ABSTRACT

Title of Thesis:	DOES VISUOSPATIAL WORKING MEMORY MATTER IN MENTAL ARITHMETIC?
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Approximate Arithmetic is a task that requires one to approximate the number of dots in dot arrays to add or subtract pairs of dot arrays. Past work has shown that Approximate Arithmetic is a significant predictor of symbolic mental arithmetic. Approximate Arithmetic is thought to engage processes like Visuospatial Working Memory. Those with higher Visuospatial Working Memory ability are better at Approximate Arithmetic. However, few studies have looked at both Visuospatial Working Memory and Approximate Arithmetic's contribution to variance in mental arithmetic performance. The current study examines the relation between Approximate Arithmetic, symbolic mental arithmetic, and Visuospatial Working Memory. Mediation analyses indicate that the relation between Approximate Arithmetic and mental arithmetic is fully mediated by individual differences in Visuospatial Working Memory. While additional analyses confirm the robustness and dominance of this full mediation model in predicting math ability.

DOES VISUOSPATIAL WORKING MEMORY MATTER IN MENTAL ARITHMETIC?

by

David Samuel Ampofo

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Advisory Committee:
Michael Dougherty, Chair
L. Robert Slevc
Tracy Riggins

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Chapter 1: Introduction

Humans have two representational systems that are thought to support symbolic mental arithmetic ability: A symbolic representation and an Approximate Number System (ANS). The symbolic representation permits domains of math such as calculus. The ANS, which is assumed to be shared across adults, infants, and non-human animals, permits approximation of quantity (Barth, Kanwisher, & Spelke, 2003; Pica, Lemer, Izard, & Dehaene, 2004; Xu & Spelke, 2000; Dehaene, 1997). Representations of numerosity (e.g., the number of pens in a holder) or another magnitude (e.g. height of the pen holder) have the formal properties of a real number. A sense of numerosity is widely thought to emerge from the ANS, which itself is assumed to permit rough calculation (Dehaene, 1997; Merritt, DeWind, & Brannon, 2012).

The ANS is commonly thought to map to exact symbols of number and summarily identified as important for promoting mathematical achievement across the lifespan (Barth et al., 2006; Gilmore & Spelke, 2010). The ANS mapping allows one to compute differences or summations of a number of objects higher than four, resulting in a continuing role of ANS in symbolic numerical representation (Dehaene & Akhavein, 1995; Knops, 2009; Moyer & Landauer, 1967; Temple & Posner, 1998). However, this ANS mapping account still fails to fully address the ‘symbol grounding problem’, in which number symbols acquire meaning (Ansari, 2016). The symbol grounding problem may be better addressed by other accounts that focus on the relationship between symbols and how we improve symbolic mental arithmetic (Reynvoet & Sasanguie, 2016b).

Species ranging from Rhesus macaque monkeys to human children have the ability to create a relative order between numerosities, this forms an integral structure of the ANS (Brannon & Terrace, 2002; Cantlon & Brannon, 2006). The relative ordering of numerosity is thought to be captured by a ‘mental number line’. The mental number line is an abstract logarithmically scaled visuospatial line. The mental number line conforms to cardinal directionality with higher number representing a further position right and vice-versa for lower numbers (Izard & Dehaene, 2008; van Dijck, Gevers, Lafosse, Doricchi, & Fias, 2011). However, as discovered by Siegler and Opfer (2003), children have difficulty estimating numerosity due to an overreliance on a logarithmic representation (Siegler & Opfer, 2003; Opfer & Siegler, 2007; Booth & Siegler, 2008). Participants were given either a 0-100 or 0-1000 number line where they placed a symbol of number to a position on the given line or gave a position to the given line. Participants in the 2^{nd} and 4^{th} grade tended to rely on a logarithmic representation when presented with a range of 0-1000, while 6^{th} grade and adult participants would rely on a linear representation when given a 0-1000 range. Their results indicate that the state of these representations themselves change, conforming to the numerical context that estranges number symbol from referents. Additionally, they provide a possible mechanism that requires to some extent the constructs of a mental number line, the Approximate Number System, and number symbols (Chen & Verguts, 2010; van Dijck et al., 2011).

However, it may be the case that the ANS, if it does exist, does not have as strong a tie to number symbol representations as the mapping account assumes. This possibility falls in line with the finding that participants systematically verbally underestimate the numerosity of dots that have been briefly presented visually (Izard & Dehaene, 2008). Izard and Dehaene (2008) argue dot numerosity underestimation is due to systematic inefficiencies in translating between representation systems, necessitated by the fact that symbolic and non-symbolic representations of number are estranged.

An alternative account of how number symbols gain meaning emphasizes an understanding of the ordinal relationship between symbols rather than symbol and magnitude. This symbol-symbol association account goes hand in hand with research by Lyons and Beilock (2009) who argued for the importance of Working Memory in learning ordinal numeric relationships. Baddeley (2000) proposed that Working Memory is a multi-component attentional system with a short-term memory storage that works on a limited amount of information. It includes a master system, the central executive, and three slave subsystems, the episodic buffer, the phonological loop, and the visuospatial sketchpad. The central executive has a supervising role regulating and controlling cognitive processes run by the slave subsystems. The phonological loop is responsible for retaining verbal information, whereas visuospatial information is maintained within the visuospatial sketchpad (Baddeley, 2000). In this study we explore the possibility that after accounting for Visuospatial Working memory (VSWM) the Approximate Number System does not explain any variance above it.

Ratio, Distance, and Size effect

The ANS is typically measured with a dot array comparison/matching task that ideally controls for continuous magnitude features such as individual dot size, density, and total occupied area. Without this control, numerosity and these continuous features are likely to be correlated. For example, the number of workers in a meeting and the total area they occupy are correlated: the more workers, the more room occupied. Additionally, ANS tasks are characterized by ratio, distance, and size effects. The ratio effect is when performance improves while the ratio between dot arrays are further from 1 (Barth et al., 2003; Cordes, Gelman, & Gallistel, 2001; Pica et al., 2004). Similar effects are observed when adults or children are asked to compare, add, or subtract digits (Moyer & Landauer, 1967; Temple & Posner, 1998). When comparing symbols of number, a distance effect can be observed where deciding whether 9 is greater than 8 (distance of 1) is harder than deciding that 8 is greater than 2 (distance of 6). Additionally, there is also a size effect where larger numbers are harder to compare (8 vs. 9) than smaller numbers (2 vs. 3). This has been interpreted as evidence in favor of the account whereby ‘nine’ and ‘9’ gain their numerical referents by mapping to the ANS.

