

## ABSTRACT

Title of Dissertation:                   DAMAGE PRECURSOR BASED  
STRUCTURAL HEALTH MONITORING  
AND PROGNOSTIC FRAMEWORK USING  
DYNAMIC BAYESIAN NETWORK

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Structural health monitoring (SHM), as an essential tool to ensure the health integrity of aging structures, mostly focus on monitoring conventional observable damage markers such as fatigue crack size. However, degradation starts and progressively evolves at microstructural levels much earlier than detection of such indicators.

This dissertation goes beyond classical approaches and presents a new SHM framework based on evolution of Damage Precursors, when conventional direct damage indicator, such as crack, is unobservable, inaccessible or difficult to measure.

Damage precursor is defined in this research as “any detectable variation in material/ physical properties of the component that can be used to infer the evolution of the hidden/ inaccessible/ unmeasurable damage during the degradation”.

Accordingly, the degradation process is to be expressed based on progression of damage precursor through time and the damage state assessment would be updated by incorporating multiple different evidences. Therefore, this research proposes a systematic integration approach through Dynamic Bayesian Network (DBN) to include all the evidences and their relationships.

The implementation of augmented particle filtering as a stochastic inference method inside DBN enables estimating both model parameters and damage states simultaneously in light of various evidences. Incorporating different sources of information in DBN entails advance techniques to identify and formulate the possible interaction between potentially non-homogenous variables. This research uses the Support Vector Regression (SVR) in order to define generally unknown nonparametric and nonlinear correlation between some of the variables in the DBN structure.

Additionally, the particle filtering algorithm is studied more fundamentally in this research and a modified approach called “fully adaptive particle filtering” is proposed with the idea of online updating not only the state process model but also the measurement model. This new approach improves the ability of SHM in real-time diagnostics and prognostics.

The framework is successfully applied to damage estimation and prediction in two real-world case studies of 1) crack initiation in a metallic alloy under fatigue and, 2) damage estimation and prognostics in composite materials under fatigue. The proposed framework is intended to be general and comprehensive such that it can be implemented in different applications.

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PROGNOSTIC FRAMEWORK USING DYNAMIC BAYESIAN NETWORK

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## Dedication

*To my beloved Mom and Dad*

*Who sacrificed so much for me,*

*and gave me faith and courage to pursue my dreams*

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Ph.D. is a long journey full of struggles, ups and downs, and unique experiences. I would have never got to the finish line, if I was alone. I would like to thank each and every one who has supported me, encouraged me and helped me throughout this path. Most importantly, I would like to express my sincere gratitude to my dear advisors Dr. Enrique Droguett and Dr. Mohammad Modarres for their support, patience and guidance. Dr. Modarres has been always an intellectual mentor for me who never withholds his advice and support in all the time of research and writing of this dissertation.

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# Chapter 1: Introduction

## *1-1 Overview*

Mechanical systems are susceptible to progressive accumulation of damage during their service life. It is crucial to detect damages at earliest possible time before they lead to system failure. Time-based scheduled maintenances have been widely used to perform Non-Destructive Inspections (NDI) on structural and mechanical systems. However, these kinds of maintenance are very labor intense and costly as they require the system to be taken out of service or disassembled; in addition, the intervals between inspections should be chosen conservatively to make sure that the existing damage would not reach the critical region before the next scheduled inspection.

Structural Health Monitoring practice exploits the Condition-Based Maintenance (CBM) to avoid costly unscheduled and/or unessential repairs that are common in traditional time-based maintenance (Rabiei and Modarres, 2013a). Structural health monitoring uses CBM to continuously monitor the health state of the system via online sensors. In addition to in-situ CBM, SHM is closely related to NDI techniques which are usually carried out off-line and with *a priori* knowledge of the damage location (Farrar and Worden, 2007).

The majority of the existing SHM frameworks use empirical damage models such as the Paris Law for fatigue crack growth to estimate the damage state; these models, however, are mostly established based on monitoring and measuring observable

damage markers or indicators of damage such as the crack length. Notwithstanding the significant development made by conventional SHM techniques in fault diagnostics and failure prognostics, by the time that the common observable damage indicator (such as crack) can be detected and measured, most of the life of the component has already passed (Cantrell, 2006). In addition, considering the detection uncertainty involved in any NDI or sensor-based method, there is always a chance that such damage remains undetected. Therefore, there is a valuable time window from the initial degradation of characteristics at material state until the emergence of measureable damage markers. Even though no observable damage indicator might be detected in this period, degradation is progressively happening inside the component. This reveals the importance of attention to indirect damage indicators referred to as damage precursors. There is no concise and universally accepted definition for damage precursor in the literature. In this research, damage precursor is defined as “any detectable variation in material/ physical properties of the component that can be used to infer the evolution of the hidden/ inaccessible/ unmeasurable damage during the degradation”. This definition applies to both homogeneous and composite materials. Contribution of information content of damage precursor into the traditional SHM frameworks provides a powerful platform to estimate the health state of a component/system even when direct signs of damage such as crack are not visible or measureable yet.

Damage estimation and prognostics in SHM are rooted in the Recursive Bayesian Estimation, also known as Bayesian filtering, problem. The Bayesian filtering problem, which involves probabilistic inference of the state of a system that changes

over time using a sequence of noisy measurements, has been around for quite a long time (almost since early 1960's) (Kalman, 1960; Chen, 2003). Different techniques, especially methods in the Kalman filtering family and the more advanced Particle Filtering (PF), have been proposed and implemented to address this problem. Unlike Kalman filtering, particle filtering does not require any restrictive assumptions and is particularly useful to address non-linear problems with non-Gaussian process/observation noise. The application of particle filtering in reliability field is quite new (Orchard and Vachtsevanos, 2009) and its popularity is rapidly increasing in the recent years because of its flexible and powerful features. However, most of the recent works in this area (Butler and Ringwood, 2010; N. Eleftheroglou and Loutas, 2016; Orchard and Vachtsevanos, 2009) only consider one observation for updating the state estimations. With recent advanced sensing and monitoring technologies, it is important to develop a framework which is capable of fusing various measurements and informative evidences from multiple sources and with different nature. It is especially important when dealing with less explored areas of study such as damage precursor because higher level of uncertainty is expected. Any piece of related information can be influential in reducing the inherent uncertainty and obtaining more robust and reliable estimations and predictions. Available information might be very diverse in nature which comes from different sources such as various online or offline sensors, physics-based or data-driven models, expert opinion, and reliability data. Accordingly, the idea of the present study is to develop a SHM framework which can integrate multiple evidences with different characteristics to reduce the uncertainty and acquire more precise damage estimation.

Dynamic Bayesian Network (DBN) has been recognized as one of the powerful approaches for integrating various complex time-dependent and uncertain variables (Dong and Yong, 2008; Medjaher et al., 2012; Iamsurang et al., 2015; Rabiei et al., 2015a). Moreover, DBN has shown capability in managing hidden or latent variables (state or parameter) which cannot be observed or measured directly. Therefore, DBN is the foundation of this research to aggregate different evidences into the SHM framework and particle filtering is chosen as the primary computational methodology for making probabilistic inference in DBN about the hidden state of damage.

Furthermore, Particle Filtering is studied more fundamentally in this research. This approach consists of two main components: state process model and measurement model. Several papers have worked on state process model and proposed some methods to update the model parameters in state process model while using a predefined measurement model (Kitagawa, 1998; Liu and West, 2001; Tulsyan et al., 2013; Hu et al., 2015a). However, to the best of our knowledge, no major study exists on updating the measurement model as well. In most of the online real world applications, the correlation between the measurements and the hidden damage is not defined in advance and therefore, presuming an offline fixed measurement model is not promising. Therefore, a modified version of particle filtering called “*Fully Adaptive Particle Filtering*” is proposed in this research which can dynamically update the measurement models in addition to state process model. The proposed particle filtering algorithm is a significant step forward toward more realistic online SHM because, in real world application, usually there is no fully known and

predefined model that can fit to all cases, and consequently, it is important to adjust the models to the particular case study.

### *1-2 Research objectives:*

Within the scope of this research, we seek an answer to the question of how to construct a new modeling and computational approach for on-line SHM framework by relying on indirect damage indicators or damage precursors? Moreover, this study investigates a systematic approach to address the challenge of fusing several sources of various information to achieve a more robust and reliable SHM framework. The approach inquires a more advance probabilistic fusion techniques for model-based and data-driven models in order to integrate different types of uncertain information.

Therefore, the main objectives of this research are as follows:

- Investigate the idea of incorporating indirect damage indicators, or more specifically damage precursors, into SHM.
- Develop a general damage precursor-based SHM framework which is capable of inferring the degradation state in the structure when direct damage indicator such as fatigue crack is inexistence, undetected or difficult to measure.
- Establish a methodology for fusing different sources of potentially non-homogeneous evidence in order to achieve a more precise damage estimation through time
- Develop a hybrid probabilistic modeling approach based on Dynamic Bayesian Network to formulate the SHM framework through time and to predict the RUL. The DBN algorithm is the combination of:

- Model-based particle filtering technique to infer the damage state based on evolution of damage precursor using various noisy measurements.
- Data-driven techniques including regular regression techniques and more advanced ones such as Support Vector Regression to learn the unknown relationship between some of the variables in DBN from data.
- Explore a methodology to learn the damage model parameters in real time when dealing with partially known degradation processes.
- Develop a fully adaptive version of particle filtering algorithm that does not require a predefined measurement model to explain the relationship between the hidden damage state and noisy measurements. This new approach should be capable of learning both the measurement model as well as the state process model in real time.

### *1-3 Dissertation's Outline*

This dissertation is arranged into the following chapters.

Chapter 2 presents a review of the background and related studies in SHM, followed by different possible sources of information that can be included in the SHM framework.

Chapter 3 focuses on dynamic Bayesian network, its representation, features and capabilities including some recent literatures that applied DBN in SHM.

Chapter 4 contains the proposed damage precursor-based SHM methodology and theoretical framework. The chapter starts by definition of damage precursor and then continues by introducing the concept of damage precursor into the SHM framework.

Two damage precursor-based SHM frameworks are developed and explained in this chapter.

Chapter 5 shows the mathematical scheme of the research. Details of standard particle filtering algorithm for estimating the hidden states with the use of observed variables are reviewed. Then the idea of combined state and parameter estimation in particle filtering is explained in which both states and model parameters can be updated online simultaneously. This chapter also presents how the prognostics and predicting the Remaining Useful Life (RUL) is handled. And finally, support vector regression is introduced.

In chapter 6 and 7 the results of applying the proposed methodology are presented in two case studies. Chapter 6 focuses on degradation of the metallic component (7075-T6 Aluminum samples) under fatigue before crack initiation. In chapter 7, a more complicated degradation process in composite material (Glass/Epoxy (G10/FR4) composite laminate) is studied in which measuring conventional damage markers such as micro-cracks is very difficult.

In chapter 8, the idea of adaptive measurement model is presented and the modified version of the particle filtering is formulated. The new algorithm is then applied on a similar case study of composite degradation.

Chapter 9 presents a complete summary of the all chapters and provides the contributions of this research. Finally, it covers some of the potential future works to improve this study.

## Chapter 2: Structural Health Monitoring

### *2-1 Overview*

This chapter presents a brief history of the Structural Health Monitoring in reliability field and then, the most common modeling approaches for formulating the SHM are discussed. Generally, these modeling techniques are classified in three groups: physics-based models, data-driven models and hybrid models. The characteristics of each group are briefly explained in this chapter and some of the recent research works in each area are reviewed.

It is very important to exploit any piece of available information for achieving more reliable and robust SHM framework. The last section of this chapter introduces potential sources in reliability field that one can extract useful information from.

### *2-2 Introduction on Structural Health Monitoring*

Structural Health Monitoring emerged as a technology built upon Condition-based maintenance (CBM) rather than time-based maintenance. The SHM framework assesses the system's health state by utilizing online sensors to monitor the condition of critical components. Therefore, SHM paves the way for more cost effective maintenance by continuous awareness of equipment health condition that results in avoiding unnecessary/unplanned maintenance (Rabiei, 2011). In general form, SHM can be seen as a pattern recognition paradigm. Farrar (Farrar and Worden, 2007) describes SHM as four-step pattern recognition process: (i) Operational evaluation to



realize what components need to be monitored and how, (ii) data acquisition, normalization and cleansing, (iii) feature extraction to identify characteristics of the collected data that can distinguish between damaged and undamaged structure and (iv) statistical model development to apply algorithms on the extracted features in order to estimate the health state of the structure. Each of these steps requires significant effort and understanding; so, most of the published research in this field have just covered some aspects of the whole process.

Rytter (Rytter, 1993) proposed a four-level damage hierarchy as a broader framework for structural health monitoring.

Level 1 – Detection: this level concerns with identifying whether or not there is damage in the system.

Level 2 – Localization: after detecting that damage is present in the system, further information is required to indicate its location.

Level 3 – Assessment: this level of the process provides estimation of the severity of damage providing information of the extent of damage

Level 4 – Prediction: The valuable information gathered in previous levels would be useless if it is not utilized to infer the safety of the structure and predict the remaining useful life.

The first three levels of Rytter’s damage hierarchy can be encapsulated in one word as “Diagnosis” (Gertler, 1998) and the forth level is the “Prognosis”. Therefore, it can be inferred from (Rytter, 1993; Gertler, 1998; Farrar and Worden, 2007; Farrar and Lieven, 2007) that the SHM problem might be seen as a combination of two concepts of “Diagnosis” and “Prognosis”.

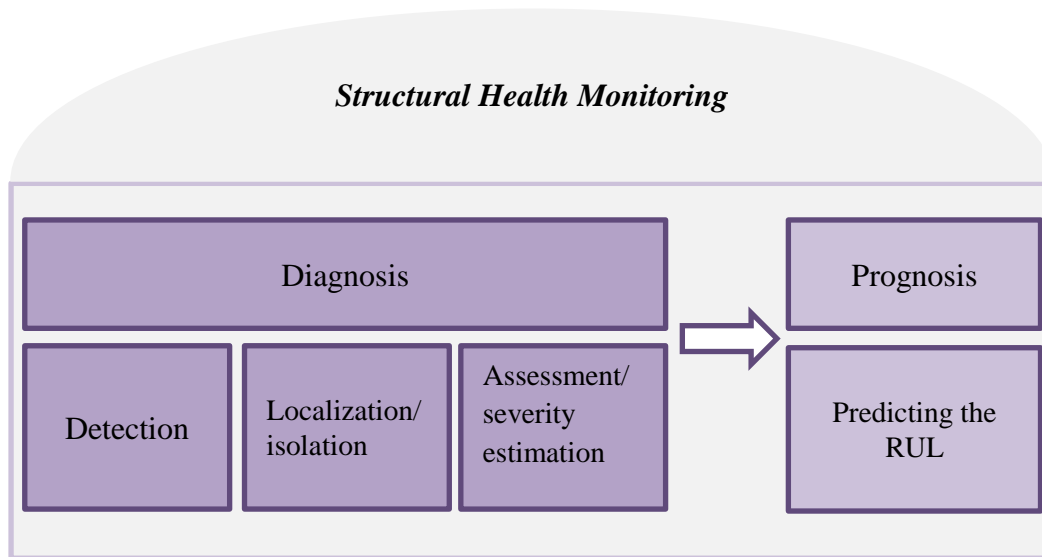


Figure 2-1: SHM as a diagnosis-prognosis procedure

In simple wording, at “Diagnosis” level of SHM, sensor technology is implemented to collect data on the condition of critical components in a system. This data will be studied to extract the important features that can relate to failure; as soon as statistical models can detect and localize any damage in the system, severity of the damage should be estimated. “Prognosis” step consists of estimation of remaining useful life of the system and inferring the health condition of the system in short term or long-term future. And finally, based on performed diagnosis and prognosis, decisions would be made in order to plan for further maintenance actions. In this research, damage localization (level 2) is not specifically studied.

With the development in advance sensor monitoring and powerful data acquisition, computing and analysis techniques, diagnostics and prognostics have become the focus of many recent researches in different fields. In general, diagnostic and prognostic methods can be mainly categorized into three major paradigms: physics-based, data-driven, and hybrid (combination of aforementioned models) approaches.

### 2-3 *Physics-based models*

The physics-based or model-based approach uses a mathematical representation of the system based on the knowledge about the underlying physical mechanisms. A major advantage of the physics-based model is that the model carries certain behavioral resemblance to the actual system, thus changes in the model output can have explainable physical meaning. However, deriving mathematical models based on first principles might not be practical for complex systems due to the lack of full understanding of all the degradation modes. If understanding the physics of failure of the component is feasible, physics-based models are preferred over other types of models (Luo et al., 2003).

Physics-based has been adopted by several researchers for diagnostics and/or prognostics. For instance, Zhao et al. (Zhao et al., 2013) developed a physical model for prognostics in gears. The physical models include the finite element model for gear stress analysis, the gear dynamics model for dynamic load calculation, and the damage propagation model described using Paris' law. This physics-based model was then applied to estimate the remaining useful life of the gear. Model parameters and uncertainty factors were updated via Bayesian inference using the simulated condition monitoring data.

In another study, Daigle and Goebel (Daigle and Goebel, 2013) developed a physics-based prognostics framework to model different damage processes occurring simultaneously within a component. They illustrated their model-based algorithm on a detailed physics-based model of a centrifugal pump. They considered both impeller wear and bearing wear as two significant damage mechanisms of pumps. Joint state-

parameter estimation was then performed in order to estimate the health state of the component and to predict the remaining useful life.

#### 2-4 Data-driven models

Data-driven models rely only on previously observed data (such as historical data, monitoring data, and field data) to estimate the health state of the system and predict the projection of system state or to match similar patterns in the history to infer RUL. The advantage of data-driven models is that detailed understanding of underlying physical degradation process is not required. Therefore, data-driven models are good approaches especially for complex systems that their failure mechanism is not fully understood or is difficult to model with mathematical equations. However, they still have some limitations in industrial applications; for example the forecasting accuracy strictly depends on if the training data are adequate and representative of all the possible application conditions. Such a requirement is usually difficult to achieve in real-world applications. Also, sometimes understanding and finding any physical meaning for the results of data-driven models is challenging.

With the recent developments in computational capacities, data-driven models got attention from many researches. Statistical models, reliability functions, and machine learning techniques are some of the methods applied in data-driven approach. Si et al. (Si et al., 2011) presented a review of the recent literatures on statistical data driven models for prognosis and estimating remaining useful life. They classified existing approaches into two broad types of models, that is, models that rely on directly observed state information of the component/system (such as regression-based and

Markovian-based models), and those do not (including stochastic filtering and hidden Markov models).

Pattipati et al. (Pattipati et al., 2013) implemented data-driven models for fault detection, isolation and severity estimation of failure in Electronic Return-less Fuel System (ERFS). They compared different machine learning algorithms for classification and regression of the available data on fuel system. Data was classified into 6 groups of possible faults in fuel system. And then regression analysis was performed to estimate the severity of the fault. They did not carry out any prognostics and RUL estimations.

## 2-5 Hybrid models

As presented above, both physics-based and data-driven models have their own strengths and weaknesses. To leverage the advantages of both models, hybrid models were developed that considers the combination of them. Liao and Kottig (Liao and Kottig, 2014) presented a comprehensive review of hybrid prognostics approaches. Besides physics-based models and data-driven models, they also considered knowledge-based or experience-based models which adopt expert knowledge and engineering experience to infer the health state of the structure. These models are less seen in the literature as independent approach for SHM and are more applied along with physics-based and/or data-driven models. Therefore, Liao and Kottig (Liao and Kottig, 2014) categorized hybrid models into five groups: 1) Experience-based model and data-driven model, 2) Experience-based model and physics-based model, 3) Data-driven model and data-driven model, 4) Data-driven model and physics-based model, 5) Experience-based model, data-driven model and physics-based model.

The following presents some of the recent publications on group 4 of the hybrid models, which is the most popular approach in the literature.

The integration of data-driven and model-based models can be carried out through different approaches. In one of the most common approaches, data-driven models are used to infer a measurement model and then RUL is predicted by applying a physics-based model. (Cheng and Pecht, 2009; Kumar et al., 2008) developed prognostics framework that incorporates both data-driven and physics-based models, and provided remaining useful life estimates for “electronics systems”. A data-driven model was first used to determine parameters to monitor, estimate the system state, and detect anomalies by comparing the system conditions with a healthy baseline. Physics-based failure modes, mechanisms, and effects analysis (FMMEA) has been used to aid the parameter identifications. The estimation of RUL however, was basically done by simple regression techniques to project the trend of parameters variation in future time.

Mohanty et al. (Mohanty et al., 2007) also combined data-driven kernel based Gaussian Process Regression (GPR) model with physics-based state space model to estimate crack growth in metallic alloys under fatigue. The crack growth equation used in the state-space model is similar to the Paris equation but is modified for crack closure.

Many more studies can be found on hybrid models for crack size estimation for example Rabiei and Modarres (Rabiei and Modarres, 2013b) used Kalman Filter along with Paris law for crack size estimation in specimens made of Al 7075-T6 , and

Smith and Modarres (Smith and Modarres, 2016) implemented GPR for estimating small crack considering the probability of detection.

Several other works have been published in hybrid modeling especially in prognosis and health management of batteries which have used different techniques such as GPR, particle filtering and Relevance Vector Machine (RVM) to estimate state-of-charge (SOC), state-of-health (SOH) or state-of life (SOL) of the batteries (Goebel et al., 2008; Saha et al., 2009, 2007).

As a summary, it is intuitive to use a hybrid approach via combining physics-based models and data-driven models to leverage both their strengths to improve damage estimation and prediction performance. Although, data-driven measurement model implicitly incorporates uncertainty into the physics-based model, it also avoids exponential error accumulation due to the Paris type formulation (Mohanty et al., 2007).

## *2-6 Different sources of information*

As George Box has said “all models are wrong, but some are useful” (Box et al., 1987), all the empirical, physical and data-driven models suffer from various uncertainties and therefore they should be used with cautious. It is very important to reduce the inherent uncertainty by including different available information coming from various sources, which is usually called heterogeneous information (Bartram and Mahadevan, 2012), (Droguett et al., 2006). For diagnostics and prognostics in a structure, one might extract relevant information from the following sources:

### 2-6-1 Offline and online data

With the enormous improvement in sensing technology, various sensors and inspection methods are now available to examine the system. Online data is collected from in-situ or built-in sensors that can monitor system in real time and measure particular characteristics. Sensors are of different types and each type of sensor has its own characteristics in terms of amount and precision of the gathered data.

Offline data, however, comes from NDI that can be applied to mechanical systems in specific intervals. Although these two sources of measurement are crucial for diagnosis and prognosis, most of the time systems are not equipped with built-in sensors and also inspection intervals might be very long. As a result, there is always a risk of insufficient or lack of measurement data. Moreover, NDI techniques have specific precision for damage detection; i.e., they can only measure the damage when it is greater than specific threshold.

### 2-6-2 Partially relevant data

Direct measurement of the system/component of interest is not always accessible. However, one can take advantage of another source of information that can provide indirect or soft evidence. The key idea is to implement inspection data gathered from other similar systems; by term “similar” we mean that data might come from systems that are not identical to the target system, for example their operating or environmental conditions could be different. We call this evidence “partially relevant data”. This goal can be achieved through advance machine learning techniques. In order to incorporate partially relevant data into the SHM modeling one needs to:

- Find related systems that have online or offline data available.



- Identify all the similar and dissimilar features between the system of interest and other comparable systems.
- Develop data-driven similarity model using machine learning methods.

### 2-6-3 Expert opinion

Field experts can add valuable and beneficial information to any part of SHM framework. Expert opinion might provide relevant information about which variables should participate in SHM structure and how they are connected. Moreover, expert's knowledge and experience can also be used to directly infer some of the parameters of the model or to interpret the functionality of component. Therefore, expert opinion is an inseparable part of the SHM framework.

### 2-6-4 Other sources

Relevant published literature, reliability handbooks and historical data can also be considered to estimate the value of some model parameters.

Consequently, dealing with mixture of various types of information requires a combination of mathematical and probabilistic techniques. The model for prediction of remaining useful life needs to be capable of integrating heterogeneous evidence in order to reduce uncertainty in RUL predictions.

## 2-7 Summary

In summary, SHM is a growing area of research that focuses on assessing the state of the structural health at every moment during the life of a structure. It involves the integration of sensors, data acquisition, feature extraction and analysis, statistical

model development and RUL prediction. Structural health monitoring can be considered as an online process of diagnostics and prognostics on engineering structures with the goal of improving the reliability and integrity of the structure.

Different diagnostics and prognostics modeling approaches have been used in the literature. The most popular techniques are physics-based, data-driven and hybrid models. physics-based models relies on underlying physics of the degradation process, while the data-driven models tries to extract meaningful results based on observed historic or monitoring data. The hybrid models are the combination of physics-based and data-driven models. These models were discussed in this chapter.

Finally in this chapter, different sources of available information for developing a comprehensive SHM framework were introduced.

## Chapter 3: Dynamic Bayesian Network

### 3-1 Overview

Bayesian Networks (BNs) have been recognized as an essential tool in modeling systems with complex dependent and uncertain variables. They are being used effectively by researchers and practitioners more broadly in science, medicine, economics and engineering. A static BN is actually a marriage between probability theory and graph theory which represents a set of random variables and their conditional dependency with a graphical characterization. Random variables are shown as nodes and directed edges are used between nodes to represent their conditional probabilities. Bayesian networks are also called directed acyclic graph (DAG), meaning loops are not allowed in the structure of the BN; hence, edge that starts from node  $x$  cannot return to the same node. Dynamic Bayesian Network extends static BN to model systems that are dynamically changing or evolving over time.

Bayesian networks and their temporal extension as dynamic Bayesian networks can be particularly useful for monitoring, diagnostics and prognostics of systems under the presence of uncertainty. Especially DBN, which incorporates time-varying properties of the system, is appropriate for monitoring and predicting the health state of the component/system through time. Moreover, BN and DBN system models can support a hybrid data-driven/model-based SHM approach due to their ability to simultaneously incorporate many types of data and serve as a system model (Bartram

and Mahadevan, 2012). Therefore, DBN provides a simply understandable representation for modeling problems in structural health monitoring.

### *3-2 Bayesian Networks in Structural Health Monitoring*

Despite the advances in Bayesian Network researches, its application for probabilistic modeling of diagnosis and prognosis with the presence of different types of information remains modest. Most of the works are confined to simple models with discrete random variables or can only partially address dynamic aspects of real world applications (Iamsurang et al., 2015). This is mainly because of the fact that the complexity and computational cost of the BN and DBN algorithms increase significantly as the number of random variables in the network grows.

In the following the review of two of the most recent and relevant studies that implement BN or DBN in SHM are presented and after that, other similar works are mentioned and explained briefly.

Ling and Mahadevan (Ling and Mahadevan, 2012) developed a probabilistic approach to integrate model-based fatigue damage prognosis with structural health monitoring data, considering different types of uncertainties. They used Modified Paris' Law equation as fracture mechanics-based fatigue model for crack growth. Then online (operational load monitoring data extracted from built-in sensors) and off-line (crack size measured by NDI technique) monitoring data were applied to update the model. The focus of the paper however is more on uncertainty quantification that is identifying sources of uncertainty such as variability, data uncertainty and model selection uncertainty and then assessing the contribution of each source of uncertainty to the overall uncertainty. They have used static Bayes

Network to show the connection between the sources of uncertainty and errors. Gaussian process model is implemented for cycle-by-cycle simulation of crack growth. Although this study tries to perform prognosis under uncertain evidence, the modified Paris' law is only applicable if the initial size of the crack is known. They introduced the concept of Equivalent Initial Flaw Size (EIFS), as a function of material properties to bypass the complication of dealing with short cracks when short cracks are of most interest in SHM and PHM.

In another recent interesting study by Bartram and Mahadevan (Bartram and Mahadevan, 2014), which has been one of the motivations for the present research, DBN is used for online diagnosis via particle filtering. Future states of the system are predicted using the DBN and sequential or recursive Monte Carlo sampling. Although the use of particle filtering for inference in DBN is quite recent, Bartram and Mahadevan have assumed that relationship between all the nodes in the DBN are known through deterministic mathematical models, while this is not the case for most of the real world applications. So, no probabilistic fusion between correlative evidences is performed in their study. Moreover, model parameters and particle weights are assumed to be constant during the diagnosis and prognosis. It would be vital to update the parameters as the characteristics of the component might change during degradation.

Here are some of the other recent works in this area:

Sheppard and Kaufman (Sheppard and Kaufman, 2005) developed a diagnostic and prognostics approach based on DBN that incorporates information on failure

probability, instrument uncertainty, and the predictions for false indication. The focus of their study is more on identifying and modeling the uncertainty.

Ferreiro et al. (Ferreiro et al., 2011) used BN model in health assessment and prognostics for aircraft line maintenances. The focus of the article is on the global framework that allows the transformation of the traditional maintenance (preventive and corrective, time based) into a predictive maintenance based on prognostic techniques. They applied the model to predict the brake wear based on parameters such as weight of the aircraft, velocity and operation of the brakes during landing. However, the proposed Bayes Net contains purely discrete random variables based on predefined probability tables.

In another recent publication, Medjahar et al. (Medjaher et al., 2012) presented a data-driven prognostics method for the estimation of the RUL of critical physical components. The model mainly uses Mixtures of Gaussian hidden Markov models (MoG-HMMs), represented by dynamic Bayesian networks, to estimate the degradation of the bearings and predict the RUL before potential failure. Their proposed methodology consists of two phases of learning and exploitation. The prognostics performance was evaluated with a set of run-to-failure accelerated life tests. The method they used however is purely statistical.

Dong and Yang (Dong and Yong, 2008) use DBNs to estimate the RUL distribution of drill bits in a vertical drilling machine. The RUL of drill-bits is defined as the number of holes that can be drilled before the failure occurs. In their constructed DBN, RUL is considered as the hidden state which is updated based on observation of thrust-force and torque.

Rosunally et al. (Rosunally et al., 2013) used a Bayesian network to predict the remaining useful life of iron structures under corrosion from physics of failure models (linear bi-logarithmic law for atmospheric corrosion). A data-driven method was used to assess the system health status, and detect anomalies based on weight, dimension, and electrical resistance. The RUL distributions from the physics of failure models and the current health status, which were derived from the distributions of the data-driven results, were used as inputs for a Bayesian network model to obtain updated RUL predictions.

