ABSTRACT

Title of dissertation: GENERALIZED NATURAL INFLATION AND THE QUEST FOR COSMIC SYMMETRY BREAKING PATTERS

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We present a two-field model that generalizes Natural Inflation, in which the inflaton is the pseudo-Goldstone boson of an approximate symmetry that is spontaneously broken, and the radial mode is dynamical. Within this model, which we designate as "Generalized Natural Inflation", we analyze how the dynamics fundamentally depends on the mass of the radial mode and determine the size of the non-Gaussianities arising from such a scenario.

We also motivate ongoing research within the coset construction formalism, that aims to clarify how the spontaneous symmetry breaking pattern of spacetime, gauge, and internal symmetries may allow us to get a deeper understanding, and an actual algebraic classification in the spirit of the so-called "zoology of condensed matter", of different possible "cosmic states", some of which may be quite relevant for model-independent statements about different phases in the evolution of our universe. The outcome of these investigations will be reported elsewhere.

GENERALIZED NATURAL INFLATION AND THE QUEST FOR COSMIC SYMMETRY BREAKING PATTERNS

by

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A mis padres, Kattya y Luis.

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I am grateful to my family, my parents, my sister, my grandmothers. They are me, and I am them.

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Chapter 1

Introduction

1.1 Some Lucubrations...

[1] The question of "**Why** is there anything at all?", or, "**Why** is there something rather than nothing?" [2] has been raised by many great philosophers, including Gottfried Wilhelm Leibniz [2], Martin Heidegger [3, 4] and Ludwig Wittgenstein [5]. It is of such importance, that Heidegger himself called it *the fundamental question of metaphysics.* For those of us who often forget what the word metaphysics is supposed to mean, the encyclopedia of our era, Wikipedia, may enlighten us:

"Metaphysics is a branch of philosophy that explores the nature of being, existence, and reality. Metaphysics seeks to answer, in a 'suitably abstract and fully general manner', the questions:

1. What *is there?*

2. And what is it *like?*

Topics of metaphysical investigation include existence, objects and their properties, space and time, cause and effect, and possibility."

It is reasonable to feel somehow overwhelmed when faced to all these highbrow concepts. Our natural instinct of looking for some firm ground compels us to drop the prefix "meta" and consider the definition of physics itself:

[1] Warning: the reader who is mainly interested in getting a quick glimpse on what this dissertation is about, may want to skip these "lucubrations" altogether, and jump directly to section 1.2.

[2] A little more "down to earth" but in the same vein kind of question has been addressed lately by Nima Arkani-Hamed and others, namely "**Why** is there a macroscopic universe?" [1]. Watch also this.

"Physics is the natural science that studies matter and its motion and behavior through space and time and that studies the related entities of energy and force. Physics is one of the most fundamental scientific disciplines, and its main goal is to understand **how** the universe behaves."

You see, we could spend our whole lives arguing about the deep meaning and "fundamentality" of each of the words entering in both definitions, which surely are definitions themselves, and to be honest, we do not know if there is ever an ending to such an endeavor. In some sense, this task seems analogous to the insipid discussion of how "pure" a field is compared to others, as the little cartoon in Figure 1.1 tries to suggest.

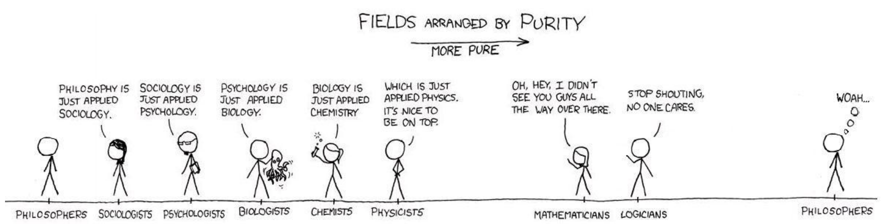


Figure 1.1: The purity of fields.

We are not interested (or, for that matter, *trained to*) delve ourselves into the deep waters of Leibniz, Heidegger, Wittgenstein, and many others. Let us just make an act of faith and trust that Sean Carroll is doing a good job "defending the flag of physics" in that trench [6]. However, one thing we would like to emphasize, is **how** the word **why** and **why** the word **how** make their appearence in each of the contexts we just revisited. To make things more interesting, let us introduce two old friends, the "pragmatic" and the "romantic", which happen to be well-trained professional physicists, and have one or two things to say about all this.

My take on this subject is quite simple - the pragmatic declares - in order to discriminate the essence of a question do as follows: in any place you have a question of the form "Why does..." you try to substitute it with "How does..."

instead. Any portion of "Why does..." not answered by "How does..." is the philosophical/non-scientific part of the question. For example, "Why does 1 + 2 = 3?" may be substituted with "How does 1 + 2 = 3?" and answered adequately. Depending on how deep and technical you may want to get, the answer may delve into the definition of addition, various axioms, Giuseppe Peano's arithmetic, and so on. If, after all of that, you still have a question about "why", then your question is of a philosophical nature. By the same token, physics, which is firmly rooted in the scientific method, mathematics, and experiment, simply cannot answer the philosophical portions of questions. Nevertheless - the pragmatic keeps arguing - sometimes why questions, within the realm of physics, do find good answers, up to a point. If a layperson asks

-"Why is the sky blue?"

We may reply

-"Because of John William Strutt's so-called Rayleigh scattering, that is, because a law of physics describes this behavior."

However, the layperson is not satisfied

-"Then, why does Rayleigh scattering work?"

-"Well, you may actually derive it from James Clerk Maxwell's equations. In other words, this law is a consequence of a more fundamental law".

The layperson, out of genuine curiosity or just because she is a smart aleck (it really does not matter), hits with a tough one

-"Why do Maxwell's equations work?"

We have reached the point where you cannot answer [3]. You can just point to ex-

[3] The expert will say, "Well, there is a remnant $U(1)$ gauge symmetry after spontaneous symmetry breaking within electro-weak theory, which itself is embedded in the fabulous Standard Model (SM) of particle physics". Then Mr.Wise-guy may ask "Why does the SM has the very specific gauge symmetry group $SU(3) \times SU(2) \times U(1)$?, Why are there three generations of leptons and quarks?, Why...?". Although some of these questions are non-trivial, insightful, and may be actually pushing forward modern theoretical advances developed by brilliant minds around the globe, we hope you see our point. It is probable that Sarah would not.

perimental evidence. These are fundamental laws from which we start, the same way mathematicians start from axioms and derive theorems. Or perhaps, these are the most fundamental laws we have worked so far.

This is indeed an interesting point of view. We still have to learn what the romantic has to say though.

Frankly, it is not that difficult to defend the role of "why", the romantic firmly states. Historically, plenty of sound scientific questions have begun with "why". Michael Faraday wondered *why* current deflects a compass needle, and *why* the galvanometer attached to a circuit jumped whenever he closed the switch of another circuit. Many scientists of the early XX century asked *why* beta decay appeared to violate otherwise sacrosanct conservation laws. Isaac Newton asked *why* Johannes Kepler's laws were so accurate. None of these questions can be adequately rephrased as "how" questions because they all arise precisely from a deep and thorough understanding of "how", an understanding left frustrated by inadequate explanation. For this reason, the questions of curiosity that challenge the limits of our theories and guess at what might lie beyond, are exactly those which allow science to grow and flourish. As Thomas Kuhn might put it [7]: "How" is *status quo* science, "journeyman" science, science "as usual"; "why" is fringe science, confrontational science, paradigm shift.

The pragmatic remains in silence for a few of seconds that feel like forever. He is quite proud, and the romantic does have several good points. However, he still wants to convey some ideas himself.

Let us start from scratch, the pragmatic insists. Physics is a science that has a large body of observations, and a limited number of mathematical models/theories that aim to organize and explain those observations and, very importantly, get validated by predicting the behavior of new observations. Mathematical theories start with axioms and some tools that develop theorems from those axioms and then various setups may be examined. For example, Euclidean geometry starts with axioms and ends with being able to predict and design complicated geometrical shapes. One may start asking why the sum of angles in a triangle is 180°, and one may prove it using the tools. If one goes further up in the why questions, one ends up with the axioms. One could as well ask "how" one gets 180° for the sum of the angles of the triangle, and then the "why" goes to "why start with these axioms". Physical theories, in addition to the mathematical construct, have equivalent to axioms; the so-called postulates. These have been postulated because of the need for the mathematical model to agree with measurements and data in general. For example, Werner Heisenberg's uncertainty principle, which is at the heart of quantum phenomena, may be derived using the axioms of quantum mechanics, such as Max Born's "rule" [4]. Therefore, in a similar manner as in the above mathematical example, all the "why" questions in physics are really answered as "how" one goes from the axioms and postulates to the specific observational data or predictions. The "why" questions end up on the axioms for the mathematics, and postulates for physics, and the answer then is "because" these basic assumptions/postulates are necessary to fit our mathematical model to the existing data, and give us confidence in predictions for new observations. The only answer to "why" is this "because", data says so. I see you frowning romantic friend, but let me add one last thing before I yield the floor to you again. Once one has a theory, and physics is really, as I stated before, a mathematical theory that organizes known data in order to be able to predict future unmeasured ones, the "how" question gives a *causal* path in our understanding of how the final data/observations happened and how the predicted ones will appear. Why questions address the existential state. When we have no theory and have an ob-

[4] Within the so-called "Copenhagen interpretation of quantum mechanics", Born rule is a postulate. There is a whole community of researchers, too many to cite, that try to derive it from more primitive principles, within other interpretations.

servation we start with “Why...”, because the observation exists. When a theory “forms” then it is the causal path that is sought and “why” goes up the mathematical ladder by “how mathematically this happens” transferring the existential question to the axioms and postulates.

The romantic is impressed, as he has never associated pragmatism, in the colloquial sense, with deep thoughts like these [5]; I have been prejudiced he thinks to himself. After taking a deep breath, he replies.

The truth is that it is disingenuous to sacrifice “why” on the altar of “how”. Forcing our language into some tightly defined pen where only “how” is allowed is simply a false dichotomy born of a mistaken appreciation of natural language. “Why” questions challenge theories or speculate about possibilities. They allow us to address the most difficult issues of science when we run up against the boundaries of our knowledge. Furthermore, they are useful when the problem at hand is characterized by hidden information or unknown parameters which are strongly affecting the experiment. For instance, in the beta decay example, it was not possible to ask “How do neutrinos affect beta decay?”, because the idea of neutrinos had not been invented yet; in fact “neutrinos” is precisely the answer to the question “Why does beta decay seem to violate conservation of momentum?”. But, of course, neutrinos could only be postulated because so many scientists asked the questions “How does beta decay behave under these conditions?” or “How does it behave under those conditions?”. By accruing data, which are the answers to “how” questions, they were able to identify discrepancies between observation and expectation which required new theories and creative thinking to adequately explain.

[5] The reader should be warned here; our pragmatic and romantic friends were dubbed this way just due to a lack of imagination, and in order to express their different views on the subject. Pragmatism is a real philosophical tradition that still lives on, while Romanticism was an artistic, literary, musical, and intellectual movement that peaked in the second half of the XIX century. Despite several coincidences, our characters are not tied to historical backgrounds.

"How" questions generally are more tightly focused and lend themselves to being phrased as implicit hypotheses which can be tested, but they necessarily live within the context of a theory and, therefore, presuppose some foreknowledge. "Why" questions, on the other hand, generally strike at the heart of a scientific issue by identifying defects or peculiarities in a theory which might lead to a new science. By doing so, "why" questions need no theory and may pursue an explanation of observation without reference to pre-established groundwork.

"How" questions are extremely important in the actual practice and study of science, but "Why" questions embody the ever-striving, almost combative quality that peer-reviewed science takes on when theories compete with one another for acceptance and dominance. "How" may be the wheels on the road, but "why" is the engine of the car. We need them both to move forward.

Before the romantic gets the chance to do anything, we suddendly jump into the conversation: Ok my friends, let us call it a night. You may now go back into the void, after all, you were just us, thinking out loud.

Now that we are alone, let us try to draw some lessons before we put out the candle for good. It seems there is an infinite dialectic between "whys" and "hows" in direct connection to how theory and experiment have been intimately intertwined during the development of modern physics [6]. The experimentalist, for example, will measure that some physical quantity is conserved; the theorist will call it charge and unveil a symmetry of the dynamics. They will write down all possible ways the relevant physical degrees of freedom, respecting such a symmetry, may show up in the lab. Some expected behavior will not occur, some unexpected

[6] The situation today is much more subtle; we will not try to say something insightful about the state of affairs of fundamental physics. An illustration of the generalized confusion within the field, a clear by-product of not finding anything but the Higgs particle [8, 9] at the Large Hadron Collider (LHC), is that noted phenomenologists are actually having imaginary conversations with late friends in order to clarify their own ideas [10]. An alternative naive hope, besides the still mythical "Nimatron" [11, 12], is that the cosmos, being the ultimate collider [13], will eventually have something to say on all these matters, and at a much lower cost, for the taxpayers' benefit.

behavior will. They might get confused, maybe they will need to check their assumptions. And there they go again, *ad infinitum*. Another possibility is that the enlightened theorist leads the way; after long, tedious calculations he may be able to tell the experimentalist "look that way" or "look this other way". Maybe they find something, maybe they do not. It is not important, as scientists are known for their stubbornness; they will keep insisting. At the end of the day the invariant lesson seems to be the same; "good" physics should always be driven by data.

It is quite remarkable that cosmology, the study of the origin, evolution, and eventual fate of the universe, has become data-driven science. It is time to get real then, and talk about some *facts* regarding the current understanding of our cosmos.

1.2 ... And Some Facts

The past two decades of advances in observational cosmology have brought about a revolution in our understanding of the universe, transforming cosmology from a largely speculative science, into a predictive science with precise agreement between theory and observation. These advances include observations of type Ia supernovæ [14, 15], measurements of temperature fluctuations in the cosmic microwave background (CMB) [16–28], and maps of the distribution of large-scale structure (LSS) [29], which have established a standard model of cosmology, the so-called ΛCDM model. This is a universe filled with $\sim 69\%$ "dark energy", $\sim 25\%$ "dark matter", and only $\sim 5\%$ ordinary atoms, as seen in Figure 1.2.

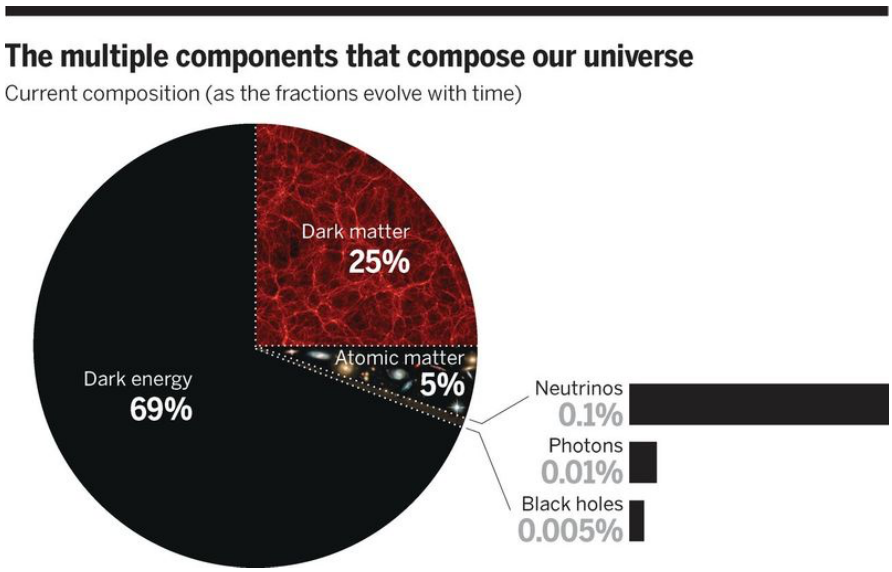


Figure 1.2: Pie chart of our universe. Figure taken from [30].

Many noted researchers have referred to modern times as the "golden age of cosmology". Moreover, there is decisive evidence that large-scale structures were formed via gravitational instability of primordial density fluctuations, and that these initial perturbations originated from quantum fluctuations [31–35], stretched to cosmic scales during a period of inflationary expansion [36–38] (an artist's impression of the evolution of the universe is shown in Figure 1.3).

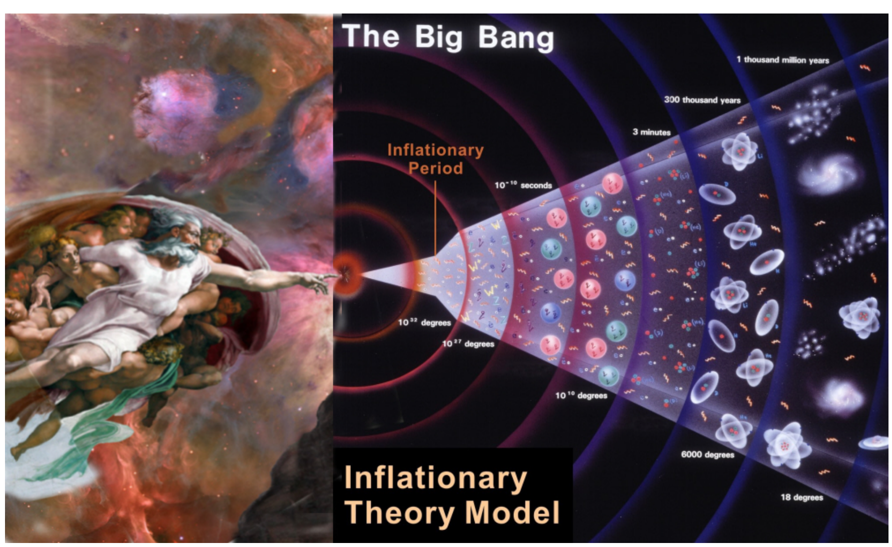


Figure 1.3: Our best theory so far.

However, it is fair to say that *"the microphysical origin of inflation remains a mystery, and it will require a synergy of theory and observations to unlock it"* [39].

In this thesis, we present our past and ongoing collaboration to both the **"hows"** and the **"whys"** within the cosmic inflation paradigm.

In chapter 2 we introduce the concept of inflation from scratch. Following a historical perspective, special emphasis is made on the most physically urgent short-coming of Big Bang cosmology, namely the horizon problem.

In chapter 3 we present the modern understanding of inflation as a symmetry breaking phenomenon [40, 41]. The effective field theory built upon this crucial insight, is quite successful in encapsulating large classes (almost all) of single-field inflation models, and drawing model-independent conclusions and predictions, by careful consideration of the symmetry breaking pattern. We go through this construction, not only because by now is "mandatory" background knowledge for any theoretical cosmologist candidate, but also because we will actually *use it* in subsequent chapters.

In chapter 4 we discuss **"how"** a well-motivated two-field model, which we have dubbed "Generalized Natural Inflation" [42], may (or may not) predict sizable new signals for future experiments in different regimes of its possible dynamics [7]. The reason behind our initial interest in this kind of model, in short, stemmed from the fact that there is a non-trivial possibility that new physics, in the form of (for example) interactions between the almost-free, very weakly-coupled "inflaton" fluctuations and other not-so-massive degrees of freedom, may leave measurable

[7] "Unfortunately" for us, but fortunately for science, during the writing process of this dissertation, Planck's latest release [28] has basically ruled out the background model over which we based our own, namely the seminal Natural Inflation model [43, 44]. These are the cons, but *also* the pros, of working in phenomenological models that lie at the frontier of experimental science, if you ask us. Even though this is a "2σ" result, things do not look good for the so-called "large-field models" class. Somewhat unexpectedly, the most popular models among string theorists, namely axion monodromy models [45–48], are not in good shape these days. Interesting times indeed. Falsifiability works, science works!

imprints in the relevant cosmological correlation functions. This new program, in which the cosmos itself is being understood as the ultimate "collider" [13], has its roots in the so-called "quasi-single-field inflation" models [13, 49–61] (which are quite interesting by themselves), and has opened several new venues of exploration [8] in the quest of observing new physics in the sky.

In chapter 5 we present the main framework and ideas (in their embryonic, heuristic form) of ongoing research [64], associated with the fundamental nature of the symmetry breaking patterns that inflation and general cosmological setups may show, in the hope of understanding **"why"** the universe picked these particular, subtle ways of evolving and becoming what it is today. In this sense, it is natural to anticipate the development of an analogous program to that introduced in [65], where the authors proposed a classification of all of condensed matter systems as specific states that spontaneously break spacetime, gauge, and internal symmetries.

Finally, chapter 6 presents some concluding remarks, leaving some technical details for appendices A, B, and C.

Fear not, dear reader, if you get the feeling that this section is not self-contained (even unintelligible) and many words and/or concepts are just alien to you at this point. As we walk through the chapters of this dissertation, we will do our best in carefully introducing and defining all the necessary ingredients, to make sense of it all. Let us invite you then, on a journey that starts by understanding "why" and "how" inflation was indeed, an unavoidable state of the primordial universe.

[8] See, for instance, [62, 63] and references therein.

Chapter 2

The Inflationary Paradigm

In this chapter we discuss the historical events that led to the understanding that "non-standard" physics, besides the naive Big Bang cosmology paradigm, must have governed the very early universe evolution, in order to explain current cosmological data. We then introduce one possible solution; cosmic inflation [36–38].

2.1 Why Do We Need Cosmic Inflation? A Quick Roadmap

The groundbreaking work of Albert Einstein between 1907 and 1915 can be summarized in a highly profound statement about the workings of nature. According to general relativity (GR), the observed gravitational attraction between masses results from the warping of space and time by those masses. In the words of John Wheeler "spacetime tells matter how to move; matter tells space how to curve" [66]. The reason for the development of GR was that the preference of inertial motions within special relativity (SR) was unsatisfactory [67]. In the article "On the Relativity Principle and the Conclusions Drawn from it" [68], Einstein argued that free fall is really inertial motion, and that for a free-falling observer the rules of special relativity must apply. This is nothing but an incarnation of the equivalence principle which itself is any of several related concepts dealing with the equivalence of gravitational and inertial mass, and to the observation that the gravitational "force" as experienced locally while standing on a massive body (such as the Earth) is the same as the pseudo-force experienced by an observer in a non-inertial (accelerated) frame of reference.

In 1917, Einstein applied GR to the universe as a whole. He discovered that his own field equations predicted a universe that was dynamic, either contracting or expanding. However, observational evidence for a dynamic universe was not known at the time, leading Einstein to introduce a "cosmological constant" term to his field equations, to allow the theory to predict a static universe of closed curvature, in accordance with his understanding of Mach's principle [1]. After Edwin Hubble discovered in 1929 the recession of nebulae, Einstein abandoned his static model of the universe. In the dynamic models that he proposed later [70, 71], he discarded the cosmological constant as it was "in any case theoretically unsatisfactory". Even though these models turned out not to be good, complete descriptions of the true dynamics of the cosmos, they are of historical significance as Einstein importantly embraced the dynamic cosmology of Alexander Friedmann in their development.

Between 1922 and 1924, Friedmann derived the main results of the so-called Friedmann-Lemaître-Robertson-Walker (FLRW) model, which describes a homogeneous, isotropic, expanding or contracting universe [72, 73]. However, his work remained relatively unnoticed by his contemporaries. In 1927, two years after Friedmann died, Georges Lemaître arrived independently at results similar to those of Friedmann, and in the face of the observational evidence for the expansion of the universe obtained by Hubble [74], his results were noticed in particular by the influential astrophysicist Arthur Eddington. In 1930-31 Lemaître's paper was translated into english and published [75]. The problem was further explored during the 1930s by Howard P. Robertson and Arthur Geoffrey Walker, who rigorously proved that the FLRW metric is the only one on a spacetime that is spatially homogeneous and isotropic [76–79]. This is a *geometric* result that is actually not tied to the equations of GR.

[1] A very general statement of Mach's principle is "local physical laws are determined by the large-scale structure of the universe" [69].

In 1927 Lemaître already had proposed that the inferred recession of the nebulae was due to the expansion of the universe [75]. In 1931 he went further and suggested that the evident expansion of the universe, if projected back in time, meant that the further in the past the smaller the universe was, until at some finite time in the past all the mass of the universe was concentrated into a single point, a "primeval atom" where and when the fabric of time and space came into existence [80]. However during those years, almost every major cosmologist preferred the eternal steady state universe [81, 82], where the density of matter in the expanding universe remains unchanged due to a continuous creation of matter, adhering to a "perfect" cosmological principle which asserts that the observable universe is basically the same at *any time* as well as at any place [2]. After World War II Lemaître's so-called Big Bang theory was advocated and developed by George Gamow, who introduced big bang nucleosynthesis, the production process of nuclei other than those of the lightest isotope of hydrogen during the early phases of the universe [83]. Contiguously, Ralph Alpher and Robert Herman predicted the cosmic microwave background (CMB), remnant electromagnetic radiation from an early stage of the universe in Big Bang cosmology [84]. For a while, support was split between the steady state and the Big Bang models. The discovery and confirmation of the CMB in 1964 by Arno A. Penzias and Robert W. Wilson settled the dispute in favor of the Big Bang theory [85]. Furthermore, in 1968 and 1970, Roger Penrose, Stephen Hawking, and George F.R. Ellis showed that mathematical singularities were an inevitable initial condition of general relativistic models of the Big Bang [86, 87]. Between the 1970s to the 1990s, cosmologists worked on characterizing the features of the Big Bang universe and resolving outstanding problems. In particular, in 1981 Alan Guth made a breakthrough in theoretical work for resolving some of the shortcomings of the Big Bang cosmology with the

[2] The cosmological principle is the notion that the spatial distribution of matter in the universe is homogeneous and isotropic when viewed on a large enough scale.

introduction of an epoch of rapid expansion in the early universe; “inflation” [36]. The usual problems that cosmological inflation is able to address are the magnetic monopoles, flatness, and horizon problems. Let us briefly state the former two as the latter will be thoroughly discussed in the next subsection [3].

The magnetic monopoles objection was raised in the late 1970s when Grand Unified Theories predicted topological defects in space that would manifest as magnetic monopoles. These theories predicted an efficient production of such objects in the hot early universe resulting in a density much higher than is consistent with observations as no magnetic monopoles have never been found. This problem is resolved by cosmic inflation, since it removes all point defects from the observable universe.

The flatness problem is an observational problem associated with a FLRW metric. Basically, the universe may have positive, negative, or zero spatial curvature, depending on its total energy density. Curvature is negative or positive if the energy density is less or greater than the so-called critical density, respectively. The universe is *flat* if the density is exactly critical. The crucial point is that any small departure from the critical density grows with time and yet the universe today is highly close to flat. For instance one can calculate that at the relatively late age of a few minutes, which is the time of nucleosynthesis, the density of the universe must have been within one part in 10^{14} of its critical value or it would not exist as it does today. Inflation drives the geometry to flatness, solving this cosmological “fine-tuning” problem. Let us now pay more attention to a somehow more fundamental issue of FLRW cosmology, the so-called horizon problem.

[3] For a comprehensive account of all these topics, see, e.g., [88].

2.1.1 The Horizon Problem

As we have already discussed, it is an historical, empirical fact that our universe, at large scales, is well-described by the spatially flat FLRW metric

$$ds^2 = -dt^2 + a^2(t)\,\mathbf{dx}^2. \tag{2.1}$$

To discuss the causal structure of this spacetime it is useful to introduce so-called conformal time τ, defined through $dt = a(\tau)d\tau$, so that (2.1) becomes

$$ds^2 = a^2(\tau)\left[-d\tau^2 + \mathbf{dx}^2\right]. \tag{2.2}$$

We see that for any $a(\tau)$, the maximum comoving distance $|\Delta\mathbf{x}|$ that a particle can travel between τ_1 and $\tau_2 = \tau_1 + \Delta\tau$ is just $|\Delta\mathbf{x}| = \Delta\tau$. In the usual Big Bang cosmology, as the energy density of radiation ρ_r goes like $\rho_r \sim a^{-4}$, it dominates the expansion at early times, and by tracing the evolution backwards, it is inevitable to find a singularity $a \to 0$. Choosing coordinates such that this singularity happens at $t = 0$, the maximum comoving distance a particle can have traversed since then is given by

$$|\Delta\mathbf{x}| = \Delta\tau = \int_0^t \frac{dt'}{a(t')} = \int_{-\infty}^{\ln a(t)} \frac{d\ln a}{aH}, \quad \text{where} \quad H \equiv \frac{\dot{a}}{a}. \tag{2.3}$$

It can be shown that during the standard Big Bang evolution, $\ddot{a} < 0 \iff \frac{d}{dt}\left(\frac{1}{aH}\right) > 0$, so the integral in (2.3) is dominated by the contributions from late times. Thus, the amount of conformal time that elapses between the singularity and the formation of the CMB, the so-called recombination event, is much smaller than the conformal time between recombination and today. Then we realize that points in the CMB that are separated by more than one degree were never in causal contact, according to the standard cosmology, as their past last cones never intersect before the spacetime terminates at the initial singularity. However, their temperatures are observed to be the same, to one part in 10^4. Not only that, but the observed temperature fluctuations are actually correlated on what seem to be acausal scales. A picture is worth a thousand words, so let us consider Figure 2.1.

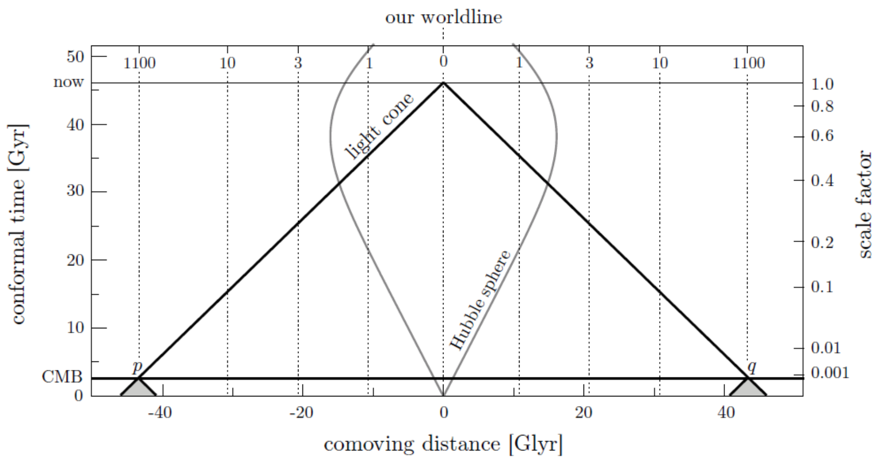


Figure 2.1: Spacetime diagram illustrating the horizon problem in comoving coordinates (figure taken from [39], which itself is an adaptation from [89]).

Here we see a spacetime diagram illustrating the horizon problem in comoving coordinates. The dotted vertical lines correspond to the worldlines of comoving objects and "we" are the central worldlines. On each worldline the current red-shifts of the comoving galaxies are labelled. Everything we currently observe lies on our past light cone and the intersection of our past light cone with the CMB spacelike slice corresponds to two opposite point on the CMB surface of last-scattering. The past light cones of these two points, which are shaded gray, do not intersect, so they appear to never have been in contact before the inevitable doom of reaching the singularity.

2.1.2 Cosmic Inflation

To address the horizon problem, we may postulate that the so-called comoving Hubble radius $(aH)^{-1}$ was actually decreasing in the early universe, meaning

$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \iff \ddot{a} > 0$, so the integral in (2.3) is dominated by the contributions from early times. Consequently there is an additional range of conformal time between the singularity and recombination. Actually, conformal time now extends to negative values [4], so if the period of decreasing comoving Hubble radius is prolonged enough, all points in the CMB do originate from a causally connected patch of space, and so the observed correlations result from ordinary causal processes at early times. Again, a picture is quite useful to get an understanding of the physics.

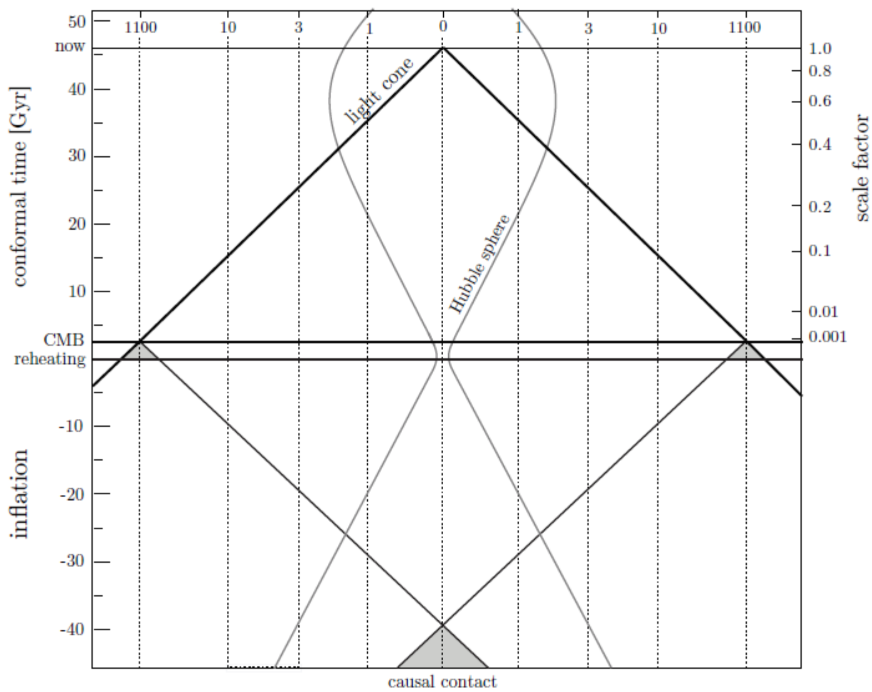


Figure 2.2: Inflationary solution to the horizon problem (figure taken from [39], which itself is an adaptation from [89]).

In Figure 2.2 we see that the so-called comoving Hubble sphere shrinks during inflation and expands during the conventional Big Bang evolution, at least until dark energy dominates. What used to be the spacelike singularity is replaced by the so-called reheating surface, and $\tau = 0$ does not mark the beginning of

[4] For example, in a de Sitter spacetime, where $a = e^{Ht} = -\frac{1}{H\tau}$ with $H = \text{constant}$, it is easy to check that $t\big|_0^\infty \to \tau\big|_{-\infty}^0$, so conformal time is negative during inflation.

spacetime but the end of inflation and the transition to the standard Big Bang cosmology. We can explicitly see that all points in the CMB do have intersecting past light cones and so can indeed originate from a causally connected region of space. Quantitatively speaking, in an expanding universe a shrinking comoving Hubble sphere implies

$$\frac{d}{dt}\left(\frac{1}{aH}\right) = -\frac{1}{a}\left[\frac{\dot{H}}{H^2} + 1\right] < 0 \iff \epsilon \equiv -\frac{\dot{H}}{H^2} < 1, \tag{2.4}$$

where we have defined the usual so-called first slow-roll parameter ϵ. We can actually take the slow evolution of the Hubble parameter as the definition of inflation. We see that the de Sitter limit is reached formally when $\epsilon \to 0$, so inflation is usually said to correspond to "quasi"-de Sitter expansion. The exact de Sitter limit obviously implies that

$$\dot{H} = 0 \Rightarrow a(t) = e^{Ht} \quad \text{with} \quad H = \text{constant}. \tag{2.5}$$

Within the slow-roll approximation the universe expands quasi-exponentially

$$a(t) \propto \exp\left(\int H dt\right) \equiv e^{-N} \tag{2.6}$$

where it is conventional to define the number of *e-folds* N with the sign convention

$$dN \equiv -H dt, \tag{2.7}$$

so that N is large in the far past and decreases as we go forward in time and as the scale factor a increases. It can be shown that in order to solve the horizon problem, we need inflation to last for a sufficiently long time, usually at least $N \sim [50, 60]$ e-folds. To achieve this requires ϵ to remain small for a sufficiently large number of Hubble times. This condition is measured by a second parameter

$$\eta \equiv \frac{\dot{\epsilon}}{\epsilon\, H}. \tag{2.8}$$

For $|\eta| < 1$, the fractional change of ϵ per Hubble time is small and inflation persists.

Einstein's equations in a spatially flat FLRW spacetime with a perfect fluid as matter content lead to Friedmann equations,

$$3M_{\rm Pl}^2 H^2 = \rho \quad \text{and} \quad 6M_{\rm Pl}^2 \left(\dot{H} + H^2\right) = -\left(\rho + 3P\right), \tag{2.9}$$

where ρ and P stand for the energy density and pressure of the fluid. These two equations can be straightforwardly combined in the form

$$\dot{H} = -\frac{1}{2M_{\rm Pl}^2}\left(\rho + P\right), \tag{2.10}$$

and then it is easy to reexpress ϵ as

$$\epsilon = \frac{3}{2}\left(1 + \frac{P}{\rho}\right), \tag{2.11}$$

so that $\epsilon < 1 \Rightarrow P < -\frac{1}{3}\rho$. This is a violation of the so-called strong energy condition (SEC), which requires that for every future-pointing timelike vector field X^μ,

$$\left(T_{\mu\nu} - \frac{1}{2} T^{\rho}{}_{\rho}\, g_{\mu\nu}\right) X^\mu X^\nu \geq 0, \tag{2.12}$$

where $T^{\mu\nu}$ is the energy-momentum tensor of the matter sector. Since for a perfect fluid, $T^{\mu\nu} = \mathrm{diag}\left(\rho, P, P, P\right)$, the SEC is equivalent to $\rho + P \geq 0$ and $\rho + 3P \geq 0$. The historical reasons for wanting such a condition is that, via

Einstein's equations, the SEC implies the so-called Ricci convergence condition, $\mathscr{R}_{\mu\nu}X^\mu X^\nu \geq 0$, with $\mathscr{R}_{\mu\nu}$ the usual Ricci tensor. This last condition is used to prove that nearby timelike geodesics are always focussed towards each other, a first critical step in proving singularity theorems and the like. However, it is not too hard to find simple *physical* systems that violate the SEC: A scalar field with negligible kinetic energy and positive potential [5]. These are basically the (now old) "new inflation" (or just "slow-roll") models of Andrei Linde [37], and Andreas Albrecht and Paul Steinhardt [38]. As the world is ultimately quantum mechanical in essence, it is crucial to appreciate the non-trivial implications of the classical inflationary background we have just discussed.

2.1.3 Cosmological "Free Lunch": Primordial Perturbations

It is quite remarkable that the inflationary paradigm not only explains the homogeneity of our universe, but also provides a mechanism to create the primordial inhomogeneities required for structure formation. This is an "automatic" consequence of quantum mechanics around the inflationary quasi-de Sitter phase of the universe [6]. The theory of fluctuations was first calculated by russian physicists Viatcheslav Mukhanov and Gennady Chibisov when analyzing Starobinsky's similar model [31, 92] [7]. In the context of inflation, they were independently calculated in 1982 by four groups: Stephen Hawking [33]; Alexei Starobinsky [34]; Alan Guth and So-Young Pi [32]; and James M. Bardeen, Paul Steinhardt and Michael

[5] Nowadays, it is the so-called averaged null enery condition $\int_C T_{\mu\nu} k^\mu k^\nu d\lambda$, where k^μ is any future-pointing null vector field and C is any flowline (integral curve) of k^μ, the one that has been proven (on Minkowski spacetime) to be satisfied even within proper quantum mechanical regimes, such as the Casimir effect.

[6] It has been stated that Paul Dirac somehow "prophesied" this fact as early as 1939 (see [90]). However, it seems that this is a 1931 Lemaître's original idea and Dirac almost surely got it from him (see [91]).

[7] As early as 1979, Alexei Starobinsky noted that quantum corrections to GR should be important for the early universe, and that such corrections generically lead to curvature-squared corrections to the Einstein-Hilbert action. The solution to the modified Einstein's equations, when the curvatures are large, leads to an effective cosmological constant. Consequently, Starobinsky proposed that the early universe went through an inflationary de Sitter era [93].

Turner [35]. The rough idea is that the exponential growth of the scale factor during inflation caused these primordial quantum fluctuations of the inflaton field to be stretched to macroscopic scales while "freezing" upon *leaving the horizon* [8]. During the later stages of radiation and matter domination, these fluctuations supposedly *re-enter the horizon*, setting the initial conditions for structure formation.

In chapter 3 we will explicitly construct a well motivated two-field inflationary model [42], with the aim of setting not only a theoretically controlled, fine-tuning free inflationary background, but also a phenomenologically attractive quantum theory of fluctuations. Before we introduce our specific two-field inflationary model, it will prove quite useful to discuss the modern understanding of inflation as a symmetry breaking phenomenon [40, 41].

[8] It is understandable that "hardcore" general relativists like our dear professor Ted Jacobson may not like this terminology due to the fact that the concept of horizon in black hole physics is quite different. However, in the cosmology community it is quite common to talk in these terms, and things will not change as far as we can see. See Wikipedia to appreciate this fact.

Chapter 3

The Effective Field Theory of Inflation

In this chapter we discuss the construction of the effective field theory of inflation, which embodies the seminal idea that cosmic inflation occurs as a consequence of the spontaneous symmetry breaking of time translational invariance [40, 41] in the early universe.

3.1 Spontaneous Symmetry Breaking

In thermodynamics, a spontaneous process is the time-evolution of a system in which it releases free energy and it moves to a lower, more thermodynamically stable energy state. On the other hand, symmetry breaking is a phenomenon in which infinitesimally small fluctuations acting on a system crossing a critical point decide the system's fate, by determining which branch of a bifurcation is taken. Spontaneous symmetry breaking (SSB) is a spontaneous process of symmetry breaking, by which a physical system in a symmetric state ends up in an asymmetric state. Technically speaking, the Lagrangian and the equations of motion still respect the symmetry but the lowest-energy vacuum solutions do not. Our aim is to revisit the seminal idea of understanding cosmic inflation as an example of SSB and discuss the associated Goldstone dynamics [40, 41]. In writing this chapter, in some parts, we closely follow [39, 94].

3.1.1 SSB of Global Symmetries

For completeness, we review the familiar cases of SSB or global and (spin 1) gauge symmetries. Needless to say, everything here is, in some way or another, much more well covered in masterpieces such as [95]. Let us start by considering a set of real scalar fields ϕ_i, $i = 1, \dots, N$, whose dynamics is determined by an action that is invariant under some global symmetry group transformation

$$\phi_i \to \phi'_i = U_{ij}\,\phi_j, \quad \text{where} \quad U = e^{i\theta^a G_a}, \tag{3.1}$$

where the G_a are the generators of the group G, and the θ^a are spacetime-independent parameters. If such fields acquire a vacuum expectation value (VEV), meaning $\langle \phi_i \rangle = v_i$, then the symmetry G is said to be spontaneously broken to the subgroup H that leaves the v_i invariant, meaning $(T_A)_{ij}\,v_j = 0$, where the T_A are the generators of H. In contrast, the transformations in the coset G/H act nontrivially on the v_i's, meaning $(X_\alpha)_{ij}\,v_j \neq 0$, where the X_α are the "broken" generators (generators of the broken symmetries). Now within SSB, spacetime-independent transformations along the directions of broken symmetry connect different vacua with the same energy and, as a result, for each broken generator there is one "flat" direction in the space of field configurations [1]. Fluctuations along the flat directions are the famous Goldstone bosons. Goldstone's theorem asserts the existence of one massless Goldstone boson π_α for every broken generator [97] [2]. The usual way of introducing the Goldstone bosons is to act on the vacuum configuration with the broken symmetry, but replacing the constant

[1] In short, we know since the seminal work of Emmy Noether, that every continuous symmetry of the action of a physical system has a corresponding conservation law [96], i.e., there exists a Noether current $J^\mu(x)$ such that $\partial_\mu J^\mu = 0$. In the quantum theory, the conserved charge $Q = \int d^3 J^0(x)$ is the operator that generates the symmetry transformation, and since it is conserved, it commutes with the Hamiltonian, i.e. $[H, Q] = 0$. The operator Q corresponds to a conserved charge no matter what vacuum we expand around. SSB occurs, by definition, if the symmetric vacuum, with $Q\,|\Omega\rangle_{\text{sym}} = 0$, is unstable and the true, stable vacuum is charged $Q\,|\Omega\rangle \neq 0$. If the vacuum has energy E_0, meaning $H\,|\Omega\rangle = E_0\,|\Omega\rangle$, then $HQ\,|\Omega\rangle = [H, Q]\,|\Omega\rangle + QH\,|\Omega\rangle = E_0 Q\,|\Omega\rangle$ and therefore the state $Q\,|\Omega\rangle$ is degenerate with the ground state.

