



SELECTING AND SCHEDULING IMPROVEMENTS IN URBAN  
TRANSPORTATION NETWORKS USING METAHEURISTICS

by

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## Dedication

I dedicate this work to my fiancée Petra who unselfishly, and lovingly supports me in my goals, and who is always an inspiration for my work and research. Secondly, I dedicate this to my mother, brother, his wife, and my little nephew who were there to provide support and encouragement. To my father as well who is forever with me in my thoughts. I know he is always proud of my achievements.

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# Chapter 1. Introduction

## Problem statement

Transportation engineers have been dealing with the problem of evaluating, selecting and scheduling infrastructure projects, which resulted in the development of various methods. Alternatives taken into consideration can be classified as follows:

- Mutually exclusive—only one alternative can be selected;
- Independent—the benefit and costs of alternatives are independent of which alternatives are selected or when those are implemented;
- Interdependent (interrelated)—alternatives that pervade transportation networks since improvements alter the flows, and hence benefits, on other network components.

The aim of the thesis is to show how a traffic assignment model can be used to evaluate the objective function of an investment planning optimization problem for an urban road network, especially by showing how intersections can be included in the traffic assignment. A method for evaluating and selecting improvement alternatives which considers intersection improvements in addition to link widening alternatives is presented. Interactions between vehicles are included in the traffic assignment by assigning pseudo links. Each of the three movements at a single approach at 4-leg intersection (left, right, through) are modeled with a pseudo-link, creating a total of 12 pseudo-links per 4-leg intersection.

As traffic increases, roads and intersections become congested, which leads to motorists experiencing increased travel time and delays. One solution, which is considered in this thesis is the construction of additional lanes. A second solution is intersection

widening, which can increase the overall capacity. After selecting the congested roads and intersections, the next step is to determine in what order the improvements should be implemented to minimize the present worth of costs. One way is to rank the roads and intersections based on congestion (delay) levels, and the project (whether it is a road improvement or intersection improvement) that has the highest value of congestion (delay) gets scheduled first. Another way is to rank them based on the benefit/cost ratio, using benefit cost analysis. These ways are not good enough because they fail to consider interrelations among projects.

Selection and scheduling of projects can become a large optimization problem as the number of considered projects in the system grows. While considering a set of improvement projects, the goal is finding a sequence of projects minimizing total system costs over the analyzed period.

The Frank Wolfe model for traffic assignment is used as the tool for evaluating network flows with changing configurations, while the Ant Colony Optimization algorithm, along with Tabu Search, and Simulated Annealing are used as optimization tools to obtain the best possible schedule of projects.

### *Research objectives and contributions*

The work presented in this thesis contributes to the relevant literature in several ways. First, we modify the Frank-Wolfe algorithm (1956) to consider intersection movements (nodes are treated as intersections) by introducing one pseudo pseudo link for each intersection movement from each approach. Moreover, we apply the Akcelik's delay model (1988, 1990) to estimate the delay on each of the pseudo links. Akcelik's model was chosen because its number of input parameters was significantly smaller than the number

of parameters required for the HCM 2010 delay model. Also, Akcelik's delay model needed fewer assumed input values than the HCM model. Assumptions in the HCM 2010 delay model that were needed were about the behavior of pedestrians, transit traffic, and possible public transportation stations near intersections.

When considering projects that need to be implemented in the near or distant future, the parameters needed for HCM model are hard to estimate. The decision maker does not have a precise information about pedestrians and future transit traffic forming assumptions about the parameters. This information can change over time, especially information about the position of the public transportation stops, which also influences the HCM delay model.

The swarm metaheuristic algorithm is applied along with other two known metaheuristic algorithms to compare different approaches for solving the problem of selecting, sequencing and scheduling projects.

A case study is also presented with the comparison of the performance of the swarm algorithm with two other known heuristics. An exhaustive enumeration test is presented which shows the goodness of solutions obtained with heuristics. A K-S statistical test is also presented in case of higher number of considered projects to estimate the likelihood that better undiscovered solutions exist than those obtained with heuristical algorithm.

### Thesis organization

After the introduction, chapter 2 provides a literature overview in which prior research is presented on delay models, project scheduling and selecting, ant colony optimization, tabu search, simulated annealing, network improvement models. Chapter 3 presents the evaluation methodology based on the Frank-Wolfe traffic assignment

algorithm. Chapter 4 describes the problem formulation as well as solution representation, objective function and assumptions made in this thesis. Chapter 5 explains delay and signalized intersections and provides the information about the delay model used in this research. Chapter 6 provides details about the optimization techniques used in the thesis, including Ant Colony Optimization (ACO), Tabu search (TS), and Simulated Annealing (SA). Chapter 7 presents the case study, the transportation network used, and the results obtained after implementing evaluation model and optimization model, and the statistical test to check the goodness of the solution optimized with the ACO algorithm. Chapter 8 includes sensitivity analysis for the ACO algorithm. In other words, the influence of ant colony size, pheromone evaporation rate and problem size on the ACO heuristic. Moreover, the changes in demand values, interest rate, and project improvement cost are also considered in the sensitivity analysis. Finally, Chapter 9 provides conclusions, closing remarks, possible future research.

## Chapter 2 Literature overview

Intersections are not only the capacity-limiting components of an urban road network, but also the most complex components within it. The reason for this is that link flows are limited by conflicting flows. At 4-leg intersections there are 12 legal vehicular movements and 4 legal pedestrian movements. As traffic demand increases on approaches, conflicts between the vehicles also increase, as well as delays which can lead to congestion not only at the intersection, but also in other parts of the network, depending on the severity of congestion.

Traffic signals assign right-of-way passages. By doing so, they can significantly reduce the number of conflicts and help regulate the traffic flow. Other advantages of traffic signal control include (Roess et al. (2015)): orderly movement of traffic, increase in traffic-handling capacity of the intersection, reduction in the frequency and severity of certain types of crashes. They also provide for continuous or nearly continuous movement of traffic at a definite speed under favorable conditions, can be used to interrupt heavy traffic at intervals to permit other traffic to cross the intersection. Disadvantages of traffic signals include the following (Roess et al. (2015)): possibility of excessive delay, excessive disobedience of the signals. Users can significantly use other routes to avoid traffic signals, thus making the network more congested. If delays occur, they not only cause congestion which leads to anxiety and nervousness amongst the drivers but also an increase in pollution and in costs. These costs consist of suppliers cost (government agencies) and user costs.

A massive part of these costs can be attributed to traffic delays, Arnott & Small (1994) suggested that U.S. annual delay costs exceeded approximately \$48 billion in the

nineties. Possibly the first papers about delays at signalized intersections were written by Wardrop (1952), and Webster (1958). Wardrop (1952) assumed that vehicles enter the intersection with uniform arrivals. Wardrop's expression can be written as:

$$d = \frac{(r - \frac{1}{2s})^2}{2C(1 - y)} \quad (1)$$

where:

$d$ -average delay [*sec*],

$r$ -the effective red time [*sec*],

$s$ -saturation flow on the approach [*vps* or *vph*],

$C$ -cycle length [*sec*],

$y$ -flow ratio,

Webster (1958), Miller (1968) and Newell (1956) proposed three more representative models that estimated delays at signalized intersections. Hutchinson (1972), Sosin (1980) and Cronje (1983) have numerically compared these delay expressions.

Webster's (1958) paper presented results of researching delays to vehicles at fixed-time traffic signals and the optimum settings of such signals. Methods developed in this paper can be, according to the author, applied both to fixed-time and to vehicle-actuated traffic signals.

Estimation of overflow delay is one of the major difficulties when developing delay models at signalized intersections. This difficulty stems from obtaining simple and easily computable formula for overflow delay and has forced researchers and analysts to search for approximations and boundary values.

Heidemann and Olszewski (1994) used probability distributions to estimate delay at signalized intersections. In their models, the probability distributions of delay were obtained from the probabilities of queue lengths.

Ban et al. (2009) estimated a delay pattern for signalized intersections using sample travel time. These patterns can provide a way to estimate the delay for any vehicle arriving at the intersection. This is useful for providing time-dependent intersection delay information to the public. Ban et al. (2009) proposed a model which was based on two observations regarding the delays: delay could be approximately represented by linear curves due to the characteristics of queue forming and discharging, and there was a meaningful increase in delay after the start of the red time.

Dion et al. (2004) introduced numerous delay models and compared delay estimates obtained from those models, specifically: deterministic queueing delay model, a model based on shock wave theory, the steady-state Webster model, the queue-based models dating from 1981 from Australian Capacity Guide, the 1995 Canadian Capacity Guide for Signalized Intersections, and the 1994 and 1997 versions of the Highway Capacity Manual (HCM), and the delays estimated from the microscopic traffic simulation software INTEGRATION. The results obtained were compared for numerous ranges of v/c ratios, specifically from 0.1 to 1.4 for consistency. The delay models from Australian Capacity Guide, the 1995 Canadian Capacity Guide, the 1997 HCM, and the INTEGRATION traffic simulation model produced similar delay estimates. Moreover, it was assessed that all delay models considered produced relatively consistent estimates of delay in under-saturated condition at signalized intersections with v/c ratios below 0.6.

Hurdle (1984) asserted that the steady-state models which do not assume completely uniform arrivals predict similar values of delay. In steady state models, delay approaches infinity as the  $v/c$  ratio increases.

Akcelik (1988) compared the performance of the HCM 1985 with his own model. Values of delay from HCM 1985 and Akcelik's delay model differed by 2 seconds (5%) at most for  $v/c$  ratios below 1. Akcelik in his paper also compared the presented model with Canadian, Australian and HCM delay models. This comparison is shown in table 2.1 (Akcelik (1988)).

**Table 2.1 Performance measures of delay models**

	Akcelik's	HCM	Australian	Canadian
Average overflow queue (veh)	4.26	4.03	3.93	4.37
Average overall delay $d$ (sec/veh)	60.1	58.4	57.7	60.9
Average stopped delay (sec/veh)	46.2	44.9	44.4	46.8

Boon et al. (2012) studied a traffic intersection with vehicle-actuated traffic control. Until all the lanes are emptied, traffic lights will remain green. Boon et al. (2012) assumed general renewal arrival processes, and based on that assumption they derived exact limiting distributions of the delays under heavy traffic conditions. Moreover, Boon et al. (2012) derived a light traffic limit of the mean delays for intersections with Poisson arrivals, and developed a heuristic adaptation of this limit. Combining these two results, they developed a closed-form approximation for the mean delays of vehicles in each lane.

Sheffi (1985) stated that the behavior of travelers and traffic control policies were two connected processes that could have two different goals: traveler's behavior influenced user optimum, and traffic control influenced system optimum. To achieve a network



optimum, traffic is controlled in many ways using, among many practices, traffic signals, traffic information and ramp metering. Changing the traffic control can change the traffic volume, for example if traffic control is changed and congestion is decreased on some routes/links, traffic might divert to other links where congestion can build up.

Traditionally, if the traffic assignment models consider intersections, they do it simplistically, mostly because of problems with data and limitations of algorithms in the assignment models. Theoretically, one can argue that the Frank Wolfe algorithm cannot guarantee to estimate delays at intersections. This is a specific problem, because in congested urban areas queues and delays at intersections often have a huge effect on total travel time and on route choice behavior. Transportation engineers are faced with two difficulties:

- Models that theoretically converge but with simplified assumptions on: speed – flow relations, interactions at intersections (Cantarella et al. (1991)) and driver behavior (Nielsen (1996)),
- Heuristic expansions of these models, which do not guarantee convergence.

Allsop (1974) was among the first authors who considered the interactions between route choice and traffic control. Allsop (1974) considered two research areas: traffic control and traffic assignment and proposed an integration by using mathematical formulation for both areas. He asserted that a common network definition was needed, and developed an approach where traffic assignment variables were considered as functions of traffic control parameters. The assumptions that Allsop (1974) made included the following: all major intersection possessed signal control, demand was fixed/static, travel time increased strictly monotonically with flow, and a fixed cycle time was used. A simple example was used to

illustrate the interaction between traffic assignment and control, while the Webster's method was used to optimize traffic control.

Allsop and Charlesworth (1977) developed an iterative approach using TRANSYT program which was used for signal settings and for estimating the relationship between link travel time and traffic flow. For a certain network Allsop and Charlesworth (1977) used two initial assessments that can lead to two different solutions of the combined control and traffic assignment problem. Later, Charlesworth (1977) continued the research in this direction but changed the cycle time and found the same results.

Smith (1979) considered the interaction between Webster's delay model and driver's route-choice decisions on a simple case (three one-way links that form two routes from  $x$  to  $y$ ). Smith (1979) simplified the model by assuming the lost times were zero and the cycle time was a fixed variable, as well as a steady demand from  $x$  to  $y$ . The paper showed that Webster's method could significantly reduce network capacity.

Nielsen et al. (1993) demonstrated a new method that could automate the task of adding data for intersection delay modelling to a network, which was developed while combining intersection delay model with assignment model. The developed method was tested on the Copenhagen metropolitan area and it used a sequential procedure to interpret the information in a pre-existing network database by implementing a set of 'expert system rules'. These rules used the existing geographic and attribute information in the link-node based traffic database.

Nielsen et al. (1998) examined the behavior of stochastic user equilibrium (SUE) traffic assignment with consideration of intersection delays and tested a version of SUE which included intersection modelling on a full-scale network to examine the convergence

and uniqueness of the solution. Nielsen et al. (1998) also analyzed if this approach yielded a better description of the traffic flows than the link-based SUE. The whole network with turns at intersections was considered as a link-node topology, which allowed the usage of all-or-nothing method for finding the paths. There were two types of links: real links and other links which represent turning movement. The model was tested on a full-scale network of Copenhagen (2369 nodes, 6108 links, 12073 pseudo-nodes and 19111 turns) and the results obtained were compared with the traffic survey of route choice which was done by Vejdirektoratet (1990).

A higher definition of intersections in networks is needed when detailed forecasts are implemented, such as travel surveys. Meneguzzer (1995) explained the framework which combined detailed models of intersection operation with user-optimal route choice model. The framework considered detailed information on intersection geometry and control to develop the cost functions, and it could be applied in urban road networks for the assessment of alternative intersection control strategies or to help with a detailed analysis and forecast.

According to Dickson (1981), if a set of signal parameters is available which can change the travel time-flow relationship on the links, it is possible to influence the equilibrium flows on the network. By exploring a simple example (one intersection, with travel time on links being a linear function of the flow plus a delay at the intersection for which the author assumed a simple delay formula), the author showed that with fixed set of flows and with the optimal signal settings, that can be obtained by iterative procedure that author applied, may lead to an increase in total network travel time.

Yang and Yagar (1995) combined traffic assignment with signal control in saturated networks, while considering queueing and congestion levels and assuming:

- a fixed level of demand,
- driver's sufficient and perfect knowledge of queueing delays and travel times via all routes,
- cycle times and
- green times.

The model is a bi-level programming problem where a lower-level problem is a network equilibrium which basically predicts how drivers will react to a specific traffic signal, while an upper-level problem determines signal splits to optimize a system objective function. Traffic delay at intersections was divided into signal delay and queueing delay, and sensitivity analysis was executed to obtain the derivatives of link flows and queue delay concerning signal splits, thus telling the authors how queueing network equilibrium behaved when signal settings were changed.

Dafermos (1971) examined how traffic assignment decision rules affect flow patterns. This paper recognized that traffic flow in many networks was controlled by conditions that lied outside the range of assumptions of the standard flow-dependent model.

On another note, many researchers dealt with extending a static framework of traffic assignment to the quasi-dynamic and dynamic cases, where at real time, control and guidance were being provided in response to real time traffic information. Gartner and Stamatiadis (1998) explored the extension by having both O-D demand and the control actions as time dependent variables, thus integrating these two interlaced variables. Gartner and Al-Malik (1996) presented the optimization model that accounted for both route choice

behavior and optimal signal settings at the intersections. Signal settings were expressed as flow variables, and it was shown that the user and system optimum was possible to calculate.