ANS and Number symbol connection contention

The Approximate Number System is dissociable from the verbal system because it does not require verbal ability (Gilmore & Spelke, 2010). Despite the intuitive connection, the relationship between the ANS and higher-order number symbols is contentious. Computational modeling has shown that overlapping representations are not required to obtain the distance and size effects and that they may be due to simple network properties (Opstal & Verguts, 2011; Verguts & Fias, 2005).

Complementing this conclusion, Briere and Campbell (2016) find that as stimuli display time decreases from 2000 ms to 500 ms, performance accuracy based on Visual Working Memory increased. They used a dot subtraction task found in Pica et al., (2004) where two dot arrays were mentally subtracted and then matched or compared. The dot subtraction task is an Approximate Arithmetic task and at face value indicates that efficiency in Visual Working Memory permits better enumeration of these dot arrays. Thus, Briere and Campbell (2016) posit that Visual Working Memory is necessary to maintain the visuospatial information to mentally subtract the dot arrays.

It should be noted that there are other strategies that accomplish the computations present in the dot subtraction task. This is seen in Butterworth et al. (2011) where participants were shown an array of tokens that were subsequently covered and then a second array of tokens was placed under the cover. Using a “pattern strategy”, participants were successfully able to reproduce the spatial pattern of the combined arrays of tokens. This strategy is presumably dependent on some visuospatial ability to organize a representation of the tokens.

Ordinal information processing

While it is possible to accomplish lower-order arithmetic using alternative strategies such the pattern strategy higher-order arithmetic requires an understanding of associations between symbols (Lyons & Beilock, 2009). Ordinal information provides the associative building blocks for the system of symbol-symbol relations that underlie complex math in general (Nieder & Dehaene, 2009). The process connecting symbol to numerical identity is nontrivial and is overshadowed by the relationship between symbols (Lyons, Ansari, & Beilock, 2012). Lyons et al. (2012) argue that numerical symbols operate primarily as an associative system where relations between symbols come to overshadow those between symbols and their numerical identity they refer to. Participants compared quantity in three conditions: dot-numeral, numeral-number word, and a mixed format. A greater performance cost was found for mixing numeral and dot array than numeral and number word. This highlights the disassociation of the approximate sense of numerosity and number symbol thought to represent it.

Similarly, Lyons and Beilock (2011) find that Number Symbol Ordering ability fully mediated the relationship between ANS and math ability (Lyons & Beilock, 2011). During a trial, digits in groups of three appeared together with three triads at any one time on the screen. The participant had to click on the triads to reorder them into the prescribed direction by order. For triads moving left to right, the goal was a descending order, and vice-versa for triads moving right to left ascending. Lyons and Beilock (2011) argue that number symbol ordering, mediates the relationship between ANS and math in part because of ordinal information processing. In sum, understanding ordinal relationships may be the main driving force behind the Approximate Number System and Working Memory predicting math ability.

Approximate Arithmetic vs. Visuospatial Working Memory

Approximate Arithmetic, a task that is assumed to measure ANS, is an important core skill necessary for engaging in symbolic mental arithmetic (Park & Brannon, 2013). Additionally, Knops' (2009) findings suggest that Approximate Arithmetic involves a spatially organized mental representation of numbers that dynamically shifts while performing an operation (Knops, 2009).

In these terms, Approximate Arithmetic is not all that different from standard measures of Visuospatial Working Memory. For example, in the symmetry span task, the participant is shown a series of grid locations one-by-one in a 4 by 4 grid. The participant must remember the grid location and the order that the presented dots appear. After each sequence of dots is presented, the participant is shown a spatial pattern that is either symmetrical along the vertical axis or not. The participants' task is to simply decide whether the array is symmetrical. After the symmetry judgment, participants are asked to recreate the sequence of dots as they appeared in the 4 by 4 grid. Similarly, in Approximate Arithmetic, the participant must retain visuospatial information while engaging in a secondary processing task. The similarity of the cognitive operations needed to perform both the Approximate Arithmetic task and the Visuospatial Working Memory task raises the possibility that both share common variance. At a minimum, it is reasonable to assume that the Approximate Arithmetic task requires Working Memory. If this is the case, then one's performance on Approximate Arithmetic will likely reflect both an underlying process in VSWM and more importantly symbolic arithmetic. Regarding symbolic arithmetic, it is necessary to tease these two sources of variance apart to understand the underlying cognitive processes responsible for mathematical ability. Although mathematical ability has been shown to relate to both ANS and VSWM, there has been relatively little work that has studied the three constructs together. This is potentially problematic because it is possible that measures of Approximate Arithmetic may share variance with measures of Visuospatial Working Memory.

Current Study

To date, the ANS mapping account has dominated numerical cognition in answering the symbol grounding problem and in describing how one develops symbolic arithmetic ability. This has resulted in an underdevelopment in alternatives that might better fit with the literature. Frank and Barner (2011;2012) provide a case for Visual Working Memory being the intermediary structure for processing of higher representations of number (Frank & Barner, 2012; Frank, Barner, Brady, Brooks, & Carey, 2011). However, few studies have examined the possibility of Approximate Arithmetic and Visuospatial Working Memory ability explaining the same variance in symbolic arithmetic. One study by Meyer et al., (2010), examined the relationship between Visuospatial Working Memory and mental arithmetic in young children. Meyer et al. (2010) found that the central executive and phonological components predicted mathematical reasoning in 2nd graders while visuospatial predicted it for 3rd graders. They suggest that early development requires the former while later development requires the latter through a cortical shift from prefrontal to parietal while acquiring mathematical ability (Meyer et al., 2010). Children and adults then may differentially represent exact symbols of number and this difference may be domain-specific to visuospatial abilities that have yet to be developed. Thus, we investigate whether Visuospatial Working Memory is useful in predicting variance in adult math ability.

Chapter 2: Method

Participants, Exclusion Criterion, and Reproducibility.

A minimum sample size of 102 undergraduate students from the University Maryland (UMD) was determined by power analysis based on suggestions by Cohen, Cohen, West, & Aiken, (2003). A total of 113 participants ultimately participated in the study. We removed one participant from the analysis for not completing the Symmetry span task, resulting in 112 usable participants. All participants received 1-hour credit toward course requirements for their participation in the study. The research protocol was approved by the University of Maryland Institutional Review Board (see Appendix A).

