

ABSTRACT

Title of dissertation: **RISK AVERSION,
PRIVATE INFORMATION AND
REAL FLUCTUATIONS**

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In this dissertation, I further explore the role of the entrepreneurial sector in creating frictions in the economy. I examine the combined effect of private information and entrepreneurial risk aversion on the dynamics of a general equilibrium macroeconomic model. I analyze the impact of these frictions both at the micro level, in terms of the optimal contract between lenders and borrowers, and at the aggregate level within the context of a dynamic stochastic general equilibrium model.

This analysis uses a model similar to Bernanke, Gertler and Gilchrist (1999), in which the entrepreneur benefits from private information. Allowing for risk aversion among entrepreneurs modifies the optimal contract by introducing insurance and a risk premium that risk-averse entrepreneurs demand due to the stochastic nature of their investment returns: the private equity premium. This premium, in general equilibrium, may become a mechanism that magnifies and propagates the effects of shocks over time. The model predicts that economies with a relatively larger privately-held sector, all else equal, should be more volatile than economies with a relatively more important corporate sector.

I first examine a closed-economy framework, which isolates the role of the private equity premium as a mechanism that magnifies and propagates shocks over time. I then consider a small open economy and examine the role of exchange rates in affecting the private equity premium and the model's dynamics. I find that the exchange rate helps alleviate the propagating feature of the private equity premium. I also execute an exchange rate regime comparison where I show that the greater volatility associated with flexible exchange rate regimes adversely impacts the private equity premium and the supply of capital, amplifying the output response to shocks. I find that fixed exchange rate regimes could be preferable under less restrictive conditions than those commonly found in the literature.

RISK AVERSION, PRIVATE INFORMATION
AND REAL FLUCTUATIONS

by

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DEDICATION

To my beloved wife, Whitney.

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TABLE OF CONTENTS

List of Figures	vi
1 Introduction	1
2 The Closed Economy Framework	12
2.1 The Partial Equilibrium Model: The Optimal Contract	12
2.1.1 Model with Risk-Neutral Entrepreneurs: The Benchmark Case	15
2.1.2 Model with Risk-Averse Entrepreneurs	21
2.2 The General Equilibrium Model	38
2.2.1 The Firms	38
2.2.2 Workers	39
2.2.3 The Entrepreneurs	40
2.2.4 Monetary Side, Market Clearing Conditions and the Rational Expectations Equilibrium	45
2.2.5 The Log-linearized Model	46
2.3 Numerical Analysis	49
2.3.1 Productivity Shock	50
2.3.2 Monetary Shock	52
3 Model under Small Open Economies	58
3.1 Introduction	58
3.2 The Optimal Contract Model	60
3.2.1 The Benchmark Case	61
3.2.2 Model with Risk Averse Entrepreneurs	63
3.3 The General Equilibrium Model	67
3.4 Numerical Analysis under Flexible Prices	73
3.5 Analysis under Nominal Rigidities	77
3.5.1 Exchange Rate Regime Comparison	80
4 Conclusion	90
5 Appendix	95
5.1 The Private Equity Premium	95
5.2 The Log-Linearized Contract	96
5.2.1 Contract under risk-neutrality	96
5.2.2 Contract under Risk Aversion	97
5.3 Parameterization	100
5.4 Impulse Response Functions	106
5.5 The Log-linearized Small Open Economy Model	108
5.5.1 Contract under risk-neutrality	110
5.5.2 Contract under Risk Aversion	111
Bibliography	115

LIST OF FIGURES

1.1 GDP Growth Gap 1995-1999: Emerging Markets and Developed Countries	1
2.1 Optimal capital stock and net worth.	20
2.2 Real Return on Capital and Optimal Investment	30
2.3 Change in μ	33
2.4 Change in Internal Net Worth	34
2.5 Change in Risk Aversion	35
2.6 Decomposition of the Risk Premium	36
2.7 Change in Variance	37
2.8 Two-Period Generation of Entrepreneurs	42
2.9 External Finance Premium and Private Equity Premium	44
2.10 Effects of a Positive Productivity Shock	50
2.11 Effects of a Negative Monetary Shock	55
3.1 Supply of capital: the risk-neutral case.	62
3.2 Supply of Capital, Marginal Cost Function and Insurance Rate.	65
3.3 Exchange Rate, the External Finance Premium and the Private Equity.	70
3.4 Effects of an Unfavorable International Interest Rate Shock	74
3.5 Effects of a Positive Export Shock	76
3.6 Exchange Rate Regime Comparison: The Case of an International Interest Rate Increase and Risk Neutrality	82
3.7 Exchange Rate Regime Comparison: The Case of a Export Demand Decrease and Risk Neutrality	83
3.8 Exchange Rate Regime Comparison: CCV Risk Premium and the Case of an International Interest Rate Increase	85

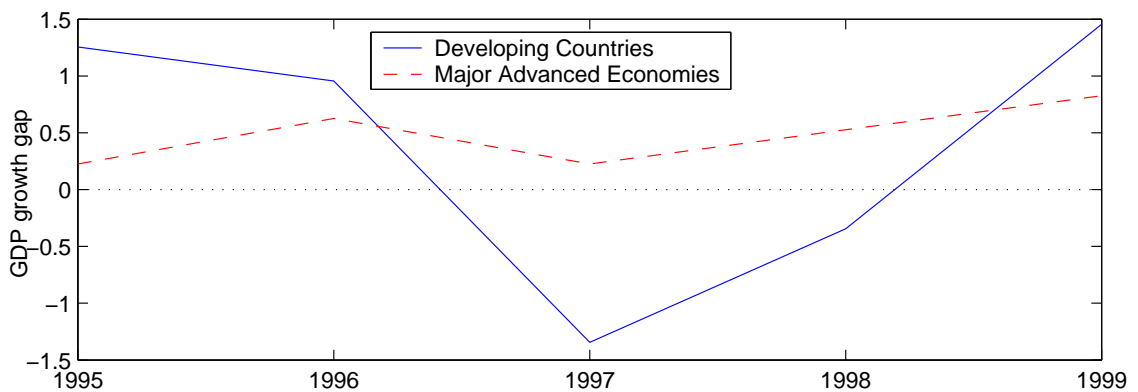
3.9	Exchange Rate Regime Comparison: CCV and the Case of a Export Demand Decrease and Risk Aversion	86
3.10	Exchange Rate Regime Comparison: The Case of an International Interest Rate Increase and Risk Aversion	87
3.11	Exchange Rate Regime Comparison: The Case of a Export Demand Decrease and Risk Aversion	88
5.1	Effects of a Favorable Productivity Shock	106
5.2	Effects of an Adverse Monetary Shock	107

Chapter 1

Introduction

Entrepreneurial activity is volatile in many economies, changing rapidly from euphoria to deep depression. Such fluctuations have become especially evident for many developing countries that fell into financial stress after the Asian crisis of the late 1990s, where economic activity has been stymied at low levels and unemployment has remained high for prolonged periods of time.

Figure 1.1: GDP Growth Gap 1995-1999: Emerging Markets and Developed Countries



Source: World Economic Outlook Database, IMF

Figure 1.1 shows the gap between GDP growth in each year and its average for

the last 25 years for both major developed countries and emerging markets.¹ One can observe that both groups of countries were growing above their long-run potential in 1995-1996, and that deviation from long-run averages was much higher for emerging economies than for developed countries. When the crisis hit, the output growth of developing countries fell well below average in 1997 and 1998. Meanwhile, developed economies experienced just a mild slow down in 1997. By 1999, emerging markets had recovered and were again growing above their average, again at a higher deviation from long-run average than that of the major advanced countries. This figure presents evidence of the more volatile output response experienced by emerging markets to the unfavorable shocks arising from the Asian crisis, as compared to the evolution of this index for major advanced countries.

One of the objectives of this dissertation is to provide an alternative explanation for the stylized fact described above by further exploring the role of the entrepreneurial sector in creating frictions in the real economy. There are numerous models explaining business cycle fluctuations as a result of financial imperfections. However, there are fewer models that describe the behavior of the entrepreneurial sector as an additional mechanism of amplification and propagation of shocks. I will focus my attention on entrepreneurs as producers of the capital necessary for the production of final goods. As we will see, imperfections affecting entrepreneurial activity will impact the GDP of an economy by directly impacting the economy's

¹Major advanced economies include only the G7 countries. As defined by the IMF, developing economies are all countries excluding the Euro area, the G7 economies and the newly industrialized Asian economies (Hong Kong, Singapore, Korea and Taiwan Province of China.)

supply of capital goods. In particular, in order to finance their capital purchases, entrepreneurs enter in a contractual relationship with lenders, and that relationship is subject to frictions. One of these imperfections is that there is private information, an assumption that is standard in the financial friction literature.²

The second problem has been much less explored in the literature and arises from entrepreneurial risk aversion. Before analyzing the impact and relevance of this assumption, one may wonder how realistic this assumption may be, as it is commonly believed that entrepreneurs are risk neutral, or even risk lovers. In fact, most existing models of financial markets and investment decisions with financial frictions assume risk-neutrality for simplicity. However, as Gale and Hellwig (1985) point out, “risk-neutrality is not an unreasonable assumption to make in the case of investors since it can be justified as a consequence of risk-pooling. It makes less sense in the case of entrepreneurs and indeed is merely a ‘simplifying’ assumption which should be relaxed if possible.” What Gale and Hellwig imply, and what I would point out, is that there is a distinction between being risk neutral and limiting risk through diversification. Lenders, even though they are risk averse, because they are able to hold diversified portfolios and can easily pool their risk, they manage to considerably reduce risk. Therefore, assuming risk neutrality for lenders is generally deemed plausible.

Although simplifying assumptions are necessary in order to develop an economic model with predictive power, the relaxation of these assumptions often provides enlightening information about economic behavior. In fact, risk aversion ap-

²See Bernanke and Gertler (1989).

pears to be particularly relevant for private entrepreneurs, or those entrepreneurs that invest in privately-owned companies. These companies are typically small and owned by few or even a single entrepreneur. As Moskowitz and Vissing-Jørgensen (2002) show, private entrepreneurs usually invest an important percentage of their wealth in private companies, often comprising at least 50 percent of their assets. Further, the vast majority of entrepreneurs invest in a single company, and are thus highly vulnerable to project-specific uninsurable fluctuations. As a consequence, private entrepreneurs are not likely to have access to complete insurance markets for their idiosyncratic risks,³ so that complete risk-pooling is not always a viable option.

In light of these observations, risk aversion is arguably a more realistic assumption in modeling the investment decisions of private entrepreneurs. In this dissertation, I study the role that imperfect information together with entrepreneurial risk aversion play in affecting the equilibrium and dynamics of a general equilibrium macro model. I analyze the effects of these frictions both at the micro level, in terms of the optimal contract between lenders and borrowers, and at the aggregate level within the context of a dynamic stochastic general equilibrium model.

This analysis follows a set-up similar to Bernanke, Gertler and Gilchrist (1999), where there are information asymmetries between lenders and borrowers. The borrower, or entrepreneur, invests in a project using both his own net worth and borrowed funds. The entrepreneur has private information on the true *ex-post* profitability of the project. Agency problems arise from the fact that there is a positive prob-

³See Angeletos and Calvet (2003) and Meh and Quadrini (2004).

ability that the project will fail, in which case the lender will not be able to recover the whole amount of the remaining revenues after default due to some bankruptcy costs. As a result, lenders optimally charge the entrepreneur an external finance premium in addition to the lender's opportunity cost of funds. As in Bernanke and Gertler (1999), this endogenously determined external finance premium (the "financial accelerator") is decreasing in the level of net worth of entrepreneurs, as investing a greater proportion of borrowed funds increases the agency costs internalized by the lender. This external finance premium becomes a mechanism that magnifies and propagates real shocks over the business cycle (see Bernanke, Gertler and Gilchrist, 1999).

Introducing risk aversion among entrepreneurs modifies the optimal contract in two ways. First, risk-averse entrepreneurs demand insurance in order to ensure a positive minimum consumption. Therefore, the external finance premium will reflect not only the lender's opportunity costs and bankruptcy costs described above, but also the cost of providing this insurance. Second, the overall risk premium, or the total rental cost of capital beyond the opportunity cost of funds, will consist of both the external finance premium and a positive risk premium that risk-averse entrepreneurs demand due to the stochastic nature of their private investment returns. This risk premium is referred to in this dissertation as the private equity premium, and implies that entrepreneurs are willing to supply less capital to final goods firms for a given rental rate of capital. The equilibrium price paid by final goods firms to rent capital from entrepreneurs will depend on the interaction between these two premia.

When the private equity premium exists, in general equilibrium it becomes a mechanism that magnifies and propagates the aggregate effects of real shocks over time. Specifically, any real shock that reduces (increases) entrepreneurial profits and wealth increases (reduces) the entrepreneur's effective risk aversion and the private equity premium in the opposite direction. In response, entrepreneurs adjust their supply of capital to final goods firms, producing a magnified impact on output, consumption and both entrepreneurial profits and net worth in subsequent periods. As a consequence, the model predicts that economies with a relatively larger privately-held private sector, all else equal, should be more volatile than economies with a relatively more important publicly-traded corporate sector.

The combined effect of entrepreneurial risk aversion and private information is studied in two alternative scenarios. I first examine a general closed-economy framework, which isolates the role of the private equity premium as an additional mechanism of amplification of shocks in a large economy framework, such as the United States or Europe. This work complements Bernanke, Gertler and Gilchrist (1999) by incorporating risk aversion and hence the private equity premium into the analysis, and differs from other recent works that deal with risk aversion and market imperfections. For instance, Angeletos and Calvet (2003) examine the implications of the risk premium on private equity, but in their work the degree of market incompleteness is exogenous, while in this model entrepreneurs endogenously determine the amount of self-insurance. As it turns out, this insurance is incomplete for incentive reasons that become apparent in section 2.1.2. This paper is also similar to Rampini (2004), in that entrepreneurs cannot fully diversify their idiosyncratic

risk, as they must bear some risk due to financial incentives. However, Rampini focuses on how investment risk affects entrepreneurial activity, disregarding the role of financial imperfections, such as constraints on external funding and the risk of default, which play a crucial role in my model.

This dissertation is also related to Meh and Quadrini (2004), as they also explicitly model investment risk that results in optimal contracts that cannot provide full insurance due to the presence of agency costs. However, they model agency problems from the exogenous probability of diversion of retained capital for private benefits, while in this paper agency costs result from asymmetries in information on the stochastic variable. Their paper also differs in how the overall risk premium and the extent of insurance are determined in the model. In Meh et al., entrepreneurs self-insure through contingent instruments, while in this paper, the risk premium and insurance arise from the interaction between risk-aversion frictions and their impact on the lender-borrower relationship. This difference is particularly relevant as the evolution of insurance will be the dynamic driving force behind the private equity premium. Finally, these authors study capital accumulation, while I analyze the response of shocks in terms of amplification and propagation.

Related empirical evidence shows that the private equity premium (or the premium on equity of privately held traded firms) is similar in many ways to the public equity premium (the premium on equity of publicly traded firms) commonly described in the literature. Moskowitz and Vissing-Jørgensen (2002), for instance, document that, on average, the market return to private equity is surprisingly no

higher than the public equity return.⁴ This result is puzzling considering that private equity is dramatically concentrated, both in terms of its importance for total entrepreneurial net worth and in terms of diversification. Additionally, they find that public and private equity returns are highly correlated⁵ and that indexes of returns are equally volatile. Nonetheless, the objective of this paper is not to explain this puzzle, but rather to explore the implications of the private equity premium in the context of a general equilibrium model.

The next chapter studies the role of domestic private entrepreneurs in the context of a small open economy. The purpose of this section is two-fold. First, I examine the role of exchange rates in affecting the private equity premium and the model's equilibrium and dynamics. In particular, changes in the real exchange rate affect not only the economy's international trade, but also capital flows. For instance, a real depreciation of the local currency benefits the economy by increasing the cost of exports. On the other hand, it raises the cost of producing capital by increasing both the value of imports (assuming that capital is partially produced using imported goods), and the value of debt repayment (assuming that part of this small economy's source of funds is foreign denominated debt). The fact that an

⁴They estimate the average return on private equity to be 12.3 percent in the period 1990-92, 17.0 percent in 1993-95, and 22.2 percent in 1996-98. Meanwhile, considering a weighted index of NYSE, AMEX and NASDAQ for firms of equivalent size, they find that over the same periods, the average returns to public equity were 11.0 percent, 14.6 percent and 24.7 percent, respectively. See also Gottschalg, Phalippou and Zollo (2004) for further evidence.

⁵Many authors find that the public equity return is countercyclical. See Mehra and Prescott (2003) for discussion and recent evidence.

economy borrows mainly in foreign currency is commonly referred to in the literature as liability dollarization.⁶ I find here that the exchange rate helps alleviate the propagating feature of the private equity premium. In particular, the exchange rate overshoots in anticipation of the amplified response of entrepreneurs due to changes in wealth and the private equity premium, and it quickly re-adjusts back when the entrepreneur's response actually takes place. This faster adjustment of the exchange rate after this overshooting implies opposite effects on entrepreneurial wealth and, therefore, quicker recovery of the private equity premium and investment to their steady state values.

Second, in this section I also execute an exchange rate regime comparison when this modified setup of risk averse entrepreneurs is considered. I study whether the existence of the private equity premium may challenge the conclusion from previous research that the expansionary effect of a nominal devaluation more than offsets its contractionary impact, even taking into account the adverse effect implied by the financial accelerator, implying that flexible exchange rate regimes are preferable to fixed regimes in terms of absorbing the effects of real shocks.⁷ I show that the greater volatility associated with flexible exchange rate regimes increases the private equity premium and, thus, the supply of capital, amplifying the output response to shocks. I find that fixed exchange rate regimes could be preferable under less restrictive conditions than, for instance, the “unrealistic” set of parameters found

⁶See Calvo and Reinhart (1999) and Chang and Velasco (2000).

⁷For analysis along these lines, see Céspedes, Chang and Velasco (2000) and Bernanke, Gertler and Natalucci (2001).

by Céspedes, Chang and Velasco (2000).

The outline of this thesis is as follows. Following this introductory chapter, Chapter 2 presents, solves and discusses the closed economy framework. The first section of this chapter examines the partial equilibrium interaction between a risk-averse entrepreneur and a risk neutral lender. I analyze the structure of this augmented risk premium, compare it with the benchmark case where all agents are risk neutral, and analyze how this risk premium is affected by different levels of net worth, the bankruptcy cost factor, risk aversion and the volatility of stochastic returns. Section 2.2 embeds these partial equilibrium results into a stochastic dynamic general equilibrium model. Following Bernanke and Gertler (1989), Carlstrom and Fuerst (1996) and Bernanke, Gertler and Gilchrist (1999), the point of departure for this analysis is a real business cycle model, where the basic financial friction involved is the external finance premium determined endogenously by the level of involvement of entrepreneurs. I depart from these papers by allowing entrepreneurs to be risk averse. Parametrization of the model and simulation exercises are presented in section 2.3.

Chapter 3 analyzes the open-economy specification of the model, following a structure similar to Chapter 2. In section 3.2, I review the optimal contract between the domestic risk averse entrepreneur and the international lender. In analyzing the equilibrium features of the model, I consider the impact of a depreciation of the local currency on the generated supply of capital, in addition to the impact of changes in other parameters, such as risk risk aversion, default costs, net worth and volatility. Section 3.3 analyzes the aggregate effects of the risk premium implied by the optimal

contract, first under flexible prices (Section 3.4) and then when there are nominal rigidities, so that monetary policy imposes real effects on the economy (Section 3.5). In the latter section, I briefly examine the effect of the private equity premium under both flexible and fixed exchange rate regimes, followed by an exchange rate regime comparison analysis, where I contrast the response to shocks under both regimes.

Chapter 4 provides some concluding remarks.

Chapter 2

The Closed Economy Framework

2.1 The Partial Equilibrium Model: The Optimal Contract

This section analyzes the implications of the optimal contract between a lender and a borrower in partial equilibrium in order to examine the risk premium that arises from bankruptcy costs and entrepreneurial risk aversion.

There are two groups of participants in this model. Risk averse entrepreneurs finance their purchase of capital using both their internal net worth as well as external, borrowed financing. Entrepreneurs then rent this capital to final goods firms. Meanwhile, risk-neutral lenders provide financing to entrepreneurs and receive a financial return for this service.

The contract between borrowers and lenders is subject to informational frictions. More specifically, the actual level of capital available for production following investment is private information and is idiosyncratic to each entrepreneur, though its distribution is common knowledge. The entrepreneur decides on the level of investment (and therefore, borrowing) prior to the realization of the stochastic return to investment. The realization is then revealed to the entrepreneur as private information. Assuming costly state verification (Townsend, 1979), lenders can observe the return to investment only if they pay an auditing cost. Since it would only be optimal for the lender to accept this cost if the entrepreneur were not fulfilling

his contractual repayment, the auditing cost can also be interpreted as a bankruptcy cost: a default allows lenders to observe the entrepreneur's private information, but they cannot recover all the remaining revenues if the entrepreneur goes bankrupt. This can be further interpreted as the cost that lenders face due to the fact that they are not familiarized with the business of the entrepreneur, and for simplicity the cost is assumed to be a constant fraction of remaining revenues. Notice that the only source of uncertainty is the idiosyncratic realization of effective capital, and thus there is no aggregate uncertainty. Therefore, throughout this dissertation the existence of an optimal contract is conditional to this assumption.

Standard literature on the lender-borrower relationship suggests that under the circumstances described above, lower involvement of the entrepreneur in the project raises the implicit agency problem due to the conflict of interest involved. In equilibrium, lenders require compensation above and beyond the opportunity cost of their funds, or an external finance premium. When entrepreneurs are risk-averse, they will also require a premium due to the stochastic nature of the return on their investments, henceforth referred to as the private equity premium. As will become evident in section 2.1.2, risk aversion also imposes an extra cost on lenders, as they must provide insurance to the entrepreneur. These costs will also be incorporated into the external finance premium.

To provide a benchmark case, and for presentational purposes, section 2.1.1 briefly analyzes the optimal contract when all agents are risk-neutral. As in previous literature, I find that informational frictions lead to a negative relationship between optimal entrepreneurial involvement and the external finance premium. This section

serves as the point of departure for section 2.1.2, which deals with the case where entrepreneurs are risk-averse.

To motivate imperfect information, let us assume that the effective units of capital available for production after entrepreneur j 's purchase of K_{t+1}^j units of raw capital is uncertain and depends on the realization of a random variable ω^j , namely

$$\omega^j K_{t+1}^j$$

with a known distribution function $H(\omega)$ with strictly positive support, in which $E_t(\omega) = 1$.

Every period, the entrepreneur makes $\omega^j K_{t+1}^j$ units of capital available to final good firms for production, charging a real rental rate of R_{t+1} , which, in this section, is assumed to be constant and known beforehand (R_{t+1} will be endogenized in section 2.2). Thus, the entrepreneur faces the equivalent of a linear production function, where the return to capital is given by

$$R_{t+1} \omega^j K_{t+1}^j$$

I assume complete depreciation of capital after one period.

Additionally, suppose that entrepreneur j 's net worth may not be enough to cover all of his investment. Then, the level of borrowing (B_{t+1}^j) is the portion of total investment not covered by the entrepreneur's internal net worth (N_t^j):

$$B_{t+1}^j = K_{t+1}^j - \lambda_t^j N_t^j \tag{2.1}$$

where λ_t is the proportion of net worth that the entrepreneur chooses to use in

the risky investment, considering that the entrepreneur can also invest in risk-free assets.

Borrowing is provided by competitive risk neutral lenders that charge a contractual nominal gross interest rate Z_{t+1}^j . Therefore, the nominal repayment for an entrepreneur j in engaging in a contract for B_{t+1}^j units of debt is given by

$$Z_{t+1}^j B_{t+1}^j$$

The optimal contract minimizes the agency problem, considering that the entrepreneur will choose the *ex-ante* optimal level of investment, taking as given for now variables that are known as of period t , such as prices, entrepreneurial net worth, and the aggregate return to capital for period $t + 1$ (R_{t+1}). These variables will be endogenized in the general equilibrium model in section 2.2. The realization of ω^j does not affect the decision of how much to invest, though it will influence the entrepreneur's repayment decisions. As in Carlstrom and Fuerst (1996), I assume that entrepreneurs can maintain enough anonymity that it is possible for them to engage in one-period contracts with lenders, regardless of their repayment history.

2.1.1 Model with Risk-Neutral Entrepreneurs: The Benchmark Case

This section looks at the structure of debt repayment when all actors are risk-neutral and there are informational frictions resulting from agency problems. This section is used as a benchmark for the case where the entrepreneur is risk-averse. Since a risk-neutral entrepreneur cares only about the mean return to his investment, he is willing to bear all of the risk. Hence, in non-bankruptcy states,

the entrepreneur is able to guarantee the lender (who does not freely observe the state of nature) a constant repayment per unit of debt, safe from all idiosyncratic risk, such that the lender is willing to participate in this contract.

Let $\bar{\omega}^j$ define the realization of ω^j such that the entrepreneur breaks even. Therefore, $\bar{\omega}^j$ solves:

$$\bar{\omega}^j R_{t+1} K_{t+1}^j = Z_{t+1}^j B_{t+1}^j \quad (2.2)$$

If $\omega^j \geq \bar{\omega}^j$, the project succeeds, and due to costly state verification, the lender optimally charges a fixed repayment per unit of capital, independent of the realization of ω^j (the true realization of ω remains private information). On the other hand, if the realization of ω^j is lower than $\bar{\omega}^j$, then the entrepreneur defaults on his contracted debt. In such a case, the lender learns the true value of ω^j and takes possession of fraction $1 - \mu$ of the remaining revenues, with μ being the bankruptcy cost rate.

Under this set-up, the resulting lender participation constraint is that the expected return from lending to entrepreneur j must not be lower than the lender's opportunity cost:

$$[1 - H(\bar{\omega}^j)] \bar{\omega}^j R_{t+1} K_{t+1}^j + (1 - \mu) \int_0^{\bar{\omega}^j} \omega R_{t+1} K_{t+1}^j dH(\omega) \geq (1 + \rho_{t+1}) B_{t+1}^j \quad (2.3)$$

Following this scheme, entrepreneur j 's expected profits can be represented by

$$(1 + \rho_{t+1})(1 - \lambda_t^j) N_t + \int_{\bar{\omega}^j}^{\infty} \omega R_{t+1} K_{t+1}^j dH(\omega) - [1 - H(\bar{\omega}^j)] \bar{\omega}^j R_{t+1} K_{t+1}^j \quad (2.4)$$

where the first term is the return from investing in the risk-free asset, and second and third term represent the risky investment's expected revenues and expected repayment, respectively. Equations (2.3) and (2.4) describe what is called a standard debt contract, which under this setup is an optimal contract (Gale and Hellwig, 1983).

