ABSTRACT

Title of dissertation: AGRICULTURAL POLICY AND PRODUCTION IN THE PRESENCE OF RISK AND INCOMPLETE FINANCIAL MARKETS

Daniel C. Voica, Doctor of Philosophy, 2017

Dissertation directed by: Professor Robert G. Chambers
Department of Agricultural and Resource Economics

Economic interventions are rarely free of debate, hence it should come as no surprise that governmental agricultural policies are usually surrounded in controversy. A topic of debate in the World Trade Organization (WTO) is how to maintain the fragile balance between two opposite objectives: the need of governments to protect their farmers and the need for a subsidy system which does not distort farmers production decisions.

Lump-sum transfers to farmers are commonly believed to affect the production choices of farmers in the presence of risk and uncertainty. Chapter 1 shows that if farmers have off-farm investment and employment opportunities, production decisions are decoupled from lump-sum subsidies in the presence of risk and uncertainty. The results are reconciled with existing results by showing that previously identified production adjustments are portfolio adjustments.

Chapter 2 contributes to the debate surrounding agricultural policy support for farmers and the potential distortionary effects of area payments. Area payments can
affect production decisions via land allocation. I show theoretically how the timing of these payments can weaken the link between area payments and production. The theoretical predictions are supported by the empirical findings.

Chapter 3 explores the trade-off between health and food consumption, and the effectiveness of health interventions such as taxing unhealthy foods. Rational agents maximize utility over health and consumption of healthy and unhealthy foods, while health is a function of discretionary and non discretionary calories and nutrients. Calories are not available for purchase in the market, thus their pricing is derived via a “household” production technology used to convert healthy and unhealthy foods into health outcomes. Additionally, consumers face a physiological constraint, a minimum calorie intake, which has further implications in terms of reducing potential health benefits associated with governmental interventions, such as taxing high-calorie foods. The future budget available to consumers depends on the consumption of discretionary calories. The theoretical model is calibrated using financial and consumption data reflecting farmworkers’s food consumption in the US.
AGRICULTURAL POLICY AND PRODUCTION IN THE PRESENCE OF RISK AND INCOMPLETE FINANCIAL MARKETS

by

Daniel Constantin Voica

Dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park in partial fulfillment of the requirements for the degree of Doctor of Philosophy 2017

Advisory Committee:
Professor Robert G. Chambers, Chair/Advisor
Distinguished University Professor Maureen L. Cropper
Assistant Professor Jorge Holzer
Distinguished University Professor Marc L. Nerlove
Professor Lars J. Olson
Foreword

The first chapter of the following dissertation is a jointly authored work with Robert G. Chambers. The Dissertation Committee acknowledges that Daniel C. Voica made substantial contributions to the relevant aspects of the chapter. The chapter was published in the American Journal of Agricultural Economics (2017), 99(3): 773-782 (doi:10.1093/ajae/aaw044).
Dedication

_Pentru parintii mei._
Acknowledgments

I am indebted to my adviser, Professor Robert G. Chambers, for his invaluable support and guidance through the process of thinking and writing this thesis. I will miss his unparalleled patience, dedication, humor, kindness and common sense. He always managed to impress on me his belief and optimism that I will succeed even when I doubted it.

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_Nu in cele din urma, parintilor mei, Mariana si Marcel Voica, le datorez spiritul si puterea care m-au ajutat sa ajung aici. Fara suportul si dragostea lor nimic nu ar fi fost possibil. In acelasi timp, bunicii mei, Irina si Mitica Giorcom, si Elena si Tanase Voica, reprezinta o sursa continua de inspiratie._

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<td>EU</td>
<td>European Union</td>
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<td>FADN</td>
<td>Farm Accountancy Data Network</td>
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<td>NAWS</td>
<td>National Agricultural Workers Survey</td>
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<td>Single Farm Payment</td>
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Chapter 1: “Decoupled” Farm Program Payments Are Really Decoupled: The Theory

1.1 Motivation

The traditional wisdom was that lump-sum agricultural subsidies do not distort production incentives. On this basis, concerted efforts were made to convert production subsidies to lump-sum payments. Just about the time that this “decoupling” started to occur, Hennessy (1998) showed that the presence of uncertainty could “recouple” lump-sum payments with production incentives. Two pathways, wealth effects and insurance effects, were identified through which recoupling might occur. Extensions, refinements, and alternative formulations of this recoupling result have been presented in Goodwin and Mishra (2005), Sckokai and Moro (2006), Serra et al. (2006), and Femenia, Gohin, and Carpentier (2010).

The intuitive story is that transferring income to farmers raises their initial wealth. If production activities are chosen to maximize expected utility of terminal wealth, wealth transfers can have several effects. First, they can change the decision maker’s marginal utility of income. This can alter risk attitudes and, thus, affect optimal choices. Second, raising initial wealth can relax existing credit constraints.
Finally, changes in initial wealth might affect on-farm, off-farm labor choices.

A large body of empirical work investigates the decoupling of existing policies and farmer decisions (among others, Burfisher, Robinson, and Thierfelder (2000); Goodwin and Mishra (2005) Goodwin and Mishra (2006); Schokai and Antón (2005); Schokai and Moro (2006); Serra et al. (2006); Femenia, Gohin, and Carpentier (2010); Just (2011); Weber and Key (2011)). In evaluating this empirical work, one need recognize that real-world “decoupling” takes different forms and rarely achieves the degree associated with lump-sum transfers. Instead, real-world “decoupled” policies often only partially sever the links between income support and production decisions. And yet, the empirical evidence suggests that coupling “...effects when measurable are small” (Bhaskar and Beghin (2009)).

This is familiar ground for agricultural economists. Theory suggests something is important. But observed farmer behavior does not. Of course, explanations exist for “decoupled” lump-sum transfers. Income effects are usually perceived as small if not negligible. Practical data and econometric limitations then could easily inhibit our ability to detect them even if present.

This paper adds another explanation. Under weak assumptions, \emph{lump-sum subsidies do not carry marginal wealth and insurance effects on production} and thus are truly decoupled. Make no mistake. Our claim is \emph{not} that existing policies are decoupled. Often they are not. Avenues exist (for example, adjusting base acres) through which program provisions affect production incentives. Rather, we show that \emph{the uncertainty-induced marginal adjustments to lump-sum transfers, first identified by Hennessy (1998), are portfolio adjustments and not production adjust-}
This is done in a framework that subsumes most existing analyses as special cases. Farmers generate income through on-farm activities, off-farm employment, and participation in competitive financial markets. Because a high percentage of farm income is from non-farm sources and modern farmers have access to well-organized futures markets, options markets, savings, and other non-farm investment opportunities, this is a realistic decision environment (Mishra and Morehart (2001); Brown and Weber (2013)).

We first introduce the model. A theorem characterizing optimal farmer behavior is then presented. It extends the Fisher separation theorem and separation results by Chambers and Quiggin (2009). Rational farmers, who as consumers prefer more to less and as producers face alternatives to farming, maximize discounted profits from their farming activities for state-claim prices beyond their control.1 Their marginal production decisions are driven by these state-claim prices, their stochastic technology, the stochastic output prices they face, and non-stochastic input prices, not by initial wealth levels. Production decisions are thus decoupled from either nonstochastic or stochastic lump-sum subsidies. Our analysis is then reconciled with existing work on decoupling under uncertainty. A final section concludes.

---

1A state-claim is an asset that pays one unit of numeraire if a particular state of Nature occurs. I am indebted to Professor Nerlove for encouraging me to clarify this concept.
1.2 The Model

The general set up follows Chambers (2007) and Chambers and Quiggin (2009). Farmers are competitive and face stochastic production and stochastic markets. There are two periods. The first period, $t$, is nonstochastic, and the second, $t + 1$, is stochastic. Uncertainty is represented by a finite state space $S$ indexed with a slight abuse of notation by $\{1, 2, ..., S\}$. Random variable space is thus the real vector space $\mathbb{R}^S$. Following the Savage tradition, for the random variable $f \in \mathbb{R}^S$, $f(s) \in \mathbb{R}$ denotes its outcome (consequence, ex post realization) in state $s \in S$. The set of probability measures is denoted by $\Delta \subset \mathbb{R}_+^S$.

Farmers use a stochastic production technology to produce $M$ stochastic outputs, $z \in \mathbb{R}_+^{S \times M}$, using labor employed on farm, $l_f \in \mathbb{R}_+$, and variable inputs, $x \in \mathbb{R}_+^N$. Inputs are chosen nonstochastically in period $t$ and are priced at $w \in \mathbb{R}_+^N$. The stochastic outputs are also chosen in period $t$ but realized or observed in period $t + 1$ after “Nature” makes a (unique) choice from $S$. Thus, if the farmer picks $z \in \mathbb{R}_+^{S \times M}$ in period $t$ and “Nature” chooses $s \in S$, the realized output vector in $t + 1$ is $(z_1(s), z_2(s), ..., z_M(s))$. The period $t + 1$ stochastic output prices are denoted $p \in \mathbb{R}_+^{S \times M}$. The farmer has a fixed endowment of labor, $L$, which can be allocated to on-farm labor, $l_f$, off-farm labor, $l_o$, and leisure, $l$. On-farm labor, off-farm labor, and leisure are all chosen nonstochastically in period $t$. The (period $t$) minimal variable cost of producing the stochastic output, $z$, is denoted $c(w, z, l_f)$ and is assumed convex in $(z, l_f)$.

Off-farm labor is compensated (stochastically) in period $t + 1$ at the rate of
$r \in \mathbb{R}^S_+$ per unit devoted to off-farm activities.\(^2\) Off-farm employment is traditionally viewed as a primary avenue by which farmers provide income assurance for their uncertain on-farm production activities. Typically, off-farm employment is treated as though its remuneration were non-stochastic. We treat the off-farm compensation as stochastic to allow for intertemporal vagaries in compensation schedules. Thus, a farmer engaging in off-farm employment effectively incorporates an asset with uncertain payoffs into his period $t+1$ income portfolio.

The main restrictions on the farmer’s *ex ante* preferences are that the preference functional be concave\(^3\) and that the farmer strictly prefers more period $t$ consumption to less, more period $t+1$ consumption to less, and more leisure to less. More formally, if the farmer’s *ex ante* preferences over period $t$ consumption, $q_t \in \mathbb{R}$, period $t+1$ stochastic consumption, $q_{t+1} \in \mathbb{R}^S_+$, and leisure are denoted by $W : \mathbb{R}_+ \times \mathbb{R}^S_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$, then

$$q^*_t > q_t \Rightarrow W (q^*_t, q_{t+1}, l) > W (q_t, q_{t+1}, l),$$

$$q^*_{t+1} \geq q_{t+1} \text{ but } q^*_{t+1} \neq q_{t+1} \Rightarrow W (q_t, q^*_{t+1}, l) > W (q_t, q_{t+1}, l),$$

and

$$l^* > l \implies W (q_t, q_{t+1}, l^*) > W (q_t, q_{t+1}, l).$$

The farmer can also transform period $t$ income into period $t+1$ consumption

\(^2\)Whether off-farm labor is compensated in period $t$ or period $t+1$ does not change our substantive results.

\(^3\)Concavity of $W$ and convexity of $c$ are assumed to ensure the existence of global solutions to the farmer’s problem. They are not strictly essential to our analysis and can be relaxed at the cost of a more complicated notation and argument.
by investing in financial markets. These markets include all financial assets available to farmers. These markets are stochastic, and the *ex ante* financial security payoffs are given by the \( S \times J \) matrix \( A \) (a matrix of \( J \) random variables). The stochastic payout on the \( j \)th financial asset is denoted \( A_j \in \mathbb{R}^S \), and its period \( t \) (nonstochastic) price is denoted \( v_j \). Thus, if state \( s \in S \) is realized the payout on the \( j \)th asset is \( A_j(s) \in \mathbb{R} \). The farm’s portfolio vector, corresponding to its period \( t \) holdings of the financial assets, is denoted \( h \in \mathbb{R}^J \). The linear sub space spanned by \( A \) is denoted \( \mathcal{M} \subset \mathbb{R}^S \) and defined by

\[
\mathcal{M} \equiv \{ y \in \mathbb{R}^S : y = Ah \}.
\]

Without any true loss of generality, \( A \) is assumed to be of full column rank, which is assumed to be strictly less than \( S \). Markets are incomplete, and there are risks that cannot be priced in the market. These risks fall in the orthogonal complement of \( \mathcal{M} \), which we denote by \( \mathcal{M}^\perp \). Define

\[
P \equiv (A^\top A)^{-1} A^\top.
\]

The government intervenes by providing a lump-sum subsidy of \( g_t \) in period \( t \) and a lump-sum, but stochastic, subsidy of \( g_{t+1} \in \mathbb{R}^S \) in period \( t + 1 \). The farmer’s
The period $t$ problem, therefore, is to choose $q_t, q_{t+1}, z, l_o, l_f, l$ and $h$ to

$$
\max \left\{ \begin{array}{l}
W(q_t, q_{t+1}, l) : Ah + p \cdot z + g_{t+1} + rl_o \geq q_{t+1}, \\
\omega_l (g_{t+1}) + g_t \geq q_t + c(w, z, l_f) + v^\top h, \\
L \geq l_o + l_f + l
\end{array} \right\},
$$

(1.1)

where $\omega_l (g_{t+1})$ with $\omega_l : \mathbb{R}^S \rightarrow \mathbb{R}$ is period $t$ wealth, $v^\top = (v_1, ..., v_J)$, $p \cdot z \in \mathbb{R}^S$ denotes the random revenue from farm production whose \textit{ex post} realization in state $s$ is $\sum_m p_m(s) z_m(s)$, where $p_m(s)$ is the \textit{ex post} realization of the $m$th stochastic output price. Period $t$ wealth is treated as a function of $g_{t+1}$ to account for the type of wealth effects emphasized by Femenia, Gohin, and Carpentier (2010). For simplicity, we restrict attention to interior solutions.

There are three constraints to optimization: budget constraints for periods $t$ and $t+1$, respectively,

$$
\omega_l (g_{t+1}) + g_t \geq q_t + c(w, z, l_f) + v^\top h,
$$

$\omega_l (g_{t+1}) + g_t \geq q_t + c(w, z, l_f) + v^\top h,$

and labor supply-demand balance

$$
L \geq l_o + l_f + l.
$$

The period $t$ budget constraint requires that pre-existing wealth plus the period $t$ lump-sum transfer from the government cover the farmer’s expenditures on period $t$
consumption, variable cost of production, and investment in financial markets. The period \( t + 1 \) budget constraint requires that income from all sources for each state \( s \in S \) be at least as large as consumption for that state.

