ABSTRACT

Title of Thesis: FOUND IN TRANSLATION: A COMPARISON OF AMERICAN, GERMAN, AND JAPANESE MATHEMATICS TEXTS AND EXERCISES

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This study compares mathematics textbooks and accompanying exercises from Germany, Japan, and the United States. Two grade-9 mathematics texts from both Germany and the United States are investigated in addition to one Japanese text. The textbooks are compared using a number of characteristics: size, weight, organizational structure, page length and certain formatting measures.

The exercises, which were taken from those chapters containing lessons with a focus on the Pythagorean Theorem, were analyzed using specific categories of criteria: sentence length and type, contextual features, response expectations, and the exercises' cognitive requirements.

Comparisons of the books and exercises were used to address two primary research questions. One, how do American textbooks cover so many more topics than those books from other countries? Two, how does the nature of the exercises vary from country to country? Following a presentation of data, the research questions are addressed and future areas of study are discussed.

FOUND IN TRANSLATION: A COMPARISON OF AMERICAN, GERMAN, AND JAPANESE MATHEMATICS TEXTS AND EXERCISES

By

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To my students, both past and future, I offer my appreciation for being a constant reminder of why I chose this field.

Lastly, I thank God for the mystery that is the human brain.

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Chapter 1: Rationale

But so many books thou readest,
But so many schemes thou breedest,
But so many wishes feedest,
That thy poor head almost turns.
Matthew Arnold (Kerber, 1968, p. 56)

Background

The average American eighth grade mathematics textbook includes thirty six topics (Schmidt, McKnight, & Raizen, 1997). It is doubtful this statement would arouse feelings of shock from the mathematical "lay person." A quick reflection on his/her own eighth grade experience might lead to quick estimation (there's one topic already) via some division (two) that thirty-six topics over a 36- week school year amounts to an average (three!) of precisely one topic per week. That seems rather reasonable. However, if the thirty-six topics in the US textbooks are compared to the five in German texts, eight in Japanese textbooks, and an international average of twenty-three a different response is sure to come (Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1993). It may or may not be safe to assume that the five topics covered in the average German text are at least slightly covered in the average US text, but for this next illustration we will suppose this is the case. Based on the data above, the five German topics would be covered for an average of about seven weeks each, while, as previously stated, a US course would average about one week per topic. There must be great organizational differences in the presentation of the mathematics between the two countries for this disparity to occur. Similar, though not as striking, comparisons could be made between the US and Japan.

In fact, those with little experience in mathematics education beyond their own education might be inclined to guess that the number of topics in Japanese texts might even be greater than that of U.S. books, and certainly not less than one-fourth the number of topics in U.S. books. This is somewhat counterintuitive because of the well-documented success of certain Asian countries in mathematics. The success of these Asian countries on mathematics assessments is well-documented and well known even outside of the mathematics education community. *How* this level of success is met, specifically with covering so many fewer topics remains, to a great extent, in question.

It is not clear what an eighth grade student or the parent of an eighth grade student would or could do armed with such information. When I first heard similar information, I was not even sure with what *I* would or could with such information, even as someone who has selected mathematics education as a career. It was those very 36-topic books that I taught out of (and learned out of, for that matter) for a number of years without knowing that anything different existed. For example, while much arithmetic and measurement in TIMSS (Third International Mathematics and Science Study) Population 2 (approximately 13 year-olds) curricula is not common internationally, such topics do persist in the US (Schmidt et al., 1997). It is logical that since we do have more topics, that there will be topics covered in US curricula that are not found in other countries' curricula. I recall in my years of teaching eighth grade mathematics the feeling of exhaustion at the end year as I pushed my students swiftly through those topics we had yet to "cover." In my assessment, both formal and informal, of my students, I felt they had achieved relative success in their mathematical experience. However, *if* success among countries was similar, regardless of the differences in curricula, it is doubtful there would be much discussion over curricular matters. But the success, based on various assessments, has *not* been the same.

Before going too much further, it is important to address a rather obvious question – just what *is* a topic? As I read the term "topic" used more and more, the more I realized that rarely was it explicitly defined. Most definitions referred to the term's use as found in the TIMSS research and related studies. Because of the prominence of TIMSS research found in this project, it is appropriate that a similar definition should be used here. The topics as defined by TIMSS curriculum analysis in each of the three population age groups were named as such based on an organization of the specific subject matter found in the texts and curriculum standards (Schmidt et al, 1997). The eventual topic list was created through a process of organizing the content into ten major categories, with each of these sub-divided into between two and seventeen subcategories. It is these subcategories that make up the list of topics. Schmidt et al acknowledge the inherent challenges of making distinctions between topics, particularly when moving from curriculum to curriculum and country to country. Though they are quick to point out that while the comparative results of the study are not as critically affected by the definition as one might expect, assuming consistency. It is interesting to note some of the significant groupings found in the topic list. It is these very groupings that may shed light on the relatively few topics covered in the German and Japanese texts. For example, the topic "Perimeter, area, and volume" is found at the same hierarchical level as "Slope and trigonometry." Having taught both of these topics at both the eighth and ninth grade level, I can see the rationale of these topics at the same organizational level, but the time spent on each in the classroom is certainly not on the same level. Nonetheless, I will echo Schmidt et al's sentiments, that while the organization of the topics themselves may be up for discussion, as long as consistency remains, the topics can be used for comparison.

So, why then this interest in textbooks? Why not class size? Or teacher education? As worthy as those subjects are of our attention, there are a number of critical reasons why I would choose to focus on a textbook comparison. Kaiser (1999) asks, "What can we learn from descriptions of mathematics teaching in totally different cultures, such as Germany and Japan, with very different value systems, social conditions, and so on?" (p. 13). I think we can learn a great deal, depending on our goals. If we are trying to figure out why we did not score as well other countries on some assessment, perhaps the comparisons can lead to frustration and resentment. But, if we enter into such comparisons with the educational well-being of students as our focus, then there is the potential to learn a great deal. I strongly agree with Willoughby (1990) when he says that the primary problem is *not* that Japan, or any other country, is doing better than us, but that "we are not doing nearly as good a job as possible to help all of our children learn and understand enough mathematics to lead productive and fulfilling lives in a modern society" (p. 2). Having a common goal when using such comparisons and also writing textbooks is a vital step in creating better learning opportunities for our students. Lappan (1999) shares her concern with the highly competitive and, at times, unprofessional antics of textbook publishers. If these concerns are well-founded, it would seem that the focus of some involved in mathematics education must be redirected towards the students.

Past ASCD President Donna Jean Carter shares her joy of becoming part of an "elite group" when the "secret of the 'nines' multiplication tables" were passed on to her (Willoughby, 1990, p. v). I still fondly recall my Aunt Nancy showing me how to multiply by nine using my fingers. I felt empowered. I have often wondered how to pass on this sense of wonder to my students, and how to keep it within myself. When faced with answering the question "Why do I teach mathematics?" I find myself agreeing with John Fujii (1979) who offers number of quality reasons, among which are, "To teach is to make ideas grow" and "Mathematics stimulates the imagination" (p. 186-187). When I began taking art courses during my undergraduate education, my father gave me a book containing pictures of sculptures by a professor emeritus of the same university. He had been a professor when my father studied architecture there, and my father asked him to sign the book. His inscription "Mike – Be better than me" has had a great effect on my view of education, and I hope I have passed this on to my students. I have begun every class I have taught with that inscription – as directions for my students to do the same, "Be better than me." I tell them if teachers only produce mathematicians (or artists or writers, etc.) that are at the same level as their instructors, our society will mathematically plateau, and consequently in other areas. Greene (1989) was inspired by Dewey when she wrote, "Too little effort might be asked of students; and they might therefore be prevented from drawing from further experiences what they have to give" (p. 32). In some regard, I think this speaks to the notion that we often sell our students short of what they can do mathematically. In a way, the obvious may seem true, the less we expect *of* our students, the less we should expect *from* our students. This notion of expectation level partially accounts for my strong interest in the use, content, and presentation of textbooks. We, as educators, need to insure that the materials we supply our students will prevent mathematical plateaus.

It is well documented that textbooks and related materials play an important role in determining what students learn in school (e.g., Lee & Zusho, 2002; Woodward & Elliott, 1990; Johnsen, 1993). In fact, studies conclude that the textbook is the most important factor in determining what content is taught in American classrooms; in almost all classrooms, essentially nothing is taught that is not found in the textbook (Willoughby, 1990). This is true to a lesser extent in other countries as internationally teachers' lessons are primarily based on the content of textbooks (Thomas, 1990). This near reliance on the textbook is certainly not a new phenomenon in American education. Woodward and Elliott (1990) report, "As early as 1913, Cubberley noted the important role that textbooks play in instruction" (p. 178). But, perhaps most importantly, not only do textbooks often define the mathematical curriculum for a school, as a result they dictate to the students *what is* mathematics (Dörfler & McLone, 1986). One of the characteristics of an ideal textbook should be that its use of language would "inspire in the majority of pupils the desire to read and use such language" (Johnsen, p.332). It has been found that the US lacks both "focus and coherence in comparison with the average patterns across all TIMSS countries; the highly repetitive nature of the US curriculum was also documented" (Valverde & Schmidt, 2000).

I suppose one way to cover more topics is to simply cover many that have already been covered. TIMSS data has shown that for the most part, US mathematics curricula were repetitive, unfocused and lacked intellectual rigor (Schmidt et al., 1999). In a study based on TIMSS research, it was found that in 12 of the higher performing countries, 70 per cent or more of the textbook space focused on the five most emphasized topics; three of the countries, including the US, had less than 45 per cent devoted to the emphasized topics (Valverde & Schmidt).

Visual concerns

As one who is certified to teach both mathematics and art, I also have a strong interest in some of the visual traits of textbooks. The visual design of a textbook is an aspect that is often overlooked by analysts and critics (Purves, 1993). This is in part because publishers are all too aware of the importance of an eye-catching cover and well-placed photographs. Regrettably, many educators are attracted to particular textbooks because of their covers and not their content (Tyson-Bernstein & Woodward, 1991). As a result, many of the visual characteristics in textbooks are seen as concern for those in the marketing departments of publishing companies. Some even say that teachers are already trained to want the flash in today's textbooks (Tyson-Bernstein & Woodward). However, research has demonstrated that textbook illustrations can have a strong, positive influence on student learning (Levin & Mayer, 1993). Levin and Mayer take this idea a step further when they write, "It follows that an effective way to improve the effectiveness of textbooks is to improve the effectiveness of textbooks illustrations" (p. 95). I agree that more effective illustrations would lead to a more effective book, but it is important to note that more effective does not necessarily *mean* effective. Purves (1993) goes on to state, "The information is embedded in a page and that very embedding may indeed shape how the information is perceived" (p. 16). For these reasons, despite Woodward's claim that much research on textbook illustrations has proven to be ambiguous, illustrations and other visual concerns have the potential to lay an important role in any textbook analysis.

The teachers' role

The teachers' role in the use of textbooks is also one that cannot be overlooked. Unfortunately, it will not be an official part of this study, but it does warrant at least some discussion here and more study in the future. Teacher-centered lessons have come to be known as "teaching by the book" (Stodolsky, 1989, p. 160). I think it is important to point out that teaching without the book does not necessarily lead to student-centered lessons. I, myself, have certainly taught lessons that were completely centered on me as well as apart from a textbook! With regard to curricular content issues, research suggests that US teachers do what is asked of them (Schmidt et al., 1997). It would seem then that change should be more easily implemented than we may think. Woodward and Elliott (1990) suggest that our system's reliance on textbooks may be "working against recruiting and retaining creative teachers" (p. 185). Dependence on textbooks and ancillary materials is somewhat a result of a teacher's lack of confidence in his/her own abilities (Dörfler & McLone, 1986).

According to surveys and interviews, US teachers report they are trying to improve, though those involved with the TIMSS Videotape Study say there is little evidence that change is occurring (Stigler & Hiebert, 1999). Chandler and Brosnan's (1994) comparison of mathematics textbooks before and after 1989 is a good indicator that change has been started in the right direction, but that it will take some time (i.e., more than the five years between 1989 and the TIMSS assessment) to see the full fruition of such changes. The study demonstrates that more emphasis is being placed on algebra, geometry, and data analysis content and less on arithmetic and measurement topics. The TIMSS assessment took place too soon after the release of the most recent reform documents for new practices to have taken any noticeable effect (US Department of Education, 1996). Due to the timing of TIMSS test with regard to new reform movements in the US, it may not be suitable as an evaluation of current US schools, teachers, and students, but should be thought of more as a baseline to compare future assessments (US Department of Education). With this mindset, it seems that not too much more comparison should be used with the TIMSS until the 'FIMSS' results are complete – whenever that might be. Schmidt et al., (1997) ask, "Are our textbooks designed to support powerful teaching even if this requires daring changes?" (p. 88) Based on what textbooks are most often used in American schools, I would say that the answer is currently "No." However, as I have stated before, change cannot be expected immediately.

Research questions

This background leads me to the questions on which this project has focused:

Research question one: How do American textbooks cover so many more topics than those books from other countries?

Food is used as nourishment for the body. Yet how and what food is prepared varies greatly from country to country. A similar train of thought can be adapted to the seemingly more specific realm of mathematics education. It appears for the most mathematics educators would agree on *what* mathematics is and what purpose it serves. However, one glance, however brief, at textbooks from around the world demonstrates that there exist vast differences in what mathematics should be emphasized and how such mathematics should be presented. One such difference highlighted in the textbooks used in countries around the world is the amount of topics included. Recall that the average American eighth-grade mathematics text includes thirty-six topics, while the respective German and Japanese texts include five and eight topics each. If one had the opportunity to read a mathematics textbook from each of the three countries, certain differences would be obvious and I shall present a selection of those differences to the reader. Similarities would not be as evident, but they would exist, primarily in those topics that are shared among the books. Does this necessarily mean that one type of textbook presentation is better or more effective than another? Much like I cannot say which is better, sushi or schnitzel, I will not pretend to know the answer to this question. What I can say and investigate is that there are differences in the number of topics covered. To do such I will look into answering the following:

Underlying questions:

- Are there physical size differences that allow for the coverage of more topics?
- Are there differences in the structure and presentation of the content (i.e., chapters) that allow for the coverage of more topics?
- What other factors contribute to the coverage of more topics? (e.g., possibly the SIMS analysis, teacher workload)

Research question two: How does the nature of the exercises vary from country to country?

Projects such as the TIMSS Videotape Study have sought to investigate both pedagogical issues and content matters as they exist in the scripts and video footage of mathematics classrooms around the world. To a lesser extent, research has also been done on the textbooks used in such classrooms. I wish to take this even a step further and focus on the exercises found in the textbooks that are used in and out of the classroom. While my first research question will focus more on structural concerns of the texts, this portion will turn to the nature of the exercises themselves, especially those found in chapters with similar content. This will attempt to keep the attention on the exercises without being distracted by the great structural differences of the books. Besides working through many of the exercises to gain a sense of their characteristics, I am particularly interested in differences in the physical makeup of the exercises, in the types of responses required of the students, and in the cognitive requirements of the exercises. Furthermore, an area that is in strong need of further exploration, without which much of this information is of much importance, is that of how the books and exercises are actually used in the classroom by both the students and the teachers. A study of the exercises in conjuncture with observations of students and teachers making use of the exercises should be of great interest to the mathematics education community. At this point, I will only be able to include a study of the exercises themselves. In doing so, I hope to address the following questions:

Underlying questions:

- Are there characteristics of the exercises in the textbooks of higher performing countries that possibly lead to a better understanding of the mathematics?

