AN INVESTIGATION OF THE RELATIONSHIP BETWEEN FIFTH-GRADE STUDENT AND TEACHER PERFORMANCE ON SELECTED TASKS INVOLVING NONMETRIC GEOMETRY

by
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ABSTRACT

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Statement of the Problem. This study investigated the relationship between teacher and student performance on selected mathematical tasks. A measure of teacher effectiveness was obtained by comparing teacher and student performance on identical geometric tasks.

Procedure. Teachers and their students from nineteen fifth-grade classes were designated as either control or experimental subjects. The six control treatment classes were presented topics in nonmetric geometry by means of self-instructional reading materials. The thirteen experimental treatment classes were presented the same topics by their teachers without the use of the reading materials. The duration of the instructional period consisted of four, fifty minute class periods. A criterion test, consisting of selected geometric tasks, was administered as a pre-test

and post-test to the students of the control and experimental classes. The same test was administered to the teachers of the experimental classes at the conclusion of the instructional period.

The hypothesis that students who read instructional materials in mathematics on their own will perform as well on selected tasks as those who have teachers explain and interpret the content for them was tested by comparing class mean scores. A second hypothesis questioned the relationship between the level of teacher performance on selected tasks and the level of performance exhibited by his students on these tasks. This hypothesis was examined by correlating the teacher scores on the criterion test with the mean scores of the classes in the experimental treatment.

The relationship between teacher and student performance on individual tasks appearing on the criterion test was examined by comparing correct and incorrect item responses selected by teachers and students. A comparison of the proportion of student incorrect responses for classes whose teachers missed an item, with the proportion of student incorrect responses for classes whose teachers correctly responded to a particular item, was made by applying the chi square statistic to response frequencies. A similar procedure investigated the relationship between particular incorrect teacher response and student response. This aspect of the study investigated the effect of the teacher on student performance by comparing teacher and student behavior on individual tasks.

Results. The reliability coefficient obtained for the criterion test was 0.72 as determined by the Kuder-Richardson formula 20. An estimate of item reliability was obtained and sixteen of the twenty-five test items exhibited acceptable reliability measures. The results of the analyses are summarized as follows: (1) An analysis of variance revealed that the mean score for the experimental classes was significantly higher than for the control classes at the 0.01 level; (2) there was a significant positive correlation between teacher test scores and class mean scores on the criterion test at the 0.02 level; (3) upon testing for independence of student and teacher selection of correct and incorrect responses to a particular item on the criterion test, ten of twenty-two items revealed a significant chi square at less than the 0.01 level. Items which exhibited a relationship between student and teacher performance either required a direct recall or application of a single definition presented in the materials; and (4) all but three of sixteen chi squares, which were not significant at less than the 0.10 level, supported the independence of teacher and student selection of a particular incorrect response to an item on the criterion test.

Conclusions. It was concluded that: (1) There is no support for the hypothesis that students who read materials in mathematics on their own will perform as well on selected tasks as those who have teachers explain and interpret the content for them; (2) there is support for the hypothesis that if a teacher performs at a certain level of success on selected mathematical tasks, then his students, following instruction, will perform at the same level on these tasks; (3) there is a relationship between student and teacher correct and incorrect performance on selected tasks involving the direct identification and application of a single definition. No evidence was found of a relationship for tasks which require a combination of the application of two or more definitions; and (4) there is no relationship between teacher and student selection of a particular incorrect response to a task on the criterion test.

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TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

INTRODUCTION

Teacher Effectiveness

The recent changes in the mathematics curriculum for the elementary school program have initiated concern over the type and amount of mathematics preparation advisable for the elementary school teacher. Recommendations have been made by such groups as the Committee on the Undergraduate Program in Mathematics[1] and The National Council of Teachers of Mathematics[2] concerning possible programs for pre- and inservice teachers of arithmetic. Institutions across the country are offering courses and workshops designed to increase the mathematical competency of elementary school teachers.

As a result of the emphasis on increasing the mathematics training of elementary school teachers, there has emerged a number of reports in the literature concerning the degree of success of various programs and the skills exhibited by teachers in mathematics. Dutton and Cheney[3] report that there was a low level of performance by a group of teachers on an arithmetic test of "understanding" which Dutton constructed. Melson[4] administered a test of selected mathematical questions to a sample of first-year teachers. The test contained specific items requiring recall of definitions and identification of symbols. The author concludes that the low level of performance indicates that institutions of higher education were not preparing teachers to teach the topics presented in many of the newer materials. Creswell reports on a situation where teachers score low on an arithmetic test and concludes:

> Much remains to be done, however, so that all elementary teachers will know and understand the concepts and processes of arithmetic. This in turn will enable them to teach arithmetic more effectively and meaningfully.[5]

The last sentence in the quotation above is one which seems to be incorporated as an underlying assumption to teacher preparation in spite of the reservations people may have concerning it. Possibly issue should not be taken with the idea that increase in subject matter competence of teachers is valuable, but attention should be given to investigate the extent to which this competence in subject matter is related to student performance. If a teacher can perform in a given area at a certain level of proficiency, then how does this effect the performance of his students on a similar task?

In 1961 an article appeared in which the author reviewed the literature on mathematical background for elementary teachers. Sparks noted that there was no research available which indicated that a better comprehension of mathematical concepts on the part of the elementary school teacher resulted in better achievement on the part of students. At the conclusion of this review Sparks offered the following as one of two questions for further research:

> What is the relationship between pupil achievement and teacher knowledge? Investigations should include experimental studies which take into consideration the specific mathematical knowledge possessed by teachers.[6]

In 1967 the Research Advisory Committee of the National Council of Teachers of Mathematics sent a questionnaire to a sample of people in the field of mathematics education asking for a response to questions concerning needed research. One of the responses offered by those canvassed concerned the need for research investigation on "the relationship between teachers' knowledge of arithmetic and pupil gains."[7]

Mitzel,[8] in his article entitled, "Teacher Effectiveness," in the _Encyclopedia of Educational Research_, described criteria to measure teacher effectiveness. While defining "Product Criteria" as those which change behavior of students, he expresses concern that so little research has used some measure of students' growth as the operational definition of teacher competence. One aspect of this student growth is achievement in the subject matter areas and this may well be related to teacher achievement.

There has been a considerable number of studies and reports made on measuring teacher effectiveness. However, most of these are concerned with variables such as personality, attitude, age, sex, number of years of education or teaching experience and a few deal specifically with a measure of teacher behavior in subject matter performance. One such set of studies is known as "The Wisconsin Studies" and was carried out under the direction of A. S. Barr.[9] Domas and Teideman[10] have compiled an extensive bibliography of studies concentrating on criteria for teacher effectiveness. Again none of them deals with specific knowledge possessed by teachers.

Perhaps the most conclusive discussion in the literature on teacher effectiveness appears in N. L. Gage's[11] chapter in the _Handbook of Research on Teaching_, entitled "Paradigm for Research on Teaching." He reports that the ultimate criterion of a teacher's effectiveness may be his effect on changing the behavior of the students with which he is associated. Gage reports that in 1952 the Committee on the Criteria of Teacher Effectiveness of the American

Educational Research Association, formulated an "Ultimacy Paradigm" for such criteria. The criteria on the chart below are placed on a continuum descending from the "ultimate" to the "proximate" and each criterion receives its place by its correlation with the criteria which appear above it.

ULTIMATE CRITERION

Teachers' effect on:
- Pupils' achievement and success in life
- Pupils' achievement in subsequent schooling
- Pupils' achievement of current educational objectives
- Pupils' satisfaction with the teacher
- Parents' satisfaction with the teacher
- Superintendents' satisfaction with the teacher

Teachers' "values" or evaluative attitudes
Teachers' knowledge of educational psychology and mental hygiene
Teachers' knowledge of methods of curriculum construction
Teachers' knowledge of the subject matter
Teachers' interest in the subject matter
Teachers' grades in practice teaching courses
Teachers' grades in education courses
Teachers' intelligence

There is a scarcity of literature reporting relationships between "Teachers' knowledge of the subject matter" and "Pupils' achievement of current educational objectives," two criteria which appear on the "Ultimacy Paradigm." Much of the research which has been conducted has investigated possible relationships between general and unrelated teacher and student performance. When commenting on the vast amount of general investigations which have been carried out to examine teacher effectiveness, Gage states:

> One solution within the 'criteria-of-effectiveness' approach may be the development of the notion of 'micro-effectiveness.' Rather than seek criteria for the overall-effectiveness of teachers in many varied facets of their roles, we may have better success with criteria of effectiveness in small, specifically defined aspects of the role.[12]

Statement of the Problem

The present investigation could be classified in Gage's study of "micro-effectiveness." This study has been designed to examine the relationship between teacher performance on a specific topic in mathematics and that of the teacher's students. It has been shown that teachers possess varying degrees of familiarity with many of the topics presented in the elementary school arithmetic programs. By obtaining a measure of teachers' performance on tasks in a particular area it is possible to examine the relationship between this performance and similar student performance. This would involve examining the correlation between two of the criteria listed in the "Ultimate Criterion" described by the Committee on

Criteria of Teacher Effectiveness. However, instead of examining the general effect of teacher knowledge on student performance an attempt has been made to analyze this effect in detail on a minute segment of the mathematics program.

A teacher is given a unit of material based on content which is new both to the teacher and to his students. He is then instructed to present this material to his class over a given period of time. After the instruction a testing instrument is administered to the teacher and to his students. The basic question is: What relationship exists between the performance of the teacher on the testing instrument and that of his students? Is there reason to believe that a teacher's ability to perform in a certain fashion or at a particular level will effect the performance of the students he instructs on a specific topic in mathematics?

The material used for this study contains selected topics from nonmetric geometry. The concepts of point, line, line segment, ray, angle and triangle were included in a unit of material. Topics from nonmetric geometry were chosen because of their "definitional nature"[13] and their relative unfamiliarity to the subjects. It is possible to identify certain common interpretations of concepts such as "triangle" and "angle" and to compare the change in pupil performance from before to after instruction. It is then feasible to relate these changes in interpretation to the performance of the particular teacher on the same item.

Teachers and their students from nineteen fifth-grade classes of three different school districts were used as subjects for this investigation. The experimental environment consisted of the regular classroom situation. The duration of the instructional period was four fifty-minute class periods. The criterion measure used to ascertain the possible relationships between teacher and student knowledge consisted of a post-test administered on the day following the final period of instruction.

Since this study investigated the possible influence of teacher "knowledge," as measured by a test, on student performance, it was felt desirable to include in the design a means of determining student performance when the teacher is not involved in the instruction. Therefore, six of the nineteen classes were exposed to the topics by means of a booklet to be read. These classes were used as a control in the experiment to determine what gain can be expected if the teacher does not have the opportunity to influence the students.

The answers to four specific questions are sought in this investigation:

(1) What correlation exists between teacher and student performance on a test of subject matter which has been presented by the teacher to the student?

(2) Is the student's ability to select the correct response to a particular item on the test independent of the teacher's ability to select the correct response on that item?

(3) If a teacher selects a particular incorrect response to a given item, then is the possible selection of that incorrect response by his students independent of his choice?

(4) Are mean scores of classes presented the selected topics in nonmetric geometry by teachers significantly higher than mean scores of classes which have read the same material written at the fifth-grade reading level?

Any answers to these questions must certainly be qualified with regard to the particular population of teachers and students and the content of the instructional material. The design and procedures employed in this study to analyze relationships between student and teacher performance on specific tasks are new. No reference to use of a similar design has been found in the literature.

The first question is investigated by the use of a correlation between mean scores of classes and the scores of their respective teachers on a post-test. A chi-square statistic will be employed to answer questions two and three. In a further attempt to investigate questions two and three, an analysis of the percent of change in student response from pre- to post-test performance will be conducted. Question four will be answered by using an analysis of covariance on mean scores of classes on the post-test while using the pre-test score means as a covariant.

Summary of the Chapters

In Chapter I the purpose of the study within the proper context is described. Chapter II contains a review of the literature pertaining to the comparison of various aspects of teacher and student performance. The experimental phase of the study is reported in Chapter III which contains a description of the experimental subjects, the materials used, the procedures employed and the statistical design. The results of the analysis are reported in Chapter IV. Chapter V contains an interpretation of the results, conclusions and recommendations.

FOOTNOTES

[1]Mathematical Association of America, _Recommendations for the Training of Teachers of Mathematics: A Summary_ (Berkeley, California: Committee on the Undergraduate Program in Mathematics, 1961), pp. 15.

[2]National Council of Teachers of Mathematics, _Mathematics for Elementary School Teachers_ (Washington, D. C.: National Council of Teachers of Mathematics, 1966), pp. 211.

[3]Wilbur H. Dutton and Augustine Cheney, "Pre-service Education of Elementary School Teachers in Arithmetic," _The Arithmetic Teacher_, XI (March, 1964), 192-198.

[4]Ruth Melson, "How Well are Colleges Preparing Teachers for Modern Mathematics?" _The Arithmetic Teacher_, XII (January, 1965), 51-53.

[5]John L. Creswell, "The Competence in Arithmetic of Prospective Georgia Elementary Teachers," _The Arithmetic Teacher_, XI (April, 1964), 250.

[6]Jack N. Sparks, "Arithmetic Understanding Needed by Elementary School Teachers," _The Arithmetic Teacher_, XIII (December, 1961), 395-403.

[7]Boyd Holtan, "Some Ongoing Research and Suggested Research Problems in Mathematics Education," _Research in Mathematics Education_ (Washington, D. C.: National Council of Teachers of Mathematics, 1967), pp. 108-114.

[8]Harold Mitzel, "Teacher Effectiveness," in _Encyclopedia of Educational Research_, ed. by Chester W. Harris (New York: The Macmillan Co., 1960), pp. 1481-1486.

[9]A. S. Barr and others, _Wisconsin Studies of the Measurement and Prediction of Teacher Effectiveness: A Summary of Investigations_ (Madison, Wisconsin: Dembar Publications, Inc., 1961), p. 156.

[10] S. J. Domas and D. V. Teideman, "Teacher Competence: An Annotated Bibliography," Journal of Experimental Education, XIX (December, 1960), 101-218.

[11] N. L. Gage, "Paradigms for Research on Teaching," Handbook of Research on Teaching (Chicago: Rand McNally Co., 1963), pp. 94-141.

[12] Ibid., p. 120.

[13] Definitions may be identified in all topics in mathematics. The term "definitional nature" is used in reference to nonmetric geometry to imply that there are an abundance of definitions associated with this topic and a person's knowledge of these definitions can easily be recognized through his performance on various geometric tasks.

CHAPTER II

TEACHER AND STUDENT PERFORMANCE

When attempting to review the literature pertaining to the comparison of teacher and student performance on subject matter tasks, the writings of such men as Barr, Ryan and Smith must be considered. These men have led the investigation on teacher competence and effectiveness in the classroom. Most of the studies reported by them have been concerned with such teacher characteristics as personality, attitudes, supervision or peer ratings and educational background and the relationship of these characteristics to teacher effectiveness. A very limited number of studies have been found which were directed at examining the relationship between teacher and student behavior on tasks requiring subject matter knowledge. Ryan points out that: "Any such characteristic is completely acceptable as a criterion of teacher effectiveness only when it is known to be related to criteria that are based on pupil behavior."[1]

B. Othanel Smith[2] classed all the variables involved in and related to teaching into three categories which are recorded in the chart below:

A Pedagogical Model

I	II	III
Independent Variable (teacher)	Intervening Variable (pupils)	Dependent Variable (pupils)
(1) Linguistic behavior (2) Performative behavior (3) Expressive behavior	These variables consist of postulated explanatory entities and processes such as memories, beliefs, needs, references and associative mechanisms.	(1) Linguistic behavior (2) Performative behavior (3) Expressive behavior

Research on "teaching" is concerned with investigating the effects of the independent (I of Smith's Model) on the dependent variable (III of Smith's Model) while controlling certain intervening variables (II of Smith's Model) in the process. The present study is centered on an investigation of a specific example of a "Teacher Performative Behavior" as it may be related to "Student Performative Behavior." As the following reports indicate, little positive support has been established for such a relationship. This may possibly be due to the general nature of the "Performative Behavior" investigated in most instances.

A Review of the Research

Student and teacher performance. Although the research pertaining to the relationship of teacher and student performance on specific academic tasks is extremely limited, there are some research studies which do provide bits of information relative to this question. Moore[3] conducted an investigation to ascertain the effects of the teacher's mathematical "understanding" on pupil achievement in arithmetic.[4]

Two hypotheses were tested in this study. The first stated that there is a positive relationship between the level of teacher understanding of mathematics and pupil gain in achievement. The second hypothesis concerned variability of gain in arithmetic achievement scores among the children. This was hypothesized to be greater when the teacher has a high level of understanding of mathematics. The criteria measures used in this study were scores obtained by testing ten fourth-grade and eleven sixth-grade teachers with the Glennon Test of Mathematical Understanding. Their respective students were administered the SRA Arithmetic Series Test, Grades 4-6, following a semester of instruction in arithmetic.

A test of the hypothesis that there would be a positive relationship between teacher and student scores on their respective tests was made by correlating the scores of the teachers with the mean scores of classes. The fourth-grade correlation coefficients did not reach significance at the 0.05 level and the sixth-grade correlation coefficients were essentially zero. The investigator does report that variability in gain in achievement among fourth-grade pupils was greater in classes taught by teachers who tested highest on the Glennon Test.

Moore attempts to explain the low correlations by referring to the small sample size and some inadequacies in the tests and programs taught by the teachers. However, he states that results of the study indicated a need for further investigation of this type of teacher-student relationship.

A similar study was conducted by Barr[5] as one of the many "Wisconsin Studies." A sample of youngsters from grades one through seven were tested with different forms of the _Stanford Achievement Test_. The mean scores for the test as a whole and for the test in arithmetic were computed for each class on both a pre- and post-test administration. The teachers of these students were subjected to a battery of tests including the _New Stanford Arithmetic Test, Form V_. One of the many criteria employed as a measure of teacher effectiveness was gain in pupil achievement as measured by the _Stanford Achievement Test_. Barr reports that contrary to what might be expected, the teachers' scores on the _New Stanford Arithmetic Test_ did not show any statistically reliable correlation with pupil gains on the Stanford Test. Actually, two of the coefficients were negative.

