Abstract

Title of Dissertation: CONNECTING THE DOTS: EXAMINING THE ROLE OF PARENTAL BELIEFS AND PRESCHOOLERS' AFFECT AND ENGAGEMENT IN PREDICTING PARENT-CHILD NUMBER EXPLORATION DURING A MEANINGFUL MATH EXPERIENCE

Erica L. Zippert, Doctor of Philosophy, 2016

Dissertation directed by: Dr. Geetha B. Ramani, Department of Human Development and Quantitative Methodology

The current study examined the frequency and quality of how 3- to 4-year-old children and their parents explore the relations between symbolic and non-symbolic quantities in the context of a playful math experience, as well as the role of both parent and child factors in this exploration. Preschool children's numerical knowledge was assessed while parents completed a survey about the number-related experiences they share with their children at home, and their math-related beliefs. Parent-child dyads were then videotaped playing a modified version of the card game *War*.

Results suggest that parents and children explored quantity explicitly on only half of the cards and card pairs played, and dyads of young children and those with lower number knowledge tended to be most explicit in their quantity exploration. Dyads with older children, on the other hand, often completed their turns without discussing the numbers at all, likely because they were knowledgeable enough about numbers that they could move through the game with ease. However, when dyads did explore the quantities

explicitly, they focused on identifying numbers symbolically, used non-symbolic card information interchangeably with symbolic information to make the quantity comparison judgments, and in some instances, emphasized the connection between the symbolic and non-symbolic number representations on the cards. Parents reported that math experiences such as card game play and quantity comparison occurred relatively infrequently at home compared to activities geared towards more foundational practice of number, such as counting out loud and naming numbers. However, parental beliefs were important in predicting both the frequency of at-home math engagement as well as the quality of these experiences. In particular, parents' specific beliefs about their children's abilities and interests were associated with the frequency of home math activities, while parents' math-related ability beliefs and values along with children's engagement in the card game were associated with the quality of dyads' number exploration during the card game.

Taken together, these findings suggest that card games can be an engaging context for parent-preschooler exploration of numbers in multiple representations, and suggests that parents' beliefs and children's level of engagement are important predictors of this exploration.

UNIVERSITY OF MARYLAND

CONNECTING THE DOTS: EXAMINING THE ROLE OF PARENTAL BELIEFS AND PRESCHOOLERS' AFFECT AND ENGAGEMENT IN PREDICTING PARENT-CHILD NUMBER EXPLORATION DURING A MEANINGFUL MATH EXPERIENCE

by

Erica Leigh Zippert

Dissertation submitted to the faculty of the Graduate School at the
University of Maryland in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2016

Advisory Committee:
Professor, Dr. Geetha B. Ramani, Chair
Professor, Natasha Cabrera
Professor, Richard Prather
Professor, Olivia Saracho
Professor, Allan Wigfield

Acknowledgements

I would like to take this opportunity to thank everyone who has helped me on my extended academic journey. First I would like to thank my advisor and soon to be colleague Dr. Geetha Ramani. I am so very grateful for all the opportunities you have given me in the almost 7 years we have worked together. My experience working with you on the teacher board game study taught me lessons that will last throughout my academic lifetime, and has inspired me to strive to make a difference in the research that I do. I have been so lucky to have your support in pursuing my interest in parental beliefs, even though it was outside of your particular area of expertise. Applying to jobs this year has made me reflect on all the things I have accomplished as your advisee, and I cannot imagine my graduate career without all of your help.

I am also grateful for all of the guidance and support I have received from my dissertation advisory committee, Drs. Allan Wigfield, Natasha Cabrera, Olivia Saracho, and Richard Prather. My dissertation study has come a long way since my proposal meeting, and in the process, I have expanded my area of expertise to include early math interest, honed my understanding of the importance of parental beliefs, and learned how to run moderation analyses. Working with you all has undoubtedly influenced the nature of the research program I hope to pursue in the future, and for that I am especially thankful.

I would also like to thank my parents, siblings, friends, and of course my partner in crime, Alon. Getting to this point would not have been possible without all of your unwavering support. Thank you for your patience, and for sharing me with the University of Maryland for so long!

Finally, I am forever indebted to Port Discovery Museum, my research assistants, and countless families who have all generously donated their time, their hard work, and their data.

Table of Contents

List of Tables

List of Figures

Chapter I: Introduction

Statement of the Problem

Mathematics is an extremely important academic subject for students, yet there are persistent problems in children's success in this domain in the United States. Over half of the nation's adolescents score below proficiency on national math assessments (NAEP, 2013), and continually underperform in the domain in comparison to their peers in other developed countries (OECD, 2012). While this could be considered to be an artifact of formal schooling, wide variation in U.S. children's mathematical knowledge also exists as early as the preschool years (Starkey, Klein, & Wakeley, 2004).

This variation has important implications, as math skills in particular are especially robust predictors of later academic achievement across academic domains (Duncan et al., 2007; Ginsburg, Lee, & Stevenson Boyd, 2008; Jordan, Kaplan, Ramineni, & Locuniak, 2009). Thus, those students who begin school behind in their math knowledge rarely catch up to their peers who began school with more developed foundational math knowledge (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004). Low proficiency in math in the preschool years has severely deleterious and far reaching implications for students' high school graduation rates, college attendance, entry into STEM majors and careers, and the ability to successfully function at home, work, and in the community in adulthood (Duncan & Magnuson, 2013; Ginsburg et al., 2008).

The early timing of variation in mathematical understanding is especially noteworthy, as it provides further evidence suggesting that the academic socialization children receive from their parents at home influences their math achievement outcomes (Eccles et al., 1983; Jacobs, Davis-Kean, Bleeker, Eccles, & Malanchuk, 2005; Wigfield, Eccles, Schiefele, Roesner, & Davis-Kean, 2006). However, far less is known about what factors affect children's development *before* they enter formal schooling, as most research examines parents' socialization influences once children have already begun school. Understanding what influences even younger children's early math development is critical for explaining the wide variation in math knowledge young children exhibit upon school entry, and future initiatives can be put forth to intervene with family's academic socialization efforts on children's behalf.

Theoretical Framework of Numerical Knowledge Development

Within the last 2 decades, there has been tremendous growth in attention on math education and learning in early childhood. The importance of early math development has gained recognition by the largest organization dedicated to the education of young children (NAEYC, 2002), and standards for learning mathematics have recently been developed for preschoolers for the first time in U.S. history (National Research Council, 2009; National Council of Teachers of Mathematics, 2000; 2006). The earliest and most important math concepts children learn is numeracy, which includes knowledge of numbers and relations between their different representations, because it is upon numbers that all subdomains in math are based (i.e., geometry, measurement and data analysis; NCTM, 2000). As such, research-based curricula and interventions focused heavily on preschool children's number development have been designed to implement these early standards into effective teaching practices in the preschool years (Griffin, 2004; Sarama & Clements, 2004; Sophian, 2004; Starkey et al., 2004; Starkey & Klein, 2000).

While the research on children's numerical knowledge development has been largely disconnected, a new framework has been recently proposed to provide the field with some organizational and theoretical insight, called the integrative theory of numerical development (Siegler, 2016; Siegler & Lortie-Forgues, 2014). In this theory, it is recognized that number exists in both non-symbolic and symbolic (e.g., number words and symbols) representations, and that children's numerical knowledge development can be characterized by a series of "acquisitions" or developmental milestones in understanding numerical magnitudes. The first involves expressing non-symbolic representations of the magnitudes of numbers in more exact ways, and the second is linking non-symbolic representations of numerical magnitudes to symbolic ones. The third involves extending understanding of the magnitudes of numbers to increasingly large whole numbers beyond 10, and the fourth involves an extension of this understanding to all rational numbers (e.g., fractions and decimals). The development of the representations of number and the corresponding first two milestones are particularly relevant for the developing preschool-aged child, and will be discussed in the following sections to help illustrate how having a strong foundation in number is important for later success in school (Duncan et al., 2007).

Non-symbolic number knowledge. Siegler and his colleague (2014) posit that humans and even some animals have the capacity to think numerically, albeit non-symbolically, from birth. Thus, young children are thought to bring with them to the classroom a capacity for mathematical cognition (Baroody, Lai, & Mix, 2006; Baroody & Wilkins, 1999; Ginsburg, Cannon, Eisenband, & Pappas, 2006; Ginsburg, Klein, & Starkey, 1998). In infancy, children demonstrate an early concept of number (the foundation of mathematical thinking), as they can differentiate representations of small quantities exactly, can approximately distinguish between larger quantities varying in number (e.g., Dehaene, 1992; Izard, Sann, Spelke, & Streri, 2009; Lipton & Spelke, 2003; Xu & Spelke, 2000), and may even recognize basic concepts of arithmetic with small sets of items (Wynn, 1992). These abilities are typically referred to as children's non-symbolic number knowledge, and the precision of these abilities sharpens over development (e.g., Cordes & Brannon, 2008; Xu & Spelke, 2000). Given the early onset of such numerical processing, it is argued that non-symbolic number knowledge is innate (Feigenson, Dehaene, & Spelke, 2004).

Symbolic number knowledge. In the toddler and preschool years, children in the U.S. and other industrialized countries with formal number systems transition to representing numbers in more exact ways through their acquisition of the symbolic number system, which includes both verbal (e.g., *four*) and written representations (e.g., written numerals). In toddlerhood, children begin to produce number words in the count string, although these skills are still developing. For example, as children are learning language, they begin to learn the names of numbers (e.g., "two") as well as the knowledge that number words are typically used in association with objects (e.g., "two shoes"). Also at this time, children may even begin to also recognize their first written numerals (Mix, 2009); however, this number knowledge is not used beyond rote reproduction, as children do not yet have the understanding that number words or numerals represent and quantify specific sets of objects (Wynn, 1990, 1992b).

Connecting symbolic and non-symbolic number knowledge. In the preschool years, children develop the ability to link together symbolic and non-symbolic representations of number (Siegler & Lortie-Forgues, 2014). For example, they begin to meaningfully associate the number words they learned in toddlerhood with the specific sets of visible objects they represent. This understanding has garnered much empirical attention, and is referred to as the cardinality principle, or *cardinal number knowledge* (Gelman & Gallistel, 1978; Wynn, 1990; 1992). Specifically, cardinal knowledge allows children to correctly use numbers to quantify sets, such as "three balls" and "four dolls," and correctly employ counting when asked for a large number of objects. Children first develop cardinal knowledge of small numbers, and then for sets of 4 and larger (Le Corre, Van de Walle, Brannon, & Carey, 2006; Wynn, 1990, 1992b). This understanding is considered one of the most foundational concepts in number development, as mastering it indicates that children understand the true meaning of numbers, at least in their verbal representation (Wynn, 1990).

More recent research and theory have also begun to recognize the importance of children's ability to understand Arabic numerals, called digit or *numeral knowledge* (Benoit, Lehalle, Molina, Tijus, & Jouen, 2013; Krajewski & Schneider, 2009; Purpura, Baroody, & Lonigan, 2013). This understanding entails both the ability to recognize written numerals and also link them with their respective quantities. Preschool children in their fourth year finish learning the names of printed numerals through 10, and most importantly link those symbols to corresponding non-symbolic sets of object arrays, first for numbers up to 3, and then for numbers of 4 and larger (Benoit et al., 2013). Numeral knowledge acquisition is the last critical step in the transition from reliance on non-symbolic number representations to competency in representing numbers entirely symbolically, and is predictive of children's success in kindergarten mathematics (Purpura et al., 2013).

As posited by the integrated number theory, numerical development involves increasing the way and range the numbers are understood (Siegler & Lortie-Forgues, 2014). The improved understanding of the links between symbolic and non-symbolic representations of quantities, namely cardinal and numeral knowledge of both small and large numbers, allows children to engage in the type of problem solving expected of them in kindergarten and first grade, such as engaging in simple arithmetic in both verbal and written formats (Common Core State Standards, 2015). Establishing these links also allows children to accurately differentiate symbolic numbers in terms of their magnitudes, which typically develops in the fourth year as well (Griffin, 2004), and is another important skill taught in kindergarten classrooms (Common Core State Standards, 2015).

Siegler and his colleague (2014) emphasize that the process of making connections between symbolic and non-symbolic representations of number, however, can be quite variable, arduous, and lengthy. For example, it can take a year or more after children become familiar with number words 1-10 (e.g., as they appear in the count list) before they link them with their respective arrays of objects (Schaeffer, Eggleston, & Scott, 1974; Wynn, 1990, 1992b), and even longer to learn that particular symbols are connected to specific sets of objects (Benoit et al., 2013). Albeit challenging for young children, the milestone of representing numbers increasingly more precisely and symbolically signifies an important transition from concrete to abstract thinking, and is critical for children to succeed in math in formal schooling (Purpura et al., 2013).

Finally, it is emphasized in the model that children’s experiences that cater to the ways in which they develop their understandings of numerical magnitudes can contribute greatly to achieving the aforementioned milestones (Siegler, in press). Games such as chutes and ladders, for example, are suggested as providing opportunities for children to link symbolic and non-symbolic representations as children point to and count the spaces on the board (Laski & Siegler, 2014). Reference to such learning opportunities is presented only briefly as part of the theory, and more research, to be described in the following section, expands upon what these experiences look like, and how math researchers have defined and studied them. Furthermore, a theory is presented later in this paper identifying what factors may contribute to variation in parent-child interactions such as these.

Factors Contributing to Early Number Development

To date, little research has examined the factors that predict variation in children's number development, especially in how they develop numeral and cardinal knowledge. However, recent work has begun to identify potential sources and pathways that predict children's developing numerical knowledge (Anders et al., 2012; Niklas & Schneider, 2014; Skwarchuk, Sowinski, & LeFevre, 2014). Three of the main contributors recognized in this work include early math experiences, parental beliefs, and children's interest and engagement in math, which will be briefly introduced in the sections that follow.

Frequency of early home math experiences. Researchers have demonstrated that children's early number related-experiences with parents are highly important for understanding young children's number development (Gunderson & Levine, 2011; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Skwarchuk et al., 2014). Along with the variation in children's early numerical knowledge, research has also revealed that there is substantial variation in the frequency with which parents and young children engage in number-related experiences at home, and this variation is largely shared and directional, in that early number-related experiences with parents are predictive of children's later number development (Levine et al., 2010). While these interactions may include formal explicit interactions in which parents directly instruct their children in counting and simple arithmetic, it is likely that informal activities that support children's number skills indirectly, such as through measuring during cooking or playing games (LeFevre et al., 2009) are far more common and beneficial for children's learning (Rogoff, 1990; Vygotsky, 1978). Evidence exists suggesting that children's cumulative experiences with numbers in the preschool and kindergarten years are predictive of both their concurrent and later general math knowledge (e.g., Blevins-Knabe & Musun-Miller, 1996; Jordan & Levine, 2009; LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; LeFevre et al., 2009; Levine et al., 2010; Melhuish et al., 2008; Skwarchuk et al., 2014). This suggests that children's early number-related experiences at home during interactions with parents are important contributors to children's currently developing math skills and later math understandings.

Quality of early home math experiences. Closer examinations of children's early number experiences have revealed, however, that not all experiences are equal in contributing to children's number knowledge development. Within the last few years, the importance of quality rather than frequency of early math experiences has gained recognition as important in understanding predictors of children's numerical knowledge development (Gunderson & Levine, 2011; Mix et al., 2012). Number experiences during which parents only emphasize children's rote number knowledge, such as through recitations of the counting sequence and naming numerals, tend to be either negatively or non-significantly associated with children's general math knowledge, though more involved experiences, such as numerical magnitude comparison and arithmetic, are typically found to be positive predictors of math understanding (Blevins-Knabe & Musun-Miller, 1996; Ramani, Rowe, Eason, & Leech, 2015; Skwarchuk, 2009). More targeted investigations have suggested that experiences that involve using symbolic representations to quantify visible arrays of objects are highly associated with gains in children's number knowledge. Specifically, research on U.S. families from a range of income backgrounds has suggested that parents' talk with their infants, toddlers, and preschoolers about number names in reference to present visible (e.g., countable) sets of objects was positively associated with children's cardinal knowledge development during the preschool years (Gunderson & Levine, 2011; Ramani et al., 2015). Experimental work further suggests that exposure to counting and labeling visible quantities of objects rapidly improves preschool children's cardinality understanding at a fraction of the time it would typically take to develop (Mix, Sandhofer, Moore, & Russell, 2012). This suggests that young children's cardinal knowledge development is especially susceptible to input that explicitly reinforces connections between symbolic (i.e., verbal) and non-symbolic number representations.

To date, research efforts have only examined children's experiences with written numerals in the context of naming them (Blevins-Knabe & Musun-Miller, 1996; Skwarchuk et al., 2014; Vandermaas-Peeler, Ferretti, & Loving, 2012); however, more recent definitions of numeral knowledge have expanded to include knowledge of both the name and the non-symbolic quantities associated with the written symbol (Purpura et al., 2013). Thus, it is likely that simply naming Arabic numerals may not be meaningful enough for children developing this understanding (Ramani et al., 2015). It is highly possible that experiences that involve both naming and linking written numerals to present sets of objects would support children's numeral knowledge as it is currently defined. Experiences that explicitly link children's pre-existing non-symbolic knowledge of quantities with symbolic representations of number are likely to allow children to transcend their concrete representations of quantity and advance to more abstract numerical cognition.

Meaningful math experiences. It is unclear exactly how often and in what contexts parents explicitly make connections between non-symbolic quantities and symbolic number representations (e.g., the number word or written numeral) for their preschoolers. In general, naturally occurring parent talk about numbers across a range of contexts within the home has been found to be quite variable, though it is on average relatively infrequent (Levine et al., 2010). Thus, there may be little opportunity for any type of number talk to occur at home, let alone talk that makes connections between number representations. Only one study to date was conducted with the goal of examining parental support for their preschooler's cardinal knowledge development, during which parents and children were observed reading a storybook that included number names in the text and corresponding illustrated pictures (Mix et al., 2012). To the researchers' surprise, parents rarely made attempts to emphasize the connections between the number words and corresponding numbers of book illustrations for their preschoolers. Similarly, when parents and children are observed interacting in other seemingly number-salient contexts, they also engage in far less number talk than would expected (Vandermaas-Peeler, Nelson, & Bumpass, 2007; Vandermaas-Peeler, Nelson, Bumpass, & Sassine, 2009). While playing with number-related toys and materials in unstructured dramatic play contexts (i.e., dramatic play toys like a cash register, or free play with a number puzzle), parents from a range of backgrounds casually comment on numbers for their children, but more generally tend to discuss topics that are culturally relevant, but inexact in regards to quantity, such as how “expensive” a toy vegetable is while playing “store” or commenting on non-numerical aspects of illustrations even in books about numbers (Mix et al., 2012; Vandermaas-Peeler et al., 2007; Vandermaas-Peeler et al., 2009). It may be that children receive very little support for their number development at home in general, and the input they typically do receive may be of less value in helping them develop their numeral and cardinal number knowledge.

Research has suggested, however, that contexts in which numbers are more salient elicit more parent number talk than those that are not (Mix et al., 2012), and in fact, number games have been recognized as an especially relevant context for parent-child number exploration more broadly (Benigno & Ellis, 2004; Bjorklund, Hubertz, & Reubens, 2004; Vandermaas-Peeler et al., 2012; Young-Loveridge, 1989). For example, parents of preschoolers have been found to engage in more number talk during number board game play than when playing with toys and books that simply have numbers printed on them (Ramani et al., 2015). Number board games may be especially beneficial in that they can keep parents and children focused on quantity, because numbers are both salient (e.g., visible on spinners and board spaces) and necessary to accomplish the goals of the game. For example, players must move their game pieces the correct number of spaces on the board based upon the number of dots that appear after rolling the dice, or the numeral to which the arrow points after players spin the spinner. Thus, number board games involving dice or spinners can expose children to single representations of number. On a spinner, symbolic number information is available (i.e., the printed numeral), and in the case of dice, non-symbolic information is presented (i.e., the dots that appear).

Research has yet to examine the frequency of parent-child playful experiences in a context that is conducive for parents and children to explore *multiple representations* of number, both symbolic and non-symbolic. Playing cards may provide one such optimal context, however. For example, typical playing cards directly display 2 representations of number (i.e., written numerals and non-symbolic arrays of suit images), and likely could directly elicit talk about the third representation (i.e., number name). Research to date has not examined how frequently parents engage their preschool children in card game play specifically.

Summary. Within the last few years, the importance of quality as opposed to the sheer quantity of early math experiences has gained recognition as important in understanding contributors to children's numerical knowledge development (Gunderson & Levine, 2011; Mix et al., 2012). While research has identified how experimenters can most optimally teach young children the concept of cardinality, it has not been possible as of yet to identify the everyday contexts in which these experiences occur between parents and their preschoolers. The literature suggests that situations in which numbers are salient and central to the activity goals undoubtedly lead to more number talk (e.g., Mix et al., 2012; Ramani et al., 2015); however, it is less necessary for children to simply receive *more* experience with numbers than it is for them to gain exposure to the right types of number-related experiences. Specifically those that make connections between children's non-symbolic and symbolic number knowledge seem to make the most impact. While parents have not yet been observed making these connections extensively for their preschool children, it is argued in the current study that appropriate contexts *do* exist, and include those in which number is not only salient and central to the activity, but is available in all representations to facilitate the connections between non-symbolic and symbolic number representations. Within these contexts, it is expected that parents and children will be encouraged to engage in the types of behaviors that ultimately support children's cardinal and numeral knowledge (i.e., emphasizing the connection between symbolic and non-symbolic quantities), and for the purposes of this study, these types of contexts will be termed *meaningful math experiences*.

Parental beliefs. Another likely source of variation in young children's number development is parents' beliefs about math. North American parents vary substantially in their reports of these beliefs, which include conceptions of their own math abilities currently and when they were in school, their values of math more generally, as well as in respect to specific skills (Blevins-Knabe & Musun-Miller, 1996; Cannon & Ginsburg, 2008; LeFevre et al., 2010; Missall, Hojnoski, & Caskie, 2014; Skwarchuk et al., 2014; Skwarchuk, 2009). When these beliefs are negative (e.g., parents rate their math ability as low and the domain as not important), it can have deleterious effects on the ability to function in their daily lives in any circumstances that involve math or numbers (Maloney & Beilock, 2012), and this could have negative consequences for how parents teach their children about math at home (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015).

Of particular concern is that some research suggests these negative beliefs are quite prevalent in the U.S. (NRC, 2009).

Negativity about math also extends to parents' math-related beliefs regarding their own children. Interviews with U.S. parents from a range of backgrounds indicate that it is common for parents to believe that their preschool children are less interested in and less able to perform well in math than in other academic areas, such as literacy (e.g., Cannon & Ginsburg, 2008). Parents have also been documented admitting that they feel it is less important for their preschool children to learn math before they begin formal schooling than other domains, such as reading, and also indicate that they lack clear expectations for their young children's math learning (Blevins-Knabe & Musun-Miller, 1996; Cannon & Ginsburg, 2008; LeFevre et al., 2010; Missall et al., 2014; Skwarchuk, 2009; Skwarchuk et al., 2014). Further, parents may hold the belief that supporting early math development at home is the role of the school and not the parents (Cannon & Ginsburg, 2008). Given these views, parental beliefs seem to be extremely important in understanding the nature of number-related interactions with children. In particular, these beliefs could help explain why U.S. parents tend not to be as highly invested in providing number-related experiences for their children during free play contexts in the lab (e.g. Vandermaas-Peeler et al., 2007) and at home (Levine et al., 2010).

As such, parental beliefs about math have been consistently considered a necessary component in the development of early home numeracy models (DeFlorio & Beliakoff, 2014; Kleemans, Segers, & Verhoeven, 2013; Missall et al., 2014; Niklas & Schneider, 2014; Skwarchuk et al., 2014), and this research has shown that these beliefs are linked to both their reports of engaging their children in math-related activities at home (which predict children's number development), as well as children's math development directly. The scope of the research on parental beliefs, however, has remained quite narrow, as it has typically been limited to only one or two types of parental beliefs on a given parent survey (e.g., LeFevre et al., 2009). In particular, most of the evidence suggesting that parents' beliefs relate to the frequency of math-related parenting behaviors refers to parents' general beliefs. It is likely that this area of research has not expanded over the years because it has not been grounded in any existing theoretical framework.

Child affect and engagement. The importance of young children's affect and engagement in understanding variation in young children's math experiences has only recently begun to garner attention in the math literature, though its relevance in children's language and literacy development has been established to a greater extent (Deckner et al., 2006). Affect and engagement in math measured in the early years, typically referred to as child interest, have been assessed via children's self-report in early childhood (Arnold, Fisher, Doctoroff, & Dobbs, 2002), but is more reliably assessed via parent or teacher report, or if children are observed by researchers as they are engaged in a math-related task (Fisher, Dobbs-Oates, Doctoroff, & Arnold, 2012). During such observations, children's behaviors are examined in terms of positivity in facial expressions, their attention towards the activity, and their sustained behavioral engagement and contributions to the task at hand. In the domain of math, older children's self-reported interests in math have been shown to be predictive of their math grades and the courses they select in school (Schiefele & Csikszentmihalyi, 1995), and early measures of affect and engagement in math significantly predict preschool children's concurrent and later math knowledge (Fisher et al., 2012). As such, interventions aimed at peaking preschool-children's interest in math have proven successful in boosting children's affect towards math play and general math knowledge in as short a time as half a year (Arnold et al., 2002). Most relevant for the current study, however, is that children's affect towards and engagement in math may be especially important in predicting the frequency of early math experiences at home with parents (Lukie, Skwarchuk, LeFevre, & Sowinski, 2014). In particular, children's shared math exploration with parents may depend on how interested and thus willing to engage in math children are during the activity.

Theoretical Framework on Academic Socialization

The parent socialization component of Eccles' and colleagues' (1983) expectancy value theory serves as an appropriate theoretical foundation for understanding the relations between parental beliefs, children's academic experiences, and their academic development. Though the theory has typically been utilized to explain school-aged children's academic achievement in math, among other domains, the framework can be used to organize and expand upon the disjointed literature on the math-related beliefs of parents of preschool-aged children. In particular, the framework can be utilized to understand the importance of parental beliefs and characteristics of the child to more broadly identify factors that may be important in predicting how parents support their children's math development in the years preceding school entry.

In this framework, different types of parental beliefs are posited to be important in predicting children's academic development. More specifically, the distinction is made between parental beliefs that are general, and those that are specific to their own children

(Jacobs et al., 2005). General beliefs include personal values of and interest in math, beliefs about parents' own math abilities, and perceptions of their role as their child's teacher at home (Jacobs et al., 2005). In contrast, parents' child-specific beliefs include perceptions of their children's abilities and interests, parents' expectations for their children's future academic learning in school, and perceptions of value of math skills for their children (Jacobs et al., 2005). These beliefs have been widely studied in research on school-aged children, but have been examined to a much lesser degree in parents of preschool-aged children. Theorists posit that parents' general and specific beliefs are both important predictors of parents' academic socialization efforts (Jacobs et al., 2005; Wigfield et al., 2006). These include parents' explicit behaviors aimed at promoting children's development, such as providing academic resources, using specific teaching strategies, and providing children with educational games, toys, and social experiences (Jacobs et al., 2005).

It is also posited in this theory that child characteristics play an important role in predicting the academic socialization they receive at home from their parents (Eccles et al., 1983). In particular, child interest is posited to moderate the association between parental beliefs and the frequency of the math activities they provide their children. While little work as empirically tested this pathway, children's interest is likely to have important impacts on interactions with parents. For example, it is likely that no matter how important parents may believe math to be, or how able they think their child is to engage in math-related problem solving, if children are uninterested in math, the dyad is likely not going to delve into math concepts as in depth if children are highly interested in math content. An abbreviated version of the theoretical model is presented in Figure 1.

Empirical support for this pathway, however, is sorely needed, as only one study to date has examined the association between children's interest in math and the frequency with which they engage in math learning experiences with their parents (Lukie et al., 2014).

The next chapter will review the relevant literature and provide a more in depth discussion of how the seminal work on parental cognitions, the parent socialization model, and the integrative theory of numerical knowledge development can be applied to understand the ways in which the parental beliefs and children's interest in math can influence the quality of children's early math experiences that contribute to their later math learning.

Summary. During early childhood, children make the critical transition from a purely non-symbolic conception of numbers to numerical cognition using abstract symbolic representations (i.e., numeral and cardinal knowledge development; Siegler & Lortie-Forgues, 2014). While research has identified ways to support this development, namely by reinforcing the connection between verbal and non-symbolic representations of number (Mix et al., 2012), it is unclear in which contexts these skills are actually supported in this way by parents during typical interactions with their preschoolers at home. Some researchers suggest that experiences with parents supporting children's cardinal knowledge are practically non-existent (Mix et al., 2012); however, a review of the literature suggests that certain contexts in which parents and children have been observed have been somewhat limited in eliciting much number talk at all, and that certain contexts may be better than others in this way (e.g., Ramani et al., 2015). It is argued in the current study that naturally occurring informal contexts *do* exist that promote parent-child exploration of numbers in ways that are supportive of children's cardinal and numeral knowledge development, and will be referred to as *meaningful math experiences*. These contexts are defined as those in which numbers are salient and central to the activity, and opportunities are available to link both symbolic and non-symbolic number representations.

Playing cards can be used in an interaction to meet the criteria for a meaningful math experience to occur. For example, many popular card games involve the use of the magnitudes of numbers as central to the objectives. In addition, cards numbered 1-10 (excluding face cards) display the printed numeral in addition to the corresponding number of images. Ultimately playing card games may provide the ideal opportunity for parents and preschoolers to explore numbers in a fun, engaging, and meaningful context. In particular, given the presence of both symbolic (Arabic numeral) and non-symbolic quantity information (dots) on the card, it is likely that parents and children will be able to make connection between the two. Thus, in the current study to be described below, this type of context will be explored in terms of how it promotes opportunities for children and parents to engage in math exploration that is supportive of children's numeral and cardinal number development, specifically exploring the connections between the symbolic and non-symbolic representations of number.

Even in the most optimal contexts, it is expected that variation will exist in how parents engage their children in talk about, and exploration of math. Theorists suggest that parental beliefs and children's interests are likely important sources of influence on parents' behaviors (Eccles et al., 1983; Jacobs et al., 2005), and research based entirely on parent surveys has suggested that parental beliefs and child interests are associated with their reports of engaging their children in math learning experiences at home (e.g., Lukie et al., 2012; Skwarchuk et al., 2014). This research is limited, however, in several ways. First, the literature examining the importance of parental beliefs for young children's math experiences and learning is relatively disjointed and is largely not driven by theory. As such, survey items assessing parent beliefs are inconsistent across studies, making it difficult to determine its importance and establish trends. Further, links between parental beliefs, children's interest in math, and the support parents provide their children at home is based entirely upon parent report, which is prone to measurement error and does not provide information about the quality of parent-child interactions nor an objective measure of children's interest in math. Thus, it is necessary to obtain more specified measures of parent-child math activity engagement and child interest, and to understand how a theoretically informed set of parental beliefs interacts with child interest to predict both the frequency and quality of children's early experiences with numbers.

Study Design

The current study employs a multi-method design, using behavioral observation, parent survey, and child assessment to examine how U.S. parents support their preschool children's developing cardinal and numeral knowledge two important areas of math knowledge for the developing preschool-aged child during meaningful math experiences, what parents believe about math more broadly, and together how these factors predict preschool children's numerical knowledge.

Participants of the study were parents and their 3- and 4-year old preschool-age children (n = 94 parent-child dyads). In the first part of the study, parents completed a survey on early learning developed for the current study including adaptations of items from an instrument developed by Eccles et al. (1983) for parents of adolescents, and recently adapted for parents of younger children (e.g., Wigfield et al., 1997). This questionnaire was administered to provide a measure of a theory-driven compilation of beliefs parents hold about math more generally, and specific to their own child. At the same time, children were tested to assess their numerical knowledge using a recently developed reliable and validated early math assessment (Purpura & Lonigan, 2015). Then, in the second part of the study, parents and their children were engaged in a 10-minute dyadic interaction that involved playing a number-related card game (i.e., a modified game of *War* with playing cards developed for the current study).

Study Aims and Predictions

Aim 1– The first study aim was to examine how parents and children explore different representations of number in terms of the frequency and quality of the math experiences they provide their children. This included examining the frequency in which families reported dedicating their time to math-related activities at home, and also included examining the quality of these experiences, specifically measured by how parents and preschoolers identify symbolic (written numerals, number words) and non-symbolic numerical information (i.e., dot arrays), and make explicit connections between these representations during a parent-child number-related interaction.

Prediction 1. It was predicted that while parents may not engage their children frequently in math-related activities such as card games and comparing the quantities of numbers when at home, parents would support children's cardinal and numeral knowledge in the context of a card game because it is a meaningful math experience that encourages the purposeful use of numbers in multiple representations in a game context.

Dyads in this context were expected to focus on identifying the quantities symbolically more so than non-symbolically, and explore their connections to the non-symbolic representations on the cards to complete each turn.

