This dissertation studies the link between global commodity price cycles, firm reallocation, and productivity dynamics. In Chapter 1, I document how commodity price fluctuations trigger a reallocation process that endogenously generates a decline in manufacturing productivity. I build a model in which firms with heterogeneous productivity decide between two technologies with different capital intensities and choose whether to become exporters. During a commodity boom, exporters lose market share due to exchange rate appreciation. Moreover, a commodity boom increases the relative cost of capital, which is used intensively in resource production, leading to additional reallocation from more capital intensive to less capital-intensive manufacturing firms. I calibrate the model to the Chilean economy and show that it can match the relevant micro and macro moments. When fed with a realistic commodity price cycle, the baseline model generates about half of the productivity decline observed in the data, a figure that is three times larger than in a counterfactual economy with no technology decision.
In Chapter 2, I study quantitatively the role of financial frictions in the U.S. business cycle. I augment an otherwise standard real business cycle model with financial intermediaries that face an occasionally binding leverage constraint. I show that the baseline model with a micro-founded friction is equivalent to a prototype economy with an exogenous intertemporal wedge. Consistent with previous literature, the results suggest that financial frictions that manifest as intertemporal wedges are relatively unimportant to understand the U.S. business cycles over the last five decades. More surprisingly, the irrelevance of intertemporal investment wedges is robust to (a) the extension to the financial crisis, (b) the introduction of a nonlinear framework able to switch between “tranquil times” and “financial crises”, (c) the solution and filtering of structural shocks using nonlinear techniques, and (d) the introduction of spread data to inform the model about the severity of the friction.
ESSAYS ON FIRM REALLOCATION
PRODUCTIVITY AND ECONOMIC GROWTH

by

Rodrigo Heresi

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Dedication

To my family
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Chapter 1: Efficient Reallocation and Productivity during Commodity Price Cycles

1.1 Introduction

This chapter revisits the link between commodity price cycles, sectoral allocations and measured productivity in resource-rich countries. Traditional narratives on the so-called Dutch disease emphasize how commodity price booms reallocate resources away from tradable sectors into both domestic services and the booming natural resource sector (see Corden and Neary 1982 [33]). This literature typically argues that such reallocation is inefficient and reduces long-run growth due to forgone productivity spillover externalities concentrated in the (non-commodity) tradable/manufacturing sector of the economy (e.g. Krugman 1987 [75], Alberola and Benigno 2017 [4], Alcott and Keniston 2018 [5]). In this dissertation, I propose an alternative framework with heterogeneous firms, that even in the absence of market failures, delivers an endogenous decline in measured productivity. The key intuition of the chapter is that commodity price booms are associated with a firm composition effect in which relatively productive firms lose market share against relatively unproductive firms, thereby rationalizing the decline in productivity typ-
ically observed during these episodes.

Why relatively productive firms lose market share during a commodity price boom? Two market-based mechanisms are key to understand this behavior. On the one hand, as the commodity producing economy becomes richer during the boom, domestic absorption increases and the real exchange rate appreciates. Within manufacturing industries, exporter firms lose market share vis à vis nonexporters, as real exchange rate appreciation reduces their competitiveness abroad and hurts their revenues from export sales. Because in the data exporters are significantly more productive than nonexporters, this exchange rate channel induce a composition effect consistent with a decline in the average efficiency of operating firms.

On the other hand, resource booms in resource-rich economies are often associated with upward pressure on input prices, as they seek to scale up aggregate supply and demand. Because commodity production uses physical capital intensively, resource booms are associated with an increase in the relative cost of capital, thereby imposing a cost disadvantage to capital-intensive firms within manufacturing sectors. Because in the data capital-intensive firms are on average more productive than labor-intensive firms, the cost of capital channel interacts with the exchange rate channel to reinforce the overall decline in average productivity.

The extent of firm-level heterogeneity in capital intensities and export intensities determines the relative importance of each channel. If all plants produce with the same capital intensity, within-firm substitution plays no role as everyone faces the same increase in their unit costs. Likewise, if all firms were equally export-intensive, exchange rate dynamics would affect all plants symmetrically and there
is no room for Dutch disease-like reallocation dynamics within sectors, in which exporters shrink relative to non-exporters. Using manufacturing firm-level data for Chile, the largest copper producer in the world, I document large within-sector variation of capital intensity in the cross-section of plants, suggesting that heterogeneous technologies with different exposures to changes in the cost of capital coexist even within narrowly-defined manufacturing industries. Moreover, only about 22% of manufacturing firms engage in exporting activities, thereby being exposed to the exchange rate channel.\(^1\)

Motivated by this evidence, I study the differential effects of commodity price booms on the relative performance of exporters versus non-exporters and more capital-intensive versus less capital-intensive firms, within Chilean manufacturing industries during the period 1995-2013. The sample period analyzed includes the commodity price super-cycle that started around 2003, which provides a unique quasi-natural experiment to test the predictions of the theory proposed in this article. I find, first, that pre-boom exporters and capital-intensive firms exhibit shrinking profits relative to their non-exporting and labor-intensive counterparts during the boom period 2003-2013. Second, I document a “missing generation of exporters”, as firms’ probability of continuing to export declines significantly during the boom. Third, firms with relatively high capital-labor ratios in the pre-boom period downsize their capital intensities significantly during the boom. Overall, as relatively

\(^1\) Chilean copper mine production accounts for 27% of worldwide production in 2017 (Cochilco, 2018). Considering pre-boom averages, the country’s mining sector accounts for roughly 10% of GDP, 50% of total exports, 21% of the economy-wide stock of physical capital, and less than 5% of the labor force.
productive exporters and capital-intensive firms shrink, exit from exporting activities and downsize their reliance on capital, the weighted average productivity of the pool of operating firms decline.

To formalize my empirical findings and quantify the relevance of the proposed channels in determining allocations and average productivity, I build a two-sector (commodity and exportable) model of a small and financially open commodity-exporting economy. The commodity sector is modeled as a representative firm that combines labor and capital to produce commodity output. The main actors in this economy are a continuum of firms with heterogeneous productivity aiming to represent the exportable/manufacturing sector. To deal with the differential effects that commodity shocks have on profits of exporters vs non-exporters (within manufacturing industries), I borrow the framework introduced by Melitz (2003) [5], in which firms trade off a fixed exporting cost against the possibility of serving the foreign market. In turn, to deal with the significant cross-sectional heterogeneity and time variation observed in capital intensities across firms within manufacturing industries, I introduce a technology choice that allows firms to adjust their capital intensity in response to changes in relative input prices (along the lines of Bustos (2011) [20], Arayachevit, Saffie and Shin (ASS14) [5], and Limão and Xu (2018) [77]). When choosing their technology, firms trade off larger fixed costs against a reduction in their variable costs (or equivalently, a productivity boost).

As is well known, this type of framework leads to self-selection, in the sense that only the most productive firm types find it profitable to pay the exporting and adoption fixed costs. Intuitively, the profitability of becoming an exporter and/or
adopting the capital-intensive technology is increasing in the firm’s productivity type, while the costs of those choices are fixed and type-independent. This ensures there are always threshold productivity levels above which exporting and upgrading technology are worthwhile for the most productive firms in the economy.

The model is calibrated to reproduce selected key macro and micro-level features of the Chilean economy, and is used to study the economy’s dynamic response to a realistic commodity price cycle. In particular, to calibrate the parameters related to the exporting and capital intensity choices, I use the observed cross-sectional variation in export and capital intensities across firms within (3-digit) manufacturing industries.

When fed with an exogenously-given commodity price boom-bust cycle, the calibrated model generates reallocation dynamics reminiscent of traditional Dutch disease narratives, but in a context in which reallocation is efficient. First, the resource sector crowds out labor and especially capital from manufacturing, consistent with the fact that mining production in Chile is substantially more capital-intensive than the typical manufacturing industry. Second, within the manufacturing sector, reallocation is shaped by firms’ initial export and capital intensities. More specifically, using a model-simulated panel of firms, I show that exporters contract significantly relative to non-exporters during the boom, while the profits of capital-intensive firms fall disproportionately, findings consistent with the microdata. Third, entry/exit and upgrade/downgrade dynamics induce a composition effect that explains about half of the decline in measured manufacturing productivity between the pre-boom period 1995-2002 and the so called super-cycle of 2003-2013. Fourth,
the amplification effect generated by the cost of capital channel via the technology decision is quantitatively relevant. I find that the baseline model generates a productivity decline two times larger relative to a counterfactual economy with no capital intensity decision.

**Related literature.** There are two closely related articles studying Dutch disease-like reallocation dynamics using micro data to uncover the transmission channels from commodity booms to the macroeconomy. First, Benguria, Saffie, and Urzua (2018) [11] exploit Brazilian regional variation in exposure to commodity price shocks and administrative firm-level data to disentangle similar channels as the ones studied here. While their emphasis is on labor markets and the role of changes in the skill premium in shaping sectoral reallocation, I focus on substitution between labor and capital. These are natural choices as commodity production in Brazil (mainly agriculture) is unskilled labor-intensive, while mining production in Chile is capital-intensive. More importantly, by introducing a technology choice, I allow for an additional margin of adjustment that takes place within establishments, as I study how firms react to input price fluctuations by adjusting their optimal mix of labor and capital.

Second, Alcott and Keniston (2018) [5] combine U.S. data on oil endowments at the county level with Census of Manufactures to estimate how oil booms affect local manufacturing firms. They find that manufacturing as a whole is not crowded out during oil booms, because negative effects on some tradables firms are offset by positive effects on upstream and locally-traded subsectors. By studying reallocation within the U.S. economy and focusing only on labor input, they abstract from the
two key mechanisms emphasized in the present chapter. On the one hand, they propose a “within-country” model of Dutch disease reallocation, thereby eliminating the differential effects of exchange rate fluctuations on the relative performance of exporters versus non-exporters. Interestingly, they do find that “tradable” manufacturing firms (those that sell outside the limits of their own county) do contract during resource booms. Their intuition for “tradable” U.S. firms has the same flavor as my results for exporters: they suffer from higher wages but do not benefit that much from the oil boom and its associated increase in local demand. On the other hand, they abstract from the capital input in the production function, which I argue is a key element to consider given the high capital intensity of oil- and metal-related extraction and production processes.

The technology choice set up introduced in this chapter blends elements from Bustos (2011) [20], Arayavechkit, Saffie, and Shin (2014) [7], and Limaõ and Xu (2018) [77]. Bustos (2011) [20] considers a single-input production function, in which firms can choose to reduce their marginal cost of production by paying a fixed cost. In my case, firms decide the capital share in a constant returns to scale Cobb-Douglas production function that combines labor and capital, as in Arayavechkit et al (2014) [7]. I discipline the fixed cost of adoption, its associated cost advantage, and the capital shares using the empirically observed differences in productivity between capital- versus labor-intensive manufacturing firms. In addition, I study the technology adoption margin in a quantitative dynamic model, and in the context of a small and open resource-dependent economy subject to persistent global cycles in commodity prices.
By studying the effects of resource booms on sectoral allocations and productivity, this article is tightly linked to the long-standing literature about the Dutch disease or “Resource Curse” (see Corden and Neary (1982) [33], Krugman (1987) [75], Sach and Warner (1997) [97], van der Ploeg (2011) [105], Frankel (2012) [45], Rodrik (2013) [95]). Alberola and Benigno (2017) [4] propose a representative-firm three-sector commodity-exporter economy model to study the effects of commodity booms on long-run growth. They show theoretically that, when dynamic productivity spillovers are concentrated in the non-resource tradable sector, the commodity boom delays convergence to the world technology frontier, and may even lead to a growth trap. While I do not consider spillover effects or endogenous growth, I extend the analysis in other important dimensions. First, I emphasize reallocation at the firm-level within the manufacturing sector, which requires a framework with firm heterogeneity and an explicit distinction between exporters and non-exporters. Second, given the importance of relative input intensities in shaping reallocation, I allow for labor and capital in the production function, and discipline their shares directly using firm-level data. I am not aware of other articles studying the capital intensity dimension in shaping reallocation dynamics during a commodity boom. Finally, this dissertation also contributes to the literature by providing an alternative explanation for persistent downturns in measured productivity during commodity price booms without relying on inefficient reallocation due to reduced-form frictions or ad-hoc spillover effects.

This chapter is also linked to the literature studying the effects of terms of trade shocks in the macroeconomy (see Mendoza (1995) [83], Kose (2002) [74], and Vegh
Motivated by the recent commodity super-cycle, several articles have focused specifically on the effects of commodity price shocks in emerging economies (Schmitt-Grohe and Uribe (2015) [98], Shousha (2016) [100], Fernandez, Schmitt-Grohe and Uribe (2017) [41]). Unlike these articles, which focus on the effects of global price fluctuations at business cycle frequencies, this dissertation seeks to understand the low-frequency dynamic effects that persistent commodity cycles have on resource allocation and productivity. This is a relevant distinction because, as emphasized by Erten and Ocampo (2013) [38] and Reinhart et al (2016) [91], commodity prices are characterized by much longer cycles (of around thirty years) than standard business cycle fluctuations, which puts significant pressure on sectoral allocations in commodity-dependent countries.

The remainder of this chapter is organized as follows. Section 1.2 presents sector-level and firm-level empirical regularities observed before and after the commodity boom that started in 2003. Section 1.3 describes the quantitative model designed to disentangle the transmission channels from commodity cycles to manufacturing productivity. Section 1.4 tests the ability of the model to replicate the empirical facts, and studies whether the model’s transitional dynamics can reproduce the most recent commodity super-cycle, and its effects on factor allocations and measured productivity. Section 1.5 concludes.
1.2 Empirical Analysis

1.2.1 Commodity Cycles and the Macroeconomy

Figure 1.1 illustrates the relationship between the recent commodity price "super-cycle", sectoral allocations and measured productivity, using aggregate data for Chile. Panel (a) shows the time paths of the real price of copper -by far the country’s main produced and exported commodity- and the country’s manufacturing share in total output. The crowding-out effect of commodity prices on manufacturing is especially marked during the persistent boom that started in 2003.

Panel (b), in turn, illustrates the relationship between aggregate TFP and the real exchange rate. During the nineties, high productivity growth led to currency appreciation, as predicted by the Balassa-Samuelson hypothesis. However, the relationship breaks down during the commodity boom period (2003-2016), when protracted exchange rate appreciation (30% between 2003 and 2017) coexisted with a medium-run slowdown in aggregate productivity growth (0% between 2003 and 2017).

How can persistent commodity booms generate productivity slowdowns in resource-rich economies? Several channels may be at play. First, the positive wealth effect raises consumption of all types of goods, which all else equal benefits domestic sales relative to export sales. Second, larger local demand induces exchange rate appreciation, which disproportionately affects exporters relative to non-exporter firms. Overall, firms face a double incentive to switch productive re-
Figure 1.1: Commodity Price Boom and Sectoral Allocations.

Notes: Author’s calculations based on data from the Central Bank of Chile. The gray area indicates the commodity boom period 2003-2017. Panel (a) reports the nominal manufacturing share in total nominal output, while the real commodity price is PPI-deflated. Panel (b) reports economy-wide measured TFP and the real exchange rate (RER). Panels (c) and (d) report real investment and real output by sector.
sources towards the domestic market as well as the booming resource-based sector. Panel (c) of Figure 1.1 illustrates this pattern. New investment flows during the commodity boom were mostly directed to the resource sector and domestic services, at the expense of the prototypical (non-commodity) tradable sector, namely manufacturing. Panel (d) confirms that while domestic services boomed and led output growth during 2003-2017, the manufacturing sector tended to lag behind. It is also noteworthy that, despite the large mining investment boom, real commodity production stayed flat during this period, partly as a consequence of a significant decrease in the quality (ore grade) of the natural resource being mined. See Appendix A.1 for a cross-country documentation of the fall in mining productivity in the period under analysis.

In this dissertation, I argue that the “between sector” reallocation dynamics illustrated in Figure 1.1 are just part of the story. There are also pervasive reallocation dynamics that take place within the manufacturing sector. On the one hand, currency appreciation shrinks exporters’ revenue, while non-exporters or “purely-domestic” firms enjoy booming local demand. I use microdata on export sales versus total sales to directly measure the exposure to exchange rate risk at the firm-year level. I argue that distinguishing between exporters and non-exporters is quantitatively important for two reasons. First, only 22% of manufacturing firms, on average, actually sell their varieties abroad, thereby being vulnerable to exchange rate risk.²

² Due to a lack of suitable data, traditional studies have focused on “tradable sectors”, relying on the strong assumption that all “tradable” producers operate in foreign markets.
Second, exporters overwhelmingly outperform non-exporters in several outcome variables such as value added, revenue productivity and capital intensity (Bernard and Jensen, 1999 [15]; De Loecker and Warzynski, 2012 [35]). More importantly for the purpose of this dissertation, I show below (both in the data and in the model simulations) that it is precisely exporters who shrink more and eventually exit from export activities during the protracted commodity price cycle illustrated in Panel (a) Figure 1.1.

On the other hand, resource booms raise the marginal products (and hence the cost) of mobile productive resources, particularly for inputs used intensively in commodity extraction and production. As emphasized by Corden and Neary (1982) [33], if the commodity sector uses relatively few resources that can be drawn from elsewhere in the economy, the crowding out or “resource movement” effect is negligible. However, commodity (mining) production in Chile uses physical capital disproportionately: while the mining sector represents about 10% of aggregate output, it uses 21% of the economy-wide capital and less than 5% of the labor force. The relative scarcity of capital induces a cost disadvantage to capital-intensive manufacturing firms. I exploit firm-level variation in capital-labor ratios within manufacturing industries in order to infer their exposure to the “cost of capital channel”.

1.2.2 Data

The data used in the present chapter comes from the Encuesta Nacional Industrial Anual (ENIA) (Annual National Industrial Survey) conducted by the Instituto
Nacional de Estadística (INE), the Chilean government statistical agency. The survey contains yearly information on establishments with more than ten employees in the period 1995-2013.\(^3\) It includes 5,000 observations per year and provides information on establishments’ characteristics such as industry, value added, domestic sales, exports sales, employment, intermediates spending, and the value of the capital stock.

Firm-level revenue total factor productivity is estimated using the method of Wooldridge (2009) [109] and, under the assumption of constant returns to scale, using cost shares (of total costs) as in Foster, Haltiwanger, and Krizan (2001) [44].

Aggregating the micro-level data, ENIA accounts for 86\% of aggregate manufacturing value added reported by the Central Bank, and 50\% of total manufacturing labor recorded by the country’s statistical office.

1.2.3 Firm characteristics and estimated productivity

In this subsection, I document several empirical regularities that are relevant for the analysis. First, I show significant heterogeneity in capital intensity across firms within 3-digit manufacturing industries. Capital-intensive firms are bigger and more productive than their labor-intensive counterparts. Second, I show that exporters are bigger, more productive, and more capital-intensive than non-exporter firms, findings that are consistent with the literature (see Bernard and Jensen, 1999 [15]).

\(^3\) Most firms in Chile are single-establishment.
Figure 1.2: Capital Intensity Moments.
Notes: Capital intensity of firm $f$ is computed as $K_{f}^{\text{int}} = \frac{K_f/L_f}{\sum_i K_i/\sum_i L_i}$, pooling all years in the sample 1995-2013. The summation is done (3-digit) industry-wise. The vertical line in panel (a) shows the frequency of firms that are as capital-intensive as their own industry average.

**Fact 1:** There is substantial cross-sectional heterogeneity in capital intensities within manufacturing industries.

The left panel of Figure 1.2 displays the distribution of capital intensities across manufacturing firms pooling all years in the sample. Each firm’s capital intensity is computed as their capital-labor ratio relative to their own (3-digit) industry average. I define as “High-K” (“Low-K”) the firms with a capital-labor ratio above (below) their industry-level average, that is, firms to the right (left) of the vertical line in the left panel. The right panel shows that exporters are significantly more capital-intensive than non-exporters.

**Fact 2:** Exporters and capital-intensive firms outperform non-exporters and labor-intensive firms.
Figure 1.3 documents the estimated productivity (revenue TFP) distribution across firm-year pairs grouped according to exporting and capital intensity status. On average, exporters outperform non-exporters, and High-K firms outperform Low-K firms. Naturally, the very selected group of High-K exporters (6% of the sample) is substantially more productive than the remaining groups, especially relative to the most numerous group of Low-K non-exporters (74% of the sample). A similar sorting pattern holds when estimating productivity using pre-boom years only. Appendix A.2 presents panel regressions documenting systematically how exporters and capital-intensive firms display significantly higher revenue TFP relative to other groups in the economy, even after controlling by sector-year fixed-effects. The quantitative model developed in the next section is calibrated to approximately replicate the average productivity levels implied by the distributions in Figure 1.3.