Our model differs from previous analyses of decoupling under uncertainty (Hennessy (1998); Goodwin and Mishra (2005); Sckokai and Moro (2006); Serra et al. (2006); and Femenia, Gohin, and Carpentier (2010)) in several ways.\(^4\) Although assumptions vary, these models are closely related. Hence, for comparison purposes, we focus on the earliest, Hennessy (1998).

Hennessy (1998) works in a single period framework and models the farmer as a Sandmovian expected-utility of profit maximizer.\(^5\) In a single-period framework, period \( t \) and \( t + 1 \) consumption, income, and expenses all occur simultaneously. The parallel assumptions here are that \( q_{t+1} \) and \( q_t \) are perfect substitutes and that \( q_{t+1} \) is not subjectively discounted relative to \( q_t \). Assuming expected utility, problem (1.1) then becomes

\[
\max \left\{ \sum_{s \in S} H(s) u(q_{t+1}(s) + q_t, l) : Ah + p \cdot z + g_{t+1} + rl_o \geq q_{t+1}, \right. \\
\left. \omega_t(g_{t+1}) + g_t \geq q_t + c(w, z, l_f) + v^T h, L \geq l_o + l_f + l \right\},
\]

\(^4\)We thank an anonymous reviewer for suggesting a direct comparison of our model with earlier contributions.

\(^5\)Hennessy (1998) treats uncertainty in the form of a continuous random variable, \( \varepsilon \), with positive support on the interval \([a, b]\) and predetermined probability distribution, \( H(\varepsilon) \). The finite-state space framework, used here, can be converted to an infinite space framework by introducing the concepts of a \( \sigma \)-algebra of \( S \), an appropriate measure, and random variables as measurable functions of \( S \). Similarly, the notion of an inner product, \( x^T y \), of random variables \( x \) and \( y \) must be redefined to accommodate the chosen measure. That also necessitates a change in the derivative concept used with random variables. So, for example, the partial derivatives of \( c(w, z, l_f) \) that appear below need to be converted to a more general derivative concept such as either a Gateaux or Fréchet derivative. Chambers (2007) contains a discussion of such issues.
where $H \in \Delta$ is a subjective probability measure and $u : \mathbb{R}^2 \to \mathbb{R}$ is a strictly increasing utility function.

To convert (1.2) to expected-utility of profit terms, assume that $q_t$, $q_{t+1}$, and $l$ exhaust, respectively, the $t$ and $t+1$ budget constraints and labor-supply demand balance. Then

$$q_{t+1}(s) + q_t = \omega_t(g_{t+1}) + g_t + g_{t+1}(s) + r(s)(L - l) + \sum_m p_m(s)z_m(s) - c(w,z,l) - r(s)l_f + Ah(s) - v^\top h,$$

where $Ah(s)$ is the $s$th realization of $Ah \in \mathbb{R}^S$. The right-hand side of (1.3) is the sum of exogenously determined wealth, $\omega_t(g_{t+1}) + g_t + g_{t+1}(s)$, and the $s$th realization of the stochastic ”profit”,

$$r(s)(L - l) + \sum_m p_m(s)z_m(s) - c(w,z,l_f) - r(s)l_f + Ah(s) - v^\top h.$$

Expression (1.2), after substituting from (1.3), parallels Hennessy (1998). The key difference is that Hennessy (1998) represents (1.4) in reduced form as a function, $\pi : S \times \mathbb{R} \to \mathbb{R}$, of the realized $s$ and a choice variable, $\alpha$ so that (1.3) is replaced by

$$q_{t+1}(s) + q_t = \omega_t(g_{t+1}) + g_t + g_{t+1}(s) + \pi(\alpha, s).$$

\footnote{See his expression (1) and the surrounding discussion.}
1.3 Equilibrium Behavior and The Result

Our main result extends Chambers and Quiggin’s (2009, Theorem 6) separation results to the current setting:

An interior optimal solution to (1.1) satisfies

\[
\max_{q_{t+1}, l} \left\{ W(\omega_t(g_{t+1}) + g_t + v^\top P g_{t+1} + v^\top P (r(L-l) - q_{t+1}) \right.

\left. + \Gamma(p, w, r; v^\top P), q_{t+1}, l) \right\},
\]

where

\[
\Gamma(p, w, r; v^\top P) \equiv \max_{z, l_f} \left\{ v^\top P (p \cdot z - rl_f) - c(w, z, l_f) \right\}.
\]

See Appendix A.

Farmers have three ways to transfer wealth, and thus consumption, between periods \( t \) and \( t+1 \). One is to invest period \( t \) resources to finance production. That generates a stochastic period \( t+1 \) revenue of \( p \cdot z \). Another is to invest in financial markets to create a period \( t+1 \) stochastic asset, \( Ah \). Finally, off-farm work yields stochastic wage earnings \( rl_o \).

For the purposes of consumption, whether period \( t+1 \) income comes from
production, investment, or wage earnings is irrelevant. Income from all three sources is equally useful in covering $q_{t+1}$. Therefore, once a particular $q_{t+1}$ is targeted, the remaining task is to generate it as cheaply as possible. That ensures maximal residual buying power for period $t$ consumption. The problem is akin to that of a multiplant producer, who must allocate output production across plants for ultimate sale at a common market price. Instead of choosing output allocations, our farmer chooses an optimal mix of activities for generating period $t+1$ income.

Because $W$ is strictly monotonic in $q_{t+1}$, $q_t$, and $l$, the farmer never foregoes a chance to raise any of the three. Thus, all available period $t+1$ income, period $t$ income, and available leisure must be consumed. Formally, that requires

\begin{align*}
q_{t+1} &= p \cdot z + Ah + g_{t+1} + rl_o, \\
q_t &= \omega_t (g_{t+1}) + g_t - c(w, z, l_f) - v^\top h, \quad \text{and} \\
l &= L - l_f - l_o.
\end{align*}

(1.6)

Rewriting the first equality in (1.6) as

\[ Ah = q_{t+1} - p \cdot z - g_{t+1} - rl_o \]

shows that $Ah$ must exactly cover the difference between $q_{t+1}$ and income from production, off-farm work, or government transfers. That means that $q_{t+1} - p \cdot z - g_{t+1} - rl_o$ must fall in $\mathcal{M}$.

If $(q_{t+1}, z, l_o)$ are fixed at their optimal levels, the farmer necessarily chooses
optimal \( h \) to solve

\[
\min \{ v^\top h : Ah = q_{t+1} - p \cdot z - g_{t+1} - rl_o \}.
\]

If the farmer chose \( h \) otherwise, an unexploited opportunity to increase consumption in period 0 without altering \( q_{t+1} \) or leisure would exist. A rational farmer would never fail to exploit such an arbitrage opportunity. The unique solution to this portfolio minimization problem is\(^7\)

\[
h = P (q_{t+1} - p \cdot z - g_{t+1} - rl_o).
\]  \hspace{1cm} (1.7)

Thus, the farmer’s period \( t \) marginal valuation of \( q_{t+1} - p \cdot z - g_{t+1} - rl_o \) is \( v^\top P(q_{t+1} - p \cdot z - g_{t+1} - rl_o) \). The valuation operator converting period \( t + 1 \) units into period

---

\(^7\)Premultiply both sides of the constraint equality by \( A^\top \) to obtain

\[
A^\top Ah = A^\top (q_{t+1} - p \cdot z - g_{t+1} - rl_o).
\]

Solving gives

\[
h = (A^\top A)^{-1} A^\top (q_{t+1} - p \cdot z - g_{t+1} - rl_o).
\]

The assumption that \( A \) is of full column rank ensures that \( A \) provides a basis for \( \mathcal{M} \), and thus that \( (A^\top A)^{-1} \) is invertible.

Here an analogy with the ordinary linear regression model may be useful. If one were to write that model as

\[
y = Ah + \varepsilon
\]

where \( \varepsilon \in \mathbb{R}^S \) is a random variable orthogonal to \( A \), the least squares estimator of \( y \) would be

\[
\hat{y} = A (A^\top A)^{-1} A^\top y,
\]

which is the orthogonal projection of \( y \) onto the subspace spanned by \( A \), our \( \mathcal{M} \). The least squares estimator of \( h \), in turn, would be

\[
\hat{h} = (A^\top A)^{-1} A^\top y,
\]

which gives the portfolio that yields the element of \( \mathcal{M} \) that is closest (in the sense of the usual Euclidean metric) to \( y \).

In the case at hand, \( y \) corresponds to \( q_{t+1} - p \cdot z - g_{t+1} - rl_o \), and because it belongs to \( \mathcal{M} \), \( \varepsilon \) is now 0 which is naturally orthogonal to \( A \).
t units is the *ideal stochastic discount factor* (pricing kernel) $v^\top P \in \mathcal{M}$, which is obviously linear.\(^8\)

Using the labor supply-demand balance condition to solve for $l_o$ and substituting into (1.7) gives

$$h = P (q_{t+1} - p \cdot z - g_{t+1} - r (L - l_f - l)),$$

and finally substitution in the period $t$ budget constraint yields after rearranging

$$q_t = \omega_t (g_{t+1} + g_t + v^\top P (g_{t+1} - q_{t+1}) + (L - l) v^\top P r + [v^\top P (p \cdot z - rl_f) - c(w, z, l_f)].$$

(1.8)

The right-hand side of (1.8) gives period $t$ resources available for period $t$ consumption. There are three sources: pre-existing wealth plus the period $t$ subsidy and the stochastically discounted difference between the period $t + 1$ subsidy and $q_{t+1}$,

$$\omega_t (g_{t+1}) + g_t + v^\top P (g_{t+1} - q_{t+1});$$

stochastically discounted income from working, $(L - l) v^\top P r$; and stochastically discounted profit from farming, $[v^\top P (p \cdot z - rl_f) - c(w, z, l_f)].$

Preferences do not depend directly upon either $z$ or $l_f$. They only depend directly upon $q_t$, $q_{t+1}$, and $l$. Thus, all else equal, the farmer chooses $z$ and $l_f$ to ensure

\(^8\)Note that $0 \in \mathcal{M}$ for $0 \in \mathbb{R}^S$. Thus, even if a rational farmer chooses

$$q_{t+1} = p \cdot z + g_{t+1} + rl_o,$$

her excess demand falls in the market span and is properly priced there.
as much \( q_t \) as possible. This requires maximizing \( v^\top P (p \cdot z - rl_f) - c(w, z, l_f) \), which is Theorem 1.3.

Treating the special case where \( W \) and \( c \) are nicely smooth may help clarify.

For the concentrated objective function,

\[
\max_{q_{t+1}, l, l_f, z} \{ W(\omega_t(q_{t+1}) + g_t - c(w, z, l_f) - v^\top P(q_{t+1} - p \cdot z - g_{t+1} - r(L - l - l_f)), q_{t+1}, l) \},
\]

the first-order necessary conditions for an interior solution are:

\[
\begin{align*}
\frac{\partial W}{\partial q_t} \left( v^\top P(s) p_m(s) - \frac{\partial c(w, z, l_f)}{\partial z_m(s)} \right) &= 0, \quad m = 1, \ldots, M, s \in S, \\
\frac{\partial W}{\partial q_t} \left( -v^\top P - \frac{\partial c(w, z, l_f)}{\partial l_f} \right) &= 0, \\
\frac{\partial W}{\partial q_{t+1}(s)} - \frac{\partial W}{\partial q_t} v^\top P(s) &= 0, \quad s \in S, \\
\frac{\partial W}{\partial l} - \frac{\partial W}{\partial q_t} v^\top P &= 0.
\end{align*}
\]

Strict monotonicity in \( q_t \) ensures that these are equivalent to

\[
\begin{align*}
v^\top P(s) p_m(s) - \frac{\partial c(w, z, l_f)}{\partial z_m(s)} &= 0, \quad m = 1, \ldots, M, s \in S, \\
-v^\top P - \frac{\partial c(w, z, l_f)}{\partial l_f} &= 0, \\
\frac{\partial W}{\partial q_{t+1}(s)} / \frac{\partial W}{\partial q_t} - v^\top P(s) &= 0, \quad s \in S, \\
\frac{\partial W}{\partial l} / \frac{\partial W}{\partial q_t} - v^\top P &= 0.
\end{align*}
\]

The first \( S \times M + 1 \) conditions in (1.9) are the first-order conditions for a profit
maximizer facing prices $v^\top P(s) p_m(s)$ for $z_m(s)$, input prices $w$ for the variable inputs, and an opportunity cost of $v^\top Pr$ for on-farm employment. The last $S + 1$ conditions in (1.9) are the first-order conditions for an individual solving his or her portfolio problem and facing Arrow state-claim prices given by $v^\top P(s), s \in S$ and off-farm employment rewarded at $v^\top Pr$.9,10

Because the first $S \times M + 1$ conditions are independent of $q_{t+1}, \omega_t (g_{t+1}), g_{t}, l,$ and $g_{t+1}$, the system can be solved recursively. First, equate the marginal costs of each state-contingent output to its respective $v^\top P(s) p_m(s)$ and the shadow price of on-farm labor to $-v^\top Pr$ to determine optimal $z$ and $l_f$. Then, use the parametrically determined $v^\top P$ and $-v^\top Pr$, along with $\Gamma(p, w, r; v^\top P)$ to solve the consumption problem. Because the farmer reacts to $v^\top P$ as both a producer and a consumer, the financial market acts as an intermediary to separate the production decision from the consumption decision. The consumption decision depends upon risk preferences, as captured by $\frac{\partial W}{\partial q_{t+1}(s)} / \frac{\partial W}{\partial q_t}$ and $\frac{\partial W}{\partial l} / \frac{\partial W}{\partial q_t}$. The production decision does not.

That agricultural production decisions, $z$, do not respond to marginal changes in $(g_t, g_{t+1})$ now follows. Theorem 1.3 actually establishes a stronger result. Not only are agricultural production decisions decoupled at the margin from $(g_t, g_{t+1})$, they are also invariant to marginal changes in initial wealth $\omega_t (g_{t+1})$.

9Because $v^\top P$ is a stochastic discount factor, the first $S \times M + 1$ conditions in (1.9), after manipulation, can also be interpreted as the first-order conditions for maximizing expected profit for a probability measure determined by $v^\top P$.

