ABSTRACT

Title of dissertation: DETERMINING OPTIMAL RELIABILITY
TARGETS THROUGH ANALYSIS OF
PRODUCT VALIDATION COST AND FIELD
WARRANTY DATA

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This work develops a new methodology to minimize the life cycle cost of a product using

the decision variables controlled by a reliability/quality professional during a product

development process. This methodology incorporates all product dependability-related

activities into a comprehensive probabilistic cost model that enables minimization of the

product's life cycle cost using the product dependability control variables. The primary

model inputs include the cost of ownership of test equipment, forecasted cost of warranty

returns, and environmental test parameters of a product validation program. Among

these parameters, an emphasis is placed upon test duration and test sample size for

durability related environmental tests. The warranty forecasting model is based on data

mining of past warranty claims, parametric probabilistic analysis of the existing field data, and a piecewise application of several statistical distributions.

The modeling process is complicated by insufficient knowledge about the relationship between product quality and product reliability. This can be attributed to the lack of studies establishing the effect of product validation activities on future field failures, overall lack of comprehensive field failure studies, and the market's dictation of warranty terms as opposed to warranties based on engineering rationale. As a result of these complicating factors an innovative approach to estimating the quality-reliability relationship using probabilistic methods and stochastic simulation has been developed. The overall cost model and its minimization are generated using a Monte Carlo method that accounts for the propagation of uncertainties from the model inputs and their parameters to the life cycle cost solution.

This research provides reliability and quality professionals with a methodology to evaluate the efficiency of a product validation program from a life cycle cost point of view and identifies ways to improve the validation test flow by adjusting test durations, sample sizes, and equipment utilization. Solutions balance a rigorous theoretical treatment and practical applications and are specifically applied to the electronics industry.

DETERMINING OPTIMAL RELIABILITY TARGETS THROUGH ANALYSIS OF

PRODUCT VALIDATION COST AND FIELD WARRANTY DATA

By

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Dissertation submitted to the Faculty of the Graduate School of the
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Dedicated to my wife Faina, who has been patiently listening to my ramblings about science and engineering and giving me all her love and support during the past 20 years
To my daughter Vicki, who is just at the beginning of her college education journey

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NOMENCLATURE

Reliability

t = time

t_T = test duration

t_L = product service life

t_S = time point where the bathtub curve transitions into 'useful life' phase

$R(t)$ = reliability as a function of time

R_0 = target reliability

C = confidence level

$\alpha = 1-C$, risk factor

N = test sample size

A_f = acceleration factor

$\chi^2_{\alpha,\ 2(k+1)}$ = Chi-square distribution.

k = number of failures

L = number of service lives the product intended to be tested

λ = failure rate

T = total test time

β = Weibull slope (or shape parameter)

η = Weibull distribution scale parameter

γ = Weibull distribution location parameter

μ = mean of the normal distribution

σ = standard deviation of the normal distribution

ξ_i = vector of statistical parameters

Warranty

$\underline{W} = \{T_0, M_0\}$ = two-dimensional warranty,

T_0 = warranty time limit (typically 36 months)

M_0 = warranty mileage limit (typically 36,000 miles)

$\underline{\theta}$ = Vector of design parameters

$F(T)_{Warranty}$ = portion of accumulated failures covered by warranty by time T

$f_{Daily}(m)$ = daily mileage distribution PDF

$f(t|M_0)$ = PDF of exceeding M_0 at time t.

n_f = number of units failed or expected to fail during warranty period

α_W = warranty cost per unit

$h_{avg}(t)$ = average hazard rate

W_C = total warranty cost

n_{sold} = units sold (approximates the total number of manufactured units)

Cost

Generally the character φ will denote an hourly rate and α will denote the cost per item, varying the subscript characters.

P' = seller's profit

α_{pv} = cost of product validation for the program

α_b = per unit cost to the buyer (customer's price)

α_d = design cost of the program

α_p = cost of producing one test sample

α_e = cost of equipping one test sample

α_m = cost monitoring one test sample

α_W = warranty cost per unit

α_{parts} = cost of the spear parts per repair – random variable

α_{PM} = cost of each preventive maintenance

N_{PM} = number of preventive maintenance actions per year

t_{repair} = duration of each repair – random variable

$t_{one\text{-}life}$ = duration of the test corresponding to one mission life

φ_{repair} = repair labor rate

W_C = total warranty cost

i = annual interest rate

$H(\alpha_W)$ = warranty cost per claim distribution function

φ_T = hourly labor rate of performing of the test

K = equipment capacity

$\lceil\ \rceil$ = is a ceiling function, indicating rounding up to the next highest integer

M = maintenance cost per year

Y = additional equipment expenses per year, such as the cost of floor space

Probability and Monte Carlo Simulation

$P(X_1<x<X_2)$ = probability of the value x falling into the range $[X_1; X_2]$

$L(Data|R)$ = likelihood of obtaining the observed test data if the reliability of each unit is R

$\pi(R)$ = prior distribution (Bayesian analysis)

ρ = knowledge factor

Q_{Corr} = correction factor (correlation coefficient between demonstrated and forecasted reliabilities at the expected mission life)

$f_i(t;\ \xi_i)$ = random function of time and statistical parameters

r = correlation coefficient

ACRONYMS

BGA – Ball Grid Array

B/C – Benefit-Cost ratio

CCNV – Customer Complaint Not Verified (see also NFF, NTF, TNF, NTI)

CDF – Cumulative Distribution Function

CFR – Constant Failure Rate

CM – Corrective Maintenance

COO – Cost Of Ownership

DFR – Decreasing Failure Rate

DPTV – Defects Per Thousand Vehicles

DV – Design Validation

FRACAS –Failure Reporting Analysis and Corrective Action System

IFR – Increasing Failure Rate

IPTV – Incidents Per Thousand Vehicles

IRR – Internal Rate of Return

LCC – Life Cycle Cost

LHS – Latin Hypercube

MCS – Monte Carlo Simulation

MIS – Months in Service

MLE – Maximum Likelihood Estimate

MTBF – Mean Time Between Failures. The term is typically used for repairable systems
following Poisson process (exponential distribution).

MTTF – Mean Time To Failure

MTTR – Mean Time To Repair

NFF – No Fault Found. (See also CCNV, NTF, TNF, NTI)

NPV – Net Present Value

NTF – No Trouble Found. (See also CCNV, NFF, TNF, NTI)

NTI – No Trouble Identified (See also CCNV, NFF, NTF, TNF)

OEM – Original Equipment Manufacturer. In this dissertation it is a common reference
for a vehicle manufacturer, such as GM, Ford, Toyota, etc.

PDF – Probability Density Function

PM – Preventive Maintenance

PPM – Parts Per Million

PTC – Power Temperature Cycling

PTH – Plated Through Hole

PV – Product Validation

QWIK – Quality With Information and Knowledge (General Motors warranty database)

RIW – Reliability Improvement Warranty

ROI – Return on Investment

RSM – Response Surface Methodology

SMT – Surface Mounted Technology

SPC – Statistical Process Control

TCO – Total Cost of Ownership

TNF – Trouble Not Found. (See also CCNV, NFF, NTF, NTI)

TNI – Trouble Not Identified. (See also CCNV, NFF, NTF, TNF, NTI)

TTR – Time To Repair, sometimes referred here as Repair Duration

VIN – Vehicle Identification Number

1. Introduction

Reliability engineering and environmental testing includes the product development activities directed at improving the reliability of the product, i.e., its ability to better survive its mission life without failures. Reliability engineering and environmental testing represent significant portions of the product development process (measured in resources and time) in many industries including the automotive. The costs of those activities are essential parts of the life cycle cost (LCC) model and should always be considered as part of a product business cycle.

Life cycle cost analysis is one of the important tools for choosing the most cost effective approach from a series of design alternatives. It is an excellent tool for finding the best design trade-off and ultimately the best product value for the customer. However since the product lifetime in the automotive industry can extend to the period of 10-15 years the process of accounting for the total cost of the product can be extremely complex due to lack of data and a random nature of many cost factors associated with automotive products. The product warranty in automotive industry is a significant contributor to the 'afterlife' portion of life cycle cost. For example, according to [Nasser *et al.* (2002)] on average General Motors spends around $3.5 billion per year (roughly 22.5 million warranty claims) paying the dealerships to repair broken parts under warranty. Various prediction models are used for the LCC analysis in the automotive industry, however since it is not known in advance how much warranty, in terms of number of claims and dollar amount, the product is expected to cause, guesswork and assumptions comprise a

significant part of these models. In addition to the random nature of the variables involved in the cost modeling, there remains a gap in determining a relationship between the product development activities and future warranties of the product. That gap also adversely affects the accuracy of the models currently used for LCC analysis in the automotive industry since warranty is a significant contributor to LCC, which is both directly and indirectly linked to product reliability

Product reliability always remains in the focus of any product development effort. Clearly, improving reliability leads to a reduction in life cycle cost through cost savings and cost avoidance during the sustainment of the product within the warranty period and beyond. However, what kind of reliability can be feasibly targeted and demonstrated during the design stage? How will the product testing activities affect the future warranty costs? How much information can be obtained from the previous models? These and other questions need to be answered in order to optimize the test and validation portion of the product development process. To keep the failure related aspect in perspective and have a realistic estimate of their causes and effects there is a clear need for the methodologies focusing on product reliability and its related engineering activities and their effects on product development as a whole. These methodologies would be primarily intended for reliability engineers and other engineering professionals involved in product test and validation and would allow the analysis of LCC and other long-term effects of these activities. To develop these methodologies we will need to concentrate on the life cycle model from the reliability engineering perspective, i.e., the model with various input variables comprising the parameters of the test and validation process

controlled by a reliability/validation engineer[1] during a product development cycle. This type of methodology should also be able to optimize the design process with the ultimate goal of reducing the overall LCC of the product.

1.1.   Key Definitions

Many reliability terms have more than one meaning. Below are the terms and their definitions the way they are applied in this dissertation.

Reliability - Reliability is the probability that the item will perform its functions without failure in specified environments for specified period of time.

Quality – Fitness for use [Juran and Gryna (1980)] or conformance to original product specifications. Quality is sometimes referred as reliability at time zero and often expressed in PPM (parts per million defects)

Dependability – A qualitative characteristic of any device that constitutes an integral view of its Reliability, Availability, Maintainability, Quality, and Safety [Fernández (2001)].

Product Validation – A formal process with legal weight confirming that the product meets defined requirements. In this dissertation the term Validation will be mostly used in the context of product reliability and environmental testing.

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[1] In this dissertation terms reliability engineer and validation engineer will be used interchangeably.

Life Cycle Cost (LCC) Analysis - A method of calculating the effective cost of a system over its entire life span. For our purposes we will bound the definition of LCC to the components essential to the original equipment manufacturer or its supplier.

Failure rate – Number of failures per accumulated time. Often expressed by λ-character and used as a sole parameter of exponential distribution.

Product warranty – The seller's assurance to a buyer that a product or service is or shall be as represented. Warranty terms may vary, but it usually includes a contractual obligation on the part of a seller to repair of refund the cost of the failed item.

Warranty claim – The customer complaint regarding the failure or malfunction of a specific vehicle system typically followed by the repair by an automotive dealer free of charge. Warranty claim typically contains all the relevant information about the vehicle including manufacturing date, repair date, vehicle mileage, etc.

Reliability-Cost curve – A graphical relationship between pursued reliability and the overall product cost required to achieve it.

Quote process (often referred as Quoting process) – An initial phase of a product business cycle, where seller and buyer negotiate the price of a new product to be designed and manufactured by the seller.

Bathtub curve – A traditional model linking failure rates with the mission life of the product. 'Classic' bathtub curve has three sections: Infant mortality, Useful life, and Wear-out period.

Validation engineer – Technical specialist ultimately responsible for the planning, conducting, and analyzing the environmental testing of the product. This term will be used here interchangeably with the term 'Reliability engineer'.

1.2.   Dependability-Related Activities in Automotive Electronics and Other Mass Production Industries

Typically in a mass production industry, including automotive electronics, a product development process goes through various stages of the design cycle. The specifics of this cycle will vary from industry to industry and even from company to company, but in general this process will include the steps shown in Figure 1.1.

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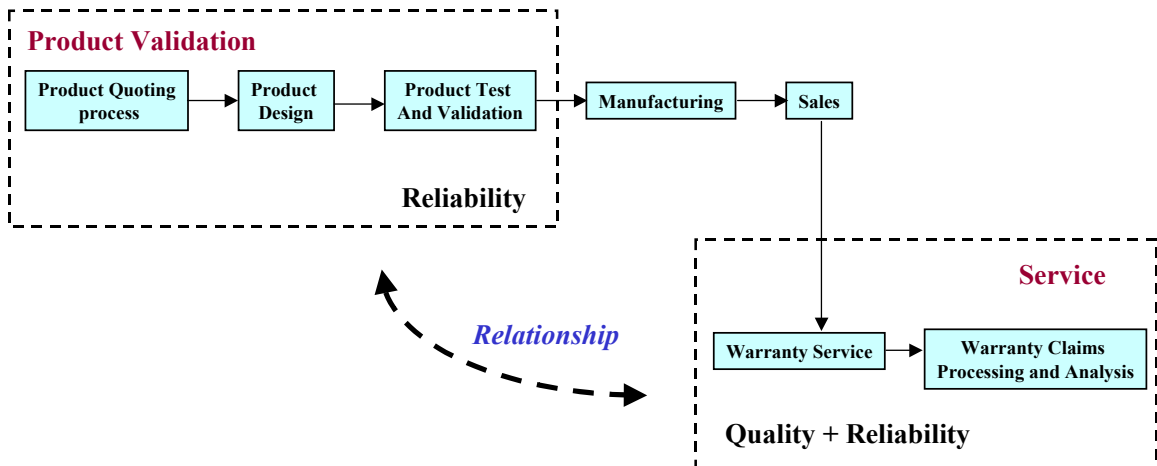
Figure 1.1. Dependability-related activities in product development process

The first three blocks of the diagram in Figure 1.1 (Quoting, Design, and Validation) are directly affected by the product validation activities and the last two are related to warranty and affected by the activities of a reliability engineer in an indirect manner. Comprehensive analysis of these relationships will help to build a model, which can subsequently be optimized to minimize the life cycle cost. In the later chapters of this

dissertation each box in this diagram will be given special attention as a contributor to the LCC model of the product.

1.3.   Motivation for this Study

A closer look at the activities presented in Figure 1.1 explains the important role dependability-related activities would play in the overall economic model of product development.

1.3.1.   Dependability Related Portion of the Life Cycle Cost of the Product

Life cycle cost (LCC) analysis is an important tool for choosing the most cost effective approach from a series of design alternatives. If a complete LCC mathematical model could be formed it would enable the optimization (minimization) of the total ownership cost, thus providing the opportunity for significant life cycle cost reduction. Clearly, minimizing the LCC will give the company a competitive advantage. According to [Kececioglu (1991)] it will affect competitive posture of the product in the marketplace, increase the profit and market share of the product, and other important business factors. Even though LCC models sometimes have credibility gaps due to lack of data [Barringer and Weber (1996)] they are effective as comparison trade-off tools and should be an integral part of the design and support process to achieve the lowest long-term product costs [Barringer and Weber (1996); Blanchard and Fabrycky (1998)].

It is important to note that from a supplier point of view the LCC of an automotive product will probably be different from the LCC of the same product from a consumer point of view. The supplier will normally be dealing with the 'truncated' version of LCC, which is different from the 'classic' cost models (see for example [Fabrycky and Blanchard (1991)]) and limited mostly to the development cost, manufacturing cost, and warranty cost of future failures or perceived failures of the product (see Chapter 2 for more details). Within that LCC structure the cost of product validation activities is a significant variable in the overall economic model. However, even within the framework of cost analysis, it appears that the issue of product validation cost and its impact on the whole program are rarely given enough attention in the early stages of business and engineering planning. In the literature, the various cost of ownership (COO) models (see [Fabrycky and Blanchard (1991); Barringer and Weber (1996); Blanchard and Fabrycky (1998)] for further details) lack the emphasis on test and validation equipment making it difficult to apply the models to estimate the overall cost of product validation. There are even fewer models, which attempt to interconnect these costs with future costs of warranty [Vintr (1999)]. The main reason for this kind of deficiency is the complexity of the task and the lack of field data. Real life LCC analysis, which includes detailed accounting for product validation cost and warranty data processing, is a task requiring unrestricted access to the industry data, which is not often available to external or even internal researchers. At the same time people in the industry, who have the necessary access, often do not have the time or expertise to approach it at a sufficient fundamental level.

There are a variety of specialists involved in the product development process, which includes designers, development engineers, reliability engineers, manufacturing engineers, materials specialists, accountants, buyers, marketing personnel, etc. All these engineering and business competencies are responsible for impacts on the life cycle cost of the product. Even though most of these activities are well coordinated, it is very difficult to develop a comprehensive mathematical model of their interactions and impacts on the LCC of the product. This also is partially due to a noticeable gap between everyday engineering practices and the latest developments in cost modeling and statistical simulation. An additional need for a comprehensive approach to cost analysis of automotive products arises from the recent trend in the automobile industry to make suppliers responsible for the partial or sometimes entire cost of a part's warranty [Ward's (1998)]. In this type of environment it is even more important to have a complete picture of a long-term cost of supplied products. Accounting for the total LCC would also provide quantitative decision support in frequent arguments between the OEM customers and their suppliers regarding specifics of various validation programs. Those disagreements often focus on test durations, schedules, sample sizes, and other engineering and business aspects of product development cycles.

One of the goals of this work is to create a methodology that enables an engineer to find a quantitative LCC-based solution to these and other related problems and subsequently optimize these solutions.

1.3.2. Quality-Reliability Relationship in the Automotive Industry

Despite the extensive coverage of various aspects of warranty in the literature reviewed by [Murthy and Djamaludin (2002)] the relationship between product quality and reliability unfortunately continues to be a gray area and analyses (if ever conducted) remain largely specific to a particular product. Even though most reliability textbooks contain the general concepts of the relationship between achieved reliability and expected warranty, and also their connection to the overall cost of the program (see Chapter 2 for details), they typically lack the specifics needed to generalize the model and make it applicable to a wide variety of product development programs.

Many if not most of the large manufacturing companies have separate quality and reliability departments, which typically have little or no interaction with one another. Even now quality and reliability professionals in the automotive industry have not established a clear connection between their activities and have not learned to place a realistic estimate on the cost of product validation in conjunction with expected costs of product warranty claims. This situation leads to certain deficiencies in a product development process such as an inability to combine all failure related activities into one comprehensive process and make a realistic cost estimate. Also it is not uncommon for each organization to blame the other for the product failures in the field, which often creates an inaccurate picture of failure root-causes and makes it difficult to identify proper corrective actions.

There is a multitude of other reasons why the relationship between quality and reliability has not been fully established, of which the difficulties in conducting comprehensive warranty data analyses would be high on the list. In the automotive industry only a portion of field failures can be associated with design problems (see Chapter 4 for more details) thus the link between product reliability and field failures can only be established on a statistical level as oppose to deterministic models favored by most engineers. Therefore, there is a need to bring a reliability-related approach to the issues of product warranty and to combine reliability and warranty into a comprehensive probabilistic model. Accomplishing this task would allow a more sophisticated approach to the comprehensive analysis of all product dependability-related activities.

1.3.3.   Contribution of Warranty Cost

At present, validation engineers in the automotive industry do not have a consistent methodology to evaluate the effect of their activities on the long-term program cost. In the initial phase of the business cycle during the product quoting, the cost of product validation is treated as a one-time expense and is rarely given enough priority and never treated in conjunction with its effect on the rest of the product life. This often leads to a customer's insistence on the highest possible reliability without any consideration for the costs involved in the process. In order for a reliability, validation, or test engineer to generate feasible reliability requirements with achievable and cost effective reliability targets, it would be beneficial to find the optimal point where the sum of validation and warranty cost as a portion of the total LCC is minimized. The current deficiency in establishing a connection between reliability and quality mentioned in Section 1.3.2

creates an enormous potential for improvement of the process of product development. The need for this kind of prediction models, design tradeoffs, and warranty estimates was previously emphasized in the literature [Economou (2004)]. Unfortunately, it is rare for the automotive designer to have any indication of predicted warranty cost for initial concept ideas [Nasser *et al.* (2002)], although the ability to estimate it would provide a certain engineering and business advantages. However due to complexity of this problem and lack of data, most authors prefer to deal with this relationship only on a theoretical level. For example [Vintr (1999)] presents the LCC minimization model assuming that the relationship between product reliability expressed in terms of the failure rate λ and its manufacturing cost is a known function $C(\lambda)$, when in reality, determining this function is expected to be the most challenging portion of the proposed effort.

Since the terms of automotive warranty are primarily dictated by marketing conditions, the future failures of the product are not part of the initial business model and only come into consideration later in the process. In the early 1990s the warranty databases in major automotive manufacturing companies existed primarily for accounting purposes. Fortunately, lately there has been a significant effort in the automotive industry to improve the process of bringing warranty analysis back to the OEM and their suppliers, both in terms of accounting and engineering data. Despite the latest improvements in this area, the process of bringing this information back into the reliability organization has been slow and inadequate for meeting all the product development needs. [Jauw and Vassiliou (2000)] list various reasons why many organizations are unable to take advantage of field product-failure or field performance data and have difficulty providing

comprehensive reliability data analyses based on quality/field data. Thus improving, or in some cases establishing a feedback loop from warranty to early design may significantly improve the process not only in terms of minimizing LCC, but also in bringing a better value to the customer.

1.3.4. Motivation Summary

As presented above, there is a need to provide reliability engineers with an approach that allows them to conduct the necessary LCC analysis and make a business case for changes in a validation program, which would also minimize a life cycle cost of the product and have a positive effect on customer's bottom line.

Therefore, the goal of this dissertation is to provide reliability professionals with a methodology to evaluate the efficiency of a product validation program from a life cycle cost point of view with the emphasis on cost of validation and product warranties, and ultimately minimize that cost by optimizing the environmental test flow of the product validation process.

1.4. Problem Formulation

Product validation activities (full-scale environmental, mechanical, electrical, and other types of testing at various stages of product development) are an important portion of the product life cycle cost and they greatly affect the warranty returns and service costs. The main goal of this work is to create a model of the life cycle cost with input variables that

can be controlled by a reliability/validation engineer during a product development process. This could be achieved by incorporating all dependability-related activities into a single comprehensive statistical cost model of a product's life cycle. After this model is created, the reliability test flow can be optimized to achieve the lowest possible cost of the dependability-related activities. The numerical optimization methods will not be the focus of this study. Instead, the emphasis here will be made on formulating the methodology and generating the model, with comprehensible inputs and outputs suitable for optimization by most of the available engineering methods.

Main Questions:

The completed model should be able to answer the following questions:

- What are the leverages available to a reliability engineer to affect the cost of the product validation process?

- How do the reliability testing activities affect the expected warranty returns of the product during its mission life?

- How can the program be optimized to achieve the minimum of dependability-related share of the LCC?

- What is the most suitable model to simulate and forecast future automotive electronics warranty claims?

1.5.   Dissertation Objectives and Tasks. Focus of the Research and Solution Strategies

1.5.1. Objectives

Research objective: To minimize the life cycle cost by utilizing the design and management options available to a reliability engineer. This will be accomplished primarily by optimizing product validation procedures (mostly in the form of reliability targets, test durations, and sample sizes) based on historical product information and the attributes of the product test flow. Solutions shall have to balance rigorous theoretical treatment and practical applications and will be specifically applied to automotive electronics products. The goal of this research is to create a statistical model to be utilized by a reliability engineer in order to minimize the dependability-related portion of the life cycle cost. To achieve these objectives the specific tasks presented below need to be accomplished.

Important Note: It is important to mention here that this work does not involve studies of risk analysis. Risk analyses involve an assessment of 'consequence,' which is outside the scope of this dissertation. The probability of failure is the focus of this dissertation, the effects of these failures will not be analyzed at a probability-consequence level beyond the scope of repair warranty costs. Therefore the methodology presented in this dissertation, though covering reliability, quality, and warranty, will not be directly involving any risk-specific terms, methods, or techniques.

1.5.2. Tasks

The following tasks should be accomplished as a part of the overall solution strategy.

1. Create statistical models to analyze product validation costs.

- Product validation cost model with inputs that include reliability targets, equipment cost of ownership, test duration, test sample size, and others

- Bayesian model of test sample size reduction with a knowledge-based mix of prior distributions.

- Account for the effect of parametric binomial relationship between test duration and test sample size.

2. Create statistical models to analyze expected warranty returns.

- Detailed warranty data analysis going back 10 years for the select automotive electronics product lines with the emphasis on audio systems (radio, cassette player, CD player)

- Analysis of the warranty trends for those product lines in terms of statistical distribution parameters based on past field performance data for each product line, key product features, years in production, novelty of the process, etc.

- Generate an innovative warranty prediction model based on best-fit statistical distribution for a 2-D warranty, estimate of expected failures, and repair cost distributions.

- Inputs include detailed warranty claims data, hazard rate stabilization time, and NTF failures

- Outputs will include expected failures and cost of unreliability with an option of accounting for the effect of a particular production lot and the model year.

3. Combine the sub-models into a comprehensive model to obtain an optimized LCC that reflects the ways and means available to a reliability engineer.

1.6.   How the Remainder of this Document is Organized

The rest of the dissertation is organized as follows. Chapter 2 will present a high level overview of the proposed cost model and will outline the direction of this research. It will describe the proposed LCC model discussing its probabilistic and deterministic inputs and outputs. Chapter 3 will focus solely on the inputs related to the cost of product validation and on reliability demonstration activities focused on achieving certain reliability levels with a significant emphasis on the role of the test duration and sample size. Chapter 4 will deal with the ways to reduce the cost of validation using Bayesian techniques. Chapter 5 will cover the model inputs related to product warranty and the costs of service and repairs associated with warranty claims. Chapter 5 will also cover the methods of forecasting future warranty based on the repair history of the existing products. Chapter 6 will cover the modeling process combining all the input variables and providing the Monte Carlo simulated outputs. It will also present a case study of an automotive electronics example for the purpose of illustrating the proposed methodology. Chapter 7 will summarize this work, outline the contribution of this research, and discuss future work and remaining problems.

2. Life Cycle Cost Model Structure

This chapter presents a high level overview of the model utilized to optimize the product validation program in order to minimize the life cycle cost of the product.

2.1. Life Cycle Cost Analysis and its Dependability-Related Variables

The objective of life cycle cost (LCC) analysis is to choose the most cost effective approach from a series of alternatives. The lowest possible long term cost of ownership can be achieved while accounting for the cost ingredients that include design, development, production, operation, maintenance, support, and final disposition of a major system over its anticipated useful life span [Barringer and Weber (1996); Landers (1996)]. LCC varies with events, time, and conditions. It is important to mention here that there is no uniform definition of what is included in LCC. The real life cycle cost of an automotive product will probably be different from its 'classic' content defined for example in [SAE (1993)]. Some manufacturing LCC categories, such as sustainment cost or performance cost [LaFrance and Westrate (1993)] would not apply to an automotive part, thus the supplier's definition of LCC becomes a truncated version of the 'classic' definition. Even though the accuracy of LCC models can vary significantly due to lack of data and consensus on how to account for it [Barringer (1996)] they are effective as comparison tools and should be an integral part of the design and support process to achieve the lowest long-term product costs [Barringer (1996); Blanchard and Fabrycky (1998)]. The benefits of LCC minimization are even greater in mass production

industries since every cost improvement will bring additional profits multiplied by high volumes.

Life cycle cost (LCC) analysis is a complicated process consisting of many steps and various inputs [Fabrycky and Blanchard (1991)]. Many cost variables are not deterministic but are probabilistic. This usually requires starting with arithmetic values for cost and then growing the cost numbers into more accurate, but more complicated, probabilistic values and their statistical distributions. In many industries including automotive, the activities directed at addressing the possible failure of the product play a significant role in the product development cycle. In Figure 1.1 these activities include new business quoting and all product validation and warranty/service related activities. Therefore product validation engineers often need to focus their activities on dependability-related variables of LCC analysis, since these are the inputs they can influence the most.

The dependability-related activities focus mostly on quality and reliability problems. However, as mentioned in Chapter 1, the link between the reliability and future warranty/service expenses is not always easy to establish, thus determining this relationship will be an important part of this model, even though it can only be done probabilistically. Also the costs of each of the activities comprising LCC are often difficult to estimate due to the random nature of quality-reliability relationship. For example, determining the cost of warranty can be quite complicated, since each repair involves the costs of parts, labor, diagnostics, removal of parts (both good and bad),

replacements, etc., and there is a significant variation of these parameters from case to case. In the same manner, the cost of equipment involved in testing will also have a certain degree of uncertainty, varying from test to test based on environment type, requirements, usage, and many other parameters.

2.2. Failure Related Activities

The relationship between the reliability/dependability of a product defined by [Fernandez (2001)] and its LCC has been occasionally discussed on a theoretical level in the literature (see for example [Kececioglu (1991); Blischke and Murthy (1994)]), and can be roughly presented by the diagram similar to the one shown in Figure 2.1. This graph presents a relationship between the reliability and cost associated with the product development. The higher the pursued reliability of the product, the higher the product development cost (the ascending curve in Figure 2.1). At the same time the higher the achieved dependability of the product, the lower the cost of the associated warranty and service (the descending curve in Figure 2.1). Thus the sum of these two costs would resemble a U-shaped curve bottoming around the value of the lowest sum of product validation and warranty cost thus minimizing the total LCC. Charts similar to Figure 2.1 have been referred to as 'Contractor's cost vs. Reliability' [Blishke and Murthy (1994)], 'Dependability vs. non-dependability cost' [Fernandez (2001)], 'Producer's Cost' [Kececioglu (1991)], and several others. For simplicity and consistency, from this point on this chart will be referred as the 'Reliability-Cost' curve.

2.2.1.   Reliability-Cost Curve

In all the mentioned literature Reliability-Cost models are presented in very general terms and often in reference to products with highly predictable cost of scheduled maintenance, like aircraft or heavy machinery with the emphasis on maintenance schedules and the cost of spare parts [Kececioglu (1991); Monga and Zuo (1998)]. In this dissertation, the concept of LCC minimization based on pursued reliability will be applied to the mass production industry with service cost largely expressed in terms of automotive warranties. In addition, the emphasis will be made on the reliability engineering costs as a significant part of the product development process.
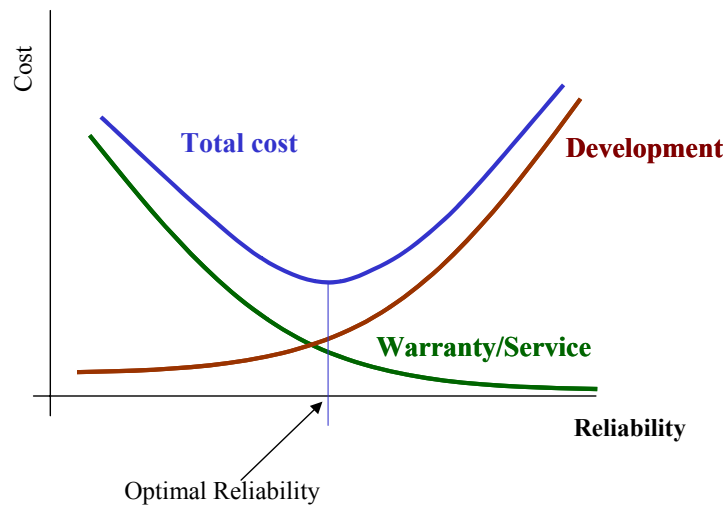
Figure 2.1. Theoretical 'Product Development Cost versus Reliability' Curve.

As mentioned previously, the concept shown in Figure 2.1 was originally developed and is most widely used for products requiring scheduled maintenance, however most automotive parts are not designed to be maintained on a regular basis. The costs and occurrences of automotive repairs are far less predictable, therefore the model presented in Figure 2.1 will have a certain number of random inputs and outputs. Therefore this

model cannot be solved on the simple deterministic level presented in Figure 2.1 and would require a probabilistic approach. In order to generate a real-life Reliability-Cost diagram it is important, among other things, to find the probabilistic relationship between future warranty returns and designed reliability. This relationship is a cornerstone of the descending curve in Figure 2.1 and its accuracy is essential to the accuracy of the whole LCC model.

2.2.2.   The Relationship between Quality and Reliability

In engineering economic analysis the cost of product validation is rarely considered in conjunction with the expected costs of product warranty claims. Typically the quality-based approaches to warranty lack any reliability focus [Blicshke and Murthy (1994); Murthy and Djamaludin (2002); Kececioglu (1991)]. The unclear relationship between reliability and expected warranty, and the random nature of that relationship add to the list of reasons why validation activities and warranty forecasting and processing are budgeted, planned, and conducted separately.

Many cost models associated with the 'Reliability – Cost' concept consider the overall cost of the design cycle, but often ignore the specific contribution of product validation cost (for further details see [Blishke and Murthy (1994)]). The knowledge about the cost of a product validation program can be a very important piece of information during a

quoting process.[2] A reliability engineer is expected to accurately estimate validation cost based on the reliability requirements presented by the OEM customer. However this kind of estimate is frequently based on the existing product information and it can often be inaccurate or outdated.

As mentioned before, quality and reliability professionals in the automotive industry are still working on the process of establishing the links between their respective activities. Since warranty data in the automotive industry accumulates all the reported incidents [Lu (1998)] and according to [Pecht (1997); Thomas *et al.* (2002); Majeske and Herrin (1995)] only a portion of the field failures can be associated with design problems, the link between product reliability and field failures can only be established on a statistical level as oppose to deterministic models[3] that are favored by engineers (see chapters 3 and 5 for more details). As a result the descending curve in Figure 2.1 is not as well defined as it appears in most textbooks. One of the objectives of this work is to bridge this gap and to improve the understanding of this relationship.

Further complicating the issue, the warranty periods are typically much shorter than the mission life (in the automotive industry it can be a 3 year warranty vs. 10-15 years

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[2] A quoting price estimate is given to the potential consumer as he/she decides which company to award the business to. A company may be legally bound to honor this quote in some jurisdictions and/or lines of business.

[3] A deterministic model or algorithm consistently produces exactly the same result for exactly the same input, where probabilistic or stochastic models have random characteristics.

mission life) and failure information beyond the standard or optional warranty period is rarely available. This makes it difficult to correlate any warranty-related prediction with the real life data and creates the need for certain experience-based assumptions, which will be covered in more details in Chapters 3 and 5.
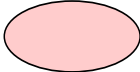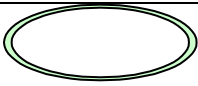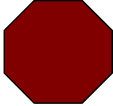
2.2.3.   Optimization of Product Validation Flow Using the Reliability-Cost Relationship

Finding the lowest point of the total cost curve in Figure 2.1 can be achieved first by constructing a realistic and practical model for both the ascending and descending parts of the curve while incorporating all dependability-related activities into a comprehensive statistical cost model of the product life cycle. In this dissertation this will be achieved by utilizing the general concepts of the Reliability-Cost relationship, while seeking a statistical solution focusing on the reliability engineering activities and their costs, thus effectively making LCC an objective function for optimization. In other words the process can be optimized by finding the minimum point of the sum of the two cost curves. It is important to note again that this dissertation will not focus on the mathematical and computational attributes of the minimization process, but rather on the reliability aspects of the model and on feasibility of finding the minimum LCC by optimizing the product validation process.