Study data in raw and processed form (CSV master data sheet), processing and analysis RMarkdown scripts, and task scripts can be found at the:

- Open Science Framework page https://osf.io/djnhz/
- Github https://github.com/davidpofo/Approximate_Arithmetic_study

Materials

Approximate Arithmetic task

Participants add or subtract large numerosities of visually presented dot arrays without counting. Then were cued to mentally add or subtract two numerical quantities, ranging from 9 to 36, represented in dot arrays. Finally, they were then asked to either compare the sum or the difference with a numerical quantity represented in a third dot array (compare trials) or to choose one of two dot arrays that matched the sum or the difference in number (match trials). The two trial types were used to minimize the development of task-specific strategies. Dot arrays were shown for 1000 ms for the first two dot arrays and 1500 ms for the dot arrays to be compared or matched. This brief timeframe is necessary to prevent participants from counting. Dot size was homogeneous within an array but differed across arrays to prevent participants from relying on total surface area to make judgments. Finally, participants respond with a mouse click, and are subsequently provided feedback after each trial.

Symmetry Span task

The participant is shown a series of grid locations one-by-one from the 4 by 4 grid in the center of the screen. The participant must remember the grid location and the order that the presented dots appear. After each grid is shown the participant is shown an 8 by 8 grid that has several grids filled black to form a pattern. The pattern was either symmetrical along the vertical axis or not, and the participant made this judgement using the left/right arrow keys before the next grid display.

Shape Builder task

In this task during 2 practice trials and 24 test trials participants see a 4 by 4 grid wherein between 2 and 4 shapes sequentially appear for 500 ms. They were tasked with remembering the order, spatial position, color, and shape of each shape presented. After the final shape is presented, they recreated the sequence by clicking on the correct colored shape and dragging it to the appropriate spatial position (Atkins et al., 2014).

Modular Arithmetic task

Modular Arithmetic is a measure of math ability that asks participants to use two operations of everyday math to solve it (i.e. subtraction, division). This truth value judgement problem provides individual differences in adult populations regardless of math expertise. This lack in expertise is important because we wanted to capture the ability to perform math, holding extraneous variables such as classes taken, work experience, or anything not related to the mental computations required to perform this task. Additionally, it has multiple levels in working memory with large numbers and equations with borrowing functions taking more capacity (Beilock & Carr, 2005).

This task involves judging the truth value of equations such as 34=22 (mod 4). To solve such equations, the second number is subtracted from the first (i.e., 34 – 22), and this difference is then divided by the last number, the mod, (i.e., 12 /4). If the resultant is a whole number (here, 3), the statement is True. Problems with remainders are considered False (i.e., 3.5). Half of the Modular Arithmetic equations presented to participants were “true,” and the rest were “false.” Additionally, each “true” problem has a “false” correlate that only differed as a function of the number involved in the mod statement. If the “true” problem 41≡19 (mod 2) was presented, then a “false” correlate problem 41≡19 (mod 3) was also presented at some point in the same problem set. There is a practice set of 8 problems half-low and half-high demand providing accuracy feedback after each answer. Problems are considered low demand if they require a single-digit no borrow subtraction operation (e.g., 7≡2 (mod5)). The difference between the first two numbers always falls between 1 and 8 ensuring each low-demand problem has no borrowing function (Beilock & DeCaro, 2007). The high-demand problems require a double-digit borrowing operation (e.g., 43≡16 (mod3)). The difference between the first two numbers is always between 15 and 49 for all high-demand problems. To ensure high-demand, no number within the equation of all the problems used contained a zero in the single digits place as no borrowing across zeroes can occur.

Participants completed the test problem set of 40 problems, each consisted of 20 low-demand and 20 high-demand separately presented for 1000 ms after the 500 ms fixation cross. The differences between the first two numbers for all problems have a similar even/odd ratio for each possible problem (i.e. 20 odd, 20 even). This prevented relying on a strategy that focuses on the difference between odd or even aiding overall performance. For both sets, problems were presented once in a different random order and counterbalanced across participants.

Procedure

This study took place in a 1-hour session at the Decision, Attention and Memory lab at the University of Maryland. During the session participants completed four tasks in the following order: (1) Approximate Arithmetic (2) Symmetry span, (3) Shape Builder and (4) Modular Arithmetic problems. Participants entered a private, sound-muted computer room to perform all tasks on a desktop. There were no breaks taken in-between except to go over instructions for each task.

Power Analysis

Using Cohen et al's (2003) method for computing power the sample size required to achieve power of 80% is 102. The power analysis was informed by the overall R^2 from the multiple regression analysis in Lyons and Beilock (2011; Table 1). However, Lyons and Beilock's sample was small (N=54) yet achieved a rarely seen partial R^2 of -.552 in the social sciences for their mediator. Thus, we assumed a conservative R^2 guided by

Cohen et al’s (2003) suggested effect sizes for power analysis. The assumed R^2 was arbitrarily set at $R^2 = .08$. Setting alpha = 0.05, and power= 0.80, we derived the estimated sample sizes for both scenarios:

$\boldsymbol{N_{needed}} = L / f^2 + k_a + k_b + 1$

$f^2 = \frac{sr^2}{1-R^2} = \frac{-.552^2}{1-.08^2} = .3066$

$\boldsymbol{N_{needed}} = \frac{7.85}{.3066} + 1 + 1 + 1 = 25.603 \rightarrow 26$

<u>Cohen suggested:</u>

Small effect $f^2 = .02$

Medium effect $f^2 = .15$

Large effect $f^2 = .35$

$$\boldsymbol{N_{needed}} = \frac{7.85}{.08}\ 1+1+1 = \mathbf{98.125 + 3 = 101.125 \rightarrow 102}$$

L is a tabled value corresponding to a specific power value, k is the number of predictors in the regression equation; f is an effect size measure for ordinary least squares regression in this case, is equal to the regression coefficients used see (Appendix B for L table). Thus, this study at minimum requires 102 participants to achieve a power level of 80% with an alpha level of 0.05.

Chapter 3: Data Analytic Approach & Results

The a priori sample goal was 102, data were analyzed both using the full dataset and using the first 102. All conclusions based on n = 102 replicated the conclusions based on n = 112 (See Appendix C for analysis with 102). Therefore, only analyses using all 112 subjects are included in the analyses below.

