ABSTRACT

Title of dissertation: TEACHING FOR INCLUSION: THE EFFECTS OF A PROFESSIONAL DEVELOPMENT COURSE FOR SECONDARY GENERAL AND SPECIAL EDUCATION MATHEMATICS TEACHERS FOR INCREASING TEACHER KNOWLEDGE AND SELF-EFFICACY IN GEOMETRY

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The current study examined the effects of a co-teacher professional development course for increasing the knowledge and self-efficacy of special education and general education geometry teachers in an inclusion setting. The professional development course included instruction on Universal Design for Learning instructional strategies as well as similarity and congruence in geometry. The course was presented in a blended learning format and included in-person and online activities. A multiple probe design across three sets of two teachers for a total of 6 participants was used in this study to demonstrate a functional relationship between the independent and dependent variables. The participants were six special education and general education geometry teachers from public charter schools in Washington, DC. Results of the study demonstrated that participants were able to improve their content and pedagogical content knowledge in
geometry as well as their self-efficacy for teaching in an inclusion setting. Specifically, special education teachers demonstrated a greater increase in content knowledge while general education teachers demonstrated a greater increase in self-efficacy for teaching students with disabilities. The study suggests that providing professional development for co-teachers can enhance collaboration as well as increase content knowledge and teacher self-efficacy.
TEACHING FOR INCLUSION: THE EFFECTS OF A PROFESSIONAL DEVELOPMENT COURSE FOR SECONDARY GENERAL AND SPECIAL EDUCATION MATHEMATICS TEACHERS FOR INCREASING TEACHER KNOWLEDGE AND SELF-EFFICACY IN GEOMETRY

by

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Chapter 1: Introduction

Mathematical literacy has critical implications for personal success in post-secondary education, career opportunities, and the ability to increase future income. The Final Report of the National Mathematics Advisory Panel (NMAP, 2008) reported a high correlation between completion of an Algebra II course with success in post secondary education and earnings from employment. Specifically, completion of Algebra II more than doubles students’ chances of graduating from college as compared to students with less mathematical preparation. Mathematics achievement is also regarded as an indicator of the nation’s ability to compete in a global economy. Global leadership is buttressed by an educated technical workforce in today’s era of technological advancements (NMAP, 2008).

Geometry, in particular, is considered to be a meaningful and vital topic in mathematics for it addresses concepts that are useful for many careers, daily living skills, and provides a critical foundation for the study of algebra (Cawley, Foley, & Hayes, 2009; NMAP, 2008). By engaging with geometry content, students have a number of opportunities to develop cognitive, language, and communication processes that develop critical thinking and problem solving skills (Seago, Driscoll, & Jacobs, 2010). In addition, geometry involves measurement and assigning characteristics to objects, skills used daily in living and employment. Congruence and similarity, in particular, are considered important topics within geometry (CCSSI, 2010; Seago, Driscoll, & Jacobs, 2010). Understanding congruence and similarity is useful for a variety of real life applications including following directions for assembling furniture, understanding the blueprint of a building, or reading a scale on a map. Given the importance of mathematical competency, it is imperative that all students have access to quality mathematics instruction.
During the past 10 years, the percentage of students in special education in inclusive classrooms has increased considerably, with over half of all identified students being educated in general education settings (National Center for Educational Statistics, 2011). Nearly 61% of students diagnosed with specific learning disability (SLD) spend 80% or more of each school day in a general education classroom and are not exempt from state and national standardized assessments (U.S. Department of Education, 2011). One of the premises of inclusion is that students with disabilities receive access to the same curriculum as their non-disabled peers, the goal being to improve the social and academic outcomes for students who with disabilities. However, students with a learning disability (LD) often experience academic failure in general education classrooms (Schumaker & Deshler, 1988). Abstract mathematical concepts, such as congruence and similarity, in particular, pose a challenge for students with LD who possess a variety of characteristics that make learning and remembering mathematics difficult (Miller & Mercer, 1997). Given that recent legislation, such as the No Child Left Behind Act (NCLB) of 2001, mandates that all students, including students with disabilities, demonstrate proficiency with the general education curriculum, it is critical that close attention is given to practices and methods for ensuring that all students achieve an understanding of content.

One of the implications for increasing the successful inclusion of students with disabilities in general education classes is that teachers must adapt their instruction to meet the needs of a more diverse group of students. Secondary content-area teachers, in particular, often feel ill prepared to teach students with disabilities or know how to provide the necessary accommodations for students to access the curriculum due to little or no training (Kosko & Wilkins, 2009; Maccini & Gagnon, 2006). In addition, special education
teachers, who are often hired to teach alongside general educators, generally find that they lack the content knowledge for helping students meet the rigorous standards for demonstrating proficiency (Eisenman, Pleet, Wandry, & McGinley, 2010). Thus, teachers need intensive and sustained professional development for adapting instruction to meet the needs of students with disabilities as well as increase their knowledge for teaching mathematical concepts.

In the section below, I will first summarize the results from international, national, and state mathematics assessments, particularly as they relate to geometry, followed by an overview of mathematics reform efforts related to teaching and learning for all students. I then discuss the characteristics of students with LD with an emphasis on the implications for educating these students with their non-disabled peers. I will also discuss the role of teacher knowledge as it relates to teacher efficacy and how in-service training may increase the teacher’s ability to meet the needs of a diverse learning community. The chapter concludes with a purpose statement, guiding research questions, and definitions of key terminology.

**Status of Mathematics Proficiency in the U.S.**

Although researchers, policymakers, and other stakeholders have acknowledged the importance of mathematical competency, students in the U.S. are not performing well as reported on international, national, and state assessments. In terms of international assessments, the Program for International Student Assessment (PISA) is sponsored by the Organization for Economic Cooperation and Development (OECD) and measures the mathematics literacy of 15-year-olds. The most recent PISA (2012) reported that U.S. 15-year-olds scored significantly below the OECD average as they were outperformed by 25 of
the 34 countries (OECD, 2013). Further, only 12.6% of U.S. students scored at or above Level 5 proficiency level, which represents the level at which students are able to select, compare, and evaluate appropriate problem-solving strategies for dealing with complex problems, skills essential in the study of geometry. A second international assessment, the Trends in International Mathematics and Science Study (TIMSS), measures the mathematics performance of students based on classroom curricula. Unlike the PISA assessment, which assesses students’ ability to apply mathematical reasoning and problem solving skills to real-life contexts, the TIMSS focuses on students’ knowledge of mathematical concepts taught in the classroom. In 2011, students’ overall mathematics performance at the eighth grade level was reported for 48 countries. Overall, the average scale score for U.S. eighth-grade students was nine percentage points higher than the international average and lagged behind 8 countries including Korea, Singapore, and Chinese Taipei (Mullis, Martin, Foy & Arora, 2012). In addition, only 6% of U.S. eighth-graders scored at or above the TIMSS advanced benchmark, which is the level at which students can make generalizations, organize information, and solve complex problems. In regard to the geometry domain, U.S. eighth-grade students scored 20 percentage points below the average range, outperforming 30 countries, and scoring lower than 17 countries (Mullis et al, 2012). The TIMSS geometry content covers congruence, similarity, and application of geometric properties. Data for students with disabilities was not disaggregated; therefore, no comparisons can be made between students with disabilities and their international counterparts.

In addition to the overall underperformance of U.S. students in mathematics on international assessments, U.S. students have not made the achievement gains expected on
national assessments, such as the National Assessment of Educational Progress (NAEP), which covers several mathematics content areas, including geometry. Overall, approximately two-thirds (64%) of eighth graders in the general education population scored below the proficient level (i.e., competency in grade level material) on the most recent NAEP assessments (National Center for Education Statistics, 2013). On the geometry portion of the same test, the average scale score was 17 percentage points below the proficient level for eighth-grade students (National Center for Education Statistics, 2013).

The performance results for secondary students with disabilities are more alarming. On the same assessment, 93% of eighth-graders with disabilities performed below the proficient level (National Center for Education Statistics, 2013). Further, the average scale score for eighth grade students with disabilities on the geometry portion of the assessment was 31 points lower than their general education peers (National Center for Education Statistics, 2013). Students have also not performed well on state and district assessments. In the District of Columbia, for example, 49% of elementary students and 46% of secondary students achieved proficiency or above on the 2013 District of Columbia Comprehensive Assessment, failing to meet the targets for Adequate Yearly Progress (Office of the State Superintendent, n.d.). For students with disabilities, only 24% of elementary students and 16% of secondary students achieved proficiency. In Maryland, while 67% of all eighth grade students demonstrated proficiency on the 2013 Maryland State Assessment, only 25% of eighth grade students with disabilities achieved the proficient level on the assessment (Maryland Department of Education, 2013). Although there may be several reasons to explain the low proficiency rates of students with disabilities in mathematics,
particular challenges associated with learning disabilities pose difficulties for children in an educational setting.

**Characteristics of Students with LD**

Students with LD possess several characteristics that hinder the learning process and impede their performance in mathematics. A learning disability is defined as a disorder in one or more of the basic psychological functions involved in understanding or in using language and manifests itself in an imperfect ability to listen, speak, read, write, spell, or do mathematical calculations (IDEA, 2004). Students with LD in mathematics often struggle with basic mathematics content due to poor short-term memory processes (Calhoon, Emerson, Flores, & Houchins, 2007). For example, students with LD may have trouble remembering the procedures for using the order of operations. Moreover, they may exhibit reading and language difficulties. Specifically, students with LD typically have reading problems, which may interfere with understanding mathematics vocabulary as well as completing homework and textbook assignments (Steele, 2010). In geometry, language deficits may also interfere with a student’s ability to associate words to the symbols of geometric terms and also to respond to questions orally in class such as explaining the difference between complementary and supplementary angles or discussing the use of the Pythagorean Theorem (Bley & Thornton, 2001; Ives, 2007; Steele, 2010). In addition, students with LD exhibit poor metacognition (Bley & Thornton, 2001). Poor metacognition impedes students’ ability to engage in problem solving as well as to solve complex, multi-step problems (Vaidya, 1999). For example, students with LD may have trouble determining the value of the variable in an unknown angle measurement of a given geometric figure.
Visual processing problems such as difficulty with visual spatial processing are also common to many students with LD (Lerner & Johns, 2009) and can interfere significantly with mathematics performance. For example, a student with visual spatial deficits may have problems with identifying or demonstrating translations, rotations, and reflections. Memory problems, typical for students with LD, can also make high school mathematics courses challenging (Lerner & Johns, 2009). These difficulties often pose a challenge for content-area teachers who are inadequately trained to provide instruction to students with disabilities.

**Reform Movement**

To address the underperformance of U.S. students in mathematics, policymakers have passed legislation and issued directives in an effort to improve mathematics education. The National Council of Teachers of Mathematics (NCTM) continues to spearhead efforts and guidance on mathematics content, pedagogy, and assessment across the K-12 grade levels. The Principles and Standards for School Mathematics (NCTM, 2000) maintains the principle of equity, demanding high expectations for mathematics learning and access to high quality instruction and resources for all students. In addition, the Professional Standards for Teaching Mathematics (NCTM, 1991) emphasizes the role of teachers in the development of positive attitudes towards mathematics. According to this document, it is essential that teachers provide a learning environment that facilitates and promotes students' flexibility in thinking, creativity, perseverance, and self-confidence in doing mathematics.

At the federal level, the reauthorization of the Elementary and Secondary Education Act (2001), also known as No Child Left Behind (NCLB), contains several mandates for
ensuring quality instruction for all students, including students with disabilities. NCLB requires that students receive instruction from a “highly qualified teacher,” defined as one who holds a "bachelor’s degree, has full state certification and has demonstrated subject area competence for each subject taught" (NCLB, 2002). This mandate supports the idea that teacher content knowledge plays a critical role in improving student achievement (Darling-Hammond & Bransford, 2005; Hill, Rowan, & Ball, 2005). NCLB also includes regulations that require states to submit plans for ensuring that students with disabilities have access to the general education curriculum (NCLB, 2002).

Most recently, the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) have led the initiative to develop a common set of English language arts and mathematics standards to be adopted by all states. Known as the Common Core State Standards Initiative (CCSSI), the standards address what students are expected to learn in order to prepare them for college and career (CCSSI, 2010). To date, forty-five states have adopted this initiative. The goal of the CCSSI for mathematics is to provide more clarity and coherence of what students should know and understand in mathematics. The standards for geometry, in particular, cover several critical areas including congruence and similarity, properties of two-dimensional and three-dimensional shapes, applying the Pythagorean theorem, and application of probability concepts (CCSSI, 2010).

Under the CCSSI are specific habits and practices that students should demonstrate as they learn mathematical concepts across grade levels. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and
proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (i.e., comprehension of mathematical concepts, operations and relations), procedural fluency (i.e., skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (i.e., habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). The eight Common Core Standards for Mathematical Practice include the following:

**Make sense of problems and persevere in solving them.** This practice standard involves students performing actions such as explaining the meaning of a problem to themselves, looking for entry points, and making conjectures and a plan to solve a problem. (CCSS, 2010).

**Reason abstractly and quantitatively.** This practice standard involves students demonstrating the ability to create coherent representations of problems, attend to the meaning of quantities, use properties of operations and objects, decontextualize situations by manipulating mathematical symbols abstractly without necessarily attending to the original referents, and to pause and contextualize the symbols during the manipulation to make sense of the symbols concretely in terms of the original context (CCSS, 2010).

**Construct viable arguments and critique the reasoning of others.** Students who demonstrate this standard are able to understand stated assumptions/definitions, make conjectures, build progressions of statements to explore the truth of conjectures, justify
conclusions, compare the effectiveness of two plausible arguments, and distinguish and use correct logic and reasoning. (CCSS, 2010).

**Model with mathematics.** This practice standard involves students applying mathematics to solve everyday problems, using multiple representations such as graphs, tables and equations and interpreting/reflecting on the mathematical results in the context of the situation (CCSS, 2010).

**Use appropriate tools strategically.** This practice standard involves students considering and using available tools (e.g., concrete model, ruler, compass, calculator, spreadsheets, dynamic geometry software) and evaluating their effectiveness for a given situation to help students explore and deepen their understanding of mathematical concepts (CCSS, 2010).

**Attend to precision.** This practice standard involves students using clear and precise definitions in discussions and explanations of their own work, stating the meaning of and/or correctly using symbols specifying units of measure, calculating with precision that is appropriate for a problem context, and giving carefully formulated explanations to each other (CCSS, 2010).

**Look for and make use of structure.** This practice standard involves students analyzing objects and examples to discern patterns and structures, extending lines in geometric figures to assist in problem solving, and “seeing” complicated things such as an algebraic expression as either a single whole or composed of several objects (CCSS, 2010).

**Look for and express regularity in repeated reasoning.** This practice standard involves students noticing repeated calculations and looking for general methods or
shortcuts, maintaining oversight of the problem solving process, and continually evaluating reasonableness of approach and results (CCSS, 2010).

As addressed in the CCSSI, students eligible for special education services under the Individuals with Disabilities Education Act (IDEA) must be able to access the general education curriculum and be prepared for post-secondary success in college and career. Although the CCSSI does not prescribe specific strategies or methods of instruction, the Standards do include guidance on accommodating students with disabilities and several organizations, including the Council for Exceptional Children, have voiced their support for this effort (CCSSI, 2010). Given the wide acceptance of these standards, it is imperative that teachers receive professional development for increasing their knowledge of geometry content as many states follow the traditional secondary mathematics curriculum sequence that includes Algebra I, Geometry, Algebra II, and Pre-Calculus (CCSSI, 2010). There are also implications for teachers to adjust their instruction to help students meet the new curriculum demands under the Common Core Standards (CCSSI, 2010).

The CCSSI reference Universal Design for Learning (UDL) as guiding principles for teachers to use for ensuring all students have access to the common core learning standards. UDL expands upon the construct of Universal Design in architecture and product design, and was established by Rose and Meyer (2002) following the 1997 reauthorization of the Individuals with Disabilities Education Act ([IDEA], Edyburn, 2010). UDL is a set of principles for curriculum development that give all individuals equal opportunities to learn. It provides a blueprint for creating instructional goals, methods, materials, and assessments that work for everyone--not a single, one-size-fits-all solution but rather flexible approaches that can be customized and adjusted for individual needs.
Thus, teachers will find these principles helpful when planning their instruction for students with diverse learning needs.

**Inclusion, Teacher Knowledge, and Self-Efficacy**

Due to recent policy reforms and legislative mandates, schools are increasingly held accountable for demonstrating a positive impact on achievement for all students (Darling-Hammond, 2004). In response, schools are increasingly educating students with disabilities alongside their non-disabled peers in an inclusive classroom setting, presenting several challenges for general education and special education teachers alike. Traditionally, personnel from these two educational fields receive separate training as general education teachers are prepared through a focus on content and pedagogy related to their expertise (e.g., mathematics) while special educators are trained on best practices for delivering specialized instruction across multiple subject areas (Buell, Hallam, Gamel-McCormick, & Scheer, 1999). However, with the increase in inclusionary practices, it is essential to address the needs and supports of both general and special educators who are often required to work in tandem to meet the diverse learning needs of the students they teach. Ensuring that students with and without disabilities receive high-quality instruction often relies on the successful preparation and support of personnel from both systems (Villa, 1996).

Teachers’ perception of their self-efficacy plays a critical role in the instructional decisions they make (Kosko & Wilkins, 2009). Researchers (Bandura, 1994; Hoy, 2007) describe self-efficacy as one’s perceived level of competency and ability to carry out specific behaviors that influence desired outcomes for student learning and note that teacher efficacy is an important variable in teacher effectiveness. Teachers must have
confidence in their ability to influence the achievement of their students. The implications for possessing high levels of efficacy are evident in research (Gibson & Dembo, 1984; Swackhamer, Koellner, Basile, Kimbrough, 2009). Specifically, it was reported that teachers with high self-efficacy were more open to using reform-based instructional methods (Czerniak & Chiarleott, 1990), more apt to work with students who have difficulties, and more open to adapting their instruction to better support their students (Gibson & Dembo, 1984; Swackhamer, Koellner, Basile, Kimbrough, 2009). However, much of the research on teacher’s self-efficacy involves pre-service teachers (Kosko & Wilkins, 2009). Future research is needed to fill the gap in the literature by focusing on inservice special education and general education mathematics teachers.

In order to improve mathematics instruction and increase the use of effective teaching practices, teachers must possess extensive knowledge of mathematics content as well as an understanding of how students think and learn (Feuerborn, Chinn, & Morlan, 2009; Heck, Banilower, Weiss, & Rosenberg, 2008). Shulman (1986) proposed that such teacher knowledge is represented in three categories: (a) general pedagogical knowledge, which involves general classroom management strategies and organization that transcend subject matter; (b) content knowledge, which includes the knowledge and understanding of mathematical facts, skills, and concepts that students need to learn; and (c) pedagogical content knowledge which involves the ability to present mathematical knowledge in an effective way for students to learn. Recent mathematics research (Krauss, et al., 2008; Shulman, 1986) has generally focused on the development of teachers’ content knowledge (CK). Teachers with a stronger conceptual understanding of mathematics content are better equipped to provide higher quality instruction when compared with teachers with
only a limited understanding of the material (Ball & Cohen, 1999; Garet et al., 2001; Krauss et al., 2008).

