Accuracy and Speed Effects of Variable Step Integration for Orbit Determination and Propagation

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Introduction

- Numerical integration is increasingly popular to improve accuracy of orbit propagation and determination.
- Fixed step integrators do not efficiently integrate elliptical orbits.
- Variable step methods are more efficient for highly elliptical orbits.
- Variable step methods have disadvantages which make them unsuitable for all orbits.
- A study is needed to find where variable step methods are advantageous.

Test Cases

- Test cases are considered with varying eccentricity and perigee height.
- All have an inclination of 40° and a ballistic coefficient of $\mathbf{0.01}\ \mathbf{m^2/kg}$.
- Epoch is 1999-10-01 00:00:00 UTC.
- Perturbations include 36 $\times$ 36 WGS-84 geopotential, Jacchia 70 drag model, and lunar/solar forces.
- Tests performed with the SPeCIAL-K orbit determination software.

Evaluations

- A 400 km circular orbit takes 35 sec to integrate 30 days with Gauss-Jackson.
- Without perturbations, the computation takes 3.33 sec.
- 90.5% of the computation time is spent evaluating perturbations.
- To be advantageous, an integrator needs to use fewer evaluations per orbit.
- Number of evaluations per step and number of steps per orbit are the only significant factors in computation time.

Gauss-Jackson Integration

- Eighth order Gauss-Jackson with time as the independent variable.
- It is a fixed step integrator, no control over the local error.
- Can use a Predict, Evaluate, Correct (PEC) implementation, or a PECEC. . . implementation.
- It is better to reduce the step size than to perform additional evaluations.
- We use a PEC implementation.

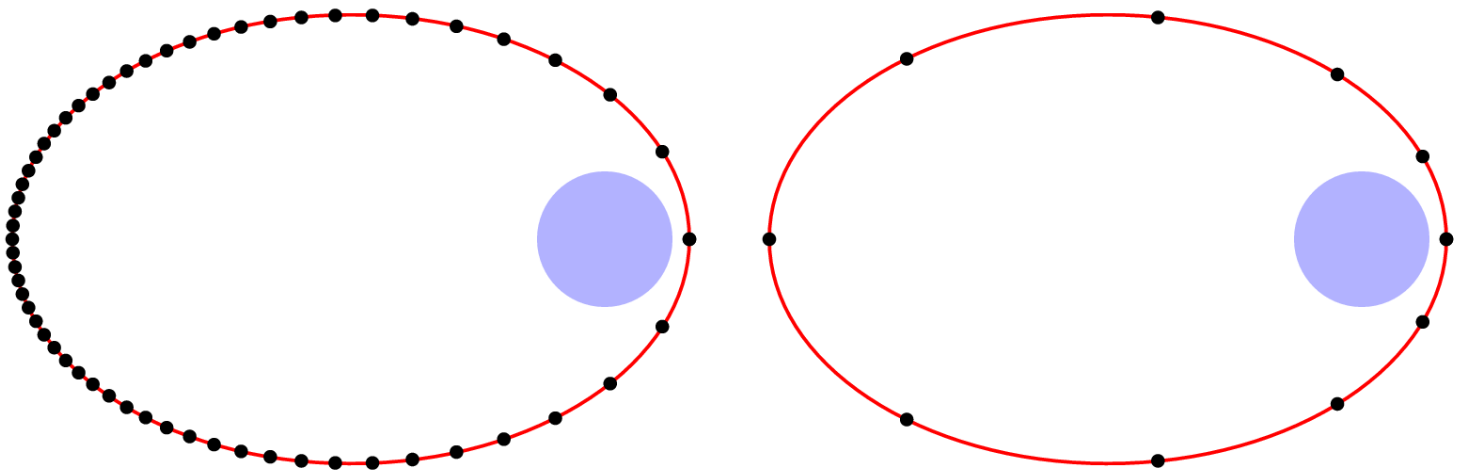
s-Integration

- Generalized Sundman transformation spreads integration points about the orbit.

$$dt = cr^n ds$$

- Still a fixed step method - no local error control.
- Step size chosen to give a certain time step at perigee.
- Unstable with a PEC implementation.
- Can use a PEC$\tilde{\text{E}}$C implementation - only re-evaluate two-body force on second evaluation.
- Cuts computation time in half with under 10mm loss in accuracy.

s-Integration



(a) t-integration with 58 steps.

(b) s-integration with 10 steps.

$$e = 0.75$$

Shampine-Gordon

- Variable step variable order multi-step integrator.
- Based on the Adams Bashforth and Adams Moulton integrators.
- Step size and order adjusted to keep local error within a user-defined tolerance.
- Performs two evaluations per step.