Proposition *A standard debt contract with maximum equity participation weakly dominates any other optimal contract.*

Proof *Consider for now that in case of bankruptcy, risk-free assets are protected from confiscation by limited liability. The optimization problem would then solve:*

$$\max_{\{\lambda_t^j, \bar{\omega}, K_{t+1}^j\}} (1 + \rho_{t+1})(1 - \lambda_t^j)N_t + \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega})R_{t+1}K_{t+1}^j dH(\omega)$$

subject to lender participation constraint

$$\int_{\bar{\omega}}^{\infty} \bar{\omega}R_{t+1}K_{t+1}^j dH(\omega) + (1 - \mu) \int_0^{\bar{\omega}} \omega R_{t+1}K_{t+1}^j dH(\omega) = (1 + \rho_{t+1})[K_{t+1}^j - \lambda_t^j N_t^j]$$

Since $E(\omega) = \int_0^{\infty} \omega dH(\omega) = \int_0^{\bar{\omega}} \omega dH(\omega) + \int_{\bar{\omega}}^{\infty} \omega dH(\omega) = 1$, then

$$\int_0^{\bar{\omega}} \omega R_{t+1}K_{t+1}^j dH(\omega) = R_{t+1}K_{t+1}^j - \int_{\bar{\omega}}^{\infty} \omega R_{t+1}K_{t+1}^j dH(\omega)$$

Then the lender participation constraint can be re-expressed as

$$\begin{aligned} \int_{\bar{\omega}}^{\infty} \bar{\omega}R_{t+1}K_{t+1}^j dH(\omega) + R_{t+1}K_{t+1}^j - \int_{\bar{\omega}}^{\infty} \omega R_{t+1}K_{t+1}^j dH(\omega) - \mu \int_0^{\bar{\omega}} \omega R_{t+1}K_{t+1}^j dH(\omega) = \\ (1 + \rho_{t+1})[K_{t+1}^j - \lambda_t^j N_t^j] \end{aligned}$$

The lender participation constraint can be re-expressed as

$$\begin{aligned} & \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}) R_{t+1} K_{t+1}^j dH(\omega) = \\ & = R_{t+1} K_{t+1}^j - \mu \int_0^{\bar{\omega}} \omega R_{t+1} K_{t+1}^j dH(\omega) - (1 + \rho_{t+1}) [K_{t+1}^j - \lambda_t^j N_t^j] \end{aligned} \quad (2.5)$$

Therefore, the objective function can be re-written as

$$(1 + \rho_{t+1})(1 - \lambda_t^j) N_t + R_{t+1} K_{t+1}^j - \mu \int_0^{\bar{\omega}} \omega R_{t+1} K_{t+1}^j dH(\omega) - (1 + \rho_{t+1}) [K_{t+1}^j - \lambda_t^j N_t^j]$$

or equivalently,

$$R_{t+1} K_{t+1}^j - \mu \int_0^{\bar{\omega}} \omega R_{t+1} K_{t+1}^j dH(\omega) - (1 + \rho_{t+1}) [K_{t+1}^j - N_t^j] \quad (2.6)$$

which is independent of λ_t^j .

Therefore, under limited liability, the entrepreneur is indifferent to how much of his wealth is kept in safe assets. Intuitively, any increase in expected utility coming from having more assets free from seizure are completely compensated by the increase in the contractual interest rate (Z_{t+1}^j) that entrepreneurs face. The latter comes from the fact that lower entrepreneurial upfront contribution to the project ($\lambda_t^j N_t^j$) are translated into higher lender opportunity cost and agency costs. If there was not limited liability, then risk-free assets would be confiscated in defaulting states, the first term of entrepreneurs objective function would drop out and λ_t^j would be strictly equal to one, as that would minimize agency costs. Therefore, for simplicity I will assume for the rest of this section that the entrepreneur invests all his net worth in the project (i.e., $\lambda_t^j = 1$), and holds no risk-free assets, so that equation (2.1) is re-expressed as:

$$B_{t+1}^j = K_{t+1}^j - N_t^j \quad (2.7)$$

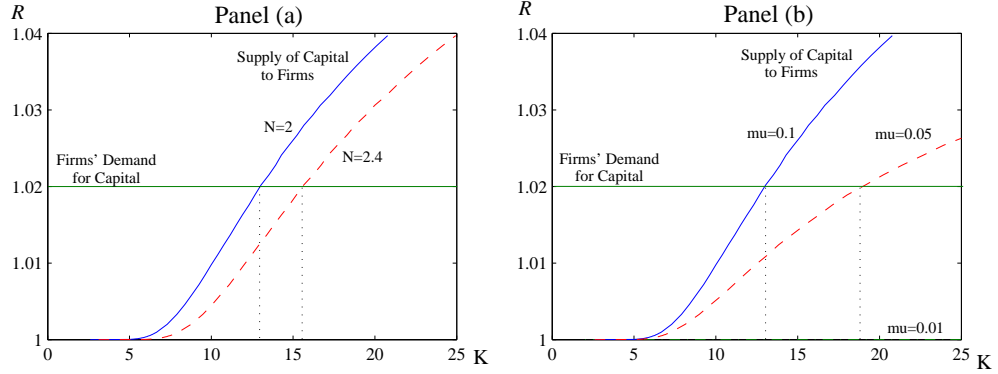
Thus, the optimal contract determines the value of capital (K_{t+1}^j) and the repayment per unit of capital ($\bar{\omega}^j$) that maximizes the entrepreneur's objective function (2.4), subject to the lender's participation constraint (2.3). The optimality conditions from this maximization problem provide the contract's optimal level of capital, given the distribution of ω , and the value of the variables R_{t+1} , N_t^j (taken as given in this section) and parameters μ and ρ_{t+1} . Specifically, the optimality conditions imply the following relationship:

$$\begin{aligned} [1 - H(\bar{\omega})] \int_{\bar{\omega}}^{\infty} \omega R_{t+1} dH(\omega) &= [1 - H(\bar{\omega})] \int_{\bar{\omega}}^{\infty} \bar{\omega} R_{t+1} dH(\omega) + \\ [1 - H(\bar{\omega})](1 + \rho_{t+1}) \frac{N_t}{K_{t+1}} &+ \mu \bar{\omega} h(\bar{\omega}) \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}) R_{t+1} dH(\omega) \end{aligned} \quad (2.8)$$

The left hand side represents the expected returns per unit of capital for the entrepreneur, which equals the expected realization of the entrepreneur's investment in the states of nature for which the entrepreneur does not default (given by the probability $[1 - H(\bar{\omega})]$). The right hand side of equation displays the cost per unit of capital for the entrepreneur, which includes the capital repayment rate for the non-bankruptcy states of nature, and the opportunity cost to the entrepreneur of investing his net worth. The term $\mu \bar{\omega} h(\bar{\omega})$ captures the fact that the bankruptcy costs change as $\bar{\omega}$ adjusts, since variations in $\bar{\omega}$ change the probability of bankruptcy.

Optimal investment K_{t+1}^j is an upward sloping function of the exogenous aggregate rental rate of capital, R_{t+1} . Put differently, final goods firms renting capital from entrepreneurs face an upward sloping supply curve of capital, in the sense that

Figure 2.1: Optimal capital stock and net worth.



higher levels of capital are available to them only if the rental rate, R_{t+1} , is greater. This relationship between R_{t+1} and K_{t+1}^j is a manifestation of the entrepreneur's cost of funds schedule (as reflected in the contracted gross interest rate, Z_{t+1}^j), which accounts for the lender's opportunity costs and increasing agency costs associated with the entrepreneur's reliance on external financing. Therefore, one would expect an upward sloping supply of capital for levels of capital above entrepreneurial net worth (as the level of investor involvement declines, or as leverage rises), and a flat supply of capital (at the risk-free rate) otherwise. The demand curve for capital from final goods firms is assumed to be flat at a rental rate R_{t+1} for the moment.

This problem can be solved numerically. As a benchmark I assume that the idiosyncratic variable ω is normally distributed with a mean of 1 and a variance $\sigma^2 = 0.1$.¹ I set the parameter that represents the costs of bankruptcy (μ) equal to

¹The problem can be solved analytically if instead we assume that ω follows a uniform distribution between 0 and 2. In this case the maximization problem may be re-expressed as

$$\max_{\{K_{t+1}^j, \bar{\omega}^j\}} \left(1 - \bar{\omega} + \frac{\bar{\omega}^2}{4}\right) R_{t+1} K_{t+1}^j$$

10 percent of revenues. The entrepreneur's net worth is assumed to be 2, and for simplicity, the risk-free rate, ρ_{t+1} , is set equal to zero. Figure 2.1 plots the optimal supply of capital, where the dashed line in panel (a) shows the impact of an increase of 20 percent in net worth. As expected, the supply of capital curve lies above the risk-free rate when investment exceeds net worth, and is always upward sloping in K_{t+1}^j . Furthermore, as expected, the lower the bankruptcy cost (μ), the greater the optimal K_{t+1}^j for each value of R_{t+1} , as shown in panel (b). In particular, note that for $\mu = 0$, the external finance premium would be equal to zero, independent of the level of entrepreneurial involvement. This result demonstrates how the external finance premium comes directly from the fact that, due to informational frictions, the lender cannot completely recover the remaining revenues when the borrower defaults.

2.1.2 Model with Risk-Averse Entrepreneurs

In this section I assess the impact of assuming that borrowers are risk-averse. As before, I use a simple one-period maximization problem, maintaining certainty for the aggregate variables of this model, but allowing for idiosyncratic risk on the subject to

$$\left[\bar{\omega} - \frac{\bar{\omega}^2}{4} (1 + \mu) \right] R_{t+1} K_{t+1}^j = (1 + \rho_{t+1}) [K_{t+1}^j - N_t^j]$$

which yields an equilibrium optimal capital investment scheduled as a function of the rental rate on capital, risk-free rate and net worth given by

$$K_{t+1}^j = \frac{\left[\frac{R_{t+1}}{1 + \rho_{t+1}} \right] \mu^2}{\left[\frac{R_{t+1}}{1 + \rho_{t+1}} \right] \mu^2 - \left[\frac{R_{t+1}}{1 + \rho_{t+1}} \right]^2 \mu + \mu + 1 + \left[\frac{R_{t+1}}{1 + \rho_{t+1}} \right]^2 - 2 \left[\frac{R_{t+1}}{1 + \rho_{t+1}} \right]} \cdot N_t^j$$

investment return of individual entrepreneur j .

Analogous to section 2.1.1, entrepreneur j 's return to capital is determined by the stochastic realization of ω^j , and the optimal contract between the lender and the entrepreneur again arises from choosing the level of investment that maximizes the entrepreneur's expected utility, subject to the lender's participation constraint. However, the repayment schedule will not have the same structure as in section 2.1.1, mainly because default risks are not shared by lender and borrower in the same manner, keeping in mind that lenders are still risk-neutral, but borrowers are now risk-averse.

For states of nature in which the entrepreneur does not default, the result is the same as in the standard contract described in section 2.1.1: the entrepreneur maintains the true realization of ω^j as private information. He optimally repays the lender a fixed amount per unit of debt Z_{t+1}^j , and due to costly state verification the lender does not monitor the true state of nature. In this case, the risk-averse entrepreneur is willing to assume the risk on the upper part of the distribution of the return to capital, and the net return to capital is given by $\omega^j R_{t+1} K_{t+1}^j - Z_{t+1}^j B_{t+1}^j$.

On the other hand, for low states of nature, the standard debt contract is no longer optimal. If we assume that for a risk averse agent the marginal utility of zero consumption is infinity or very large², then the optimal contract must ensure the entrepreneur positive consumption in any state. One way to accomplish that is for the entrepreneur to invest in risk-free assets. However, that is not the only possibility. For default states, which are observable by both parties, the risk-neutral lender can

²See Gale and Hellwig (1985).

provide a positive return to the risk-averse entrepreneur, equal to $X_t^j \lambda_t^j N_t^j \leq \lambda_t^j N_t^j$. That is, the lender can insure a fraction $X_t^j \leq 1$ of the entrepreneur's initial contribution to the project.³ Consequently, the entrepreneur can receive insurance directly from the lender, can invest part of his net worth in risk-free assets, or both.

Proposition *With risk-averse entrepreneurs, a debt contract with maximum equity participation weakly dominates any other optimal contract.*

Proof *As in the risk-neutral case, let us consider for now that in case of bankruptcy, risk-free assets are protected from seizure by limited liability. In such a case, the optimization problem solves:*

$$\begin{aligned} \max_{\{\lambda_t^j, \hat{\omega}, K_{t+1}^j, X_t^j\}} \int_0^{\hat{\omega}^j} U\{(1 + \rho_{t+1})(1 - \lambda_t^j)N_t + \lambda_t^j X_t^j N_t^j\} dH(\omega) + \\ \int_{\hat{\omega}^j}^{\infty} U\{(1 + \rho_{t+1})(1 - \lambda_t^j)N_t + \omega^j R_{t+1} K_{t+1}^j - Z_{t+1}^j B_{t+1}^j\} dH(\omega) \end{aligned}$$

subject to lender participation constraint

$$\begin{aligned} \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1} K_{t+1}^j dH(\omega) + (1 - \mu) \int_0^{\hat{\omega}} \omega R_{t+1} K_{t+1}^j dH(\omega) - \lambda_t^j X_t^j N_t^j = \\ (1 + \rho_{t+1})[K_{t+1}^j - \lambda_t^j N_t^j] \end{aligned}$$

that can be re-expressed as

$$\begin{aligned} \lambda_t^j X_t^j N_t^j = \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1} K_{t+1}^j dH(\omega) + (1 - \mu) \int_0^{\hat{\omega}} \omega R_{t+1} K_{t+1}^j dH(\omega) \\ - (1 + \rho_{t+1})[K_{t+1}^j - \lambda_t^j N_t^j] \end{aligned} \tag{2.9}$$

³If there are no information asymmetries so that states are always observable, the standard contract theory result calls for a complete insurance from the risk neutral agent to the risk averse counterpart.

Additionally, $\hat{\omega}$ solves

$$\lambda_t^j X_t^j N_t^j = \hat{\omega}^j R_{t+1} K_{t+1}^j - Z_{t+1}^j B_{t+1}^j$$

or

$$Z_{t+1}^j B_{t+1}^j = \hat{\omega}^j R_{t+1} K_{t+1}^j - \lambda_t^j X_t^j N_t^j \quad (2.10)$$

Replacing equations (2.9) and (2.10) into the objective function, we get

$$\begin{aligned} & \int_0^{\hat{\omega}^j} U \left\{ \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1} K_{t+1}^j dH(\omega) + (1 - \mu) \int_0^{\hat{\omega}} \omega R_{t+1} K_{t+1}^j dH(\omega) \right. \\ & \quad \left. - (1 + \rho_{t+1}) [K_{t+1}^j - N_t^j] \right\} dH(\omega) \\ & + \int_0^{\hat{\omega}^j} U \left\{ (\omega^j - \hat{\omega}^j) R_{t+1} K_{t+1}^j + \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1} K_{t+1}^j dH(\omega) + \right. \\ & \quad \left. (1 - \mu) \int_0^{\hat{\omega}} \omega R_{t+1} K_{t+1}^j dH(\omega) - (1 + \rho_{t+1}) [K_{t+1}^j - N_t^j] \right\} dH(\omega) \end{aligned}$$

which is independent of λ_t^j .

Thus, as in the risk neutral case, the entrepreneur invests all his net worth in the project (i.e., $\lambda_t^j = 1$), and retains no part of his wealth in risk-free assets. Therefore, the entrepreneur maximizes

$$\int_0^{\hat{\omega}^j} U(X_t^j N_t^j) dH(\omega) + \int_{\hat{\omega}^j}^{\infty} U(\omega^j R_{t+1} K_{t+1}^j - Z_{t+1}^j B_{t+1}^j) dH(\omega) \quad (2.11)$$

where $\hat{\omega}^j$ is the cut-off default/non-default state value.

Entrepreneur j will be indifferent between defaulting or not when the utility of defaulting equals that of not defaulting. Therefore, the cut-off default/non-default state value, $\hat{\omega}^j$, solves

$$X_t^j N_t^j = \hat{\omega}^j R_{t+1} K_{t+1}^j - Z_{t+1}^j B_{t+1}^j \quad (2.12)$$

or equivalently,

$$Z_{t+1}^j B_{t+1}^j = \hat{\omega}^j R_{t+1} K_{t+1}^j - X_t^j N_t^j \quad (2.13)$$

Thus, the entrepreneur's expected utility can be re-expressed as

$$\int_0^{\hat{\omega}^j} U(X_t^j N_t^j) dH(\omega) + \int_{\hat{\omega}^j}^{\infty} U(X_t^j N_t^j + (\omega^j - \hat{\omega}^j) R_{t+1} K_{t+1}^j) dH(\omega) \quad (2.14)$$

where $\hat{\omega}^j$ also represents the repayment rate per unit of capital in the non-default states.

The entrepreneur receives $X_t^j N_t^j$ regardless ω . In addition, if $\omega^j > \hat{\omega}^j$, the entrepreneur gets an extra random return $\omega^j R_{t+1} K_{t+1}^j$ and pays to the lender a fixed payment $\hat{\omega}^j R_{t+1} K_{t+1}^j$. This insurance can be interpreted as a hedging instrument available to entrepreneurs, and thus one can think of entrepreneurs as operating in an environment of incomplete markets, as they do not completely eliminate the idiosyncratic risk they face. However, one should be cautious with such an interpretation, as the lack complete markets is not exogenously imposed, but rather is a result of the incentives given to lenders and entrepreneurs. That is, entrepreneurs are willing to face some uncertainty because it allows them to receive higher returns by maintaining the realization of ω as private information in good states.

Lenders, for their part, have to pay a positive price to learn the true realization of ω , and are willing to charge a constant repayment rate to avoid such costs. Hence,

the lender's participation constraint is given by

$$\int_0^{\hat{\omega}^j} [(1 - \mu) \omega R_{t+1} K_{t+1}^j] dH(\omega) + \int_{\hat{\omega}^j}^{\infty} [\hat{\omega}^j R_{t+1} K_{t+1}^j] dH(\omega) - X_t^j N_t^j \geq (1 + \rho_{t+1}) B_{t+1}^j \quad (2.15)$$

That is, the expected return from lending must be greater than or equal to the opportunity cost of the lender's funds.

Therefore, the optimal contract is given by the choice of K_{t+1}^j , $\hat{\omega}^j$ and X_t^j that maximize the entrepreneur's expected utility (2.14), subject to the lender's participation constraint (2.15), taking as given for now N_t^j , R_{t+1} , ρ_{t+1} , and the distribution of ω ⁴ The first order conditions with respect to K_{t+1} and $\hat{\omega}$ are given by equations (2.16) and (2.17), respectively.

$$\begin{aligned} \int_{\hat{\omega}}^{\infty} (\omega - \hat{\omega}) dH(\omega) R_{t+1} &= \left[1 + \frac{E(U'(\cdot) | \omega < \hat{\omega})}{E(U'(\cdot) | \omega > \hat{\omega})} \right] \\ \left[(1 + \rho_{t+1}) - (1 - \mu) \int_0^{\hat{\omega}} \omega R_{t+1} dH(\omega) + R_{t+1} \hat{\omega} [1 - H(\hat{\omega})] \right] \\ &\quad - \frac{R_{t+1}}{E(U'(\cdot) | \omega > \hat{\omega})} Cov\{U'(\cdot), \omega\} \end{aligned} \quad (2.16)$$

$$\left[1 + \frac{E(U'(\cdot) | \omega < \hat{\omega})}{E(U'(\cdot) | \omega > \hat{\omega})} \right] = \frac{1}{[1 - H(\hat{\omega})] - \mu \hat{\omega} h(\hat{\omega})} \quad (2.17)$$

which after some algebraic manipulation imply the following optimality condition:

⁴Gale and Hellwig (1985) briefly analyzed an analogous version of this contract and proved that it is the optimal contract.

$$\begin{aligned}
\frac{X_t N_t}{K_{t+1}} + [1 - H(\hat{\omega})] \int_{\hat{\omega}}^{\infty} \omega R_{t+1} dH(\omega) &= [1 - H(\hat{\omega})] \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1} dH(\omega) \\
+ (1 + \rho_{t+1}) \frac{N_t}{K_{t+1}} + \mu \hat{\omega} h(\hat{\omega}) \int_{\hat{\omega}}^{\infty} (\omega - \hat{\omega}) R_{t+1} dH(\omega) \\
- \frac{[1 - H(\hat{\omega}) - \mu \hat{\omega} h(\hat{\omega})]}{E(U'(\cdot) | \omega > \hat{\omega})} \cdot Cov\{U'(\cdot), \omega R_{t+1}\} & \quad (2.18)
\end{aligned}$$

Optimal investment decisions in equilibrium equate the entrepreneur's expected returns capital to its marginal cost. This condition is captured by the left and right hand sides of equation (2.18). Specifically, the left hand side is given by the sum of the insurance that the entrepreneur receives in any state of nature, and the expected net return to investment in states where the entrepreneur does not default.

The right hand side of equation (2.18) shows the marginal cost for the entrepreneur, which is captured by four components. As in the risk-neutral case, the first two costs are given by, respectively, the per unit of capital repayment in the states of nature where the entrepreneur does not default, and the opportunity cost for the entrepreneur of investing his net worth. The third term, as in the risk-neutral case, captures the change in expected default costs due to changes in $\hat{\omega}$. The risk-averse entrepreneur, however, faces a fourth cost, associated with the risky nature of his investment's returns. This last component depends on the covariance between the return to capital and the entrepreneur's marginal utility of consumption. Since the entrepreneur is risk-averse, this covariance is negative. When entrepreneurs are risk-neutral, the covariance is zero. This additional cost borne by the risk-averse

entrepreneur due to the cost of facing uncertain returns is what I call the private equity premium.

The equilibrium rental rate of capital depends first on the external finance premium, or the component of the pecuniary borrowing cost that corresponds to the markup over the risk-free interest rate that lenders charge entrepreneurs due to agency problems and insurance costs. Secondly, the equilibrium rental rate of capital also depends on the premium required by the entrepreneur due to the stochastic nature of his investments. This second component depends on the covariance between the entrepreneur's return and the entrepreneur's marginal utility of consumption. As a result, a risk-averse entrepreneur requires a higher expected return on capital than a risk-neutral entrepreneur in order to invest. In other words, for a given return to capital, the risk-averse entrepreneur is willing to supply a lower quantity of capital than the risk-neutral entrepreneur, as uncertainty implies a decrease in his utility.

Further, this endogenous private equity premium will create an additional financial friction that causes business cycle fluctuations to become stronger and sharper and more persistent over time in response to real shocks. This feature will be analyzed in more detail in the general equilibrium framework in section 2.2. Before embarking upon that analysis, however, the next section numerically explores the main attributes of the optimal contract from this section. Specifically, I show how investment, the insurance rate and the repayment schedule behave in response to changes in exogenous variables such as the aggregate return to capital (exogenous at this point, though it will be endogenized in the general equilibrium model), the

bankruptcy cost parameter, the coefficient of risk aversion and the volatility of the idiosyncratic return to investment.

Numerical Approach

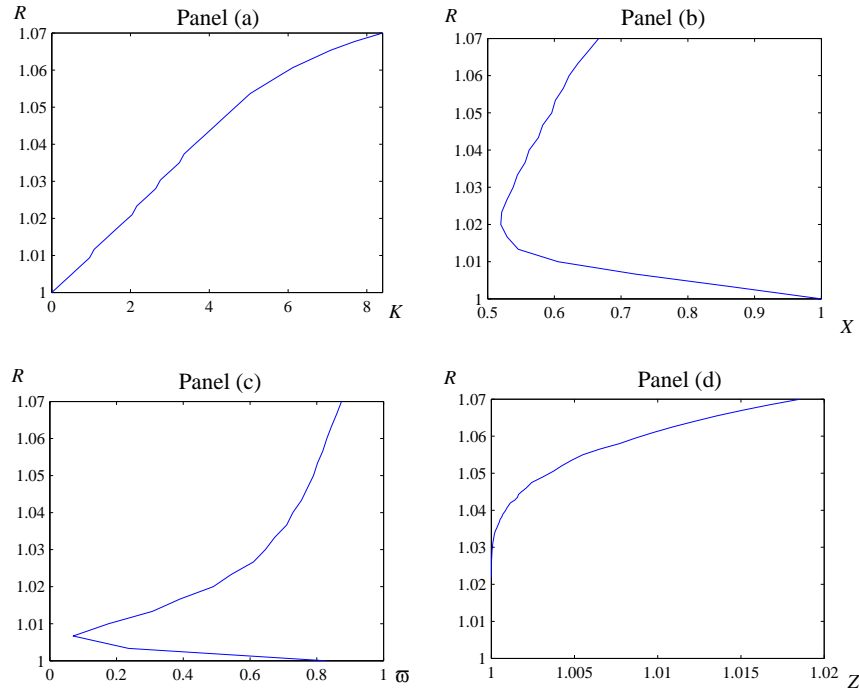
In order to analyze the main effects of entrepreneurial risk aversion, I select parameters and solve for the optimal contract numerically.⁵ As in section 2.1.1, the goal of this section is to identify the most important features and qualitative results of the modified financial contract, and to present results on the quantitative impact of parameter changes.

Figure 2.2 shows the simulated behavior of some variables of interest for different values of the exogenous rental rate on capital. Panel (a) shows the supply of capital to final goods firms as a function of the exogenous rental rate on capital. As explained earlier, this upward sloping supply of capital reflects not only the default cost for the lender, as in section 2.1.1, but also (1) the extra return that risk-averse entrepreneurs require in order to expose their own net worth by investing in a risky project, and (2) the cost of insurance that lenders provide to the risk-averse entrepreneur. In contrast to section 2.1.1, capital will be supplied only when the rental rate on capital strictly exceeds the opportunity cost of funds.

Risk-averse entrepreneurs demand a real return above the risk-free rate for any pos-

⁵The benchmark assumptions of section 2.1.1 are repeated with respect to the distribution of the idiosyncratic variable ω , the cost of bankruptcy (μ), the entrepreneur's net worth and the risk-free rate. Additionally, the entrepreneur is assumed to have a CRRA utility function with a coefficient of relative risk aversion equal to 2.

Figure 2.2: Real Return on Capital and Optimal Investment



itive level of investment, even if the investment does not require external financing ($K_{t+1}^j \leq N_{t+1}^j$).

Panels (b) and (c) show how the insurance per unit of net worth (X_t^j) and the non-default repayment rate per unit of capital ($\hat{\omega}$) behave as the return to capital rises. For levels of R_{t+1} where borrowing is needed (when $K_{t+1}^j > N_{t+1}^j$), X_t^j and $\hat{\omega}$ increase, together with capital, as the return to capital rises. Basically, a rise in R_{t+1} increases the returns from capital investment. The corresponding increase in K_{t+1}^j is translated into higher agency costs for the lender that more than offset the increase in revenues coming from the rise in R_{t+1} . This can be compensated through a combination of an increase in debt repayment rate ($\hat{\omega}$) and the insurance rate (X_t^j) (the latter due to the increase in defaulting states, that are compensated

through higher insurance.) For the parameters chosen, the lender is ensured a minimum consumption as soon as the entrepreneur relies on external financing for his investment.

