

## ABSTRACT

Title of Thesis: ANALYSIS OF SLACK TIME FOR A REAL-TIME RIDESHARE SYSTEM

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Ridesharing and carpooling are effective traffic demand management strategies with many benefits comparable to other highway management methods. As a safety factor can increase the reliability of transit systems, a slack time can be added to the passenger pickup schedule to increase the system stability and reliability. This thesis focuses on a driver-passenger system, modeling three objective cost functions using similar steps and assumptions. These modeled cost functions yield the optimal slack time for passenger and vehicle and reflect the user's and supplier's behavior towards changes in different model components.

The numerical results of the cost models are presented to show the relations between different model components and to test the behavior of the cost models. The sensitivity analysis of these relationships reveal that factors such as the value of time, maximum waiting time, the penalty of missed pickup, and the standard deviation of the distribution of arrivals, affect the optimum slack time for the driver and

passenger. These cost models may be integrated with matching algorithms for use in real-time ridesharing applications. Although the arrival distributions for both passengers and vehicles are assumed to be normal in this study, other probability distributions can be substituted to investigate the costs associated with any connection among multiple vehicles or modes at a transfer point. The method presented in this study is applicable when passengers schedule pickups in advance and is especially suitable when penalties may be charged for passenger and vehicle lateness.

# **ANALYSIS OF SLACK TIME FOR A REAL-TIME RIDESHARE SYSTEM**

by

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## Dedication

This thesis is dedicated to my parents, and my beloved husband, Reza.

Without you, I would not be able to accomplish this...

## Acknowledgements

I would like to express my sincere gratitude to my advisor, Professor Paul Schonfeld for the continuous support of my thesis study, for his patience, motivation, and immense knowledge.

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## List of Abbreviations

$S_p$	Passenger's slack time (minute)
$S_v$	Vehicle's slack time (minute)
$T_i$	Scheduled pickup time (minute)
$t_p$	Passenger's expected arrival (minute)
$t_v$	Vehicle's expected arrival (minute)
$V_p$	Value of time for passenger (\$/minute)
$V_v$	Value of time for the vehicle (\$/minute)
$O_c$	Unit operating cost for vehicle (\$/minute)
$M_p$	Maximum waiting time for passenger (minute)
$M_v$	Maximum waiting time for Vehicle (minute)
$P$	Penalty of missed pickup (\$/pickup)
$\sigma_p$	Standard deviation of passenger's arrival time (minute)
$\sigma_v$	Standard deviation of driver's arrival time (minute)
$C_T$	Total system cost function (\$/pickup)
$C_s$	Supplier cost function (\$/pickup)
$C_u$	User cost function (\$/pickup)

## **Chapter 1: INTRODUCTION**

Today, many transportation professionals believe that ridesharing and carpooling are among the most effective solutions to reduce dependency on fossil fuel, decrease traffic congestion during peak hours and enhance the parking facilities in metropolitan areas. Ridesharing and carpooling are cost-effective traffic demand management strategies to improve road systems efficiency and maintain an acceptable level of service on the roads.

The cost of a well-implemented ridesharing program, that meets user needs while lowering single-occupancy vehicle (SOV) use, is significantly lower than road construction, and can have potentially comparable benefits (Fellows and Pitfield 2000).

In a carpooling system, passengers share a private or public vehicle and it is prescheduled (often) by means of many available websites or online applications. On the other hand, dynamic ridesharing is a derivative of normal carpooling as it provides pickup for passengers on an as-needed basis. Also, carpooling was known to be used for specific trip purposes (work, education) while advanced mobile technology makes the ridesharing and carpooling very cost-effective and accessible for many trip purposes.

While smartphones allow people to offer and request trips whenever they want and wherever they are, a system with such real-time characteristics calls for a real-time schedule optimization algorithm to ensure successful pickups and timely drop-

offs for the users. Just as transit planners typically add some slack time to the schedule for system stability and reliability (Zhao et al., 2006), having a slack time in the driver-passenger system can also improve the reliability of the ridesharing service.

For a real-time ridesharing system, designing a slack time in the schedule will increase the probability of having a successful pickup. Rideshare systems that use new smartphone technology as their platform, strongly rely on providing a reliable service to the costumers. Because the market for rideshare applications and services is becoming more and more competitive, public review of a service provider is of great importance for the customer's choice.

Based on the definition in McGraw-Hill Dictionary of Scientific & Technical Terms (6th Edition, 2003), slack time is the amount of time a task can be delayed without causing disruption in the system and delay propagation problems. In this study however, slack time is defined as the amount of time (in minutes) that the passenger or driver add to their schedule in order to decrease the chance of delays and missed pickups.

To further elaborate on the concept of slack time in this study, we use a hypothetical scenario in which a passenger calls for a pickup at a specific time and location. Assuming that the arrival of the vehicle at the designated pickup location follows a probabilistic distribution with the Probability Density Function of  $f(x)$ , the expected (average) arrival time of vehicle will be  $\mu_V$ . Suppose the driver (or the rideshare system operator) decides that the vehicle can leave its origin to pick up a passenger at  $\mu_V - TT$  ( $TT$ : vehicle's travel time from its origin to the pickup location).

If we assume a normal distribution for the arrival of the driver, there is a 50 percent chance that the vehicle will arrive at the pickup location later than expected.

Now assume that the vehicle leaves its origin,  $S$  minutes earlier than  $\mu_{V-TT}$ . Then, the probability of arriving later than the scheduled pickup time decreases from 50 percent to a smaller probability equal to  $P(\text{Late})$ , as shown in figure 1.

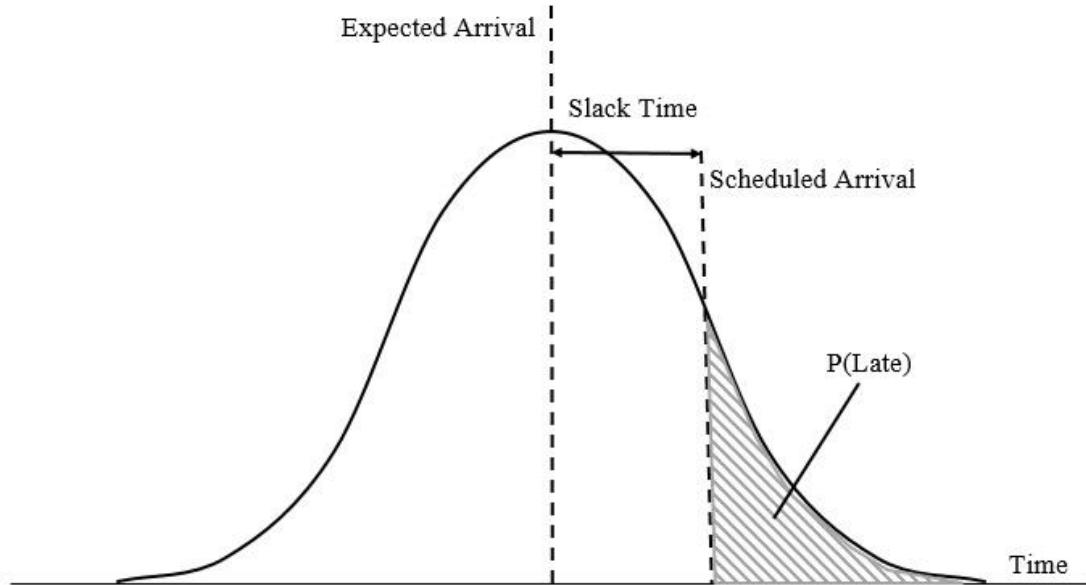


Figure 1: the probability of arriving at the pickup location, later than scheduled time

As the probability of arriving at the pickup point later than scheduled time decreases, the probability that the passenger can reach the destination on an acceptable time window and the probability for driver to have a successful pickup increase. This means the ridesharing system will become more reliable and therefore more appealing for the customers.

In this study, an objective function is modeled that addresses the optimal slack time problem for a simple driver-passenger scenario. A probabilistic distribution of

arrivals is considered for both vehicle and passenger and numerical analysis is used to obtain optimum slack times. One of the main purposes of this research is to investigate the benefits of having such optimized slack time in the rideshare schedule.

Slack time optimization has been explored in many previous studies. Most of these studies focus on coordinating multiple routes and/or transportation modes while adding a slack time to the transfer schedule. The methodology used in this study follows similar steps that Lee and Schonfeld (1991) used to optimize slack times for coordinated transit routes. In their study, the total transfer cost is minimized, considering stochastic vehicle arrivals at a single transfer station. Their results showed that when service headways are small and the standard deviations of arrivals exceed a certain level, the optimal slack time becomes zero. In another similar study, Chien and Schonfeld (1998) optimized headways, station/stop locations, and route spacing for an integrated bus and rail network, while minimizing the total cost, consisting of user and supplier costs.

Chowdhury and Chien (2001) also formulated a model to optimize a coordinated service provided by multiple transit modes including a train line and its feeder bus system. Their objective function was total system cost, including supplier and user costs, which yielded the optimized headways and slack times. As an extension of their previous study, later in 2002, Chowdhury and Chien developed a four-stage procedure to optimize coordination among routes at multiple transfer stations in an intermodal transit network, which again, minimized the total cost.

A review of these similar studies showed that the deterministic and stochastic models for transfer coordination have received considerable attention and the steps

taken to develop those models can be used as guidelines for this research. However, stochastic arrivals of passengers have never been an objective in any of the above studies. This study focuses on modeling a probabilistic objective function for passenger and driver and optimizing the slack times simultaneously. In a simple call for pickup scenario (which is the case for many rideshare systems using online applications), passenger's behavior has significant effect on the outcome of the call and therefore, probabilistic arrival distribution is assumed for passenger as well as driver, to fill this void.

Although the simple rideshare system introduced in this study consists of only a driver and a passenger, the concept of a passenger being pickup after a call, is very similar to the scenario in which passengers are transferring from one transit vehicle to another. In both cases, the arrival of the passengers at the terminal or (in this study) pickup location can be probabilistic or deterministic. Also, the single vehicle in this study, can be replaced by any transit vehicle that has a schedule to stop at specific times and locations.

The objective function modeled in this study is the total system cost, which includes supplier (driver or rideshare company) cost and user's (passenger) cost. A great part of this research focuses on the formulation of theses cost functions to be minimized by changing factors such as slack times, value of time and penalty for missed pickups.

From the suppliers' point of view, besides the operating costs, the total cost should incorporate some penalties for an unsuccessful ride (in sense of timely pickup or drop-off or missing the passenger), as well as the cost of waiting, to ensure a non-

zero slack time for the driver. The driver's (or operator's) decision of adding a slack time to the schedule to increase reliability, depends on many factors such as value of time for driver, penalty that should be paid in case of missed pickups, maximum waiting time allowed for the driver and the characteristics of vehicle's probabilistic arrival.

On the other hand, the user's total cost function includes the cost of adding a slack time to the schedule, waiting cost and the expected penalty cost. The passenger, may be obliged to pay a price for no shows, as it will affect rideshare company's schedule to have a missed pickup. This penalty amount is an important component of the (supplier and user's) cost functions and is further analyzed in the sensitivity analysis chapter of this study.

After developing the cost functions in chapter 3 and testing their behavior in chapter 4, numerical analysis is used in chapter 5 to analyze the sensitivity of slack times related to changes in different components of the cost functions. As mentioned, the penalty for missed pickup is one major factor affecting the amount of optimized slack time. The analysis of this penalty can be used to find the break-even point at which the real cost of missed pickup for the driver (or passenger) is equal to the penalty paid by the party responsible for the unsuccessful ride.

Another important factor affecting the amount of optimum slack time in the total cost function is the value of time. Since Becker's (1965) explicit treatment of time as an input in the production of household goods, the concept of value of time has been the topic of extended studies in transportation economics as it is a crucial factor to investigate the effects and monetary values of transportation developments. In 1976,

Gronau modeled travel as an intermediate good to prove that the cost of traveling only partially depends on travel time and value of time varies between individuals depending on income, wage rate, trip purpose and urgency of it. In the same year, Reichman discussed two various methods for economic analysis of the value of time and showed that the value of time is related to individual's perception of alternative use of saved time and disutility of traveling.

Many of the methods for value of time analysis in transportation economics are based on stated or revealed preference data achieved by surveys (Meloni, et al., 2004; Tilahun & Levinson, 2006; Palmquist et al., 2007) but simulation methods have also been used to estimate value of time and value of travel time reliability (Lam, 2004; Tilahun & Levinson, 2006; Li et al., 2010; Carrion & Levinson, 2012).

In this study, value of time for vehicle and passenger are incorporated in the objective functions (supplier cost, user cost and total cost) and the sensitivity of slack time for vehicle and passenger with respect to changes in value of vehicle's time and value of passenger's have been investigated in chapter 5.

Chapter 5 also discusses the sensitivity of slack times to other components of the objective functions. Besides the parameters of the normal distributions for arrival of vehicle and passenger, maximum waiting time window for vehicle and passenger are also included in the cost models.

These maximum waiting times are introduced in the model to ensure that passenger will not leave the pickup location immediately if the driver is not already there. Also, if the driver arrives earlier than passenger, she/he has to wait for passenger up to the maximum waiting time. The waiting time can change based on

the characteristics of the pickup location, weather and the purpose of trip for the passenger. For example, the vehicle may be able to stop at the side of a local street for 10 minutes, while maximum waiting time for vehicle at the side of a busy one-way street may be less than 3 minutes. Although the characteristics of the street may not affect the maximum waiting time for passenger, weather/air quality and trip purpose for the passenger may be some of the limitations for it. In chapter 5, the sensitivity of optimal slack times to changes in maximum waiting time for driver and passenger will also be examined.

The results of the sensitivity analysis in chapter 5 of this study reflect the relation between different components of the presented cost models determine the consequences of changing model components on the optimal slack times for passenger and vehicle. Chapter 6, reviews the results of this analysis, together with a discussion of the benefits and limitations of the models presented in this study.

## **Chapter 2: REVIEW OF THE LITERATURE**

In this chapter, a review of previous studies related to the topic of this research is presented. To discuss the different aspects of the present discussion, the review literature are classified into different subjects as follows. This study aims to capture the steps taken in previous studies and present a model for schedule coordination between driver and passenger in a real-time rideshare system.

### **Benefits of Ridesharing and Carpooling**

Carpooling is a system in which the passengers, share a private or public vehicle with other individuals. On the other hand, ridesharing is often a pickup service in which a person arranges for a ride (by using a website or smartphone app) in a (usually) privately owned vehicle. The main difference between carpooling and ridesharing is that in carpooling, usually the same group of people take it in turns to drive each other rather than exchanging money for getting a ride.

Advanced mobile technology makes the ridesharing and carpool system cost effective and accessible for everyone owning a smart phone. Many transportation professionals believe, that ridesharing and carpooling are among the most effective means to sustain fuel, decrease traffic jams during peak hours and enhance the parking facilities in metropolitan areas.

Therefore, the body of literature available for the topic of ridesharing is so vast and diverse that there are many papers solely dedicated to categorizing the study of

ridesharing. For instance, Amey et al. (2011) focused on the lessons learned about ridesharing and provided guidance for future studies. The overall goal of their paper was to provide a foundation for further real-time ridesharing research by identifying and discussing the potential benefits and obstacles of real-time ridesharing. The authors presented a definition of real-time ridesharing, followed by a comprehensive categorization of challenges that hinder greater rideshare participation. The information they gathered suggests that “the rideshare challenge, rather than being a single challenge to be overcome, is a series of economic, behavioral, institutional, and technological obstacles to be addressed”. The authors also provided several recommendations toward next steps to understand the users’ behavior towards ridesharing.

Chan and Shaheen (2012) provides a comprehensive study of the history of ridesharing in North America and its potentials for the future. In the light of growing concerns about climate change, congestion, and dependency on fossil fuels, more studies are focused on better understanding the effects of ridesharing on infrastructure, congestion, and environmental emissions. Chan and Shaheen, believe that our time is “likely to include greater interoperability among services, technology integration, and stronger policy support”.

One of the benefits of ridesharing and carpooling systems is providing affordable everyday trips for people who live farther from employment centers. Due to residential affordability factors, more and more commuters are trading travel time to afford better housing options farther from central business districts. In addition, many public transit systems are either over or under used. During peak hours, crowding

creates discomfort for users and low ridership at other times makes many services financially unsustainable (especially in suburban areas).

In spite of significant subsidies and cross-financing (e.g. tolls) in developed countries, almost no public transit system can generate sufficient income to cover its operating and capital costs (Rodrigue, 2017). While many cities have long failed to provide adequate transportation options to suburban commuters, and Park-n-Ride facilities beside the highways often sit empty, the platform-based uberPOOL pairs individuals with common locations and destinations and has already proven popular at making ridesharing easy.

Moreover, according to Miller (2016), uberPOOL technology is being provided in a way people are willing to use. If widely implemented, such ridesharing platforms can reduce the cost of infrastructure, provide cheap transportation options in suburban areas for low income individuals and assist with reducing environmental emissions.

Li et al. (2016) investigated the effects of Uber (or any peer-to-peer mobile ridesharing platform), on traffic congestion and environment (carbon emissions) of US urban areas. This study, provided empirical evidence that ridesharing services such as Uber, can significantly decrease the traffic congestion and showed that on-demand ride sharing services can be part of a solution to traffic congestion in major urban areas.

One of the main appeals of real-time ridesharing, is its flexibility over transit systems such as bus or light rail. A rideshare systems can provide pickup/drop off for passengers at any designated meeting point. The potential benefits of introducing meeting points is investigated by Stiglic et al. (2015). The result of their study

showed that the increased flexibility resulted by introducing meeting points, led to additional feasible matches between drivers and passengers and allowed a driver to be matched with multiple riders without increasing the number of stops.