Aim 2- The second aim of this study was to examine a wider range of parents' math-related beliefs through a theoretical lens (Eccles et al., 1983; Jacobs et al., 2005; Wigfield et al., 2006) and determine their role in predicting the frequency and quality of the number-related experiences they provide their preschool children at home.

Prediction 2- It was expected that there would be strong associations among parents' individual child-specific and general belief items, respectively, and minimal association between the two belief types given that in the academic socialization model, these beliefs are represented as distinct constructs (Jacobs et al., 2005). Further, it was expected that more positive parental beliefs specific to their own children would be positively associated with the quality of children's number experiences, while more positive general parental beliefs would be positively associated with more frequent number-related experiences shared with their preschool children.

Aim 3- The third aim was to examine children's levels of child engagement and affect exhibited during the game, and determine its role in predicting the frequency and quality of the number-related experiences they provide their children.

Prediction 3- It was predicted that children would be highly interested in the game, and that math interest would moderate the association between parental beliefs and the frequency and quality of preschoolers' early number experiences. In particular, it is predicted that for children with high levels of positive affect and engagement, there will be a positive association between parents' beliefs and the frequency and quality of children's math experiences. However, for children with low positive affect, high negative affect, and engagement, this relation is expected to be non-significant, assuming that regardless of parents' values and intentions, children who are uninterested in math (show low levels of engagement and positive affect, and high levels of negative affect) are less likely to engage in related activities in depth.

Contributions

The current study expands upon the research on early math development in several ways. First, while much research suggests that parents in the U.S. talk very little about numbers with their young children overall, it is of interest in the current study to demonstrate instead that parents *do* support the development of their preschool children's cardinal knowledge in the right contexts. The current study introduces the novel concept of a *meaningful math experience*, which represents activities in which numbers are salient, central to the interaction, and available in all visual forms (i.e., numeral and visible object arrays). Identification of such contexts is sorely needed, as the importance of early number development has become widely recognized as critical for later academic success (Duncan et al., 2007), and efforts are needed to identify contexts in which math development can be supported in the home by parents of young children in developmentally appropriate ways, just as it is understood that reading at home supports children's literacy development. Findings from this study will identify at least one activity (i.e., card games), in which parents of young children can be encouraged to participate that promotes talk and number exploration that specifically supports young children's cardinal knowledge development.

There has been a rise in empirical attention as of late to the development of numeral knowledge, identifying it as a central skill for children to transcend from their foundational sense of quantity to thinking abstractly about numbers, and is an important understanding or milestone for children to acquire once they begin school (Benoit et al., 2013; Krajewski & Schneider, 2009; Purpura et al., 2013; Siegler & Lortie-Forgues, 2014); however, research has yet to address how this skill in particular is supported in the preschool years beyond numeral naming. Given the skills upon which this ability is based (i.e., knowledge of the name and non-symbolic quantities associated with each numeral), it is highly likely that experiences that familiarize children with both the name and corresponding non-symbolic representations of printed numerals are likely to directly facilitate the connections that children must make to acquire this skill. The current study provides the first investigation of how parents emphasize the link between written numerals and their non-symbolic quantities in a meaningful, number-relevant informal context.

Even in the most optimal contexts, parents vary in how they support their young children's numerical knowledge development. Theorists suggest that parental beliefs and children's interests, are paramount for understanding variation in parenting behaviors (Eccles et al., 1983; Sigel et al., 1992; Sigel, 1985), and supporting research has developed instruments that show them to be significant predictors of children's academic outcomes (e.g., Parsons, Adler, & Kaczala, 1982). Thus, when parental beliefs are considered in research to explain variation in children's early math experiences, they are not examined consistently across studies or with good theoretical rationale. Growth in this area will continue to be stymied until theory is utilized to inform item selection, analyses, and discussion of results. Thus, the current study adopts one framework, the parent socialization model (Jacobs et al., 2005), to conceptualize and expand our understanding of the role of parents' math-related beliefs, and their role in explaining preschool children's number development indirectly as well as directly.

The current study also expands upon existing research on early math development by examining children's interest in math by separately examining engagement and affect, as well as how these measures interact with parental beliefs in predicting the frequency of at-home math experiences (measured by parental report) *and* quality of number-related experiences parents and children share (as measured by parent-child exploration of number during a specific number-related playful number experience). This is an important contribution because little research has examined child interest in the context of children's early math experiences and their math development, and has done so using parents' reports of children's interests more broadly, as opposed to a measure that is obtained objectively (via behavioral observation), in terms of affect and engagement separately, and regarding numbers specifically. Further, most of the relevant work on children's math development has focused on adult contributions to the interactions, with only a few exceptions (e.g., Ramani et al., 2012). As such, no study to date has assessed the interaction between parent and child contributions to the interaction (e.g., parents' math-related beliefs as they interact with child engagement and affect). The current study will thus provide evidence of associations between the math-related beliefs of parents of preschoolers, child engagement and affect, and the frequency and quality of parental support of preschoolers' number development.

The next chapter will describe the research that has led to the development of the arguments presented above, and provide more specific rationale for the proposed study.

Chapter II: Literature Review

Over the past several decades, there has been an increase in empirical attention paid to math education and learning in early childhood. This area of research has revealed that young children are surprisingly adept at processing numerical properties of sets, and develop the ability to think symbolically about numbers during the preschool years. Children's development of symbolic number knowledge has been recognized as especially dependent upon input, thus the early home environment is now understood as strongly impactful on children's numerical knowledge development; however, wide variation exists in the types of experiences children are provided at home, and research has begun to identify the key aspects of these experiences that are most important in promoting children's number development, or rather, defining what quality math activities entail. However, little is known about how common quality experiences are, and what contexts and parent and child variables are most optimal for promoting them.

Theoretical models explaining the predictors of young children's math experiences at home with parents have begun to emerge, and the literature has identified that parental beliefs and children's interest in math can both contribute to how frequently math experiences occur (Lukie et al., 2014; Skwarchuk et al., 2014). However, this body of research is limited in 2 important ways. First, while parental beliefs about math have been identified as important for school-aged children's academic outcomes, very little is still known about these beliefs and how they might predict the ways in which parents might provide math-related experiences for their preschool children, given that the literature, unguided by any particular theoretical framework, has been inconsistent in the parent surveys and belief items utilized. Secondly, research on the role of child interests in determining the nature of parent-child math-related interactions has been largely absent, as most work has focused solely on understanding the influences of the parent. This chapter provides a review of the research on children's early numerical knowledge development, the importance of preschool children's early home experiences with numbers that align with their math development, and introduces a theoretical framework to help assess the role that parental beliefs and children's interest in math play in explaining variation in the frequency and quality of children's math-related experiences. This work is especially critical as families are increasingly becoming expected to promote math development prior to school entry.

Theory and Research on Early Numerical Knowledge Development

Given the large accumulation of research on children's math development, scholars have begun to organize findings from the literature to develop theoretical frameworks for understanding this development, particularly in the domain of number (Geary, 2006; Siegler, 2016; Siegler & Lortie-Forgues, 2014). The integrative theory of numerical development by Siegler and colleagues is the first of such models, and depicts children's numerical knowledge development as a series of developmental processes or acquisitions. The first involves non-symbolic number knowledge and processing such representations, such as groups of visible objects or audible tones, in increasingly exact ways. This is typically assessed in terms of how children differentiate two groups of objects that differ in number, and given how early this skill can be detected, it is posited that the ability to think numerically is inherent in both humans and even some non-human animals (Feigenson et al., 2004). Our earliest numerical thinking is likely non-symbolic, meaning that this type of cognition precedes verbal language and written notation. This knowledge is important because it is believed to lay the groundwork for more advanced mathematics. The second acquisition involves linking non-symbolic representations of numerical magnitudes to symbolic ones. During the toddler years, children develop the precursors for symbolic understanding of numbers, such as learning the names and the written notation of the numbers in the number system. However, children develop more advanced understanding of these symbols as they integrate their non-symbolic number knowledge with their understanding of number symbols. More specifically, children must attach meaning to the number words and the written numerals by linking them with the specific quantities they represent. Once children have made these connections, they can use numbers in meaningful ways to solve more advanced number problems because they have an understanding of their underlying properties, and can thus understand transformations on them (e.g., addition and subtraction). In older children, final acquisitions include extending this magnitude understanding to numbers beyond 10, a wider range of integers (i.e., negative as well as positive numbers) and to include rational numbers such as fractions decimals. The next sections will illustrate the research methods and findings that provide the foundation of our understanding of children's developing number skills in accordance with the first two acquisitions associated with the integrative theory of numerical development, as they are the most relevant for children during the preschool years (Siegler & Lortie-Forgues, 2014).

Non-symbolic number knowledge. Within a few hours of birth, infants are already sensitive to number (e.g., Izard, Sann, Spelke, & Streri, 2009). In studies testing this early ability, babies are familiarized with a particular quantity of objects, and are then shown sets that differ in number. Increases in infants' looking times are measured to determine if they have perceived that the new set differs in size. Studies examining this paradigm have shown that infants' looking times increase when presented with different sets of small arrays, such as 1 object versus 3 objects (Antell & Keating, 1983; Starkey & Cooper, 1980; Starkey, Spelke, Elizabeth, & Gelman, 1983; Strauss & Curtis, 1981). Infants may also be able to perceive exact differences in small visual quantities whose set size has been transformed. When they watch transformations being made on small sets of objects, such as addition or subtraction of one object, they look longer when the results of the transformation are impossible (Wynn, 1992a). By the age of 12 months, children can correctly choose the larger of two containers that they have witnessed being filled with one versus two, and two versus three crackers (Feigenson, Carey, & Hauser, 2002), and make the correct number of individual reaches to retrieve each one from a container (Feigenson & Carey, 2003). In the preschool years, children are able to watch transformations of existing sets of small discs and reproduce the resulting set with their own set of discs (Jordan et al., 1992). These results suggest that as early as infancy, children are able to accurately recognize and perceive small sets of objects, and understand results of transformations of them.

Infants are thought to process large sets of objects differently than small sets. Sets of objects that are large (i.e., 4 or greater) are too numerous for children to track item-by-item, and thus, they are processed with far less precision. One perspective (Dehaene, 1997; Feigenson et al., 2004) suggests the existence of an analog magnitude system that allows us to *approximately* differentiate larger quantities in a ratio-dependent manner, with very large ratios required for young infants but smaller ratios required as children develop. This means that children cannot automatically detect the difference between sets of objects that are large and too closely similar in number, though precision does improve over time (e.g., Xu & Spelke, 2000). Initial research aiming to determine infants' abilities to process large numbers initially found that babies were completely unable to differentiate between large numbers at all, later research suggested that infants in these studies paid attention to properties of the stimuli other than number, such as the area taken up by the dots, the length the dots spanned, and how close together the dots were (Clearfield & Mix, 1999). Once researchers controlled for these confounding factors, infants were indeed found to be able to distinguish between large quantities, as long as they differed by sufficiently large enough ratios. For example, when presented with pairs of large quantities, newborns could distinguish groups of items differing by a ratio of 3:1 (Izard et al., 2009), 6-month-olds were able to distinguish sets differing by 2:1 (Xu, 2003), 9-month-olds required sets to differ only by 3:2 (Xu & Arriaga, 2007), and adults require a ratio of as little as 9:10. Research and the first acquisition of the integrative number theory suggest that in the preschool years, the ability to compare the magnitudes of large sets varies among children, and is still in the process of developing (Halberda & Feigenson, 2008; Siegler, in press).

Thus, as early as infancy, young children are able to distinguish among small numbers exactly, as well as some large numbers, albeit only approximately. It is not until children learn their culture's formal number system, if one exists for them, that they are able to quantify large numbers, and distinguish between them in exact ways (Feigenson et al., 2004).

Symbolic number knowledge. Beginning in the toddler years, children start to develop the foundations of abstract numerical cognition, namely through becoming familiar with number words and their written notations. The process of learning the symbolic system begins as rote memorization of the words and written representations, and becomes more meaningful as they are connected to the quantities they represent. Specifically, according to the integrative number theory, children begin to link their pre-existing non-symbolic number knowledge to the symbolic number representation in their culture's formal number system (Siegler & Lortie-Forgues, 2014). The next section summarizes the research that details this development.

Number words and written numerals. Beginning in the toddler years, children start to develop the foundations of abstract numerical cognition, namely through becoming familiar with number words. In fact, numbers are amongst children's first words as they are developing language (Fuson, 1988; Mix, 2009). Usually these words are used in reference to objects (e.g., *two cookies*), but are also as a part of the count sequence (Wynn, 1990). Researchers posit that young children first learn the number names in a rote manner, without attaching specific meaning to them, similar to memorizing a nursery rhyme (Briars & Siegler, 1984; Fuson, 1988). This is evidenced by toddlers' inability to distinguish between different count words when reciting the counting string, similar to the way young children blur the distinction between individual letters when reciting the alphabet (e.g., "elemenopee"; Fuson, 1988). Similarly, even when children use numbers in reference to objects, this is often done in a repetitive and context dependent way, such as "Two shoes" (Mix, 2009). Children also begin to recognize Arabic numerals as early as 18 months (Fuson, 1988). One case study demonstrated that some children even learn to name numerals before they have discussed number words in other contexts, such as in rote counting or in reference to objects (Mix, 2009), though children at this age do not know what these written symbols represent. By age 4, on average, about one quarter of children are able to name numbers up to 9 (Ginsburg & Baroody, 2003), and by age 5, the majority of preschoolers can name numerals up to 10 (Benoit et al., 2013).

Connecting symbolic and non-symbolic representations. Children move beyond rote understanding of the symbolic representations of number (e.g., names and written numerals) through linking them with their respective quantities of objects. When children connect non-symbolic quantities and number words is termed cardinal number knowledge (Gelman & Gallistel, 1978; Wynn, 1990), and when they do so with Arabic numerals, this is termed numeral knowledge (Purpura et al., 2013). The research on the development of both of these skills is presented next.

Cardinal number knowledge development. Once children have begun developing the words for numbers, they must then come to learn that these number words represent specific sets of objects. This knowledge is termed the cardinality principle, or *cardinal knowledge*. For number words, the process of making connections with arrays is disjointed at best, and typically initially involves children applying their rote understanding of the counting string directly to sets of objects. Children's early attempts at making these connections reflect their underdeveloped understanding, as they haphazardly sweep their pointer fingers across arrays of objects asynchronously with their count words (Gelman & Gallistel, 1978). Additionally, when 2- and even young 3-year-old children are asked to identify the set size of a group of objects after counting them, they often give wrong answers by incorrectly providing the next number word in a sequence, a random incorrect number, or begin to count the items again instead of providing the last count word they recited (Wynn, 1990), even after the set has been covered to prevent re-counting (Schaeffer et al., 1974). Although children do correctly repeat the last number recited in counting a set, they do not always conceptually understand its significance and may simply do it out of habit. For example, children will repeat the last word used in the count string whether the count is correct or incorrect (Fuson, 1988; Wynn, 1990, 1992b). Researchers determined that children's repetition of the last count word may thus not appropriately represent their cardinal knowledge, and using a different task, asked 2½-year-olds to give an experimenter a certain number of objects from a larger pile, called the Give-N task (Wynn, 1990). Instead of providing the correct number of objects, young children tended to grab nondescript handfuls of items regardless of the set size requested. Toddlers and young preschool children's emerging symbolic number skills may not yet be meaningful; however, knowledge of individual pieces of symbolic numerical information (e.g., number words, Arabic numerals) is still important to learn before connections can be made to non-symbolic quantities.

During the preschool years, children begin to demonstrate a stronger understanding of the meaning of numbers words. By age 3½, they begin to associate or map familiar number words onto their respective arrays of objects (e.g., they are able to label 3-candies with the number word *three*). Children make these mappings for smaller numbers before larger ones, and do so for number words first within their counting range (e.g. the number words with which they are familiar and can recite in order), and then outside of it (Fuson, 1988; Wynn, 1990). This occurs first for smaller numbers, likely because of children's inherent ability to perceive small sets exactly (Wynn, 1990, 1992b). However, it typically still takes about a year for children to produce each of the small numbers of objects 1 through 3 upon request. For example, there is a 6-month lag between when children can successfully produce "one" and "two" objects upon request, and about 9-months before children are able to do so for "three" (Wynn, 1992b). Nevertheless, by the time children are mid-way through their third year, they are able to more consistently produce the correct number of objects in their count list as compared to 2½-year-olds, and can often do so by grabbing the appropriate handful of objects almost automatically (Wynn, 1990). When children are able to produce their first item, they are called *one-knowers*, and subsequently *two-knowers* and *three-knowers* once they can produce 2 and 3 objects upon request, respectively (Le Corre et al., 2006).

After developing cardinal knowledge of small numbers, children do so for large number words (Le Corre et al., 2006). However, to do this successfully for more numerous quantities, children must adhere to specific counting rules, or principles to enumerate sets accurately (Gelman & Gallistel, 1978), rules which they often break earlier in development. In particular, they must understand the concept of *one-to-one correspondence*, or that they must attribute one number name per object when counting a set; that counting has a *stable order*, for which counting tags must maintain a particular sequence in how they are applied; and finally, the *cardinality* principle, which entails an understanding that the last number word recited when counting objects represents the set size. Typically after about a year of being *subset knowers*, by age 4½, children are able to successfully use counting to produce 4 objects upon request, and are considered cardinal-principle-knowers because they understand the purpose and meaning of counting (Le Corre & Carey, 2007). Once children have gained cardinal knowledge of their first large number (i.e., 4), they are able to link the names and quantities of the remaining numbers within their counting range seemingly all at once (Wynn, 1990). With this knowledge, children can solve verbal number-related story problems, such as addition. For example, once a child has a concept of the number *four*, she can correctly judge precisely whether and by how much it is larger than two, and understand transformations on the set (Krajewski & Schneider, 2009; Purpura et al., 2013).

Interestingly, the same ability that allows children to compare large sets of objects approximately may also help children develop their cardinal knowledge. One study suggests that the ability to identify the larger of two sets of quantities in the preschool years is associated with children's cardinal number knowledge, solving verbal arithmetic problems, and comparing the magnitudes of two number words at age 6 (Libertus, Feigenson, & Halberda, 2013).

Numeral knowledge development. Shortly after children have acquired the names of number words in connection with their respective quantities, they begin to develop a deeper understanding of written numerals. This entails linking their knowledge of numeral names and quantities to their written representations, and this skill seems to emerge beginning in children's fourth year (Benoit et al., 2013). Numeral knowledge in preschool is thought to be especially important for later mathematics achievement because children are tasked in kindergarten to use Arabic numerals to solve addition, subtraction, multiplication, and division problems (Common Core, 2015). One study even found that in assessing a range of preschool children's numerical skills, the ability to both name written numerals and associate them with their respective non-symbolic quantities was a significant mediator of the relation between children's informal number knowledge (e.g., non-symbolic and strictly verbal understanding of numbers) and their formal number knowledge, or children's ability to solve arithmetic problems using written numerals and formal notation such as the "plus" sign (Purpura, Baroody, & Lonigan, 2013). This suggests that numeral knowledge is critical for success in elementary school math.

The importance of developmentally appropriate math input. Siegler and his colleague (2014) emphasized as an extension to their theory that children's experiences that align with the theory's acquisitions can lead to more developed numerical magnitude knowledge (Siegler, in press). In accordance with the second acquisition, experiences that explicitly link symbolic and non-symbolic magnitude representations such as board games like chutes and ladders are suggested to provide opportunities for children to point to and count the spaces on the board (Laski & Siegler, 2014; Ramani & Siegler, 2008). Some researchers suggest that children's non-symbolic number knowledge can be enhanced through practice in discriminating groups of objects (Hyde, Khanum, & Spelke, 2014). Reference to such learning opportunities is presented only briefly as part of the theory, however, and more research unconnected to the math development body of research, to be reviewed in the following section, has expanded upon what children's actual home math experiences might look like, and how researchers have defined and studied them.

Summary. Young children vary substantially in their symbolic number knowledge (DeFlorio & Beliakoff, 2014; Starkey et al., 2004). However, theorists suggest that children's math knowledge develops by achieving particular milestones (Siegler & Lortie-Forgues, 2014). Most importantly in the preschool years, children make the transition from their purely non-symbolic understanding of quantities to representing non-symbolic quantities in symbolic and abstract ways by linking their knowledge of the two representations; however, this process varies from child to child. For example, while some four-year-olds can already make the connections between number words and their respective quantities for both small and large numbers, other children the same age are still attempting to link the number words and quantities for numbers "one" and "two" (e.g., Klibanoff et al., 2006). This wide variation has become a topic of particular interest for many researchers, especially the timing of this variation. Wide individual differences found *prior* to children's entry into formal schooling indicates that children's informal experiences outside of school are likely contributing to the wide individual differences in children's development of their symbolic number knowledge. In addition, it is posited that such experiences must be aligned with children's development to make the most optimal impact on their number learning (Siegler, in press). The following section provides empirical evidence suggesting that children's early mathematical experiences contributes to their number development.

Frequency of Early Math Experiences

Findings from a plethora of studies, typically involving middle- to high-income families, have revealed that young children whose parents, through surveys or interviews, self-report exposing their preschoolers and kindergarteners to more number-related experiences at home have more advanced concurrent and later number knowledge (Blevins-Knabe & Musun-Miller, 1996; LeFevre et al., 2009; Levine et al., 2010; Musun-Miller & Blevins-Knabe, 1998; Niklas & Schneider, 2014; Skwarchuk et al., 2014). These results are based upon questionnaire data in which parents of preschoolers and kindergarteners are asked to rate the frequency of occurrence of specific early home-based math activities thought to be beneficial for children's math learning. Specified in these items are the types of skills emphasized during a given experience, which typically include activities and direct efforts to foster learning of foundational concepts for preschool and kindergarten children such as the number names (e.g., during rote counting), their symbolic notation (e.g., recognizing Arabic numerals), and cardinality (e.g., counting objects and labeling a set's size). Additionally, items typically also include more advanced activities that involve using number magnitudes to solve more complex problems for these age groups, such as which number is larger during magnitude comparison, and the sum or difference of two numbers during simple arithmetic. The distinction between math content supported in these activities has been made in several studies (Blevins-Knabe & Musun-Miller, 1996; Ramani et al., 2015; Skwarchuk, 2009; Zippert & Ramani, 2016).

Additionally, the context or goal of the activity is sometimes made salient, such as whether the interaction is explicitly didactic in nature (i.e., where parents teach their children specific skills like arithmetic) or informal, such as during game play, cooking, and singing number songs (LeFevre et al., 2009). During informal math experiences, children learn about numbers through their participation in regularly occurring family activities, where number learning is an unintended outcome (LeFevre et al., 2009). This is theoretically considered to be the most common and beneficial type of early math learning experience for children (Rogoff, 1990; Vygotsky, 1978). Typically, parents report engaging most often in helping their children practice learning the number names and applying them to quantify sets of objects, but focus on little else. For example, parents of preschoolers report infrequently comparing numbers and solving arithmetic problems with their children, though they frequently report engaging in rote practice of numbers, such as counting out loud and naming Arabic numerals at home (Blevins-Knabe & Musun-Miller, 1996; Skwarchuk, 2009).

Observations of parent-child number talk mirror the findings of studies utilizing parental report. In the first study of its kind, Levine et al. (2010) utilized naturalistic observation methodology to obtain a more objective and realistic measure of the number talk to which young children were exposed in their homes from infancy to toddlerhood. A representative sample of 44 Chicago-resident primary caregivers and their children were visited in their homes every 4 months for 90 minutes each when children were between 14- and 30-months-old, and were audio-recorded while engaging in their everyday routines. Transcript analyses of their talk revealed that parents engaged in relatively little discussion of numbers 1-10 with their children; however, there was also substantial variation in the amount of talk parents produced. For example, parents spoke an average of only 90 number words across almost 8 hours of observations with their children, with some parents eliciting as little as four words, while others elicited over 250. These findings suggest that wide individual differences exist not only in children's numerical knowledge, but in their early home number-related experiences as well.

When parents in this study did discuss numbers, they did so in a range of informal contexts, such as book reading, playing with number toys, as well as during every day routines. Additionally, parents' number talk was typically in reference to small numbers (1 through 3), and most often involved labeling the set size of objects (50% of total number talk) and counting (about 32% of total number talk), while the little talk that remained (about 18%) involved naming written numerals, making number comparisons, and using number words in ambiguous situations (e.g., “high five” or “I’ll start from disc two”). Parents’ total number talk (total talk about numbers 1-10 summed into a composite) was a significant predictor of children’s cardinal knowledge as measured by their performance on the “What’s on this card?” task assessed at 46 months, which measured children’s ability to accurately match number words with the corresponding printed non-symbolic quantities of objects printed on cards. These relations were upheld after controlling for parents’ total talk (number and non-number-related talk) as well as SES (which was a composite of parent education and income). This was the first study to examine the prevalence of naturally occurring number-related experiences of young children in their homes, and revealed that there was much variation between families, and that talk about numbers, rather than simply more overall parental talk, contributed to the development of children’s cardinal number knowledge in the preschool years.

Quality of Early Math Experiences

While Levine et al. (2010) examined parents’ *total* number talk, more fine-grained analyses of these data by Gunderson and Levine (2011) revealed that certain types of parent number talk were more important, or higher in quality than others in predicting children’s cardinal number knowledge in the preschool years. To examine this, researchers divided the total parent number talk into discussion of visible objects of large and small set sizes, and number talk about small and large sets of non-visible objects, or of number words referencing no objects in particular (e.g., during rote counting or discussion of quantities that could not be observed, such as “we sang four songs’). Across all five sessions, parents most often discussed small quantities of visible objects with their children (45% of instances of number talk), followed by slightly fewer references to small objects that were not present, and minimal discussions of large sets, present or not. Of all the categories, parent number talk about objects that were both present and observable significantly predicted children's cardinal number knowledge of a range of numbers, whereas none of the other types of talk significantly predicted children's cardinal knowledge in preschool. The authors suggested that parents' use of number words to quantify visible sets of objects allowed children the opportunity to link number words with their corresponding quantities, thus contributing to their developing cardinal number knowledge. In contrast, talk about numbers without concrete referents, such as through reciting the count sequence out loud, or using number words that have nominal values (e.g., phone numbers and addresses) may not help children make connections between their non-symbolic and symbolic representations of number, because children are being exposed to only one type of number format.

Experimental research has revealed that number talk used in reference to present objects is indeed causally related to the development of children's cardinal number knowledge. In a recent study, Mix et al. (2012) recruited 60 3-½-year-old predominantly Caucasian middle-class children, and had experimenters engage the children once a week for 6 weeks in reading a picture book that contained numbers and corresponding pictures of objects ranging from 1-9 on the book's pages. While some children counted objects on the book pages with an experimenter, others only labeled those objects. Another group was asked to both label and count the sets of objects, and the last group alternated between labeling and counting objects with an experimenter. Results indicated that children who were exposed to consecutive counting and labeling of set sizes of the pictures in the book significantly improved in their cardinal number knowledge after only

3-weeks, whereas simply being exposed to either counting or labeling set sizes alone or in alternation did not even impact children's cardinal knowledge after 6 weeks. This finding illustrated the importance of using number words in connection with present sets of objects, and also suggested that it is especially important to reinforce the labeling of the set size of observable objects with counting of those objects for preschool children. This is because labeling and counting objects in succession doubly emphasizes the connection between symbolic number words and the arrays of objects they represent, and also provides meaning to children's initially rote understanding of counting, namely that number words are used in order to determine the set sizes of groups of objects.

Determining contexts conducive to quality number exploration. As a follow-up to this training study, Mix et al. (2012) examined the extent to which parents actually engaged their children in talk about numbers with present sets of objects during a number-related activity. To do so, they recruited fourteen White, middle-class parents and their 3-½-year-olds to the lab to read a book that had number words in the text and corresponding numbers of illustrations of babies engaging in various activities (e.g., clapping) as well as printed numerals on the pages. Excluding word-for-word reading of the text, parents' non-text elaborations were found to be predominantly non-number related topics (e.g., commenting on the types of things babies were doing, or labeling things appearing in the pictures that were non-numerical) than actual talk about numbers. When parents did speak about numbers, the majority of their talk regarded labeling the set size without making explicit connections to the illustrations ("oh look, 3 babies"), or asking their child "how many?" without further indicating that this request was in reference to the illustrations themselves. Parents spent only a small amount of time counting, or encouraging their child to count, the illustrated pictures, and spent even less time engaged in both counting *and* set size labeling with their children. Doing so would have further emphasized the connection between the symbolic number words and the non-symbolic arrays. Further, as the researchers suggested, counting and labeling would communicate that the purpose of counting was to specifically identify the set size. Parents and children may have engaged in counting as a part of a fun and enjoyable activity, rather than a goal driven action to determine how many objects there were, and this may have been because the overall context or goal of the activity to read the book, was not as conducive to in depth number exploration, as perhaps other activities would be. This suggests that parents may not emphasize the connection between symbolic and non-symbolic number information for their preschool children in the most optimal ways, at least in the context of reading a book. It was unclear from the study whether parents and children actually talked about the printed numerals on the pages, perhaps because the researchers were more interested in children's cardinal knowledge as opposed to numeral knowledge development. However, this study was important in determining what contexts may be less conducive to supporting the second acquisition, or the connection between children's non-symbolic and their symbolic number knowledge (Siegler & Lortie-Forgues, 2014).

To date, while, no other research has examined how parents specifically connect symbolic and non-symbolic representations of number for their children in activities other than reading a number book, research has identified that certain contexts may be more conducive to eliciting at least *more* parent number talk in general than others. In particular, studies examining free play with a specific set of toys between parents and preschoolers from a range of SES backgrounds have found that parents rarely reference symbolic number representations at all. Vandermaas-Peeler et al. (2009) recruited 37 families of 4-year-olds of both low and middle-high SES backgrounds into a lab setting and asked them first to read a storybook about shopping, and then to engage in free play with toy food, money, and a cash register. Instead of engaging in talk about exact numbers, parents talked more vaguely about the price of the produce, and how generally "expensive" the products were. The authors termed these types of conversations cultural exchanges because they are relevant to everyday life, and somewhat informative about relative quantity, though these statements do not demonstrate for the child the meaning and use of symbolic numbers. Similar results were found in a previous study of U.S. middle-income Caucasian parents and their 4-year olds, who were given toys that resembled a post-office theme such as a mailbox, a cash register, play money, envelopes, and stamps (Vandermaas-Peeler et al., 2007).

Number games as a context conducive to quality number talk. When parents and children are observed playing games with explicitly number-related rules and objectives, however, the frequency of their number talk tends to increase. For example, a study of play interactions between Head Start parents and children involving three number-related activities revealed that the number-related board game yielded over twice the amount of parent number talk (11% of total talk) than did the number-related book (3% of total talk) and puzzle (5% of total talk). This was likely because the number-related game, which was a modified version of Chutes and Ladders, was far more inherently number-relevant than the puzzle or the book (Ramani et al., 2015), both of which could be played with whether or not numbers were discussed extensively. For example, the game involved using a spinner to move game pieces across the board, thus parents and children were likely engaged in both identifying numbers on the spinner and talk about the number of spaces they and their children had to move.

Others have also found the context of number games to be especially beneficial for promoting parent number talk. For example, during play with another modified version of "Chutes and Ladders" with North American Caucasian middle-class mothers and older preschoolers (age 5), the use of dice provided opportunities for children and parents to identify the number of dots on the face of dice through counting or subitizing (i.e., judging set size without counting), engage in adding simple sums (i.e., to determine the total number of spaces to move on the two dice) and also use the number of dots displayed on the dice to determine the number of spaces to move on the board game (Bjorklund et al., 2004). Another board game used in the early math literature with younger preschoolers, "The Ladybug Game," involves cards with numbers and corresponding sets of pictures of objects instead of dice to indicate the number of spaces players should move across the board. Vandermaas-Peeler et al. (2012) found that their sample of 28 middle- to high-income Caucasian mothers and their 4-year-olds who played this game 3 times over the course of 3 weeks engaged frequently in counting objects and naming written numerals, as per transcribed audio recordings of their talk. Further, engaging in this number game play benefited children's abilities to complete their game turns using numbers more on their own and in more accurate ways, as their errors and the need for more parent guidance over the course of the 3 weeks significantly decreased. This suggests game play to be both rich in number talk and a context that promotes children's learning of numbers.

Parent report studies also show that the context of playing number games is especially strongly associated with children's number development. For example, one study of 122 Canadian parents and their children aged 5, 6, and 7 found that parents' reports of number-related game play (e.g., playing card games or board games with dice or spinners) with their preschooler predicted children's general concurrent number knowledge, while reports of engagement in other informal number-related experiences (e.g., cooking, reading number storybooks) and teaching number skills in direct instruction contexts were not significantly associated with children's general number knowledge, as measured by a standardized early numeracy assessment (LeFevre et al., 2009). Two recent studies have shown that parental reports of playing number games contribute to more specific measures of children's concurrent number knowledge. Parents' (183 Canadian middle-income parents of 5- and 6-year-olds) reports of playing number games with their children such as monopoly, dominoes, and go fish cards predicted their children's non-symbolic arithmetic problem-solving abilities (Skwarchuk et al., 2014). Further, in a study of a diverse sample of German parents of preschoolers, parents' reports of playing number games with their children right before they began kindergarten predicted children's number knowledge (especially when those games involved dice) at the end of their kindergarten year, which included rote counting, written numeral naming, cardinality, and non-symbolic quantity comparison, as well as children's numeral knowledge at the end of first grade (Niklas & Schneider, 2014). Together these studies suggest that parent-child game play to be an especially beneficial context for children to improve in their numerical knowledge; however, neither observational research nor parent report studies have thus far provided insight whether parents use games as an opportunity to make connections between symbolic and non-symbolic number representations for their children.