2.2.4.   LCC Optimization Process Influence Diagram

This section presents an influence diagram for the LCC optimization process. Influence

diagrams can be useful tools in describing inter-system relationships including all the

factors that affect the process of modeling and decision-making. An influence diagram is

a graphical tool that shows the relationships among the decision elements of a system

[Ayyub (2003)].   The influence diagram in Figure 2.2 shows the relationship of the

factors influencing the modeling of the dependability-related portion of LCC.   The

influence diagram symbols used in this dissertation are presented in Table 2.1 and are

consistent with those used in [Ayyub (2003)].

Table 2.1. Influence diagram symbols used in this dissertation (based on [Ayyub (2003)])

	Decision Node: Indicates where a decision must be made
	Chance Node: Represents a probabilistic or random variable
	Deterministic Node: Determines from the other nodes or other non-deterministic variables
	Value Node: Defines consequences over the attributes measuring performance
	Arrow: Denotes influence among nodes and the direction of the decision process flow

The influence diagram in Figure 2.2 shows all the factors affecting this LCC decision

making process. Those factors include the variety of inputs affecting the process from

the new business quoting event through design, validation, and warranty.   All the

influence factors fall under the following major categories: (1) Business-Finance, (2)

Design and Validation, (3) Service and Warranty, and (4) Assumptions and Models. The

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first three represent the flow of product development from business contract to design, validation, and consequent repair/service. The fourth group (Assumptions and Models) influences all of the above blocks since the modeling process incorporates a number of engineering assumptions, utilized models, and equations (both previously existing and developed in this dissertation). Each of the four categories has at least one major decision-making block and a variety of probabilistic and deterministic node inputs. All of these inputs will directly and indirectly affect the outcome value node, where the final dependability-related portion of LCC is calculated and minimized.
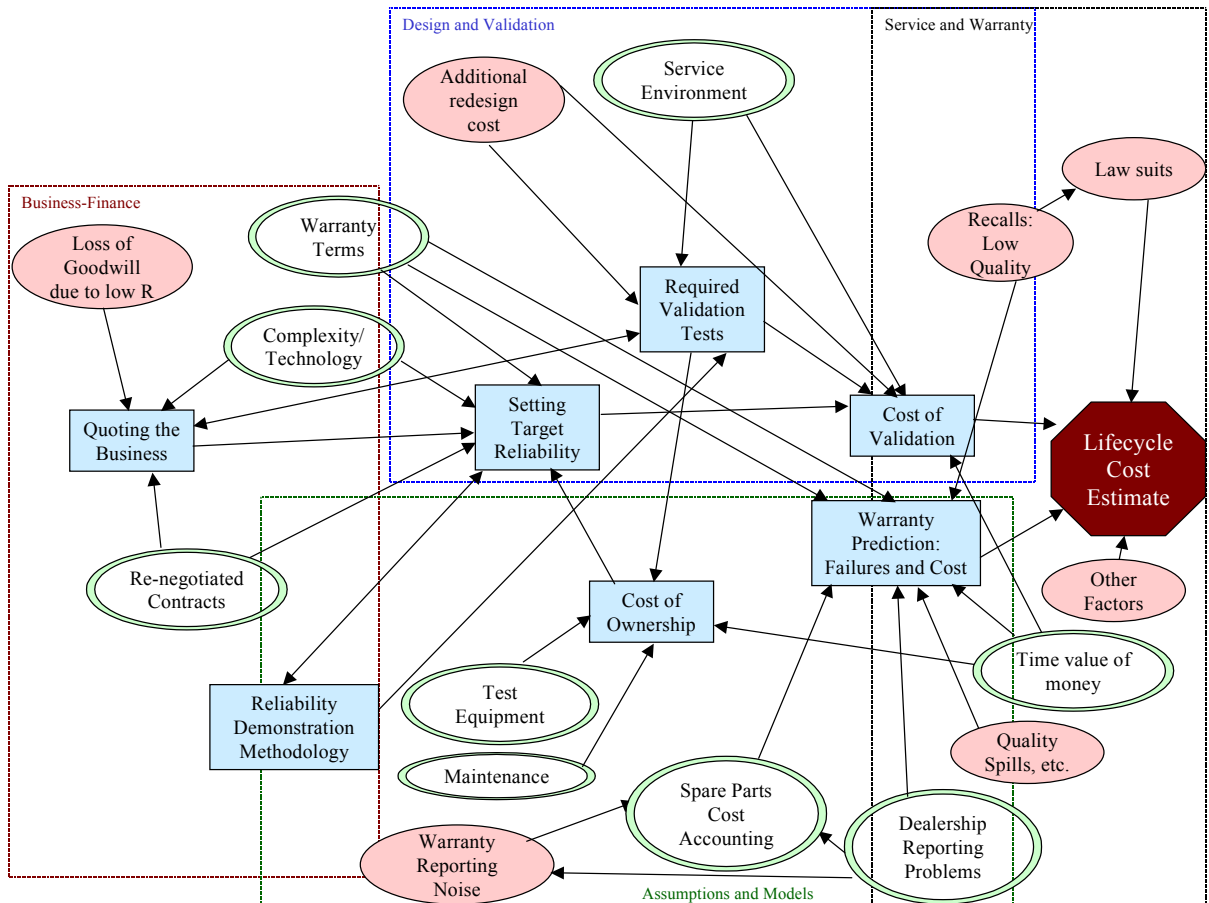
Figure 2.2. Influence diagram. All potential factors are included.

It is important to mention that not all the nodes shown in the influence diagram Figure 2.2 can be effectively accounted for in the model developed in this dissertation, therefore the truncated version of this diagram is presented in Figure 2.3, which reflects only the factors that will actually be included in this work. The removed blocks in this diagram are bypassed for various reasons including minimal influence on the product design with rare or unpredictable occurrence (e.g., 'Law suites' and 'Loss of goodwill'). Even though these two factors can have a profound effect on the LCC of the product, their financial impact is rarely taken into consideration by the design and validation team with the exception of the passenger safety related products, such as airbags or vehicle breaks.

Other influence factors, such as re-negotiated contracts, spare parts cost accounting, quality spills, and recalls are eliminated from the original diagram because they fall outside the scope of the problem addressed in this work, i.e., they are not within the realm of responsibility of the professionals who work the optimization procedure addressed here. In other words, these items are addressed by different engineering and business competencies and are out of control of the reliability engineer. Regarding the additional redesign cost; the data supporting this particular node is virtually non-existent, however based on the Delphi institutional knowledge, the effect of this node is believed to be low. Also this effect is mitigated by partial allocation of separate funds specifically to address this during the initial quoting process.

Despite the fact that several influence factors have been eliminated in Figure 2.3, the diagram in Figure 2.2 acknowledges their existence, legitimacy, and importance, even though they were not reflected in the final model developed in this dissertation. The removed factors still remain important and could be used to improve the accuracy of LCC analysis in future modeling work, which could expand the scope of the problem to include a more comprehensive inter-functional approach.
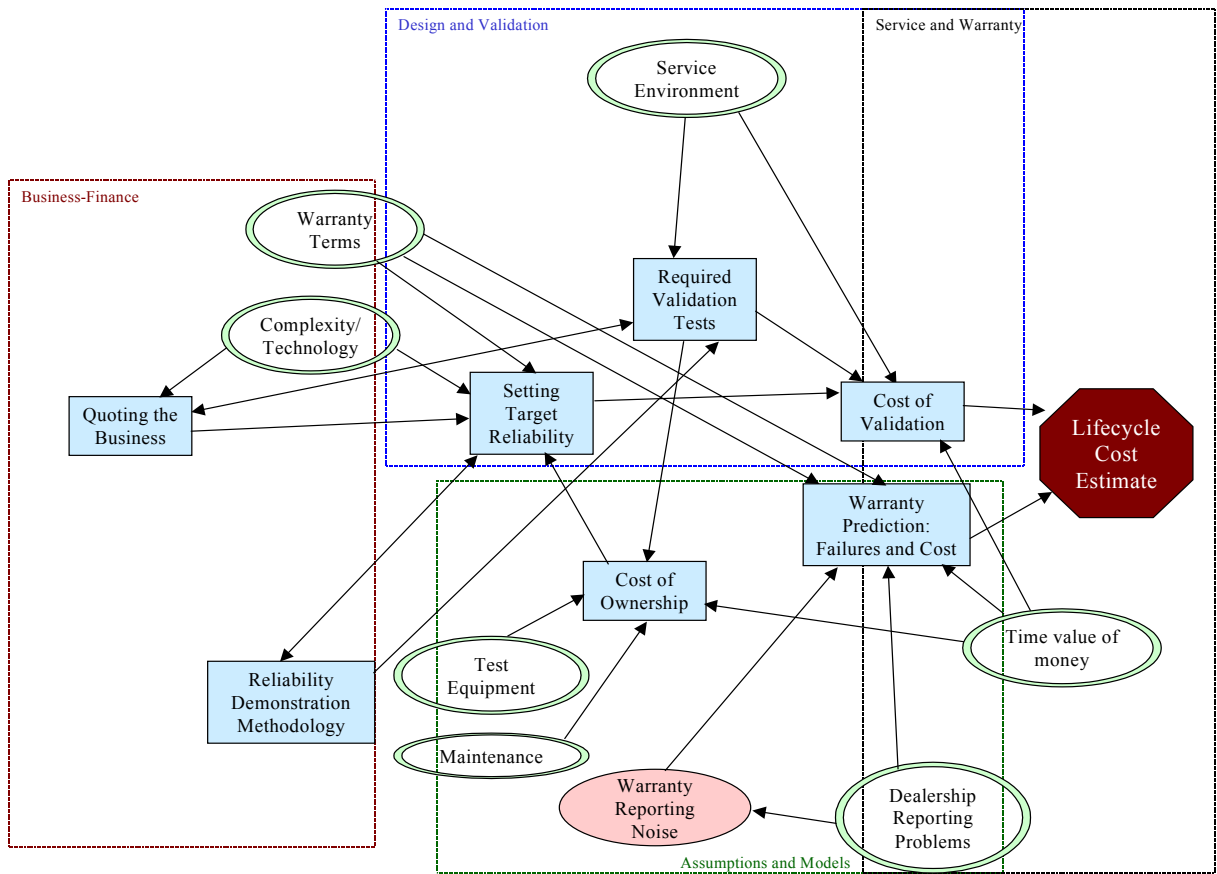
Figure 2.3. Truncated influence diagram reflecting the content of the model developed in this dissertation

The final diagram in Figure 2.3 shows the decision/solution process presented in this work and contains only the factors included for consideration in this dissertation.

2.3.   Block Diagram of the LCC Methodology Flow

The following is the high-level overview block diagram, which will be discussed in detail in Section 2.4.   Each box in this diagram represents a combination of several sub-systems, which will be discussed in details in the later chapters.
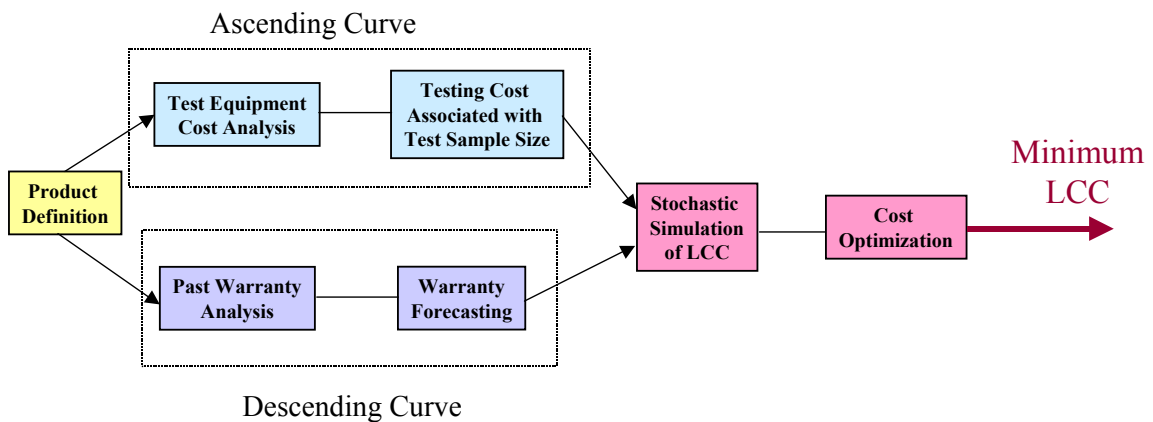
Figure 2.4. Block Diagram of the LCC Methodology Flow

The diagram in Figure 2.4 provides the outline of the LCC model. The top path corresponding to the ascending curve of Figure 2.1 includes the cost of ownership (COO) model (for more information on COO see [Dance (1996)] and Chapter 3 of this dissertation) of the test equipment required to conduct particular environmental tests as part of a product validation program. The second important contributor to the ascending part of the curve is the costs associated with test units, which is highly influenced by a test sample size.   This includes the cost of producing each test sample (which can be

quite high in some industries), equipping each sample with monitoring equipment, adequate test capacity equipment to accommodate all the required samples, and many others. The bottom path (Descending curve) deals solely with the expenses related to future product failures, such as warranty and service costs (see Chapter 5 for details). The main source for this type of information would typically be a company's warranty database and other types of failure and repair related information. All these inputs are incorporated into the total cost model by the means of stochastic simulation with the random variable inputs. Minimization of total costs will allow finding an optimal duration of the most expensive tests and the respective sample sizes, while at the same time satisfying reliability requirements for the product.

2.4.   LCC Model Inputs

There are a variety of LCC computation methods. The SAE model is considered as one of the most comprehensive LCC methods in the automotive industry [SAE (1993)]. However, as mentioned before, most of these models apply to maintenance intensive products, which do not include the majority of automotive components and automotive electronics in particular. The variety of cost inputs needed to populate the model developed in this dissertation is obtained from various sources such as automotive warranty databases, COO analysis for the test lab equipment, costs associated with test and validation lab, etc. Specific data inputs will be detailed in their respective chapters and will include various expenses required to take a product through a complete series of environmental and functional tests. The following sections describe the categories of model inputs with many of the inputs being random variables.

2.4.1. Product Definition and Reliability Requirements

Typically the first step in product development includes some form of product definition, which requires a wide variety of information including complete product specification, functionality, usage, and others attributes. However the main focus for a validation engineer usually remains on the technology utilized in the product and the usage conditions. Both items are very critical to defining the validation part of the product development sequence. However in most of industries, including automotive, OEM customers often provide the requirements pertinent to the reliability performance of the product in the field. It is important to collect and understand all the reliability specifications in terms of required environmental and functional tests and also in statistical terms of reliability and confidence level. Reliability requirements are typically specified by the OEM customer in terms of percent survival, cumulative failures, MTBF, MTTF, failure rate, B_X-life[4], and various others.

Reliability requirements usually come in a variety of shapes and forms. More information on automotive reliability requirements and how they are derived can be found in [Krasich (2003); Lu and Rudy (2000)]. Most of the environmental tests for automotive electronics can be divided into two major categories: durability and capability tests (see Chapter 3 for more details). Durability tests are intended to simulate the field environment as applied to a product mission life. Usually some form of fatigue failure

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[4] B_X-life is the product's service life where X% of the population is expected to fail.

mechanism is caused by those types of environments. Capability tests do not simulate the mission life, but instead are used to verify that the product is capable of functioning under certain environmental conditions. This work concentrates on environmental test formats most typical to automotive electronics requirements, although it is important to note that other mass production industries, especially consumer electronics, have similar product validation procedures. Typical reliability requirements contain the detailed information about the types of environmental tests to be conducted on the product with some specific parameters like 10 hours at high temperature of 125° C or 3 hours of random vibration with specified profile. In addition, the test sample sizes for each test are often specified in order to demonstrate certain target reliability – this is one of the key variables where validation engineer can affect the ultimate LCC of the product. Durability testing is more involving and takes longer time, therefore the potential cost savings can be more substantial compared to much shorter capability tests targeted to discover the immediate design flaws. Since a big part of the equipment costs are driven by the type of tests and their durations, understanding the reliability requirements is the first critical step in determining the cost inputs associated with the test equipment involved in product validation.

2.4.2.   Cost of Ownership for Product Validation

After finalizing the reliability requirements and determining the types of environmental tests needed we can start the process of calculating a test equipment cost of ownership (COO). COO relates to the total cost of acquiring, installing, using, maintaining, changing, upgrading, and disposing of a piece of equipment over its predicted useful

lifespan (for further details see [LaFrance and Westrate (1993); McKenzie (2004); Dance (1996)]). Sometimes it is referred as TCO (Total Cost of Ownership), however the term COO will be primarily used in this dissertation. COO analysis usually includes the equipment depreciation, installation, sustainment cost (energy, repair and maintenance, etc.), disposal cost, and various other contributions. However, despite the uncomplicated math COO models have a substantial degree of uncertainty. In many cases the process of accounting for these costs can be complicated by lack, incompleteness, or inaccuracy of the equipment data pertinent to the maintenance (both scheduled and unscheduled), cost of replacement parts, duration of repairs, etc., thus contributing to the uncertainty of the cost model. A method of dealing with the common problem of missing and incomplete equipment maintenance records and its effect on the process of calculation of the equipment COO will also be discussed in this dissertation.

As mentioned earlier, certain portions of the equipment cost are driven by the type of tests, their durations, and test sample sizes. The cost associated with test equipment can reach millions of dollars, especially in large manufacturing or testing organizations. The effect of these variables on life cycle cost has not been fully studied in either reliability or warranty literature. More detailed analysis of these and other related issues will be discussed in Chapter 3. COO analysis will be the main source of cost input associated with the ascending part of the Reliability-Cost curve in Figure 2.1.

2.4.3. Test Sample Size

Since demonstrated reliability is typically a function of test sample size, the latter becomes one of the control factors available to a reliability engineer (also referred here as validation engineer) whose function is to detect a potential nonconformance to the specification of the product and to communicate this information to a design engineer.

When the result of a test has only two outcomes (in the case of reliability testing it is pass or fail) the Binomial distribution is often applied to calculate the reliability (see Appendix A). In their pursuit of high quality and high reliability in a mass production environment, the automotive manufacturers require their suppliers to prove target reliability with an assigned confidence level on a supplied product. This is often done through a reliability demonstration test by running a specific number of samples under conditions simulating the mission life sometimes called *test to a bogey*. Most of the time the number of samples is determined by the required reliability and the confidence level. Test sample size in turn can be affected by a test duration (see Appendix A) or application of knowledge-based techniques, such as Bayesian analysis (see Chapter 4 and Appendix B for details). Test sample size carries the cost of producing each test sample (which can be quite high in some industries), equipping each sample with monitoring equipment, and adequate test capacity equipment to accommodate all the required samples. The last contribution to test sample size can present a significant cost problem, since a large sample may require additional capacities of expensive test equipment, such as temperature/humidity chambers or vibration shakers costing tens or hundreds of thousands of dollars.

Several different approaches to determining and consequently reducing test sample size and overall cost of product validation is considered and analyzed in this dissertation.

2.4.4. Warranty/Service Cost

Warranty cost is a significant part of the overall product's cost. As mentioned in Chapter 1, on average General Motors spends around $3.5 billion on warranty [Nasser *et al.* (2002)]. Most companies maintain some form of FRACAS reporting system, where they collect and analyze past field and test failures. All automotive manufacturers and most of their suppliers maintain internal and external warranty databases. There are a variety of warranty database formats, but generally they are organized in a similar fashion and contain information specific to a FRACAS reporting system. For example, the structure of a DaimlerChrysler automotive warranty database was described by [Hotz *et al.* (1999)] and General Motors database by [Walters (2003)]. A typical automotive warranty claim contains all the relevant information about the vehicle and the failed system including manufacturing date, repair date, vehicle mileage, problem description, some geographical data, repair code, cost of repair, and many others. Some general information about how the General Motors warranty database is organized can be found in [Walters (2003)] and DaimlerChrysler warranty reporting system in [Hotz *et al.* (1999)]. Also a typical warranty database being a large entity contains a certain amount of noise and unusable data, like inaccurate reporting, wrong codes, and NTFs (No Trouble Found) as described in [Thomas *et al.* (2002); Salzman and Liddy (1996)]. The issues of statistical analysis of warranty data will be discussed in more details in Chapter 5.

It is important to remember that product warranty is an inseparable part of a business model. Market conditions have traditionally been the main factor that determines the terms of automotive warranties. While expected reliability and quality of the product is considered an important supporting factor, in reality, the actual warranty terms are most often determined by marketing pressures. [Mitra and Patankar (1997)] analyze the effect of warranty decisions on market share, examine market share as a function of warranty, and analyze the option of extending the warranty at the end of the base warranty period. Currently the terms of the standard automotive warranty, often referred to as the manufacturer's basic warranty are 36 months or 36,000 miles, whichever comes first [Auto Warranty Advise (2004)] on all of the vehicle systems with additional optional extended warranties or standard longer term warranties on selected sub-systems, such as catalytic converters or engine controllers.

Warranty history and warranty expectations greatly affect the market value of new and used cars sold and lease residual values. Because of these and other financial and marketing considerations, a multitude of business decisions are being made based on the forecasted number of warranty returns for the overall warranty period and subsets thereof. More detailed warranty information will be presented in Chapter 5.

As mentioned above, the information presented in warranty databases is extensive and can be used for various types of statistical analysis both parametric and non-parametric.

In this dissertation warranty claims information is used extensively to calculate the after shipment factor of automotive part LCC.

In many industries quality and reliability engineers who are involved in the warranty forecasting process often use empirical models based on past warranty claims of products with similar design and complexity adjusted by certain, experience-based correction factors accounting for the design and technology changes in the product. A reasonably accurate, scientific, and user-friendly model could help to accomplish these types of forecasting with better precision and improve the overall quality of business decisions requiring estimates of future warranty claims.

Warranty terms are not determined by the reliability of the product, but rather by financial and marketing considerations. In addition to practical reasons, longer warranty periods are often used as an enhanced marketing tool. Clearly product validation activities affect both cost of the product development and future service cost (mostly in the form of the cost of warranty returns), but how can it be quantified?
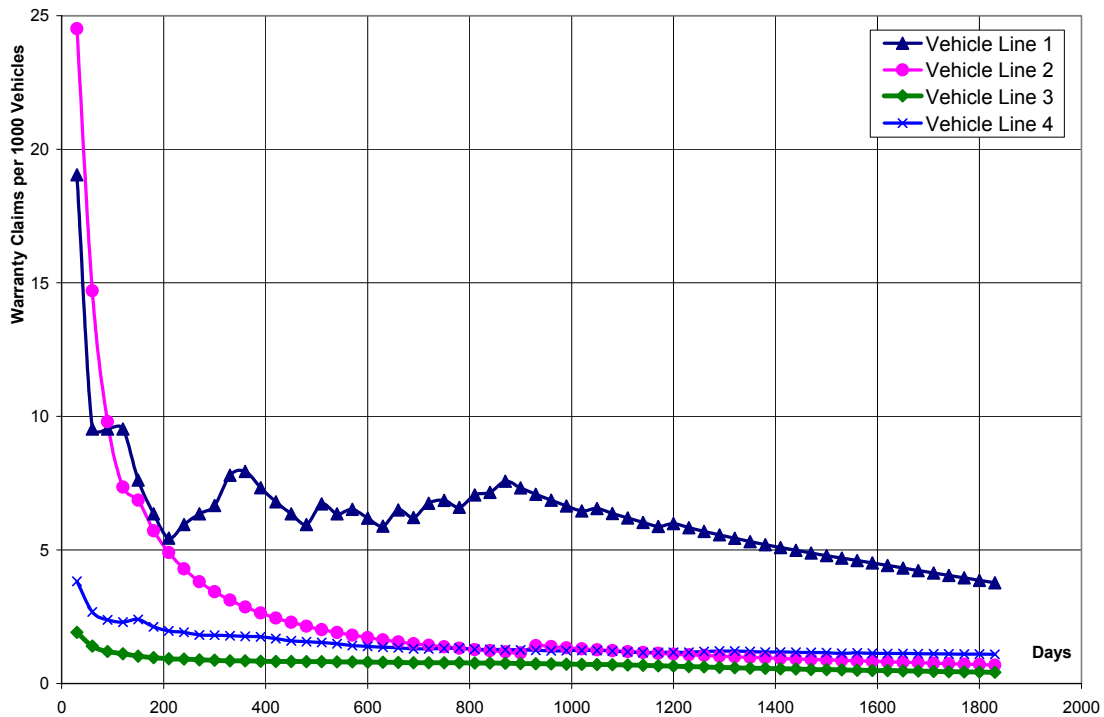
Figure 2.5. Extended warranty charts compiled from Delphi Corporation's 5-year warranty data for the four different automotive radios mounted on several vehicle lines. The data shows no wear-out mode for at least 5 years of service.

The diagram in Figure 2.5 suggests that in the majority of the cases the warranty failure model is sufficiently represented by the infant mortality and useful life phases of bathtub[5] curve. A detailed study of the existing warranty of various product lines of automotive parts performed at Delphi Electronics & Safety showed a clear trend of diminishing failure rate for the first 8 to 18 months (see also Figure 5.2) followed by a flattening of

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[5] The reliability of electronic devices has often been represented by an idealized plot called a bathtub curve, which consists of three regions: infant mortality, useful life, and wear-out [O'Connor (2003)]

the failure rate curve for the remainder of the time period that warranty and extended warranty data were available.

Field failures can result from inadequate design, defects generated during component manufacturing, errors in the assembly process, and other effects mentioned by [Majeske (2003)]. There is a variety of warranty analysis and prediction methods including both parametric (see for example [Yang and Zaghati (2002); Majeske and Herrin (1995); Oh and Bai (2001)]) and non-parametric ([Lawless (1998); Kalbfleisch *et al.* (1991)]). In this dissertation the focus is made on parametric methods due to the emphasis on forecasting, where parametric models can typically do a better job of extrapolating the results of warranty analysis. The prediction model is based on probabilistic analysis of the existing warranty data and will be discussed in more details in Chapter 5. The forecasted cost of warranty claims will be the main input corresponding to the descending part of the Reliability-Cost curve in Figure 2.1.

2.5.   Proposed Cost Model

One of the objectives of this work is to create a cost model to be used by reliability engineers and which can be optimized based on decision variables controlled by these engineers. The ideal cost model in our case would be practical and intuitively obvious to reliability practitioners and at the same time mathematically descriptive and conducive to optimization.

In general terms, the cost model for a mass production automotive component can be described by the equalities below presented by [Kleyner *et al.* (2004)]. Equations (2.1), (2.2), and (2.3) show the cost components of the buyer's cost for the products in general and automotive products in particular.

Buyer's Cost = Design Cost + Validation Cost + Manufacturing Cost + Warranty Cost + Seller's Profit

$$(2.1)$$

Writing equation (2.1) more explicitly,

$$n\alpha_b(\underline{\theta}, \underline{W}) = \alpha_d(\underline{\theta}, \underline{W}) + \alpha_{pv}(\underline{\theta}) + n\alpha_m(\underline{\theta}) + n_f(\underline{\theta}, \underline{W})\alpha_W(\underline{\theta}) + P'$$

$$(2.2)$$

Where:

α_b = per unit cost to the buyer (customer's price) α

α_d = design cost of the project

α_{pv} = total cost of product validation

α_m = manufacturing cost on per unit basis

α_w = cost of warranty on per unit basis

P' = seller's profit

$\underline{W} = \{T_0, M_0\}$ = two-dimensional warranty, where T_0 is the warranty time limit (typically 36 months) and M_0 is the warranty mileage limit (typically 36,000 miles).

$\underline{\theta}$ = vector of design parameters

Equation (2.2) assumes that the number of manufactured units n approximates the number of units sold, which is usually true for high-volume products. Equation (2.2) can also be regrouped the following way:

$$n\alpha_b(\underline{\theta},\underline{W}) - \alpha_d(\underline{\theta},\underline{W}) - n\alpha_m(\underline{\theta}) - P' = \alpha_{pv}(\underline{\theta}) + n_f(\underline{\theta},\underline{W})\alpha_W(\underline{\theta})$$

(2.3)

On the left-hand side of equation (2.3), the cost of design α_d represents the value, which is most difficult to estimate, since it often involves engineering time, prototype fabrication, testing, training, overhead, and many other factors. However most of α_d is estimated prior to the beginning of new product quoting process, i.e., during product specification phase. The quoting process specifically consists of documenting technical characteristics, cost estimates with risk analysis, engineering requirements, manufacturing plan, and preliminary product price. The cost of product development that is included in product quotes is usually based on forecasting methods, such as analogy models, expert judgment, prototype models, top-down calculations, and others (see for example [Rush and Roy (2000); Bashir and Thompson (2001)]). Thus, the first order approach will associate α_d with the value, which based on historical development cost of similar product lines and assume it is not significantly affected by product validation activities and therefore will be considered as constant of test sample size and test duration. Other left-hand side components of equation (2.3) also will not be affected by either test sample size or test duration.

In addition to the 'traditional' costs listed above, companies should consider the 'intangible' factors, such as the cost of tarnishing brand image associated with poor product quality and reliability. Also the cost of future lawsuits can significantly increase the LCC (see influence diagram in Figure 2.2). However these aspects of cost will not be covered in this dissertation due to their extreme unpredictability.

Now let's look at the terms on the right-hand side of equation (2.3). Assuming the validation procedures will be similar across products with similar application conditions, which for automotive electronics are largely dictated by product location in a vehicle. The requirement of reliability and associated confidence level submitted by the OEM customers are linked to reliability demonstration procedures, which are in turn related to a sample size and test duration. Thus, the main factor, affecting the variable cost of product validation will again be the test sample size, and test duration,

$$\alpha_{pv}(\underline{\theta}) \cong \alpha_{pv}(N, t_T)$$

(2.4)

For a given rate of defects, the number of products that reach the market and trigger warranty claims will be approximately proportional to the number of products shipped. Therefore, the number of units, n_f, expected to fail under warranty, will be proportional to unreliability *(1-R)* of the product and thus partially dependent on validation procedures. In fact, assuming that the demonstrated reliability would be reflected in product

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performance in the field, n_f will also become dependent on not only the warranty terms, but also on demonstrated reliability and thus the test sample size: $n_f = n_f(W, N, t_T)$

Thus equation (2.3) will take form:

$$n\alpha_b(\theta, \underline{W}) - \alpha_d(\theta, \underline{W}) - n\alpha_m(\theta) - P' = \alpha_{pv}(N, t_T) + n_f(\underline{W}, N, t_T)\alpha_W(\theta)$$

(2.5)

The left-hand side of this equation is primarily determined during the new product quoting process and often based on previous cost data as well as competitive pressures. Therefore, we assume, to first order that the left-hand terms of the equation (2.5) cannot be significantly affected by product validation efforts. Thus the right-hand side of the equation (2.5) would be used to optimize the life cycle cost if only the variable cost of validation can be controlled.

$$Dependability\ Cost = \alpha_{pv}(N, t_T) + n_f(\underline{W}, N, t_T)\alpha_W(\theta)$$

(2.6)

In automotive electronics applications the biggest share of product validation expense generally comes from various environmental testing and durability-temperature related testing in particular. Environmental type testing will remain largely (but not exclusively) in the focus of this analysis.

The portion $\alpha_{pv}(N, t_T) + n_f(\underline{W}, N, t_T)\alpha_W(\theta)$ of equation (2.6) would be consistent with the classical Reliability-Cost model (Figure 2.1) where it can be optimized based on

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the inverse relationship between target reliability and expected warranty cost. As mentioned before, most of the models presented in the literature (e.g., [Blischke and Murthy (1996); Rush and Roy (2000)]), typically lack specifics due to unavailability of the real cost data (which can be quite extensive). In this dissertation the general relationship represented by equation (2.6) will remain in the focus of the probabilistic cost model.

2.6. Model Development and Solutions

The LCC modeling will focus on the mathematical equations described in Section 2.5. Though seemingly simple, equation (2.6) comprises all the probabilistic and deterministic inputs described in Section 2.4.

2.6.1. Model Description

The model includes the calculation of the total dependability-related LCC, which includes the inputs from both descending and ascending parts of the cost curve Figure 2.1. Each variable in equation (2.6) is considered in detail and included into the model described in detail in Chapter 3 and Chapter 5.

Cost of product validation α_{pv} will be comprised of the inputs described in the Sections 2.4.1 through 2.4.3 and warranty cost α_W will come from the Section's 2.4.4 inputs. The information needed to populate this model will be obtained from a combination of

warranty databases and the COO for the test laboratory at Delphi Electronics & Safety. The final cost model will be implemented using Monte Carlo simulation in order to account for all the probabilistic and deterministic input variables. As with any probabilistic model, the uncertainty will be a factor in the calculations, thus confidence intervals will accompany any optimization solution.

2.6.2.   Monte Carlo and Other Stochastic Simulation Techniques

There are various ways to generate and analyze a probabilistic model, mostly with some form of stochastic simulation. It is important to mention here again that it is outside the scope of this work to determine the best and most mathematically sound stochastic simulation approach. In this dissertation the Monte Carlo technique will be utilized as a tool to process random data as an input to the probabilistic LCC model.

As mentioned in Section 2.4 the model has inputs of both deterministic and probabilistic nature. The random inputs will be modeled using Monte Carlo techniques. The data sources for the necessary statistical distributions, such as daily vehicle mileage, repair cost, failure rates, and many others will be obtained from the analysis of the existing field data, most of which comes from the automotive dealerships. The stochastic simulation will be carried out using @Risk® 4.5 along with certain programming features of Microsoft Excel. Mathcad® will be utilized for numerical and analytical integration and other types of mathematical calculations.

2.6.3. Model Outputs

The following outputs are expected as a result of the model simulation. The outputs can be subdivided into two groups: interim and final:

Interim Outputs:

- Warranty forecasting model and the failure function $F(t)$ for the mission life of the product

- Expected number of warranty claims for the warranty period.

- Statistical distribution of daily/yearly mileage for the products under consideration.

- Expected cost of warranty for any product under consideration based on 2-D warranty model.

- Per unit cost of product validation.

- Equipment cost and statistical distribution of its maintenance schedule and cost.

Final Outputs:

- Optimal sample size and duration of the test requiring to achieve lowest possible LCC value for the product.

- Target reliability to be pursued to obtain the lowest possible share of LCC associated with failure related activities of product development.

Chapters 3, 4, and 5 will detail the process of obtaining these outputs.

2.7.   Application of the Model to Optimization

The dependability-related portion of the LCC will play the role of the objective function

in the optimization procedure performed using the models developed in this dissertation.

A direct search method will be used to find the minimum of this function. However, as

mentioned before, it is outside the scope of this dissertation to determine the best

numerical optimization approach to solving the optimization problem. Rather in this

dissertation we are interested in demonstrating that an optimum point exists and can

actually be achieved. The important aspect of this approach is that the output of the

model will be minimized utilizing the decision variables available to a validation

engineer. The mathematical aspects of numerical optimization will not be the focus of

this study. Instead, the emphasis here will be made on formulating the methodology and

compiling the model with comprehensible inputs and outputs suitable for optimization by

most of the available engineering and mathematical methods. More emphasis will be

made on the existence of the solution and its finding, rather than on determining the most

efficient mathematical aspects of this process. Thus the overall goal is to find the input

parameters of the model delivering the lowest output of the model, i.e., the total LCC of

the product.

The proposed methodology is intended to be applied mostly by reliability engineers and

project managers involved in the new business quoting process and following product

development. Also this model can be used by reliability engineers negotiating validation

programs with the OEM customer and trying to find the optimal solution. It can be

effectively applied during the conceptual stage of the product as well as at the later development stages. However the benefits of this model will be considerably higher at the product planning, development, and validation stages, since these are the stages, where the program outcome and the future LCC can be influenced the most. An additional advantage of this methodology is that it can be split into independent segments, thus enabling parts of this model be applied independently of other model segments or on the other hand be eliminated from the model altogether depending on the field of application and the user's choice.

