ABSTRACT

Title of Dissertation: MARRIAGE MARKETS,
DIFFERENTIAL FECUNDITY AND SEARCH

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It is commonly observed that over time and across societies, women tend to marry older men. The traditional explanation for this phenomenon is that wages increase with age and hence older men are more attractive in the marriage market. The model developed in Chapter 2 of this dissertation shows that a marriage market equilibrium where women marry earlier in life than men can be achieved without making any assumptions about the wage process or gender roles. The only driving force in this model is the asymmetry in fecundity horizons between men and women. When the model is calibrated with Census Data, the average age at first marriage and the pattern of the sex ratio of single men to single women over different age groups mimics the patterns observed in developed countries during the last decade.

Chapter 3 extends the model in order to analyze assortative mating. In this case people belong to one of two groups and prefer to marry someone within the group. In this chapter it is shown that, given constant preferences, the limited horizon for searching for a mate affects the likelihood of intermarriage through ages, and the dynamic is different for men and women.

Chapter 4 is an empirical study and uses 1970 and 1980 US Census data to study how the local sex ratios of single men to single women affect several aspects of the marriage market. Unlike earlier literature, this work also investigates other margins over which individuals can substitute in the marriage market - specifically the choice of spouse's characteristics. These new results suggest that a shortage of single men leads women (and also men) to marry earlier. This suggests a more elastic response for women to a tight marriage market than the one for men. This is consistent with a marriage model where the search horizon for women is shorter than the one for men, as the one developed in the previous chapters. The results also suggest that an adverse change in the sex ratio can lead both men and women to marry outside of their own racial or educational group.

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DIFFERENTIAL FECUNDITY AND SEARCH

by

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DEDICATION

To my mother and in memory of my father, who always wanted to be a full-time professor

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Chapter 1

Introduction

It is commonly observed that, over time and across societies, women tend to marry older men. In the economics literature, the conventional explanation is that wages rise with age and hence men, who are the breadwinners in specialized families, are more attractive to women at older ages. Knowing this, young men will wait for the better marriage opportunities that come along with higher salaries at older ages. Historically this rationale played a role. For most of human history gender specialization in marriage was strong and many models rightfully reflected this strong historical specialization (e.g. Bergstrom and Bagnoli, 1993).[1] However, with the tremendous rise in the labor force participation of women over the last four decades and the increasing fraction of families in which women earn more than men, the compelling nature of the conventional economic argument begins to break down.

If the conventional economic argument was the only explanation, the rise in women's economic independence should have relaxed the necessity of younger women marrying older men. In fact, between 1960 and 1990, female labor force

[1] Ted Bergstrom (1996) recognizes this fact when referring to his own theoretical model of marriage (Bergstrom and Bagnoli (1993)).

participation rose from approximately 35% to approximately 60% (leveling off in the 1990's). During this time the wage gap (adjusting for skills) between women and men declined. Moreover, by the 1990s more than one-third of dual income families had women earning more than men.[2] However, the age difference at marriage between men and women barely moved. According to the US Census, the difference in the median age between men and women at first marriage was 2.5 years in 1960. Thirty years later, in spite of tremendous social changes the difference in the median age at first marriage between men and women was still 2.3 years.

The important changes in gender roles observed in the last decades occurred along side a delay of marriage for *both* sexes rather than a decrease in the age difference at first marriage between men and women. This is inconsistent with the purest version of the conventional economic model and is one reason to revisit our marriage models. A second reason is that, even when women specialized in home production, the economic model may not have been as important as has been suggested. For example, the common occurrence of a young woman marrying her high school sweetheart who is two years older, seems hard to explain using a purely "gains-from-trade type" argument. With wages continuing to rise steeply with age among the young it is reasonable that even more gains to trade could occur if women married even older men. Finally, from a theoretical point of view, these models implicitly assume the myopia of women, imperfect capital markets or imperfect information about men's ability. It seems desirable to understand whether the age difference can be derived in a model without these auxiliary assumptions.

[2] See Winkler (1998)

A handful of recent studies investigate a second potentially important factor leading younger women to marry older men - biology. It is a biological fact that women are fertile for less of their lives than are men. The consequence of this asymmetry in the fecundity horizons is that there will always be more fertile men than fertile women at any given point in time. Thinking of this imbalance as relative scarcity implies more bargaining power for the sex in short supply (in this case women) and competition among agents for the sex in abundant supply (in this case males). When this is true, one way that men may compete for women is through the resources they can bring to the marriage. When this insight is added to the upward sloping profile of wages with age, men may rationally wait to marry to compete better with younger men. (Siow (1998)). While this argument incorporates biology in a serious way, it is the age-wage profile that remains the underlying driving force of the market equilibrium. Biology, in this case, is simply the reason that justifies women as the scarce resource and justifies women choosing among men according to what they bring to the marriage and receiving the rent from their scarcity. One question that has not been addressed is whether the scarcity of fertile women arises in a world where individuals have rational expectations and are fully aware of the asymmetry between men and women that Siow describes.

The point of departure for this dissertation is exactly this question: How does the equilibrium in the marriage market look when both men and women behave optimally and recognize that men have more time than women to search for the right mate and still have children? Therefore, the main objective through the following chapters will be to study how this differences in horizons affect marital behavior in general and how this behavior can change over ages.

Even assuming that asymmetric fecundity horizons play an important role in marriage market behavior, it remains unclear which aspects of the marriage market equilibrium are due to biology itself and which depend on a wage process where wages rise with age. The objective of the model developed in the following chapters of this dissertation is limited to addressing the question: What can be explained exclusively by biology? Another reason for isolating the effects of biology independent of the wage processes is that while the latter varies considerably across societies, biology varies very little. Therefore, any prediction gleaned from a model that does not rely on upward sloping wage profiles is likely more applicable in a variety of social contexts. Furthermore, this framework does not need any additional assumptions about the perception of future earnings.

The model developed in Chapter 2 is a two-sided general equilibrium search model where (as in most of the labor-related search literature) men and women are *ex ante* homogenous and utility is non-transferable.[3] Only after a random meeting do the man and the woman receive signals about the match quality (match-specific heterogeneity). The purpose of the paper is to analyze how the agents' behavior and opportunity sets are affected by the asymmetry in the fecundity and (in the generalized version of this model) life horizons. Therefore, the age heterogeneity will be the crucial element here.[4] In this model utility depends both on the quality of the match and on the joy derived from having

[3] Using non-transferable utility is helpful in order to provide a framework that is able to explain stylized facts about marriage independently of the potential gains of specialization, as it is common in the literature.

[4] There is an increasing theoretical literature about ex ante heterogeneous agents (for example Burdett and Coles (1997) and Smith (2002).

children within marriage[5]. Unlike most of the previous marriage market literature, neither employment decisions nor capital accumulation is analyzed here. The total number of single men and women, and therefore the sex ratio, is determined endogenously in the model. To the best of my knowledge, this is the first attempt in the economic literature to analyze age heterogeneity in a search-matching framework.

One of the major findings in Chapter 2 is that biology alone can provide an alternative explanation of the age difference between men and women at first marriage. Here, biology has two countervailing effects. First, as in Siow (1998), when women are young both older men and younger men compete for them making them scarce and hence raising the minimum acceptable match quality for marriage. By itself this would tend to make women marry at ages older than men. But offsetting this, forward-looking women, who know about their shorter fecundity horizon, reduce their optimal reservation value. The net result for reasonable parameter values is that, at most ages, women set an optimal reservation value that is relatively lower than the one a man of the same age sets. Therefore, women marry relatively younger than men because the biological clock induces them to accept a lower match quality even in the face of their relative scarcity at a given point in time. That result differs substantially from previous literature where relatively scarce fertile women are able to choose from a larger set of fertile men who "compete" for them. In addition, using the generalized version of the model, solved numerically, this work is able to quantify the age difference in marriage, the age composition of single males and females and the

[5] The underlying assumption here is that people derive more utility from having biological children than from either having them out of wedlock or through adoption.

pattern of the sex ratio of singles along the life cycle. One of the features of these results is that a relatively large difference in the fecundity horizon (say, 20 years) leads to an age difference at first marriage that is much smaller (1.5 years). These results are then compared with micro census data for the US and other selected countries, from 1960 to 2000.

Chapter 3 extends the work of Chapter 2 to analyze assortative mating. In this case people belong to one of two groups and prefer to marry to someone within the group. In this chapter is shown that, given constant preferences, the limited horizon for searching for a mate affects the likelihood of intermarriage through ages, and the dynamic is different for men and women. Assuming a proportional discount in utility for marrying someone of the other group, this barrier decreases with time as reservation values of men and women do. Therefore the model implies that the fraction of intermarriage increases with age.[6] Moreover, the model suggests women tend to increase the rate of intergroup marriage relatively to men right before fecundity starts to decline. Assuming, as before, random matching, one crucial element here will be the relative size of the two groups, issue extensively discussed in the Sociology literature specially after the work of Blau (1977).[7] Here the model suggests that, due to increasing search frictions, the smaller the size of the group, the higher the rate of intermarriage, the lower the marriage rate and the older the age of marriage. All the implications of the model are then compared with US data.

Chapter 4 is an empirical study and uses 1970 and 1980 US Census data to

[6] Kalmijn (1993) notices that late marriers are more likely to intermarry than people who marry young.

[7] As Blau (1977) states "...in the relation between any two groups, the rate of intergroup associations of the smaller group exceeds that of the larger."

study how the local sex ratios of single men to single women affect several aspects of the marriage market. It begins by addressing how a shortage of "marriageable men" changes the rate and timing of marriage. Unlike earlier literature, this work also investigates other margins over which individuals can substitute in the marriage market - specifically the choice of spouse's characteristics. In this paper, the sex ratio is defined as the ratio of single men over single women in the same geographical area, belonging to the same ethnic/educational group. The empirical approach is similar to that in Angrist (2002) analyzing the effects of sex imbalances over the behavior of the children of immigrants. Unlike previous empirical results (e.g. Brien (1997)), the results of the empirical study of Chapter 4 suggest that *a shortage of single men leads women (and also men) to marry earlier*. This result suggests that the behavior of men and women respect to the timing of marriage is not driven only by the direct effect of the availability of partners but also by the reaction of the rest of the market to a given sex imbalance In addition, the result may imply a more elastic response for women to a tight marriage market than the one for men.

Chapter 5 is a conclusion.

Literature Review

Since the publication of Gary Becker's first paper on marriage (Becker 1973) there has been growing interest in investigating decisions about marriage as if they occurred in a market. Becker argues that marriage has many aspects that are similar to trade of any other good in a market. Marriage is a voluntary contract between two people, or two families, who believe that they will be better off married than remaining single. Further, like buyers and sellers, many men and women compete to find mates. These aspects make marriage amenable to investigation as voluntary trade in a competitive market.

The compelling logic of this argument has spawned a large volume of research in both economics and sociology of both a theoretical and empirical nature. Much of the early literature followed up on Becker's insight and thought about one person, usually the woman, "purchasing" a mate in the marriage market. In this literature, women made decisions on marriage based on "meeting" men from the available pool and choosing whether to marry them or remain single. Men were passive agents and the bilateral nature of the marriage market was ignored. Heer and Grossbard-Shechtman (1981) outlined the idea of a negative relationship between the marriage squeeze for women and the proportion of women able to choose husbands with (1) high income and (2) a high inclination toward having children.

The spatial dimension of marriage market, and the effects of "local sex ratios" was analyzed by Lichter (1991), where local marriage markets are defined over 382 data of Labor Market Areas. As most part of the empirical literature of the subject, his analysis is based in prevalence marriage rates. Another study by South and Lloyd (1992), analyze incidence rates in the 50 states, using annual statistics for marriages and divorces.

While these models were perhaps unrealistic in construct, they have had a major impact on the literature and on public policy. One well cited example is the work of Wilson and Neckerman (1986) who argue that the rise in out-of-wedlock childbearing among African-Americans is primarily a result of African-American women increasingly choosing not to marry from a shrinking pool of African-American men as they are deemed to be of insufficient quality to be "marriageable" (i.e. close to the women's age and education level, not in prison and employed). In the last decade, several empirical studies contrast this hypothesis with data, like Wood (1995) and its implications, for example in teen childbearing crime, as in Barber (2001).

Investigating this theory, Brien (1997) finds that while the pool of marriageable men does affect the age at first marriage, this mechanism explains very little of the difference in the timing of marriage (and fertility) between African-Americans and Whites. Other empirical examples where women are seen as choosing from a pool of available men include Fitzgerald (1991), Lichter et al. (1992), Wood (1995) and Schmidt (2002).

While empirical work has largely ignored the bilateral nature of marriage, theoretical work has had a rich tradition of investigating marriage as a bilateral process (either in a stable matching context (i.e. Gale and Shapley (1962), Roth and Sotomayor (1990)), or a dynamic search context (i.e. Mortensen (1988)).[8]

Most of the literature views utility of marriage arising solely from the quality of the match between the husband and the wife.[9] However, clearly one of the main reasons that marriage occurs is for the production of children. While more recently, bearing children outside of marriage has become more common in developed countries, there are still reasons to believe that it is less costly, or of higher utility for parents to raise a child within marriage. For example, Willis and Weiss (1993) argue that children are a public good within marriage and as such both parents can derive utility from the child at the same time while sharing

[8] For detailed surveys about the search and matching literature, see Burdett and Coles (1999) and also Pissarides (2000)

[9] An exception is Siow (1998).

the cost of raising the child. This advantage is lost when a child's time needs to be divided between a custodial and non-custodial parent outside of marriage. In a real sense, the distinction of utility arising both from the marriage itself as well as from the children produced by it is unimportant when the marriage market is viewed as static (as the utility from the marriage can simply be redefined as the marriage's intrinsic value plus the expected utility from children produced from it at the time of marriage). But as we discuss below, when men and women are forward looking, and when fecundity falls with age, this distinction becomes important.

Bergstrom and Bagnoli (1993) present a model with incomplete information where men who expect to be successful delay marriage until they are able to give a signal that allows them to attract more desirable women. The equilibrium of this model is that, while all women marry early in life, the most desirable women marry successful older men and the less desirable women marry young men who do not expect to prosper.

The interaction between marriage, labor market and human capital accumulation has also been addressed in the literature. Recent examples are Aiyagari et al. (2000), Seitz (2002) and Greenwood, Guner and Knowles (2002). Also, in a recent paper, Brien, Lillard and Stern (2002) analyze cohabitation before marriage as a learning process about match quality.

Also in the last few years a growing theoretical literature on assortative matching has shed light on some of the issues raised by Becker's pioneering work, for example Shimer and Smith (2000) and Fernandez, Guner and Knowles (2001). Even though most of this literature is focused in assortative mating by education and income, the issue of interracial marriage has also been studied recently, for example by Wong (2003).

As noted above, Siow (1998) introduced the issue of the shorter fecundity period of women.[10] In a model with capital accumulation and where utility comes exclusively from having children, old and young men (all fertile) compete for young women as by assumption infertile women do not participate in the market. Young men would always marry young women except that Siow allows wages to rise with age as well. Because of this some old men, those who successfully obtain a higher wage, are able to marry. This displaces some of the young men in the competition over scarce fertile women. Moreover, Siow argues that there is a relationship between the scarcity of fertile women and the fact that men are more likely to remarry after divorce.[11] While it is hard to argue that, at any point of time, the stock of single fertile women is smaller than the stock of single fertile men, it is not clear whether this will be true in a dynamic framework. What this paper shows is that a market with more single men than women could be the equilibrium outcome where women, aware of their relatively limited fecundity horizon reduce their reservation value over the quality of a mate in order to ensure they marry when they are still in their fertile period.

[10] Tertilt (2002) uses a similar frameworrk to analyze the effects of polyginy.

[11] As Siow(1998) states in the introduction (pg. 335) "First, in monogamous societies with divorce and remarriage, fecund women are relatively scarce. For example, in North America, at least 30 percent of first marriages fail. Twenty percent of divorced women and 60 percent of divorced men will remarry. This differential in remarriage rates suggest that 12 percent of women who marry for the first time will marry divorced men. There are at least 12 percent fewer never-married to match with never-married men. Women will behave differently than men in response to this relative scarcity."

Chapter 2

A Search Model of Marriage with Differential Fecundity

2.1 A Simple 2-period Model

In this section we develop a simple overlapping generations model where people live two periods, women are fertile only in the first period and men are fertile in both periods. This simplification will allow us to obtain closed form solutions of the strategies and to prove existence and uniqueness. In the next section we will generalize this model allowing people to live a larger number of periods, and where the fecundity horizon for women is shorter than the one for men. The numerical solution for the generalized model is then compared with census data.

2.1.1 Assumptions

There is a continuum of single women of measure $F(t)$, and of men, $M(t)$. We will focus on the steady state, so $F(t) = F$ and $M(t) = M$.

In the spirit of Pissarides (1990), the number of contacts between single women and men is determined by a constant return to scale meeting function, as follows

$$\eta = \mu M^{\theta} F^{1-\theta} \tag{2.1}$$

where $0 < \theta < 1$ and μ a constant lower than 1.[1]

Women will meet at most one man per period and vice versa. The probability of meeting someone of the opposite sex each period will depend the relative scarcity of each sex. For that reason, the probability that a single woman meets a single man is

$$\eta^f = \frac{\eta}{F} = \mu \left(\frac{M}{F} \right)^{\theta} = \mu S^{\theta} \tag{2.2}$$

Similarly, the probability that a single man meets a single woman is

$$\eta^m = \frac{\eta}{M} = \mu \left(\frac{M}{F} \right)^{\theta - 1} = \mu S^{\theta - 1} \tag{2.3}$$

where S is the ratio single men/single women (sex ratio).

All singles are ex-ante homogeneous except for their (observable) age and potential fecundity. The preferences over the opposite sex are idiosyncratic. As stated above, this paper focuses in how time affects marriage behavior; therefore this assumption, along with the one of random matching, are for simplicity and does not affect generality.

Men and women differ in potential fecundity by age. While men are fertile at all ages, women are only fertile at age 1.

Both men and women live two periods, ages 1 (young) and 2 (old) . At any moment, there will be a number of women from both generations, f_1 of age 1 and f_2 of age 2 looking for a husband. Similarly these women will face a market of m_1 (young) and m_2 (old) bachelors.

Since men and women get married in pairs we need the number of young and old women that get married each period (w_1 and w_2) to be equal to the total of

[1] This constant is merely a time scaling parameter introduced to ensure that the probability of meeting is lower than 1 and to allow a replication of the model in an arbitrary number of periods.

men (h_1 young plus h_2 old) who enter into marriage. That is:

$$w_1 + w_2 = h_1 + h_2$$

Each period, an exogenous flow of single young people of age 1, f_1 women and m_1 men (we assume $m_1 = f_1$) enter the market.[2] The men and women who have not married in the previous period will remain in the market. In the steady state, this flow of young people entering the market will be equal to the number of people who exit the market through marriage at any age plus the number that die single after period 2 (f^s women and m^s men).[3]. That is,

$$\begin{aligned} f_1 &= w_1 + w_2 + f^s \\ m_1 &= h_1 + h_2 + m^s \end{aligned}$$

Since the motivations of an eventual divorce and remarriage could be very different that the ones for first marriage, this topic is not investigated in this paper.[4] We assume that people who divorce or whose spouse die do not re-enter the market. The meaning of this assumption is that, when single, people plan to marry only once in life. In other words, that at the moment people decide to marry the first time they believe that their marriage will last for the rest of their lives.

[2] As in Burdett and Coles (1997).

[3] Here we implicitly assume that the actual number of children that people have is the quantity needed to ensure the steady state with no population growth. Since the goal of this paper is to explain only the decision of marriage we assume the decision about the number of children as exogenous.

[4] For a model of marriage with "on the job" search and therefore endogenous separations, see Cornelius (2003)

The stock of single female of each age will be

$$\begin{aligned} f_2 &= f_1 - w_1 \\ f^s &= f_2 - w_2 \\ F &= f_1 + f_2 \end{aligned}$$

Similarly, the stock of single men of each age will be

$$\begin{aligned} m_2 &= m_1 - h_1 \\ m^s &= m_2 - h_2 \\ M &= m_1 + m_2 \end{aligned}$$

The discount factor is equal to $\beta \in (0, 1)$

The age composition of the marriage market is endogenously determined in the model. The fraction of young women and men,

$$p = \frac{f_1}{F} \tag{2.4}$$

$$q = \frac{m_1}{M}$$

are simultaneously determined as a function of the reservation strategies of men and women.

2.1.2 Payoffs

Given that a man and a woman meet, their potential payoffs come from mutual compatibility and the utility of having children within marriage. We assume that both men and women will receive zero utility if they do not marry either in period one or two.

The specific utility that a woman receives from a man and vice versa are considered as independent random draws from the distribution $G_m(y)$ and $G_f(x)$, respectively. Assume that $G_m(y)$ has support $[0, ymax]$ and mean $\overline{y}$, and $G_f(x)$ has support $[0, xmax]$ and mean $\overline{x}$.[5] Both distributions are strictly increasing on x and y respectively.

In addition, if a fertile man and a fertile woman meet, the utility is increased by a multiplicative parameter $k > 1$ because of the possibility of having children together. For example, if a fertile man marries a fertile woman he will receive kx per period and she will receive ky per period. If either the man or the woman involved is infertile, both of them will only receive x or y respectively, that is, only the love of the other person.

The rationale for the parameter k is that the value of a "having a family" will be a function of the attraction to their significant other. That is, people enjoy having children more with a person they care about. If we assume that people always receive utility from having children, we can separate it into two components, one coming from parenthood, and the other component coming from who the agent are having children with. Since it is possible to have children without a stable relationship, the specific joy of having children (and thus the utility out-of-wedlock parenthood) is normalized to 0 in this model. We assume further that the multiplicative parameter k has a maximum such that the utility

[5] In theory, y or x could take on negative values if the mean of both distributions were strictly positive. It sounds perfectly plausible that any man or woman could find that marrying certain canditates to be worse than staying single, and having children with these potential mates as a discount over having them out of wedlock. However, since the utility of being single is equal to 0, the reservation values set by men and women will be always nonnegative and that assumption will become irrelevant.

of marrying and have children with an average person can not be higher than the joy of finding a perfect match. That is,

$$k\overline{x} \leq xmax \quad \text{and} \quad k\overline{y} \leq ymax \tag{2.5}$$

Thus, the payoffs of marriage for men and women are the following:

Women	Husband Age 1	Husband Age 2
Marry at age 1	$ky\,(1+\beta)$	ky
Marry at age 2	y	

Men	Wife Age 1	Wife Age 2
Marry at age 1	$kx\,(1+\beta)$	x
Marry at age 2	kx	x

2.1.3 The Man's Optimization Problem

Probability of a Marriage Offer for Men

Let us first analyze the Male Problem.[6] In each period a man will meet a woman with probability η^m (by Equation (2.3)). The man will meet a single young woman with probability p . This probability is equal to $\frac{f_1}{F}$ (the fraction of single women who are young), and this fraction (while endogenous to the market) is exogenous to each individual. However, the fact that he meets a young woman does not mean that he has a concrete opportunity to marry her. Even though all men are fertile, a given young woman will not be indifferent between a man of age 1 and of age 2, because if she marries a senior bachelor she will enjoy his company for

[6] Unless note otherwise I use the term man and woman in this Section to refer to single man and single woman.

only one period. Hence she will set two different reservation values, $R^f(i,j)$ for young men and R^f_{old} for men of age 2. In other words, a senior bachelor will have a probability of a marriage offer from a young woman (that is, to meet and also being accepted by a young woman) of

$$\alpha_2^{young} = \eta^m \left[p \left(1 - G_m \left(R^f_{old} \right) \right) \right]$$

and a young man will receive an offer from a young woman with probability

$$\alpha_1^{young} = \eta^m \left[p \left(1 - G_m \left(R^f(i,j) \right) \right) \right]$$

Since old women will have reservation utility equal to 0 , they will accept any proposal. Then the probability that a given male receives and offer from an old woman will be

$$\alpha^{old} = \eta^m (1-p)$$

Given that a marriage offer is available, the man receives a signal drawn from the distribution $G_f(x)$ and decide to marry or not.

Utility of Marriage for Men of Age 2 (Old)

Old men who do not marry will die single, earning zero utility. The reservation value for an old man is therefore equal to 0. He would be willing to marry any woman who makes him a marriage offer. If he meets a woman age 2 (who also has a reservation utility equal to 0), they will marry with certainty. If he meets a young woman (age 1) and he marries her, he will enjoy the extra utility from the prospect of having children (k times the type of the woman).

Therefore, the value of marrying at age 2 will be

$$U_2^m = V_2^m = \left(\alpha_2^{young} k + \alpha^{old} \right) \overline{x} \qquad (2.6)$$

Utility of Marriage for Men of Age 1 (Young)

Since young men are able to wait until they are old in order to find the right mate, in period 1 men set a reservation value for accepting a woman taking into account next period prospects. As before, they can meet young or old women. Of course, if a young man marry a young woman, he will enjoy having children and live with his wife for two periods. If he marries an old woman he will be married for only one period and without children. Consequently, the reservation values of match quality a young man will set for marrying a young or an old woman will not be the same. Call these two reservation values $R^m(j,i)$and R^m_{old}, respectively. Moreover, in order to marry a young woman, he has to be accepted by her. This will happen with probability $\left(1 - G_m\left(R^f(i,j)\right)\right)$. The utility that a man derives from marrying at period 1 is then

$$U_1^m = \alpha_1^{young} k\left(1+\beta\right) \int_{R^m(j,i)}^{xmax} x dG_f(x) + \alpha^{old} \int_{R^m_{old}}^{xmax} x dG_f(x)$$

Optimization Problem for Young Men

The problem that a young man faces is to choose to marry or not in order to maximize

$$U_1^m + \left(1-\Phi_1\right)\beta U_2^m$$

where

$$\Phi_1 = \alpha_1^{young}\left(1 - G_f\left(R^m(j,i)\right)\right) + \alpha^{old}\left(1 - G_f\left(R^m_{old}\right)\right) \tag{2.7}$$

is the probability that a man marries at age 1 with a young or an old woman.