Hypotheses

The following three hypotheses where of interest:

Hypothesis 1: Without taking into account Visuospatial Working Memory there will be a significant relationship between Approximate Arithmetic and Modular Arithmetic. This hypothesis is tested with a simple regression model, regressing Modular Arithmetic on Approximate Arithmetic.

Hypothesis 2: The relationship between Approximate Arithmetic and Modular Arithmetic will be fully mediated by Visuospatial Working Memory. This hypothesis is tested in two ways. First, we use Baron and Kenny's mediation approach and estimate paths between Approximate Arithmetic, Modular Arithmetic, and Visuospatial Working Memory. Full mediation is obtained when the effect between Modular Arithmetic and Approximate Arithmetic is not significant (c' path), while the paths between Approximate Arithmetic and Visuospatial Working Memory (a path) and Visuospatial Working Memory and Modular Arithmetic (b path) are significant.

Hypothesis 3: There will be a positive correlation between Modular Arithmetic and the other three tasks.

Mediation Analyses

Baron and Kenny's Approach

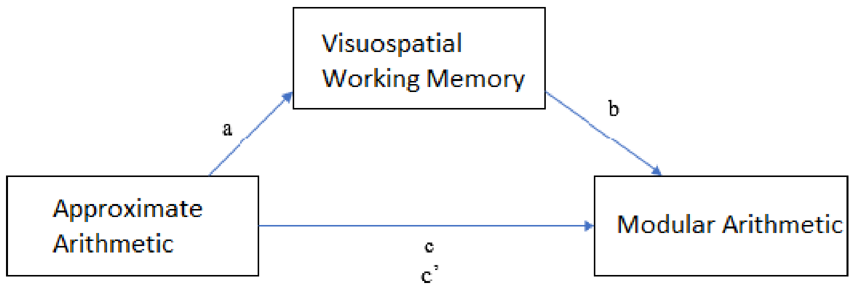

Figure 1. Mediation Path Diagram between Approximate Arithmetic, Visuospatial Working Memory, and Modular Arithmetic

To test mediation, we employed Baron and Kenny's (1986) causal steps approach that comprise three conditions to establish mediation: Condition 1: The independent variable must affect the mediator. (path a, Table 2) Condition 2: The independent variable must affect the dependent variable. (path c, Table 1) and Condition 3: The mediator must affect the dependent variable when the dependent variable is regressed on the mediator and independent variable together (path b and c', Table 3; Baron & Kenny, 1986). Through this mediation analysis we seek to explain the mechanism underlying the positive relation between approximate and symbolic arithmetic. In this framework, we ask whether there is a significant indirect effect (quantified as the product of the unstandardized path coefficients, a and b) of the mediator (Visuospatial Working Memory) that accounts for some portion of the direct effect c observed between the original predictor (Approximate Arithmetic) and the outcome (Modular Arithmetic) variables. The remaining (unmediated) direct effect is denoted c'. The model is constrained by the assumption that c = ab + c'. Unlike in a standard multiple regression analysis, we are explicitly asking what portion of the relation between Approximate

Arithmetic acuity and Modular Arithmetic can be accounted for by the mediating variable (Visuospatial Working Memory). The expected result should indicate full (ab is significant but c' is not) as opposed to partial (when both ab and c' remain significant) mediation. The construct of Visuospatial Working Memory was created by Z-score compositing the proportion correct from Symmetry Span and standardized percent correct from Shape Builder with the 'composite' function in the R package 'multicon'. (Ryne A. Sherman 2015)

Using the 'apa.reg.table' function from the R package 'apaTables' we produced three regression tables (Tables 1-3) that makeup the mediation paths for Baron and Kenny's approach (Stanley, 2018). For Condition 1 we find at the alpha level criterion of .05 the direct effect between Approximate Arithmetic and Modular Arithmetic to be significant (t (110) = 2.484, p = 0.01448, *B* =.39, se=0.16). For Condition 2 we find at the alpha level criterion of .05 the direct effect between Approximate Arithmetic and Visuospatial Working Memory to be significant (t (110) = 3.947, p = 0.00014, *B* =3.88, se=0.98). For Condition 3 we find at the alpha level criterion of .05 the direct effect between Approximate Arithmetic and Modular Arithmetic to be non-significant (t (109) = 0.658, p = 0.512, *B* =0.10, se=0.15) whereas the mediation path ab is significant (t (109) = 5.585, p =1.74e-07, *B* =0.07, se=0.01) indicating full as opposed to partial mediation. The full mediation was expected given previous work by Lyons and Beilock (2012) that also showed full-mediation by a similar task that depends on active manipulation of numerical representations (number ordering) (Lyons & Beilock 2012).

Table 1.
Path c, Regression results using Modular Arithmetic Accuracy as the criterion

	B	SE	Beta	sr2	r	Fit
(Intercept)	0.41**	0.12				
Approximate Arithmetic Accuracy	0.39*	0.16	0.23	.05	.23*	
						R2 = .053 F (1, 110) = 6.17

Table 2.
Path a, Regression results using Visuospatial Working Memory Accuracy as the criterion

	B	SE	Beta	sr2	r	Fit
(Intercept)	-3.07**	0.78				
Approximate Arithmetic Accuracy	3.88**	0.98	0.35	.12	.35**	
						R2 = .124 F (1, 110) = 15.58

Table 3.
Path c' & b, Regression results using Modular Arithmetic Accuracy as the criterion

	B	SE	Beta	sr2	r	Fit
Model 1						
(Intercept)	0.64**	0.12				
Approximate Arithmetic Accuracy	0.10	0.15	0.06	.00	.23*	
Visuospatial WM Accuracy	0.07**	0.01	0.49	.21	.51**	
						R2 = .264 F (2, 109) = 19.53

Note. * indicates $p < .05$; ** indicates $p < .01$. A significant *b*-weight indicates the beta-weight and semi-partial correlation are also significant. *b* represents unstandardized regression weights; *SE* represents the standard error of the unstandardized regression weights; *beta* indicates the beta-weights or standardized regression weights; sr^2 represents the semi-partial correlation squared; *r* represents the zero-order correlation.