[2] Using $J_0(x)$ and E_0 as defined in footnote 1, we can construct states of 3-momentum $\mathbf{p}$ from the vacuum as $|\pi(\mathbf{p})\rangle = -\frac{2i}{f}\int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}}\, J^0(x)\,|\Omega\rangle$ which have energy $E(\mathbf{p}) + E_0$. Here, f is a constant of mass dimension 1 and the prefactor is just conventional. We see that since $|\pi(0)\rangle = -\frac{2i}{f} Q\,|\Omega\rangle$ has energy E_0 we may conclude that $E(\mathbf{p}) \to 0$ as $\mathbf{p} \to 0$, so the Goldstones are gapless. Note that this reasoning has not required us to assume a Lorentz-invariant dispersion relation.

transformation parameters θ^α with spacetime-dependent parameters $\pi^\alpha(x)$, that is

$$\phi_i' = \left(e^{i\pi^\alpha(x)X_\alpha}\right)_{ij} v_j. \tag{3.2}$$

One crucial fact is that, generically, the remaining directions in field space are not flat, while the $\pi^\alpha(x)$ parametrize massless excitations. Thus, the "massive" directions decouple from the Goldstone dynamics, making the latter the natural degrees of freedom of the low-energy effective field theory (EFT). The EFT is determined by the symmetry-breaking pattern to a large degree, and is such that the symmetries in H are *linearly* realized while those in G/H are *nonlinearly* realized. To construct the EFT, one introduces the field

$$U(x) = e^{i\pi(x)\cdot X}, \quad \text{where} \quad \pi(x)\cdot X \equiv \pi^\alpha(x)X_\alpha. \tag{3.3}$$

Then at lowest order in the derivative expansion, the unique G-invariant Lagrangian is given by

$$\mathscr{L}_{\text{eff}}^{(0)} = -\frac{f_\pi^2}{4}\,\mathrm{Tr}\,\partial_\mu U^\dagger \partial^\mu U, \tag{3.4}$$

where f_π is a mass dimension 1 parameter. It is clear that there can be no terms without derivatives as $\mathrm{Tr}\,U^\dagger U$ = constant. Let us specialize to the case $G/H = SU(2)$, so that $X_\alpha = \frac{1}{2}\tau_\alpha$ where τ_α are the usual Pauli matrices, in order to expand (3.4). In such a case we can think of the π_α as the triplet of pions of quantum chromodynamics (QCD). Using the normalization $\pi_c \equiv f_\pi\,\pi$,

$$\mathscr{L}_{\text{eff}}^{(0)} = -\frac{1}{2}\partial_\mu\pi_c\cdot\partial^\mu\pi_c - \frac{1}{6f_\pi^2}\left\{(\pi_c\cdot\partial_\mu\pi_c)^2 - \pi_c^2(\partial_\mu\pi_c\cdot\partial^\mu\pi_c)\right\} + \dots, \tag{3.5}$$

where use has been made of the "Killing form" $2\,\mathrm{Tr}\,X_\alpha X_\beta = \delta_{\alpha\beta}$. We see the appearance of an infinite series of non-renormalizable interactions. The symmetry breaking pattern dictates relations among all these operators, with all couplings determined by the single parameter f_π. This is the so-called "universal" part of the action.

At higher order in the derivative expansion there exist additional non-universal operators involving only single derivatives

$$\mathscr{L}^{(0)}_{\text{eff}} = -\frac{f_\pi^2}{4}\,\mathrm{Tr}\,\partial_\mu U^\dagger \partial^\mu U + c_1\left(\mathrm{Tr}\,\partial_\mu U^\dagger \partial^\mu U\right)^2 + c_2\left(\mathrm{Tr}\,\partial_\mu U^\dagger \partial^\mu U\,\mathrm{Tr}\,\partial_\nu U^\dagger \partial^\nu U\right) + \ldots, \tag{3.6}$$

where c_1 and c_2 are model-dependent dimensionless constants. Expanding the previous expression in terms of π_c we would find new structures where individual operators are related by the non-linearly realized symmetry. If we know a so-called "UV-completion", coefficients like c_1 and c_2 can be calculated in terms of the parameters of the completion after integrating out the heavy modes of the high-energy theory. When UV-completions are not known, the effective action provides a model-independent description of the low-energy dynamics and coefficients like c_1 and c_2 are, in principle, fixed (mostly bounded) by experiments.

The symmetry-breaking scale can be read off from the state

$$|\pi(\mathbf{p})\rangle = -\frac{2i}{f}\int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}}\, J^0(x)\,|\Omega\rangle\,, \tag{3.7}$$

which is gapless, as proven in footnote 2. Taking the inner product with $\langle\pi_c(\mathbf{q})|$ and integrating over $\int d^3p\, e^{-i\mathbf{p}\cdot\mathbf{y}}$, one finds that

$$\langle\pi_c(\mathbf{q})|\, J^0(t,\mathbf{y})\,|\Omega\rangle = iE(\mathbf{q}) f e^{-i\mathbf{q}\cdot\mathbf{y}}, \tag{3.8}$$

where we have used the relativistic normalization $\langle \pi_c(\mathbf{q})|\pi_c(\mathbf{p})\rangle = 2E(\mathbf{q})(2\pi)^3\delta(\mathbf{q}-\mathbf{p})$. The current interpolates between the vacuum state $|\Omega\rangle$ and the Goldstone boson state $|\pi_c\rangle$, with a strength set by the scale f. In other words, f is the "order parameter" of the symmetry breaking, i.e., symmetry breaking occurs around f. Now the current associated with the effective Lagrangian (3.5) is given by $J^\mu = -f_\pi\,\partial^\mu\pi_c + \ldots$, so if $|\pi_c\rangle$ is the state created by acting with the operator π_c on the vacuum state $|\Omega\rangle$, then

$$\langle \pi_c(q)|\, J^\mu(y)\,|\Omega\rangle = iq^\mu f(q^2)e^{iqy}, \tag{3.9}$$

where $f(q^2) = f_\pi + \ldots$, is the Lorentz-invariant version of (3.8), and f_π plays the role of f. The symmetry is restored when the right-hand side of the above equation vanishes, and this happens when higher-order corrections cancel the leading term in $f(q^2)$, i.e., at energies of order f_π. Below f_π, weakly-coupled Goldstone bosons is an appropriate description of the physics, while above f_π other degrees of freedom become relevant. A fairly reliable method for identifying the cutoff of the effective theory is to determine the so-called strong coupling scale, Λ. At this scale the perturbative expansion breaks down. More formally, we may define the strong coupling scale as the energy scale at which the loop expansion breaks down or perturbative unitarity of Goldstone boson scattering is violated. One can then derive that $\Lambda = 4\pi f_\pi$ [95], as long as the non-universal interactions do not have large coefficients $c_n \gg 1$; otherwise $\Lambda \lesssim 4\pi f_\pi$. Let us now consider the case of broken gauge symmetries.

3.1.2 SSB of Gauge Symmetries

It is usually stated that the term gauge symmetry is something of a misnomer, as it is actually related to a redundancy of description which is introduced to have a local (Lagrangian) description of the physics, and not to a true physical symmetry with an associated conserved charge. Therefore, one may ask the legitimate question of what does it mean to break a gauge symmetry. Roughly speaking, we can always split the set of gauge transformations G intro those that approach the identity at spatial infinity, $G_\star$, and those that do not, "$G/G_\star$" [3]. We call the latter the global part of the gauge transformation and it is only this part that spontaneously breaks. Therefore, when we speak about the SSB of gauge symmetries, it should be understood that $G_\star$ is not broken as it still represents the gauge redundancy of the system, while $G/G_\star$ is a physical symmetry with a corresponding non-trivial Noether current.

Now let us recall that the usual recipe to gauge a global symmetry is to replace all partial derivatives with covariant derivatives,

$$D_\mu \equiv \partial_\mu + i g_{\mathrm{YM}} A_\mu, \tag{3.10}$$

when acting on charged fields. Here $A_\mu = A_\mu^\alpha X_\alpha$ are the gauge fields and g_{YM} the gauge coupling [4]. As usual, the gauge fields transform as a connection

$$A_\mu(x) \to U(x) \left(A_\mu(x) - \frac{i}{g_{\mathrm{YM}}} \partial_\mu \right) U^\dagger(x). \tag{3.11}$$

The low-energy effective theory is now given by

$$\mathscr{L}_{\mathrm{eff}} = -\frac{1}{4} \operatorname{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{f_\pi^2}{4} \operatorname{Tr} D_\mu U^\dagger D^\mu U + \dots \tag{3.12}$$

or

$$\mathscr{L}_{\mathrm{eff}} = -\frac{1}{4} \operatorname{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial_\mu \pi_c \cdot \partial^\mu \pi_c - \frac{1}{2} m^2 A_\mu \cdot A^\mu + m\, \partial_\mu \pi_c \cdot A^\mu + \dots, \tag{3.13}$$

[3] Here, $G/G_\star$ is *not* a quotient group, but our abuse of notation does no harm as we only use it in this paragraph in order to make a point.

[4] We use g_{YM} (Yang-Mills) instead of the usual g to avoid confusion with the determinant of the spacetime metric, namely g.

where $m^2 \equiv f_\pi^2\, g_{\rm YM}^2$. Even though the gauge fields have a mass, the Lagrangian in (3.13) still possesses a gauge symmetry that relates physically equivalent configurations $\left\{\pi', A'_\mu\right\} \sim \{\pi, A_\mu\}$. As usual, this redundancy of description can be removed by fixing the gauge. There exists a gauge called the *unitary gauge*, where $\pi \equiv 0$, so the theory is described in terms of massive vector bosons A_μ. The reverse process of introducing the Goldstones and the associated gauge redundancy into the theory of massive vector bosons is usually known as the *Stueckelberg trick* [98]. The advantage of describing the physics in terms of the Goldstone bosons is that it makes the high-energy behavior of the theory manifest. In particular, at high energies the scattering of the longitudinal modes of the gauge fields is well described by the scattering of the Goldstone bosons; this is the essence of the so-called Goldstone boson equivalence theorem [99] (see, e.g. [100], for a nice discussion about the equivalence theorem). Indeed, since the mixing operator $\partial_\mu \pi_c \cdot A^\mu$ has one less derivative than the kinetic operator $(\partial_\mu \pi_c)^2$, it is expectable that the mixing becomes irrelevant at sufficiently high energies. One easy way of explicitly seeing this is by taking the so-called *decoupling limit*,

$$g_{\rm YM} \to 0, \quad m \to 0, \quad \text{for} \quad f_\pi = \frac{m}{g_{\rm YM}} = \text{constant}, \tag{3.14}$$

in which there is no mixing between π and A^μ, so the Goldstone boson part of the action (3.13) becomes just (3.4). In the decoupling limit, what was a local (gauge) symmetry effectively becomes a global one, and for energies above $E > E_{\rm mix} = m$, the Goldstone bosons are the simplest way to describe the scattering of the massive vector fields. If we were to restore finite g and m, corrections of $\mathcal{O}\left(\frac{m}{E}, g_{\rm YM}^2\right)$ become perturbatively important.

3.2 Symmetry Breaking in Cosmology

3.2.1 Broken Time Translations

Einstein's GR can be thought of as a gauge theory, with general covariance, that is invariance under space-time diffeomorphisms (diffs) $x^\mu \to x'^\mu = x'^\mu(x^\nu)$, as the gauge symmetry. In this setting, we may think that the metric tensor $g_{\mu\nu}$ plays the role of the gauge fields sector of the theory [5]. In cosmology, one is primarily interested in looking at time reparametrizations, $t \to t' = t'(x^\nu)$. As we have argued in the introductory paragraph of subsection 3.1.2 above, a gauge symmetry is spontaneously broken if its "global part" is. It is well-known that any time-like Killing vector defines a global symmetry. For example, time-translations is a global symmetry of Minkowski space, which is generated by the time-like Killing vector $\xi = \partial_t$. In general, as we are interested in local dynamics, it does not mater that such a vector is defined globally or not. For example, let us think about de Sitter space, with the line element

$$ds^2 = -dt^2 + e^{2Ht}\mathbf{dx}^2. \tag{3.15}$$

In spite of its appearence, this spacetime does not break time-translation symmetry as the dilation isometry [6]

$$t \to t + \lambda, \quad \mathbf{x} \to e^{-H\lambda}\,\mathbf{x}, \tag{3.16}$$

[5]Attempts to quantizing gravity in exact analogy with Yang-Mills gauge theory are well-known to be doomed to fail. Here we will deal with a quantum field theory on a classical gravitational background, and thinking in analogy with gauge theory is quite helpful to build intuition.

[6]Recall that by definition the isometries of a spacetime endowed with a metric $g_{\mu\nu}$ are generated by Killing vector fields that satisfy Killing's equation $\pounds_\xi\, g_{\mu\nu} = \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu = 0$, where $\pounds_\xi$ is the Lie derivative along the flow of ξ^μ, and ∇_μ is the spacetime covariant derivative.

implies the existence of a time-like Killing vector, at least in a patch of size H^{-1}. In this sense, a patch of perfect de Sitter spacetime has no preferred "time slice", as all slices are related by gauge transformations. In other words, de Sitter space is "a state of gravity with unbroken time-translations" [94].

The de Sitter example is relevant as it represents the limiting case of most inflationary models. The expansion during inflation is of a quasi-de Sitter nature because of the empirical evidence of a red tilt in the primordial power spectrum [24] and because we need to exit the accelerating phase at some point, and it is quite natural to think about this transition happening smoothly. Therefore, an order parameter, or "clock", measuring how much inflationary expansion remains, needs to be present in the dynamics. One example could be the expansion rate H as it is a monotonically decreasing function during inflation, $\dot{H} < 0$. However it proves to be more convenient to consider time-dependent expectation value(s) $\psi_m(t)$ of some matter field(s) ψ_m, such as an inflaton field ϕ or the inflationary vacuum energy density ρ. Such expectation value(s) $\psi_m(t)$ defines *a* [7] preferred time slicing, and different time slices are defined and labeled by distinct values of ψ_m, breaking the time-translation symmetry (3.16). This implies that a transformation of the form

$$t \to t + \pi(t, \mathbf{x}), \tag{3.17}$$

is not a symmetry of the relevant action anymore [8]. Hence, following the gauge theory analogy, we should expect a Goldstone excitation, corresponding to a

[7] This assumption is necessary in order to describe so-called "single-clock" cosmologies.

[8] In other words, as we realize that inflation cannot be understood as a perfect de Sitter background, there should be a Goldstone excitation upon application of the Stueckelberg trick. This is actually a pragmatic way of realizing SSB for gauge symmetries; the gauge symmetry is spontaneously broken if by applying the Stueckelberg trick to the action written in unitary gauge, interacting Goldstone bosons are produced. Since a quasi-de Sitter spacetime does not have a time-like Killing vector, such a transformation does not leave the action invariant and therefore we expect Goldstone bosons in the spectrum of the theory.

spacetime-dependent transformation along the broken generator, meaning

$$U(t,\mathbf{x}) = t + \pi(t,\mathbf{x}). \tag{3.18}$$

In chapter 5 we revisit the well-known fact that in the case of SSB of spacetime symmetries, the counting of degrees of freedom is subtle in the sense that the number of Goldstone bosons #, does not have to match the number of broken symmetry generators, i.e. $\# \neq \dim(G/H)$ in general. In the de Sitter case, even though four isometries are being broken (one dilation and three so-called special conformal transformations), there is only one Goldstone boson, namely $\pi(x)$ as introduced in (3.18).

3.2.2 Intermission I: Adiabatic Fluctuations

Before we show the construction of the EFT in unitary gauge, it is important to stop for a second and understand the link between the Goldstone boson of SSB of time-translations and the so-called *adiabatic fluctuations* of time-dependent FLRW backgrounds. By definition, an adiabatic fluctuation is a specific type of perturbation induced by a local, common shift in time of the homogeneous fields

$$\delta\psi_m(t,\mathbf{x}) \equiv \psi_m(t+\pi(t,\mathbf{x})) - \psi_m(t). \tag{3.19}$$

We see that, at linear order, adiabatic fluctuations are proportional to the Goldstone π, $\delta\psi_m = \dot{\psi}_m\,\pi$. The crucial fact that observations do not show any signs of departures from purely adiabatic initial conditions, allows us to use the Goldstone language to describe the data. In spatially flat gauge, $g_{ij} \equiv a^2(t)\,\delta_{ij}$, all metric perturbations are related to the Goldstone mode through Einstein's equations. For purely adiabatic fluctuations, we can perform a time shift $t \to t - \pi(t,\mathbf{x})$, to remove all matter fluctuations, $\delta\psi_m \to \delta\psi_m \equiv 0$. This transformation induces an isotropic perturbation to the spatial part of the metric, $\delta g_{ij} = a^2(t)\, e^{2\,\mathcal{R}(t,\mathbf{x})}\, \delta_{ij}$, where $\mathcal{R} = -H\pi + \ldots$ and the ellipsis denotes terms that are higher order in π. In other words, the so-called curvature perturbation in comoving gauge $\mathcal{R}$ is proportional to the Goldstone boson π in spatially flat gauge, so for nearly constant H we can think of $\mathcal{R}$ and π interchangeably. Crucially, $\mathcal{R}$ is a massless field, which implies that it is conserved on superhorizon scales [9] [101]. In short, the free action for $\mathcal{R}$ reads

$$S_{\mathcal{R}}^{(2)} = M_{\rm Pl}^2 \int d^4x\, a^3 \frac{\epsilon}{c_s^2} \left\{ \dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2} \left(\nabla \mathcal{R}\right)^2 \right\}, \tag{3.20}$$

which leads to the so-called Mukhanov-Sasaki equation, which in momentum space is given by

$$\ddot{v}_k + 3H\dot{v}_k + \frac{c_s^2\, k^2}{a^2} v_k = 0, \tag{3.21}$$

where $v \equiv \sqrt{\frac{2M_{\rm Pl}^2 \epsilon}{c_s^2}}\,\mathcal{R}$ is a canonically normalized field. Here, c_s is the so-called speed of sound of the primordial perturbations that, in principle, may differ from unity $c_s \neq 1$, since Lorentz symmetry is broken by the time-dependence of the background. The Mukhanov-Sasaki equation (3.21) is the equation of a simple harmonic oscillator with a friction term provided by the expanding background. The oscillation frequency depends on the physical momentum and is therefore time-dependent,

$$\omega_k(t) \equiv \frac{c_s\, k}{a(t)}. \tag{3.22}$$

[9] The Fourier modes of this field are the frozen primordial perturbations that play the role of initial conditions for the subsequent Big Bang evolution, as mentioned by the end of subsection 2.1.3.

At early times (small a), $\omega \gg H$ for all modes of interest. In this limit, the Hubble friction is irrelevant and the modes oscillate as in Minkowski space. However, due to the quasi-de Sitter expansion during inflation, the frequency of any given mode drops exponentially, and at late times (large a), the dynamics is dominated by friction, and the amplitude of the mode is constant. Then we say that the mode "freezes at horizon crossing", i.e. when $\omega_k(t_\star) = H$ or $c_s\, k = aH(t_\star)$. Again, a picture is quite useful here.

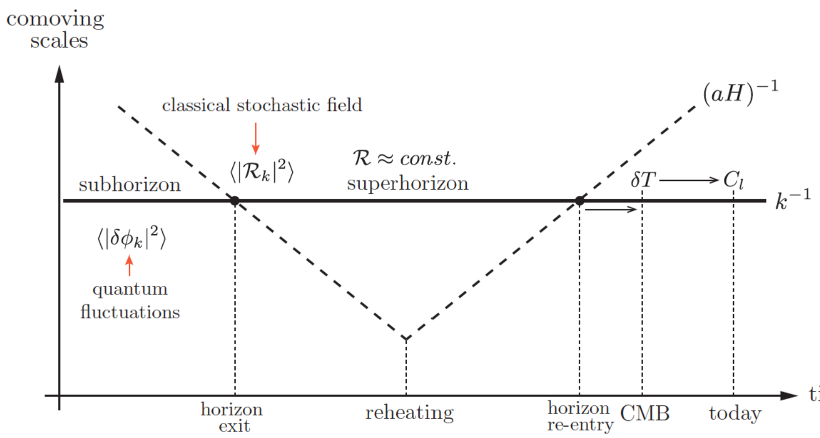


Figure 3.1: The evolution of curvature perturbations during and after inflation (Figure taken from an updated version of [102]).

In Figure 3.1 we see that the comoving horizon $(aH)^{-1}$ shrinks during inflation and grows in the subsequent FLRW evolution, implying that the comoving scales $(c_s\, k)^{-1}$ exit the horizon at early times and re-enter the horizon at late times. In physical coordinates, the Hubble radius H^{-1} is constant and the physical wavelength grows exponentially, $\lambda \propto a(t) \propto e^{Ht}$. For adiabatic fluctuations, the curvature perturbations $\mathcal{R} = -H\pi = -H\frac{\delta\phi}{\dot{\phi}_0}$, do not evolve outside of the horizon, so the power spectrum $P_{\mathcal{R}}(k)$ at horizon exit during inflation can be related directly to CMB observables at late times.

3.2.3 Exploring the EFT of Inflation

After this intermission, and to get some intuition, we will quickly and not systematically explore the EFT of the Goldstone mode. Let us come back to the transformation function $U(x) = t + \pi(x)$. This is the building block of the EFT, and it should be a scalar under time-diffs. Therefore, under a time-diff we impose

$$t \to t + \xi(x), \quad \pi(x) \to \pi(x) - \xi(x) \quad \text{such that} \quad U(x) \to U(x). \tag{3.23}$$

Now we should write the most general Lorentz-invariant action for the field $U(x)$ [10], meaning

$$S_\pi = \int d^4x \sqrt{-g}\, \mathscr{L}\left[U, (\partial_\mu U)^2, \Box U, \dots\right]. \tag{3.24}$$

The low-energy expansion should unify all known single-field models of inflation and allow a systematic classification of interactions. Let us consider, for example, the theory with the minimal set of operators

$$\mathscr{L}_{\text{s.r.}} = \Lambda^4(U) - f^4(U) g^{\mu\nu}\partial_\mu U \partial_\nu U, \tag{3.25}$$

where $\Lambda(U)$ and $f(U)$ are a priori free functions of the "invariant time" $U = t + \pi$ [11]. Considering such a system as the matter content coupled to dynamical Einstein gravity, Friedmann's equations on a flat FLRW spacetime demand that

$$\Lambda^4(U) = -M_{\text{Pl}}^2(3H^2(U) + \dot{H}(U)) \quad \text{and} \quad f^4(U) = -M_{\text{Pl}}^2 \dot{H}(U). \tag{3.26}$$

[10] Given that the background breaks Lorentz invariance, there is no reason why the EFT should respect such a symmetry. However, this "quick and dirty" way of deriving the EFT does work and it is good enough for entering the problem.

[11] Dear reader, please allow me not to explicitly write spacetime-dependence where obvious.

It can be shown that these equations can also be reached by demanding tadpole cancellation for the Goldstone perturbation, i.e. $\langle\Omega|\,\pi(x)\,|\Omega\rangle = 0$, so that the action starts quadratic in π. Thus, at leading order the coefficients of the action are therefore completely fixed by the FLRW background, so that (3.25) is necessarily given by

$$\mathscr{L}_{\text{s.r.}} = M_{\text{Pl}}^2 \dot{H}(U) g^{\mu\nu}\partial_\mu U \partial_\nu U - M_{\text{Pl}}^2(3H^2(U) + \dot{H}(U)). \tag{3.27}$$

Moreover, in unitary gauge, which can always be reached, $U = t$ and we see that

$$\mathscr{L}_{\text{s.r.}} = M_{\text{Pl}}^2 \dot{H} g^{00} - M_{\text{Pl}}^2(3H^2 + \dot{H}) \to -\frac{1}{2}\dot{\phi}_0^2(t)\, g^{00} - V(\phi_0(t)) \to -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi), \tag{3.28}$$

where we have assumed a homogeneous field configuration $\phi = \phi_0(t)$ in a flat FLRW background and we have used Friedmann's equations in the form

$$\dot{H} = -\frac{1}{2M_{\text{Pl}}^2}\,\dot{\phi}_0^2(t) \quad \text{and} \quad M_{\text{Pl}}^2(3H^2 + \dot{H}) = V(\phi_0(t)). \tag{3.29}$$

In other words, we realize that the theory defined by (3.27) is nothing but slow-roll inflation in disguise, hence the $_{\text{s.r.}}$ subscript in $\mathscr{L}_{\text{s.r.}}$.

This is a good place to note something rather generic. The slow-roll theory (3.27) couples metric fluctuations $\delta g^{\mu\nu}$ with the Goldstone π. This is analogous to the couplings between π and A_μ in the gauge theory example of subsection 3.1.2. Moreover, just like in the gauge theory case, we may find a limit in which π alone controls the dynamics, i.e., we may define a decoupling limit

$$M_{\text{Pl}} \to \infty, \quad \dot{H} \to 0, \quad \text{for} \quad M_{\text{Pl}}^2\dot{H} = \text{constant}, \tag{3.30}$$

which is formally the same limit as in (3.14) once we make the identifications $g_{\rm YM} \Leftrightarrow M_{\rm Pl}^{-1}$ and $m^2 \Leftrightarrow \dot{H}$. Indeed, considering the leading mixing operator $M_{\rm Pl}^2 \dot{H}\dot{\pi}\,\delta g^{00}$, which after canonical normalization of the fields $\pi_c \equiv \sqrt{M_{\rm Pl}^2|\dot{H}|}\pi$ and $\delta g_c^{00} \equiv M_{\rm Pl}\delta g^{00}$ becomes $|\dot{H}|^{1/2}\,\dot{\pi}_c\,\delta g_c^{00}$, we see that gravitational perturbations decouple from the Goldstone mode for frequencies above $\omega_{\rm mix} \sim \epsilon^{1/2}H$, recalling the definition $\epsilon \equiv |\dot{H}|H^{-2}$. Therefore, for frequencies $\omega^2 \gg |\dot{H}|$ we can evaluate the action for the Goldstone in the unperturbed de Sitter background $\overline{g}^{\mu\nu}$, so e.g.,

$$g^{\mu\nu}\partial_\mu U\partial_\nu U \to \overline{g}^{\mu\nu}\partial_\mu(t+\pi)\partial_\nu(t+\pi) = -1-2\dot{\pi}+(\partial_\mu\pi)^2, \quad \text{with} \quad (\partial_\mu\pi)^2 \equiv \overline{g}^{\mu\nu}\partial_\mu\pi\partial_\nu\pi. \tag{3.31}$$

As we basically care about correlation functions evaluated at freeze-out, where $\omega \sim H$, the decoupled Lagrangian for π will give accurate answers up to fractional corrections of $\mathcal{O}\left(\frac{H^2}{M_{\rm Pl}^2}, \frac{|\dot{H}|}{H^2}=\epsilon\right)$ [12]. In the decoupling limit, the EFT given by (3.27) defines a massless, perfectly Gaussian theory for the fluctuations

$$\mathscr{L}^{(2)}_{\rm s.r.\pi} = M_{\rm Pl}^2\dot{H}(\partial_\mu\pi)^2. \tag{3.32}$$

Had we kept the mixing with gravity, and by using one of Einstein's constraint equations, whose solution is [103]

$$\delta g^{00} = 2\epsilon H\pi, \tag{3.33}$$

we would find a small mass term for the Goldstone boson so that the Lagrangian is instead given by

$$\mathscr{L}^{(2)}_{\rm s.r.\pi} = M_{\rm Pl}^2\dot{H}\left\{(\partial_\mu\pi)^2+3\epsilon H^2\pi^2\right\}, \tag{3.34}$$

[12] Actually, since the characteristic energy scale is $\omega \sim H$, then $\omega^2 \gg |\dot{H}| \Leftrightarrow \epsilon \ll 1$, so in the context of inflation, the decoupling limit is a very good approximation.

where we are keeping next-to-leading order in slow-roll parameters. Crucially, such a mass term is exactly what is needed to get, by using the relation $\mathcal{R} = -H\pi$, the free massless action (3.20) with $c_s = 1$. Let us emphasize that the masslessness of $\mathcal{R}$, which implies its conservation outside the horizon, is an exact result even though (3.34) involves a slow-roll approximation.

The near-perfect Gaussianity is not maintained once we consider higher orders in the derivative expansion. For example we can add the next-to-leading order single-derivative operator

$$\mathscr{L}_{M_2} = \frac{1}{2} M_2^4 \left\{ g^{\mu\nu} \partial_\mu U \partial_\nu U + 1 \right\}^2 . \tag{3.35}$$

Here, "-1" is substracted to cancel the tadpole and ensure that such a contribution starts quadratic in π, and we assume a slow-roll condition, $|\dot{M}_2| \ll H M_2$, for the a priori time-dependent coefficient $M_2(t)$ [13]. In the decoupling limit this operator becomes

$$\mathscr{L}_{M_2} = M_2^4 \left\{ \dot{\pi}^2 - \dot{\pi} (\partial_\mu \pi)^2 \right\} + \mathcal{O}(\pi^4). \tag{3.36}$$

We appreciate that the non-linearly realized symmetry relates *dispersion* to *interactions*, as the size of the kinetic operator $\dot{\pi}^2$ and the strength of the interaction operator $\dot{\pi}(\partial_\mu \pi)^2$ are related to the same EFT coefficient M_2 [40]. We may add

[13] Indeed, as we are interested in quasi-de Sitter spacetimes, it is natural to assume that the fractional change per Hubble time is small, not only for H and $\dot{H}$, but for all the couplings of the EFT. Assuming that all coefficients vary slowly implies that the action for the fluctuations is approximately time-translation invariant. This additional global symmetry should not be confused with the broken time-translation symmetry of the background. More to the point, in the unitary gauge, and by assuming monotonicity, the authors of [40] claim that one can always perform a field redefinition such that $\phi_0(t) = t$, so invariance under time-translation is implied by the *approximate* shift symmetry $\phi \to \phi +$ constant. This symmetry and the time-translation symmetry of the ϕ Lagrangian are broken down to the diagonal subgroup by the background. This residual symmetry is the *approximate* time shift symmetry in the unitary gauge Lagrangian. We will have more to say about all this in the coset approach discussed in chapter 5.

now (3.32) and (3.36) to get

$$\mathscr{L}^{(2)}_{\mathrm{s.r.}\pi} + \mathscr{L}_{M_2} = \frac{-M_{\mathrm{Pl}}^2\dot{H}}{c_s^2}\left\{\dot{\pi}^2 - c_s^2\frac{(\nabla\pi)^2}{a^2} - (1-c_s^2)\dot{\pi}(\partial_\mu\pi)^2\right\}, \tag{3.37}$$

where we have defined the speed of sound as

$$\frac{1}{c_s^2} \equiv 1 - \frac{2M_2^4}{M_{\mathrm{Pl}}^2\dot{H}}. \tag{3.38}$$

We see that as $\dot{H} < 0$ during inflation, we need that $M_2^4 > 0$ to avoid superluminal propagation for the fluctuations. We can arrive to the same condition by noting that even before defining c_s, the time-kinetic operator would have a coefficient given by the combination $-M_{\mathrm{Pl}}^2\dot{H} + 2M_2^4$ which should be positive (> 0) in order to avoid instabilities, and again, since $\dot{H} < 0$ during inflation, we are led to demand $M_2^4 > 0$. This single condition then ensures that our Goldstone π is neither a tachyon nor a ghost. The non-trivial relation between dispersion and interactions is explicit in (3.37), where we see that if $c_s^2 \ll 1$ we have large interactions, and this is related to the size of M_2^4, since $M_2^4 \gg M_{\mathrm{Pl}}^2|\dot{H}| \Leftrightarrow c_s^2 \ll 1$. In chapter 4 we will extensively discuss a two-field model from which we will be able to explicitly calculate a non-trivial speed of sound for the adiabatic perturbations in terms of the couplings of such a UV-completion.

Let us end this heuristic exploration by noting that adding higher powers of $(g^{\mu\nu}\partial_\mu U\partial_\nu U + 1)$ reproduces the so-called $P(X)$-theories [104, 105], where the inflaton Lagrangian is a functional of the inflaton kinetic operator $X \equiv -\frac{1}{2}(\partial_\mu\phi)^2$, meaning

$$\mathscr{L}_{P(X)} = P(X,\phi). \tag{3.39}$$

Going to unitary gauge, where $\phi = \phi_0(t)$, we find

$$\mathscr{L}_{P(X)} = P\left(-\frac{1}{2}\dot{\phi}_0^2\, g^{00}, \phi_0\right) = M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2(3H^2 + \dot{H}) + \sum_{n=2}^{\infty} \frac{M_n^4(t)}{n!} \left(\delta g^{00}\right)^n , \tag{3.40}$$

where

$$M_n^4(t) \equiv \left(-\frac{1}{2}\dot{\phi}_0^2\right)^n \frac{\partial^n P}{\partial X^n}. \tag{3.41}$$

Let us now go through the systematic, geometrical approach, for the construction of the EFT of inflation.

3.2.4 EFT Construction in the Unitary Gauge

Instead of following the usual construction for the EFT that we have been discussing so far, i.e. equation (3.24), we will now uncover the EFT from a geometrical perspective [40, 41]. The goal is to write the most general EFT for the metric perturbations around a FLRW background. We have already discussed that for purely adiabatic fluctuations, we can remove any matter perturbations $\delta\psi_m$ by a local time shift that takes us to the unitary gauge $\pi = 0$; the scalar degree of freedom is "eaten" by the metric $g_{\mu\nu}$ [14]. Logically, after we fix the gauge, the action needs only be invariant under spatial diffs $x^i \to x^i + \xi^i(x)$. We can still

[14] Here, and in many places, analogies with usual gauge theory break down. For instance, in electroweak theory (EW), by "eating" the Goldstones the EW gauge bosons actually get mass, which is what one explictly sees in unitary gauge, as in the little picture in Figure 3.2 below.

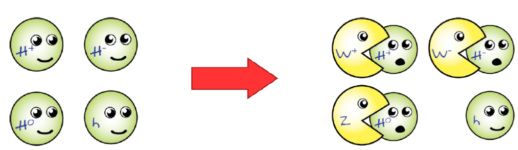


Figure 3.2: Three poor little Goldstones get eaten by bottomless gauge bosons while a lucky Higgs lives to tell the tale.

write down operators that are invariant under all diffs, like curvature invariants $\mathscr{R}, \mathscr{R}_{\mu\nu\rho\sigma}\mathscr{R}^{\mu\nu\rho\sigma}$ and the like [15], but the reduced symmetry allows us now to write several new terms in the effective action. As we have discussed in subsection 3.2.1, the time-dependence of the background fields $\psi_m(t)$ picks a preferred foliation of the spacetime into a series of spacelike hypersurfaces Σ_t. Metric perturbations on Σ_t then describe adiabatic fluctuations. It is natural to introduce a unit four-vector n_μ orthogonal to Σ_t. A natural candidate for such a normal would be given by

$$n_\mu \equiv -\frac{\partial_\mu \psi_m(t)}{\sqrt{-g^{\nu\rho}\partial_\nu\psi_m(t)\partial_\rho\psi_m(t)}}, \tag{3.42}$$

where the overall minus sign is convention and we are explicitly normalizing the vector. Assuming monotonicity, one can always use a field redefinition to set $\psi_m(t) = t$, so that

$$n_\mu = -\frac{\delta^0_\mu}{\sqrt{-g^{00}}}. \tag{3.43}$$

Therefore, by contracting covariant tensors with n_μ we will produce objects with uncontracted upper "0" indices, such as g^{00} and $\mathscr{R}^{00}$. Actually, the latter are scalars under spatial diffs [16] so functions of such objects are allowed in the EFT

This is *not* what happens in the gravitational case, since the scalar degree of freedom cannot be understood as corresponding to a mass term for the graviton (in 4d a massive spin 2 particle has 5 degrees of freedom, 3 more than the massless case). The usual lore is that "gravity cannot be Higgsed".

[15]Dear reader, please excuse me for using such a cumbersome notation for the Ricci scalar, the Riemann tensor, and so on. I am explicitly saving the familiar letter "R" for later use in the manuscript.

[16]For an arbitrary tensor field, say $\mathcal{X}^{\mu\nu}(x)$, its transformation rule under a diff implies

$$\begin{aligned}\mathcal{X}^{\mu'\nu'}(x') = \frac{\partial x'^\mu}{\partial x^\rho}\frac{\partial x'^\nu}{\partial x^\sigma}\mathcal{X}^{\rho\sigma} \Rightarrow \mathcal{X}^{0'0'}(x') &= \frac{\partial x'^0}{\partial x^\rho}\frac{\partial x'^0}{\partial x^\sigma}\mathcal{X}^{\rho\sigma}(x) \\ &= \mathcal{X}^{00}(x) + \partial_\mu\xi^0(x)\mathcal{X}^{\mu 0}(x) + \partial_\mu\xi^0(x)\mathcal{X}^{0\mu}(x) \\ &+ \partial_\mu\xi^0(x)\partial_\nu\xi^0(x)\mathcal{X}^{\mu\nu}(x),\end{aligned} \tag{3.44}$$

so $\mathcal{X}^{00}(x)$ is indeed a scalar under spatial diffs $t \to t,\ x^i \to x^i + \xi^i(x)$.

action. More generally, products or any four-dimensional covariant tensors with free upper 0 indices, but with all spatial indices contracted, are allowed operators. Finally, we may have three-dimensional quantities describing the geometry of the hypersurfaces Σ_t, such as the *induced metric*,

$$h_{\mu\nu} \equiv g_{\mu\nu} + n_\mu n_\nu, \tag{3.45}$$

the *extrinsic curvature*,

$$K_{\mu\nu} = h_\mu{}^\rho \nabla_\rho n_\nu, \tag{3.46}$$

and the Riemann curvature $\widehat{\mathscr{R}}_{\mu\nu\rho\sigma}$ of the induced metric, i.e. the Riemann curvature on Σ_t. Note however, that using $\widehat{\mathscr{R}}_{\mu\nu\rho\sigma}$ is redundant as it can be rewritten using the well-known Gauss-Codazzi relation (see, e.g., [106])

$$\widehat{\mathscr{R}}_{\alpha\beta\gamma\delta} = h_\alpha{}^\mu h_\beta{}^\nu h_\gamma{}^\rho h_\delta{}^\sigma \mathscr{R}_{\mu\nu\rho\sigma} - K_{\alpha\gamma}K_{\beta\delta} + K_{\alpha\delta}K_{\beta\gamma}. \tag{3.47}$$

Thus, we can forget about the 3d Riemann tensor altogether. Also, we may avoid using the induced metric explicitly, as through (3.45), it can be expressed in terms of the normal and the 4d metric. Furthermore, we may avoid the use of covariant derivatives with respect to the induced metric as the 3d covariant derivative of a projected tensor may be obtained as the projection of the 4d covariant derivative. Finally, the determinant of the induced metric h is related to the one of the full metric 4d metric by $h = g^{00}g$ and the completely antisymmetric 3d tensor can be rewritten in terms of the 4d one as $\left(\sqrt{h}\right)^{-1}\varepsilon^{ijk} = \left(\sqrt{-g}\right)^{-1}\left(\sqrt{-g^{00}}\right)^{-1}\varepsilon^{0ijk}$. We conclude then, that the most general action in unitary gauge is given by

$$S = \int d^4x\sqrt{-g}\,\mathscr{L}\left[\mathscr{R}_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu; t\right], \tag{3.48}$$

where all the free indices inside $\mathscr{L}$ must be upper 0's and spacetime indices are contracted with the 4d metric $g^{\mu\nu}$.

3.2.4.1 Universal Part of the Action

A flat FLRW spacetime has the following associated background quantities:

$$\overline{g}^{00} = -1, \quad \overline{\mathscr{R}} \equiv \overline{\mathscr{R}}^{\mu}{}_{\mu} = 12H^2 + 6\dot{H}, \quad \overline{K} \equiv \overline{K}^{\mu}{}_{\mu} = 3H. \tag{3.49}$$

Expanding (3.48) around this background leads to

$$S = \int d^4x\sqrt{-g}\left\{\Lambda_0(t) + c_1(t)\left(g^{00} - \overline{g}^{00}\right) + c_2(t)\left(K - \overline{K}\right) + c_3(t)\left(\mathscr{R} - \overline{\mathscr{R}}\right) + \ldots\right\}, \tag{3.50}$$

where we have only explicitly written operators that are linear in the perturbations. The breaking of time-diff invariance allows us to put arbitrary time-dependence in the coefficients of the EFT. It is easy to convince ourselves that we can always absorb most time-dependent pieces into the zeroth-order operator $\Lambda_0(t)$ and, through a conformal transformation of the metric, get rid of the $c_3(t)$ coefficient in front of the Ricci scalar, effectively picking the so-called "Einstein frame". Furthermore, the term linear in K can be traded for a function of g^{00} [17]. Therefore we can rewrite our action in the form

$$S = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\mathscr{R} - \Lambda(t) - c(t)g^{00}\right\} + \Delta S, \tag{3.51}$$

[17] Indeed, under the integrated volume-form $\int d^4x\sqrt{-g}$, we see that $f(t)K^{\mu}{}_{\mu} = f(t)\nabla_\mu n^\mu = -n^\mu\partial_\mu f(t) = +\sqrt{-g^{00}}\dot{f}(t)$. If you are missing the possible allowed term $\mathscr{R}^{00}$, consider the geometrical identity [106] $(-g^{00})\mathscr{R}^{00} = \mathscr{R}_{\mu\nu}n^\mu n^\nu = K^2 - K_{\mu\nu}K^{\mu\nu} - \nabla_\mu(n^\mu\nabla_\nu n^\nu) + \nabla_\nu(n^\mu\nabla_\mu n^\nu)$. The last two terms, under the integrated volume-form give, respectively, $f(t)\nabla_\mu(n^\mu\nabla_\nu n^\nu) = -\partial_\mu f(t)n^\mu K^\nu{}_\nu$ and $f(t)\nabla_\nu(n^\mu\nabla_\mu n^\nu) = -\partial_\nu f(t)n^\mu\nabla_\mu n^\nu = 0$, where in the last equality use has been made of $\partial_\nu f(t) \propto n_\nu$.

where ΔS denotes terms of quadratic order and higher and we have set the correct normalization of the good old Einstein-Hilbert term. The functions $\Lambda(t)$ and $c(t)$ are determined by the FLRW background which we have expanded around. As previously emphasized, by expanding around the correct background solution we automatically take care of the annoying tadpoles, i.e., terms linear in fluctuations vanish. Varying the linear terms in (3.51) with respect to the metric gives the Friedmann equations

$$3M_{\rm Pl}^2 H^2 = c(t) + \Lambda(t) \quad \text{and} \quad 3M_{\rm Pl}^2(\dot{H} + H^2) = \Lambda(t) - 2c(t), \tag{3.52}$$

or

$$\Lambda(t) = M_{\rm Pl}^2(3H^2 + \dot{H}) \quad \text{and} \quad c(t) = -M_{\rm Pl}^2 \dot{H}. \tag{3.53}$$

This way, we set once and for all the "universal" part of the total action, which now reads

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_{\rm Pl}^2}{2} \mathscr{R} - M_{\rm Pl}^2(3H^2 + \dot{H}) + M_{\rm Pl}^2 \dot{H} g^{00} \right\} + \Delta S. \tag{3.54}$$

3.2.4.2 Higher-Order Terms

It is wise to write $\Delta\mathscr{L}$, where $\Delta S \equiv \int d^4x \sqrt{-g}\, \Delta\mathscr{L}$, as an expansion in powers of fluctuations, i.e.,

$$\begin{aligned}\Delta\mathscr{L} =& \frac{1}{2!} M_2^4(t) \left(\delta g^{00}\right)^2 + \frac{1}{3!} M_3^4(t) \left(\delta g^{00}\right)^3 + \frac{1}{4!} M_4^4(t) \left(\delta g^{00}\right)^4 + \dots \\ & - \frac{1}{2}\bar{M}_1^3(t) \delta g^{00} \delta K - \frac{1}{2}\bar{M}_2^2(t) (\delta K)^2 - \frac{1}{2}\bar{M}_3^2(t) \delta K^\mu{}_\nu \delta K^\nu{}_\mu + \dots, \end{aligned} \tag{3.55}$$

where $\delta g^{00} \equiv g^{00} + 1$ and $\delta K_{\mu\nu} \equiv K_{\mu\nu} - a^2 H h_{\mu\nu}$.