1.2.4 Firm-Level Implications of a Commodity Boom

In this subsection, I present evidence that commodity booms disproportionately affect the profitability of export-oriented and capital-intensive firms.\footnote{To ease exposition, I use High-K vs Low-K and capital-intensive vs labor intensive interchangeably.}

Second, they induce a large decline in net entry rates into exporting, as well as a significant increase in the probability of exit from exporting during periods of high commodity prices. Similarly, the probability of using capital-intensive technologies also shrink during commodity price booms, suggesting that firms do react
Figure 1.3: Firm-level productivity distribution by groups. Pooling years.
Notes: Capital intensity of firm $f$ is computed as $K_{int}^f = \frac{K_f/\sum L_i}{\sum K_i/\sum L_i}$, pooling all years in the sample 1995-2013. The summation is done (3-digit) industry-wise. The vertical line in panel (a) shows the frequency of firms that are as capital-intensive as their own industry average.
to changes in relative input prices and substitute towards labor.

**Fact 3:** Exporters and capital-intensive firms lose market share during a commodity boom (intensive margin).

In order to document how firm characteristics shape the intensive margin of adjustment during commodity booms, I estimate the following specification:

$$
\ln(Y_{ft}) = \alpha X_{f0} \cdot \bar{P}^{Co}_{t-1} + \beta K_{f0}^{int} \cdot \bar{P}^{Co}_{t-1} + \gamma X_{f0} \cdot K_{f0}^{int} \cdot \hat{P}^{Co}_{t-1} + \delta' Z_{ft} + \varphi_f + \varphi_{st} + \varepsilon_{ft}(1.1)
$$

where $Y_{ft}$ denotes an outcome variable (such as real value added or real profits) for firm $f$ in year $t$, $X_{f0}$ is a dummy variable that takes the value 1 if firm $f$ exports in its first period $t = 0$ in the sample (conditional on $t = 0$ being in the pre-boom period 1995-2003), $K_{f0}^{int}$ denotes the capital intensity of firm $f$ in period $t = 0$, and $\bar{P}^{Co}_{t} = P^{Co}_{t} - \bar{P}^{Co}$ is the demeaned real commodity price shock. Finally, the vector variable $Z_{ft}$ collects firm-level controls, while $\varphi_{st}$ and $\varphi_f$ represent (3-digit) sector-year and firm fixed effects, respectively. The coefficient $\alpha$ in 1.1 measures the relative effect of commodity shocks on the subsample of exporting firms. Similarly, $\beta$ is the relative effect of commodity price fluctuations on the subsample of capital-intensive firms. Finally, $\gamma$ is the incremental relative effect of commodity price shocks on High-K exporters relative to Low-K non-exporter firms.$^5$

Table 1.1 presents the results. Columns (1) and (2) report the results for the period 1995-2007, to avoid concerns about the Great Recession being a potential

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$^5$ Note that the baseline impact of commodity price shocks on Low-K non-exporters is absorbed by the sector-year fixed effects.
confounding factor. Columns (3) and (4) report the baseline results for the full sample 1995-2013. It is clear from the table that exporters and capital-intensive firms shrink significantly during periods of high commodity prices. The double interaction is also negative (and significant for the full sample 1995-2013), suggesting that High-K exporters, the most productive firms in the economy, suffer a double hit in the form of decreasing revenues due to currency appreciation and disproportionately larger variable costs through the cost of capital channel. Overall, they face a $13\% = 100 \cdot (0.079 + 0.023 + 0.031)$ larger decrease in their real profits relative to Low-K non-exporter firms. A potential concern is the possibility that financial frictions are partly driving these patterns. Appendix A.3 presents robustness analysis which shows that my main results survive even after controlling by that channel using firm-level size measures interacted with the commodity price shock.

**Fact 4:** Exporters and capital-intensive firms are more likely to exit from foreign markets and downsize their capital-labor ratios during a commodity boom (extensive margin).

This subsection documents the extensive margin of adjustment. More specifically, when the commodity shock is persistent enough, the protracted real appreciation of the exchange rate induces some pre-boom exporters to exit from foreign markets. Similarly, some pre-boom capital-intensive firms are not able to bear the increase in the cost of capital and are forced to downsize to less capital-intensive technologies. Figure 1.4 illustrates these patterns. Panel (a) presents net entry
<table>
<thead>
<tr>
<th></th>
<th>VA Profits</th>
<th>VA Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{f0} \cdot \bar{P}_{t-1}$</td>
<td>-0.122*** (0.0331)</td>
<td>-0.092*** (0.0291)</td>
</tr>
<tr>
<td>$K_{f0}^{int} \cdot \bar{P}_{t-1}$</td>
<td>-0.015* (0.0077)</td>
<td>-0.021*** (0.0073)</td>
</tr>
<tr>
<td>$X_{f0} \cdot K_{f0}^{int} \cdot \bar{P}_{t-1}$</td>
<td>-0.017 (0.0180)</td>
<td>-0.032** (0.0157)</td>
</tr>
<tr>
<td>Firm FE</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Sector×Year FE</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.139</td>
<td>0.176</td>
</tr>
<tr>
<td>N. obs.</td>
<td>49,178</td>
<td>59,945</td>
</tr>
</tbody>
</table>

Table 1.1: Panel Regressions: Commodity Booms and Outcome Variables.

Notes: Results for regression 1.1. ***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.1$. Columns (1) and (2) present results for the sample 1995-2007, while columns (3) and (4) for 1995-2013. Columns (1) and (3) use real value added as dependent variable, while columns (2) and (4) use real profits. All specifications include controls for firm size.

rates into foreign markets, while Panel (b) displays analogous net entry rates into the capital-intensive technology as defined in Figure 1.2. Panel (a) shows a protracted decline in net entry into foreign markets that coincides with the commodity price super-cycle period. Overall, it is noteworthy the positive correlation between the exchange rate appreciation induced by the commodity boom and the plummeting of net entry rates, as predicted by the exchange rate channel. In turn, Panel (b) illustrates how manufacturing firms massively switch away from physical capital during the period in which the real commodity price skyrocketed.

To document systematically the effects of commodity booms on firms’ decisions to exit from exporting and downsize their capital-labor ratios, I follow the literature
Figure 1.4: Net Entry Rates.
Notes: Net entry rates are defined as the difference between entry rates and exit rates. Panel (a): Entry into the foreign market is defined as the number of firms exporting in year $t$ that did not export in year $t - 1$ divided by the total number of firms that export in year $t - 1$. The exit rate is defined as the number of firms that export in year $t$ but do not export in year $t + 1$ relative to the number of firms exporting in year $t$. Panel (b): The entry rate into the capital-intensive technology is defined as the number of firms with $K_{ft}^{int} > 1$ and $K_{ft-1}^{int} < 1$ divided by the total number firms with $K_{ft-1}^{int} > 1$. The exit rate from the Capital-Intensive technology is defined as the number of firms with $K_{ft}^{int} > 1$ and $K_{ft+1}^{int} < 1$ relative to the number of firms with $K_{ft}^{int} > 1$. 

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and specify a dynamic linear probability model. I estimate:

$$Y_{ft} = \alpha_1 Y_{ft-1} + \alpha_2 Y_{ft-2} + \beta_1 Y_{ft-1} \cdot Z_t + \beta_2 Y_{ft-2} \cdot Z_t + \varphi_{st} + \varphi_f + \varepsilon_{ft}$$ (1.2)

where $Y_{ft}$ can take the form of an export dummy $Y_{ft} = X_{ft} = 1$ if firm $f$ exports in year $t$ or a capital intensity dummy $Y_{ft} = K_{ft} = 1$ if firm $f$ classify as High-K in year $t$ (according to the definition in Figure 1.2), $Z_t$ is a commodity cycle measure, and $\varphi_{st}$ and $\varphi_f$ are sector-year and firm fixed effects. I use two alternative measures for the commodity cycle. First, I use a “boom” dummy variable that takes the value $Z_t = 1$ in years 2004-2013 and zero otherwise. Second, I use a continuous variable given by the demeaned real commodity price $Z_t \equiv \bar{P}_t^{Co} = P_t^{Co} - \bar{P}^{Co}$. The regression also includes controls for firm-level size and productivity (not shown in equation 1.2). The lagged dependent variable is included as fixed costs induce state-dependence in the exporting and capital-intensity decisions. I interact lags of the dependent variable with the commodity cycle measure in order to understand to what extent the probabilities of continuing to export and using the capital-intensive technology are affected by commodity price fluctuations. I introduce two lags in order to capture the idea that the negative effects of persistent commodity booms take some time to build up.

Table 1.2 reports the results. The coefficient on $Y_{ft-j}$ is the marginal increase in the probability of exporting in period $t$ if firm $f$ exported in $t-j$. The interaction terms are interpreted as the incremental/detrimental effect of the commodity boom on the probability of continuing to export. For instance, from column (1) we have

---

6 See Lincoln, McCallum, and Siemer (2017) [78].
\[ Y_{ft} = X_{ft} = \{0, 1\} \]
\[ Z_t = \{0, 1\} \]
\[ (1) \]
\[ Y_{ft-1} \]
\[ 0.330^{***} \]
\[ (0.0148) \]
\[ Y_{ft-2} \]
\[ 0.077^{***} \]
\[ (0.0137) \]
\[ Y_{ft-1} \cdot Z_t \]
\[ 0.023 \]
\[ (0.0181) \]
\[ Y_{ft-2} \cdot Z_t \]
\[ -0.043^{**} \]
\[ (0.0182) \]
\[ \]
\[ Y_{ft} = K_{ft} = \{0, 1\} \]
\[ Z_t = \{0, 1\} \]
\[ (3) \]
\[ 0.234^{***} \]
\[ (0.0124) \]
\[ 0.1030^{***} \]
\[ (0.0118) \]
\[ 0.0443^{***} \]
\[ (0.0142) \]
\[ -0.0757^{**} \]
\[ (0.0144) \]
\[ \]
\[ Z_t = \tilde{P}_{t}^{C_{o}} \]
\[ (4) \]
\[ 0.258^{***} \]
\[ (0.0096) \]
\[ 0.0627^{***} \]
\[ (0.0085) \]
\[ 0.0524^{**} \]
\[ (0.0141) \]
\[ -0.0805^{***} \]
\[ (0.0138) \]

| Firm FE | yes | yes | yes | yes |
| Sector×Year FE | yes | yes | yes | yes |
| Adj. $R^2$ | 0.150 | 0.150 | 0.140 | 0.140 |
| N. obs. | 49,439 | 49,439 | 49,439 | 49,439 |

Table 1.2: Panel Analysis: Dynamic Linear Probability Model.
Notes: Results for regression 1.2. ***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.1$. The dependent variable $X_{ft} = \{0, 1\}$ is a dummy equal to 1 if firm $f$ exports in year $t$. Columns (1) and (3) use a binary commodity price cycle variable, $Z_t = \{0, 1\}$, that takes the value 1 in 2004-2013 and 0 in 1995-2003. Alternatively, columns (2) and (4) use the continuous real commodity price (demeaned). All specifications include controls for firm size and revenue TFP (not reported).

that an exporter in $t - 1$ has a 30% higher probability of being an exporter in period $t$; if the firm also exported in $t - 2$, the probability increases by about 5%.

Regarding the interactions, for firms that exported last year, the commodity boom has a positive (sometimes not significant) effect on the probability of exporting today. But the negative effects are significant for firms that exported two years ago.

Moreover, across specifications, the negative effect on $t - 2$ dominates the positive effect on $t - 1$ in absolute value and significance. Similar correlations hold in columns (3) and (4) for the probability of using capital-intensive technologies.
1.3 A Trade Model with Capital Intensity Choice

Consider a small and financially-open commodity-exporting economy with three goods: exportables (X), importables (M), and commodity (Co) goods. Households only consume exportables and importables. Commodity production is sold abroad at international price $p^{Co}$, the only exogenous driving force in the model. Exportable varieties are produced by a continuum of firms with heterogeneous productivity using labor and capital, while commodity goods are produced by a representative firm, using labor, capital, and a fixed natural resource. For simplicity, investment goods are fully imported. Capital accumulation is subject to quadratic adjustment costs.

1.3.1 Household

Time is discrete and indexed by $t$. There is an infinitely-lived representative household that maximizes lifetime utility given by:

$$U = \Sigma_{t=0}^{\infty} \beta^{t} \left( C_t - \varphi \frac{L_t}{\zeta} \right)^{1-\upsilon},$$

(1.3)

where $C$ and $L$ are consumption and labor supply, while the parameters $\beta$, $\upsilon$, $\zeta$, and $\varphi$ govern time discounting, the intertemporal elasticity of substitution, the Frisch elasticity of labor supply, and the marginal rate of substitution between consumption and leisure. The consumption bundle $C$ is defined as a CES aggregator of exportable $C^X$ and importable $C^M$ goods:

$$C_t = \left[ \chi^{\frac{1}{\varepsilon}} \left( C^X_t \right)^{\frac{\varepsilon - 1}{\varepsilon}} + \left( 1 - \chi \right)^{\frac{1}{\varepsilon}} \left( C^M_t \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}},$$

(1.4)
where \( \chi \) and \( \epsilon \) control the weights and the elasticity of substitution between goods. Exportable consumption is, in turn, a bundle over a continuum of manufacturing varieties indexed by \( \omega \):

\[ C^X_t = \left[ \int_\omega (q_{dt}(\omega))^\rho \, d\omega \right]^{\frac{1}{\rho}}, \quad (1.5) \]

where \( \sigma = 1/(1 - \rho) > 1 \) is the elasticity of substitution among varieties.

The household supplies labor, accumulates capital, smooths consumption via foreign borrowing, and owns firms. The budget constraint can be written as:

\[ p_tC_t + I_t + B_{t+1} = w_tL_t + r^K_tK_t + (1 + r^*)B_t + \Pi_t, \quad (1.6) \]

where \( p \) is the price of the consumption bundle, which is also a model-based proxy for the real exchange rate (RER); \( B \) is the country’s net foreign asset position that pays exogenous interest rate \( r^* \), \( w \) is the wage, \( I \) and \( K \) are investment and capital with rental rate \( r^K \), and \( \Pi = \Pi^X + \Pi^{Co} \) collects profits from the ownership of firms in both sectors. Investment goods are fully imported at price \( p^M_t = 1 \) (the numeraire).

The aggregate stock of capital evolves according to:

\[ K_{t+1} = (1 - \delta^K)K_t + I_t - \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 K_t. \quad (1.7) \]

where \( \delta^K \) is the depreciation rate and \( \phi \) governs the capital adjustment cost. The price of the exportable bundle consumed domestically is given by:

\[ p^X_t = \left[ \int_\omega (p_{dt}(\omega))^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}}. \quad (1.8) \]

The household’s cost minimization determines the following demands for each composite good:
\[ C_t^X = \chi \left( \frac{p_t}{p_X^t} \right)^\epsilon C_t \tag{1.9} \]
\[ C_t^M = (1 - \chi) \left( \frac{p_t}{p_M^t} \right)^\epsilon C_t \tag{1.10} \]

Note that plugging demands 1.9-1.10 in 1.4 yields an expression for the domestic basket price or real exchange rate (RER):

\[ p_t = \left[ \chi \left( \frac{p_t^X}{p_t^X} \right)^{1-\epsilon} + (1 - \chi) \right]^\frac{1}{1-\epsilon} . \]

The domestic demand for each variety in the exportable sector is given by:

\[ q_{dt}(\omega) = \left[ \frac{p_{dt}(\omega)}{p_t^X} \right]^{-\sigma} C_t^X . \tag{1.11} \]

Household’s optimal behavior is characterized by demands (1.9)-(1.11), the flow budget constraint (1.6) (with Lagrange multiplier \( \beta^t \lambda_t \)), and the following optimality conditions:

\[ \frac{1}{1 + r^*} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \left[ \frac{C_t - \varphi \frac{L_t^e}{\zeta}}{C_t - \varphi \frac{L_{t+1}^e}{\zeta}} \right]^{\nu} \left( \frac{p_t}{p_{t+1}} \right) \tag{1.12} \]
\[ 1 + \phi \left( \frac{K_{t+1}}{K_t} - 1 \right) = \beta \frac{\lambda_{t+1}}{\lambda_t} \left[ \nu \frac{K_{t+1}}{K_t} + 1 - \delta^{k+1} + \text{adj}_{t+1} \right] \tag{1.13} \]
\[ \text{adj}_{t} \equiv \phi \left( \frac{K_{t+1}}{K_t} \right) \left( \frac{K_{t+1}}{K_t} - 1 \right) - \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 \]
\[ \varphi L_t^{e-1} = \frac{w_t}{p_t} . \tag{1.14} \]

1.3.2 Exportable Sector

This subsection augments the model of Melitz (2003) [82] with physical capital and technology choice. There is an infinite pool of forward-looking potential entrants that consider making an initial investment, modeled as a one-time sunk entry cost \( f_e \), in order to draw a permanent productivity type \( z \) from a distribution \( g(z) \) with
positive support over \((0, \infty)\) and continuous cumulative distribution \(G(z)\). After observing \(z\), firms with sufficiently low draws optimally decide to exit and never produce. In turn, successful entrants decide (i) between two constant-returns-to-scale technologies that combine labor and capital but differ in their capital share \(\alpha\), and (ii) whether to serve the foreign market and become an exporter. The market structure is monopolistic competition. To ease notation, I drop time subscripts in this subsection.

**Technology choice.** The basic technology with low capital intensity \((\alpha_l)\) entails a (per-period) fixed operational cost \(f_d\), while adopting the capital-intensive technology \((\alpha_h > \alpha_l)\) requires a larger fixed cost \(f_d + f_a\). Henceforth, I refer to these as “Low-K” and “High-K” technologies. For each \(j = \{l, h\}\), the unit cost function is given by \(c_j(z) = \frac{\phi_j}{z}\) where \(\phi_j = \left(\frac{r_k}{\alpha_j}\right)^{\alpha_j} \left(\frac{w}{1-\alpha_j}\right)^{1-\alpha_j}\) is the weighted average price of the composite input. In essence, firms trade off lower variable cost (via the distance between \(\alpha_h\) and \(\alpha_l\)) with larger fixed operation cost (via \(f_a\)).

**Exporting choice.** Firms serving only the domestic market pays fixed (per-period) cost \(f_d\) and face residual demand given by (1.11). To serve the foreign market firms have to pay an additional fixed (per-period) exporting cost \(f_x\). Foreign demand is given by \(q_x(z) = \gamma \left(p_x(z)\right)^{-\sigma}\), where \(\gamma\) controls the size of the foreign market. Firms trade off a larger market (via \(\gamma\)) with a larger fixed cost (via \(f_x\)).\(^7\)

\(^7\) I assume the same price elasticity for domestic and foreign demand.
Pricing rule. Each firm charges a constant markup \((1/\rho)\) over unit cost. Then, for any \(s = \{d, x\}\) and \(j = \{l, h\}\), we have \(p_{sj}(z) = \frac{\phi_j}{\rho z}\).

Profits. All fixed costs are valued in units of the numeraire. Depending on their productivity type, firms self-select into one of the following four groups: (a) purely domestic Low-K firms, (b) purely domestic High-K firms, (c) Low-K exporters, and (d) High-K exporters. After some manipulation, profits can be written as follows:

\[
\pi_{sj}(z) = \begin{cases} 
\frac{1}{\sigma} p^\sigma C \left[ \frac{\phi_l}{\rho z} \right]^{1-\sigma} - f_d & \text{if } s = d \text{ and } j = l \\
\frac{1}{\sigma} p^\sigma C \left[ \frac{\phi_h}{\rho z} \right]^{1-\sigma} - f_d - f_a & \text{if } s = d \text{ and } j = h \\
\frac{1}{\sigma} p^\sigma C \left[ \frac{\phi_l}{\rho z} \right]^{1-\sigma} + \frac{1}{\sigma} \gamma \left[ \frac{\phi_h}{\rho z} \right]^{1-\sigma} - f_d - f_a - f_x & \text{if } s = x \text{ and } j = l \\
\frac{1}{\sigma} p^\sigma C \left[ \frac{\phi_h}{\rho z} \right]^{1-\sigma} + \frac{1}{\sigma} \gamma \left[ \frac{\phi_h}{\rho z} \right]^{1-\sigma} - f_d - f_a - f_x & \text{if } s = x \text{ and } j = h
\end{cases}
\]

(1.15)

where I use the convention that adoption fixed costs are assigned to domestic profits. Note that, for any \(j = \{l, h\}\), the total profits of exporters are the sum of domestic and foreign profits \((\pi_{dj}(z) + \pi_{xj}(z))\).