### 3-3 Summary

Dynamic Bayesian network has been known as a powerful and suitable framework in SHM diagnosis and prognosis because of some beneficial features (Bartram and Mahadevan, 2014; Iamsurang et al., 2015; Rabiei et al., 2016):

- DBN facilitates understanding of complex systems by providing a graphical representation of all the variables and their temporal and functional dependencies.
- DBN framework enables us to consider different sources of uncertainty that might exist in the system such as measurement uncertainty, detection error, modeling error, etc. (Ling and Mahadevan, 2012)
- DBN is efficiently capable of integrating information from different sources such as physical model, historical data, operational data, expert opinion, etc. (Bartram and Mahadevan, 2012)
- As soon as new information in terms of evidence or observation becomes available, DBN incorporates that piece of knowledge to update the belief state

of all the variables. This feature makes it favorable in damage diagnosis and prognosis in time.

Therefore, DBN can be a competent approach with favorable characteristics for modeling a comprehensive SHM framework.

This chapter provided the foundation for the modeling approach in the rest of the dissertation. The overview of BN and DBN and their application in SHM frameworks were presented and some state-of-the-art papers in this area were surveyed. And finally, the strengths of DBN in dealing with uncertain dynamic systems were discussed.



## Chapter 4: Proposed Structural Health Monitoring Framework Based on the Evolution of Damage Precursor

### 4-1 Overview

Although many papers have been published with the focus on damage diagnosis and prognosis in different fields, most of the proposed methodologies are established based on estimating conventional “direct damage indicators”, which are regarded as “observable markers of damage” in some studies, such as the fatigue crack size; see for instance (Orchard and Vachtsevanos, 2009; Zio and Pelsoni, 2011; Ling and Mahadevan, 2012; Rabiei and Modarres, 2013a; Keshtgar and Modarres, 2013). This chapter focuses on developing a new SHM framework based on alternative “indirect damage indicators”, as opposed to direct damage indicators, that can be used for early damage detection, when conventional observable damage markers are absent, inaccessible or difficult to measure.

This chapter begins with the definition of damage precursor followed by its important role in early detection of degradation. Then, some examples of identified damage precursors for fatigue loading are presented. Most importantly, the idea of damage precursor-based SHM framework is proposed and elaborated later in this chapter. The framework is formulated for two scenarios:

- 1) when there is a chance to detect, measure and model the direct damage indicator some times before the failure of the component (for example in fatigue of metallic components). In this situation, SHM can be modeled first

based on evolution of damage precursor and then based on variation of direct damage indicator as soon as it emerges.

- 2) when measurement of conventional direct damage indicator is very difficult or impractical throughout the lifetime of the component (for example in fatigue of composite materials). So, the SHM framework only relies on evolution of damage precursor.

Later, the idea of each framework is expressed systematically through general DBN structure.

In order to apply the proposed SHM framework for diagnostics and prognostics, a combination of different mathematical techniques is required. In the last section of this chapter, an integrated mathematical modeling approach is proposed that is desired to be flexible enough with minimum restrictions for applying in various case studies.

#### *4-2 Damage Precursor Definition*

It is crucial to detect damage at earliest possible time. The concept of damage is abstract to some extent, and its definition relies on the variables used as damage indicators or markers of damage to describe the aging or degradation process (Arson, 2012; Imanian and Modarres, 2015). In fact, definitions of damage due to physical mechanisms vary for different materials, geometries and scales (Imanian and Modarres 2015). At the very basis, some researchers looked for a microscopically consistent definition in the context of the Continuum Damage Mechanics (CDM) (Lemaitre, 1996). For example, in CDM the damage,  $D$ , as an internal variable is

defined as the effective surface density of intersections of micro-cracks and cavities in most damaged sections:

$$D = \frac{S_D}{S_i} \quad (4-1)$$

where,  $S_D$  is the damage surface area and  $S_i$  is the initial (pre-damage) cross section area. Due to the difficulty in direct measurement of the density of defects on the surface or volume of materials, alternative methods were investigated. For example, the more common damage marker widely used to describe fatigue in metallic materials is the crack size; similarly, the wear volume is used for wear damage (Archard, 1953) and the delamination for the degradation of composite (Suh, 1977). These categories of common observable damage markers are called “direct damage indicators” in this study.

However, by the time that the conventional NDI techniques can detect common direct damage indicators, most of the life of the component has already expended and it would be too late to save the degraded component/system (Cantrell, 2006).

Therefore, it is desirable to be able to estimate the degradation much earlier than the emergence of direct damage indicators. On the other hand, in some cases such as fatigue in composites, although direct damage indicators might appear early in the degradation process, measuring them directly would be very difficult or even impossible. In general, in many cases, it is not efficient to rely on conventional damage indicators to develop models for damage estimation and prediction and therefore, widely used empirical or physics-based models, which are derived based on such direct damage indicators, would not be valid for accurate early estimation. Hence, it would be beneficial to search for “indirect damage indicators” represented

more specifically as “damage precursors”. There are no formal studies on this concept and different interpretations can be found in the literature. Weiss and Ghoshal (Weiss and Ghoshal, 2014) defined damage precursor as “the progression of structural material property degradation or morphology that can evolve into damage”. In this description, damage precursor refers to flaws or defects that happen at material level “before damage” that can grow and propagate through time and ultimately “evolve into damage”. However, the term damage in this definition might be confusing because, as explained earlier in this chapter, damage itself is an abstract concept and different interpretations can be perceived based on scale, geometry and material of the problem in hand. In other words, deterioration of microstructural properties – which is called “damage precursor” by (Weiss and Ghoshal, 2014) – itself can be regarded as damage of the component at microscale. Therefore, a broader definition of damage precursor is more appropriate to convey the idea. Accordingly, we define *Damage precursor* representing the *indirect damage indicator* as “any detectable variation in material/ physical properties of the component that can be used to infer the evolution of the hidden/ inaccessible/ unmeasurable damage during the degradation”.. This comprehensive definition can be adjusted to describe different degradation processes. In this research, damage precursor and indirect damage indicator terms are used interchangeably.

#### 4-3 *Damage precursors in fatigue:*

Some microstructural changes, which are currently known as damage precursor, are identified in laboratory settings for metals and composites under fatigue. They include increase in dislocation density, crazing (i.e., a network of fine cracks on the

surface of a material), inhomogeneity of strain, shear localization, variation of electrical resistivity and conductivity, change in chemical composition, electrical signal and acoustic response (Habtour et al., 2016; Hall et al., 2013; Ingo Weber, 2001; Irving and Thiagarajan, 1998; N. E. Bedewi, 1997; Rabiei et al., 2015a; Wang et al., 1998).

For fatigue in metallic components in particular, Weiss et.al (Weiss and Ghoshal, 2014) discussed about the possibility of observing linear behavior in progression of some damage precursors such as micro-strain and particle size, and electrical resistivity, which are all related to dislocation processes that precede crack formation. This linear property opens new doors to SHM procedure because while the crack size initiates and starts to grow exponentially, linear growth of some microstructural properties can be used for early stage diagnostics (prior to formation of cracks detectable by traditional nondestructive methods) and also for more precisely prognostics and estimating remaining useful life of the component.

Therefore, it is advantageous to look for damage precursors that change nearly linear with time or cycle. However, even if such precursors are not recognized in degradation process, monitoring any other type of damage precursor is extremely beneficial as it provides valuable awareness about the health of the material state. As the trade off, more advance analysis techniques should be implemented in order to track and predict non-linear behaviors through time.

In summary, the idea of considering indirect damage indicators (more specifically, damage precursors) in SHM frameworks (Rabiei et al., 2016, 2015a, 2015b) provides the opportunity to estimate the damage state of a component when direct signs of

damage (e.g., crack) have not developed yet or are difficult to measure (e.g., complex degradation processes in composites). Therefore, with the help of damage precursors, one can assess and ultimately predict the degradation state.

#### *4-4 Damage precursor-based SHM framework*

The objective is to develop a new structural health monitoring framework based on evolution of damage precursor utilizing different available sources of information considering their associated uncertainties.

In order to identify and then implement the damage precursor in SHM framework, detailed understanding of material properties under loading at micro or even nano scale is required. Studying micro-mechanical properties of materials is not a new area of research, however, incorporating their contribution into the SHM process is quite novel. Indeed, this approach leads to Health Conscious Structures technology which is based on material state awareness. Material state awareness is defined as reliable nondestructive quantitative material/damage characterization regardless of scale (Lindgren E and Buynak C, 2011). The ultimate goal of Health Conscious Structures is to achieve a zero maintenance systems by detecting and measuring damage precursors at the very early stage and to integrate them into RUL estimation (Le et al., 2014). This will require several steps as following:

1. Perform thorough research on possible failure mechanisms of the mechanical structure of interest.
2. Develop advance sensor technology that is capable of monitoring evolution of microstructure defects over time.

3. Develop advance data interpretation procedure to encode relationship of sensor response to microstructural changes.
4. Investigate and identify damage precursors.
5. Develop methods to quantify damage precursors.
6. Incorporate damage precursor measurements into SHM framework
7. Predict the remaining useful life of the structure

Each of these steps can be a separate research topic. The current study mostly focuses on the last four steps and briefly touches on the third step.

In the following, two damage precursor based SHM frameworks are proposed depending on whether the direct damage indicator can be measured at some point during the useful life or not. Each methodology is explained thoroughly with details on how and when they can be applied.

#### *4-5 Proposed SHM framework: two-stage approach*

As described earlier, in many cases when a component is under load, unseen microstructural changes occur inside the component that gradually evolve into visible and measureable damage markers. Therefore, there is a time period when, although the component seems quite healthy and does not reveal any conventional recognizable damage signs, degradation is actively happening inside the component.

The idea here is to be able to monitor the health of the component even when visible direct damage indicator such as crack does not exist. This situation might relate to many cases such as degradation of the metallic components prior to crack initiation.

Consider a metallic component in operation under load when a conventional direct damage indicator (such as crack length) does not exist, is very small or difficult to

detect. Figure 4-1 illustrates the proposed two-stage SHM framework for estimating the health state of the component based on monitoring the evolution of a proper damage precursor up to the time that a direct damage indicator is recognized. Then, when a direct damage indicator (e.g., fatigue crack length) becomes measurable by conventional sensor or NDI tools, the focus of the methodology can shift to tracking propagation of the direct damage indicator instead and then, the widely used empirical models such as the Paris Law can be implemented. Otherwise, if necessary, health assessment can be continued based on the evolution of both damage precursor and direct damage indicator. In Figure 4-1, *DP* represents Damage Precursor, *DDI* is Direct Damage Indicator, *Z* shows all the available measurements, and *D* is the damage parameter defined based on the evolution of either damage precursor or direct damage indicator provided which one is available. Subscript *k* refers to the  $k^{th}$  time step or cycle.

The criteria for prognostics and predicting the RUL is completely application-dependent. In one system, as soon as the crack is initiated, the component might be considered as already failed. Therefore, prognostic can be seen as predicting the “crack initiation time”. Whereas, in another case, component can be still operational until the damage size reaches some specific threshold. The proposed damage precursor-based SHM framework, Figure 4-1, tends to be general to represent both scenarios. In the following, the main elements of the proposed methodology are discussed in detail.



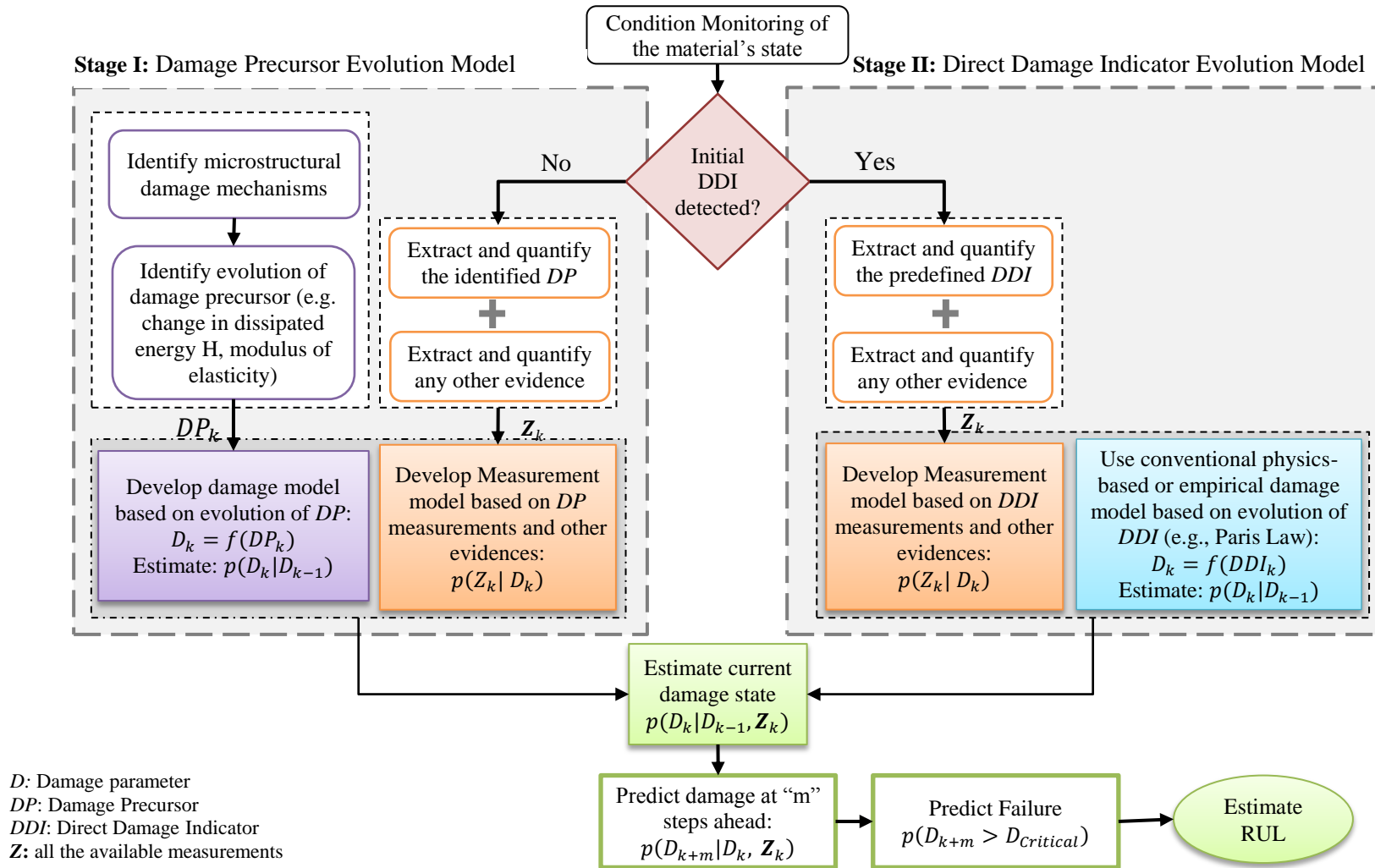


Figure 4-1: Proposed two-stage SHM Framework for monitoring and prognostics

#### 4-5-1 Stage I: Damage Precursor modeling

The first stage is based on identifying and monitoring the evolution of the proper damage precursors. It requires a deep understanding of the possible failure mechanisms, system's functionality, relevant components and their interactions, and other influential factors. As shown in Figure 4-1, stage I of the proposed framework starts with:

- Identifying microstructural damage mechanisms<sup>1</sup> for the material under consideration and in a particular application.
- Looking for a set of precursors that collectively represent the highest information content about the progression of the damage.
- Developing a damage model based on evolution of identified damage precursor

When the proper damage model is established, continuous monitoring of the material's health state is required for extracting and quantifying the measurements of damage precursor. These measurements will be used recursively to update the damage estimation in time based on variation of damage precursor. Moreover, if any other type of monitoring data (such as measurements of other properties or related conditions) exists, the framework will integrate them all in a fused measurement model and use all the available sources of information to update the damage estimations.

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<sup>1</sup> Although it is not the focus of current research, this step can be performed in different scale of nano, meso or micro based on different case studies. In fact, it does not violate the framework.

#### 4-5-2 Stage II: Direct damage indicator modeling

Microstructure defects grow over time and evolve into observable damage in the component. When a direct damage indicator is recognized and measured, one can switch the modeling approach to more traditional damage evolution models that already exist in the literature. Modeling the component's degradation process at this stage has been the focus of study for years and different approaches exist in the literature. For example, underlying physical mechanism of crack growth in metallic components under fatigue has been widely studied and several models have been proposed including Paris Law (Paris and Erdogan, 1963), modified Paris Law (Donahue et al., 1972) and Weertman's equation for fatigue crack growth (Weertman, 1969). Therefore, one of these predefined damage models based on evolution of the direct damage indicators can replace the damage precursor-based degradation model developed in stage I. Then continuous monitoring and measuring the progression of direct damage indicator is required to update the damage estimations in stage II. Similar to stage I, the framework considers all the other available measurements as well.

The methodology then can be used to estimate the damage state in future and consequently to predict the RUL.

#### 4-5-3 Integrate other evidences:

As seen in Figure 4-1, the idea of integrating other available evidences into the SHM framework is graphically represented by an independent box in developing the measurement model.

#### 4-5-4 General DBN representation of the proposed two stage SHM framework

As discussed in previous chapter, DBN is a proper methodology to model the complex time dependent relationships between uncertain variables of the proposed SHM framework. Figure 4-2 shows a general form of the DBN which is required for representing the proposed 2-stage SHM framework. Two time slice of the DBN are presented for each of the stages I and II. The underlying true hidden damage which is not directly accessible is depicted by dash line. Arrows from left to right show the progression of damage in time and other arrows demonstrate the causality or correlation relationship between variables. In stage I, damage model is developed based on variation of damage precursor in time considering the inherent uncertainty  $\sigma_k$  and model parameters  $\theta_k$ . Then, all the available measurements are used to recursively update the damage estimations through a proper measurement model which depends on the measurement uncertainty  $\varepsilon_k$  and model parameters  $\varphi_k$ . In stage II, however, it is assumed that a conventional direct damage indicator such as crack size is detected and measured at some point in time e.g. at  $k+l$  time step, therefore, widely used empirical damage growth models can be applied instead. And the measurements of that direct damage indicator along with any other available evidence are integrated to update the estimations of the damage state. Notice that the error terms ( $\sigma'_{k+l}$  and  $\varepsilon'_{k+l}$ ) and the model parameters ( $\theta'_{k+l}$  and  $\varphi'_{k+l}$ ) are different than those in stage I and need to be assigned accordingly.

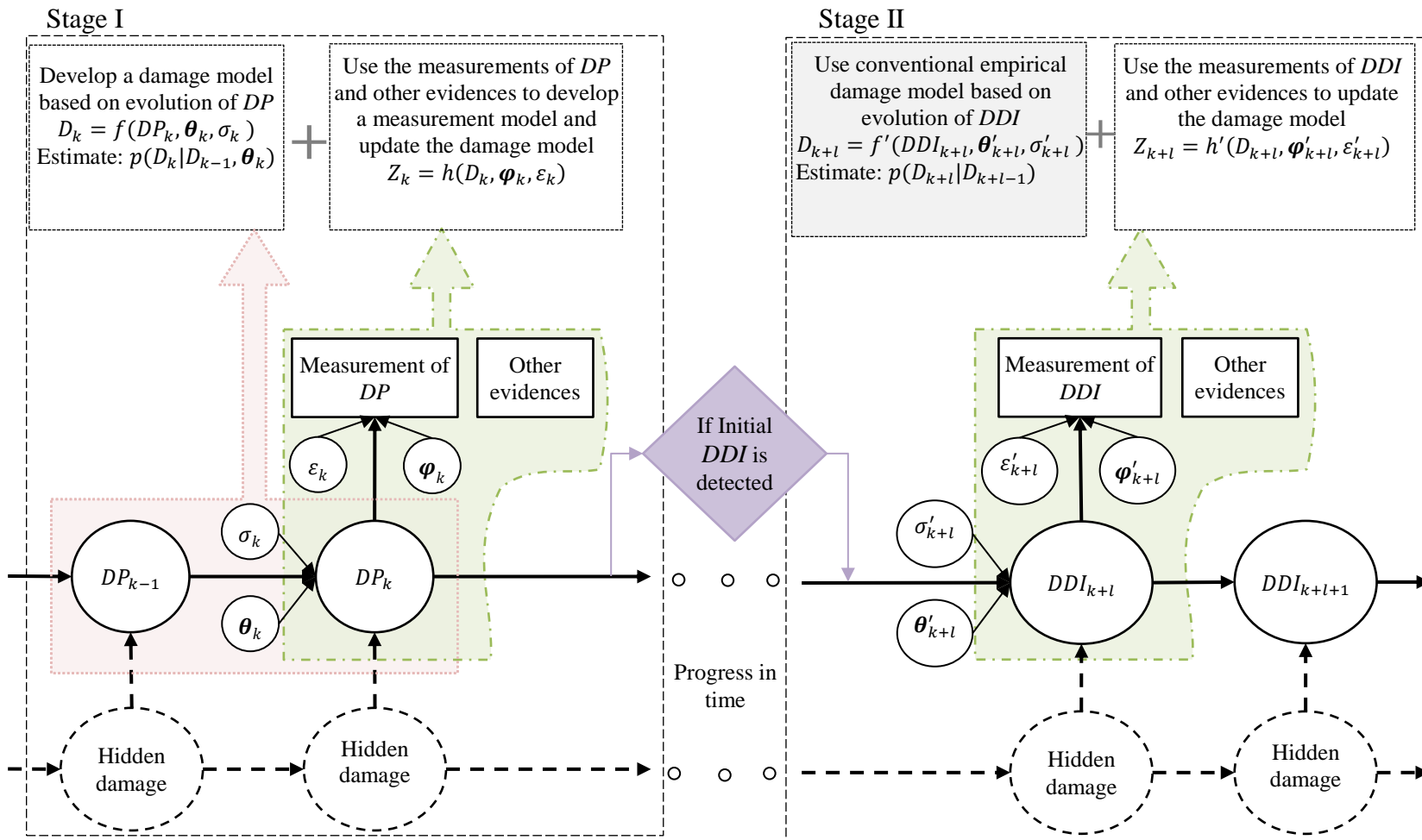


Figure 4-2: General DBN representation of the proposed two-stage SHM framework

Again, it is important to notice that the DBN in Figure 4-2 is the high-level overview of the idea of the proposed SHM with the capability of integrating different sources of information and it needs to be customized based on the problem at hand. That is why the rectangle for other evidences is not specifically linked to other variables. In 0, the proposed SHM framework is applied on a real world case study and more details on the DBN structure are presented.

#### *4-6 Proposed SHM framework: one-stage approach*

In addition to cases such as fatigue in metals that the concept of damage precursor can be used for damage estimation prior to any detectable crack initiation, another class of damage precursor relates to cases when the degradation process itself is very complicated in a way that a conventional direct damage indicator is not well-defined or is very difficult to measure with regular inspection techniques. It might happen in complex degradation process such as in composite material. In such events, damage markers (such as matrix of mirco-cracks) might emerge very early in the process, however, effective assessment of them could be burdensome in practice. Therefore, it is useful to search for alternative indirect damage indicators that can model the damage evolution more fundamentally. Here, assuming two separate stages (as explained in previous section) for evolution of first indirect damage indicators and then direct damage indicator might not be legitimate. Therefore, the idea of proposed damage precursor based SHM is formulated here as a new one-stage SHM framework. Schematic diagram of this diagnosis-prognosis framework is presented in Figure 4-3.

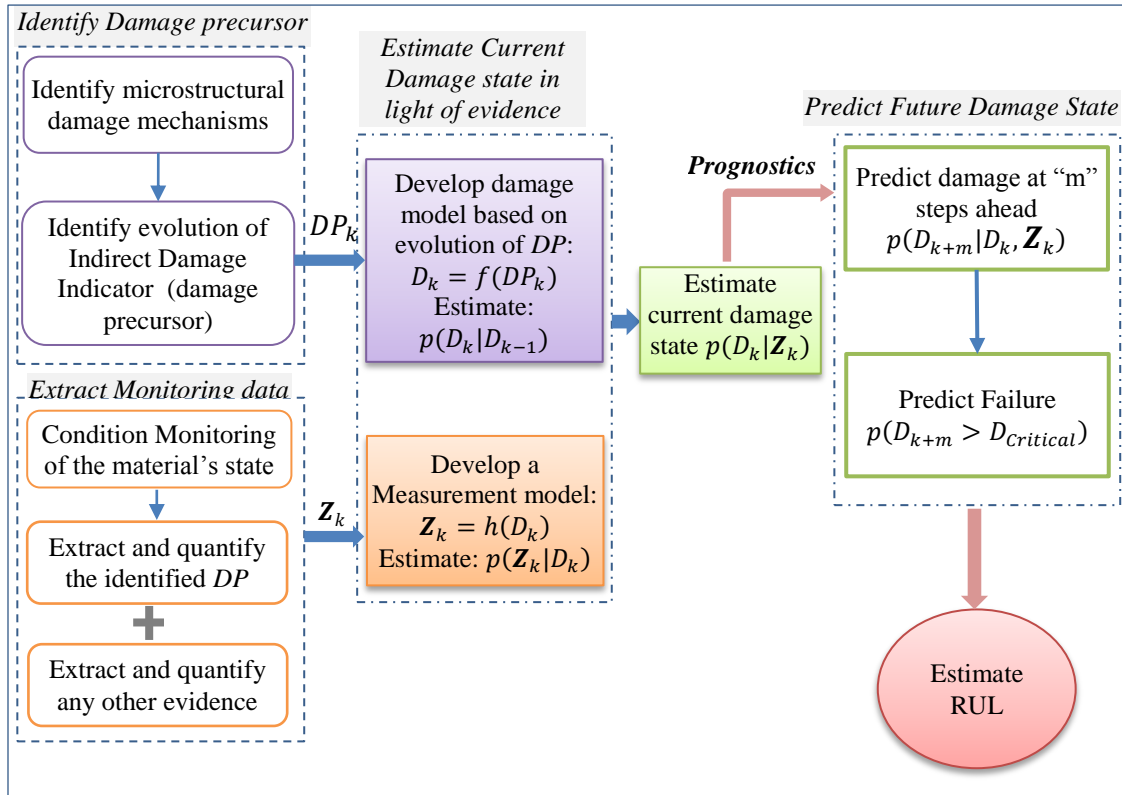


Figure 4-3: Proposed one-stage SHM Framework for diagnostics and prognostics based on damage precursor evolution

Thus, similar to the two-stage framework, the process starts with identifying microstructural damage mechanisms and then search for possible measurable damage precursors that can provide some insights about the damage state. With the use of sensor-based monitoring, one should be able to extract and quantify the variation of indirect damage indicators. This information is required to develop a damage model based on the evolution of the damage precursor. When the damage model is defined, continuous condition-based monitoring measurements can be used to update the damage estimation in time. If any other type of evidence is also available, a method (usually a data-driven method) is needed to map their correlation with the evolution of the damage precursor. Estimated current state of damage will be

applied in prognostic techniques to predict the future state of the system and estimate the remaining useful life based on critical threshold. Note that the model for damage evolution based on damage precursor evolution in time can be attained by either physics-based or data-driven method regarding the complexity of the degradation process.

#### 4-6-1 General DBN representation of the proposed one-stage SHM framework

The proposed one-stage SHM framework is formulated with DBN in Figure 4-4 which is basically the first stage in the two-stage DBN (Figure 4-2).

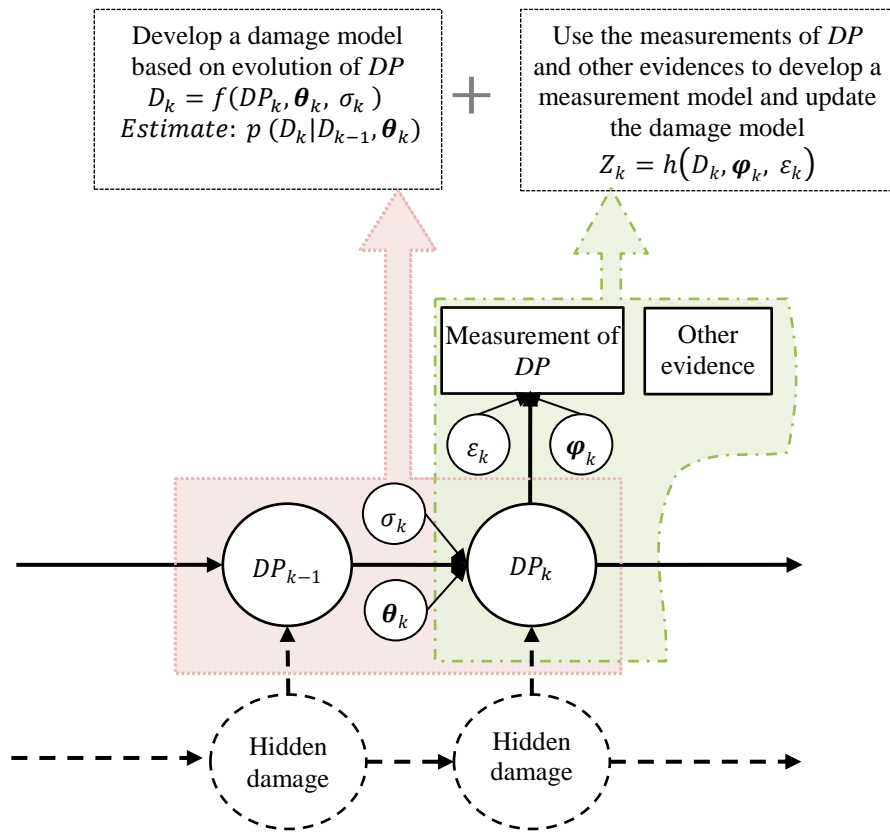


Figure 4-4: General DBN representation of the proposed one-stage SHM framework

Later in Chapter 7, the damage precursor concept is used to study the complex degradation process of composites under fatigue, and the proposed damage precursor-



based SHM framework will be presented more formally with details on damage in composites.

#### 4-7 Integrated mathematical approach to formulate the proposed SHM:

As demonstrated in sections 4-5-4 and 4-6-1, DBN is the fundamental approach to represent the proposed SHM framework. Therefore, the first step is to construct the DBN of the problem considering all the variables including hidden variables, observed variables, model parameters, related uncertainties and any other sources of evidence. A combination of different physics-based models, empirical models, data-driven models (regular regression or more flexible SVR) and expert opinion models is required to define the underlying relationship between variables.