For levels of R_{t+1} where borrowing is not needed (when $K_{t+1}^j \leq N_{t+1}^j$), there is still risk sharing among lenders and borrowers. Specifically, in those cases the entrepreneur lends money to the lender—right hand side of the lender participation constraint (equation 2.15) becomes negative—and thus, insurance increases and approaches to 1 (complete insurance). Additionally, as R_{t+1} decreases, given that the lender is effectively borrowing from the entrepreneur, the repayment rate initially decreases, and then, as the insurance rate increases the repayment rate also needs to be higher.

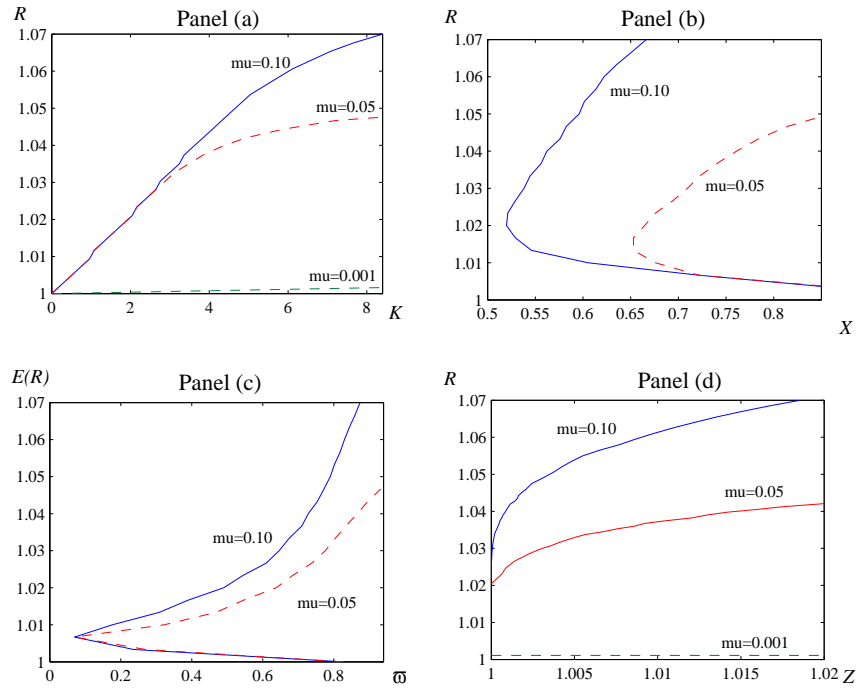
The supply curve of capital depends in part on the entrepreneur's marginal contractual cost of funds schedule, which is given by Z_{t+1}^j from equation (2.13), and is shown in panel (d) of Figure 2.2. The entrepreneur's marginal contractual cost of funds increases as the return on capital rises, due to increasing opportunity costs, agency costs and insurance costs associated with levels of investment above entrepreneurial net worth (recall that K_{t+1}^j increases when R_{t+1} rises). This upsurge in the marginal cost of funds occurs despite the fact that increases in R_{t+1} have a favorable impact on lender profits.

Changes in the Default Cost Parameter

Figure 2.3 reproduces Figure 2.2 for different levels of the bankruptcy parameter (μ). As shown in panel (a), a lower μ increases the optimal K at each level of R_{t+1} . This result can be intuitively explained as follows: a reduction in μ reduces the cost of bankruptcy, increasing the returns from lending. This change makes the lender's participation in the project less costly, where by participation I mean an increase in $\hat{\omega}$ and a higher provision of insurance X_t^j to the entrepreneur. The lender is therefore willing to bear more risk and allow more default. These effects can be seen in panels (b) and (c) of Figure 2.3, where for each level of R_{t+1} , the optimal $\hat{\omega}$ and X_t^j are higher. The lender's greater involvement, together with the corresponding increase in the insurance provided, make the marginal contractual cost of funds increase for each level of R_{t+1} , as seen in panel (d).

As in the risk-neutral case, in the extreme situation that μ approaches zero, the optimal K diverges. This means that if this financial friction is "shut-down", that is, if the lender can recuperate all revenues in the case of entrepreneurial default, then the return to capital that borrowers and lenders require in order to carry out the contract equals the risk-free rate. This is true in spite of the risk-aversion frictions coming from the borrower. The intuition is straightforward: if there are no costs of verifying revenue for the lender, risk-neutrality allows the lender to completely insure the borrower, ensuring him the level of his initial net worth for any state of nature. In other words, the lender increases his involvement in the project to the maximum and buys the project from the risk-averse entrepreneur at price N_t^j ,

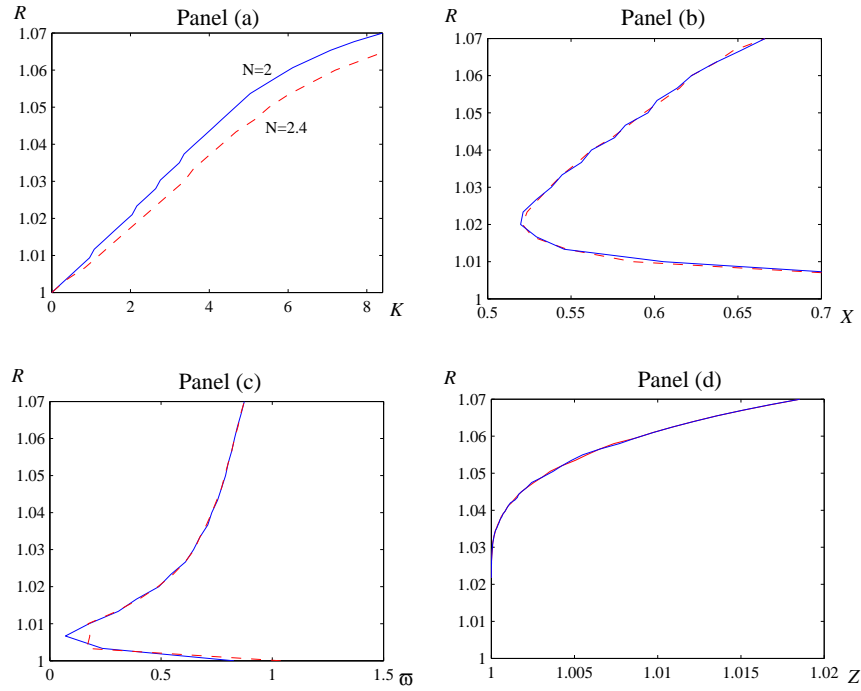
Figure 2.3: Change in μ



executing the project himself in a frictionless environment.⁶ This shows that the existence of private information is crucial for frictions coming from entrepreneurial risk aversion to have real effects.

In general, however, the lender does not provide full insurance when the cost of bankruptcy (μ) is positive. Therefore, the risk-averse entrepreneur requires a return to capital above the risk-free rate, even for levels of investment below his net worth (that is, when the entrepreneur does not need to rely on external finance).

Figure 2.4: Change in Internal Net Worth



Change in Internal Net Worth

Figure 2.4 analyzes the effect of a 20 percent increase in net worth (from 2 to 2.4). From the perspective of entrepreneurs, higher net worth implies higher guaranteed consumption, which in turn implies a decrease in the private equity premium. Therefore, the entrepreneur will be willing to invest a greater amount of capital for each level of R_{t+1} . This can be seen in panel (a) of Figure 2.4.

The next question is what effect the increase in entrepreneurial net worth has on the lender. On the one hand, an increase in N_t^j reduces the lender's opportunity cost by $(1 + \rho_{t+1})N_t^j$ (see equation 2.15), as the entrepreneur's need for external

⁶This means that X_t^j becomes 1 and $\hat{\omega}$ diverges to infinity. These results cannot be seen in the graphs because there are no equilibria for levels of R_{t+1} above the risk-free rate.

financing decreases. However, higher N_t^j raises the insurance cost $X_t^j N_t^j$. The partial net effect of higher N_t^j on the lender's profit is positive, but, as it turns out, this is completely offset by the increase in the agency cost resulting from the rise in optimal K_{t+1}^j at each level of R_{t+1} . Therefore, neither X_t^j nor $\hat{\omega}$ are affected by the increase in N_t^j (panels (b) and (c) of figure 2.4), and thus the marginal cost of funds remains constant (panel (d) of figure 2.4).

Changes in Risk Aversion

Figure 2.5: Change in Risk Aversion

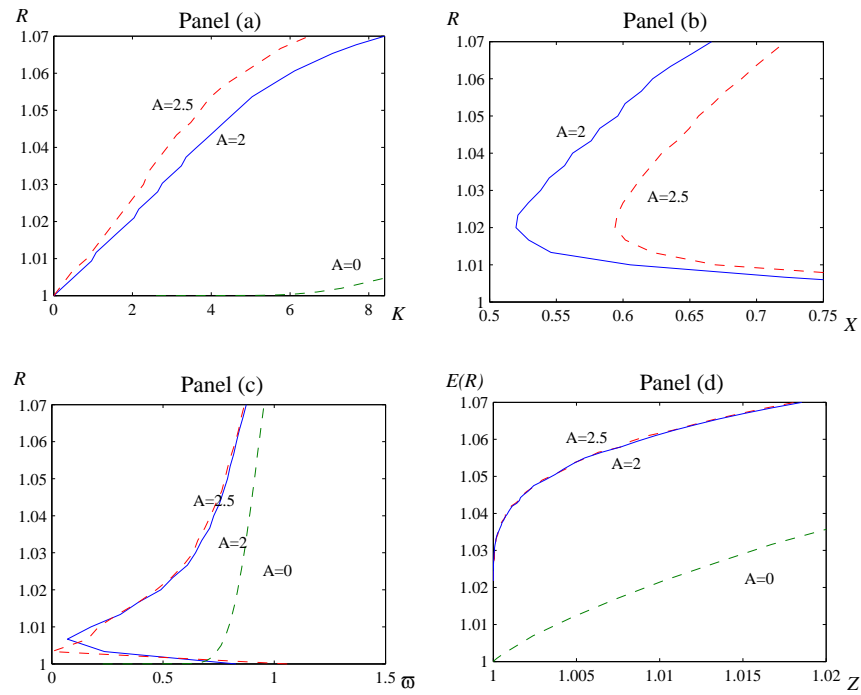


Figure 2.5 examines the impact of varying the coefficient of risk-aversion. If entrepreneurs are more risk-averse, they require a higher return in order to invest in a risky project. That is, for each level of R_{t+1} , they invest less capital, as demonstrated

in panel (a).

Higher risk aversion makes the entrepreneur demand a higher level of insurance X_t^j for each level of R_{t+1} (panel (b) of Figure 2.5). This costly insurance negatively impacts lender profits. However, the decrease in K_{t+1}^j resulting from a higher level of risk aversion causes the agency cost to fall, which more than offsets the negative impact of the increase in X_t^j on lender profits. Overall, this allows a small decrease in $\hat{\omega}$ and, thus, in the marginal cost of funds (panels (c) and (d) of Figure 2.5).

Figure 2.5 also illustrates the impact of risk aversion relative to risk-neutrality. As expected, risk-neutrality generates impacts similar to lower levels of risk aversion, although naturally there is no insurance in the case of risk-neutrality.

Figure 2.6: Decomposition of the Risk Premium

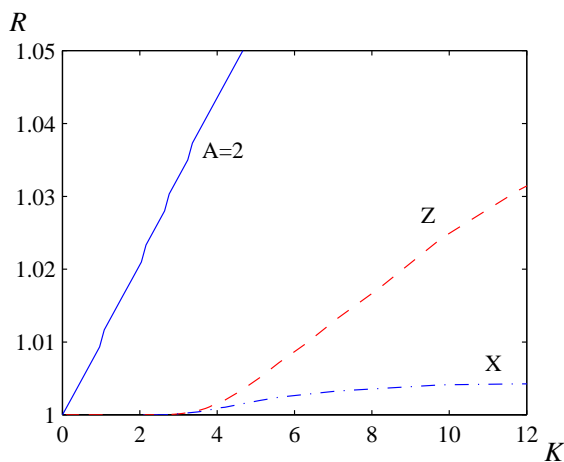
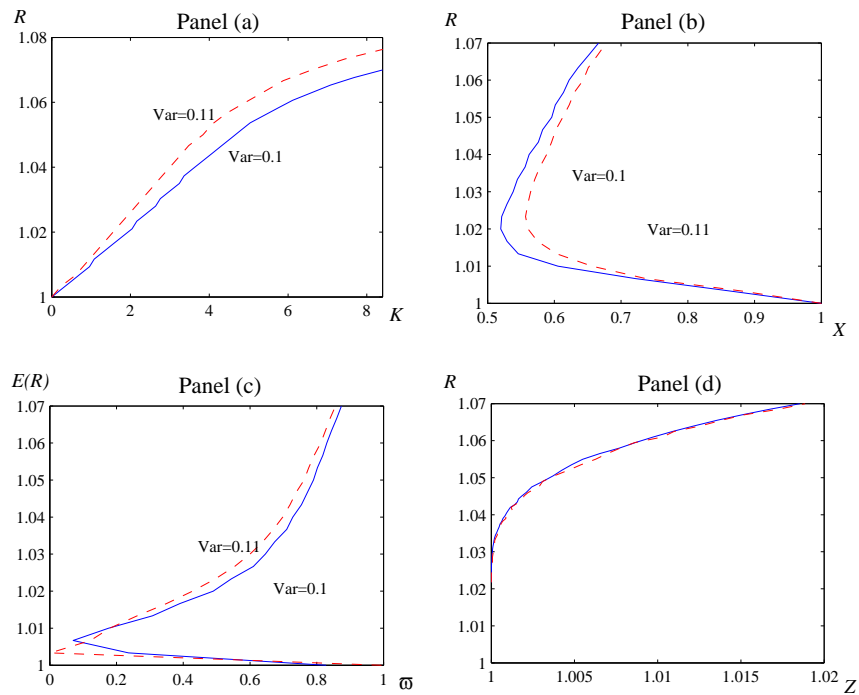


Figure 2.6 shows the decomposition of the overall wedge between the return of capital and the economy's risk-free rate (given by the solid line) into components due to the private equity premium, agency costs and insurance. In particular, the middle line and the bottom line plot the equilibrium contractual interest rate (Z_t^j) and the

lender cost of providing insurance, for different levels of the equilibrium K_{t+1}^j . The interpretation is as follows: the gap between the middle line and the bottom line corresponds to the part of the cost of borrowing that excludes insurance costs, that is, agency costs. The gap between the solid line and the middle line captures the return of capital on top of the borrowing costs, which is determined by the private equity premium.

Changes in the Volatility of Returns

Figure 2.7: Change in Variance



The effect of an increase in the volatility of returns to capital is qualitatively similar to that of higher risk aversion. That is, higher variance makes risk-averse entrepreneurs demand higher returns to capital in order to invest their net worth in

a uncertain project. That is, as in the case of higher risk aversion, greater volatility makes the entrepreneur demand a higher level of insurance X_t^j and a lower K_{t+1}^j for each level of R_{t+1} . The overall effect on lender profits is positive, allowing the lender to reduce the non-default repayment schedule $\hat{\omega}$ (and thus, the marginal cost of funds) for each level of the rental rate on capital, as shown in Figure 2.7.

2.2 The General Equilibrium Model

This section analyzes the dynamics and aggregate effects of the optimal contract by incorporating the modified supply of capital, derived in the previous section, into a stochastic dynamic general equilibrium model. I consider a closed economy that produces and consumes one good in an infinite horizon framework. This aggregate good is manufactured with labor (L) supplied by workers, and capital (K) supplied by entrepreneurs, combined through a standard constant returns to scale Cobb-Douglas production function

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} \tag{2.19}$$

where A_t is aggregate multifactor productivity.

2.2.1 The Firms

There is a representative firm that maximizes profits by optimally choosing capital, labor and total output.

$$\Pi_t = P_t Y_t - R_t K_t - W_t L_t$$

where P_t is the price of output, R_t is the domestic nominal rental rate on capital and W_t is the nominal wage paid to the representative worker.

The standard optimality conditions for capital and labor, respectively, from the firm's profit maximization are given by:

$$\frac{R_t}{P_t} = \alpha \frac{Y_t}{K_t} \quad (2.20)$$

$$\frac{W_t}{P_t} = (1 - \alpha) \frac{Y_t}{L_t} \quad (2.21)$$

Equation (2.20) represents the aggregate demand for capital. This, together with the supply of capital coming from the optimal contract, given an aggregate level of net worth, it will jointly determine the economy's optimal level of capital investment (K_{t+1}) and the rental rate of capital (R_{t+1}).

The firm's profits are zero in equilibrium.

2.2.2 Workers

Workers in this model work, consume and save. The representative worker maximizes his lifetime utility over consumption and leisure,

$$E_{t-1} \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{1}{\nu} L_t^\nu \right] \quad (2.22)$$

subject to the budget constraint

$$P_t C_t + D_{t+1} = W_t L_t + (1 + \rho_t) D_t \quad (2.23)$$

where $v > 0$ is the elasticity of labor supply, C_t is the level of consumption chosen by the representative worker and D_{t+1} stands for deposits (in nominal terms) held at a financial intermediary earning a risk-free nominal interest rate ρ_t . Workers' savings will finance entrepreneurs' investments through these financial intermediaries.

The representative worker's optimality conditions yield the following Euler equation and labor supply condition:

$$\frac{1}{C_t} = \beta(1 + \rho_{t+1}) E_t \left[\frac{P_t}{P_{t+1}} \frac{1}{C_{t+1}} \right] \quad (2.24)$$

$$\frac{W_t}{P_t} \frac{1}{C_t} = L_t^{v-1} \quad (2.25)$$

2.2.3 The Entrepreneurs

In contrast to section 2.1, which focused on the entrepreneur as an individual agent, this section describes the aggregate behavior of the entrepreneurial sector as a supplier of capital. Entrepreneurs are risk-averse players that supply capital partly financed by their own net worth (N) and the rest by external borrowing (B). Therefore, analogous to equation (2.7),

$$N_t + B_{t+1} = K_{t+1} \quad (2.26)$$

The optimal amount of capital supplied to firms is determined by the first order conditions of the contract (equations 2.16 and 2.17), given the aggregate return to capital, the amount of net worth they hold in each period, and the value of

parameters. As seen in section 2.1, these conditions capture both the external finance premium and the private equity premium that the entrepreneur optimally requires for engaging his net worth in risky investments.

The condition describing the aggregate equity accrued by the entrepreneurial sector from renting capital to operating firms is standard, except that the repayment rate per unit of capital $\hat{\omega}$ also incorporates the cost to the lender of providing insurance to entrepreneurs (that is, $\hat{\omega}$ is greater than $\bar{\omega}$ from section 2.1.1). From equation (2.14), the entrepreneurial sector aggregate equity (V_{t+1}) is given by

$$V_{t+1} = X_t N_t + \int_{\hat{\omega}}^{\infty} (\omega - \hat{\omega}) R_{t+1} K_{t+1} dH(\omega)$$

which is always positive, though not necessarily greater than the entrepreneur's initial net worth. Rearranging terms from the lender's aggregate participation constraint (see equation 2.15), one can show that the entrepreneurial sector aggregate expected equity (V_{t+1}) satisfies the following condition:⁷

$$V_{t+1} = R_{t+1} K_{t+1} - (1 + \rho_{t+1}) B_{t+1} - \mu \int_0^{\hat{\omega}} \omega R_{t+1} K_{t+1} dH(\omega) \quad (2.27)$$

which represents the profits from investment, where the second term shows the opportunity cost and the term $\mu \int_0^{\hat{\omega}} \omega R_{t+1} dH(\omega)$ captures both the bankruptcy cost as well as the insurance cost.

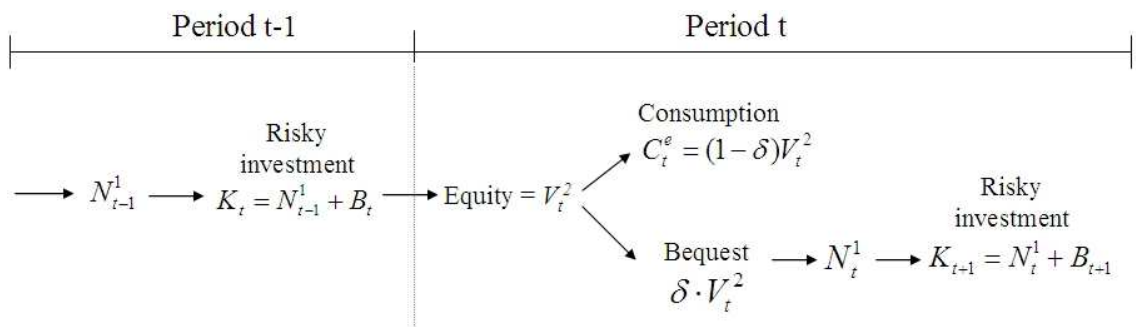
⁷Note that from the lender's participation constraint, one can use the following expression:

$$\begin{aligned} X_t N_t - \int_0^{\hat{\omega}} \omega R_{t+1} K_{t+1} dH(\omega) - \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1} K_{t+1} dH(\omega) = \\ -\mu \int_0^{\hat{\omega}} \omega R_{t+1} K_{t+1} dH(\omega) - (1 + \rho_{t+1})(K_{t+1} - N_t) \end{aligned}$$

Evolution of Aggregate Net Worth

In order to simplify the dynamics of aggregate net worth, I assume overlapping generations of entrepreneurs that live for two periods. As shown in Figure 2.8, an entrepreneurial generation is born in period $t - 1$ and receives a bequest N_{t-1}^1 , where the subscript denotes the period and the superscript indicates the age of entrepreneurs. This generation invests its endowment (plus an amount B_t financed by external borrowing) in a risky project that yields aggregate equity V_t^2 , as defined in equation (2.27), in period t .

Figure 2.8: Two-Period Generation of Entrepreneurs



I assume that entrepreneurs, subsequently consume a fraction $1 - \delta$ of their equity and bequeath the rest to the next generation of entrepreneurs. This new generation receives this bequest, $N_t^1 = \delta V_t^2$, and carries out risky investment in the same period. Note that bequests are put into a common pool so that any particular entrepreneur receives the same bequest as any other entrepreneur from the same generation. That is, *ex-ante*, there is homogeneity among entrepreneurs, therefore all entrepreneurs make the same investment decisions. Heterogeneity emerges from

the idiosyncratic realization of returns from the production of capital.

This environment allows us to account for the constant creation and destruction of firms, avoiding the possible scenario that firms accumulate sufficiently high assets in order to finance their investment without the need to borrow, which would obviate the purpose of this discussion. It also limits dynamic decision making in the model to consumers for simplicity.

As a result, the entrepreneurial sector as a whole will bequeath only a fraction δ of its wealth V_t each period, and will consume the rest (C_t^E). Therefore, net worth and consumption of young entrepreneurs at period t can be defined respectively as

$$N_t = \delta \left\{ (1 + \rho_t) N_{t-1} + \left[R_t - (1 + \rho_t) - \mu \int_0^{\hat{\omega}} \omega R_t dH(\omega) \right] K_t \right\} \quad (2.28)$$

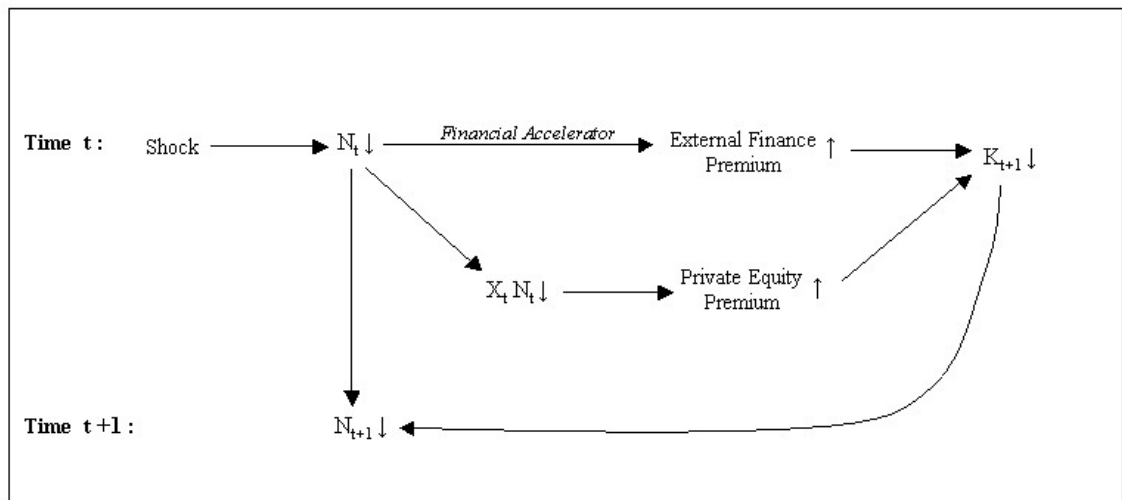
$$C_t^E = \frac{1 - \delta}{\delta} N_t \quad (2.29)$$

The impact of shocks on the dynamics of entrepreneurial net worth and its effects and interaction with the external finance premium and the private equity premium are schematically summarized in Figure 2.9.⁸ If there is a shock that negatively affects entrepreneurs' profits (V_t), it will reduce the level of net worth available for the next period, and thus, for subsequent periods. Also, a lower level of net worth is translated into a lower level of investment for a given R_{t+1} , since the agency problems discussed in section 2.1 lead to a higher external finance premium

⁸Note that this picture does not account for the dynamics of other variables triggered by shocks, such as the labor market, goods prices, etc.

charged by the lender. This reduced level of investment is translated into lower entrepreneurial profits, further reducing the level of net worth for subsequent periods. This is the standard “financial accelerator” effect discussed by Bernanke and Gertler (1999). In addition, the effect of the drop in net worth will also be amplified due to risk aversion frictions described in section 2.1.2, as lower N_t decreases the level of insurance $X_t N_t$ (recall that X_t does not change following changes in N_t .) A lower level of guaranteed consumption leads entrepreneurs to require a higher private equity premium for each level of investment. Investment further decreases for each level of R_{t+1} , which further reduces future net worth, amplifying the initial shock over time.

Figure 2.9: External Finance Premium and Private Equity Premium



Propagation of the shock works through lower entrepreneurial profits due to the decrease in investment. This effect eventually dies out as a depressed supply of capital increases the rental rate of capital. Note that these model, as other agency

cost models, is able to generate persistence without having to assume long-lived capital or persistence in shocks.

2.2.4 Monetary Side, Market Clearing Conditions and the Rational Expectations Equilibrium

Monetary decisions are not explicitly modeled here and are assumed to be made by a monetary authority that uses its policy instruments to keep the short-term interest rate (ρ_t) fixed, allowing the price level (P_t) to fluctuate.