To visualize our results we used the 'corrgram' package to graphically display a correlation matrix and 'mediate.diagram' function from the R 'psych' package to view the unstandardized regression paths (Figure 2 and 3; Friendly, 2002, Hayes, 2013, respectively)

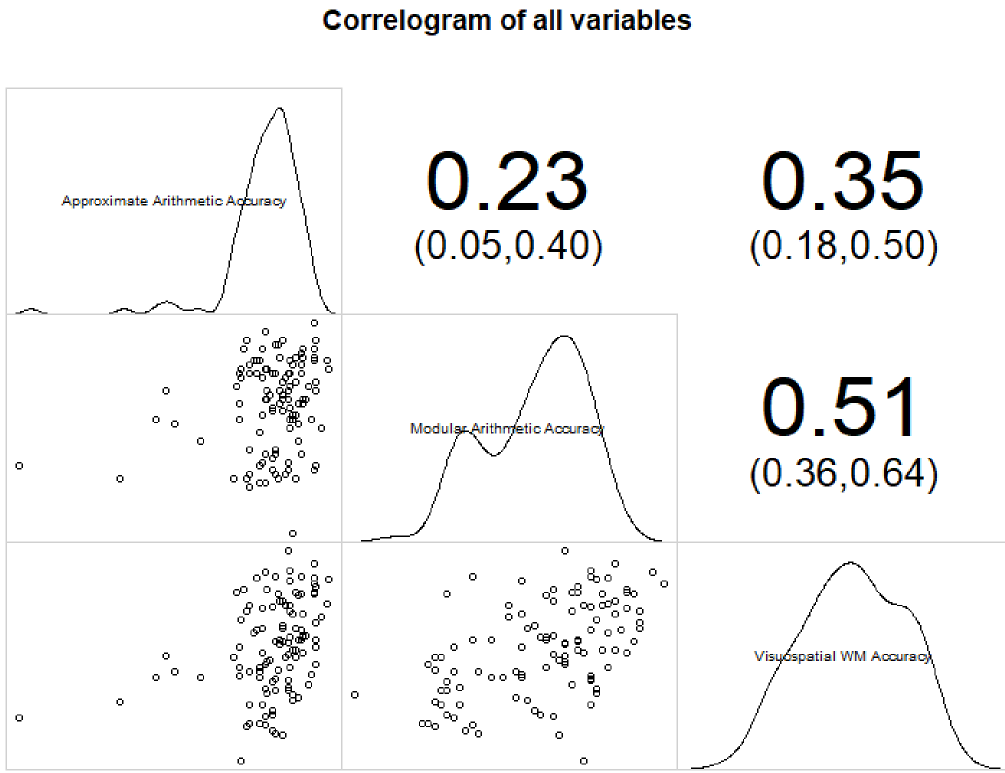

Figure 2. Correlations and their distributions for all variables.

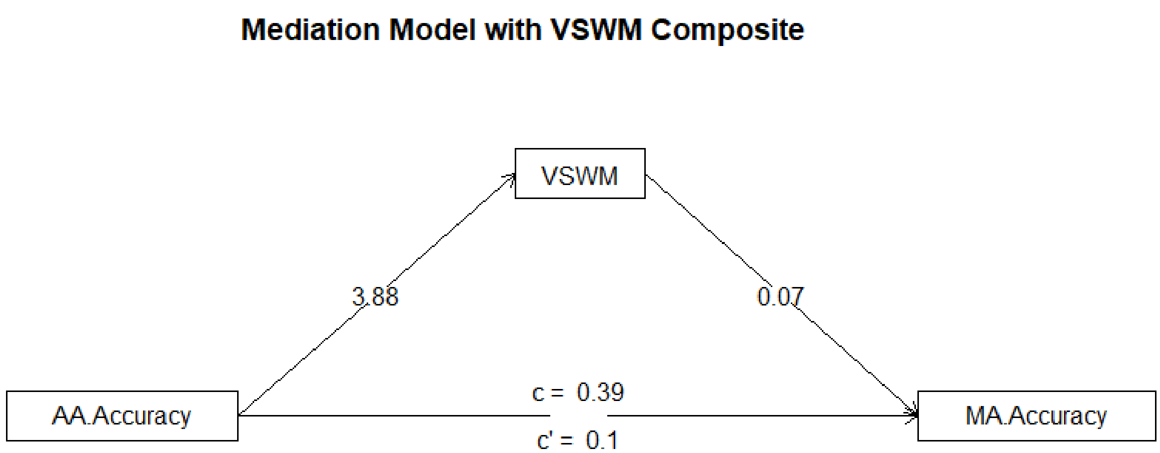

Figure 3. Mediation model with Approximate Arithmetic (AA Accuracy) as a predictor, Visuospatial Working Memory (VSWM) as a mediator, and Modular Arithmetic (MA Accuracy) as an outcome with unstandardized regression coefficients.

Empirical M-test

Baron and Kenny's approach has fallen out of favor because of its bad balance of power and Type 1 error control. To overcome these obstacles, we took the a and b coefficients and standard errors from the analysis and performed an Empirical M-test that determines the empirical sampling distribution of the ab product (not assuming normality). The direct effect between Approximate Arithmetic and Modular Arithmetic accuracy was completely mediated by Visuospatial Working Memory (VSWM). As Table 2 indicates, the unstandardized regression coefficient between Approximate Arithmetic accuracy and VSWM was statistically significant, as was the unstandardized regression coefficient between VSWM and Modular Arithmetic. The unstandardized indirect effect was (3.88) (.07) = .29. We tested the significance of this indirect effect using the Empirical M-test bootstrapping procedure (Figure 4). The bootstrapped unstandardized indirect effect was .289 (SE=0.091). The indirect estimate had a 95% confidence interval lower limit of 0.129 and upper limit of 0.483. Thus, the indirect effect was statistically significant as the confidence interval range did not contain zero.