The focus of the DBN in the context of proposed SHM framework would be inferring the damage state and parameters of the damage model at any time step  $k$  in the light of all the available observations, i.e.,  $p(D_k, \theta_k | \mathbf{Z}_k)$  as presented in Figure 4-2 and Figure 4-4. The presence of different types of random variables in the DBN makes the inference more challenging. Therefore, particle filtering as the stochastic state-space model is used to make the inference inside the DBN. Particle filtering as an independent filtering approach has captured significant research attentions for performing diagnostics and prognostics in recent years. However, its application for inference in DBN when more than one type of observation exists has not been studied profoundly. Moreover, when dealing with less explored areas such as damage models based on evolution of damage precursor, the state-space model of interest also depends on unknown static parameters that need to be estimated from the data. In this

context, standard particle filtering methods fail and it is necessary to rely on more sophisticated algorithms. Accordingly, the "Augmented particle filtering" with the ability of estimating both the model parameters and the damage states is required to make inferences in the proposed DBN.

The proposed integrated mathematical framework is presented in Figure 4-5 and each component with all the mathematical details will be thoroughly described in the next chapter.

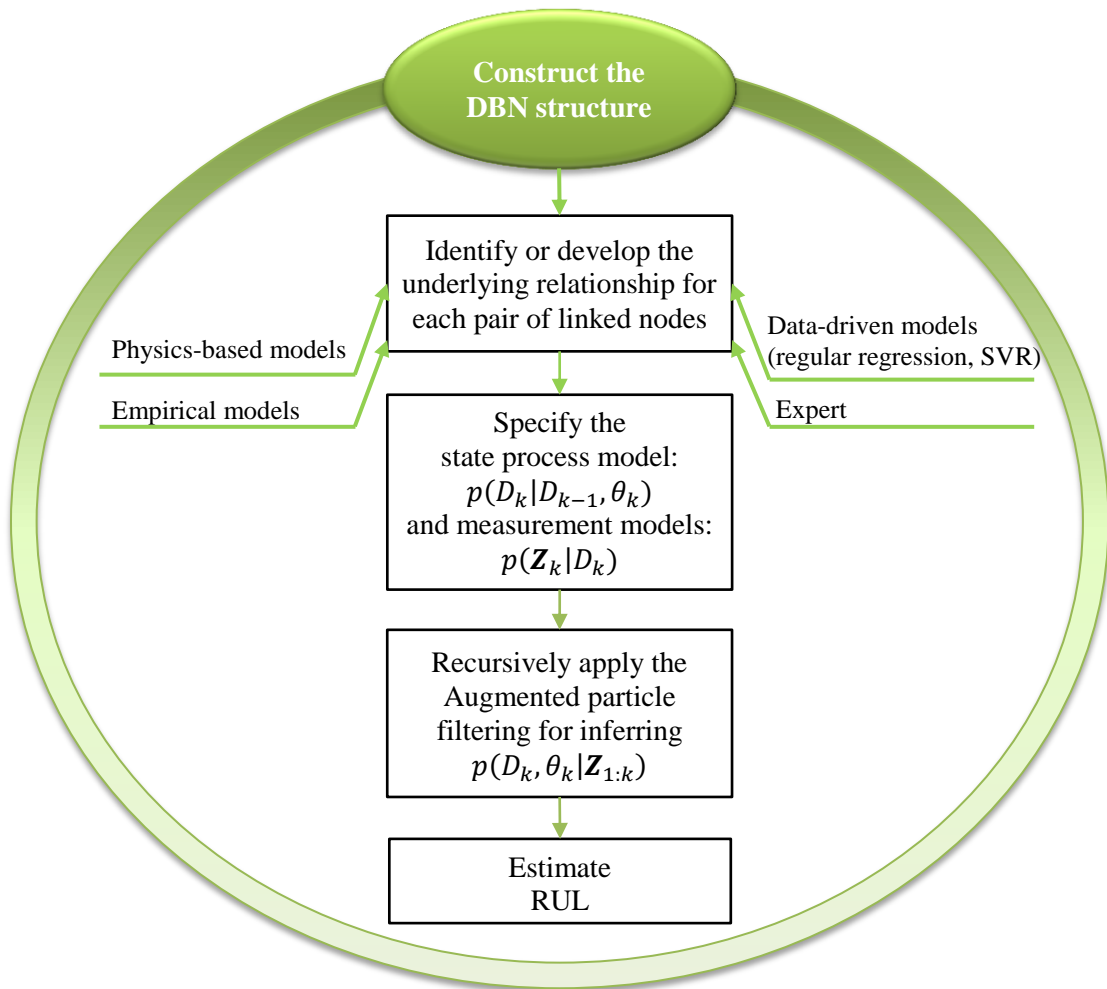


Figure 4-5: Integrated mathematical approach required for the proposed SHM framework

#### 4-8 Summary:

The concept of indirect damage indicator and more specifically damage precursor was presented in this chapter and some details and examples were provided. Then the idea of incorporating the damage precursor into SHM framework, which is the principle idea of this research, was explained and two different frameworks were presented. The first framework was the two-stage approach in which there is a chance that a direct damage indicator such as fatigue crack can be detected and measure at some point in time before the failure. As a result, the degradation state can be estimated by monitoring the evolution of the damage precursors in the first stage of the proposed SHM when direct damage indicator cannot be observed yet. And later, if such direct damage indicators emerged, the SHM can switch to stage two where it relies on growth and propagation of direct damage indicator instead.

The second SHM framework focused on complex degradation processes (such as those in composite materials) when a well-known direct damage indicator is not defined or is not easily measureable. The framework assesses the degradation state based on variation of damage precursors.

Both of these frameworks were formulated through DBN and a general DBN structure was presented that can be customized for the particular problem under consideration.

At last, an integrated mathematical framework was proposed to demonstrate the underlying mathematical elements required for the proposed SHM framework to work. each technical component will be elaborated further in the next chapter.

## Chapter 5: Mathematical Model Development

### 5-1 Overview

The proposed approach encompasses several techniques that must be integrated to achieve the desired goal of monitoring and prediction. In this section, the underlying mathematical details of each technique are provided. The section starts with short introduction on representation of DBN and making inference using particle filtering approach for state estimation. Later on, the augmented particle filtering for combined state and parameter estimation is introduced followed by the procedure of damage prognostics and prediction of time to failure with augmented particle filtering. And finally, a brief introduction on SVR is presented to demonstrate how it fits into our mathematical framework.

### 5-2 Dynamic Bayesian Network Representation

As explained earlier, DBN is the main approach underpinning the proposed framework considering correlation among the variables and their uncertainties. Dynamic Bayesian Network extends static BN to model systems that are dynamically changing or evolving over time. Dynamic Bayesian Network can be recognized as an important member of a bigger family called State-Space Models (SSM), which are represented as probabilistic graphical models to handle time-series data.

Dynamic Bayesian Network is a powerful tool to model probability distribution of collection of random variables  $Z_t$  over time. Random variable  $Z_t$  can be representative

of input ( $u_t$ ), hidden ( $x_t$ ), and observed ( $y_t$ ) random variables. When the DBN is constructed over the sets of suitable random variables  $Z_t$  and their conditional correlations, the joint probability distribution for all the variables can be represented as:

$$p(Z_1, \dots, Z_n) = \prod_{t=1}^T \prod_{i=1}^n p(Z_t^i | Pa(Z_t^i)) \quad (5-1)$$

$Z_t^i$  is the  $i^{\text{th}}$  node at time  $t$  that could be any of  $u_t$ ,  $x_t$ , or  $y_t$ , and  $Pa(Z_t^i)$  shows all the parents of node  $Z_t^i$  that could be either in the same time slice or in previous one. In principle, we can use the Bayes rule to “infer” any probability of interest. Inference is effectively computing the probability of each state of a node (or subset of nodes) in a Bayesian network when other (or some of the) variables are known. In other words, when random variables and their relationship are modeled in DBN structure, one can employ the model to deduce the belief distribution over random variables of interest. Inference in DBN is a challenging task and the exact inference only exists for particular cases with simplifying assumptions. Murphy (Murphy, 2002) presents a comprehensive review of exact, approximate and stochastic inference methods for DBN. Kalman Filter and its extensions (Extended Kalman Filter, Unscented Kalman Filter) are well-known inference methods that are widely used. However, they are only applicable when random variables are Gaussian and the system is linear (the linearity assumption is relaxed for EKF and UKF). Unfortunately, this is not the case in most of the real world applications. In this context, one can resort to particle filtering: a stochastic algorithm capable of inferring the unknown belief state when the system is non-linear and non-Gaussian.

### 5-3 Particle Filtering:

Particle filtering is a stochastic computational technique, also referred to as Sequential Monte Carlo, which uses Bayesian recursive estimation to address the filtering problem especially when dealing with nonlinear and/or non-Gaussian processes. In the context of this research, suppose  $x_k$  refers to the underlying progressive damage at any given time step  $k$ , which is not observable directly. And,  $y_{1:k}$  represent all the available noisy measurements or evidences that can be observed and tracked through time (see Figure 5-1).

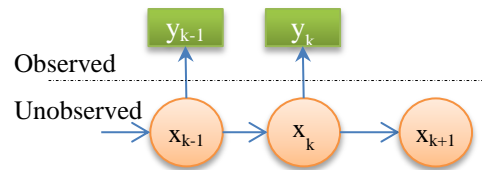


Figure 5-1: representation of Bayesian recursive estimation

In Recursive Bayesian Estimation:

- The unknown of interest is the state of the system ( $x_k$ )
- Stochastic evolution of system state in time is presented by the “process model” or “transition model”:

$$x_k = f(x_{k-1}, \omega_k) \rightarrow p(x_k | x_{k-1}) \quad (5-2)$$

In which  $x_k$  is state at time step  $k$ ,  $\omega$  is called process noise and  $f$  is the evolution function.

- When measurement  $y$  gets available, a probabilistic “measurement model” is needed to link that measurement with the system state  $x$ :

$$y_k = h(x_k, v_k) \rightarrow p(y_k | x_k) \quad (5-3)$$

$v$  is called measurement noise and  $h$  is the measurement function.

Equations (5-2) and (5-3) are not deterministic and the uncertainty is introduced by noise terms  $\omega_k$  and  $v_k$ . The goal is to estimate the unobserved state of the system  $x_k$  based on all the observations available up to time  $k$ ; that is inferring  $p(x_k|y_{1:k})$ . It can be obtained recursively in two steps called prediction and update.

*Prediction step:* in this step, one tries to predict  $p(x_k|y_{1:k-1})$ , supposing that prior probability  $p(x_{k-1}|y_{1:k-1})$  at time  $k-1$  is available.

*Update step:* as a new measurement  $y_k$  becomes available at time  $k$ , predicted probability will be updated via Bayes' rule:

$$p(x_k|y_{1:k}) = p(x_k|y_{1:k-1}, y_k) = \frac{p(y_k|x_k)p(x_k|y_{1:k-1})}{p(y_k|y_{1:k-1})} \quad (5-4)$$

Where the prior and the normalizing factor are:

$$p(x_k|y_{1:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|y_{1:k-1})dx_{k-1} \quad (5-5)$$

$$p(y_k|y_{1:k-1}) = \int p(y_k|x_k)p(x_k|y_{1:k-1})dx_k \quad (5-6)$$

Computing the posterior  $p(x_k|y_{1:k})$  through such integrals is very difficult and does not lead to analytical solution except in special cases (for example when all the distributions are Gaussian). The key idea of particle filtering is to represent the required posterior density function  $p(x_k|y_{1:k})$  by a set of random samples (called particles  $\{x^i\}$ ) with associated weights  $\{\omega^i\}$  :

$$p(x_k|y_{1:k}) \approx \sum_{i=1}^N w_k^i \delta(x_k - x_k^i) \quad (5-7)$$

$\delta$  is Dirac's delta function and  $\omega_k^i$  is the normalized weight of the  $i^{th}$  particle at time  $k$ . Practically, in particle filtering approach, the *prediction* step starts by recursively propagating the particles forward in time using the state process model Eq. (5-2). And

then, each particle will be weighted based on the likelihood of observed measurement  $y_k$  using the measurement model Eq. (5-3); that is the *update* step.

Weights are chosen using a Monte Carlo based method called the principle of sequential importance sampling (SIS) (Arulampalam et al., 2002; Doucet et al., 2001). SIS is based on this assumption that target posterior density  $p(x)$  is unknown or difficult to draw samples from; therefore, particles are sampled from importance density or proposal density  $q(x)$  instead. The particle weights are then introduced as the ratio of  $\frac{p(x)}{q(x)}$  which is called the importance weight:

$$w_k^i \propto \frac{p(x_k^i | y_{1:k})}{q(x_k^i | y_{1:k})} = \frac{\text{Target distribution}}{\text{Proposal distribution}} \quad (5-8)$$

Importance weight is calculated for each particle and shows how important that particular particle is in building the posterior distribution. Both target and proposal distributions can be expanded by applying the Bayesian and chain rule as following:

$$w_k^i \propto \frac{p(x_k^i | y_{1:k})}{q(x_k^i | y_{1:k})} = \frac{p(y_k | x_k^i) p(x_k^i | x_{k-1}^i)}{q(x_k^i | x_{k-1}^i, y_{1:k})} \cdot \frac{p(x_{k-1}^i | y_{1:k-1})}{q(x_{k-1}^i | y_{1:k})} \quad (5-9)$$

$$w_k^i \propto \frac{p(y_k | x_k^i) p(x_k^i | x_{k-1}^i)}{q(x_k^i | x_{k-1}^i, y_{1:k})} \cdot \omega_{k-1}^i$$

Now, the challenge is how to choose the proposal density or importance density function  $q(x_k | x_{k-1}^i, y_{1:k})$ . Some approaches are proposed in the literature. Ideally, we would like the importance function to be the posterior distribution itself,  $p(x_k | x_{k-1}^i, y_{1:k})$ . This is called the “optimal importance function” proposed by (Akashi and Kumamoto, 1977) based on the strategy of minimizing the variance of the importance weights. However, there is no analytical closed-form solution in general cases when using optimal importance function. Therefore, some suboptimal



methods were proposed to approximate the importance function for instance by using MCMC or local linearization techniques.

The most common and convenient choice implemented in the literature is to use the prior distribution of  $p(x_k|x_{k-1}^{(i)})$  as the importance function (Handschin, 1970). That is sampling the particles from transition equation (or process model):

$$q(x_k^i | x_{k-1}^i, y_{1:k}) = p(x_k^i | x_{k-1}^i) \quad (5-10)$$

Substituting (5-10) back into (5-9), the particle weights at each time step  $k$  would be calculated as:

$$w_k^i \propto w_{k-1}^i \cdot p(y_k | x_k^i) \quad (5-11)$$

Basically, given that the state of the particle is at  $x_k^i$ , its weight at time step  $k$  is the multiplication of its previous weight at time step  $k-1$  and the likelihood of observing  $y_k$ . When the weights are calculated, each particle will carry two pieces of information  $\{x_k^i, w_k^i\}$ . Roughly speaking, each particle is a representative of one possible hidden state of the system at time step  $k$  along with its probability of happening. Interested readers can refer to (Arulampalam et al., 2002) and (Doucet et al., 2001) for more detailed information about the importance density function and the procedure of weighing the particles. Figure 5-2 shows the weighing procedure schematically.

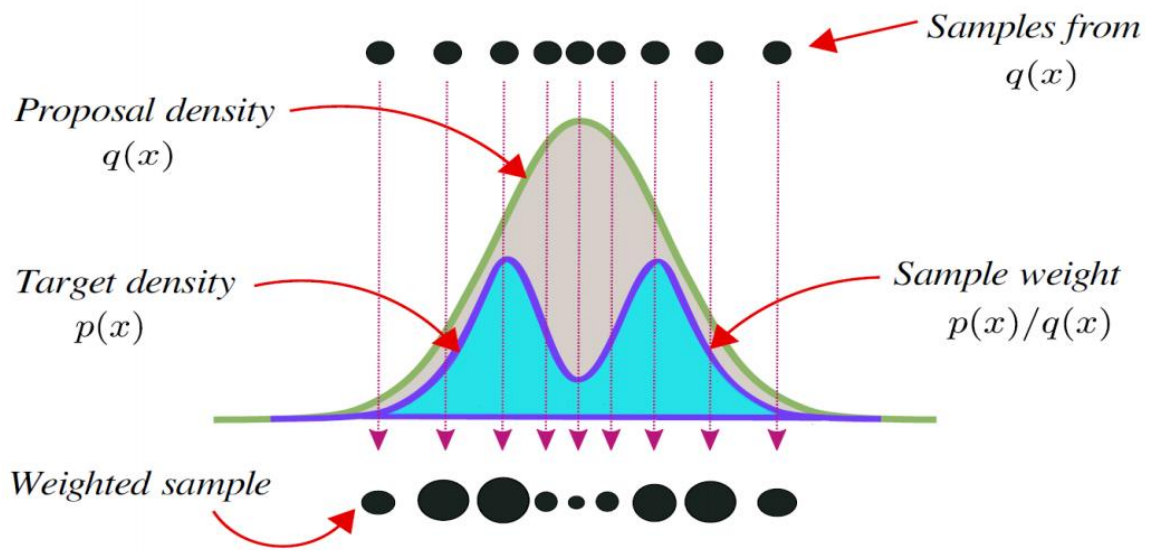


Figure 5-2: Weighing procedure

The original SIS algorithm has a common problem called “*Degeneracy Problem*”, where the distribution of the importance weights becomes more and more skewed and its variance increases as we progress in time. So, after a few time steps, the weights of all but one particle will drop dramatically to zero. One of the intuitive proposed techniques to resolve this issue is “Resampling” based on the calculated weights (Gordon et al., 1993). The key idea of resampling is to eliminate particles that have small weights and to concentrate on particles with large weights. Therefore, at each iteration, the new sets of particles will be drawn from the proposed distribution based on calculated weights. This approach makes sure that more samples would be selected from higher probability areas.

The final expected estimation of the state at each time step will be then calculated as the weighted average of all the particles:

$$\bar{x}_k = \sum_{i=1}^N w_k^i \cdot x_k^i \quad (5-12)$$

A lot of attention has been paid recently toward particle filtering approach because of its flexibility in dealing with nonlinear and non-Gaussian problems. Other versions of the standard particle filtering have also been proposed in the literature to improve the performance of the algorithm usually at the cost of more computation. Among them one can refer to Auxiliary Particle Filter (Pitt and Shephard, 1999), Regularized Particle Filter (Musso et al., 2001) and Unscented Particle Filter (Doucet et al., 2000). Researchers have used the standard PF algorithm and its variants for diagnostics and prognostics in many different fields such as life prediction of batteries and fuel cells (Dalal et al., 2011; Goebel et al., 2008; He et al., 2011; Jouin et al., 2014; Miao et al., 2013), degradation assessment and prediction in gears and bearings (Chen et al., 2011; Zhou et al., 2011; Yoon and He, 2014), Health monitoring and prognostics of gas turbines (Sun et al., 2012a), machine tools (Wang et al., 2015), and pumps (Daigle and Goebel, 2013; Wang and Tse, 2015), and also, damage estimation and prediction of composite materials (Chiachío et al., 2015a; Corbetta et al., 2016; Rabiei et al., 2015b). More application examples can be found in the most recent review paper on PF algorithm by (Jouin et al., 2016).

#### 5-4 Combined estimation of model parameters and states in Particle Filtering

The original particle filtering is established based on the assumption that state process model is fully defined with fix known parameters in advance. However, in many cases especially when dealing with uncertain dynamic systems, even if the form of

state model is known, all or some of the parameters might be unknown. This aroused an interest in combining parameters and states together and estimating both of them simultaneously through augmented particle filtering (Carvalho et al., 2010; Kitagawa, 1998; Liu and West, 2001; Storvik, 2002). In this regard, it is required to learn the state process model by online tuning its parameters as well as estimating the damage states.

The extension of standard particle filtering to augmented-particle filtering is not trivial. One of the conventional proposed strategies (Liu and West, 2001) is to treat the model parameters the same way as states, which results in estimating the augmented state space problem  $P(x_k, \theta_k | y_{1:k})$ . Therefore, the original state process model Eq. (5-2) needs to be modified to consider parameter evolution as well:

$$\begin{aligned}\theta_k &= g(\theta_{k-1}, \gamma_{k-1}) \rightarrow p(\theta_k | \theta_{k-1}) \\ x_k &= f(x_{k-1}, \theta_k, \omega_k) \rightarrow p(x_k | x_{k-1}, \theta_k)\end{aligned}\tag{5-13}$$

where  $g$  and  $\gamma$  are the transition function and random noise for model parameters  $\theta$ , respectively. Furthermore, when measurements are available, both state and parameters should be updated with respect to newly arrived observation. Therefore, based on Bayes' rule, the final joint posterior distribution of interest (presented in Eq. (5-4)) can be rewritten as (Liu and West, 2001):

$$P(x_k, \theta_k | y_{1:k}) \propto P(y_k | x_k, \theta_k) P(x_k | \theta_k, y_{1:k-1}) P(\theta_k | y_{1:k-1})\tag{5-14}$$

The last term in Eq. (5-14) is the main modification in the posterior of the original particle filtering and it should be estimated accordingly. In the following, the procedure of computing the distribution  $P(\theta_k | y_{1:k-1})$  is briefly explained.

Going back to the augmented state process model, it is suggested in (Kitagawa, 1998; Liu and West, 2001) that a Gaussian random walk with mean 0 and variance  $\gamma$  can satisfy the parameter transition model as:

$$\theta_k = g(\theta_{k-1}, \gamma_{k-1}) = \theta_{k-1} + \mathcal{N}(0, \gamma_{k-1}) \quad (5-15)$$

Note that the parameters are not time variant, i.e., they are not supposed to dynamically evolve in time. Therefore, adding random noise results in more diffused posterior relative to the theoretical posterior of the actual fixed parameters. This issue was recognized very early and an approach based on Kernel Smoothing was proposed by (Liu and West, 2001) to control the variance. The idea of kernel smoothing is to reduce the variability in particles by shrinking them (with shrinkage parameter  $h$ ) towards the current estimated mean  $\bar{\theta}$ , and then to add controlled reduced noise ( $h^2\gamma$ ) for the next step in the estimation process. In this sense, the smooth kernel probability density is proposed to estimate the last term in Eq. (5-14) as following:

$$P(\theta_k | y_{1:k-1}) \approx \sum_{i=1}^N \omega_{k-1}^i \mathcal{N}(\theta_k | m_{k-1}^i, h^2\gamma_{k-1}) \quad (5-16)$$

where  $m$  is the kernel location calculated for each particle ( $i$ ) with the following shrinkage rule:

$$m_{k-1}^i = \left(\sqrt{1-h^2}\right) \theta_{k-1}^i + \left(1-\sqrt{1-h^2}\right) \bar{\theta}_{k-1} \quad (5-17)$$

The value of  $h \in [0,1]$  is suggested in (Chen et al., 2005; Liu and West, 2001) to be less than 0.2 for slowly varying particles and more than 0.8 for highly stochastic process. Some works also have been published recently on optimizing the value of  $h$  using historical data or online observations (Hu et al., 2015a, 2015b; Tulsyan et al., 2013).

A pseudocode for the augmented particle filtering technique is presented in Table 5-1.

Table 5-1: Augmented Particle filtering algorithm

---

(1) Initiation step:

- Sample  $N$  particles from initial distributions of states and parameters:

$$x_0^i \sim p(x_0) \quad i = 1, 2, \dots, N$$

$$\theta_0^i \sim p(\theta_0) \quad i = 1, 2, \dots, N$$

- Assign initial equal weights to all the particles

$$w_0^i \sim \frac{1}{N} \quad i = 1, 2, \dots, N$$

(2) Recursive steps:

Prediction:

- Estimate  $m_{k-1}^i$  for each parameter using shrinkage rule Eq.(5-17)
- Draw new samples for parameter vector from:

$$\theta_k^i \sim \mathcal{N}(\cdot | m_{k-1}^i, h^2 \gamma_{k-1})$$

- Propagate each particle one step forward using state process model with new sampled parameter:

$$x_k^i \sim p(\cdot | x_{k-1}^i, \theta_k^i)$$

Update:

- Calculate the weights for each particle as new measurement  $y_k$  gets available:

$$w_k^i = w_{k-1}^i \cdot P(y_k | x_k^i, \theta_k^i)$$

- Normalize the weights:
-

---

$$w_k^i = \frac{W_k^i}{\sum_{i=1}^N W_k^i}$$

Estimate:

- Estimate the expected state:

$$\bar{x}_k = \sum_{i=1}^N w_k^i \cdot x_k^i$$

Resample:

- Resample (with replacement) new set of particles for states and process model parameters  $\{x_k^i, \theta_k^i\}_{i=1}^N$  based on calculated weights  $w_k^i$
- 

One way to perform the resampling step in coding can be as following:

Having separate vectors of  $N$  particles for representing the states  $[x_k^1, x_k^2, \dots, x_k^N]$  and the model parameters  $[\theta_k^1, \theta_k^2, \dots, \theta_k^N]$  at each time step  $k$ , we can randomly choose  $N$  integers “with replacement” from  $1, 2, \dots, N$  based on the distribution of the calculated weights. These  $N$  numbers would be used as indices to reconstruct (resample) the vector of states and parameters in a way that particles with higher probability of occurrence would appear more and particles with very low likelihood would be disregarded.

Consequently, selecting the augmented-particle filtering algorithm as an inference technique along with kernel smoothing for handling unknown parameters will make DBN more flexible and powerful to model complex nonlinear dynamic systems. In our case, it would be specifically useful for studying less explored degradation processes when the damage model itself is not well-defined.

### 5-5 Prognostics with Augmented Particle Filtering

The ultimate goal of the SHM framework is to predict the future health state of the system and estimate the RUL. Prognostic with Bayesian recursive approaches such as augmented-particle filtering deals with the challenge of making long-term predictions without having any further observations to update the estimated states. Some methods are proposed in (Liu et al., 2012; Orchard and Vachtsevanos, 2009; Zio and Peloni, 2011) to handle this issue. In (Orchard, 2007) three methods are described in details. The first approach is based on updating the initial weights of the particles by integrating over discretized domain of particle population, while the second approach aims at reducing the computational cost by resampling the predicted state pdf rather than updating the weights. The third and simplest method is to keep the weights of the particles constant when predicting long-term state. It is shown in (Orchard and Vachtsevanos, 2009) that keeping the particles weights invariant during the long-term prediction can provide satisfactory results. In this approach, the weights of the particles are updated based on the last available observations in the current time instance and then these weights are stored and kept constant during the long-term predictions. We implemented this approach in the current research.

Each weighted particle can be considered as a hypothesis of the hidden state (i.e., state of damage in this study), which we desire to estimate and also predict in future. Having a predefined threshold for damage,  $t_f^{(i)}$  corresponds to the time when particle ( $i$ ) crosses this threshold and represents a possible failure time for the component. When the failure time of all the  $N$  particles are recorded  $\{t_f^{(i)}\}_{i=1:N}$ , the distribution of



the component's time to failure can be obtained (Rabiei et al., 2015a) and therefore, Mean-Time-To-Failure (MTTF) of the component would be determined by:

$$MTTF = \sum_{i=1}^N t_f^{(i)} \cdot w^{(i)} \quad (5-18)$$

Note that long-term prediction heavily relies on state process model. Since in augmented-particle filtering algorithm the parameters of the state model are not known in advance and are supposed to be learned during the process, the accuracy of prognostics depends on the state of convergence or maturity of the parameters at the time of prediction. Predictions cannot be reliable when variation in model parameters is still large.

#### 5-6 Support Vector Regression

When a complex DBN is adopted to represent an unknown and complicated degradation process, the relationship between all the involved variables needs to be defined. Although a field expert would be necessary to learn the structure of the DBN and identify the possible links between the nodes at the first place, physical or empirical or data-driven models are then required to quantify such links. However, there might not be a proper predefined model to interpret the relationship between some of the variables. It is worth noting that the relationship between the nodes in DBN can be based on either causality or statistical correlation. In cases where underlying physical causality is missing or unexplored, more flexible technique such as regression with Support Vector Machine (SVM) can play an important role to define the possible correlation.

Support Vector Regression (SVR) (Smola and Schölkopf, 2004) is an extension of a supervised machine learning technique called SVM which was originally developed by (Cortes and Vapnik, 1995) for binary classification. Support vector regression implements the nonparametric kernel-based method to model the relationship between input (regressors) and output (response) variables. The regression algorithm is to be trained based on available data (training data) and then will be utilized to estimate the system's output when new input variables get available. Examples of successful application of SVM in different fields such as risk and reliability can be found in (Droguett et al., 2014; Lins et al., 2015; Moura et al., 2011).

Support vector regression has flexible features that make it a very good candidate to perform regression to describe the relationship between some variables:

- It is especially powerful to model generally unknown nonparametric and nonlinear mapping between input and output variables.
- It is particularly useful when the underlying functional relationship between random variables is not fully known.
- It does not require any hypothesis on the distribution of the variables.
- It does not require any assumption on the distribution of noise.
- It guarantees to find the global optimum.

Brief mathematical background of regression with Support Vector Machine is presented in the following.

Suppose we are given “ $l$ ” pairs of observed data  $\{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \dots, (\mathbf{x}_l, y_l)\}$ , which can be considered as training set for the supervised SVR. In general, each  $\mathbf{x}_i$  denotes the space of the input variables that can be  $p$ -dimensional real vector

$(\mathbf{x}_i \in \mathbb{R}^p)$  and  $y_i$  belongs to the corresponding output variable. Considering the response variable  $Y$  generated by the model:

$$Y = \mu_y(x) + \epsilon(x) \quad (5-19)$$

where  $\mu_y(x)$  is the unknown expected value of  $Y$  and  $\epsilon(x)$  is a random error with zero mean and non-zero variance  $\sigma_\epsilon^2$ . SVR tries to estimate  $\mu_y(x)$  by utilizing training set. More formally it can be written in form of the regression:

$$\mu_y(x) \equiv f(x) = \mathbf{W}^T \boldsymbol{\phi}(x) + b \quad (5-20)$$

$\boldsymbol{\phi}(x)$  is an implicit mapping of the input data into a higher-dimensional feature space that will be explained in more details later in this section. The weight vector  $\mathbf{W}$  and linear coefficient  $b$  should be adjusted regarding the training data that leads to solving a quadratic and convex optimizing problem (Droguett et al., 2014; Kecman, 2005; Lins et al., 2012):

$$\text{Minimize} \quad \frac{1}{2} \langle \mathbf{W}, \mathbf{W} \rangle + C \cdot \sum_{i=1}^l (\xi_i + \xi_i^*) \quad (5-21)$$

$$\text{Subject to} \quad \begin{cases} y_i - \langle \mathbf{W}, \boldsymbol{\phi}(x_i) \rangle - b \leq \epsilon + \xi_i & \forall i \in \{1, \dots, l\} \\ \langle \mathbf{W}, \boldsymbol{\phi}(x_i) \rangle + b - y_i \leq \epsilon + \xi_i^* & \forall i \in \{1, \dots, l\} \end{cases} \quad (5-22)$$

$C$  is the control parameter (regularization factor) that adjusts between two parts of the optimization problem. The first part in Eq. (5-21) relates to SVR prediction of unseen data, while the second part tries to minimize error on training data. Parameters  $\xi_i$  and  $\xi_i^*$  are called slack variables correspond to measurements “above” and “below” an  $\epsilon$ -tube respectively. The  $\epsilon$ -tube, defined by Vapnik’s  $\epsilon$ -insensitivity loss function (Vapnik, 2000), is a tube with the width of  $\epsilon$  around the output values (Figure 5-3).

