

ABSTRACT

Title of Dissertation: THERMAL GENERATION ASSET VALUATION
PROBLEMS IN A COMPETITIVE MARKET

Wei Zhu, Doctor of Philosophy, 2004

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With deregulation in the electric power industry, traditional approaches for minimizing production costs have become unfit for the present competitive environment. Owners of generation assets must now consider price uncertainty in solving unit commitment problems for scheduling and operating their power plants. Operation flexibility of the generating assets, such as fuel switching and overfire, becomes an important issue. Because in a competitive market with volatile electricity prices, these flexibility may add significant values. On the other hand, operational constraints, such as ramp and minimum uptime/downtime constraints, present physical limits for the generating assets to flexibly react to rapid price changes, which have a negative effect on the asset value. Both of the operational flexibility and operational constraints must be considered simultaneously so as to achieve optimal operation under uncertainty. This dissertation devotes to this very important subject.

Deregulation in the power industry allows new firms to freely enter the generation markets. As a result, capacity expansion is no longer the responsibility of local utility companies and has become a pure investment problem. Overestimating the value of a power may result in stranded capital for a long time period. Therefore, to ensure a successful investment a fair valuation method is essential. The generation asset valuation must fully account for market uncertainty, which results in not only risks but also opportunities. To minimize the risks, one must first have sound models for market uncertainties. In this research, we consider not only the uncertainties of electricity price and fuel price, but also environment temperature because some characteristics of power plants may be sensitive to the temperature. To fully capitalize on profitable opportunities arising in the marketplace due to price spreads of different commodities, such as fuel and electricity, a real options approach is considered, in which different options are exercised at different but ‘optimal’ timings.

Overall, this research is expected to contribute a new methodology for fair generation valuation that accounts for multiple and interdependent uncertainties and complex physical constraints. The proposed approach can help operators achieving optimal operation and investors making appropriate investment decisions. In the long run, customers also benefit from the improved societal efficiency.

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IN A COMPETITIVE MARKET

by

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Chapter 1

Introduction

1.1 Motivation

With deregulation in the electric power industry, traditional approaches for minimizing production costs have become unfit for the present competitive environment. Owners of generation assets must now consider price uncertainty in solving unit commitment problems for scheduling and operating their power plants. Operation flexibility of the generating assets, such as fuel switching and overfire, becomes an important issue. Because in a competitive market with volatile electricity prices, these flexibility may add significant values. On the other hand, operational constraints, such as ramp and minimum uptime/downtime constraints, present physical limits for the generating assets to flexibly react to rapid price changes, which have a negative effect on the asset value. Both of the operational flexibility and operational constraints must be considered simultaneously so as to achieve optimal operation under uncertainty. This dissertation devotes to this very important subject.

Deregulation in the power industry allows new firms to freely enter the gen-

eration markets. As a result, capacity expansion is no longer the responsibility of local utility companies and has become a pure investment problem. Overestimating the value of a power may result in stranded capital for a long time period. Therefore, to ensure a successful investment a fair valuation method is essential. The generation asset valuation must fully account for market uncertainty, which results in not only risks but also opportunities. To minimize the risks, one must first have sound models for market uncertainties. In this research, we consider not only the uncertainties of electricity price and fuel price, but also environment temperature because some characteristics of power plants may be sensitive to the temperature. To fully capitalize on profitable opportunities arising in the marketplace due to price spreads of different commodities, such as fuel and electricity, a real options approach is considered, in which different options are exercised at different but ‘optimal’ timings.

Overall, this research is expected to contribute a new methodology for fair generation valuation that accounts for multiple and interdependent uncertainties and complex physical constraints. The proposed approach can help operators achieving optimal operation and investors making appropriate investment decisions. In the long run, customers also benefit from the improved societal efficiency.

1.2 Summary of contributions

The research in this dissertation has the following original contributions:

- It uniquely develops an efficient method to solve ramp-constrained self-scheduling unit commitment problem with and without price uncertainties.

A polynomial-time algorithm is proposed for solving the self-scheduling problem when the electricity prices are known with certainty.

- It is the first one to propose a general framework to value a thermal generation unit with fuel-switching options considering operational constraints. In this framework, three correlated uncertainties including electricity price and two different fuel prices are considered.
- It uniquely integrates both overfire options and maintenance constraints to value a thermal generation unit in competitive power market.
- It is the first one to incorporate a stochastic ambient temperature model with the asset valuation of power plants.

1.3 Organization of the dissertation

The remaining sections of this dissertation are structured as follows. Chapter 2 presents some background knowledge related to this research, including power plant operations, multivariate linear regression, discounted cash flow (DCF), and options theory. Chapter 3 provides a thorough literature review in unit commitment, real options, and asset valuation of power generation units. A self-scheduling unit commitment problem, subject to ramp constraints and price uncertainties, is discussed in Chapter 4. A regression-based method is devised to determine the optimal commitment and dispatch decision rule under price uncertainty in terms of reductions of fuel economy, heat-electricity transformation efficiency, and available generation capacity. Chapter 5 presents a Monte Carlo (MC) simulation approach to value fuel-switching units. Chapter 6 values

a gas turbine considering overfire options, maintenance constraints and environment temperature in the competitive power market based on a MC method. Finally, Chapter 7 concludes this dissertation and suggests some future research directions.

An overview of this dissertation is illustrated in Fig. 1.1. It can be seen that this dissertation is composed of three main parts. Part I focuses on the optimal self-scheduling problem in the deregulated electric market (Chapter 4). Part II discusses the value of a power generating unit with fuel switching options (Chapter 5). In Part III, overfire options, maintenance constraints, and environment temperature are considered to value a thermal generation unit in the competitive power market (Chapter 6).

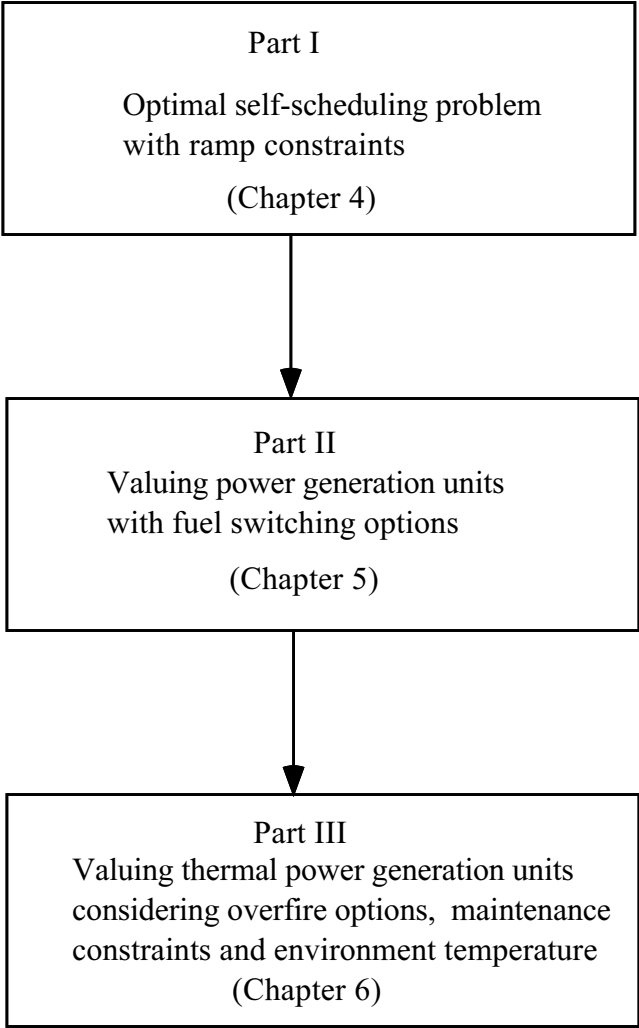


Figure 1.1: Overview of the research.

Chapter 2

Background

In this Chapter, a brief introduction of background knowledge related to this research will be given, including power plant operation, multivariate linear regression, DCF analysis, options theory and least squares Monte Carlo approach.

2.1 Power plant operation

In terms of the load and size, power plants can be classified into two groups: base-load units and peak-load units. The base-load units are normally with large size, including nuclear units, hydro units and large-scale thermal units etc. Such units may always stay on-line whether in regulated or deregulated environment except for scheduled maintenance or unexpected malfunctions. On the other hand, the peak-load units are smaller. One representative is gas turbine (GT). Because a GT can start up within minutes, it is preferred in a volatile competitive power market. In this dissertation, I focus on thermal power plants. Some important concepts relative to the operation of thermal power plants are provided next.

2.1.1 Unit commitment

Unit commitment (UC) is an important optimization problem for power utilities to economically schedule generating resources to achieve cost minimization under the traditional regulatory regime. Although the electricity industry is moving toward deregulation, the importance of the UC does not diminish along with the restructuring trend.

The objective of the traditional UC problem is to minimize the total generating cost over the planning horizon subject to system constraints, such as demand and spinning reserve constraints, and operational constraints, such as capacity constraints, minimum up/down time constraints and ramp rate constraints.

In this dissertation, only the gas-fired or oil-fired units are considered. The dynamic processes of a gas unit operation are relatively simple. For such a generation unit, the startup (or shutdown) cost can be estimated by a constant or a linear function of startup (or shutdown) rate. The fuel cost can be captured by a quadratic function of power amount produced in the corresponding time period.

There are many constraints considered in the typical UC problem. They are stated as follows.

- Demand constraint

The amount of power produced must equal the power consumed in each time period. This is due to the unique characteristic of electricity that it cannot be stored in inventory.

- Spinning reserve constraint

Spinning reserve refers to capability that the system can quickly make up

for loss of some unit(s) and may be modeled as the total capacity of all on-line units less the present load. The amount of spinning reserve at any time must follow certain rules, usually set by regional reliability councils.

- Capacity constraint

There exists an upper bound (q^{\max}) and lower bound (q^{\min}) for the production level of each unit, which is determined by the physical characteristics of each unit.

- Minimum up/down time constraint

A thermal unit is turned on (resp. shut down), it is required to stay on-line (resp. off-line) for a minimum period, known as the minimum uptime (resp. downtime), before it can be shut down (resp. turned on) again. In other words, a thermal unit cannot switch frequently between the on-line and the off-line mode, due to the unit's response time and the damaging effects of stress.

- Ramp rate constraint

Ramp rate limits the capability of a unit to move between scheduled operating levels over short time periods. With the ramp rate constraints, the generation level of a unit becomes interdependent in all hours, which complicates the solution procedure of the UC problem.

More detailed information for UC can be found in Tseng (1996), Baldick (1995) and Lai (1999).

2.1.2 Fuel switching

Generally speaking, fuel switching of a generating unit refers to the ability to convert alternate fuels, such as nature gas (gas, for short) or fuel oil, to electricity. Switching occurs when one fuel out-of-the-money is replaced by another in-the-money (or less out-of-the-money), and can be categorized based on the rate of occurrence:

- Short-term: switch between gas and oil in dual-fired units within hours;
- Mid-term: switch from one unit which can only burn gas or oil to another dual-fired unit through device modifications within days or weeks;
- Long-term: switch from a gas-fired unit to an oil-fired unit (or from an oil-fired unit to a gas-fired unit) through new equipment installation within months or years.

In power industry, players may prefer nature gas to fuel oil because nature gas is much ‘cleaner’ than fuel oil under environment restriction on emission of waste. Therefore, fuel switching has been considered as an emission abatement means. On the other hand, nature gas prices are much more expensive than fuel oil prices based on per British thermal unit. Power producers may have to react to the soaring price of natural gas by switching to cheaper, more environmentally harmful fuel sources. Overall, to reduce power producers’ exposure to price volatility or possible supply disruptions in the present deregulated environment, industrial users are expected to increasingly seek the flexibility to switch fuels using hybrid technologies. Motivated by this, this dissertation focuses on the short-term switchable units.

2.1.3 Overfire

Overfire is an ability to generate power over the upper bound of a unit's production capacity (q^{\max}) under the normal conditions. The disadvantage of overfire is that the unit may wear out more quickly because the overfiring temperature inside the unit is much higher than the designed one in normal conditions. This is also the reason why a unit is not allowed to overfire continuously for a long time period. Therefore, a unit that overfires frequently will require more frequent maintenance. In addition, there also exists an upper limit (q^{over}) for the overfired production level due to the damage effects of high temperature. Normally, q^{over} is different from unit to unit and it is usually about 5 ~ 20% over q^{\max} . In a volatile market, overfire can be viewed as a real option (the concept of real options is reviewed in Section 2.4), which can be exercised when the benefit outweighs the cost of outage due to maintenance. Fig. 2.1 shows a typical production schedule of a unit with overfire capacity.

Furthermore, overfire can be regarded as a compound options (an option of options), because the a unit can be overfired only when it has been on-line. In other words, overfire options cannot be isolated from the real option of turning on/off the power plant.

In a regulated world, overfire is only used as a temporary remedy in emergency. For example, when a large unit is shut down suddenly due to a serious malfunction, and operators cannot obtain enough spinning-reserved capacity or turn on the other units in a short time to meet the demand load, a simple and efficient way is to overfire some on-line units to meet the demand without delay and startup costs. After deregulation, the demand constraints do not exist any more especially for some peak-load units controlled by individual owners, who

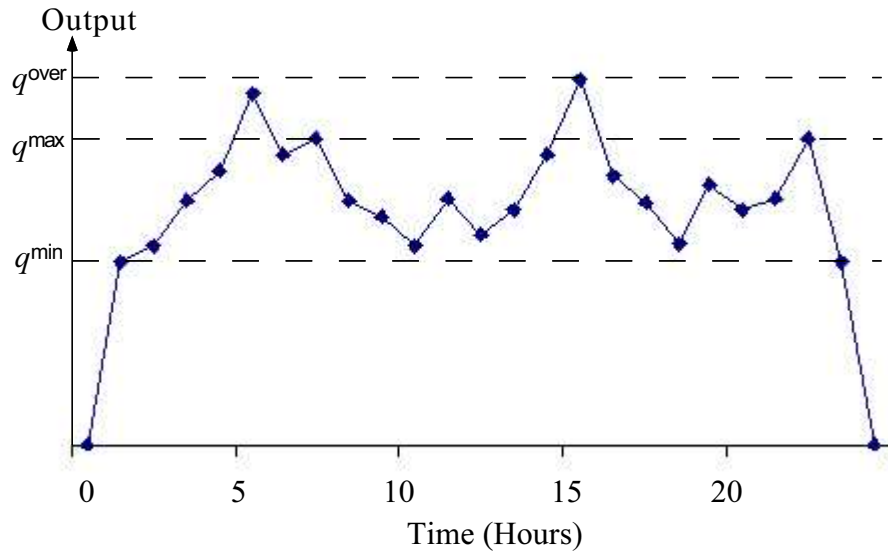


Figure 2.1: An example of production schedule for an overfired unit.

produce for profit. In this situation, overfire can be viewed as a useful option if exercised properly.

There are two types of overfire processes in thermal power generation units as follows:

1. Similar to the fuel switching process (especially for large-scale units), two fuels can be burned to generate power. One fuel is assumed to be considerably more expensive and provides additional capability. In this sense, the fuel cost function will also change correspondingly during overfire.
2. Only one kind of fuel is burned, but additional capacity is provided by additional fuel (particularly for small-scale units). The fuel cost function will not change.

In this dissertation, overfire is assumed to be the latter case for small-size power

generation units. The capacity of overfired units can be improved around 5 ~ 15% over q^{\max} by adding the same additional fuel.

The overfire decisions involve the amount of power to be added and the times at which they should be added. Typically, the overfire options should be exercised without any hesitation once they are profitable in the short run. However, in the long run, overfire may increase maintenance cost and maintenance frequency at the same time, which may cost more than the profit made from overfiring. Therefore, overfire options must be considered in conjunction with maintenance constraints. That is, the value of the overfire options should be incorporated in the asset valuation framework to maximize the total profit of a thermal generation unit. Omission of the option value of overfire may result in over or under valuation of power plants.

2.1.4 Maintenance of thermal units

For a thermal generation unit, whether a steam turbine (ST) or a gas turbine (GT), maintenance in time is always one necessary operation to keep the unit working economically and reliably. A typical maintenance process of thermal unit includes: (1) let the unit cool down; (2) clean all working parts; (3) test hot section components; (4) replace ineffective ones with new parts; (5) reassemble the unit and lubricate it.

In this dissertation, we do not consider the detailed maintenance process and how to improve the reliability of thermal units, but only focus on how to minimize maintenance costs while preserving service safety.

The maintenance cost consists of the following two parts:

1. Direct cost: including the salary of maintenance crews, the cost of new

parts and some resources and utilities, such as electricity, gas and water.

2. Indirect cost: during the time period of maintenance interval, the unit has to stay off. That means loss of many hours for generating power and the corresponding revenue.

In power industry, maintenance itself is a complicated problem, especially for a large unit (Shahidehpour and Marwali 2000). A maintenance manager must consider many factors, including crew availability, resource availability, seasonal limitations, desirable schedule, and reliability check, before making any maintenance plan. In order to simplify maintenance constraints, we transform all these constraints into a maintenance contract, which is common in small-scale units and will be discussed in detail in Chapter 6. The payment of the maintenance contract can be viewed as maintenance cost. In this dissertation, a valuation framework is developed for determining the value of thermal generating unit while considering maintenance contracts, overfire options and other operation constraints in the competitive power market.

2.2 Multivariate linear regression

The multiple regression model is typically generalized to handle the prediction of several dependent variables. To illustrate the model, we may let x_1, x_2, \dots, x_n be n dependent variables related to a response variable y . The linear regression model with a single response takes the following form:

$$y = a_0 + a_1x_1 + \dots + a_nx_n + \varepsilon \tag{2.1}$$

The term *linear* regression refers to that the right hand side of (2.1) is a linear

where $A = (a_0, a_1, \dots, a_n)^T$, $Y = (y_1, y_2, \dots, y_m)^T$ and

$$X = \begin{pmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1n} \\ 1 & x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix}$$

Let $\hat{Y} = (\hat{y}_1, \hat{y}_2, \dots, \hat{y}_m)^T = XA$ denote the fitted value of Y , $\bar{y} = \sum_{i=1}^m y_i/m$ be the average value of Y . Thus, the quality of the model fit can be measured by the coefficient of determination

$$R^2 = \frac{\sum_{i=1}^m (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^m (y_i - \bar{y})^2} \quad (2.4)$$

Here R is called the *multiple correlation coefficient*. When R is close to 1, it means that the fitted equation passes through most of the data points. On the other hand, when R is close to 0, it represents the dependent variables x_1, x_2, \dots, x_n have no influence on the response.

2.3 DCF analysis

Before the electricity industry move toward deregulation, electricity prices were set by the regulators based on the cost of service. At that time, the economic viability of investment in power generation units could be determined by a traditional economic evaluation, so called the discounted cash flow (DCF) method (e.g., Riggs et al. 1996). The DCF method identifies estimated cash flows over time, and then discount the cash flows back to the present with criterion of net present value (NPV). NPV represents the difference between the present value of the discounted cash flows and the investment costs. If the NPV

is positive, the investment is accepted, otherwise, it is rejected. In the research, the corresponding NPV can be determined by this DCF analysis coupled with a simulation model that produces the projected cash flow of a generation unit under some loads. For example, a cash flow stream (x_0, x_1, \dots, x_T) that spans over time period $[0, T]$, the NPV is as follows:

$$\text{NPV} = \sum_{t=0}^T \frac{x_t}{(1+r)^t} \quad (2.5)$$

where r is the discount rate per time period.

DCF method assumes that the investment opportunity is not reversible and is a now or never opportunity. That means new information and future opportunities are ignored. Therefore, DCF often underestimates the value of investment strategies. Although DCF method tends to undervalue assets in the presence of uncertainty since that approach tends to ignore the value of real options, such as turning off a plant when the price is too low, this analysis can still be helpful and play a role in valuing a power plant in the current electric industry.

2.4 Options theory

A brief introduction to options theory will be given in this section. A more thorough introduction can be found in classical textbooks, such as Luenberger (1998). Some basic concepts about options theory in finance are stated as follows:

- Option: an *option* is defined as the right, but not the obligation, to buy (or sell) an asset under specified terms (e.g., Luenberger 1998; Hull 1999). Usually there are a specified price and a specified period of time over which the option is valid.

- Call option: A *call option* gives the right to purchase something.
- Put option: A *put option* gives the right to sell something.
- Exercise price (or strike price): For each option, usually there is a specified price (called an *exercise price* or *strike price*) at which the underlying asset can be purchased or sold upon exercise of the option and within a specified period of time over which the option is valid.
- American option: An *American option* allows exercise at any time before and including the expiration date.
- European option: A *European option* allows exercise only on the expiration date.
- In the money or out of money: A call option is called *in the money*, *at the money*, or *out of the money*, depending on whether the underlying asset's price is greater than, equal to, or less than the exercise price; whereas put options have reverse terminology, since the payoff at exercise are positive only if the asset's price is less than the strike price.

For example, consider a call option that allows you to buy a specified stock at exercise price K at some future time T . This option will be valuable if the stock price at T , denoted by $S(T)$, turns out to be higher than K . The call option can be exercised by buying the stock at K and then reselling the stock back to the market at a profit of $S - K$. However, if the stock price falls below K at T , this call option is virtually worthless then.

In general, an option provides an opportunity for the decision-maker to take some action after uncertainties are revealed. For example, the owner of a call

option will exercise the option only after learning that the stock price $S(T)$ is greater than K . Again, to determine whether holding this option is worthy the expected option payoff should be compared with the cost to acquire the option. Note that risk preference can be taken into account through either the discount rate or the probability measure.

In the recent, the concept of options has been extended toward a variety of areas other than financial contracts. One of the most popular subjects is known as *real options* valuation. Real options allows evaluating the investment taking into account the value of flexibility embedded in real operational processes, activities, or investment opportunities that are not financial instruments (e.g., Trigeorgis 1996). Typically, a real option gives the option holder the right but not the obligation to take an action in the future. An embedded real options in valuing electric generating units is one immediate example of real options because turning on/off a power plant is an obvious real option in the volatile marketplace. It is no doubt that a power plant owner would only exercise the operational right at time t when the electricity price less generating fuel cost is positive at that time. A *spark spread call option* is an option that yields its holder the positive part of electricity price less the production cost at its maturity time. Therefore, the value of the underlying power plant can be estimated by summing up a set of spark spread call options with maturity time spanning the lifetime of the plant (e.g., Deng 1999).

Another recognition of real options theory is that the most valuable investment opportunities may come with uncertainty (Ameram 1999). Its focus is strategic investment and operating decisions. Some types of real options are given below (Trigeorgis 1996):

- *Option to defer*: enables management to defer investment and benefit from more information. It is an American call option where the investment is the exercise price and value of underlying asset is the operating cash flow.
- *Option to expand*: is a call option to acquire an additional part of the base project, paying the investment as exercise price.
- *Option to contract*: is to reduce the production if market conditions turn bad. It is a put option on part of the base sale project with exercise price equal to the potential cost savings.
- *Option to abandon*: is to abandon a project when the market conditions turn bad in order to avoid loss. It is an American put option on the project's current value with an exercise price of the salvage value.
- *Option to switch*: is the possibility to switch one input to a cheaper input or one output to a more profitable output as the prices of input and output fluctuate.

More discussions on real options can be found in Chapter 3 of literature review.

2.5 Least squares Monte Carlo method

The least squares Monte Carlo (LSMC) approach was first introduced by Longstaff and Schwartz (2001) to value American-like financial options. As well known that standard simulation programs follow forward algorithms to generate the paths of state variables over time. It is not enough to value American-like options purely by the forward algorithms, and generally a backward algorithm

is required to look for the optimal exercise strategy due to the property of *early exercise*.

At any exercise time, the holder of an American option compares the payoff of immediate exercise with the expected payoff from retaining it, then exercise if the immediate payoff is higher. The holder can get the highest expected payoff by doing so, thus it is the optimal exercise strategy, which is fundamentally determined by the conditional expectation of the payoff from continuing to keep the option alive. The key characteristic is that the conditional expected function can be estimated from the representational information in the simulation by using least squares regression. The realized payoff from retaining the option are regressed to estimate the parameters of the conditional expected functions. The fitted value of this regression is an efficient unbiased estimate of the conditional expected function. By estimating the conditional expectation function for each exercise time period, we can obtain the optimal exercising rule along each path.

To apply the LSMC method to generation asset valuation problem, consider the following (generic) multi-stage stochastic program. Assume that at unit state x_t at time t , the uncertainty \mathbf{Q}_t is revealed. After observing \mathbf{Q}_t the decision maker (i) must realize the current asset value $f_t(x_t, \mathbf{Q}_t)$; and (ii) can maximize the expected value of the unit for the rest time periods by making decision v_t . Let $F_t(x_t, V_t; \mathbf{Q}_t)$ be the value-to-go function of the unit for the remaining period at state x_t at time t with operational option set V_t . Then this problem can be formulated as the following recursive relations:

$$F_t(x_t, v_t; \mathbf{Q}_t) = f_t(x_t, \mathbf{Q}_t) + \max_{v_t \in V_t} E_t[F_{t+1}(x_{t+1}, v_{t+1}; \mathbf{Q}_{t+1})] \quad (2.6)$$

where E_t denotes the expectation operator, and the subscript t indicates that the expectation is based on the available information for uncertainty at time t ,

and $t \in [0, T - 1]$, T represents the length of the unit's lifetime. The last term in (2.6) with the expectation operator defines another stochastic program to be considered in the subsequent time period. Also the optimization problem in (2.6) is subjected to a set of operational constraints. When boundary conditions F_T and initial conditions x_0 , v_0 and \mathbf{Q}_0 are given, the optimal value of F^* , representing the maximal expected unit value with operational option set V_t over the period $[0, T]$, can be obtained from the last step of recursive relation as $F_0(x_0, v_0; \mathbf{Q}_0)$.

The difficulty for solving (2.6) comes from the last term $E_t[F_{t+1}(x_{t+1}, v_{t+1}; \mathbf{Q}_{t+1})]$, which in general cannot be expressed in an analytical form. This term, however, can be approximated using the LSMC method. Through simulating a set of random variables, the expected value yields the least square error. Thus the expected value of $F_{t+1}(x_{t+1}, v_{t+1}; \mathbf{Q}_{t+1})$ can be approximated by a conditional expected function $\theta_t(x_t, \mathbf{Q}_t)$ that regresses F_{t+1} on the revealed uncertain data \mathbf{Q}_t . The functional form $\theta(\cdot)$ can be obtained by the linear regression method treating x_t and \mathbf{Q}_t as the *predictor* variables and θ_t as the corresponding *response* variable. The Monte Carlo method is used to generate the *observations* of θ_t and the associated values of \mathbf{Q}_t . Once \mathbf{Q}_t is realized, for any state x_t at time t , one would know how to make optimal decisions for the next time period based on the above regression function $\theta_t(x_t, \mathbf{Q}_t)$. This is the basic idea of the LSMC method.

The above LSMC approach will be used to solve other complex asset valuation problems similar to (2.6) in this dissertation. Like other regression-based methods, the most difficult part of LSMC method is to find the suitable basis functions of the relevant state variables, which may affect the regression quality significantly.

Chapter 3

Literature Review

In this section, a complete review will be given the subjects related to the valuation of a thermal power plant in the present electric industry, including research on unit commitment, real options, and asset valuation etc.