Smith (1980) promoted a combined assignment-control model based on  $P_0$  policy (maximize the ‘travel capacity’ of the road network, assuming all drivers look for the best route) which considered signal settings on route choice. Furthermore, Ghali and Smith (1994) conducted the necessary computation and found out that  $P_0$  policy better performed than the two policies: delay minimization at each intersection, and Webster’s equisaturation method where green times needed to equalize the degree of saturation on intersection approaches.

The critical role of traffic control systems in the operation of urban street networks was also recognized by Gartner and Malik (1996). To develop an effective signal strategy, it was necessary to evaluate and optimize signal timings and to optimize the traffic flow patterns, which presented combined model that could be used to develop advance traffic strategies that would lower congestion and avoid bottlenecks. Flow dependent traffic signal control model was developed which was used to model the interrelated activities between signal control and route choice. Model used delay equation developed by Webster while Frank Wolf algorithm was used to solve the traffic assignment problem. Two example networks were presented with the results obtained from the optimization model. Both system optimum and user optimum can be computed, and the model presented can also be applied to traffic-adaptive signals with minor modifications.

Often when there are many suggested projects but not enough resources, money or time the projects need to be implemented in a certain order. Project selection is the process

of assessing each project and selecting those with the highest priority. Priority can be based on:

- Benefits: a measure of positive outcomes when implementing a certain project.

Types of benefit include:

- Biodiversity,
  - Economic,
  - Social and cultural,
  - Fulfilling obligations/commitments as a part of national, regional, international or worldwide plans and agreements
- Feasibility: a measure of likelihood of a successful project (if the project will achieve its goal/objective).

Organizations and people involved in selecting projects and project management are:

- Agency management,
- Stakeholders,
- Project Manager

Selecting and scheduling projects is a difficult and time-consuming task. Many studies in the past dealt with this topic, among many approaches used, two approaches were:

- Integer programming: Weingartner (1966), Cochran et al (1971), Clark et al (1989),
- Dynamic programming: Weingartner (1966), Nemhauser and Ulman (1969), Moring and Esogbue (1971), Erlenkotter (1973)

One of the many notable papers about interrelated projects is Weingartner's (1966), which used an integer programming approach to solve the problem of interrelations.

Mehrez et al (1983) used a multi-attribute function to specify the decision maker's preference with a zero-one budget model to solve the problem of selection of interrelated multi objective long-range projects.

Thompson (1976) suggested a capital budgeting model for individual project selection that followed the capital asset pricing model. Approaches discussed in the paper were mathematical programming with inter-related project sets and market value maximization point of view with independent projects. The first approach treated the objective function in an unsatisfactory way, ignoring the market mechanism for dealing with risk as developed in portfolio theory. Furthermore, Thompson (1976) asserted that a programming approach was useful when considering project interrelationships, but it must relate to the market value maximization approach. Thompson (1976) also presented a way of relating the two approaches using a single-period model with project inter-relationship and market value determined by the capital asset pricing model. Martinelli (1993) explored a heuristic method for selecting and scheduling interdependent waterway investment projects by comparing their combinations. Martinelli (1993) began with an initial sequence of projects which was adjusted with the help of a heuristic that swapped the projects if the costs were improving.

A network design problem is a decision-making problem in urban transportation planning to select improvements or additions to an existing network to decrease traffic congestion. This definition is the most common in literature, but this problem is also named Urban Transportation Network Design Problem (UTNDP) or Road Network Design Problem (RNDP). Urban Transportation Network Design Problem considers the reactions of travelers.

The road network design problem (RNDP) is a subset of Urban Transportation Network Design Problem and it is mostly concerned with street networks while assuming all vehicle flows are homogenous. RNDP can be classified into following groups:

1. Discrete network design - only deals with discrete decisions: constructing new roads, adding roads, lanes, determining the direction of one-way streets
2. Continuous network problems-concerned with continuous design decisions: expanding the capacity of the streets, scheduling traffic signals, determining tolls.
3. Mixed network design problems-combination of the previous two.

In the literature, RNDP has been generally formulated as a bilevel programming problem, where the upper level is the design problem (decision-maker's problem) and the lower level problem decide whether to travel and which routes to take.

Inputs to RNDP can be enumerated as follows:

- Network topology,
- Travel demand between each O/D pair
- Street characteristics, such as capacity, free flow travel time, travel time function
- The set of candidate projects
- The available budget
- The cost of each candidate project

LeBlanc (1975) first used a branch-and-bound algorithm to solve the discrete network optimal design problem of a fixed investment budget. Link addition was considered on a given road network, with projected increase in demands for road travel between various pairs of nodes. The decisions considered building new links to counter congestion or to increase the capacity of some existing links as well as the various possible



types of network improvement. An important part of tackling this problem was predicting the number of vehicle flows along the links of a road network in some future period. Assumptions on driver's behavior were necessary to do this prediction. These assumptions were based on Wardrop's principles. Leblanc (1975) showed that the discrete network design problem can be solved by a sequence of shortest route problems and one-dimensional searches. The technique was tested on a numerical example, network used was the Sioux-Falls network, and the results showed the efficiency of the technique.

Abdulaal and LeBlanc (1979) presented a network design model with continuous investment decision variables. The budget constraint, after being converted to travel time units, was incorporated into the objective function. Continuous network design problem with convex investment costs increased the practical capacity of the arcs that were proposed for improvement. This could be desirable if the purpose was to improve or maintain the transportation network instead of just constructing new roads (links), and computational results obtained by Abdulaal and LeBlanc (1979) corroborated this. The additions to practical capacity were higher for congested links.

When considering improving or adding new links to an existing network, the method of exhaustive enumeration has the crucial disadvantages of being limited to relatively small sets of projects. To overcome this disadvantage, many authors in the past have studied a 0-1 mixed integer approach for this problem. Commonly, 0-1 decision variables were ascribed for each proposed new link or improvement. Branch and bound and branch and backtrack procedures for 0-1 network problem were explored in various publications. Boyce et al. (1973) considered a problem of selecting links for improvement to minimize the shortest path distances between all O/D pairs, while considering budget

constraints on total link length. Modification of the branch-and-bound procedure was assessed for optimal variable selection.

Tzeng and Tsaur (1994) used multiple criteria decision making for a metropolitan network improvement plan. A bilevel multiple objective network design model was considered: minimizing government budget and minimizing user's total travel time. Link improvements were considered in an existing network and in fixed travel demand. To formulate a continuous network design problem, a multiple objective mathematical programming was used, and it was tested on the network of metropolitan Taipei.

Tzeng and Tsaur (1994) proposed a continuous network design model with a nonlinear objective function, while the improvements affected the equilibrium flow assignment. The origin-destination trip matrix was assumed to be fixed. Feasible alternatives to bottleneck link under existing network structure and travel demands were being sought in the paper. Alternatives included an increase in link capacity. Multicriteria decision making was employed to select and evaluate a compromise alternative from a set of feasible projects, thus solving the discrete network design problem. The concept of bi-level programming was used to solve project searching stage of the problem.

The term "Swarm Intelligence" was first used by Beni, Hackwood and Wang (1988, 1989, 1991, 1992, 1991, 1992) in the context of cellular robotic systems, where many simple agents occupy one or two – dimensional environments to generate patterns and self – organize through nearest – neighbor interactions. Bonabeau et al (1999) extended this term to include any attempt to design algorithms or distributed problem – solving devices inspired by the collective behavior of social insect colonies and other animal societies.

Complex collective behavior is one of the main characteristics of the animal realm and it is based on the biological needs of individuals to stay together. Different animals, from bacteria to ants and caterpillars exhibit such a behavior. These animals not only increase their probability of surviving since predators usually attack only isolated individuals, but can build objects, organize, maintain order and communication among the colony which is highly adaptable and flexible. Bonabenau et al (1999) stated that this flexibility enabled these colonies to be robust and to endure and survive in conditions that suffered great disorders. In biology, insect interaction was highly explored and described, such as secretion of pheromones by ants, or dancing of bees while gathering food for the colony. This type of communication greatly contributes to the creation of collective intelligence (group intelligence), and many researchers dealt with this type of intelligence in recent decades: Beni (1988), Beni and Wang (1989,1991), Beni and Hackwood (1992), Bonabeau et al (1999).

Swarm intelligence is the branch of artificial intelligence which explores individuals' actions in different decentralized systems (multi agent systems). These multi agent systems are constituted from physical individuals (robots) or virtual (artificial) ones that communicate, cooperate, collaborate, exchange information and knowledge, and perform tasks in their environment.

Deneubourg et al (1990), Franks et al (1992), Beckers et al (1992), Nonacs and Soriano (1998), Vienne et al (1998) postulated that basic rules of behavior constitute the ant self – organization. Self – organization relies on four basic ingredients (Bonabeau et al. (1999)):

- Positive feedback (amplification) – rules of thumb that promote the creation of structures (examples are recruitment and reinforcement). Two examples of recruitment in the animal realm are ants relying on pheromone trail in search for food and bees dancing for the same reason.
- Negative feedback serves as a counterbalance for positive feedback, and stabilizes the collective pattern.
- Self – Organization is based on fluctuations (random walks, task – switching). Randomness is crucial for the development of structures and in finding new solutions. It can also be an initial point from which structures grow.
- All cases of Self – Organization are based on multiple interactions and SO requires a minimal density of mutually tolerant individuals who make use of their own activities as well as others’ activities.

Most of the ant species have a certain number of “Scouts” that leave the colony in search for food (Deneubourg et al (1990)), and those who have been successful in finding food secrete pheromone and leaving a trail of it behind them. Pheromone provides information to other ants about the path they need to take. As the number of ants on a certain path increase so does the level of pheromone on that same path, thus increasing the signal to other ants in search for food. Secretion of pheromones is a characteristic of the processes that are self – enhancing and quickly converging. The mechanism of pheromone evaporation exists to prevent the “explosion” of information. The decision which path to take mostly depends on the behavior of other ants that leave the pheromone trail behind them. At the same time one ant’s behavior and path selection is influencing the other ants

that must yet choose the path which they will take. In other words, the decisions of one ant depend on the decision and behavior of other ants.

Many researchers have used ant systems to solve many problems that are classified as NP hard problems. Probably the first authors who considered an ant system in optimization were Dorigo et al (1992).

The most notable artificial systems based on swarm intelligence are Ant System and Ant Colony Optimization (Coloni et al (1991), Dorigo et al (1992), Dorigo (1992), Dorigo et al (1996), Dorigo and Gambardella (1997a, 1997b), Bonabeau et al (1999)).

Dorigo et al. (1992,1996,1997,1991,1999,2004) introduced ACO (Ant Colony Optimization) algorithm in the early 1990's. These algorithms were, as previously stated, inspired by the observation of the real ant colonies.

Blum (2004) dealt with the biological aspects of ant colony optimization algorithms and showed how they could be transferred into algorithms that could be used for optimization. This author presented this type of optimization in general, and presented some of the variants of this type of algorithm. An ACO algorithm initially was used for solving the traveling salesman problem (TSP) and was later used for many other problems in optimization, specifically for NP hard problems. Most of the problems that were solved fall into one of the following categories: routing assignment, scheduling and subset problems.

The ACO metaheuristic was successfully used in tackling the following problems:

- Routing problems (Dorigo, Maniezzo, and Coloni (1991), Dorigo and Gambardella (1997a, 1997b), Stützle and Hoos (1998)),
- Sequential ordering (Gambardella and Dorigo (2000)),

- Vehicle routing (Gambardella, Taillard, and Agazzi (1999), Reimann, Doerner, and Hartl (2004)),
- Assignment problems
  - Quadratic assignment (Colomi et al (1994)),
  - Frequency assignment (Maniezzo, and Carbonaro (2000)),
- Scheduling problems (Stützle (1998), den Besten, Stützle, and Dorigo (2000), Gagne, Price, and Gravel (2002), Merkle, Middendorf, and Schmeck (2002), Blum and Sampels (2004)),
- Subset problems
  - Set covering (Leguizamón, and Michalewicz (1999, 2000)),
  - Weight Constrained Graph Tree Partition Problem (Cordone and Maffioli (2001)),
  - Arc – Weight  $l$  – Cardinality Tree Problem (Blum and Blesa (2003)),
  - Multiple knapsack problem (Leguizamón, and Michalewicz (1999, 2000)),
  - Maximum Independent Set problem (Leguizamón, and Michalewicz (1999, 2000)),
  - Maximum Clique Problem (Bui and Rizzo Jr (2004)),
- Shortest Common subsequence problem (Michael and Middendorf (1998)),
- Bin Packing (Levine and Ducatelle (2003)),
- Machine learning problems
  - Learning the structure of Bayesian networks (De Campos, Fernandez – Luna, Gamez and Puerta (2002)),
  - Data mining (Parpinelli, Lopes, and Freitas (2002))

- Network routing
  - Connection – oriented network routing (Schoonderwoerd, et al (1997), White et al (1998))
  - Connectionless network routing (Di Caro et al (1997), Subramanian et al (1997))
  - Optical network routing (Navaro, Sinclair (1999))

Tabu Search was first suggested by Glover (1986), while Hansen (1986) further explored this technique. Tabu search originated as a method for implementing the oscillating assignment strategy. Glover (1986) asserted that this type of technique, from an artificial intelligence point of view, slightly deviated from a normal human behavior. Four main developments were crucial in developing Tabu search:

- Strategies that combined decision rules based on logical restructuring and non – monotonic (variable depth) search, came from a study of decision rules for job shop scheduling problems.
- Systematic violation and restoration of feasibility, which became imprinted on associated strategies of tabu search that was included in a method for solving integer programming problems by reference to corner polyhedral relaxations.
- Flexible memory based on recency and frequency, which involved an exact approach for integer programming problems.
- Selective process for combining solutions, applied to a systematically maintained population which involved surrogate constraints methods being introduced for integer programming.

Tabu search has been effectively applied to many combinatorial optimization problems in many fields including planning and scheduling problems, telecommunications, parallel computing, transportation, routing and network design, optimization on structures, optimization on graphs, neural networks and learning, Manufacturing, and financial analysis.

In the field of transportation, routing and network design there are many researchers who applied the principles of tabu search to solve such problems. Sun et al. (1998) developed a tabu search process for the fixed charge transportation problem, using recency-based and frequency-based memories and a network implementation of the simplex method as the local search method. Tabu search obtained optimal and near-optimal solutions much faster than the (exact) simplex algorithm for simple problems. The same authors (1998) also proposed a heuristic procedure based on tabu search to solve the transportation problem with exclusionary side constraints. Attractiveness of a move was evaluated by net changes in total cost and in total infeasibility, and strategic oscillation is used to implement the intensification and diversification functions. This procedure found optimal or near optimal solutions using a small fraction of the CPU time.

Gendreau, Hertz, and Laporte (1986) developed a tabu search heuristic for the vehicle routing problem with capacity and route length restrictions. The developed procedure considered adjacent solutions which were obtained by repeatedly removing a vertex from its current route, and reinserting it into another. Results, which were gathered from many benchmark problems and compared with a dozen of other procedures, indicated better performance for Tabu Search.



Barnes and Carlton (1995) presented a reactive tabu search procedure for solving the vehicle routing problem with time windows, while Chiang and Russell (1977) developed a reactive tabu search method for the VRPTW that dynamically changes the list of forbidden moves to avoid cycles.

Rochat and Semet (1994) considered a real-life vehicle routing and distribution problem for a Swiss company producing pet food and flour. The main constraints that the authors considered are accessibility and the time windows at customers, the capacity of the vehicles, the total duration of the route and the driver's breaks. Computational results showed that procedure yielded better solutions than the solutions of constructive heuristics, with reasonable CPU time.