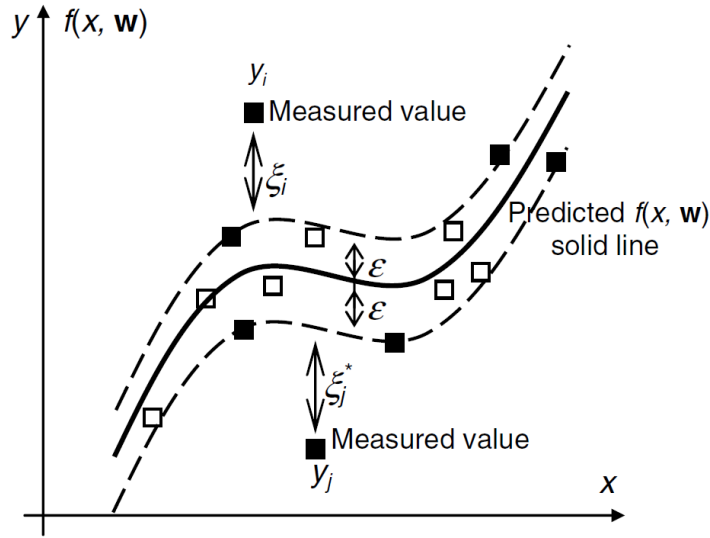


Figure 5-3: The parameters used in (1-dimensional) support vector regression. (Kecman, 2005)

If the predicted value is within the tube the loss is zero. For all other predicted points outside the tube, the loss equals the magnitude of the difference between the predicted value and the radius  $\varepsilon$  of the tube, i.e. measured  $\xi_i$  and  $\xi_i^*$ . Therefore, only the points outside this tube contribute to the regression function and all points inside the tube are neglected.

By applying Lagrange multiplier, the dual formulation of this optimization problem can be written as Eq. (5-23), in which parameters  $\alpha_{i,j}$  and  $\alpha_{i,j}^*$  are  $l$ -dimensional Lagrange multiplier correspond to measurements above and below the  $\varepsilon$ -tube.

$$\begin{aligned}
 \text{Max}_{\alpha, \alpha^*} \left\{ -\frac{1}{2} \sum_{i=1}^l \sum_{j=1}^l (\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j) \right. \\
 \left. - \sum_{i=1}^l [\varepsilon(\alpha_i + \alpha_i^*) + y_i(\alpha_i - \alpha_i^*)] \right\} \quad (5-23)
 \end{aligned}$$

Subjects to conditions:

$$\sum_{i=1}^l (\alpha_i - \alpha_i^*) = 0, \quad 0 \leq \alpha_i, \alpha_i^* \leq C, \quad \forall i \quad (5-24)$$

Therefore, the SVR equation for nonlinear predictions for the optimal value “o” becomes:

$$f_o(x) = \langle \mathbf{W}_o \cdot \boldsymbol{\phi}(x) \rangle + b = \sum_{i=1}^l (\alpha_{io} - \alpha_{io}^*) \boldsymbol{\phi}(x_i)^T \boldsymbol{\phi}(x) + b_o \quad (5-25)$$

Regression with SVR is different than other regression techniques because of the term  $\boldsymbol{\phi}(x)$ , which is an implicit mapping of the input data into a higher-dimensional feature space. This mapping facilitates dealing with possible nonlinear correlation between the input and output variables. However, defining a proper function and calculating the dot product is usually tedious in practice. Therefore, SVR uses a kernel function in form of  $K(x, x') = \boldsymbol{\phi}(x)^T \boldsymbol{\phi}(x')$  which can compute the dot product implicitly in the original space. The learning then takes place in the feature space, and the data points only appear inside dot products with other points. This is often referred to as the “kernel trick” (Schölkopf et al., 2000). Different kernel functions are proposed in the literature such as the linear, polynomial, and Gaussian radial basis function (RBF). The Gaussian radial basis function,  $K(x, x') = \exp(-\gamma \|x - x'\|^2)$  is the most popular kernel function (Droguett et al., 2014; Hsu et al., 2003).

In this paper, SVR with RBF kernel function is implemented inside DBN to define the unknown nonparametric and nonlinear correlation relationships between some of the hidden and/or observed variables. The trained SVR then will be introduced into

augmented- particle filtering algorithm to infer the model parameters and damage states.

There are some concerns related to the SVM and SVR such as being deterministic or using a lot of kernels to perform the regression/classification. Hence, recently in the literature more sophisticated techniques such as Relevance Vector Machine (RVM) (Tipping, 2000) and Bootstrapped SVR (Lins et al., 2015) have been introduced to remedy these concerns by proving uncertainty quantification via different approaches. However, for the problem at hand, at this stage of the framework development, SVR presents promising results based on our experimental data. Therefore, SVR was implemented in this research to capture the relationship between some of the key variables in the DBN.

## *5-7 Summary*

Different mathematical elements which are required to formulate the proposed SHM have been presented in this chapter.

First, fundamentals of DBN structure were presented and then the details of standard particle filtering as the main underlying inference techniques were discussed. Later, it was described how the standard particle filtering can be extended to augmented particle filtering to update both states and model parameters at the same time. The augmented particle filtering algorithm along with kernel smoothing technique supplies a flexible stochastic inference technique to infer the hidden states and unknown parameters in the DBN structure with the presence of multiple uncertain evidences. The prognostics then can take place by projecting the particles in time based on the trained DBN when no more observation exists.

The last but not the least, the regression with SVR was explained which can be particularly handy in modeling the unknown relationship between some variables in the DBN.

The proposed integrated mathematical framework presented in previous chapter (section 4-7) provides the foundation to fuse all these elements in a DBN structure. In the following two chapters, the proposed integrated mathematical framework will be applied for investigating the degradation process and predicting the RUL in two different real world case studies: degradation of metallic components and degradation of composite components.

## Chapter 6: Damage estimation and prediction of RUL in

### Metallic component prior to crack initiation (Case Study I)

#### *6-1 Overview*

In this chapter, degradation of the metallic components (7075-T6 Aluminum alloy) under fatigue prior to crack initiation is studied from the new perspective of damage precursor. The experimental setup and procedures used for fatigue testing will be explained. The proposed SHM and prognostics framework presented in Chapter 4: section 4-5 is employed here to develop a damage model based on evolution of damage precursor when the crack is not detected yet. This damage model is updated with the existence of two different uncertain sensor data. The proposed SHM framework integrates all these information via DBN structure and presents accurate damage estimation through time. The framework is then used to predict the crack initiation time.

#### *6-2 Introduction*

Fatigue is one of the most common and well-studied failure mechanisms in mechanical and structural systems. Although many published papers in the reliability field have effectively studied diagnosis and prognosis in systems under fatigue, there is a significant gap for considering damage precursors for fatigue related failures in the SHM framework. Widely used empirical models (e.g., Paris Law) are applicable when an initial crack is already present, while damage may start much earlier than



onset of crack formation. Life estimation based on crack length measurements using empirical models and very early service data would result in immature and inaccurate estimation, while on the other hand, it would be too late if we wait until easily measureable crack is detected.

### 6-3 Experimental setup:

An accelerated life testing is designed and run in the Laboratory of the Center for Risk and Reliability at the University of Maryland, College Park. In this set of experiments, dog-bone 7075-T6 Aluminum samples undergo cyclic load with frequency of 5Hz and stress ratio of  $R=0.1$ . Samples contain a small notch in order to localize the stress intensity and accelerate the degradation process. Extension of the specimen is measured by extensometer which is placed around the notch area. Moreover, two Acoustic Emission sensors are also employed on the sample in order to capture any acoustic wave emitted during the fatigue test. A high resolution microscopic camera is adjusted and zoomed on the notch area that captures photos every 5 seconds. When setup is complete, the specimen experiences fatigue under cyclic load with maximum load of 11 KN. Experimental setup and schematic of the test specimen are shown in Figure 6-1.

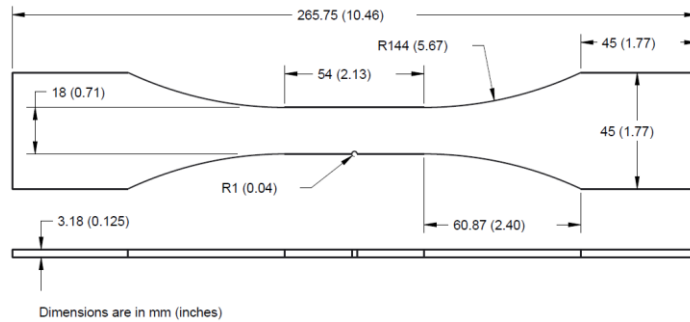
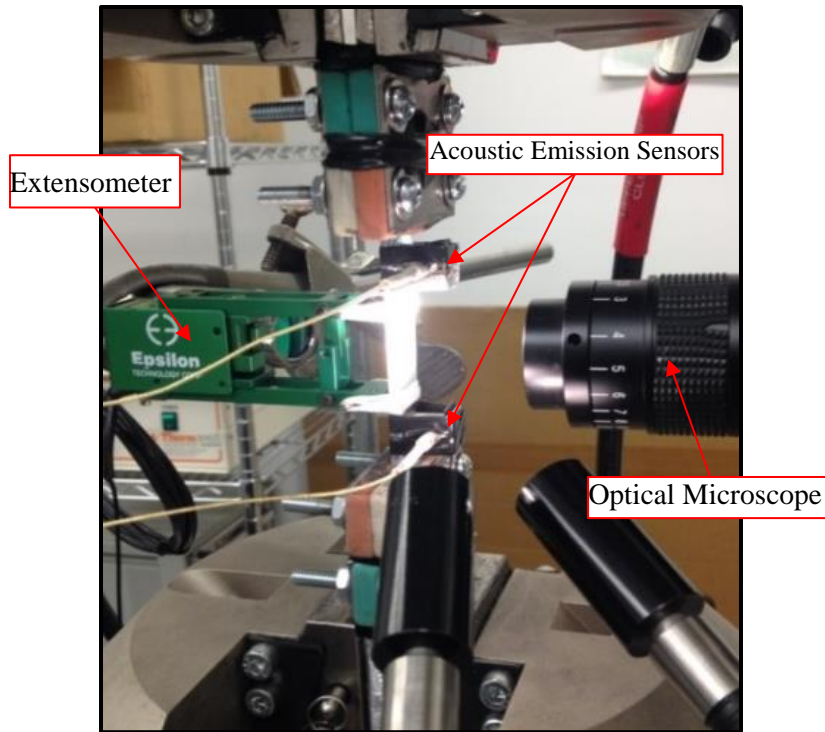


Figure 6-1: Experimental set up (top), schematic Dog-bone specimen used for the fatigue test (bottom)

#### 6-4 Define the damage precursor:

Since the focus of the experiment is on crack initiation, the test will be stopped as soon as first signs of crack can be recognized by microscopic camera. Such experiment relates to the time period before crack initiation in the component, so conventional damage models (e.g., Paris Law) are not valid and cannot be applied to

estimate the damage level. Therefore, without loss of generality and because of the nature of the experiment, only stage I of the proposed two-stage SHM framework (Figure 4-1) can be employed in this case study. If the experiment were to continue after crack initiation into crack growth until some predefined threshold is reached, then stage II of the framework would be applicable.

Having experimental results, the challenge is to define damage precursors which can explain the microstructural degradation happening in the component prior to crack initiation. Referring to the proposed SHM framework in Figure 4-1, the procedure of state estimation starts in stage I by identifying the microstructural damage mechanisms and proper damage precursor. When a component undergoes fatigue loading, microstructural changes such as micro deformation, slipping and micro-cracks at grain's boundaries happen at material scale which can be treated as underlying damage mechanisms. Such phenomena, although unobservable, make the component weak and reduce its resistance to deformation. Modulus of elasticity, as the measure of substance's resistance to deformation, has been reported in literature (Lemaitre, 1996, 1985) as one of the microstructural properties that changes during degradation. Therefore, variation of modulus of elasticity can be considered as a damage precursor that would provide insight about the undergoing damage within the component in advance to any visible crack on the surface of the component.

Lemaitre (Lemaitre, 1985) proposed that it is possible to estimate damage through the variations of the modulus of elasticity. If  $E_0$  is the modulus of elasticity of undamaged material, then damage parameter  $D$  can be expressed by:

$$D = 1 - \frac{E}{E_0} \quad (6-1)$$

where  $E$  is the modulus of elasticity for the degraded material. As soon as damage occurs and propagates in the material, modulus of elasticity decreases. This relationship is simply presented by Eq. (6-1). In this model, damage would reach 1 only if  $E$  reduces to 0. This situation might not be obtained in reality, not even at breakage point. A modified version of the Lemaitre damage parameter (Lemaitre, 1996, 1985) was introduced by Mao and Mahadevan (Mao and Mahadevan, 2002):

$$D = \frac{E_0 - E}{E_0 - E_f} \quad (6-2)$$

where,  $E_f$  is the Young's modulus when the failure occurs. Eq. (6-2) presents a damage parameter based on variation of modulus of elasticity, which is scaled between 0 and 1, so that damage would be 1 when  $E$  reaches a predefined threshold on modulus of elasticity  $E_f$ . Therefore, by measuring the modulus of elasticity as a damage precursor during degradation process, one can use Eq. (6-2) to estimate a normalized damage parameter  $D$  with respect to  $E_0$  and  $E_f$ .

Having defined a suitable damage precursor, the next step is to identify its variation in order to develop a damage evolution model (refer to Figure 4-1). Modulus of elasticity is basically the slope of stress-strain curve in elastic region. Since we are dealing with cyclic load, stress-strain curves will form hysteresis loop throughout the loading process. Variation of stress-strain can be monitored by extensometer. To quantify  $E$ , the slope of the linear portion of stress-strain loop is calculated for each loading cycle in the hysteresis loop. Figure 6-2 shows how modulus of elasticity changes during the aforementioned experiment.

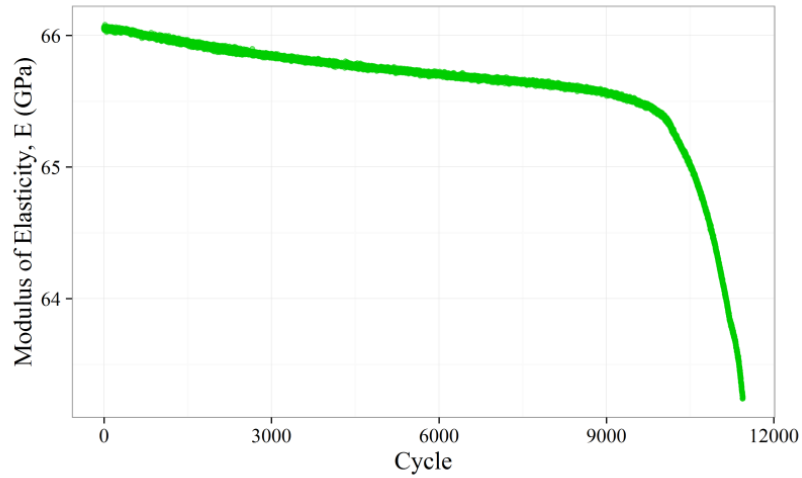


Figure 6-2: Variation of the modulus of elasticity during fatigue test until first signs of crack is detected

#### 6-5 DBN representation: general overview

Figure 6-3 represents a high-level DBN for modeling the degradation process in this experiment. As explained above, in the hidden layers of the material, damage mechanisms take place that result in change in modulus of elasticity,  $E$ . Referring to general DBN structure which was presented in chapter 4 Figure 4-2, the node for damage precursor  $DP_k$  would be  $E_k$  in this case.  $E$  can be measured and tracked through monitoring variations of hysteresis loop in each cycle. Calculated modulus of elasticity (i.e., the measurement of  $DP_k$  in Figure 4-2) is then entered into Eq. (6-2) to acquire a normalized damage index  $D$  used as a proper representative of hidden damage evolution. On the other hand, acoustic emission sensors can capture signals from underlying progressive degradation. AE signals can be treated as another observed evidence of hidden damage (equivalent to node “other evidences” in Figure 4-2).

Having the DBN topology ready, probability distribution of all the correlated nodes should be defined in order to make inference in DBN. Each arrow in Figure 6-3 indicates the probabilistic relationship between the nodes that needs to be modeled by physical or data-driven methods. Hence, in the DBN shown in Figure 6-3, probability distributions  $P(E_k|D_k^*)$ ,  $P(D_k|E_k)$ ,  $P(AE_k|D_k^*)$  and  $P(D_k^*|D_{k-1}^*)$  need to be explicitly provided at each time step  $k$ . Following section presents how these probabilistic relationships are estimated, and then a more elaborate DBN with all the contributing factors can be constructed.

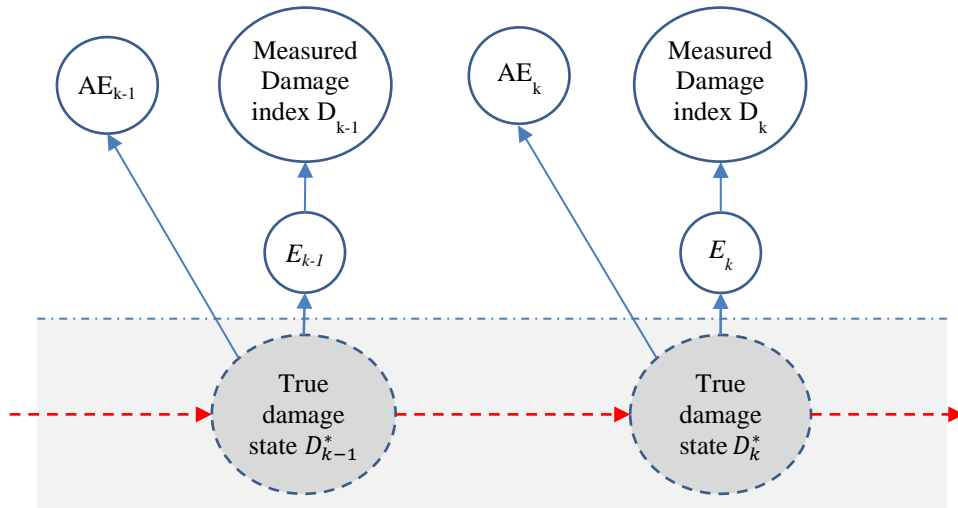


Figure 6-3: General DBN representation of the damage evolution considering actual hidden underlying damage mechanisms

### 6-6 Inference in DBN using Augmented-Particle Filtering

A combination of physics-based and data-driven models is required to represent the relationships between the nodes in the DBN in Figure 6-3. To make an inference about the hidden variables (true damage parameter  $D^*$ ) in the DBN, state process model and observation models need to be identified. The details are presented in the following.

### 6-6-1 Online learning of both state and parameters in the degradation model:

Ideally, it is advantageous to apply physics-based model to describe the system degradation in the form of an analytical system equation (degradation model). The standard particle filter state estimation process was retained as the model-based technique. Mao and Mahadevan (Mao and Mahadevan, 2002) proposed a versatile empirical model for explaining the evolution of damage index in composite material (see Eq. (6-3)). Although the model was first suggested for damage parameter in composites, the form of the model fits our experimental results very well. This is because of the two-part format of the model as the first term controls the damage accumulation at the beginning of the degradation, and the second term captures the fast damage growth toward the end of the life. Therefore, based on our experimental results, the model is quite satisfactory for explaining the behavior of damage evolution in terms of decrease in modulus of elasticity prior to crack initiation in our case study.

$$D^* = q \left( \frac{n}{N_f} \right)^{m_1} + (1 - q) \left( \frac{n}{N_f} \right)^{m_2} \quad (6-3)$$

In this equation,  $q$ ,  $m_1$  and  $m_2$  are model parameters that need to be estimated online during the experiment and  $n$  is the elapsed cycle which is normalized with respect to number of cycles at failure threshold  $N_f$ . However, in online monitoring of the component/system, the value of  $N_f$  is not known in advance. In fact, estimating the maximum number of cycles to failure is the final objective of the whole diagnostics and prognostics framework. Therefore,  $N_f$  would be treated as another unknown model parameter that needs to be updated in real time.

State of damage at each time step  $k$  relates to not only the elapsed cycle, but also its previous damage level  $D_{k-1}^*$ . With Eq. (6-3) and for small enough  $\Delta n$ , the state process model can be discretized in the form of (Zio and Pelsoni, 2011):

$$D_k^* = D_{k-1}^* + \frac{\Delta D^*}{\Delta n} |_{k-1} \times \Delta n \times e^{\omega_k} \quad (6-4)$$

Where  $\frac{\Delta D^*}{\Delta n}$  is derivative of  $D^*$  with respect to cycle ( $n$ ) in degradation model (Eq. (6-3)) and the stochastic behavior of the state process model is represented by  $e^{\omega_k}$ . There is no restriction on the noise term distribution  $\omega$ . Here, a white Gaussian noise with mean zero and standard deviation  $\sigma$  is considered  $\omega \sim \mathcal{N}(0, \sigma)$ , which will result in a lognormal process noise when embedded in exponent.  $k$  denotes the  $k^{\text{th}}$  cycle. Accordingly, based on Eqs. (6-3) and (6-4), the state process model would be:

$$D_k^* = D_{k-1}^* + \left[ \frac{m_1 \times q}{N_f} \times \left( \frac{n}{N_f} \right)^{m_1-1} + \frac{m_2 \times (1-q)}{N_f} \times \left( \frac{n}{N_f} \right)^{m_2-1} \right]_{k-1} \times \Delta n \times e^{\omega_k} \quad (6-5)$$

Eq.(6-5) is used to describe the probability distribution  $P(D_k^* | D_{k-1}^*, \theta_k)$  in which  $\theta_k = [q, m_1, m_2, N_f]_k$ . The idea here is to update both damage states and model parameters simultaneously as time goes on. Since the number of state and parameters that need to be estimated is relatively large, high level of uncertainty is expected. Thus, the more observation/information gathered and employed, the more precise estimation can be obtained.



## 6-6-2 Observation models

Real time observations will be used to weigh the projected particles. In this case study, two types of observation are available: one is AE signals and the other is the measurements of variation of modulus of elasticity  $E$  as damage precursor. In order to demonstrate the importance of incorporating different observations into the framework, three cases are studied here:

- Case 1: only consider AE signals, i.e.,  $P(AE_k|D_k^*)$
- Case 2: only consider measured modulus of elasticity  $E$ , i.e.,  $P(E_k|D_k^*)$
- Case 3: consider both AE signals and measurements of reduction in modulus of elasticity, i.e.,  $P(E_k, AE_k| D_k^*)$

In case 1, the likelihood of observing AE signals at time step  $k$  given underlying damage state should be estimated. Acoustic emission signal acquisition system captures and reports different features of AE signals. In the present study, cumulative absolute energy of signals is calculated and employed as one of the observations. Ideally, in an online monitoring process, it is preferred to implement new observations as soon as they get available to update a predefined empirical or physical model. However, not all the time a well-defined physical or empirical relationship exists between the variables. To the best of our knowledge, there is not a pre-determined model to correlate the cumulative absolute energy of AE signals to underlying hidden damage parameter prior to crack initiation. Moreover, no common type of regression family can efficiently model this relationship. Therefore, to overcome this challenge, more flexible regression approach based on SVM is applied. A SVR model is trained offline based on 60% of captured AE signals (Figure 6-4).

This model, including some measurement noise  $\vartheta_k$ , will be then used online to estimate  $P(AE_k|D_k^*)$  when updating the states and parameters when the rest of AE signals (test data) get available. Therefore, an offline data-driven model based on a portion of data (train data) is developed to relate acoustic emission signals AE to damage parameter  $D^*$ .

On the other hand, in case 2, we assume that only in-situ measurement of modulus of elasticity as indirect damage indicator prior to crack initiation is incorporated in the likelihood equation. Measured  $E$  will be used in Eq. (6-2) to calculate damage level  $D$ . Considering measurement uncertainties ( $v_k$ ), online evaluated  $D$  can be then used through the following observation model to update the estimations of state process model.

$$D_k^* = D_k + v_k$$

$$P(E_k|D_k^*) \propto P(D_k|D_k^*) \tag{6-6}$$

Expert opinion and specification of measurement instruments (Acoustic Emission acquisition system and extensometer) are used to decide about the observation noises  $\vartheta_k$  and  $v_k$ .

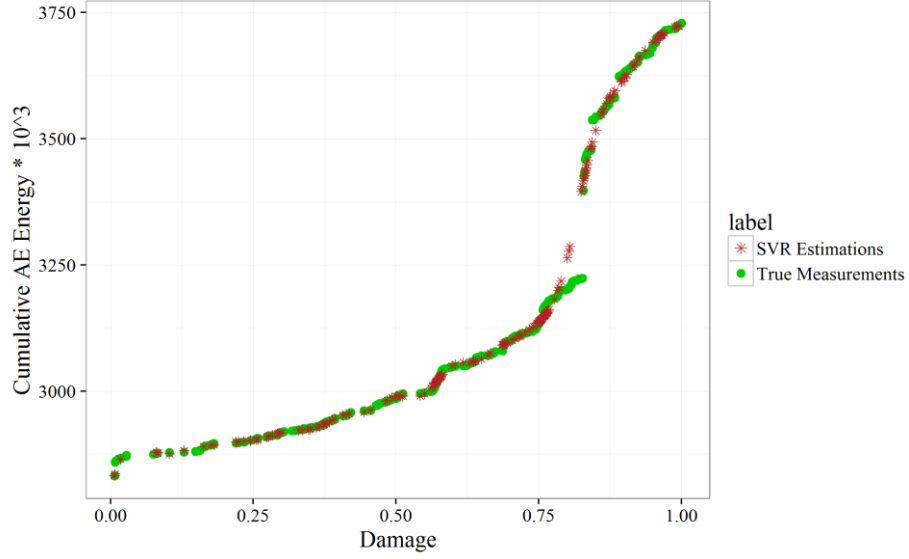


Figure 6-4: Correlation of damage index and cumulative AE energy using SVR (model is trained based on 60% of data)

In case 3, however, when both AE and  $E$  are integrated simultaneously, one fused measurement model exists as  $P(E_k, AE_k | D_k^*)$ . Based on the probability chain rule, we will have:

$$P(E_k, AE_k | D_k^*) = P(E_k | AE_k, D_k^*) \cdot P(AE_k | D_k^*) \cdot P(D_k^*) \quad (6-7)$$

In this case study, it is reasonable to assume that variation of modulus of elasticity is independent of captured acoustic emission as they are different in nature, therefore:

$P(E_k, AE_k | D_k^*) \propto P(E_k | D_k^*) \propto P(D_k | D_k^*)$ . This results in the integrated measurement model as:

$$P(E_k, AE_k | D_k^*) \propto P(D_k | D_k^*) \cdot P(AE_k | D_k^*) \cdot P(D_k^*) \quad (6-8)$$

More details on estimating these 3 cases as well as existing challenges are presented in the next section.

### 6-6-3 DBN representation: detailed model

Now that all the elements of the DBN and the details of state process and observation models are explained, a more elaborate version of Figure 6-3 can be constructed.

Figure 6-5 demonstrates the detailed DBN of this case study:

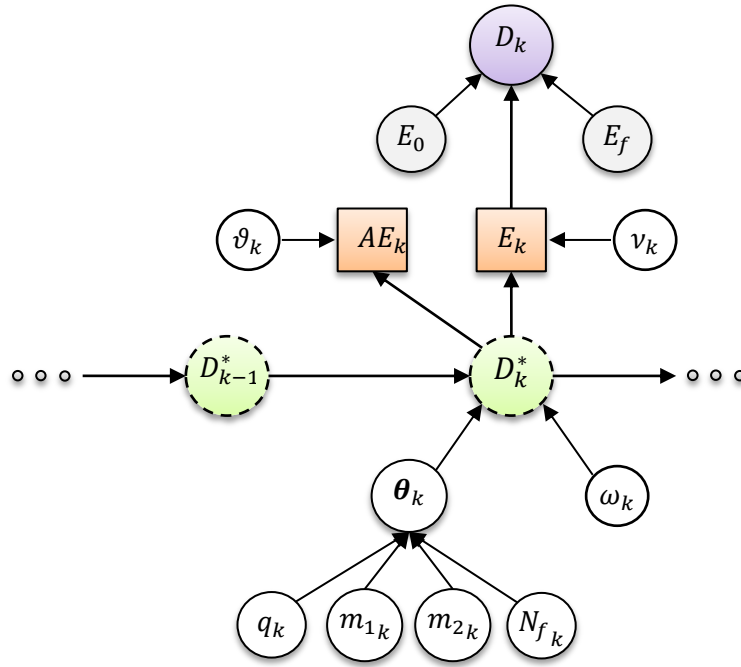


Figure 6-5: Detailed DBN representation considering all the factors

In Figure 6-5, observations (i.e., AE signals and measured damage precursor  $E$ ) are shown with rectangular. Recalling from section 6-6,  $\omega_k$ ,  $v_k$  and  $\vartheta_k$  are process noise, noise in  $E$  measurements and noise in captured AE signals respectively.  $\theta_k$  is the vector of parameters for state process model Eq. (6-5) consists of  $q$ ,  $m_1$ ,  $m_2$  and  $N_f$  at time step  $k$ . In the context of this case study, only  $E_0$  and  $E_f$  are considered to be constant. The DBN presented in Figure 6-5 is used in the rest of this paper for estimating and predicting damage in the component.