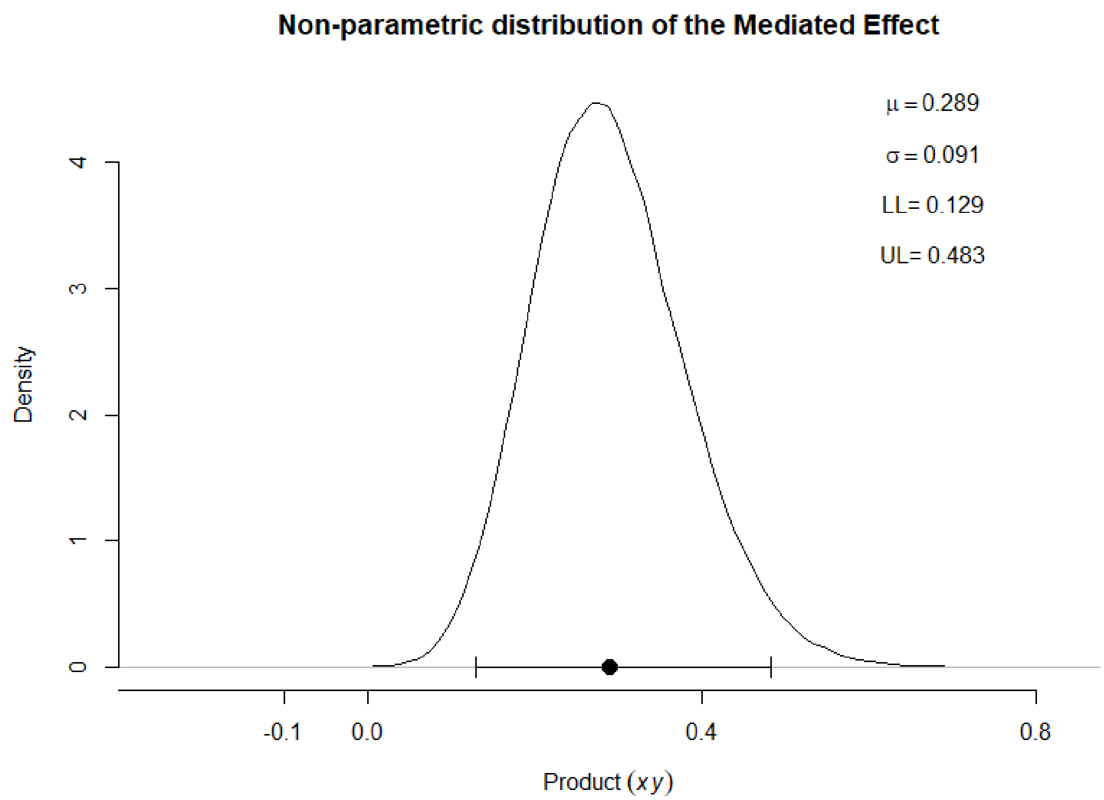

Figure 4. Non-parametric bootstrapping of the ab product with a Confidence Interval (Empirical M-test) where μ is the point estimate of the indirect effect (ab product), α is the standard error of the indirect effect, LL is the lower limit, and UL is the upper limit.

Bayesian Model Comparison

Using the Bayesian model comparison, we contrasted models allowing the strength of the evidence indicated by the Bayes Factor to speak for itself. The Bayes factor is a measure of the relative evidence from the data allowing for null and essentially infinite alternative hypotheses to be tested. This allows us to state the strength of evidence for the null, that in the Frequentist framework is not possible (only the retention or rejection of the null). The Bayes factor provides the probability of the data occurring under any specific hypothesis relative to any alternative hypothesis, including the null. A general guideline for interpretation puts a Bayes Factor ratio above 1 to 3 as not worth a mention, 3 to 20 as positive evidence, 20 to 150as strong evidence, and 150+ as very strong evidence in favor of the numerator model ((Kass & Raftery, 2012), Table 4).

Table 4.

Bayes Factor Interpretation where K is Bayes Factor[a]

2 ln *K*	*K*	Strength of evidence
0 to 2	1 to 3	Not worth more than a bare mention
2 to 6	3 to 20	Positive
6 to 10	20 to 150	Strong
>10	>150	Very strong

[a](Kass & Raftery, 2012)

For our purposes, the following comparison sets are necessary. The first comparison was between the direct effect model (M1) containing Approximate Arithmetic as a predictor over the null model (M0) predicting Modular Arithmetic (BF_{M1}/BF_{M0}). The ratio of these models indicate positive evidence for the direct effect of Approximate Arithmetic when predicting Modular Arithmetic ($BF_{M1}/BF_{M0} = 3.10$) confirming the c path in the mediation analysis. The second comparison against the null model contains only Visuospatial Working Memory as a predictor (M2). M2 achieved a BF ratio well above 150 in favor of M2 ($BF_{M2}/BF_{M0} =1135001.86$) indicating very strong evidence in favor of the single predictor model.

For our third comparison we used both predictors in a third model (M3) over the null model. For the comparison between M3 and M0 we find very strong evidence in favor of M3 ($BF_{M3}/BF_{M0} = 243409.22$). Against the single predictor model M1 there was strong evidence against the two-predictor model ($BF_{M1}/BF_{M3} = 2.70e-06$) confirming the 'c' path. In line with the 'b path' of the mediation analysis there is a positive evidence in favor for the model using only Visuospatial Working Memory as a predictor of Modular Arithmetic compared to a model that incorporated Approximate Arithmetic ($BF_{M2}/BF_{M3} = 4.66$). Our last comparison confirms the 'a path' with very strong evidence that

Approximate Arithmetic predicts Visuospatial Working Memory performance (BF=162.71).

Table 5.
Bayesian Model Comparisons for Modular Arithmetic (BF10) where Null is the Null Model and VSWM is the Visuospatial Working Memory predictor

Comparison	BFRatio
Approximate Arithmetic / Null	BFM1 / BFM0 = 3.10
VSWM / Null	BFM2 / BFM0 =1135001.86
Approximate Arithmetic + VSWM / Null	BFM3 / BFM0 = 243409.21
Approximate Arithmetic / Approximate Arithmetic + VSWM	BFM1 / BFM3 = 2.70e-06
VSWM / Approximate Arithmetic + VSWM	BFM2 / BFM3 = 4.66

Robustness Analysis

A robustness analysis was run to verify that the above conclusions were not dependent on extreme scores. The ‘gvlma’ package provides both a global test statistic and tests for violations of normality of errors (skewness, kurtosis, heteroscedasticity) and linearity (link function) for ordinary least-squares regression. The results of these tests are in Table 6. None of the tests were significant, indicating that model assumptions were not obviously violated. However, two problematic subjects (14 and 84) were identified using the ‘deletion.gvlma’ function in the ‘gvlma’ R package (Table 7; Pena & Slate, 2006). After reviewing study notes there was nothing remarkable noted during the procedure that might indicate why these observations violated linear model assumptions. Nevertheless, all statistical conclusions provided above were unchanged when data were re-analyzed without these two problematic scores.

Table 6.
Assessment of the linear model assumptions. Using the Global test on 4 degrees of freedom: level of significance 0.05.

	Value	P-value	Decision
Global Stat	3.77	0.43	Assumptions Acceptable.
Skewness	2.66	0.10	Assumptions Acceptable.
Kurtosis	0.80	0.37	Assumptions Acceptable.
Link Function	0.03	0.86	Assumptions Acceptable.
Heteroscedasticity	0.32	0.57	Assumptions Acceptable.