Theory on Parental Beliefs and Behaviors

Until recently, no theoretical framework has been utilized to guide the study of young children's number learning, though contemporary theorists have synthesized the literature on number development, and proposed recommendations as to how this learning should be supported (Siegler, in press). In line with these recommendations, observational and parent survey research has suggested that certain contexts may be most optimal for encouraging quality number experiences for children. However, given that variation inevitably exists in parenting behaviors regardless of context, research has also been conducted to better understand the factors that affect this variation in children's early math-related learning experiences (Anders et al., 2012; Niklas & Schneider, 2014; Skwarchuk et al., 2014). Findings suggest that parental beliefs may be one such factor, and parents have been documented to hold varying beliefs about math in terms of their conceptions about their own abilities, the abilities and interests of their children, as well as their beliefs about the importance of math in their and their children's lives (e.g., Cannon & Ginsburg, 2008). When parents hold negative beliefs about their abilities, this can have consequences for how efficacious they feel introducing math concepts to their children, and ultimately in terms of the quality of such interactions with their children. When parents devalue math's importance in their own or their child's lives, they may choose to emphasize learning in other domains, such as literacy instead of supporting children's math development. Ultimately, parental beliefs about math may be especially important to consider in determining both the frequency and the quality of the math experiences they share with their children (e.g., Cannon & Ginsburg, 2008; NRC, 2009). Little research has examined these associations in the domain of number specifically; however, the seminal work that that has been done to develop theory and empirical work associated with parents' beliefs and behaviors will be reviewed in the following section to help frame the inclusion of parental beliefs for the current study.

Much of the research on the importance of parental beliefs in understanding children's cognitive skills began in the 1980's and 1990's. The publishing of Sigel's (1985) first edited volume on parental belief systems represented an important shift in psychological research from a predominant focus on observable parent behaviors to a concentration on how their beliefs about children could influence their overt actions of childrearing. He asserted that parental beliefs were especially worthy of study because they are based upon experience, reality and truth, and are vital to understanding the specific nature of their interactions with their children, and subsequently their child's development. Sigel (1985) defined beliefs as knowledge that a person has about an object, and then extended this definition to apply to parents specifically through an analysis of survey items administered to families of preschool-aged children. Specifically, he suggested a range of possible types of parental beliefs as relevant for parents, including those about the origins of development, such as how nature and nurture affect children's learning and growth; the idea of readiness, or the requirement that certain skills be in place for children to accomplish specific developmental tasks; how best to structure children's environments to promote healthy development; and how to motivate their children in various ways. Sigel (1985) suggested that beliefs affect behaviors only when certain factors are also considered. In particular, he explained that parents must have intentionality, or the willingness and ability to engage in the act associated with one's belief; they must have positive feelings or attitude about the action; and finally, they must believe the action is important or has value in being carried out. Finally, Sigel (1985) incorporated these concepts into a preliminary theoretical model of the belief-behavior connection as they apply to parent-child relationships.

Sigel's model is based upon the work he conducted examining how preschool children's cognitive development may be influenced by how parents believe children learn and develop, and how parents act upon these beliefs to promote their children's learning (McGillicuddy-DeLisi, 1982, 1985; Sigel, McGillicuddy-DeLisi, & Johnson, 1980). More specifically, Sigel and his colleagues examined how parents' beliefs about development, or their adoption of a constructivist perspective on children's learning, affected their children's representational competence, or their ability to use symbols to solve problems through the distance from their child during joint problem solving, in lieu of providing direct guidance. Their research somewhat supported their hypotheses in that on average, parents' beliefs positively predicted their distancing strategies.

In later work, Sigel & McGillicuddy-Delisi (2002) developed a more updated model, called dynamic belief systems, guided by Bronfenbrenner (1979). This theory depicted parental beliefs as encompassing multiple levels and domains that differ in the proximity in which they affect the child, with higher levels being more distal. The first level includes parents' general beliefs, or worldviews. Level 2 regards parents' specific beliefs about particular content areas (e.g., cognitive development and social skills). The third level includes specific beliefs within different domains. In the cognitive realm, parents' beliefs include their conceptions of children's thinking, planning and reasoning, and in the social domain include getting along with others at home and school and appropriate social behaviors. The fourth level entails how parents believe they should act upon their beliefs, such as parental guidance provided, or toys they provide to promote certain skills in their children. Finally, the fifth level involves acting upon these beliefs accordingly.

McGillicudy-Delisi (1985) expanded upon the ways in which beliefs are reflected in parents' actions. She first acknowledged and affirmed that parental beliefs motivate parenting behaviors during one-on-one interpersonal interactions with their children. This link can be measured in terms of the association between parenting beliefs and the levels of guidance parents provide their children during joint tasks, and can be measured via analysis of observations of parent-child interactions, such as during free play or semi-structured activities. This connection between parental beliefs and their behaviors has important implications for the quality of children's learning experiences they share with their parents. She also suggested that parents' beliefs are reflected more generally in how they structure the home environment. Examples include the toys parents buy for their children and the activities they provide. It was suggested that evidence for this link can be examined in terms of the association between parental beliefs and questionnaires or interviews about the home environment, and recommended that data obtained from parent surveys serves as an important supplement to observations of parent-child interactions. The use of both of these methods within a single study was suggested to be especially important in addressing the measurement problems of both approaches, namely that neither captures a child's vast history of interactions with her parents. However, McGillicuddy-DeLisi's work largely employs observational methods, implying that direct associations between parental beliefs and children's outcomes are due to the mediating role of variation in the home learning environment (1982, 1985; Sigel, McGillicuddy-DeLisi, & Johnson, 1980).

Eccles and her colleagues (1983) also considered parental beliefs to be an important source of their socialization practices; however, these theorists focused on how beliefs are important in predicting school-aged children's development, specifically in terms of their academic outcomes in the domain of math. Most relevant to the current study, they examined the effects of parents' beliefs and socialization practices on children's math achievement behaviors (Eccles et al., 1983). Similarly to Sigel's dynamic belief systems model, Eccles and colleagues posit that parents hold both general beliefs, or world views, as well as beliefs more specific to their own children, and suggest that these beliefs affect parents' socialization practices at different levels.

The following sections include descriptions of each of the constructs in the parent socialization model (Jacobs et al., 2005) as they were initially conceptualized for the math achievement of school-aged children, as well as suggestions on how the model can be used to understand the math development of children of the preschool age.

Parent characteristics. The demographic background of parents is posited in this model to play an important role in predicting the nature of their beliefs and behaviors, and children's academic achievement outcomes as well (Jacobs et al., 2005). Parent characteristics are considered to include factors such as education, income, ethnicity, and the characteristics of one's community. Much of the research on children's math development, especially for preschool children, has focused on differences in development by SES (income and education) as opposed to ethnicity and culture (e.g., Klein & Starkey, 1995; Saxe et al., 1987; Starkey et al., 2004); however, overall, there is little research on how these factors affect parenting behaviors in regards to teaching their children about math.

In regards to community characteristics, U.S. families live in a society in which beliefs about math vary widely; however it is not uncommon for adults to express negative views towards math regarding their competence, and devalue its importance for daily life (NAEP, 2013; NRC, 2009; OECD, 2012). Commonly negative views shared by those around them are thought to inform adults' general math-related beliefs (beliefs about their own ability and interest in math) as well as their value of math more generally, or even how they think about math in reference to their own children (e.g., whether their children like math and are competent in it). Importantly, theorists posit that beliefs commonly held in one's community are thought to impact one's general beliefs as well as their behaviors.

Child characteristics. It is posited by Jacobs et al. (2005) that child characteristics are especially important, as they serve as the basis for parents' child-specific beliefs and behaviors, and are posited to moderate the pathway between parental beliefs and parents' academic socialization of their children. A few child characteristics typically studied associated with this model include the sex of the child, children's past performance in school, and their interest in a given domain (e.g., math), which is a subcomponent of what researchers associated with the model call subjective task values (Eccles et al., 1983). School-aged children's past performance is typically measured as grades on assignments, or end of the year report card marks (Parsons et al., 1982), and their grades in school are thought to influence how parents perceive their children's abilities in different domains, and may ultimately influence how they support their children's math development (i.e., providing help with homework, seek tutoring). Because parents of preschoolers do not yet receive formal feedback on their children's math skills, this variable is not relevant for the current study. School-aged children's math interest has typically been assessed via children's self reports of their preferences for math as a school subject or through parent or teacher reports of children's domain specific interests (Wigfield et al., 1997). Further, parent- and teacher report methods have been quite successful at measuring this construct in areas such as math for young elementary school students (e.g., Wigfield et al., 1997). In early childhood, interests can be measured using observational methods of child engagement, attention, and affect (e.g., Deckner et al., 2006), or parental reports of their children's preferences for relevant activities (Lukie et al., 2014).

General beliefs and behaviors. Parents' general beliefs include the extent to which they value math as relevant in their lives, how much they personally enjoy it, and perceptions of their own competence in the subject (Eccles et al., 1983; Jacobs et al., 2005). These beliefs are directly expressed via parents' math-related routines and activities in which they engage apart from their children. Thus, in the theoretical framework (Jacobs et al., 2005), general beliefs and behaviors (i.e., personal math activity involvement) are modeled to be a single unified construct. These routines are particularly important because children can perceive and internalize parents' personal values based upon how much time and resources they see their parents dedicate to math-related hobbies and other activities, and in terms of the enthusiasm they express regarding the subject. In addition, parents' involvement in personal routines provides opportunities for children to model their parents' behaviors, and ultimately learn from them (Bandura & Walters, 1963). Parents who do not value math, or who dislike it, may refrain from providing math-related guided learning opportunities for their children.

Parents who value and positively evaluate their own abilities in math, and who subsequently hold math-relevant careers are likely to be positive role models and resources for their school-aged children. For example, school-aged children may identify with parents and wish to align their values and career paths with them if they believe their parents are interested in and value math (Eccles, 1993). In addition, parents who see math as important and valuable in their daily lives may use it often to solve problems, and ultimately provide children with observational learning opportunities. In a restaurant, a parent who is confident in their math skills could show their school-aged children how to calculate the tip, while a parent who is more insecure may avoid this situation entirely.

Parent modeling of math may be especially important for preschool children to learn early number concepts (Skwarchuk et al., 2014). For example, children can learn the names of numbers from overhearing parents' conversations about them at home, such as in the context of cooking or discussing finances. Fuson (1988) suggested that number permeates all aspects of our daily lives, and thus, many opportunities exist in the home and elsewhere for children to hear and observe their parents and other adults using numbers to solve daily problems and make decisions. At the same time, Fuson (1988) suggested that hearing parents speak about numbers might provide children with conflicting information. For example, children may not understand how numbers used in different contexts vary in meaning (e.g., numbers referenced as part of a phone number hold different meanings than number words used in reference to actual quantities). When numbers are used both in reference to quantity as well as in other contexts, this may confuse children, especially in regards to developing the link between number words and their respective quantities. However, in line with this theory, math development researchers in the past have posited that young children may develop their initial rote number knowledge by watching and imitating their parents engaging in counting and labeling sets of objects (Mix, 2009), and naming written numerals, since their knowledge in toddlerhood appears scripted and inflexible (e.g., Briars & Siegler, 1984). Thus, observational learning may be a more complicated process for young children than school-aged children, albeit possible.

Child-specific beliefs. In the parent socialization model, parents' are posited to hold math-related beliefs regarding their own children specifically (Jacobs et al., 2005). Relevant beliefs that have been studied include parents' perceptions of their children's abilities in math (Eccles et al., 1983) as well as beliefs about children's interests, expectations for what children will be learning in math once they enter school and the value of math for their own child. Parents who perceive their children as highly capable and interested in an area may provide their children with toys and activities thought to further foster their math development and further pique their interests (Eccles et al., 1983). Further, parents' child-specific beliefs are likely to be extremely important in understanding the nature of interactions between parents and their children. For example, parents who feel their children are interested in math may provide them with further encouragement to engage in math-related activities. Research in the early math domain suggests that parents may cater the math experiences of their children according to what they believe their children are capable of accomplishing. For example, parents of preschool children report frequently engaging their children in rote exercises such as reciting the count sequence or naming written numerals, but report infrequently engaging them in more advanced operations like adding numbers together (e.g., Skwarchuk, 2009). This could be because they do not think the latter skills are interesting or feasible for their preschool children to learn before school, but believe that counting and naming numbers are both important and feasible for young children to learn. Parents of young children, as opposed to parents of school-aged children, likely do not yet receive objective feedback and evaluations of their children's development in math, thus they must make assumptions based upon their own reasoning about what math concepts their children are able to learn, and this will likely vary significantly from parent to parent. In addition, a parent who thinks her child is not very interested in math may be less likely to talk about numbers with her child during their everyday interactions, where young children's number learning occurs most (LeFevre et al., 2009). Similarly, if parents do not expect that their children will need to learn about math in the near future, they may not be concerned with supporting their children's development in this area during the preschool years (Musun-Miller, & Blevins-Knabe, 2000). Thus, parents' child-specific beliefs are likely important in understanding the ways in which parents engage their preschool children in number exploration at home.

Parent-specific behaviors. Parent-specific behaviors involve the types of math-related experiences they directly and explicitly provide for their children (Jacobs et al., 2005). Ultimately, these experiences are thought to contribute to children's academic development because they provide direct instruction in a math concept, or provide children with firsthand parent-guided experiences during ongoing activities that informally facilitate their learning. For school-aged children, these activities may involve parents' homework-helping behaviors, seeking a tutor for their children, enrolling their children in extracurricular activities that foster math development and including them in cultural events and daily household activities that involve the practical application of math (Jacobs et al., 2005).

Parent-specific behaviors directed towards their younger children may involve spending more time engaged in one-on-one interactions with parents than older children. Thus, as previously discussed in the early math experiences section, these activities include informal joint activities where the goal may not be to directly teach their children about math, though math learning may result from participating in the activity. For example, parents may engage their children in informal math-related games (Bjorklund et al., 2004), read books about numbers (Bjorklund et al., 2004), sing counting songs, or discuss quantities while cooking or shopping at the grocery store (Vandermaas-Peeler et al., 2007; Vandermaas-Peeler et al., 2012). These activities involve applying math concepts to different situations, and thus children can learn about math by simply participating in them. Parents may also directly engage their young child in formal learning opportunities in which the direct goal of the interaction is to teach them about math, such as learning the names of numbers or doing simple arithmetic using a more direct instruction approach, such as with flashcards (LeFevre et al., 2009); however, the former types of activities are thought to be far more beneficial from a theoretical standpoint (e.g., Rogoff, 1990).

The next section will briefly summarize the math-related literature assessing parents' general beliefs as previously defined.

Parents' General Beliefs and Behaviors

General beliefs and behaviors include parents' math ability beliefs, values, and involvement in their own math activities in the home, and have been commonly considered in research aimed at building home numeracy models (e.g., Anders et al., 2012; Niklas & Schneider, 2014; Skwarchuk et al., 2014). These beliefs are especially important as they are thought to be directly tied to the math-related experiences parents provide their young children. The next section presents findings from research examining the relevant general beliefs of parents of young children.

Math ability beliefs, values, and involvement in activities. Survey research of parents of preschoolers of middle- to upper income backgrounds range in their beliefs about being good at math themselves, though on average, tend to report only low- to moderate levels of enjoyment of math when they were in school (Skwarchuk, 2009) and currently (Skwarchuk et al., 2014; Sonnenschein et al., 2012). Similarly, they report believing that they are more skilled (Skwarchuk, 2009; Vandermaas-Peeler & Pittard, 2014) and interested in literacy than math (Blevins-Knabe, Austin, Musun, Eddy, & Jones, 2000; Skwarchuk et al., 2014). Similarly, in interviews with U.S. middle- and low-income White and Latina mothers indicated that both groups reported being more interested in and able to teach their preschool children language than mathematics at home (Cannon & Ginsburg, 2008). Parents who hold these negative beliefs may engage less frequently in math-related activities with their children. For example, while Canadian parents from varied SES backgrounds ranged in their tendencies to avoid math-related situations, such as calculating the bill at a restaurant, they did show a tendency on average to endorse these statements (Skwarchuk et al., 2014).

Parents have also reported on their past and present math-related experiences and preferences. One study more broadly examining the social context of young children's math development found that middle class and low-income U.S. mothers of 4-year-olds reported that they were highly interested in number-related activities (Saxe et al., 1987). The extent to which parents' interest in math is associated with the frequency and quality of parent-child math experiences is not yet known.

Parents' Child-Specific Beliefs

Parents' specific beliefs about their children have received less attention in the literature on early math development, however research has somewhat addressed beliefs related to parents' value of math for their children, expectations of their children's later math learning in school, and perceptions of their children's abilities and interests in math.

Ability beliefs. Parents' beliefs about their children's numerical abilities are likely important to consider in understanding parent-child number-related interactions, though little work has been done exploring the math ability beliefs of parents of young children. The two studies conducted in this area suggest that parents of preschoolers both from Canada and the UK are knowledgeable of their children's rote counting skills, but to a much lesser extent their children's cardinality knowledge. English-speaking Canadian parents of 3- to 5-year-old preschoolers (n = 38) from middle- to high- income backgrounds were asked in a survey of home numeracy and literacy practices to indicate the highest number to which their preschooler could count (LeFevre et al., 2002). In comparison to researchers' direct assessments of the child's actual counting skills, while a small number of parents grossly overestimated how many number names their children knew (e.g., children could count to 60, though parents believed they could count almost twice as high), the majority of parents' reports were quite accurate (differing only by an average of 3 numbers). This suggests that at least in some areas of early numerical development, parents are aware of how their children's number abilities are developing, and parents may be able to use this knowledge to scaffold later learning.

Another investigation of parents' beliefs about their children's numerical abilities confirmed the aforementioned results. Fluck, Linnell, and Holgate (2005) surveyed 35 middle- and working class parents of 3- and 4-year-old preschoolers from Southern England about their children's counting abilities, and their answers were also compared to the experimenters' direct assessments of children's number knowledge (Fluck, Linnell, & Holgate, 2005). They found that parents' estimates of the highest number to which their children could count was also comparable to children's actual rote counting skills (parents' estimates on average differed by 5 count words in comparison to children's counts); however, when researchers asked parents to estimate their child's cardinal number knowledge, parents were found to grossly overestimate their children's number knowledge in this area. Specifically, the majority of parents believed that their children were able to provide 5 items out of a pile of 10 upon request, though very few children could actually do so. Parents had similar misconceptions about their children's interpretation of the last word reached in the count sequence as representing the set size of a group of objects counted. This suggests that parents of preschoolers seem to be familiar with their children's counting abilities in the rote and conventional sense (parents probably know that counting is age appropriate for preschoolers), but may have misconceptions about children's deeper number knowledge, such as cardinal number knowledge. The researchers suggested that this could have consequences for how parents support their preschoolers' number development at home. For example, if parents think their children already have a developed knowledge of cardinality, they may be less likely to recognize the need to reinforce the connections between the number words and objects, as is suggested by at least one study (Mix et al., 2012). Thus, while parents are accurate in their ability beliefs specific to certain types of their children's number skills, they hold misconceptions of other skills, and this has implications for how parents support their children's development at home.

There is some evidence to suggest that parents are vaguely aware of other math skills their young children are developing. Semi-structured interviews with 37 Caucasian and Latina middle- and low-SES mothers of preschoolers about their children's language and math development revealed that parents could recall instances in which their children engaged in numerical cognition beyond counting (Cannon & Ginsburg, 2008). For example, mothers reported previously observing their children engaging in simple arithmetic, magnitude comparison, as well as exploration of spatial concepts; however, mothers did admit being surprised at seeing their children engage in such advanced number cognition, (Cannon & Ginsburg, 2008). This again stresses that parents' understandings of their children's math development in the early years are quite narrow and often overinflated (Wigfield et al., 2006).

Expectations for child's later math learning. Similar to their limited understanding of children's current number skills, parents of preschoolers in the U.S. may also be less clear about what types of number-related skills they should anticipate their children developing in the future. When U.S. middle- and low-income mothers of preschoolers were interviewed about their beliefs about early math and literacy development, the majority admitted having little clear expectations or goals for their child's mathematics learning at all, though they did cite goals and expectations for their child's early language learning (Cannon & Ginsburg, 2008). Expectations are especially important for parents when setting goals for their children and challenging them in certain situations to develop their knowledge. If parents do not have any math goals in mind, or those that are limited, this may negatively affect how parents incorporate number into interactions that would develop children's foundational math knowledge.

Value of math. As parents are limited in the time they have available to interact with their children, values are especially important in understanding whether certain interactions, specifically joint parent-child number-related activity engagement, happen at all. On average, in comparison to other domains, parents of preschoolers report having low priorities for children's early math development. Specifically, parents from both middle- and low-SES backgrounds interviewed about their children's early math and literacy development tended to regard learning literacy skills as more important for their child than learning math skills in the preschool years (Cannon & Ginsburg, 2008). Parents from diverse SES backgrounds also tend to consider the development of social skills, such as getting along with others, as more important than their children's math skills in the years before formal school (Rescorla, 1990). This could be because it is not entirely common knowledge that children are able to learn academic content before formal school, or that such content is the responsibility of the schoolteacher. Social skills, on the other hand, are largely understood to be the responsibility of the parent, at least in the early years, and are perhaps more intuitive to teach. It has been documented that parents often lack goals for their children's math development (Cannon & Ginsburg, 2008), thus they are less likely to support this development at home. Thus, with parents' limited time and knowledge of how to support young children's school readiness, they are likely to choose to engage children in literacy practices, or work on their children's social skills instead of engaging in math activities, though parental involvement tends to decline once children enter formal schooling in the U.S.

Regarding more specific number skills, U.S. and Canadian parents from a range of SES backgrounds report preferring that their preschoolers acquire basic, rather than advanced number skills before starting school. For example, parents report identifying the ability to count to 10 and recognize printed numerals as important for their children to acquire before starting school (Belfield & Garcia, 2014; LeFevre et al., 2010; 2002), while less importance is typically placed on more advanced number skills, such as knowing simple sums (Belfield & Garcia, 2014; LeFevre et al., 2002; 2010; Rescorla, 1990; Saxe et al., 1987; Skwarchuk et al., 2014). U.S. parents demonstrate this trend not only on parent surveys, but also during open-ended interviews about children's early academic development (Barbarin et al., 2008). When middle class parents were asked to explain what school readiness skills were important for preschoolers to acquire before school, they overwhelmingly mentioned "nominal" skills, which the researchers defined as rote abilities such as naming numerals and counting. In contrast, parents hardly ever spontaneously recognized the importance of "relational" skills, which the researchers defined as more advanced number operations, such as numerical magnitude comparison and using numbers to solve problems such as arithmetic. Advanced number skills are important because they require a more conceptual understanding of numbers, and are more likely to provide children with meaningful math experiences. For example, when parents and children compare number names or numerals in terms of their magnitudes, this requires that they identify what each symbolic number representation stands for before meaningful comparisons could be made. Engaging in simple arithmetic also requires that connections be made between symbolic number representations and non-symbolic quantities in order to transform sets of objects. Engaging in rote counting and naming numerals, however, may not afford children opportunities to develop their numeral and cardinal number knowledge because in these activities, non-symbolic quantities are not involved.

Beliefs about child's interests. A smaller number of studies have examined parents' beliefs about their young children's interests in math and numbers. In one study, low-income mothers of kindergarteners who previously attended Head Start indicated in a parent survey that they believed their children were more interested in learning the alphabet, making friends, and doing sports than learning about numbers (Galper, Wigfield, & Seefeldt, 1997). A mix of low- and middle-income U.S. mothers seemed to also negatively view their preschool children's interest in math, as they reported feeling the need to make math more enjoyable for their preschool children, likely because they did not think their preschoolers were particularly interested in math (Cannon & Ginsburg, 2008).

While parents of young children may overall think their children are more interested in subjects other than math more broadly, they seem to recognize that some aspects of number may be enjoyable for children. In particular, both British and U.S.

parents of preschoolers reported that they believed that their children enjoyed counting (Fluck et al., 2005; Saxe et al., 1987). It may be that parents differentiate counting from conventional notions of mathematics, because counting can be considered an enjoyable and interesting activity for children as opposed to arithmetic, especially when done during a nursery rhyme, walking up stairs, or in other playful interactions between parents and children.

Associations Between Parents' General and Child-Specific Beliefs

In their model, Eccles and her colleagues posit that parents' general beliefs predict their child-specific beliefs (Eccles et al., 1983; Jacobs et al., 2005; Wigfield et al., 2006); however, research in early math development has provided little support for this pathway. Only one study to date has found that parents' general beliefs (i.e., a composite measure of how much parents liked and excelled in math in school, and reports of their competence in math) predicted parents' child-specific beliefs about their children's interest and ability in math (Huntsinger, Jose, & Fong-Ruey, 1997). In this study, 80 Chinese and European American parents of preschool- and kindergarten-aged children were surveyed assessing their beliefs about their children's interests in abilities in math in addition to parents' general beliefs about their math abilities and values. When parents' general beliefs and child-specific beliefs were entered into a path analysis, the general beliefs composite was a significant predictor of child-specific beliefs. Other research that has examined these relations has only shown moderate associations between individual items representing parents' general and child-specific beliefs. For example, Chinese and European American middle-income parents who emphasized the importance of daily living activities as the best way for their child to learn math (child specific belief about math learning) were only moderately more likely to report greater personal involvement in and enjoyment of math (Sonnenschein et al., 2012).

In some studies, evidence suggested the two types of beliefs are not significantly related. LeFevre et al. (2010), for example, found no significant correlation between their measures of child-specific (e.g., expectations of what their children would be learning in kindergarten) and general beliefs (such as personal attitudes about math such as their enjoyment of math and their perceptions of their math ability when they were in school) of Canadian parents of children about to start kindergarten. Further, Canadian parents' expectations for their kindergarten children's arithmetic skills by first grade were not significantly correlated with their general beliefs, which included their enjoyment of and past experiences with math (Skwarchuk et al., 2014). Thus, this suggests that in understanding young children's number development, parents' general beliefs about math may likely be considered separately from their child-specific beliefs in terms of their contributions to the early home numeracy environment. However, this research is limited in that parents' child-specific and general beliefs have not been assessed consistently enough to make conclusions about how these two types of cognitions are associated on a larger scale.

Parental Beliefs as Predictors of Children's Number Experiences

This section presents evidence suggesting that the pathways depicted in the parent socialization model are reflected in the literature on early number development. In particular, evidence will be presented suggesting that parents' beliefs about math influence the types of early math experiences they provide their children.

Evidence suggests that parents' general beliefs about math are associated with the frequency with which they engage their children at home in math activities. Evidence from a recent study indicates that the presence of math disability (thought to be associated with negative parental beliefs about math) in German families with children entering kindergarten predicted parental reports of at-home numeracy engagement (Niklas & Schneider, 2014). Similarly, a study of U.S. low-income families with preschoolers found that parents' who believed they played an important role in their children's math learning more often engaged their children in math activities at home (Klein et al., 2008). A study of Canadian parents showed that parents' general beliefs (i.e., beliefs about being good at math in school and their current enjoyment of math) predicted how often they reportedly taught their preschool children various math concepts, such as doing mental math, memorizing math facts, learning simple sums, and counting money (LeFevre et al., 2010). Together, these studies provide evidence, albeit limited, suggesting that parents' general beliefs predict the frequency of the early math experiences they provide their children.

Correlational evidence also exists suggesting a link between parents' child-specific and general math beliefs and the frequency of at-home math engagement. For example, 40 middle-income U.S. parents' general beliefs (i.e., reports of enjoyment of engaging in math activities with their preschooler at home) were found to be positively correlated with their reports of engaging their preschoolers in math-related experiences at home (Blevins-Knabe et al., 2000). Additionally, the importance parents reportedly placed on early math development for their kindergarteners' performance in first grade (child-specific belief) positively correlated with their reports of engaging their children in math activities such as rote counting, counting objects, and using number words (Musun-Miller & Blevins-Knabe, 1998). Similarly, 183 Canadian parents' reports of the importance they placed on their preschool children's attainment of certain math skills by the first grade (e.g., counting to 100, reading Arabic numerals to 100, doing arithmetic) positively predicted their reports of directly teaching their children a range of number skills, including learning simple sums, rote counting, and counting on their fingers (Skwarchuk et al., 2014).

The aforementioned studies provide evidence of a link between parents' beliefs and the frequency with which they engage their young children in math-related activities at home. However, they are limited in that the methods rely on parental report alone, which can only indicate the frequency as opposed to the quality of math activities. Further, parent report is not without bias, thus these data were collected via a convenient, but not a truly objective manner (response bias could have affected parents' survey item responses). Research examining the association between parents' beliefs and observations of parent-child number-related activities are needed to gain a better understanding of the association between parents' beliefs and their academic socializing behaviors towards their children.

Child Affect and Engagement

Research has suggested that children's affect and engagement, both of which are typically called child interest, can be detected surprisingly early in development in a variety of domains including math, and measured using a variety of methods. For example, Arnold and colleagues (2002) used a Likert scale with a series of 5 faces that ranged from having large smiles to large frowns to assess Head Start children's concurrent affect (referred to as interest) exhibited in reaction to numbers more generally, and in reaction to playing with specific math toys that for 10 minutes, such as sorting animals, felt numbers and a felt board, and a matching card game with numbers (the reliability of the measure was acceptable, with a Coefficient alpha = .62). However, as has been documented in past research (e.g., Wigfield et al., 1997), children's self-reports of this kind in specific domains at young ages may not be highly stable. Arnold et al. (2002) found that their child self-report affect towards math (termed interest) had a test-retest reliability of only .33 (when children self-reported about their affect towards numbers after only 6-weeks). Even in elementary school, self-reports of first-, second-, and fourth-grade children's interests in specific domains are low (alpha for math interest = .36), though reliability of theses children's self-reports of interest in specific domains such as math does improve with age (alpha values for math and other domains 2 and 3-years later ranged from .73 to .92; Wigfield et al., 1997).

While young children may not be able to reliably and consistently communicate their affect towards and engagement in math, such measures have been assessed more reliably via global ratings made by adults. Specifically, parents and other caregivers such as teachers have rated children's interests in various domains, including math. In the research conducted by Eccles and colleagues, survey items are administered to parents of school-aged children and teachers assessing how much their child likes a range of domains using a Likert scale ranging from 1 = *not at all*, to 7 = *very much* (Eccles et al., 1983). Research with parents of younger children typically also involves survey items with Likert scales, but asks about their children's affect towards specific activities, likely to make the questions more relatable and less abstract (e.g., Lukie, Skwarchuk, LeFevre,

& Sowinski, 2014). Not surprisingly, parent and teacher reports tend to have better reliability than child report (alphas range between .80 to .90; Lukie et al., 2014; Wigfield et al., 1997).

In addition, researchers have reliably measured young children's affect and engagement in math directly through short-term observations of children partaking in a particular math-related task. Measures aimed at assessing child interest typically include positive affect exhibited (Hidi, Renninger, & Krapp, 2004, Schiefele, 2001), and engagement measures such as sustained attention to the task (Krapp, 2002), and children's individual contributions, or their persistence at the task (Ainley, Hidi, & Berndorr, 2002). Children's affect towards and engagement in storybook reading as early as toddlerhood has been assessed in the area of language and literacy development, and have included measures of affect, and engagement (i.e., attention and participation) through applying behavioral codes at successive short time intervals (e.g., 30 seconds; Deckner et al., 2006; Malin et al., 2014).

Similar observational methods have been used to assess children's solitary free play with educational math-related toys. For example, Fisher et al. (2012) utilized a semi-structured task involving free play with modified playing cards with numbers and pictures of corresponding quantities after a brief presentation of their potential uses (i.e., pairing cards with numbers and cards with appropriate matching quantities of printed objects). The researchers assessed children's math engagement and affect through analyzing the videotaped play sessions for the total time children were involved in playing with the cards, a global rating of children's affect, called enjoyment, on a Likert scale from 1 = *did not enjoy at all*, to 7 = *complete enjoyment* (i.e., codes were based on engagement, eagerness, and positive affect while playing with the cards), and an addition al measure of engagement, called goal directed play, was rated on a Likert scale from 1 = *random approach to activity* to 7 = *consistently purposeful approach to activity* (e.g., whether children were doing mathematically meaningful things with the cards, such as grouping the cards with the numbers and pictures together).

