This dissertation describes a study of Raman coherence effects using superconducting quantum circuits. Raman coherence can occur in a three-level system driven by two coherent electromagnetic fields. In a suitable system with a metastable state, the effect is typically manifest as coherent population trapping (CPT) and electromagnetically induced transparency (EIT). I derive the theoretical framework and show experimentally that in the case of a cascade three-level system based on transmon superconducting qubit states, an effect known as the Autler-Townes doublet (ATD), rather than CPT or EIT, occurs. I propose, model, and implement a quasi-Λ system made of combined transmon-cavity levels, which has a meta-stable state required for CPT and EIT. I measure CPT, and demonstrate coherence of the dark state in the time domain. Instead of EIT, I observe a new phenomenon – electromagnetically suppressed transmission (EST). The large negative dispersion accompanying EST leads to superluminal pulse propagation in the system. My results suggest that quantum superconducting circuits provide a viable platform for studying quantum optics of multi-level systems.
RAMAN COHERENCE EFFECTS IN A SUPERCONDUCTING JAYNES-CUMMINGS SYSTEM

by
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<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3.3 Hamiltonian, mode, and reservoir engineering</td>
<td>10</td>
</tr>
<tr>
<td>1.4 Overview of the thesis</td>
<td>10</td>
</tr>
<tr>
<td><strong>2 Theory</strong></td>
<td>13</td>
</tr>
<tr>
<td>2.1 Transmon theory</td>
<td>14</td>
</tr>
<tr>
<td>2.2 Coupling a transmon to a cavity – the Jaynes-Cummings Hamiltonian</td>
<td>23</td>
</tr>
<tr>
<td>2.3 Cascade and ( \Lambda ) systems</td>
<td>26</td>
</tr>
<tr>
<td>2.3.1 Driven Hamilton</td>
<td>27</td>
</tr>
<tr>
<td>2.3.2 Adding decoherence</td>
<td>31</td>
</tr>
<tr>
<td>2.3.3 Dark state fidelity</td>
<td>36</td>
</tr>
<tr>
<td>2.4 Quasi-( \Lambda ) system</td>
<td>39</td>
</tr>
<tr>
<td>2.4.1 Model assumptions</td>
<td>40</td>
</tr>
<tr>
<td>2.4.2 Driven Hamilton</td>
<td>43</td>
</tr>
<tr>
<td>2.4.3 Adding decoherence</td>
<td>43</td>
</tr>
<tr>
<td>2.4.4 Dark state fidelity</td>
<td>45</td>
</tr>
<tr>
<td><strong>3 Device design and fabrication</strong></td>
<td>48</td>
</tr>
<tr>
<td>3.1 Cavity</td>
<td>48</td>
</tr>
<tr>
<td>3.1.1 Design considerations</td>
<td>49</td>
</tr>
<tr>
<td>3.1.2 Improving the internal quality factor of cavities</td>
<td>55</td>
</tr>
<tr>
<td>3.1.3 Summary of cavity parameters</td>
<td>56</td>
</tr>
<tr>
<td>3.2 Qubit</td>
<td>58</td>
</tr>
<tr>
<td>3.2.1 Design considerations</td>
<td>58</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.2.2 Fabrication</td>
<td>62</td>
</tr>
<tr>
<td>3.2.3 Summary of transmon devices built</td>
<td>68</td>
</tr>
<tr>
<td>3.3 Full-system simulation with black-box quantization</td>
<td>71</td>
</tr>
<tr>
<td>4 Experimental setup</td>
<td>75</td>
</tr>
<tr>
<td>4.1 Leiden dilution refrigerator cryogenic setup</td>
<td>76</td>
</tr>
<tr>
<td>4.1.1 Filtering and isolation</td>
<td>78</td>
</tr>
<tr>
<td>4.2 Continuous-wave transmission measurements</td>
<td>80</td>
</tr>
<tr>
<td>4.3 Spectroscopic measurements</td>
<td>81</td>
</tr>
<tr>
<td>4.4 Time-domain measurements</td>
<td>85</td>
</tr>
<tr>
<td>4.5 Nanosecond vector pulse control</td>
<td>85</td>
</tr>
<tr>
<td>4.5.1 DAC specifications</td>
<td>86</td>
</tr>
<tr>
<td>4.5.2 Basic commands</td>
<td>88</td>
</tr>
<tr>
<td>4.5.3 LabVIEW implementation</td>
<td>91</td>
</tr>
<tr>
<td>4.5.4 IQ mixer</td>
<td>92</td>
</tr>
<tr>
<td>4.5.5 Experimental integration</td>
<td>103</td>
</tr>
<tr>
<td>4.6 Pulsed transmission measurements</td>
<td>107</td>
</tr>
<tr>
<td>4.7 Signal demodulation and acquisition</td>
<td>108</td>
</tr>
<tr>
<td>5 Characterization of transmons</td>
<td>110</td>
</tr>
<tr>
<td>5.1 High-power readout</td>
<td>110</td>
</tr>
<tr>
<td>5.2 Spectroscopy</td>
<td>114</td>
</tr>
</tbody>
</table>
# Table of Contents

5.3 Relaxation, Rabi, Ramsey, and Spin-echo .......................... 118  
  5.3.1 Relaxation measurements ........................................ 119  
  5.3.2 Rabi measurements ............................................. 121  
  5.3.3 Ramsey measurements ........................................... 123  
  5.3.4 Spin-echo measurements ....................................... 125  
5.4 State tomography ...................................................... 126  
5.5 Conclusions ............................................................ 130  

6 Observation of Autler-Townes doublet in a 3D transmon 131  
  6.1 Introduction .......................................................... 131  
  6.2 Device and setup ................................................... 132  
  6.3 Modeling ............................................................. 134  
  6.4 High-power readout ................................................ 136  
  6.5 System characterization .......................................... 137  
  6.6 ATD data .............................................................. 141  
  6.7 Conclusions .......................................................... 146  

