 1.

The medians of the maximum likelihood estimators were not consistent with previous studies on value and weighting functions. The medians of parameter values of α and β in the current study were higher than 1 while the reported values in

previous studies were usually lower than 1. Comparing parameter values in prospect theory across studies is more meaningful than across models because prospect theory was used to fit different sets of data. The dependent models proposed in this study have not been used in other studies. High values of α and β in $v(x)$ and $w(p)$, respectively, in the current study suggest that the goal of the task may guide people's attention and affect the subjective evaluation of gamble payoffs and probabilities. I emphasized this point in the *Results and Discussion* section of Experiments 1 and 2. There are at least three reasons for higher α and β values. First, different combinations of functional forms of $v(x)$ and $w(p)$ were used in this study than in others. Second, parameter domains were relaxed to accommodate individual differences, which were ignored in most previous studies that estimated value and weighting functions. They usually report medians of parameters in a group, which blur the variety of individual decisions. The empirical distributions of α and β showed that some participants did have parameter values lower than 1 even though they could not represent the group. Third, other studies assumed that payoffs and probabilities are independent before estimating the parameters, but models other than prospect theory in the current experiments include possible cross-attribute interaction. So, direct parameter comparisons are less valuable than model comparisons, as the latter provide a big picture about whether certain patterns exist in both loss and gain domains and whether dependence between attributes is real and in what a way

The parameters γ and δ varied among participants, showing different degrees of influence from cross-attribute contrasts. The common feature among all participants was that γ was negative and δ was positive and that the absolute

magnitude of γ was much smaller than that of δ . Parameter θ played a role in managing potential fluctuations from trial to trial. Since it was not a key parameter in this study, I leave the parameter estimation in Table 3, 6 and 7 without further discussion.

Previous studies expressed the functional forms of $v(x)$ and $w(p)$ in various ways. And contrasts along probability and payoff also can be expressed in many ways. I used only one form for each because my interest was on the existence of the interaction between payoff and probability, and not necessarily its best functional form.

Parameter estimation by sequential testing (PEST) (Bostic, Herrnstein and Luce, 1990) is a psychophysical method to obtain the certainty equivalent of a gamble. It reduces the impact of cognitive fluctuation and avoids one-shot decision mistakes. Ideally, PEST requires hundreds or even thousands of trials to repeat choices in order to obtain stable results. Fewer numbers of trials may cause test-retest inconsistency, which means results may vary from test to test. Hence, I examined the data from Experiment 1 for this issue. Recall that certainty equivalents (CEs) were derived by taking the midpoints of two values: the highest rejected sure outcome and the lowest accepted sure outcome. The former value is supposed to be smaller than the latter one. Any reversion indicates fluctuation in choice behaviors. Only 1 out of 41 participants showed this phenomenon for only one gamble. Overall, the data from relays were reliable regarding the quality of CEs, which helped prevent errors from proliferating along the relays. To quickly converge to CEs, one method called QUICKINDIFF, introduced by von Winterfeldt, Chung, Luce and Cho (1997), can be

used in the future study. It is similar to PEST, except that respondents rate their degree of preference among stimulus pairs. This extra requirement avoids lengthy experiments.

One result found in this study is particularly interesting. Participants underweighted lower probabilities, and overweighted higher probabilities; and they were more sensitive to higher payoff than to lower ones. This phenomenon may be due to motivation for the task, in which the goal was to seek maximum gain and minimum loss. A future study will look closely at the effect of motivation on choice, in particular at the situation under which the payment is contingent on the performance.

Table 1. The Results of Minimum Window of 41 Participants in Experiment 1

	Minimum	Maximum	Mean	Variance
x	78 (0.8%)	4938 (49%)	2030 (20.3%)	$2.1 \cdot 10^{-6}$
$v'(x)$	$1.04 \cdot 10^{-5}$ (0.001%)	0.009 (0.9%)	0.003 (0.3%)	$4.1 \cdot 10^{-6}$

Note. The values in the parenthesis are the percentage of the full scale

Table 2. Pair-Wise Model Comparisons between Higher Ranked and Nested Models in Experiment 1

		Higher rank models		
		Interaction	Probability-weighted	Value-weighted
	Prospect theory	52 (94.6%)	52 (94.6%)	52 (94.6%)
Nested models	Probability-weighted	24 (43.6%)	N/A	N/A
	Value-weighted	10 (18.2%)	N/A	N/A

Note. The entries are the number (percentage) of participants who showed significant improvement of higher order models over the model(s) nested under them. All data are out of a total of 55 subjects. Significance level is $\alpha = 0.05$

Table 3. The Median (and Inter-Quartile) Values of Parameter Estimates in Four Models in Experiment 1