3.1 Unit commitment problem

3.1.1 Deterministic unit commitment problem

In power industry electricity is a non-storable commodity and needs to be produced and consumed at the same rate. It is very important to determine when to switch generating units between on and off mode, in order to schedule a group of generators over a set of time periods to satisfy a series of constraints at least cost (Baldick 1995). The decision problem of optimally scheduling the operation of generating units is known as the unit commitment (UC) problem (Wood and Wollenberg 1996). In a typical UC problem, especially for thermal units, the objective is usually to minimize the total generating cost of a power system over the planning horizon and to come up with the detailed generating schedule

involving when to start up or shut down and generation levels in each time period for each generator simultaneously (Bard 1988). The constraints usually include demand constraints, spinning reserve constraints, capacity constraints, minimum up/down time constraints and ramp constraints etc. These restrictions make the unit commitment complicated and become a difficult mixed integer program (Lai and Baldick 1999). The NP-hardness of the unit commitment problem has been rigorously proved in literature (Tseng 1996).

UC is a short-term planning problem with a time horizon ranging from a day to a week and considers unit operation on an hourly basis. As opposed to the short-term UC problem, there are also mid-term and long-term planning problems. The time horizon for a mid-term planning problem could be a month or a quarter, which is suitable to manage hydro units. For a long-term problem, the time horizon could be a year or a decade, especially useful for the electric utility planning and the study of the cyclical natures of electricity markets (Marin and Salmeron 1998; Louveaux 1988).

Generally speaking, the UC problem is usually very complex to solve because it has both on/off combination and continuous production level optimization components. Some coupling constraints, especially ramp constraints, make this problem even more complicate.

3.1.2 Stochastic unit commitment problem

In a regulated environment, the UC problem is independent of the price of electricity, which is predetermined by the customer demand and the total generating capacity. In this sense, the decisions on how to operate the generating units have no effect on the total revenue of the company. The maximum profit is

guaranteed by the minimization of the production costs (Sheble and Fahd 1994).

However, the price of electricity is not predetermined any more under deregulation (Galiana and Ilic 1998). When restructuring is completed, the utility company will be under no obligation to serve any or all of the demand. For example, the power generating utility has the option to turn off the units and refuse to deliver the power when the electricity price in the spot market is so low that its production cost cannot be recovered by its revenue.

Since the spot prices of electricity are highly volatile, the power industry will no longer rely on traditional rules. The UC problem needs to incorporate a new stochastic formulation considering load uncertainty and price uncertainties in the spot market.

In literature, Allen and Ilic (1999) provided a new formulation of the unit commitment problem in the deregulated environment. They discussed the stochastic characteristics of the spot market prices derived from available data on market-clearing prices, load, and covariates (temperature). Then Takriti et al. (2000) developed a stochastic model for the UC problem in which the demand and price uncertainties are introduced via a set of possible scenarios generated by the MC method. Tseng (2001) mentioned that the prices of electricity and fuel can be modelled by Ito processes. However, it is still a challenge to find an efficient way to generate representative scenarios and to forecast price trend in the future, because we may need many years of data, which may not be available. Besides, the electricity price under deregulation could be highly volatile and the history data may not affect the future price in some sense.

3.1.3 Methodologies

Researchers have developed different methods for solving the UC problem. Among these methods, priority list-based methods (Lee 1988; Chandler et al. 1953) are typical heuristic methods, which can commit available units in a pre-determined order, but may yield a far from optimal, or even infeasible solution. Dynamic programming methods can solve the UC problem efficiently (Hobbs et al. 1987), but still suffer from the familiar curse of dimensionality especially when the size of state space is large. Similar to the dynamic programming methods, branch-and-bound methods can schedule a small power system within reasonable time, but fail to solve real-world power system scheduling problems due to the large size of the search space (Cohen et al. 1983; Dillon 1978). Lagrangian relaxation (LR) based technique (Ferreire et al. 1989; Guan et al. 1992) seems to be the most efficient and popular, because it attempts to solve the problem indirectly by solving the dual problem. Zhuang and Galiana (1988) proposed a simulated annealing approach to obtain dual optimal solutions, but the corresponding reserve-feasible commitments are not guaranteed to be dispatchable. To ensure the feasibility of solution, a new unit decommitment method is introduced to not only improve the solution quality, but also be able to relieve unpredicted heuristic effects and make the LR approach more robust (Tseng 1996). As to other methods, genetic algorithms (Kazarlis et al. 1996; Cheng et al. 2000) have also been tried, but the results are not encouraging. A hybrid method of genetic algorithms and neural networks, reported by Huang et al. (1997), seems to produce reasonable results. Further references regarding the UC can be found in the book of Wood and Wollenberg (1996).

Compared with the deterministic UC problem, there were few attempts to

solve the stochastic unit commitment (SUC) problem in earlier periods, but subject to limitation in computer hardware. Recently, the SUC problem has been solvable with advanced computers. Valenzuela et al. (2000) report on the statistical analysis of hourly demand covering a region of the Eastern United States. They have shown that when the effect of temperature is suitably subtracted from the hourly load, such a process can represent the demand quite accurately. Both Carpentier et al. (1996) and Takriti et al. (1996, 2000) use a set of scenarios with associated probabilities to model the random electric load over the planning horizon. Thus the SUC problem is solved by minimizing the expected cost of running the electric system while meeting the load of the different scenarios. To decompose the stochastic model, Carpentier et al. (1996) relax the demand constraints. As a result, each generating unit has its own separate stochastic dynamic programming. In Takriti et al. (1996, 2000), both electric and fuel price uncertainty are incorporated in their model in addition to load uncertainty. The cost function of each generator is allowed to change with fuel prices. That means this model is a multi-stage, mixed-integer, stochastic program and each node in the scenario tree carries information regarding the spot market prices of electricity and fuel as well as the electric demand. Another approach also incorporates uncertainties into the UC problem in a two-stage stochastic program (Caroe et al. 1997). In the first stage, the on/off decisions are made, while the generating level of each unit is determined by the second stage.

3.2 Real options

3.2.1 Overview

In literature, many applications of real options (or flexibility) have been identified and valued by researchers. It is well recognized that flexibility has value because of uncertainty, and the higher uncertainty in the payoffs of an investment, the higher value of real options is (Dixit and Pindyck 1994).

The real options approach applies derivative pricing theory to the analysis of options opportunities in real assets. For example, suppose an investment would be profitable at the average price, but not at the lower price. The decision maker can postpone her decision and collect more information about actual price movement. She invests if the price has gone up, but not if it has gone down. Thus it avoids the loss if she had invested in the very beginning and then seen the price go down. This value of waiting option must be traded off against the loss of profit flow during the waiting time period (e.g., Dixit and Pindyck 1994).

As an extension of financial theory, the real options theory may use the methods in financial options, which cannot always be applicable because the option pricing method requires that the underlying asset is tradable while the real assets are not always tradable. However, according to Copeland and Antikarov (2001), the options valuation and the portfolios of traded securities are still usable, if we adopt same assumptions used by DCF approach which attempts to determine what a project would be worth if it were to be traded.

Besides, most of investments in real life are not restricted to only one type of real options. Typically, there exist complicated interdependencies between

options, such as compound options, which can not be valued individually or separately, because the exercise of an option may influence other existing options or even incur a number of new options. It is difficult for researchers to find analytical expressions for these complicated options, but approximate solutions could be achieved through numerical techniques. To evaluate interrelated options, Herath and Park (2002) develop a binomial lattice framework to model a multi-stage investment as a compound real options. Kulatilaka (1995) shows that the value of a collection of interdependent options is less than the sum of individual options. Similarly, Trigeorgis (1993) finds that the value of a set of options generally deviates from the sum of the values of each individual option. The incremental value of an additional option when other options are present is generally less than the value of the options when it is valued individually.

3.2.2 Applications of real options in electric industry

Since Black and Scholes (1973) and Merton (1973) presented their work on option pricing theory, many application areas, including valuing complex financial securities and valuing real options, have been found. Especially after the electric industry is moving toward restructuring, in which electricity prices become extremely volatile and significant financial risks are introduced, the real options approach can be a useful tool to take into account the flexibility in strategic decision-making and operation management in power industry.

In the context of electricity producing firms, real options theory can be used as a method of identifying and quantifying the contingent decisions embedded in owning generation assets (Hlouskova et al. 2002). Applications of the real options approach fall into two broad categories:

- Operational options - those related to the UC problem such as fuel switching capabilities, overfire options, locational arbitrage on regional prices, arbitrage between energy and ancillary services markets (Griffes et al. 1999). For example, in the operation stage, the option value of a generating unit can be modelled as a series of call options on the spread between electricity prices and variable costs (Deng et al. 1999).
- Capital investment options - those related to the acquisition of new plants or the extension of current plants. The investors will exercise such options if the market value of the generation unit exceeds the construction cost or if the extra revenue after the expansion exceeds the expansion cost.

Edleson et al. (1995) analyzed the actual investment decision of a utility using real options, in order to find a suitable alternative from buying or selling pollution allowances, installing scrubbers or switching to low-sulphur coal for a particular coal-fired power plant to account for the pollution levels. Hsu (1998) compared the spark spread call and put options with financial instruments in order to mitigate exposure to both electricity and gas price risks. Thus the spark spread options conveniently tie the power and gas price movements. Pereira et al. (1999) developed some basic ideas, such as methodologies and computational tools on real options theory to handle risk and uncertainty in investment planning both in traditional power sector and in the competitive environment. Chaton et al. (1999) discussed how uncertainty affects investment in power market, if given the uncertainty of demand, the natural gas price and the possibility of electricity sales between regions. Min (2000) illustrated various options of capacity expansion and capacity reduction, and evaluated the values of options for a utility that has two inter-related generation units. Frayer et al. (2001)

calculated the value of two plants, one of which is peaking gas-fired plant, the other is mid-merit coal-fired plant, and shown that the gas-fired plant has a higher value taking into account its operation flexibility using real options theory. Similarly, Ajax et al. (2000) applied real options theory to analyze the investment decision on a hydropower and a thermal power plant. The exercise price is the investment of each plant and the utility's revenue is collected by selling electricity both in forward contracts and in spot market. For each plant, the investor calculates the option value and decides whether to keep the option alive or delaying the investment, or to exercise it.

3.3 Asset valuation of power generation units

3.3.1 Overviews

The asset value of a power plant can be regarded as the total profit (revenue less cost) of generating power over its life time. The revenue is dependent upon the electricity prices, which are determined by the electric spot market. The cost consists of mainly two parts: one is the production cost (90 ~ 95%), the other is maintenance cost (5 ~ 10%). In order to maximize the total profit of a unit, we need to find optimal commitment, production and maintenance schedules under price uncertainties considering both production constraints and maintenance constraints.

Before deregulation, the method of the net present value (NPV) is widely used in valuing a power generating asset or investment. The economic viability of such investment can be determined by means of a DCF method (e.g., Dixit and

Pindyck, 1994), but the result tends to underestimate the value of power plant because of ignoring the value of real options associated with the investment.

With the development of financial and physical markets for electricity after deregulation, the value of a power plant can be modelled as a set of financial instruments on electricity (Deng et al. 1998; and Hsu 1998). The idea of their approaches is as follows: a power plant can generate electricity by consuming a particular fuel with its associated heat rate. When the electricity price is high but the fuel price is low, the power plant should turn on to collect the profitable price spread between the electricity price and the generation cost of the unit. If the price spread is negative, then the unit should stay off. Therefore, the value of a power plant can be regarded as a series of call options of spark spreads, defined as the electricity price less the product of the heat rate associated with the generator and the fuel price. Although the option-based valuation provides a much better result than does the traditional DCF valuation, it overlooks the power plant's operational constraints, such as minimum up/down time constraints and ramp constraints. This may lead to overvalue a power plant.

In Tseng and Barz (1999, 2001), a MC method is introduced to value the power generating units with physical constraints including ramp constraints over a short-term period. Although the computational speed of this approach is not fast, its result of valuation is closer to the "true" value than the financial options methods. Similarly in Gardner and Zhuang (2000), employs a real options-based stochastic dynamic programming method to value power plants considering operational characteristics including minimum up/down time, ramp rate, non-constant heat rate and response rate. Their numerical results also show that operating constraints have a significant impact on power plant values and

optimal operating policies.

Understanding the value of the operating flexibility is important for making investment decisions in power industry, especially when the environment is highly volatile and new technology also emerges quickly. For example, when facing unexpected stochastic prices, a generator with operating options can protect itself against some of the adverse price movements by switching to an alternate model of operation (e.g., Kulatilaka 1993). Cohen and Ostrowski (1996) discussed different operating modes including combined cycles, fuel switching, overfire and duel boiler, but did not consider them from the perspective of operational options, and did not consider their values. To my best knowledge, there are relatively few papers in the domain of valuing the operating options of power generating units especially incorporating the operational constraints. The fuel-switching flexibility of a duel-fuel steam boiler has been valued in Kulatilaka (1993), which may shed some light on future research about the valuation of the operating options. As to the overfire options, no literature has been found.

Although there is a great deal of literature on maintenance issues, most of them are regarding maintenance scheduling. A recent comprehensive review for maintenance scheduling can be found in Shahidehpour and Marwali (2000). They provide a general framework for achieving trade-off between minimizing the maintenance cost and preserving the service reliability.

3.3.2 Valuation methods

Currently methods for asset valuation can be roughly classified into two types: DCF methods and options-based methods. Because the traditional DCF method ignores the value of operating flexibility and strategic option, the real options-

based methods have become increasingly popular (Trigeorgis and Mason 1987).

Numerous methodologies (see, for example, Dixit and Pindyck 1994; Trigeorgis 1995; and Feurstein 2000) in the domain of real options have been developed to calculate the value of options. Although some valuation techniques in finance can be used directly in some specific cases, most of real option pricing problems need to resort to approximation methods because the underlying stochastic process becomes multivariate due to the interaction of several types of uncertainties. Among these numerical methods, discrete tree/lattice models, finite difference approaches, and the MC simulation are widely used.

Cox et al. (1979) introduced discrete tree/lattice models to represent stochastic processes in option valuation. To generate a binomial price lattice, we need to know initial price of the asset, which is assumed to be multiplied by either the up factor u , or the down factor d , the choice should be made according to the corresponding possibilities p and $1 - p$. The option value at expiration is computed by boundary conditions and all the other nodes in the option lattice are calculated sequentially by working backward through the periods (e.g., Luenberger 1998). The trinomial models can also be developed similarly for generation asset valuation problems (Tseng and Lin 2004). They develop a framework for generating discrete-time two-factor price lattices for two general correlated Ito processes: electricity and fuel prices. In Gardner and Zhuang (2000) and Hull and White (1994), the price processes for electricity and fuel are also modelled as a two-factor lattice, which allow fuel prices to be stochastic and make it possible to model the spark spread directly if the power plant heat rate is assumed to be constant. Although the lattice models are easy to be implemented, they still need to face the curse of dimensionality, especially when the option is path

dependent and the time horizon is long. If extended to three-factor problems, the lattice model becomes almost infeasible due to the increased dimensionality.

Parallel to lattice/tree model, there exists an alternate approach, finite difference methods (FDM) (e.g., Schwartz 1977; Brennan and Schwartz 1978). The basic idea of FDM is to approximate the solution of the Black-Scholes partial differential equation to calculate the option prices using numerical techniques. All state variables are discretized, the boundary conditions are determined by the value of contingent claim at expiration, and the first and second derivatives are replaced by a finite difference approximation. Then the option price is solved backward by using a discretized partial differential equation that represents the valuation equation. In the domain of valuing electric power generators, a general framework for the valuation based on partial-integro-differential equations is developed by Thompson et al. (2004). This method can handle both European-like options and American-like options, and achieve high levels of computational speed, but it cannot treat path-dependent options, and cannot be extended to multi-factor problems either.

MC simulation is one of the most popular methods to value options. Based on the idea that the probability distributions of underlying asset values are approximated by simulating price trajectories, Boyle (1977) proposes MC simulation procedures for European option valuation. Recently, several efforts have been taken to extend MC simulation techniques to value real American-like options. Tseng and Barz (1999, 2002) provide a valuation model for short-term generation asset based on MC simulation. Longstaff and Schwartz (2001) discussed the solving process of American-like options, in which the criteria of the optimal option exercise in backward induction was introduced and a detailed review on

valuing American-like options via MC simulation was also provided. The main drawback of the MC technique is that the simulation process is time consuming, but its major advantage is that MC methods can accommodate general price processes and can be extended to multi-factor problems easily. For example, in Chapter 5 a MC-based approach, integrated with stochastic dynamic programming, is proposed to value a thermal generation unit with fuel-switching options involving three uncertainties: electricity, gas and oil prices.

Chapter 4

Optimal Self-Scheduling Problem

4.1 Problem description

Unit commitment (UC) is an important optimization problem for power utilities to economically schedule generating resources to achieve cost minimization in the regulated environment (e.g., Tseng 1996). Although the electricity industry is moving toward deregulation, the importance of the UC does not diminish along with the restructuring trend. On the other hand, new features and requirements are now considered in UC models such as price uncertainty, which significantly increase the complexity of the problem (e.g., Hobbs et al 2001). Some centralized markets (such as the PJM) still perform UC-like optimization to conduct electricity auctions.

Among the methods that have been proposed to solve the UC problems, the Lagrangian relaxation (LR) methods are the most widely used ones. One nice feature of the LR approach for solving the UC problem is its decomposition of the centralized problem into decentralized unit subproblems, which are linked by Lagrange multipliers. Under the assumption of competitive market, the mul-

multipliers can be viewed as the market prices and each unit subproblem represents a profit-maximizing commitment decision-making for the unit in response to the market prices. In this chapter, we focus on thermal units and discuss optimization of a *decentralized* unit subproblem, called the *self-scheduling problem* (SSP), which refers to optimal scheduling of a single unit in response to price changes in the spot market, subject to operational constraints, where the demand constraint is left out, because nobody needs to take the responsibility to satisfy the electric demand in the competitive market. In an SSP, the operator of a power plant intends to maximize total profit by optimally committing the unit to generate power to sell in a spot market, since no participants have the capabilities to alter spot prices. The unit commitment is subject to physical constraints including the ramp constraints. These operational constraints can impact the capability of how the unit can quickly respond to profitable opportunities. The solution procedure developed in this chapter can be used to help independent power producers achieve optimal commitment and dispatch decisions in the competitive marketplace. It can also contribute to the traditional UC optimization.

Ramp constraints limit the capability of a unit to the change generation levels over a short period of time. They can have significant impacts on the solution of the SSP. When subject to the ramp constraints, the generation levels of a unit become interdependent in all hours, which complicates the solution procedures. Thus far, in the LR approach the ramp constraints have only been dealt with by further relaxation (Svoboda et al 1997), discretization of the generation levels (Bard 1988), or other methods (Guan et al 1992 and Lai and Baldick 1999). None of these approaches can achieve optimality of the SSP, or achieve it efficiently. More discussions of these approaches will be given in the next section.

Arroyo and Conejo (2000) proposed a rigorous approach using a mixed linear integer programming (MILP) method to solve the SSP with ramp constraints. Their approach is general and does not assume convexity of operating costs. However all nonlinear cost structures need to be linearized. Although the method produces a satisfactory result for a 24-hour test case, it may not be applicable to larger cases, such as 7-day ones, in unit commitment. This is because the MILP solvers (e.g., CPLEX used in their paper) are based on branch-and-bound or branch-and-cut searches, whose computational complexity normally increases exponentially with the problem size.

In this paper, we first tackle the SSP by assuming the spot prices are known with certainty, same as in Arroyo and Conejo (2000). A network graph method is proposed to solve the SSP with the ramp constraints. We will show that under the convexity assumption, the ramp-constrained SSP is not NP-hard by presenting an efficient polynomial-time algorithm. Our proposed algorithm can guarantee the optimality of the solution without discretizing the generation levels and linearizing the cost functions. Because of its efficient polynomial-time complexity, the proposed method can be incorporated in large-scale UC related problems.

Price uncertainty is then introduced to the SSP via price scenarios generated by the Monte Carlo method. The optimal (deterministic) ramp-constrained SSP is solved for each scenario. A linear regression- based method is developed to devise the optimal unit response strategy under the ramp constraints and the price uncertainty.

4.1.1 The mathematical formulation

This section presents a mathematical model for the optimal unit response problem to spot prices. The following standard notation will be used. Additional symbols will be introduced when necessary.

t : index for time (in hours), $t = 0, \dots, T$, where T is the number of hours of the operating period.

τ : unit startup time.

ν : unit shutdown time.

u_t : zero-one decision variable indicating whether unit is up or down in time period t .

\tilde{u}_0 : initial condition of u_t at $t = 0$.

x_t : state variable indicating the status of the unit in time period t (length of time unit has been up or down).

\tilde{x}_0 : initial condition of x_t at $t = 0$.

t^{on} : the minimum number of periods the unit must remain on after it has been turned on.

t^{off} : the minimum number of periods the unit must remain off after it has been turned off.

t^{cold} : the number of periods required to cool the unit from shutdown ($t^{\text{cold}} \geq t^{\text{off}}$).

q_t : variable indicating the amount of power the unit is generating in time period t .

\tilde{q}_0 : initial condition of q_t at $t = 0$.

q^{\min} : minimum rated capacity of the unit.

q^{\max} : maximum rated capacity of the unit.

Δ : ramp rate of the unit.

$C(q_t)$: fuel cost for operating the unit at output level q_t in time period t .

$S(x_t)$: startup cost associated with turning on the unit at state x_t in time period t .

λ_t : spot price at time t for electricity.

μ_t : spot price at time t for spinning reserve.

The SSP is formulated as a mixed-integer programming problem. The objective is to maximize the total profit:

$$(P_0) \min_{\mathbf{u}, \mathbf{x}, \mathbf{q}} \sum_{t=1}^T (\lambda_t q_t - C(q_t)) u_t + \mu_t (q^{\max} - q_t) u_t - S(x_{t-1}) u_t (1 - u_{t-1}) \quad (4.1)$$

where revenue is collected from selling electricity to the spot market and providing spinning reserve. Note that the symbols in bold face are vectors. We assume that the spot prices and are deterministic and are known. This assumption will be relaxed in Section § 4.3.

To describe the power plant operation, let Φ be the set of states that is composed of four subsets of states: Φ_1 for the startup period, Φ_2 for the normal operation period, Φ_3 for the shutdown period, and Φ_4 for the normal off-line period.

$$x_t \in \Phi = \Phi_1 \cup \Phi_2 \cup \Phi_3 \cup \Phi_4 \quad (4.2)$$

where $\Phi_1 \equiv \{1, 2, \dots, \tau\}$ (4.3a)

$$\Phi_2 \equiv \{\tau + 1, \tau + 2, \dots, \tau + t^{\text{on}}\} \quad (4.3b)$$

$$\Phi_3 \equiv \{-1, -2, \dots, -\nu\} \quad (4.3c)$$

$$\Phi_4 \equiv \{-\nu - 1, -\nu - 2, \dots, -\nu - t^{\text{cold}}\} \quad (4.3d)$$

The optimization is subject to the following constraints.

- State transition constraints:

$$x_{t+1} = \begin{cases} \min(\tau + t^{\text{on}}, x_t + 1), & \text{if } x_t \in \Phi_1 \cup \Phi_2 \text{ and } u_{t+1} = 1, \\ x_t - 1, & \text{if } x_t \in \Phi_3 \text{ and } u_{t+1} = 1, \\ \max(-\nu - t^{\text{cold}}, x_t - 1), & \text{if } x_t \in \Phi_4 \text{ and } u_{t+1} = 0, \end{cases} \quad (4.4)$$

and

$$u_t = \begin{cases} 1, & \text{if } -\nu \leq x_t < \tau + t^{\text{on}}, \\ 0, & \text{if } -\nu - t^{\text{off}} < x_t \leq -\nu - 1 \\ 0 \text{ or } 1, & \text{otherwise,} \end{cases} \quad (4.5)$$

- Ramp constraints:

If $x_t \in \Phi_1$,

$$q_t = q^{\min} x_t / \tau \quad (4.6)$$

If $x_t \in \Phi_2$,

$$|q_{t+1} - q_t| \leq \Delta \quad (4.7)$$

$$q^{\min} \leq q_t \leq q^{\max} \quad (4.8)$$

If $x_t \in \Phi_3$,

$$q_t = q^{\min} (\nu + x_t + 1) / \nu \quad (4.9)$$

If $x_t \in \Phi_4$,

$$q_t = 0 \quad (4.10)$$

- Initial conditions:

$$u_0 = \tilde{u}_0, x_0 = \tilde{x}_0, q_0 = \tilde{q}_0. \quad (4.11)$$

Remark

Note that the equation (4.6) implies that when the unit is turned on, it takes τ hours with a fixed increasing rate in generation q^{\min}/τ to reach the minimum rated capacity of the unit. Therefore, at $x_t = \tau$, $q_t = q^{\min}$. Similarly, (4.9) implies that at $x_t = -1$, $q_t = q^{\min}$. That means when an on-line unit is turned off, its generation level has to be reduced to q^{\min} first, then from q^{\min} it takes ν hours with a fixed decreasing rate in generation q^{\min}/ν to reduce to zero. To sum up, during the startup and shutdown periods, the unit continues to generate power, but not as much as in the normal on-line period Φ_2 . Therefore, $u_t = 1$ when x_t is in Φ_1 , Φ_2 or Φ_3 .

Problem (P_0) is similar to a LR unit subproblem of the unit commitment, in which λ and μ are Lagrange multipliers of the demand and spinning capacity constraints, respectively. In the following section, we discuss some existing methods for solving (P_0) .

4.1.2 Related methods

Wang and Shahidepour (1993, 1994) introduced an artificial neural network (ANN) method incorporating dynamic programming (DP) to solve the ramp-constrained unit commitment problems. Although the authors claimed that they can obtain good results, the solution quality is significantly affected by the training examples and categories. Also When the size of training set is increased,

additional memory and training time required correspondingly. Also, using peak-load units to accommodate the unit ramping characteristics may introduce errors and solution feasibility is not guaranteed, let alone the solution optimality.

Lee et al. (1995) claimed that ramp-rate constraints can be reflected by hourly marginal ramp-rate price of generators and these marginal prices can be achieved by a simple iteration algorithm. It seems that their method can improve the solution quality to some extent, but cannot avoid infeasible or bad results either.

In Bard (1988), a LR unit subproblem similar to (P_0) was tackled. The author discretized the generation level q_t to accommodate the ramp constraints. With the discretized generation levels and the discrete states x_t , the subproblem is then solved using dynamic programming. The more refined the discretization of the generation level is, the more precise the solution is and the greater state space for the dynamic programming is required. A tradeoff between optimality and computational feasibility must be made. Since the discretization of the generation level cannot be made to infinitely refined, this approach obtains only a sub-optimal (or near-optimal) solution. However, the duality theory of the LR approach holds only when each subproblem is solved to the exact optimality. Therefore, the approach using discretization to solve the ramp-constrained subproblem is somewhat questionable in theory, although it is recognized to be practically feasible (Peterson et al 1995).

To avoid solving a ramp-constrained subproblem, Svoboda et al. (1997) further relaxed ramp constraints in the LR scheme, which incurred additional $2TI$ Lagrange multipliers to the original $2T$ ones, where I is the number of units. For example, a common test case with 10 units over a 7-day period would re-

quire updating 3,696 multipliers simultaneously (as opposed to 336 multipliers otherwise), which makes the dual optimization much harder to converge. Furthermore, it can be shown that the duality gap increases when more constraints are relaxed, indicating a potentially worse solution quality. Therefore, a method that can directly solve a ramp-constrained LR unit subproblem to optimality is very much in need.