Barr states: "This fact is difficult to explain inasmuch as the knowledge of the subject taught is ordinarily assumed as a prerequisite for successful instruction."[6]

Bassham[7] found results contrary to those in the last study. He reports that level of teachers' understanding of basic concepts in mathematics was significantly associated with pupils' efficiency in learning and that this efficiency was correlated positively with level of pupil intelligence.

Sixth-grade students and their teachers were used for this study. In early fall of a school year pre-experimental measures of pupil difference in arithmetic achievement, reading achievement, interest in arithmetic and mental ability were obtained by tests. In March of the same year, the tests were readministered and teachers were administered a test of understanding of basic mathematical concepts and an inventory of attitudes towards methods of teaching arithmetic. The teachers' scores on these tests were compared with post-experimental achievement scores of their pupils while controlling statistically for pupil differences found to exist at the onset of the experiment. The test administered to the teachers was the <u>Glennon Test of Mathematical Understanding</u> and a standardized arithmetic test was used with the students. A correlation coefficient significant at the one percent level of significance was found between teacher understanding of concepts and weighted pupil gain in arithmetic for pupils classified as above class mean intelligence. The correlation coefficient was not significant when youngsters classified as below class mean intelligence were considered.

In a study conducted with an available group of instructors at the Air Force Technical School at Sheppard Air Force Base, Morsh, Burgess and Smith[8] investigated possible relationships between teacher characteristics and student achievement. Comparisons were made between student achievement gains with student and fellow instructors' ratings of teachers, general instructor intelligence and instructor subject matter knowledge measured by a proficiency examination. One hundred and twenty-one instructors were used in this study and correlations were made between the instructor characteristics and pupil gain. The instructional material consisted of an eight-day unit which was part of an aircraft mechanics course. Instructors were tested with a proficiency examination developed for airplane hydraulic specialists. Intelligence scores were also obtained for all instructors. The students were pre- and post-tested with a test battery covering the material presented during the experimental period. It was reported that neither instructors' hydraulic subject matter knowledge nor intelligence scores correlated significantly with any of the student gain criteria. The only positive correlation coefficients obtained were those involving ratings by fellow instructors and students.

Student performance and teacher characteristics. There have been some studies which considered relationships between pupil achievement and general assessment of teacher knowledge and background. The conclusions that differences in specialized subject-matter preparation of teachers had no apparent influence on the mean achievement in arithmetic of eighth-grade students were reported by Smith[9] in summarizing research he conducted. Students were given a standardized test at the beginning and conclusion of a school year and gain scores were analyzed for possible relationships with teachers' preparation background in mathematics and professional education courses.

In 1924, Taylor[10] conducted an extensive study to investigate the question of whether changes in the proficiency of elementary school students in the areas of reading and arithmetic correspond with the estimates of the teaching abilities of their teachers. Students from nine different schools were tested with standardized arithmetic and reading achievement tests twice during the school year. The difference between initial and final scores of each child constituted the measure of pupil progress in "arithmetic fundamentals" during a four-month period of teacher instruction. Teachers were ranked by principals and supervisors according to estimates of their general effectiveness. Correlations were made between variables, and extremely low coefficients between teacher ratings and student achievements were obtained.

Shim[11] investigated the cumulative effect of teachers' college grade-point average, type of degree and certification and years of teaching experience on pupil achievement over a period of five years. The study was conducted with 214 white sixth-grade pupils ranging from below to above average I.Q. levels. The criteria of teacher achievements in arithmetic, language and reading according to the California Achievement Tests, Form W, Elementary. He found that students taught by teachers having below 2.50 grade-point average achieved significantly more in arithmetic and language than students taught by teachers who obtained above 2.50 average in college. In general, Shim reported that there was little evidence to support the hypothesis that differences in achievement of students were related to the teacher variables employed in his study.

A report of comparing adults' and children's performance on tasks involving the estimation of quantity was made by Corle.[12] The adults consisted of teachers and college students while the children were intermediate grade school students. It was reported that teachers exhibited a higher level of success in estimating than did the children, but lower level than the college students. The author implies that some of the difficulties exhibited by the children in appraising quantitative values may have been due to the poor showing of the teachers. No attempt was made to relate the performance of the teachers to that of the students in this investigation.

Ediger[13] conducted a study which, although it did not relate teacher and student performance, measured a teaching effect by using student achievement. The goal was to measure the effect on students' achievement, as measured by The Iowa Test of Basic Skills, of having student-teachers in selected fifth- and sixth-grade arithmetic classes. No significant differences were reported between mean gain of classes with or without student-teacher instruction.

Effect of inservice programs. In recent years there have been numerous inservice programs in mathematics and science designed to increase the competence of elementary school teachers. The assumption has been that this increase in knowledge, if it in fact occurs, will assist the teacher in changing behavior of students exposed to mathematical and scientific materials. There have been a few studies designed to examine this assumption and three of these are reported in the following paragraphs.

The premise that inservice education in arithmetic for teachers will bring about increase in achievement of learning in their pupils was the center of concern for a study conducted by Houston and DeVault.[14] A thirteen-hour inservice program on mathematical topics was presented to 102 intermediate grade school teachers. The teachers were tested for achievement in the selected aspects of mathematics content presented in the program by an instrument entitled <u>Understanding Mathematics Test, Form A and B</u>, which was developed by the investigators. The teachers' general mathematics achievement was measured by the <u>Sequential Test of Educational Progress - Mathematics, Level 2</u>. The investigators constructed a test of "mathematical understandings" to be administered to the pupils which was similar in content to that presented to the teachers and they also administered the <u>Sequential Test of Educational Progress - Mathematics, Level 4</u> to the children. These tests were administered to the students as a pre-test before the first inservice meeting and as a post-test after the termination of the program.

Teachers and students were reported to have made statistically significant gains on all tests at the one percent level. Correlation coefficients were computed to relate teachers' achievement before and after the inservice activity with pupils' mathematics achievement during the program. Also, "changes" in the teachers' and students' achievement during the program were computed and correlated. The correlation coefficients of teacher post-test and teacher change scores with student change scores were 0.31 and 0.21, respectively. It was also reported that teachers' pre-test and student-change scores correlated with a coefficient of 0.05.

The authors note that, since the first two coefficients were significant at the one percent level, these findings indicate change in mathematics understanding among teachers was related to change in understanding among pupils. They go on to conclude that the inservice program was worthwhile when evaluated in light of student gain in subject matter. However, they qualify this statement by raising the question of whether this change was actually a product of an exchange of content between teacher and pupil or the result of the pupils' taking of the test and method of the teachers.

A second study involving teacher-student performance in evaluating the effect of an inservice program in arithmetic was conducted by Hammond.[15] An experimental and control group of intermediate grade school teachers and their students were established. The experimental group of teachers was exposed to an inservice program on mathematical concepts felt to be necessary for teaching elementary school arithmetic. The control group of teachers had no such program. Dutton's Test of Elementary Arithmetic Concepts was administered to all teachers on a date before and following the inservice program. The proper form of Dutton's Arithmetic Comprehension Test was administered to students in all classes at two different times corresponding to the times the classroom teachers were tested. Results from t-tests computed for teachers and students for the pre- and post-tests showed that significant differences existed. The results showed that there was no significant difference between scores made by the control and experimental students and only a slight difference in favor of the experimental teachers when comparing teachers. The author concludes that more research is needed to determine the value of such inservice programs.

A third attempt to evaluate an inservice activity was conducted by Morh[16] in connection with a science program. This particular study lasted two years and involved the testing of pupils during a year when the teachers did not participate in an inservice program and again the following year when some of the teachers did participate in a science program for teachers. The experimental classes consisted of those whose teachers did attend the inservice classes and a control group consisted of classes whose teachers did not attend.

Analysis of results was made by testing the significance of within-year changes in means and variances by the use of appropriate t-tests allowing for correlation between measures. A comparison between years was made by analysis of variance. The null hypothesis that there was no difference in science achievement among fifth- and sixth-grade pupils between control and experimental groups was rejected. The author concludes that this seems to indicate that teachers may, through the given test results of their pupils, show an increased effectiveness in instruction as an outcome of such an inservice program.

Teacher performance on standardized tests. On two occasions teachers have been tested with standardized tests of arithmetic for which norms have been established for elementary school students. On the basis of the teachers' performances on these tests, statements have been made concerning their possible effectiveness in teaching arithmetic. Although no direct comparison is made between teachers and students, the following two studies report such endeavors.

Carroll[17] administered the Sequential Test of Educational Progress in Mathematics to 358 prospective elementary school teachers from twenty-four colleges. Of the twenty-four college groups, only one obtained a median score as high as the ninth-grade norm and some of the groups had median scores which were below the seventh-grade norm.

Creswell[18] conducted a similar study by testing 313 prospective elementary teachers with the same _Sequential Test of Educational Progress in Mathematics_ and, in addition, the _Metropolitan Achievement Test, Advanced Arithmetic_. He reported that 9.6 percent of these subjects failed to achieve at the ninth-grade level on both computation and concepts. The hypothesis investigated was that competency in arithmetic was not inadequate for any substantial portion of the prospective elementary teachers involved in the study. Creswell felt that the findings left reason to reject this hypothesis. Of course, issue can be taken with the use of the term "inadequate." Creswell implies that "inadequate" is related to effectiveness in working with students, but little evidence has shown any relationship between teacher and student knowledge as measured by a testing situation.

Readability of mathematics materials. There have been reports of attempts to examine the readability of selected mathematics materials for elementary school students. Since one aspect of the design of this study involves having fifth-grade students read instructional materials it was decided to review some of these reports.

As part of an examination of the reading levels of some of the recent experimental mathematics materials for the elementary grades Smith and Heddens[19] applied the Dale-Chall reading formula to intermediate grade materials. The authors report that most of these instructional materials were written at a grade level above that for which the materials were recommended. They conclude that there is a need for revision of the materials in an attempt to place them at a more appropriate level. In a subsequent report Heddens and Smith[20] present similar findings after examining the reading levels of some selected commercial arithmetic textbooks. They recommend that mathematics materials should be written at a reading level below the grade level for which they are intended. It would seem advisable to accompany a recommendation of this nature with the results of an investigation relating student performance and reading levels of the materials concerned.

Fay[21] administered the _Gates Basic Reading Test_, _Stanford Achievement Test: Reading Section_, and the _Iowa Every-Pupil Test of Basic Skills, Test B_ to obtain measures of reading ability for 384 sixth-grade pupils. The _Stanford Achievement Test_ for arithmetic, social studies and science was also administered to the same sample of students. Fay classified the top one-third of the students as superior readers and the bottom one-third as inferior readers. The two groups were compared on arithmetic, social studies and science achievement by means of the Johnson-Neyman technique with mental and chronological ages statistically controlled. The null hypothesis tested was that the difference in arithmetic achievement, as measured by the *Stanford Achievement Test*, between superior and inferior readers would be zero. Fay reports that superior readers were found to achieve no better in arithmetic than inferior readers.

Reports were found which related students' reading levels with their performance on computational tasks and the findings indicate a low correlation between reading ability and computational skills in arithmetic. No research was found where the design involved comparing students possessing different reading abilities with respect to their comprehension of specific instructional materials in mathematics.

Summary and Generalizations

After examining the literature reported in this chapter it becomes apparent that there is little support for the hypothesis that student achievement on various testing instruments is dependent upon teachers' knowledge of the content. In fact, although Shim's study considered a general measure of teacher knowledge, namely grade-point average, he reported that students of teachers with the lower averages seem to achieve higher on the criteria measures than those taught by teachers who obtained higher averages. Of the research reported, only that conducted by Bassham reveals any positive relationship between teacher and student performance. Corle and Ediger both express concern over the low level of arithmetic performance exhibited by teachers when compared to student norms. As Barr pointed out, this might be a major concern in light of the assumption that knowledge of the subject taught is a prerequisite for successful instruction.

In light of the evidence reported one might wonder if a teacher needs more than a superficial knowledge of a particular subject to be in a position to assist students in increasing their achievement level. However, a more detailed investigation of the teacher-student relationship in achievement in specific subject matter topics is needed. This might be done by selecting certain topics in the various subject matter areas as a center of concentration for the type of research being conducted.

All of the research reviewed has been concerned with comparing the degree of success at a task between teachers and students. There seems to be a notable lack of investigation concerned with the influence of teacher knowledge on children's performance which results in success or failure.

Suppose a teacher performs a given task incorrectly. What influence, then, does this have on the manner in which the student responds? Does he tend to just fail at the task or does he fail in the same manner as his teacher? It would seem that questions of this type would be as enlightening to investigate as those merely comparing levels of success. If a teacher possesses certain misconceptions concerning a specific topic, does this increase the probability of his students forming the same misconceptions? The author was unable to locate any literature concerned with questions such as these, which represent a major goal of the present study.

Special note should be made of the three studies employed as a means to evaluate inservice programs for teachers. Two of the three studies show increase in achievement for students of teachers who had participated in the inservice programs. These endeavors indicate an effort towards evaluating the effect of inservice programs by investigating changes in behavior of teachers, indirectly through student performance. It is encouraging to witness this effort in light of the common practice of assuming that inservice activities automatically have some effect on teachers' behavior in the classroom.

FOOTNOTES

[1]David G. Ryans, "The Investigation of Teacher Characteristics," The Educational Record, XXXIV (October, 1953), 376.

[2]B. Othanel Smith, "A Concept of Teaching," Teachers College Record, LXI (1960), 229-241.

[3]Robert E. Moore, "The Mathematical Understanding of the Elementary School Teacher as Related to Pupil Achievement in Intermediate-Grade Arithmetic" (unpublished Ph.D. dissertation, Stanford University, 1965).

[4]It should be noted that the term "understanding" appears in this chapter only when it has appeared in the literature reviewed. It is apparent that the phrase "understanding of arithmetic" seems to be a popular label to be placed on an otherwise undefined state of possessing some type of enlightenment in arithmetic.

[5]A. S. Barr and others, Wisconsin Studies of the Measurement and Prediction of Teacher Effectiveness: A Summary of Investigation (Madison, Wisconsin: Dembar Publications, Inc., 1961), pp. 1-156.

[6]Ibid., p. 125.

[7]Harrell C. Bassham, "Relationship of Pupil Gain in Arithmetic Achievement to Certain Teacher Characteristics" (unpublished Ph.D. dissertation, University of Nebraska, 1961).

[8]Joseph Morsh, George Burgess and Paul Smith, "Student Achievement as a Measure of Instructor Effectiveness," The Journal of Educational Psychology, XLVII (January, 1956), 79-88.

[9]Wallace R. Smith, "The Achievement of Eighth-Grade Students in Arithmetic with Respect to Selected Patterns of Teacher Preparation," Dissertation Abstracts, XXV (January, 1965), 3947.

[10]Howard R. Taylor, "Teacher Influence on Class Achievement: A Study of the Relationship of Estimated Teaching Ability to Pupil Achievement in Reading and Arithmetic," Genetic Psychology Monographs, VII (February, 1930), 81-175.

[11]Chung-Phing Shim, "A Study of the Cumulative Effect of Four Teacher Characteristics on the Achievement of Elementary School Pupils," The Journal of Educational Research, LIX (September, 1965), 83-90.

[12]Clyde Corle, "Estimates of Quantity by Elementary Teachers and College Juniors," The Arithmetic Teacher, X (October, 1963), 347-353.

[13]Marlow Ediger, "The Student Teacher and Pupil Achievement in Elementary School Arithmetic," School Science and Mathematics, LXV (November, 1965), 697-700.

[14]Robert W. Houston and M. Vere DeVault, "Mathematics In-service Education: Teacher Growth Increases Pupil Growth," The Arithmetic Teacher, X (May, 1963), 243-247.

[15]Harry R. Hammond, "Developing Teacher Understanding of Arithmetic Concepts Through In-service Education" (unpublished Ph.D. dissertation, University of California, 1965).

[16]Gordon M. Morh, "Effects of an Inservice Teaching Training Program on Pupil Outcomes in Fifth and Sixth Grade Science" (unpublished Ph.D. dissertation, University of Minnesota, 1953).

[17]Edward M. Carroll, "Competencies in Mathematics of Certain Prospective Elementary School Teachers," Dissertation Abstracts, XXV (March, 1965), 5134.

[18]John L. Creswell, "An Analysis of the Relationship of Selected Factors to Mathematics and Arithmetic Competency of Prospective Elementary Teachers in Georgia" (unpublished Ph.D. dissertation, University of Georgia, 1964).

[19]Kenneth J. Smith and James W. Heddens, "The Readability of Experimental Mathematics Materials," The Arithmetic Teacher, XI (October, 1964), 391-394.

[20]James W. Heddens and Kenneth J. Smith, "The Readability of Elementary Mathematics Books," The Arithmetic Teacher, XI (November, 1964), 466-468.

[21] Leo C. Fay, "The Relationship Between Specific Reading Skills and Selected Areas of Sixth Grade Achievement," The Journal of Educational Research, XLIII (March, 1950), 541-547.

CHAPTER III

THE DESIGN

The Nature of the Content

The specific content chosen for this study centers on selected topics from nonmetric geometry. Nonmetric geometry is the study of sets of points in space and does not involve the use of measurement. From the undefined terms, point, line, plane and space and some selected definitions of additional sets of points, a detailed description can be made of various geometric figures and their relationships in space. The specific geometric concepts utilized in the instructional materials employed in this study were point, line, line segment, ray, angle, and triangle.

Nonmetric geometry was selected for this study for three reasons. First, it is a subject area which is new to most teachers and students in the elementary grades. Secondly, it is a topic which has become popular in many of the more recent arithmetic curriculum materials for elementary schools. Of sixteen elementary arithmetic textbook series examined by the author, nine contained materials devoted to topics in nonmetric geometry in grades one through six. The third reason is the definitional nature of the materials and the degree to which many of the definitions are different from the everyday interpretation people give to the terms.