2.8.   Summary and the Remaining Chapters

This chapter presented a high level overview of the proposed cost model, its critical inputs and expected outcomes. Each block of the cost model in Figure 2.4 will be covered in detail in Chapters 3, 5, and 6. Chapter 3 will give a detailed analysis of the development and product validation cost represented by ascending curve of the diagram Figure 2.1. Chapter 5 will focus on warranty cost and other aspects of the field life of the product (descending curve in Figure 2.1). Chapter 6 will cover details of the modeling process, model integration, and Monte Carlo simulation with uncertainty analysis. It will also present a case study of the existing product and a step-by-step procedure of practical application of this model. Chapter 4 will discuss the ways to reduce the cost of validation using Bayesian techniques. Chapter 7 will summarize the work, outline the contribution of this research, and discuss future directions of this research.

3. Product Validation Cost

The important part of Life Cycle Cost (LCC) analysis is forecasting the values of business variables, such as future sales, expected failure-related expenses, future service costs, etc. This chapter addresses the issue of estimating the cost of product validation with an emphasis on environmental tests. Even though the business forecasting has a certain degree of art [Verzuh (1999)] and may have various degrees of uncertainty, the goal is to increase the accuracy of every aspect of forecasting, since most projects are viewed as investments [Verzuh (1999)]. Test and validation of the product is an integral part of the product development cost. In the automotive industry the cost of product validation can easily reach several million dollars depending on the type of the product, its geometry, technology, functional requirements, reliability specifications, and many other parameters.

3.1. Validation Cost

Considering that a product is normally designed to survive a predetermined service life (e.g., 10 years and/or 100,000 miles for automotive products) it is not always possible to predict accurately its expected failures, which are often a result of variations in the design characteristics of the product. Thus testing to demonstrate the particular reliability would reveal the adequacy of the design as well the consistency of the product parameters across the production lot. Thus, even a properly designed product may or may not demonstrate the required reliability depending on the amount of variation from product-to-product. Based on this consideration, it is very difficult to predict in advance the

additional cost required to make improvements in a non-conforming product so that it will adhere to reliability requirements. Variability of product parameters usually belongs to the realm of manufacturing engineering and is only partially addressed by reliability engineers via the size of an environmental test sample. The typical focus of a reliability organization remains on a test plan and its execution, which would include the set of environmental tests, appropriate sample size, and test duration. These are parameters that can be controlled by the reliability organization and they will be the main focus of this study.

3.1.1. Quoting Activities – The Role of Reliability Organization

Knowledge about the cost of the validation program, which is usually application specific, can be a very important piece of information during a quoting process, where a validation engineer is expected to estimate the validation cost based on the reliability requirements presented by the OEM customer. If the cost of product validation is estimated incorrectly it may render the project unprofitable (the case of a low estimate) or generate high bidding quotes resulting in a loss of business (the case of a high estimate). Thus, accurate modeling of validation cost (as well as the total LCC) would allow the company to increase the accuracy of the bidding process and, among other benefits, to increase the company's chances of obtaining profitable business contracts (see [Barringer and Weber (1996); Verzuh (1999)] for further details).

During the quoting process the marketing organization often assigns the lowest possible value on the development cost following the competitive marketing strategy described in

[O'Shaughnessy (1988)], thus making the future profit margin vulnerable to variations in actual product development costs. Therefore the importance of achieving accuracy in estimating the cost of product validation during the product quoting stage increases as the pricing strategy becomes more aggressive.

3.1.2. Product Validation Cost Estimate Diagram

The diagram in Figure 3.1 presents the steps required to estimate the cost of validation. This diagram is a detailed version of the upper branch of the "LCC methodology flow" previously shown in Figure 2.4.

A typical validation cost model would include the steps and transitions presented in the Figure 3.1. The steps would begin with product definition followed by the analysis of reliability requirements and other relevant product specifications. The next step would be a selection of the types of environmental tests and their durations, the equipment required to conduct them, and the required test sample sizes along with environmental test durations.
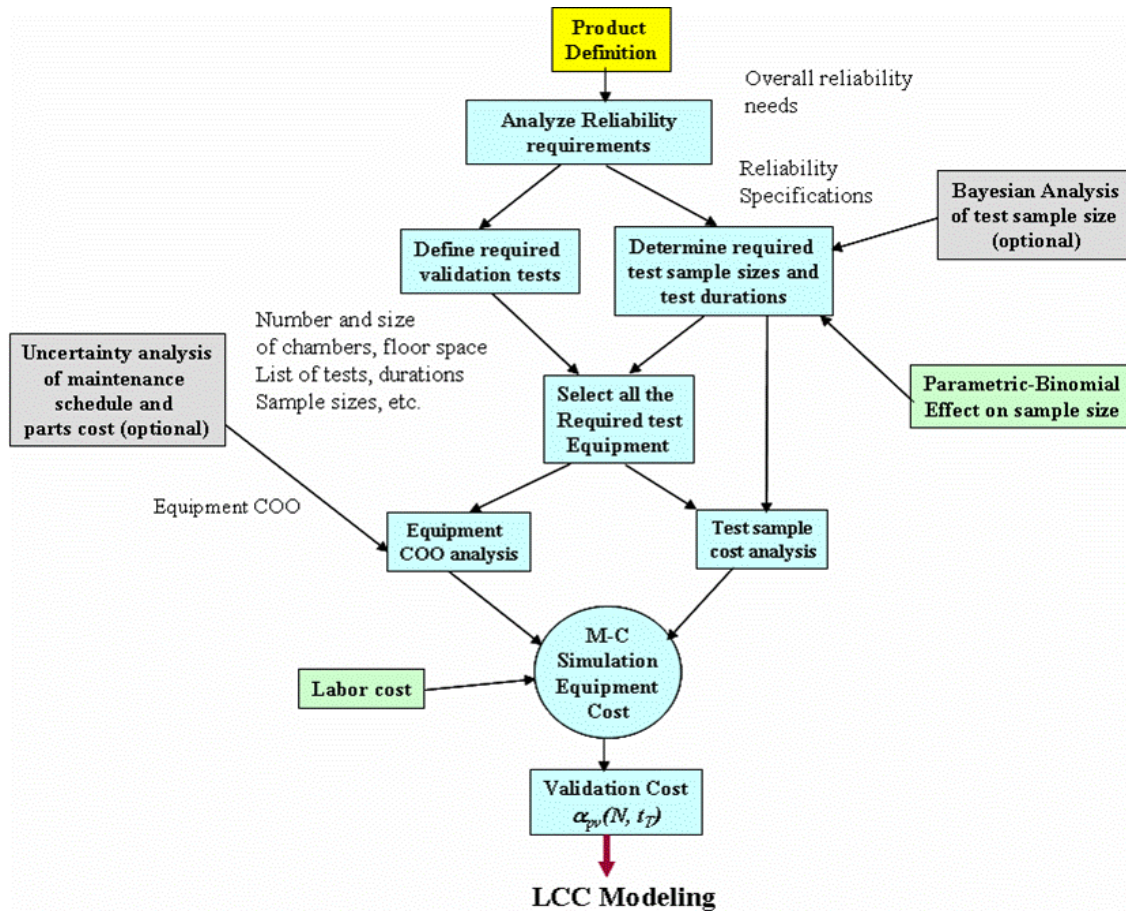
Figure 3.1. Validation cost calculation diagram

The following parallel steps of analysis of equipment Cost of Ownership (COO) and the
test sample cost analysis are the primary inputs to the total validation cost simulation,
which is performed using Monte Carlo or other stochastic simulation techniques. The
noise parameters, such as incompleteness of input data will be the factors affecting the
uncertainty of this cost analysis model. Most of the modeling blocks presented in Figure
3.1 will be discussed in detail in the following sections of this chapter.

3.2.   Product Specifications and Requirements

Analysis of product specifications and the resulting development of the test plan is a critical stage of product validation, since it is where most of its engineering and business decisions are made. These decisions will affect the overall product LCC and especially its ascending branch - cost of product validation, Figure 2.1. The majority of the products designed to be used by consumers in the real world are validated using a series of environmental tests. A classic example of environmental test specifications is the General Motors standard for validation of electrical and electronic products [GMW 3172 (2004)]. This standard covers a wide variety of environmental tests including temperature, humidity, vibration, mechanical shock, dust, electrical overloads, and many others. An example test flow based on GMW3172 is presented in Figure 3.2 showing the wide variety of tests required for automotive electronics products organized in various groups and sequences. Due to the large variety of required test procedures it would take a long time to do all the required tests sequentially on the same set of units. Therefore in the majority of the cases the tests are done in parallel as presented in the example Figure 3.2. The test flow has four major parallel test legs, which helps to reduce the total test time, but increases the size of the sample population, since each leg would require its own set of test units.

As mentioned before, most of the environmental tests for automotive electronics can be divided into two categories: Durability tests and Capability tests, [Lewis (2000)]. The durability tests are intended to simulate a full mission life and may trigger some fatigue failure mechanisms. The most common automotive durability tests are vibration, high

temperature endurance, low temperature endurance, and power temperature cycling (PTC). These types of tests require costly test equipment and are often lengthy and expensive to perform. For example the PTC test in Figure 3.2 takes 17 days and is also sequenced with other environmental tests. The capability tests do not simulate the mission life, but instead are used to verify that the product is capable of functioning under certain environmental conditions. Failures in capability tests can be a permanent damage or a temporary loss of function that can be 'reset' after the environmental stressing condition is withdrawn. The examples of these tests can be found in legs 4, 5, and 6 of Figure 3.2 and include dust tests, over-voltage, certain types of a humidity test, and several others.

The durability testing is where the potential cost savings can be substantial due to the longer tests intended to represent the total mission life as opposed to capability tests that are targeted at discovering more easily detectable design flaws. Since most of the validation cost in the automotive industry is driven by the durability tests, they will remain in the focus of this study; therefore the main effort of minimizing LCC will be directed at the two most expensive types of tests, i.e., PTC and vibration shown in legs 1, 2, and 3 of Figure 3.2.
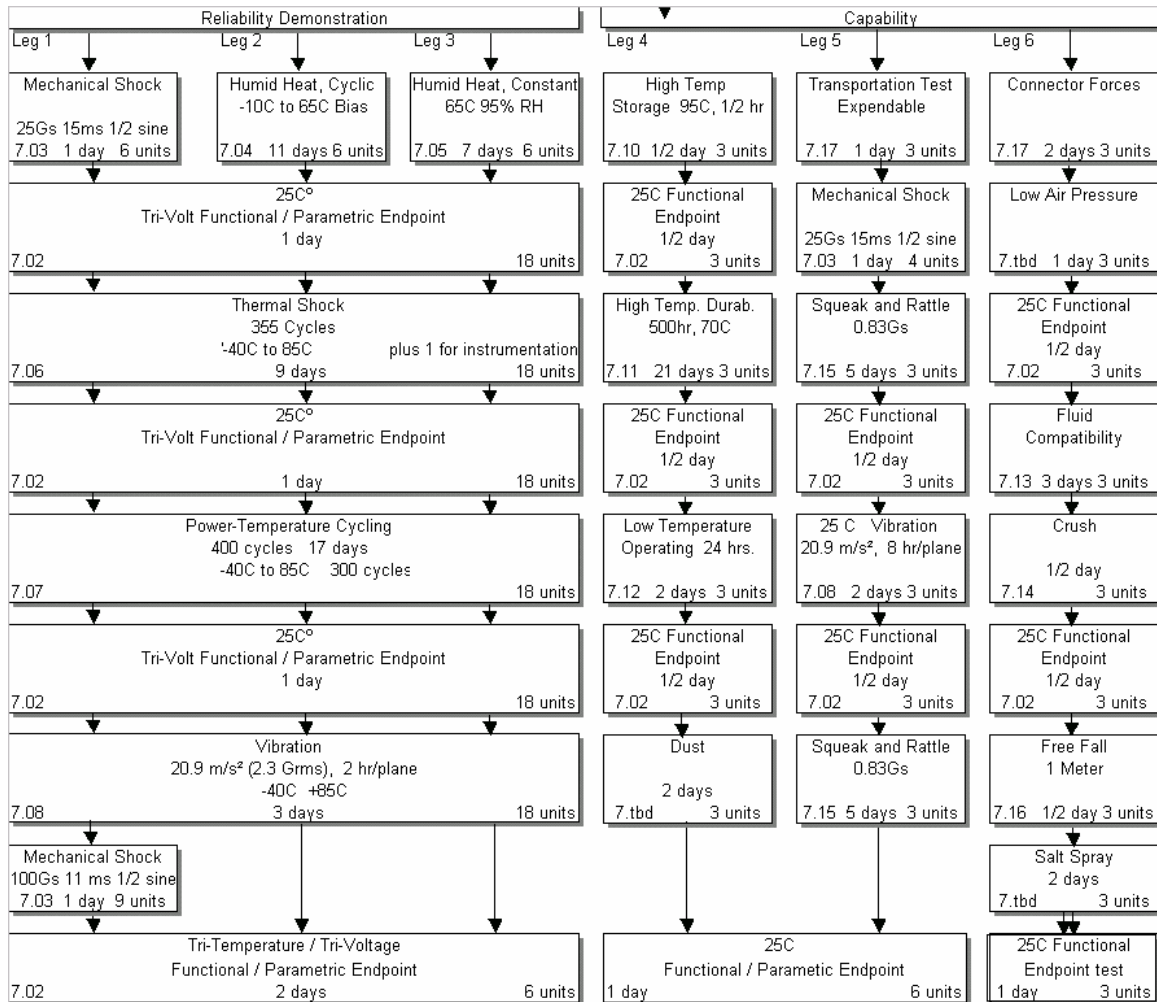
Figure 3.2. Example of an automotive validation test flow per [GMW 3172 (2004)]

3.3. Approaches to Validation Cost Estimate

Most of the product life cycle accounting models presented in the literature (e.g., [Blischke and Murthy (1996); Fabrycky and Blanchard (1991)]) consider the overall cost of the design cycle, but often ignore its specific components such as product validation cost. In the automotive industry however this cost can be quite substantial and should be addressed in all stages of LCC analysis.

Most of the approaches considered in the literature that account for the cost of reliability and its variations include the cost of preventive maintenance [Kececioglu (1991)] and the overall cost of the design cycle [Blischke and Murthy (1994)], but do not sufficiently address the input of validation activities as well as the cost of ownership of test and validation equipment.
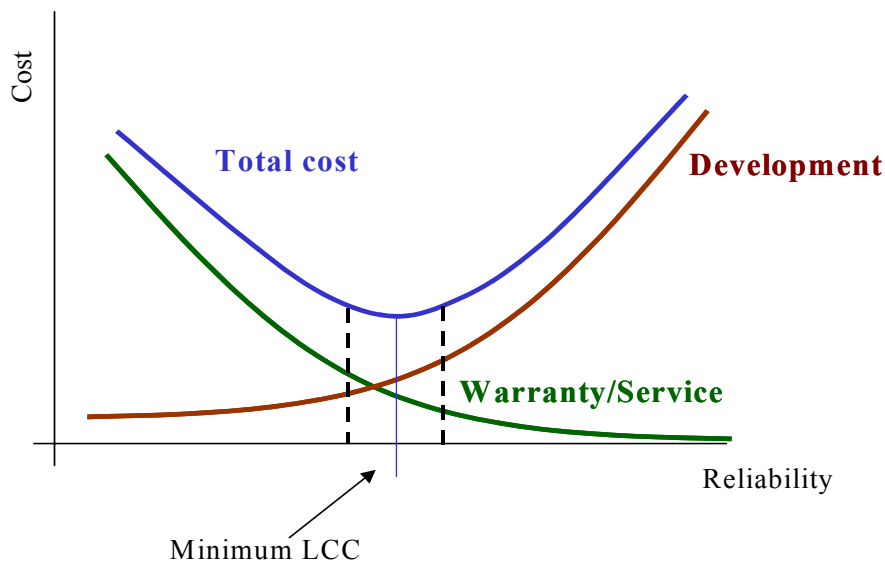
Figure 3.3. Life cycle cost versus reliability (with solution confidence bounds)

The chart presented in Figure 3.3, which already appeared in Chapter 2 illustrates the theoretical approach to minimization of the product LCC. This chart appears again here to emphasize the contribution of product validation into the whole LCC model. It shows the growth in product development cost with increasing reliability and decrease in warranty/service cost. It also shows that the minimum LCC can be presented in form of an interval due to uncertainties in the model.

The cost of validation is an integral part of the ascending curve in Figure 3.3 and it can be significant even when compared to the rest of the design and development cost. However most of the literature sources mention it as a built-in part of the overall development cost and overlook the significant input of test laboratory cost and the ways they can affect the location of the optimal reliability area on the X-axis in Figure 3.3.

3.4.   Proposed Approach

One of the main objectives of this work is to provide a realistic methodology to specify the relationships presented in Figure 3.3 and use this model to minimize the life cycle cost of a product using the validation cost input variables. As mentioned before, this work presents an analysis of life cycle cost from the viewpoint of a reliability organization and suggests ways to optimize the validation procedures with the controls available to a reliability engineer as oppose to a product design or any other engineering or business competency. The methodology proposed in this section concentrates on estimating the cost required to validate the product according to environmental and mission life specifications including meeting the required target reliability.

3.4.1.   Main Contributors to the Cost of Product Validation

There are a variety of cost contributors to the test and validation process, some of them also depend on the product applications. The main cost contributors to the typical automotive validation program are:

- Test equipment cost of ownership (COO)

- Labor cost

- Test sample population related costs

Secondary cost contributors:

- Floor space

- Laboratory overheads

- Other miscellaneous costs

Each contributor will be analyzed in the sections below and compiled within the total validation cost model in Section 3.4.5.

3.4.2. Effect of a Test Sample Size and Test Duration

Some of the cost categories listed in the Section 3.4.1 could be considered as a fixed cost of reliability demonstration and some can be categorized as variable cost. The expenses linked to the test sample size could be qualified as variable costs. Their effect has been consistently overlooked in everyday product validation practice. Needless to say, the larger the test sample size the greater the cost of validation. Despite that, the cost effect of the number of samples required to be tested is usually not given enough attention. Meanwhile, each test sample carries the following costs associated with the sample population:

- Cost of producing a test sample.

- Cost of equipping each test sample. In the electronic industry it would include harnesses, cables, test fixtures, connectors, etc.

- Cost of monitoring each sample during the test. In the electronics industry this would include the labor cost of:

  - designing and building the load boards simulating the inputs to the electronic units
  - connecting and running the load boards
  - recording the data
  - visual and other types of inspection

Considering that some tests may run for weeks or even months, these expenses can be significant

Most of the time the number of the required test samples is determined from the reliability and the confidence level defined by the customer specifications mentioned in the Section 3.2. This process is called *Reliability Demonstration*, which is most often based on the binomial distribution, requiring a particular test sample size in order to demonstrate the desired reliability number with required confidence level [Meeker *et al.* (2004)], e.g., 97% reliability with 80% confidence. The basic relationship between reliability and confidence level is provided by equation (3.1), derivation of which is presented in detail in Appendix A.

$$C = 1 - R^N$$

(3.1)

Equation (3.1) can be solved for the test sample size *N* as:

$$N = \frac{\ln(1-C)}{\ln R}$$

Based on equation (3.2) the demonstration of reliability R approaching 1.0 requires the sample size N to approach infinity. Table 3.2 shows an example of reliability sample sizes based on equation (3.2) calculations.

Table 3.1. Examples of reliability sample sizes

Reliability, R	Confidence Level, C	Sample Size, N
90%	90%	22
95%	90%	45
99%	90%	229
99.9%	90%	2,301

Considering geometric size and complexity of automotive electronic units, and what is involved in testing and validating them, test sample sizes above certain level become impractical due to the rapidly growing 'variable' cost of validation. With ever-increasing reliability requirements, the sample population to be tested would require more and more of human resources and capital equipment. Since reliability demonstration is one of the metrics controlled by a reliability engineer, it is only natural to use it as one of the metrics in quantifying the future reliability of the product. Furthermore, it would be advantageous to find the optimal target reliability delivering the lowest possible product LCC, which is one of the objectives of this dissertation.

It is also important to note that the increase in sample size may actually cause the growth of the equipment related costs as a step-function due to the discrete nature of the

equipment capacity. For example, if the capacity of a chamber is 25 units of a particular geometric size, then a test sample of 26 units would require two chambers instead of one needed for 25 samples. This trend will be reflected in the equation for the overall validation cost in Section 3.4.5.

As mentioned before, the test sample size grows exponentially according to equation (3.2) with the increasing target reliability. Table 3.1 shows that the demonstration of 99.9% reliability with 90% confidence would require the impractical 2,301 samples. Based on the fact that sometimes customer requirements do contain this kind of reliability target, other mathematical methods would be required to achieve those high numbers. For example, in the cases where prior knowledge about the product's dependability is available, certain methods of sample size reduction based on Bayesian approach can be utilized. These approaches can help to bring the number of test samples within practical limits and will be discussed in detail in Chapter 4.

Another factor, which can significantly affect the test sample size is test duration. There is a relationship between the test sample size and test duration referred as *Parametric Binomial*, which allows the substitution of test samples for an extended test time and visa versa. This relationship is sometimes called Lipson equality [Lipson and Sheth (1973)] and presented here in the equation (3.3).

$$C = 1 - R^{NL^{\beta}}$$

(3.3)

Where:

L = number of service lives the product intended to be tested for

β = Weibull slope for primary failure mode

It is important to note here that equation (3.3) is derived under assumption of *Success Run* testing (see Appendix A), i.e., no failures are experienced during the test. However as L increases (increased test duration) the probability of the failure occurrence is increasing. Therefore the value of L should be limited to provide a reasonable duration within the framework of *Success Run* testing. Also the Weibull slope in equation (3.3) should not be confused with the β-values used for warranty prediction in Chapters 5 and 6. The β-values in equation (3.3) are corresponding to the end-of life conditions and therefore correspond to wear-out mode with $\beta > 1$. Therefore the higher the β the sooner the product will fail (smaller L) and the higher the probability that the *zero-failure* assumption will be violated.

Based on equation (3.3) the required number of test samples can be reduced L^{β} times in the cases where tests duration is longer than the equivalent of one service life ($L > 1$). Therefore this approach allows an additional flexibility in minimizing the cost of testing by adjusting the test sample size up or down according to this relationship. The detailed derivation of equation (3.3) and its applications are presented in [Kleyner and Boyle (2005)] and also reproduced here in this dissertation Appendix B.

3.4.3.   Cost of Ownership Model for Product Test and Validation

Despite the fact that fixed costs of equipment are covered extensively in the business and accounting literature, the cost of ownership has not received the attention it deserves. The cost of environmental laboratory equipment is still often calculated based on acquisition costs rather than cost of ownership [Avamar Technologies (2004)].

The concept of cost of ownership (COO) is more complex than just depreciation of the equipment and maintenance cost and relates to the total cost of acquiring, installing, using, maintaining, changing, upgrading, and disposing of a piece of equipment over its predicted useful lifespan. The concept of COO applied to the semiconductor industry are discussed in [LaFrance and Westrate (1993); Dance (1996)] and were later summarized and further developed in [Sandborn (2005)]. Most of the manufacturing COO concepts are listed in the Table 3.2 and include the major cost categories such as Capital, Sustainment, and Performance costs. The details of these costs can be obtained from the listed references, and will not be discussed here. Even though most of these concepts were developed for the wafer fabrication industry, most of them are transferable to COO of a validation test laboratory (Table 3.2, column 1), when others are less suitable for those purposes (Table 3.2, column 2).

Table 3.2. Cost of ownership concepts application to test laboratory equipment

Transferable concepts	Less applicable concepts
Capital cost ▪ Acquisition ▪ Installation ▪ Depreciation ▪ Floor space	Sustainment Cost ▪ Lost production (non-service organization)
Sustainment cost: ▪ Personnel training ▪ Scheduled maintenance ▪ Unscheduled maintenance ▪ Indirect maintenance cost ▪ Utilities (energy, water, CO_2, etc.) ▪ Insurance	Performance cost: ▪ Change-over cost ▪ Repairable defect cost ▪ Scrap cost ▪ Lost production due to scrap ▪ Cycle time penalty

Due to the objective of this dissertation to concentrate on the engineering and statistical aspects of the model without undue complication of the methodology, the equipment COO will be combined into three major groups:

1. Capital and depreciation cost (D), which would include acquisition, installation, and cost of scraping, all spread over the useful life of the equipment.

2. Maintenance cost (M) including both scheduled and unscheduled maintenance, plus indirect maintenance cost. Indirect maintenance may include technician training, lost revenue due to the equipment idle time, etc.

3. Miscellaneous costs (*Y*). Include energy cost, floor space, upgrades, insurance, etc.

Based on this grouping, the hourly cost of operation can be calculated as:

$$Hourly\ Cost = \frac{D + M + Y}{365 days \times 24 hours}$$

(3.4)

Where *D*, *M*, and *Y* are yearly costs.

3.4.4.   Maintenance Cost

Maintenance cost per year (including both corrective and preventive maintenance) can be calculated as the total cost of parts and labor multiplied by the number of maintenance actions per year [Wortman and Dovich (2002)] or in simplified form:

M = Number of maintenance actions × [repair duration × labor rate + parts cost per repair]

(3.5)

However, the only deterministic variable in equation (3.5) is the labor rate, while the remaining variables can be defined as random variables and presented in mathematical form:

$$M(yearly) = \frac{365\ days}{MTBF_{EQ}} \left(t_{repair} \varphi_{repair} + \alpha_{Parts} \right)$$

(3.6)

Where:

$MTBF_{EQ}$ = Mean Time Between Failures of the test equipment (repairable system) – random function $f_1(t;\ \xi_1)$

φ_{repair} = repair labor rate

α_{parts} = cost of the spear parts per repair – random function $f_2(x;\ \xi_2)$

t_{repair} = duration of each repair – random function $f_3(t;\ \xi_3)$

ξ_i = vector of statistical parameters. These parameters can be obtained from statistical analysis of the repair and failure data of a particular test facility

Note: in this dissertation most of the cost variables will be expressed by two Greek characters α and φ. Where, with various subscript characters φ will denote an hourly rate and α will denote the cost per item.

Equation (3.6) groups together both preventive and corrective maintenance. However when the costs of CM and PM differ significantly or in the cases, where it is warranted for other reasons [Thevik (2000)], equation (3.6) should separate CM and PM as:

$$M(yearly) = \frac{365\ days}{MTBF_{EQ}} \left(t_{repair} \varphi_{repair} + \alpha_{Parts} \right) + N_{PM} \alpha_{PM}$$

(3.7)

Where:

N_{PM} = number of preventive maintenances per year

α_{PM} = cost of each preventive maintenance

Due to the random nature of the variables in equations (3.6) and (3.7) it is practical to involve statistical analysis methods such as Monte Carlo or some other form of stochastic simulation.

3.4.5.   Total Validation Cost

A simplified version of the product validation cost model can be found in [Kleyner *et al.* (2004)]. Below presented is the more detailed version of it, consistent with the above COO model. The total cost of product validation per test is given by,

$$\alpha_{pv} = t_{test}\left(\varphi_T + \frac{(M+D+Y)}{365 \times 24}\right)\left[\frac{N}{K}\right] + N(\alpha_p + \alpha_e + \alpha_m)$$

(3.8)

Where:

α_{pv} = total cost of product validation per test

D = equipment depreciation cost per year

M = maintenance cost per year – random variable

Y = additional equipment expenses per year

φ_T = hourly labor rate of performing the test

α_p = cost of producing one test sample

α_e = cost of equipping one test sample

α_m = cost of monitoring one test sample

t_{test} = test duration

K = equipment capacity

$\lceil \; \rceil$ = ceiling function, indicating rounding up to the next highest integer

The information needed to populate this model will be obtained from an automotive electronics environmental test laboratory case study utilizing the sanitized data from Delphi Electronics & Safety (see the case study in Section 3.5 and Chapter 6)

3.5.   Effect of Incomplete Test Equipment Data on the Cost of Ownership

The maintenance cost is an inextricable part of a test laboratory cost of ownership. Though not a major COO expense, the maintenance cost for a large environmental test laboratory can approach the order of magnitude of the depreciation cost. However, one of the common problems with maintenance accounting in industry is incomplete or missing maintenance records. This is especially true when the company has a blanket maintenance contract with an outside vendor or has its own maintenance staff paid independently from the actual time spent performing the maintenance. The maintenance staff in these cases have little incentive to maintain good records of repair dates and times. In addition, the cost of spare parts is not always accurately recorded and often the purchase orders are not explicitly linked to particular repairs. Parts data is often stored

together with non-maintenance related purchases, which makes it difficult to calculate an exact cost of any particular repair. Keeping track of these expenses is important, even in the cases of maintenance contracts or salaried laboratory personnel. It is still important to estimate these costs in order to allocate the correct dollar amount associated with a particular product or program. In this section we will present the approaches to evaluate parameter M (the maintenance cost per year) in equation (3.8) in the cases where the maintenance records are missing, incomplete, or accounted for in the wrong databases.

3.5.1.   Accounting for Missing and Incomplete Data

Missing data can often include repair dates, repair durations, the cost of spare parts, and their association with particular repairs. This dissertation will present a case study of a real validation test laboratory and will show the methods of calculating the maintenance costs based on incomplete records. This case study will analyze a large test laboratory with the equipment ranging in ages from 1 to 22 years, but with only four years of existing maintenance records (2000-2004).

There is a variety of methods to process and analyze missing and incomplete data, most of which are covered in [Little and Rubin (2002)]. In addition, the problem of incomplete/missing data for parameter estimation has been widely discussed in the literature (see for example [Baxter and Tortorella (1994); Oh and Bai (2001), Nelson (2003), Zhao *et al.* (2000), Parthasarathy and Aggarwal (2003); Rai and Singh (2003)]), and also specifically in application to maintenance records [Celeux *et al.* (2002))].

Among those sources [Rai and Singh (2003)] present an especially good review of the methodologies of dealing with highly truncated data, both left and right censored.

Since all the missing records are related to the past, our data set can be classified as a left-censored (see Figure 3.4) with univariate[6] missing data. In this case study, we are dealing with a large amount of missing data, which may cause a high degree of uncertainty. This can be partially compensated for by the general knowledge of the nature of the data and the expectation of the failure trends for this type of maintenance equipment.
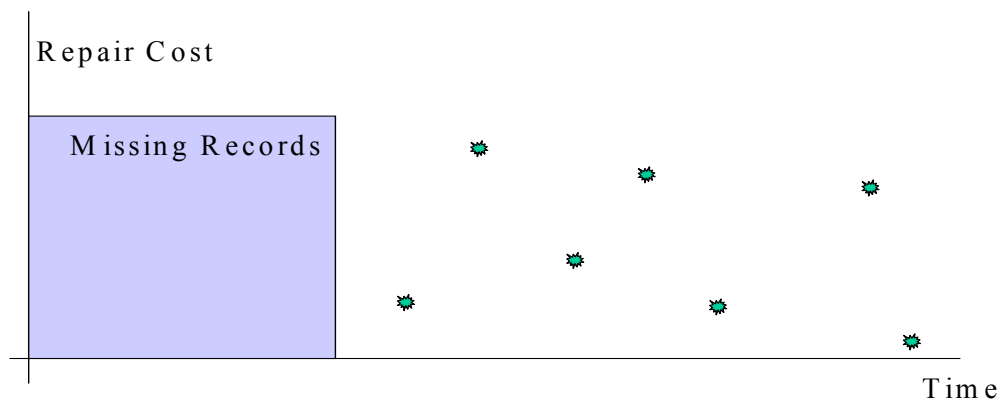
Figure 3.4. Left censored repair records

The common sense approaches to this kind of data restoration are considered in [Parthasarathy and Aggarwal (2003)], which is based on the natural conceptual structure of the data. Since the methodology for this kind of analysis could be a separate research topic, the amount of time spent here will be just enough to explain the author's engineering approach and its incorporation into the overall LCC model.

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[6] In univariate cases the missing data is confined to a single variable. In our case it is the exact time of the equipment failure.

3.5.2.   Maintenance Cost Case Study

The case study discussed in this section is based on data from an automotive validation test laboratory and resembles the operation of Delphi Corporation's environmental test facility. The test laboratory has 25,000 ft^2 of floor space and contains 42 temperature chambers and twelve pieces of other test equipment including vibration shakers, dust chambers, thrusters, turntables, and others. As mentioned before the maintenance data was available only for the last four years. Table 3.3 presents a summary of the known and unknown parameters for this analysis.

Table 3.3. Known and unknown parameters for each piece of equipment:

Known	**Unknown**
• Acquisition cost including transportation, delivery, and installation • Date of the purchase/installation. • Date of each repair made in the past four years. • Cost of the parts purchased in the past 4 years • Duration of each repair in 0.5-hour increments. • Repair dates and durations for each chamber for the past 4 years.	• Number of repairs and their dates going back more than four years. • There is no clear indication in the record which repair was attributed to preventive and which to corrective maintenance • In many cases it was impossible to determine which part was purchased for which piece of equipment. Therefore, not all the spare parts purchases can be correctly allocated to the appropriate repairs.

This type of left-censored univariate data should first be analyzed using a common sense reliability engineering approach. Speaking in terms of MTTF and consequently the failure rate on a bathtub curve, there can potentially be three major data trends: increasing failure rate (IFR), decreasing failure rate (DFR), and constant failure rate (CFR).

Therefore, based on the characteristics of this repair data and the approaches presented in [Parthasarathy and Aggarwal (2003); Rai and Singh (2003); Celeux *et al.* (2002)] the following analysis steps can be suggested:

Table 3.4. Uncertainty analysis of test equipment

1. Determine the types and attributes of the data to be analyzed and set the expectation as if the data set was complete, e.g., when the equipment is old enough the expected failure rates would be CFR or IFR.
2. Analyze the available data and determine their MTTF and failure rates. Determine which category they belong to (CFR, DFR, or IFR)
3. Determine the possible types of statistical distributions appropriate for this kind of data
4. Make assumptions about the missing data based on the existing data trends
5. Parametric statistical analysis of the existing data
6. Filling the gaps by 'reconstructing' the original data using the methods discussed in the literature above.
7. Conclusions and applications

Following this procedure, the first step in our case study was to analyze the available equipment data sheets in order to evaluate the nature and the character of the data. The ages of each piece of equipment in months were obtained from the original equipment list. The purchase order list provided the dates and costs of parts purchases and a separate spreadsheet was used for the labor records (dates and durations). Due to the non-homogeneous nature of the equipment ages, the procedure determining correlation between the equipment service times and the number of repairs made between 2000 and 2004 was conducted as a second step. A highly positive correlation would have indicated

the increasing failure rate, i.e., the wear-out state. A highly negative correlation would have indicated the decreasing failure rate, typical for the infant mortality stage. And low correlation failures would have indicated that the random nature of the failures, typical for the useful life stage would follow the Poisson process. The obtained correlation coefficient in this case study was $r = 0.19$, which implied little correlation between the age and number of repairs. That suggested two possibilities: (A) The majority of the chambers are still in their useful life period and have not yet entered the wear-out stage or (B) the repair concept called 'as good as new' is applicable for this type of repair. Though it would not make any difference from statistical standpoint, the hypothesis (A) would imply that this stage is only temporary and the failure rate trend may change at any time in the future, therefore restricting this hypothesis to a particular time limit. Whereas the hypothesis (B) would suggest a more stationary statistical process thereby simplifying the future analysis. For simplicity purpose we will assume the hypothesis (B) and the continuous use of the Poisson process and exponential distribution for the steps 3 and 4 in Table 3.4. Step 5 (parametric statistical analysis) combines all the time and cost data available for spare parts and repairs and finds a best distribution for those. The time interval (2000-2004) produced the following distribution of spare parts cost: lognormal with parameters $\mu = 5.845$ and $\sigma = 1.371$, which translates to the mean of approximately $800 with standard deviation of approximately $1400.