The Bellman Equation of this Problem is

$$V_1^m = \underset{D^m}{Max}\left[U_1^m + \left(1-\Phi_1\right)\beta V_2^m\right] \tag{2.8}$$

where

$$D^m = \begin{cases} 1 & \text{if } x \geq R^m(j,i) \text{ or } x \geq R^m_{old} \\ 0 & otherwise \end{cases}$$

where D^m is the decision of marrying at at age 1.

The reservation value set for an old woman is exactly equal to the discounted value that a man has if remains in the market at age 2. Notice that, the to the linearity assumed in the utility function, the reservation value that men set for older women is $k(1+\beta)$ times the reservation value for young women. The reason for this is if he marries an old woman he will live with his wife only one period and without children. That is,

$$R^m_{old} = \beta V^m_2 = k(1+\beta)R^m$$

2.1.4 The Woman's Optimization Problem

Probability of a Marriage Offer for Women

Now we can analyze the female problem. In each period , given the probability of meeting rate η^f (by Equation (2.2)), a given woman will meet a young man with probability q and an old man with probability $(1-q)$. She will marry him if the utility of marrying the man she meets, drawn from the distribution $G_m(y)$ is greater than the value of search for a better mate for one more period.

Utility of Marriage for Women of Age 2 (Old)

A woman is age 2 knows two things: first, she will die at the end of the period, and therefore her reservation value will be $= 0$; second, she is not fertile. This means that she will not receive the extra utility of having children, nor will she be able to provide that extra utility to any man she marries.

We can define the offer rates that a senior woman faces in the following way. A woman will meet a man each period with probability η^f. If she happen to meet an old man (with probability $(1-q)$) he will propose with probability 1, and so she will have a concrete offer from an old bachelor with probability

$$\lambda^{old} = \eta^f (1-q)$$

If she meets a young man (with probability q) she will only marry him if her type x is as least as large as his reservation utility for a senior bachelorette, R^m_{old}. For that reason, the probability that a young man proposes to a senior woman will be $(1 - G_f(R^m_{old}))$,what means that a senior bachelorette will receive a proposal from a young man with probability

$$\lambda_2^{young} = \eta^f q \left(1 - G_f\left(R^m_{old}\right)\right)$$

Then the value of being single for an old woman is

$$U_2^f = V_2^f = \left(\lambda_2^{young} + \lambda^{old}\right) \overline{y}$$

Utility of Marriage for Women of Age 1 (Young)

A young woman sets a reservation value taking into account that she may have future opportunities to find a better spouse. However, if she doesn't marry young she will not be able to have children. Even though men of all ages are fertile, a young woman will not be indifferent between marrying a young man or an old man of the same match quality because a marriage with the old man lasts only for one period. Of course, while any old man will accept her, she will only be able to marry a young man if her match quality is higher than the reservation value set by him, $R^m(j,i)$. A young man proposes to a young woman with probability

$(1 - G_f (R^m (j,i)))$. Thus, a young woman will receive a proposal from a young man with probability

$$\lambda_1^{young} = \eta^f q (1 - G_f (R^m (j,i)))$$

The expected utility a woman receives from marrying when young is then

$$U_1^f = \lambda_1^{young} k (1+\beta) \int_{R^f(i,j)}^{ymax} y dG_m(y) + \lambda^{old} k \int_{R^f_{old}}^{ymax} y dG_m(y)$$

Optimization Problem for Young Women

Hence, the problem facing a young woman is to choose to marry or not at Age 1 in order to maximize

$$U_1^f + (1 - \Gamma_1) \beta V_2^f$$

where

$$\Gamma_1 = \lambda_1^{young} \left(1 - G_m \left(R^f (i,j)\right)\right) + \lambda^{old} \left(1 - G_m \left(R^f_{old}\right)\right) \quad (2.9)$$

is the probability that a woman marries at age 1.

The Bellman Equation for this problem is then

$$V_1^f = \underset{D^f}{Max} \left[U_1^f + (1 - \Gamma_1) \beta V_2^f\right] \quad (2.10)$$

$$D^f = \begin{cases} 1 & \text{if } y \geq R^f(i,j) \text{ or } y \geq R^f_{old} \\ 0 & otherwise \end{cases}$$

where D^f is the decision of marrying at at age 1.

The reservation value set for an old man is equal to the discounted value that a woman has if remains in the market at age 2 divided by k. Since men are fertile at all ages, if a woman marry at age 1 will have children with probability one, but if she waits until the second period she will not be able to bear children.

Notice that, because all men are fertile, the reservation value that women set for older men is $(1+\beta)$ times the reservation value for young men (old men die first). That is,

$$R^f_{old} = \beta \frac{1}{k} V^f_2 = (1+\beta) R^f$$

2.1.5 Steady State Equilibrium

Reaction Functions

Solving the problems stated in Equations (2.8) and (2.10) , the reaction functions for men and women, respectively are

$$\begin{aligned} R^m(j,i) &= \frac{\eta^m \beta \left[(1-p) + kp\left(1 - G_m\left(R^f_{old}\right)\right)\right]}{k(1+\beta)} \overline{x} && (2.11) \\ R^m_{old} &= \eta^m \beta \left[(1-p) + kp\left(1 - G_m\left(R^f_{old}\right)\right)\right] \overline{x} \\ R^f(i,j) &= \frac{\beta \eta^f \left(1 - qG_f\left(R^m_{old}\right)\right)}{k(1+\beta)} \overline{y} && (2.12) \\ R^f_{old} &= \frac{\beta \eta^f \left(1 - qG_f\left(R^m_{old}\right)\right)}{k} \overline{y} \end{aligned}$$

Clearly, the higher a young man's reservation value, the greater the probability that he will still be in the market when old. Therefore, the higher the probability of being accepted by a women when he is older, the higher the minimum match quality he requires when young.

For women, the intuition is as follows. The reservation value of a woman depends positively on the average "match quality" of the available men, the degree of patience and the meeting rate. Women will decrease their reservation value the higher the value of having children and the higher the reservation value that men set for older women, times the fraction of young men in the market. The explanation for this last factor is the following: the more choosy are young men about old women (and the greater the fraction of young men in the market), the larger the incentives of young women to worry about their future and marry young.

Stock of Singles in the Market

Given the existence of an equilibrium, we can characterize the steady state number of single men and women using equations (2.2), (2.3), (2.7),(2.9) , (2.11) and (2.12). The number of man and women that marry at the young age is

$$h_1 = m_1\Phi_1 \tag{2.13}$$

$$w_1 = f_1\Gamma_1 \tag{2.14}$$

respectively, leaving the number of remaining (old) singles in the market as

$$m_2 = m_1 - h_1 = m_1(1-\Phi_1) \tag{2.15}$$

$$f_2 = f_1 - v_1 = f_1(1-\Gamma_1) \tag{2.16}$$

In the same way, given the probabilities of marrying for old people are

$$\Phi_2 = \left(\alpha_2^{young} + \alpha^{old}\right) \text{ for men and}$$

$$\Gamma_2 = \left(\lambda_2^{young} + \lambda^{old}\right) \text{ for women}$$

the number of people who marry when old are

$$h_2 = m_2\Phi_2 = m_1(1-\Phi_1)\Phi_2$$

$$w_2 = f_2\Gamma_2 = f_1(1-\Gamma_1)\Gamma_2$$

Theorem 1 *An Equilibrium exist in the system formed by equations* (2.11), (2.12) , (2.15) *and* (2.16) .

Proof. See Appendix. ■

Theorem 2 (Uniqueness in the Equilibrium Strategies) *Assume that* $G_f(x)$ *and* $G_m(y)$ *have the same support* $[0, xmax]$. *Assume further that there exists a constant* $C < \frac{1}{\overline{x}\overline{y}}$ *such that the distributions' densities* g_f *and* g_m *satisfy*

$$g_f(x)\, g_m\left(R^f_{old}(x)\right) \leq C$$

for all $x \in [0, xmax]$. *Then there exists a unique equilibrium for the system formed by equations* (2.11) *and* (2.12). *This equilibrium will be an interior solution, that is, both men and women will marry either at age 1 or 2 with positive probability.*

Proof. [Outline of Proof] To facilitate the following proof, we define

$$\begin{aligned} T_1(R^f_{old}) &= \eta^m \beta \left[(1-p) + kp\left(1 - G_m\left(R^f_{old}\right)\right)\right] \overline{x} \\ T_2(R^m_{old}) &= \frac{\beta \eta^f \left(1 - qG_f\left(R^m_{old}\right)\right) \overline{y}}{k} \end{aligned}$$

In this notation, an equilibrium is characterized simply by the following equations:

$$R^m_{old} = T_1(R^f_{old}) \tag{2.17}$$

$$R^f_{old} = T_2(R^m_{old}) \tag{2.18}$$

Define

$$H(x) = T_1(T_2(x)) \tag{2.19}$$

In Equations (2.17) and (2.18) we show that every steady state equilibrium of the model corresponds to a fixed point of $H(.)$. A long calculation, relegated to the Appendix, shows that under the hypothesis of the theorem

$$| H'(x) | < 1$$

Consequently, $H(x)$ is a contraction mapping.

By the contraction mapping theorem, there exists a unique fixed point of $H(.)$. Call it R^m_{old}. A short argument in the Appendix shows that $R^m_{old} \in [0, xmax)$, and the associated $R^f_{old} \in [0, xmax)$, and that the uniqueness is ensured for R^m and R^f. We conclude that the unique fixed point of $H(.)$ corresponds to a steady state equilibrium of the model. ■

Remark 1 *The hypothesis of Theorem 1 is trivially satisfied if* $G_f(x)$ *and* $G_m(y)$ *are uniformly distributed with support* $[0, 1]$

Proposition 3 *Assume that the distributions of men and women are equal. Provided that people derive utility for having children within marriage* ($k > 1$)*, then men will be choosier than women, that is,* $R^m > R^f$ *.*

Proof. See Appendix ■

2.1.6 Example: Uniform Distribution

In order to gain further intuition on the model, in this section we will solve the model assuming that the distribution of men and women of ages 1 or 2 is uniform with support $[0, 1]$. That is:

$G_f(x) = G_m(y) \sim U[0, 1]$

With this specification, the unique equilibrium is:

$$R^m(j,i) = \frac{1}{k(1+\beta)\left(4 - pq\eta^m\beta^2\eta^f\right)}\eta^m\beta\left[2 + p\left(2(k-1) - \beta\eta^f\right)\right] \quad (2.20)$$

$$R^m_{old} = \frac{1}{\left(4 - pq\eta^m\beta^2\eta^f\right)}\eta^m\beta\left[2 + p\left(2(k-1) - \beta\eta^f\right)\right]$$

$$R^f(i,j) = \frac{1}{k(1+\beta)\left(4 - pq\eta^m\beta^2\eta^f\right)}\eta^f\beta\left[2 - q\eta^m\beta\left(1 + p(k-1)\right)\right] \quad (2.21)$$

$$R^f_{old} = \frac{1}{k\left(4 - pq\eta^m\beta^2\eta^f\right)}\eta^f\beta\left[2 - q\eta^m\beta\left(1 + p(k-1)\right)\right]$$

Equations (2.15) and (2.16) can be solved numerically in order to find the steady state equilibrium of the model. Figure 2.1 show the equilibrium values of selected variables as a function of $k \in [1, 2]$, considering the following values for the parameters:

$\theta = 0.5 \quad \beta = 0.9 \quad \mu = 0.9 \quad m_1 = f_1 = 100$

Figure 2.1 shows how the reservation values of men and women, the fraction of single men (women) who are young, the probability of marriage at Age 1 and the ratio Single Men/Single Women change with the value of children (k). As shown, the higher the increase in the utility of marriage for having children, the lower the reservation values for women, and the higher the probability that a woman marry young. Because women marry younger, the fraction of young women over the total of single women is increasing with k. On the other hand, men's behavior is the opposite to the one of women but the patterns seem to be relatively more stable. Therefore, the predictions of this simple 2-period model are the following:

- The higher the value of having children within marriage, women tend to marry younger and men older. For that reason, the age difference in marriage tend to increase with higher values of k.

- Single women in the marriage market tend to be younger than single men. That is, a given man is more likely to meet a young woman than is a woman to meet a young man.

- As their reservation values decrease with higher values of k, match qualities for women also tend to decrease.

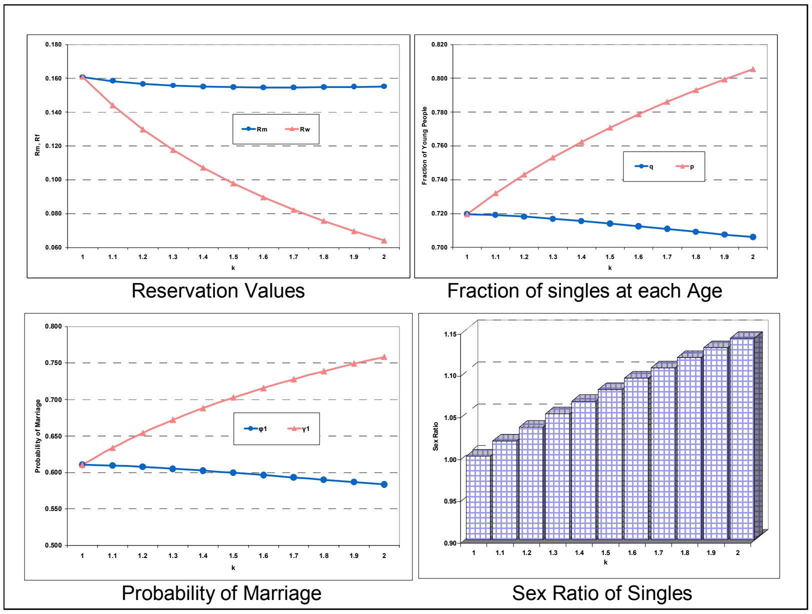

Figure 2.1: Equilibrium of the 2 Period Model for Different Values of k.

2.2 A Generalized Model

In this section we extend the simple two period model to a more general finite horizon model. This general model is solved numerically. The following are the modified assumptions:

Both men and women live T periods. Women are fertile for L periods, men are fertile for N periods, with $L \leq N \leq T$. While women's fecundity decrease linearly form period L_1 to L, men keep their full fecundity until they lose it completely at period N.

As in the two period model above, women will meet at most one man per period and vice versa. The probability of meeting is determined by equations (2.2) and (2.3) .

As before, an exogenous flow of single young people of age 1, f_1 women and m_1 men enter the market each period and the men and women who do not marry will remain in the market. Hence, the total number of single women and men will be the sum of the stock of single men and single women ages i and j respectively, $i,j \in [1,T]$

$$\begin{aligned} M &= \sum_{j=1}^{T} m(j) \\ F &= \sum_{i=1}^{T} f(i) \end{aligned}$$

Therefore, the fraction of single and men and women of ages i,j will be

$$p(i) = \frac{f(i)}{F} \tag{2.22}$$

$$q(j) = \frac{m(j)}{M} \tag{2.23}$$

which are endogenously determined.

We redefine the extra utility for having children for a man who marries at age j with a woman of age i as k_{ji}^{m} where

$$k_{ji}^{m} = \begin{cases} k & \text{if } \; j \leq N \text{ and } i \leq L_1 \\ k - \frac{(k-1)(i-L_1)}{(L+1-L_1)} & \text{if } \; L_1 < i \leq L \text{ and } j \leq N \\ 1 & \text{otherwise} \end{cases} \tag{2.24}$$

Similarly, for a woman of age i who marries a man of age j,

$$k_{i,j}^{f} = \begin{cases} k & \text{if } \; i \leq L_1 \text{ and } \; j \leq N \\ k - \frac{(k-1)(i-L_1)}{(L+1-L_1)} & \text{if } \; L_1 < i \leq L \text{ and } \; j \leq N \\ 1 & \text{otherwise} \end{cases} \tag{2.25}$$

where $k > 1$ and is subject to the condition established in (2.5).

Given that the number of periods will be large enough in order to calibrate the results of the model with census data by age, it will be convenient to relax the assumption that people die only in the last period. For that reason we introduce a probability of dying each period that depends on the agent's age. The probability of dying in a given period for women and men of ages i, j will be

$$d_j^m \quad \text{for a man of age } \; j < T$$
$$d_i^f \text{ for a woman of age } i < T$$
$$d_T^m = d_T^f = 1$$

In the same way, the probability of that the marriage ends in a given period because of death will depend on the ages of husband and wife, as follows

$$\Delta_{i,j} = d_i^f \left(1 - d_j^m\right) + \left(1 - d_i^f\right) d_j^m + d_i^f d_j^m$$

The characteristics of the utility functions for men and women remain as in the model of the previous section.

2.2.1 The Man's Optimization Problem

Each period a man of age j will meet a woman of age i with probability $\eta^m p(i)$ (by equations (2.3) and (2.22)). The probability of being accepted by that woman depends on the age of both the man and the woman. A man of age j will receive an offer from a woman of age i with probability

$$\alpha(j,i) = \eta^m p(i) \left(1 - G_m(R^f(i,j))\right)$$

where $R^f(i,j)$ will be the reservation value that a woman of age i set for a man of age j.

Then the probability that a man of age j has a concrete concrete opportunity of marriage will be

$$\alpha_j = \sum_{i=1}^{T} \alpha(j,i) = \sum_{i=1}^{T} \eta^m p(i) \left(1 - G_m(R^f(i,j))\right) \tag{2.26}$$

Expected Utility of Marrying at age j

The expected utility that a man of age j derives from marrying a woman of age i depends on both the man's age and the woman's age. This occurs not only because fecundity matters, but also because the age of each partner will determine how long they will enjoy each other company. Consider first a man of age j where $j \leq L_1$ (a fertile man such that a woman of the same age is still completely fertile). In this case he will be indifferent between any woman his age or younger because he will spend with her the rest of his life. If he marries an older woman, he will survive her and receive zero utility from the moment he become a widower until his own death. The case of an infertile ($j > N$) man is similar, because the only utility of marriage is derived from the quality of the match and the length of the marriage. In the case that $L_1 < j \leq N$ (a fertile man who is older than the age at which women start losing fecundity), a man will not be indifferent between any woman younger than himself because he will receive extra utility from a fully fertile spouse. Therefore, the expected utility of a man of age j who marries a woman of age i will be the discounted sum of the flows of expected payoffs of marriage through the length of the marriage. That is,

$$u^m(j,i) = \sum_{s=0}^{Min(T-i,T-j)} \beta^s \left(1 - \Delta_{i+s,j+s}\right) k_{j,i}^m \int_{R^m(j,i)}^{xmax} x dG_f(x) \tag{2.27}$$

Given that the probability of a marriage offer from women of different ages differ, the expected utility of marrying at age j will be

$$
\begin{aligned}
U^{m}(j) &= \sum_{i=1}^{T} \alpha(j,i)\, u^{m}(j,i) \\
&= \sum_{i=1}^{T} \alpha(j,i) \sum_{s=0}^{Min(T-i,T-j)} \beta^{s}\left(1-\Delta_{i+s,j+s}\right) k_{j,i}^{m} \int_{R^{m}(j,i)}^{xmax} x dG_{f}(x)
\end{aligned}
\tag{2.28}
$$

Probabilities of Marriage for Men

Now we define the hazard rate for a man to marry at age j, as follows

$$
\Phi(j) = \sum_{i=1}^{T} \alpha(j,i)\left(1-G_{f}\left(R^{m}(j,i)\right)\right) \tag{2.29}
$$

Objective Function for Men

Given Equations (2.28) and (2.29), the objective function for any man at a given age j is the following

$$
\sum_{j=t}^{T} \beta^{j-t} U^{m}(j) \prod_{s=t+1}^{j}\left(1-d_{s-1}^{m}\right)\left(1-\Phi(s-1)\right)
$$

The Bellman Equation for the problem above is

$$
\begin{aligned}
V^{m}(j) &= \underset{D_{j}^{m}}{Max}\left[U^{m}(j)+\left(1-d_{j}^{m}\right)\left(1-\Phi(j)\right)\beta V^{m}(j+1)\right] \\
V^{m}(T) &= U^{m}(T)
\end{aligned}
\tag{2.30}
$$

$$
D_{j}^{m} = \begin{cases} 1 & \text{if } x > R^{m}(j,i) \\ 0 & otherwise \end{cases}
$$

where D_{j}^{m} is the decision of marrying at age j with a woman of age i.

Men's Reservation Values

The Reservation Values set by men can be obtained recursively given that

$$R^m(j,i)\left(\sum_{s=0}^{Min(T-i,T-j)}\beta^s\left(1-\Delta_{i+s,j+s}\right)k^m_{j,i}\right)=\beta V^m_{j+1}\ \text{ if }\ 1\leq j\leq T-1$$

The reservation value that a man of age j sets for a given woman of age i is

$$R^m(j,i)=\begin{cases}\dfrac{\beta V^m_{j+1}}{\left(\sum\limits_{s=0}^{Min(T-i,T-j)}\beta^s(1-\Delta_{i+s,j+s})k^m_{j,i}\right)} & \text{if }\ 1\leq j\leq T-1\\ \\ 0 & \text{if }\ \ j=T\end{cases} \tag{2.31}$$

Stocks of Single and Married Males

The stocks of singles of age j will be equal to the surviving singles of age $j-1$ who did not married during the last period. That is

$$m(j)=m(j-1)\left(1-d^m_j\right)\left(1-\Phi(j-1)\right) \tag{2.32}$$

Similarly to the 2-period Model (see Equation (2.13)), the total men who marry at age j will be

$$h(j)=m(j)\,\Phi(j)$$

and the stock of married men of age j will be the sum of the surviving males who married at age j or younger. That is,

$$H(j)=\sum_{t=1}^{j}h(t)\prod_{s=t}^{j-1}\left(1-d^m_s\right) \tag{2.33}$$

2.2.2 The Woman's Optimization Problem

Each period a woman of age i will meet a man of age j with probability $\eta^f q(j)$ (by Equations(2.2) and (2.23)). As above, the probability of being accepted by that bachelor will depend on the age of both of the woman and the man she meet. The probability that the woman of age i receives an offer from a man of age j is

$$\lambda\left(i,j\right)=\eta^{f}q\left(j\right)\left(1-G_{f}(R^{m}\left(j,i\right))\right)$$

where $R^{m}\left(j,i\right)$ will be the reservation value that a man of age j set for a woman of age i.

The probability of receiving an offer from any man at age i will be

$$\Lambda\left(i\right)=\sum_{j=1}^{T}\lambda\left(i,j\right)=\sum_{j=1}^{T}\eta^{f}q\left(j\right)\left(1-G_{f}(R^{m}\left(j,i\right))\right). \tag{2.34}$$

Expected Utility of Marrying at age i

In the same way as for men, the expected utility that a woman of age i derives from marrying a man of age j depends on the expected length of the marriage

$$u^{f}\left(i,j\right)=\sum_{s=0}^{Min(T-i,T-j)}\beta^{s}\left(1-\Delta_{i+s,j+s}\right)k_{i,j}^{f}\int_{R^{f}(i,j)}^{ymax}ydG_{m}(y) \tag{2.35}$$

and her expected utility of marrying at age i

$$\begin{aligned}U^{f}\left(i\right) &= \sum_{j=1}^{T}\lambda\left(i,j\right)u_{ij}^{f} \qquad (2.36)\\ &= \sum_{j=1}^{T}\lambda\left(i,j\right)\sum_{s=0}^{Min(T-i,T-j)}\beta^{s}\left(1-\Delta_{i+s,j+s}\right)k_{i,j}^{f}\int_{R^{f}(i,j)}^{ymax}ydG_{m}(y)\end{aligned}$$

Probabilities of Marriage for Women

The hazard rate of marriage for a woman at age i is defined as

$$\Gamma\left(i\right)=\sum_{j=1}^{T}\lambda\left(i,j\right)\left(1-G_{m}\left(R^{f}\left(i,j\right)\right)\right) \tag{2.37}$$

Objective Function for Women

Given Equations (2.36) and (2.37) , the objective function of a single woman at age i is the following

$$\sum_{i=t}^{T} \beta^{j-t} U^f(i) \prod_{s=t+1}^{i} \left(1 - d_{s-1}^f\right)(1 - \Gamma_{s-1})$$

As is the case of the man above, the Bellman Equation for the woman's problem is then

$$\begin{aligned} V^f(i) &= \underset{D_i^f}{Max} \left[U^f(i) + \left(1 - d_i^f\right)(1 - \Gamma(i)) \beta V^f(i+1)\right] \qquad (2.38) \\ V^f(T) &= U^f(T) \end{aligned}$$

$$D_i^f = \begin{cases} 1 & \text{if } \; y > R^f(i,j) \\ 0 & otherwise \end{cases}$$

where D_i^f is the decision of marrying at age i with a man of age j.

Reservation Values for Women

As in the case for men, the reservation values for women can be obtained recursively. Given that

$$R^f(i,j) \left(\sum_{s=0}^{Min(T-i,T-j)} \beta^s (1 - \Delta_{i+s,j+s}) k_{i,j}^f \right) = \beta V^f(i+1) \qquad \text{if } \; 1 \leq i \leq T-1$$

The reservation value for a woman of age i with respect to a man of age j will be

$$R^f(i,j) = \begin{cases} \dfrac{1}{\left(\sum_{s=0}^{Min(T-i,T-j)} \beta^s (1-\Delta_{i+s,j+s}) k_{i,j}^f \right)} \beta V^f(i+1) & \text{if } \; 1 \leq i \leq T-1 \\ \\ 0 & \text{if } \; i = T \end{cases}$$

Stocks of Single and Married Females

Similarly to the previous case, we define the number of single women of age i as follows

$$f(i) = f(i-1)\left(1 - d_{i-1}^{f}\right)(1 - \Gamma(i-1)). \tag{2.39}$$

The total of women who marry at age i will be

$$w(i) = f(i)\,\Gamma(i)$$

As for men, the stock of married women of age i is

$$W(i) = \sum_{t=1}^{i} w(t) \prod_{s=t}^{i-1} \left(1 - d_s^{f}\right) \tag{2.40}$$

2.2.3 Numerical Solution

Now we can solve numerically the system formed by Equations (2.30), (2.32), (2.38) and (2.39). The distribution functions $G_f(x)$ and $G_m(y)$ are both uniform with support $[0, 1]$ and the values given to the parameters will be the following[7]:

$T = 60$ (75 years old)	$N = 45$ (60 years old)
$L_1 = 20$ (35 years old)	$L = 30$ (46 years old)
$\mu = 0.9$	$k = 1.3$
$\beta = 0.915$	$\theta = 0.5$
$m_1 = 100$	$f_1 = 100$

Reservation Values and Marriage Offers

[7] The Mortality data is obtained from the US Life Tables for whites for 1995.