- Are there such characteristics that could plausibly be incorporated into US textbooks?
- What other factors should be considered when investigating textbook exercises? (e.g., use, teachers' manuals)

Chapter 2: Methodology

A relatively brief history of research

Looking back, the selection of the actual textbooks *to* study, was a much easier decision than *how* to study the books of interest. Despite the previously stated reality that textbooks have been and continue to be a vital component in today's educational systems, it was interesting to find that until the last few years, there had been relatively little research done with regard to textbooks. In the third edition of the *Handbook on Research on Teaching* there is only one entry in reference to textbooks (Tanner, 1986). A more recent check in the 2003 printing of the 1992 edition of the *Handbook of Research on Mathematics Teaching and Learning* would find a section dedicated to textbook analyses, but with only three studies referenced, including one that was in press at the time. There have been studies done since these handbooks were published, as well as several that were not included, but there is, nonetheless, a feeling that this has been an overlooked area in educational research. Lumsdaine (1963) offered the following explanation for the lack of textbook research in the first edition of the *Handbook on Research on Teaching*:

> The usual textbook does not control the behavior of the learner in a way which makes it highly predictable as a vehicle of instruction or amenable to experimental research. It does not in itself generate a describable and predictable process of learner behavior, and this may be the reason why there has been little experimental research on the textbook (p. 608).

These may not be the consensus views of today, but they may partially indicate why the research has been delayed. For example, twenty-three years later, in the 1986 *Handbook of Research on Teaching*, there is only one entry on textbooks. Research with regard to *how* textbooks are used, as opposed to their content, is even more rare (Tanner, Stodolsky, 1989). With the abundance of literature concerning curricula and the content of textbooks, the use of texts seems perhaps even more important at this time. And, unfortunately for this particular project, Johnsen (1993) notes that primary school textbooks have been the focus for such research much more often than secondary texts.

I believe it is not coincidental that since the well-publicized results from the Third International Math and Science Study (TIMSS) much more research has been done with respect to textbooks, both their content and use. The amount of research that involves international comparisons (e.g., Li, 1998; Samimy & Liu, 1997) certainly adds credence to the TIMSS assertion. The Michigan Studies compared the mathematical achievement of students in the US to those in Japan and Taiwan through a series of text analyses and classroom observations (Kaiser, 1999). And while the focus seems to remain on primary textbooks, there have been a number of studies in recent years (e.g., Mesa, 2000) that have investigated secondary mathematics textbooks from different countries.

Country selection

One of my original goals was to investigate my research questions for a number of different countries, focusing on more than just three. However, the more I read *about* textbooks, the more I felt it was critical to read the textbooks themselves and not only about them. As I was charged with the task of gathering and examining textbooks, it become quite clear that there were two primary obstacles – language differences and, more significantly, availability. That said, I should be forthright and let it be known that a great deal of my selection process had to do with availability of textbook materials.

Thanks to the University of Chicago School Mathematics Project Textbook Translation, I was able to obtain translated copies of *New Mathematics 1, 2, and 3* (1992). This is a Japanese series intended for grades 7, 8 and 9. Due to the well-documented mathematical success of Japanese students, I wanted to include a Japanese textbook in my study; I subsequently focused my search on those textbooks covering grades 7, 8, and 9. Through a series of connections in our department, I was able to obtain six German mathematics textbooks from a grammar teacher at the German School in Washington, D.C. Four of these books were intended for grade 9 while the others were written for an analytic geometry course intended for grade 11, and a second year probability and statistics course intended for grade 12.

At this point, I was somewhat curious as to a) what I would do with six German mathematics books (written in German) and b) how would I select exactly which books to study. These two concerns ultimately led me to the decision to focus on Japanese, German, and American textbooks aimed at 9th grade mathematics students. Despite a certain lack of choice in country selection, I thought it important to be able to justify the countries on which I settled. My interest in American textbooks is obvious, but why should we be concerned specifically with Japanese and German textbooks? I have already spoken to the success of the Japanese in terms of mathematical assessments as a reason for studying their product, but there are other successful countries (e.g., the Czech

Republic, Singapore), for which literature and translated materials are not as readily available.

Censoring history: citizenship and memory in Japan, Germany, and the United States (Hein & Selden, 2000) is an example of how Japan, Germany, and the United States have frequently been linked in educational studies. This particular study focuses on the content of history textbooks in the three countries. The roles these three countries have had in international affairs in the last 60+ years may be one indicator of why they are so often linked. Germany and Japan were chosen for the TIMSS Videotape Study because both are viewed as "important economic competitors of the US," and also due to Japan's repeated success in international mathematical assessments (Kawanaka, Stigler, & Hiebert, p. 89). One need not look past the automobile industry (Toyota, Volkswagen, Nissan, BMW, etc.) to see why the mathematical achievement of Japan and Germany would be of some interest to many Americans.

It would seem logical that educational factors would also go into the decision to linking these three countries. Japan and Germany are often referred to as having superior educational systems than the US, as well as having vastly different social and cultural customs (McAdams, 1993). Perhaps more can be gained by looking at those cultures most different from our own. It is interesting to note, that despite their educational and cultural differences, Germany and the US scored quite similarly on the TIMSS assessment. Two examples of this similarity are found in the average raw scores, Germany 509 versus US 500, and in the percent of eighth-grade students among the top ten per cent of all TIMSS countries' eighth-graders, Germany 6% versus US 5% (Schmidt, et al., US Department of Education). These figures make Germany and the US appear to be even more similar when compared to similar data from Japan (605 and 32%, respectively). Educators often like to see how our 'best' would match up against Japan's 'best.' Based on the TIMSS assessment, American students scoring at the 95th percentile in the US essentially performed at the same level as those Japanese students who scored at the 75th percentile (US Department of Education, 1996). These figures reinforced my 'selection' even more. How interesting to look at textbooks from a country, Germany, whose educational framework is so different from ours, yet whose achievement levels, according to TIMSS, have been quite similar. One last, important result of working with these particular countries is that each has a different form of curricular control. Japan's educational system is almost exclusively by the Ministry of Education, German states have most of the power over educational matters, and in the US, local school districts are in control of nearly all educational matters.

Development of study

Stodolsky outlined three areas which textbook use and influence should be analyzed: i) topics, ii) actual material in the textbooks, and iii) activities suggested in teachers' editions. Due to my limited access to teachers' editions, I chose to focus on the first two items. There are a number of drawbacks to text analyses that focus more on content than on use. Most of the TIMSS text analysis dealt with content and topical issues. Howson (1999) comments on the cautious importance to be placed on the text analysis of the TIMSS, "I believed that the methods used could throw only limited light upon a very difficult to describe, yet enormously important, factor in mathematics education – the textbook" (p. 171). Keitel and Kilpatrick (1999) present a discussion that looks at potential areas of concern with the TIMSS and similar studies. While I will not go into the details of these concerns at this time, I do feel it is important to mention that there are people who see limitations in the TIMSS studies. Because of their wide scope and seeming depth, the TIMSS studies have been used as the ultimate reference when it comes to international comparisons in mathematics and science education. What is done with such studies, beyond a reference for projects such as mine, will certainly contribute to both their legacy and future. Another content study of secondary history books was the *Japan/United States Textbook Project: Perceptions in the textbooks in each country about the history of the other* (1983), which essentially ended up as a list of recommendations for revisions (Johnsen). It was a goal of mine, which I hopefully have achieved, to not be critical of the textbooks I chose to study, but ultimately to gain strategies from each of the books that will benefit both my students and students of others.

To a great extent, it did not seem practical to investigate every page of each of the books. I decided to focus on the portion of the textbook most utilized by students. The research indicates that students most commonly use textbooks for homework assignments and preparing for tests (Posamentier, Hartman, & Kaiser 1998). Other studies have shown mathematics textbooks to be used primarily as workbooks (Stodolsky, 1989). As a result, I decided to focus on the exercises portion of the textbooks. I believe that there can be useful information taken from the 'lesson' portions of the text, but Stodolsky's work also suggests that, in general, three out of five pages went unread by mathematics students. I found this phenomenon to be quite unlikely with respect to the *Contemporary Mathematics in Context* and the *New Mathematics* (Japanese) texts, but as a matter of consistency, I elected to focus on the exercises. A related issue would be selecting the actual problems to study. As has been previously referenced, both the number and specific topics vary greatly from country to country. I thought it would be best to find a common topic and investigate the exercise found in the chapter containing that topic. A topic that could provide a variety of problem types, as well as the opportunity for image use and practical context would be ideal. As I read through the books, one topic that seemed to fit the aforementioned criteria was the Pythagorean Theorem.

It is critical to define an "exercise" as used in this study. First, it should be said that the terms exercise and task are used interchangeably in this document. Secondly, in all but a few cases, which will be addressed later, an exercise is considered to be any numbered task whose apparent intention is for the item to be completed independently by the student. There was a great range of length in the verbal portions of such exercises, as evident in Table 5, but also in more obvious distinctions such as attached lettered items. The exceptions I mentioned earlier were limited to the American Prentice Hall text, and were a result of a characteristic unique to this text. When collecting data on sentence structure of the verbal portions of the exercises, the Prentice Hall book posed a challenge in that on a number of occasions one set of instructions would be followed by a series of five to ten numbered exercises. Consequently, the data for the Prentice Hall text in Table 5 is based on its separate verbal directions and questions, *not* on the number of numbered exercises. Beyond this table, the notion of exercise remains constant for each book.

Of the German books I had available, each of the ninth grade textbooks covered the Pythagorean Theorem. I thought it would be most useful to select the two most recent editions. When it came to deciding on two US books to study, I first elected to select one traditional and one reform text. I had hoped to gather information on which texts are most widely used in the US to help justify my selection. Unlike the issues I faced with the foreign texts, it would be much easier to come across almost any widely used American book. However, as Stevenson and Bartsch point out, there are no comprehensive, national statistics regarding the frequency with which certain textbooks are used in the US (1992 in JEP). I did come across the California approved adoption list, and used it to make a book choice. According to the US Census (2000), approximately 12% of the US population lives in California, and the California Textbook Adoption Committee approved the Prentice Hall *Pre-Algebra* textbook I have used in this study. This does *not* mean that at least 12% of *Pre-Algebra* use this text, as there are similarly leveled books on the adoption list. It does mean that there is a good chance that the book is used by a large number of students in California and across the country. Tsuchida and Lewis (2002) performed a study comparing Japanese and US primary science textbooks. They encountered similar selection issues as I did, and used similar criteria for their American textbook selection. In opting for the *Contemporary Mathematics in Context, Course1*, I chose one of the more widely used reform books on the market. My selection of *Contemporary Mathematics* was also influenced by my familiarity with the material as I have both taught from the series (Courses 2 and 3) and am currently working as a project collaborator with the revision of the series.

Now armed with both a topic and chapters on which to focus, I needed to decide how to analyze the content of the problems in these chapters. The more literature I read in the area of textbook analysis and the more I read the textbooks themselves, there were many issues that seemed to be both importance and interest to both me and my teaching.

As the scope of these issues continued to grow, I was forced to make decisions as to which categories to focus. Eventually the areas on which I settled were as follows: i) sentence structure of the tasks, ii) use of images in relation to the tasks, iii) number of tasks contained in the chapters, iv) number of items in context and v) questioning styles. Li's (2000) study of Chinese and American primary textbooks' approach to addition and subtraction, though covering quite different subject matter, seemed to be easily adaptable to most of the issues of my concern. Of particular use from Li's study was his analysis of exercises, which will be discussed later.

Before beginning any analysis, I needed to translate the chapters of interest from the two German books. This was a rather laborious task, but one that caused me to pay close attention to the text and, consequently, gain a familiarity with the books that I might not have otherwise gained. My translation process was as follows: Step 1) scan German text using character reading software (with software set on 'German Text' to read special characters), which enters scanned text into a word processing document, Step 2) copy a few lines at a time of the German text into the online translator at www.worldlingo.com (with the machine translator subject indicator set on 'Mathematics'), and, finally, Step 3) copy translated English text to a new word processing document, making obvious language corrections (e.g., "thighs of a triangle" to "legs of a triangle"). I repeated this process for three chapters from three German texts, though only two ended up being used in the study.

Now that all of the text of interest was available in English I could begin an analysis modeled after Li's study. It soon become apparent that I would have to make alterations to Li's format, most of which came in the form of the additions of categories that piqued my interest as I studied the textbooks through the data gathering. Examples of such additions include total sentences covered by the tasks, total words in the tasks, chapter pages containing at least one problem task, and one I would not have anticipated before the study – punctuation. The interest in punctuation came primarily from noticing an inordinate number of exclamation points in some of the German text. One of the German books, not one in the study, contained thirteen exclamation points on a page that contained only eleven problems. As I looked through each of the German texts I had at my disposal, it seemed as if the use of the exclamation point was fading as time passed. In Li's study, there was a section entitled 'Response Type' which allowed for three types of responses: Numerical Answer, Numerical Expression, and Explanation or Solution Required. I decided to break the last type into three subsections of Proof, Conjecture or Other. It became obvious that the secondary textbooks I was using were asking for a wider range of explanation options than the primary books in Li's study. Li also called for the categorizing of the tasks as either one-step computations or multi-step computations. I began my data collection using the computation categories. But because of the level of mathematics with which I was dealing, I did not pay much attention to this category, as nearly all of the computations were multi-step.

As I began to notice the great variety in sentence length and number of words per sentence and task, I did some outside investigating on reading levels. I found a number of readability formulas that are commonly used. The four that I ended up employing each required the use of three 100-word passages of continuous text. In a couple of the books, this was a difficult procedure, as I did not include any tasks in the passages. While I did end up using the readability formulas and have included the results in the following chapter, due to the language differences and the use of the translated Japanese text, I am not sure how much importance can be placed in the results. When comparing mathematics textbooks, even translated copies, some differences may be related to differences in language (Cai, Lo, & Watanabe, 2002). This is not to say that such studies cannot provide valuable information, but it clearly prevents different challenges than comparing textbooks of the same language origin. Willoughby (1990) goes as far as to say that reading-level formulas have destroyed the quality of the verbal text in mathematics book; he also sites the NCTM as saying that "formula-determined reading levels are generally inappropriate as selection criteria" (p. 96). I suppose I would agree with regard to using the formulas as selection criteria, but I think they can be used in comparisons. Tyson-Bernstein and Woodward (1991) suggest that books are judged more on their learnability rather than their readability. According to Sewall (1987), contrary to what some teachers are lead to believe, the use of readability formulas can sometimes "dumb material down" (p. 74). Aside from the oft-used formulas, there are other criteria that can be used to compare readability between texts. According to readability expert Keith Johnson (1998), there are four layout aspects that affect the readability of text: i) the size of type, ii) the length of typed lines, iii) the spacing between the lines, and iv) the weight of the print. For the purposes of this study, only the first and third will be of interest.

Chapter 3: Educational issues in Japan, Germany, and the United States

Rather than give the reader a detailed description of the educational goings on in each country, I will give a brief background of each country's educational framework, followed by a series of findings regarding the countries' accommodations of differences, views of mathematics, students within the countries as well as other curricular and textbook issues. Although a great deal of the information in Chapter 3 is seemingly far removed from textbook issues, I think it is important that the reader gain a sense of the educational background from which the books in this study come. While it is difficult to paint an accurate picture of cultural differences between the countries in such a document, my goal is to offer the reader a sense of educational context for the data and descriptions to follow in Chapter 4.