Thus, overall several methodologies have been used to measure children's engagement and affect, all in efforts to assess early interests. Direct observations are the most preferred method of assessment because they are more objective than children's self-reports and reports by adults made about children. However, the ratings are typically focused on children's engagement in and affect towards a single or a limited number of activities. Thus, less is known about children's interests in math in a broad range of contexts.

Child Affect and Engagement as Moderators

Children's interest in math is especially important to consider given its implications for later math outcomes. For example, school-aged children's math interests have been found to be predictive of students' academic achievement in addition to the courses they select to take (Schiefele & Csikszentmihalyi, 1995). This is especially important in the context of STEM education, as math courses often act as gateways to more challenging math and science classes required to pursue STEM majors in college. Thus, interest in math could very well influence the extent to which careers students pursue are math-related. In the earlier years, preschool-children's affect towards and engagement in math predicts both their concurrent and later math knowledge, as assessed by a standardized assessment of early mathematics understandings 5-months later (Fisher et al., 2012). As such, interventions aimed at improving Head Start children's interest through playful math-related activities in the classroom have proven successful in increasing both children's math interests and their general math knowledge (Arnold et al., 2002). This research is limited, however, in that it does not provide insight into how such interventions affects children's math development. It is posited, however, that improving child engagement and interest in the math activities, makes the nature of the activities more meaningful, and thus maximizes the benefits they received from the experiences, such as math learning.

One study with preschool children suggests that child affect and engagement in math are tied to the frequency of the math-related experiences they share with their parents at home. Lukie and her colleagues (2014) surveyed 170 Canadian parents of preschool-aged children from a range of income backgrounds about the math-related activities they engaged in with their children at home, including advanced (e.g., arithmetic) and foundational (e.g., rote counting) numeracy activities. Regression analyses revealed that children's affect towards various activities measured via parental report (e.g., card and board games, reading, building, and printing letters and numbers) significantly and positively predicted the frequency of advanced math activities preschoolers shared with their parents at home, such that the more positive affect parents perceived their children to have towards the activities included in the parent report measure, the more frequent their parents reported engaging their children in math problem-solving activities at home. This suggests that parents who view their children as having more positive affect towards math and literacy activities are more likely to engage their children in these activities. However, additional work is needed to assess children's affect *and* engagement more objectively and in a wider range of contexts (e.g., with different math toys and during game as opposed to free play), the extent to which these measures are associated with both the frequency (parent report of at-home math activity) *and* quality (observations of parent-child number exploration) of children's math-related experiences with their parents during these activities. Research in the domain of literacy suggests a strong relation between child affect and engagement and quality of early reading experiences (observations of parents' metalingual talk to their children during book reading), suggesting that child affect and engagement in a number-related parent-child activity could also be associated with the quality of that experience in terms of number concepts explored, such that higher engagement and affect in the game could be associated with higher quality parent-child number-related experiences (Deckner et al., 2006; Malin et al., 2014). Further, little is known about how both parent- and child-factors may interact in these contexts.

Summary

Children transition during the preschool years from a purely non-symbolic conception of numbers to numerical cognition using abstract symbolic representations, and this development is critical for their later success in school (e.g., Duncan et al., 2007). Early math experiences with parents have been shown to contribute to children's math development (e.g., Skwarchuk et al., 2014), and even more fine-grained analyses have suggested that certain experiences are higher quality in comparison to others in promoting children's number development. In particular, experiences that reinforce the connection between symbolic and non-symbolic representations of number are theoretically and empirically shown to be critical in developing children's cardinal number knowledge (Mix et al., 2012; Siegler & Lorti-Forgues, 2014), and similar experiences of connecting written numerals to their respective quantities may also improve children's numeral knowledge as well.

However, it is unclear if parents actually support children's numeral and cardinal knowledge in these ways during typical interactions with their preschoolers at home. Parents' number-related input seems to be overall minimal at best (Levine et al., 2010), even in contexts that seem relevant for parents and children to explore numbers (e.g., Vandermaas-Peeler et al., 2007). Some number-related contexts, however, seem to be better at eliciting number-related talk between parents and children than others (Ramani et al., 2015). Number games may be particularly beneficial because they encourage parents and children to work directly with numbers to solve problems in fun and motivating contexts. A likely optimal number game for reinforcing children's cardinal and numeral knowledge development is card games. This is because in addition to being explicitly number-related, multiple representations of numbers are presented in clear ways that allows parents and children to talk about number names, connect the number name to the corresponding suit images (non-symbolic number representations), and identify the printed numerals on the cards in connection with the other representations. Research is needed, however, to determine if parents do support their children's cardinal and numeral knowledge in this type of context, and if so, parents more broadly could be encouraged to engage in card games to support children's number knowledge, just as they are encouraged to read books to support their young children's emerging language and literacy skills.

It is expected that variation in parent-preschooler exploration of quantity likely exists in even the most optimal contexts, and theory suggests that parental beliefs and child interest are likely important sources of influence on parents' academic socializing behaviors (Eccles et al., 1983; Jacobs et al., 2005; Wigfield et al., 2006), such as in the domain of math. Research on parents' child-specific beliefs suggest that many parents lack goals and expectations for what their preschooler should be learning in math, demonstrate having inaccurate understandings of their children's number development (e.g., especially about their children's cardinal number knowledge), and report that they believe their children, like themselves, are also less interested in math than in other areas, such as reading (e.g., Cannon & Ginsburg, 2008). These beliefs, if associated with the quality of the experiences parents provide their children, likely have consequences for children's number learning in their early years, and later in development, as they could negatively impact how often and in what ways parents engage their young children in number-related activities.

Parent survey research provides preliminary evidence that some parents' beliefs and children's affect towards and engagement in numbers during typical interactions are associated with their reports of their shared number learning experiences at home (e.g., Lukie et al., 2014; Skwarchuk et al., 2014); however, this evidence is limited in that parental beliefs have been examined inconsistently and preschool children's affect in math has largely been assessed via parental report. Research is thus needed in which a range of parental beliefs are included in parent surveys, so their relations can be assessed and compared to the frequency of at-home math activity engagement *as well as* the quality of these experiences, particularly their support of children's numeral and cardinal number knowledge, as this is currently entirely missing from this literature. Further, objective measures of preschoolers' math engagement and affect are sorely needed. Addressing these research gaps is important given the rising recognition of the quality of math-related input on children's math development (Gunderson & Levine, 2011; Mix et al., 2012), and the urgency to develop appropriate standards and curriculum to promote math readiness for young children in the U.S.

Chapter III: Methods

Participants

Participants were a sample of 121 parents and their 3- to 4-year-old children (i.e., 121 parent-child dyads; *n* = 242 participants total) were recruited from a children's museum in an urban city in a mid-Atlantic state. Twelve dyads (10% of sample) were eliminated from the final sample either because they did not participate in the interaction or the parent preferred not to be videotaped, 2 families (2%) were accidentally made aware of the study's focus on math and thus could not be included, 5 families (4%) involved guardians other than a parent, making them ineligible for retention in study, 4 dyads (3%) did not complete enough of the game (the minimum number of pairs used for retention was 4 for substantial analyses on the math exploration to be conducted), 2 children were not the appropriate age for the study (2%), 1 child (1%) demonstrated comprehension issues on the math assessment which would have hindered his or her participation in the game, and 1 parent (1%) participated with both of her children, and the older of the two (who participated after her younger sibling) was eliminated. The final sample included 94 parents and their 3- and 4-year-old children evenly divided by age (i.e., 94 parent-child dyads; *n* = 188 participants total; see Table 4 for more detailed demographic information). Gender of both participating parents and children were 60% and 55% female, respectively. Ethnicity of parents was 74% White, 11% African American, and 10% Asian, while children were reportedly 72% White, while 10% were multiple ethnicities. On average, parents held Bachelor's degrees, reported a household annual income of $76,000-$100,000, and on average *never* or *rarely ever* attended the children's museum where data were being collected; however, 31% of parents did report holding annual memberships at the museum. Anecdotally, parents said they visited the museum about once or twice a year; though this was not reflected in the rating scale used for the study.

Study Design

This study employed a multi-method design, using behavioral observation, parent survey, and direct child assessment methodologies. These methods were used to examine how parents of preschoolers guide their children's development of cardinal and numeral knowledge within the context of a meaningful math experience, examine how this exploration varies by parents' general and child-specific beliefs about math and children's interest and ability.

Procedure

On days when data were collected, families were approached in the museum and invited to participate (see Appendix A for the recruitment script). If they agreed, they were taken to a designated area in the museum blocked off for the study and given a consent form to complete (see Appendix B). After completing the consent form, parents filled out the parent survey, while the child met with the researcher for the math assessment. Verbal assent was obtained from the child before the math assessment began (see Appendix C). Then, the parent-child dyad took part in the interaction, which involved playing a card game together. The initial testing and survey administration lasted about 10-15 minutes, while the card game lasted around 5-10 minutes, so the total study time was between 20-25 minutes.

Parent-child interaction. The interaction was videotaped and involved parents and preschoolers playing a modified version of the popular card game *War*. In this game, an experimenter told parents and children to help Ernie and Cookie Monster, two stuffed Sesame Street characters sitting on the tabletop to follow the rules and play the card game. Pilot testing suggested that children were too distracted by winning to enjoy the game and play it correctly if they believed that they themselves were the recipient of the cards each turn. Each character was assigned a stack of 15 cards each, with values ranging from 1-10. The cards resemble typical poker cards in size (3.5 x 2.5 inches) and placement of the numbers (see Figure 2 for examples images of cards), but contained simple blue dots instead of suit images to make it easier for children to view the non-symbolic quantities on the cards. An image of Sesame Street characters was attached to the backs of the cards to make them more fun and engaging (see Figure 2 for example images of the back of the cards). After giving the instructions, the experimenter invited the parent-child dyad to begin the game, encouraging the parents to help their child as they normally would at home, and then left the immediate area while remaining accessible for parents to contact the researchers. The area where the interaction took place was closed off from other museum visitors. A camera placed approximately 3 feet away videotaped the parent-child interaction.

In the interaction, parents and children were told by the experimenter that they would be helping the characters play a card game, and were shown 2 example turns by the experimenter drawn from the two decks of 15 cards in front of the characters (see Appendix E for protocol and detailed instructions for playing the game). For each turn, both parents and children placed a single card from each character's pile next to each other, face up in the middle of the table. Parents and children then decided which card was more, and the character with the higher card received both cards in its own separate pile. Each subsequent turn involved the same procedures. Once parents and children drew all of the remaining 13 cards each, they finished the game and proceeded to determine which character won the game by comparing whose pile of captured cards was larger.

Behavioral observation coding. Parent-child interactions were coded to examine the types of behaviors in which parents and children engaged during game play. These included math codes, which captured math-relevant talk and gestures; guidance codes, which captured how parents assisted their children during each turn; and child-centered codes, which assessed children's attention, their participation or contributions during the game, and how they expressed interest while playing. In the following paragraphs, these codes are defined in more detail.

Quality of parent-child number experiences. Math codes were applied for every turn, which began when parents and children placed the characters' cards face up on the table, and ended when the cards were collected and given to one of the characters. Each turn was examined in terms of how parents and children talked about quantity in order to decide who was to collect the cards. This was divided into 2 steps: the first involved *quantity identification*, or identifying the quantities on the cards, and the second involved *quantity comparison,* which involved comparing the quantities on the cards to determine which is more.

Quantity identification. A behavioral coding system was adapted from Mix et al. (2012) to examine how parents identified quantities with their preschool children (see Table 1 for detailed definitions and examples), an experience that Siegler and Lortie-Forgues (2014) suggest is especially important for young children's numerical magnitude learning. One of the following codes (0-6) was applied in a mutually exclusive and hierarchical manner for each of the 26 cards played, in that higher-level behavior would be coded even if it co-occurred with lower-level behavior (see Table 1 for code definitions). Levels included 0) No explicit quantity reference, 1) Relative/approximate quantity identification, 2) Count dots only, 3) Counting the dots and labeling the final quantity with a number word, 4) Connecting the numeral with the dots, 5) Stating the number name without referencing the dots, and 6) Identifying the numeral without referring to the dots. Explicit cues were relied upon to make coding judgments, thus children may have used the numerals to identify the quantity, but if there were no accompanying gestures such as pointing, this was coded as Level 5. The codes are similar to those of Mix et al. (2012) in that they assess rote counting separately from labeling sets of objects, however this coding system further aims to specify whether families are labeling non-symbolic or symbolic quantities. However, the coding system by Mix et al. (2012) specified the order in which parents and children labeled and counted, while the system for the current study did not make this distinction.

Quantity comparison. Parents' and children's strategies for comparing the magnitudes of the quantities were also coded using a system developed for the current study, since no detailed examination of parent-child exploration of magnitude comparison has been conducted to date (see Table 2 for examples of codes). Coding categories ranged from levels 0 to 3 and include the following: Level 0 = no explicit comparison of magnitudes, Level 1 = comparing dot arrays, Level 2 = comparing quantities using number words, Level 3 = comparing printed numerals. These codes were applied in a mutually exclusive and hierarchical manner, in that higher-level behavior would be coded even if it co-occurred with lower level behavior. Explicit cues were relied upon to make coding judgments, thus children may have used the numerals to compare the quantities, but if there were no accompanying gestures such as pointing, this was coded as Level 2.

Guidance codes. The extent to which dyads' quantity exploration was guided by the parent was also coded during the interaction. For each of the three math codes applied for every turn (i.e., *quantity identification* of both cards and *quantity comparison* of each card pair), the guidance parents provided was analyzed using a coding system adapted from previous observational research on adult-child game play and problem solving during informal activities (Bjorklund et al., 2004; Ramani, Siegler, & Hitti, 2012; Wood & Middleton, 1975). These codes range from 0-5 representing the extent to which parents assisted and involved their child in completing each turn (see Table 3 for examples of codes). Guidance codes include the following: Level 0 = *no parental guidance*, Level 1 = g*eneral prompting*, Level 2 = *instruction/specific prompt*, Level 3 = *modeling or co-participation*, Level 4 = *explanation or correction*, Level 5 = *direct instruction*. The codes were applied in a mutually exclusive and hierarchical manner for each card/pair of cards compared such that parental guidance that was more direct was coded even if less direct prompts were also offered. Further, as was done by Ramani et al. (2012), code composites were created for medium-level guidance codes = level 1 and 2, and high-level guidance codes = Level 3-5, and these combined variables in addition to instances of no guidance were used in subsequent analyses.

Child affect and engagement. Children's affect and engagement in the card game was assessed using a behavioral coding scheme adapted from Deckner, Adamson, and

Bakeman (2006) and Malin, Cabrera, and Rowe (2014). Child engagement consisted of two components: child's *attention* and *active participation* during the card game. Paying attention during the card game was rated from 1 = *not paying attention* (child not attending to the game for the whole interval), to 5 = *constant paying attention* for the game. Active participation was rated based on behavioral or vocal contributions to the card game from 1 = *no participation* (child made no contributions during the interval), to 5 = *high participation* (more than 4 physical acts, gestures, or manipulations or verbal utterances during interval; e.g. pointing to cards or characters, flipping the cards, labeling quantities on the cards). Both of these codes were applied for every 30-second interval. Mean scores for attention and active participation were created across intervals. Scores for the attention and active participation scales were correlated strongly for the interactions (r = .58). Cronbach's alpha coefficient was .73.

While child affect is typically included in the aforementioned coding scheme (Deckner, et al., 2006; Malin, Cabrera, & Rowe, 2014), it could not be coded at each interval due to limited visibility of children's faces. Thus an additional coder watched each interaction entirely, and separately assigned global affect codes, which were rated using separate 7-point scales for positive and negative affect (Deater-Deckard, 2000; Deater-Deckard, Pylas, & Petrill, 1997). The codes were based on facial, vocal, and behavioral cues from 1 (no positive or negative affect), to 7, extremely positive or negative affect (child laughing or smiling frequently or crying or protesting during interval).

Reliability

For all of the behavioral codes, interobserver reliability was established among the independent observers. For the math and engagement codes, one observer (the first author) served as the master coder, and the other observers established reliability with a set of videos coded by the master coder. For the affect codes (positive and negative), one undergraduate research assistant served as the master coder, and a second research assistant established reliability with a set of videos coded by the student master coder. Each observer independently coded and achieved reliability with the master coders on 20% of the total number of dyadic interactions. For the *quantity identification,* and *quantity comparison*, *guided participation*, and child *attention* and *participation* codes, the interobserver reliability was calculated between each observer's and the master coders' codes using Cohen's Kappa for each coding scheme, which is a common method for calculating reliability, and is appropriate for use with coding systems that are categorical and mutually exclusive (Roberts, 2008). Kappa values between .6 and .8 indicate substantial reliability (Landis & Koch, 1977), and all reliabilities were within this range. For the *affect* codes (*positive* and *negative*), Cronbach's alpha was used to calculate reliability because they are measured on a continuous scale.

For the quantity identification codes, coder 1 had a Kappa coefficient of .89, and coder 2 achieved a reliability of .83 with the main coder. For magnitude comparison codes, coder 1's Kappa coefficient was .78 and coder 2's Kappa coefficient was .77. For guidance codes for quantity ID, coder 1 and 2 Kappa coefficient was .80, and quantity comparison guidance .81 and .71. The two coders for engagement (participation and attention) achieved a Kappa coefficient of .72 and .73 for the active participation and attention codes, respectively, on 20% of the videotaped interactions. Internal reliability was also calculated for the engagement codes (Cronbach's alpha = .73). Two coders of child affect achieved interrater reliability of α = .93 and .95 for negative and positive affect, respectively. Further, with a 1-point discrepancy allowance on these 7-point scales, reliability was perfect for both codes.

Measures

Demographic measures. Parents completed a demographic questionnaire attached to the initial consent form (see Appendix B). The questionnaire included questions pertaining to the age, race, and ethnicity of the child, language(s) spoken at home, household income, number and ages of additional children, education and race/ethnicity of mother and father, museum membership status, and frequency with which the family visited the museum.

Children's number knowledge measure. Children's number knowledge was assessed using select scales from a recently developed and validated instrument for preschool children called the Preschool Early Numeracy Scales, or PENS (Purpura & Lonigan, 2015; see Appendix G for details about the measure items). The portion of the measure selected contains 9 scales with 41 questions that focus on children's numerical knowledge, assessing children's strictly non-symbolic (dot array comparison) and symbolic representations of number (numeral naming and magnitude comparison), as well as children's associations between non-symbolic and symbolic representations (e.g., cardinality and numeral knowledge). One total score (max 46) was calculated to determine children's general numerical knowledge. Administration took approximately 10-15 minutes. The instrument showed good internal consistency with high reliabilities of subscales (Purpura & Lonigan, 2015) as well as concurrent and predictive validity.

Parent survey. Parents were administered a survey assessing their beliefs about math and the math activities they engage in with their children at home. Items were also included pertaining to literacy, social, and motor development, and non-math-related activities as distractors. In accordance with the parent socialization model (Jacobs et al., 2005; Wigfield et al., 2006), parents' general beliefs about math, in addition to their math-related child-specific beliefs were examined. Items were adapted from a questionnaire consisting of items previously developed by Eccles and colleagues and administered to parents of 5-year-old kindergarteners in the first wave of a longitudinal study called Childhood and Beyond (CAB; Eccles, Wigfield, & Blumenfeld, 1984) to understand the effects of academic socialization on children's self-perceptions, task values, and activity choices. While factor analyses on the CAB items regarding parents' beliefs about their children's abilities have not been performed, the items have strong psychometric properties when responses from parents of children in primary grades have been used as scales (e.g., internal consistency reliabilities range from Cronbach's $\alpha = .82$ to .92; Wigfield et al., 1997). Additional items were adapted from survey research examining young children's early mathematical experiences to examine parents' beliefs about math (LeFevre et al., 2010; LeFevre et al., 2009; Musun-Miller & Blevins-Knabe, 1998; Skwarchuk et al., 2014; Skwarchuk, 2009), also with high internal reliabilities (e.g., Cronbach's $\alpha = .89$). Additional items were adapted from a common early math measure (TEMA; Ginsburg & Baroody, 2003) of preschoolers to assess parental beliefs about their children's specific number skills, but will not be included in the current study (see Appendix D only math-related items used in the analyses). Parental belief items were transformed and consolidated via the data reduction technique principal components analysis, and standardized regression factor scores were saved and used in subsequent analyses.

General beliefs. General parental beliefs included parents' conceptions about the role of the parent, school, and child's ability in children's learning, the extent to which they value certain domains, and parent's perceptions of their own ability in math. Items, all of which had 7-point scales, included 2 questions about parents' personal involvement in math-related activities (i.e., math and finances), 2 questions about their own ability (in school and presently), 1 question about their value of math, and 1 question about their role in promoting their children's math development. Five other questions were included in the survey regarding parents' perceptions of their role in children's development more generally, but were not included because they were not specific to math development.

Child-specific beliefs. As illustrated by the parent socialization model (Jacobs et al., 2005; Wigfield et al., 2006), parents' child-specific beliefs include parents' perceptions of their children's abilities and interests, expectations about later number learning, and extent to which parents recognize numbers as valuable for their children. In total, 4 questions with 7-point scales each asked about parents' perceptions of their child's number ability more generally, interest in numbers, expectations, and value of number learning for their child. Seven items using a 7-point scale asked about parents' beliefs about the importance of various number-related benchmarks for children's success in school. An additional 14 items, created for the current study assessed parents' beliefs about their child's specific number-related competencies, including items assessing parents' perceptions of children's counting, cardinality, non-symbolic set comparison, and numeral naming and comparison, but will not be reported for the current study.

Frequency of at-home math engagement. Parents indicated on the parent survey the frequency with which they engaged their children at home in math-related activities on a 5-point scale, 1 = Rarely or Never, 2 = Monthly, 3 = Weekly, 4 = Several days per week, 5 = Every day or almost every day (see Appendix F for survey items). Literacy-related activities were also included as distractor items. These items were based upon prior survey research on how parents' engage their children in at-home math activities (e.g., LeFevre et al., 2009). Composites were created by averaging frequency values (on a 5-point scale) for math-related experiences that emphasized rote practice (i.e., count out loud, identifying numbers, counting items, and determine how many in a group of objects), and advanced number activities which are considered more challenging but meaningful (compare written numbers and identify which is bigger, compare 2 or more quantities of objects to determine which is more, perform simple addition, play board games, and play card games). Similar item groupings have been used in prior research to distinguish between foundational and advanced at-home math activities, and have had good internal reliability (Cronbach's $\alpha = .83$; Skwarchuk, 2009; Zippert & Ramani, 2016).

Aims and Analysis Plan

Preliminary analyses. To determine whether socioeconomic status and child gender should be controlled for in the main analyses, bivariate correlations were conducted between education of the participating parent, and child gender (with male as the reference group) with the quantity exploration codes. If significant correlations were found, SES and gender would be controlled in the subsequent analyses. Further, experimenter effects were assessed on the PENS measure and the math codes for the 2 primary experimenters who administered the measures, and provided instructions to parents and children for the game.

Aim 1. The first aim was to determine the frequency and quality of preschoolers' number-related experiences with their parents.

Frequency of at-home number-related experiences. Descriptive statistics for all math-related at-home activity items and activity composites (i.e., foundational and advanced activities) were calculated to determine the frequency with which parents engaged their children at home in math-related activities. One paired samples t-test assessed the extent to which families engaged children in advanced versus foundational math activities more frequently at home. In addition, age differences between 3- versus 4-year-olds were assessed for both foundational and advanced at-home activity frequencies via two independent samples t-test.

Quantity identification descriptive analyses and means tests. Descriptive statistics were calculated to determine the proportion of total cards played (max 26) that were identified at each *quantity identification* level (see Table 1 for code descriptions at each level). In addition, descriptive statistics for broader code categories (see Table 1) were calculated after creating summed composites of the proportions to determine: the amount out of the total cards that quantities were identified in any representation (Levels 1 through 6), how often quantities were identified symbolically (Levels 2-6), how often connections were made between non-symbolic arrays and symbolic number representations (Levels 2-4), and how often dyads identified symbolic number representations without reference to non-symbolic arrays (codes 5 and 6). A repeated measures multivariate analysis of variance and subsequent post-hoc analyses with a

Bonferonni correction were conducted to determine whether quantities were identified more often than not identified (codes 1-6 vs. 0), whether more symbolic versus non-symbolic quantities were identified (codes 1 vs. 2-6), and whether dyads more often made connections between symbolic quantities and non-symbolic quantities than identifying symbolic quantities without their non-symbolic referents (2-4 vs. 5-6). Finally, differences in dyads' quantity identification for those with 3- versus 4-year-old children were assessed via a one-way multivariate analysis of variance.

Quantity comparison descriptive analyses and means tests. Descriptive statistics were calculated to determine the proportion of card pairs played (max 13) for which *quantity comparison* codes were applied at each level. Descriptive statistics were also calculated to identify the proportion of card pairs played for which dyads explicitly compared quantities of any representation (levels 1-3), and symbolic quantities specifically (levels 2-3). A repeated measures multivariate analysis of variance and subsequent post-hoc analyses with a Bonferonni correction were conducted to determine whether quantities were compared explicitly more often than not (level 0 vs. 1-3), and whether dyads compared symbolic quantities more than non-symbolic quantities (level 1 vs. 2-3). Finally, differences in dyads' quantity comparison for those with 3- versus 4-year-old children were assessed via a one-way multivariate analysis of variance.

Parental guidance descriptive analyses and means tests. Descriptive statistics were calculated to determine the level of parental guidance for quantity identification and quantity comparison. Proportions were calculated for each level of guidance (e.g., no guidance = 0, direct guidance = 5) provided during quantity comparison out of the total number of card pairs (arrays, number words, numerals), and provided during quantity identification out of the total number of cards played to determine the levels of parental guidance provided while dyads were identifying and comparing quantities specifically. A repeated measures multivariate analysis of variance and subsequent post-hoc analyses were calculated to determine the predominant composite level of guidance for quantity identification and comparison. Finally, differences in parental guidance of quantity identification and comparison for dyads with 3- versus 4-year-old children were assessed via a one-way multivariate analysis of variance.

Correlations between PENS and quantity exploration. Bivariate correlations were conducted between all individual quantity identification codes with children's scores on the PENS measure. Correlations were also run between PENS scores (proportion correct out of 46) and all individual quantity comparison codes to examine how dyads' *quantity identification* and *quantity comparison* were associated with preschoolers' numerical knowledge. Correlations were also examined between children's numerical knowledge and broader category codes for *quantity identification* and *quantity comparison*.

Aim 1 predictions. It is predicted that meaningful math experiences will occur relatively infrequently for preschool children at home. In terms of quality of these experiences, in the context of this meaningful math experience of playing a card game, parents and their preschoolers will explicitly identify quantities on the cards in significantly more of the turns during the game in comparison to not explicitly identifying the quantities on the cards. They will also identify the symbolic quantities on the cards significantly more than commenting only on the non-symbolic representation of the quantities. Dyads will spend more time exploring symbolic quantities in connection with the non-symbolic arrays (counting and labeling the quantities or numerals) than symbolic number representations on their own (i.e., just naming numerals or labeling the cards with a number word). For quantity comparison, dyads are predicted to compare the quantities on the two cards more than not comparing the quantities explicitly. Dyads are also predicted to compare the quantities on the cards using the symbolic representations more than the non-symbolic representations of the numbers. Math knowledge was predicted to be positively associated with explicit quantity exploration.

Aim 2. The second aim of this study was to examine a wider range of parents' math-related beliefs through a theoretical lens (Eccles et al., 1983; Jacobs et al., 2005; Wigfield et al., 2006) and determine their role in predicting the frequency and quality of the number-related experiences they provide their preschool children at home.

Descriptive statistics were calculated for responses of the entire sample on the parental belief items. Then, bivariate correlations were conducted in order to assess the relations among and between items associated with parents' general beliefs and those specific to their children. A principal components analysis was then conducted and factor scores were saved and labeled according to the items with the highest loading on each component. This procedure was chosen because it was of interest to transform the original set of variables into a much smaller set that represents most of the information in the original items (Dunteman, 1989). Unlike factor analysis, this technique does not infer an underlying statistical model of the observed variables, and it attempts to explain the maximum amount of total, as opposed to the common variance. Orthogonal rotation of the original components was employed to aid interpretation of the final variables. To determine associations between parents' beliefs and behaviors, bivariate correlations were run between parental belief components, the at-home math activity frequency variables (foundational and advanced), and the math code composites for quantity identification (no quantity identification proportion, non-symbolic quantity identification proportion, symbolic quantity identification proportion, and quantity identification involving symbolic and non-symbolic quantity identification proportion) and comparison (no quantity comparison proportion, non-symbolic quantity comparison proportion, and symbolic comparison proportion). Then separate regressions were run with parental beliefs as predictors of the aforementioned at-home math activity frequency variables and the math code composites.

Aim 2 predictions. It was expected that there would be strong associations among parents' individual child-specific and general belief items, respectively, and minimal association between the two types of belief items. In addition, a two-factor solution was expected to result from the PCA, representing the two types of parental beliefs represented in the parent socialization model (Jacobs et al., 2005). It was also expected that more positive parental beliefs specific to their own children would be positively associated with the quality of children's number experiences, while more positive general beliefs (e.g., math-related ability beliefs and values) would be associated with more frequent number-related experiences shared with their preschool children at home. These specific predictions stem from the work of McGillicuddy-Delisi (1985) and Sigel (1985) in that parental beliefs are thought to be reflected in the nature of the home environment that can be measured via parent surveys (frequency), and also in terms of observations of specific interactions between parents and children (quality). They also stem from Sigel & McGillicuddy-Delisi's (2002) theory of dynamic belief systems, which, guided by Bronfenbrenner (1979), suggests that parents hold beliefs that are both proximal and distal to the child. Thus it is suggested that proximal beliefs like child-specific beliefs are thought to affect the nature of parent-child interactions (quality), while parents' more distal beliefs, like general cognitions, are likely to affect children's broader math-related environment at home (frequency).

Aim 3. The third aim is to examine children's levels of affect and engagement in the game, and determine its role in predicting the frequency of at home math experiences and quality of the number-related experiences parents provide their children during a particular number-related playful and meaningful experience.

Descriptive statistics were calculated to determine means and standard deviations in children's levels of interest throughout the game. Then, bivariate correlations were conducted between all individual *quantity identification* codes with children's levels of engagement (composite of attention and active participation) and affect (positive and negative separately). Correlations were also run between these variables and all quantity exploration codes to examine how dyads' *quantity identification* and *quantity comparison* was associated with preschoolers' engagement and affect in the game. Correlations were also examined between children's engagement and affect and broader composite codes for *quantity identification* and *quantity comparison*. Additionally, a one-way MANOVA examined age differences in affect and engagement between 3- and 4-year-olds during the game, and subsequent post-hoc tests were conducted.

Moderation analyses. Then, analyses were run to determine the role of child affect and engagement in moderating the association between parents' beliefs and the frequency and quality of children's experiences with numbers.

Aim 3 prediction. It is predicted that children's affect and engagement in math would moderate the association between the frequency and quality of children's experiences with numbers and parents' child-specific and general beliefs. In particular, it is predicted that for children with high levels of affect and engagement, there would be a positive association between parents' beliefs and the frequency and quality of children's math experiences. However, for children with low math affect and engagement, this relation was expected to be non-significant, assuming that regardless of parents' values and beliefs about children's abilities, preschoolers who are uninterested in math are less likely to engage extensively in math activities.

Power analysis. For the main moderation analyses, in order to achieve 80% power to detect a medium effect size of .15 with 5 total predictors but only 3 tested predictors (including the interaction effect) in the model and an alpha level of .05, a minimum sample size of 77 was needed.

Missing data. Two children's ages were not provided. Thus, based on their math test scores, one child was assigned as age 4, and the other as age 3. For the PENS measure, (3%) of children were missing rote counting scores, so mean scores for children corresponding in age were imputed. One subitizing score for one child was missing, so the mean score corresponding with the child's age (3yrs) was imputed. Two percent of number comparison verbal scores were missing, so the mean score for 3-year-olds was imputed for both cases. Two percent of numeral identification scores were missing, thus the means corresponding to children's ages were imputed. Four parents did not fill out the household income question, two parents omitted the education questions, and thus means for the sample were imputed for the missing values. Between 2-4% of the data for the parental belief items were missing. Parent gender differences were non-significant for all but one question (parents' beliefs about how much influence they have over their children's math development; $M = 6.28$ vs. 5.76, $t(89) = 52.10$, $d = .47$), thus, for the remaining parental belief items with missing data, sample means were imputed. For the item in which parental beliefs differed by parent gender, 3 means were imputed for the corresponding parent genders. Between 2-7% of data were missing from the home math activity variables. Many of the mean difference tests for age were significant, thus values were imputed based upon averages for the child's corresponding age group.

Chapter IV: Results

Preliminary Analyses

Associations between demographic variables and quantity exploration. Bivariate correlations suggested no significant association between participating parents' education, and children's number-related experiences (frequency of at-home math engagement and math codes applied during the interaction). Child gender was non-significantly related to dyads' math exploration or at-home math activities. Further, means tests to assess experimenter effects for the math measure (*ps* > .91) and the math codes were non-significant (*p's* > .19). Thus, SES, experimenter effects, and child gender were not considered further in the analyses.