The list of repair dates, times, and their durations was based on manual entries to the maintenance journal. The distribution for repair durations came out also as lognormal, which was consistent with the conventional notion that most of the downtimes associated

with repairs are distributed in a lognormal fashion [Ebeling (1997)]. The parameters of this distribution were $\mu = 0.5037$ and $\sigma = 0.6159$, which translated to the mean of approximately 2 hours and the standard deviation of 1.36 hours. The rest of the data was treated as a homogeneous pool of failures distributed over four years and 42 pieces of similar temperature chambers. Total time from January 2000 until September 2004 covered 56 months. The 225 repairs recorded during those 1680 days implies MTTF = 1680/225 = 7.47 for the equipment pool of 42 chambers, resulting in MTTF_{EQ} = 7.47 × 42 = 313 days per unit. In statistical terms it can be expressed by the equation below,

$$MTBF_{EQ} = \frac{2 \times 42 \times 1680 days}{\chi^2_{2 \times 225}}$$

(3.9)

3.5.3. Uncertainties in the Cost Model

There is an extensive amount of literature dealing with uncertainties and uncertainty propagation in economic and engineering problems. For example [Morgan and Henrion (1992); Schjaer (2002); Serrano (2001)] present high-level overviews of major techniques of how to account for uncertainty propagation in the analysis. The incompleteness of data and high degree of its censoring raises the level of data uncertainty in our case study; however the low correlation between the equipment age and failures was a positive factor reducing the uncertainty. The Poisson distribution chosen based on this correlation is a

'memory-less' process; therefore the unknown failure data will have less impact on the accuracy of the analysis than that in the case of other types of failure processes.

The major analysis-associated uncertainties in the calculation of our maintenance cost are:

- Uncertainty of calculation of MTTF according to equation (3.9) based on the repair history of the equipment

- The uncertainty of MTTR also obtained from the repair history and presented here in a parametric form of a distribution in a case study Section 3.5.2

Besides the uncertainty associated with the data itself, there is an issue of model uncertainty. Even though the correlation between the equipment age and the number of repairs was low, the probability still exists that some of the equipment already entered the wear-out stage of their service life therefore questioning the accuracy of the chosen process. The model uncertainty will not be estimated here due to the high complexity of the subject and relatively low contribution of maintenance to the overall LCC value (less than 5% in our case). Detailed information on model uncertainty and its estimation can be found in [Mosleh (1985); Droguett and Mosleh (2002)] and other relevant sources. The total uncertainty will be estimated for the stochastic simulation of overall LCC in Chapter 6.

3.6. Summary

This chapter presented the analysis of cost of the product validation involving an automotive environmental test lab. It derived the equations for product validation costs and defined the major inputs required to populate this cost model. It also discussed the ways to deal with incomplete and missing maintenance records, which is common occurrence in an industrial environment. In particular, Section 3.5 shows the techniques to estimate the cost parameter M, in equation (3.8) with a limited amount of accurate maintenance data and presents a real life example of this type of analysis. The model presented in this chapter will be integrated with the other inputs in the overall LCC model later in Chapter 6 for the purpose of cost optimization. The data from the case study presented in this chapter will also be used in the modeling example in Chapter 6.

Even though the main focus of this validation cost model remains on automotive electronics industry, most of the concepts presented above would be applicable to test and validation procedures for variety of products outside automotive industry. However it is important to understand certain limitations of this model. For example, the effect of test sample size (Section 3.4.2) may not apply to the products with high cost and low production volume, such as airplanes, satellites, heavy machinery, and others, due possible imbalance in the right-hand side of equation (3.8). Also this approach may not work in the cases of low cost products, where reliability is not one of the key objectives. In these cases the validation cost will be artificially low due to the cost saving efforts at the expense of product reliability. More on application boundaries for this model will be presented in Chapter 6.

4.   Bayesian Approach to Test Sample Size Reduction

This chapter discusses the Bayes theorem related approaches to calculating the test sample size needed to demonstrate required reliability and confidence level during the durability-type environmental testing. These approaches are useful in the cases where the required reliability and confidence level are too high to be practical from the cost and test facilities stand point. The use of a prior knowledge about a product can demonstrate a significant sample size reduction when used where applicable.

With increasing demands for development cost reduction, and shortening of the product development cycle time, the modern validation program should accommodate all the available knowledge about the product under development. Most of the automotive products are created through a development cycle of evolutional rather than revolutionary changes. Thus a certain amount of the existing product information can be incorporated into a validation program. One of the possible ways of incorporating this information is by utilizing the Bayesian approach of analyzing priors and obtaining posteriors.

This section will present a brief survey of the Bayesian models and their applications to test sample size reduction in product development, when prior history is available in form of laboratory and/or field testing, warranty data, or some other data formats. One of the ways to achieve a reduction in the cost of a validation program is by reducing the number of units subjected to reliability testing. The importance of cost and development time reduction cannot be overstressed in the current competitive environment of automotive parts business. This section will also discuss perspectives and challenges of practical

applications of Bayesian techniques in industry in general, and the automotive industry in particular.

4.1.   Background

Despite the fact that the concept of Bayesian inference has been known for many years, it only started attracting the attention of automotive reliability engineers in the past 15-20 years. The interest was caused by an increasing number of automotive specifications requiring higher reliability demonstration in automotive parts testing. The predominant use of the binomial distribution in determining test sample sizes caused a steady growth in the number of units required to test without failure, due to the fact that under the *Success Run* concept (Appendix A), the number of units tested successfully would be calculated from the equation (4.1), which is currently used by the majority of automotive manufacturers and their suppliers (see derivations in Appendix A).

$$C = 1 - R_0{}^N$$

(4.1)

Where R_0 = target reliability

C = required confidence level.

N = test sample size

Equation (4.1) can be solved for N as:

$$N = \frac{\ln(1 - C)}{\ln R_0}$$

(4.2)

In the automotive industry, C and R_0 are usually stipulated by the OEM customer; and the Success Run formula (4.1) is then used for the determination of the required test sample size N.

The general approach to this problem can be described as the calculation of a confidence level that the reliability of the product lies above minimum required reliability R_0 (or $R_0 \leq R \leq 1$). For the Bayesian form of the Success Run formulae please refer to Appendix B.

The Bayesian form of this derivation (after processing test data, often consisting of N test samples, of which k have failed) can be presented in general form:

$$C = P(R_0 \leq R \leq 1 \mid Data) = \frac{\int_{R_0}^{1} L(Data \mid R)\pi(R)dR}{\int_{0}^{1} L(Data \mid R)\pi(R)dR}$$

(4.3)

Where $L(Data|R)$ = likelihood of obtaining the observed test data if the reliability of
each unit is R

$\pi(R)$ = prior distribution of that reliability R

One of the accepted forms of representing reliability prior $\pi(R)$ is a Beta distribution:

$$\pi(R) = \frac{R^{A-1}(1-R)^{B-1}}{\beta(A,B)} \quad \text{if } 0 \le R \le 1$$

<div align="right">(4.4)</div>

Where $\beta(A,B) = \dfrac{\Gamma(A)\Gamma(B)}{\Gamma(A+B)}$ $\quad$ *and* $\quad \Gamma(a) = \displaystyle\int_0^\infty x^{a-1}e^{-x}dx$

The constants *A* and *B* (sometimes called hyper-parameters) have a convenient interpretation - *A* being thought of, sometimes, as the number of successes out of *A+B* trials in a similar pre-experiment, real or imaginary. More importantly, the beta prior distribution is conjugate to binomial sampling, that is, the posterior is a beta distribution as well. This allows for a continuous updating of the posterior within the same general class of distributions (for further details see the Appendix B)

One can see that in the equation (4.2) the sample size *N* grows very rapidly with *R* approaching 1.0. For example, to demonstrate 99% reliability with 90% confidence would require 229 test samples, which would be practically impossible to do, considering today's realities of automotive development programs with their high competitive cost and time-to-market pressures.

4.1.1. Single Point Estimate

In the early 1990-s [Bayer and Lauster (1990)] presented a Bayesian method, which instead of depending on a complete prior distribution, required only one value as a prior information, namely the value of R_0 at the confidence level $C = 63.2\%$. Even though it

has all the features of a single point estimate, their method was based on the earlier work [Martz and Waller (1982)] and conjugate properties of beta and binomial distributions. The main concept was based on beta prior distribution with the fixed parameter $B = 1$, where parameter A of the posterior was increased by the number of successfully tested samples m, thus becoming $A+m$. The m was indirectly obtained from the success run theorem by substituting $C = 0.632$ into equation (4.1):

$$m = \frac{\ln(1 - 0.632)}{\ln R_0} = -\frac{1}{\ln R_0}$$

(4.5)

Thus m, which was calculated according to (4.5), represented the number by which the original number of test samples N could be reduced, based on the knowledge of R_0 at the confidence level of $C = 63.2\%$. The new required number of test samples will be:

$$N_{new} = N_{original} - m = N_{original} + \frac{1}{\ln R_0 \, (at \ C = 63.2\%)}$$

(4.6)

This method presented a technique, which was practical and convenient for reliability practitioners, especially those accustomed to 'test to success' reliability approaches.

4.1.2.   Mixed Priors

One of the difficulties of applying traditional Bayesian methods to a calculation of sample sizes is caused by continuous product development and never ending design

changes introduced to a product. Often this means that the prior distribution obtained

from the warranty information or prior test result is applicable to the previous models of

the product. Naturally, it raises the question of the relevance of the existing data to the

current version of the product, which is different from the original. The major concern in

the industry was that while applying Bayesian technique a reliability engineer might miss

the problems caused by newly introduced product changes, since they were not

incorporated in the prior distribution. In order to address this problem, [Kleyner *et al.*

(1997)] suggested combining beta prior distribution as suggested by equation (4.4),

constructed from the product history with the uniform prior, which would account for the

lack of knowledge on the newly introduced product changes. This work therefore

proposed to use a two-component mixture of beta and uniform distributions, with density:

$$\pi(R) = \rho \frac{R^{A-1} \times (1-R)^{B-1}}{\beta(A,B)} + (1-\rho)$$

(4.7)

The [Kleyner *et al.* (1997)] paper is reproduced here in Appendix B. The first component

of the mixture is a beta prior with parameters A and B to be derived from failure data.

The second component of the mixture is a uniform prior (a special case of the beta)

representing uncertainty about the new product reliability. The two components are

combined according to weights ρ and $(1-\rho)$, where ρ is a *knowledge factor* representing

how similar the new product is to the old one, and $(1-\rho)$ is an *innovation factor*, reflecting

the proportion of new content in the new product. For further details of this method see

Appendix B. It should be noted that the use of a uniform prior alone would lead to the

Bayesian version of the Success Run formula; the use of mixtures represents therefore a practical compromise between Bayesian approach and binomial distribution. Table 4.1 presents an example of the data with the 'favorable prior' obtained from warranty analysis (beta distribution parameters $A = 770$ and $B = 2.5$). Table 4.1 shows the required number of test samples to satisfy $R = 99\%$ and $C = 90\%$ requirement based on the value of knowledge factor ρ.

Table 4.1. Required test sample size for $R = 99\%$ $C = 90\%$ with different values of ρ

Knowledge Factor (ρ)	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Sample Size, N	0	1	2	4	6	9	13	19	30	54	229

The idea of using mixture priors in the context of product reliability was generalized to the case of heterogeneous prior information, in particular to the case where failure data is available for different past products, some more similar than others to the new product. In this case, the analysis could be generalized to the consideration of prior densities of the form

$$\pi(R) = \sum_i \left(\rho_i \frac{R^{A_i-1} \times (1-R)^{B_i-1}}{\beta(A_i, B_i)} \right) + (1-\rho)$$

(4.8)

where $\rho = \sum_i \rho_i$

and the different knowledge coefficients ρ_t reflect different degrees of similarity between the new and the old products.

In addition to the general concept [Kleyner *et al.* (1997)] presented an improved procedure for the computation of the beta parameters *A* and *B* based on failure data obtained from automotive warranty databases based on IPTV (Incidents per Thousand Vehicle) values available for the series of 30-day intervals. The modified procedure introduced by [Martz and Waller (1976)] was utilized for determining the beta-distribution parameters *A* and *B*.

4.1.3. Effect of Lifetime Ratio and Acceleration Factors

In the past several years a number of interesting articles have been published by the group of researchers from the Institute of Machine Components, affiliated with University of Stuttgart, Germany. [Krolo *et al.* (2002a, b, c)] introduced the effect of *Lifetime Ratio* to both [Bayer and Lauster (1990)], and [Kleyner *et al.* (1997)] methods. The Lifetime Ratio *L* in this case is the ratio of the test time t_T to the test time equivalent to one life in the field, sometimes referred as *bogey*. Combined with the Weibull slope β, assumed to be known for the tested product, the factor L^β similar to that in equation (3.3) was embedded into reliability prior and final posterior computations according to equations (4.7) and (4.3) reflect the effect of L^β further affecting the solution, i.e., the sample size required to demonstrate the stipulated reliability. The lifetime ratio approach, sometimes referred as *Parametric Binomial* (see Appendix A) has been commonly used

before to account for the effect of shortened or extended lab testing on fatigue life of tested parts, however in combination with Bayesian approach it presented a comprehensive model able to account simultaneously for prior failure data, existing testing results, and the known Weibull slopes. In [Krolo *et al.* (2002a)] this model was extended to cover accelerated tests, now accounting simultaneously for lifetime ratio, acceleration factors, and prior reliability distribution in order to obtain Bayesian solution for the test sample size. The likelihood for the data containing k failures out of N trials under conditions of accelerated test with the effect of lifetime ration was expressed in modified binomial form:

$$L(R \mid k, N) = C_N^k R^{(A_f L)^\beta (N-k)} (1 - R)^{(A_f L)^\beta k}$$

(4.9)

Where A_f = acceleration factor

$$C_N^k = \frac{N!}{k!(N-k)!}$$

One of the sources to obtain prior distribution of reliability is an existing test data, which is often gathered in form of life data (time to failure format), which is typically fit by a Weibull distribution. As a further step [Krolo and Bertsche (2003)] applied the Weibull distribution to the procedure of determining A and B beta parameters. Based on the beta-binomial conjugate properties, linking parameters A and B with N, total number of test samples, and k, number of failed items, ($A = N - k+1$, $B = k$), the authors suggest using median ranks in defining the time-dependent number of failed units by the form of:

$$A = N - (N + 0.4)\left(1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}\right) + 0.7$$

<div align="right">(4.10)</div>

$$B = (N + 0.4)\left(1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}\right) + 0.3$$

Where β and η are parameters of the Weibull distribution and t is a time duration, which can be a field life or its 'bogey equivalent'.

In addition, the authors introduced the decrease factor δ, which was applied directly to the beta distribution parameters A and B in form of:

$$\pi(R) = \frac{R^{\delta(A-1)}(1-R)^{\delta(B-1)}}{\beta(\delta A,\ \delta(B-1)+1)}$$

<div align="right">(4.11)</div>

Where δ reflects the uncertainty of the information on the prior reliability and can be assigned the value between 0 and 1. $\delta = 0$ reverts the distribution to the uniform prior. Note that δ is similar to the knowledge factor ρ presented in [Kleyner *et al.* (1997)], equation (4.7) reflects the user's confidence in prior information and its relevance to the product, but it is applied directly to the prior distribution, instead of being used as a 'mixing ratio'. This approach, combined with the earlier introduced lifetime ratio and acceleration factor, creates a comprehensive and flexible model able to account for various inputs associated with product specifications and reliability demonstration techniques.

4.1.4.   Additional Engineering Methods of Obtaining a Bayesian Prior

In the past 10 years there have been numerous articles published in the field of Bayesian analysis and its implications for reliability testing. Therefore we will only mention references, which in the author's opinion have a bearing on the topic of the sample size reduction. [Campodonico (1993)] summarized the work undertaken by several individuals at the Institute for Reliability and Risk Analysis of George Washington University. This paper presented a summary of prior distributions and data collection procedures, which are often associated with those distributions. To an engineering practitioner it offered a better understanding of associations between failure count data and non-homogeneous Poisson prior, life data and Weibull prior, elicitation of expert opinion and gamma distribution, and several others. In general, this information can be useful in categorizing the choices of prior distribution $\pi(R)$ (see equation (4.3)) based on the types of data available for analysis, before actually running that analysis.

In a different development [Giuntini and Giuntini (1993)] suggested the means of deriving a reasonable estimate of reliability prior distribution for situations where there is no applicable data available. One of the sources for obtaining a prior distribution would be a combination of component data from military standard [MIL-HDBK-217 (1991)] and Monte Carlo simulation. Even though the data in MIL-HDBK-217 is presented in form of failure rates, which automatically assumes an exponential distribution, the authors suggest obtaining several data points by applying MIL-HDBK-217 methodology. The next step is to fit them with a Weibull distribution, obtaining parameters β and η, and

then use Monte Carlo to generate data points for numerical calculation of a posterior distribution. Note, that while a statistician would likely disagree with performing a Weibull best fit on exponentially distributed data, an engineer would most likely object to the use of MIL-HDBK-217, which has demonstrated a rather low accuracy in predicting failure rates for new technologies and is generally a discounted reliability prediction approach at this time. Despite these shortcomings, this approach can certainly be considered as an engineering alternative to the use of uniform prior distribution or no prior at all.

In the cases where the product is brand new and utilizes new technologies, the prior information is often unattainable due to the fact that the product has not been available for testing and data collection. In these cases the data can be collected in the form of less certain evidence. For example, instead of product warranty or previous test data, information regarding the expected failure rates provided by industry or technology experts can be utilized. This kind of information contains a certain degree of uncertain evidence. In addition, due to limited choices, the information about the prior can be obtained from the product, which is noticeably different from the current model. In these cases certain methods of processing the uncertain evidence can be applied, e.g., [Groen and Mosleh (2001)].

4.2. Current State of Bayesian Methods in Automotive Industry

At present there still exists a certain level of misgivings regarding Bayesian approach to reliability demonstration and some automotive customers even view it as a 'supplier's

trick'. The sole fact that suppliers may end up testing fewer samples than originally planned makes some OEM customers uneasy. However, since business cost considerations became increasingly important, some more cost conscious companies started to look more favorably toward the use of prior reliability knowledge when it assures them certain cost benefits without violating major product validation integrity. The following are the major concerns regarding the use of Bayesian techniques in developing product validation programs.

4.2.1.   Concerns on Customer's Side

It is a responsibility of a supplier to address product test and validation with mathematical rigor and engineering diligence. However, the application of Bayesian techniques in the eyes of the customer opens the door to some potential inadequacies. The customer concerns might include the following:

1. The product will not receive an adequate amount of testing due to the reduced test sample size

2. Since the new product is not an exact carry-over of the old one (for which the prior information is obtained), the product modifications might introduce the changes, which could produce serious reliability problems and the reduced amount of testing would potentially miss those problems.

3. Reduced sample will not adequately represent variations in product design and manufacturing characteristics.

4. The amount of 'newness' in the product is not adequately reflected by the prior distribution, therefore producing results, which will not be conservative enough.

4.2.2.   Concerns on Supplier's Side

Even though the supplier is the party generally benefiting from the use of Bayesian techniques, there is still some uneasiness on the supplier's side about use of prior knowledge. These concerns are:

1. Potential negative effect of a prior distribution. If the history of the product performance provides an 'unfavorable' prior distribution, the test sample size may actually become greater than that defined by the Success Run method described in Appendix A. Thus the result may increase product validation costs, instead of decreasing it.

2. Lack of consensus on how to quantify the innovation portion of the product while estimating the knowledge factor ρ in equation (4.7) may lead to disagreements between different functional units within the supplier's organization.   For example, the design engineers may want to use a comparative analysis of bills of material in order to compare the number of carry-over parts from the old to the new design and based on that, calculate the knowledge factor.   In contrast, validation engineer might want to take a look at the differences, which might directly affect the product reliability. For example replacing 90Ω resistor with a geometrically identical 150Ω resistor, mounted in the same location, will not make the product any riskier from a validation stand point, but may change the overall performance of the electrical circuit.

4.3.   Summary of Bayesian Approach to Reliability Demonstration Testing

There are certainly some advantages as well as challenges in utilizing Bayesian techniques in reliability testing. The challenges include data interpretation, relevance of prior data to the posterior, alternative choices of prior distribution, embedded uncertainty, interpretation of the posterior, degree of confidence, and some others. The advantages are obviously in potential cost reduction due to decrease in test sample size. However, the following can be acknowledged in regards to the automotive and some other mass production industries:

1. Bayesian modeling is a valuable statistical tool, which can be utilized in reliability demonstration, especially where high reliability requirements are stipulated by the OEM customer. It can provide a significant sample size reduction in the cases where traditional Success Run testing or life-data analysis would yield a prohibitive validation cost.

2. A certain amount of care and expertise should be exercised in applying this technique. Understanding the product, its development process, and design changes is critical in proper statistical application of the Bayesian method.

3. Knowledge of the method limitations should provide application boundaries and help to understand an appropriateness of the method to a particular validation program.

4. The development changes to the product should be quantified and further carefully considered before deriving prior distribution or even making decision about applying Bayesian models.

5. The choice of a Bayesian prior should be based on the type of data available for analysis as well as the reliability demonstration targets. For example the [Krolo *et al.* (2002a, b, c)] methods would be more applicable in the cases of medium reliability targets ($0.90 \leq R \leq 0.98$) with the prior obtained from the previous test data or other type of data with the medium number of data points (under 100). Consequently, [Kleyner *et al.* (1997)] would be more appropriate where high reliability demonstration targets are desired ($R \geq 0.98$) and large amount of field data with low failure rates is available.

In this dissertation the practical applications of Bayesian analysis will be based on the method presented in [Kleyner *et al.* (1997)]. However other methods discussed in this section and beyond can also be applied to reduce the test sample size and therefore the overall cost of reliability demonstration testing.

5.   Warranty/Service Cost

This chapter addresses the cost analysis methods associated with the descending portion of Reliability-Cost curve (Figure 3.3) as it applies to the automotive industry in general and automotive electronics in particular. This chapter will deal specifically with the issues of automotive warranties and methods of their accounting and prediction.

Automotive warranties amount to a whooping $12 billion per year for North American manufacturers alone [Warranty Week (2004a)] and that is not even including any of the brands associated with the third biggest contributor DaimlerChrysler, since the company is now technically foreign-owned. According to [Warranty Week (2004b)], this amount constitutes more than half of all the warranties for all US manufacturers worldwide. Therefore finding the best possible ways of predicting future warranty claims and more accurately accounting for the existing warranties can have a great engineering and financial impact on the whole process of planning and analyzing warranties in the automotive industry.

5.1.   Automotive Warranty Overview

Market conditions have traditionally been the main factor that determines the terms of warranties in general [Mitra and Patankar (1997)] and automotive warranties in particular. While expected reliability and quality of the product is considered an important supporting factor, in reality, the actual warranty terms are most often

determined by marketing pressures. Currently the terms of the standard automotive warranty, often referred to as the manufacturer's basic warranty are 36 months or 36,000 miles (whichever comes first) on the majority of vehicle parts [Auto Warranty Advise (2004)] with additional extended warranties on selected subsystems. Longer warranty periods are often used as an enhanced marketing tool. Warranty history and warranty expectations greatly affect the market value of new and used cars sold and lease residual values. Because of these and other financial and marketing considerations, a multitude of business decisions are being made based on the forecasted number of warranty returns for the overall warranty period and subsets thereof. All the aforementioned makes the process of improving warranty claims forecasting even more important, further increasing the need for models that provide an acceptable accuracy for business decision making. A parallel need for warranty forecasting also arises when the first few months of warranty claims are being analyzed for the purpose of forward extrapolation and development of appropriate corrective actions.

The warranty literature is vast and it is beyond our needs to review it completely here. An extensive warranty literature survey was presented in [Murthy and Djamaludin (2002)] covering warranty publications between 1987 and 2002; hence this work will not attempt to replicate it. Despite the extensive coverage, the choice of comprehensive warranty prediction engineering models capable of addressing practical problems is limited, and there is a clear lack of accurate, comprehensive, and application-specific models consistent with industry data formats. The material in this chapter will attempt to fill that gap as well as to enhance the statistics arsenal of reliability and quality engineers.

In many industries quality and reliability engineers who are involved in the warranty forecasting process use empirical models based on past warranty claims of products with similar design and complexity adjusted using experience-based correction factors accounting for the design and technology changes in the product. A reasonably accurate, scientific, and user-friendly model could help to accomplish these types of forecasting with better precision and improve the overall quality of business decisions requiring estimates of future warranty claims.

5.1.1.   Warranty Contributors

Analysis of automotive warranty problems shows that the range of warranty claims contains a wide mix of different types of problems. It contains various types of failures, which are qualitatively presented in Figure 5.1. The failure rate curves shown in this diagram reflect the general trends in automotive electronics warranty observed at Delphi Electronics & Safety, but do not represent any particular set of hard data.
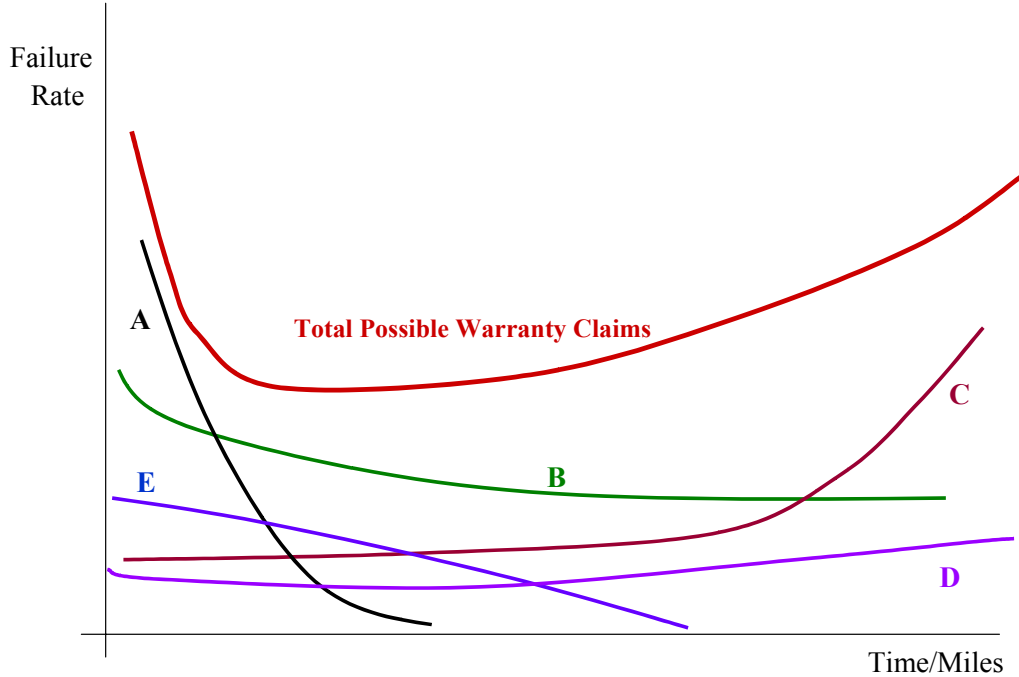
Figure 5.1. Conceptual breakdown of warranty claims by problem type

Following is a non-exhaustive list of problems comprising a typical automotive warranty

mix per Figure 5.1.

A: Initial performance or quality

B: Manufacturing or assembly related

C: Design-related failure or unacceptable performance degradation due to applied

       stresses (environment, usage, shipping, etc.)

D: Service damage, misdiagnosis, etc.

E: Software related problems

The sum of these types of failures makes up total warranty claims (top curve in Figure 5.1) and based on the collected data for automotive electronics presented in Figure 5.2 the total warranty curve approximately follows the first two sections of the bathtub curve.

Even though the product validation mostly deals with design problems, other types of failures as shown in Figure 5.1 will be included in the warranty claims mix. Due to variability in manufacturing process, some of the items do not conform to design specifications and these are termed 'nonconforming'. The higher the number of 'conforming' units, the higher the manufacturing quality is. A subset of the existing literature is dedicated to statistical analysis of quality problems as part of a product warranty, including [Juran and Gryna (1980)].

Despite the variety of sources, warranty claims tend to follow the first two sections of the bathtub curve as can be deduced from Figure 2.5 and Figure 5.2.
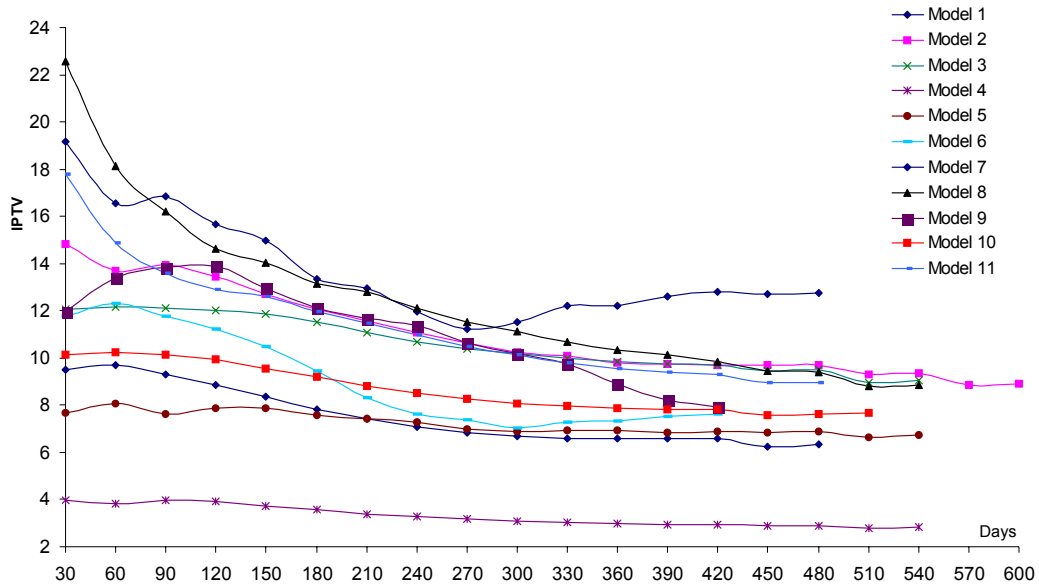
Figure 5.2. Failure rates, expressed in IPTV (Incidents per Thousand Vehicles) for selected passenger compartment mounted electronic products recorded by Delphi Electronics & Safety. Note, the actual IPTV values have been modified to protect the proprietary nature of the data.

Typical automotive warranty claims data would also contain a variety of noise factors, the biggest of which is undoubtedly unidentified failures, often referenced as *No Trouble Found* (NTF), Customer Complaint Not Verified (CCNV), and other terms listed in acronyms section including misdiagnosed data, duplicate records, and some others [Salzman and Liddy (1996); Thomas *et al.* (2002)]. These factors present a separate problem for statistical data analysis since for most automotive electronics product NTFs are often 50% of all warranty claims, sometimes reaching 90%. There are three common ways of approaching this problem. One is to ignore the NTFs and account only for the failures with determined root causes. The second is to include all the failure data in a statistical analysis. And the third is to model the NTF percentage and use it as a random

variable for all statistical simulations involving warranty analysis. In this dissertation the NTFs will be included in the total count of failures due to the fact that both NTFs and the true failures represent the real expenses to the manufacturer, however their influence will be specifically analyzed in the automotive electronics example Chapter 6.

5.1.2.   Two-Dimensional Aspects of Warranty

Since automotive warranty is usually expressed in both time and mileage terms, e.g., 36 months or 36,000 miles whichever comes first [Auto Warranty Advise (2004)], it can be described as a two-dimensional warranty [Blischke and Murthy (1996)]. A two-dimensional warranty is characterized by a region on a two-dimensional plane as opposed to an interval in one dimension. Different shapes for the region characterize different policies. Even though most of the 2-D policies have rectangular regions, other variations are possible, such as triangular shape, where the boundary of that region will be defined as an arithmetic combination of time and mileage or other usage parameters analogous to cumulative damage models [Ebeling (1997)]. For more information on 2-D shapes see [Blischke and Murthy (1994); Singpurwalla and Wilson (1998); Krivtsov and Frankstein (2004); Yang and Zaghati (2002); Majeske and Herrin. (1995)]. Higher dimensional warranties are also theoretically possible, but they are not common.

Most automotive manufacturers sell vehicles with a basic two-dimensional (time and mileage) warranty coverage and provide customers the option to buy an extended

coverage. Most often automotive warranty is specified in terms of $\{T_0, M_0\}$ with T_0 being a specified maximum time period and M_0 a specified maximum mileage Figure 5.3.
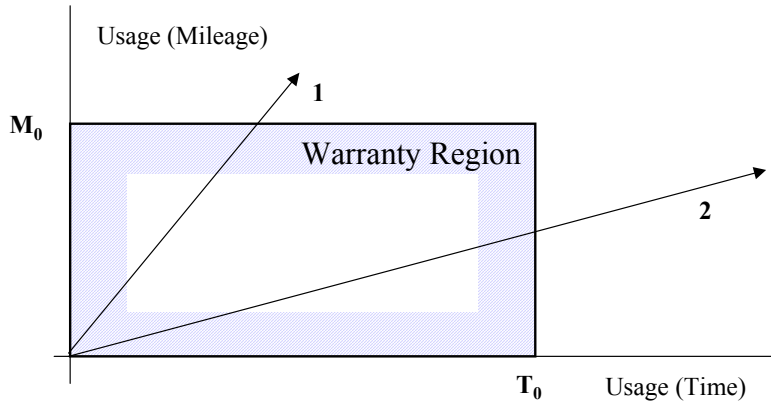
Figure 5.3. Warranty region for two-dimensional automotive warranty

Usage path 1 in Figure 5.3 shows the case where maximum warranty mileage M_0 is reached first and path 2 where maximum service life T_0 is reached before M_0. Age is known for all sold vehicles all the time, but mileage is only observed for a vehicle with a claim and only at the time of the claim. However, for automotive electronic parts it is more appropriate to use time as the primary usage variable since there are no moving parts involved in the process of wear-out, though the mileage variable is also important in estimating the expected warranties.

A generic problem is that warranty information is restricted to failure events occurring inside the warranty period and very little or no information about mileage accumulation is available for vehicles that have not experienced any failures [Campean *et al.* (2001)]. Thus, certain assumptions need to be made about the mileage accumulation in order to be

able to properly account for the 2-D aspect of an automotive warranty. An approach for this based on the daily mileage distribution will be presented later in the Section 5.2.3.