In principle, this approach also encodes high energy effects for gravity, specifically in operators containing perturbations in the Riemann tensor $\delta\mathscr{R}_{\mu\nu\rho\sigma}$. However, as these corrections are of higher order in derivatives, they are not usually taken into account.

3.2.4.3 Stueckelberg Trick for the EFT of Inflation

The unitary gauge action in (3.54) describes three degrees of freedom, namely, the two graviton helicities and a scalar mode. We will now make use of the Stueckelberg trick; we perform a broken time-diff so that the Goldstone degree of freedom becomes manifest. Moreover, through such a procedure, the Goldstone π actually restores the gauge-invariance of the theory. Concretely, we perform the spacetime-dependent time reparametrization

$$t \to t' = t + \pi(x), \quad x^i \to x'^i = x^i. \tag{3.56}$$

Under this transformations, good old quantities like the four-dimensional Ricci scalar $\mathscr{R}$ and the volume form $d^4x\sqrt{-g}$ are invariant under general four-dimensional diffs, so they are invariant under (3.56), thus making no contribution to the Goldstone action. On the other hand, a generic function of time $f(t)$ transforms as

$$f(t) \to f(t+\pi) = f(t) + \dot{f}\pi + \frac{1}{2}\ddot{f}\pi^2 + \ldots, \tag{3.57}$$

i.e., we just "Taylor-expand" in powers of π around t. This implies that, e.g., $\delta\mathscr{R} \equiv \mathscr{R} - \overline{\mathscr{R}}$ transforms as

$$\delta\mathscr{R} \to \delta\mathscr{R} - \dot{\overline{\mathscr{R}}}\pi + \ldots. \tag{3.58}$$

Furthermore, contravariant and covariant tensor components transform as

$$\mathcal{X}^{\mu\nu} \to \frac{\partial x'^{\mu}}{\partial x^{\rho}} \frac{\partial x'^{\nu}}{\partial x^{\sigma}} \mathcal{X}^{\rho\sigma} = \left(\delta^{\mu}_{\rho} + \delta^{\mu}_{0} \partial_{\rho}\pi\right) \left(\delta^{\nu}_{\sigma} + \delta^{\nu}_{0} \partial_{\sigma}\pi\right) \mathcal{X}^{\rho\sigma}, \tag{3.59}$$

$$\mathcal{X}_{\mu\nu} \to \frac{\partial x^{\rho}}{\partial x'^{\mu}} \frac{\partial x^{\sigma}}{\partial x'^{\nu}} \mathcal{X}_{\rho\sigma} = \left(\delta^{\rho}_{\mu} + \delta^{\rho}_{0} \partial_{\mu}\pi\right)^{-1} \left(\delta^{\sigma}_{\nu} + \delta^{\sigma}_{0} \partial_{\nu}\pi\right)^{-1} \mathcal{X}_{\rho\sigma}\,. \tag{3.60}$$

Therefore, for the contravariant components of the metric we find that

$$g^{00} \to g^{00} + 2\partial_{\mu}\pi\, g^{0\mu} + \partial_{\mu}\pi\, \partial_{\nu}\pi\, g^{\mu\nu}, \tag{3.61}$$

$$g^{0i} \to g^{0i} + \partial_{\mu}\pi\, g^{\mu i}, \tag{3.62}$$

$$g^{ij} \to g^{ij}, \tag{3.63}$$

while covariant components can be written as an expansion in π.

Finally, when dealing with three-dimensional quantities characteristic of the $t =$ constant surface, such as the extrinsic and intrinsic curvatures $K_{\mu\nu}$ and $\widehat{\mathscr{R}}_{\mu\nu}$, it is important to note that under a change of coordinates, they do not just transform covariantly, as the surface Σ_t relative to which they are defined changes too. The spatial components of the extrinsic curvature orthogonal to the constant time hypersurface are given by

$$K_{ij} = \frac{1}{2}\sqrt{-g^{00}} \left(\partial_0 g_{ij} - \partial_i g_{0j} - \partial_j g_{i0}\right), \tag{3.64}$$

which, by using (3.60) for the metric $g_{\mu\nu}$, transform to linear order as

$$\begin{aligned} K_{ij}(x^{\mu}) \to K'_{ij}(x'^{\mu}) &= \frac{1}{2}\sqrt{-g^{00}}\,(1+\dot{\pi}) \left[(1-\dot{\pi})\,\partial_0 g_{ij} - \partial_i \left(g_{0j} + \partial_i\pi\right) - \partial_j \left(g_{i0} + \partial_i\pi\right)\right] \\ &= K_{ij} - \partial_i\partial_j\pi, \end{aligned} \tag{3.65}$$

where the K_{ij} in the last line is the extrinsic curvature orthogonal to the constant t hypersurfaces of the new coordinates [18]. A similar agument holds for $\widehat{\mathscr{R}}_{ij}$. Now, even if we just consider, for simplicity, the terms coming from powers of g^{00}, the Goldstone action is quite complicated

$$S=\int d^4x\sqrt{-g}\Big\{\frac{M_{\rm Pl}^2}{2}\mathscr{R}-M_{\rm Pl}^2\left(3H^2(t+\pi)+\dot{H}(t+\pi)\right)$$
$$+M_{\rm Pl}^2\dot{H}(t+\pi)\left(g^{00}+2\partial_\mu\pi g^{0\mu}+\partial_\mu\pi\partial_\nu\pi g^{\mu\nu}\right)$$
$$+\sum_{n=2}^{\infty}\frac{M_n^4(t+\pi)}{n!}(1+g^{00}+2\partial_\mu\pi g^{0\mu}+\partial_\mu\pi\partial_\nu\pi g^{\mu\nu})^n\Big\}. \quad (3.67)$$

Here, the Goldstone mixes with the metric perturbations in a highly non-trivial way so one may wonder: What have we gained by using the Stueckelberg trick to get out of the unitary gauge? This is when we remember the good old equivalence theorem for the Goldstone of time-translations that we already discussed in subsection 3.2.3. To repeat, under the decoupling limit, $M_{\rm Pl}\to\infty$ and $\dot{H}\to 0$ with $M_{\rm Pl}^2\dot{H}$ = constant, gravitational fluctuations decouple from the Goldstone mode for frequencies above $\omega_{\rm mix}^2=|\dot{H}|$. Therefore, for frequencies $\omega>\omega_{\rm mix}$, we can evaluate (3.67) in the *unperturbed* spacetime $\bar{g}^{\mu\nu}$ to get

$$S=\int d^4x\sqrt{-g}\Big\{\frac{M_{\rm Pl}^2}{2}\mathscr{R}-M_{\rm Pl}^2\left(3H^2(t+\pi)+\dot{H}(t+\pi)\right)$$
$$+M_{\rm Pl}^2\dot{H}(t+\pi)\left(-1-2\dot{\pi}+(\partial_\mu\pi)^2\right)+\sum_{n=2}^{\infty}\frac{M_n^4(t+\pi)}{n!}(-2\dot{\pi}+(\partial_\mu\pi)^2)^n\Big\}, \quad (3.68)$$

[18] A somewhat more formal approach [39] would be to first write $K_{\mu\nu}$ in terms of the four-dimensional metric

$$K_{\mu\nu}=\frac{\delta^0_\nu\partial_\mu g^{00}}{2\left(-g^{00}\right)^{3/2}}+\frac{\delta^0_\mu\delta^0_\nu g^{0\sigma}\partial_\sigma g^{00}}{2\left(-g^{00}\right)^{5/2}}-\frac{g^{0\sigma}\left(\partial_\mu g_{\sigma\nu}+\partial_\nu g_{\sigma\mu}-\partial_\sigma g_{\mu\nu}\right)}{2\left(-g^{00}\right)^{1/2}}, \quad (3.66)$$

and now use the transformation of the metric to determine how $K_{\mu\nu}$ transforms. Of course, using this prescription, ${K'}_{ij}(x'^\mu)$ coincides with (3.65) at linear order in π.

which is good up to errors of $\mathcal{O}\left(\frac{\omega_{\rm mix}^2}{\omega^2}\right)$.

In the EFT we have constructed so far, all the coefficients may have, a priori, generic time-dependence, but we will assume slow-roll conditions on them, i.e. $|\dot{M}|M^{-1} \ll H$ for a generic coefficient M (see footnote 13). With this assumption the Lagrangian is approximately time-translation invariant so the time dependence is suppressed in a "technically natural" way [19]. This last fact is quite important because the rapid time dependence may win against the Hubble friction, so that inflation may cease to be a dynamical attractor, which is necessary to solve the homogeneity problem of standard FLRW cosmology [20]. In this limit, the action for the relevant degrees of freedom dramatically simplifies to

$$S = \int d^4x\sqrt{-g}\Big\{\frac{M_{\rm Pl}^2}{2}\mathscr{R} - M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right) \\ + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\nabla\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\,\dot{\pi}^3 + \dots\Big\}. \tag{3.69}$$

With this action one is able to compute all the observables which are not dominated by the mixing with gravity. Recalling that $\mathcal{R} = -H\pi$ is conserved outside the horizon, the problem of computing predictions for present cosmological observations is reduced to calculating correlation functions just after horizon crossing, meaning we are interested in studying the theory with an IR cut-off of $\mathcal{O}(H)$. If the decoupling scale $\omega_{\rm mix}$ is smaller than H, the action (3.69) gives correct predictions up to terms suppresed by $\frac{\omega_{\rm mix}}{H}$. When the mixing with gravity is important, the full action (3.54) does contain everything we need to calculate accurate predictions.

By the way, we could also "explore" several limits of the general theory (3.54)

[19] In words of Matthew Schwartz [100], "It is technically natural for a parameter to be small if quantum corrections to the parameter are proportional to the parameter itself. This happens if the theory has an enhanced symmetry when the parameter is zero". The notion of technical naturalness was introduced by Gerard 't Hooft in [107].

[20] "Ultra slow-roll" inflation seems to be a counter-example to this statement. See [108] and references therein.

as we did in subsection 3.2.3, but we would arrive basically to the same findings, e.g.:

- The background operators recover the slow-roll inflation "class".
- If we keep the mixing with gravity we find a small mass term for the Goldstone, exactly such that $\mathcal{R}$ is massless, therefore conserved after horizon-crossing.
- Having a non-vanishing M_2 coefficient is equivalent to the existence of a non-trivial speed of sound given by $c_s^2 = \frac{M_{\rm Pl}^2|\dot{H}|}{M_{\rm Pl}^2|\dot{H}|+2M_2^4}$. In order to avoid superluminal propagation for the fluctuations, we must impose $M_2^4 > 0$. Furthermore, the nonlinearly-realized time-translation symmetry relates a small sound speed to large interactions.
- The theory defined by $\sum_{n=2}^{\infty} \frac{M_n^4(t)}{n!} \left(\delta g^{00}\right)^n$ is equivalent to a $P(X)$ theory.

Let us just mention, for completeness, that operators involving the extrinsic curvature fluctuation in (3.55) are relevant in dealing with more exotic models such as ghost inflation [109] and Galileon inflation [110], and the phenomenology of these contributions to the Goldstone dynamics was worked out in [111] and [112], respectively. For us, the action given in (3.69) will be more than enough. In the next chapter we will motivate and introduce our own two-field model of inflation, dubbed "Generalized Natural inflation".

Chapter 4

Generalized Natural Inflation

4.1 Motivation

As we have discussed so far, inflation is a well established framework that resolves several puzzles in big bang cosmology. The well known flatness, horizon and monopole problems can successfully be tackled by demanding a period of quasi-exponential expansion of the early universe [36–38]. While this classical picture is quite nice by itself, the quantum implications of this idea are also far reaching. Roughly speaking, all the structure in the universe can be understood as arising from primordial quantum fluctuations of the inflaton field [31]. To successfully match the percent-level deviation from perfect scale-invariance of the power spectrum of gauge-invariant primordial curvature perturbations that current observations demand, a considerable exponential growth of the scale factor $a(t)$ of the flat FLRW geometry is required, which is translated into several "slow-roll" conditions over the potential for the inflaton field. Many models that realize this slow-roll inflation scenario have been proposed over the years [113, 114].

The power spectrum contains all the information about the primordial perturbations *if* the initial conditions are drawn from a Gaussian distribution function. However, higher-order correlations may encode a significant amount of new information, as they are sensitive to non-linear interactions, while the power spectrum only probes the free theory. In the early 2000s, Maldacena proved that so-called non-Gaussianities for primordial scalar fluctuations in the simplest [1] inflationary models are generically suppressed by slow-roll parameters [115], meaning

[1]By simplest we mean single scalar field slow-roll inflation with a canonical kinetic term plus Einstein gravity using the so-called Bunch-Davies vacuum.

$f_{\rm NL} \sim \mathcal{O}(\epsilon, \eta)$, where the non-linear parameter $f_{\rm NL}$ is a measure of the amplitude of non-Gaussianities, $\epsilon \equiv -\frac{\dot{H}}{H^2}$ and $\eta \equiv \frac{\dot{\epsilon}}{\epsilon H}$ are the usual slow-roll parameters of inflation [2]. Consequently, since slow-roll conditions demand $\{\epsilon, \eta\} \ll 1$, we should abandon the possibility of observing such features if Nature really picked up this single-field slow-roll scenario as it is highly unlikely that we will ever be able to disentangle these "quantum" non-Gaussianities from "classical" ones that arise from CMB evolution [118] and from LSS [119] (due to the non-linear gravitational evolution or the galaxy bias), with $f_{\rm NL} \sim \mathcal{O}(1)$ as the natural size of these effects.

One way out of this "no-go" situation is to consider the so-called $P(X)$ theories [104, 105], where $X \equiv -\frac{1}{2}(\partial_\mu \varphi)^2$ and φ denotes the inflaton field. These theories may produce large non-Gaussianities without disrupting the inflationary background solution by respecting a mildly broken shift symmetry $\varphi \to \varphi + \text{constant}$, though it is important to keep in mind that it is a challenge to find a radiatively stable $P(X)$ scenario [3]. It has been found that $P(X)$ theories generically predict that $f_{\rm NL} \sim c_s^{-2}$, where c_s is the "speed of sound" of adiabatic fluctuations. Consequently, in principle, a non-trivial (small) speed of sound can lead to observable non-Gaussianities.

Another logical possibility is to consider additional fields during inflation. One crucial property of these fields is their mass, collectively denoted as m, compared to the Hubble scale H. There is an extensive literature regarding the case when these extra fields are light or even massless so that $m^2 \ll H^2$ (see [122, 123] for a review). This range of masses implies that non-Gaussianities will be effectively generated from non-linearities *after* horizon crossing, when all modes have become classical. At the other end, in the very massive case, meaning $m^2 \gg H^2$, we can always "integrate out" the heavy fields, leading to a simplified theory by

[2] It is now understood that in the local subcase, $f_{\rm NL}^{\rm local} = 0$, for *all* single-field inflation models [116, 117]. See appendix A for an exact definition of $f_{\rm NL}^{\rm local}$.

[3] One example of a radiatively stable UV-completion, where the form of the action is protected by a "higher-dimensional boost symmetry" is the case of DBI inflation [120, 121].

producing new (non-slow-roll) operators in the effective field theory (EFT) for the inflaton. The so-called Quasi-Single-Field (QSF) inflation models [13, 49–61] explore the third relevant regime, $m^2 \sim H^2$, where the new particles can in principle be produced by quantum fluctuations during the inflationary stage and then decay into inflatons, leaving a statistical imprint on the spectrum of primordial fluctuations. Importantly, the production of these particles gives rise to non-local effects which cannot be captured by a single-field EFT and can potentially give rise to observable non-Gaussianities.

There are several arguments for why it is reasonable to expect that the inflationary paradigm should naturally incorporate particles with such masses [4] and how they may show up in the "cosmological experiment", as has been recently emphasized in [13].

In this chapter, we will introduce and explore a two-field model that we unimaginatively dub "Generalized Natural Inflation" (GNI), a well-motivated generalization or "UV-completion" of the influential Natural Inflation (NI) scenario [43, 44]. Let us recall that single-field NI originally conceived the seminal idea that the inflaton is a *pseudo*-Nambu-Goldstone boson (pNGB) so it naturally has an exceptionally flat potential, which is a slow-roll requirement. In our model the inflaton plays the role of the phase θ of a complex scalar field $\chi \sim \sigma\, e^{i\theta}$, and the radial mode σ is taken to be dynamical, with a mass m_σ determined by the spontaneous breaking of a global $U(1)$ symmetry. To give a small mass to the would-be Goldstone (inflaton) field, so slow-roll conditions are satisfied, we softly break the $U(1)$ symmetry by a relevant operator. We will consider the cases $m_\sigma^2 \gg H^2$ and

[4] For example, consider the case when supersymmetry (SUSY) is invoked to tame the quantum corrections to the inflationary potential. Under the assumption that SUSY is not broken at energies higher than the inflationary scale H, the vacuum energy during inflation will surely break it as there is no supersymmetric theory in de Sitter space. This implies that additional fields which are not protected by global symmetries will inherit Hubble scale masses from SUSY breaking (this is related to the so-called "eta problem" of supergravity inflation models [124]). See [51] for details.

$m_\sigma^2 \sim H^2$ and find estimates for the non-Gaussianities that may arise in these scenarios [5]. The latter QSF regime is specially interesting, as we are effectively able to constrain an a priori arbitrary potential for the so-called "isocurvature" mode of the original (vanilla) QSF model of Chen and Wang [49, 50].

This chapter is structured as follows. In section 4.2 we introduce our model and go through the analysis of its associated inflationary background solution. We discuss how suitable initial conditions can lead to observable non-Gaussianities by dynamically decreasing the speed of sound of adiabatic fluctuations. We calculate the observables of the inflationary model and discuss its current viability given updated bounds coming from Planck 2015 [24] and Planck/Bicep [125] missions. In section 4.3 we discuss the theory of inflationary perturbations. First, we analyze the case when the radial field is very massive so it can be naively integrated out. We contrast the predictions for non-Gaussianities of our model when neglecting [126], as opossed to taking into account [55], the self-interactions of the heavy field. Then we address the QSF regime and obtain quantitative estimates for the size of non-Gaussianities. We find that, contrary to naive expectations, due to the tight observational constraints on the parameters of the model, non-Gaussianities are unobservably small.

[5] We briefly consider the case $m_\sigma^2 \ll H^2$ in subsection 4.3.2.3, where we demonstrate why this case is rather uninteresting for our particular model.

4.2 The Inflationary Background

4.2.1 Multifield Inflation

Let a "multifield" theory [6] be described by the following action [128]

$$S[g,\phi] = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\mathscr{R} - \frac{1}{2}\gamma^{ab}(\phi)g^{\mu\nu}\partial_\mu\phi_a\partial_\nu\phi_b - V(\phi)\right\}, \tag{4.1}$$

where $\gamma(\phi)$ is a "field metric", ϕ^a is a "vector" in field space, and $V(\phi)$ is some potential for the scalar fields. From $\gamma(\phi)$ we can construct a Christoffel symbol

$$\Gamma^a{}_{bc} = \frac{1}{2}\gamma^{ad}\left(\partial_b\gamma_{cd} + \partial_c\gamma_{bd} - \partial_d\gamma_{bc}\right), \tag{4.2}$$

and a corresponding curvature tensor

$$\mathbb{R}^a{}_{bcd} = \partial_c\Gamma^a{}_{bd} - \partial_d\Gamma^a{}_{bc} + \Gamma^a{}_{ce}\Gamma^e{}_{db} - \Gamma^a{}_{de}\Gamma^e{}_{cb}. \tag{4.3}$$

Varying (4.1) with respect to ϕ_a we get the field equations

$$\Box\,\phi^a + \Gamma^a{}_{bc}\,g^{\mu\nu}\partial_\mu\phi^b\partial_\nu\phi^c = V^a, \tag{4.4}$$

where $V^a \equiv \gamma^{ab}\,\partial_b V$. It is amusing to note the resemblance of this set of equations with the geodesic equation of motion of a relativistic particle in a non-trivial spacetime background under the influence of external (non-gravitational) forces. Now if we assume that $\phi^a = \phi^a(t)$ and $ds^2 = -dt^2 + a^2(t)\,\mathbf{dx}^2$, the field equations

[6] Usually the so-called multifield inflation scenario is understood to be one equipped with a shift symmetry, i.e., $\sigma^i \to \sigma^i + \text{constant}$ for the non-adiabatic (isocurvature) directions σ^i, so they remain light [127]. We do not assume such a constraint in the present multifield formalism.

for ϕ^a along with Einstein's equation for the spacetime metric read

$$\frac{D}{dt}\dot{\phi}^a + 3H\dot{\phi}^a + V^a = 0, \tag{4.5}$$

$$H^2 - \frac{1}{3M_{\mathrm{Pl}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V\right) = 0, \tag{4.6}$$

$$\dot{H} + \frac{\dot{\phi}^2}{2M_{\mathrm{Pl}}^2} = 0, \tag{4.7}$$

where $DX^a \equiv dX^a + \Gamma^a{}_{bc}\, X^b\, d\phi^c$ is a field space covariant derivative and $\dot{\phi}^2 \equiv \gamma_{ab}\, \dot{\phi}^a \dot{\phi}^b$ is the squared norm of $\dot{\phi}^a$. It is easy to show that (4.7) is not independent but can actually be derived from (4.5) and (4.6).

4.2.2 The Model

The model we want to introduce is motivated by the idea that the inflaton field can be identified as the pseudo-Goldstone boson associated with the spontaneous breaking of an approximate symmetry. Thus, we are led to choose the following potential for a complex scalar field χ,

$$V\left(\chi^\dagger, \chi\right) = \lambda\left(|\chi|^2 - v^2\right)^2 - M^2\left(\chi^\dagger \chi^\dagger + \chi\,\chi\right) + \mathbb{C}, \tag{4.8}$$

where λ, v, M and $\mathbb{C}$ are constants of mass dimension 0, 1, 1 and 4, respectively. The first term in (4.8) spontaneously breaks a global $U(1)$ symmetry while the second one is a soft explicit breaking [7]. Denoting as $\widehat{\psi}$ the vacuum expectation value (VEV) of any field ψ, the extrema of the potential, defined through $V_{\chi^\dagger}\big|_{(\chi^\dagger = \widehat{\chi}^\dagger,\, \chi = \widehat{\chi})} = V_{\chi}\big|_{(\chi^\dagger = \widehat{\chi}^\dagger,\, \chi = \widehat{\chi})} = 0$, are such that $|\widehat{\chi}|^2 = v^2 \pm \frac{M^2}{\lambda}$. We will parametrize the complex scalar χ in the polar form so the (broken) symmetry is

[7] In principle one should also consider the lower-dimensional symmetry breaking operator $\Upsilon(\chi^\dagger + \chi)$. However, if we impose a $\mathbb{Z}_2$ symmetry such that $\chi \to -\chi$ leaves the action invariant, $\Upsilon = 0$ naturally. In this work we are choosing this latter option.

manifest, meaning

$$\chi \equiv \frac{1}{\sqrt{2}}\left(\widetilde{R}+\sigma\right)e^{i\theta}, \tag{4.9}$$

where $\widetilde{R}$ is a constant of mass dimension 1. In the effective theory, after integrating the radial field σ, we want to recover a chaotic (concave) potential for the "inflaton" field θ. The minimum is then taken to be

$$|\widehat{\chi}|^2 = v^2 + \frac{M^2}{\lambda} = \frac{1}{2}\,(\widetilde{R}+\widehat{\sigma})^2 \equiv \frac{1}{2}\,R^2, \quad \widehat{\theta}=0. \tag{4.10}$$

Now we fix $\mathbb{C}$ by demanding a vanishing "cosmological constant" at the minimum

$$V\left(\widehat{\chi}^\dagger,\widehat{\chi}\right) = -M^2\left(2\,v^2+\frac{M^2}{\lambda}\right)+\mathbb{C}=0. \tag{4.11}$$

The potential $V\left(\chi^\dagger,\chi\right)$ can then be written as

$$V(\sigma,\theta) = \mu^4\left\{\left(1-\left(\frac{\widetilde{R}+\sigma}{\sqrt{2}\,v}\right)^2\right)^2 - \beta\left(\frac{\widetilde{R}+\sigma}{\sqrt{2}\,v}\right)^2\cos(2\theta)+\beta\left(1+\frac{\beta}{4}\right)\right\}, \tag{4.12}$$

where

$$\mu^4 \equiv \lambda\,v^4 \quad \text{and} \quad \beta \equiv \frac{2\,M^2}{\lambda\,v^2}. \tag{4.13}$$

Note that this potential is "non-separable", meaning $V(\sigma,\theta)\neq V(\sigma)+V(\theta)$. Since μ^4 is an overall constant that is fixed by the amplitude of the 2-point function of the inflaton fluctuation, β and v are the only parameters that determine the dynamics of the theory. It is easy to see that in the limit $\beta\to 0$, the "masses" of the radial and angular fields (evaluated at the minimum of the potential) are

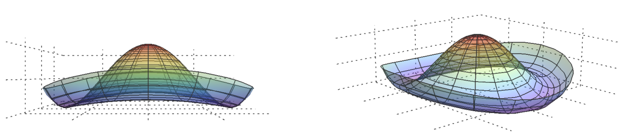
$4\,\lambda\,v^2$ and 0, respectively. At $\mathcal{O}(\beta)$ we find that they are given by $4\,\lambda\,v^2-2M^2$ and $4M^2$. From now on we will pick coordinates such that, without loss of generality,

$$\widehat{\sigma}=0 \rightarrow \widetilde{R}=R=\sqrt{2\,v^2+\frac{2M^2}{\lambda}}=\sqrt{2+\beta}\,v. \tag{4.14}$$

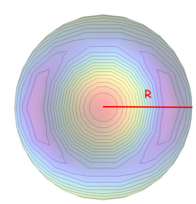
Finally we can rewrite (4.12) as

$$V(\sigma,\theta)=\mu^4\left\{\left(1-\left(\sqrt{1+\frac{\beta}{2}}+\frac{\sigma}{\sqrt{2}\,v}\right)^2\right)^2-\beta\left(\sqrt{1+\frac{\beta}{2}}+\frac{\sigma}{\sqrt{2}\,v}\right)^2\cos(2\theta)+\beta\left(1+\frac{\beta}{4}\right)\right\}. \tag{4.15}$$

In Figure 4.1 we plot the potential $V(\sigma,\theta)$ for a suitable choice of couplings. We see that it can be thought of as a "deformed" Mexican Hat.



(a) Front and aerial views of $V(\sigma,\theta)$. We see that the brim of the hat has sinusoidal behavior due to the explicit symmetry breaking.



(b) Top view of $V(\sigma,\theta)$. We see that the contour lines are ellipses and there are different extrema with different radii.

Figure 4.1: Deformed Mexican Hat.

The canonical Lagrangian for the χ field is given by

$$\mathscr{L}=-\partial_\mu\chi^\dagger\partial^\mu\chi-V\left(\chi^\dagger,\chi\right), \tag{4.16}$$

which, when written in the polar coordinates (4.9), takes the following form [8]

$$\mathscr{L} = -\frac{1}{2} g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}(R+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - V(\sigma,\theta). \tag{4.17}$$

Defining $\phi^a(t) = (\sigma(t), \theta(t))^T$ and $\gamma_{ab}(\phi) = \mathrm{diag}\,(1, (R+\sigma)^2)$ we may cast this class of models in the geometric language of multifield inflation. The non-vanishing Levi-Civita connection components are then given by $\Gamma^\sigma{}_{\theta\theta} = -(R+\sigma)$ and $\Gamma^\theta{}_{\theta\sigma} = (R+\sigma)^{-1}$, and since this is a polar coordinatization of a plane, $\mathbb{R}^a{}_{bcd} = 0$ trivially. Consequently the field equations (4.5) become

$$\ddot{\sigma} + 3H\dot{\sigma} - (R+\sigma)\,\dot{\theta}^2 + V_\sigma = 0, \tag{4.18}$$

$$(R+\sigma)^2\,\ddot{\theta} + 2(R+\sigma)\,\dot{\sigma}\,\dot{\theta} + 3H(R+\sigma)^2\,\dot{\theta} + V_\theta = 0, \tag{4.19}$$

where, given the potential in (4.15),

$$V_\theta = 2M^2(R+\sigma)^2\sin(2\theta) \quad \text{and} \quad V_\sigma = (R+\sigma)\left[\lambda\left\{(R+\sigma)^2 - 2\,v^2\right\} - 2M^2\cos(2\theta)\right]. \tag{4.20}$$

From (4.17) and (4.15) we see that when *naively* setting σ to its VEV, $\widehat{\sigma} = 0$, we are left with an effective NI theory [43, 44] for the canonically normalized field $\varphi \equiv R\,\theta$, whose Lagrangian is given by

$$\mathscr{L}_{\mathrm{eff}\,\varphi} = -\frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \widetilde{V}\left(1 - \cos\left(\frac{\varphi}{f}\right)\right), \tag{4.21}$$

[8] Let us note that the original QSF model [49, 50] is indeed determined by a two-field system with a Lagrangian seemingly identical to the one given in (4.17) but with $V(\sigma,\theta) = V(\sigma) + V_{\mathrm{sr}}(\theta)$, i.e., the potential is assumed to be "separable". Moreover, $V(\sigma)$ is a potential that traps the "isocurvaton" at some $\sigma = \widehat{\sigma}$ but remains otherwise arbitrary while $V_{\mathrm{sr}}(\theta)$ is an unspecified slow-roll potential. Our model instead, has a very specific non-separable potential given by (4.15). The motivation behind the original QSF model was the fact that when the inflaton trajectory moves along an arc, the action can be conveniently written in terms of polar coordinates of a circle with radius $\widetilde{R}$.

where $\widetilde{V} \equiv m_\varphi^2 f^2$, $m_\varphi \equiv 2M$ and $f \equiv \frac{R}{2}$ [9]. It may be argued that this procedure is rather incomplete, as care must be taken of the remnant equation of motion for σ, which now becomes a constraint equation. To prove that the single-field EFT of the inflationary background is indeed, to a very good approximation NI, we proceed as follows. As we neglect the dynamics of the radial field, σ becomes a Lagrange multiplier, so we can solve algebraically its own equation of motion (which is now a constraint equation) to give $\sigma = \sigma(\theta, \dot{\theta})$. Neglecting time-derivatives of σ in (4.18) we find that

$$(R+\sigma)\,\dot{\theta}^2 = V_\sigma = (R+\sigma)\left[\lambda\left\{(R+\sigma)^2 - 2\,v^2\right\} - 2M^2\cos(2\theta)\right], \tag{4.22}$$

where we have used (4.20). Recalling that $R \equiv +\sqrt{2\,v^2 + \frac{2M^2}{\lambda}}$, it is clear that $\sigma = -R$ is *not* a sensible solution, so (4.22) can be solved for σ to give

$$\sigma(\theta, \dot{\theta}) = -R + \left(2\,v^2 + \frac{2M^2}{\lambda}\cos(2\theta) + \frac{\dot{\theta}^2}{\lambda}\right)^{1/2}. \tag{4.23}$$

Now we plug this back into the single-field Lagrangian

$$\mathscr{L}_{\text{eff}\,\theta} = -\frac{1}{2}(R+\sigma(\theta,\dot{\theta}))^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - V(\sigma(\theta,\dot{\theta}),\theta), \tag{4.24}$$

[9]NI, at least before Planck's 2018 latest release [28], could be succesfully fit to data. In particular for $N_{\text{e-folds}} > 50$ and $n_s \approx 0.96$ one finds that $f \gtrsim 10\,M_{\text{Pl}} \approx 2.43 \times 10^{19}$ GeV and $\widetilde{V} \gtrsim (10^{-2}\,M_{\text{Pl}})^4 \approx (2.43 \times 10^{16}\,\text{GeV})^4$. Saturating these bounds implies that $m_\varphi \approx 2.43 \times 10^{13}$ GeV and $H \approx 1.4 \times 10^{14}$ GeV during the slow-roll regime, so indeed $m_\varphi^2 \ll H^2$, which is a requirement of the slow-roll approximation.

with $V(\sigma,\theta)$ as given in (4.15). After straightforward algebra one finds that

$$\mathscr{L}_{\text{eff}\,\varphi} = \frac{1}{2}\,\dot\varphi^2 - \widetilde{V}\left(1-\cos\left(\frac{\varphi}{f}\right)\right) \\ + \frac{\beta}{8(2+\beta)}\frac{\dot\varphi^4}{\widetilde{V}} - \frac{\beta}{2(2+\beta)}\,\dot\varphi^2\left(1-\cos\left(\frac{\varphi}{f}\right)\right) + \frac{2\beta}{(2+\beta)}\,\widetilde{V}\sin^4\left(\frac{\varphi}{2f}\right). \tag{4.25}$$

The first line in (4.25) reproduces NI as given in (4.21). Since $\beta\to 0$ as $\frac{m_\sigma^2}{H^2}\to\infty$, where $m_\sigma^2 \equiv 4\,\lambda\,v^2$ is the "mass" of the radial mode that is being integrated out, it is clear that the second line in (4.25) contains operators that are highly suppressed compared to this background theory, so they can be safely neglected, justifying the naive conclusion that the single-field effective background theory is NI to a very good approximation.

4.2.3 EFT for the Slowly-Rolling Field θ

Consider the set of equations (4.6), (4.18) and (4.19). Let us study the regime in which time derivatives of σ can be neglected. This would naively imply that the background trajectory is a circle in field space. We then impose that

$$\{\ddot\sigma,\, 3H\dot\sigma\} \ll \{(R+\sigma)\,\dot\theta^2,\, V_\sigma\} \quad \text{and} \quad 2(R+\sigma)\,\dot\sigma\,\dot\theta \ll \{(R+\sigma)^2\,\ddot\theta,\, 3H(R+\sigma)^2\,\dot\theta,\, V_\theta\}. \tag{4.26}$$

It has been argued [129] that the kinetic coupling $\mathscr{L} \ni -\frac{1}{2}(R+\sigma)^2(\partial_\mu\theta)^2$ manifests itself through the fact that the radial field will have a minimum at $\bar\sigma \neq \widehat\sigma$ where $\widehat\sigma$ is a solution to (4.10) (and $\widehat\sigma = 0$ is our "good choice of coordinates"). The inequalities in (4.26) imply that [10]

$$\frac{1}{(R+\bar{\sigma})}\frac{d\bar{\sigma}}{d\theta} \ll 1. \tag{4.27}$$

During the slow-roll regime, meaning $(R+\sigma)^2\,\ddot{\theta} \ll \{3H(R+\sigma)^2\,\dot{\theta},\, V_\theta\}$ as usual, the independent equations of motion become

$$3H(R+\sigma)^2\,\dot{\theta} + V_\theta = 0, \tag{4.28}$$

$$(R+\sigma)\,\dot{\theta}^2 = V_\sigma, \tag{4.29}$$

$$3M_{\rm Pl}^2 H^2 = V. \tag{4.30}$$

If σ = constant, (4.28) and (4.30) are the well-known equations that govern the slow-roll regime of a genuine single-field theory, whereas (4.29) can be thought of as a remnant "constraint", after ignoring the isocurvature field dynamics, that enforces "centripetal equilibrium" during an almost constant angular speed turn in field space. In appendix C we discuss the self-consistency of the above slow-roll approximation.

Using the set of equations (4.28), (4.29) and (4.30) it is easy to show that the algebraic relation,

$$(R+\sigma)^3\,V_\sigma\,V = \frac{M_{\rm Pl}^2}{3}\,V_\theta^2, \tag{4.31}$$

is a consistency requirement that should hold during the slow-roll evolution. We will define $\bar{\sigma} \neq 0$ as the time-dependent "solution" to this equation. It will be useful to state that, without making any assumptions about the "displaced" value

[10] Recall that the single field description is possible provided the kinetic energy is dominated by the angular field, or more specifically that $\dot{\sigma}^2 + (R+\sigma)^2\,\dot{\theta}^2 = \left(\left(\frac{d\sigma}{d\theta}\right)^2 + (R+\sigma)^2\right)\dot{\theta}^2 \sim (R+\sigma)^2\,\dot{\theta}^2$, which is indeed equivalent to demand $\frac{1}{(R+\sigma)}\frac{d\sigma}{d\theta} \ll 1$.

of σ, (4.20) and (4.14) imply that (4.29) becomes

$$\begin{aligned}\dot{\theta}^2 &= \lambda \left[\beta\,(1-\cos(2\theta))\,v^2 + 2\sqrt{2+\beta}\,v\,\sigma + \sigma^2\right] \\ &\approx \lambda \left[2\sqrt{2}\,v\,\sigma + \sigma^2 + \beta\left((1-\cos(2\theta))\,v^2 + \frac{\sqrt{2}}{2}\,v\,\sigma\right)\right] \quad \text{to} \quad \mathcal{O}(\beta).\end{aligned} \tag{4.32}$$

Let us consider now two limiting cases for the displaced value of σ.

4.2.3.1 Small Radial Displacement

If we assume that $\bar{\sigma}(\theta) = \sigma_1(\theta)$, where σ_1 is a "small" departure from the naive VEV, the solution to (4.31) after linearizing with respect to σ is found to be given by

$$\sigma_1(\theta) \approx \frac{\beta}{6\sqrt{2}} \frac{\left[M_{\text{Pl}}^2 - 3\,v^2 + (M_{\text{Pl}}^2 + 3\,v^2)\cos(2\theta)\right]}{v} \quad \text{to} \quad \mathcal{O}(\beta)\,. \tag{4.33}$$

In principle we can plug this solution back in the potential and find a canonical variable so that we have a single-field effective potential. However, in situations in which the solution $\bar{\sigma}(\theta)$ is a complicated function of θ, it may be too difficult to follow this procedure, the main reason being that we need to find a canonical variable ϕ such that $(R + \bar{\sigma})\,\dot{\theta} = \dot{\phi}$. However, the system can still be solved "semi-analytically" as was argued in [129].

4.2.3.2 Big Radial Displacement

As we will see in section (4.3), if we consider the perturbations around the background model in a regime where the dynamics of the fluctuation $\delta\sigma$ is negligible in comparison with its effective mass M_{eff}, the so-called $M_{\text{eff}}^2 \gg H^2$ regime, the EFT for the inflaton perturbation $\delta\theta$ develops a non-trivial speed of sound c_s given by

$$c_s^{-2} = 1 + 4\frac{\dot{\theta}_0^2}{M_{\rm eff}^2} \quad \text{where} \quad M_{\rm eff}^2 \equiv V_{\sigma\sigma}(\sigma_0, \theta_0) - \dot{\theta}_0^2, \tag{4.34}$$

and ψ_0 denotes the background value of any field ψ [128, 130, 131]. When the potential $V(\sigma, \theta)$ is given by (4.12) we find that

$$V_{\sigma\sigma}(\sigma_0, \theta_0) = 3\,\lambda\,(R + \sigma_0)^2 - 2(\lambda\,v^2 + M^2\cos(2\theta_0)). \tag{4.35}$$

Using (4.34), (4.35), (4.14) and (4.32) the effective mass is given by

$$\begin{aligned} M_{\rm eff}^2 &= 2\,\lambda\left[(2+\beta)\,v^2 + 2\sqrt{2+\beta}\,v\,\sigma + \sigma^2\right] \\ &\approx \lambda\left[4\,v^2 + 4\sqrt{2}\,v\,\sigma + 2\,\sigma^2 + \beta\left(2\,v^2 + \sqrt{2}\,v\,\sigma\right)\right] \quad \text{to} \quad \mathcal{O}(\beta). \end{aligned} \tag{4.36}$$

Looking at (4.34) we see that $c_s^2 \ll 1 \iff 4\frac{\dot{\theta}^2}{M_{\rm eff}^2} \gg 1$. This condition, using (4.32) and (4.36), is equivalent to

$$c_s^2 \ll 1 \iff \sigma^2 + 2\sqrt{2+\beta}\,v\,\sigma + [\beta(1 - 2\cos(2\theta)) - 2]\,v^2 \gg 0, \tag{4.37}$$

which is satisfied whenever

$$\sigma \gg \left[\sqrt{4 + 2\beta\cos(2\theta)} - \sqrt{2+\beta}\right]v \approx \sqrt{2}\left(\sqrt{2} - 1\right)v + \frac{\beta}{2}\left(\cos(2\theta) - \frac{\sqrt{2}}{2}\right)v \quad \text{to} \quad \mathcal{O}(\beta). \tag{4.38}$$

Neglecting $\mathcal{O}(\beta)$ terms we see that when the radial field is considerably displaced from its trivial minimum, i.e., $\sigma \gg \sqrt{2}\left(\sqrt{2}-1\right)v \approx 0.585\,v > \widehat{\sigma} = 0$, it is possible, "dynamically", to get $c_s^2 \ll 1$. This fact has been previously understood and emphasized [132]. Though interesting, we will not consider this big radial field displacement scenario any further. Additional developments along these lines can be found in [129].

4.2.4 Intermission II: Vacuum Fluctuations, Curvature Perturbations, Gravitational Waves

To compare the predictions of the model with data in a self-contained way, we need to digress a little bit again and introduce some definitions [39]. Recall from (3.21) that $v \equiv \sqrt{\frac{2M_{\rm Pl}^2\epsilon}{c_s^2}}\mathcal{R}$, the so-called Mukhanov-Sasaki variable, is the field to be quantized in the classical inflationary background. As usual, one promotes the Fourier modes to quantum operators

$$\hat{v}_{\mathbf{k}} = v_k(t)\hat{a}_{\mathbf{k}} + \text{h.c.} \tag{4.39}$$

At sufficiently early times, all modes of cosmological interest were deep inside the Hubble radius and behave as ordinary harmonic oscillators. The annihilation operators $\hat{a}_{\mathbf{k}}$ define the vacuum $|\Omega\rangle$ through $\hat{a}_{\mathbf{k}}|\Omega\rangle = 0$, and the "zero-point fluctuations" of the oscillation amplitude are the same as those of an oscillator in flat space, meaning

$$\langle\Omega|\,\hat{v}_{\mathbf{k}}\hat{v}_{\mathbf{k}'}\,|\Omega\rangle = (2\pi)^3|v_k|^2\delta(\mathbf{k}+\mathbf{k}') \quad \text{where} \quad |v_k|^2 = \frac{1}{a^3}\frac{1}{2\omega_k}. \tag{4.40}$$

As long as the physical wavelength of the mode is smaller than the Hubble radius, the ground state evolves adiabatically, so (4.40) holds through time. Once a given mode gets stretched outside the Hubble radius, adiabaticity breaks down and the fluctuation amplitude freezes at

$$|v_k|^2 = \frac{1}{2a_\star^3}\frac{1}{\left(\frac{c_s k}{a_\star}\right)} = \frac{1}{2}\frac{H^2}{(c_s k)^3} \tag{4.41}$$

where $a_\star$ is the value of the scale factor at horizon crossing $\frac{c_s k}{a_\star} \equiv H$.
Using the relation between v and $\mathcal{R}$, we find the power spectrum of primordial perturbations

$$P_\mathcal{R} \equiv |\mathcal{R}_k|^2 = \frac{1}{4}\frac{H^4}{M_{\rm Pl}^2 \dot{H} c_s}\frac{1}{k^3}. \tag{4.42}$$

The variance in real space is $\langle \mathcal{R}^2 \rangle = \int d\ln k\, \Delta^2_\mathcal{R}(k)$, where we have defined the dimensionless power spectrum

$$\Delta^2_\mathcal{R}(k) \equiv \frac{k^3}{2\pi^2} P_\mathcal{R}(k) = \frac{1}{8\pi^2}\frac{H^4}{M_{\rm Pl}^2 |\dot{H}| c_s}. \tag{4.43}$$

Any time dependence of H and c_s translates into a scale dependence of the power spectrum because the right-hand side of (4.43) is supposed to be evaluated at horizon crossing, $c_s k = aH$. By definition, scale-invariant fluctuations correspond to $\Delta^2_\mathcal{R}(k) =$ constant, and deviations from scale invariance are quantified by the so-called spectral tilt

$$n_s - 1 \equiv \frac{d \ln \Delta^2_\mathcal{R}}{d \ln k} = -2\,\epsilon - \eta - s, \tag{4.44}$$

where we have defined a new expansion parameter

$$s \equiv \frac{\dot{c}_s}{c_s\, H}. \tag{4.45}$$

Inflationary backgrounds typically satisfy $\{\epsilon, \eta, s\} \ll 1$ and so predict $n_s \approx 1$. However, inflation would not end if the slow-roll parameters vanished, so we do expect a finite deviation from perfect scale invariance, $n_s \neq 1$.
It may be argued that the cleanest prediction of inflation is a spectrum of primor-

dial gravitational waves as these are tensor perturbations to the spatial metric,

$$g_{ij} = a^2(t)\left(\delta_{ij} + h_{ij}\right), \tag{4.46}$$

where h_{ij} is transverse and traceless. Expanding the Einstein-Hilbert action leads to the quadratic action for the tensor fluctuations

$$S_h^{(2)} = \frac{M_{\rm Pl}^2}{8}\int d^4x\, a^3 \left\{ \dot{h}_{ij}{}^2 - \frac{\left(\nabla h_{ij}\right)^2}{a^2} \right\} \tag{4.47}$$

The structure of the action is identical to that of the scalar fluctuations (3.20), except that tensors do not have a non-trivial speed of sound and the canonically normalized field does not have an ϵ-dependence since at linear order tensors do not "feel" the symmetry breaking due to the background evolution. The quantization of tensor fluctuations then proceeds exactly as for the scalar fluctuations case, with a Mukhanov-Sasaki variable for each polarization mode of the gravitational field. Adding the power spectra of the two polarization modes Starobinsky found [93]

$$\Delta_h^2(h) \equiv \frac{k^3}{2\pi^2} P_h(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\rm Pl}^2}, \tag{4.48}$$

where the right-hand side is evaluated at horizon crossing, $k = aH$. We see that the power spectrum of tensor fluctuations is only a function of the de Sitter expansion rate H, in contrast with the scalar fluctuations case in (4.43), where there is dependence on H, $\dot{H}$ and c_s. Tensor fluctuations are therefore said to be a direct probe of the energy scale at which inflation took place. The scale dependence of the tensor modes is determined by the time dependence of H, parametrized through [11]

$$n_t \equiv \frac{d \ln \Delta_h^2}{d \ln k} = -2\,\epsilon. \tag{4.49}$$

Observational constraints on tensor modes are usually expressed in terms of the tensor-to-scalar ratio,

$$r \equiv \frac{\Delta_h^2}{\Delta_\mathcal{R}^2} = 16\,\epsilon\, c_s. \tag{4.50}$$

As the amplitude of scalar fluctuations has been measured, the tensor-to-scalar ratio quantifies the size of the tensor fluctuations, and by using (4.48) one can write

$$\frac{H}{M_{\rm pl}} = \pi\,\Delta_\mathcal{R}(k_\star)\sqrt{\frac{r}{2}} \Rightarrow H \approx 3\times 10^{-5}\left(\frac{r}{0.1}\right)^{1/2} M_{\rm Pl}, \tag{4.51}$$

when using that $\Delta^2_\mathcal{R}(k_\star) = 2.14\times 10^{-9}$ at the pivot scale $k_\star = 0.05\,\mathrm{Mpc}^{-1}$ [24]. Detecting inflationary tensor perturbations at the level of $r \sim 0.1$ would imply then that the expansion rate during inflation was about $10^{-5} M_{\rm Pl}$. This is sometimes expressed in terms of the "energy scale of inflation",

$$E_{\rm inf} \equiv \left(3H^2 M_{\rm Pl}^2\right)^{1/4} = 8\times 10^{-3}\left(\frac{r}{0.1}\right)^{1/4} M_{\rm Pl}. \tag{4.52}$$

From this last expression we see that reducing r by four orders of magnitude reduces $E_{\rm inf}$ by only one order of magnitude. As of now, gravitational waves from inflation are only observable if inflation occurred near the grand unified theory (GUT) scale, $E_{\rm inf} \sim 10^{-2} M_{\rm Pl} \sim 10^{16}\,\mathrm{GeV}$.