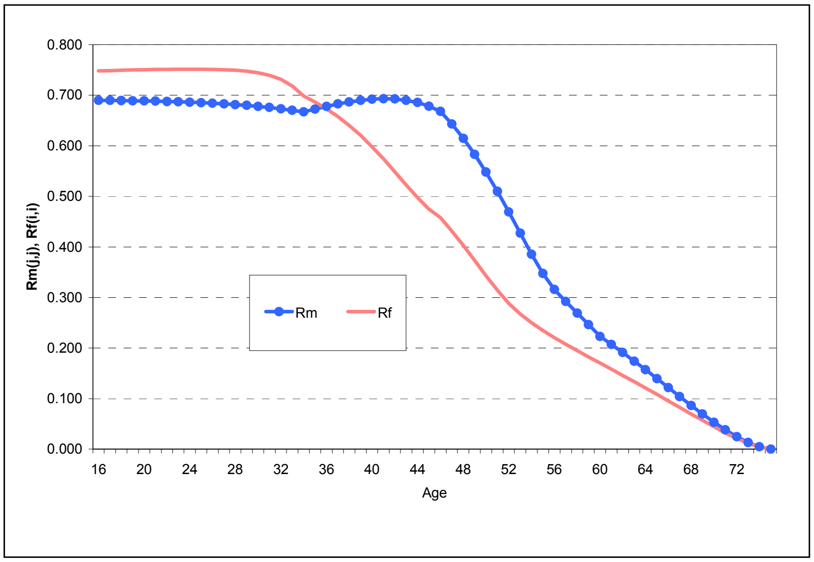

Figure 2.2: Reservation Values for People of the Same Age

Figure 2.2 shows the reservation values for men and women with respect to people of the opposite sex of the same age and Figure 2.3 shows the probability of receiving a marriage offer at each age (by Equations (2.26) and (2.34)). Interpreting both graphics will help to summarize several of the predictions of the model about marriage behavior.

As shown in Figure 2.2, women younger than age 34 set a higher reservation value than men of the same age. The reason for this is the traditional one: fertile women are outnumbered by fertile men. As in Siow (1998), a young woman faces relatively better market conditions than a man of her age. This is what has been emphasized by the literature. In principle there is a counter balancing force lowering the reservation values of women that is their relatively shorter fecundity horizon. However, for women in their late teens or early twenties the distant end of their fecundity years is sufficiently removed that the better marriage market conditions for women are large enough to make them more choosy than men.

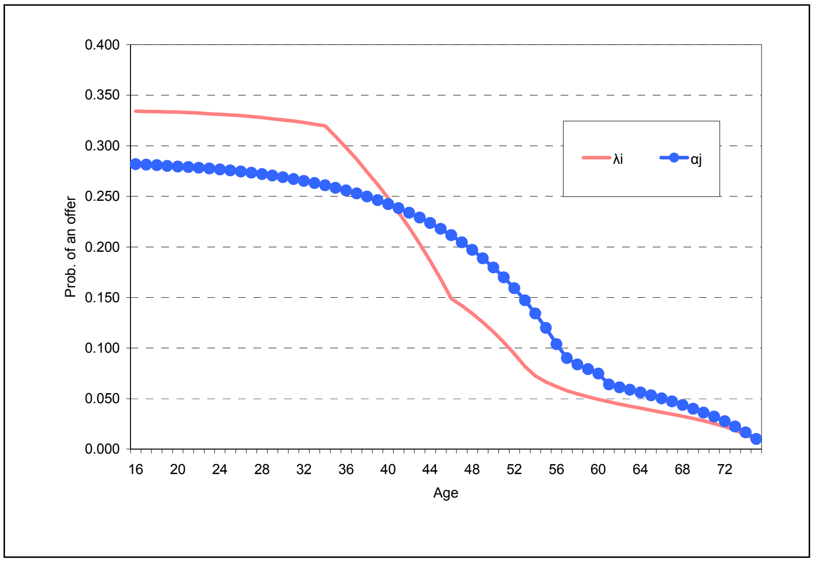

Figure 2.3: Probability of Receiving a Marriage Offer for Men (α_j) and Women (λ_i)

When the decline in fecundity is nearer, the reservation values of women start to decrease sharply. In this example we assume that fecundity start to decline at age 36, and this causes the reservation value to start to decrease in the mid twenties and continue through the end of their fertile years. Note that after fecundity ends, reservation values continue to decline. This however is driven by mortality as the shorter life expectancy is cause fewer chances to marry

The behavior of men is different than the one of women. The reservation value for men younger than age 34 is flat and lower than the one for women because, as stated above, fertile men outnumber fertile women. When women's fecundity start to decline around 35 years old, men of the same age raise their reservation value for those women in light of their increasing probability of being barren. Around age 50, the reservation values for men decrease sharply in light of the foreseeable loss of fecundity (in this example at age 60).

The behavior of men and women in this general model can be compared to the results established in the 2-period model. One of the principal results of the model was that women (in period 1) set a reservation value lower than the one set by men. This is because the 2-period model does not give women sufficient distance from the end of fecundity to take advantage of their relative scarcity at a point in time.

Figure 2.3 reflects how reservation values of men and women affect the opportunities of the other side of the market. In the case of women, the probability of receiving a marriage offer drops when they are in their middle 30's (when fecundity starts to decline). Note that offers start to decline later than when reservation values begin to decline (at age 25). The reason for this is that the decline in offers from men to women is a function of men's reservation values. Women who are younger than 35 remain completely acceptable to men as they are still completely fertile. This occurs for two important but different reasons. First, women worry about their own ability to bare children and the utility they will receive for this. Second, women rationally anticipate their worsening position in the marriage market knowing that men will begin increasing their reservation value for women over 35 as younger women will remain as a viable substitute. For men, the probability of receiving a marriage offer decreases at an increasing rate between ages 35 and 60.

Hazard Rates

Figure 2.4 shows the hazard rates of marriage for men and women at each age. Observe that there is a sharp increase in the probability that single women marry from their late 20's to their middle 30's. That increase is due to lower women's reservation values as the decline in fecundity is approaching. Assuming that fecundity starts to decline at age 35, the figure shows how the hazard rate for women increase at a diminishing rate from 35 to 40 and then decrease trough the end of her life. As women's fecundity declines, men are more reluctant to marry them due to the increasing risk of not having a child (observe in Figure 2.2 how men from 35 to 46 increase their reservation value for women of the same age). When women lose fecundity completely (here at age 47), notice the kink in the hazard rate curve. The explanation for this is that a new market appears: infertile women are now much less choosy about marrying infertile men.

The pattern of the hazard rates for men is similar to that of women, but the timing is different. As the reservation values of men decrease with time, their hazard rates for marriage are increasing through their fertile period (until 60 years old).Thereafter the reservation values decrease sharply. Then, as infertile women become acceptable, the drop stops to then continue as a slower pace during the last few years of their life.

Stocks of Men and Women by Marital Status

Figure 2.5 shows the stocks of single men, single women, married men (by Equation (2.33)) and married women (Equation (2.40)) at each age. The actual pattern of the 2000 US census data (whites) is displayed in Figure 2.6. Note that single men outnumber single women from their early 20's until near age 60, and that the number of married women is greater than the number of married men in most of the life cycle. The reason for this is that, since more women marry at a young age, this affects the stocks of people of all ages. One interesting feature of the model is that by age 55 the sex ratio of married men to married women

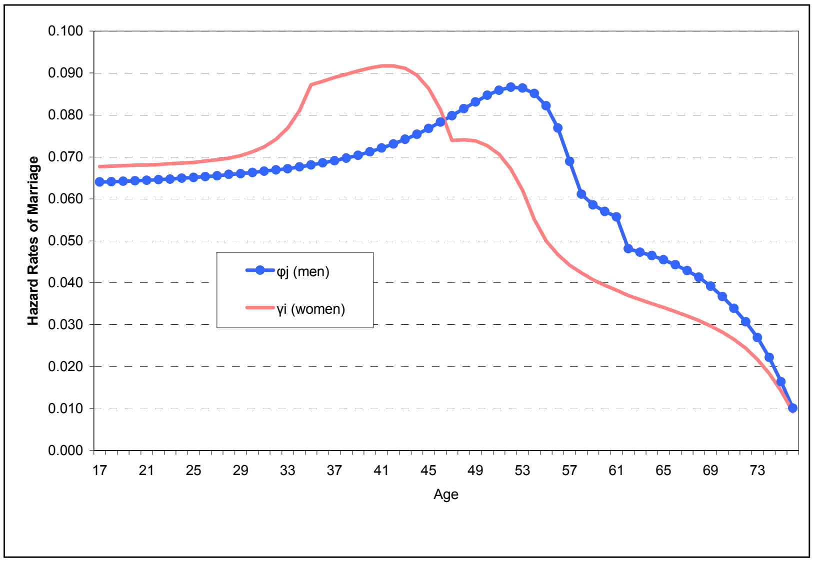

Figure 2.4: Hazard rates of Marriage for Men (ϕ_j) and Women (γ_i)

approaches one, but before reaching parity the number of married men starts to decline more quickly than the number of married women. because men marry later than women, for to attain parity with women there must be ages at which men marry with higher probability than women. This does occur (after age 42) but the higher mortality rates of men offset the higher marriage rates leaving the stock of married women to be greater than the stock of married men. Stocks of Single Men $(m(j))$, Single Women $(f(i))$, Married Men (H_j) and Married Women (W_i)

Comparative Statics

This model has two key parameters -the value of having children within marriage, k and the discount factor, β. Here we analyze how do the model predictions change with changes in these parameters. Figures 2.7 and 2.8 the comparative statics results on k while Figures 2.9 and 2.10 show results for β. Take example for

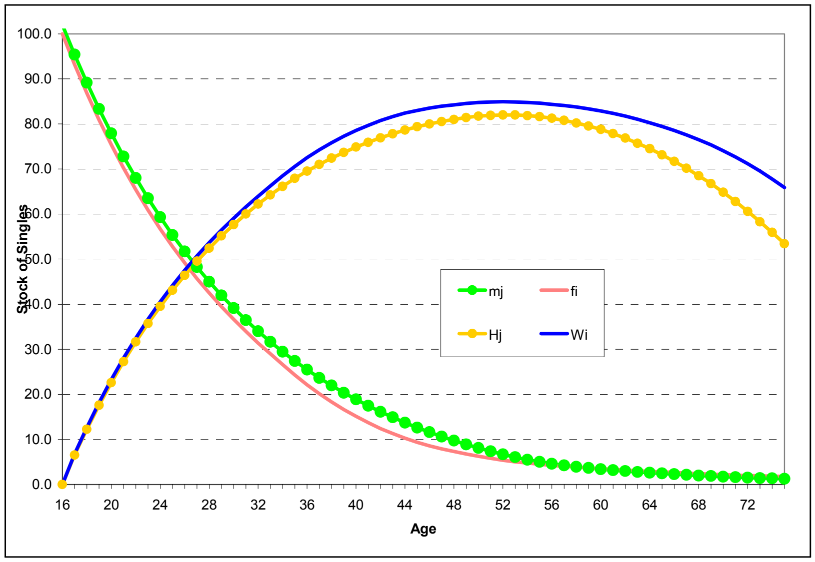

Figure 2.5: Stocks of single men (m_j), single women (f_i), married men (H_j) and married women (W_i)

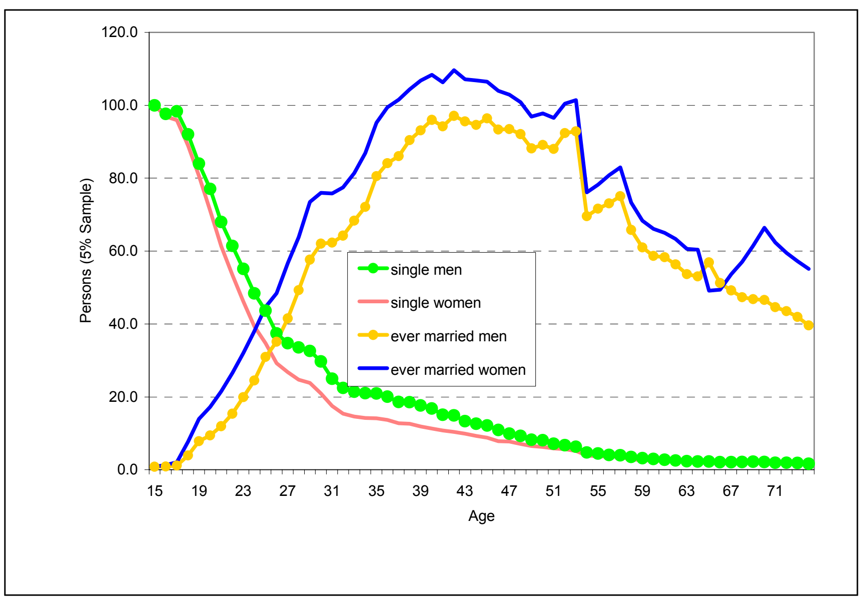

Figure 2.6: Stock of single and ever married men and women in US Census 2000 (Whites - 5% Sample)

$k = 1.1$, a low premium for having children within marriage; in this case we have that men marry at age 28 and women at 27.5. The pattern of the sex ratio is almost flat, with a small increase during the period of declining fecundity and then a more pronounced decline from mid 50's (due to higher male mortality rates). That is, if having children does not play a big role in the decision of marriage, the age difference tend to disappear, and the sex ratio of singles differs from parity only because differential mortality of men and women. As the value of having a family increases, men marry at older ages and women at younger ages. This causes the sex ratio to have an inverted U-shape that peaks during the decline in women's fecundity. One interesting case is what happen when $k = 1$. As shown in Figure 2.7, men marry on average younger than women. The sex ratio of singles is then decreasing during the entire life cycle (Figure 2.8). If $k = 1$, it is only mortality that causes the sex ratio to differ from parity and cause any age difference at marriage between men and women. The higher male mortality causes a scarcity of men increasing the probability of an offer per period and improving their marriage prospects. Given that this solution is calculated with the mortality rates for whites in the U.S. in 1995, an even greater imbalance in young mortality between men and women (for example Blacks in the U.S.) could have a big effect on the composition of the marriage market and age at marriage for men and women.

Figures 2.9 and 2.10 show the age at marriage and the pattern of the sex ratio for singles at different values of β. As one can imagine, people tend to marry later when they are more patient (higher levels of β). Also, as shown in Figure 2.10, the sex ratio tends to be flatter for levels of β within the usual range (0.90 to 0.99). However, what is striking is what happen at high discount rates

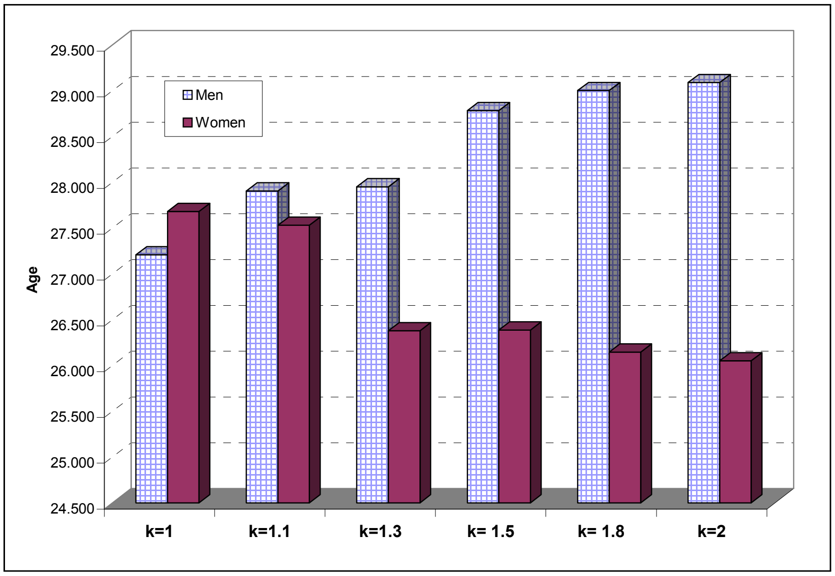

Figure 2.7: Average age at marriage for different values of k ($\beta = 0.91$)

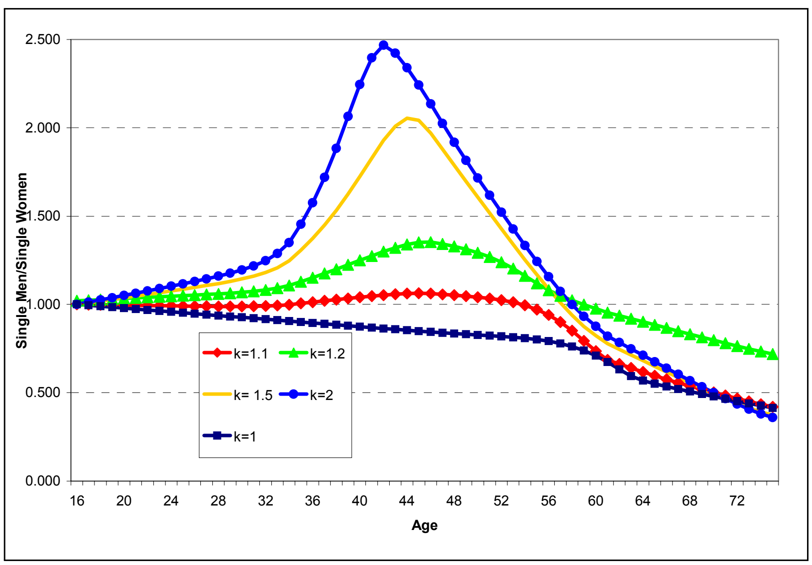

Figure 2.8: Ratio single men/single women by age for different levels of k ($\beta = 0.91$)

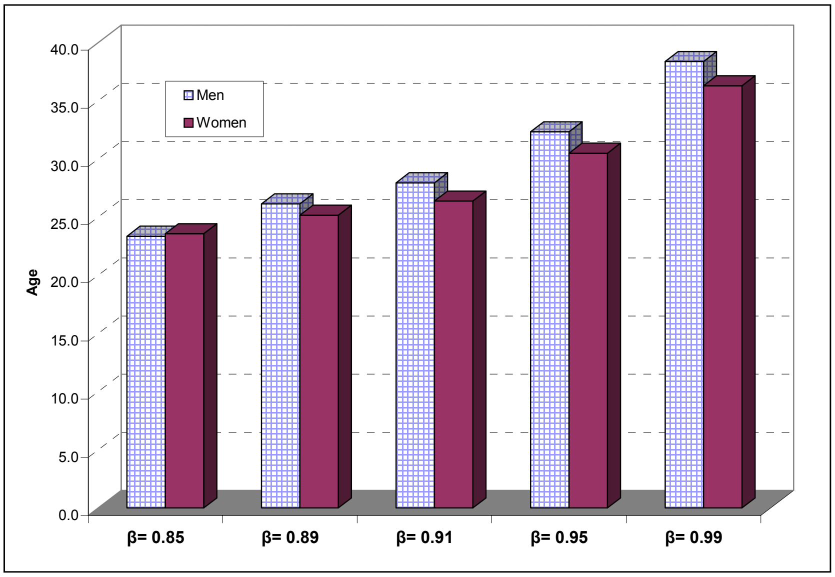

Figure 2.9: Average Age at Marriage for Different Values of β ($k = 1.3$)

(say $\beta = 0.85$). Here again men marry younger than women and the sex ratio is decreasing at all ages. When people discount future utility heavily enough, the differential mortality rates between men and women, even when very small, cause men to marry younger than women (same effect as when children within marriage are not valuable).

2.3 Comparison with Census Data

2.3.1 US Census 2000

In order to compare the model results with US Census data two sources of data are used. The data on age at first marriage is from the 1989-95 Marriage Detail File (MDF) of the U.S. Vital Statistics Registry.[8] For all other statistics the

[8] The Marriage Detail File has not been released since 1995 and is the closest data to the U.S Census 2000. The Census stopped asking age at first marriage in 1980.

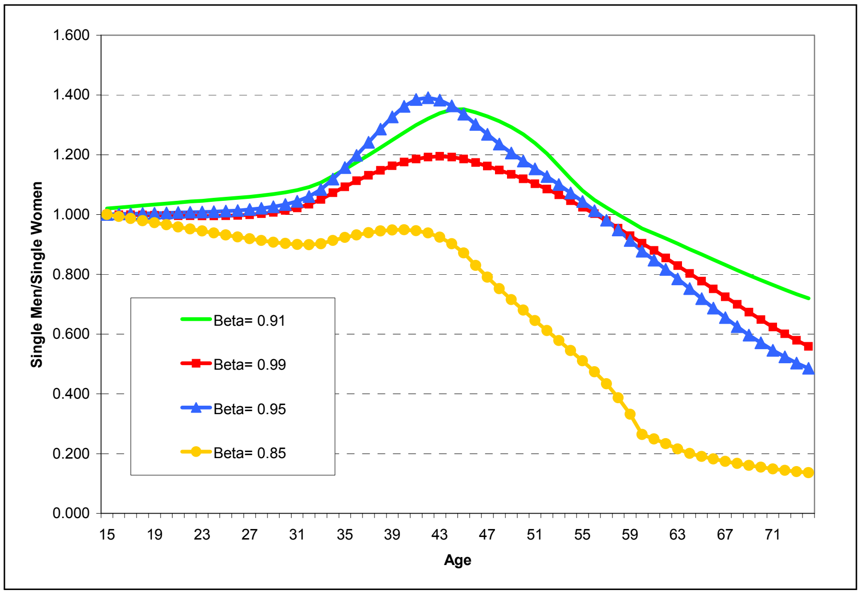

Figure 2.10: Ratio Single Men/Single Women by Age for Different Levels of β ($k = 1.3$)

data is from the 2000 IPUMS 5% data. People in institutions are excluded from the sample, and the analysis is limited to people born in the US. Despite pooling across cohorts, the fact that most marriages occur by age 40 minimizes that problem. Of particular interest is the different marriage markets for White and Black Americans. Therefore a separate analysis is conducted for Blacks and Whites.

Whites

Figure 2.11 and Table 2.1 show a comparison between the model results and the data when $\beta = 0.92$ and $k = 1.2$. The model predicts men and women marrying later (men at age 27.4 and women at 26.6 years old compared with the actual 26.1 and 25 respectively). Also the model suggests a smaller age difference at

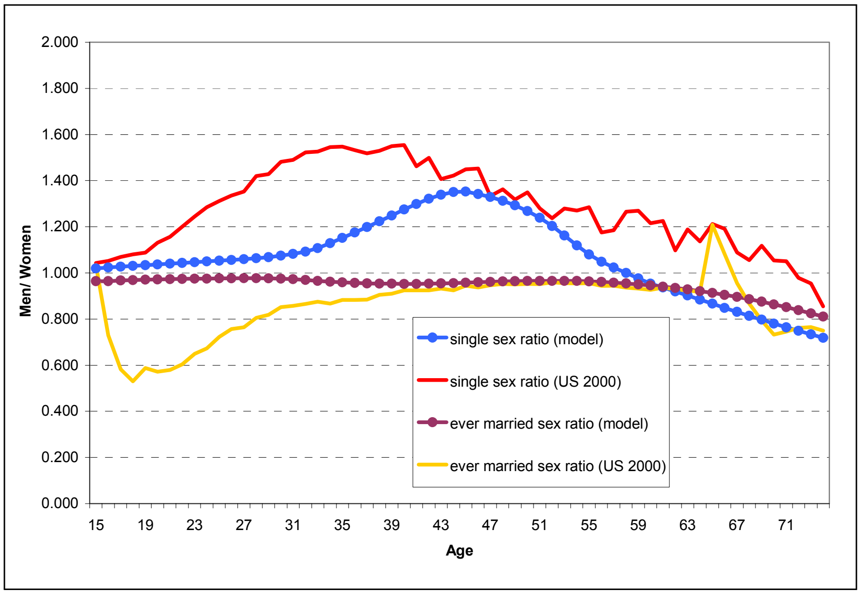

Figure 2.11: Sex Ratio for Single and Ever Married. US Census 2000 (Whites) and Model ($k = 1.2,\ \beta = 0.92$) with mortality rates for whites.

first marriage (0.8 year versus 1.6 years in the data). Table 2.1 also displays a comparison between the model and the census data and MDF for the average age of singles and ever married, the ratio single men/single women, the fraction of ever married and widows/ers (only widows in the model) and the sex ratio of ever married. Figure 2.11 shows the pattern of the sex ratio by age for singles and ever married. Observe that the ratio single male/single female peaks at around age 40 in the data and around the assumed end of fecundity (age 47) in the model results.