Table 7.
Dominance Analysis Deletion table: two outliers identified with the global stat criterion.

Subject number	Delta Global Stat (%)	Global Stat p-value
14	48.33	0.23
84	33.78	0.28

<u>Dominance Analysis</u>

A dominance analysis was run to evaluate each predictor's validity across possible models using the 'dominance' function from the 'yhat' package (Nimon & Oswald, 2013). Dominance analysis assesses the individual importance of each variable by estimating the independent contribution of each variable in terms of total variance accounted for. Dominance analysis is useful when there is collinearity amongst predictor variables (Budescu, 1993; Nathans, Oswald, & Nimon, 2012). Of the total variance explained (.263) Approximate Arithmetic uniquely explained .028, whereas Visuospatial Working Memory explained the lion's share at .235. Visuospatial Working Memory has achieved complete dominance over Approximate Arithmetic because its contribution is greater across the average of all sub models as compared with the Approximate Arithmetic.

Chapter 4: Discussion

The goal of the present study was to determine if the variance in Modular Arithmetic that is attributed to Approximate Arithmetic is unique from that of Visuospatial Working Memory. It was hypothesized that Approximate Arithmetic would significantly predict variance in Modular Arithmetic (Hypothesis 1). This hypothesis is based on literature suggesting a mapping of numerical meaning to symbol is accomplished through the Approximate Number System that this task measures (Libertus, Odic, Feigenson, & Halberda, 2016; Reynvoet & Sasanguie, 2016a). This hypothesized effect, as predicted, was confirmed by the observation that a sizable amount of variance in Modular Arithmetic was explained by Approximate Arithmetic. Though, as Hypothesis 2 predicted this relationship was fully mediated by Visuospatial Working Memory rendering variance explained by Approximate Arithmetic non-significant. This follows given the positive correlation found between the three tasks with the highest being between VSWM and MA (.51), the next between VSWM and AA (.35), and finally between AA and MA (.23) as seen in Figure 2. Overall participants were more accurate in Approximate Arithmetic than the other tasks, while the other two had slight bi-modal or nearing normal distributions (Figure 2).

As reviewed in the introduction, previous studies have found significant individual differences in the ability to approximate the numerosity of dots (Izard & Dehaene, 2008). However, this ability to approximate dots has been suggested to be grounded in low-level visual parameters (Gebuis & Reynvoet, 2012). This suggests then, that all its variance explained can be covered by a more complex system such as Working Memory while further predicting performance in Modular Arithmetic.

Past numerical cognition work has relied heavily on parametric approaches to null hypothesis significance testing. We address this issue in two ways. First, we non-parametrically bootstrapped the product of the a and b paths with the addition of a confidence interval (Empirical M-test) to test for mediation. Second, we supplemented our analysis with a Bayesian Model Comparison providing robustness through differing school of inferences. Both supplemental analyses confirm the finding that the model with only Visuospatial Working Memory best explains variance in Modular Arithmetic as seen in the simple mediation analysis.

Aside from these analytical differences, a methodological advantage that Approximate Arithmetic has over most ANS tasks is a control for continuous magnitude features (i.e., individual dot size, density, and total occupied area). This is important because as Gebius and Reyvonet (2012) find, non-symbolic number processes rely on multiple visual cues calling into question the necessity of the ANS in number processing (Gebuis & Reynvoet, 2012). Concurrently, the current study has provided clear evidence that Visuospatial Working Memory robustly explains variance in Modular Arithmetic above and beyond Approximate Arithmetic. This conclusion follows the symbol-symbol association account in which understanding the relationship between number symbols is more integral than between symbol and magnitude (Lyons et al., 2012).

Domain-Generality of Mediators

This study was inspired by Lyons and Beilock's (2011) work in which they found that Approximate Arithmetic significantly predicted everyday mental arithmetic and was subsequently fully mediated by Number Symbol Ordering ability. The Number Symbol Ordering task carries the integral function of ordering information found in virtually all tasks considered to measure working memory. The construct of Visuospatial Working Memory, we suggest, captures this domain-general function.

Limitations and Future Directions

The present study suggests that Visuospatial Working Memory mediates the relationship between Approximate Arithmetic and Modular Arithmetic. One limitation of this study, however, is that math was measured using the Modular Arithmetic task. While this task has been used previously in other studies of math ability, it is an unusual task that does not reflect everyday mathematics. Thus, one question concerning the results is whether the observed effects would generalize to other measures of math ability. Future research should attempt to replicate this study using a set of three or more different measures of math ability. A second limitation is that math ability and Approximate Arithmetic were each measured with only a single task. One problem with this approach is that relationship between task performance can reflect task-specific characteristics, rather than latent cognitive abilities. A final limitation was the lack of a formal measure of Number Symbol Ordering that could be used to make concrete comparisons between the current work and Lyons and Beilock's (2011) related work. Where we lacked a Number Symbol Ordering task Lyons and Beilock (2011) lacked measures of Working Memory. Although we accounted for a different mediator between the direct relationship between the ANS and math, a measure of Number Symbol Ordering would ensure a more complete comparison between studies. The Number Symbol Ordering task inclusion would allow for independent assessment of possible task-dependent variance explained. In this instance it would allow the investigator to test whether their results were separately mediated by Number Symbol Ordering ability or if that variance explained is eclipsed by Visuospatial Working Memory.

Summary

In sum, the present work demonstrated that Visuospatial Working Memory does matter in mental arithmetic. Visuospatial Working Memory fully mediated the direct effect between Approximate Arithmetic and Modular Arithmetic. The full mediation by Visuospatial Working Memory provides support for the importance of domain-general cognitive abilities in mental arithmetic. However, this is conclusion limited by the scope of tasks measured. Future research could address these limitations by using a factor-analytic approach that involves multiple measures for Working Memory, mathematical ability, Approximate Arithmetic, and possibly other cognitive abilities (e.g. Number Symbol Ordering).