Research demonstrates a positive correlation between teacher CK and student achievement (Hill, Rowan, & Ball, 2005; Mullens & Murnane, 1996). For example, in a four-year study of 115 elementary schools, Hill, Rowan, and Ball (2005) found that teacher’s mathematical knowledge was significantly related to student achievement at first and third grades. Researchers defined teacher’s mathematical knowledge as the teacher’s ability to solve, explain, and interpret mathematical concepts, which goes beyond the teacher’s computational skills or years of coursework. In a one-year study of tenth grade students and their mathematics teachers in Germany, Bruner and colleagues (2009) investigated the importance of CK and pedagogical content knowledge (PCK) for improving student outcomes in mathematics based on a test covering the federal states curriculum standards for tenth grade. Although researchers determined that PCK had a greater effect on student achievement, CK was highly correlated with PCK and it was concluded that CK helps to form the basis for the development of PCK.

Given the importance of increasing teacher knowledge of mathematics content and pedagogy for improving student outcomes, researchers and policy makers have sought to improve the quality and number of mathematics courses required in pre-service teacher education programs (National Commission on Teaching and America’s Future, 1996). However, there remains a need for increased training opportunities for practicing mathematics teachers (Feuerborn, et al., 2009; Kosko & Wilkins, 2009). Inservice teachers need meaningful professional development that expands their knowledge and
understanding of mathematics content and pedagogy for increasing their ability to meet the needs of all learners.

**Teacher Professional Development**

A growing body of research demonstrates that teacher professional development (PD) can be an effective means for improving teacher CK and PCK in mathematics (Garret, Porter, Desimone, Birman & Kwang Suk, 2001; Heck, Banilower, Weiss, & Rosenberg, 2008). For instance, Anderson and Hoffmeister (2007) found that the development of a PD course that addressed mathematics content for middle school mathematics teachers helped to increase their content knowledge and conceptual understanding of mathematical concepts based on pre- and post-tests. Also, Feuerborn, Chinn, and Morlan (2009) reported that teachers’ CK significantly improved after participating in a week-long professional development course emphasizing algebra concepts, probability, and geometry concepts.

Although PD is a general term for a variety of activities and interactions designed to improve teacher practices, there are several core features of effective PD programs (Desimone, 2011; Garet, Porter, Desimone, Birman, & Yoon, 2001; Leko & Brownell, 2009). Specifically, effective PD should be: (a) content focused and address knowledge and pedagogy specific to the targeted subject matter (Garet et al., 2001; Yoon, Duncan, Lee, Scarloss, & Shapley, 2007); (b) coherent and aligned to teacher needs, school-wide goals, and curriculum standards (Leko & Brownell, 2009); (c) collaborative, allowing teachers to discuss their practices and work collectively to address student achievement (Garet et al., 2001); and (d) active and engaging, affording teachers the opportunity to actively participate in the PD session and practice what they learned (Desimone, Porter, Garet, Yoon, & Birman, 2002). Therefore, when designing effective PD opportunities, it is
important to incorporate these features by focusing on specific content, addressing identified teacher goals and needs, and promoting collaboration and active teacher engagement. Given the increase of including students with disabilities in general education classrooms, it is imperative that secondary general and special education teachers receive professional development for improving their CK and PCK in mathematics as well as their ability to meet the needs of a diverse classroom.

When designing PD opportunities, an important feature to consider is the format of the PD experience. Emerging technologies, such as video and online instruction, make it possible to enhance PD content, providing an advantage over more traditional instructor directed PD formats (Frey, 2009). One particularly promising example is using computer animated representations which involve computer generated representations of actual videotaped classroom interactions which can provide an advantage over more traditional PD formats (Moreno & Ortegano-Layne, 2008). Specifically, the researchers found that use of video animations and narrative exemplars (written accounts of teacher-student interactions during instruction) are a “promising tool for instruction and research” (p. 463), and reported that animations offer greater flexibility for aligning instructional objectives and provide an efficient means for capturing a variety of learning situations. As a joint program between the University of Maryland College Park and the University of Michigan under a National Science Foundation (NSF) funded research and development project, the Thought Experiments in Mathematics Teaching (ThEMaT) project created animated representations of mathematics teaching based on models of content specific classroom interactions (Center for Math Education, n.d.). These animations are also being considered for use in teacher education courses as a tool for increasing teacher CK as well
as for analyzing student learning. In a study of 21 inservice and preservice teachers, Moore-Russo and Viglietti (2010) analyzed teachers’ interpretations of and responses to a set of animated representations of geometry content created under the ThEMaT project. Results from this study indicated that teachers found the animations useful for considering the effects of their pedagogical decisions on student learning and engagement and helped to improve their content knowledge. However, there is currently no research on the use of animations as a vehicle for helping teachers adapt their instruction to meet the needs of students with disabilities or those experiencing mathematics difficulties, particularly in geometry. Thus, future studies are needed to examine the effects of animated exemplars as a means for increasing the content and pedagogical content knowledge of general and special education as well as the impact on the teachers’ ability to adjust their instruction for teaching in an inclusive setting.

Statement of Purpose

Improving the performance of secondary students in geometry is critical in order to prepare students for success in post-secondary education, daily living skills, and career attainment. However, many students, particularly students with LD struggle with learning geometry content. Ensuring that all students receive quality instruction and have access to the general education curriculum is critical for addressing the challenges and difficulties that hinder the learning process for many students with disabilities. Given that a majority of students with special needs are educated in the general education classroom, general and special educators are being encouraged to collaborate, often teaching alongside one another or being responsible for instruction to a diverse group of students. The self-efficacy beliefs of teachers play a significant role in teacher’s ability to adapt their instruction to
meet the needs of a diverse student population. Research shows that general educators generally feel underprepared to provide appropriate instruction for students with special needs. Likewise, special education teachers report that they often lack the necessary content knowledge and understanding needed to help students meet rigorous academic standards. In order to ensure that all students improve their conceptual knowledge of geometry content, teachers must have a deep understanding of geometry content knowledge as well as strong pedagogical content knowledge.

Teacher inservice PD is an effective way to increase teacher CK and PCK. Although using emerging technologies, such as computer animated representations of teaching, can be an effective tool for facilitating teacher PD, the research base for using such animations as a means for increasing teacher CK and the use of effective instructional practices is limited. Further, there is no research on the use of emerging technologies in PD for general and special education teachers of students with disabilities. Thus, the purpose of this study is to extend a growing body of PD research involving use of animated representations for increasing teacher’s self-efficacy, content knowledge, and pedagogical content knowledge of geometry topics involving congruence and similarity for inservice special education and general education mathematics teachers within an inclusive setting.

**Research Questions and Hypotheses**

The current study is guided by the following research questions:

1. What is the effect of a teacher professional development course involving use of animated representations addressing congruence and similarity on special education and mathematics education teacher self-efficacy, content knowledge, and pedagogical content knowledge in an inclusion setting?
2. How do general education and special education teachers compare in their understanding of congruence and similarity as well as their self-efficacy for teaching geometry in an inclusive setting?

3. To what extent do general and special education teachers find a blended learning professional development course as an effective means for increasing teacher content and pedagogical content knowledge in geometry?

The hypotheses for this study are:

1. It is predicted that the PD course will be effective for increasing teacher content knowledge, pedagogical content knowledge, and self-efficacy.

2. It is anticipated that general education teachers will demonstrate a greater self-efficacy for teaching geometry and special education teachers will have a greater self-efficacy for teaching in an inclusive setting.

3. It is predicted that general education teachers will report a greater increase in self-efficacy for teaching in an inclusive setting compared to special education teachers;

4. It is predicted that special education teachers will report a greater increase in content and pedagogical content knowledge in geometry compared to general education teachers.
Definition of Key Terms

*Animated Representations* – manufactured episodes of classroom interactions whereby the classroom instruction is depicted in detail with vague details of the setting and individuals (Herbst & Miyakawa, 2008).

*Congruence* – the concept in geometry that two shapes are equivalent by a combination of translations, rotations, and/or reflections (Seago, Jacobs, & Driscoll, 2010).

*Content Knowledge* – the understanding of mathematical facts and concepts used to carry out mathematics instruction (Krauss et al., 2008).

*Inclusive Setting* – the learning environment where students with disabilities meaningfully participate in the general education classroom with their non-disabled peers (Bateman & Bateman, 2002).

*Learning Disability* – a disorder in one or more of the basic psychological processes involved in understanding or in using language, which may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations (IDEA, 2004).

*Pedagogical Content Knowledge* – teacher’s ability to present mathematical knowledge in an effective way for students to learn (Shulman, 1986).

*Professional Development* – a range of formal and informal learning activities or experiences intended to increase teachers’ knowledge and skills, improve their practice, and contribute to their personal, social, and emotional growth (Cohen, McLaughlin, and Talbert 1993).

*Self-efficacy* – an individual’s perception of their ability to organize and execute actions that will bring about desired results (Bandura, 1977).
Similarity – the concept in geometry that states that two figures are similar if one is congruent to a dilation of the other (Seago, Jacobs, & Driscoll, 2010).

Universal Design for Learning – an educational framework used in helping to improve educational outcomes through targeting support to individuals and by providing multiple methods for instruction, assessment and engagement (CAST, 2006).
Chapter 2: Review of the Literature

Achieving mathematical literacy is significant for students to realize success academically as well as increase their opportunities to obtain meaningful employment. Students who participate in higher level mathematics courses in high school are more likely to have a higher income, use new technology, vote, and engage in civic leadership (National Mathematics Advisory Panel [NMAP], 2008). Geometry, in particular, is a critical strand of the school mathematics curriculum (Cawley, Foley, & Hayes, 2009). Although there is less emphasis within the secondary mathematics curriculum, as compared to algebra and number sense concepts, learning geometry concepts offers students numerous opportunities to improve their cognitive ability, as well as reason and communicate about mathematics in multiple ways (Seago, Driscoll, & Jacobs, 2010). Understanding congruence and similarity in particular connects a variety of important mathematical topics including linear functions, transformations, proportional reasoning, and modeling (Seago, Jacobs, & Driscoll, 2010).

Due to recent legislation, such as No Child Left Behind (NCLB) and the Individuals with Disabilities Education Improvement Act (IDEIA), as well as the increase in inclusionary practices, a majority of students with learning disabilities (LD) take their geometry course in the general education classroom (Steele, 2010). A major premise of the 1997 and 2004 Reauthorizations of IDEIA is to ensure that students with disabilities have access to grade appropriate curriculum and requires schools to educate this student population with their non-disabled peers to the greatest extent possible (IDEIA, 2004). In compliance with NCLB, all students are assessed in mathematics annually in grades 3-8, and at least once more between grades 10 – 12. These assessments must align with the
state’s academic content and student academic achievement standards and must assess higher-order thinking skills and understanding. Results from assessments are reported with disaggregation of scores for students with disabilities, therefore, students with disabilities are being held accountable for the general education secondary mathematics content. As such, 62% of secondary students with LD participate in mathematics courses in the general education classroom (Newman, 2006); however, only 13.6% of students with LD perform on grade level during secondary school (Wagner et al., 2003).

In this chapter, I will first summarize the achievement results of U.S. students in mathematics, particularly geometry. I then discuss the characteristics of students with learning disabilities and the role of education reform, including policies for including students with disabilities in the general education classroom. This is followed by a review of the literature, that supports the design of this study. Specifically, the literature review begins with an overview of the research on teacher self-efficacy, content knowledge and pedagogical content knowledge in mathematics, followed by a review of the current literature of professional development as it relates to increasing instruction and learning in mathematics. I also examine the application and use of animated representations of teaching in mathematics professional development. The chapter concludes with a summary of findings from the literature and implications for the current study.

**Proficiency of U.S. Students in Mathematics**

In general, students in the U.S. have not performed well on assessments of their understanding of geometry concepts. Data from the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) indicate that students in the U.S. are performing below the level of many other
industrialized countries in the skills that are critical for understanding geometry concepts such as their ability to use visual and spatial skills to solve problems, make generalizations, and organize information to complete sequential tasks (Mullis et al, 2012). U.S. students have also not made the achievement gains expected on national assessments. On the geometry portion of the National Assessment of Educational Progress (NAEP), the average scale score was 17 percentage points below the proficient level for eighth grade students and 21 percentage points below the proficient level for twelfth grade students (National Center for Education Statistics, 2013). The average scale score for eighth and twelfth grade students with disabilities on the geometry portion of the assessment was 31 and 38 points lower, respectively, than their general education peers (National Center for Education Statistics, 2013).

**Characteristics of Students with Learning Disabilities**

Certain characteristics of students with LD may impede their geometry performance, including difficulties with calculation fluency and procedural knowledge (Flores, Houchins, & Shippen, 2006; Garnett, 1998; Geary, 2004). In addition, students with disabilities in mathematics tend to lack an understanding of mathematical concepts, which hinders their problem solving and reasoning abilities (Miller & Mercer, 1997; Parmar, Cawley, & Frazita, 1996). Poor strategy knowledge and strategy use also contribute to difficulties across mathematics achievement (Flores, Houchins, & Shippen, 2006; Montague & van Garderen, 2003). Further, they may experience poor self-esteem, decreased motivation, and/or passivity in the classroom (Gagnon & Maccini, 2001).

**Mathematics Reform and the Role of Inclusion**
In response to the lagging student achievement in mathematics, the National Council of Teachers of Mathematics (NCTM, 2000) addressed the need for more rigorous standards by publishing the *Principles and Standards for School Mathematics*. The NCTM Standards focus on conceptual understanding and real-world problem solving and reflect a belief in the importance of mathematics for all students, including students with disabilities. The *Content Standards* describe the content that all students should learn from pre-kindergarten through twelfth grade (i.e., number and operations, algebra, geometry, measurement, data analysis, and probability). The *Process Standards* (i.e. problem solving, reasoning and proof, communication, connections, and representations) describe ways of learning the content knowledge. Additionally, the high school mathematics curriculum should emphasize mathematical reasoning (i.e. drawing conclusions based on evidence) and sense making (i.e. developing an understanding of a concept by connecting it to existing knowledge) in all courses, for students of varying abilities (NCTM, 2009).

Findings from research studies of mathematics education programs in high-performing countries and suggestions from organizations such as NCTM, NCR, and ADP, pointed to the need for more focused and coherent mathematics standards in the U.S. Together, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) collaborated to create a national set of standards in both language arts and mathematics called the Common Core States Standards (CCSS, 2010). The CCSS in mathematics focus on understanding key concepts by reviewing and building on the organizing principles of mathematics and how the properties of operations lead up to more advanced concepts (CCSS, 2010). The CCSS for mathematics also provide a set of practices that students should demonstrate to show depth and breadth of mathematical knowledge.
The sections below describe the eight Common Core Standards for Mathematical Practice and how each relates to geometry instruction.

**Make sense of problems and persevere in solving them.** This practice standard involves students performing actions such as explaining the meaning of a problem to themselves, looking for entry points, and making conjectures and a plan to solve a problem. This standard also includes considering analogous problems, monitoring and evaluating progress, checking answers with alternative methods and generally insuring if steps taken or derived answers make sense in the context of the problem (CCSS, 2010). For example, students may be asked to develop a congruence statement explaining how two shapes or objects are related to each other. Students will identify the corresponding parts that match each other as well as map certain transformations to prove their statement.

**Reason abstractly and quantitatively.** This practice standard involves students demonstrating the ability to create coherent representations of problems, attend to the meaning of quantities, use properties of operations and objects, decontextualize situations by manipulating mathematical symbols abstractly without necessarily attending to the original referents, and to pause and contextualize the symbols during the manipulation to make sense of the symbols concretely in terms of the original context (CCSS, 2010). Babai et al (2009) investigated an intervention for helping students solve geometric tasks by raising student awareness to relevant information. The results of the study demonstrated that students improved their fluency and accuracy for solving geometry problems by attending to the relevant variables and overcoming irrelevant information presented in the tasks. This is an example of students’ ability to reason abstractly and quantitatively.
**Construct viable arguments and critique the reasoning of others.** Students who demonstrate this standard are able to understand stated assumptions/definitions, make conjectures, build progressions of statements to explore the truth of conjectures, justify conclusions, compare the effectiveness of two plausible arguments, and distinguish and use correct logic and reasoning. They are also able to communicate findings, respond to the arguments of others, decide if an argument makes sense, and ask clarifying questions (CCSS, 2010). Problem posing activities is a way for students to construct reasonable arguments and make conjectures about geometry concepts. Problem posing activities involve the generation of new problems and questions aimed at exploring a given situation, as well as the reformulation of given problems (Silver, 1994). For example, students may generate new problems when formulating a geometric proof. Providing students with opportunities to pose their own problems can foster more diverse and flexible thinking, enhance students’ problem solving skills, broaden their perception of mathematics, and enrich and consolidate their knowledge of basic concepts (Brown & Walter, 1993; English, 1996).

**Model with mathematics.** This practice standard involves students applying mathematics to solve everyday problems, using multiple representations such as graphs, tables and equations and interpreting/reflecting on the mathematical results in the context of the situation (CCSS, 2010). Cihak and Bowlin (2009) examined the effect of students using video modeling to solve geometry problems. Teachers created video clips of step-by-step processes for solving problems, which were linked to handheld devices used by three students diagnosed with LD. The researchers reported that each participant acquired and maintained new geometry skills.
**Use appropriate tools strategically.** This practice standard involves students considering and using available tools (e.g., concrete model, ruler, compass, calculator, spreadsheets, dynamic geometry software) and evaluating their effectiveness for a given situation to help students explore and deepen their understanding of mathematical concepts (CCSS, 2010). A recent study that was conducted on this practice was by Guven (2012), who examined the effect of Dynamic Geometry Software (DGS) on students’ learning of transformation geometry. Study participants were 68 eighth grade students (36 in the experimental group and 32 in the control group). Students were taught how to use the *Cabri* software and used the DGS to explore and test the characteristics of transformations. Students were also able to check their understanding by completing practice problems. Pre- and post-test results demonstrated that the experimental group outperformed the control group in terms of academic achievement and the level of understanding of transformation geometry.

**Attend to precision.** This practice standard involves students using clear and precise definitions in discussions and explanations of their own work, stating the meaning of and/or correctly using symbols specifying units of measure, calculating with precision that is appropriate for a problem context, and giving carefully formulated explanations to each other (CCSS, 2010). Poon (2011) report the use of a study of students using a simple trigonometric problem that produces several answers depending on the method used to solve the problem. Through this activity, students learned the importance of having precise diagrams when solving geometric problems.

**Look for and make use of structure.** This practice standard involves students analyzing objects and examples to discern patterns and structures, extending lines in
geometric figures to assist in problem solving, and “seeing” complicated things such as an algebraic expression as either a single whole or composed of several objects (CCSS, 2010). For example, students may be required to analyze and construct the relationship between three-dimensional geometric figures and the related two-dimensional representations to solve problems.

**Look for and express regularity in repeated reasoning.** This practice standard involves students noticing repeated calculations and looking for general methods or shortcuts, maintaining oversight of the problem solving process, and continually evaluating reasonableness of approach and results (CCSS, 2010). For example, students may use ratios to solve problems involving similar figures. Another example is for students to describe the effect on area, perimeter and volume when or more of the dimensions are changed.

The standards and practices inform what and how teachers should instruct students for learning mathematical skills and concepts. While the Geometry standards are outlined through all grade levels beginning with kindergarten, students at the secondary level begin to formalize their geometry experiences from the earlier grades, using more precise definitions and developing careful proofs. The concepts of congruence and similarity, in particular, are understood through geometric transformations, which is consistent with the NCTM Principles and Standards for Mathematics (CCSSI, 2010; NCTM 2000). Thus the CCSS are important for considering helping teachers meet the needs of all students.