Children's Number Knowledge

Children's numerical knowledge was assessed using the Preschool Early Numeracy Scales (Purpura & Lonigan, 2015). As shown in Table 5, across subscales, 4-year-olds performed significantly better than 3-year-olds. Three-year-olds tended to do best comparing the magnitudes of sets of dots (suggesting that non-symbolic number skills were the areas of strength in this game), though their performance was only slightly above chance. Further, children in this younger age group tended to do most poorly on problems involving numerals and number words. In contrast, 4-year-olds were quite successful on most of the numeracy tasks, but did struggle with subitizing (labeling a quantity of dots with a number word) and numerical magnitude comparison tasks involving number words and numerals. This suggests that the card game activity would likely be at least somewhat challenging for the majority of the children in the study.

Parent-child Interaction

Dyads played the game for an average of about four and a half minutes, which was much shorter than expected. Dyads with 3-year-olds played the game significantly longer than those with 4-year-olds, averaging about 5 minutes, and this varied by about 2.5 minutes (range = 2-12 minutes), while dyads with 4-year-old children played the game for around 3-minutes, and this varied by about 1.5 minutes on average (range = 1.5-9), $t(92) = 1.75$, $p < .001$, $d = 1.03$. Most of the dyads finished the game, averaging 12 out of the 13 possible turns, and 3-year-olds and 4-year-olds did not differ significantly in the number of turns they took.

Aim 1: Examining the Frequency and Quality of Preschoolers' Experiences with Numbers

This section provides descriptive statistics of preschool children's experiences with numbers in terms of both the frequency (i.e., parental report of at home engagement with number activities) as well as the quality (i.e., through observations of parent-child quantity exploration while playing the card game).

Frequency of meaningful math experiences. Parents reported engaging their children in math-related activities at home frequently (see Table 6 for descriptive statistics). Specifically, parents reported foundational math activities such as counting out loud, counting items, and identifying numbers as occurring several days per week on average in the home. However, they reported engaging their children significantly less frequently (i.e., on a monthly basis) in more complex or advanced activities such as comparing the magnitudes of written numbers or objects, performing simple addition, and playing card and board games ($M = 4.27$ vs. 2.82, $t(93) = 17.07$, $p < .01$, $d = 1.80$).

While foundational number experiences occurred with comparable frequency for both 3- and 4-year-olds, advanced math experiences occurred more frequently (on a weekly basis on average) for 4-year-olds than 3-year-olds (M = 3.13 vs. 2.51, $t(92) = 3.25$, $p = .002$, $d = .67$).

Quality of number-related experiences between parents and preschoolers. To investigate the quality of preschoolers' number experiences, means and standard deviations for dyads' quantity identification and comparison during the card game for the whole sample and separated by child age were calculated and are reported in Table 7. Quantity identification code frequencies were transformed into proportion of total cards identified (max 26, 13 for each player) at each *quantity identification* level to account for variation in the number of cards played. In addition, composites were created to determine the proportion of total card quantities explicitly identified in any representation (Levels 1 through 6), symbolically (Levels 2-6), via connecting symbolic and non-symbolic representations of quantity (Levels 2-4), and how often dyads identified symbolic number representations without referencing the non-symbolic arrays on the cards (Levels 5 and 6). Frequencies for *quantity comparison* codes applied at each level were transformed into proportion pairs of card played (max 13 pairs) to account for variation in the amount of cards compared. Proportions were also created to determine the amount of card pairs played in which dyads explicitly compared quantities of any representation (levels 1-3), and symbolic quantities specifically (levels 2-3).

Quantity identification descriptive statistics. Dyads on average completed over half of their turns without explicitly identifying the quantities on the cards. A repeated measures multivariate analysis of variance (MANOVA) was conducted comparing the frequency of quantity identification for each type of number representation, and the omnibus test was significant $F(3, 91) = 32.21$, $p < .001$ partial $\eta^2 = .41$. Post-hoc tests with a Bonferonni correction revealed that dyads were significantly more likely to not explicitly identify quantities than explicitly identify them in any representation (p's < .001). In contrast, dyads identified the cards by commenting on the non-symbolic representations of the quantities (e.g., "That's a lot of dots!") significantly less frequently than they identified quantities symbolically only ($t(93) = 13.10$, $p < .001$, $d = 1.89$), or symbolically and non-symbolically ($t(93) = 6.24$, $p < .001$, $d = .86$). This suggests that when dyads identified quantities on the cards, they did so in exact and symbolic ways rather than approximately.

Much of dyads' symbolic identification of quantities involved stating the symbolic representation without connecting it to the non-symbolic representation (30%). The majority of this exploration involved verbally labeling the quantities (30%) without gesturing to the Arabic numeral. In fact, limited explicit evidence existed that dyads identified the numerals on the cards, such as parents or children pointing or gesturing to the numerals and naming them (~1%). In contrast, dyads did on average make connections between symbolic and non-symbolic quantity information on 15% of their turns, and it occurred overall significantly less often (about half as much) as dyads' exploration of symbolic quantities only ($M = .15$ vs. .30, $t(93) = -3.49$, $p < .001$, $d = -.36$). The most common way to make connections between non-symbolic and symbolic quantity information was through counting the dots (12%), and to a minimal extent labeling after counting (3%). Dyads made minimal direct explicit connections between the dots and the numerals, however (< 1%).

Further, while a one-way MANOVA run to assess age differences in quantity identification was not significant $F(3, 90) = 2.27$, $p < .09$, partial $\eta^2 = .07$, trends in age differences were observed. Trends suggested that dyads with 3-year-olds tended to explore quantity identification more explicitly than 4-year-olds ($M = .52$ vs. .38). This was especially the case for the frequency with which dyads made explicit connections between symbolic and non-symbolic representations of number on the cards, as dyads with 3-year-olds tended to make these connections more explicitly ($M = .21$) than dyads with 4-year-olds ($M = .10$).

Quantity identification guidance analyses. Descriptive analyses and results of means tests for guidance codes for quantity identification for the entire sample and broken down by age are reported in Table 7. Frequencies of quantity identification guidance codes applied at each level were transformed into proportions of card pairs played (max 26). A repeated measures MANOVA and subsequent post-hoc tests comparing the frequency of each composite level of guidance of quantity identification (i.e., none, medium, high) was conducted, and the omnibus test was significant, $F(2, 92) = 57.35$, $p < .001$, partial $\eta^2 = .56$. Post-hoc tests with a Bonferonni correction revealed that very little parental guidance for quantity identification was detected, as evidenced by the fact that the percentage of cards in which no parent guidance was provided for quantity identification (64% of cards played) was significantly greater than the high and medium guidance provided by parents for identification (p's $< .001$), though there was no significant difference between how often parents provided medium versus high guidance for quantity identification ($p < .69$). Parents provided medium levels of guidance for quantity identification in the form of specific prompts on 11% of cards played, and high levels of guidance in the form of direct instruction on 12% of the cards played. Modeling and explanations or corrections tended to be minimally frequent forms of guidance (4% each).

A one-way MANOVA was run to examine age differences in the parental guidance children received when identifying the card quantities, and the omnibus test was significant $F(3, 90) = 4.68$, $p < .01$, partial $\eta^2 = .14$. Post-hoc analyses with a Bonferonni correction suggested that 4-year-olds identified quantities significantly more without parent help than 3-year-olds ($M = .74$ vs. .55, $t(92) = 3.14$, $p = .002$, $d = .68$), and parents of 3-year-olds provided high levels of quantity identification guidance on a higher proportion of cards than parents of 4-year-olds ($M = .28$ vs. .11, $t(92) = 3.67$, $p < .001$, $d = .77$). Age differences were not significant for incidences of medium guidance.

Quantity comparison descriptive analyses and means tests. Means, standard deviations and results of means tests for quantity comparison codes for the entire sample and broken down by age are reported in Table 7. Composites were calculated to identify the proportion of card pairs played for which dyads explicitly compared quantities of any representation (levels 1-3), and symbolic quantities specifically (levels 2-3). A repeated measures MANOVA was conducted comparing the frequency of quantity comparison for each type of number representation, and the omnibus test was significant $F(2, 92) = 16.54$, $p < .001$, partial $\eta^2 = .26$. Dyads did not explicitly compare the quantities on slightly over half of the card pairs played (53%). Quantity comparisons using non-symbolic representations of the quantities (24%), and symbolic representations (23%), such as indicating that 4 is greater than 3, occurred in comparable proportions of card pairs played (i.e., neither occurred significantly more frequently, $p < .99$). When dyads compared quantities symbolically, they mostly discussed number words during their comparison without explicitly pointing to the numerals (23%) instead of gesturing to them (1%).

A one-way MANOVA was also run to examine age differences in dyads' magnitude comparisons, but the omnibus test was not significant $F(2, 91) = 1.79$, $p < .17$, partial $\eta^2 = .04$. Thus, post-hoc analyses were not conducted.

Quantity comparison guidance analyses. Descriptive analyses and results of means tests for guidance levels during quantity comparison for the entire sample and broken down by age are reported in Table 7. Parental guidance during quantity comparison occurred more frequently than their guidance of quantity identification (M = .77 vs. .36, $t(93) = -12.69$, $p < .001$, $d = 1.28$). For example, the proportion of pairs in which no parent guidance was provided at all amounted to only 29% of card pairs played. Much of the parental guidance provided, however, involved medium forms of guidance such as general prompts, which involved parents asking who has more, without specifying a strategy for the comparison (39% of card pairs played). The other type of medium guidance provided by parents was specific prompts, which involved asking children to compare quantities while giving them a strategy, such as to refer to the dots or the numerals (16% of card pairs played). The next most common form of guidance was higher-level assistance, such as explanation and corrections after a child completed a turn. While direct instruction was comparable to explanation and correction, there was practically no modeling or co-participation in dyads' quantity comparisons. A repeated measures MANOVA was conducted comparing the frequency of quantity comparison guidance at each level (i.e., none, medium, high), and the omnibus test was significant

$F(2, 92) = 29.13$, $p < .001$, partial $\eta^2 = .39$. Post-hoc comparisons with Bonferonni corrections suggested that medium guidance was the most common level of parent assistance ($p < .001$), though there were no differences in how often parents provided high versus no guidance ($p < .99$).

In addition, a one-way MANOVA was conducted comparing the amount of parental guidance to 3-year-olds versus 4-year-olds, and the omnibus test was significant $F(2, 91) = 11.30$, p < .001, partial $\eta^2 = .20$. Post-hoc comparisons with a Bonferonni adjustment suggested that suggested that 4-year-olds compared quantities significantly more without parent help than 3-year-olds ($M = .33$ vs. .12, $t(92) = 4.16$, $p < .001$, $d = .88$), and parents of 3-year-olds provided high levels of quantity comparison guidance on a higher proportion of cards than parents of 4-year-olds ($M = .30$ vs. .15, $t(92) = 3.36$, $p < .001$, $d = .70$). Age differences were not significant for incidences of medium guidance.

Correlations between math knowledge and the frequency and quality of number experiences, and guidance. Correlations were run assessing the relations between math knowledge (proportion correct on the PENS measure) and dyads' quantity exploration (*quantity identification* and *comparison* composites; see Table 8), and *quantity identification* and *comparison guidance* composites. Children's number knowledge was significantly positively correlated with the frequency of advanced at-home activity engagement ($r = .23$, $p < .02$) and marginally significantly correlated with foundational at-home math activities ($r = .20$, $p < .06$). For the quality measures, math knowledge was negatively significantly correlated with dyads' identification of quantities via symbolic and non-symbolic representations ($r = -.22$, $p < .03$) and non-symbolic quantity comparison ($r = -.24$, $p < .02$). While math knowledge was not associated with medium guidance for quantity identification($r = .10$, $p < .35$) and comparison ($r = -.05$, $p < .63$), correlations were positive and significant for incidences of no parental guidance for both quantity identification ($r = .27$, $p < .01$), and quantity comparison ($r = .44$, $p < .001$) and negatively associated with high guidance for both quantity identification ($r = -.41$, $p < .001$) and ($r = -.44$, $p < .001$).

Results from aim 1 suggest that while families were engaging their children in math at home, they tended to do so in the form of rote number practice such as counting out loud and number identification as opposed to advanced and meaningful at-home math experiences, such as card and board games, and exploring the magnitudes of numbers through comparing and combining them. In examining the quality of these experiences, much of dyads' number exploration was implicit, though dyads with younger children were more explicit in their number use than dyads with older children, and math knowledge was negatively associated with dyads' explicit exploration of the connection between symbolic and non-symbolic quantity identification, suggesting that number talk of this kind may have been more necessary when children needed to solve the problems out loud such as through counting the dots, or parents needed to provide the answer for their children. Older children and those with more math knowledge tended to receive less guidance in quantity identification and comparison from their parents and tended to discuss numbers out loud much less often, particularly during quantity exploration involving non-symbolic number representations.

Aim 2: Examining The Role of Parental Beliefs in Predicting the Frequency and Quality of Preschoolers' Experiences with Numbers

Descriptive statistics for parental beliefs. Descriptive statistics of parents' general and child-specific math-related beliefs and at-home math engagement for the entire sample are reported in Table 6. On average, parents reported spending only minimal amounts of time involved in math activities and finances at home (on a monthly basis on average). In terms of their beliefs about their own abilities, parents reported that they were moderately good at math both currently and when they were in school. In contrast, parents reportedly believed that it was important to be good at math and felt that in general, parents have a good deal of influence over their children's math development.

Parents reported upon their math-related beliefs specific to their own children in a series of items with 7-point scales (see Table 6). Children were viewed by their parents as having moderately good number skills in general ($M = 5.45$, $SD = 1.29$), though parents' ratings of their children's abilities were significantly lower when comparing them to other children ($M = 5.45$ vs. 5.13, $t(93) = 2.65$, $p < .01$, $d = .26$) and higher when these ratings reflected children's number skills a year later ($M = 6.07$ vs. 5.45, $t(93) = -5.87$, $p < .01$, $d = -.53$). Parents also rated their children as having moderate interest in numbers ($M = 5.46$, $SD = 1.27$), and rated both the value of numbers to their children currently and the utility of numbers to their child in the future very highly ($M = 6.37$, $SD = .85$; $M = 6.54$, $SD = .83$, respectively).

Intercorrelations and correlations between parent beliefs and the frequency, and quality of children's number-related experiences. Correlations among and between parental beliefs, home engagement (frequency), and math exploration (quality) are presented in Table 9. Bivariate correlations suggest that items regarding parents' general beliefs and involvement were moderately correlated. For example, the frequency with which parents reported time spent doing math was correlated with their reports of time spent on finances ($r = .30$, $p < .01$). Parents' conceptions of their ability in math currently were correlated with their perceptions of their ability when they were in school ($r = .80$, $p < .01$). Finally, parents' beliefs about how important they felt it was to be good at math ability was correlated with both their beliefs about their math ability currently ($r = .40$, $p < .01$) and when they were in school ($r = .54$, $p < .01$).

Intercorrelations among child-specific belief items are also presented in Table 9. In comparison to parents' general beliefs, their child-specific beliefs tended to be much more strongly intercorrelated (most *r's* were in the .5-.6 range), especially parents' beliefs about their children's number skills currently, number skills in the next year, and ability compared to others.

Relations between child-specific and general beliefs were varied (significant *r's* ranged from .21 to .40), particularly among parents' general value and utility beliefs about math and their beliefs about the value of math for their own child, as well as their beliefs about their child's interest in math. These correlations are presented in Table 9.

Relations between parental beliefs and home math engagement are also presented in Table 9. The frequency with which parents reported engaging in their own math-related activities was associated with the frequency with which they engaged their children in foundational math activities like counting and number naming ($r = .24$, $p < .05$). However, other reports about parents' general beliefs and behaviors were non-significantly correlated with their reports about their at-home math engagement with their children. In contrast, parents' child-specific beliefs were more widely correlated with their at-home engagement reports. Specifically, the frequency with which parents reported engaging their children in foundational math activities at home was significantly correlated with their beliefs about their children's interest in numbers ($r = .35, p < .01$), number ability currently ($r = .34, p < .01$), in the future ($r = .23, p < .05$), and compared to other children ($r = .25, p < .05$). Advanced home math activity engagement was correlated with parents' beliefs about their children's current ability ($r = .27, p < .01$) and their ability compared to other children ($r = .29, p < .01$).

Relations between parental beliefs and the quality of dyads' quantity exploration during the card game were also assessed. As shown in Table 9, while parents' general beliefs were not significantly associated with proportion of total cards in which dyads identified quantities using symbolic and non-symbolic number representations, parents' child-specific beliefs about their children's abilities compared to other children were, however, significantly negatively associated with this type of quantity exploration ($r = -.20, p < .05$). Finally, at-home math engagement (math frequency) was not significantly associated with dyads' math exploration (quality) during the card game.

Parental belief items were then grouped via principal components analysis. This procedure was chosen because it was of interest to transform the original set of variables into a much smaller set that represents most of the information in the original items (Dunteman, 1989). Unlike factor analyses, this technique does not infer an underlying statistical model of the observed variables, and it attempts to explain the maximum amount of total, as opposed to the common variance. This analysis, which also involved an orthogonal Quartimax rotation to aid in interpretation, resulted in 5 components (standardized component scores were saved and used in subsequent analyses) that together explain 83% of the total variance. The analysis confirmed that parents' beliefs tended to be associated with other beliefs of the same type (e.g., child-specific or general beliefs; see Table 10 for item loadings onto components and communalities). For example, parents' child-specific beliefs about the importance of 6 different number-related benchmarks for their child (e.g., counting to 10 and 20, naming numbers to 10 and to 20, and comparing and adding numbers) strongly loaded onto the first component (with loadings ranging from .97 to .98). On the second component, parents' child-specific beliefs about their children's current and future number abilities, number skills in comparison to other children, and their children's interest in numbers had component loadings ranging from .81 to .84, while parents' general beliefs about their own math abilities in school and currently loaded onto a separate fourth factor at .91 and .92. Further, parents' child-specific beliefs about the value and utility of numbers for their children loaded onto a single factor at .90 and .88, and finally, parents' personal involvement with math and numbers had loadings onto a fifth factor of .66 and .89, respectively.

PCA regression factor scores were saved for the 5 orthogonal (uncorrelated) rotated principal components, and then their relations with dyads' math exploration were assessed (see Table 10 for a list of the components). The first component (parents' child-specific value of number benchmarks), fourth component (parents' child-specific beliefs about the value and utility of numbers to their child) and fifth component (parents' personal involvement with numbers and math) were not significantly correlated with dyads' quantity exploration or the frequency of their at-home math activity engagement.

The second component, called child-specific ability and interest beliefs were non-significantly correlated with dyads' quantity exploration during the game, but significantly correlated with foundational at-home number activities ($r = .35$, $p < .01$) and advanced home math activities ($r = .27$, $p < .01$). The third component, parents' general beliefs about their own abilities and value of math was not significantly correlated with at-home math engagement, but was significantly negatively correlated with dyads' exploration of symbolic quantity (e.g., identifying numerals and using number words; $r = -.35$, $p < .01$) and negatively correlated with dyads' symbolic quantity comparison ($r = -.25$, $p < .01$).

As shown in Table 11, regression analyses controlling for children's age and number knowledge (proportion correct on PENS) were run to determine the relations between parental beliefs and at-home math engagement. Results showed that parents' beliefs about their child's ability and interest in math (the second principal component) predicted at-home math engagement in both foundational ($R^2 = .15$, $F(4, 93) = 3.81$, $p < .001$) and advanced math activities ($R^2 = .18$, $F(4, 93) = 4.78$, $p < .001$), while child-specific beliefs about children's math ability and interest was not a significant predictor in either analysis. Multiple regressions analyses were also conducted, controlling for children's age and number knowledge, to determine the associations between parents' beliefs and the quality of children's early number-related experiences (see Table 12). Results showed that parents' general beliefs about their own abilities and value of math explained 13% of the total variance in dyads' symbolic quantity exploration, or identifying numerals and using number words to label the card quantities ($R^2 = .13$, $F(4, 93) = 3.41$, $p < .001$). The relations were comparable for the proportion of cards dyads chose *not* to identify. Specifically, parents' general beliefs about their own abilities and value of math, controlling for parents' child-specific beliefs and children's age and math knowledge, explained 13% of the total variance in the proportion of intervals in which dyads did not explicitly identify quantities ($R^2 = .13$, $F(4, 93) = 3.41$, $p < .001$). Parents' beliefs did not significantly predict dyads' quantity identification using both symbolic and non-symbolic quantity information on the cards, however. Parents' general beliefs did significantly predict dyads' symbolic magnitude comparisons.

Aim 3: Examining the Role of Child Affect and Engagement in Predicting the Frequency and Quality of Preschoolers' Experiences with Numbers

As shown in Table 13, overall, there was very little negative affect exhibited by children during their play, as they scored 1.23 on a 7-point scale. Positive affect was moderately prevalent; as children exhibited smiles or laughing a few times during the game, but children's smiling did not amount to half of the duration of the interaction (earning a global score of about 2.5). Descriptive analyses for child engagement for the entire sample and broken down by age are also reported in Table 13. Overall, children were highly participatory in playing the game with their parents. Out of 5 possible average active participation points, preschoolers averaged 4.4 (making 4 or more individual contributions, verbally or nonverbally, to the game for every 30-seconds). These could have included turning over the cards for the characters, counting or labeling the quantities on the cards, indicating the winner of a given round, or awarding the cards to the appropriate character. Children were also highly attentive to the game, scoring above 4.4 in attention on average (staying focused on the game for almost all of the interaction, with the exception of minor distractions).

In addition, a one-way MANOVA was conducted comparing affect and engagement between 3- and 4-year-olds during the game, and the omnibus test was significant $F(3, 90) = 11.30$, $p < .001$, partial $\eta^2 = .20$. Post-hoc comparisons with a Bonferonni adjustment suggested that 4-year-olds were more engaged in the game on average than 3-year-olds ($M = 4.59$ vs. 4.22, $t(92) = 4.49$, $p < .01$, $d = .93$), though children's affect did not differ significantly by age.

Correlations between child affect and engagement and the frequency and quality of preschoolers' number experiences. As shown in Table 8, engagement and positive affect were not significantly correlated; however, negative affect was significantly negatively correlated with engagement ($r = -.32$, $p < .01$). This suggests that lower instances of negative affect were associated with higher attention and participation in the game.

To measure the association between child variables and the frequency of preschoolers' number-related experiences, correlations were run between the at-home engagement survey variables as well as child affect and engagement variables in the card game (see Table 8). The only significant correlation found was between the frequency of advanced experiences and children's engagement in the card game ($r = .20$, $p < .05$).

To measure the association between child characteristics and the quality of children's number experiences, correlations were run between child variables and the math codes. Positive affect was non-significantly associated with dyads' quantity identification or comparison during the card game and thus was not considered in subsequent analyses. In contrast, children's negative affect was significantly positively associated with dyads' exploration of the connections between symbolic and non-symbolic quantity information ($r = .32$, $p < .01$) and significantly negatively associated with dyads' total symbolic quantity comparison (comparisons via number name and numerals; $r = -.21$, $p = .04$). Child engagement was positively correlated with the proportion of intervals in which no quantities were identified ($r = .30$, $p < .01$) and significantly negatively associated with dyads' exploration of the connections between symbolic and non-symbolic quantity information during quantity identification ($r = -.23$, $p = .03$).

Together, these results suggest that children's affect and engagement may be important in understanding variation in the frequency (parent-report of at-home math activities) and quality (observations of parent-child number exploration during the card game) of children's number experiences. More specifically, children's negative affect may be important in understanding variation in the frequency and quality of children's number experiences, and child engagement may additionally be important in understanding the quality, but not the frequency, of children's number experiences. These associations were further assessed in regression analyses with negative affect and engagement as moderators of the association between parental beliefs (child-specific and general) and dyads' quantity exploration.

Examining the role of child engagement in dyads' quantity exploration. The aforementioned correlations suggest that the proportion of cards in which dyads did not identify quantities explicitly is associated with child engagement and parental beliefs (general parent ability beliefs), and meaningful math experiences were positively associated with child engagement and predicted by parental beliefs (child-specific ability beliefs). Thus, two regressions were run to determine if child engagement moderated the association between parental beliefs and the frequency (parent report of at-home math experiences) and quality (observations of parent-child number exploration during the card game) of children's number-related experiences.

Moderation analyses were conducted using the PROCESS macro add-on in SPSS, version 2.15 (Hayes, 2013). Parent-belief and child variables were mean centered, number knowledge and age were entered as covariates, and both analyses used Model 1 provided by the PROCESS package, which tests for moderation (see Table 14 for results). Results suggested that child engagement was neither a significant predictor of frequency of advanced home number experiences, nor did it significantly moderate the relation between parents' child-specific ability beliefs and frequency of children's meaningful math experiences at home (R^2 = .17, $F(5, 88) = 3.62$, $p < .01$). Engagement was also not a significant moderator of the relation between parents' general beliefs about their math ability and values, and the proportion of intervals in which dyads did not explore quantity; however, it remained a significant predictor along with parents' general beliefs (R^2 = .16, $F(5, 88) = 3.32$, $p < .01$). To aid interpretation, an additional moderation analysis was run with the proportion of intervals in which quantity identification was done explicitly at all (this includes non-symbolic, symbolic, and linking symbolic with non-symbolic). Proportion of variance explained and F values were equivalent to the aforementioned regression analysis results predicting the proportion of intervals in which quantities were not explicitly identified, but beta coefficients had opposite signs (see Table 14 for coefficients for this analysis). Results suggested that the more able and interested parents believed their preschooler to be, the more often they engaged them in advanced math activities at home. Further, more explicit quantity identification was associated with less child engagement and lower parental math-related ability beliefs and values.

Chapter V: Discussion

The current study examined the frequency (i.e., parental report) and quality (observations of parent-child behaviors during a number-related card game) of how preschool children and their parents explore the relations between symbolic and non-symbolic quantities in the context of an informal yet meaningful math experience, as well as how parent and child factors are associated with this quantity exploration. Parents and 3- to 4-year-old children were videotaped playing a modified version of the card game *War*. Further, preschool children's understanding of symbolic and non-symbolic quantities was assessed while parents filled out a survey about the number-related experiences they share with their children at home, and their math-related beliefs.

Results showed that, in the context of this number-related card game, parents and children explored quantity explicitly on only half of the cards and card pairs played, and dyads of young children and those with lower number knowledge tended to be most explicit in their quantity exploration. Dyads with older children, on the other hand, often completed their turns without explicitly discussing the numbers. When dyads did explore the quantities explicitly during the game, they focused on identifying numbers symbolically. However, dyads used non-symbolic card information just as often as symbolic information to make the quantity comparison judgments, and in some instances, emphasized the connection between the symbolic and non-symbolic information on the cards, though they largely ignored the Arabic numeral in favor of using just the number words (e.g., *four*). Parents reported that advanced math experiences such as card game play and quantity comparison occurred relatively infrequently at home compared to activities geared towards more rote practice of number, such as counting out loud and naming numbers. However, parental beliefs were important in predicting both parental report of the frequency of at-home math engagement as well as the quality of these experiences. In particular, parents' specific beliefs about their children's abilities and interests (which were strongly correlated) were associated with the frequency of home math activities, while parents' general beliefs about their own abilities and value of math (which were also strongly correlated) were associated with the quality of dyads' number exploration while playing the card game. Finally, both parental beliefs and child engagement during the card game were significant predictors of the quality of dyads' quantity exploration during the card game (observations of parent-child number exploration during the card game). The following section discusses these results in terms of the aims of the study.

Aim 1: Examining the Frequency and Quality of Preschoolers' Experiences with Numbers

Frequency of number experiences. Findings from aim 1 suggest that advanced number activities in which parents and preschoolers use magnitudes of numbers meaningfully to solve problems, such as card games, occur far less frequently at home (on a monthly basis) than rote practice of numbers (occurring several times per week), such as naming numerals and rote counting. This was expected given that prior research has also found that number practice of foundational number skills to be the predominant type of number-related activity to occur in the home for parents and preschoolers (Blevins-Knabe & Musun-Miller, 1996; Skwarchuk, 2009; Zippert & Ramani, 2016). This is likely because these types of activities are conventionally understood to support early numeracy, while this is less of the case for some of the activities considered more meaningful and challenging for preschoolers (Zippert & Ramani, 2016).

Quality of number experiences. The behavioral coding used in the present study provided insight into the nature and quality of one such meaningful math experience, specifically parents and their preschoolers number exploration while playing a number card game together, which involved identifying and comparing pairs of card quantities. While dyads engaged in explicit quantity identification or comparison on only half of the cards or card pairs, when quantities were discussed explicitly, they were done so in exact symbolic ways to a greater extent than approximate estimations (e.g., “That’s a lot a of dots on your card!”). Further, on average, quantities identified and compared explicitly were substantial (i.e., dyads identified approximately 12 cards and compared the quantities of about 7 pairs of cards). This was expected, and suggests that this experience was meaningful in that it encouraged dyads to discuss the magnitudes of the quantities of the cards. Other activities may be more conducive to rote number practice such as counting out loud or labeling numerals, as is the case with number books and puzzles. While necessary for developing foundational math knowledge, it is important for children to also problem solve with numbers in ways that involve exploring the connections between symbolic and non-symbolic number representations, and these are thought to represent quality number-related experiences (e.g., Gunderson & Levine, 2011; Mix et al., 2012). Siegler and Lorti-Forgues (2014) emphasize that integrating symbolic and non-symbolic number representation is a critical milestone for children during the preschool years, and recommend experiences that encourage children to make these connections (Siegler, in press).

While it was not clear from the current analyses why half of the cards and card pairs were not explicitly identified and compared, there are several possible reasons. One reason could be that some of the quantities were easily identifiable because they were small enough to be subitized (i.e., identified without counting). Other potential reasons are that children knew the Arabic numerals but simply did not name them out loud, or another reason could be that identifying quantities was not of priority for some families while playing the game. Similarly, there are several possible reasons as to why some of dyads' card comparisons were done less explicitly. Certain pairs were likely easily differentiable, as is the case with numbers differing by large quantities (e.g., 1 vs. 6), thus eliminating the need to discuss the quantities at all and proceed to determining the winner of the round (e.g., "Ernie wins!"). It is also possible that a combination of these reasons influenced dyads' interactions during the game. To better understand the factors that influenced the dyads' number exploration while playing the game, future analyses should examine the identification and comparison of quantities that dyads make based on the size of each number on the card and the ratio between the pairs of the numbers being compared.

One of the beneficial features of the current study's parent-child math activity was that it involved playing cards that contained both symbolic and non-symbolic quantity information, thus providing opportunities for dyads to make connections between both representations of numbers. Surprisingly, the majority of dyads' explicit symbolic talk, however, involved identification of quantities without discussing the adjacent non-symbolic arrays on the cards. Further, this type of talk was largely coded as exploration of number words as opposed to naming the Arabic numerals, as few families made corresponding gestures to the corners of the cards (where numerals were located) along with their number talk. It is possible that dyads were using the numerals to identify the quantities, especially when numbers outside of the subitizing range (e.g., 4 to 5 or greater) were identified rapidly. Measuring reaction time of quantities identified may provide insight into which representation of number children are using to identify the card quantities. For example, children who rapidly identify the value of a large number without evidence of counting are probably likely to be using the numeral to make their decisions, since large quantities cannot be easily subitized. The same could also be true for comparing the magnitudes of two numbers that are large and highly similar in value (e.g., 8 vs. 10). Children who generate the correct response quickly to a magnitude comparison problem are less likely to have used the non-symbolic number representations to make their decisions, as many younger children pointed out during the game, two large adjacent quantities tend to look the same.

Further, when dyads made connections between symbolic and non-symbolic information, they tended to link the dot arrays to their corresponding number words rather than the Arabic numerals (i.e., through pointing to each dot and saying the number words in order through counting), even though the numerals were printed on each card. This connection was made significantly more explicit for children with lower scores on the PENS, and dyads with 3-year-olds tended to do so more than 4-year-olds. In fact, math knowledge was significantly negatively correlated with this type of quantity identification. Thus, explicit discussion of numbers could have been more frequent for children who were less knowledgeable about numbers, and ultimately, counting was a way for them to use an easier strategy to first identify them. Benoit et al. (2013)

suggested that knowledge of Arabic numerals tends to develop after children have mastered the number words (e.g., *four*). Thus, as indicated by the integrated number theory, the second "acquisition" or developmental milestone of linking non-symbolic representations to symbolic representation seems to include both the development of number words and numerals, which may need to be considered separately in understanding this particular milestone (Siegler & Lortie-Forgues, 2014).

Dyads also rarely labeled the quantities after they counted them. Mix et al. (2012) suggest that this particular talk in addition to counting is especially important for young preschoolers who are developing cardinal number knowledge. However, perhaps this was redundant for the participating children, as their performance on the cardinality subscale of the PENS indicated that neither age group necessarily needed help mastering this particular skill. For those children still learning this concept, however, Siegler and his colleague would likely emphasize the importance of this exercise to integrate non-symbolic quantities with symbolic number representations (2014). Thus, further work may be needed to determine if this pattern of behavior occurs for children who have yet to develop their concept of cardinality, which could include low-income children of the same age group, and how to encourage parent-child dyads to explore these connections further.