Blacks

Figure 2.12 and Table 2.2 show a comparison between the model results and the census data when $\beta = 0.92$ and $k = 1.2$. In this case while the model predicts

Model Predictions and US Census 2000: Whites

	Model (1)		US 2000 (2)	
	Men	**Women**	**Men**	**Women**
ß	0.92			
k	1.20			
Age of First Marriage	27.4	26.6	26.1	24.1
Average Age of Singles	27.2	26.9	28.1	27.2
Sex Ratio of Singles	1.07		1.25	
Ever Married (%)	0.721	0.765	0.740	0.790
Fraction of Singles 45 and Over	0.059	0.049	0.065	0.048
Fraction of Widows/ers		0.057	0.015	0.063
Average Age of Ever Married	48.1	48.6	47.9	47.6
Sex Ratio for Ever Married	0.94		0.90	
Total Sex Ratio	0.98		0.97	

Table 2.1: Comparison Between Model and US Census 2000 (Whites)

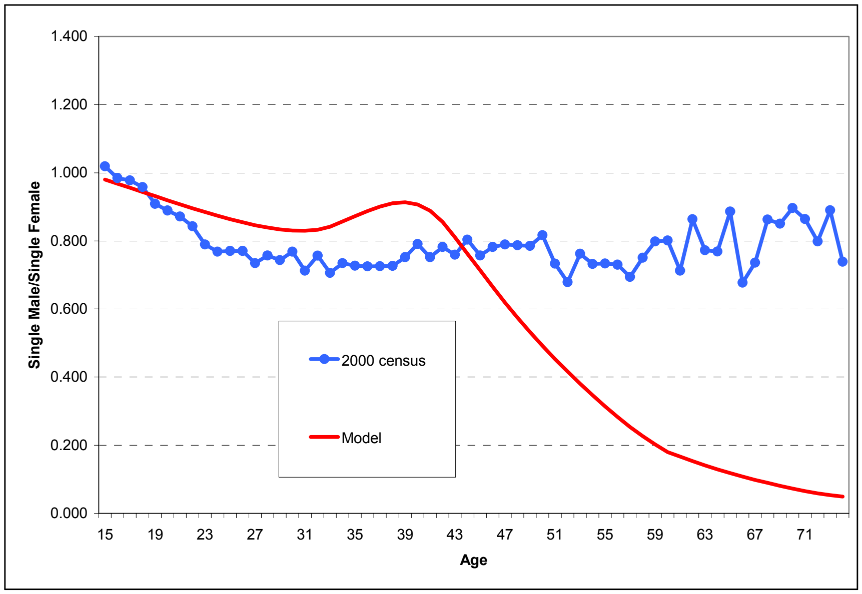

Figure 2.12: Ratio Single Males/Single Females. US Census 2000 (Blacks) and Model ($k = 1.2,\ \beta = 0.92$) with mortality rates for blacks.

women marrying older than men and the age at first marriage for black men falls short with respect to the data (26.3 versus 27.9)

How important is differential mortality? What is particularly important in the model is the *relative* mortality of men and women in a racial group. Figure 2.13 shows the relative mortality rate for men versus women at each age. Notice that while the mortality rate of men is greater than the mortality rate of women for both Blacks and whites, the mortality rate for Black men relative to Black women is extremely large between ages 15 and 28. [9]

[9] It is well known that the fraction of men in federal or state prison or local jails at these ages is also differentially high for Black men. Statistics from the Bureau if Justice Statistics suggest that approximately 9-12% of Black men between ages 18 and 29 are in federal or state prison or in a local jail. This would tend to reinforce the results discussed here.

Model Predictions and US Census 2000: Blacks				
	Model		**US 2000**	
	(1)		**(2)**	
	Men	**Women**	**Men**	**Women**
ß	0.920			
k	1.20			
Age of First Marriage	26.26	26.37	27.9	26.7
Average Age of Singles	25.7	28.0	28.9	29.7
Sex Ratio of Singles	0.880		0.828	
Ever Married (%)	0.726	0.718	0.562	0.579
Fraction of Singles Over 45	0.043	0.076	0.087	0.140
Fraction of Widows/ers		0.086	0.024	0.081
Average Age of Ever Married	46.3	47.8	46.8	47.2
Sex Ratio for Ever Married	0.914		0.771	
Total Sex Ratio	0.90		0.79	

Table 2.2: Comparison Between Model and US Census 2000 (Blacks)

How does the higher mortality of Black men effect the marriage market equilibrium? To analyze this, Figure 2.14 shows the reservation values for Black men and women as well as white men and women at each age. Here the same parameter values apply to both races but each race is calculated according to their own race (and gender) specific mortality rates. In Figure 2.14 it is clear that unlike whites, where the reservation value for men is higher than for women only at older ages, for Blacks, the reservation value for men is higher than the reservation for women at all ages. That is, if children within marriage are not very valuable (relative to outside of marriage), then the relative scarcity of fertile women plays only a weak role in the market. Conversely, the higher mortality rates for Black men give them the bargaining power in the market. For that reason, Black men receive relatively more offers than women reducing their waiting time to marriage from what it would be with lower mortality. The net result is that Black men and women tend to marry around the same age.

Figure 2.15 shows that unlike for white Americans, the ratio of single men to single women falls below parity at all ages for Black men. Higher male mortality would seem to almost mechanically cause the sex ratio of single men to single women to fall below parity (because there are generally fewer men alive than women). However, white men also have higher mortality than white women. The mortality of white men is not sufficiently high relative to white women to offset the natural scarcity that young women enjoy because of women's fecundity is limited and children within marriage are valuable ($k > 1$). For whites, the scarcity of fertile women (driven by limited fecundity) dominates the scarcity of men (driven by differential mortality). Thus white women are choosier at young ages causing young men to wait to marry until the terms of trade change in their favor (as women's fecundity declines). Thus for whites, even though male mortality is greater than female mortality, the sex ratio of single men to single women remains above parity for much of the life cycle.

Blacks are different. Black women face the same scarcity producing effect of a limited fecundity horizon as white women. However, Black male mortality is sufficiently greater than Black female mortality to offset the natural scarcity women usually enjoy at young ages. With differentially high Black male mortality, it is men that are scarce over the entire life cycle. Thus Black men do not face the same incentives to delay marriage as they do not have growing scarcity over time (as do white men). For this reason, Black men and women marry at close to the same age and the fewer Black men that survive mortality end up driving the sex ratio below parity at all ages. Two factors play a role in making the sex ratios for Blacks decrease over the entire life cycle: First the higher mortality rates for Black men and second, the fact that Black men do not wait more than women in order to marry.

The consequence of this behavior is that a larger fraction of Black women never marry. As shown in Tables 2.1 and 2.2, in 2000, 13% of Black women aged 45 and over never married compared to only 5 % of White women. The predictions of the model is qualitatively similar. However the fraction of women predicted never to marry are lower for both races (9% for Blacks versus 5% for Whites).[10]

This finding is related to the Wilson Hypothesis, although the mechanism leading to lower marriage rates is different. Wilson's model is typically interpreted as Black women rejecting Black men (who have made offers) because they do not

[10] Remember that for simplicity the model assumes that there is no utility of remaining single.

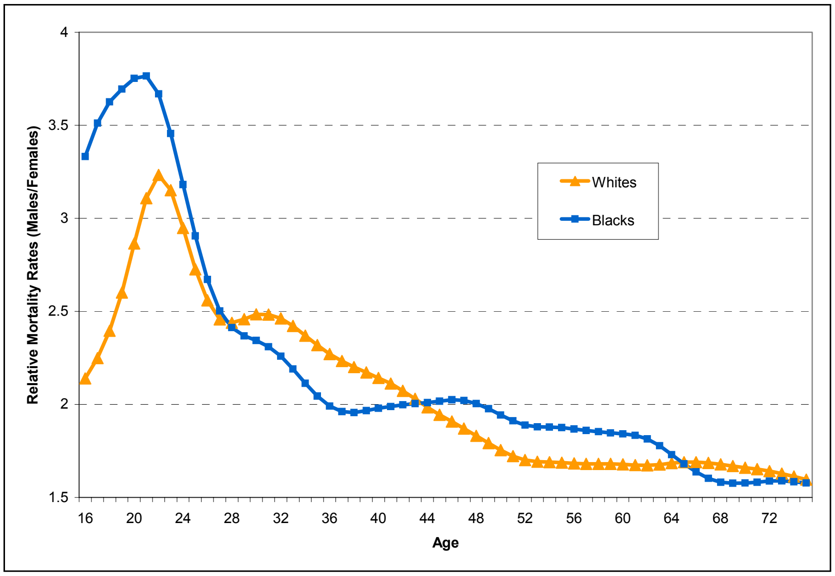

Figure 2.13: Relative Mortality Rates (Males/Females) for Blacks and Whites under Age 75

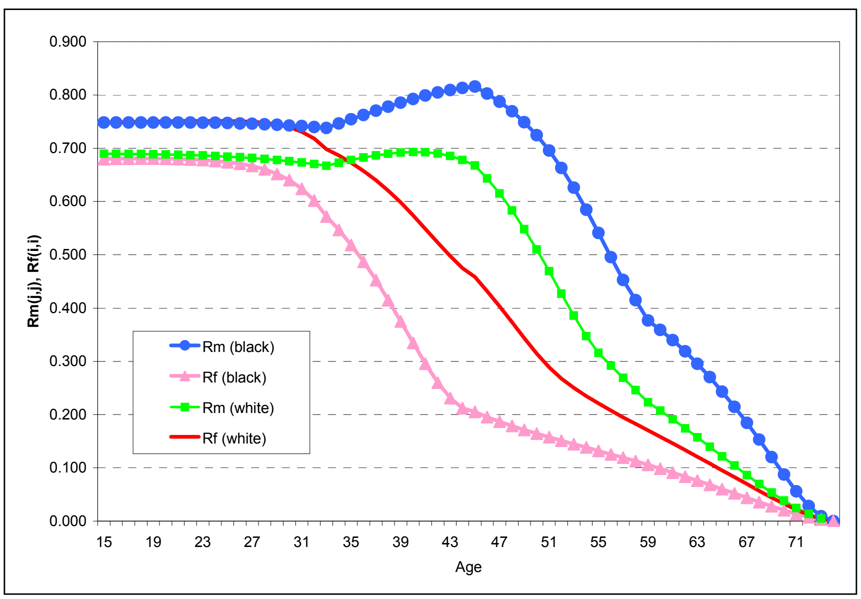

Figure 2.14: Reservation Values for Potential Spouses of the Same Age. Model with $k = 1.2$, $\beta = .92$ and Mortality for Blacks and Whites in 1992.

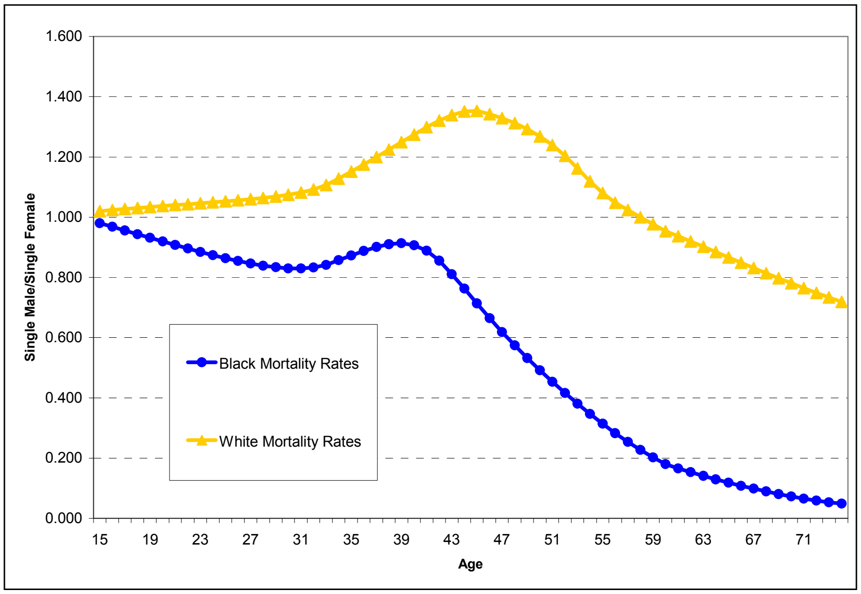

Figure 2.15: Ratio Single Men/Single Women. Model with $k = 1.2$, $\beta = .92$ and Mortality for Blacks and Whites in 1992.

bring enough to the marriage (i.e. are not "marriageable" because of poor job prospects). In my model Black women also delay marriage (or do not marry) but it is because men reject women. Men reject women because the higher male mortality makes them scarce and the low value of children within marriage make women not as valuable to men.

2.3.2 France

Table 2.3 and Figure 2.16 compare the model results with the 1999 France census. Data is from the National Institute for Statistics and Economic Studies - France for the entire population who lives in metropolitan areas. The model results are for $\beta = 0.925$ and $k = 1.25$. Notice than in this case the predictions of the model match quite accurately the data.

	Model		France 1999	
	(1)		**(2)**	
	Men	**Women**	**Men**	**Women**
β	0.925			
k	1.25			
Age of First Marriage	29.4	27.9	30.2	28.1
Sex Ratio of Singles	1.13		1.16	
Ever Married (%)	0.69	0.74	0.61	0.67
Fraction of Widows		0.07	0.02	0.10
Sex Ratio for Ever Married	0.90		0.87	
Total Sex Ratio	0.96		0.97	

Table 2.3: Comparison Between Model and France Census 1999

2.3.3 Sweden

Here we compare the model results with data for the year 2000 in Sweden. Data is form Statistics Sweden for the entire population. The model results are for $\beta = 0.95$ and $k = 1.25$. Here the model predicts accurately the age at first marriage for men (32.3 years) and women marry in average one year earlier in the data than when the model results predict (30.1 versus 31.3 years). Thus, the model predict a smaller age difference than the actual mean age difference (1 year versus 2.3 years).

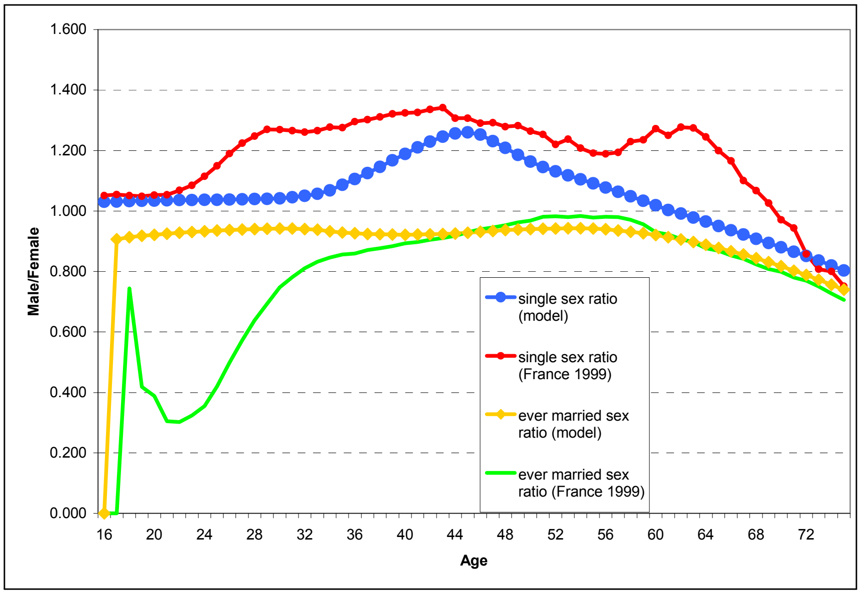

Figure 2.16: Sex Ratio for Single and Ever Married. Census 1999 (France) and Model ($k = 1.25,\ \beta = 0.925$) with mortality rates in France.

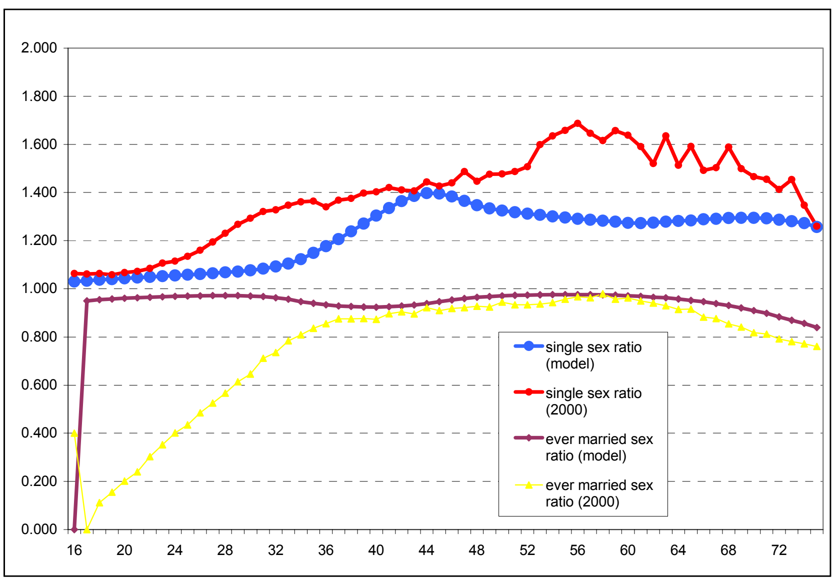

Figure 2.17: Sex Ratio for Single and Ever Married. Census 2000 (Sweden) and Model ($k = 1.25,\ \beta = 0.95$) with mortality rates in Sweden.

	Model		Sweden 2000	
	(3)		(4)	
	Men	**Women**	**Men**	**Women**
β	**0.95**			
k	**1.25**			
Age of First Marriage	32.3	31.3	32.4	30.1
Sex Ratio of Singles	1.12		1.24	
Ever Married (%)	0.64	0.68	0.55	0.64
Fraction of Widows/ers		0.05	0.03	0.08
Sex Ratio for Ever Married	0.95		0.87	
Total Sex Ratio	1.00		1.01	

Table 2.4: Comparison Between Model and Census 2000 (Sweden)

2.3.4 US in Previous Decades

Figure 3.3 shows the median age at first marriage in US since 1940. The tendency to delaying marriage over the last decades is clear. This reflects an increase in both mean and variance of the age at marriage for men and women. For example the mean age at first marriage for white males married between 1960 and 1965 was 23.5 years, and 21 years for white females. In 1975-80 the mean for white males was 23.9 and 21.8 for white females. Note that the age difference between men and women appear to be quite stable.[11]

Figure 2.19 shows the sex ratio (single males/single females) for whites in

[11] For an empirical study about the change in marriage patterns in the US in last decades, see Rose (2001).

the US for 1960, 1980 and 2000. The age pattern of sex ratios appear to have changed over time. First, the pattern of the sex ratio by age is flatter in 1980 with respect to 1960 and even more flat in 2000. Second, the "peak" sex ratio in 1960 and 1980 was in the mid 20's. This peak moved to the mid 30's in 2000 census. Figure 2.20 shows the pattern of the sex ratio (single males/single females) for blacks in the US for 1960, 1980 and 2000. As in the case of whites, the pattern appear to have changed over time. While in 1960 the graphic shows a very similar pattern to the one for whites, that is not the case for 1980 and 2000.

In the case of 1960, in order to achieve a similar pattern to that of the data it would be necessary to assume that women's fecundity starts to decrease at age 24. This is a signal that this 2 parameter model is not enough in order to explain the behavior of the marriage market 40 years ago.

The last few decades observed more similar roles for men and women. For example, in the US, the level of education have become increasingly similar and women's labor participation have increased dramatically in the last 20 years. Moreover, marriage specialization have consequently decreased[12], and traditional roles in marriage are not so common as they were in the past. Even though social norms have changed making that roles of men and women became more and more similar, the fecundity horizon differences will persist and that can be an explanation of why women still tend to marry older men.

2.3.5 Developing Countries

Figure 2.21 shows the sex ratio by age for Kenya and Vietnam in 1999 and Mexico in 2000. It easy to tell that the data for developed countries match better with the

[12] For a study on the decline in marriage specialization, see Lundberg and Rose (1998)

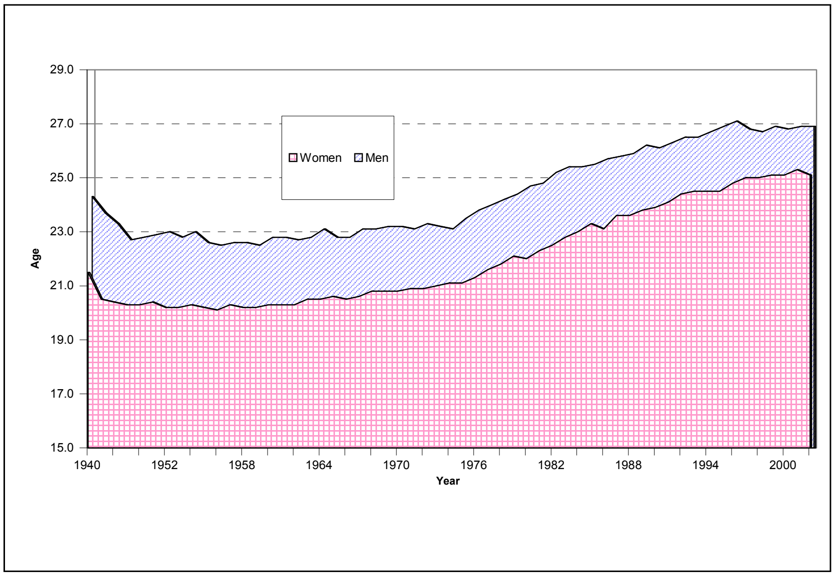

Figure 2.18: Median Age at First Marriage in the US

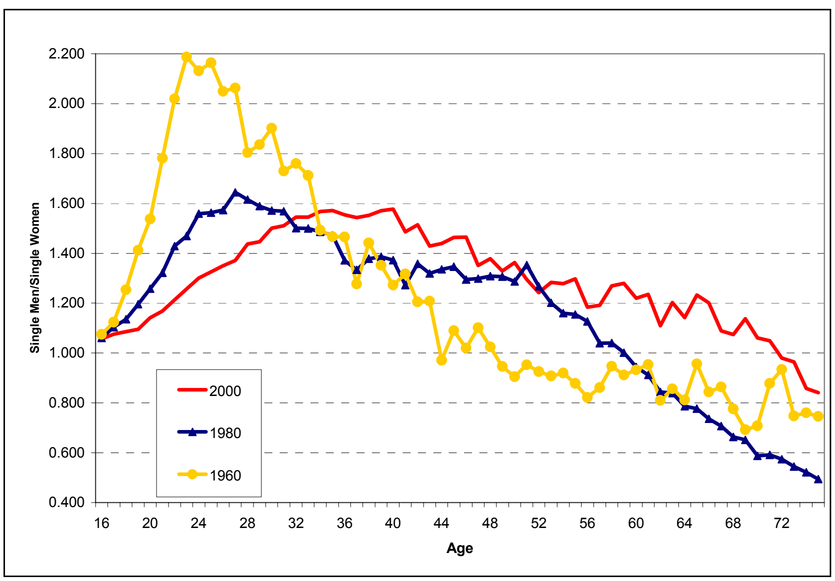

Figure 2.19: Sex Ratio (Single Men/Single Women) in the US (Whites)

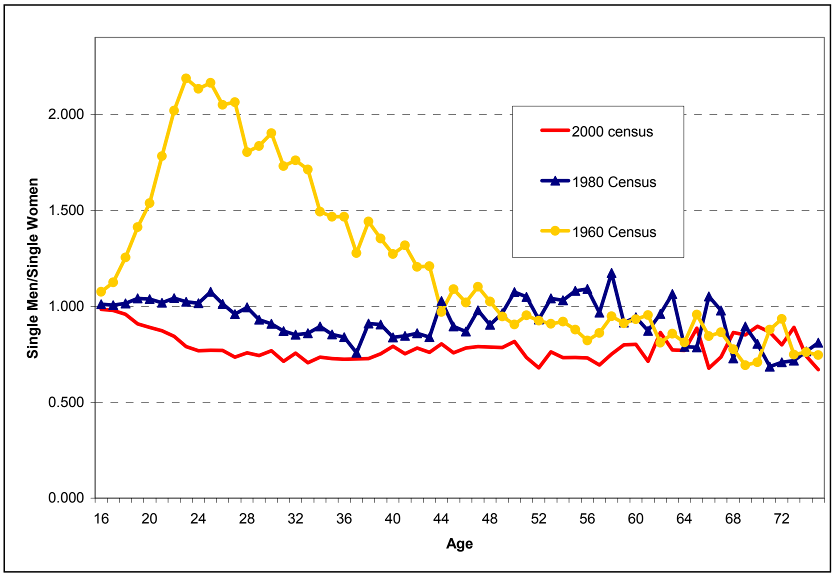

Figure 2.20: Sex Ratio (Single Men/Single Women) in the US (Blacks)

model that the one for developing countries. Notice than, for these countries the pattern is very different than the in the case of the US or the European countries described above. In the three countries of the figure the sex ratio reaches a maximum at ages 24-27 and then decreases sharply. This is somewhat similar to the pattern in the US in 1960 (Figure 2.19 above).

In summary, this model does a better job explaining the patterns in developed countries than in developing countries, and in recent times compared to previous decades. In this model the evolution of the sex ratio with age is entirely determined by the different fecundity horizon of men and women. For that reason, all other differences between sexes intentionally excluded in this model, obviously also play a role in marriage behavior. Social norms may also be important. For example, when we find a peak in the sex ratio around 25 years old, as in Figure 2.21, or the sharply increase in the sex ratio in the early twenties in US in 1960, perhaps we are talking about some "social limit" to the age when women should

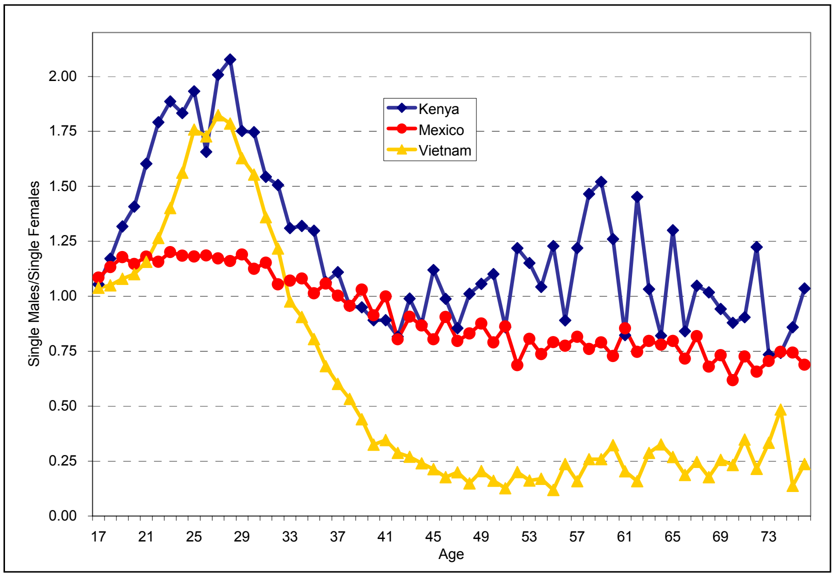

Figure 2.21: Ratio Single Men/Single Women. Kenya and Vietnam, 1999 and Mexico 2000

marry. What is important is that his social norm appear to be more significant in societies where traditional marital roles are still well defined.