Educational leadership and organization

Woodward and Elliott (1990) report, "As early as 1913, Cubberley noted the important role that textbooks play in instruction" in American schools (p. 178). Even this realization is relatively recent when compared to the age of some Japanese educational traditions. For example, by the late 1880s the Ministry of Education already controlled all aspects of education, including textbook selection (McAdams, 1993). To put that in perspective, by the late 1880s, twelve of the fifty US states had yet to achieve statehood. Japan has 50% of its population contained on 2% of its land (McAdams). At the risk of sounding defensive, this seems to make a centralized educational system a much more realistic undertaking. As populous as California is, it only has about one-fourth the population of Japan, in a slightly bigger area. There are three principle factors that have been credited with the success of Japanese primary and secondary students: i) the curricula, ii) educational tradition, dating back to the Tokugawa Era (1600 – 1868 A.D.), and iii) social pressure for academic achievement (Arimoto, 1992).

Appendix A presents the typical educational pathways taken by students in Japan, Germany, and the Untied States. While there are certainly exceptions found in all three countries, these paths represent the great majority of the students. The shaded cells represent compulsory years in the educational system. I encourage the reader to carefully "follow" a student through each of the possible paths found in each country. It is interesting to note that despite the fact that our educational system is most closely based on the German system, there remain tremendous differences between the two countries (Marlow-Ferguson, 2002). Also worth noting, is that while some may propose implementing a Japanese mathematics curriculum as a way of improving the mathematics performance of our students, it is clear that there are significant differences in the educational framework between the United States and Japan that may contribute to past performance, perhaps even more than curricular issues.

Aside from the educational paths their respective students can take, there are other differences worth reporting. Those areas that present such differences include: population issues, educational governance, and educational opportunities. According to the CIA *World Factbook* (2004), the estimated populations for Japan, Germany, and the United States are 127,333,002, 82,424,609, and 293,027,571, respectively. While these are each substantial populations, the United States' population seems even larger, educationally speaking, when data such as population growth rate and the percent of the population between birth and fourteen years of age are taken into account. The current population growth rate for the three countries is 0.08% in Japan, 0.02% in Germany, and 0.92% in the US (CIA). Some quick calculations would show that the US growth rate is more than eleven times that of Japan and 46 times that of Germany. The US also has a higher percentage of children. Estimates made in 2004 place the population percentages for those between the ages of zero and fourteen in Japan, Germany, and the US at 14.3, 14.7, and 20.8 respectively. These figures represent additional challenges faced by the United States – larger, faster growing populations that contain a higher percentage of school children. One could argue that we have the most room to grow, however, with the latest population density estimates (in persons per square kilometer) at 337, 231, and 30 for Japan, Germany and the US, respectively. With these figures in mind, how the educational systems of these countries are governed will be presented.

In Germany, each of the 16 *Landers* (states) has its own constitution and has control of educational legislation and administration (Simon, 2003). This is in great contrast to Japan, whose Ministry of Education oversees all national, prefectural, and municipal components (Simon). The US is under a different system altogether. Its constitution makes no mention of education, and while each of the fifty states is ultimately responsible for the education of its citizens, every state, save Hawaii, has handed over a great deal of the control of public education to local school districts. The federal government does have a Department of Education, though most of its responsibilities, listed below, do not focus directly on curricular issues:

1. providing leadership in addressing critical issues in American education
2. assisting in the collection of ideas for the improvement of education
3. helping students pay for their education beyond high school
4. helping communities and schools meet the needs of their students
5. preparing students for employment
6. working to ensure equal educational opportunities for all Americans (Simon, 2003, p14).

Conversely, Japan's Ministry of Education not only sets the standard number of hours per subject for elementary and junior high schools, the Ministry also oversees the curricular standards, textbooks, and all school fiscal matters (Simon, 2003). In Germany, curricular decisions are made by the state, while the federal government sets teacher-training guidelines. Needless to say, because of the variety within these countries, it either makes them an interesting sample to compare educationally, as far as areas like textbooks are concerned, or they are a rather useless sample. I will encourage the former.

Adjusting for different level students

The German Hauptschul (considered the least academically challenging of the three levels of German secondary schools) teachers are more apt to change both their lessons and their use of the textbook according to their students' abilities (Haggarty & Pepin, 2002). This feature is unique to the Hauptschul. Haggarty and Pepin note that in Hauptschul classrooms with higher achieving students, teaching styles more closely resembled those found in the Gymnasium (considered the most academically challenging of the three).

The American books in the study seemed to contain questions that were intended to be answered by everyone. This did not seem to be the case in the Japanese textbooks (Stevenson & Bartsch, 1999). One of the greatest differences among the three countries was the separation of students mathematically before students reach ninth grade. In both Japan and Germany, all eighth grade students are enrolled in the same course. However, in the US, between fifteen and twenty per cent of US eighth graders are enrolled in an Algebra I course (Schmidt, et al., 1997).

Mathematics

The approach to mathematics itself I read to be quite different in Germany and Japan. In German math books, mathematics is primarily presented as a "pre-discovered body of knowledge, a static discipline developed abstractly" (Haggarty & Pepin, 2002, p. 586). Whereas according to the video study, it was found the Japanese teachers act as if the mathematics presented is inherently interesting and that they expect the students to be interested in the mathematics without having to be convinced of its use or practicality (Stigler & Hiebert, 1999). These two essentially contrasting approaches would be worth further investigation. My experience tells me that the US would fall somewhere between the two.

Stigler and Hiebert (1999) share the thoughts of a mathematics education professor, a participant in the video study, in the form of his summary of the three countries' teaching styles:

> In Japanese lessons, there is the mathematics on one hand, and the students on the other. The students engage with the mathematics, and the teacher mediates the

relationship between the two. In Germany, there is the mathematics as well, but the teacher owns the mathematics and parcels it out to the students as he sees fit, giving facts and explanations at just the right time. In US lessons, there are the students and there is the teacher. I have trouble finding the mathematics; I just see interactions between students and teachers. (p. 25)

I am quite certain this description does not fit all classrooms in the US, or the other countries, for that matter. However, if in this particular study there were enough teachers to give this professor the impression that there is not any, or at least very little, mathematics happening in US classrooms, then we are certainly in need of greater change. It has been five years since this study was published, it would be worthwhile to at least revisit the *same* teachers used in the study. While this may not be the most scientific of studies, it may give some indication as to how much change has actually taken place.

Textbook use

The chart below shows the average percentage of seatwork time spent in three kinds of tasks. These values certainly seem to reinforce the data of my study.

Table 1
Average percentage of seatwork time spent on three kinds of tasks

	practice procedure	apply concept	invent/think
Germany	89.2	6.3	4.5
Japan	42.5	13.8	43.8
United States	94.9	4.9	0.2

(Schmidt et al., 1997)

Johnsen (1993) sites the 1988 Freeman and Porter study “Does the Content of Classroom Instruction Match the Content of Textbooks?” when he reports their findings of teachers in seven countries. The lower secondary teachers in the study were found to spend an average of about one-third of their lessons with the textbook. Additionally, about 80 per cent of the material covered in the classes was material found in the textbooks.

Based on investigations of teaching style and textbook use, one study found that a teacher's use of textbooks in the classroom follows his teaching as opposed to dictates it (Zahorik, 1991). This notion places more responsibility on the teacher, but also speaks to the autonomous nature of the teaching profession. Even in situations in which I had a detailed curriculum in which I was to follow, I have never felt the demand to *teach* the material in a certain way.

As to the use of specific parts of the textbook, Freeman and Porter's case studies presented a believable set of figures. The data represent the percentage of lessons that made use of the given section of the textbook (Stodolsky, 1989):

Student Exercises: 92.2%
Review: 86.3%
Teacher-directed: 73.5%
Enrichment: 56%
Additional practice: 23.6%

A Freeman and Porter study found that a teacher's use of a textbook and the accompanying materials greatly influenced the topics covered, but had a much less effect on the manner in which such content was covered (Stodolsky). Some attention can be paid to the idea that the five most emphasized topics in American eighth grade curricula take up a much smaller percentage of the books than those in other countries (Schmidt, et al., 1997). I am not sure that I take away the intended message. In the US non-algebra textbooks, for example, in order for the percentages to be comparable to the German and Japanese books, the US books would have to be longer than they already are (if the topics were to remain) and those top-five topics would just need to be covered more in depth. It is possible that the potential for similar coverage is there, but is currently cluttered by many, many other topics.

Three key points have been identified with regard to textbook dominance in the classroom (Johnsen, 1993):

i) Textbooks dominate instruction

ii) Teachers acknowledge this dominance

iii) It is only *assumed* that textbooks have an influence on teaching methods

TIMSS survey data indicates that US teachers use textbooks in their daily lessons at about the same rate as their counterparts in Germany and Japan (Schmidt, et al., 1997). This runs slightly contrary to a study by the US Department of Education that found during many researcher observations of junior high Japanese classrooms students' textbooks remained closed during the entire class period (US Department of Education, 1999). I suspect that this would not be the case in most US junior classrooms.

In fourth grade, and I assume in other grades as well, students are asked to buy one or two drill workbooks that the students are to use at home and in the classroom (Schmidt, et al., 1996). The use of the drill workbooks is one way teachers meet the needs of individual students (US Department of Education). Teachers also often provide worksheets (called 'printouts') to provide practice problems during class.

Students

In a study comparing US and Japanese students' problem solving behaviors, it is interesting to note that, in general, the American students thought it was unusual to be asked to find more than one way to solve a problem, and also to be asked to provide explanations for their work.

A higher proportion of US 13-year-old students feel hard work is more important than talent or luck when it comes to success in mathematics than those students in Japan and Germany (Schmidt, et al., 1993). Perhaps not coincidentally, more than 40 per cent of students in Japan and Germany reported disliking mathematics in the TIMSS study (Beaton & Robitaille, 1999). Furthermore, thirty-five per cent of US eighth-grade students had highly positive attitudes towards mahematics based on the TIMSS surveys, as opposed to nine per cent for Japanese eighth-graders (US Department of Education, 1999).

Students in Japan are taught, and have been since about 700 A.D., that it is their duty to study, as a way to repay their debts to heaven and their parents (McAdams, 1993). On the other hand, by the age of 18, the average American has watched 25,000 hours of television while receiving 12,000 hours of classroom instruction (McAdams). As for high school students, German exchange teachers noticed that students in America seem to place greater importance on sports and cars than German students (McAdams). They say that German students are aware that education should be a teenager's first priority. It seems difficult for American students to academically compete with such a different set of priorities and motivational level.

Curriculum

When looking at lists of the top ten most commonly taught topics in eighth grade mathematics, it stands out that not one algebra topic is in the US's top ten. On the contrary, both Germany and Japan contain at least one algebra topic in their respective top three (Schmidt, et al., 1997). While it may seem that reform has not lead to the trimming down of topics in American textbooks, I feel it is too early to recognize drastic changes.

When looking at the averages of the number of periods devoted to specific topics in the three countries, a few differences are quite striking. One, Japanese teachers devote 31.3 periods to similarity and congruence while the US devotes an average of 4.1 in their eighth grade classrooms. Similarly, when it comes to patterns, relations, and functions, Japan averages 16.8 periods compared to 2.4 for the US. The values are somewhat reversed when it comes to fractions and decimals, as Japanese teachers average 2.4 periods, while US teachers devote and average of 24 periods (Schmidt, et al., 1997).

The following table displays five pieces of information for each country: 1) the number of TIMSS topics listed in the country's content standards, 2) the number of these topics covered by the country's textbooks, 3) the number of topics covered by teachers, based on a condensed list of TIMSS topics, 4) the number of topics in the country's standards on the TIMSS assessment, and 5) the number of topics in the country's textbook that were also on the TIMSS assessment.

Table 2
TIMSS Standards compared to those covered by the teachers, found in the textbooks, and outlined in the curricular standards of the three countries

	Standards	**Textbook**	**Teacher**	**Test/Standards**	**Test/Text**
Total Possible	44	44	21	26	26
Japan	12	15	21	8	10
Germany	22	15	21	17	9
US	44	41	21	26	26

(Schmidt, et al., 2001)

Of the countries that appear to be our 'mathematics curricular peers', 5 of the 9 scored significantly higher than the US on the Population 2 TIMSS assessment (Schmidt, et al., 1997). This may indicate that perhaps too much attention is paid to curricular concerns. Along the same lines, it is interesting to note that all of the curricular peers of Japan except for one were significantly higher than the US on the Population 2 TIMSS assessment. It would be interesting to look at other non-curricular factors in this one country (Portugal), which scored significantly lower than the US.

Textbooks

In the Stevenson and Bartsch (1999) study comparing US and Japanese mathematics textbooks, 18 US books and 21 Japanese books were studied, with both countries represented by grades 7-12. There are certain quantitative results that seem to be consistent with my data. For example, the range of the page amounts for the American books was from 400 to 856 pages, with a mean of 540 pages; while the Japanese ranged from 120 to 230 pages with a mean of 178 pages. The Japanese books tended to be much more abstract than the American books. There seemed to be "little effort [in the Japanese textbooks] to place the problems in concrete, everyday settings" (p. 125). While the CorePlus textbook had certain structural similarities to the Japanese text, this was an area where the two had glaring differences.

It seems as if one of the obvious reasons for the greater length of the American books is our spiraling curriculum. This is not meant to discount the positive factors of the spiraling effect, just to offer an additional reason as to the length of US books. Stevenson and Bartsch (1999) discovered that 72 per cent of the concepts covered in the

American books were repeated again. Twenty-four per cent were repeated twice and almost ten per cent were repeated three times. In great contrast is the 38 per cent of the concepts repeated in the Japanese textbooks, as well as the 6 percent that were repeated more than once. Part of this deals with the fact that most Japanese students are required to do additional practice outside of their textbooks (Schmidt, et al., 1993). The items in these practice workbooks would certainly alter much of the data collected in text analyses (page length, e.g.).

Another interesting difference between the American and Japanese teacher manuals was the length of the introduction sections at the beginning of the manuals. The Japanese introductions were each about five pages in length, which is in great contrast to the length of the American introductions, which ranged in length from 20 to 35 pages (Lee & Zusho, 1999).

US textbooks use a spiraling technique for a number of reasons. One potential reason is as a way to hold students attention by breaking topics down into smaller parts (Schmidt, et al., 1997). But, perhaps as a result, more than 94% of US textbooks are devoted to the simplest performance expectations (Schmidt, et al.). It would seem that the higher number of topics in the US textbooks almost ‘force’ the use of more efficient, lower-level activities and questions. Also common to the US books were the solutions to selected exercises in the back of the book. This was not a feature common to the Japanese books (Stevenson & Bartsch, 1999).

The Lee and Zusho (1999) study found that there were four aspects of the Japanese teacher manuals that set them apart from the American manuals:

i) Japanese materials are concise and informative with clear adherence to the Ministry of Education curriculum guidelines.

ii) The manuals provide systematic and solid mathematics content knowledge for teachers.

iii) The lesson plans offer principles of conceptual organization and realistic suggestions for instruction.

iv) The information in the manuals is highly mathematics-specific with little discussion of the broader learning context (p. 70).

The merits of each of these aspects could be debated, but the acknowledgement of these differences is certainly important for comparison's sake.

Japan's textbooks are commercially produced, but commercial publishers must adhere to the comprehensive national standards (Schmidt, et al. 1996). In 1983, 0.7% of American school budgets were spent on basal textbooks and related materials. This was a decline of 50% from 1966 (Tanner, 1988). Perhaps this amount is too insignificant to ponder, but it would be worthwhile to investigate current figures. The typical Japanese junior textbook costs (with adjustment for inflation from 1993 price) cost a little over $4 USD (Tani et al., 1993). This is in sharp contrast to the *Pre-Algebra* text used in this study which is priced at around $52 USD (www.prenticehall.com).