In the fall of 1963 at an inservice workshop on mathematics for elementary school teachers, a group of approximately sixty teachers were asked to describe various geometric figures in their own words.[1] This was done prior to any presentation of material on nonmetric geometry. One of the questions was, "What do you think an angle is?" Approximately thirty different descriptions were given, but most of these can be classified into one of five different types.

Approximately one-third of the teachers described an angle as the space between two lines. A good portion of the teachers felt that the angle was a meeting point of two lines while others looked at an angle as being the lines themselves. Another general category was that an angle was the rotation of a line about a point or a portion of a circle. All of the responses made by the teachers before a formal presentation of the material were different than the definition of an angle found in most elementary textbooks. This definition is expressed something like: "An angle is made up of a set of points which contains the points on two distinct rays which have a common endpoint."

Similar questions were asked concerning additional geometric figures and similar results were obtained. After a few days of instruction the teachers were asked to state the various definitions presented during the instructional period. A large percent of the teachers recited the definitions verbatim, but on problems where they were asked to apply the definition by identifying a particular figure in a given situation it was apparent that many of them reverted back for their response to their previously stated description of the figures.

Weaver found similar results while developing an inventory to assess levels or degrees of geometric understanding among elementary school children in grades four to six. In an attempt to refine a series of geometric tasks involving the identification of selected geometric figures, Weaver administered the inventory to a group of forty-five elementary school teachers. On the basis of Weaver's findings he states that:

> Some teachers do have a very low level of understanding of rather simple aspects of non-metric geometry . . . These teachers also have 'misunderstandings' pertaining to some crucial facets of geometric content.[2]

In an article by Rutland and Hosier[3] there appears an attempt to familiarize elementary school teachers with some basic geometric ideas in terms of sets of points. The authors state one definition after another in a fashion which implies that now the teacher is "getting the scoop" and if he learns these, then he will be up to date. The implication of material such as this is that the mere presentation of these topics to teachers will erase previous misconceptions and enable them to change behavior in the students they instruct.

It was felt that nonmetric geometry, because of its definitional nature, would be excellent content for examining the possible effect of teacher "knowledge" on change of behavior in his students. Also, its popularity in the various curriculum programs makes it worthy of careful consideration for future presentation.

Experimental Materials

Four instructional booklets (Appendix A) were written for fifth-grade students on the geometric topics listed in the previous section. Booklet one contains material presenting activities involving points, lines and line segments. Booklet two describes the many attributes of a triangle. The third booklet develops the concept of a ray and the last booklet is devoted to the presentation of an angle. Each set of materials is designed to be read by a student during one instructional period of approximately fifty minutes. The materials attempt to review items presented on previous days and to tie together the various geometric topics. Although the material is not programmed in the usual sense, it is made self-instructional by actively involving the reader by asking him to answer questions and draw pictures as he proceeds. Each child was supplied with a red and blue pencil to be used for his responses. All of the questions in the written materials are discussed in the reading which follows the particular question. This offers the reader an opportunity to go back to his response and change it in light of the given discussion.

A further attempt to keep the reader active in the instructional process was made by presenting a set of four or five exercises to follow each instructional booklet. The exercises, which appear in Appendix A following each individual booklet, consist of applications of the topics appearing in the written material. For example, following the presentation of the material on a triangle there is an exercise which asks the reader to identify triangles in a series of diagrams. Another exercise requests that the student draw a picture of a triangle and color it red.

The correct response to each exercise and a brief discussion concerning that response is written on a five by eight card. This facilitates the immediate knowledge of correctness of response for the reader. Each student who reads the booklets and responds to a given question is instructed to look at the "answer card" immediately after recording his response to that particular item. Hence, a set of "answer cards" is supplied with each day's booklet.

The instructional materials are patterned in content after the fourth- and fifth-grade geometric materials appearing in textbook series such as those of the School Mathematics Study Group and the American Book Company Publishers. After the initial writing, the materials were read by various people on the mathematics-education staffs at the Universities of Maryland and Delaware. Following revisions based on the suggestions made by those reviewing the materials, a class of fifth-grade students was administered the complete set of booklets and exercises. A group of elementary school teachers and undergraduate education majors were also allowed to read the materials and record comments. On the basis of these trials the materials were revised to their final form.

Since the materials were to be read by fifth-grade youngsters, it was felt desirable to test the reading level of the booklets. This was done by applying the Dale-Chall[4] "Formula for Predicting Readability" to five random selections of approximately 150 words each of the written materials. The corrected grade level for the instructional materials was fifth-sixth grade on the Dale-Chall chart.

In summary, the instructional materials consist of four booklets with accompanying sets of exercises and "answer cards," to be presented to fifth-grade students over four class periods of approximately fifty minutes in duration.

The Population

The population for this study consisted of fifth-grade teachers and their students from three different school districts. The Oxford School District, Oxford, Pennsylvania, the Smyrna Special School District, Smyrna, Delaware, and the Elkton School District, Elkton, Maryland were selected to supply the nineteen fifth-grade teachers and classes used in this study. All seven fifth-grade classes from Oxford and Smyrna and five classes from two elementary schools in Elkton were used. These subjects were involved in the study during various two-week periods in April and May of the 1966-67 school year.

The three school districts used in this study are located in similar types of communities. The communities are rural and agricultural in nature and the majority of the students come to school in buses from outlying developments and farms. All of the school districts employ a homogeneous grouping procedure for the elementary grades; however, for the duration of this study the students remained in heterogeneously organized homerooms.

Four teachers and their respective classes from each of the Oxford and Smyrna School Districts were randomly assigned the "Experimental Treatment Group" of the study. The remaining three teachers and classes from each of these districts were designated the "Control Treatment Group." All of the teachers and classes from the Elkton School District were assigned to the "Experimental Treatment Group" in an attempt to increase the size of this group. This procedure secured a total of thirteen teachers and classes for the "Experimental Treatment Group" and six teachers and classes for the "Control Treatment Group."

With the exception of two of the teachers involved in this study, all had at least four years of teaching experience. Only one of the teachers had been exposed to the topic of nonmetric geometry previous to the experiment.

All of the classes involved in this study were heterogeneously grouped and the instruction was conducted in their regular classroom. Reading levels, intelligence quotients and scores on standardized arithmetic tests were obtained for all students. Those students with intelligence quotients below eighty-five or reading levels measured at below the second half of grade four were eliminated from the statistical analysis of the study. Also, any student who was absent from any of the instructional periods was not considered in the experiment.

The Experimental Phase

The experimental treatment. Thirteen teachers and their respective classes were designated the "Experimental Treatment Group" for this study. Each teacher was asked to present the content contained in the instructional booklets to their students on four consecutive arithmetic classes from Monday through Thursday of a school week. One week prior to the instructional period each teacher was given a complete set of the instructional materials and told that it was his responsibility to cover all of the topics in the four booklets. The experimenter met with all of the teachers before the instruction began and answered questions which the teachers had after reading the materials. During these sessions with the teachers there was a discussion of the reasons for teaching these selected topics in nonmetric geometry to the students.

The teachers were allowed to present the material to their classes in whatever manner they wished. All of them employed a technique which could be classified as a "lecture procedure." Although these students were not given any written materials on the subject, each teacher was supplied with the sets of exercises for each day's work. The teacher was instructed to use these exercise sheets during the individual class periods with the students, and the experimenter collected them at the end of the week. Each student answered the exercises on a particular day's presentation and then the teacher discussed the responses with them. However, it should be noted that the teachers were not given the correct responses to the exercises in preparation for these presentations, but were asked to choose the correct responses. This was done in an effort to keep the teachers' presentation of the materials flavored with their interpretation of the content.

Four of the treatment classes were obtained from Oxford, four from Smyrna, and the remaining five from Elkton. The procedure employed with these thirteen classes consisted of situations where the teacher studied and interpreted content which was new to him and prepared and presented it to his individual class over a four-day instructional period.

The experimenter considers this to be a "typical" situation faced by many teachers in the field when confronted with new curriculum materials. This is, of course, disregarding the "Hawthorne Effect" which may have entered into the situation.

The control treatment. The "Control Treatment Group" consisted of a total of six classes of fifth graders, three from Oxford and three from the Smyrna School District. The teachers of these classes were not involved in the study although the regular classroom environment was used for the presentation.

The instructional materials were presented to the control treatment students in the booklet form to be read individually. For the four consecutive arithmetic class periods, from Monday through Thursday, these classes were allowed to read the booklets and respond to the same exercise sheets administered to the experimental students. However, these students were supplied the correct responses to the exercises by means of the "answer card" decks.

The students were asked to read a booklet carefully, answering questions and drawing pictures with a red and blue pencil supplied, and then to answer the items on the exercise sheets. A deck of "answer cards" was placed face down on each desk and immediately after responding to a given item the students were instructed to turn the card over for that item and compare their response with the correct one given. If their response was incorrect they were instructed to reread the question and change the incorrect response. They were allowed to go back and reread any of the instructional material while they were working on an exercise if they so desired.

This treatment was administered by the experimenter and an assistant and no discussion was allowed during the class periods. No time limit was placed on the reading period, and all youngsters completed each program within fifty minutes. The materials were passed out and collected as a group to help eliminate the possibility of developing a "racing atmosphere" among the students.

Essentially, the presentation of the content differed with this group in that the students interpreted the materials from their own reading without assistance from a teacher and, although they responded to the same exercise sheets, they were supplied the correct responses from cards rather than from the teacher's choice.

Pre- and post-tests. A twenty-five item testing instrument was constructed on the selected geometric topics appearing in the instructional materials (Appendix B). The instrument requires the identification of definitions, construction of selected geometric sets of points and the application of definitions in various situations. Sixteen of the items on the test are of a multiple-choice variety while the remaining nine require short answers or the drawing of pictures.

One recall item is included in the test for each definition which is in the instructional materials and there are also two additional types of items pertaining to most of the definitions. One of these types of items requires an identification of some attribute of the particular concept in a geometric situation, while the other type of item relates the concept to a physical situation not usually included in mathematics. The following three items pertaining to the definition of a triangle appear on the test.

Item 6. A triangle is made up of
a) three angles and three line segments
b) three points
c) three points not on the same line and the line segments between the points
d) three line segments and all of the points "inside" the three segments

Item 21. Draw a picture of a triangle with your regular pencil. Now color the triangle you have drawn red.

Item 1. A man places 3 fence posts in the ground and builds a fence between each pair of posts. He then digs up all the ground inside the fence.

Which of the following would be the best example of what we mean by a triangle?

a) the three fence posts
b) the fence and posts
c) the ground dug up
d) the fence, posts and ground dug up

The correct response for item 6 is selection c. This item requires a straight recall of the definition as it is described in the material. From the subject's response to item 21 it is possible to determine whether he has applied the definition correctly and did not color the interior of the triangle red, but just colored the three segments. Item 1 will offer an opportunity to investigate whether the subject will use the definition correctly in a situation where he must transfer the application of the definition to a non-mathematical situation. Items such as these appear for most of the concepts presented to the subjects during the instructional period.

In an attempt to establish content validity, the test was examined by the same people in mathematics education who had read the instructional materials and revised in light of the many suggestions received. Also, the questions were answered by the sample class of fifth-grade students and college students after they had read the instructional materials. After two trial runs and subsequent revisions the test was constructed in its final form.

An estimate of the test's reliability was obtained by using the Kuder-Richardson formula number 20. The reliability coefficient obtained in this procedure was 0.72. The experimental design of this study requires an examination of the results for each of the items on the criterion test. To obtain a measure of item reliability, the test was administered to a sample of fifth graders and then readministered to the same students one week later. The percent of students who responded correctly to each item on both administrations of the test was found and used as a measure of item reliability.

When the scores on particular items are correlated positively with the total scores on the test, the item is said to be discriminating. Estimates of discriminating coefficients were calculated for the twenty-five items on the test by using Flanagan's table of normalized biserial coefficients of correlation. All of the coefficients obtained were positive and ranged from 0.11 to 0.79 with a mean coefficient of 0.46 for the twenty-five items.

This test was administered as a pre-test a week before the instructional period began to all of the fifth-grade students used in the study. The same form of the test was readministered as a post-test to the subjects on the day following the last period of instruction. Subjects were given as much time as they desired to respond to the items. The administration of the test took approximately thirty minutes and was administered by the experimenter.

The teachers were not permitted to look at the test when it was administered to their students as a pre-test. With no prior warning the teachers of the experimental groups were asked to respond to the test at the time when it was administered as a post-test to the students. Each teacher answered all of the items on the test, thus making it possible to compare teacher and student responses on the same tasks.

When asked to respond to the items on the criterion test, three of the teachers originally assigned to the "Experimental Treatment Group" refused to cooperate. They expressed the opinion that the experimenter and their administers were conspiring to evaluate them by making a comparison between their performance and that of their students and other teachers. This situation was remedied by replacing these teachers and classes in the study. These teachers were in the same building and exhibited the only instance of unfavorable cooperation during the study.

The Experimental Design

The purpose of this experimental study was to select specific subject matter and investigate the possible effect of teacher knowledge on student performance on tasks related to the particular content. Specifically, degree of teacher knowledge, as measured in terms of performance on the criterion test, was compared to performance of their students on the same test. There are several aspects of this possible relationship which have been investigated in this study by employing various statistical techniques.

To gain a measure of the effect of teacher presentation of the content to the students, a comparison was made between test performance of teacher-taught classes and the classes which were administered the self-study reading instruction. The null hypothesis to be investigated is that students who read instructional materials in mathematics on their own will perform as well on selected tasks as those students who have teachers explain and interpret the content for them.

The thirteen teacher-taught classes were designated the "Experimental Treatment Group" while the six self-study classes were assigned to the "Control Treatment Group." The statistical hypothesis being tested for this aspect of the study is stated as follows:

(1) The adjusted mean scores for the Experimental classes and the Control classes are equal; or $H_o: \mu_{EC} = \mu_{CC}$.

This aspect of the study fits "The Nonequivalent Control Group Design" described by Campbell and Stanley[5] in the chapter entitled, "Experimental and Quasi-Experimental Design for Research on Teaching," of the Handbook of Research on Teaching. As suggested by Campbell and Stanley, the hypothesis for this design is tested by employing an analysis of covariance procedure where the mean scores of classes on the post-test will be adjusted for the effect of the mean scores of the classes on the pre-test. It will be necessary to test the assumption of homogeneity of regression.

Since the teachers and the students in the Experimental classes of this study took the same criterion test, it is possible to investigate the predictability of student performance from teacher performance. Is there evidence to support the hypothesis that if a teacher performs at a certain level on selected tasks, then his students, following instruction, will perform at the same level on these tasks? The statistical hypothesis to be tested will be stated as follows:

(2) There is no significant positive correlation between teacher and student performance on the criterion test on nonmetric geometry.

This hypothesis will be examined by applying the formula for the Pearson Product-Moment Coefficient of

Correlation, commonly designated r_{XY}, where the X's are the teacher scores and the Y's are the class mean scores on the test for the students.

Due to the limitation of implications which can be made from a correlation coefficient it was decided to examine more carefully the possible relationships between teacher and student performance on the test. If it is assumed that the teacher's response to a particular item on the test is an indication of his interpretation of the content, then it would seem worthwhile to investigate the possible relationship between the teacher's response to a particular item and that of the student. Two questions are directed at investigating this relationship on individual items.

First, if a teacher answers a particular item incorrectly, then is there a tendency for his students to respond incorrectly to that item? By comparing the proportion of student-incorrect responses for classes whose teachers missed an item with the proportion of student-incorrect responses for classes whose teachers correctly responded to a particular item, it was possible to investigate the following hypothesis.

(3) The selection of a correct or incorrect response to a particular item on the criterion test was independent of the selection of the correct response by the teacher on that item.

This hypothesis will be tested by applying the chi square statistic to the frequencies of correct and incorrect responses of students to particular items on the criterion test. Employing this statistic involves determining whether a set of observed frequencies is consistent with a set of expected frequencies.

The second question is concerned with the particular incorrect response selected by those students who miss an item. Suppose it is assumed that a teacher's selection of a particular incorrect response is a reflection of his interpretation of the content involved. There may be reason to believe that the teacher's misconception of the content involved in that item, as indicated by his incorrect response, was present in his classroom instruction. Its possible effect on the performance of his students would be interesting to examine.

Consider the following example as an illustration of the discussion in the preceding paragraph.

Item 6. A triangle is made up of
- a) three angles and three line segments
- b) three points
- c) three points not on the same line and the line segments between the points
- d) three line segments and all of the points "inside" the three segments

Suppose a teacher selects the incorrect response _d_ when answering this item. Then there is reason to hypothesize that in his classroom presentation he might have described the triangle as being all of the points inside the segments. It is possible that a person might verbally state the definition of a triangle as it appears in the instructional material, but when it came to applying the definition to a situation they would interpret the triangle in a different manner. If this in fact does occur, then what effect does it have on the students' performance?

The author fully realizes the limitations of the present study in investigating this aspect of the effect of the teacher on student performance. However, it is feasible to examine whether students select particular incorrect responses to items independent of their teachers' selection of that response. If a direct relationship exists between teacher and student selection, then further research should be designed to investigate its cause.

The hypothesis being examined for this aspect of the present study is stated as follows:

> (4) The student selection of a particular incorrect response to an item on the criterion test was independent of the same incorrect selection by the teacher on that item.

Changes in student selection of responses on test items from the pre-test to the post-test will be the criterion measure for examining this hypothesis. If a teacher answers a certain item incorrectly by selecting a particular response, then what proportion of his students selects that same incorrect response on the post-test when they had originally selected a different response to that item on the pre-test? This proportion will be compared with a similar one computed for students of teachers who answered the item correctly. The chi square statistic will be used to make this comparison.

Item 6, which was previously referred to in this chapter, will be used to illustrate this process. Suppose a teacher selects the incorrect response d, then how many of his students, who selected responses a, b or c on the pre-test, have changed to d on the post-test? The same process will be carried out on the classes of teachers who correctly answered item 6 and a comparison will be made of the proportions. This will be conducted for any of the sixteen multiple-choice items appearing on the testing instrument which are answered incorrectly by a teacher.