In addition to the NCTM and Common Core standards, reports from the National Mathematics Advisory Panel (NMAP; 2008) and the American Diploma Project (ADP, 2004) emphasize a rigorous curriculum for all learners. Authors of both reports suggest
mathematics benchmarks, which include foundational skills to prepare elementary age learners for algebra (i.e., fluency and conceptual understanding of whole numbers, fractions, and certain aspects of geometry; NMAP, 2008) and algebra skills necessary for completion of Algebra during secondary education (i.e., linear equations, quadratic equations, functions, and polynomials; ADP, 2004; NMAP, 2008). Considering that the No Child Left Behind (NCLB) Act of 2001 mandates that all students, with few exceptions, master the general education curriculum, participate in standardized assessments, and achieve passing levels of performance, it becomes even more imperative to study the effectiveness of inclusion programs from a variety of perspectives. Furthermore, proportionately, students with LD are the largest special education group to be included in general education classes. Forty-nine percent of students classified with specific LD spent 80% or more of each school day in a general education classroom. These students are not among the groups exempt from state and national standardized tests (U.S. Department of Education, 2003).

**Co-teaching**

Shifts in policy, which place an emphasis on educating students with disabilities in the general education classroom, have increased the need for modified forms of instruction, such as co-teaching (Palmer, 2005). Co-teaching is a service delivery model in which two or more professionals share responsibility for a group students in the same workspace (Murawski, 2005). The history of co-teaching can be traced back to the 1960’s, when the concept was popularized as an example of progressive education (Villa, Thousand & Nevin, 2008). Co-teaching was further advanced in the 1970’s by legislated school reforms and teachers’ increasing need to modify instruction for a more diverse student population.
Today, co-teaching is highly regarded as a primary answer to federal legislative changes to NCLB and IDEA, as well as questions arising from the challenges dealing with the increase in students requiring special education services (Villa, Thousand, & Nevin, 2008).

Research into the effectiveness of co-teaching has been limited and demonstrated mixed results. Researchers Wendy Murawski and H. Lee Swanson (2001) examined data-based studies on the effectiveness of co-teaching. The report found that only six of 89 reviewed articles provided sufficient quantitative information for an effect size to be calculated. Special education students were primarily students with learning disabilities and/or low achievement and the studies encompassed all grade levels. Walther-Thomas (1997) did a study on co-teaching models in 23 schools across eight school districts. Positive outcomes were documented including improved academic and social skills of low-achieving students, improved attitudes and self-concepts reported by students with disabilities, and more positive peer relationships. Students perceived that these improvements were the result of more teacher time and attention (Walther-Thomas, 1997). The co-teachers also reported gains in professional growth, personal support, and enhanced sense of community within the general education classrooms. However, the most frequently mentioned drawback was the lack of staff development to learn how to be more effective teachers.

Research suggests that one of the greatest issues faced by co-teachers has to do with content knowledge and expertise (Mastropieri, Scruggs, & Graetz, 2005). Special education teachers at the secondary level tend to be trained in learning differences and accommodations but not content mastery, while their general education counterparts are typically trained in content mastery at a high level. This study will address this particular
concern by providing professional development to special education and general education co-teachers in geometry to increase content knowledge as well their efficacy for teaching in an inclusion setting.

**Universal Design for Learning**

Universal design for learning is an educational framework used in helping to improve educational outcomes through targeting support to individuals by reducing barriers to learning (Rose & Meyer, 2002). UDL expresses the idea that standard educational goals, materials, methods, and assessments may cause barriers to learning or hinder students from accessing the curriculum (CAST, 2009; Edyburn, 2010). For example, print resources (such as mathematics textbooks) contain mathematical terminology that may be challenging for students with reading difficulties. Providing alternative ways to present information (i.e. virtual manipulatives) may help students overcome their reading challenges, while allowing them to access the curriculum. The organization of the UDL framework focuses on three cognitive networks: (a) recognition network, (b) strategic network, and (c) affective network (CAST, 2009). The recognition network is the **what** of learning, which occurs in the occipital lobe or back part of the brain (CAST, 2009; Jensen, 2000; Restak, 2004). The type of knowledge dealing with applications is declarative knowledge, in which learners identify and interpret patterns using their five senses. The strategic network is the **how** of learning (CAST, 2009). The type of knowledge applied is procedural knowledge in which learners plan, execute, and monitor their actions and skills. The affective network is the **why** of learning and involves the application of affective knowledge in which learners evaluate and set priorities using their emotions and their interest for learning (CAST, 2009; Gordon, n.d.). Through strategic planning and training
using the UDL teaching principles, teachers become engaged in how to include the learning modalities of their students, regardless of the student’s cognitive development and personal interests (CAST, 2009; Furner, Yahya, & Duffy, 2005; Rose & Meyer, 2002).

Learning modalities are the ways in which learners process information into memory from learning by seeing, learning by hearing, and learning by doing (CAST, 2009). The recognition principle applies to presenting information in multiple formats (CAST, 2009). The strategic principle develops the idea that multiple pathways are available for learners to conceptualize and demonstrate what they needed to learn (CAST, 2009). The affective principle provides multiple ways for learners to engage in real life learning experiences using emotions (CAST, 2009). By applying these principles, teachers can effectively plan and implement instruction for all learners.

Current trends and policies support the adoption of inclusion or the practice of educating students with disabilities in the general education setting with their non-disabled peers (Idol, 2006). Inclusive practices allow students with special learning and/or behavioral needs to have access to the general education curriculum, which supports IDEA mandates for educating students with disabilities in the least restrictive environment (IDEA, 2004). However, inclusion often poses a challenge for general education teachers who often feel ill-prepared to teach students with LD (Borasi, Fonzi, Smith, & Rose, 1999; Jenkins & Ornelles, 2009; Maccini & Gagnon, 2007). In a recent study of seven in-service middle school mathematics teachers (Desimone & Parmar, 2006), each teacher reported that they felt that their undergraduate and graduate teaching programs did not adequately prepare them to teach mathematics in an inclusive classroom. These results are consistent with a study conducted by Rao and Lim (1999) in which general education teachers
reported that they needed, among other supports, training on curriculum adaptations and instructional strategies to be successful in an inclusion classroom. Due to inadequate teacher training and relevant PD for teaching students with LD in mathematics, general education teachers may lack confidence in their ability to provide accommodations for those students (Jordan, Stanovich, & Roach, 1997). Further, special education teachers, who receive training for meeting the needs of students with LD, may feel inadequate with regard to their knowledge of the subject area content (Leko & Brownell, 2009). For instance, in a study of 34 beginning special education teachers (less than 3 years teaching experience), Brownell et al (2009) found that special education teachers struggled with domain specific content and pedagogical knowledge and, instead, relied more on general classroom management practices when providing instruction. The implications of these results suggest that special education teachers may rely on general instruction practices, which may limit their ability to provide quality instruction to their students.

Frey (2009) reported that the shortage of special education teachers also contributed to the lack of knowledge in the profession. The increased demand for qualified special education teachers often requires schools to hire teachers who are less trained and experienced to work with students with disabilities. Special education teachers completing short, intensive alternative training programs for certification may find it difficult to help general education teachers meet the needs of students with diverse learning needs. General and special education teachers need quality preparation and ongoing training to increase their ability to provide instruction to all students.

**Teacher Self-Efficacy**

In order to meet the diverse learning needs of students, teachers must believe in
their own ability to do so. Self-efficacy, in regards to education, plays a critical role in the instructional decisions teachers make on a consistent basis (Kosko & Wilkins, 2009). Bandura (1986) defined self-efficacy as, “beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (p. 2). Bandura postulated that the theory of self-efficacy contains two expectancies, self-efficacy and outcome efficacy. Self-efficacy expectation provides individuals a way to decide whether they have the ability to perform the required task at the desired level of competency, whereas outcome expectancy provides individuals a way to decide if they have accomplished a task at a desired level (Hoy & Woolfolk, 1993).

With regard to education research, Ashton and Webb (1986) focused the self-efficacy construct on teachers by defining teachers’ sense of self-efficacy as a “situation-specific expectancy” that they can affect or influence student learning. Similar to Bandura (1986), Ashton and Webb recognized that these beliefs affect a teacher’s choice of classroom activities, the amount of effort the teacher is willing to expend, and his or her persistence when experiencing difficulties. When teachers believe in their ability to meet the learning needs of their students, they design and deliver instruction which provides students access to the skills and concepts they are learning while enabling them to construct new knowledge and understanding (Ashton & Webb, 1986).

Teacher efficacy is a self-perception and not necessarily an outcome of teaching effectiveness (Ross & Bruce, 2007). However, researchers have linked teachers’ self-efficacy to a variety of positive outcomes. For example, high school teacher self-efficacy has been positively correlated with improved student achievement (Ashton & Webb, 1986; Dembo & Gibson, 1985; Tschannen-Moran, Hoy, & Hoy, 1998). Researchers claim that
teacher efficacy relates to student achievement as it results in teachers’ efforts to adapt instructional practices that support student learning (Allinder, 1995; Ashton & Webb, 1986; Caprara, Barbaranelli, Steca, & Malone, 2006; Dembo & Gibson, 1985). In addition, teachers with higher efficacy levels are more apt to plan engaging lessons and interact with students to encourage their participation in the lesson (Schunk, 2008). In regards to teachers working in inclusive settings, teachers with high levels of efficacy are willing to involve students with disabilities in class discussions and persist in educating them (Brownell & Pajares, 1996; Nunn, Jantz, & Butikofer, 2009), while maintaining best practices for classroom management (Woolfson & Brady, 2009). Teachers with higher efficacy also tend to provide increased support to struggling students in their classes such as one-one guidance and additional opportunities for practice (Dembo & Gibson, 1985).

Recent research has further defined Bandura’s efficacy theory in the context of education. Individual efficacy and collective efficacy are two independent measures often used in research regarding teacher efficacy (Bandura, 1998; Tschannen-Moran, Hoy, & Hoy, 1998). Although individual efficacy is generally defined as a teacher’s belief in his or her skills and abilities to positively impact student achievement, collective efficacy describes the teacher’s belief about his or her colleagues’ effectiveness to positively impact student achievement (Zambo & Zambo, 2008). Given that personal teaching efficacy is influenced by the additional education teachers receive (Hoy & Woolfolk, 1993), general educators who are faced with the challenges of having students with varying levels of ability and behavioral responses in their classrooms should be provided the knowledge to understand the needs of such students and the skills to teach these students in their classrooms. Therefore, future research should focus on the needs of general and special education
teachers who teach in an inclusive setting.

Teacher PD can influence self-efficacy when the knowledge and tasks that are covered are relevant and connected to the teacher’s classroom situation (McLaughlin & Berman, 1977; Scribner, 1998). In a study of 12 third to sixth grade teachers who participated in an intensive PD program, researchers Bruce and Ross (2008) found that the participants shifted to more standards based instruction, resulting in higher teacher self-efficacy for facilitating student discussion and improving the quality of the tasks assigned. The PD program spanned six months and consisted of four sessions that involved collaborative lesson planning, examining student work samples, and modeling effective teacher practices. In a more extensive study (Powell-Moman & Brown-Schild, 2011), researchers investigated the impact of a two-year professional development program on teachers’ self-efficacy for adopting inquiry-based instruction (e.g., cognitive processes of scientists). Participants were 23 practicing science and mathematics teachers at the elementary and secondary levels who participated in a fellowship program that used a scientist-teacher partnership to help teachers use inquiry-based instruction as well as develop their teacher leadership skills. Results from this study support prior research for using PD to increase teacher knowledge and self-efficacy.

**Content and Pedagogical Content Knowledge**

The literature on the content and pedagogical content that mathematics teachers need to know is substantial. Over the past two decades, policymakers and national organizations, such as the National Council of Teachers of Mathematics (NCTM), have called for improvements to the teaching and learning of mathematics. They argue that traditional, procedural-based approaches to mathematics teaching do not prepare enough
students for higher-level mathematics. Success in higher-level mathematics is dependent on students’ understanding of concepts as well as procedures. In order for students to receive mathematics instruction that attends to concepts as well as procedures, teachers must receive additional training and support to deliver instruction in a more conceptually based way (National Council of Teachers of Mathematics [NCTM], 1989; 1991; 2000; Kilpatrick et al., 2001).

Though mathematicians and mathematics education experts may differ over the relative emphasis of particular mathematics concepts, they agree that conceptual understanding is a necessary condition for effective teaching. Teachers need to know why, not just how, so that they can deliver meaningful instruction to students. Teachers need to be able to make connections among mathematical concepts so that students obtain a coherent, rather than a fragmented, view of mathematics (National Math Panel Advisory Panel [NMAP], 2008). Further, in order for teachers to help students demonstrate the CCSS Standards for Mathematical Practice, teachers must have a deep understanding of the mathematics concepts.

Related to strong conceptual understanding of the topics they teach, experts also agree that teachers need pedagogical content knowledge, which is knowledge about how to teach the concepts in a way that is effective for students to understand (Shulman, 1986). For example, a teacher with conceptual understanding about the meaning of fractions might use that knowledge to improve the precision or coherence of an explanation about fractions.

Pedagogical content knowledge is viewed as a set of attributes that facilitate the transfer of the teacher’s knowledge of the content to their students (Geddis, 1993).
Shulman (1987) further posited that these attributes that the teacher possesses allows presentation of content in a way that is personally meaningful for students. This includes what makes learning geometry concepts, for example, particularly more accessible or challenging for students with diverse learning needs.

**Organization of the Review of the Literature on Professional Development**

In this section, I present a comprehensive review of current research involving professional development studies on teachers' self-efficacy, content, and pedagogical content knowledge in geometry. Specifically, I examine the salient features of professional development models that are effective for increasing teachers' self-efficacy as well as their content and pedagogical content knowledge in geometry.

**Search Procedures**

For this literature review, I conducted an electronic search using the following databases: Education Research Premier (EBSCO); ERIC; PsychINFO; and Professional Development Collection. Potentially relevant studies published between 1989 and 2012 were identified using these descriptors in varying combinations: *teacher self-efficacy, inclusion, math*, *geometry, professional development, content knowledge,* and *pedagogical content knowledge*. This initial search yielded a total of 116 studies. Selected studies were narrowed for this review using the following criteria: (a) published in a peer-reviewed journal; (b) included general and/or special education teachers, and (c) either examined the effects of a PD program on teacher self-efficacy in mathematics or studied the effects of a PD program on teachers’ content and pedagogical content knowledge in geometry. *Secondary* was defined as students in grades 6-12 or approximately ages 11-18. Applying these conditions, 10 studies met the criteria for inclusion (Arbaugh, 2003; Borasi, Fonzi,
Overview of the Studies

A summary of each of the studies is presented in Table 1. Overall there were 476 teacher participants, with 469 general education math teachers and 7 special education teachers. Two of the ten studies reported the gender of the participants, which totaled 6 female and 4 male teachers (Arbaugh, 2003; Brown & Benken, 2009). A third study reported that teacher participants were mostly female (Feuerborn et al., 2009). Six of the ten studies feature PD programs aimed at increasing teacher content and pedagogical content knowledge in geometry (Arbaugh, 2003; Borasi et al., 1999; Brown & Benken, 2009; Kimmel & Deek, 1999; Merrill et al., 2010; Powell-Moman et al., 2011), two studies focus on increasing teacher knowledge and self-efficacy (Feuerborn et al., 2009; Swackhamer et al., 2009), and two studies address the effects of a PD program on teacher efficacy alone (Ross & Bruce, 2007; Zambo & Zambo, 2008). Five of the ten studies (Arbaugh, 2003; Borasi et al., 1999; Brown & Benken, 2009; Kimmel & Deek, 1999; Merrill et al., 2010), are qualitative in nature, (Arbaugh, 2003; Borasi et al., 1999; Brown & Benken, 2009; Kimmel & Deek, 1999; Merrill et al., 2010), three studies (Powell-Moman et al., 2011; Swackhamer et al., 2009; Zambo & Zambo, 2008) used correlational analysis (Powell-Moman et al., 2011; Swackhamer et al., 2009; Zambo & Zambo, 2008), one study (Ross & Bruce, 2007) used a randomized field trial, and the remaining while another study (Feuerborn et al., 2009) used a mix of qualitative and quantitative methods. (Feuerborn et al., 2009). Eight of the ten studies included only general education mathematics teachers.
(Arbaugh, 2003; Brown & Benken, 2009; Feuerborn et al., 2009; Merrill et al., 2010; Powell-Moman & Brown-Schild, 2011; Ross & Bruce, 2007; Swackhamer et al., 2009; Zambo & Zambo, 2008). The remaining two studies included both general and special education teachers (Borasi et al., 1999; Kimmel & Deek, 1999).

**Content Analysis**

Since professional development is a broad term that encompasses a variety of structures and delivery types, it is useful to frame a discussion about the more effective types of professional development around a few, core structural features that apply to most, if not all, professional development models. The professional development literature identifies specific features of professional development models that are deemed to be more promising or effective for improving teacher learning (Garet et al., 2001; Hawley & Valli, 1999). As part of a nationally representative sample of more than 1,000 teachers who participated in professional development (mostly mathematics and science), researchers identified several critical features that contextualize professional development (Birman et al., 2000). These features included the form or type of PD activity, the duration of the PD program, and how the participants were selected and arranged. In this section, I discuss the selected studies in regards to four features that have been deemed important for evaluating PD programs: 1) source; 2) purpose; 3) format; and 4) duration (Birman et al., 2000; Garet et al., 2001). Each feature is defined, including best practices according to the literature, followed by an analysis of the selected studies. The section is concluded with a summary of the implications for the current study.

**Source.** The source, or origin of the PD program represents why the participants engaged in the PD activity. For example, did the teachers voluntarily participate in the PD
or was it required by school administration or district. In general, professional
development activities can be described as more or less teacher-initiated, which is an
important distinction to make when categorizing the level of teacher ownership of learning
opportunities. In their review of the literature on effective types of professional
development, Hawley and Valli (1999) suggested that effective professional development
programs, to the greatest extent possible, involve teachers in the identification and
development of what they need to learn and how they can learn it. It is also important that
PD programs are connected to school and/or district-wide goals (Little, 1993).

With regard to the selected studies, seven of the ten reported that the participants
engaged in the PD program on a voluntary basis (Arbaugh, 2003; Brown & Benken, 2009;
Feuerborn et al., 2009; Merrill et al., 2010; Powell et al., 2011; Swackhamer et al., 2009; and
Zambo & Zambo, 2008). Generally, involving teachers in the process of selecting the PD
activity increases their sense of ownership and the amount of motivation and effort they
bring to learning situation (Hawley & Valley, 1999). Brown and Benken (2009) reported on
a PD program that offered two courses in content and pedagogy that were designed based
on the participant's responses to surveys, current classes taught, and the curriculum
participants used. Although offering the PD on a voluntary basis increases teacher
participation, researchers also employ other methods. For example, Feuerborn et al. (2009)
reported that the participating teachers in the PD institute received incentives and
continuing education credits for completing the program. Research also supports the use of
school district and university partnerships as a means for increasing teacher knowledge in
a targeted subject area. Merrill et al. (2010) investigated the influence of a PD partnership
between the Peoria Public School District and Illinois State University. Eight mathematics
teachers participated in the partnership aimed at improving teachers’ trigonometric and geometric knowledge. In another example, researchers Powell-Moman and Brown-Schild (2011) reported on a study of the Kenan Fellows Program, an intensive PD program for science, technology, engineering, and mathematics teachers. Participating teachers were required to apply for acceptance into the program. Selected teachers were trained to increase their responsibilities at their respective schools, deliver PD, and received six graduate level credits.