Solving (P_0) to optimality was first reported by Arroyo and Conejo (2000). The authors made efforts to convert the minimum uptime/downtime constraints to standard integer programming formulations and solved the problem to optimality using a commercial mixed-integer linear programming (MILP) solver CPLEX. In order to use the MILP solver, the cost function C and the start up cost S must be linearized. Their method does not assume the cost function C to be convex. The authors reported a satisfactory test result for a 24-hour instance. Although Arroyo and Conejo (2000) provide a means to solve the ramp-constrained subproblem to optimality, their method may not be applicable to larger-scale problems in UC. As we will show in the next section, under the convexity assumption for the cost function C the ramp-constrained unit subproblem is not NP-hard. We demonstrate a polynomial-time algorithm for solving it. Given the fact that a polynomial-time algorithm exists, the branch-and-bound (or branch-and-cut) based MILP algorithms become less appealing for solving such a problem because its computational complexity normally increases exponentially as the problem size (such as T) increases.

4.2 Algorithm development

In this chapter, we assume the cost function C to be a quadratic function:

$$H(q_t) = c_0 + c_1 q_t + c_2 q_t^2, \quad (4.12a)$$

$$C(q_t) = H(q_t) \times P^F, \quad (4.12b)$$

where $H(\cdot)$ is the heat rate function, P^F is the fuel price and is assumed fixed. In the sequel, without loss of generality we will focus on solving the following problem (P) , equivalent to (P_0) .

$$(P) \quad \min_{\mathbf{u}, \mathbf{x}, \mathbf{q}} \sum_{t=1}^T \pi_t(q_t) u_t + S(x_{t-1}) u_t (1 - u_{t-1}) \quad (4.13)$$

where π_t is a concave quadratic function and (P) is subject to (4.4) - (4.11) including the ramp constraints. Note the spot price information is now implied in the subscript t of π_t . Next we will propose a method for solving (P) that does not require discretization of the generation level or linearization of the objective function, and has a polynomial-time complexity.

4.2.1 A network graph approach

Consider a directed graph $G(\mathcal{N}, \mathcal{A})$, where \mathcal{A} and \mathcal{N} are the sets of nodes and arcs, respectively. \mathcal{N} is defined as $\cup_{t=0}^T W_t$, where W_t is the set of states at time t :

$$W_t \equiv \Phi_1 \cup \Phi_3 \cup \Phi_4 = \Phi \setminus \Phi_2 \quad (4.14)$$

That is, each node in \mathcal{N} corresponds to a state of generating status. Note that Φ_2 has been suppressed. In the remainder of this chapter, a node (of the network) and a state y_t (a counterpart of x_t) are used interchangeably. \mathcal{A} contains directed

arcs only, each of which is incident from a state of some time period to another state of a later time period. Detailed configuration is as follows.

- Arc from y_t to y_{t+1}

$$y_{t+1} = \begin{cases} y_t + 1, & \text{if } y_t \in [1, \tau), \\ \max(-\nu - t^{\text{cold}}, y_t - 1), & \text{if } y_t < 0, \\ 1, & \text{if } y_t \in [-\nu - t^{\text{cold}}, -\nu - t^{\text{off}}]. \end{cases} \quad (4.15)$$

- Arc from y_{t_1} to y_{t_2} for every $t_1, t_2 \in [0, T]$ such that $t_2 - t_1 \geq t^{\text{on}}$

$$y_{t_1} = \tau, \quad y_{t_2} = -1 \quad (4.16)$$

Since an arc satisfying (4.16) represents some normal operation period, it is called an *operating arc*. Basically, the states in Φ_2 are replaced by all possible operating arcs in the proposed approach.

1. Nodal properties

Each node in \mathcal{N} is associated with a generation level Q_t (a counterpart of q_t), which is a function of the corresponding state y_t and is subject to the following constraints.

$$Q_t(y_t) = \begin{cases} y_t q^{\text{min}} / \tau, & \text{if } y_t \in \Phi_1, \\ (\nu + y_t + 1) q^{\text{min}} / \nu, & \text{if } y_t \in \Phi_3, \\ 0 & \text{if } y_t \in \Phi_4, \end{cases} \quad (4.17)$$

except that $Q_0(\tau) = \tilde{q}_0$ and $Q_T(-1)$ is a free variable between q^{min} and q^{max} .

Each node is associated with a profit(receipt) $\pi_t(Q_t)$, which is collected when the node is visited.

2. Arc properties

- For each arc incident into $y_{t+1} = 1$ from y_t , a transition cost $S(y_t)$ is incurred, which represents a unit startup cost.
- For each operating arc, incident from y_{t_1} to y_{t_2} satisfying (4.16), a profit $f(t_1, t_2)$ is applied, where the profit $f(t_1, t_2)$ is the optimal objective function value of the following parametric optimization problem, denoted by $(Q_{t_1}^{t_2})$:

$$(Q_{t_1}^{t_2}) \quad f(t_1, t_2) \equiv \max_{\mathbf{q}} \sum_{t=t_1+1}^{t_2-1} \pi_t(q_t) \quad (4.18a)$$

$$\text{s.t.} \quad q_{t_1} = Q_{t_1}(\tau) \quad (4.18b)$$

$$q_{t_2} = Q_{t_2}(-1) \quad (4.18c)$$

$$-\Delta \leq q_{t+1} - q_t \leq \Delta, \quad \forall t \in [t_1, t_2 - 1] \quad (4.18d)$$

$$q^{\min} \leq q_t \leq q^{\max}, \quad \forall t \in [t_1, t_2] \quad (4.18e)$$

If $(Q_{t_1}^{t_2})$ is not feasible, $f(t_1, t_2)$ is assigned to be $-\infty$.

3. Start node

A start node is the node in W_0 that corresponds to the initial state \tilde{x}_0 . If $\tilde{x}_0 \in \Phi_2$, the start node is $y_0 = \tau$.

Clearly, each path in G from the start node to any node in W_T corresponds to a feasible ramp-constrained unit schedule, and vice versa. The corresponding commitment and dispatch of a node on a path in G is defined by (4.5) and (4.17). An operating arc between t_1 and t_2 implies that $u_t = 1$ and q_t are obtained from the optimal solution of $f(t_1, t_2)$ for all $t \in [t_1, t_2]$. Therefore, solving (P) is equivalent to finding a longest (or shortest) path from the starting node to any node in W_T .

Given a directed network, finding a longest (or shortest) path problem is an

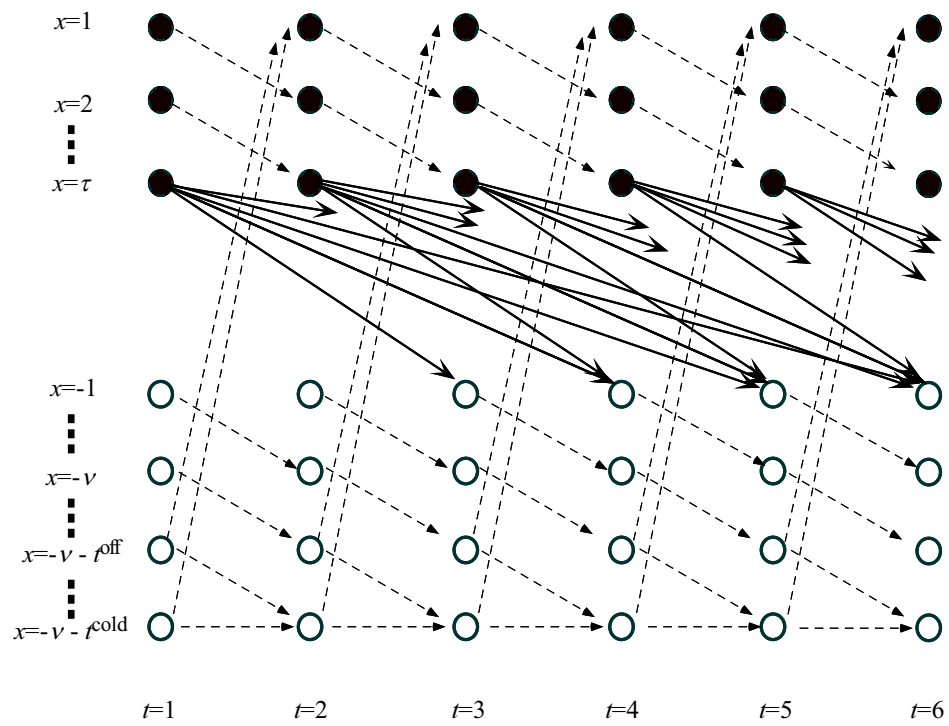


Figure 4.1: An example network graph with five time periods.

easy task, which can be done efficiently using the dynamic programming. The most efficient algorithm for finding a shortest path is the Dijkstra method with $O(T^2)$ arithmetic operations required (Murty 1992). Solving (P) , however, includes developing the network graph G and evaluating *all* costs associated with arcs. The part that demands the most computational effort is the evaluation of $(Q_{t_1}^{t_2})$ of *all* operating arcs. There are as many as $O(T^2)$ of problem $(Q_{t_1}^{t_2})$ to be solved. Since $\pi_t(\cdot)$ is assumed to be a convex quadratic function, each $(Q_{t_1}^{t_2})$ is a standard quadratic programming problem with linear constraints, which can be solved in polynomial-time. The best known result for solving a $(Q_{t_1}^{t_2})$ is $O(|t_2 - t_1|^3 K)$ (e.g., Kojima et al. 1987; Monteiro and Adler 1989), where L is the problem size (Papadimitriou and Steiglitz 1982). Therefore, the optimal solution of (P) can be solved with no more than $O(T^5 K)$ arithmetic operations using the proposed network graph approach.

Theorem The proposed network graph approach solves (P) within a polynomial time. ■

Remark

Problem $(Q_{t_1}^{t_2})$ can be solved using commercial nonlinear programming solvers such as LSQRG2 and MINOS. The former solver is incorporated in our numerical tests. Note the algorithm complexity discussed defines algorithm performance in the worst case. Therefore, in practice, a better complexity may be sought by a more efficient implementation. In view of this, note that all operating arcs are not necessarily unrelated. For example, $f(t_1, t_2)$ and $f(t_1, t_2 + 1)$ share the same multipliers (or spot prices) from t_1 to t_2 . Therefore, the optimal solution

of $f(t_1, t_2)$ can be used to create an initial feasible solution for $f(t_1, t_2 + 1)$, e.g., incorporating with $q_{t_2+1} = q^{\min}$. Since the number of $(Q_{t_1}^{t_2})$ to be evaluated is large, such a “warm start” strategy can make significant savings of CPU time (see Section 4 for numerical tests). In the next section, I will show how to take advantage the special structure of the problem and develop a more efficient algorithm.

In the next section, I will focus on solving $(Q_{t_1}^{t_2})$ that is subject to (4.4) – (4.11) including the ramp constraints. I will propose a method for solving $(Q_{t_1}^{t_2})$ that does not require discretization of the generation level, linearization of the objective function, and is polynomial-time.

4.2.2 An active-set method for solving $(Q_{t_1}^{t_2})$

In this section, I will use an example to illustrate the basic idea of a new algorithm For solving $(Q_{t_1}^{t_2})$. Consider solving $f(0, 15)$ denoted by (Q_0^{15}) , which is summarized below.

(Q_0^{15})

$$\max_{q_1, \dots, q_{14}} \sum_{t=1}^{14} \pi_t(q_t) \quad (4.19a)$$

$$\text{s.t. } q_{t-1} - q_t - \Delta \leq 0, \quad t = 1, \dots, 15 \quad (4.19b)$$

$$q_t - q_{t-1} - \Delta \leq 0, \quad t = 1, \dots, 15 \quad (4.19c)$$

$$-q_t + q^{\min} \geq 0, \quad t = 1, \dots, 14 \quad (4.19d)$$

$$q_t - q^{\max} \geq 0, \quad t = 1, \dots, 14 \quad (4.19e)$$

$$q_{15} = q^{\min} \quad (4.19f)$$

$$q_0 = q^{\min} \quad (4.19g)$$

Let μ_t , ξ_t , α_t and β_t be the Lagrange multipliers associated with (4.19b), (4.19c),

(4.19d) and (4.19e), respectively. The Lagrangian of the problem is defined as follows:

$$\begin{aligned} \mathcal{L}(q_t, \mu_t, \xi_t, \alpha_t, \beta_t) = & \sum_{t=1}^{14} \pi_t(q_t) + \sum_{t=1}^{15} \left(\mu_t(q_{t-1} - q_t - \Delta) + \xi_t(q_t - q_{t-1} - \Delta) \right) + \\ & \sum_{t=1}^{14} \left(\alpha_t(-q_t + q_t^{\min}) + \beta_t(q_t - q^{\max}) \right) \end{aligned} \quad (4.20)$$

The optimality condition ensures that

$$\frac{\partial \mathcal{L}}{\partial q_t} = \frac{\partial \mathcal{L}}{\partial \mu_t} = \frac{\partial \mathcal{L}}{\partial \xi_t} = \frac{\partial \mathcal{L}}{\partial \alpha_t} = \frac{\partial \mathcal{L}}{\partial \beta_t} = 0, \quad \forall t \quad (4.21)$$

and the complementary slackness condition for each constraint.

Starting with a feasible solution, the basic idea of the active-set method is to determine the values of the Lagrange multipliers corresponding to the *active* (or binding) constraints that satisfy the optimality conditions (4.21). If all the multipliers for the active constraints (inequalities) are nonnegative, then the feasible solution must be optimal. Note that $(Q_{t_1}^{t_2})$ is a convex problem and its optimal solution is unique. If the multiplier of an active constraint is negative, it implies that the objective function value could have been improved, had that constraint not been active. Therefore, one can search along the direction that is strictly feasible with respect to that constraint alone to locate a better feasible solution.

For illustrative purpose, consider a feasible solution $\{\tilde{q}_0, \dots, \tilde{q}_{15}\}$, which corresponds to the sixteen discrete generation levels shown in Fig. 4.2. If a ramp constraint is binding between two consecutive hours, a solid line is depicted to connect these two corresponding generation levels, otherwise a dash line is used. In this example, the two dash segments separate the interval of concern into

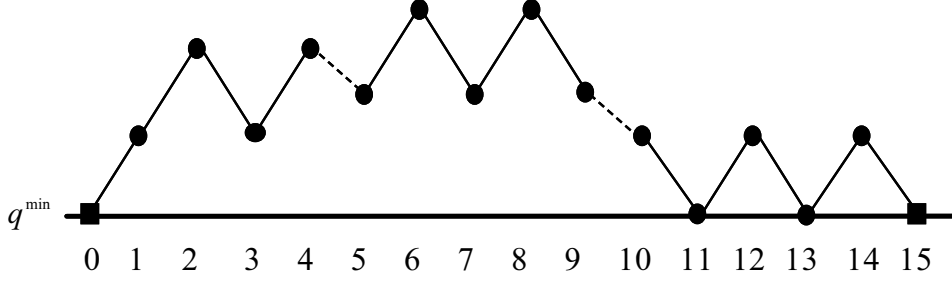


Figure 4.2: A feasible solution to Q_0^{15} .

three subintervals, $[0, 4]$, $[5, 9]$, and $[10, 15]$. Each of these three subintervals is not coupled with any adjacent subinterval and can be treated independently. These three subintervals also illustrate three categories of subintervals that can cover all possible scenarios. They are discussed below.

- One-end fixed and nondegenerate ($t \in [0, 4]$)

This subinterval has one end point ($t = 0$) whose generation must be set to q^{\min} . Because no other points except the end point has its q_t equal to q^{\min} or q^{\max} , this subinterval is called *nondegenerate*. The issue of degeneracy will be discussed in more detail later. The corresponding optimality conditions are as follows, with the multipliers corresponding to inactive constraints ignored:

$$\partial\mathcal{L}/\partial q_1 = \pi'_1(\tilde{q}_1) - \mu_1 + \mu_2 = 0 \quad (4.22a)$$

$$\partial\mathcal{L}/\partial q_2 = \pi'_2(\tilde{q}_2) - \mu_2 - \xi_3 = 0 \quad (4.22b)$$

$$\partial\mathcal{L}/\partial q_3 = \pi'_3(\tilde{q}_3) + \xi_3 - \mu_4 = 0 \quad (4.22c)$$

$$\partial\mathcal{L}/\partial q_4 = \pi'_4(\tilde{q}_4) - \mu_4 = 0 \quad (4.22d)$$

One can solve μ_4 from (4.22d), and then plug it to (4.22c) for ξ_3 , (4.22b) for μ_2 , and (4.22a) for μ_1 . If a multiplier is negative, say $\xi_3 < 0$, it means

that it is an improving direction by further dividing this subinterval to two subintervals, $[0, 2]$ and $[3, 4]$, i.e., replacing the corresponding solid line to a dash line between $t = 2$ and 3. The subinterval $[3, 4]$ then becomes one with “both-end free,” to be discussed next.

- Both-end free and nondegenerate ($t \in [5, 9]$)

In this subinterval, both end points are not tied to q^{\min} and none of the points is either q^{\min} or q^{\max} . We will also call such an subinterval as a free subinterval. The optimality conditions are as follows.

$$\partial\mathcal{L}/\partial q_5 = \pi'_5(\tilde{q}_5) + \mu_6 = 0 \quad (4.23a)$$

$$\partial\mathcal{L}/\partial q_6 = \pi'_6(\tilde{q}_6) - \mu_6 - \xi_7 = 0 \quad (4.23b)$$

$$\partial\mathcal{L}/\partial q_7 = \pi'_7(\tilde{q}_7) + \xi_7 + \mu_8 = 0 \quad (4.23c)$$

$$\partial\mathcal{L}/\partial q_8 = \pi'_8(\tilde{q}_8) - \mu_8 - \xi_9 = 0 \quad (4.23d)$$

$$\partial\mathcal{L}/\partial q_9 = \pi'_9(\tilde{q}_9) + \xi_9 = 0 \quad (4.23e)$$

In this case, one may solve μ_6 first from (4.23a), then solve other multipliers from top-down; or solve ξ_9 first from (4.23e) and then move from bottom-up. However, both results may not necessarily coincide. To avoid this, for a free subinterval, one should always determine its optimal position by performing parallel shift. For example, in this case one can solve the following one-dimensional optimization problem.

$$\min_{\delta q} \quad \sum_{t=5}^9 \pi_t(\tilde{q}_t + \delta q) \quad (4.24a)$$

$$\text{s.t.} \quad |\tilde{q}_5 + \delta q - \tilde{q}_4| \leq \Delta \quad (4.24b)$$

$$|\tilde{q}_9 + \delta q - \tilde{q}_{10}| \leq \Delta \quad (4.24c)$$

$$q^{\min} \leq \tilde{q}_t + \delta q \leq q^{\max}, \quad 5 \leq t \leq 9. \quad (4.24d)$$

After a free subinterval is positioned optimally, it may no longer be free if it becomes binding with its adjacent subinterval(s). If this subinterval remains to be free, solving the multipliers in (4.23a) to (4.23e) either from top-down or bottom-up would yield the same result.

- A degenerate subinterval ($t \in [10, 15]$)

First, let's examine the optimality conditions for this subinterval.

$$\partial\mathcal{L}/\partial q_{10} = \pi'_{10}(\tilde{q}_{10}) - \xi_{11} = 0 \quad (4.25a)$$

$$\partial\mathcal{L}/\partial q_{11} = \pi'_{11}(\tilde{q}_{11}) + \xi_{11} + \mu_{12} - \alpha_{11} = 0 \quad (4.25b)$$

$$\partial\mathcal{L}/\partial q_{12} = \pi'_{12}(\tilde{q}_{12}) - \mu_{12} - \xi_{13} = 0 \quad (4.25c)$$

$$\partial\mathcal{L}/\partial q_{13} = \pi'_{13}(\tilde{q}_{13}) + \xi_{13} + \mu_{14} - \alpha_{13} = 0 \quad (4.25d)$$

$$\partial\mathcal{L}/\partial q_{14} = \pi'_{14}(\tilde{q}_{14}) - \mu_{14} - \xi_{15} = 0 \quad (4.25e)$$

$$\partial\mathcal{L}/\partial q_{15} = \pi'_{15}(\tilde{q}_{15}) + \xi_{15} = 0 \quad (4.25f)$$

In this subinterval, because $\tilde{q}_{11} = \tilde{q}_{13} = q^{\min}$, two additional multipliers α_{11} and α_{13} appear in the optimality conditions. This creates linear dependence among the linear constraints. Therefore, the system of equations may have infinitely many solutions. Such a situation is normally termed as *degeneracy*. To deal with the degeneracy, it is necessary to drop some dependent variables, i.e., to make some active constraints inactive while retaining enough active constraints to identify the existing feasible solution. In this example, two variables may be dropped corresponding to the two additional active constraints. They are one from $\{\xi_{11}, \mu_{12}, \alpha_{11}\}$ and one from $\{\xi_{13}, \mu_{14}, \alpha_{13}\}$. For example, if μ_{12} would be dropped, it is equivalent to further dividing the subinterval into two subintervals, $[10, 11]$ and

[12, 15], where the former becomes a free section. It is likely that after dropping a variable, no improvement in the objective function value can be made without violating one of the dependent constraints that have been excluded. In that case, such a constraint must be added back to the active constraints. One can then try to drop another dependent variable. Finally, note that the linear dependence among the linear constraints here is not a unique product of the proposed method. It is, however, “*commonplace in practical problems, but it is not a serious impediment to practical computation*” (P. 201, Gill et al. 1989). The interested reader is directed to Gill et al. (1989) for more detailed discussions on how to deal with degeneracy.

4.2.3 A numerical test on ramp-constrained SSP

Numerical tests are conducted to measure the performance of the proposed approach. The network graph method for solving (P) is implemented in FORTRAN on a Pentium IV PC. Two methods for solving ($Q_{t_1}^{t_2}$) are implemented: LSGRG2 and the proposed active-set-based method. For LSGRG2, the FORTRAN source code of the commercial software is integrated with our programs. A natural gas-fueled generating unit with the input-output characteristics following (4.12a) and (4.12b) is considered. Assume P^F is \$ 2.2/MMBtu, and $q^{\min}=250$ MW, $q^{\max}=750$ MW, $c_0=600$, $c_1=9.121$, and $c_2=0.00131$. We also assume that $\tau = \nu = 2$, $t^{\text{on}} = t^{\text{off}} = 5$, and $t^{\text{cold}} = 10$ to fully capture the influence of the physical constraints. Assume the startup cost

$$S_u(x_t) = 2300(1 - \exp(x_t/4)) + 950 \quad (4.26)$$

and a constant shutdown cost \$1000. Hourly electricity prices λ_t are generated

by a uniform random variable between \$16/MWh and \$25/MWh. Note that in reality electricity hourly prices may follow some pattern, such as on-peak vs. off-peak. Without considering any price pattern, our test cases here are actually harder in the sense that degeneracy is more likely to occur. Without loss of generality, the spinning reserve market is not considered by setting $\mu_t = 0$. To fully observe the algorithm performance in terms of the complexity analysis, seven cases are tested corresponding to seven operating periods, ranging from 1 day (24 hours) to 7 days (168 hours). For each case, we randomly generate 100 instances of the electricity hourly prices. The LSGRG2 method and proposed active-set method are used to solve $(Q_{t_1}^{t_2})$ for each instance of (P) . The average CPU times are recorded in Table 4.1.

Table 4.1: Average CPU time (seconds)

(T)	24	48	72	96	120	144	168
LSGRG2	0.16	2.42	15.79	57.90	180.77	463.63	972.23
Active-Set	0.03	0.46	1.49	2.92	4.96	7.54	12.19

Note that in the implementations of both methods for solving $(Q_{t_1}^{t_2})$, the warm-start feature, discussed in Section § 4.2.1, has been included, which reduced the CPU times as much as 60% on average. Based on the performance data in Table 1, both approaches reveal a polynomial-time complexity with approximately $O(T^{5.0})$ for LSGRG2 and $O(T^{2.5})$ for the proposed active-set-based method. The complexity of the proposed active-set-based method is much better than the theoretic result $O(T^5K)$, as shown in Section § 4.2.1.

To sum up, the numerical testing result verifies the polynomial-time perfor-

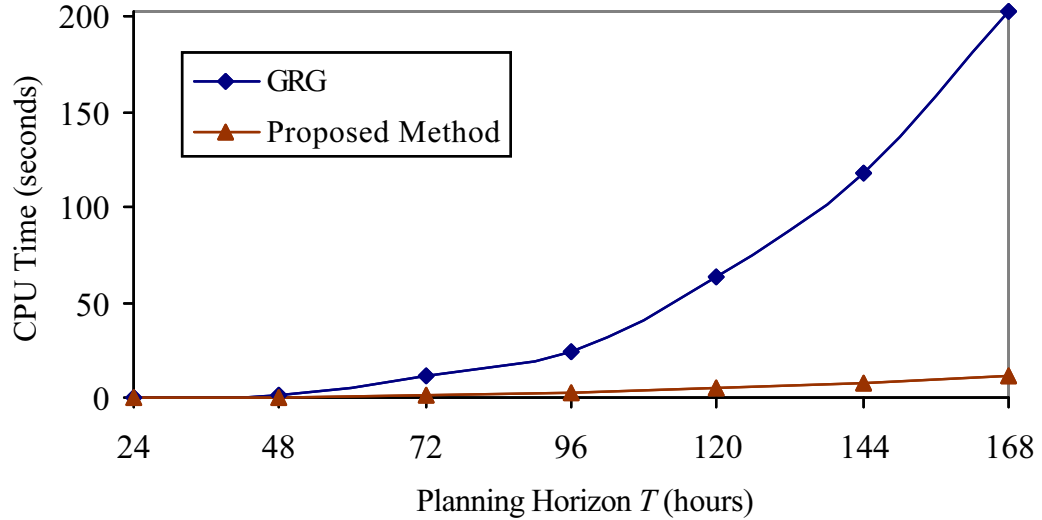


Figure 4.3: Comparison of average CPU times for solving (P) .

mance for the proposed methods for solving the optimal unit response problem. The proposed method also guarantees the optimality of the solution. Because of the polynomial-time complexity, the proposed method can be incorporated in large-scale unit commitment related problems.

4.3 Stochastic ramp-constrained SSP

In this section, we consider the SSP under price uncertainty in the spot markets. Let $\mathbf{P}_t = (\lambda_t, \mu_t)$ be the market price information at time t . At time t , the operator observes the price \mathbf{P}_t and he needs to make a commitment decision subject to the physical constraints. We assume a one-hour lead time for commitment decisions. Therefore, the operator determines u_{t+1} at time t . If the unit is on-line at time t (i.e., $u_t = 1$), the operator needs to dispatch the

generator. It is assumed that the dispatch decision can be made instantaneously after the price \mathbf{P}_t is observed. The stochastic SSUCP can be formulated as the following multi-stage recursive equation, denoted by (SP) :

$$(SP) \quad F_t(u_t|\mathbf{P}_t) = \max_{q_t, u_{t+1}} (\pi_t(q_t)u_t + S(x_{t-1})u_t(1 - u_{t-1}) + E_{t+1}[F_{t+1}(u_{t+1}|\mathbf{P}_{t+1})]) \quad (4.27)$$

where $F_t(u_t|\mathbf{P}_t)$ represents expected total profit of the unit for the remainder of the operating period starting at commitment u_t at time t , and E_t is the expectation operator given the available price information at time t . The maximization in (SP) is subject to constraints (4.4) - (4.11). Without considering the ramp constraints, (SP) can be solved using the Monte Carlo simulation (Tseng and Barz 2002) or a lattice approach (Tseng 2001). With the presence of the ramp constraints, solving (SP) to optimality becomes very difficult because each stage is interdependent and the dispatch variable q_t is a *continuous* variable. We consider a *two-stage* approximate formulation for (SP) , in which the first stage is $t = 1$ and the second stage covers from $t = 2$ to T . First, consider the dispatch decision problem at time $t = 1$ (given that has been made available at $t = 0$).

$$F_t(u_t|\mathbf{P}_t) \approx \max_{q_1} \theta(q_1|\mathbf{P}_1) \quad (4.28)$$

where

$$\theta(q_1|\mathbf{P}_1) \equiv (\pi_1(q_1)u_1 + S(\tilde{x}_0)u_1(1 - \tilde{u}_0) + E_1[\max_{\mathbf{u}, \mathbf{x}, \mathbf{q}} \sum_{t=2}^T \pi_t(q_t)u_t + S(x_{t-1})u_t(1 - u_{t-1})|\mathbf{P}_1]) \quad (4.29)$$

Apparently, $q_1 = 0$ if $u_1 = 0$ and the dispatch decision in (4.28) is made only when $u_1 = 1$. The first stage decision q_1 (as well as u_1) will become the initial

condition of the second stage problem. We are interested in the *optimal dispatch rule* at $t = 1$ under ramp constraints and price uncertainty if $u_1 = 1$. That is,

$$q_1^*(\mathbf{P}_1) = \arg \max_{q_1} \theta(q_1 | \mathbf{P}_1) \quad (4.30)$$

Since we assume that the dispatch can be performed immediately, $q_1^*(\mathbf{P}_1)$ represents the optimal dispatch decision after the operator observes the spot price \mathbf{P}_1 at $t = 1$. This optimal dispatch rule has accounted the ripple effect of the ramp constraints to the future hours imposed by $q_1^*(\mathbf{P}_1)$.