Rochat and Taillard (1995) proposed a probabilistic TS procedure to diversify, intensify and parallelize almost any local search for almost any vehicle routing problem making it more robust.

Voss (1990), Domschke et al. (1992), Daduna and Voss (1995) gave a dynamic tabu search approach for the quadratic semi-assignment problem in a series of applications for modeling and solving a schedule synchronization problem in a mass transit system, the goal being to minimize the overall transfer waiting passenger time. The outcomes showed better schedules were produced than those obtained by previous approaches, which were based on simulated annealing.

Chiang and Kouvelis (1994) addressed the flow path design issue of AGVs and concentrated on the design of unidirectional flow paths (vehicles were restricted to travel only on one direction along a given segment of flow path), and developed different versions of TS and SA procedure.

Crainic, Gendreau and Farvolden (1996) presented an efficient TS approach to find feasible solutions to realistically sized capacitated multicommodity fixed cost network design problems. This method was the first attempt to develop an efficient TS procedure for a mixed integer programming problem. Results obtained showed the robustness of the procedure in terms of relative importance of the fixed costs and capacities, size and the number of commodities.

Based on this literature overview, we can see that, firstly only link improvement and link addition but not intersections were considered in the network improvement model, while the improvement of intersections in this thesis is also considered. Secondly, traffic assignment models that consider intersections exist in the literature. Such traffic assignment models are incorporated into this thesis. Thirdly, ant colony optimization algorithms were used to solve many engineering problems. Research presented in this thesis tries tackle the problem of how to schedule network improvements (link and intersection improvements) using this type of algorithm along with two other known metaheuristics: Tabu Search, and Simulated Annealing.

### **Chapter 3    Evaluation model**

Traffic assignment can be formulated as the problem of finding the equilibrium flow pattern over a given graph representation of a transportation network, the associated link performance function and an origin-destination matrix. A specification of how travelers choose a route is needed to solve the problem. Moreover, a reasonable assumption is that every traveler will try to minimize its own travel time when traveling from origin to destination. Another assumption is that the travel time may change with the flow on each link and that all individuals behave identically. User Equilibrium (stable condition) is

achieved when no traveler can improve its travel time by changing route. Assignment of traffic flows on network links is a result of equalizing transportation demand (O/D matrix) and transportation supply (link and node capacity, management actions). Notable publications that dealt with traffic assignment include Florian (1976), Sheffi (1985), Ran and Boyce (1994,1996). However, they did not consider intersections in traffic assignment.

This thesis applies the convex combination algorithm developed by Frank and Wolfe (1956) to evaluate link and intersection expansion projects upon their implementation in the network. This algorithm is an iterative algorithm that is used for solving traffic assignment problem which is a nonlinear programming problem with convex objective function and linear constraints. Current travel time for link  $a$ ,  $t_a^{n-1}$  is given and the  $n$ th iteration of the Frank Wolfe algorithm can be written as follows:

1. Initialization: perform all or nothing assignment assuming  $t_a^{n-1}$ , which yields flows  $x_a^n$ , set counter  $n = 1$ ,
2. Update: Set link travel time (BPR function)  $t_a^n = t_a(x_a^n) = t_0(1 + 0.15 \left(\frac{v}{c}\right)^4)$
3. Direction finding: Perform all or nothing assignment based on  $\{t_a^n\}$ , which will yield a set of (auxiliary) flows  $\{y_a^n\}$
4. Line search: find  $\alpha_n$  that solves the following problem:

$$\max_{0 \leq \alpha \leq 1} \sum_a \int_0^{x_a^n + \alpha(y_a^n - x_a^n)} t_a(\omega) d\omega \quad (2)$$

5. Move: set  $x_a^{n+1} = \alpha(y_a^n - x_a^n)$ ,  $\forall \alpha$
6. Convergence test: if a convergence criterion is met, stop. Otherwise, set  $n = n + 1$  and go back to step 1.

A major concern with the Frank Wolfe algorithm is that it does not consider intersection interactions. Therefore, in this study, pseudo links are introduced for each of the movement at signalized intersection. The evaluation model determines traffic flow volumes on each of the link and intersection (pseudo-link), as well as travel time, speed, delay and waiting time.

## Chapter 4 Problem formulation

The problem considered here is an NP hard problem with the non-convex objective function. The problem grows rapidly as the number of candidate projects increases, and is classified as a combinatorial optimization problem. Such problems involve finding values for discrete variables in such a way that the optimal solution is found with respect to the objective function. Many practical problems can be classified as combinatorial optimization problems (such as the shortest-path problem, optimal assignment of employees to tasks). Dorigo et al (2004) formulated a combinatorial optimization problem  $\Pi$  as a triple  $(S, f, Q)$ , where  $S$  was a set of candidate solutions,  $f$  was the objective function which assigned an objective function value  $f(s)$  to each candidate solution  $s \in S$ , and  $\Omega$  was a set of constraints. The solution belonging to the set  $\tilde{S} \subseteq S$  of candidate solutions that satisfied the constraints  $\Omega$  were called feasible solutions. The goal, according to Dorigo et al (2004), was to find a globally optimal feasible solution  $s^*$ .

A naïve and straightforward approach when trying to solve a combinatorial optimization problem would be an exhaustive enumeration of all possible solutions. If we opt for this approach, the computational time increases rapidly as the problem grows in complexity. It is useful, when solving this type of problem, to know how difficult it would be to find an optimal solution.

Dorigo et al. (2004) asserted that the difficulty of combinatorial problems is that of  $NP$  – completeness, and they could be classified as following:

- The class  $P$  for which an algorithm's output in polynomial time was the correct answer (“yes” or “no”),

- The class *NP* for which an algorithm exists that verifies for every instance, independently of the way it was generated, in polynomial time whether the answer ‘yes’ is correct.

Algorithms can be classified as:

- Exact algorithms—they are guaranteed to find the optimal solution and to prove its optimality for every finite size instance of the problem. In the case of *NP* hard problems, in the worst-case scenario, exact algorithms need a lot of time to find the optimum.
- Approximate (heuristic) algorithms can obtain good, or near – optimal solutions at relatively low computational cost and time without being able to guarantee the optimality of solutions. These types of methods can be further classified as: constructive or local search methods.

According to Dorigo et al. (2004) a metaheuristic is a set of algorithmic concepts that can be used as a general heuristic method to solve problems that are different in nature, with few modifications.

The objective function used in this research minimizes the total costs: supplier cost, which was defined as the present value of all project costs, and user cost, defined as the delay multiplied by the value of time, during the analysis period subject to budget constraint. Supplier’s costs are represented by the first and third term of the equation, while the user’s costs are represented by the second and fourth term. The objective function can be formulated as follows:

$$\begin{aligned}
Z = & \sum_{j=1}^T \frac{v}{(1+r)^j} \sum_{i=1}^{n_l} w_{ij} + \sum_{j=1}^T \frac{v}{(1+r)^j} \sum_{i=1}^{n_{pl}} c_i X_i(t) + \sum_{j=1}^T \frac{v}{(1+r)^j} \sum_{i=1}^{n_l} d_{ij} \\
& + \sum_{j=1}^T \frac{v}{(1+r)^j} \sum_{i=1}^{n_{pl}} C_i X_i(t)
\end{aligned} \tag{3}$$

where:

$w_{ij}$ -waiting time on link  $i$  in year  $j$ ,

$c_i$ -present worth of the cost of link project  $i$ ,

$n_{pl}$ -number of link projects (link improvements),

$n_l$ -total number of links,

$n_I$ -total number of intersections,

$n_{pl}$ -number of intersection projects (intersection improvements),

$C_i$ -present worth of the cost of intersection project  $i$ ,

$v$ -value of time,

$r$ -interest rate,

The present worth of the cost of intersection project  $i$  consists of capital cost of improvement and cost of pavement maintenance. It can be written as:

$$C_i = \sum_{j=1}^T \frac{1}{(1+r)^j} (C_{c_i} + C_{p_i}) = \sum_{j=1}^T \frac{1}{(1+r)^j} (A_{I_i} \cdot 200 + A_i \cdot 50) \tag{4}$$

where:

$C_{c_i}$ -capital cost of improvement of intersection  $i$ , value is 200 \$/ft<sup>2</sup>

$C_{p_i}$ -cost of pavement maintenance of intersection  $i$ , value is 50 \$/ft<sup>2</sup>

$A_{I_i}$ -area of the land that is needed to improve intersection  $i$ ,

$A_i$ -overall area of the intersection  $i$

The objective function is bound by the following cumulative budget constraint (Jong, Schonfeld, 2001):

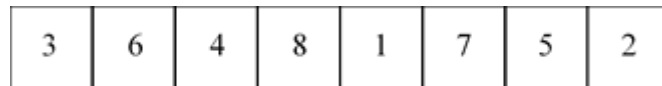
$$\sum_{i=1}^{n_p} c_i x_i(t) \leq \int_0^t B(t) dt, \quad 0 \leq t \leq T \quad (8)$$

$$\begin{cases} x_i(t) = 0 & \text{if } t < t_i \\ x_i(t) = 1 & \text{if } t > t_i \end{cases}$$

where  $t_i$  is the time when project  $i$  is finished and  $x_i(t)$  is a binary variable specifying whether project  $i$  is finished by time  $t$ . Since in most realistic problems the cumulative budget constraint is binding, i.e. there is never enough funding for all the available projects that are worth implementing, the optimized project sequence represented by the set of all  $t_i$ s uniquely determines the schedule of projects (Jong and Schonfeld (2001), Shayanfar et al. (2016)).

#### Solution representation

Solutions obtained by heuristics are represented as the sequence of projects that should be implemented, each project occurring after its predecessors and before its successors, which is shown in figure 4.1.



**Figure 4.1 Solution representation**

#### Objective function

The objective function minimizes the present worth of the total cost subject to budget constraint.



### Stopping criterion

Two stopping criteria were considered in this research: number of iterations and running time of the algorithms. Each algorithm was tested for 200 iterations.

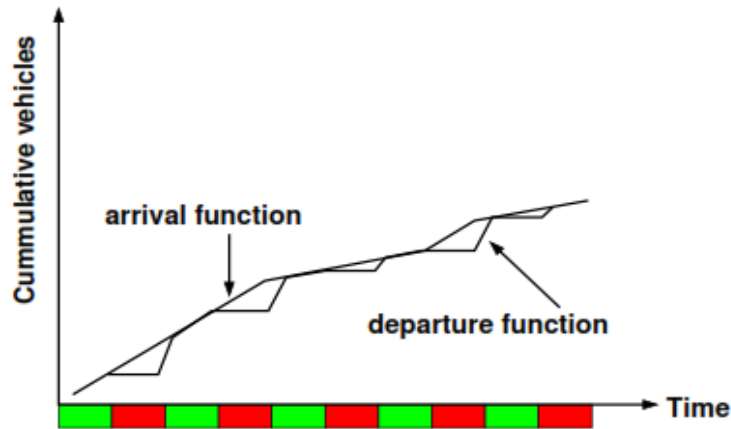
## **Chapter 5 Delay at signalized intersections**

Delay relates to the amount of lost travel time, fuel consumption, and the frustration and discomfort of drivers. It can be estimate in several ways such as measurements in the field, simulation, and analytical models. The last method is the most practical and convenient. Numerous analytical models have been developed using numerous assumptions for different conditions in traffic.

Many stochastic steady state models use two major simplifications (Roess et al. (2015)):

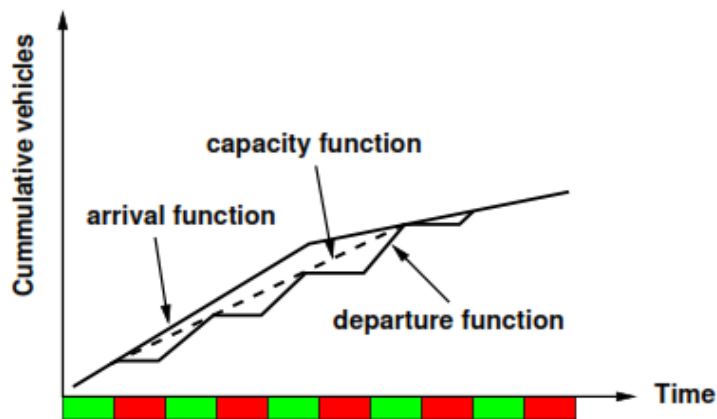
- The assumption that the arrival rate is uniform. Actual arrivals are random even when an isolated signalized intersection is concerned. However, inter-vehicle arrival times would vary around an average rather than being constant
- Another assumption is that the queue is forming up at a point location.

All analytic models of delay begin with a plot of cumulative vehicles arriving and departing versus time at a given location. Stable flow throughout the analysis period can be seen on figure 5.1. No signal cycle fails and during every green phase, the departure function at some point intersects the arrival function. Total aggregate delay is defined as the area of a triangle between the two curves: the departure curve and the arrival curve.



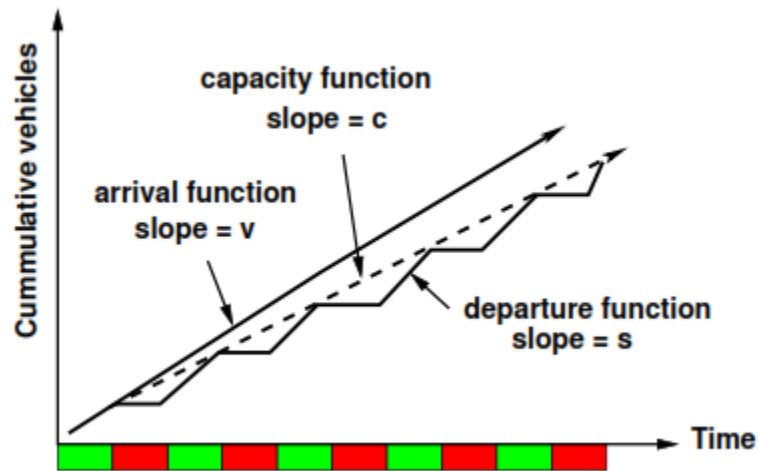
**Figure 5.1 Delay in case of stable flow**

In figure 5.2 we can see that some signal phases fail. Because of this some vehicles are not served, and must to wait for the next green phase. In this case, departure function intersects with the arrival function after some time and there is no residual queue. In this case the analysis period is a stable one. Besides uniform delay, another component of delay emerges: overflow delay. This delay represents the area between the dashed line and the arrival curve. Dashed line also represents the capacity of the intersection.



**Figure 5.2 Individual cycles fail, operation is stable**

The worst possible case can be seen in Figure 5.3. In this case, every green interval fails for a period that is significant. The residual queue, because of this, is growing throughout the analysis period.

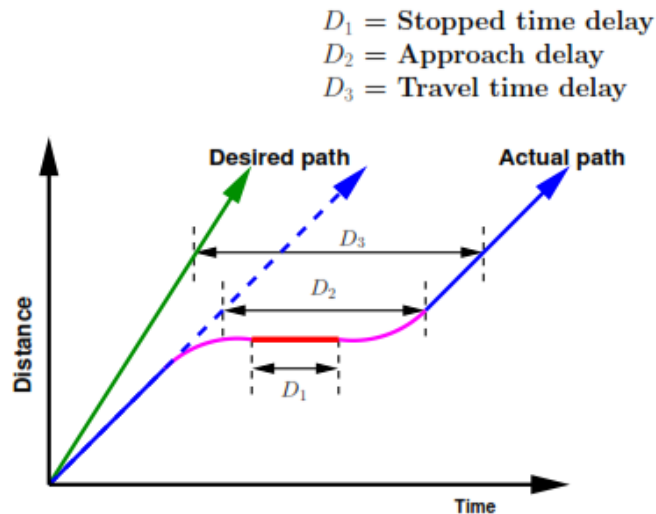


**Figure 5.3 Demand exceeds capacity for a significant period**

Delay can be quantified in many ways. Forms of delay that are mostly used can be seen on figure 5.4, and are as follows (Roess et al. (2015)):

1. **Stopped time delay**-time a vehicle is stopped in queue while waiting to traverse the intersection;
2. **Approach delay**-includes the previous delay and the time that is lost because of deceleration from the approach speed to a stop, plus the time lost in accelerating again;
3. **Travel time delay**-represents the difference between the expected driver's travel time through intersection and the actual time.
4. **Time-in-queue delay**-total time from a vehicle joining an intersection queue to when a discharge across the STOP line on departure.