Although many of the benefits of ridesharing are obvious and straightforward (such as traffic congestion improvements and savings on fuel consumption), many researchers have tried to demonstrate the benefits of ridesharing to the society. Xu et al. (2015) propose a traffic assignment model that explicitly represented ridesharing as a mode of transportation. Their objective is to analyze the impact of ridesharing on traffic congestion, methods of motivating people to participate in ridesharing and effects of congestion on ridesharing including prices and the number of drivers and passengers. Their computational results show that the ridesharing base price, influence the congestion level and within a certain price range, an increase in price reduce the traffic congestion. Also, the utilization of ridesharing in their model increases as the congestion increases.

### **Ridesharing Technology**

A real-time ridesharing system mainly incorporates two technologies: (1) GPS, which is used for determining the location and route for both drivers and passengers (2) smart phone application, which provides a user interface for both drivers and passengers. The success story of uberPOOL is known to all but many similar online applications and website have been developed for real-time ridesharing in the past but they had a relatively smaller community of users.

In some cases, a social network can be the means to build trust between the passengers and drivers and make it easier to schedule a certain trip (home-work-

home) repeatedly. Aarthi (2015) introduced a smart phone application to be used as rideshare interface in the city of Delhi, India. Their proposed system used information from the social networking contacts derived from the user's Gmail account with the promise of having benefits in form of reducing congestion, fuel consumption and pollution.

Whatever platform used for development, the role of mobile technology is undeniable in the development of ridesharing systems. Siddighi and Buliung (2013) discussed the relationship between dynamic ride share (DRS) and information and communication technology (ICT), identifying the evolution of this relation over time and its potential developments in the future. For this purpose, they reviewed a series of case studies within their historic-technical context focusing on the underlying technologies and their contributions to the success or failure of the projects.

The new technology can be used to improve the reliability of real-time rideshare and reduce the risk for unsuccessful rides. Passenger's and driver's attitude towards a reliable system also reflects their value of time as will be discussed next.

### **Value of Time and Travel Time Reliability**

Discussing the concept of reliability and risk in transportation systems is never complete without a study of value of time and value of travel time reliability. Since the introduction of time as an input in the production function of households (Becker, 1965), the concepts of value of time and value of travel time reliability have been discussed in many studies in transportation economics.

The value of time in transportation, is used to investigate the effects of travel time savings and monetary values of transportation developments for the public. In 1976, Gronau modeled travel as an intermediate good to prove that the cost of traveling only partially depends on travel time and value of time varies between individuals depending on income, wage rate, trip purpose and urgency of it. Reichman (1976) also discussed the calculation of value of time using two different methods of economic analysis. The results of his study showed that the value of time is not only related to individual's perception of alternative use of saved time, but also the disutility of traveling.

On the concept of travel time reliability, Knight (1974) investigated the trade-off, made by commuters, if they allow an extra time for their travelling in order to avoid unpredictable delays. This extra time included in the travel schedule has effects similar to what is defined as slack time which we discuss in this study.

Knight also review other approaches to the evaluation of travel unreliability, and provided an outline to test the applicability of the “safety margin” in London commuters' timing of the trips that they take to work.

In 1998, Noland et al. presented a simulation model based on the model presented by Small (1982) on the scheduling of activities by consumers. They designed their model with purpose of determining the impact of travel time uncertainty on congestion. Their model was a combination of a supply side model of congestion delay and a discrete choice econometric demand model.

Many of the methods for value of time analysis in transportation economics are based on stated or revealed preference data obtained from surveys (Meloni, et al.,

2004; Tilahun & Levinson, 2006; Palmquist et al., 2007) but simulation methods have also been used to estimate value of time and value of travel time reliability (Lam, 2004; Tilahun & Levinson, 2006; Li et al., 2010; Carrion & Levinson, 2012).

Later in 2012, Small admitted that after decades of study of the value of travel time, the topic remains incompletely understood and further theoretical and empirical investigation is needed to completely capture the characteristic of value of time in transportation. Small (2012) addressed the connections between willingness to pay for time savings and other economic factors including time of day choice, aversion to unreliability, labor supply, taxation, activity scheduling, intra-household time allocation, and out-of-office productivity.

Wu et al. (2015) considered a bus timetabling problem with stochastic travel times. They added slack times into the timetable to mitigate the randomness in bus travel times. Their objective function was to minimize the total waiting time cost for transferring, boarding and through passengers using a genetic algorithm with local search (GALS). Their numerical results based on a small bus network showed that the obtained timetable can reduce the total waiting time cost by an average of 9.5%. The authors also discuss that adding slack time into timetable can greatly benefit transferring passengers by reducing the rate of transferring failure.

Beaud et al. (2016) studied the impact of travelers' attitude towards risk, on the value of time. In their study, measures of travelers' willingness to pay to save travel time and improve the reliability of a given trip are modeled using a simple microeconomic mode choice algorithm. They characterized the trips based on price of

the mode and the statistical distribution of random travel time, assuming that travelers have expected utility preferences over the latter.

Vodopivec and Miller-Hooks (2017) use the theory of optimal stopping in dynamic vehicle routing to determine the optimal timing of a recourse action if the customer's deadlines for pickup is due. The factors involved in their model are the probability that the first vehicle arrives late, the location of the backup vehicle, and value of time in waiting for additional travel-time information. The two-stage stochastic optimization model in this study, integrated dynamic vehicle recourse into a priori scheduling and routing. The framework is demonstrated for a stochastic dial-a-ride application in which taxis serve as backup to ridesharing vehicles.

As will be discussed in chapter 3 of this study, the value of time for passenger and vehicle are separately included in the cost models. The value of time for driver can affect her/his decision of adding a safety factor (i.e., slack time) to the schedule. Same for the passenger, the value of time affects the amount of slack time added to her/his schedule. Different values of time for driver and passenger can be entered in the final cost models for sensitivity analysis.

### **The Matching Problem**

On the subject of modeling a rideshare system, extensive search on ridesharing and carpooling papers shows that many studies focus on the problem of matching the driver and passenger in a dynamic setting, providing an optimization algorithm based on cost, vehicle miles traveled and etc. (Agatz et al., 2011; DiFebbraro & Gattorna, 2013; Herbawi & Weber, 2012).

Effective and efficient optimization technology that matches drivers and riders in real time is one of the necessary components for a successful dynamic rideshare system. Agatz et al. (2012), presented a study in which they systematically outline the optimization challenges that arise when developing technology to support ridesharing. They surveyed the related operations research models in the academic literature, hoping that their paper will encourage more research by the transportation science and logistics community in this emerging area of public transportation.

Although many of these studies use the same well-known matching algorithms, they each focus on specific scenarios and methodologies to achieve different sets of goals. For instance, the matching problem provided in a study by Ghoseiri et al. (2011) is based on promoting programs. The paper presents a Dynamic Rideshare Matching Optimization (DRMO) model which aims at identifying suitable matches between passengers that request for rideshare services and available drivers. The promoting system in this research encourages the drivers to carpool for credits and HOV-lane privileges rather than being paid by the passengers.

Fast shortest path and matching algorithms can be used together with a smartphone application interface to provide a reliable ridesharing system. Besides the fast growing *uberPOOL*, another example is *Noah* which enables the users to submit requests from a smartphone and choose specific parameters such as number of taxis in the system, service constraints and matching algorithms to explore the internal functionalities and implementations of *Noah*. The system analyzer shows the system performance including average waiting time, average detour percentage, average response time and average level of sharing (Tian et al., 2013).

Using a smartphone interface can also make the matching problem more interactive. The user's profile in the real-time carpooling system introduced by Pukhovskiy and Lepshokov (2011), contains all the information needed for traveling. The interactive credit systems in their application help the carpooling system know the attitudes of each user and cooperation with local police provide the users with a trust in the system for their safety.

The authors' believe that unlike other research projects, their system affects all aspects of carpooling because it works with the latest technologies that provides major advantages over previous works. Although such online background check systems can be appealing to some of the opponents of new rideshare applications, the implication of this model to a vast network of users may not be cost effective. The interesting aspect of their work is how they connected the carpooling system to the transit schedules in a way that the user can specify her/his schedule and time limitations, and in case there is no match available for carpooling, the system will provide the best transit options for traveling.

Another approach to provide matching in a ridesharing system is an online information platform that informs every user (driver or passenger) of other's status and route. In most of the current ride-sharing portals over the internet, the users must explicitly enter information about origin, destination, route, time and date when searching for riders who fulfill their mobility needs. But Bravo-Torres et al. (2016), explored new opportunities of ride-sharing to proactively discover the most frequent trips of each user and automatically selecting trip mates for each itinerary. In this study, the people gathered together in heavily trafficked zones at certain times

deployed a smart Vehicular Ad-Hoc Network (VANET) over their handheld devices. The smart VANET enabled the vehicles to exchange the information necessary for matching the users' itineraries and particular preferences, and identifying riders for common routes. Such new technologies are the next step in providing rideshare for people living in congested areas of the cities.

### **Schedule Optimization**

While matching the drivers and passengers in a dynamic rideshare system is of great importance, the question of scheduling is also another aspect of rideshare systems. In a real-time rideshare system, the scheduling should also be real-time which is normally provided by an operator or an online automatic scheduling platform working based on optimization algorithms.

The research by Herbawi and Weber (2012) focused on solving the schedule optimization problem with a multi criteria objective function. The authors considered minimizing the total travel distance and time of the drivers and the total travel time of the passengers while maximizing the number of the transported passengers. Their proposed genetic and insertion heuristic algorithm for solving the addressed problem modifies the solution of the genetic algorithm to do ride-matching in real time. In addition, the authors provided datasets for the ride-matching problem, derived from realistic data, to test the algorithm and their results indicated that the algorithm can successfully solve the problem by providing answers in real time, and it can be easily tuned between response time and solution quality.

Ma et al. (2013) presented a large-scale real-time taxi ridesharing algorithm in which the real-time requests are sent by passengers, generating ridesharing schedules that reduce the total travel distance. In their method, the authors first propose a taxi-searching algorithm by using a spatio-temporal index to quickly retrieve candidate taxis that are likely to satisfy a passenger's query. A scheduling algorithm is then proposed that checks each candidate taxi and inserts the query's trip into the schedule of the taxi, which satisfies the query with minimum additional incurred travel distance.

Yotsutsuji et al. (2016) focused on a scheduling a short-trip rideshare system including nonmonetary rewards supported by other-regarding preferences of altruistic drivers in a small community. Using numerical simulation, they studied the ways to improve the sustainability of the system by providing informational guidance by a system operator which induced each driver to take action towards delayed matching.

The concept of real-time ridesharing attempts to add flexibility to rideshare arrangements by allowing drivers and passengers to arrange occasional shared rides ahead of time or on short notice. The addition of this service innovation not only presents opportunities to overcome existing rideshare challenges, but also leads to new challenges.

A rideshare system is similar to a bus system in many aspects. Rideshare systems can have a designated pickup location for passengers similar to bus stops and as Fonzone et al. (2015) discussed, if bus service departure times are not completely unknown to the passengers, non-uniform passenger arrival patterns can be expected. In their paper, they proposed that passengers' arrival patterns can significantly

influence the bus bunching process, which should be considered when analyzing service control measures. Unlike in a bus transit system, no shows for passengers can be costly (in terms of lost time to find another mode of transportation or call for another carpool pickup or a designated penalty) and therefore, passengers may consider a slack time for their timely arrival at the pickup location.

### **Slack Time, Risk and Reliability**

Scheduling in a real-time system needs some measure of reliability to ensure the quality of service for passengers. As transit planners typically add some slack time to the schedule for system stability and reliability, assuming a slack time when scheduling a real-time ride-share system can make the system more reliable.

Optimizing the mentioned slack time for a transportation system, assuming a designated slack have been the objective of many studies. Furthermore, many studies have focused on optimizing intermodal coordination while optimizing a designated slack time in the scheduling system. Lee and Schonfeld (1991), proposed a numerical approach to optimize slack times in transit schedules. The designated slack time in the system was to minimize the operational cost and decrease the chance for having a missed connection at the transfer station. The results of this paper showed that the optimal slack time for a bus-train transfer system varies with variables such as headway and variance of bus arrivals as well as transfer volume, value of passenger time and operating costs.

Slack times in transit schedules were also optimized by Chien and Schonfeld (1998). In their study, the headways, station/stop locations and route spacing are also optimized for an integrated bus and rail network. The objective function of this

system was minimizing the total cost, consisting of user and supplier costs. They examined the sensitivity of the characteristics of transit service to changes in travel time and cost parameters and presented numerical examples of integrated transit systems for coordinating the rail and bus schedules. The results of this study showed that after using the proposed optimization method, rail transit ridership increases and total passenger travel time decreases.

Chowdhury and Chien (2001) optimized a coordinated service provided by multiple transit modes including a train line and its feeder bus system using modeled total system cost as objective function. The total cost function in this research included supplier and user costs, which yielded the optimized headways and slack times.

Later in 2002, Chowdhury and Chien developed a model to optimize coordination among routes at multiple transfer stations in an intermodal transit network, again, using the minimized total cost function. Their presented four-stage procedure was developed for determining the optimal coordination status among routes at every transfer station. They considered stochastic arrivals for the feeder vehicle at the transfer stations, and calculated the optimal slack times by balancing the savings from transfer delays, additional cost from slack delays and operating costs.

In a similar approach, Ting and Schonfeld (2005) used a heuristic algorithm to jointly optimize the headways and slack times by minimizing the total costs of operating a multiple-hub transit network. The authors used headways equal to integer multiples of a base cycle to ensure that vehicles arrive nearly simultaneously at the

transfer stations and showed that as demand decreases, optimized headways and the net benefits of coordinated operation increase. The results of this study also showed that the optimized slack times for routes vary with changes in headways, standard deviation of vehicle arrival time, transfer volumes and passenger's value of time. They also concluded that it is not worth attempting schedule coordination for routes with high standard deviations of arrivals.

Zhao et al. (2006) presented an analytic model that addressed the optimal slack time problem for a schedule-based transit operation on a single loop with a single checkpoint. Their system was associated with a D/G/c queue model and the results showed that for general cases, it is difficult to obtain closed form solutions. This study also provided approximation approaches for multiple buses and different travel time distributions. They showed that compared to simulation results, their approximation methods work well for the interval of appropriate slack time, which often contains the optimal value.

In 2011, as an extension of their previous work, Chowdhury and Chien developed a mathematical model to minimize the total cost including the supplier and user costs in a transfer hub that consists of multiple transit routes, subject to capacity constraints. The results of this study showed that the level of transit service may be elevated by efficient timed transfer, which reduces travel time and increases productivity. On the other hand, they concluded that timed transfer may be costly because of the stochastic nature of vehicle arrivals. They used bus size, headway, and slack time as decision variables and jointly optimized those through consideration of various levels of coordination using numerical examples.

Liu et al. (2014) also used a slack time in their coordination model. Their paper focused on developing a mathematic model to optimize coordination among lines in a real-world large scale metro network. The results of their work showed a substantial reduction in travel time for transferring passengers.

A review of the studies mentioned above showed that although many researchers have focused on the schedule optimization and transfer coordination problems, the assumption of stochastic arrivals for passengers as the users of transit system have only been considered in a few studies. The passenger's arrival pattern in a rideshare system can significantly affect the schedule for the vehicle as well as other passengers.

This research, aims to introduce a slack time for the vehicle as well as passengers in a real-time rideshare system. As in many previous schedule optimization papers, the objective function is the cost (supplier cost, user cost and total system cost) which includes slack time for passenger and vehicle. First, the optimal slack times will be determined by minimizing relevant objective functions and then the sensitivity of slack times to changes in other model components will be explored.

## **Chapter 3: MODEL FORMULATION**

In this chapter, objective functions will be introduced as costs related to the passenger-vehicle system. First, an optimization function will be modeled based on the supplier cost function. Although the total cost to the driver-passenger system and user's (passenger's) cost will also be calculated, the main focus of this study is on optimizing the slack times in passenger's and driver's schedule by minimizing the supplier cost function.

As a smartphone application or any user interface for ride-sharing is normally provided by the supplier, we first optimize the slack time from the point of view of the supplier. It is also good for passengers to know the actions that they can take to increase the reliability of travel time, but it is usually the service provider (rideshare company or the driver) who will suffer monetary loss if the service is unreliable for the passengers. However, each time there is an analysis of the supplier cost function in this study, the user cost function is also analyzed.

The development of the cost functions in this research is based on a flowchart that divides the timeline of a scheduled pickup into different time windows. This is done for simplification of the modeling process, since the behavior of passengers and drivers towards a scheduled pickup can create many possible outcomes that change dynamically.

Before starting the formulation of supplier cost and user cost functions by adding a slack time to the system, the relation between the slack time for vehicle and the

standard deviation of distribution of vehicle's arrival at the pickup location is discussed in the next section.

Later, to develop the model further, we will assume a probabilistic distribution for the arrival of passenger at the pickup point as well. This leads to several scenarios that are discussed further in this chapter.

To develop the model step by step, this chapter considers the characteristics of slack time in two scenarios. In the first scenario, a passenger calls for a pickup when only the distribution of vehicle's arrival is probabilistic and in the second scenario, the distribution of arrival for both passenger and vehicle are probabilistic.

The results of this chapter are supplier cost, user cost and total cost functions that yield the optimal slack times in a numerical analysis approach in the next chapter. The behavior of cost functions and their components related to changes in amount of slack times will also be discussed in next chapter.