5.1.3. Warranty Data Reporting Formats

There are a multitude of data formats used for warranty data reporting. Without loss of generality, this dissertation will emphasize automotive warranties as they commonly appear in the United States. Most common warranty data formats are based on monthly failure reporting, where the number of product failures is presented on monthly basis. For example Ford reports the number of failures for each month in service (MIS) in the form of a table with failures versus month of occurrence [Yang and Zaghati (2002)]. Since vehicle sale dates are not linked to a particular calendar month, the '30-day buckets' formats are more common among automotive OEMs and their suppliers. In this format, the failure data is divided into 30-day service time intervals counted from the date of vehicle sale, where all the failures occurring within each 30-day time interval are reported in failed quantities or IPTV. The '30-day buckets' format presented in Table 5.1 is an easier, faster, and more common form of data reporting and is usually sufficient for the first-level approach to data analysis. Along with IPTV numbers, many companies also report DPTV (defects per thousand vehicles):

$$DPTV = IPTV - NTF$$

(5.1)

The DPTV metric helps to reflect the actual failures versus the cases where trouble was not found,[7] which also helps to report better quality figures. The raw warranty data typically contains additional information including vehicle identification number (VIN), vehicle mileage, geographical information, cumulative costs, cumulative IPTV, and many other parameters.

Table 5.1. '30-day Bucket' data format

Days in service	Vehicles in the field during the time period	Reported failures	IPTV
1-30	10,000	8	0.80
31-60	9,000	2	0.22
61-90	7,000	9	1.29

If the failed units can be traced to a specific production lot, this data can be converted into a more comprehensive format sometimes referred by quality professionals as 'layer cake', which usually combines all sold and failed units on a monthly basis, as presented in. This format provides information, which allows the user to trace each failure to a particular production group and can be used to conduct more sophisticated statistical analyses. Some commercially available software packages, such as ReliaSoft have this format as one of the data entry option for warranty analysis [ReliaSoft (2002)].

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[7] Trouble not found means that the unit was functioning normally in the laboratory environment and the failure could not be replicated.

Table 5.2. 'Layer cake' data format

Month	New Vehicles Sold	Number of Failures by Month			
		Month 1	Month 2	Month 3	Month 4
1	15,980	5	3	12	1
2	23,340		5	7	12
3	26,541			6	1
4	18,510				2

The numbers in Table 5.1 and Table 5.2 were created for example purposes and are not linked to any real product or to each other. The format in Table 5.2 is easier to understand and the data in this format can be processed with Weibull++ [ReliaSoft (2002)] and be easily converted into interval-censored life data.

Both formats discussed above are acceptable for the statistical data analysis, however the '30-day bucket' data can be analyzed only on a percentage-failed basis and is thus unusable for the calculation of confidence bounds. In contrast, the 'layer cake' data provides more options for determining a best-fit distribution including the estimation of confidence bounds. However, it is important to mention, that the '30-day bucket' format can be considered as a cost/time saving version of 'layer cake,' because it involves fewer data processing steps. Since most of the electronic units during the time period of interest remain in the functional state, this data can be considered right censored. Even though some of the automotive electronics units, such as radios, CD, players, engine controllers, etc., are often repaired at remanufacturing centers, a vehicle owner receives a different

electronic unit from the failed one (new or refurbished) after the claim is submitted, therefore the replacement is considered 'as good as new'. Also, automotive electronics products are typically not subject to preventive maintenance (PM), therefore the repairs are treated as an unscheduled corrective maintenance (CM).

Considering the practical applications, it is important to know that each automotive manufacturer has its own warranty database system and each system, despite commonalities, has its own data formatting and processing specifics. For example, General Motors has the database called QWIK (Quality With Information and Knowledge). Some of the details on its architecture and interfaces can be found in [Walters (2003)]. Similarly, DaimlerChrysler has a system called QUIS (Quality Information System) see [Hipp and Lindner (1999)]. Ford in North America has been cultivating the AWS database (Analytical Warranty System) for years and Toyota has a supplier focused warranty system called SQIDS (Supplier Quality Information Data System), which is a limited version of its complete quality/reliability database. This dissertation will not discuss the specifics of those systems, due to their proprietary nature, but instead will focus on commonalities as they apply to the warranty analysis and prediction. The author of this dissertation has a direct access to the QWIK database and will use its data for statistical analysis. This data will appear in form of statistical distributions only; the actual warranty claim numbers will not be included due to their proprietary nature.

5.1.4. Current Techniques of Forecasting Warranty Claims in the Industry

A forecast of a product warranty often becomes an important input in the decision making process associated with awarding automotive component business. Therefore, since warranty prediction is usually a part of a general business model; there exists a multitude of warranty cost models. However, due to a contribution of various factors (see Figure 5.1) warranties are difficult to predict and the accuracy of these predictions are usually poor.

The majority of the existing models involve the existing warranty databases for the products already in the field. The most simplistic methods use the recent warranty numbers for the similar products multiplied by an empirical 'fudge factor'. For example, the last year's percent failed increased by 25% to account for the newness of the design, technology, and production. Needless to say that these methods are too crude and apply a 'one size fits all' approach to the problem. More advanced approaches are utilized for example at DaimlerChrysler [Hotz *et al.* (1999)] where the warranty cost prediction has been realized by using a conventional planning method based on the amount of warranty cost observed in the last budget year. This amount is modified by information available about the expected inflation, quality index of the vehicles, and the development of the sales figures for the different vehicle series.

Many warranty analysts who process and analyze warranty data do it at an accounting level, i.e., reporting numbers without a comprehensive analysis. Often the personnel involved in warranty reporting produce a large number of tables, bar graphs, Pareto

charts, warranty summaries, etc., without doing proper statistical analysis, root cause analysis, or any other type of in-depth analysis of product failures. In the majority of the cases, the warranty data is analyzed as homogeneous pool of data without any regard for the non-stationary nature of the data, such as possible changes in the trends of warranty claims or special attention to unusual patterns of failures occasionally leading to a product recall. Fortunately, there is an enormous existing data cache associated with vehicles and their parts warranty claims, which potentially allow more sophisticated approaches to the warranty prediction to be performed.

Some companies utilize Weibull analysis to process failure data and to make a prediction of future warranty claims based on the obtained Weibull distribution parameters. This approach, though more sophisticated, also has its pitfalls. The biggest pitfall is the fact that most of the electronics failure trends follow the bathtub curve; therefore the prediction based on the declining failure rate (infant mortality phase) would be an oversimplification underestimating failures for the time periods exceeding the initial phase. Alternatively, detailed statistical approaches addressing the trend change in the failure rates [Haupt and Schabe (1992); Xie and Lai (1995); Baskin (2002); Wang (2000); Yang and Zaghati (2002)] adequately represent the bathtub curve, but are not formulated for forecasting and are generally not practical for use with real data and its associated uncertainties. Several interesting mixture models are presented in [Majeske and Herrin (1995); Majeske *et al.* (1997); Majeske (2003)], however they use additional 'tuning' variables, which unduly complicate the process of analysis and simulation.

Classic warranty literature [Lawless *et al.* (1995); Kalbfleisch (1991); Lawless (1998); Robinson and McDonald (1991)] concentrate mostly on Poisson-based and non-parametric empirical models as opposed to Weibull life data analysis. The main reason for this is the fact that warranty repairs approximately follow the renewal process, which is better described by Poisson models, where Weibull is more applicable to life data (time duration until failure). However on an electronic component level only a small portion of repaired or replaced parts fail again. From the author's experience at several Delphi Corporation's remanufacturing centers, less than 5% of the returned parts have been repaired before. Therefore the use of the Weibull distribution can be justified for the electronic parts warranty analysis. The following section will describe the warranty forecasting method proposed by the author of this dissertation. Some of the aspects of this approach were presented in [Kleyner and Sandborn (2005)].

5.2.   Proposed Method of Warranty Analysis and Prediction

Figure 5.4 presents a step-by-step procedure of predicting the warranty and organizing it as an input variable to the overall LCC modeling and optimization process.
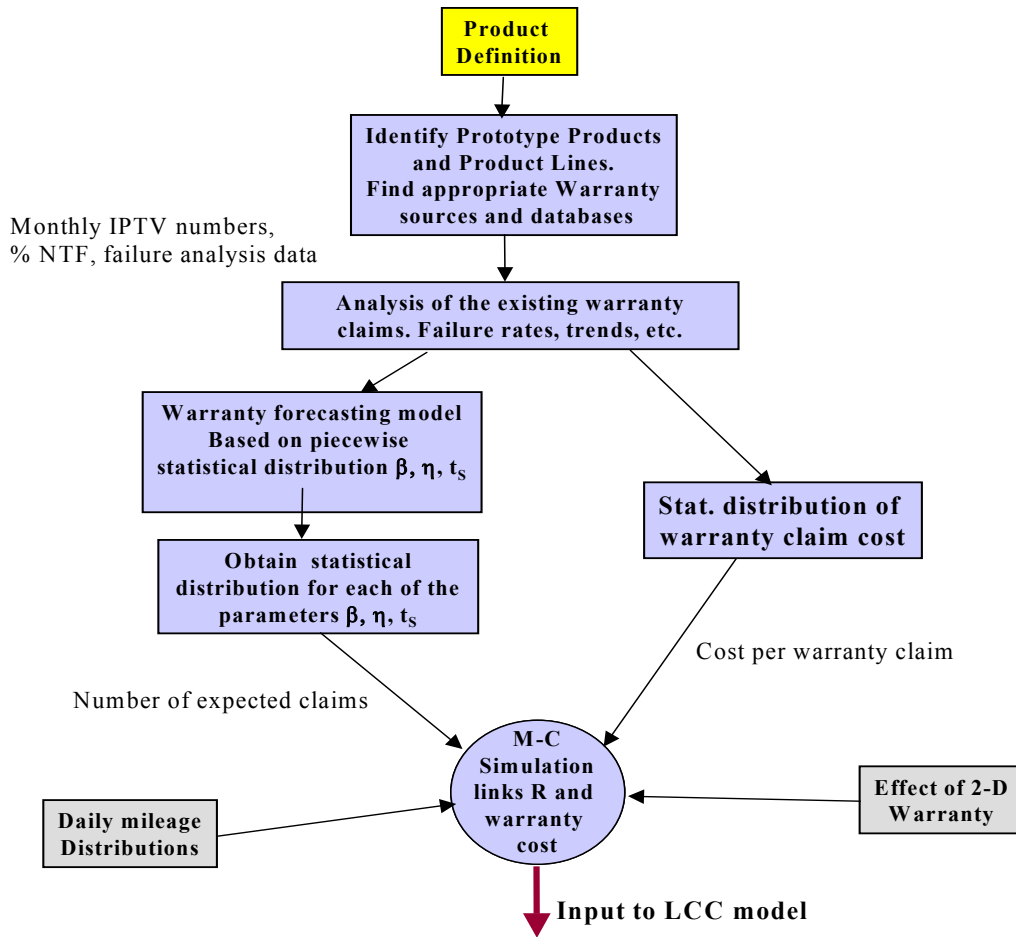
Figure 5.4. Warranty cost analysis flow

The process starts with product specifications, where the main design characteristics of the product should be defined. Based on the knowledge of geometry, utilized technology, applications, and other parameters (see Section 5.2.1 for more details) we can determine the products that can be identified as prototypes for the product under development. The warranty numbers for the prototypes can be analyzed for failure rates, trends, statistical distributions, and other properties. This data can be utilized for the warranty analysis and prediction described in detail in Section 5.2.2. Expected warranty will be mathematically linked to a product validation process, Section 5.3 and included into the final stochastic simulation of the LCC analysis described in Chapter 6.

5.2.1.   Utilization of the Existing Warranty Data

Most of the warranty prediction methods are based on the product's past history. It is important to have a database of past warranty claims in order to determine what products are applicable based on design and usage similarities. The existence of the reference base, ideally in the form of FRACAS or other type of warranty database, is critical to the success of this method and its applications.

The product families should be divided into the groups with similar features. The similarity criteria may include the following:

- Vehicle platform the product is mounted on (passenger cars, light trucks, heavy duty trucks, etc.)

- Mounting location: (passenger compartment, underhood, on-engine)

- System function: (powertrain, entertainment, safety, ignition, etc.)

- Manufacturing site: (USA, Mexico, China, Poland, etc.)

- Existence of the moving parts inside the unit versus pure electronics

- Critical parts: (playback mechanisms, capacitors, large microprocessors, etc.)

- Packaging technology utilized: (flip chip, BGA, leaded SMT, PTH, etc.)

- Time already in production (new product, first year production, second year, etc.)

The criteria for choosing the appropriate prototypes for warranty prediction may be based on a simple engineering judgment, recommendation from a designer or a quality engineer, or a more sophisticated sorting technique such as similarity analysis. Formal

similarity analysis will not be discussed here in detail due to the scope of this work, but the relevant information can be found in [Yan and Forbus (2004)], [Cuberos *et al.* (2002)], or other similar sources. In this dissertation the choice of the warranty prototype will be based on similarity in the type of product, component types, technology, and design.

5.2.2. Proposed Warranty Forecasting Model

Warranty data usually contains information on all incidents reported during the warranty period. As mentioned previously, the product failure behavior can be partially modeled by a bathtub curve. There exist a variety of mathematical models that adequately represent the reliability bathtub curve [Haupt and Schabe (1992); Xie and Lai (1995); Baskin (2002); Wang (2000); Yang and Zaghati (2002)]. For our purposes we are interested in a model's ability to fit the data presented in the automotive warranty reporting formats described in the Section 5.1.3. Many bathtub-curve models are mathematically expressed in terms of hazard rate, while validation engineers are usually more accustomed to working with reliabilities and percentages of failures. Also since reliability forecasting is usually the ultimate goal of this kind of analysis, a model expressed in terms or reliability would typically be easier to apply directly in engineering calculations.

Based on the fact that a typical automotive part is designed for a mission life of 10-15 years it is very unlikely that it would be subjected to wear-out failures during either warranty or even extended warranty period of 3 to 7 years.
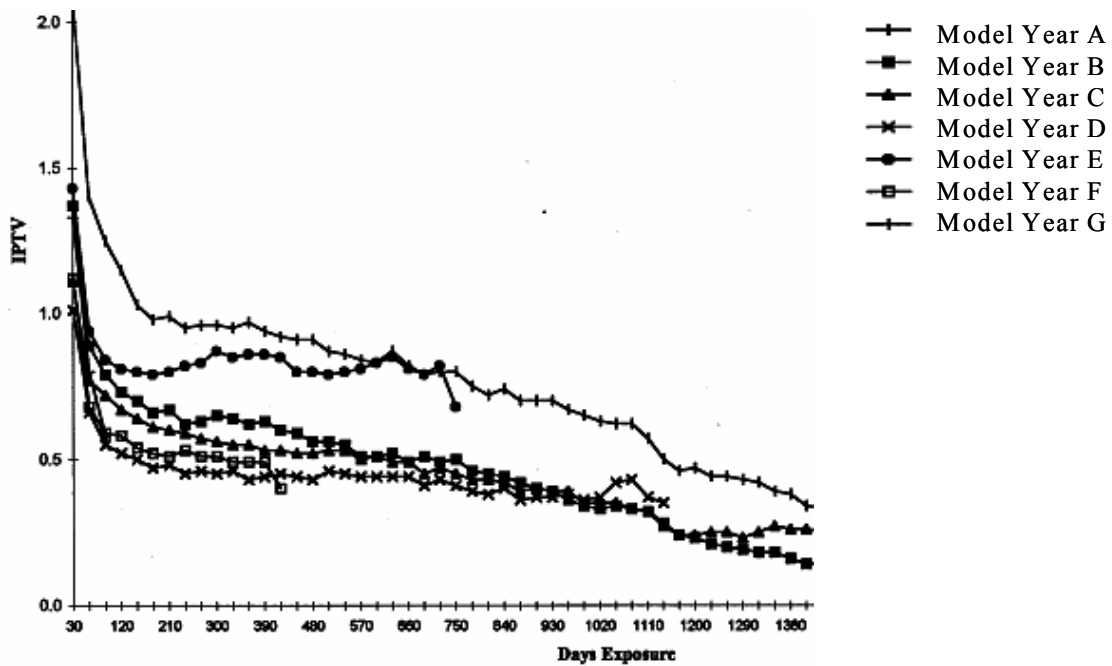
Figure 5.5. Extended warranty charts compiled from Delphi Corporation warranty data for the several model years of the same electronic product mounted in the engine compartment. The data shows no wear-out mode for at least 4 years of service.

The data shown in Figure 5.5 provides an illustration of an automotive electronics product family recorded in terms of IPTV according to equation (5.2) for seven different model years[8] of the same automotive electronics family (model years 'A' through 'G')

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[8] Model year is a manufacturer's annual production period. In automotive industry new model year production may start as early as July of the previous calendar year.

$$IPTV(t) = \frac{Claims(t)}{N(t)} \times 1000$$

<div align="right">(5.2)</div>

Where *Claims(t)* = number of claims reported in the period t

$N(t)$ = number of vehicles in the field in the period t

The data suggests that in the majority of cases the warranty failure model is sufficiently represented by the infant mortality and useful life phases of bathtub curve. A detailed study of the existing warranty of various product lines of automotive parts performed at Delphi Electronics & Safety showed a clear trend of diminishing failure rate for the first 8 to 18 months (see Figure 5.5) followed by a flattening of the failure rate curve for the remainder of the time period that warranty and extended warranty data were available.

To combine the first two sections of the bathtub curve and to provide a best fit for the warranty data in Figure 5.2 and Figure 5.5 a conditional reliability equation is suggested:

$$R(t) = R(t_S)\, R(t_S \rightarrow t) \qquad (t > t_S)$$

<div align="right">(5.3)</div>

Where *R(t)* = reliability at the time interval t

t_S = predetermined time coordinate

R(t_S) = reliability at the time t_S

$R(t_S \rightarrow t)$ = probability of reaching the time point t, under the condition that time

t_S has already been reached.

As mentioned earlier, many reliability and quality engineers are more accustomed to working with reliabilities expressed in terms of commonly used distributions: Weibull, exponential, normal, and lognormal. Analysis of the existing data (Figure 5.2) shows that t_S can be determined as the time coordinate where hazard rate stabilizes, the failure data with decreasing failure rate in the interval $[0; t_S]$ could be fit with Weibull distribution. Similarly the failure data in the interval $[t_S; t]$ could be fit with exponential distribution, since the failure rate would remain relatively constant in this range. Methodologies of detecting the changes in the pattern of the data over time and estimating the points where these changes occur in application to Statistical Process Control (SPC) was presented in [Hawkins and Qiu (2003)]. Under these assumptions, (5.2) becomes:

$$R(t) = e^{-\left(\frac{t_S}{\eta}\right)^{\beta}} e^{-\lambda(t - t_S)}$$

(5.4)

Where η = Weibull scale parameter

λ = constant failure rate after t_S

β = warranty Weibull slope (not to be confused with β used in the parametric binomial equation (3.3), where it represents wear-out mode and typically $\beta > 1$). In equation (5.4) it represents the infant mortality mode and the expected value $\beta < 1$)

Time t_S can be referred as a *change point,* the coordinate where the pattern of data changes requires a different data-fitting model, [Hawkins and Qiu (2003)]. The continuity at the junction point t_S can by achieved by equating the hazard rates at the point t_S. The hazard rate for Weibull distribution $h_{Weibull}$ is:

$$h_{Weibull} = \frac{\beta}{t_S}\left(\frac{t_S}{\eta}\right)^{\beta}$$

(5.5)

Thus equating $h_{Weibull}$ with the constant failure rate λ past the point t_S would produce:

$$\lambda = \frac{\beta}{t_S}\left(\frac{t_S}{\eta}\right)^{\beta}$$

(5.6)

The overall reliability expressed in (5.4) has four parameters β, η, t_S, and λ, using (5.6) to eliminate λ, (5.4) can be transformed into:

$$R(t) = e^{-\left(1+\frac{\beta(t-t_S)}{t_S}\right)\left(\frac{t_S}{\eta}\right)^{\beta}} \qquad t \geq t_S$$

(5.7)

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Equation (5.7) is in a suitable format for a stochastic simulation such as Monte Carlo method, which has been successfully applied in a variety of parametric studies of reliability, e.g., [Chen *et al.* (1999)]. Each of the parameters, β, η, t_S is a random variable and could be represented by a statistical distribution. The best way of obtaining those distributions is by observing the past history of the product. The author of this work studied warranty returns for several automotive electronics product families including Radio-CD players, engine controllers, and climate control modules and identified some common trends in the data. While the variation of statistical parameters between these groups was significant, parameter variation within the same group was far less apparent. An important factor governing variation within a product family was found to be the number of years in production with a tendency for the first year to have the highest number of warranty claims.

Besides forecasting the expected warranty returns for the future products, this model can also be used for ongoing forecasting of current products, where the final warranty prediction is based on the number of claims reported after the product's first several months in the field and is subject to continuous updates. This type of forecasting is often used to compile monthly reports to the management as well as to detect potentially serious field reliability problems.

The procedure for determining distribution parameters of the forecasting variables β, η, t_S starts with obtaining the change point estimation t_S. Since any real data would demonstrate some form of variation between consecutive 30-day intervals, it can be

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suggested to use the Bayesian smoothed hazard function described in [Campean *et al.* (2001)]. It would modify the stepwise pattern of the interval-based hazard function and would provide a continuous transition between adjacent 30-day intervals using Bayesian estimation of hazard rates. For simplicity purposes the average hazard rate $h_{avg}(t)$, given by equation (5.8) can also be used for this type of analysis:

$$h_{avg}(t) = \frac{Number\ of\ accumulated\ failures\ (t)}{Total\ accumulated\ time\ in\ service\ (t)}$$

(5.8)

Graphic analysis of the average hazard rate shows the general trend of saturation starting at t_S. One of the criteria used for determining the exact change point t_S could be the flattening of the curve fit to within ± 10% of the boundaries of the hazard rate value as illustrated in Figure 5.6 (other criteria may be practical depending on the specific nature of the data).
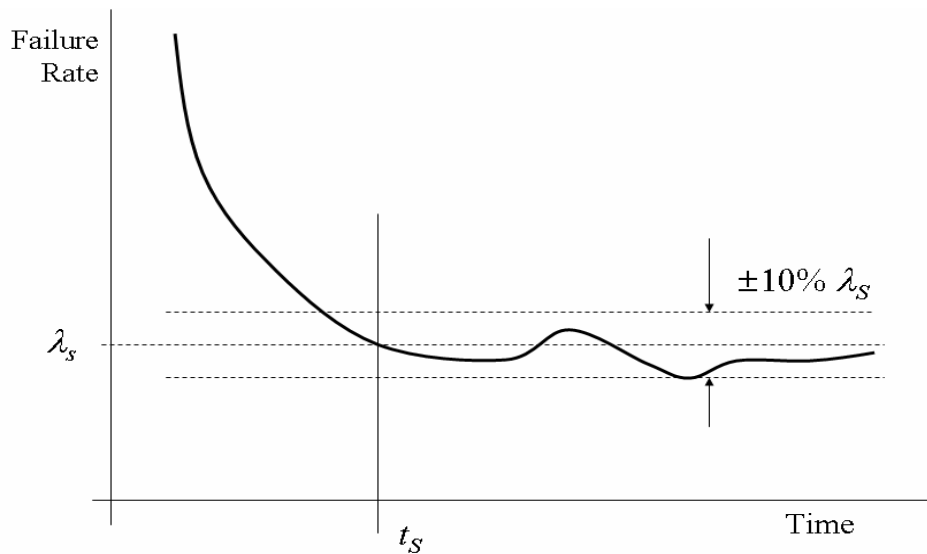
Figure 5.6. Change point estimation for t_S, λ_S is the failure rate at t_S

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If the characteristics of the data are different from that presented in Figure 5.6 and do not

have a pattern of decreasing failure rate followed by stabilization, then the parameter t_S

can be estimated from visual observation of the plotted data or the data set can be

considered as an outlier and be removed from the analysis pool. For each set of data the

failure numbers should be split between pre-t_S and post-t_S intervals. Each of the two data

sets should be Weibull-fit as a separate group for determining Weibull parameters β and

η. Analysis of the product groups mentioned previously, demonstrated stable trends,

showing that pre-t_S Weibull slope β (we will refer to it as β_1) typically stays in the range

of 0.65 – 0.85. The statistical analysis of more than forty different data sets with @Risk,

the risk analysis and simulation add-in for Microsoft Excel [Palisade Corporation (2002)]

demonstrated that a two-parameter Weibull distribution was indeed the best-fit

distribution for pre-t_S data in almost half of the cases. For the remainder of the datasets

Weibull was in a top five out of 28 different distribution options thus supporting the

choice of Weibull distribution for this procedure. The similar analysis of post-t_S data

showed that Weibull slopes β_2 in all forty cases were within $\pm$ 10% of $\beta_2 = 1.0$, thus

confirming the constant failure rate assumption for the post-infant-mortality stage.

Different procedures corresponding to the two different data formats discussed in Section

5.1.3 can be performed using commercially available reliability analysis software.[9] The

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[9] When using ReliaSoft Weibull++ with the '30-day bucket' format, it is best to use a "free form data"

format, which is made up of X time to failure data and Y position data in % in which ranks are not assigned

data presented in 'Layer cake' format allows more sophisticated data processing, since the user would be able to obtain exact failure time intervals and the number of suspended items. This more detailed information would allow the implementation of MLE (Maximum Likelihood Estimate) Weibull analysis (or other distribution best fit) and provide the confidence intervals for the results of the best-fit approximation. It is also important to address the effect of the production year. For example, it has been observed that quality usually improves with the years in production due to continuous improvement of manufacturing procedures.

5.2.3. Effect of Two-Dimensional Warranties

As mentioned in Section 5.1.2, the automotive industry mostly deals with two-dimensional warranties usually specified in terms of $\{T_0, M_0\}$ with T_0 being a specified maximum time period and M_0 a specified maximum mileage. For automotive electronic parts it is more appropriate to use time as the primary usage variable since there are no moving parts involved in the process of wear-out, though the mileage variable is also important in estimating the expected warranties. Any of the methodologies described in the literature referenced in the Section 5.1.2 can be applied to the proposed model in order to add an additional dimension of warranty. The method utilized in this dissertation is slightly different and will be based on constructing $CDF(t|M_0)$ for the probability of exceeding the maximum mileage M_0 at any particular time t.

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to the time interval (30-day, 60-days, etc.) would represent X-axis and 0.1×IPTV (percent failed) would be plotted on Y-axis.

First, using the dealership repair data containing the dates and mileages associated with each warranty claim, we can construct a probability distribution function of daily mileage $f_{Daily}(m)$. The daily mileage distribution was obtained from the dealership data of more than 1000 data points, each containing the number of days to failure and the corresponding vehicle mileage. At each particular time t_i in Figure 5.7 the cumulative probability distribution function of exceeding M_0 can be calculated as:

$$CDF(t_i \mid M_0) = \int_{\frac{M_0}{t_i}}^{\infty} f_{Daily}(x)dx$$

(5.9)

where M_0/t_i is the daily mileage required to reach M_0 at the time t_i. For each arbitrarily selected t_i, the $CDF(t_i|M_0)$ can be calculated and consequently fit into the analytical distribution. Based on a statistically sufficient number of points t_i providing the refinement of $[0; T_0]$ we can run the best fit to determine the PDF: $f(t|M_0)$, which would be a continuous function of time characterizing the probability of running out of warranty at any particular time t. The obtained best fit for $f_{Daily}(m)$ was two-parameter Weibull with the shape parameter $\beta = 1.55$ and the scale parameter $\eta = 41.1$ miles. The next step was to plot the probability of running out of warranty for the number of time periods t_1, t_2, t_3, etc., similar to that presented in Figure 5.7.
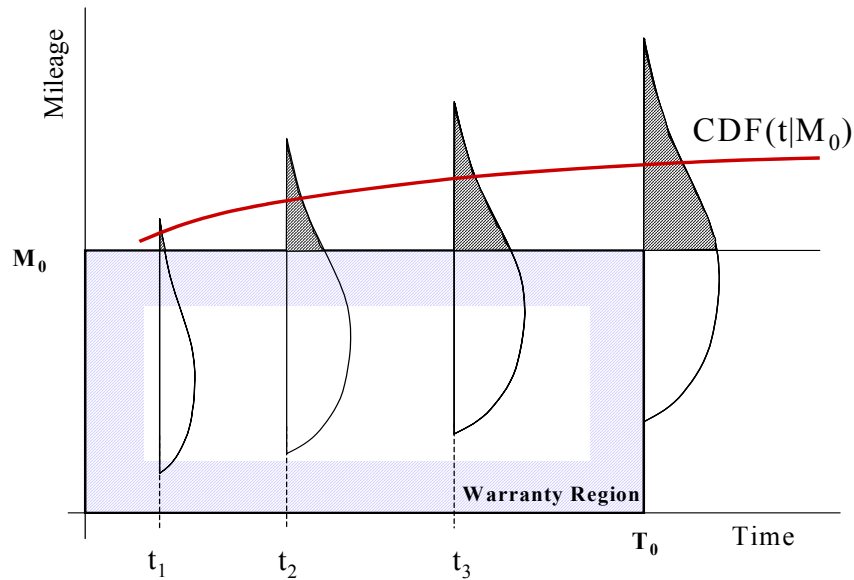
Figure 5.7. Statistical distribution for daily mileage accumulation

The t_i values were chosen arbitrarily every 100 days in order to provide a sufficient number of points to plot the $CDF(t|M_0)$ similar to Figure 5.7. Table 5.3 presents the probabilities of exceeding 36,000 miles for the first 1000 days with 100-day increments. The criteria for sufficient data points was based on the convergence of the resulting distribution. The best-fit PDF based on 100-day increments (36 data points) overlapped 98% with the best-fit PDF based on the 200-day increments (22 data points), which indicates they were at the state of convergence.

Table 5.3. Probabilities of exceeding 36,000 miles based on daily mileage distribution

Service time, days	100	200	300	400	500	600	700	800	900	1000
% probability of exceeding 36,000 miles	1E-11	0.0051	0.51	3.42	9.17	16.51	24.3	31.4	38.3	44.22

Due to lack of space, the numerical data beyond 1000 miles is not shown in Table 5.3. Performing a best-fit distribution analysis with the complete data set produced the

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f(t|36,000) as a lognormal distribution with $\mu = 7.34$, $\sigma = 0.675$, which corresponds to the

mean of 1930 days with standard deviation of 1465 days, Figure 5.8.

Therefore, to approximate the percent of failures, which occurred before T_0, causing a

warranty claim ($t \leq T_0$, $m \leq M_0$) the unreliability would have to be multiplied by the CDF

of not exceeding M_0):

$$F(T)_{Warranty} = [1 - R(T)] \int_T^\infty f(t|M_0)dt \qquad T \leq T_0$$

(5.10)

Where $F(T)_{Warranty}$ = failures covered by warranty for the time period T.

After substitution equation (5.7) into equation (5.10) and considering the lognormal

character of the mileage distribution *f(t|36,000)*, Figure 5.8, the resulting failures can be

calculated as:

$$F(T)_{Warranty} = \left[1 - e^{-\left(1 + \frac{\beta(T-t_S)}{t_S}\right)\left(\frac{t_S}{\eta}\right)^\beta} \right] \times \left[1 - \Phi\left(\frac{\ln T - \mu}{\sigma}\right) \right] \qquad t_S \leq T \leq T_0$$

(5.11)
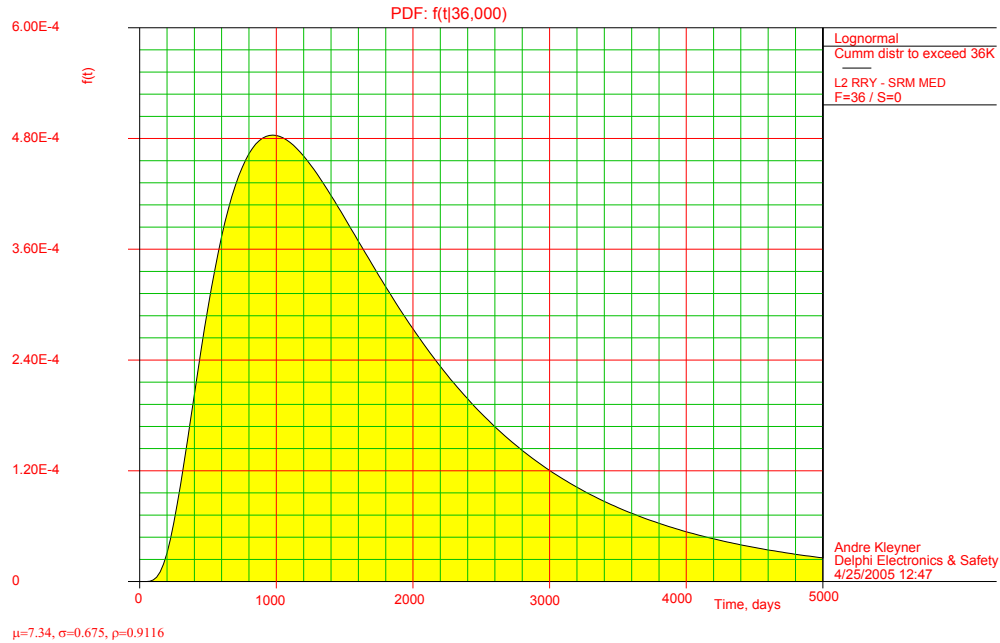
Figure 5.8. Plot of *f(t|36,000)* based on the automotive dealership data

On a separate note, equation (5.7) can be used for ongoing warranty forecasting for current products already in production. Direct application of equation (5.7) in conjunction with equation (5.11) would allow using pre-t_S data (data from several months of warranty return) to predict the post-t_S data expanding to the full warranty period, extended warranty period, and beyond.

5.2.4. Automotive Electronics Example

In order to illustrate the warranty forecasting method discussed in Section 5.2.2, an automotive electronics example will be presented in this section. For simplicity, only the data stored in 30-day bucket format will be considered here. Let's assume that we must forecast the 5-year/50,000 miles extended warranty of the new automotive radio with CD

player and let's also consider the effect of production start (usually the first year production) on the rate of returns for this part. The warranty data is available for four different radio models with similar features and complexities. Due to limited space we will present the initial data for only one model called Radio 1, 1st year production lot (Table 5.4) and show the rest of the data in a statistical distribution format. As before, the presented warranty numbers will be altered due to proprietary nature of the data.

Table 5.4. Radio 1, 1st year production lot. '30-day bucket' warranty data for 960 days of service

Days in Service	IPTV	Total % Failed	Days in Service	IPTV	Total % Failed
0 -30	3.03	0.30	481 - 510	0.92	2.40
31- 60	1.50	0.45	511– 540	2.80	2.68
61- 90	1.41	0.59	541- 570	0.30	2.71
91- 120	1.39	0.73	571- 600	1.20	2.83
121- 150	1.32	0.87	601- 630	0.20	2.85
151- 180	1.31	1.00	631- 660	0.15	2.87
181- 210	1.37	1.13	661- 690	0.55	2.92
211- 240	0.49	1.18	691- 720	2.40	3.16
241- 270	0.36	1.22	721- 750	0.60	3.22
271- 300	1.70	1.39	751- 780	2.00	3.42
301- 330	0.45	1.43	781- 810	2.50	3.67
331- 350	1.70	1.60	811- 840	0.90	3.76
361- 390	1.76	1.78	841- 870	5.00	4.26
391- 420	1.74	1.95	871- 900	3.27	4.59
421- 450	0.65	2.02	901- 930	1.12	4.70
451- 480	2.90	2.31	931- 960	0.21	4.72

Since the data comes in '30-day bucket' format it is best to apply the free form data format (percentages failed) to pre-t_S (β_1) and post-t_S (β_2) separately.