5. **Control Delay** – this type of delay is a consequence of a control device, either a traffic light or STOP sign. This concept was developed in the 1994 *Highway Capacity Manual*, and is included in the current version of the *HCM* as well (Highway Capacity Manual 2010).



**Figure 5.4 Types of Delay**

Time delay models generate more realistic results when estimating the delays at signalized intersections. They are a combination of steady state and the deterministic models. Time dependent delay models are better in predicting the delay for under saturated and oversaturated conditions.

Numerous delay models have been developed during the last few decades, including: Webster delay model (1958), Akcelik delay model, HCM (Highway Capacity Manual 1994, 2010) delay models, Australian, and Canadian. The delay model used in this thesis is the Akcelik's delay model and is expressed with Equations 5 and 6 as (1988, 1990):

$$d = 0.5 \frac{C(1 - \lambda)^2}{(1 - \lambda x)} + 900T x^n [(x - 1) + \sqrt{(x - 1)^2 + \frac{m(x - x_0)}{cT}}] \quad (5)$$

and:

$$x_0 = a + bsg \quad (6)$$

where:

$d$ -average overall delay,

$C$ -cycle time (sec),

$\lambda$ -proportion of the cycle which is effectively green for the phase under consideration,

$x$ - $v/c$  ratio,

$T$ -flow period in hours, typical value is  $0.25h$

$c$ -capacity in vehicles per hour, (this is the capacity of a link's approaching lane)

$m, n, a, b$ -calibration parameters,

$s^*g$ -capacity per cycle,

Parameters  $n, m, a$  and  $b$  according to Akcelik's papers (1988, 1990) have the following values respectively: 0, 8, 0.5, and 0. Therefore, the equations (5) and (6) become:

$$d = 0.5 \frac{C(1 - \lambda)^2}{(1 - \lambda x)} + 900T [(x - 1) + \sqrt{(x - 1)^2 + \frac{8(x - 0.5)}{cT}}] \quad (7)$$

$$x_0 = 0.5 \quad (8)$$

The link interactions among different traffic movements can be categorized as follows:

- Direct interactions (within-phase interactions) that occur during the same phase.

Examples are: left turns against an opposing flow, and all-movement single-lane

intersection approaches in which blocking effects on through traffic caused by left turns are often observed.

- Indirect interactions (between-phase interactions) caused by one of more traffic movements to other traffic movements.

## Chapter 6 Optimization techniques

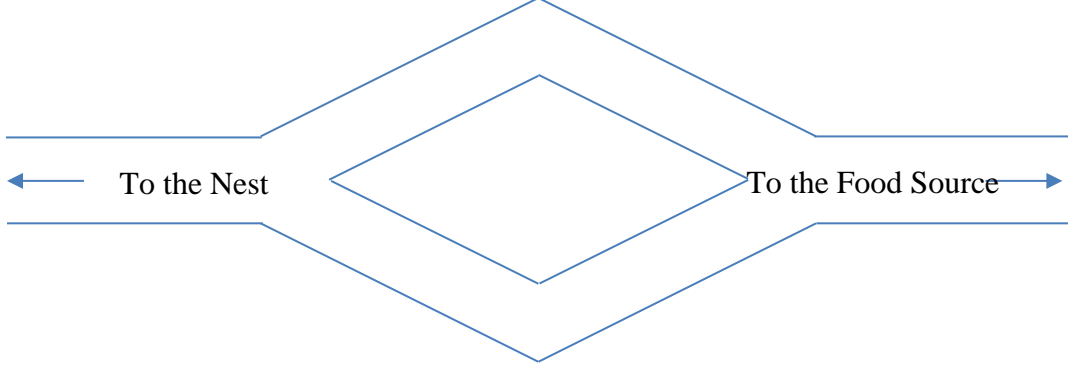
### Ant Colony Optimization (ACO) metaheuristic

In the past, experiments indicated how ants managed to find the shortest path between the two points. Deneubourg et al. (1990) conducted an experiment with the ants by separating the nest from the food source with a double bridge and the length of both the bridge was equal, which figure 6.1 shows.

Ants, although significantly blind, exhibit a remarkable space orientation and can reach their food supplies using the shortest path. One of the metaheuristics that have gained increased popularity in recent years, beside the Genetic Algorithm, Tabu Search and Simulated Annealing, is the Ant Colony Optimization.

At the start of the experiment all ants were in the nest, and there were no pheromones deposited on the bridges. One assumption was that there was some probability that defined the choice of route of each ant that was increasing as the level of pheromone increased.

A pheromone serves as an indication to other ants about which path to take and as the number of ants on one path increases so does the level of deposited pheromone, and thus the signal that other ants receive when searching for food. Randomly, one of the two paths will have a slightly higher number of ants that will lead to the increase in the level of pheromone, thus increasing the probability of choosing the path and eventually will lead to all ants using only one path.



**Figure 6.1 A two-bridge experiment**

Similarly, many authors experimented with posting an obstacle, and in these cases, ants exhibited a remarkable characteristic of finding the shortest path very quickly as well.

Ant colony optimization was used in solving traveling salesman problem (TSP). Time is discrete in the case of artificial ants. Pheromone intensity is  $\tau_{ij}(t)$ , where  $i$  and  $j$  are nodes that constitute a certain path. At the start of the experiment,  $t = 0$  and the variable  $\tau_{ij}(t)$  is equal to some small value ( $c$ ). At time  $t$  every ant moves from the starting node to some other node, and at time  $t + 1$  ants arrive at new nodes. The probability that an ant  $k$  which is located at node  $i$  will at time  $t$  go to node  $j$  will be  $p_{ij}^k(t)$  which can be formulated as follows (Deneubourg et al. (1990)):

$$p_{ij}^k(t) = \begin{cases} \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{h \in \Omega_i^k(t)} [\tau_{ih}(t)]^\alpha [\eta_{ih}]^\beta}, & j \in \Omega_i^k(t) \\ 0, & j \notin \Omega_i^k(t) \end{cases} \quad (9)$$

where:

$\Omega_i^k(t)$ -set of permissible nodes that can be visited by ant  $k$  (this set is being updated for every ant after each ant's movement),

$$\eta_{ij} = \frac{1}{d_i} \text{ - 'visibility',}$$



$\alpha$ ,  $\beta$ -parameters that define the relative importance of the intensity of pheromones and visibility.

Visibility is based on local information, and by increasing of importance of visibility, the probability of choosing the node that is in proximity of the node where the ant is located is also increasing. Similarly, by increasing the importance of the amount of pheromone deposited, the probability of choosing the link which was used by many ants is also increasing.

Ant movement is considered an action in which an ant travels from one node to another during the time interval. Bonabeau et al. (1999) asserted that  $n$  ants would move  $n$  times during the time interval  $(t, t + 1)$ , and the whole cycle was comprised of  $m$  iterations.

Colomi et al. (1994), Dorigo et al. (1992), in their research proposed a way to calculate the pheromone quantity on each link:

$$\tau_{ij}(t) \leftarrow \rho\tau_{ij}(t) + \Delta\tau_{ij}(t) \quad (10)$$

where:

$\rho$ -coefficient such that  $1-\rho$  is the intensity of evaporation of pheromone during one cycle,  $(0 < \rho < 1)$ ,

Total increase of the pheromone quantity after one cycle on link  $(i, j)$  can be calculated (Deneubourg et al. (1990)):

$$\Delta\tau_{ij}(t) = \sum_{k=1}^n \Delta\tau_{ij}^k(t) \quad (11)$$

where:

$\Delta\tau_{ij}^k(t)$ -pheromone quantity left by ant  $k$  on link  $(i, j)$  during the cycle

The variable  $\Delta\tau_{ij}^k(t)$  can be calculated (Deneubourg et al. (1990)):

$$\Delta\tau_{ij}^k(t) = \begin{cases} \frac{Q}{L_k(t)}, & \text{If } k\text{th ant goes through link } (i, j) \text{ during the cycle} \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

where:

$Q$  -constant

$L_k(t)$ -length of the route that was developed by the  $k$ th ant during the cycle

Artificial ants cooperate to find good solutions and this cooperation stems from leaving the pheromone trails that other ants can follow, if they so choose.

Development of artificial systems should not be solely based on imitation of natural systems alone. On the contrary, these systems should act as a source of different ideas how to tackle complex problems and how to develop artificial systems. Multi-agent systems represent systems that consist on many individuals that use communication, cooperation, knowledge to perform certain tasks in their environment.

Dorigo et al. (1999) tried to improve the ant system and developed ant colony optimization which, at that time, was a new heuristic technique for solving complex combinatorial optimization problems.

In the case of ACO, the level of pheromone on links is changing based on local rules and global rules of changing the pheromone level. Local rules of pheromone change say that artificial ant deposits the pheromone on each link that he visits while searching for food, and this rule can be written as (1999):

$$\tau_{ij}(t) \leftarrow (1 - \rho)\tau_{ij}(t) + \rho\tau_0 \quad (13)$$

where:

$\rho$ -parameter that can take value  $0 < \rho < 1$ ,

$\tau_0$ -amount of pheromone which ant deposits on link  $(i, j)$

A global rule of pheromone change is applied only when all the ants create a route to food source, and it can be written as:

$$\tau_{ij}(t) \leftarrow (1 - \alpha)\tau_{ij}(t) + \alpha\Delta\tau_{ij}(t) \quad (14)$$

where:

$$\Delta\tau_{ij}(t) = \begin{cases} (L_{gb}(t))^{-1}, & \text{if } (i, j) \in \text{best created route of all the route to the food source} \\ 0, & \text{otherwise} \end{cases}$$

$L_{gb}(t)$ -length of the best route to the food source discovered from the beginning,

$\alpha$ -parameter which regulates the evaporation of the pheromones ( $0 < \alpha < 1$ )

The pseudo code of ant colony optimization proposed by Dorigo et al. (1999) can be written as follows:

**Procedure** ACO\_Meta\_Heuristic

**While** (termination\_criteria\_not\_satisfied)

**Schedule\_activities**

ants\_generation\_and\_activity;

pheromone\_evaporation;

**end schedule\_activities**

**end while**

**end procedure**

**procedure** ants\_generation\_and\_activity

**while** (available\_resources)

*schedule\_the\_creation\_of\_a\_new\_ant();*

*new\_active\_ant();*

**end while**

**end procedure**

```

procedure new_active_ant()

    initialize_ant();

     $M = \text{update\_ant+memory}()$ ;

    While (current_state  $\neq$  target_state)

         $A = \text{read\_local\_ant\_pheromone\_table}()$ ;

         $P = \text{compute\_transition\_probabilities}(A, M, \text{problem\_constraints})$ ;

        next_state = apply_ant_decision_policy(P, problem_constraints);

        move_to_next_state(next_state);

        if (online_step – by – step_pheromone_update)

            deposit_pheromone_on_the_visited_arc();

            update_ant_pheromone_table ();

        end if

         $M = \text{update\_internal\_state}()$ ;

    end while

    if (online_delayed_pheromone_update)

        Evaluate_solution ();

        Deposit_pheromone_on_all_visited_arcs ();

        Update_ant_pheromone_table ();

    end if

end procedure

```

Basically, the simulated ants exchange information on the quality of the solutions found using a way of communicating similar to that of real ants. An ant from one solution chooses

the next solution via a stochastic mechanism: If one of the next solutions has not been previously visited, it can be selected with a probability that is proportional to the pheromone associated with that solution.

Moreover, the amount of pheromone deposited depends on the quality of the solution found. Pheromone information is used by subsequent ants as a guide towards promising regions of the search space. Pheromone update aims to increase the values of pheromone associated with promising solutions. Furthermore, at the end of each iteration, depending how good the solutions are, the pheromone values are modified to bias ants in future iterations to construct good solutions.

### Tabu Search

Root of the word “Taboo” comes from the island of Tonga. The natives of that island in the Pacific Ocean used this word to denote holy things and objects that cannot be touched. Taboo technique was suggested by Glover (1986) and significant contributions gave Hansen (1986). This type of heuristic begins in the same way as ordinary local or neighborhood search and goes iteratively from one solution to another until some terminating criteria is met. This procedure is based on the following:

1. The use of flexible attribute-based memory structures designed to permit evaluation criteria and historical search information to be exploited more thoroughly than by rigid memory structures or by memoryless systems
2. An associated mechanism of control-for employing the memory structures-based on the interplay between conditions that contain and free the search process (embodied in Tabu restrictions and aspiration criteria),

3. The incorporation of memory functions of different time spans, from short term to long term, to implement strategies for intensifying and diversifying the search.

Short-term memory process is the foundation of this procedure. It constitutes an aggressive exploration that seeks to make the best move possible, subject to available choices to satisfy limitations or constraints. Constraints are imposed to prevent the reversal and/or repetition of certain moves by marking the certain moves as forbidden (tabu), making the primary goal of these restrictions to allow the method to go beyond the points of local optimum while making sure that the quality of the solutions is high at each or most of the steps. An important stage in this procedure is choosing the best admissible candidate. Every move from the list of candidates is evaluated per turn.

A Tabu search procedure can be summarized as follows:

1. **Begin with a starting current solution.** Obtain the solution from initialization or from an intermediate or long-term memory component
2. **Create a candidate list of moves.** Each move generates a new solution from the current solution.
3. **Choose the best admissible candidate.** Admissibility is based on the Tabu restrictions and aspiration criteria. Designate the solution obtained as the new current solution, and record it as the new best solution if it improves the fitness function.
4. **Stopping criterion.** Stop if a specified number of iterations has elapsed in total or since the last best solution was found.

- a. **Stop. Terminate globally or transfer.** A transfer initiates an intensification or diversification phase embodied in an intermediate or long-term memory component.
- b. **Continue. Update Admissibility conditions.** Update Tabu restrictions and aspiration criteria.

### Simulated annealing

Simulated annealing is a random search technique used to tackle complex combinatorial optimization problems, which is motivated by an analogy to the statistical mechanics of annealing in solids. The annealing process involves tempering certain alloys of metal, glass, or crystal by heating above its melting point (this leads to a high energy state of the atoms, and a high possibility to re-arrange the crystalline structure), holding its temperature, and then cooling it very slowly (the atoms have a lower and lower energy state and a smaller possibility to re-arrange the crystalline structure) until solidification is achieved, which produces high-quality materials with superior structural integrity.

By successfully lowering the temperature and running this algorithm, we can simulate the material coming into equilibrium at each newly reduced temperature, and thus effectively simulate the physical annealing.

Thermodynamic behavior and the search for the global minima for a discrete optimization problem is connected by the simulated annealing technique. At each iteration of the algorithm the objective function generates values of two solutions (current one and the newly created one) that are compared. The search starts with a random state and in a polling loop we move to neighboring states. Improved solutions are always accepted, while some of the non-improving solutions are accepted to possibly escape

local optima in search of global optima. The probability of accepting non-improving solutions depend on temperature parameter, which typically does not increase along with each iteration of the algorithm.