Value functions. Regardless of their productivity type, all operating firms are subject to a constant probability \(\delta\) of a bad shock that forces them to exit the market. Firms can also exit endogenously when their present discounted value becomes negative. Type-\(z\) firm chooses the technology and exporting decisions yielding the largest present discounted value:

\[
V(z) = \max\{V_{dl}(z), V_{dh}(z), V_{xl}(z), V_{xh}(z)\},
\]

(1.16)
\[
V_{dj}(z) = \max \left\{ 0, \pi_d(z) + \frac{(1 - \delta)}{(1 + r^*)} V'(z) \right\}, \quad j = l, h
\]
\[
V_{xj}(z) = \max \left\{ 0, \pi_d(z) + \pi_x(z) + \frac{(1 - \delta)}{(1 + r^*)} V'(z) \right\}, \quad j = l, h.
\]

**Cutoffs.** This well-known environment gives rise to productivity cutoff rules that determine firms’ entry/exit into domestic \((z_d)\) and foreign markets \((z_x)\) as well as adoption of the capital-intensive technology \((z_a)\). The least productive but successful entrants \((z_d \leq z < z_x)\) serve the domestic market using the Low-K technology. Then, the marginal condition to pin down the domestic cutoff is given by:

\[
V_{dl}(z_d) = 0.
\]

(1.17)

If \(z_a < z_x\) (case 1), the marginal type that optimally chooses to upgrade technology is a purely domestic firm, while the marginal exporter uses the high-capital technology. Conversely, if \(z_x < z_a\) (case 2), the marginal exporter uses the low-capital technology, while the marginal adopter is an exporter type. As in Bustos (2011) [20] and Limão and Xu (2018) [77], I calibrate the model to be consistent with case 2, because it is closer to the data. The cutoffs for the case \(z_x < z_a\) are pinned down by:

\[
V_{dl}(z_x) = V_{xl}(z_x)
\]

(1.18)
\[
V_{xt}(z_a) = V_{zh}(z_a).
\]

**Distribution.** Let \(\mu(z)\), \(\mathcal{M}\), and \(\mathcal{M}_e\) denote the distribution of types, the mass of incumbent firms, and the mass of entrants firms in the current period. The
distribution of types evolves as follows:

\begin{equation}
\mathcal{M}'(z) = \begin{cases} 
(1 - \delta)\mathcal{M}(z) + \mathcal{M}'(z) & \text{if } z \geq z' \\
0 & \text{otherwise}
\end{cases}
\end{equation}

Equation 1.19 tells us that the number of firms of each type tomorrow equals the number of firms that survive both exogenous exit ($\delta$) and endogenous exit ($z \geq z_d$) today plus the number of new entrants of each type. By the law of large numbers, the latter is simply given by the unconditional distribution $g(z)$. Note also that successful entrants are allowed to produce immediately upon entry. Finally, integrating over all active types, the law of motion for the mass of producers can be written as:

\begin{equation}
\mathcal{M}' = (1 - \delta)\mathcal{M} \int_{z_d}^{z'} \mu(z)dz + \mathcal{M}' \int_{z_d}^{z'} g(z)dz.
\end{equation}

**Free entry.** Sunk entry costs are valued in terms of the numeraire. They combine a fixed with a convex component that captures congestion effects in firm creation, help to match the empirical entry rate, and are computationally convenient. The assumed functional form is:

\begin{equation}
f_e (\mathcal{M}_e) = \overline{f}_e + \phi_e \left[ \exp \left( \mathcal{M}_e - \overline{\mathcal{M}}_e \right) - 1 \right]
\end{equation}

where $\overline{f}_e$ is the fixed component, $\mathcal{M}_e$ is the mass of entrants (with steady state value $\overline{\mathcal{M}}_e$) and $\phi_e$ controls the degree of congestion effects. Because firms learn $z$ after paying the sunk entry cost, prospective entrants consider the expected present value of entering net of entry cost:

\begin{equation}
\int_{z_d}^{\infty} V(z)g(z)dz = f_e (\mathcal{M}_e).
\end{equation}
Optimal input demands. With this production structure, we can derive the cost function that combines labor and capital spending used both directly in production and to cover the fixed operational costs:

\[
TC_{sj}(z) = \frac{q_{sj}(z)}{z} \phi_j + \mathcal{F}, \quad s = d, x, \quad j = l, h
\]

(1.23)

where \( \mathcal{F} = [f_d + f_a \mathbf{1} (\alpha = \alpha_h)] \mathbf{1} (s = d) + f_x \mathbf{1} (s = x) \) collect fixed costs for any \( sj \) pair. By Sheppard’s Lemma, demands for labor and capital by a type-\( z \) firm are:

\[
l_{sj}(z) = \frac{\partial TC_{sj}(z)}{\partial w} = \frac{(1-\alpha_j) \phi_j}{w} \cdot \left[ (p^X)^\sigma \left( \frac{\rho}{\phi_j} \right)^\sigma (z)^{\sigma-1} \right] \quad \text{if } s = d
\]

(1.24)

and

\[
k_{sj}(z) = \frac{\partial TC_{sj}(z)}{\partial r^k} = \frac{\alpha_j \phi_j}{r^k} \cdot \left[ (p^X)^\sigma \left( \frac{\rho}{\phi_j} \right)^\sigma (z)^{\sigma-1} \right] \quad \text{if } s = x
\]

(1.25)

Aggregation. Aggregate labor and capital used in the exportable sector (both for domestic and foreign sales) can be computed as follows:

\[
L^X = \mathcal{M} \left[ \int_{\pi_d} l_{dl}(z) + \int_{\pi_a} l_{dh}(z) + \int_{\pi_l} l_{lx}(z) + \int_{\pi_h} l_{lxh}(z) \right] \mu(z) dz
\]

\[
K^X = \mathcal{M} \left[ \int_{\pi_d} k_{dl}(z) + \int_{\pi_a} k_{dh}(z) + \int_{\pi_l} k_{lx}(z) + \int_{\pi_h} k_{lxh}(z) \right] \mu(z) dz
\]

Aggregate exportable output sold in the domestic market is:

\[
Y^X = \left[ \mathcal{M} \left( \int_{\pi_d} (q_{dl}(z))^\rho \mu(z) dz + \int_{\pi_a} (q_{dh}(z))^\rho \mu(z) dz \right) \right]^\frac{1}{\rho}.
\]

Similarly, the total value of exported varieties is:

\[
X^X = \mathcal{M} \left[ \int_{\pi_l} p_{lx}(z) q_{lx}(z) \mu(z) dz + \int_{\pi_h} p_{lxh}(z) q_{lxh}(z) \mu(z) dz \right].
\]

(1.26)
1.3.3 Commodity Production

There is a representative firm in the commodity sector that hires labor and rents capital from the representative household in order to maximize profits. The technology is given by:

\[ Y_t^C = \bar{R} \left[ \left( K_t^C \right)^{\alpha^C} \left( L_t^C \right)^{1-\alpha^C} \right]^\eta. \]

where \( \eta < 1 \) induces decreasing returns to scale, and the constant \( \bar{R} \) is set to target the empirical share of commodity output in total GDP.

1.3.4 Market Clearing

In equilibrium, the domestic market for exportable varieties clear:

\[ C_t^X = \mathcal{M}_t \left( \int_{z_{dt}}^{\infty} (\mu_t(z))^{\rho} \mu_t(z) dz \right)^{\frac{1}{\rho}} \equiv Y_t^X \]

Labor and capital market clearing require:

\[ L_t = L_t^X + L_t^C \]
\[ K_t = K_t^X + K_t^C \]

See Appendix A.4 for details about these aggregation terms. Finally, plugging several equilibrium conditions into the household’s budget constraint, the balance of payments condition can be written as follows:

\[ B_{t+1} = (1 + r^*) B_t + TB_t, \]
where the following definitions for the trade balance, total exports, manufacturing exports and total imports apply:

\[ TB_t \equiv X_t - M_t \]
\[ X_t \equiv p_t^{Ca} y_t^C + X_t^X \]
\[ M_t \equiv C_t^M + I_t + \Phi_t + F_t \]

where \( X_t^X \) denotes the value of manufacturing exports given by (1.26), and \( \Phi_t = \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \) and \( F_t = M_t f_d + M_t p_{xt} f_x + M_t p_{at} f_a + M_{et} f_e (M_{et}) \) collect capital adjustment and fixed costs, respectively. Variables \( p_{zt} = \left[ \frac{1 - G(z_{xt})}{1 - G(z_{dt})} \right] \) and \( p_{at} = \left[ \frac{1 - G(z_{at})}{1 - G(z_{dt})} \right] \) denote the fraction of exporters and the fraction of capital-intensive firms, respectively. Appendices A.4, A.5 and A.6 contain details including the full set of dynamic and static equilibrium conditions as well as computational algorithms.

1.4 Quantitative Analysis

In this section, I present the calibration strategy designed to match certain key macro- and micro-level features of the Chilean economy. Next, I assess the model fit along both targeted and untargeted moments. Finally, I test the model’s ability to reproduce the pattern of reallocation observed during the commodity super-cycle that started in 2003, and examine how the composition dynamics affect the average measured productivity in the manufacturing sector.
Table 1.3 reports a set of parameters set a priori either using standard values in the literature or based on direct firm-level data. All data moments used in the calibration are averages over the pre-commodity boom period 1995-2003. The model period is one year. I set the time preference parameter $\beta = 0.96$ to target a long-run interest rate of 4%. I set the inverse of the intertemporal elasticity of substitution equal to $\nu = 1$ (log utility), and the Frisch elasticity equal to the baseline value documented by Rios-Rull et al (2011) [93], which is 0.72 ($\zeta = 1 + 1/0.72 = 2.4$).

The elasticities of substitution between $C^X$ and $C^M$ goods ($\epsilon$) and among exportable varieties ($\sigma$) are set to standard values used in the literature. Capital depreciation is set at $\delta^k = 0.08$, while the exogenous exit shock probability is set to $\delta = 0.08$, so that the model’s steady state reproduces the average between entry and exit rates observed in the data.

The remaining parameters, listed in Table 1.4, are chosen to match several key data moments. Certain parameters are set to match selected macroeconomic targets. I normalize the initial state of the economy to have a zero net-foreign asset
position, $\bar{B} = 0$. I set the fixed resource parameter $\bar{R}$ in the commodity sector to match the share of mining in total output in Chile, $p^{C}Y^{C}/Y = 0.1$. The scale parameter of labor supply is chosen to normalize the initial steady state nominal output $Y = 1$.

The middle block of Table 1.4 composed by parameters $\{\chi, \mu^z, \sigma^z, \gamma, f_d, f_x, f_a, \alpha_l, \alpha_h, \alpha^C\}$ is jointly estimated by minimizing a loss function given by the sum of squared residuals associated with the following set of moments: (a) nontraded share in total output $Y^{X}/Y$, (b) the (log) value-added ratio between percentiles 50th and 25th, 75th and 50th, (c) 90th and 10th, (d) 95th and 5th, (e) 99th and 1th, (g) fraction of exporters, (h) fraction of High-K firms, (i) capital cost share for High-K firms, (j) capital cost share for Low-K firms, (k) labor in commodity sector (% of total $L$), (l) capital in commodity sector (% of total $K$). Note that I estimate 10 parameters targeting 12 moments so the system is over-identified.

Finally, the last block of Table 1.4 composed by parameters $\{\eta, \phi, \phi_e\}$ is calibrated to match moments from the transition dynamic equilibrium. The level of decreasing returns in commodity production $\eta$ is set to match the peak-to-through change in the share of commodity output during the commodity boom ($\Delta Y^{C}/Y$). The capital adjustment cost parameter $\phi$ is set to target the economy-wide investment boom in the data, measured as the ratio between average investment in the pre-boom 1995-2003 and the commodity boom 2004-2013. The congestion cost at entry parameter $\phi_e$ is set to match the observed entry rate volatility in the manufacturing sector.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{B} )</td>
<td>0</td>
<td>SS NFA</td>
<td>( TB/Y )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \overline{R} )</td>
<td>0.34</td>
<td>fixed resource</td>
<td>( Y^C/Y )</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>69.3</td>
<td>labor supply</td>
<td>( Y )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \chi )</td>
<td>0.83</td>
<td>share ( C^X ) in ( C )</td>
<td>( Y^X/Y )</td>
<td>0.55</td>
<td>0.56</td>
</tr>
<tr>
<td>( \mu^z )</td>
<td>0.35</td>
<td>( \ln z \sim N(\mu^z, \sigma^z) )</td>
<td>( \ln(\text{VA50}/\text{VA25}) )</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>( \sigma^z )</td>
<td>0.58</td>
<td>( \ln z \sim N(\mu^z, \sigma^z) )</td>
<td>( \ln(\text{VA75}/\text{VA50}) )</td>
<td>1.23</td>
<td>1.05</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1.04</td>
<td>foreign size</td>
<td>( \ln(\text{VA90}/\text{VA10}) )</td>
<td>4.26</td>
<td>4.12</td>
</tr>
<tr>
<td>( f_d )</td>
<td>0.0023</td>
<td>operational cost</td>
<td>( \ln(\text{VA99}/\text{VA01}) )</td>
<td>8.63</td>
<td>7.54</td>
</tr>
<tr>
<td>( f_x )</td>
<td>0.0452</td>
<td>exporting cost fraction exporters</td>
<td></td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>( f_a )</td>
<td>1.3842</td>
<td>adoption cost fraction High-K</td>
<td></td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>( \alpha_l )</td>
<td>0.12</td>
<td>K share Low-K cost share Low-K</td>
<td></td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>( \alpha_h )</td>
<td>0.33</td>
<td>K share High-K cost share High-K</td>
<td></td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>( \alpha^C )</td>
<td>0.76</td>
<td>K share C sector ( K^C/K )</td>
<td></td>
<td>0.21</td>
<td>0.16</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.49</td>
<td>DRS C sector</td>
<td>( \Delta Y^C/Y )</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td>( \phi )</td>
<td>20</td>
<td>K adjustment cost</td>
<td>( \Delta I )</td>
<td>1.23</td>
<td>1.22</td>
</tr>
<tr>
<td>( \phi_e )</td>
<td>10</td>
<td>congestion cost entry volatility</td>
<td></td>
<td>0.04</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 1.4: Internally Calibrated Parameters.
1.4.2 Transition Dynamics during Commodity Cycles

In this section, I solve for a perfect foresight transition equilibrium in which the commodity-producing economy is subject to an exogenously given global cycle in commodity prices. The economy is assumed to be in the steady state (with a zero initial net foreign asset position) up until period $t = 0$, assumed to be the year 2003 in the data. In period $t = 1$ (2004 in the data), the exogenous commodity price cycle illustrated in Panel (a) of Figure 1.5 is revealed once-and-for-all to all the agents. I feed the model with a commodity price boom-bust cycle similar to the one observed in the period 2003-2013.

Panels (b)-(d) of Figure 1.5 report the dynamic response of the endogenous prices directly related with the two key channels emphasized in this chapter. Each panel display the time paths for the baseline model (solid lines), a counterfactual simulation without technology choice, and data counterparts when available. Panel (b) shows that the real exchange rate $p$ appreciates by about 15% from through to peak (panel (b)), thereby hurting exporters’ revenues relative to non-exporters. Similarly, the cost of capital relative to the cost of labor $r^k/w$ increases (panel (c)), inducing a cost disadvantage to the High-K types in the sense that their variable costs increase relatively more than for Low-K types during the boom phase ($\phi_h/\phi_l$ increases in panel (d)).
Figure 1.5: Exogenous trigger and endogenous price responses. 
Notes: The solid lines depict the time series in the baseline model, while the dotted lines correspond to a counterfactual without technology decision. The dark and light gray shades represent the exogenous boom and bust cycle path fed to the model, illustrated in panel (a). Panels (b)-(d) are endogenous prices responses.
Table 1.5: Panel Regressions on Model-Simulated Data.

Notes: Results for regression 1.1. ***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.1$. For comparability with the model, I replace the continuous capital-intensity variable from the data with a dummy equal to one when the firm classifies as “High-K” as defined in Figure 1.2. Moreover, because in the model-based panel all High-K firms are exporters, the triple interaction in 1.1 is removed from the specification.

1.4.2.1 Firm-Level Implications of Commodity Booms

To validate the model’s ability to reproduce the micro-level empirical regularities, I simulate a panel of artificial firms based on the transition equilibrium, and then I re-estimate the panel regressions reported in Section 1.2. Table 1.5 shows that the model does a good job in reproducing the untargeted correlations between export status and capital intensity with firm-level performance measures during the commodity boom.

Figure 1.6 illustrates how the main channels of the model operate at the firm level. Column (a) in the figure displays the evolution of total profits for different key productivity types in the economy. Columns (b) and (c) break down total profits among domestic and foreign components. In turn, the first row in the figure
compares profits for the average exporter ($\tilde{z}_x$) versus the average purely-domestic type ($\tilde{z}_d$). The second row compares the average Low-K firm ($\tilde{z}_l$) with the average High-K type ($\tilde{z}_h$), while the third row compares the average exporter with Low-K ($\tilde{z}_{x_l}$) against the average exporter with High-K technology ($\tilde{z}_{x_h}$).

The first row confirm that relatively low productivity firms (represented here by the average purely-domestic type $\tilde{z}_d$) enjoy high domestic demand, which more than compensates them for the economy-wide increase in input costs. The average exporter ($\tilde{z}_x$), in turn, exhibits a similar increase in its profits from domestic markets, but because the value of their export sales plummet, they approximately break-even when regarding aggregate profits. The second row shows that the average Low-K type ($\tilde{z}_l$) experience a similar pattern than the average purely-domestic firm ($\tilde{z}_d$).\(^8\)

However, the average High-K firm, which is also an exporter, exhibits a strong decline in its total profits as a consequence of their plummeting export sales and large increase in variable costs. The third row illustrates that within exporters, those using capital-intensive technologies are worst-off, as predicted by the model.

Regarding the extensive margin, Figure 1.7 display the change in the cutoffs that determine the exporting and technology decisions. As emphasized above, the commodity boom induces a composition effect by shifting the aggregate productivity thresholds that determine firm selection into exporting and the capital-intensive technology. In particular, the cutoff that determines entry/exit into the domestic market shifts to the left (panel (b) of Figure 1.7), allowing some previously unprofitable low-type firms to enter and enjoy an environment with richer households.

\(^8\) Note that the average Low-K firm is a purely-domestic type, so that $\Pi_x(\tilde{z}_l) = 0$. 

40
Figure 1.6: Profit Responses for average group types.
Notes: The columns report total profits and its break down. The first row compares profits for the average exporter ($\tilde{z}_x$) versus the average purely-domestic type ($\tilde{z}_d$). The second row compares the average Low-K firm ($\tilde{z}_l$) with the average High-K firm ($\tilde{z}_h$). The third row compares the average exporter with Low-K ($\tilde{z}_{xl}$) against the average exporter with High-K technology ($\tilde{z}_{xh}$). The dark and light gray shades represent the boom and bust cycle path fed to the model. All series are in percent deviation from the initial steady state.
Figure 1.7: Self Selection: Cutoff Dynamics.
Notes: The solid lines depict the time series in the baseline model, while the dotted lines correspond to a counterfactual without technology decision. The dark and light gray shades represent the exogenous boom and bust cycle path fed to the model, illustrated in panel (a).
Table 1.6: Panel Regressions on Model-Simulated Data.
Notes: Results for regression 1.2. ***: p < 0.01, **: p < 0.05, *: p < 0.1. Sample: 1995-2013. Columns (1) and (2) present results for the exporting dummy, while columns (3) and (4) for the capital-intensive dummy. All specifications include controls for firm size and revenue TFP (not reported).

In turn, both the exporting and adoption cutoffs shift to the right (panels (c) and (d)), forcing some exporters to exit the foreign markets as well as some adopters to downgrade their technology. While there are not data analogs for these cutoffs, Table 1.6 shows that the model also does a good job in replicating the untargeted coefficients related to the dynamic linear probability model presented in Section 1.2.

1.4.2.2 Productivity Measures

I follow Foster, Haltiwanger, and Krizan (2001) [44] (FHK henceforth) in computing the model-based average productivity using employment weights:

$$ Z_t = \sum_f \omega_{ft} z_f $$

(1.27)
Figure 1.8: Productivity Measures: Composition Effect.
Notes: The solid lines depict the time series in the baseline model, while the dotted lines correspond to a counterfactual without technology decision. The dark and light gray shades represent the exogenous boom and bust cycle path fed to the model.

where $\omega_{ft}$ is the time-varying (employment- or value-added-based) weight for firm $f$ in year $t$, and $z_f$ is the model-based time-invariant productivity of firm $f$. Alternatively, I construct a Solow residual-based productivity measure as follows:

$$A_t^X = \frac{p^X Y_t^X + X_t^X}{(K_t^X)^{\alpha^X} (L_t^X)^{1-\alpha^X}} \quad (1.28)$$

where $\alpha^X$ is the average capital-intensity of the manufacturing sector as a whole.