7 Raman coherence in a cavity-transmon \( \Lambda \) system 148  
  7.1 Introduction .......................................................... 148  
  7.2 Device and setup ................................................... 149  
  7.3 Modeling ............................................................. 154  
  7.4 Coherent population trapping .................................... 155  
  7.5 Dark state coherence .............................................. 159
# Table of Contents

7.6 Dressed state blockade .................................................. 163
7.7 3-photon Rabi oscillations ............................................. 165
7.8 Electromagnetically suppressed transmission ..................... 166
7.9 Superluminal pulse propagation ....................................... 172
7.10 Conclusions .............................................................. 175

8 Conclusions ................................................................. 177
8.1 Summary of main findings .............................................. 177
8.2 State stabilization ....................................................... 179
8.3 Off-resonant control ..................................................... 179
8.4 EIT with a notch-style cavity .......................................... 181
8.5 Raman coherence in the ultra-strong coupling limit ............... 182

Appendix A Fabrication recipes ........................................... 183
A.1 Cavity cleaning, etching, and polishing ............................... 183
   A.1.1 Recipe 1 .......................................................... 183
   A.1.2 Recipe 2 .......................................................... 184
   A.1.3 Recipe 3 .......................................................... 184
A.2 Optical lithography ..................................................... 185
   A.2.1 Spinning resist ................................................... 185
   A.2.2 Development ....................................................... 185
   A.2.3 Lift-off ............................................................ 185
# Table of Contents

A.3 Electron-beam lithography ........................................... 186
  A.3.1 Spinning resist .................................................... 186
  A.3.2 Development ...................................................... 186
  A.3.3 Lift-off ............................................................ 187

Appendix B Leiden CF-450 operation ................................. 188
  B.1 Cooldown ............................................................ 188
    B.1.1 Pumping out the dilution unit and the traps .............. 188
    B.1.2 Pumping out the OVC and the IVC ......................... 188
    B.1.3 Pre-cooling to 77 K ........................................... 189
    B.1.4 Cooling down to 3 K ......................................... 189
    B.1.5 Condensing the mixture .................................... 189
  B.2 Warmup .............................................................. 190
    B.2.1 Recovering the mixture .................................... 190
    B.2.2 Warming up to 300 K ....................................... 190
    B.2.3 Cleaning the nitrogen traps ............................... 190