Summary

This chapter has dealt with the design of the study. Specific topics considered are the instructional materials which included the presentation of selected topics in non-metric geometry and accompanying exercise sheets with "answer cards"; the experimental and control treatments; the population which was defined to be nineteen fifth-grade classes and their teachers; and the experimental phase of the study was described.

In the experimental phase, a twenty-five item testing instrument, covering the content of the instructional material, was administered as a pre-test and following four class periods of instruction as a post-test. The same instrument was administered once to the teachers at the end of the instructional sequence. A detailed analysis of possible relationships between teacher and student performance on this instrument has been outlined in this chapter. The statistical hypotheses to be investigated in this study were then stated and statistical methods used to test these hypotheses were discussed.

FOOTNOTES

[1]This was part of an informal questionnaire administered by the author during an inservice workshop on elementary school arithmetic held for the Clayton School District, Clayton, Delaware.

[2]J. F. Weaver, "Nonmetric Geometry and the Mathematical Preparation of Elementary School Teachers," The American Mathematical Monthly, LXXIII (December, 1966), 1115-1121.

[3]Leon Rutland and May Hosier, "Some Basic Geometric Ideas for the Elementary Teacher," The Arithmetic Teacher, VIII (November, 1961), 357-362.

[4]Edgar Dale and Jeanne S. Chall, "A Formula for Predicting Readability," Educational Research Bulletin, XXVII, (January, 1948), 11-20.

[5]Donald T. Campbell and Julian C. Stanley, "Experimental and Quasi-Experimental Design for Research on Teaching," Handbook of Research on Teaching (Chicago: Rand McNally and Co., 1963), pp. 171-246.

CHAPTER IV

THE DATA

Comparison of Class Means

The twenty-five item test, which appears in Appendix B, was administered as a pre-test and post-test to all students involved in this study. Each item on the test was assigned a scoring weight of one point. A point was given only for items in which the entire response was acceptable.

The mean scores for the experimental and control classes on both pre-test and post-test administrations are given in Table 1. The number of students in each class and the standard deviations of post-test scores also appear in this table. The notation used to denote the classes consists of two symbols. The capital letters O, S and E denote that the class was from the Oxford, Pennsylvania, Smyrna, Delaware, or Elkton, Maryland school districts. The numerals are used to identify particular classes and their teachers within each district. For example, O-4 denotes a particular fifth-grade class and teacher from the Oxford School District, Oxford, Pennsylvania.

Table 1. Pre-Test and Post-Test Means, Post-Test Standard Deviations and the Number of Subjects for Each Class.

Class	Number	Pre-Test Mean	Post-Test Mean	Post-Test Standard Deviation
		Experimental Treatment Classes		
0-1	25	6.04	11.28	4.21
0-2	28	5.00	10.39	3.36
0-3	24	6.35	9.96	4.42
0-4	29	6.04	7.90	4.11
S-5	30	5.40	12.17	4.55
S-6	24	5.21	10.97	3.14
S-7	26	5.65	9.62	3.49
S-8	23	5.30	7.57	2.46
E-9	23	5.32	11.70	4.52
E-10	14	4.92	10.71	2.53
E-11	28	5.32	10.61	3.63
E-12	32	5.06	10.31	4.13
E-13	31	5.55	8.01	2.69
		Control Treatment Classes		
0-14	27	5.96	8.64	4.14
0-15	26	4.77	7.65	4.22
0-16	25	5.56	7.16	2.92
S-17	22	5.41	7.32	2.96
S-18	27	4.96	7.26	2.69
S-19	24	5.33	6.79	2.41

The district post-test mean scores for the experimental treatment group from each of the three school districts--Oxford, Smyrna and Elkton--are 9.83, 10.21 and 10.13 respectively. The district post-test mean scores for the control treatment group from Oxford and Smyrna are 7.82 and 7.12 respectively. There is no significant difference between the district means within each group.

The pre-test mean scores for classes appearing in Table 1 were used to investigate the first hypothesis presented in Chapter III. This hypothesis is stated as follows:

> (1) The mean scores for the experimental classes and the control classes are equal; or $H_o: \mu_E = \mu_C$.

In Chapter III it is stated that hypothesis 1 would be tested by conducting an analysis of covariance where the mean scores of classes on the post-test were adjusted for the effect of the mean scores of the classes on the pre-test. However, the correlation coefficient obtained between pre-test and post-test mean scores was approximately zero. Therefore, it would be more appropriate to test this hypothesis by employing an analysis of variance on the post-test mean scores. Table 2 presents the mean and standard deviation of the class post-test scores for each treatment.

The variances for the means of the two treatment groups were tested for homogeneity using Bartlett's Test.

Table 2. Mean and Standard Deviation of the Class Post-Test Scores and the Number of Classes for each Treatment.

Treatment	Post-test Mean	Standard Deviation	Number of Classes
Experimental	10.09	1.40	13
Control	7.47	0.58	6

The observed F for this test was 3.64, which is below $F_{(1,\ 545.6)} > 3.84$ at the 0.05 level of significance. This result gives support to the assumption that the variances are homogeneous.

The appropriate adjustments for unequal cells were made to the analysis of variance computational formulas before applying them to the data. A summary of this analysis of variance is reported in Table 3.

Table 3. Analysis of Variance of Post-Test Mean Scores for Experimental and Control Treatment Classes.

Source of Variation	Degrees of Freedom	Mean Square	F-Ratio	Action Taken ($\alpha = 0.01$)
Treatment	1	23.23	17.32	Reject H_o
Error	17	1.63		
Total	18			

The critical value for a 0.01 level test in this case is $F_{.99}(1, 17) = 8.40$. Thus, the experimental data indicate a statistically significant difference between post-test scores for the groups.

Relationship Between Teacher and Student Performance

Teachers' score and classes' mean. The same criterion test, administered as a post-test to the students, was given to the teachers of the experimental classes. The teachers' scores and their corresponding class mean scores for the post-test appear in Table 4.

Table 4. Teacher Test Scores and Class Post-Test Mean Scores.

Class	Teacher Score	Class Post-test Mean
0-1	25	11.28
0-2	25	10.39
S-7	25	9.62
0-3	24	9.96
E-9	23	11.70
E-12	23	10.31
S-5	22	12.17
E-11	21	10.61
E-10	20	10.71
S-6	16	10.96
S-8	16	7.57
0-4	14	7.90
E-13	13	8.01

In an attempt to investigate the relationship between teacher and student performance on the criterion test, the Pearson Product-Moment Coefficient of Correlation was determined between the teacher scores and class means. This was done to examine the second hypothesis stated in Chapter III.

(2) There is no significant positive correlation between teacher and student performance on the criterion test on nonmetric geometry.

The correlation coefficient obtained between the teacher scores and class means in Table 4 is 0.64. A sample correlation coefficient is significant if it leads to rejection of the hypothesis that the population coefficient is zero. Employing the t-test of significance on the coefficient obtained, it was found that this correlation coefficient is significant, where $0.02 > P > 0.01$. Therefore hypothesis 2, as stated above, is rejected.

<u>Item reliability</u>. It was reported in Chapter III that an estimate of the criterion test's reliability was 0.72. This coefficient was obtained by applying the Kuder-Richardson Formula 20 to the data. Since the analyses for this study included an examination of the individual items on the testing instrument, it was desirable to obtain a measure of item reliability.

The technique employed to obtain an estimate of item reliability involved two separate administrations of the test to a randomly selected sample of fifty fifth-grade students. The time interval between administrations was one week and no discussion of the material used on the test was conducted during this period. These students were selected from a population of fifth-grade students which had previously been exposed to the selected topics from nonmetric geometry in their regular arithmetic program. Hence, it is assumed that this sample population is much like that employed for the experimental and control classes after instruction.

The number of students who responded correctly to each item on the first administration of the test was obtained. The proportion of these students who continued to respond correctly to a given item on the second administration of the test was computed and used as a measure of the particular item's reliability. These proportions, together with the corresponding response frequencies, are reported in Table 5.

Test items 1, 4, 6, 9, 10, 11, 13, 18b and 20a exhibited proportions which are below 0.700. The reliability proportions for these items are lower than might be desirable and this should be taken into consideration when

Table 5. The Frequencies and Corresponding Proportion of those Students in the Reliability Sample Who Responded Correctly to a Test Item on the Pre-test and also Responded Correctly to that Item on the Post-test.

	Correct Responses				Correct Responses		
Item	Pre-test	Post-test	Proportion	Item	Pre-test	Post-test	Proportion
1	20	11	.550*	14	24	18	.750
2	36	34	.765	15	33	32	.973
3	36	31	.874	16	26	19	.731
4	9	4	.444*	17	11	8	.727
5	32	29	.906	18a	24	19	.792
6	9	2	.222*	b	14	9	.643*
7	25	21	.840	19	12	9	.750
8	17	14	.824	20a	9	5	.555*
9	3	1	.333*	b	26	22	.846
10	21	11	.524*	c	47	47	1.000
11	12	1	.083*	21	15	14	.933
12	5	4	.800	22	42	38	.905
13	4	2	.500*				

* Item proportions below 0.700.

examining the data obtained with these items. However, it should be noted that in many of these nine cases few students answered the item correctly on the first administration of the test and therefore the proportions are based on small numbers. Most of the remaining proportions were based on twenty-five or more students responding correctly on the initial administration. Since the remaining sixteen proportions are above 0.700, it is assumed that the corresponding items possess a satisfactory degree of reliability.

Student continuance to select a correct response. Proportions, computed in the same manner as those used for an item reliability estimate, were found for the item responses of the students of the experimental classes. These proportions are reported in Table 6. A comparison was made between the proportions obtained from the reliability responses and the response of the students in the experimental classes. The procedure employed is presented by Dixon and Massey[1] and involves estimating the difference between two proportions by determining approximate confidence intervals. Those proportions in Table 6 accompanied by an asterisk are significantly lower than the corresponding proportions for the reliability estimates appearing

Table 6. The Frequencies and Corresponding Proportion of those Students in the Experimental Treatment Classes Who Responded Correctly to a Test Item on the Pre-test and also Responded Correctly to that Item on the Post-test.

	Correct Responses				Correct Responses		
Item	Pre-test	Post-test	Proportion	Item	Pre-test	Post-test	Proportion
1	32	11	.344	14	97	62	.618
2	221	107	.484*	15	3	0	.000*
3	98	69	.704*	16	70	35	.500*
4	48	21	.438	17	37	18	.487
5	56	33	.589*	18a	10	8	.800
6	50	23	.460	b	17	7	.412
7	44	33	.750	19	3	1	.333
8	63	38	.603*	20a	66	25	.379
9	18	2	.111	b	89	56	.627*
10	65	26	.400	c	164	124	.756*
11	35	7	.200	21	34	25	.735*
12	13	8	.615	22	44	38	.864
13	22	5	.227				

* Proportion significantly lower than reliability proportion, appearing in Table 5, at the 0.05-level of significance.

in Table 5. In all of the remaining cases there is no reason to conclude that the proportions are different.

It should be noted that none of the proportions from the teacher-taught experimental treatment classes were significantly higher than the proportions obtained from students who received no immediate instruction. In fact, approximately one-third of the proportions from the experimental classes indicate a significantly lower rate of continuance to select the correct response to an item. The low magnitude of the proportions appearing in Table 6 indicate that the rate of change of response from pre-test to post-test was high.

Correctness of student response. The third hypothesis stated in Chapter III concerns the students' selection of correct or incorrect responses to the individual test items. It is stated as follows:

(3) The selection of a correct or incorrect response to a particular item on the criterion test was independent of the selection of the correct response by the teacher on that item.

To facilitate the testing of this hypothesis, a comparison was made between students of teachers who answered correctly and students of teachers who answered incorrectly to each item on the test. The frequency of student correct and incorrect responses was tabulated for each of these two classifications of teachers. The chi square statistic was employed to test the independence of the student's correctness of response to his teacher's response.

Table 7 presents the frequencies for each item together with the corresponding chi square. None of the teachers responded incorrectly to items 7, 8 and 17 which explains the lack of frequencies for these items on the table.

Ten of the twenty-two test items have chi squares associated with them which are significantly below the 0.10 level. According to Tate, "When P is less than 0.10, the hypothesis is in doubt; and when P is about 0.05 or less, the hypothesis ordinarily is considered untenable."[2] Therefore, for these ten individual items, hypothesis 3 is rejected. However, it should be noted that items 1, 4, 6 and 9, which possess significant chi squares, are items which indicate a low level of reliability as reported in Table 5.

A close examination of the ten items which are accompanied by significant chi squares reveals that nine of them are items which require a recognition of a definition or the direct application of a definition to a problem. Items which required a synthesis of the definitions or concepts presented in the instructional material did not

Table 7. Frequencies of Correct and Incorrect Student Responses and Corresponding Chi Squares for Students of Teachers who Responded Correctly and Incorrectly to each Item.

	Students of Teachers with Correct Responses		Students of Teachers with Incorrect Responses		
Item	Correct	Incorrect	Correct	Incorrect	Chi^2
1	79	176	9	45	3.69**
2	139	107	29	39	3.57**
3	191	96	14	11	.71
4	88	166	12	48	4.14**
5	117	164	14	15	.24
6	94	163	9	45	7.11*
9	8	168	15	123	3.67**
10	105	182	9	16	.02
11	15	109	26	161	.05
12	51	165	13	82	3.39**
13	27	179	19	87	.94
14	128	132	24	28	.06
15	96	180	2	27	8.12*
16	117	156	9	20	1.05
18 a	126	165	3	20	6.85*
b	68	125	36	85	.77
19	14	116	23	161	.08
20 a	55	154	27	78	.00
b	161	96	29	28	2.23
c	196	93	19	6	.38
21	221	59	0	31	80.76*
22	214	68	16	15	7.24*

*p <0.01
**p <0.10

produce significant chi squares. For example, response to items 21 and 22, which require the subject to color a triangle and angle with a red pencil, each produced a chi square which was significant at the 0.01 level. However, items 11 and 20, each requiring the application of the same definitions in a more complex situation, did not produce frequencies that obtained a significant chi square.

Student selection of a particular incorrect response. The last hypothesis stated in Chapter III concerns the particular incorrect responses which the students select when failing the tasks. Was the choice of a particular incorrect response by students independent of whether the teacher responded incorrectly to that item by selecting the same response? This question was examined by testing the following hypothesis:

> (4) The student selection of a particular incorrect response to an item on the criterion test was independent of the same incorrect selection by the teacher on that item.

The proportion of students who changed their selection of a response to an item on the pre-test to a particular incorrect response on the post-test was used to examine this hypothesis. These frequencies were tabulated for each incorrect response to each item on the test answered incorrectly by one of the experimental teachers. The frequencies were compared to those obtained by counting the number of students who did not change to the particular incorrect response given by the teacher. A similar count was made for the same items, with respect to the particular incorrect response, for students of teachers who selected the correct response to each item. The chi square statistic was employed to test the independence of the students' and teachers' selections of these incorrect responses. The frequencies, together with the corresponding chi squares, appear in Table 8.

To elaborate on the procedure used to obtain the frequencies appearing on Table 8, item 1 will be considered. Teachers have responded incorrectly to item 1 by selecting responses A and D. An attempt was made to determine whether students in the teacher's class, who incorrectly responded to item 1 by selecting A and D, did so independently of their teacher's choice. Eight students from the class of the teacher who selected response D, changed from a non-D response on the pre-test to a D response on the post-test while twenty-three did not take this action. For students of teachers who responded correctly to item 1, twenty-three changed to D while 183 did not change. The chi square obtained from these frequencies was 3.86, which

Table 8. Teacher Incorrect Responses, Frequencies of Student Response Selections Related to the Teacher Responses and Corresponding Chi Squares.

		Teacher Incorrect Response		Teacher Correct Response		
		Student		Student		
Item	Teacher Incorrect Response Y	Change to Response Y	Did not Change to Response Y	Change to Response Y	Did not Change to Response Y	Chi^2
1	A	5	18	53	159	.00
1	D	8	23	23	183	3.86*
2	A	22	46	59	139	.05
3	B	1	23	12	233	.11
4	C	13	47	20	190	5.30*
5	D	3	26	38	204	1.32
6	D	15	39	33	179	3.50*
9	D	32	105	34	89	.42
10	B	6	25	52	197	.00
11	A	5	24	14	96	.11
11	C	10	91	11	99	.04
11	D	3	25	29	81	2.25
12	B	7	51	10	158	1.52
13	D	25	82	39	120	.01
14	D	6	45	13	200	1.21
16	A	8	21	65	177	.02

*$p < 0.05$

is significant at less than the 0.05 level. Hence, hypothesis 4, as applied to this particular item, was rejected.

The data reported in Table 8 indicate that only three of the sixteen chi squares computed were significant at less than the 0.10 level. Hence, there is evidence to support the acceptance of hypothesis 4.

Summary

This chapter contains the results of the analyses of the data collected in this study. A comparison was made between the means of the teacher-taught experimental classes and the self-instructed control classes. It was found that the experimental treatment classes scored significantly higher than the control treatment classes. A correlation of the teacher test scores and class post-test mean scores revealed a significant positive coefficient. Hence, hypotheses 1 and 2 of this study were rejected.

A measure of reliability was determined for each item on the testing instrument. The reliability measure for each item was compared with a similar measure for the experimental and control treatment groups. In an attempt to test hypotheses 3 and 4, chi squares were computed from the frequencies of student responses to particular items. As a result of chi squares obtained, hypothesis 4 was accepted. However, the data establish reason to question hypothesis 3.

FOOTNOTES

[1]Wilfrid J. Dixon and Frank J. Massey, Jr., Introduction to Statistical Analysis (New York: McGraw-Hill Book Co., 1957), p. 232.