	α	β	γ	δ	θ
Interaction	2.005 (1.817, 2.259)	1.833 (0.960, 4.098)	-0.053 (-0.250, -0.004)	4.618 (2.087, 5.000)	6.844 (0.583, 37.161)
Probability-weighted	2.038 (1.817, 2.322)	2.087 (1.534, 5.349)	-0.152 (-0.364, -0.045)	N/A	6.438 (1.219, 102.817)
Value-weighted	1.9772 (1.791, 2.199)	1.309 (0.895, 1.978)	N/A	5.640 (2.206, 19.660)	3.6158 (0.614, 42.082)
Prospect theory	2.133 (1.863, 2.298)	0.9611 (0.542, 1.292)	N/A	N/A	26.782 (3.659, 138.709)

Note. The results are over 55 participants. N/A means not applied.

Table 4. Pair-Wise Model Comparisons between Higher Ranked and Nested Models in Gain Domain in Experiment 2

		<u>Higher rank models</u>		
		Interaction	Probability-weighted	Value-weighted
Nested models	Prospect theory	23 (85.2%)	20 (77.1%)	22 (81.5%)
	Probability-weighted	18 (66.7%)	N/A	N/A
	Value-weighted	8 (29.6%)	N/A	N/A

Note. The entries are the number (percentage) of participants who show significant effect between pair wised models. All data are out of total 27 subjects in Experiment 2. Significance level $\alpha = 0.05$. N/A means not applied.

Table 5. Pair-Wise Model Comparisons between Higher Ranked and Nested Models in Loss Domain in Experiment 2

		<u>Higher rank models</u>		
		Interaction	Probability-weighted	Value-weighted
Nested models	Prospect theory	24 (88.9%)	24 (88.9%)	20 (74.1%)
	Probability-weighted	11 (40.7%)	N/A	N/A
	Value-weighted	5 (18.5%)	N/A	N/A

Note. The entries are the number (percentage) of participants who show significant effect between pair wised models. All data are out of total 27 subjects in Experiment 2. Significance level $\alpha = 0.05$. N/A means not applied.

Table 6. The Median (and Inter-Quartile) Values of Parameter Estimates in Four Models in the Gain Domain in Experiment 2

	α	β	γ	δ	θ
Interaction	1.888 (1.742, 1.925)	2.360 (1.081, 4.472)	-0.013 (-0.149, -0.004)	5.010 (2.389, 5.011)	1.780 (0.992, 5.070)
Probability-weighted	1.845 (1.747, 1.919)	2.165 (1.178, 4.887)	-0.119 (-0.334, -0.007)	N/A	1.469 (0.661, 3.982)
Value-weighted	1.827 (1.752, 2.049)	1.865 (0.781, 6.528)	N/A	9.689 (1.820, 315.783)	1.737 (0.309, 6.174)
Prospect theory	1.821 (1.664, 1.909)	1.939 (0.733, 3.390)	N/A	N/A	3.336 (1.283, 9.310)

Note. The results are over 27 participants. N/A means not applied.

Table 7. The Median (and Inter-Quartile) Values of Parameter Estimates in Four Models in the Loss Domain in Experiment 2

	α	β	γ	δ	ξ	θ
Interaction	1.800 (1.735, 1.802)	2.124 (0.568, 5.021)	-0.027 (-0.729, -0.007)	2.571 (0.450, 5.020)	2.759 (1.963, 2.990)	2.227 (0.776, 4.688)
Probability -weighted	1.957 (1.043, 2.102)	2.131 (0.778, 3.704)	-0.120 (-1.424, -0.005)	N/A	9.226 (2.134, 21.658)	32.864 (5.361, 56.883)
Value- weighted	1.950 (1.581, 2.106)	1.238 (0.553, 3.094)	N/A	6.153 (3.369, 140.553)	4.213 (1.677, 19.563)	38.085 (7.109, 95.764)
Prospect theory	1.732 (0.940, 1.801)	1.307 (0.500, 3.470)	N/A	N/A	2.945 (2.236, 2.999)	2.425 (0.979, 6.594)

Note. The results are over 27 participants. N/A means not applied.