Market clearing conditions that must be satisfied each period close the model. For the goods market, production must equal the sum of investment and consumption goods purchased each period:

$$Y_t = K_{t+1} + C_t + C_t^E \quad (2.30)$$

Furthermore, workers' savings must equal the total debt required by entrepreneurs:

$$D_t = B_t \quad (2.31)$$

Therefore, the risk-neutral rational expectations stochastic dynamic general equilibrium is given by equations (2.3), (2.8), (2.19), (2.20), (2.21), (2.23), (2.25), (2.26), (2.28), (2.29), (2.30), (2.31) and assumptions on the processes for stochastic variables, that solve for P_t , Y_t , L_t , K_{t+1} , R_t , $\bar{\omega}_t$, W_{t+1} , N_t , B_{t+1} , D_{t+1} , C_t , and C_t^E . Similarly, the risk-averse rational expectations stochastic dynamic general equilibrium is defined by equations (2.15), (2.16), (2.17), (2.19), (2.20), (2.21), (2.23),

(2.25), (2.26), (2.28), (2.29), (2.30), (2.31), along with an assumption on the processes for the stochastic variables, solving for $P_t, Y_t, L_t, K_{t+1}, R_t, \hat{\omega}_t, W_{t+1}, X_t, N_t, B_{t+1}, D_{t+1}, C_t$, and C_t^E .

2.2.5 The Log-linearized Model

This section presents the general equilibrium model in terms of variables' log deviations from the stochastic steady-state to analyze the local behavior of the model in response to small shocks.⁹ The bulk of the derivation is standard.

Aggregate demand

$$p_t + c_t + \frac{D}{C}d_{t+1} = \frac{W}{C}(w_t + l_t) + (1 + \rho)\frac{D}{C}(d_t + \rho_t) \quad (2.32)$$

$$\frac{n_t}{\kappa} + (1 - \frac{1}{\kappa})b_{t+1} = k_{t+1} \quad (2.33)$$

$$\xi_R^{foc} r_{t+1} - \xi_{\bar{\omega}}^{foc} \bar{\omega}_t + \xi_{\rho}^{foc} \rho_{t+1} = 0 \quad (2.34)$$

$$\xi_R^{PC} r_{t+1} + \xi_K^{PC} k_{t+1} + \xi_{\bar{\omega}}^{PC} \bar{\omega}_t + \xi_N^{PC} n_t - \xi_{\rho}^{PC} \rho_{t+1} = 0 \quad (2.35)$$

$$\varepsilon_{XN}^{\hat{\omega}}(x_t + n_t) + \varepsilon_R^{\hat{\omega}} r_{t+1} + \varepsilon_K^{\hat{\omega}} k_{t+1} - \varepsilon_{\hat{\omega}}^{\hat{\omega}} \hat{\omega}_t = 0 \quad (2.36)$$

$$\varepsilon_{XN}^K(x_t + n_t) + \varepsilon_R^K r_{t+1} + \varepsilon_K^K k_{t+1} - \varepsilon_{\hat{\omega}}^K \hat{\omega}_t + \varepsilon_{\rho}^K \rho_{t+1} = 0 \quad (2.37)$$

⁹Since I am incorporating the first order conditions and lender participation constraint from the optimal contract in section 2.1 into the general equilibrium model, a log-linear approximation is sufficient to capture all second-order effects of risk aversion, in particular the private equity premium. The log-linear approximation ignores third-order attributes of the entrepreneur utility function, which are not of central importance in this model. However, higher order approximations are necessary in case where quantitative welfare analysis is carried out.

$$\varepsilon_{XN}^{PC}(x_t + n_t) = \varepsilon_R^{PC} r_{t+1} - \varepsilon_K^{PC} k_{t+1} + \varepsilon_{\hat{\omega}}^{PC} \hat{\omega}_t + \varepsilon_N^{PC} n_t - \varepsilon_{\rho}^{PC} \rho_{t+1} \quad (2.38)$$

$$y_t = \frac{K}{Y} k_{t+1} + \frac{C}{Y} c_t + \frac{C^E}{Y} c_t^e \quad (2.39)$$

$$d_t = b_t \quad (2.40)$$

$$c_t^e = n_t \quad (2.41)$$

Aggregate supply

$$y_t = a_t + \alpha k_t + (1 - \alpha) l_t \quad (2.42)$$

$$r_t - p_t = y_t - k_t \quad (2.43)$$

$$w_t - p_t = y_t - l_t \quad (2.44)$$

$$w_t - p_t - c_t = (v - 1) l_t \quad (2.45)$$

Evolution of State Variables

$$\begin{aligned} Nn_t = \delta \{ & [K - \mu E(\omega|\omega < \hat{\omega})] R r_t + [R - \mu E(\omega|\omega < \hat{\omega})] K k_t \\ & + (1 + \rho) B (b_t + \rho_t) + \mu R \hat{\omega}^2 h(\hat{\omega}) \hat{\omega}_t \} \end{aligned} \quad (2.46)$$

The first block of equations represents aggregate demand. Equation (2.32) is the log-linearized version of the workers' aggregate budget constraint (2.23).¹⁰ In the steady state, consumption (C) must equal the sum of income from deposits (ρD) and labor income (W). Therefore, parameters $\rho \frac{D}{C}$ and $\frac{W}{C}$ stand for the weights that explain the change in consumption over time. Equation (2.33) describes the entrepreneurs' borrowing needs given investment and available net worth, where the parameter κ is the steady-state total investment to net worth ratio.

¹⁰I normalize the steady state price level to equal 1.

Equations (2.34) and (2.35) are the log-linearized form of the risk-neutral contract's first order condition and the lender participation constraint, respectively. Similarly, equations (2.36), (2.37) and (2.38) are the log-linearized form of the risk-averse contract's first order conditions with respect to $\hat{\omega}$ and K_{t+1} , and the lender's participation constraint, respectively. Given an expectation of a change in R_{t+1} , these equations jointly determine the deviation from the steady state of capital investment and the repayment rate ($\hat{\omega}$). Parameters ξ_i^{foc} and ξ_i^{PC} , $\varepsilon_i^{\hat{\omega}}$, ε_i^K and ε_i^{PC} are constants at the steady state that accompany the endogenous variables. More details about these constants can be found in appendix 5.2.

Equations (2.39) and (2.40) are the economy-wide resource constraints. Output changes are explained by variation in investment and consumption from workers and entrepreneurs, weighted by their importance in total output at the steady state ($\frac{K}{Y}$, $\frac{C}{Y}$ and $\frac{C^E}{Y}$, respectively), while debt and deposits are equal every period. Finally, equation (2.41) shows the evolution of entrepreneurial consumption, which corresponds to the fraction of profits not saved as net worth, so that both net worth and c^e will vary in the same proportion.

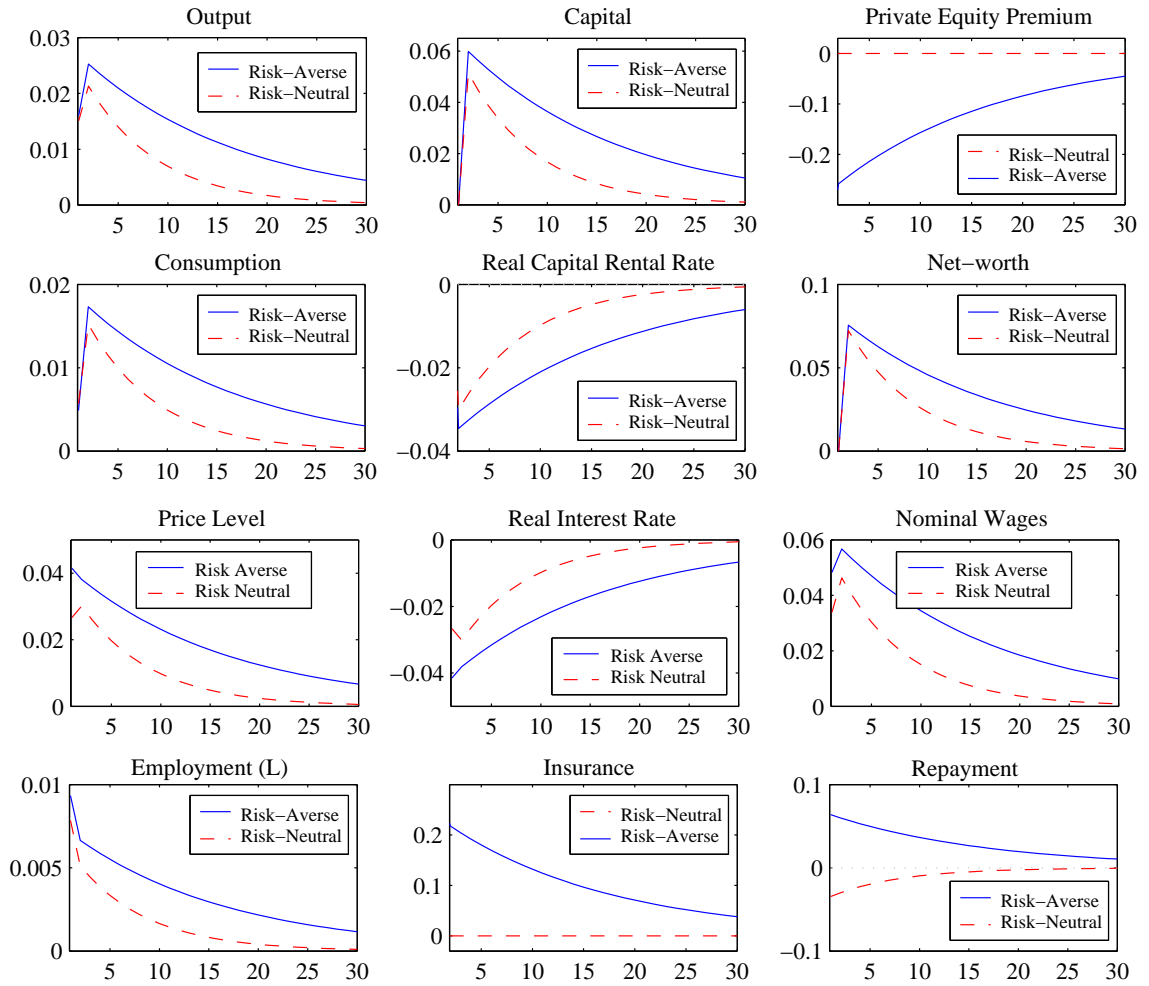
The second block of equations describes the aggregate supply for this economy. Specifically, equation (2.42) presents the log-linearized version of the production function, while equations (2.43) and (2.44) are the first order conditions from the firm's profit maximization problem with respect to capital and labor, respectively. Lastly, equation (2.45) is the linearized version of equation (2.25), and shows workers' optimal substitution between consumption and work, taking into account changes in the price level and wages.

Finally, equation (2.46) introduces the evolution of the model's state variable, net worth, as the log-linearized form of equation (2.28).

2.3 Numerical Analysis

This section will explore the effects of small aggregate shocks on some macroeconomic variables of interest. Given that only idiosyncratic risk is allowed in this dissertation, it is only possible to treat one-time shocks to this economy, followed by perfect foresight dynamics of aggregate variables back to steady state. In particular, a one-time one percent increase in aggregate multifactor productivity and a one-time one percent increase in the nominal safe interest rate are separately considered. The analysis is carried out by examining the impulse response functions that result from numerically solving the complete system, differentiating between the case where the effect of risk-aversion is incorporated using an endogenously determined private equity premium (illustrated in the graphs as a continuous line), and the benchmark case where entrepreneurs are assumed to be risk-neutral (denoted by a dashed line). For the benchmark case, equations (2.15), (2.16) and (2.17) are replaced by the first order condition and participation constraint of the risk-neutral contract, or equations (2.3) and (2.8) respectively. The rest of the equations are the same for both cases, except for the equation describing the evolution of net worth (equation (2.28)), where the repayment rate is given by $\bar{\omega}$ for the risk-neutral case, and $\hat{\omega}$ for the risk-averse case. Details on the parametrization of the model is included in appendix 5.3

Figure 2.10: Effects of a Positive Productivity Shock



2.3.1 Productivity Shock

Impulse response analysis shows that the private equity premium amplifies the impact of a small real aggregate productivity shock and makes its effects significantly more persistent. In particular, Figure 2.10 shows impulse response functions for output, capital, consumption, the private equity premium, and other variables,

resulting from an unexpected, one-time, positive technology shock.¹¹

Following a positive productivity shock, output as well as consumption rise in the first period as a result of higher productivity and labor employment, but there is no initial difference between the benchmark case and the risk-aversion case, as the private equity premium affects investment decisions only with a one-period lag. In subsequent periods, however, the increase in output is higher for the risk-averse case. The amplification can be explained by the behavior of the private equity premium. In the short-term, the increase in entrepreneurial profits and net worth produces an increase in entrepreneur's guaranteed consumption, which in turn generates a decrease in the private equity premium. As a result, capital investment increases in the second period, intensifying the initial effect of the positive technological shock. The effects of this one-time shock die out as higher capital implies a decrease in the real rental rate of capital, which decreases profits, as one may observe in equation (2.28). In addition, in the risk averse case, the greater increase in the aggregate demand driven by the surge in investment and consumption, push up the economy's price level and thus decrease the real interest rate.

Higher levels of entrepreneurial profits and net worth produce a slower convergence of the private equity premium to the steady state. As a result, capital investment, and thus output and consumption, remain persistently higher in the risk-averse case as compared to the risk-neutral case.

Notice that, as a response of the shock, the repayment rate $\hat{\omega}$ behaves qualitatively different between the risk-neutral and the risk-averse cases. In particular,

¹¹Additional impulse response functions can be found in appendix 5.4.

following the favorable shock, $\hat{\omega}$ decreases for the case of risk-neutral entrepreneurs, while it increases for the risk-averse case. Put differently, given that an increase in $\hat{\omega}$ makes default more likely, entrepreneurial default turns out to be countercyclical under risk neutrality and procyclical under risk aversion. Note that, unlike for the risk-neutral case, debt default from risk-averse entrepreneurs is not equivalent to bankruptcy. Changes in the default cutoff ($\hat{\omega}$) mean that risk is shared differently in the sense that, as shocks impact net worth, they are also changing the insurance $X_t N_t$. More explicitly, after a favorable shock, the corresponding increase in net worth implies that for entrepreneurs to provide maximum equity participation, they require a higher level of insurance. For lenders, this extra cost can only be achieved through higher debt repayment. Recall that $\hat{\omega}$ includes the cost of providing insurance, therefore when the value of the insured good increases, so does the insurance premium and, thus, the repayment rate $\hat{\omega}$ charged by the lender.

2.3.2 Monetary Shock

For monetary shocks to have real effects, the model must display some sort of nominal rigidity. I assume that nominal wages are determined one period in advance. In order to provide a framework for nominal wage rigidities, I assume that workers can distinguish the services they provide at no significant cost and therefore engage in monopolistic competition (Dixit and Stiglitz, 1977).

Assume that there is a unit mass of workers defined by a CES aggregate:

$$L_t = \left[\int_0^1 L_{it}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1 \quad (2.47)$$

where σ denotes the intratemporal elasticity of demand for each worker's services. Additionally, each worker i maximizes his lifetime utility over consumption and leisure, as in the homogeneity case,

$$E_{t-1} \sum_{t=0}^{\infty} \beta^t \left[\log C_{it} - \frac{\sigma-1}{\sigma v} L_{it}^v \right]$$

subject to worker i 's budget constraint

$$P_t C_{it} + D_{it+1} = w_{it} L_{it} + (1 + \rho_t) D_{it}$$

I assume that work is indivisible, therefore under this context, workers choose either to work or not, depending on whether the heterogeneous wages they face are greater or lower than the value of their marginal disutility of working. If wages were flexible, each worker would sign a contract for one period at the wage w_{it} that satisfies his individual labor optimality condition. Since each would commit himself to work at that wage rate, everyone would work, and it would be therefore true that aggregate employment $L_t = 1$.

However, under sticky wages each worker signs a fixed nominal wage contract every period before the realization of stochastic shocks. Workers choose their wages for the next period $w_{i,t+1}$ and commit to providing their services at that wage rate. I assume workers set wages so that their optimality conditions are satisfied in expectation. However, given that workers face an downward sloping demand for labor, an unfavorable shock would reduce the demand for workers would, at fixed

wages there would be unemployment and L would be less than 1. In the following period wages would adjust to the new situation, and under the absence of further shocks unemployment would disappear. This implies that aggregate employment only in expectation would be equal to the fixed unit mass of workers:

$$E_{t-1}L_t^v = 1 \quad (2.48)$$

The aggregate version of the workers' budget constraint is the same as in the homogeneity case.¹²

On the production side, the representative firm's maximization problem is given by

$$P_t Y_t - R_t K_t - \int_0^1 w_{it} L_{it} di$$

where w_{it} is the nominal wage paid to worker i . The optimality condition for hiring an individual worker is given by:

$$P_t(1 - \alpha) \frac{Y_t}{L_t} L_t^{1/\sigma} L_{it}^{-1/\sigma} = w_{it} \quad (2.49)$$

Aggregating over workers, I obtain

$$L_t \left[\int_0^1 w_{it}^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} = (1 - \alpha) P_t Y_t$$

which implies that the aggregate wage (W_t) in this model can be expressed as

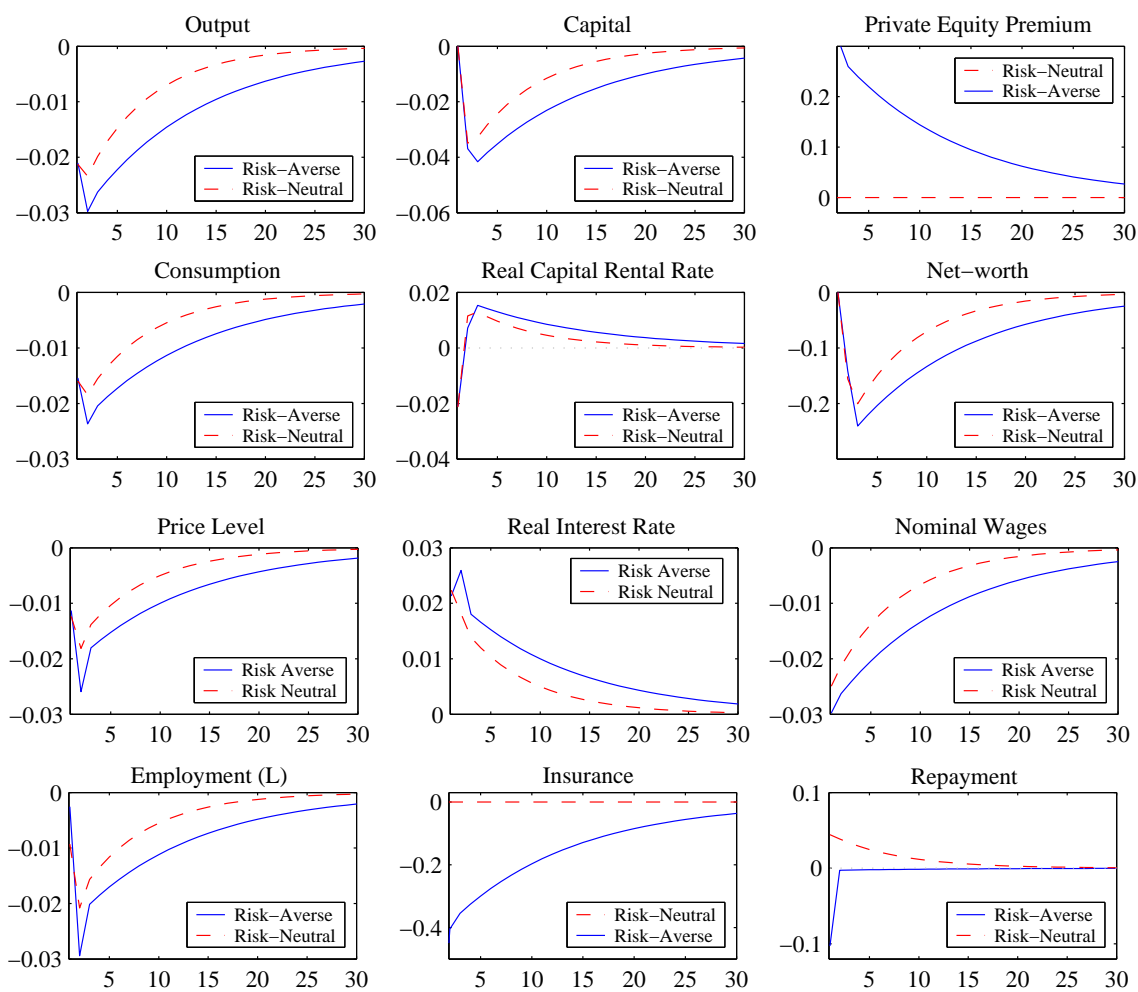
¹²This can be obtained by summing equation (2.23) over i ,

$$P_t C_t + D_{t+1} = \int_0^1 w_{it} L_{it} di + (1 + \rho_t) D_t$$

where C_t and D_t are aggregate consumption and aggregate savings, respectively.

$$W_t = \left[\int_0^1 w_{it}^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \quad (2.50)$$

Figure 2.11: Effects of a Negative Monetary Shock



Therefore, the firm's optimality conditions for capital and aggregate labor demand are the same as in the homogenous case (equations (2.20) and (2.21)), while the optimal level of labor demanded for an individual worker i is given by:

$$L_{it} = \left[\frac{w_{it}}{W_t} \right]^{-\sigma} L_t \quad (2.51)$$

Figure 2.11 presents impulse responses for this model to a sudden, unexpected increase in the safe nominal interest rate (ρ_{t+1}).¹³ An increase in ρ_{t+1} pushes up the lender's opportunity cost per unit of capital borrowed. This implies that the lender requires a higher external finance premium. As a result, entrepreneur's profits, and thus entrepreneurial net worth, are reduced as the value of debt repayment increases, as shown by equations (2.27) and (2.28). Hence, in subsequent periods, entrepreneurs' guaranteed consumption falls and the private equity premium increases. Consequently, capital investment decreases in the second period, amplifying the initial effect of the adverse monetary shock. The effect dies out as the lower supply of capital and its resulting higher real rental partly offset the impact of the negative shock by increasing the entrepreneurial sector's net worth.

Similar to what we observed in the case of a productivity shock, the increased investment and consumption arising from the effects of the shock imply a persistent drop in the price level and an increase in the real interest rate on top of the initial rise in the risk-free rate. Additionally, the monetary shock generates more persistence and volatility in the risk averse case than in the risk neutral case. Therefore, this model suggests that business cycles should be longer and more volatile in economies where non-publicly traded firms are relatively more important in the private sector, as compared to economies where publicly-traded private firms are relatively

¹³As before I find that entrepreneurial risk aversion amplifies and propagates this shock over time, relative to the benchmark model with risk neutrality.

more important. Though beyond the scope of this work, it would be an interesting empirical exercise to test this prediction.

Chapter 3

Model under Small Open Economies

3.1 Introduction

This section extends the model above by analyzing risk aversion and financial frictions in a small open economy. The main differences arise from the existence of international capital flows and international trade. Also, the domestic economy takes international prices as given, including the interest rate and goods prices. As commonly observed in emerging markets, I assume that this economy exports the goods that it produces and imports the goods necessary for the production of capital. In addition, domestic entrepreneurs engage in debt contracts with international lenders, a phenomenon commonly referred to in the literature as liability dollarization. Therefore, exchange rates play a key role, both in the financial sector and the real sector. For instance, a nominal depreciation of the local currency has both the positive effect of boosting exports and the negative impact of increasing the nominal value of the outstanding debt.

I analyze these topics under two scenarios. First, I look at an economy with flexible prices, so that alternative exchange rate regimes impose no real effects, and examine the effect of exchange rate depreciation or appreciation on the dynamics of the model, and in particular on the private equity premium. The second scenario assumes nominal rigidities, so that shocks affect the economy differently under a

flexible as opposed to a fixed exchange rate regime.

The motivation behind this model is that some studies¹ have found that even while incorporating the financial accelerator introduced by Bernanke and Gertler (1989), a floating exchange rate is superior to a pegged regime in terms of absorbing real shocks, because it allows faster real exchange rate adjustment. These studies point out that, in general equilibrium, the favorable impact on exports of devaluations offsets the adverse effect on net worth in the long run. In particular, Céspedes, Chang and Velasco (2001) assert that a converse result (i.e., where a fixed exchange rate is preferable to free floating) is only possible under unrealistic assumptions on the parameters of their model.

On the other hand, when the private equity premium is incorporated into the analysis, the decrease in the entrepreneur's net worth from devaluations will increase this sector's effective risk aversion and the private equity premium. In response, entrepreneurs adjust their supply of capital to final goods firms, producing a magnified impact on output, consumption and both entrepreneurial profits and net worth in subsequent periods.² I find that fixed exchange rate regimes can be preferable under conditions less restrictive than those found in previous studies.

In both scenarios I use a setup similar to that used in the closed economy

¹See for example Céspedes, Chang and Velasco (2001) or Gertler, Gilchrist and Natalucci (2001).

²In addition to these two aspects, the higher volatility associated with flexible exchange rate regimes—channeled through higher volatility of the value of the debt in domestic currency, and thus on the level of leverage and risk-premium—can also be translated into a decrease in investment, since risk-averse entrepreneurs care about volatility in their investment decisions, requiring a higher return in order for them to finance firm investments.

framework. In particular, I introduce entrepreneurial risk aversion into a model of the open economy financial system, and study how this assumption affects both the optimal contract between foreign lenders and domestic borrowers, and the dynamics of an open economy dynamic stochastic general equilibrium model.

3.2 The Optimal Contract Model

In this section, I quickly review the optimal contract between an international lender and a domestic entrepreneur in the context of a small open economy. As before, the model assumes that there is no aggregate uncertainty, and that risk averse domestic entrepreneurs that finance their investments with both internal net worth and foreign borrowing from international risk neutral lenders. That is, foreign debt (B_{t+1}^j) equals total investment minus the firm's net worth (N_{t+1}^j):

$$S_t B_{t+1}^j = Q_t K_{t+1}^j - P_t N_t^j \quad (3.1)$$

where B_{t+1}^j is denominated in foreign currency, N_t^j is denominated in terms of the domestic good (whose price is P_t), Q_t is the domestic price of capital (explained in more detail in Section 3.3), and S_t is the nominal exchange rate.

The contract between borrowers and lenders is subject to the informational frictions described before. The optimal contract therefore maximizes the entrepreneur's utility, subject to the foreign lender's participation constraint, taking as given prices, the nominal exchange rate, entrepreneurial net worth in that period, and the rental return to capital for period $t + 1$ (R_{t+1}), which are endogenized in the general equi-

librium model. I will again start with the standard case where all agents are risk neutral, then analyze the case where entrepreneurs are risk-averse.

3.2.1 The Benchmark Case

As a benchmark case, this section looks at the optimal contract when there are only informational frictions, that is, when all actors are risk neutral. The realization of ω^j such that the entrepreneur breaks even is determined by:

$$\frac{\bar{\omega}^j R_{t+1} K_{t+1}^j}{S_{t+1}} = Z_{t+1}^j B_{t+1}^j \quad (3.2)$$

where Z_{t+1}^j is the contractual interest rate denominated in foreign currency.

This implies that the entrepreneur j 's expected profits in units of consumption can be represented by

$$\int_{\bar{\omega}^j}^{\infty} \omega \frac{R_{t+1} K_{t+1}^j}{S_{t+1}} dH(\omega) - [1 - H(\bar{\omega}^j)] \frac{\bar{\omega}^j R_{t+1} K_{t+1}^j}{S_{t+1}} \quad (3.3)$$

where S_{t+1} is also the price of entrepreneurial consumption as entrepreneurs are assumed to consume only imports (explained in more detail in Section 3.3).