It can be shown easily that the *optimal dispatch rule* without considering the ramp constraints (or $\Delta q = \infty$) is a piecewise-linear function of $\mathbf{P}_1 = (\lambda_1, \mu_1)$:

$$q_{1,\Delta q=\infty}^*(\mathbf{P}_1) = \max \left(q^{\min}, \min \left(q^{\max}, \frac{1}{2c_2} \left(\frac{\lambda_1 - \mu_1}{PF} - c_1 \right) \right) \right) \quad (4.31)$$

where the cost function of the unit is assumed to follow (4.12a) and (4.12b). If $q_1^*(\mathbf{P}_1)$ is available, then the optimal commitment decision u_1 (made at $t = 0$) can be determined by

$$u_1^* = \arg \max_{u_1 \in \{0,1\}} E_0[F_1(u_1 | \mathbf{P}_1)] \quad (4.32)$$

The two-stage formulation in (4.29) can then be performed sequentially in time over a rolling horizon to make optimal commitment and dispatch decisions.

To evaluate $\theta(q_1 | \mathbf{P}_1)$ and $q_1^*(\mathbf{P}_1)$, we employ the least squares Monte Carlo (LSMC) method, originally introduced by Longstaff and Schwartz (2001) for financial options valuation.

4.3.1 Solving ramp-constrained SSP using LSMC

To determine $\theta(q_1 | \mathbf{P}_1)$, recognize that $\theta(\cdot)$ can be viewed as some conditional probability of q_1 and \mathbf{P}_1 . Therefore, the functional form $\theta(\cdot)$ can be obtained by

the linear regression method treating q_1 and \mathbf{P}_1 as the *predictor* variables and θ as the corresponding *response* variable. The Monte Carlo method is used to generate the *observations* of θ and the associated values of q_1 and \mathbf{P}_1 . This is the basic idea of the LSMC method.

Based on the initial condition (q_0, \mathbf{P}_0) at time 0, randomly generate N samples of $(q_1^{(j)}, \mathbf{P}_1^{(j)})$, $j = 1, \dots, N$ where $q_1^{(j)} \in [q_0 - \Delta q, q_0 + \Delta q] \cap [q^{\min}, q^{\max}]$. For each $\mathbf{P}_1^{(j)}$, generate a price scenario (vector) for the rest of the operating period $(\mathbf{P}_2^{(j)}, \dots, \mathbf{P}_T^{(j)})$, based on which evaluate the corresponding observation of θ , denoted by $\theta^{(j)} = \theta(q_1^{(j)} | \mathbf{P}_1^{(j)})$. Note that to evaluate $\theta^{(j)}$, a deterministic SSUCP (over $[2, T]$) must be solved, which is handled by the proposed network graph method in Section §4.2.

Finally, $\theta(q_1 | \mathbf{P}_1)$ can be approximated by regressing $\{\theta^{(j)}\}_{j=1}^N$ on $\{(q_1^{(j)}, \mathbf{P}_1^{(j)})\}_{j=1}^N$. What remains to show is an appropriate functional form for the regression. Note the analytic form for $\theta(q_1 | \mathbf{P}_1)$ may not exist. Our experience shows that the following polynomial form works well for the regression.

$$\theta(q_1 | \mathbf{P}_1) \approx a_1 + a_2 q_1 + a_3 q_1^2 + a_4 \hat{q} q_1 + a_5 \lambda_1 + a_6 \mu_1 \quad (4.33)$$

where $\hat{q} = \max(q_0 - \Delta q, \min(q_0 + \Delta q, ((\lambda_1 - \mu_1)/P^F - c_1)/2c_2))$ is similar to the optimal dispatch rule without the ramp constraints in (4.31), and $a_1 \sim a_6$ are the parameters to be fitted in the regression. Here we assume that the heat rate function of the unit follows the quadratic function given in (4.12a) and c_1 and c_2 are the coefficients.

Note that $\theta(q_1 | \mathbf{P}_1)$ is a highly nonlinear function in q_1 and \mathbf{P}_1 and $\theta(q_1 | \mathbf{P}_1)$ may not be smooth everywhere because it involves \hat{q} . However, $\theta(q_1 | \mathbf{P}_1)$ is a linear function of the regression parameters a_1, \dots, a_6 . It is well known that linear regressions can be done efficiently, which only involves solving a system of

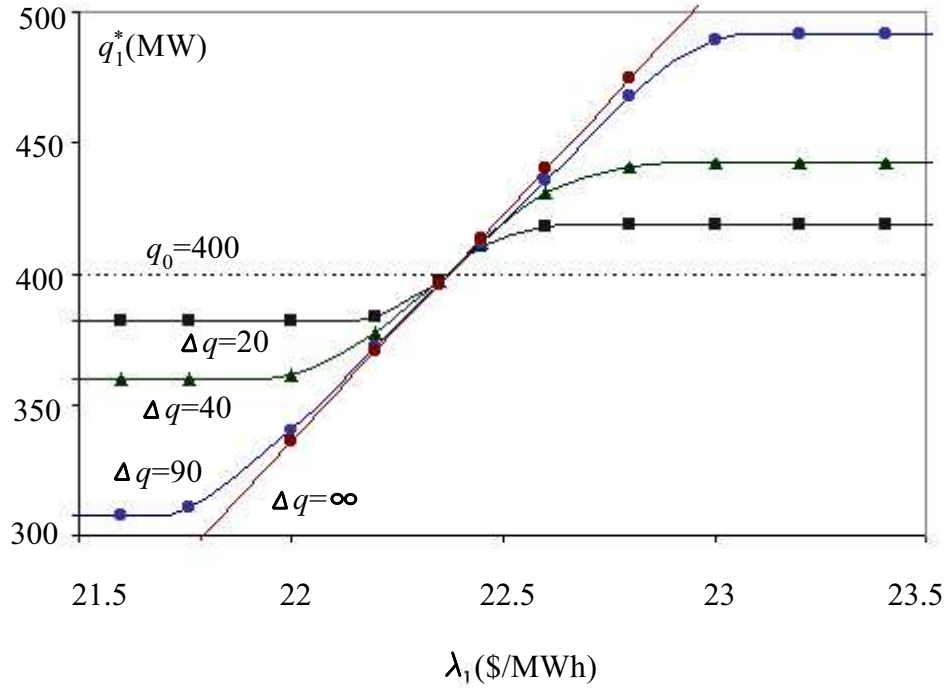


Figure 4.4: Optimal dispatch rule $q_1^*(\lambda)$ for different ramp rates.

linear equations (a 6×6 linear system in this case.)

Once $\theta(q_1|\mathbf{P}_1)$ is obtained, the ramp constrained optimal dispatch rule $q_1^*(\mathbf{P}_1)$ in (4.30) can be determined using standard nonlinear programming methods.

4.3.2 Numerical tests

The same natural gas-fired generating unit used in Section §4.2.3 is considered, and the initial conditions are $\lambda_0 = \$20/MWh$, $q_0 = 400MW$. For simplicity, the spinning reserve market is not considered by setting $\mu_t = 0$ at all times. Assume a 7-day (168-hour) operating period and the hourly electricity spot prices λ_t are generated based on the mean reverting process reported in (Tseng and Barz 2002).

The expected profit function $\theta(q_1|\lambda_1)$ varies with ramp rates Δq , and so does the optimal dispatch rule $q_1^*(\lambda_1)$. In Fig. 4.4, the optimal dispatch rule $q_1^*(\lambda_1)$ under the ramp constraints and price uncertainty is displayed for four different cases: $\Delta q = \infty$ (no ramp), $\Delta q = 90$, $\Delta q = 40$ and $\Delta q = 20$. As shown in (4.31), $q_1^*(\lambda_1)$ is linear when there is no ramp constraint. From Fig. 4.4, when the ramp constraints exist, is a s-curve bounded by $\tilde{q}_0 \pm \Delta q$. When the ramp rate Δq decreases, the slope of the s-curve decreases. Again assume the fuel cost function can be represented by a quadratic function in (4.4) and (4.11). A less-steep s-curve can be viewed as an equivalent unit with (i) greater c_1 and c_2 ; (ii) reduced available capacity; and (iii) no ramp constraints. A greater c_2 indicates a more curved (upward sloping) cost function representing a worse fuel economy; while a greater indicates a higher (incremental) heat rate representing a less efficient heat-electricity transformation. Adjusting the cost function to deal with ramp rates is commonly practiced in industry. Our test result validates this practice, but also indicates that it is only an approximation because the s-curve is nonlinear.

Finally, Fig. 4.5 illustrates the LSMC approach for determining $\theta(q_1|\lambda_1)$. Around 1000 ($= N$) samples are taken to estimate the conditional expectation function θ .

4.4 Summery and conclusions

In this chapter, the optimal unit response of a thermal unit to an electricity spot market is studied. When the spot prices are known with certainty, a polynomial-time algorithm based on network graph is proposed for solving the

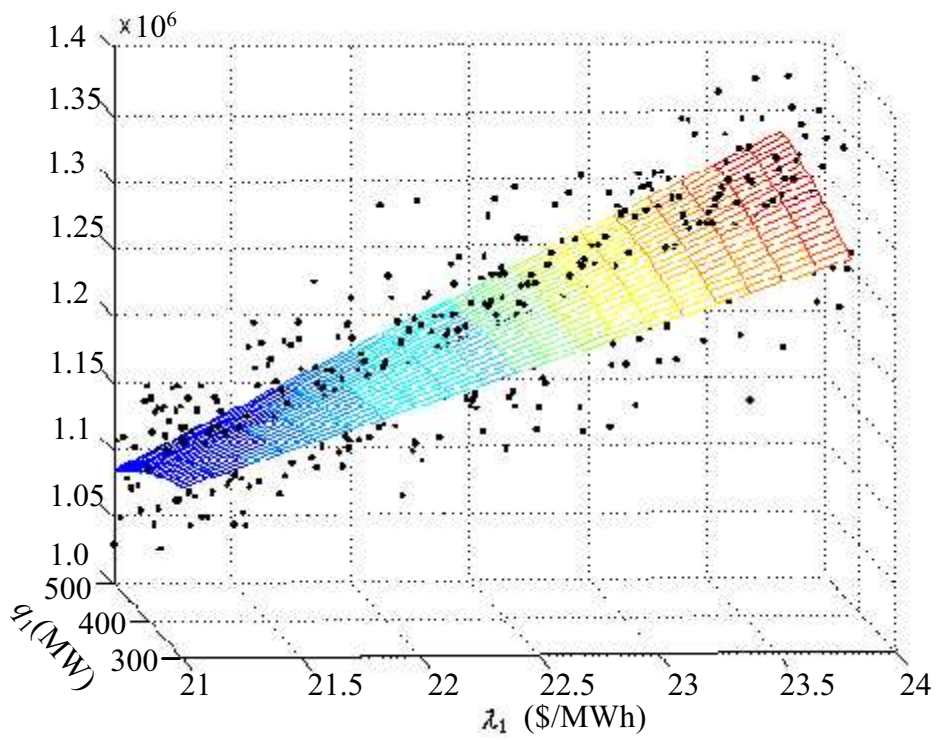


Figure 4.5: An example of $\theta(q_1|\lambda_1)$ with $\Delta q=100$ MW.

problem. Using the developed algorithm to evaluate the unit responses in various price scenarios generated by the Monte Carlo method, we obtain the optimal commitment and dispatch rule under price uncertainty. Our test results indicate that the ramp constraints impact a thermal unit on its capability of responding to price uncertainty by reducing its fuel economy, heat-electricity transformation efficiency, and available generation capacity. Our proposed method provides a way to quantify these effects.

Chapter 5

Valuing Power Generation Units with Fuel Switching Options

The deregulation of the US electric power market in the last few years has replaced the vertical utility by a series of independently operating units with a more horizontal relationship. After the restructuring, the electric utility industry throughout the US has been facing pressure to increase its efficiency, to reduce operational costs, and to lower purchase cost of power equipment. In order to adapt well in the new business environment, generation asset investors must consider market uncertainty in appraising the asset value.

In addition to market uncertainty, the investors must also consider an asset's *operating flexibility* that enables the unit to respond to changing exogenous economic conditions. The importance of such *operating options* becomes critical when the market environment is highly volatile. For example, when facing exogenous stochastic prices a generator with *operating options*, such as fuel switching can protect itself against adverse price movements, with the capability of switching into an alternative fuel that may be less affected by the adverse price realizations (e.g. Kulatilaka 1993).

In the present competitive environment, a generator will be invested only when an adequate return on the investment is expected. Since electricity is non-storable in nature, the generation asset value may be replicated on future spark spread options (Deng et al. 1999). In addition to pure spark spread options, physical constraints such as minimum uptime and downtime constraints may also affect the value. Tseng and Barz (1999, 2002) proposed a Monte Carlo (MC) simulation to formulate the power plant valuation problem as a multi stage stochastic problem. In addition, discrete-time price trees for correlated price processes for both electricity and fuel, such as geometric mean reverting processes are employed to value power asset. The computational efficiency of the valuation problem may be improved by using stochastic dynamic programming via a price tree (Tseng 2000). Although the method produces a satisfactory result for a two-factor case (referring to two uncertainties such as electricity price and gas price), it may not be applicable to a three-factor case if an additional uncertainty must be considered (e.g., for a unit capable of switching fuels). The tree approach is especially prohibitive when the time horizon is long because of the ‘curse of dimensionality’. Therefore, valuing a generation asset with fuel switching option (considering three price uncertainties) is a challenging task. In this chapter, we explore the LSMC approach and use it to value the fuel switching option.

This chapter is organized as follows. In Section 5.1, we provide an overview of the fuel switching unit including physical constraints and fuel switching options. We then formulate and solve the generation asset valuation problem in Section 5.2. Numerical results are presented in Section 5.3. This chapter concludes in Section 5.4.

5.1 Problem statement

5.1.1 Overview of fuel-switching units

A thermal generation unit with fuel-switching options can consume two different fuels, which can be switched within a short time period, and then converts the fuels into electricity. This conversion involves three commodities with different market prices. The following conditions are implicitly assumed: (1) fuel switching does not affect normal operation of a unit; (2) fuel switching can be finished within a reasonable time. In general, the fuel-switching options can provide the following two advantages:

- Fuel-switching capabilities may help stabilizing the operation of a unit under limited resources available to a country or region;
- Fuel-switching options may help solving some pollution issues, such as reducing emissions of waste.

In power industry, players may prefer nature gas to fuel oil because nature gas is much ‘cleaner’ than fuel oil under environment restriction on emission of waste. Therefore, fuel switching has been considered as an emission abatement means. On the other hand, nature gas prices are much more expensive than fuel oil prices based on per British thermal unit. Power producers may have to react to the soaring price of natural gas by switching to cheaper, more environmentally harmful fuel sources. To reduce power producers’ exposure to price volatility or possible supply disruptions in the present deregulation environment, industrial users are expected to increasingly seek the flexibility of switching fuels using hybrid technologies.

5.1.2 Notations

The following standard notation will be used to formulate the valuation of a fuel-switching unit mathematically. Additional symbols will be introduced when necessary.

t : index for time horizon in hours ($t = 0, \dots, T$, where T is the length of time horizon).

j : index for fuel ($j = 1, 2$, where “1” represents gas and “2” represents oil).

x_t : state variable indicating the commitment status of the unit in time period t .

u_t : zero-one generation unit commitment decision variable in time period t .

v_t : zero-one variable indicating which fuel the unit choose in time period t (where “1” represents gas and “0” represents oil).

t_j^{on} : the minimum number of periods the unit must remain on after it has been turned on for fuel j .

t_j^{off} : the minimum number of periods the unit must remain off after it has been turned off for fuel j .

t_j^{cold} : the minimum number of periods required to cool down the boiler of a unit after it has been turned off for fuel j .

q_t : decision variable indicating the amount of power the unit is generating in time period t .

q_j^{min} : minimum rated capacity of the unit for fuel j .

q_j^{\max} : maximum rated capacity of the unit for fuel j .

P_t^E : electricity price (\$/MWh) in time period t .

P_t^{F1} : fuel 1 price (\$/MMBtu) in time period t .

P_t^{F2} : fuel 2 price (\$/MMBtu) in time period t .

$\pi_t(x_t, v_t, q_t; P_t^E, P_t^{F1}, P_t^{F2})$: profit for operating the unit at state x_t at output level q_t when the electricity and fuel prices are P_t^E , P_t^{F1} and P_t^{F2} in time period t respectively.

J_t : the asset value of unit in time period t .

$h_t^{1,1}$: the regression function when the unit is on in both time periods t and $t+1$.

$h_t^{1,0}$: the regression function when the unit is on in time period t and off in time period $t+1$.

$h_t^{0,1}$: the regression function when the unit is off in time period t and is on in time period $t+1$.

$h_t^{0,0}$: the regression function when the unit is off in both time periods t and $t+1$.

$h_t^{0,0,n}$: the regression function when the unit is cold in both time periods t and $t+1$ without switching fuel.

$h_t^{0,0,s}$: the regression function when the unit is cold in both time periods t and $t+1$ with fuel switched.

$S_t(u_t, u_{t-1}, v_t)$: startup/shutdown cost when the unit is turned on/off in time period t .

5.1.3 Fuel-switching options

Generally speaking, fuel switching options of a generating unit refers to the ability to burn alternate fuels, such as nature gas or fuel oil. Switching occurs when one fuel out-of-the-money is replaced by another in-the-money (or less out-of-the-money).

Since a fuel-switching unit can operate using different fuels, the cost characteristics of the unit depend on the fuel used. Different fuel may have a different amount of MMBtu required to produce a MW (Cohen et al. 1996; Zhai et al. 2001). This property differs from unit to unit and fuel to fuel. The value of a fuel-switching unit is affected by the operational constraints such as the minimum up/down time constraints.

In reality, fuel switching is an transitional process, which may take from minutes to hours dependent upon different units. In this chapter, we assumed that the fuel switching could be done instantaneously under other conditions to be addressed next.

There are normally two ways to switch fuels: on-line switching and off-line switching. The switch takes place when the unit has already been on-line for at least its minimum uptime periods (i.e., state t_1^{on}). During the switching, it transits to the shutdown status using another fuel. For the off-line switching, the unit must be off-line while the fuel switching takes place. Therefore, the state transits (from t_1^{cold} to t_2^{cold} or from t_2^{cold} to t_1^{cold}). Fig. 5.1 illustrates the state transitions of both the on-line and off-line switching. In this dissertation, we focus on the off-line switching model.

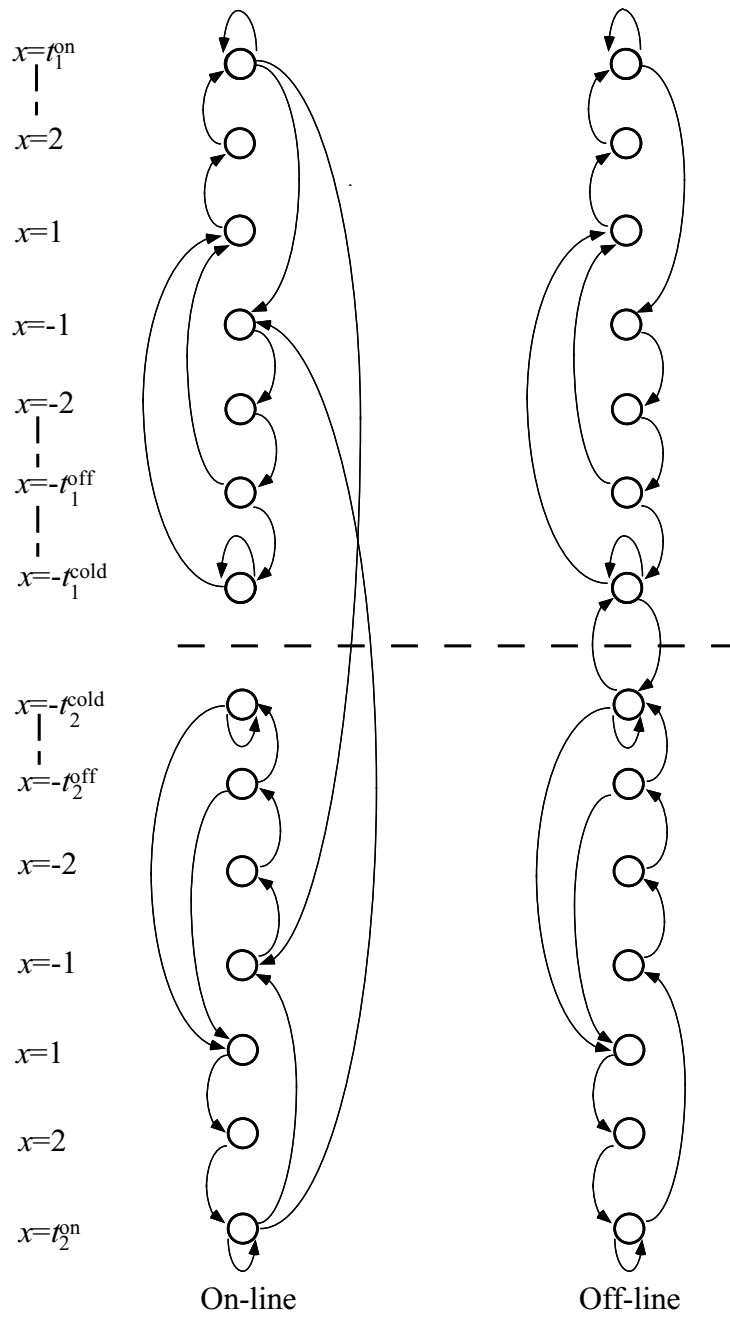


Figure 5.1: A state transition diagram between two fuels (on-line and off-line).

5.1.4 Profit function of a fuel-switching unit

As in the previous chapters, the input-output characteristic of a generating unit is captured by $H(q)$ (MMBtu), which is a function described required to generate q (MW) of power. Now this function is dependent on the fuel type used, denoted by $H_j(q)$ for fuel j . The profit at time t may be represented as follows:

$$\begin{aligned} \pi_t(x_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) \\ = \begin{cases} P_t^E q_t - H_1(q_t) P_t^{F_1} v_t - H_2(q_t) P_t^{F_2} (1 - v_t), & \text{if } x_t > 0, \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (5.1)$$

where q_t is the generation level at time t and we assume that q_t is dispatched *instantaneously* and *optimally* after the prices $(P_t^E, P_t^{F_1}, P_t^{F_2})$ are revealed.

5.1.5 Operational constraints of a fuel-switching unit

The operational constraints are formulated as follows.

- Minimum up/downtime constraints:

$$u_t = \begin{cases} 1, & \text{if } 1 \leq x_t < t_1^{\text{on}} v_t + t_2^{\text{on}} (1 - v_t), \\ 0, & \text{if } -t_1^{\text{off}} v_t - t_2^{\text{off}} (1 - v_t) < x_t \leq -1 \\ 0 \text{ or } 1, & \text{otherwise.} \end{cases} \quad (5.2)$$

- Switch constraints:

$$v_t = \begin{cases} 1 - v_{t-1} \text{ or } v_{t-1}, & \text{if } x_t = t_1^{\text{cold}} \text{ or } t_2^{\text{cold}} \\ v_{t-1}, & \text{otherwise.} \end{cases} \quad (5.3)$$

switch happens only when the unit is *cold* at time t .

- State transition constraints:

$$x_t = \begin{cases} \min(t^{\text{on}}, \max(x_{t-1}, 0) + 1), & \text{if } u_t = 1, \\ \max(-t^{\text{cold}}, \min(x_{t-1}, 0) - 1), & \text{if } u_t = 0 \end{cases} \quad (5.4)$$

where

$$t^{\text{on}} = t_1^{\text{on}}v_t + t_2^{\text{on}}(1 - v_t) \quad (5.4a)$$

$$t^{\text{off}} = t_1^{\text{off}}v_t + t_2^{\text{off}}(1 - v_t) \quad (5.4b)$$

$$t^{\text{cold}} = t_1^{\text{cold}}v_t + t_2^{\text{cold}}(1 - v_t) \quad (5.4c)$$

- Unit capacity constraints:

$$u_t(q_1^{\text{min}}v_t + q_2^{\text{min}}(1 - v_t)) \leq q_t \leq u_t(q_1^{\text{max}}v_t + q_2^{\text{max}}(1 - v_t)) \quad (5.5)$$

- Startup/shutdown cost:

$$S_t(u_t, u_{t-1}, v_t) = \begin{cases} S_1^u v_t + S_2^u (1 - v_t), & \text{if } u_t = 1 \text{ and } u_{t-1} = 0, \\ S_1^d v_t + S_2^d (1 - v_t), & \text{if } u_t = 0 \text{ and } u_{t-1} = 1, \\ 0, & \text{otherwise.} \end{cases} \quad (5.6)$$

where S_1^u and S_2^u represent constant startup cost for fuel 1 and fuel 2; S_1^d and S_2^d are constant shutdown cost for fuel 1 and fuel 2.

The operational constraints including the switch constraints have been fully captured in the state transition diagram in Fig. 5.1.

5.2 Problem formulation and solution procedure

The method to be introduced in this section can be viewed as an extension of the method proposed in Longstaff and Schwartz (2001) for valuing American-style options. We extend their approach to a more complicated situation involving multi-stage decision making and fuel switching options.

Let $J_t(x_t, u_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$ be the so-called value-to-go function indicating the total value of the unit for the remaining period at state x_t at time t , and assume a finite time horizon $[0, T]$. The asset valuing problem can be formulated as the following recursive relation:

$$J_t(x_t, u_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = \pi_t(x_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) + \max_{u_t, v_t} \{E_t[J_{t+1}(x_{t+1}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] - S_t(u_t, u_{t-1}, v_t)\} \quad (5.7)$$

where E_t denotes the expectation operator given the price information available at time t .

5.2.1 Solution procedure

From (5.7), it can be seen at time t to make an optimal commitment decision, one must know $E_t[J_{t+1}(x_{t+1}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})]$, which implicitly is a function of current price information, P_t^E , $P_t^{F_1}$, and $P_t^{F_2}$. The main idea of the LSMC method is to approximate such a function by regression. In terms of the states and the switching option, different functions are to be approximated using regression. They are defined below.