Chapter 3

A Model of Assortative Mating

In this chapter we extend the model of the previous chapter to a two market framework in order to analyze the effect of limited fecundity, mortality and size of the market on assortative mating. As before, after a description of the model, a numerical solution of the model and a discussion of its implications, the model implications will be compared with US data. The following are the modified assumptions:

There are two groups of men and women, for example black and white men and women. The total single population will be the sum of single people in the two groups:

$$M = M_1 + M_2$$

$$F = F_1 + F_2$$

So, for example M is the number of single men as before while M_1 might be the number of single white men and M_2 the number of single black men.

As before, an exogenous flow of single people of each group and age 1, $f_1(1)$, $f_2(1)$, $m_1(1)$ and $m_2(1)$ enter the market each period and the men and women who do not marry will remain in the market.

Now the total number of single men of all ages t from Group $i \in \{1, 2\}$ is

$$M_i = \sum_{t=1}^{T} m_i(t)$$

and the total number of women of all ages s

$$F_i = \sum_{s=1}^{T} f_i(s)$$

Therefore, the fraction of single men and women from Group i and of ages s, t respectively becomes

$$\begin{aligned} q_i(t) &= \frac{m_i(t)}{M_i} \\ p_i(s) &= \frac{f_i(s)}{F_i} \end{aligned}$$

The assumptions of the previous chapter about value of children and declining fertility remain untouched here. However, we make an additional assumption about intergroup marriage. Even though people can marry across groups, we assume, as in Wong (2003) that people receive a discount in utility for marrying someone of a different group. This discount will be proportional at a rate $\tau \in [0, 1)$[1,2]. Therefore the payoff per period that a woman or a man (ages s, t respectively) will receive for marrying a person of the same or different group will be the following:

[1] Wong (2003) assumes that the discount in utility is a lump sum.

[2] This is a model of "own group preference". However, the model is general enough to allow for example that both groups of women prefer the same group of men or vice-versa, or that the preferences differ between groups or between men and women. Suppose for example that Group 1 and 2 are index of potential income. Therefore all women regardless of their earnings will prefer men of Group 1.

	Spouse of the same group	Spouse of different group
Women	$k^f_{s,t}y$	$(1-\tau)\, k^f_{s,t}y$
Men	$k^m_{t,s}x$	$(1-\tau)\, k^m_{t,s}x$

where $k^m_{t,s}$ and $k^f_{s,t}$ follow the patterns described in (2.24) and (2.25).

As before, we assume random matching and at most one meeting per period. However here the matching functions will be slightly different from those described in equations (2.1), (2.3) and (2.2). Now the probability of meeting someone of the opposite sex and of the certain group will depend on the relative size of the group and of the relative scarcity of each sex within the group. Therefore one man will meet one woman with probability

$$\eta^m = \Psi_1 \left(\frac{M_1}{F_1}\right)^{\theta-1} + \Psi_2 \left(\frac{M_2}{F_2}\right)^{\theta-1} \tag{3.1}$$

and a woman will meet a man with probability

$$\eta^f = \Psi_1 \left(\frac{M_1}{F_1}\right)^{\theta} + \Psi_2 \left(\frac{M_2}{F_2}\right)^{\theta} \tag{3.2}$$

where $\Psi_1 = \frac{M_1+F_1}{M+F}$ and $\Psi_2 = \frac{M_2+F_2}{M+F}$ are the relative weight of each group in the total population of singles. The fecundity horizon is assumed equal for both groups, but the mortality rates can differ. Therefore,

$$\delta^m_{i,t} \quad \text{for a man of age } t < T$$

$$\delta^f_{i,s} \quad \text{for a woman of age } s < T$$

$$\delta^m_{i,T} = \delta^f_{i,T} = 1$$

and

$$\Delta ij_{s,t} = \delta^f_{i,s}\left(1-\delta^m_{i,t}\right) + \left(1-\delta^f_{j,s}\right)\delta^m_{i,t} + \delta^f_{j,s}\delta^m_{i,t}$$

is the probability that a marriage between a husband of Group i and age t and a wife from Group j and age s ends in the current period because of the death of one of the spouses, where $i, j \in \{1, 2\}$.

3.1 The Man's Problem

Each period a man from Group i and of age t will meet at most a woman from the same group and of age s with probability $\Psi_i \left(\frac{M_i}{F_i}\right)^{\theta-1} p_i\left(s\right)$. Therefore, the probability that a man from Group i and of age t receives a proposal from a woman from the same group and age s is

$$\alpha_{ii}\left(t,s\right) = \Psi_i \left(\frac{M_i}{F_i}\right)^{\theta-1} p_i\left(s\right) \left(1 - G_m(R_{ii}^f\left(s,t\right))\right) \tag{3.3}$$

and the probability of an offer from a woman from a different Group $j \neq i$ and age s is

$$\alpha_{ij}\left(t,s\right) = \Psi_j \left(\frac{M_j}{F_j}\right)^{\theta-1} p_j\left(s\right) \left(1 - G_m(R_{ji}^f\left(s,t\right))\right) \tag{3.4}$$

where $R_{ii}^f\left(s,t\right)$ and $R_{ji}^f\left(s,t\right)$ are the reservation values that a woman of age s from Group $i, j \in \{1,2\}$ set for a man of age t from Group i, respectively

3.1.1 Expected Utility of Marrying at age t

The expected utility that a man from Group i of age t derives from marrying a woman of age s will be similar to that established in equation (2.27) with respect to age, but with a discount in utility when marrying someone of Group $j \neq i$.

$$u_{ii}^m\left(t,s\right) = \sum_{l=0}^{Min(T-s,T-t)} \beta^l \prod_{r=0}^{l} \left(1 - \Delta ii_{s+r,t+r}\right) k_{t,s}^m \int_{R_{ii}^m(t,s)}^{xmax} x g_f(x) dx \tag{3.5}$$

$$u_{ij}^m\left(t,s\right) = (1-\tau) \sum_{l=0}^{Min(T-s,T-t)} \beta^l \prod_{r=0}^{l} \left(1 - \Delta ij_{s+r,t+r}\right) k_{t,s}^m \int_{R_{ij}^m(t,s)}^{xmax} x g_f(x) dx \tag{3.6}$$

Given that the probability of a marriage offer from women of different ages differ, the expected utility of marrying at age t for a man of Group i will be

$$U_i^m(t) = \sum_s \alpha_{ii}(t,s)\, u_{ii}^m(t,s) + \sum_s \alpha_{ij}(t,s)\, u_{ij}^m(t,s) \tag{3.7}$$

3.1.2 Probabilities of Marriage for Men

A single man of Group i and age t will marry a woman from the same group of age s with probability

$$\phi_{ii}(t,s) = \alpha_{ii}(t,s)\left(1 - G_f\left(R_{ii}^m(t,s)\right)\right) \tag{3.8}$$

and a woman of the other group with probability

$$\phi_{ij}(t,s) = \alpha_{ij}(t,s)\left(1 - G_f\left(R_{ij}^m(t,s)\right)\right) \tag{3.9}$$

Therefore, the hazard rate that a man of Group i marries at age t will be

$$\begin{aligned} \Phi_i(t) &= \sum_s \phi_{ii}(t,s) + \sum_s \phi_{ij}(t,s) \qquad (3.10) \\ &= \sum_s \alpha_{ii}(t,s)\left(1 - G_f\left(R_{ii}^m(t,s)\right)\right) + \sum_s \alpha_{ij}(t,s)\left(1 - G_f\left(R_{ij}^m(t,s)\right)\right) \end{aligned}$$

3.1.3 Objective Function for Men

Given Equations (3.7) and (3.10), the objective function for any man at a given age t is the following

$$\sum_{t=t}^{T} \beta^{t-t} U_i^m(t) \prod_{s=t+1}^{t} \left(1 - \delta_{i,s-1}^m\right)\left(1 - \phi_i(s-1)\right) \tag{3.11}$$

The Bellman Equation for the problem above is

$$\begin{aligned} V_i^m(t) &= \underset{D_t^m}{Max}\left[U_i^m(t) + \left(1 - \delta_{i,t}^m\right)\left(1 - \phi_i\right)\beta V_i^m(t+1)\right] \qquad (3.12) \\ V_i^m(T) &= U_i^m(T) \end{aligned}$$

$$D_{i,t}^m = \begin{cases} 1 & \text{if } x > R_{ii}^m(t,s) \text{ or } x > R_{ij}^m(t,s) \\ 0 & otherwise \end{cases}$$

where $D_{i,t}^m$ is the decision of marrying at age t for a man from Group i.

3.1.4 Men's Reservation Values

The Reservation Values set by men can be obtained recursively given that

$$R_{ii}^m(t,s)\left(\sum_{l=0}^{Min(T-s,T-t)} \beta^l \prod_{r=0}^{l} (1-\Delta ii_{s+r,t+r})\, k_{t,s}^m\right) = \beta V_i^m(t+1) \qquad (3.13)$$

$$\text{if } 1 \le t \le T-1$$

and

$$(1-\tau)\, R_{ij}^m(t,s)\left(\sum_{l=0}^{Min(T-s,T-t)} \beta^l \prod_{r=0}^{l} \left(1-\Delta ij_{s+r,t+r}\right) k_{t,s}^m\right) = \beta V_i^m(t+1) \qquad (3.14)$$

$$\text{if } 1 \le t \le T-1$$

The reservation value that a man of age t from Group i sets for a given woman from the same group of age s is

$$R_{ii}^m(t,s) = \begin{cases} \dfrac{1}{\left(\sum_{l=0}^{Min(T-s,T-t)} \beta^l \prod_{r=0}^{l} (1-\Delta ii_{s+r,t+r}) k_{t,s}^m\right)} \beta V_i^m(t+1) & \text{if } 1 \le t < T \\ 0 & \text{if } t = T \end{cases} \qquad (3.15)$$

and for a woman of Group $j \neq i$

$$R_{ij}^{m}(t,s) = \begin{cases} \frac{1}{(1-\tau)\left(\sum\limits_{l=0}^{Min(T-s,T-t)} \beta^{l} \prod\limits_{r=0}^{l} \left(1-\Delta ij_{s+r,t+r}\right) k_{t,s}^{m}\right)} \beta V_{i}^{m}(t+1) & \text{if } 1 \leq t < T \\ 0 & \text{if } t = T \end{cases} \quad (3.16)$$

3.2 The Woman's Problem

The probability that a woman of age s from Group i receives an marriage offer from a man of age t from the same group is

$$\lambda_{ii} = \Psi_{i} \left(\frac{M_{i}}{F_{i}}\right)^{\theta} q_{i}(t)\left(1 - G_{f}(R_{ii}^{m}(t,s))\right) \quad (3.17)$$

and the probability of receiving an offer from a man from a different group j

$$\lambda_{ij} = \Psi_{j} \left(\frac{M_{j}}{F_{j}}\right)^{\theta} q_{j}(t)\left(1 - G_{f}(R_{21}^{m}(t,s))\right) \quad (3.18)$$

3.2.1 Expected Utility of Marrying at age s

In the same way as for men, the expected utility that a woman of age s and Group i derives from marrying a man of age t depends on the group of the spouse and on expected length of the marriage

$$u_{ii}^{f} = \sum_{l=0}^{Min(T-s,T-t)} \beta^{l} \prod_{r=0}^{l} \left(1 - \Delta ii_{s+r,t+r}\right) k_{s,t}^{f} \int_{R_{ii}^{f}(s,t)}^{ymax} y g_{m}(y) dy \quad (3.19)$$

for a husband of the same group i and

$$u_{ij}^{f} = (1-\tau) \sum_{l=0}^{Min(T-s,T-t)} \beta^{l} \prod_{r=0}^{l} \left(1 - \Delta ij_{s+r,t+r}\right) k_{s,t}^{f} \int_{R_{ij}^{f}(s,t)}^{ymax} y g_{m}(y) dy \quad (3.20)$$

for a husband from Group $j \neq i$.

As in the case of men, the expected utility for a woman from Group i of marrying at age s is

$$U_i^f(s) = \sum_t \lambda_{ii}(s,t)\, u_{ii}^f(s,t) + \sum_t \lambda_{ij}(s,t)\, u_{ij}^f(s,t) \tag{3.21}$$

3.2.2 Probabilities of Marriage for Women

A single woman of Group i and age s will marry a man from the same group and age s with probability

$$\gamma_{ii}(s,t) = \lambda_{ii}(s,t)\left(1 - G_m\left(R_{ii}^f(s,t)\right)\right) \tag{3.22}$$

and a man from Group $j \neq i$ with probability

$$\gamma_{ij}(s,t) = \lambda_{ij}(s,t)\left(1 - G_m\left(R_{ij}^f(s,t)\right)\right) \tag{3.23}$$

The hazard rate of marriage for a woman at age s is defined as

$$\begin{aligned} \Gamma_i(s) &= \sum_t \gamma_{ii}(s,t) + \sum_t \gamma_{ij}(s,t) \\ &= \sum_t \lambda_{ii}\left(1 - G_m\left(R_{ii}^f(s,t)\right)\right) + \sum_t \lambda_{ij}\left(1 - G_m\left(R_{ij}^f(s,t)\right)\right) \end{aligned} \tag{3.24}$$

3.2.3 Objective Function for Women

Given Equations (3.21) and (3.24), the objective function of a single woman at age s is the following

$$\sum_{s=t}^{T} \beta^{s-t} U_i^f(s) \prod_{s=t+1}^{s}\left(1 - \delta_{i,s-1}^f\right)\left(1 - \Gamma_i(s-1)\right) \tag{3.25}$$

As is the case of the man above, the Bellman Equation for the woman's problem is then

$$\begin{aligned} V_i^f(s) &= \underset{D_{i,s}^f}{Max}\left[U_i^f(s) + \left(1 - \delta_{i,s}^f\right)\left(1 - \Gamma_i(s)\right)\beta V_i^f(s+1)\right] \\ V_i^f(T) &= U_i^f(T) \end{aligned} \tag{3.26}$$

$$D^f_{i,s} = \begin{cases} 1 & \text{if } y > R^f_{ii}(s,t) \text{ or } y > R^f_{ij}(s,t) \\ 0 & otherwise \end{cases}$$

where $D^f_{i,s}$ is the decision of a woman of Group i to marry at age s a man of age t.

3.2.4 Reservation Values for Women

As in the case for men, the reservation values for women can be obtained recursively. Given that

$$R^f_{ii}(s,t)\left(\sum_{l=0}^{Min(T-s,T-t)} \beta^l \prod_{r=0}^{l} (1-\Delta ii_{s+r,t+r})\, k^f_{s,t}\right) = \beta V^f_i(s+1) \quad (3.27)$$

$$\text{if } 1 \leq s < T$$

The reservation value for a woman of age s with respect to a man of age t from Group i will be

$$R^f_{ii}(s,t) = \begin{cases} \dfrac{1}{\left(\sum_{l=0}^{Min(T-s,T-t)} \beta^l \prod_{r=0}^{l}(1-\Delta ii_{s+r,t+r})k^f_{s,t}\right)} \beta V^f_i(s+1) & \text{if } 1 \leq s < T \\ 0 & \text{if } s = T \end{cases} \quad (3.28)$$

and with respect a man of Group $j \neq i$

$$R^f_{ij}(s,t) = \begin{cases} \dfrac{1}{(1-\tau)\left(\sum_{l=0}^{Min(T-s,T-t)} \beta^l \prod_{r=0}^{l}\left(1-\Delta ij_{s+r,t+r}\right)k^f_{s,t}\right)} \beta V^f_i(s+1) & \text{if } 1 \leq s < T \\ 0 & \text{if } s = T \end{cases} \quad (3.29)$$

3.3 Stocks of Single and Married People

3.3.1 Men

The stocks of singles of age t and Group i ($i \in \{1,2\}$)will be equal to the surviving singles of age $t-1$ who did not marry during the last period. That is

$$m_i(t) = m_i(t-1)\left(1-\delta^m_{i,t-1}\right)(1-\Phi_s(t-1)) \tag{3.30}$$

Similarly, the total number of men who marry at age t will be

$$h_i(t) = m_i(t)\Phi_i(t) \tag{3.31}$$

and the stock of married men of age t will be the sum of the surviving males who married at age t or younger. That is,

$$H_i(t) = \sum_{t=1}^{t} h_i(t) \prod_{r=t}^{t-1}\left(1-\delta^m_{i,r}\right) \tag{3.32}$$

We will also define the fraction of men of age t and Group i who marry a woman of the other group. Therefore, the fraction of men of Group i and age t who marry women of Group $j \neq i$ (using equations (4.5) and (3.10)) is

$$\varphi_{ij}(t) = \frac{\sum_s \phi_{ij}(t,s)}{\Phi_i(t)} \tag{3.33}$$

3.3.2 Women

Similarly to the previous case, we define the number of single women of age s and Group i ($i \in \{1,2\}$) as follows

$$f_i(s) = f_i(s)\left(1-\delta^f_{i,s-1}\right)(1-\Gamma_i(s-1)). \tag{3.34}$$

The total of women who marry at age t will be

$$w_i\left(s\right)=f_i(s)\Gamma_i(s) \tag{3.35}$$

As for men, the stock of married women of age s is

$$W_i(s)=\sum_{t=1}^{s}w_i\left(t\right)\prod_{r=t}^{s-1}\left(1-\delta_{i,r}^{f}\right) \tag{3.36}$$

Similarly as before, the fraction of women of Group i and age s who marry men of Group $j\neq i$ (using equations (4.8) and (3.24)) is

$$\varpi_{ij}\left(s\right)=\frac{\sum_t\gamma_{ij}(s,t)}{\Gamma_i(s)} \tag{3.37}$$

3.4 Numerical Solution

Now we can solve numerically the system formed by Equations (3.12), (3.30), (3.26) and (3.34). The distribution functions $G_f\left(x\right)$ and $G_m\left(y\right)$ are uniform with support $[0,1]$ (as in the previous chapter) and the values given to the parameters will be the following:

$T=60$ (75 years old)	$N=45$ (60 years old)
$L_1=20$ (35 years old)	$L=30$ (46 years old)
$\mu=0.9$	$k=1.2$
$\beta=0.92$	$\theta=0.5$
$m_1\left(1\right)=m_2\left(1\right)=100$	$f_1\left(1\right)=f_2\left(1\right)=100$

For now we assume that the mortality rates for men and women are the same for the two groups (U.S. 1992 for whites).

3.4.1 Different Levels of Within Group Preference (τ)

Figures 3.1 and 3.2 display the evolution of the reservation values that people of Group $i \in \{1, 2\}$ have for people of the same and different groups when people prefer to marry within a group. Since in this example the two groups have the same size the reservation values will be the same. The solid line represent the case when people are indifferent about marrying within or across groups $(\tau = 0)$, which is equivalent to the baseline case of the previous chapter. The preference for within group marriage $(\tau > 0)$ acts as a friction in the market. Therefore men and women lower the reservation values for people of the same group (they will only meet them 50% of the time) and raise them for people of the other group. However, after comparing figures 3.1 and 3.2 one may notice that for women, reservation values tend to converge more rapidly after the age of 30 than for men. This has direct consequences on the dynamics of intergroup marriage of men and women as will be shown below.

The effect of the added search friction introduced by adding a preference for marrying within a group is shown by Table 3.1. As the rate of group preference (τ) increases, the fraction of intergroup marriage and the fraction of people ever married decreases. In addition, people tend to marry later in general if they end up marrying someone outside their group. For example, if $\tau = 0.2$, the average age of marriage for a woman is 26.7 years but if she marries outside their group the average age increases to 27.7 years.

One interesting result of the model is that the willingness to marry someone outside ones own group increase with age. Figure 3.3 shows the fraction of women and men marrying someone outside their own group by age for different levels of τ (equations (3.33) and (3.37)). The solid line represents the case when people

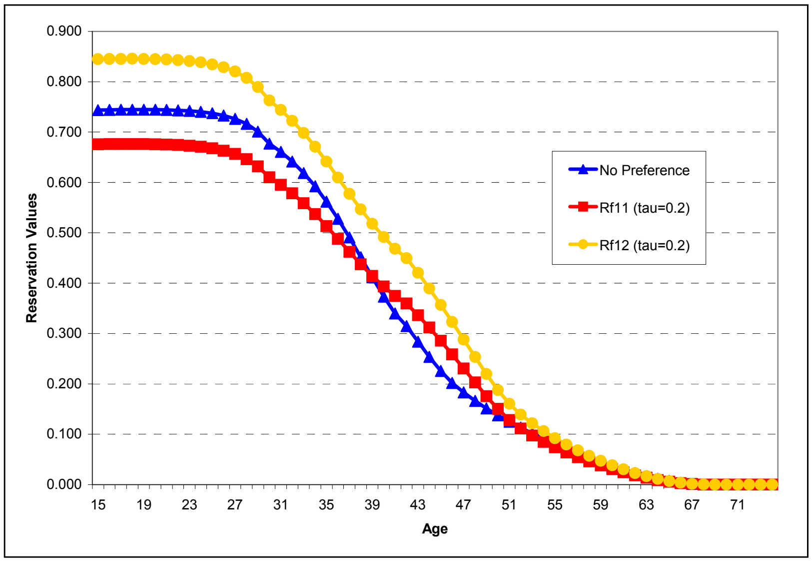

Figure 3.1: Reservation values of women for different levels of for same group preference

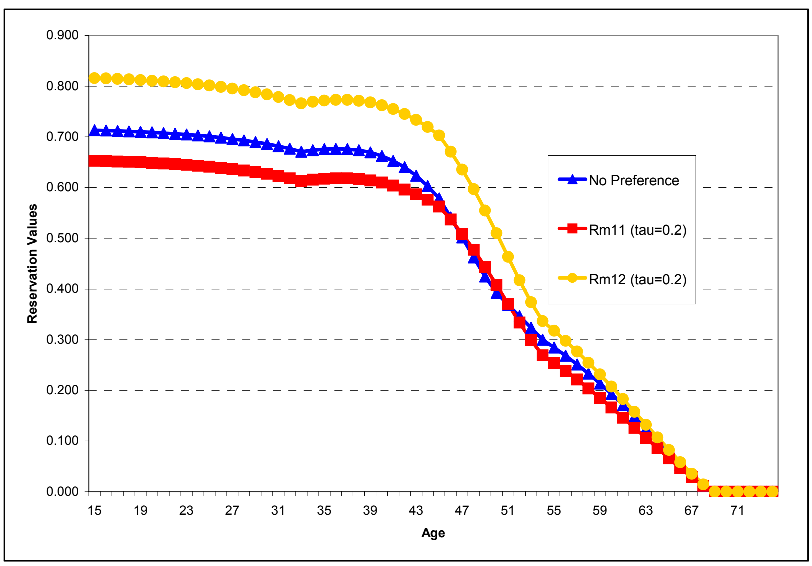

Figure 3.2: Reservation values of men for different levels of for same group preference

	$\tau = 0$		$\tau = 0.2$		$\tau = 0.4$	
	Men	*Women*	*Men*	*Women*	*Men*	*Women*
Size	*0.50*		*0.50*		*0.50*	
Age at Marriage (same)	27.3	26.3	27.5	26.7	28.1	27.3
Age at Marriage (other)	27.3	26.3	28.8	27.7	61.4	56.9
Ever Married	0.73	0.75	0.72	0.74	0.70	0.72
Intergroup Marriage	0.50	0.50	0.20	0.20	0.01	0.01
Sex Ratio	1.02		1.01		1.01	

Table 3.1: Selected Variable Means. Different Levels of Same Group Preference

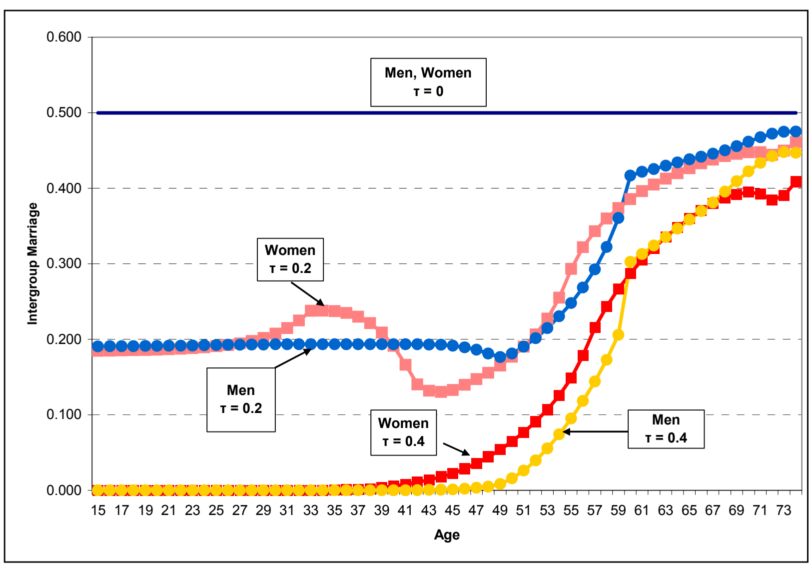

Figure 3.3: Fraction of men and women who marry people outside their own group

are indifferent respect who to marry $(\tau = 0)$. Not surprisingly, in this case half of all marriages are between people of different group, regardless of age. That is not the case when $\tau > 0$. For example, for $\tau = 0.2$, Figure 3.3 shows that the fraction of women who marry someone outside their group increases relative to the fraction of men marrying outside their group during the late 20's and early 30's (right before the decline in fecundity starts); then the fraction for women declines during their 40's. From the mid 40's intermarry increases for both men and women monotonically.