[11] The absence of the "-1" in the definition of the tensor spectral tilt n_t, cf. equation (4.44), seems to be due to untraceable historical reasons.

4.2.5 Semi-analytical Approach

After this necessary intermission, let us come back to our GNI model. There is a semi-analytical way of dealing with the system of equations (4.28)-(4.30) [129]. Recalling the usual definitions of slow-roll parameters $\epsilon \equiv -\frac{\dot{H}}{H^2}$ and $\eta \equiv \frac{\dot{\epsilon}}{\epsilon H}$, and defining

$$\delta \equiv \frac{\dot{\bar{\sigma}}}{(R+\bar{\sigma})H} = \frac{1}{(R+\bar{\sigma})}\left(\frac{d\,\bar{\sigma}}{d\,\theta}\right)\left(\frac{\dot{\theta}}{H}\right) \approx -\frac{M_{\rm Pl}}{(R+\bar{\sigma})^2}\left(\frac{d\,\bar{\sigma}}{d\,\theta}\right)\sqrt{2\,\epsilon}, \tag{4.53}$$

it is straightforward to show that

$$\epsilon \approx \frac{M_{\rm Pl}^2}{2(R+\bar{\sigma})^2}\left(\frac{V_\theta}{V}\right)^2, \tag{4.54}$$

$$\eta \approx -\frac{2M_{\rm Pl}^2}{(R+\bar{\sigma})^2}\left(\frac{V_{\theta\theta}}{V}\right) - \frac{2M_{\rm Pl}^2}{(R+\bar{\sigma})^2}\left(\frac{d\,\bar{\sigma}}{d\,\theta}\right)\left(\frac{V_{\theta\sigma}}{V}\right) + 4\,\epsilon - 2\,\delta. \tag{4.55}$$

Finally, recalling that $dN \equiv -H dt$, we get that the number of e-folds before the end of inflation is given by

$$N = \frac{1}{M_{\rm Pl}^2}\int (R+\bar{\sigma})^2\left(\frac{V}{V_\theta}\right)d\theta, \tag{4.56}$$

stressing again that $\bar{\sigma}$ is defined as the solution to (4.31). The deviations from NI are due to the implicit time dependence of $\bar{\sigma} = \bar{\sigma}(\theta(t))$. We see that even if the reduced equations of motion demand $\delta \ll 1$, δ may still be $\mathcal{O}(\epsilon, \eta)$. Thus, even if we can neglect the derivatives of σ at the level of the equations of motion, they may still play an important role in determining the observables of the model. Using (4.53), (4.54) and (4.55) with $\bar{\sigma}(\theta) = \sigma_1(\theta)$ as given by (4.33) and the potential given by (4.15), we find that

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \approx \frac{M_{\rm Pl}^2\cot^2(\theta)}{v^2}\left\{1 - \frac{\beta}{72}\frac{\left[3M_{\rm Pl}^4 - 3M_{\rm Pl}^2\,v^2 + 18\,v^4 + 2(2M_{\rm Pl}^4 - 9\,v^4)\cos(2\theta) + M_{\rm Pl}^2(M_{\rm Pl}^2 + 3\,v^2)\cos(4\theta)\right]\csc^2(\theta)}{v^4}\right\}, \tag{4.57}$$

$$\eta \equiv \frac{\dot{\epsilon}}{\epsilon\, H} \approx \frac{2M_{\rm Pl}^2 \csc^2(\theta)}{v^2}\left\{1 - \frac{\beta}{18}\frac{\left[(M_{\rm Pl}^2+3\,v^2)(6M_{\rm Pl}^2-3\,v^2+M_{\rm Pl}^2\cos(2\theta))\sin^2(\theta)-8M_{\rm Pl}^4-12M_{\rm Pl}^2\,v^2+9\,v^4+3M_{\rm Pl}^4\csc^2(\theta)\right]}{v^4}\right\}. \tag{4.58}$$

We see that to $\mathcal{O}(\beta^0)$, $\{\epsilon, \eta\} \sim \frac{M_{\rm Pl}^2}{v^2}$ which implies that in order to have $\{\epsilon, \eta\} \ll 1$ we need $v^2 \gg M_{\rm Pl}^2$, as is usually the case for NI. In our model $\{\dot{\sigma} \approx 0 \Rightarrow \dot{\theta} \approx \text{constant}\} \Longrightarrow \{c_s \approx \text{constant} \Rightarrow s \approx 0\}$ to $\mathcal{O}(\beta^0)$. We pick parameter values λ, v and M so they are compatible with the set of relations

$$\begin{gathered} \widetilde{V} \approx \frac{3\,\pi^2}{2}\, r\, \Delta_{\mathcal{R}}^2\, M_{\rm Pl}^4, \quad v \approx \sqrt{\frac{16\,c_s}{r}} M_{\rm Pl}, \quad M \approx \sqrt{\frac{\widetilde{V}}{2\,v^2}}, \\ H \approx \sqrt{\frac{\widetilde{V}}{3}}\, M_{\rm Pl}^{-1}, \quad \lambda \equiv \frac{\widetilde{\alpha}\,(1+3\,c_s^2)H^2}{16\,v^2}, \quad \dot{\theta} \approx \sqrt{\frac{4\,\lambda\,v^2\,(1-c_s^2)}{1+3\,c_s^2}}, \end{gathered} \tag{4.59}$$

where $\widetilde{V}^{1/4}$ is the energy scale of inflation, $\widetilde{\alpha} \equiv \frac{\widetilde{M}_{\rm eff}^2}{H^2}$ is the ratio between the "Hamiltonian effective mass squared" $\widetilde{M}_{\rm eff}^2 \equiv M_{\rm eff}^2\, c_s^{-2}$ and H^2 and we are neglecting $\mathcal{O}(\beta)$ terms [12]. Indeed, $\beta \equiv \frac{2\,M^2}{\lambda\,v^2} \approx \frac{3}{1+3\,c_s^2}\frac{r}{\widetilde{\alpha}\,c_s} \approx \frac{48}{1+3\,c_s^2}\frac{\epsilon}{\widetilde{\alpha}}$ within the above approximations, so β is always a very small number due to slow-roll. Then, we saturate the current constraint $r < 0.07$ [125] in (4.59) to build up Figure 4.2 and Table 4.1. Note that when $\widetilde{\alpha} \sim \mathcal{O}(1)$ the EFT for a single-field theory is not really justified since $\widetilde{M}_{\rm eff}^2 \sim H^2$. Nevertheless, it is illuminating to "extrapolate" our results since, in particular, the value of $\frac{\dot{\theta}}{H}$ is quite important for the theory of fluctuations exactly in this limit, as we will see in the next section.

[12] In subsection 4.3.2.1 and appendix B the introduction of the more "physical" effective mass $\widetilde{M}_{\rm eff}$ is justified. $\widetilde{M}_{\rm eff}$ has been also referred to as the "entropy mass" [133] and it really corresponds to the mass of a particle belonging to the spectrum of the theory, which is not the case for $M_{\rm eff}$.

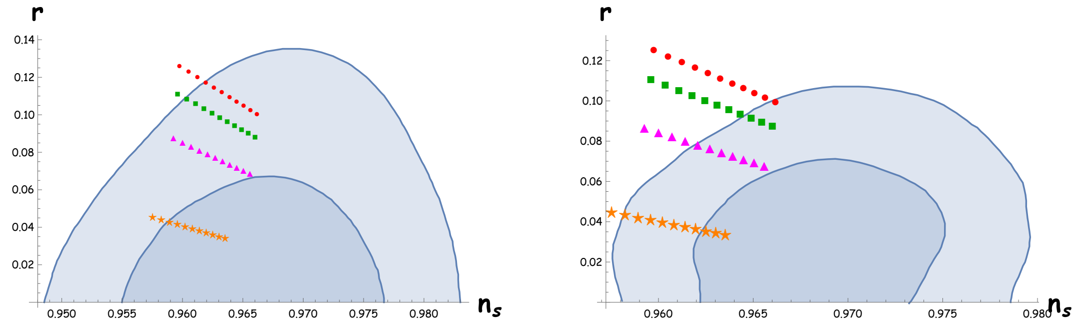


Figure 4.2: The (n_s, r) plane for the "Natural" potential, when the mass of the heavy field is given by $\widetilde{M}^2_{\rm eff} = 100\,H^2$. The blue regions are the 1-σ and 2-σ allowed regions from *Left:* Planck 2015 (Planck TT+lowP) [24] and *Right:* Planck/Bicep (Planck TT+lowP+BKP+lensing+ext) [125].
We plot the predictions for $N = [50, 60]$ when $c_s = 0.999$ (●), $c_s = 0.9$ (■), $c_s = 0.75$ (▲) and $c_s = 0.47$ (★).

	λ_{100}	$\dot\theta_{100}$ (GeV)	$(\dot\theta/\mathbf{H})_{100}$	λ_{10}	$\dot\theta_{10}$ (GeV)	$(\dot\theta/\mathbf{H})_{10}$	λ_1	$\dot\theta_1$ (GeV)	$(\dot\theta/\mathbf{H})_1$
$\mathbf{c_s = 0.999}$	8.081×10^{-11}	1.477×10^{13}	0.224	8.081×10^{-12}	4.671×10^{12}	0.071	8.081×10^{-13}	1.477×10^{12}	0.022
$\mathbf{c_s = 0.9}$	7.704×10^{-11}	1.440×10^{14}	2.179	7.704×10^{-12}	4.553×10^{13}	0.689	7.704×10^{-13}	1.440×10^{13}	0.218
$\mathbf{c_s = 0.75}$	7.243×10^{-11}	2.185×10^{14}	3.307	7.243×10^{-12}	6.910×10^{13}	1.046	7.243×10^{-13}	2.185×10^{13}	0.331
$\mathbf{c_s = 0.47}$	7.151×10^{-11}	2.916×10^{14}	4.413	7.151×10^{-12}	9.221×10^{13}	1.396	7.151×10^{-13}	2.916×10^{13}	0.441

Table 4.1: $\lambda_{\widetilde\alpha}$, $\dot\theta_{\widetilde\alpha}$ and $\left(\frac{\dot\theta}{H}\right)_{\widetilde\alpha}$, where $X_{\widetilde\alpha} \equiv X(\widetilde\alpha)$, with $\widetilde\alpha \equiv \frac{\widetilde{M}^2_{\rm eff}}{H^2} = \{100, 10, 1\}$ for different values of c_s.

Looking at Figure 4.2 we see that for $N = 60$, this model is alive and well, meaning the current constraint $r < 0.07$ is satisfied [125], when $c_s = 0.75$. The only parameter of the model which depends upon $\widetilde\alpha$ is λ, which only influences the slow-roll parameters (therefore the predictions for the observables) at a negligible order way beyond the current experimental sensitivity. In other words, taking $\widetilde\alpha = \{100, 10, 1\}$ gives the same predictions depicted in Figure 4.2. However it is interesting to note from Table 4.1 that for fixed c_s, as $\widetilde\alpha$ decreases, $\frac{\dot\theta}{H}$ decreases too, since $\frac{\dot\theta}{H} \approx \frac{1}{2}\,\widetilde\alpha^{1/2}\,(1 - c_s^2)^{1/2}$ according to (4.59). This feature is doubly reassuring:

1. It is consistent with the fact that according to the EFT analysis it has been understood that $\dot\theta^2 \ll H^2$ is *not* a restriction for the EFT to be valid as

some authors initially argued in the literature [13].

2. When the heavy field is not super heavy, like in the QSF scenario, $\frac{\dot{\theta}}{H}$ plays the role of a time-dependent coupling between the adiabatic and isocurvature perturbations, so $\dot{\theta}^2 \ll H^2$ is a standard perturbative condition one needs to impose to do perturbative physics. Even if the limit $\widetilde{\alpha} \to 1$ is ill-defined from the single-field EFT point of view, we believe this extrapolation sheds some light on the perturbative limitations that the theory of fluctuations has in the two-field regime (see 4.3.2.1 below). Let us now study the theory of fluctuations.

4.3 Inflationary Perturbations in the GNI Model

In this section we will study the theory of fluctuations of the GNI model in order to calculate the non-Gaussianities that arise due to the presence of the isocurvature mode. We will address the regimes $M_{\rm eff}^2 \gg H^2$ and $M_{\rm eff}^2 \sim H^2$ separately, as the physics is quite different.

To study the inflationary perturbations defined as $\delta\phi^a(t,\mathbf{x}) \equiv \phi^a(t,\mathbf{x}) - \phi_0^a(t)$ it is useful to consider vectors tangent and normal to the trajectory $\phi_0^a(t)$ given by

$$T^a \equiv \frac{\dot{\phi}_0^a}{\dot{\phi}_0}, \quad N^a \equiv -\frac{D_t T^a}{|D_t T|}. \tag{4.60}$$

The fluctuations along the direction T^a define the curvature perturbations as $\mathcal{R} \equiv -\frac{H}{\dot{\phi}_0} T_a\, \delta\phi^a$ whereas the fluctuations along N^a correspond to the isocurvature perturbations [133, 135]. The introduction of T^a and N^a allows us to define Ω,

[13] In [134] the "adiabaticity" condition $|\ddot{\theta}| \ll M_{\rm eff}\, |\dot{\theta}|$ has been identified as a requirement for the heavy field to not become excited during the turn.

the angular velocity with which the inflationary trajectory bends, via

$$D_t T^a \equiv -\Omega\, N^a. \tag{4.61}$$

Comparing (4.60) with (4.61) we see that $\Omega = |D_t T|$ is positive definite by construction. It is clear that in the two-field case $\{T^a, N^a\}$ is an orthonormal basis that spans the vector space, implying that $V_a = V_\phi\, T_a + V_N N_a$, where $V_\phi \equiv T^a V_a$ and $V_N \equiv N^a V_a$. The equation resulting from projecting (4.5) along T^a is

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_\phi = 0, \tag{4.62}$$

resembling the equation of motion of a single scalar field in a FLRW spacetime. On the other hand, the equation obtained from projecting (4.5) along N^a is given by

$$\Omega = \frac{V_N}{\dot{\phi}_0}. \tag{4.63}$$

Whenever the trajectory is subjected to a bend, it moves up towards the outer wall of the potential. The angular velocity Ω plays a crucial role in the dynamics of fluctuations, as it couples together curvature and isocurvature modes. From (4.60) we see that the normal vector is constructed such that $T_a N^a = 0$ and $N_a N^a = 1$. In the two-field case it can be taken as $N_a = (\det\gamma)^{1/2}\, \varepsilon_{ab}\, T^b$, where ε_{ab} is the two-dimensional Levi-Civita symbol with $\varepsilon_{11} = \varepsilon_{22} = 0$ and $\varepsilon_{12} = -\varepsilon_{21} = 1$. Then for our model we get that [136]

$$T^a = \frac{(\dot{\sigma}_0,\, \dot{\theta}_0)}{\Big[(R+\sigma_0)^2\, \dot{\theta}_0^2 + \dot{\sigma}_0^2\Big]^{1/2}}, \quad N^a = \frac{(R+\sigma_0)(\dot{\theta}_0,\, -(R+\sigma_0)^{-2}\dot{\sigma}_0)}{\Big[(R+\sigma_0)^2\, \dot{\theta}_0^2 + \dot{\sigma}_0^2\Big]^{1/2}}. \tag{4.64}$$

Considering (4.63) we see that since $V_N \equiv \Omega\,\dot\phi_0 = N^\sigma V_\sigma + N^\theta V_\theta$, Ω is given by

$$\Omega = \frac{1}{\left[(R+\sigma_0)^2\,\dot\theta_0^2 + \dot\sigma_0^2\right]}\Big\{(R+\sigma_0)\,\dot\theta_0\left(-\ddot\sigma_0 - 3H\dot\sigma_0 + (R+\sigma_0)\,\dot\theta_0^2\right) \\ - (R+\sigma_0)^{-1}\dot\sigma_0\left(-(R+\sigma_0)^2\,\ddot\theta_0 - 2(R+\sigma)\,\dot\sigma_0\,\dot\theta_0 - 3H(R+\sigma_0)^2\,\dot\theta_0\right)\Big\}, \tag{4.65}$$

where use has been made of (4.18) and (4.19). Thus,

$$\text{if} \quad \sigma_0 = \text{constant}, \quad \text{meaning} \quad \dot\sigma_0 = 0, \quad \text{then} \quad \Omega = \dot\theta_0, \tag{4.66}$$

without assuming slow-roll conditions on θ_0.

The theory of fluctuations of the polar fields is determined by the expansion [14]

$$S[g_0,\phi_0,\delta\phi] = S^{(0)}[g_0,\phi_0] + S^{(2)}[g_0,\phi_0,\delta\phi] + S^{(3)}[g_0,\phi_0,\delta\phi] + \dots, \tag{4.67}$$

$$S^{(0)}[g_0,\phi_0] = \int d^4x\, a^3\left\{\frac{1}{2}(R+\sigma_0)^2\,\dot\theta_0^2 + \frac{1}{2}\dot\sigma_0^2 - V(\sigma_0,\theta_0)\right\}, \tag{4.68}$$

$$S^{(2)}[g_0,\phi_0,\delta\phi] = \int d^4x\, a^3\Big\{-\frac{1}{2}(R+\sigma_0)^2 g^{\mu\nu}\partial_\mu\delta\theta\partial_\nu\delta\theta - \frac{1}{2}V_{\theta\theta}(\sigma_0,\theta_0)(\delta\theta)^2 \\ + 2(R+\sigma_0)\,\dot\theta_0\delta\dot\theta\delta\sigma - V_{\theta\sigma}(\sigma_0,\theta_0)\delta\theta\delta\sigma - \frac{1}{2}g^{\mu\nu}\partial_\mu\delta\sigma\partial_\nu\delta\sigma - \frac{1}{2}M_{\text{eff}}^2(\delta\sigma)^2\Big\}, \tag{4.69}$$

$$S^{(3)}[g_0,\phi_0,\delta\phi] = \int d^4x\, a^3\Big\{-(R+\sigma_0)(g^{\mu\nu}\partial_\mu\delta\theta\partial_\nu\delta\theta)\delta\sigma + \dot\theta_0\delta\dot\theta(\delta\sigma)^2 - \frac{1}{6}V_{\theta\theta\theta}(\sigma_0,\theta_0)(\delta\theta)^3 \\ - \frac{1}{2}V_{\theta\sigma\sigma}(\sigma_0,\theta_0)\delta\theta(\delta\sigma)^2 - \frac{1}{2}V_{\theta\theta\sigma}(\sigma_0,\theta_0)(\delta\theta)^2\delta\sigma - \frac{1}{6}V_{\sigma\sigma\sigma}(\sigma_0,\theta_0)(\delta\sigma)^3\Big\}, \tag{4.70}$$

where $M_{\text{eff}}^2 \equiv V_{\sigma\sigma}(\sigma_0,\theta_0) - \dot\theta_0^2$ as in (4.34) and the $\dots$ in (4.67) stem from higher order terms in the expansion. Let us now consider the $M_{\text{eff}}^2 \gg H^2$ scenario.

[14] As usual, the $S^{(1)}[g_0,\phi_0,\delta\phi]$ term in this expansion vanishes due to the background equations of motion (4.18) and (4.19).

4.3.1 $M_{\text{eff}}^2 \gg H^2$ Regime

4.3.1.1 Effective Theory for the Adiabatic (Inflaton) Fluctuation

In this subsection we will show how the naive expectation, that when the mass of the isocurvature mode is very heavy we can integrate it out to obtain an effective single-field description with non-trivial coefficients for non-slow-roll operators, is realized. We will match our findings with the general parametrization introduced in the so-called EFT of inflation developed by Cheung et al. [40], that we thoroughly reviewed in chapter 3, and for which the relations between coefficients of the EFT and the amplitudes of non-Gaussianities are well-known. Following Gong et al. [55] we vary (4.69) and (4.70) with respect to $\delta\sigma$ to obtain

$$\delta\ddot{\sigma} + 3H\delta\dot{\sigma} - \left(\frac{\nabla^2}{a^2} - M_{\text{eff}}^2 + 2\dot{\theta}_0\,\delta\dot{\theta} - V_{\theta\sigma\sigma}\delta\theta\right)\delta\sigma + \frac{V_{\sigma\sigma\sigma}}{2}(\delta\sigma)^2$$
$$= 2(R+\sigma_0)\,\dot{\theta}_0\,\delta\dot{\theta} - V_{\theta\sigma}\,\delta\theta + (R+\sigma_0)\left((\delta\dot{\theta})^2 - \frac{(\nabla\delta\theta)^2}{a^2}\right) - \frac{V_{\theta\theta\sigma}}{2}(\delta\theta)^2. \tag{4.71}$$

Assuming that the effective mass of $\delta\sigma$ is very large (so the term $M_{\text{eff}}^2\,\delta\sigma$ dominates in the above equation) and neglecting its dynamics, we can find a perturbative solution given by [15]

$$\delta\sigma \approx \frac{2R\,\dot{\theta}_0}{M_{\text{eff}}^2}\delta\dot{\theta} + \left(\frac{R}{M_{\text{eff}}^2} - \frac{2R^2\,\dot{\theta}_0^2}{M_{\text{eff}}^2}\frac{V_{\sigma\sigma\sigma}}{M_{\text{eff}}^4}\right)(\delta\dot{\theta})^2, \tag{4.72}$$

where we have taken $\sigma_0 = \widehat{\sigma} = 0$.

Plugging (4.72) back into (4.69) and (4.70), and keeping only the leading order

[15] Here we have neglected both time derivatives and gradients of $\delta\sigma$. In principle, one can keep the gradients to obtain an effective theory that captures the regime of non-linear dispersion relations [137, 138], the so-called "new physics window" dubbed by Baumann and Green [139] (see appendix B to get a quick understanding of how non-linear dispersion relations generically arise when integrating out a heavy field). In (4.72) we are also neglecting terms proportional to M^2 since $M^2 \ll M_{\text{eff}}^2$.

terms in slow-roll parameters, we find the effective single field fluctuation action

$$S^{(2)}_{\text{eff}\,\delta\theta}[g_0,\theta_0,\delta\theta] = \int d^4x\, a^3 \left\{ \frac{1}{2} R^2 (\delta\dot\theta)^2 \left(1 + 4\frac{\dot\theta_0^2}{M_{\text{eff}}^2}\right) - \frac{1}{2} R^2 \frac{(\nabla\delta\theta)^2}{a^2} \right\}, \tag{4.73}$$

$$S^{(3)}_{\text{eff}\,\delta\theta}[g_0,\theta_0,\delta\theta] = \int d^4x\, a^3 \left\{ \left(\frac{R^2\,\dot\theta_0}{M_{\text{eff}}^2} + \frac{R^2\,\dot\theta_0}{M_{\text{eff}}^2}\left(1 + 4\frac{\dot\theta_0^2}{M_{\text{eff}}^2}\right) - \frac{4}{3}\frac{R^3\,\dot\theta_0^3}{M_{\text{eff}}^6} V_{\sigma\sigma\sigma} \right)(\delta\dot\theta)^3 \right.$$
$$\left. - \frac{2R^2\,\dot\theta_0}{a^2 M_{\text{eff}}^2}\delta\dot\theta(\nabla\delta\theta)^2 \right\}. \tag{4.74}$$

Indeed we see that if we define the speed of sound c_s through (4.34), the quadratic action is equivalent to that of general single-field inflation. To evaluate the observable quantities, we have to transfer this action into that of the curvature perturbation. It is well known that the curvature perturbation on the comoving slices $\mathcal{R}$ is given in terms of the field fluctuation on the flat slices along the trajectory $\delta\theta$ as

$$\mathcal{R} = -\frac{H}{\dot\theta_0}\,\delta\theta. \tag{4.75}$$

A straightforward calculation shows that

$$S^{(2)}_{\text{eff}\,\mathcal{R}}[g_0,\theta_0,\mathcal{R}] = M_{\text{Pl}}^2 \int d^4x\, a^3 \frac{\epsilon}{c_s^2} \left\{ \dot{\mathcal{R}}^2 - c_s^2 \frac{(\nabla\mathcal{R})^2}{a^2} \right\} \tag{4.76}$$

$$S^{(3)}_{\text{eff}\,\mathcal{R}}[g_0,\theta_0,\mathcal{R}] = M_{\text{Pl}}^2 \int d^4x\, a^3 \left\{ -\frac{H^2\epsilon}{c_s^2}\left[\frac{c_s^2}{2}\left(\frac{1}{c_s^4} - 1\right) - c_s^2 \frac{R\,V_{\sigma\sigma\sigma}}{6M_{\text{eff}}^2}\left(\frac{1}{c_s^2} - 1\right)^2 \right] \frac{\dot{\mathcal{R}}^3}{H^3} \right.$$
$$\left. + \epsilon\left(\frac{1}{c_s^2} - 1\right)\frac{\dot{\mathcal{R}}}{H}\frac{(\nabla\mathcal{R})^2}{a^2} \right\}, \tag{4.77}$$

where $\epsilon \equiv -\frac{\dot H}{H^2} = \frac{R^2\,\dot\theta_0^2}{2M_{\text{Pl}}^2 H^2}$. We see that $\mathcal{R}$ is indeed massless which implies that $\dot{\mathcal{R}} \approx 0$ at super-sound-horizon crossing scales, $k\,c_s \ll aH$ [101].

Now, from equation (3.69) of chapter 3, the EFT for the Goldstone boson π of gravity in a de Sitter background reads

$$S_\pi = \int d^4x\, a^3 \Big\{ -M_{\rm Pl}^2 \dot H \left(\dot\pi^2 - \frac{(\nabla\pi)^2}{a^2}\right) + 2M_2^4 \left(\dot\pi^2 + \dot\pi^3 - \dot\pi \frac{(\nabla\pi)^2}{a^2}\right) - \frac{4}{3} M_3^4\, \dot\pi^3 + \dots \Big\}, \tag{4.78}$$

where $M_2(t)$ and $M_3(t)$ are (a priori) undetermined time-dependent coefficients of mass dimension 1. From (4.78) we see that the speed of sound of π fluctuations is given by

$$(c_s^\pi)^{-2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2 \dot H}, \tag{4.79}$$

so the Goldstone action can be rewritten at cubic order as

$$\begin{aligned} S_\pi = \int d^4x\, a^3 \Big\{ & -\frac{M_{\rm Pl}^2 \dot H}{(c_s^\pi)^2} \left(\dot\pi^2 - (c_s^\pi)^2 \frac{(\nabla\pi)^2}{a^2}\right) \\ & + M_{\rm Pl}^2 \dot H \left(1 - \frac{1}{(c_s^\pi)^2}\right) \left(\dot\pi^3 - \dot\pi \frac{(\nabla\pi)^2}{a^2}\right) - \frac{4}{3} M_3^4\, \dot\pi^3 + \dots \Big\}. \end{aligned} \tag{4.80}$$

Using the fact that $\mathcal{R} = -H\pi + \mathcal{O}(\pi^2)$ and identifying $c_s^\pi = c_s$, we find, comparing (4.77) with (4.80) that

$$M_2^4 = \frac{1}{2}\, \epsilon\, M_{\rm Pl}^2 H^2 \left(\frac{1}{c_s^2} - 1\right), \tag{4.81}$$

$$M_3^4 = \frac{3}{4}\, \epsilon\, M_{\rm Pl}^2 H^2 \left(\frac{1}{c_s^2} - 1\right)^2 \left[\frac{R}{6M_{\rm eff}^2} V_{\sigma\sigma\sigma} - \frac{1}{2}\right]. \tag{4.82}$$

It can be shown that in the limit when self-interactions of the heavy field σ are ignored while "solving" its own (constraint) equation of motion, the sound speed

c_s and the couplings M_n^4 are uniquely related by [126]

$$M_n^4 = (-1)^n \, n! |\dot{H}| M_{\rm Pl}^2 \left(\frac{c_s^{-2} - 1}{4} \right)^{n-1} . \tag{4.83}$$

Indeed, we see from (4.81) and (4.82) that if the $V_{\sigma\sigma\sigma}$ term is dropped we agree with this result. Comparing the coefficient $M_3^4 \sim M_{\rm eff}^{-4}$ coming from (4.83) to the one calculated in (4.82), which has an additional $\sim M_{\rm eff}^{-6}$ behavior, we realize that M_3^4 reflects the non-linear self-interaction of the heavy field during inflation as was stressed in [55]. This is based on the fact that the $V_{\sigma\sigma\sigma}$ term actually dominates M_3 even if it is naively further suppressed by one more power of $M_{\rm eff}^2$. We provide a proof of this last statement by the end of appendix A. Let us now estimate and calculate non-Gaussianities arising in this particular limit or our model.

4.3.1.2 Non-Gaussianities

In subsection 4.2.4 we computed the two-point function of primordial curvature perturbations,

$$\langle \Omega | \, \widehat{\mathcal{R}}_{\mathbf{k}_1} \widehat{\mathcal{R}}_{\mathbf{k}_2} \, | \Omega \rangle = (2\pi)^3 P_{\mathcal{R}}(k_1) \delta^3(\mathbf{k}_1 + \mathbf{k}_2). \tag{4.84}$$

In principle, there is more information in the VEV of higher-order n-point functions. Schematically, we can write these as the path integral

$$\langle \Omega | \, \widehat{\mathcal{R}}_{\mathbf{k}_1} \ldots \widehat{\mathcal{R}}_{\mathbf{k}_n} \, | \Omega \rangle \propto \int [\mathscr{D} \mathcal{R}] \, \mathcal{R}_{\mathbf{k}_1} \ldots \mathcal{R}_{\mathbf{k}_n} \, e^{iS[\mathcal{R}]}. \tag{4.85}$$

For a free field theory, the action is a quadratic functional $S^{(2)}$ and the e^{iS} weighting of the path integral is a Gaussian after Wick rotating to Euclidean time. As such, all correlation functions with n odd vanish, while those with n even are completely determined by the two-point function (4.84). Nonetheless, if we include non-trivial interactions in the action, meaning $S^{\rm int} = S^{(3)} + S^{(4)} + \ldots$, makes the weighting of the path integral non-Gaussian. This allows non-trivial n-point functions for all n.

The primary diagnostic for primordial non-Gaussianities is the three-point function or bispectrum which is defined as [16]

$$\langle \mathcal{R}_{\mathbf{p}_1} \mathcal{R}_{\mathbf{p}_2} \mathcal{R}_{\mathbf{p}_3} \rangle \equiv (2\pi)^7 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) \, (\Delta^2_{\mathcal{R}}(p_\star))^2 \, \frac{\mathcal{S}(p_1, p_2, p_3)}{(p_1 \, p_2 \, p_3)^2}, \tag{4.86}$$

where $p_\star$ is a fiducial momentum scale, $\Delta^2_{\mathcal{R}}(k)$ is the dimensionless power spectrum given by

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle \equiv (2\pi)^5 \delta^3(\mathbf{k} + \mathbf{k}') \, \frac{1}{2k^3} \, \Delta^2_{\mathcal{R}}(k) \tag{4.87}$$

and $\mathcal{S}(p_1, p_2, p_3)$ is the so-called "shape function". The Dirac delta-function in (4.87) is a consequence of statistical homogeneity; it enforces that the three-momentum vectors form a closed triangle. The momentum dependence of the bispectrum determines the amount of non-Gaussianity associated with triangles of different shapes. A useful measure of the size of the non-Gaussianity is the parameter $f_{\rm NL}$, defined as

$$f_{\rm NL} \equiv \frac{10}{9} \, \mathcal{S}(p_1 = p_2 = p_3). \tag{4.88}$$

There are several shapes that authors have studied thoroughly over the years. One first historical example is the so-called local shape, which is defined through

$$\mathcal{S}^{\rm local}(p_1, p_2, p_3) \equiv \frac{3}{10} f^{\rm loc}_{\rm NL} \left(\frac{p_1^2}{p_2 \, p_3} + 2 \, \text{perm.} \right) \tag{4.89}$$

[16] As usual, $\mathcal{R}_{\mathbf{k}} \equiv \int d^3x \, \mathcal{R}(\mathbf{x}) \, e^{i\mathbf{k}\cdot\mathbf{x}}$.

and follows from an ansatz (in real space) of the form

$$\mathcal{R}(\mathbf{x}) = \mathcal{R}_g(\mathbf{x}) + \frac{3}{5} f_{\rm NL}^{\rm local} \left[\mathcal{R}_g^2(\mathbf{x}) - \langle \mathcal{R}_g^2 \rangle\right] , \tag{4.90}$$

where $\mathcal{R}_g(\mathbf{x})$ is a Gaussian random field [140–142]. In momentum space, the signal peaks for squeezed triangles, $k_1 \ll k_2 \sim k_3$. The local shape arises in models of multifield inflation. On the other hand, in single-field inflation the signal vanishes in the squeezed limit. This is the famous Maldacena's consistency condition [115, 143] which reads

$$\lim_{k_3 \to 0} \langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)(1 - n_s) \mathcal{P}_{\mathcal{R}}(k_1) \mathcal{P}_{\mathcal{R}}(k_3), \tag{4.91}$$

where $\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{2\pi^2}{k^3} \Delta^2_{\mathcal{R}}(k)$. In other words, for single-field inflation, the squeezed limit of the three-point function is suppressed by $(1 - n_s) \sim \mathcal{O}(\epsilon, \eta)$, so a detection of non-Gaussianities in the squeezed limit can therefore rule out *all* models of single-field inflation.

In order to first estimate the non-Gaussianities associated with the effective action for π it is convenient to absorb the sound speed into a redefinition of the spatial coordinates $x^i \to \tilde{x}^i \equiv c_s^{-1} x^i$ so that "fake" Lorentz invariance is restored [39, 139]. Then the effective theory Lagrangian $\widetilde{\mathscr{L}_\pi} \equiv c_s^3 \mathscr{L}_\pi$ can be casted like

$$\widetilde{\mathscr{L}_\pi} = -\frac{1}{2}(\widetilde{\partial}_\mu \pi_c)^2 - \frac{1}{2\Lambda^2} \left(\dot{\pi}_c \frac{(\widetilde{\nabla} \pi_c)^2}{a^2} + \mathcal{A} \dot{\pi}_c^3 \right) , \tag{4.92}$$

where $\widetilde{\partial}_\mu \equiv (\partial_t,\, c_s\,\partial_i)$, $\pi_c \equiv f_\pi^2\,\pi$ is a canonically normalized field and

$$f_\pi^4 \equiv 2M_{\rm Pl}^2|\dot{H}|c_s, \tag{4.93}$$
$$\Lambda^4 \equiv \frac{c_s^4}{(1-c_s^2)^2}\,f_\pi^4 = \frac{2M_{\rm Pl}^2|\dot{H}|c_s^5}{(1-c_s^2)^2}, \tag{4.94}$$
$$\frac{\mathcal{A}}{c_s^2} \equiv -1 + \frac{2}{3}\frac{M_3^4}{M_2^4}. \tag{4.95}$$

Here, f_π^4 and Λ^4 are the so-called "symmetry breaking" and "strong coupling" [17] scales respectively. A simple "back-of-the-envelope" estimate for the amplitude of the non-Gaussianity can be found by comparing the non-linear (cubic) terms with the quadratic terms in the Lagrangian, around freezing time $\omega \sim H$. This is because the interaction operators have derivatives acting on the fluctuations so they effectively are shut down after freezing. Using our fake Lorentz-invariant Lagrangian we find that

$$f_{\rm NL}^{\dot{\pi}(\nabla\pi)^2}\mathcal{R} \equiv \left.\frac{\widetilde{\mathscr{L}_3}^{\dot{\pi}(\nabla\pi)^2}}{\widetilde{\mathscr{L}_2}}\right|_{\omega\sim H} \sim \frac{1}{2\,\Lambda^2}\frac{\dot{\pi}_c\,(\widetilde{\partial}\,\pi_c)^2}{(\widetilde{\partial}\,\pi_c)^2} \sim \left(\frac{f_\pi}{\Lambda}\right)^2\mathcal{R} \sim \left(\frac{1-c_s^2}{c_s^2}\right)\mathcal{R}, \tag{4.96}$$
$$f_{\rm NL}^{\dot{\pi}^3}\mathcal{R} \equiv \left.\frac{\widetilde{\mathscr{L}_3}^{\dot{\pi}^3}}{\widetilde{\mathscr{L}_2}}\right|_{\omega\sim H} \sim \frac{\mathcal{A}}{2\,\Lambda^2}\frac{\dot{\pi}_c^3}{(\widetilde{\partial}\,\pi_c)^2} \sim \mathcal{A}\left(\frac{f_\pi}{\Lambda}\right)^2\mathcal{R} \sim \mathcal{A}\left(\frac{1-c_s^2}{c_s^2}\right)\mathcal{R}. \tag{4.97}$$

Then it is easy to estimate non-Gaussianities once the matching between a particular model and the EFT of inflation has been made. With M_2 and M_3 as given by (4.81) and (4.82) respectively, we find that [18]

$$\mathcal{A} \equiv -c_s^2 + \frac{2}{3}\frac{M_3^4}{M_2^4}c_s^2 = -c_s^2 + c_s^2\left(\frac{1}{c_s^2}-1\right)\left[\frac{R}{6M_{\rm eff}^2}V_{\sigma\sigma\sigma} - \frac{1}{2}\right]. \tag{4.98}$$

[17] It can be shown that the breakdown of perturbative unitarity of Goldstone boson scattering occurs when $\omega^4 > \frac{24\pi}{5}(1-c_s^2)\Lambda^4 \equiv \Lambda_u^4$ [139, 144] . Λ_u is referred to as the "unitarity bound". These definitions rely on the linear dispersion relation that we have assumed throughout this work.

[18] If the $V_{\sigma\sigma\sigma}$ term is neglected, $\mathcal{A} = -\frac{1}{2}(1+c_s^2) \to f_{\rm NL}^{\dot{\pi}^3} \sim -\frac{1}{2}\frac{(1-c_s^4)}{c_s^2}$. Some authors [111, 145] argue that in order not to have an unnatural hierarchy between the scales associated with the two distinct operators $\dot{\pi}_c\,(\widetilde{\partial}_i\pi_c)^2$ and $\dot{\pi}_c^3$, one must require $\mathcal{A} \sim \mathcal{O}(1)$.