10An Arrow state price is the price of a state-claim. I am indebted to Professor Nerlove for encouraging me to clarify this concept.
1.4 Reconciliation and Generalization

To reconcile earlier analyses in an expected-utility framework with ours, note that strict monotonicity of \( u \) ensures that (1.8) holds. Completely parallel arguments then reveal that interior solutions to the expected-utility maximizer’s problem must satisfy:

\[
\max_{l, q_{t+1}} \left\{ \sum_{s \in S} H(s)u(q_{t+1}(s)) - v^\top Pq_{t+1} + (L - l) v^\top Pr + \omega_t(q_{t+1}) + g_t + v^\top Pg_{t+1} + \Gamma(p, w, r; v^\top P, l) \right\},
\]

which requires choosing \( l \) and \( q_{t+1} \) to maximize the expected utility of stochastic profit.\(^{11}\) “Stochastic profit” consists of three components: the realized value of \( q_{t+1} \) less its period \( t \) valuation, \( q_{t+1}(s) - v^\top Pq_{t+1} \); the period \( t \) valuation of labor not devoted to leisure, \( (L - l) v^\top Pr \); and \( \omega_t(q_{t+1}) + g_t + v^\top Pg_{t+1} + \Gamma(p, w, r; v^\top P) \), period \( t \) discounted profit, government subsidies, and pre-existing wealth. The first and the second terms depend upon \( q_{t+1} \) and \( l \), respectively. The final does not.

Now compare (1.10) with (1.5). Hennessy’s (1998) core “decoupling” result is that the optimal choice of \( \alpha \) for \( \pi(\alpha, s) \) typically depends upon \( g_t + v^\top Pg_{t+1} \).\(^{12}\) This

\(^{11}\)Residual is likely more apt than profit, but our expositional purposes are best served by the latter terminology.

\(^{12}\)By invoking basic supermodularity theory, Hennessy (1998) also signs the direction of the effect. Once the direction of change for \( \alpha \) is determined, the model specification determines whether the result is a first-order stochastic dominant shift in stochastic profit.

While important, these results are not germane to the point at issue. But such results are readily available for \((q_{t+1}, q_t, l)\) in our model by imposing proper supermodularity conditions on \( W(q_{t+1}, q_t, l) \).
is undoubtedly correct. It manifests the well-known result that wealth endowments affect optimal portfolio choices (Arrow 1965).

The fly-in-the-ointment is the inference drawn from this observation. Because $\alpha's$ optimal value depends upon $g_t + v^T P g_{t+1}$, the presumption was that production choices must as well. But Theorem 1.3 as well as (1.10) confirm that optimal $z$ are characterized by $\Gamma (p, w, r; v^T P)$, which is independent of $g_t + v^T P g_{t+1}$. As the last two expressions in (1.9) show, what are not decoupled from lump-sum income-support are the decisions for $q_{t+1}$ and $l$. Farmers derive no direct value from $z$. It is produced to be sold to finance consumption of $q_t$. As (1.10) shows, that decision is captured by $\Gamma (p, w, r; v^T P)$. The wealth and insurance effects identified by Hennessy (1998) are real. But they occur for $q_t$, $q_{t+1}$, and $l$, and not $z$.

Working in a nonstochastic, two-period framework with perfect foresight using an extension of the life-cycle model of consumption adapted to accommodate agricultural production, Phimester (1995) has suggested that direct payments can affect production incentives by relaxing credit constraints.\footnote{We thank an anonymous reviewer for drawing our attention to this paper.} His setting is different from ours and that studied by Hennessy (1998), Goodwin and Mishra (2005), Sckokai and Moro (2006), and Serra et al. (2006). To investigate the effects of credit constraints in the latter, we must first establish the mechanism via which they might occur. There are two budget constraints. The one relevant to credit constraints is that for period $t$:

$$\omega_t (g_{t+1}) + g_t \geq q_t + c (w, z, l_f) + v^T h.$$
At present, no domain restrictions have been placed on either $A$ or $h$. Thus, for example, if all $A_j$ were restricted to $\mathbb{R}^S_+$, borrowing today against future income would be associated with choosing elements of $h$ to be negative (short selling). Thus, our analysis clearly permits borrowing to finance period $t$ production expenses, and no constraints (beyond budgetary) have been placed on access to credit.

To ensure credit constraints have maximal bite, we investigate the polar case where current-period borrowing against future income is prohibited. To that end, assume that $A \in \mathbb{R}^{S \times J}_+$ and $h \in \mathbb{R}^J_+$. The first ensures that only positive payouts are permissible in $t+1$ from investment activities. The second (no short selling) ensures that the decision maker cannot effectively borrow by selling a claim against future income. Together the assumptions guarantee two things: credit constraints for financing agricultural production exist, and increasing direct payments relaxes those constraints.

The reformulated version of our model is now.

$$
\max \begin{cases}
W(q_t, q_{t+1}, l) : Ah + p \cdot z + g_{t+1} + rl_o \geq q_{t+1}, \\
\omega_t (g_{t+1}) + g_t \geq q_t + c(w, z, l_f) + v^\top h, \\
L \geq l_o + l_f + l, \\
L \geq l_o + l_f + l, \\
h \in \mathbb{R}^J_+.
\end{cases}
$$

(1.11)

Now suppose that $h$ has been chosen optimally for (1.11). Because the farmer’s preferences are strictly increasing in $q_{t+1}$, it must remain true that $q_{t+1}$ consumes
all income generated regardless of which \( s \in S \) is realized. That is,

\[
q_{t+1} = Ah + p \cdot z + g_{t+1} + rl_o,
\]

but for an interior solution this requires as before

\[
Ah = q_{t+1} - p \cdot z - g_{t+1} - rl_o,
\]

so that, despite the domain restriction on \( h \), \( q_{t+1} - p \cdot z - g_{t+1} - rl_o \) still must fall in \( \mathcal{M} \) and be price there. The argument proceeds as before.

The essential point is that requiring \( h \in \mathbb{R}^J_+ \) does not prevent the farmer from interacting with the market. Rather it restricts the form that interaction can take. So, for example, the farmer can still deposit some of \( \omega_t (g_{t+1}) + g_t \) in a savings account to earn interest to finance \( q_{t+1} \). Stocks could be purchased. Before \( q_{t+1} - p \cdot z - g_{t+1} - rl_o \) could fall anywhere within \( \mathcal{M} \). Now it must fall in

\[
\mathcal{M}_+ = \{ z : z = Ah, h \in \mathbb{R}^J_+ \}.
\]

But, clearly, \( \mathcal{M}_+ \subset \mathcal{M} \) so that \( q_{t+1} - p \cdot z - g_{t+1} - rl_o \) still remains in \( \mathcal{M} \) and thus must be accurately priced there.
1.5 Concluding Remarks

This paper reconsiders whether lump-sum income support policies are, in fact, decoupled from production decisions under uncertainty. We find that they are. This contrasts markedly with received wisdom. Naturally, our analysis depends crucially upon our maintained assumptions. As is usual in economic analysis, we operate in a stylized setting and to the extent that stylized setting departs from reality, so may real-world results. But that criticism necessarily applies to all previous studies.

We model a farmer operating a sole-owner operation who, in the absence of government subsidies, generates consumption income via three sources: stochastic production activities, stochastic investment opportunities, and participation in a labor market with uncertain rewards. This farmer’s preferences are strictly monotonic in non-stochastic current period consumption, stochastic future consumption, and leisure consumed. The key assumption is that of a single residual claimant with strictly monotonic preferences with access to off-farm income opportunities.

Virtually all the conceptual studies cited maintain a single residual claimant with monotonic increasing preferences facing stochastic production. Thus, the fundamental departure from existing work is the presence of off-farm income opportunities. Existing studies have either ignored these opportunities or embedded them in reduced-form models. Adding structure identifies a previously overlooked avenue for market valuation of income opportunities. The added structure is very plausible. Indeed, it is hard to envision a farmer in the developed world who does not participate in financial markets or receive income from nonfarm sources. For exam-
ple, Pope, LaFrance, and Just (2011) explore the application of portfolio theory to the estimation of risk preferences in agriculture. A vast literature investigates how farmers can properly exploit opportunities in futures and off-farm labor markets. But if a rational farmer exploits these opportunities, the farmer is also bound by their accompanying market disciplines (for example, the law of one price (LeRoy and Werner 2001)). And these market disciplines ensure, at the margin, that the farmer faces parametrically determined state-claim prices for on-farm labor and production. On the other hand, if these opportunities do not exist, farmers do not face parametrically determined state-claim prices and production decisions will not be decoupled from lump-sum transfers.

The analysis does not imply that lump-sum subsidies are non-distortionary under uncertainty. As noted, the effects isolated by Hennessy (1998) are real. But marginal consumption and leisure choices are the ones affected and not marginal production choices. In a general-equilibrium setting, those consumption and leisure effects can impinge upon other economic choices and thus upon the givens of our model \((p, w, A, v, r)\). That, in turn, could induce changes in \(z\). But because such effects are second-order, econometric attempts to measure their marginal impact on production may well fail.
Chapter 2: The Effect of the Single Farm Payment Timing on Production Incentives

2.1 Motivation

In 2003, the European Union (EU) introduced the Single Farm Payment (SFP) scheme, a per hectare subsidy that is independent of production level and crop planted. The SFP was primarily motivated as a means to decrease distorted production incentives. The new area payments were expected to fulfill two previously mutually exclusive roles. First, it would continue to augment farmers’ income, which is acknowledged as a political dictate. Second, production distortions induced by previous interventions would be reduced, if not eliminated.

If land supply is perfectly inelastic, area payments act as lump-sum transfers to landowners with no distortionary effects (Just, Hueth, and Schmitz (2004)). This remains true even in the presence of uncertainty (Chambers and Voica (2016)). But land supply, although typically inelastic, is not perfectly inelastic. Area payments encourage land use and likely result in increased acreage.

Previous empirical work has found that although area payments are distortionary, the impact is small. Acreage tends to be affected more than the output (see
among others, (Goodwin and Mishra, 2005), Sckokai and Antón (2005), Sckokai and Moro (2006), Bhaskar and Beghin (2009), Weber and Key (2011)). This suggests that the “coupling” link between area payments and production, while present, is small as well.

One possible explanation is that area payments are thoroughly capitalized into land values, benefiting landowners rather than producers. While plausible, previous empirical works suggest that the incidence of subsidy payments favor tenants rather landowners. For example, Kirwan (2009) reports that tenants capture 75% of the subsidy in the US.

Another potential explanation relates to the timing of the area payments. The SFP scheme stipulates that area payments are to be made to farmers between December 1st of the current calendar year and June 30th of the following year (Europe, 2011). That means that farmers receive the payments after all, or at least part, of their production decisions are made. The timing adds an additional intertemporal component to the decision environment producers face. Unless there is no intertemporal discounting and consumption today and consumption tomorrow are perfect substitutes, the timing of the payments may affect production decisions. Area payments create new production incentives. But the timing of those payments could temper those incentives especially if producers have limited ability to transfer income between time periods.

Attempts to measure time preferences have revealed that farmers have average discount rates as high as 34% Duquette, Higgins, and Horowitz (2011), which is consistent with discount rates estimates found in other studies (Coller and Williams
This paper examines theoretically and empirically how the timing of the SFPs affect agricultural production incentives. It is organized as follows. First, a theory of how timing affects the coupling between area payments and production is presented. Then the timing effect is estimated using several years of data from Romanian EU Farm Accountancy Data Network (FADN). A final section concludes.

2.2 Theoretical Model

There are three time periods. The first period (the decision period), 0, involves no uncertainty. The second and the third periods, 1 and 2, are uncertain. Uncertainty is modeled by a finite state space, described by a finite set, Ω, where each element of Ω, referred to as a state, is a complete and mutually exclusive description of the world. ¹ For example, in a two-states representation of the world, a state could be “rain” and another could be “no rain”. Uncertainty is resolved by Nature, choosing from Ω. That choice, however, is only revealed to the farmer after the farmer’s choices have been made in period 0. ²

¹The theoretical framework used here is the state-contingent approach to uncertainty. Although most treatments within this framework assume that agents with subjective probability beliefs and Von Neumann-Morgenstern preferences maximize expected utility, additive separability is unnecessary in my application and in many others. Indeed, the classical treatment due to Debreu (1959) makes no use of the expected utility hypothesis and subjective probability. As in my case, an agents preferences are defined over the vector of dated, state-contingent consumptions. An accessible treatment to the state-contingent approach is Chambers and Quiggin (2000).

²To interpret later results in terms of expectations, I assume that agents have well defined subjective probability vectors over the realization of the states of the Nature.
Preferences over consumption in the three periods, $k^0 \in \mathbb{R}_+$, $k^1 \in \mathbb{R}^2_+$, and $k^2 \in \mathbb{R}^2_+ \setminus \mathbb{R}^2_+ \setminus \mathbb{R}^2_+$ are continuous and strictly increasing in each argument, and represented by $W(k^0, k^1, k^2)$. The initial wealth endowment, $\omega > 0$, is nonstochastic. In period 0, the farmer can undertake production and financial activities that generate state-contingent income in periods 1 and 2. Production is characterized by a stochastic technology. In period 0, the farmer chooses the level of state-contingent period 2 output $z \in \mathbb{R}^S_+$ and the amount of land devoted to farming, $l$. The associated variable cost is $c(w, z; l)$ where $w \in \mathbb{R}^N_+$ is the vector of variable input prices in period 0. Cost is assumed to be convex in both $z$ and $l$. There is a rental market for farm land which pays in period 0 the rental rate $r$. The farmer is endowed with $L$ units of land.

The farmer can also buy and sell assets in financial markets. In period 0, the farmer can purchase two types of financial assets, one that pays off in period 1 and $J$ that pay off in period 2. The asset paying off in period 1 sells for a period 0 price of $v_1 > 0$ and pays off $A \in \mathbb{R}^{++}$ for each unit of the asset purchased. The number of units of this asset purchased in period 0 is denoted $h_1$. Since the payoff from this asset will be the consumers only source of income in period 1 whenever the subsidy happens to be delayed, he cannot sell this asset (borrow short term) in period 0 and repay the loan in period 1 since he will lack the wherewithal to pay. Hence, $h_1 \geq 0$.

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3 Readers are free to specialize $W$ to additively separable (following Von Neumann-Morgenstern) scaling functions weighted by subjective probabilities. They can further restrict the scaling functions to concave functions reflecting risk aversion; but such assumptions are unnecessary for my results.

4 $S$ is a subset of $\Omega$. As such, $S$ represents uncertainty corresponding to period 2, while $\Omega$ represents uncertainty corresponding to periods 1 and 2.