Appendix C E-beam exposure matrices ............................... 191

Bibliography ............................................................... 199
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Variations of a driven three-level system.</td>
<td>4</td>
</tr>
<tr>
<td>2.1</td>
<td>Schematic of an Al/AlO$_x$/Al Josephson junction.</td>
<td>15</td>
</tr>
<tr>
<td>2.2</td>
<td>Schematic of the transmon.</td>
<td>16</td>
</tr>
<tr>
<td>2.3</td>
<td>Energy bands of the Cooper-pair box as $E_J/E_C$ is increased.</td>
<td>20</td>
</tr>
<tr>
<td>2.4</td>
<td>Energy dispersion and anharmonicity of the transmon.</td>
<td>21</td>
</tr>
<tr>
<td>2.5</td>
<td>Conceptual diagram of a Jaynes-Cummings system.</td>
<td>24</td>
</tr>
<tr>
<td>2.6</td>
<td>Dispersive shift of the cavity.</td>
<td>25</td>
</tr>
<tr>
<td>2.7</td>
<td>Energy level diagram for cascade and $\Lambda$ systems.</td>
<td>27</td>
</tr>
<tr>
<td>2.8</td>
<td>Simulation results of the cascade system with a metastable $</td>
<td>2\rangle$.</td>
</tr>
<tr>
<td>2.9</td>
<td>Simulation results of the cascade system with a fast-decaying $</td>
<td>2\rangle$.</td>
</tr>
<tr>
<td>2.10</td>
<td>Simulation results of the $\Lambda$ system with a metastable $</td>
<td>2\rangle$.</td>
</tr>
<tr>
<td>2.11</td>
<td>Simulation results of the $\Lambda$ system with a fast decaying $</td>
<td>2\rangle$.</td>
</tr>
<tr>
<td>2.12</td>
<td>Dark state fidelity simulation of cascade and $\Lambda$ systems with a metastable and a fast decaying $</td>
<td>2\rangle$.</td>
</tr>
<tr>
<td>2.13</td>
<td>Level diagram for the quasi-$\Lambda$ system.</td>
<td>40</td>
</tr>
<tr>
<td>2.14</td>
<td>Simulation results of the quasi-$\Lambda$ system with a metastable state $</td>
<td>2\rangle$.</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>2.15</td>
<td>Simulation results of the quasi-Λ system with a fast-decaying state</td>
<td>45</td>
</tr>
<tr>
<td>2.16</td>
<td>Dark state fidelity simulation of Λ and quasi-Λ systems with a metastable</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>or fast-decaying</td>
<td>2</td>
</tr>
<tr>
<td>2.17</td>
<td>Simulated dark state fidelity ( F_D ) for the maximum superposition state in</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>the quasi-Λ systems versus ( \Omega_p/(2\pi \times \Gamma) ) and ( \kappa/(2\pi \times \Gamma) ).</td>
<td></td>
</tr>
<tr>
<td>3.2</td>
<td>HFSS simulation of the electric field inside the cavity.</td>
<td>51</td>
</tr>
<tr>
<td>3.3</td>
<td>Two schemes of mounting a transmon inside a cavity.</td>
<td>52</td>
</tr>
<tr>
<td>3.4</td>
<td>Evolution of my transmon designs.</td>
<td>61</td>
</tr>
<tr>
<td>3.5</td>
<td>Junction fabrication steps.</td>
<td>62</td>
</tr>
<tr>
<td>3.6</td>
<td>Optical image of the junction after development.</td>
<td>65</td>
</tr>
<tr>
<td>3.7</td>
<td>SEM images of the junction before lift-off.</td>
<td>66</td>
</tr>
<tr>
<td>3.8</td>
<td>Images of the junction after lift-off.</td>
<td>67</td>
</tr>
<tr>
<td>4.1</td>
<td>Photographs of the dilution refrigerator setup.</td>
<td>77</td>
</tr>
<tr>
<td>4.2</td>
<td>Schematic of the microwave lines in the Leiden refrigerator.</td>
<td>78</td>
</tr>
<tr>
<td>4.3</td>
<td>Schematic of the transmission measurement setup with a VNA.</td>
<td>80</td>
</tr>
<tr>
<td>4.4</td>
<td>Schematic of the spectroscopy and typical time domain measurements</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>setup.</td>
<td></td>
</tr>
<tr>
<td>4.6</td>
<td>Schematic of the DAC setup for spin-echo and tomography measurements.</td>
<td>86</td>
</tr>
<tr>
<td>4.7</td>
<td>Schematic of main DAC components.</td>
<td>87</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.8</td>
<td>LabVIEW &quot;code&quot; for Write_DAC_reg subVI.</td>
<td>92</td>
</tr>
<tr>
<td>4.9</td>
<td>Mixer calibration setup.</td>
<td>94</td>
</tr>
<tr>
<td>4.10</td>
<td>Localization of ((I_0, Q_0)) offset point for the IQ mixer.</td>
<td>95</td>
</tr>
<tr>
<td>4.11</td>
<td>LO leakage through an un-calibrated and calibrated IQ mixers.</td>
<td>97</td>
</tr>
<tr>
<td>4.12</td>
<td>Polar plot of (R_{out}(\phi_{in})) before and after IQ mixer circle correction.</td>
<td>100</td>
</tr>
<tr>
<td>4.13</td>
<td>Deviation from perfect circle for the calibrated mixer in the band of interest.</td>
<td>100</td>
</tr>
<tr>
<td>4.14</td>
<td>IQ mixer circle linearity with input power.</td>
<td>101</td>
</tr>
<tr>
<td>4.15</td>
<td>PCB drawing of the PECL-to-TTL converter.</td>
<td>104</td>
</tr>
<tr>
<td>4.16</td>
<td>High-frequency feedthrough of the IQ mixer.</td>
<td>105</td>
</tr>
<tr>
<td>4.17</td>
<td>Photograph of the vector pulse shaping setup.</td>
<td>106</td>
</tr>
<tr>
<td>4.18</td>
<td>Schematic of the setup used to measure propagation of Gaussian-modulated coupler pulses in presence of a probe tone.</td>
<td>107</td>
</tr>
<tr>
<td>4.19</td>
<td>Schematic of the demodulation setup used to mix down the measurement signal.</td>
<td>108</td>
</tr>
<tr>
<td>5.1</td>
<td>Cavity transmission versus drive frequency and power for different prepared transmon states.</td>
<td>112</td>
</tr>
<tr>
<td>5.2</td>
<td>Linecuts from Fig. 5.1 showing cavity transmission versus drive frequency and power for different prepared transmon states.</td>
<td>113</td>
</tr>
<tr>
<td>5.3</td>
<td>Transmon spectroscopy measurement.</td>
<td>115</td>
</tr>
<tr>
<td>5.4</td>
<td>Photon number splitting measurement.</td>
<td>117</td>
</tr>
<tr>
<td>5.5</td>
<td>Transmon relaxation measurement.</td>
<td>120</td>
</tr>
<tr>
<td>5.6</td>
<td>Rabi oscillations measurement.</td>
<td>122</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.7</td>
<td>Ramsey oscillations measurement.</td>
<td>124</td>
</tr>
<tr>
<td>5.8</td>
<td>Spin-echo measurement.</td>
<td>125</td>
</tr>
<tr>
<td>5.9</td>
<td>State tomography measurement.</td>
<td>129</td>
</tr>
<tr>
<td>6.1</td>
<td>Optical micrograph and energy diagram of the transmon in the ATD experiment.</td>
<td>133</td>
</tr>
<tr>
<td>6.2</td>
<td>High-power readout and spectroscopy of $\omega_{01}$ and $\omega_{12}$ in the ATD experiment.</td>
<td>137</td>
</tr>
<tr>
<td>6.3</td>
<td>Relaxation and Ramsey decay measurements for device C3DQ29.</td>
<td>139</td>
</tr>
<tr>
<td>6.4</td>
<td>Data and simulations of the ATD for swept coupler and probe frequencies and different coupler powers.</td>
<td>142</td>
</tr>
<tr>
<td>6.5</td>
<td>Data and simulations of the ATD for zero coupler detuning, swept probe frequencies and different coupler powers.</td>
<td>144</td>
</tr>
<tr>
<td>6.6</td>
<td>Dark state fidelity inferred from simulations versus $\Omega_c/\Omega_p$.</td>
<td>145</td>
</tr>
<tr>
<td>7.1</td>
<td>Experimental arrangement for CPT measurements and energy levels for the quasi-$\Lambda$ system.</td>
<td>150</td>
</tr>
<tr>
<td>7.2</td>
<td>Coherent population trapping measurement and simulation.</td>
<td>156</td>
</tr>
<tr>
<td>7.3</td>
<td>Simulated dark state populations and coherences.</td>
<td>158</td>
</tr>
<tr>
<td>7.4</td>
<td>Dark state coherence measurement in time domain.</td>
<td>160</td>
</tr>
<tr>
<td>7.5</td>
<td>Dressed state blockade of probe transitions.</td>
<td>164</td>
</tr>
<tr>
<td>7.6</td>
<td>Off-resonant three-photon Rabi oscillation measurement.</td>
<td>166</td>
</tr>
<tr>
<td>7.7</td>
<td>Experimental arrangement for EST measurements for the quasi-$\Lambda$ system.</td>
<td>167</td>
</tr>
</tbody>
</table>
## List of Figures