Table 3 displays the differences between the number of topics intended to be covered by Population 2 mathematics teachers in a school year and the number of topics found in the average textbook. What message does this send to students? How aware are they of the topics that are not 'touched" in their textbooks? Unfortunately, I think it may be telling students that some topics are not as important as others.

Table 3
Number of topics in Population 2 textbooks versus number of intended topics

	in textbook	**intended**	**difference**
Germany	4	15	–11
Japan	8	25	–17
United States	36	29	+7

(adapted from Schmidt, et al., 1993)

The truth to that thinking is certainly debatable, but it seems as if the ability to learn *everything* in one's book not only sends a good message, but creates a sense of accomplishment and closure not often felt in American classrooms. In great contrast are the German and Japanese textbooks that cover far fewer topics than their respective curricular intentions.

Schools

There are some that might see additional instructional hours as the answer for increased math success in American students. This may be of some help, but as the US proves by having more mathematics instructional hours than both Germany and Japan, greater instructional time does not *necessarily* lead to greater success in mathematics (Schmidt, et al., 1993).

School structures with regard to grade: For the eighth grade alone, US schools have eight different structures (e.g., K-8, 9-12 and K-5, 6-8, 9-12) that encompass at least 4% of the schools, German schools have 5, and Japanese schools have 1 (Schmidt, et al., 1993). The average eighth grade class size is around twenty-five for both the US and Germany, but around thirty-seven in Japan (Stigler & Hiebert, 1999). Most Japanese students are also required to wear some sort of school uniforms, where such rules are definitely not the norm in either the US or Germany (Stigler & Hiebert). German parents are expected to purchase their children's textbooks, barring financial trouble (Haggarty & Pepin, 2002). The Japanese junior high textbooks are property of the students (US Department of Education, 1998). One reason they are encouraged to take their books home every day is that lockers for overnight storage are not made available for students (US Department of Education).

Teaching

In a series of interviews and surveys with foreign exchange teachers, it was made known that the exchange teachers noticed quickly that their US counterparts worked a longer day and were required to perform seemingly unprofessional tasks such as bus and bathroom duties (McAdams, 1993). These teachers also noticed that parents had a greater say in their students' education and that these parents are most likely responsible for grade inflation. US students also seemed to expect more formal assessment opportunities. When I think of the time spent preparing for and administering tests and quizzes…

Additional comments by the exchange teachers included the inordinate amount of paperwork required of American teachers, the unresponsiveness of American students, and the little amount of time American teachers have to interact with their colleagues (McAdams, 1993). This is not to say that teachers in the other countries do not face their own sets of challenges. As little literature as there is regarding non-primary mathematics education, the amount that does exist seems to primarily focus on grades 7 and 8. It would be worthwhile to investigate secondary mathematics education at the international to the degree that primary education has been investigated. I came across some thoughts of two Japanese teachers that would warrant research, if anything to check their validity:

> The object of the curriculum [at the high school level] is to give everyone exposure to the curriculum, not to demand that they have to achieve up to a certain level. I think that every school adjusts what it teaches according to level of the school. There are schools that do a whole lot of the curriculum and there are schools that only do the simple problems. (Stevenson & Nerison-Low, 2000, p. 9)

Teachers in the German Hauptschul find themselves with other challenges often found in US classrooms. They are charged with the task of teaching a highly structured mathematics curriculum to a "low-achieving and de-motivated audience of children, where about one-third have difficulties reading German, and have had life experiences that teachers feel they cannot attend to in class" (Haggarty & Pepin, p. 588). When these comments are paired with the Japanese teachers' concerns, it would seem as if there are perhaps more similarities between the three countries than one might guess. Perhaps these shared challenges should be immediate concerns in the educational community, rather than an abundance of concern about mathematical rankings.

It is also interesting to compare differences in the area of teacher education and salary issues. German teachers receive extensive teacher training. It is not uncommon for a teacher to be 28 or 30 before getting his/her first teaching assignment (McAdams, 1993). The German teachers' workdays are rather different than their American counterparts. Most start their day around 7:30 or 8:00 and finish around 1:00 in the afternoon (McAdams). It is typical for a teacher to go home at this time to continue to work. The issue of trust seems to fit here, too. McAdams writes that, “Japanese teachers have attained the highest level of professional respect and practice” (p. 207). He also mentions that they are known for their “zest and enthusiasm” (p. 209). Particularly interesting was the fact that it was written in to Japanese law in 1974 that teachers are to be paid higher than any other national public service personnel (McAdams). Despite a much longer school year (240 days versus 180), Japanese schoolchildren spend about the same number of hours on academic subjects as Americans schoolchildren (McAdams). In both Germany and Japan secondary teachers’ salaries are greater than primary teachers’ salaries (McAdams). I am not sure what to make of that, but it is interesting to note.

Lessons and Homework

What follows are the summaries of typical lessons, based on Kawanaka, Stigler and Hiebert’s (1999) work on the TIMSS Videotape Study:

United States: Lessons are broken up into two phases, acquisition and application. During the acquisition phase the teacher leads a discussion in how to solve a particular problem or demonstrates how to do such. In the application phase, students have the opportunity to practice solving similar problems.

Japan: Lessons are broken up into three major parts: first, the major concepts from the previous lesson are reviewed, and then a new type of problem is introduced on which the students will work. The teacher will solicit responses as to the methods used in solving the problem. The teacher will summarize the methods given by the students. If there is a third phase, it involves the presentation of another problem followed by a similar process of sharing and summarizing.

Germany: Lessons were discussion based, focusing on a particular goal. The teacher will typically start the lesson with a situation or concept on the chalkboard. Following the introduction of the concept, the teacher then guides the class through a collaborative discussion that leads to the general principle of the day. This goal is often unstated at the beginning of the lesson.

The average grade level of the content in 8^{th} grade lessons, according to the TIMSS Videotape Study, was as follows: US, 7.41; Japan, 9.14; Germany 8.67 (Kawanaka, Stigler, & Hiebert, 1999). This also speaks to the expectations we have set forth for our students.

I found it somewhat surprising that Japanese eighth graders are typically not assigned homework (Stigler & Hiebert, 1999). Though their teachers follow a prescribed curriculum and textbook, Japanese students primarily find answers for themselves and receive reinforcement from their teachers (McLean, 1995). However, both US and German eighth-grade teachers typically see the objective of their lessons is that students would attain specific skills (US Department of Education, 1996). This difference by itself may be as significant a reason as any for the great difference between mathematical achievement between Japan and both Germany and the United States.

Hopefully, the information in this chapter has offered the reader a better sense of context with which to digest the data and information to follow with regard to the textbooks from the individual country. The information also indicates that there are many issues beyond textbook content that should be continually investigated, some of which will be commented on in Chapter Five of this study.

Chapter 4: Investigation Findings and Presentation of Data

In this chapter I will first present descriptions of the books used in this study followed by a series of tables containing figures obtained from the collection and analysis of data from the books. Following each table is a description of the table's contents and each of its categories. I urge the reader to investigate the tables closely and note any unexpected or interesting relationships between the countries or within the categories themselves. I will first offer a reminder of the research questions:

Research question one: How do American textbooks cover so many more topics than those books from other countries?

Research question two: Are there characteristics within the textbooks of higher performing countries that might lead to a better understanding of the mathematics?

Physical Descriptions and Formatting Details

Before an analytical look at their contents, I will present a physical description of the texts of interest. For each book I will include measurements such as page dimensions, weight, and total number of pages. Other features to be incorporated are the layout of each book, the structure of the chapters, and the number of authors. Lastly, I will call on my experience and interest in graphic design to comment on various visual characteristics of the text: use of color, font (size and variety), and typical imagery use. The quantitative information in this section can be found in the table that follows the last description. One objective of this section is that the reader would get a sense of the physicality of each text, such that one could pick each book out based on the descriptions to follow, language issues aside.

Lambacher Schweitzer Ninth-Grade Mathematics Instruction for the Gymnasium (1996), a German text, weighs 23.1 ounces and contains 207 pages that each measure 10.25 inches by 7.75 inches. The book is divided into eight chapters and four appendices. Its appendices include the solutions of the chapter reviews, a brief reference on probability, a guide to symbols and terminology used in the book, and an index. Each chapter is divided into between 3 and 10 sections. Towards the end of each chapter is either a lesson where the topic is demonstrated in application or an enrichment lesson based on the section's topic (though three of chapters contain at least one of each); following these 'extra' lessons is a brief chapter review that contains one page of information and a facing page consisting of review exercises.

Each of the sections in this German book begins with a couple of problems for the students to solve. Examples and necessary vocabulary follows the opening problems. After the necessary skills have been covered, a set of between 10 and 20 problems is found. The number of problems is somewhat misleading, as many of the numbered exercises have lettered sub-problems – sometimes up through the letter q! The numbering of these problems continues where the opening problems in the section ended. This process continues throughout each section. Two primary authors and six secondary authors were acknowledged in the text. Full-color images are used in the book, though most of the pages make use of two or three colors. The body text is in a ten-point serif font that is used for all text in the book. The only variety found in the text is the use of different sizes, as well as boldfaced and italicized type. A variety of image-types (photographs, technical drawings, period art, cartoons) make the book quite visually pleasing.

The other German text in the study, ***Base Mathematics for Ninth-Grade, Practice-Understand-Use*** (1994), weighs 22.3 ounces and contains 359 pages that each measure 9 inches by 6 inches. The book is divided into nine chapters and one lengthy appendix. Its appendix contains theorems and other resources, as well as brief proofs for many of the theorems. Two tables are also included in the appendix; one is a factorial table and the other a table of combinations. Overall, the structure is much looser than in the other German book.

Each of the sections in this book begins with a couple of simple examples and vocabulary pertaining to the new topic. This introduction is followed by between five and ten bright yellow pages containing very detailed, worked-out examples, as well as other related information, such as calculator tips. After the necessary skills have been covered, a set of anywhere from 26 to over 100 problems is found. Despite the great number of exercises, many of the numbered exercises have lettered sub-problems – sometimes up through the letter z! This process continues throughout each section. Three primary authors and one contributor were acknowledged in the text. Full-color photographs are used in the book mainly to demonstrate the topic in some context, while most of the pages make use of two or three colors. The body text is in a nine-point sans-serif font that is used for all text in the book. Most pages contain two typefaces, one for the body and a different sans-serif variety for the headings. Aside from the effective, artistic photography, the images used are primarily simple line drawings.

New Mathematics, Book 3 (1992), a Japanese book, weighs 9.6 ounces and contains 118 pages that each measure 8.25 inches by 5.75 inches. The book is divided into eight chapters and six appendices. Its appendices include additional computation and review exercises, the solutions to these exercises, a table of square roots, and ends with glossy, full-color depictions of many important geometric principles, including a couple of proofs of the Pythagorean Theorem. Each chapter is divided into two sections, save the first, which has three. The sections themselves are made up of a series of examples and problems to be solved by the students. Examples and information of particular importance are highlighted by either use of color or placement in a textbox, or both in some cases. Most sections are followed by a short set of exercises, 2 to 5 problems. At the end of each chapter is a longer set, around 10, of Chapter Exercises. Following three of the Chapter Exercises are sections entitled “Advanced Topics for Individual Study,” which are related enrichment lessons. Only one editor is acknowledged in the text.

Full-color images are used only in the very front and back portions of the book, and these sections have been glued into the book. About one-third of the pages use black and one other color. The body text is in eleven-point Japanese characters. There are two distinct styles of characters used throughout the book, one for the body text and one for headings. Roman letters and numerals are used throughout the book in their common mathematical contexts. The full-color photographs in the opening section of the book are quite interesting, with obvious mathematical connections. Throughout the rest of the book, a few small photographs are used, primarily to mark sections and chapters, but for the most part, simple line drawings, diagrams, and cartoons are implemented.

The American text ***Pre-Algebra, Tools for a Changing World*** (2001) weighs 68.5 ounces and contains 858 pages that each measure 10 inches by 8.25 inches. The book is divided into thirteen chapters and six appendices. Its appendices include an Extra Practice section (organized by chapter), a Skills Handbook (arithmetic skills), a series of tables (symbols used, squares and square roots, basic trigonometric values), a glossary, answers to selected exercises, and an index. Each chapter is divided into between 7 and 10 new content sections. Aside from these sections, each chapter also contains two or three "Math Toolbox" features that focus on a particular skill related to the chapter. There are also six distinct review and assessment segments in each chapter, as well as a chapter project at the beginning of all chapters.

Each of the sections in this American book begins with a set of two objectives for the lesson, followed by an explanation or application of the first objective and then by an example of the skill or concept. This process is then repeated for the second objective. Each section contains a set of exercises divided into three series of problems, "Check Understanding," "Practice and Problem Solving," and "Mixed Review." These problems typically number between 50 and 60. Four primary authors and twelve reviewers and consultants were acknowledged in the text. Full-color images are used extensively throughout the book, with most of the pages making use of either full-color images or 4-color color combinations. The body text is in an eleven-point serif font, with up to six different typefaces used on a page. A variety of image-types (photographs, technical drawings, graphs, cartoons) make the book quite visually interesting.

Contemporary Mathematics in Context, A Unified Approach Course 1 Book B (2003), the other American text in the study, weighs 28 ounces and contains 229 pages

(555 pages in Books A and B) that each measure 11 inches by 8.5 inches. The books are divided into seven "Units" and one "Capstone" project found at the end of Unit 7. Following the Capstone, there are two indexes, one is an index of mathematical topics and the other of contexts used throughout the book. Each Unit is divided into four or five "Lessons," which are further divided into between two and four "Investigations." The Investigations lead the user through a situation in which specific mathematical skills are needed to solve problems relating to the context. At the end of each Investigation is a section called the "Checkpoint." The Checkpoint serves as a review for the mathematical ideas covered in the Investigation. After each Checkpoint are one or two exercises under the heading "On Your Own" which serve as practice for the concepts discussed in the Checkpoint.

Twice during each Lesson of this American text, sets of problems, entitled MORE (acronym for Modeling, Organizing, Reflecting, and Extending), offer a chance for additional practice of skills gained in the Lessons. The last Lesson in each of the Units is named "Looking Back" and serves as a review for the entire unit. Seven primary authors, three secondary authors, and thirty-four contributors and collaborators were acknowledged in the text. Two colors, black and green, are used on every page of the text. The secondary color is used to highlight and accent key points. The body text is in an eleven-point serif font and up to four typefaces are used on a page. A variety of image-types (photographs, technical drawings, calculator screenshots, cartoons) make the book visually crisp and inviting.