3.4.2 Different Size of Each Group

Now we allow the two groups to have different relative sizes, for a given rate of same group preference $(\tau = 0.2)$. The way to introduce different sizes in the model will be through the initial flow of men and women to the market. Keeping the number of people in Group 1 constant, in the following examples the values of $f_2(1)$ and $m_2(1)$ will be set in a way to make Group 2 be first 10% and then 35% of the market. This is another way to introduce friction in the market, because the minority group will now face increasing search frictions in order to meet people of the same group. On the other hand, the majority group will face less friction compared with the case where both groups are the same size.

Table 3.2 shows the means of the principal variables of the model in three cases: the top panel shows the results when both groups are of the same size, the middle panel when the size of Group 2 is 35 % of the market, and the bottom the case when the minority group is only 10% of the population. Also, Figure 3.4 shows the evolution of the intergroup marriage over ages when Group 1 is 65% and Group 2 is 35% of the population.

Same Size				
	Men		*Women*	
Size	0.5			
Age at Marriage	27.7		26.9	
Age at Marriage (same group)	27.5		26.7	
Age at Marriage (other group)	28.8		27.7	
Ever Married	0.72		0.74	
Intergroup Marriage	0.20		0.20	
Sex Ratio	1.01			
Different Size				
	Group 1		*Group 2*	
	Men	*Women*	*Men*	*Women*
Size	*0.65*		*0.35*	
Age at Marriage	27.5	26.7	28.1	27.2
Age at Marriage (same group)	27.2	26.3	27.9	27.3
Age at Marriage (other group)	29.4	29.1	28.6	26.9
Ever Married	0.72	0.74	0.71	0.73
Intergroup Marriage	0.14	0.14	0.28	0.28
Sex Ratio	1.014		1.014	
Size	*0.90*		*0.10*	
Age at Marriage	27.3	26.5	28.8	27.6
Age at Marriage (same group)	27.0	26.1	29.0	29.2
Age at Marriage (other group)	31.9	33.9	28.7	26.5
Ever Married	0.73	0.75	0.69	0.70
Intergroup Marriage	0.06	0.06	0.58	0.57
Sex Ratio	1.02		1.00	

Table 3.2: Groups of Different Size. Solution of the model when t=0.2

The principal characteristics of the results are the following:

1. The fraction of the people who marry someone of the other group increases by age. For women, this fraction increase more rapidly before fecundity starts to decline, decreases in their 40's and then increases up to the end of their lives.

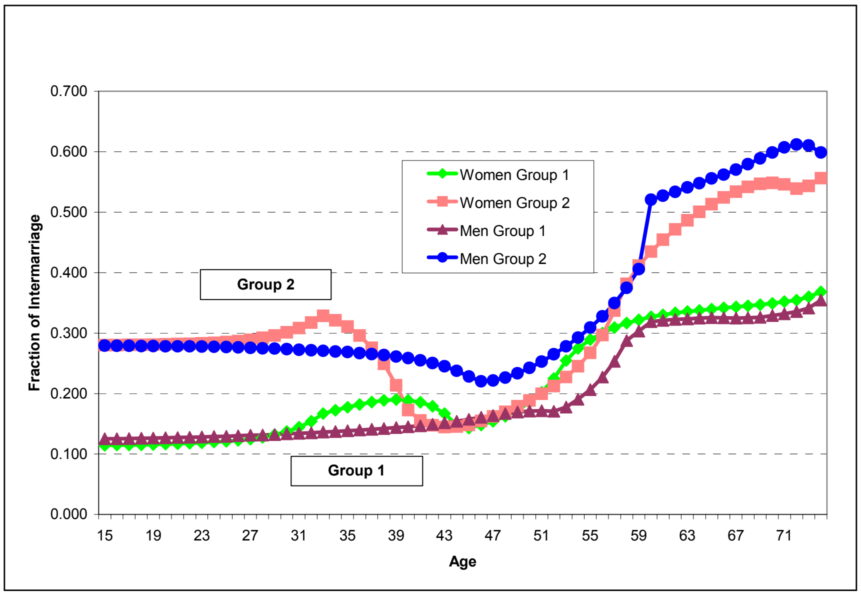

Figure 3.4: Fraction of intergroup marriage by age for groups of different size.

2. Minority groups tend to marry later than the majority group. As shown in Table 3.2, if 35% of the population belong to a certain group, the average age of marriage is higher than the group comprising 65% of the population, for both men (28.1 versus 27.5 years) and women (27.2 versus 26.7 years).

3. When groups are of the same size, the average age at first marriage is higher for both men and women if their spouse is in the other group. If the size of the groups is different, the same is true for people of the majority group.

Women of minority groups, however, tend to marry younger if they marry someone of the majority group than if they marry a man of their same group. This is true also for men when the size of the minority group is 0.1.

4. If people marry within their group, age difference at marriage is lower for people of the minority group. In the particular case that the size of the minority group is 0.1, women marry older than men.

5. Due to search frictions, the fraction of ever married is lower for the minority group.

6. The rate of intergroup marriage is always higher for the minority group (Blau (1977)).[3]

3.5 Comparison with U.S. Data

In order to compare the model results with US Census data two sources of data are used. The data on age at first marriage and intergroup marriage from the 1984-88 and 1989-95 Marriage Detail File (MDF) of the U.S. Vital Statistics Registry.[4] Also, data on couples married between 1975 to 1980 from the 1980 Census PUMS 5% file is also used. For the fraction ever married the data is from the 2000 Census PUMS 5% file.

[3]For example, Davidson and Widman (2002) find that Catholics are most likely to marry outside their group when they comprise a relatively small percent of the population in their dioceses.

[4]The data on race in the MDF is reported in 35 states.

3.5.1 Interracial Marriage

Figures 3.5 and 3.6 show the fraction of white women and white men aged 16-42 who married in 1975-80 (from 1980 Census), 1984-88 and 1989-95 (MDF) who are in interracial marriages. The pattern in both cases appears to be consistent with point 1 above (interracial marriage rises with age). In the data, however, the fraction of white men in interracial marriages increases relatively faster than the model predicts.

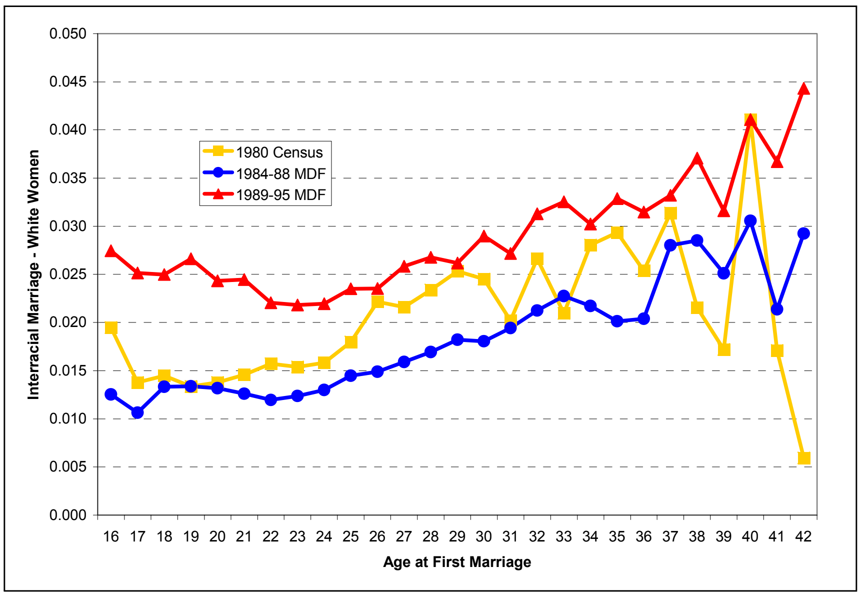

Figure 3.5: Fraction of white women in interracial marriages by age at first marriage. 1980 Census, 1984-88 and 1989-95 MDF

For a more detailed comparison between men and women, Figure 3.7 plots the fraction of interracial marriage of white men and white women from the 1989-95 MDF. The figure shows two characteristics that resemble Figures 3.3 and 3.4 (for Group 1). First, women tend to engage in more interracial marriage in their 20's and 30's, and second, that the difference in the fractions tend to increase from

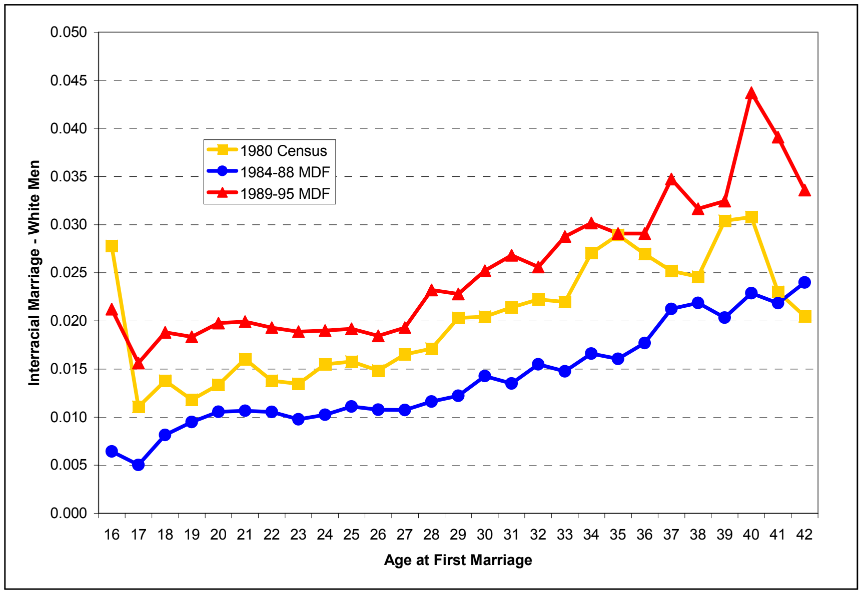

Figure 3.6: Fraction of white men in interracial marriages by age at first marriage. 1980 Census, 1984-88 and 1989-95 MDF

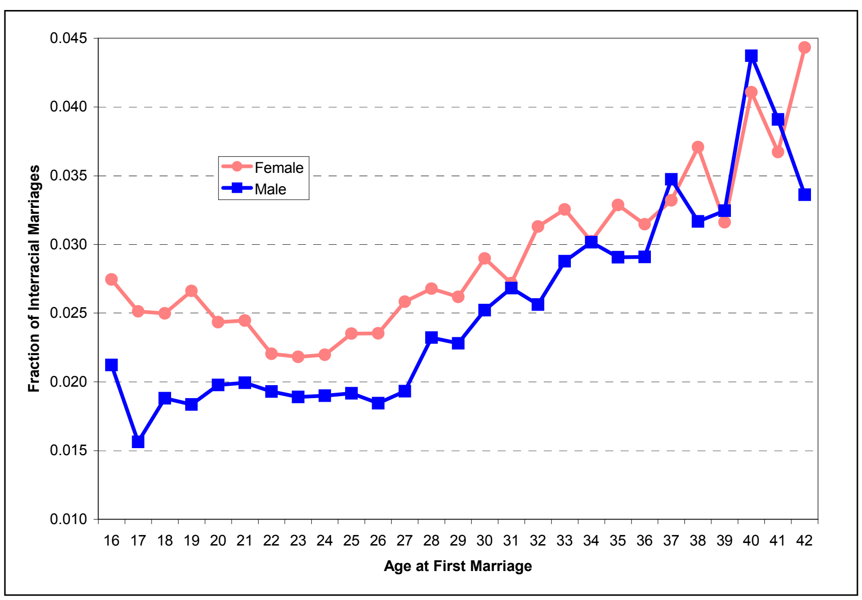

Figure 3.7: Fraction of white men and women in interracial marriages by age at first marriage. 1989-95 MDF

the mid 20's to the early 30's.

It can be argued that one cause for people to intermarry more with age could be that people tend to interact more with people of other races at work after finishing school. However, it is still surprising that the fraction of interracial marriage increase even at a higher rate well after the age of 30. Moreover, this pattern appears to hold for whites of all levels of education, as shown in Figures 3.8 and 3.9.

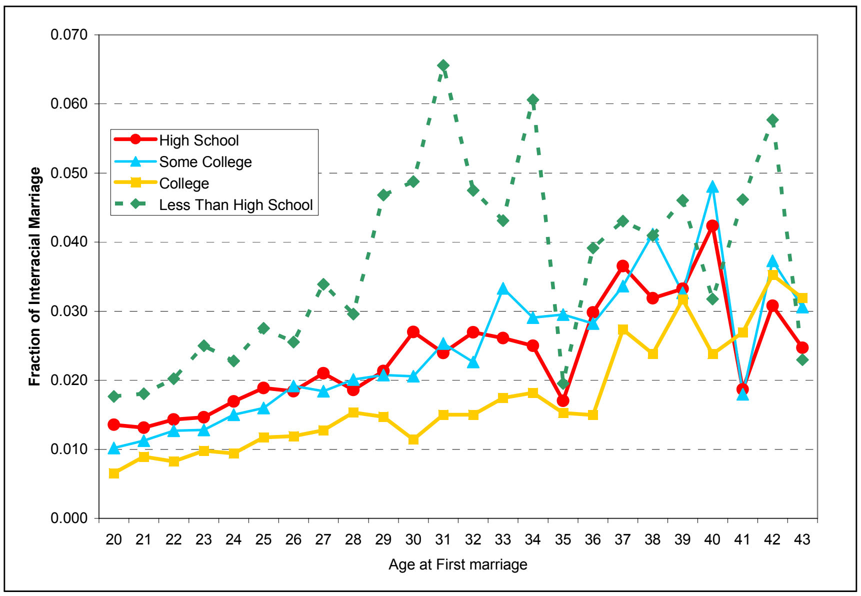

Figure 3.8: Fraction of white women in interracial marriages by age at first marriage and level of education. 1984-88 MDF

The pattern for black men and women for is shown in Figures 3.10 and 3.11. The fraction of interracial marriage of black women appears to increase after the age of 30 (late 30's in 1989-95). However, for men the pattern of interracial marriage does not appear to change over ages. More importantly, the fraction of interracial marriage for both men and women is much lower than what this model

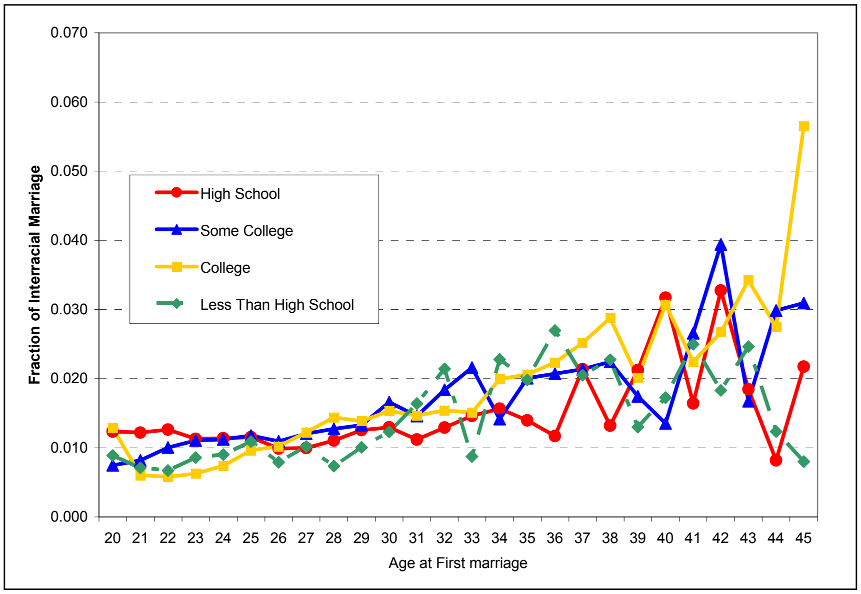

Figure 3.9: Fraction of white men in interracial marriages by age at first marriage and level of education. 1984-88 MDF

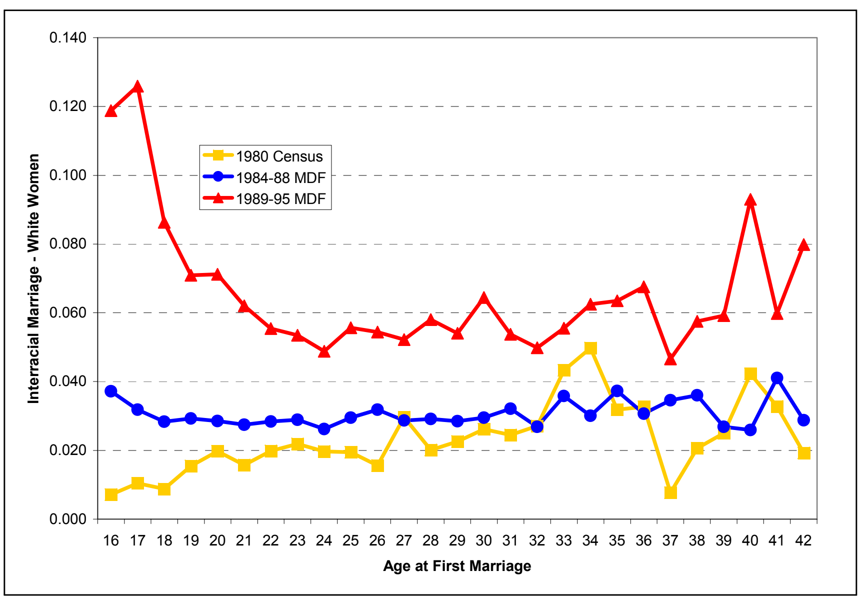

Figure 3.10: Fraction of black women in interracial marriages by age at first marriage. 1980 Census, 1984-88 and 1989-95 MDF

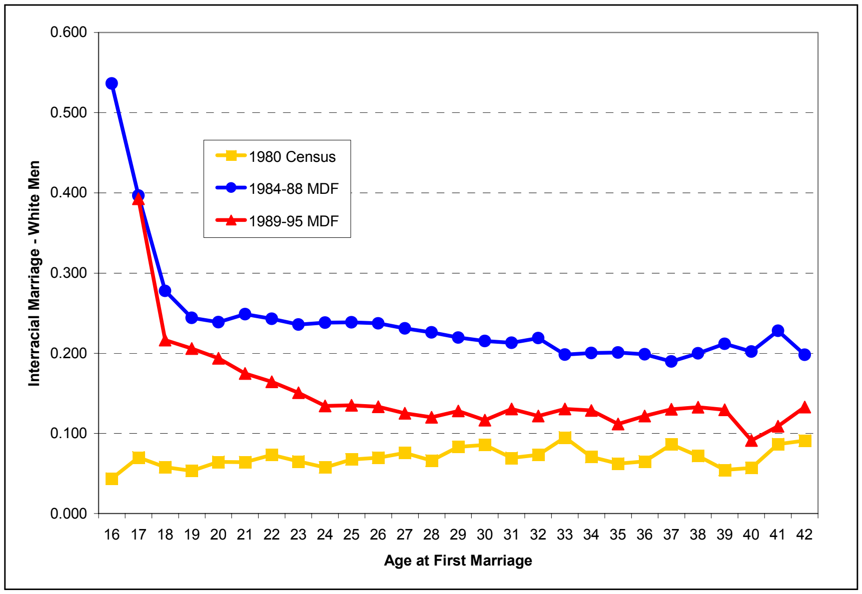

Figure 3.11: Fraction of black men in interracial marriages by age at first marriage. 1980 Census, 1984-88 and 1989-95 MDF

predicts for a minority group[5]. Notice also that the fraction of intermarriage is remarkably higher in teenage years, specially for black men.[6]

3.5.2 Husband with Lower Level of Education (Women "Marrying Down")

Now we analyze the case where women marry men with lower levels of education. Hypergamy, or the tendency of women of "marrying up" in education is a fact well documented by the literature.[7] Therefore, in this section we treat having a husband with lower education as an "intergroup marriage" in terms of the model.

[5] This issue is analyzed in detail in a recent paper by Wong (2003).

[6] For a detailed study on the trends in black/white intermarriage see Kalmijn (1993).

[7] For a recent empirical analysis about this topic, see Rose (2004).

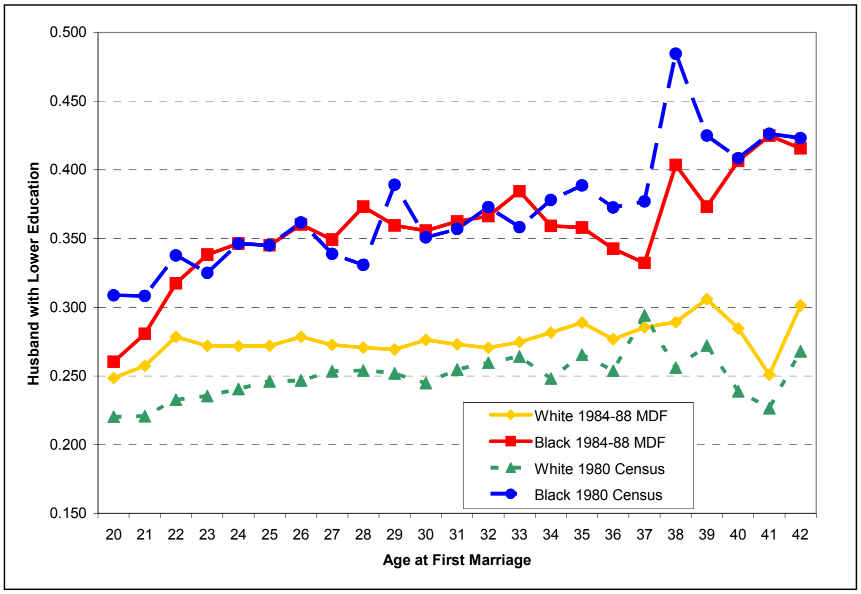

Figure 3.12: Fraction of marriages with wives more educated than husband. 1980 Census and 1984-88 MDF.

That is, as if women have a loss in utility for "marrying down". For the reason above, it is very difficult to analyze the case for men, so we will restrict this discussion to the case of women.

Figure 4.4 shows the fraction of white and black women aged 20-42 marrying men of lower levels of education in 1975-80 (1980 Census) and 1984-88 (MDF)[8]. While in the case of whites it appears to slightly increase in their mid 30's, the pattern of black women shows an unambiguous increase from age 35 to age 42.

	Age at First Marriage					
	1980 Census		*1984-88 MDF*		*1989-95 MDF*	
Same Race	*Men*	*Women*	*Men*	*Women*	*Men*	*Women*
White	24.3	22.4	25.2	23.2	26.0	24.0
Black	25.7	23.6	27.0	25.2	28.1	26.7
Other	27.0	24.4	27.2	24.8	27.8	25.6
Total	24.5	22.5	25.4	23.5	26.2	24.3
Interracial						
White	25.2	23.0	26.3	24.0	27.2	24.5
Black	25.9	24.5	26.5	25.3	27.0	26.2
Other	25.3	24.1	26.3	25.1	26.8	25.5
Total	25.4	23.6	26.4	24.5	27.0	25.1

Table 3.3: Age at First Marriage. Men and women marrying people of the same race and in interracial marriages

3.5.3 Age at First Marriage

This section is devoted to comparing the age at first marriage in different groups in the data with the analysis of different sizes of groups in Section 3.4.2. Table 3.3 shows the average age at first marriage by race[9] for people married in 1975-80, 1984-88 and 1989-95. A simple inspection to Table 3.3 is enough to show the consistency of the data with point 2. on page 78: the average age at first

[8] The information on the level of education of groom and bride in the MDF is reported in 21 states and ends in 1988.

[9] MDF uses a the traditional definition of race without disclosing people of hispanic descent.

marriage for whites (majority group) is lower than in the case of blacks and other races. A closer look over the age at marriage for people in interracial marriages allows us to illustrate point 3. On average, whites marry in average older if the spouse is of another race, but the reverse is true for blacks and people of other races (1984-88 (men) and 1989-95). For the samples in MDF the age difference at marriage between blacks is remarkably lower than for whites (point 4.).

3.5.4 Size of the Group and Fraction of Ever Married

Size of Ethnic Group by Education (Age 22-40)			
	White	*Black*	*Hispanic*
Less than High School	38.96	11.02	44.41
High School	66.71	13.81	14.13
Some College	71.52	11.94	10.14
College or More	77.16	6.49	5.69
Total	***67.45***	***11.09***	***14.44***

Fraction of Ever Married (Age 22-40)						
	White		*Black*		*Hispanic*	
	Men	*Women*	*Men*	*Women*	*Men*	*Women*
Less than High School	0.66	0.79	0.39	0.42	0.65	0.78
High School	0.68	0.82	0.49	0.49	0.62	0.74
Some College	0.66	0.77	0.55	0.54	0.61	0.70
College or More	0.65	0.70	0.56	0.52	0.61	0.67
Total	***0.67***	***0.77***	***0.51***	***0.50***	***0.63***	***0.73***

Table 3.4: Size of the group and fraction of ever married. US Census 2000

To complete the comparison of the implications of the model regarding the size of the groups with the data, we now discuss the fraction of ever married. The top panel of Table 3.4 shows, by level of education, the relative size of the population of whites, blacks and Hispanics in the US Census 2000, where we note the large number of Hispanics among people with less than high school education. The bottom panel displays the fraction of people ever married, aged 22-40 for the described groups[10]. Since the ever married rates for blacks are low in all cases it is difficult to observe any pattern for this group, but the case of Hispanics deserves consideration. For people with less than high school education the fraction of ever married Hispanics (44.4% of the people) is very similar to that for whites (0.65 for men and 0.78 for women). In all other cases the ever married rate for Hispanics falls short with respect to the fraction for whites (for women the difference is 8% for high school education and 7% for some college education). Overall, the marriage rates are always higher for whites (majority group). This result consistent with point 5. on page 5.

3.6 Different Mortality Rates

As an exercise to capture the effects of different mortality rates on the behavior of the model, in this last section the model is solved for using mortality rates for whites ($size = 0.9$) and blacks. In this example the rate of preference for the same group (ι) is set in 0.3. Table 3.5 summarize the main results. The model averages for interracial marriage of whites and rate of ever married is close to the data (0.72 for men and 0.75 for women versus 0.73 and 0.79 in the data

[10] The minimum age in this case is 22 years old to avoid including people currently in school in the groups of less educated people.