7.8 EST measurements versus probe detuning. .................................. 168
7.9 EST measurements versus coupler power. ................................. 170
7.10 EST measurement transmission difference and data fits. .......... 171
7.11 Superluminal propagation measurement. ................................. 173

8.1 STIRAP protocol. ............................................................... 180
8.2 Proposed storage of microwave photons in a Λ system. .............. 182

C.1 Optical images of 10µm × 10µm squares written with e-beam at different exposures. .............................................................. 193
C.2 Optical images of 1-junction test patterns written with e-beam at different exposures, from 150µC/cm² to 430µC/cm². ......................... 194
C.3 Optical images of 1-junction test patterns written with e-beam at different exposures, from 450µC/cm² to 730µC/cm². ......................... 195
C.4 Optical images of 2-junction test patterns written with e-beam at different exposures, from 150µC/cm² to 430µC/cm². ......................... 196
C.5 Optical images of 2-junction test patterns written with e-beam at different exposures, from 450µC/cm² to 730µC/cm². ......................... 197
C.6 Optical images of 2-junction test patterns written with e-beam at different exposures, from 750µC/cm² to 1030µC/cm². ......................... 198
List of Tables

3.1 $\text{TE}_{101}$ internal quality factor for aluminum cavities C2A and C3A under various preparation conditions at 300 K. ........................................ 56

3.2 $\text{TE}_{101}$ internal quality factor for aluminum cavities C2A and C3A under various preparation conditions at 350 mK. ............................... 57

3.3 $\text{TE}_{101}$ internal quality factor for aluminum cavities C2A and C3A at 20 mK. 57

3.4 $\text{TE}_{101}$ internal quality factor for OFHC copper cavity C3C under various chemical etches. ................................................................. 57

3.5 Summary of measured transmon and cavity device parameters. ........ 70

3.6 Summary of simulated transmon devices, with a comparison to experimentally extracted values. ................................................................. 74

4.1 Hexadecimal commands for DAC memory operations. ..................... 89

6.1 Parameters for the simulation of ATD data in Fig. 6.5. ..................... 140

7.1 Parameters for the simulation of CPT data in Fig. 7.2 (b)-(e). ............ 158

7.2 Parameters for dark state Ramsey fits in Fig. 7.4. ............................ 162
### List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFG</td>
<td>arbitrary function generator</td>
</tr>
<tr>
<td>ATD</td>
<td>Autler-Townes doublet</td>
</tr>
<tr>
<td>AWG</td>
<td>arbitrary waveform generator</td>
</tr>
<tr>
<td>BBQ</td>
<td>black-box quantization</td>
</tr>
<tr>
<td>CPB</td>
<td>Cooper-pair box</td>
</tr>
<tr>
<td>CPT</td>
<td>coherent population trapping</td>
</tr>
<tr>
<td>cQED</td>
<td>circuit quantum electrodynamics</td>
</tr>
<tr>
<td>DA</td>
<td>differential amplifier</td>
</tr>
<tr>
<td>DAC</td>
<td>digital-to-analog converter</td>
</tr>
<tr>
<td>DAQ</td>
<td>data acquisition</td>
</tr>
<tr>
<td>DIP</td>
<td>dual in-line package</td>
</tr>
<tr>
<td>DR</td>
<td>dilution ratio</td>
</tr>
<tr>
<td>EIA</td>
<td>electromagnetically induced absorption</td>
</tr>
<tr>
<td>EIT</td>
<td>electromagnetically induced transparency</td>
</tr>
<tr>
<td>ESD</td>
<td>electrostatic discharge</td>
</tr>
<tr>
<td>EST</td>
<td>electromagnetically suppressed transmission</td>
</tr>
<tr>
<td>FPGA</td>
<td>field-programmable gate array</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>FWHM</td>
<td>full-width at half-maximum</td>
</tr>
<tr>
<td>HEMT</td>
<td>high electron mobility transistor</td>
</tr>
<tr>
<td>IF</td>
<td>intermediate frequency</td>
</tr>
<tr>
<td>IP</td>
<td>internet protocol</td>
</tr>
<tr>
<td>IVC</td>
<td>inner-vacuum can</td>
</tr>
<tr>
<td>LNA</td>
<td>low-noise amplifier</td>
</tr>
<tr>
<td>LO</td>
<td>local oscillator</td>
</tr>
<tr>
<td>LVDS</td>
<td>low-voltage differential signaling</td>
</tr>
<tr>
<td>NIC</td>
<td>network interface card</td>
</tr>
<tr>
<td>NMP</td>
<td>N-Methyl-2-pyrrolidone</td>
</tr>
<tr>
<td>OFHC</td>
<td>oxygen-free high-conductivity</td>
</tr>
<tr>
<td>OVC</td>
<td>outer-vacuum can</td>
</tr>
<tr>
<td>PECL</td>
<td>positive emitter-coupled logic</td>
</tr>
<tr>
<td>PLL</td>
<td>phase-locked loop</td>
</tr>
<tr>
<td>QED</td>
<td>quantum electrodynamics</td>
</tr>
<tr>
<td>RF</td>
<td>radio frequency</td>
</tr>
<tr>
<td>RPM</td>
<td>revolutions per minute</td>
</tr>
<tr>
<td>SEM</td>
<td>scanning electron microscope</td>
</tr>
<tr>
<td>SMA</td>
<td>SubMiniature version A</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>SRAM</td>
<td>static random-access memory</td>
</tr>
<tr>
<td>TTL</td>
<td>transistor-transistor logic</td>
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</table>
List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>VCO</td>
<td>voltage-controlled oscillator</td>
</tr>
<tr>
<td>VNA</td>
<td>vector network analyzer</td>
</tr>
<tr>
<td>WD</td>
<td>working distance</td>
</tr>
</tbody>
</table>
List of Symbols