5 I assume Inada conditions hold.

6 For an axiomatic study of cost functions see Chambers and Quiggin (2000).

7 To preserve simplicity, I model the period 1 financial market as riskless. A more complex
The period 2 payoffs for the $J$ assets are given by the payoff matrix $D \in \mathbb{R}^{S \times J}$ and the period 0 price of the $j$th asset is denoted $v_{2j}$. The portfolio vector for the assets paying off in period 2 is denoted $h_2 \in \mathbb{R}^J$. The agent can sell asset $j$ in period 0 in exchange for his making state-contingent payments in period 2. It is assumed that $D$ is of full column rank and that $J < S$.

The government pays the farmer a fixed subsidy $a$ per unit of land $l$ either in the second period, or in the third period, but not in both. The timing of this payment is only revealed to the farmer in the second period, after the farmer has made the land-allocation decision. Thus, from the farmer’s perspective in period 0 its timing is stochastic.

Figure 2.1 - inserted here.

In the second period, the farmer receives the payoff $Ah_1$ plus the subsidy $al$, if the subsidy is paid in period 1 and $Ah_1$ if the subsidy is paid in period 2. Thus, the farmer state-contingent consumption is either $k^1_E = Ah_1 + al$ or $k^1_L = Ah_1$, where subscript $E$ denotes an “early” subsidy payoff and $L$ a “late” subsidy payoff.

In the third period, the farmer receives the revenue from farming $p_s z_s$, where $p_s$ is the output price in state $s \in S$, the revenue from financial markets participation, $D_s h_2$, where $D_s \in \mathbb{R}^J$ is the vector of assets payoffs in state $s$, and the subsidy $al$ if the subsidy was not paid in period 1.\(^8\)

Because there are $S$ possible realizations of the output $z$ (i.e. $z_s \in \mathbb{R}_{++}, s \in S$) financial market for period 1 could be modeled, but this would bring notational clutter without substantially changing results.

\(^8\)The row vector $D_s = (D_{s1}, D_{s2}, \ldots, D_{sj})^T$, where $D_{sj}$ is the payoff of $j$-th asset in state $s$. Later, I introduce the column payoffs vector $D_j \in \mathbb{R}^S$, the payoffs vector of the $j$-th asset, where $D_j = (D_{1j}, D_{2j}, \ldots, D_{sj})$. 

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{1, \ldots, S} \)), and two possible realizations of subsidy payments (i.e. late or early), the total number of states of Nature in the third period is $2S$ (i.e. the dimension of $\Omega$ is $2S$). For example, $k^2_{sE} = p_s z_s + D_s h_2$ is the consumption in state $s$ given the subsidy is paid in the second period, and $k^2_{sL} = p_s z_s + D_s h_2 + al$ is the consumption in state $s$ given the subsidy is paid in the third period. Figure 2, illustrates the timing of the subsidy payments to the farmer.

Figure 2.2 - inserted here.

The farmer’s period 0 problem is to choose $k^0 \in \mathbb{R}_{++}$, $k^1 = (k^1_E, k^1_L) \in \mathbb{R}^2_{++}$, $k^2 = (k^2_{1E}, \ldots, k^2_{SE}, k^2_{1L}, \ldots, k^2_{SL}) \in \mathbb{R}^{2S}_{++}$, $z \in \mathbb{R}^S_+$, $l \in \mathbb{R}_{++}$, $h_1 \in \mathbb{R}^2_{++}$ and $h_2 \in \mathbb{R}^J$ to

$$\max \left\{ W(k^0, k^1, k^2) : \right. \begin{array}{l}
k^0 \leq \omega - c(w, z; l) + r(L - l) - v_1 h_1 - v_2^T h_2, \\
k^1_E \leq A h_1 + al, \quad k^1_L \leq A h_1, \\
k^2_E \leq p z + D h_2 \text{ and } k^2_L \leq p z + D h_2 + al 1^S \end{array} \} \right.$$  

where $v_2^T = (v_{21}, \ldots, v_{2J})$ and $1^S$ is a $S$ dimensional vector with each entry equal to 1.

The farmer faces three sets of budget constraints. Period 0 consumption can be no larger than the difference between initial wealth $\omega$ plus the income from renting out farm land $r(L - l)$ and the costs of production and operating in financial

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9With a slight abuse of notation $S$ is used to denote both the set of states of nature associated with the production uncertainty, but also the state $S$. As such, $S = \{ s : 1 \leq s \leq S \}$. 

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markets, $c(w, z; l)$, $v_1 h_1$, and $v_2^T h_2$, respectively. The consumption in period 1 is bounded by the revenue generated in the financial market, $Ah_1$, plus the subsidy, $al$, if the subsidy is paid on time. While period 2 consumption in the state of nature $s$, conditional on the subsidy payment being made late, can be no larger than the agricultural revenue in that state, $p_s z_s$, plus the return in the financial markets, $D_s h_2$, and the subsidy $al$.

Strict monotonicity of $W$ ensures that at the optimum:

$$ k^0 = \omega - c(w, z; l) + r(L - l) - v_1 h_1 - v_2^T h_2 $$  \hspace{1cm} (2.2)

$$ Ah_1 = k_E^1 - al, \quad Ah_1 = k_L^1 $$  \hspace{1cm} (2.3)

$$ Dh_2 = k_E^2 - pz \quad \text{and} \quad Dh_2 = k_L^2 - pz - al1^S $$  \hspace{1cm} (2.4)

For any given level of consumption, the budget constraints (2.3) and (2.4) ensure there exist unique portfolios $h_1$ and $h_2$ such that

$$ h_1 = \frac{k_E^1 + k_L^1 - al}{2A} $$

and

$$ h_2 = (D^T D)^{-1} D^T \left( \frac{k_E^2 + k_L^2}{2} - pz - \frac{al1^S}{2} \right) $$
Substituting out of the portfolios $h_1$ and $h_2$, the period 0 budget constraint yields:

$$
\max_{k_E^1,k_L^1,k_E^2,k_L^2,z,l} W\left( \omega - c(w, z; l) + r(L - l) - \frac{v_1}{2A} (k_E^1 + k_L^1 - al) \right)
- v_2^T (D^T D)^{-1} D^T \left( \frac{k_E^2 + k_L^2}{2} - pz - \frac{a l S}{2} \right), k_E^1, k_L^1, k_E^2, k_L^2
$$

(Note in (2.5) that (a) $z$ and $l$ appear only in the first argument of this unconstrained maximization problem and that (b) although first and second-period state-contingent consumptions do appear in the first argument, the optimal choice of $z, l$ does not depend on them since they appear as additive constants.

Intuitively, the farmer is indifferent between consumption that is produced via the agricultural technology and consumption produced via the financial markets. Assuming strictly increasing preferences, at the margin, the farmer will price the consumption produced with the agricultural technology using the opportunity cost of assembling the same consumption via the financial markets. Hence, this implies separability between consumption and production decisions.  

$$
\Pi(p, w, r, v_1/A,v_2^T (D^T D)^{-1} D^T ) = \max_{z,l} \left\{ v_2^T (D^T D)^{-1} D^T pz 
- c(w, z; l) - \left[ r - \frac{a}{2} \left( \frac{v_1}{A} + v_2^T (D^T D)^{-1} D^T 1 S \right) \right] l \right\}
$$

The state-contingent output level, $z$, is determined independent of consumption or farmer’s preferences for risk.

\[10\]Chambers and Voica (2016) contains a detailed discussion of this result.
Result 1. An interior solution to (2.1) satisfies \(^{11}\)

\[
\max_{k_1^E, k_1^L, k_2^E, k_2^L} W\left(\omega + rL - \frac{v_1}{2A} (k_1^E + k_1^L) - v_2^T (D^T D)^{-1} D^T \left( \frac{k_2^E + k_2^L}{2} \right) \right) \\
+ \Pi(p, w, r, v_1/2A, v_2^T (D^T D)^{-1} D^T), k_1^E, k_1^L, k_2^E, k_2^L)
\]

where

\[
\Pi(p, w, r, v_1/A, v_2^T (D^T D)^{-1} D^T) = \max_{z,l} \left\{ v_2^T (D^T D)^{-1} D^T p z \right. \\
-c(w, z; l) - \left[ r - \frac{a}{2} \left( \frac{v_1}{A} + v_2^T (D^T D)^{-1} D^T 1^s \right) \right] l \}
\]

Assuming \(c(w, z; l)\) is differentiable, the first order conditions for determining the optimal output \(z\) require

\[
\frac{\nabla_z c(w, z; l)}{p} D_j = v_{2j}, \quad j = 1, \ldots, J
\]

(2.6)

where \(\nabla_z c(w, z; l)/p \in \mathbb{R}^S\) and its \(s\)-th entry equals \((\partial c(w, z; l)/\partial z_s)/p_s\), \(D_j = (D_{1j}, D_{2j}, \ldots, D_{sj})^T\) is the payoffs vector of the \(j\)-th asset, \(D_{sj}\) is the payoff of asset \(j\) in state \(s\), and \(v_{2j}\) is the price of the asset \(j\) in period 0. For the subjective probability measure, \(\pi = (\pi_1, \ldots, \pi_s)\) (2.6) can be written in expectation form as

\[
E\left[ \frac{\nabla_z c(w, z; l)}{p} \tilde{D}_j \right] = v_{2j}, \quad j = 1, \ldots, J
\]

(2.7)

\(^{11}\)This result extends Theorem 1 of Chambers and Voica (2016) to the present context. The strand of literature on which this result is based can be traced to Chambers and Quiggin (2009).
where expectation is take over the discrete subjective probability measure $\pi$ and $\tilde{D}_j = D_j/\pi$.\textsuperscript{12} Expressions (2.6) and (2.7) show that $\nabla_z c(w, z; l)/p \in \mathbb{R}^S$ can be interpreted as a stochastic discount factor that ensures that the discounted return on period 2 payouts for each asset equals its period 0 acquisition cost.

Similarly, the first order condition for using farm on land requires

$$\frac{\partial c(w, z; l)}{\partial l} + r = a \frac{v_1}{2A} \left[ 1 + \frac{A}{v_1} v_2(D^T D)^{-1} D^T 1^S \right]$$

(2.8)

$$= a \frac{v_1}{2A} \left[ 1 + \frac{A}{v_1} P 1^S \right]$$

$$= a \frac{v_1}{2A} \left[ 1 + \frac{A}{v_1} E[\tilde{P} 1^S] \right]$$

where $P = v_2(D^T D)^{-1} D^T$, $\tilde{P} = P/\pi$. Expression (2.8) demonstrates how the acreage allocation decision depends upon the timing of the SFP payment.

If the subsidy is paid in the first period with certainty, the optimal amount of farmland, $l_a$, is the implicit solution to

$$\frac{\partial c(w, z; l)}{\partial l} + r = a$$

(2.9)

For given $z$, the convexity of $c$ in $l$ ensures that $l_a$ will be greater than the optimal amount farmed, $l_{a=0}$, in the absence of a subsidy determined as the implicit solution to

$$\frac{\partial c(w, z; l)}{\partial l} + r = 0$$

(2.10)

The difference between the two optimal choices of farmland reflects the change in

\textsuperscript{12}For the complete derivation please see the Appendix B.
the opportunity cost of diverting land from the rental market to farming caused by
the area payment.

Delaying the subsidy payment, however, weakens this effect because an area
payment of \( a \) paid in period 2 is valued less by a farmer who intertemporally dis-
counts consumption than an area payment of \( a \) paid in period 1. So long as farmers
have positive intertemporal discount factors delaying the subsidy mitigates its dis-
tortionary effect on production.

This can be seen as follows. If the subsidy is paid in period 1 with certainty,
its period 0 discounted value equals \( av_1/2A \). If the subsidy is paid in period 2 with
certainty, its period 0 discounted value is \( aE[\bar{P}_1^S]/2 \). Thus, so long as \( v_1/A > E[\bar{P}_1^S] \), the delay decreases the distortionary impact of the subsidy. But this latter
condition is always satisfied if farmers have positive intertemporal discount factors.

2.3 Estimation

Data limitations do not allow to discriminate between consumption activities
that occur within a given agricultural production cycle. For empirical work, the
three-period model was collapsed into a two period model. The formal justification
that would permit this is to assume \( A/v_1 = 1 \). Then period 0 and period 1 are
perfect substitutes in consumption and there is no discounting between period 0
and period 1. In short, the new first period includes the old first and second periods
and the second period is previous third period. This also changes the cardinality
of the state space \( \Omega \) from \( 2S \) to \( S \). Optimal output, \( z \), and farmland, \( l \), values are
determined by solving the profit maximization problem:

$$\Pi(p, w, r, 1, P) = \max_{z,l} \left\{ Ppz - c(w, z; l) - \left[ r - \frac{a}{2}(1 + P1^S) \right] l \right\}$$

Hence, the new first order conditions for the profit maximization now require:

$$E \left[ \nabla_z c(w, z; l) \frac{\tilde{p} R_j}{\tilde{p}} \tilde{D} \right] = v_2$$  \hspace{1cm} (2.11)

$$\frac{\partial c(w, z; l)}{\partial l} + r = \frac{a}{2} \left[ 1 + E[\tilde{P}1^S] \right]$$  \hspace{1cm} (2.12)

If the theory is descriptive of farmer behavior, the system of equations (3.1) and (3.9) should permit estimation a parametric specification of $c(w, z, l)$ while allowing one to determine the effect of delaying subsidy payments on production and farmland. It proves convenient to rewrite them in terms financial returns.

Letting $R_j = \tilde{D}_j/v_{2j}$, $R_j \in \mathbb{R}^S$ for all $j$ expression (3.1) becomes:

$$E \left[ \nabla_z c(w, z; l) \frac{\tilde{p} R_j}{\tilde{p}} \right] - 1 = 0.$$  \hspace{1cm} (2.13)

If $R_1$ is assumed to be the return on the riskless (that is, 1$ today returns 1 + i$ tomorrow), $R_1 = 1 + i$, and (2.13) becomes

$$E \left[ \nabla_z c(w, z; l) \frac{1}{\tilde{p}} \right] = \frac{1}{1 + i}$$  \hspace{1cm} (2.14)
Using this result in equation (3.9) gives

\[
\frac{\partial c(w, z; l)}{\partial l} + r = \frac{a}{2} \left[ 1 + \frac{1}{1 + i} \right]
\]  

(2.15)

Or in expectation format, equation (3.9) becomes

\[
E \left[ \left( \frac{\partial c(w, z; l)}{\partial l} + r \right) \frac{2(1 + i)}{a(2 + i)} \right] - 1 = 0
\]  

(2.16)

The econometric strategy is to use the generalized method of moments (GMM) to estimate a parametric representation of these equilibrium relationships (Hansen and Singleton (1982), Cochrane (2001), Chambers (2007), Pope, LaFrance, and Just (2011)). The analysis focuses on Romanian wheat production. This crop has an important share of the agricultural farmland area in Romania and is planted in the fall. As such, it is a good candidate for testing the timing effect. The cost function \( c(w, z; l) \) for producing wheat is assumed to take the general form:

\[
c_t(w_t, z_{t+1}; l_t) = \tau(w_t) + \phi(w_t) \left[ \alpha z_t E_t(z_{t+1} - z_t) + \frac{\beta z}{2} E_t[(z_{t+1} - z_t)^2] + \right.
\]

\[
+ \eta z_t E_t(z_{t+1} - z_t) l + \gamma z E_t[(z_{t+1} - z_t)(y_{t+1} - y_t)] + \alpha l_t + \frac{\beta l_t^2}{2} \right]
\]

(2.17)

where \( z \) now represents wheat output and \( y \) is an output index corresponding to all crops output. While simple, this cost structure is sufficiently flexible to capture the effects of mean and dispersion shifts in \( z \) on the production cost \( c(w, z; l) \), the effect of the covariance between \( z \) and other crops on the cost, as well as, the interaction
between land and output.