Overall, this literature supports that effective professional development models provide opportunities for teachers to be active participants in the selection of the types of PD offered to allow for greater participation and learning engagement. Providing incentives, such as course credits or teacher leadership opportunities are also effective ways for increasing teacher buy-in and completion of PD programs that are involuntary. While the proposed study will be voluntary, teacher participants will receive a modest monetary incentive for their participation. In addition, the online activities will provide teachers greater flexibility, which may enhance their participation by making them active learners.

Purpose. The purpose of a professional development activity, which includes the focus or emphasis of the PD, is another distinguishing characteristic of any professional development activity or model. The purpose varies across activities and models. For example, one might focus on deepening teachers’ understanding of the subject matter; another might emphasize pedagogical skills; and another might focus on aligning teaching practices with district standards and assessments.

Professional development activities that focus on deepening teachers’ content
and/or pedagogical content knowledge are thought to be more effective than other models. For example, Cohen and Hill (1998) conducted a study of mathematics teachers in California and found that student achievement was higher in schools where teachers had participated in extensive, content-focused professional development. Garet et al. (2001), utilizing a national probability sample of over 1,000 math and science teachers, came to a similar conclusion. They found that content-focused professional development had a significant positive direct effect on teacher self-reported knowledge and skills and a significant positive indirect effect on changes in teacher practice. These findings are consistent with Kennedy’s (1998) review of studies that linked various types of professional development to student achievement and pedagogical content-focused professional development had larger positive effects on student achievement – particularly on students’ conceptual understanding – than more general types of professional development. Publications by the National Council of Teachers of Mathematics (NCTM 1989; 1991; 2000) and the National Research Council (2001) also highlight the importance of content and pedagogical content-focused professional development activities. Both organizations recommend sustained, intensive professional learning opportunities for teachers in these areas.

In a review of the selected studies, six of the ten featured PD programs that focused on increasing teacher content and pedagogical content knowledge in geometry (Arbaugh, 2003; Borasi et al., 1999; Brown & Benken, 2009; Kimmel & Deek, 1999; Merrill et al., 2010; Powell-Moman et al., 2011). For example, Merrill et al. (2010) developed a partnership between Illinois State University and the Peoria Public School District to provide a series of PD courses that aimed at improving teachers’ geometric and
trigonometric knowledge and their ability to teach mathematics using 3-D modeling software. In another study, seven high school geometry teachers participated in PD study groups that focused on developing a geometry curriculum that was more student-centered and inquiry based (Arbaugh, 2003). The topics covered in the study groups included triangle congruency, student reasoning, and engaging activities using *The Geometer’s Sketchpad*.

This literature suggests that professional development models that focus on building teachers’ content or pedagogical content knowledge are promising in terms of promoting teacher learning that can improve the quality of instruction. The literature also suggests that when the professional development is coherent with district standards and assessments, it is likely to impact the quality of teaching and learning. Therefore, the proposed study will focus on building and increasing teacher content and pedagogical content knowledge in congruence and similarity concepts. These concepts are aligned with the Common Core Standards and Practices for mathematics and teachers will be able to apply their learning experience.

**Format.** The format, or how the PD is organized and delivered, can take many forms. Garet et al. (2001) noted that the most widely used format, and the most evaluated in the literature, is the traditional workshop, which typically occurs at structured times outside of the classroom and is led by a content expert (Loucks-Horsley, Hewson, Love, & Stiles, 1998). Traditional workshop formats have generally been found to be ineffective for increasing teacher knowledge and improving teacher practice (Garet et al., 2001; Loucks-Horsley et al., 1998). Other formats include collaborative study groups, and individualized coaching. For example, many teachers in Japan participate in lesson study groups that meet...
regularly over the course of a school year to focus on improving a lesson or series of interconnected lessons (Stigler & Hiebert, 1999). As a reference for this review, Garet et al. (2001) coded PD formats into two broad categories: 1) traditional activities, which include institutes, college courses, and out-of-district workshops or conferences, and 2) reform activities (i.e., study groups, individualized coaching and mentoring, and school-university partnerships).

In regards to studies selected for this review, six of the ten (Feuerborn et al., 2009; Kimmel & Deek, 1999; Powell-Moman et al., 2011; Ross & Bruce, 2007; Swackhamer et al., 2009; Zambo & Zambo, 2008) featured PD programs that fall under Garet et al.’s (2001) characterization of traditional PD activities. For example, Zambo and Zambo (2008) implemented a summer PD workshop led by an outside mathematics expert for 63 4th – 10th grade mathematics teachers. Similarly, Feuerborn et al. (2009) offered teachers two five-day PD institutes that focused on mathematics content and pedagogy. The institutes were led by an expert mathematics teacher employed by the participating school district. Swackhamer et al. (2009) examined whether participation in a series of college level courses increased the self-efficacy of secondary mathematics teachers. Kimmel & Deek (1999) engaged general and special education teachers through inservice workshops and a summer institute, which included a classroom practicum experience that included twelve students with learning disabilities. The PD model included a focus on collaborative practices, increasing teacher knowledge, and integrating teaching practices that address the needs of special education students.

Four of the ten studies reviewed meet the Garet et al. (2001) criteria for reform PD activities (Arbaugh, 2003; Borasi et al., 1999; Brown & Benken, 2009; Merrill et al., 2010).
Arbaugh (2003) examined the value of teacher participation in a school-based study group, which was defined as a group of educators who meet on a regular basis to study varying aspects of education. Specifically, seven geometry teachers met approximately once every two weeks to discuss a range of topics related to geometry instruction and student understanding. Analysis of teacher interviews and study group transcripts show that the study groups had a positive impact on teacher efficacy as well as increased their content knowledge. In another study, researchers developed a school-university partnership to create a series of courses aimed at improving teacher content knowledge for implementing three-dimensional modeling in the classroom. Results from formative and summative evaluations conducted by the external evaluator indicated that teacher participants found the program valuable for promoting innovative teaching strategies and for using practices that lead to greater student understanding. Brown and Benken (2009) reported on a study of a researcher and teacher collaboration as participants in two PD courses aimed at increasing their content and pedagogical knowledge as well as building a community of collaborative practice. Results from the study revealed that teacher educators must know more about and be responsive to teacher needs and challenges before engaging them in PD. Further, PD should provide a mechanism for participants to feel empowered to implement change. In another study with middle school special education and general education teaching teams, Borasi, Fonzi, Smith, and Rose (1999) found that a PD program that involved engaging teachers in teaching and learning experiences for implementing an inquiry approach to geometry instruction was effective for promoting reform based instructional methods in mathematics as well as increasing teachers’ reflective practices. A promising feature of this study is the benefit of collaboration among special and general
education teachers, which is significant for successful teaching in inclusive classrooms (Buell, Hallam, & Gamel-McCormick, 1999).

The literature contains considerable support for professional development models that promote collective and collegial participation among teachers. Garet et al. (2001) found that collective participation had modest positive effects on the core features of coherence and active learning. They defined collective participation as group participation in professional development, such as participation by a department or grade-level group of teachers. Talbert and McLaughlin (1994) found that teacher participation in active learning communities enhanced professional knowledge and overall professionalism. Little (1993) argued that teachers should have regular opportunities to engage intellectually with colleagues both inside and outside of teaching. Thus, teacher learning communities should include not only teachers within a department or within a school, but also content experts and university researchers who are capable of infusing the learning communities with relevant professional knowledge. The proposed study will allow general and special education teachers to work together in face to face sessions as well as collaborate through online discussions to promote collective participation and enhance the shared learning experience.

**Duration.** Garet et al. (2001) defined duration as including both the total number of teacher contact hours and the time span of the professional development. The researchers found that both dimensions of duration had substantial positive direct effects on active learning and coherence and modest positive effects on content focus. This finding suggests that both “how much” and “how long” are important characteristics of professional development. Yoon et al.’s (2007) meta-analysis of professional development and student
achievement found that the professional development models that showed gains in student achievement averaged about 70 hours of professional development for one year.

For the studies selected for this review, only three of the ten reported PD activities that incorporated over 70 hours of total duration (Kimmel & Deek, 1999; Merrill et al., 2010; Powell-Moman & Brown-Schild, 2011). The general and special education teachers who participated in the Kimmel and Deek (1999) study participated in a multiyear PD program designed for teachers who work with student with disabilities. The program consisted of a 3-week summer institute followed by 3-4 one-day follow up meetings during the school year. The school-university partnership PD program used in the Merrill et al. (2010) study featured over 80 hours of PD activities between the Summer 2009 semester and the Spring 2010 semester. Participants in the Powell-Moman and Brown-Schild (2011) study were part of a two-year PD institute made up of a four-week externship followed by a two-week summer session during the first year, and a five-week externship followed by a one-week summer institute during the second year. Participants completed the externship at a corporate or university laboratory to develop a curriculum project under the guidance of a field expert.

Recent scholarly work has included a focus on more intensive, content-focused professional development programs (Scher & O’Reilly, 2009). The remaining seven studies report shorter, intensive PD models. For example, Feuerborn et al. (2009) evaluated the effects of a 1-week teacher PD on probability and geometry concepts. Researchers reported that the PD led to a significant increase in teachers’ content knowledge as evidenced by the results in pre- and post-test scores. Sixty-three teachers participated in the Zambo and Zambo (2008) study, which examined the effects of a two-week PD workshop on the self-
efficacy of teacher participants. According to teacher self-reported pre- and post-surveys, researchers noted an increase in teacher efficacy. Borasi et al. (1999) reported a study in which general and special education teachers participated in a six-day summer institute followed by three to four meetings during the school year. The PD program was part of a National Science Foundation (NSF) grant project aimed at supporting secondary teachers with mathematics reform instruction. Results of this study suggested that the PD program was successful in supporting teachers for providing reform mathematics instruction.

The literature suggests that short, intensive PD programs along with several follow up PD meetings can be effective for allowing for greater teacher participation as well as promote the transfer of PD activities into classroom practice (Garet et al., 2001). Further, PD experiences that incorporate activities for promoting collaboration enhance teacher learning and model effective practices for teaching in inclusive classrooms (Borasi et al, 1999; Clark, 1994) Therefore, future research should include an intensive PD course, with follow up learning experiences that promote collaboration amongst participants.

**Professional Development for Special Education Teachers**

Very little research specific to PD for special education mathematics teachers exists. The two studies that were identified are reviewed here. Schumm and Vaughn (1995) reported the findings of a series of seven PD programs during a three-year period offered to special education teachers (elementary through high school) for the purposes of improving their instructional practices for teaching in a general education setting. Researchers reported several key findings including the importance of determining teachers’ needs prior to PD implementation, focusing on specific content and effective instructional strategies, and the benefit of collaborating with general education teachers. In
a qualitative case study involving four in-service special education teachers, Frey (2009) investigated the effects of a project-based online PD experience on teacher practice and student learning. The focus of the PD was helping teachers develop and implement individualized academic interventions for students. The PD was offered as a 3-credit hour, practicum experience during a 16-week semester and was primarily delivered through a university-supported online management system. Teachers engaged in online discussion groups and submitted regular journal entries detailing their implementation of assigned tasks as well as student progress. Results of the study indicated that the teachers increased their ability to provide targeted interventions and students were able to achieve academic goals. Teachers also reported their plans to continue the practices and strategies they learned through the PD. The gap in the literature base for providing effective PD for special education teachers adds to the significance of the current study.

**Video Cases**

In addition to traditional PD courses, there has been an increase in the use of video cases or video taped representations of teaching in teacher education programs (Kurz & Kokic, 2012). Alsawaie and Alghazo (2010) used video lesson-analysis methodology to measure preservice teachers’ ability to notice noteworthy classroom interactions. The results indicated that when teachers used an online forum to discuss the video cases, they performed better than the control group. Furthermore, the experimental group paid closer attention to student learning and provided deeper evidence in relation to what they saw in the video case (Alsawaie & Alghazo, 2010). Recent studies (Koc, 2011; So, Pow & Hung, 2009) have focused on video case implementation in the university classroom and their influence on preservice teachers’ growth. The results of the study, in which preservice
teachers developed and analyzed video cases, indicated that video case implementation can improve preservice teachers’ motivation, learning, empathy, and the construction of professional identity (Koc, 2011).

**Animated Representations of Teaching**

Although there are various mechanisms for promoting the teacher’s ability to apply teaching and instructional principles in the classroom, the use of classroom exemplars or scenarios of expert teachers in practice, is a promising technique for fostering the implementation of teacher learning to classroom practice (Moreno, 2009). These classroom scenarios are helpful for demonstrating how knowledge about teaching and learning can be transferred to classroom practice. According to social cognitive theory of learning (Bandura, 1977), people tend to learn and imitate behaviors they observe, known as observational learning. Thus, when student teachers are presented with a classroom narrative, video, or animation modeling how an expert teacher applies principles of teaching to his or her classroom, they will be more likely to imitate the displayed behaviors in the future (Moreno & Ortega-Layne, 2008). Research supports the idea of using classroom exemplars in teacher education (Beck, King, & Marshall, 2002). Recent studies have examined the use of video taped classroom interactions, or video cases, in teacher education programs and inservice PD. For example, Santagata (2009) investigated the effects of a two-year video-based PD program with secondary mathematics teachers in low-performing schools. Throughout the program, teachers viewed and analyzed selected portions of video taped classroom instruction. In addition, they participated in online and face-to-face discussions and analyzed instructional lessons. Findings of this study suggest three principles for designing video-based professional development tasks for teachers: (a)
attending to content-specific understanding, (b) scaffolding analysis of student thinking, and (c) modeling a discourse of inquiry and reflection on the teaching and learning process. In another study of 26 preservice and inservice teachers, researchers explored the outcomes of implementing an online video-based PD program aimed at improving the teaching of mathematics at the elementary and secondary level (Koc, Peker, & Osmanoglu, 2009). Teachers viewed and provided reflections of video cases provided through an online forum. Findings from the study indicated that the video case discussion helped teachers make theory to practice connections and implement what they learned in their classroom instruction.

Although there is support for using video cases in teacher education and PD, the research is limited for the use of animated representations of classroom experiences. However, this tool provides a unique way to engage teachers to increase their content and pedagogical content knowledge as well as provide a mechanism for building collaboration amongst teacher teams. In a study of 80 student teachers enrolled in a psychology course, participants were randomly assigned to receive one of four conditions: a) control group (textbook narrative); b) text narrative; c) video exemplar; and d) animated exemplar. Teachers were allowed to choose a subject (i.e., math or science) and were presented with a classroom exemplar. Teachers were then assessed on how well they received their respective exemplar and if the exemplar helped them to connect theory to practice. Based on statistical results, researchers cited strong support for Bandura’s social cognitive theory (Bandura, 1977). In addition, visual representations of classroom experiences resulted in more positive attitudes and motivation in teachers over textual narratives. Animated exemplars in particular were found to be a more promising tool over video as animations
provide greater flexibility in that they can be created to meet instructional needs with relative ease. Moreover, the animations can be designed to address specific content and instructional strategies while eliminating distractors such as teaching style and student behaviors.

The proposed study uses video cases of mathematics instruction as well as allow teachers to view and create animated representations of classroom instruction through the Thought Experiments in Mathematics Teaching (ThEMaT) project. Several animated experiences on geometry instruction have been created and available for viewing online, which will serve as a basis for increasing teacher knowledge. In addition, the ThEMaT offers an online tool called LessonSketch, which allows users to create their own animated classroom experience. This provides a unique way to capture and demonstrate instruction that is geared towards students with learning disabilities. As research is limited for visual representations of teaching in inclusive classrooms, creating animations of instructional best practices provides the opportunity to address specific learning strategies for students and teachers. Further, teachers will be developing the animations in teams, which promotes collaboration.

Study Implications

This literature review supports the development of the current study in several ways. In terms of the source of the professional development, the current study offers incentives to selected teachers. In addition, the PD provider and researcher are non-evaluative in terms of teacher performance in the current setting so teachers feel empowered to implement and practice what they learn.

The purpose of the professional development in the current study is to increase
teachers’ content and pedagogical content knowledge in congruence and similarity as well as increasing their self-efficacy for teaching geometry in an inclusion setting. The literature supports professional development models that seek to deepen teachers’ content and pedagogical content knowledge, particularly when the content is aligned with district standards and teachers’ daily work. The content covered during the PD is aligned to the Common Core Standards, which is a statewide goal for the proposed school district. Thus, the purpose of the PD is content specific and connected to teacher’s current practice, elements of good practice as outlined by the literature.

The format of the current PD model is consistent with what the research indicates is most promising in terms of promoting teacher learning. Professional development activities that include opportunities for collective participation and collaboration, that are linked to teachers’ daily work, and that promote active learning are more likely to be perceived by teachers as beneficial (Birman et al., 2000; Garet et al., 2001). This PD model addresses these organizational aspects in that general and special education teachers will participate as co-teaching teams, develop curriculum and instructional materials that are applicable to their current practice, and participate in engaging learning activities such as group discussions and presentations of their work.

The literature is clear that one-time workshops or other short professional development opportunities rarely promote improvements in teacher learning or instruction. The proposed PD model features a 5-day summer session followed by 3 follow-up meetings totaling approximately 30 hours of targeted learning opportunities spanning over several months. Overall, the current study will incorporate many of the most widely accepted features of effective professional development.
Chapter 3: Methodology

This chapter outlines the methodology used in this single-subject design study examining the effects of a professional development course for increasing teacher self-efficacy as well their content and pedagogical content knowledge in geometry. The professional development course consisted of an in-depth exploration of congruence and similarity concepts including the development of and proof of geometric theorems, describing transformations, and applying trigonometric ratios to solve problems. The course also instructed participants on implementing Universal Design for Learning (UDL) concepts, which is a set of curriculum principles used to instruct and meet the needs of diverse learners. The course was delivered in a blended learning format consisting of online as well as in-person participation. A multiple probe design across three sets of two teachers for a total of 6 participants was used in this study to demonstrate a functional relationship between the independent and dependent variables. Qualitative data were collected through a focus group interview at the end of the study. Additionally, social validity data on the value of the intervention were obtained from the participants teachers. Descriptions and justifications of the sample, design, measurement techniques, instrumentation, and data analysis procedures are described in the sections to follow.

Participant Eligibility and Selection

Six teachers from three public charter schools in Washington, DC participated in this study. Teachers participated as pairs and were co-teachers of a geometry course at their respective school. To meet eligibility criteria established prior to the study, eligible participants were: a) co-teaching partners of a secondary geometry course; and b) teaching a classroom with at least 25% of students having an IEP. Table 2 displays demographic
information from the Survey on Teaching Mathematics to Students with Learning Disabilities in Middle School, which was completed by each teacher.