\( C_\Sigma \) \hspace{1cm} \text{transmon shunt capacitance}

\( E_C \) \hspace{1cm} \text{transmon charging energy}

\( E_J \) \hspace{1cm} \text{Josephson tunneling energy}

\( \omega_{\text{cav}} \) \hspace{1cm} \text{cavity resonance frequency}

\( \omega_q \) \hspace{1cm} \text{qubit transition frequency}

\( g \) \hspace{1cm} \text{vacuum Rabi splitting}

\( \Omega_c \) \hspace{1cm} \text{coupler amplitude}

\( \Omega_p \) \hspace{1cm} \text{probe amplitude}

\( \mathcal{F}_D \) \hspace{1cm} \text{dark state fidelity}

\( Q_L \) \hspace{1cm} \text{cavity loaded quality factor}

\( Q_{\text{in}} \) \hspace{1cm} \text{cavity external input quality factor}

\( Q_{\text{out}} \) \hspace{1cm} \text{cavity external output quality factor}

\( Q_i \) \hspace{1cm} \text{cavity internal quality factor}

\( T_\phi \) \hspace{1cm} \text{qubit dephasing time}

\( T' \) \hspace{1cm} \text{Rabi decay time of the qubit}

\( a^\dagger \) \hspace{1cm} \text{boson creation operator}
### List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>transmon anharmonicity</td>
</tr>
<tr>
<td>$A_{r}(q)$</td>
<td>characteristic value for the even Mathieu functions with exponent $r$ and parameter $q$</td>
</tr>
<tr>
<td>$a$</td>
<td>boson annihilation operator</td>
</tr>
<tr>
<td>$C_B$</td>
<td>transmon shunt capacitance</td>
</tr>
<tr>
<td>$C_g$</td>
<td>transmon gate capacitance</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Jaynes-Cummings dispersive shift</td>
</tr>
<tr>
<td>$C_J$</td>
<td>junction capacitance</td>
</tr>
<tr>
<td>$</td>
<td>D\rangle$</td>
</tr>
<tr>
<td>$\Delta_c$</td>
<td>coupler detuning</td>
</tr>
<tr>
<td>$\Delta_{gap}$</td>
<td>Al superconducting gap energy</td>
</tr>
<tr>
<td>$\Delta_p$</td>
<td>probe detuning</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>qubit-cavity detuning</td>
</tr>
<tr>
<td>$\mathcal{D}$</td>
<td>Lindblad dissipation operator</td>
</tr>
<tr>
<td>$\epsilon_k$</td>
<td>energy dispersion of $k$-th transmon level</td>
</tr>
<tr>
<td>$\eta_\phi$</td>
<td>maximum phase for IQ mixer</td>
</tr>
<tr>
<td>$\eta_R$</td>
<td>maximum normalized deviation from mean circle radius for IQ mixer</td>
</tr>
<tr>
<td>$f_{IF}$</td>
<td>intermediate mix-down frequency</td>
</tr>
<tr>
<td>$\gamma_\phi$</td>
<td>qubit dephasing rate</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>qubit relaxation rate</td>
</tr>
<tr>
<td>$I_c$</td>
<td>critical current of the junction</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
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<tr>
<td>$n_g$</td>
<td>gate offset charge</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>probe frequency</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density matrix</td>
</tr>
<tr>
<td>$S_{21}$</td>
<td>transmission scattering parameter</td>
</tr>
<tr>
<td>$\sigma_j$</td>
<td>$j$-th Pauli matrix</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>dark state mixing angle</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Rabi coupling</td>
</tr>
<tr>
<td>$R_J$</td>
<td>room-temperature junction resistance</td>
</tr>
<tr>
<td>$T_1$</td>
<td>lifetime of the qubit excited state</td>
</tr>
<tr>
<td>$T_{2}^*$</td>
<td>Ramsey decay time of the qubit</td>
</tr>
<tr>
<td>$T_2$</td>
<td>spin-echo time of the qubit</td>
</tr>
</tbody>
</table>
CHAPTER 1

Introduction

1.1 Motivation

The study of interactions between light and matter has long been one of the major drivers that has produced advances in our understanding of nature. The field of quantum optics involves the physics of light-matter interactions at the most fundamental level \[ CT98; Ger05; Har06 \]. A great deal of physics has been discovered in quantum optics using systems made of naturally-occurring atoms \[ Nob14a; Nob14b; Nob14c; Nob14d \].

In recent years, quantum superconducting circuits have become a rather widely used platform for studying quantum mechanics and, in particular, quantum optics in the microwave range. Because superconducting circuits are fabricated, their Hamiltonians can be designed with a high degree of precision. Dipole moments of superconducting qubits can be made large, allowing for highly non-linear behavior at the single-photon and single-qubit level \[ Ast07; Dep08; Fin08; Hof11; Kya15; Nie10; Sch07; Wal04 \]. Improvements in these devices have enabled physicists to control single low-energy
1.1 Motivation

microwave photons with unprecedented subtlety and accuracy [Bia09a; DiC09; Hof09; Luc08].

Interest in building a quantum computer with superconducting circuits has led to much work on improving the coherence of driven two-level superconducting systems. Although the physics of two-level systems is remarkably rich, there is an even wider range of phenomena associated with quantum systems with three or more levels. Except for a handful of experiments [Abd10; Bau09; Kel10; Nov13; Sil09; Sur13], many quantum effects that can occur in three-level systems driven by two electromagnetic fields have not been studied. Raman coherence [Aga93; Ari96; Bol91; Mar98], an almost ubiquitous phenomenon in three-level atomic systems, had apparently never been explicitly shown with superconducting qubits prior to the work described in this thesis. Given the design flexibility and the potential for strong light-matter coupling, achieving Raman coherence in a superconducting architecture could further the ability to control single microwave photons. It could also lead to interesting new phenomena, previously unseen in other realizations of three-level systems.