The parameters $\alpha_z$ measure the change in the cost due to a change in the mean output by the farmer, $\beta_z$ measure the effect on the cost of a change in the output dispersion as captured by the second moment, $\gamma_z$ measures the change in the cost due to a change in the covariance between $z$ and $y$, while $\eta_z$ measures the interaction between land and the output. The land parameters $\alpha_l$ and $\beta_l$ measure the effect on the cost of a change in the amount of land used. Given the subsidy $a$ is paid per unit of land $l$, $\alpha_l$ and $\beta_l$ can also be interpreted as measuring the effects of a change in the amount of subsidy $a$ over the marginal cost of producing output $z$.

Because the timing of the subsidy payments influence the subsidy amount received, when discounted to the first period, a change in $\alpha_l$ and $\beta_l$ measure the effect of the payments timing on the coupling between subsidy and production decisions.

Given this representation of the production cost function,

$$\frac{\nabla_z c_t(w_t, z_{t+1}; l_t)}{p_{t+1}} = \frac{\phi(w_t)}{p_{t+1}} \left[ \alpha_z + \beta_z (z_{t+1} - z_t) + \gamma_z (y_{t+1} - y_t) + \eta_z l_t \right]$$

(2.18)

and

$$\frac{\partial c_t(w_t, z_{t+1}, y_{t+1}; l_t)}{\partial l_t} = \phi(w_t) \left[ \alpha_l + \beta_l l_t + \eta_z E(z_{t+1} - z_t) \right]$$

(2.19)

Using suitable instruments ensures that the number of moment conditions is at least as large as the number of parameters to be estimated and helps identify those parameters. If conditional on information available at time $t$ (2.13) and (2.16) hold as identities, then for any set of instruments $Z_t$ predetermined at time $t$, the
law of iterated expectations requires

\[ g(d_t, \theta) = E \left[ Z_t^T \left( \frac{\nabla Z_c (w_t, z_{t+1}, l_t; \theta)}{p_{t+1}} R_{jt+1} - 1 \right) \right] = 0 \quad (2.20) \]

where \( d_t = (w_t, z_{t+1}, l, p_{t+1}, R_{t+1}) \), \( \theta = (\alpha_z, \beta_z, \gamma_z, \alpha_l, \beta_l) \) is the vector of parameters to be estimated, and

\[ h(d_t, \theta) = E \left[ Z_t^T \left( a \left( 1 + \frac{1}{1+i} \right) + \frac{c(w, z_{t+1}, l)}{c} - r \right) \right] = 0 \quad (2.21) \]

The GMM procedure estimates \( \theta \) as the solution to the minimization problem

\[ J_T(\theta) = \left[ g(d_t, \theta), h(d_t, \theta) \right]^T W \left[ g(d_t, \theta), h(d_t, \theta) \right] \quad (2.22) \]

where \( W \) is a positive definite weighting matrix.

### 2.3.1 Data and Empirical Strategy

The data for this paper come from the Romanian EU Farm Accountancy Data Network (FADN). These data are part of a short unbalanced panel which covers the period 2007 to 2010, and includes data on agricultural production, prices, land use, subsidy amount, labor, farm financial assets. The number of farmers sampled in this survey has increased yearly from approximately 1,000 farmers in 2007 to around 6,000 in 2010. In order to create a balanced panel, the data were aggregated to the county level. To account for differences between micro and aggregate levels, I use county level fixed effects. Pope, LaFrance, and Just (2011) provide a detailed
explanation of the potential gains and losses from data aggregation.

A second issue is the lack of description regarding the sequence of decisions. The approach used here is to assume that the first period stochastic discount factor equals 1. Hence, periods one and two are perfect substitutes in consumption. While restrictive, this assumption allows testing for the timing effects on subsidy.

The data used in the estimation of the equations above are as follows. \( \phi(w_t) \) corresponds to an index of agricultural input prices for Romania obtained from the Eurostat. Output and output prices for period are available from the FADN. For the first year, \( z_t \) is taken as the national county average. The instruments used are the unemployment rates, lagged observed output prices and the county fixed effects. The financial assets used are Romanian Stock Exchange return and the national interest rates obtained from the Romanian National Bank.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsidy (RON/ha)</td>
<td>267.51</td>
<td>162.65</td>
<td>137.26</td>
<td>2133.93</td>
</tr>
<tr>
<td>Input Price Index</td>
<td>132.23</td>
<td>7.34</td>
<td>121.90</td>
<td>141.30</td>
</tr>
<tr>
<td>Wheat Price (RON/tonne)</td>
<td>649.66</td>
<td>229.70</td>
<td>271.33</td>
<td>1267.00</td>
</tr>
<tr>
<td>Wheat Production (tonnes)</td>
<td>256.00</td>
<td>338.07</td>
<td>0.21</td>
<td>1539.62</td>
</tr>
<tr>
<td>Land Rent (RON/ha)</td>
<td>8891.64</td>
<td>34004.37</td>
<td>456.75</td>
<td>332462.60</td>
</tr>
<tr>
<td>Area (ha)</td>
<td>233.47</td>
<td>270.38</td>
<td>2.86</td>
<td>1736.10</td>
</tr>
<tr>
<td>Interest Rate - national</td>
<td>1.09</td>
<td>0.02</td>
<td>1.07</td>
<td>1.12</td>
</tr>
<tr>
<td>Romanian Stock Exchange Return</td>
<td>0.99</td>
<td>0.41</td>
<td>0.55</td>
<td>1.53</td>
</tr>
<tr>
<td>Unemployment</td>
<td>6.60</td>
<td>0.57</td>
<td>5.80</td>
<td>7.30</td>
</tr>
</tbody>
</table>

\(^1\) Tonne, also referred to as the metric ton, is a metric unit of mass equal to 1,000 kilograms. I am indebted to Professor Nerlove for encouraging me to clarify this concept.
2.3.2 Empirical Results and Discussion

Table 2.2: Estimated Pricing Kernel and Subsidy Delay Effect for County Level with Fixed Effects and Panel Newey-West Standard errors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Arbitrage Wheat</th>
<th>Timing Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_z$</td>
<td>$\beta_z$</td>
</tr>
<tr>
<td>Estimate</td>
<td>14289.31</td>
<td>-770.99</td>
</tr>
<tr>
<td>t</td>
<td>2.40</td>
<td>-5.82</td>
</tr>
<tr>
<td></td>
<td>$\alpha_i$</td>
<td>$\beta_i$</td>
</tr>
<tr>
<td>Estimate</td>
<td>-67.19</td>
<td>0.35</td>
</tr>
<tr>
<td>t</td>
<td>-67.1958</td>
<td>-0.43</td>
</tr>
<tr>
<td>TJt</td>
<td>66.01</td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td>Observation</td>
<td>164</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 summarizes the results of the GMM estimation. The estimated value of the J statistics suggests that this cost function specification reasonably approximates the underlying data.

The estimated value of $\alpha_z$ is positive and significant. The positive sign suggests that keeping other moments constant, the cost of producing wheat increases as the expected value of the crop increases. The estimated value of $\beta_z$ is negative suggesting that allowing the variance of wheat to increase would decrease cost suggesting that production of wheat is inherently risky in the sense of Chambers and Quiggin (2000). Keeping other moments constant, decreasing variability as measured by second moment of the crop distribution, raises costs. Hence, eliminating production risk is costly. The estimate of $\gamma_z$ is also positive suggesting that the cost of producing wheat increases as the covariance between wheat production and
other crop production increases. Keeping other moments constant, increasing \( y \) will
determine a reduction in the optimal level of \( z \), hence \( z \) and \( y \) are substitutes in
production. The estimate of \( \eta_z \) is negative and insignificant.

The estimated land coefficients, \( \alpha_l \) and \( \beta_l \), imply that the cost of producing
wheat is indeed convex in land farmed and will be decreasing in the amount of land
farmed over an appropriate interval of the data. Convexity is particularly important
because it implies that, holding \( z \) constant, land use will be increasing in the SFP
subsidy.

To illustrate, suppose that the estimated cost function is realistic for the representa-
tive Romanian farmer. If there is no delay in the subsidy payments the optimal
choice of land, \( l_{a=r,ND} \), satisfies

\[
\phi(w_t)(-67.19 + 0.35l_{a=r,ND} - 0.08E(z_{t+1} - z_t)) = a - r \tag{2.23}
\]

which can be compared to the optimal amount of land in the absence of any area
subsidy, \( l_{a=0} \), which solves

\[
\phi(w_t)(-67.19 + 0.35l_{a=0} - 0.08E(z_{t+1} - z_t)) = -r. \tag{2.24}
\]

Convexity ensures that the amount of land will depend upon both the magnitude
and timing of the subsidy payments, because in the presence of delays the optimal
amount of land, \( l_{a=r,D} \) is

\[
\phi(w_t)(-67.19 + 0.35l_{a=r,D} - 0.08E(z_{t+1} - z_t)) = a\frac{1}{1+i} - r
\]  

(2.25)

from where it is clear that \( l_a=0 < l_{a=r,D} < l_{a=r,ND} \), hence the coupling effect between production and subsidy decreases with the delay in the payments of the subsidy.

The estimated elasticity of land with respect to land rent, calculated at sample means, in the absence of a subsidy is \(-0.823\), which compares with an estimated elasticity of land with respect to the land rent when the subsidy is paid on time is \(-0.798\). The mean of the subsidy is approximately 3\% of the average rental rate of land. Thus, holding \( z \) constant, one would expect that a subsidy paid on time would result in a 2.5\% increase in land utilization by the representative farmer. Delaying the payment of the subsidy, given existing interest rates, by one period is equivalent to reducing the subsidy by approximately 10\% which translates into about a .25\% change in land utilization by the farmer. For example, for our sample, delaying the subsidy by one period at a 10\% discount rate translates into a change of 0.58 ha for the average farmed area, 0.007 ha for the minimum farmed area and 4.34 ha for the maximum farmed area.  

The estimated yield effects associated with the SFP subsidy are legibly small. The estimated elasticity of expected output with respect to the land rent is insignificantly different of the elasticity of expected output in the presence of the subsidy. Hence the change in the expected output due to the subsidy delay is negligible and

\[\text{\begin{footnotesize}
\label{fn:13}
\begin{footnotesize} \text{I am indebted to Professor Olson for encouraging me to illustrate the timing effect.} \end{footnotesize} \end{footnotesize}}\]

40
2.4 Conclusions

This paper contributes to the debate surrounding agricultural policy support for farmers and the potential distortionary effects of area payments. It shows how the timing of subsidy payments decreases the intensity of the distortionary effect associated with subsidies. Measured at current levels, the area based SFP causes the representative Romania wheat farmer to increase acreage by approximately 2.5%. Delaying the subsidy has negligible yield effects but reduces acreage by approximately .25%.

Following Chambers and Voica (2016), the paper also shows that the production distortion associated with area payments is independent of the farmer’s risk preferences. In the presence of off-farm opportunities, which are exogenous to the government intervention, separation between farmers consumption and production decisions occurs. However, area payments can affect production via land allocation. I show that the timing of subsidy payments can weaken the link between area payments and production decisions. The theoretical predictions are supported by the empirical analysis. The results of the paper provide an alternative explanation to why previous empirical studies have found little evidence of coupling between area payments and agriculture production.
Farmer chooses consumption $(k^0, k^1, k^2)$, stochastic output $z$, financial portfolio $(h_1, h_2)$, and area farmed $l$.

Farmer learns if the subsidy is deferred to period 2. If subsidy now, the realized consumption is $k^1 = Ah_1 + a l$. If not, consumption is $k^2 = Ah_1$

Nature chooses a state of the world.

If the subsidy paid in period 1, consumption is $k^2_{SE} = p_s z_s + D_s h_2$. If not, consumption is $k^2_{SL} = p_s z_s + D_s h_2 + a l$

Figure 2.1: Decisions Timeline
<table>
<thead>
<tr>
<th>Nature</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Late Payments</td>
<td>( p_1 z_1 + \sum_{j=1}^{l} D_{1j} h_{2j} + al )</td>
<td>( p_2 z_2 + \sum_{j=1}^{l} D_{2j} h_{2j} + al )</td>
<td>...</td>
<td>( p_s z_s + \sum_{j=1}^{l} D_{sj} h_{2j} + al )</td>
</tr>
<tr>
<td>Early Payments</td>
<td>( p_1 z_1 + \sum_{j=1}^{l} D_{1j} h_{2j} )</td>
<td>( p_2 z_2 + \sum_{j=1}^{l} D_{2j} h_{2j} )</td>
<td>...</td>
<td>( p_s z_s + \sum_{j=1}^{l} D_{sj} h_{2j} )</td>
</tr>
</tbody>
</table>

Figure 2.2: Subsidy Payments
Figure 2.3: Subsidy Timing Effect
Chapter 3: Are We Spending or Wasting Health When Eating?

The World Health Organization defines health as a “resource”. In its words, “Health is, therefore, seen as a resource for everyday life, not the objective of living. Health is a positive concept emphasizing social and personal resources, as well as physical capacities.” (WHO, 2017a). This definition is consistent with observed behaviour. People trade health for consumption in a way that resembles any other resource. For example, immigrant farmworkers in the US often find themselves in poor health despite harvesting healthy products (Fuller, 2016). The main reason is budgetary constraints, that prevent purchasing healthy foods, thus effectively inducing a trade off between health and current consumption. On the other hand, there are plenty of examples when budget constraints are not binding, but consumers prefer to overindulge (Rashad, 2006).