Table 5.5. Results of Weibull analysis of each data set for four radios

Product	t_S (days)	β_1 (pre-t_S)	η_1 (days)	β_2 (post-t_S)
Radio 1. 1st year production	**390**	**0.668**	**205,781**	**1.21**
Radio 1. 2nd year production	270	0.761	378,248	0.961
Radio 1. 3rd year production	420	0.872	501,320	1.03
Radio 2. 1st year production	**330**	**0.890**	**290,258**	**0.920**
Radio 2. 2nd year production	420	0.793	483,692	0.986
Radio 3. 1st year production	**240**	**0.731**	**242,725**	**1.06**
Radio 3. 2nd year production	180	0.903	618,440	1.02
Radio 4. 1st year production	**270**	**0.912**	**252,551**	**0.946**

Typically the type of information presented in Table 5.5 would contain a much larger amount of data with more automotive product categories due to the large number of parts and applications. For example the radio models can be subdivided by vehicle platforms, where the same radios would be considered as a different group if they were installed on light trucks as opposed to mid-size cars. The larger the number of similar product lines the better the confidence intervals for the results obtained with Monte Carlo simulation.

There are several possible ways of processing the data presented in the Table 5.5. All the data can be analyzed together by finding the best distributions for each of the three parameters β_1, η, t_S, and based on the obtained distributions, model these values for Monte Carlo simulation with equation (5.7). However, if we are for example interested in the warranty of the product manufactured within the first year after the start of production, only the data pertinent to the first year of production will be analyzed (see the four bold rows in Table 5.5). Based on these four data groups the following distributions were obtained:

t_S – lognormal distribution: $\mu = 5.71$, $\sigma = 0.186$

β_I – 2 parameter Weibull distribution: $\beta = 6.62$, $\eta = 0.815$

η_I – normal distribution: $\mu = 247{,}830$, $\sigma = 30{,}069$

In order to account for the effect of two-dimensional characteristics of warranty we need to estimate the probability distribution function *f(t|50,000 miles)* of mileage reaching 50,000 miles at time *t*, analogous to that presented in Figure 5.7. Applying the method described in Section 5.2.3 for 50,000 miles mark we can obtain the conditional probability distribution *f(t|50,000)*, which is best represented by Lognormal distribution with parameters: $\mu = 7.53$ and $\sigma = 0.904$, which corresponds to the mean of 2804 days with standard deviation of 3152 days.

A 10,000 sample Monte Carlo simulation of expected warranty returns at the 5-year mark (1825 days) produced the following results according to equation (5.4). Mean value for cumulative return of claims covered by warranty was $F(5\text{yr}) = 2.2\%$ (50% confidence). With upper 80% confidence this value reaches $F_{80\%}(5\text{yr}) = 3.1\%$.

This example demonstrates the use of equation (5.11) with real data to perform a reliability/warranty prediction. A common simplistic method to treat the data associated with this example would have been a Weibull analysis of early failures for existing parts with similar design features. In our case, a simple Weibull analysis of early failure data accounted for 2-D aspect of warranty would produce $F(5\text{yr}) = 0.74\%$, which is

significantly lower than the result obtained from Monte Carlo simulation using equation (5.11).

All stochastic simulations in this work are performed with the software @Risk v. 4.5, which is the Monte Carlo simulation add-in for Microsoft Excel [Palisade Corporation (2002)]. Random inputs for equations (5.7) and (5.11) are generated using the Latin Hypercube sampling technique.

5.2.5. Warranty Prediction Modeling Summary

The model presented here offers a straightforward solution to a complex two-dimensional warranty prediction problem. The solution is easy to implement within Monte Carlo or other types of stochastic simulations because it is represented by a single closed-form equation. The procedure is a practical means of accomplishing two major reliability prediction tasks: 1) the forecasting of future product warranty at a product planning stage, and 2) the ongoing forecasting for current products, where the warranty returns are known for the first several months of production. This method can be used to predict the number of failed parts, which would not be reflected by warranty claims due to mileage exceeding the warranty limit. In addition, the methodology also enables the accurate calculation of various life cycle cost components.

Advantages of the presented warranty prediction model:
1. Used as a prediction model it is based on the production launches of the similar models

2. For prediction purposes only the first year productions can be used to simulate the launch of the new product. The model can be continuously improved by including other years if the forecast over several years is of interest. Since a manufacturing process is always associated with the learning curve process, the failure rates usually drop each year of production for the same model with the visible drop from the first year to the second year.

3. Forecast can also be done based on the first months of production. Based on obtained β and η the rest of the life cycle can be predicted. Also it can be easily identified if new product launch has problems – the first months will be quite different from the historically obtained distribution parameters.

The approach developed and demonstrated in this section represents a balance between correctly modeling the failure rate trend changes and analysis practicality for real world reliability analysis organizations. The automotive electronics example in Section 5.2.4 clearly showed that simplistic data fitting approaches do not adequately model the real application data.

Unlike anything published in the warranty literature, this predictive model is tuned to the existing automotive warranty reporting formats and mathematically accommodates the projected change point in the failure rate pattern. It is understandable by decision makers while at the same time maintaining statistical rigor. Mapping the automotive supplier warranty data to existing models would be a cumbersome procedure, while this model fits naturally into the existing data reporting structures. This approach is also oriented

towards reliability engineering applications and more practical and specific for product validation tasks. It is also important to note here that the stochastic simulation in this work is done on the parameters of the observed distributions unlike some other warranty prediction methods where Monte Carlo simulation is applied directly to the failure times and repair times [Kaminskiy and Krivtsov (1997)].

5.2.6.   Warranty Cost Simulation

In order to calculate the expected cost of warranty returns it is necessary to estimate the number of units expected to fail within the 2-D warranty box, Figure 5.3 along with the cost of each warranty claim. Based on equation (2.5) the total warranty cost will be:

$$W_C = n_f \alpha_W$$

(5.12)

Where W_C = total cost of warranty

n_f = number of units expected to fail

α_W = warranty cost per repair, charged by automotive dealership to the vehicle

manufacturer

Number n_f can be estimated as a function of time based on the expected reliability at the end of warranty period as follows:

$$n_f(t) = [1 - R(t)] n_{sold} \qquad t \leq T_0$$

(5.13)

Where n_{sold} = number of units sold, which approximates the total number of manufactured units.

Cost of each repair is an input to the cost model. In general, the cost to repair a failed item is a random variable that can be characterized by a distribution function $H(\alpha_W)$. The cost of the past warranty repairs will be analyzed and assembled into the statistical distribution based on best fit. This function will be used for Monte Carlo simulation as one of the warranty cost inputs later in the Chapter 6.

Since warranty expenses are spread over the period of time (3 years for a standard warranty and longer for the extended warranty) the LCC solution may be affected by the time value of money. Warranty cost can be calculated in today's dollars using the present value of money and compounded interest [Ayyub (2003)]. Assuming that warranty payments to dealerships are distributed approximately equally over the warranty period the equal payment capital recovery approach [Ayyub (2003)] can be applied. The total amount of money spent on warranty can be divided over the total number of months and each monthly payment can be approximated as an equal payment. Therefore the net present value (NPV) of warranty cost can be calculated as:

$$NPV(W_C) = \frac{W_C\left[\left(1+\dfrac{i}{12}\right)^{T_0} - 1\right]}{T_0\left[\dfrac{i}{12}\left(1+\dfrac{i}{12}\right)^{T_0}\right]}$$

(5.14)

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Where i = annual interest rate

T_0 = warranty period expressed in months (in our case $T_0 = 36$)

The net present value of the warranty cost given in equation (5.14) will be simulated for the total LCC calculations and added to the cost of product validation in equation (2.6).

5.3. Connecting Reliability Demonstration with Future Warranty

In this section the mathematical link between product validation and the future warranties will be defined. Most of the time warranty reporting systems and product validation activities deal with different time horizons. Product validation is normally intended to simulate the product mission life, which in automotive industry is 10-15 year. However warranty mostly deals with shorter time intervals, typically 3 years or in the cases of extended warranties 5-7 years. Therefore the current warranty reporting system does not provide enough information to evaluate the failure rates corresponding to the product mission life. Therefore it is not possible to suitably verify if the warranty prediction model (5.7) is entirely accurate beyond warranty period. Therefore the best way to link this model with reliability at mission life is to map the projected numbers with the target reliabilities demonstrated during product validation. In order to tie the two models at the service life of a product, t_L the correction factor Q_{Corr}, can be introduced

$$R_{Forecast}(t_L) = Q_{Corr} R_0(t_L)$$

(5.15)

Where Q_{Corr} = Correction factor (random variable)

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$R_0(t_L)$ = a demonstrated reliability according to equation (5.16)

$$R_0(t_L) = (1 - C)^{\frac{1}{N}}$$

<div align="right">(5.16)</div>

Where C = confidence level

N = the number of test samples (see Appendix A for more details)

Therefore substituting equation (5.7) into (5.15) will equate the predicted and the demonstrated reliabilities at the time of a service life t_L via correction factor Q_{Corr}:

$$e^{-\left(1 + \frac{\beta(t_L - t_S)}{t_S}\right)\left(\frac{t_S}{\eta}\right)^{\beta}} = Q_{Corr} R_0(t_L)$$

<div align="right">(5.17)</div>

Since that correction factor Q_{Corr} is also a random variable, it will be modeled using Monte Carlo simulation. Solving equation (5.17) for η gives

$$\eta = \frac{t_S}{\left[\dfrac{-\ln[Q_{Corr} R_0(t_L)]}{1 + \dfrac{\beta(t_L - t_S)}{t_S}}\right]^{\frac{1}{\beta}}}$$

<div align="right">(5.18)</div>

Equation (5.18) links the scale parameter η of the warranty distribution model with the validation target reliability R_0. It has been noticed from the warranty data analysis that η fluctuates significantly more than the shape parameter β. The shape of the warranty

distribution remains reasonably consistent within the same product line, where the scale parameter η is more volatile due to the fact that it directly linked with the expected life of the failed part. Parameter Q_{Corr} in equation (5.18) will be used for mapping η with R_0. As a result Q_{Corr} will be generated as one of the random inputs for Monte Carlo simulation.

5.4. Conclusions

This chapter presented an extended overview of the automotive warranties with an emphasis on vehicle electronics. It covered the analysis of the warranty/service cost part of the LCC (the descending curve in Figure 2.1). The chapter focuses on the introduction of a new warranty prediction model, two-dimensional aspect of warranties, warranty reporting formats, warranty cost calculations, and the mathematical links of future warranties with certain aspects of product validation programs. This chapter concluded the analysis and description of all product validation cost inputs (Chapters 3, 4, 5) comprising the total LCC value. The next two chapters (6 and 7) will focus on combining together the inputs presented in the previous chapters in order to model the total LCC and consequently minimize its value.

6. Stochastic Simulation of Life Cycle Cost. Case Study and Results.

This chapter will cover the methodology of the life cycle cost (LCC) simulation, inputs and outputs of the model, a case study for a typical automotive electronics validation program, results, and associated uncertainty analysis.

6.1. Stochastic Simulation Methods

Among the variety of stochastic simulation techniques [Nelson (1995)] the most commonly used are Monte Carlo Simulation (MCS) [Craney (2003)], Response Surface Methodology (RSM) [Myers and Montgomery (2002)], and Discrete Probability Tree [Morgan and Henrion (1992)]. In this dissertation the preference will be given to Monte Carlo simulation due to its robustness and wide acceptance in the engineering community. Monte Carlo simulation, though sometimes slow and arduous, has proven its robustness and ability to deal with a wide variety of uncertainty types and values caused by mathematical and real life engineering models. For the LCC minimization methodology presented in this dissertation more efficient solutions and ways to accommodate the uncertainties of this problem may exist, however the target of the work in this dissertation is to demonstrate that a solution to the engineering problem can be obtained as well as the value of that solution. Finding the most efficient implementation of the solution is left to future work.

6.2.   Monte Carlo Simulation

There are numerous books and other sources written on Monte Carlo simulation, see for example [Nelson (1995); Craney (2003); Morgan and Henrion (1992), etc.] Monte Carlo simulation requires that the key inputs be assigned a probability distribution that characterizes the expected variability in the parameters. Then, random values from these distributions are selected and used in the LCC modeling to arrive at a final cost [Brennan (1994)]. Monte Carlo simulation is a very practical method and in most cases produces the 'true' output distribution. On the downside, the Monte Carlo method is a 'brut-force' type of simulation that can sometimes be computationally intensive. However all the computations for the model presented in this dissertation could be completed within 30 minutes or less, making Monte Carlo approaches practical for the types of applications presented herein.

  6.2.1.   General Simulation Info

The stochastic simulation and optimization process presented in this dissertation consists of two distinct analysis steps:

Step 1. Deterministic analysis of LCC intended to find the combinations of the test parameters R (reliability), C (confidence level), and L (number of test service lives) that deliver the lowest LCC value. This step targets finding the lowest dollar value for LCC based purely on the mean values of each distribution. This step also helps to narrow the search for the optimal values of these test parameters R, C, and L to avoid extensive

calculations. However if the computation resources are not limited, one can run a stochastic simulation for each set of the input variables in order to enhance the search for the optimal set of *R*, *C*, and *L*. For more detail see the case study Section 6.3.

Step 2. Monte Carlo simulation for the optimal combinations selected during Step 1 to estimate the uncertainties associated with the LCC solution. It is important to note that the deterministic value of LCC is based on the mean value of each distribution and, in general, it will not coincide with the mean of the output distribution obtained with Monte Carlo simulation. This is caused by the fact that most of the input distributions are not generally symmetrical and that the problem is non-linear.

There are a variety of commercially available software packages designed to perform Monte Carlo analysis. [Morgan and Henrion (1992)] emphasize the importance of the choice of the appropriate uncertainty propagation software. Availability, ease-of-use, and flexibility are mentioned as the top criteria for the software choice. All the Monte Carlo simulations for this dissertation were performed with @Risk 4.5, the uncertainty analysis and simulation add-in for Microsoft Excel [Palisade Corporation (2002)]. The simulation model was prepared in Microsoft Excel. For performing the uncertainty analysis on the LCC by itself, random inputs were generated from the input uncertainty distributions using the @Risk Latin Hypercube (LHC) stratified sampling technique [Palisade Corporation (2002)]. This technique is more efficient than random sampling in that it achieves a given level of precision with a smaller size sample. LHC can introduce slight bias in the estimate of moments, but in practice the bias is negligible [Morgan and

Henrion (1992)]. The dependencies between random variables (correlated inputs) were also considered in the simulations (details appear in Section 6.3.1.)

6.2.2.   Block Diagram and the Equations

The LCC simulation block diagram is presented in Figure 6.1.
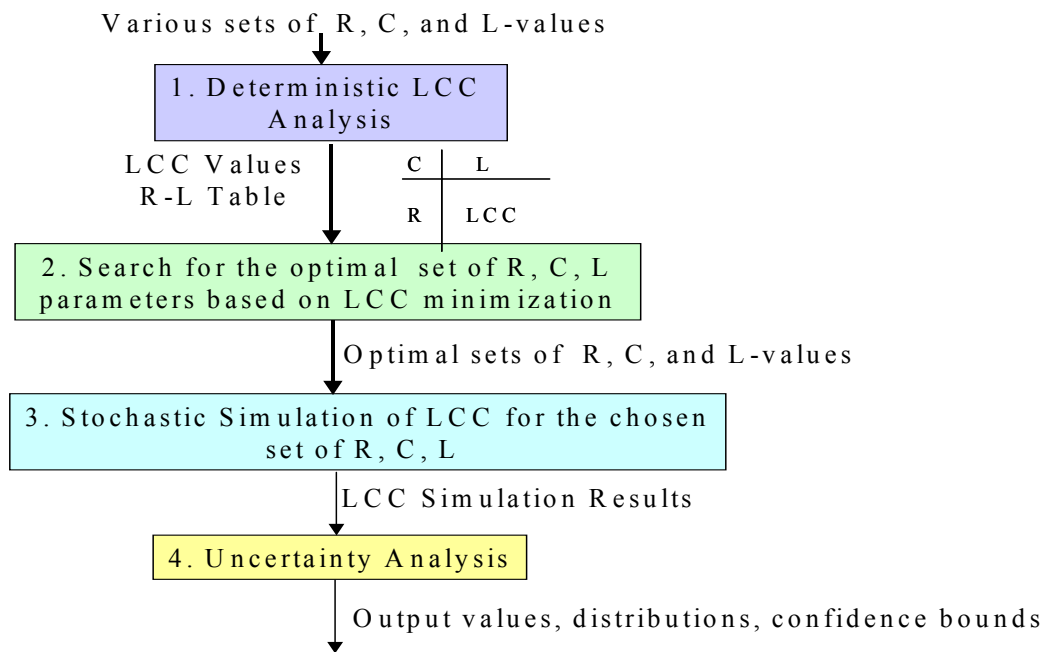
Figure 6.1. LCC simulation block diagram

The diagram shows the four major steps in obtaining the uncertainty analysis solution for the desired LCC value. Step 1 from Section 6.2.1 combines together blocks 1 and 2 and Step 2 combines blocks 3 and 4 in Figure 6.1. Both the deterministic analysis (Block 1) and stochastic simulation (Block 3) use virtually the same set of equations. The main difference between the two blocks is that the first calculation is based on the means of the distribution inputs and the second is the actual Monte Carlo simulation. The key

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calculations for the product validation portion include the following equations that will not be replicated here entirely: total product validation cost equation (3.8), maintenance cost equation (3.7), expected equipment failures equation (3.9), and the dependency between test mission lives and demonstrated reliability test lives (3.3). The key equations for warranty cost include the sum of the claims cost (2.6), the forecasted warranty claims equations (5.7), (5.10), (5.18) and NPV of the total warranty cost equation (5.14). However, in the simulation procedure the warranty cost W_C in equation (5.14) is represented by the sum (6.1) in order to account for variations in repair costs:

$$W_C = \sum_{j=1}^{n_f} \alpha_{Wj}$$

(6.1)

Where n_f is the number of failed units, which is calculated according to equation (5.13).

6.2.3.   Uncertainty Analysis

As any result of stochastic simulation, the LCC probabilistic solution of the problem will have a certain level of uncertainty inherent within it. The task of uncertainty analysis is to determine the uncertainty features of the system model itself and the stochastic variables involved [Morgan and Henrion (1992)]. There are a variety of types and sources of uncertainty. The list of uncertainty categories according to [Tung (1996)] includes:

- Natural uncertainties

    - Inherent randomness of natural processes (e.g., unforeseen failures or expenses)

- Model uncertainties

  - Reflects inability of a model or design technique to represent precisely the system's true behavior

- Parameter uncertainties

  - Resulting from the inability to quantify accurately the model inputs and parameters (associated with distribution parameters and best fit functions)

- Data uncertainties

  - Measurement errors
  - Inconsistency and non-homogeneity of data
  - Data handling and transcription errors
  - Inadequate representation of data sample due to time and space limitations

- Operational uncertainties

  - Factors including construction, manufacture, deterioration, maintenance, and human interfaces.
  - Knowledge of the environment, how the system will operate in this environment

- Computational uncertainties

  - Include rounding errors, convergence, etc.

In practice, it is often difficult to separate different types of uncertainties [Hall and Strutt (2003)], therefore they are often treated together as a combined effect of contributing uncertainties of the LCC solution.

According to [Schjaer-Jacobsen (2002)] the best representation of uncertainty is the one that is able to handle all relevant information available. Therefore, based on the available

data, the most straightforward and complete description of uncertainty is a PDF. It provides all the necessary information for the user to be able to study the output, to determine all possible confidence intervals, and if need be, to use this output as an input for another stochastic simulation. If a PDF cannot be obtained, an alternative measure of uncertainty can be expressed in terms of a probability domain, such as the confidence interval. According to [Nelson (1995)] a confidence interval is a numerical interval that captures the quantity subject to uncertainty with a specific probabilistic confidence. In automotive electronics the statistical inputs can be obtained for the existing data, therefore in this dissertation the uncertainty will be expressed in terms of a PDF. Since the output results for this model are represented by skewed distributions (see Section 6.3.2), 80% confidence intervals will most often be used for the numerical representation of the results. 80% confidence intervals offer a reasonable spread of the output range while providing enough confidence to make an engineering or business decision.

6.3.   Case Study

This automotive electronics case study illustrates the methodology discussed in the previous chapters and demonstrates the steps required to perform the analysis and optimization. Since the proposed model requires a large number of calculation steps and many input variables, it is easier to explain this methodology by working an example in a step-by-step manner. This case study is an example of the product validation practice typical for an automotive electronics supplier. It contains many similarities to the operation of the product validation and quality departments of the Electronics & Safety division of Delphi Corporation, however with certain modifications due to the issues of

propriety. The case study will consider the same automotive radio with CD player discussed in the Section 5.2.4 with the total production volume of 500,000 units sold to the automotive OEM for $150 each. The remaining cost variables will be presented in Section 6.3.1.

6.3.1.　　　Inputs and Outputs

Table 6.1. Model inputs for the cost of product validation

Input	Symbol, units	Value
Confidence level (search variable)	C	0.9
Target Reliability (search variable)	R_0	0.97
Number of lives tested (search variable)	L	2
Test sample size adjusted for L	N_l, units	19
Depreciation of test chamber	D, $/year	$25,000
Additional equipment expenses	Y, $/year	$10,000
Hourly labor rate for equipment maintenance	φ_{repair}, $/hr	$35.00
Hourly labor rate for product testing	φ_T, $/hr	$30.00
Cost: spare parts (random)	α_{parts}, $/year /chamber	$836.21
Time of maintenance repair (random)	t_{repair}, hr	2.30
Maintenance MTBF, χ^2-distr (random)	days	313.6
Number of PM	N_{PM}/year /chamber	2
Cost of each PM	α_{PM}, $/year /chamber	$2,000
Maintenance cost	M, $/year /chamber	$5,067
Test duration (one mission life)	$t_{one-life}$, hr	800
Chamber capacity, units	K, units	25
Cost of producing one test sample	α_p, $/unit	$2,000
Cost of equipping one test sample	α_e, $/unit	$450
Cost of monitoring one test sample	α_m, $/unit	$500

The model inputs include the mix of probabilistic and deterministic inputs presented in Table 6.1 for the cost of product validation and Table 6.2 for the cost of warranty and service.

Please note that the random input variables simulated as probability distributions are marked "(random)" in the first column of each table titled "Input". Also C, R, and L are marked as "(search variable)" since they are used for the direct search in the process of LCC optimization.

According to the LCC diagram in Figure 2.1, the total dependability-related cost function is a result of the sum of the ascending and descending curves. The ascending curve represents all the cost inputs of product validation activities and the descending curve has all the cost inputs related to future failures covered by product warranty.
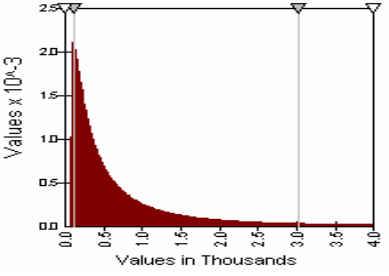
Table 6.2. Model inputs for the cost of warranty and service
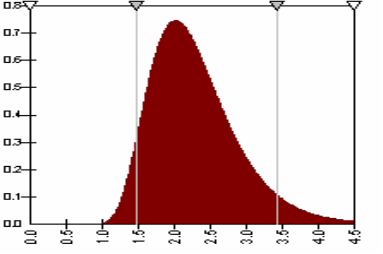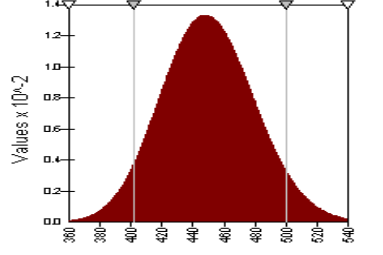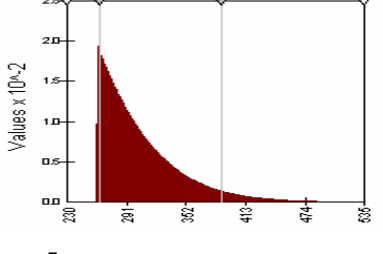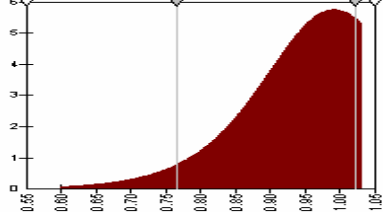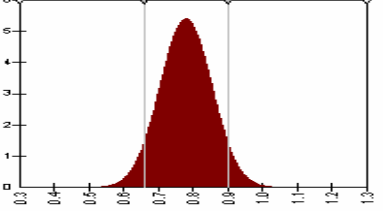
Input	Symbol	Value
Production volume	n, units	500,000
Service life	t_L, years	10
Failure rate change point (random)	t_S, days	305.4
Correlation factor: Warranty to Reliability (random)	Q_{Corr}	0.9
Shape parameter (random)	β	0.780
Scale parameter	η, days	101,953
Percent of NTF	NTF, %	0
Cost of one warranty claim (random)	α_W, \$/unit	\$504.47
Warranty period	T_0, days	1095
Warranty covered percent failed,	$[1-R(T_0)][1-\Phi(T_0)]$	0.120%
Number of failed parts within T_0 time	$n_f = n[1-R(T_0)]$, units	601
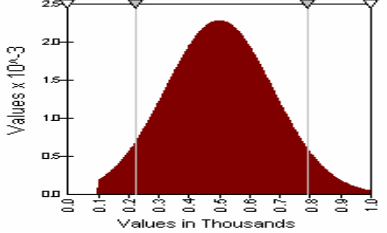
One of the inputs requiring special attentions is the percent NTF (Not Trouble Found), Table 6.2. Depending on how the NTFs are viewed within the organization and by the OEM customer they may or may not be included into the LCC analysis (in this case study they were initially set to 0%). In this dissertation NTFs are discussed in detail in Section 6.3.7.

The random inputs used for this model were obtained from the analysis of the existing automotive data for each of the inputs by finding the best analytical distribution fitting the original data. Goodness of fit of the existing data was used to determine the distribution that best describes the analyzed data. The obtained distributions for each random input are presented analytically and graphically in Table 6.3.

Table 6.3. Random inputs and their distributions used in Monte Carlo simulation

Cost of the yearly maintenance spare parts per chamber. α_{parts}, \$/year Equation (3.7)	Lognormal distribution ($\mu = 843.71$, $\sigma = 1767.8$, Shift +30)	

Time of maintenance repair. t_{repair}, hours Equation (3.7)	Lognormal distribution ($\mu = 1.8$, $\sigma = 0.616$, Shift +0.5)	
Maintenance MTBF, days Equation (3.7)	Chi-square distribution $\dfrac{141{,}120}{\chi^2_{(450)}}$, per equation (3.9)	
Time change point, t_S, days Equation (5.7)	Weibull distribution ($\beta = 1.63$, $\eta = 117$, $\gamma=60$) Truncated [200; 500] days	
Warranty-Reliability Correlation factor, Q_{Corr} Equation (5.18)	Logistic distribution ($\alpha = 0.9916$, $\beta = 0.0682$) Truncated [0.6; $1/R_0$]	
Shape parameter for the pre-t_S portion of Weibull distribution, β Equation (5.7)	Normal distribution ($\mu = 0.78$, $\sigma = 0.074$) Truncated: [0.4; 2.0]	

Cost of a single warranty repair, α_W, \$/unit Equation (2.6)	Normal distribution ($\mu = \$499$, $\sigma = \$177$) Truncated: [\$50; \$3000]	

Consideration of the dependencies between random variables is an important part of any Monte Carlo simulation. Not accounting for the correlation between the random inputs can lead to the wrong estimates of the output variances; therefore correlated inputs should be modeled as such. The most common way to express the correlation between the random inputs is through the correlation coefficient r. The correlation coefficient may vary between 1 (perfect correlation) and –1 (perfect negative correlation). $r = 0$ would mean non-correlated, fully independent variables [Hines and Montgomery (1990)]. In order to address this issue, a correlation analysis was performed on the original data used for determining the input distributions.

The analysis indicates that the cost of the equipment spare parts and the duration of their corrective maintenance are correlated. Therefore they were simulated as correlated inputs with the correlation factor $r = +0.4$ found from the analysis of the data presented in Figure 6.2. Similarly, the data analysis showed some positive correlation between β and t_S. Based on the available data it was modeled with the correlation factor $r = +0.2$. No correlation was found between other model inputs.
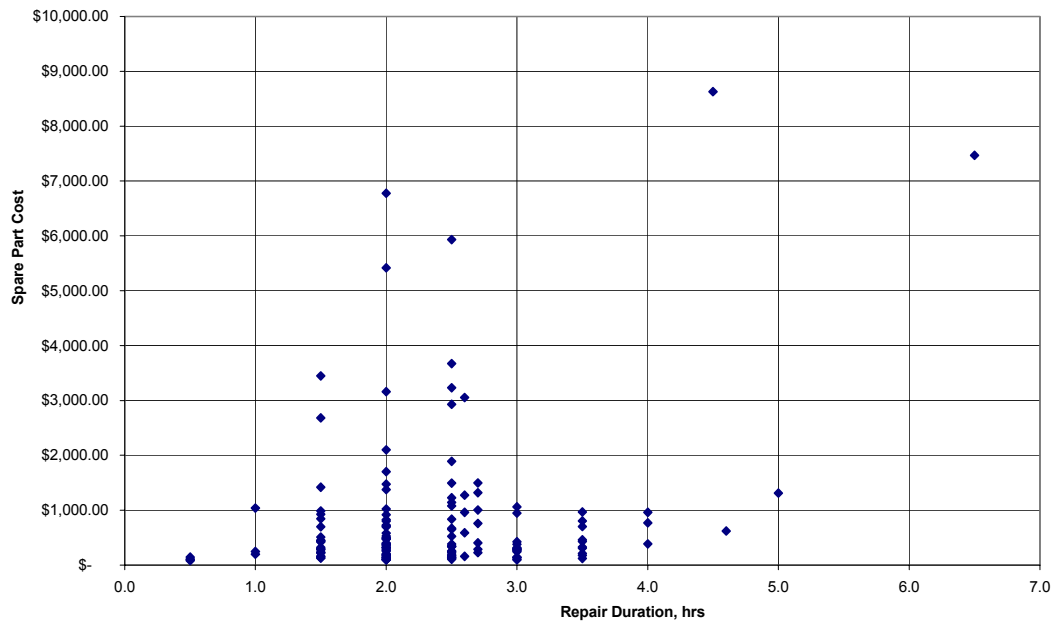
Figure 6.2. Correlated inputs: repair duration and spare parts cost

The correlation of input distributions in @Risk is based on the rank order correlations [Morgan and Henrion (1992)]. This method is based on rearranging the random numbers prior to simulation to achieve the required level of correlation. This type of correlation is known as a 'distribution-free' approach because any distribution types may be correlated. Although the samples drawn for the two distributions are correlated, the integrity of the original distribution is maintained [Palisade Corporation (2002)]. The resulting samples for each distribution reflect the input distribution functions from which they are drawn.

6.3.2. Results of the Simulation

Each simulation run was conducted with 10,000 iterations sometimes referred as samples.[10] The choice of 10,000 iterations was based first on the guidelines presented in [Garvey (1999)] and second on the convergence characteristics of the simulation run. The process demonstrated 3% convergence with 1,000 iterations; therefore 10,000 appeared to be sufficient. More on model convergence will be discussed in Section 6.3.5.

Table 6.4. LCC values for deterministic analysis

| R_0	L, number of test mission lives										
	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
0.8	1,172,425	1,169,291	1,169,107	1,168,922	1,168,738	1,168,554	1,171,320	1,171,136	1,173,902	1,173,718	1,176,484
0.85	820,934	814,850	811,716	811,531	811,347	811,163	810,979	810,795	813,561	813,377	816,143
0.9	596,117	590,033	583,949	577,865	577,681	574,546	574,362	574,178	573,994	576,760	576,576
0.91	568,030	558,996	549,962	546,828	543,694	540,559	540,375	540,191	540,007	539,823	542,589
0.92	546,875	531,232	525,148	519,064	515,930	512,795	509,661	509,477	509,293	509,109	508,925
0.93	528,751	517,133	504,074	495,040	491,906	488,772	485,638	482,504	482,320	482,136	481,951
0.94	519,801	502,283	490,664	480,189	471,155	468,021	464,887	461,753	461,569	461,385	461,201
0.95	516,605	499,087	484,519	472,901	459,110	453,026	449,892	446,758	443,624	443,439	443,255
0.96	534,157	504,130	486,611	472,043	460,425	454,707	443,134	437,050	433,916	430,782	430,598
0.97	575,164	536,652	510,650	485,791	471,223	459,605	450,936	445,218	432,914	426,830	423,696
0.98	674,091	618,245	577,859	541,566	518,514	498,411	478,354	466,736	458,068	449,400	443,682
0.99	1,020,092	900,101	812,537	741,942	685,365	645,344	608,319	582,683	553,410	536,258	522,056
0.995	1,724,978	1,486,255	1,304,337	1,160,380	1,052,533	964,603	890,692	827,848	776,073	739,369	698,296
0.996	2,077,462	1,777,178	1,549,157	1,373,848	1,235,425	1,124,628	1,027,893	950,666	894,043	839,318	789,762
0.997	2,671,732	2,273,000	1,969,400	1,732,529	1,543,613	1,390,441	1,265,304	1,165,575	1,079,864	1,008,170	944,595
0.998	3,861,674	3,259,072	2,796,606	2,443,588	2,158,443	1,931,908	1,741,529	1,584,674	1,459,493	1,347,277	1,255,665
0.999	7,424,507	6,219,487	5,296,180	4,580,037	4,009,930	3,548,196	3,170,525	2,860,316	2,600,966	2,377,771	2,192,148
0.9999	71,585,675	59,496,903	50,277,385	43,085,320	37,365,786	32,744,545	28,949,191	25,798,630	23,153,869	20,911,337	18,989,556

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[10] In this dissertation the term "iteration" rather than "sample" will be used when referring to Monte Carlo runs in order to avoid confusion with test samples.

The results of the deterministic LCC analysis (see Figure 6.1) are presented in Table 6.4. The minimum value of LCC was achieved at $C = 90\%$, $R = 0.97$, $L = 2.0$ and equal to $423,696 (value in bold).