### **Slack Time Optimization for Probabilistic-Deterministic Arrivals**

For any vehicle scheduled for a pickup, the question is how the supplier should decide the vehicle's departure time to ensure a successful ride and reduce the probability of missing the customer. To achieve this, a slack time  $S_v$  is introduced as the difference between the scheduled pickup time and the expected arrival time of the vehicle.

On the other hand (as shown in figure 1 of chapter 1), if the passenger who calls for a pickup at a specific time, departs from the origin  $S_p$  minutes earlier to get to the pickup location, then the probability of having a successful ride will increase.

To simplify the problem, we assume that the passenger and vehicle both have a maximum wait margin and they inform each other or the operator about it at the time of scheduling. As discussed in chapter 1, these maximum wait margins ( $M_v$ ,  $M_p$ ) depend on many factors such as weather conditions and the trip purpose for the passenger, stopping restrictions and limitations at the street side and existence of a next passenger scheduled to be picked up.

As the main purpose of this study is to optimize the slack times for vehicle and passenger ( $S_v$ ,  $S_p$ ), such a decision can be evaluated with an objective function that includes all the corresponding cost components affected by adding slack times to the system.

At first, let us assume a scenario in which a vehicle is scheduled to pick up a passenger at time  $T_i$ . Assume that vehicle's arrival time follows a Normal distribution but the passenger is definitely present at the pickup location at the scheduled time  $T_i$  (deterministic arrival) and will wait up to  $M_p$  minutes to be picked up. If the vehicle arrives after  $T_i + M_p$ , then the ride is unsuccessful and a penalty of  $$P$  should be paid to the passenger. The passenger, then, can decide to wait for another ready vehicle to be picked up or choose another provider or mode of transport. If the passenger asks for another vehicle, then the probabilistic-deterministic scenario repeats for another vehicle and same passenger.

Here, the objective is to find the optimal slack time for the vehicle to minimize the cost. The cost function formulated in here, is the supplier or the service provider's cost, as it is desirable for rideshare companies or the drivers to minimize their cost

while providing a reliable service for the customers and avoiding the penalty cost.

Equation 1 shows the structure of the supplier cost used in this study.

$$\begin{aligned} \text{Supplier cost} = & \text{Vehicle operating cost} + \text{Cost of waiting for passenger} \\ & + \text{Expected penalty cost for missed pickup} \end{aligned} \quad (1)$$

Here, because we assumed that the passenger is present at the pickup location exactly at time  $T_i$ , the cost of waiting for passenger being late is eliminated from the formula and we rewrite (1) as:

$$C_S = O_C \cdot S_V + P \cdot \int_{T_i+M_P}^{\infty} f(v) \cdot dv \quad (1a)$$

where:

$C_S$  = supplier cost (\$/scheduled trip)

$O_C$  = unit operating cost for vehicle (\$/minute)

$S_V$  = slack time for vehicle (minutes)

$P$  = penalty for unsuccessful pickup (\$/pickup)

$f(v)$  = probability density function for vehicle's arrival

$T_i$  = scheduled pickup time (minutes)

$M_P$  = maximum wait margin for the passenger (minutes)

Based on this simple model, the optimal slack time for vehicle and the resulting minimum supplier cost have been calculated using a standard deviation of  $\sigma_V = 10$  minutes for the normal distribution of vehicle's arrival (figure 2). Note that the mean of normal distribution of vehicle's arrival, is  $S_V$  minutes less than scheduled pickup time  $T_i$ .

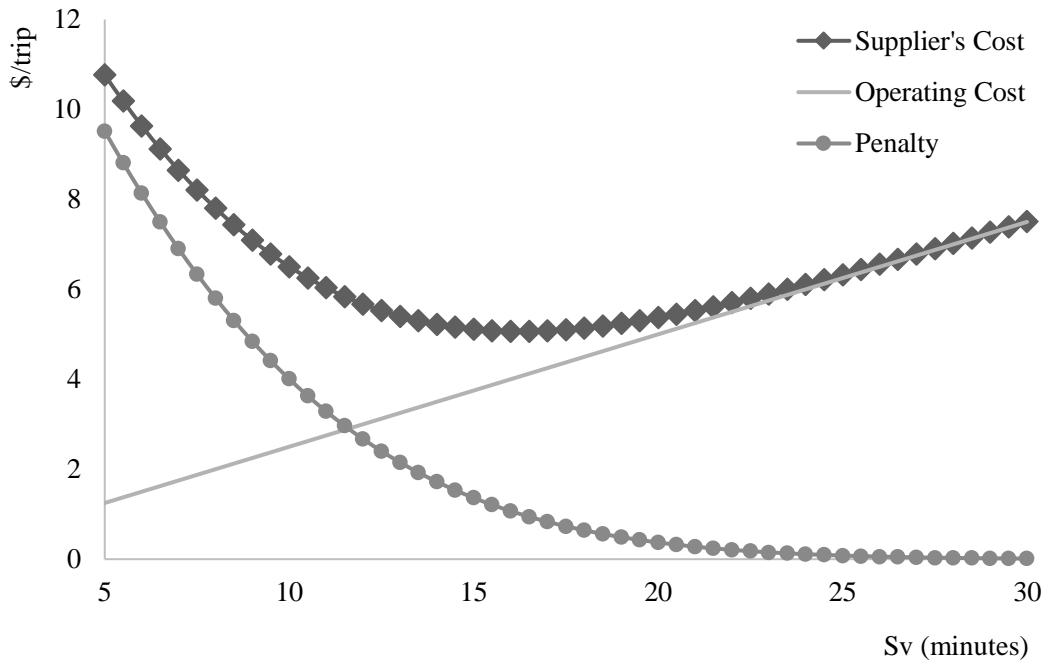


Figure 2: Changes in supplier cost and its components when  $S_v$  increases

Figure 2 shows that at  $\sigma_v = 10$  minutes, the minimum supplier cost is \$5.07 which occurs when  $S_v$  is 16.3 minutes. Here, we can also change the value of the standard deviation and show the changes in optimum slack time for the vehicle as the standard deviation of its arrival increases (figure 3).

As figure 3 shows, when the standard deviation of the normal distribution for vehicle's arrival increases, the probability of having a missed pickup increases as well, and the penalty cost curve in figure 2 moves upward. As the operating cost has a linear curve with constant positive slope, the optimum slack time will increase as long as additional uncertainty (larger standard deviation) in vehicle's arrival justifies a larger slack time.

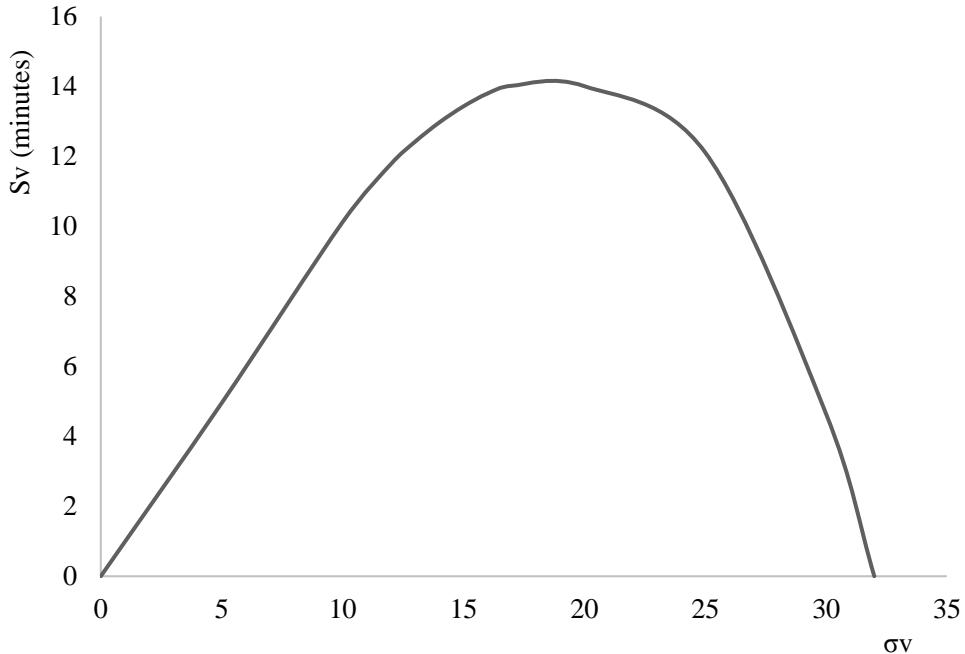


Figure 3: Optimum  $S_v$  vs the standard deviation of vehicle arrival time

After the optimum slack time reaches its maximum amount (for this numerical example, in around 19 minutes of standard deviation), the penalty cost dominates the operating cost and the optimum slack time decreases until  $S_v = 0$ , where the operating cost is also zero. Here, the maximum slack time of 14 minutes, occurs when the standard deviation of distribution for vehicle's arrival is around 19 minutes.

Lee and Schonfeld observed same behavior in their 1991 study of timed transfers between trains and buses in a transit terminal. When the train arrivals were deterministic and bus arrivals followed normal distribution, as standard deviation of arrival times increased, the optimum slack time increased and then decreased back to zero implying that when vehicle's arrival is relatively uncertain, no schedule coordination between the two modes is worth trying.

The cost function in Lee and Schonfeld (1991) included headway for the next available bus and the optimal slack time was introduced as a fraction of this headway. This present study, however, uses the penalty for missed pickup to justify adding a safety factor (i.e. slack time) to the schedule system. As figure 3 shows, beyond a certain amount of standard deviation for vehicle's arrival, the optimal slack time for vehicle is equal to zero, indicating that when the uncertainty of vehicle's arrival time is relatively high, coordinating the vehicle-passenger schedule is not economically feasible and it becomes preferable to have a higher probability of missed pickup (in the tail of the distribution for vehicle's arrival).

### **Model Formulation for Probabilistic Arrivals**

In further development of our simple model, assume that the arrival of passenger at the pickup location also follows a Normal distribution with a mean of  $\mu_p$  minutes which is  $S_p$  minutes less than the scheduled pickup time  $T_i$ . Here, the objective is to find the optimum slack time for the vehicle and passenger to minimize the supplier and user cost.

The rideshare system modeled in this step, consists of a single vehicle scheduled for a pickup and drop-off. In this kind of real-time ridesharing system, normally the operator should decide the dispatching of the vehicles after receiving a call for a pickup.

Here we assume that arrival times for passenger and vehicle are normally distributed and independent from each other. This assumption, simplifies the calculation of joint probabilities. In addition, we assume that the mean and the

standard deviation of arrival distributions are available and known for the system operator.

It is assumed in this problem that vehicles are not allowed to leave the pickup location earlier than the maximum wait margin unless the passenger is picked up. This is also true for passengers as they must wait until they reach their specified maximum wait margin unless the vehicle arrives before that time.

In the real world, normally a call for pickup is made when the passenger is already at the pickup location or very close to it. Also, if we assume that the main modes of traveling to the pickup location for passenger, only include walking and biking, the variance of the vehicle's arrival should usually be greater than the passenger's.

It is also reasonable to assume that  $M_v$  is usually smaller than  $M_p$ , because cars, unlike busses, do not have a dedicated stopping location.

As previously discussed, the slack time for passenger ( $S_p$ ), defined as the difference between scheduled pickup and expected arrival time, increases the probability that passenger will be at the pickup location before vehicle arrives. If  $S_p$  is too small to ensure a successful pickup, a penalty is considered for the rideshare company and the passenger will call for another vehicle or use another mode of transport.

To simplify the calculation of probabilities, a flowchart is presented in figure 4. The flowchart shows different possibilities when the driver arrives at the pickup location at time  $t_v$ . There are two main scenarios because the driver's arrival time can be before or after the scheduled pickup  $T_i$ .

$\mu_V$  is the expected arrival time of the vehicle which is calculated as:

$$\mu_V = T_i - S_v \quad (2a)$$

$\mu_P$  is the expected arrival time of the passenger which can be defined as:

$$\mu_P = T_i - S_p \quad (2b)$$

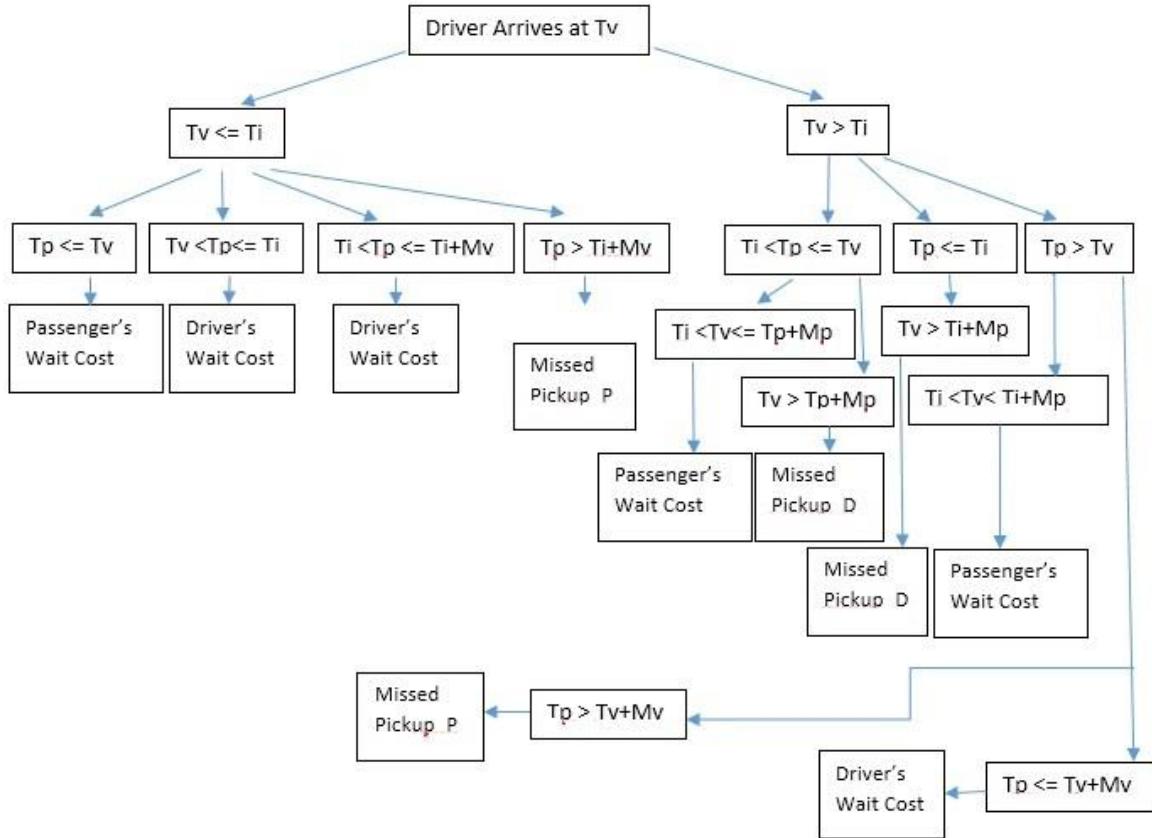


Figure 4: Flowchart of probabilities

When the vehicle arrives at the pickup location before the scheduled pickup time  $T_i$ , four scenarios are possible:

1. Passenger arrives before the vehicle and waits; in this case the ride is successful and vehicle can leave almost as soon as arriving at the pickup location.
2. Passenger is early but arrives shortly after the vehicle; here the vehicle waits for passenger and can pick up the passenger as soon as she/he arrives.
3. Passenger arrives later than scheduled pickup time but before the maximum waiting time for vehicle is due; in this case the pickup is successful but the vehicle should wait for passenger.
4. Passenger arrives later than maximum waiting time for vehicle and the vehicle leaves without picking up the passenger; here the penalty should be paid to the driver or the provider company.

These four scenarios each yield a different part of the probability functions discussed later. When the vehicle arrives after the scheduled pickup time  $T_i$ , five different outcomes are possible according to the flowchart.

1. Passenger arrives earlier than  $T_i$  but the driver arrives after maximum wait time for passenger; in this case the ride is unsuccessful and the penalty should be paid to the passenger.
2. Passenger arrives earlier than  $T_i$  and the driver arrives before maximum wait time for passenger; in this case the passenger waits for the driver.
3. Passenger arrives after the vehicle but before the maximum waiting time for vehicle is due; here the driver waits for the passenger and pickup is successful.
4. Passenger arrives after maximum waiting time for vehicle and the pickup is unsuccessful. The penalty should be paid to the driver or service provider.

5. Passenger arrives later than scheduled pickup time  $T_i$ , but before vehicle arrives. If vehicle's arrival is before maximum waiting time for passenger, then the pickup is successful and passenger waits for the vehicle.
6. Passenger arrives later than scheduled pickup time  $T_i$ , but before vehicle arrives. If vehicle's arrival is later than maximum waiting time for passenger, then the pickup is unsuccessful and the penalty should be paid to the passenger.

As the flowchart in figure 4 shows, the cost of waiting is added to the supplier cost function if the driver is late. The waiting cost component is added to the user cost function if the passenger should wait for the driver. Also, the penalty for missed pickup is added to the supplier cost if the driver is responsible for it (shown as Missed Pickup\_D in the flowchart), otherwise, the passenger is responsible for the missed pickup and this cost will be added to the user cost function.