If $x_t = x_{t+1} = t^{\text{on}}$,

$$h_t^{1,1}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(t^{\text{on}}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (5.8a)$$

If $x_t = t^{\text{on}}$ and $x_{t+1} = -1$,

$$h_t^{1,0}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(-1, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (5.8b)$$

If $-t^{\text{cold}} \leq x_t \leq -t^{\text{off}}$ and $x_{t-1} = 1$,

$$h_t^{0,1}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(1, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (5.8c)$$

If $-t^{\text{cold}} < x_t \leq -t^{\text{off}}$ and $x_{t+1} < 0$,

$$h_t^{0,0}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(x_t - 1, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (5.8d)$$

If $x_t = x_{t+1} = -t^{\text{cold}}$,

$$h_t^{0,0,n}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(-t^{\text{cold}}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (5.8e)$$

If $x_t = -t^{\text{cold}}$ and $x_{t+1} = -(t_1^{\text{cold}}(1 - v_t) + t_2^{\text{cold}}v_t)$,

$$h_t^{0,0,s}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(x_{t+1}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (5.8f)$$

If the above regression functions $h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$ are available at time t , one could know the expected unit value for the next time period when the uncertainty prices $(P_t^E, P_t^{F_1}, P_t^{F_2})$ are revealed. Then, one could also know how to make optimal decisions at t . Especially when the unit stay at the *cold* state and $h_t^{0,0,n} < h_t^{0,0,s}$, fuel switching may happen at this time period. Since analytical forms of $h_t(\cdot)$ are nonexistent in general, we can only use numerical methods based on Monte Carlo simulation to approximate $h_t(\cdot)$. Through simulating a set of random variables, the expected value yields the least square error. Therefore, to approximate the above expected function, we generate N data samples of prices based on the mean reverting uncertainty model (5.10). Thus, the expected value of $J_{t+1}(x_{t+1}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})$ can be approximated by the function that best regressions J_{t+1} on the data of price $(P_t^E, P_t^{F_1}, P_t^{F_2})$ and possible decision values of (u_t, v_t, q_t) . That means any realization of (u_t, v_t, q_t) at time t , one would know how to optimally make decisions for the next time period based on the above regression functions $h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$.

The expected value of $J_{t+1}(\cdot)$ can be approximated through *forward-moving* simulation and *backward-moving* dynamic programming iterations from $T - 1$ to $T - 2, \dots, 0$. Thus, the asset value can be obtained by $J_0(x_0, u_0, v_0, q_0; P_0^E, P_0^{F_1}, P_0^{F_2})$, where $(x_0, u_0, v_0, q_0; P_0^E, P_0^{F_1}, P_0^{F_2})$ are determined by the initial conditions of the unit. As to J_T , it is determined by the boundary conditions as follows.

5.2.2 Boundary conditions

At time T , there is no commitment decision to make, because the power plant value is only conditioned on the state x_T as follows:

$$\begin{aligned}
& J_T(x_T, u_T, v_T, q_T^*; P_T^E, P_T^{F_1}, P_T^{F_2}) \\
&= \begin{cases} P_T^E q_T^* - H_1(q_T^*) P_T^{F_1} v_T - H_2(q_T^*) P_T^{F_2} (1 - v_T), & \text{if } x_T > 0, \\ 0, & \text{otherwise.} \end{cases} \quad (5.9)
\end{aligned}$$

where q_T^* represents the optimal generation level in time T within the capacity range $[q_j^{\min}, q_j^{\max}]$ for fuel j , which is determined by v_T .

At time $T - 1$, for the remaining two time periods, one can use the LSMC to estimate $J_{T-1}(x_T, u_{T-1}, v_{T-1}, q_{T-1}; P_{T-1}^E, P_{T-1}^{F_1}, P_{T-1}^{F_2})$ for every data sample. The decision maker can choose whether to switch fuel immediately or not and revisit the exercise decision at the next time period when the unit is at *cold* state. The value of $J_{T-1}(\cdot)$ is maximized path-wise, and hence the value of fuel switching option is greater than or equal to 0 unconditionally. This procedure can be repeated to $T-2, \dots, 0$. The last iteration, starting with the initial conditions at time 0, provides the optimal planning and asset estimation for the whole time horizon.

5.2.3 Price processes

In this paper, we assume that price of electricity P_t^E , price of fuel-1 P_t^{F1} , and price of fuel-2 P_t^{F2} are all functions of y_1 , y_2 , and y_3 , respectively, which are governed by the following (mean-reverting) stochastic differential equations.

$$dy_l = -\mu_l[y_l(t) - m_{t,l}(t)]dt + \sigma_l dB_l, \quad (5.10)$$

where $l=1, 2$, and 3 represents electricity, fuel-1, and fuel-2 respectively; μ_l is a drift function; σ_l is a constant volatility and B_l is a Wiener process with correlation ρ_{lm} ($l, m=1, 2, 3$). There exists a one-to-one transformation between y_1 and P_t^E , between y_2 and P_t^{F1} , and between y_3 and P_t^{F2} . Therefore, $(P_t^E, P_t^{F1}, P_t^{F2})$ can be obtained through the corresponding (y_1, y_2, y_3) , and vice versa.

For the purpose of carrying out the simulation process, the time horizon of fuel-switching options is divided into T subintervals of length Δt ($=1$ hour). The discrete version of the process for y_l is

$$\Delta y_l = -\mu_l(y_l - m_{t,l})\Delta t + \sigma_l \epsilon_l \sqrt{\Delta t} \quad (5.11)$$

where Δy_l is the change in y_l in time Δt ; ϵ_l is a random sample from a standardized normal distribution. The coefficient of correlation between ϵ_l and ϵ_m is ρ_{lm} for $1 \leq l, m \leq 3$. One simulation trial involves obtaining T samples of the ϵ_l ($1 \leq l \leq 3$) from a multivariate standard normal distribution. These are substituted into equation (5.11) to produce simulated paths for each y_l and enable a sample value for the real option to be calculated.

In (5.11), to generate three correlated normal random variables, ϵ_1 , ϵ_2 , and ϵ_3 , first we generate three mutually independent standard normal random variables,

Z_1 , Z_2 , and Z_3 . We then consider the following linear transformation:

$$\epsilon_l = \sum_{k=1}^l \alpha_{lk} Z_k \quad (l = 1, 2, 3) \quad (5.12)$$

To meet the variances and covariances of $\epsilon_l (l = 1, 2, 3)$, the following two constraints are imposed.

$$\sum_k \alpha_{lk}^2 = 1 \quad (5.13)$$

and

$$\sum_k \alpha_{lk} \alpha_{mk} = \rho_{lm} \quad (5.14)$$

The first sample ϵ_1 is set equal to Z_1 . Then the above equations can be solved, ϵ_2 is calculated from Z_1 and Z_2 , ϵ_3 is calculated from Z_1 , Z_2 and Z_3 .

From (5.12), we have

$$\begin{aligned} \epsilon_1 &= \alpha_{11} Z_1, \\ \epsilon_2 &= \alpha_{21} Z_1 + \alpha_{22} Z_2, \\ \epsilon_3 &= \alpha_{31} Z_1 + \alpha_{32} Z_2 + \alpha_{33} Z_3. \end{aligned} \quad (5.15)$$

In matrix form, it is

$$\epsilon = \alpha Z \quad (5.16)$$

where $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3)^T$, $Z = (Z_1, Z_2, Z_3)^T$ and

$$\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

Then, according to (5.13) and (5.14), we obtain

$$\alpha = \begin{pmatrix} 1 & 0 & 0 \\ \rho_{12} & \sqrt{1 - \rho_{12}^2} & 0 \\ \rho_{13} & \frac{\rho_{23} - \rho_{12}\rho_{13}}{\sqrt{1 - \rho_{12}^2}} & \sqrt{1 - \frac{\rho_{13}^2 - 2\rho_{12}\rho_{13}\rho_{23} + \rho_{23}^2}{1 - \rho_{12}^2}} \end{pmatrix} \quad (5.17)$$

Define new uncertainty variables z_l , $l = 1, 2, 3$ that satisfy the following linear relation.

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \alpha^{-1} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (5.18)$$

It can be shown that z_l , $l = 1, 2, 3$ have been ‘decoupled’. That is, given a realization of z_l , $l = 1, 2, 3$ at time t , their increments dz_l , $l = 1, 2, 3$ are uncorrelated. We generate sample path data (over time) for z_l , $l = 1, 2, 3$ first, which are then converted to y_l , $l = 1, 2, 3$, and then P_t^E , $P_t^{F_1}$, and $P_t^{F_2}$ to evaluate the payoffs. This procedure preserves the correlation among the price data that is critical to the value of the generation assets.

5.2.4 Algorithm development

Now, we can use the above regression functions and boundary conditions to value the fuel switching unit through backward dynamic programming based on the pre-generated price database. The detailed algorithm is as follows:

Data: Initial conditions $(x_0, u_0, v_0, q, 0, P_0^E, P_0^{F_1}, P_0^{F_2})$ are given, and data set size $N > 0$ is also given.

Step 0: Set $t \leftarrow T-1$, $k \leftarrow 1$, $j \leftarrow 1$, $i \equiv x_t \leftarrow t_j^{\text{on}}$, $J_T(x_T, u_T, v_T, q_T^*; P_T^E, P_T^{F_1}, P_T^{F_2})$ get from (5.9).

Step 1: Obtain a set of sample prices $(P_t^{E(k)}, P_t^{F_1(k)}, P_t^{F_2(k)})$.

Step 2: Regress $J_{t+1}^{(k)}$ on $(P_t^{E(k)}, P_t^{F_1(k)}, P_t^{F_2(k)})$ to obtain $h_t(\cdot)$.

Step 3: If $x_t = t_j^{\text{on}}$,

$$J_t^{(i,j,k)} \leftarrow \pi_t(x_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) + \max(h_t^{1,1}, h_t^{1,0} - S_t);$$

else if $0 < x_t < t_j^{\text{on}}$,

$$J_t^{(i,j,k)} \leftarrow \pi_t(x_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) + J_{t+1}^{(i+1,j,k)};$$

else if $x_t = 0$,

$$J_t^{(i,j,k)} \leftarrow -\infty;$$

else if $-t_j^{\text{off}} < x_t < 0$,

$$J_t^{(i,j,k)} \leftarrow J_{t+1}^{(i-1,j,k)};$$

else if $-t_j^{\text{cold}} < x_t \leq -t_j^{\text{off}}$,

$$J_t^{(i,j,k)} \leftarrow \max(h_t^{0,0}, h_t^{0,1} - S_t);$$

else if $x_t = -t_j^{\text{cold}}$,

$$J_t^{(i,j,k)} \leftarrow \max(h_t^{0,0,n}, h_t^{0,0,s}, h_t^{0,1} - S_t).$$

Step 4: If $i \geq -t_j^{\text{cold}}$, $i \leftarrow i - 1$, go to Step 3.

Step 5: If $j < 2$, $j \leftarrow j + 1$, $i \leftarrow t_j^{\text{on}}$, go to Step 3.

Step 6: If $t > 0$, $t \leftarrow t - 1$, $j \leftarrow 1$, $i \leftarrow t_j^{\text{on}}$, go to Step 1.

Step 7: Stop.

Note that the fuel switching will happen when $x_t = -t^{\text{cold}}$, and $h_t^{(0,0,n)} < h_t^{(0,0,s)}$. Now, according to the initial unit status (x_0, u_0, v_0, q_0) , and initial prices $(P_0^E, P_0^{F_1}, P_0^{F_2})$, the expected value of the fuel-switching unit is

$$J_0(x_0, u_0, v_0, q_0; P_0^E, P_0^{F_1}, P_0^{F_2}) \approx \sum_{k=1}^N J_0^{(k)} / N \quad (5.19)$$

Among the above algorithm, the most difficult part is to obtain the regression function $h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$. What remains to show is an appropriate functional form for $h_t(\cdot)$. Our experience shows that the following polynomial form works well for the regression.

$$h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) \approx a_1 + a_2 q_t + a_3 q_t^2 + a_4 q_t^3 + a_5 q^* q_t + a_6 P_t^E + a_7 P_t^F$$

$$+a_8q^*P_t^F + a_9(P_t^E)^2/P_t^F \quad (5.20)$$

where

$$\begin{aligned} q^* = & \max(q_1^{\min}, \min(q_1^{\max}, (P_t^E/P_t^{F_1} - c_{1,1})/2c_{2,1}))v_t \\ & + \max(q_2^{\min}, \min(q_2^{\max}, (P_t^E/P_t^{F_2} - c_{1,2})/2c_{2,2}))(1 - v_t) \end{aligned} \quad (5.21)$$

q^* is determined by the optimal dispatch rule and the present fuel, $P_t^F = P_t^{F_1}v_t + P_t^{F_2}(1 - v_t)$, and a_1 to a_9 are the parameters to be fitted in the regression. Here we assume that the heat rate function of the unit follows the quadratic function given in (15a) and $c_{1,j}$ and $c_{2,j}$ are the coefficients for different fuel j . As a result, the multiple correlation coefficient R^2 is around 0.89, which means the regression on the above polynomial form is good.

Although $h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$ is a highly nonlinear function in (v_t, q_t) and $(P_t^E, P_t^{F_1}, P_t^{F_2})$, and it may not be smooth everywhere because it involves q^* , $h_t(\cdot)$ is still a linear function of the regression parameters a_1, \dots, a_9 , which can be figured out efficiently by solving a system of linear equations (a 9×9 linear system in this case.)

5.3 Numerical results

5.3.1 Baseline: a non-switching case

Without considering the fuel-switching option, the valuation problem only involves two uncertainties and can be solved using a two-factor lattice method (Tseng 2000). This serves a baseline case, using which we can calibrate the performance of the LSMC method.

Table 5.1: Mean reverting process coefficients of electricity, gas and oil

Coefficients	σ	m_u	m_t	P_0
Electricity	0.27	0.072	2.878	20
Gas	0.24762	0.010570	1.01945	2.2
Oil	0.10209	0.003704	0.48054	0.58

Consider a natural gas-fired generating unit with the following input-output characteristics.

$$H_1(q_t) = c_{0,1} + c_{1,1}q_t + c_{2,1}q_t^2 \quad (5.22a)$$

$$C_1(q_t) = H_1(q_t) \times P_t^{F_1} \quad (5.22b)$$

where the cost function $C_1(\cdot)$ is assumed to be a quadratic function of q_t , $H_1(\cdot)$ is the heat rate function, $P_t^{F_1}$ is the fuel price at time period t . Assume $P_0^{F_1}$ is \$2.2/MMBtu, P_0^E is \$20/MW, and $q_1^{\min} = 225\text{MW}$, $q_1^{\max} = 700\text{MW}$, $c_{0,1} = 540$, $c_{1,1} = 9.223$, and $c_{2,1} = 0.00234$. We also assume that $t_1^{\text{on}} = 5$, and $t_1^{\text{off}} = t_1^{\text{cold}} = 10$ to fully capture the influence of the physical constraints. Let the startup cost be \$2300 and shutdown cost be \$1000. Hourly electricity prices and gas prices are generated by two mean reverting processes following (5.10). The corresponding coefficients are in Table 5.1. The correlation coefficient between electricity and gas is 0.078744. And electricity hourly prices also follow a certain on-peak vs. off-peak pattern as Table 5.2.

To compare the algorithm performance of LSMC with the lattice method in (Tseng 2000), seven cases are tested corresponding to seven operating periods using the same gas-fueled generating unit, ranging from 1 day (24 hours) to 7 days (168 hours). The average CPU times are recorded in Table 5.3.

Table 5.2: Mean log Price of electricity: $m_t(t)$

t	1	2	3	4	5	6	7	8
$m_t(t)$	1.8874	2.6557	1.9348	2.3402	3.5027	3.8568	3.7583	4.6602
t	9	10	11	12	13	14	15	16
$m_t(t)$	4.8613	4.71	5.8114	4.7363	5.044	5.7383	5.9166	4.7126
t	17	18	19	20	21	22	23	24
$m_t(t)$	3.7233	1.4573	1.322	2.5106	3.6167	0.6446	1.6033	1.8328

Table 5.3: Average CPU time (seconds)

T (hours)	24	48	72	96	120	144	168
Lattice	0.78	1.64	2.83	4.22	5.86	7.86	10.09
LSMC	0.23	0.47	0.69	0.94	1.16	1.38	1.61

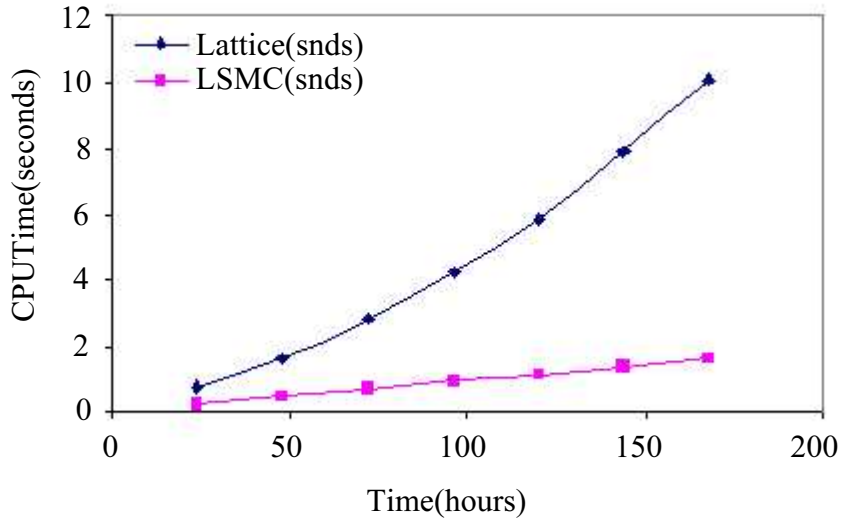


Figure 5.2: CPU time of the lattice model and the LSMC approach.

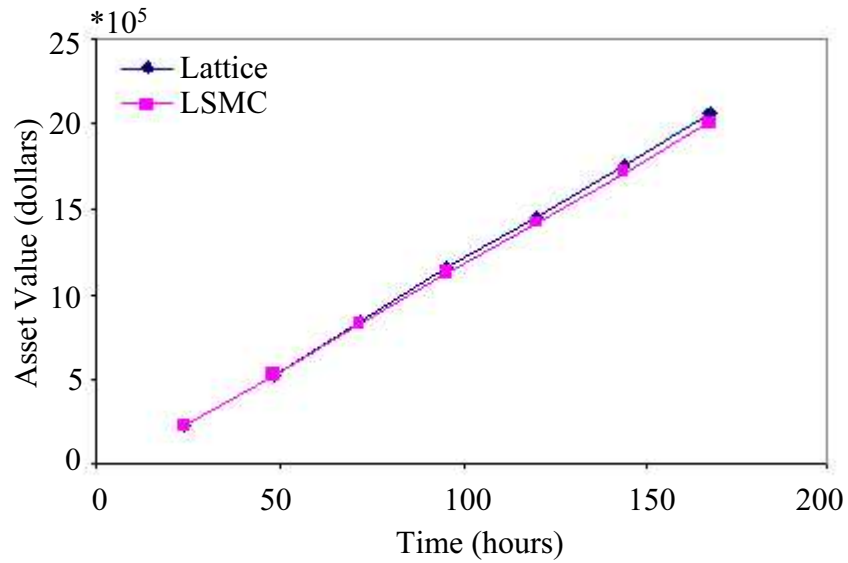


Figure 5.3: The asset value of the lattice model and the LSMC approach.

Fig. 5.2 shows that the LSMC method is much more efficient than the lattice method. However, the LSMC method obtains 1 to 3% lower value than that from the lattice method in Fig. 5.3. This discrepancy increases as the time horizon increases. This is because the regression function is only an approximation, not an exact fit. So there are errors in assessing the asset values. In addition, the errors may add up during the forward and backward iterations. Nevertheless, this test serves a calibration of the LSMC method for valuing a generation asset.

5.3.2 A fuel-switching unit

To value a fuel-switching unit, an additional (fuel) price uncertainty must be considered. As mentioned previously, the lattice approach is prohibitive because of huge memory space and the corresponding large CPU time required.

For simplicity, we assume that when the generating unit is fired by oil, the

Table 5.4: Asset value (dollars)

T (hours)	24	48	72	96	120	144	168
Lattice (w/o FSO)	221343	525641	835970	1145564	1453553	1759987	2065077
LSMC (w/o FSO)	218998	518436	821987	1122878	1420875	1715641	2006559
LSMC (w FSO)	229492	547244	873881	1202886	1535541	1868028	2201012

input-output characteristics remain the same as in (5.22a) and (5.22b), although in reality the coefficients of the quadratic function in (5.22a) should vary. That is,

$$H_1(q_t) = H_2(q_t) \quad (5.23)$$

and

$$C_2(q_t) = H_2(q_t) \times P_t^{F_2} \quad (5.24)$$

Hourly electricity prices, gas prices and oil prices are generated by three mean reverting process following (5.10). The correlation coefficient between electricity and oil is 0.033024 and the correlation between gas and oil is 0.19704.

In Fig. 5.4 and Table 5.4, it can be seen that the fuel-switching capability can increase the asset value. There exist a 7% (when $T=168$) additional value due to the fuel switching option, comparing with the baseline using the lattice method. Since the LSMC approach underestimates the asset value (3%) in the baseline case (compared with the lattice method), it may also underestimate the asset value with the fuel-switching options. If this argument is true, than the fuel switching option may increase the asset value as high as 10%. To verify this conjecture, we build a three-factor lattice (with a small T , $T = 12$). We compare the asset values using both the (3-factor) lattice method and the LSMC method, the result of lattice method is 1% higher than that of the LSMC method. Since

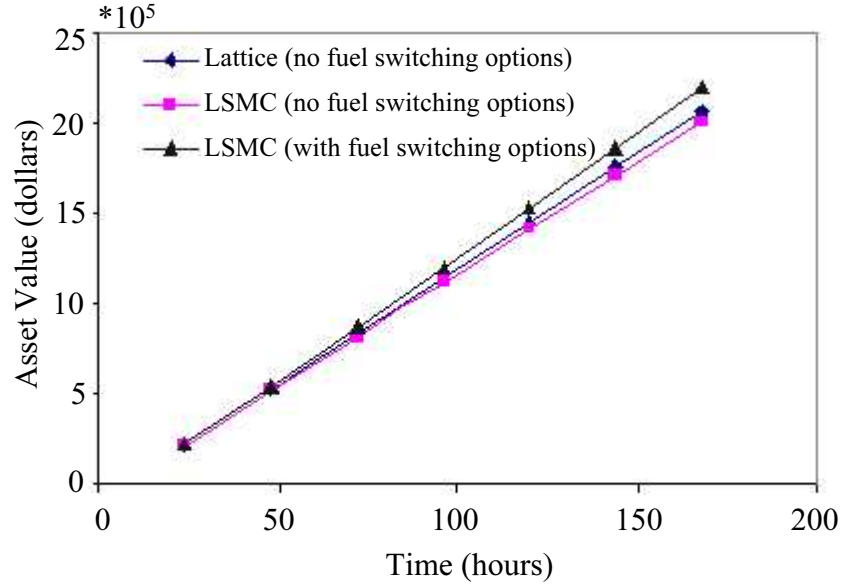


Figure 5.4: The asset value with and without fuel switching options (FSO).

Table 5.5: Asset Value (dollars)

ρ	0.0	0.1	0.2	0.3	0.4	0.5
Asset value	2208745	2202111	2195412	2188820	2182207	2175539

the error increases over T , it is fair to estimate that the error in the 1-week case ($T = 168$), the error (underestimate) is at least 3%.

5.3.3 Sensitivity analysis on $\rho_{gas,oil}$

Intuitively the value of the fuel-switching option increases as the correlation between the fuel prices decreases. That means more chances for one fuel to be in-the-money and the other out-of-the-money.

To study the impacts of fuel price correlation on the option value, we take

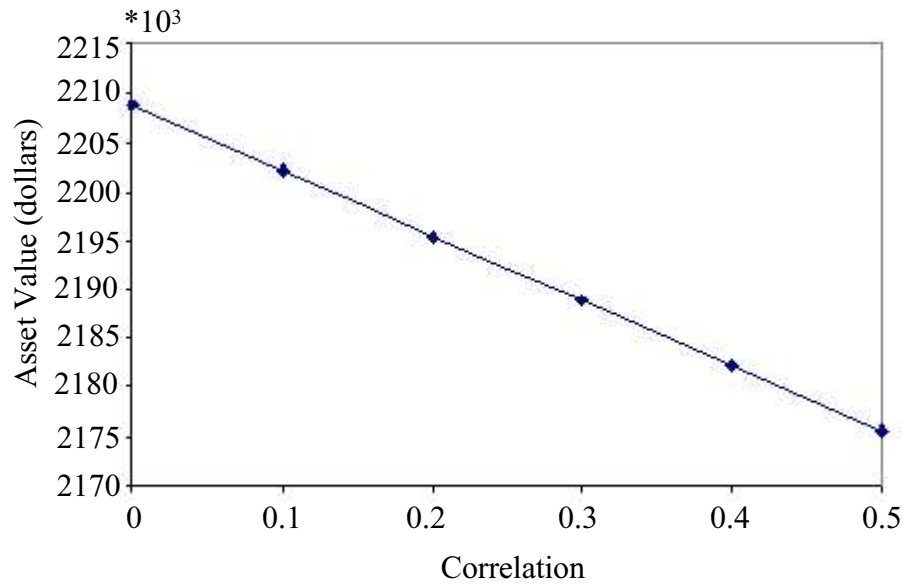


Figure 5.5: The asset value vs. $\rho_{gas,oil}$.

the same fuel-switching unit and repeatedly run the program with different ρ between gas and oil prices. The following results for one-week case ($T=168$) are obtained.

It can be seen that the asset value decreases linearly as the correlation coefficient $\rho_{gas,oil}$ increases. For an example, under an extreme case, the correlation between gas and oil price is -1. That is the increase of gas price will cause the decrease of oil price correspondingly. Under this condition, the power producer will make decisions in switching fuel from nature gas to fuel oil to cut down fuel cost, so as to increase the asset value of the fuel-switching unit. In the present energy market, changes in oil prices in this country are almost irrelevant to natural gas. Our recent analysis based on futures price data, indicates that the price correlation between the oil and gas is only 0.033024, which means the fuel-switching units are still in great need in the current power market. This has

not considered the value as being emission abatement.

5.4 Conclusions

In this chapter, we use the LSMC method to value the fuel-switching option of a generation asset considering the operational constraints. We estimate that the option value is very significant (at 10% over a 1 week period). It provides another evidence that operational flexibility should not be overlooked when valuing an generation asset.

Chapter 6

Valuing Thermal Generation Units with Overfire Options and Maintenance Constraints

6.1 Problem statement

As the power industry moves toward deregulation, more and more small investors are attracted to buy gas turbines (GTs) to participate the competitive energy market. Compared with steam turbines (STs), GTs have the following advantages:

- A GT has much lower installment cost than a ST.
- The construction time of a GT is much shorter than that of a ST.
- A GT may cause less environment pollution problems than any other thermal units.

The above are also the reasons why a small or medium size GT working in peak hour is an ideal choice of small investors, who may not wait for years to be paid off.

In the current restructured market, the existence of stochastic electricity prices allow each individual generator to be decoupled from that original collective power pools and to optimize its own generation schedules based on the available market information and its operating conditions without considering whether the demand side is satisfied by the supply side or not. The main concern of individual owners is profitability. For example, if turning off his generator is profitable from the perspective of individual owner, he will do so without hesitation regardless of how much the power is in need in energy market.