Using the minimum given by (4.10) and the definition $\widetilde{M}^2_{\text{eff}} \equiv M^2_{\text{eff}}\, c_s^{-2}$ we find through (4.34) and (4.98) that

$$c_s^2 \simeq \frac{4\,\lambda\,v^2 - \dot{\theta}_0^2}{4\,\lambda\,v^2 + 3\,\dot{\theta}_0^2} \approxeq \frac{M^2_{\text{eff}}}{\widetilde{M}^2_{\text{eff}}}, \tag{4.99}$$

$$\mathcal{A} \simeq \frac{\dot{\theta}_0^4 + 8\,\lambda\,v^2\,\dot{\theta}_0^2 - 16\,\lambda^2\,v^4}{(4\,\lambda\,v^2 - \dot{\theta}_0^2)(4\,\lambda\,v^2 + 3\,\dot{\theta}_0^2)} \approxeq \frac{M^2_{\text{eff}}\,\widetilde{M}^2_{\text{eff}} - 4(2\sqrt{2}\,\lambda\,v^2 - \dot{\theta}_0^2)(2\sqrt{2}\,\lambda\,v^2 + \dot{\theta}_0^2)}{M^2_{\text{eff}}\,\widetilde{M}^2_{\text{eff}}}. \tag{4.100}$$

Then, using (4.99) and (4.100) in (4.96) and (4.97), we find the following estimates for the amplitude of non-Gaussianities

$$f_{\text{NL}}^{\dot{\pi}(\nabla\pi)^2} \sim \frac{4\,\dot{\theta}_0^2}{4\,\lambda\,v^2 - \dot{\theta}_0^2} \sim 4\frac{\dot{\theta}_0^2}{M^2_{\text{eff}}}, \tag{4.101}$$

$$f_{\text{NL}}^{\dot{\pi}^3} \sim \frac{4\,\dot{\theta}_0^2\,(\dot{\theta}_0^4 + 8\,\lambda\,v^2\,\dot{\theta}_0^2 - 16\,\lambda^2\,v^4)}{(4\,\lambda\,v^2 - \dot{\theta}_0^2)^2(4\,\lambda\,v^2 + 3\,\dot{\theta}_0^2)} \sim \frac{4\,\dot{\theta}_0^2}{M^4_{\text{eff}}\,\widetilde{M}^2_{\text{eff}}}\left[M^2_{\text{eff}}\,\widetilde{M}^2_{\text{eff}} - 4(2\sqrt{2}\,\lambda\,v^2 - \dot{\theta}_0^2)(2\sqrt{2}\,\lambda\,v^2 + \dot{\theta}_0^2)\right]. \tag{4.102}$$

The precise analysis using the so-called "in-in" formalism [19] gives the numerical coefficients we are missing for the exact prediction. With f_π and Λ defined through (4.93) and (4.94) respectively, it can be shown that (see appendix A)

$$f_{\text{NL}}^{\dot{\pi}(\nabla\pi)^2} = -\frac{85}{324}\left(\frac{f_\pi}{\Lambda}\right)^2 = -\frac{85}{81}\frac{\dot{\theta}_0^2}{M^2_{\text{eff}}} = -\frac{85}{324}\left(\frac{1}{c_s^2} - 1\right), \tag{4.103}$$

$$\begin{aligned} f_{\text{NL}}^{\dot{\pi}^3} &= +\frac{5\,\mathcal{A}}{81}\left(\frac{f_\pi}{\Lambda}\right)^2 = \frac{20}{81}\frac{\dot{\theta}_0^2}{M^4_{\text{eff}}\,\widetilde{M}^2_{\text{eff}}}\left[M^2_{\text{eff}}\,\widetilde{M}^2_{\text{eff}} - 4(2\sqrt{2}\,\lambda\,v^2 - \dot{\theta}_0^2)(2\sqrt{2}\,\lambda\,v^2 + \dot{\theta}_0^2)\right] \\ &= \frac{5}{81}\left[-\frac{5}{8} + \frac{1}{8\,c_s^4} - \frac{3}{8\,c_s^2} + \frac{7\,c_s^2}{8}\right], \end{aligned} \tag{4.104}$$

[19]See appendix A and [146]. For a highly improved covariant prescription to calculate cosmological correlators see the Schwinger-Keldysh formalism recently presented in [61].

where, in order to get the last line in (4.104), use has been made of the expression for $\dot\theta_0^2$ as given in (4.59), which also implies

$$M_{\rm eff}^2 \approx \frac{16\,\lambda\,v^2\,c_s^2}{1+3\,c_s^2}, \quad \widetilde{M}_{\rm eff}^2 \approx \frac{16\,\lambda\,v^2}{1+3\,c_s^2} \quad \text{and} \quad \mathcal{A} = -\frac{1}{4} + \frac{1}{8\,c_s^2} - \frac{7\,c_s^2}{8}. \tag{4.105}$$

We should compare the last expression in (4.104) with the naive prediction that one gets when using (4.83) in (4.95) instead,

$$f_{\rm NL\,(naive)}^{\dot\pi^3} \equiv f_{\rm NL}^{\dot\pi^3}\big|_{V_{\sigma\sigma\sigma}=0} = +\frac{5\,\mathcal{A}}{81}\left(\frac{f_\pi}{\Lambda}\right)^2\bigg|_{V_{\sigma\sigma\sigma}=0} = \frac{5}{81}\left[-\frac{1}{2\,c_s^2} + \frac{c_s^2}{2}\right], \tag{4.106}$$

which is clearly negative when $c_s < 1$ and tends to $-\infty$ as c_s decreases. The behavior of $f_{\rm NL}^{\dot\pi^3}$, on the other hand, is quite different as can be anticipated by looking at (4.104). Indeed, it possesses a zero-crossing point around $c_s \approx 0.51$, a global minimum around $c_s \approx 0.67$ (where $f_{\rm NL}^{\dot\pi^3} \approx -2.76 \times 10^{-2}$) and tends to zero as c_s approaches 1, as it should. Also, due to the presence of the positive c_s^{-4} term, $f_{\rm NL}^{\dot\pi^3}$ tends to $+\infty$ as c_s approaches zero. All this can be seen in Figure 4.3 where we plot $f_{\rm NL}^{\dot\pi^3}$, $f_{\rm NL\,(naive)}^{\dot\pi^3}$, $f_{\rm NL}^{\dot\pi(\nabla\pi)^2}$ and $\mathcal{A}$ vs. c_s.

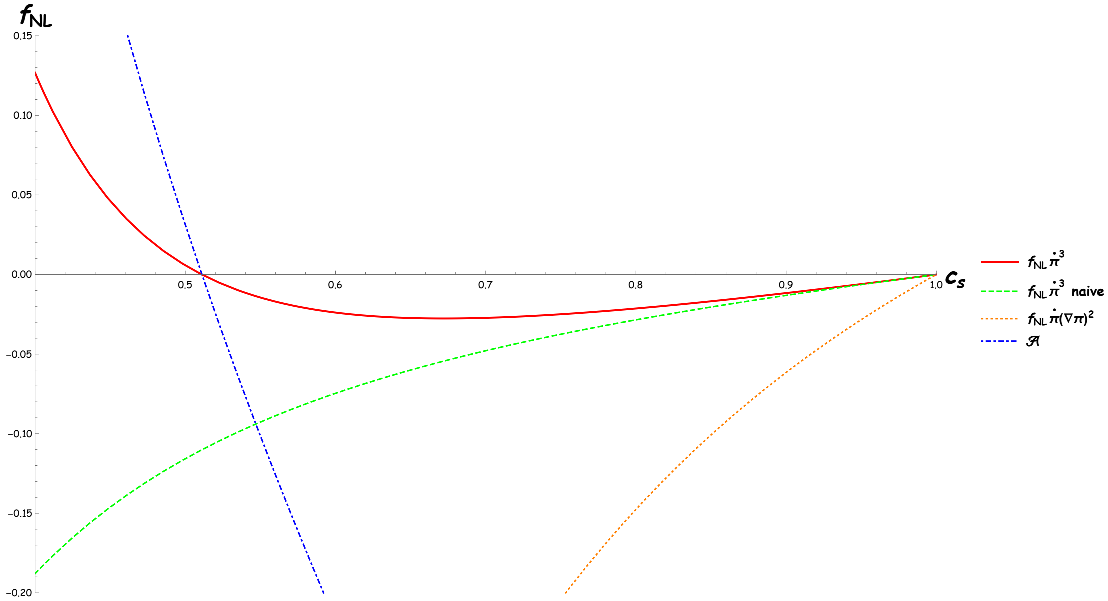


Figure 4.3: $f_{\rm NL}^{\dot\pi^3}$, $f_{\rm NL\,(naive)}^{\dot\pi^3}$, $f_{\rm NL}^{\dot\pi(\nabla\pi)^2}$ and $\mathcal{A}$ vs. c_s.

Let us just comment that the scaling $f_{\rm NL}\sim c_s^{-4}$ is not usual for non-canonical models like DBI [120, 121] or k-inflation [104, 105], where it is a familiar result that $f_{\rm NL}\sim c_s^{-2}$ [147]. This peculiar scaling does arise in Galileon models of inflation [110] based on the so-called "Galilean symmetry" introduced in [148] [20].

Planck 2015 [26] puts bounds on two specific linear combinations of $f_{\rm NL}^{\dot\pi(\nabla\pi)^2}$ and $f_{\rm NL}^{\dot\pi^3}$, namely the "equilateral" $f_{\rm NL}^{\rm equil}$ and the "orthogonal" $f_{\rm NL}^{\rm ortho}$ [21]. The mean values of the estimators for $f_{\rm NL}^{\rm equil}$ and $f_{\rm NL}^{\rm ortho}$ are given by

$$f_{\rm NL}^{\rm equil}=\left[-\frac{11}{40}+\frac{39}{500}\,\mathcal{A}\right]\left(\frac{f_\pi}{\Lambda}\right)^2=\frac{181}{800}+\frac{39}{4000\,c_s^4}-\frac{1217}{4000\,c_s^2}+\frac{273\,c_s^2}{4000}, \tag{4.107}$$

$$f_{\rm NL}^{\rm ortho}=\left[\frac{159}{10000}+\frac{167}{10000}\,\mathcal{A}\right]\left(\frac{f_\pi}{\Lambda}\right)^2=-\frac{2107}{80000}+\frac{167}{80000\,c_s^4}+\frac{771}{80000\,c_s^2}+\frac{1169\,c_s^2}{80000}, \tag{4.108}$$

[20] In [110] the c_s^{-4} behavior appears since, due to symmetry, the dimension seven operator (after canonical normalization) $\partial^2\pi(\partial\pi)^2$ is naturally of comparable "size" with the usual $\dot\pi(\partial\pi)^2$ and, only for the latter, the non-linearly realized Lorentz invariance "requires" $f_{\rm NL}\sim c_s^{-2}$.

[21] $f_{\rm NL}^{\rm equil}$ and $f_{\rm NL}^{\rm ortho}$ are "defined" as the result of projecting the shapes associated with $f_{\rm NL}^{\dot\pi(\nabla\pi)^2}$ and $f_{\rm NL}^{\dot\pi^3}$ into the equilateral and orthogonal templates using the shape inner product introduced in [149]. For details see [111].

where we have used $\mathcal{A}$ as given in (4.105). These are the "physical" constrained amplitudes of interest.

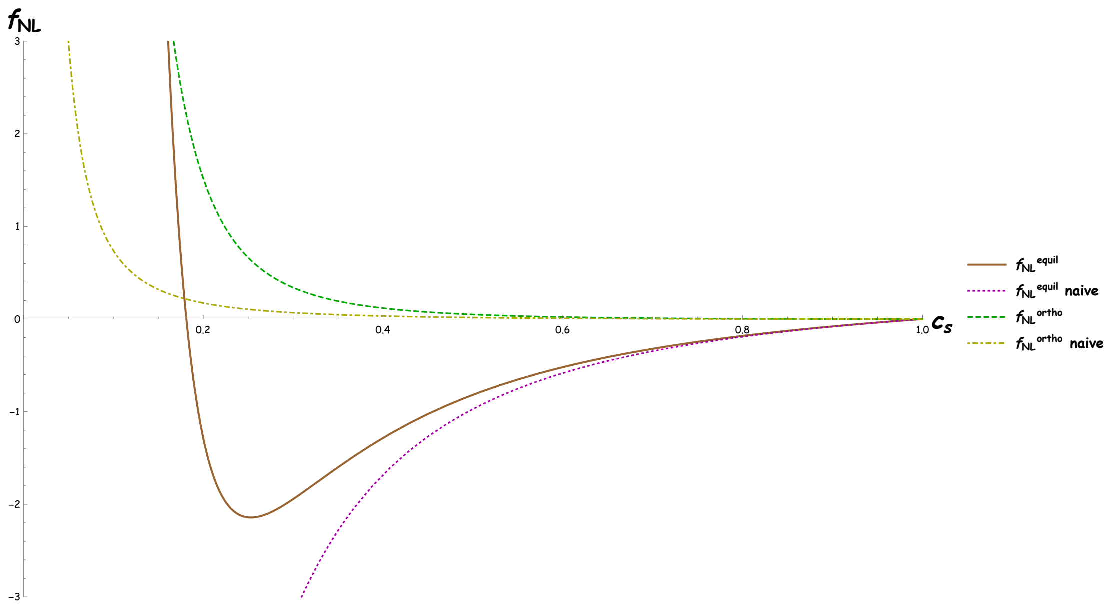


Figure 4.4: $f_{\rm NL}^{\rm equil}$, $f_{\rm NL\,(naive)}^{\rm equil}$, $f_{\rm NL}^{\rm ortho}$ and $f_{\rm NL\,(naive)}^{\rm ortho}$ vs. c_s.

In Figure 4.4 we plot $f_{\rm NL}^{\rm equil}$ and $f_{\rm NL}^{\rm ortho}$ vs. c_s using the last expressions in (4.107) and (4.108) along with the naive result of using $\mathcal{A}_{\rm naive} = -\frac{1}{2}(1+c_s^2)$ instead of $\mathcal{A}$. We observe again that, due to the presence of the c_s^{-4} term, the behavior of the $f_{\rm NL}$'s is quite different from the naive expectation when ignoring the non-linear self-interactions of the heavy field. In particular, we see that since

$$f_{\rm NL\,(naive)}^{\rm equil} \equiv f_{\rm NL}^{\rm equil}\big|_{V_{\sigma\sigma\sigma}=0} = \left[-\frac{11}{40} + \frac{39}{500}\,\mathcal{A}\right]\left(\frac{f_\pi}{\Lambda}\right)^2\Bigg|_{V_{\sigma\sigma\sigma}=0} = \frac{11}{40} - \frac{157}{500\,c_s^2} + \frac{39\,c_s^2}{1000}, \tag{4.109}$$

$$f_{\rm NL\,(naive)}^{\rm ortho} \equiv f_{\rm NL}^{\rm ortho}\big|_{V_{\sigma\sigma\sigma}=0} = \left[\frac{159}{10000} + \frac{167}{10000}\,\mathcal{A}\right]\left(\frac{f_\pi}{\Lambda}\right)^2\Bigg|_{V_{\sigma\sigma\sigma}=0} = -\frac{159}{10000} + \frac{151}{20000\,c_s^2} + \frac{167\,c_s^2}{20000}, \tag{4.110}$$

$f_{\rm NL\,(naive)}^{\rm equil}$ ($f_{\rm NL\,(naive)}^{\rm ortho}$) tends to $-\infty$ ($+\infty$) as c_s decreases. $f_{\rm NL\,(naive)}^{\rm equil}$ does not have a global minimum while $f_{\rm NL}^{\rm equil}$ does have one around $c_s \approx 0.25$ (where $f_{\rm NL}^{\rm equil} \approx -2.14$)

and tends to $+\infty$ as c_s decreases with a zero-crossing point around $c_s \approx 0.18$. $f_{\rm NL}^{\rm ortho}$ ($f_{\rm NL\,(naive)}^{\rm ortho}$) tends to $+\infty$ as c_s^{-4} (c_s^{-2}) for small c_s but otherwise stays very close to zero all the way up to $c_s = 1$, having a zero-crossing point around $c_s \approx 0.98$ ($c_s \approx 0.95$) and a global minimum at $c_s \approx 0.99$ ($c_s \approx 0.98$) where $f_{\rm NL}^{\rm ortho} \approx -9.79 \times 10^{-6}$ ($f_{\rm NL\,(naive)}^{\rm ortho} \approx -2.01 \times 10^{-5}$). We summarize the values of the different $f_{\rm NL}$'s (for the same c_s's that we considered in Figure 4.2 and Table 4.1) in Table 4.2.

	$\mathbf{f_{NL}^{equil}}$	$\mathbf{f_{NL\,(naive)}^{equil}}$	$\mathbf{f_{NL}^{ortho}}$	$\mathbf{f_{NL\,(naive)}^{ortho}}$
$\mathbf{c_s = 0.999}$	-7.067×10^{-4}	-7.069×10^{-4}	-1.536×10^{-6}	-1.569×10^{-6}
$\mathbf{c_s = 0.9}$	-7.922×10^{-2}	-8.106×10^{-2}	5.785×10^{-4}	1.845×10^{-4}
$\mathbf{c_s = 0.75}$	-2.454×10^{-1}	-2.613×10^{-1}	5.613×10^{-3}	2.219×10^{-3}
$\mathbf{c_s = 0.47}$	-9.362×10^{-1}	-1.138	6.330×10^{-2}	2.012×10^{-2}

Table 4.2: $f_{\rm NL}^{\rm equil}$, $f_{\rm NL\,(naive)}^{\rm equil}$, $f_{\rm NL}^{\rm ortho}$ and $f_{\rm NL\,(naive)}^{\rm ortho}$ for different values of c_s.

The current Planck constraints at 2σ are [26]

$$
\begin{aligned}
-156 < f_{\rm NL}^{\rm equil} < 124, \qquad -100 < f_{\rm NL}^{\rm ortho} < 32 \quad &\text{(temperature data only)},\\
-90 < f_{\rm NL}^{\rm equil} < 82, \qquad -68 < f_{\rm NL}^{\rm ortho} < 16 \quad &\text{(temperature + polarization data)}.
\end{aligned}
\tag{4.111}
$$

Looking at Table 4.2 we see that current observations are not sensitive enough to rule out the equilateral and orthogonal non-Gaussianities of our model. Needless to say, probing non-Gaussianities down to $f_{\rm NL} \sim \mathcal{O}(1)$ or smaller is an important target for future experiments.

For completeness, let us mention that the local shape with size $f_{\rm NL}^{\rm local}$ is much more well constrained. At 2σ Planck found that [26]

$$
\begin{aligned}
&-8.9 < f_{\rm NL}^{\rm local} < 13.9, \text{(temperature data only)},\\
&-9.2 < f_{\rm NL}^{\rm local} < 10.8, \text{(temperature + polarization data)}.
\end{aligned}
\tag{4.112}
$$

In [126] the EFT for single-field inflationary models descending from a "parent theory" containing several scalar fields was derived. Besides the cubic operators $\dot{\pi}^3$ and $\dot{\pi}(\nabla\pi)^2$ that we have found within our approximations, the following two terms were found in the decoupling limit

$$S_\pi \ni \int d^4x\, a^3 M_{\rm Pl}^2 \dot{H} \left\{ 2\,\frac{\dot{c}_s}{c_s^3}\,\pi\,\dot{\pi}^2 + 2H\eta_\parallel\,\pi\left(\frac{\dot{\pi}^2}{c_s^2} - \frac{(\nabla\pi)^2}{a^2}\right)\right\}, \tag{4.113}$$

where $\eta_\parallel \equiv -\frac{\ddot{\varphi}_0}{H\dot{\varphi}_0}$ and $\varphi_0 \equiv R\,\theta_0$. These two operators lead to non-Gaussianities that satisfy the so-called Maldacena's consistency relation [115] in the sense that $f_{\rm NL}^{\rm local} \sim \mathcal{O}(\epsilon, \eta)$, confirming the fact that the $M_{\rm eff}^2 \gg H^2$ limit is indeed a single-field scenario. In other words, even if the constraints in the local subcase are tighter, local non-Gaussianities are negligible in the $M_{\rm eff}^2 \gg H^2$ limit, in agreement with the equivalence principle (see [143] for a general discussion of these points). Now we will consider the $M_{\rm eff}^2 \sim H^2$ case, which is quite different from the $M_{\rm eff}^2 \gg H^2$ one as the heavy field cannot be integrated out anymore.

4.3.2 $M_{\text{eff}}^2 \sim H^2$ Regime

4.3.2.1 The Single-Field EFT Breaks Down

Let us come back to (4.69) and (4.70). In our specific model, taking $\sigma_0 = \widehat{\sigma} = 0$ so R is determined by the naive VEV given by (4.10), we have

$$S^{(2)}[g_0, \phi_0, \delta\phi] = \int d^4x\, a^3 \Big\{ -\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 2M^2\cos\left(\frac{2\varphi_0}{R}\right)\varphi^2 + \frac{2}{R}\dot{\varphi}_0\,\dot{\varphi}\,\mathcal{F} - 4M^2\sin\left(\frac{2\varphi_0}{R}\right)\varphi\,\mathcal{F} - \frac{1}{2}g^{\mu\nu}\partial_\mu\mathcal{F}\partial_\nu\mathcal{F} - \frac{1}{2}M_{\text{eff}}^2\mathcal{F}^2 \Big\}, \tag{4.114}$$

$$S^{(3)}[g_0, \phi_0, \delta\phi] = \int d^4x\, a^3 \Big\{ -\frac{1}{R}(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi)\,\mathcal{F} + \frac{1}{R^2}\dot{\varphi}_0\,\dot{\varphi}\,\mathcal{F}^2 + \frac{4}{3R}M^2\sin\left(\frac{2\varphi_0}{R}\right)\varphi^3 - \frac{2M^2}{R}\sin\left(\frac{2\varphi_0}{R}\right)\varphi\,\mathcal{F}^2 - \frac{4M^2}{R}\cos\left(\frac{2\varphi_0}{R}\right)\varphi^2\mathcal{F} - \lambda\, R\,\mathcal{F}^3 \Big\}, \tag{4.115}$$

where $\varphi_0 \equiv R\,\theta_0$, $\varphi \equiv R\,\delta\theta$ and we have used the definition $\mathcal{F} \equiv N^a\delta\phi_a = \delta\sigma$ which holds as long as $\dot{\sigma}_0 = 0$ [22]. In appendix B we review, for completeness, the general conditions under which we can integrate out the high frequency degrees of freedom to get an effective single field theory [132]. It is clear though that when $M_{\text{eff}} \sim H$ integrating out the heavy mode is not justified as the cosmological experiment actually probes exactly this energy scale regime. We then need to consider the dynamics of the isocurvature perturbation $\mathcal{F}$ and its influence on the correlation functions of the adiabatic mode φ. Thus, we are interested in the

[22] The change in "notation" $\delta\sigma \to \mathcal{F}$ makes contact with the literature and also aims for notational clarity.

action

$$S^{(0)}[g_0,\phi_0,\delta\phi] \equiv \int d^4x\, a^3 \left\{ -\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2} g^{\mu\nu}\partial_\mu\mathcal{F}\partial_\nu\mathcal{F} - \frac{1}{2} M_{\rm eff}^2\,\mathcal{F}^2 \right\}, \tag{4.116}$$

$$S^{\,\rm int}[g_0,\phi_0,\delta\phi] \equiv \int d^4x\, a^3 \left\{ 2\,\dot\theta_0\,\dot\varphi\,\mathcal{F} - \sqrt{2}\,\lambda\,v\,\mathcal{F}^3 - \frac{1}{4}\,\lambda\,\mathcal{F}^4 \right\}, \tag{4.117}$$

where we have neglected $\mathcal{O}(\beta)$ terms since we are dealing with the theory of fluctuations, we have taken $R \approx \sqrt{2}\,v$ and we have included the fourth-order term $\mathscr{L}^{\,\rm int} \supset -\frac{1}{4!}V_{\sigma\sigma\sigma\sigma}(\sigma_0,\theta_0)(\delta\sigma)^4$ which is also not suppressed by any slow-roll conditions. Note that among the rest of the interaction terms in (4.115) we have also neglected the "irrelevant" operators $(\partial\,\varphi)^2\,\mathcal{F}$ and $\dot\varphi_0\,\dot\varphi\,\mathcal{F}^2$ as they are suppressed by $\left(\frac{H}{v}\right)$ and $\left(\frac{H}{v}\right)^2$, respectively, while keeping the "relevant" operator $\mathcal{F}^3$. This is consistent with the analysis made in the original "vanilla" QSF model where it has been emphasized that the only operator that may (in principle) make $f_{\rm NL} \gg 1$ is exactly the cubic term $\mathcal{F}^3$ (see Tables 1 and 2 of [50] and the discussion therein). Note also that the operator $\dot\theta_0\,\dot\varphi\,\mathcal{F}$ in (4.117) is second order in field fluctuations but still we treat it as an interaction (mixing) term. This is crucial for the perturbative Hamiltonian analysis that we now briefly review.

Starting from the full action $S[g_0,\phi_0,\delta\phi]$ we define the canonical momenta $\pi_{\delta\phi} \equiv \frac{\delta S}{\delta\dot{\delta\phi}}$ as usual. Then we construct the Hamiltonian as $\mathscr{H} = \sum_{\delta\phi}\pi_{\delta\phi}\dot{\delta\phi} - \mathscr{L}$ where the $\dot{\delta\phi}$ are expressed in terms of the $\pi_{\delta\phi}$ and the $\delta\phi$. We now divide $\mathscr{H}$ into a free-field $\mathscr{H}^{(0)}$ and an interacting part $\mathscr{H}^{\,\rm int}$ and replace the $\pi_{\delta\phi}$ by $\pi^I_{\delta\phi}$, satisfying Hamilton's equations of the free-field Hamiltonian, meaning $\dot{\delta\phi}_I = \frac{\delta\mathscr{H}^{(0)}}{\delta\pi_{\delta\phi}}\Big|_{\pi_{\delta\phi}=\pi^I_{\delta\phi}}$. We finally use this last definition to get rid of the $\pi^I_{\delta\phi}$ in terms of the $\delta\phi$ and $\dot{\delta\phi}$ (see [146, 150] for more details). In the case at hand, the free and interaction

Hamiltonian densities $\mathscr{H}^{(0)}$ and $\mathscr{H}^{\text{int}}$ are then respectively given by

$$\mathscr{H}^{(0)} \equiv \frac{a^3}{2}\left\{\dot{\varphi_I}^2 + \frac{(\nabla\varphi_I)^2}{a^2} + \dot{\mathcal{F}}_I^2 + \frac{(\nabla\mathcal{F}_I)^2}{a^2} + \widetilde{M}^2_{\text{eff}}\,\mathcal{F}_I^2\right\}, \tag{4.118}$$

$$\mathscr{H}^{\text{int}} \equiv \mathscr{H}_2^I + \mathscr{H}_3^I = a^3\left\{-2\,\dot{\theta}_0\,\dot{\varphi}_I\,\mathcal{F}_I + \sqrt{2}\,\lambda\,v\,\mathcal{F}_I^3 + \frac{1}{4}\,\lambda\,\mathcal{F}_I^4\right\}, \tag{4.119}$$

where the "$_I$" subscript highlights the fact that we now deal with interaction picture fields and

$$\widetilde{M}^2_{\text{eff}} \equiv V_{\sigma\sigma} + 3\,\dot{\theta}_0^2 = M^2_{\text{eff}}\,c_s^{-2}, \tag{4.120}$$

where use has been made of (4.34). It is interesting to note that $\widetilde{M}^2_{\text{eff}}$ is nothing but the low-energy effective theory cut-off discussed in appendix B, cf. equation (B.11). In Figure 4.5 below, we draw the "Feynman rules" associated with the interaction Hamiltonian (4.119).

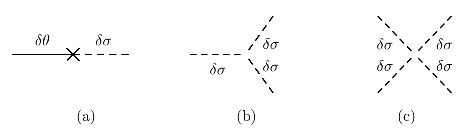


Figure 4.5: "Feynman rules" for the interaction Hamiltonian $\mathscr{H}^{\text{int}}$. (a) is the so-called "transfer function" between adiabatic and isocurvature modes while (b) and (c) represent the three and four-point self-interaction terms of the isocurvature mode.

In order to rely on perturbation theory using $\mathscr{H}^{\rm int}$ we will demand that [23]

$$\frac{\dot{\theta}_0^2}{H^2} \ll 1 \tag{4.121}$$

and

$$|V_{\sigma\sigma\sigma}|\, H^3 \ll 3\, V_{\sigma\sigma}\, H^2. \tag{4.122}$$

Condition (4.121) is necessary since the correction to the leading power spectrum is suppressed by the factor $\frac{\dot{\theta}_0^2}{H^2}$ as we will see in (4.134) below. Condition (4.122) reflects the fact that, in the potential, the quadratic term should dominate over the cubic one when $\mathcal{F} \lesssim H$. In the QSF scenario, corresponding to $\widetilde{M}^2_{\rm eff} = V_{\sigma\sigma} + 3\,\dot{\theta}_0^2 \equiv \widetilde{\alpha}\, H^2$ with $\widetilde{\alpha} \sim \mathcal{O}(1)$, condition (4.122) is equivalent to

$$\frac{|V_{\sigma\sigma\sigma}|}{H} \ll 3\,\widetilde{\alpha} \tag{4.123}$$

as long as (4.121) simultaneously holds [50]. Within our model $V_{\sigma\sigma\sigma} = 6\,\lambda\, R \approx 6\sqrt{2}\,\lambda\, v$, so using a "benchmark point" compatible with (4.59), where we pick

$$v \approx 15.1\, M_{\rm Pl} \approx 3.67 \times 10^{19}\,{\rm GeV} \quad \text{and} \quad H \approx 6.6 \times 10^{13}\,{\rm GeV}, \tag{4.124}$$

condition (4.123) implies that $\lambda \ll (6.35 \times 10^{-7})\,\widetilde{\alpha}$. This last constraint on λ is trivially satisfied since

$$\lambda \approx \frac{\widetilde{\alpha}\, H^2}{4\, v^2} \approx (8.08 \times 10^{-13})\,\widetilde{\alpha}, \tag{4.125}$$

[23] Let us emphasize that this perturbativity condition is *not* tied to the Hamiltonian analysis. Within the SK formalism, the generating functional $Z[J]$ is put into useful form by splitting the classical Lagrangian into free and interacting parts $\mathscr{L}[\phi] = \mathscr{L}_0[\phi] + \mathscr{L}_{\rm int}[\phi]$, such that $Z[J] \sim \exp\left(i \int \mathscr{L}_{\rm int}\left[\frac{\delta}{i\delta J}\right]\right) Z_0[J]$ where $Z_0[J] \sim \int \mathscr{D}\phi \exp\left(i \int \{\mathscr{L}_0[\phi] + J\phi\}\right)$. Since $Z_0[J]$ is a Gaussian integral, it can be carried out explicitly. Then the interaction piece is expanded *perturbatively* to get the desired correlators. Barring unimportant subtleties, this is not different than good old QFT à la Feynman. See [61] for a modern review of SK, its applicability on QSF inflation and original references.

in agreement with the hierarchy $\dot{\theta}_0^2 \ll H^2 \ll v^2$.

Hamilton's equations deriving from the free Hamitonian (4.118) read

$$\varphi_I'' + 2\,\mathcal{H}\,\varphi_I' + k^2\varphi_I = 0, \tag{4.126}$$

$$\mathcal{F}_I'' + 2\,\mathcal{H}\,\mathcal{F}_I' + k^2\mathcal{F}_I + a^2\widetilde{M}^2_{\text{eff}}\,\mathcal{F}_I = 0, \tag{4.127}$$

recalling that $f' \equiv \partial_\tau f$ and conformal time τ is defined through the relation $dt = a\,d\tau$, so in particular $\mathcal{H} \equiv \frac{a'}{a}$. Working in the de Sitter approximation ($\dot{H} = 0$) for simplicity [24] one finds that $\mathcal{H} = -\frac{1}{\tau}$ and $a = -\frac{1}{\mathcal{H}\tau}$. It is straightforward to show that if we define $u_k \equiv a\,\varphi_I$ and $v_k \equiv a\,\mathcal{F}_I$, the equations of motion (4.126)-(4.127) can be put in the form

$$y_k'' + \left(k^2 - \frac{\nu_y^2 - \frac{1}{4}}{\tau^2}\right) y_k = 0, \quad \nu_y^2 \equiv \frac{9}{4} - \frac{m_y^2}{H^2}, \tag{4.128}$$

where m_y stands for the mass of the modes $y_k = \{u_k, v_k\}$. In the massless case, meaning $\nu_u = \frac{3}{2}$, the solutions to (4.128) are the Bunch-Davies mode functions which are given by [151]

$$u_k(\tau) = \frac{H}{\sqrt{2k^3}}\,(1 + ik\tau)\,e^{-ik\tau}. \tag{4.129}$$

For $m_v \equiv \widetilde{M}_{\text{eff}} \neq 0$ the more general solutions to (4.128) need to be considered.

[24] This is equivalent to neglect slow roll corrections to the Mukhanov-Sasaki equation (4.128).

These are given by [25]

$$v_k(\tau) = \begin{cases} -i\, e^{i\left(\nu+\frac{1}{2}\right)\frac{\pi}{2}} \frac{\sqrt{\pi}}{2} H(-\tau)^{\frac{3}{2}} \mathbb{H}_\nu^{(1)}(-k\tau), & \text{for} \quad \frac{\widetilde{M}_{\text{eff}}^2}{H^2} \leq \frac{9}{4}, \\ -i\, e^{-\frac{\pi}{2}\mu + i\frac{\pi}{4}} \frac{\sqrt{\pi}}{2} H(-\tau)^{\frac{3}{2}} \mathbb{H}_{i\mu}^{(1)}(-k\tau), & \text{for} \quad \frac{\widetilde{M}_{\text{eff}}^2}{H^2} > \frac{9}{4}, \end{cases} \tag{4.132}$$

where $\mathbb{H}_\nu^{(1)}$ is the Hankel function of the first kind and $\mu \equiv \sqrt{\frac{\widetilde{M}_{\text{eff}}^2}{H^2} - \frac{9}{4}}$. The normalization of the mode functions are chosen so that when the physical momentum $\frac{k}{a}$ is much larger than the Hubble parameter H and the mass m_y, we get back the Bunch-Davies vacuum, i.e., u_k as given in (4.129) and $v_k \approx i \frac{H}{\sqrt{2k}} \tau e^{-ik\tau}$. We see from (4.130) that when $0 \leq \widetilde{M}_{\text{eff}} \leq \frac{3}{2}H$ the amplitude of the mode v_k decays as $(-\tau)^{\frac{3}{2}-\nu}$ after horizon exit, so the lighter the isocurvaton is, the slower it decays. In the limit $\widetilde{M}_{\text{eff}} \to 0$ $\left(\nu \to \frac{3}{2}\right)$ the amplitude is frozen. On the other hand, when $\widetilde{M}_{\text{eff}} > \frac{3}{2}H$, we see from (4.131) that v_k not only contains a decay factor $(-\tau)^{\frac{3}{2}}$ but an oscillation factor $\tau^{\pm i\mu}$ as well. While this oscillation is marginal for $\widetilde{M}_{\text{eff}} \sim H$, it causes cancellations in the integrals of the correlation functions and is equivalent to factors of Boltzmann-like suppression $\sim e^{-\frac{\widetilde{M}_{\text{eff}}}{H}}$ in the $\widetilde{M}_{\text{eff}} \gg H$ limit [26]. This is the reason behind the fact that most authors originally considered

[25] It is worth considering the behavior of the mode functions after horizon exit, namely, as $k\tau \to 0$.
When $\frac{\widetilde{M}_{\text{eff}}^2}{H^2} \leq \frac{9}{4}$

$$v_k(\tau) \to \begin{cases} -e^{i\left(\nu+\frac{1}{2}\right)\frac{\pi}{2}} \frac{2^{\nu-1}}{\sqrt{\pi}} \Gamma(\nu) \frac{H}{k^\nu} (-\tau)^{-\nu+\frac{3}{2}}, & \text{for} \quad 0 < \nu \leq \frac{3}{2}, \\ e^{i\frac{\pi}{4}} \frac{1}{\sqrt{\pi}} H(-\tau)^{\frac{3}{2}} \ln(-k\tau), & \text{for} \quad \nu = 0. \end{cases} \tag{4.130}$$

When $\frac{\widetilde{M}_{\text{eff}}^2}{H^2} > \frac{9}{4}$

$$v_k(\tau) \to -ie^{-\frac{\pi}{2}\mu + i\frac{\pi}{4}} \frac{\sqrt{\pi}}{2} H(-\tau)^{\frac{3}{2}} \left[\frac{1}{\Gamma(i\mu+1)} \left(\frac{-k\tau}{2}\right)^{i\mu} - i\frac{\Gamma(i\mu)}{\pi} \left(\frac{-k\tau}{2}\right)^{-i\mu} \right], \tag{4.131}$$

where $\mu \equiv \sqrt{\frac{\widetilde{M}_{\text{eff}}^2}{H^2} - \frac{9}{4}}$.

[26] In analogy to thermal field theory, the contributions of massive states to correlation functions are exponentially suppressed by a Boltzmann factor if the mass is much higher than the temperature. In de Sitter space there is a "Gibbons-Hawking" temperature given by $T_{\text{GH}} = \frac{H}{2\pi}$ [152] and hence the corresponding Boltzmann factor reads $e^{-\frac{\widetilde{M}_{\text{eff}}}{T_{\text{GH}}}} = e^{-\frac{2\pi\widetilde{M}_{\text{eff}}}{H}}$.

the $0 \leq \nu \leq \frac{3}{2}$ regime only. However it has been recently understood that the regime $\widetilde{M}_{\text{eff}} \gtrsim \frac{3}{2}H$ has very peculiar features in the so-called "squeezed limit" that however, realistically, will only be disentangled after finding some first evidence of non-Gaussianities [13, 153] [27]. We are interested in the perturbative corrections to the 2, 3 and 4-point functions of the adiabatic fluctuation. In Figure 4.6 we draw the (tree-level) correlators along with the perturbative corrections due to the presence of the isocurvature mode.

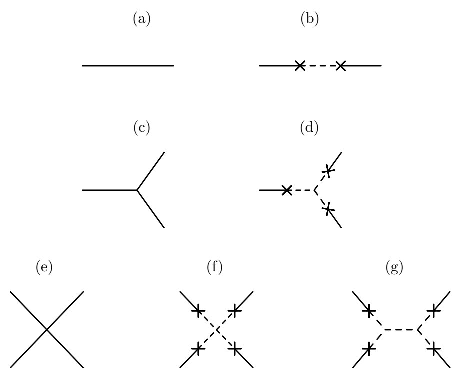


Figure 4.6: Leading (tree-level) diagrams for the 2, 3 and 4-point functions of the curvature perturbation in QSF inflation models. (a), (c) and (e) represent the naive (tree-level) correlators while (b), (d), (f) and (g) are the leading corrections through isocurvature (tree-level) mediation.

The standard tool to calculate cosmological correlators is the in-in formalism, as reviewed in appendix A. The master formula of in-in applied to the two-point function of φ is given by

$$\langle \varphi^2 \rangle = \left\langle 0 \middle| \left[\overline{T} \exp\left(i \int_{-\infty^-}^{t} dt''\, H_I(t'') \right) \right] \varphi_I^2(t) \left[T \exp\left(-i \int_{-\infty^+}^{t} dt'\, H_I(t') \right) \right] \middle| 0 \right\rangle, \tag{4.133}$$

where $H_I = \int d^3\mathbf{x}\, \mathscr{H}_2^I$, $\overline{T}$ is the anti-time-ordering symbol and $\infty^{\pm} \equiv \infty(1 \pm i\,\varepsilon)$. Then, recalling that $\mathcal{R} \approx -\frac{H}{\dot{\varphi}_0}\varphi$, the dimensionless power spectrum of curvature

[27] The regime $\widetilde{M}_{\text{eff}} \gg H$ is not trivial. The time-dependent inflationary background implies that integrating out a heavy mode leaves an imprint in the speed of sound of adiabatic fluctuations, as we have discussed thoroughly in subsection 4.3.1.1 and (in some generality) appendix B. See [132] and references therein.

fluctuations $\Delta^2_{\mathcal{R}}$ as defined in (4.87) and the scalar tilt, defined as $n_s \equiv 1+\frac{d\ln\Delta^2_{\mathcal{R}}}{d\ln k}$, are given by [28]

$$\Delta^2_{\mathcal{R}} = \frac{H^4}{4\pi^2\dot{\varphi}_0^2}\left\{1+\mathfrak{C}(\nu)\left(\frac{\dot{\theta}_0}{H}\right)^2\right\}, \quad n_s-1=-2\,\epsilon-\eta+\eta\,\mathfrak{C}(\nu)\left(\frac{\dot{\theta}_0}{H}\right)^2. \tag{4.134}$$

The explicit analytic calculation of $\mathfrak{C}(\nu)$ for arbitrary $\widetilde{M}_{\rm eff}$ can be found in [53] and it is not enlightening (see [61] for a "quick" derivation). The result in (4.134) justifies the necessity of the first perturbative condition in (4.121) [29].

4.3.2.2 Non-Gaussianities

As for the bispectrum it can be shown [50] that in the squeezed limit $p_3 \ll p_1 \simeq p_2$, when $\nu \neq 0$ $\left(0\leq \widetilde{M}_{\rm eff}<\frac{3}{2}H\right)$, the curvature scalar bispectrum that one gets through the $V_{\sigma\sigma\sigma}$ interaction of the isocurvature mode has a momentum dependence that scales as

$$\langle \mathcal{R}_{\mathbf{p}_1}\mathcal{R}_{\mathbf{p}_2}\mathcal{R}_{\mathbf{p}_3}\rangle \sim \frac{1}{p_1^3\,p_3^3}\left(\frac{p_3}{p_1}\right)^{\frac{3}{2}-\nu} \quad \text{when} \quad p_3 \ll p_1 \simeq p_2 \quad \text{and} \quad \nu\neq 0. \tag{4.135}$$

Looking at (4.130) we can understand this momentum dependence by recalling that under Bunch-Davies initial conditions a correlation between long and short

[28] In [53, 154] it was proven that, in the large effective mass limit, $\mathfrak{C}(\mu) \approx \frac{2}{\mu^2}$ with $\mu^2 \equiv \frac{\widetilde{M}^2_{\rm eff}}{H^2}-\frac{9}{4}\approx\frac{\widetilde{M}^2_{\rm eff}}{H^2}$. Using this in (4.134) we get that $\Delta^2_{\mathcal{R}} \approx \mathring{\Delta}^2_{\mathcal{R}}\left(1+2\frac{\dot{\theta}_0^2}{\widetilde{M}^2_{\rm eff}}\right)$, where $\mathring{\Delta}^2_{\mathcal{R}}$ stands for the single field ($c_s=1$) power spectrum. This should be compared with the effective single field ($c_s\neq 1$) prediction which in this case is given by $\Delta^2_{\mathcal{R}} \approx \mathring{\Delta}^2_{\mathcal{R}}\,c_s^{-1} \approx \mathring{\Delta}^2_{\mathcal{R}}\left(1+2\frac{\dot{\theta}_0^2}{M^2_{\rm eff}}\right) = \mathring{\Delta}^2_{\mathcal{R}}\left(1+2\frac{\dot{\theta}_0^2}{\widetilde{M}^2_{\rm eff}}\,c_s^{-2}\right)$, so both predictions coincide to $\mathcal{O}\left(\frac{\dot{\theta}_0^2}{\widetilde{M}^2_{\rm eff}}\right)$.

[29] Strictly speaking, the perturbative condition one needs to impose is $\mathfrak{C}(\nu)\,\dot{\theta}_0^2 \ll H^2$. $\mathfrak{C}(\nu)$ is a very slowly growing function that stays $\mathcal{O}(1)$ until it diverges as $\nu\to\frac{3}{2}$, as can be seen in FIG.6 of reference [50] and equation (3.15) of reference [52]. The divergence represents the massless limit, and since we want to focus in the regime where $\widetilde{M}^2_{\rm eff}\gtrsim H^2$, we can safely take $\mathfrak{C}(\nu)\sim\mathcal{O}(1)$ for all our purposes.

wavelengths can only be generated once the short wavelength modes approach horizon scales. The amplitude of the long wavelength will have decayed according to the factor $\left(\frac{\tau_1}{\tau_3}\right)^{\frac{3}{2}-\nu} = \left(\frac{p_3}{p_1}\right)^{\frac{3}{2}-\nu}$ by that time, explaining the behavior in (4.135). The shape function (4.135) has been dubbed "intermediate" [49] since it interpolates between local and equilateral shapes as $\nu \to \left\{\frac{3}{2}, 0\right\}$, respectively (cf. (4.89) and (A.34); see [150] for standard definitions). Indeed, the more massive the isocurvature mode is, the faster it decays on super-horizon scales, so the largest contribution to non-Gaussianities is generated around horizon-crossing scales, i.e. in the equilateral configuration. On the other hand, if the isocurvature mode is lighter, the super-horizon isocurvature fluctuations survive longer and can contribute to correlations between long and short modes, i.e. in the so-called local configuration.

We can estimate the size of non-Gaussianities, i.e. the order of magnitude of $f_{\rm NL}$, by realizing that the dimensionless coupling constants for the cubic isocurvature interaction and the transfer vertex go like $\left(\frac{V_{\sigma\sigma\sigma}}{H}\right)$ and $\left(\frac{\dot{\theta}_0}{H}\right)$, respectively [50]. Thus, since $\mathcal{R} \sim \sqrt{\Delta^2_{\mathcal{R}}}$, we find through inspection of diagram (d) in Figure4.6 that

$$\langle \mathcal{R}^3 \rangle \sim \left(\frac{V_{\sigma\sigma\sigma}}{H}\right)\left(\frac{\dot{\theta}_0}{H}\right)^3 (\Delta^2_{\mathcal{R}})^{3/2} \sim f_{\rm NL}\,(\Delta^2_{\mathcal{R}})^2 \to f_{\rm NL} \sim \frac{1}{\sqrt{\Delta^2_{\mathcal{R}}}}\left(\frac{V_{\sigma\sigma\sigma}}{H}\right)\left(\frac{\dot{\theta}_0}{H}\right)^3 . \tag{4.136}$$

In our model, $V_{\sigma\sigma\sigma} = 6\,\lambda\,R \approx 6\sqrt{2}\,\lambda\,v$ and $\lambda \approx \frac{\widetilde{\alpha}\,H^2}{4\,v^2}$ thus $\frac{V_{\sigma\sigma\sigma}}{H} \approx \frac{3\sqrt{2}\,\widetilde{\alpha}}{2}\frac{H}{v} \approx (3.81 \times 10^{-6})\,\widetilde{\alpha}$, where we have used the benchmark point defined in (4.124). Taking $\sqrt{\Delta^2_{\mathcal{R}}} \approx 4.63 \times 10^{-5}$ (from observations)

$$f_{\rm NL} \sim \left(8.23 \times 10^{-2}\right)\,\widetilde{\alpha}\left(\frac{\dot{\theta}_0}{H}\right)^3 . \tag{4.137}$$

If we assume a non-conservative value $\frac{\dot{\theta}_0}{H} \approx \frac{1}{\sqrt{10}}$ (so we get an $\mathcal{O}(10^{-1})$ correction to the power spectrum in (4.134)) we find using (4.137) that

$$f_{\rm NL} \sim \left(2.6 \times 10^{-3}\right) \widetilde{\alpha}. \tag{4.138}$$

The estimation above lacks a numerical factor (and a sign) that Chen and Wang originally obtained. Quoting their result,

$$f_{\rm NL} \approx \frac{\vartheta(\nu)}{\sqrt{\Delta_{\mathcal{R}}^2}} \left(\frac{-V_{\sigma\sigma\sigma}}{H}\right) \left(\frac{\dot{\theta}_0}{H}\right)^3, \tag{4.139}$$

where $\vartheta(\nu)$ is a positive numerical coefficient which is expected to be $\mathcal{O}(1)$ [30]. Then our estimation (4.137) is slightly modified to finally give

$$f_{\rm NL} \approx -\left(2.6 \times 10^{-3}\right) \widetilde{\alpha}\, \vartheta(\nu) \lesssim \mathcal{O}\left(\epsilon, \eta\right), \quad (\widetilde{\alpha},\, \vartheta(\nu) \sim \mathcal{O}(1)\,\text{numbers}) \tag{4.140}$$

which is (still) unobservably small.