Appendices

Appendix A

UNIVERSITY OF MARYLAND
INSTITUTIONAL REVIEW BOARD

1204 Marie Mount Hall
College Park, MD 20742-5125
TEL 301.405.4212
FAX 301.314.1475
irb@umd.edu
www.umresearch.umd.edu/IRB

DATE:	February 12, 2018
TO:	David Ampofo, B.A. Psychology & Biology
FROM:	University of Maryland College Park (UMCP) IRB
PROJECT TITLE:	[951779-4] A Cognitive Symbol Study
REFERENCE #:	
SUBMISSION TYPE:	Amendment/Modification
ACTION:	APPROVED
APPROVAL DATE:	February 12, 2018
EXPIRATION DATE:	November 7, 2018
REVIEW TYPE:	Expedited Review
REVIEW CATEGORY:	Expedited review category # 7

Thank you for your submission of Amendment/Modification materials for this project. The University of Maryland College Park (UMCP) IRB has APPROVED your submission. This approval is based on an appropriate risk/benefit ratio and a project design wherein the risks have been minimized. All research must be conducted in accordance with this approved submission.

Prior to submission to the IRB Office, this project received scientific review from the departmental IRB Liaison.

This submission has received Expedited Review based on the applicable federal regulations.

This project has been determined to be a Minimal Risk project. Based on the risks, this project requires continuing review by this committee on an annual basis. Please use the appropriate forms for this procedure. Your documentation for continuing review must be received with sufficient time for review and continued approval before the expiration date of November 7, 2018.

Please remember that informed consent is a process beginning with a description of the project and insurance of participant understanding followed by a signed consent form. Informed consent must continue throughout the project via a dialogue between the researcher and research participant. Unless a consent waiver or alteration has been approved, Federal regulations require that each participant receives a copy of the consent document.

Please note that any revision to previously approved materials must be approved by this committee prior to initiation. Please use the appropriate revision forms for this procedure.

All UNANTICIPATED PROBLEMS involving risks to subjects or others (UPIRSOs) and SERIOUS and UNEXPECTED adverse events must be reported promptly to this office. Please use the appropriate reporting forms for this procedure. All FDA and sponsor reporting requirements should also be followed.

All NON-COMPLIANCE issues or COMPLAINTS regarding this project must be reported promptly to this office.

- 1 -

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University of Maryland Institutional Review Board approval letter.

Appendix B

L Values for $\alpha = .05$

k_B \ Power	.10	.30	.50	.60	.70	.75	.80	.85	.90	.95	.99
1	.43	2.06	3.84	4.90	6.17	6.94	7.85	8.98	10.51	13.00	18.37
2	.62	2.78	4.96	6.21	7.70	8.59	9.64	10.92	12.65	15.44	21.40
3	.78	3.30	5.76	7.15	8.79	9.77	10.90	12.30	14.17	17.17	23.52
4	.91	3.74	6.42	7.92	9.68	10.72	11.94	13.42	15.41	18.57	25.24
5	1.03	4.12	6.99	8.59	10.45	11.55	12.83	14.39	16.47	19.78	26.73
6	1.13	4.46	7.50	9.19	11.14	12.29	13.62	15.26	17.42	20.86	28.05
7	1.23	4.77	7.97	9.73	11.77	12.96	14.35	16.04	18.28	21.84	29.25
8	1.32	5.06	8.41	10.24	12.35	13.59	15.02	16.77	19.08	22.74	30.36
9	1.40	5.33	8.81	10.71	12.89	14.17	15.65	17.45	19.83	23.59	31.39
10	1.49	5.59	9.19	11.15	13.40	14.72	16.24	18.09	20.53	24.39	32.37

The L table allows one to compute sample size needed by hand. L is a function of the number of tested predictors k_B and power desired. (Cohen et al., 2003)

Appendix C

Table 1.

Regression results using Modular Arithmetic as the criterion

	B	SE	beta	sr^2	r	Fit
Model 1						
(Intercept)	0.41**	0.13				
Approximate Arithmetic	0.40*	0.16	0.24	.06	.24*	
						R^2 = .058 F (1, 100) = 6.15

Table 2.

Regression results using Visuospatial Working Memory as the criterion

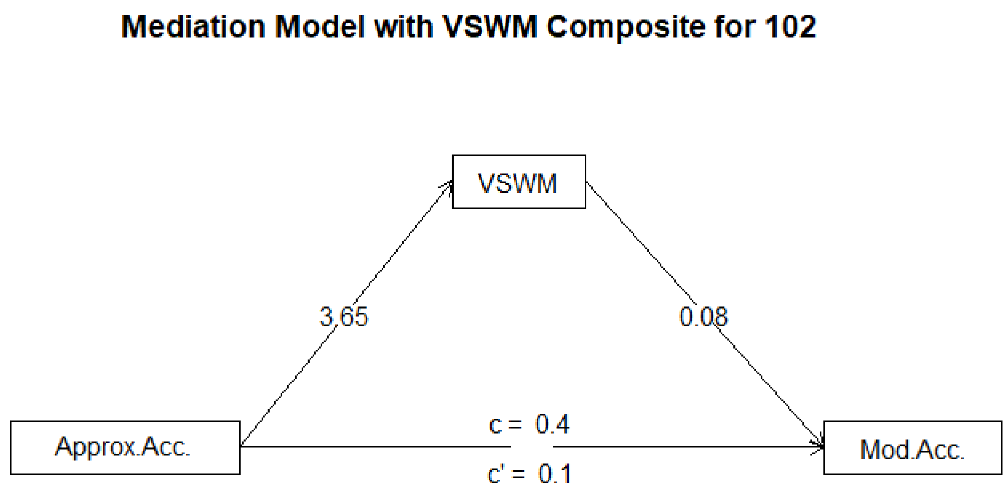
	B	SE	beta	sr2	r	Fit
Model 1						
(Intercept)	-2.89**	0.80				
Approximate Arithmetic	3.65**	1.00	0.34	.12	.34**	
						R2 = .117 F (1, 100) = 13.29

Table 3.

Regression results using Modular Arithmetic as the criterion

	B	SE	beta	sr2	r	Fit
Model 1						
(Intercept)	0.64**	0.12				
Approximate Arithmetic	0.10	0.15	0.06	.00	.24*	
Visuospatial Working Memory	0.08**	0.01	0.53	.24	.55**	
						R2 = .301 F (2, 99) = 21.35

Note. * indicates p < .05; ** indicates p < .01. A significant *b*-weight indicates the beta-weight and semi-partial correlation are also significant. *b* represents unstandardized regression weights; *SE* represents the standard error of the unstandardized regression weights; *beta* indicates the beta-weights or standardized regression weights; sr^2 represents the semi-partial correlation squared; *r* represents the zero-order correlation.



Baron and Kenny’s (1986) approach to mediation analysis for the first 102 participants.

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