Table 8. The Median (and Inter-Quartile) Values of Parameter Estimates for subgroup 2 in the Gain Domain in Experiment 2

	α	β	γ	δ	θ
Interaction	1.788 (1.761, 1.958)	2.352 (1.926, 7.577)	-0.5834 (-1.979, -0.271)	4.946 (2.256, 4.964)	4.194 (1.224, 6.878)
Value-weighted	1.983 (1.673, 2.041)	0.963 (0.617, 2.111)	N/A	138.03 (1.801, 254.13)	2.438 (0.660, 28.127)
Prospect theory	1.848 (1.554, 1.905)	1.939 (0.991, 4.821)	N/A	N/A	3.336 (4.166, 9.951)

Table 9. The Median (and Inter-Quartile) Values of Parameter Estimates for subgroup 3 in the Gain Domain in Experiment 2

	α	β	γ	δ	θ
Interaction	1.917 (1.840, 1.956)	2.258 (1.48, 3.938)	-0.030 (-0.070, -0.003)	5.000 (4.889, 5.014)	1.413 (1.148, 2.291)
Probability-weighted	1.887 (1.792, 1.938)	2.137 (1.508, 3.517)	-0.124 (-0.249, -0.047)	N/A	1.329 (0.673, 1.693)
Prospect theory	1.871 (1.788, 1.919)	2.010 (0.996, 2.935)	N/A	N/A	1.540 (1.043, 9.372)

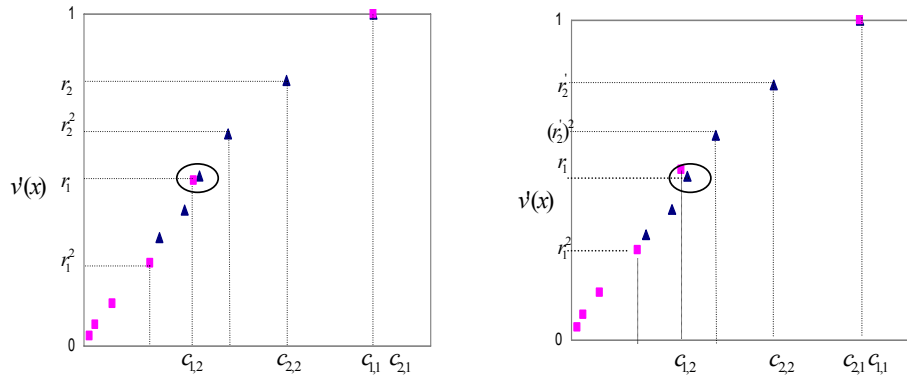


Figure 1. An example of monotonic (left panel) and non-monotonic (right panel) value functions. The square points came from one relay and the triangles from another. The circled pairs of points differ in their monotonicity relation.

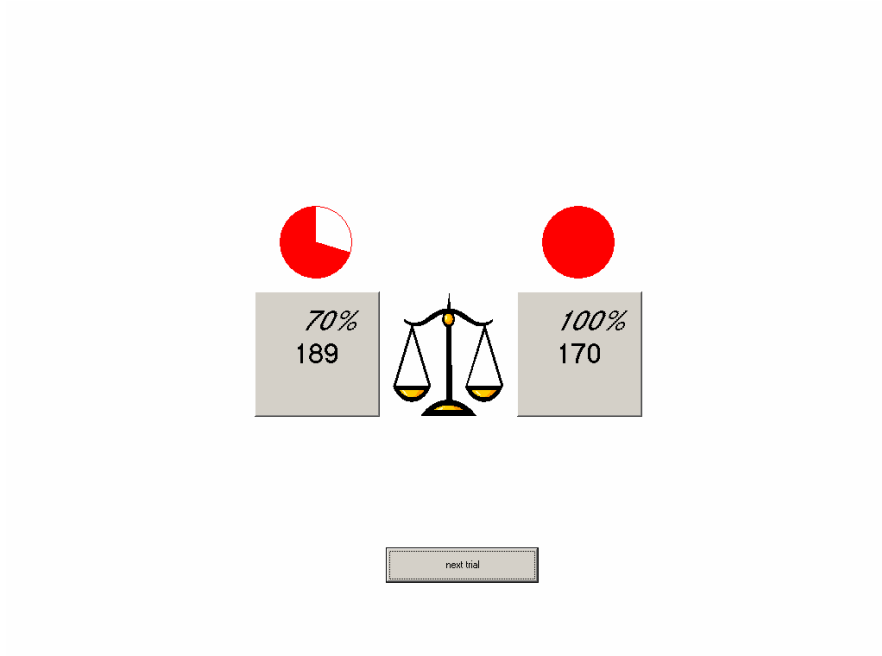


Figure 2. Display of an experimental trial.