The first row of Figure 1.8 presents the FHK measures while the second row displays the Solow Residual measures. All figures compare the baseline model against the counterfactual economy without the technology choice. While the distance between the zero line and the dotted line reflects the exchange rate channel and its
impact on export exit, the distance between the dotted line and the solid line isolates the pure additional amplification effect given by the cost of capital channel and its impact on capital downsizing. Panels (a)-(d) of Figure 1.8 illustrate how both channels induce composition dynamics that combine to generate a decline in average productivity in the exportable/manufacturing sector. The baseline model generates about half of the productivity decline observed in the data, a figure that is two times larger than in a counterfactual economy with no technology decision.

1.5 Concluding Remarks

This dissertation uses Chilean manufacturing firm-level data to study the effects of commodity price cycles on factor reallocation across heterogeneous firms, and their consequences for measured productivity. I argue that in heterogeneous firm models that are consistent with the observed micro-level variation in firms capital intensity and exporting decisions, the symptoms of the old-fashioned Dutch disease, including a decline in manufacturing productivity, emerge endogenously in a context of purely efficient reallocation.

In addition to the usual channel that hurts exporters due to the appreciation of the exchange rate, I conjecture that the commodity boom might crowd out capital from manufacturing, given that copper extraction is a capital-intensive activity. The data confirms both effects. Interestingly, I document large variations of firm-level capital intensity within manufacturing industries, suggesting that different technologies coexist even within narrowly-defined sectors.
I provide a dynamic general equilibrium model in which firms with heterogeneous productivity decide whether to enter the domestic market, whether to become an exporter, and whether to adopt a (more productive) capital-intensive technology. Thereby, three productivity thresholds arise endogenously, the first one determining endogenous entry/exit, the second one governing the choice of a capital-intensive technology, and the third one governing the exporting decision. These thresholds change endogenously during the boom. Less productive firm enter the domestic market, while the thresholds for technology adoption and exporting become more stringent, thereby implying exit from exporting and capital downsizing.

These composition dynamics are able to rationalize a decrease in measured average productivity of the manufacturing sector, consistent with firm-level data during the commodity boom started in 2003. Notably, unlike most of the literature that opens the door to inefficient reallocation through reduced-form market failures, my model generates Dutch disease-like chain of events, including crowding-out of exporters and productivity declines in manufacturing, in a framework in which reallocation is purely efficient and welfare-improving.
Chapter 2: Quantifying the Role of Financial Factors during the Great Recession

2.1 Introduction

There is a long debate in macroeconomics about the ultimate drivers of business cycles. Under the lens of stochastic general equilibrium frameworks, economic fluctuations arise from disturbances to the model’s equilibrium conditions, which can be interpreted as structural shifts in preferences and technology (e.g. Smets and Wouters (2007) [101]), as reduced-form representations of frictions that manifest as time-varying wedges (e.g. Chari, Kehoe, and McGrattan (2007) [23]), or simply as convenient representations of model misspecification (e.g. Primiceri, Schaumburg, and Tambalotti (2006) [89]).

The Real Business Cycle literature pioneered by Kydland and Prescott (1982) [76] and Long and Plosser (1983) [79] strikingly illustrated that economic fluctuations can be largely accounted for by random technology shocks to the production function (King and Rebelo (1999) [70]). On the downside, the neoclassical growth model predicts that the (tax-adjusted) household’s marginal rate of substitution (MRS) between consumption and leisure should equal the marginal product of la-
bor (MPL), an equilibrium condition that fails miserably in the U.S. post-war data. The resulting labor wedge, the gap between MRS and MPL, varies significantly over the business cycle in a countercyclical way (Shimer (2009) [99], Karabarbounis (2014) [68]), and is one of the key driving forces to explain U.S. business cycles (Chari, Kehoe, and McGrattan (2007) [23]). Hall (1997) [55] decomposes the labor wedge, emphasizing the distinction between intratemporal and intertemporal channels, finding that most of the movements in employment over the business cycle are due to intratemporal preference shocks.

On the other hand, Primiceri, Schaumburg, and Tambalotti (2006) [89], and Justiniano, Primiceri, and Tambalotti (2010, 2011) [66] [67] find that intertemporal disturbances are the key source of macroeconomic fluctuations. They reach this conclusion using a New Keynesian setup with a rich set of real and nominal frictions. In recent years, motivated by the arguably prominent role of explicit financial frictions during the Great Recession episode of 2007-2009, there has been a renewed interest on the role of intertemporal disturbances in shaping business cycles. In fact, most prominent models with microfounded financial frictions used to study the recent financial crisis can be mapped into prototype economies with intertemporal wedges. For instance, in the present chapter I show that a real version of the Gertler and Karadi (2011) [48] model is equivalent to an economy with intertemporal investment shocks. Likewise, Chari, Kehoe, and McGrattan (2007) [23] show that an economy with the type of credit market frictions considered in Bernanke, Gertler, and Gilchrist (1999) [14] is equivalent to a growth model in which there is a wedge in the Euler equation for capital. More recently, Ajello (2016) [2] sets up a model with
Kiyotaki and Moore’s (2012) [72] type of friction, in which financial intermediation disturbances act as intertemporal wedges. He finds that these financial shocks were a key business cycle driver not only at the onset of the Great Recession but also during most of the Great Moderation period.

This chapter contributes to this literature by assessing quantitatively the importance played by financial frictions and financial shocks in the U.S. business cycle, with a particular focus on the Great Recession. The financial crisis of 2007-2009 is a particularly relevant episode to study the role of intertemporal disturbances, because as mentioned above, most financial frictions emphasized in the literature manifest themselves as intertemporal investment wedges. At the core of the analysis is a real business cycle model augmented to include financial intermediaries (banks, for short) facing endogenously determined balance sheet constraints. Banks take deposits from households and combine them with their own net worth to produce state-contingent loans to firms. Following Gertler and Kiyotaki (2011) [50] and Gertler and Karadi (2011) [48] (GK henceforth), the relationship between banks and households is characterized by a moral hazard problem, which ultimately limits banks’ ability to raise funds (borrow), and hence to acquire assets (lend). In equilibrium, a contraction of banks’ net worth may activate a financial accelerator effect, in which banks delever through fire sales of assets, credit spreads rise, investment plummets, and the economy may face a protracted recession. Following much of the RBC literature, the competing shocks include standard productivity, labor wedge and government spending shocks, as well as a less standard disturbance to the quality of capital held by the banking sector. The latter shock is often used
in the literature to trigger asset price dynamics, and is also interpreted here as a financial shock that mimics the significant losses from toxic assets observed in the U.S. banking sector during the Great Recession.

In the spirit of Chari, Kehoe, and McGrattan (2007) [23] (CKM henceforth), I show that the baseline RBC model with GK frictions maps into a prototype economy with labor and investment wedges. Unlike that paper, however, and given the size of the shocks hitting the economy during the financial crisis, I use nonlinear techniques not only to solve the model but also to uncover the structural driving forces behind economic fluctuations. Moreover, in order to capture the intrinsic nonlinear nature of this type of crises, I allow the balance sheet constraint associated with the GK friction to bind only occasionally, typically when banks’ leverage is sufficiently high. In the spirit of Mendoza (2010) [84], occasionally binding constraints can potentially capture the idea of infrequent financial crises nested within typical business cycles.

In a nutshell, the model has the ability to generate conditional amplification, giving rise to an asymmetry in the relationship between the net worth of the banking sector and economic activity. During tranquil times, when the balance sheet constraint is slack, credit spreads are low and the economy (conditional on the realization of other relatively benign shocks) is booming. I show that under a fully nonlinear solution the economy spends most of the time in the slack regime, because banks have the incentive to act cautiously and hold precautionary equity capital. In other words, forward-looking banks anticipate the possibility that future shocks may push them into a vulnerable zone (dangerously near the constraint), leading to precautionary deleveraging, a mission that is accomplished by cutting lending
to firms. In this environment, the sensitivity of the financial system to shocks is relatively small and the economy behaves like a frictionless neoclassical benchmark. However, in some states (say, a 2007-2009 scenario), an unlikely but possible combination of bad shocks can push the banks to hit the leverage limit, and the economy shifts into a financial crisis regime. Credit spreads rise sharply, and non-financial firms respond by borrowing less, so the equilibrium amount of credit drops. Along the way, the financial accelerator mechanism embedded in the model amplifies the initial shock. Less borrowing translates into less investment, which in turn leads to a fall in output, consumption, and the price of capital. The fall in the return to capital feeds back into the balance sheets of banks, propagating the effects even after the initial shock has dissipated.

**Related literature.** The present chapter is related to the literature that explores quantitatively the main driving forces behind macroeconomic fluctuations. Stochastic general equilibrium models imply three broad classes of equilibrium conditions: intratemporal first-order conditions, intertemporal first-order conditions, and accounting relationships between inputs and outputs. Business cycles originate from disturbances hitting these equilibrium relationships.

Chari, Kehoe, and McGrattan (2007) [23] use their so-called Business Cycle Accounting method to conclude that neutral technology shocks (efficiency wedges in their nomenclature) and intratemporal preference shocks (labor wedges) together account for essentially all of the economic fluctuations during the Great Depression and the 1982 recession in the United States. More recently, Brinca, Chari, Kehoe, and McGrattan (2016) [18] apply the same CKM method to update their
results including the Great Recession, obtaining similar results: the intratemporal labor wedge played the dominant role during the recent financial crisis, while the intertemporal investment wedge played a decidedly tertiary role. These conclusions are reminiscent of classic results in the real business cycle literature emphasizing the role of productivity and preference shocks (e.g. King and Rebelo (1999) [70], Hall (1997) [55]).

Primiceri, Schaumburg, and Tambalotti (2006) [89] (PST henceforth) emphasize two main problems about the studies mentioned above emphasizing the role of the labor wedge: first, they study environments in which physical capital is the only asset; second, they disregard asset market returns data to inform the model. Therefore, the only Euler equation implies very smooth dynamics on both the return to capital in the economy (as a function of the stable output-to-capital ratio) and the stochastic discount factor (measured through consumption growth), thus fitting very small intertemporal disturbances. By considering an economy in which a short-term nominal bond is traded along with physical capital, and by exploiting the bond pricing implications of an estimated state-of-the-art business cycle model, they obtain a prominent role for intertemporal disturbances. Similarly, Christiano, Eichenbaum, and Trabandt (2015) [28] find that the vast bulk of movements in aggregate real activity during the Great Recession were due to intertemporal wedges introduced in the households’ Euler equations associated with both nominal risk-free bonds and capital accumulation.

This dissertation borrows insights from both strands of the literature. First, in the spirit of CKM, I build a detailed economy with microfounded financial frictions
that is observationally equivalent to a prototype economy with investment wedges. I allow for the financial constraint to bind only occasionally, and show that the associated intertemporal wedge in the prototype model is a function of the multiplier on the bankers’ inequality constraint. Intuitively, if the financial constraint in the baseline model never binds, then the investment wedge is always zero, and both models behave like a frictionless RBC benchmark. Unlike CKM, who allow for correlated shocks, I assume that the model’s exogenous innovations are independent, a necessary condition for a meaningful structural interpretation of the shocks.

Second, following PST’s advice, I build a model in which a risk-free government bond is traded along with physical capital, giving rise to two intertemporal Euler equations, and inform them with data on credit spreads. Unlike PST, and given the focus on the Great Recession, I filter the structural innovations using a nonlinear filter that enforces the occasionally binding constraint.

This chapter also builds on the growing body of literature that studies the role of financial frictions and financial shocks for business cycles. Much of the earlier research about financial frictions emphasized the role of non-financial firms’ balance sheets in the propagation of shocks. Seminal articles by Bernanke and Gertler (1989) [13], Carlstrom and Fuerst (1997) [22], and Bernanke, Gertler, and Gilchrist (BGG) (1999) [14] state that credit-market imperfections may significantly amplify shocks and hinder investment by worsening the terms at which firms can borrow. As asset values typically fall during downturns, the initial shock may be further amplified in subsequent periods through tightening collateral constraints (Kiyotaki and Moore (1997) [71]). Recently, Christiano, Motto, and Rostagno (2014) [30] use
a BGG framework in which the volatility of cross-sectional idiosyncratic uncertainty fluctuates stochastically over time, and show that fluctuations in risk are the most important shock driving the cycle, including during the 2008 financial crisis.

On the other hand, the Great Recession gave rise to renewed research emphasizing frictions and shocks that originate directly in the financial sector (see Gertler and Kiyotaki (2011) [50], Christiano, Motto, and Rostagno (2010) [29], Del Negro et al. (2010) [36], Gertler and Karadi (2011, 2012) [48] [49], Jermann and Quadrini (2012) [63], Kiyotaki and Moore (2012) [72], Iacoviello (2015) [61], Bigio (2015) [16], among others). For recent surveys on financial frictions, see Brunnermeir, Eisenbach, and Sannikov (2012) [19] and Quadrini (2011) [90]. In the presence of financial frictions, fluctuations in credit spreads and overall lending standards may reflect shifts in the effective supply of funds offered to firms, with important spillovers to the real economy (Gilchrist and Zakrajsek (2012) [52]). The so-called bank lending channel states that banks’ losses from toxic assets during the financial crisis forced them to delever by fire-selling securities and reducing lending, therefore shrinking the effective supply of credit available to non-financial firms. This chapter builds on the insights of the latter articles, but allowing for an occasionally binding financial constraint. By doing so, I attempt to explain the mechanisms that caused small losses in the mortgage market (relative to the size of the economy) to amplify into such large dislocations in the financial markets as the ones observed in the summer of 2008.

By building a model with an occasionally binding constraint, this chapter is also related to the new body of literature studying nonlinear models with endoge-
nous switching between normal times and financial crisis regimes, as in Mendoza (2010) [84]. Unlike the latter paper, the occasionally binding constraint is derived from a micro-founded moral hazard problem and is imposed on the banking sector rather than the entrepreneurial sector, as in Akinci and Queralto (2014) [3] and Bocola (2016) [17]. Unlike the present chapter, Akinci and Queralto (2014) [3] use a small open economy setup in which the interest rate evolves mechanically as an autoregressive process with debt-elastic feedback from the country’s international debt-to-output ratio. In the present dissertation, the interest rates (and hence the lending-deposit spread) are fully determined in the general equilibrium. In that sense, this chapter is closer to He and Krishnamurthy (2014) [58], who also build a model with an occasionally binding constraint on the banks’ equity capital, but using a setup similar to the one proposed by Holmstrom and Tirole (1997) [60]. They calibrate the model for the U.S. economy and use it to characterize the transition from a normal state to what they label as a systemic risk state that apparently took place during the Great Recession episode. In turn, Bocola (2016) [17] focuses on the effects of a sovereign default risk shock on financial intermediation, in the context of the Italian debt crisis of 2011.

The remainder of this chapter is organized as follows. Section 2.2 describes the baseline model with an occasionally binding GK friction. Section 2.3 presents an alternative model with exogenous wedges. Section 2.4 shows the mapping between the baseline model and a prototype economy with an intertemporal investment wedge. Section 2.5 discusses the main results of the chapter and Section 2.6 concludes.
2.2 The Model

I consider a real business cycle model augmented with a financial friction that limits the ability of the banking sector to acquire funds from savers, in the spirit of Gertler and Kiyotaki (2011) [50] and Gertler and Karadi (2011) [48]. Such a friction ultimately gives rise to an endogenous limit on banks’ leverage ratio that may restrict their ability to channel funds efficiently from savers to bank-dependent agents. Unlike the above mentioned articles, the bank’s constraint binds only occasionally, typically when a sequence of bad shocks hits a relatively vulnerable and highly leveraged banking sector. I assume there is no friction in the relationship between banks and the corporate sector.

The economy is populated by four types of private agents: households, banks, capital goods producers, and final goods producers. There is also a government that finances its purchases of the final goods by levying lump-sum taxes and by issuing bonds. Regarding households, I use the “large family” metaphor in order to maintain the tractability of the representative agent approach. More specifically, there are constant fractions of workers and bankers within each household. Workers supply labor and return the wages they earn to the household. Bankers manage financial intermediaries and transfer any earnings back to their household. Within the family there is perfect consumption insurance. Final good producers combine labor and capital in order to produce the single final good in the economy. They need external finance from banks in order to buy physical capital from capital producers, which in turn combine old left-over capital with investment in order to produce new
capital. Both types of firms are owned by households.

2.2.1 Households

Households consume, supply labor, and save. Households do not hold capital directly. Rather, they save by making deposits in competitive financial intermediaries or by purchasing government bonds.Both bank deposits and government debt are non-state-contingent, one-period real bonds that pay the gross return $R_t$ from $t-1$ to $t$. In the equilibrium considered here, both instruments are riskless and thus perfect substitutes. Therefore, I impose this condition in the budget constraint from the outset. The typical household solves the following problem:\footnote{It is best to think of them as making deposits in banks other than the ones they own. The implicit assumption is that banks are specialists at evaluating, monitoring and enforcing loan contracts, which is why firms rely exclusively on banks to obtain funds. Gertler and Kiyotaki (2015) [51] consider a model in which banks and households may extend loans to firms, but the latter are less efficient in doing so.}

$$\max_{\{C_t, H_t, D_t\}} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\gamma}}{1-\gamma} - \varphi_t \chi \frac{H_t^{1+\zeta}}{1+\zeta} \right]$$

subject to \[ C_t + D_t + T_t = W_t H_t + R_{t-1} D_{t-1} + \Sigma_t \quad (2.1) \]

where $C_t$ is consumption, $H_t$ is hours worked, $D_t$ is total savings, $W_t$ is the real wage, $T_t$ is lump-sum taxes, $\Sigma_t$ is real dividends from the ownership of firms and banks (net of start-up transfers that households give to its members entering banking activities, as described below), and $\varphi_t$ is an exogenous preference (labor wedge) shock. Parameters $\beta$, $\gamma$, $\zeta$, and $\chi$ are the discount factor, the risk aversion, \footnote{Since all households solve an identical problem I omit household subscripts.}
the inverse of the Frisch elasticity of labor supply, and a scale parameter that affects
the marginal rate of substitution between consumption and leisure, respectively.
The first-order conditions are fairly standard:

\[
\varphi_t \chi H^e_t = C^{-\gamma} W_t
\]

\[
1 = \mathcal{E}_t [\Lambda_{t,t+1} R_t]
\]

where \(\Lambda_{t,t+i} \equiv \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\gamma}\) is the household’s marginal rate of substitution.

2.2.2 Banks

Banks use their own net worth together with one-period deposits from house-
holds to provide equity finance to the final goods producers. In particular, they buy
claims on the returns of physical capital that final goods producers purchase, period
by period, from capital goods producers. Let \(N_{jt}\) be the end-of-period \(t\) net worth
in the hands of bank \(j\), \(D_{jt}\) be deposits received from households, and \(S_{jt}\) be the
number of claims purchased from firms at market price \(Q_t\). The balance sheet of
bank \(j\) at the end of period \(t\) is given by

\[
Q_t S_{jt} = D_{jt} + N_{jt}.
\]

Financial intermediaries accumulate net worth through retained earnings. Banks’
liabilities pay the non-state-contingent real gross return \(R_t\), and its assets earn the
state-contingent real gross rate \(R^K_{t+1}\). Accordingly, banks’ net worth evolves as fol-

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\[ N_{jt+1} = R^K_{t+1} Q_{jt} - R_t D_{jt} = (R^K_{t+1} - R_t) Q_{jt} + R_t N_{jt} \quad (2.5) \]

where the second equality follows from equation (2.4). Intuitively, any increase in net worth above the riskless return is a function of the spread \((R^K_{t+1} - R_t)\) and the market value of the securities purchased from firms. Under frictionless financial markets, as in the standard neoclassical model, banks always have enough funds to arbitrage away differences between the risk-adjusted lending and deposit rates:

\[ \mathcal{E}_t \Lambda_{t+1} R^K_{t+1} = \mathcal{E}_t \Lambda_{t+1} R_t, \quad i \geq 0. \]

The key to the notion of financial factors affecting real activity is the existence of limits to this arbitrage, so that credit spreads may arise. Following Gertler and Kiyotaki (2011) [50] and Gertler and Karadi (2011) [48], I assume limited enforcement of contracts in the relationship between savers (households) and bankers. In particular, in each period, after portfolio decisions but before financial payouts are made, the banker can choose to divert a fraction \(\mu\) of total assets \(Q_{jt} S_{jt}\). The cost is that depositors can then force the intermediary into bankruptcy and recover the remaining fraction \((1 - \mu)\) of the assets. Because rational households recognize the bank’s option to divert assets, they will only be willing to supply funds conditional on an incentive compatibility constraint: the continuation value of operating the bank, \(V_{jt}\), cannot be less than the outside option:

\[ V_{jt} \geq \mu Q_{jt} S_{jt}. \quad (2.6) \]

Because bankers may face a binding financial constraint in their ability to obtain
deposits from households (that is, when (2.6) holds with equality), they will retain earnings in order to accumulate net worth until escaping the constraint indefinitely. To allow financial frictions to remain a relevant threat over time, I assume that bankers have finite lifetimes. Specifically, each period a banker continues operating with exogenous i.i.d. probability \( \theta \) which is independent of history.\(^3\) This mechanism motivates dividend payouts upon exit, while the financial constraint is still binding or (with some positive probability) expected to be binding in the near future. Then, a mass \( (1 - \theta) \) of bankers exit each period (and become workers), and are replaced by an equal mass of workers that become new bankers, keeping the mass of agents in each occupation constant over time.\(^4\)

Therefore, given that the bank pays dividends only upon exit, the objective of bank \( j \) at the end of period \( t \) is to maximize the expected present value of terminal wealth:

\[
V_{jt} = \mathcal{E}_t \left\{ \sum_{i=1}^{\infty} (1 - \theta) \theta^{i-1} \Lambda_{t,t+i} N_{jt,t+i} \right\}.
\]  

(2.7)

where \( \Lambda_{t,t+i} \) is the appropriate stochastic discount factor because the banker is ultimately a member of the household.