Finally, we can estimate the trispectra $\tau_{\rm NL}$ (4-point function) by considering diagrams (f) and (g) in Figure4.6. We get that

$$\tau_{\rm NL} \sim \max\left\{\tau_{\rm NL}^{\rm SE} \cong \frac{1}{\Delta_{\mathcal{R}}^2}\left(\frac{\dot{\theta}_0}{H}\right)^4 \left(\frac{V_{\sigma\sigma\sigma}}{H}\right)^2, \tau_{\rm NL}^{\rm CI} \cong \frac{1}{\Delta_{\mathcal{R}}^2}\left(\frac{\dot{\theta}_0}{H}\right)^4 V_{\sigma\sigma\sigma\sigma}\right\}, \tag{4.141}$$

[30] It can be numerically shown that $\vartheta(\nu)$ blows up as $\nu \to \frac{3}{2}$ ($\widetilde{M}_{\rm eff} \to 0$). The divergence occurs because we use the constant turn assumption. However when $\widetilde{M}_{\rm eff} = 0$, a $\delta\sigma$ fluctuation never decays at super-horizon so the transfer from isocurvaton to curvaton lasts forever. As [50] points out, if the horizon crossing time of a perturbation mode is N_f e-folds before the end of inflation (or the time when the inflaton trajectory becomes straight), one needs to impose a cut-off in the conformal time integrals of the exact in-in formula for $\langle \mathcal{R}^3 \rangle$. All in all we could naively conclude that the integrals are dominated by a N_f^4 behavior. However, we need to realize that in this limit, $\mathfrak{C}(\nu)$ in (4.134) scales as N_f^2 for the same reason. For large N_f, the perturbativity condition becomes $N_f^2\left(\frac{\dot{\theta}_0^2}{H^2}\right) \ll 1$ instead. Thus, in the perturbative regime, the effective "enchancement" factor is only N_f (which in principle can be as large as 60). Since we are not interested in the "multifield" inflation limit [127], $\vartheta(\nu)$ is $\mathcal{O}(1)$ for our purposes.

where, following [50], SE and CI in $\tau_{\rm NL}^{\rm SE}$ and $\tau_{\rm NL}^{\rm CI}$ stand for "scalar-exchange" and "contact-interaction", respectively. Recalling that in our model, $V_{\sigma\sigma\sigma\sigma} = 6\,\lambda \approx \frac{3}{2}\,\widetilde{\alpha}\left(\frac{H}{v}\right)^2$, we find that

$$\tau_{\rm NL} \sim \max\left\{\tau_{\rm NL}^{\rm SE} \sim 6.78\times 10^{-3}\,\widetilde{\alpha}^2\left(\frac{\dot\theta_0}{H}\right)^4,\ \tau_{\rm NL}^{\rm CI} \sim 2.27\times 10^{-3}\,\widetilde{\alpha}\left(\frac{\dot\theta_0}{H}\right)^4\right\}. \tag{4.142}$$

Assuming again that $\frac{\dot\theta_0}{H} \approx \frac{1}{\sqrt{10}}$ this becomes

$$\tau_{\rm NL} \sim \max\left\{\tau_{\rm NL}^{\rm SE} \sim 6.78\times 10^{-5}\,\widetilde{\alpha}^2,\ \tau_{\rm NL}^{\rm CI} \sim 2.27\times 10^{-5}\,\widetilde{\alpha}\right\}. \tag{4.143}$$

Considering (4.136) and (4.141) we see that

$$\tau_{\rm NL}^{\rm SE} \sim \left(\frac{H}{\dot\theta_0}\right)^2 f_{\rm NL}^2 \quad \text{and} \quad \tau_{\rm NL}^{\rm CI} \sim \left(\frac{H}{\dot\theta_0}\right)^2\left(\frac{V_{\sigma\sigma\sigma\sigma}\,H^2}{{V_{\sigma\sigma\sigma}}^2}\right) f_{\rm NL}^2. \tag{4.144}$$

As as consequence of perturbativity, we find that

$$\tau_{\rm NL}^{\rm SE} \gg \left(\frac{6}{5}f_{\rm NL}\right)^2, \tag{4.145}$$

so that the so-called "Suyama-Yamaguchi bound" [155] is satisfied as expected in the QSF scenario [52] [31]. We also see that for our specific model (4.144) implies

[31] The Suyama-Yamaguchi bound reads

$$\tau_{\rm NL}^{\rm SE} \geq \left(\frac{6}{5}f_{\rm NL}\right)^2. \tag{4.146}$$

The inequality is saturated for single-field inflation while multifield inflation satisfies (4.146). The case for QSF is in principle distinguishable as $\tau_{\rm NL}^{\rm SE} \gg \left(\frac{6}{5}f_{\rm NL}\right)^2$ is expected to hold instead. See [52] for a discussion.

that

$$\tau_{\rm NL}^{\rm CI} \sim \left(\frac{H}{\dot{\theta}_0}\right)^2 \left(\frac{1}{3\,\widetilde{\alpha}}\right) f_{\rm NL}^2 \quad \rightarrow \quad \tau_{\rm NL}^{\rm SE} > \tau_{\rm NL}^{\rm CI} \quad \text{when} \quad \widetilde{\alpha} \sim \mathcal{O}(1), \tag{4.147}$$

which is the case in (4.142). Interestingly, this "hierarchy" reverses when $\widetilde{\alpha} \leq \frac{1}{3}$. This fact could, in principle, be used to pin down the mass range of the isocurvatons of the QSF scenario once the "cosmological collider physics" program is up and running [13][153]. Needless to say, measuring the trispectra of primordial density perturbations is way beyond our current experimental expectations.

4.3.2.3 Comments on the $M_{\rm eff}^2 \ll H^2$ Regime

As has been previously stressed, when approaching the isocurvaton light mass limit, the squeezed limit of the bispectrum in the QSF scenario is of "quasi-local" type, and the fluctuations decay much slower than in the heavy mass case. This situation has been originally discussed in [50], where the following two instances have been distinguished:

- If $V_{\sigma\sigma\sigma}$ is still "large", the QSF analysis does apply, so we can use (4.139) to estimate the size of non-Gaussianities, but with an infrared e-folds cutoff as discussed in footnote 30.

- It is possible that in this limit the isocurvature background solution slow-rolls as well as the inflationary one, implying through slow-roll conditions that the coupling $\left(\frac{V_{\sigma\sigma\sigma}}{H}\right)_{\rm sr}$ is $\sim \mathcal{O}\left(\epsilon^{3/2}\right)\frac{H}{M_{\rm Pl}}$. As is well known [156], this scenario does not produce sizable non-Gaussianities.

Let us then analyze the isocurvaton light mass limit of our model to see into which of the above cases it falls. The light mass condition, $\widetilde{M}_{\rm eff}^2 \approx 4\,\lambda\,v^2 + 3\,\dot{\theta}_0^2 \ll H^2$, amounts to $\lambda \ll \frac{H^2}{4\,v^2}$, as $\frac{\dot{\theta}_0^2}{H^2} \ll 1$ due to perturbativity. Using the benchmark point (4.124), which is required by the background NI theory, this implies that $\lambda \ll 8.08 \times 10^{-13}$. Since $V_{\sigma\sigma\sigma} = 6\sqrt{2}\,\lambda\, v$, we find that $\frac{V_{\sigma\sigma\sigma}}{H} \ll 3.81 \times 10^{-6}$. On the other hand, assuming $\epsilon \sim 10^{-2}$, we see that this last constraint on $\frac{V_{\sigma\sigma\sigma}}{H}$ takes us quite close to the slow-roll regime as $\left(\frac{V_{\sigma\sigma\sigma}}{H}\right)_{\rm sr} \sim \epsilon^{3/2}\frac{H}{M_{\rm Pl}} \sim 2.72 \times 10^{-8}$. We then realize that due to the tight symmetry constraints on the parameters of our model, non-Gaussianities are much more suppressed in the isocurvaton light mass scenario when compared to the QSF regime ones, which are already quite small. For this reason we do not further discuss this particular limit.

4.4 Discussion and Conclusions

We have considered a generalization of Natural Inflation [43, 44] where the dynamics of the radial mode σ is included. To this end we have carried out an educated field-theoretic construction of a "UV-complete" two-field model undergoing spontaneous as well as explicit symmetry breaking of a global $U(1)$ symmetry. The (soft) explicit symmetry breaking operators of our model give the (inflaton) pseudo-Goldstone field θ a naturally small mass in accordance with slow-roll requirements and makes the potential for the two-field system $V(\sigma,\theta)$ non-separable. We analyzed the dynamics of the background solution assuming an almost constant angular speed circular motion in (flat) field space. As for the theory of fluctuations, the results depend crucially on whether the effective mass squared of the radial field $M^2_{\rm eff}$ is very heavy ($\gg$) or not ($\sim$) with respect to the cosmological collider experiment energy scale squared, H^2.

We have found that effective single-field Natural Inflation ($M^2_{\rm eff} \gg H^2$) has a better fit to current bounds in the (n_s, r) plane [25] if the speed of sound of adiabatic fluctuations c_s is mildly smaller than one [32]. However the amplitudes of non-Gaussianities, collectively denoted as $f_{\rm NL}$, are negligible unless $c_s^2 \ll 1$.

[32]See [129] for previous developments along these lines.

In particular, we have noticed that the assumptions on the relative "weight" of the heavy field operators when neglecting its dynamics changes the behavior of $f_{\rm NL}$ as a function of c_s, especially in the small c_s regime. Indeed, keeping the $V_{\sigma\sigma\sigma}$ contribution in the constraint equation for $\delta\sigma$ changes the predictions of the model quite dramatically, as was argued in [55]. This "free parameter" (from the single-field EFT of inflation [40] point of view) is constrained by the symmetry or our model and feeds into the functional dependence of $f_{\rm NL} = f_{\rm NL}(c_s)$ leaving a characteristic behavior [33], that in the small c_s regime, scales like $f_{\rm NL} \sim c_s^{-4}$ instead of the usual $f_{\rm NL} \sim c_s^{-2}$ scaling that is naively expected in this class of models [26, 147] (the $f_{\rm NL} \sim c_s^{-4}$ scaling does arise, for example, in Galileon models of infation [110] [34]). In our model, to get small c_s such that the $f_{\rm NL}$'s get any chance of being observable, requires a bit of tuning of initial conditions which is obviously unappealing from the theoretical point of view.

The other possibility that we have analyzed is the $M_{\rm eff}^2 \sim H^2$ scenario, i.e. the Quasi-Single-Field regime [49, 50]. A quick estimate shows that $f_{\rm NL}$ becomes unobservably small given the observational constraints on the parameters of the model; in short, the Natural Inflation background requires super-Planckian values of the VEV v, which entails that in order to have $M_{\rm eff}^2 \approx 4\,\lambda\, v^2 \sim H^2$ we need λ to be quite small, implying that $\frac{V_{\sigma\sigma\sigma}}{H} \approx \frac{6\sqrt{2}\,\lambda\, v}{H} \sim \frac{3\sqrt{2}}{2}\frac{H}{v}$ is just too small to produce sizable non-Gaussianities through the use of (4.139). This somehow "negative" result is at odds with the original naive expectations that through a $(\delta\sigma)^3$ interaction, non-Gaussianities for the adiabatic mode can become large. Although this conclusion is also based on the perturbative assumption that the mixing coupling $\frac{\dot\theta_0}{H}$ is small in the QSF regime, we have seen from the single-field EFT point of

[33] Indeed our model is quite peculiar in the sense that in (4.98), all terms are related by symmetry in such a way that $\frac{R}{6M_{\rm eff}^2}V_{\sigma\sigma\sigma} \approx \frac{3}{8} + \frac{1}{8}c_s^{-2}$, so $f_{\rm NL} = f_{\rm NL}(c_s)$ ultimately. This kind of simplification does not occur in a generic model.

[34] It would be interesting to clarify the connection between such a scenario and the case where we integrate out the radial mode of a pseudo-Goldstone model without neglecting its self-interactions. This, however, lied beyond the scope of our work.

view that this is indeed the case as we lower down M_{eff}^2. Even if the single-field EFT does not make sense in the QSF limit, this might shed some light on the real limitations of this particular perturbative condition. Recently there has been renewed interest in non-perturbative (strongly-coupled) QSF models [157–159]. It would be interesting to see if, through these new developments, we could find less supressed signatures of our model. Another avenue worth exploring would be to introduce a new scale in the problem, like for example, a non-trivial curvature tensor in field space $\mathbb{R}^a{}_{bcd}$. One way of naturally doing this would be to extend the symmetry group of our model to a non-abelian one, say $SU(2)$ for definiteness [35]. All these ideas will be investigated elsewhere.

[35] As manifolds, $SU(2)$ and the 3-sphere S^3 are diffeomorphic, implying that the spectrum of this non-abelian model would consist of three angular (Goldstone) directions plus a radial one. It is not easy to anticipate the phenomenology of such a "Multi-Quasi-Single-Field" model.

Chapter 5

Cosmological Cosets

In this chapter we will revisit the seminal work of Curtis Callan, Sidney Coleman, Julius Wess, and Bruno Zumino (CCWZ) on the rigorous construction of phenomenological Lagrangians, the so-called "coset construction" [160, 161]. For the whole review of SSB of internal symmetries we will closely follow [162]. Subsequently, we will learn how to generalize such a construction to the case of local gauge and spacetime symmetries, following the approach of [163]. Finally we will derive (some limit of) the EFT of inflation using these formal algebraic, geometric tools, and we will discuss several subtle points that arise on the way.

5.1 Phenomenological Lagrangians: SSB of Internal Global Symmetries as a Warm-Up

5.1.1 Generalities

To be concrete, let us consider again the example that we introduced in the SSB of global symmetries section of chapter 3. We have a set of scalar fields ϕ_i, $i = 1, \ldots, N$, whose kinetic term is invariant under some global symmetry group, e.g. the orthogonal group in N dimensions $O(N)$, defined by elements O that satisfy $O^T O = \mathbb{1}$. The minimal non-trivial Lagrangian for such a system would be

$$\mathscr{L} = -\frac{1}{2}\partial_\mu \phi^T \partial^\mu \phi - V(\phi), \tag{5.1}$$

where ϕ is a column vector and we are omitting its vector indices for simplicity of notation. The potential $V(\phi)$ need not be invariant under $O(N)$ but may only preserve some subgroup of these, $G \subset O(N)$, meaning, if $g \in G$, then $V(g\phi) = V(\phi)$. Within the semiclassical approximation, if the potential is minimized by some field configuration $\phi = v \neq 0$, then the symmetry group G may be spontaneously broken to some subgroup, $H \subset G$, which is defined by

$$hv = v, \quad \forall\ h \in H. \tag{5.2}$$

As usual, we assume that any group element $g \in G$ may be written as the exponential of parameters times generators of the corresponding Lie algebra $\mathfrak{g}$,

$$g = \exp\left(i\,\theta^a\, G_a\right). \tag{5.3}$$

We now split the generators G_a into $G_a = \{T_A, X_\alpha\}$, where the T_A's generate the Lie algebra of H, denoted by $\mathfrak{h}$. In particular, (5.2) implies that $T_A\, v = 0$. Closure of H under group multiplication implies that

$$[T_A, T_B] = i\, c_{ABC}\, T_C, \tag{5.4}$$

so in particular $c_{AB\alpha} = 0$.

The X_α's do not lie in $\mathfrak{h}$, and so satisfy $X_\alpha\, v \neq 0$. Furthermore, they are said to generate the space of *cosets*, denoted by G/H. In short, a coset is an equivalence class which is defined to contain all of the elements of G that are related by the multiplication by an element of H. Physically, the X_α's represent the generators of the symmetry group which are spontaneously broken. For a compact group, the structure constants c_{abc} can be shown to be completely antisymmetric, so that

$c_{AB\alpha} = 0 \Rightarrow c_{A\alpha B} = 0$, i.e.,

$$[T_A, X_\alpha] = i\, c_{A\alpha\beta}\, X_\beta. \tag{5.5}$$

This last commutator implies that the X_α's fall into a representation of H, meaning that the exponentiation of the above commutator is equivalent to the statement

$$h\, X_\alpha\, h^{-1} = L_\alpha{}^\beta X_\beta, \tag{5.6}$$

for any h and some coefficients $L_\alpha{}^\beta$. For completeness, let us state that in contrast, the commutator of the X_α's among themselves does not have a simple form, since

$$[X_\alpha, X_\beta] = i\, c_{\alpha\beta\gamma}\, X_\gamma + i\, c_{\alpha\beta A}\, T_A, \tag{5.7}$$

and $c_{\alpha\beta A} \neq 0$, otherwise the X_α's would form a subgroup as well. Moreover, when $c_{\alpha\beta\gamma} = 0$ the coset G/H corresponds, in math parlance, to a "symmetric space". As we have thoroughly reviewed in chapter 3, Goldstone's theorem implies the existence of one massless scalar for each broken symmetry generator. Intuitively, Goldstone modes are obtained by performing symmetry transformations on the ground state. Since an infinitesimal symmetry transformation on the ground state corresponds to the directions $X_\alpha\, v$ in field space, we expect that the components of ϕ in this direction, $v^T\, X_\alpha\, \phi$, to be the Goldstone bosons. In order to make the low-energy decoupling of the Goldstone modes manifest we require that they do not appear at all in the scalar potential. It is therefore convenient to make a change of variables $\phi(x) \to \{U(\theta(x)), \chi(x)\}$ such that

$$\phi = U(\theta)\chi, \quad U(\theta) = \exp\left(i\, \theta^\alpha\, X_\alpha\right). \tag{5.8}$$

Indeed, $U(\theta)$ can be thought as a spacetime-dependent symmetry transformation in the direction of the broken generators X_α. χ is taken to be perpendicular, in field space, to the Goldstone directions, $X_\alpha\, v$, meaning [1]

$$v^T X_\alpha \,\chi = 0. \tag{5.9}$$

This last fact, together with the antisymmetry of the generators [2], which implies that $v^T X_\alpha\, v = 0$, precisely ensures the vanishing of the "cross terms" $\sim \partial_\mu\theta\, \partial^\mu \delta\chi$ in the kinetic sector when expanding around the ground state configuration $\chi = v + \delta\chi$. As G-invariance requires the potential to satisfy $V(U\chi) = V(\chi)$, the variable θ is guaranteed to drop out of the scalar potential, so that all terms in $\mathscr{L}$ involving the Goldstones θ^α vanish when $\partial_\mu \theta^\alpha = 0$.

5.1.2 The Non-linear Realization

The field ϕ transforms linearly under G, meaning $\phi : \phi \to \tilde{\phi} = g\, \phi$, where $g = e^{i\,\alpha^a\, G_a}$. After performing the change of variables from $\phi(x) = \{\phi^i\}$ to $\chi(x) = \{\chi^n\}$ and $\theta(x) = \{\theta^\alpha\}$, we would like to find the transformation rules for the new variables $\theta \to \tilde{\theta}$ and $\chi \to \tilde{\chi}$. The natural guess $\tilde{\phi} = U(\tilde{\theta})\, \tilde{\chi}$ implies then that

$$g\, U(\theta)\, \chi = U(\tilde{\theta})\, \tilde{\chi}. \tag{5.10}$$

Equation (5.10) can be rewritten as $\tilde{\chi} = \gamma\, \chi$, with $\gamma \equiv \tilde{U}^{-1}\, g\, U$. It can be shown that this condition implies that γ must lie within the subgroup H of unbroken transformations, and as such, may be written in the form $\gamma = e^{i\, u^A\, T_A}$, for some functions $u^A = u^A(\theta, g)$. Therefore the transformation laws $\theta^\alpha \to \tilde{\theta}^\alpha(\theta, g)$ and

[1] It can be shown that condition (5.9) can always be satisfied starting from any field configuration. See [162].

[2] The generators G_a are necessarily antisymmetric after demanding unitarity and reality of the group elements g.

$\chi \to \tilde{\chi}(\theta, g, \chi)$ are given by

$$g\, e^{i\,\theta^\alpha\, X_\alpha} = e^{i\,\tilde{\theta}^\alpha\, X_\alpha}\, e^{i\,u^A\, T_A}, \tag{5.11}$$

$$\tilde{\chi} = e^{i\,u^A\, T_A}\,\chi. \tag{5.12}$$

Equation (5.11) defines the non-linear functions $\tilde{\theta}^\alpha(\theta, g)$ and $u^A(\theta, g)$. In other words, we find the element $g\, e^{i\,\theta\cdot X} \in G$, and then decompose the matrix into the product of a factor $e^{i\,\tilde{\theta}\cdot X} \in G/H$, times an element $e^{i\,u\cdot T} \in H$. Then (5.12) defines the transformation rule for the non-Goldstone fields χ.

There is a interesting special case that can be explicitly worked out for γ and $\widetilde{U}$; when $g = h \in H$. It is easily seen that in this case $\gamma = h$ and $\widetilde{U} = h\, U\, h^{-1}$, in other words both χ and θ transform linearly under the unbroken subgroup H,

$$\theta^\alpha\, X_\alpha \to \tilde{\theta}^\alpha\, X_\alpha = h\,\theta^\alpha\, X_\alpha\, h^{-1}, \tag{5.13}$$

$$\chi \to \tilde{\chi} = h\,\chi, \tag{5.14}$$

$\forall\; h \in H$.

For the broken symmetries, $g \in G/H$, things are not so simple, but we can specialize to infinitesimal transformations by taking $g = 1 + i\,\omega^\alpha\, X_\alpha + \ldots$, $\gamma = 1 + i\,u^A(\theta, \omega)\, T_A + \ldots$, and $U(\tilde{\theta}) = U(\theta)\,[1 + i\,\Delta^\alpha(\theta, \omega)\, X_\alpha + \ldots]$, where ω^α, $u^A(\theta, \omega)$, and $\Delta(\theta, \omega)$ are infinitesimal quantities. Using the Killing form (inner product) that satisies $\mathrm{Tr}(X_\alpha X_\beta) = \delta_{\alpha\beta}$, $\mathrm{Tr}(T_A T_B) = \delta_{AB}$ and $\mathrm{Tr}(T_A X_\alpha) = \delta_{A\alpha} = 0$ we find that

$$\Delta_\alpha = \mathrm{Tr}\left[X_\alpha\, e^{-i\,\theta\cdot X}\,(\omega\cdot X)\, e^{i\,\theta\cdot X}\right] \approx \omega_\alpha - c_{\alpha\beta\gamma}\,\omega^\beta\theta^\gamma + \mathcal{O}(\theta^2), \tag{5.15}$$

$$u_A = \mathrm{Tr}\left[t_A\, e^{-i\,\theta\cdot X}\,(\omega\cdot X)\, e^{i\,\theta\cdot X}\right] \approx -c_{A\alpha\beta}\,\omega^\alpha\theta^\beta + \mathcal{O}(\theta^2). \tag{5.16}$$

It can be shown [162] that there exists a linear relation between Δ_α and $\Xi^\alpha(\theta,\omega) \equiv \tilde{\theta}^\alpha - \theta^\alpha$, i.e., $\Delta_\alpha = M_{\alpha\beta}(\theta)\,\Xi^\beta$, where $M_{\alpha\beta} = \int_0^1 ds\,\mathrm{Tr}\left[X_\alpha\, e^{-i\,s\,\theta\cdot X}\, X_\beta\, e^{i\,s\,\theta\cdot X}\right]$ [3] such that

$$\delta\theta^\alpha = \omega^\alpha - c^\alpha{}_{\beta\gamma}\,\omega^\beta\theta^\gamma + \mathcal{O}(\theta^2). \tag{5.18}$$

We see that the transformation rule for the θ^α's under elements of G/H is inhomogeneous, and it is this property which enforces the decoupling of the Goldstone bosons at low energies. Moreover, for the non-abelian case ($c_{abc} \neq 0$), the symmetries act non-linearly on the fields θ^α. The fact that the transformation of θ^α and χ^n are both field dependent implies that the action of these symmetries are spacetime-dependent. So, for example, even if the transformation parameters ω^α are constants, as G is a global symmetry, the transformation matrix $\gamma = e^{i\,u\cdot T}$ is *not* a constant, $\partial_\mu\gamma \neq 0$. This fact makes the construction of invariant Lagrangians quite non-trivial. Happily for us, smart people like CCWZ realized how to proceed.

5.1.3 Invariant Lagrangians

It does not seem easy to unravel how to construct invariant Lagrangians which can describe the low-energy interaction of the Goldstone bosons, given the complicated spacetime-dependent transformation laws (5.11) and (5.12). A heuristic way to proceed is the following. Consider the kinetic term of our toy model $\partial_\mu\phi^T\partial^\mu\phi$, which is manifestly G-invariant. Now perform the change of variables from ϕ to θ and χ. This trivial operation cannot change the physics, so

[3] To arrive to this result one uses the matrix identity

$$e^{-i\,A}e^{i(A+B)} = \mathbb{1} + i\int_0^1 ds\, e^{-i\,s\,A}\, B\, e^{i\,s(A+B)} = \mathbb{1} + i\int_0^1 ds\, e^{-i\,s\,A}\, B\, e^{i\,s\,A} + \mathcal{O}(B^2), \tag{5.17}$$

taking $A = \theta\cdot X$ and $B = \Xi\cdot X$.

the new kinetic sector needs to be G-invariant as well. So, how does this happen specifically?

Consider the derivative of ϕ under the change of variables $\partial_\mu \phi \to \partial_\mu (U\chi) = U\left(\partial_\mu \chi + (U^{-1}\partial_\mu U)\,\chi\right)$. Our experience with gauge theories and gravity leads us to define a covariant derivative acting on χ

$$\mathcal{D}_\mu \chi \equiv \partial_\mu \chi + \left(U^{-1}\partial_\mu U\right)\chi. \tag{5.19}$$

We do this because equation (5.11) may be rewritten as $\tilde{U} = g\,U\,\gamma^{-1}$, which implies that $U^{-1}\partial_\mu U$ transforms as a connection

$$U^{-1}\partial_\mu U \to \tilde{U}^{-1}\partial_\mu \tilde{U} = \gamma\,U^{-1}\,g^{-1}\,\partial_\mu\left(g\,U\,\gamma\right) = \gamma\left(U^{-1}\partial_\mu U\right)\gamma^{-1} - \left(\partial_\mu \gamma\right)\gamma^{-1}, \tag{5.20}$$

where we have used the fact that g is a spacetime-independent group element and $\gamma\,\partial_\mu \gamma^{-1} = -\left(\partial_\mu\gamma\right)\gamma^{-1}$. Moreover, it is wise to separate out the parts of the so-called "Maurer-Cartan one-form" $U^{-1}\partial_\mu U$ which are proportional to X_α from those which are proportional to T_A. That is, defining

$$U^{-1}\partial_\mu U = -i\,\mathcal{A}_\mu^A\,T_A + i\,e_\mu^\alpha\,X_\alpha, \tag{5.21}$$

the transformation law in (5.20) implies that

$$-i\,\mathcal{A}_\mu^A(\theta)\,T_A \to -i\,\mathcal{A}_\mu^A(\tilde{\theta})\,T_A = \gamma\left(-i\,\mathcal{A}_\mu^A(\theta)\,T_A\right)\gamma^{-1} - \partial_\mu\gamma\,\gamma^{-1}, \tag{5.22}$$

$$i\,e_\mu^\alpha(\theta)\,X_\alpha \to i\,e_\mu^\alpha(\tilde{\theta})\,X_\alpha = \gamma\left(i\,e_\mu^\alpha(\theta)\,X_\alpha\right)\gamma^{-1}, \tag{5.23}$$

under G transformations. We see that $\mathcal{A}_\mu^A$ transforms like a gauge potential, while e_μ^α transforms covariantly. Using the infinitesimal version of g and $\gamma(\theta, g)$ one can derive the infinitesimal transformation laws

$$\delta\mathcal{A}_\mu^A(\theta) = \partial_\mu u^A(\theta,\omega) - c^A{}_{BC}\, u^B(\theta,\omega)\,\mathcal{A}_\mu^C(\theta), \tag{5.24}$$

$$\delta e_\mu^\alpha(\theta) = -c^\alpha{}_{A\beta}\, u^A(\theta,\omega)\, e_\mu^\beta(\theta). \tag{5.25}$$

It is natural to "extract" the overall factor $\partial_\mu\theta^\alpha$ in $\delta\mathcal{A}_\mu^A$ and $\delta e_\mu^\alpha(\theta)$, so that

$$\mathcal{A}_\mu^A(\theta) = \mathcal{A}_\alpha^A(\theta)\partial_\mu\theta^\alpha \Rightarrow \mathcal{A}_\alpha^A(\theta) = -\int_0^1 \mathrm{Tr}\left[T_A\, e^{-i\,s\,\theta\cdot X}\, X_\alpha\, e^{i\,s\,\theta\cdot X}\right] \approx \frac{1}{2}\, c_{A\alpha\beta}\,\theta^\beta + \mathcal{O}(\theta^2), \tag{5.26}$$

$$e_\mu^\alpha(\theta) = e_\beta^\alpha(\theta)\partial_\mu\theta^\beta \Rightarrow e_\beta^\alpha(\theta) = \int_0^1 \mathrm{Tr}\left[X_\alpha\, e^{-i\,s\,\theta\cdot X}\, X_\beta\, e^{i\,s\,\theta\cdot X}\right] \approx \delta_{\alpha\beta} - \frac{1}{2}c_{\alpha\beta\gamma}\,\theta^\gamma + \mathcal{O}(\theta^2), \tag{5.27}$$

where to get the first equalities (after the arrows) one follows analogous reasoning and identities that one uses to get the expression for $M_{\alpha\beta}$ right above equation (5.18).

We are ready to build G-invariant couplings among the θ^α, and between the θ^α's and other fields, such as the χ field from the scalar fields example. As "building blocks" for the construction of G-invariant operators for the EFT Lagrangian we have found that:

- We may combine the covariant quantity $e_\mu^\alpha = e_\beta^\alpha\,\partial_\mu\theta^\beta$ in all possible H-invariant ways. This is of course quite simple as they transform homogeneously under G, namely $G:\ e_\mu\cdot X \to \gamma\, e_\mu\cdot X\,\gamma^{-1}$.

- We may also combine covariant derivatives of e_μ^α, $(\mathcal{D}_\mu e_\nu)^\alpha \equiv \partial_\mu e_\nu^\alpha + c^\alpha{}_{A\beta}\,\mathcal{A}_\mu^A e_\nu^\beta$, in H-invariant ways since they transform as they should, i.e. in the same way as does e_μ^α, $G:\ (\mathcal{D}_\mu e_\nu)\cdot X \to \gamma\ (\mathcal{D}_\mu e_\nu)\cdot X\,\gamma^{-1}$.

The Lagrangian $\mathscr{L}\left[e_\mu, \mathcal{D}_\mu e_\nu, \ldots\right]$, where the ellipses denote terms that involve higher covariant derivatives, is "automatically" globally G-invariant provided it is

H-invariant

$$\mathscr{L}\left[h\,e_\mu\,h^{-1}, h\,\mathcal{D}_\mu e_\nu\,h^{-1}, \dots\right] = \mathscr{L}\left[e_\mu, \mathcal{D}_\mu e_\nu, \dots\right]. \tag{5.28}$$

For Poincaré-invariant systems, the first term in the derivative expansion is given by

$$\mathscr{L} = -\frac{1}{2}\,f_{\alpha\beta}\,\eta^{\mu\nu} e^\alpha_\mu\, e^\beta_\nu + \text{higher-derivative operators}, \tag{5.29}$$

where positivity of the kinetic energy demands the matrix $f_{\alpha\beta}$ to be positive definite, while G-invariance demands that it also satisfies $f_{\gamma\beta}\,c^\gamma{}_{A\alpha} + f_{\alpha\gamma}\,c^\gamma{}_{A\beta} = 0$. Moreover, from equation (5.25) we realize that the matrices X_α furnish a representation $\mathbb{R}$ of the unbroken group H with representation matrices given by $(\mathcal{T}_A)_{\alpha\beta} = c_{\alpha A\beta}$, and in terms of these matrices G-invariance requires $[\mathcal{T}_A, f] = 0$, $\forall\ \mathcal{T}_A$. If the representation $\mathbb{R}$ is irreducible, then by Schur's lemma, $f_{\alpha\beta} = F^2\delta_{\alpha\beta}$. If $\mathbb{R}$ is reducible into n irreducible diagonal blocks, then $f_{\alpha\beta} = \mathrm{diag}\,(F_1^2\,\mathbb{1}, F_2^2\,\mathbb{1}, \dots, F_n^2\,\mathbb{1})$, for n independent constants, F_n^2. Therefore, the lowest-dimension operators in the most general low-energy Goldstone boson self-coupling Lagrangian is parametrizable in terms of these n constants.

Finally, if other fields, that we collectively denote by χ, also appear in the low-energy theory, we ensure G-invariance by assigning the transformation rule $\chi \to \gamma\,\chi$, where $\gamma = \gamma(\theta, g) \in H$ is the field-dependent H matrix which is defined through the non-linear realization, equations (5.11) and (5.12), and use covariant derivatives $\mathcal{D}_\mu\,\chi = \partial_\mu\chi - i\,\mathcal{A}^A_\mu\,T_A\,\chi$ as they also transform nicely under G, $\mathcal{D}_\mu\,\chi \to \gamma\,\mathcal{D}_\mu\,\chi$.

The final general Lagrangian is therefore given by $\mathscr{L}\left[e_\mu, \chi, \mathcal{D}_\mu\, e_\nu, \mathcal{D}_\mu\,\chi, \dots\right]$ where

G-invariance is guaranteed as long as it is globally H-invariant, meaning

$$\mathscr{L}\left[h\, e_\mu\, h^{-1}, h\, \chi, h\, \mathcal{D}_\mu e_\nu\, h^{-1}, h\, \mathcal{D}_\mu\, \chi, \ldots\right] = \mathscr{L}\left[e_\mu, \chi, \mathcal{D}_\mu e_\nu, \mathcal{D}_\mu\, \chi, \ldots\right]. \tag{5.30}$$

It is remarkable that such construction can be proven to be unique [162].

5.2 The General Case: Local (Gauge) Internal and Spacetime Symmetries

We would like to discuss now the general case of local gauge symmetries, and even more generally, spacetime symmetries. It is crucial to realize that unlike internal symmetries which may or may not be gauged and/or spontaneously broken, spacetime symmetries are always gauged by gravity and moreover, any conceivable physical system other than the vacuum is bound to break at least some of them. We will see that in order to couple gravity with the Goldstone fields that non-linearly realize the spontaneous breaking of space-time symmetries we need to weakly gauge the Poincaré group in the context of the coset construction.

5.2.1 Review of a Generalized Coset Construction

The coset construction was extended to the case of SSB of spacetime symmetries by Dmitri Volkov, Victor Ogievetsky and Evgeny Ivanov [164–166]. Several subtleties, some of them seeds of ongoing active research areas, arise in such a context: the number of Goldstone modes does not need to equal that of broken symmetries [166, 167]; Goldstone excitations do not need to be either massless nor stable [168–171]; UV completions may occur in non-standard ways [172]; the issue of superluminality may become devious [173–176]. The SSB of spacetime symmetries is not only an academic endeavor, as any state of matter other than the vacuum must break at least some of them. Consider the example of a state with a single point particle at rest. It certainly breaks boosts by selecting a preferred reference frame. In contrast, if said particle is charged under a $U(1)$ symmetry, the corresponding state is an eigenstate of the charge and does not break $U(1)$. The point is that even if we may consider states that spontaneously break any internal symmetry, we are not forced to do so. In some sense, we realize that while Nature has provided us with both global and gauge internal symmetries, there is no such ambiguity when it comes to spacetime symmetries, as they are gauged by gravity.

Consider the symmetry group G which now includes Poincaré as a subgroup and assume that the ground state spontaneously breaks it down to a subgroup H. We subdivide the generators of G into three groups:

$X_\alpha =$ broken generators, $\bar{P}_a =$ unbroken translations, $T_A =$other unbroken generators.

Both the X_α's and the T_A's do in general contain some spacetime and some internal generators. Even though the effective action for the Goldstone bosons must be invariant under the whole symmetry group G, the broken symmetries generated by the X_α's *and* the unbroken translations $\bar{P}_a$'s will be non-linearly realized on the Goldstone fields.

We define a "local parametrization" of the coset G/H_0, where H_0 is the subgroup of H generated by the T_A's, given by

$$\Omega(y, \pi) \equiv \exp\left(i\, y^a(x)\, \bar{P}_a\right) \exp\left(i\, \pi^\alpha(x)\, X_\alpha\right). \tag{5.31}$$

Ω can be thought of as the most general group element generated by the X_α's and the $\bar{P}_a$'s using coordinate-dependent parameters. The transformation properties of the Goldstones under a generic element $g \in G$ is derived from the relation

$$g\,\Omega(y,\pi) = \Omega(y',\pi')\,h(y,\pi,g), \tag{5.32}$$

where $h(y,\pi,g) \in H_0$ depends on the Goldstones and the coordinates in such a way that it guarantees that the "form" of Ω in (5.31) is preserved under the action of g. The Goldstones π will usually transform non-linearly, while the y's transform like cartesian coordinates under unbroken Poincaré transformations. Indeed, say g is an unbroken translation, meaning $g = e^{i\,\epsilon^a\,\bar{P}_a}$ for some constant parameters ϵ^a, then from (5.31) one can derive that $y'(x) = y(x) + \epsilon$, $\pi'(x) = \pi(x)$ and $h(y,\pi,g) = 1$. In a Minkowski background one can trivially take $y^a(x) \equiv x^a$ everywhere. On a curved background, however, the y's need to be thought of as locally inertial coordinates at some point within the patch described by the x coordinates, and such a trivial picking may not be possible. We now introduce the Maurer-Cartan (MC) one-form $\Omega^{-1}d\,\Omega$, whose components are calculated explicitly using only the Lie algebra $\mathfrak{g}$ associated with the Lie group G. By the group property, the MC form may be expressed a linear combination of all the generators

$$\Omega^{-1}\partial_\mu\Omega = E_\mu^a\left(\bar{P}_a + \nabla_a\pi^\alpha\,X_\alpha + A_a^B\,T_B\right). \tag{5.33}$$

Let us summarize the properties of the different objects appearing in the expression above:

- It can be shown [165] that the object E_μ^a plays the role of a vielbein as it defines a "volume form" (or integration measure) $d^dx\,\det E$, which is a scalar under all symmetries and is covariant under arbitrary x-coordinate transformations, ensuring the fact that the coset construction can be carried out in an arbitrary coordinate system. When all X_α's are internal and we take the x's to be Cartesian coordinates, the "coset vielbein" E_μ^a is trivial, i.e. $E^a_\mu = \delta^a_\mu$.

- On the other hand, the objects $\nabla_a \pi^\alpha$ should be thought of as covariant derivatives for the Goldstone fields, as they transform covariantly under all symmetries, meaning

$$\nabla_a \pi^\alpha(x) \xrightarrow{g} \nabla_a \pi'^\alpha(x) = h_a{}^b(y,\pi,g) h_\beta{}^\alpha(y,\pi,g) \nabla_b \pi^\beta(x), \tag{5.34}$$

where the $h_a{}^b$ and $h_\beta{}^\alpha$ matrices are some representations of the group element $h(y,\pi,g) \in H_0$. Thus, the covariant derivatives $\nabla_a \pi^\alpha$ transform according to a field-and coordinate-dependent representation of the unbroken subgroup H_0.

- Finally, the coefficients A^B_a transform like a connection and may be used to define covariant derivatives

$$\nabla^H_a \equiv E^\mu_a \, \partial_\mu + i \, A^B_a \, T_B, \tag{5.35}$$

acting on the Goldstone fields and additional matter fields transforming in some linear representation of the unbroken subgroup H_0. Here E^μ_a is defined through $E^\mu_a E^b_\mu = \delta^b_a$ and $E^\mu_a E^a_\nu = \delta^\mu_\nu$.

The most general G-invariant x-coordinates independent Lagrangian density $\mathscr{L}$ is then constructed by simply contracting all covariant derivatives, e.g. $\nabla_a \pi^\alpha, \nabla^H_a \nabla_b \pi^\alpha, \ldots$, that are manifestly H_0-invariant.

5.2.2 Gauging Some Symmetries: The General Case and GR as an Example

If a subgroup $G' \subseteq G$ with generators V_I is gauged, we simply replace the partial derivative in the definition of the MC form with a covariant derivative

$$\Omega^{-1}\partial_\mu\Omega \to \Omega^{-1}\mathscr{D}_\mu\Omega \equiv \Omega^{-1}\left(\partial_\mu + i\tilde{A}^I_\mu V_I\right)\Omega. \tag{5.36}$$

Such a modified MC form may also be decomposed as in equation (5.33).

$$\Omega \to g(x)\Omega, \quad \tilde{A}_\mu \to g(x)\tilde{A}_\mu g^{-1}(x) - ig(x)\partial_\mu g^{-1}(x), \quad \text{where} \quad g(x) \in G'. \tag{5.37}$$

Crucially, if the gauged generators V_I contain some of the broken generators X_α, we may always perform a gauge transformation and set to zero some of the Goldstones π^α, which amounts to be working in the unitary gauge.

We will use the procedure outlined in (5.36) to introduce dynamical gravity into systems of interest by gauging the Poincaré group. Actually, armed with all these tools, let us review how GR may be derived from a coset construction by gauging the Poincaré group $ISO(3,1)$ with non-linearly realized translations [177]. In other words, we are interested in the coset $G/H = ISO(3,1)/SO(3,1)$, which is parametrized by

$$\Omega = \exp\left(i\, y^a(x)\, P_a\right). \tag{5.38}$$

Now we introduce the gauge connections for translations $\tilde{e}^a_\mu$ and Lorentz transformations ω^{ab}_μ so that the MC form is given by

$$\Omega^{-1}\mathscr{D}_\mu\Omega \equiv e^{-i\,y^a(x)P_a}\left(\partial_\mu + i\,\tilde{e}^a_\mu P_a + \frac{i}{2}\,\omega^{ab}_\mu J_{ab}\right)e^{i\,y^a(x)P_a} = i\,e^a_\mu P_a + \frac{i}{2}\,\omega^{ab}_\mu J_{ab}\,, \tag{5.39}$$

where we have defined

$$e^a_\mu \equiv \tilde{e}^a_\mu + \partial_\mu y^a + \omega^{ab}_\mu\, y_b. \tag{5.40}$$

According to the previous discussion, the fields e^a_μ are a coset vielbein from which we may construct volume form $d^dx\,\det e$. Moreover, they also define the spacetime metric through the usual identity $g_{\mu\nu} = e^a_\mu\, e^b_\nu\,\eta_{ab}$, so e^a_μ is indeed the usual vielbein field of the tetrad formalism. Now we understand why the covariant derivative acting on matter fields such as fermions “only” carries the “spin connection” for the local Lorentz group, i.e.

$$\nabla^L_a \equiv e^\mu_a\left(\partial_\mu + \frac{i}{2}\,\omega^{bc}_\mu J_{bc}\right). \tag{5.41}$$

The vielbein and the spin connection are the only necessary building blocks to describe the non-linear realization of translations and the local action of the Poincaré group. If we do not consider additional matter fields, the most general Poincaré and diff-invariant theory is defined with an action of the form

$$S = \int d^4x\,\det e\,\mathscr{L}\left[\nabla^L_a\right], \tag{5.42}$$

where, as always, indices of covariant derivatives must appear in suitable contracted ways. We now proceed as usual, recalling that, in analogy with gauge theories, the “field strengths” from which we define gauge invariant kinetic operators for the gauge fields are defined through the commutators of covariant derivatives, which acting on a test vector V^a gives

$$\left[\nabla^L_a, \nabla^L_b\right] V^c = \mathscr{R}^c{}_{dab} V^d - \mathcal{T}_{ab}{}^d \nabla^L_d V^c, \tag{5.43}$$

where $\mathscr{R}^c{}_{dab} = \mathscr{R}^c{}_{dab}(\omega)$ and $\mathcal{T}_{ab}{}^d = \mathcal{T}_{ab}{}^d(e,\omega)$ are the components of the usual Riemann curvature and torsion tensors, respectively. Finally, at lowest order in the derivative expansion the effective action reads

$$\begin{aligned} S =& \frac{1}{16\pi G} \int d^4x\, e \Big\{ \mathscr{R} + b_1 \mathscr{R}^2 + b_2\, \mathscr{R}_{\mu\nu} \mathscr{R}^{\mu\nu} + \dots \\ &+ c_1 \mathcal{T}_{ab}{}^c \mathcal{T}^{ab}{}_c + c_2 \mathcal{T}_{abc} \mathcal{T}^{acb} + c_3 \mathcal{T}_{ab}{}^b \mathcal{T}^{ac}{}_c + \dots \Big\}, \end{aligned} \tag{5.44}$$

where $\{G, b_1, b_2, \dots, c_1, c_2, c_3, \dots\}$ are constants, while the ellipses in (5.44) stand for higher-order terms in the derivative expansion [4].