This issue is naturally linked to the problem of obesity, a major health issue, that is pervasive in today’s world. Since 1980 obesity has doubled, with more than 1.9 billion adults, 18 years and older being overweight in 2014, while almost a third of these were obese (WHO, 2017b). In the US alone the prevalence of obesity was over 36% in adults and 17% in youth between 2011 – 2014 (Ogden et al., 2015), while in Europe, obesity was responsible for more than 1 million deaths
and 12 million life-years of health reduction in 2010 (Cuschieri and Mamo, 2016). Attempts to reduce the obesity epidemic and to improve the health outcomes of the population, have prompted governmental interventions, such as taxes on high-calorie foods (Jacobson and Brownell, 2000).¹ Proponents of these interventions argued that taxes will change the relative prices of healthy and unhealthy foods, inducing rational consumers to substitutes towards a healthier diet while improving the health outcomes.

The effectiveness of these interventions has been debated in literature, where potential factors that could reduce the health benefits of these interventions were proposed. Among them, the substitution between taxed and non-taxed high-calorie foods (Schroeter, Lusk, and Tyner, 2008; Fletcher, Frisvold, and Tefft, 2010), an inverse relation between healthier foods consumption and physical exercise (Yaniv, Rosin, and Tobol, 2009), taste and inventory decisions (Wang, 2015), or addiction (Becker and Murphy, 1988; Becker, Grossman, and Murphy, 1991; Richards, Patterson, and Tegene, 2007).

In order to explore the trade-off between health and consumption and the effectiveness of health interventions such as taxing unhealthy foods, this paper proposes a theoretical model where rational agents maximize utility over health and consumption of healthy and unhealthy foods, while health is a function of discretionary and non discretionary calories and nutrients. Calories are not available for purchase in the market, thus their pricing is derived via a “household” produc-

¹A calorie is the amount of heat required at a pressure of one atmosphere to raise the temperature of one gram of water one degree Celsius (Merriam-Webster).
tion technology used to convert healthy and unhealthy foods into health outcomes.\footnote{The “household” production was popularized by Deaton and Muellbauer (1993), while Chambers (2017) provides a set-theoretical approach.} Additionally, consumers face a physiological constraint, a minimum calorie intake, which has further implications in terms of reducing potential health benefits associated with governmental interventions, such as taxing high-calorie foods. We model the future budget of consumers as depending directly on the consumption of discretionary calories.

The structure of the paper is as follows. The next section places the paper in the literature, while the third section presents the theoretical model. The fourth section reports empirical results from a calibrated version of the theoretical model. The final section concludes.

3.1 Comparison with the Literature

The observation that rational economic agents trade health for the consumption of other goods is not new. For example, addiction (reinforcement and tolerance) was investigated by Becker and Murphy (1988) and Becker, Grossman, and Murphy (1991), counter cyclical consumption effects on health were examined by Dockner and Feichtinger (1993). The economics of consumption of foods with negative effects on health were investigated, among others, by Grossman (1972a), Forster (2001), Chavas (2013) and Bolin and Lindgren (2016), while Chavas (2015) provides a benefit function treatment.

Furthermore, the health economics literature, in particular the economic lit-
ature concerning the health effects of food consumption, has investigated theoretically and empirically the effects of various government interventions, such as taxing calorie-rich foods, on the rational agents’s food choices and health outcomes. While not always in agreement, the general consensus is that governmental interventions targeting improvements in health outcomes are rarely fully successful. For example, with some exceptions (Richards, Patterson, and Tegene, 2007), the attempts to improve the health outcomes of consumers by taxing high-calorie foods had mixed results, and the taxes proved to be regressive. Ignoring consumers’ inventory behaviors and the persistence of their tastes overestimates the benefits of soda taxes (Wang, 2015), in addition to offsetting some of the benefits due to the substitution between various types of high-calorie foods (Schroeter, Lusk, and Tyner, 2008; Fletcher, Frisvold, and Tefft, 2010). Assuming a Leontief technology for the production of healthy foods and perfect substitution in consumption between healthy and unhealthy foods, Yaniv, Rosin, and Tobol (2009) show that a “fat” tax can decrease the health outcomes of consumers, if the reduction in the unhealthy food due to the tax is not enough to compensate the reduction in time allocation to physical exercise, now needed for cooking the healthy food. Alternatively, taxing or subsidizing calories rather than foods avoids potential substitution effects between similarly perceived items increasing the health benefits of the interventions (Richards, Patterson, and Tegene, 2007; Okrent and Alston, 2012).

The contribution of this paper to the previous literature is as follows. First, I account for physiological constraints imposed by biological processes. There is a minimum calorie intake a person needs to fulfill in order to maintain the energy
level required to engage in productive activities. ³ This intake is independent of
the nutrient density of calories consumed, however health is not. Thus, a potential
trade off dictated by budgetary constraints is apparent. Furthermore, as long as this
minimum calorie intake is binding, taxing high-calorie foods will have the opposite
effect of decreasing the nutrient density of the foods consumed, while increasing the
intake of high-calorie foods. This provides an alternative explanation to justify the
mixed results of tax interventions on improving health outcomes, especially in the
case of poor consumers.

Second, previous literature has modeled the budget available to consumers
as an increasing function of health (Grossman, 1972b; Chavas, 2013; Bolin and
Lindgren, 2016). While intuitively correct, people tend to be more productive if
they are healthy, in the short run people might find it in their best interest to push
the health “envelope” so to speak. For example, diabetes will impact a person’s
ability to generate income in the long run, but in the initial stages of the disease
the person’s ability to perform is less likely to be affected.⁴ On the other hand,
consumption of unhealthy foods that provide a boost in energy is more likely to
increase productivity. Consider farmworkers. Their income is contingent on how
much they harvest at a given time (i.e., piece rate payment). An unhealthy boost in
energy will increase their productivity and so their income at the expense of health.

Another example can be found in academics. Tenure track assistant professors will,

³I am indebted to Professor Holzer for pointing out the correct meaning of the physiological
constraint. A previous version of the chapter argued the constraint is the minimum level of calories
needed to survive.

⁴Diamond (2003) argues that changes in diets and life style can precede negative health out-
comes by as much as two decades.
at times, find themselves in the position of trading health in order to increase the
time budget available to beat the tenure clock. In both instances, budget is a
function of unhealthy food consumption, rather than health. This avenue seems
worth exploring. Finally, I calibrate the theoretical model with data pertaining to
the US farm-workers food choices.

3.2 Theoretical Model

There are two time periods. The agent has preferences over health, $h \in \mathbb{R}^{++}$,
and two food products, $t_i \in \mathbb{R}^+_+$ and $s_i \in \mathbb{R}^+_+$, where $i = 1, 2$ represents consumption
of food products in the first and second periods. Preferences are represented by the
utility function $u : \mathbb{R}^5_+ \rightarrow \mathbb{R}^{++}$, where $u$ is assumed to be increasing and concave
in health, $s_i$ and $t_i$. The agent can buy the food product $t_i$ for a price $q_i \in \mathbb{R}^{++}$,
which for simplicity is assumed to be equal across periods and is normalized to 1,
and the good $s_i$ for a price $p_i \in \mathbb{R}^{++}$, which for simplicity is assumed to equal $p$
across periods. Health can not be purchased from the market, it is a nonmarketable
good.

However, the agent can produce health using a technology where the inputs
are calories and nutrients delivered by the two food products. Calories come in two
flavours: discretionary calories, also known as “empty calories”, $c_{e_i} \in \mathbb{R}^+_+$, which
are produced by nutrient deficient food products, $t_i$ in this case, and full calories,
$c_{f_i} \in \mathbb{R}^+_+$, which are produced along side nutrients by healthy food products, $s_i$ in this
case. The difference between empty and full calories is that the latter calories deliver
energy and nutrients while the former deliver only energy.\textsuperscript{5} The health technology is represented by the health production function $H(c_{e_1}, c_{f_1}, c_{e_2}, c_{f_2})$, where\textsuperscript{6}

$$H(c_{e_1}, c_{f_1}, c_{e_2}, c_{f_2}) = \max\{h : (c_{e_1}, c_{f_1}, c_{e_2}, c_{f_2}) \text{ can produce } h\}$$

The health production function $H(c_{e_1}, c_{f_1}, c_{e_2}, c_{f_2})$ is increasing in $c_{f_i}$ and non-increasing in $c_{e_i}$. The full calories technology is represented by the input requirement function $f(c_{f_i})$, where

$$f(c_{f_i}) = \min\{s_i : s_i \text{ can produce } c_{f_i}\}$$

and the empty calories technology is represented by the input requirement function $e(c_{e_i})$, where

$$e(c_{e_i}) = \min\{t_i : t_i \text{ can produce } c_{e_i}\}$$

In the first period, the agent faces the budget constraint, $p s_1 + t_1 \leq m$, and, in the second period, the budget constraint $p s_2 + t_2 \leq (1 + \beta(c_{e_1}))m$, where $m$ is the agent’s income and $\beta(c_{e_1})$ is a productivity bonus (i.e., agricultural workers can harvest more products in the field if they consume $c_{e_1}$). I assume that $\beta(c_{e_1})$ is concave in $c_{e_1}$ (i.e., $\beta'(c_{e_1}) > 0, \beta''(c_{e_1}) < 0$).

Additionally, in each period, the agent must reach a minimum level of calories

\textsuperscript{5}I am indebted to Professor Olson for encouraging me to clarify the distinctions between the two types of calories.

\textsuperscript{6}The health production function refers to the ability of the body to transform calories and nutrients in health, keeping constant genetics and environment, exercise, life style, and other contributing factors.
consumption, \( \bar{c} \), in order to insure a minimum level of energy necessary to sustain life. I assume that the income \( m \) is sufficiently large to allow the consumer to cover the minimum calories requirement, \( \bar{c} \), at least from the consumption of empty calories \( c_{e1} \). Technically, this requires \( e(\bar{c}) \leq m \). Similarly, the consumer will be able to cover \( \bar{c} \) only from the consumption of full calories \( c_{f1} \) if \( pf(\bar{c}) \leq m \).

The consumer chooses \( s_i, t_i, c_{f1} \) and \( c_{e1}, i = 1,2 \), to solve:

\[
\max \left\{ u_1(t_1, s_1) + \delta u_2(h, t_2, s_2) : h \leq H(c_{e1}, c_{f1}, c_{e2}, c_{f2}), s_i \geq f(c_{f1}), t_i \geq e(c_{e1}), \right. \\
p_s + t_1 \leq m, ps + t_2 \leq (1 + \beta(c_{e1}))m, c_{e1} + c_{f1} \geq \bar{c}, i = 1,2 \} 
\]

where \( \delta \) is an intertemporal discount factor.

In words, the consumer maximizes consumption of health and food products over two periods conditional on technology, budget and physiology constraints. The technology constraints are the health constraint, \( h \leq H(c_{e1}, c_{f1}, c_{e2}, c_{f2}) \), and the production of calories, \( s_i \geq f(c_{f1}) \) and \( t_i \geq e(c_{e1}) \) for \( i = 1,2 \). The budget constraints are \( ps + t_1 \leq m \) in the first period, and \( ps + t_2 \leq (1 + \beta(c_{e1}))m \) in the second period. Finally, the physiology constraints are \( c_{e1} + c_{f1} \geq \bar{c}, i = 1,2 \), the consumer must achieve at least the minimum calorie intake in each period.

It is convenient to recast (3.1) exclusively in terms of health inputs, \( c_{e1} \) and \( c_{f1} \). Because the agent is the residual claimant, at optimum, \( h = H(c_{e1}, c_{f1}, c_{e2}, c_{f2}) \), \( s_i = f(c_{f1}) \) and \( t_i = e(c_{e1}) \) for all \( i \). Thus, the first period budget constraint becomes \( pf(c_{f1}) + e(c_{e1}) \leq m \), and the second period budget constraint becomes \( pf(c_{f2}) + \)
\( e(c_{e_2}) \leq (1 + \beta(c_{e_1}))m. \) Both budget constraints are binding at the optimum.

The consumer chooses \( c_{f_i} \) and \( c_{e_i} \), \( i = 1, 2 \), to solve:

\[
\begin{align*}
\max \{ & u_1(e(c_{e_1}), f(c_{f_1})) + \delta u_2(H(c_{e_1}, c_{f_1}, c_{e_2}, c_{f_2}), e(c_{e_2}), f(c_{f_2})) : \\
& pf(c_{f_1}) + e(c_{e_1}) = m, \\
& pf(c_{f_2}) + e(c_{e_2}) = (1 + \beta(c_{e_1}))m, \\
& c_{e_i} + c_{f_i} \geq \bar{c}, i = 1, 2 \} \\
\end{align*}
\]  

Figure 3.1 illustrates the optimal consumption of calories and the optimal choice of health for one period in the calories space. For a budget \( m \) and prices \( (p, 1) \), the maximum amount of full and empty calories available for consumption are \( f^{-1}(m/p) \), point \( B \), and \( e^{-1}(m) \), point \( J \), respectively. The isocost connecting \( B \) and \( J \) represents all possible combinations of full and empty calories \( (c_{f_i}, c_{e_i}) \) available for consumption given the budget \( m \) and prices \( (p, 1) \). The indifference curve represents all possible combinations of full and empty calories \( (c_{f_i}, c_{e_i}) \) that deliver the same level of satisfaction to the consumer. The optimal consumption of calories, point \( D \), is given by the tangency between the indifference curve and the isocost curve. The optimal health corresponding to point \( D \) is given by the health isoquant at \( D \). The minimum caloric intake is represented by the line of slope \(-1\) (i.e. line \([\bar{c}, \bar{c}]\)).
3.2.1 Equilibrium Behavior

In order to characterize the optimal decisions of the agent in (3.2), first we need to consider whether the minimum caloric intake constraints are binding or not. To focus the analysis, we consider only the polar cases: both constraints are binding and none of the constraints are binding. The mixed cases are similar.