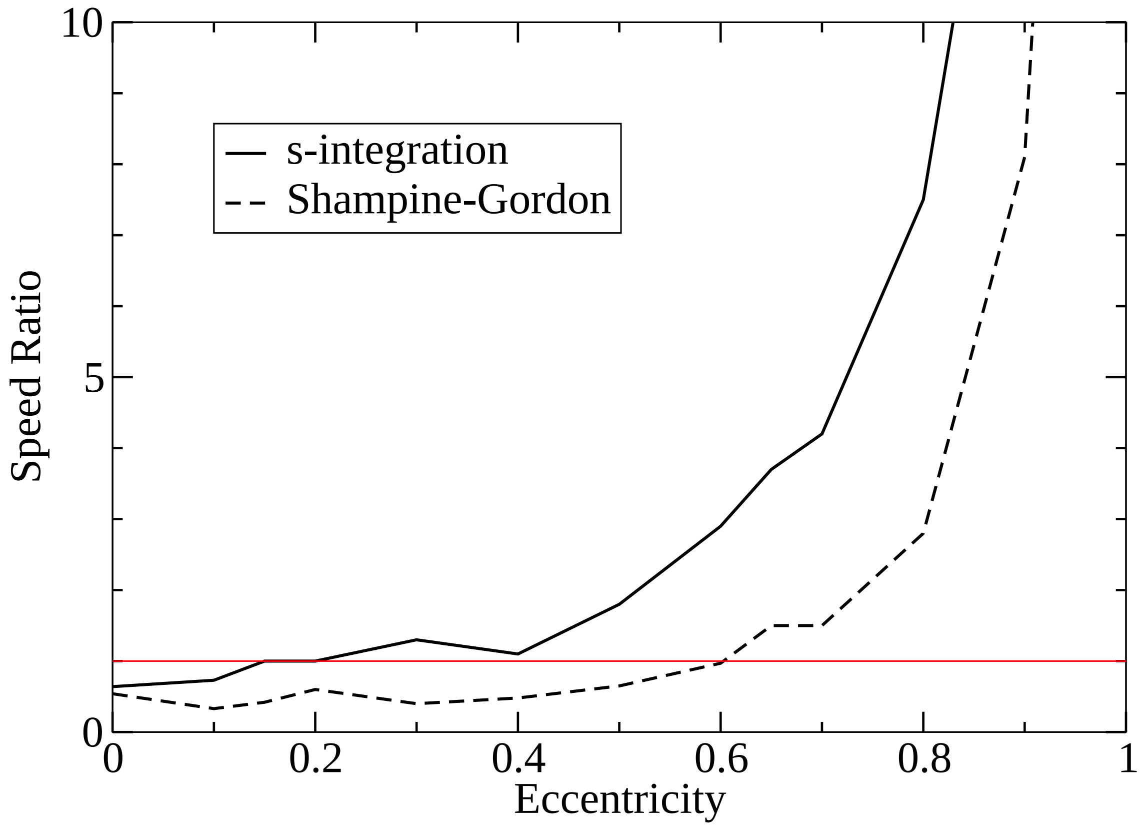
Accuracy Tests

- Define an error ratio:

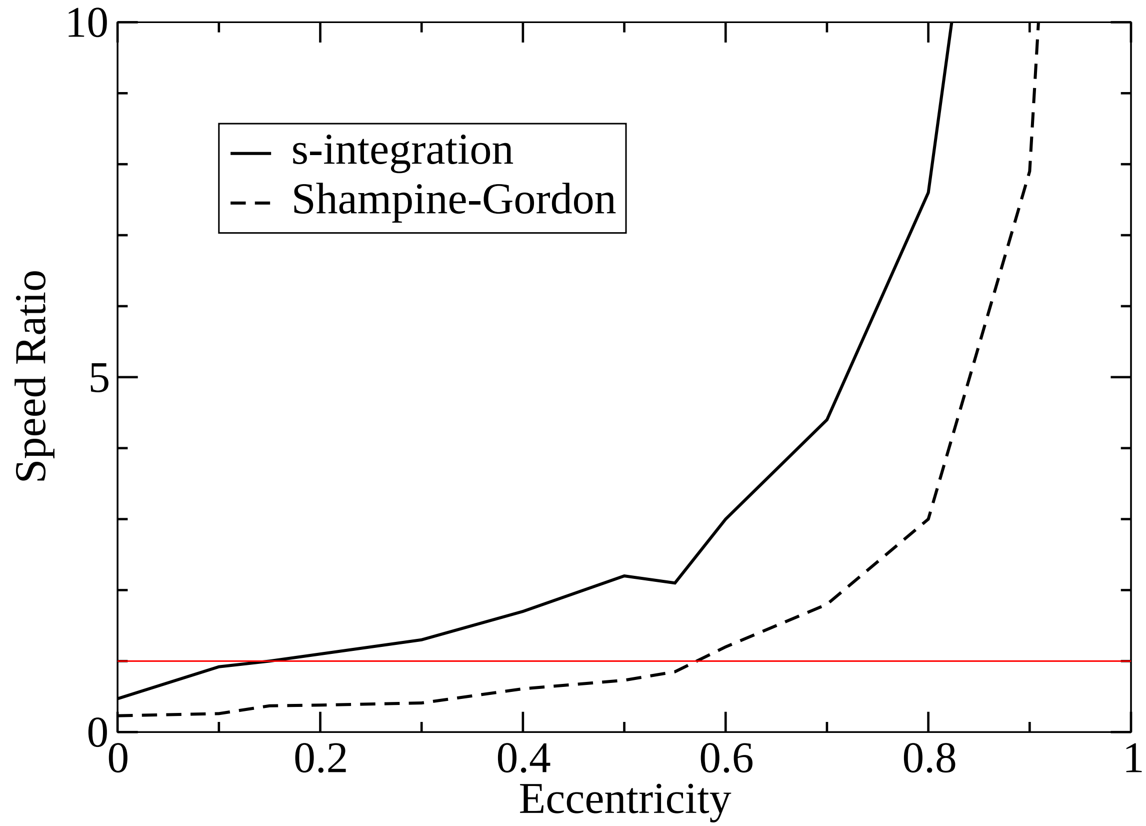
$$\rho_r = \frac{1}{r_A N_{\text{orbits}}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\Delta r_i)^2}$$