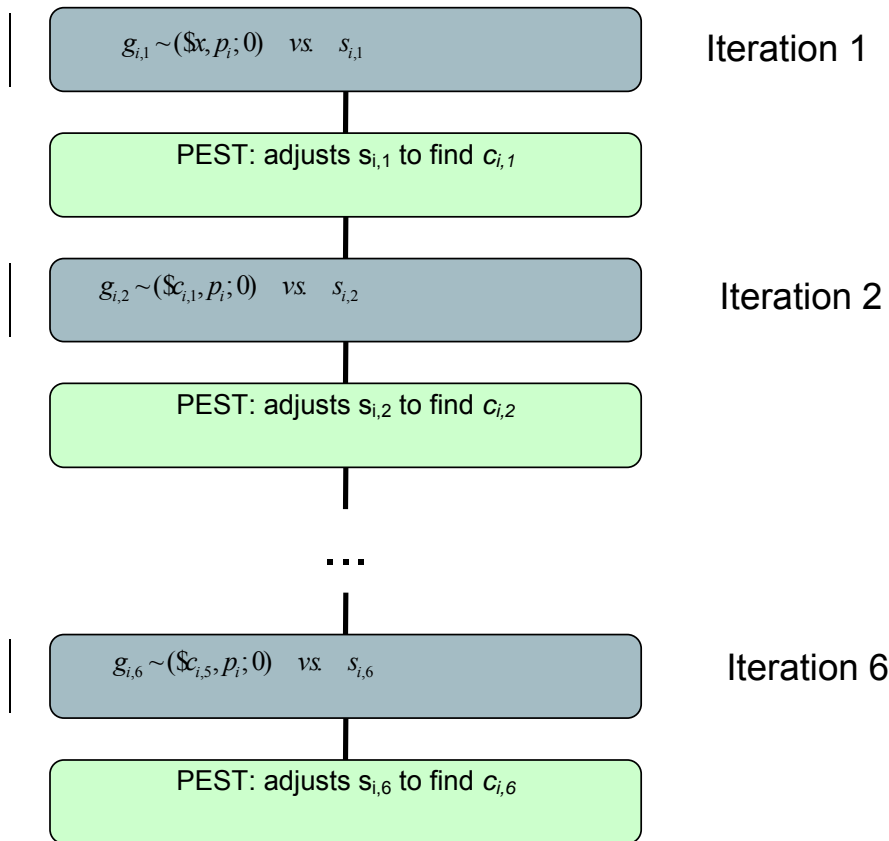


Figure 3. The flowchart of the dynamic procedure of relays. For relay i , each iteration includes a gamble with a fixed probability and a payoff obtained from CE of the previous gamble within that relay (except for the first gamble, which has a payoff 10,000). The initial sure outcome of the new gamble is a random number between 0 and the gamble payoff. Then paired sure outcome is adjusted up and down through PEST in the next several trials until the certainty equivalent of the gamble is obtained. There are six interations in one relay (except for the relay with p of 5%).

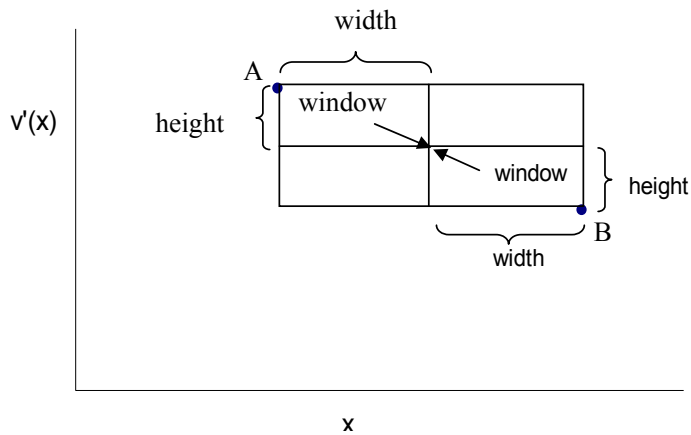


Figure 4. Illustration of minimum window size. Data points A and B represent minimized maximal inverse relationship. Weak monotonicity is satisfied only when the two data points move to the center of the rectangle, indicated by the arrows. Strong monotonicity is achieved for any two data points if either moves further beyond the center. Thus, $\frac{1}{2}$ of the full height and width of the rectangle are the two dimensions of the minimum window).

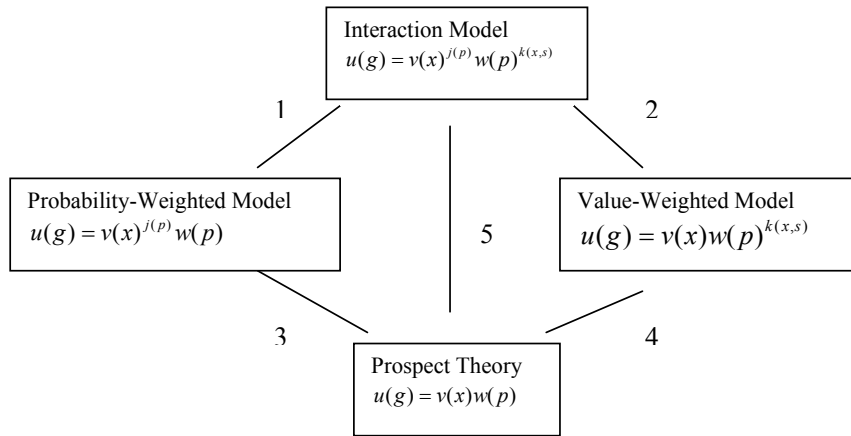


Figure 5. The structure of models with highest ordered model on the top and lowest at the bottom.