## 6-7 Results and discussion:

In each case mentioned in previous section, 5000 particles that included 1000 particles for each variable ( $q$ ,  $m_1$ ,  $m_2$ ,  $N_f$  and damage states  $D$ ) were randomly selected. Any prior information about the initial value of the parameters would be very helpful in achieving the faster convergence of the technique. Such prior information might come from relevant literature, similar experiments or expert opinion. In our case, the suggested range (Mao and Mahadevan, 2002) for parameters  $m_1$  and  $m_2$  are  $m_1 < 1$  and  $m_2 > 1$ . However, more information on the range of the parameters was obtained from fitting the same model to other experiments under similar test conditions. Therefore, model parameters were initialized as following:  $q =$  uniform [0.01, 1],  $m_1 =$  uniform [0.1, 0.8],  $m_2 =$  uniform [18, 25],  $N_f =$  uniform [10000, 14000]. Also, for damage states  $D$ , initial particles should be selected very close to zero as it is assumed that the component is completely healthy and no damage exists in the component before loading. Randomly selected particles are propagated in time based on the proposed state model with unknown model parameters, Eq.(6-5), following the procedure explained in section on augmented particle filtering. The estimation of the model parameters as well as states will be updated once any measurement gets available. The methodology will be validated by comparing the estimation and prediction results of DBN with the true damage evolution. The following results present that the proposed methodology is able to effectively track the true damage evolution based on variation of modulus of elasticity along with captured AE signals.

### 6-7-1 Damage state monitoring

In case 1, when only one observation model exists  $P(AE_k|D_k^*)$ , the challenge is the scarcity of observations due to lack of significant AE events at preliminary stages of fatigue. Very few AE signals were received in almost 80% of the experiment, while the number of observations increases dramatically toward the end of test. Therefore, there are not enough data points at the beginning to both learn the model parameters and estimate the damage state. Relying only on AE signals to update the model leads to significant errors in damage estimations. However, as it is presented in Figure 6-6 (bottom), particles begin to move toward true damage state when more observations are captured at the end of experiment. Model parameters (Figure 6-6(top)) also cannot be trusted as they behave very randomly and do not follow any particular pattern.

In case 2, however, modulus of elasticity can be measured in each cycle, therefore there will be plenty of observations to both learn the model and estimate the damage state. Measurement model  $P(D_k|D_k^*)$  comes from Eq.(6-6), in which  $D_k$  is calculated based on measured modulus of elasticity  $E_k$ . It can be seen in Figure 6-7 that measurement of modulus of elasticity is obviously more informative than AE signals before crack initiation. High level of uncertainty was observed at the beginning of the process because of the fact that all the model parameters were selected randomly. This uncertainty decreases through time when more  $E$  measurements are obtained. The particle filtering approximation of damage state nicely follows the true damage evolution especially in the middle part. Toward the end of the experiment, however, slight discrepancy (overestimating of damage) can be seen. Examining the model parameters (Figure 6-7) shows that the variation of parameters reduces after almost

4000 cycles and they tend to converge to some particular values, but parameters  $m_2$  and  $N_f$  suddenly starts to increase near the end of the estimation and this is the reason for slight inconsistency appeared in state approximations (Figure 6-7(bottom)). In fact, recalling from Eq.(6-3), parameter  $m_2$  is responsible for the sharp increasing slope of the degradation model near the end of the experiment.

In order to reduce the computational cost and increase the practicality of the proposed method, modulus of elasticity was calculated at some specific interval (for example every 30 cycles) instead of every cycle.

Figure 6-8 shows the results of damage estimation in case 3 when both AE signals and  $E$  are incorporated in the DBN for the updating process. It was noted that results were improved and more precise estimation can be achieved. It is interesting that even though AE signals seem to be very ineffective and incapable for updating the states and parameters once used alone in case 1, they can improve the results when combined with  $E$  measurements. Compared to case 1 and 2, DBN approximations of damage state in case 3 follow the trend of true damage evolution more accurately through time, and even the uncertainty of estimation (dispersion of particles) was reduced especially at the tail where many AE signals are available.

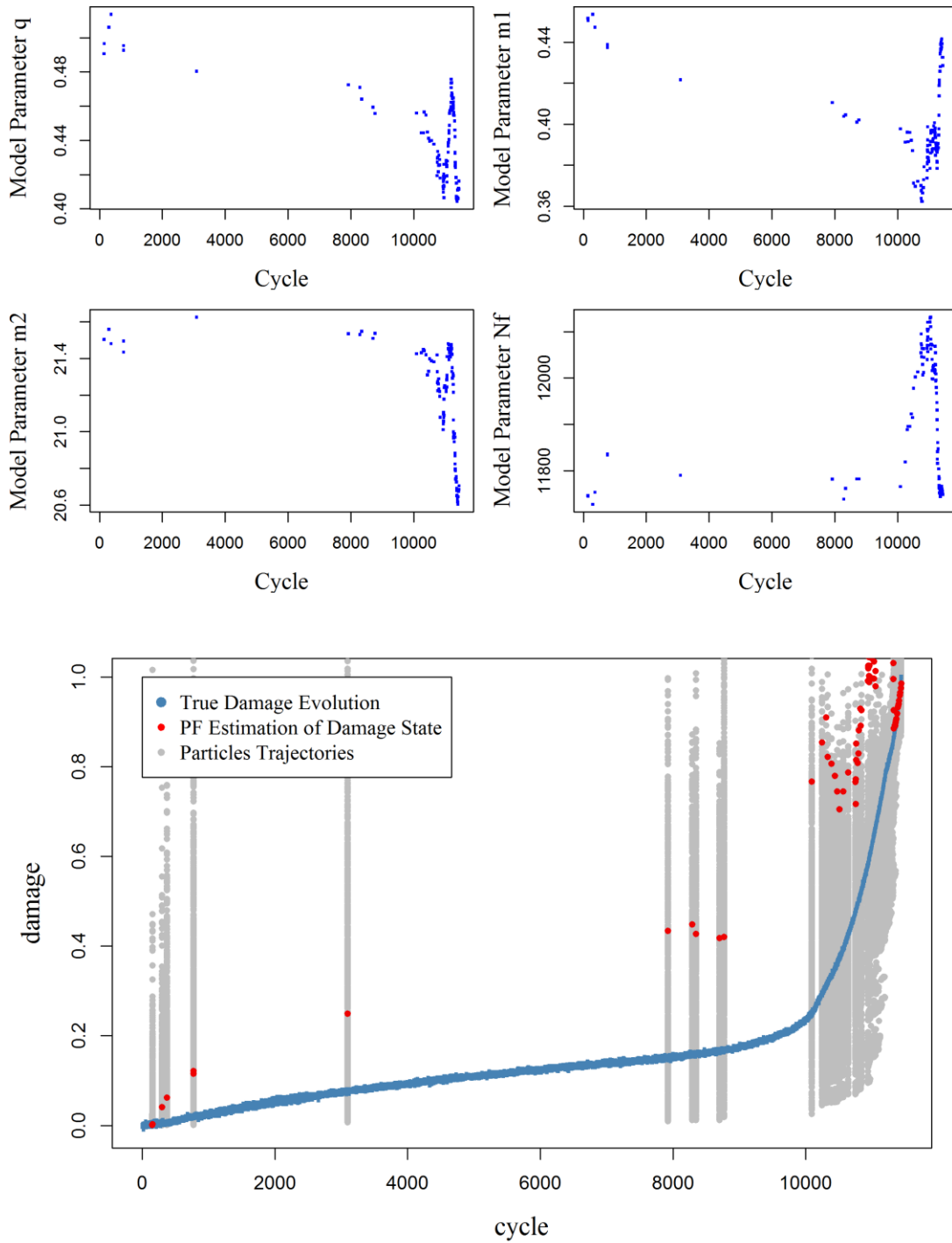


Figure 6-6: Estimation of damage evolution in time using only AE signals for case 1 (bottom) and corresponding model parameters (top)



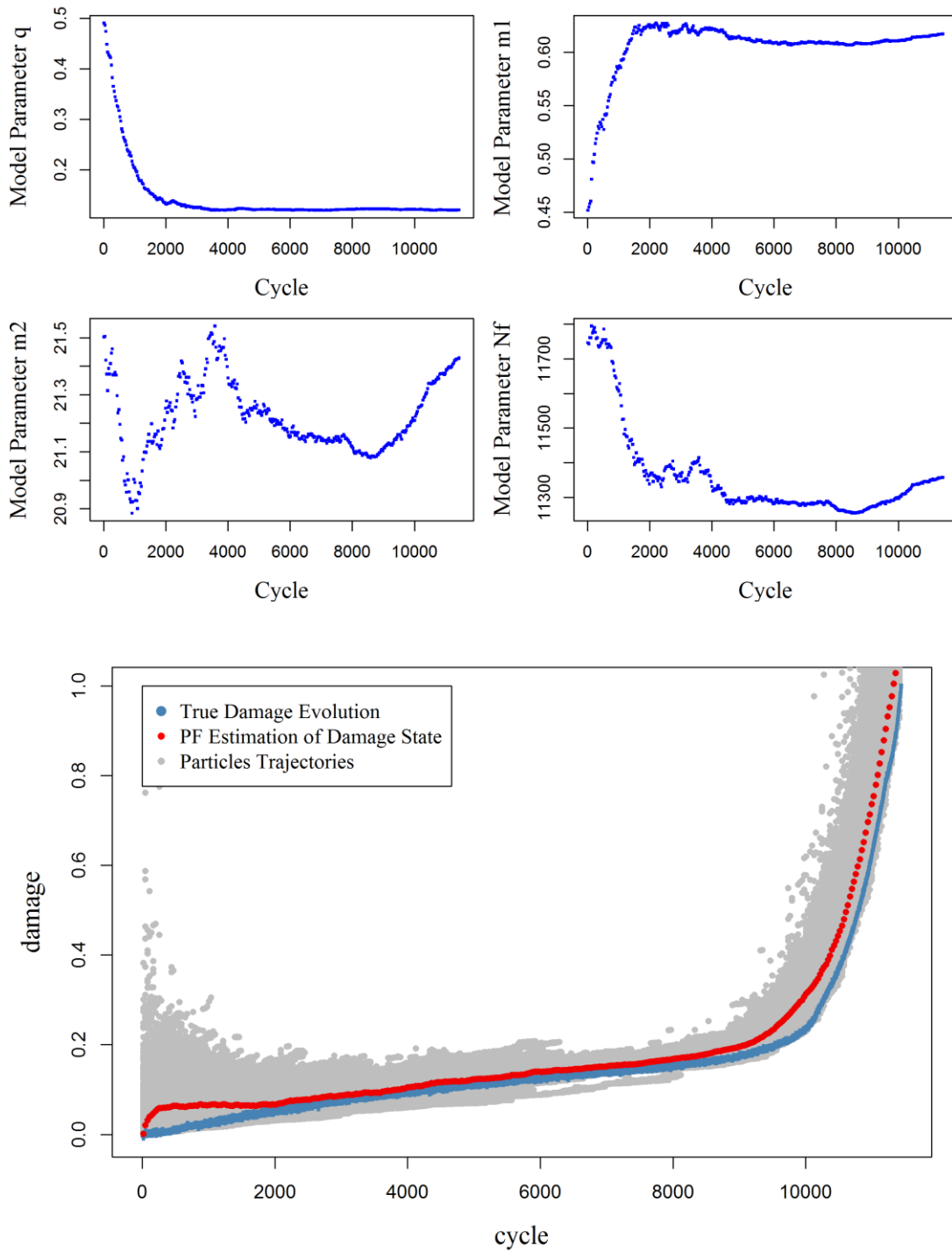


Figure 6-7: Estimation of damage evolution in time using only measured E for case 2 (bottom) and corresponding model parameters (top)

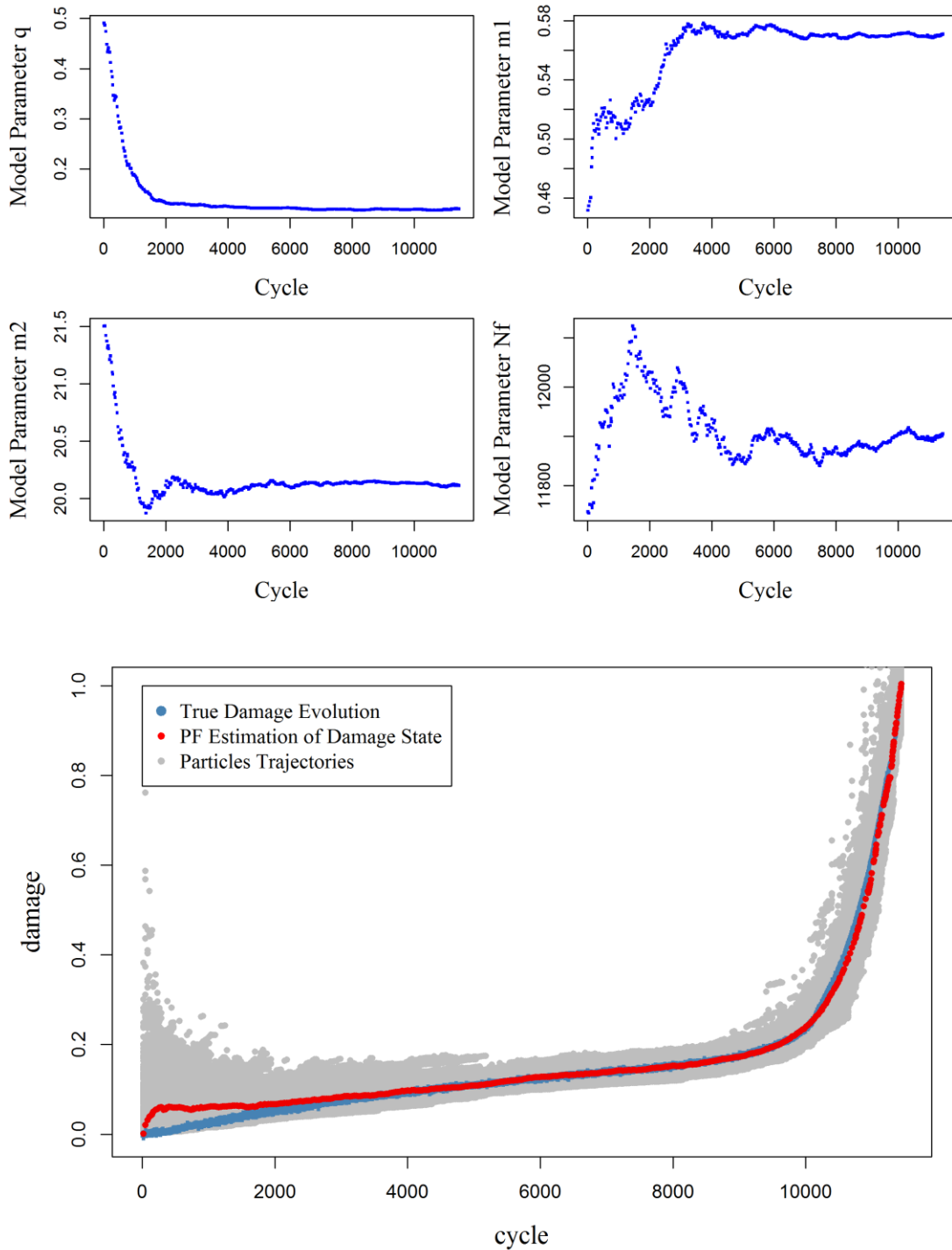


Figure 6-8: Estimation of damage evolution in time using both AE signals and measured  $E$  for case 3 (bottom) and corresponding model parameters (top)

Since the observations (AE signals and  $E$  measurements) are not necessarily synchronized, they do not enter into the DBN simultaneously. Therefore, the

procedure for considering both evidences is as follows: as soon as any of the observations (AE signals or  $E$ ) is captured independently, the model parameters and damage state are updated with corresponding measurement model similar to cases 1 and 2. And if both AE and E were captured simultaneously, then Eq. (6-8) should be used to consider the fused measurement model. In other words, unlike regular filtering techniques, in this approach time step  $\Delta n$  is not fixed to a predefined value and it will change adaptively based on availability of the observation as we progress in time.

Another important improvement in fusing different observations is illustrated in convergence of model parameters. Figure 6-8 shows smoother and more stable convergence in all the model parameters  $q$ ,  $m_1$ ,  $m_2$  and  $N_f$ . This feature is especially significant in prognostics. Since in dual updating particle filtering algorithm, model parameters are not known in advance, prognostics results are not reliable unless fluctuations of model parameters subside.

Although the improvement of estimation results is clearly apparent in Figure 6-6 to Figure 6-8, in order to mathematically show the increase in accuracy of the approach, the Root Mean Square Error (RMSE) is calculated for each case by Eq. (6-6) and presented in Table 6-1.

$$RMSE = \sqrt{\frac{\sum_{n=0}^{N_f} (\hat{y}_n - y_n)^2}{N_f}} \quad (6-9)$$

$\hat{y}_n$  is the true damage coming from (6-3), and  $y_n$  is the estimated damage by augmented-particle filtering (the red dots in Figure 6-6 to Figure 6-8). Then their

difference is squared and averaged through time from the beginning of life to the failure time.

Table 6-1: RMSE calculation for the three cases presented above

	<i>Case 1: Only consider AE</i>	<i>Case 2: Only consider measured modulus of elasticity E</i>	<i>Case 3: Consider both AE and E</i>
<b>RMSE</b>	0.5195	0.0438	0.016

As it was anticipated, the error in Table 6-1 for case 1 is significantly large. Case 2 shows much better RMSE results because of the substantial number of monitoring data coming from E measurements at each cycle. Nevertheless, the results of RMSE are improved further for case 3. Therefore, Table 6-1 mathematically confirms that more accurate damage estimation is obtained by integrating both evidences.

However, the value of RMSE is not solely illustrative of the precision of the results which relates to the uncertainty in the damage estimation. The dispersion of the particles is the representation of the uncertainty, and in order to measure and compare this uncertainty mathematically, the credibility interval or more specifically the Highest Posterior Density (HPD) interval can be computed for each case through time. The HPD is an interval in which most of the distribution of the particles lies. A  $100 * (1 - \alpha)\%$  HPD is the region that satisfies the following two conditions:

- The posterior probability of that region is  $100 * (1 - \alpha)\%$ .
- The minimum density of any point within that region is equal to or larger than the density of any point outside that region.

Figure 6-9 shows the 95% HPD intervals for all the three cases which are calculated at their corresponding time step and plotted on top of each other. For better illustration of the uncertainty bounds, the scale on y axis is extended to 1.4. As expected, the 95% HPD in the first case, shown by grey lines, are extremely wide because of the very few measurements of AE signals.

The green and red lines relate to 95% HPD for case 2 and 3, respectively. While the green area is deviated from the true damage trend especially toward the last 20% of the life, it is clearly evident that in case 3, this deviation is resolved by fusing both AE and E measurements and the reduced 95% HPD always surrounds the true damage trend. Therefore, more precise damage estimations are achieved through integrating two sources of evidence.

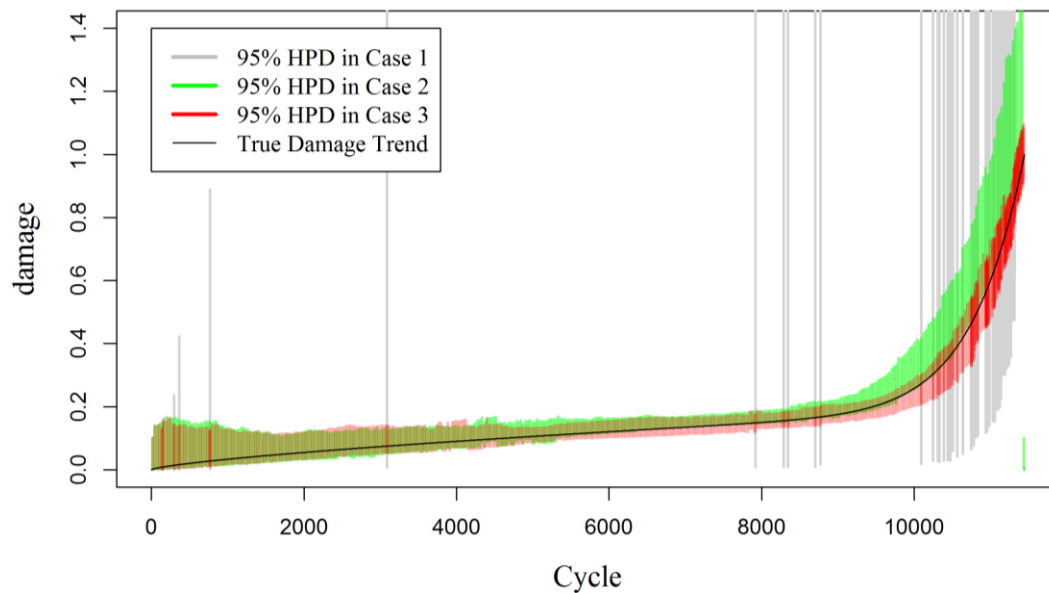


Figure 6-9: Comparison of 95% HPD intervals in all the three cases.

### 6-7-2 Prognostics and crack initiation prediction:

Without loss of generality, suppose that at a particular time  $T_p$ , one intends to look  $p$ -step ahead and predict the remaining useful life. In this case study, the experiment stops immediately after detection of crack by the optical camera. Therefore, RUL here relates to remaining useful life before the crack initiation, and prognostics encompass predicting the crack initiation time. As explained earlier, prognostics in augmented-particle filtering algorithm is more challenging because not only no other observation exists to update the estimation, but also the degradation model itself is not fully known. Therefore, prognostics should be postponed until variation of model parameters decreases.

Figure 6-10 shows the result of prognostics at  $T_p= 4000$  cycles.  $E$  measurements and AE signals were employed to learn the model parameters and damage states (like case 3) up to 4000 cycles and after that nothing was observed. Prognostic was performed by propagating the particles in time relying only on state process model without any more updating in model parameters and/or states. It is evident that in this case the particles disperse more and more through time. However, the final approximation of particle filtering,  $(\sum_1^N \omega_i \delta(x - x_i))$ , is remarkably close to the true damage. To better illustrate the uncertainty in particles, the y-axis is scaled to the maximum value of 1.6 in Figure 6-10.

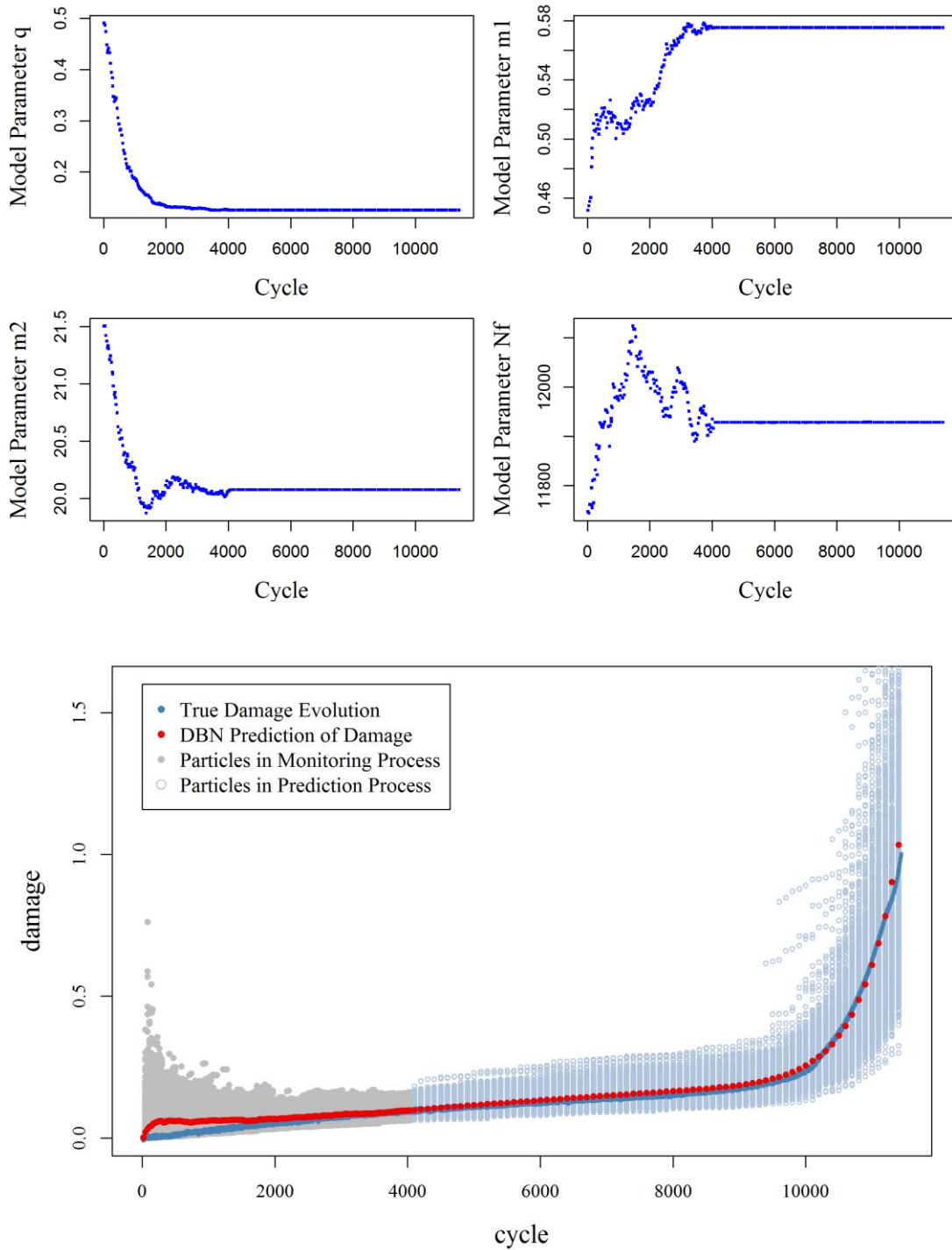


Figure 6-10: Variation of model parameters(top). DBN prediction of damage evolution until crack initiation (bottom). Note that model parameters do not change during prognostics.

Prediction of the Time to Failure (TTF) and Remaining Useful Life (RUL), as the ultimate goal in prognostics, is based on a predefined damage threshold.

Characterizing the failure threshold depends on the features of the problem in hand. In our case study, “failure” relates to observing first signs of direct damage indicator (crack initiation), so TTF means time to crack initiation. In that sense, the threshold in the present study is defined as when evolution of damage parameter reaches 1.

As described in the Section 5-5, each particle is tracked from the start of prognostics ( $T_p$ ) until it passes the threshold at cycle  $t_f$ . The process continues until all the particles cross the limit and fail. The TTF corresponds to a distribution over all the  $t_f^{(i)}$  ( $i$  from 1 to  $N$ ) and the MTTF is estimated based on Eq.(5-18). The long-term prediction starting at cycle  $T_p=4000$  and the distribution of TTF are also shown in Figure 6-11.

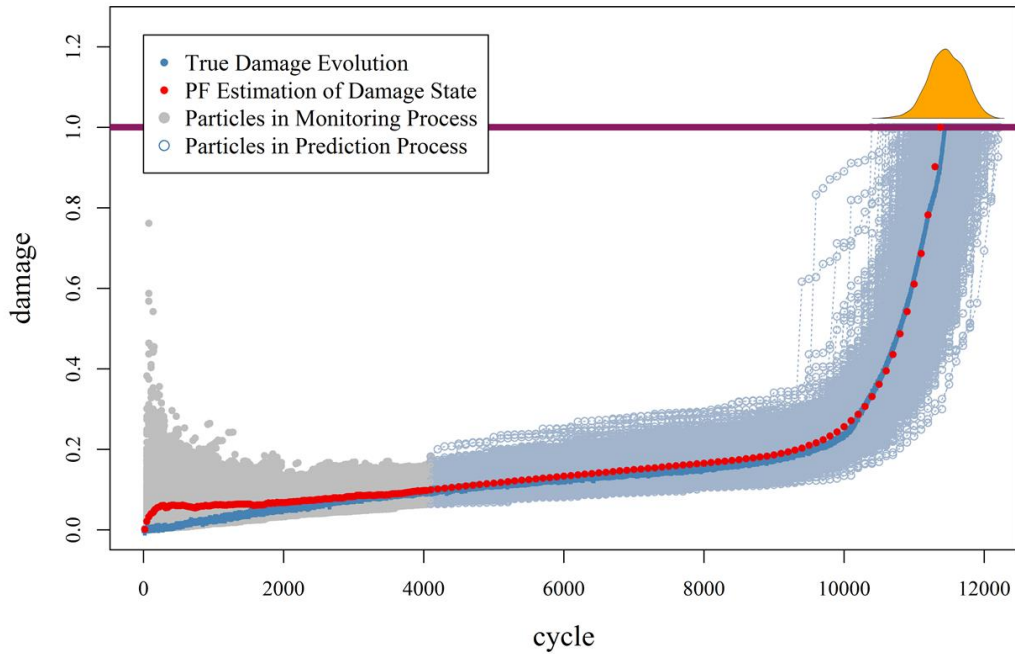


Figure 6-11: Long-term prediction of life before crack initiation and distribution of TTF

Referring again to Figure 6-8, it can be seen that even though variation of model parameters decreases after 4000 cycles, there is still some noise especially in parameters  $m_1$  and  $N_f$ . So, it is expected to get different prediction results if the



prognostics begin at different cycle  $T_p$ . In order to show how the accuracy of the prognostics will change with respect to variation of parameters, the prognostic procedure was repeated with different starting points namely 4000, 4200, 4400, ... , 7000 cycles. The noise in the model parameters at the aforementioned prediction starting times  $T_p$  results in slightly different distributions for TTF (shown in Figure 6-12). However, the average of their MTTF is 11415 cycles which is in 0.2% error with respect to true TTF = 11444 cycles. The true TTF is when crack initiation is detected by the microscopic camera and the experiment stops.