Switching to a recursive formulation, the bank problem can be written as

\(3\) This implies an average survival time equal to \( \frac{1}{1-\theta} \).

\(4\) The retained capital of exiting bankers \( (1 - \theta) \left[ (R^K_{t+1} - R_t) Q_t A_{jt} + R_t N_{jt} \right] \) is transferred back to households (as “dividends”), which in turn use part of it to provide new bankers with small start-up funds. These transactions are accounted for in the households’ budget constraint through the term \( \Sigma_t \). See Appendix B.1 for more details on these transactions and how they wash out in deriving the aggregate resource constraint of the economy.
follows:

\[ V_{jt} = \max_{\{S_{jt}, D_{jt}\}} \{ \mathcal{E}_{jt} A_{t,t+1} [(1 - \theta) N_{jt,t+1} + \theta V_{jt+1}(N_{jt,t+1})] \} \]

subject to

\[ V_{jt} \geq \mu Q_{jt} S_{jt} \quad (2.8) \]

\[ Q_{jt} S_{jt} = D_{jt} + N_{jt} \quad (2.9) \]

\[ N_{jt,t+1} = \mu K_{jt+1} Q_{jt} S_{jt} - R_{jt} D_{jt} \quad (2.10) \]

In order to solve the dynamic program, we guess (and then verify) that the value function is linear in net worth, \( V_{jt}(N_{jt}) = \psi_{jt} N_{jt} \).\(^5\) Combining (2.9) and (2.10) to eliminate deposits \( D_{jt} \), and given the conjectured value function, the problem can be conveniently written as follows:

\[ V_{jt}(N_{jt}) = \psi_{jt} N_{jt} = \max_{\{S_{jt}\}} \{ \mu K_{jt} Q_{jt} S_{jt} + \mu N_{jt} N_{jt} \} \]

subject to

\[ \psi_{jt} N_{jt} \geq \mu Q_{jt} S_{jt} \]

where

\(^5\) Gertler and Kiyotaki (2015) [51], Akinci and Queralto (2014) [3], and Bocola (2016) [17] follow a similar strategy. Ultimately, this linearity result implies that banks’ heterogeneity does not affect aggregate dynamics, and therefore, we do not need to keep track of the wealth distribution. This feature of the model helps to maintain tractability in the numerical analysis below.
\[ \mu_{K,t} = \mathcal{E}_t \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1}^K - R_t) \quad (2.11) \]
\[ \mu_{N,t} = \mathcal{E}_t \Lambda_{t,t+1} \Omega_{t+1} R_t \quad (2.12) \]
\[ \Omega_t = (1 - \theta) + \theta \psi_t \quad (2.13) \]

Note that \( \psi_t = \frac{V_{jt}}{N_{jt}} \) corresponds to bank’s \( j \) value per unit of net worth and can be interpreted as the “Tobin’s Q” ratio of the franchise. The variable \( \Omega_t \) represents the value to the bank of an extra unit of net worth in period \( t \), which equals \( \psi_t \) if the bank survives (with probability \( \theta \)), and one otherwise (probability \( 1 - \theta \)). Given the financial constraint, the Tobin’s Q ratio \( \psi_t \) will always exceed unity (see Gertler and Kiyotaki (2015) [51]). We can think of \( \mu_{N,t} \) and \( \mu_{K,t} \) as the expected discounted marginal cost of funds, and the excess marginal return on assets over liabilities, respectively.

Letting \( \xi_t \) be the multiplier on the incentive compatibility (IC) constraint, the first-order and slackness conditions are:

\[ \mu_{K,t} = \mu \xi_t \quad (2.14) \]
\[ \xi_t [\psi_t N_{jt} - \mu Q_t S_{jt}] = 0 \quad (2.15) \]

Combining equations (2.11)-(2.14) it can be shown that:

\[ \psi_t = \frac{\mu_{N,t}}{1 - \xi_t} = \frac{\mathcal{E}_t [\Lambda_{t,t+1}] [1 - \theta + \theta \psi_{t+1}] R_t}{1 - \xi_t} \quad (2.16) \]
From equations (2.14) and (2.15), when the IC constraint binds, $\xi_t > 0$ and:

$$
\phi_t \equiv \frac{Q_t S_{jt}}{N_{jt}} \begin{cases} 
= \frac{\psi_t}{\mu} = \frac{\mu N_{jt}}{\mu (1 - \xi_t)} = \overline{\phi}_t & \text{if constraint is binding, } \xi_t > 0 \\
< \frac{\psi_t}{\mu} = \frac{\mu N_{jt}}{\mu} & \text{if constraint is slack, } \xi_t = 0.
\end{cases} (2.17)
$$

The above expressions lie at the heart of the GK financial accelerator. Equation (2.16) tells us that the marginal value of wealth ($\psi_t$) is increasing in the IC multiplier ($\xi_t$). Furthermore, even when the constraint is slack ($\xi_t = 0$), we have $\psi_t > 1$ because the bank recognizes the possibility of a binding constraint in subsequent periods and would like to hold “precautionary capital”.

From equation (2.17), when the IC constraint binds there is an endogenous upper bound $\overline{\phi}_t$ on the bank’s leverage ratio $\phi_t$. Notice that from the linearity property of (2.17), we can easily aggregate to get $Q_t S_t \leq \overline{\phi}_t N_t$: total credit provided by the banking sector depends positively on aggregate net worth. A negative shock to banks’ wealth triggers an endogenous decline in their lending capacity, which reinforces itself in subsequent periods through fire-sale asset price declines and the law of motion for aggregate net worth (equation (2.20), to be described below).

Also note that combining (2.11) and (2.14) we can write an Euler equation of the form:

$$
\mathcal{E}_t \left[ \Lambda_{t+1} \Omega_{t+1} R^K_{t+1} \right] = \mathcal{E}_t \left[ \Lambda_{t+1} \Omega_{t+1} R_t \right] + \mu \xi_t. (2.18)
$$

Two features are noteworthy in equation (2.18). First, a binding leverage constraint introduces a wedge between the expected discounted return on loans and the risk-free rate on deposits. The tighter the constraint binds (the higher $\xi_t$), the
more financial distress in the economy (the higher $\mu_{K,t}$) and the higher the lending-deposit spread (via (2.11)). Second, the bank’s stochastic discount factor (SDF) is “augmented” by the factor $\Omega_{t+1}$, which is ultimately a function of the bank’s leverage ratio. Intuitively, if the leverage constraint was never to bind in the future, we would have $\xi_{t+i} = 0$ and $\psi_{t+i} = \Omega_{t+i} = 1$ for all $i \geq 0$, and equation (2.18) would collapse to the neoclassical benchmark.

Defining the augmented SDF of banks as $\hat{\Lambda}_{t,t+1} = \Lambda_{t,t+1}\Omega_{t+1}$ we obtain an intuitive expression for the risk and liquidity premium required by banks as compensation for holding the claims issued by the corporate sector:

$$E_t[R_{K,t+1} - R_t] = \frac{\mu_{\xi_t}}{E_t[\hat{\Lambda}_{t,t+1}]} - \frac{COV_t(\hat{\Lambda}_{t,t+1}, R_{K,t+1})}{E_t[\hat{\Lambda}_{t,t+1}]}.$$  \hspace{1cm} (2.19)

Expected excess returns on capital may arise for two reasons. First, as in canonical equity premium models, high excess returns reflect a fair compensation that bankers demand for holding assets whose payouts covary negatively with the (augmented) SDF. Second, positive spreads may reflect the inability of bankers to increase their portfolio of assets (raise new profitable lending) due to the leverage constraint ($\xi_t > 0$).

The financial intermediaries characterization is closed with the law of motion for bankers’ net worth. Aggregate net worth in each period is the sum of the net worth of “surviving” bankers ($N^s_t$) and the net worth of “new” bankers ($N^n_t$). Since the fraction of surviving bankers from $t-1$ to $t$ is $\theta$, we have:

$$N^s_t \equiv \theta N_t = \theta \left[ (R^K_t - R_{t-1})Q_{t-1}S_{t-1} + R_{t-1}N_{t-1} \right].$$
As noted earlier, new bankers receive a start-up transfer of funds from households, corresponding to a small share $\iota$ of the value of the assets that bankers have intermediated in the previous period:

$$N_t^n \equiv \iota(1 - \theta)Q_{t-1}S_{t-1}.$$ 

Accordingly, the aggregate net worth of banks evolves according to:

$$N_t = N_t^s + N_t^n = \left\{ \theta \left[ (R^K_t - R_{t-1})Q_{t-1}S_{t-1} + R_{t-1}N_{t-1} \right] + \iota(1 - \theta)Q_{t-1}S_{t-1} \right\}.$$  

(2.20)

Equation (2.20) can be conveniently rewritten as:

$$N_t = \theta R^K_t Q_{t-1}S_{t-1} + P_{t-1}$$  

(2.21)

$$P_t = \theta [R_t(N_t - Q_tS_t)] + \iota(1 - \theta)Q_tS_t$$  

(2.22)

where the state variable $P_{t-1}$ measures the interest on deposits that bankers pay to households at the beginning of the period (net of startup transfers), and is sufficient to keep track of the evolution of aggregate net worth.

### 2.2.3 Final Goods Producers

There is a large set of competitive final goods producers that combine labor and capital to produce the single final good in the economy, using a constant returns-to-scale Cobb-Douglas technology. I introduce two relatively non-standard features on these agents. First, they need external financing from banks (described above)
to purchase new physical capital from capital producers (to be described below). At the beginning of each period, they issue perfectly state-contingent claims to bankers in exchange for funds, which are used to purchase the capital to be used in production in the current period. A no-arbitrage condition implies that the latter two transactions are made at the capital market price $Q_t$. That is, in equilibrium, final goods firms pay $Q_t$ for each unit of physical capital to capital producers. In turn, for each security, bankers also pay $Q_t$ to final goods firms. Second, after purchasing the capital stock, the realization of the (aggregate) “quality of capital” shock $\Psi_t$ determines the effective amount of physical capital available for production. Therefore, the realized return on the securities issued by firms and purchased by banks is given by:

$$R_{t+1}^K = \left[ \frac{Z_{t+1} + (1 - \delta)Q_{t+1}}{Q_t} \right] \Psi_{t+1}$$  \hspace{1cm} \text{(2.23)}$$

where $Z_t$ denotes the net revenue from production per unit of effective capital. Anticipating the labor market clearing condition, profit maximization gives rise to the following first-order conditions:

$$W_t = (1 - \alpha) \frac{Y_t}{H_t}$$ \hspace{1cm} \text{(2.24)}$$
$$Z_t = \alpha \frac{Y_t}{\Psi_t K_{t-1}}.$$ \hspace{1cm} \text{(2.25)}$$

with

$$Y_t = A_t (\Psi_t K_{t-1})^\alpha H_t^{1-\alpha}.$$ \hspace{1cm} \text{(2.26)}$$
where $A_t$ is a standard technology shock. Note that in the aggregate, $Q_t K_t$ is the total value of capital acquired and $Q_t S_t$ is the total value of claims against this capital (total credit in the economy). Then, by arbitrage, the capital market clearing condition implies $K_t = S_t$.

2.2.4 Capital Producers

There is an arbitrarily large set of competitive capital producers that operate the technology to increase the economy-wide stock of capital. At the end of each period $t$, capital producers purchase from final goods producers the stock of undepreciated capital already used in production $(1 - \delta) \Psi_t K_{t-1}$, repair it, and then combine it with new investment $I_t$ to produce new capital $K_t$. The latter will be available for production next period. The newly produced capital is then sold to firms and any profit is transferred back to the households. Since the marginal rate of transformation (the repair stage) from previously used capital to new capital is unity, the market price of new and used capital are both equal to $Q_t$. Accordingly, the period $t$ profit of capital producers is given by:

$$\Pi^K_t = Q_t K_t - Q_t (1 - \delta) \Psi_t K_{t-1} - I_t$$ (2.27)

where the market price of capital $Q_t$ is taken as given. The implied law of motion for capital is given by:

$$K_t = (1 - \delta) \Psi_t K_{t-1} + \Gamma \left( \frac{I_t}{\Psi_t K_{t-1}} \right) \Psi_t K_{t-1}$$ (2.28)
where the function $\Gamma(.)$ is of the form:

$$\Gamma(x) = a_1 x^{1-\varrho} + a_2. \quad (2.29)$$

The constants $a_1$ and $a_2$ are set in order to ensure that in the steady state $\Gamma(\delta) = \delta$ and $\Gamma'(\delta) = 1$ (so that $Q = 1$). The parameter $\varrho \in [0, 1]$ governs the amount of adjustment costs in the economy through the elasticity of Tobin’s $Q$ with respect to the investment-capital ratio, as in Bernanke, Gertler, and Gilchrist (1999) [14].

Capital producers choose $I_t$ in order to maximize (2.27) subject to (2.28). The first-order condition is:

$$Q_t = \left[ \Gamma' \left( \frac{I_t}{\Psi_t K_{t-1}} \right) \right]^{-1} = \left[ \frac{I_t}{\delta \Psi_t K_{t-1}} \right]^\varrho. \quad (2.30)$$

### 2.2.5 Government

The government finances its purchases of the final good ($G_t$) by levying lump-sum taxes ($T_t$), and by issuing one-period risk-free bonds ($B_t$). Lump-sum taxes adjust every period to compensate any difference between spending and net bond issuance. The budget constraint is of the form:

$$G_t + R_{t-1} B_{t-1} = T_t + B_t. \quad (2.31)$$

The ratio of government spending-to-output evolves exogenously as follows:

---

$^6$ The required constants in (2.29) are $a_1 = \frac{\delta \varrho}{1-\varrho}$ and $a_2 = -\frac{\delta \varrho}{1-\varrho}$, expressions that are used to obtain the second equality in (2.30).
\[ G_t = \left(1 - \frac{1}{g_t}\right) Y_t \]  
(2.32)

where the spending shock \( g_t \) follows an AR(1) process.

2.2.6 Market Clearing and Driving Forces

Appendix B.1 shows that the aggregate resource constraint can be written as follows:

\[ Y_t = C_t + I_t + G_t. \]  
(2.33)

The labor market clearing condition requires that hours supplied by households equal hours demanded by final producers. In turn, the capital market clearing condition states that:

\[ K_t = S_t. \]

The driving forces in the model are the TFP shock \( A_t \), the labor wedge shock \( \varphi_t \), the government-spending shock \( g_t \), and the quality of capital shock \( \Psi_t \). These variables follow stationary AR(1) processes in logs. The competitive equilibrium and the full system of equilibrium conditions are described in Appendix B.2.

2.3 Prototype RBC Model with Wedges

In the spirit of Chari, Kehoe, and McGrattan’s (2007) [23] business cycle accounting method, in what follows I show that the GK financial friction described
above maps into an agnostic intertemporal wedge distorting the capital Euler equation in a prototype RBC model. In turn, as is well-known, the preference shock to the disutility of work maps into a labor wedge distorting the intratemporal optimality condition between the household’s marginal rate of substitution and the firm’s marginal product of labor.

The productive sector in the prototype economy with wedges is the same as in the baseline model. The difference is that there are no financial intermediaries. Rather, households provide loans directly to final goods producers. As before, households can also save by purchasing government securities. Therefore, the representative household solves the following problem:

$$\max_{\{C_t, H_t, L_t, B_t\}} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ C_t^{1-\gamma} \frac{H_t^{1+\zeta}}{1-\gamma} - \chi \frac{H_t^{1+\zeta}}{1+\zeta} \right]$$

subject to

$$C_t + L_t + B_t = (1 - \tau_t^H) W_t H_t + (1 - \tau_t^K) R_{t+1}^{K} L_{t-1} + R_{t-1} B_{t-1} + \Sigma_t + T_t$$

(2.34)

where $C_t$ is consumption, $H_t$ is hours worked that earn the real wage $W_t$, $L_t$ is loans to final goods producers that earn the state-contingent real gross rate $R_{t+1}^{K}$, $B_t$ is savings in government bonds that pay the non-state-contingent real gross return $R_t$, $\Sigma_t$ are real dividends from the ownership of firms, and $T_t$ are lump-sum transfers received from the government. $\tau_t^H$ and $\tau_t^K$ are exogenous processes that resemble taxes on labor and capital income, and play the role of labor and investment wedges, respectively. Redefining the wedges as $\eta_t^H = 1 - \tau_t^H$ and $\eta_t^K = 1 - \tau_t^K$, the first-order conditions are:
\[
\chi H_t^\zeta = \eta_t^H C_t^{-\gamma} W_t
\]
\[
1 = \mathcal{E}_t \Lambda_{t,t+1} R_t
\]
\[
1 = \mathcal{E}_t \Lambda_{t,t+1} \eta_{t+1}^K R_{t+1}^K
\]

where \(\Lambda_{t,t+i} \equiv \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\gamma}\) as in the baseline model. In equilibrium, total credit from households to final goods producers are \(L_t = Q_t K_t\).

The government finances its purchases of the final good \((G_t)\) by levying capital \((\tau_t^K)\) and labor \((\tau_t^H)\) income taxes, and by issuing one-period risk-free bonds \((B_t)\). Any difference between spending, tax revenues, and net bonds issuance is rebated back to households in a lump-sum fashion via \(T_t\). The budget constraint is of the form:

\[
G_t + T_t + R_{t-1} B_{t-1} = \tau_t^H W_t H_t + \tau_t^K R_t^K L_{t-1} + B_t.
\]

Tax rates \(\tau_t^K\) and \(\tau_t^H\) follow exogenous AR(1) processes. Public spending evolves as described in equation (2.32).

### 2.4 Mapping from frictions to wedges

Comparing equation (2.2) in the baseline model with microfounded frictions with equation (2.35) in the prototype RBC model, it is clear that the labor wedge maps as follows:
\[ \eta_t^H = 1 - \tau_t^H = \frac{1}{\varphi_t} . \] (2.39)

Combining equations (2.11)-(2.14) in the baseline model, we can write:

\[ 1 = \mathcal{E}_t \left[ \Lambda_{t,t+1} \left( \frac{1 - \theta + \theta \psi_{t+1}}{\mu \xi_t + \psi_t (1 - \xi_t)} \right) R_{t+1}^K \right] . \] (2.40)

Comparing the resulting Euler equation for capital in the baseline model (2.40) with equation (2.37) in the prototype RBC model, the investment wedge maps into the financial friction as follows:

\[ \eta_{t+1}^K = 1 - \tau_{t+1}^K = \left( \frac{1 - \theta + \theta \psi_{t+1}}{\mu \xi_t + \psi_t (1 - \xi_t)} \right) . \] (2.41)

The agnostic intertemporal investment wedge in the prototype model is a function of the banks’ Tobin’s Q ratio and the multiplier on the occasionally binding constraint. Intuitively, if the leverage constraint in the model with explicit financial frictions never binds (so that the multiplier on the incentive constraint \( \xi_t = 0 \ \forall t \), and the banks’ marginal utility of net worth \( \psi_t = 1 \ \forall t \)), then \( \eta_t^K = 1 \) (and \( \tau_t^K = 0 \) \ \forall t \). In other words, both models converge to the frictionless RBC benchmark, in which the expected discounted returns on all assets in the economy are equalized:

\[ 1 = \mathcal{E}_t \left[ \Lambda_{t,t+1} R_t \right] = \mathcal{E}_t \left[ \Lambda_{t,t+1} R_{t+1}^K \right] . \]

This result will be used in the quantitative section below, in order to provide a direct structural estimate of the impact of the financial friction on business cycle fluctuations.
2.5 Quantitative Results

This section presents a series of numerical experiments that shed light on the main features of the model dynamics, its ability to account for U.S. business cycles, and the role of financial factors (the “financial shock” $\Psi_t$ and the GK-type financial friction itself) during the Great Recession. The first subsection briefly describes the computational strategy. The second subsection presents the calibration/estimation strategy and results. Next, I present several experiments to illustrate the dynamics of the model. Finally, the last subsection reports the main results of the chapter.