My thesis work combines the fields of three-level systems and superconducting circuits, and therefore introductions to both are in order. In Section 1.2, I describe the physics of driven three-level systems and the relevant experiments. In Section 1.3, I present a brief historical perspective on the progress of superconducting qubits. I also review some of the quantum optics experiments with superconducting qubits, outlining past difficulties in achieving Raman coherence. Finally, I briefly review results in engineering both the Hamiltonian and dissipation of superconducting circuits. These results helped inspire
1.2 Quantum effects with three levels and two fields

me to create a superconducting system where Raman coherence was possible.

1.2 Quantum effects with three levels and two fields

1.2.1 The role of coherence

Consider a system consisting of three atomic levels. Two coherent electromagnetic tones (probe and coupler) can drive transitions between the levels in three possible ways: cascade, Λ, and V [see Fig. 1.1 (a)-(c)]. The drives have frequencies $\omega$, amplitudes $\Omega$, and are detuned by $\Delta$ from the respective transitions that they drive.

The behavior of a driven three-level system can be understood using a dressed state picture [CT98; Mar98]. In the simple case of the coupler resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition, the dressed states are symmetric and anti-symmetric superpositions of bare states $|1\rangle$ and $|2\rangle$ [see Fig. 1.1 (d)]. The energy separation of the dressed states is given by the coupler amplitude $\Omega_c$. The probe can drive transitions to these states, as they have contributions from $|1\rangle$. For large $\Omega_c$, the dressed states are well-separated, resulting in no probe absorption in the region of energies between them. This effect is known as the Autler-Townes doublet (ATD) [Aut55], and is a simple consequence of dressing the bare states with the coupler. At small $\Omega_c$, probe absorption at the midpoint between the dressed states can be reduced due to destructive interference of the transition probability amplitudes [Mar98]. The resulting reduction in probe absorption is called electromagnetically induced transparency (EIT) [Bol91; Fle05]. The interference comes from the fact that $|1\rangle$ appears with opposite signs in the dressed states, so $|1\rangle \leftrightarrow |2\rangle$ transition amplitudes contribute with opposite signs to the total probability amplitude.
1.2 Quantum effects with three levels and two fields

Fig. 1.1: (a) Cascade system, (b) Λ-system, and (c) V-system consisting of states $|0\rangle$, $|1\rangle$, and $|2\rangle$ driven by probe (blue) and coupler (red) tones. (d) Dressed state picture when the coupler is resonant with $|1\rangle \leftrightarrow |2\rangle$ transition. Green dashed line represents the midpoint between dressed states where interference can occur.

This phenomenon requires $|2\rangle$ to be metastable, i.e. decaying much slower than other states in the system [AS10; Mar98]. It has been shown that EIT cannot occur in the V system [AS10].

Because EIT relies on the coherence of energy levels, it has been proposed as a sensitive probe of decoherence in superconducting qubits [Mur04]. Unfortunately, unlike naturally-occurring atoms, cascade and Λ superconducting systems do not typically possess a metastable state. Designing a superconducting Λ system was one of the main goals of my thesis.
1.2.2 Coherent population trapping

When a three-level system displays the quantum interference discussed above, it can be driven into a so-called *dark state* [Ste14], a zero-eigenvalue eigenstate of the driven system. In the simple case of resonant probe and coupler ($\Delta_p = \Delta_c = 0$), the dark state $|D\rangle$ is

$$|D(\Theta)\rangle = \cos \Theta |0\rangle - \sin \Theta |2\rangle,$$  \hspace{1cm} (1.1)

where the mixing angle $\Theta = \tan^{-1}(\Omega_p/\Omega_c)$ depends on the probe and coupler amplitudes $\Omega_p$ and $\Omega_c$. The drives create a coherent superposition of $|0\rangle$ and $|2\rangle$ without populating the intermediate state $|1\rangle$. This effect is called coherent population trapping (CPT) [Aga93; Ari96]. For small $\Omega_c$, the system population is inverted into $|2\rangle$ provided that state is metastable. At large $\Omega_c$, in the ATD regime, the system is driven into the ground state $|0\rangle$ regardless of the coherence of the states. For that reason, quantum interference leading to CPT can only be observed in the low-coupler limit [Ani11; AS10].

CPT is a feature of Raman coherence that pertains to the atomic dynamics of the system. As long as $|2\rangle$ is metastable, any superposition $|D(\Theta)\rangle$ can be achieved by varying the ratio of coupler to probe amplitudes. The coherence of the superposition is maintained as long as the drives are present and are coherent. This enables superposition generation and stabilization with only continuous-wave tones, something that is not possible in a two-level system. The only requirement is a cascade or a $\Lambda$ system with a metastable state $|2\rangle$. 
1.2.3 Electromagnetically induced transparency and absorption

When the system is in the dark state, it is rendered transparent to both probe and coupler. At low coupler powers, this phenomenon is called EIT [Bol91; Mar98]. The transparency has been demonstrated in a variety of atomic and other systems [Müc10; SN11; Xu07], and it can be used to control photon propagation properties through a system in situ [Din13; Hei13].

Because EIT arises due to quantum interference of excitation pathways, the transparency window can be much narrower than the overall width of the atomic transition [AS10]. The narrow width is one of the major features that qualitatively distinguishes EIT from ATD [Ani11]. Quantitatively, an EIT spectrum is described by a difference of two Lorentzian profiles, whereas ATD is described by a sum. Although inside the EIT window the transparency is restored, large changes in the absorption coefficient lead to large changes in the refractive index as required by the Kramers-Kronig relations [Big03].

1.2.4 Slow and fast light

The large positive change in the refractive index accompanying EIT means the group velocity $v_g$ of light pulses propagating through the system is reduced. If the pulses are narrow enough in frequency to fit inside the transparency window, they travel through the system undistorted. This feature has been used to slow down light to $v_g = 17 \text{ m s}^{-1}$ [Hau99], as well as to store and retrieve light pulses [Hei13; Liu01; Phi01].