If the minimal calories intake constraints are not binding, \( c_{e_i} + c_{f_i} > \bar{c} \), (3.2) can be written as:

\[
\max \left\{ u_1(e(c_{e_i}), f(c_{f_i})) + \delta u_2(H(c_{e_1}, c_{f_1}, c_{e_2}, c_{f_2}), e(c_{e_2}), f(c_{f_2})) \right. \\
+ \gamma_1 (m - pf(c_{f_1}) - e(c_{e_1})) \\
+ \left. \gamma_2 ((1 + \beta(c_{e_1}))m - pf(c_{f_2}) + e(c_{e_2})) \right\} (3.3)
\]

where \( \gamma_1 \) and \( \gamma_2 \) are shadow prices, and the optimal consumption of discretionary and full calories, assuming interior solutions, are characterized by the first order conditions:

\[
\frac{\partial u_1}{\partial s_1} \frac{\partial f}{\partial c_{f_1}} + \frac{\partial u_2}{\partial H} \frac{\partial H}{\partial c_{f_1}} - \gamma_1 p \frac{\partial f}{\partial c_{f_1}} = 0 (3.4)
\]

\[
\frac{\partial u_1}{\partial t_1} \frac{\partial e}{\partial c_{e_1}} + \frac{\partial u_2}{\partial H} \frac{\partial H}{\partial c_{e_1}} - \gamma_1 \frac{\partial e}{\partial c_{e_1}} - \gamma_2 \beta' m = 0 (3.5)
\]

\[
\frac{\partial u_2}{\partial s_2} \frac{\partial f}{\partial c_{f_2}} + \frac{\partial u_2}{\partial H} \frac{\partial H}{\partial c_{f_2}} - \gamma_2 p \frac{\partial f}{\partial c_{f_2}} = 0 (3.6)
\]
\[
\frac{\partial u_2}{\partial t_2}\frac{\partial e}{\partial c_{e_2}} + \frac{\partial u_2}{\partial H}\frac{\partial H}{\partial c_{e_2}} - \gamma_2\frac{\partial e}{\partial c_{e_2}} = 0 \quad (3.7)
\]

In Figure 3.1, an example of the range of possible solutions is the isoquant map \((A, B]\), where \(A\) represents behaviour consistent with zero health, and \(B\) is maximum health given the budget \(m\) and prices \((p, 1)\). In this case, the minimum caloric constraint is not binding, and the optimal mix between empty and full calories, \((c_e, c_f)\) is given by the preferences over health, and the consumption of other goods. The optimum choice is at point \(D\), where the indifference curve is tangent to the isocost curve.

Alternatively, if the constraints are binding, \(c_{e_i} + c_{f_i} = \bar{c}\), the empty calories consumed \(c_{e_i} = \bar{c} - c_{f_i}\), and the optimum consumption of full calories is determined by the budget constraints \(pf(c_{f_i}) + e(\bar{c} - c_{f_i}) = m\) in the first period, and \(pf(c_{f_i}) + e(\bar{c} - c_{f_i}) = (1 + \beta(c_{e_1}))m\) in the second period.

Denote the first period optimal consumption of full calories by \(\hat{c}_{f_1}\), where \(\hat{c}_{f_1}\) represents the implicit solution for \(c_{f_1}\) in terms of the budget constraint \(pf(c_{f_1}) + e(\bar{c} - c_{f_1}) = m\), and the second period optimal consumption of full calories by \(\hat{c}_{f_2}\), where \(\hat{c}_{f_2}\) represents the implicit solution for \(c_{f_2}\) in terms of the budget constraint \(pf(c_{f_2}) + e(\bar{c} - c_{f_2}) = (1 + \beta(c_{e_1}))m\). Then, the first period optimal consumption of empty calories is \(\bar{c} - \hat{c}_{f_1}\), the second period optimal consumption of empty calories is \(\bar{c} - \hat{c}_{f_2}\) and the consumer reaches the indifference curve corresponding to the utility.
level

\[ u_1(e(\bar{c} - \hat{c}_f), f(\hat{c}_f)) + \delta u_1(H(\bar{c} - \hat{c}_f, \hat{c}_{f_1}, \bar{c} - \hat{c}_{f_2}, \hat{c}_{f_2}), e(\bar{c} - \hat{c}_f), f(\hat{c}_{f_2})) \]

In Figure 3.1, points \( F \) and \( G \) are examples of such an optimum. These points are the intersection between the minimum calorie intake constraint, and the corresponding isocost curves and the health isoquants. Furthermore, they are consistent with health maximization behaviour. Contingent on the minimum caloric-intake, \( F \) and \( G \) are the highest health outcomes feasible for the budget \( m_1 \) and prices \((p, 1)\) in the case of \( F \), and prices \((p, 1/(1 + \mu))\) in the case of \( G \).

### 3.2.2 The Nutritionist Standard and The Health Economist

Before analyzing the agent’s optimal choices in the presence of a “fat” tax, additional insights can be gathered from two related problems. The first problem is that of an agent who maximizes health given the constraints imposed by the health technology, \( H(c_{e_1}, c_{f_1}, c_{e_2}, c_{f_2}) \), and the minimum calorie intake, \( \bar{c} \). Mnemonically, we refer to it as the nutritionist’s standard. This is a technology driven problem, and its solution is independent of income and prices considerations. Thus, it will generate the highest level of health. In Figure 3.1, this corresponds to the point \( N \).

The second problem is derived by adding the budget constraints, \( ps_1 + t_1 \leq m \) and \( ps_2 + t_2 \leq (1 + \beta(c_{e_1}))m \), to the nutritionist’s problem. Mnemonically, we refer to it as the health economist’s problem because this agent maximizes health conditional on the budget constraints, available health technology and the minimum
calorie intake constraints. The difference between the optimal level of health of the nutritionist and that of the health economist is due to the income and prices pressure.

In short, the nutritionist’s standard is:

$$\max \left\{ H(c_{e_1}, c_{f_1}, c_{e_2}, c_{f_2}) \right\}$$  \hspace{1cm} (3.8)

The nutritionist can always cover the minimum calorie intake requirement, because her decisions are not subject to budget constraints. Considering that health is decreasing in empty calories, the optimal level of $c_{e_i}$ is zero. \(^7\) Let the optimal health level of this problem be $h^*$, then $h^*$ is the highest level of health feasible given the technology $H(c_{e_1}, c_{f_1}, c_{e_2}, c_{f_2})$. Denote by $\star$ the inputs associated with $h^*$. The minimum budget required to reach the health level $h^*$ is $m^*$ and $pf(c_{f_i}^\star) = m^*$.

For budgets $m > m^*$, the health level decreases independent of the type of calories consumed (i.e., malnutrition due to overnutrition). The case where too many nutrients are consumed is consistent with a negative health marginal product of full calories $c_{f_i}$ (i.e., the inverse “U” shape curve discuss by Bolin and Lindgren (2016)).

\(^7\)According to the Dietary Guidelines released jointly by the U.S. Departments of Agriculture (USDA) and Health and Human Services (HHS), a healthy diet allows for a certain proportion of the total calories consumed to be discretionary calories. They provide energy but without nutrients, hence inducing a nonincreasing effect on health. Without lost of generality, in this analysis I assume that health is decreasing rather that nonincreasing in empty calories.
Next, consider the health economist’s problem:

\[
\text{max} \left\{ H(c_{e1}, c_{f1}, c_{e2}, c_{f2}) : pf(c_{f1})+e(c_{e1}) \leq m, \ pf(c_{f2}) + e(c_{e2}) \right. \\
\left. \leq (1 + \beta(c_{e1}))m, c_{f1} + c_{e1} \geq \bar{c}, i = 1, 2 \geq \bar{c} \right\}
\] (3.9)

Problem (3.9) mirrors (3.8) with the addition of the budget constraints, where \(m(1 + \beta(c_{e1}) \leq m^*,\) and the minimum calories intake constraints. For expositional convenience, we assume \(\beta(c_{e1}) = 0.\) Later, this assumption will be relaxed. Depending on the income, the minimum caloric intake requirement could be binding. Hence, two cases emerge depending on whether this constraint is binding or not.

First, if \(pf(\bar{c}) \geq m,\) the health economist can cover the minimum caloric intake requirement exclusively from the consumption of full calories. The minimum caloric intake is not binding. Hence, \(c_{e1} = 0, i = 1, 2,\) and the optimal health level is \(H(0, c_{f1}(m, p, 1, \bar{c}), 0, c_{f2}(m, p, 1, \bar{c})).\) For \(m < m^*\), the optimal health level in (3.9) is lower than in (3.8), because the budget constraint forces the health economist to consume a lower amount of full calories than the nutritionist. For a visual representation, compare point \(B\) (i.e. health economist choice) with point \(N\) in Figure 3.1.

An observation is in order. For comparison purposes, \(\beta(c_{e1})\) was assumed to be 0. However, for \(\beta(c_{e1}) > 0\) (i.e., the productivity of empty calories in the first period is strictly positive), the health economist will find it optimal to consume a strictly positive amount of empty calories in the first period, if the loss in health is offset by the gain in health from consuming additional full calories \(c_{f2}\) in the second
period. The nutritionist, however, will always choose to consume only full calories because she does not face any budget constraints.

Second, if \( pf(\bar{c}) < m \), the health economist can not cover the minimum calories intake requirement, \( \bar{c} \), exclusively from the consumption of full calories, \( c_{f_2} \). Thus, the amount of empty calories consumed will be strictly positive, \( c_{e_2} > 0 \). However, because health is decreasing in the consumption of \( c_{e_2} \), the amount of empty calories consumed will be just enough to cover the minimum consumption level \( \bar{c} \). Hence, the minimum caloric intake constraint will be binding \( c_{e_2} + c_{f_2} = \bar{c} \). Furthermore, \( \hat{c}_{f_2} < \bar{c} \) implies that \( \hat{c}_{f_1} < \bar{c} \), thus \( c_{e_1} > 0 \) even if \( \beta(c_{e_1}) = 0 \), in which case \( c_{e_1} + c_{f_1} = \bar{c} \). Visually, these choices are represented by points such as \( F \) and \( G \) in Figure 3.1.

Because the budget does not cover the minimum calorie intake requirement exclusively from the consumption of full calories, the amount of empty calories consumed is strictly positive. Furthermore, the health economist has no preference over the consumption of \( c_{e_2} \), hence the additional consumption of empty calories is just enough to cover the minimum calorie intake requirement \( \bar{c} \). In Figure 3.1, for the budget \( m_1 \) and prices \((p, 1)\), the health economist chooses the health level corresponding to point \( F \). The consumer, however, can choose anything between \( C \) and \( F \), the consumer’s optimum health level being decided by the preferences over health and the consumption goods. Thus, the health economist’s optimal health derived from problem (3.9) will serve as a higher bound for the health level derived by the consumer in (3.1).
3.2.3 Tax Effects on Health

Consider a tax $\mu > 0$ on the consumption of the nutrient deficient food products $t$ (i.e., sugar tax, fat tax). The tax raises the price of $t_i$ from 1 to $1+\mu$, changing the relative prices of $t_i$ and $s_i$. It is expected that the tax will induce the consumers to change the mix of calories in the favor of $c_f$, increasing the health level.

However, this may not occur when the minimum calorie intake is binding. In this case, the optimal consumption of full calories in the first and second periods must satisfy the budget constraints $pf(c_{f1}) + (1+\mu)e(\bar{c} - c_{f1}) \leq m$ in the first period, and $pf(c_{f2}) + (1+\mu)e(\bar{c} - c_{f2}) \leq (1 + \beta(c_e))m$ in the second period, respectively.

By the implicit function theorem, it follows that

$$\frac{\partial c_{fi}}{\partial \mu} = \frac{e(\bar{c} - c_{fi})}{(1+\mu)\partial e/\partial c_e - p\partial f/\partial c_f}, \quad i = 1, 2 \quad (3.10)$$

Because $p\partial f/\partial c_f - (1 + \mu)\partial e/\partial c_e_i$ equals the inverse of the shadow price of the budget constraint $i$, it follows that $\partial c_{fi}/\partial \mu < 0$. Increasing the price of empty calories, by imposing a tax on $t_i$, will decrease health if the minimum calorie intake is binding. Visually, this is equivalent to moving from a health level $F$, in Figure 3.1, to a lower health level $G$. Clearly, a health conscious consumer will be made worse off by the tax. If $(1 + \beta(m))m/p < \bar{c}$, then for any level of the tax $\partial c_{f2}/\partial \mu < 0$. 

60
3.3 Empirical Analysis

This section presents calibration results of the theoretical model based on data characterizing immigrant farmworkers in the US. Beside providing a vital service to the agriculture and food system in the U.S., immigrant farmworkers and their families are vulnerable to health issues resulting from food insecurity (Weigel et al., 2007; Kilanowski and Moore, 2010). Specific data used consists of farm workers income, daily wage, daily calories needs and optimal consumption of full calories.

According to the findings of the National Agricultural Workers Survey (NAWS), the average age of a farmworker is 38, and males comprised 72% of the hired crop labor force in 2013 – 2014.\(^8\) Based on the Dietary Guidelines for Americans for 2015 – 2020, the estimated calorie needs per day for a physically active individual in the age group 36 – 40 are 2,800 for men and 2,200 for a women.\(^9\)^10 Thus, the weighted average calorie needs of a farmworker is 2,632 calories. Borre, Ertle, and Graff (2010), surveying migrant and seasonal farmworkers families, find the median calorie intake of food insecure farmworkers is around 1,500, which considering the level of physical activity is consistent to minimum intake required to sustain life.\(^11\)

I use available data to characterize the optimal behaviour of a health economist

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\(^9\)Active means a lifestyle that includes physical activity equivalent to walking more than 3 miles per day at 3 to 4 miles per hour, in addition to the activities of independent living.

\(^10\)Jointly released by the U.S. Departments of Agriculture (USDA) and the U.S. Department of Health and Human Services (HHS), the Dietary Guidelines for Americans is designed as a resource for health professionals and policymakers. The calories needs are provided in Appendix 2 of the guidelines (https://health.gov/dietaryguidelines/2015/guidelines/appendix-2/#table-a2-1)

\(^11\)A low-calorie diet requires reducing daily calorie intake to 1,200 to 1,500 for women, but no less than 1,000, and 1,500 to 1,800 for men, but no less than 1,200 (http://www.webmd.com/diet/lowlcaloriediet).
agent facing budget and minimum calorie intake constraints. Health is assumed to be an increasing function of full calories consumption, with a maximum health reached at a consumption of 2,632 full calories, and a minimum calorie intake of at least 1,500 calories. Health is assumed to be decreasing in the consumption of empty calories.

Specifically health is assumed to follow the quadratic equation\(^{12}\)

\[
H(c_{f1}, c_{e1}, c_{f2}, c_{e2}) = -c_{f1}^2 + 5,264c_{f1} - c_{e1} + \delta(-c_{f2}^2 + 5,264c_{f2} - c_{e2}) \tag{3.11}
\]

where \(\delta\) is a discount factor equal to 95%, which is consistent with discount rates used in other studies (Laibson, Repetto, and Tobacman, 2007) and 5,264 is chosen to ensure that the production function of health reaches a maximum at a consumption of 2,632 full calories. I assumed that the consumption of empty calories in the first period, \(c_{e1}\), increases the budget available in the second period by a factor \(\beta\), where \(\beta\) is the marginal effect of consuming an empty calorie in the first period on the second period budget. For each \(c_{e1}\) consumed, the second period budget increases by \(\beta \times 0.00176\), where 0.00176 is the price of \(c_{e1}\) (Monsivais and Drewnowski, 2007).