2.5.1 Computational Strategy

The empirical strategy combines both calibrated and estimated parameters. Some parameters are calibrated before the estimation step because the likelihood function is not informative about their value. On the other hand, estimating the fully nonlinear model subject to the occasionally binding financial constraint is computationally challenging because it requires the solution of the nonlinear model to be computed for a large number of parameter vectors. Instead I estimate the parameters using a log-linearized approximation of the model equilibrium conditions, and characterize the posterior distribution using a Random Walk Metropolis-Hastings algorithm, as described in An and Schorfheide (2007) [6]. Conditional on the estimated parameter vector, I solve the model and extract the underlying states and structural shocks enforcing the occasionally binding constraint by means of fully nonlinear methods.
In particular, the model is solved using a global method based on Chebyshev approximations of the decision rules along the lines of Judd (1992) [64]. Given the minimum set of state variables associated with the DSGE model, $S_t$, the solution algorithm requires choosing a grid of points $G = \{S_1, ..., S_M\}$ in the model’s state-space and determining the coefficients on the Chebyshev polynomials by minimizing the unweighted sum of squared residuals associated with the Euler equations of the model. Following Aruoba, Cuba-Borda, and Schorfheide (2018) [8], the solution algorithm involves two non-standard tools. First, because the occasionally binding constraint potentially introduces kinks in the policy functions, I use a piecewise smooth representation of the approximated decision rules. Second, the solution grid $G$ is chosen using an iterative procedure based on a simulation-based clustered-grid-algorithm (CGA) first proposed by Judd, Maliar, and Maliar (2010) [65]. The solution algorithm and the accuracy of the numerical approximations are described in detail in Appendix B.3. Given the model solution, I uncover the hidden states and disturbances that best fit the data over the sample using the Bootstrap Particle Filter, as described in Herbst and Schorfheide (2016) [59].

2.5.2 Model Estimation

Table 2.1 reports the parameters and targeted steady state values used in the experiments. The model includes four conventional parameters ($\beta, \gamma, \alpha, \delta$), for which I choose standard values and steady state targets used in related studies. I target a steady state risk-free annual interest rate of 4%. I set the inverse of the
intertemporal elasticity of substitution $\gamma$ equal to one, which implies log utility. I set the capital share $\alpha = 0.33$ and the quarterly depreciation rate of capital $\delta = 0.025$, as in GK. I set $g$ to target the empirically observed share of government consumption in total output. There are three parameters that are specific to bankers. First, the divertable share $\mu$ is calibrated to generate a frequency of financial crises of about 0.5% (two systemic financial crises every 100 years). Second, the start-up share $\iota$ is set to target an ergodic mean leverage ratio of four, which is the typical value used in the literature (Gertler and Karadi (2011) [48]). Finally, a value of $\theta = 0.96$ for the banks’ survival probability is used, following Bocola (2016) [17] who uses a similar GK model subject to an occasionally binding constraint.

The remaining parameters which govern the dynamics of the model are estimated using Bayesian techniques. Namely, I estimate the Frisch elasticity of labor supply, the adjustment costs, and the persistence and standard deviations of all exogenous AR(1) processes. The sample period is 1954:I-2015:IV. The observables used are real output, real investment, hours worked, and a measure of the credit spread. Real variables are in per capita terms, and are constructed scaling nominal variables by the GDP deflator. Consumption includes nondurable goods and services, while investment includes durable consumption and gross private domestic investment. Because I work with a closed economy model, the data on output excludes net exports. The spread measure is the difference between the Moody’s seasoned Baa and Aaa corporate bond yields.\footnote{Similar results can be found using the spread between the Baa corporate bond yield and the yield on long-term Treasury bonds, or using the Gilchrist and Zakrajsek’s (2012) [52] excess bond premium. I choose to use the Baa-Aaa spread} All observables are HP-filtered, and
<table>
<thead>
<tr>
<th>Calibrated</th>
<th>Description</th>
<th>Source or Target</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Disc. factor</td>
<td>$R = 4%$ annual</td>
<td>0.99</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Risk aversion</td>
<td>Gertler and Karadi (2011)</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation</td>
<td>Gertler and Karadi (2011)</td>
<td>0.025</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>Gertler and Karadi (2011)</td>
<td>0.33</td>
</tr>
<tr>
<td>$g$</td>
<td>Steady State $G/Y$</td>
<td>$G/Y$ Avg. 1955-2015</td>
<td>0.22</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Diversion share</td>
<td>Freq. Fin. Crises 0.5%</td>
<td>0.256</td>
</tr>
<tr>
<td>$\iota$</td>
<td>Start-up share</td>
<td>$LEV = 4.$</td>
<td>0.007</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Survival rate</td>
<td>Bocola (2016)</td>
<td>0.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimated</th>
<th>Prior</th>
<th>Posterior [5% 95%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta$</td>
<td>Inverse Frisch</td>
<td>Gamma(1.4, 1)</td>
</tr>
<tr>
<td>$\varrho$</td>
<td>Adjustment Cost</td>
<td>Beta(0.25, 0.1)</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>AR TFP</td>
<td>Beta(0.5, 0.2)</td>
</tr>
<tr>
<td>$\rho_\phi$</td>
<td>AR Labor Wedge</td>
<td>Beta(0.5, 0.2)</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>AR Gov. Spending</td>
<td>Beta(0.5, 0.2)</td>
</tr>
<tr>
<td>$\rho_\Psi$</td>
<td>AR Quality K</td>
<td>Beta(0.5, 0.2)</td>
</tr>
<tr>
<td>$100\sigma_A$</td>
<td>Std. TFP</td>
<td>InvGamma(1, 2)</td>
</tr>
<tr>
<td>$100\sigma_\eta$</td>
<td>Std. Labor Wedge</td>
<td>InvGamma(1, 2)</td>
</tr>
<tr>
<td>$100\sigma_g$</td>
<td>Std. Gov. Spending</td>
<td>InvGamma(1, 2)</td>
</tr>
<tr>
<td>$100\sigma_\Psi$</td>
<td>Std. Quality K</td>
<td>InvGamma(1, 2)</td>
</tr>
</tbody>
</table>

Table 2.1: DSGE Model Parameters.
Notes: Priors and posteriors based on 100,000 draws from the Metropolis-Hastings algorithm (discarding the first 50,000) applied to the log-linearized model in which the constraint always binds. Sample period: 1954:1-2015:IV.

the associated HP-cycle time series are matched with the model-implied variables expressed as percentage deviations in percent deviation from their respective ergodic means.

I adopt fairly agnostic priors for the autoregressive processes. I assume a Beta distribution with mean 0.5 and standard deviation 0.2 for the autoregressive coefficients. The prior for the inverse Frisch elasticity is a Gamma distribution centered at 1.4 ($\approx 1/0.72$), which is the baseline figure used by Rios-Rull et al (2012) [93], who extensively study appropriate values for this parameter using U.S. because it allows me to extend the data until 1947 (the beginning of the NIPA sample), and have several years of pre-sample data that is useful to train the filters in the experiments below. The effective sample used is 1954:1-2015:IV.
data in the context of DSGE models. The prior for the elasticity of Tobin’s $Q$
with respect to the investment-to-capital ratio (the adjustment cost parameter) is
assumed to be a Beta distribution centered at 0.25, which is the value calibrated
by Bernanke, Gertler, and Gilchrist (1999) [14]. Overall the data proved to contain
significant information about the estimated parameters, which is reflected in the
considerably different posterior densities relative to the chosen priors.

2.5.3 Model Dynamics

2.5.3.1 Goodness of fit

Table 2.2 compare some selected model-implied second moments with their
analogs in the data over the sample period 1954:I-2015:IV. Overall, the model is able
to reproduce several key volatilities and correlations observed in U.S. business cycles.
The model slightly underpredicts the absolute volatility of output (1.5%) relative
to the data (1.7%), because it generates less volatility in investment. However,
the relative standard deviation of output over investment is reasonably well-aligned
around three in both the model and the data, consistent with standard results in the
business cycle literature. The model also does a fair job in capturing the moments
of the financial variable included in the estimation (the Aaa-Baa spread), and also
those for consumption which is not observable. As expected, the model is able to
produce countercyclical spreads, but the absolute level of the correlation is only half
the size observed in the data. As explained above, positive spreads may arise in the
model (and in the data) because of two reasons: first, a standard countercyclical
\[ \sigma(X) \]
\[ \sigma(X)/\sigma(Y) \]
\[ corr(X, Y) \]
\[ corr(X, X_{-1}) \]
\hline
Data & Model & Data & Model & Data & Model & Data & Model \\
\hline
Output (Y) & 1.73 & 1.53 & 1 & 1 & 1 & 1 & 0.87 & 0.83 \\
Consumption & 0.85 & 0.91 & 0.49 & 0.60 & 0.80 & 0.67 & 0.84 & 0.99 \\
Investment & 5.79 & 4.14 & 3.34 & 2.71 & 0.94 & 0.90 & 0.86 & 0.76 \\
Hours & 1.88 & 1.49 & 1.08 & 0.98 & 0.85 & 0.78 & 0.92 & 0.83 \\
Spread & 0.07 & 0.05 & 0.04 & 0.03 & -0.58 & -0.29 & 0.75 & 0.63 \\
\hline
Table 2.2: Second Moments.
Notes: Model-implied moments compared to data. Sample period: 1954:I-2014:IV.

risk-premium, and second, the financial constraint. As will become clear below, the model presented here displays a very low risk premium (a canonical result in Euler-equation based macro frameworks), preventing the model from generating the full countercyclicality of spreads observed in the data.

2.5.3.2 Sample Decision Rules

In order to illustrate how the occasionally binding constraint drives an asymmetry in the economy, this subsection presents decision rules for selected variables and the three most important shocks analysed in the model, the technology shock, the labor wedge shock, and the disturbance to the quality of capital.

Figure 2.1 shows the results for slices of the decision rules in which only one exogenous state variable varies along a fine grid. The range of the grids is wide enough (+ and - three standard deviations) to cover cases in which the IC constraint is both slack and binding. Each column of the figure moves along the corresponding exogenous state variable (TFP, labor wedge, quality of capital), while keeping all other states at their ergodic means. Each row contains the decision rules for investment...
(\(I_t\)), the bank Tobin’s Q ratio (\(\psi_t = V_t/N_t\)), and the spread (\(E_t\left[R_{t+1}^{K} - R_t\right]\)).

Each panel includes the policy functions for three different solution methods. “Nonlinear” corresponds to the piecewise nonlinear solution algorithm described in detail in Appendix B.3. “OccBin” is the piecewise linear solution using the OccBin toolkit described in Guerrieri and Iacoviello (2015) [54]. “Linear” corresponds to a first-order perturbation solution which does not respect the occasionally binding constraint. The comparison with the other solution methods commonly used in the literature sheds light on the importance of the nonlinear solution method in capturing both the kinks in the policy functions (not captured in the linear solution) and precautionary motives (not captured in the piecewise linear solution).

Figure 2.1 shows that the economy experiences a binding financial constraint after large negative (positive) TFP (labor wedge) shocks. Not surprisingly, the economy tends to switch into a “financial crisis” regime after relatively small negative quality of capital shocks, given the fact that these disturbances directly affect the net worth of the banking sector, thereby increasing the leverage ratio and activating

---

8 Guerrieri and Iacoviello (2015) [54] provide a fast an efficient Dynare-based toolkit (OccBin) to solve dynamic models with occasionally binding constraints. They adapt a first-order perturbation approach and applies it in a piecewise fashion. Importantly, the piecewise solution is not just linear - with two different set of policy functions depending on whether the constraint is binding or not - but rather, it can be highly nonlinear. The dynamics in each regime depends on how long one expects to be in that regime, which in turn depends on the state vector. This interaction produces the high nonlinearity, and allows to capture the kinks in the decision rules accurately. However, there is a limitation of such a method. Just as any linear approximation method, the algorithm discards all information regarding the realization of future shocks. Therefore, the algorithm is not able to capture precautionary behaviour linked to the possibility that a constraint become binding in the future, as a result of shocks yet unrealized. I use the piecewise linear solution in order to find a reasonable initial guess for my fully nonlinear algorithm.
Figure 2.1: Policy Functions for Selected Variables. Pooling years.
Notes: In each panel, only one exogenous state variable varies on the horizontal axis. The other state variables are fixed at their ergodic means. “Nonlinear” corresponds to the piecewise nonlinear solution algorithm described in detail in Appendix B.3. “OccBin” is the piecewise linear solution using the OccBin toolkit described in Guerrieri and Iacoviello (2015) [54]. “Linear” corresponds to a first-order perturbation solution which does not respect the occasionally binding constraint.
the GK financial accelerator. Likewise, for high positive values of both TFP and quality of capital shocks (or negative values for the labor wedge shock) the economy is in a slack regime. In this regime, the level of banks’ net worth is high, and the leverage ratio tends to be low relative to the mean of the ergodic distribution and relative to the leverage limit imposed by the financial friction ($\phi_t$ in equation (2.17)). Because the multiplier of the incentive compatibility constraint is equal to zero and the model generates a small risk premium when the constraint is not binding, the lending-deposit interest rate spread is around zero (see equation (2.19)). Cheap credit fuels a rise in both investment and asset prices, and the economy booms.

Notice that the marginal value of wealth in the slack regime is always lower than in the binding regime. Intuitively, due to the bank’s financial constraint, the bankers’ marginal value of an extra unit of net worth is higher during periods of financial distress. It is also noteworthy that the bank’s Tobin’s Q ratio in the fully nonlinear decision rule is always greater than (or equal to) the values under the piecewise linear (OccBin) solution in both regimes. This is precisely the “precautionary capital” motive embedded in the model: under the fully nonlinear solution, even under a slack regime, banks realize that future shocks might push them into the leverage limit. Hence, the marginal value of net worth is higher in every state of the economy. This is an important mechanism because it implies the economy will spend less time under the binding regime, as is clear in the last row of the figure that shows a wider range of shocks in which the spread (a direct function of the multiplier on the IC constraint, see equation (2.19)) is near zero. Therefore, the model has the potential to capture the idea of financial crises being relatively rare.
events nested within typical business cycles.

2.5.3.3 Crisis Experiments: Impulse Responses

This section presents several experiments to show how the model dynamics work. Figure 2.2 shows the responses of key variables in the model to the three main disturbances: the TFP shock, the labor wedge shock, and the “financial” (capital quality) shock. In each case, the direction of the shock is designed to generate a recession, and the size of the impulse is two standard deviations, which is consistent with the size of the filtered innovations uncovered during the Great Recession (see below). The rows in the figure present the responses of output, consumption, investment, the banking sector’s leverage ratio, and the spread.

The negative shocks generate an immediate increase in the leverage ratio until the point at which the financial constraint is activated in every case. Therefore the multiplier on the incentive compatibility constraint becomes increasingly positive, implying an increase in the spread between the lending and the risk-free deposit interest rate. In a context of severe financial distress, banks are forced to delever through a protracted cut in lending to the corporate sector in order to escape from the binding constraint, which in turn translates into a significant investment slump at impact. Consumption also falls persistently in all cases but in a much smoother fashion. While the initial recession is much smaller under the quality of capital shock, the economy tends to stay below trend for a larger period. As expected, the financial shock has the largest impact on financial variables (such as leverage and
Figure 2.2: Impulse Responses
Notes: Responses to negative 2-standard deviation shocks. All variables are in percent deviation from the baseline unshocked path.
spread) as well as smaller impacts on real variables.

2.5.4 Inference of Unobserved States from a Nonlinear Filter

2.5.4.1 Quantifying the role of the financial shock

In this section I use a nonlinear filter to back out the hidden states and structural innovations that best fit the U.S. data over the sample 1954:I-2015:IV, under the lens of the model subject to the occasionally binding financial constraint. The DSGE model has a nonlinear state-space representation of the form

\[
y_t = \Psi(s_t) + u_t, \quad u_t \sim N(0, \Sigma_u) \quad (2.42)
\]

\[
s_t = \Phi(s_{t-1}, \epsilon_t), \quad \epsilon_t \sim N(0, \Sigma_\epsilon) \quad (2.43)
\]

where \( y_t \) is the vector of observables in period \( t \), \( s_t \) stacks the hidden state vector, \( \epsilon_t \) is the vector of structural shocks, while \( u_t \) are measurement errors. (2.42) is the measurement equation that links the model state variables with the observable time series used to inform the model. (2.43) is the transition equation given by the piecewise nonlinear solution of the model represented here by the nonlinear function \( \Phi(\cdot) \). I use the bootstrap particle filter to conduct inference about the unobserved state \( (s_t) \) and shocks \( (\epsilon_t) \) over the sample. The details of the algorithm can be found in Herbst and Schorfheide (2016) [59].

Figure 2.3 presents the filtered i.i.d. innovations \( (\epsilon_t) \) uncovered from the filter, using the same observable variables as in the estimation step (real output, real
investment, per capita hours, and the spread). By construction, feeding these structural shocks back to the nonlinear state-space system recovers the observable data used to inform the model (up to a small measurement error assumed to be 10% of the sample variance of the respective observable time series). Likewise, counterfactual experiments can be run by turning on and off one or more of these driving forces at a time.

In general, the sequence of shocks extracted from the filter are consistent with historical accounts and previous findings: productivity shocks are procyclical (Basu and Fernald (2002) [9]), while labor wedge shocks are highly countercyclical (Hall (1997) [55], Shimer (2009) [99], Karabarbounis (2014) [68]), and government spending shocks tend to be less important for business cycle fluctuations in the U.S. (Chari, Kehoe, and McGrattan (2007) [23]). As expected, the quality of capital or “financial” shock seems to be especially relevant during the Great Recession, and also during previous recessions arguably caused by other unmodeled disturbances (e.g. the oil price shocks during the seventies). With the exception of some spikes around particular recession episodes, all the innovations tend to be within two standard deviations.

One way to externally validate the results of the filter is to compare some of the extracted states to analogous objects in the data. Figure 2.4 compares the underlying autoregressive processes for TFP and the labor wedge implied by the filter, with available empirical counterparts provided by Fernald (2012) [39] and Karabarbounis (2014) [68], respectively. For both TFP and the labor wedge the comovement between the model-implied states and their external data counterparts
Figure 2.3: Structural Innovations (in number of standard deviations)
Notes: Structural innovations scaled by the standard deviation of each shock. The gray areas indicate NBER recession dates.
is striking (correlation of 0.88 and 0.91, respectively), despite the fact of being obtained through completely different methodologies and using different observable time series.

What were the main driving forces behind the Great Recession? Figure 2.5 presents the evolution of output, investment, hours, and the spread in the data and decomposes each quarterly observed realization into the positive (above the x axis) and negative (below the x axis) contributions of the structural shocks in the model. To focus attention on the Great Recession episode, the figures show a zoom of the period 2000-2015. The long-run analogs starting at the beginning of the sample are available in Appendix B.4.

According to the structural model, the economic downturn during the Great Recession was mainly a result of negative productivity and labor wedge shocks. On the one hand, TFP shocks and to a lesser extent the quality of capital or “financial” shock were key to explaining the investment slump at the very onset of the crisis. Interestingly, the impact of previous positive productivity shocks on output was fading a couple of quarters before the recession began, when the economy was already showing significant signals of distress. On the other hand, the labor wedge was key to explaining the slow recovery of output and investment. In fact, the long-run shock decompositions reported in Appendix B.4 are consistent with this same pattern: the TFP process tends to lead the cycle while the labor wedge tends to lag the cycle, a result reminiscent of the findings by CKM in the context of the 1982 crisis.