In the next step, the different probabilities are calculated based on different outcomes shown in the flowchart in figure 4:

1. Passenger waiting for vehicle:

$$P(p) = \int_{-\infty}^{T_i} F(tv) dtv \cdot \int_{-\infty}^{T_i-Sv} F(tp) dtv + \int_{T_i}^{T_i+Mp} F(tv) dtv \cdot \int_{-\infty}^{T_i} F(tp) dtv + \\ \int_{T_i}^{T_i-Sp+Mp} F(tv) dtv \cdot \int_{-\infty}^{T_i-Sv} F(tp) dtv \quad (3)$$

2. Vehicle waiting for passenger:

$$P(v) = \int_{-\infty}^{T_i} F(tv) dtv \cdot [\int_{T_i-Sv}^{T_i} F(tp) dtv + \int_{T_i}^{T_i+Mv} F(tp) dtv] + \\ \int_{T_i}^{\infty} F(tv) dtv \cdot \int_{T_i-Sv}^{T_i-Sv+Mv} F(tp) dtv \quad (4)$$

3. Missed pickup and passenger is responsible:

$$P(mp) = \int_{-\infty}^{T_i} F(tv) dtv \cdot \int_{T_i+M_v}^{\infty} F(tp) dtv + \\ \int_{T_i}^{\infty} F(tv) dtv \cdot \int_{T_i-S_v+M_v}^{\infty} F(tp) dtv \quad (5)$$

4. Missed pickup and driver is responsible:

$$p(md) = \int_{T_i-S_p+M_p}^{\infty} F(tv) dtv \cdot \int_{T_i}^{\infty} F(tp) dtv + \\ \int_{T_i+M_p}^{\infty} F(tv) dtv \cdot \int_{-\infty}^{T_i} F(tp) dtv \quad (6)$$

where;

$S_V$  = slack time for vehicle (minutes)

$S_P$  = slack time for passenger (minutes)

$f(tv)$  = probability density function for vehicle's arrival

$f(tp)$  = probability density function for passenger's arrival

$T_i$  = scheduled pickup time (minutes)

$M_V$  = maximum wait margin for the vehicle (minutes)

$M_P$  = maximum wait margin for the passenger (minutes)

Based on the formulas provided to calculate the probabilities (equations 3 to 6) and using the equation 1, the supplier cost function is formulated as:

1) The additional operating cost of adding slack time to schedule equal to:

$$C_{S1} = S_V \times O_C \quad (7)$$

2) The cost of waiting for passenger calculated as the product of value of time for vehicle, difference between arrival time of passenger and vehicle, and probability of vehicle waiting for passenger (this probability is calculated based on the flowchart in figure 4 and shown in equation 4):

$$C_{S2} = V_V \times (\text{Waiting time}) \times \text{Probability of passenger being late}$$

$$C_{S2} = V_V \cdot (t_p - t_v) \left\{ \int_{-\infty}^{T_i} F(tv) dtv \cdot \left[ \int_{T_i-Sv}^{T_i} F(tp) dtv + \int_{T_i}^{T_i+Mv} F(tp) dtv + \right. \right. \\ \left. \left. \int_{T_i}^{\infty} F(tv) dtv \cdot \int_{T_i-Sv}^{T_i-Sv+Mv} F(tp) dtv \right] \right\} \quad (8)$$

3) The expected penalty cost in case in which the driver is responsible for the missed pickup calculated as the product of the penalty P, and the probability of missed pickup as the responsibility of driver (this probability is calculated based on the flowchart in figure 4 and shown in equation 6):

$$C_{S3} = P \times \text{Probability of missed pickup - driver responsible}$$

$$C_{S3} = P \cdot \left\{ \int_{T_i-Sp+Mp}^{\infty} F(tv) dtv \cdot \int_{T_i}^{\infty} F(tp) dtv + \int_{T_i+Mp}^{\infty} F(tv) dtv \cdot \int_{-\infty}^{T_i} F(tp) dtv \right\} \quad (9)$$

Based on Eq. 1, the resulting supplier cost function will be:

$$C_S = C_{S1} + C_{S2} + C_{S3} \quad (10)$$

By substituting equations 7, 8 and 9 into equation 10 we obtain equation 11:

$$\begin{aligned}
 C_S = & S_V \cdot O_C + V_V \cdot (t_P - t_V) \left\{ \int_{-\infty}^{T_i} F(tv) dtv \cdot \left[ \int_{T_i-S_V}^{T_i} F(tp) dtp + \right. \right. \\
 & \left. \int_{T_i}^{T_i+M_V} F(tp) dtp + \int_{T_i}^{\infty} F(tv) dtv \cdot \int_{T_i-S_V}^{T_i-S_V+M_V} F(tp) dtp \right] + \\
 & P \cdot \left\{ \int_{T_i-S_P+M_P}^{\infty} F(tv) dtv \cdot \int_{T_i}^{\infty} F(tp) dtp + \int_{T_i+M_P}^{\infty} F(tv) dtv \cdot \int_{-\infty}^{T_i} F(tp) dtp \right\} \quad (11)
 \end{aligned}$$

where:

$C_S$  = supplier cost (\$/scheduled trip)

$O_C$  = unit operating cost for vehicle (\$/minute)

$V_V$  = value of time for vehicle (\$/minute)

$V_P$  = value of time for passenger (\$/minute)

$P$  = penalty for missed pickup (\$/pickup)

### User Cost Function

Now that the supplier cost function is formulated, we can develop a function for user (passenger). This  $C_U$  function can be developed based on the flowchart in figure 4 with a similar method that was used for developing the  $C_S$  function.

Similarly to the supplier cost, the user cost function includes three components:

- 1) Slack time cost, which is the additional cost of adding a slack time for passenger to the schedule and is the product of slack time for passenger and the value of passenger's time ( $V_P$ ):

$$C_{U1} = S_P \times V_P \quad (12)$$

- 2) Passenger waiting cost, which is the cost of passenger waiting for driver and is calculated using the probability formula presented as equation 3 in previous section:

$$\begin{aligned}
 C_{U2} &= V_p \times (\text{Waiting time}) \times \text{Probability of vehicle being late} \\
 C_{U2} &= V_p \cdot (t_v - t_p) \{ [\int_{-\infty}^{T_i} F(tv) dtv \cdot \int_{-\infty}^{T_i-Sv} F(tp) dtv] + \\
 &\quad [\int_{-\infty}^{T_i} F(tp) dtv \cdot \int_{T_i}^{T_i+Mp} F(tv) dt] v + \\
 &\quad [\int_{T_i}^{T_i-Sp+Mp} F(tv) dtv \cdot \int_{-\infty}^{T_i-Sv} F(tp) dtv] \}
 \end{aligned} \tag{13}$$

- 3) Expected penalty cost, which is the cost of probable missed pickup in the case in which the passenger responsible is responsible for the unsuccessful ride. This cost is calculated using the probability equation 5, formulated in the previous section.

$$\begin{aligned}
 C_{U3} &= P \times \text{Probability of missed pickup - passenger responsible} \\
 C_{U3} &= P \cdot \{ \int_{-\infty}^{T_i} F(tv) dtv \cdot \int_{T_i+Mv}^{\infty} F(tp) dtv + \\
 &\quad \int_{T_i}^{\infty} F(tv) dtv \cdot \int_{T_i-Sv+Mv}^{\infty} F(tp) dtv \}
 \end{aligned} \tag{14}$$

Based on the Eq. 1, the resulting supplier cost function will be:

$$C_U = C_{U1} + C_{U2} + C_{U3} \tag{15a}$$

The resulting User Cost function is expressed in equation 15:

$$\begin{aligned}
 C_U = & S_P \cdot V_P + V_P \cdot (t_V - t_P) \{ [\int_{-\infty}^{T_i} F(tv) dtv \cdot \int_{-\infty}^{T_i-Sv} F(tp) dtv] + \\
 & [\int_{-\infty}^{T_i} F(tp) dtv \cdot \int_{T_i}^{T_i+Mp} F(tv) dtv] v + [\int_{T_i}^{T_i-Sp+Mp} F(tv) dtv \cdot \int_{-\infty}^{T_i-Sv} F(tp) dtv] \} + \\
 & P \cdot \{ \int_{-\infty}^{T_i} F(tv) dtv \cdot \int_{T_i+Mp}^{\infty} F(tp) dtv + \int_{T_i}^{\infty} F(tv) dtv \cdot \int_{T_i-Sv+Mp}^{\infty} F(tp) dtv \} \quad (15)
 \end{aligned}$$

where;

$C_U$  = user cost (\$/scheduled trip)

$V_V$  = value of time for vehicle (\$/minute)

$V_P$  = value of time for passenger (\$/minute)

$P$  = penalty for missed pickup (\$/pickup)

### **Total Cost Function**

In this section, the total system cost function  $C_T$  is modeled using the supplier and user cost functions formulated previously. As described before, for every pickup, the system consists of the driver (or the rideshare company) and the passenger. As in previous sections, we assume that the passenger calls for a pickup at a designated pickup location and at a specific time.

Also, we assume that the rideshare operator, knowing about the travel time from the origin to the pickup point, decides to dispatch the driver  $S_V$  minutes earlier than dictated by travel time to ensure a successful pickup. On the other hand, the passenger, knowing about the travel time to the pickup point, decides to move towards the pickup location  $S_P$  minutes earlier to ensure a successful ride.

If the passenger misses the pickup, a penalty should be paid to the driver or the rideshare supplier. Also, in the event that the driver is unsuccessful in picking up the passenger, the penalty should be paid to the user.

In previous sections, these penalty costs were part of the supplier cost and user cost functions, but when modeling the total system cost, the penalty cost should not be taken into account.

The penalty cost component of the  $C_S$  function is revenue for user and the penalty cost component of the  $C_U$  function is revenue for the supplier and therefore these will not be included in the total system cost function. Here we introduce the real user cost function ( $C_{UR}$ ), which only includes the costs associated with the user's value of time. It is assumed that the expected cost of penalty for missed pickup (passenger's responsibility) is equal to the expected revenue of missed pickup (driver's responsibility) and therefore, the real user cost is expressed as:

$$C_{UR} = C_{U1} + C_{U2} \quad (16a)$$

formulated as;

$$\begin{aligned} C_{UR} = & S_P \cdot V_P + V_P \cdot (t_V - t_P) \{ \left[ \int_{-\infty}^{T_i} F(tv) dtv \cdot \int_{-\infty}^{T_i - Sp} F(tp) dtv \right. \right. \\ & \left. \left. + \left[ \int_{-\infty}^{T_i} F(tp) dtv \cdot \int_{T_i}^{T_i + Mp} F(tv) dtv \right] v + \left[ \int_{T_i}^{T_i - Sp + Mp} F(tv) dtv \cdot \int_{-\infty}^{T_i - Sp} F(tp) dtv \right] \right] \} \end{aligned} \quad (16b)$$

where;

$C_{UR}$  = real user cost (\$/scheduled trip)

$V_V$  = value of time for vehicle (\$/minute)

$V_P$  = value of time for passenger (\$/minute)

$S_V$  = slack time for vehicle (minutes)

$S_P$  = slack time for passenger (minutes)

$f(tv)$  = probability density function for vehicle's arrival

$f(tp)$  = probability density function for passenger's arrival

$T_i$  = scheduled pickup time (minutes)

$M_P$  = maximum wait margin for the passenger (minutes)

The real supplier cost function includes the costs associated with value of time for vehicle and the expected cost of penalty for missed pickup (driver's responsibility) is equal to the expected revenue of missed pickup (passenger's responsibility). Therefore, the real supplier cost is expressed as:

$$C_{SR} = C_{S1} + C_{S2} \quad (17a)$$

formulated as:

$$C_{SR} = S_V \cdot O_C + V_V \cdot (t_P - t_V) \{ \int_{-\infty}^{T_i} F(tv) dtv \cdot [ \int_{T_i - S_V}^{T_i} F(tp) dtp + \int_{T_i}^{T_i + M_V} F(tp) dtp + \int_{T_i}^{\infty} F(tv) dtv \cdot \int_{T_i - S_V}^{T_i - S_V + M_V} F(tp) dtp ] \} \quad (17b)$$

where;

$C_{US}$  = real supplier cost (\$/scheduled trip)

$M_V$  = maximum wait margin for the vehicle (minutes)

The resulting total cost function  $C_T$  is expressed in equation 18 as the summation of real user cost and real supplier cost functions.

$$C_T = C_U + C_S \quad (18)$$

The numerical results of formulation of supplier, user's and total cost functions are examined in the next chapter.

## **Chapter 4: NUMERICAL RESULTS**

This chapter focuses on the numerical analysis of the three cost functions formulated in previous chapter as: supplier cost, user cost and total cost. The numerical results in this chapter are mainly intended for examining the behavior of cost functions and explaining the effects of changes in slack times on the final results of the models. Although the optimum values for safety factors (in this study, slack times) can be calculated using simulation methods, such solutions are very difficult to obtain given the complexity of the cost models. In this study, solutions are sought by numerical analysis of the formulated cost functions.

This approach seems acceptable for the purpose of this study because it is more practical and reliable and less time consuming for real-time optimization. Although the numerical results shown in this chapter are limited to a few relations between different variables and components of the three formulated cost functions, they are selected in a way that displays the general behavior of cost functions.

The baseline values for parameters used in this study are selected as they seemed reasonable.

$T_i$  = scheduled pickup time = 9:00 am,

$O_C$  = vehicle's operating cost = 0.25 \$/minute

$V_V$  = vehicle's value of time = 0.3 \$/minute

$V_P$  = passenger's value of time = 0.3 \$/minute

$M_v$  = vehicle's maximum waiting time = 3 minutes

$M_p$  = passenger's maximum waiting time = 5 minutes

$P$  = penalty for missed pickup = 10 \$/pickup

$\sigma_v$  = standard deviation of vehicle's arrival distribution = 5 minutes

$\sigma_p$  = standard deviation of passenger's arrival distribution = 2 minutes

The arrival of vehicle and passenger at the pickup location are normally distributed and independent. The standard deviation for vehicle's arrival distribution and passenger's arrival distribution are selected based on the behavior of total cost function, which will be discussed at the total cost analysis section of this chapter.

### **Numerical Results for the Supplier Cost Function**

With the baseline values presented above, the supplier cost function formulated as equation 1b, is used to plot figure 5. Figure 5 shows the supplier cost function components vs the slack time for vehicle ( $S_v$ ). The expected penalty cost, waiting cost and vehicle's operating cost are the components of the supplier cost function that are also presented in this figure.

We can see from figure 5 that as the slack time for vehicle increases, the operating cost also increases as it is a function of value of time for vehicle and slack time for vehicle.

Also, the cost of waiting for vehicle has an increasing trend because as the vehicle leaves its origin earlier and earlier to avoid missed pickup, the probability that it arrives before the passenger and waits at the pickup location increases.

The expected penalty cost is the cost considered when the driver is responsible for the missed pickup. Therefore, this cost decreases as the vehicle slack time increases because the probability of its occurrence decreases.

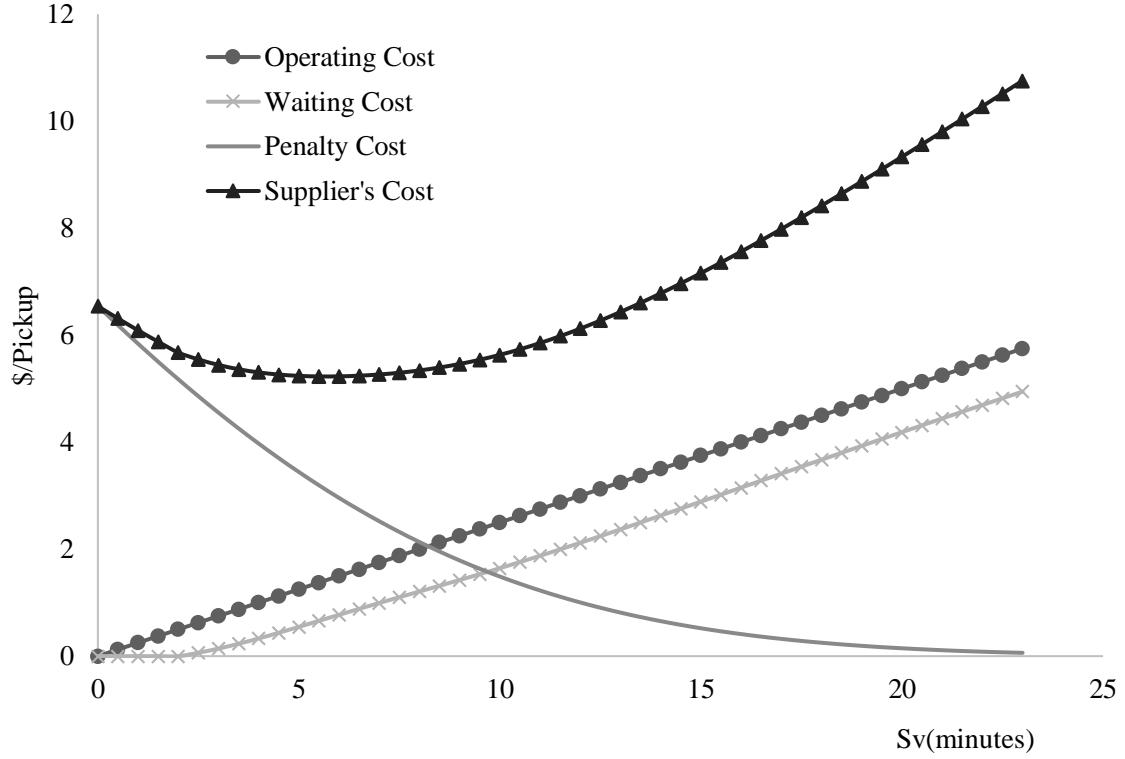


Figure 5: Supplier cost and its components

Here, the minimum supplier cost occurs at  $C_s = 5.23$  \$/pickup, when the slack time for vehicle is equal to 5.5 minutes. If adding a slack time to the schedule is not considered by the operator or the driver, the supplier cost will be equal to the penalty for a missed pickup (6.54 \$/pickup). To avoid this cost, the driver can add 5.5 minutes to her/his pickup schedule and decrease her/his cost by more than \$1.3 per pickup.