Starting from an initial solution ( $S$ ), the value of the objective function is calculated for the new solution ( $S'$ ) in the neighborhood. The algorithm then attempts to move to a neighborhood solution based on a specified criterion. In minimization problems, transition is allowed when:  $\Delta=f(S')-f(S)<0$ . A transition to the new solution is also allowed based on the probability function  $exp(-\Delta/T)$ , where  $T$  is the temperature (control parameter). Allowing such transitions enables diversification and enables the algorithm to escape local optimum. After each iteration, the temperature decreases with a cooling function ( $T=T*\alpha$ ) where  $\alpha$  is a constant parameter by which the temperature decreases after each iteration. The algorithm stops when the stopping criterion is met.

## **Chapter 7 Case study**

The Sioux Falls network configuration is used for a case study. Sioux Falls is situated in Minnehaha County in South Dakota. It has 24 nodes and 76 links and in the figure below we can see the graphical representation of the Sioux Falls network.

The Sioux Falls network presented in this paper has been used to examine and compare results on the networks, starting with the paper from LeBlanc et al. (1975). After we run the traffic assignment model, lanes and intersections (nodes) that have critical volume/capacity (high volume of vehicles) are identified and set as an initial set of project improvements. The model allows volume-capacity ratios to exceed 1.0, due to the BPR function used as a link performance function. Project alternatives that were being considered are link expansions, which is assumed to be in both directions between the two



connecting nodes because the O/D table is symmetrical, and vertical, horizontal or vertical and horizontal improvements of intersections. These assumptions save not only costs but also the use of construction equipment. The input data, besides the volume for each origin-destination pair is the link free flow travel time.

Figure 7.1 shows the map of Sioux Falls city in South Dakota, with all the major arterials labeled. In figure 7.2 we see the coded graphical representation of the Sioux Falls network based on the map from figure 7.1.

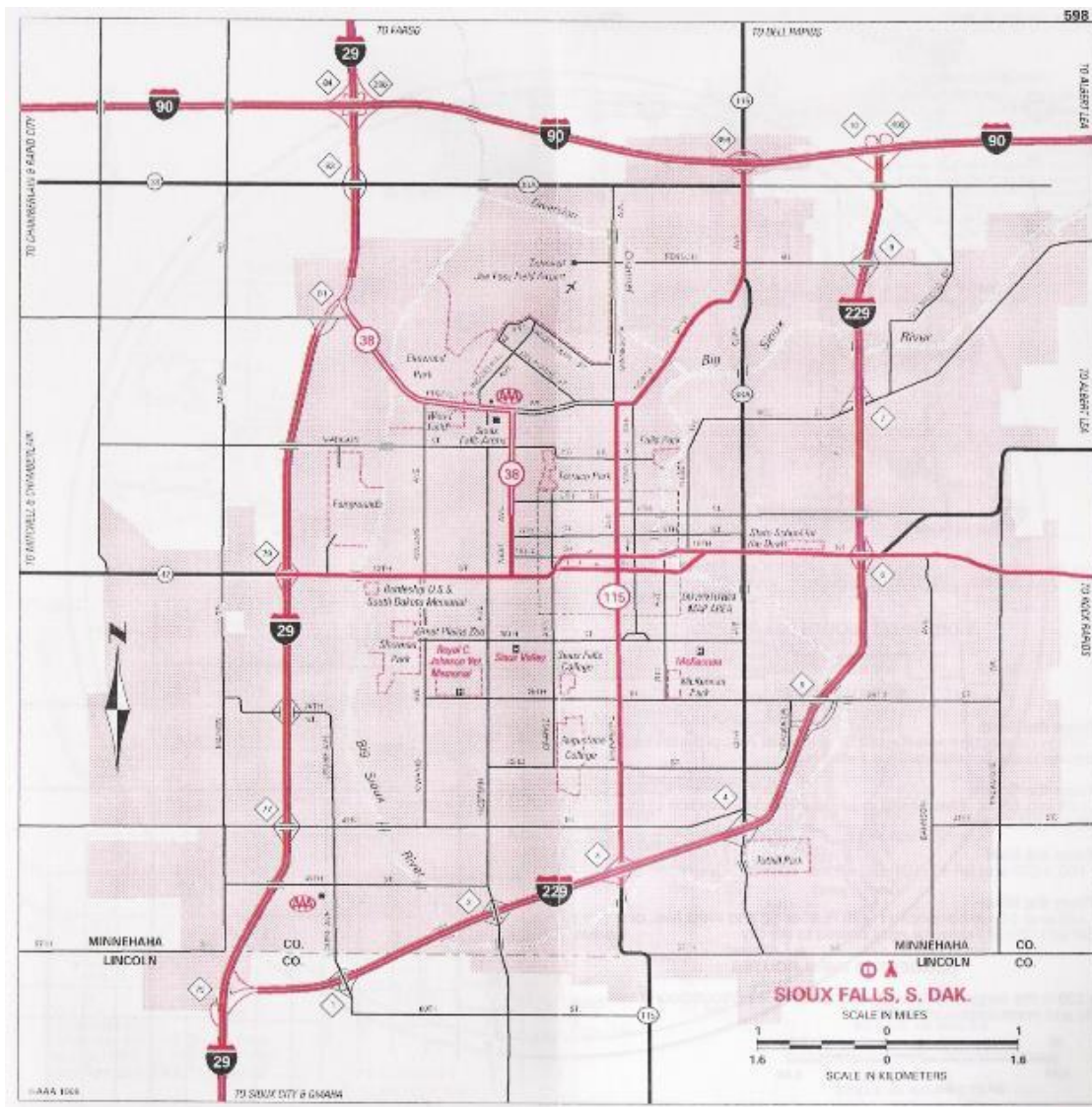
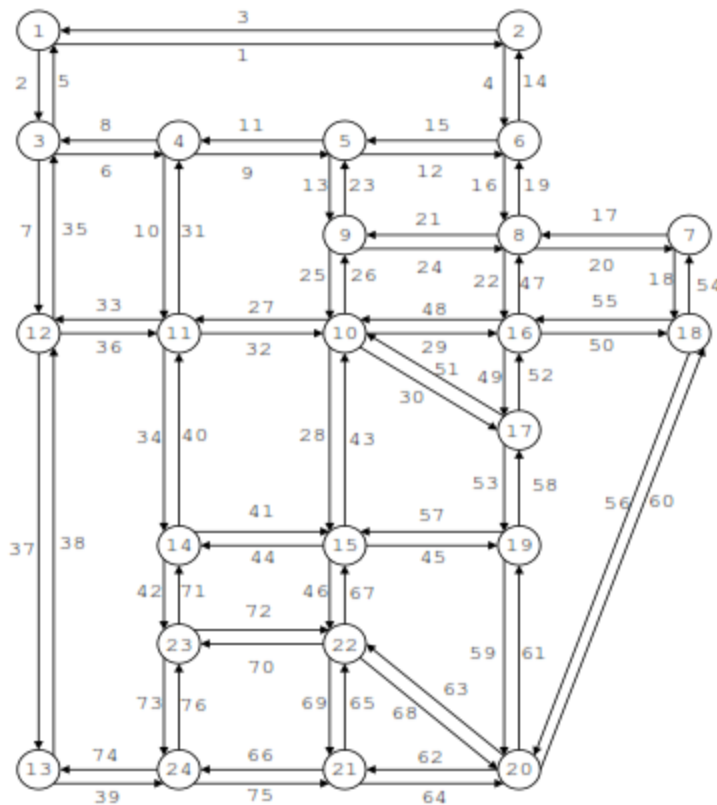


Figure 7.1 Map of the Sioux Falls, South Dakota



**Figure 7.2 Graphical representation of the Sioux Falls network**

Nodes 8, 11 and 16 represent intersections in the Sioux Falls network. Intersections were added by adding pseudo-links, each one for each movement for each link, i.e. for intersection 8, link 47 we have 3 pseudo-links for 3 separate movements (two turning movements, and through movement). Overall, for these three intersections 36 pseudo-links added to the network. In table 7.1, we can see the pseudo links for intersections, last digit represents the movement (2 is for left turning movement, 4 is for through movement and 6 is for right turning movement), and their capacity and free flow travel time ( $t_0$ ).

**Table 7.1 Pseudo – links for intersections 8, 11, and 16, their capacity and free flow travel time**

<b>1st node</b>	<b>2nd node</b>	<b>Pseudo Link ID</b>	<b>Capacity</b>	<b>t<sub>0</sub></b>	<b>Intersection</b>
4	14	10004	731	2.652	11
4	10	10002	736	3	
4	12	10006	736	3.876	
6	16	16004	734	1.302	8
6	9	16006	734	1.302	
6	7	16002	734	1.302	
7	9	17004	757	1.5	8
7	6	17006	734	1.302	
7	16	17002	756	1.5	
8	17	22004	756	1.002	16
8	10	22006	728	2.7	
8	18	22002	756	1.614	
12	4	36002	736	3.876	11
12	10	36004	736.32	3	
12	14	36006	731	2.652	
14	12	40002	731	2.652	11
14	4	40004	731	2.652	
14	10	40006	731	2.652	
16	9	47002	756	2.892	8
16	6	47004	734	1.302	
16	7	47006	756	1.5	
17	10	52002	728	1.002	16
17	8	52004	756	1.002	
17	18	52006	756	1.002	
18	17	55002	784	1.002	16
18	10	55004	728	1.614	
18	8	55006	756	1.614	

**Table 7.2 Origin/Destination demand table**

O/D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	200	120	360	180	240	300	0	360	840	0	180	360	180	300	0	300	120	180	180	60	240	180	120
2	200	0	6	18	6	30	12	0	18	36	0	12	18	6	12	0	18	6	6	12	6	12	6	6
3	120	6	0	18	6	18	6	0	12	18	0	18	12	6	6	0	6	0	6	6	6	6	6	6
4	360	18	18	0	30	30	30	0	48	72	0	42	36	30	30	0	30	6	18	24	12	24	30	18
5	180	6	6	30	0	18	12	0	48	60	0	12	12	12	18	0	18	6	12	12	6	12	12	6
6	240	30	18	30	18	0	24	0	24	48	0	18	18	12	18	0	36	6	18	24	6	18	12	6
7	300	12	6	30	12	24	0	0	36	114	0	48	30	18	30	0	60	60	30	36	18	36	12	6
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	360	18	12	48	48	24	36	0	0	168	0	42	36	36	60	0	60	12	30	42	24	42	36	12
10	840	36	18	72	60	48	114	0	168	0	0	126	114	132	240	0	234	42	108	156	78	162	108	54
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	180	12	18	42	12	18	48	0	42	126	0	0	84	42	48	0	42	12	18	30	24	48	42	30
13	360	18	12	36	12	18	30	0	36	114	0	84	0	36	42	0	36	6	24	42	36	78	48	48
14	180	6	6	30	12	12	18	0	36	132	0	42	36	0	45	0	42	6	24	30	24	72	66	24
15	300	12	6	30	18	18	30	0	60	240	0	48	42	45	0	0	90	18	48	66	48	156	60	30
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	300	18	6	30	18	36	60	0	60	234	0	42	36	42	90	0	0	42	102	102	42	102	36	18
18	120	6	0	6	6	6	60	0	12	42	0	12	6	6	18	0	42	0	24	100	6	24	6	6
19	180	6	6	18	12	18	30	0	30	108	0	18	24	24	48	0	102	24	0	78	30	78	24	12
20	180	12	6	24	12	24	36	0	42	156	0	30	42	30	66	0	102	100	78	0	78	150	42	30
21	60	6	6	12	6	6	18	0	24	78	0	24	36	24	48	0	42	6	30	78	0	114	42	36
22	240	12	6	24	12	18	36	0	42	162	0	48	78	72	156	0	102	24	78	150	114	0	132	72
23	180	6	6	30	12	12	12	0	36	108	0	42	48	66	60	0	36	6	24	42	42	132	0	4.8
24	120	6	6	18	6	6	6	0	12	54	0	30	48	24	30	0	18	6	12	30	36	72	4.8	0

As we can see from table 7.2, there are no trips originating and ending at nodes 8, 11, 16, because we consider them as intersections in the Sioux Falls network. In table 7.3, the values of delays on intersection pseudo-links are presented, the volumes on each of the link and their pseudo  $v-c$  ratios.

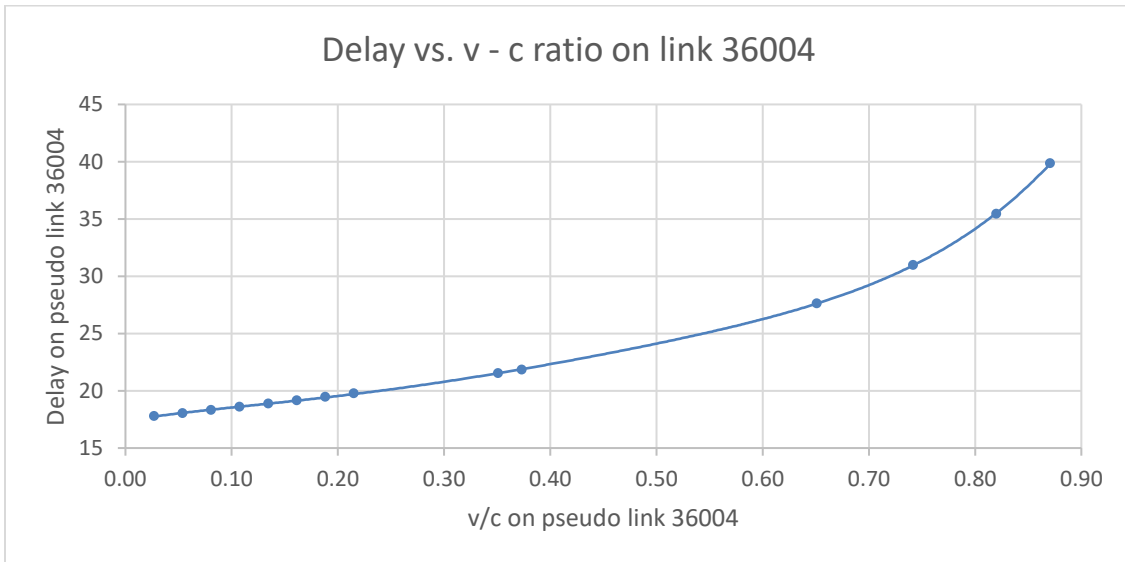
**Table 7.3 Values of delay [sec/veh], volume [veh] and v – c ratios for pseudo links for intersections 8, 11, and 16**

Intersection 8				Intersection 16				Intersection 11			
Pseudo Link	Delay	Volume	v/c	Pseudo link	Delay	Volume	v/c	Pseudo Link	Delay	Volume	v/c
47002	17.97558	30.00016	0.039637	52002	27.64979792	473.9998	0.650886	40002	20.85631	222	0.303496
47004	21.64109	263.9999	0.359287	52004	19.34905034	132.0003	0.174402	40004	18.64288	83.99998	0.114836
47006	20.77005	222	0.293312	52006	18.5155287	72	0.095128	40006	19.44368	138	0.18866
24004	18.10124	39.8841	0.05265	55002	18.55803339	72	0.09178	27002	21.19155	240	0.328104
24002	18.56317	78	0.106153	55004	18.22056692	54	0.074152	27004	19.72321	156	0.211863
24006	17.97557	29.99998	0.039637	55006	19.52566035	144	0.190256	27006	17.542	0	0
17004	18.09034	72.16643	0.051511	22004	19.13900122	117.4108	0.155126	10004	18.81465	95.99994	0.131241
17006	26.32588	60.00023	0.604255	22006	17.51455175	0	0	10002	17.542	0	0
17002	20.46663	474.9461	0.26953	22002	19.61531938	150	0.198184	10006	19.91571	168	0.22816
16004	18.48216	39.02082	0.098214	29002	18.22056744	54.00004	0.074152	36002	19.62853	150	0.203715
16006	18.31556	443.9997	0.081657	29004	18.22056692	54	0.074152	36004	21.52258	258.416	0.350954
16002	27.49526	204	0.646371	29006	29.63141306	515.9998	0.70856	36006	20.85631	222	0.303496