**Table 2 Participant Demographic Information**

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male: 4</th>
<th>Female: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of years teaching</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2 years:</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3-8 years:</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>9-14 years:</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Number of years teaching in an inclusive classroom</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2 years:</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3-5 years:</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6-10 years:</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Type of school where you teach</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Urban:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Suburban:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Rural:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Private:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Public:</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Number of students in your school</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-200:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>201-500:</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>501-800:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>801-1100:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>More than 1100:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Average number of students in your inclusive classes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 15:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>15-20:</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>21-25:</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>26-30:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>31-35:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>The number of professional development workshops related to teaching students with learning disabilities I have been exposed to has been:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-2:</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3-4:</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5-6:</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7-9:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>10 or more:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>The level of administrative support for teaching an inclusive class in my school is:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extremely Low:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Low:</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Average:</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>High:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Extremely High:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>The level of additional support services (e.g., counseling, resource room or teacher, instructional materials, etc.) for teaching an inclusive class in my school is:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extremely Low:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Low:</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Average:</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>High:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Extremely High:</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

I used the following steps to select potential participants. First, I contacted and met with the Principal of each school to discuss the study. During each discussion, I presented the Principal with a letter (see Appendix A) detailing a description of the study, the duration of the course, and any surveys and assessments each participant may be requested to complete. If the principal agreed for me to speak to their teachers, I then discussed the study with the general and special education geometry co-teachers and
presented them with a Participant Consent form (see Appendix B).

**Informed Consent.** Written consent was obtained from each participant using the Participant Consent form prior to initiating the study. Each participant was informed of their right to withdraw from the study at any time.

**Human Subjects Review.** Permission was obtained from the Human Subjects Review Committee at the University of Maryland, College Park prior to beginning the investigation.

**Instructor and setting.** The intervention was facilitated by the investigator. The study took place after regular school hours inside a conference room at three different public charter schools in Washington, DC. Each school supplied an LCD projector, and a projection screen. The instructor also used a portable whiteboard during the intervention. The actual intervention included four three-hour sessions after school, two six-hour sessions on Saturdays, and two online sessions designed to take four hours each to complete. The intervention was designed to take 32 hours to complete over a two-week period. All instructional lessons and assessments were preplanned prior to the start of the intervention.

**Professional Development Resources**

This section provides an overview of the instructional materials that were used during the intervention to increase teacher’s self-efficacy and knowledge of geometry teaching. Lesson plans for each session were created by the investigator using research based evidence for providing teachers with effective PD experiences.

**UDL Guidelines.** The UDL guidelines (CAST, 2011) were used to provide a framework for understanding how to create curricula that meets the needs of all learners.
The guidelines are designed to help teachers plan and create learning experiences that address students’ recognition, strategic and affective learning styles. During the intervention, the investigator presented the UDL guidelines to the participants and demonstrated how the guidelines could be used to develop differentiated lesson plans. Participants used the guidelines to develop lesson plans for teaching congruence and similarity concepts. See Appendix C for a sample of the guidelines.

**Videocases.** During the intervention, the investigator used videocases developed by the Learning and Teaching Geometry project (Seago et al., 2013) to engage participants in congruence and similarity activities. The videocases offered insight into what an emerging understanding of similarity looks like, and encouraged participants to consider specific instructional strategies that can foster this understanding. The clips are unedited segments selected from real classroom footage of unstaged mathematics lessons, representing a range of grade levels, geographic locations and student populations. These clips typically represent a conceptual hurdle or portray some degree of mathematical confusion, based on the expectation that they are likely to provoke inquiry and discussion within the PD setting.

**LessonSketch.** *LessonSketch* is education software that helps researchers and educators create a learning environment. Features of *LessonSketch* include lesson scripts, content discussions, media authoring tools, user tracking, and assessment and course creation tools. It is built on the idea that representations of mathematics instruction—that is, depictions of interactions between a teacher, their students, and the content—can anchor discussions about the teaching, learning, and mathematics, and provide grounds for practice-based teacher development.
Experimental Design and Study Procedures

The current research study utilized both quantitative and qualitative methods. Quantitative data were collected through pre- and post-measures as well as a single-subject multiple probe design across three sets of two teachers for a total of 6 participants. The single-subject design was used given its importance in developing evidence-based practice in special education. Single-subject designs provide an opportunity for researchers to test and identify an educational or behavioral intervention as evidence-based practice (Odom et al., 2005). In the current study, a multiple probe design was selected over a multiple baseline design because continual use of probes for teachers in baseline would be impractical (Kazdin, 2011) given the academic nature of the intervention. Additionally, collecting infrequent probes rather than continuous baseline reduced the chances of participants improving their results through repeated practice (Kazdin, 2011) and was necessary due to the short duration of the intervention. The study consisted of 4 phases conducted in the following order: a) pre-intervention measure administration, b) baseline phase, c) intervention phase, and d) post-intervention measure administration.

The independent variable was introduced across each pair of teachers using a staggered method (Horner & Odom, 2013) by collecting a baseline series of data (consisting of at least 3 data points) for all pairs then introducing the intervention to Stephanie and Talib only. After Stephanie and Talib reached a predetermined level of criteria, all 3 pairs were probed (consisting of at least 3 data points). Nicole and Tiano were then introduced to the intervention only, followed by probing all three pairs. This pattern continued until all three groups had completed the intervention and met the
required criteria (Horner & Odom, 2013). This staggered approach was used to address internal validity (Horner & Odom, 2013).

**Pre- and Post-Study Instruments**

The current study utilized a survey and a geometry assessment to measure teacher self-efficacy, and content and pedagogical content knowledge in geometry both before and after the single subject study. See Appendix D for table of measures and instruments. Each teacher participant completed the pre- and post-measures individually in their classroom. I reserved the classroom for several hours and contacted each teacher to arrange a time for them to complete the measures. During the assessment, participants were given a copy of the measure and pencil. I then explained the purpose of the measures and asked if the participants had any questions. Participants were allowed to work on each measure independently for as much time as needed before I collected the measure for scoring. Teachers completed the measures in approximately 90 minutes on average.

**Teacher Efficacy Measures.** The Teachers’ Sense of Efficacy Scale (TSES), developed by Tschannen-Moran and Woolfolk Hoy (2001), was used to capture teacher self-efficacy beliefs as pre- and post-test measures. The TSES assesses the major aspects of teacher self-efficacy through a cyclical feedback loop for efficacy judgments demonstrating a more balanced picture of teacher self-efficacy without over generalization or deep levels of specificity. The short form version of the TSES was selected for the current study to decrease the total number of survey items since participants will be completing two separate instruments to assess their efficacy beliefs in addition to completing demographic information.

The short form consists of twelve questions focusing on three domains: self-efficacy
in student engagement, self-efficacy in instructional strategies, and self-efficacy in classroom management. Each item of the survey instrument reflects pedagogical activities, which regularly occur in an inclusive classroom. The twelve questions allow the respondent to select the level of his or her belief along a nine-point Likert scale. Question stems begin with the words, “How much can you do?” or “To what extent can you …” followed by a specific pedagogical activity. Following Bandura’s (1997) nine-point response scale, the odd numbers are labeled as follows: one is “Nothing”, three is “Very Little”, five is “Some Influence”, seven is “Quite A Bit”, and nine is “A Great Deal”. Each point on the scale expresses how much or how well the respondent felt he or she could do regarding the specific task or activity (Tschannen-Moran & Woolfolk Hoy, 2001).

The short form of the TSES is scored in a manner that provides an overall self-efficacy score for each respondent as well as an individual score on three subscales among samples of practicing teachers. The overall self-efficacy score is computed by summing the numeric value for the recorded responses for each of the twelve items on the self-report instrument. The minimum number of points achievable on the TSES instrument is 12, which represents the lowest level of efficacy a respondent can possess. The maximum number of points achievable on the instrument is 108, which represents the highest level of self-efficacy a respondent can possess. See Appendix E for the TSES form including a permission letter from the developer.

**Geometry Content and Pedagogical Content Knowledge Measure.** The Geometry Assessment for Secondary Teachers (GAST) assessment was used to measure changes in teacher’s content and pedagogical content knowledge in geometry. The development of the GAST began in October of 2008 as part of an NSF grant designed to study high school
geometry teacher's geometry knowledge, teaching practices, and their relationship to student achievement. The test was designed to be predictive in nature- a high score on the GAST assessment was meant to signify the likelihood of student success in the high school classroom. As requested by the NSF, geometry content of the GAST assessment was restricted to four main areas: similarity, congruence, area, and volume. The intent of the request was to make the scope of the project less overwhelming by concentrating on these specific areas, which are fundamental to geometry, generating results representative of high school geometry content in general. To enhance construct validity of the assessment, GAST team members first analyzed secondary and college geometry textbooks, state standards, and national assessments (e.g., NAEP, ACT) to better understand the current geometry used for teaching. Following this analysis, a team of mathematics educators, mathematicians, high school mathematics teachers, and doctoral students developed a blueprint for the assessment, incorporating ideas from Webb's depth of knowledge framework. The blueprint of the assessment included three principal content areas: Teacher Knowledge of Mathematics (30%), Teacher Knowledge of Geometric Reasoning and Problem Solving (25%), and Teacher Knowledge of Student Learning (45%). The test did not specifically measure all of the mathematical knowledge for teaching domains as categorized by Ball and colleagues, but the three major sub-domains-Common Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching were well represented. After item production, regional reviewers were assembled to validate items for section, domain, topic, item type, and DOK level. To further ensure construct validity for the test, both the blueprint and test items were reviewed by a panel of national experts in geometry and mathematics teaching. See Appendix F for a sample
version of the GAST assessment.

**Qualitative Measure**

At the conclusion of the study, the six participants were invited to participate in a focus group to discuss the outcomes of the intervention. The focus group was facilitated and audiotaped by an independent facilitator. The researcher was not present for the focus group. The facilitator selected was a licensed school psychologist from a nearby school district. The focus group session lasted 30 minutes. The facilitator used an interview questionnaire developed by the researcher to guide the session (see Appendix G).

**Single Subject Data Collection**

*Probes.* Items on the probes were created by the investigator and were directly related to the objectives of the instructional unit. Domain probes, and objective probes were given to participants in the study. Domain probes provided a baseline and post-intervention assessment of participant knowledge of co-teaching principles, UDL concepts and knowledge for teaching geometry. Objective probes provided a daily progress monitoring of participant performance on individual objectives presented in the lessons. Domain and objective probes were scored for the percent of accurate responses.

*Domain Probes.* During the baseline phase and after the intervention, participants completed randomly chosen parallel versions of an investigator created domain probe made up of items related to the information covered during the instructional units. Each domain probe consisted of 10 – 15 items related to UDL principles, teaching students with disabilities, and geometry pedagogical and content knowledge for teaching. A minimum of two domain probes were given to participants during baseline. A minimum of three domain probes were given at the end of the intervention phase, after a participant
completed all lessons and reached criterion (80%) on all objective probes in the intervention. Three parallel versions of the domain probe were used. See Appendix H for a sample domain probe.

**Objective Probes.** The objective probe included items related to the objectives covered in a specific lesson. Objective probes were given only during the intervention phase, at the end of the 2nd, 3rd, 5th and 6th class sessions. Each objective probe consisted of a variety of multiple-choice questions for assessing concepts and skills covered in a specific lesson. Objective probes totaled 8-10 points each. If a participant met criterion (80%) on the objective probe at the end of the class session, the next lesson was presented. If criterion was not met, the same lesson was administered the following session. See Appendix I for a sample objective probe.

**Baseline phase.** The baseline condition consisted of participants completing domain probes measuring their knowledge of geometry concepts, universal design for learning, and pedagogical knowledge for teaching in an inclusion setting. I collected at least three domain probes per participant during baseline. During probe sessions, participants were given a probe, a pencil and scratch paper. I read the purpose of the probe followed by the directions, along with asking if the participants had questions. Each participant was allowed to work on the domain probe independently for as much time as needed before I collected the probe for scoring. If a participant had a question about the probe items, I responded, “Do the best you can.” When both participant’s performance reflected stability in level and trend during baseline (i.e., at or below 60% percent criterion for at least two consecutive data points) I began the intervention with the first pair of teachers. Participants in the baseline condition were presented with no instruction during that
period.

**Intervention.** The independent variable in a single case design study must be actively manipulated to document experimental control (Horner et al., 2005). In the current study, the independent variable (i.e. the professional development course), included features of professional development cited in the literature base and incorporated UDL practices. The professional development course consisted of an in-depth exploration of congruence and similarity concepts including the development of and proof of geometric theorems, describing transformations, and applying trigonometric ratios to solve problems. The course was comprised of online as well as in-person participation. A course outline is included in Appendix J.

The in-person portion of the course consisted of three lessons covered over a three day period and instructed participants on best practices for co-teaching in inclusive settings as well as featured instruction on two modules created through the Learning and Teaching Geometry Project (Seago et al., 2013). Lesson 1 of the professional development course provided instruction on co-teaching and inclusive practices focused on effective co-teaching models, and incorporating UDL principles into teacher lesson plans. Participants viewed video examples, participated in role-play, and modified actual lesson plans. Lesson 2 examined the meaning of defining congruence and similarity through transformations as articulated in the Common Core State Standards. I guided teachers through a constructed problem similar to what they could present to their students. Teachers then discussed and developed strategies for how they would teach it in their classroom. Lesson 3 investigated the connection between similarity, slope, and the graphs of linear functions through mathematical tasks, analyzing student thinking, and exploring a computer-based applet.
Each of the two geometry instructional modules presented in Lessons 2 and 3 contained a sequence of videocases in which specific and increasingly complex mathematical ideas were presented within the dynamics of classroom practice. The videocases offered insight into what an emerging understanding of similarity looks like, and encouraged teachers to consider specific instructional strategies that can foster this understanding. Appendix K contains the lesson plans for the in-person sessions.

The online section of the course was facilitated through the use of the LessonSketch tool provided by the Thought Experiments in Mathematics Teaching (ThEMaT) project. LessonSketch is a website for practice-based professional development of secondary mathematics teachers as well as a forum for ongoing conversations about mathematics instruction. It supports the creation, examination, and discussion of scenarios that depict the practice of mathematics instruction. Teacher participants viewed and discussed animated sketches about geometry concepts as well as created lesson plans using the online tool. The online portion of the course consisted of two days. Each day, participants viewed an animated representation of geometry instruction and responded to a series of questions through an online discussion board or animated script. Figure 1 displays two screenshot examples of an animation from LessonSketch. Using the animated scripts, participants provided feedback and commented on the animations as well as offered suggestions for specific teacher dialogue that would enhance instruction. For example, in response to a geometry animation on congruent triangles, Stephanie suggested that the teacher help student tend to precise measurement by matching the vertices of two congruent triangles.
Inter-assessor reliability. Inter-assessor reliability was obtained for 100 percent of the data points from the domain probes. A trained graduate student and I independently scored each probe. The percentage of scorer agreement was calculated by dividing the number of agreements of correct by the number of disagreements and agreements and multiplying the result by 100 percent (Kennedy, 2005). Reliability was 100 percent across all domain probes for each teacher.

Fidelity of treatment. Two independent observers reviewed the video recorded PD lessons for treatment fidelity using a checklist (Appendix L) containing critical features of the intervention (O’Donnell, 2008). I trained the independent observers by explaining the checklists and reviewing the written lesson plans, assessments, and recorded sessions. I conducted two mock instructional sessions where I followed my lesson script and the two observers were instructed to note either a “1” (if the component was present) or a “0” (if the component was not present) for each component on the fidelity checklist during the session. I compared the two checklists and reviewed any discrepancies. Observers
maintained 100% agreement over the mock sessions.

Fidelity observations were conducted on 33% of the instructional and assessment sessions using video recordings of the sessions by two independent observers (Kennedy, 2005). The fidelity of treatment for each session was calculated by dividing the number of observed components by the number of total components on the checklist and multiplying the result by 100 (O’Neill, et al., 2011). The fidelity of treatment for the study was obtained by adding the percentages for each session and dividing the sum by the number of observations. Treatment fidelity was 100 percent.

Inter-assessor agreement on the objective probes was obtained by collecting the scores of the two assessors and calculating the percentage obtained by dividing the smaller score by the larger score and multiplying by 100, yielding a total percent agreement (Gast, 2010). Interobserver agreement for the study was 100 percent.

**Data analysis procedures**

In single-subject research, experimental control is established when visual analysis of the data demonstrates a functional relationship between independent and dependent variables. A functional relationship must be established where systematic manipulation of an independent variable has a consistent effect on a dependent variable (Kennedy, 2005). In the present study, a functional relationship was determined based on the improvement of participants’ content and pedagogical content knowledge for teaching geometry concepts, and whether the change was caused by the treatment. Visual analysis of graphed data was used to determine the: (a) stability of the baseline conditions, (b) rapidity of changes in the variables between conditions (baseline and post-intervention), (c) changes in the mean performance and changes in pattern of individual data points between
conditions, and (d) variability in the level and/or trend within and across conditions
(Kennedy, 2005).

Statistical tests selected to analyze the collected data are frequency counts and non-
parametric tests. Data analysis for the three research questions was conducted using the
data sources outlined in Table 3.

**Table 3 Data Sources and Analyses**

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Data Sources</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the effect of a teacher professional development course involving use of animated representations addressing congruence and similarity on special education and mathematics education teacher self-efficacy, content knowledge, and pedagogical content knowledge in an inclusion setting?</td>
<td>Geometry Assessments for Secondary Teachers (GAST); Domain and Objective Probes; Teachers Sense of Efficacy Scale (TSES)</td>
<td>Single-subject analysis Non-parametric tests Frequency counts</td>
</tr>
<tr>
<td>How do general education and special education teachers compare in their understanding of congruence and similarity as well as their self-efficacy for teaching geometry in an inclusive setting?</td>
<td>Geometry Assessments for Secondary Teachers (GAST); Teachers Sense of Efficacy Scale (TSES)</td>
<td>Non-parametric tests</td>
</tr>
<tr>
<td>To what extent do general and special education teachers find a blended learning professional development course as an effective means for increasing teacher content and pedagogical content knowledge in geometry?</td>
<td>Social Validity Measure Focus Group</td>
<td>Mean scores Focus Group Notes</td>
</tr>
</tbody>
</table>

**Social validity.** Social validity refers to the degree of social acceptance of the importance, effectiveness, and appropriateness of an intervention (Carter, 2010; Foster & Mash, 1999).
Horner and colleagues (2005) suggested that the social validity of research goals can be enhanced through design features that include: a) dependent variables that have high social importance, b) independent variables that can be applied with fidelity by typical intervention agents, c) procedures that are acceptable, feasible, and effective as reported by typical intervention agents, and d) an intervention that meets a defined need. At the conclusion of the study, participants completed an investigator-developed survey (Appendix M) based on other social validity measures (Mulcahy, 2007; Strickland, 2011) which assessed general likes and dislikes about the intervention along with perceptions of specific design features (i.e. video cases and online discussions).
Chapter 4: Results

The purpose of this study was to investigate the effects of a professional development course for increasing teacher self-efficacy as well their content and pedagogical content knowledge in geometry. This chapter presents the profile of participants used for the research and a summary of the findings relative to each of the research questions posed in Chapter 1. Finally, I report the results from social validity measures across participants.

The research questions included the following:

1. What is the effect of a teacher professional development course involving use of animated representations addressing congruence and similarity on special education and mathematics education teacher self-efficacy, content knowledge, and pedagogical content knowledge in an inclusion setting?

2. How do general education and special education teachers compare in their understanding of congruence and similarity as well as their self-efficacy for teaching geometry in an inclusive setting?

3. To what extent do general and special education teachers find a blended learning professional development course as an effective means for increasing teacher content and pedagogical content knowledge in geometry?

Participant Profile

Participant demographic data were collected using Part I of the Survey on Teaching Mathematics to Students with Disabilities in Middle School Survey, which was summarized in Chapter 3. Teacher participants were co-teachers in their respective schools and participated in the intervention in pairs. Stephanie is a special education teacher and has
taught with Talib, a general education mathematics teacher, for two years. Nicole is a first year special education teacher and this is her first year working with Tiano, a mathematics teacher. Reggie is a special education teacher and this is his first year working with Gregory, a general education mathematics teacher.