As the numerical analysis in figure 5 shows, adding a slack time as a safety factor to the schedule in order to avoid costs of waiting and expected missed pickup,

decreases the supplier cost initially. As we increase the slack time, the supplier cost keeps decreasing. After a certain amount of slack time (which is at the point where minimum supplier cost occurs), the supplier cost starts increasing because the tradeoffs in the penalty cost function which yield the slack time become increasingly dominated by the operating cost and waiting cost functions. After that point, it would not be reasonable to increase the slack time for vehicle.

To better understand the concept of expected penalty cost, figure 6 is used to show the probability of having a missed pickup cost in the supplier cost function.

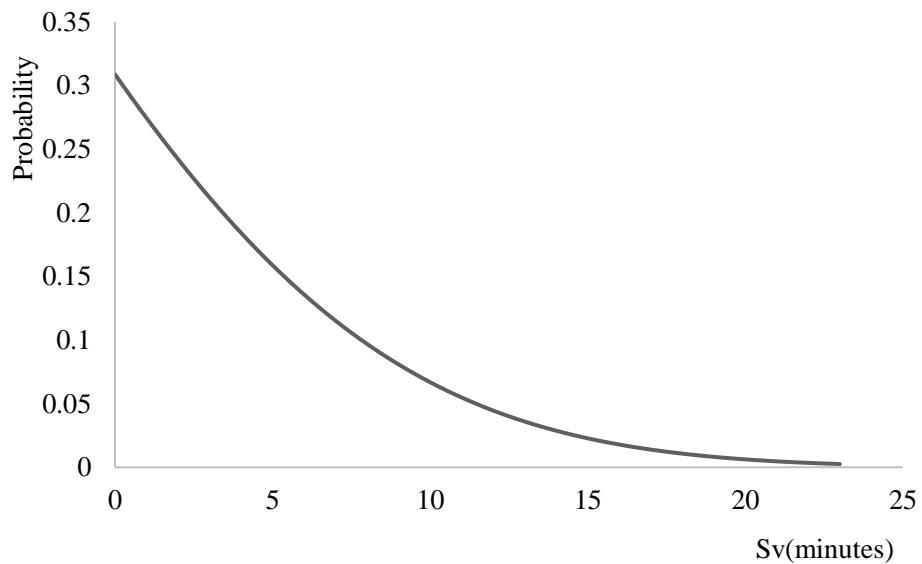


Figure 6: Missed connection probability

The probability function showed in figure 6 is the result of two mixed probabilities:

- 1) Passenger is early or on time but the driver arrives after the maximum wait time of the passenger:  $t_P < T_i$  and  $t_v > T_i + M_P$
- 2) Passenger is late but the driver is also late:  $t_P > T_i$  and  $t_v > t_P + M_P$

The expected penalty cost for vehicle is the product of penalty and above probabilities.

The decreasing trend in the probability of missed connection in the supplier cost function yields the slack time for vehicle to avoid the expected missed pickup. As this probability nears zero for larger slack times, the other components of supplier cost function (operating cost and waiting cost) become dominant over the expected penalty cost and result in a positive slope for the supplier cost function.

In the next step of model formulation, the user cost function will be modeled using the same approach used for formulating the supplier cost function.

### **Numerical Results for the User cost Function**

Now that the results of numerical analysis for supplier cost function conform to expectations, we analyze the behavior of user cost function.

Note that the user's behavior in both  $C_U$  and  $C_S$  functions is based on the timeline defined for the vehicle's arrival (as shown in the flowchart in figure 4). If the flowchart in figure 4 was based on user's arrival time, the resulting  $C_U$  and  $C_S$  functions would have a different appearance while leading to the same results.

Figure 7 shows graphically the  $C_U$  function and its components. In the numerical example here, the optimum slack time for passenger is 1.5 minutes leading to about 2.5 \$/pickup cost for the passenger. The  $C_U$  function has an upward trend after this point because the increase in wait cost and slack time cost for passenger dominate over the expected penalty cost for a missed pickup.

The user cost function and its components in figure 7 behave as expected because the cost of waiting and slack cost increase as the slack time for passenger increases and the probability of having a missed pickup because of passenger no-show decreases.

The decreasing trend in the first part of the user cost function shows that adding a safety factor (here; slack time) in the passenger's schedule can increase the reliability of the pickup service by decreasing the probability of missed pickup. The passenger does not want to pay the penalty for missing the ride, so she/he decides to leave earlier to increase the probability of a timely arrival at the pickup location. However, extending the slack time after a certain point is not economically justified for the passenger as it increases the additional cost of slack time as well as the probability of the passenger having to wait for the driver.

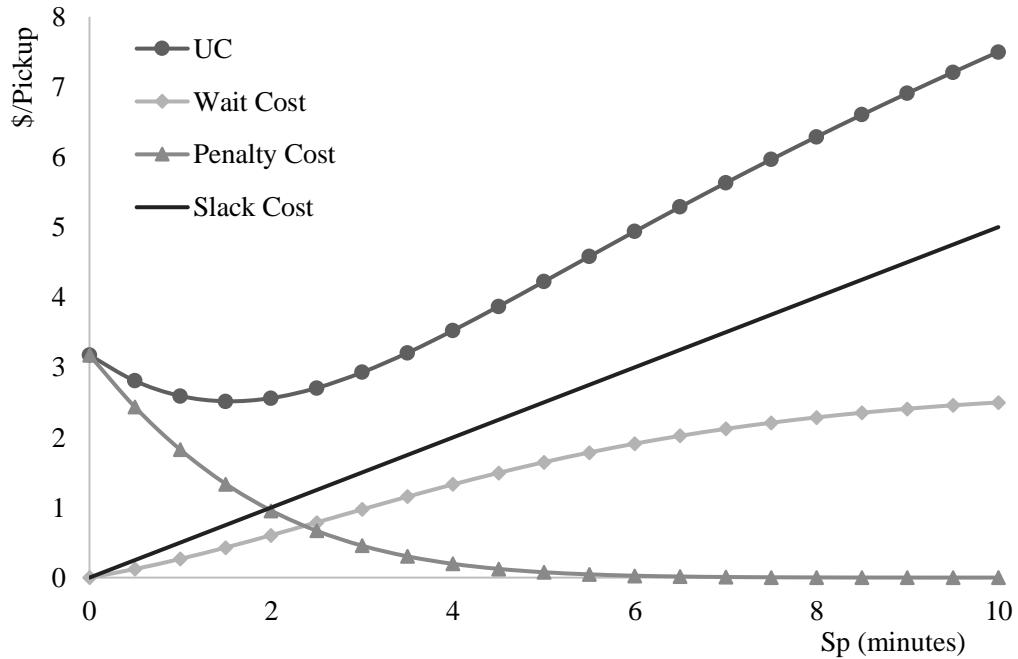


Figure 7: User cost and its components vs the slack time for passenger

Unlike the wait cost for the supplier, the wait cost function for the user is non-linear in the resulting graph. Beyond  $S_P = 4.5$  minutes, the slope of wait cost function decrease with a trend approximately similar to the expected penalty cost function. Note that the standard deviation of passenger's arrival for this numerical analysis is assumed to be 3 minutes and the standard deviation of vehicle's arrival is assumed to be 10 minutes.

To investigate this property of the wait cost function for users, three graphs are drawn for three different standard deviations of passenger's arrival. Figure 8 shows that as the standard deviation of passenger's arrival increases, the changes in slope of the wait cost curve decrease.

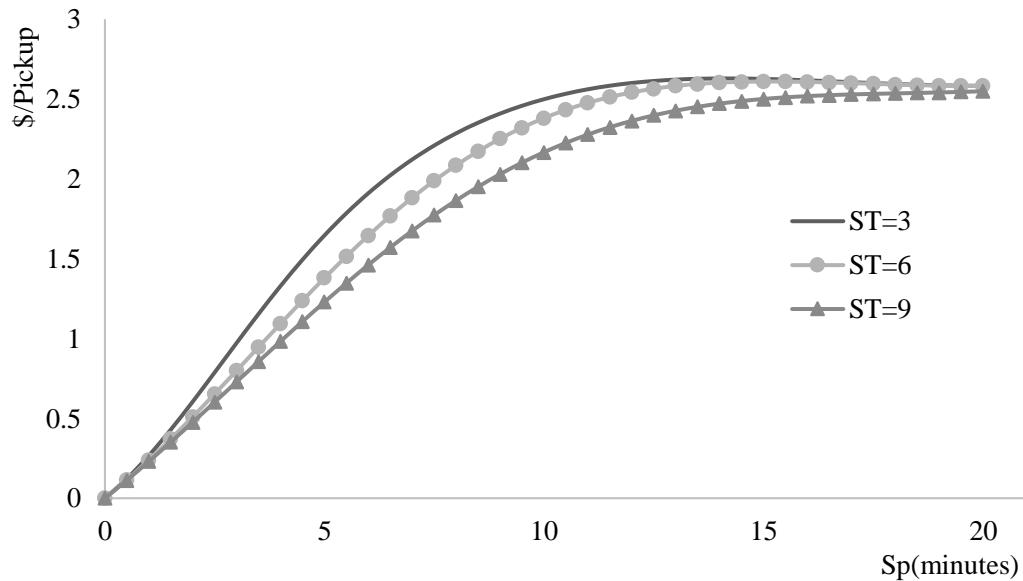


Figure 8: User wait cost vs the slack time for passenger (different standard deviations of passenger's arrival)

As the standard deviation of the normal distribution function increases, the probability density function becomes shallower and more spread. This reduces the

changes in the slope of the user's waiting cost function. In the next section, total cost function is formulated using the user's and supplier cost formulas.

### **Numerical Results for the Total Cost Function**

In this section, the behavior of total system cost function  $C_T$  is examined using numerical analysis. As mentioned earlier in this chapter, the base line values for standard deviation of arrivals are selected according to the behavior of total cost function.

As figure 9 shows, keeping the standard deviation of passenger's arrival constant, increase in the standard deviation of vehicle's arrival leads to increase and then decrease in the optimal total cost. The same situation holds for the standard deviation of passenger's arrival as an increase in it first, increases and then decreases the value of the optimal total cost.

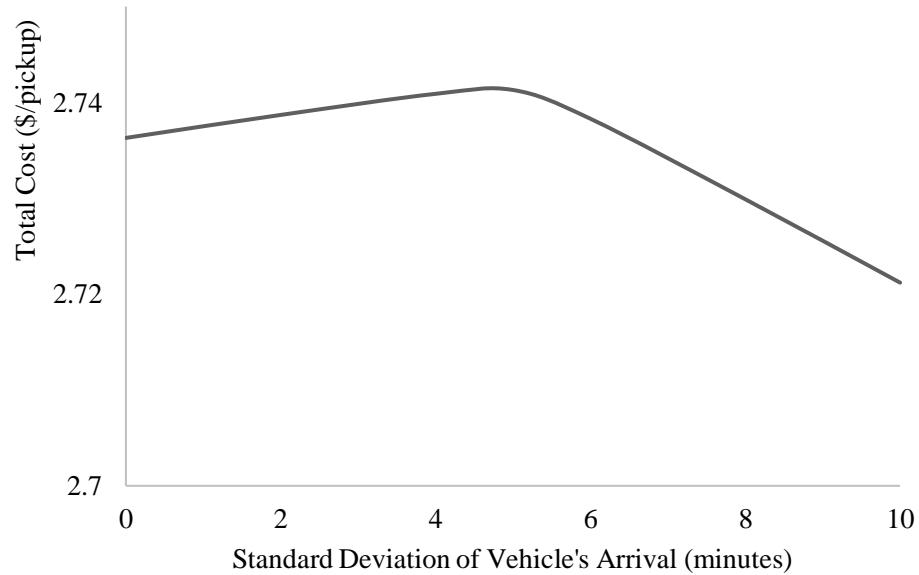


Figure 9: Optimal total cost vs the standard deviation of arrivals for vehicle

For the numerical analysis in this study, the standard deviations leading to maximum optimal total cost are chosen to display the behavior of different functions and model components. Figure 10 and 11 show the total cost function vs slack time for passenger and vehicle. The real user cost and real supplier cost are also incorporated in the graphs below to show their relation with the total cost function. As expected, when the slack time for passenger increases, the real supplier cost decreases and the real user cost and total cost increase.

As shown in figures 10 and 11, the total cost function is the result of adding up the real user cost and real supplier cost functions. Because the penalty cost is eliminated when formulating the total cost function, when the slack time (for passenger or vehicle) is zero, the total cost function only reflects the waiting costs for the passenger and vehicle.

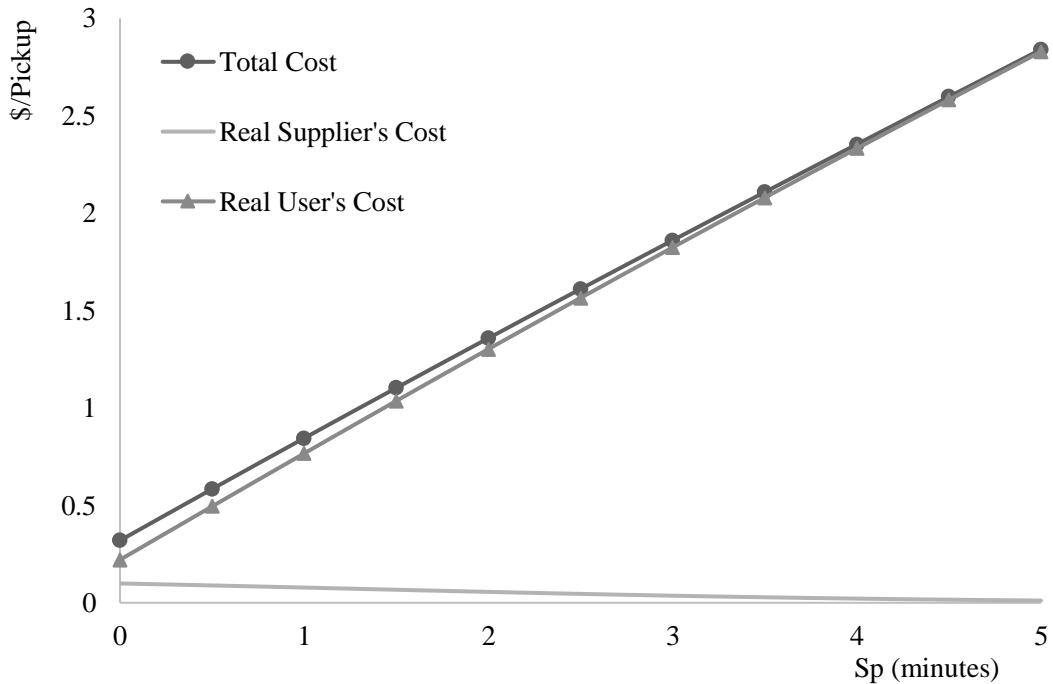


Figure 10: Total cost, real user cost and real supplier cost vs the slack time for passenger

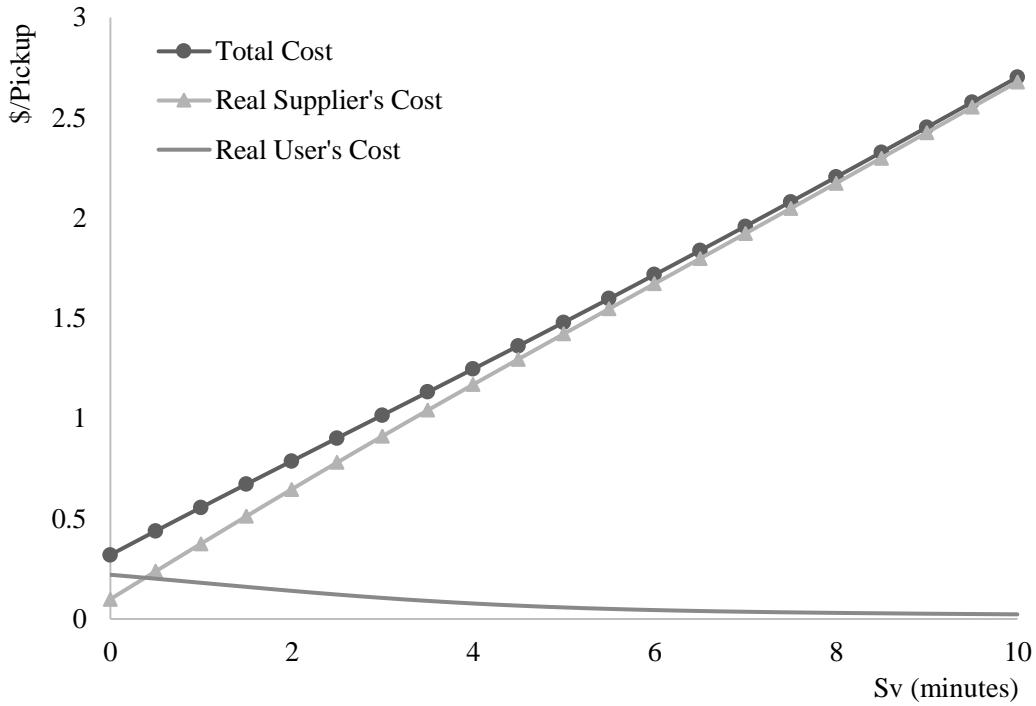


Figure 11: Total cost, real user cost and real supplier cost vs the slack time for vehicle

Based on the numerical example in figure 10, the minimum total cost for system occurs when the slack time for passenger is equal to zero. However, note that when  $S_p=0$  minutes, the real supplier cost is maximum because the probability that the vehicle waits for passenger is maximum at this point. When the passenger increases the slack time in her/his schedule, the real supplier cost decreases until it is close to zero (less than 1 cent per pickup) at  $S_p = 5$  minutes. Without considering the penalty cost for a missed pickup, 5 minutes added to the passenger's schedule can reduce the real supplier cost to zero \$/pickup. The real user cost at that point is around 3 \$/pickup for this numerical case.