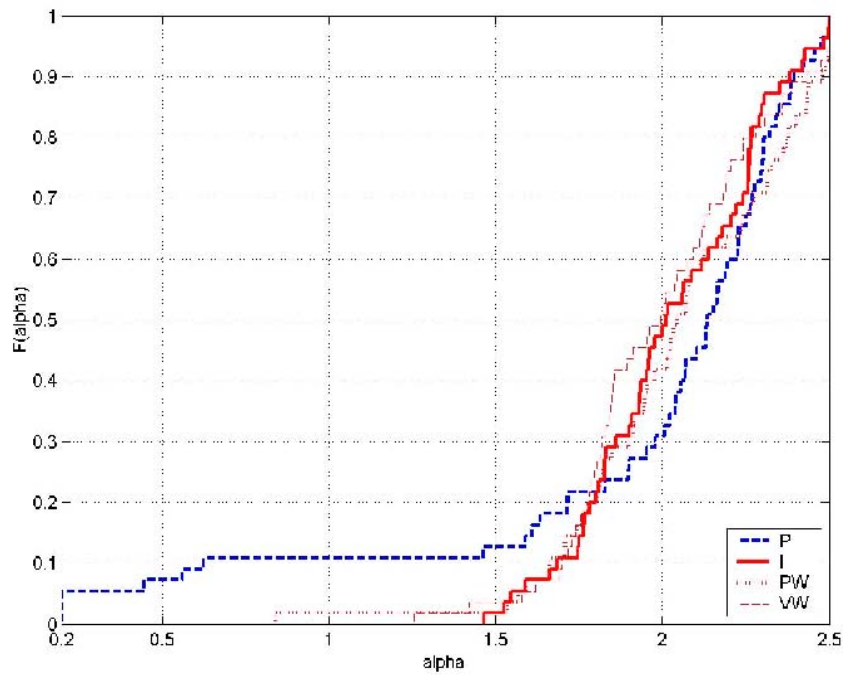


Figure 6. The empirical distribution of the m.l.e. of parameter α in Experiment 1. P represents for prospect theory; I for interaction model; PW for probability-weighted mode and VW for value-weighted model.

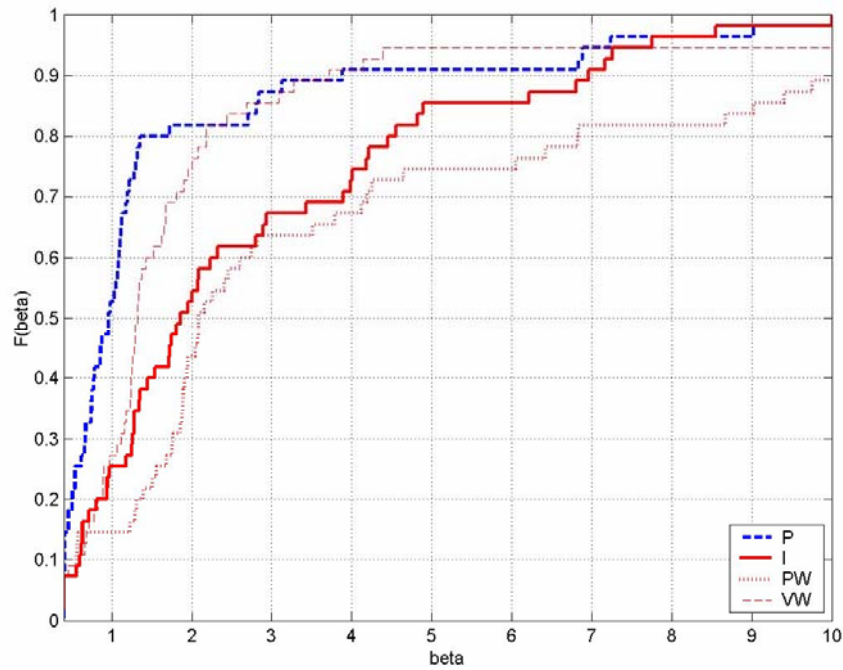


Figure 7. The empirical distribution of the m.l.e. of paramete β . P represents for prospect theory; I for interaction model; PW for probability-weighted mode and VW for value-weighted model.