This is consistent with a piece rate payment (i.e. payment is contingent on the quantity harvested) that characterized farm workers remuneration. In order to boost their productivity, and so increase their income, farm workers can consumed energy-dense, but nutrient poor, food products. For the analysis, different values for \(\beta\) are

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\(^{12}\)The functional representation of the health technology was selected because of its tractability and simplicity. I am indebted to Professor Olson for encouraging me to justify the choice of this functional representation.
While health is positively correlated with the consumption of nutrient-dense foods (i.e., whole grains, lean meats, low fat dairy products, vegetables and fruits) and negatively correlated with the consumption of energy-dense, but nutrient poor foods (i.e., refined grains, sweets and fats), the energy cost of foods (i.e., the price of calories ($/kcal)) increases with the nutritional content (Drewnowski, 2010). Furthermore, the energy-density of foods, measured as kilocalories per gram (kcal/g), is negatively correlated with the nutrient content and the energy cost (Monsivais, Mclain, and Drewnowski, 2010). Estimation based on retail food prices puts the energy cost of foods, in the lower quintile of energy density, at $1.8/100kcal compared to $0.17/100kcal in the top quintile (Monsivais and Drewnowski, 2007). Similarly, the average cost of foods in the top quintile of nutrient density is $2.7/100kcal compared to an average of $0.33/100kcal in the lower quintile (Monsivais, Mclain, and Drewnowski, 2010). Because these food prices correspond to bottom and top quintile of nutrient and energy densities foods, and to insure that farmworkers can purchase foods with a caloric content of 2,632 full calories, the cost of healthy calories is decreased by 30%.

According to the Farm Labor Survey of the National Agricultural Statistics Service, the wage for a non-supervisory farm laborer was $10.80 per hour in 2012, and according to the Bureau of Labor Statistics the average annual wage for agricultural workers is $25,650. Based on the The National Agricultural Workers Survey, the average daily number of hours worked by migrant farm workers is 8 hours. While farm workers shifts may be longer than 8 hours, the 8-hour figure is reasonable if
we think in terms of total number of hours worked per year divided by the total number of days available in a year.

I use available data to calibrate the optimal consumption of calories by a consumer with Cobb-Douglas utility facing budget and minimum caloric intake constraints. Specifically, the consumer maximizes:

\[
Ac_1^{\alpha} c_1^{1-\alpha} - c_2^2 + 5, 264c_1 - c_1 + \delta(Ac_2^{\alpha} c_2^{1-\alpha} - c_2^2 + 5, 264c_2 - c_2) \tag{3.12}
\]

where \( A = 100 \) to allow utility values to be comparable with health values (i.e. health reaches a maximum at a consumption of 2,632 full calories). The parameter \( \alpha \) takes three distinct values (i.e. 0.5, 0.1 and 0.9), in order to allow for variation in taste over consumption of full and empty calories. A higher value of \( \alpha \) suggests a higher taste preference for the consumption of full calories, and a lower preference for the consumption of empty calories.

Table 1 provides estimates of the optimal consumption of calories for the health economist agent and the consumer under different scenarios. First, \( \beta \) is assumed to take four distinct values: 0, 0.5, 1, and 1.5. A value \( \beta = 0 \) means the consumption of empty calories in the first period has no effect on the second period budget. Anecdotal evidence suggests farmworkers, especially illegal immigrants who are socially more vulnerable, spend most of their resources on rent, off-season savings and to support their families back home. Thus, I assumed that farmworkers spend between 10% and 40% of their daily income on food purchases. Based on
data available, this is equivalent to an expenditure in the range of $8.64 to $34.56 per day. This is consistent with Borre, Ertle, and Graff (2010), who find that the minimum daily grocery spending per person by the food insecure families is $9.52 while the maximum spending by migrant farmworkers is $22.86.

Calibration results suggest that health is not decreasing in $\beta$ for the health economist. As long as the minimum caloric intake constraint is binding, the health level is increasing. The necessary consumption of empty calories in the first period relaxes the budget constraint in the second period allowing a higher consumption of full calories. This holds independent of the value of $\beta$.

For sufficiently large values of $\beta$, the health economist will consume additional empty calories in the first period, reducing first period health, in order to increase the second period consumption of full calories, and overall health. This is the case for $\beta = 1.5$, when the consumption of empty calories increases from 0 to 570 in the case of a 30% food expenditure allocation. For the 40% food expenditure, the health economist already reaches the maximum health, thus $\beta$ has no effect on empty calories consumption.

For lower values of $\beta$ (i.e., 0 and 0.5) and income allocation (i.e., 10% and 20%), consumer's health choices coincide with those of the health economist independent of the taste (i.e., value of $\alpha$). Nonetheless, it suggests that information campaigns, tailored at decreasing the weight consumers place on the utility derived from consumption of foods versus the effect on health, could have beneficial results. Outside these values, for a fixed level of $\beta$ and income allocation, consumer’s health is increasing in $\alpha$ (i.e., the taste preference for the consumption of full calories,
For a fixed level of income and taste, the consumption of both type of calories increases in $\beta$, but the consumption of empty calories increases at higher rate than the consumption of full calories. For example, for income allocation of 30% and $\alpha = 0.5$, consumption of empty calories increases by 44% for a change in $\beta$ from 1 to 1.5, while the full calories consumption increases only by 2%.

Finally, keeping constant both $\beta$ and $\alpha$, the consumption of full calories increases at a decreasing rate in income, while the consumption of empty calories initially decreases, but afterwards increases at an increasing rate in income. For example, for $\beta = 1$ and $\alpha = 0.5$, the consumption of full calories in the first period increases from 499, for a 10% budget allocation, to 1,262, for 20% budget allocation, to 1,912, for a 30% budget allocation, to finally 2,358 calories, for a 40% budget allocation. While, empty calories consumption in the first period decreases from 1,308 to 714 between a 10% and 20% budget allocation, to increase afterwards to 913 and 2,616 in the case of a 30% and 40% budget allocations. This provides some evidence that budget relaxing policies (i.e., food stamps) might have an effect only on the very poor consumers.

Table 2 provides estimates of the tax effect on the optimal consumption of full and empty calories for the health economist and the consumer under four different tax regimes (i.e. 0%, 5%, 10%, and 15%), and $\beta = 0$. Previous studies have employed tax regimes of 10% (Fletcher, Frisvold, and Tefft, 2010; Wang, 2015), while lower tax regimes were reported in other studies (Jacobson and Brownell, 2000; Schroeter, Lusk, and Tyner, 2008).
As long as the minimum calorie intake is binding, the tax raises the consumption of empty calories and decreases the consumption of full calories. This result provides evidence that changing the relative prices of full and empty calories is not sufficient to increase the consumption of full calories. Furthermore, this result holds independent of the taste preferences, since even the health economist responds to the tax regimes by increasing the consumption of empty calories, despite a decrease in health.

Alternatively, for the case where the minimum caloric intake is not binding, taxes decrease the consumption of empty calories, but they have a small effect on the consumption of full calories. Part of the reason is the big gap between the price of full calories and the price of empty calories.

3.4 Conclusion

This paper provides an alternative explanation to the observed trend of unhealthy food consumption. To accomplish this a theoretical model of unhealthy food consumption is provided, where the consumer maximizes utility over health and consumption of food over two periods. The consumer faces a minimum caloric intake constraint and benefits from a productivity boost derived from the consumption of unhealthy foods. If the minimum caloric intake is binding, a tax on the unhealthy food will have the opposite effect of decreasing the consumption of healthy foods and increasing the consumption of unhealthy foods. Another contribution of the paper is to allow the productivity boost (i.e. $\beta$) to be a function of empty calories,
as compared to full calories which was the norm in previous studies. Potential extensions of the model could allow $\beta$ to be a function of both empty and full calories.\footnote{I am indebted to Professor Olson for suggesting this extension of the model.}

The theoretical model predictions are calibrated with data on food consumption patterns of immigrant farmworkers in the U.S. In terms of robustness, the calibration’s results can be improved by testing alternative functional forms for the health technology and the consumer’s utility. For example, the currently Cobb-Douglas utility function can be replaced by a more general constant elasticity of substitution utility. Also as mentioned above, the theoretical predictions can be expanded by allowing $\beta$ to be a function of both type of calories.\footnote{I am indebted to Professor Olson for encouraging me to discuss the robustness of the results derived in this chapter.} Future directions of research could also explore in detail the effect of information and subsidy on the optimal consumption of calories considering the above constraints imposed by minimum calorie requirement and intertemporal productivity.
Table 3.1: Full and Empty Calories Consumption for a Health Economist Agent and Consumer with Cobb-Douglas Utility

<table>
<thead>
<tr>
<th>Income (%)</th>
<th>Health Economist</th>
<th>Consumer (α = 0.5)</th>
<th>Consumer (α = 0.1)</th>
<th>Consumer (α = 0.9)</th>
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</thead>
<tbody>
<tr>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>40%</td>
<td>10%</td>
</tr>
<tr>
<td>Income ($)</td>
<td>8.64</td>
<td>17.28</td>
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1 Percentage of the daily income spent on food purchases. 2 Daily dollar amount spent on food purchases. 3 α is utility power associated with the consumption of full calorie $c_{fi}, i = 1, 2$. 4 β is the marginal effect of consuming an empty calorie in the first period on the second period budget. For each $c_{ei}$ consumed the second period budget increases by $\beta \times 0.00176$, where 0.00176 is the price of $c_{ei}$. 
Table 3.2: Full and Empty Calories Consumption for a Health Economist Agent and Consumer with Cobb-Douglas Utility under Empty Calories Taxation

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<th>Income (%)&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Health Economist</th>
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<th>Consumer (α = 0.1)</th>
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<td>975</td>
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<sup>1</sup> Percentage of the daily income spent on food purchases.  
<sup>2</sup> Daily dollar amount spent on food purchases.  
<sup>3</sup> α is utility power associated with the consumption of full calorie $c_{fi}, i = 1, 2$. 
Figure 3.1: Health Choices
Appendix A: Proof Theorem 1.3

Theorem 1.3: By Bellman’s principle, the farmer’s problem can be written:

\[
\max_{q_t, q_{t+1}, l} \left\{ \begin{array}{l}
W(q_t, q_{t+1}, l) : Ah + p \cdot z + g_{t+1} + rl_o \geq q_{t+1}, \\
\omega_l(g_{t+1}) + g_t \geq q_t + c(w, z, l_f) + v^\top h, \\
L \geq l_o + l_f + l
\end{array} \right\}.
\]

Because \( W \) is strictly increasing in \( q_t \), following Chambers (2007) and Chambers and Quiggin (2009), this problem reduces to

\[
\max_{q_{t+1}, l} \left\{ \begin{array}{l}
W(q_{t+1}, l) : c(w, z, l_f) + v^\top h : \\
\omega_l(g_{t+1}) + g_t - \min_{l_o, l_f, h, z} \left\{ Ah + p \cdot z + g_{t+1} + rl_o \geq q_{t+1}, q_{t+1}, l \right\}, q_t, l
L \geq l_o + l_f + l
\end{array} \right\}.
\]

Strict monotonicity of \( W \) in \( q_{t+1} \) and \( l \) ensures in the optimum that

\[
Ah = q_{t+1} - p \cdot z - g_{t+1} - rl_o \quad \text{(A.1)}
\]

\[
L = l_o + l_f + l.
\]
Expression (A.1) requires that \( q_{t+1} - p \cdot z - g_{t+1} - r l_o \in M \). By the Projection Theorem (see, for example, Luenberger 1969, Theorem 3.3.2), the unique interior portfolio solving (A.1) is

\[
h = P(q_{t+1} - p \cdot z - g_{t+1} - r l_o).
\]

Using \( L = l_o + l_f + l \) to eliminate \( l_o \) gives

\[
h = P(q_{t+1} - p \cdot z - g_{t+1} - r (L - l - l_f)).
\]

Thus, the decision maker’s problem concentrates as

\[
\max_{q_{t+1}, l} \left\{ W(\omega_t(g_{t+1}) + g_t) - \min_{l_f, z} \{ c(w, z, l_f) + v^\top P(q_{t+1} - p \cdot z - g_{t+1} - r (L - l - l_f)) \} \right\}.
\]

But

\[
- \min_{l_f, z} \begin{bmatrix} c(w, z, l_f) \\ + v^\top P(q_{t+1} - p \cdot z - g_{t+1} - r (L - l - l_f)) \end{bmatrix} = -v^\top P(q_{t+1} - g_{t+1} - r (L - l)) + \max_{l_f, z} \left\{ v^\top P(p \cdot z - rl_f) - c(w, z, l_f) \right\},
\]

which is the desired result. Concavity of \( W \) and convexity of \( c \) ensure a global optimum.
Appendix B: Pricing

The optimal stochastic agricultural output $z$ can be determined as a solution to the profit maximization problem

$$
\Pi(p, w, r, v_1/A, v_2^T(D^TD)^{-1}D^T) = \max_{z,l} \left\{ v_2^T(D^TD)^{-1}D^Tp z - c(w, z; l) - \left[ r - \frac{s}{2} \left( \frac{v_1}{A} + v_2^T(D^TD)^{-1}D^T 1^s \right) \right] l \right\}
$$

Assuming the profit function is differentiable, the first order conditions for the output $z$ are

$$(z_s): \frac{\partial c(w, z; l)}{\partial z_s} = v_2^T(D^TD)^{-1}D_s^T p_s, \forall s \in S$$

from where

$$(z_s): \frac{\partial c(w, z; l)}{\partial z_s} \frac{1}{p_s} = v_2^T(D^TD)^{-1}D_s^T, \forall s \in S$$

or in vector notation

$$\nabla_z c(w, z; l) = v_2^T(D^TD)^{-1}D^T, \forall s \in S$$
post multiply by $D$ to obtain

$$\frac{\nabla z c(w, z; l)}{p} D = v_2$$

which must hold for any asset $j$

$$\frac{\nabla z c(w, z; l)}{p} D_j = v_{2j}, \ j = 1, \ldots, J$$

multiply and divide every states $s$ by its associate probability $\pi_s$

$$\frac{\nabla z c(w, z; l)}{p} \frac{\pi_s}{\pi_s} D_j = v_{2j}, \ j = 1, \ldots, J$$

given the expectation form

$$E\left[\frac{\nabla z c(w, z; l)}{p} \tilde{D}_j\right] = v_{2j}, \ j = 1, \ldots, J$$
Bibliography