The lender participation constraint is given by:

$$[1 - H(\bar{\omega}^j)] \frac{\bar{\omega}^j R_{t+1} K_{t+1}^j}{S_{t+1}} + (1 - \mu) \int_0^{\bar{\omega}^j} \omega \frac{R_{t+1} K_{t+1}^j}{S_{t+1}} dH(\omega) \geq (1 + \rho_{t+1}) B_{t+1}^j \quad (3.4)$$

where $(1 + \rho_{t+1})$ is the international risk-free interest rate.

The optimality conditions imply the following relationship:

$$[1 - H(\bar{\omega})] \int_{\bar{\omega}}^{\infty} \omega R_{t+1} dH(\omega) = [1 - H(\bar{\omega})] \frac{S_{t+1} (1 + \rho_{t+1}) P_t N_t}{S_t K_{t+1}} + [1 - H(\bar{\omega})] \int_{\bar{\omega}}^{\infty} \bar{\omega} R_{t+1} dH(\omega) + \phi \quad (3.5)$$

where the left hand side of equation 3.5 represents the expected returns in domestic currency per unit of capital for the entrepreneur, and the right hand side displays the expected cost per unit of capital for the entrepreneur. This includes the opportunity cost to the entrepreneur of investing his net worth and the capital repayment rate ($\bar{\omega}$) for the non-bankruptcy states of nature. The opportunity cost is adjusted by real depreciation or appreciation of the local currency as realized in period t . The term ϕ is negligible and captures the fact that the expected bankruptcy costs change as $\bar{\omega}$ adjusts.³

Figure 3.1: Supply of capital: the risk-neutral case.

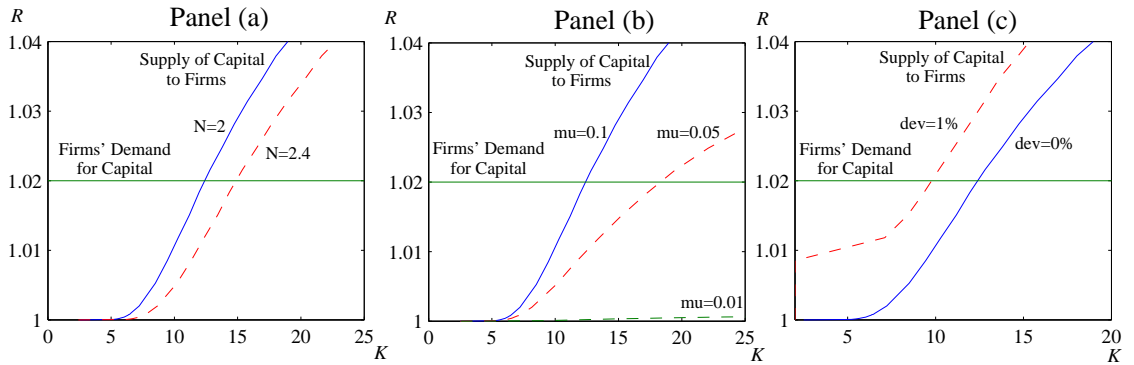


Figure 3.1 graphs the upward sloping supply of capital, and how it behaves

³ $\phi = \mu \bar{\omega} h(\bar{\omega}) \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}) R_{t+1} dH(\omega)$.

before exogenous changes in net worth, bankruptcy costs and a one percent forecastable depreciation, measured by the term (S_{t+1}/S_t) .

3.2.2 Model with Risk Averse Entrepreneurs

In this section I explore the implications of the optimal contract that arise from a simple one-period contract when the domestic borrower is risk-averse.

Let $\hat{\omega}^j$ denote the default cut-off state such that if $\omega^j < \hat{\omega}^j$, the entrepreneur optimally decides to default, and the lender receives the residual revenues and observes the realized ω^j after paying a positive cost. As in the closed economy case, the risk-neutral lender provides the risk-averse entrepreneur with a state invariant insurance $X_t P_t N_t$, where X_t is the fraction of the entrepreneur's initial nominal net worth that is guaranteed. If instead $\omega^j > \hat{\omega}^j$, there is no default, and the entrepreneur keeps the true realization of ω^j as private information. Therefore, the entrepreneur maximizes

$$\int_0^{\hat{\omega}^j} U\left(\frac{X_t^j P_t N_t^j}{S_{t+1}}\right) dH(\omega) + \int_{\hat{\omega}^j}^{\infty} U\left(\frac{\omega^j R_{t+1} K_{t+1}^j - Z_{t+1}^j B_{t+1}^j}{S_{t+1}}\right) dH(\omega)$$

where $\hat{\omega}^j$ solves

$$U\left(\frac{X_t^j P_t N_t^j}{S_{t+1}}\right) = U\left(\frac{\hat{\omega}^j R_{t+1} K_{t+1}^j - Z_{t+1}^j B_{t+1}^j}{S_{t+1}}\right)$$

Thus, the entrepreneur's expected utility can be re-expressed as

$$\int_0^{\hat{\omega}^j} U\left(\frac{X_t^j P_t N_t^j}{S_{t+1}}\right) dH(\omega) + \int_{\hat{\omega}^j}^{\infty} U\left(\frac{X_t^j P_t N_t^j + (\omega^j - \hat{\omega}^j) R_{t+1} K_{t+1}^j}{S_{t+1}}\right) dH(\omega) \quad (3.6)$$

where $\hat{\omega}^j$ also represents the repayment rate per unit of capital in the non-default states.

The lender's participation constraint is given by

$$\int_0^{\hat{\omega}^j} (1-\mu) \left[\frac{\omega R_{t+1} K_{t+1}^j}{S_{t+1}} \right] dH(\omega) + \int_{\hat{\omega}^j}^{\infty} \left[\frac{\hat{\omega} R_{t+1} K_{t+1}^j}{S_{t+1}} \right] dH(\omega) - \frac{X_t^j P_t N_t^j}{S_{t+1}} \geq (1+\rho_{t+1}) B_{t+1}^j \quad (3.7)$$

Taking as given N_t^j , R_{t+1} , S_t , S_{t+1} , and P_t , the first order conditions of the optimal contract with respect to K_{t+1} and $\hat{\omega}$ are given by equations (3.8) and (3.9), respectively.

$$\int_{\hat{\omega}}^{\infty} (\omega - \hat{\omega}) dH(\omega) R_{t+1} = \left[1 + \frac{E(U'(\cdot) | \omega < \hat{\omega})}{E(U'(\cdot) | \omega > \hat{\omega})} \right] \left[\frac{(1 + \rho_{t+1}) P_t N_t S_{t+1}}{K_{t+1} S_t} - \frac{X_t P_t N_t}{K_{t+1}} \right] - \frac{R_{t+1}}{E(U'(\cdot) | \omega > \hat{\omega})} Cov\{U'(\cdot), \omega\} \quad (3.8)$$

$$\left[1 + \frac{E(U'(\cdot) | \omega < \hat{\omega})}{E(U'(\cdot) | \omega > \hat{\omega})} \right] = \frac{1}{[1 - H(\hat{\omega})] - \mu \hat{\omega} h(\hat{\omega})} \quad (3.9)$$

which, after some algebraic manipulation, imply the following optimality condition:

$$\begin{aligned} & \frac{X_t P_t N_t}{K_{t+1}} + [1 - H(\hat{\omega})] \int_{\hat{\omega}}^{\infty} \omega R_{t+1} dH(\omega) = \\ & \frac{S_{t+1}}{S_t} \frac{(1 + \rho_{t+1}) P_t N_t}{K_{t+1}} + [1 - H(\hat{\omega})] \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1} dH(\omega) \\ & - \frac{[1 - H(\hat{\omega}) - \mu \hat{\omega} h(\hat{\omega})]}{E(U'(\cdot) | \omega > \hat{\omega})} \cdot Cov\{U'(\cdot), \omega R_{t+1}\} + \varphi \end{aligned} \quad (3.10)$$

In equilibrium, the marginal benefit of investing in capital (left hand side) equals its marginal cost (right hand side). As in the closed-economy framework, the marginal cost includes the covariance between the return to capital and the entrepreneur's marginal utility of consumption: the private equity premium. The term φ , as in the risk-neutral case, captures the change in expected default costs due to changes in $\hat{\omega}$.⁴

⁴ $\varphi = \mu \hat{\omega} h(\hat{\omega}) \left[\int_{\hat{\omega}}^{\infty} (\omega - \hat{\omega}) R_{t+1} dH(\omega) + \frac{Cov\{U'(\cdot), \omega\}}{E(U'(\cdot) | \omega > \hat{\omega})} \right]$.

Numerical Approach

Here, as in section 2.1.1, I try to identify the most important features of the modified financial contract, and present results on the quantitative impact of parameter changes.

Figure 3.2: Supply of Capital, Marginal Cost Function and Insurance Rate.

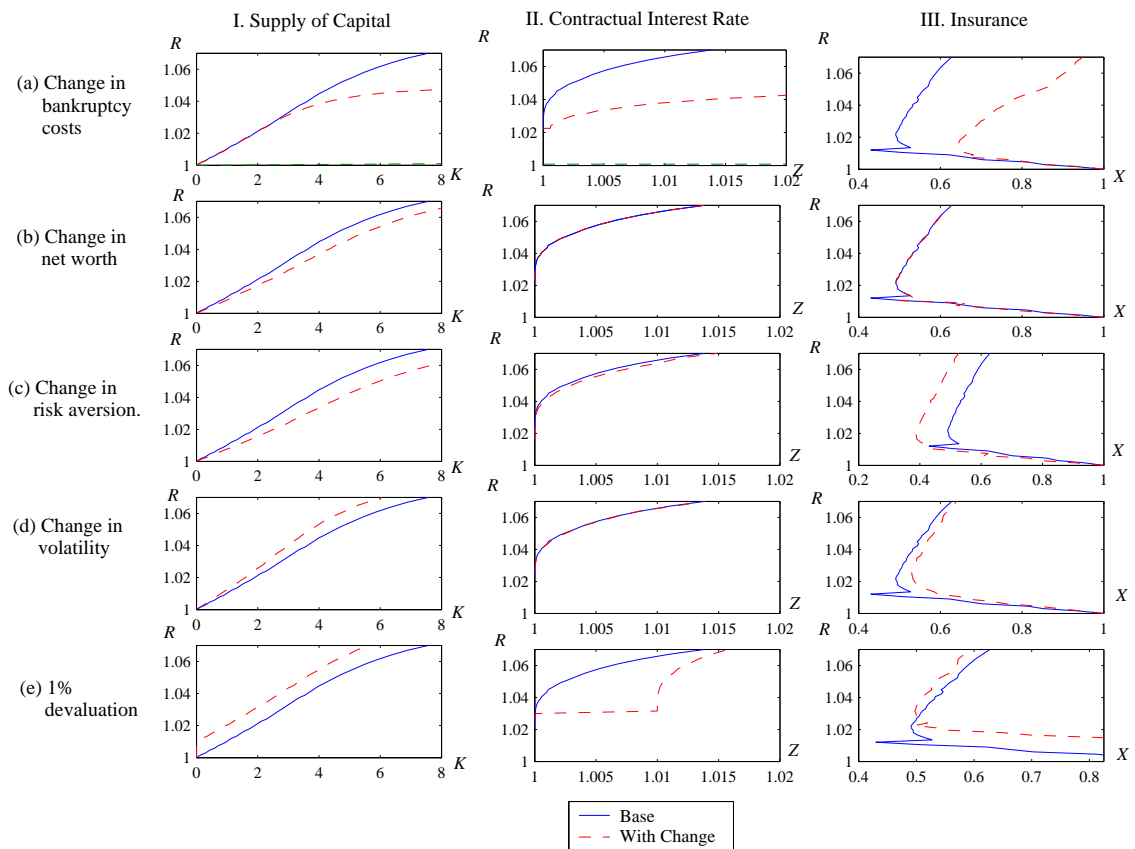


Figure 3.2 shows the simulated behavior of some variables of interest for different values of the exogenous rental rate of capital under the benchmark parameter values (solid lines), and the impact of changes in these parameters (dashed lines). Column I shows the supply of capital to final goods firms as a function of the ex-

ogenous rental rate on capital. The supply curve of capital depends in part on the entrepreneur contractual cost of funds Z_{t+1}^j as a function of the return to capital, arising from equation (3.2), and is shown in column II of Figure 3.2. Finally, column III shows insurance per unit of net worth (X_t^j) for different values of the return of capital.

Row (a) of Figure 3.2 shows the effects of different levels of the bankruptcy parameter (μ); row (b) analyzes the effect of a 20 percent increase in net worth (from 2 to 2.4); while row (c) examines the impact of varying the coefficient of risk-aversion. The interpretation of the impact of these changes on the equilibrium features of the optimal contract are the same as in the closed economy framework discussed in section 2.1.

Row (e) of Figure 3.2 shows the impact of a one percent forecastable depreciation, measured by the term (S_{t+1}/S_t) . Note that while the contract calls for payment in foreign currency, the lender still has to care about the value of local currency, since the lender provides the domestic entrepreneur with financing in period t and receive revenues (including default revenues) in period $t + 1$. In particular, an increase in the exchange rate in period $t + 1$ relative to the previous period reduces the value of lender revenues relatively to the opportunity costs of funds. As a consequence, the supply of capital (column I) shift up by one percent. In addition, the contractual gross interest rate Z_{t+1}^j equals 1 for values of R_{t+1} where there is no borrowing, an increases 1% with respect to the base case (starts at 1.01 instead of 1.00) as soon as the entrepreneur relies on external financing. A higher cost of funds implies lower insurance provided by the lender for each level of R_{t+1} .

3.3 The General Equilibrium Model

This section analyzes the aggregate effects of the optimal contract described in last section on the dynamics of a small open economy. As noted earlier, I first consider an economy with flexible prices, followed by an economy with nominal rigidities. By introducing price rigidities I can analyze the different effect of shocks on the model's dynamics under flexible or fixed exchange rate regime.

Consider a small open economy whose domestic firms produce one good through a standard constant return to scale Cobb-Douglas production function:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} \quad (3.11)$$

The good can either be consumed inland or exported. The representative firm maximizes nominal profits denominated in domestic currency by optimally choosing capital, labor and total output.

$$P_t Y_t - R_t K_t - W_t L_t$$

where P_t is the price of domestic output, R_t is the nominal domestic rental rate of capital and W_t is the nominal wage paid to workers. All prices are denominated in local currency.

The optimality conditions are standard:

$$\frac{R_t}{P_t} = \alpha \frac{Y_t}{K_t} \quad (3.12)$$

$$\frac{W_t}{P_t} = (1 - \alpha) \frac{Y_t}{L_t} \quad (3.13)$$

Workers maximize utility over consumption and leisure,

$$E_{t-1} \log C_t - \frac{\sigma - 1}{\sigma v} L_t^v$$

where $C_t = [\theta^\theta (1 - \theta)^{(1-\theta)}]^{-1} (C_t^H)^\theta (C_t^F)^{(1-\theta)}$ is a CES aggregate of domestic goods (C^H) and foreign goods (C^F), θ is the weight of the domestic good in total consumption, and v is the elasticity of labor supply.

I assume for simplicity that workers cannot save or borrow. This assumption is necessary to avoid the possibility that entrepreneurs could borrow from workers (through banks) in domestic currency. Workers solve a static problem, subject to a budget constraint where the only source of income are labor proceeds that is entirely consumed within the period.

$$W_t L_t = P_t C_t^H + S_t C_t^F$$

Assuming that the law of one price holds and taking the price of the foreign good P_t^* is taken as a numeraire, the domestic price of imports is also the nominal exchange rate (that is, $P_t^F = S_t$).

From consumption cost minimization, taking into account the definition of aggregate consumption C_t , the following optimality conditions can be obtained:

$$P_t C_t^H = \theta P_t^\theta S_t^{1-\theta} C_t, \quad S_t C_t^F = (1 - \theta) P_t^\theta S_t^{1-\theta} C_t$$

Therefore, the relevant cost of consumption can be expressed as the following

function.

$$Q_t = P_t^\theta S_t^{1-\theta} \quad (3.14)$$

From the worker's maximization problem, the aggregate budget constraint can be expressed as:

$$Q_t C_t = W_t L_t \quad (3.15)$$

The labor supply condition is given by:

$$\frac{W_t}{Q_t C_t} = \left(\frac{\sigma - 1}{\sigma} \right) L_t^{\nu-1} \quad (3.16)$$

The entrepreneurial sector is modeled similarly to the closed economy case. Therefore, analogous to equation (2.7) and assuming for simplicity that entrepreneurs transform domestic and foreign consumption goods to produce capital in the same proportion as workers purchase goods for consumption, it is true that

$$P_t N_t + S_t B_{t+1} = Q_t K_{t+1} \quad (3.17)$$

The resulting condition describing the aggregate equity in domestic currency accrued by the entrepreneurial sector from renting capital to operating firms similar to that in the closed economy case:

$$V_{t+1} = R_{t+1} K_{t+1} - (1 + \rho_{t+1}) S_{t+1} B_{t+1} - \mu \int_0^{\hat{\omega}} \omega R_{t+1} K_{t+1} dH(\omega) \quad (3.18)$$

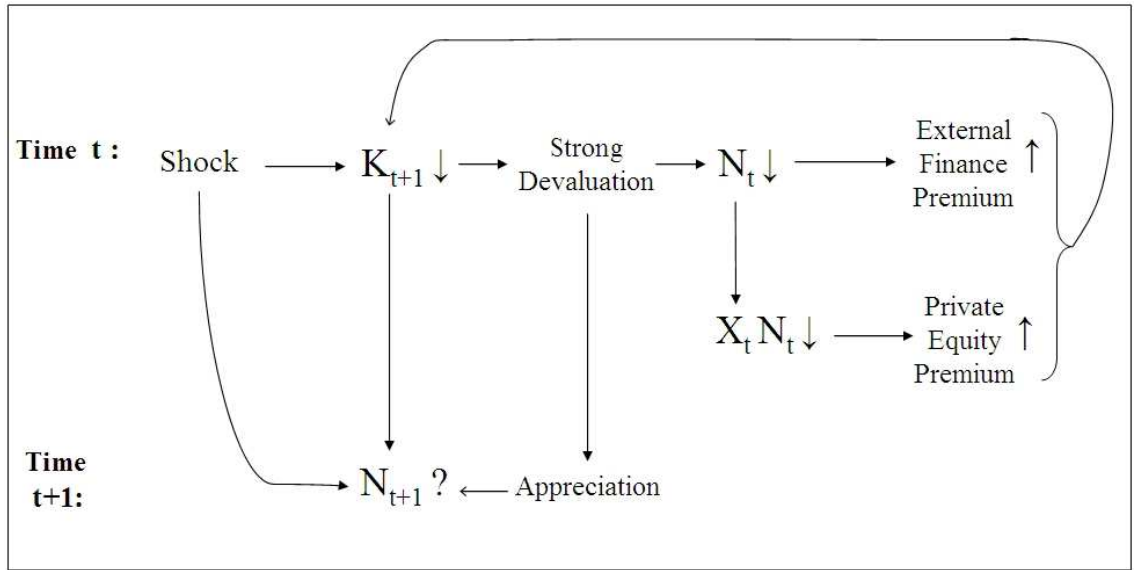
Notice that under these assumptions of preferences, the labor supply is constant. In addition, assuming the same two-period overlapping generation model for the entrepreneurial sector as above, the real net worth and consumption of young entrepreneurs at period t can be defined as

$$N_t = \delta \left\{ (R_t/P_t) K_t - (1 + \rho_t) \frac{S_t/P_t}{S_{t-1}/Q_{t-1}} B_t - \mu \int_0^{\hat{\omega}} \omega (R_t/P_t) K_t dH(\omega) \right\} \quad (3.19)$$

$$S_t C_t^E = \frac{1 - \delta}{\delta} P_t N_t \quad (3.20)$$

assuming for simplicity that entrepreneurs consume only imports.

Figure 3.3: Exchange Rate, the External Finance Premium and the Private Equity.



The effect of a shock on the net worth dynamics and, thus, on the evolution of the external finance premium and the private equity premium are schematically presented in Figure 3.3. An international interest rate shock, for instance, will be translated in an immediate decrease in planned capital investment, as lenders increase their external finance charge to entrepreneurs due to higher opportunity costs. The implied capital outflows is translated into a devaluation of the local currency, which reduces the entrepreneur's net worth by increasing the domestic currency value of debt repayment (captured by $\frac{S_t}{P_t}$ in equation (3.19)), due to the

fact that debt is denominated in foreign currency. As a consequence, both the external finance premium and the private equity premium increase, further impacting planned investment decisions and the real exchange rate. That is, in the first period, the shock together with entrepreneurial risk aversion produce a magnified effect on capital investment and the real exchange rate.

In the second period, once the drop in capital inflows occur, the real exchange rate quickly adjusts back, which for the risk averse case, it produces an overshooting response due to the stronger depreciation in the first period. This appreciation, in turn, positively affects entrepreneurs' profits (V_t) by decreasing the value of the debt repayment. This effect works against the direct effect of the shock and of lower capital investment, on net worth, thus reducing the propagating effect of shocks. Therefore, the impact of shocks coupled with the presence of the private equity premium is that it produces magnified, however less persistent, responses of capital investment and output.

The monetary side of the benchmark flexible-price model is given by a monetary authority that uses its policy instruments to keep the price level constant while letting the nominal exchange rate fluctuate.

The model is closed with a market clearing condition that must be satisfied each period. Recalling that $\theta Q_t C_t$ and $\theta Q_t K_{t+1}$ correspond to the domestically produced part of consumption and investment, then the home goods market is in equilibrium when:

$$P_t Y_t = \theta Q_t (K_{t+1} + C_t) + S_t X_t \quad (3.21)$$

where X_t corresponds to exports, which are assumed to be exogenous to the model. Note that having the domestic goods market in equilibrium guarantees an equilibrium in the trade balance:

$$(1 - \theta)Q_t(K_{t+1} + C_t) = S_t X_t$$

Equivalently,

$$Q_t(K_{t+1} + C_t) = \theta Q_t(K_{t+1} + C_t) + S_t X_t \quad (3.22)$$

To see that this condition holds, summing equations (3.15) and (3.15), one gets:

$$W_t L_t + P_t N_t + Q_t K_{t+1} = Q_t(K_{t+1} + C_t)$$

Note that the left hand side of the last equation captures all sources of income, therefore it is true that

$$P_t Y_t = Q_t(K_{t+1} + C_t)$$

which by using equation (3.22), we get

$$P_t Y_t = \theta Q_t(K_{t+1} + C_t) + S_t X_t$$

which is the same as equation (3.21).

Therefore, the risk-neutral rational expectations stochastic dynamic general equilibrium is given by equations (3.4), (3.3), (3.11), (3.12), (3.13), (3.14), (3.15), (3.16), (3.17), (3.19) and (3.21) and assumptions on the processes for stochastic variables, that solve for Y_t , L_t , K_{t+1} , Q_t , S_t , R_t , $\bar{\omega}_t$, W_t , N_t , D_{t+1} , and C_t . Similarly, the

risk-averse rational expectations stochastic dynamic general equilibrium is defined by equations (3.7), (3.8), (3.9), (3.11), (3.12), (3.13), (3.14), (3.15), (3.16), (3.17), (3.19) and (3.21), along with an assumption on the processes for the stochastic variables, solving for Y_t , L_t , K_{t+1} , Q_t , S_t , R_t , $\hat{\omega}_t$, W_t , X_t , N_t , D_{t+1} , and C_t .

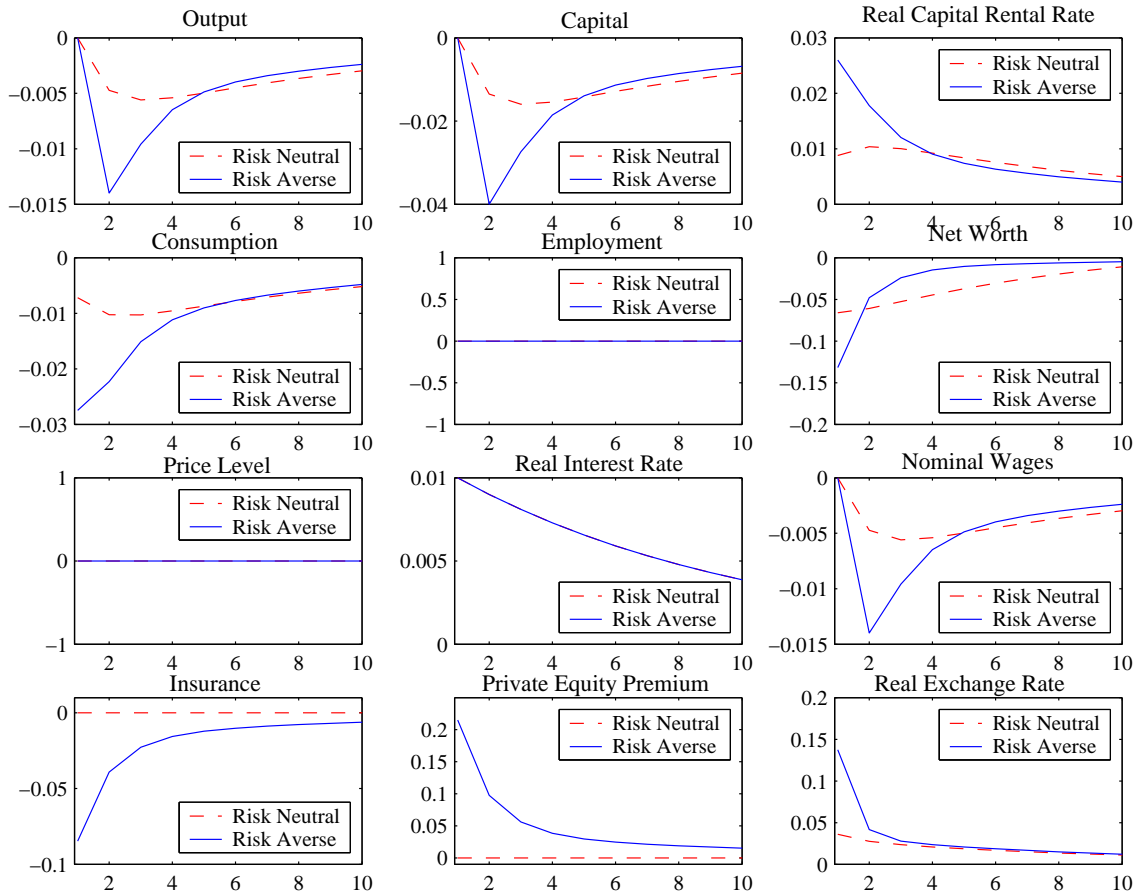
3.4 Numerical Analysis under Flexible Prices

In this section I will study the effects of small one-time shocks to international interest rates and export demand on some macroeconomic variables of interest. I assume that these shocks follow an AR(1) process with an autocorrelation coefficient of 0.9, which is known by all agents. As in previous sections, I analyze the impulse response functions that result from numerically solving the complete system, differentiating the case where the private equity premium is considered (continuous line) from the benchmark risk-neutral case (dashed line).