- Step size found for Gauss-Jackson with t- and s-integration which give error ratios of 1×10^{-9} in step-size halving test.
- Tolerance found for Shampine-Gordon which gives an error ratio of 1×10^{-9} in two-body test.
- Time found to run for 30 days with perturbations using this step size or tolerance.

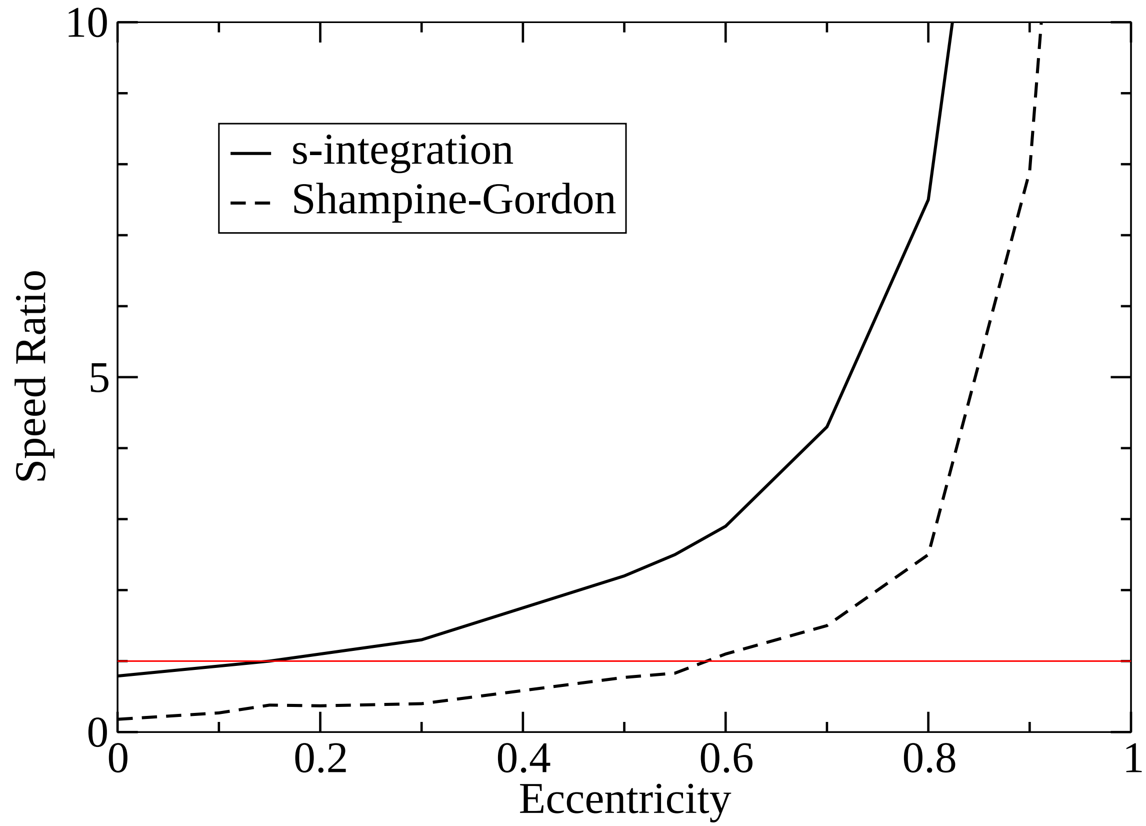
Speed Ratios at 300 km Perigee



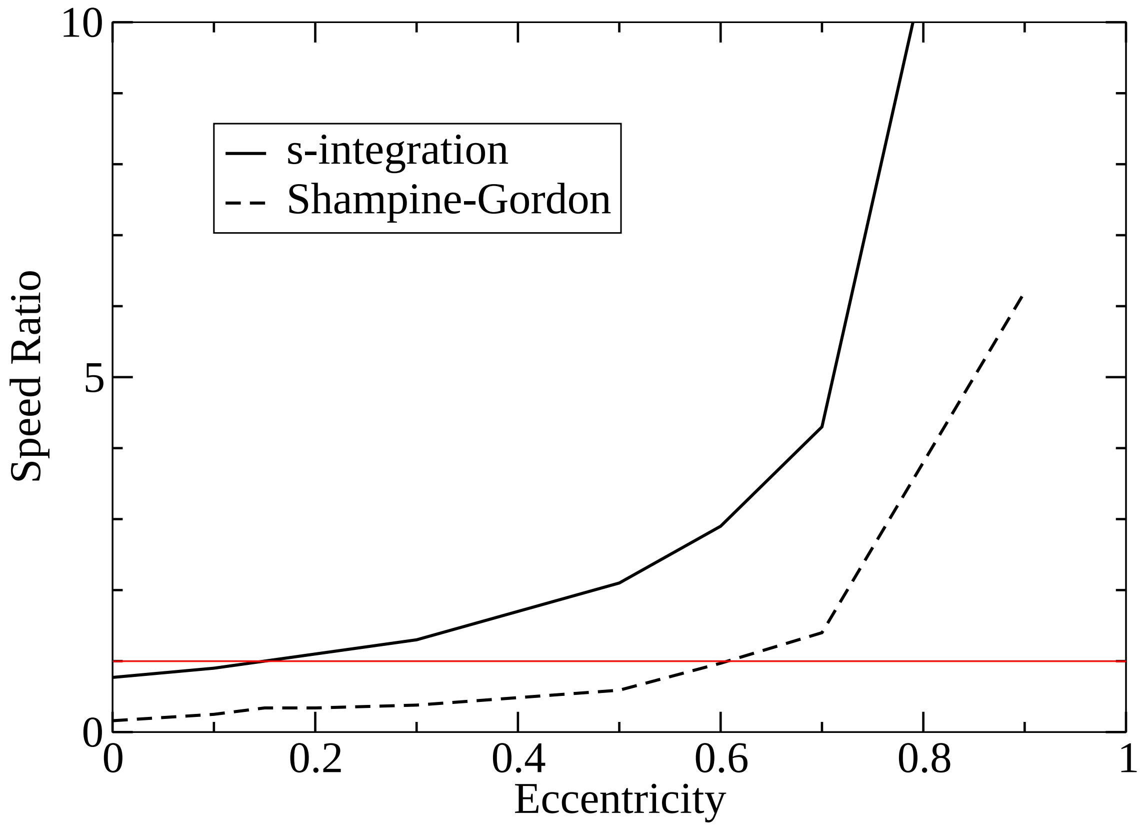
Speed Ratios at 400 km Perigee



Speed Ratios at 500 km Perigee



Speed Ratios at 1000 km Perigee



Orbit Determination Testing

- Test performed on set of cataloged objects from 1999-09-29.
- 8003 objects in catalog, 1000 randomly selected for test.
- Perform 3 tests:
  - Time all 1000 objects with t-integration.
  - Use both t-integration and s-integration on objects with $e > 0.15$.
  - Use both t-integration and Shampine-Gordon on objects with $e > 0.60$.

Orbit Determination Results

- Takes 11.0 hrs to fit 1000 objects.
- s-integration is 1.59 hrs faster than t-integration. 14.5% improvement.
- Shampine-Gordon is 0.77 hrs faster than t-integration. 7.0% improvement.
- s-integration and Shampine-Gordon give comparable results to Gauss-Jackson - position differences are within the accuracy of the observations.

Conclusions

- Use the PEC method for Gauss-Jackson with $\boldsymbol{t}$-integration.
- Use the PEC$\tilde{\text{E}}$C method for $\boldsymbol{s}$-integration.
- $\boldsymbol{s}$-integration is more efficient than $\boldsymbol{t}$-integration at eccentricities above 0.15, with a 14.5% improvement for OD.
- Shampine-Gordon is more efficient than $\boldsymbol{t}$-integration at eccentricities above 0.60, with a 7.0% improvement for OD.
- $\boldsymbol{s}$-integration is more efficient than Shampine-Gordon.
- Shampine-Gordon could benefit from the pseudo-evaluation.