Not surprisingly, hours worked are overwhelmingly explained by the labor wedge shock, while around half of the spike in the spread in 2008-2009 was due to
Figure 2.4: Selected Filtered States: Model vs Data
Notes: Model-implied filtered states extracted from the Particle Filter are in percent deviation from their steady state values. The TFP data is from Fernald (2012) [39], while the labor wedge data is from Karabarbounis (2014) [68]. Both data series are in percent deviation from a Hodrick-Prescott trend. The gray areas indicate NBER recession dates.
the effects of the financial shock. However, the incidence of the financial shock in the spread does not translate into a significant effect of this structural force into output and investment, a result that is in part due to the quick mean reversion of the observable spread time series.

2.5.4.2 Assessing the role of the financial friction

What was the role of the GK financial friction itself during the Great Recession? The relatively small role of the financial shock does not mean that the effects of the financial friction were not important. In fact, the GK financial constraint was binding during the crisis, likely triggered by the full combination of disturbances reported in Figure 2.3. One way to estimate the direct effect of the friction in the economy is to use the prototype RBC model with wedges and the equivalence result presented in Section 4. In that model, we find that all the endogenous effect of the friction (captured in the baseline model by the marginal value of wealth $\psi_t$, and the multiplier on the occasionally binding constraint $\xi_t$) can be structurally captured by an agnostic exogenous wedge in the spirit of CKM. In contrast to CKM whose shocks are allowed to be correlated, I use the particle filter to uncover the model’s independent innovations, a necessary condition for a meaningful structural interpretation of the shocks.

Figure 2.6 presents the results. Overall, the results are consistent with the baseline model. Productivity and labor wedge shocks are still the main drivers of the economy during the boom-bust cycle. However, the financial forces appear to
Figure 2.5: Baseline Model: Historical Decomposition: Real Variables and Spread

Notes: Output, investment, and hours worked are in quarterly terms, expressed in percent deviations from their ergodic means (solid line). Spread is expressed in annualized percent terms. The solid lines correspond to the model-implied filtered observable variables, which are up to a small measurement error equal to the data described above. The bars decompose each filtered variable into the contributions of each structural shock. The gray areas indicate NBER recession dates.
Figure 2.6: Pure RBC Model: Historical Decomposition: Real Variables and Spread
Notes: See notes from Figure 2.5.
be more important under this metric, explaining around one third of the slump in investment by the end of 2008 and the beginning of 2009.

Another difference in the RBC model with exogenous frictions is that now the investment wedge absorbs the full effect not only of the financial friction but also the financial shock. This result can be better explained by combining equations (2.36) and (2.37):

\[ 1 = \mathcal{E}_t \Lambda_{t,t+1} R_t = \mathcal{E}_t \Lambda_{t,t+1} \eta^K_{t+1} R^K_{t+1}. \]

Whenever there is a difference between the risky and the riskless return in the economy, the model can explain it directly through the investment wedge $\eta^K_{t+1}$ or through the financial shock embedded in $R^K_{t+1}$. In the prototype RBC model, the data tend to favor the direct investment wedge effect because the financial shock implies a sudden decrease in the supply of capital in the economy, causing a counterfactual increase in the price of capital ($Q_t$) during the recession.

2.6 Conclusions

This chapter studies quantitatively the role of financial factors in U.S. business cycles, with a particular focus on the Great Recession. To do so, I augment an otherwise standard real business cycle model with a non-trivial banking sector, in which financial intermediaries face an occasionally binding endogenous limit on their leverage ratio. The asymmetry induced by the occasionally binding constraint generates non-monotone dynamics, therefore capturing the idea of infrequent financial crises nested within typical business cycles. At the same time, the framework
is still tractable enough to allow for the introduction of several standard features typically used in the DSGE literature.

In the spirit of Chari, Kehoe, and McGrattan (2007) [23], I show that the baseline model with the GK friction is equivalent to a prototype economy with an intertemporal investment wedge that distorts the Euler equation for capital. Moreover, the investment wedge is shown to be a function of the key endogenous variables associated with the friction in the microfounded model: that is, the multiplier associated with the occasionally binding constraint, and the marginal value of net worth (a measure of the “precautionary capital” motive that arises in the nonlinear solution). Unlike that paper, however, I back out the structural economic shocks that drove the economy into the Great Recession using the bootstrap particle filter.

Consistent with previous literature, the results suggest that financial frictions that manifest as intertemporal wedges are relatively unimportant to understanding U.S. business cycles over the five decades previous to the crisis. More surprisingly, while the GK friction (or investment wedge) was indeed quantitatively more relevant during the Great Recession, its effects are still of second-order importance relative to other driving forces such as productivity or labor wedge shocks.
Appendix A: Appendices for Chapter 1

A.1 Cross-country decline in mining productivity

Table A.1 compares labor productivity growth in the mining sector, before and after the commodity super cycle started in 2003, across countries with 5% or higher mining share in total output. With the exception of China, all countries experience a significant decline in real value added per worker, thereby suggesting some inefficient rent-seeking behavior.

<table>
<thead>
<tr>
<th>Mining Share in Total Output</th>
<th>Labor Productivity Mining</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a) 1990-2003</td>
<td>(b) 2004-2012</td>
</tr>
<tr>
<td>Argentina</td>
<td>6.8</td>
<td>2.0</td>
</tr>
<tr>
<td>Brazil</td>
<td>6.4</td>
<td>4.6</td>
</tr>
<tr>
<td>Chile</td>
<td>17.8</td>
<td>7.8</td>
</tr>
<tr>
<td>Colombia</td>
<td>10.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Mexico</td>
<td>12.5</td>
<td>5.2</td>
</tr>
<tr>
<td>Peru</td>
<td>11.2</td>
<td>4.3</td>
</tr>
<tr>
<td>Indonesia</td>
<td>17.6</td>
<td>2.5</td>
</tr>
<tr>
<td>Malaysia</td>
<td>19.5</td>
<td>5.2</td>
</tr>
<tr>
<td>Russia</td>
<td>13.4</td>
<td>4.3</td>
</tr>
<tr>
<td>SouthAfrica</td>
<td>13.0</td>
<td>5.9</td>
</tr>
<tr>
<td>Australia</td>
<td>10.1</td>
<td>3.8</td>
</tr>
<tr>
<td>Canada</td>
<td>13.1</td>
<td>1.8</td>
</tr>
<tr>
<td>USA</td>
<td>5.0</td>
<td>1.8</td>
</tr>
<tr>
<td>China</td>
<td>7.8</td>
<td>9.2</td>
</tr>
</tbody>
</table>

Table A.1: Mining Countries: Labor Productivity.
Notes: Author’s calculations based on National Accounts data by economic sector from United Nations, combined with information on employment by sector from the 10-Sector-Database.~
A.2 Firm Characteristics and Estimated Revenue TFP

To document in a systematic fashion how exporters and capital-intensive firms outperform their non-exporters and labor-intensive counterparts, I run the following panel regression:

\[
\ln(Y_{ft}) = \alpha X_{f0} + \beta K_{f0}^{int} + \delta X_{f0} \cdot K_{f0}^{int} + \gamma' Z_{ft} + \varphi_{st} + \varepsilon_{ft} \quad \text{(A.1)}
\]

where \(Y_{ft}\) denotes a productivity measure for firm \(f\) in year \(t\), \(X_{f0}\) is a dummy variable that takes the value of 1 if firm \(f\) exports in its first period \(t = 0\) in the sample (conditional on \(t = 0\) being in the pre-boom period 1995-2003), \(K_{f0}^{int}\) denotes firm \(f\) period \(t = 0\) capital intensity, \(Z_{ft}\) are firm-level controls, and \(\varphi_{st}\) represents sector-year fixed effects. Firm-level multi-factor productivity is estimated using the method of Wooldridge (2011) and, under the assumption of constant returns to scale, using cost shares as in Foster, Haltiwanger, and Krizan (2001).

Table A.2 presents the results. Pre-boom exporters and capital-intensive firms are significantly more revenue-productive than their non-exporters and labor-intensive analogs. Similar results emerge when using alternative firm-level outcome variables such as real value added and real profits.
<table>
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<th>CRS</th>
<th>WLP</th>
<th>CRS</th>
<th>WLP</th>
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<tr>
<td>$X_{f0}$</td>
<td>0.569***</td>
<td>0.691***</td>
<td>0.611***</td>
<td>0.657***</td>
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<tr>
<td></td>
<td>(0.0258)</td>
<td>(0.0281)</td>
<td>(0.0253)</td>
<td>(0.0281)</td>
</tr>
<tr>
<td>$K_{f0}^\text{int}$</td>
<td>0.084***</td>
<td>0.108***</td>
<td>0.098***</td>
<td>0.111***</td>
</tr>
<tr>
<td></td>
<td>(0.0061)</td>
<td>(0.0066)</td>
<td>(0.0060)</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>$X_{f0} \cdot K_{f0}^\text{int}$</td>
<td>0.155***</td>
<td>0.159***</td>
<td>0.149***</td>
<td>0.172***</td>
</tr>
<tr>
<td></td>
<td>(0.0142)</td>
<td>(0.0155)</td>
<td>(0.0140)</td>
<td>(0.0157)</td>
</tr>
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<td>Firm FE</td>
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<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Sector×Year FE</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.080</td>
<td>0.030</td>
<td>0.116</td>
<td>0.066</td>
</tr>
<tr>
<td>N. obs.</td>
<td>52,138</td>
<td>52,138</td>
<td>63,687</td>
<td>63,687</td>
</tr>
</tbody>
</table>

Table A.2: Panel Regressions: Firm Characteristics and Productivity.
Notes: Results for regression A.1. ***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.1$. Control by size included (not reported). CRS: Elasticities obtained using cost shares (constant returns to scale). WLP: Wooldridge (2011) estimation (decreasing returns to scale).

A.3 Robustness

Table A.3 augments the baseline panel regressions presented in Section 1.2 with an interaction between firm-level size and the commodity price shock. The purpose of this interaction is to check the robustness of my main results to a financial friction channel that affects differentially firms with different sizes. Column (1) in Table A.3 displays the baseline result. Columns (2)-(4) shows that the results survives to the introduction of these interactions.
Table A.3: Panel Regressions: Commodity Booms and Outcome Variables.
Notes: Results for regression ?? with additional controls. ***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.1$. The variables $SIZE_{f0}$ and $TFP_{f0}$ are constructed as firm $f$ quintile in the size and productivity distributions in its first period $t = 0$ in the sample. Size is measured as the number of workers, while firm-level productivity is estimated using the method of Wooldridge (2009).
A.4 General Equilibrium System

Endogenous (34) = \{C, C^X, C^M, Y^X, Y^C, r^k, w, p, p^X, p^R\} = 10


= \{V(z), V_{ij}(z), \mu(z), \bar{z}_d, \bar{z}_a, \bar{z}_x, \mathcal{M}, \mathcal{M}_e, \phi(\cdot), \Phi(\cdot), \mathcal{F}\} = 12

A.4.1 Household

\[ p_t = \left[ \chi \left( \frac{p_t^X}{p_t} \right)^{1-\epsilon} + (1 - \chi) \left( \frac{p_t^M}{p_t^M} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \] (A.2)

\[ \varphi L_t^i = C_t^{-\psi} w_t \] (A.3)

\[ C_t^X = \chi \left( \frac{p_t}{p_t^X} \right)^{\epsilon} C_t \] (A.4)

\[ C_t^M = (1 - \chi) \left( \frac{p_t}{p_t^M} \right)^{\epsilon} C_t \] (A.5)

\[ C_{t+1} = \left( \frac{p_{t+1}}{p_{t}} \right)^{1/\nu} C_t \] (A.6)

\[ 1 + \phi \left( \frac{K_{t+1}}{K_t} - 1 \right) = \beta \left[ r_{t+1}^k + 1 - \delta^k + \text{adj}_{t+1} \right] \] (A.7)

\[ \text{adj}_t = \phi \left( \frac{K_{t+1}}{K_t} \right) \left( \frac{K_{t+1}}{K_t} - 1 \right) - \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 \]

\[ K_{t+1} = (1 - \delta^k) K_t + I_t \] (A.8)

A.4.2 Exportable Goods

\[ \phi_{jt} = \left( \frac{r^k_{t}}{\alpha_j} \right)^{\alpha_j} \left( \frac{w_t}{1 - \alpha_j} \right)^{1-\alpha_j}, \quad j = l, h \] (A.9)
\[ \begin{align*}
V_t(z) &= \max\{V_{dlt}(z), V_{dht}(z), V_{xlt}(z), V_{xht}(z)\} \tag{A.10} \\
V_{dj}(z) &= \max\{0, \pi_{dj}(z; \alpha) + (1 - \delta)\beta V_{t+1}(z)\}, j = l, h \tag{A.11} \\
V_{xj}(z) &= \max\{0, \pi_{xj}(z) + \pi_{xj}(z) + (1 - \delta)\beta V_{t+1}(z)\}, j = l, h \tag{A.12} \\
V_{dlt}(z) &= 0 \tag{A.13} \\
V_{dlt}(z) &= V_{xlt}(z) \tag{A.14} \\
V_{xlt}(z) &= V_{xht}(z) \tag{A.15} \\
\int_{-\infty}^{\infty} V_t(z) g(z) dz &= \overline{f}_c + \phi_c \left[ \exp(\mathcal{M}_t - \overline{\mathcal{M}}_e) - 1 \right] \tag{A.16} \\
\mathcal{M}_{t+1} &= \left\{ \begin{array}{ll}
(1 - \delta)\mathcal{M}_t \mu_t(z) + \mathcal{M}_{t+1} g(z), & \text{if } z \geq z'_{dt+1} \\
0, & \text{otherwise}
\end{array} \right. \tag{A.17} \\
Y^X_t &= \left[ \mathcal{M}_t \left( \int_{-\infty}^{z_{at}} (q_{dlt}(z))^{\rho} \mu_t(z) dz + \int_{-\infty}^{\infty} (q_{dlt}(z))^{\rho} \mu_t(z) dz \right) \right]^{\frac{1}{\rho}} \tag{A.19} \\
X^X_t &= \mathcal{M}_t \left[ \int_{-\infty}^{z_{at}} p_{xlt}(z) q_{xlt}(z) \mu_t(z) dz + \int_{-\infty}^{\infty} p_{xlt}(z) q_{xlt}(z) \mu_t(z) dz \right] \tag{A.20} \\
L^X_t &= L^X_{dlt} + L^X_{dht} + L^X_{xlt} + L^X_{xht} \tag{A.21} \\
K^X_t &= K^X_{dlt} + K^X_{dht} + K^X_{xlt} + K^X_{xht} \tag{A.22}
\end{align*} \]

A.4.3 Commodity Goods

\[ \begin{align*}
w_t &= p_t^C (1 - \alpha^R - \alpha^C) \frac{Y_t^C}{L_t^C} \tag{A.23} \\
r_t^k &= \frac{p_t^C \alpha^C Y_t^C}{K_t^C} \tag{A.24}
\end{align*} \]
\[ p_t^R = p_t^{Co} \alpha R Y_t^C \]  
\[ Y_t^C = \left[ \left( \frac{R}{\alpha R} \right)^{\alpha R} \left( K_t^C \right)^{\alpha C} \left( L_t^C \right)^{1-\alpha R-\alpha C} \right]^\eta \] (A.25-26)

A.4.4 Aggregation

\[ L_t = L_t^X + L_t^C \] (A.27)
\[ K_t = K_t^X + K_t^C \] (A.28)
\[ Y_t^X = C_t^X \] (A.29)
\[ B_{t+1} = (1 + r^*)B_t + TB_t \] (A.30)
\[ TB_t = X_t - M_t \] (A.31)
\[ X_t = p_t^{Co}Y_t^C + X_t^X \] (A.32)
\[ M_t = C_t^M + I_t + \Phi_t + \mathcal{F}_t \] (A.33)
\[ \Phi_t = \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \] (A.34)
\[ \mathcal{F}_t = \mathcal{M}_t f d + \mathcal{M}_t p x f x + \mathcal{M}_t p a f a + \mathcal{M}_t e f e (\mathcal{M}_e t) \] (A.35)

A.4.5 Transition Algorithm

- **Setup**: Economy is in steady state until \( t = 0 \). Boom-bust cycle \( \{p_t^{Co}\}_{t=1}^T \) is revealed in \( t = 1 \).

- **Initial State**: \( \{B_1, K_1, \mathcal{M}_1, \mu_1(z)\} \) is given.

- **Outer Loop**: Guess \( C_1 \). Bisection update using transversality condition.
• **Inner Loop**: Guess \( \{w_t, p_t^X, K_{t+1}\}_{t=1}^T \).

  - **Households**:
    * Get \( \{p_t\}_{t=1}^T \) using (A.2).
    * Get \( \{L_t\}_{t=1}^T \) using (A.3).
    * Get \( \{C_{t+1}\}_{t=1}^T \) using (A.6).
    * Get \( \{C_t^X, C_t^M\}_{t=1}^T \) using (A.4), (A.5).
    * Get \( \{r_{t+1}\}_{t=1}^T \) using (A.7).
    * Get \( \{I_t\}_{t=1}^T \) using (A.8).
  
  - Get \( \{\phi_t(\alpha)\}_{t=1}^T, \alpha = \alpha_l, \alpha_h \), using (A.9).
  
  - Set period \( t = T \) (final steady state) value function vector \( V_T(z) \).

  - **Iterate Backward**: For \( t = T - 1 \colon -1 : 1 \)
    
    * Compute value functions and cutoffs via (A.10)-(A.15).
    * Use (A.16) to get the mass of entrants \( M_{et} \).

  - **Iterate Forward**: For \( t = 1 : T \)
    
    * Get mass \( M_t \) and distribution \( \mu_t(z) \) using (A.17)-(A.18).

  - **Aggregation**:
    
    * Get \( \{X_t^X, K_t^X, L_t^X, Y_t^X\}_{t=1}^T \) using (A.20), (A.21), (A.22), and (A.29).
    * Get \( \{r_t^R, L_t^C, K_t^C, Y_t^C\}_{t=1}^T \) using (A.23)-(A.26).

  - **Model-Implieds**:
    
    * \( \{w_t\}_{t=1}^T \) using (A.27). (Solver)
\* \{p_t^X\}_{t=1}^T \text{ using (A.19). (Analytic)}

\* \{K_{t+1}\}_{t=1}^T \text{ using (A.28). (Analytic)}

- **Iterate** over \{w_t, p_t^X, K_{t+1}\}_{t=1}^T until convergence.

- **Fixed Costs:** Get \{\Phi_t, \mathcal{F}_t\}_{t=1}^T \text{ using (A.34), (A.35)}.

- **Trade Balance:** Get \{X_t, M_t, TB_t\}_{t=1}^T \text{ using (A.32), (A.33), (A.31), respectively.}

- **NFA:** Get \{B_{t+1}\}_{t=1}^T \text{ from (A.30).}

- **Iterate:** over \(C_1\) until \{B_{t+1}\}_{t=1}^T \text{ is stable in the long run.}

### A.5 Steady State System

\text{Endogenous (35)} = \{C, C^X, C^M, Y^X, Y^C, Y, r^k, w, p, p^X, p^R\} = 11

\begin{align*}
= \{V(z), V_d(z; \alpha), V_x(z; \alpha), \mu(z), \bar{z}_d, \bar{z}_a, \bar{z}_x, \mathcal{M}, \mathcal{M}_e, \phi(\alpha), \mathcal{F}\} & = 11
\end{align*}

Given \(r^*\), \(\beta = \frac{1}{(1+r^*)}\) from Euler. Solve given \(B\).