International Interest Rate Shock

Figure 3.4 shows the impact of a 1% increase in the international interest rate ρ_{t+1} at time t . As implied by equation (3.21), the direct effect is that there is an instantaneous real depreciation due to the immediate fall in entrepreneurs' planned capital investment K_{t+1} as a response to the interest rate shock. This implies that there is a decrease in entrepreneurs' net worth at time t since the domestic currency value of foreign debt repayment increases when the real exchange rate rises. For the risk averse case, the resulting fall in entrepreneurial net worth translates into

Figure 3.4: Effects of an Unfavorable International Interest Rate Shock



a rise in the private equity premium, and thus a reinforced decrease in the supply of capital and foreign borrowing. This depreciates the local currency even more, further impacting net worth and capital investment. In the following period, there is a quicker and stronger exchange rate recovery due to the large drop in output in period $t + 1$ under risk aversion (see equation 3.21).

This feature of the real exchange rate can be observed in Figure 3.4. In addition, as explained before, such a shock increases directly decreases the entrepreneurial net worth, since it raises the opportunity cost of lending, thereby

reducing the supply of funds available for entrepreneurs (recall that entrepreneurs face an upward sloping marginal cost of funds function).

For the risk aversion case, the described path of the real exchange rate implies that entrepreneurial net worth experiences a sharper initial decrease and a more rapid return to the steady state. Capital and output respond accordingly. That is, for the risk aversion case, there is a sharper decrease of both capital and output in the second period following the increase in the private equity premium, and a quick recovery afterwards.

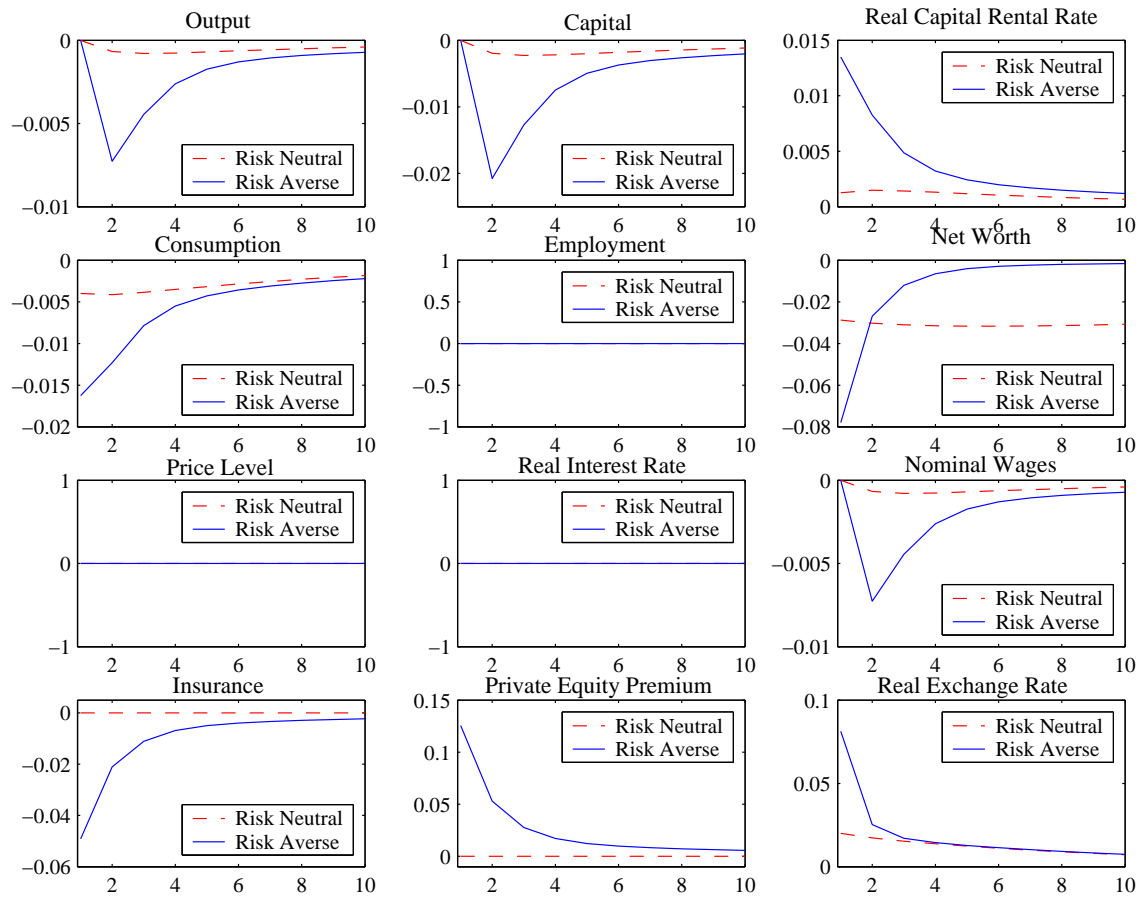
To sum up, the private equity premium amplifies the impact of the interest rate shock. However, quick recoveries fostered by exchange rate overshooting and its effect on the value of debt, net worth and, thus, the private equity premium, imply that shocks have less persistent effect than in the risk neutral case.

Export Demand Shock

In this section, I analyze the effect of a 1% decrease in the demand from the rest of the world for the domestic good. A decrease in the demand for exports produces an initial real depreciation of the domestic currency, as implied by equation (3.21). This real depreciation decreases the level of entrepreneurs' net worth, as it increases the domestic currency value of the foreign stock of debt. This, in turn, raises the private equity premium and decreases the supply of capital, further depreciating the exchange rate. Given that for the risk-averse case output contracts rapidly in period $t + 1$, the exchange rate adjusts quickly to its steady state path, producing

a rapid recovery of net worth to its steady state value. Consequently, the private equity premium, the capital supply and the rental rate of capital converge rapidly to their steady state path, as observed in Figure 3.5.

Figure 3.5: Effects of a Positive Export Shock



It is also possible to observe in this exercise that shocks are amplified due to the presence of the private equity premium. However, the exchange rate plays a stabilizing role by allowing for a faster convergence to the steady state path, decreasing the persistence due to the private equity premium.

3.5 Analysis under Nominal Rigidities

In this section I look at an economy with nominal rigidities, so that alternative exchange rate regimes impose real effects on the economy. Specifically, I examine how the faster real exchange rate adjustment associated with a flexible exchange rate regime may impact the economy, in particular entrepreneurs' net worth and on the private equity premium. This analysis will help further explore under what conditions a certain exchange rate regime might outperform the other.

In this section, I use the same structure of nominal rigidities described in Section 2.3.2. That is, nominal wages are determined one period in advance and there is heterogeneity in workers so that they enjoy some monopolistic competition in the labor services they provide. I assume that there is a unit mass of workers defined by a CES aggregate $L_t = \left[\int_0^1 L_{it}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$, where each worker i maximizes lifetime utility over an aggregation of domestic and foreign goods consumption C_{it} (as defined in section 3.3) and leisure,

$$E_{t-1} \left[\log C_{it} - \frac{\sigma-1}{\sigma v} L_{it}^v \right]$$

subject to worker i 's budget constraint

$$W_t L_{it} = P_t C_{it}^H + S_t C_{it}^F$$

Consumption decisions follow the same logic as in Section 3.3 and are governed by equations (3.14) and (3.15). On the other hand, workers' labor decisions are simpler. They decide whether to work or not depending on how the heterogeneous wages they face and the value of their marginal disutility of working compare one

to the other. As explained in section 2.3.2, in the absence of shocks, all workers work at their desired wage rates. However, when wages are sticky and the economy is subject to an unexpected shock, each worker's wage and his marginal disutility of working are not necessarily equal, at least in the period in which the shock occurs. Therefore, only in expectation it is true that all workers decide to provide their services.

$$E_{t-1}L_t^v = 1 \tag{3.23}$$

The difference between exchange rate regimes stems from the monetary authority's choice to either let the nominal exchange rate (S_t) fluctuate and pursue price targeting, or to aim for a constant nominal exchange rate, while letting domestic prices fluctuate.

Flexible Exchange Rate Regime

Price targeting implies that the monetary authority maintains P_t at its steady state value. Therefore, in period 0 (when the shock takes place), nominal wages are fixed, and aggregate labor demand is determined by

$$L_t = (1 - \alpha) \frac{PY_t}{W_t} \tag{3.24}$$

where P is the steady state value of P_t .

For later periods, however, in the absence of further unexpected shocks, workers find jobs that offer wages that satisfy their optimality conditions. Under these

conditions, aggregate labor is constant and equal to one. In addition, wages are defined as

$$W_{t+1} = (1 - \alpha)PY_{t+1} \quad (3.25)$$

Therefore, the rational expectations stochastic dynamic general equilibrium for the risk-neutral case is given by equations (3.3), (3.4), (3.11), (3.12), (3.14), (3.15), (3.17), (3.19), (3.21), (3.24) and (3.25), and assumptions on the processes for stochastic variables, that solve for $Y_t, L_t, K_{t+1}, Q_t, S_t, R_t, \bar{\omega}_t, W_{t+1}, N_t, D_{t+1}$, and C_t . For the risk-averse case, these are given by equations (3.7), (3.8), (3.9), (3.11), (3.12), (3.14), (3.15), (3.17), (3.19), (3.21), (3.24) and (3.25), the assumption on the processes for the stochastic variables, solving for $Y_t, L_t, K_{t+1}, Q_t, S_t, R_t, \hat{\omega}_t, W_{t+1}, X_t, N_t, D_{t+1}$, and C_t .

Fixed Exchange Rate Regime

Under this regime, the monetary authority maintains the nominal exchange rate S_t constant at its steady state value, allowing prices to freely fluctuate. Thus, as of period 0, aggregate labor demand is given by

$$L_t = (1 - \alpha) \frac{P_t Y_t}{W_t} \quad (3.26)$$

For later periods, there is equilibrium without unemployment, where $L = 1$ and wages are defined as

$$W_{t+1} = (1 - \alpha)P_{t+1}Y_{t+1} \quad (3.27)$$

The rational expectations stochastic dynamic general equilibrium for the risk-neutral case is given by equations (3.3), (3.4), (3.11), (3.12), (3.14), (3.15), (3.17), (3.19), (3.21), (3.26) and (3.27), and assumptions on the processes for stochastic variables, that solve for $Y_t, L_t, K_{t+1}, Q_t, P_t, R_t, \bar{\omega}_t, W_{t+1}, N_t, D_{t+1}$, and C_t . Likewise, for the risk-averse case the equations that define the rational expectations stochastic dynamic general equilibrium are (3.7), (3.8), (3.9), (3.11), (3.12), (3.14), (3.15), (3.17), (3.19), (3.21), (3.26) and (3.27), the assumption on the processes for the stochastic variables, solving for $Y_t, L_t, K_{t+1}, Q_t, P_t, R_t, \hat{\omega}_t, W_{t+1}, X_t, N_t, D_{t+1}$, and C_t .

3.5.1 Exchange Rate Regime Comparison

In this section I compare how the two alternative exchange rate regimes impact the response and dynamics of aggregate variables after a real shock. I do this by analyzing the impulse response functions for both a fixed exchange rate regime and a flexible exchange rate regime, subject to separate shocks to international interest rates and export demand.

In order to contrast the results for risk-averse entrepreneurs to those found in the literature, I also examine the dynamics of the model if the supply of capital defined by the optimality conditions from the risk neutral contract (equations 3.3 and 3.4) are replaced by the specification followed by Céspedes, Chang and Velasco

(2000) (CCV, henceforth). These authors assume that the supply of capital is determined by the following relationship, which was derived by Bernanke, Gertler and Gilchrist (1999).

$$\frac{R_{t+1}}{\rho_{t+1}} = F\left[1 + \frac{S_t B_{t+1}}{P_t N_{t+1}}\right] \quad (3.28)$$

That is, the wedge between the domestic rental rate of capital and the risk-free rate, referred to as the risk premium, is a positive function of the ratio debt to net worth (or leverage).

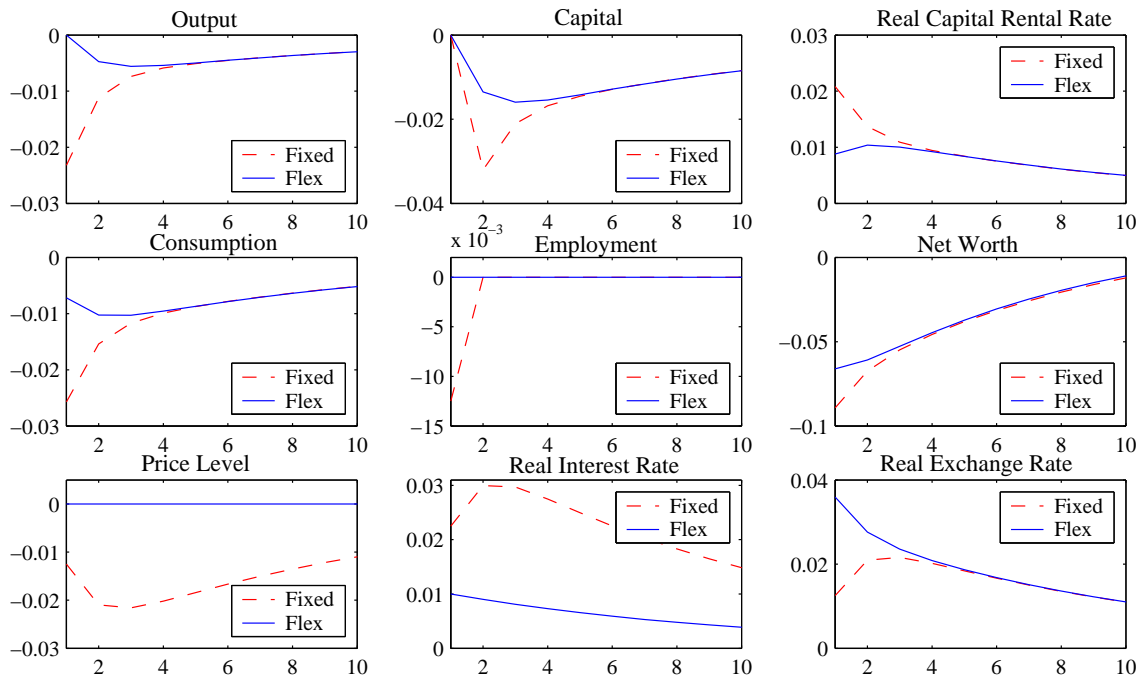
Due to the assumptions on preferences, under flexible wages the labor supplied by workers is constant regardless of the exchange rate regime. However, for the case of sticky wages and a fixed exchange rate regime, the demand for labor, represented by the log-linear version of the firm's first order condition with respect to employment, is given by

$$l_t = \frac{1}{\alpha} p_t$$

Therefore, the initial decrease in prices after an adverse shock (recall that real depreciation under fixed exchange rate regimes is accomplished through domestic price deflation) would imply a drop in the demand for labor. At constant wages in the first period, the consequence is that there is unemployment and a fall in output. In the second period, wages adjust down and unemployment disappears. As we can see, under fixed exchange rate regimes the initial drop in output comes from the nominal wage rigidity assumption, not from frictions resulting from imperfect information and entrepreneurial risk aversion. Given that the effect of shocks on

investment decisions can be observed from the second period on, I will focus my analysis there.

Figure 3.6: Exchange Rate Regime Comparison: The Case of an International Interest Rate Increase and Risk Neutrality

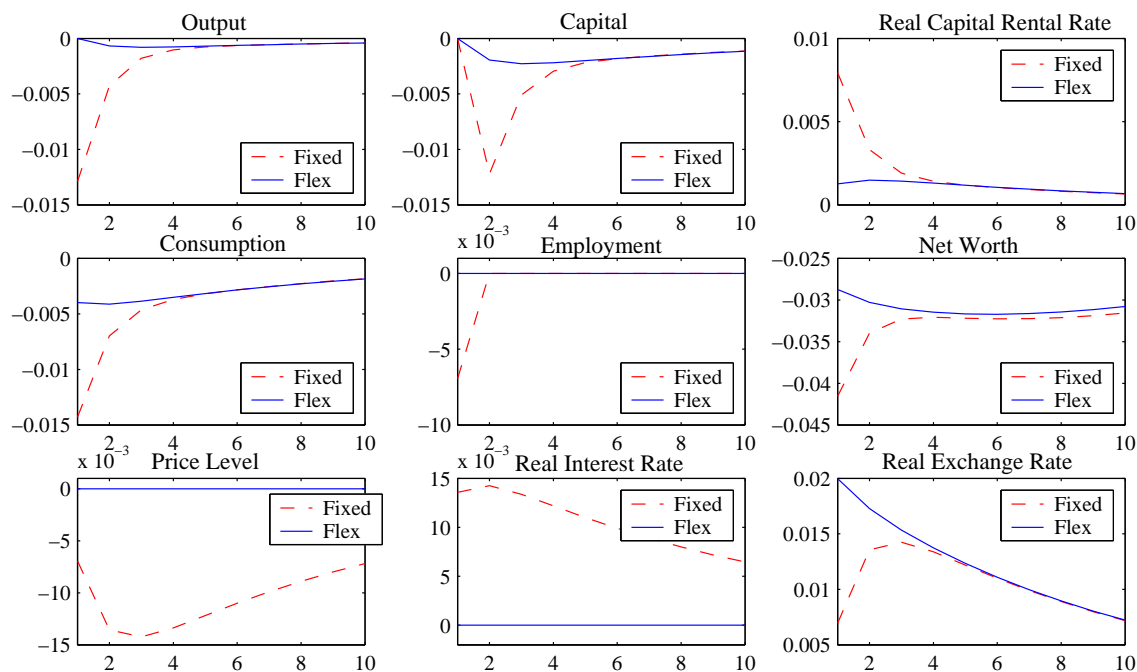


The Risk-Neutral Case

Figure 3.6 examine the effect of an international interest rate increase for the benchmark case of risk-neutral entrepreneurs. As explained before, regardless of whether the nominal exchange rate is held fixed or floats, an increase in the risk-free interest rate results in a decrease in investment demand, and thus results

in real depreciation of the local currency. Recall that real depreciation is accomplished through nominal depreciation and domestic price deflation for free floating and pegged exchange rates, respectively.

Figure 3.7: Exchange Rate Regime Comparison: The Case of a Export Demand Decrease and Risk Neutrality



The initial drop in output under fixed exchange rate regimes results from the real depreciation via deflation, which increases real wages, and thus decreases employment and output. In addition, although real depreciation negatively impacts net worth, deflation imposes a greater impact on real net worth turns than that of nominal devaluation. As a consequence, the drop in net worth is greater for fixed exchange rate regimes. This produces a larger increase in the external finance

premium and this a greater decrease in capital investment.

Figure 3.7 present the dynamic effects of an unfavorable drop in the external demand for exports. As noted in previous sections, a drop in exports negatively affects consumption, generates an immediate decrease in capital investment, and thus a real depreciation of local currency under both regimes.

Results are similar to those found for the international interest rate shock. Specifically, depreciation through deflation decreases employment and output. Additionally, the greater drop in net worth under fixed exchange rates produces a larger increase in the external finance premium, and thus an amplified response of capital and output.

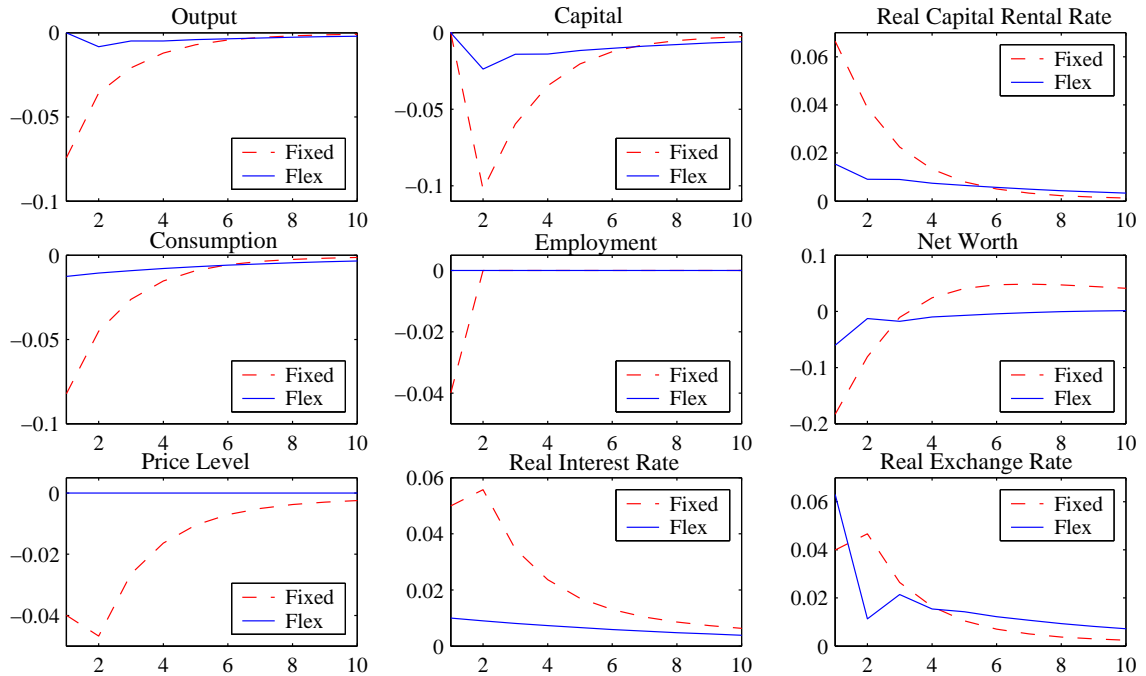
To sum up, the optimal response of risk-neutral entrepreneurs as developed in section 3.2.1 implies that the conventional wisdom stands, as flexible exchange rate regimes are better insulator of real shocks than fixed exchange rate regimes. This result occurs as domestic deflation imposes a direct impact on real wages and relatively greater effects on real net worth, than nominal depreciation.

The CCV Risk-Premium Case

This section discusses the effect of the shocks analyzed in previous sections when the risk-neutral specification is instead given by equation (3.28). The resulting impulse response functions are shown in Figures 3.8 and 3.9.

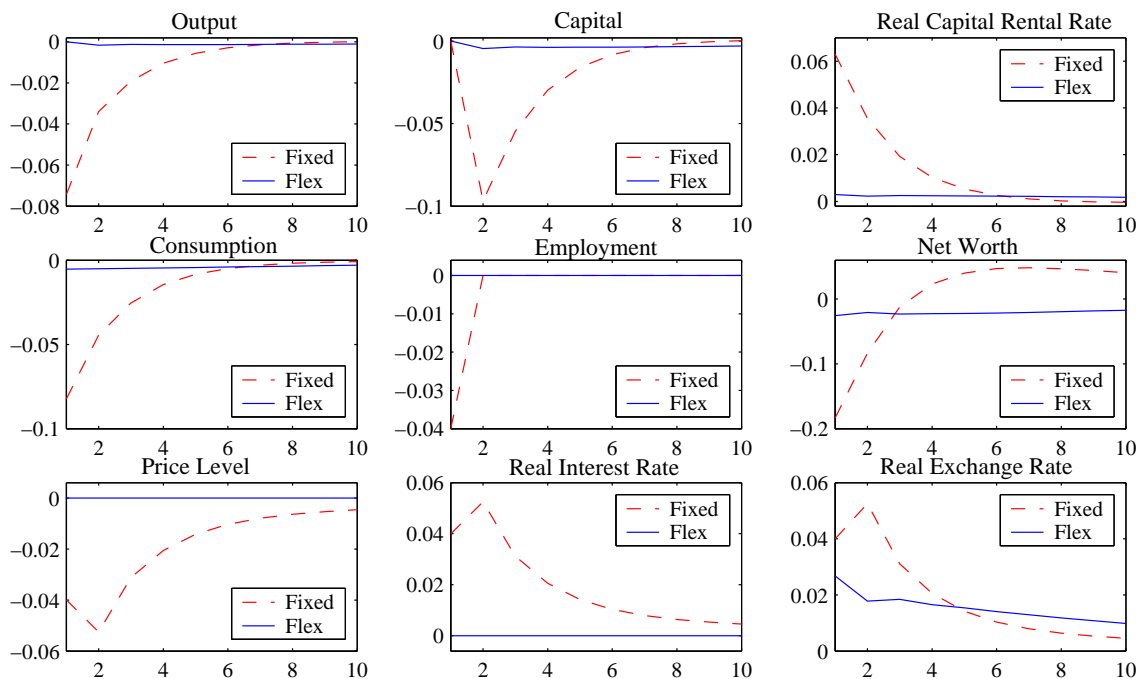
Results are qualitatively equivalent in the sense that the real depreciation arising from these shocks negatively affect the entrepreneurial net worth, and this

Figure 3.8: Exchange Rate Regime Comparison: CCV Risk Premium and the Case of an International Interest Rate Increase



drop in net worth is greater for fixed exchange rate regimes. Therefore, for pegged exchange rates, the larger increase in the external finance premium together with the initial drop in employment as a consequence of domestic price deflation, generates an amplified response of output to shocks. Consequently, as also shown by Céspedes et al (2000), the conventional wisdom holds for this case as well.