	Model (1)		Data (2)	
	Men	**Women**	**Men**	**Women**
ß	0.920			
k	1.25			
t	0.30			
Whites				
Size	*0.9*			
Age at First Marriage	27.6	26.5	26.1	24.1
Sex Ratio of Singles	1.04		1.22	
Interracial Marriage	0.022	0.023	0.025	0.022
Ever Married (%)	0.72	0.75	0.73	0.79
Blacks				
Size	*0.1*			
Age at First Marriage	30.1	30.0	28.1	26.7
Sex Ratio of Singles	0.903		0.839	
Interracial Marriage	0.247	0.238	0.060	0.141
Ever Married (%)	0.63	0.63	0.57	0.59

Table 3.5: Model with mortaility rates of whites (size=0.9) and blacks. Data from 2000 US Census and MDF 1989-95

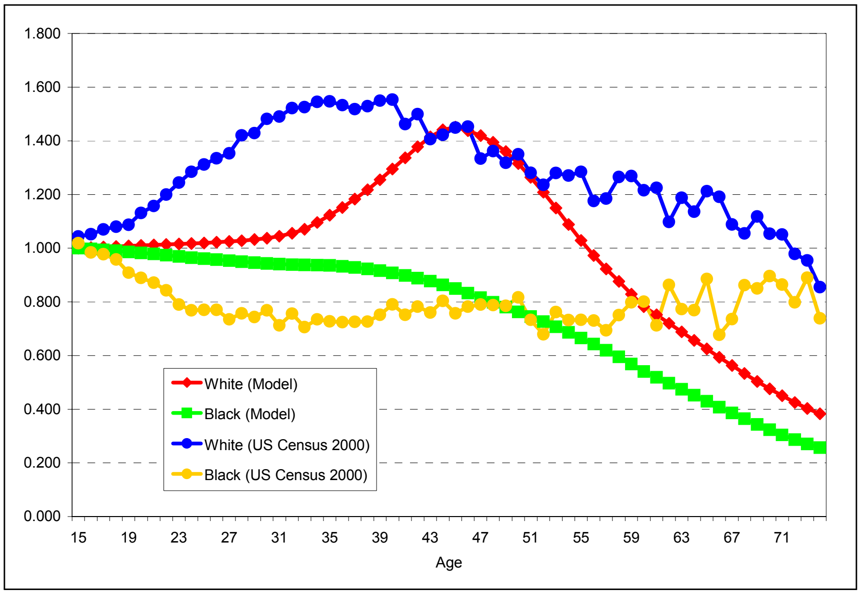

Figure 3.13: Ratio single men/single women. US Census 2000 for blacks and whites and model using mortality rates for each race

respectively). It also captures the lower marriage rates for blacks (0.63 in the model versus 0.59 in the data) and the differences in the sex ratio. However, it does a poor job in predicting the fraction of interracial marriage for blacks (0.24 in the model versus 0.14 for women and only 0.06 for men in the data).

The pattern in the ratio single men/single women over ages is displayed in Figure 3.13, where the very different behavior of the sex ratio can be appreciated by simple inspection.

3.7 Summary

The model of this chapter has several predictions that are consistent with some patterns observed in US data. First, at least for whites (Figures 3.5 and 3.6), the fraction of interracial marriage appear to increase as people marry older.

The rationale that this model provides for this fact comes from the preference of marrying within the group combined with the limited search horizon. As time passes, reservation values of men and women decrease and so the premium for marrying within the group. Therefore, people is more willing to propose to someone from a different group. This is especially true in the case of women, whose limited fecundity horizon causes the difference in reservation values for marrying within and outside the group to converge more rapidly than in the case of the reservation values of men. Since the different fecundity horizon of men and women affects their willingness to accept potential partners in a different way through ages, the dynamic of intergroup marriage also differs. As shown in Figure 3.3, when people prefer to marry within a group ($\tau > 0$) the fraction of women marrying outside the group increases relatively with respect to the fraction of men in intermarriage from the late 20's to near age 40. When people have no preference about the spouse's group, the fraction of intermarriage is constant over ages.

Another regularity found in the data concerns the size of the group and the age of first marriage. As shown in Table 3.3 for the period 1980-95, ethnic minorities in the US marry consistently later than the white majority. Moreover, for the periods 1984-88 and 1989-95, while the average age at first marriage for whites is higher if they are in an interracial marriage, this is not the case for blacks and other races. This model is able also to provide an explanation for that. If people prefer to marry within the group, minority groups will face frictions that delay the age of marriage. In addition, from the viewpoint of the minority group, intermarriage will ease the process of finding a mate, even given a within group preference. But for the majority group, intermarriage means more frictions in the search process, and therefore a delay in the age of marriage.

Chapter 4

Changes on the Sex Ratio, Timing of Marriage and Spouse Characteristics

The purpose of this chapter is to analyze the effects of an exogenous change in the sex ratio over marital behavior of men and women. As a theory background, the following section uses the two period model of Chapter 2 (considering the sex ratio as exogenous) and show the effects on the probability of marriage of these changes. The rest of the chapter is devoted to an empirical analysis over Census data.

4.1 Comparative Statics: Effects of an Exogenous Change in the Sex Ratio

Remember from Chapter 2 (equations (2.2) *and* (2.3)) that

$$S = \frac{M}{F}$$

and by equation (2.4)

$$\begin{aligned} p &= \frac{f_1}{F} \\ q &= \frac{m_1}{M} \end{aligned}$$

Assume S, p and q are exogenous. In this section we analyze the effect of a change in the sex ratio at age 1 on the reservation values and the probability of marriage at age 1 for men and women:

4.1.1 Reservation Values

Differentiating R^m_{old} (equation (2.11)) and R^f_{old} (equation (2.12)) with respect to S and solving we get:

$$\frac{\partial R^m_{old}}{\partial S} = \frac{\begin{pmatrix} -\mu S^{\theta-1}\beta\overline{x}\left(1-\theta\right)\left(1-p+kp\left(1-G_m\left(R^f_{old}\right)\right)\right) \\ -pS^{\theta}\beta\theta\mu\left(1-qG_f\left(R^m_{old}\right)\right)\overline{y}g_m\left(R^f_{old}\right) \end{pmatrix}}{S-pqS^{2\theta}\beta^2\mu^2\overline{xy}g_f\left(R^m_{old}\right)g_m\left(R^f_{old}\right)} \tag{4.1}$$

whose sign is negative and

$$\frac{\partial R^f_{old}}{\partial S} = \frac{S\theta^{\theta-1}\beta\mu\overline{y}\begin{pmatrix} S\theta\left(1-qG_f\left(R^m_{old}\right)\right) \\ +\mu qS^{\theta}\beta\left(1-\theta\right)\left(1-p+kp\left(1-G_m\left(R^f_{old}\right)\right)\right)\overline{x}g_f\left(R^m_{old}\right) \end{pmatrix}}{S-pqS^{2\theta}\beta^2\mu^2\overline{xy}g_f\left(R^m_{old}\right)g_m\left(R^f_{old}\right)} \tag{4.2}$$

with a positive sign.

The intuition of the signs are straightforward: the higher the sex ratio, women become choosier and vice-versa for men.

4.1.2 Probability of Marriage

As will be shown below, an exogenous variation in the sex ratio has three different effects on the probability of marriage (equations (2.7) and (2.9)) in a given period:

1. A direct effect in the availability of mates.

2. An indirect effect via reservation values of men.

3. An indirect effect via reservation values of women.

Differentiating Φ_1 (equation (2.7)) and Γ_1 (equation (2.9))with respect to S and solving simultaneously we get the following system:

$$\begin{pmatrix} \frac{\partial \Phi_1}{\partial S} \\ \frac{\partial \Gamma_1}{\partial S} \end{pmatrix} = \begin{pmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{\partial R^m_{old}}{\partial S} \\ \frac{\partial R^f_{old}}{\partial S} \end{pmatrix} \tag{4.3}$$

where

$$\Phi_{11} = -(1-\theta)\mu S^{\theta-2} \begin{pmatrix} (1-p)(1-G_f(R^m_{old})) \\ +p(1-G_f(R^m))(1-G_m(R^f)) \end{pmatrix} \tag{4.4}$$

$$\Phi_{12} = -\frac{S}{k(1+\beta)}\mu S^{\theta-2} \begin{pmatrix} k(1-p)(1+\beta)g_f(R^m_{old}) \\ +p(1-G_m(R^f))g_f(R^m) \end{pmatrix} \tag{4.5}$$

$$\Phi_{13} = -\mu S^{\theta-2}kp(1-G_f(R^m))g_m(R^f) \tag{4.6}$$

$$\Gamma_{11} = \mu q S^{\theta-1}\theta \begin{pmatrix} 1-qG_m(R^f)-qG_f(R^m_{old})(1-G_m(R^f)) \\ -(1-q)G_m(R^f_{old}) \end{pmatrix} \tag{4.7}$$

$$\Gamma_{12} = -\mu S^{\theta}\left(\frac{q(1-G_m(R^f))g_f(R^m)}{k(1+\beta)}\right) \tag{4.8}$$

$$\Gamma_{13} = -\mu S^{\theta}\left(\mu(1-q)+\frac{q(1-G_f(R^m))g_m(R^f)}{(1+\beta)}\right) \tag{4.9}$$

The three coefficients above (equations $(4.4), (4.5)$ and (4.6)) refer to the different effects on the probability of marriage for men. The first coefficient (unambiguously < 0) indicates the direct effect of the change in the sex ratio at age 1 on the probability that a man marries at age 1. Since an increase in the sex ratio implies a relative scarcity of women, it has a negative effect in the probability of marriage. The second coefficient reflects the effect in the probability of marriage via the change in the reservation value of men. Clearly, the negative coefficient multiplying a negative derivative (see equation(4.1)) indicates that this effect is positive. The decline in the reservation value of men increases the probability of marriage. The third coefficient is the indirect effect through the change of the reservation value of women. As women become choosier the higher the ratio male/female (by equation(4.2)), this effect is negative for the probability that a man marries at age 1.

Equations $(4.7), (4.8)$ and (4.9) show the coefficients for each effect on the probability of marriage for women. In this case we have that the direct effect is positive, that is, since there are more men available so the probabilities of marriage increase for a woman. The second effect is also positive (a negative coefficient multiplying a negative derivative), and reflect the declining on the reservation value of men when the sex ratio increases. The third effect is the one related to the change in the reservation value for women, and will be the only with negative sign.

For both men and women, the net effect of a change of the sex ratio on the probability of marriage is ambiguous, depending on the magnitude of the three effects mentioned above. The empirical analysis of the following sections is an attempt to explore further the interaction of these effects.

4.2 Data and Descriptive Statistics

The data used in this chapter come from the 1970 and 1980 Census IPUMS files. Combining the two samples of 1970 Form 1 (Metro and State), we have a 2% sample of the population in 1970, the 1980 IPUMS file is a 5 % sample. With this files three samples are constructed in the following way:

1. A sample of men and women who were in the 1970 Census and were single in 1965 or were in the 1980 Census and were single in 1975. This sample contains observations on men and women age 21-35 in 1970 and 1980. Therefore we have a sample of people who had never married and were aged 16-30 in 1965 or 1975.

2. A sub-sample women, from the above group who were in their first marriage at the time of the survey married between 1965 and 1970 or 1975-1980. Using the Spousal links provided by the IPUMS file, these women are matched with their husbands. The age range for husbands is 16-50.

3. A sub-sample of men who were in their first marriage at the time of the survey but also had not previously married 5 years prior to the survey. As before, the spousal links were used to match wives to their husbands for this group of men. Wives were aged 16-50.

These two last samples allow constructing outcome variables that reflect the marriage characteristics for the selected groups of men and women: differences in education level, race, etc., between spouses. For the purpose of estimation, the pooled samples are weighted by the relative size of the Census samples.

Descriptive statistics for Black, Hispanic and the whole sample are shown in Tables 4.1- 4.6. Sex Ratios by Race and education level are shown in Table 4.1 and the relative size of the different ethnic/educational group in Table 4.2.The level of the sex ratio vary considerably by race and education. For example, for low-education whites the ratio is very high (1.77 in 1975), and the opposite is true for blacks. Blacks particularly with college education have a low sex ratio demonstrating the shortage of black men that rises with education, fact extensively discussed in previous research. Table 4.3 shows a much lower marriage rates for Black over the 5 year period previous to the Census. This fact has been extensively discussed in previous literature (Wilson 1987; Wood 1993; Barber 2001). Tables 4.4 and 4.5 show the fraction of marriage with education differentials between spouses both for sample if married women and for the sample of married men. In both samples, near one half of the couples have the same education level. Finally, table 4.6 shows the proportions of interracial marriages for both the sample of men and the sample of women.

4.3 Empirical Framework

The strategy is to capture the effects differential sex ratios faced by men and women when they are unmarried 5 years prior to the census. Since the way the local marriage market is defined in this paper, the identification of the model comes from spatial variation within the same race/education cell. The estimating equation for individual i in ethnic group j , education level k and living in State s is:

$$Y_i = \theta Year + \alpha_s State_s + \beta_k Educ_k + \gamma_j Race_j + \delta_g Age_g + \varpi Size_{skj} + \mu Ratio_{skj} + \epsilon_i \tag{4.10}$$

where θ is a year fixed effect (1970 or 1980), α_s is a state fixed effect, β_k an educational level effect, γ_j a race effect, δ_g an age effect, $Size_{skj}$ is the relative size of the ethnic educational group within the state s and $Ratio_{skj}$ is the Sex

Ratio of race j and educational level k in state s at the beginning of the period. The list of dependent variables denoted Y_i will be a latent index, such be the case of several demographic discrete choice outcomes like marital status, interracial marriage, etc.

Here the two variables of interest are the ones regarding the size of the group and the sex ratio. The variable $Size_{skj}$ is the fraction of the population of the same race and education level in a given state. The variable $Ratio_{skj}$ is the ratio of the number of single men aged 16-30 in 1965 or 1975 to the number of single women of the same age group, by race, educational level and state. Since the samples contain observations for people who were never married in 1965, in one case, and 1975, in the other, this ratio is an approximation of the "virtual sex ratio" in 1965 and 1975 respectively. The meaning of "virtual" is that the ratio is calculated only for available men and women at that date. The structure of the data make impossible to include divorced people in those years.

The simultaneous effects of these two variables on marital behavior can help in the analysis of assortative mating. First, sex imbalances in a given group can affect timing of marriage and also who to marry. Second, as largely discussed in the previous chapter, the behavior can be also affected by the search friction generated by a relative small size of the group in the population.

Through all previous chapters the sex ratio has been treated as endogenous. However, from the perspective of an individual the sex ratio can be viewed as exogenous. That is, past decisions of other people determine the sex ratio a specific individual faces but those decisions are beyond the control of that individual. A specific individual must act in the marriage market taken the sex ratio he or she faces as given. With this framework, the sex ratios are calculated in certain point of time (the starting points in this case are 1965 and 1975) and the dependent variables would be a "flow" of decision outcomes on a period of time (1965-70 or 1975-80).

Even though the way that the sample was constructed allows avoiding a great part of the risk of endogeneity of the explanatory variable, there is still a potential problem of "Sample Selection Bias". Taking for example, the sample of people who were single in 1975, the sample omit observations on people married before 1975. Because less educated people and people living in the South and the Midwest tend to marry earlier, those groups are underrepresented in the sample. Unfortunately, is impossible to apply the known procedures to correct for this problem because, in general, the variables that could explain the presence of the observation in the sample are the same ones that explain the outcome variable, the propensity to marry.

Another problem of sample selection may arise in the samples of married women and married men. Since the census show only current marital status, the sample will drop the couples married during the period 5 year prior to the census but divorced before the Census. If there is any degree of correlation between divorce and intergroup marriage this could cause a misrepresentation of the fraction of this kind of marriage in the samples. However, more than 90% of marriages that occur in the 5 year prior to the census are intact at the census. This selection problem should be small.

The empirical study is divided in two parts: First, we study the effects of a variation in the local sex ratio and the size of the ethnic/educational group on the timing of marriage, using the "general sample" described above and estimating a separate equation for men and women. Second, we analyze the consequences of variation in the sex ratio and size on "spouse's characteristics" for the sample of people who choose to marry in the period 5 year previous to the census. In all cases the equations were estimated using probit models.

4.4 Basic Results

4.4.1 Probability of Marriage

The first part of the empirical study analyses is with respect to the likelihood of first marriage between 1965 and 1970 or between 1975 and 1980. One of our main questions of interest is how the sex ratio affects the propensity to marry differently at different ages. We modify equation (4.10) adding interactions between age and sex ratio to capture this, in the following way:

$$\begin{aligned} Y_i &= \theta Year + \alpha_s State_s + \beta_k Educ_k + \gamma_j Race_j + \varpi Size_{skj} + \\ &\quad \delta_g Age_g + \mu Ratio_{skj} + \xi_g Age_g * Ratio_{skj} + \epsilon_i \end{aligned} \tag{4.11}$$

where the dependent variable is $= 1$ if the person was ever married by 1970 or 1980.

Tables 4.7 and 4.8 show the results for women and men without year fixed effects and Tables 4.9 and 4.10 display the probit coefficients adding census year fixed effects. The significant coefficients in the model with fixed effects are in similar to those in Tables 4.7 and 4.8 but the standard deviations are in general smaller.

Table 4.9 shows the marginal effects from the probit model for women single in 1965 or 1975 for four different samples (4.11). The first column of Table 4.9 presents estimates for the whole sample, column (2) presents estimates for the sample of black women, column (3) presents estimates for a sample of college graduates aged 22-25 in 1965 or 1975 and column (4) presents estimates for the sample of high school graduates aged 16-19 in 1965 or 1975.The effect of a change in the size of the group is small but significantly negative for the whole sample (-0.062) and bigger for High School graduates (-0.21). In the case of College graduates the effect is reversed: an increase of the size of the ethnic and educational group by 0.1 implies an increase in the probability of marriage in the next five years by 0.06. This last result is in line with the implication of the model of the previous chapter, in the sense that a decrease in search friction leads to an increase in the probability of marriage in the current period. An explanation of the results in the other two cases could be that the increase in reservation values that come with a bigger size more than neutralize the effect of the decrease in search friction. Moreover, it seems reasonable that the effect of a decrease in friction is more important for a smaller group than for the whole sample.

To analyze the effects of a change in the sex ratio, we look first at the results for the whole sample (column (1)) of Table 4.9. The coefficient on "Sex Ratio", applies to all age groups but specifically address the effect on the youngest age group. This implies that for women aged 16-17 in 1965 or 1975 the effect of sex ratio on marriage is positive (0.1). The coefficient suggests that an increase in the sex ratio by 25% implies an increase in the probability of marriage in the next 5 years by 2.5%. The interactions between age group and sex ratio then estimate the difference between the overall effect and the effect of a specific age group. For example from ages 26 to 30 the coefficient is around -0.2, which implies that the net effect of this is -0.1. This means that in this age range if the sex ratio increases by 25%, the probability of marriage in the next five years decreases by

2.5%. This result, at odds with the literature,[1] indicates that *an adverse change in the market conditions can make women tend to marry faster, at least at older ages.*

The comparative statics of Section 4.1.2 can help us to explain why. Provided a decrease in the sex ratio, both the direct effect (availability of males) and one of the indirect effects (higher reservation values of men) act together lowering women's probability of marriage. The third effect (indirect effect via lower reservation values of women) is the only that may act in the reverse direction, that is, increasing the probability of marriage when the sex ratio decreases. Therefore, the explanation of the negative coefficient (age 18 and beyond) could be that the third effect more than compensate the other two.

The second column of Table 4.9 repeat the analysis for a sample of black women. The net coefficient is positive for age 16-17 (0.2) and 18-19 (0.068) and then turns negative to reach a net effect of -0.18 at age 26-27. In the case of college graduates (column (3)) the only significative coefficient is at age 24-25 (-0.08) and for high school graduates (column (4)) the coefficient is 0.2 at age 16-17 and 0.14 at 19-20.

Table 4.10 shows a similar analysis for males. The coefficient for size is significant and negative for the while sample (-0.044) and significant and positive for blacks (0.234). Again, the effect of a decrease in friction appear to be more important in a minority group than in the whole sample.

Not surprisingly, the effects of a change in the sex ratio for men are in line with the results for women. The first column of Table 4.10 shows that the net coefficients for a change in the sex ratio are positive for ages 16-17 (0.1) and 18-19

[1] See for example Brien (1997)

(0.06) but turns negative from age 22-23 and beyond. In this case, there are two effects that explain the negative coefficients: if the sex ratio increases, both the lower availability of males and the higher reservation values of women appear to be more important than the reduction in the reservation values of men.

In the case of black men (column (2)) the coefficient are significantly negative from age 22-23, but less important in magnitude than the results for black women.

It is widely recognized that when men become in short supply (reducing the sex ratio) this changes the "terms of trade" inducing women to change their optimal time at marriage. The "terms of trade" argument alone would suggest that the same forces that delay marriage for women when men are in short supply make men marry earlier. However, the mechanisms discussed above help to "coordinate the market".

4.4.2 Spouse's Characteristics

This part of the study consists in the probit estimation of equation (4.10) to a selected set of outcome variables that represent intergroup marriage for both the sample of married women and the sample of married men. These outcome variables are binary variables for interracial marriage, for spouse with same, lower or higher level of education, for spouse previously married and for spouse with own children.

Panel (a) of Table 4.11 shows the result of the estimation of the different outcome variables for the sample of married women and Panel (b) for a sub-sample of black women. For interracial marriage the only significant coefficient is the one relative to the size of the group for the whole sample. In this case, an increase of the size of the group by 0.1 implies a decrease in the fraction of interracial marriage of a 0.02.

The coefficient for having a husband with the same level of education (that can be considered a "good") are, not surprisingly, significantly positive for both size and sex ratio in the whole sample (0.2 and 0.6 respectively) and also in the sample of black women (2.3 and 0.105). The interpretation of these coefficients are straightforward: the probability that a woman marries a man with the same level of education depends positively on the fraction of the people with the same level of education in the population and in the relative availability of males within that group. The case of husbands with more education ("marrying up") is similar than the previous case but the coefficients are bigger in all cases. For husbands with less education ("marrying down") the sign of the coefficient is positive and very high in all cases. In the case of size the coefficient is close to -1 for the whole sample and around -2.5 for blacks. The effect of an decrease in the sex ratio by 0.10 implies an increase in the probability of marrying someone less educated by 0.03 in the whole sample and by 0.06 in the case of blacks.

For the case of husband previously married the coefficient of size is significant and negative (-0.08) and for husband with children the significant coefficient is the one for sex ratio (-0.05) in both cases for the whole sample. There's no coefficient significantly different from 0 in the sub-sample of black women for this two variables.

Table 4.12 shows the results of the same analysis for the sample of married men. In the case of interracial marriage the coefficients for the whole sample (Panel (a)) and for a sub-sample of black men (Panel (b)) have different sign. The coefficient of the variable size is negative for the whole sample (-0.1) and in the case of black men an increase in the size of the group implies a higher probability of interracial marriage (0.52). The coefficient of sex ratio is only significant for the whole sample, but very low (0.006).

The different outcome variables of education don't show discrepancies between blacks and the whole sample. The probability of marrying someone of the same education level depend positively on the size of the group (1.1 for the whole sample and 3.02 for blacks) and negatively on the sex ratio (a lower sex ratio means relatively that women are relatively abundant). In the case of a wife with less education the result are somewhat surprising: in both cases depend negatively on the size of the group (-1.5 for the whole sample and -4.07 for blacks) but also are negative on the sex ratio (-0.34 and -0.2 respectively). For wives with more education a the results are similar: all the coefficients are positive.

Finally, the variable wife previously married shows a negative coefficient for size (-0.066) and a positive for the sex ratio (0.015) and wife with children shows reverse sign, only for the whole sample. As in the case of the sample of women, there is no significant coefficients in the sample of black men in these two variables.

4.5 Summary

The results of the empirical study of this chapter are generally consistent with the models of the previous chapters. The results of Section 4.4.1 suggest that the behavior of men and women respect to the timing of marriage is not driven only by the direct effect of the availability of partners but also by the reaction of the rest of the market to a given sex imbalance. Moreover, this behavior is not constant over time and may change over ages, as largely discussed in Chapter 2. The main result in this section is that an adverse change in the sex ratio may increase the probability of marriage of women. To the best of my knowledge, this is the first empirical work with these findings, especially given that most of the literature has focused on analyzing only the direct effect of mate availability.

The findings of Section 4.4.2 are consistent with the model of Chapter 3: the probability that people marry outside their own group increases when there is an adverse sex imbalance in their group or when the size of their own group is small enough to produce frictions that compensate for the benefits of marrying within their group.

Table 4.1: Means of Sex ratios by race and education

Descriptive Statistics - Sex Ratios by Race and Education					
	Less Than HS	*High School*	*Some College*	*College*	*Total*
Single in 1965					
White	1.741	1.001	1.320	1.429	1.276
Black	1.141	0.908	0.980	0.801	0.997
Hispanic	1.334	0.951	1.269	1.298	1.168
Total	1.541	0.986	1.296	1.398	1.236
Single in 1975					
White	1.776	1.221	1.172	1.226	1.263
Black	1.134	0.968	0.840	0.745	0.953
Hispanic	1.341	1.134	1.187	1.327	1.235
Total	1.547	1.179	1.133	1.198	1.219

Table 4.2: Relative Size of the group by Race and Education

Descriptive Statistics - Relative Size by Race and Education					
	Less Than HS	*High School*	*Some College*	*College*	*Total*
Single in 1965					
White	0.112	0.311	0.192	0.161	0.777
Black	0.042	0.045	0.015	0.007	0.109
Hispanic	0.035	0.045	0.021	0.013	0.114
Total	0.189	0.401	0.229	0.182	1.000
Single in 1975					
White	0.079	0.300	0.229	0.197	0.805
Black	0.033	0.055	0.034	0.014	0.137
Hispanic	0.020	0.019	0.013	0.005	0.058
Total	0.133	0.375	0.276	0.217	1.000

Table 4.3: Fraction of People Married the next 5 Years. People never married by 1965 or 1975.