Similar to figure 10, graphs in figure 11 reflect the same results from the user's point of view. Without considering the expected penalty cost of missed pickup,

adding around 10 minutes to the vehicle's schedule reduces the real user cost to zero.

The real supplier cost at this point is around 3 \$/pickup.

Figure 12 shows the joint effects of slack time for passenger and vehicle on the total cost function in three dimension display mode. Figure 13 is a display of equal total cost isochrones vs slack time for the passenger and driver.

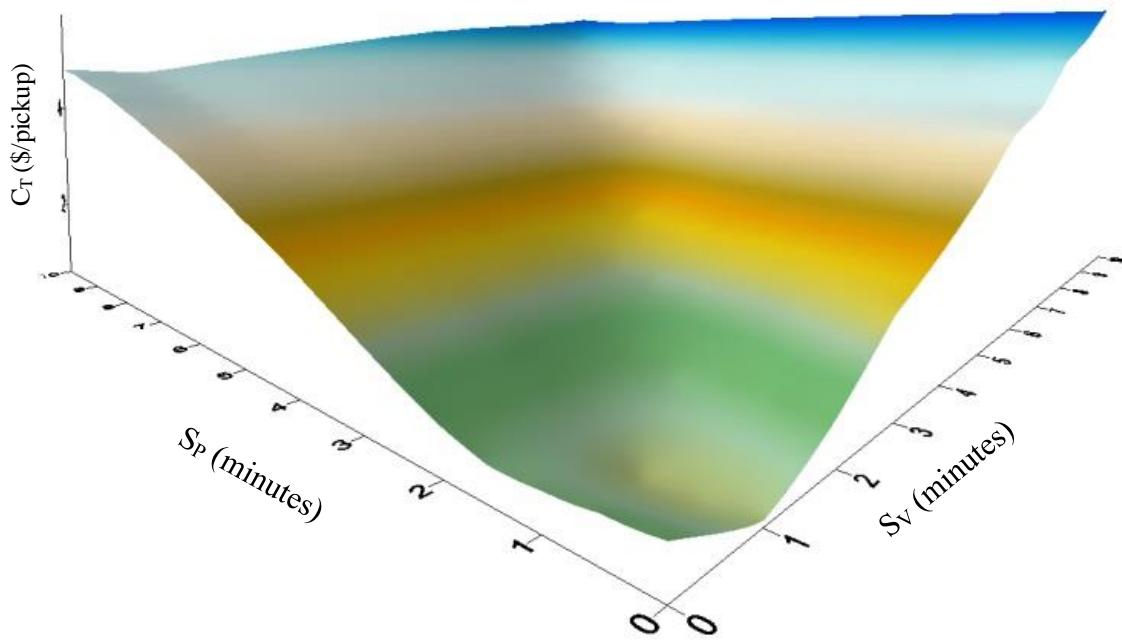


Figure 12: Total cost function vs slack time for passenger ( $S_p$ ) and vehicle ( $S_v$ )

Figure 13 can be used to obtain different pairs of slack time for vehicle and passenger for any particular total system cost. At the point where slack times are both equal to zero, total system cost has non zero values as the cost components associated with waiting time are non-zero. As a result of base values used for numerical analysis in this study, the minimum total system cost occurs (approximately) when  $S_v = S_p = 1$

minutes. The resulting total cost for this point is equal to 0.6 \$/pickup. To explain the behavior of total cost function in figure 13, cutting planes at  $S_V = 3$  minutes and  $S_P = 1$  minutes are shown in figure 14 and 15 respectively.

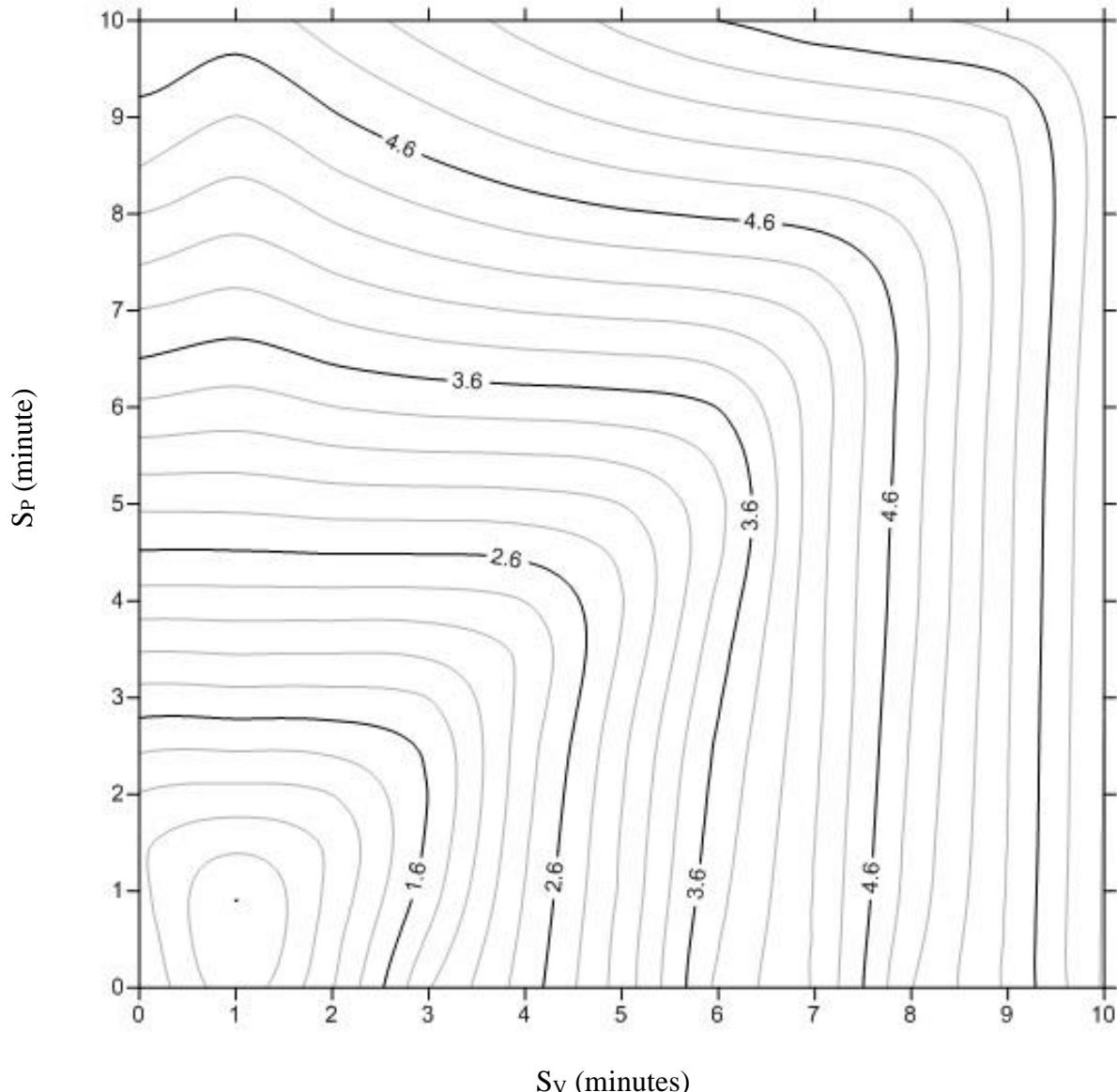


Figure 13: Total cost isochrones for different values of slack time for passenger ( $S_P$ ) and vehicle ( $S_V$ )  
(the total cost is expressed in \$/pickup for one driver-passenger system)

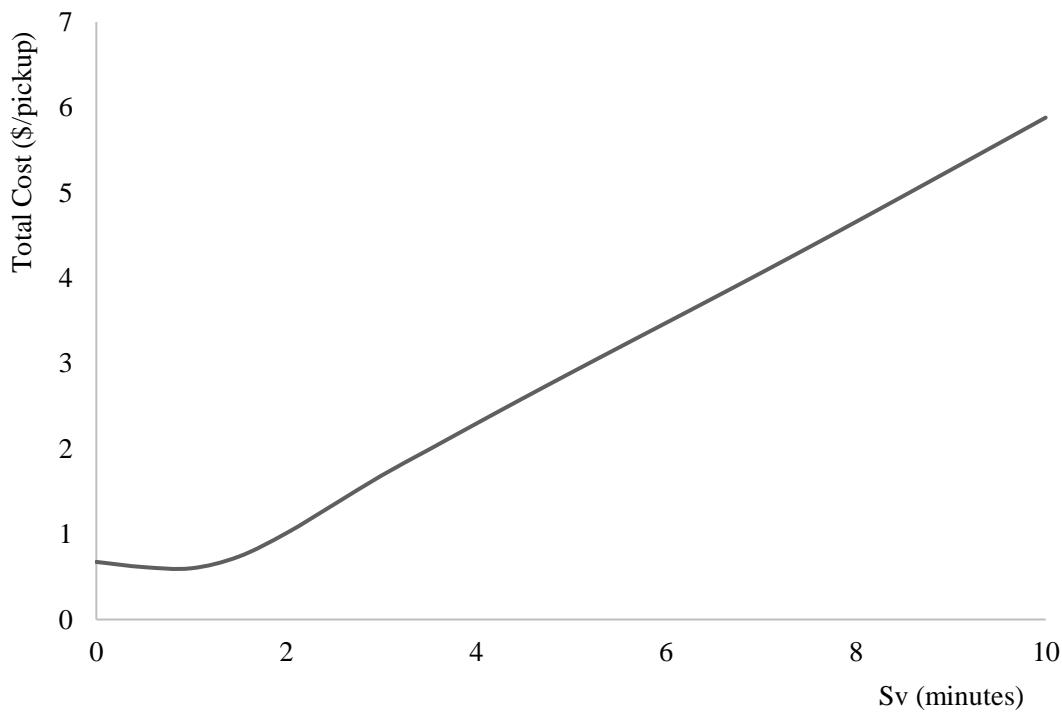


Figure 14: Total cost vs slack time for vehicle at  $S_p = 1$  minutes

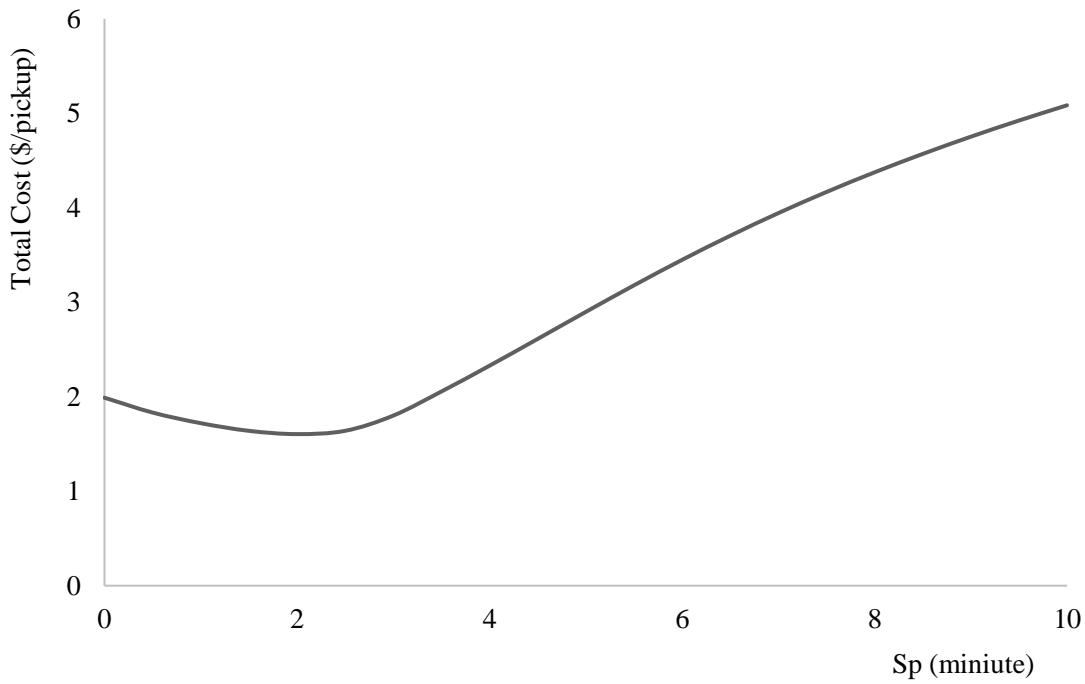


Figure 15: Total cost vs slack time for passenger at  $S_v = 3$  minutes

Figures 14 and 15 show the total cost function behavior vs changes in slack time for the passenger and vehicle. At the point where slack time for the vehicle is equal to zero, the total cost is non-zero as there are costs associated with waiting times and slack time for passenger. The total cost function (as expressed in equation 18, chapter 3) consists of two components: additional cost of adding  $S_v$  and  $S_p$  to the schedule, and the waiting cost for passenger and driver.

In figure 14, as the slack time for the vehicle increases, the probability of having passenger waiting for the vehicle decreases while it becomes more probable that the vehicle should wait for passenger. Initially, the first probability is dominant over the latter probability and as a result, the total cost function keeps decreasing down to the minimum point ( $C_T = 0.6 \text{ \$/pickup}$ ). After that point, the tradeoffs between the additional cost of slack times and waiting cost make it unreasonable to increase the slack time.

The same explanation is true for the total cost function behavior in figure 15. As the slack time for the passenger increases, the probability of having vehicle waiting for the passenger decreases, increasing the probability of passenger waiting for vehicle. After  $C_T = 1.6 \text{ \$/pickup}$ , the tradeoffs between the additional cost of slack times and waiting cost make it unreasonable to increase the slack time.

## **Chapter 5: SENSITIVITY ANALYSIS**

After formulating the user cost, supplier cost and total cost functions and analysis of their behavior relative to the changes in slack times, the sensitivity of results to changes in values of different model components should be examined. Because the main purpose of this study is the analysis of slack time in a simple real-time rideshare system, the sensitivity of the optimized slack times for the passenger and vehicle with respect to changes in different components of the different cost functions is presented in this chapter.

In this study, numerical optimization is used by changing the value of inputs and getting a different optimum slack time as result. As in the previous chapter, the cost functions for supplier and user, contain joint probabilities which can be solved by simulation or numerical analysis. Compared to simulation method, numerical analysis approach needs far less computations (which can be crucial for real-time applications) and seems more practical for the purpose of this study.

In this chapter, first, the optimum slack times for passenger and vehicle will be calculated for different values of time. Also, values of the minimum user's and supplier costs for different values of time for vehicle will be displayed in a graph.

Next, the sensitivity of slack times to other components of the objective functions will be discussed. Other than parameters of the normal distributions for arrival of vehicle and passenger, maximum waiting time window for vehicle and passenger are also included in the cost models.

In the next step, the sensitivity of optimized slack times to changes in waiting time windows ( $M_P$  and  $M_V$ ) will be tested and finally, the penalty ( $P$ ) will be the subject of sensitivity analysis.

The results of sensitivity analysis in this chapter are presented in multiple graphs that provide information on the relations between different cost model components. The base parameter values used in this chapter are the same as values expressed in previous chapter unless stated otherwise.

The results of sensitivity analysis presented in this chapter, show optimal values unless it is stated differently.

### **Optimal Slack Time vs the Value of Time**

The value of time is the dollar amount assigned to value the benefit of a change in expected travel time or unscheduled delay resulting from transportation projects (TRB's ACRP Document 22, 2015). In essence, it reflects the amount that the driver (or rideshare company) and passenger are willing to pay in order to save a unit of time. The unit used in this research is \$ per minute.

According to the revised version of the Departmental Guidance for Conducting Economic Evaluations (Revision 2, 2014), the value of travel time is a critical factor in evaluating the benefits of transportation investments and rule making initiatives, when the reduction of delays in passenger or freight transportation is the major purpose.

Furthermore, the value of time can also be defined as the amount a person will accept to be paid as a compensation to her/his lost time. Here in this study, when a pickup is unsuccessful, estimating the real value of lost time can be complicated. The

value of time with such definition can range from near to 0 \$/minute for one passenger, to very high amounts for the other. As discussed previously in this study, the penalty of a missed pickup  $P$  is considered for driver and passenger, which is different from value of time for vehicle ( $V_v$ ) and for passenger ( $V_p$ ).

To analyze the sensitivity of the Slack times with respect to the value of time, the optimum slack times  $S_v$  and  $S_p$  are calculated by minimizing the supplier cost function and changing the value of time of the vehicle.