The value of delay for intersection 16 which is 24.231 seconds. The values of delay for the other two intersections are 20.756 sec and 20.37 sec for intersections 8 and 11, respectively. These values were obtained using the following formula:

$$d_I = \frac{\sum d_A v_A}{\sum v_A}$$

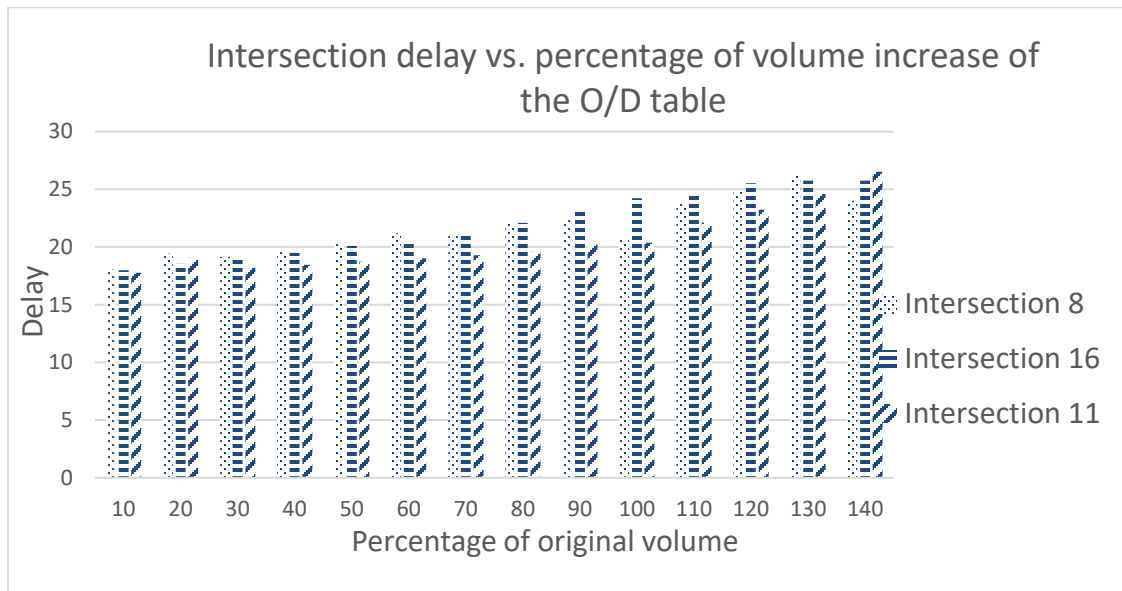
As we can see from figure 7.3, the values of delay increases as the v/c ratio increases as well, for the pseudo link 36004. This pseudo link was chosen because of the high values of delay that change as the O/D volumes change from 10% to 100% in 10% increments. The traffic assignment model was run with 3 intersections and with added pseudo links for all Sioux Falls network for different values of O/D tables (from 10% to 100% with a 10% increment). Most of the links have a value of v/c ratio below 0.5, which indicates that the traffic assignment works well, and assigns the traffic and volumes to other links, distributing almost evenly the volumes throughout the network. This can also be said for the pseudo links. The most congested intersection pseudo link is 36004, the through movement from link 36 and the delay there is approximately 21 sec/veh with the value of v-c ratio of 0.889 at most (when the values of the flow from the O/D matrix reach 140%).



**Figure 7.3 Delay vs v-c ratio for pseudo link 36004**

Figure 7.4, shows the overall intersection delay for the three intersections as the function of the percentage of increase of the original O/D volumes. As we can see, the

intersection delay increases as the percentage of volume increases and the two intersections with the highest increase in delay are intersections 8 and 16.



**Figure 7.4 Intersection delay vs. percentage increase of the original volume of the O/D table**

Two intersections that are being considered for improvement are 8 and 16 based on the value of delay. More specifically, both intersections will be widened vertically and horizontally. Moreover, links considered for improvement are 30, 51, 62 and 29. The links were chosen because of the high  $v-c$  ratio, which is above 0.6. The projects that are considered, with their costs and descriptions are shown in table 7.4.

**Table 7.4 Project ID and descriptions**

<b>Project ID</b>	<b>Project Description</b>	<b>Project cost (\$)</b>
1	Improvement of link 69&65	1,800,000
2	Improvement of link 30&51	4,800,000
3	Improvement of link 62&64	3,900,000
4	Improvement of link 68&63	4,200,000
5	Horizontal improvement of intersection 8 (improvement of pseudo link (16,47))	921,600
6	Vertical improvement of intersection 8 (improvement of pseudo link (17,24))	921,600
7	Horizontal improvement of intersection 16 (improvement of pseudo link (22,52))	921,600
8	Vertical improvement of intersection 16 (improvement of pseudo link 29,55)	921,600

Using the ACO and TS metaheuristic, the sequence of these projects that will be implemented will be identified. Two variables that are important for the problem are: the cost of the project and his time it takes to complete it. The time of the completion of the projects is one variable, and costs of the projects were calculated using the objective function that was presented in chapter 4. The baseline interest rate is assumed to be 2% per year (0.02 in other words) in calculations and the number of years is 20. Table 7.5 shows the parameters of the algorithms used.

**Table 7.5 Parameters used for metaheuristics**

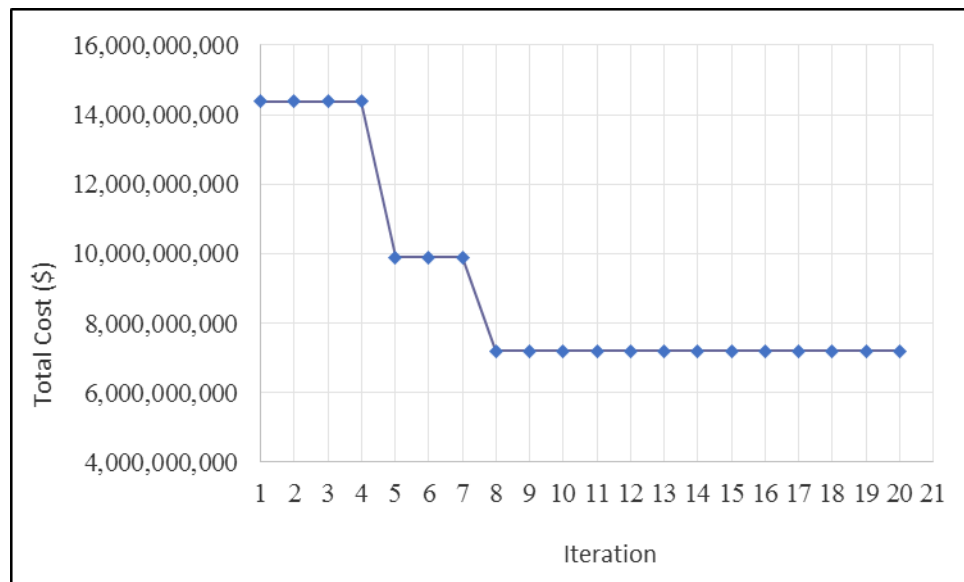
<b>ACO</b>	
Number of iterations	50
Number of ants	50
Initial pheromone	0.000000000372
Evaporation rate	0.7
<b>SA</b>	
Temperature start	10000
Cooling factor	0.001
<b>TS</b>	
Number of iterations	50



In figure 7.5, we see how the cost function changes over successive iterations. The maximum number of iterations was at first set at 200, but later was changed to 50 because the algorithm did not show any improvement at all after the 50 iterations.

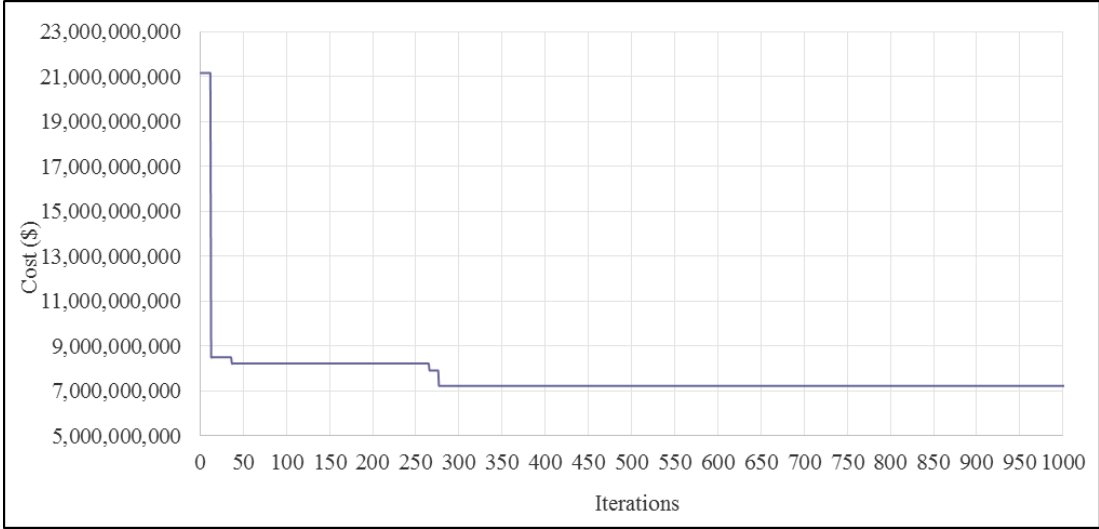
The parameters used to calculate the values of costs for each of the projects individually are:

- $c_i=3,000,000$  \$/lane mile,
- $A_i$ -area needed for improvement is 2880 ft<sup>2</sup> (12 ft is the width of the lane),
- overall area of the intersection is 6912 ft<sup>2</sup>.



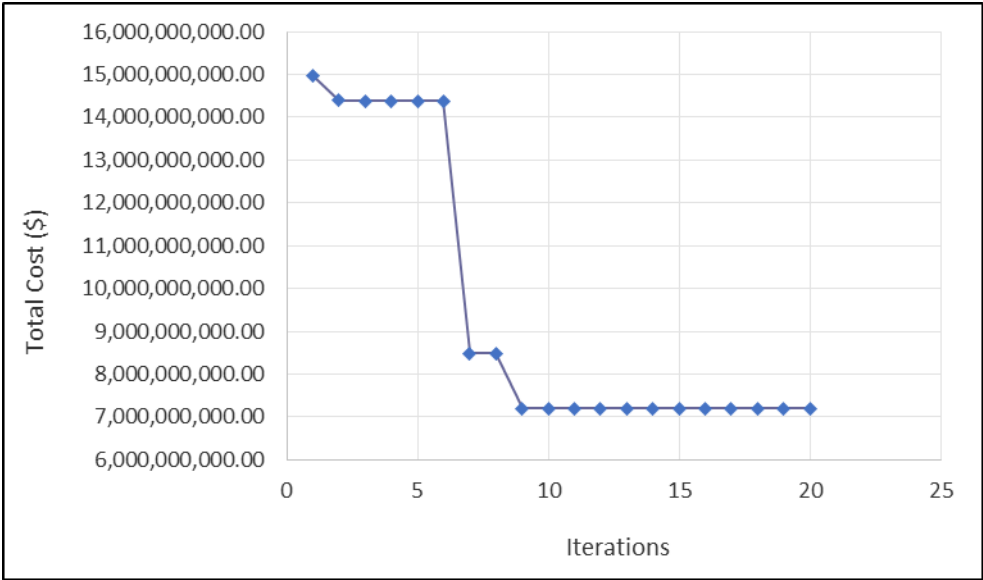
**Figure 7.5. Total Cost vs. number of iterations for the ACO**

From figure 7.5, we can see that ACO algorithm finds the best value of the cost function relatively fast (after 40 minutes of running time).



**Figure 7.6 Cost vs. Iteration number for simulated annealing**

Figure 7.6 shows the behavior of the simulated annealing for the first 1000 iterations. The best value of the cost function is found around the 43<sup>rd</sup> minute.



**Figure 7.7 Cost vs Iteration number for Tabu Search heuristic procedure**

Figure 7.7 shows the performance of the Tabu Search procedure, and that the optimal solution of the cost function is obtained after approximately 44 minutes.

**Table 7.6 Projects schedule and the value of the cost function for implemented algorithms**

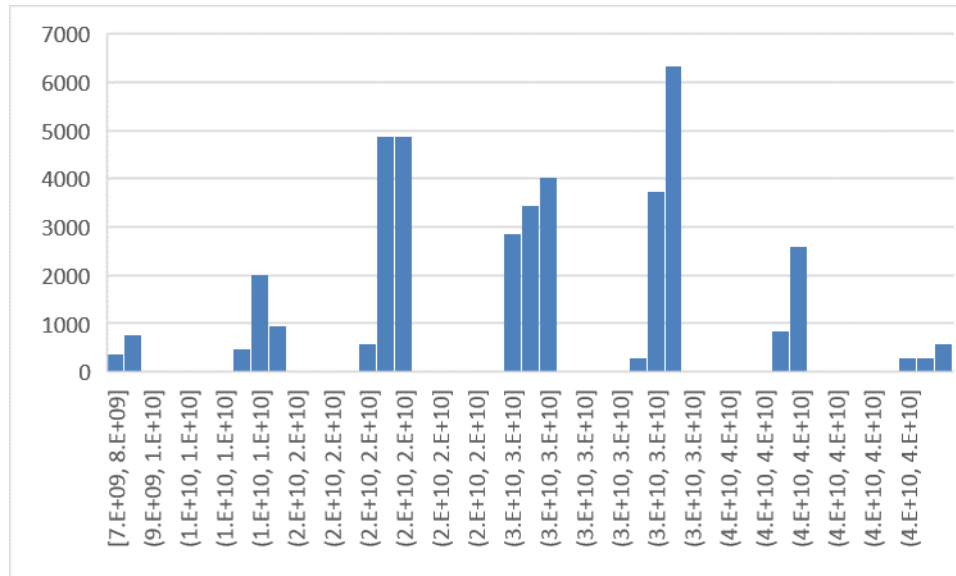
<b>Algorithm</b>	<b>Project schedule</b>	<b>Cost (\$)</b>
<b>ACO</b>	8,6,7,5,2,1,3,4	7,202,111,619.36
<b>SA</b>	8,6,7,5,2,1,3,4	7,202,111,619.36
<b>TS</b>	8,6,7,5,2,1,3,4	7,202,111,619.36

Table 7.6 shows the schedule of projects and the value of the cost function. Results that are obtained with heuristic algorithms are not guaranteed to be optimal, and it is difficult to assess the quality of the obtained solution. To check the algorithms a complete enumeration test was conducted for a small number of projects. There are 40 320 possible permutations of the projects (8 projects) and each of the permutation has its own value of the objective function. Table 7.7 shows the number of possible permutations, the minimum and maximum value of the objective function. As can be seen from the table, the minimum values correspond to the minimum value of the objective functions obtained by Simulated Annealing (SA), Ant Colony Optimization (ACO) algorithm, and Tabu Search (TS).