Descriptive Statistics - *Married in the Next 5 years*						
	White		*Black*		*Hispanic*	
Age in 1965 or 1975	*Male*	*Female*	*Male*	*Female*	*Male*	*Female*
16	0.274	0.466	0.208	0.320	0.353	0.547
17	0.381	0.567	0.294	0.390	0.459	0.633
18	0.481	0.631	0.357	0.420	0.534	0.673
19	0.536	0.642	0.440	0.435	0.586	0.673
20	0.578	0.647	0.464	0.439	0.625	0.680
21	0.607	0.645	0.499	0.432	0.631	0.640
22	0.613	0.626	0.496	0.414	0.653	0.637
23	0.603	0.586	0.497	0.390	0.631	0.599
24	0.583	0.541	0.493	0.362	0.610	0.553
25	0.557	0.494	0.460	0.340	0.570	0.520
26	0.532	0.457	0.471	0.344	0.583	0.511
27	0.502	0.423	0.443	0.324	0.556	0.472
28	0.479	0.373	0.429	0.294	0.517	0.428
29	0.447	0.354	0.419	0.263	0.473	0.391
30	0.409	0.325	0.376	0.261	0.475	0.384
Total	*0.493*	*0.571*	*0.400*	*0.384*	*0.541*	*0.609*

Table 4.4: Fraction of marriages with education differentials. Women married between 1965 and 1970 or between 1975 and 1980.

Descriptive Statistics- Sample of Wives				
Husband with Less Education				
Age in 1965 or 1975	*White*	*Black*	*Hispanic*	*Total*
16-17	0.214	0.257	0.221	0.218
18-19	0.244	0.294	0.252	0.248
20-21	0.246	0.315	0.267	0.253
22-23	0.252	0.321	0.262	0.259
24-25	0.251	0.303	0.238	0.256
26-27	0.253	0.325	0.219	0.259
28-30	0.254	0.312	0.211	0.256
Total	0.238	0.295	0.243	0.243
Husband with More Education				
Age in 1965 or 1975	*White*	*Black*	*Hispanic*	*Total*
16-17	0.281	0.231	0.286	0.278
18-19	0.264	0.219	0.251	0.260
20-21	0.258	0.208	0.258	0.254
22-23	0.232	0.224	0.250	0.233
24-25	0.233	0.222	0.261	0.234
26-27	0.241	0.235	0.277	0.244
28-30	0.255	0.232	0.255	0.252
Total	0.261	0.222	0.265	0.259

Table 4.5: Fraction of marriages with education differentials. Men married between 1965 and 1970 or between 1975 and 1980.

Descriptive Statistics - Sample of Husbands				
Wife with More Education				
Age in 1965 or 1975	*White*	*Black*	*Hispanic*	*Total*
16-17	0.236	0.302	0.255	0.243
18-19	0.201	0.288	0.222	0.209
20-21	0.185	0.279	0.228	0.196
22-23	0.175	0.280	0.219	0.187
24-25	0.170	0.286	0.233	0.186
26-27	0.163	0.277	0.226	0.180
28-30	0.172	0.292	0.213	0.190
Total	0.190	0.285	0.228	0.201
Wife with Less Education				
Age in 1965 or 1975	*White*	*Black*	*Hispanic*	*Total*
16-17	0.252	0.193	0.241	0.247
18-19	0.293	0.212	0.263	0.284
20-21	0.320	0.222	0.279	0.308
22-23	0.344	0.245	0.299	0.332
24-25	0.355	0.242	0.277	0.338
26-27	0.361	0.252	0.279	0.342
28-30	0.353	0.223	0.251	0.328
Total	0.317	0.226	0.271	0.306

Table 4.6: Fraction of Men and Women with Interracial Marriages. People married between 1965 and 1970 or between 1975 and 1980.

Descriptive Statistics - Interracial Marriage				
Women Single in 1965 or 1975				
Age in 1965 or 1975	*White*	*Black*	*Hispanic*	*Total*
16-17	0.051	0.142	0.389	0.094
18-19	0.049	0.122	0.382	0.086
20-21	0.046	0.117	0.386	0.082
22-23	0.045	0.110	0.375	0.081
24-25	0.047	0.106	0.358	0.086
26-27	0.047	0.106	0.331	0.087
28-30	0.051	0.120	0.359	0.102
Total	0.048	0.122	0.380	0.087
Men Single in 1965 or 1975				
Age in 1965 or 1975	*White*	*Black*	*Hispanic*	*Total*
16-17	0.085	0.289	0.401	0.131
18-19	0.068	0.184	0.348	0.100
20-21	0.063	0.162	0.351	0.094
22-23	0.060	0.156	0.346	0.090
24-25	0.057	0.143	0.325	0.087
26-27	0.062	0.170	0.311	0.096
28-30	0.065	0.161	0.297	0.099
Total	0.066	0.177	0.348	0.099

Table 4.7: Marginal probit effects of a change in the sex ratio and in the size of the group to the likelihood of first marriage between 1965 and 1970 or 1975-80.

	Marry in the Next 5 Years - Female			
	Whole Sample	*Black*	*College*	*High School*
	(1)	*(2)*	*(3)*	*(4)*
Size	-0.206	-1.28	-1.555	-0.209
	(0.014)**	(0.103)**	(0.171)**	(0.038)**
Sex Ratio	0.068	0.202	0.212	0.199
	(0.006)**	(0.020)**	(0.029)**	(0.019)**
(age=18-19)*Sex Ratio	-0.1	-0.127		-0.056
	(0.007)**	(0.026)**		(0.016)**
(age=20-21)*Sex Ratio	-0.099	-0.246		
	(0.008)**	(0.027)**		
(age=22-23)*Sex Ratio	-0.087	-0.367		
	(0.010)**	(0.031)**		
(age=24-25)*Sex Ratio	-0.128	-0.287	-0.076	
	(0.012)**	(0.036)**	(0.028)**	
(age=26-27)*Sex Ratio	-0.183	-0.357		
	(0.014)**	(0.046)**		
(age=28-30)*Sex Ratio	-0.182	-0.252		
	(0.014)**	(0.045)**		
Observations	496,093	71,565	28,106	126,828

Standard errors in parentheses

* significant at 5%; ** significant at 1%

Table 4.8: Marginal probit effects of a change in the sex ratio and in the size of the group to the likelihood of first marriage between 1965 and 1970 or 1975-80.

	Marry in the Next 5 Years - Male			
	Whole Sample	*Black*	*College*	*High School*
	(1)	*(2)*	*(3)*	*(4)*
Size	-0.188	-0.806	-2.96	0.076
	(0.013)**	(0.107)**	(0.136)**	(0.043)
Sex Ratio	0.075	0.043	0.091	0.036
	(0.005)**	(0.016)**	(0.019)**	(0.014)*
(age=18-19)*Sex Ratio	-0.029	0.018		0.023
	(0.006)**	(0.019)		(0.014)
(age=20-21)*Sex Ratio	-0.075	-0.015		
	(0.007)**	(0.019)		
(age=22-23)*Sex Ratio	-0.093	-0.042		
	(0.008)**	(0.022)		
(age=24-25)*Sex Ratio	-0.135	-0.111	-0.031	
	(0.009)**	(0.024)**	(0.023)	
(age=26-27)*Sex Ratio	-0.153	-0.069		
	(0.011)**	(0.037)		
(age=28-30)*Sex Ratio	-0.174	-0.101		
	(0.011)**	(0.032)**		
Observations	593,672	67,454	40,578	120,741

Standard errors in parentheses

* significant at 5%; ** significant at 1%

Table 4.9: Marginal probit effects of a change in the sex ratio and in the size of the group to the likelihood of first marriage between 1965 and 1970 or 1975-80. Census year fixed effects.

Marry in the Next 5 Years - Female (Year Fixed Effects)				
	Whole Sample	*Black*	*College*	*High School*
	(1)	*(2)*	*(3)*	*(4)*
Size	-0.062	0.027	0.625	-0.209
	(0.014)**	(0.107)	(0.208)**	(0.038)**
Sex Ratio	0.1	0.2	0.019	0.199
	(0.006)**	(0.020)**	(0.029)	(0.019)**
(age=18-19)*Sex Ratio	-0.117	-0.132		-0.056
	(0.007)**	(0.026)**		(0.016)**
(age=20-21)*Sex Ratio	-0.13	-0.266		
	(0.008)**	(0.028)**		
(age=22-23)*Sex Ratio	-0.123	-0.396		
	(0.010)**	(0.032)**		
(age=24-25)*Sex Ratio	-0.159	-0.333	-0.082	
	(0.012)**	(0.037)**	(0.029)**	
(age=26-27)*Sex Ratio	-0.206	-0.378		
	(0.014)**	(0.046)**		
(age=28-30)*Sex Ratio	-0.197	-0.287		
	(0.014)**	(0.045)**		
Observations	496,093	71,565	28,106	126,828

Standard errors in parentheses

* significant at 5%; ** significant at 1%

Table 4.10: Marginal probit effects of a change in the sex ratio and in the size of the group to the likelihood of first marriage between 1965 and 1970 or 1975-80. Census year fixed effects.

Marry in the Next 5 Years - Male (Year Fixed Effects)				
	Whole Sample	*Black*	*College*	*High School*
	(1)	*(2)*	*(3)*	*(4)*
Size	-0.044	0.234	0.338	0.076
	(0.013)**	(0.110)*	(0.186)	(0.043)
Sex Ratio	0.099	0.025	-0.022	0.036
	(0.005)**	(0.016)	(0.018)	(0.014)*
(age=18-19)*Sex Ratio	-0.042	0.017		0.023
	(0.006)**	(0.019)		(0.014)
(age=20-21)*Sex Ratio	-0.107	-0.026		
	(0.007)**	(0.019)		
(age=22-23)*Sex Ratio	-0.136	-0.057		
	(0.008)**	(0.022)**		
(age=24-25)*Sex Ratio	-0.179	-0.132	-0.036	
	(0.009)**	(0.024)**	(0.023)	
(age=26-27)*Sex Ratio	-0.197	-0.092		
	(0.011)**	(0.037)*		
(age=28-30)*Sex Ratio	-0.211	-0.126		
	(0.011)**	(0.032)**		
Observations	593,672	67,454	40,578	120,741

Standard errors in parentheses

* significant at 5%; ** significant at 1%

Table 4.11: Marginal Probit effects of a change of the Sex Ratio on husbands' characteristics. Women married the first time between 1965-70 or 1975-80.

Spouse's Characteristics - Sample of Wives (Year Fixed Effects)				
	Whole Sample		*Black*	
	(a)		*(b)*	
Dependent Variable	*Size*	*Sex Ratio*	*Size*	*Sex Ratio*
Husband of Different Race	-0.178	0.002	0.025	0.025
	(0.011)**	(0.003)	(0.159)	(0.015)
Husband with Same Education	0.194	0.056	2.299	0.105
	(0.012)**	(0.005)**	(0.161)**	(0.020)**
Husband with Less Education	-0.982	-0.336	-2.527	-0.59
	(0.015)**	(0.005)**	(0.148)**	(0.021)**
Husband with More Education	0.452	0.175	1.241	0.394
	(0.015)**	(0.005)**	(0.143)**	(0.016)**
Husband Previously Married	-0.077	0.006	0.13	-0.006
	(0.012)**	(0.004)	(0.146)	(0.016)
Husband with Own Children	-0.006	-0.048	0.165	-0.022
	(0.021)	(0.007)**	(0.228)	(0.023)
Observations	251,660		19,546	

Standard errors in parentheses

* significant at 5%; ** significant at 1%

Table 4.12: Marginal Probit effects of a change of the Sex Ratio on wives' characteristics. Men married the first time between 1965-70 or 1975-80.

Spouse's Characteristics - Sample of Husbands (Year Fixed Effects)				
	Whole *Sample*		*Black*	
	(a)		*(b)*	
Dependent Variable	*Size*	*Sex Ratio*	*Size*	*Sex Ratio*
Wife of Different Race	-0.1	0.006	0.521	-0.018
	(0.011)**	(0.003)*	(0.157)**	(0.014)
Wife with Same Education	1.109	-0.067	3.022	-0.244
	(0.017)**	(0.005)**	(0.143)**	(0.018)**
Wife with Less Education	-1.523	-0.336	-4.074	-0.209
	(0.016)**	(0.005)**	(0.115)**	(0.013)**
Wife with More Education	0.295	0.327	1.763	0.427
	(0.013)**	(0.004)**	(0.138)**	(0.015)**
Wife Previously Married	-0.067	0.015	-0.086	0.003
	(0.012)**	(0.003)**	(0.108)	(0.011)
Wife with Own Children	0.07	-0.043	0.304	-0.024
	(0.021)**	(0.006)**	(0.192)	(0.019)
Observations	247,823		20,880	

Standard errors in parentheses

* significant at 5%; ** significant at 1%

Chapter 5

Conclusion

The model developed in Chapter 2 that asymmetric fecundity horizons between men and women alone are sufficient to generate a stylized fact that holds across many societies – on average younger women marry older men. The 2-parameter model developed to address this fact also accounts for other stylized facts about the marriage market. The contribution of this paper is to provide a framework where age of marriage is determined by biological concerns, ignoring potential gains of specialization. In the last few decades, men and women have become more alike in their social roles. Female labor force participation has increased dramatically in many societies and differences in education level are disappearing in more developed countries. The model fits the data well for recent decades in the United States, France and Sweden. A second contribution of this work is to show how the effect of asymmetries in mortality between men and women can affect the structure of the marriage markets. This plays a particularly important role for Black Americans.

Countries with advanced post-industrial demographics (e.g. France, Sweden and the U.S.) have ages at first marriage and an age pattern of sex ratios that closely resemble the model's predictions. In developing countries, however, where traditional gender roles are still important the model predictions fail to explain the patterns in the data. This failure suggests that in these contexts the model is incomplete.

The model of Chapter 3 has several implications that appear to be consistent with US data. First it suggests that the fraction of intermarriage increases with age. This fact is an empirical regularity for interracial marriage of whites in the US, at least from 1975 to 1995. For blacks, however, data does not show a similar increasing pattern in interracial marriage. However, the data on women marrying lower educated men does indeed shows this patter for African- American women married during their 30's. The second implication of the model is that, due to increasing search frictions, the smaller the size of the group, the higher the rate of intermarriage, the lower the marriage rate and the older the age of marriage. Here even qualitative consistent, the model predicts a much a higher rate of interracial marriage for African-Americans than the pattern observed in the data.

Now that we have a better understanding of the role of biology, one natural extension of this work is to incorporate features of the labor market that do vary across countries and over time. Of particular importance are both the earnings ability of men in a society and the relative earnings ability of men versus women. This model is designed to incorporate these extensions.

The results of the empirical study of Chapter 4 suggest that *a shortage of single men leads women (and also men) to marry earlier*. This result suggests that the behavior of men and women respect to the timing of marriage is not driven only by the direct effect of the availability of partners but also by the reaction of the rest of the market to a given sex imbalance. In addition, the result may imply a more elastic response for women to a tight marriage market than the one for men. This is consistent with a marriage model where the search horizon for women is shorter than the one for men, as the one developed in the previous chapters. The results also suggest that an adverse change in the sex ratio or a relatively small size of their ethnic/educational group, in line with Chapter 3, can lead both men and women to marry outside of their own racial or educational group.

Appendix A

Complete Proof of Theorems 1 and 2

A.1 Existence

By Equations (2.11) and (2.12), we know that

$$\begin{aligned} R^m &= \frac{R^m_{old}}{k(1+\beta)} \\ R^f &= \frac{R^f_{old}}{k(1+\beta)} \end{aligned}$$

Then, it will be sufficient to show existence and uniqueness for R^m_{old} and R^f_{old} to show them for R^m and R^f.

We need to show that the following functional relationship is a fixed point

$$T \begin{pmatrix} R^m_{old} \\ R^f_{old} \\ m_2 \\ f_2 \end{pmatrix} \longrightarrow \begin{pmatrix} T_1\left(R^m_{old}, R^m_{old}, m_{2,} f_2\right) \\ T_2\left(R^m_{old}, R^m_{old}, m_{2,} f_2\right) \\ T_3\left(R^m_{old}, R^m_{old}, m_{2,} f_2\right) \\ T_4\left(R^m_{old}, R^m_{old}, m_{2,} f_2\right) \end{pmatrix}$$

where

$$
\begin{aligned}
T_1 &= \beta\mu\left(\frac{m_1+m_2}{f_1+f_2}\right)^{\theta-1}\left[k\frac{f_1}{f_1+f_2}\left(1-G_m\left(R^f_{old}\right)\right)+\frac{f_2}{f_1+f_2}\right]\overline{x} \\
T_2 &= \frac{1}{k}\beta\mu\left(\frac{m_1+m_2}{f_1+f_2}\right)^{\theta}\left(1-\left(\frac{m_1}{m_1+m_2}\right)G_f\left(R^m_{old}\right)\right)\overline{y}
\end{aligned}
$$

$$
\begin{aligned}
T_3 &= m_1\left(1-\mu\left(\frac{m_1+m_2}{f_1+f_2}\right)^{\theta-1}\left(\begin{array}{l}\frac{f_1}{f_1+f_2}\left(1-G_m\left(\frac{R^f_{old}}{1+\beta}\right)\right)\left(1-G_f\left(\frac{R^m_{old}}{k(1+\beta)}\right)\right)+ \\ \frac{f_2}{f_1+f_2}\left(1-G_f\left(R^m_{old}\right)\right)\end{array}\right)\right) \\
T_4 &= f_1\left(1-\mu\left(\frac{m_1+m_2}{f_1+f_2}\right)^{\theta}\left(\begin{array}{l}\frac{m_1}{m_1+m_2}\left(1-G_m\left(\frac{R^f_{old}}{1+\beta}\right)\right)\left(1-G_f\left(\frac{R^m_{old}}{k(1+\beta)}\right)\right)+ \\ \frac{m_2}{m_1+m_2}\left(1-G_m\left(R^f_{old}\right)\right)\end{array}\right)\right)
\end{aligned}
$$

is a fixed point.

Lets define a compact set $X=[0,xmax]\times[0,ymax]\times[0,m_1]\times[0,f_1]$ .The next step is to show that $T(.)$ is continuous and maps from X to itself. Lets pick the point $x_0=(R^m_{old0},R^m_{old0},m_{20,}f_{20})$ where $R^m_{old0}\in[0,xmax]\,,R^f_{old0}\in[0,xmax]\,,m_{20}\in 0,m_2\in[0,m_1]$ and $f_2\in[0,f_1]$. Then

$$
\begin{aligned}
T_{10}\left(R^m_{old0},R^m_{old0},m_{20,}f_{20}\right) &\in [0,xmax] \\
T_{20}\left(R^m_{old0},R^m_{old0},m_{20,}f_{20}\right) &\in [0,ymax] \\
T_{30}\left(R^m_{old0},R^m_{old0},m_{20,}f_{20}\right) &\in [0,m_1] \\
T_{40}\left(R^m_{old0},R^m_{old0},m_{20,}f_{20}\right) &\in [0,f_1]
\end{aligned}
$$

Lets show first the case of T_{10}. By definition $0<\mu\left(\frac{m_1+m_{20}}{f_1+f_{20}}\right)^{\theta-1}<1, 0\leq G_m\left(R^f_{old0}\right)\leq 1$, $\frac{f_1}{f_1+f_{20}}+\frac{f_{20}}{f_1+f_{20}}=1$ and $0<\beta<1$. Also, by condition $(k1)$ $k\overline{x}<xmax$. Therefore, $T_{10}\in[0,xmax]$.

Similar is the case of T_{20}. Since $0.5<\frac{m_1}{m_1+m_{20}}<1, k>1$ and $0\leq G_f\left(R^m_{old0}\right)\leq 1$, it is clear that $T_{20}\in[0,ymax]$

In the case of T_3 , the expression

$\left(\frac{f_1}{f_1+f_{20}}\left(1-G_m\left(\frac{R^f_{old0}}{1+\beta}\right)\right)\left(1-G_f\left(\frac{R^m_{old0}}{k(1+\beta)}\right)\right)+\frac{f_{20}}{f_1+f_{20}}\right)$ is clearly between 0 and 1. The probability of meeting a woman $\left(\frac{m_1+m_{20}}{f_1+f_20}\right)^{\theta-1}$ is also between 0 and 1. Therefore m_2 must belong to the interval $[0, m_1]$. Similar explanation can be used to justify that $T_{40}\left(R^m_{old0}, R^m_{old0}, m_{20,}f_{20}\right) \in [0, f_1]$.

The continuity of $T(.)$ is ensured given the boundaries given to the ratio single men/single women. Discrete jumps are prevented given $\left(\frac{m_1+m_2}{f_1+f_2}\right) \in [0.5, 2]$.

A.2 Uniqueness in the Equilibrium Strategies

Lets start defining the reaction functions

$$R^m_{old} = T_1(R^f_{old}) = \eta^m\beta\left[(1-p)+kp\left(1-G_m\left(R^f_{old}\right)\right)\right]\overline{x} \tag{A.1}$$

$$R^f_{old} = T_2(R^m_{old}) = \frac{\beta\eta^f\left(1-qG_f\left(R^m_{old}\right)\right)\overline{y}}{k} \tag{A.2}$$

By definition, $p \in [0.5, 1]$ and $q \in [0.5, 1]$. Differentiating $(A.1)$ and $(A.2)$ we see that T_1 and T_2 are strictly decreasing in R^f_{old} and R^m_{old} respectively.

$$T_1'(R^f_{old}) = -kp\eta^m\beta g_m\left(R^f_{old}\right)\overline{x} < 0 \tag{A.3}$$

$$T_2'(R^m_{old}) = -\frac{q\beta\eta^f g_f\left(R^m_{old}\right)\overline{y}}{k} < 0 \tag{A.4}$$

Now, using the condition in (2.5), it is easy to show that the intercepts are

$$\begin{aligned} T_1(0) &= (1+p(k-1))\,\eta^m\beta\overline{x} < xmax \\ T_2(0) &= \frac{\beta\eta^f\overline{y}}{k} < ymax \\ T_1(ymax) &= (1-p)\,\eta^m\beta\overline{x} \geq 0 \\ T_2(xmax) &= \frac{\beta\eta^f(1-q)\,\overline{y}}{k} \geq 0 \end{aligned}$$

The above conditions, summarized in the figure, rule out any corner solution and at the same time guarantee the existence of at least one interior solution.

The next step will be to show that there is a unique equilibrium in the model. We know that

$$\begin{aligned} R_{old}^{m} &= T_1(R_{old}^{f}) = T_1(T_2\left(R_{old}^{m}\right)) \\ &\equiv H\left(R_{old}^{m}\right) \end{aligned}$$

where

$$H\left(R_{old}^{m}\right) = \eta^{m}\beta\left[(1-p) + kp\left(1 - G_m\left(T_2\left(R_{old}^{m}\right)\right)\right)\right]\overline{x}$$

To ensure uniqueness, we need a fixed point of $H\left(.\right)$. Thus, we need to show that $H\left(R_{old}^{m}\right)$ is a contraction mapping. that is

$$\begin{aligned} \|H\left(R_{old}^{m}\right) - H\left(R_{old}^{m}\right)\| &\leq \partial\|R_{old}^{m} - R_{old}^{m}\| \\ \text{where } 0 &< \delta < 1 \end{aligned}$$

To show that H is a contraction mapping, it is enough to prove that

$$H'\left(R_{old}^{m}\right) \leq \delta < 1 \quad \forall R_{old}^{m} \in [0, xmax]$$

But

$$H'\left(R_{old}^{m}\right) = T_1'(T_2\left(R_{old}^{m}\right)).T_2'\left(R_{old}^{m}\right) \tag{A.5}$$

Using (2.2), $(2.3)\,(2.4)$, $(A.3)$, $(A.4)$and the assumption that $m_1 = f_1$, substituting in $(A.5)$ and manipulating, we get

$$H'\left(R_{old}^{m}\right) = \left(\mu\left(\frac{M}{f}\right)^{\theta}\right)^{2}\beta^{2}\left(\frac{m_1}{M}\right)^{2} g_f\left(R_{old}^{m}\right) g_m\left(T_2\left(R_{old}^{m}\right)\right)\overline{yx}$$

By assumption, we know that $\left(\mu\left(\frac{M}{f}\right)^{\theta}\right) \leq 1$, that $\frac{m_1}{M} \leq 1$ and that $\beta < 1$. Hence, it will be sufficient for $H'\left(R_{old}^{m}\right)$ to be a contraction mapping if we have

$$g_f\left(R_{old}^{m}\right) g_m\left(T_2\left(R_{old}^{m}\right)\right) \leq C$$

where

$$C < \frac{1}{y\overline{x}}$$

Therefore, we have that

$$H'\left(R^m_{old}\right) < 1$$

A.3 Proof of Proposition 3

By the assumption above, $G_f(.) = G_m(.)$. First, by (2.11) and (2.12), and using (2.2), (2.3) and (2.4), we define the difference between reservation values of men and women, R^m and R^f, as follows

$$R^m - R^f = \frac{\beta\overline{x}\left(\frac{M}{f}\right)^{\theta}\left(m_1\left(1 - G_f\left(R^m_{old}\right)\right) + kf_1\left(1 - G_f\left(R^f_{old}\right)\right) + m_2 - f_2\right)}{k(1+\beta)M}$$

taking derivatives with respect to k

$$\frac{\partial}{\partial k}\left(R^m - R^f\right) = \frac{\beta\overline{x}\left(\frac{M}{f}\right)^{\theta}\left(m_1\left(1 - G_f\left(R^m_{old}\right)\right) + m_2 - f_2\right)}{k^2(1+\beta)M}$$

For $k = 1$, we know that men and women face exactly the same problem. Then $R^m = R^f, h_1 = w_1$ and $m_2 = f_2$.By the proof of Theorem 1 also $R^m_{old} < 1$. So,

$$\frac{\partial}{\partial k}\left(R^m - R^f\right)_{k=1} = \frac{\beta\overline{x}m_1\left(1 - G_f\left(R^m_{old}\right)\right)}{(1+\beta)M} > 0$$

Hence, $R^m > R^f$ if $k > 1$, which completes the proof.

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