So the new challenge offered by the restructured market is how individual market participants respond to the stochastic prices optimally considering various maintenance constraints, such as crew availability, resource availability, seasonal limitations, desirable schedule; and operating constraints, such as capacity constraints, ramping constraints, overfire constraints and minimum up/down time constraints.

Among these constraints, the capacity constraint remains the same as in the previous chapters, except that q^{\max} may be sensitive to the ambient temperature. In general, q^{\max} decreases when the environment temperature increases.

As for the ramp constraints, they will be ignored in this chapter, because we focus on gas turbines only, which can be turned on or shut down within minutes.

To the contrary of the ramp constraints that may limit power generation (change), overfire process improves the production capacity of the unit, although it may have adverse effect to the unit such that it may require more frequent

maintenance. There are two types of overfire process in thermal power generation industry. The first is similar to fuel switching process. Two fuels can be burned to generate power, and the second fuel is assumed to be considerably more expensive and provides additional capability. In this sense, the fuel cost function will also change correspondingly. The second type is simpler. Only one kind of fuel is burned and additional capacity is provided by burning additional fuel. The fuel cost function will not change because of overfiring. In this paper, for simplicity we only consider the second type.

Although overfire can bring additional profit for the short run, it also creates problems. This is because overfiring a unit can cause the temperature inside the combustor to be significantly higher than its normal value. Under this condition, the unit's hot section components are subjected to overstressing and high-temperature corrosion, which may then create further damages, such as degradation, deformation, and/or even cracks. Therefore, the duration for overfiring a unit must be restricted based on metal characteristics of the gas turbine.

Maintenance is an efficient way to correct/prevent such potential damages due to overfire. During a maintenance process, the following steps are taken:

- Let the unit cool down;
- Clean all working parts;
- Test hot section components;
- Replace ineffective ones with new parts;
- Reassemble the unit and lubricate it.

Sometimes the above steps may take weeks to complete. After the maintenance, the gas turbine can work efficiently and safely again.

Some basic definitions about maintenance are introduced here:

- A *maintenance interval* refers to the time of length required to finish maintenance.
- A *operation period* refers to the time period between any two maintenance intervals that the unit is in operation.
- An *outage* refers to a period of power system non-function due to supply failure or scheduled maintenance.

6.2 A valuation framework considering overfire options and maintenance constraints

In this section, the following standard notation will be used to formulate the asset valuation problem considering the overfire option and maintenance constraints. Additional symbols will be introduced when necessary.

t : index for time (in hours), $t = 0, \dots, T$, where T is the number of hours of the planning horizon.

u_t : zero-one decision variable indicating whether the unit is up or down in time period t .

v_t : zero-one decision variable indicating whether the unit is overfired or not in time period t .

w_t : zero-one decision variable indicating whether the unit is shutdown for maintenance or not in time period t .

x_t : state variable indicating the operation status of the unit in time period t .

\tilde{x}_0 : initial condition of x_t at $t = 0$.

y_t : state variable indicating how many operation hours remained at time t before the next maintenance interval for the unit.

\tilde{y}_0 : initial condition of y_t at $t = 0$.

z_t : state variable indicating how many startup numbers remained at time t before the next maintenance interval for the unit.

\tilde{z}_0 : initial condition of z_t at $t = 0$.

t^{on} : the minimum number of periods the unit must remain on after it has been turned on.

t^{off} : the minimum number of periods the unit must remain off after it has been turned off.

t^{cold} : the number of periods required to cool the unit from shutdown.

t^{over} : maximum number of periods the unit can be overfired continuously.

q_t : variable indicating the amount of the unit is generating in time period t .

q^{min} : minimum rated capacity of the unit.

q^{max} : maximum rated capacity of the unit.

q^{over} : maximum rated capacity of the unit when it is overfired ($q^{\text{over}} > q^{\text{max}}$).

$C_t(q_t, P_t^F)$: fuel cost for operating the unit at output level q_t with fuel price at P_t^F in time period t .

$S_t(u_t, u_{t-1})$: startup/shutdown cost associated with turning on/off the unit in time period t .

P_t^E : spot price at time t for electricity.

P_t^F : spot price at time t for fuel.

T_t^e : environment temperature at time t .

T^d : designed operating temperature of this gas turbine.

ΔT^d : designed operating temperature range of the gas turbine.

$\beta_f^e(T_t^e)$: fuel cost adjusting factor, which is dependent on environment temperature T_t^e at time t .

$\beta_q^e(T_t^e)$: q^{\max} adjusting factor, which is dependent on environment temperature T_t^e at time t .

R_f : fuel cost adjusting range (%) due to environment temperature variation.

R_q : q^{\max} adjusting range (%) due to environment temperature variation.

$n_t(u_t, u_{t-1}, x_t)$: equivalent startup number at time t determined by the unit status x_t in time period t before turning on.

t^{mt} : the minimum number of periods the unit must be shutdown for maintenance.

M^{mt} : direct maintenance cost which are determined by the maintenance contracts.

N^{op} : the maximal operating time periods between any two maintenance intervals, which are determined by the maintenance contracts.

N^{start} : the maximal number of startups between any two maintenance intervals, which are determined by the maintenance contracts.

\mathbf{X}_t : a state vector including x_t , y_t and z_t indicating the status of the unit in time period t .

\mathbf{V}_t : a decision vector including u_t , v_t and w_t indicating the decisions of the unit in time period t .

\mathbf{Q}_t : an uncertainty vector including P_t^E , P_t^F and T_t^e indicating all the uncertainties in time period t .

$f_t(\mathbf{X}_t; \mathbf{Q}_t)$: value function of the unit in time period t under state \mathbf{X}_t and uncertainty \mathbf{Q}_t .

6.2.1 The mathematical model

The objective is to value the thermal generation unit with overfire options and maintenance contracts subject to environment temperature uncertainty. This valuation is a typical multi-stage stochastic problem. Assume that at any time t and state \mathbf{X}_t , uncertainty vector \mathbf{Q}_t is observed. The operator can realize the asset value in the current time period first of all, then maximize the expected value of the unit for the rest time periods by taking suitable actions \mathbf{V}_t . Let $J_t(\mathbf{X}_t, \mathbf{V}_t; \mathbf{Q}_t)$ be the so-called value-to-go function indicating the total value of

the unit for the remaining period at state \mathbf{X}_t at time t , and assume a certain time horizon $[0, T]$. The asset valuation problem can be formulated as the following recursive relation:

Maximize the expected total profit:

$$(P_0) \quad J_t(\mathbf{X}_t, \mathbf{V}_t; \mathbf{Q}_t) = \pi_t(x_t, q_t; \mathbf{Q}_t) + \max_{u_t, v_t, w_t} \{E_t[J_{t+1}(\mathbf{X}_{t+1}, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] - S_t(u_t, u_{t-1}) - M^{\text{mt}}w_t\} \quad (6.1)$$

where

$$\pi_t(x_t, q_t; \mathbf{Q}_t) = \begin{cases} P_t^E q_t - C_t(q_t, P_t^F), & \text{if } x_t > t^{\text{on}}, \\ 0, & \text{otherwise.} \end{cases} \quad (6.2)$$

Assume that q_t can be dispatched *instantaneously* and *optimally*, when uncertainty \mathbf{Q}_t is realized.

$$C_t(q_t, P_t^F) = \beta_f^e(T_t^e)(c_0 + c_1 q_t + c_2 q_t^2) P_t^F \quad (6.3)$$

$$\beta_f^e(T_t^e) = 1 + \left(\frac{T_t^e - T^d}{\Delta T^d} \right) R_f \% \quad (6.4)$$

In (6.3), fuel cost $C(q_t, P_t^F)$ is dependent on the environment temperature T_t^e at time t . Equation (6.4) shows that when the environment temperature deviates from its designed temperature T^d , the adjusting factor β_f^e will have value and apply to the fuel cost.

$$S_t(u_t, u_{t-1}) = \begin{cases} S^{\text{up}}, & \text{if } u_{t-1} = 0 \text{ and } u_t = 1 \\ S^{\text{down}}, & \text{if } u_{t-1} = 1 \text{ and } u_t = 0 \\ 0, & \text{otherwise,} \end{cases} \quad (6.5)$$

where S^{up} and S^{down} represent constant startup cost and constant shutdown cost.

The last term $M^{\text{mt}}w_t$ in the objective function represents a fixed maintenance cost, incurred only when maintenance takes place. Finally, the above optimization is subject to several constraints, each of which will be detailed next.

Maintenance contract

Proper maintenance can keep a GT to work efficiently and reliably. However, it may not be economic for an owner to staff full-time crews to maintain the units. This is true especially for owners of small GTs, who usually prefer to sign maintenance contracts with some maintenance companies, responsible for on-site unit maintenance.

A complete maintenance contact is normally complicated. Since frequent up and down and violent temperature variation are the main causes for early fatigue or outage of a unit, we consider a simplified contract such that maintenance must be performed (therefore, the unit must be shutdown) if at least one of the following conditions is met: (i) the aggregated unit operation hours (y_t) exceeds some prespecified level N^{op} ; (ii) the aggregated unit startup numbers z_t exceeds some prespecified level N^{start} . These two conditions are detailed next.

- The aggregated operation hours is tracked by y_t , whose initial value is 0. A *normal operation hour* refers to an hour during which the generation level is within the rated capacities, i.e., $[q^{\min}, q^{\max}]$. As opposed to the normal operation hour, an *overfire hour* is one in which the unit overfires. Since overfiring a unit results in more fatigue than normal operation, an overfire hour is considered equivalent to $W > 1$ normal operation hours (specified in the maintenance contract). Therefore,

$$y_{t+1} = y_t + (Wv_t + (1 - v_t))u_t. \quad (6.6)$$

- The aggregated startup numbers is tracked by z_t , initially set to 0. Similar to the operation hours, different levels of startups are categorized depending on the temperature of turbines. They are: warm start, normal start,

cold start, and very cold start. From the perspective of maintenance, the range of temperature variation is the less, the better. The startup of a gas turbine is a typical process of increasing temperature. Generally, the longer a unit has been shutdown, the lower the temperature of the turbine is. A warm start causes less temperature variation and fatigue than a normal start; and a cold and very cold starts cause more temperature variation and fatigue than a normal start. The following is an example how the startup number is considered, depending on how long the unit has been down ($x_t < 0$).

$$n_t(x_t) = \begin{cases} 0.5, & \text{if } x_t > -4 \text{ (warm start),} \\ 1, & \text{if } -4 \geq x_t > -20 \text{ (normal start),} \\ 1.5, & \text{if } -20 \geq x_t > -40 \text{ (cold start),} \\ 2, & \text{if } x_t \leq -40 \text{ (very cold start),} \\ 0, & \text{otherwise.} \end{cases} \quad (6.7)$$

In (6.7), a warm start is considered a half normal start if the unit has been down for less than 4 hours, and a cold start (down for 20 to 40 hours) is considered one and a half normal start. Overall, the startup number is aggregated as follows.

$$z_{t+1} = z_t + n_t(x_t)(1 - u_{t-1})u_t, \quad (6.8)$$

As aforementioned, once the aggregated operation hours or aggregated startup numbers reaches some levels, specified in the maintenance contract, shutting down the unit for maintenance ($w_t = 1$) is enforced.

Maintenance condition

$$w_t = \begin{cases} 1, & \text{if } y_t \geq N^{\text{op}} \text{ or } z_t \geq N^{\text{start}}, \\ 0 \text{ or } 1, & \text{otherwise.} \end{cases} \quad (6.9)$$

Maintenance interval constraints

Once a maintenance decision $w_t = 1$ is made at time t , the unit goes into a maintenance interval with a duration t^{mt} time periods. That is, the unit must stay off for at least t^{mt} time periods (approximately two weeks in reality). That is, if $w_t = 1$ and $w_{t-1} = 0$, then

$$x_{t+t^{\text{mt}}} = -t^{\text{cold}}, \quad (6.10)$$

$$y_{t+t^{\text{mt}}} = 0, \quad (6.11)$$

$$z_{t+t^{\text{mt}}} = 0. \quad (6.12)$$

A sample state transition diagram is shown in Fig. 6.1, in which it can be seen that once a maintenance decision is made at t , the state is then transitioned to time $t + t^{\text{mt}}$, and the counters $y_{t+t^{\text{mt}}}$ and $z_{t+t^{\text{mt}}}$ are reset to 0. In (6.10), $x_{t+t^{\text{mt}}} = -t^{\text{cold}}$ implies that this generator can be viewed as staying at the *cold* status and ready to put back on-line again.

Next we incorporate the maintenance constraints into the state transition (see Fig. 6.1). Two phases are discussed: (i) operation phase ($w_t = 0$); (ii) maintenance phase ($w_t = 1$).

State transition: operation phase ($w_t = 0$)

$$x_{t+1} = \begin{cases} \min(t^{\text{on}} + t^{\text{over}}, \max(t^{\text{on}}, x_t) + 1), & \text{if } u_t = 1 \text{ and } v_t = 1, \\ \min(t^{\text{on}}, \max(x_t, 0) + 1), & \text{if } u_t = 1 \text{ and } v_t = 0, \\ \max(-t^{\text{cold}}, \min(x_t, 0) - 1), & \text{if } u_t = 0 \text{ and } v_t = 0. \end{cases} \quad (6.13)$$

and

$$u_t = \begin{cases} 1, & \text{if } 1 \leq x_t < t^{\text{on}} \text{ and } t^{\text{on}} < x_t \leq t^{\text{on}} + t^{\text{over}}, \\ 0, & \text{if } -t^{\text{off}} < x_t \leq -1, \\ 0 \text{ or } 1, & \text{otherwise.} \end{cases} \quad (6.14)$$

$$v_t = \begin{cases} 0, & \text{if } x_t = t^{\text{on}} + t^{\text{over}} \text{ or } x_t < t^{\text{on}}, \\ 0 \text{ or } 1, & \text{otherwise.} \end{cases} \quad (6.15)$$

Equations (6.14) and (6.15) represent the minimum uptime/downtime and overfire constraints, respectively. Equation (6.15) implies that the unit can be overfired only when after the minimal uptime constraint has been satisfied, i.e., after at least t^{on} hours of normal operation. This restriction can prevent significant temperature variation within a short time period due to overfire. In addition, the unit cannot overfire for more than t^{over} hours continuously. When it is turned back to normal operation status from overfire, it must remain normal operation for at least an hour before it can overfire again.

State transition: maintenance phase ($w_t = 1$)

$$x_{t+1} = \max(-t^{\text{cold}} - 1, \min(0, x_t) - 1) \quad (6.16)$$

$$y_{t+1} = 0 \quad (6.17)$$

$$z_{t+1} = 0 \quad (6.18)$$

and

$$u_t = 0 \quad (6.19)$$

$$v_t = 0 \quad (6.20)$$

Equations (6.16) to (6.20) describe the values of the states when the unit is in the maintenance phase (i.e., shut down). Apparently, the unit cannot overfire in a maintenance interval.

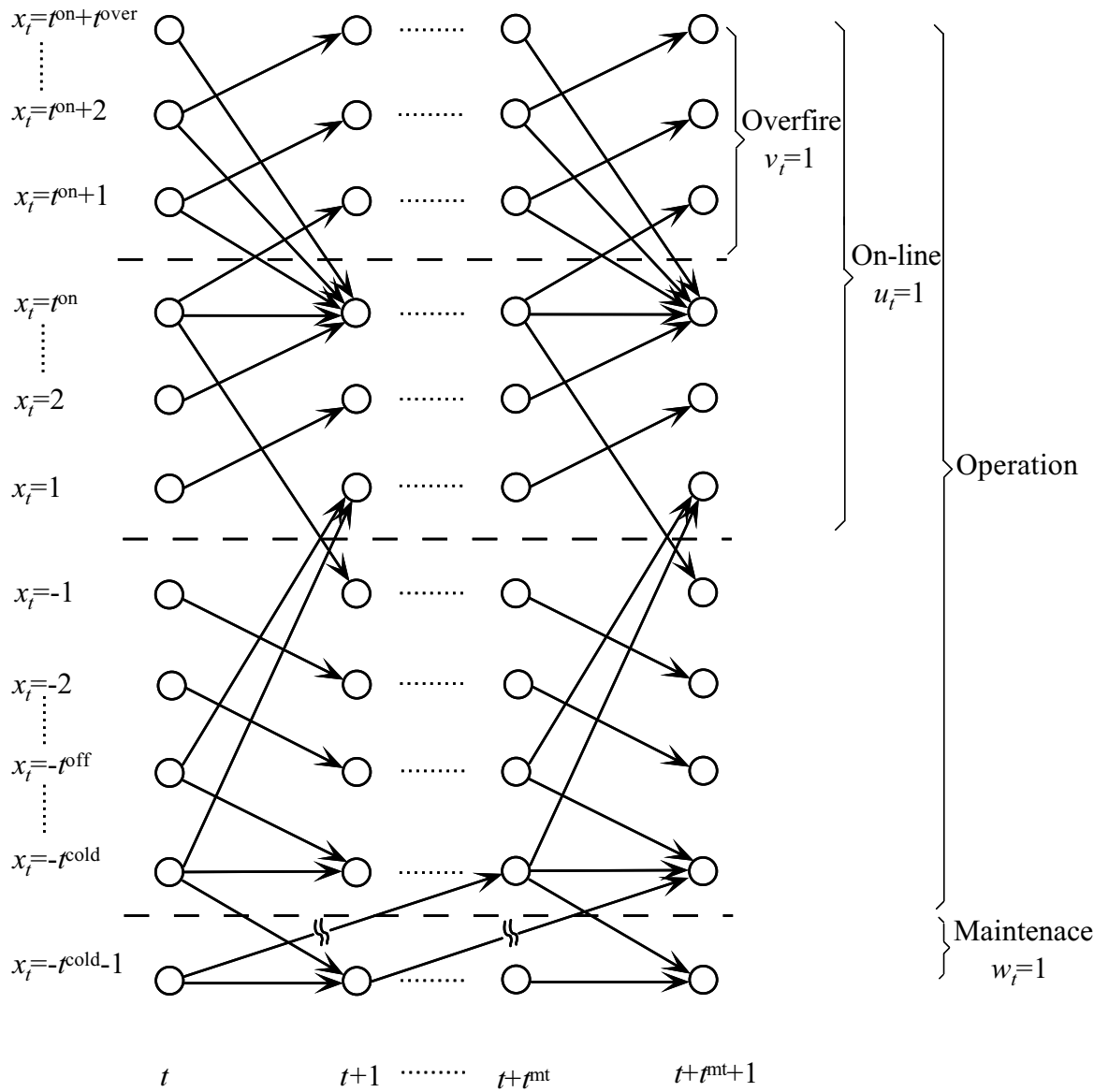


Figure 6.1: State transition diagram including operation and maintenance processes.

Capacity constraint

As aforementioned, the production capacity of a unit may be dependent on environment temperature. The capacity constraint is modeled below.

$$q^{\min} \leq q_t \leq (q^{\max}(1 - v_t) + q^{\text{over}}v_t)\beta_q^e(T_t^e) \quad (6.21)$$

where

$$\beta_q^e(T_t^e) = 1 - \left(\frac{T_t^e - T^d}{\Delta T^d} \right) R_q \% \quad (6.22)$$

Equations (6.21) and (6.22) state the maximum rated capacity can be increased from q^{\max} to q^{over} . Regardless of overfire, both q^{\max} and q^{over} are sensitive to the environment temperature T_t^e . For instance, when T_t^e increases, both q^{\max} and q^{over} will decrease.

6.2.2 Temperature model

In the formulation, we have shown that GT's operational characteristics (including $C(q_t, P_t^F)$, q^{\max} , and q^{over}) are sensitive to the environment temperature T_t^e . It is, therefore, important to study temperature variation over time.

Cao and Wei (2000) suggested that future temperature deviation on day $\tau + 1$ can be forecasted based on temperature deviations over the three previous time periods, τ , $\tau - 1$ and $\tau - 2$. Later Baldick et al. (2003) claim that the temperature deviations on $\tau - 2$ are statistically insignificant to the future temperature deviations on $\tau + 1$. Thus, in their model the temperature deviation from the historical average on $\tau + 1$ is a function of temperature deviations on τ and $\tau - 1$ only. The stochastic fluctuations around historical average can be further described by the following equations:

$$\Delta_{\tau+1}^T = \rho_1^T \Delta_{\tau}^T + \rho_2^T \Delta_{\tau-1}^T + \sigma_{\tau}^T \epsilon_{\tau}^T \quad (6.23)$$

Table 6.1: Value of the parameters of the temperature model

Parameters	Value
ρ_1^T	0.831
ρ_2^T	-0.194
σ_0^T	8.322
$\sigma_{1,1}^T$	5.697
$\sigma_{1,2}^T$	5.774
$\sigma_{1,3}^T$	5.741
$\sigma_{1,4}^T$	5.753
ϕ	-14.2

$$\sigma_\tau^T = \sigma_0^T - \sigma_1^T \left| \sin\left(\frac{\pi(\tau + \phi)}{365}\right) \right| \quad (6.24)$$

where $\Delta_\tau^T = T_t - \bar{T}_\tau$, T_τ is the actual temperature for day τ , \bar{T}_τ is the average temperature for day τ , and ρ_1^T, ρ_2^T represent the autocorrelation coefficients for deviations from average temperature on day τ and $\tau - 1$, respectively. The magnitude of the random fluctuations is seasonal, with a fixed term σ_0^T and a seasonal term of magnitude σ_1^T . Each ϵ_t^T is a standardized normal random variable.

Following the model of Baldick et al. (2003), we collect weather data (from January 2000 through December 2002) at the National Climatic Data Center website (www.ncdc.noaa.gov), and use which to calibrate the model. We further categorize σ_1^T into seasonal values: $\sigma_{1,1}^T$ (spring), $\sigma_{1,2}^T$ (summer), $\sigma_{1,3}^T$ (fall) and $\sigma_{1,4}^T$ (winter). The estimated values of the parameters are summarized in table 6.1.

6.2.3 Stochastic price process

In this chapter, it is assumed that the prices for electricity P_t^E and fuel P_t^F follow some mean reverting processes and are functions of α_1 and α_2 respectively, which are governed by the following stochastic differential equations.

$$d\alpha_l = \mu_l(\alpha_l, t)dt + \sigma_l dB_l, \quad (6.25a)$$

$$\mu_l(\alpha_l, t) = -m_{ul}[\alpha_l(t) - m_{ul}(t)], \quad (6.25b)$$

where $l = 1$ and 2 represent the prices for electricity and fuel, respectively, μ_l is a drift function, σ_l is a constant volatility and B_l is a Wiener process with correlation $\rho_{1,2}$. Assume that there exists one-to-one transformations between α_1 and P_t^E , and between α_2 and P_t^F . Therefore, (P_t^E, P_t^F) can be obtained through the corresponding (α_1, α_2) . Price samples are generated in the same way used in Chapter 5.

6.3 Algorithm development

The problem (P) is a difficult mixed-integer multi-stage program. In order to solve the problem (P), which is subject to all the above operating constraints (6.6) to (6.22) including overfire and maintenance constraints, we need to find a suitable method.

In this chapter, we use the LSMC approach described in the previous chapter to solve the problem (P) based on the graphic network in Fig. 6.1.

6.3.1 Solution procedure

At any time t , one must know how to approximate $E_t[J_{t+1}(\cdot)]$ before making

optimal decisions. The regression functions $h_t(\cdot)$ of current uncertainty \mathbf{Q}_t and production level q_t are regressed as follows:

- Stay overfire

$$h_t^{o,o}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}(\min(t^{\text{on}} + t^{\text{over}}, x_t + 1), y_t - W, z_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26a)$$

- From overfire to normal

$$h_t^{o,n}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}(t^{\text{on}}, y_t - 1, z_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26b)$$

- From normal to overfire

$$h_t^{n,o}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}(t^{\text{on}} + 1, y_t - W, z_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26c)$$

- Stay normal

$$h_t^{n,n}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}(\min(t^{\text{on}}, x_t + 1), y_t - 1, z_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26d)$$

- From up to down

$$h_t^{u,d}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}(-1, y_t, z_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26e)$$

- From down to up

$$h_t^{d,u}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}(1, y_t - 1, z_t - n_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26f)$$

- Stay down

$$h_t^{d,d}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}(\max(-t^{\text{cold}}, x_t - 1), y_t, z_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26g)$$

- From cold to maintenance

$$h_t^{c,m}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}(-t^{\text{cold}} - 1, 0, 0, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26h)$$

- Stay maintenance

$$h_t^{m,m}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}(-t^{\text{cold}} - 1, 0, 0, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26i)$$

- From maintenance to cold

$$h_t^{m,c}(q_t; \mathbf{Q}_t) = E_t[J_{t+1}^{\text{mt}}(-t^{\text{cold}}, N^{\text{op}}, N^{\text{start}}, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})] \quad (6.26j)$$

The aforementioned expected value of $J_{t+1}(\cdot)$ can be approximated through *forward-moving* simulation and *backward-moving* dynamic programming iterations. Since analytical forms of $h_t(\cdot)$ are nonexistent in general, we can only use numerical methods based on Monte Carlo simulation to approximate it. If the above regression functions $h_t(\cdot)$ are available at time t , one could know the expected unit value for the next time period when the uncertainty prices (P_t^E, P_t^F) and environment temperature T_t^e are revealed. Then, one could determine: (i) whether to do maintenance or not; (ii) whether to turn on or off the unit; or (iii) whether to overfire or not. That means for any realization of \mathbf{Q}_t at time t , one would know how to optimally make decisions for the next time period based on the above regression functions $h_t(\cdot)$.

6.3.2 Overfire options

Overfire may be viewed as a real option. However, when and how to exercise the overfire option may not be an easy task. This is because that overfire operation affects maintenance schedule of gas turbine. Therefore, optimally exercising the overfire option must be considered over the entire planning horizon. Assume that the length of time horizon is T and all boundary conditions are given. Whether one should overfire or not can be determined by the following

equations:

$$v_t = \begin{cases} 1, & \text{if } 1 \leq x_t < t^{\text{on}} + t^{\text{over}} \text{ and } J_{t+1}(\max(t^{\text{on}}, x_t) + 1, y_t - W, z_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1}) \\ & > J_{t+1}(\min(t^{\text{on}}, x_t + 1), y_t - 1, z_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1}), \\ 0, & \text{otherwise.} \end{cases} \quad (6.27)$$

6.3.3 Maintenance options

From maintenance constraints (6.9), it can be seen that one may still decide to shutdown the unit for maintenance even when any of the two conditions specified in the contract is not met. Therefore, maintenance may also be viewed as a real option. For example, one may intentionally maintain the unit even when it is not required in order to make the unit available for some forecasted heat wave in the near future. Since the maintenance requires shutting down the unit for some period, it is not obvious whether early exercise the option is optimal. At any time t when $x_t = t^{\text{cold}}$, one may have three options: (i) turn on the unit; (ii) stay off; or (iii) do the maintenance. The optimal decision can be determined based on the expected value of J_{t+1} as follows:

$$w_t = \begin{cases} 1, & \text{if } x_t = t^{\text{cold}}, y_t > 0, z_t > 0, \text{ and } J_{t+1}(-t^{\text{cold}} - 1, 0, 0, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1}) > \\ & \max(J_{t+1}(-t^{\text{cold}}, y_t, z_t, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1}), J_{t+1}(1, y_t - 1, z_t - 1, \mathbf{V}_{t+1}; \mathbf{Q}_{t+1})), \\ 0, & \text{otherwise.} \end{cases} \quad (6.28)$$

6.3.4 Boundary conditions

Boundary conditions refer to the values of J_T at time T . If T represents the life time of a GT, J_T must be zero for sure; otherwise, J_T should be equal to the

rest value of the GT.