An opposite effect to EIT is electromagnetically induced absorption (EIA) [Lez99]. It
1.3 Advent of circuit QED

is accompanied by large negative change in the refractive index, which had been used to achieve superluminal and negative group velocities [Big03; Chu82; Wan00]. It should be noted that in this case $v_g$ is not representative of the information propagation speed in the system, and neither relativity nor causality is violated [Boy09; Gar98; Geh06; Ste03].

1.3 Advent of circuit QED

1.3.1 Historical perspective on charge qubits

Over the past decade and a half, qubits based on superconducting circuits have become leading candidates for studying quantum mechanics. Much progress has been made since the first observation of quantum oscillations in a Cooper-pair box (CPB) [Nak99]. The CPB was based on charge states that were coupled by the tunneling of Cooper pairs through a Josephson junction. The typical coherence times of early CPB designs were $\sim 1$ ns. By enabling readout of a CPB at a charge-insensitive bias point (the “sweet spot”), a qubit called quantronium was made [Cla08; Vio02]. Since the quantronium could be operated at a bias where it was insensitive to charge to first order, quantum coherence was improved. This allowed an increase in the coherence times to $\sim 0.5 \mu$s.

The next major development in CPBs was to couple the CPB to an on-chip microwave resonator, and use this to achieve a dispersive (non-dissipative) readout [Bla04; Bla07]. This so called circuit quantum electrodynamics (cQED) architecture, similar to cavity QED in atomic physics, also enabled the study of Jaynes-Cummings physics [Jay63; Sho93] with superconducting quantum circuits.
Despite numerous improvements and operation at the charge sweet spot, charge-based qubits such as the CPB and quantronium were still plagued by charge noise. The qubit transition energy could change whenever charges in the surrounding environment moved, making these devices unstable and difficult to measure. To avoid these difficulties, the transmon was invented [Koc07]. It was based on a remarkable fact that reducing the charging energy of a CPB exponentially suppressed charge sensitivity while only polynomially reducing anharmonicity (much needed for qubit operation). Transmons were made by shunting a CPB with a large capacitor to reduce the charging energy, pushing them into the phase qubit regime. With charge noise no longer an issue, their coherence times rose to a few microseconds [Hou09].

It was soon discovered transmon coherence time could be limited by the dielectric loss of the substrate on which the transmon was fabricated [Mar05] and coupling to other modes of the microwave environment [Hou08; Kim11]. The participation ratio of the lossy materials in the electric fields of the transmon shunt capacitor was large. By placing the transmon in a three-dimensional resonant microwave cavity, the fields could be spread out [Pai11]. This reduced the participation ratio, pushing transmon coherence times to \( \sim 100\,\mu\text{s} \) [Rig12]. The trade-off was larger devices but the 3D transmon architecture became a great test-bed for single- and multi-qubit systems because of its long coherence time [Gee13; Pet15; Pol12; Ris13; Sea12; Vij12].
1.3 Advent of circuit QED

1.3.2 Quantum optics with superconducting qubits

With cQED came new physics associated with the Jaynes-Cummings Hamiltonian. The large dipole moment of a transmon allowed for strong resonant \cite{Bis08} and dispersive \cite{Boi09} interactions between qubit excitations and cavity photons. These interactions could be studied at the single-photon and single-qubit level. Experiments demonstrating photon number states of a resonator \cite{Sch07}, photon blockade \cite{Hof11}, creating Fock states \cite{Hof09}, and observing the Jaynes-Cummings non-linearity \cite{Fin08} are representative of what could be achieved with cQED.

Three-level system physics also received attention in the superconducting qubit community. Spectroscopic signatures of CPT were shown with a phase qubit \cite{Kel10}, ATD was demonstrated with transmons \cite{Bau09} and phase qubits \cite{Sil09}, and attempts at EIT were made \cite{Abd10} (although were later deemed inconclusive \cite{Ani11}). Even though the potential for quantum routing and storage of EIT-capable systems was understood, superconducting circuits presented difficulties in creating such systems. Because typical devices like transmons and phase qubits had the level structure and decay times of weakly anharmonic (nearly harmonic) oscillators, creating a metastable excited state by using qubit energy levels alone was impossible. Despite being very flexible in terms of Hamiltonian design, these qubits had well-defined decay rate scaling for each level \cite{Pet15}. It was almost as if nature conspired to ban EIT physics from the realm of superconducting qubits.

Fortunately, the development of superconducting quantum devices and cQED came alongside our ability to design and control various aspects of their energy spectrum, as
well as their electromagnetic and dissipation environments.

### 1.3.3 Hamiltonian, mode, and reservoir engineering

The design of quantronium and transmon qubits came from the understanding of the CPB Hamiltonian and its dependence on charge. Once the ways to decrease or suppress charge sensitivity were understood theoretically, appropriate devices were implemented.

The understanding of the dielectric [Mar05] and microwave environment [Kim11] loss as a major contributor to qubit relaxation led to the design of the 3D transmon architecture. The 3D cavities presented a well-defined electromagnetic environment while reducing the dielectric participation ratio.

Recently, results of reservoir engineering [Poy96] have been adapted to superconducting circuits as well [Ino14; Mur12; Sha13]. Dissipation was set up and used as a resource, providing yet another degree of control.

Combining these three approaches led me to create a superconducting system with a metastable excited state that can display three-level physics of quantum interference.

### 1.4 Overview of the thesis

Although a three-level system with a metastable excited state does not occur “naturally” in superconducting qubits, combining two or more superconducting quantum circuits can increase the number of independent parameters enough to enable such a system.

In Chapter 2, I lay out the theory behind the transmon and cQED. I then derive the steady-state density matrix equations for a three-level system and show that a metastable
1.4 Overview of the thesis

state is needed to observe CPT, and that ATD but not EIT can be observed with just a transmon. I define the dark state fidelity, which serves as a figure of merit for how well the system can display Raman coherence effects. I propose a quasi-Λ system based on the combined transmon-cavity levels, and model it to show that CPT is possible with experimentally achievable device parameters.