**Results on Professional Development Course**

In this section, I report results as they relate to each research question. First, I provide the research question, and then I present each participant’s results, including mean percent accuracy increases from baseline to post-intervention, and the range of scores for each participant. Results are presented graphically for each participant for percent accuracy on domain probes, which were administered during baseline and post-intervention phases, as well as percent accuracy on daily objective probes, which were administered during the intervention phase.

**Research Question 1**

*What is the effect of a teacher professional development course involving use of animated representations addressing congruence and similarity on special education and mathematics education teacher self-efficacy, content knowledge, and pedagogical content knowledge in an inclusion setting?*

As noted in Chapter 3, increases in teacher efficacy, content knowledge and pedagogical content knowledge in an inclusion setting were measured by pre-post surveys as well as participant performance on domain and objective probes.

**Pre-/Post- Survey**

The TSES was used in order to discover if any significant changes occurred in teacher self-efficacy for participants from pre-test to post-test. For the TSES, survey items
are factored by one of three categories: Efficacy in Student Engagement, Efficacy in Instructional Practices, and Efficacy in Classroom Management. To determine the subscale score for each factor, the unweighted means that load on each factor are computed. Table 4 shows the pretest and posttest unweighted means for each participant in each factor.

### Table 4 TSES Results

<table>
<thead>
<tr>
<th></th>
<th>Efficacy in Student Engagement</th>
<th>Efficacy in Instructional Practices</th>
<th>Efficacy in Classroom Management</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-Mean Score</td>
<td>Post</td>
<td>Pre</td>
</tr>
<tr>
<td>Stephanie</td>
<td>8</td>
<td>8.25</td>
<td>8</td>
</tr>
<tr>
<td>Talib</td>
<td>7.5</td>
<td>8</td>
<td>6.75</td>
</tr>
<tr>
<td>Nicole</td>
<td>6.5</td>
<td>6.75</td>
<td>6</td>
</tr>
<tr>
<td>Tiano</td>
<td>7</td>
<td>7.5</td>
<td>6.75</td>
</tr>
<tr>
<td>Reggie</td>
<td>7.5</td>
<td>7.5</td>
<td>7.75</td>
</tr>
<tr>
<td>Gregory</td>
<td>6.25</td>
<td>6.5</td>
<td>6</td>
</tr>
</tbody>
</table>

### Domain and Objective Probes

All participants improved their mean percent accuracy from baseline to post-intervention on the domain probes, and each participant met criterion of 80%. Figure 1 displays the percent accuracy on domain and objective probes during baseline, intervention, and post-intervention phases for each pair of participants. Visual analysis of each graph reveals marked changes in level from baseline to post-intervention for all participants. Further, there is low variability in data points in both the baseline and post-intervention phases, reflecting stable data for each participant. Stephanie increased 76 percentage points in mean percent accuracy, Talib increased 68 percentage points, Nicole increased 83 percentage points, Tiano increased 80 percentage points, Reggie increased 77 percentage points, and Gregory increased 77 percentage points.
Figure 1 Percentage of Accurate Responses on Domain Probes
*Stephanie.* During baseline, Stephanie earned a mean score of 17% on domain probes, \((r= 10\% - 20\%)\). Following the intervention, she earned a mean score of 92% \((r = 85\% - 100\%)\).

*Talib.* During baseline, Talib earned a mean score of 27% \((r = 20\% - 30\%)\). Following the intervention, he earned a mean score of 94% \((r = 90\% - 95\%)\).

*Nicole.* During baseline, Nicole earned a mean score of 7.5% on domain probes \((r = 5\% - 10\%)\). Following the intervention, she earned a mean score of 90% \((r = 85\% - 95\%)\).

*Tiano.* During baseline, Tiano earned a mean score of 16% \((r =10\% - 20\%)\). Following the intervention, he earned a mean score of 96% \((r = 90\% - 100\%)\).

*Reggie.* During baseline, Reggie earned a mean score of 8% \((r =5\% - 10\%)\). Following the intervention, he earned a mean score of 85% \((r = 80\% - 90\%)\).

*Gregory.* During baseline, Gregory earned a mean score of 16% \((r =10\% - 20\%)\). Following the intervention, he earned a mean score of 93% \((r = 90\% - 100\%)\).

Daily objective probes were administered throughout the intervention phase. Figure 2 displays the results of the objective probes for each participant pair. Visual analysis reveals stability across all data points for all participants. In addition, the graphs demonstrate high levels of performance across all participants.
Figure 2 Percentage of Accurate Responses on Objective Probes

- Stephanie
- Talib

- Nicole
- Tiano

- Reggie
- Gregory
Research Question 2

How do general education and special education teachers compare in their understanding and knowledge of congruence and similarity as well as their self-efficacy for teaching geometry in an inclusive setting?

Participant understanding and knowledge of congruence and similarity concepts was measured using the GAST assessment. The assessment consists of 15 multiple choice and 11 constructed response items. The maximum score obtained on the assessment is 48 points. The assessment was scored by an independent researcher who was trained by the assessment developer. Table 5 reports the pre and post mean scores within groups for special education teachers and general education teachers based on their responses on the GAST assessment.

Table 5 – Within Group Pre and Post Mean Scores on GAST Assessment of Congruence and Similarity

<table>
<thead>
<tr>
<th></th>
<th>Pre-Test Score</th>
<th>Post-Test Score</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Special Education Teachers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stephanie</td>
<td>20</td>
<td>29</td>
<td>+9</td>
</tr>
<tr>
<td>Nicole</td>
<td>16</td>
<td>22</td>
<td>+6</td>
</tr>
<tr>
<td>Reggie</td>
<td>13</td>
<td>15</td>
<td>+2</td>
</tr>
<tr>
<td><strong>Special Education Group Mean</strong></td>
<td>16.33</td>
<td>22</td>
<td>+6.33</td>
</tr>
<tr>
<td><strong>General Education Teachers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Talib</td>
<td>32</td>
<td>35</td>
<td>+3</td>
</tr>
<tr>
<td>Tiano</td>
<td>36</td>
<td>37</td>
<td>+1</td>
</tr>
<tr>
<td>Greg</td>
<td>27</td>
<td>28</td>
<td>+1</td>
</tr>
<tr>
<td><strong>General Education Group Mean</strong></td>
<td>31.67</td>
<td>33.33</td>
<td>+1.66</td>
</tr>
</tbody>
</table>
To examine how special education and general education teachers compare in their self-efficacy, the average subscale score for each dimension on the TSES were calculated for special education and general education teachers separately (see Table 6). Based on participant responses, special education teachers generally had higher self-efficacy scores compared to their general education counterparts in two out of the three dimensions.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Special Education Teachers</th>
<th>General Education Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficacy for Student Engagement</td>
<td>7.4</td>
<td>7.1</td>
</tr>
<tr>
<td>Efficacy for Instructional Practices</td>
<td>7.8</td>
<td>7.4</td>
</tr>
<tr>
<td>Efficacy for Classroom Management</td>
<td>7.4</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Research Question 3

To what extent do general and special education teachers find a blended learning professional development course as an effective means for increasing teacher content and pedagogical content knowledge in geometry?

Based on focus group notes participants felt that there were three aspects from this professional development experience that were different than other professional development experiences they shared. They were a) the opportunity to work through the video cases, b) the use of research to support the content in the course, and c) the exposure to the LessonSketch tool.

Participants felt that the use of video cases in the course enabled them to use these same experiences in their classroom. Participants said that while taking the class they were often placed in the role as the student. “The instructor often wanted us to see how this strategy works through the eyes of a student,” one teacher remarked. Another teacher commented, “the videos provided examples that could be brought right back to the classroom”.

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The use of research supporting the UDL content was also important to the participants. In several responses participants said that the use of the UDL guidelines validated the strategies that was being discussed. One teacher commented, “people think often that we just pick up strategies and they are not research-based and I think the research that has been shared in this course proves that it can work.”

Participants also found the LessonSketch tool to be a “creative way to write lessons and learn new concepts”. The LessonSketch tool allowed teachers to work together to create a lesson script as well as view online animated representations of geometry instruction. One teacher commented, “I think my students would enjoy using this tool to design math problems in class”.

**Social Validity**

The average scores on the social validity measure ranged from 4 to 5 (see Table 7). Overall, participants responded that they strongly agreed that the professional development course relevant and useful and that the activities were engaging. While the special education teachers were somewhat neutral that the course increased their understanding and use of Universal for Design for Learning strategies \((M=3.33)\), general education teachers strongly agreed that the intervention increased their knowledge and understanding of UDL concepts \((M=5)\).
Table 7 – Social Validity Measure Results

<table>
<thead>
<tr>
<th></th>
<th>Stephanie</th>
<th>Talib</th>
<th>Nicole</th>
<th>Tiano</th>
<th>Reggie</th>
<th>Gregory</th>
<th>Special Ed Teacher Mean</th>
<th>General Ed Teacher Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In-Person Sessions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The professional development content was relevant and useful</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>The activities were engaging and connected to the professional development objectives</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4.67</td>
<td>4.67</td>
</tr>
<tr>
<td>I gained knowledge about Universal Design for Learning concepts</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>3.33</td>
<td>5</td>
</tr>
<tr>
<td>I gained knowledge about using Universal Design for Learning concepts in my lessons</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>3.67</td>
<td>5</td>
</tr>
<tr>
<td>I gained knowledge about similarity and congruency concepts in geometry</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3.67</td>
</tr>
<tr>
<td>I will be able to apply the strategies discussed in this professional development immediately in my teaching</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4.33</td>
<td>4.67</td>
</tr>
<tr>
<td>The video case studies were effective for learning about teaching geometry concepts</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4.67</td>
</tr>
<tr>
<td><strong>Online Sessions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The animated representations of teaching were effective for facilitating pedagogical discussions</td>
<td>Stephanie</td>
<td>4</td>
<td>Talib</td>
<td>5</td>
<td>Nicole</td>
<td>5</td>
<td>Tiano</td>
<td>5</td>
</tr>
<tr>
<td>The online activities were effective for facilitating collaboration</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>4.33</td>
<td>4.67</td>
</tr>
</tbody>
</table>

Participants also responded favorably to the intervention in open-ended questions.

For example, Nicole stated that the best part of the course was “completing a variety of activities.” Gregory reported that “learning to incorporate UDL strategies” was the best part of the course. When comparing the in-person and online sessions, Tiano reported that the animated representations “was a unique way to learn a concept.”
Chapter 5: Discussion

The purpose of this study was to examine the effects of a co-teacher professional development course for increasing the knowledge and self-efficacy of special education and general education geometry teachers in an inclusion setting. The participants learned best practices for co-teaching, how to apply UDL strategies and concepts, and gained knowledge for teaching congruence and similarity concepts.

This chapter summarizes the major findings of the study, presents the study limitations, provides recommendations that result from this study, and offers opportunities for future research.

Study Design and Rationale

A multiple probe design across three sets of two teachers for a total of 6 participants was used in this study to demonstrate a functional relationship between the independent and dependent variables. In addition, the study utilized several instruments to gather teacher demographic data, and measure teacher self-efficacy, content and pedagogical content knowledge in geometry both before and after the single subject study.

The major themes that guided this study were the following: the effectiveness of various professional development delivery systems; the importance of teachers’ perceptions of their effectiveness in the classroom; and the mathematical knowledge teachers need to effectively teach congruence and similarity concepts.

The literature concerning professional development for educators from Garet et al (2001), the National Council for Teachers of Mathematics, the National Research Council, and the Common Core Standards for Mathematics provided the frameworks for the researcher’s design in the professional development delivery activities used in the
study. Changing teachers’ perceptions of mathematics instruction and keeping teachers current in mathematics practices has been a major issue in education in the United States (TIMSS, 2007). In addition, teachers are instructing students with diverse learning needs. In order for teachers to meet the diverse learning needs of students, teachers must believe in their own ability to do so. Self-efficacy, in regards to education, plays a critical role in the instructional decisions teachers make on a consistent basis (Kosko & Wilkins, 2009).

The content delivered in the professional development treatment was based on the Common Core State Standards Mathematical Practices for Teaching, and the work of Ball et al (2008) on investigating what mathematical knowledge is needed for effective teaching, and the research base for best practices for teaching in an inclusive setting. The testing instruments used in this study consisted of a pre and post assessment two surveys as well as researcher developed probes for single-subject analysis. The GAST assessment was used to measure participant knowledge and understanding of geometry concepts before and after the intervention. The first survey, the Teachers’ Sense of Efficacy Scale (Tschannen-Moran and Woolfolk Hoy 2001), was used to capture teacher self-efficacy beliefs as pre- and post-test measures in three dimensions; student engagement, instructional strategies, and classroom management. The Survey on Teaching Mathematics to Students with Learning Disabilities in Middle School (DeSimone & Parmar, 2006) is a second survey used measure teacher beliefs related to teaching mathematics to students in inclusive settings.

The professional development course consisted of an exploration of congruence and similarity concepts including the development of and proof of geometric theorems, describing transformations, and applying trigonometric ratios to solve problems. In addition, participants received instruction on the best practices for teaching in an inclusive
setting. The course was delivered in a blended learning format, consisting of online as well as in-person participation.

**Interpretation of Findings**

**Research question one:** What is the effect of a teacher professional development course involving use of animated representations addressing congruence and similarity on special education and mathematics education teacher self-efficacy, content knowledge, and pedagogical content knowledge in an inclusion setting? In regards to teacher content and pedagogical content knowledge, the success of this intervention is demonstrated in the results each participant experienced from baseline to post-intervention phase on the domain probes. Stephanie increased 75 percentage points from baseline to post-intervention. Talib increased 67 percentage points from baseline to post-intervention. Nicole, who is a first year special education teacher, increased 82.5 percentage points from baseline to post-intervention. Tiano, an experienced mathematics teacher, increased 80 percentage points from baseline to post-intervention. Reggie increased 77 percentage points from baseline to post-intervention. Gregory, who has been teaching mathematics for two years, increased 77 percentage points from baseline to post-intervention.

The ability of each of the participants to perform at such high levels both during instruction and during the post-intervention domain probe sessions suggests that the professional development course, presented through blended learning activities, positively affected teacher content and pedagogical content knowledge for teaching geometry concepts in an inclusive setting. Furthermore, the consistent changes in outcomes for Stephanie and Talib demonstrated the independent variable had a positive impact on the dependent variable, which establishes a functional relationship (Kennedy, 2005).
Replication of these findings with the remaining teacher pairs demonstrated robustness of the experimental control and generality to other participants (Kennedy, 2005).

Further, the post-test mean scores on the GAST assessment, which measures teacher content and pedagogical content knowledge in geometry, increased for the participants by 13 points. This supports the evidence that the professional development course was effective for increasing teacher content and pedagogical content knowledge.

In regards to teacher self-efficacy, post survey results on the TSES demonstrate an increase in each of the three dimensions: Efficacy for Student Engagement, Efficacy for Instructional Practices, and Efficacy for Classroom Management. However, the increase in Efficacy for Instructional Practices was more measurable compared to the other two dimensions. This suggests that while the professional development course was beneficial for improving instructional practices, the course was not as effective for improving participants’ ability to improve their classroom management skills or increase student engagement.

**Research question two:** How do general education and special education teachers compare in their understanding and knowledge of congruence and similarity as well as their self-efficacy for teaching geometry in an inclusive setting? In regards to teacher knowledge of congruence and similarity concepts, general education teachers outperformed special education teachers, based on GAST assessment results. This supports the literature, which suggests that special education teachers generally lack content specific knowledge. It should be noted that while Tiano, a mathematics teacher obtained the highest score on the GAST assessment (37 out of 48), this score was less than 80% proficiency. In addition, the special education teachers demonstrated higher
increases from pre to post-test compared to general education teachers. Yet, the average score for special education teachers on the GAST post assessment was less than 50% proficiency. Special education and general education teachers alike reported that they felt that the GAST assessment was particularly challenging.

With regard to teacher self-efficacy, special education teachers reported higher self-efficacy in student engagement and instructional practices. This suggests that special education teachers feel more confident in their ability to motivate students as well as provide a variety of instructional strategies. As special education teachers receive specific training in these areas, it makes sense that these teachers would report a higher self-efficacy.

**Research question three:** To what extent do general and special education teachers find a blended learning professional development course as an effective means for increasing teacher content and pedagogical content knowledge in geometry? Based on focus group notes participants noted several positive aspects of the professional development including the use of video cases, the application of research-based strategies and the use of the LessonSketch tool. In addition, teacher participants felt that the collaborative piece that allowed teacher pairs to work together was the major support in making their professional development experience a positive one. The fact that the participants were already co-teachers and participating with their partner may have enhanced the collaborative nature of the course. During the course, I observed Talib and Stephanie’s willingness to work through the activities together with less hesitation than the other co-teaching partners. This is indicative of their current working relationship. The other co-teaching partners had only been together for the current school year. In addition,
Reggie, the most experienced teacher, was more reticent in his approach to co-teaching, often allowing Gregory to take the lead on collaborative activities in the course. This was an indication of the type of role Reggie perceives of himself as a special education co-teacher.

The participants also felt that having a common experience as participating in the same professional development allowed conversations to be geared toward their new learning. The use of the LessonSketch helped to facilitate collaboration as teachers worked together to discuss the animated representations.

Summary
This study suggests that a co-learner blended learning professional development course can improve teacher content and pedagogical knowledge in geometry as well as increase teacher efficacy for teaching in an inclusive setting. Each participant dramatically improved his or her performance on domain probes from baseline to post-intervention. In addition, participants improved their scores on post assessments as well as reported favorable outcomes in the focus group interview and social validity measure.

Study Limitations
There are several limitations to this study. One limitation was the length of the professional development course. The average duration of the intervention was 30 hours. Although the literature suggests that short, intensive PD programs can be effective for allowing for greater teacher participation as well as promote the transfer of PD activities into classroom practice (Garet et al., 2001), the short duration of the current study may warrant the need for additional follow up sessions. Another limitation of the study is that student achievement was not measured as a dependent variable. The research is limited
for studies that link student performance to professional development outcomes. Further, this study did not include observations of teachers in their normal setting after completing the professional development course. Follow-up observations would help determine teacher application of the strategies they learned during the course.

Given that the investigator implemented the intervention, my own biases may have influenced the study. I attempted to control for those biases through the use of scripted lessons, explicit instructional procedures, and fidelity of treatment procedures. It is also noted that while participant scores increased significantly on domain and objective probes, there was less of an increase in scores on the GAST post-test. This may indicate that the probes were heavily aligned to the intervention.

**Directions for Future Research**

This study could advance the body of knowledge on the effects of a co-teacher professional development model in relation to teachers’ specialized mathematics knowledge for teaching. While the current study's design was intended to examine how a blended format professional development delivery model could affect teachers’ self-efficacy and knowledge in geometry. The co-teacher model was found to allow collaborative inquiry to occur; a major component in the institutionalization of any practice (Guskey, 2000). This study could have the potential to direct future research on educating teacher partners to build collaboration and promote sustainability in their current setting.

The targeted population for this study was current teachers. However, it is suggested that future research include pre-service teachers as part of their teacher preparation program. Including special education and general education pre-service teachers in the same course may enhance collaboration and increase content and
pedagogical knowledge for teachers. For example, a study could be conducted to find if there is a significant difference in influencing preservice teachers’ self-efficacy of mathematics instruction between taking courses with peers and conducting traditional field work requirements, or taking the course and conducting fieldwork requirements with a co-teacher candidate. While the current study focused on mathematics content, future research could find the feasibility of using the co-learner model in teaching other disciplines.