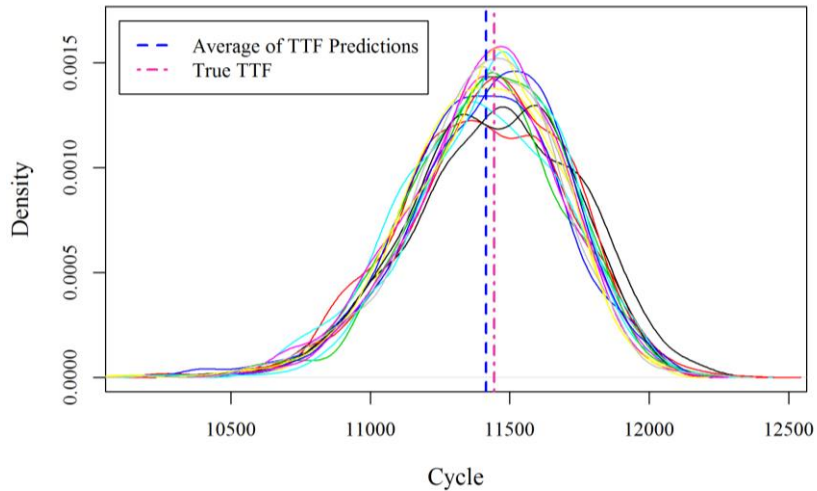


Figure 6-12: Different distributions of TTF when prediction is started at different cycles in [4000, 4200, 4400, ..., 7000]

### 6-8 Summary:

Widely used empirical damage models for fatigue of metallic components such as Paris Law can be used only when an initial size of crack is known. However, degradation process starts way before emergence of measurable crack size and it is important to monitor the structural health even when crack is not detected.

This chapter demonstrated the application of the proposed damage precursor-based SHM framework on fatigue of metallic components prior to crack initiation. A set of accelerated life testing experiments were conducted in which Al 7075-T6 specimens underwent fatigue loading until detection of measureable cracks. The tests were stopped as soon as the crack was seen by optic microscopic camera. Stress-strain and acoustic emission signals were captured and recorded during the experiment. A DBN was established to represent the related variables and their causal or correlation relationships. Variation of modulus of elasticity  $E$  was selected as damage precursor to describe the underlying active damage state in the component while crack had not detected yet. Then, online measurements of  $E$  and cumulative absolute energy of Acoustic emission signals were applied through augmented particle filtering to update the damage estimation and damage model parameters.

This chapter presented how the proposed SHM can be applied to estimate the current damage state and to predict the RUL in metallic component by focusing on damage precursor evolution instead on direct damage indicators. The method could successfully predict the crack initiation time in this case study.

## Chapter 7: Damage estimation and prediction of RUL in Composites (Case Study II)

### 7-1 Overview

In previous chapter, the proposed damage-precursor based SHM framework was successfully applied to monitor the damage state in metallic components before onset of cracks. This chapter attends to the cases when degradation process is very complex and measuring direct damage indicators such as crack is very difficult throughout the life of the component. The proposed one-stage SHM framework is employed to damage estimation and prognostics in composite materials under fatigue.

### 7-2 Introduction

Popularity of using composite materials in industry is increasing rapidly because of their low weight, high strength and long fatigue life. However, inhomogeneity of the composite material makes the degradation process very complicated especially when subjected to cyclic loading. Therefore, early-stage detection of damage and continuous assessment of degradation during the lifespan of the component are of critical importance in reliability estimation of the composite structures (Chiachío et al., 2015a). Several experimental, numerical and theoretical researches have been published recently on characterizing the fatigue damage in composites for example (Kahirdeh et al., 2013; Montesano et al., 2015; Naderi and Khonsari, 2012; Reis et al., 2010), just to name a few.

Majority of the proposed degradation models for composites are deterministic empirical formulations usually developed for particular material properties under specific testing conditions, whereas, damage evolution in composite is a complex stochastic and uncertain phenomenon. Moreover, lack of complete understanding of the degradation process and its corresponding failure modes add extra uncertainty and complexity to the modeling approach. Therefore, it is crucial to take the stochasticity of the process into consideration. Probabilistic approach toward damage estimation and prediction in composites is getting attention in recent years; however, the number of contributions in this context is still very limited (Chiachío et al., 2015b, 2015c, 2016; N. Eleftheroglou and Loutas, 2016; Peng et al., 2015). Among the most recent works, Manuel and Juan Chiachío and their colleagues in a series of similar papers (Chiachío et al., 2015b, 2015c, 2016; Chiachío et al., 2013) studied the progression of fatigue damage in composites by focusing on two characteristics: the matrix-cracks density, and the normalized effective stiffness. To obtain the evolution of matrix-crack density during the fatigue life, they implemented the energy release rate (ERR) that represents the energy released due to the formation of a new crack between two existing cracks at specific stress amplitude. The damage prognostic is then performed via particle filtering. They measured the same variables (micro-cracks density and stiffness) through the experiment to update the particle filtering estimations. In a similar research, (Corbetta et al., 2016) extended the same energy-based approach by (Chiachío et al., 2013) to consider not only the matrix-cracking but also the delamination stage in order to enable the real-time monitoring and prediction of a structure's RUL with multiple co-existing damage-mechanisms. Although the

aforementioned studies present physics-based approaches, which are preferred in general, their proposed energy-based models for damage estimation and prediction in composite are very complicated and might have limitations on other types of composites or more complex geometries (N. Eleftheroglou and Loutas, 2016). Also, the proposed models require precise measurement of many physical parameters that can be difficult or may need specific tools. Moreover, in their particle filtering algorithm, the measurements are exactly the same as the variables of interest which is not the case most of the times.

In comparison to the aforementioned works in SHM-based prognostics, Eleftheroglou and Loutas (Nick Eleftheroglou and Loutas, 2016) proposed a purely data-driven approach that does not rely on stiffness measurements or measurements of the load to define the energy release rate within the fatigue cycle. They modeled the damage evolution in composite as stochastic hidden Markov process by implementing only acoustic emission data. They used a generalized nonhomogeneous hidden semi Markov approach is to model the hidden damage process in a composite under fatigue loading. They took SHM data i.e., acoustic emission as the only input data. Nevertheless, purely data-driven approaches usually suffer from the lack of physical meaning and sometimes it is difficult to interpret the results.

The complication of damage evolution in composites and lack of a general and well-defined SHM framework for estimation and prediction of composite's degradation were motivations to this study. The current chapter presents new hybrid SHM framework for damage estimation and prediction of composites in which not only physical properties of the degradation process are considered as indirect damage

indicator, but also other sources of structural health monitoring data such as acoustic emission are integrated into the model in order to reduce the uncertainty and obtain more robust and reliable damage estimations and predictions. Moreover, in contrast with similar works, in this research, the measurements captured during the degradation are different than the hidden damage state.

### 7-3 *Damage in Composite*

Unlike the fatigue in metals which starts with crack initiation and then propagates in time until the component fails, fatigue in composite structures is a more complex process. Although understanding detailed failure mechanism of composite is out of the scope of this research, brief summary of the damage propagation in composites would be helpful.

Damage evolution in composite materials is a multistep procedure that involves micro-cracks formation and progression until failure occurs. The idealized trend of damage evolution in composite (shown in Figure 7-1) is proposed by Toubal et al. (Toubal et al., 2006) and Mao and Mahadevan (Mao and Mahadevan, 2002) as a three-stage process. Matrix cracking starts in Stage I in the weak points of the laminate and continues in Stage II where cracks coalesce and the damage takes place at the matrix–fiber interface. Also, in the second stage II, debonding and fiber matrix delamination take place and as a result damage accumulates and stiffness is reduced. In Stage III, fiber breakage is the dominant damage mechanism which results in the failure of the specimens. However, it is worth noting that fiber breakage may start earlier than Stage III (Kahirdeh et al., 2013; Natarajan et al., 2005; Toubal et al., 2006; Wu and Yao, 2010).

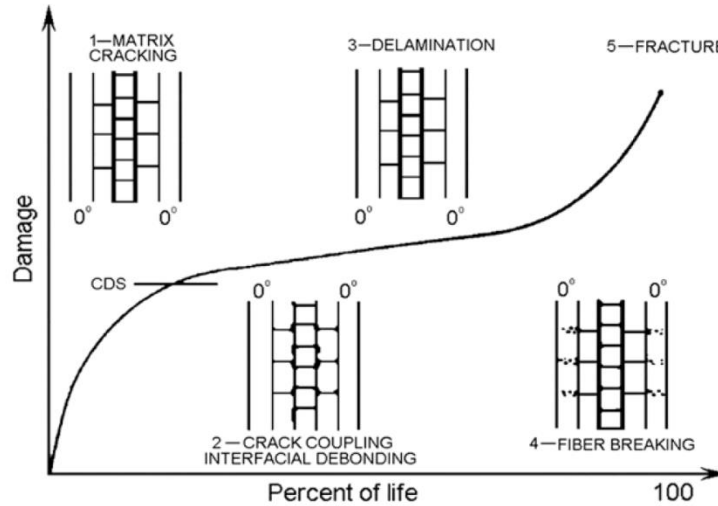


Figure 7-1: Fatigue damage evolution in composite laminates (Wu and Yao, 2010)

#### 7-4 Damage parameter or damage index:

In fatigue of metals, the crack size is a commonly accepted direct damage indicator that shows the progression of degradation in the component. However, as explained earlier, such well-defined and easily measurable direct damage indicator does not exist or is very difficult to monitor in composite materials because of the complexity of the degradation process. Therefore, researchers have tried to explain the degradation of composite using the normalized “damage index” or “damage parameter”, which varies from 0 – no damage, to 1 – completely failed.

Table 7-1 shows a number of the existing damage indices related to fatigue of composite materials. The damage parameters shown in Table 7-1 are all derived from experimental studies; the first four parameters are defined based on physical properties of the composite during the fatigue, while the last two rows are more analytical (curve fitting) damage parameters. Equation (7-1) is a modified version of the Lemaitre damage parameter (Lemaitre and Chaboche, 1994) introduced by Mao

and Mahadevan (Mao and Mahadevan, 2002) where  $E_0$  is the elastic modulus of the material in its untouched (undamaged) condition,  $E$  is the elastic modulus in the damaged state of the material and  $E_f$  represent the modulus of elasticity at the failure point of the data.

Dissipated energy during the fatigue degradation of the composite laminate is also employed by Giancane et al. (Giancane et al., 2010) to represent the damage profile. The  $H$  in Eq. (7-2) represents the dissipated energy per cycle,  $H_0$  is the dissipated energy in the initial cycle of the operation and  $H_f$  is the final condition, respectively.

Azouaoui et al. (Azouaoui et al., 2010; Azouaoui et al., 2001) also defined a damage parameter based on the variations in the bending stiffness of the composite laminates during the cyclic impact-bending loading which characterizes the three stages of the life of the laminates. Their damage parameter is defined in Eq. (7-3).  $R_0$  represents the initial stiffness,  $R_f$  represents the final stiffness and  $R$  is the bending stiffness at any time.

Most recently, Longbiao (Longbiao, 2016) proposed a new damage parameter Eq. (7-4) based on variation of the hysteresis dissipated energy  $U$  during the fatigue degradation. As defined in Eq.(7-4),  $U_0$  is the initial hysteresis energy released at the first cycle of the fatigue.  $U_e$  denotes the elastic strain energy given by  $U_e = \frac{1}{2}(\sigma_{max} - \sigma_{min})(\varepsilon_{max} - \varepsilon_{min})$  which is based on min and max of stress and strain in the loading process.

Equations (7-5) and (7-6) are analytical models to characterize the damage profile as a function of time in composites.  $n$  and  $N$  are the number of applied loading cycles



and the fatigue life at a load level, respectively. The  $q$ ,  $m_1$ ,  $m_2$ ,  $A$  and  $B$  are model parameters.

Table 7-1: Some of the existing damage models for composites

Damage indicators by:	Definition	Physical parameter to define damage parameter
(Mao and Mahadevan, 2002)	$D = \frac{E_0 - E}{E_0 - E_f}$ (7-1)	Modulus of elasticity
(Giancane et al., 2010)	$D = \frac{H - H_0}{H_f - H_0}$ (7-2)	Dissipated thermal energy
(Azouaoui et al., 2010; Azouaoui et al., 2001)	$D = \frac{R_0 - R}{R_0 - R_f}$ (7-3)	Bending stiffness
(Longbiao, 2016)	$D = \frac{U - U_0}{U_e}$ (7-4)	Hysteresis dissipated energy
(Mao and Mahadevan, 2002)	$D = q \left(\frac{n}{N}\right)^{m_1} + (1 - q) \left(\frac{n}{N}\right)^{m_2}$ (7-5)	-
(Wu and Yao, 2010)	$D = 1 - \left(1 - \left(\frac{n}{N}\right)^B\right)^A$ (7-6)	-

### 7-5 Application example

The complexity of damage propagation in composites under fatigue loading is a motivation to search for alternative indirect damage metrics. Naderi et al. (Naderi et al., 2012), in an experimental research, utilized dissipated energy approach to evaluate fatigue damage in a woven Glass/Epoxy (G10/FR4) laminate. They showed that during the low cycle fatigue, most of the applied mechanical work converts into dissipated thermal energy which, in turn, gives rise to temperature.

In the current research, results of Naderi's experiment (Naderi et al., 2012) on Glass/Epoxy (G10/FR4) composite laminate during bending fatigue with displacement amplitude of 38.1 at 10 Hz, are used to demonstrate the proposed damage precursor-based SHM framework. For composite material under low-cycle fatigue, damage starts as soon as load applies on the component. In this context, underlying damage mechanisms such as matrix cracking, debonding and delamination occur in the composite material. Even though the direct measurement of actual damage such as density of micro-cracks at early stages can be very difficult in practice, damage might be revealed by change in some physical properties (indirect damage indicators) such as dissipating energy and variation in material's elasticity or stiffness. In this case study, based on the published experimental data in (Naderi et al., 2012) dissipated energy is considered as an indirect damage indicator. This property can be used to define the proper damage parameter as it is suggested in the literature (Table 7-1, Eq. (7-2)). Dissipated energy causes temperature increase in the material which can be monitored continuously and measured more easily by thermography techniques. Therefore, temperature is treated as one of the observations related to the indirect damage indicator.

As discussed before, the proposed SHM framework is meant to be able to take advantage of other sources of information. Naderi et al (Naderi et al., 2012) have also reported the cumulative count of acoustic emission (AE) signals in addition to temperature measurements. Some features of captured AE signals such as count or energy are also informative about the underlying damage evolution and should be

integrated into the SHM framework as another source of evidence to reduce the uncertainty.

### 7-6 DBN representation: general overview

Figure 7-2 shows two time-slices of the high level DBN representation of the problem. Similar to section 6-5 for metallic component, the same story can be explained for composite as well. Inaccessible actual damage mechanisms occur in the material and lead to dissipation of thermal energy ( $H$ ). Variation of  $H$  as an indirect damage indicator can be used to define the normalized damage parameter ( $D$ ) which is the representative of the actual hidden damage happening in the component. Dissipated thermal energy or its normalized form as damage index causes temperature rise in the component. On the other hand, the underlying progression of damage results in acoustic waves as well which are captured through AE signal acquisition system. Therefore, AE measurements are also available as another sensory data that provides information about the damage. Figure 7-2 can be compared to Figure 4-4 where  $H_k$  is equivalent to  $DP_k$  and its measurement will be used to compute a damage index  $D_k$  and then develop the damage model.  $T_k$  and  $AE_k$  are the measurements (denoted as  $Z_k$  in Figure 4-4) which are used to update the damage model. Each arrow in Figure 7-2 shows the relationship between the nodes which needs to be obtained either from the underlying physical dependency between the variables or from mathematical data driven methods.

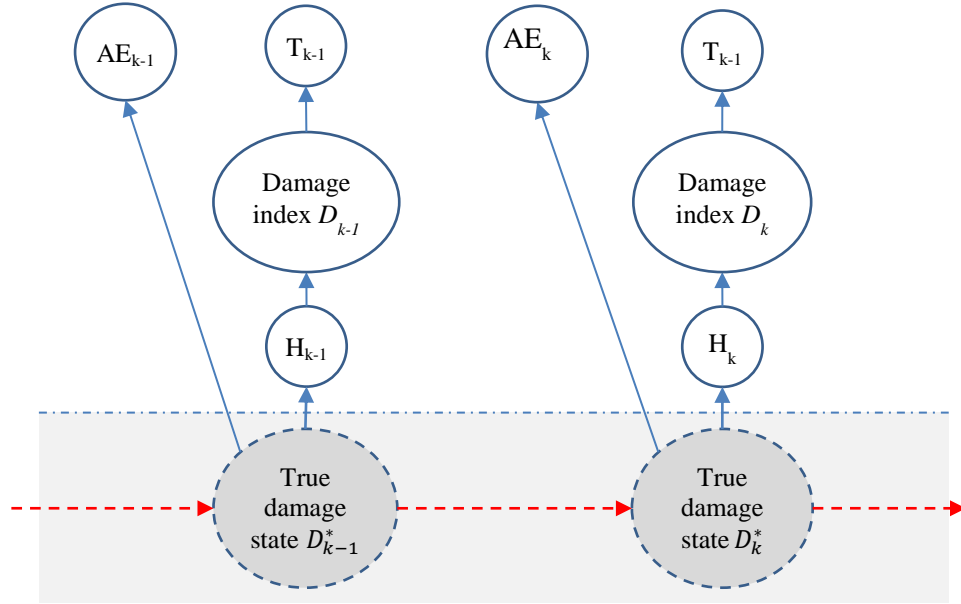


Figure 7-2: General DBN representation of the SHM framework in composite degradation

### 7-7 Inference in DBN using Augmented-Particle Filtering

Similar to the case study in previous chapter, the same procedure would be followed here for damage estimation and prediction in composite material with integrating different sources of information through DBN.

#### 7-7-1 Online learning of both state and parameters in the degradation model:

The first step is to define a proper state process model to explain the degradation process in composite material. A combination of damage indexes shown in table 1 is utilized to develop the process model. More formally, experimentally measured dissipated energies  $H$  are entered in Eq.(7-2) to compute the damage indices  $D = \frac{H}{H_f}$ .

Note that in Eq.(7-2)  $H_0$  is zero because when the applied load is of the fully reversed bending type, the initial value of heat dissipation is zero (Naderi et al., 2012).

Having the calculated damage parameter, Eq. (7-6) will then be used to demonstrate the evolution of damage index in time. Naderi (Naderi et al., 2012) followed an experimental procedure to measure dissipated energy at a few specific time steps during the test. Figure 7-3 demonstrates the damage parameter calculated based on reported dissipated energy and the fitted model derived from Eq. (7-6).

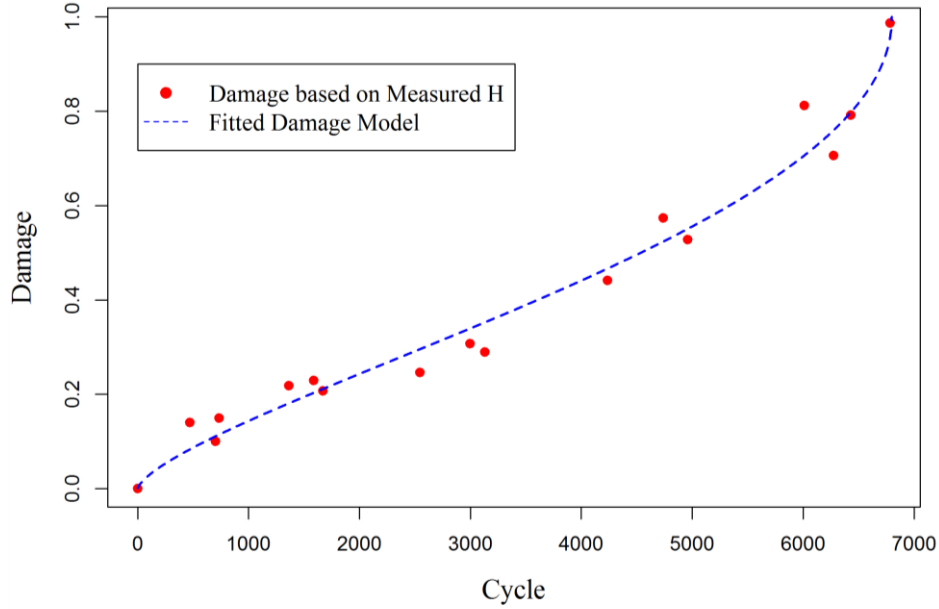


Figure 7-3: Damage parameter based on evolution of dissipated thermal energy

In order to convert it to the form of state process model, one can discretize Eq. (7-6) with sufficiently small  $\Delta N$ :

$$D_k = D_{k-1} + \frac{\Delta D}{\Delta N} \Big|_{k-1} \times \Delta N \times e^{\omega_k} \quad (7-7)$$

$\Delta D/\Delta N$  is derivative of  $D$  with respect to  $N$ .  $e^{\omega_k}$  is used to show the random behavior of the state process model;  $\omega$  can be a white Gaussian noise with mean zero and standard deviation  $\sigma$ , and  $k$  indicates the  $k^{\text{th}}$  cycle. Therefore, based on Eq. (7-6) the final state process model  $P(D_k | D_{k-1}, A, B, N_f)$  can be developed based on:

$$D_k = D_{k-1} + \left[ \frac{A \cdot B}{N_f} \left( \frac{n}{N_f} \right)^{B-1} \times \left( 1 - \left( \frac{n}{N_f} \right)^B \right)^{A-1} \right]_{k-1} \times \Delta n \times e^{\omega_k} \quad (7-8)$$

in which  $A$ ,  $B$  and  $N_f$  are unknown model parameters to be estimated by augmented-particle filtering along with damage states  $D_k$ .

### 7-7-2 Observation models

Again, here two types of measurements are monitored throughout the experiment: AE signals and temperature rise. Similar to metallic example, three cases are considered to discuss the fusion of different information:

- Case 1: only incorporate temperature data, i.e.,  $P(T_k | D_k^*)$
- Case 2: only incorporate acoustic emission data, i.e.,  $P(AE_k | D_k^*)$
- Case 3: combine both sources of data, i.e.,  $P(T_k, AE_k | D_k^*)$

Temperature is related to damage through the damage precursor  $H$  and its normalized form as damage index  $D$ . This relationship between temperature  $T$  and the damage index can be expressed by data-driven polynomial regression as:

$$T_k = \varphi_0 + \varphi_1 \cdot D_k + \varphi_2 \cdot D_k^2 + \varphi_3 \cdot D_k^3 + v_k \quad (7-9)$$

$\varphi_0, \varphi_1, \varphi_2, \varphi_3$  are model parameters and  $v_k$  is the measurement noise. Figure 7-4 shows the correlation between temperature and damage parameter following the regression model in Eq. (7-9).

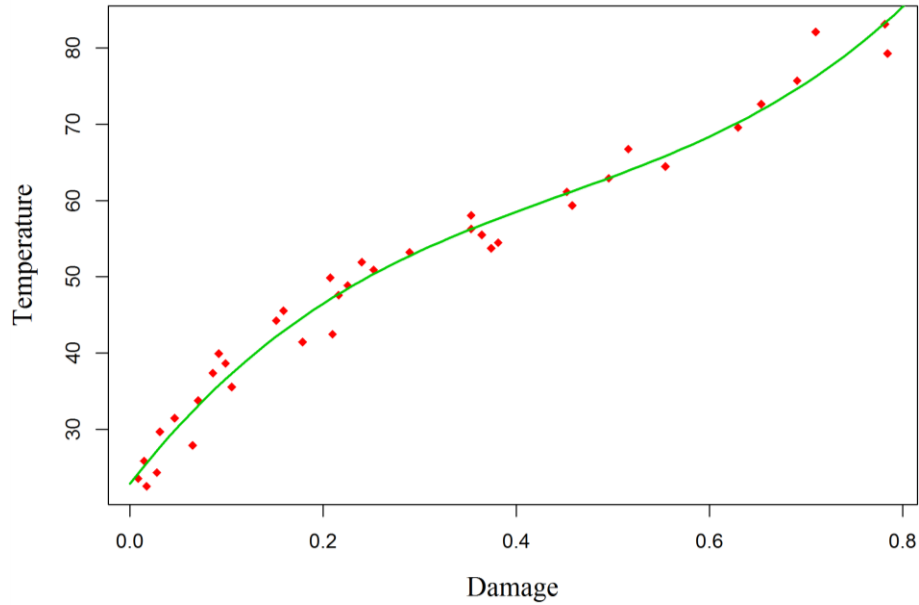


Figure 7-4: Correlation of damage index and temperature

Unlike relationship between temperature and damage that is nicely explained by regular regression techniques, the underlying link between acoustic emission and damage index cannot be captured by any known type of regression family. That is mainly because of the steep slope at stage III of damage evolution. Interested readers might refer to (Naderi et al., 2012) for more details. Therefore, a more flexible SVR algorithm is used to generate a data-driven correlation between AE signals and estimated damage index. SVR model is trained based on 53 data points in cycles where AE measurements were available. This model is then used to predict the value of D, at any other data points in time. Figure 7-5 demonstrates the results of SVR presenting the correlation between AE cumulative counts vs damage parameter.

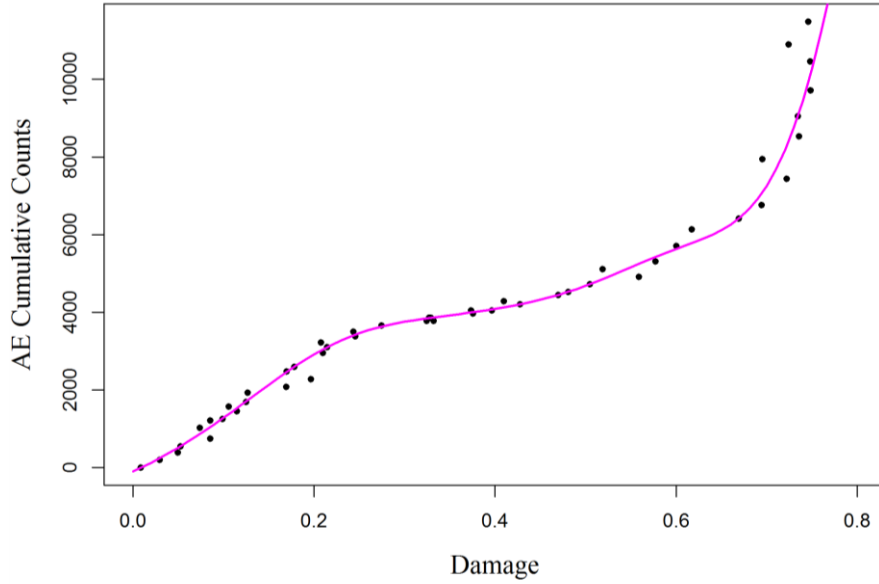


Figure 7-5: Correlation of damage index and cumulative AE counts using SVR

For the third case, the challenge remains as how to combine two measurement models and obtain one merged measurement model as  $P(T_k, AE_k | D_k^*)$ . Again, based on the probability chain rule:

$$P(T_k, AE_k | D_k^*) = P(T_k | AE_k, D_k^*) \cdot P(AE_k | D_k^*) \cdot P(D_k^*) \quad (7-10)$$

Since Acoustic emission and temperature are conditionally independent in nature given the  $D^*$ , i.e.,  $T \perp\!\!\!\perp AE | D$ , therefore:  $(T_k | AE_k, D_k^*) = (T_k | D_k^*)$ . This will result in the integrated measurement model as:

$$P(T_k, AE_k | D_k^*) = P(T_k | D_k^*) \cdot P(AE_k | D_k^*) \cdot P(D_k^*) \quad (7-11)$$

Equation (7-11) is used to update the weight of each particle when both temperature and acoustic emission signals are measured simultaneously.

### 7-7-3 DBN representation: detailed model

More detailed DBN considering all the random variables of the problem is shown in Figure 7-6:



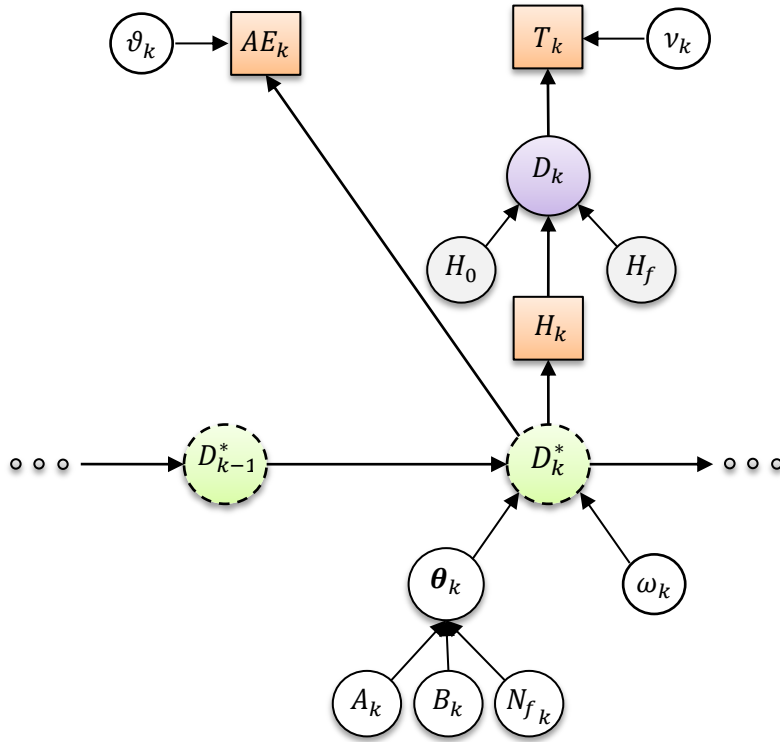


Figure 7-6: Detailed DBN representation considering all the factors

Observations (i.e., AE signals, measured damage precursor H, and measured temperature T) are shown with rectangular because they can be measured through the experiment.  $\omega_k$ ,  $v_k$  and  $\vartheta_k$  are process noise, noise in T measurements and noise in captured AE signals respectively.  $\theta_k$  is the vector of parameters for state process model Eq. (7-8) consists of  $A$ ,  $B$  and  $N_f$  at time step  $k$ . In the context of this case study, only  $H_0$  and  $H_f$  are considered to be constant. The DBN presented in Figure 6-5 is used in the rest of this chapter for estimating and predicting damage in the component.

### 7-8 Results and discussion:

One of the main challenges in this case study was the scarcity of both measurements. Only 38 data points for temperature and 51 measurements for AE were reported

during the 6800 cycle of fatigue life which is quite few. Moreover, the last data point in both temperature and AE measurements was captured at 90% of life; meaning no observation was available to train the measurement models in the last 10% of life. One suggested way to handle this issue is to synthetically generate more data in between, based on the trend of measurement. This method would be satisfactory for the sake of illustration; however, it is contradictory with real time monitoring, because in online SHM, the trend of measurements is not known in advance. Therefore, we decided to use the original raw data as is and show how the proposed SHM framework would perform in case of insufficient measurement data. However, it is important to notice that the state process model as expressed in Eq. (7-7) and (7-8) is valid for sufficiently small time step, so whenever the interval between two consecutive measurements is big, intermediate steps would be taken to propagate the particles forward in time without updating. And as soon as any measurement is captured, the states of the particles would be updated. In other words, we consider a maximum allowable time step  $\Delta n$ , here it is every 50 cycle or  $\frac{50}{6800} = 0.007$ . If the measurement is available during this  $\Delta n$ , the *update step* of the particle filtering algorithm would take place as usual, otherwise only the particles are transmitted forward in time using the state process model while carrying their last calculated weights. It is equivalent to applying the prognostics approach between two successive measurements. This helps the validity of the methodology and decrease the uncertainty of the estimations.