Table 4
Summary of the five textbooks' attributes

	dim (in)	weight (oz.)	total pages	chptr + app	authors
German/LS	10.25 × 7.75	23.1	207	8 + 4	2 + 6
German/BM	9 × 6	22.3	359	9 + 1	3 + 1
Japanese/NM	8.25 × 5.75	9.6	118	8 + 6	1 + 0
American/PA	10 × 8.25	68.5	858	13 + 6	4 + 12
American/CM	11 × 8.5	28.0	555	7 + 2	7 + 3
Mean	9.7 × 7.25	30.3	419	9 + 3.8	3.4 + 4.4

Key: dim (in): the page dimensions in inches; weight (oz.): weight in ounces; total pages: total pages in book; chptr + app: number of chapters and number of appendices; authors: number of primary authors and number of secondary authors

Presentation of Data

Table 5
Task sentence counts and words per task and sentence

	verbal tasks	total sentences	sentences per task	total words	words per task	words per sent.
German/LS	110	285	2.591	2,750	25.000	9.649
German/BM	141	400	2.837	4,820	34.184	12.050
Japanese/NM	65	132	2.031	1,886	29.015	14.288
American/PA	87	181	2.080	1,693	19.460	9.354
American/CM	134	873	6.515	10,414	77.716	11.929
Mean	537	1931	3.430	22,454	39.883	11.628

In Table 5, data related to sentence structure can be found. The "verbal tasks" column represents the total number of verbal tasks in the chapter containing lessons on the Pythagorean Theorem. Note that in each of the books except for the American *Pre-Algebra: Tools for a Changing World*, the "verbal tasks" entry also represents the total number of exercises in the chapter. In the *Pre-Algebra* textbook, the 87 refers to the number of different verbal instructions *or* problems; in some cases a verbal task referred to one exercise, in other cases directions were given for a set of numbered exercises. The actual number of exercises, 237, is used in future tables. The column titled "total sentences" indicates the total number of sentences given by the tasks included in the first column. The mean number of sentences per task is given in the "sentences per task" column. Similar calculations are found in the fourth and fifth columns. The final column states the mean number of words per sentence.

Table 6
Frequency of image use, contextualization, and punctuation use

	images per task	% of items in context	% of sentences with "?"	% of sentences with "!"
German/LS	0.755	0.118	0.126	0.004
German/BM	0.560	0.312	0.135	0.065
Japanese/NM	0.631	0.077	0.250	–
American/PA	0.264	0.074	0.210	–
American/CM	1.060	0.463	0.284	0.001
Mean	0.654	0.193	0.212	0.015

Table 6 contains an eclectic collection of categories. In the "images per task" column, for instance, the Japanese book averaged 0.631 images per task. In some ways this can be thought of as a percentage, but it is not the percentage of items of tasks. For example, the American *Contemporary Mathematics* book averaged 1.060 images per task, but not all tasks contained images. The second column represents the percentage of the items that were presented in some context outside the realm or classroom mathematics. The punctuation entries are simply the percentage of the task sentences containing either a "?" or a "!" (a "–" signifies none were found in the text).

Table 7
Response type and number per item

	numerical	numerical expression	proof	conjecture	other (non-num)	resp. per item
German/LS	2.009	0.136	0.173	0.073	0.291	2.682
German/BM	1.567	0.199	0.177	0.199	0.206	2.348
Japanese/NM	1.123	0.338	0.092	0.015	0.062	1.631
American/PA	0.836	0.048	–	0.037	0.144	1.066
American/CM	0.918	0.358	0.045	0.948	1.470	3.739
Mean	1.158	0.169	0.075	0.234	0.561	2.834

Table 7 represents the data gathered on different response types. The "numerical" column displays how many purely numerical responses (e.g., $x = 4$) and the "numerical expression" column displays how many numerical expressions (e.g., $x(x + 2)$) were asked per task. If an explanation was required, it was coded as either a "proof," "conjecture," or "other." Examples of 'other' explanations include a list, a drawing, or a graph. The final column is the sum of the previous five columns and represents the total number of responses stipulated for each task.

Table 8
Cognitive requirement type and number per item

	procedural	conceptual	prob. solv.	other	total req. per item
German/LS	2.000	0.664	0.182	0.036	2.882
German/BM	1.624	0.539	0.156	0.043	2.362
Japanese/NM	0.815	0.708	0.108	0.092	1.723
American/PA	0.840	0.182	0.007	0.041	1.070
American/CM	1.597	1.328	0.440	0.306	3.671
Mean	1.264	0.566	0.148	0.104	2.082

Data concerning the cognitive requirements for each task are found in Table 8. The first column displays the mean number of times procedural practice was required of the student. The mean number of times a student was to depend on their conceptual understanding is found in the "conceptual" column. In column three are the mean number of times a student would need to rely on his or her problem-solving skills. Requirements earning an "other" coding include prior knowledge of another topic (e.g., circles), links to a previous exercise or lesson, and a specialized artistic ability. Once again, the last column is the total of the first columns and represents the mean number of cognitive requirements for each task.

Table 9
Task density, as determined by examining tasks per page

	task pages	chapter pages	% pages as task pages	total responses	items per task page	responses per task page	resp. per chap. pg.
German/LS	16	25	.64	295	6.875	18.439	11.801
German/BM	24	33	.73	331	5.875	13.795	10.032
Japanese/NM	23	29	.79	106	2.826	4.609	3.656
American/PA	16	46	.35	289	16.813	18.055	6.280
American/CM	59	93	.63	501	2.271	8.492	5.387
Mean	27.6	45.2	.63	304	6.932	12.678	7.431

Because of the variety of question formats from book to book, I felt it was important to find additional measures that would reflect the amount of "actual work" found in the exercises. Recall that the German *Base Mathematics* book often had numbered exercises that had many lettered sub-problems, as does the American *Contemporary Mathematics*. Table 9 contains a collection of some possible additional measures to help paint a better picture of how many responses are being asked for in each book. In the first column "task pages" are defined as any page that contains one or more of the tasks of interest. The third column shows what percentage of the pages in the chapter are task pages (e.g., thirty-five per cent of the *Pre-Algebra* pages contained at least one task). The "total responses" are the total responses in each chapter as defined in

Table 5. The fifth and sixth columns demonstrates that in each of the books, the 'number' of exercises can be a misleading indicator of how many responses are asked to be given. The final column spreads the responses out over the entire chapter, even to those pages not containing any tasks.

Table 10

Projected area measurements and numbers of tasks

	lines per page	potential lines	in^2 of pages	projected tasks
German/LS	61.50	12,730.50	16,443.56	880
German/BM	60.00	21,450.00	19,386.00	1,410
Japanese/NM	30.50	6,588.33	10,246.50	455
American/PA	53.33	45,757.14	70,785.00	3,497
American/CM	58.67	32, 561.85	51,892.50	938
Mean	52.8	17,305.19	33,750.71	1,436

The entries in Table 10 are primarily projections of data made on the entire textbook, the exception being the first column data, which is an actual measurement based on the body font size and line spacing. The "potential lines" is found by multiplying the lines per page by the total number of pages. The "in^2 of pages" is found similarly, but multiplying the *square inches* of each page by the total number of pages. The final column is found by multiplying the number of tasks found in the sample chapters by the total number of chapters in the book (or in the series in the case of the American *Contemporary Mathematics*).

Table 11

Reading levels of three 100 word passages from each of the five textbooks

	Raygor	Fry	Flesch	Lix
German/LS	College*	n/a	n/a	58.05
German/BM	Professional*	n/a	n/a	58.08
Japanese/NM	8	9	59.92	45.33
American/PA	8	7	65.23	44.43
American/CM	10	10	54.49	49.96
Mean				51.17

Table 11 contains the results of four readability tests done on each of the five textbooks. The Raygor and Fry tests provide the corresponding grade levels of the texts. While the Flesch and Lasbarhetsindex (Lix) tests give a score based on a range of readability (100 – 0 and 20 – 60, respectively). For the Flesch test, a score of 0 would represent the most difficult text, while a 60 would be the most difficult on the Lix scale.

The Fry and Flesch tests require a syllable count, and due to my lack of German speaking abilities, I was unable to make the necessary syllable counts.

Chapter 5: Discussion

This study was motivated by the wealth of research that indicates that mathematics textbooks used by students in the US contain many more topics than those used by students in other countries. And while the numbers of topics in a book do not always (rarely, in fact) exactly dictate the numbers of topics to be covered over the course of a school year, the great correlation between the two certainly justifies an investigation of the texts themselves. The purpose of this chapter is to present the original research questions within the new context of the data, to address the research questions by relating the data to the specific underlying questions, and to offer further questions for future research and dialogue. It will not take the reader long to realize that a great deal of questions remain in the area of textbook research, particularly in their use.

Research questions

Research question one asked how American textbooks can cover so many more topics than those books from nearly all other countries. Readers may have begun to form their own opinions based on their own experiences and the data presented in chapter 4. In the following section, the underlying questions of the first research question will be addressed by analysis of the data and connections made with information that was previously presented. In addition, other questions with regard to the number of topics in textbooks that have come up as a result of this study will be offered as possible fodder for research and discussion.

Research Question 1: How do American textbooks cover so many more topics than those books from other countries?

Are there physical size differences that allow for the coverage of more topics?

Based on the book descriptions from Chapter 3, a quick answer would be a resounding, "Yes!" The extent of the differences is even more revealing. Before looking at these differences, it is certainly important to recall that this study bases many of its findings on five representative books. And regardless of their representativeness, they are nonetheless five books. In Table 4, one can see that the traditional American text contains 858 pages and the American reform 555. To Americans familiar with such texts, this may not be very surprising. We have simply become used to big books (see Harry Potter, etc.). However, when compared to the lengths of the texts from Japan (118) and Germany (207 and 359), 858 pages do seem as if they allow for much greater topical coverage.

An important category to be paired with page length when addressing physical size is that of actual page dimensions. When the area of each of the pages is multiplied by the number of pages, the result is the total amount of potential area. The American traditional text, for example contains 70,785 square inches of potential (the most) versus 10,246.50 square inches for the Japanese text. In fact, the American traditional text has almost exactly the same amount of potential area as both of the German texts in this study combined. I suppose the answer remains a resounding, "Yes!"

Are there differences in the structure and presentation of the content (e.g., chapter organization, task construction) that allow for the coverage of more topics?

In Table 4, it is clear that the American texts' chapters seem to be longer, which is logical given the longer books. Though it can be further said that the American *Pre-Algebra* had the second longest chapter, and by far the most number of chapters (thirteen) of the five. The American *Contemporary Mathematics* chapter was roughly 3 times as long as those from Japan and Germany. This is in part due to the length of the *Contemporary Mathematics* lessons, but also because of the number of topics covered in the U.S. chapters. The longer chapter lengths are amplified when the physical size of the text is taken into account. The American books have greater dimensions than the other three. One reason for the *Contemporary Mathematics* length is the high percentage of items presented in context. It simply takes more words to set up mathematical exercises, as evident of *Contemporary Mathematics'* 77.716 words per task and the next closest at 34.184.

It would seem, particularly based on the last column in Table 9, that the German textbooks are the most economical, in terms of getting the most "mathematical work" for the given number of pages. This does not by any means imply that the German book would cover more topics, only that more exercises fit into less space. On a related note, it is astonishing how much the *Pre-Algebra* tasks per page value dropped when the comparison was switched from "per task page" to "per chapter page." This reiterates the idea that the traditional American texts, this one in particular, contain a great deal of text and writing that is not likely to be actively utilized by the students. The collections of the exercises themselves also lent more topics to be covered in the American textbooks. In the foreign texts, for example, if an exercise set was related to applications of the Pythagorean Theorem, then every exercise in the set dealt with applications of the Pythagorean Theorem. However, in both of the American texts, there were opportunities, in some cases many, to revisit earlier, typically unrelated topics. The US Saxon books are an extreme case of this idea, as every exercise set reviews many previous mathematical concepts. It is important to note that this type of review may be occurring in other countries, but the evidence is simply not found in the textbooks.

What other factors contribute to the coverage of more topics? (e.g., possibly the SIMS analysis, teacher workload)

Flanders (1987) study of American textbooks reiterates the notion that our books are repetitive. His study reported the amount of material that was new to particular textbooks. The books for grades 6, 7, and 8 contained around 38%, 36%, and 31% new material, respectively. My initial thought was that this repetitiveness allows US books to cover more topics by revisiting them year after year. But, in some ways, it would seem that time spent covering old material actually takes away the ability to cover more and more topics. Again, how “topic” is defined may vary from study to study, therefore making it difficult to determine if “Flanders’ topics” and the “TIMSS’ topics” were similar. A similar repetition study would be interesting to perform on books from other countries, as well as obtaining more current data from US books to see if this repetition remains, in spite of current reform movements.

According to Flanders (1994) the content of the early 1980s’ SIMS assessment did not represent the content covered in US SIMS classes. Perhaps this observation caused an ‘*over*-correction’ in US curricula and as a result perhaps *too* much was covered. That may not necessarily be correct, but other countries have certainly demonstrated that success, in the eyes of the SIMS and TIMSS assessments, can happen while not including all of the SIMS and TIMSS content in their curricula. I can certainly speak from my own experience as a student throughout the 1980s and as a teacher in the mid to late 1990s that there has definitely been material added to both the junior high and high school curricula. For example, the only statistics I was introduced in junior high or high school were the ideas of mean, median, and mode. These ideas were not even referred to as being part of a larger strand known as statistics. I remember seeing a statistics book on a shelf in my eleventh grade mathematics teacher’s and could not contain my curiosity as I asked Ms. Welling if there really existed whole courses on sports statistics. Thus began my first introduction to the *true* meaning of statistics. Little did I know, the mathematics education climate would change greatly in the next eleven years and I would find myself guiding tenth graders as they built 95% confidence intervals through the use of random number generators found on their graphing calculators. It is interesting as I look back that it was Ms. Welling who had the *only* graphing calculator in our entire school. Other topics, such as matrices and linear programming, that I only encountered as a college student have found their way regularly into high school curricula across the country. Again, this is based on my own experiences as student and teacher, but the additions are both obvious and substantial.

Lee and Zusho (1999) point out that one possible factor for success in mathematics, often overlooked by researchers, is the role of teacher manuals in the classroom and the learning process. While Lee and Zusho’s work focused on comparing early primary teacher manuals, I think it would be beneficial to perform similar studies on lower and upper secondary manuals. Based on the characterizations of teacher manuals set forth by Lee and Zusho, it would seem as there is greater trust placed in Japanese teachers. This idea of trust brings up a number of questions in the realm of teacher professionalism: Have Japanese teachers somehow earned more trust from their governing body? What more can US teachers do to earn such trust? Do American teachers even wish to have such trust placed with them? Furthermore, what does the research say about US students whose teachers' subject-matter preparation and knowledge are comparable with those teachers in countries deemed more mathematically successful than the US?

Some research suggests that experience is a factor in the degree of textbook use by a teacher (Woodward & Elliott, 1990). Is this true for other countries? I believe that our teachers are prepared, but they are left 'alone' too early. Beginning teachers are more likely to depend on textbooks than those with more experience (Tanner, 1988). To what extent is this affected by Japan's mentoring program and Germany's extensive training program?

There are more curricular factors that I have come across that seem to be worthy of investigation. For example, I wonder if there are reasons, beyond remediation, that topics such as arithmetic are included in US eighth grade curricula? I venture to guess that at some level, the fact that most students can find relative success with such topics is a reason for their inclusion. It seems to me that there are more worthwhile areas that we can turn our students on to where they can find success while learning richer mathematics. The data in the Chandler and Brosnan (1994) study suggests to me that there are still changes to be made. The post 1989 eighth-grade textbooks in their study still had 39.9 per cent of their pages devoted to arithmetic concepts. Perhaps a study of books before and after *1999* would be a good indicator of progress made in the past ten years.

I wonder to what extent the variety of school structures, with regard to grade levels, are taken into account when deciding on curricula and curricula materials. Schmidt et al. (1993), refer to the great differences in structure when looking at how many countries see eighth grade as the beginning of secondary school as opposed to the end of primary school. This can be seen in the school structure statistics discussed earlier. In the US, around 83% of the schools containing 7^{th} or 8^{th} grade levels had 8^{th} grade as their final grade. This is in stark contrast to the 0% for both Japanese and German schools. It might be valuable to look at the mathematical performance of the 17% of the schools that have 8^{th} grade as a non-terminal grade.