[2]Merle W. Tate, Statistics in Education and Psychology (New York: The Macmillan Co., 1965), p. 293.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The population for this study consisted of nineteen classes of fifth-grade students and their teachers from three rural school districts. The conclusions presented in this chapter are based upon data collected from this population and the instructional materials employed in the study.

Prior to stating any conclusions, the results of the analyses reported in the previous chapter can be summarized as follows:

(1) The mean scores for the teacher-taught experimental classes were significantly higher than those of the self-instructed control classes at the 0.01 level.

(2) There was a significant positive correlation between teacher test scores and class mean scores on the criterion test at the 0.02 level.

(3) Of the twenty-five test items, responses by fifty fifth-grade students to sixteen items produced reliability measures above the 70 percent level of agreement.

(4) Independence of student and teacher selection of correct and incorrect responses to a particular item on the criterion test was tested by application of the chi square statistic. In ten of twenty-two cases a significant chi square was obtained at less than the 0.10 level. Items which exhibited a relationship between student and teacher performance either required a direct recall or application of a single definition presented in the materials.

(5) The independence of teacher and student selection of a particular incorrect response to an item on the criterion test was supported by the data. Of sixteen cases, only three items produced chi squares which were significant at less than the 0.10 level. The three items were concerned with the incorrect identification of a triangle or rectangle as including the "interior" as well as the boundary of the figure.

Conclusions

On the basis of the results reported in Chapter IV and summarized above, the following conclusions can be drawn:

(1) The data, which lead to the rejection of statistical hypothesis 1, do not support the research hypothesis stated as follows:

> Students who read instructional materials in mathematics on their own will perform as well on selected tasks as those who have teachers explain and interpret for them.

(2) The data, which lead to the rejection of statistical hypothesis 2, support the research hypothesis stated as follows:

> If a teacher performs at a certain level of success on selected tasks, then his students will perform at the same level on these tasks following instruction.

(3) The data do not support hypothesis 3 stated as follows:

> The student selection of a correct or incorrect response to a particular item on the criterion test was independent of the selection of the correct response by the teacher on that item.

(4) The data support hypothesis 4 stated as follows:

> The student selection of a particular incorrect response to an item on the criterion test was independent of the same incorrect selection by the teacher on that item.

Discussion

Teacher-taught students perform better. The first two conclusions stated deal generally with the types of instruction employed. It was found that when presenting the mathematics topic to the fifth-grade students, having teacher instruction enabled students to perform better on test items than having the materials to read.

As reported in Chapter III, the instructional materials used in this study were tested for readability level, which indicated a suitable level for the majority of the population. People familiar with elementary arithmetic curricula had read the materials and verified that they were similar in content and style to the many presentations of the topic in various commercial textbooks. However, it was shown that the fifth-grade students did not achieve at as high a level after reading these materials as those students who had been presented the same content by teachers. This finding seems to support the recommendation of Smith and Heddens[1] calling for mathematics materials having a reading level below the grade level for which they are intended. The reading level which Smith and Heddens were referring to would be one measured by a formula such as that presented by Dale and Chall.[2]

There may be more to the comprehension of reading materials, such as those pertaining to topics in nonmetric geometry, than indicated by a reading level quotient. Reading levels are determined by counting the number of words which have been found to be unfamiliar to certain grade level students. It may be that while materials may not contain too many unfamiliar words for a certain grade level student, many of the "familiar" words take on different meanings than the reader associates to them. As an example, the word "triangle" appears in the Dale-Chall list of words familiar to fifth graders. However, the reader may associate a different meaning to the word "triangle" than presented in the instructional materials. It then becomes a question of whether the written materials can communicate this difference in meanings to the reader.

Teacher and student performance is related. When examining the teacher effectiveness in this study, the data indicate that the higher the level of achievement on the criterion test of the teacher, the higher the level of performance of his students on the same test. This finding does not imply that the higher level of teacher performance caused the higher student performance, but the fact that this relationship exists emphasizes the need for further investigation into causation.

The last two conclusions are concerned with the nature of the particular teacher effectiveness in the study. Conclusion three concerns the hypothesis that a student is more likely to perform a task incorrectly if his teacher performs the same task incorrectly. Support is found for this hypothesis with respect to tasks of a "direct definitional nature." This suggests that the tasks required a recognition of a particular definition or the identification of a defined object. When the subject is asked to color a picture of a triangle red or identify which of a group of objects is the best representation of a triangle, this action is interpreted as an application of "direct definitional nature." The results of the study indicated that the correctness of student response to a task of this type was not independent of the teacher's response to the same task. However, none of the items that required the combination of two applications of the same or different definitions indicated dependence on the teacher's performance. An illustration of this type of task requires the subject to find the "points in common" to a triangle and angle, or in a given situation to identify a point which is part of an angle and triangle.

Consider the results obtained from the response dependence test on tasks 10 and 15 of the criterion test.

Task 15 pictures a line segment and asks the subject to indicate whether it is made of two points, ten points, more points than we can count or no points. This task is an application of the definition of a line segment which is defined as a "part of a line which includes two endpoints and all of the points between them." Student performance on this task was related to the teacher's response. Indication of a relationship was determined when thirty-five percent of the students of teachers who responded correctly to this task also selected the correct response, while only six percent of the students of teachers who performed incorrectly selected the correct response.

Task 10 asks the subject to identify the points which are common to two "overlapping" segments on the same line. This task requires more than identifying a figure. The student responses were independent of the teacher responses for this task as indicated by the lack of a significant difference between the proportions of correct student response for teachers who responded correctly and incorrectly.

The difference in dependence found for responses to these two types of tasks may be a result of their roles in the classroom presentation. The definitions, as they appear in the materials, are explicit and the teacher's knowledge of them is likely to be apparent in his presentation to the students. However, the various applications of the definitions to situations are numerous. When faced with a task involving the application of the definitions, the student is less likely to be able to rely on his exposure to a similar situation presented during the teacher's instruction.

Although a teacher's ability to perform a certain task correctly may have had some effect on his students' correct or incorrect performance, the particular incorrect response a student selected was not related to the particular incorrect selection made by his teacher. This finding is indicated by the support established for the fourth hypothesis of this study.

The following question was raised in Chapter III: If a teacher possesses certain misconceptions concerning a specific topic, does this increase the probability of his students forming the same misconceptions? Conclusion four indicates that the student performance on the criterion tasks did not reflect the particular misconceptions exhibited by the teacher. Only three of the first sixteen tasks on the test produced response frequencies which indicated a relationship between teacher and student selection of a particular incorrect response. All three of these tasks involved the recognition of a plane geometric figure as consisting of the "boundary" instead of the "interior" of the figure. When a teacher selected the response which identified the figure as including the "interior" it was found that a larger proportion of his students had selected the same incorrect response than those students of teachers who correctly responded to the task.

<u>A Related Finding</u>

The percent of student continuance to respond correctly to a task on the post-test after doing so on the pre-test was low for the teacher-taught classes. To obtain an estimate of item reliability, a sample of fifty fifth-grade students were administered the criterion test twice with an interval of one week between administrations. The percent of students who continued to respond correctly on the second administration of the test was determined for each item. A similar procedure was used to establish percents for continuance to respond correctly to an item on pre-test and post-test for the teacher-taught experimental classes. A comparison was made of the percents obtained for these two groups of student responses.

Although the reliability group of students did not receive the same instruction presented to the experimental treatment students, they had been exposed to the topics of nonmetric geometry in their regular arithmetic program prior to the first administration of the test. This provided for a comparison of test performance between a group of students who were exposed to the teacher instruction and a group who did not receive instruction between two administrations of the same test.

None of the twenty-five item percents computed for teacher-instructed students who continued to respond correctly to a test item on the post-test, after doing so on the pre-test, were significantly higher than those percents for students not receiving instruction. In fact, eight of these response pairs of percents were significantly higher for the noninstructed students.

It might be assumed that if a student can perform a task correctly before teacher instruction, then he should retain this behavior after instruction. However, the data do not support this assumption and in fact indicate that the student is less likely to continue this acceptable behavior if exposed to the teacher's presentation. This finding emphasizes the importance of identifying tasks which the student can successfully perform prior to instruction. It implies that the practice of presenting instruction to an entire class when some of the students already possess the desired behavior may not only be a misuse of their time, but detrimental to the students' ability to perform the desired tasks.

Implications for Teacher Effectiveness

The conclusions and discussion in the previous sections pertain to a limited population and to the teaching of the particular topic of nonmetric geometry. The results of this study imply that a teacher's ability to perform some tasks, related to nonmetric geometry, may have a direct influence on his students' performance of these tasks. However, this influence was found to hold for behavior on recalling definitions and identifying geometric figures, but not necessarily for behavior associated with the application and transfer of these competencies to different situations.

The results of this study seem to imply that, except for the ability to recall definitions and identify defined objects, the ability of the teacher to perform the behavior he is attempting to shape in his students is not related to the students' performance. This implication questions the assumption that a teacher must possess a high level of proficiency in the content area he is teaching. Hence, a teacher who exhibits the ability to successfully perform the many applicational tasks related to a particular mathematics course may not be any more effective at producing desired behavior on the part of his students than the teacher who cannot perform these tasks.

An important implication of this study is that it suggests that teacher effectiveness can be analyzed by relating the performance of the teacher directly to that of his students. As reported in Chapters I and II, investigations have been conducted in which student performance was related to such teacher characteristics as personality, years of teaching experience, number of college courses completed and exposure to inservice programs. The present study has investigated the relationship between teacher and student performance on specific tasks.

Recommendations for Further Study

The effectiveness of teacher instruction and possible relationships between teacher and student performance can only be determined by knowledge gained through accumulated research. Additional studies employing the design used in this investigation, involving the use of different tasks, would assist in determining possible relationships between teacher and student performance. The use of a topic in mathematics where the definitions are not as explicit as they are in nonmetric geometry might reveal a different degree of student performance dependence on teacher performance. Perhaps the use of a topic which centers on arithmetic computation might yield results of interest.

A limiting factor of the present study was the small number of teachers who performed incorrectly on individual items on the criterion test. A modification of the design which would eliminate this weakness might involve the establishment of various predetermined levels of performance on the part of the teachers employed in the investigation. A group of teachers could be assisted until they can perform at a given level of achievement on selected tasks. This would facilitate a comparison, involving a greater number of teachers than the present study, of the effect of student performance of those teachers who can and cannot perform selected tasks.

It seems desirable to design studies to test the possible teacher effect on student performance of tasks which are of an applicational or problem-solving nature. Task 10, referred to in the discussion of the conclusions presented in this chapter, might be an illustration of such a task. The present study indicated that student performance was related to teacher performance when considering tasks involving the recognition of definitions and the identification of geometric figures. However, no relationship was indicated for tasks requiring what might be referred to as a "problem-solving attack." This aspect of the design should be investigated in a situation providing for a longer instructional time period and in which the types of tasks involved were clearly defined. There is constant reference in literature pertaining to the teaching of "problem solving." However, little has been done to investigate the effect of the teacher on "problem-solving" performance of his students.

A possibility for further study, suggested by the results of this investigation, concerns those students who enter a period of teacher instruction already possessing the behavior which the teacher is attempting to shape. The results reported in Chapter IV exhibited a relatively low level of performance maintenance during teacher instruction. In fact, in some cases, this level was lower for teacher-instructed students than those not exposed to instruction over the same length of time. An investigation concentrated on this phenomenon would seem essential to the study of teacher effectiveness.

Further research should be conducted to investigate the nature of the geometric content which might be presented to elementary school students. The general level of achievement exhibited by the majority of the students employed in this study was not as high as might be desired. Perhaps a development of geometric topics from an informal approach, based on the students' intuition and experience, would be more effective than presenting definitions and situations emphasizing the application of the definitions.

Summary

From the results reported in Chapter IV, support was obtained for the hypothesis that students achieve at a higher level when exposed to teacher instruction pertaining to topics in nonmetric geometry than they did after reading instructional materials without teacher assistance. It was also shown that the performance of students on selected geometric tasks was directly related to the performance of their teachers. A further investigation of this relationship showed an independence of student and teacher performance on individual tasks requiring an application of the geometric definitions presented by the teacher. However, the student response frequencies on items requiring recognition of definitions and identification of geometric figures did indicate a relationship to teacher performance. An investigation of possible relationships between the particular incorrect responses selected by the students and those selected by teachers did not indicate a significant dependence.

The present study in teacher effectiveness was limited to the teaching of one topic in the mathematics curriculum for the elementary school. Such a limitation is necessary if specific aspects of teacher effectiveness, as related to teacher performance, are to be investigated. The chapter concluded by presenting some suggestions for research studies that might be undertaken to further investigate possible teacher effect when measured by student performance.

FOOTNOTES

[1]Kenneth J. Smith and James W. Heddens, "The Readability of Experimental Mathematics Books," The Arithmetic Teacher, XI (November, 1964), 466-468.

[2]Edgar Dale and Jeanne S. Chall, "A Formula for Predicting Readability," Educational Research Bulletin, XXVII (January, 1948), 11-20.

APPENDIX A

INSTRUCTIONAL MATERIALS

BOOKLET ONE

What do you think of when you hear the word "point"? Some of us might picture the tip of a sharpened pencil, the sharp end of a pin, the corner of a box or a dot on our paper. All of these objects can be used to picture what we think of as a point, even though we cannot find an object which is actually a point.

We would like to draw pictures of points and give them names so we may talk about them. The dots drawn below are pictures of two points which are named by the capital letters A and B.

.A

.B

Look around your classroom and select three objects which might suggest a point. What are the objects you have selected?

1. ____________ 2. ____________ 3. ____________

Draw a picture of four dots on your paper. Name the points you have pictured with four capital letters.

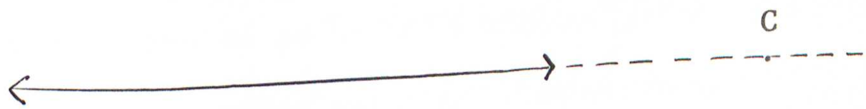
Another idea which we can picture by using objects and drawings is that of a "straight line." The edge of a desk, a tightly drawn string, the joining of two walls in the room and the edge of our paper all suggest straight lines. Think of a straight line as extending on and on in opposite directions and never stopping. Of course none of the objects we use to picture straight lines go on and on, but remember they just make us think of a straight line. To draw a picture of a straight line on our paper we use a figure like the one below.

The arrow on each end of the picture reminds us that the line does not stop, but goes on beyond that which our eye can see.

From now on when we use the word "line" we will mean a straight line. We can think of a line as being made up of many, many points. More points than you can ever count. Look at the picture of the line below.

One of the many points on this line is named point A. Mark with dots and name three other points on the line pictured above.

Look at the picture of the line below.



Is point C on the line? ____ Since the line extends on and on in both directions, then point C does belong to the line. Remember, the picture does not show the whole line.

We can draw many lines on our paper. By naming different points on these lines it is possible for us to name

lines so that we may talk about one or the other of them. Therefore, if we write line AB, then you should know which of the two lines in the picture above we are talking about.

Draw a picture of a point on your paper and label it X.

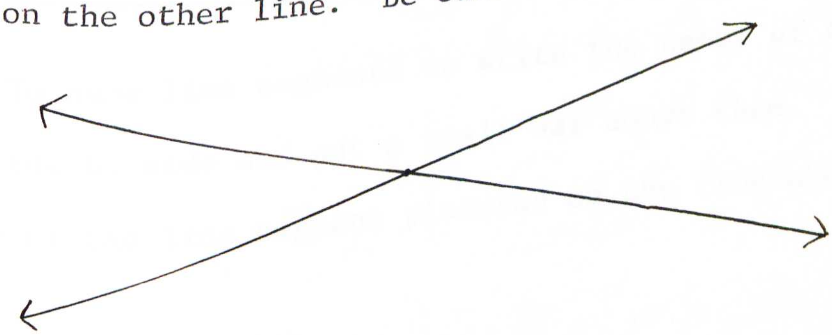
Now draw a picture of a line containing point X. Draw several other lines which will contain point X. How many lines did you draw which contain point X? ____ You could draw as many as you wish because many, many lines contain each point.

Draw two points on your paper and name them P and Q.

Now draw a picture of a line containing both points P and Q. Try to draw a second line containing both points P and Q. Give up? You should be having trouble because there is only one line which will contain two different points. This is why when you talk about line PQ we will know exactly which line you mean.

When we draw pictures of points and name them with capital letters, we should not use the same letter for two different points. This would be as confusing as having two people in our class with the same name.

Look at the drawing below. Mark with a dot and name two points A and B on one of the lines. Next name two points A and C on the other line. Be careful where you name point A.

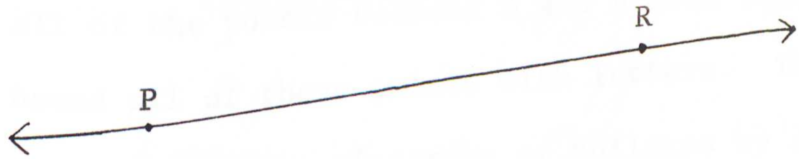


You should have labeled the point where the lines cross A, because point A has to be on both lines. Could lines AB and AC cross at any other point besides point A? ____ If they did then they would be the same line.

On the line pictured below name two points A and B.

Now name four points using E, F, G and H which lie between points A and B on line AB. There are many, many points on the line AB which lie between points A and B. If we think of the points A and B, and all of the points on the line between A and B, then this will be a part of the line which we will call a <u>line segment</u>. The points A and B are called the endpoints of the line segment. Remember that lines do not have endpoints because they go on and on, but line segments <u>do</u> have two endpoints.

To name line segments we write the names of the endpoints side by side and put a small bar above them. $\overline{PR}$ is the name of the line segment pictured on the line above.



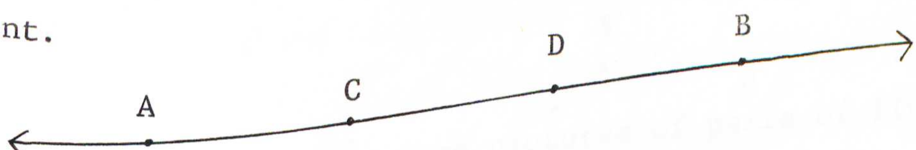
Look at the picture of the line above. Color line PR with your red pencil. Color line segment $\overline{PR}$ with your blue pencil. See that the red mark should color the whole picture, while the blue mark should color the whole picture, while the blue mark should end on points P and R. This is because a line segment differs from the line in that it has endpoints.