The 3-D chart corresponding to the results in Table 6.4 is presented in Figure 6.3.
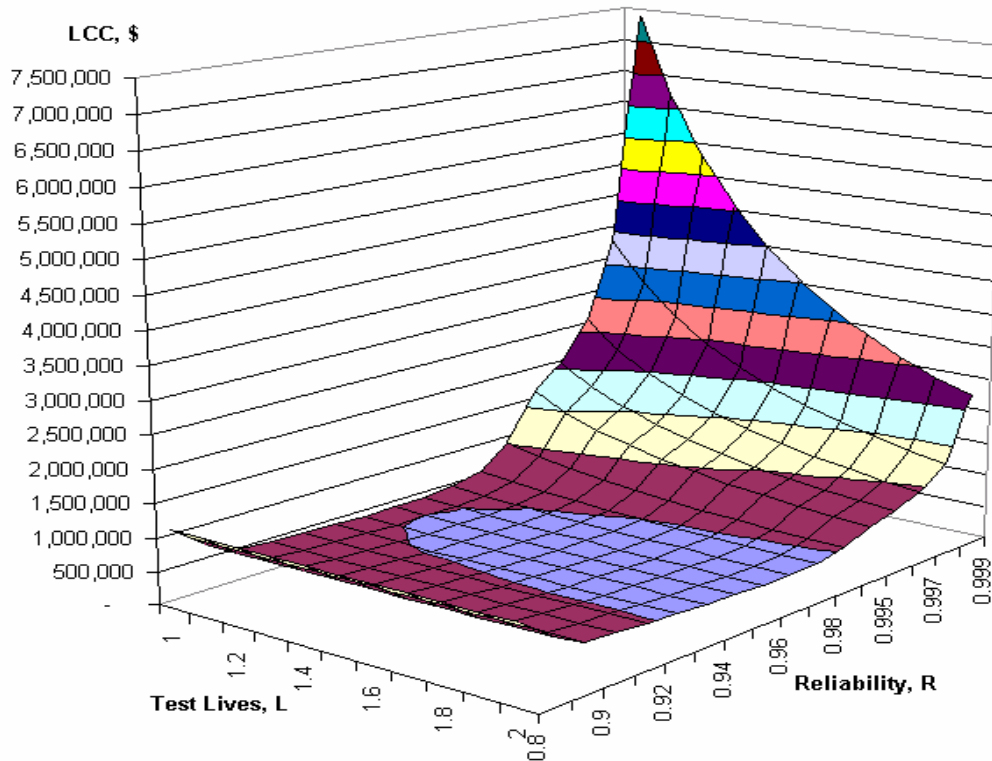
Figure 6.3. 3-D Plot of LCC Deterministic analysis with $C = 90\%$

The 2-D slices of the Figure 6.3 plot are presented in Figure 6.4 for a standard bogey testing (1× mission life, $L = 1$) and an extended bogey testing (2× mission lives, $L = 2$).
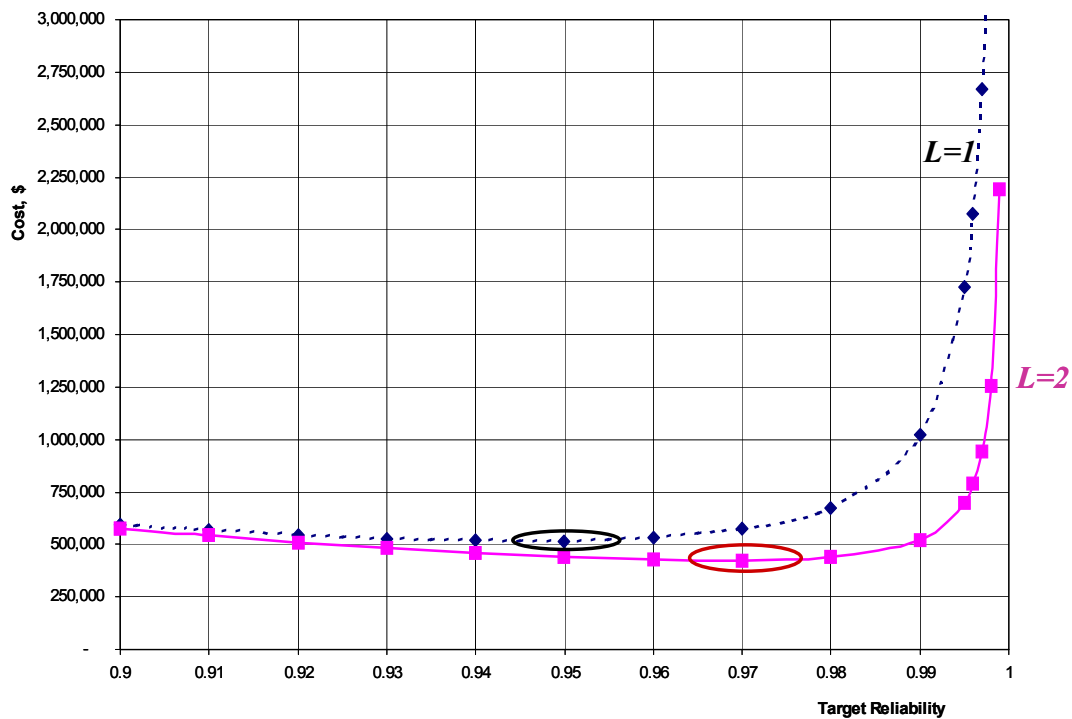
Figure 6.4. LCC comparison charts for $L = 1$ and $L = 2$. Lowest cost data points are circled

An increase in L reduces the LCC value due to the fact that in this particular example the hourly cost of testing is lower than the cost of additional test units. Therefore it is more cost effective to test fewer test samples for the longer period of time. The optimal reliabilities R_0 are circled on the plot and situated in the ranges of R_0 [0.95; 0.98].

After the optimal input set is found per diagram Figure 6.1-step 2 (in this case study $C = 90\%$, $R = 0.97$, $L = 2.0$, Figure 6.4) the stochastic simulation is run with these three variables (Figure 6.1, step 3) and the output results in the form of the histogram are presented in Figure 6.5.
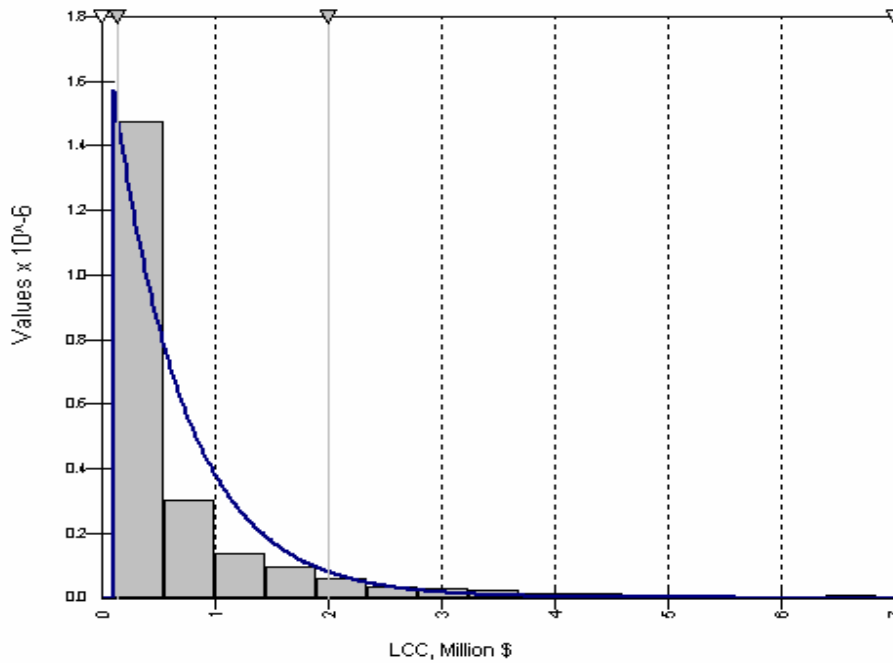
Figure 6.5. LCC output distribution histogram and the best fit distribution.

The histogram Figure 6.5 can be statistically best approximated by the 3-Parameter Weibull distribution (dark curve over the histogram) with $\beta = 0.574$ (shape parameter), $\eta = \$415,750$ (scale parameter), and $\gamma = \$110,510$ (location parameter). Other close best fit choices included lognormal and exponential statistical distributions.

6.3.3. Results of the Uncertainty Analysis

Any predictive model can be significantly affected by uncertainty propagation. The existence of uncertainty implies the existence of a range of possible solutions [Garvey (1999)]. In order to recreate the confidence bounds for the whole LCC optimization

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curve, fifteen additional simulation runs were conducted for target reliability ranging from $R = 0.8$ to 0.999. For each run the multiple percentile LCC solutions were obtained ranging from 0% to 100% with 5% increments; and three data points 25%-tile, median (50%-tile), and 75%-tile were plotted for each solution establishing the effective 50% double-sided confidence bounds presented in Figure 6.6. 50% confidence bounds provide sufficient engineering data range consistent with a traditional Box and Whisker diagram [Hines and Montgomery (1990)] focusing on the two middle quartiles of the distribution (50% double-sided). It was also within the scope of this study to observe if the various percentile values of the solution follow the shape of the deterministic solution; and see if they yield the same optimization parameters R, C, and L.
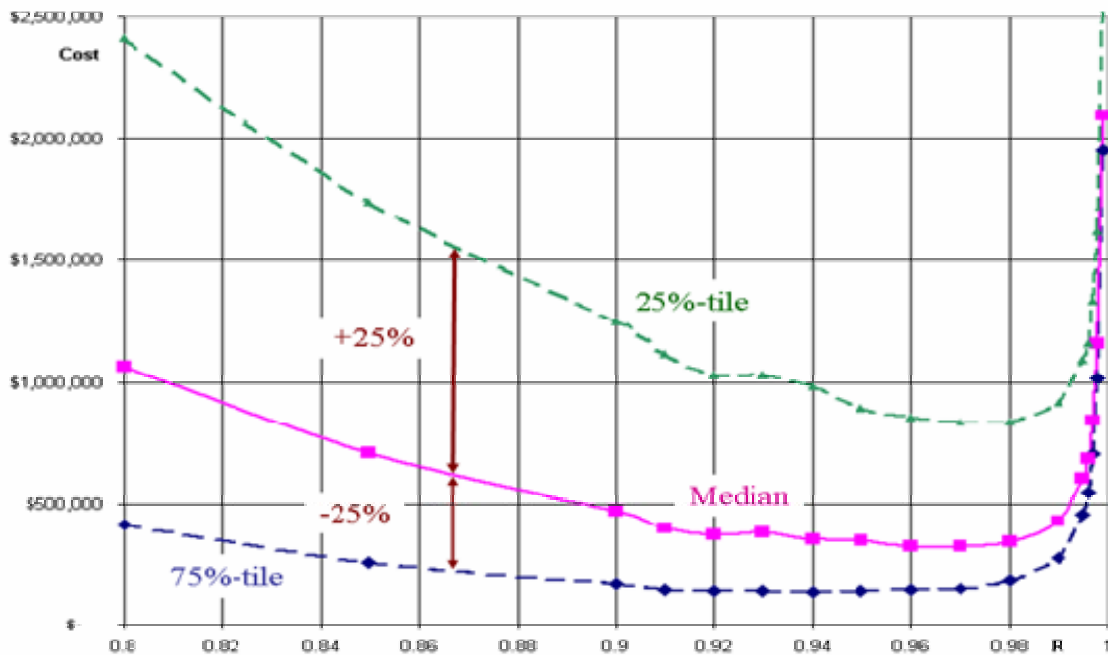
Figure 6.6. Results of LCC uncertainty analysis

The LCC chart in Figure 6.6 shows the 50% confidence bounds of the solution. The median value from Monte Carlo simulation is matching the optimal value of target

reliability R_0 = 0.97. It was also noticed that the confidence bounds are becoming narrower along the X-axis showing that the uncertainty of the solution is decreasing with the increasing reliability R_0.

6.3.4.   Sensitivity Analysis

Sensitivity analysis is an important part of any stochastic simulation modeling process. It identifies the inputs, which are significant in determining output variables values. With this analysis, correlation coefficients are calculated between the output values and each set of sampled input values. For our case study the results of sensitivity analysis are displayed in Figure 6.7 as a 'Tornado' type chart, with longer bars at the top representing the most significant input variables. As follows from the sensitivity chart Figure 6.7, the output solution is most sensitive to Q_{Corr}, which links the expected warranty with demonstrated reliability, β - the Weibull parameter of warranty prediction, and α_W, the cost of a warranty repair.
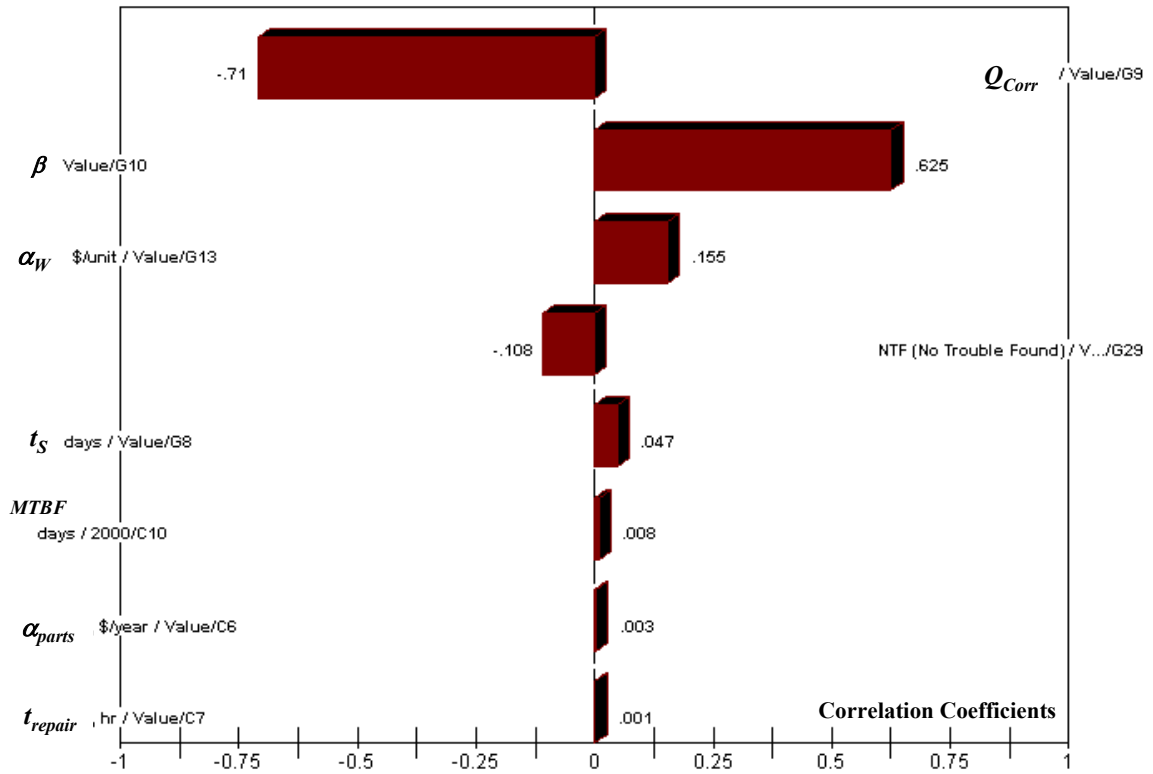
Figure 6.7. Sensitivity analysis results in form of the Tornado graph

The significant influence of these top three parameters can be explained by the fact that expected warranty cost is a major contributor to the LCC, and these three parameters are the modeling parameters of the future warranty claims, which has a large effect on the LCC output and therefore on the whole mathematical model.

6.3.5.   Additional Checks on the Stochastic Model

Convergence Monitoring

Convergence monitoring was utilized to evaluate the stability of the output distributions during a simulation. As more iterations are run, output distributions become more stable as the statistics describing each distribution changes less with each additional iteration. The statistics monitored on each output distribution are:

1) The average percent change in percentile values 0% to 100% in 5% increments,

2) The mean

3) The standard deviation

The above statistics are calculated on the data generated for each output cell at regular intervals throughout the simulation. The 10,000 iterations run in the case study was sufficient to achieve 2.5% stability on the statistics generated on the LCC and other monitored outputs. In fact $< 2.5\%$ precision error on all percentile values was noticeable between 1,000 and 3,000 iterations in the majority of the simulation runs, much earlier than 10,000 iterations conducted in this study.

The Effect of the Production Volume

Warranty cost is roughly proportional to the production volume, therefore the value of optimal target reliability will be affected by n, the number of units manufactured and sold to the OEM customer. One of the model checks includes the study of the relationship between n, and R_0 including the model behavior near the extremes of $n = 0$ and $n = \infty$. The optimal target reliability is expected to increase as the production volume increases due to the fact that LCC is driven up by the warranty cost. Several additional simulation runs were conducted for the production volume ranging from 1,000 units to the unrealistically high volume of 1 billion. The results of those simulations are presented in Figure 6.8. As can be seen from that graph, the optimal target reliability R_0 increases from 0.8 to 0.999 with the rising production volume.
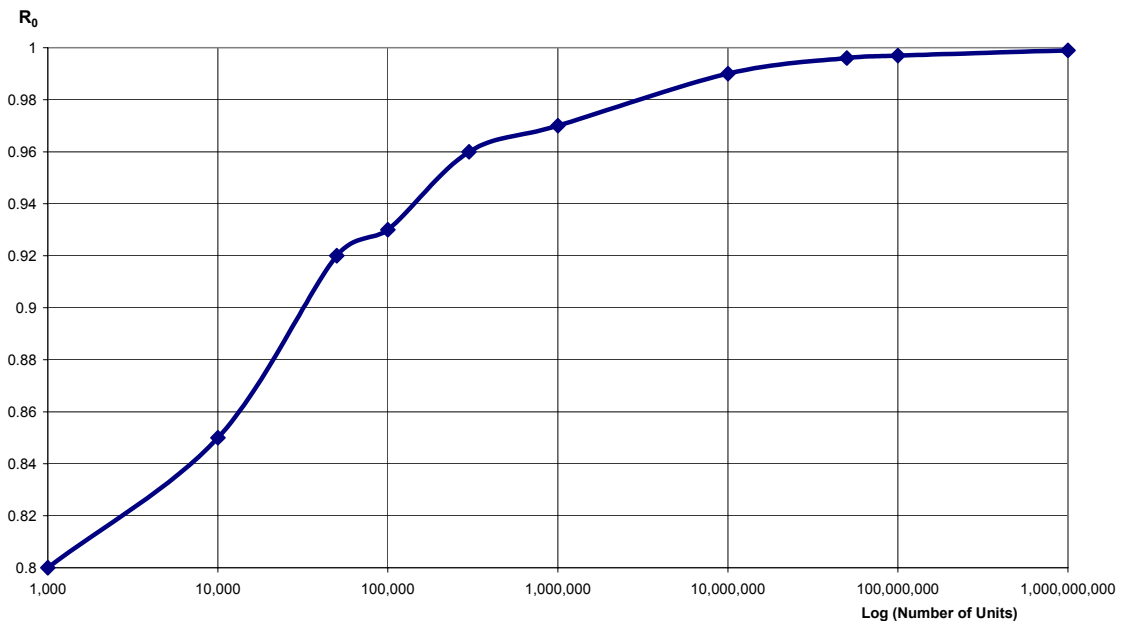
Figure 6.8. Dependency of the optimal target reliability R_0 on production volume n

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The model clearly shows the expected behavior at the extreme values of production volume, where $n \to 0$ would cause $R_0 \to 0$ and $n \to \infty$ would make $R_0 \to 1$.

6.3.6.   Application Limits

Like most of the mathematical models, this simulation has its application limits. There are two reasons why this model's inputs (and therefore outputs) have well defined limitations. The first reason is caused by the fact that certain model assumptions cease working when the inputs exceed their acceptable levels, i.e., the model assumptions become violated outside the set input ranges. This happens when the model for example loses its linearity or its imbedded mathematical equations no longer work for the inputs exceeding predetermined levels. The second reason is based on the window of feasibility, where the model restrictions are based on the practical considerations, such as the real world application boundaries.

The input boundaries listed below belong to one of those two categories or to the combination of both.

- Production volume for automotive electronics is limited to n [20,000; 40 million] for the practical reason. The maximum possible volume of the vehicles sold defines the upper limit, where the low limit is typically the smallest volume acceptable for an automotive supplier. The model will however work correctly outside these limits, (see the chart R_0 versus n, Figure 6.8).

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- The number of test mission lives in these applications is limited to L [1.0; 2.0]. The lower limit is based on the fact that automotive customers usually will not allow testing that is shorter than the equivalent of one mission life. On the other extreme $L > 2$ would constitute a rare event in the automotive business and could only be prompted by the availability of excess capacity of the test lab, allowing long test time durations without undue delays on other products waiting in the test jobs pipeline. Even though reliability specifications do not set strict limits on the value of L, see for example [GMW 3172 (2004)], with today's tight delivery schedules it would be difficult to justify the excessively long test procedures. In this dissertation the search of minimum LCC is conducted under the assumption of full utilization of the facilities, i.e., the test facilities do not stay idle. The chances of this assumption being violated increase as L becomes larger than 2. However it is important to note that there is a possibility of special cases, where lax delivery schedules and additional cost benefits may prompt longer test times with $L > 2$. Some of these special cases, which go against practical wisdom may suggest certain economical advantages of the extended test time - these will be discussed in Chapter 7. An additional reason to limit the value of L is mentioned in Section 3.4.2 and related to the *zero-failure* assumption for the derivation of the parametric binomial equation (3.3). With the increasing value of L the probability of failure during the extended life test will also be increasing.

- Test sample sizes in this study are limited to N [3; 100]. Sample size below this range will not provide statistically significant information about the product and

that above this range will be too costly and impractical to implement. The practical sample sizes will probably be well below the upper limit of 100 listed here.

- The target reliability should remain above 90%, i.e., R_0 [0.9; 1.0) due to the customer's demands for higher quality and reliability. For example, $R_0 = 0.8$ would not be acceptable to an automotive OEM. In addition, low R may violate the model assumptions by adding certain un-quantified though very real costs, such as tarnishing of a brand name, potential law suits, recalls, costs of additional marketing efforts associated with poor quality, etc. In order to be able to eliminate these cost items from the influence diagram Figure 2.2 reliability demonstration target R_0 must stay above 90%.

- Confidence limits are typically restricted between C [50%; 95%], although $C = 90\%$ seem to be dominant in automotive reliability specifications.

The input ranges above should not be considered as explicitly 'rigid', since it is not always possible to determine the exact boundary value where the model loses its validity. Instead, those limits should be considered as 'soft boundaries' and used as guidelines for the model's practical applications. Also it is not the intention of this work to embrace these limits and therefore to constrain the solution, but rather to acknowledge their existence in order to obtain a better understanding of the modeling process. The limits discussed above are generally associated with automotive industry and could be modified or even eliminated when this model is applied to other industries.

6.3.7.   Effect of the Unverified Warranty Claims
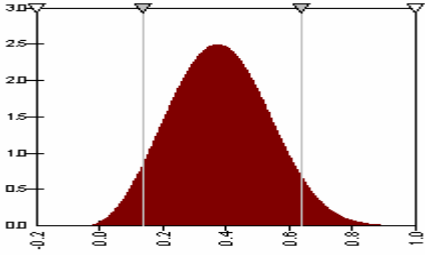
Warranty claims classified as NTF (No Trouble Found) make up sizable part of reported automotive electronics warranty problems and have been previously discussed in the literature [Kaminskiy and Krivtsov (1997); Salzman and Liddy (1996); Williams *et al.* (1998)]. Most warranty reporting systems report NTFs separately and often exclude them from the reported numbers of product failures [Thomas *et al.* (2002)]. Some suppliers tend to discount NTFs since they are often caused by the problems outside the part in question, such as failure of other systems communicating with the electronic unit under consideration. However, on the warranty cost side, NTFs are often accounted for the same way as 'true' claims, since OEMs and their suppliers still pay the dealerships for the repair work done on those units including extraction of the 'faulty' unit and the consequent replacement work. Therefore, in the majority of cases, NTF-related claims will be part of the total LCC dollar value and therefore need to be included in the overall LCC model. 'Included' in this context means that all the unconfirmed failures are still considered as failures and therefore NTF = 0% (case study Section 6.3.2). From the LCC and statistics point of view it does not make any difference if the claim is real or 'imaginary', therefore NTFs are included and counted as a part of the warranty expense, making NTF = 0%.

With that said it is important to note here that there is no uniformity in the way OEMs are treating NTF warranty claims, therefore there are cases where NTFs would need to be subtracted from the total pool of warranty claims. Subtracting the NTF values will modify equation (5.13) into

$$n_f(t) = (1 - NTF)\left[1 - R(t)\right]n_{sold}$$

<div style="text-align:right">(6.2)</div>

In these cases the percent NTF should also be simulated in the form of a statistical distribution and included in the model as one of the random inputs. For the purpose of studying the effect of NTF failures, the case study presented in Section 6.3.1 was modified by subtracting the NTF percentages based on numbers obtained for the Radio/CD player used in the case study. Historical percentage of NTF relative to the total number of warranty claims tend to fluctuate and therefore was also modeled as a random variable input. Based on historical NTF data for this type of products it was simulated by the distribution presented in Table 6.5.

Table 6.5. NTF input distribution used in Monte Carlo simulation.

NTF percentage, *NTF, decimal value.* Equation (6.2)	Weibull distribution ($\beta = 3.02$, $\eta = 0.473$, $\gamma = -0.0398$) Truncated $[0; 1]$	

The 2-D results of the simulation are presented here in Figure 6.9 and similar in format to that in Section 6.3.2. Comparing Figure 6.4 and Figure 6.9, it can be seen that the minimum LCC point occurs at a different target reliability value. The numerical differences between the simulation results with and without NTF are presented in Table

6.6. As you can see, the optimal values of R and L have shifted due to the reduced warranty cost caused by the exclusion of NTF warranty claims.

Undoubtedly, NTFs are the important contributors to the model.
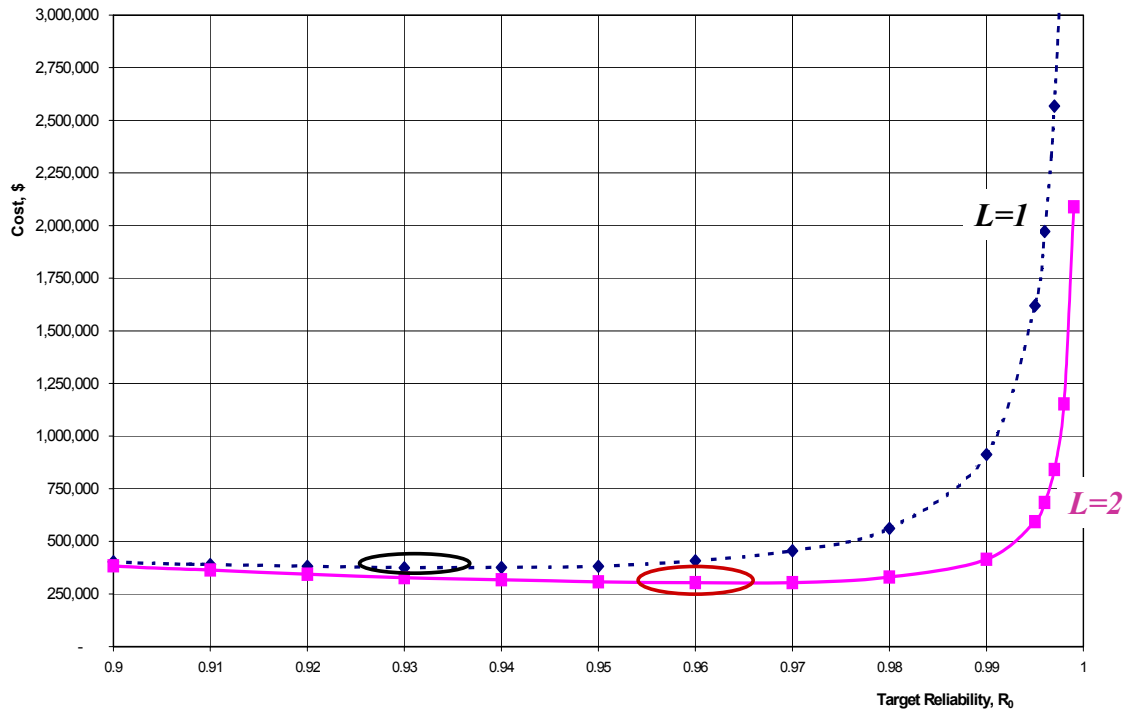
Figure 6.9. LCC comparison charts for the model excluding NTF failures for $L = 1$ and $L = 2$. Lowest cost data points are circled. NTF percent assumed for this modeling is the distribution described in Table 6.5

As can be seen from the sensitivity chart, Figure 6.7, the random NTF input is the fourth most influential input affecting the LCC solution. NTF exclusion is occasionally

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requested when design improvements focused on elimination of NTF are planned and the positive economic impact of these modifications needs to be assessed.

Table 6.6. Target reliability values minimizing LCC: NTF counted vs. NTF subtracted

Test mission lives, L	NTF counted as failures, NTF=0% (Figure 6.4)	NTF subtracted from the failures (Figure 6.9)
$L = 1$	$R_0 = 0.96$	$R_0 = 0.93$
$L = 2$	$R_0 = 0.97$	$R_0 = 0.96$

6.4.   Cost Benefit Analysis

Cost benefit analysis is designed to determine the feasibility of a project or plan by quantifying its costs and benefits. It is not always practical to couple a monetary value to every implementation benefit, however it is always beneficial to be able to justify the project from the standpoint of engineering economics. A high return on investment (ROI) is often a strong argument to move the project implementation forward. According to [Hoisington and Menzer (2004)] quality and reliability professionals should use this measurement to show the impact on the organization of investing money to fix or prevent a problem or improve the process and the expected return. Basic information on calculating ROI can be found in [Short and Welsch (1990)]. Generally, ROI is an amount, expressed in terms of percentage or a ratio, of the profit or loss resulting from a transaction or investment. The application of ROI concepts to engineering projects and process improvements is described in [Westcott (2005)]. In the cases of quality or

reliability improvement, the ROI is the ratio of the sum of the improvement benefits divided by the total cost of the improvement. Even though the ROI is often just a rough estimate, it can be a powerful argument in managerial decision-making.

[Rico (2004)] suggests using a simple set of metrics and models for ROI, as such we will ignore taxes and compounded interest in our simple treatment here. Therefore in the case of LCC improvement, ROI can be represented by equation (6.3).

$$ROI = \frac{LCC_{Before} - LCC_{After}}{Cost\ of\ implementation}$$

(6.3)

Some engineering fields such as risk analysis, civil engineering, and some others use *Benefit-to-Cost Ratio* [Ayyub (2003)] to evaluate the financial benefits of the project. Benefit-to-Cost Ratio (B/C) an can be presented by equation:

$$B/C = \frac{Project\ Benefits}{Project\ Cost}$$

(6.4)

In the cases of process optimization projects similar to that described in this dissertation, equation (6.4) will produce the same cost benefit value as equation (6.3), therefore they can be used interchangeably.

In the case study presented in Section 6.3 the 'conventional' reliability demonstration parameters were $C = 90\%$, $R = 0.90$, $L = 1.0$. According to the deterministic results presented in Table 6.4, the LCC value corresponding to those parameters is $LCC_{Before} =$

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$596,117. For the optimized parameters C = 90%, R = 0.97, L = 2.0, this value is LCC_{After}= $423,696. The engineering expenses for collecting all the required information, compiling the model, and running the simulations can be conservatively estimated at around $15,000. Substituting those values into equation (6.3) will produce ROI exceeding 12× The stochastic simulation of ROI value according to equation (6.3) using simultaneous iterations of two LCC values corresponding to the old and improved sets of *C-R-L* values is shown in Figure 6.10 and can be best represented by the *Extreme Value* distribution with the parameters a = 6.24 and b = 25.11, which produces the 50% confidence interval for ROI [0.1×; 24.6×].
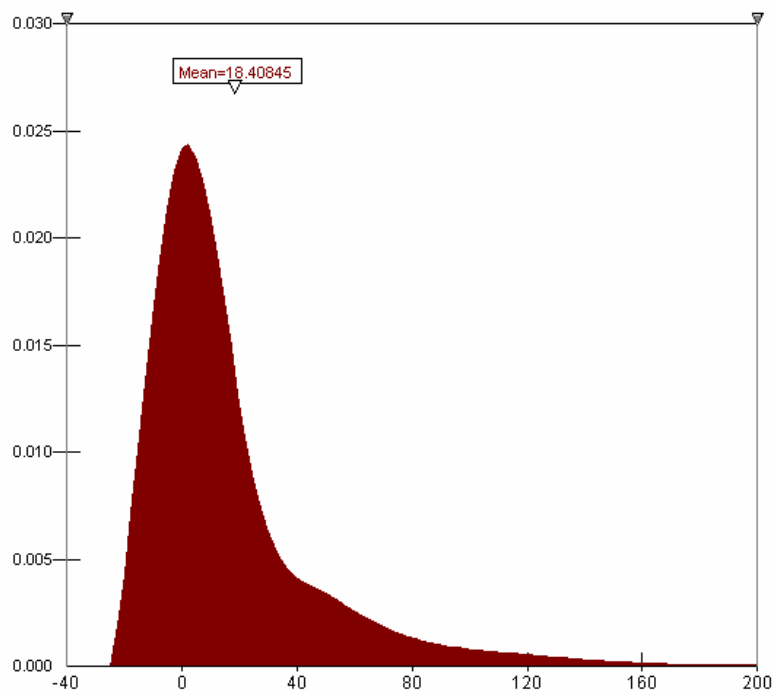
Figure 6.10. ROI simulation results

This magnitude of ROI could be considered common for a project involving process improvement [Rico (2004)]. In the case of LCC stochastic modeling application the ROI can be further increased by improving the process and reducing the expenses requiring

for the model implementation in each particular product line. Other economic analysis criteria, such as internal rate of return (IRR) or payback period [Ayyub (2003)] could also be applied to determine the cost benefits of the LCC improvement project.

6.5.   Case Study Conclusions

The following are the conclusions drawn from the analysis of this model in general and the case study in particular.

- This LCC minimization model has a suitable format for optimization. For the ranges of values of *R, C*, and *L*; their relationship in the LCC model, and the analysis cases considered herein a minima LCC point, which is not an extreme always exists.

- Despite the non-symmetrical nature of most of the distributions used in the model, the optimal solution set of *R, C,* and *L* for deterministic model based on the means of each distribution matches the solution set for the stochastic simulation based on the median of the outputs. However the 10%-tile and 90%-tile solutions are different from the deterministic solution. Therefore the random LCC values retain the same rank as the deterministic LCC values. Therefore the stochastic simulation provides an additional level of insight into the internal dependencies between LCC and the model inputs and enhances the process of cost optimization.

- According to [Chauhan and Bowels (2003)], deterministic estimates, which are derived from the best estimate inputs (as opposed to the parameters of the input

distributions) do not necessarily yield outputs that are equal to the mean or median due to nonlinearities in the process that relate outputs to inputs. Therefore it is very important to use Monte Carlo simulation even for the deterministic analysis. The deterministic analysis based on mean or median of the distribution provides a more reliable minimization scheme than the same analysis based on the input best estimates.

- The increase in test duration (higher L-value) favorably affects the LCC value: the higher the L the lower the total cost. It can be explained that the cost of each additional hour of testing is less that the cost associated with adding more test samples. However assuming that the test laboratory is running close to its full capacity and all test jobs should be completed within the timeframe dictated by the normal production schedule the L has to be constrained within the reasonable limits (see Section 6.3.6). Scheduling issues are often specific to a particular test laboratory and therefore are very difficult to quantify. Hence the decision makers will need to look at the auxiliary costs associated with the extension of the test times and make decisions regarding the test duration based on the impact of the increased L on the current test schedules and the timing of the other projects currently in the test pipeline. However it is entirely possible that the extended test times with $L > 2$ can carry economic benefits in some particular business cases.

- As mentioned in Section 6.3.5 the model simulation is most sensitive to the value of Q_{Corr}. Therefore it places the additional importance on the process of mapping the forecasted warranty with the reliability demonstration targets as described in Chapter 5.

- Although the size of the production volume is an important factor, which significantly drives up the cost of warranty, the general behavior of the model is not markedly sensitive to the number of the units in production (see Figure 6.8). In the analysis performed in Section 6.3.5 the optimal target reliability R_0 has risen from 0.8 to 0.999, while the production volume increased 10,000 fold. Production volume analysis can also serve as one of the additional model checks proving its viability.

- From the graph Figure 6.6, uncertainty of the solution diminishes with growth of R_0. This can be explained by the fact that the contribution of the warranty cost to overall LCC diminishes with the growing target reliability, therefore reducing the uncertainty. As mentioned before, the warranty cost is the main source of uncertainty.