There are also a number of economic issues that I believe may indirectly contribute to curricular matters. Schmidt et al. (2001) note that American textbooks on the whole tend to be conservative because they are produced in a market-driven economy. How closely related is the extent to which a country's economy is market-driven to the content of its textbooks? Also, in the United States, 32 per cent of the teachers in high poverty schools agreed that materials such as textbooks were not available as needed ("To Close the Gap, Quality Counts," 2003). This percentage drops to 25 per cent, which still seems a bit high, for all schools. This made me curious as to what similar percentages would be in other countries. This may or may not speak highly to the priority placed on education in a society.

Research question two: *How does the nature of the exercises vary from country to country?*

Are there characteristics of the exercises in the textbooks of higher performing countries that possibly lead to a better understanding of the mathematics?

The reading level of texts may seems as an area that might help increase student understanding of mathematics. However, as stated before, the language differences and translation issues certainly take away from the credibility of the data in Table 11. However, it is curious to see that the two American books differ by three complete grade levels according to the Fry test. It is also incredible how close the two German Lix test scores were.

While the use of the exclamation point in the German books was the inspiration for the inclusion of the punctuation categories in Table 6, perhaps even more intriguing was the much higher occurrence of questions in the three non-German books. As I looked back through the books, I found representative cases when a proof was called for in an exercise and the German text would instruct the student to “Prove this” (or “Prove!”) and the non-German texts would ask the student, “Can you show how you would prove this?” The high occurrence of exclamatory sentences in the German *Base Mathematics* chapter is even more glaring when one considers that 6.5 % of the sentences in that particular chapter were exclamatory while the other titles’ combined percentage was one-half of a percentage point (!).

There were three entries in Table 7 that stood out as I analyzed the data. First, was the fact that there were zero proofs called for in the *Pre-Algebra* text, while both

German texts contained a significant amount of proof tasks. Secondly, I noticed that the *Contemporary Mathematics* chapter called for far more conjectures than the other four books combined (0.948 per item versus 0.324 combined). The apparent lack of conjectures in the Japanese textbook is also quite surprising. Based on some video clips from the TIMSS Videotape Study, it seems as if Japanese students offer conjectures at the request of their teachers rather than their textbooks. The third item I will point out from this table is the disparity in the number of numerical expressions per item between the *Pre-Algebra* responses and the other four. This possibly indicates much less exposure to variables and might also be worth further investigation.

When comparing the inherently related columns of "numerical responses" with "procedural practice," in Table 7, I found it interesting that for the Japanese tasks, the numerical response value was quite a bit higher than the procedural practice value, especially in light of the pairs of data from the other countries. This made me wonder if the Japanese text asks its users to find numerical answers to questions that are not exclusively procedural practice. I found a number of examples that illustrate this notion and will include such exercises in subsequent versions of this study.

Concerning the *pages with tasks* data found in Table 9, aside from the traditional U.S. book (*Pre-Algebra*), the others seemed to have homework tasks on a similar percentage of pages in their respective chapters. More than sixty-five per cent of the pages in the Prentice Hall chapter did not contain homework tasks. This seemed to create the potential for more reliance on "the text as teacher." This also reflects well the previously referenced research indicating that students in the US do not read about sixty per cent of their mathematics texts.

Focusing on the sentence structure data in Table 5, one can notice that the *Contemporary Mathematics* text has a significantly higher number of sentences per item. This leads to a greater amount of words; however, when looking at the average number of words per sentence, *Contemporary Mathematics* is right in the middle. This is perhaps more significant, as most readability tests are concerned with the *lengths* of the sentences rather than the *number* of sentences. While an initial glance of the books themselves would indicate that the *Pre-Algebra* authors seem to have put an emphasis on visual factors, the number of images per item was least with the *Pre-Algebra* text. On the other hand, the seemingly "wordy" *Contemporary Mathematics* tasks lead all five books in the category of images per item with an average of over one image per item. In the somewhat related realm of placing the mathematics in context, it was not surprising that highest percentage of items in context was the forty-six per cent of items given in context by the *Contemporary Mathematics* book. I was actually expecting that percentage to be higher. However, I was interested to see only just over seven per cent in the Japanese *New Mathematics* book. It would take further investigation to determine if this relative lack of contextualization is consistent throughout the Japanese curriculum, as well as in-class lessons.

Are there such characteristics that could plausibly be incorporated into US textbooks?

Since American mathematics students are not reading more than half of the pages textbooks, it would seem there is a quick way to make our books smaller. Another way to approach this problem is to design more books along the models set forth by books such as the *New Mathematics* and *Contemporary Mathematics in Context* texts in this study whose lesson pages require action and thought by the students. However, if one were to simply pare down the “unused” pages from our traditional texts, how would this affect the learning of mathematics in those classrooms where such books are being used? I suspect the books would either become more like the German texts, with a short lesson section and extensive problem sets or resemble a hardback workbook, primarily used for practice problems, much like the Japanese practice books. This has caused me to ask if students are really reading as little of the textbooks as the past research would indicate? The Freeman and Porter case studies, as cited in Johnsen (1993), referenced earlier indicate that teachers, too, ‘skip’ some of the material in the textbooks. This sends a potentially dangerous message to our students. As the mathematical authority in my classroom, I felt that including relevant lessons that were not part of the textbook added to this authority. Whereas, I feel that *not* covering material in the text may detract from a teacher’s mathematical authority, or at least cast doubt on the thoroughness of the course. Student interviews from both classes where teaches supplement topics not found in the text and those where a substantial amount of material in the texts was not covered might provide insight in this area.

Additionally, TIMSS studies have found there to be little difference in the content coverage of 8^{th} grade courses regardless of different titles, with those students in Algebra courses the only exception (Schmidt et al., 1993). This seems to be an area that could either use research or streamlining, or both, in the future. Such questions to be asked could include: Are there advantages to having such differently titled courses, with allegedly different curricula, whose content is essentially the same? How much effort is put into creating such courses and could such efforts be put into creating one better course?

What other factors should be considered when investigating textbook exercises and materials? (e.g., use, teachers' manuals)

Again, there are a number of issues concerning teachers and teacher materials that relate to exercises in both direct and indirect fashions. Johansson, as quoted in Johnsen (1993), asserts that, "Textbooks are hopeless no matter how you approach them" (p. 158). Is this how many teachers feel? Both student *and* teacher attitudes towards textbooks are critical given their widespread use; it is important that teachers feel confident in the books they employ and the exercises within those texts. On a related note, there is an increasing amount of alternative instructional material available to teachers. To what extent is this material being used? With what success is the use being met?

There are a number of clear visual differences among the books in this study. These differences have created a series of related questions and possible avenues for further study. First, I found that type size is a relatively untested area as far as what sizes are most readable by different ages (Purves, 1993). Is type size an area worth studying? I feel it may have an effect on student attitudes towards the text. Though, in American textbooks, the sizes are kept rather standard, and, aside from the one German text, the typeface sizes in this study are essentially the same. Secondly, Levin and Mayer (1993) bring up the idea of picture quality and how little research has been done investigating to what extent the quality of a text's pictures has on its effectiveness. What an interesting study this would be, though time-consuming and challenging to setup. Lastly, because visual attractiveness is an important factor in the adoption process, publishers feel compelled to match their competitors in terms of quality and quantity of illustrations (Woodward, 1993). Is this still true in an age when the ability to create more visually interesting material is easily accessible to all? Woodward charges textbook authors to strike a better balance "between the competing functions of textbooks" (1988, p. 20). These functions Woodward speaks of include learning and providing visual appeal. He also notes that because of these many functions, books have "begun to resemble the media that define what it means to be literate in our society" (p. 20). Otte (1983) speaks to the belief that, at times, illustrations can serve as a distraction, "It may be pleasant in a market place to be all the time distracted by a new optical stimulus, but the excitement lasts only as long as the reacting attention is appropriately rewarded" (p. 23). Noonan (1990) also suggests that the "visual appearance of text can make some difference to the enthusiasm with which pupils will read it" (p. 61). The investigation of visual appearance's effect on enthusiasm seems to be a worthwhile venture. How important is the visual appearance of a text when it comes the reader's enthusiasm? Does this translate into enthusiasm towards the material? Given the chance, it might be revealing to interview some students to get reactions to the formatting of different books from Germany, Japan and the U.S. This would probably be somewhat slanted, as US students might be intimidated by the foreign language, but interesting nonetheless. As I look through the different texts from the three countries of interest, it is clear there are some striking differences in these areas on which I have not found much literature. I wonder if it would be worthwhile to look at these differences, particularly as I question how much teachers really encourage students to read the 'non-homework parts' of their texts?

Perhaps if we are to expect students to read more of their mathematics texts, page layout needs to be given consideration (Noonan, 1990).

Johnsen (1993) notes that the implementation of homework leads to the textbook's role as a tool outside of the classroom. There have been few studies done investigating the textbook as a homework tool, and, perhaps more importantly, of its use when an instructor is not present. Furthermore, as mathematics reform entered the 70s and 80s, mathematics textbooks contained more verbal text. Some have wondered if American decline has coincided with this increase in text as a result of poor reading ability (Johnsen). How can we study the effect of such issues on students' use of exercises found in the text? Of this, I am not certain.

I suppose going in to this study, I was aware that there were significant differences in the *content* of textbooks from various countries. I have now become aware of differences in other areas of textbook development (perhaps even greater differences than content). In graduate courses of mine in which content differences have been discussed, the result of content can be argued, however, one question that persisted was *how* could this great difference in topics occur? How can it occur physically? I have started to gain a sense of the realities of these differences, particularly with regards to presentation. Not only physical presentation (image use, font selection and size, size of texts, etc.), but also how homework tasks are presented (what is expected of the student, context, readability of the texts themselves, etc). Purves (1993) states eloquently, "Textbooks are indeed a kaleidoscope, and we should not see them as being a single image or even a single refraction of the light of instruction. How we view them depends on who we are, what our view of curriculum and instruction may be, and what our view of knowledge and learning may be" (p. 16). This sentiment has caused me to rethink the idea of textbook analysis. It has been shown that the content of textbooks is a more common focus of research than their use by teachers and students (Johnsen, 1993). I feel that in the future research should almost exclusively focus on how students use textbooks and their reaction to and success with different formats. How a researcher views a text may be helpful in some respects, but ultimately it is the user of the book whose reaction we should be seeking.

This last idea may seem quite a shift from the focus of this particular study. However, as I reflected on the textbooks in the study and the information collected from them the more I thought of the importance of their use. What in my eyes may seem to be an extremely effective presentation method may not be well received by students. This is particular true when there is an attempt to transfer certain methods from other cultures. The information found in Chapter 3 speaks to the great differences under the umbrella of education alone, not to mention tremendous cultural differences (of which the educational differences are a by-product). This is not to say that there are not textbook characteristics or teaching methods that can be implemented in the US that are in use in other countries. I do see it more important for the United States' educational community to reflect on our own teaching practices, student reaction and achievement as a result of such practices, goals in both of these areas for the future, and whether the textbooks in use provide the opportunity for these goals to be met.

Appendix A

The Educational Framework of Germany, Japan, and the United States for K-12 Public Education

Typical Educational Paths for the United States, Germany, and Japan

Age	United States		Germany					Japan	
3	Pre-primary		Kindergarten					Pre-Primary	
4	Pre-primary		Kindergarten					Pre-Primary	
5	Pre-primary		Kindergarten					Pre-Primary	
6	Elementary		Primary			Primary		Elementary	
7	Elementary		Primary			Primary		Elementary	
8	Elementary		Primary			Primary		Elementary	
9	Elementary		Primary			Primary		Elementary	
10	Elementary		Orientation			Primary		Elementary	
11	Elementary	Middle	Orientation			Primary		Elementary	
			Students then proceed to one of four types of Lower Secondary Schools						
12	Junior High	Middle	Gymnasium	Hauptschule		Realschule	Gesamtschule	Lower Sec	Unified
13	Junior High	Middle	Gymnasium	Hauptschule		Realschule	Gesamtschule	Lower Sec	Unified
14	Junior High	Senior High	Senior High	Hauptschule		Realschule	Gesamtschule	Lower Sec	Unified
15	Senior High		Gymnasium	Vocational	Dual	Realschule	Gesamtschule	Upper Sec	Unified
16	Senior High		Gymnasium	Vocational		Dual System		Upper Sec	Unified
17	Senior High		Gymnasium	Vocational		Dual System		Upper Sec	Unified
18	Community and Junior Colleges, Universities, Colleges, Technical and Professional Schools		Gymnasium	Vocational		Dual System		Universities, Vocational and Technology Colleges	
19			Vocational, Non-University Institutions			University			

Appendix B

Exercise Data Collection Form

	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Procedural Practice	Conceptual Understanding	Problem Solving	Special Requirement
1																			
2																			
3																			
4																			
5																			
6																			
7																			
8																			
9																			
10																			
11																			
12																			
13																			
14																			
15																			
16																			
17																			
18																			
19																			
20																			
21																			
22																			
23																			
24																			
25																			

Appendix C

Recorded Exercise Data from

Each of the Five Textbooks

LS	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	53	4	14	2	.:.			IMG							Justify		1		
2		5	19	2	..			IMG							Constr		1		
3		6	20	2	..			1							Draw	1			
4		7	15	2	..			IMG							Constr	3			
5		8	30	2	..			1							Draw	1			
6		9	23	3	…			IMG							Draw		1		
7		10	17	1	.			1							Draw	1			
8		11	22	1	.			1							Draw			2	
9	55	2	14	2	..			1							Draw	3			
10		3	13	2	..			1							Draw	5			
11		4	11	1	.			1							Draw	5			
12		5	21	3	..?	1		1			1				Draw	2			
13		6	22	3	..?	1		1			1				Draw	2			
14	56	7	21	2	..		1	IMG			1						1		
15		8	18	2	..		6	IMG			6					6			
16		9	24	2	..		1	1			1						1		
17		10	7	1	.		1	IMG			1					1			
18		11	12	1	.		7	IMG			7					7			
19		12	38	4	….			IMG					1				1		
20		13	32	3	.:.		2	IMG					1		Justify		2		
21	59	3	22	2	?.			5IMG				5				5			
22		4	11	1	.		5	5IMG			5					5			
23		5	7	1	.		9	1			9					9			
24		6	20	2	..	1	1	1			2					2			
25		7	10	1	?		2	1			2					2			

LS	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	59	8	15	1	.		2	IMG			2						2		
2		9	16	1	.		2	(IMG)			2						2		
3		10	17	2	..		1	(IMG)			1					1			
4		11	9	1	.		4	(IMG)			4					4			
5		12	23	2	..		2	(IMG)			2					2			
6	60	13	21	2	..		2	IMG			2					2			
7		14	19	2	.?		1	IMG			1					1			
8		15	38	3	..?		4	1			4					3	1		
9		16	24	2	.?		2	1			2							2	
10		17	18	2	..		2	1			2						2		
11		18	17	2	..		4	IMG			4					4			
12		19	24	3	;.?		2	(IMG)			2					2			
13		20	26	2	..		2	1			2						1	1	
14		21	10	2	..		2	1			2							2	
15		22	19	3	…		4	1							Justify	4			
16	61	23	40	4	:…			IMG					1		Compar		2		
17		224	37	4	:?..			IMG					1		Disect		1	1	
18		25	21	3	:..		1	IMG					1				1		
19		26	46	4	:?.?			IMG							Disect			1	
20		27	33	6	::….			IMG					1		Disect			1	
21		28	25	4	:;..		1	IMG					1				1		
22	63	3	13	1	.		5	1							Justify	5			
23		4	11	1	!			1						2	Draw		2		
24		5	28	4	:…		5	IMG			5		1			5	1		
25		6	29	4	::.?		2	IMG					2				2		