It may be argued that the action in (5.44) is not GR; e.g., it does not represent the degrees of freedom of gravity, and the Ricci scalar operator is a function of the connection only. However, the equations of motion for ω^{ab}_μ to lowest order in derivatives are such that the solution is trivially $\omega = \omega(e)$, meaning ω is the torsion-free "Levi-Civita" connection compatible with metricity. Moreover, even in the presence of additional matter fields, the equation of motion for the spin connection may still be solved algebraically at lowest order in the derivative expansion, and even if the solution differs from the naive Levi-Civita combination, upon plugging such solution into the effective action we would obtain a torsion-free theory with shifted coefficients in the matter effective action. Therefore, in this context, treating the spin connection ω as an independent variable is anyway equivalent to imposing the torsion-free condition. One may also take the pragmatical view-point that it is simply consistent with all the symmetries to enforce

[4]There is no need of a $\mathscr{R}_{\mu\nu\rho\sigma}\mathscr{R}^{\mu\nu\rho\sigma}$ operator due to the Chern-Gauss-Bonnet identity in 4-d.

the torsion tensor to be zero, $\mathcal{T}_{ab}{}^{c} = 0$, and the solution to this "constraint" is that the connection is the Levi-Civita one. Actually, there is people that do the opposite consistent picking of setting the good old Riemann tensor to zero, $\mathscr{R}^{a}{}_{bcd} = 0$, obtaining so-called "teleparallel theories of gravity" [178, 179], but we will not do such heresy [5] here. Assuming then that the torsion-free condition holds, (5.44) collapses to the good old Einstein-Hilbert action at lowest order in derivatives, and comparison with experiments would reveal that G is nothing but Isaac Newton's gravitational constant G_{N}.

Before we move on and use the coset construction to study some relevant examples involving the SSB of spacetime symmetries, we need to introduce the concept of so-called "inverse Higgs contraints" which arise as a consequence of the possible mismatch between the number of broken generators and the actual number of Goldstone degrees of freedom belonging to the spectrum of the theory.

5.2.3 Inverse Higgs Constraints

We have naively assigned one Goldstone field to each broken symmetry generator X_α. However, it is well known that when spacetime symmetries are spontaneously broken, the usual Goldstone theorem does not apply, meaning that the number of Goldstones denoted by $\#$ is not given by the dimension of the coset space, i.e. $\# \neq \dim(G/H_0)$ in the general case. The possible mismatch follows from what is known as the inverse Higgs mechanism [166].

We may summarize the inverse Higgs mechanism as follows:

[5] All jokes aside, there is good compelling "bottom-up" arguments involving Stueckelberg tricks and the like that, roughly speaking, show that "...the Lagrangian for general relativity is given uniquely as the only Lagrangian that can couple a massless spin 2 particle to matter..." (see section 8.7 - "Higher-spin fields" of chapter 8 - "Spin 1 and gauge invariance" in Schwartz's book [100]).

If

1. the commutator between an unbroken translation $\bar{P}$ and a broken generator X contains another broken generator X', in other words $\left[\bar{P}, X\right] \supset X'$, and

2. X and X' do not belong to the same multiplet under H_0,

then we may set to zero the covariant derivative of the Goldstone of X' in the direction of $\bar{P}$, denoted as $\bar{\nabla}\pi'$, an solve such a condition, which is known as an inverse Higgs constraint (IHC), to eliminate the Goldstones of X from the low-energy spectrum of excitations in a way that is consistent with all the symmetries. We may understand, from a UV perspective, that when provided with an explicit order parameter Φ, it may happen that the Goldstones associated with the broken generators do not describe independent degrees of freedom. Namely, it may occur that

$$\left(\pi X + \pi' X'\right) \langle \Phi \rangle = 0, \tag{5.45}$$

where $\langle \Phi \rangle$ is the VEV of the order parameter [167]. Imposing the IHC is then, in some way or another, analogous to the action of "fixing a gauge" which effectively eliminates redundant degrees of freedom [171]. However it has been understood that such an interpretation is not always quite accurate as these additional degrees of freedom need not be redundant in the general case [172]. It seems that the state of the art interpretation is that there is not really an overcounting of degrees of freedom going on, and the IHC may arise in a dynamical way in the low-energy limit. More to the point, the covariant derivative of the Goldstone of X' contains a term linear in π and with no derivatives, implying that a generic action contains a mass term for π, i.e. π is gapped, receiving the inventive name "gapped Goldstone". At energies below this gap we may integrate out π and obtain an EFT for the remaining Goldstones. The IHC may be interpreted as coming from the equation of motion for π, even though such an equation of motion may be much more complicated that the condition $\bar{\nabla}\pi' = 0$ [166]. The EFT does not care about such subtleties though; once the derivative expansion is correctly implemented, the difference between imposing some kind of generalized IHC or just the simplest version, which we have discussed, amounts to redefinitions of the coupling constants of the EFT. Hence, there is no loss of generality in working with the simplest possible IHC. Let us now scrutinize a couple of relevant examples exhibiting SSB of spacetime symmetries.

5.2.4 Some Relevant Examples

5.2.4.1 Membranes

The effective action for a $(d-1)$-brane embedded in $(d+1)$-dimensions was derived by Raman Sundrum in [180]. Let us now use the coset construction to rederive it [163]. We will use notation that has become standard in the literature on extra-dimensions:

- $A, B, C, D, \dots$ and $M, N, P, Q, \dots$ denote **Lorentz** and **spacetime** indices in $d+1$ dimensions, respectively.
- $\alpha, \beta, \gamma, \delta, \dots$ and $\mu, \nu, \rho, \sigma, \dots$ denote **Lorentz** and **spacetime** indices in d dimensions, respectively.

The symmetry breaking pattern associated with a $(d-1)$-brane in $(d+1)$-dimensions is determined by the following classification of the Poincaré generators:

$$\text{Unbroken} = \begin{cases} P_\alpha \\ J_{\alpha\beta} \end{cases}, \quad \text{Broken} = \begin{cases} P_d \\ J_{\alpha d} \end{cases}, \tag{5.46}$$

where the subscript $_d$ denotes the spatial direction which is broken by the brane. The coset element reads

$$\Omega = e^{i\, y^\alpha(x) P_\alpha} e^{i\, \pi(x) P_d} e^{i\, \xi^\alpha(x) J_{\alpha d}} \equiv e^{i\, Y^A(x) P_A} e^{i\, \xi^\alpha(x) J_{\alpha d}}, \tag{5.47}$$

where $Y^A(x) \equiv (y^\alpha(x), \pi(x))$.

The MC form of the full spacetime is then given by

$$\Omega^{-1} \mathscr{D}_M\, \Omega \equiv \Omega^{-1} \left(\partial_M + i\, \tilde{e}_M^A P_A + \frac{i}{2}\, \omega_M^{AB} J_{AB} \right) \Omega, \tag{5.48}$$

where, as previously discussed, we are gauging the Poincaré algebra to include dynamical gravity. Using the Poincaré algebra one can show that

$$\begin{aligned} \Omega^{-1} \mathscr{D}_M\, \Omega &= e^{-i\, \xi^\alpha J_{\alpha d}} \left(\partial_M + i\, e_M^A P_A + \frac{i}{2}\, \omega_M^{AB} J_{AB} \right) e^{i\, \xi^\alpha J_{\alpha d}}, \\ &= i\, e_M^A\, \Lambda_A{}^B\, P_B + \frac{i}{2} \left\{ \left(\Lambda^{-1} \partial_M \Lambda \right)^{AB} + \omega_M^{CD} \Lambda_C{}^A \Lambda_D{}^B \right\} J_{AB}, \end{aligned} \tag{5.49}$$

where

$$e_M^A \equiv \tilde{e}_M^A + \partial_M Y^A + \omega_M^{AB}\, Y_B, \tag{5.50}$$

as in (5.40), and

$$\Lambda^A{}_B\, (\xi) \equiv (e^{i\, \xi^\alpha J_{\alpha d}})^A{}_B. \tag{5.51}$$

It is crucial to realize that the position of the brane in the local Lorentz frame $Y^A(x)$ and in the "global" spacetime $Y^M(x)$ need to be differentiated when dealing with curved spacetime. The "induced" (or "projected") MC form, which is the relevant object of study for this system, is then given by

$$\Omega^{-1}\mathscr{D}_\mu\,\Omega \equiv \partial_\mu Y^M\,\Omega^{-1}\mathscr{D}_M\,\Omega \equiv iE^\alpha_\mu\left(P_\alpha + \nabla_\alpha\pi\,P_d + \nabla_\alpha\xi^\beta J_{\beta d} + \frac{1}{2}J_{\beta\gamma}A^{\beta\gamma}_\alpha\right), \tag{5.52}$$

and so we find, using (5.49), that

$$E^\alpha_\mu = \partial_\mu Y^M e^A_M\,\Lambda_A{}^\alpha, \tag{5.53}$$

$$\nabla_\alpha\pi = E^\mu_\alpha\,\partial_\mu Y^M e^A_M\,\Lambda_A{}^d, \tag{5.54}$$

$$\nabla_\alpha\xi^\beta = E^\mu_\alpha\,(\Lambda^{-1})^\beta{}_C\left(\eta^{CD}\partial_\mu + \partial_\mu Y^M\omega^{CD}_M\right)\Lambda_D{}^d, \tag{5.55}$$

$$A^{\beta\gamma}_\alpha = E^\mu_\alpha\,(\Lambda^{-1})^\beta{}_C\left(\eta^{CD}\partial_\mu + \partial_\mu Y^M\omega^{CD}_M\right)\Lambda_D{}^\gamma, \tag{5.56}$$

where E^μ_α is the inverse of E^α_μ such that $E^\alpha_\mu E^\nu_\alpha = \delta^\nu_\mu$ and $E^\alpha_\mu E^\mu_\beta = \delta^\alpha_\beta$. Now we notice that

$$\left[P_\alpha, J_{\beta d}\right] = -iP_d\,\eta_{\alpha\beta}, \tag{5.57}$$

so there is redundancy within the Goldstone modes and a IHC must be imposed. The simplest IHC that one comes up with is

$$\nabla_\alpha\pi = E^\mu_\alpha\,\partial_\mu Y^M e^A_M\,\Lambda_A{}^d \equiv E^\mu_\alpha\,\partial_\mu Y^M e^A_M\,n_A = 0 \longrightarrow \partial_\mu Y^M e^A_M\,n_A = 0, \tag{5.58}$$

where we have defined $n_A \equiv \Lambda_A{}^d(\xi)$, the "unit vector" perpendicular to the brane in the local Lorentz frame [6]. Now solving the IHC allows us to express the Goldstones ξ^β in terms of derivatives of π, so the covariant derivative $\nabla_\alpha\xi^\beta$ enters the action only at higher order in the derivative expansion.

[6]Since $\Lambda_A{}^B$ is a Lorentz transformation it satisfies $(\Lambda^{-1})^A{}_C\,\eta^{CD}\Lambda_D{}^B = \eta^{AB}$ so that $\Lambda_A{}^d\Lambda^{Ad} = \eta^{dd} = 1$, i.e., $\Lambda_A{}^d$ indeed has unit norm.

At lowest order in derivatives, the effective action for the brane is given by

$$
\begin{aligned}
S &= -T\int d^dx\ \det E = -T\int d^dx\sqrt{-\det\left(EE^T\right)\det\left(\eta\right)} \\
&= -T\int d^dx\sqrt{-\det\left(\partial_\mu Y^M e^A_M\,\Lambda_A{}^\alpha\partial_\nu Y^N e^B_N\,\Lambda_{B\alpha}\right)} = -T\int d^dx\sqrt{-\det h_{\mu\nu}},
\end{aligned}
\tag{5.59}
$$

where T is the "brane tension" with physical units of $\left[\frac{\text{energy}}{\text{area}}\right]$ and we have used the fact that, as long as (5.58) holds,

$$
\begin{aligned}
\partial_\mu Y^M e^A_M\,\Lambda_A{}^\alpha\partial_\nu Y^N e^B_N\,\Lambda_{B\alpha} &\overset{\text{IHC}}{=} \partial_\mu Y^M e^A_M\,\Lambda_A{}^C\partial_\nu Y^N e^B_N\,\Lambda_{BC}, \\
&= \partial_\mu Y^M e^A_M\,\partial_\nu Y^N e^B_N\,\Lambda_A{}^D\,\eta_{DC}\left(\Lambda^{-1}\right)^C{}_B \\
&= e^A_M e^B_N\,\eta_{AB}\,\partial_\mu Y^M\partial_\nu Y^N \equiv g_{MN}\,\partial_\mu Y^M\partial_\nu Y^N \equiv h_{\mu\nu},
\end{aligned}
\tag{5.60}
$$

where we have introduced the so-called induced metric $h_{\mu\nu}$. Moreover, defining the covariant derivative

$$
\left[\mathbb{D}_\mu\right]^{CD} \equiv \eta^{CD}\partial_\mu + \partial_\mu Y^M\omega_M^{CD}, \tag{5.61}
$$

equation (5.55) reads

$$
\nabla_\alpha\xi_\beta = E^\mu_\alpha\left(\Lambda^{-1}\right)_\beta{}^C\left[\mathbb{D}_\mu\right]_C{}^D\,n_D. \tag{5.62}
$$

Using (5.53) and the IHC it is easy to verify that ${(\Lambda^{-1})_\alpha}^A = E^\mu_\alpha \, \partial_\mu Y^M e^A_M$, and therefore

$$\begin{aligned}
\nabla_\alpha \xi_\beta &= E^\mu_\alpha \, E^\nu_\beta \, \partial_\nu Y^M e^C_M \, {[\mathbb{D}_\mu]_C}^D \, n_D \\
&= E^\mu_\alpha \, E^\nu_\beta \, \partial_\mu Y^M e^A_M \, \partial_\nu Y^N e^B_N \, {[\mathbb{D}_A]_B}^C \, n_C \\
&= E^\mu_\alpha \, E^\nu_\beta \, \partial_\mu Y^M e^A_M \, \partial_\nu Y^N e^B_N \, \nabla_A \, n_B \\
&\equiv E^\mu_\alpha \, E^\nu_\beta \, K_{\mu\nu} \,,
\end{aligned} \tag{5.63}$$

where we have made the identifications ${[\mathbb{D}_A]_B}^C \, n_C = \nabla_A \, n_B$ and

$$K_{\mu\nu} \equiv \partial_\mu Y^M e^A_M \, \partial_\nu Y^N e^B_N \, \nabla_A \, n_B . \tag{5.64}$$

We thus effectively see that the higher derivative covariant objects $\nabla_\alpha \xi_\beta$ are proportional to the extrinsic curvature $K_{\mu\nu}$.

Finally, the covariant derivative of matter fields living on the brane is of the form

$$\begin{aligned}
\nabla_\alpha \psi &= E^\mu_\alpha \left(\partial_\mu \psi + \frac{i}{2} \left\{ {(\Lambda^{-1})^\beta}_C \left(\eta^{CD} \partial_\mu + \partial_\mu Y^M \omega^{CD}_M \right) {\Lambda_D}^\gamma \right\} J_{\beta\gamma} \, \psi \right) \\
&= E^\mu_\alpha \left(\partial_\mu \psi + \frac{i}{2} \left\{ {(\Lambda^{-1})^\beta}_C \, [\mathbb{D}_\mu]^{CD} \, {\Lambda_D}^\gamma \right\} J_{\beta\gamma} \, \psi \right) \\
&= E^\mu_\alpha \, \partial_\mu \psi + \frac{i}{2} \, A^{\beta\gamma}_\alpha J_{\beta\gamma} \, \psi .
\end{aligned} \tag{5.65}$$

It is possible to show that the combination $E^\alpha_\mu A^{\beta\gamma}_\alpha \equiv \varpi^{\beta\gamma}_\mu$ is indeed the spin connection associated with the induced metric.

To gain some insight within the construction we have just gone through, and for simplicity, let us consider the gravityless case, which amounts to setting $\tilde{e}$ and ω to zero, effectively "undoing" the gauging of the Poincaré symmetry. In such a limit, $g_{MN} \to \eta_{AB}$, and by picking $y^\alpha(x) = x^\alpha$, $\partial_\mu Y^M \to \partial_\mu Y^A = \left(\delta^\alpha_\mu, \partial_\mu \pi\right)$, so that (5.60) yields

$$h_{\mu\nu} = \eta_{AB}\partial_\mu Y^A \partial_\nu Y^B = \eta_{\mu\nu} + \partial_\mu \pi \, \partial_\nu \pi, \tag{5.66}$$

which is a familiar result. Moreover, let us recall that the IHC implies that ${\Lambda_B}^d$ is a unit normal to the brane. For a codimension-1 brane there is only one unit vector n_B which is perpendicular to all the $\partial_\mu Y^A$, namely $n_B \sim \epsilon_{A_1 \dots A_d B}\, \epsilon^{\mu_1 \dots \mu_d}\, \partial_{\mu_1} Y^{A_1} \dots \partial_{\mu_d} Y^{A_d} = \delta^d_B - \delta^\mu_B \partial_\mu \pi$. Therefore, upon normalization one finds that

$${\Lambda_A}^d(\xi) \equiv n_A = \frac{\delta^d_A - \delta^\mu_A \partial_\mu \pi}{\sqrt{1 + (\partial\pi)^2}}. \tag{5.67}$$

With such a normal, the extrinsic curvature defined in (5.64) is then given by

$$K_{\mu\nu} = \partial_\mu Y^A \partial_\nu Y^B \partial_A n_B = -\frac{\partial_\mu \partial_\nu \pi}{\sqrt{1 + (\partial\pi)^2}} + \dots, \tag{5.68}$$

which is clearly of higher order in the derivative expansion compared to the induced metric.

Let us now consider another relevant example of a system that shows a peculiar pattern of SSB of spacetime and internal symmetries: the perfect superfluid.

5.2.4.2 Perfect Superfluid

A zero-temperature perfect superfluid is a system with a finite density of a spontaneously broken $U(1)$ charge Q which embodies a very interesting example of the interplay between spontaneously broken internal and spacetime symmetries. The low-energy description of a perfect superfluid was worked out by Dam Thanh Son in [181]. The gravityless coset construction derivation was performed in [182], while the case with non-trivial gravitational interactions was introduced in [163].

In short, the ground state of a perfect superfluid breaks local boosts, time translations and the global $U(1)$ symmetry, but remains invariant under the action of a "diagonal" combination [7]

$$\bar{P}_0 \equiv P_0 + \mu\, Q, \tag{5.69}$$

where μ is the so-called chemical potential [181, 183]. To proceed, let us first set some notation:

- 4d and 3d **Lorentz** indices will be denoted by $a, b, \ldots$ and $m, n, \ldots$, respectively.
- 4d and 3d **spacetime** indices will be denoted by $\mu, \nu, \ldots$ and $i, j, \ldots$, respectively.
- The time-like **Lorentz** and **spacetime** index will be denoted by 0 and t, respectively.

The symmetry breaking pattern of the superfluid is then given by

$$\text{Unbroken} = \begin{cases} \bar{P}_0 \equiv P_0 + \mu\, Q & \text{(time translations)} \\ \bar{P}_m \equiv P_m & \text{(spatial translations)} \\ J_{mn} & \text{(spatial rotations)} \end{cases} \tag{5.70}$$

$$\text{Broken} = \begin{cases} K_m \equiv J_{0m} & \text{(boosts)} \\ Q & \text{(shift symmetry)} \end{cases} \tag{5.71}$$

[7] The situation when a state spontaneously breaks both an internal symmetry and time-translation invariance in such a way that time-evolution moves the system along the symmetry direction has been dubbed by Alberto Nicolis and Federico Piazza the "spontaneous symmetry probing" (SSP) scenario [183].

For such a symmetry breaking pattern the coset element reads

$$\Omega = e^{i\,y^a(x)\bar{P}_a} e^{i\,\pi(x)Q} e^{i\,\eta^m(x)K_m}. \tag{5.72}$$

Consequently, the MC form is given by

$$\begin{aligned}
\Omega^{-1}\mathscr{D}_\mu\Omega &\equiv e^{-i\,\eta^m(x)K_m} e^{-i\,\pi(x)Q} e^{-i\,y^a(x)\bar{P}_a}\left(\partial_\mu + i\,\tilde{e}^a_\mu P_a + \frac{i}{2}\,\omega^{ab}_\mu J_{ab}\right) e^{i\,y^a(x)\bar{P}_a} e^{i\,\pi(x)Q} e^{i\,\eta^m(x)K_m} \\
&= i\,e^a_\mu\,\Lambda_a{}^b\bar{P}_b + i\left(\partial_\mu\psi - \mu\,e^a_\mu\,\Lambda_a{}^0\right)Q + \frac{i}{2}\,J_{ab}\left\{\left(\Lambda^{-1}\partial_\mu\Lambda\right)^{ab} + \omega^{cd}_\mu\Lambda_c{}^a\Lambda_d{}^b\right\} \\
&\equiv i E^a_\mu\left(\bar{P}_a + \nabla_a\pi\,Q + \nabla_a\eta^m K_m + \frac{1}{2}\,A^{mn}_a J_{mn}\right),
\end{aligned} \tag{5.73}$$

where in order to get the second line we have used the Poincaré algebra plus the fact that Q belongs to the "center" of the Lie algebra $\mathfrak{g}$, i.e. $[Q,\mathscr{A}] = 0\ \forall\ \mathscr{A} \in \mathfrak{g}$, the third line is a definition, and we have introduced

$$e^a_\mu \equiv \tilde{e}^a_\mu + \partial_\mu y^a + \omega^{ab}_\mu\,y_b, \tag{5.74}$$

$$\psi \equiv \mu\,y^0 + \pi, \tag{5.75}$$

and

$$\Lambda^a{}_b(\eta) \equiv \left(e^{i\eta^m K_m}\right)^a{}_b. \tag{5.76}$$

Comparing the second and third lines in (5.73) we arrive to the following expressions

$$E^a_\mu = e^b_\mu\,\Lambda_b{}^a, \tag{5.77}$$

$$\nabla_a\pi = e^\mu_b\,\Lambda^b{}_a\partial_\mu\psi - \delta^0_a\,\mu, \tag{5.78}$$

$$\nabla_a\eta^m = e^\mu_b\,\Lambda^b{}_a\left\{\left(\Lambda^{-1}\partial_\mu\Lambda\right)^{0m} + \omega^{cd}_\mu\Lambda_c{}^0\Lambda_d{}^m\right\}, \tag{5.79}$$

$$A^{mn}_a = e^\mu_b\,\Lambda^b{}_a\left\{\left(\Lambda^{-1}\partial_\mu\Lambda\right)^{mn} + \omega^{cd}_\mu\Lambda_c{}^m\Lambda_d{}^n\right\}. \tag{5.80}$$

Now we notice that

$$\left[\bar{P}_m, K_n\right] = \left[P_m, K_n\right] = i P_0\, \delta_{mn} = i\left(\bar{P}_0 - \mu\, Q\right) \delta_{mn} \Rightarrow \left[P_m, K_n\right] \supset Q, \tag{5.81}$$

so there is redundancy within the Goldstone modes and an IHC may be imposed. The simplest IHC that one comes up with is

$$\nabla_m \pi = e^\mu_b\, \Lambda^b{}_m\, \partial_\mu \psi = \eta_{ab}\, \Lambda^a{}_m\, e^b_\mu\, \partial^\mu \psi = 0. \tag{5.82}$$

As is the case in many other examples, including the membrane case discussed in 5.2.4.1, this IHC may be understood as a statement of orthogonality. Indeed, the very definition of Lorentz transformations $\eta_{ab}\, \Lambda^a{}_c\, \Lambda^b{}_d = \eta_{cd}$ implies that there are three orthonormal vectors Λ_m satisfying $\Lambda_m \cdot \Lambda_n \equiv \eta_{ab}\, \Lambda^a{}_m\, \Lambda^b{}_n = \delta_{mn}$, which in turn by acknowledging (5.82), are orthogonal to the direction $e^\mu_b\, \partial_\mu \psi$, naturally leading us to the definition of a time-like unit normal vector given by

$$n^a \equiv -\frac{e^a_\mu \partial^\mu \psi}{\sqrt{-\partial_\nu \psi \partial^\nu \psi}}. \tag{5.83}$$

Let us now introduce the velocity vector

$$\beta_m \equiv \frac{\eta_m}{\eta} \tanh \eta, \tag{5.84}$$

where $\eta \equiv \sqrt{\eta^m \eta^n \delta_{mn}}$ is the so-called "rapidity angle" in the relativistic jargon. With such a parametrization the Lorentz factor is given by

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} = -\frac{e^0_\mu\, \partial^\mu \psi}{\sqrt{-\partial_\nu \psi \partial^\nu \psi}}. \tag{5.85}$$

Moreover, since high-school we are supposed to know that a boost matrix may then be decomposed as

$$\Lambda^{0}{}_{0} = \gamma, \quad \Lambda^{0}{}_{m} = \gamma\,\beta_m, \quad \Lambda^{m}{}_{0} = \gamma\,\beta^m, \quad \Lambda^{m}{}_{n} = \delta^m_n + (\gamma - 1)\,\frac{\beta^m \beta_n}{\beta^2}. \tag{5.86}$$

Consequently one can show that (5.82) implies that

$$\beta_m \approx -\frac{e^{\mu}_{m}\partial_\mu \psi}{e^{\nu}_{0}\partial_\nu \psi}, \tag{5.87}$$

which together with (5.85) amount to the conclusion that

$$n^a = \Lambda^{a}{}_{0}, \quad n \cdot \Lambda_m = n_a\,\Lambda^{a}{}_{m} = 0, \quad \text{and} \quad \Lambda_m \cdot \Lambda_n \equiv \eta_{ab}\,\Lambda^{a}{}_{m}\,\Lambda^{b}{}_{n} = \delta_{mn}. \tag{5.88}$$

This solution of the IHC allows us to isolate the building block of the EFT for the relevant degree of freedom π,

$$\nabla_0 \pi = e^{\mu}_{b}\,\Lambda^{b}{}_{0}\,\partial_\mu \psi - \mu = \sqrt{-g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi} - \mu, \tag{5.89}$$

where the usual identification of the spacetime metric $g^{\mu\nu} = e^{\mu}_{a}\,e^{\nu}_{b}\,\eta^{ab}$ has been used. The coset vielbein determinant $\det E = \det\sqrt{-g}$ defines an invariant measure to build the effective action, which then reads

$$S = \int d^4x \det E\left\{a_0 + a_1\nabla_0\pi + a_2\left(\nabla_0\pi\right)^2 + \ldots\right\} = \int d^4x\sqrt{-g}\,\mathbb{F}\left(\sqrt{-g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi}\right), \tag{5.90}$$

where $\mathbb{F}$ is a function such that $\mathbb{F}^{(n)}(\mu) = a_n$.

One of the main advantages of going through the coset construction relies on the fact that it yields a systematic derivative expansion. In particular, the first higher derivative corrections to the low-energy EFT given by (5.90) are $\nabla_0\nabla_0\pi$ and $\nabla_i\eta^i$.

It can be shown [163] that such operators may be written in terms of ψ and its derivatives,

$$\nabla_0\nabla_0\pi = \frac{\partial_\mu\psi\partial^\mu\partial_\rho\psi\partial^\rho\psi}{2\partial_\lambda\psi\partial^\lambda\psi}, \quad \nabla_i\eta^i = -\frac{\left(\partial_\rho\psi\partial^\rho\psi\Box\psi + \frac{1}{2}\partial_\mu\partial_\rho\psi\partial^\rho\psi\partial^\mu\psi\right)}{\left(-\partial_\lambda\psi\partial^\lambda\psi\right)^{3/2}}, \tag{5.91}$$

and these expressions are indeed just particular linear combinations of the expected additional higher derivative terms discussed in [181].

After this enlightening review of relevant examples of the coset construction in the presence of gravity, let us come back to our main interest, which is cosmic inflation.

5.3 Coset Construction of the EFT of Inflation

Relativistic fluids coupled to gravity have been used in the past to generate inflation [109] or to modify the large distance behavior of gravitational interactions [184]. In reference [163] the interest of the authors in perfect superfluids stems from the fact that they may be the simplest systems in which a combination of spacetime and internal symmetries is broken down to a diagonal subgroup. Very recently, in the context of addressing the inclusion of light particles with spin during inflation, the authors of [63] have argued that as the induced foliation breaks the local Lorentz symmetry down to the rotation subgroup, it is natural to use the CCWZ approach. Moreover, they claim that the coset construction of single-clock inflation in the presence of an "approximate" [8] shift symmetry of the inflaton coincides exactly with the gravitating perfect superfluid introduced in [163]. We independently worked out the exact mapping between these two

[8] The right wording, in our opinion, should be "... in the presence of an **exact** spontaneously broken shift symmetry...", because in the coset construction, as we know it, there is no room for approximate symmetries, but this is not what they say.

seemingly different systems before [63] hit the arXiv [9], and we shall go through it, for completeness, and to motivate current ongoing research [64].

Our job is to make the connection between (5.90) and the EFT of inflation. To do so let us notice that if we set $y^0(x) = t$ and $\mu = 1$ [10], we get that

$$\nabla_0 \pi = \sqrt{-g^{\mu\nu}\partial_\mu(t+\pi)\partial_\nu(t+\pi)} - 1 \equiv \sqrt{-\tilde{g}^{tt}(\tilde{x}(x))} - 1, \tag{5.93}$$

where the tilde is associated with an active diffeomorphism $t \to \tilde{t} = t - \tilde{\pi}(\tilde{x})$, so we see that our building block is already "Stueckelberged". In the decoupling limit, where we neglect metric fluctuations, $\sqrt{-g^{\mu\nu}\partial_\mu(t+\pi)\partial_\nu(t+\pi)} \to \sqrt{1+2\dot{\pi}-(\partial\pi)^2}$, and therefore the action will generically contain a "cosmological constant" and a linear term in π that should be absent after demanding tadpole cancellation, which is equivalent to imposing the correct background evolution. Thus, we find that the g^{tt} dependence in the usual EFT of inflation is accounted for by the building block $\nabla_0\pi$ within the coset construction.

What about the extrinsic curvature?

We observe that the "tensor" part of (5.79) can be casted as

$$\nabla_n \eta_m = E^\mu_n E^\nu_m e^b_\nu \left[\mathcal{D}_\mu\right]_b{}^c n_c, \tag{5.94}$$

[9] It is not fair to say that they "scooped" us, as we were informed, through private communication, that they were working on this before already.

[10] Note that, in this context, μ is expected to be the time-derivative of an "order parameter" of the SSB of the shift symmetry [181, 183], implying the identification $\mu = \dot{\phi}_0(t) =$ constant. Therefore, using a trivial field redefinition, it is easy to see that we may always set $\mu = 1$. If we do not identify μ in this way, we may still get a expression similar to (5.93) by going to unitary gauge, namely

$$\nabla_0\pi = \mu\left(\sqrt{-g^{tt}} - 1\right), \tag{5.92}$$

which, up to a sign (clearly a typo), coincides with equation (15) in [63].

where $[\mathcal{D}_\mu]^{bc} \equiv \eta^{bc}\partial_\mu + \omega_\mu^{bc}$ and $n_a \equiv -\Lambda_a{}^0 = (-\gamma, \gamma\,\beta_m)$.

To prove that (5.94) corresponds to the extrinsic curvature, which is usually defined as

$$K_{\mu\nu} \equiv h_\mu{}^\rho \nabla_\rho n_\nu = h_\mu{}^\rho h_\nu{}^\sigma \nabla_\rho n_\sigma, \tag{5.95}$$

where $h_{\mu\nu}$ is the induced metric and $n_\mu = -\frac{\partial_\mu \phi}{\sqrt{-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}}$ is a normal to the foliation, we proceed as follows. Using the fact that by definition $e_c^\mu = E_a^\mu\,(\Lambda^{-1})^a{}_c$ and η^{ab} is invariant under boosts we find that

$$g^{\mu\nu} = e_c^\mu\,e_d^\nu\,\eta^{cd} = E_a^\mu\,(\Lambda^{-1})^a{}_c\,E_b^\nu\,(\Lambda^{-1})^b{}_d\,\eta^{cd} = E_a^\mu E_b^\nu\,(\Lambda^{-1})^a{}_c\,\eta^{cd}\Lambda_d{}^b = E_a^\mu E_b^\nu\,\eta^{ab}. \tag{5.96}$$

Therefore

$$g^{\mu\nu} = -E_0^\mu E_0^\nu + E_m^\mu E_n^\nu\,\delta^{mn} = -n^\mu n^\nu + E_m^\mu E_n^\nu\,\delta^{mn}, \tag{5.97}$$

where we have identified $E_0^\mu = e_b^\mu\,\Lambda^b{}_0 = e_b^\mu\,n^b \equiv n^\mu$, and consequently the definition

$$h^{\mu\nu} \equiv E_m^\mu E_n^\nu\,\delta^{mn} \tag{5.98}$$

is quite natural. With this in mind, the correspondence between $K_{\mu\nu}$ as given by (5.95) and $\nabla_n\eta_m$ is now complete as it is straightforward to show that (5.94) can be rewritten as

$$\begin{aligned}\nabla_m \eta_n &= E_m^\rho E_n^\sigma h_\rho{}^\mu h_\sigma{}^\nu e_\nu^b \left[\mathcal{D}_\mu\right]_b{}^c n_c \\ &= E_m^\rho E_n^\sigma h_\rho{}^\mu h_\sigma{}^\nu \nabla_\mu n_\nu \\ &\equiv E_m^\rho E_n^\sigma K_{\rho\sigma} ,\end{aligned} \tag{5.99}$$

where we have made the identification $e_\nu^b \left[\mathcal{D}_\mu\right]_b{}^c n_c = \nabla_\mu n_\nu$, a geometric relation that clearly holds on a torsionless spacetime, and we have used the definition in (5.95).

What about $\nabla_0 \eta_m$?

It can be shown that the "vector" part of (5.79) can be written as

$$\nabla_0 \eta_m = E_0^\mu E_m^\nu e_\nu^b \left[\mathcal{D}_\mu\right]_b{}^c n_c = n^\mu E_m^\nu e_\nu^b \left[\mathcal{D}_\mu\right]_b{}^c n_c = E_m^\nu n^\mu \nabla_\mu n_\nu . \tag{5.100}$$

The structure above indeed contains the so-called "acceleration" vector

$$A^\mu \equiv n^\nu \nabla_\nu n^\mu , \tag{5.101}$$

which obviously satisfies $A^\mu n_\mu = 0$, implying it really corresponds to a 3-vector living in the hypersurface whose normal is n^μ. Noncrucially, this building block was not considered in the original EFT of inflation [40] but its effects have been discussed subsequently (see, e.g., [185, 186]). This exhausts the list of building blocks for the EFT that we obtain from the coset construction [11]. If additional matter fields were to be included, the algorithm should be familiar by now; they must enter the action in suitable $SO(3)$-invariant combinations using the connection defined in (5.80).

Several comments are in order:

- As of today it seems that nobody knows how to proceed with the coset construction of systems which undergo SSB of *time*-translations with *no* diagonal unbroken time-translation generator. Indeed, in several references [12] it is stated that the generalized CCWZ construction was developed by Volkov, Ogievetsky, and Ivanov (VOI), for situations where translational invariance is not spontaneously broken, and how nice it would be to find a "generalized" VOI approach that can in principle deal with this situation.

- On the other hand, as discussed thoroughly in 5.2.4.1, a consistent "analogous" system, the membrane, is an example of SSB of a *spatial* translation with no diagonal unbroken combination, that admits a sensible dynamical theory with Goldstones that only propagate in the unbroken spacetime directions within the brane. The authors of [167] actually suggest, providing no formal proof though, that the non-propagation of Goldstones in broken spatial directions is a rather obvious feature of Goldstone theory. Their reasoning is that as Goldstones have a gapless dispersion relation, meaning $E(k) \to 0$ as $k \to 0$, and k is only defined in the translationally invariant

[11] For completeness, let us just state that the building blocks $\nabla_0\eta_m$ and $\nabla_m\eta_n$, in the unitary gauge, may be put in the form

$$\nabla_0\eta_m \approx -e^\mu_m\partial_\mu \ln\sqrt{-g^{tt}} \quad \text{and} \quad \nabla_m\eta_n \approx e^\mu_m e^\nu_m K_{\mu\nu}\,, \tag{5.102}$$

once one makes the approximation that $\beta_m \approx 0 \Rightarrow \{\gamma \approx 1, \Lambda^a{}_b \approx \delta^a_b\}$, thus coinciding with equations (16) and (17) in [63], respectively. In the unitary gauge $\beta_m = -\frac{e^t_m}{e^t_0}$, so that $\beta_m \approx 0 \Leftrightarrow e^t_m \approx 0$.

[12] See, for instance, [187, 188].

directions, they may only propagate in these unbroken spatial directions. The Goldstone in the brane example does indeed satisfy this condition. Ultimately, it all boils down to the fact that the brane has an intrinsic $ISO(d-1,1)$ preserved symmetry, which in particular contains unbroken spacetime translation generators, the latter being the fundamental required structure for the VOI prescription to work.

- A Goldstone associated with the breaking ot time-translational invariance like the one appearing in the EFT of inflation, from this perspective, is quite different in nature, as $\pi = \pi(t, \mathbf{x})$ obviously *does* progagate in time, and in principle, does not have a gapless dispersion relation, as we learned in the EFT construction of chapter 3 (see equation (3.34)).

- Within the geometrical construction of the EFT of inflation [40], the authors assume the existence of an implicit *approximate* shift symmetry on the inflaton ϕ, such that upon the SSB of time-diffs, there remains a diagonal *approximate* time-translation invariance governing the dynamics of the π fluctuations, effectively implying that the a priori time-dependent coefficients of the EFT are slow-roll suppressed. From this point of view, the perfect superfluid interpretation stemming from the coset construction is necessarily only a well-motivated limiting case. At the risk of being pedantic, let us summarize these last arguments in an explicit way:

  - In the heuristic approach, within the unitary gauge where $\pi = 0$, we have, e.g., that $\mathscr{L} \supset M_n^4(t)\,(\delta g^{00})^n$, and the $M_n^4(t)$ coefficients are a priori, arbitrary time-dependent objects. However, according to [40], by *assuming* that $\phi_0(t)$ is a monotonic function of time, a field redefinition may always set $\phi_0(t) = t$ [13], which in the unitary gauge implies

[13] This is of course equivalent to taking $\dot{\phi}_0 = \text{constant}$, which through the use of $\dot{H} = -\frac{1}{2}\frac{\dot{\phi}_0^2}{M_{\rm Pl}^2}$,

that $\phi(t, \mathbf{x}) = t$. Crucially, assuming an *approximate* shift symmetry $\phi(t, \mathbf{x}) \to \phi(t, \mathbf{x}) + c$, in the unitary gauge is quite clear that it is natural to find a diagonal combination of broken time-diffs and broken approximate shift symmetry which ultimately implies an *approximate* time-translation invariance of the π Lagrangian. This remnant *approximate* symmetry implies slow-roll conditions of the form $\dot{M}_n(t) \ll M_n(t)H$, which as discussed in chapter 3, in most scenarios, is a required feature to solve the homogeneity problem of standard FLRW cosmology.

– In the coset construction of the perfect superfluid time-translation invariance and an *exact* shift symmetry are broken down to an *exact* time-translation invariance. In other words, in such a construction there is no room for approximate symmetries. In [188] the authors have explored the EFT of "shift-symmetric cosmologies", where assuming exact shift symmetry, they derived model-independent consequences for single-clock cosmologies. Importantly they realize, at least in the "flat limit", which can be thought of as the decoupling limit $M_{\mathrm{Pl}} \to \infty$ around a Minkowski background , that still assuming a monotonic time-dependent order parameter associated with the SSB of shift symmetry, there are several possibilities for its exact time-dependence, and only in the case when it is linear, meaning $\phi_0(t) = t$, the Goldstone associated with shift symmetry and the Goldstone associated with time-translation invariance coincide. This is the case of the *perfect* superfluid [14], thoroughly discussed in 5.2.4.2, which having

implies that $\dot{H} =$ constant. This last condition may be argued to be a very good approximation for a *quasi*-de Sitter background, but still as $\epsilon \equiv -\frac{\dot{H}}{H^2}$, it is a nontrivial condition on the hierarchy of slow-roll parameteres, starting with $\eta \equiv \frac{\dot{\epsilon}}{\epsilon H}$. While the assumption of monotonicity is quite natural, *linearity* is a further extra condition.

[14] As opposed to the cases of *imperfect* (or "braided") and *driven* superfluids where $\phi_0(t) \neq t$ generically.

a diagonal preserved Hamiltonian, is a SSP system that admits a coset construction, and has been proposed [63] as the right framework to describe the EFT of inflation in the CCWZ/VOI approach. However, for the perfect superfluid in the flat limit, due to exact shift symmetry invariance, the authors of [188] show that the coefficients of the EFT are necessarily time-*independent*. On the other hand, in [63] the authors have not discussed this fact. Even if in the context of spacetime symmetries the coset construction of VOI, with the restriction of having unbroken spacetime translational invariance, has not been proven to yield the most general EFT with all the symmetries [15], this is a clear hint that the perfect superfluid cannot reproduce the EFT of inflation in the general case.

In practical calculations, the time-dependence of the coefficients in the EFT of inflation are usually neglected, so this whole discussion may seem redundant for some authors, and indeed it may be so, phenomenologically speaking. However, these arguments may shed some light to a very interesting program that may be developed in direct analogy with the so-called "zoology of condensed matter" [65], where the authors classify condensed matter systems in terms of the spacetime symmetries they spontaneously break. In that spirit, imposing an exact shift symmetry is only the simplest case of symmetry breaking patters that one could consider. The generalized case, involving additional internal, gauge, and spacetime symmetries, should lead to different "states of cosmology", unveiling a "zoology of cosmology". For instance, in the zoology classification of [65] the so-called "type-I framid", which is a static, homogeneous, and isotropic scenario that spontaneously breaks only boost symmetry not requiring any additional symmetry beyond the Poincaré group, upon coupling to gravity gives rise to a very well known Lorentz-

[15] As opposed to the good old CCWZ construction case. See [189].

violating modification of GR introduced by Ted Jacobson and David Mattingly; the so-called Einstein-æther theory [190]. Another interesting example is the case when one promotes the $U(1)$ shift symmetry of the "type-I superfluid" (for us, the perfect superfluid) to internal monotonic diffeomorphisms of the form $\psi \rightarrow f(\psi)$. Naively, imposing such a symmetry implies the existence of infinitely many Goldstone modes in the spectrum of the theory, and however this is not the case as there is still just one Goldstone mode. A field enjoying such an internal symmetry arises for instance in the infrared limit of Horăva-Lifshitz gravity [191, 192] and has been dubbed "khronon" in the gravity/cosmology literature [185].