Once the boundary values of J_T are known, one can use the LSMC to estimate J_{T-1} for every data sample. The decision maker can choose whether to do maintenance or not, whether to startup or not, whether to overfire or not, and revisit the exercise decision at the next time period. This procedure can be repeated to $T - 2, \dots, 0$. The last iteration, starting with the initial conditions at time 0, provides the optimal planning and asset estimation for the whole time horizon.

6.3.5 Algorithm for the asset valuation

Next we show how to use the LSMC method to value the GT with overfire capacity and maintenance contracts through backward dynamic programming based on the pre-generated price database and temperature data. Two algorithms will be presented. The first one is the generic algorithm, which, however, requires a huge state space. The second one is an improved method.

Algorithm 1: Generic algorithm

Data: Initial conditions $(x_0, y_0, z_0, P_0^E, P_0^R, P_0^F, T_0^e)$ are given, and data set size $N > 0$ is also given.

Step 0: Set $t \leftarrow T - 1$, $i \leftarrow t^{\text{on}} + t^{\text{over}}$, $j \leftarrow N^{\text{op}}$, $k \leftarrow N^{\text{start}}$, $J_T(\cdot)$ get from boundary conditions.

Step 1: Obtain a set of sample prices $(P_t^{E(n)}, P_t^{R(n)}, P_t^{F(n)})$ and environmental temperatures $T_t^{e(n)}$ ($n = 1, \dots, N$).

Step 2: Regress $J_{t+1}^{(n)}$ on $(P_t^{E(n)}, P_t^{R(n)}, P_t^{F(n)})$ to obtain $h_t(\cdot)$.

Step 3: If $x_t = t^{\text{on}} + t^{\text{over}}$,

$$J_t^{(i,j,k;n)} \leftarrow \pi_t(\cdot) + J_{t+1}^{(t^{\text{on}},j-1,k;n)};$$

else if $t^{\text{on}} < x_t < t^{\text{over}}$,

$$J_t^{(i,j,k;n)} \leftarrow \pi_t(\cdot) + \max(h_t^{o,o}, h_t^{o,n});$$

else if $x_t = t^{\text{on}}$,

$$J_t^{(i,j,k;n)} \leftarrow \pi_t(\cdot) + \max(h_t^{n,o}, h_t^{n,n}, h_t^{u,d} - S^{\text{down}});$$

else if $0 < x_t < t^{\text{on}}$,

$$J_t^{(i,j,k;n)} \leftarrow \pi_t(\cdot) + \max(J_{t+1}^{(i+1,j-1,k;n)}, h_t^{n,o});$$

else if $x_t = 0$,

$$J_t^{(i,j,k;n)} \leftarrow -\infty;$$

else if $-t^{\text{off}} < x_t < 0$,

$$J_t^{(i,j,k;n)} \leftarrow J_{t+1}^{(i-1,j,k;n)};$$

else if $-t^{\text{cold}} < x_t \leq -t^{\text{off}}$,

$$J_t^{(i,j,k;n)} \leftarrow \max(h_t^{d,d}, h_t^{d,u} - S^{\text{up}});$$

else if $x_t = -t^{\text{cold}}$,

$$J_t^{(i,j,k;n)} \leftarrow \max(h_t^{c,m} - M^{\text{mt}}, h_t^{d,d}, h_t^{d,u} - S^{\text{up}});$$

else if $x_t = -t^{\text{cold}} - 1$,

$$J_t^{(i,j,k;n)} \leftarrow \max(h_t^{m,m}, h_t^{m,c}).$$

Step 4: If $i > -t^{\text{cold}} - 1$, $i \leftarrow i - 1$, go to Step 3.

Step 5: if $j > 0$,

$$j \leftarrow j - 1, i \leftarrow t^{\text{on}} + t^{\text{over}}, \text{ go to Step 3};$$

else

$$J_t^{(i,j,k;n)} \leftarrow \max(J_t^{(i,j,k;n)}, J_{t+t^{\text{mt}}}^{(i,N^{\text{OP}},k;n)}).$$

Step 6: If $k > 0$,

$k \leftarrow k - 1, j \leftarrow N^{\text{op}}, i \leftarrow t^{\text{on}} + t^{\text{over}},$ go to Step 3;

else

$$J_t^{(i,j,k;n)} \leftarrow \max(J_t^{(i,j,k;n)}, J_{t+t^{\text{mt}}}^{(i,j,N^{\text{start}};n)}).$$

Step 7: If $t > 0, t \leftarrow t - 1, k \leftarrow N^{\text{start}}, j \leftarrow N^{\text{op}}, i \leftarrow t^{\text{on}} + t^{\text{over}},$ go to Step 1.

Step 8: Stop.

While in theory the above algorithm can work, it is not solvable from the perspective of implementation due to the huge state space ($I * J * K * T$) required. For example, when the number of unit status x_t is 10, the number of total operation hour y_t is 1000, the number of total startup z_t is 100, and the number of total hours T (in a year) is 8760, the state space becomes $10 * 1000 * 100 * 8760 = 8.76 * 10^9$. Further considering N sample paths of several uncertainties and the least squares regression in each state, this problem cannot be handled within reasonable time by the current computer hardware. One alternative method is to absorb the state variables y_t and z_t into each regression function so as to reduce the state space significantly. The algorithm is as follows:

Algorithm 2: An improved version

Data: Initial conditions $(x_0, y_0, z_0, P_0^E, P_0^R, P_0^F, T_0^e)$ are given, and data set size $N > 0$ is also given.

Step 0: Set $t \leftarrow T - 1, i \leftarrow t^{\text{on}} + t^{\text{over}}, J_T(\cdot)$ get from boundary conditions.

Step 1: Obtain a set of sample prices $(P_t^{E(n)}, P_t^{R(n)}, P_t^{F(n)})$ and environmental temperatures $T_t^{e(n)}$ ($n = 1, \dots, N$).

Step 2: Regress $J_{t+1}^{(n)}$ on $(P_t^{E(n)}, P_t^{R(n)}, P_t^{F(n)})$ to obtain $h_t(\cdot)$.

Step 3: If $x_t = t^{\text{on}} + t^{\text{over}}$,

$$J_t^{(i;n)} \leftarrow \pi_t(\cdot) + J_{t+1}^{(t^{\text{on}};n)};$$

$$y_t^{(i;n)} \leftarrow y_{t+1}^{(t^{\text{on}};n)} - W;$$

$$z_t^{(i;n)} \leftarrow z_{t+1}^{(t^{\text{on}};n)};$$

else if $t^{\text{on}} < x_t < t^{\text{over}}$,

if $y_{t+1}^{(i+1;n)} > 0$ and $z_{t+1}^{(i+1;n)} > 0$;

$$J_t^{(i;n)} \leftarrow \pi_t(\cdot) + \max(h_t^{o,o}, h_t^{o,n});$$

$$y_t^{(i;n)} \leftarrow \begin{cases} y_{t+1}^{(i+1;n)} - W, & \text{if } h_t^{o,o} > h_t^{o,n}; \\ y_{t+1}^{(t^{\text{on}};n)} - W, & \text{otherwise.} \end{cases}$$

$$z_t^{(i;n)} \leftarrow \begin{cases} z_{t+1}^{(i+1;n)}, & \text{if } h_t^{o,o} > h_t^{o,n}; \\ z_{t+1}^{(t^{\text{on}};n)}, & \text{otherwise.} \end{cases}$$

else

$$J_t^{(i;n)} \leftarrow \pi_t(\cdot) + h_t^{o,n};$$

$$y_t^{(i;n)} \leftarrow y_{t+1}^{(t^{\text{on}};n)} - W;$$

$$z_t^{(i;n)} \leftarrow z_{t+1}^{(t^{\text{on}};n)};$$

else if $x_t = t^{\text{on}}$,

if $y_{t+1}^{(i+1;n)} > 0$ and $z_{t+1}^{(i+1;n)} > 0$;

$$J_t^{(i;n)} \leftarrow \pi_t(\cdot) + \max(h_t^{n,o}, h_t^{n,n}, h_t^{u,d} - S^{\text{down}});$$

$$y_t^{(i;n)} \leftarrow \begin{cases} y_{t+1}^{(i+1;n)} - 1, & \text{if } h_t^{o,o} > \max(h_t^{n,n}, h_t^{u,d} - S^{\text{down}}); \\ y_{t+1}^{(i;n)} - 1, & \text{else if } h_t^{n,n} > h_t^{u,d} - S^{\text{down}}; \\ y_{t+1}^{(-1;n)} - 1, & \text{otherwise.} \end{cases}$$

$$z_t^{(i;n)} \leftarrow \begin{cases} z_{t+1}^{(i+1;n)}, & \text{if } h_t^{o,o} > \max(h_t^{n,n}, h_t^{u,d} - S^{\text{down}}); \\ z_{t+1}^{(i;n)}, & \text{else if } h_t^{n,n} > h_t^{u,d} - S^{\text{down}}; \\ z_{t+1}^{(-1;n)}, & \text{otherwise.} \end{cases}$$

else if $y_{t+1}^{(i;n)} > 0$ and $z_t^{(i;n)} > 0$;

$$\begin{aligned}
J_t^{(i;n)} &\leftarrow \pi_t(\cdot) + \max(h_t^{n,n}, h_t^{u,d} - S^{\text{down}}); \\
y_t^{(i;n)} &\leftarrow \begin{cases} y_{t+1}^{(i;n)} - 1, & \text{if } h_t^{n,n} > h_t^{u,d} - S^{\text{down}}; \\ y_{t+1}^{(-1;n)} - 1, & \text{otherwise.} \end{cases} \\
z_t^{(i;n)} &\leftarrow \begin{cases} z_{t+1}^{(i;n)}, & \text{if } h_t^{n,n} > h_t^{u,d} - S^{\text{down}}; \\ z_{t+1}^{(-1;n)}, & \text{otherwise.} \end{cases}
\end{aligned}$$

else

$$\begin{aligned}
J_t^{(i;n)} &\leftarrow \pi_t(\cdot) + h_t^{u,d} - S^{\text{down}}; \\
y_t^{(i;n)} &\leftarrow y_{t+1}^{(-1;n)} - 1 \\
z_t^{(i;n)} &\leftarrow z_{t+1}^{(-1;n)}
\end{aligned}$$

else if $0 < x_t < t^{\text{on}}$,

if $y_{t+1}^{(t^{\text{on}}+1;n)} > 0$ and $z_{t+1}^{(t^{\text{on}}+1;n)} > 0$;

$$\begin{aligned}
J_t^{(i;n)} &\leftarrow \pi_t(\cdot) + \max(h_t^{(n;n)}, h_t^{n,o}); \\
y_t^{(i;n)} &\leftarrow \begin{cases} y_{t+1}^{(t^{\text{on}}+1;n)} - 1, & \text{if } h_t^{n,o} > h_t^{n,n}; \\ y_{t+1}^{(i+1;n)} - 1, & \text{otherwise.} \end{cases} \\
z_t^{(i;n)} &\leftarrow \begin{cases} z_{t+1}^{(t^{\text{on}}+1;n)}, & \text{if } h_t^{n,o} > h_t^{n,n}; \\ z_{t+1}^{(i+1;n)}, & \text{otherwise.} \end{cases}
\end{aligned}$$

else

$$\begin{aligned}
J_t^{(i;n)} &\leftarrow \pi_t(\cdot) + h_t^{(n;n)}; \\
y_t^{(i;n)} &\leftarrow y_{t+1}^{(i+1;n)} - 1 \\
z_t^{(i;n)} &\leftarrow z_{t+1}^{(i+1;n)}
\end{aligned}$$

else if $x_t = 0$,

$$J_t^{(i;n)} \leftarrow -\infty;$$

else if $-t^{\text{off}} < x_t < 0$,

$$\begin{aligned}
J_t^{(i;n)} &\leftarrow J_{t+1}^{(i-1;n)}; \\
y_t^{(i;n)} &\leftarrow y_{t+1}^{(i-1;n)} \\
z_t^{(i;n)} &\leftarrow z_{t+1}^{(i-1;n)}
\end{aligned}$$

else if $-t^{\text{cold}} < x_t \leq -t^{\text{off}}$,

if $y_{t+1}^{(1;n)} > 0$ and $z_{t+1}^{(1;n)} > 0$;

$$\begin{aligned} J_t^{(i;n)} &\leftarrow \max(h_t^{d,d}, h_t^{d,u} - S^{\text{up}}); \\ y_t^{(i;n)} &\leftarrow \begin{cases} y_{t+1}^{(1;n)}, & \text{if } h_t^{d,u} - S^{\text{up}} > h_t^{d,d}; \\ y_{t+1}^{(i-1;n)}, & \text{otherwise.} \end{cases} \\ z_t^{(i;n)} &\leftarrow \begin{cases} z_{t+1}^{(1;n)} - n_t(i), & \text{if } h_t^{d,u} - S^{\text{up}} > h_t^{d,d}; \\ z_{t+1}^{(i-1;n)}, & \text{otherwise.} \end{cases} \end{aligned}$$

else

$$\begin{aligned} J_t^{(i;n)} &\leftarrow h_t^{d,d}; \\ y_t^{(i;n)} &\leftarrow y_{t+1}^{(i-1;n)} \\ z_t^{(i;n)} &\leftarrow z_{t+1}^{(i-1;n)} \end{aligned}$$

else if $x_t = -t^{\text{cold}}$,

if $y_{t+1}^{(1;n)} > 0$ and $z_{t+1}^{(1;n)} > 0$;

$$\begin{aligned} J_t^{(i;n)} &\leftarrow \max(h_t^{c,m} - M^{\text{mt}}, h_t^{d,d}, h_t^{d,u} - S^{\text{up}}); \\ y_t^{(i;n)} &\leftarrow \begin{cases} y_{t+1}^{(1;n)}, & \text{if } h_t^{d,u} - S^{\text{up}} > \max(h_t^{c,m} - M^{\text{mt}}, h_t^{d,d}); \\ y_{t+1}^{(i;n)}, & \text{else if } h_t^{d,d} > h_t^{c,m} - M^{\text{mt}}; \\ y_{t+1}^{(i-1;n)}, & \text{otherwise.} \end{cases} \\ z_t^{(i;n)} &\leftarrow \begin{cases} z_{t+1}^{(1;n)} - n_t(i), & \text{if } h_t^{d,u} - S^{\text{up}} > \max(h_t^{c,m} - M^{\text{mt}}, h_t^{d,d}); \\ z_{t+1}^{(i;n)}, & \text{else if } h_t^{d,d} > h_t^{c,m} - M^{\text{mt}}; \\ z_{t+1}^{(i-1;n)}, & \text{otherwise.} \end{cases} \end{aligned}$$

else if $y_{t+1}^{(i;n)} > 0$ and $z_{t+1}^{(i;n)} > 0$;

$$\begin{aligned} J_t^{(i;n)} &\leftarrow \max(h_t^{c,m} - M^{\text{mt}}, h_t^{d,d}); \\ y_t^{(i;n)} &\leftarrow \begin{cases} y_{t+1}^{(i;n)}, & \text{if } h_t^{d,d} > h_t^{c,m} - M^{\text{mt}}; \\ y_{t+1}^{(i-1;n)}, & \text{otherwise.} \end{cases} \\ z_t^{(i;n)} &\leftarrow \begin{cases} z_{t+1}^{(i;n)}, & \text{if } h_t^{d,d} > h_t^{c,m} - M^{\text{mt}}; \\ z_{t+1}^{(i-1;n)}, & \text{otherwise.} \end{cases} \end{aligned}$$

else

$$J_t^{(i;n)} \leftarrow h_t^{c,m} - M^{\text{mt}};$$

$$y_t^{(i;n)} \leftarrow y_{t+1}^{(i-1;n)}$$

$$z_t^{(i;n)} \leftarrow z_{t+1}^{(i-1;n)}$$

else if $x_t = -t^{\text{cold}} - 1$,

$$J_t^{(i;n)} \leftarrow \max(h_t^{m,m}, h_t^{m,c});$$

$$y_t^{(i;n)} \leftarrow \begin{cases} N^{\text{op}}, & \text{if } h_t^{m,c} > h_t^{m,m}; \\ y_{t+1}^{(i;n)}, & \text{otherwise.} \end{cases}$$

$$z_t^{(i;n)} \leftarrow \begin{cases} N^{\text{start}}, & \text{if } h_t^{m,c} > h_t^{m,m}; \\ z_{t+1}^{(i;n)}, & \text{otherwise.} \end{cases}$$

Step 4: If $i > -t^{\text{cold}} - 1$, $i \leftarrow i - 1$, go to Step 3.

Step 5: If $t > 0$, $t \leftarrow t - 1$, $i \leftarrow t^{\text{on}} + t^{\text{over}}$, go to Step 1.

Step 6: Stop.

Based on the initial unit status (x_0, y_0, z_0) , initial prices (P_0^E, P_0^R, P_0^F) and the initial temperature T_0^e , the asset value of the GT at $t = 0$ is

$$J_0(x_0, y_0, z_0, \mathbf{V}_0; \mathbf{Q}_0) \approx \sum_{n=1}^N J_0^{(x_0, y_0, z_0; n)} / N \quad (6.29)$$

Since the analytical form of $h_t(\cdot)$ is unavailable in general, we can only try to find an appropriate functional form to approximate it. Based on our numerical experience, the following polynomial form including state variable y_t and z_t works well for the regression.

$$\begin{aligned} h_t(q_t, y_t, z_t; \mathbf{Q}_t) \approx & a_1 + a_2 q_t + a_3 q_t^2 + a_4 q^* q_t + a_5 P_t^E + a_6 P_t^R + a_7 P_t^F + a_8 T_t^e \\ & + a_9 q^* (P_t^E)^2 / P_t^F + a_{10} y_t + a_{11} z_t + a_{12} y_t q^* \end{aligned} \quad (6.30)$$

where

$$q^* = \begin{cases} \max(q^{\min}, \min(c_q^e(T_t^e)q^{\text{over}}, \frac{P_t^E/P_t^F - c_1}{2c_2})), & \text{if } x_t > t^{\text{on}}, \\ \max(q^{\min}, \min(c_q^e(T_t^e)q^{\text{max}}, \frac{P_t^E/P_t^F - c_1}{2c_2})), & \text{if } 0 < x_t \leq t^{\text{on}}, \\ 0, & \text{otherwise.} \end{cases} \quad (6.31)$$

q^* is determined by the present status x_t , and the current environmental temperature T_t^e ; and $a_1 \sim a_{12}$ are the parameters to be fitted in the regression.

6.4 Numerical results

Consider a small-size gas-fired turbine with the input-output characteristics following (6.3) and (6.4). Assume that P_0^F is \$2.2/MMBtu, P_0^E is \$20/MW, and $q^{\min} = 75\text{MW}$, $q^{\text{max}} = 200\text{MW}$, $q^{\text{over}} = 230\text{MW}$, $c_0 = 200$, $c_1 = 8.149$, and $c_2 = 0.00452$. We also assume that $t^{\text{on}} = 2$, $t^{\text{off}} = t^{\text{cold}} = 1$, and $t^{\text{over}} = 1$ to fully capture the influence of the physical constraints. The corresponding state transition diagram is given in Fig. 6.2. Let the startup cost be \$1000 and shutdown cost be \$500. Hourly electricity prices and gas prices are generated by two mean reverting process following (6.25a) and (6.25b) The corresponding parameters of the price models are given in Table 6.1 and Table 6.2. Assume that the correlation coefficient between electricity and gas prices is 0.078744.

For the maintenance contracts, we assume that the maximum number of operation hours $N^{\text{op}} = 1600$ hours and the maximum startup numbers is $N^{\text{start}} = 120$ between any two maintenance intervals. Assume $W = 4$, i.e., an overfire hour is equivalent to 4 normal operation hours. Assume the duration of each maintenance interval T^{mt} is two weeks, i.e., 336 hours. As to the environment temperature model, the corresponding parameters are showed in Table 6.1, calibrated

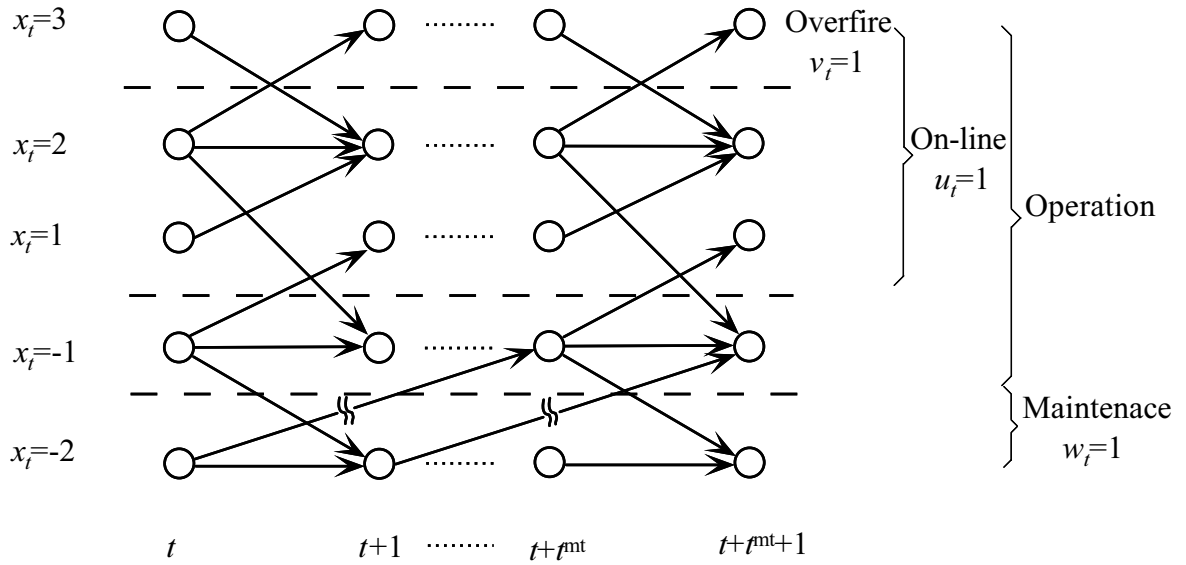


Figure 6.2: The state transition diagram for this case.

Table 6.1: Mean reverting process coefficients of electricity and gas

Coefficients	σ	m_u	m_t	P_0
Electricity	0.27	0.072	2.878	20
Gas	0.2476	0.01057	1.0195	2.2

from real data. Assume the designed operating environment temperature T^d to be 66°F and the allowable variation range of environment temperature ΔT^d to be 60°F. Both fuel cost adjusting range R_f and q^{\max} adjusting range R_q are 2%.

6.4.1 Valuing a GT with overfire capacity and maintenance contract

A set of price and temperatures scenarios are generated to estimate the ca-

Table 6.2: Mean log Price of electricity: $m_t(t)$

t	1	2	3	4	5	6	7	8
$m_t(t)$	1.887	2.656	1.935	2.340	3.503	3.857	3.758	4.660
t	9	10	11	12	13	14	15	16
$m_t(t)$	4.861	4.710	5.811	4.736	5.044	5.738	5.917	4.713
t	17	18	19	20	21	22	23	24
$m_t(t)$	3.723	1.457	1.322	2.511	3.617	0.645	1.603	1.833

capacity value of the same GT considering overfire capacity and maintenance constraints using Algorithm 2. Assume that the time horizon is 8760 hours (1 year) and after which the remaining value of the GT is zero. (This does not mean that the GT is worthless in a year, but a simplified ‘mid-term’ asset valuation and planning.)

Asset value vs. time T

First we consider the capacity value of the GT vs. the length of the time horizon T , ranging from one month to one year. The result is depicted in Fig. 6.3. It can be seen that the capacity value of the GT with overfire capacity and maintenance contract is around 150\$/KW for one year. In general, the capacity value increases with the length of time horizon T monotonically and approximately linearly. The relation is bumpy when T is between 3 and 7 months. This is because that maintenance intervals start to exist within such durations. Since each maintenance interval takes two weeks, it impacts the overall asset value and creates the bumpiness of the seemingly linear relation when T is not big. However, as T exceeds 8 months, which is considerably much longer than the

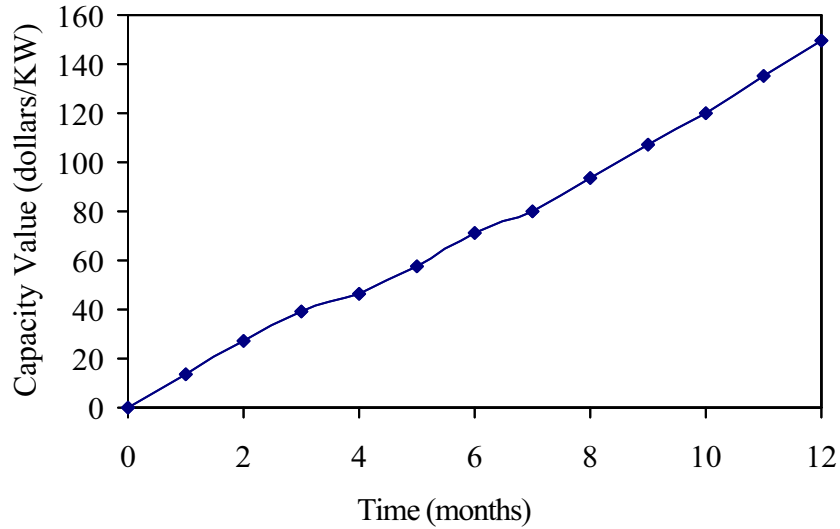


Figure 6.3: Capacity value of the GT over time.

duration of the maintenance interval, the asset value returns to an approximately linear function of T .

Asset value affected by environment temperature

In the proposed model, a GT's operational characteristics (including $C(q_t, P_t^F)$, q^{\max} , and q^{over}) are sensitive to the environment temperature T_t^e . To test how the asset value is sensitive to the environment temperature, we design a baseline test case, in which the environmental temperature $T_t^e = T_t^d$ for all t such that $\beta_f^e = \beta_f^q = 1$. That is, in the baseline case, these operational characteristics are not temperature dependent. The baseline case is then compared with two cases with uncertain temperatures following the model in Section 6.2.2. For simplicity, we choose T to be 8 weeks to exclude the effect of maintenance. One test case is in summer and one in winter. Generally, $T_t^e > T_t^d$ in the summer, and both

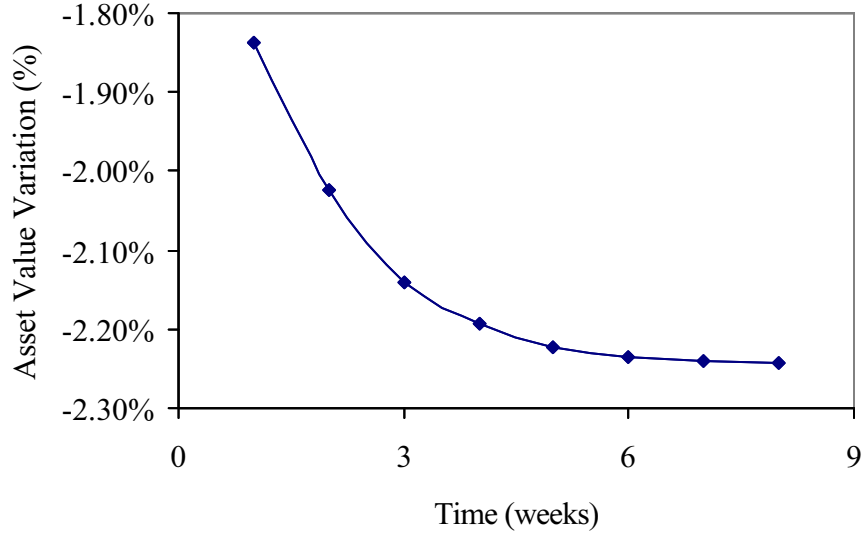


Figure 6.4: Asset value affected by environment temperature in summer.

q^{\max} , and q^{over} decrease. Therefore, the asset value also decreases. The situation is reversed in the winter case. The result of the summer case is depicted in Fig. 6.4. It can be seen that as $T = 8$, temperature uncertainty accounts for approximately 2.2% decrease of the asset value (compared with the baseline.) The result of the winter case is given in Fig. 6.5, where the asset value is higher than that of the baseline.