Similar to previous case study in chapter 6, the number of the particles and the initial values of the parameters (here  $A$ ,  $B$ , and  $N_f$ ) need to be set in advance. Here, 500

particles are selected randomly for each model parameters as well as damage states (2000 particles in total). And, the state model parameters were initialized regarding (Naderi et al., 2012; Wu and Yao, 2010) as following:  $A = \text{uniform } [0.1, 0.6]$ ,  $B = \text{uniform } [0.1, 0.6]$ , and  $N_f = \text{uniform } [6000, 7000]$ .

### 7-8-1 Damage state monitoring

Figure 7-7 to Figure 7-9 show the three cases discussed earlier. Part (a) of each figure presents the evolution of the state model parameters and in part (b) the result of damage estimation with augmented particle filtering is demonstrated. In Figure 7-7, only temperature measurements are used for updating. Without loss of generality and for better illustration of the effect of fusion technique, we considered more uncertainty in temperature measurements because it is measured less frequently than AE signal. Therefore, a wider bound of particle trajectories is expected in estimation of damage. Considering AE signals (Figure 7-8) results in better estimation and lower uncertainty because of the presence of more data points and consequently more accurate measurement model. However, in both cases in Figure 7-7 and Figure 7-8, the model parameters (specially parameters  $A$  and  $N_f$ ) are very noisy and their variation does not subside through time and because of that, both methods perform poorly in the last 10% of life. Particles start to disperse more and more at the end. In Figure 7-9, however, when both temperature and AE observations are integrated into the framework, state process model parameters converge more smoothly and rapidly that leads to more precise damage estimation with less diffused particles. As discussed before, the convergence of the model parameters is specifically important for prognostic purposes.

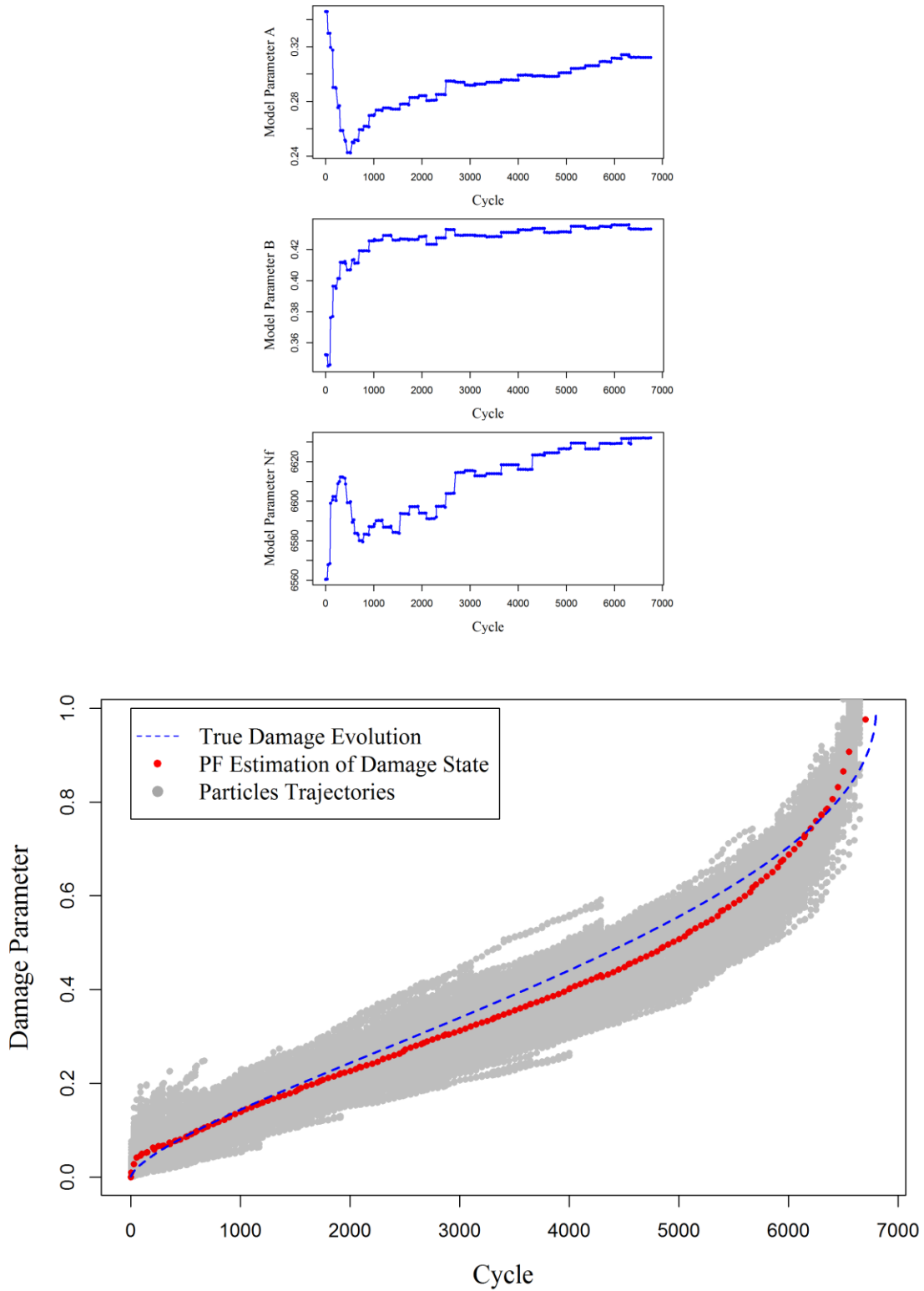


Figure 7-7: Estimation of damage evolution in time using only temperature measurement for case 1 (bottom) and corresponding model parameters (top)

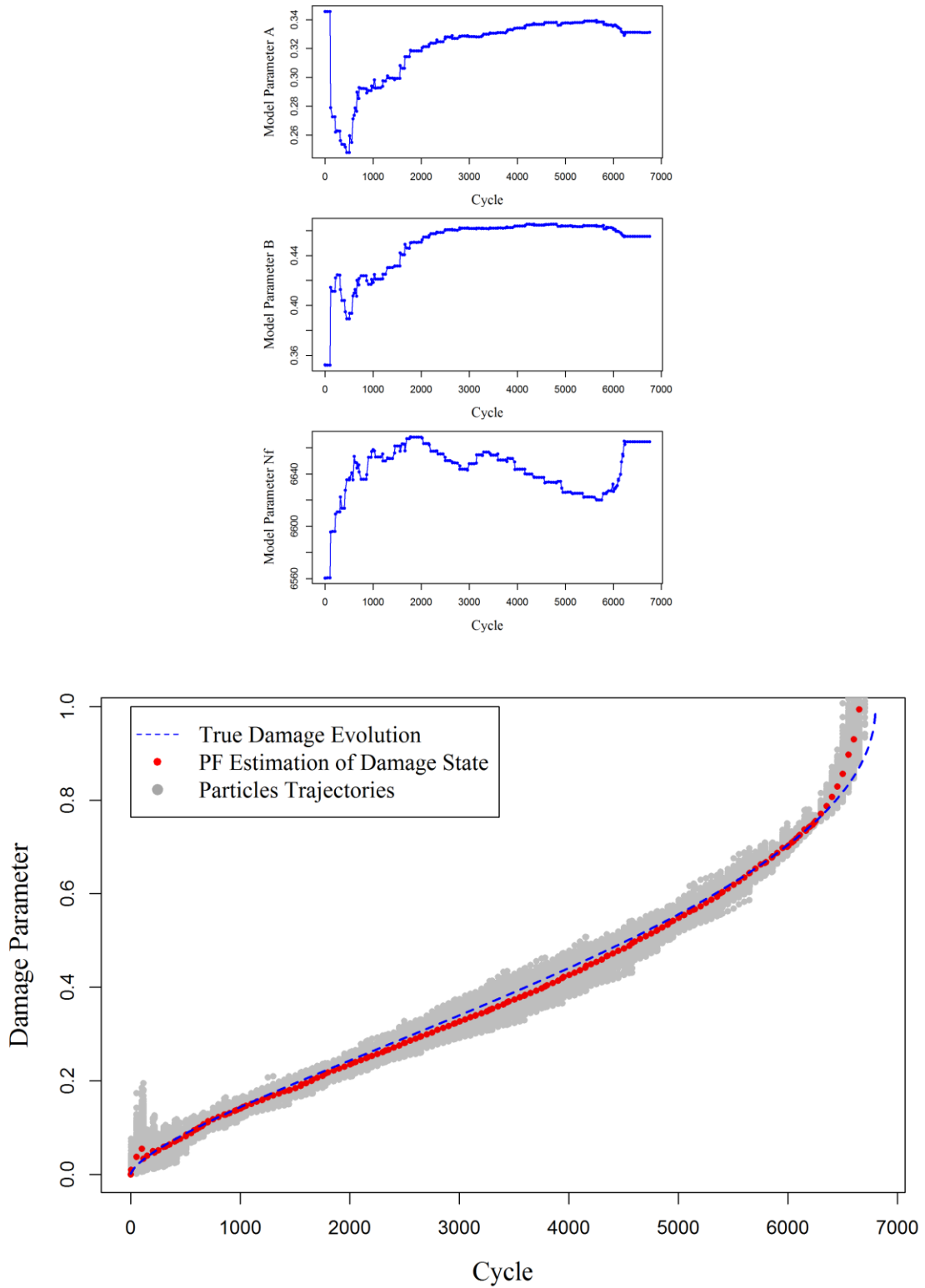


Figure 7-8: Estimation of damage evolution in time using only AE signals for case 2 (bottom) and corresponding model parameters (top)

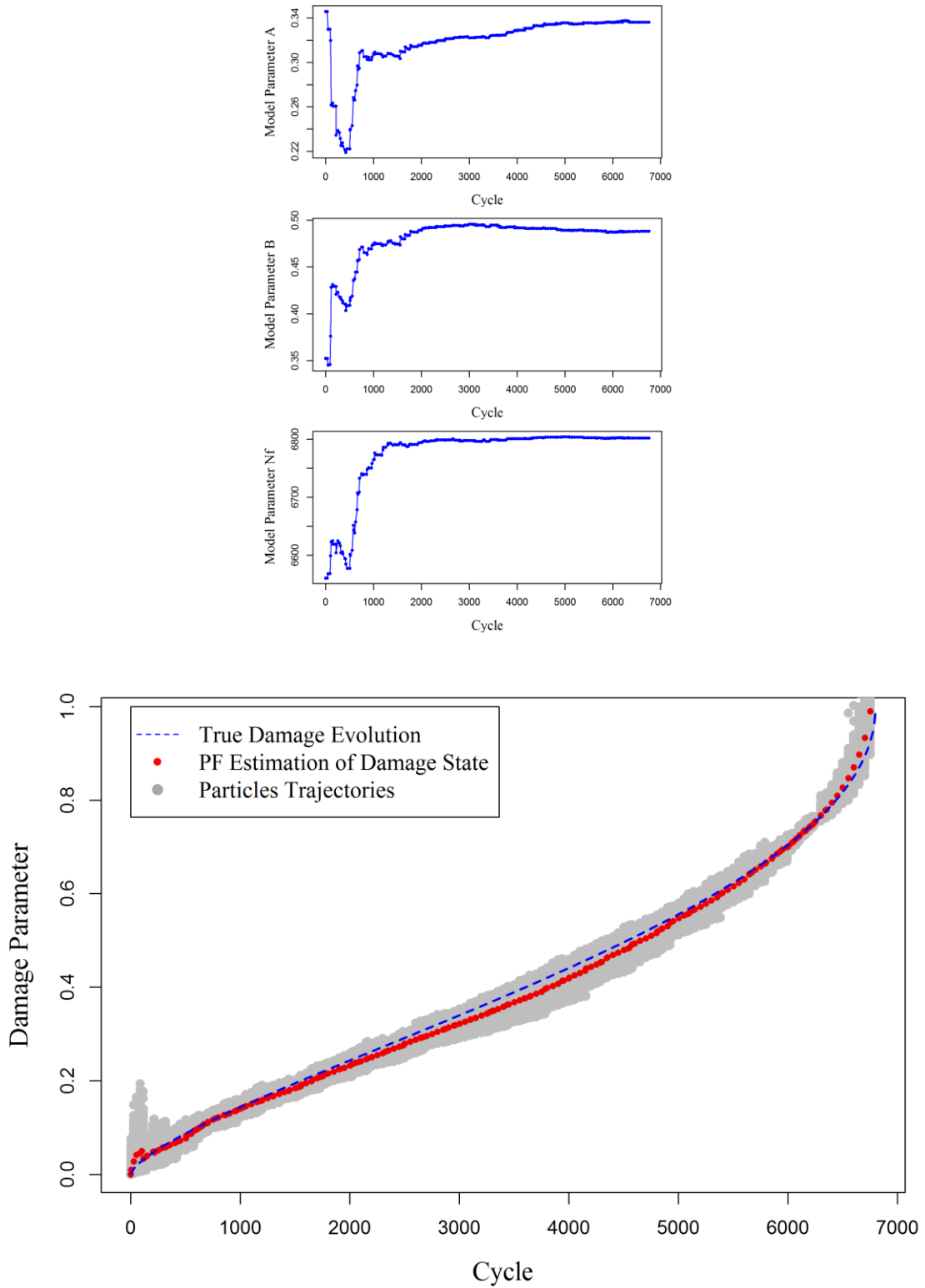


Figure 7-9: Estimation of damage evolution in time using both AE signals and temperature measurements for case 3 (bottom) and corresponding model parameters (top)

The root mean square error RMSE is calculated for each case and is presented in Table 7-2. As expected, the RMSE for case 3 is less than other two cases. In comparison of case 1 (Figure 7-7) and 2 and (Figure 7-8), it is apparent that case 2 outperforms case 1 during almost 90% of the life, however, significant dispersion of the particles in the last few steps cause the slightly bigger RMSE in case 2.

Table 7-2: RMSE calculation for the 3 cases presented above

	<b>Case 1: Only consider T</b>	<b>Case 2: Only consider AE</b>	<b>Case 3: Consider both AE and T</b>
<b>RMSE</b>	0.0397	0.0408	0.0128

Similar to chapter 6, the HPD intervals are also computed at 95% and depicted in Figure 7-10. As discussed, the uncertainty bounds in case 1 is wider than the other two cases, mainly because of insufficient temperature measurements during the experiment. So, case 1 is less successful to damp the state process noise and control the dispersion of particles.

The results are more satisfactory for case 2 because more AE data points with less measurement error were used to update the estimations. Again similar to case 1, the HPD intervals diffuse and deviate from true damage more and more toward the end.

In case 3, no significant improvement can be seen before 6000 cycle compared to case 2, however, the results are apparently improved in the last 20% of life. The uncertainty is decreased and the true damage trend is enclosed in the HPD intervals from the beginning to the end of damage estimation.

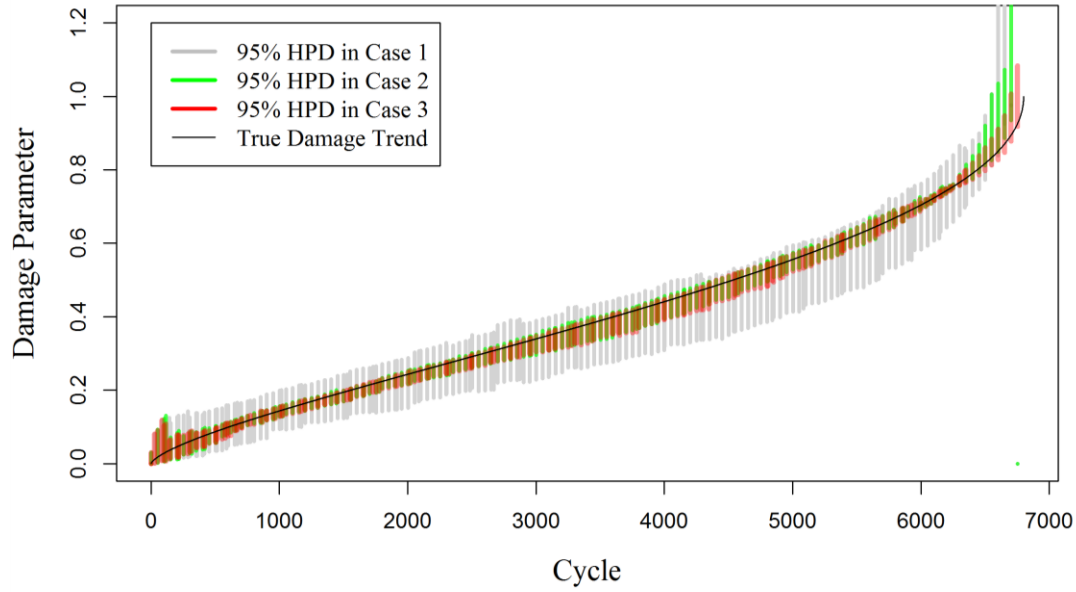


Figure 7-10: Comparison of 95% HPD intervals in all the three cases.

#### 7-8-2 Prognostics and prediction of RUL:

Similar to previous chapter, the prognostics and life prediction are performed on case 3 when both observations are considered and the results are presented in Figure 7-11. Prediction is started around 3000 cycle; it is when the model parameters are almost converged and no more significant variation is seen. In order to estimate the RUL, a critical hazard zone or threshold needs to be defined, above which the system fails or its performance is no longer acceptable. In the case study of fatigue of composite material, the critical threshold might be defined based on the level of dissipated energy that corresponds to stage III of damage evolution which is identified in (Naderi et al., 2012) by a drastic increase in both temperature and acoustic emission signals. It happens when almost 80-90% of the component life has expended. Therefore, dissipated energy at 85% of the life is selected as critical threshold  $H_{crt}$  for estimating the  $D_{crt}$ . This will result in the critical damage threshold as  $D_{crt} =$



0.67. As it is shown in Figure 7-11, particles are tracked from the beginning of the prognostics until they pass cross the critical threshold.

The distribution of the TTF is also generated based on the time at which each particle reaches the threshold and the MTTF can be calculated using Eq.(5-18). In this case, the estimated MTTF via DBN with augmented particle filtering is 5829 cycle and the true MTTF is 5780. The absolute error is 49 cycles which is equivalent to %0.8 error.

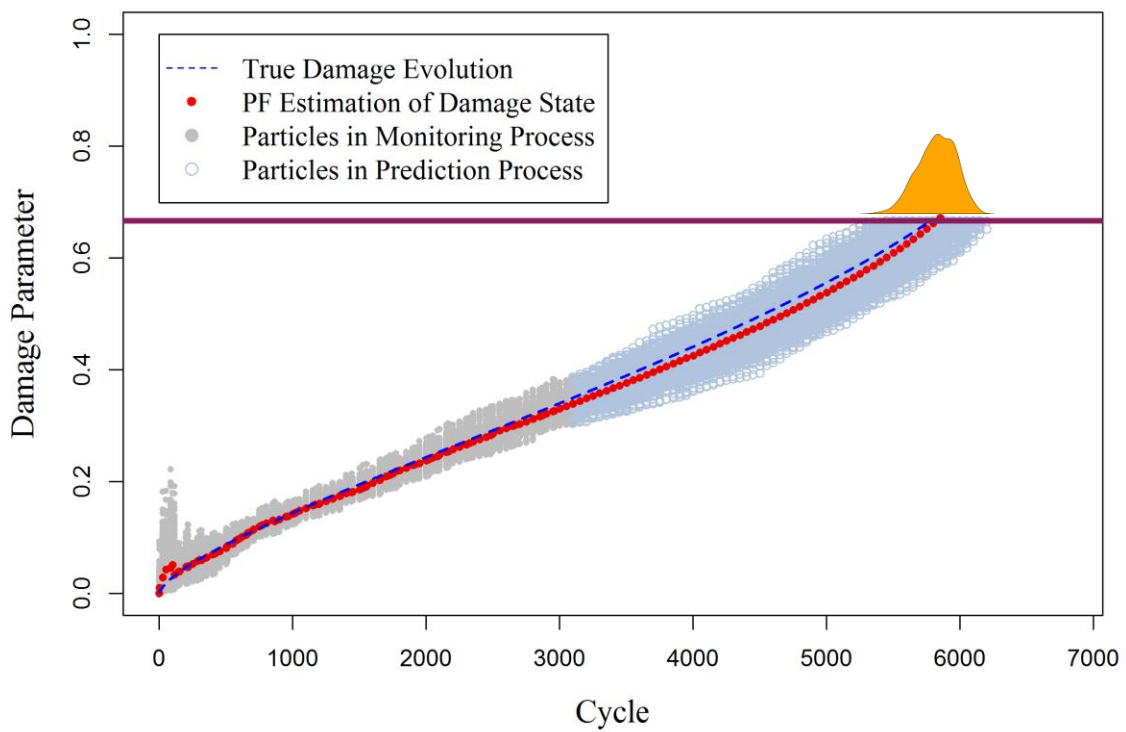


Figure 7-11: Long-term prediction of life in composite material and distribution of TTF

### 7-9 *Summary:*

This chapter presented how the proposed SHM framework can be applied in cases with complicated degradation processes where fully defined or well-studied damage models do not exist and effectively measuring a conventional direct damage indicator is very difficult. Indeed, the application of the proposed damage precursor-based SHM framework was illustrated for bending fatigue test of woven Glass/Epoxy (G10/FR4) laminate composite. While the direct state of the damage was not accessible, the dissipated energy was considered as the damage precursor. A DBN was built as the underlying model to fuse two types of observations (temperature and AE signals) in order to update the knowledge about the state of damage. Regular regression techniques and also SVR were used to model the relationship between damage index and the observations. The augmented particle filtering technique was then used to infer the evolution of damage given the available monitoring information during the lifetime of the composite. In addition, the framework was utilized to perform the prognostics and promising prediction of TTF.

## Chapter 8: Proposed Fully Adaptive Particle Filtering Algorithm

### 8-1 Motivation

The details of the particle filtering approach were discussed thoroughly in chapter 5 sections 5-3 and 5-4. To recall, particle filtering, like any other state-space model, is defined based on two elements: the state process model that shows the progression of the hidden state of interest  $x_k$  through time, and the measurement model that presents the relationship between the observed variables  $y_k$  and the hidden states  $x_k$  at each time step. In the recent literature, particular attention has been paid to developing and updating the state process model and, as discussed earlier in 5-4, different methods with varying degrees of success, such as augmented particle filtering and kernel smoothing (Hu et al., 2015a, 2015b; Liu and West, 2001; Storvik, 2002), have been proposed for updating the model parameters when dealing with partially known process models. However, to the best of our knowledge, no formal study has been published with the focus on improving the likelihood calculation method and updating the measurement model.

Measurement model plays a very important role in updating both states and the model parameters. In the literature, usually two types of measurements  $y_k$  and, consequently, two types of measurement models are considered:

- 1) The most common technique is that “ $y_k$  is the same quantity as  $x_k$ ” that can be measured with some error. In other words, it is assumed that we can observe the hidden variable with some measurement error. A few examples

can be found in (Dalal et al., 2011; Liu et al., 2012; Orchard and Vachtsevanos, 2007; Saha and Goebel, 2009; Sun et al., 2012b). This leads to a very basic and simple measurement model as the following:

$$y_k = x_k + v_k \quad (8-1)$$

- 2) The second group can be considered as the cases when “ $y_k$  is NOT necessarily the same quantity as  $x_k$ ”; for example in (Zio and Peloni, 2011). Therefore,  $y_k$  has a different nature and needs to be related to the hidden variable through some physical or data-driven models:

$$y_k = h(x_k, v_k) \quad (8-2)$$

For example the hidden variable is the crack size, but we observe acoustic emission or ultrasonic waves instead. This case is more close to real world situations; however, the relationship between the hidden variable and observed variables is not always easy to develop.

Although one of the powerful features of the particle filtering is dealing with non-linear measurement models, the majorities of the published papers are based on case (1) in which the measurement model is a purely linear function that is simply constructed by adding noise to the hidden variable.

Also, in cases when observed variable is different than the hidden state, i.e., case (2), a fixed predefined measurement model has been used in the literature. However, in real world application, most of the time no predefined function exists to exactly explain the relationship between the state of the system and the observed variable for that particular study. It is especially noteworthy for online monitoring, diagnostics

and prognostics. Since there is no accurate predefined measurement model, it is important to adjust the measurement model to map the real-time streaming monitoring data.

This is the motivation to consider cases where not only the state process model, but also the exact measurement model is not completely known. Although it is expected to add additional layers of uncertainty to the estimation that needs to be handled accordingly, the approach would be a significant step forward to SHM in real world application; because, when performing the damage monitoring and prognostics in real time, the underlying state process and measurement models are not fully defined in advance for the particular component/system under specific testing or operating condition. For example recalling the composite degradation case study in Chapter 7:, the state process model that represents the damage evolution in composite was partially known and its parameters needed to be learnt online based on new online measurements. Now suppose that the measurement model also has not been predefined, which is actually a very reasonable and logical assumption for online SHM. Therefore, it would be critical to adjust the parameters of the measurement model in real time as well.

## *8-2 Proposed adaptive measurement model for particle filtering approach*

The idea is to establish an adaptive measurement model that can be trained online as time goes by. Since fully online-learning of an unknown function without any prior knowledge is impractical, we need to impose some restrictions. Therefore, similar to the strategy for updating the state process model, it is supposed here that the

measurement model is also “partially known”. That is, only its functional form is known in advance but its model parameters need to be adjusted for the particular case study at hand. This requirement can be met in practice by generating an offline data-driven or a physics-based model based on partially relevant data. For example, suppose that the results of a set of fatigue experiments are available in which the fatigue crack length is the hidden variable of interest  $x$  and the acoustic emission signals are the observations  $y$ . So, a priori observation model can be built offline to correlate the acoustic emission signal to crack size based on the historic experiments. Now, if this model is to be applied in the particle filtering algorithm for damage estimation in a new experiment with different loading condition, the priori fitted model needs to be updated online to adjust to the condition of the new experiment as measurement data comes in. More specifically, both the state process model and measurement model are condition-based and need to be adjusted to the particular case under study.

#### 8-2-1 How the approach is different from augmented-particle filtering?

As presented in section 5-4, in augmented-particle filtering or dual updating particle filtering the state process model is partially known, so the measurements are used to update both states and process model parameters simultaneously. However, it is assumed that the measurement model is predefined and completely known in advance.

The proposed adaptive measurement model, however, deals with the situation when the parameters of the measurement model are also unknown in addition to the state process model. Therefore, nothing is fixed or predefined. This is a more complex

problem because in order to update the states and process model parameters, a measurement model is required, while in this case, the measurement model itself is partially known and is adaptively changing based on new observations.

Table 8-1 compares the proposed fully adaptive particle filtering with standard and augmented particle filtering in terms of knowledge about the state process model and the measurement model.

Table 8-1: Comparison of standard, augmented and fully adaptive particle filtering

<i>Filtering Method</i>	<i>State process model</i>		<i>Measurement model</i>	
	<i>Functional form</i>	<i>Parameters</i>	<i>Functional form</i>	<i>Parameters</i>
<i>Standard particle filtering</i>	Known	Known	Known	Known
<i>Augmented particle filtering</i>	Known	Unknown/ Learned online	Known	Known
<i>Fully adaptive particle filtering</i>	Known	Unknown/ Learned online	Known	Unknown/ Learned online

### 8-3 Proposed Likelihood Adaptation Approach

The proposed approach for adaptive likelihood is based on minimizing the Kullback–Leibler divergence or KL-divergence or KLD for short (Kullback and Leibler, 1951). KL-divergence, also known as relative entropy, is an effective evaluation method to compare two distributions  $P$  and  $Q$ :

$$D_{KL}(P||Q) = \int P \cdot \log \frac{P}{Q} \quad (8-3)$$

Usually  $P$  is the true distribution of the data (the objective distribution) and  $Q$  comes from the model used to approximate that true distribution.

In order to apply the idea of KLD to adaptive measurement model,  $P$  would be the distribution of the real measurement which is captured online at each time step  $k$  with

some degree of measurement uncertainty, whereas  $Q$  would be the distribution built based on predicted measurements at that time. Predicted measurements in particle filtering algorithm are calculated by applying the partially known (approximate) measurement model  $\tilde{h}$  on the propagated particles. Since the exact values of the parameters in  $\tilde{h}$  are not known, the measurement model might be very inaccurate which will lead to false predicted measurements. In this situation, the distribution  $Q$ , fitted to the predicted observations, might be significantly different than the distribution of the real measurement  $P$ . The main idea is to optimize the parameters in the measurement model with the objective of minimizing the KL-divergence between  $Q$  and  $P$ .

Recently, KL-divergence was introduced to particle filtering algorithm as a statistical approach in the context of mobile robot localization (Fox, 2003, 2001) to increase the efficiency of particle filters by “adapting the sample size” during the estimation process. The proposed approach in our research, however, implements the concept of KL-divergence for adjusting the measurement model on the fly while the number of samples is fixed. In what follows, we will determine how the KL-divergence is used to dynamically adapt the measurement model in particle filtering through time.

#### *8-4 Introducing the KLD into the particle filtering algorithm*

This section presents how to incorporate the idea of adaptive measurement model into the particle filtering algorithm.

As explained in the procedure of augmented-particle filtering algorithm, the state process model with unknown model parameters can be demonstrated by Eq.(5-13) which is presented here again for convenience:



$$\begin{aligned}\theta_k &= g(\theta_{k-1}, \gamma_{k-1}) \rightarrow p(\theta_k | \theta_{k-1}) \\ x_k &= f(x_{k-1}, \theta_k, \omega_k) \rightarrow p(x_k | x_{k-1}, \theta_k)\end{aligned}\quad (8-4)$$

In addition, it is assumed that the measurement model is also partially defined. Suppose that the analyst's belief about the approximate prior measurement model is presented by:

$$\tilde{y}_k = \tilde{h}(x_k, \varphi_k, \tau_k) \rightarrow p(\tilde{y}_k | x_{k-1}, \varphi_k) \quad (8-5)$$

in which  $\varphi$  is the measurement model parameters that need to be updated through time as new real-time monitoring data gets available.  $\tau$  is the uncertainty in model  $\tilde{h}$  based on parameters uncertainty and measurement uncertainty.