Finally, let us point out that exact global symmetries are an idealization and are argued to no exist in consistent quantum gravitational theories [193–195]. Moreover, in inflation theory, we do not really expect an exact shift symmetry to be realistic, not only due to these theoretical considerations but also because it leads to an scenario which is in tension with slow-roll backgrounds, at least in the simplest shift-symmetric models [188]. It is quite important then to understand how to incorporate "soft" symmetry breaking effects into the game, analogous to the pseudo-Goldstone boson analysis of QCD, in which the squared pion masses are linked to the symmetry-breaking quark mass terms. In principle, one could expect to get an interpretation of cosmological observables in terms of the scale characterizing the breaking of the shift symmetry, which is distinct from the scale associated to the breaking of time-translation invariance [196]. Needless to say, it would be quite enlightening to generalize the CCWZ/VOI coset construction to the case of broken time-translational invariance. Some (if not all) of these questions are being investigated and will be reported elsewhere [64].

Chapter 6

Concluding Remarks

As mentioned in the the introductory chapter, during the writing process of this dissertation, Planck's latest release [28] has basically ruled out the Natural Inflation model, which we picked as the single-field EFT background on which to build upon our two-field model. This is disappointing, among other reasons, because we have learned that the sizes of non-Gaussianites we estimated for the QSF regime, may not be as small as we initially thought, in light of futuristic so-called "21-cm cosmology" [153] [1]. Furthermore, the crucial n_s vs. r plot that we see in Figure 6.1, shows that all so-called large-field models of inflation are in pretty bad shape. In particular, Linde's ϕ^2 "Chaotic Inflation" [197] is plain dead, while axion monodromy models $(\phi^{4/3}, \phi, \phi^{2/3})$ [45–48] are just marginally alive.

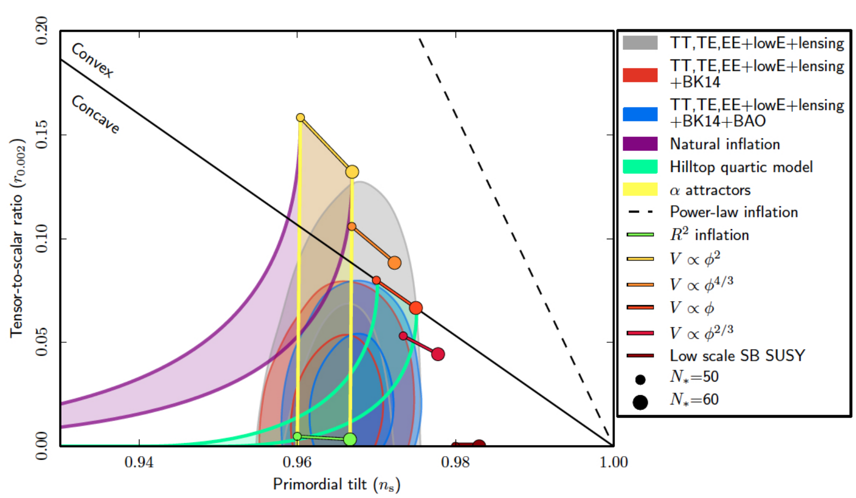


Figure 6.1: n_s vs. r plot from Planck's 2018 release [28].

Starobinsky's $\mathscr{R}^2$ model [198] is still the best fit, while so-called "α-attractors" [199, 200] and the "quartic hilltop" model [200] span a good compatible portion of

[1]We thank Soubhik Kumar for a conversation.

parameter space. As we thoroughly discussed in chapter 4, a subluminal inflaton speed of sound due to a non-standard kinetic term [105], may lower the prediction for the tensor-to-scalar ratio r of the GNI model, and of *any* single-field model for that matter, as this conclusion is based on the robust prediction that $r \approx 16\,\epsilon\, c_s$. However, we also realized that such a scenario requires finely-tuned initial conditions, which is quite unappealing. Other mechanisms that can "do the trick", meaning lowering the predictions for r within any single-field model include, for example, introducing a non-minimal coupling to gravity [201–204], or adding a damping term for the inflaton due to dissipation in other degrees of freedom, as in "warm inflation" [205, 206].

If you had asked us a couple of months ago, we would have said that such avenues of exploration are quite interesting, and the pains involved in going through complicated calculations are totally worth it. However, we are not who we used to be; we have grown to feel that any model that smells of contrivance is, in some way or another, doomed to fail. You may say we are getting very narrow-minded in this sense, and maybe you are right. Probably it has to do with the dialectics of the "hows" and "whys" within oneself. A safe approach will always be to try to make model-independent statements. This last point leads to the second topic we presented in this dissertation.

By the end of chapter 5 we have discussed at length about the subtleties related to trying to make sense, within the coset construction, of systems that spontaneously break time translations with no unbroken diagonal combination. We may only speculate at this point, since it is really a subject of ongoing research. For example, one naive possibility is that the construction may be carried by analogy with the membrane case analyzed in 5.2.4.1. Such a setting is reminiscent of the so-called "initial value problem" of GR [2] (see, e.g. chapter 10 in [106]).

[2]In short, one is given initial data on a 3d manifold Σ, in the form (h_{ab}, K_{ab}), where h_{ab} is the induced metric and K_{ab} the extrinsic curvature, and attemp to construct a globally hyperbolic

Without entering into details, let us just say that this approach, as of now, is far from being satisfactory. Other approaches are under investigation and should be reported elsewhere [64]. In the meantime, the authors of [63] have claimed that the EFT of inflation is simply represented by a perfect superfluid. While this may be a very good approximation, it is clearly not the general case, as for example, the a priori time-dependent coefficients, when assuming an exact shift symmetry, are necessarily constants [188]. Moreover, the very necessity of internal shift symmetries is "worrisome", in light of theoretical arguments that imply their non-compatibility with consistent quantum gravity [193–195]. We expect that by thinking hard about these puzzles, we will eventually have something sensible to say about them.

spacetime $(\mathscr{M}, g_{ab})$, where $\mathscr{M}$ is a 4d manifold endowed with a metric g_{ab}, and for which Σ is a "Cauchy surface" on which the initial data are induced. When the initial data is subject to certain initial value contraints, on may prove that the spacetime $(\mathscr{M}, g_{ab})$ satisfies Einstein's equations.

Appendix A

In-In Formalism and Equilateral Bispectra

In a quantum field theory in an asymptotically Minkowski spacetime one is usually interested in calculating the so-called S-matrix describing the transition probability for a state $|in\rangle$ in the far past to become some state $|out\rangle$ in the far future. This is the standard tool to determine the relevant correlation functions of the theory. The scattering particles are taken to be non-interacting at very early and very late times, when they are far from the interaction region, and the asymptotic states can be taken to be the vacuum states of the free Hamiltonian H_0.

In cosmology instead, we are interested in calculating expectation values of (products of) operators $\widehat{Q}_k$ at a fixed time. For example, $\hat{Q}$ could be the product of n copies of the Goldstone boson, i.e. $\hat{Q} = \hat{\pi}_{\mathbf{k_1}}\hat{\pi}_{\mathbf{k_2}} \ldots \hat{\pi}_{\mathbf{k}_n}$ so that

$$\langle \hat{Q} \rangle \equiv \langle \Omega_{\rm int}| \, \hat{Q}(\tau) \, |\Omega_{\rm int}\rangle \tag{A.1}$$

is the n-point function of π, where $|\Omega_{\rm int}\rangle$ is the interacting vacuum at some moment τ_0 in the far past, and $\tau > \tau_0$ is some later time, such as horizon crossing or the end of inflation. We only impose boundary conditions at very early times, where the wavelengths of all relevant modes are much smaller than the horizon. In this limit, by the equivalence principle, the interaction picture fields have the same form as in Minkowski space. This defines the Bunch-Davies vacuum. Let us now review the Schwinger-Keldysh or in-in (for short) formalism.

In the Heisenberg picture, the time evolution of the operators is determined by

Heisenberg's equations

$$\frac{d\hat{\pi}}{d\tau} = i[\hat{H}, \hat{\pi}], \quad \frac{d\hat{p}_\pi}{d\tau} = i[\hat{H}, \hat{p}_\pi], \tag{A.2}$$

where $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$ is the perturbed Hamiltonian. In order to deal with the complicated non-linear equations that arise from the interacting part of $\hat{H}$ one introduces the interaction picture in which the leading time-dependence of the fields is determined by the free (quadratic) Hamiltonian, so that

$$\hat{\pi}'_I = i[\hat{H}_0, \hat{\pi}_I], \quad \hat{p}'_{\pi,I} = i[\hat{H}_0, \hat{p}_{\pi,I}], \tag{A.3}$$

are linear equations with initial conditions $\hat{\pi}_I(\tau_0) = \hat{\pi}(\tau_0)$ and $\hat{p}_{\pi,I}(\tau_0) = \hat{p}_\pi(\tau_0)$. The solution to these equations can be written as

$$\hat{\pi}^I_{\mathbf{k}}(\tau) = \pi^I_k(\tau)\hat{a}_{\mathbf{k}} + \text{h.c.}, \tag{A.4}$$

where $\pi^I_k(\tau)$ is the solution to the free-field equation of motion (say the Mukhanov-Sasaki equation), and the operators $\hat{a}_{\mathbf{k}}$ define the free-field vacuum $|\Omega\rangle$ through $\hat{a}_{\mathbf{k}}|\Omega\rangle = 0$. Heisenberg's equations (A.2) can be rewritten as

$$\hat{\pi}(\tau) = U^{-1}(\tau,\tau_0)\hat{\pi}(\tau_0)U(\tau,\tau_0), \quad \hat{p}_\pi(\tau) = U^{-1}(\tau,\tau_0)\hat{p}_\pi(\tau_0)U(\tau,\tau_0), \tag{A.5}$$

where $U(\tau,\tau_0)$ is defined by the differential equation

$$\frac{d}{d\tau}U(\tau,\tau_0) = -i\hat{H}U(\tau,\tau_0), \quad U(\tau_0,\tau_0) = 1. \tag{A.6}$$

Analogously, we can re-write (A.3)

$$\hat{\pi}_I(\tau) = U_0^{-1}(\tau,\tau_0)\hat{\pi}_I(\tau_0)U_0(\tau,\tau_0), \quad \hat{p}_{\pi,I}(\tau) = U_0^{-1}(\tau,\tau_0)\hat{p}_{\pi,I}(\tau_0)U_0(\tau,\tau_0), \tag{A.7}$$

where $U_0(\tau, \tau_0)$ is defined by the differential equation

$$\frac{d}{d\tau} U_0(\tau, \tau_0) = -i\hat{H}_0 U_0(\tau, \tau_0), \quad U_0(\tau_0, \tau_0) = 1. \tag{A.8}$$

Using (A.6) and (A.8) we get that

$$\frac{d}{d\tau}\left[U_0^{-1}(\tau, \tau_0) U(\tau, \tau_0)\right] = -iU_0^{-1}(\tau, \tau_0)\hat{H}_{\text{int}} U(\tau, \tau_0), \tag{A.9}$$

and using (A.3), this implies that

$$U(\tau, \tau_0) = U_0(\tau, \tau_0) F(\tau, \tau_0), \tag{A.10}$$

where

$$\frac{d}{d\tau} F(\tau, \tau_0) = -i\hat{H}_{\text{int},I} F(\tau, \tau_0), \quad F(\tau_0, \tau_0) = 1, \tag{A.11}$$

and $\hat{H}_{\text{int},I}$ is the interaction Hamiltonian in the interaction picture,

$$\hat{H}_{\text{int},I}(\tau) = U_0(\tau, \tau_0)\hat{H}_I(\tau_0) U_0^{-1}(\tau, \tau_0). \tag{A.12}$$

The solution of (A.11) is given by

$$F(\tau, \tau_0) = T \exp\left(-i \int_{\tau_0}^{\tau} \hat{H}_{\text{int},I}(\tau'') d\tau''\right). \tag{A.13}$$

Therefore, we find that an operator in the Heisenberg picture in terms of operators in the interaction picture is written as

$$\hat{Q}(\tau) = F^{-1}(\tau, \tau_0)\hat{Q}^I(\tau) F(\tau, \tau_0). \tag{A.14}$$

We may think of $F(\tau, \tau_0)$ as an operator evolving quantum states in the interaction picture,

$$|\Omega_{\text{int}}(\tau)\rangle = F(\tau, \tau_0)\,|\Omega_{\text{int}}(\tau_0)\rangle\,, \tag{A.15}$$

where $|\Omega_{\text{int}}(\tau_0)\rangle \equiv |\Omega_{\text{int}}\rangle$. We would now like to relate the vacuum of the interacting theory, $|\Omega_{\text{int}}\rangle$, to the vacuum of the free theory, $|\Omega\rangle$. To do so we insert a complete set of energy eigenstates $\{|\Omega_{\text{int}}\rangle\,, |n\rangle\}$ of the full theory, where $|n\rangle$ are the excited states, in order to cast $|\Omega\rangle$ as

$$|\Omega\rangle = |\Omega_{\text{int}}\rangle\,\langle\Omega_{\text{int}}|\Omega\rangle + \sum_n |n\rangle\,\langle n|\Omega\rangle\,. \tag{A.16}$$

Correspondingly, we see that

$$e^{-i\hat{H}(\tau-\tau_0)}\,|\Omega\rangle = e^{-i\hat{H}(\tau-\tau_0)}\,|\Omega_{\text{int}}\rangle\,\langle\Omega_{\text{int}}|\Omega\rangle + \sum_n e^{-iE_n(\tau-\tau_0)}\,|n\rangle\,\langle n|\Omega\rangle\,. \tag{A.17}$$

Now we add a small imaginary part to the initial time, $\tau_0 \to -\infty\,(1 - i\varepsilon) \equiv -\infty^-$ so the excited states get projected out, $e^{-iE_n(\tau-\tau_0)} \to e^{-\infty\times\varepsilon E_n}\,(\ldots) \to 0$. With this $i\varepsilon$ prescription we then find that

$$F(\tau, -\infty^-)\,|\Omega_{\text{int}}\rangle = \frac{F(\tau, -\infty^-\,|\Omega\rangle}{\langle\Omega_{\text{int}}|\Omega\rangle}, \tag{A.18}$$

so we effectively turn off the interaction in the far past and project the interacting vacuum Ω_{int} onto the free vacuum $|\Omega\rangle$. Setting $|\langle\Omega_{\text{int}}|\Omega\rangle| \to 1$, we arrive to the in-in master formula

$$\langle\hat{Q}(\tau)\rangle = \left\langle\Omega\,\middle|\,\left[\overline{T}\exp\left(i\int_{-\infty^+}^{\tau} d\tau''\,\hat{H}_I(\tau'')\right)\right]\hat{Q}^I(\tau)\left[T\exp\left(-i\int_{-\infty^-}^{\tau} d\tau'''\,\hat{H}_I(\tau''')\right)\right]\,\middle|\,\Omega\right\rangle, \tag{A.19}$$

where $\overline{T}$ is the anti-time-ordering symbol and $\infty^{\pm} \equiv \infty\,(1 \pm i\varepsilon)$. By the expanding the exponentials in (A.19), we may compute the correlation function perturbatively in $\hat{H}_{\text{int}}$. At leading order, i.e. tree-level, we find

$$\langle \hat{Q}(\tau) \rangle = -i \int_{-\infty^-}^{\tau} d\tau''\, \langle \Omega |\, [\hat{Q}^I(\tau), \hat{H}^I_{\text{int}}(\tau'')]\, | \Omega \rangle\,. \tag{A.20}$$

Let us re-write the interacting Goldstone Lagrangian from (4.80)

$$\mathscr{L}_\pi = -\frac{M_{\text{Pl}}^2 \dot{H}}{c_s^2} \left\{ \left(\dot{\pi}^2 - c_s^2 \frac{(\nabla\pi)^2}{a^2} \right) - \left(1 - c_s^2\right) \left(\dot{\pi} \frac{(\nabla\pi)^2}{a^2} + \frac{\mathcal{A}}{c_s^2}\, \dot{\pi}^3 \right) \right\}, \tag{A.21}$$

as our goal is now to compute the non-Gaussianities stemming from the operators $\dot{\pi}^3$ and $\dot{\pi}(\nabla\pi)^2$. Using the definition

$$\langle \hat{\pi}_{\mathbf{k}_1} \hat{\pi}_{\mathbf{k}_2} \hat{\pi}_{\mathbf{k}_3} \rangle \equiv (2\pi)^3\, \delta^{\,3}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, \mathcal{B}_\pi(k_1, k_2, k_3), \tag{A.22}$$

we are interested in calculating the bispectrum of Goldstone fluctuations after horizon crossing,

$$\mathcal{B}_\pi(k_1, k_2, k_3) \equiv \lim_{\tau \to 0} \langle \hat{\pi}_{\mathbf{k}_1}(\tau) \hat{\pi}_{\mathbf{k}_2}(\tau) \hat{\pi}_{\mathbf{k}_3}(\tau) \rangle', \tag{A.23}$$

where the prime indicates that an overall delta-function, $(2\pi)^3\, \delta^{\,3}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$, has been omitted. Using (A.20) we find that

$$\mathcal{B}_\pi(k_1, k_2, k_3) = -i \int_{-\infty^-}^{0} d\tau\, \langle [\hat{\pi}_{\mathbf{k}_1}(0)\, \hat{\pi}_{\mathbf{k}_2}(0)\, \hat{\pi}_{\mathbf{k}_3}(0), \hat{H}_{\text{int}}(\tau)] \rangle, \tag{A.24}$$

where we have suppressed the index I on the interaction picture operators to avoid the clutter. At leading order, $H_{\text{int}} = -\int d^3x\, a^4 \mathscr{L}_\pi^{(3)}$, where $\mathscr{L}_\pi^{(3)}$ is given by the cubic part of (A.21). To keep moving forward we note that, as we are interested in the limit of large non-Gaussianity, we may ignore slow-roll corrections to the mode functions and use the solution for de Sitter space of the Mukhanov-Sasaki equation associated with the quadratic sector in (A.21). Therefore, we insert the Bunch-Davies mode functions

$$\pi_k(\tau) = \pi_k^{(0)}\,(1 + ic_s\,k\tau)\,e^{-ic_s\,k\tau}, \quad \pi_k^{(0)} \equiv \frac{i}{2M_{\rm Pl}\sqrt{\epsilon\,c_s}}\frac{1}{k^{3/2}}, \tag{A.25}$$

in (A.24) and perform the appropriate Wick contractions [1] to get

$$\mathcal{B}_\pi(k_1,k_2,k_3) = M_{\rm Pl}^2|\dot{H}|\left(\frac{1}{c_s^2}-1\right)\mathrm{Re}\left[\pi_{k_1}^{(0)}\pi_{k_2}^{(0)}\pi_{k_3}^{(0)}\,\mathcal{I}(k_1,k_2,k_3)\right] + \text{perms.}, \tag{A.26}$$

where $\mathcal{I} \equiv \mathcal{I}_{\dot{\pi}(\nabla\pi)^2} + \mathcal{I}_{\dot{\pi}^3}$, with

$$\mathcal{I}_{\dot{\pi}(\nabla\pi)^2} \equiv \int_{-\infty^-}^{0}\frac{d\tau}{H\tau}\left(\pi_{k_1}^*\right)'\pi_{k_2}^*\pi_{k_3}^*\left(\mathbf{k}_1\cdot\mathbf{k}_2\right), \tag{A.27}$$

$$\mathcal{I}_{\dot{\pi}^3} \equiv \frac{\mathcal{A}}{c_s^2}\int_{-\infty^-}^{0}\frac{d\tau}{H\tau}\left(\pi_{k_1}^*\right)'\left(\pi_{k_2}^*\right)'\left(\pi_{k_3}^*\right)'. \tag{A.28}$$

To perform analytically the above integrals, e.g. by using Mathematica, one can formally deform the contour by a Wick rotation $\tau \to i\tau$, since there are no poles in the complex τ plane.

We may obtain the bispectrum for the curvature pertubation $\mathcal{R}$ by a simple rescaling $\mathcal{B}_\mathcal{R}(k_1,k_2,k_3) = -H^3\mathcal{B}_\pi(k_1,k_2,k_3)$. Moreover, we may the express the bispectrum $\mathcal{B}_\mathcal{R}(k_1,k_2,k_3)$ in terms of the amplitude $f_{\rm NL}$, and the "normalized" shape function $\widetilde{\mathcal{S}}(k_1,k_2,k_3)$,

$$\frac{(k_1k_2k_3)^2}{(2\pi)^4\,\Delta_\mathcal{R}^4(k_\star)}\,\mathcal{B}_\mathcal{R}(k_1,k_2,k_3) = \frac{9}{10}\,f_{\rm NL}\,\widetilde{\mathcal{S}}(k_1,k_2,k_3), \tag{A.29}$$

[1] See [150] for details.

where $\widetilde{\mathcal{S}}(k_1,k_2,k_3) \equiv \frac{1}{f_{\rm NL}}\frac{10}{9}\mathcal{S}(k_1,k_2,k_3)$ with $\widetilde{\mathcal{S}}(k,k,k) = 1$ [2]. The normalized shape functions are then given by

$$\widetilde{\mathcal{S}}_{\dot{\mathcal{R}}(\nabla\mathcal{R})^2} = \frac{\hat{k}_1^2 - \hat{k}_2^2 - \hat{k}_3^2}{\hat{k}_1\hat{k}_2\hat{k}_3}\left(-1 + \sum_{i>j}\frac{\hat{k}_i\hat{k}_j}{9} + \frac{\hat{k}_1\hat{k}_2\hat{k}_3}{27}\right) + \text{perms.}, \tag{A.30}$$

$$\widetilde{\mathcal{S}}_{\dot{\mathcal{R}}^3} = \hat{k}_1\hat{k}_2\hat{k}_3, \tag{A.31}$$

where $\hat{k}_i \equiv \frac{k_i}{\mathcal{K}}$ and $\mathcal{K} \equiv \frac{1}{3}(k_1+k_2+k_3)$. Finally, the amplitudes associated with these two shapes are

$$f_{\rm NL}^{\dot{\mathcal{R}}(\nabla\mathcal{R})^2} = -\frac{85}{324}\left(\frac{1}{c_s^2} - 1\right), \tag{A.32}$$

$$f_{\rm NL}^{\dot{\mathcal{R}}^3} = \frac{5\,\mathcal{A}}{81}\left(\frac{1}{c_s^2} - 1\right). \tag{A.33}$$

The non-Gaussianities associated with a perturbative action like the one in (4.77) are then well known [147, 150]. In the limit $M_{\rm eff} \to \infty$ the bispectrum is of equilateral shape and the contribution from the $\dot{\mathcal{R}}^3$ term has the shape

$$\mathcal{S}_{\dot{\pi}^3}(p_1,p_2,p_3) = -6\frac{\dot{\theta}_0^2}{M_{\rm eff}^2}\left[1 - 2\frac{\dot{\theta}_0^2}{M_{\rm eff}^2}c_s^2\left(1 + \frac{R}{3M_{\rm eff}^2}V_{\sigma\sigma\sigma}\right)\right]\frac{p_1\,p_2\,p_3}{(p_1+p_2+p_3)^3}, \tag{A.34}$$

with an associated non-linear parameter given by

$$f_{\rm NL}^{\dot{\pi}^3} = -\frac{20}{81}\frac{\dot{\theta}_0^2}{M_{\rm eff}^2} + \frac{40}{81}c_s^2\frac{\dot{\theta}_0^4}{M_{\rm eff}^4} + \frac{40}{243}\frac{R}{M_{\rm eff}^2}V_{\sigma\sigma\sigma}\,c_s^2\frac{\dot{\theta}_0^4}{M_{\rm eff}^4}, \tag{A.35}$$

which is just another way of writing (4.104). Let us estimate how big may the last term in (A.35) be, since at first glance it just looks further suppressed by an additional $M_{\rm eff}^2$ factor. First of all, we rely in perturbativity, so convergence of the

[2] In explicitly "extracting" $f_{\rm NL}$ in the definition of $\widetilde{\mathcal{S}}(k_1,k_2,k_3)$, we have implicitly assumed a scale-invariant bispectra.

perturbative series should imply that

$$\frac{V_{\sigma\sigma\sigma}\,\delta\sigma}{3\,V_{\sigma\sigma}} = \frac{V_{\sigma\sigma\sigma}\,\delta\sigma}{3\left(M_{\rm eff}^2+\dot\theta_0^2\right)} \leq 1. \tag{A.36}$$

Using (4.72) to first order along with $\epsilon = \frac{R^2\,\dot\theta_0^2}{2M_{\rm Pl}^2H^2}$, we can write

$$\delta\sigma = 2\sqrt{2\,\epsilon}\,\frac{H M_{\rm Pl}}{M_{\rm eff}^2}\,\delta\dot\theta. \tag{A.37}$$

Furthermore, considering the expression $\mathcal{R} = -\frac{H\delta\theta}{\dot\theta_0}$ and the conservation of $\mathcal{R}$ on large scales, $\dot{\mathcal{R}} \approx 0$, one can prove that

$$\delta\dot\theta = \frac{H\eta}{2}\,\delta\theta \sim \frac{H^2\eta}{4\pi R\,c_s^{1/2}} \Longrightarrow \delta\sigma \sim \sqrt{2\,\epsilon}\,c_s\,\eta\,\frac{H^3 M_{\rm Pl}}{2\pi M_{\rm eff}^2 R}. \tag{A.38}$$

Finally, using (A.36) and (A.38) it is straightforward to show that the last term in (A.35) satisfies

$$\frac{R}{3M_{\rm eff}^2}V_{\sigma\sigma\sigma}\,c_s^2\frac{\dot\theta_0^4}{M_{\rm eff}^4} \lesssim \frac{1}{\eta\,(\Delta_{\mathcal{R}}^2)^{1/2}}\left(\frac{\dot\theta_0^2}{M_{\rm eff}^2}\right), \tag{A.39}$$

so it dominates in a "natural" model where $\left(\frac{\dot\theta_0^2}{M_{\rm eff}^2}\right)$ is required to be small, having the chance of giving a non-negligible contribution to $f_{\rm NL}$ due to the $(\eta\,(\Delta_{\mathcal{R}}^2)^{1/2})^{-1}$ prefactor.

Appendix B

Effective Single-Field Theory Regime

Neglecting $\mathcal{O}(\beta)$ terms as we are dealing with the theory of fluctuations, we can rewrite action (4.114) as

$$S^{(2)}[g_0, \phi_0, \delta\phi] = \frac{1}{2}\int d^4x\, a^3 \Bigg\{ \left(\frac{\dot{\varphi}_0^2}{H^2}\right)\left(\dot{\mathcal{R}}^2 - \frac{(\nabla\mathcal{R})^2}{a^2}\right) + \dot{\mathcal{F}}^2 - \frac{(\nabla\mathcal{F})^2}{a^2} - M_{\rm eff}^2\,\mathcal{F}^2 - 4\dot{\theta}_0\left(\frac{\dot{\varphi}_0}{H}\right)\dot{\mathcal{R}}\,\mathcal{F}\Bigg\}, \tag{B.1}$$

where we have used the fact that $\mathcal{R} \approx -\frac{H}{\dot{\phi}_0}T_a\,\delta\phi^a = -\frac{H}{\dot{\theta}_0}\delta\theta = -\frac{H}{\dot{\varphi}_0}\varphi$ which holds as long as $\dot{\sigma}_0 = 0$ and we have taken $R \approx \sqrt{2}\,v$. Now varying the quadratic action (B.1) we get the equations of motion for $\mathcal{R}$ and $\mathcal{F}$ after Fourier transforming spatial coordinates

$$\ddot{\mathcal{R}} + (3 + 2\,\epsilon - 2\,\eta_{\|})\,H\,\dot{\mathcal{R}} + \frac{k^2}{a^2}\mathcal{R} = 2\dot{\theta}_0\left(\frac{H}{\dot{\varphi}_0}\right)\left\{\dot{\mathcal{F}} + \left(3 - \eta_{\|} + \epsilon + \frac{\ddot{\theta}_0}{H\dot{\theta}_0}\right)H\,\mathcal{F}\right\}, \tag{B.2}$$

$$\ddot{\mathcal{F}} + 3H\,\dot{\mathcal{F}} + \frac{k^2}{a^2}\mathcal{F} + M_{\rm eff}^2\,\mathcal{F} = -2\dot{\theta}_0\left(\frac{\dot{\varphi}_0}{H}\right)\dot{\mathcal{R}}, \tag{B.3}$$

where $\epsilon \equiv -\frac{\dot{H}}{H^2}$ and $\eta_{\|} \equiv -\frac{\ddot{\varphi}_0}{H\dot{\varphi}_0}$. As it has been emphasized before $\mathcal{R} =$ constant and $\mathcal{F} = 0$ are non-trivial solutions to these equations due to the background isometries [126]. Given that $\mathcal{F}$ is heavy, $\mathcal{F} \to 0$ shortly after horizon exit while $\mathcal{R} \to$ constant as in single-field inflation theory. If we consider the short wavelength limit we can neglect the "Hubble friction" terms and take $\frac{\dot{\varphi}_0}{H} =$ constant. We also take the physical wave number $p \equiv \frac{k}{a}$ to be a constant in this regime. In this approximation

$$\ddot{\mathcal{R}}_c + p^2\,\mathcal{R}_c = 2\dot{\theta}_0\,\dot{\mathcal{F}}, \tag{B.4}$$

$$\ddot{\mathcal{F}} + p^2\,\mathcal{F} + M_{\rm eff}^2\,\mathcal{F} = -2\dot{\theta}_0\,\dot{\mathcal{R}}_c, \tag{B.5}$$

where $\mathcal{R}_c \equiv \left(\frac{\dot{\varphi}_0}{H}\right)\mathcal{R}$. The solutions of this system are given by

$$\mathcal{R}_c = \mathcal{R}_+\,e^{i\omega_+ t} + \mathcal{R}_-\,e^{i\omega_- t}, \tag{B.6}$$

$$\mathcal{F}_c = \mathcal{F}_+\,e^{i\omega_+ t} + \mathcal{F}_-\,e^{i\omega_- t}, \tag{B.7}$$

where the frequencies $\omega_\pm$ read [132]

$$\omega_\pm^2 = \frac{M_{\rm eff}^2}{2\,c_s^2} + p^2 \pm \frac{M_{\rm eff}^2}{2\,c_s^2}\sqrt{1 + \frac{4\,p^2\,(1-c_s^2)}{M_{\rm eff}^2\,c_s^{-2}}}. \tag{B.8}$$

Here $(\mathcal{R}_-, \mathcal{F}_-)$ and $(\mathcal{R}_+, \mathcal{F}_+)$ represent the amplitudes of low and high frequency modes respectively and satisfy

$$\mathcal{F}_- = \frac{-2\,i\,\dot{\theta}_0\,\omega_-}{M_{\rm eff}^2 + p^2 - \omega_-^2}\,\mathcal{R}_-, \tag{B.9}$$

$$\mathcal{R}_+ = \frac{-2\,i\,\dot{\theta}_0\,\omega_+}{\omega_+^2 - p^2}\,\mathcal{F}_+. \tag{B.10}$$

We see that the fields in each pair oscillate coherently.

Demanding that the high frequency degrees of freedom do not participate in the dynamics of the adiabatic modes, is only justified in the presence of a hierarchy of the form $\omega_-^2 \ll \omega_+^2$, which is equivalent to demand that $p^2 \ll M_{\rm eff}^2\,c_s^{-2}$ by the use of (B.8). Under these circumstances we get that

$$\omega_+^2 \approx M_{\rm eff}^2 \, c_s^{-2} \approx M_{\rm eff}^2 + 4\,\dot\theta_0^2, \tag{B.11}$$

$$\omega_-^2 \approx p^2 c_s^2 + (1-c_s^2)^2 \frac{p^4}{M_{\rm eff}^2\, c_s^{-2}}. \tag{B.12}$$

As far as low energy frequencies are concerned, the condition $p^2 \ll M_{\rm eff}^2 \, c_s^{-2}$ is equivalent to $\omega_-^2 \ll M_{\rm eff}^2 \, c_s^{-2}$ so $\omega_+^2 \approx M_{\rm eff}^2 \, c_s^{-2}$ behaves as the cut-off of the low energy effective theory regime. In this approximation $\mathcal{F}$ is completely determined by $\mathcal{R}_c$ through the relation $\mathcal{F} = \frac{-2\,\dot\theta_0\,\dot{\mathcal{R}}_c}{M_{\rm eff}^2 + p^2 - \omega_-^2}$. When linear perturbations evolve, their physical wave number $p \equiv \frac{k}{a}$ decreases and the modes enter the long wavelength regime $p^2 c_s^2 \lesssim H^2$, where they become strongly influenced by the background and no longer have a simple oscillatory behavior. However, the low energy contributions to $\mathcal{F}$ satisfy $\dot{\mathcal{F}} \sim H\mathcal{F}$ and since we assume $H^2 \ll M_{\rm eff}^2$, we can neglect time derivatives in (B.3) so we can solve $\mathcal{F}$ in terms of $\mathcal{R}$ as

$$\mathcal{F} = -\left(\frac{\dot\varphi_0}{H}\right) \frac{2\,\dot\theta_0\,\dot{\mathcal{R}}}{\frac{k^2}{a^2} + M_{\rm eff}^2}. \tag{B.13}$$

Plugging this algebraic relation back into the action (B.1) , we get an effective (tree-level) action for the curvature perturbation which at quadratic order reads [1]

$$S_{\rm eff}^{(2)}[g_0, \varphi_0, \mathcal{R}] = \frac{1}{2}\int dt\, đ^3k\, a^3 \left(\frac{\dot\varphi_0^2}{H^2}\right)\left\{\frac{\dot{\mathcal{R}}^2}{c_s^2(k)} + \frac{k^2\,\mathcal{R}^2}{a^2}\right\}, \tag{B.14}$$

[1] Here $đ^3k \equiv \frac{d^3k}{(2\pi)^3}$.

where $c_s^{-2}(k) = 1 + 4\left(\frac{\dot{\theta}_0^2}{\frac{k^2}{a^2}+M_{\text{eff}}^2}\right)$ is a k-dependent speed of sound.

Appendix C

Slow-Roll Approximation Consistency

We start by reviewing the slow-roll approximation consistency for single-field inflation. The field equations for single-field inflation are given by

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0, \tag{C.1}$$

$$3M_{\text{Pl}}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V. \tag{C.2}$$

The slow-roll approximation consists, at this level, in neglecting $\ddot{\phi}$ and $\frac{1}{2}\dot{\phi}^2$ in (C.1) and (C.2), respectively. Therefore, we are led to the system

$$3H\dot{\phi} + V_\phi = 0, \tag{C.3}$$

$$3M_{\text{Pl}}^2 H^2 = V. \tag{C.4}$$

From (C.3) we may express $\dot{\phi} = -\frac{V_\phi}{3H}$, and using the chain rule $\dot{V}_\phi = V_{\phi\phi}\,\dot{\phi}$, we get that

$$\ddot{\phi} = -\frac{\left(3H\dot{V}_\phi - 3\dot{H}V_\phi\right)}{9H^2} = -\frac{V_{\phi\phi}}{3H}\dot{\phi} + \frac{\dot{H}}{3H^2}V_\phi = -\frac{m_\phi^2}{3H}\dot{\phi} - \frac{1}{3}\,\epsilon\, V_\phi, \tag{C.5}$$

where $V_{\phi\phi} \equiv m_\phi^2$ and $\epsilon \equiv -\frac{\dot{H}}{H^2}$. Moreover, within the same approximation

$$\dot{\phi}^2 = \frac{V_\phi^2}{9H^2}. \tag{C.6}$$

Plugging (C.5) and (C.6) into (C.1) and (C.2), respectively, yields

$$-\frac{m_\phi^2}{3H^2}\dot{\phi} - \frac{1}{3}\,\epsilon\,\frac{V_\phi}{H} + 3\,\dot{\phi} + \frac{V_\phi}{H} = 0, \tag{C.7}$$

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\left(\frac{V_\phi^2}{9H^2}\right) + V \simeq \frac{1}{6}M_{\rm Pl}^2\left(\frac{V_\phi}{V}\right)^2 V + V = \frac{1}{3}\,\epsilon_V V + V. \tag{C.8}$$

As $m_\phi^2 \ll H^2 \iff \left\{\eta_V \equiv M_{\rm Pl}^2 \frac{V_{\phi\phi}}{V}\right\} \ll 1$, $\epsilon \ll 1$, and $\left\{\epsilon_V \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V_\phi}{V}\right)^2\right\} \ll 1$, we see, comparing (C.7) and (C.8) with (C.3) and (C.4), that the slow-roll approximation is self-consistent, as within such a limit, the neglected terms in the equations of motion become very suppressed "corrections" to the approximate equations.

Let us now consider the two-field system of chapter 4, whose equations of motion are given by

$$\ddot{\sigma} + 3H\dot{\sigma} - (R+\sigma)\,\dot{\theta}^2 + V_\sigma = 0, \tag{C.9}$$

$$(R+\sigma)^2\,\ddot{\theta} + 2(R+\sigma)\,\dot{\sigma}\,\dot{\theta} + 3H(R+\sigma)^2\,\dot{\theta} + V_\theta = 0, \tag{C.10}$$

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}(R+\sigma)^2\dot{\theta}^2 + \frac{1}{2}\dot{\sigma}^2 + V. \tag{C.11}$$

In this case the slow-roll approximation we consider is determined by

$$\{\ddot{\sigma},\, 3H\dot{\sigma}\} \ll \left\{(R+\sigma)\,\dot{\theta}^2,\, V_\sigma\right\}, \tag{C.12}$$

$$\left\{2(R+\sigma)\,\dot{\sigma}\,\dot{\theta}, (R+\sigma)^2\,\ddot{\theta}\right\} \ll \left\{3H(R+\sigma)^2\,\dot{\theta},\, V_\theta\right\}, \tag{C.13}$$

$$\left\{\frac{1}{2}(R+\sigma)^2\dot{\theta}^2, \frac{1}{2}\dot{\sigma}^2\right\} \ll V. \tag{C.14}$$

More to the point, we are interested in the case when $\sigma \simeq$ constant and $\dot{\theta} \simeq$ constant. Therefore we are led to the system

$$-(R+\sigma)\,\dot{\theta}^2 + V_\sigma = 0, \tag{C.15}$$

$$3H(R+\sigma)^2\,\dot{\theta} + V_\theta = 0, \tag{C.16}$$

$$3M_{\rm Pl}^2 H^2 = V. \tag{C.17}$$

Equations (C.12) and (C.13) imply, respectively, that

$$\frac{1}{(R+\sigma)}\frac{\dot{\sigma}}{\dot{\theta}} \ll \frac{\dot{\theta}}{3H} \quad \text{and} \quad \frac{1}{(R+\sigma)}\frac{\dot{\sigma}}{\dot{\theta}} \ll \frac{3H}{2\dot{\theta}}, \tag{C.18}$$

which together demand that

$$\varsigma \equiv \frac{1}{(R+\sigma)}\frac{d\sigma}{d\theta} \ll 1. \tag{C.19}$$

Equation (C.19) is a consequence of the slow-roll approximation, and in this sense, it is also an slow-roll requirement. Moreover, the natural slow-roll parameter $\delta \equiv \frac{\dot{\sigma}}{(R+\sigma)H} \ll 1$ may be written as

$$\delta = \frac{\dot{\sigma}}{(R+\sigma)H} = \frac{1}{(R+\sigma)}\frac{\dot{\sigma}}{\dot{\theta}}\frac{\dot{\theta}}{H} = \frac{1}{(R+\sigma)}\frac{d\sigma}{d\theta}\frac{\dot{\theta}}{H} = \varsigma\frac{\dot{\theta}}{H} \ll 1, \tag{C.20}$$

so we see that the ratio $\frac{\dot{\theta}}{H}$ is *not* slow-roll suppressed as one could naively guess, and may actually be $\mathcal{O}(1)$, as long as $\varsigma \ll 1$. Now using (C.16) we get that

$\dot{\theta} = -\frac{V_\theta}{3H(R+\sigma)^2}$. Therefore

$$\begin{aligned}
(R+\sigma)^2\ddot{\theta} &= -\frac{\left(3H(R+\sigma)^2\dot{V}_\theta - 3V_\theta\left(\dot{H}(R+\sigma)^2 + H(R\dot{+}\sigma)^2\right)\right)}{9H^2(R+\sigma)^2} \\
&= -\frac{\left(V_{\theta\theta}\,\dot{\theta} + V_{\theta\sigma}\,\dot{\sigma}\right)}{3H} + \frac{V_\theta}{3}\frac{\dot{H}}{H^2} + \frac{2V_\theta}{3H}\frac{\dot{\sigma}}{(R+\sigma)} \\
&= -\frac{V_{\theta\theta}}{3}\frac{\dot{\theta}}{H} - \frac{V_{\theta\sigma}(R+\sigma)}{3}\,\delta - \frac{V_\theta}{3}\,\epsilon + \frac{2V_\theta}{3}\,\delta.
\end{aligned} \tag{C.21}$$

To find $\dot{\sigma}$ we take an implicit time-derivative of (C.15), which yields

$$\dot{\sigma} = \frac{2}{M_{\text{eff}}^2}\frac{\dot{\theta}}{(R+\sigma)}(R+\sigma)^2\ddot{\theta} - V_{\sigma\theta}\frac{\dot{\theta}}{M_{\text{eff}}^2}, \tag{C.22}$$

where $M_{\text{eff}}^2 \equiv V_{\sigma\sigma} - \dot{\theta}^2$. Using (C.21) in (C.22) we find that (C.10) reads

$$\left(-\frac{V_{\theta\theta}}{3}\frac{\dot{\theta}}{H} - \frac{V_{\theta\sigma}(R+\sigma)}{3}\,\delta - \frac{V_\theta}{3}\,\epsilon + \frac{2V_\theta}{3}\right)\frac{1}{c_s^2} - 2(R+\sigma)V_{\sigma\theta}\frac{\dot{\theta}^2}{M_{\text{eff}}^2} + 3H(R+\sigma)^2\dot{\theta} + V_\theta = 0, \tag{C.23}$$

where $c_s^{-2} \equiv 1 + 4\frac{\dot{\theta}^2}{M_{\text{eff}}^2}$. Moreover, inserting (C.22) in (C.9) we find

$$\frac{6}{M_{\text{eff}}^2}\frac{\dot{\theta}H}{(R+\sigma)}\left(-\frac{V_{\theta\theta}}{3}\frac{\dot{\theta}}{H} - \frac{V_{\theta\sigma}(R+\sigma)}{3}\,\delta - \frac{V_\theta}{3}\,\epsilon + \frac{2V_\theta}{3}\right) - 3V_{\sigma\theta}\frac{\dot{\theta}H}{M_{\text{eff}}^2} - (R+\sigma)\dot{\theta}^2 + V_\sigma = 0. \tag{C.24}$$

Finally, (C.11) becomes

$$3M_{\text{Pl}}^2H^2 = \frac{1}{3}\epsilon_V V + \frac{1}{2}\left[\frac{2}{M_{\text{eff}}^2}\frac{\dot{\theta}}{(R+\sigma)}\left(-\frac{V_{\theta\theta}}{3}\frac{\dot{\theta}}{H} - \frac{V_{\theta\sigma}(R+\sigma)}{3}\,\delta - \frac{V_\theta}{3}\,\epsilon + \frac{2V_\theta}{3}\,\delta\right) - V_{\sigma\theta}\frac{\dot{\theta}}{M_{\text{eff}}^2}\right]^2 + V, \tag{C.25}$$

where ϵ_V, in this context, is defined as $\epsilon_V \equiv \frac{M_{\rm Pl}^2}{2(R+\sigma)^2}\left(\frac{V_\theta}{V}\right)^2$. As θ is a pseudo-Goldstone field, terms involving V_θ (and higher derivatives such $V_{\theta\theta}, V_{\theta\sigma}$, and so on) are proportional to the small explicit symmetry breaking parameter. Moreover, $M_{\rm eff}$, the "effective mass" of the heavy field, is taken to be at least $\gtrsim H$. All in all, it is clear, by comparing (C.23), (C.24), and (C.25) with (C.16), (C.15), and (C.17), respectively, that the slow-roll approximation in the two-field system is self-consistent, as within such a limit, the neglected terms in the equations of motion become very suppressed "corrections" to the approximate equations.

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