LS	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	65	2	21	2	..	4	4	1			4					4			
2		3	21	1	.		2	1			2					1	1		
3		4	7	1	.		5	1							Draw	5			
4		5	24	2	..	6	18	IMG			24					24			
5		6	19	2	..		2	1							Draw		2		
6		7	10	2	..		2	1							Draw		2		
7		8	39	5	..::.		2	(IMG)					1	1			2		
8		9	15	2	..		1	1							Draw			1	
9		10	15	1	.		1	IMG					1				1		
10	67	3	22	2	..		9	1			9					9			
11		4	23	2	??		2	1			2					2			
12		5	22	2	??		2	1			2					2			
13		6	16	2	..		1	IMG			1					1			
14		7	20	3	…		1	IMG			1					1			
15		8	43	5	…..		4	IMG			1	3				2	2		
16	68	9	23	2	..		2	IMG			2					2			
17		10	4	1	.		4	IMG			4					4			
18		11	10	1	.		1	IMG				1					1		
19		12	18	2	..		6	1			6						6		
20		13	20	4	::..		1	1					1				1		
21		14	20	1	.		4	(IMG)			4					4			
22		15	10	1	.		4	(IMG)			4					4			
23		16	25	3	…		2	IMG			2					1	1		
24		17	87	10	8.s 2?s		4	IMG			4					2	2		
25	69	18	16	1	?		1		1	Moving				1				1	

LS	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	69	19	13	2	..		2		IMG	Arch	2							2	
2		20	35	3	..?		1			Arch	1							1	
3		21	61	5	….?	1	2		IMG	Arch	3					3			
4		22	39	3	…		1		IMG	Bell	1						1		
5		23	64	7	.:…??		3		IMG	Navig		2		1	Justify	1	2	1	
6	70	1	16	2	..	5	15	IMG			20					20			
7		2	21	2	..		2				2					2			
8		3	8	1	.		1	IMG			1					1			Circle
9		4	12	1	?		2	IMG			2						2		Circle
10		5	13	1	.		6				6					3	3		
11		6	37	4	….		5	IMG			5			1		3	2		
12		7	27	2	..		1	IMG				1						1	
13	71	8	27	3	…	2	2	IMG			2	2				3	1		
14		9	17	2	..		2				2					2			
15		10	72	8	……..		5	IMG			5				Draw	2	3		
16		11	44	4	….		4	IMG			4					4			
17		12	55	5	.?:..		1	IMG			1						1		Visual
18	72	13	44	4	:..?		1	IMG			1			1			1	1	
19		14	37	4	.??.			IMG						1	Draw			1	
20		15	27	3	…		3				3				Compare	2	1		
21		16	67	7	..:..:.		1	IMG			1		2			1	2		
22		17	33	4	.:..			IMG					1				1		
23		18	18	2	:.			IMG					1				1		
24	73	19	57	6	..?..?		2		2IMG	Maps	2						2		
25		20	44	2	??		2		IMG	Geom	2					2			

LS	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	73	21	34	3	…		1		IMG	Electr	1					1			
2		22	18	2	.?		1		IMG	Rope	1					1			
3		23	22	3	..?		1		IMG	Trees	1					1			
4		24	50	3	..?		2		IMG	Dist	2					2			
5	77	1	43	4	…?		2		IMG	Bridge	2					2			
6		2	11	1	.		2	2IMG			2					2			
7		3	7	1	.		1	IMG			1					1			
8		4	19	2	..			IMG					1				1		
9		5	32	2	..		1	IMG				1					1		Visual
10		6	34	4	:…		2	IMG					1		Draw		1	1	
11																			
12																			
13																			
14																			
15																			
16																			
17																			
18																			
19																			
20																			
21																			
22																			
23																			

NM	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	120	1	15	1	?		1	IMG				1					1		
2		2	29	2	??	2		IMG			2						2		
3		3	29	2	..		1	1					1				1		
4		4	34	4	….	1	2	1			3					3			
5	121	5	9	1	?		1	IMG				1					1		
6		6	8	1	?		1	IMG				1					1		
7		7	31	2	??	2		IMG				2					2		
8		8	22	3	…	1	1	1			2					2			
9	122	9	19	1	.		1	IMG			1					1			
10		10	28	1	.		1	IMG				1					1		
11	123	1	29	1	.		1	IMG				1				1			
12		2	44	4	….		3	1			3					3			
13	124	3	24	1	.		1	IMG				1				1			
14		4	27	2	??		2	1				2					2		
15	125	5	33	2	..		1	1				1					1		
16		6	48	3	.??	2		1				2				2			
17	126	7	66	3	..?	1	1	IMG			2					2			
18		8	42	2	..		1	IMG			1					1			
19		1	19	2	.?				1	Dist			1				1		
20	127	2	10	1	.		1		1	Scale	1					1			
21		3	38	2	..		1		1	Dist	1					1			
22	128	1	48	3	..?		1	IMG			1					1			
23		2	40	2	..		1		1	Dist	1					1			
24	129	1	46	5	..?.?	1	1	IMG			2			1		1	1	1	

NM	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	131	3	7	1	.		4	4IMG			4					4			
2	133	4	13	1	?		4	1							Yes/no	4			
3	134	1	16	1	.		1	1				1					1		
4		2	22	1	.		1	1				1				1			
5	135	3	9	1	.		1	1			1					1			
6		4	18	3	..?		3	1			3				Def	1			Vocab
7	136	5	14	1	.		2	1			2					2			
8		6	22	2	..		1	IMG					1				1		Prev ex
9	137	7	20	1	?		1	1			1						1		
10		8	33	2	..		2	1				2					2		
11		9	22	1	?		1	1			1						1		Vocab
12	138	10	24	1	.		1	IMG			1							1	
13		1	16	1	:		2	1			2					2			
14		2	15	1	?		1	1				1					1		
15	139	3	66	6	……	2	2	IMG			4						4		Vocab
16		4	18	1	.		2	IMG			2						2		Visual
17	140	5	26	2	..		1	1				1					1		
18		1	18	1	?		1	IMG			1						1		
19		2	37	2	..		2	1				2					2		
20		3	25	2	..		3	1			3					3			
21	141	4	42	4	….		3	IMG			3					3			
22		5	32	1	N/A		1		IMG	Dist					Verify		1		
23	142	1	55	4	:?.?		2	1			2						2		
24		2	33	2	.?		1	IMG			1						1		

NM	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	142	4	35	3	.??		2	1			2					1	1		
2		5	16	1	.		1	1			1					1			
3	143	6	8	1	:		2	2IMG			2						1	1	
4		7	29	1	.		1	1			1						1		
5		8	34	2	..		1	IMG					1				1		
6		9	21	2	.?		1	1			1						1		
7		10	32	2	..		1	IMG			1							1	
8		11	36	2	.?		1	IMG			1					1			
9		12	33	3	.??	1	1	1			2					1	1		
10	144	1	33	2	.?		1	1				1					1		
11		2	37	2	..		1	IMG					1				1		
12		3	36	2	..		1	IMG					1				1		
13		4	24	1	.		1	IMG			1							1	Visual
14		5	47	3	..?		1	IMG			1							1	
15		6	63	6	.:….	4	3	IMG			7					5	1	1	
16																			
17																			
18																			
19																			
20																			
21																			
22																			
23																			
24																			

CP	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
	Pg.	#	Structure			Single Comp	Multiple Comp	Purely Math	In Context		Response Type					Cognitive Requirement			
			# Wrd	# Snt	Punct				Image	Topic	Numerical Answer	Numerical Expression	Explanation Required: Proof	Explanation Required: Conj	Explanation Required: Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	328	o	29	9	6.s 3?s		1			Columns	3			1	Plot	1	3		
2	332	o	21	2	.?				IMG	Arch					Def		1		Spatial
3	334	o	18	3	…	1		1						1	Descr		1	1	
4	335	1	83	5	?…?					Columns				1	Graph	1	1		Experiment
5		2	90	6	….??					Columns				2	Graph	1	1		Experiment
6		3	138	11	10.s 1?				3IMG	Arch					Descr	2	1		
7	336	4	87	5	.?..?				IMG	Dice	1			2			1	1	Dice
8		1	118	7	.?….?	6	1	IMG			9	1		1		1	1		
9	337	2	89	7	.??.???	1			IMG	Mirror	1			3		1	1		
10		3	83	6	..??.?			IMG						2	Descr		1	1	
11		4	260	16	8.s 8?s			2IMG			3	2		5	Draw	2	2		
12	338	1	24	3	.??					Pyramid					Cultural				Cultural
13		2	48	3	..?			1							Draw		1		Drawing
14	339	3	56	7	.?.?.??					Packing					2Draw				Consumer
15		4	26	2	.?					Science					Examp		1		Science
16		1	55	3	…			8							Descr			8	
17		2	79	7	.?…..				IMG	Column					Design				Prev ex
18	340	3	120	11	6.s 5?s			1						2	Observ			1	Dictionary
19		4	22	2	.?		1	1			1						1	1	
20		5	54	3	.?.		1	1							Draw		1	1	Cubes
21	344	o	29	2	..					Arch					Applic		1		Visualize
22	346	o	31	3	??.			1			1			1	Draw		1		
23	347	1	95	11	7.s 4?s		1		IMG	Construct	1			1	Draw		1		Drawing
24		2	78	9	5.s 4?s					Construct	2			1	Draw	2	1	1	Prev ex

CP	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	348	4	52	6	..?…			3IMG						1	2Draw		2	1	Cubes
2		5	40	3	.?.				IMG	Chair				1	Recog		1	1	
3		1	148	9	9.s			5IMG						1	3Draw	1	3		Spatial
4	349	2	137	11	9.s 2?s			1				3			Patterns	1	3		Prev ex
5		3	160	21	13.s 8?s			1			3			4	Draw	2	3		Prev ex
6	350	4	88	7	…….				4IMG	Bridge				1	Draw		1	1	Euler
7	351	5	101	9	9.s			IMG			1				Descr	1	2		
8		6	94	10	8.s ;?	10		IMG			10			1	Graph	10	1		
9		1	114	10	4.s 6;s			6							6Descr		6		
10	352	2	14	2	??			1							Reflect				Reflect
11		3	15	2	?.			1							Reflect				Reflect
12		4	27	2	??			1						1			1		
13		5	27	2	.?					Construct				1			1		
14		1	54	4	…?			IMG						1	Sketch		1	1	
15		2	42	4	.?..			1						1			1		
16		3	111	10	6.s 4?s					Hotel	2				2Symm	2	2		Prev ex
17	353	4	109	13	9.s 4?s				2IMG	Geodesic				1	Rsrch				Def
18	354	5	117	10	10.s			2IMG							Weave			2	Dexterity
19	359	o	26	2	..		1		IMG	Drawing	1					1			
20	361	o	55	4	.??.	1	1	1			2				Observ	2		1	
21	366	o	61	4	….		2		IMG	Garden	2			2		2		2	Gardening
22		1	64	7	….???		5			TV		5	1			1	2	1	
23		2	148	11	6.s 5?s	3	2		IMG	Garden	4			2	Table	4	2		

CP	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	368	5	73	4	.??.			1					1	2			1	2	
2		1	100	9	6.s 3?s			IMG						2	Draw			1	
3	369	2	78	6	……		1		IMG	TV	1	2			Check	2	2		
4		3	109	15	10.s 5?s		1	1			2	1		2	Table	4	4		
5		4	143	17	13.s 4?s	6	1	IMG			9			1	Plot	2	2		Cylinder
6	370	5	63	4	….		2	1			2			1	2Descr		1	1	Visualize
7		6	114	12	9.s 3?s			1				1		2	Support	1	2		Prev ex
8		1	32	3	…				IMG	Arch				1					Arch
9	371	2	18	2	?.		2	1						1	Justif	1	1		
10		3	29	2	??			1						1				1	
11		4	110	11	9.s 2?s			IMG						2	Check	1	1	1	
12		1	68	6	.?.?.?			1						2			1		Prev ex
13	372	2	70	6	..???.			2IMG			1	1		2		1	1	1	
14		3	114	9	4.s 5?s	2	3		IMG	Garden	4			1		2	2	1	
15		4	48	3	…		1		IMG	Kite	1							1	
16	376	O	68	4	..??		2		IMG	Package	2						1	1	
17	377	1	179	12	8.s 4?s		6		IMG	Farm	6			1	Justif	6	1		
18		2	78	7	….???		3		IMG	Package	3				Draw	4			
19	378	3	172	13	11.s 2?s				IMG	Package	1	4				2	2		Calculator
20		4	140	12	6.s 6?s	1	4		IMG	Pool	5				Draw	3	2		
21	379	1	88	7	…??..		20		IMG	Giftwrap	20			1	Observ	20	1		
22		2	80	4	….		2	1				2				2			
23		3	113	10	8.s 2?s			1				1			2Plot	2	2		

CP	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
	Pg.	#	Structure			Single Comp	Multiple Comp	Purely Math	In Context		Response Type					Cognitive Requirement			
													Explanation Required						
			# Wrd	# Snt	Punct				Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	380	2	18	2	?.		2	1						1	Reason	1		1	
2		3	44	2	..			1						1			1		
3		4	41	4	..??			1						2			2		
4	381	1	86	6	…??.		1		IMG	Shipping		1		1			1	1	
5		2	64	6	…??.		2	1			2	1			Sketch	1	2		
6		3	106	7	….?..			IMG				1		1	Comp		2		
7		4	271	14	13.s ?		1	5IMG				2	2	1			3	1	
8	382	4	109	7	..?.???		3			Package		3		1			4		
9		5	49	3	.??		2					2				2			
10	386	O	36	7	…….			2IMG							T/F		5		
11	389	O	81	8	.?.?.?..	1	1	IMG			2					1	4		
12	391	O	34	3	…			IMG							Patterns			1	
13	394	O	33	3	…				IMG	Package					Draw	1			
14	395	1	82	8	.?.?.?.?				IMG	Rugs					Sketch		2		
15		2	77	8	..?.?.?.				IMG	Quilt	1				Sketch		3		
16	396	3	148	12	11.s ?				IMG	Game				3			1	1	Rules
17		4	113	11	10.s ?				9IMG	Nature					9Symm	9			
18	397	5	74	8	……?.		2		IMG	Candy	2				2Sketch	3	2		
19		1	123	12	7.s 5?s		1	1			1	1	2	2		2	3	1	
20	398	2	58	7	…?.?.			IMG				2		2	Symm	2	4		
21		3	67	7	…..?.			4IMG				7			4Symm	8	1		
22	399	4	16	2	??			1						2			2		
23		5	69	5	..?.?			1						2	Describe		3		
24		6	28	2	..			2IMG							Describe		2		