Do you see a line segment pictured on the line below? ____ Do you think $\overline{AB}$ and $\overline{BA}$ should be thought of as

different line segments? ____ If you use your red pencil to color $\overline{AB}$ and your blue pencil to color $\overline{BA}$ you will find that the same part of line AB is colored twice and nowhere on the line would points be colored just one color. Therefore, we will say that they are the same line segments. The order in which the endpoints appear in the name for a line segment does not make a difference. We can call the segment pictured above $\overline{AB}$ <u>or</u> $\overline{BA}$.

When we talk about segment $\overline{AB}$ we are talking about many, many more points besides points A and B. The segment includes all of the points between A and B even though we have not named all of these points with letters. This is just like talking about a classroom of children by saying this is "Bobby Smith's and Joe Brown's class." We don't mean that Bobby and Joe are the only ones in the class, but we use their names to label the whole class of children. Remember that when we talk about lines and segments we are talking about sets of many points, even though we only name a few of these points to use as labels.

Look at the line AB pictured below. See how many different line segments you can name using as endpoints the points marked on the line. List your segments on the blanks below. Remember that you only use two letters to name each segment.



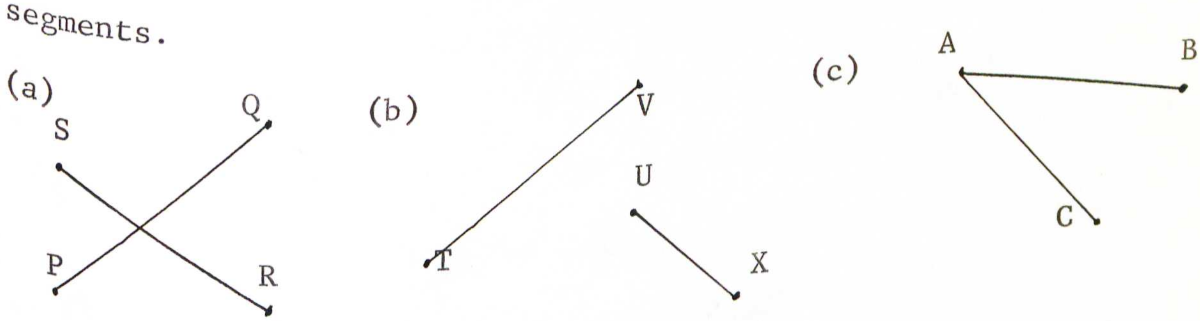
1. ________
2. ________
3. ________
4. ________
5. ________
6. ________

Did you find all six line segments on the line pictured above? You should have named the following segments: $\overline{AB}$, $\overline{AD}$, $\overline{AC}$, $\overline{CD}$, $\overline{CB}$ and $\overline{DB}$. Remember that the order in which the letters appear does not make any difference when we name line segments. So if you said $\overline{BA}$ that would be the same segment as $\overline{AB}$. There are many more line segments on line AB, but using just the points marked on the line we can only name six of them.

We may picture a line segment without showing the whole line of which it is a part. A picture of $\overline{RS}$ might be drawn like this,

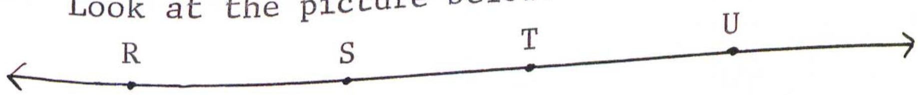
without showing the whole line RS. Remember that even though the line is not pictured when showing the segment above, we still think of the segment as part of a line. Using the points X and Y marked below, draw $\overline{XY}$. Remember this line segment could also be called $\overline{YX}$.

The following are some pictures of pairs of line segments.



The segments $\overline{RS}$ and $\overline{PQ}$ in picture (a) meet at a point between the endpoints of the segments. They have a point in common to both. The segments in picture (b) have no common points. In picture (c) the segments meet at an endpoint of each of the two segments. Segments $\overline{AB}$ and $\overline{AC}$ have the point A in common. Point A is on both segments.

Look at the picture below.



Do $\overline{RT}$ and $\overline{SU}$ have any points in common? ____ Let us see if we can name the common points. Color $\overline{RT}$ red and color $\overline{SU}$ blue. The part of the line which you have colored <u>both</u> blue and red is on both segments $\overline{RT}$ and $\overline{SU}$. Points S and T should be colored red and blue. Also all of the points between S and T are colored red and blue. This should show you that the set of points which are common to the two segments can be called line segment $\overline{ST}$.

Exercise I

1. Use the point R which is named below.

   a. Draw five different lines through the point R.

   b. Mark and name another point on each line.

R ·

   c. Name the lines you have drawn.

   1. _____ 2. _____ 3. _____ 4. _____ 5. _____

   d. Can many more lines be drawn through R? _____

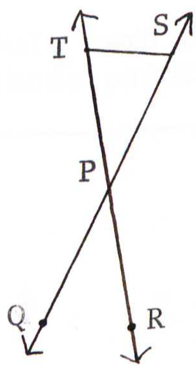
2. Draw a picture of three line segments which have point K as an endpoint of each.

K .

3. Draw a picture of three line segments which have point S as a common point, but S is <u>not</u> an endpoint of any of the three segments.

. S

4. Use the drawing below to answer the questions.



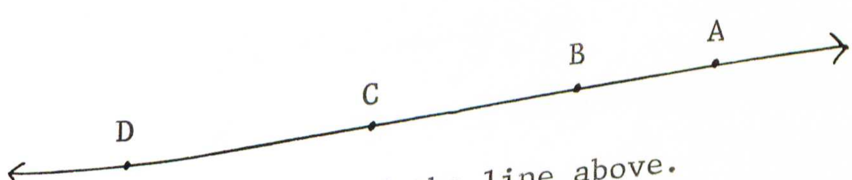
a. Name two lines pictured above. ____ and ____

b. What part of line $\overline{ST}$ is pictured above? ____

c. Name five line segments pictured above. ____, ____, ____, ____ and ____.

d. In the drawing above draw $\overline{RQ}$.

e. Name three segments which contain P. ____, ____ and ____.

5. 

a. Look at the picture of the line above.

1. Color $\overline{CA}$ red.

2. Color $\overline{DB}$ blue.

b. Put an "X" next to the correct way to finish the following sentence.

The points which lie on <u>both</u> $\overline{CA}$ and $\overline{DB}$ should <u>now</u> be

____ a. colored red only.

____ b. colored blue only.

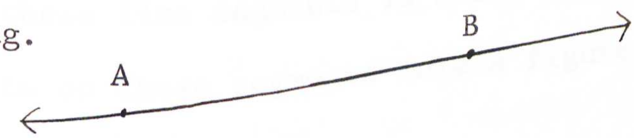
____ c. colored red and blue.

____ d. not colored at all.

c. We could name the set of points which is made up of all those points which are on both $\overline{DB}$ and $\overline{CA}$ as ________________.

BOOKLET TWO

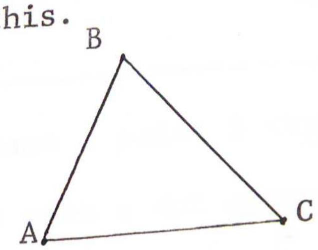
Yesterday we talked about points, lines and line segments. Dots on our paper were used to picture what we think of as a point and capital letters were used as names for the points. Straight lines, which can be thought of as many, many points, are pictured with a drawing like the following.



Remember that we would call this line AB or BA. A segment is part of a line which is made up of two endpoints and all of the points on the line between the endpoints. Looking at the picture of the line AB above you should remember that the line goes on and on, while the segment $\overline{AB}$ ends at points A and B.

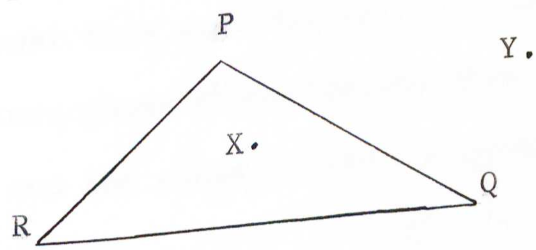
On the picture below name the three points with the letters A, B and C. See that the three points would not be on the same line.

Using only the points A, B and C above, draw all of the line segments you can which have two of the three points as end-points. Your picture should show three line segments and look <u>something like</u> this.



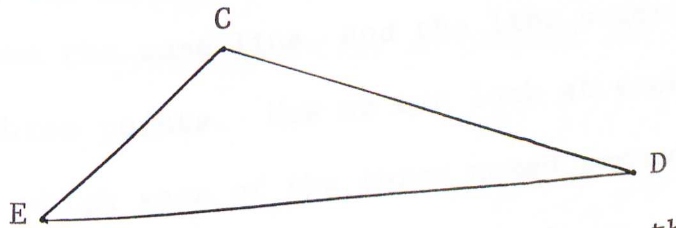
Each of these line segments is made up of many points. All of the points on these segments make a figure which we will call a <u>triangle</u>.

Look at the line segments pictured below and color the segments $\overline{PQ}$, $\overline{PR}$ and $\overline{QR}$ with your red pencil.



The points which you have colored red make a triangle. Is point X or Y on the triangle pictured above? ____ Since the triangle is made up of three segments, we could ask this question by asking: Are the points X or Y on any of the three line segments $\overline{PQ}$, $\overline{PR}$ or $\overline{QR}$? We can see that neither X nor Y are on any of these segments, therefore they are not on the triangle. The triangle includes only the points on the three segments.

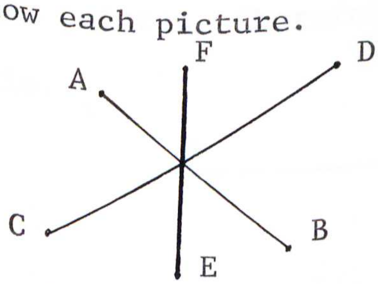
Here is a picture of three line segments. Do the segments $\overline{CD}$, $\overline{DE}$ and $\overline{CE}$ make a triangle? ____



Mark with a dot and name a point S which is on the triangle pictured above. Mark with a dot and name a point T which is not on the triangle pictured above. Point S can be named on any of the three segments of the triangle. The point which you named T can be "inside" or "outside" of the three segments, but not on the line segments.

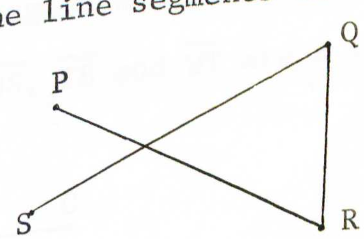
Does this mean that any three segments will make a triangle? To answer this question we will look at the pictures below. Each picture shows three labeled line segments which cross each other and the names of the line segments are listed below each picture.

(a)



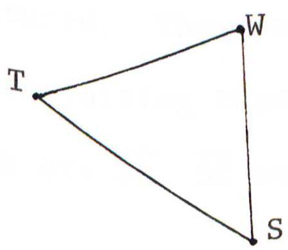
$\overline{AB}$, $\overline{CD}$ and $\overline{EF}$

(b)



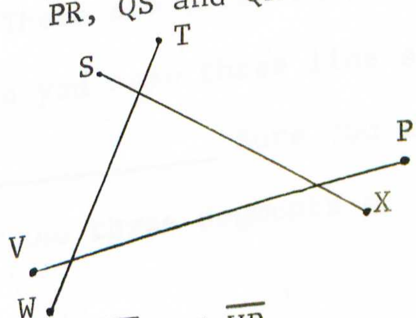
$\overline{PR}$, $\overline{QS}$ and $\overline{QR}$

(c)



$\overline{TW}$, $\overline{WS}$ and $\overline{TS}$

(d)

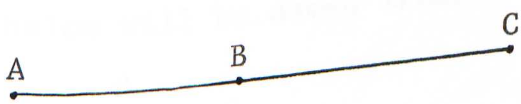


$\overline{TW}$, $\overline{SX}$ and $\overline{VP}$

Which of these sets of three segments shall we call a triangle? Let us say that a triangle is made up of three points, not on the same line, and the line segments which join these three points. Now we can look at each of the pictures to see which sets of the three named segments are triangles.

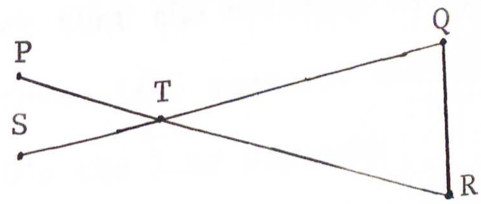
Picture (a) shows three labeled segments which clearly do <u>not</u> make a triangle since there are six endpoints pictured. How many endpoints can you count in pictures (b) and (d)? There are four endpoints named in picture (b) and six endpoints in picture (d). Therefore, the line segments named in pictures (b) and (d) do not make triangles. Look at picture (c) and count the endpoints named.

In picture (c) we see three endpoints, W, S and T, not on the same line and the three segments which join these three points. Therefore, segments $\overline{WS}$, $\overline{TS}$ and $\overline{WT}$ are a triangle.



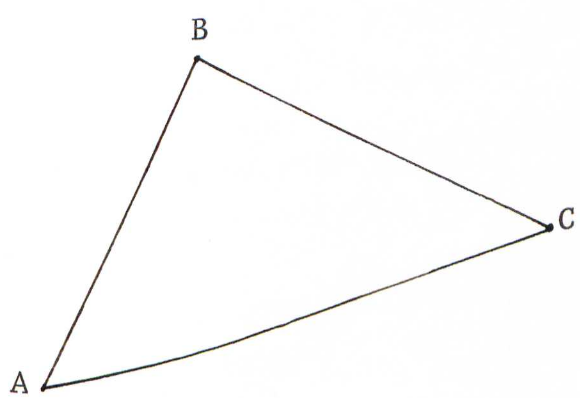
Look at the picture above. There are three endpoints pictured. These are A, B and C. Can you name three line segments joining these three points? ______________ Sure you can! They are $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$. Well, are these three segments a triangle? ____ Definitely not, because the points A, B and C are on the same line.

Let us take another look at picture (b) from the last page.



We were asked if line segments $\overline{PR}$, $\overline{QS}$ and $\overline{QR}$ make a triangle and the answer was no. If we were to name the point where $\overline{PR}$ and $\overline{QS}$ cross with the letter T and consider the segments $\overline{QT}$, $\overline{QR}$ and $\overline{TR}$, then we could name a triangle which is shown by part of the picture. This triangle will be made up of the three endpoints Q, T and R and the segments joining these points.

To name a triangle we will use a small picture of a triangle followed by the letters of the endpoints. The triangle shown below will be named $\triangle ABC$.

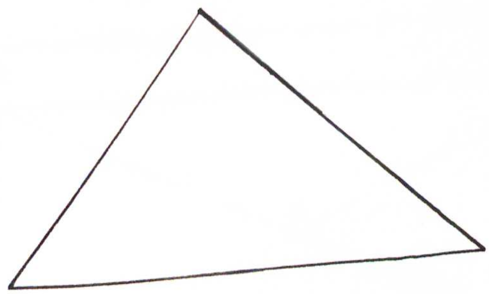


The order in which the letters appear will not make any difference. We could name the triangle on the last page as $\triangle$ BAC, $\triangle$ CAB or $\triangle$ BCA.

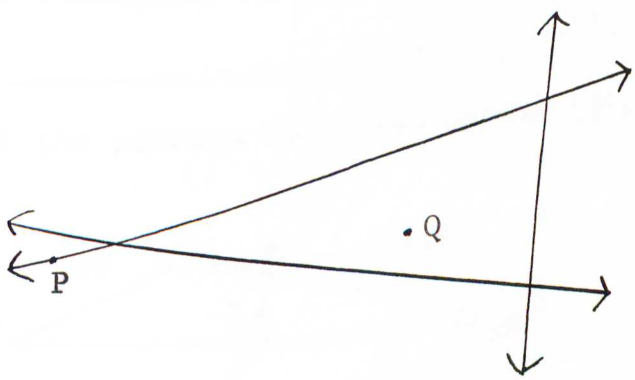
Remember that the triangle is made up of only those points on the three line segments and does not include points inside or outside the line segments.

Exercise II

1. Use your red pencil and color just the triangle below.



2. Look at the picture and label points on it so that you can name a triangle which is part of the picture.



a. Name your triangle. ____________

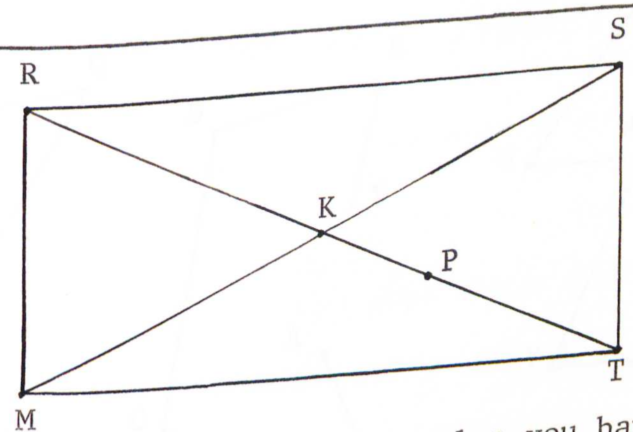
b. Is point P on the triangle named above? ________

c. Is point Q on the triangle named above? ________

3. a. Look at the picture below and name as many triangles as you can see pictured. Write the names on the blank below.

____

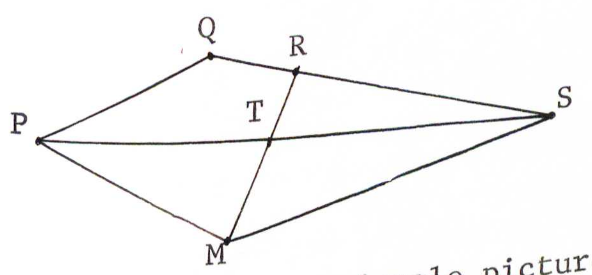
____



b. Look at point P and the triangles you have named. Triangles are made up of many points. To which of the triangles that you have named does point P belong?