**Table 7.7 Number of possible permutations and minimum and maximum value of the objective function**

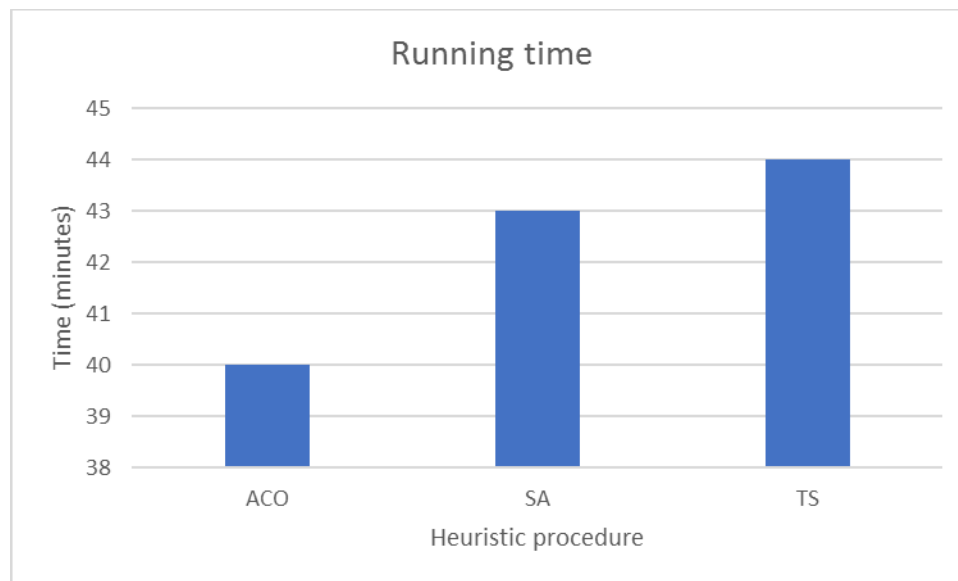
<b>Number of possible permutations</b>	<b>Smallest value of the cost function (\$)</b>	<b>Highest value of the cost function (\$)</b>
40320	7,202,111,619.36	45,945,312,044.59

Figure 7.8 shows the values of the objective functions for the generated 40320 solutions.



**Figure 7.8 Value of the cost function for the 40320 exhaustively generated solutions**

Figure 7.9 shows the running time of three heuristic procedures. We can see that the ACO has the lowest running time and the sensitivity analysis will be conducted for ACO because it has the lowest running time.



**Figure 7.9 Running time for ACO, SA and TS heuristic procedures**

To further test the goodness of solutions obtained by the heuristic procedure, we increase the number of projects considered for implementation to 12, thus increasing the number of possible permutations of projects to more than 400 million. The projects being considered are summarized in table 7.8.

**Table 7.8 Project ID's and their descriptions**

<b>Project ID</b>	<b>Project Description</b>	<b>Project cost (\$)</b>
1	Improvement of link 69&65	1,800,000
2	Improvement of link 30&51	4,800,000
3	Improvement of link 62&64	3,900,000
4	Improvement of link 68&63	4,200,000
5	Horizontal improvement of intersection 8 (improvement of pseudo link (16,47))	921,600
6	Vertical improvement of intersection 8 (improvement of pseudo link (17,24))	921,600
7	Horizontal improvement of intersection 16 (improvement of pseudo link (22,52))	921,600
8	Vertical improvement of intersection 16 (improvement of pseudo link 29,55)	921,600
9	Improvement of link 9&10	3,000,000
10	Improvement of link 29&48	3,300,000
11	Improvement of link 42&71	2,700,000
12	Improvement of link 5&22	6,900,000

Table 7.9 shows the order of the projects that should be implemented, and the value of the objective function obtained by three heuristic procedures.

**Table 7.9 Project schedule and the value of the objective function for different heuristic procedures for the testing purposes**

<b>Algorithm</b>	<b>Project schedule</b>	<b>Cost (\$)</b>
<b>ACO</b>	8,6,7,5,12,1,2,4,11,9,10,3	7,890,249,717.66
<b>SA</b>	8,6,7,5,12,1,2,4,11,9,10,3	7,890,249,717.66
<b>TS</b>	8,6,7,5,12,1,2,4,11,9,10,3	7,890,249,717.66

Table 7.10 presents the maximum, minimum, average value and the value of standard deviation of the 50 000 randomly generated solutions.

**Table 7.10 Minimum, maximum, average value and the value of standard deviation**

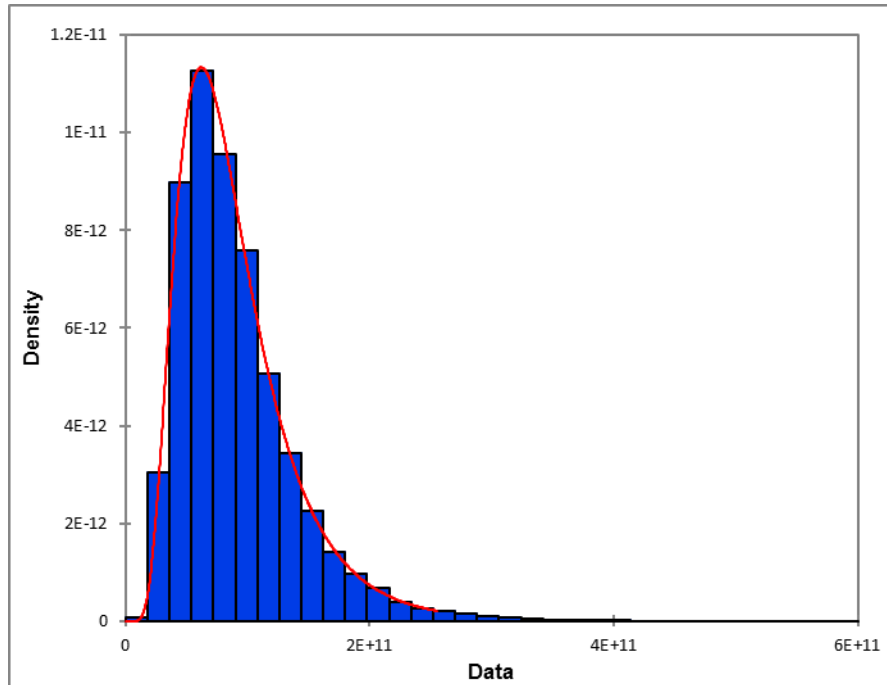
<b>Minimum</b>	<b>Maximum</b>	<b>Mean</b>	<b>Std. deviation</b>
8,526,207,979	537,461,226,823	90,386,329,436	48,043,531,069

In case of 12 projects, complete enumeration is harder, because it takes a lot more time than in the case of 8 projects. Therefore, we apply the Kolmogorov-Smirnov (K-S) goodness of fit test (Chakravart, Laha, and Roy, (1967)) which is used to decide if a sample comes from a population with a specific distribution, and to estimate the probability that better or much better solutions can be found.

The K-S test compares two cumulative frequency distributions. A cumulative frequency distribution (CDF) is useful for finding the number of observations above or below a value in a data sample. It can be calculated by taking a given frequency and adding all the preceding frequencies in the list. The observed CDF and empirical CDF allow us to find the point at which these two distributions show the largest divergence, and the test uses this parameter to identify two-tailed probability estimate  $p$  to determine if the samples are statistically similar or different. There are two hypotheses:

- $H_0$ -null hypotheses-there is no difference between the observed distribution of the sample and the empirical distribution,
- $H_A$ -research hypotheses-there is a difference between the observed distribution and the empirical distribution.

The sample data were tested for various distributions. It was found that the Lognormal distribution best fitted the data, which is shown in figure 7.10.



**Figure 7.10 Fitted lognormal distribution**

The cumulative probability distribution for the Lognormal distribution can be calculated by Eq. 15:

$$p = F(x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^x \frac{e^{-\frac{(\ln(t)-\mu)^2}{2\sigma^2}}}{t} dt \quad (15)$$

This gives us the value of  $p = 0.000000009796913329673100000000$ . The solution shows us that the solution obtained by the ACO procedure is better than 99.999% of the solutions randomly generated in the distribution. Hence, the solution obtained by ACO procedure is very good compared to other randomly generated solutions. Table 7.11 shows the estimated parameters for Lognormal distribution obtained with the K-S test, as well as the parameters of the Kolmogorov-Smirnov test.

**Table 7.11 Parameters of the Lognormal distribution and the K-S parameters**

<b>Parameter</b>	<b>Value</b>
$\mu$	25.102
sigma	0.5
D	0.004
p-value	0.492
alpha	0.05

## **Chapter 8    Sensitivity analysis**

This chapter studies how the uncertainty about various input parameters of the optimization model can influence the overall goodness of solution obtained by the ACO procedure. It is useful to conduct sensitivity analysis because it helps us understand the model's behavior. Sensitivity analysis is carried on showing the effects based on ACO parameters and network specifications. Factors considered are

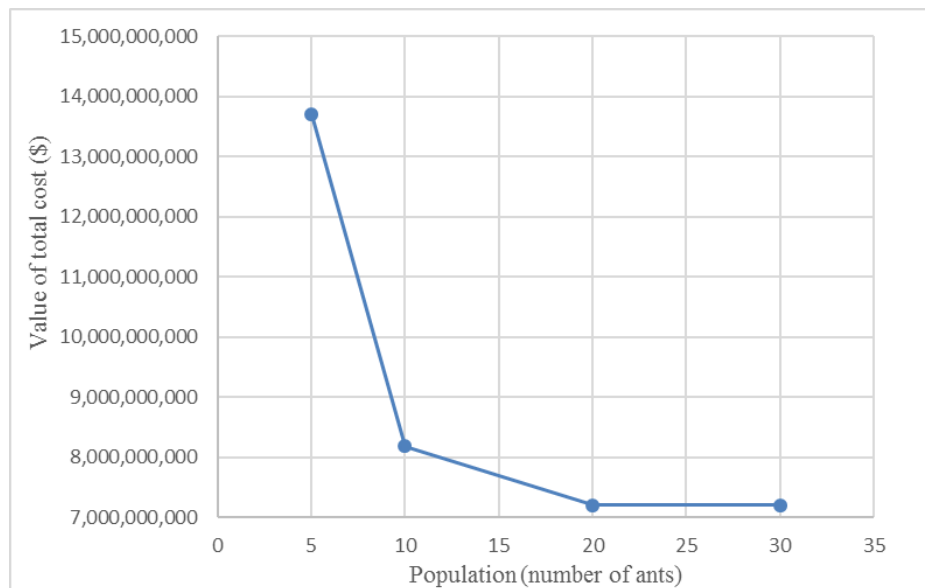
- ACO parameters
  - Ant population size,
  - Problem size,
  - Evaporation rate,
- Problem parameters
  - Improvement cost,
  - Different demand,
  - Interest rate

### Ant population size

A crucial parameter of the Ant colony optimization algorithm is the size of ant colony. By increasing the colony size, the optimization speed should increase to a certain point, however increasing it too much could result in an oversaturation of the system. By



having a small size of the colony, the computational time increases. Basically, it all comes down to the magnitude of the problem. Simple, straightforward problems could be handled using the smaller colony and by using the larger one would sometimes lead to oversaturation. On the other side, complex problems, with huge variables and a high number of possible solution would require a larger colony to save computation time, but not too large. Figure 8.1 shows how the present worth of total cost changes as the colony's population changes. Table 8.1 shows how the sequence of projects changes as the population is changed.



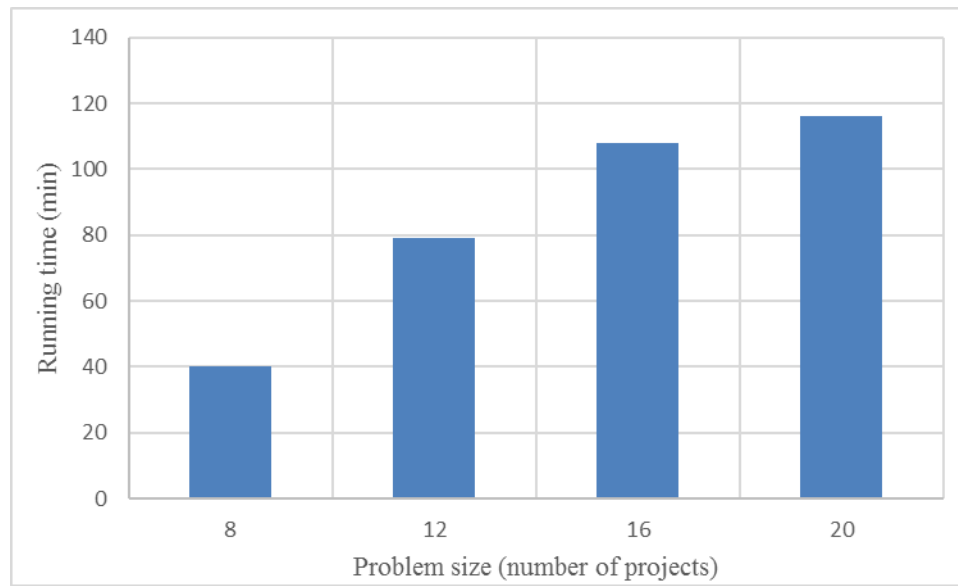
**Figure 8.1 Colony population (number of ants) vs value of total cost**

**Table 8.1 Population size vs sequence of projects**

Population size	Sequence
5	4,5,1,6,7,2,3,8
10	8,6,7,4,5,1,3,2
20	8,6,7,5,2,1,3,4
30	8,6,7,5,2,1,3,4

### Problem size

Problem size and computation time are two important and related parameters. Computation time increases as the problem size increases as we can see from figure 8.2. Problem size, in this case study, is related to the number of projects that are considered for implementation. As the problem size increases, so should the size of the ant colony to guarantee a reasonable exploration of the solution space. A network with the same characteristics is tested, and the only variable that is subject to change is problem size.



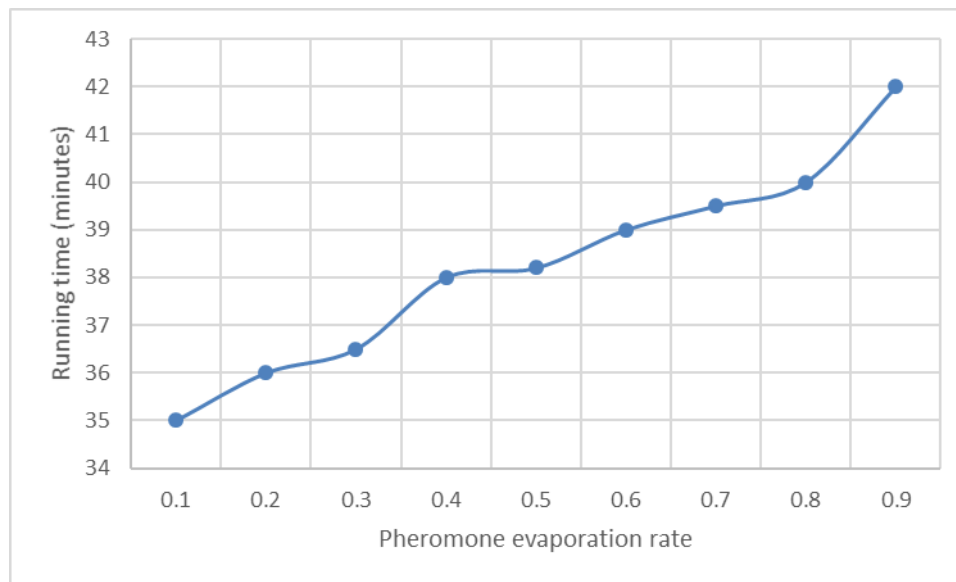
**Figure 8.2 Problem size vs Running time**

### Pheromone evaporation rate

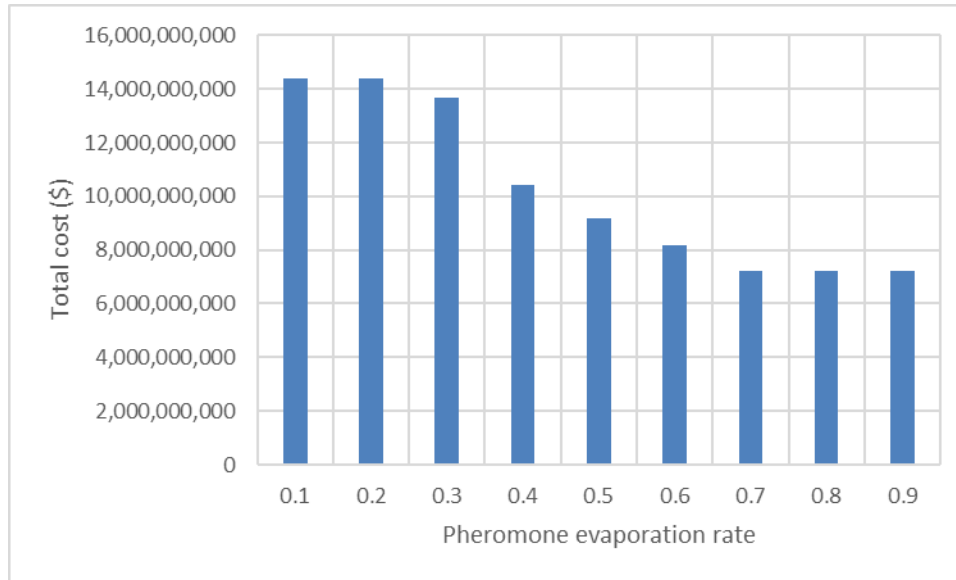
Evaporation rate of pheromones is a parameter of ACO procedure used to avoid unlimited accumulation of pheromone trail over some components of the solution. The evaporation rate influences the running time of the algorithm and the value of the total cost.