Figure 3.9: Exchange Rate Regime Comparison: CCV and the Case of a Export Demand Decrease and Risk Aversion

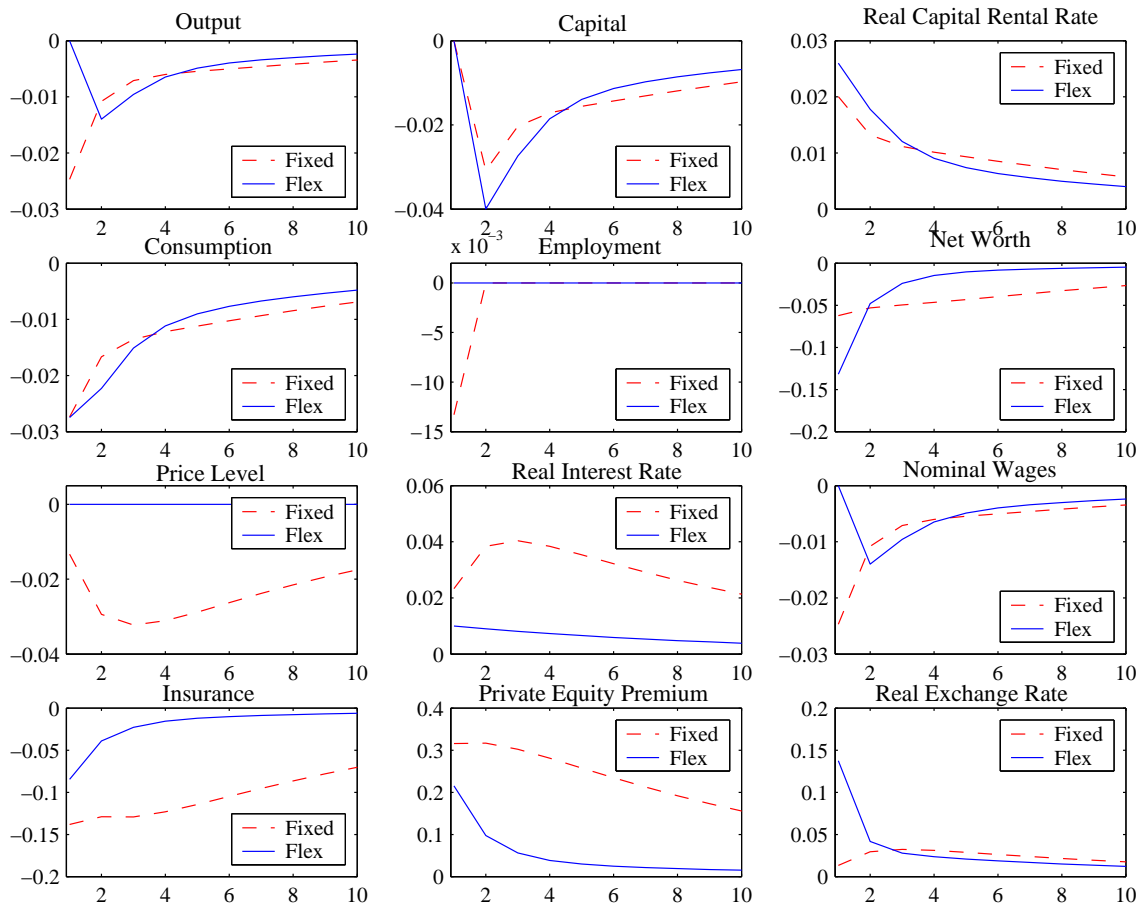


The Risk-Averse Case

Figures 3.10 and 3.11 show the impulse response functions resulting from the same unfavorable shocks, when the entrepreneurs are risk averse. Both shocks produce a real depreciation of the local currency as a consequence of the drop in capital inflows due to the lower domestic investment demand. The drop in net worth as a consequence of depreciation not only produces an increase in the external finance premium, but also raises the private equity premium, and thus reduces the capital supply. This amplified response of capital investment produces a much larger real depreciation under flexible exchange rates than under fixed rates, further decreasing

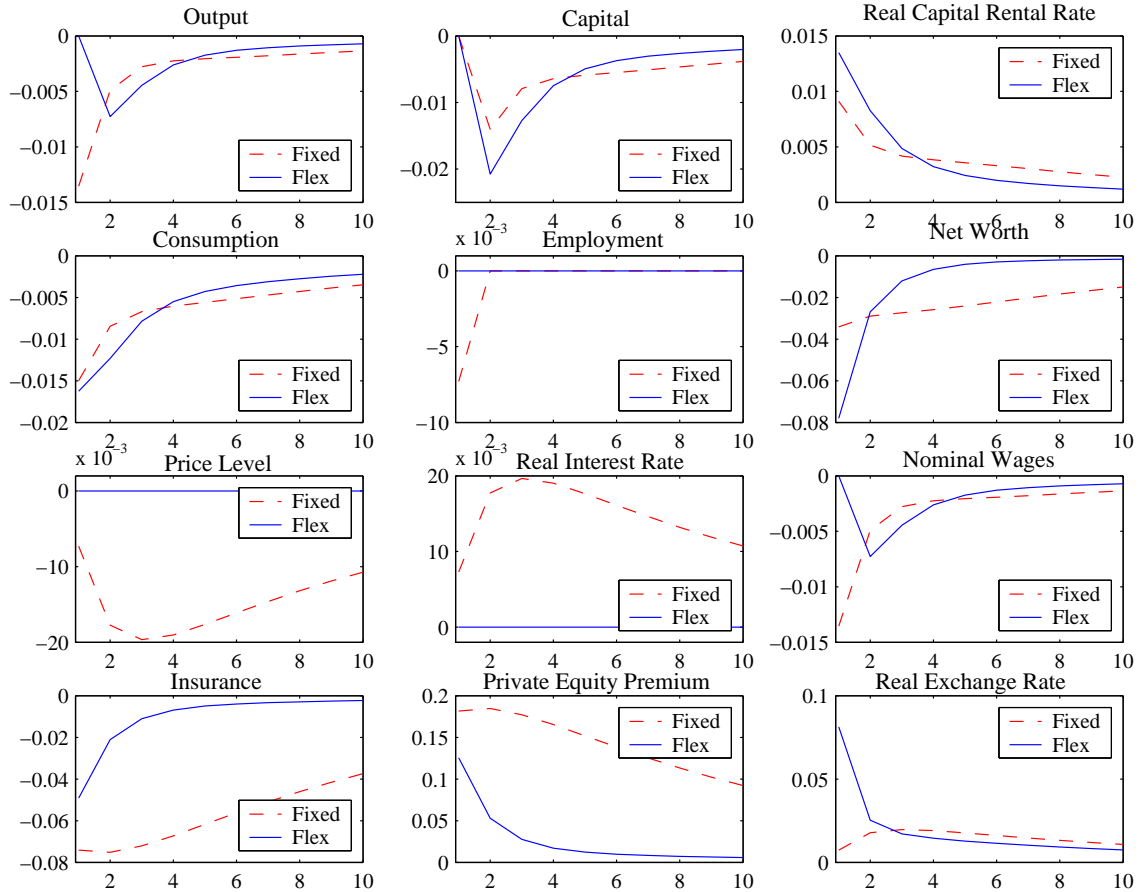
net worth and capital.

Figure 3.10: Exchange Rate Regime Comparison: The Case of an International Interest Rate Increase and Risk Aversion



As a result, the real depreciation and the drop in net worth is significantly larger under flexible rates, producing an sharper response of capital and output than when the exchange rate is pegged. Therefore, output and investment volatility is considerably higher when the exchange rate freely floats. On the other hand, given

Figure 3.11: Exchange Rate Regime Comparison: The Case of a Export Demand Decrease and Risk Aversion



that the real exchange rate responses to shocks is much larger for the floating than for fixed exchange rate regimes, the subsequent faster exchange rate recovery fosters quick output and capital recovery. This implies that for these cases, persistence of shocks is significantly reduced.

To sum up, real exchange rate flexibility may be beneficial for an economy in the sense that it helps absorb the negative effects of unfavorable real shocks through the expansionary effect of real depreciation on exports. However, this model suggests

that this higher flexibility comes at a cost, which is that there is higher real exchange rate volatility. This volatility negatively impacts the profits of the capital producing entrepreneurial sector through a higher private equity premium, and thus produces more pronounced responses of capital and output to real shocks. Therefore, when entrepreneurial risk aversion is considered, the conventional wisdom does not hold.

In welfare terms as measured by the path followed by consumption, notice that it is no longer clear that flexible exchange rates outperform fixed rates, since consumption experiences a larger decrease under free floating, although it more quickly returns to the steady state. The policy implications are that fixed exchange rate regimes may have benefits for economies with deficient levels of information technology, or with a relatively less active corporate sector.

Chapter 4

Conclusion

As we have seen, the behavior of the entrepreneurial sector can act as an additional mechanism that magnifies and propagates shocks of an economy. Relaxing the simplifying assumption that entrepreneurs engaging in debt contracts are risk-neutral may explain why entrepreneurs in some economies rapidly move from euphoria during booms to deep depression and stagnation during (even mild) recessions.

In the microeconomic model, the inclusion of risk aversion has two main consequences. First, the entrepreneur demands insurance as an incentive for taking on the risk of a new investment. Therefore, the external finance premium that the lender charges reflects both the cost imposed by the standard agency problems coming from asymmetric information, as well as the cost of providing insurance to the entrepreneur. Second, the total rental cost paid by final goods firms to use capital produced by entrepreneurs incorporates not only the external finance premium, but also the risk premium required by risk-averse entrepreneurs due to the stochastic nature of their investment returns, or the private equity premium. As a result, for a given return to capital, risk-averse entrepreneurs are willing to supply less capital, as the risky nature of such investments implies a decrease in their expected utility.

Sensitivity analysis reveals that the lower the informational frictions, the lower

the private equity premium, as the lender is more willing to participate in the project when default costs are low. In the extreme case in which there are no informational frictions, the lender takes over the project, and executes it in a frictionless environment. When net worth is higher, entrepreneurs obtain more insurance in the optimal contract and the private equity premium is lower, as the effective degree of risk aversion is lower. As expected, the more volatile the economy and the higher the level of risk aversion of the entrepreneur, the higher the private equity premium. In addition, in the small open economy model, a real depreciation of the local currency reduces lender revenues in dollar terms, holding the opportunity costs of funds constant. As a result, both the supply of capital and the marginal cost of funds curves shift up.

The main conclusions arising from the analysis of the optimal contract can be summarized as follows: (i) entrepreneurial risk aversion limits the economy's capital supply through the action of the private equity premium; (ii) the private equity premium is countercyclical since changes in entrepreneurial profits and net worth affect the effective level of risk aversion and therefore the private equity premium in the opposite direction; (iii) the presence of asymmetric information is crucial for the private equity premium to exist. Absence of such frictions implies that risk-neutral agents, such as lenders, would be willing to execute the entrepreneur's projects.

In the closed-economy dynamic general equilibrium model with risk averse entrepreneurs, I show that the effect of real productivity and monetary shocks is magnified and propagated over time through the private equity premium. A shock resulting in an increase in profits and in net worth will decrease effective risk aver-

sion, thereby lowering the private equity premium. In response, entrepreneurs are willing to supply more capital to final goods firms, producing a positive impact on output, consumption and both entrepreneurial profits and net worth in subsequent periods. The opposite occurs if the shock is negative. The endogenous private equity risk premium causes business cycle fluctuations to be stronger and more persistent over time. As shocks are propagated and amplified by the private equity premium when private entrepreneurs are risk averse, this model predicts that economies with an important private entrepreneurial sector will show more volatile and persistent business cycles than economies with a private sector composed largely of publicly traded companies.

When analyzing the effect of asymmetric information and entrepreneurial risk-aversion in the context of a small open economy and liability dollarization, two interesting results are found. First, flexible exchange rates alleviate the propagating feature of the private equity premium. Specifically, the exchange rate overshoots in anticipation of the expected increase in the private equity premium. As the exchange rate quickly adjusts back in the following periods, we encounter effects in the opposite direction on entrepreneurial wealth, and faster recovery of the private equity premium and investment to their steady state values. Second, by carrying out an exchange rate regime comparison, I find that the greater volatility associated with flexible exchange rate regimes adversely impacts the private equity premium, and thus amplifies the supply of capital and output responses to shocks. In this context, fixed exchange rate regimes may be preferable under less restrictive conditions than those conventionally found in the literature.

The most important conclusions resulting from the general equilibrium analysis are (i) the private equity premium amplifies business cycles, because it becomes less relevant during booms and more important during recessions; (ii) in a large closed economy, the private equity premium also reinforces itself and becomes a mechanism that helps business cycles become more prolonged; (iii) in a small open economy, the presence of flexible exchange rates somewhat alleviates the autocorrelation of shocks over time, at the expense of an even more pronounced effect of shocks on capital and output; (iv) flexible exchange rate regimes, by allowing faster real exchange rate adjustment, accentuate the volatility feature of the private equity premium in small open economies; (v) in terms of economic performance, fixed exchange rate regimes could be preferable under less restrictive conditions than, for instance, the “unrealistic” set of parameters found by Céspedes, Chang and Velasco (2000).

To sum up, this dissertation suggests an additional reasons for procyclical entrepreneurial activity, which, in turn, helps explain the magnitude of business cycle fluctuations that information frictions alone have failed to rationalize. It is known that business cycles in some countries are stronger than in others. This dissertation predicts that economies with a relatively higher share of private companies should present sharper business cycles, since in this context there is room for a more active role for entrepreneurs. That is, two economies with equal financial health and real sector robustness may have different cyclical volatility due to differences in the ownership structure of the productive sector.

In terms of policy implications, any improvement in information technology

and transparency in the privately-owned private sector necessarily implies an alleviation in the volatility produced by asymmetric information in the context of private entrepreneurs. Policies encouraging more established businesses to become public would accomplish a similar effect. Finally, economies with deficient levels of information technologies or with a relatively low share of public companies could benefit from lower volatility under fixed exchange rate regimes.

An interesting extension of this model would be to test empirically whether industries, geographic regions, or countries with relatively larger privately-held private sectors exhibit more volatile business cycles. Some economies enjoying both a healthy financial system and a robust private sector may remain stymied in periods of prolonged recessions following a negative shock while experiencing protracted periods of euphoria during booms. Such behavior on the part of an economy could be explained in part by risk-aversion on the part of entrepreneurs.

Chapter 5

Appendix

5.1 The Private Equity Premium

$$U'(C_{t+1}^e) = \left[X_t N_t + (\omega - \hat{\omega}) R_{t+1} K_{t+1} \right]^{-\gamma}$$

If we define $\alpha = X_t N_t - \hat{\omega} R_{t+1} K_{t+1}$ and $\beta = \omega R_{t+1} K_{t+1}$, then

$$\text{Cov}[U'(C_{t+1}^e), \omega] = \text{Cov}[(\alpha + \beta\omega)^{-\gamma}, \omega] = \text{Cov}[f(\omega), \omega] \quad (\text{B.1})$$

By a first-order Taylor approximation,

$$f(X) \approx (\alpha + \beta\omega_0)^{-\gamma} - \gamma(\alpha + \beta\omega_0)^{-(1+\gamma)}(\omega - \omega_0)$$

where ω_0 is the steady-state value of ω .

Therefore,

$$\text{Cov}[f(\omega), \omega] \approx -\gamma(\beta\omega_0 + \gamma)^{-(1+\gamma)} \sigma_\omega$$

and so,

$$\text{Cov}[U'(C_{t+1}^e), \omega] \approx -\gamma \left[X_t N_t + (1 - \hat{\omega}) R_{t+1} K_{t+1} \right]^{-(1+\gamma)} \sigma_\omega^2 \quad (\text{B.2})$$

Clearly, $\text{Cov}(\cdot) < 0$. Also, $\frac{\partial EP}{\partial N_t} < 0$.

5.2 The Log-Linearized Contract

5.2.1 Contract under risk-neutrality

First Order Condition of Contract with Respect to $\bar{\omega}$.

$$\xi_R^{foc} r_{t+1} - \xi_{\bar{\omega}}^{foc} \bar{\omega}_t + \xi_{\rho}^{foc} \rho_{t+1} = 0 \quad (\text{C.1})$$

where

$$\begin{aligned} \xi_R^{foc} &= R \left\{ [1 - H(\bar{\omega}^{ss})] \left[(1 - \mu) E(\omega | \omega < \bar{\omega}) + E(\omega | \omega > \bar{\omega}) \right] \right. \\ &\quad \left. - \mu \bar{\omega}^{ss} h(\bar{\omega}^{ss}) \left[E(\omega | \omega > \bar{\omega}) - \bar{\omega}^{ss} [1 - H(\bar{\omega}^{ss})] \right] \right\} \\ \xi_{\bar{\omega}}^{foc} &= \bar{\omega}^{ss} \left\{ \mu R h(\bar{\omega}^{ss}) \bar{\omega}^{ss} [1 - H(\bar{\omega}^{ss})] - (1 + \rho) h(\bar{\omega}^{ss}) \right\} \\ \xi_{\rho}^{foc} &= (1 + \rho) [K - N] \end{aligned}$$

Lender Participation Constraint.

$$\varepsilon_R^{PC} r_{t+1} + \varepsilon_K^{PC} k_{t+1} + \varepsilon_{\bar{\omega}}^{PC} \bar{\omega}_t + \varepsilon_N^{PC} n_t - \varepsilon_{\rho}^{PC} \rho_{t+1} = 0 \quad (\text{C.2})$$

where

$$\begin{aligned} \varepsilon_R^{PC} &= R \left\{ (1 - \mu) E(\omega | \omega < \bar{\omega}) K + K [1 - H(\bar{\omega}^{ss})] \bar{\omega}^{ss} \right\} \\ \varepsilon_K^{PC} &= K \left\{ (1 - \mu) E(\omega | \omega < \bar{\omega}) R + R [1 - H(\bar{\omega}^{ss})] \bar{\omega}^{ss} - (1 + \rho) \right\} \end{aligned}$$

$$\xi_{\bar{\omega}}^{PC} = \bar{\omega}^{ss} \left\{ RK[1 - H(\bar{\omega}^{ss})] - \mu RK h(\bar{\omega}^{ss}) \bar{\omega}^{ss} \right\}$$

$$\xi_N^{PC} = N(1 + \rho)$$

$$\xi_{\rho}^{PC} = (1 + \rho)[1 - H(\bar{\omega}^{ss})]$$

5.2.2 Contract under Risk Aversion

First Order Condition of Contract with Respect to $\hat{\omega}$.

$$\varepsilon_{XN}^{\hat{\omega}}(x_t + n_t) + \varepsilon_R^{\hat{\omega}} r_{t+1} + \varepsilon_K^{\hat{\omega}} k_{t+1} - \varepsilon_{\hat{\omega}}^{\hat{\omega}} \hat{\omega}_t = 0 \quad (\text{C.3})$$

where

$$\varepsilon_{XN}^{\hat{\omega}} = XN \left\{ [XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} [1 - H(\hat{\omega}^{ss})]^2 - (XN)^{-(1+\gamma)} H(\hat{\omega}^{ss})^2 \right\}$$

$$\varepsilon_R^{\hat{\omega}} = R \left\{ \gamma(1 - \hat{\omega}^{ss})K[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} [1 - H(\hat{\omega}^{ss})]^2 \right\}$$

$$\varepsilon_K^{\hat{\omega}} = K \left\{ \gamma(1 - \hat{\omega}^{ss})R[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} [1 - H(\hat{\omega}^{ss})]^2 \right\}$$

$$\begin{aligned} \varepsilon_{\hat{\omega}}^{\hat{\omega}} = & \hat{\omega}^{ss} \left\{ \gamma(1 - \hat{\omega}^{ss})RK[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} [1 - H(\hat{\omega}^{ss})]^2 \right. \\ & \left. - (XN)^{-(1+\gamma)} H(\hat{\omega}^{ss})^3 h(\hat{\omega}^{ss}) - (XN)^{-\gamma} H(\hat{\omega}^{ss}) \right\} \end{aligned}$$

First Order Condition of Contract with Respect to K_{t+1} .

$$\varepsilon_{XN}^K(x_t + n_t) + \varepsilon_R^K r_{t+1} + \varepsilon_K^K k_{t+1} - \varepsilon_{\hat{\omega}}^K \hat{\omega}_t + \varepsilon_{\rho}^K \rho_{t+1} = 0 \quad (\text{C.4})$$

where

$$\begin{aligned} \varepsilon_{XN}^K &= XN\gamma \left\{ [(1-\rho) - (1-\mu)E(\omega|\omega < \hat{\omega})R - E(\omega|\omega > \hat{\omega})R] \right. \\ &\quad [XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma} [1 - H(\hat{\omega}^{ss})] \\ &\quad + [(1-\rho) - (1-\mu)E(\omega|\omega < \hat{\omega})R - R\hat{\omega}^{ss}[1 - H(\hat{\omega}^{ss})]](XN)^{-(1+\gamma)} H(\hat{\omega}^{ss}) \\ &\quad \left. + R[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} \frac{1 + \gamma}{XN + (1 - \hat{\omega}^{ss})RK} \sigma_{\omega}^2 \right\} \\ \varepsilon_R^K &= R \left\{ [(1-\rho) - (1-\mu)E(\omega|\omega < \hat{\omega})R - E(\omega|\omega > \hat{\omega})R][XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma} \right. \\ &\quad [1 - H(\hat{\omega}^{ss})](1 - \hat{\omega}^{ss})\gamma K \\ &\quad + \left((XN)^{-\gamma} H(\hat{\omega}^{ss}) + [XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma} [1 - H(\hat{\omega}^{ss})] \right) (1 - \mu)E(\omega|\omega < \hat{\omega}) \\ &\quad + (XN)^{-(1+\gamma)} \hat{\omega}^{ss} [1 - H(\hat{\omega}^{ss})] \\ &\quad \left. - [\gamma(XN + (1 - \hat{\omega}^{ss})RK)]^{-(1+\gamma)} \left[1 - \frac{1 + \gamma}{XN + (1 - \hat{\omega}^{ss})RK} \right] \sigma_{\omega}^2 \right\} \\ \varepsilon_K^K &= K \left\{ [(1-\rho) - (1-\mu)E(\omega|\omega < \hat{\omega})R - E(\omega|\omega > \hat{\omega})R][XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma} \right. \\ &\quad [1 - H(\hat{\omega}^{ss})](1 - \hat{\omega}^{ss})R \\ &\quad \left. + R^2\gamma[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} \frac{1 + \gamma}{XN + (1 - \hat{\omega}^{ss})RK} (1 - \hat{\omega}^{ss}) \sigma_{\omega}^2 \right\} \end{aligned}$$

$$\begin{aligned}
\varepsilon_{\hat{\omega}}^K &= \hat{\omega}^{ss} \left\{ [(1 - \rho) - (1 - \mu)E(\omega|\omega < \hat{\omega})R - E(\omega|\omega > \hat{\omega})R] \right. \\
&\quad \left. [XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma} [1 - H(\hat{\omega}^{ss})] \gamma RK + \right. \\
&\quad \left. [(XN)^{-\gamma} H(\hat{\omega}^{ss}) - ((XN)^{-\gamma} H(\hat{\omega}^{ss}) + [XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma} [1 - H(\hat{\omega}^{ss})]) (1 - \mu)] \right. \\
&\quad \left. Rh(\hat{\omega}^{ss})\hat{\omega}^{ss} - (XN)^{-\gamma} H(\hat{\omega}^{ss})R[1 - H(\hat{\omega}^{ss}) - h(\hat{\omega}^{ss})\hat{\omega}^{ss}] \right. \\
&\quad \left. + R^2 K \gamma [XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} \frac{1 + \gamma}{XN + (1 - \hat{\omega}^{ss})RK} \sigma_{\omega}^2 \right\} \\
\varepsilon_{\rho}^K &= (1 + \rho) \left((XN)^{-\gamma} H(\hat{\omega}^{ss}) + [XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma} [1 - H(\hat{\omega}^{ss})] \right)
\end{aligned}$$

Lender Participation Constraint.

$$\varepsilon_{XN}^{PC}(x_t + n_t) = \varepsilon_R^{PC} r_{t+1} - \varepsilon_K^{PC} k_{t+1} + \varepsilon_{\hat{\omega}}^{PC} \hat{\omega}_t + \varepsilon_N^{PC} n_t - \varepsilon_{\rho}^{PC} \rho_{t+1} \quad (\text{C.5})$$

where

$$\varepsilon_{XN}^{PC} = XN$$

$$\varepsilon_R^{PC} = R \left\{ (1 - \mu)E(\omega|\omega < \hat{\omega})K + K[1 - H(\hat{\omega}^{ss})] \hat{\omega}^{ss} \right\}$$

$$\varepsilon_K^{PC} = YK$$

$$\varepsilon_{\hat{\omega}}^{PC} = \hat{\omega}^{ss} RK [1 - H(\hat{\omega}^{ss})]$$

$$\varepsilon_N^{PC} = N(1 + \rho)$$

$$\varepsilon_{\rho}^{PC} = (1 + \rho)[K - N]$$

5.3 Parameterization

In this section I present the values of the parameters used to numerically simulate this model. Note that this is an exercise intended to shed light on the qualitative impact of the private equity premium on the model dynamics, and thus its purpose is not to perform a calibration exercise.

Table 5.1: Parametrization.

Parameter	Description	Value
α	share of capital to output	0.35
γ	coefficient of risk aversion	2
β	discount factor	0.99
ν	labor supply elasticity	1.2
μ	bankruptcy cost	0.11
δ	entrepreneur saving rate	0.95
κ^{RN}	risk-neutral capital to net worth ratio	2

I select parameter values in a standard fashion, according to previous literature when possible. Following Bernanke, Gertler and Gilchrist (1999), the share of capital in production is set at 0.35, the discount factor β is set to 0.99,¹ and the labor supply elasticity ν is set at 1.2. In line with Rampini (2003), the constant relative risk-aversion coefficient is taken to be 2. From Céspedes, Chang and Velasco (2000), I set the default cost parameter to be equal to 0.11. Finally, consistent with both Bernanke et al. (1999) and Céspedes et al (2000), I use an risk-neutral capital to net worth ratio of 2.

In order to get the implied steady state value of other parameters, I solve for

¹Since in the steady state $1 + \rho = \beta^{-1}$ (see equation (2.24)), the risk-free interest rate $1 + \rho$ equals $\frac{1}{0.99}$.

the steady state of the model. Results turn out to be sensitive to the capital to net worth, $\frac{K}{N}$, that is, proportional changes in K and N do not affect the model. Having said that, without loss of generality I arbitrarily set $N^{SS} = 2$. Then, for the risk-neutral case $K^{SS} = 4$. The rest of the variables are determined by the following equations coming from the steady state solution of the model:

$$R^{SS} = \frac{1}{1 - \mu * E(\omega|\omega < \hat{\omega})} \left[1 + \rho + \frac{1 - \delta(1 + \rho)}{\delta \frac{K^{SS}}{N^{SS}}} \right] \quad (C.6)$$

$$B^{SS} = K^{SS} - N^{SS} \quad (C.7)$$

$$C_E^{SS} = \frac{1 - \delta}{\delta} N^{SS} \quad (C.8)$$

$$C^{SS} = \frac{1}{\alpha} \left[(1 - \alpha)K^{SS} + \frac{(1 - \alpha)(1 - \delta)}{\delta} N^{SS} + \rho B^{SS} \right] \quad (C.9)$$

$$Y^{SS} = K^{SS} + C^{SS} + C_E^{SS} \quad (C.10)$$

$$L^{SS} = \left[\frac{(1 - \alpha)Y^{SS}}{C^{SS}} \right]^{\frac{1}{\nu}} \quad (C.11)$$

$$W^{SS} = (1 - \alpha) \frac{Y^{SS}}{L^{SS}} \quad (C.12)$$

For the risk-averse case, since there is no known capital to net worth ratio, I obtain it by following this procedure. Without loss of generality, I maintain the assumption that $N^{SS} = 2$. Then I arbitrarily chose a value for the capital to net worth ratio (say 2, to start). That gives me the corresponding values of K^{SS} and R^{SS} from equation (C.6). Then I check whether the resulting (R^{SS}, K^{SS}) pair coincides with one of the points on the capital supply coming from the risk-averse optimal contract. I adjust the arbitrary value of K^{SS} until converging to the equilibrium (R^{SS}, K^{SS}) coordinate. This results in an equilibrium capital to net worth ratio equal

to 1.55, which is expectedly lower than the risk neutral ratio due to the additional costs of supplying capital by risk averse entrepreneurs.

Table 5.2 summarizes the implied steady state values of some of those parameters.

Table 5.2: Implied steady state values.