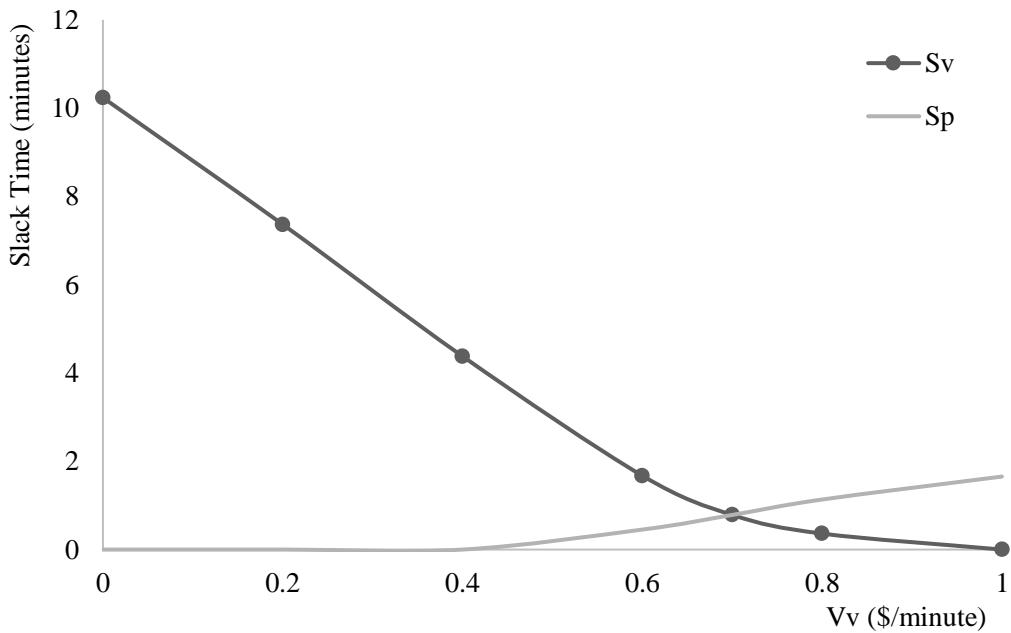


Figure 16: Value of vehicle's time Vs the optimal slack times for vehicle and passenger (minimizing supplier cost)

As figure 16 shows, the optimum slack times for passenger and driver are both equal to 0.8 minutes at  $V_v = 0.7$  \$/minute. Here, the value of time for passenger is assumed to be fixed at 0.5 \$/minute while the value of time for vehicle is changing from zero to 2 times the value of time for passenger (1 \$/minute). As the value of

time for vehicle increase, the optimum slack time for the driver will decrease to avoid the cost of waiting.

On the other hand, decreasing the slack time for vehicle increases the probability of having a missed pickup. Therefore, when the value of time for vehicle reaches to around 0.4 \$/minute, the slack time for passenger increases from zero to make up for the reduction in slack time for vehicle. Based on the results of the sensitivity of the optimum slack times to changes in value of time for vehicle, the changes in the supplier cost and user cost functions are shown in figure 17.

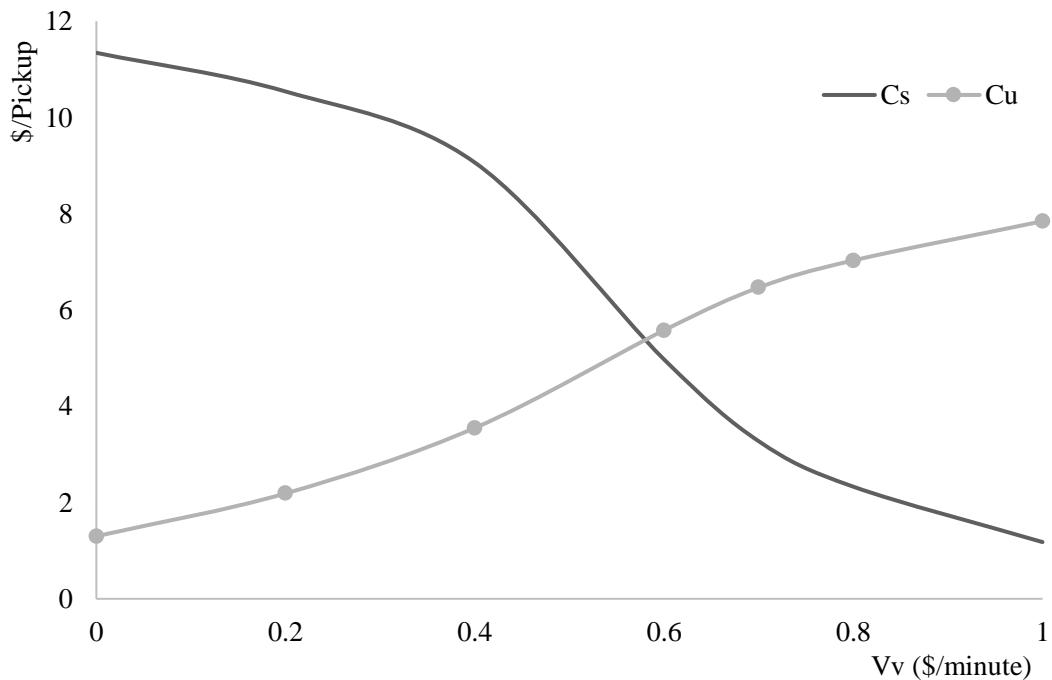


Figure 17: Value of vehicle's time vs the supplier cost and user cost

The Supplier cost and user cost for different optimal slack times for vehicle and passenger follow the same trend as the graphs shown in figure 16. As the value of time for vehicle increases, the investment in safety factor becomes more infeasible for the driver (or the operator). As a result, the driver decreases the amount of safety factor in the schedule, but to avoid the waiting cost and penalty cost, the user (passenger) should add a safety factor to the schedule. The user cost increase as the optimal slack time for vehicle increase.

Considering that the main parameter affecting the cost function is the related slack time, the behavior of supplier cost and user cost functions seems reasonable in figure 17.

### **Value of Time and Total Cost Function**

As mentioned previously, the main objective of this study is to analyze the slack time and coordinate the pickup schedule from point of view of the vehicle or the service provider. However, the total cost function is also analyzed to show the sensitivity of user's behavior to changes in different model components (figure 18).

Similarly to the behavior of slack times presented in figure 16, graphs in figure 18 show that when the value of time for vehicle is zero, the slack time for vehicle becomes equal to the largest amount necessary to avoid additional waiting cost. For relatively high values of time for vehicle, adding more slack time to the vehicle's schedule is not cost effective. On the other hand, the slack time for passenger increases to ensure minimum total system cost.

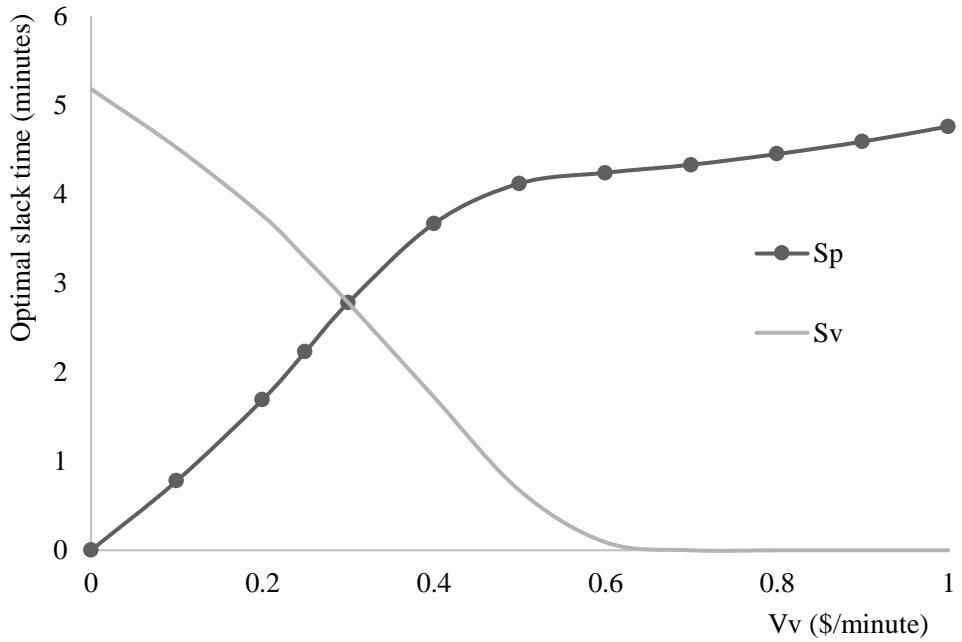


Figure 18: Value of vehicle's time vs the optimal slack times for vehicle and passenger (minimizing total cost)

### Optimal Slack Time vs the Waiting Time Window

In this study, the maximum time that vehicle or passenger wait after the scheduled pickup is defined in the model as maximum waiting time window ( $M_v$  and  $M_p$  in minutes). To simplify the formulations, we assumed that the vehicle should not leave the pickup location (without picking up the passenger) before waiting  $M_v$  minutes after the scheduled pickup.

Also, the passenger is obliged to wait for the vehicle up to  $M_p$  minutes after the scheduled pickup unless the driver arrives before that time. This maximum wait time is one of the decision factors affecting the probabilities in the model formulations (see flowchart in figure 4).

To analyze the sensitivity of the optimum slack times to changes in the maximum waiting time, the supplier cost functions is minimized to get different pairs of

optimum slack times. In figure 19, all conditions are kept unchanged while  $M_V$  is changing from 0 to 15 minutes (Three times the  $M_P$ ).

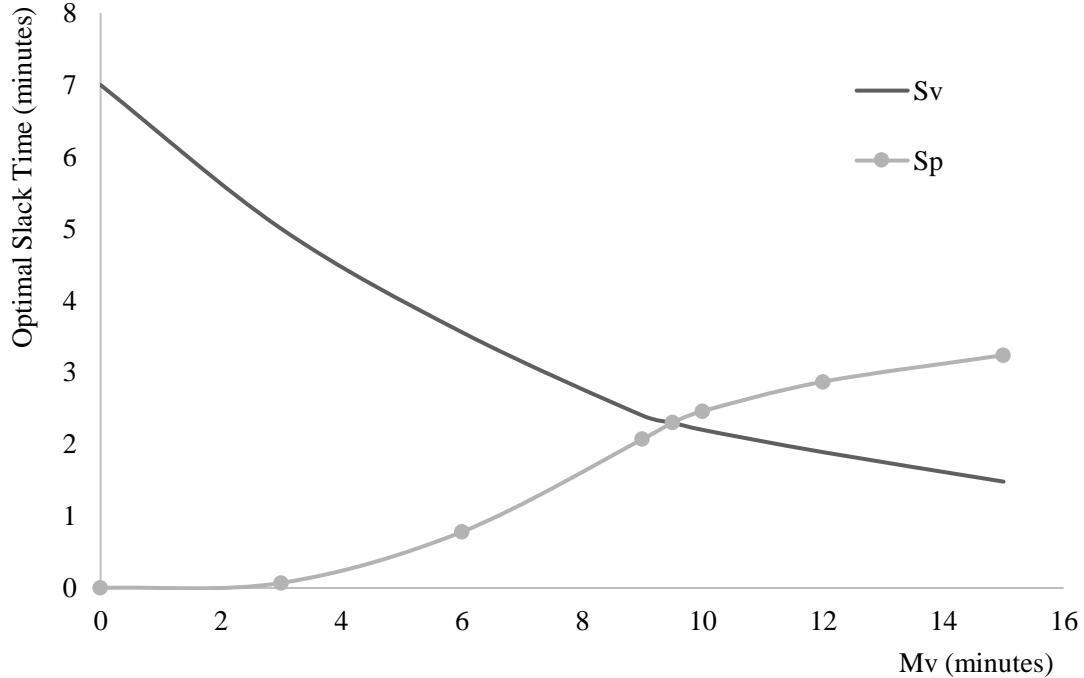


Figure 19: Maximum waiting time for vehicle vs the optimal slack times for vehicle and passenger

When the waiting time window for vehicle is small (close to zero minutes), the probability of having an unsuccessful pickup increases. Increase in the probability of missed pickup leads to increase in the amount of penalty cost component of the supplier cost function. As the objective is to minimize the supplier cost, the slack time for vehicle increases to avoid the penalty cost. As figure 19 shows, the slack time for vehicle is the highest at  $M_V = 0$  (where  $S_v$  is approximately 7 minutes).  $M_V$  decreases the waiting time window for vehicle increases.

On the other hand, decrease in the slack time for vehicle leads to increase in the slack time for passenger in order to reduce the probability of having an unsuccessful pickup.

The main reason for such behavior is the penalty cost and the fact that when minimizing the supplier cost function, the penalty cost can be avoided if slack times suffice to reduce the probability of an unsuccessful pickup. When the waiting time window is very large, the probability of having a missed pickup becomes very small and therefore, the operator would not invest in adding a safety factor to the schedule.

### **Optimal Slack Time vs the Penalty**

As explained in the model formulation chapter, adding the penalty cost component to the supplier (or user) cost function causes that the driver or passenger to consider adding a slack time to their schedule.

Figure 20 shows the behavior of supplier (driver or service provider), relative to changes in penalty (P). The vertical axis shows the optimum values of slack time for vehicle, when minimizing the supplier cost function for different penalties. The amount for penalty changes from 0 \$/pickup to 20 \$/pickup. For the supplier cost function, the penalty represents the amount that should be paid to the passenger, if the driver is responsible for a missed pickup.

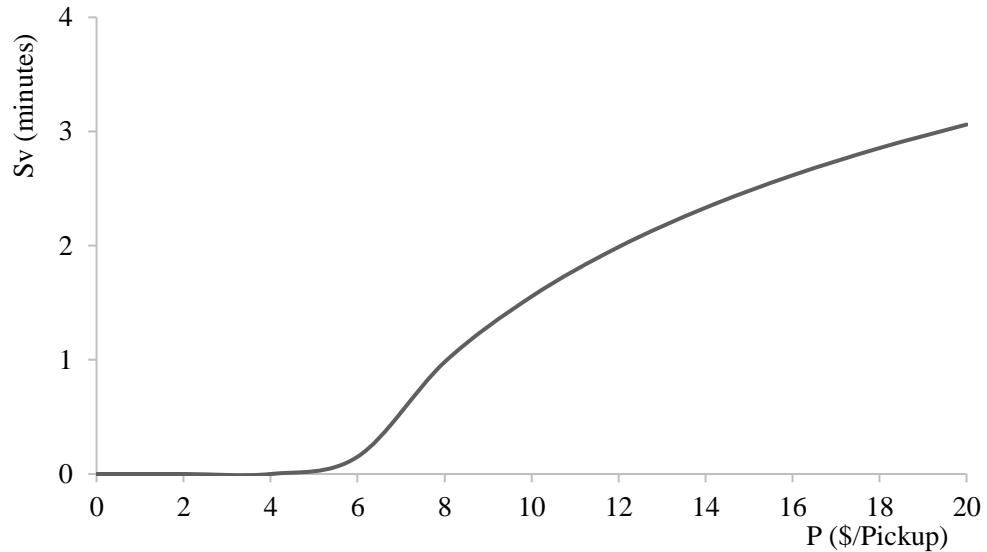


Figure 20: Optimum slack time for vehicle vs the penalty

As figure 20 shows, at  $P = 5.73$  \$/pickup, the supplier (or driver) will see fit to add a slack time to the schedule to ensure a successful pickup. The slack time for vehicle at this price is 0.006 minutes and as the penalty increase, the slack time for vehicle also increase until it is 3 minutes for a penalty of 20 \$/pickup.

The minimum supplier costs together with corresponding user costs for different penalties is shown in figure 21. Until the point in which is equal to 5.73 \$/pickup, both  $S_V$  and  $S_P$  are zero and therefore the supplier and user costs are approximately equal.

Note that the reason for this equal part is that the value of time for vehicle, value of time for passenger and vehicle's operating cost are all assumed to be 0.25 \$/minute for this numerical example. The only factor that causes slight difference in values for supplier and user costs when both slack times are to zero, is the difference in standard deviations of arrival distributions for the vehicle and passenger.

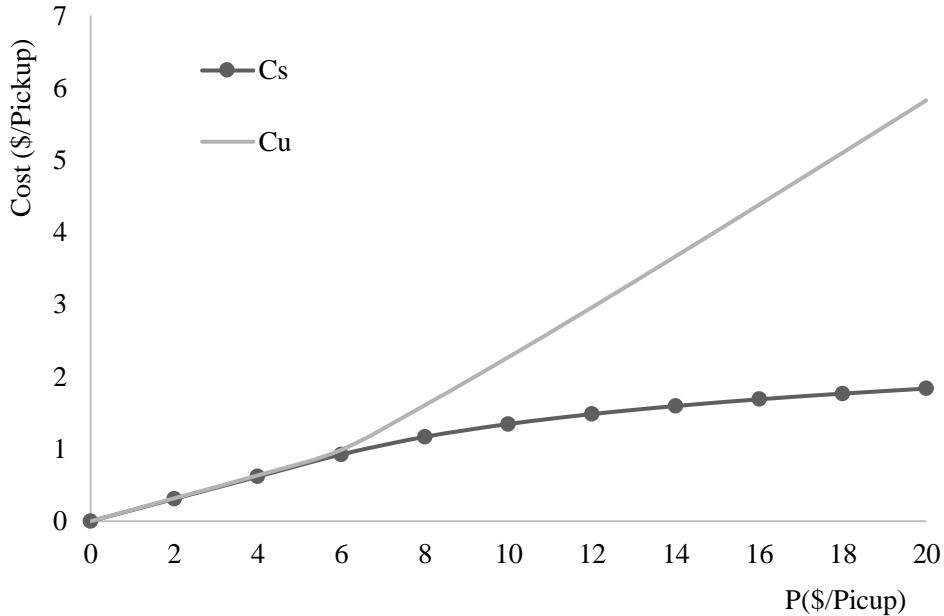


Figure 21: Minimum supplier cost and user cost vs the penalty

After  $P = 5.73$  \$/pickup, slack time for the passenger stays zero (as only the supplier cost function is being minimized) while the slack time for the vehicle is increasing to minimize the supplier cost by reducing the probability of missed pickup.