In addition to examining the ways parents explicitly identified quantities while playing the game, the present study also examined the guidance parents provided during this step. This guidance associated with children's quantity identification reflected the frequency with which explicit quantity identification occurred. For example, no parental guidance was provided for well over half (60%) of the cards played in which quantity identification could have explicitly occurred. When parents provided quantity identification assistance, they mostly either provided specific prompts to identify the quantities, or identified the numbers for their children (e.g., “This is a 5”), and most of this direct instruction was given by the parents of 3-year-old children, suggesting that parents of younger preschoolers recognized that their children needed more help than parents of 4-year-olds. This help often came in the form of explanations or corrections, ensuring that children knew the correct answer, or why their answer was correct (e.g., if a child pointed to the correct card, a parent may have further explained that the child was correct because the number pointed to was larger than the number on the other card).

These data align with the quantity identification code frequencies in that parents provided very little guidance for identifying quantities, however, when they did provide guidance, it was mostly direct, such as when parents needed to either correct children’s answers or extend or explain their responses. Throughout the game, parents took the opportunity to extend a child’s non-numerical response into a symbolic one. For example, after a child pointed to the correct card, parents often provided additional number-related information to complete the turn (e.g., in response to a child point, the parent may say, “right, that’s an 8”). In this way, among others, parents played the important role in making the interaction explicitly more number-related.

The corresponding aspect of the behavioral coding of the parent-child interaction involved examining how parents and preschoolers compared the quantities on the cards, either via non-symbolic representations, symbolic number information, or in non-detectable ways. This coding showed that when dyads explicitly discussed comparing the quantities, they spent half of those card pairs doing so non-symbolically (e.g., indicating which card had more dots) and symbolically for the remaining half of pairs (indicating, for example, that 4 is bigger than 3). As previously mentioned, this could have been driven by the ratio of the two numbers, as some pairs were likely very easy to compare if the numbers differed by a large range. Further, the majority of symbolic card pair comparisons were strictly verbal, without accompanying gestures to the numerals. Though again, dyads could have been using the numerals to make comparisons implicitly.

It was often the case that children depended on the non-symbolic comparison strategy when they had not yet developed a solid understanding of symbolic magnitudes. This was reflected in the negative correlation between children's number knowledge and the extent to which they engaged in non-symbolic magnitude comparison. This suggests that dyads with children who were still learning their numbers engaged in more non-symbolic comparison since it was easier, or more feasible for them. However, even when children's number knowledge was more advanced, and thus advanced strategies for comparing the magnitudes of the numbers were available to them (e.g., such as using number words), children made comparable decisions to use a simpler strategy, such as comparing the dots on the cards if it lead to an answer more easily and efficiently. Previous work investigating strategy selection in the area of math development in preschoolers often focuses on their strategy use while solving arithmetic problems and completing Piagetian conservation tasks (for review see Siegler, 1996); however, the present study extends this previous work in strategy selection by also demonstrating its importance in understanding how children use symbolic and non-symbolic representations in their everyday problem solving, whether or not they have surpassed a given acquisition or milestone (Siegler, in press).

In contrast to quantity identification, parents provided more guidance to their children when comparing the quantities of the cards during the game. This may have been due to the nature of the activity, in which the object of the card game was to directly compare the quantity of the cards, rather than only identifying the quantities. It is also likely that quantity comparison was especially challenging for children. Over half of the card pairs involved at least some form of guidance, the majority of which (55%) involved lower levels of guidance, which likely helped to move the game along, as opposed to providing help to the children in making their quantity comparison judgments. In contrast, parents did provide some higher-level guidance to their children, especially parents of 3-year-olds, also in the form of explanations and corrections of errors and direct instruction. The fact that parents and preschoolers spent time talking about magnitude comparisons is an especially important feat, as research has yet to capture parents and children discussing numbers in this way. Ramani and her colleagues (2015), for example, found that magnitude comparison between Head Start parents and preschoolers during free play with math toys, such as a number puzzle, a board game, and a number-related book was non-existent, and parents of preschoolers in Canada and in the U.S. have reported comparing numbers to be minimal in comparison to more rote practice activities such as naming numbers and counting out loud (Blevins-Knabe & Musun-Miller, 1996; Skwarchuk, 2009; Zippert & Ramani, in press).

Overall, findings from aim 1 demonstrate the importance of the design of the number activity used in the present study (e.g., such as the object of the game), the age and number knowledge of the child, and strategy selection theory (e.g., in terms of which representation makes the most sense to use to make identification and comparison judgments as opposed to using those that are more advanced, such as number words or symbols; Siegler, 1996) in attempts to encourage parents and preschoolers to explore the connections between symbolic and non-symbolic quantity information. Further, as parents reported advanced math experiences like the one in the present study as occurring infrequently at home, parents' experience with such activities should be considered as well.

Aim 2: Examining the Role of Parental Beliefs in Predicting the Frequency and Quality of Preschoolers' Early Number Experiences

In accordance with the parent socialization model, parent factors are considered especially important in understanding the learning experiences they provide their children (Eccles et al., 1983; Jacobs et al., 2005; Wigfield et al., 2006). In particular, two important characteristics include general beliefs and behaviors, and child-specific beliefs. Parents' general and child-specific beliefs are further differentiated into subtypes, including beliefs about their own and their children's abilities and interests in math, as well as beliefs about the importance of math in their own lives and for their children. Adapting and administering these items to preschool-aged children was particularly novel as the parent survey associated with the work of Eccles et al. (1983) has typically been administered to parents of school-aged children from the kindergarten age and older. Results related to parents' general beliefs suggest that they were reportedly not highly involved in math or activities involving numbers such as finances in the home. However, parents did believe that they were moderately competent in math, that it was important to be good at math, and that parents in general can influence children's math development. In terms of parents' child-specific beliefs, they viewed their children's number skills, interest in numbers, and the value and utility of numbers for their child positively. Their ability ratings tended to be lower in reference to other children, but higher when thinking about their children's number skills the following year. These results are in slight contrast with sources that document adults in the U.S. to have a tendency to hold negative beliefs about math (e.g., NRC, 2009). Perhaps, however, negative beliefs are prevalent more broadly in the U.S., but not as much in the select sample recruited.

The results also showed that questions related to child-specific beliefs and general beliefs were most strongly correlated within each of these categories rather than between these two types of categories of beliefs. Dimension reduction techniques suggested that there were multiple subgroups of belief items within these belief categories that were even more strongly associated. Specifically, there were five separate subgroups of child-specific and general beliefs. These five subgroups of belief items were created using saved factor scores from data reduction analysis and then their relations to the behavioral coding from the parent-child interaction math codes (quality) and parental reports of at-home engagement of math activities (frequency) were assessed. These findings are somewhat aligned with the constructs included in the parent socialization model (Jacobs et al., 2005) and the associated study predictions. First, they provided evidence for a distinction between the types of beliefs measured in the parent survey. Specifically, general and specific belief items did not load significantly onto the same components, which was in support of the study's predictions, but not all parental beliefs of the same type loaded onto the same component. Thus, it is suggested that the parent socialization model may not represent a parallel correlational structure for survey responses from parents of young children, which is an important contribution to the literature given this new age group.

Further, in line with the model, parents' child-specific and general beliefs were both significantly related to the nature of children's math-learning experiences. For the current study, the frequency (parent-reported math experiences at home) and the quality (observations of parent-child number exploration during the card game) of children's number experiences were separately examined, which also remain undifferentiated in the parent socialization model. Interestingly, certain subcomponents of the two belief types were predictive of different types of number experiences. Specifically, parents' general beliefs about their own abilities and values of math were associated with quality of early number experiences, or observations of parent-child number exploration during the card game, while parents' child-specific beliefs about their children's abilities and interest in math were associated with the frequency of parent-reported math experiences at home.

The current study findings align with past research in demonstrating that parents' child-specific beliefs are predictive of the frequency with which they engage their children in math at home as reported by parents (e.g., Skwarchuk et al., 2014), and additionally contributes to the literature in providing evidence that parental beliefs are also associated with the *quality* of parent-child math-related interactions, or objective observations of parent-child number exploration during a card game. A recent study provides some insight into the association between parents' general beliefs about their abilities, their interactions with their children, and subsequent math development. Maloney, Ramirez, Gunderson, Levine, and Beilock (2015) examined 529 Chicago area parents' reported levels of math anxiety, and found that parent anxiety moderated the association between homework help and first and second-graders' math achievement gains across the school year. Specifically, children made significantly lower gains during the year in their math knowledge when their highly anxious parents helped them frequently with their homework. In contrast, children with highly math anxious parents who helped them with homework less often made gains that were comparable to their peers with parents with low levels of math anxiety. The researchers suggested that parents with negative math-related beliefs could provide lower quality math guidance during math-related interactions with their children, which has the potential to negatively affect children's math learning. However, data on the actual quality of the parent-child homework helping were not collected, thus this interpretation is speculative. As parents' math anxiety has garnered a good deal of empirical attention, it should be studied in future research along with parents' actual beliefs about their math abilities, and how they both operate to influence both the actual quality of the math experiences parents provide their children.

The aforementioned correlations showed that parents' general beliefs about their ability and value of math were correlated with the quality of number exploration during the card game, whereas child-specific beliefs about children's abilities and interest in numbers were associated with the frequency of at-home engagement in advanced math activities. Regression analyses further corroborated these findings. These results are in partial support of the study predictions. For example, while they suggest that parental beliefs are differentially reflected both in the ways in which they interact with their children directly (quality) as well as the home environment they create for their children (frequency; McGillicuddy-Delisi, 1985), it was instead hypothesized in that parents' general beliefs would predict the nature of the home environment because it represents a set of beliefs that are more distal from the child, while parents' child-specific beliefs were posited to predict parents' one-on-one interactions with their children (quality of interactions), because these beliefs are more proximal to them (Bronfenbrenner, 1979; Sigel & McGillicuddy-Delisi, 1992). While misaligned with the study hypotheses, reasons for these findings could be that choosing activities and resources that shape the home environment for children (including buying games and resources for math learning) should likely depend on what parents believe their child is able to accomplish and is interested in doing. In contrast, the ways in which parents help their children during a specific task, especially the extent to which numbers are central to the interaction, may depend on the value they place on the domain (in this case math) and parents' beliefs in their abilities to provide assistance to their child and promote exploration of numbers. This argument is plausible because parents could easily shy away from supporting their children's math learning if they do not feel confident and capable in helping their children, and if they feel math is not an important domain in which to be competent.

The study's predictions about the directions of the belief-behavior associations were also only partially supported. The pathway between parental beliefs and at-home math frequency, as reported by parents, were in the expected direction, such that more positive parental beliefs about their children's math abilities and interest were associated with increased engagement in both foundational and advanced math experiences at home. However, associations between parents' general beliefs and quality (observations of parent-child number exploration during the game) of children's number experiences were not found to be in the expected direction. For example, parents' general beliefs were negatively associated with the proportion of intervals in which symbolic quantity identification and comparison occurred. While unexpected, these findings may suggest that while parents may have had negative experiences with math in school, feel negatively about their math abilities currently, and may believe that it is not important to be good at math themselves, they may feel motivated to emphasize math more with their children to ensure they have a strong foundation in number.

Overall, the strength of the associations between parents' beliefs and the quality of parent-child number-related interactions were weaker than those between beliefs and parents' reports of frequency of at-home math engagement. This is likely due to how children's number-related experiences were measured. Sigel (1992) theorized that when beliefs and behaviors are measured at the same level (e.g., via self-report), associations tend to be higher, which was certainly the case for the current study. Sigel also posited that low belief-behavior associations can exist when contexts of the two measures differ. In his study, Sigel assessed links between parents' general views about development (e.g., the extent to which children are active participants in their learning, or passive recipients of knowledge), and their guidance strategies during problem solving with their preschool-aged children. In finding low and non-significantly correlations with his measures of beliefs and behaviors, he explained these results by suggesting that his measure of beliefs was too general to apply to parents' behaviors in one specific context. Thus, one source of measurement error in the current study may also pertain to the abstractness of the parental belief items (i.e., parents' beliefs about their own abilities and values of math) and how parents assisted their child with math during one particular number-related activity. As such, parents' beliefs about math more generally may not be at play in influencing their behaviors with their child during a specific card game.

Overall, results from aim 2 suggest that it may not be appropriate to group all aspects of parents' child specific and general beliefs, respectively, into single categories, at least for parents of young children. Thus work should be conducted to determine the existence of two distinct latent constructs in explaining variation in parents' general beliefs, and those they hold that are specific to their children. Research from the current study suggests that there may not be especially strong associations between all child-specific items, and all general belief items, nor between the two types. However, in alignment with the parent socialization model (Jacobs et al., 2005), it is suggested that these beliefs are tied to the ways in which parents support their children's math development, albeit in unexpected ways, and broadens the applicability of the model in helping to explain parents' academic socialization of their younger children.

Aim 3: Examining the Role of Child Affect and Engagement in the Frequency and Quality of Number-related Experiences

While much of the work on young children's early math development has focused upon parent factors (e.g., beliefs) as predictors of how parents support their young children's math development through activity engagement (e.g., Blevins-Knabe & Musun-Miller, 1996; LeFevre et al., 2009; Skwarchuk et al., 2014), much less work has examined child contributions to the interactions, such as children's interest in the frequency and quality of engagement of math-related activities between parents and preschoolers. To date, only one known study has examined the association between child affect and the frequency with which parents engage their children in math activities at home. This study assessed children's affect towards play and exploration more broadly as opposed to math activities specifically and the data were collected via parent report, which is prone to measurement error (Lukie et al., 2013). The current study is thus the first to assess the role of children's interest in math measured through observations of children's affect as well as their behavioral engagement (i.e., attention and participation) during a number game with their parents in predicting the quality of children's math experiences. While previous research has combined all three-interest measures (Deckner et al., 2006; Malin et al., 20014), the current study found lower correlations between children's affect and measures of their engagement. As a result, these variables were examined separately.

Overall, the results showed that during the game, children exhibited little negative affect, a moderate amount of smiles (positive affect), and were highly engaged in the task as evidenced by a large amount of contributions and focused attention towards the game. The extent of children's engagement in the game is very encouraging and important to consider because mothers of preschool-aged children have reported being apprehensive about engaging their children in number-related activities (Cannon & Ginsburg, 2008). The observations in the present study showed that even in a more challenging number-related game, preschool children could remain engaged, and infrequently exhibit negative affect. It should be noted that children did not exhibit a large amount of positive affect during the game, which was not supported by the study's hypothesis. When children lacked positive affect, however, this may have been due to children's concentration on the numbers and problem solving during each turn. Anecdotally, positive affect tended to occur most frequently at the end of each turn if the child's favorite character won, as opposed to during the initial part of the turn (i.e., identifying and comparing the quantities on the cards). It is likely the case that positive affect occurs more frequently during free play than it does during tasks that may be game-like, but require concentration and problem solving. Even still, children enjoyed the activity.

Interestingly, parents' beliefs about their children's interest in numbers were not associated with children's levels of affect during the card game. Previous research has shown stronger correlations between parental beliefs and children's interests when assessed via self- and parent-report in parents of older children (Wigfield et al., 1997). This could suggest that assessing children's math interest via two methodologies (parent report and observation) may measure two different things. For example, parents' judgments about their children's interests may refer to a broader array of number-related experiences in which they have seen their child engage, which may or may not include card games. As such, children's interests in this particular activity may not be as generalizable to the experiences he or she has had. Future research that includes measures of children's interest through both parent and self-report, as well as observations of affect, attention, and engagement during a larger array of activities over time could provide better insight into how these methodologies relate, and how best to measure child interest more broadly.

The relations between children's engagement and affect and number experiences were not in the hypothesized direction. Children's negative affect, for example, was positively associated with the quality of children's number-related experiences. Specifically, incidences of negative affect was positively associated with dyads' identification of card quantities through connecting symbolic and non-symbolic quantity information, and negatively associated with dyads' symbolic quantity comparison. This is likely because children who relied upon connecting symbolic and non-symbolic representations were more likely to exhibit frustration during the game, because having to count and label most of the quantities required children to work harder at the game, and possibly cause them to make mistakes. Ultimately, this could have elicited minor disappointment from children. While infrequent, small instances of negative affect on the part of the child could very well deter families from engaging in more advanced math activities at home, and may explain why families of preschoolers tend to engage their children less in advanced activities such as these on average. The game may have also felt more monotonous for children who depended upon connecting symbolic and non-symbolic quantities, as their engagement scores tended to be lower, meaning that they made fewer numbers of unique contributions to the game, and perhaps also had lower degrees of attentiveness. This is because counting large quantities could take considerable time, especially if the child committed an error, causing them to have to repeat their counting to get the right answer. As such, children who did not spend time identifying the quantities could get through a large number of cards in a short amount of time, and thus had higher engagement scores. Similarly, children who were able to utilize more advanced symbolic representations were less likely to need to work hard, and thus were less likely to encounter any difficulties while playing the game.

It was depicted in the parent socialization model (Jacobs et al., 2005) that child characteristics, particularly their engagement in the card game, moderated the association between parents' beliefs and the children's math learning experiences with parents. Therefore, in line with the parent socialization model, regressions were conducted in the present study to examine if child interest (measured as affect and engagement) moderated the association between parental beliefs and the frequency and quality of children's number-related experiences with their parents. After controlling for children's age and math ability, child engagement in the game, was not a significant moderator of the relation between parents' specific beliefs about their child's abilities and interests and the frequency of advanced math-related activities at home. Child engagement also did not significantly moderate the association between parents' general beliefs about their own abilities and values of math and the proportion of cards identified explicitly in any representation; however, parents' math-related ability beliefs and values and child engagement significantly predicted dyads' explicit identification of quantities, but in unexpected ways. In particular, both parents' ability beliefs and child engagement negatively predicted dyads' quantity identification. As mentioned above, the negative association with child engagement could indicate that when dyads made the effort to identify numbers explicitly, this took away from the amount of time children could progress quickly through the game and earn a higher average on the engagement code. The negative association between parents' ability beliefs and values and dyads' quantity identification was especially unexpected. As discussed as a part of aim 2, these findings could suggest, however, that while parents may have had negative experiences with math in school, and feel negatively about their math abilities currently, they may feel motivated to emphasize math more with their children to ensure they have a strong foundation in number to avoid the difficulties with math in school that parents experienced. Even though interaction terms were not significant in either model, these findings provide initial evidence suggesting that certain types of parental beliefs and child engagement are associated with the quantity and quality of preschool children's math experiences.

Future Directions and Implications

The card game activity was particularly successful in encouraging parents and children to focus on the magnitudes of numbers, though future work is needed to determine how to encourage middle-income parents in this activity to pay particular attention to supporting their children's numeral knowledge development during card games. This would include prompting parents to make connections between the Arabic numeral printed on the card and the corresponding quantity of dots, such as through the game's instructions. Making connections between the symbolic and non-symbolic representations could also help support the development of children's cardinal number knowledge when it is warranted, though future research should assess how parents make these connections for their children who are currently learning this concept. Further research is also needed in determining children's processes of making the quantity identification and comparison judgments when they did not verbally indicate their strategies. Future analyses could begin by examining children's identification of each individual card and the comparison of quantities by each pair of cards. This would help to determine if the larger quantities were identified differently than smaller quantities, as well as whether pairs of numbers that differed to a greater degree in quantity were compared differently than pairs of numbers that differed by a small amount. Research suggests that reaction times in answering magnitude comparison questions are associated with numbers that span a great distance, and longer RT is associated with numbers that span a short distance (Moyer & Landauer, 1967). Thus, future work could also directly assess children's reaction times or ask them to self-report on how they made their identification and comparison decisions (e.g., which representations they used).

Relatedly, modifications to the game itself as it is conventionally played could enhance children's experiences. First, the addition of characters to the game may help children focus on numbers as opposed to winning and losing. Further, modifications to the game instructions or to the cards themselves may be warranted to encourage parents and children to more frequently explore the connection between the non-symbolic quantities and the Arabic numerals. Numerals and the cards themselves could be enlarged to encourage more focus on them; however in at least some instances, children showed higher preferences for counting if their numeral knowledge was less developed (e.g., especially for 3-year-olds). Further, to engage children who have developed numeral and cardinal knowledge, the object of the game could be shifted to determining by how much a card quantity is larger or smaller than the other. To help children develop later acquisitions, card quantities could involve numbers larger than 10, or fractions or decimals instead of small whole numbers (Siegler, in press).

Another area for future research is to focus on better defining parental beliefs, since, as demonstrated in the present study, they were significantly associated with both the frequency and quality of preschool children's math experiences. One line of this research could be to conduct further research using parent surveys with a large sample of families, and expanding upon the number of parental belief items assessed in the survey. This would allow for conducting analyses examining these beliefs in the latent domain, rather than being limited to explaining the observed variance of the variables. Another line of research could be to examine existing datasets that include measures of parent beliefs. For example, conducting factor analyses on parental beliefs measures from rich and relevant datasets, such as the Childhood and Beyond study, would provide some insight into the structure and relations between both parents' child-specific and general beliefs, and confirm the existence of two distinct latent constructs posited in the parent socialization model (i.e., child-specific and general beliefs; Jacobs et al., 2005). The model fit can also be compared for younger and older children, albeit all children in the data set are school-aged. Establishing a more specified measure of parental beliefs in this way (based on factor analytic techniques) is an important next step in that it can then be used to determine how parental beliefs predict the quality of preschool-aged children's other number-related experiences, and in other domains of math (e.g., spatial activities) for children both before and after school entry.

A final area for future research could be to examine how interventions can target improving the quantity and quality of family engagement in math-related activities for young children. This study suggests that intervening in the home to promote higher frequency and quality math activities may need to target parents' child-specific beliefs and general beliefs to achieve these two goals, respectively. However, this assumes the relations represent a causal association between parental beliefs and the math-related experiences they provide their children. As such, future research must test this assumption first. Further, in designing interventions attempting to improve quality of early math experiences, results from the current study show that children's levels of engagement in number tasks must be taken into consideration, as it significantly predicts the kinds of math concepts families explore.

Limitations

While the research design had many advantages (e.g., behavioral observations, in depth parent survey), the sample was still of convenience, and the nature of the study was correlational in that all data were collected at a single time point. Thus, causation could not be inferred from the relations found, for example, between parental beliefs, child interest, and dyads' quantity exploration during the game and in terms of families' at-home math activity engagement. The sample was also limited in that it represented mostly middle-income families who could afford the admission fee to the museum. Typically, children from middle-income families tend to have more advanced symbolic number knowledge than low-income children (e.g., Jordan et al., 1992). This mostly impacted our results in that children did not need much assistance in understanding the cardinality principle, since the majority of the children scored well on this subscale. It is likely that low-income families may benefit from this game more, as their children might have more to gain, and a higher need to practice with numbers, given that their number skills on average tend to trail behind their middle-income peers (e.g., Ramani & Siegler, 2008).

The study also was limited in the ways in which dyads' quantity exploration could be captured. As was previously discussed, many of the dyads' turns were completed without explicitly focusing on number. Due to the nature of the coding scheme, it could not be deciphered on which turns families were using symbolic numbers, albeit implicitly, or disregarded quantities altogether. This was somewhat evident when children made more or less random points to characters to indicate the winner of each round; however to indicate that number cognition was occurring would be an overstatement in at least some cases. Even when dyads were discussing numbers, it was often unclear whether they were subitizing or were labeling the numerals. The results show that dyads' numeral identification was likely underestimated due to the limited gestural cues provided by the dyads to infer they were being identified. However, the present study provides an initial step in demonstrating the kinds of behavioral coding that can be used to provide in-depth insight into the nature of parent-children interactions while engaging in a card game focusing on numbers on their magnitudes.

Finally, while a wealth of demographic information was obtained from parents and children, we did not include in our survey a question about children's childcare arrangement. This may have impacted children's math knowledge, their ability to be engaged and attentive in the game, and their overall number talk. Future work should examine this variable and its effects on the quality of parent-child number-related interactions.

Conclusions

Playing card games such as *War* are highly conducive to joint exploration of numbers between parents and their preschoolers. This fun and engaging game provides parents and children with a meaningful context for practicing identifying numbers, and allows them to make connections between symbolic and non-symbolic quantity representations. Whether children have adept knowledge of numbers or are still acquiring their understandings of them, children of this age remain attentive and engaged during the activity, and can rely on the non-symbolic quantities for most of the card pairs to make their quantity comparison judgments.

Since families reported advanced math activities such as card games as occurring very infrequently at home, they should be encouraged to engage their children in such activities more so during their free time. Card game play could be a simple and engaging way for parents to support core concepts of number at home. These games could also be easily adopted into homes because they are relatively inexpensive to buy and easy to make. Further, the need to encourage families to support children's numeral and cardinal knowledge in particular may be accomplished through teaching families about how early numerical magnitude knowledge develops and how it could be supported through playing games such as *War*. Informing families and providing more tailored game instructions could be based on recent theoretical frameworks of number development (e.g., Siegler, in press), as well as previous empirical work on using numerical games to support young children's numerical magnitude knowledge development (Ramani et al., 2012).

Both parent and child contributions arose as salient factors to consider in how parent-preschooler dyads explored numbers during the card game, and in the frequency with which parents support their children's math knowledge at home. Thus, researchers and practitioners should focus their efforts to encourage early math learning through more frequent and higher quality experiences with numbers for children, but to also consider that children's engagement and affect must be taken into account to achieve sustained attention and participation in the game. Further, parents must hold both positive beliefs about their own abilities and values of math, as well as perceive their children's affect and abilities as high. Future research that intervenes on these two factors may make an important contribution to how we understand how children can learn to connect the dots between mathematical concepts at home.

Tables and Figures

Tables

Table 1: Quantity Identification Codes

Category	Number Representation	Behavior	Example
No quantity	None	0 = No explicit quantity reference	No explicit identification of quantities
Non-symbolic	Array	1 = Relative/approximate quantity ID	Indicates that a card contains "a lot of dots"
Symbolic *with* non-symbolic referent	Number word to array	2 = Counting dots	Points to each dot and counts them out loud (e.g., "1, 2, 3, 4" while pointing to each dot)
		3 = Label set size and count dots	Labels dots *and* counts them consecutively (e.g., "1, 2, 3, 4, that's four dots" or the reverse)
	Numeral to array	4 = Connecting a numeral with dots	Indicating that the printed number 4 represents 4 dots (e.g., counting the dots and then pointing and labeling the written numeral *4*)
Symbolic *without* non-symbolic referent	Number word	5 = Stating number name without referent	Identifies number name without referring to the written numeral or the dots (e.g., 'Your card is a "5" and mine is a "3"')
	Numeral	6 = Numeral naming without referent	Discusses the numeral on the card (e.g., "this is a 4", while pointing to the 4)

Note. Coding system adapted from Mix et al., 2012

Table 2: Quantity Comparison Codes

Category	Number Representation	Definition
No quantity	Level = 0 None	Dyad completes the turn without talk about comparing magnitudes ("Ernie gets both cards!"), or evidence for quantity comparison is unclear
Non-symbolic	Level = 1 Arrays	Dyad compares the relative size of the arrays on the two cards (e.g., which is more, these dots or those?"), or evidence exists that the dots are being explored through nonverbal gesture
Symbolic	Level = 2 Number words	Dyad discusses which number word is bigger (e.g., "4 is bigger than 2")
	Level = 3 Numerals	Discussing which numeral is bigger (e.g., "4 is bigger than 2" while pointing to each Arabic numeral)

Table 3: Guidance Codes

Behavior	General Definition	Quantity ID Example	Quantity Comparison Example
Level 0: No guidance	Turn is completed entirely by the child.		
Level 1: General prompt	The parent encourages the child to participate by asking general questions without providing strategies to answer them.	"Turn over your card and see what you get" "Let's get the next card" "What card do you have?"	"Whose card has more, mine or yours?"
Level 2: Instruction/Specific prompt	Parent provides a specific prompt or strategy to complete the step, without indicating exactly how to complete it.	"Why don't you try counting the dots on the card?" "What number is this?" "What comes after 8?" "Point to every dot you count." "How many dots on your card?"	"Are there more dots on my card or your card? "Which number is bigger?" "Which is bigger 8 or 9?" "Which of these is bigger 8 or 9?" while pointing to Arabic numerals
Level 3-Modeling/Co-participation	Parents demonstrate how to take the turn and allow the child to imitate, or parents and children complete the task together	"Watch me count first, *one, two, three, four. Now you try."*	"8 is bigger than 4., so which is more?"

Level 4-Explanation/Correction	Parent extends/explains or corrects child's answer	Child counts 5 dots and stops. Then Parent says "Right, you got 5 dots", or child errs and counts only 4, then parent points to the dot missed and says, "No there are 5 dots."	Child points to the correct card in comparing who has more, and mom says, "Right, because 5 is bigger than 4."
Level 5: Direct guidance	Parent gives answer without child participation	"This is the number 8"	"Bert has the bigger number so he gets to keep both of them!"

Table 4: Demographic Data

Child	
Age	
3	47(50%)
4	47(50%)
Gender	
Male	42(45%)
Female	52(55%)
Race/ethnicity	
White/Caucasian	68(72%)
African American	8(9%)
Hispanic	2(2%)
Asian/Pacific Islander	7(7%)
Biracial/Mixed race	9(10%)
Math knowledge [a]	
3-year-olds	19.17(8.05)
4-year-olds	31.87(7.06)
All ages	25.52(9.87)
Parent	
Gender	
Father	38(40%)
Mother	56 (60%)
Race/ethnicity	
White/Caucasian	69(74%)
African American	10(11%)
Hispanic	2(2%)
Asian/Pacific Islander	9(10%)
Mixed race	3(3%)
Education of both of children's parents [b]	
Father	4.93(1.25)
Mother	4.99(1.05)
Education of participating parent only [b]	
Father	5.08(1.02)
Mother	4.96(1.00)
Income [c]	6.26(1.67)
PD Attendance [d]	1.36(.62)
PD members	31%

[a] =Out of 46 points, [b]=On a scale from 1 = some education, 2 = HS/GED, 3= Some College, 4 = Associates, 5= BA/BS, 6 = Postgrad. [c] = 1= On a scale from <$15k to 8= $151k+. [d] on a scale from 1= rarely/never to 5 = everyday or almost everyday.

Table 5: Descriptive Statistics of Children's Numerical Knowledge

	Age		
	3	4	Total
Numeracy scale	(n =47)	(n =47)	(n =94)
Total Numeracy Score	.42(.17)	.69(.15)	.55(.21)
Nonsymbolic			
Set comparison (which has most dots)	.58(.40)	.88(.25)	.73(.36)
Set comparison (which has least dots)	.52(.37)	.82(.25)	.67(.35)
Symbolic with non-symbolic			
Structured counting (count objects)	.43(.26)	.69(.25)	.56(.29)
Cardinality (label after count)	.50(.43)	.90(.25)	.70(.40)
Set to numeral (match dots to numeral)	.34(.29)	.75(.23)	.55(.33)
Subitizing (match number word to dots)	.47(.16)	.65(.47)	.56(.18)
Symbolic without non-symbolic			
Rote counting (number words)[a]	.40(.19)	.62(.21)	.51(.23)
Magnitude comparison (number words)	.11(.21)	.46(.37)	.28(.35)
Numeral identification (numerals)	.46(.28)	.69(.26)	.58(.29)
Magnitude comparison (numerals)	.23(.27)	.52(.29)	.37(.31)

Note. [a] = Rote counting was originally scored as the highest number reached during rote counting without error, and then was re-coded into a 5-point scale in which counting to 5 = 1pt, counting to 10 = 2pts, counting to 15 = 3pts, counting to 20 = 4pts, and counting to 40 = 5pts.