Parameter	Description	Value
κ^{RN}	risk-averse capital to net worth ratio	1.55
R^{RN}	risk-neutral capital rental rate	1.0318
R^{RA}	risk-averse capital rental rate	1.0378
PEP	private equity premium	0.6%
$\left(\frac{D}{C}\right)^{RN}$	risk-neutral share of debt to consumption	0.26%
$\left(\frac{D}{C}\right)^{RA}$	risk-averse share of debt to consumption	0.19%
$\left(\frac{WL}{C}\right)^{RN}$	risk-neutral share of labor income to consumption	99.74%
$\left(\frac{WL}{C}\right)^{RA}$	risk-averse share of labor income to consumption	99.81%
$\left(\frac{K}{Y}\right)^{RN}$	risk-neutral capital to output ratio	0.34
$\left(\frac{K}{Y}\right)^{RA}$	risk-averse capital to output ratio	0.33
$\left(\frac{C^E}{Y}\right)^{RN}$	risk-neutral entrepreneur consumption to output ratio	0.89%
$\left(\frac{C^E}{Y}\right)^{RA}$	risk-averse entrepreneur consumption to output ratio	1.15%

The resulting steady state capital rental rate for the risk-neutral case is equal to 1.0318, while for the risk-averse case it corresponds to 1.0378. This implies a differential of 0.6%, mainly explained by the private equity premium (although it also includes the insurance cost). The steady state capital to output ratio, $\frac{K^{SS}}{Y^{SS}}$, turns out to be 0.34 and 0.33 for the risk-neutral and the risk-averse cases, respectively. Likewise, respectively for both cases, the entrepreneurial consumption to output ratio are 0.89% and 1.15%. Finally, the resulting share of debt to consumption for

the risk-neutral case is 0.26%, whereas for the risk-averse case, it is equal to 0.19%.

Robustness Check

I carried out a robustness check to test how sensitive these results are to changes in some of the parameter values, namely the default cost parameter (μ), the weight of domestic goods on total consumption (θ), the entrepreneurial saving rate (δ), the share of capital in production (α) and the risk aversion coefficient (γ). I do this for the closed economy framework and for a small open economy under both exchange rate regimes.

Results in general are very insensitive to changes in μ . By examining the parameter values of the contracts, one may notice that μ is generally multiplied by very small numbers, considerably reducing its role in determining the equilibrium variables of the contract. On the other hand, while results are also insensitive to changes in the entrepreneurial savings rate δ in the risk neutral case, the opposite is true for the risk averse case. In particular, increases (decreases) in δ produce a larger (smaller) impact of shocks on output. Intuitively, the higher the savings rate, the more direct the impact of entrepreneurs' profits on net worth, adding volatility that matters more to risk-averse entrepreneurs. For instance, decreases in profits as a consequence of an adverse shock are translated into larger changes in net worth the higher the saving rate. Therefore, low values of δ can potentially reverse the result that risk aversion amplifies shocks. In particular, I find that if δ lies around 0.65 or lower, shocks are stronger if entrepreneurs are risk neutral rather than risk

averse for the interest rate shock (0.15 for the export shock). Conventional wisdom does not hold under any value of δ , and the difference between exchange rate regimes widens as δ rises.

As expected, increases (decreases) in the share of capital in production (α) increase (decrease) the impact of shocks on output, as capital plays a more (less) important role. However, variations in endogenous variables due to changes in α occur in the same direction and similar magnitude so that qualitative results are not affected. Also as expected, higher levels of risk aversion magnify the impact of shocks on capital and output. Therefore, potentially low levels of risk aversion can reverse the result that risk aversion magnifies the effects of shocks. In fact, if the coefficient of risk aversion is below 0.54, output more strongly respond to shocks under risk neutrality than under risk aversion for the interest rate shock (0.335 for the export shock).

Finally, results are sensitive to variations in the share of domestic goods in total consumption (θ) only under flexible exchange rate regimes. That is, for both the risk neutral case and the risk averse case, the impact of shocks on output under free floating exchange rate regimes is greater (less) the lower (higher) θ (or equivalently, the higher (lower) the share of imports in consumption). Under flexible exchange rates, real depreciation occurs through increases in the nominal exchange rate, which increases yet more the cost of consumption the higher the importance of imports in consumption. On the other hand, under pegged rates, real depreciation is accomplished through domestic deflation, which is slower and imposes a neutralizing effect as it reduces the cost of consumption. Therefore, lower levels of θ under risk

neutrality can cause conventional wisdom not to hold, however it does not occur for any value of θ . In addition, a high θ can potentially result in the opposite outcome under risk aversion. That is, for conventional wisdom to still hold when risk-averse entrepreneurs are considered, the value of θ has to be above 0.87 for the interest rate shock and above 0.92 for the export demand shock.

5.4 Impulse Response Functions

Figure 5.1: Effects of a Favorable Productivity Shock

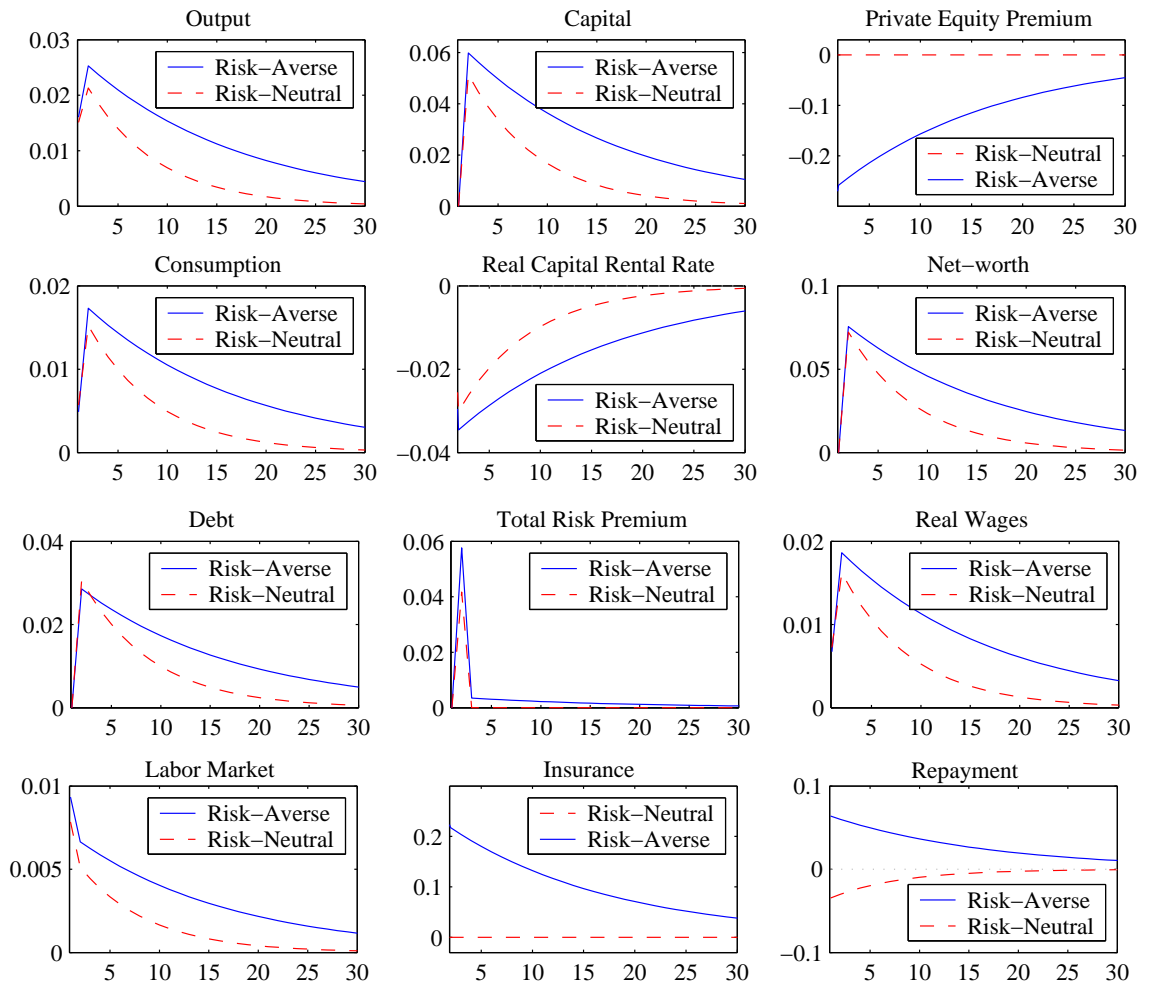
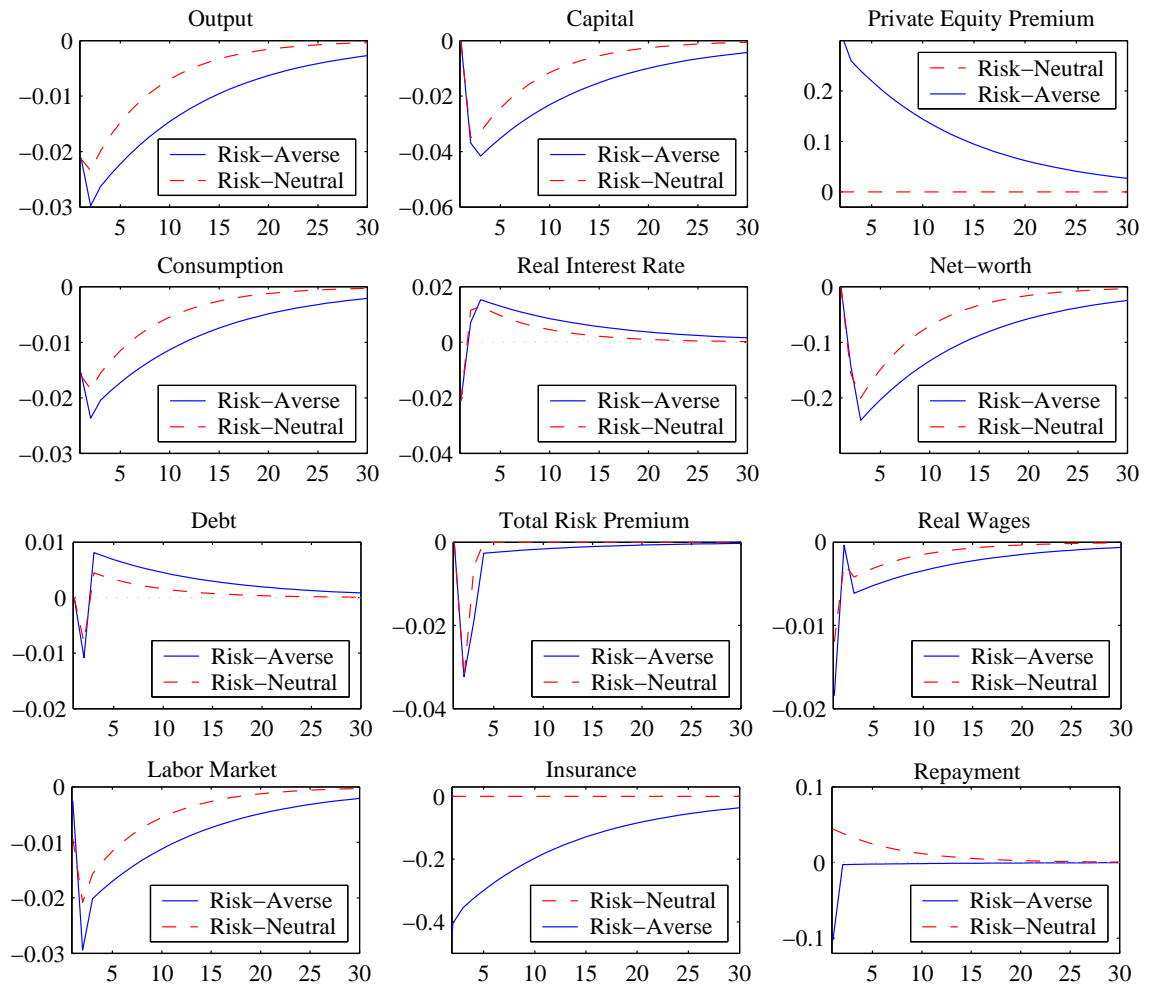


Figure 5.2: Effects of an Adverse Monetary Shock



5.5 The Log-linearized Small Open Economy Model

Aggregate demand

$$q_t + c_t = w_t + l_t \quad (\text{D.1})$$

$$q_t = (1 - \theta)s_t \quad (\text{D.2})$$

$$\frac{n_t}{Q_K} + (1 - \frac{1}{Q_K})(s_t + b_{t+1}) = (1 - \theta)(s_t + k_{t+1}) \quad (\text{D.3})$$

$$\xi_R^{foc} r_{t+1} - \xi_{\bar{w}}^{foc} \bar{w}_t + \xi_K^{foc} k_{t+1} + \xi_N^{foc} n_t + \xi_{S_{t+1}}^{foc} s_{t+1} - \xi_{S_t}^{foc} s_t = 0 \quad (\text{D.4})$$

$$\xi_R^{PC} r_{t+1} + \xi_K^{PC} k_{t+1} + \xi_{\bar{w}}^{PC} \bar{w}_t + \xi_N^{PC} n_t - \xi_{S_{t+1}}^{PC} s_{t+1} + \xi_{S_t}^{PC} s_t = 0 \quad (\text{D.5})$$

$$\varepsilon_{XN}^{\hat{w}}(x_t + n_t) + \varepsilon_R^{\hat{w}} r_{t+1} + \varepsilon_K^{\hat{w}} k_{t+1} - \varepsilon_{\hat{w}}^{\hat{w}} \hat{w}_t = 0 \quad (\text{D.6})$$

$$\varepsilon_{XN}^K(x_t + n_t) + \varepsilon_R^K r_{t+1} + \varepsilon_K^K k_{t+1} - \varepsilon_{\hat{w}}^K \hat{w}_t + \varepsilon_S^K (s_{t+1} - \theta s_t) = 0 \quad (\text{D.7})$$

$$\varepsilon_{XN}^{PC}(x_t + n_t) = \varepsilon_R^{PC} r_{t+1} - \varepsilon_K^{PC} k_{t+1} + \varepsilon_{\hat{w}}^{PC} \hat{w}_t + \varepsilon_N^{PC} n_t - \varepsilon_{S_{t+1}}^{PC} s_{t+1} + \varepsilon_{S_t}^{PC} s_t \quad (\text{D.8})$$

$$y_t = \theta \frac{QK}{Y} (q_t + k_{t+1}) + \theta \frac{QC}{Y} (q_t + c_t) + \frac{SX}{Y} (s_t + x_t) \quad (\text{D.9})$$

Aggregate supply

$$y_t = a_t + \alpha k_t + (1 - \alpha)l_t \quad (\text{D.10})$$

$$r_t - p_t = y_t - k_t \quad (\text{D.11})$$

$$w_t - p_t = y_t - l_t \quad (\text{D.12})$$

$$w_t - q_t - c_t = (v - 1) l_t \quad (\text{D.13})$$

Evolution of State Variables

$$Nn_t = \delta \left\{ [1 - \mu E(\omega | \omega < \hat{\omega})] RK (r_t + k_t) - (1 + \rho)B b_t - \mu R \hat{\omega}^2 h(\hat{\omega}) \hat{\omega}_t \right\} \quad (D.14)$$

The first block of equations represents aggregate demand. Equation (D.1) is the log-linearized version of the workers' aggregate budget constraint (3.15).² In the steady state, the value consumption (C) must equal the nominal labor income, where the definition of consumption cost is given by Equation (D.2). Equation (2.33) describes the entrepreneurs' borrowing needs given investment and available net worth, where the parameter κ is the steady-state total investment to net worth ratio.

Equations (D.4) and (D.5) are the log-linearized form of the risk-neutral contract's first order condition and the lender participation constraint, respectively. Similarly, equations (D.6), (D.7) and (D.8) are the log-linearized form of the risk-averse contract's first order conditions with respect to $\hat{\omega}$ and K_{t+1} , and the lender's participation constraint, respectively. Given changes in R_{t+1} , N_t , S_{t+1} , and S_t these equations jointly determine the deviation from the steady state of capital investment and the repayment rate ($\hat{\omega}$). Parameters ξ_i^{foc} and ξ_i^{PC} , $\varepsilon_i^{\hat{\omega}}$, ε_i^K and ε_i^{PC} are constants at the steady state that accompany the endogenous variables. More details about these constants can be found in the next appendix.

Finally, equation (D.9) is the economy-wide resource constraints. Output changes are explained by changes in investment, consumption and exports, weighted by their importance in total output at the steady state.

²I normalize the steady state price level to equal 1.

The second block of equations describes the aggregate supply for this economy. Specifically, equation (D.10) presents the log-linearized version of the production function, while equations (D.11) and (D.12) are the first order conditions from the firm's profit maximization problem with respect to capital and labor, respectively. Lastly, equation (D.13) is the linearized version of equation (3.16), and shows workers' optimal substitution between consumption and work, taking into account changes in the price level and wages.

Finally, equation (D.14) introduces the evolution of the model's state variable, net worth, as the log-linearized form of equation (3.19).

5.5.1 Contract under risk-neutrality

First Order Condition of Contract with Respect to $\bar{\omega}$.

$$\xi_R^{foc} r_{t+1} - \xi_{\bar{\omega}}^{foc} \bar{\omega}_t + \xi_K^{foc} k_{t+1} + \xi_N^{foc} n_t + \xi_{S_{t+1}}^{foc} s_{t+1} - \xi_{S_t}^{foc} s_t = 0 \quad (\text{E.1})$$

where

$$\xi_R^{foc} = R \left\{ [1 - H(\bar{\omega}^{ss})] \left[(1 - \mu) E(\omega | \omega < \bar{\omega}) + E(\omega | \omega > \bar{\omega}) \right] \right.$$

$$\left. - \mu \bar{\omega}^{ss} h(\bar{\omega}^{ss}) \left[E(\omega | \omega > \bar{\omega}) - \bar{\omega}^{ss} [1 - H(\bar{\omega}^{ss})] \right] \right\}$$

$$\xi_{\bar{\omega}}^{foc} = \bar{\omega}^{ss} \left\{ \mu R h(\bar{\omega}^{ss}) \bar{\omega}^{ss} [1 - H(\bar{\omega}^{ss})] - (1 + \rho) h(\bar{\omega}^{ss}) \right\}$$

$$\xi_K^{foc} = (1 - \mu) R K E(\omega | \omega < \bar{\omega}) + R \bar{\omega}^{ss} [1 - H(\bar{\omega}^{ss})] - (1 + \rho) Q$$

$$\xi_N^{foc} = N(1 + \rho)$$

$$\xi_{S_{t+1}}^{foc} = (1 + \rho)(QK - N)$$

$$\xi_{S_t}^{foc} = (1 + \rho)(\theta QK - N)$$

Lender Participation Constraint.

$$\xi_R^{PC} r_{t+1} + \xi_K^{PC} k_{t+1} + \xi_{\bar{\omega}}^{PC} \bar{\omega}_t + \xi_N^{PC} n_t - \xi_S^{PC} (s_{t+1} - s_t) = 0 \quad (\text{E.2})$$

where

$$\xi_R^{PC} = RK[1 - H(\bar{\omega}^{ss}) - \mu\bar{\omega}^{ss}h(\bar{\omega}^{ss})][1 - H(\bar{\omega}^{ss})] \bar{\omega}^{ss}$$

$$\xi_K^{PC} = RK[1 - H(\bar{\omega}^{ss}) - \mu\bar{\omega}^{ss}h(\bar{\omega}^{ss})][1 - H(\bar{\omega}^{ss})] \bar{\omega}^{ss}$$

$$\xi_N^{PC} = N(1 + \rho)[1 - H(\bar{\omega}^{ss})]$$

$$\begin{aligned} \xi_{\bar{\omega}}^{PC} = & \bar{\omega}^{ss} \left\{ RK[1 - H(\bar{\omega}^{ss}) - \mu h(\bar{\omega}^{ss})\bar{\omega}^{ss}][1 - H(\bar{\omega}^{ss}) - h(\bar{\omega}^{ss})\bar{\omega}^{ss}] \right. \\ & \left. + E(\omega|\omega > \bar{\omega}) - \bar{\omega}^{ss}[1 - H(\bar{\omega}^{ss})] * (1 + \mu)h(\bar{\omega}^{ss}) \right\} - (1 + \rho)Nh(\bar{\omega}^{ss}) \end{aligned}$$

$$\xi_S^{PC} = N(1 + \rho)[1 - H(\bar{\omega}^{ss})]$$

5.5.2 Contract under Risk Aversion

First Order Condition of Contract with Respect to $\hat{\omega}$.

$$\varepsilon_{XN}^{\hat{\omega}}(x_t + n_t) + \varepsilon_R^{\hat{\omega}} r_{t+1} + \varepsilon_K^{\hat{\omega}} k_{t+1} - \varepsilon_{\hat{\omega}}^{\hat{\omega}} \hat{\omega}_t = 0 \quad (\text{E.3})$$

where

$$\varepsilon_{XN}^{\hat{\omega}} = XN \left\{ [XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} [1 - H(\hat{\omega}^{ss})]^2 - (XN)^{-(1+\gamma)} H(\hat{\omega}^{ss})^2 \right\}$$

$$\varepsilon_R^{\hat{\omega}} = R \left\{ \gamma(1 - \hat{\omega}^{ss})K [XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} [1 - H(\hat{\omega}^{ss})]^2 \right\}$$

$$\begin{aligned}\varepsilon_{\hat{\omega}}^{\hat{\omega}} &= K \left\{ \gamma(1 - \hat{\omega}^{ss})R[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)}[1 - H(\hat{\omega}^{ss})]^2 \right\} \\ \varepsilon_{\hat{\omega}}^{\hat{\omega}} &= \hat{\omega}^{ss} \left\{ \gamma(1 - \hat{\omega}^{ss})RK[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)}[1 - H(\hat{\omega}^{ss})]^2 \right. \\ &\quad \left. - (XN)^{-(1+\gamma)}H(\hat{\omega}^{ss})^3h(\hat{\omega}^{ss}) - (XN)^{-\gamma}H(\hat{\omega}^{ss}) \right\}\end{aligned}$$

First Order Condition of Contract with Respect to K_{t+1} .

$$\varepsilon_{XN}^K(x_t + n_t) + \varepsilon_R^K r_{t+1} + \varepsilon_K^K k_{t+1} - \varepsilon_{\hat{\omega}}^K \hat{\omega}_t + \varepsilon_S^K (s_{t+1} - \theta s_t) = 0 \quad (\text{E.4})$$

where

$$\begin{aligned}\varepsilon_{XN}^K &= XN\gamma \left\{ [(1 - \rho) - (1 - \mu)E(\omega|\omega < \hat{\omega})R - E(\omega|\omega > \hat{\omega})R] \right. \\ &\quad \left. [XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma}[1 - H(\hat{\omega}^{ss})] \right. \\ &\quad \left. + [(1 - \rho) - (1 - \mu)E(\omega|\omega < \hat{\omega})R - R\hat{\omega}^{ss}[1 - H(\hat{\omega}^{ss})]](XN)^{-(1+\gamma)}H(\hat{\omega}^{ss}) \right. \\ &\quad \left. + R[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)} \frac{1 + \gamma}{XN + (1 - \hat{\omega}^{ss})RK} \sigma_{\omega}^2 \right\} \\ \varepsilon_R^K &= R \left\{ [(1 - \rho) - (1 - \mu)E(\omega|\omega < \hat{\omega})R \right. \\ &\quad \left. - E(\omega|\omega > \hat{\omega})R][XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma}[1 - H(\hat{\omega}^{ss})](1 - \hat{\omega}^{ss})\gamma K \right. \\ &\quad \left. + ((XN)^{-\gamma}H(\hat{\omega}^{ss}) + [XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma}[1 - H(\hat{\omega}^{ss})]) \right. \\ &\quad \left. (1 - \mu)E(\omega|\omega < \hat{\omega}) + (XN)^{-(1+\gamma)}\hat{\omega}^{ss}[1 - H(\hat{\omega}^{ss})] \right. \\ &\quad \left. - [\gamma(XN + (1 - \hat{\omega}^{ss})RK)]^{-(1+\gamma)} \left[1 - \frac{1 + \gamma}{XN + (1 - \hat{\omega}^{ss})RK} \right] \sigma_{\omega}^2 \right\} \\ \varepsilon_K^K &= K \left\{ [(1 - \rho) - (1 - \mu)E(\omega|\omega < \hat{\omega})R \right.\end{aligned}$$

$$\begin{aligned}
& -E(\omega|\omega > \hat{\omega})R[XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma}[1 - H(\hat{\omega}^{ss})](1 - \hat{\omega}^{ss})R \\
& + R^2\gamma[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)}\frac{1 + \gamma}{XN + (1 - \hat{\omega}^{ss})RK}(1 - \hat{\omega}^{ss})\sigma_\omega^2 \Big\} \\
\varepsilon_{\hat{\omega}}^K &= \hat{\omega}^{ss} \left\{ [(1 - \rho) - (1 - \mu)E(\omega|\omega < \hat{\omega})R \right. \\
& \left. - E(\omega|\omega > \hat{\omega})R][XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma}[1 - H(\hat{\omega}^{ss})]\gamma RK + \right. \\
& \left. [(XN)^{-\gamma}H(\hat{\omega}^{ss}) - ((XN)^{-\gamma}H(\hat{\omega}^{ss}) + [XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma}[1 - H(\hat{\omega}^{ss})])](1 - \mu) \right] \\
& \left. Rh(\hat{\omega}^{ss})\hat{\omega}^{ss} - (XN)^{-\gamma}H(\hat{\omega}^{ss})R[1 - H(\hat{\omega}^{ss}) - h(\hat{\omega}^{ss})\hat{\omega}^{ss}] \right. \\
& \left. + R^2K\gamma[XN + (1 - \hat{\omega}^{ss})RK]^{-(1+\gamma)}\frac{1 + \gamma}{XN + (1 - \hat{\omega}^{ss})RK}\sigma_\omega^2 \right\} \\
\varepsilon_S^K &= (1 + \rho)Q[(XN)^{-\gamma}H(\hat{\omega}^{ss}) + [XN + (1 - \hat{\omega}^{ss})RK]^{-\gamma}[1 - H(\hat{\omega}^{ss})]]
\end{aligned}$$

Lender Participation Constraint.

$$\varepsilon_{XN}^{PC}(x_t + n_t) = \varepsilon_R^{PC}r_{t+1} - \varepsilon_K^{PC}k_{t+1} + \varepsilon_{\hat{\omega}}^{PC}\hat{\omega}_t + \varepsilon_N^{PC}n_t - \varepsilon_{S_{t+1}}^{PC}s_{t+1} + \varepsilon_{S_t}^{PC}s_t \quad (\text{E.5})$$

where

$$\varepsilon_{XN}^{PC} = XN$$

$$\varepsilon_R^{PC} = R \left\{ (1 - \mu)E(\omega|\omega < \hat{\omega})K + K[1 - H(\hat{\omega}^{ss})]\hat{\omega}^{ss} \right\}$$

$$\varepsilon_K^{PC} = YK$$

$$\varepsilon_{\hat{\omega}}^{PC} = \hat{\omega}^{ss}RK[1 - H(\hat{\omega}^{ss})]$$

$$\varepsilon_N^{PC} = N(1 + \rho)$$

$$\varepsilon_{S_{t+1}}^{PC} = (1 + \rho)[QK - N]$$

$$\varepsilon_{S_t}^{PC} = (1 + \rho)[\theta QK - N]$$

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