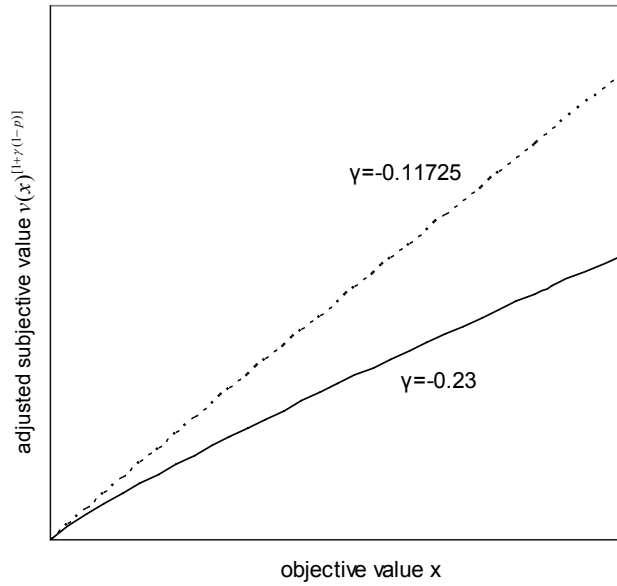


Figure 8. The subjective value $v(x)$ is adjusted by the power $(1 + \gamma(1 - p))$ in the interaction and probability-weighted models. Higher values of γ increase the subjective feeling of the payoff. (The curves are based on $\alpha = 1.0743$ and fixed $1 - p = 0.95$)

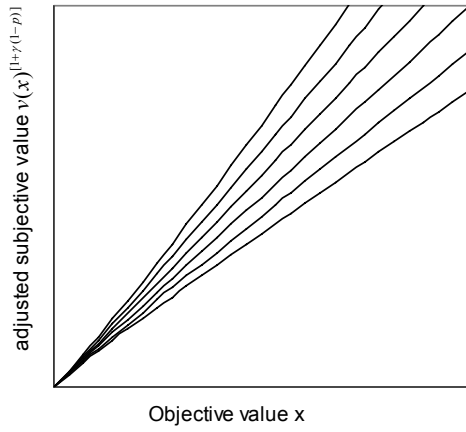


Figure 9. The subjective value of payoff is adjusted by the probability p of the gamble (Take $\gamma = -0.11725$ as one example, The lines from low to high represent for the subjective payoff values adjusted by probability p equals 0, 0.2, 0.4, 0.6 0.8 and 1, respectively).

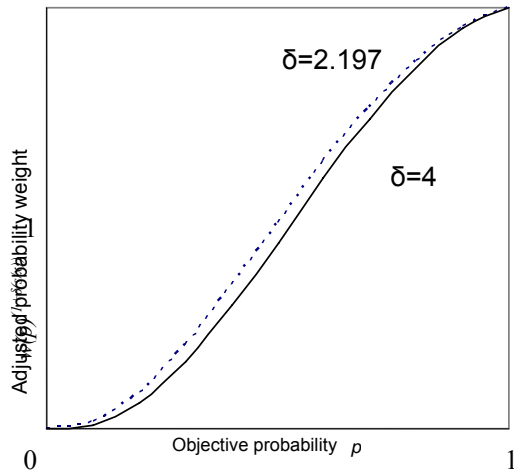


Figure 10. The probability weight is adjusted by the power $(1 + \delta \frac{S}{x})$. Higher δ tends to decrease the evaluation of the probability (Take $\beta = 1.4717$ and ratio $\frac{S}{x} = 0.1$ as one example).

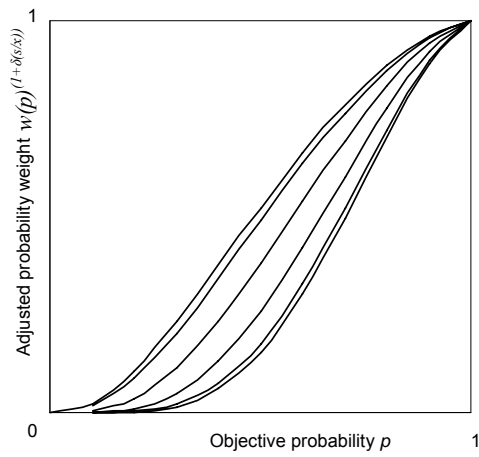


Figure 11. The weighting function adjusted by the ratio of the sure outcome to the gamble payoff. (Take $\beta = 1.4717$ and $\delta = 2.197$ as one example, the lines from left to right represent the probability weight adjusted by ratios of 0.05, 0.1, 0.3, 0.6, 0.9, 1, respectively.)