____

4. Look at the picture below and answer the question.

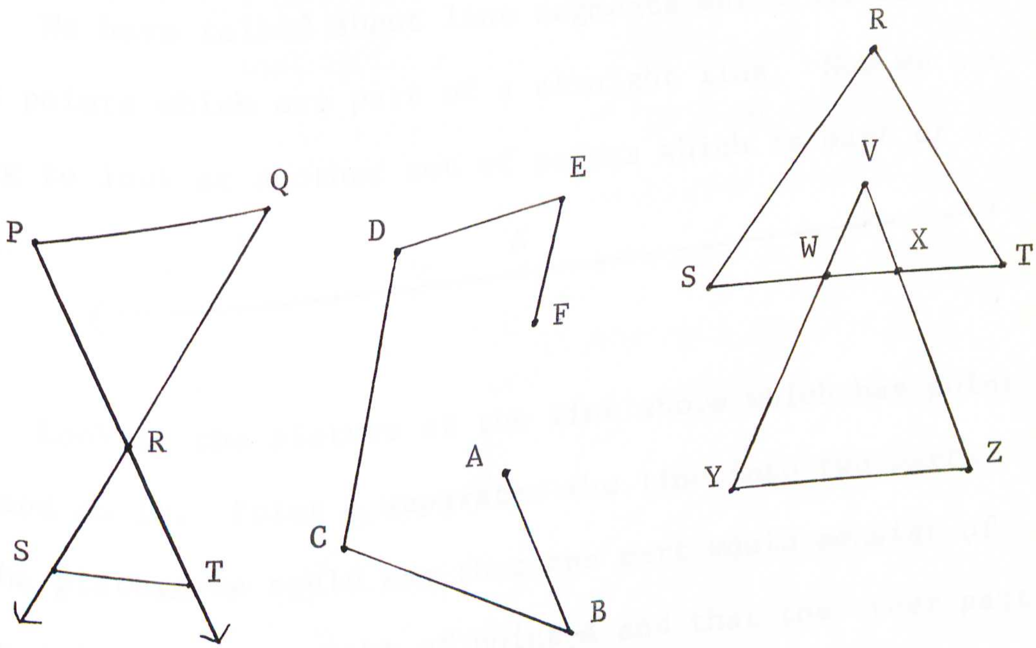


Write the name of <u>one</u> triangle pictured above which will have all of the following.

a. Point S on it.
b. A part of line segment $\overline{PS}$ on it.
c. Line segment $\overline{MT}$ on it.

Answer ____________

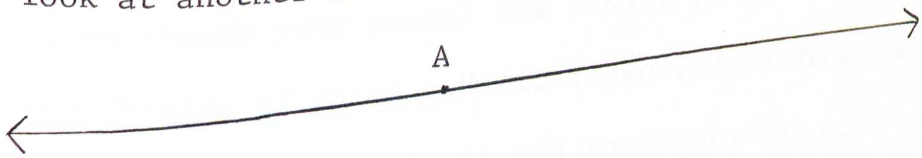
5. a. Look at the pictures below and color all of the triangles you see with your red pencil.



b. Write the name of each triangle under the picture to which it belongs.

BOOKLET THREE

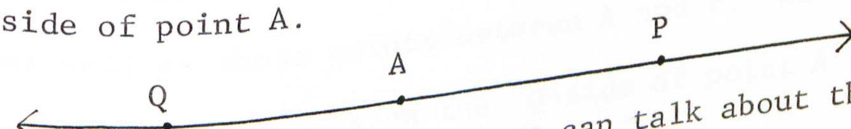
We have talked about line segments which are made up of many points which are part of a straight line. Now we are going to look at another set of points which is part of a line.



Look at the picture of the line above which has point A named on it. Point A separates the line into two parts. In the picture we could say that one part would consist of those points to the right of point A and that the other part might be those points to the left of A. This would be fine until the picture was drawn like this.

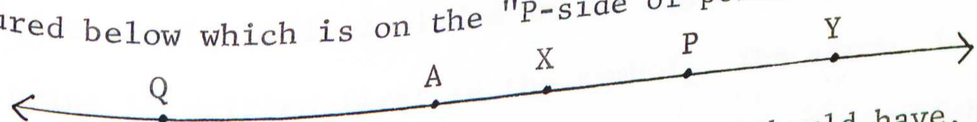
Now the words left and right would be confusing, so let us attempt to find a way to name the different parts no matter how the picture of the line appears on the paper.

Suppose we name two additional points on the line, one on each side of point A.



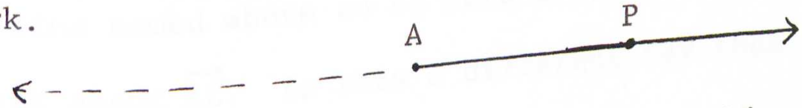
How might this help us? Well, now we can talk about the part of the line which is made up of the points on the "P-side of point A." Place your pencil on the line between points A and P. Is your pencil on the "P-side of point A"? ____ Move you pencil to point P. You are still on the "P-side of A." Now move your pencil in the same direction past point P. Even though your pencil has passed point P you are still on the "P-side of point A." If a man drives his car from New York to Washington, D. C. and then goes on to Florida, could we say that he is on the "Washington-side" of New York? ____ Sure, any city which is south of New York could be thought of as being on the "Washington-side" of New York.

Take your red pencil and color that part of the line pictured below which is on the "P-side of point A."



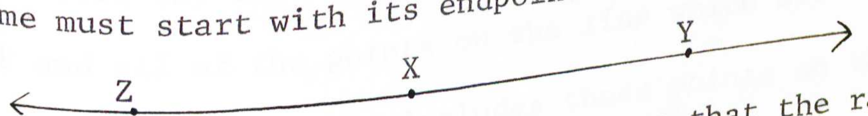
Have you colored the point X red? ____ You should have, because X is on the "P-side of A." What about the point Y? ____ Point Y should also be colored red because it too is on the "P-side of A." See that the points on the "P-side of point A" will include all of the points on the line beyond point P as well as those points between A and P. We could also think about the points on the "Q-side of point A" which would be a different set of points.

Suppose we think about all of the points on the "P-side of point A" <u>and</u> the point A. We then have a set of points which looks like the one below shown by the solid pencil mark.



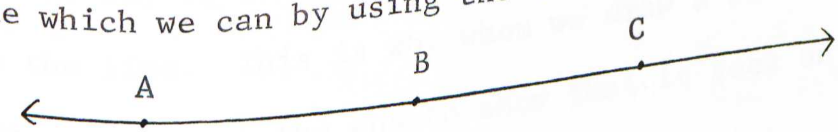
See that it includes the point A. We will give this set of points the name <u>ray</u>. The symbol we will use for the ray will be made up of the letters of point A with some point on the desired side of point A. These letters will be written side-by-side with an arrow drawn above them. In other words, the ray pictured above with a solid mark will be named $\overrightarrow{AP}$.

Two of the rays which could be named in the picture below might be $\overrightarrow{XY}$ and $\overrightarrow{YX}$. Remember that the point X, which separates the line, is included in the rays. The letter for this point is written first in the symbol. The arrow in the name must start with its endpoint over the point which

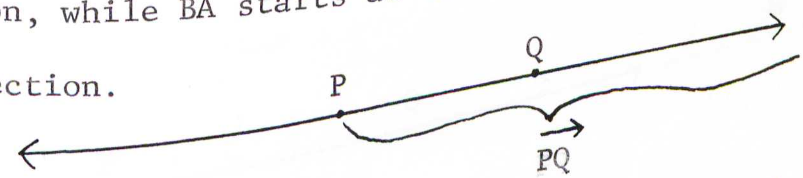


separates the line. This arrow reminds us that the ray goes on and on in the second letter's direction.

Suppose we consider the following line with the three named points A, B and C. Let us name all of the parts of the line which we can by using the three points A, B and C.



We can name segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$. Also, the following rays: $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CB}$ and $\overrightarrow{BA}$. Use your pencil and trace each of the parts of a line named above to be sure that you see them. You might ask about $\overrightarrow{AC}$. Is this a different ray than those we have listed? _____ First color $\overrightarrow{AC}$ with your red pencil. Be sure that you color beyond point C. Now color $\overrightarrow{AB}$ with your blue pencil. You should have colored exactly the same part of the line blue and red. In other words, the "B-side" and the "C-side" of point A are the same. $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays. $\overrightarrow{AB}$ starts at the point A and goes on in the B direction, while $\overrightarrow{BA}$ starts at the point B and goes on in the A direction.



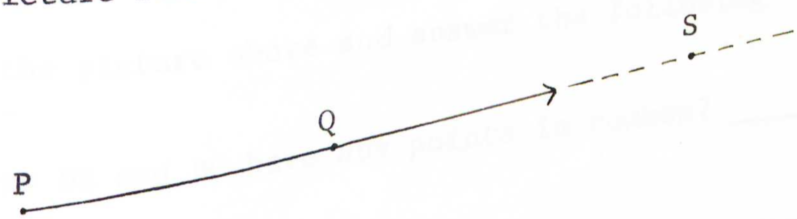
We will say that the ray $\overrightarrow{PQ}$ will be made up of the point P and all of the points on the line which are on the "Q-side of point P." This includes those points on the line beyond point Q so don't let your "mind" stop at point Q. If in the picture above you are just thinking about the points from P to Q, then this part of the line is a segment and not a ray. The ray $\overrightarrow{PQ}$ does not stop at point Q, but goes on and on like the line. This is why when we draw a picture of a ray we place an arrow at the end to show that it goes on and on.

Exercise III

1. Draw three different rays which have point X as an end-point.

X ·

2. Use the picture below to answer the questions.

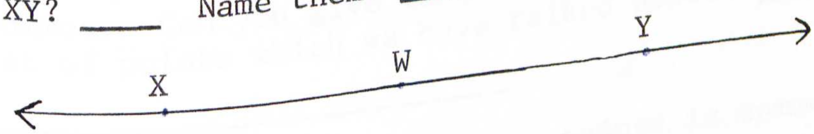


a. Is point S on line PQ? ____

b. Is point S on $\overline{PQ}$? ____

c. Is point S on $\overrightarrow{PQ}$? ____

3. Using the picture below, how many rays can you name on line XY? ____ Name them. ____________



How many rays can you name on line XY which have point W as an endpoint? ____

4. Consider the picture above and answer the following questions.

   a. Do rays DE and BC have any points in common? ________________

   b. Do ray DE and segment $\overline{AB}$ have any points in common? ____

   c. Do rays AB and CB have any points in common? ____

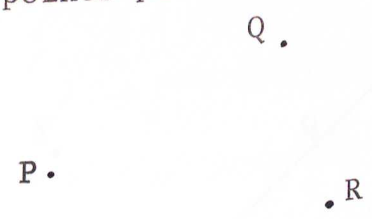
   d. Describe the points which rays AB and CB have in common. Can you give them a name using a name of a set of points which we have talked about? ________________

   e. Do segments $\overline{AB}$ and $\overline{BC}$ have any points in common? ____

   f. Color the ray ED using your red pencil.

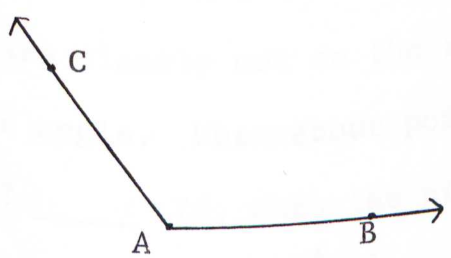
BOOKLET FOUR

Using the points pictured below draw $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

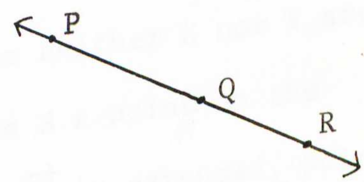


You have now drawn two rays, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$, which have the same endpoint P. We will call the set of points you have pictured above an <u>angle</u>. The angle is made up of just the points on the two rays.

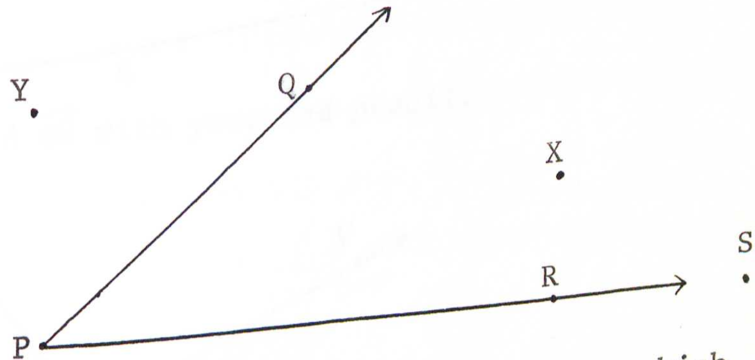
(a)



(b)

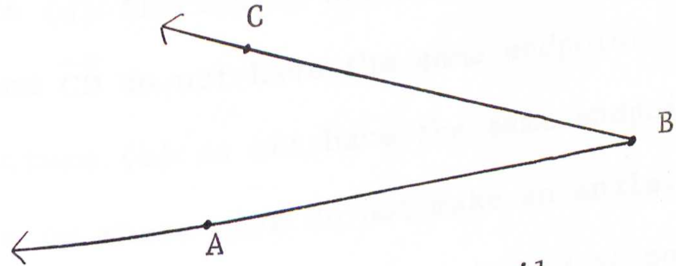


Look at picture (a). Here we have $\overrightarrow{AC}$ and $\overrightarrow{AB}$ which have the same endpoint A. All of the points on these two rays make up an angle. Picture (b) also shows two rays, $\overrightarrow{QR}$ and $\overrightarrow{QP}$, which have the same endpoint A. Should we say that the set of points in picture (b) is an angle? We already have a name for the set of points shown in picture (b). We call it a straight line. Since these two rays make a straight line and there is no need to have two different names for the same set of points, we will not <u>call</u> these two rays an angle. Two rays, with the same endpoint, will be called an angle only when they are not on the same line.

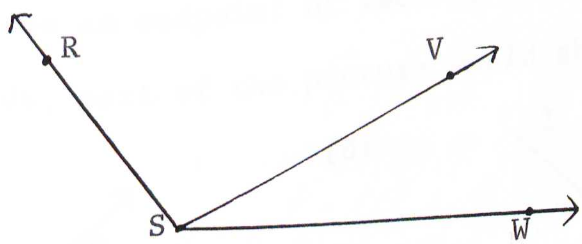


Look at the picture above which shows two rays which are an angle. Are points X and Y on the angle? ____ First, we must ask if X and Y are on either of the two rays. Since they are clearly not on the rays, then neither X nor Y are on the angle. What about point S? Is S a point on the angle? ____ If, when the picture of $\overrightarrow{PR}$ is extended, point S is shown to be on $\overrightarrow{PR}$, then S is a point of the angle pictured.

We can name an angle by using a small picture of an angle, $\angle$, followed by three of the letters of the points on the angle. The angle pictured above is $\angle QPR$. See that the common endpoint of the rays of the angle appears between a point of each of the two rays. In other words, $\angle ABC$ might look like the picture below, where B is the common endpoint of $\overrightarrow{BA}$ and $\overrightarrow{BC}$.



Color $\overrightarrow{SR}$ and $\overrightarrow{SW}$ with your red pencil.

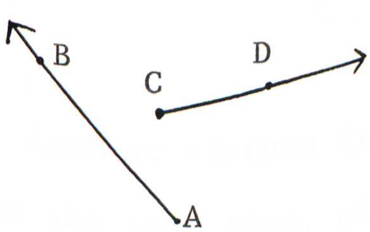


The points which you have colored could be named $\angle$RSW. Is point V on $\angle$RSW which you have colored? No, because V is not on either of the rays $\overrightarrow{SR}$ or $\overrightarrow{SW}$.

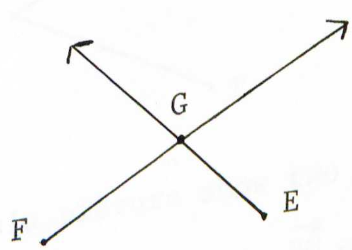
Suppose we name as many different angles pictured above as possible. We could name $\angle$WSR, $\angle$WSV and $\angle$VSR. See that $\angle$WSR and $\angle$RSW are the same angles. It makes no difference in which order we place the W and R in our name just so long as we put the S in the middle.

Now we will look at some pictures and see which of them shows pictures of angles.

(a)

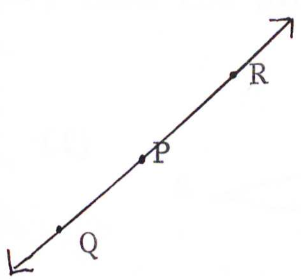


(b)

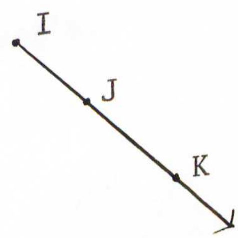


Picture (a) is clearly not a picture of an angle because $\overrightarrow{AB}$ and $\overrightarrow{CD}$ do not have the same endpoint. Rays $\overrightarrow{EG}$ and $\overrightarrow{FG}$ in picture (b) do not have the same endpoint, so all of the points on these rays do not make an angle. If, however, we just look at the rays beginning at point G, then G would be an endpoint of two rays which are an angle. In other words, part of the picture would show an angle.