The procedure of the fully adaptive particle filtering is as following:

At each time step  $k$ , the state process model Eq. (8-4) is used to propagate the particles one step ahead in time. In standard augmented-particle filtering, the predefined measurement model would be applied at this stage to estimate the weights of the particles regarding the true observation at that time. However, here, the measurement model is not fully defined in advance. Hence, before assigning the weights to the particles, the measurement model needs to be adjusted. Therefore, the tentative prior measurement model Eq. (8-5) will be used to generate a set of predicted observations  $\tilde{y}_k$  based on particles state. Distribution  $Q(\tilde{y}_k^i | x_k^i), i = 1, \dots, n$  is the probability distribution that can be fitted to the set of predicted observations  $\tilde{y}_k^i$ . On the other hand,  $P(y_k | x_k)$  would be the probability distribution of the real measurement  $y_k$  at time step  $k$  assuming a known measurement noise. Since, the model parameters  $\varphi$  in  $\tilde{h}$  are unknown, predicted measurements might be very far from the real observation at time  $k$ . This will result in assigning negligible weights to

the particles and, consequently, the algorithm will collapse because of the very low likelihood. However, the root of this problem is the fact that the measurement model was not completely defined at the first place.

Therefore, to address this issue, the proposed approach is to update the measurement model by adjusting its parameters  $\varphi$  in a way that the discrepancy between  $Q(\tilde{y}_k^i | x_k^i)$  and  $P(y_k | x_k)$  is minimized. Using the KL-divergence in Eq. (8-3), the optimum  $\varphi$  can be adapted based on real measurement data at time k. Then, the updated measurement model is to be used to calculate the weights of the particles as before. This procedure will be repeated every step as a new measurement is captured.

Table 8-2 presents the algorithm for the fully adaptive particle filtering:

Table 8-2: Proposed fully adaptive particle filtering algorithm

---

(1) Initiation step:

- Sample N particles from initial distributions of states and parameters:

$$x_0^i \sim p(x_0) \quad i = 1, 2, \dots, N$$

$$\theta_0^i \sim p(\theta_0) \quad i = 1, 2, \dots, N$$

- Assign initial equal weights to all the particles

$$w_0^i \sim \frac{1}{N} \quad i = 1, 2, \dots, N$$

(2) Recursive steps:

*Prediction:*

- Estimate  $m_{k-1}^i$  for each parameter using shrinkage rule Eq.(5-17))
-

- 
- Draw new samples for parameter vector from:

$$\theta_k^i \sim \mathcal{N}(\cdot | m_{k-1}^i, h^2 \gamma_{k-1})$$

- Propagate each particle one step forward using state process model with new sampled parameter:

$$x_k^i \sim p(\cdot | x_{k-1}^i, \theta_k^i)$$

*Update:*

- Compute the predicted observations using the tentative measurement model:

$$\tilde{y}_k = \tilde{h}(x_k, \varphi_k, \tau_k)$$

- Update the parameters of  $\tilde{h}$  to minimize the KL-divergence:

$$\varphi_k = \arg \min \{D_{KL}(P||Q)\}$$

$$= \arg \min \left\{ \int P(y_k | x_k^i, \theta_k^i, \varphi_k) \cdot \log \left[ \frac{P(y_k | x_k^i, \theta_k^i, \varphi_k)}{Q(\tilde{y}_k^i | x_k^i, \theta_k^i, \varphi_k)} \right] \right\}$$

- Calculate the weights for each particle as new measurement  $y_k$  gets available:

$$w_k^i = w_{k-1}^i \cdot p(y_k | x_k^i, \theta_k^i, \varphi_k)$$

- Normalize the weights:

$$w_k^i = \frac{w_k^i}{\sum_{i=1}^N w_k^i}$$

*Estimate:*

- Estimate the expected state:

$$\bar{x}_k = \sum_{i=1}^N w_k^i \cdot x_k^i$$

*Resample:*

- Resample (with replacement) new set of particles for states and process model parameters  $\{x_k^i, \theta_k^i\}_{i=1}^N$  based on calculated weights  $w_k^i$ .
-

### 8-5 Challenges of the proposed approach:

The main concern about the proposed fully adaptive particle filtering is setting the initial values for model parameters  $\varphi$  and  $\theta$ . If the state process model has  $m_1$  parameters and the measurement model consists of  $m_2$  parameters,  $\theta$  and  $\varphi$  will be vectors of size  $m_1$  and  $m_2$ , respectively. Also, each of the model parameters is a random variable and needs the mean and standard deviation to be defined. Therefore, it will result in  $2 \times (m_1 + m_2)$  unknown variables. In addition, there would be also the process noise  $\omega$  to represent the stochasticity of the process and the measurement noise  $\nu$ , which relates to the uncertainty in measured data. Therefore, there are  $2 \times (m_1 + m_2) + 2$  random variables (hyper-parameters) in total that should be addressed accordingly.

The existence of multiple uncertain random variables results in additional uncertainty in the SHM. However, the proposed approach is mainly recommended to be applied in online monitoring context, where a high sampling rate, big data sets, and continuous streaming of monitoring data are expected. More frequent observation data will increase the convergence rate in the model parameters and, therefore, the concern of uncertain estimations can be managed.

On the other hand, it might be argued that the method introduces some bias toward the measurement. In another words, minimizing the KL-divergence can be considered as shifting the probability distribution  $Q$  toward  $P$  before weighing the particles. Therefore, the approach is sensitive to the true measurements. Indeed, this is a favorable property that makes sure the tentative inaccurate measurement model gets updated based on real-time true measurement before being used for weighing the

particles. However, if the coming real-time measurements are sparse, or too noisy or not informative enough, there is a risk that the method fails or skewed toward unreliable measurements. To overcome this problem, it can be beneficial to use a regularization parameter to control how much distribution  $Q$  is allowed to shift towards  $P$ , that is regularizing the KL-divergence. Consequently, this method will prevent the distribution  $Q$  from getting too close to distribution  $P$ , if the true measurement is not reliable enough. This idea needs to be validated in future studies. And, finally, obviously the trade-off of dynamically updating the parameters of both process model and measurement model along with estimating the states would be more expensive computations. The approach requires solving an optimization problem at each time step which comes at a high computational cost. This problem is also deferred to future research.

8-6 *Real-time damage estimation in composite material using the proposed Fully adaptive particle filtering algorithm:*

Recall the case of composite degradation presented in chapter 7. Now, for simplicity, suppose that only temperature measurements are available. It was discussed that a 3-degree polynomial (Eq. 7-9) is a good fit to represent the relationship between the damage and the temperature in the composite. This information can be mainly obtained from similar experiments performed previously. Thus, the tentative function for the measurement model can be expressed as:

$$\tilde{T}_k = \tilde{h}(D_k, \boldsymbol{\varphi}_k) = \varphi_{3k} \times D_k^3 + \varphi_{2k} \times D_k^2 + \varphi_{1k} \times D_k + \varphi_{0k} \quad (8-6)$$

In chapter 7, similar to other existing works on particle filtering, this measurement model was assumed to be completely known for the particular composite under study and, therefore, it was applied as a predefined fix model to explain the correlation between hidden damage and observed temperature. However, when dealing with real-time SHM, no exact predefined model exists for the particular problem under study. On the other hand, any prior measurement model that might exist usually comes from previous similar experiments or related literature and does not necessarily fit to the upcoming monitoring data for the particular component under study. The reason is that even identical components do not behave exactly the same under equal loading condition. Moreover, there is no guarantee that the historic data used to develop the tentative measurement model has been extracted under identical testing conditions. In this situation, other filtering techniques in the literature such as Kalman filter, standard particle filter and even augmented particle filter fail to accurately estimate the state of the new system, because they are all based on fixed predefined measurement model and cannot adapt a preexisting model to fit the upcoming monitoring data on the fly.

In the rest of this section, the capability of the proposed adaptive particle filtering is compared with augmented particle filtering in handling the real world situation when both state process and measurement models are inaccurate and need to be adjusted for the particular case under study.

Accordingly, we revisited the (Naderi et al., 2012) and used another set of published data on the same type of specimen but under different loading condition to define a prior measurement model. More specifically, the experimental results of the same

composite specimen (G10/FR4) subjected to the frequency of 10 Hz and displacement amplitude of 40.64 mm were used to define a measurement model similar to Eq. (8-6). As reported in (Naderi et al., 2012), this specimen failed at 4000 cycles of bending fatigue.

Now, suppose that this developed model is the only available information that we have in advance about the possible relationship between the damage and the online monitoring data. If the predefined model with fixed model parameters obtained from the first experiment is adopted in the augmented particle filtering (similar to chapter 7) for damage estimation in a new experiment under different testing condition (displacement amplitude of 38.1 mm), the results of damage estimation would be as presented in Figure 8-1.

As it was expected and demonstrated in Figure 8-1, the augmented particle filtering and any other existing filtering method which are based on fixed predefined measurement models, fail to correctly track the damage evolution in the component under study because the existing measurement model was derived from different experiment and the method was not able to adjust it for the particular problem in hand. This shows the necessity of learning/updating the measurement model in addition to the state process model.

Therefore, in real-time SHM, it is necessary to adaptively update the parameters of the measurement model which are encapsulated in  $\boldsymbol{\varphi}_k$  for this case study as follows:

$$\boldsymbol{\varphi}_k = [\varphi_0, \varphi_1, \varphi_2, \varphi_3]_k \quad (8-7)$$

The state process model is the same as before Eq. (7-8). Thus, the process model parameter vector would be:  $\boldsymbol{\theta}_k = [A, B, N_f]_k$

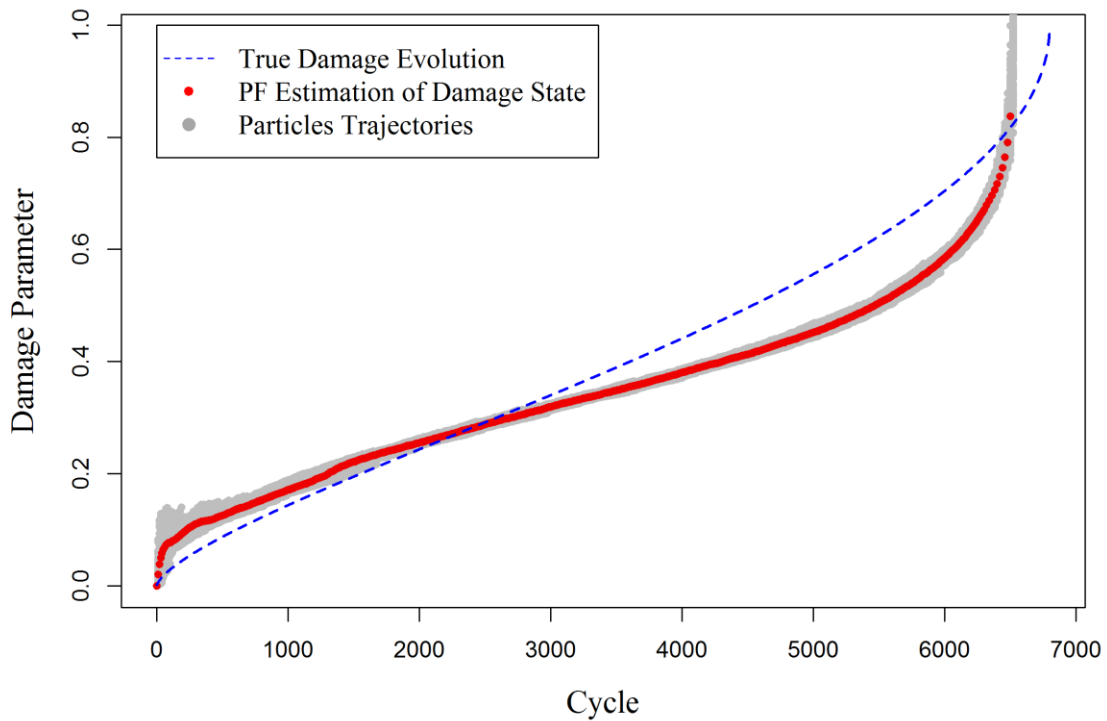


Figure 8-1: Damage estimation in composite specimen under fatigue loading (with 38.1 displacement amplitude) using augmented particle filtering with inaccurate fixed predefined measurement model

As explained in the previous section, there would be 16 hyper-parameters that affect the result of particle filtering estimation and need to be tuned. These variables consist of 4 parameters in measurement model and 3 parameters in process model (each one with unknown mean and standard deviation), plus state process noise and measurement noise.



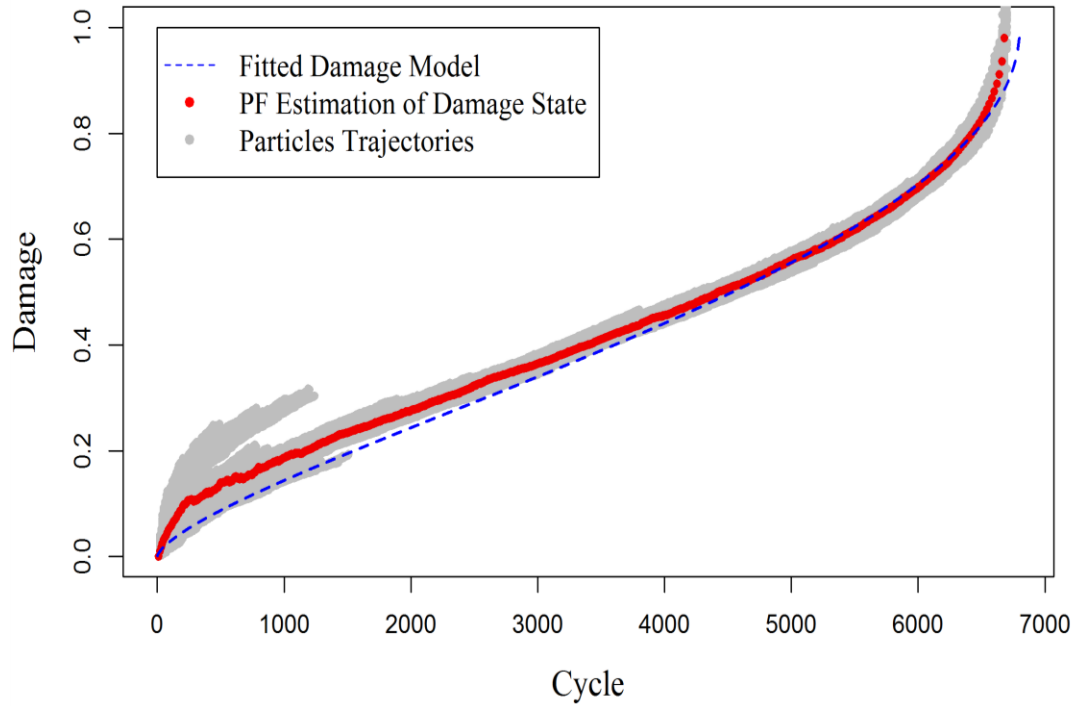


Figure 8-2: Damage estimation in composite specimen under fatigue loading (with 38.1 displacement amplitude) using fully adaptive particle filtering which updates the parameters of the inaccurate preexisting measurement model

Figure 8-2 shows that the proposed fully adaptive particle filtering approach can successfully estimate the damage in the composite when both state process and measurement models are partially known. Compare to Figure 8-1, Figure 8-2 presents how the proposed fully adaptive particle filtering outperforms the augmented particle filtering in situations when the preexisting measurement model does not exactly fit to the particular case under study.

As before, Figure 8-3 shows how the state process model parameters change over time until their variations decrease.

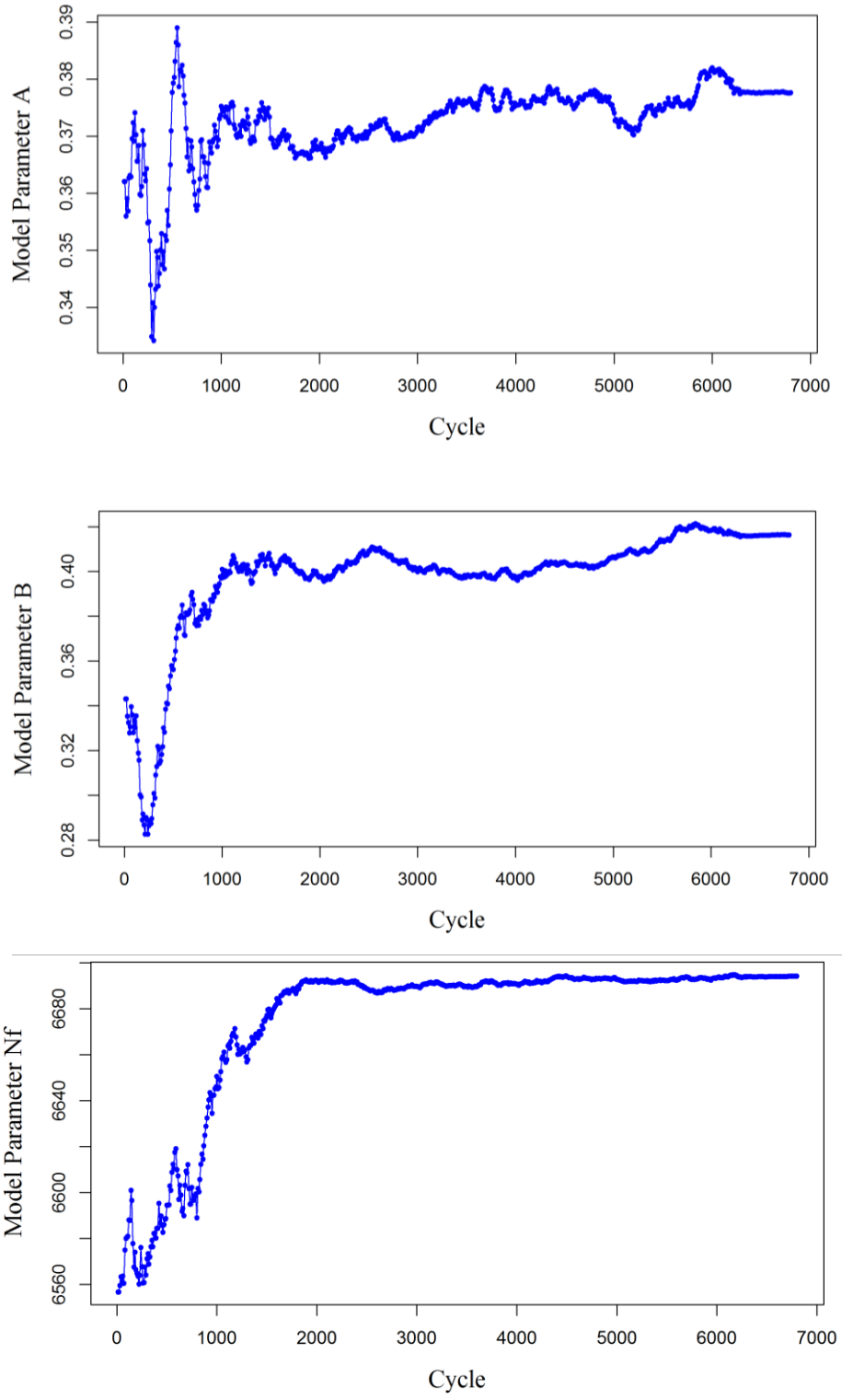


Figure 8-3: Updating the parameters of the state process model in fully adaptive particle filtering

Figure 8-4 tracks the convergence of measurement model parameters  $[\varphi_0, \varphi_1, \varphi_2, \varphi_3]$ . The red line in each graph is the expected value for that parameters coming from curve fitting. The expected values can be derived offline after completion of the experiment by fitting the measurement model (Eq. (8-6)) to all the recorded measurement data. Note that even though the parameters in both state and measurement models are noisy and do not necessarily converge to their expected values (especially, parameter  $\varphi_3$ ), the combination of them is able to successfully estimate the damage as presented earlier in Figure 8-2. It seems that updating both state process model and measurement model through fully adaptive particle filtering gives more flexibility to the method in tracking the true damage evolution.

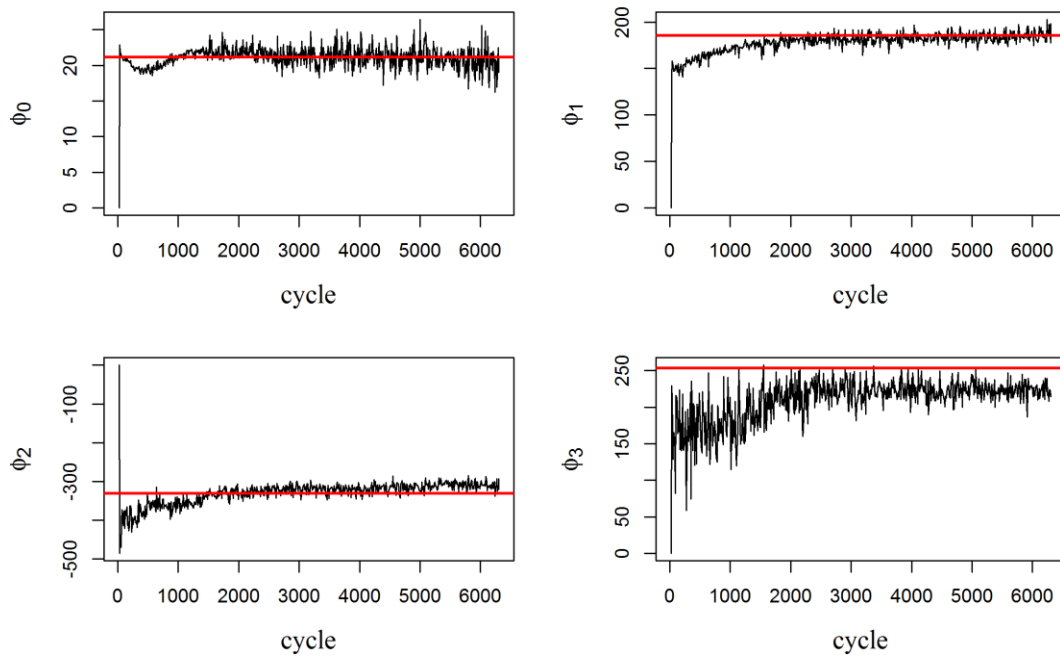


Figure 8-4: parameters of measurement model in fully adaptive particle filtering

### 8-7 *Summary:*

In this chapter the algorithm of fully adaptive particle filtering was proposed. The idea of the proposed approach is to develop a filtering technique that requires neither the fully known state process model nor a predefined measurement model. Thus, both of the state process and measurement models would be updated online through time. It is particularly useful for performing fully online structural health monitoring. The proposed algorithm incorporates the concept of KL-divergence to update the parameters of the measurement model based on real-time upcoming measurements while the parameters of the process model would be learnt via augmented particle filtering as before.

The mathematical details of the algorithm along with the potential concerns and implementation challenges were discussed. And finally, the method was applied on a simplified version of composite case study in chapter 7.

## Chapter 9: Conclusion, Contributions and Suggested Future

### Works

#### 9-1 *Summary*

In this dissertation, a new structural health monitoring framework was proposed based on monitoring and estimating the evolution of damage precursors or indirect damage indicators when conventional direct damage indicator such as crack is unobservable, inaccessible or difficult to measure. It was shown that unlike the traditional widely used empirical damage models, the proposed framework does not have to wait until a known direct damage indicator such as a fatigue crack is observed, whereas it is able to inform about the underlying damage much earlier by monitoring the evolution of some predefined damage precursors. Hence, there would be more time for decision makers to perform corrective actions. The proposed framework is intended to take advantage of various sources of available information in order to reduce the inherent uncertainty and achieve more precise estimation of the system's health state. Dynamic Bayesian Network was adopted as the main modeling technique to materialize the proposed SHM framework. Moreover, the present research showed how incorporating various observations in the framework leads to more precise and more accurate estimations and predictions. Also, integration of different computational approaches (i.e., DBN, augmented- particle filtering with Kernel smoothing and SVR) in the proposed framework provides a more general and flexible methodology that can be applied in many different contexts of application.

To demonstrate and validate the proposed approach, two different case studies were presented. In the first one, the results of a fatigue test on Aluminum specimen prior to crack initiation were used. A model based on variation of modulus of elasticity  $E$  as a damage precursor was developed to describe the underlying active damage state in the component while crack had not emerged yet. A DBN was established to represent the related variables and their causal or correlation relationships. Since the degradation model based on damage precursor was not completely known, the model parameters also needed to be learned during the monitoring process. Augmented-particle filtering along with kernel smoothing technique was applied to infer both the model parameters and the damage state in the component prior to crack initiation. Support Vector Regression technique, which is a powerful and flexible method especially for describing an unknown nonlinear correlation, was also implemented inside the DBN to incorporate AE signals. The results of the proposed framework in real-time estimating the damage state are in good agreement with the experimental observations. Consequently, the methodology described in this dissertation was able to successfully track the true damage evolution and predict the crack initiation in Aluminum specimen when no direct damage indicator existed.

In the second case study, degradation of composite specimen was investigated. Inhomogeneity of the composite material makes the degradation process very complicated in which measuring the conventional damage indicators is quite difficult. Variation of the dissipated thermal energy was selected as the damage precursor to represent the damage evolution during the fatigue life of the component. We used DBN to model the correlation between different elements of the problem and then

applied the augmented-particle filtering approach to estimate the state model parameters and damage states simultaneously. Two types of measurements, i.e., temperature and acoustic emission counts, were adopted to update the damage states. And at the end, the results of damage estimation and TTF prediction were presented. In both case studies, the uncertainty is reduced by integrating multiple evidence and more accurate and precise results were achieved for parameter and state estimations as well as RUL predictions. Furthermore, a new version of fully adaptive particle filtering algorithm was proposed that is capable of learning the parameters of the measurement model in addition to the parameters of the state process model. The proposed adaptive particle filtering is based on minimizing the Kullback–Leibler divergence between the distribution of the predicted measurements by particle filtering and the distribution of the actual measurement at each time step. The algorithm would be particularly useful when the measurements have different nature than the state of interest and their relationship is not fully known in advance. For example, it can be used for estimating and predicting the life of newly designed products or new experiments with different conditions.

It is important to mention that all the analyses are conducted in “*R*”, version 3.2.3 (2015-12-10) -- "Wooden Christmas-Tree", which is a powerful open-source language and environment for statistical computing and graphics. An *R* code is developed throughout this research for performing diagnostics and prognostics with augmented particle filtering inside DBN considering more than one type of measurement. Also a separate code is built for fully adaptive particle filtering. Some of the *R* packages which are installed additionally and used in this research are

kernlab (Karatzoglou et al., 2016), e1071 (Friedrich, 2015) and CEoptim (Benham et al., 2015).

## 9-2 Contributions and possible benefits of this work:

As discussed above, the contributions of this research can be summarized into following categories:

- 1) The concept of indirect damage indicator or more specifically damage precursor was formally studied and its incorporation into SHM framework as an alternative way for degradation estimation was investigated.
- 2) A new SHM framework for diagnostics and prognostics was proposed based on evolution of damage precursors when direct damage indicator such as fatigue crack is inexistence, undetected or difficult to measure.
- 3) A methodology was established for fusing different sources of potentially non-homogeneous evidences via DBN structure including multiple online non-contemporary monitoring data captured from different sensors.
- 4) A hybrid probabilistic approach was developed based on DBN structure for damage estimation and RUL prediction by monitoring the evolution of damage precursor through time. The algorithm of DBN was the combination of:
  - Model-based Particle filtering technique to infer the damage state recursively over time using incoming noisy measurements.
  - Data-driven techniques (such as Support Vector Machine (SVM)) to learn the unknown relationship between some of the variables in DBN from data



- 5) Founded on the technique of augmented particle filtering with kernel smoothing, a solid theoretical scheme for joint parameter and state estimation was examined through DBN structure. The approach is beneficial for modeling partially known degradation process when dealing with less explored area of damage precursor evolution.
- 6) Particle filtering algorithm was modified and a new version of fully adaptive particle filtering was proposed that does not require a predefined measurement model to explain the relationship between the hidden damage state and noisy measurements. This new approach is capable of learning both the measurement model as well as the state process model in real time.
- 7) The proposed methodology was validated and demonstrated in two real world applications:
  - Estimation and prediction of fatigue damage in metallic component prior to crack initiation
  - Estimation and prediction of fatigue damage in complex degradation process such as degradation of Composite component

Therefore main advantages of this research can be:

- 1) Enhancements in real-time decision making for system maintenance when direct damage indicator (such as crack) is yet undetected or difficult to be measured.
- 2) Improvement in uncertainty reduction and achieving more accurate and more precise damage estimation and RUL prediction by integrating different types of information such as various monitoring data

- 3) Advancement in performing SHM when there is no existing predefined models for both the damage evolution as well as correlation of sensor measurements to the damage. This might happen in newly designed products
- 4) Systematic general SHM approach with possible application in different areas such as oil and gas, automobile, and any critical systems that requires online health monitoring under multiple uncertain observations.

### 9-3 Suggestions for future research

In this section some recommendations for extending this research are presented:

- Additional work should be done to confirm the validity and capability of the proposed framework in damage estimation and prognostics for cases with more complicated degradation processes and in the presence of more types of sensor measurements.
- One of the important challenges in this research was optimizing the hyper-parameters such as initial range or mean and standard deviation of the model parameters, noise in state process model, noise in different measurement models, shrinkage parameter in augmented particle filtering, hyper-parameters in SVR. The method is sensitive to the value of each of these parameters and it is important to be managed in a systematic way. As a future research, it is required to develop an optimization algorithm on top of the proposed approach to define the optimum hyper-parameters.
- Although the results are promising at this stage, it would be interesting to apply more advanced versions of SVR such as RVM and Bootstrapped SVR in future work and compare the results with original SVR.

- The proposed idea of fully adaptive particle filtering is in its infancy and needs additional research for validation. Even though the presented results are satisfactory, the performance of the new algorithm should be tested and assessed in other real world applications when for example the measurement model is more complicated or other types of evidence also exist.
- More work and deeper study are required to address the challenges mentioned for the fully adaptive particle filtering approach in section 8-5:
  - More research should be done to control the bias toward the true real-time measurements. The suggested idea of using a regularization parameter needs to be applied and validated.
  - The computational cost of the approach needs to be managed.

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