Table 6: Descriptive Statistics of Frequency of Home Math Engagement and Parental Beliefs

Math Activities and Parental Beliefs	Age 3	Age 4	Total
At-home Math Engagement[a]			
Foundational Math Activities	4.17(.72)	4.38(.54)	4.27(.64)
Count out loud	4.51(.72)	4.57(.54)	4.54(.63)
Count items	4.57(.65)	4.66(.52)	4.62(.59)
Determine 'how many' in a group of objects	3.43(1.38)	4.02(.94)	3.72(1.21)
Identify or name numbers	4.17(.87)	4.26(.82)	4.21(.84)
Advanced Math Activities	2.51(.89)	3.13(.95)	2.82(.97)
Compare written numbers and identify which is bigger	2.23(1.42)	3.04(1.27)	2.64(1.40)
Compare 2 or more quantities of objects to determine which is more	3.02(1.45)	3.36(1.24)	3.19(1.35)
Perform simple addition	2.38(1.26)	3.32(1.32)	2.85(1.37)
Play card games	2.19(1.17)	2.87(1.23)	2.53(1.23)
Play board games	2.74(1.13)	3.06(1.13)	2.90(1.14)
Others activities (not included in analysis)			
Play educational video games	3.06(1.37)	3.32(1.40)	3.19(1.38)
Play educational computer games	2.66(1.39)	3.19(1.38)	2.92(1.41)
Sing nursery rhymes	4.47(.78)	4.26(.90)	4.36(.84)
Name shapes	4.09(.88)	4.13(.80)	4.11(.84)
Parental Beliefs			
General beliefs and personal math involvement			
Time spent on activities[b]			
Math			2.66(1.56)
Finances			2.22(1.10)
Ability beliefs[c]			
Good in math in school			5.07(1.71)
Good at math currently			4.94(1.62)
Value of math[d]			
Importance of being good at math			5.50(1.52)
Role perception			
Influence parents have on children's math learning[e]			6.05(1.09)
Child-specific beliefs			
Ability and interest beliefs			
How good is your child at numbers[c]			5.45(1.29)
Child's innate number ability compared to other children[f]			5.13(1.19)
How well do you think your child will do with numbers next year [g]			6.07(1.03)
How much does your child like numbers [h]			5.46(1.27)
Values			
Importance of child doing well with numbers [i]			6.37(.85)
Usefulness of numbers to child in future [j]			6.54(.83)
Importance of counting to 10 [d]			6.64(.88)
Importance of counting to 20 [d]			6.25(1.31)
Importance of naming numbers to 10 [d]			6.42(1.12)

Importance of naming numbers to 20 [d]	5.90(1.46)
Importance of comparing numbers [d]	5.85(1.34)
Importance of doing addition and subtraction [d]	5.03(1.76)

[a] On a scale from 1 = Rarely or never, 2 = monthly 3 = weekly 4 = Several days per week 5 = Every day or almost every day [b] = Scale 1 = 0 hours, 2 = 1-3 hours, 3 = 4-6 hours, 4 = 7-10 hours, 5 = 11-15 hours, 6 = 16-20 hours, 7 = more than 20 hours. [c] On a scale from 1 = Not good at all, 7 = very good. [c] On a scale from 1 = Not good at all, 7 = Very good. [d] On a scale from 1 = Not important at all, 7 = very important. [e] on a scale from 1 = very little, 7 = a great deal. [g] On a scale from 1 = much less than other children, 7 = much more than other children. [g] on a scale from 1 = not well at all to 7 = very well. [h] On a scale from 1 = not at all, to 7 = very much. [i] On a scale from 1 = not very important, to 7 = very important. [j] On a scale from 1 = not at all useful, to 7 = very useful

Table 7: Descriptive Statistics of Quantity Identification, Comparison, and Associated Guidance

	Age		
	3	4	Total
Category	(n =47)	(n =47)	(n =94)
Length of interaction (minutes)	5.35(2.52)	3.30(1.46)	4.48(2.23)
Number of turns	11.98(2.31)	12.53(1.57)	12.26(1.98)
Quantity Identification[a]	.52(.34)	.38(.31)	.45(.33)
No quantity	.48(.34)	.62(.31)	.55(.33)
Non-symbolic	.00(.01)	.00(.01)	.00(.01)
Symbolic with non-symbolic	.21(.27)	.10(.17)	.15(.23)
Count dots	.15(.22)	.08(.14)	.12(.19)
Label number name and count	.05(.10)	.01(.04)	.03(.08)
Label numeral and count	.00(.03)	.00(.00)	.00(.02)
Symbolic without non-symbolic	.32(.30)	.28(.29)	.30(.29)
Number name	.24(.24)	.23(.25)	.30(.29)
Numeral name	.08(.15)	.05(.10)	.06(.12)
Quantity Comparison[b]			
No quantity comparison	.47(.33)	.59(.31)	.53(.33)
Non-symbolic	.26(.26)	.22(.28)	.24(.27)
Symbolic	.27(.28)	.19(.23)	.23(.26)
Number name	.27(.28)	.19(.22)	.23(.26)
Numeral	.00(.02)	.01(.02)	.01(.02)
Guidance codes			
Quantity identification [a]			
No guidance	.55(.31)	.74(.25)	.64(.30)
Medium guidance	.17(.20)	.15(.17)	.16(.19)
General prompt	.04(.07)	.05(.10)	.04(.09)
Specific prompt	.13(.18)	.10(.14)	.11(.16)
High guidance	.28(.27)	.11(.17)	.20(.24)
Modeling/Co-participation	.06(.14)	.02(.04)	.04(.11)
Explanation and correction	.07(.13)	.01(.03)	.04(.10)
Direct instruction	.15(.19)	.08(.16)	.12(.18)
Quantity comparison [b]			
No guidance	.12(.20)	.33(.28)	.23(.26)
Medium guidance	.58(.28)	.52(.28)	.55(.28)
General prompt	.37(.23)	.41(.23)	.39(.24)
Specific prompt	.21(.23)	.11(.18)	.16(.21)

High guidance	.30(.25)	.15(.18)	.22(.23)
Modeling/Co-participation	.00(.01)	.00(.02)	.00(.01)
Explanation and correction	.18(.20)	.07(.11)	.12(.17)
Direct instruction	.12(.19)	.07(.14)	.10(.17)

[a] = Means represent proportion of cards played. [b] = Means represent proportion of pairs compared

Table 8: Correlations of Demographics, Quantity Identification and Comparison Codes, and Home Math Engagement

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1. Length of interaction	-																		
2. Age	-.39**	-																	
3. Child gender	.06	-.04	-																
4. Parent gender (mother)	.04	.00	.04	-															
5. Parent education	-.01	.14	.07	.06	-														
6. Household income	.05	.06	.18	-.04	.39**	-													
7. Number Knowledge	-.25*	.65**	.06	.04	.29**	.20*	-												
8. Engagement	-.38**	.42**	-.01	-.04	-.04	.05	.35**	-											
9. Positive affect	.15	.19	.00	-.06	-.09	-.19	.05	.08	-										
10. Negative affect	.18	-.07	-.22*	.03	-.05	-.18	-.14	-.30**	-.03	-									
11. No ID	-.65**	.22*	.13	-.03	.01	.00	.17	.30**	-.17	-.11	-								
12. ID Non-Symbolic	-.07	.00	-.02	.03	.15	.02	-.01	.04	-.06	-.07	-.03	-							
13. ID Symbolic	.17	-.06	-.08	.05	-.08	-.04	-.01	-.16	.09	-.14	.73**	.08	-						
14. ID Symbolic and Nonsymbolic	.72**	-.24*	-.09	-.02	.08	.05	-.22*	-.23*	.14	.32**	-.52**	-.08	-.21	-					
15. No Comparison	-.31**	.19	-.10	-.25*	.06	-.05	.14	.11	.05	.18	.12	.01	-.10	-.05	-				
16. Comparison Non-symbolic	.07	-.09	.06	.29**	-.11	-.01	-.23*	.01	-.12	-.02	.19	.04	-.28**	.08	-.63**	-			
17. Comparison Symbolic	.32**	-.15	.06	.02	.04	.07	.07	-.15	.06	-.21*	-.34**	-.05	.41**	-.02	-.61**	-.23*	-		
18. Home Foundational	-.06	.16	-.08	.00	.12	.00	.20	.18	.05	-.11	.03	.02	-.03	-.02	.08	-.11	.01	-	
19. Home Advanced	-.23**	.32**	-.06	-.03	-.09	-.10	.23*	.20*	.04	-.13	.14	.09	-.04	-.15	.13	-.01	-.15	.54**	

** = p <.01, * = p < .05

Table 9: Correlations Between Beliefs, Frequency of Home Math Engagement, Quantity Identification and Comparison Codes

	Home Freq.			General beliefs/involvement					Child-specific beliefs						Quantity Exploration			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	...
1. Foundational math at home	-																	
2. Advanced math at home	.54**	-																
3. Time spent doing math	.24*	.10	-															
4. Time spent on finances	.10	.06	.30**	-														
5. Parent math ability in school	-.10	-.11	.29**	-.03	-													
6. Parent math ability currently	.00	-.03	.28**	-.03	.80**	-												
7. Import. of being good at math	-.05	.02	.18	.06	.40**	.54**	-											
8. Parent influence on child math	.03	-.01	.21*	.18	.18	.05	.23*	-										
9. Child number ability currently	.34**	.27**	-.03	-.05	.10	.15	-.02	-.13	-									
10. Child number ability next year	.23*	.18	.11	.00	.16	.16	.14	.10	.62**	-								
11. Child number ability vs. others	.25*	.29**	.02	-.08	.10	.15	.18	-.02	.56**	.58**	-							
12. Child interest in number	.35**	.17	.10	-.05	.12	.24*	.34**	.06	.56**	.58**	.62**	-						
13. Importance of numbers to child	.10	.13	.21*	.07	.02	.07	.31**	.27**	.01	.24*	.31**	.24*	-					
14. Future use of number to child	.12	.12	.27**	.07	.15	.19	.41**	.29**	.01	.23*	.20*	.35**	.73**	-				
15. No Quantity ID	.03	.09	.11	.16	.28**	.26*	.33**	-.03	.09	-.05	.16	.19	.06	-.04	-			
16. Non-symbolic Quantity ID	.02	.08	-.11	-.10	.04	.01	.00	-.08	.06	-.01	-.08	.06	.02	.08	-	-		
17. Symbolic Quantity ID	-.03	-.04	-.14	-.21*	-.33**	-.28**	-.36**	-.08	-.01	-.02	-.02	-.10	-.01	.02	-	-	-	
18. Symbolic and Non-symbolic ID	-.02	-.15	.03	.05	.01	-.02	-.02	.14	-.11	.10	-.20*	-.15	-.07	.03	-	-	-	...
19. No Quantity Comparison	.08	.13	.10	.16	.11	.13	.15	.10	-.05	-.13	-.08	.04	.01	.06	-	-	-	...
20. Comparison Non- Symbolic	-.11	-.01	-.12	-.01	.10	.08	-.01	.09	-.12	-.06	.00	-.14	.03	.00	-	-	-	...
21. Comparison Symbolic	.01	-.15	-.010	-.20	-.24*	-.25*	-.18	-.22*	.19	.23*	.09	.09	-.04	-.07	-	-	-	...

** = p <.01, * = p < .05

Table 10: Parental Belief Variable Loadings onto Principal Components, and Communalities

Variable Principal Components Loading with Quartimax Rotation						
	Child-Specific Value of Number Activities	Child-specific Ability and Interest Beliefs	General beliefs about abilities and value	Child-Specific Values	Parent Personal Involve-ment	Comm-unalities
Time spent on activities						
Math					.69	.64
Finances					.86	.76
General Parent Ability beliefs						
Good in math in school			.89			.82
Good at math currently			.92			.88
Value of math						
Importance of being good at math			.60			.61
Child **Ability and interest beliefs**						
How good is your child at numbers		.85				.77
Child Innate ability compared to others		.82				.71
Child number ability next year		.84				.72
How much does your child like numbers		.80				.70
Values						
Importance of child doing well with numbers				.81		.83
Usefulness of numbers to child in future				.88		.85
Importance of counting to 10	.80					.69
Importance of counting to 20	.90					.81
Importance of naming numbers to 10	.86					.78
Importance of naming numbers to 20	.92					.86
Importance of comparing numbers	.83					.76
Importance of doing addition and subtraction	.72					.60

Note. The item about parents' general beliefs about their role in their children's math development was removed because it did not load substantially on any factor, loading = .37 and .41 on components 3 and 4.

Table 11: Regression Models Predicting At-Home Math Engagement

	Foundational Math Experience		*Advanced Math Experience*	
Predictors	β	ΔR^2	β	ΔR^2
Step 1		.04		.10
Age (in years)	.06		.29*	
PENS	.16		.04	
Step 2		.01		.02
Age (in years)	.07		.30*	
PENS	.18		.07	
General beliefs	-.12		-.14	
Step 3		.09		.05
Age (in years)	.13		.35**	
PENS	.02		-.05	
General beliefs	-.10		-.13	
Child-specific	.33**		.25*	
Total R^2		.15		.18
F	3.81**		4.78**	
F df's	(4, 93)		(4, 93)	

** = p <.01, * = p < .05

Table 12: Regression Models Predicting Dyads' Quantity Exploration

	Quantity Identification						*Quantity Comparison*					
	None		*Symbolic*		*Symbolic and Nonsymbolic*		*None*		*Nonsymbolic*		*Symbolic*	
Predictors	β	ΔR^2	β	ΔR^2	β	ΔR^2	β	ΔR^2	β	ΔR^2	β	ΔR^2
Step 1		.05		.01		.07*		.04		.06		.07*
Age	.19		-.09		-.17		.17		.11		-.33**	
PENS	.04		.04		-.11		.03		-.31*		.28**	
Step 2		.07*		.13*		.00		.02		.01		.07**
Age	.19		-.08		-.17		.17		.11		-.33**	
PENS	.00		.10		-.12		.01		-.33*		.33**	
General beliefs	.27**		-.36**		.06		.13		.11		-.27**	
Step 3		.01		.00		.01		.01		.00		.02
Age	.21		-.09		-.19		.15		.11		-.30**	
PENS	-.05		.12		-.08		.06		-.33*		.26**	
General beliefs	.28**		-.36**		.06		.12		.11		-.26**	
Child-specific	.10		-.05		-.08		-.12		.00		.14	
Total R^2	.13		.13		.07		.06		.07		.07	
F	3.27*		3.41*		1.78		1.50		1.77		4.08*	
F dfs	(4, 93)		(4, 93)		(4, 93)		(4, 93)		(4, 93)		(4, 93)	

Note. Non-symbolic quantity identification was not included as a dependent variable because of its infrequent occurrence. Quantity identification via any representation, or the sum of symbolic, non-symbolic, and symbolic with non-symbolic identification was also included as a dependent variable, and ΔR^2 values were the same, though beta values had opposite signs. ** = p <.01, * = p < .05

Table 13: Descriptive Statistics and Reliability of Child Affect and Engagement Measures

	Age		
Category	3	4	Total
Engagement	4.22(.47)	4.59(.33)	4.40(.44)
Active participation[a]	4.13(.55)	4.68(.37)	4.40(.54)
Attention[b]	4.30(.48)	4.50(.39)	4.40(.45)
Correlation	.61**	.47**	.58**
Cronbach's Alpha	.76	.62	.73
Affect[c]			
Positive	2.30(1.21)	2.79(1.38)	2.54(1.32)
Negative	1.26(.49)	1.19(.40)	1.22(.44)

Note. Codes were applied in 30-second intervals. **$p < .01$ [a] = On a scale from 1 = no participation to 5= more than 4 contributions to the game. [b] = On a scale from 1= not paying attention for entire 30-second interval 5= paying attention for entire interval. [c] = On a scale from 1 = No positive or negative affect displayed to 7 = Constant smiling and laughing or always scowling/frowning voice in harsh tones.

Table 14: Moderation Analyses

	Advanced Home Math Activity (Frequency)		Quantity Identification (Quality)
Predictors	β	Predictors	β
Age	.61**	Age	-.07
PENS	-.41**	PENS	-.04
Child-Specific Beliefs[a]	.26**	General Beliefs[b]	-.08*
Engage[c]	.19	Engage[c]	-.17*
Belief x Interest	-.11	Belief x Interest	.00
Total R^2	.17**	Total R^2	.16**
F	3.62	F	3.32
F dfs	(5, 88)	F dfs	(5, 88)

[a] Child-specific beliefs is measured as the regression scores for the principal component representing parents' beliefs about children's abilities and interest in numbers. [b] General beliefs are measured as the regression scores for the principal component representing parents' beliefs about their own math abilities and value of math. [c] Child interest was measured by children's engagement levels, or their average participation and attention scores during the game.

Figures

Figure 1: Adapted Version of the Parent Socialization Model

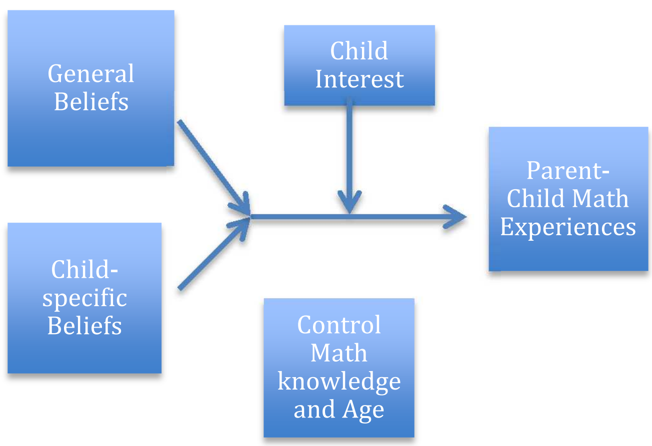
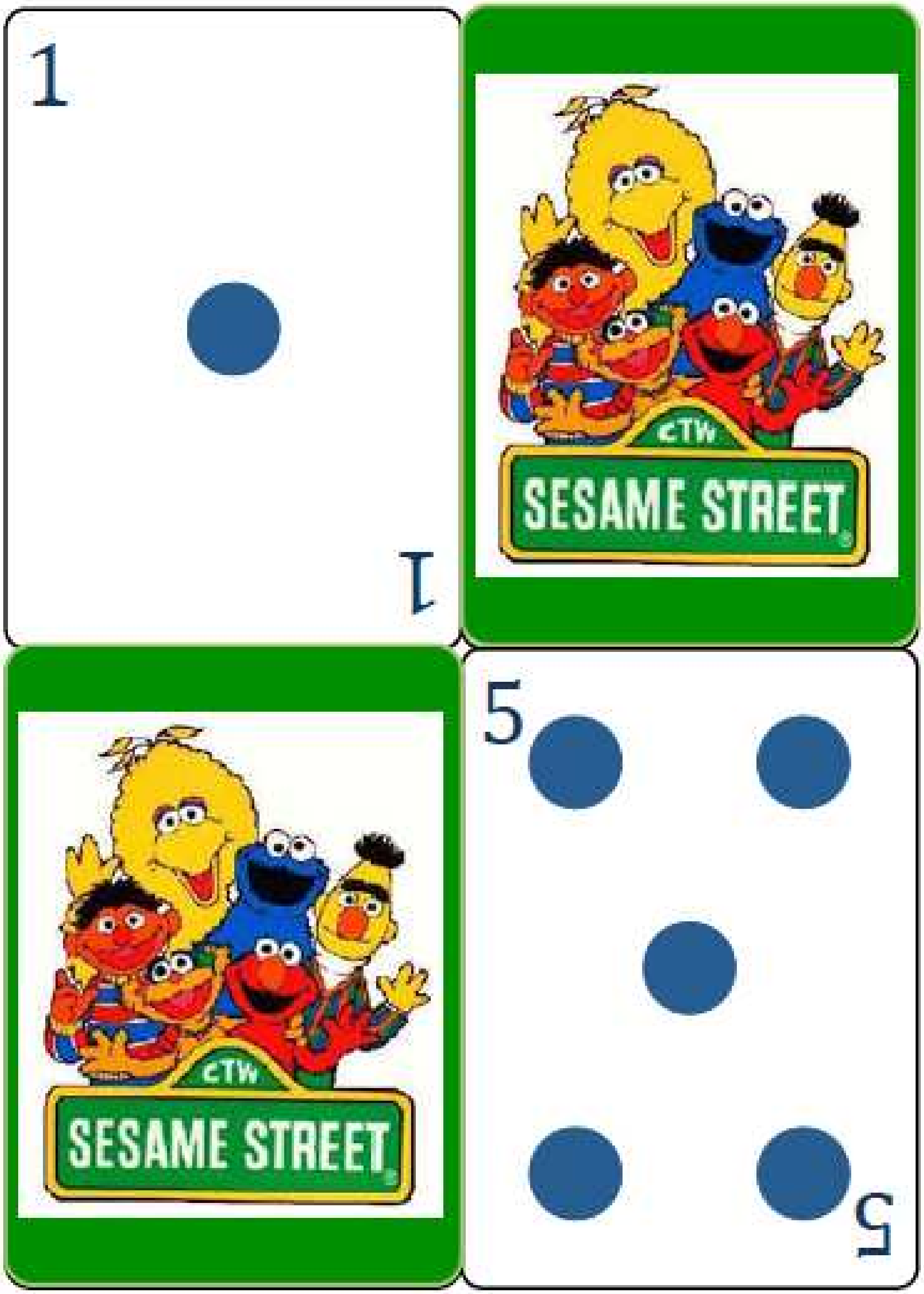


Figure 2: Example of Playing Cards



Appendix

Appendix A: Script for Recruiting Museum Visitors to Participate in the Study

"Hello, my name is XX and I am a doctoral student/research assistant at the University of Maryland College Park. You may have seen Port Discovery's list of events for the day, but in case you haven't, Port Discovery has given us permission to conduct a research study today, right here in the museum with visiting families of preschool-aged children. If you have a minute, I would love to tell you about the study and see if you and your child may be interested in participating."

If the parent is interested in learning more, continue to describe the study purpose and rationale.

"We are currently conducting a project studying children's early academic development, and how this may come about during play. There is reason to believe that much development occurs in children's academic skills before they even enter school, and think that children's early experiences such as play may contribute to their learning.

"We are inviting preschool children and their parents to participate in this project, and we would really like for you and your child to take part. If you agree to participate, you will meet a researcher in a designated area of the museum and partake in a 2-part research study. First, your child will complete several age-appropriate academic games in a one-on-one session with a researcher. During this time, we will ask you to fill out a brief questionnaire about early learning. Second, you and your child will participate in a videotaped parent-child interaction where you will play a short card game. Each part will last approximately 15 minutes, so the entire visit will last approximately 30 minutes."

"To thank you for participating, you will receive a flyer listing fun learning activities you can engage in with your child at home. Also, your child will be given a small toy."

"Information on all the participants in this research will remain confidential. There are no risks to those who participate in this study. There is no penalty for deciding not to participate in the study. If you and your child agree to participate but change your mind during the sessions, there is no penalty. You or your child can decide to stop participating at any time."

"Does this sound interesting to you?" If yes, give parents a consent form to fill out and discuss when they would like to begin.

Appendix B: Consent Form and Demographic Survey

UNIVERSITY OF MARYLAND

College of Education
3304 Benjamin Building
College Park, Maryland 20742-1311

INSTITUTE FOR CHILD STUDY
DEPARTMENT OF HUMAN DEVELOPMENT AND QUANTITATIVE METHODOLOGY

TEL: 301.405.8777
EMAIL: gramani@umd.edu

Dear Parents,
Have you ever noticed how fast your pre-school child learns and develops? Have you ever marveled at their ability to learn from any situation or activity?

We are currently conducting a project studying how children learn academic skills through play. There is reason to believe that much development occurs in children's academic skills before they even attend school. This research is very important in examining how and why children come to school already with a wealth of academic knowledge.

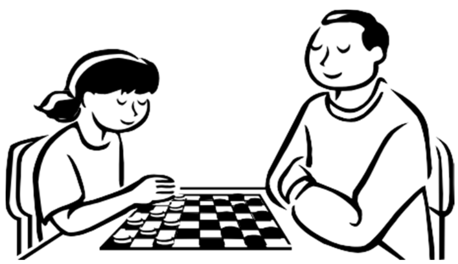
We are inviting preschool children and their parents to participate in this project, and we would really like for you and your child to take part. If you agree to participate, you will meet a researcher in a designated area of the museum and partake in a 2-part research study. First, your child will complete several age-appropriate academic games in a one-on-one session with a researcher. During this time, we will ask you to fill out a brief questionnaire about early learning. Second, you and your child will participate in a videotaped parent-child interaction where you will play a short card game. Each session will last approximately 10-15 minutes, so the entire visit will last no longer than 30 minutes. This study is being conducted to learn more about early play and learning, so you and your child may not benefit from the study beyond the enjoyment you feel playing our games with us. To thank you for participating, your child will receive a small toy, and you will receive a flyer listing fun learning activities you can engage in with your child at home.

Information on all the participants in this research will remain confidential. All of the records for the research study will be stored in locked cabinets, and only the research team will have access to the records.

There are no risks to those who participate in this study. There is no penalty for deciding not to participate in the study. If you and your child agree to participate but changes his or her mind during the session, there is no penalty. You or your child can decide to stop participating at any time.

Attached you will find a consent form and a brief questionnaire on at-home play and learning activities. If you are interested in having your child participate, please sign and return both the consent form and questionnaire to the researcher on site at the museum. If you have any questions, please feel free to contact me by phone at (301) 405-8777, or by email at gramani@umd.edu.

Thank you for your cooperation. Your help is greatly appreciated!



Geetha Ramani, Ph.D.
Associate Professor
Department of Human Development and Quantitative Methodology, University of Maryland

Erica Zippert, Doctoral Candidate
Department of Human Development and Quantitative Methodology, University of Maryland

INSTITUTE FOR CHILD STUDY
DEPARTMENT OF HUMAN DEVELOPMENT
College of Education
3304 Benjamin Building
College Park, Maryland 20742-1311

Project Title	Early Play and Learning Study
Purpose of the Study	This research is being conducted by Geetha Ramani and Erica Zippert at the University of Maryland, College Park. We are inviting you to participate in this research project because your child is of the preschool age. The purpose of this research project is to examine how children could learn about academic skills when interacting with their parents during informal activities.
Procedures	The procedures involve the following and will last about 30 minutes to complete during your visit to the museum. 1) A researcher will contact you to participate in the study. 2) If you agree to participate, you will fill out this consent form, and then a researcher will lead you and your child to a designated location in the museum. Your child will meet one-on-one with a researcher to play academic games and you will be given a brief demographic survey and questionnaire to complete about early learning. 3) Lastly, you will participate in a videotaped parent-child interaction playing a card game with your child. Afterwards, your child will receive a small toy, and you will receive an information sheet about ideas for educational games you can play at home.
Potential Risks and Discomforts	There are no known risks associated with participating in this research project.
Potential Benefits	There are no direct benefits from participation in this research. However, possible benefits include helping expand our knowledge about what parents think about early academic development and the types of experiences they provide for their children. We hope that, in the future, other people might benefit from this study through improved understanding of how parents engage their children in play and think about early learning.

Confidentiality	Any potential loss of confidentiality will be minimized by: (1) storing records and data in locked cabinets; (2) not including yours or your child's name on the collected data; (3) a code will be placed on the collected data; (4) through the use of an identification key, we will be able to link our data to you and your child's identity; and (5) only the researchers will have access to the identification key. If we write a report or article about this research project, your identity will be protected to the maximum extent possible. Your information may be shared with representatives of the University of Maryland, College Park or governmental authorities if you or someone else is in danger or if we are required to do so by law.

Right to Withdraw and Questions	Your participation in this research is completely voluntary. You may choose not to take part at all. If you decide to participate in this research, you may stop participating at any time. If you decide not to participate in this study or if you stop participating at any time, you will not be penalized or lose any benefits to which you otherwise qualify. If you decide to stop taking part in the study, if you have questions, concerns, or complaints, or if you need to report an injury related to the research, please contact the investigator: *Geetha Ramani at 3304 Benjamin Building; Human Development Department; University of Maryland; College, Park, MD 20742, 301-405-8777 or gramani@umd.edu.*
Participant Rights	If you have questions about your rights as a research participant or wish to report a research-related injury, please contact: **University of Maryland College Park** **Institutional Review Board Office** **1204 Marie Mount Hall** **College Park, Maryland, 20742** **E-mail: irb@umd.edu** **Telephone: 301-405-0678** This research has been reviewed according to the University of Maryland, College Park IRB procedures for research involving human subjects.
Statement of Consent	Your signature indicates that you are at least 18 years of age; you have read this consent form or have had it read to you; your questions have been answered to your satisfaction and you voluntarily agree to participate in this research study. You will receive a copy of this signed consent form. If you agree to participate, please sign your name below.
Signature and Date	**NAME OF CHILD [Please Print]** | **NAME OF PARENT [Please Print]** | **RELATION TO CHILD (e.g., mother, father, aunt, uncle, grandmother)** |

	SIGNATURE OF PARENT/CAREGIVER	
	DATE	

Educational Uses of Video Records	Occasionally, we use video records for classroom demonstrations, training purposes, and presentations in scientific meetings. This project may be of interest to graduate or undergraduate classes, to research staff and/or other researchers. Your or your child's name will not be associated with these additional uses of the video records.
Participant Rights	The consent for you and your child's video records to be used for additional uses is optional. You and your child will still be videotaped as part of the study, but we will only use these video records for educational and demonstration uses if you give consent for us to do so. The permission for you and your child's video records to be used for non-research purposes is not required for participation in the current study and it will not impact your child's eligibility for the current study.
Please check <u>one</u> box.	If you agree that you and your child's video records can be used for educational and demonstration purposes, you or your child will not be identified by name in any publication or presentation. ___Additional **consent given** for you and your child's video records to be used for educational and demonstration purposes. ___Additional **consent not given** for you and your child' records to be used for educational and demonstration purposes.
Statement of Age of Subject and Consent	Your signature indicates that: you are a parent or guardian of the child; the use of you and your child's video records for non-research purposes has been explained to you; your questions have been fully answered; and you freely and voluntarily choose to give your consent for your child's video records to be used for non-research purposes.
	NAME OF PARENT (Print)
	SIGNATURE OF PARENT
	DATE

Please tell us a little about you and your child. The information will be kept confidential and will not influence you and your child's eligibility to participate in the study.

1. Your child's age (in years)_________and date of birth (month, day, year) ____/_____/_____

2. Your child's gender (circle one): MALE FEMALE

3. Your child's race/ethnicity (Check the <u>one</u> that best describes your child)
   - ❑ African-American or Black
   - ❑ Caucasian/White
   - ❑ Hispanic or Latino
   - ❑ Asian or Pacific Islander
   - ❑ American Indian or Alaska Native
   - ❑ Biracial/Mixed Race (please list all groups that apply) ___________________________

4. Your race/ethnicity (Check the <u>one</u> that best describes you)
   - ❑ African-American or Black
   - ❑ Caucasian/White
   - ❑ Hispanic or Latino
   - ❑ Asian or Pacific Islander
   - ❑ American Indian or Alaska Native
   - ❑ Biracial/Mixed Race (please list all groups that apply) ___________________________

5. What is your relation to the child?
   - ❑ Mother
   - ❑ Father
   - ❑ Aunt
   - ❑ Uncle
   - ❑ Grandmother
   - ❑ Grandfather
   - ❑ Other

6. Language(s) spoken at home (Check all that apply)
   - ❑ English
   - ❑ Spanish
   - ❑ French
   - ❑ Chinese
   - ❑ Other (please specify) ___________________________

7. Please indicate the highest level of education completed by the child's mother:
   - ❑ Some High School Coursework
   - ❑ High School Diploma/GED
   - ❑ Some College Coursework/Vocational Training

- ❑ 2-year College Degree (Associates)
- ❑ 4-year College Degree (BA/BS)
- ❑ Postgraduate or Professional degree (MA, PhD, MD, JD)

8. Please indicate the highest level of education completed by the child's father:
   - ❑ Some High School Coursework
   - ❑ High School Diploma/GED
   - ❑ Some College Coursework/Vocational Training
   - ❑ 2-year College Degree (Associates)
   - ❑ 4-year College Degree (BA/BS)
   - ❑ Postgraduate or Professional degree (MA, PhD, MD, JD)
9. Please indicate your annual household income:
   - ❑ Less than $15,000
   - ❑ $15,000 - $30,000
   - ❑ $31,000 - $45,000
   - ❑ $46,000 - $59,000
   - ❑ $60,000 - $75,000
   - ❑ $76,000 - $100,000
   - ❑ $101,000 - $150,000
   - ❑ $151,000 or more

10. Please indicate if you are currently a resident of a nearby state or are from out of town.
   - ❑ Maryland
   - ❑ District of Columbia
   - ❑ Another U.S. state (please list) ______________________
   - ❑ Virginia
   - ❑ Pennsylvania
   - ❑ Another country ______________________

11. Do you have a season pass currently or in the past (circle one)? YES NO

12. Do you have any other children? If so, please list their age(s) and gender(s).

Siblings' Date of Birth (M/D/YYY)	Siblings Age (in years)	Sibling's Gender (M or F)

Appendix C: Verbal Assent for Children

Before beginning the study, the experimenter will obtain verbal assent from the child. This will be done in the following manner:

The experimenter will ask the child in the classroom whether s/he would like to play games with the experimenter. **"Hello. My name is _______. Today I have some fun games that I would like you to play. Would you like to play my games with me today?"**

If the child says no, the experimenter will thank the child. **"Okay. That is fine that you do not want to play today. Thanks for letting me know."** If the child expresses concern that their parent/guardian would not want them to participate, the experimenter can explain the child's parent/guardian has given their consent. **"Your mom/dad/grandparent said that it was okay for you to play these activities with me."**

If the child agrees, s/he walks with the experimenter to the place in the museum where the study is being run. After they arrive, the experimenter will give the child instructions on how to play the games. Periodically during the instructions, the child will be asked if s/he understands. If the child indicates that something is not clear, that part will be explained further. After all of the instructions have been given, the child will be asked if s/he would like to play the games.

If the child says yes, the study will begin.

If the child's answer is no or s/he appears unsure whether to participate (at any point during session), the experimenter tells the child that it is okay to go back to the regular museum activities. **"Thank you for letting me know that you do not want to play our activities today. We can be finished. We can go back to your parent/guardian now."**

In such cases (which are rare but occasionally occur), the experimenter will walk the child back to his or her guardian and thank them both again.