The user cost in figure 21, maintains its upward trend with an increase in slope after  $P = 5.73$  \$/pickup. The reason for this behavior is that an increase in slack time for the vehicle, decreases the probability of a missed pickup caused by driver while increasing the probability of a missed pickup caused by the passenger.

The same process is used to obtain the graph in figure 22 that displays the changes in slack time for passenger related to changes in penalty. This time, the user cost function is minimized for different penalties to obtain optimum slack times for passenger. Once again, the penalty changes from 0 \$/pickup to 20 \$/pickup.

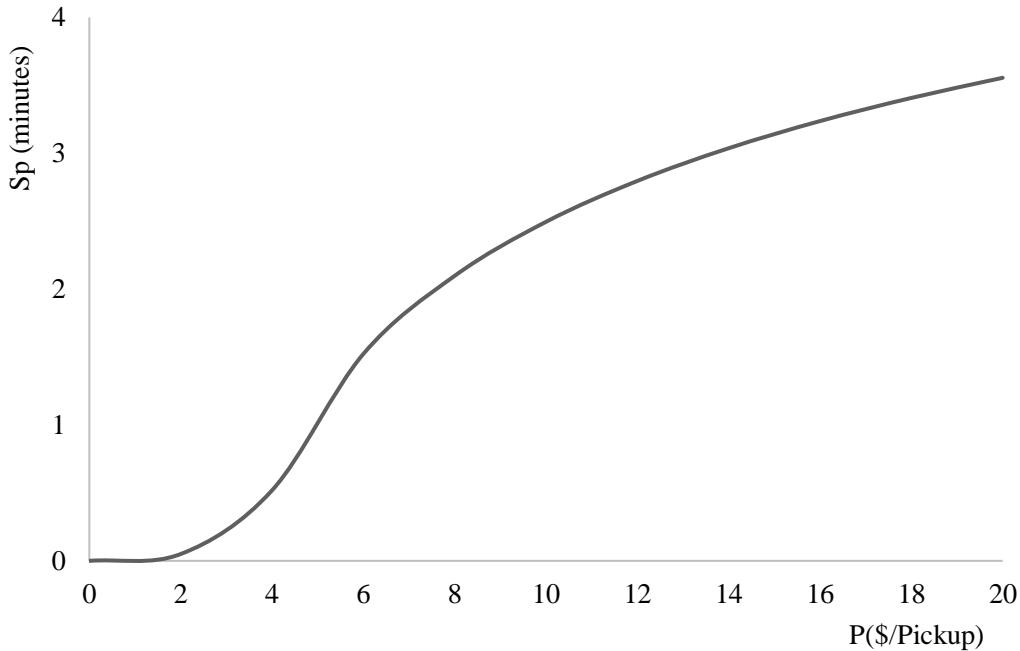


Figure 22: Optimum slack time for passenger vs the penalty

As shown in figure 22, at  $P = 3.4$  \$/pickup, the passenger will start to consider a slack time in the schedule to ensure a successful pickup. The slack time for passenger at this price is 0.004 minutes and as the penalty increase, the slack time for passenger also increase until it is about 3.55 minutes for 20 \$/pickup.

In comparing figure 20 and figure 22, we can conclude that, all other factors being equal, changes in penalty affect the passenger's more than driver (or rideshare company). If the penalty exceeds 3.4 \$/pickup, the passenger is willing to leave the house, office, etc. earlier to get to the pickup point while the driver (or operator) would only be willing to add a slack time to the schedule if the penalty is equal to or more than 5.73 \$/pickup.

As the penalty keeps increasing, both passenger and driver are willing to invest in a larger safety factor to arrive earlier and earlier and avoid a missed pickup. As expected, the penalty cost components in the supplier and user cost functions, ensure

that either driver or passenger or both will consider adding a slack time to their schedule to decrease the probability of having a missed pickup.

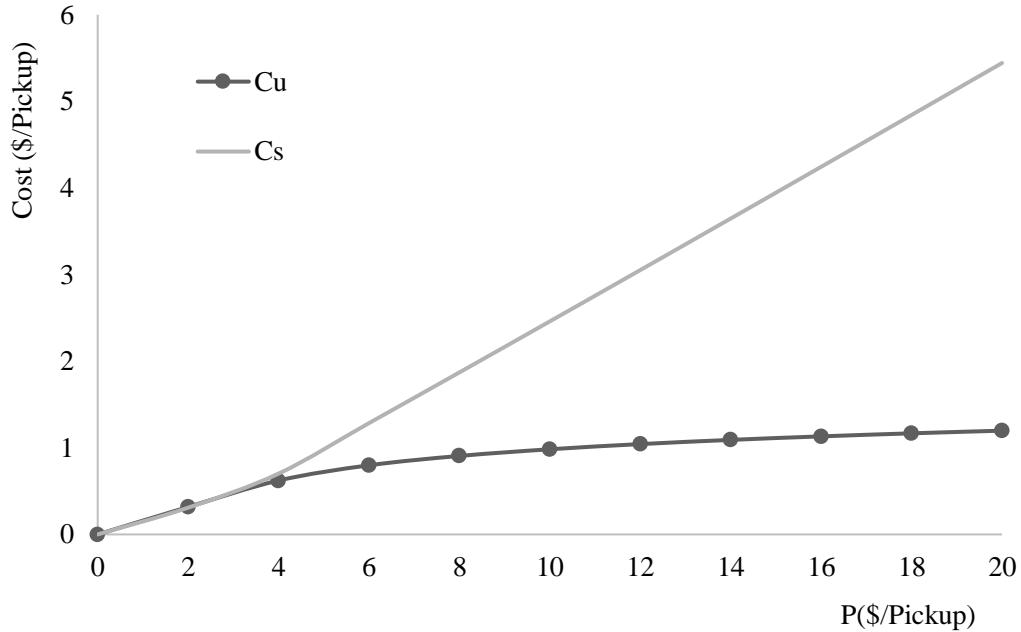


Figure 23: Minimum user cost and supplier cost vs the penalty

The minimum user costs and related supplier costs for different penalties are shown in figure 23. Similarly to behavior of cost functions shown in figure 21, until the point in which penalty is equal to 3.4 \$/pickup, both  $S_v$  and  $S_p$  are equal to zero and therefore the supplier and user costs are approximately equal.

After  $P = 3.4$  \$/pickup, slack time for vehicle stays zero (as only the user cost function is being minimized) while the slack time for passenger is increasing to minimize the user cost by reducing the probability of missed pickup.

### Optimal Slack Time vs the Standard Deviation of Arrivals

One important factor affecting the amount of slack time for stochastic arrivals, is the standard deviation of the arrival distribution. Similar to the discussion of effects of standard deviation on the optimal slack time in chapter 3, the effects of standard deviation of arrivals for vehicle and passenger on optimal slack time for vehicle is investigated in figure 24.

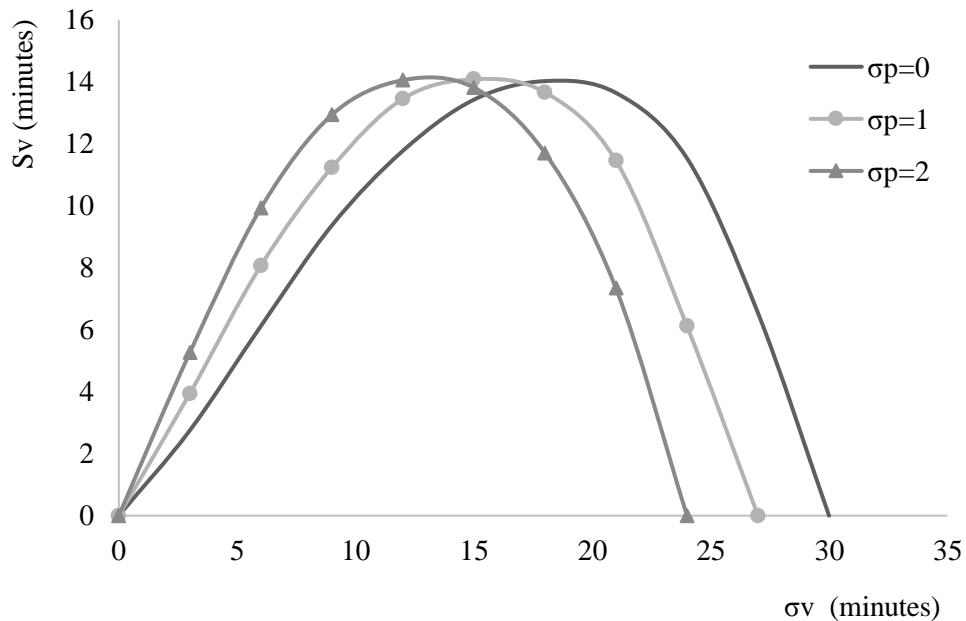


Figure 24: Optimal slack times for vehicle for various standard deviations of arrival times

The slope of the curves are defined by the slope of the normal distributions as the standard deviations for vehicle and passenger change. As shown in figure 24, increase in the standard deviation of vehicle's arrival, first increase and then decrease the optimal slack time.

Higher value for standard deviation of vehicle's arrival, contributes to higher probability of having a missed pickup (tail of the vehicle's arrival distribution), justifying a larger optimal safety factor. Beyond a certain standard deviation, the optimal safety factor is zero which indicates that the vehicle's arrival is very uncertain and investment in safety factor (slack time) for the vehicle's schedule is not economically justified.

Increase in the standard deviation of passenger's arrival also increase the overall uncertainty of the system. Therefore, the optimal slack time occurs for smaller standard deviation of vehicle's arrival and the critical standard deviation (for which the optimal slack time is zero), decreases as the standard deviation of passenger's arrival increases.

## **Chapter 6: CONCLUSIONS**

Slack time optimization for coordinating the connections between different transportation modes has been the topic of some recent studies. Mostly, these studies focused on coordinating an optimized transfer schedule to increase the reliability of transit systems.

As transit planners often consider adding a slack time to the schedule, in order to increase system stability, such slack (or safety factor) can be considered for a simple vehicle-passenger system as well. A major purpose of this study was to optimize the slack time for both passenger and driver in a real-time rideshare system. A great part of this research focused on the formulation of the supplier, user's and total system cost functions.

The behavior of the modeled cost functions was explored using numerical analysis. Model parameters such as slack times for vehicle and passenger, value of time for vehicle and passenger, maximum waiting time window, and penalty for missed pickups were the subjects of sensitivity analysis in chapter 5.

This study also investigates the probabilities and outcomes in scheduling a simple passenger-vehicle rideshare system. This simple system, is called real-time as in practice, an operator or an online application should run the schedule optimization each time there is a call for a pickup.

## **Formulation Process**

In the rideshare scenarios presented here, it is assumed that the passenger and the driver are both aiming to arrive at a designated pickup location at a scheduled pickup time. The final models for supplier cost, user costs and total system cost functions are based on probabilistic arrivals for both vehicle and passenger. If the passenger is already at the pickup location at the time of the request call, then the standard deviation for the arrival of passenger will be equal to zero and it will not affect the structure of the cost models.

To introduce the effects of having a designated slack time in a schedule for pickup, first a simple scenario is presented in which the passenger is already at the pickup location and the arrival of the driver at the pickup point follows normal distribution. The supplier cost function was introduced at this section which was further developed so that the arrival of the passenger at the pickup point was also probabilistic, following a normal distribution.

At this point, two slack times were incorporated in each cost model: one for vehicle and one for passenger. To simplify the calculation of probabilities, a chart of all the possible outcomes of the developed scenarios was presented in figure 4. All the subsequent formulations were based on this chart.

To further develop the supplier cost model, the user cost and the total cost functions were developed based on the structure of supplier cost function with reference to the chart in figure 4.

### **Analysis of Numerical Results and Sensitivity**

In this study, numerical analysis was used to test the cost functions and the sensitivity of slack times relative to changes in different model components. Given that the cost models in this study included several joint probabilities, model optimization was possible by means of empirical analysis or simulation methods. However, numerical analysis is less time-consuming compared to simulation methods which is a crucial quality for real-time systems, so numerical analysis method, is more practical for the purpose of this study.

The optimal slack time for passenger and vehicle are the results of tradeoffs among the components of the user's and supplier cost functions. Especially, the expected penalty cost component in both cost user's and supplier functions, is a determining factor as it ensures that the driver or passenger would consider investing a safety factor (slack time) in their schedule to avoid paying the penalty.

To further develop this study, a total cost function was introduced that consisted of additional slack time cost and the waiting cost components. The total cost components associated with waiting cost for vehicle and passenger, ensured that the driver and/or passenger would consider a slack time in their schedule to minimize the total system cost.

The sensitivity analysis of the supplier and user cost functions with respect to changes in value of time, waiting time window and the penalty, demonstrated the usefulness of the presented cost models.

According to the numerical results, from the supplier's point of view, adding a safety factor (slack time) to the schedule, decreases the waiting cost and the cost

associated with possible missed pickup, leading to decrease in the resulting supplier cost. With numerical analysis, it is also shown that after a certain value for slack time, the tradeoffs in the penalty cost function which yield the slack time become increasingly dominated by the operating cost and waiting cost functions. Beyond that point, increasing the vehicle's slack time is not cost effective.

From the user's point of view, adding a safety factor to the coordinated schedule leads to decrease in the probability of missed pickup. Investing in a safety factor to avoid the penalty cost, is not economically justified for the passenger after a certain point after which, the additional cost of slack time becomes dominant over the waiting cost and penalty cost.

In addition, we analyzed the costs from the point of view of the whole vehicle-passenger system. The numerical analysis of the total system cost model showed that at the point where slack time for vehicle is equal to zero, the total cost is non-zero because the costs associated with waiting times and slack time for passenger are non-zero. When the slack times for vehicle and passenger increased, the total cost function decreased to its minimum point and then started increasing. After the minimum total cost point, the tradeoffs between the additional cost of slack times and waiting cost did not justify increases in the slack times.

Numerical results show that changes in value of time also affect the optimal slack time for vehicle and passenger. When the vehicle's value of time is zero, the vehicle's slack time becomes equal to the largest amount necessary to avoid additional waiting cost but when the value of time is relatively high, adding more slack time to the vehicle's schedule is not preferable.

Analysis of the waiting time window showed that small waiting times for vehicle increase the penalty cost component of the supplier cost function. In this case, the penalty cost can be avoided if the passenger's slack time suffices to reduce the probability of an unsuccessful pickup. Beyond a certain amount for waiting time window, adding a safety factor to the vehicle's schedule is no more cost effective for the supplier.

In this study, the expected penalty cost was an effective component of both supplier and user cost functions which ensured reasonable safety factors in the vehicle's and passenger's schedules. Relatively high penalties made passenger and driver (or operator) more willing to invest in slack time while low values for penalty led to smaller optimal slack times.

The expected penalty cost affects the amount of optimal slack time; the user(s) and supplier(s) are then aware that they can reduce expected penalties by adding safety factors to their schedules. The effects of exploring penalties in a rideshare system we can obtain a deeper understanding of the users' and suppliers' responses to such incentives.

### **Policy Implication and Guidelines**

The main focus of this study was on exploring the benefits of including slack times in the schedules of drivers and passengers in a rideshare system. Optimized slack times in the schedules can increase the reliability of connections in the system and decrease the probability of missed pickups.

On the other hand, developments in mobile communications have enabled persons using an online rideshare system to have access to real-time information. We can now assume that the passengers can also estimate their travel time to the pickup location using the same rideshare application. We can also assume that the application can provide the users with a suggested slack time to avoid a missed pickup. The following are some benefits of an optimized rideshare schedule presented in this study:

- Reliable ridesharing and carpooling can appeal to more users and more commuters can depend on them for their daily trips. The relations developed in this study can significantly increase the reliability of rideshare system by coordinating drivers and passengers to avoid delays and no shows.
- The simple cost functions modeled based on probabilities in figure 4, showed reasonable behavior and no surprises. They can be used for optimizing slack times in more complicated schedule optimization models.
- The distributions of arrivals for the passenger and vehicle are both assumed to be normal in this study, but the developed models are flexible and other probability distributions can be used to investigate the costs associated with any connection among vehicles or modes.
- The method presented in this study is applicable when passengers take advantage of advanced scheduling for pickups. A rideshare system might provide some incentives for users to schedule their pickup with substantial lead time rather than requesting a ride with minimal advanced notice, on an as-needed basis.

### **Limitations of the Method**

The specific numerical analyses presented in this paper are limited to a few of the important relations between the variables in the cost models. Also, using probability distributions based on real-time data can significantly improve the behavior of the final cost models.

The dynamic characteristics of a simple ridesharing scenario are simplified in this study and a flowchart of different possibilities and outcomes is used to simplify the formulation of joint probabilities. As a result, while the model may be generally applicable to other mode transfer scenarios, some changes to probability functions are required to adjust the results.

### **Future Extensions**

The model presented in this study might be further improved by jointly considering multiple vehicles and multiple passengers per vehicle. Thus, additional passengers may wait downstream for pickup. The model might be developed further to include the choice of vehicle, passenger preferences, and different probability distributions, including empirical ones, for vehicle and passenger arrivals.

Although the rideshare model presented here is designed to deal with individual call for pickups, it can also be improved to deal with real-time decision making problems such as changes in routing, destination or preferred arrival time.

To further develop the model, more factors such as waiting time for upstream and downstream passengers, probabilistic headways (if another vehicle is called because of a missed pickup) and value of time for upstream and downstream

passenger(s) can be added to the cost models to apply in more complicated connection coordination scenarios.

A final and comprehensive real-time rideshare cost model can be combined with a matching algorithm for multiple passengers on different routes and multiple vehicles providing service in a complex network. Such comprehensive algorithm can be the basis of routing and scheduling systems for rideshare operators.

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