(c)

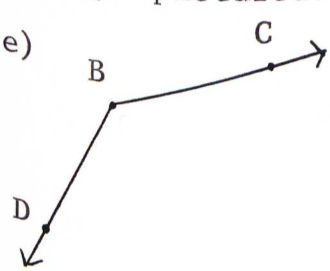


(d)

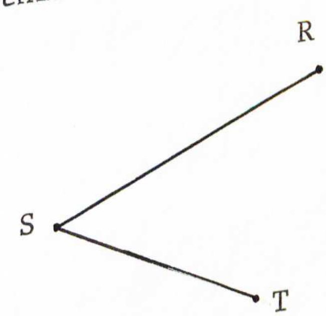


In picture (c) $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ are on the same line, so this will not be called an angle. Picture (d) does not show an angle because only part of one line is pictured. Rays $\overrightarrow{IJ}$ and $\overrightarrow{IK}$ are the same and this means we do not have two rays pictured.

(e)



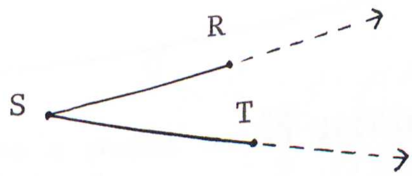
(f)



Look at picture (e). Does this picture show two rays, not on the same line, with a common endpoint? Yes, $\overrightarrow{BC}$ and

$\overrightarrow{BD}$ are the two rays with the endpoint B. Therefore, picture (e) shows $\angle CBD$. What about picture (f)? The picture shows two line segments, $\overline{SR}$ and $\overline{ST}$, which have the same endpoint. But angles are made up of rays and not segments. We would have to say that the picture of the two segments in (f) does not show an angle. If we were to change the drawing to picture points on $\overrightarrow{SR}$ and $\overrightarrow{ST}$, beyond segments $\overline{SR}$ and $\overline{ST}$, then the picture would show an angle.

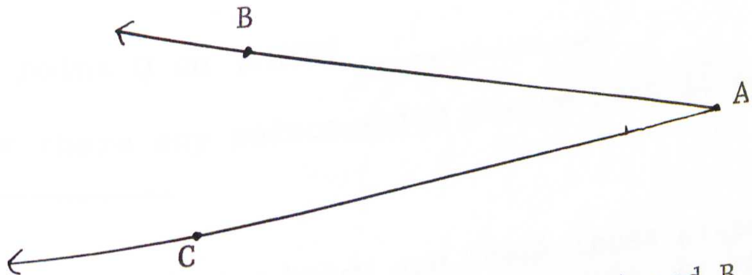
(f)



Exercise IV

1. Draw an angle and label it $\angle SRT$.

2. Use the picture below to answer the following questions.



a. Choose a point on $\overrightarrow{AB}$ different from A and B and label it D.
Choose a point on $\overrightarrow{AC}$ different from A and C and label it E.

b. Is $\overrightarrow{AB}$ the same ray as $\overrightarrow{AD}$? ____________

c. Is $\angle BAC$ the same angle as $\angle DAE$? ____________

d. Draw segment $\overline{DE}$.

e. Are all of the points in $\overline{DE}$ also points of $\angle BAC$?

____

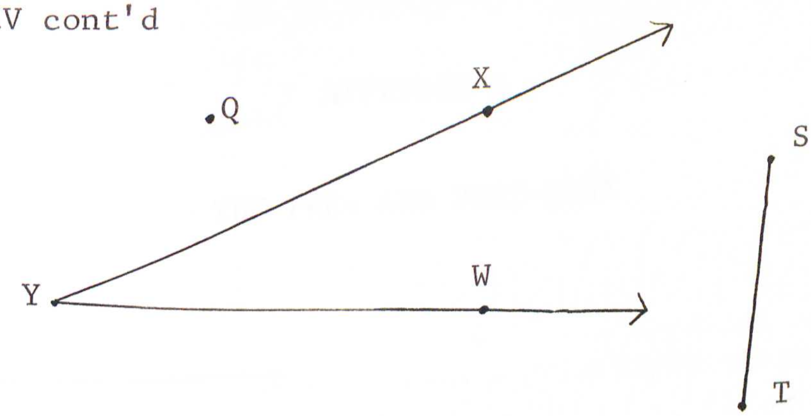
3. a. What set of points do the hands of a clock picture when it is 3 o'clock? ____________.

b. Each hand of a clock can be thought of as representing a part of a ____________, when answering question a.

c. What set of points do the hands of a clock picture when it is 6 o'clock? ____________.

Exer. IV cont'd

4.



a. Is point Q on $\angle XYW$? ______________

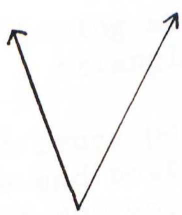
b. Are there any points which are on both $\overline{ST}$ and $\angle XYW$?

____

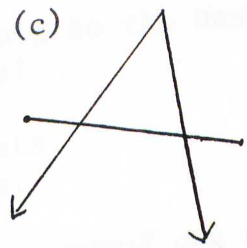
5. Look at the pictures below and check those pictures where the part colored in red is picturing an angle.

(a)

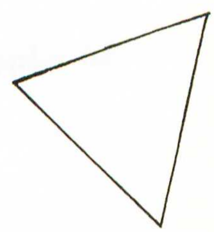
(b)



(c)



(d)



APPENDIX B

THE PRE- AND POST-TEST

Name____________________

Teacher____________________

Place a circle around the letter of the best answer for each of the following. Read each question carefully. Circle just one answer for each question.

1. A man places 3 fence posts in the ground and builds a fence between each pair of posts. He then digs up all of the ground inside the fence.

   Which of the following would be the best example of what we mean by a triangle?

   a. the three fence posts.
   b. the fence and posts.
   c. the ground dug up.
   d. the fence, posts and ground dug up.

2. Which of the following might be the best example of something which makes us think of a ray?

   a. a long straight road across a desert.
   b. a yard stick.
   c. the light beam coming from a flashlight.
   d. a wall in your room.

3. For a tug-a-war two knots are tied in a long piece of rope. The two teams are told to hold the rope so that no one is holding a knot or the part of the rope between the two knots.

We could think of that part of the rope which no one can hold as being an example of a

a. segment.
b. ray.
c. line.
d. point.

4. This sheet of paper is shaped like a "rectangle." Which of the following parts of the paper would be the best example of a rectangle?

a. the four corners of the paper.
b. the four edges of the paper.
c. the whole sheet of paper.

5. Two airplanes take off from the Wilmington Airport and one flies straight towards Philadelphia while the other plane flies straight toward New York City.

Which of the following would be the best example of something which would make us think of an angle?

a. the sky between the paths of the two planes.
b. the airport at Wilmington.
c. the airport at Wilmington together with the paths of the two planes.
d. the airports at Wilmington, Philadelphia and New York City.

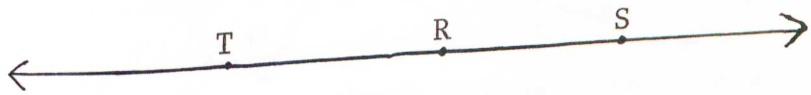
6. A triangle is made up of

a. three angles and three line segments.
b. three points.
c. three points not on the same line and the line segments between the points.
d. three line segments and all of the points "inside" the three segments.

7. An angle is made up of

a. two rays with a common endpoint.
b. the point where two rays meet.
c. all of the points between two rays.
d. two line segments with a common endpoint.

8. The point S pictured on the line below



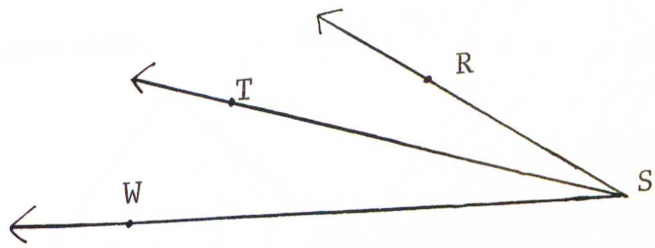
a. is on ray $\overrightarrow{TR}$.

b. is on segment $\overline{TR}$.

c. is not on ray $\overrightarrow{ST}$.

d. is not on ray $\overrightarrow{TS}$.

9.



Angle RST and angle RSW have points in common. All of these common points could be called

a. segment $\overline{RS}$.

b. angle RST.

c. ray $\overrightarrow{SR}$.

d. point S.

10.

The points which are on both segments $\overline{AC}$ and $\overline{BD}$ could be called

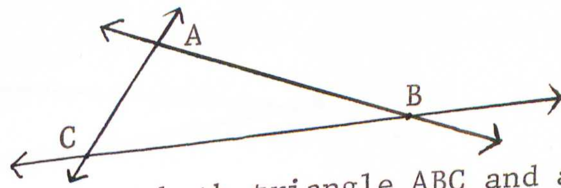
a. segment $\overline{BC}$.

b. line BC.

c. points B and C only.

d. There are no points on both $\overline{AC}$ and $\overline{BD}$.

11.



The points which are on both triangle ABC and angle ABC could be named

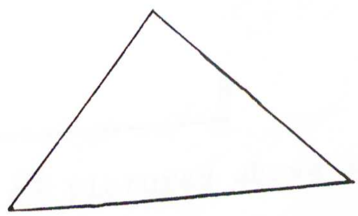
a. angle ABC.

b. segments $\overline{AB}$ and $\overline{BC}$.

c. points A, B and C only.

d. triangle ABC.

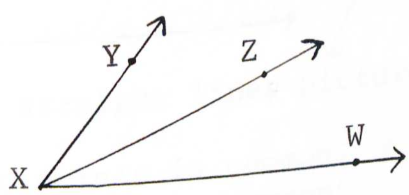
12.



How many angles have all of their points in the triangle pictured above?

a. six.
b. three.
c. more than we can count.
d. none.

13.



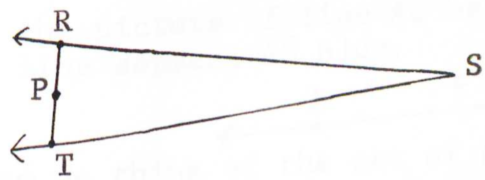
Angle YXZ and angle WXZ have points in common. All of these common points could be called

a. segment $\overline{XZ}$.

b. angle YXW.

c. ray $\overrightarrow{XZ}$.

d. point X.

14. 

Point P in the picture above is

a. on angle RST.

b. on $\overline{RS}$.

c. on triangle RST.

d. on triangle RST and angle RST.

15. 

The segment $\overline{AB}$ pictured above is made up of

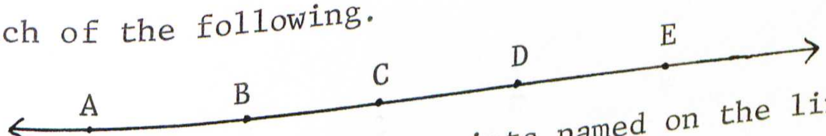
a. two points.
b. ten points.
c. more points than we can count.
d. no points.

16. 

The two straight lines pictured above have

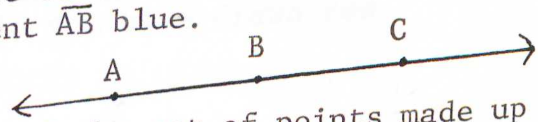
a. no points in common.
b. one point in common.
c. more than one point in common.
d. it is impossible to tell how many points in common.

Answer each of the following.

17. 

Write the letters of <u>three</u> points named on the line pictured above which are on ray $\overrightarrow{BC}$. _____, _____, and _____.

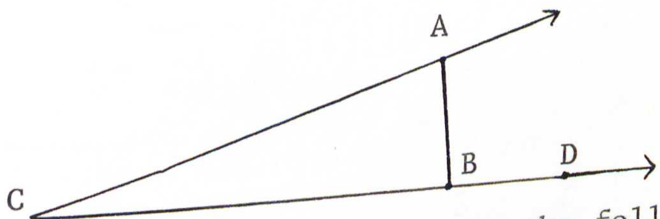
18. Look at the picture of line AC below. Color ray $\overrightarrow{BC}$ red and line segment $\overline{AB}$ blue.



Suppose we think of the set of points made up of the points on ray $\overrightarrow{BC}$ and segment $\overline{AB}$ together. What could we name this? ________.

19. Mark with dots and name four points with the letters A, B, C and D so that all of the points on segment $\overline{CD}$ are also on segment $\overline{AB}$.

20.



Look at the picture above and answer the following.

a. Are all of the points of angle ACB on triangle ACB? ________.

b. Is point D a point of angle ACB? ________.

c. Is point D a point of triangle ACB? ________.

21. Draw a picture of a triangle with your regular pencil. Now color the triangle you have drawn red.

22. Draw a picture of an angle with your regular pencil. Now color the angle you have drawn red.

SELECTED BIBLIOGRAPHY

American Educational Research Association, The. Encyclopedia of Educational Research. Edited by Chester W. Harris. New York: The Macmillan Co., 1960.

________. Handbook of Research on Teaching. Edited by N. L. Gage. Chicago: Rand McNally Co., 1963.

Barr, A. S., et al. Wisconsin Studies of the Measurement and Prediction of Teacher Effectiveness: A Summary of Investigations. Madison, Wisconsin: Dembar Publications, Inc., 1961.

Bassham, Harrell C. "Relationship of Pupil Gain in Arithmetic Achievement to Certain Teacher Characteristics." Unpublished Ph.D. dissertation, University of Nebraska, 1961.

Carroll, Edward M. "Competencies in Mathematics of Certain Prospective Elementary School Teachers." Dissertation Abstracts, XXV (March, 1965), 5134.

Corle, Clyde. "Estimates of Quantity by Elementary Teachers and College Juniors." The Arithmetic Teacher, X (October, 1963), 347-353.

Creswell, John L. "An Analysis of the Relationship of Selected Factors to Mathematics and Arithmetic Competency of Prospective Elementary Teachers in Georgia." Unpublished Ph.D. dissertation, University of Georgia, 1964.

________. "The Competence in Arithmetic of Prospective Georgia Elementary Teachers." The Arithmetic Teacher, XI (April, 1964), 248-250.

Dale, Edgar, and Chall, Jeanne S. "A Formula for Predicting Readability." Educational Research Bulletin, XXVII (January, 1948), 11-20.

Dixon, Wilfrid J., and Massey, Frank J., Jr. Introduction to Statistical Analysis. New York: McGraw-Hill Book Co., 1957.

Domas, S. J., and Teideman, D. V. "Teacher Competence: An Annotated Bibliography." Journal of Experimental Education, XIX (December, 1960), 101-218.

Dutton, Wilbur H., and Cheney, Augustine. "Pre-service Education of Elementary School Teachers in Arithmetic." The Arithmetic Teacher, XI (March, 1964), 192-198.

Ediger, Marlow. "The Student Teacher and Pupil Achievement in Elementary School Arithmetic." School Science and Mathematics, LXV (November, 1965), 697-700.

Fay, Leo C. "The Relationship Between Specific Reading Skills and Selected Areas of Sixth Grade Achievement." The Journal of Educational Research, XLIII (March, 1950), 541-547.

Hammond, Harry R. "Developing Teacher Understanding of Arithmetic Concepts Through In-service Education." Unpublished Ph.D. dissertation, University of California, 1965.

Heddens, James W., and Smith, Kenneth J. "The Readability of Elementary Mathematics Books." The Arithmetic Teacher, XI (November, 1964), 466-468.

Houston, Robert W., and DeVault, M. Vere. "Mathematics In-service Education: Teacher Growth Increases Pupil Growth." The Arithmetic Teacher, X (May, 1963), 243-247.

Mathematical Association of America. Recommendations for the Training of Teachers of Mathematics: A Summary. Berkeley, California: Mathematical Association of America, 1961.

Melson, Ruth. "How Well are Colleges Preparing Teachers for Modern Mathematics?" The Arithmetic Teacher, XII (January, 1965), 51-53.

Moore, Robert E. "The Mathematical Understanding of the Elementary School Teacher as Related to Pupil Achievement in Intermediate-Grade Arithmetic." Unpublished Ph.D. dissertation, Stanford University, 1965.

Morh, Gordon M. "Effects of an Inservice Teaching Training Program on Pupil Outcomes in Fifth and Sixth Grade Science." Unpublished Ph.D. dissertation, University of Minnesota, 1953.

Morsh, Joseph; Burgess, George; and Smith, Paul. "Student Achievement as a Measure of Instructor Effectiveness." The Journal of Educational Psychology, XLVII (January, 1956), 79-88.

National Council of Teachers of Mathematics. Mathematics for Elementary School Teachers. Washington, D. C.: National Council of Teachers of Mathematics, 1966.

________. Research in Mathematics Education. Washington, D. C.: National Council of Teachers of Mathematics, 1967.

Rutland, Leon, and Hosier, May. "Some Basic Geometric Ideas for the Elementary Teacher." The Arithmetic Teacher, VIII (November, 1961), 357-362.

Ryans, David G. "The Investigation of Teacher Characteristics." The Educational Record, XXXIV (October, 1953), 371-396.

Shim, Chung-Phing. "A Study of the Cumulative Effect of Four Teacher Characteristics on the Achievement of Elementary School Pupils." The Journal of Educational Research, LIX (September, 1965), 83-90.

Sparks, Jack N. "Arithmetic Understanding Needed by Elementary School Teachers." The Arithmetic Teacher, XIII (December, 1961), 395-403.

Smith, B. Othanel. "A Concept of Teaching." Teachers College Record, LXI (February, 1960), 229-241.

Smith, Kenneth J., and Heddens, James W. "The Readability of Experimental Mathematics Materials." The Arithmetic Teacher, XI (October, 1964), 391-394.

Smith, Wallace R. "The Achievement of Eighth-Grade Students in Arithmetic with Respect to Selected Patterns of Teacher Preparation." Dissertation Abstracts, XXV (January, 1965), 3947.

Tate, Merle W. Statistics in Education and Psychology. New York: The Macmillan Co., 1965.

Taylor, Howard R. "Teacher Influence on Class Achievement: A Study of the Relationship of Estimated Teaching Ability to Pupil Achievement in Reading and Arithmetic." Genetic Psychology Monographs, VII (February, 1930), 81-175.

Weaver, J. F. "Nonmetric Geometry and the Mathematical Preparation of Elementary School Teachers." The American Mathematical Monthly, LXXIII (December, 1966), 1115-1121.

Winer, B. J. Statistical Principles in Experimental Design. New York: McGraw-Hill Book Co., 1962.