- The unverified failures, referred here as NTF have a substantial effect on the production economic model. The case study in this chapter showed that subtraction of NTFs changes the optimal points for the key test parameters C, R, and L. In addition, subtraction of NTF failures from the total number of warranty claims has a dual effect on the uncertainty of the model. On one hand it decreases the uncertainty by reducing the total contribution of the warranty cost forecasted with the higher degree of uncertainty. However on the other hand it increases the uncertainty adding another random input (percent NTF) to the model. The case of exclusion or inclusion of NTF into consideration is specific to the product, design process, and the needs of a particular customer; therefore it should be decided on a case-by-case basis.

- The cost benefit analysis of the proposed methodology yields a potentially high return on investment; therefore besides engineering benefits, the implementation of this method makes a good business sense. High ROI numbers on the order of magnitude of 1,000%, similar to that presented in the case study would be expected for this type of project. Moderate investments with high returns are usually anticipated for the projects involving process improvements. It is also clear from the model that potentially achievable returns can be even higher than those presented in this case study. The engineering cost of implementing the method usually goes down when the process becomes more developed and sophisticated, while the economic advantage of LCC improvement can be potentially much higher as can be inferred from Table 6.4.

The remaining Chapter 7 contains general conclusions, contributions, remaining issues, and future directions of the research.

7. Summary and Contributions

7.1. Summary

This dissertation develops a methodology for minimizing a product's life cycle cost using the decision variables controlled by a reliability/quality professional during a product development process. The methodology developed in this dissertation incorporates all dependability-related activities into a comprehensive probabilistic cost model that enables minimization of the product's life cycle cost. The mathematical model utilizes the inverse relationship between the cost of product validation activities and expected cost of repair service and warranty returns. Among the key input parameters, an emphasis was placed on the test duration and sample size for the environmental tests performed in a product validation program. The overall stochastic cost model and its minimization are done with Monte Carlo simulation in order to account for uncertainties in model inputs and parameters.

The results of this work provide reliability professionals with a methodology to evaluate the efficiency of a product validation program from a life cycle cost point of view with an emphasis on the cost of validation and product warranties, and ultimately minimize that cost by optimizing the environmental test flow of the product validation process.

7.2.   Discussion and Conclusions

Development of this methodology and its consequent application in the automotive industry generated several general conclusions regarding various aspects of design validation, environmental testing, modeling, and cost analysis.

7.2.1.   Life Cycle Cost Model

The life cycle cost analysis model is introduced in Chapter 2 and implemented in the real life example in Chapter 6. The major model inputs are discussed in Chapters 3, 4, and 5. As with any analytical model, this LCC method has its application limits, which are discussed in Section 6.3.5. Some of those limits are based on the automotive industry's rules and preconceptions.   It was important to examine each real life situation and determine if it would be beneficial to reconsider those conventionally set boundaries in the cases where business conditions call for it.

Among other items, this model suggests the application of extended life testing, where the duration of the environmental test exceeds the predetermined one-life test bogey in order to reduce the test sample size. This work shows that the relationship between the cost of test sample size and the cost of running those tests is critical in determining the economic benefits of extending the test duration. The relationship is driven by the parametric binomial distribution (see Appendix A) defining the connection between test duration and test sample size, which enables minimization of validation cost. From the analysis of the case study presented in Section 6.3 and from other applications of this

model <u>it was determined that extending the automotive durability tests beyond one bogey life is economically beneficial only when the cost of producing and equipping one test sample exceeds approximately one day labor cost of running the tests</u>. In the case of less expensive test samples it is better to limit the durability testing to one bogey life or even shorter if customer requirements allow it.

7.2.2. Warranty Forecasting

A new warranty forecasting model based on a piecewise statistical distributions and stochastic simulation was presented in Section 5.2. This model is currently being implemented in the procedures for new business quoting at Delphi Electronics & Safety and will also be used for expanded warranty forecasting for future products. In addition it will be used to detect alarming trends in current products warranty claims during the initial months of production.

7.2.3. Bayesian Analysis

Application of Bayesian analysis to the test sample size reduction was presented in Chapter 4. In cases where customer requirements for target reliability exceed the practical range of R [0.9; 0.98], certain statistical techniques of sample size reduction can be used to make product validation economically feasible. One of those practical measures can be an application of statistical priors derived from the past reliability or product's performance in the field, where favorable product history allows sample size reduction while maintaining the required demonstrated reliability and confidence. However it is

important to understand that unfavorable priors may adversely affect the outcome and require a sample size that is larger than that obtained without application of Bayesian method.

7.2.4. Stochastic Simulation

The stochastic simulation of the presented LCC model was implemented using a Monte Carlo method and is discussed in Chapter 6. It is important to use stochastic simulation techniques in order to analyze the propagation of data uncertainties through the model and obtain the required confidence bounds of the solution. It is especially important in cases where the random inputs are represented by skewed statistical distributions, such as lognormal, exponential, Weibull, or others. In these cases the mean or median of the output will be different from the deterministically obtained output, even when the inputs are represented by the means of their respective PDF functions.

7.2.5. Cost Benefit Analysis

An economic analysis of the developed methodology is presented in Section 6.4. The cost benefit analysis utilizes ROI as a measure of its economic feasibility. This analysis shows that reasonably high ROI on the order of 10 are achievable as the result of the methodology developed in this dissertation. Moderate investments with high returns are usually expected for the projects involving engineering process improvements since the ROI analysis is an estimate with a high degree of uncertainty for this type of project. In the case of LCC stochastic modeling, the ROI can be further increased by improving the

process and reducing the expenses requiring for the model implementation in each particular product line.

7.3. Contributions

The research work presented in this dissertation can be divided into major and minor contribution categories.

7.3.1. Major Contributions

- First known comprehensive application of statistical modeling approaches to life cycle cost analysis covering all product dependability activities and comprising the cost of product validation and the consequent warranty/service cost. This work presents a mathematical formulation of the probabilistic version of the 'Reliability-Cost' relationship and addresses many shortcomings of the currently existing deterministic models. This work also introduces a new approach to account for the cost of product validation and its relation to the expected warranty and service cost.

  - The methodology developed in this work enables optimization of an environmental test flow in order to minimize the life cycle cost of a product. The methodology makes use of the input controls and variables available to a reliability/validation engineer, such as reliability demonstration targets, test sample sizes, and environmental test durations. This methodology also includes a statistical analysis of the cost relationship between product reliability and quality and establishes statistical links between product validation activities and expected warranty returns. Previous efforts have

failed to establish this link due to the complexity and product specificity of this relationship.

- This methodology will provide a system supplier with economic justification for a business case supporting a chosen validation program and will help to avoid the issues of unreasonably high reliability targets and therefore unnecessary high costs of product validation or potentially delayed delivery schedules.

- This dissertation developed and mathematically formulated the warranty prediction model based on a piecewise application of Weibull and exponential distributions. The prediction model has three parameters, which are the characteristic life and shape parameter of the Weibull distribution and the time coordinate of the junction point of the two distributions. The values of the parameters are obtained by data mining past warranty claims for products with similar design characteristics.

- Applications of the developed methodology provided the following insights: This work demonstrated the importance of the relationship between variable cost of testing and cost of a test sample needed to make an educated business decision about extending the duration of the environmental tests. It showed that for electronics products, where the product validation involves durability testing, such as temperature cycling and random vibration simulating 10-15 years of mission life, certain simple criteria apply. When the cost of a test sample exceeds the labor cost of approximately one day of validation testing, it would be beneficial to extend the environmental testing beyond one mission life as shown in the case study Chapter 6. Similarly, under reversed conditions where the test sample cost is lower than the one day labor cost it is better to limit the test time to one bogey mission life. In other words, more expensive test samples warrant the extension of the test time

while the extended validation of less expensive test samples would not provide any cost benefits. It is important to remember that this 'one-day labor rule is just a rule of thumb and may need to be verified for each particular model for better accuracy.

7.3.2.   Minor Contributions

- Development of a comprehensive validation laboratory equipment cost of ownership model addressing missing repair data and incomplete maintenance records

- Formulation and introduction of a *knowledge factor* into the process of generating mixed Bayesian priors and suggesting a procedure for its assessment

- Introduction of a unique method of analyzing the existing warranty data by presenting and storing them in the form of statistical distribution parameters.

- Connecting product reliability and quality by establishing statistical links between product validation activities and warranty returns. Mapping the warranty forecasting model to the expected percent failures at the mission life period.

- Addressed the issue of unverified failures and their economic impact on the overall life cycle cost model

7.4.   Future Work

Although this dissertation can be considered as a completed research activity, certain steps could be done to further this study. The future work can be divided into two major parts: model enhancement and data improvement.

7.4.1.   Model Enhancement

This model was created specifically for automotive electronics applications; therefore the next logical step would be to expand this model to non-automotive applications. Despite obvious differences there are many commonalities between product validation programs in different industries. Careful analysis of these specifics would help to adjust the model to make it usable for alternative applications.

Also, certain steps can be taken to make this model more robust. At present the output of stochastic simulation is highly sensitive to the value of the correlation coefficient Q_{Corr}. Reduction of the model dependency on its value would increase the robustness and stability of this method. Future work may also expand the original model by including the factors presented in the influence diagram Figure 2.2 and later eliminated from the consideration for various reasons (see Figure 2.3). Taking the factors such as additional redesign cost or potential product recalls into consideration may enhance the model and make it more versatile.

While this methodology has proven to be practical and beneficial, much remains to be done in promoting in the engineering and management community the wider acceptance and use of stochastic simulation as opposed to deterministic calculations. Even though Monte Carlo warranty prediction based on the methodology presented in this dissertation is currently in the process of being implemented into every day forecasting practices at

Delphi Electronics & Safety, most of the analysis and simulation activities are still performed using deterministic methods.

On the topic of stochastic simulation, alternative simulation techniques, such as response surface methodology could be used in lieu of Monte Carlo in order to increase the speed and efficiency of the computation processes. Also, this model can be expanded by including additional input variables, such as equipment utilization or cost of the schedule delays, although it is important not to complicate the model beyond the level where it remains practical.

7.4.2. Data Improvement

At present, the availability of warranty data beyond the standard automotive 3-year warranty is very limited. Even with extended warranties on selected systems, such as engine controllers or restraint systems, many owners take their vehicles for repair to places other than dealerships and therefore data is not captured. With the general trend of increasing the standard automotive warranty (e.g., Hyundai Automotive is expanding its standard warranty to 10 years) more data is expected to be available in the future allowing the analysis of the correlation between the predictive models and the actual warranty and also to provide better Q_{Corr} for the mapping of the warranty prediction model.

Also this model would require future updates due to continuous technological developments in the automotive industry and especially in automotive electronics. Those

developments include miniaturization of electronic units, increased functionality, and continuous insertion of new packaging technology. Since warranty prediction is based on the existing warranty claims, one of the challenges to this methodology is to address the continuous changes in automotive electronics technology based on warranty prediction for the old technology.

In certain cases it is important to account for human factors while processing the field return data. For example, even with the extended warranties the number of claims drops significantly after 3 years because customers forget, unaware, or not sure whether to report the problem to the dealership as opposed to an independent auto mechanic. In addition, the number of warranty claims jumps up shortly before the expiration of warranty, since people are trying to repair the old problems before the warranty runs out. That brings aberrations to the data patterns and complicates the warranty data processing and analysis. All those factors will need to be taken into consideration to improve the accuracy of the model.

Appendix A.

Reliability Demonstration Fundamentals

From [Kleyner and Boyle (2005)]

Material in this section can be used as a supplement to the topic of reliability demonstration. It provides additional information in the form of the definitions and derivations of certain equations, which have been used in the main body of this dissertation.

Success Run Formulae

The following applies to the cases where a test has only two outcomes: pass or fail. The random variable x, that denotes the number of successes in Bernoulli trials [O'Connor (2003)] has a binomial distribution given by $p(x)$, where

$$p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \qquad x = 0, 1, 2, ..., n$$

(A.1)

Let's consider p is a probability of the product to fail. The probability of obtaining x bad items and $(n-x)$ good items $(R=1-p)$ if applied to reliability) is:

$$F(k) = \sum_{i=0}^{k} \frac{n!}{i!(n-i)!} R^{n-i} (1-R)^i$$

(A.2)

Where *F(k)* is cumulative binomial distribution, the probability of obtaining *k* or fewer failures in *n* trials is also called cumulative reliability.

If *n* items are tested and *k* have failed, the reliability of the sample is

$$R_c \approx 1 - [C - rank\ of\ the(k+1)th\ ordered\ value\ in\ (n+1)]$$

(A.3)

Where *C* denotes the confidence level required

Therefore, based on equations (A.2) and (A.3)

$$C = 1 - \sum_{i=0}^{k} \frac{n!}{i!(n-i)!} R^{n-i}(1-R)^i$$

(A.4)

Where *n* = total number of samples

If *k* = 0 (no units failed) the (turns to the well-known 'Success Run Formula'

$$C = 1 - R^n$$

(A.5)

Alternative Solution for Success Run with Failures

In the case of failures during a bogey test there is an alternative solution utilizing Chi-square distribution. Assuming that the failures follow exponential distribution pattern

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R(t)=exp(-λt), one-sided estimate for MTBF [O'Connor (2003)] based on time to failure will be:

$$MTBF \geq \frac{2T}{\chi^2_{\alpha,2(k+1)}}$$

(A.6)

or

$$\lambda \leq \frac{\chi^2_{\alpha,2(k+1)}}{2T}$$

(A.7)

$$\ln R = -\frac{\chi^2_{\alpha,2(k+1)}}{2N} \Rightarrow R = \exp\left(-\frac{\chi^2_{\alpha,2(k+1)}}{2N}\right)$$

(A.8)

This (can be used as a simplified form of more complicated binomial equation.

Parametric Binomial Equation (Lipson Equality)

The relationship between reliability and test time for the two-parameter Weibull cumulative distribution failure function is given by [Lipson and Sheth (1973)]

The probability of survival (reliability) then is:

$$R(t) = 1 - F(t) = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$

(A.9)

Now suppose that n_1 items are run without failure to t_1 time, and R_1 is the reliability at t_1 with a confidence C. Combining equations (A.5) and (A.9) will produce

$$R_1 = (1 - C)^{\frac{1}{n_1}} = \exp\left[-\left(\frac{t_1}{\eta}\right)^{\beta}\right]$$

(A.10)

Or

$$\ln(1 - C) = -\frac{1}{\eta^{\beta}} n_1 t_1^{\beta}$$

(A.11)

Next suppose that n_2 items are run without failure to t_2 time, and R_2 is the reliability at t_2 with the same confidence C.

Now as before:

$$\ln(1 - C) = -\frac{1}{\eta^{\beta}} n_2 t_2^{\beta}$$

(A.12)

Thus equating right hand sides of equations (A.11) and (A.12) would produce

$$-\frac{1}{\eta^{\beta}} n_1 t_1^{\beta} = -\frac{1}{\eta^{\beta}} n_2 t_2^{\beta}$$

(A.13)

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Or

$$\frac{n_2}{n_1} = \left(\frac{t_1}{t_2}\right)^{\beta}$$

(A.14)

In some cases we want to test the product to L number of lives in order to reduce the number of test samples, that would make

$$t_2 = Lt_1$$

(A.15)

Thus combining equations (A.14) and (A.15):

$$n_1 = L^{\beta} n_2$$

(A.16)

With the use of equation (A.16) the classical Success Run formula (A.5) will transform into:

$$C = 1 - R^{nL^{\beta}} \ \text{or} \ R = (1-C)^{\frac{1}{nL^{\beta}}}$$

(A.17)

Equation (A.17) is often referred as *parametric binomial* equation or *Lipson equality*.

Appendix B.

Bayesian Techniques to Reduce the Sample Size in Automotive Electronics Attribute Testing. Reproduced from [Kleyner *et al.* (1997)]

Introduction

In the pursuit of high quality and high reliability in a mass production environment, the automotive manufacturers require their suppliers to prove a target reliability with an assigned confidence level on a supplied product. This is usually done through a reliability demonstration test by running a certain number of samples under conditions simulating the mission life, an experiment, which is sometimes called test to a bogey. Most of the time the sample size is determined only by the required reliability and the confidence level. Most of the methods currently used in the industry presume no prior information about the product or its predecessors, though very often this information is available. With the ever increasing reliability requirements the number of samples to be tested is growing out of proportion and out of economical sense, requiring larger and larger amounts of human resources and capital equipment. Based on the fact that many new automotive products are evolutionary and not revolutionary, Bayes method can be one of the approaches to incorporate prior knowledge about the product, thus reducing the number of test samples and the amount of resources dedicated to the test programs.

Existing Techniques for Sample Size Determination

Statistical experiments are generally performed to learn more about unknown parameters characterizing our material of interest. In an automotive setup, the unknown parameter is the product reliability R, that is, the probability of surviving a specified mission life under standard condition: an attribute reliability experiment is performed to learn more about it. The experiment consists of observing N successes out of N reliability test trials. A peculiar feature is that most often no less than a 100% success rate is required - failing which corrective actions are to be taken- whereas in the usual reliability trials the success rate, albeit usually high, is random.

Techniques commonly utilized to calculate sample sizes for reliability demonstration of a product when a 100% success rate is required are generally referred to as Success Run Formulae [Johnson (1960); Benedict (1967)]. The likelihood function, that is the probability of observing all successes given a certain value of the unknown product reliability R, is

$$L(data|R) = R^{N}$$

(B.1)

Based on this equation, the classical Success Run Formula

$$C = 1 - R_{L}^{N}$$

(B.2)

has been obtained in [Benedict (1967)]. In equation (B.2), R_L is the lower bound of a one-sided $C \times 100\%$ confidence interval for the unknown reliability R. R_L is referred to from now on as the *demonstrated reliability*. In the automotive industry C and R_L are usually stipulated by the customer; the Success Run formula is then used for the determination of the required sample size N.

In a Bayesian approach instead, we use prior distributions on the unknown parameters of a statistical experiment to exploit useful pre-experimental information, for example the data from previous test results or similar product usage. For Success Run experiments, the likelihood (B.1) has to be combined with the prior distribution on R to obtain a posterior distribution on R. Such a posterior distribution summarizes all available information about the unknown product reliability R.

The Bayesian version of the classical Success Run Formula uses a *Uniform Prior*, also called a Rectangular Prior, which presumes an equal likelihood for the reliability value to fall anywhere between 0 and 1 and expresses the idea of 'vague' prior information. In other words, since this prior assigns the same weight to every value of R, we expect it to produce results similar to the classical Success Run formula. The uniform prior density is simply the constant 1 between 0 and 1, 0 otherwise and is plotted in Figure B.1.

Figure B.1. Uniform Distribution

Combining the uniform prior and the likelihood using Bayes theorem we obtain the

Bayesian version of the Success Run formula from the posterior probability

$$C \;=\; p(R_L < R < 1) \;=\; \frac{\displaystyle\int_{R_L}^{1} R^N \, dR}{\displaystyle\int_{0}^{1} R^N \, dR}$$

(B.3)

or,

$$C \;=\; 1 - R_L^{N+1}$$

(B.4)

where C is a *(Bayesian) confidence* and R_L is a *(1-C)* quantile of the posterior distribution

of R, still referred to as the *demonstrated reliability*. For practical reasons we use here

the word 'confidence' for the quantity C, but this is different from the standard use in

classical statistics. An interpretation of (B.4) is that, after the successful completion of a

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Success Run experiment with N units, there is a $C \times 100\%$ Bayesian confidence that the unknown reliability is greater than R_L.

The sample size calculated using equation (B.2) is one sample more than what we would get using equation (B.4). Some common reliability demonstration requirements and the sample sizes for Success Run of these demonstrations are given in Table B.1. Equation (B.4) has been used in the calculation of these sample sizes.

Table B.1. Some Common Reliability Demonstration Requirements

Reliability to be Demonstrated	Confidence Level	Sample Size (Success Run formula)
0.95	0.9	45
0.97	0.7	40
0.99	0.5	69
0.99	0.9	229

From Beta Priors To Mixtures of Beta Priors For Product Reliability

A generalization of the Success Run formula (B.4) can be obtained from priors other than the uniform. In Bayesian statistics, it is well known that for a binomial likelihood such as (B.1), a beta prior distribution on R, with density

$$\pi^*(R) = \frac{R^{A-1}(1-R)^{B-1}}{\beta(A,B)} \qquad if \quad 0 \le R \le 1$$

(B.5)

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Where $\quad \beta(A,B) = \dfrac{\Gamma(A)\Gamma(B)}{\Gamma(A+B)}$

is particularly convenient; the constants A and B (sometimes called hyper-parameters) have a nice interpretation - A being thought, sometimes, as the number of successes out of $A+B$ trials in a similar pre-experiment, real or imaginary. Figure B.2 shows examples of beta distributions with different combinations of A and B. More importantly, the beta prior distribution is conjugate to binomial sampling, that is, the posterior is a beta distribution as well. This allows for a continuous updating of the posterior within the same general class of distributions. The uniform prior is a special case of (B.5) for $A = B = 1$.

The posterior density on R obtained by combining equations (B.1) and (B.5) through Bayes theorem is

$$\pi^*(R|data) \;=\; \dfrac{\left(\dfrac{R^N R^{A-1}(1-R)^{B-1}}{\beta(A,B)}\right)}{\left(\dfrac{\int S^N S^{A-1}(1-S)^{B-1}dS}{\beta(A,B)}\right)} \;=\; \dfrac{R^{A+N-1}(1-R)^{B-1}}{\beta(A+N,B)}$$

(B.6)

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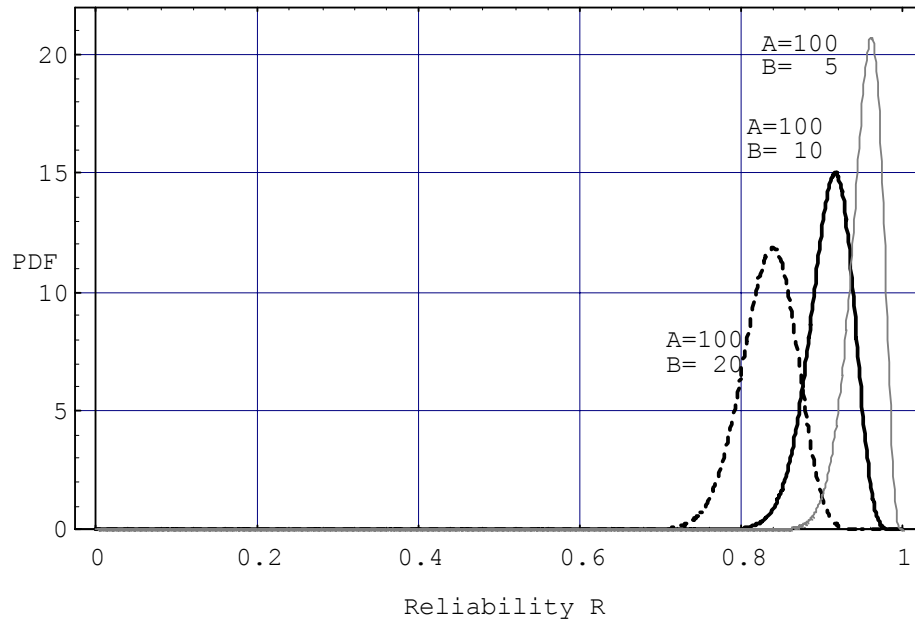
Figure B.2. Typical Beta Distributions

that is, a beta density with parameters *(A+N)* and *B*. The use of beta priors for binomial sampling has a long history, starting somewhere in the prehistory of modern Bayesian statistics. For an account of the uses of beta distributions in attribute reliability trials, see for example [Martz and Waller (1976, 1982)]. As in the case of the standard Success Run formula (B.4), the immediate use of posterior (B.6) is to establish a reliability level R_L above which there is a high Bayesian confidence *C* that the reliability *R* will be met. For this purpose, we use equation

$$C = \int_{R_L}^{1} \frac{R^{A+N-1}(1-R)^{B-1}}{\beta(A+N,B)} dR$$

(B.7)

which tells us that there is a *C* posterior probability that *R* will be greater than R_L.

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If, before the experiment, we require a certain Bayesian confidence C based on the contractual specifications, for given A and B the only unknown in expression (B.7) is the sample size N. For a given prior (B.5) we have to solve equation (B.7) numerically for N, in order to know how large a sample size we have to observe, with 100% success rate, to satisfy the required C and R_L.

The choice of the parameters of the prior A and B is a crucial one. It seems reasonable, in automotive reliability applications, to base such a choice on failure data, which are easily available and contain a lot of relevant information on past models or similar products. In the presence of information on the success rate of n previous life tests, a possible way to obtain A and B is based on an empirical Bayes approach discussed in [Copas (1972)]. See, for example, [Martz and Waller (1976)] where empirical Bayes estimates of A and B are derived as follows:

$$A + B = \frac{n^2 (\sum_{j=1}^{n} R_j - \sum_{j=1}^{n} R_j^2)}{n(n\sum_{j=1}^{n} R_j^2 - K\sum_{j=1}^{n} R_j) - (n-K)(\sum_{j=1}^{n} R_j)^2}$$

(B.8)

and

$$A = (A + B)\overline{R}$$

Where n is a number of life tests

l_j is a number of units in j^{th} test.

R_j is the jth observed failure rate $= \dfrac{Number\ of\ failures\ in\ the\ j^{th}\ test}{l_j}$

$$K = \sum_{j=1}^{n} l_j^{-1}$$

$$\overline{R} = \frac{\sum_{j=1}^{n} R_j}{n}$$

When n is small, sampling error may cause equation (B.8) to yield negative estimates. If this occurs, [Martz and Waller (1976)] suggest using another form of this equation.[11]

These equations can be applied to processing real life data, where n would be the number of test sets for the similar products and R_j would be the reliability data from each set.

Beta priors of the form (B.5) have a long history and are mathematically convenient, but for our purposes they are too restrictive. The best way to understand this is observing that an industrial product is in continuous evolution and, although a lot of similarity exists between old and new models, we always have a margin of novelty, which should

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[11] For small n, the following equation may be used in place of equation (B.8):

$$A + B = \left(\frac{n-1}{n} \right) \left(\frac{n \sum R_j - (\sum R_j)^2}{n \sum R_j^2 - (\sum R_j)^2} \right) - 1$$

be accounted for. On the other hand, we do want to use prior information on similar products in our research; this is the reason why we want to use Bayesian methods in the first place. The right compromise between these conflicting goals seems to be generalizing the class of beta priors to the larger class of finite mixtures of beta priors. The plan is then to put together a prior distribution derived from failure data, and a margin of uncertainty intrinsic to the new model. The latter margin of uncertainty can be expressed as a uniform prior on the reliability.

Our proposal is therefore the use of a two-component mixture of beta distributions, with density

$$\pi(R) = \rho \frac{R^{A-1} \times (1-R)^{B-1}}{\beta(A,B)} + (1-\rho) \qquad if \quad 0 \le R \le 1$$

(B.9)

The first component of the mixture is a beta prior with parameters A and B to be derived from failure data. The second component of the mixture is a uniform prior (a special case of the beta) representing uncertainty about the new product reliability. The two components are combined according to weights ρ and $(1-\rho)$, where ρ is a '*knowledge factor*' representing how similar the new product is to the old one, and $(1-\rho)$ is an '*innovation factor*', reflecting the proportion of new content in the new product. Notice that the use of a uniform prior alone would lead to the Bayesian version of the Success Run formula; the use of mixtures represents therefore a reasonable compromise between Bayesian and classical methods.

The idea of using mixture priors in the context of product reliability could be generalized to the case of heterogeneous prior information, in particular to the case where failure data is available for different past products, some more similar than others to the new product. In that case, the analysis could be generalized to the consideration of prior densities of the form

$$\pi(R) = \sum_i \left(\rho_i \frac{R^{A_i-1} \times (1-R)^{B_i-1}}{\beta(A_i, B_i)} \right) + (1-\rho)$$

(B.10)

where $\rho = \sum_i \rho_i$

and the different knowledge coefficients ρ_i reflect different degrees of similarity between the new and the old products. Another reference to the use of mixture priors in Bayesian reliability is [Savchuk and Martz (1994)].

For now, we consider only mixtures with two components of the form given in the equation (B.9). Combining equations (B.1) and (B.9) using Bayes theorem we obtain the posterior density

$$\pi(R|data) = \frac{(1-\rho)R^N + \rho \dfrac{R^{A+N-1} \times (1-R)^{B-1}}{\beta(A,B)}}{\dfrac{(1-\rho)}{N+1} + \rho \dfrac{\beta(A+N,B)}{\beta(A,B)}}$$

(B.11)

and the corresponding expression

$$C = \int_{R_L}^{1} \pi(R|data)\,dR$$

(B.12)

where a required demonstrated reliability R_L and confidence coefficient C can be achieved. The solution of equation (B.12) has to be found, in general, by numerical methods.

An Example to Demonstrate Application of the Technique

The sample size determination technique described in previous sections of this paper has been applied to a real life example to demonstrate a significant reduction in sample size. Table B.2 shows failure data for an electronic vehicle control product (slightly modified from actual data for security reasons) in terms of IPTV (Incidents Per Thousand Vehicles). Table B.2 shows breakdown by model years and body styles, totally constituting 12 test sets ($n = 12$). The observed failure rates R_j, are calculated from the IPTV data using:

$$R_j = 1 - \frac{IPTV}{1000}$$

(B.13)

Using equation (B.8) the values of A and B for the data in Table B.2 are found to be 769.34 and 2.53 respectively. The cumulative distribution functions (CDFs) of the uniform, beta, and mixture distributions are shown in Figure B.3 for the crucial range of $0.98 \leq R \leq 1$

Using equation (B.12) and solving numerically for the sample size, N, for a demonstrated reliability of $R_L = 0.99$ with $C = 90\%$, the sample sizes for various knowledge factors are as shown in Table B.3. Using the classical Success Run formula (no prior knowledge about the product or knowledge factor $\rho = 0$), 229 samples of the product A will have to be tested with no failures to demonstrate a 0.99 reliability with 90% confidence. From Table B.3 it is seen that with only a 10 % prior knowledge of the product (knowledge factor $\rho = 0.1$), the sample size reduces to 54 and as the knowledge factor increases, the sample size decreases.

Table B.2. Calculation of Coefficients A and B from IPTV / Reliability Data

	Product	Model	Body Style	IPTV	Volume Sold l_j	Reliabili
1	Product A	19XX	Type I	2.94	170	0.9971
2			Type II	3.57	121	0.9964
3			Type III	2.45	206	0.9976
4			Type IV	5.32	35	0.9947
5			Type V	2.38	52	0.9976
6			Type VI	8.68	38	0.9913
7		19YY	Type I	1.75	306	0.9983
8			Type II	1.12	113	0.9989
9			Type III	4.06	87	0.9959
10			Type IV	1.61	27	0.9984
11			Type V	1.12	173	0.9989
12			Type VI	4.41	156	0.9956
n = 12					**K** $= \Sigma l_j^{-1} = $ **1.66E-04**	**$\Sigma R_j =$**

Table B.3. Sample sizes for various knowledge factors at $R = 0.99$ and $C = 90\%$

| Knowledge Factor	Sample Size N
1	0
0.9	1
0.8	2
0.7	4
0.6	6
0.5	9
0.4	13
0.3	19
0.2	30
0.1	54
0	229
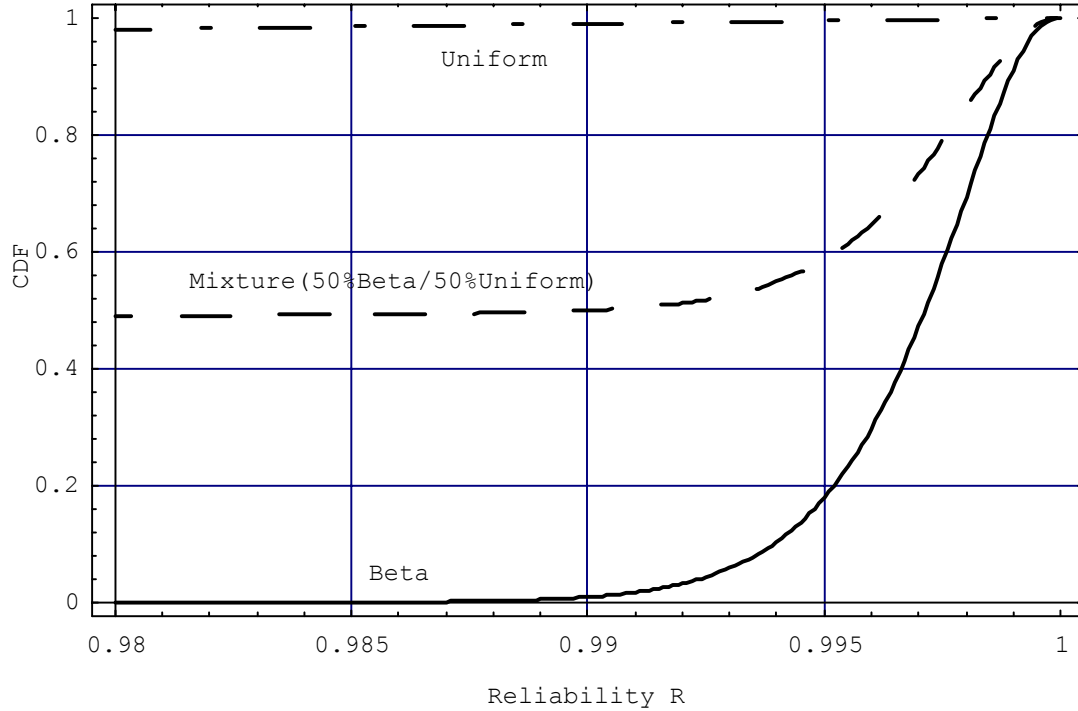
Figure B.3. CDFs for Beta, Mixture and Uniform Distributions with $A = 769.34$, $B = 2.53$

Conclusion

The method presented in this paper has great potential for cost reduction in reliability demonstration testing in a mass production environment like an automotive electronics industry. The failure data on similar products used to build a prior can significantly decrease the number of test items to a bogey. Even in cases with a low knowledge factor such as 0.2 or 0.3 (20-30% prior knowledge about the product), the method may present significant sample size reductions.

In cases with a favorable prior, the number of samples may sometimes go down to zero or even become negative. The zero or negative sample sizes would mean that the required reliability has already been demonstrated during the previous stages of product development and no further testing is needed.

In instances with an unfavorable prior the number of samples to be tested may actually exceed the number computed using the classical method. This means that the product's prior has already shown that the product's reliability is most likely less than the desired outcome and no further testing should be performed without appropriate design corrections.

Acknowledgments

Appendix C. List of Scientific Papers Published up to Date from this Dissertation

- Kleyner, A., Bhagath, S., Gasparini, M., Robinson, J., Bender, M. (1997), "Bayesian Techniques to Reduce the Sample Size in Automotive Electronics Attribute Testing". Microelectronics and Reliability, vol. 37, No. 6, June, 879-883.

- Kleyner A., Sandborn P., Boyle J. (2004), "Minimization of Life Cycle Costs Through Optimization of the Validation Program – A Test Sample Size and Warranty Cost Approach". Proceedings of Annual Reliability and Maintainability Symposium: Los Angeles, 553-557.

- Kleyner, A., and Boyle, J. (2004), "The Myths of Reliability Demonstration Testing", TEST Engineering and Management, August/September, 16-17.

- Kleyner A., and Sandborn P. (2005) "A Warranty Forecasting Model Based on Piecewise Statistical Distribution and Stochastic Simulation", Reliability Engineering and System Safety, vol. 88, No 3, 207-214.

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