Although the duration of study may have limited the intervention’s influence on student achievement, it may have been beneficial to include student performance data for the teachers before and after the intervention. The single-subject design of the study is a limitation as most professional development courses involve groups of teachers. In the current student study, there were limited opportunities for distractions by other individuals, and since the room contained only the materials and equipment used for the intervention, there were no other distracting items in the room. Additionally, the teachers had the sole attention of the instructor, which is atypical of a professional development course. Under these controlled conditions, the participants demonstrated success. However, replication in a larger setting is necessary to generalize the results to common professional development courses.

Opportunities to determine any effects of teacher professional development on student achievement are needed as well. As noted in the literature review, there is limited research linking professional development to student achievement. This particular study could be extended to include student performance scores for those teachers participating in the professional development course.
Conclusion

The self-efficacy beliefs of teachers play a significant role in teacher's ability to adapt their instruction to meet the needs of a diverse student population. In order to ensure that all students improve their conceptual knowledge of geometry content, teachers must have a deep understanding of geometry content knowledge as well as strong pedagogical content knowledge. The current study investigated the effects of a blended professional development course for increasing teacher self-efficacy as well their content and pedagogical content knowledge in geometry. The results of this study provide initial evidence that a short, intensive professional development course can be effective for improving teacher self-efficacy and knowledge in geometry concepts. The co-teacher participant design of the course also promoted collaborative inquiry, which is a major component for effective professional development outcomes (Guskey, 2000). Continued research is critical to identify opportunities to provide additional courses that involve general and special education co-teachers learning together in other disciplines.
<table>
<thead>
<tr>
<th>Authors (Date)</th>
<th>Purpose</th>
<th>Sample/Participants</th>
<th>Research Design and Intervention (IV)</th>
<th>Dependent Variable</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arbaugh (2003)</td>
<td>Increase content knowledge and pedagogical content knowledge in geometry</td>
<td>7 high school mathematics teachers</td>
<td>Qualitative Design Study groups 10 Bi-weekly study groups sessions</td>
<td>Individual interviews, audio tape accounts of study group sessions</td>
<td>Teachers reported increase self-efficacy and confidence in teaching geometry concepts</td>
</tr>
<tr>
<td>Borasi, Fonzi, Smith &amp; Rose, (1999)</td>
<td>Increase content knowledge and pedagogical content knowledge in geometry</td>
<td>39 general and special education teachers in mathematics</td>
<td>Qualitative Design 6-day PD summer institute followed by at least two follow up meetings during the school year.</td>
<td>Qualitative measures (surveys, journals, interviews, transcriptions of audio recordings</td>
<td>Teachers reported increase in use of inquiry-based instruction. Heterogeneous groups of general and special education teachers was a benefit.</td>
</tr>
<tr>
<td>Brown &amp; Benken (2009)</td>
<td>Increase content knowledge and pedagogical content knowledge in geometry</td>
<td>3 high school mathematics teachers</td>
<td>Qualitative Design Two integrated courses on content and pedagogy. Courses met weekly</td>
<td>Transcripts of instructional sessions and informal meetings, field notes and participant course assignments</td>
<td>Pre-tests showed teachers had gaps in content knowledge. Focus on content and collaboration is needed in professional development.</td>
</tr>
<tr>
<td>Study</td>
<td>Intervention</td>
<td>Participants</td>
<td>Design/Methodology</td>
<td>Pre/post data</td>
<td>Findings</td>
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<tr>
<td>Feuerborn, Chinn, &amp; Morlan, 2009;</td>
<td>Increase content knowledge and pedagogical content knowledge in</td>
<td>55 middle school mathematics teachers</td>
<td>Mixed Study, Two 1-week professional development courses given over the summer.</td>
<td>Pre/post content test. Self-report survey data</td>
<td>Significant increase in teacher content knowledge. Teachers reported minimal change in self-efficacy.</td>
</tr>
<tr>
<td>Kimmel &amp; Deek (1999)</td>
<td>Increase teacher ability to meet the needs of students with</td>
<td>84 secondary teachers (3 cohorts of 23 each)</td>
<td>Comprehensive professional development model delivered during the summer and</td>
<td>Teacher self-reported data</td>
<td>The program led to development of coping skills and persistence in the teaching of science and math for all students.</td>
</tr>
<tr>
<td>Merrill, Devine, &amp; Brown, 2010</td>
<td>Increase content knowledge and pedagogical content knowledge in</td>
<td>8 mathematics teachers</td>
<td>More than 80 hours of professional development on using 3-D solid modeling software for instructing geometry concepts.</td>
<td>Teacher self-reported evaluation data</td>
<td>Teacher participants reported the value of the PD experience 4.5/5.0. Teachers also reported that the PD improved their instructional practice.</td>
</tr>
<tr>
<td>Study</td>
<td>Increase teacher self-efficacy</td>
<td>Participants</td>
<td>Program Details</td>
<td>Demographics</td>
<td>Outcome Measures</td>
</tr>
<tr>
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<tr>
<td>Powell-Moman &amp; Brown-Schild (2011)</td>
<td>Increase teacher self-efficacy</td>
<td>23 mathematics teachers</td>
<td>Two-year STEM professional development program taught by science expert.</td>
<td>Demographics; self-efficacy pre/post survey</td>
<td>Post test results show significant improvement in teacher self-efficacy</td>
</tr>
<tr>
<td>Ross &amp; Bruce (2007)</td>
<td>Increase teacher self-efficacy</td>
<td>106 grade 6 mathematics teachers</td>
<td>Professional development consisted of 1 full day followed by three 2-hr after school sessions. Focus was on communicating mathematical ideas.</td>
<td>Teacher Sense of Efficacy Scale</td>
<td>Treatment group outperformed control group in 3 out 4 measures of self-efficacy. Results were statistically significant for only teacher efficacy in classroom management</td>
</tr>
<tr>
<td>Swackhamer, Koellner, Basile, &amp; Kimbrough, 2009</td>
<td>Increase teacher self-efficacy</td>
<td>88 secondary mathematics teachers</td>
<td>Teachers completed at least one of 15 content based math and science courses designed to increase content and pedagogical content knowledge.</td>
<td>Science Teaching Efficacy Belief Instrument</td>
<td>Survey results report that teacher's efficacy for teaching was higher for those teachers who attended at least 4 courses.</td>
</tr>
<tr>
<td>Zambo &amp; Zambo, 2008</td>
<td>Increase teacher self-efficacy</td>
<td>63 4th through 10th grade mathematics teachers</td>
<td>Two-week summer professional development workshop on mathematics problem solving.</td>
<td>Collective Efficacy Questionnaire (Goddard, Hoy, &amp; Woolfolk-Hoy, 2000); Enoch &amp; Riggs</td>
<td>Post survey results report increase in teacher personal competence compared to collective</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>Elementary Science Efficacy Questionnaire (1990)</td>
<td>Competence (belief in their colleagues)</td>
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</tbody>
</table>

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October 1, 2014

Dear School Leader,

I am writing to request permission to provide targeted professional development to several of your teachers as part of a research project I completing under my doctorate program at the University of Maryland College Park. My advisor is Dr. Peter Leone and the purpose of this research project is to study the effects of a professional development course for secondary geometry teachers and special education teachers for the purpose of increasing teacher self efficacy as well as their content and pedagogical content knowledge in geometry.

The professional development course will take place during the 2014 Fall semester. The course will be delivered in a blended format consisting of both in-person and online participation. The total duration of the course is 32 hours over a two-week period. Teachers will be provided with learning materials and resources needed to participate in the course. Lesson topics will focus on the teaching of similarity and congruence in geometry. During the intervention, teachers will be asked to complete several brief assessments to measure their understanding of the material presented. Each assessment should take no more than 15 minutes to complete. Three 90-minute follow-up sessions will be conducted for intervention maintenance during the semester.

At the beginning and conclusion of the study, teachers will be asked to complete an assessment to measure their understanding of Geometry concepts as well as complete two surveys. The surveys will collect specific demographic data about their teaching experience as well as their beliefs about their personal teaching effectiveness. Altogether, the assessment and surveys should take no more than 90 minutes to complete. I have enclosed a Teacher Participation Consent form for more detailed information.

I hope you will allow me to offer this professional development opportunity to your teachers. Please let me know if you have any questions and I look forward to meeting with you to further discuss my research project.

Sincerely,

Kenneth Wright
Principal Research Investigator
## Appendix B: Participant Consent Form

<table>
<thead>
<tr>
<th>Project Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching for Inclusion: The Effects of a Professional Development Course for Secondary General and Special Education Mathematics Teachers for Increasing Teacher Knowledge and Self-Efficacy in Geometry</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Purpose of the Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>This is a research project being conducted by Mr. Kenneth Wright, Vice-Principal at Meridian Public Charter School, as part of his doctoral studies at the University of Maryland: College Park, under the supervision of Dr. Peter Leone. The purpose of this research project is to study the effects of a professional development course for secondary geometry teachers and special education teachers for the purpose of increasing teacher self efficacy as well as their content and pedagogical content knowledge in geometry. You are being asked to participate to serve as a typical intervention agent, or someone who would possibly receive this professional development as a non-researcher.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>What will I be asked to do?</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the beginning and conclusion of the study, you will be asked to complete an assessment to measure your understanding of Geometry concepts as well as complete two surveys. The surveys will collect specific demographic data about your teaching experience and your beliefs about your teaching effectiveness. Altogether, the assessment and surveys should take no more than 90 minutes to complete. During the intervention, you will be asked to complete several brief assessments to measure your understanding of the material presented. These assessments should take no more than 15 minutes to complete.</td>
</tr>
</tbody>
</table>

The course will be delivered in a blended format consisting of both in-person and online participation. The total duration of the course is 32 hours over a two-week period. You will be provided with learning materials and resources needed to participate in the course. Lesson topics will focus on the teaching of similarity and congruence in geometry. Three 90 minute follow-up sessions will be conducted for intervention maintenance during the semester.

Additionally you will be asked your opinion of the intervention. For example you will be asked if you think the intervention helped you as a teacher to provide better instruction to your students and what you liked most and least about the intervention. Participants will receive $100 as reimbursement for your time and travel expenses upon conclusion of the study.

During the study, we will be video recording the instructional sessions. We will use portions of this video to 1) determine if the intervention is being implemented as planned; and 2) provide information for research presentations, publications, and/or teacher trainings.

<table>
<thead>
<tr>
<th>Potential Risks</th>
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</thead>
<tbody>
<tr>
<td>Risks include possible frustration with some of the assessments and tasks and the possibility of teachers’ image being viewed in future research presentations and/or teacher trainings. In addition, while I am a Vice Principal for Meridian Public Charter School, I will have no influence on participant employment or evaluations.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Potential Benefits</th>
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</thead>
<tbody>
<tr>
<td>Potential benefits include acquiring new knowledge and strategies for improving instructional practices</td>
</tr>
</tbody>
</table>
related to teaching similarity and congruence concepts.

Confidentiality

All information collected by this study will be kept confidential to the extent permitted by law. All data collected will be kept in a locked file cabinet in my office at Meridian Public Charter School or digitally on a password protected hard drive. If we write a report or article about this research project, your identity will be protected to the maximum extent possible and your name will not be used. Data will be identified using false names or an identification code. Your information may be shared with representatives of the University of Maryland: College Park, or governmental authorities if we are required to do so by law.

Do I have be in this research? Can I stop participating at any time?

Your participation in this research is completely voluntary. You may choose not to take part at all. If you decide not participate in this study you will not be penalized. If you decide to participate in this research, you may request to stop participating at any time. If you withdraw from the study prior to the conclusion, you will still be reimbursed the same amount/your reimbursement will be pro-rated based on the number of days you participated in the study.

What if I have Questions?

This is a research project being conducted by Mr. Kenneth Wright, a Vice-Principal at Meridian Public Charter School, as part of his doctoral studies at the University of Maryland, College Park, under the supervision of Dr. Peter Leone. If you have any questions about the research study itself, please contact Mr. Kenneth Wright at: 4279 South Capitol St SW, Washington, DC 20032 (telephone) 301-706-0001, (e-mail) wright1@umd.edu or Dr. Peter Leone at (email) leonep@umd.edu.

If you have questions about your rights as a research participant, please contact: University of Maryland College Park, Institutional Review Board Office, 1204 Marie Mount Hall College Park, Maryland, 20742 (Telephone) 301-405-0678 (E-mail) irb@umd.edu

This research has been reviewed according to the University of Maryland, College Park IRB procedures for research involving human subjects.

Statement of Consent

Your signature indicates that you are at least 18 years of age and you hereby give permission to participate in this educational study; the research has been explained to you; your questions have been fully answered; and you freely and voluntarily choose to participate in this research project.

| I agree to:                              | ___ be video recorded for internal use to ensure the intervention is being implemented as planned. |
|                                          | ___ be video recorded for external use in research presentations and/or teacher trainings.     |

<table>
<thead>
<tr>
<th>PRINTED NAME</th>
<th>SIGNATURE</th>
<th>DATE</th>
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</thead>
</table>
Appendix C: UDL Guidelines

Universal Design for Learning Guidelines

I. Provide Multiple Means of Representation
   1. Provide options for perception
      1.1 Offer ways of customizing the display of information
      1.2 Offer alternatives for auditory information
      1.3 Offer alternatives for visual information
   2. Provide options for language, mathematical expressions, and symbols
      2.1 Clarify vocabulary and symbols
      2.2 Clarify syntax and structure
      2.3 Support decoding of text, mathematical notation, and symbols
      2.4 Promote understanding across languages
      2.5 Illustrate through multiple media
   3. Provide options for comprehension
      3.1 Activate or supply background knowledge
      3.2 Highlight patterns, critical features, big ideas, and relationships
      3.3 Guide information processing, visualization, and manipulation
      3.4 Maximize transfer and generalization

II. Provide Multiple Means of Action and Expression
   4. Provide options for physical action
      4.1 Vary the methods for response and navigation
      4.2 Optimize access to tools and assistive technologies
   5. Provide options for expression and communication
      5.1 Use multiple media for communication
      5.2 Use multiple tools for construction and composition
      5.3 Build fluencies with graduated levels of support for practice and performance
   6. Provide options for executive functions
      6.1 Guide appropriate goal-setting
      6.2 Support planning and strategy development
      6.3 Facilitate managing information and resources
      6.4 Enhance capacity for monitoring progress

III. Provide Multiple Means of Engagement
   7. Provide options for recruiting interest
      7.1 Optimize individual choice and autonomy
      7.2 Optimize relevance, value, and authenticity
      7.3 Minimize threats and distractions
   8. Provide options for sustained effort and persistence
      8.1 Heighten salience of goals and objectives
      8.2 Vary demands and resources to optimize challenge
      8.3 Foster collaboration and community
      8.4 Increase mastery-oriented feedback
   9. Provide options for self-regulation
      9.1 Promote expectations and beliefs that optimize motivation
      9.2 Facilitate personal coping skills and strategies
      9.3 Develop self-assessment and reflection

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### Appendix D: Table of Measures

<table>
<thead>
<tr>
<th>Measure</th>
<th>Purpose</th>
<th>Timeframe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers’ Sense of Self Efficacy Scale (Tschannen-Moran &amp; Woolfolk Hoy, 2001)</td>
<td>Pre and Post measurement of teacher self-efficacy</td>
<td>Prior to baseline measures and at the conclusion of the intervention</td>
</tr>
<tr>
<td>Survey on Teaching Mathematics to Students with learning Disabilities in Middle School (DeSimone &amp; Parmar, 2006)</td>
<td>Gather teacher demographic data and measure teacher beliefs related to teaching mathematics in an inclusive setting</td>
<td>Prior to baseline measures</td>
</tr>
<tr>
<td>Geometry Assessment for Secondary Teachers (GAST)</td>
<td>Measure teacher content and pedagogical content knowledge of geometry concepts</td>
<td>Prior to baseline measures and at the conclusion of the intervention</td>
</tr>
<tr>
<td>GAST parallel probes</td>
<td>Intervention progress measure</td>
<td>During the intervention phase of the study</td>
</tr>
</tbody>
</table>
### Appendix E: Teacher Sense of Efficacy Scale – Short Form and Permission Letter

<table>
<thead>
<tr>
<th>Teacher Beliefs</th>
<th>Nothing</th>
<th>Very Little</th>
<th>Some Influence</th>
<th>Quite A Bit</th>
<th>A Great Deal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How much can you do to control disruptive behavior in the classroom?</td>
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<tr>
<td>2. How much can you do to motivate students who show low interest in school work?</td>
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<tr>
<td>3. How much can you do to get students to believe they can do well in school work?</td>
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<tr>
<td>4. How much can you do to help your students value learning?</td>
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<tr>
<td>5. To what extent can you craft good questions for your students?</td>
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<tr>
<td>6. How much can you do to get children to follow classroom rules?</td>
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<tr>
<td>7. How much can you do to calm a student who is disruptive or noisy?</td>
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<tr>
<td>8. How well can you establish a classroom management system with each group of students?</td>
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<tr>
<td>9. How much can you use a variety of assessment strategies?</td>
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<tr>
<td>10. To what extent can you provide an alternative explanation or example when students are confused?</td>
<td></td>
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</tr>
<tr>
<td>11. How much can you assist families in helping their children do well in school?</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>12. How well can you implement alternative strategies in your classroom?</td>
<td></td>
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</tr>
</tbody>
</table>
Dear

You have my permission to use the *Teachers’ Sense of Efficacy Scale* in your research. A copy of both the long and short forms of the instrument as well as scoring instructions can be found at:

[http://www.coe.ohio-state.edu/ahoy/researchinstruments.htm](http://www.coe.ohio-state.edu/ahoy/researchinstruments.htm)

Best wishes in your work,

Anita Woolfolk Hoy, Ph.D.
Professor
Appendix F: Sample GAST Items

1. Identify four ways a teacher can use these images of pyramids to introduce the topics of surface area and volume. (Knowledge of Content and Teaching/Knowledge of Content and Students)

2. In promoting reasoning about triangle congruence, a teacher might do the following classroom activities with students: (Knowledge of Content and Teaching)

   1. Have students draw a triangle with two sides of given length and included angle of given measure. Have students cut out the triangle and compare it to other students’ triangles and describe what they notice about the triangles.

   2. Draw two congruent triangles and ask students to explore the triangles to discover the relationship between the triangles.

   3. Have students explore pairs of triangles to identify which three corresponding congruent measurements guarantee triangle congruence.

   4. Draw two congruent triangles on a coordinate grid, and ask students to measure the length of three sides in one triangle and the corresponding sides in the other triangle to illustrate SSS congruence. Ask students to describe the relationship about the corresponding angles of the triangles.

   How should these activities be ordered so that they represent a developmentally appropriate learning sequence for students?

   A. 1, 2, 3, then 4
   B. 2, 4, 1, then 3
   C. 3, 2, 4, then 1
   D. 4, 1, 2, then 3
Appendix G: Focus Group Questions

**Introductory Questions**

Tell us who you are, what grade level you teach, and what you enjoy doing when you are out of school.

What are your general thoughts about professional development?

**Transition Question**

Speak about a professional development experience (other than this one) that you will always remember? (This can be a negative or positive experience.)