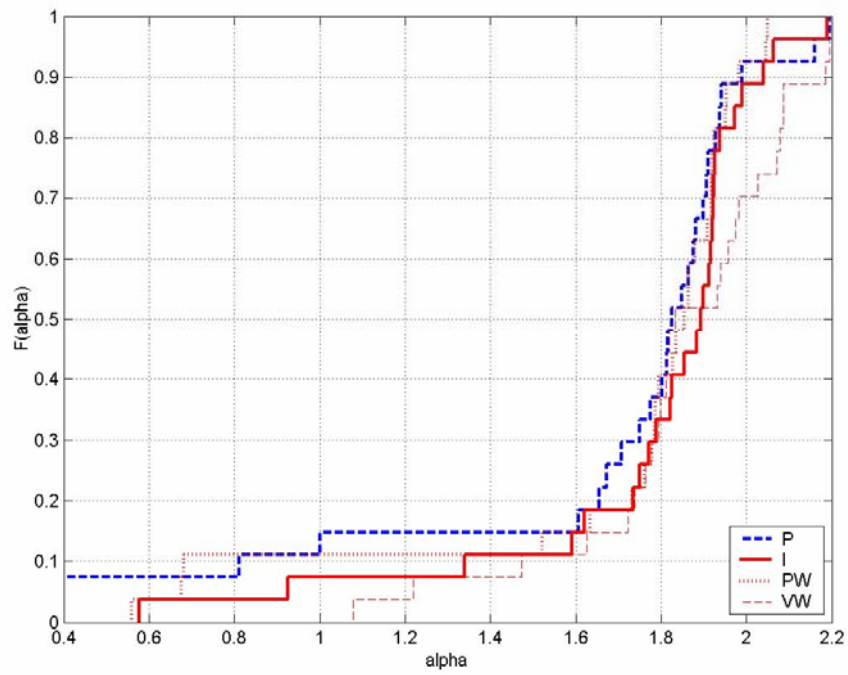


Figure 12. The empirical distribution of the m.l.e.s of α in the gain domain. P represents the prospect theory estimates; I the interaction model estimates; PW the probability-weighted model estimates and VW the value-weighted model estimates.

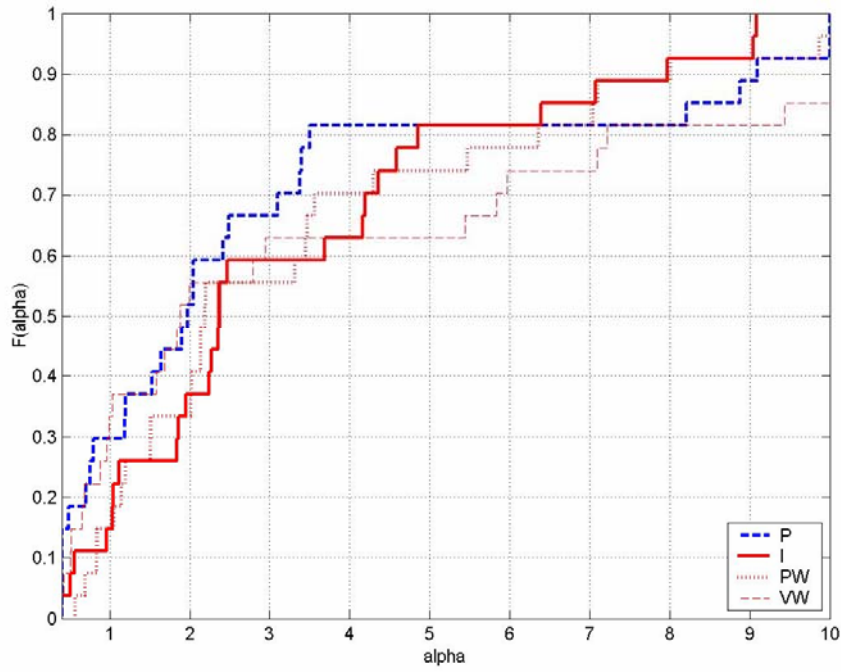


Figure 13. The empirical distribution of the m.l.e.s of β in the gain domain. P represents the prospect theory estimates; I the interaction model estimates; PW the probability-weighted model estimates and VW the value-weighted model estimates.

Appendices

Appendix A

Instructions for Experiment 1:

There are two options on each trial, one is a simple gamble and the other is a sure outcome. Your task is to choose one option by clicking on that square. The selected square turns to pink. Your goal is to accumulate as many points as you can. And you can switch your option before going to the next trial. Your final bonus is determined by your performance.

Instruction for Experiment 2:

There are two blocks in this experiment. One block has positive payoffs, and the other has negative payoff only. See specific instructions at the beginning of each block. Your final bonus will be determined by your performance.

For the gain domain: There are two options on each trial, one is a simple gamble and the other is a sure outcome. Your task is to choose one option by clicking on that square. The selected square turns to pink. Your goal is to accumulate as many points as you can. And you can switch your option before going to the next trial.

For the loss domain: There are two options on each trial, one is a simple gamble and the other is a sure outcome. Your task is to choose one option by clicking on that square. The selected square turns to pink. Your goal is to avoid to lose as few points as you can. And you can switch your option before going to the next trial.