### A.5.1 Household

\[p = \left[\chi \left(p^X\right)^{1-\epsilon} + (1 - \chi) \left(p^M\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \quad \text{(A.36)}\]
\[ C^X = \chi \left( \frac{p}{p^X} \right)^\epsilon C \]  
\[ C^M = (1 - \chi) \left( \frac{p}{p^M} \right)^\epsilon C \]  
\[ \varphi L^e = C^{-\nu}w \]  
\[ r^k = r^* + \delta \]  
\[ I = \delta^kK \]  

A.5.2 Exportable Goods

\[ \phi_j = \left( \frac{r^k}{\alpha_j} \right)^{\alpha_j} \left( \frac{w}{1 - \alpha_j} \right)^{1 - \alpha_j} \]  
\[ V(z) = \max\{V_{dl}(z), V_{dh}(z), V_{xl}(z), V_{xh}(z)\} \]  
\[ V_{dj}(z) = \max \left\{ 0, \frac{(1 + r^*)}{(\delta + r^*)} \pi_{dj}(z) \right\} \]  
\[ V_{xj}(z) = \max \left\{ 0, \frac{(1 + r^*)}{(\delta + r^*)} [\pi_{dj}(z) + \pi_{xj}(z)] \right\} \]  
\[ V_{dl}(\overline{z}_d) = 0 \]  
\[ V_{dl}(\overline{z}_x) = V_{xl}(\overline{z}_x) \]  
\[ V_{xl}(\overline{z}_a) = V_{xh}(\overline{z}_a) \]  
\[ \int_{\overline{z}_d}^{\infty} V(z)g(z)dz = \overline{f}_e + \phi_e \left[ \exp \left( M_e - \overline{M}_e \right) - 1 \right] \]  
\[ \mu(z) = \begin{cases} \frac{g(z)}{1 - G(\overline{z}_d)}, & \text{if } z \geq \overline{z}_d \\ 0, & \text{otherwise} \end{cases} \]  
\[ \delta M = [1 - G(\overline{z}_d)] M_e \]
\( Y^X = \left[ M \left( \int_{z_a}^{a} (q_{dl}(z))^\rho \mu(z) dz + \int_{z_a}^{\infty} (q_{dh}(z))^\rho \mu(z) dz \right) \right]^{\frac{1}{\rho}} \) \hspace{1cm} (A.52)

\( p^X = \frac{1}{\rho} \phi_i^{1-\sigma} \cdot \left( M \int_{z_a}^{a} z^{\sigma-1} \mu(z) dz + \left( \frac{\phi_h}{\kappa} \right)^{1-\sigma} \cdot \left( M \int_{z_a}^{\infty} z^{\sigma-1} \mu(z) dz \right) \right)^{\frac{1}{1-\sigma}} \)

\( X^X = M \left[ \int_{z_a}^{a} p_{xl}(z) q_{xl}(z) \mu(z) dz + \int_{z_a}^{\infty} p_{xh}(z) q_{xh}(z) \mu(z) dz \right] \) \hspace{1cm} (A.53)

\( L^X = L_{dl}^X + L_{dh}^X + L_{xl}^X + L_{xh}^X \) \hspace{1cm} (A.54)

\( K^X = K_{dl}^X + K_{dh}^X + K_{xl}^X + K_{xh}^X \) \hspace{1cm} (A.55)

### A.5.3 Commodity Goods

\( w = p^{Co}(1-\alpha^R-\alpha^C) \frac{Y^C}{L^C} \) \hspace{1cm} (A.56)

\( r^k = p^{Co} \alpha^C \frac{Y^C}{K^C} \) \hspace{1cm} (A.57)

\( p^R = p^{Co} \alpha^R \frac{Y^C}{R} \) \hspace{1cm} (A.58)

\( Y^C = \left[ (R)^{\alpha^R} (K^C)^{\alpha^C} (L^C)^{1-\alpha^R-\alpha^C} \right]^\eta \) \hspace{1cm} (A.59)

### A.5.4 Aggregation

\( L = L^X + L^C \) \hspace{1cm} (A.60)

\( K = K^X + K^C \) \hspace{1cm} (A.61)

\( Y^X = C^X \) \hspace{1cm} (A.62)

\( TB = -r^* \cdot B \) \hspace{1cm} (A.63)
\[ TB = X - M \]  \hspace{1cm} (A.64)

\[ X = p^{Co}Y^C + X^X \]  \hspace{1cm} (A.65)

\[ M = C^M + I + \mathcal{F} \]  \hspace{1cm} (A.66)

\[ \mathcal{F} = \mathcal{M}_{fd} + \mathcal{M}_{p_x f_x} + \mathcal{M}_{p_o f_o} + \mathcal{M}_{x f_x}(\mathcal{M}_e) \]  \hspace{1cm} (A.67)

\[ Y = p^X Y^X + X^X + p^{Co}Y^C - \mathcal{F} \]  \hspace{1cm} (A.68)

\[ YCY = \frac{p^{Co}Y^C}{Y} \]  \hspace{1cm} (A.69)

\[ TBY = \frac{TB}{Y} \]  \hspace{1cm} (A.70)

A.5.5 Steady State Solution Algorithm

Targets = \{r^*, TBY, YCY, L\}

Parameters = \{\beta, B, R, \varphi\}

- \( p^{Co} \) is exogenously given.

- Assumption: No congestion cost in this steady state.

- Given \( r^* \), \( \beta = \frac{1}{1+r^*} \) from Euler equation (not listed).

- Guess \( (w, p^X, C) \).

- Residuals: Free entry condition in X sector (A.49), GDP definition (A.68), and balance of payments (A.64).

- Get implied \((r^k)\) directly from (A.40).
• Get \( (Y^C, TB) \) from (A.69) and (A.70). Get \( \overline{B} \) from (A.63).

• Get \( (L^C, K^C) \) from (A.57) and (A.56), respectively.

• Get \( \overline{R} \) from (A.59) and \( p^R \) from (A.58).

• Get \( (p, C^X, C^M, Y^X) \) from (A.36), (A.37), (A.38), and (A.62).

• Get \( (\phi(\alpha)) \) from (A.42).

• Get values and cutoffs from (A.43)-(A.48).

• Get distribution from (A.50).

• Get \( (\mathcal{M}, \mathcal{M}_e) \) from (A.52), (A.51).

• Get \( (X^X, L^X, K^X) \) from (A.53), (A.54), (A.55).

• Get \( K \) from (A.61).

• Get \( \overline{L} \) from (A.60).

• Get implied \( \psi \) from (A.39).

• Get \( I \) from (A.41)

• Get \( (\mathcal{F}) \) from (A.67).

• Get \( (X, M) \) from (A.65), (A.66).

• Residuals: Free entry condition in \( X \) sector (A.49), GDP definition (A.68), and balance of payments (A.64).

• Iterate over \( (w, p^X, C) \) until convergence.
A.6 Analytical Cutoffs

Combining the static versions of equations 1.15-?? in the main text, we can obtain analytical expressions for the long-run productivity thresholds that determine self-selection into the capital-intensive technology and into exporting. The domestic cutoff can be written as:

$$z_d = \left[ \frac{\sigma f_d}{p^\sigma \cdot C} \right]^{\frac{1}{\sigma-1}} \left( \frac{\phi_l}{\rho} \right). \quad \text{(A.71)}$$

Given the proposed sorting pattern $z_d < z_x < z_a$, the marginal exporter uses the Low-K technology. The condition that determines the exporting cutoffs is given by $\pi_{dl}(z_x) = \pi_{xl}(z_x)$, which implies:

$$z_x = \left[ \frac{\sigma f_x}{\tau^{1-\sigma \gamma}} \right]^{\frac{1}{\sigma-1}} \left( \frac{\phi_l}{\rho} \right). \quad \text{(A.72)}$$

Finally, the lowest productivity type that is able (and willing) to use the High-K technology is an exporter. Thereby, the adoption cutoff satisfies $\pi_{xl}(z_a) = \pi_{xh}(z_a)$. Solving for $z_a$ yields:

$$z_a = \left[ \frac{\sigma f_a}{p^\sigma C + \tau^{1-\sigma \gamma}} \right]^{\frac{1}{\sigma-1}} \left[ \left( \frac{\phi_h}{\kappa} \right)^{1-\sigma} - \phi_l^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \frac{1}{\rho}. \quad \text{(A.73)}$$

**Testable Predictions.** Positive commodity price (windfall) shocks induce higher consumption ($\frac{\partial C}{\partial p}>0$), currency appreciation ($\frac{\partial p}{\partial p^\sigma}>0$), and an increase in the rental rate of capital relative to the cost of labor $\frac{\partial (r^k/w)}{\partial p^\sigma}>0$. These are, the
wealth or demand channel, the substitution or exchange rate channel, and the cost of capital channel, respectively. These well-known basic correlations can be easily shown in the full general equilibrium model presented in Section 1.3. In this section, I take them as given.

**Prediction 1 (Intensive margin).** For any given exporting type, export sales shrink relative to domestic sales after a positive windfall shock.

From equation 1.15, it can be seen that, for any given exporting type (either Low-K or High-K), the ratio of export sales to domestic sales, $\frac{R_{xd}}{\gamma _{1}}$, is given by:

$$R_{xd} = \frac{\gamma _{1}^{1-\sigma}}{(p)^{\sigma}C}$$

(A.74)

Given that $\frac{\partial C}{\partial p^{\sigma}} > 0$ and $\frac{\partial p}{\partial p^{\sigma}} > 0$, then it must be the case that $\frac{\partial R_{xd}}{\partial p^{\sigma}} < 0$.

Note that the shrinking of exporters relative to non-exporters is a combination of both demand and exchange rate channels. The intuition is as follows. First, the economy is richer, so the household increases demand immediately to smooth consumption (income effect). Second, higher demand pushes domestic prices up (currency appreciation), thereby leading to further adjustments in favor of purely-domestic producers and at the expense of exporters (substitution effect).¹

**Prediction 2 (Extensive margin: Exporters vs Non-exporters).** The exporting cutoff increases after a positive windfall shock.

From equation A.72 it is direct that higher costs during the boom ($\phi_t$) unam-

---

biguously leads to a one-to-one increase in the exporting cutoff $z_x$, therefore inducing some previously profitable exporter types to stop selling their varieties abroad. Notice that, under monopolistic competition, increasing composite costs are passed through to consumers via the pricing rule $p$, ultimately raising the average basket price $p = \left[ \int_z (p_d(z))^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}$, that is, a real appreciation of the domestic currency.

In essence, firm exit from exporting is associated with the exchange rate channel.

**Prediction 3 (Extensive margin: High-K vs Low-K).** The adoption cutoff increases after a positive windfall shock.

From equation A.73 it is direct that both larger aggregate demand ($\uparrow C$) and currency appreciation ($\uparrow p$) push $z_a$ down. Then, the overall effect depends crucially on the cost of capital channel, that is, on the term $\Omega \equiv \left( \phi_h^1 - \phi_l^1 \right)$ in equation A.73. It can be shown that $\Delta \Omega$ is proportional to $(\alpha_h - \alpha_l) (\Delta r^k - \Delta w)$, which is positive as long as $\Delta r^k > \Delta w$. Intuitively, if the cost of capital increases more than the cost of labor, then capital-intensive firms face a cost disadvantage, and some of them will be forced to downgrade into the less profitable technology. If the cost channel is large enough, that is, if $(\alpha_h - \alpha_l)$ is large, it may offset the effects of wealth and substitution channels, leading to an increase in $z_a$. 

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Appendix B: Appendices for Chapter 2

B.1 Aggregate Resource Constraint

In the aggregate, $Q_tK_t$ is the total value of capital acquired by final goods producers in period $t$, while $Q_tS_t$ is the total value of claims issued against that capital (total credit in the economy). By arbitrage, the capital market clearing condition implies:

$$K_t = S_t$$  \hspace{5em} (B.1)

To get the economy-wide resource constraint, we start with the budget constraint of the households:

$$C_t + D_t + T_t = W_tH_t + R_{t-1}D_{t-1} + \Sigma_t$$ \hspace{5em} (B.2)

where $\Sigma_t$ includes the profits from the ownership of productive firms (final good producers, $\Pi^C_t$, and capital goods producers $\Pi^K_t$) and the net transfers between households and banks (exiting and new). Therefore, we have:

$$\Sigma_t = \Pi^C_t + \Pi^K_t + (1 - \theta) \left[ R^K_{t-1}Q_{t-1}K_{t-1} - R_{t-1}D_{t-1} \right] - (1 - \theta)Q_{t-1}K_{t-1}$$ \hspace{5em} (B.3)
\[
\Pi_t^C = Y_t - W_t H_t - Z_t \Psi_t K_{t-1}
\]
\[
\Pi_t^K = Q_t K_t - Q_t (1 - \delta) \Psi_t K_{t-1} - I_t.
\]

Next, combining the balance sheet and the law of motion for net worth of the aggregated banking sector, and imposing the capital market clearing (B.1), we have:

\[
Q_t K_t = D_t + N_t = D_t + \theta \left[ R^K_t Q_{t-1} K_{t-1} - R_{t-1} D_{t-1} \right] + (1 - \theta) \iota Q_{t-1} K_{t-1} (B.4)
\]

where \( R^K_t = \left[ \frac{Z_t + (1 - \delta) Q_t}{Q_{t-1}} \right] \Psi_t \). Combining (B.2), (B.4), the budget constraint of the government (2.31), and (B.3) yields:

\[
C_t + I_t + G_t = [Z_t + (1 - \delta) Q_t] \Psi_t K_{t-1} + Y_t - Z_t \Psi_t K_{t-1} - Q_t (1 - \delta) \Psi_t K_{t-1}
\]

which implies:

\[
C_t + I_t + G_t = Y_t. \tag{B.5}
\]

**B.2 Competitive Equilibrium**

The rational expectations equilibrium of the model is a set of sequences for the 17 endogenous variables

\[
\{ C_t, I_t, Y_t, H_t, K_t, N_t, P_t, R_t, R^K_t, Q_t, \phi_t, \psi_t, \xi_t, SPR_t, G_t, W_t, Z_t \}_{t=0}^\infty
\]

such that for given initial conditions and exogenous sequences

\[
\{ A_t, \varphi_t, g_t, \Psi_t \}_{t=0}^\infty
\]
the following conditions are satisfied:

- Households maximize utility subject to their budget constraint, that is, the following equations hold:

\[ 1 = \mathcal{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_t \right] \quad (B.6) \]
\[ \varphi_t \chi H_t^\zeta = C_t^{-\gamma} W_t \quad (B.7) \]

- Banks maximize their expected terminal wealth subject to the IC constraint, that is, the following equations hold:

\[ \phi_t = \frac{Q_t K_t}{N_t} \quad (B.8) \]
\[ \psi_t = \frac{\mathcal{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_t \right] [1 - \theta + \theta \psi_{t+1}]}{1 - \xi_t} \quad (B.9) \]
\[ \mu \xi_t = \mathcal{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] [1 - \theta + \theta \psi_{t+1}] \left[ R_{t+1}^K - R_t \right] \quad (B.10) \]
\[ 0 = [\psi_t - \mu \phi_t] \xi_t \quad (B.11) \]
\[ N_t = \theta R_t^K Q_{t-1} K_{t-1} + P_{t-1} \quad (B.12) \]
\[ P_t = \theta [R_t (N_t - Q_t K_t)] + (1 - \theta) I_t Q_t K_t \quad (B.13) \]

- Capital producers maximizes profits subject to their technology, that is, the following equations hold:

\[ K_t = (1 - \delta) \Psi_t K_{t-1} + \left[ a_1 \left( \frac{I_t}{\Psi_t K_{t-1}} \right)^{1-e} + a_2 \right] \Psi_t K_{t-1} \quad (B.14) \]
\[ Q_t = \left[ \frac{I_t}{\delta \Psi_t K_{t-1}} \right]^e \quad (B.15) \]

- Final good producers maximizes profits subject to their technology, and markets clear, that is:
\[ R_t^K = \left[ \frac{Z_t + (1 - \delta)Q_t}{Q_{t-1}} \right] \Psi_t \] (B.16)

\[ Y_t = A_t (\Psi_t K_{t-1})^\alpha H_t^{1-\alpha} \] (B.17)

\[ W_t = (1 - \alpha) \frac{Y_t}{H_t} \] (B.18)

\[ Z_t = \alpha \frac{Y_t}{\Psi_t K_{t-1}} \] (B.19)

\[ Y_t = C_t + I_t + G_t \] (B.20)

\[ G_t \equiv \left( 1 - \frac{1}{g_t} \right) Y_t \] (B.21)

\[ SPR_t \equiv \varepsilon_t R_{t+1}^K - R_t \] (B.22)

### B.3 Solution Algorithm

I solve the model using a global approximation method based on Chebyshev approximations of decision rules along the lines of Judd (1992) [64]. Following Aruoba, Cuba-Borda, and Schorfheide (2018) [8], the solution algorithm involves two non-standard tools: (i) a piecewise smooth representation of the approximated decision rules, and (ii) an iterative procedure of choosing grid points based on a clustered-grid-algorithm (CGA) proposed by Judd, Maliar, and Maliar (2010) [65].

The set of equilibrium conditions for \{C_t, I_t, Y_t, H_t, K_t, N_t, P_t, R_t, Q_t, \phi_t, \psi_t, \xi_t\} can be written as follows:

\[ 1 = \varepsilon_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_t \right] \] (B.23)

\[ \psi_t = \frac{\varepsilon_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_t \right] [1 - \theta + \theta \psi_{t+1}]}{1 - \xi_t} \] (B.24)
\[ \mu \xi_t = \mathcal{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \left[ 1 - \theta + \theta \psi_{t+1} \right] \left( \frac{\alpha Y_{t+1}}{\Psi_{t+1} K_t} + (1 - \delta)Q_{t+1} \right) \Psi_{t+1} - (B.25) \]

\[ 0 = \left[ \psi_t - \mu \phi_t \right] \xi_t \quad (B.26) \]

\[ H_t = \left[ \frac{(1 - \alpha) A_t (\Psi_t K_{t-1})^\alpha}{\chi \varphi_t C_t^\gamma} \right] \frac{1}{\alpha + \gamma} \quad (B.27) \]

\[ \phi_t = \frac{Q_t K_t}{N_t} \quad (B.28) \]

\[ N_t = \theta R_t^K Q_{t-1} K_{t-1} + P_{t-1} \quad (B.29) \]

\[ P_t = \theta [R_t (N_t - Q_t K_t)] + (1 - \theta) t Q_t K_t \quad (B.30) \]

\[ K_t = (1 - \delta) \Psi_t K_{t-1} + \left[a_1 \left( I_t \frac{L_t}{\Psi_t K_{t-1}} \right)^{1 - \theta} + a_2 \right] \Psi_t K_{t-1} \quad (B.31) \]

\[ Q_t = \left[ \frac{I_t}{\delta \Psi_t K_{t-1}} \right]^\theta \quad (B.32) \]

\[ Y_t = A_t (\Psi_t K_{t-1})^\alpha H_t^{1 - \alpha} \quad (B.33) \]

\[ Y_t = C_t + I_t + \left( 1 - \frac{1}{g_t} \right) Y_t \quad (B.34) \]

The model has two endogenous and four exogenous state variables: \( S_t = \{ K_{t-1}, P_{t-1}; A_t, \eta_t, g_t, \Psi_t \} \). I approximate the decision rules for consumption \( C_t \), the risk-free interest rate \( R_t \), and the banking sector Tobin’s q ratio \( \psi_t \), in a piecewise fashion as follows. Define the set of approximated control variables in period \( t \) as \( \mathcal{X}_t = \{ C_t, R_t, \psi_t \} \). The piecewise smooth functions are parametrized by the set \( \Theta = \{ \Theta_s^X, \Theta_b^X \} \) as follows:

\[ \mathcal{X}_t = (1 - I_b) \cdot \Theta_s^X \mathcal{T}(S_t) + I_b \cdot \Theta_b^X \mathcal{T}(S_t) \]

where \( I_b \) is an indicator function that takes the value of one when the economy is under the binding regime \( (\xi_t > 0) \) and zero otherwise, and \( \mathcal{T}(.) \) is a vector collecting complete combinations of Chebyshev polynomials. The controls \( \mathcal{X}_t = \)
\{C_t, R_t, \psi_t\} solve the set of residual functions given by:

\begin{align*}
\mathcal{R}_{1t} &= \mathcal{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_t \right] - 1 \\
\mathcal{R}_{2t} &= \mathcal{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \left[ 1 - \theta + \theta \psi_{t+1} \right] \left[ \left( \frac{\alpha \psi_{t+1} K_t}{Y_t} + (1 - \delta) Q_{t+1} \right) \Psi_{t+1} - R_t \right] - \mu \xi_t \\
\mathcal{R}_{3t} &= \mathcal{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \left[ 1 - \theta + \theta \psi_{t+1} \right] - \psi_t
\end{align*}

I assess the accuracy of the numerical solution by computing Euler equations errors. Figure B.1 show a histogram of the three Euler equation residuals computed over a CGA grid based on a long simulation of the model. I express the errors in decimal log scale as is common in the literature. The Euler errors are small, averaging -4.9, -4.8, and -4.5 for \( \mathcal{R}_1, \mathcal{R}_2, \) and \( \mathcal{R}_3 \), respectively.
Figure B.1: Histogram of Euler Equation Errors (log10(abs(EE)) scale). Pooling years.
Notes: The histograms report the Euler equation errors over a simulation of 10,000 periods in decimal log basis. The dotted vertical line corresponds to the mean of the residuals over the simulation.
Figure B.2: Baseline Model: Historical Decomposition: Real Variables and Spread

Notes: Output, investment, and hours worked are in quarterly terms, expressed in percent deviations from their ergodic means (solid line). Spread is expressed in annualized percent terms. The solid lines correspond to the model-implied filtered observable variables, which are up to a small measurement error equal to the data described above. The bars decompose each filtered variable into the contributions of each structural shock. The gray areas indicate NBER recession dates.
Figure B.3: Pure RBC Model: Historical Decomposition: Real Variables and Spread

Notes: See notes from Figure B.2.
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