**Focus Questions**

What is one thing that you found different from this current professional development experience than from other professional development experiences you have had.

What strengths do you see in a blended learning type of professional development delivery for teachers?

Has this professional development experience enhanced your ability to collaborate with other teachers? Please explain.

How has this professional development delivery help you use and maintain the strategies and principles that were taught during the class?

**Summary Question**

Are there any other points about the professional development experience that haven’t been discussed.
Appendix H: Sample Domain Probe

Co-Teaching Items

Label the following co-teaching strategies as:

\[1/1 = \text{One Teach/One Support}\]
\[\text{PT} = \text{Parallel Teaching}\]
\[\text{ST} = \text{Station Teaching}\]
\[\text{AT} = \text{Alternative Teaching}\]
\[\text{TT} = \text{Team Teaching}\]

1. _______ In a history class studying the Civil War, the class is divided into two heterogeneous groups. Teacher A supervises one group in writing letters home to explain to their families why they have joined the Confederate army. Teacher B works with a group completing a mapping outline on reasons for individuals to support the Union. Half way through the class, the groups rotate.

2. _______ In an English class, Teacher B leads the class in an activity on correct punctuation in letter writing. Teacher A observes two students that have problems completing their work in order to collect data for the teacher’s afternoon planning session.

Multiple Choice

1. Teachers need to collaborate when developing
   a. accommodations.
   b. modifications.
   c. lesson planning.
   d. All the answers are correct.

2. In co-teaching, the teachers need to
   a. agree on and understand each other's role.
   b. communicate only occasionally with each other.
   c. choose one teacher to be in charge.
   d. have separate groups of students for whom they are responsible.

3. Who is ultimately responsible for all the students in a collaborative teaching model?
   a. the special education teacher
   b. the general education teacher
   c. the paraprofessional assigned to a student with disabilities
   d. the general and special education teachers

4. Which of the following is an important component of co-teaching?
   a. being friends with your co-teacher
   b. when both teachers have been teaching for 5 or more years
   c. interpersonal communication
   d. having identical educational philosophies.

5. Which model of co-teaching is where both teachers share the same space and students, and they share responsibilities, but one teacher teaches while the other teacher supports the students by wandering among the students and monitoring their work and behavior?
UDL Questions

1. Which brain network helps us identify this image as a table?
   a. Recognition
   b. Strategic
   c. Affective
   d. None of the above

2. If the assignment is to read Cask of Amontillado, what is an example of integrating UDL into the lesson to support many learners?
   a. Provide the text in at least three languages
   b. Provide a SMART board, a document camera, and multiple iPads in the classroom.
   c. Provide a Co-Teacher to read the text aloud to any of the students who would like that as a support.
   d. Provide a paper copy, a digital copy with text-to-speech software, and a closed caption video

3. The ________ network of the brain enable us to plan, execute, and monitor actions and skills.
   a. recognition
   b. strategic
   c. affective
   d. reflective

Online discussion questions (graded with a rubric)

1. How can using a variety of materials and methods reach more of the students within your classroom? What are the benefits of doing so? What are the challenges?

2. What barriers are inherent in traditional assessments? What are the challenges in offering varied options for assessment?
Appendix I: Sample Objective Probe

Multiple Choice

1. Which of the following is an important component of co-teaching?
   a. being friends with your co-teacher
   b. when both teachers have been teaching for 5 or more years
   c. interpersonal communication
   d. having identical educational philosophies.

2. Which model of co-teaching is where both teachers share the same space and students, and they share responsibilities, but one teacher teaches while the other teacher supports the students by wandering among the students and monitoring their work and behavior?
   a. parallel teaching
   b. station teaching
   c. alternative teaching
   d. one teach, one drift

3. Which model of co-teaching is where both teachers share the same space, students, and responsibilities, but one teaches a separate heterogeneous group of students (for example, to provide additional instruction on a concept) while the other works with the remaining larger group?
   a. one teach, one observe
   b. shared teaching
   c. alternative teaching
   d. one teach, one drift

Rectangle Problem

Which rectangles are similar to rectangle a? Explain the method you used to decide.
### Appendix J: Course Outline

<table>
<thead>
<tr>
<th>Session/Time</th>
<th>Session Goals</th>
<th>Materials/Tools</th>
<th>Outline/Activities</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 1 (In Person)</td>
<td>Session Goals:</td>
<td>Samples of Coteaching models</td>
<td>Outline</td>
<td>Big Ideas:</td>
</tr>
<tr>
<td>8 hours</td>
<td>• To explore the role of coteaching as a service delivery model.</td>
<td>Virtual manipulatives</td>
<td>• Introduction</td>
<td>1) Coteaching is a service delivery model effective for meeting the needs of all learners.</td>
</tr>
<tr>
<td></td>
<td>• To examine the principles of UDL.</td>
<td></td>
<td>• Goals of the Session</td>
<td>2) Incorporating UDL is aligned with the CCSS.</td>
</tr>
<tr>
<td></td>
<td>• To examine the relationship between UDL and the Common Core Standards for Mathematical Practice</td>
<td></td>
<td>• Introduction to Co-teaching and Incorporating UDL Practices</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>o Teaching in Inclusive Classrooms</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>o Co-teaching Models</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Introduction to UDL and the relationship with CCSS</td>
<td></td>
</tr>
<tr>
<td>Day 2 (In Person)</td>
<td>Session Goals:</td>
<td>Module video clip: “Randy” <a href="http://www.mathedleadership.org/">http://www.mathedleadership.org/</a></td>
<td>Outline</td>
<td>Big Ideas:</td>
</tr>
<tr>
<td>8 hours</td>
<td>• Examine the meaning of defining congruence and similarity through transformations as articulated in the Common Core State Standards.</td>
<td></td>
<td>• Congruence and Similarity Through Transformations</td>
<td>To gain an understanding of the CCSS 8th grade transformation based definition of congruence and similarity as well as the mathematical practices that deal with precise language, viable arguments, appropriate tools, and structure.</td>
</tr>
<tr>
<td></td>
<td>• Examine the use of precise language, viable arguments, appropriate tools, and geometric structure</td>
<td></td>
<td>• Application of the Common Core Standards</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Incorporating Mathematical Practices</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Video Case Study</td>
<td></td>
</tr>
</tbody>
</table>

Day 3 (In Person)
<table>
<thead>
<tr>
<th>Day</th>
<th>Session Goals:</th>
<th>Lessonsketch online interface</th>
<th>N/A</th>
<th>Big Idea: Lessonsketch is an effective media authoring tool that facilitates coplanning for general and special ed teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 4</td>
<td><strong>Session Goals:</strong></td>
<td></td>
<td></td>
<td><strong>Big Idea:</strong> Lessonsketch is an effective media authoring tool that facilitates coplanning for general and special ed teachers</td>
</tr>
<tr>
<td>(Online)</td>
<td>• Respond to animated lesson via discussion board.</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>• Develop a lesson plan using the Lessonsketch interface</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Day 5</td>
<td><strong>Session Goals:</strong></td>
<td></td>
<td></td>
<td><strong>Big Idea:</strong> Lessonsketch is an effective media authoring tool that facilitates coplanning for general and special ed teachers</td>
</tr>
<tr>
<td>(Online)</td>
<td>• Respond to animated lesson via discussion board.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Develop a lesson plan using the Lessonsketch interface</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Day 6</td>
<td><strong>Session Goals:</strong></td>
<td>Teacher Sense of Efficacy Scale</td>
<td>Review of material and lesson plans</td>
<td>Assessments given: GAST TSES (Short Form)</td>
</tr>
<tr>
<td>(In Person)</td>
<td><strong>Session Goals:</strong></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>• Respond to animated lesson via discussion board.</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>• Develop a lesson plan using the Lessonsketch interface</td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td>• Review lessons created by teachers</td>
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<tr>
<td></td>
<td>• Complete survey</td>
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</tbody>
</table>
### Appendix K: Sample Lesson Plan

**Illustrating the Standards for Mathematical Practice: Congruence and Similarity through Transformations**

**FACILITATOR'S AGENDA**

**Session 1: Congruence and Similarity through Transformations**

**2 hours**

**Session Goals:**
- To explore the Standards for Mathematical Content and Practice through video of classroom practice
- To consider how the Standards are likely to impact your mathematics program and to plan next steps
- To examine congruence and similarity defined through transformations
- To examine the use of precise language, viable arguments, appropriate tools, and geometric structure

**What is the point of this session?**
To gain an understanding of the CCSS 8th grade transformation based definition of congruence and similarity as well as the mathematical practices that deal with precise language, viable arguments, appropriate tools, and structure.

*The Notes section of each session agenda includes guidance and notes to support you in facilitating the activities. P Indicates mathematics content information and support.*

<table>
<thead>
<tr>
<th>Time</th>
<th>Materials</th>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 min</td>
<td>PPT slides 1-5 Standards for Mathematics Practice Handout</td>
<td>Introduction &amp; Goals</td>
<td>Welcome the group and introduce the session, noting that it is situated in both the content and practice standards included in the CCSS (slide 2). Share the goals of the session (slide 3) and explain that the session will explore both types of standards through video of classroom practice. If the participants do not already know one another, also take this opportunity for everyone to introduce himself or herself. Distribute the Standards for Mathematics Practice Handout. Review the 2 slides (slides 4 &amp; 5) on the Standards for Mathematics Practice, and ask if the participants have questions.</td>
</tr>
</tbody>
</table>

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NCSM Module 1 Congruence & Similarity through Transformations

Learning and Teaching Geometry Project, WestEd
<table>
<thead>
<tr>
<th>25 min</th>
<th>PPT slides 6-14 Static &amp; Transformation-Based Conceptions Handout</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Defining Congruence &amp; Similarity through Transformations</strong></td>
</tr>
<tr>
<td></td>
<td>- Introduction to Congruence &amp; Similarity through Transformations (slide 6)</td>
</tr>
<tr>
<td></td>
<td>- Definition of Congruence &amp; Similarity</td>
</tr>
<tr>
<td></td>
<td>- Reflective writing: Your definitions of congruence &amp; similarity (slide 7)</td>
</tr>
<tr>
<td></td>
<td>- Definitions used in the standards (slide 8)</td>
</tr>
<tr>
<td></td>
<td>- Static and Transformation-Based Conceptions</td>
</tr>
<tr>
<td></td>
<td>- Static &amp; transformation-based conceptions of similarity (slides 9-10)</td>
</tr>
<tr>
<td></td>
<td>- Static &amp; transformation-based conceptions handout (slide 11)</td>
</tr>
<tr>
<td></td>
<td>- Categorize your definitions (slide 12)</td>
</tr>
<tr>
<td></td>
<td>- Standards for Mathematics Content &amp; Practice (slides 13-14)</td>
</tr>
</tbody>
</table>

|        | **Definition of Congruence & Similarity (slides 7-8)** |
|        | Have the participants individually reflect on and write about their definitions of congruence and similarity. Do not have them share their definitions at this point. They will share and classify their definitions after they discuss static & transformation-based conceptions (see slide 12). |

|        | Then go over the definitions of congruence & similarity used in the standards. Note that the sequences of transformations pictured in slide 8 are not the only sequences possible to establish the congruence and similarity of figures. |

|        | **Static & Transformation-based Conceptions of Similarity (slides 9-12)** |
|        | There are various accurate conceptions of similarity. These conceptions can be categorized as either static or transformation-based. Static conceptions are likely to be more familiar, whereas transformation-based conceptions are closely aligned with those used in the standards. |

|        | Note that Slide 9 has two parts. Click ONE TIME to show a comparison of the ratios of corresponding parts between two geometric figures. Click ONE MORE TIME to show the comparison between the ratio of two parts within a geometric figure and the corresponding ratio within another figure. |

|        | Slide 9 illustrates a traditional, static presentation of similarity, which is a comparison of two discrete figures. Our experiences with similarity as learners and as teachers have most likely been from a static/discrete perspective—examining two or more figures as discrete and comparing corresponding side lengths and angle measures. This differs from the Common Core State Standards transformation-based definition which utilizes a dynamic, continuous perspective. |

|        | The given triangles share two different relationships; namely, a "between-figure" relationship and a "within-figure" relationship. The "between-figure" relationship reveals that the pairs of corresponding sides are in the same proportion. The "within-figure" relationship reveals that the ratios of lengths within a figure are equal to ratios of corresponding lengths in a similar figure. The within-figure relationship is sometimes referred to as the aspect ratio and the between-figure ratio |

NCSM Module 1 Congruence & Similarity through Transformations

Learning and Teaching Geometry Project, WestEd
is the scale factor.

Slide 10 contains an animation. Click ONCE to begin the animation that will run through the creation of several similar triangles.

Slide 10 illustrates a transformation-based perspective, in which the focus is on enlarging & reducing figures proportionally to create a class of similar figures. Slide 10 starts with the same 1/3 triangle that we just encountered in Slide 9, and when it passes through the similar 2/3 triangle from Slide 9 that triangle turns purple to match the one from Slide 9. The animation highlights the fact that this discrete triangle belongs to an infinite class of similar triangles, which include all of the ones for which the scale factor and side lengths are named in the animation, but also all of the triangles in between.

In response to the question "What do you notice about the geometric structure of the triangles," teachers might notice:
- All of the triangles are part of a class of similar triangles.
- As the scale factor changes, the height and base change proportionally.
- We can still examine the within and between relationships among the triangles.
- The triangles are dilation images.
- The triangles are bounded by the 2 given dilation lines (the hypotenuses & the bottom legs).
- The angles remain constant (i.e., their measures are preserved)

If teachers are not sure how to answer the question, ask them to list anything they notice or wonder about the set of triangles that the animation shows, and then point out the ways that they are attending to the geometric structure of those triangles. For example, the relationships between parts of the triangles, the relationships among the set of triangles, and specific geometric features of the triangles (such as the angles or dilation lines used to create them) are all parts of the geometric structure of the set of triangles.

Slide 11 defines the terms "static" and "transformation-based," and provides a summary of these 2 conceptions of similarity. There is also a handout that matches this slide, which may be helpful to teachers as a reference: the Static & Transformation-Based Conceptions Handout.
| min | PPT Slide 15-16 | Rectangle problem handout | Hannah's Rectangle Problem (Slide 15-16 and handout) | Working on the Task
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>Rectangle problem handout</td>
<td>Tracing paper,</td>
<td>Have teachers work on Hannah's Rectangle Problem individually or in small groups, and then discuss it as a whole group using the focus questions on slide 16. Have tracing paper, rulers, and scissors available. Try not to allot more than 10 minutes for teachers to solve the problem.</td>
</tr>
</tbody>
</table>

Note that the mathematical practice of “look for and make use of structure” emerges in this slide as geometric structure can be applied here, especially to the transformation-based conception.

Slide 12 prompts teachers to share and categorize the definitions they wrote for congruence & similarity. Starting with congruence, have teachers share, classify, & provide rationale for each idea as static or transformation-based. Then do the same for similarity.

Be sure to prompt for teachers’ reasoning behind their classifications. For example, are their definitions focused on numeric or geometric relationships? Do their definitions attend to discrete figures or a class of similar figures? It is possible that some definitions may not contain enough information to readily classify. You might want to add a “not sure” category.

It might be helpful to do this exercise on chart paper (using two charts— one for congruence and one for similarity) This allows for a public record of teachers’ definitions, which we will return to at the end of the session. It is not necessary to discuss accuracy or precision of the definitions at this point, that will come at the end of the session.

Standards for Mathematics Content & Practice (slides 13-14)
The 8th grade geometry content standards that define congruence and similarity from a transformation-based perspective frame the mathematical work in this session. Of particular relevance are the transformation-based definitions of congruence and similarity. Other content standards are also connected such as 7.G.1 “Solve problems involving scale drawings of geometric figures.”

Of the 8 standards for mathematical practice, the 4 that we will highlight in the remainder of this session are: constructing viable arguments & critiquing the reasoning of others, using appropriate tools strategically, attending to precision, and making use of structure.
Appendix L: Fidelity of Treatment Checklist

Name of Observer __________________________________________________ Date(s) Observed ____________
Time(s) Observed __________________________________________ Total Elapsed Time Observed ________

Directions: Indicate your response to the observed behaviors by shading in the circles under “Yes” or “No”.

<table>
<thead>
<tr>
<th>The professional development provider:</th>
<th>Observed?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Preparation**

1. Provides a description of the training with learning goals prior to training. 0 0 0
2. Provides readings, activities, and/or questions to think about prior to the training. 0 0 0

**Notes:**

**Introduction**

3. Provides an agenda before or at the beginning of the training. 0 0 0
4. Connects content to participants’ context (e.g., community, school, district, state). 0 0 0
5. Includes the empirical research foundation of the content (e.g., citations, verbal references to research literature, key researchers). 0 0 0
6. Engages the participant in a preview of the content (e.g., material knowledge or practice). 0 0 0
7. Builds on or relates to participants’ previous professional development. 0 0 0
8. Aligns with school/district/state standards or goals. 0 0 0
9. Emphasizes improving student learning outcomes. 0 0 0

**Notes:**

**Demonstration**

10. Builds shared vocabulary required to implement and sustain the practice. 0 0 0
11. Provides examples, demonstrates, or otherwise illustrates the content/practice. 0 0 0
12. Illustrates the use or applicability of the material, knowledge or practice for the participant. 0 0 0

**Notes:**

**Engagement**
<table>
<thead>
<tr>
<th>13. Includes opportunities for participants to practice and/or rehearse new skills.</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>14. Includes opportunities for participants to express personal perspectives (e.g., experience, thoughts on concept).</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15. Includes opportunities for participants to interact with each other related to training content.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16. Adheres to agenda and time constraints.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes:

**Evaluation**

<table>
<thead>
<tr>
<th>17. Includes opportunities for participants to reflect on learning.</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. Includes discussion of specific indicators—related to the knowledge, material, or skills provided by the training—that would indicate a successful transfer to practices.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>19. Engages participants in assessment of their acquisition of knowledge and skills.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Mastery**

<table>
<thead>
<tr>
<th>20. Includes follow-up activities that require participants to apply their learning in a new setting or context.</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>21. Provides continued feedback through technical assistance and resources.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>22. Includes coaching to improve fidelity of implementation.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes:

Appendix M: Social Validity Measure

Please read each statement below and circle your response.

<table>
<thead>
<tr>
<th><strong>In-person Sessions:</strong></th>
<th><strong>Strongly Disagree</strong></th>
<th><strong>Disagree</strong></th>
<th><strong>Neutral</strong></th>
<th><strong>Agree</strong></th>
<th><strong>Strongly Agree</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The professional development content was relevant and useful</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>The activities were engaging and connected to the professional development objectives</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>I gained knowledge about Universal Design for Learning concepts</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>I gained knowledge about Universal Design for Learning concepts in my lessons</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>I gained knowledge about similarity and congruency concepts in geometry</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>I will be able to apply the strategies discussed in this professional development immediately in my teaching</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>The video case studies were effective for learning about teaching geometry concepts</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Online Sessions:</strong></th>
<th><strong>Strongly Disagree</strong></th>
<th><strong>Disagree</strong></th>
<th><strong>Neutral</strong></th>
<th><strong>Agree</strong></th>
<th><strong>Strongly Agree</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The animated representations of teaching were effective for facilitating pedagogical discussions</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>The online activities were effective for facilitating collaboration</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
Open Response Questions:

What did you like best about the intervention?

What did you like least about the intervention?

How would you compare the in-person sessions and the online sessions in terms of preference?

Please provide any other specific comments about the intervention:
References


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