Appendix B

Parameter estimation by sequential testing (PEST) (Bostic, Herrnstein and Luce, 1990) is a psychophysical method to obtain the certainty equivalent of a gamble. It reduces the impact of cognitive fluctuation and avoids one-shot decision mistakes. PEST was implemented in the current study in following ways:

1. The initial value of a sure outcome, s , was a random integer drawn from uniform distribution $(0, x)$, where x was the payoff of the simple gamble.

2. If the participant chose s , s decreased when the gamble next appeared; otherwise, it increased. The initial step size for the increment or decrement was $\frac{1}{5}x$.

3. When the choice reversed for any identical gamble, that is choosing g right after choosing s or vice versa, the step size was halved. If the choice did not change three times in a row, the step size doubled. Otherwise it was kept the same size.

4. The certainty-equivalent searching procedure was over when the step size was less than $\frac{1}{50}x$ or 1, whichever came first.

To make it more convenient for participants to process the information, the computer rounded up the sure outcomes to the nearest integers. The certainty equivalent c was calculated by averaging the highest rejected and the lowest accepted values of s .

Appendix C

Optimization procedure for testing independence assumption

The procedure was programmed in MatLab. There are 8 parameters r_i , where $i = 1, 2, \dots, 8$. The Optimization procedure searched for a set of solutions that can satisfy two monotonicity criteria.

First, the computer generated a set of initial value to r_1 by randomly selecting a random number from a uniform distribution (0, 1). Then, it assigned r_2 a value by generating a random number from a uniform distribution $(r_1, 1)$, given $p_1 < p_2$. The parameters r_3 through r_8 were assigned values by the similar procedure which guarantees the monotonicity of the weighting function. Then, the computer conducted SQP (sequential quadratic programming), which finds a set of parameters that can minimize the sum of the inverse magnitudes, $\sum_{i,j} s_{i,j}$, where $s_{i,j}$ is defined as

$$s_{i,j} = \begin{cases} v'(c_i) - v'(c_j) & \text{if } c_i < c_j \text{ and } v'(c_i) > v'(c_j) \\ 0 & \text{otherwise} \end{cases}$$

SQP is carried out by a built-in function *fmincon* in Matlab. Briefly, a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function is calculated at each iteration. Then a quadratic programming (QP) problem is solved. Finally, the solution to the QP sub-problem used to form a new iteration (Coleman, Branch and Grace, 1999).

Once the minimum value of $\sum_{i,j} s_{i,j}$ is found, the optimization terminates. To avoid a local minimum, the computer repeated the optimization procedure 500 times per participant, each time with a new set of randomly generated initial values. The

Comment [MSOffice1]: HONG – NOTE THAT YOU HAD THE LOWER LIMIT AS r_2

computer identified the parameters that produced the minimum value of $\sum_{i,j} s_{i,j}$. For the purpose of testing monotonicity, failing to obtain $\sum_{i,j} s_{i,j} = 0$ is regarded as violation of monotonicity of the value function.

Optimization procedure for maximum likelihood estimates (m.l.e.)

The optimization for maximizing the joint likelihood is very similar to that mentioned above. The differences include (1) maximize the joint probability of all trials, (2) constraint the parameters in models, and (3) optimize 100 times. The parameters that produce greatest likelihood are regarded as m.l.e.

Appendix D

Ranges of parameters $\alpha, \beta, \gamma, \delta, \theta$ are $(0, +\infty), (0, +\infty), (-\infty, +\infty), (-\infty, +\infty)$ and $(0, +\infty)$, respectively. To make the range practical in the optimization procedure, the parameter ranges were set as follows:

$$0 \leq \alpha \leq 10; 0 \leq \beta \leq 10; -10 \leq \gamma \leq 10; -10 \leq \delta \leq 10; 0 < \theta \leq 150.$$

The practical boundaries above were determined by the values of parameters estimated within much broader ranges.

Footnotes

1. There is a form of sign-dependence in cumulative prospect theory (CPT) (Tversky and Kahneman, 1992), where two weighting functions exist, one for probabilities of gain and one for loss. Also, CPT is rank-dependent, as the decision weight of each outcome is decided by the rank of the outcome in addition to the corresponding probability. However, CPT yields independence in two-outcome gambles.

2. $v'(c_{k,l}) = \frac{v(c_{k,l})}{v(x)}$. Since $v(x)$ is a constant, the order held by

$v(c_{k,l})$ and $v(c_{g,h})$ also held for normalized values $v'(c_{k,l})$ and $v'(c_{g,h})$.

3. The domain of parameter α was not defined in Tversky & Kahneman's (1992) paper. I added one reasonable domain to do parameter estimation.

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