6.4.2 Overfire option value

To extract the value of the overfire real option, we introduce an additional constraint that limits the overall number of overfire hours. First, let o_t be a new state variable that tracks the aggregated overfire hours, which is reset to 0 after a maintenance is performed. An upper bound N^{over} is then imposed to o_t . The

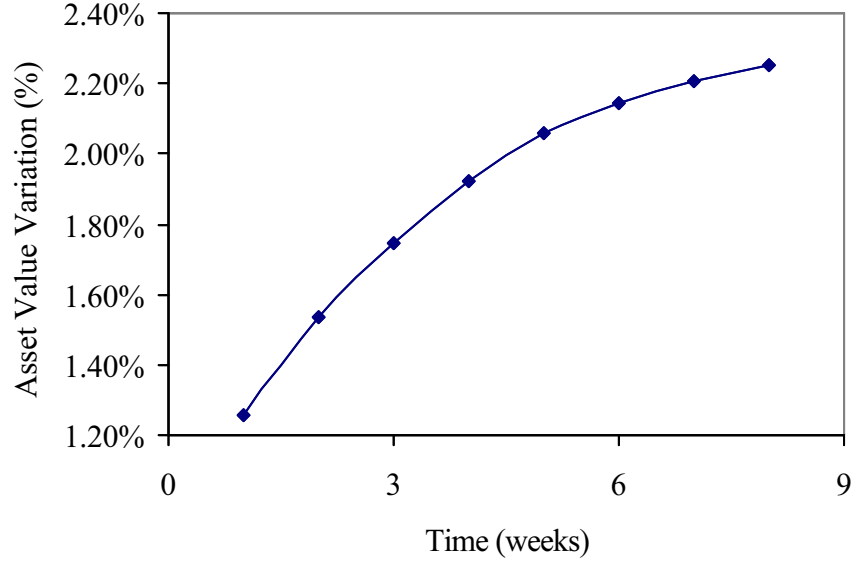


Figure 6.5: Asset value affected by environment temperature in winter.

new and modified constraints as follows.

$$v_t = \begin{cases} 0 \text{ or } 1, & \text{if } 0 < x_t < t^{\text{on}} + t^{\text{over}} \text{ and } o_t \leq N^{\text{over}}, \\ 0, & \text{otherwise.} \end{cases} \quad (6.32)$$

$$o_{t+1} = o_t + v_t \quad (6.33)$$

$$o_t = 0 \text{ if } w_t = 0 \text{ and } w_{t-1} = 1 \quad (6.34)$$

We then run the algorithm for determining the asset value ($T = 1$ year) repeatedly with the value of N^{over} increased gradually. The capacity value of the GT vs. N^{over} is depicted in Fig. 6.6. Apparently, the asset value increases as N^{over} increases, since overfiring the GT is a real option that has value. When the value of N^{over} reaches 100 hours, the asset value is at a maximum and does not increase even N^{over} is further increased. Therefore, $N^{\text{over}} = 100$ hours can be viewed as the optimal number of overfire per year. Overfiring the GT less

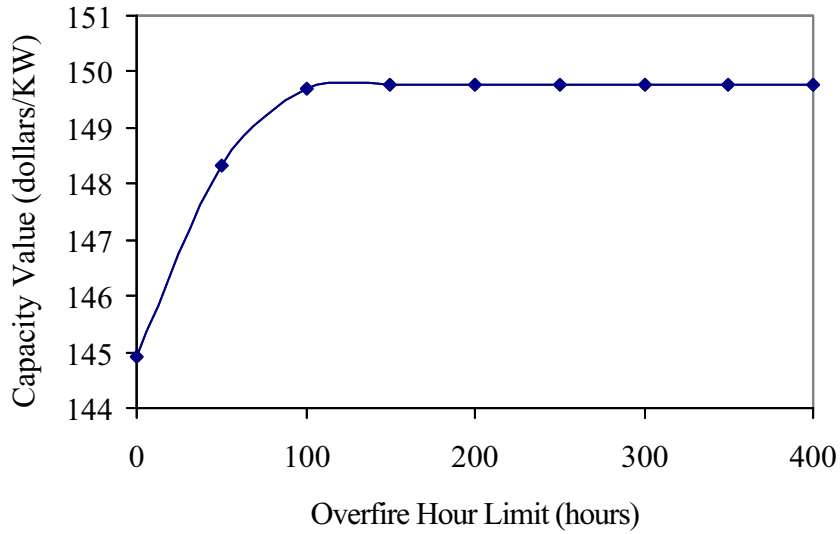


Figure 6.6: Capacity value affected by overfire limit.

than 100 hours is equivalent to giving away this option value. On the other hand, overfiring the unit more than 100 hours is not economical, because the maintenance costs outweigh the benefit.

6.4.3 Maintenance option value

Note that (6.9) states that maintenance is mandatory if either one of the two maintenance conditions is met. This equation, however, allows to perform the maintenance even when any of the two conditions is not met. That is, the maintenance is an (American style) option. To measure the value of such an option, we manage to take away the option modify by modifying (6.9) as follows.

$$w_t = \begin{cases} 1, & \text{if } x_t = -t^{\text{cold}} \text{ and } y_t \geq N^{\text{op}} \text{ or } z_t \geq N^{\text{start}}, \\ 0, & \text{otherwise.} \end{cases} \quad (6.35)$$

In (6.35), a slight modification appears in the second line, which implies that

maintenance is performed only when it is required. We then run the algorithm for determining the asset value (with $T = 1$ year) and compare it with the original asset value with the maintenance option. The capacity value of the GT is 146.7 \$/KW without the maintenance option, and was 149.8 \$/KW with the option. That is, approximately 2 percent of capacity value is due to the maintenance. This shows that maintenance can be part of a profitable strategic plan to increase the asset value.

6.5 Conclusions

In this chapter, a general valuation framework for thermal units considering overfire option, maintenance constraints and environment temperature is developed. The numerical results show that the overfire option can increase the capacity value (about 3.3% per year) and the maintenance option can also increase the capacity value (about 2% per year). We also show that the environment temperature may affect the asset value, which tends to be decreased in summer and increased in winter. The proposed model and the numerical results provide some new insights in the valuation and operation of the GT.

Chapter 7

Conclusions and Further Research

Directions

This dissertation advocates that all relative operational options, constraints and interdependent uncertainties should be considered when a thermal generation unit is valued in the competitive power market. Failing to do may significantly overestimate a power plant, which may result in stranded capital for a long time period.

In Chapter 4, the optimal self scheduling of a thermal unit in an electricity spot market was studied. When the spot prices are known with certainty, a polynomial-time algorithm based on network graph is proposed for solving the problem. Using the developed algorithm to evaluate the unit responses in various price scenarios generated by the LSMC method, we obtain the optimal commitment and dispatch rule under price uncertainty. The test results indicate that the ramp constraints impact a thermal unit on its capability of responding to price uncertainty by reducing its fuel economy, heat-electricity transformation efficiency, and available generation capacity. The proposed method provides a way to quantify these effects. Although the ramp-constrained self-scheduling

problem can be solved using the proposed two-stage approximation method, to solve the multi-stage recursive equation directly with ramp constraints remains very challenging. The major difficulty lies in the ramp constraints. Further research direction may include new multi-stage recursive formulation and new techniques for reducing state space.

In Chapter 5, a LSMC approach was proposed to value a power plant with fuel-switching options. The numerical results show that the LSMC is much more efficient than lattice model especially when time horizon is long and more than two uncertainties are involved. Our test shows that the fuel-switching options may increase the power plant value from 4 to 7% or even higher, depending on the market condition. This shows that fuel-switching is an important operational real option in determining the asset value. This, however, has not included the fact that fuel-switching is also a useful emission abatement means. Although the initial tests show that the LSMC approach is a useful tool for handling multiple uncertainties, it remains unclear how a suitable regression functional form is determined. This problem, however, is not unique to our case, but general to all regression-based methods.

In Chapter 6, a general valuation framework for thermal units considering overfire option, maintenance constraints and environment temperature has been developed. The numerical results show that the overfire option can increase the capacity value (about 3.3% per year). We also showed that the environment temperature may affect the asset value, which tends to be decreased in summer and increased in winter. The proposed model and the numerical results provide some new insights in the valuation and operation of the GT. Further research includes development of better temperature and price models.

In conclusion, this dissertation contributes a new methodology for fair generation asset valuation that accounts for multiple and interdependent uncertainties and complex physical constraints. The proposed research will certainly help unit operators achieving optimal operation and investors making appropriate investment decisions. In the long run, customers also benefit from the improved societal efficiency.

REFERENCES

- Ajax, R. B. M, K. Rocha, and P. A. David, 2000, "Effects of new Brazilian electrical power regulatory framework on generation investment", (www.puc-rio.br/marco.ind/pdf/generation.pdf).
- Allen, E. H., and M. D. Ilic, 1999, *Price-Based Commitment Decisions in the Electricity Market*, Springer-Verlag, London.
- Amaram, M., and N. Kulatilaka, 1999, *Real Options: Managing Strategic Investments in an Uncertainty World*, Harvard Business School Press, Boston.
- Arroyo, J. M. and A. J. Conejo, 2000, "Optimal response of a thermal unit to an electricity spot market," *IEEE Transactions on Power Systems*, 15, pp. 1098 -1104.
- Baldick, R., S. Kolos, and S. Tompaidis, 2003, "Valuation and optimal for interruptible electricity contracts," working paper, University of Texas.
- Baldick, R., 1995, "The generalized unit commitment problem," *IEEE Transactions on Power Systems*, 10, pp. 465-475.
- Bard, J. F., 1988, "Short-term scheduling of thermal-electric generators using Lagrangian relaxation," *Operations Research*, 36, pp. 756-766.
- Barlow, M. T., 2002, "A diffusion model for electricity prices," *Mathematical Finance*, 12, pp. 287-298.

- Bellman, R. E. and S. E. Dreyfus, 1962, *Applied Dynamic Programming*, Princeton University Press, Princeton, NJ.
- Birge, J. R. and J. M. Mulvey, 1996, "Stochastic programming," in *Mathematical Programming for Industrial Engineers*, Dekker, M. eds., New York, NY., pp. 543-574
- Birge, J. R., 1985, "Decomposition and partitioning methods for multi-stage stochastic linear programs," *Operations Research*, 33, pp. 989-1007.
- Boyle, P., 1977, "Options: A Monte Carlo approach," *Journal of Financial Economics*, 4, pp. 323-338.
- Breipohl, A.M., and F.N. Lee, 2000, "Generation adequacy or asset valuation," *the 33rd Annual Frontiers of Power Conference*, Stillwater, OK, USA, 30-31.
- Brennan, M. J. and E. S. Schwartz, 1978, "Finite difference method and jump processes arising in the pricing of contingent claims," *Journal of Financial and Quantitative Analysis*, 13, pp. 461-474.
- Cao, M., and J. Wei, 2000, "Pricing the weather: an intuitive and practical approach," *Risk*, pp. 67-70.
- Caroe, C. C., A. Ruszczyński, and R. Schultz, 1997, "Unit commitment under uncertainty via two-stage stochastic programming," *Proceeding of NOAS 97*, University of Copenhagen, (<http://www.math.ku.dk/caroe/cccresearch.html>).
- Carpentier, A. G., G. C. Cohen, and A. R. Culioli, 1996, "Stochastic optimization of unit commitment: a new decomposition framework," *IEEE Trans.*

on Power Systems, 11(2), pp. 1067-1073.

Carriere, J., 1996, "Valuation of early-exercise price of options using simulation and nonparametric regression," *Insurance: Mathematics and Economics*, 19, pp. 19-30.

Chandler, W. G., P. L. Dandeno, A. F. Gilman and L. K. Kirchmayer, 1953, "Short-range operation of a combined thermal and hydroelectric power system," *AIEE, Trans.* Paper no. 53-326.

Chaton, C., J. A. Doucet, 1999, "Uncertainty and investment in electricity generation: the case of hydro-Quebec," (ideas.repec.org/p/lvl/laeccr/9914.html).

Cohen, A. I., and G. Ostrowski, 1996, "Scheduling units with multiple operation modes in unit commitment," *IEEE Trans. on Power Systems*, 11, pp. 497-503.

Cohen, A. I., and M. Yoshimura, 1983, "A branch and bound algorithm for unit commitment," *IEEE Trans. on Power Apparatus and Systems*, 102(2), pp. 444-451.

Cox, J. S., S. Ross, and M. Rubinstein, 1979, "Option pricing: A simplified approach," *Journal of Financial Economics*, 7, pp. 229-264.

Deng, S.J., B. Johnson, and A. Sogomonian, 1998, "Exotic electricity options and the valuation of electricity generation and transmissions," Presented at the *Chicago Risk Management Conference*, Chicago, IL.

Deng, S.J., B. Johnson, and A. Sogomonian, 1999, "Spark spread options and the valuation of electricity generation assets," *the 32nd Annual Hawaii*

International Conference on System Sciences, Maui, HI, USA, 5-8.

Dillon, T. S. and V. R. Sherkat, 1978, "Integer programming approach to the problem of optimal unit commitment with probabilistic reserve determination," *IEEE Trans. Power Apparatus and systems*, vol. PAS-97, no.6, pp. 2154-2164.

Dixit, A. K. and R. S. Pindyck, 1994, *Investment Under Uncertainty*, Prentice Hall, Englewood Cliffs, NJ.

Edleson, M. E., and F. L. Reinhardt, 1995, "Investment in pollution compliance options: the case of Georgia power, Chapter 14 in the book," in the book *real options in capital investment*, edited by Lenos Trigeorgis.

Fine, C. H., and R. M. Freund, 1990, "Optimal investment in product- flexible manufacturing capacity," *Management Science*, 36(4), pp. 449-466.

Ford, A., 1999, "Cycles in competitive electricity markets: a simulation study of the western United States," *Energy Policy*, 29, pp. 637-658.

Ferreira, L. A., T. Andersson, C. F. Imparato, C. K. Pang, A. Svoboda and A. F. Vojdani, 1989, "Short-term resource scheduling in multi-area hydrothermal power systems," *Electrical Power & Energy Systems*, vol.11, no.3.

Freidenfelds, J., 1981, *Capacity expansion: analysis of simple models with applications*, North Holland, New York.

Frayar, J. and N. Z. Uludere, 2001, "What is it worth? Application of real options theory to the valuation of generation assets", *The Electricity Journal*, 14(8), pp. 40-51.

- Fu, M. C., and J. Q. Hu, 1995, "Sensitivity analysis for monte carlo simulation of option pricing," *Probability in the Engineering and Information Sciences*, 9, 417-446.
- Galiana, F. D. and M. Ilic, 1998, "A mathematical framework for the analysis and management of power transactions under open access," *IEEE Transactions on Power Systems*, 13, pp. 681-687.
- Garcia, D., 2000, "A Monte Carlo method for pricing American options," working paper, University of California, Berkeley.
- Gardner, D. and J. Rogers, 1999, "Planning electric power systems under demand uncertainty with different technology lead times," *Management Science*, 45(10), pp. 1289-1306.
- Gardner, D. and Y. Zhuang, 2000, "Valuation of power generation assets: a real options approach," *ALGO Research Quarterly*, 3(3), pp. 9-20.
- Gill, P. E., W. Murray, and M. H. Wright, 1981, *Practical Optimization*, London: Academic Press.
- Gnansounou, E., A. Sxhmutz and J. Dong, 2003, "Investment decisions with flexibility in the liberalized electricity markets: an approach using Real Options theory," *SAEE Annual Conference*, ETH Zurich.
- Grant, D., G. Vora and D. Weeks, 1997, "Path-dependent options: extending the Monte Carlo simulation approach," *Management Science*, 43(11), pp. 1589-1602.

- Griffes, P., M. Hsu and E. Kahn, 1999, "Real options, ancillary services and environmental risks," in Jamson, R. (ed), *The New Power Markets, Corporate Strategies for Risk and Reward*, Risk books.
- Guan, X., P. B. Luh, H. Yan and J. A. Amalfi, 1992, "An optimization-based method for unit commitment," *Electrical Power & Energy Systems*, 14, pp. 9-17.
- Herath, H. S. and C. S. Park, 2002, "Multi-stage capital investment opportunities as compound real options," *The Engineering Economist*, 47(1), pp. 1-27.
- Hlouskova, J., S. Kossmeier, M. Obersteiner and A. Schnabl, 2002, "Real option models and electricity portfolio management," *OSCOGEN* discussion paper no. 9.
- Hobbs, B., M. Rothkopf, R. O'Neill and H.-P. Chao, 2001, *The Next Generation of Unit Commitment Models*, Boston, MA: Kluwer.
- Hobbs, W. J., G. Hermon, S. Warner and G. B. Sheble, 1987, "Dynamic programming approach to unit commitment," *IEEE Trans. Power Systems*, 2, pp. 339-350.
- Hsu, M., 1998, "Spark spread options are hot!," *The Electricity Journal*, 11(2), pp. 28-39.
- Huang, S. J. and C. L. Huang, 1997, "Application of genetic-based neural networks to thermal unit commitment," *IEEE Trans. on Power Systems*, 12(2), pp. 654-659.

- Hull, J. and A. White, 1994, "Numerical procedures for implementing term structure models II: two-factor models," *Journal of Derivatives*, 2(2), pp. 37-48.
- Hull, J. C., 1993, *Options, Futures, and Other Derivative*, 2nd edition, Prentice Hall, Englewood Cliffs, NJ.
- Hull, J. C., 1999, *Options, Futures, and Other Derivative*, 4th edition, Prentice Hall, Englewood Cliffs, NJ.
- Kazarlis, S. A., A. G. Bakirtzis, V. Petridis, 1996, "A genetic algorithm solution to the unit commitment problem," *IEEE Trans. on Power Systems*, 11(1), pp. 83-92.
- Kojima, M., S. Mizuno and A. Yoshise, 1987, "A polynomial time algorithm for a class of linear complementarity problems. Research report," Department of Information Sciences, Tokyo Institute of Technology, Tokyo, Japan.
- Kulatilaka, N., 1993, "The value of flexibility: the case of a dual-fuel industrial steam boiler," *Financial Management*, 22(3), pp. 271-279.
- Kulatilaka, N., 1995, "Operating flexibilities in capital budgeting: Substitutability and complementarity in real options," in *Real Options in Capital Investment: Models, Strategies, and Application*, Trigeorgis, L. eds., Praeger, New York, NY.
- Kumar, R. L., 1995, "An options view of investments in expansion-flexible manufacturing systems," *International Journal of Production Economics*, 38(2-3), pp. 281-291.

- Lai, S. Y., and R. Baldick, 1999, "Unit commitment with ramp multipliers," *IEEE Trans. Power Systems*, 14, pp. 58-64.
- Lee, F. N., L. Lemonidis and K. C. Liu, 1994, "Price-based ramp-rate model for dynamic dispatch and unit commitment," *IEEE Transactions on Power Systems*, 9(3), pp. 1233-1242.
- Lee, F. N., 1988, "Short-term unit commitment: a new method," *IEEE Trans. Power Systems*, 3(2), pp. 421-428.
- Longstaff, F. A. and E. S. Schwartz, 2001, "Valuing American options by simulation: A simple lease-squares approach," *The Review of Financial Studies*, 14(1), pp. 113-147.
- Louveaux, F. V., 1980, "A solution method for multistage stochastic programs with recourse with application to an energy investment problem," *Operations Research*, 28, pp. 892-902.
- Louveaux, F. V. and Y. Smeers 1988, "Optimal investments for electricity generation: a stochastic model and a test-problem," *Numerical Techniques for Stochastic Programming*, Y. Ermovliev and R J-B. Wets eds., Springer, Berlin.
- Luenberger, D. G., 1998, *Investment Science*, Oxford, New York, NY.
- Majd,S. and R.S. Pindyck, 1987, "Time to build, option value, and investment decisions," *Journal of Financial Economics*, 18(1), pp. 7-27.
- McClanahan, R.H., 2002, "Electric deregulation", *IEEE Industry Applications Magazine*, 8(2), p. 11-18.

- Min, K.J. and C. Wang, 2000, "Generation planning for inter-related generation units: a real options approach", *IEEE Power Engineering Society Summer Meeting*, 4, p. 2261-2265.
- Monteiro, R. D. C. and I. Adler, 1989, "Interior path following primal-dual algorithms-Part II: Convex quadratic programming," *Mathematical Programming*, 44, pp. 43-66.
- Murty, K. G., 1992, *Network Programming*, New Jersey: Prentice Hall.
- Myers, S. C., 1996, "Fischer Black's contribution to corporate finance," *Financial Management*, 25(4), pp.95-100.
- Papadimitriou, C. H. and K. Steiglitz, 1982, *Combinatorial optimization: algorithms and complexity*, Prentice-Hall, Englewood Cliff, New Jersey.
- Park, C. S. and H. S. B. Herath, 1999, "Exploiting uncertainty-investment opportunities as real options: A new way of thinking in engineering economics," *The Engineering Economist*, 45(1), pp. 1-36.
- Peterson, W. L., and S. Brammer, 1995, "A capacity based lagrangian relaxation unit commitment with ramp rate constraints". *IEEE Transactions on Power Systems*, 10(2), pp. 1077-1084.
- Pereira, M. V. F. and L. M. V. G. Pinto, 1985, "Stochastic optimization of a multi-reservoir hydroelectric system—A decomposition approach," *Water Resources Research*, 21, pp. 779-792.
- Pereira, M. V. F. and L. M. V. G. Pinto, 1999, "Planning risks," *IEEE PICA Tutorial*.

- Rendleman, R. J. and B. J. Barter, 1979, "Two state option pricing," *Journal of Finance*, 34, pp. 1092-1110.
- Schwartz, E. S., 1977, "The valuation of warrants: implementing a new approach". *Journal of Finance Economics*, 4, pp. 79-93.
- Shahidehpour, M., and M. Marwali, 2000, *Maintenance schedule in restructured power systems*, Kluwar Academic Publishers Norvell, Massachusetts.
- Sharpe, W. F., 1978, *Investment*, Prentice Hall, Englewood Cliffs, NJ.
- Sheble, G. B. and G. N. Fahd, 1994, "Unit commitment literature synopsis," *IEEE Transactions on Power Systems*, 9, pp. 128-135.
- Svoboda, A. J., C. L. Tseng, C. A. Li, R. B. Johnson, 1997, "Short term resource scheduling with ramp constraints," *IEEE Transactions on Power Systems*, 12, pp. 77-83.
- Takriti, S., B. Krasenbrink, and L. Wu, 1996, "A stochastic model for the unit commitment problem," *IEEE Trans. on Power Systems*, 11(3), pp. 1497-1508.
- Takriti, S., B. Krasenbrink, and L. Wu, 2000, "Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem," *Operations Research*, 48, pp. 268-280.
- Thompson, M., M. Davision, and H. Rasmussen, 2004, "Real options valuation and optimal operations of electrical power plants in competitive markets," *Operations Research*, forthcoming.

- Trigeorgis, L. and S. P. Mason, 1987, "Valuing managerial flexibility," *Midland Corporate Financial Journal*, 5(1), pp. 14-21.
- Trigeorgis, L., 1991, "A log-transformed binomial numerical analysis method for valuing complex multi-option investment," *Journal of Financial and Quantitative Analysis*, 26(3), pp. 309-326.
- Trigeorgis, L., 1993, "The nature of option interactions and the valuation of investments with multiple real options," *Journal of Financial and Quantitative Analysis*, 28(1), pp. 1-20.
- Trigeorgis, L., 1996, *Real Options: Managerial Flexibility and Strategy in Resource Allocation*, the MIT Press, Cambridge, MA.
- Tseng, C. L., 1996, *On power system generation unit commitment problems. Ph.D. Dissertation*, Department of Industry Engineering and Operations Research, University of California at Berkeley.
- Tseng, C. L., G. Barz, 1999, "Short-term generation asset valuation," Presented at *the 32nd Hawaii International Conference on System Sciences*, Maui, Hawaii.
- Tseng, C. L., G. Barz, 2002, "Short-term generation asset valuation: a real options approach," *Operations Research*, 50, pp. 297-310.
- Tseng, C. L., K. Y. Lin, 2004, "A framework using two-factor price lattices for generation asset valuation" (accepted by *Operations Research*).
- Tseng, C. L., 2000, "Exercising real unit operational options under price uncertainty," *IEEE Power Engineering Society Winter Meeting, Conference*

Proceedings, pp. 436-40 vol.1, Piscataway, NJ, USA.

Tseng, C. L., 2001, "A stochastic model for the price-based unit commitment problem and its application to short-term generation asset valuation," *The Next Generation of Electric Power Unit Commitment Models*. Kluwer Academic Publisher, Boston, B. Hobbs, M. Rothkopf, R. O'Neill and H. Chao, eds., 117-138.

Tsitsiklis, J., and B. Van Roy, 1999, "Optimal stopping of Markov processes: Hilbert space theory, approximation algorithms, and an application to pricing high-dimensional financial derivatives," *IEEE Transactions on Automatic Control*, 44, pp. 1840-1851.

Valenzuela, J., M. Mazumadar and A. Kapoor, 2000, "Influence of temperature and load forecast uncertainty on estimates of power generation production costs," *IEEE Trans. on Power Systems*, 15, pp. 668-674.

Van Roy, B. and J. N. Tsitsiklis, 2001, "Regression methods for pricing complex American-style options," *IEEE Transaction on Neural Networks*, 12(4), pp. 694-703.

Wang, C., and S. M. Shahidehpour, 1993, "Effects of ramp-rate limits on unit commitment and economic dispatch". *IEEE Transactions on Power Systems*, 8(3), pp. 1539-1545.

Wang, C., and S. M. Shahidehpour, 1994, "Ramp-rate limits in unit commitment and economic dispatch incorporating rotor fatigue effect". *IEEE Transactions on Power Systems*, 9(3), pp. 1539-1545.

- Wagner, H., 1975, *Principles of operations research, 2nd edition*, Englewood Cliffs, NJ:Prentice-Hull.
- Wood, A. J., and B. F. Wollenberg, 1984, *Power generation, operation and control*, John Wiley, New York.
- Wood, A. J., and B. F. Wollenberg, 1996, *Power generation, operation and control*, 2nd edition, John Wiley, New York.
- Wu, R. W., 2002, “Applications of Monte Carlo simulation in derivative securities pricing,” Ph.D. Dissertation, Business School, University of Maryland at College Park.
- Zhai, D., W. Snyder, J. Waight, J. Farah, A. Gonzalez and P. Vallejo, 2001, “Fuel constrained unit commitment with fuel mixing and allocation,” *IEEE Trans. on Power Systems*, 11, pp. 11-16.
- Zhuang, F. and F. D. Galiana, 1988, “Towards a more rigorous and practical unit commitment by Lagrangian relaxation,” *IEEE Trans. on Power Systems*, vol. PWRS-3, no. 2, pp. 763-770.