Figure 8.3 shows that as we increase the pheromone evaporation rate, the running time of the algorithm increases, as expected, because when the evaporation rate is small,

the pheromone will evaporate slowly, and the algorithm will converge faster because the ants do not explore the solution space that much. On the other hand, if the evaporation rate is high, the pheromone will evaporate more quickly, thus ants will explore a much larger space of solutions, leading to an increase in overall running time of the algorithm. As we can see from the figure 8.4, the total cost is also influenced by pheromone evaporation rate. If the rate is low, the algorithm converges more quickly, and the solution obtained could be locally optimal. However, setting the rate higher will ensure that the ants search a much larger space, thus finding the optimal solution. We can also note that for the values 0.8 and 0.9 the objective function does not change, but the running time increases slightly.



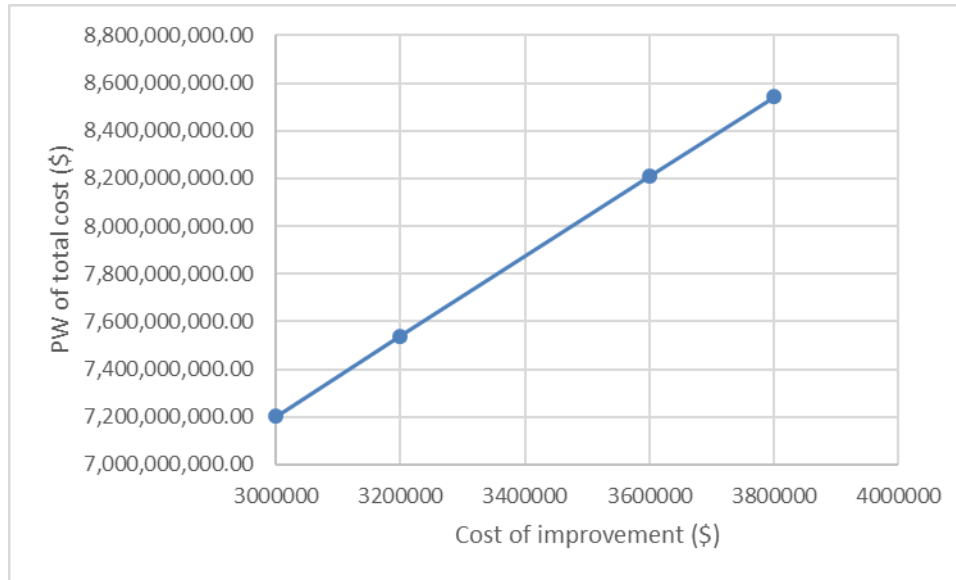
**Figure 8.3 Pheromone evaporation rate vs the running time of the algorithm**



**Figure 8.4 Pheromone evaporation vs the total cost**

Improvement cost

In this study, the cost of link improvement is a function of its length, and the cost of intersection improvement is a separate function. This subsection explores how variation of project cost may affect the optimization results by increasing the cost per lane mile and the cost per square foot by 5%, 10%, and 20%. Figure 8.5 shows the optimizing results for different costs for lane improvement. We can note that as the cost of lane improvement increase, so does the present worth of total cost, and that the increase appears to be linear.



**Figure 8.5 Cost of link improvement vs present worth of total cost**

Table 8.2 shows the sequence of projects, when the cost of improvement is changed.

**Table 8.2 Cost of link improvement and the sequence of projects**

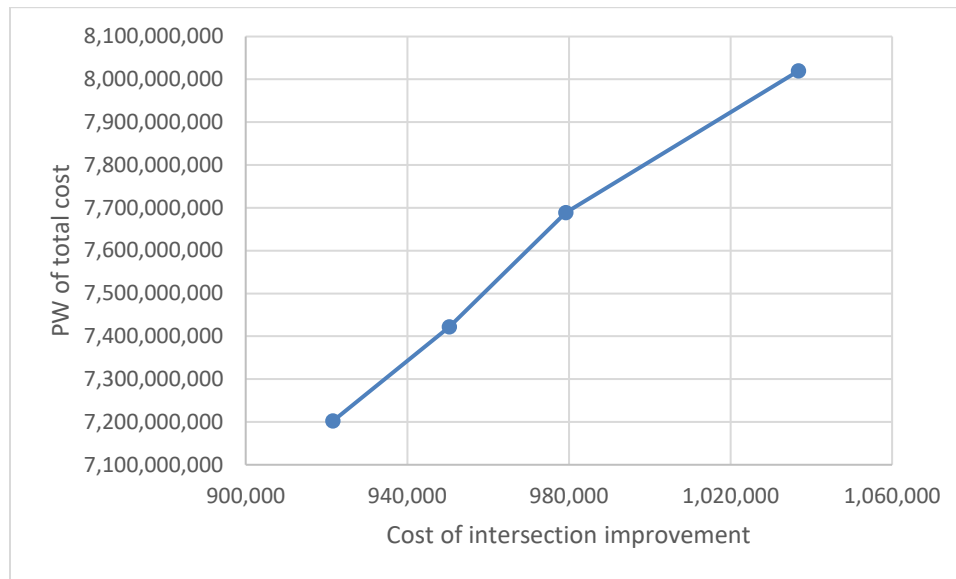
Cost of Improvement (\$)	Sequence of projects
3,000,000	8,6,7,5,2,1,3,4
3,200,000	8,4,2,5,7,3,1,6
3,600,000	4,8,7,3,6,5,2,1
3,800,000	7,8,6,1,4,5,3,2

A similar analysis was done for intersection improvements. The cost of land per square foot is 200\$. This influences the cost of intersection improvement. The value is increased, similarly, by 5%, 10%, and 20%. Table 8.3 shows the sequence of projects and different values of cost of improvement.

**Table 8.3 Cost of intersection improvement and the sequence of projects**

Cost of Improvement (\$)	Sequence of projects
921,600	8,6,7,5,2,1,3,4
950,400	1,6,4,8,2,5,3,7
979,200	2,8,5,1,3,6,4,7
1,036,800	7,5,1,8,4,6,3,2

The change in total cost function is shown in figure 8.6, and we see that as the cost of intersection improvement increases, the present worth of total cost also increases.



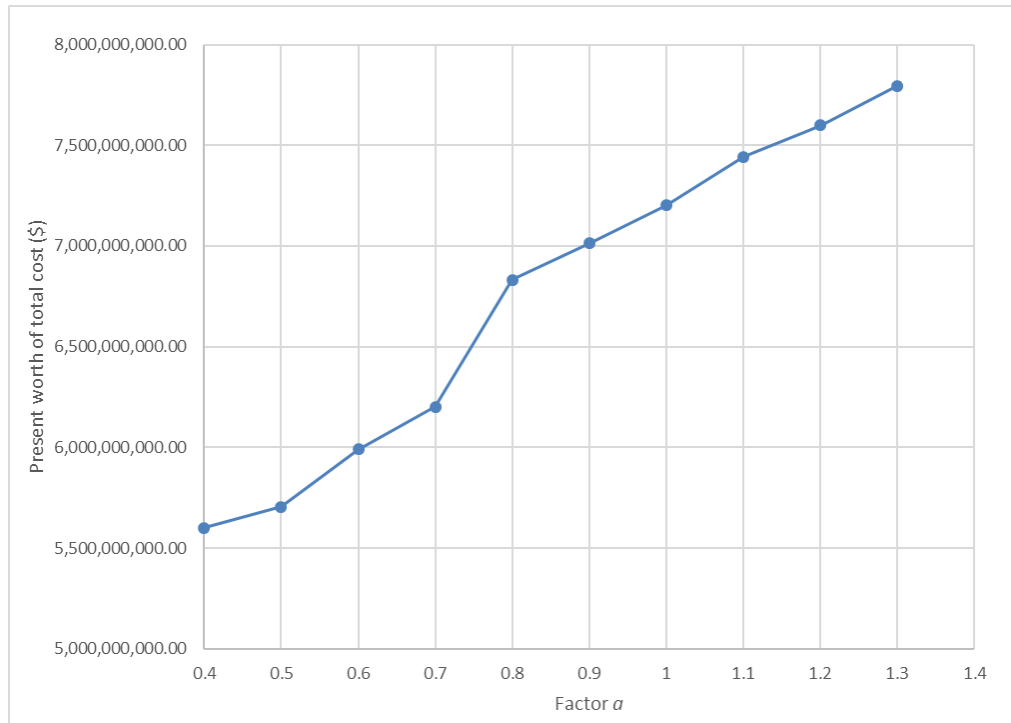
**Figure 8.6 Cost of intersection improvement vs the present worth of total cost**

### Demand variations

The demand for all origin-destination pairs is changed (multiplied) by a factor  $a$ . The values of  $a$  range from 0.4 to 1.3, and we can see the change in present worth of total in figure 8.7. As the demand increases, so does the present value of total cost. Table 8.4 shows how the sequence of projects is changed as demand changes.

**Table 8.4 Demand change and the sequence of projects**

Factor $a$	Sequence
0.4	8,3,4,5,2,7,1,6
0.5	3,4,2,1,5,7,6,8
0.6	6,4,3,8,7,5,1,2
0.7	3,8,5,7,4,6,1,2
0.8	8,5,2,6,1,4,7,3
0.9	6,2,4,8,7,1,3,5
1	8,6,7,5,2,1,3,4
1.1	4,5,6,3,1,2,7,8
1.2	4,1,7,8,2,6,3,5
1.3	7,1,6,2,8,4,3,5

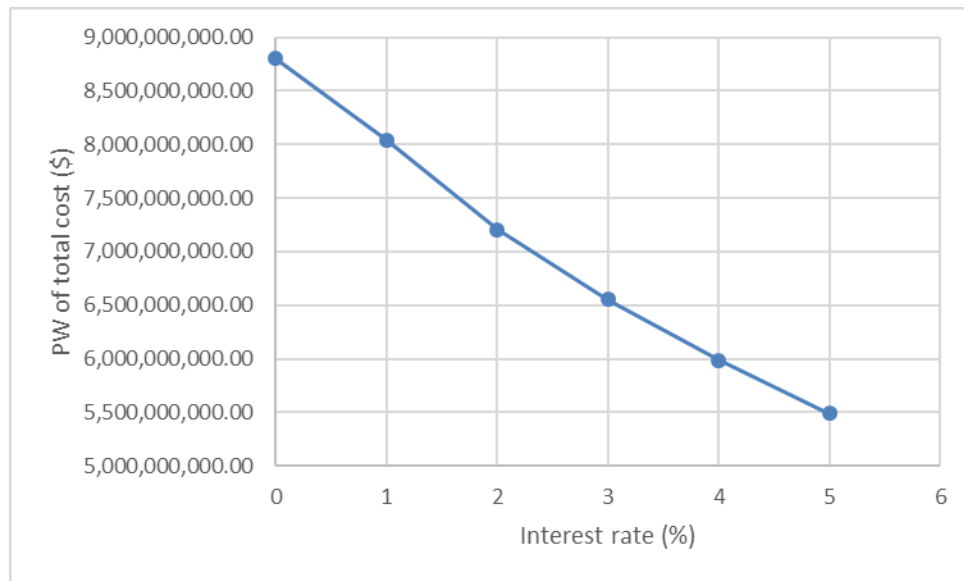


**Figure 8.7 Factor  $a$  vs present value of total cost**

Interest rate

In this subsection, interest rate is changed to see how it affects the behavior of the optimization model, and how that change affects the present worth of total cost. Figure 8.8 shows the change in present worth of total as the interest rate changes. We can note that as

the interest rate increases, the present worth of total cost decreases because future costs are more heavily discounted.



**Figure 8.8 Interest rate vs present worth of total cost**

Table 8.5 shows the change in sequence of the projects subject to interest rate change.

**Table 8.5 Interest rate change vs project sequence change**

Interest rate (%)	Sequence
0	6,2,3,4,8,7,5,1
1	1,5,7,2,8,3,4,6
2	8,6,7,5,2,1,3,4
3	5,2,6,8,1,7,3,4
4	3,7,4,2,6,8,1,5
5	6,7,1,4,2,8,5,3

## Chapter 9 Conclusion

Planning organizations, policy makers and transportation planners deal with the problem of selecting and scheduling of projects on a regular basis. This problem encompasses several disciplines, such as: economics, management, transportation and operations research. In this study traffic assignment is modified to consider intersection



interactions by introducing pseudo links for each intersection movement (left, right, through). The objective function used is the cost function which incorporates link improvement cost, intersection improvement cost, and delay (waiting time) cost with budget constraint. Moreover, three different heuristic algorithms were used to optimize the schedule of the projects considered for improvement while minimizing the overall cost function.

The contributions of this thesis include:

- Application of the swarm metaheuristic algorithm along with other two known metaheuristic algorithms to compare different approaches for solving the selection and scheduling of projects.
- Case study where we compare the performance of the swarm algorithm with two other known heuristics, we present an exhaustive test which shows the goodness of solutions obtained by heuristics, and a K-S statistical test is also ran to show the goodness of fit of the solution.
- Consideration of intersection improvements in the selection and scheduling of problems that are interrelated.

All three heuristic procedures yield the best possible solutions which minimizes the overall cost function. Ant colony optimization algorithm finds the best possible solution earliest (computation time is 40 minutes approximately), while Tabu Search and Simulated Annealing find it latter (approximately 43 and 44 minutes for SA and TS).

The overall objective of this study is to present an example how the project selection and schedule can become a combinatorial optimization problem, which becomes complex when the number of projects increase. It also aims to show how a new heuristic

procedure, specifically, Ant Colony Optimization algorithm can be used in project scheduling while optimizing the cost function. The methods presented in this study can be easily applied to any other road network with minor changes in coding, or any other transportation infrastructure application (waterways, transit).

Future research might focus on introducing a new type of alternative such as re-surfacing the pavement. Bus traffic could be traced along with passenger vehicles in the traffic assignment method to see how the transit traffic affects the assignment procedure. Moreover, different cost rates could be assumed for different types of improvements. For example, the cost of intersection improvements ( $\$/\text{ft}^2$ ) could be higher than the cost of link improvements ( $\$/\text{ft}$ ), because intersections are more complex components of urban transportation networks. Another instance that could be tackled in the future is that individual improvements could be grouped to form a project. For example, several link improvements could be grouped, or two intersections could be grouped for improvement. Furthermore, a more complex traffic assignment could be used to better estimate the volumes on intersections and links considering bus traffic, while incorporating more detailed evaluation methods (such as simulation models) could help to capture dynamic effects in congested networks missed by the Frank Wolfe algorithm. Future work might also focus on further researching how the combination of BPR function for link travel time and Akcelik's delay model for pseudo links behaves on a larger network. Also, this combination could be compared with some microsimulation model to capture more complex interactions, such as intersection turning movements.

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