CP	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	399	2	23	2	??			1						1					Reflect
2	400	3	110	6	….?.					Psych				1	Study		1	1	
3		4	25	3	.??					Tiling				1			1		
4		5	82	7	..???.?				IMG	Sports				3			2		Spatial
5		1	45	3	..!					Quilt					Draw		1		
6		2	37	5	?.???			1						1	Symm		1	1	
7		3	25	3	..!			1							Draw			1	
8	401	4	74	10	9.s !			IMG			1			1	Hist	1	1		Dice
9		5	47	4	..?.			1				1			Symm		2		
10		6	263	23	17.s 6?s				IMG	Quilt				4	Sketch		4	2	
11	404	O	17	1	.			1							Design		1		
12	407	O	37	3	.?.			1						1			1		
13	408	1	96	8	…??.?.	14			3IMG	Archeo				5		14		5	
14	409	2	53	5	…..				5IMG	Cloth					4Symm	4			
15		3	26	3	…				14IMG	Japan					Sort		1		
16	410	4	65	7	……?				IMG	Art					Draw	2			
17		5	49	3	…					Art					Draw			1	
18		1	52	2	..			1							6Draw	6			
19		2	82	6	….?.			6IMG						1	6Draw	6		1	
20	411	3	25	3	.?.			4IMG							4Symm	4			
21		4	35	3	…			4						4			4		
22	412	1	30	3	..?			1						1				1	
23		2	31	2	.?			IMG						1			1		
24		3	36	4	….					Patterns					Draw		1		

CP	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	412	5	89	3	.??					Escher				1			1		
2		1	72	7	…????			IMG			3			1		2	2		
3	413	2	117	7	..;….				28IMG	Design					Label	28			Prev ex
4	414	3	160	11	6.s 5?s				2IMG	Design				4	Sketch		4	2	
5	415	4	60	5	.?…				IMG	Design				1	Draw		1	1	
6		5	113	9	5.s 4?s				IMG	Design				2	Draw		2		
7	416	1	140	16	14.s 2?s			IMG						1	Draw	10	1		
8	417	2	161	14	11.s 3?s				IMG	Art				2	Label	1	3		
9	418	O	35	2	..			1							Outline		1		
10																			
11																			
12																			
13																			
14																			
15																			
16																			
17																			
18																			
19																			
20																			
21																			
22																			
23																			
24																			

PH	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
	Pg.	#	Structure			Single Comp	Multiple Comp	Purely Math	In Context		Response Type					Cognitive Requirement			
													Explanation Required						
			# Wrd	# Snt	Punct				Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	557	1.10	3	1	.	10		1			10					10			
2		11.18	7	1	.			IMG							10Pts	8			
3		19.26	6	1	.			(IMG)				10				8			
4		27.42	3	1	.		16	1			16					16			
5	562	1.5	4	1	.	5		1			5					5			
6		5.10	5	1	.		5	1			5						5		
7		11.14	7	1	.			1							Vocab				Vocab
8		15	12	1	?			1						1			1		
9		16.20	4	1	.	5		1			5					5			
10		21.30	5	1	.		10	1			10					10			
11		31.35	7	1	.			1							Vocab				Vocab
12		36	15	2	..			1							Proc		1		
13		37	21	4	..?.			1			1			1			2		
14		38	18	2	.?	1		1			1						1		
15		39.42	8	1	.		4	1			4						4		
16		43.45	11	2	..		3	1			3					3			
17		46	14	1	.	1		1			1					1			
18		47	14	2	..		1	1			1					1			
19		48	36	2	..	1			1	Arch	1					1			
20	567	1.6	14	3	…	6		1			6					1			
21		7.9	6	1	.			3IMG							Name	3			
22		10.12	11	1	.		3	3IMG			3					3			
23		13	14	1	?		1	1							Verif		1		
24		14.19	11	1	.		6	6IMG			6					6			

PH	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	568	27.32	17	2	..		4	IMG			4					4			
2		33.36	14	2	..		4	4IMG			4					4			
3		37.39	38	5	…?.			3								3			
4		40	32	2	.?		1			Carpet	1					1			
5		41	30	3	..?		1			Ladder	1					1			
6		42	65	4	…?		1			Lndscp	1					1			
7	569	43	61	5	…..		4	IMG			3					2	2		
8		44	23	2	.?	1		1			1						1		
9		45	29	3	..?		1	1			1						1		
10	575	1.3	8	1	.		3	3IMG			3						3		
11		4.6	10	1	.		3	3			3					3			
12		7	16	2	.?			1						1			1		
13		8.13	13	2	..		6	6			6					6			
14		14.19	10	1	.		6	6			6					6			
15		20.21	6	1	.		2	2			2						2		
16	576	22	25	2	.?			IMG						1				1	
17		23	16	3	..?		1	1				1					1		
18		24	18	2	..		3	1							Vocab	1			Vocab
19		25	19	2	..	1	2	1			1				Verify	1	1		
20	585	1.3	16	2	..		3	3							Yes/no		3		
21		4.6	4	1	.	5		3			5						5		
22		7	22	3	.?.		1	1						1			1		
23		8.14	23	2	..	14		7			14					14			
24		15	21	1	.			1							Steps		1		

PH	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	586	17	63	7	…????	3	1	IMG			4					3	1		
2		18	94	10	10.s		1	1			1				Constr	1	1		Prev ex
3	591	1	25	3	???			1			3								Vocab
4		2	7	1	.	3		1			3								Vocab
5		3.7	8	2	..	5		1			5					5			
6		8.10	10	2	..	3		1			3						1		Spec tri
7		11	7	1	.	3		1			3					3			
8		12	7	1	.	3		1			3					3			
9		13.24	8	2	..	12		1			12					12			
10		25	36	3	.?.		1			Balloon	1						1		
11		26	32	4	..?.			1						1			1		
12		27	25	3	…	3		1						1			1		
13	592	28	24	2	.?		1		IMG	Arch	1					1			
14		29	19	1	.		1		(IMG)	Arch	1					1			
15	597	1.2	11	1	.			1				4			Names				Vocab
16		3.4	5	1	.		2		2IMG	Boat	2					2			
17		5	23	2	.?			1						1			1		
18		6.7	5	1	.		2		2IMG	Bird	2					2			
19		8	35	2	.?		1			Navig	1					1			
20		9	36	3	..?		1			Meteor	1					1			
21	598	10	53	3	..?				IMG	Astron	1					1			
22		11	44	4	..?.		1			Canyon	1				Draw	1			Vocab
23		12	13	1	?			1						1			1		
24		13	51	5	…?.		1			Bird	1				Draw	1			

PH	Pg.	#	Structure			Single Comp	Multiple Comp	Purely Math	In Context		Response Type					Cognitive Requirement			
			# Wrd	# Snt	Punct				Image	Topic	Numerical Answer	Numerical Expression	Explanation Required			Proc Prac	Concept Und	Prob Solv	Spec Req
													Proof	Conj	Other				
1	600	1.5	4	1	.	5		5			5					5			
2		6.10	5	1	.		5	5			5					5			
3		11.15	7	1	.			5							Vocab				Vocab
4	601	16	7	1	.			1						1			1		Vocab
5		17.20	14	2	?.		4	4							Verify	1			
6		21.26	13	2	..		6	6			6					6			
7		27.32	10	1	.		6	6			6					6			
8		33	37	3	..?		1		IMG	Engin	1							1	
9	602	34.36	6	1	.	5		3IMG			5					5			
10		37.46	8	2	..	10		10			10					10			
11		47	25	2	.?		1			Loading	1					1			
12		48	39	4	…?		1			Kites	1					1			
13																			
14																			
15																			
16																			
17																			
18																			
19																			
20																			
21																			
22																			
23																			
24																			
25																			

BM	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure				Multiple Comp	Purely Math	In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp			Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	219	1	6	1	:	8		IMG			8					8			
2	220	2	6	1	:	5		IMG			5					5			
3		3	21	2	!!	35		IMG			35					35			
4		4	27	3	…			1							Just		1		
5		5	6	1	?		5	5							Y/N	5			
6		6	12	2	..		3	IMG			3				Verif	3	1		
7		7	20	1	?		1	1							Verif	1			
8		8	26	4	…!		1	1			1					1			
9	221	9	29	3	.?!		1	1			1					1			
10		10	41	3	.!:		2	2					2				2		
11		11	4	1	:		2	2							Cnstr		2		
12		12	15	2	.?		4	IMG			4			1		4	1		
13		13	5	1	:		9	9			9					9			
14		14	21	3	.		6	1			6					6			
15		15	13	2	.!		1	1			1					1			
16		16	31	2	.?		1			Ladder	1					1			
17		17	44	4	…		1		IMG	River	1					1			
18		18	12	1	.		1	1						1			1		
19		19	42	3	?..		1	1						1			1		
20	222	20	51	4	….		2	IMG				1	2				2		
21		21	84	6	…..?		2	IMG				1	1	1			2		
22		22	27	3	.:;;		1	IMG					1				1		
23		23	25	2	?.		1	1					1				1		
24		24	45	3	.:;		1	(IMG)					1				1		

BM	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	223	26	54	3	.?.			1			1								
2		27	10	1	.		1	1											
3		28	44	3	…		1	1											
4		29	13	2	.		2	IMG											
5		30	27	4	…		1	IMG											
6	224	31	29	2	..		6	IMG			6								
7		32	26	2	.:		3	1			3								
8		33	34	4	..?.		3	1											
9		34	21	3	…		3	IMG			3								
10		35	25	3	:..		2	IMG			2								
11		36	46	3	…		3	1			3								
12		37	13	2	..		2	IMG			2								
13	225	38	27	3	..?			IMG											
14		39	10	2	..		1	1			1								
15		40	17	2	..		1	1			1								
16		41	19	3	..?		2	1			2								
17		42	19	2	..		2	1			2								
18		43	37	2	.?		1	1			1								
19		44	16	2	..		3	3IMG			3								
20		45	13	2	?.		9	1											
21		46	29	5	:.?.?		3	1			3								
22	226	47	27	3	…		6	IMG			6								
23		48	49	3	…		4	IMG											
24		49	17	3	…		2	2IMG			2								

BM	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	226	51	19	2	..		4	IMG			4					4			
2	227	52	17	3	:..		6	1			6					6			
3		53	28	2	..		2	1			2					2			
4		54	19	1	..		2		1	Roofing	2					2			
5		55	27	2	..		2	1			2					2			
6		56	35	2	..		5	1			5					5			
7		57	19	2	..		1	1			1					1			
8		58	22	3	..!		2	1			2					1		1	
9		59	32	3	…		1	IMG			1					1		1	
10	228	60	21	2	..		1	1			1					1			
11		61	16	2	..		1	1				1					1		
12		62	29	3	..!		1	1				1					1		Prev Ex
13		63	39	4	….		2	1				2					2		
14		64	32	2	..		1	IMG				1				1			
15		65	26	3	…		2	1			1				Verif	1	1		
16		66	50	4	….		2	1			2				Draw	2	1		
17		67	46	4	…!		2	IMG				2					2		
18	229	68	34	3	..?		2	IMG			1				Verif	1	1		
19		69	18	2	..		1	1			1					1			
20		70	17	2	..	1	2	1			3					3			
21		71	58	6	…?.?		3	1				3		2		3	2		
22		72	47	2	..		2	IMG			2					2			
23		73	51	3	…		2	2IMG					2				2		
24	230	74	27	3	…		1			Weather	1					1			

BM	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	230	76	39	3	..?		1			Cables					Verif	1			
2		77	17	2	..		2		IMG	Wires	2					2			
3		78	51	3	…		1		IMG	Physics	1						1		
4		79	48	4	..??		1	1			1			1		1		1	
5		80	37	2	..		1		IMG	Physics	1					1			
6	231	81	58	5	…?.		1		IMG	Physics	1					1			
7		82	19	2	.?		1		IMG	Pendelum	1					1			
8		83	36	2	..		1		IMG	Particles		1					1		
9		84	74	4	….		1		IMG	Optics	1					1			
10	232	85	47	3	…		1		IMG	Optics	1					1			
11		86	61	3	…		1		IMG	Particles	1						1		
12		87	63	4	….		1		2IMG	Atoms		1						1	
13	233	88	90	5	.:…		2		IMG	Skiing	2		1				1		Physics
14		89	25	2	..		1		IMG	Pipes	1					1			
15		90	58	5	…..		1		IMG	Metals	1					1			
16		91	34	2	.?		1		IMG	Tubes				1		1			
17		92	28	3	:..	1	1		IMG	Cable		1				1	1		
18	234	93	45	3	…		2		2IMG	Construct	2					2			
19		94	67	4	…?		1		IMG	Ladder	1					1			
20		95	51	4	….		1		2IMG	Construct					Verif			1	
21	235	96	43	4	…!		1		IMG	Luggage					Verif	1			
22		97	59	4	..?!		1			Construct					Verif	1			
23		98	58	6	..???!.		1		IMG	Rods				1	Verif	1		1	
24		99	25	1	.		1		IMG	Cabinets					Verif	1			

BM	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	236	101	53	5	?….		5		IMG	Geog	4	1				4			
2		102	19	1	?		1			Satellite	1								Prev Ex
3		103	16	1	?		1			Climb	1					1			Prev Ex
4		104	18	2	..		1		IMG	Sound	1					1			
5		105	20	3	..?		1			Ski lift	1					1			
6		106	66	5	..?..		1		IMG	Ski lift	1					1			
7		107	38	4	..??		2			Maps	2					2			
8		108	32	4	.???		2		IMG	Gradient	2			1		2	1		
9		109	40	3	…		2		IMG	Land	2					1		1	
10	237	110	19	2	..		2		IMG	Survey	2					2			
11		111	39	6	..!.!.		2		IMG	Survey	2					2			
12		112	26	3	.?!		1		IMG	Survey	1					1			
13		113	33	3	..?		1		IMG	Survey				1				1	
14	238	114	19	1	.		2	1				2			Draw		2		
15		115	10	1	.	2		1				2			Draw	2			
16		116	13	1	.		1	1				1			Draw	1			
17		117	15	1	!		1	1				1			Draw	1			
18		118	11	1	.	1	1	1			2					1		1	
19		119	28	3	.!!		3	1			3				Draw	1	2		
20		120	17	1	.		1	1			1				Draw		1		
21		121	34	2	.!		1	1			1				Draw		1		
22		122	33	3	…		1	1					1				1		
23		123	30	2	..		1	IMG					1				1		
24		124	19	1	.		3	1			3				Draw	3			

BM	Problem					Mathematical Feature		Contextual Feature			Performance Requirements								
			Structure						In Context		Response Type					Cognitive Requirement			
													Explanation Required						
	Pg.	#	# Wrd	# Snt	Punct	Single Comp	Multiple Comp	Purely Math	Image	Topic	Numerical Answer	Numerical Expression	Proof	Conj	Other	Proc Prac	Concept Und	Prob Solv	Spec Req
1	239	126	19	1	.		1	1					1				1		
2		127	25	2	!?		1	1					1				1		
3		128	18	2	..		4	1			4					4			
4		129	89	7	..?.?!.		1	2IMG					1	2			3		
5	240	130	36	3	:..		1	IMG				1		1	Verif	2	1		
6		131	87	7	….!..			IMG					1	1			1	1	
7		132	31	1	.		1	IMG					1				1		
8		133	21	1	!		1	1			1						1		
9		134	20	1	.		1	1			1				Draw			1	
10		135	71	5	?.!..		5		3IMG	Navigat				3		5	1	1	
11	241	136	65	4	.:.?		1	IMG						1				1	Biograph
12		137	97	6	:…..		1	1						1				1	Biograph
13	242	138	91	4	…?		1		IMG	Ladder					Verif	1			
14		139	65	4	:.??		1	IMG						1	Verif	1	1		
15		140	62	4	….		1	1				1			Verif	1	1		
16		141	49	4	..?!		1	1						1				1	
17																			
18																			
19																			
20																			
21																			
22																			
23																			
24																			

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