Appendix D: Survey Items Used in Analyses

Items Adapted from Eccles and Colleagues (1983)

General Beliefs

Personal involvement

1. How much time did you spend on the following particular activities last week? (1 = 0 hours, 2 = 1-3 hours, 3 = 4-6 hours, 4 = 7-10 hours, 5 = 11-15 hours, 6 = 16-20 hours, 7 = more than 20 hours)
    a. Math-related activities
    b. Managing family finances

Perception of one's abilities

2. Please indicate how Good or Not Good you were when you were in school in each of the areas below. (1 = Not good at all, 7 = Very good)
    a. Math
3. Please indicate how Good or Not Good you are currently in each of the areas below. (1 = Not good at all, 7 = Very good)
    a. Math

Value of math

4. How important is it to you that you are good at each of these areas listed below? (1 = Not important at all, 7 = Very important)
    a. Math

Perception of role as child's teacher

5. How much influence do parents have over their children in each of these different areas? (1 = Very little, 7 = A great deal)
    a. Math

Child-specific beliefs

Ability beliefs

6. How good is your child in each area listed below? (1 = Not good at all, 7 = Very good)
    a. Counting and naming numbers
7. Compared to other children, how much innate ability or talent does your child have in each of these areas? (1 = Much less than other children, 7 = Much more than other children)
    a. Numbers

Expectations for future performance

8. How well do you think this child will do in each of these areas next year? (1 = Not at all well, 7 = Very well)
    a. Counting and naming numbers

Perceptions of child's interest

9. How much does your child like each of the following activities? (1 = Not at all, 7 = Very much)
    a. Counting and naming numbers

Value of math for child

10. How important is it to you that your child do well in each of these activities? (1 = Not very important, 7 = Very important)
    a. Counting and naming numbers
11. How useful do you think each of these kinds of activities will be to your child in the future? (1 = Not at all useful, 7 = Very useful)
    a. Counting and naming numbers

Items Adapted from the Math Literature

Child-specific beliefs

Expectations

1. In your opinion, how important is it for your preschooler to reach the following benchmarks before entering kindergarten? (1 = Not important, 7 = Very important)
    a. Count to 10
    b. Count to 20
    c. Name numbers 1-10
    d. Name numbers 11-20
    e. Compare two written numbers to see which is bigger
    f. Associate Arabic numerals with corresponding number of items
    g. Do simple addition and subtraction problems

Appendix E: War Card Game Instructions Script

Set up:
Place Bert and Ernie facing across from each other at the opposite side of the table as the parent and child. Take out the pre-sorted stacks of 20 cards each (18 pairs plus 2 practice pairs). Place one set of the cards in a stack face-down in front of Bert, and the other half in front of Ernie.

Introduction to game:
Today you are going to help our friends play a game with these cards. Before we get started, let me introduce you to our friends and go over the rules. Just like in other games, this game is just for fun. What really matters is that we have fun playing the game together.

This is Bert and this is Ernie. They are both friends who want to play the game together. However, they sometimes have trouble following the rules, so they would like you to help them to play the game the right way.

They each have a stack of cards – Bert has a stack (point to Bert's stack of cards) **and Ernie has a stack** (point to Ernie's stack). **In this game, we leave the cards in the stack so that the pictures are not showing and the blank side faces up. I'm going to show you how to play the game. To play, you will turn over one card at the top of Bert and Ernie's stacks and put them here in the middle** (demonstrate putting the top card from each stack in the middle of the space). **Then you decide whose card is bigger.**

Let's start with the first pair. It looks like in this case Ernie's card has more.

In this game, the friend who put in the card with more gets to take both of the cards and put them in another stack, like this (demonstrate taking the cards from the center and turning them face down in a second stack to the side of Ernie).

After that, we flip over the next cards and do the same thing. Let's practice! (Guide the child in turning over the top card.) **So remember, we turn the cards over, and then we figure out whose card has more, and that friend gets both cards!**

Flip over the second set of top cards. **It looks like in this case Bert's card has more** (demonstrate taking the cards from the center and turning them face down in a second stack to the side of Bert). **At the end of the game, whoever has more cards in their stack** (gesture to the secondary stacks), **wins the game.**

Okay, you can continue helping Bert and Ernie play the game! Allow parents and children to begin playing the game as described above.

After all 20 cards have been played by each player, move the secondary stacks of cards over to the middle and tell the parent and child: **Now you can see who has more cards, Bert, or Ernie.**

Appendix F: At-home Math Activity Engagement Survey

Please check how frequently you and your child typically engage in the following activities together.

Activities	Rarely or Never	Monthly	Weekly	Several days per week	Every day or almost every day
Count out loud	☐	☐	☐	☐	☐
Identify letters	☐	☐	☐	☐	☐
Play educational video games	☐	☐	☐	☐	☐
Count items	☐	☐	☐	☐	☐
Name shapes	☐	☐	☐	☐	☐
Play educational computer games	☐	☐	☐	☐	☐
Identify or name numbers	☐	☐	☐	☐	☐
Compare written numbers and identify which is bigger	☐	☐	☐	☐	☐
Play board games	☐	☐	☐	☐	☐
Compare 2 or more quantities of objects to determine which is more	☐	☐	☐	☐	☐
Perform simple addition	☐	☐	☐	☐	☐
Play card games	☐	☐	☐	☐	☐
Determine ‘how many’ in a group of objects	☐	☐	☐	☐	☐
Write letters in child’s name	☐	☐	☐	☐	☐
Sing nursery rhymes	☐	☐	☐	☐	☐
Read with your child	☐	☐	☐	☐	☐
Visit Port Discovery	☐	☐	☐	☐	☐

Appendix G: Preschool Early Numeracy Scales

Task name	Task Description	Maximum Total Points (Total 46)
Verbal (Rote) Counting	Child counts out loud to 40	1pt for counts to 5 2pts for counts to 10 3pts for counts to 15 4pts if child counts to 20 5pts (max) if child counts to 40
Structured Counting (α = .79) Cardinality (α = .75)	Child counts dots and then is asked to indicate how many dots they counted. This is one task but points are applied separately for counting and cardinality in case the child counts incorrectly but correctly repeats the last number counted to. Quantities include 3, 16, 6, 11, and 14 dots. Cardinality is only assessed for 3, 16, and 6 dots.	5pts (max), 1pt. for each quantity counted 3pts (max), 1pt for each quantity
Subitizing Dots (α = .69)	Child is shown a picture of rabbits for 2 seconds on a page, then the page is flipped and the child is asked how many rabbits they saw. Quantities include 3, 2, 5, 1, 4, 7, and 6	7pts (max), 1 pt. for each quantity
Numeral comparison (α = .74)	Child sees sets of 4 numbers and is asked to indicate which number in each set of 4 is the *most (*2 sets*)* and the *least (*2 sets*)*. Sets for indicating which is *most* include (1, 3, 4, 2; 5, 3, 8, 1) and *least* include (*3, 6, 2,*	4pts (max), 1 pt for each set

	8; 9, 7, 6, 12)	
Number word comparison (α = .74)	Child hears 2 sets of 4 numbers spoken to them (1 set at a time) and is asked to indicate which number means the most (5, 2, 7, 1) and least (4, 3, 10, 13)	2pts (max), 1 pt for each set
Non-symbolic quantity comparison (α = .77)	Child is asked to examine 4 different groups of dots at a time and asked which is the most and least. Children are asked for 3 sets of groups of dots which is the most (1, 4, 3, 2; 3, 1, 2, 0; 5, 3, 8, 1), and for 3 sets of groups of dots which is the least or lowest (3, 6, 7, 5;, 9, 2, 11, 7; 8, 7, 6, 12)	6pts (max), 1 pt. for each set
Numeral Identification (α = .90)	Child is asked to name written numerals on a series of flashcards. Numbers include 1, 2, 3, 7, 8, 10, 12, 14, 15	9pts (max), 1pt. for each number named
Set-to-Numeral (α = .80)	Child sees a numeral and 5 groups of dots and is asked which group means the same thing as the numeral above. Dot groups for the numeral 3 include (7, 5, 8, 3, 9), dot groups for numeral 1 include (1, 8, 3, 12, 2), for 7 include (10, 7, 11, 1, 4), for 5 (1, 5, 3, 2, 4), and for 8 (3, 7, 6, 4, 8)	5pts (max), 1pt. for each numeral

Note. Measure adapted from Purpura & Lonigan, 2015. Alpha values represent internal consistency for the normed sample.

Appendix H: IRB Approval Letter

UNIVERSITY OF MARYLAND

INSTITUTIONAL REVIEW BOARD
1204 Marie Mount Hall College Park, MD 20742-5125 TEL 301.405.4212
FAX 301.314.1475
irb@umd.edu www.umresearch.umd.edu/IRB

DATE: February 22, 2016

TO: Geetha Ramani, Ph.D.
FROM: University of Maryland College Park (UMCP) IRB

PROJECT TITLE: [662743-3] Early Play and Learning Study REFERENCE #:
SUBMISSION TYPE: Continuing Review/Progress Report

ACTION: APPROVED APPROVAL DATE: February 22, 2016
EXPIRATION DATE: March 1, 2017
REVIEW TYPE: Expedited Review

REVIEW CATEGORY: Expedited review category # *8(a)(c)*

Thank you for your submission of Continuing Review/Progress Report materials for this project. The University of Maryland College Park (UMCP) IRB has APPROVED your submission. This approval is based on an appropriate risk/benefit ratio and a project design wherein the risks have been minimized. All research must be conducted in accordance with this approved submission.

Prior to submission to the IRB Office, this project received scientific review from the departmental IRB Liaison.

This submission has received Expedited Review based on the applicable federal regulations.

This project has been determined to be a Minimal Risk project. Based on the risks, this project requires continuing review by this committee on an annual basis. Please use the appropriate forms for this procedure. Your documentation for continuing review must be received with sufficient time for review and continued approval before the expiration date of March 1, 2017.

Please remember that informed consent is a process beginning with a description of the project and insurance of participant understanding followed by a signed consent form. Informed consent must continue throughout the project via a dialogue between the researcher and research participant. Unless a consent waiver or alteration has been approved, Federal regulations require that each participant receives a copy of the consent document.

Please note that any revision to previously approved materials must be approved by this committee prior to initiation. Please use the appropriate revision forms for this procedure.

All UNANTICIPATED PROBLEMS involving risks to subjects or others (UPIRSOs) and SERIOUS and UNEXPECTED adverse events must be reported promptly to this office. Please use the appropriate reporting forms for this procedure. All FDA and sponsor reporting requirements should also be followed.

All NON-COMPLIANCE issues or COMPLAINTS regarding this project must be reported promptly to this office.

Please note that all research records must be retained for a minimum of seven years after the completion of the project.

If you have any questions, please contact the IRB Office at 301-405-4212 or irb@umd.edu. Please include your project title and reference number in all correspondence with this committee.

This letter has been electronically signed in accordance with all applicable regulations, and a copy is retained within University of Maryland College Park (UMCP) IRB's records.

References

Anders, Y., Rossbach, H. G., Weinert, S., Ebert, S., Kuger, S., Lehrl, S., & von Maurice, J. (2012). Home and preschool learning environments and their relations to the development of early numeracy skills. *Early Childhood Research Quarterly*, *27*, 231–244. doi:10.1016/j.ecresq.2011.08.003

Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development*, *54*, 695–701. doi: 10.2307/1130057

Aunola, K., Leskinen, E., Lerkkanen, M.-K., & Nurmi, J.-E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology*, *96*, 699–713. doi:10.1037/0022-0663.96.4.699

Bandura, A., & Walters, R. H. (1963). *Social learning and personality development*. New York: Holt, Rinehart & Winston.

Barbarin, O. A., Early, D., Clifford, R., Bryant, D., Frome, P., Burchinal, M., … Pianta, R. (2008). *Parental conceptions of school readiness: Relation to ethnicity, socioeconomic status, and children's skills*. *Early Education & Development* (Vol. 19). doi:10.1080/10409280802375257

Baron, R. M., & Kenny, D. a. (1986). The moderator-mediator variable distinction in social psychological research: conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology*, *51*, 1173–1182. doi:10.1037/0022-3514.51.6.1173

Belfield, C., & Garcia, E. (2014). Parental notions of school readiness: How have they changed and has preschool made a difference? *The Journal of Educational Research*, *107*, 138–151. doi:10.1080/00220671.2012.753863

Benigno, J. P., & Ellis, S. (2004). Two is greater than three: effects of older siblings on parental support of preschoolers' counting in middle-income families. *Early Childhood Research Quarterly*, *19*, 4–20. doi:10.1016/j.ecresq.2004.01.006

Benoit, L., Lehalle, H., Molina, M., Tijus, C., & Jouen, F. (2013). Young children's mapping between arrays, number words, and digits. *Cognition*, *129*, 95–101. doi:10.1016/j.cognition.2013.06.005

Bjorklund, D., Hubertz, M., & Reubens, A. (2004). Young children's arithmetic strategies in social context: How parents contribute to children's strategy development while playing games. *International Journal of Behavioral Development*, *28*, 347–357. doi:10.1080/01650250444000027

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting*, *5*, 35–45. doi: 10.1002/(SICI)1099-0917(199603)5:1<35::AID-EDP113>3.0.CO;2-0

Blevins-Knabe, B., Austin, A. B., Musun, L., Eddy, A., & Jones, R. M. (2000). Family Home Care Providers' and Parents' Beliefs and Practices Concerning Mathematics with Young Children. *Early Child Development and Care*, *165*, 41–58. doi:10.1080/0300443001650104

Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology*, *20*, 607–618. doi:10.1037/0012-1649.20.4.607

Cannon, J., & Ginsburg, H. P. (2008). "Doing the Math": Maternal Beliefs About Early Mathematics Versus Language Learning. *Early Education & Development*, *19*, 238–260. doi:10.1080/10409280801963913

Children, N. A. for the E. of Y. (2002). *Early childhood mathematics: Promoting good beginnings*. *National Association for the Education of Young Children*. Retrieved from http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Early+Childhood+Mathematics+:+Promoting+Good+Beginnings#4

Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science*, *10*, 408–411. doi: 10.1111/1467-9280.00177

Common Core State Standards. (2015). Retrieved from http://www.corestandards.org/Math/

Cordes, S., & Brannon, E. (2008). The difficulties of representing continuous extent in infancy: Using number is just easier. *Child Development, 79,* 476-489. doi: 10.111/j.1467-8624.2007.01137.x

Council, N. R. (2009). *Mathematics Learning in Early Childhood*. *Annals Of The New York Academy Of Sciences*.

Deater-Deckard, K. (2000). Parenting and child behavioral adjustment in early childhood: A quantitative genetic approach to studying family processes. *Child Development*, *71*, 468-484. Retrieved from http://www.jstor.org.proxy-um.researchport.umd.edu/stable/1132003

Deater-Deckard, K., Pylas, M. V., & Petrill, S. A. (1997). Parent-Child Interaction System (PARCHISY). London, UK: Institute of Psychiatry.

Deckner DF, Adamson LB, Bakeman R. Child and maternal contributions to shared reading: Effects on language and literacy development. Applied Developmental Psychology. 2006;27:31–41. doi: 10.1016/j.appdev.2005.12.001

DeFlorio, L., & Beliakoff, A. (2014). Socioeconomic status and preschoolers' mathematical knowledge: The contribution of home activities and parent beliefs. *Early Education and Development*, *26*, 319–341. doi:10.1080/10409289.2015.968239

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York, NY: Oxford University Press.

Duncan, G. J., Claessens, A., Magnuson, K., Klebanov, P., Pagani, L. S., Feinstein, L., … Japel, C. (2007). School Readiness and Later Achievement, *43*, 1428–1446. doi:10.1037/0012-1649.43.6.1428

Dunteman, G. H. (1989). *Principal components analysis.* Newberry Park, CA: Sage Publications.

Eccles, J. S. (1993). School and family effects on the ontogeny of children's interests, self-perceptions, and activity choices. In J. E. Jacobs (Ed.), *Nebraska Symposium on Motivation, 1992: Devleopmental perspectives on motivations* (Vol. 40, pp. 97-132). Linoln: University of Nebraska Press.

Eccles, J., Adler, T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J., & Midgley, C. (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Ed.), *Achievement and Achievement Motives*. San Francisco: W. H. Freeman.

Feigenson, L., & Carey, S. (2003). Tracking individuals via object-files: Evidence from infants' manual search. *Developmental Science*, *6*, 568–584. doi:10.1111/1467-7687.00313

Feigenson, L., Carey, S., & Hauser, M. (2002). The representations underlying infants' choice of more: Object files versus analog magnitudes. *Psychological Science : A Journal of the American Psychological Society / APS*, *13*, 150–156. doi:10.1111/1467-9280.00427

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*, 307–314. doi:10.1016/j.tics.2004.05.002

Fluck, M., Linnell, M., & Holgate, M. (2005). Does counting count for 3- to 4-year-olds? Parental assumptions about preschool children's understanding of counting and cardinality. *Social Development*, *14*, 496–513. doi:10.1111/j.1467-9507.2005.00313.x

Fuson, K. C. (1988). *Children's Counting and Concepts of Number*. (C. Brainerd, Ed.). New York, NY: Springer-Verlag New York Inc.

Galper, A., Wigfield, A., & Seefeldt, C. (1997). Head Start parents' beliefs about their children's abilities, task values, and performances on different activities. *Child Development*, *68*, 897–907. doi:10.1111/j.1467-8624.1997.tb01969.x

Gelman, R., & Gallistel, C. R. (1978). *The Child's Understanding of Number*. Cambridge, MA: Harvard University Press.

Ginsburg, H. P., & Baroody, A. J. (2003). *The Test of Early Mathematical Ability* (3rd ed.). Austin: Pro-Ed.

Ginsburg, H. P., Lee, J. S., & Stevenson Boyd, J. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report*, *12*, 3–22.

Griffin, S. (2004). Building number sense with Number Worlds: A mathematics program for young children. *Early Childhood Research Quarterly*, *19*, 173–180. doi:10.1016/j.ecresq.2004.01.012

Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: Relations between parents' input and children's number knowledge. *Developmental Science*, *14*, 1021–1032.

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "Number Sense": The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, *44*, 1457–1465. doi:10.1037/a0012682

Holloway, S. D., Rambaud, M. F., Fuller, B., & Eggers-Piérola, C. (1995). What is "appropriate practice" at home and in child care?: Low-income mothers' views on preparing their children for school. *Early Childhood Research Quarterly*, *10*, 451–473. doi:10.1016/0885-2006(95)90016-0

Huntsinger, C. S., Jose, P. E., & Fong-Ruey, L. (1997). Cultural differences in early mathematics learning: A comparison of Euro-American, Chinese-American, and Taiwan-Chinese families. *International Journal of Behavioral Development*, *21*, 371–388. doi:10.1080/016502597384929

Huntsinger, C. S., Jose, P. E., Larson, S. L., Balsink Krieg, D., & Shaligram, C. (2000). Mathematics, vocabulary, and reading development in Chinese American and European American children over the primary school years. *Journal of Educational Psychology*, *92*, 745–760. doi:10.1037/0022-0663.92.4.745

Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences of the United States of America*, *106*, 10382–10385. doi:10.1073/pnas.0812142106

Jacobs, J. E., Davis-Kean, P., Bleeker, M., Eccles, J. S., & Malanchuk, O. (2005). I can, but I don't want to. In A. M. Gallagher & J. C. Kaufman (Eds.), *Gender Differences in Mathematics; An Integrative Psychological Approach* (pp. 246–263). Cambridge: Cambridge University Press.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential Calculation Abilities in Children From Middle- and Low-Income Families. *Developmental Psychology*, *28*, 644–653.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1994). Assessing early arithmetic abilities: Effects of verbal and nonverbal response types on the calculation performanc eof middle- and low-income children. *Learning and Individual Differences*, *6*, 413–432.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*, 850–867. doi:10.1037/a0014939

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews*, *15*, 60–68. doi:10.1002/ddrr.46

Kleemans, T., Peeters, M., Segers, E., & Verhoeven, L. (2012). Child and home predictors of early numeracy skills in kindergarten. *Early Childhood Research Quarterly*, *27*, 471–477. doi:10.1016/j.ecresq.2011.12.004

Kleemans, T., Segers, E., & Verhoeven, L. (2013). Relations between home numeracy experiences and basic calculation skills of children with and without specific language impairment. *Early Childhood Research Quarterly*, *28*, 415–423. doi:10.1016/j.ecresq.2012.10.004

Klein, A., & Starkey, P. (1995). Preparing for the transition to school mathematics: The Head Start Family Math Project. In *School readiness and early achievement of impoverished children*. Indianapolis: Society for Research in Child Development.

Klein, A., Starkey, P., Clements, D., Sarama, J., & Iyer, R. (2008). Effects of a Pre-Kindergarten Mathematics Intervention: A Randomized Experiment. *Journal of Research on Educational Effectiveness*, *1*, 155–178. doi:10.1080/19345740802114533

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk.". *Developmental Psychology*, *42*, 59–69. doi:10.1037/0012-1649.42.1.59

Krajewski, K., & Schneider, W. (2009). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, *19*, 513–526. doi:10.1016/j.learninstruc.2008.10.002

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics*, *33*, 159–174. doi:10.2307/2529310

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, *105*, 395–438. doi:10.1016/j.cognition.2006.10.005

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology*, *52*, 130–169. doi:10.1016/j.cogpsych.2005.07.002

LeFevre, J.-A., Polyzoi, E., Skwarchuk, S., Fast, L., & Sowinski, C. (2010). Do home numeracy and literacy practices of Greek and Canadian parents predict the numeracy skills of kindergarten children? *International Journal of Early Years Education*, *18*, 55–70. doi:10.1080/09669761003693926

LeFevre, J.-A., Clarke, T., & Stringer, A. P. (2002). Influences of language and parental involvement on the development of counting skills: Comparisons of French- and English-speaking Canadian children. *Early Child Development and Care*, *172*, 283–300. doi: 10.1080/03004430212127

LeFevre, J.-A., Skwarchuk, S.-L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science/Revue Canadienne Des Sciences Du Comportement*, *41*, 55–66. doi:10.1037/a0014532

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. a. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, *46*, 1309–19. doi:10.1037/a0019671

Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Numerical approximation abilities correlate with and predict informal but not formal mathematics abilities. *Journal of Experimental Child Psychology*, *116*, 829–838. doi:10.1016/j.jecp.2013.08.003

Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Science, 12,* 404-406. doi: 10.1016/j.tics.2012.06.008

Maloney, E. A., Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2015). Intergenerational effects of parents' math anxiety on children's math achievement and anxiety. *Psychological Science, 26,* 1480-1488.

Mathematics, N. C. of T. of. (2000). *Principles and standards for school mathematics*. Reston: National Council of Teachers of Mathematics.

McGillicuddy-DeLisi, A. V. (1982). Parental beliefs about developmental processes. *Human Development*, *25*, 192–200.

McGillicuddy-DeLisi, A. V. (1985). The relationship between parental beliefs and children's cognitive level. In I. E. Sigel (Ed.), *Parental Belief Systems* (pp. 7–24). Hillsdale, MI: Lawrence Erlbaum Associates.

Melhuish, E. C., Phan, M. B., Sylva, K., Sammons, P., Siraj-Blatchford, I., & Taggart, B. (2008). Effects of the Home Learning Environment and Preschool Center Experience upon Literacy and Numeracy Development in Early Primary School. *Journal of Social Issues*, *64*, 95–114. doi:10.1111/j.1540-4560.2008.00550.x

Missall, K., Hojnoski, R. L., & Caskie, G. I. L. (2014). Home numeracy environments of preschoolers: Examining relations among mathematical activities, parent mathematical beliefs, and early mathematical skills. *Early Education and Development*, *0*, 1–21. doi:10.1080/10409289.2015.968243

Mix, K. S. (2009). How Spencer made number: first uses of the number words. *Journal of Experimental Child Psychology*, *102*, 427–44. doi:10.1016/j.jecp.2008.11.003

Mix, K. S., Sandhofer, C. M., Moore, J. A., & Russell, C. (2012). Acquisition of the cardinal word principle: The role of input. *Early Childhood Research Quarterly*, *27*, 274–283. doi:10.1016/j.ecresq.2011.10.003

Musun-Miller, L., & Blevins-Knabe, B. (1998). Adults' beliefs about children and mathematics: How important is it and how do children learn about it? *Early Development and Parenting*, *202*, 191–202.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics*. Reston: National Council of Teachers of Mathematics.

Niklas, F., & Schneider, W. (2014). Casting the die before the die is cast: The importance of the home numeracy environment for preschool children. *European Journal of Psychology of Education*, *29*, 327–345. doi:10.1007/s10212-013-0201-6

OECD. (2012). PISA 2012 results in focus: What 15-year-olds know and what they can do with what they know. Retrieved from http://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf

Parsons, J. E., Adler, T. F., & Kaczala, C. M. (1982). Socialization of Achievement Attitudes and Beliefs: Parental Influences. *Child Development*, *53*, 310. doi:10.2307/1128973

Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods*, *40*, 879–891. doi:10.3758/BRM.40.3.879

Purpura, D. J., Baroody, A. J., & Lonigan, C. J. (2013). The transition from informal to formal mathematical knowledge: Mediation by numeral knowledge. *Journal of Educational Psychology*, *105*, 453–464. doi:10.1037/a0031753

Purpura, D. J., & Lonigan, C. J. (2015). Early Numeracy Assessment: The Development of the Preschool Early Numeracy Scales. *Early Education and Development*, *26*, 286–313. doi:10.1080/10409289.2015.991084

Roberts, C. (2008). Modelling patterns of agreement for nominal scales. *Statistics in Medicine*, *27*, 810-830. doi: 10.1002/sim.2945

Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. a. (2015). Math talk during informal learning activities in Head Start families. *Cognitive Development*, *35*, 15–33. doi:10.1016/j.cogdev.2014.11.002

Ramani, G. B., Siegler, R. S., & Hitti, A. (2012). Taking it to the classroom: Number board games as a small group learning activity. *Journal of Educational Psychology*, *104*, 661–672. doi:10.1037/a0028995

Rescorla, L. (1990). Academic expectations in mothers of preschool children: A psychometric study of the Educational Attitude Scale. *Early Education and Development*, *1*, 165–184.

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. New York: Oxford University Press.

Sarama, J., & Clements, D. H. (2004). Building Blocks for early childhood mathematics. *Early Childhood Research Quarterly*, *19*, 181–189. doi:10.1016/j.ecresq.2004.01.014

Saxe, G. B., Guberman, S. R., & Gearhart, A. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development*, *52*, 1–136. Retrieved from http://www.jstor.org/action/showPublication?journalCode=monosociresechil

Schaeffer, B., Eggleston, V. H., & Scott, J. L. (1974). Number development in young children. *Cognitive Psychology*, *6*, 357–379. doi:10.1016/0010-0285(74)90017-6

Sénéchal, M., & Lefevre, J.-A. (2002). Parental involvement in the development of children's reading skill: A five-year longitudinal study. *Child Development*, *73*, 445–460.

Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking.* New York, NY: Oxford University Press.

Siegler, R. S. (in press). Magnitude knowledge: The common core of numerical development. *Developmental Science*. doi: 10.1111/desc.12395

Siegler, R. S. & Lortie-Forgues, H. (2014). An integrative theory of numerical development. *Child Development Perspectives, 8*, 144-150. doi: 10.1111/cdep.12077

Sigel, I. E. (1985). *Parental belief systems*. (I. E. Sigel, Ed.). Hillsdale: Hillsdale.

Sigel, I. E., McGillicuddy-DeLisi, A. V., & Goodnow, J. J. (Eds.). (1992). *Parental belief systems: The psychological consequences for children* (2nd ed.). Hillsdale: Lawrence Erlbaum Associates.

Simpkins, S. D., Fredericks, J. A., & Eccles, J. S. (2012). Charting the Eccles' expectancy-value model from mothers' beliefs in childhood to youths' activities in adolescence. *Developmental Psychology, 48*, 1019-1032. doi: 10.1037/a0027468

Skwarchuk, S.-L. (2009). How do parents support preschoolers' numeracy learning experiences at home? *Early Childhood Education Journal*, *37*, 189–197. doi:10.1007/s10643-009-0340-1

Skwarchuk, S.-L., Sowinski, C., & LeFevre, J.-A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: the development of a home numeracy model. *Journal of Experimental Child Psychology*, *121*, 63–84. doi:10.1016/j.jecp.2013.11.006

Sonnenschein, S., Galindo, C., Metzger, S. R., Thompson, J. A., Huang, H. C., & Lewis, H. (2012). Parents' beliefs about children's math development and children's participation in math activities. *Child Development Research*, *2012*, 1–13. doi:10.1155/2012/851657

Sophian, C. (2004). Mathematics for the future: Developing a Head Start curriculum to support mathematics learning. *Early Childhood Research Quarterly*, *19*, 59–81. doi:10.1016/j.ecresq.2004.01.015

Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science*, *210*, 1033–1035.

Starkey, P., & Klein, A. (2000). Fostering Parental Support for Children's Mathematical Development : An Intervention with Head Start Families, *11*, 659–680. doi: 10.1207/s15566935eed1105_7

Starkey, P., Klein, A., Chang, I., Dong, Q., Pang, L., & Zhou, Y. (1999). Environmental suports for young children's mathematical development in China and the United States. Albuquerque: Sociey for Research in Child Development.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, *19*, 99–120. doi:10.1016/j.ecresq.2004.01.002

Starkey, P., Spelke, Elizabeth, S., & Gelman, R. (1983). Detection of intermodal numerical correspondences by human infants. *Science*, *222*, 179–181.

Stipek, D., Milburn, S., Clements, D., & Daniels, D. H. (1992). Parents' beliefs about appropriate education for young children. *Journal of Applied Developmental Psychology*, *13*, 293–310. doi: 10.1016/0193-3973(92)90034-F

Strauss, M. S., & Curtis, L. E. (1981). Infant perception of numerosity. *Child Development*, *52*, 1146–1152. doi:10.1111/1467-8624.ep8860444

Vandermaas-Peeler, M., Nelson, J., & Bumpass, C. (2007). "Quarters are what you put into the bubble gum machine": Numeracy interactions during parent-child play. *Early Childhood Research and Practice*, *9*. Retrieved at http://ecrp.uiuc.edu/v9nl/vandermaas.html

Vandermaas-Peeler, M., & Pittard, C. (2014). Influences of social context on parent guidance and low-income preschoolers' independent and guided math performance. *Early Child Development and Care*, *184*, 500–521. doi:10.1080/03004430.2013.799155

Vandermaas-Peeler, M., Ferretti, L., & Loving, S. (2012). Playing the ladybug game: Parent guidance of young children's numeracy activities. *Early Child Development and Care*, *182*, 1289–1307. doi:10.1080/03004430.2011.609617

Vandermaas-Peeler, M., Nelson, J., Bumpass, C., & Sassine, B. (2009). Numeracy-related exchanges in joint storybook reading and play. *International Journal of Early Years Education*, *17*, 67–84. doi:10.1080/09669760802699910

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds.). Cambridge: Harvard University Press.

Wigfield, A., Eccles, J. S., Schiefele, U., Roesner, R. W., & Davis-Kean, P. (2006). Development of Achievement Motivation. In W. Damon & R. M. Lerner (Eds.), *Handbook of Child Psychology* (6th ed., pp. 933–1002). John Wiley & Sons.

Wigfield, A., Eccles, J. S., Yoon, K. S., Harold, R. D., Arbreton, A. J., Freedman-Doan, C., & Blumenfield, P. C. (1997). *Journal of Educational Psychology, 89,* 451-469. doi: 10.1037/0022-0663.89.3.451

Wood, D., & Middleton, D. (1975). A study of assisted problem-solving. *British Journal of Psychology*, *66*, 181–191. doi: 10.1111/j.2044-8295.1975.tb01454.x

Wynn, K. (1990). Children's understanding of counting. *Cognition*, *36*, 155-193. doi: 10.1016/0010-0277(90)90003-3

Wynn, K. (1992a). Addition and subtraction by human infants. *Nature*, *358*, 749–750. doi: 10.1038/358749a0

Wynn, K. (1992b). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, *24*, 220–251. doi:10.1016/0010-0285(92)90008-P

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition*, *89*, 15–25. doi:10.1016/S0010-0277(03)00050-7

Xu, F., & Arriaga, R. I. (2007). Number discrimination in 10-month-old infants. *British Journal of Developmental Psychology*, *25*, 103–108. doi:10.1348/026151005X90704

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*,
74, B1-B11. doi: 10.1016/S0010-0277(99)00066-9

Young-Loveridge, J. (1989). The relationship between children's home experiences and their mathematical skills on entry to school. *Early Child Development and Care*, *43*, 45–59. Retrieved from http://www.tandfonline.com/doi/abs/10.1080/0300443890430105

Zippert, E. L. & Ramani, G. B. (2016). Parents' Estimations of Preschoolers' Number Skills Relate to at-Home Number-Related Activity Engagement. *Infant and Child Development*, doi: 10.1002/icd.1968