I proceed to discuss fabrication of 3D transmons in Chapter 3. I discuss design considerations and improvements that can be made to both cavities and qubits. Fabrication details are presented along with a summary of my devices and their relevant parameters.

Details of the experimental setup are presented in Chapter 4. I discuss both the cryogenic and the room-temperature setup, show various pulsing techniques for qubit manipulation and measurement, and describe in detail how I obtained vector control of the qubit to perform quantum gates and tomography measurements.

I show typical measurements of transmons in Chapter 5. These include setting up the readout, doing single- and multi-tone spectroscopy, performing time-domain measurements to extract device coherence times, and obtaining state tomography.

In Chapter 6, I discuss my use of the three lowest levels of the transmon to observe the Autler-Townes effect. After characterizing the system independently, I was able to obtain excellent agreement between my model and the ATD data without any fitting parameters. I estimated the dark state fidelity in my experiment, and showed that CPT and EIT cannot be achieved with just a transmon. The results I present in this chapter were published elsewhere [Nov13].

Based on my model of the quasi-Λ system, I describe my measurement of a transmon-
1.4 Overview of the thesis

cavity device in Chapter 7. I demonstrate both atomic and photon propagation aspects of Raman coherence, show CPT spectroscopically, and prove that the dark states are coherent by performing a time-domain free evolution experiment. Instead of EIT, I observe a new phenomenon – electromagnetically suppressed transmission (EST). I show that the accompanying dispersion is negative, and pulses sent through the EST window propagate superluminally. These results are observed at a single-photon and single-qubit level, further solidifying the place of cQED as a platform for quantum optics, and suggesting that new phenomena can be observed with superconducting quantum circuits.

I conclude with Chapter 8 by summarizing my main findings, and then outlining several avenues of further exploration. These include state stabilization with continuous tones, off-resonant control, EIT and information storage, and exploring Raman coherence in the ultra-strong limit of qubit-cavity interactions.
CHAPTER 2

Theory

This chapter provides the theoretical foundation upon which I designed and analyzed my experiments. First, in Section 2.1, I discuss how to obtain the energy spectrum of the transmon qubit by using Mathieu functions. Other parameters, such as the transmon anharmonicity, charge dispersion, and inter-level transition matrix elements, are also defined. Next, in Section 2.2, I discuss coupling of a transmon to a cavity.

In Section 2.3, I discuss the theory behind cascade and Λ systems and the difference between EIT and ATD effects, with applications to my experimental system. Incorporating decoherence into the theory sheds light on how the coherence of the states and quantum interference is destroyed. I define the dark state fidelity and use it as the metric for quantifying whether a three-level system displays quantum interference effects when irradiated by two coherent fields. I find that in the case of a cascade transmon system, the interference is impossible due to transmon decay rate scaling. This theory provides the basis for the experiment in Chapter 6.

In Section 2.4 I consider a quasi-Λ system consisting of both transmon and cavity
levels. I show that it is possible to engineer the decay rates in such a system to have quantum interference effects. I model the dynamics of the transmon-cavity system, and show how it is possible to create a system where a coherent dark state can be observed experimentally (see Section 7.4). The same interference results in exotic light dynamics, as evidenced in the electromagnetically suppressed transmission experiment I discuss in Section 7.8.

Mathematica notebooks I made for some of this chapter’s calculations are available in an online repository [Nov15], or can be requested from the author by email (snovikov AT gmail DOT com).

2.1 Transmon theory

A simplest quantum-mechanical circuit one can build is a harmonic oscillator made from an inductance $L$ and a capacitance $C$. The Hamiltonian for a harmonic oscillator can be written as:

$$\hat{H}_{\text{HO}} = \hbar \omega \left( a^\dagger a + \frac{1}{2} \right),$$  \hspace{1cm} (2.1)

where $\omega = 1/\sqrt{LC}$ is the resonance frequency and $a^\dagger(a)$ is the bosonic creation (annihilation) operator. This circuit by itself, however, cannot be used as a qubit precisely because it is harmonic, and no pair of its energy levels can be isolated and addressed.

To create a qubit, one must have anharmonicity from a non-linearity in the circuit, so that adding a single excitation changes the amount of energy required to add another
2.1 Transmon theory

one. The best-known non-linear virtually dissipation-less circuit element [Dev04] is the Josephson junction [Jos62; Jos74]. Physically, it consists of a thin insulating dielectric layer sandwiched between two superconducting plates (see Fig. 2.1). Cooper pairs can tunnel from one side of the junction to the other, and the energy cost associated with this tunneling is the Josephson energy $E_J$. Since the tunneling involves moving charges between two metal plates, there will also be an associated capacitive energy $E_C$.

The non-linearity of the Josephson junction is due to the Josephson relations for the voltage $V$ across the junction:

$$V(t) = \frac{\hbar}{2e} \frac{d\phi}{dt}$$  \hspace{1cm} (2.2)

and the super-current $I$ through the junction:

$$I(t) = I_c \sin(\phi)$$  \hspace{1cm} (2.3)

where $\phi$ is phase difference across the junction and $I_c$ is the critical current of the

![Fig. 2.1: (a) Schematic of an Al/AlO$_x$/Al Josephson junction showing the two Al superconductor leads separated by the AlO$_x$ insulator. (b) The corresponding circuit element.](image-url)
2.1 Transmon theory

One can think of the junction as a non-linear inductance. Connecting the junction to other circuit elements imparts non-linearity onto the overall circuit, allowing one to create anharmonic systems that can be used in qubits [Nak99]. As I discussed in Section 1.3, several flavors of Josephson junction-based superconducting qubits exist. The transmon superconducting qubit [Koc07] consists of a Josephson junction shunted by a large capacitance $C_B$ (see Fig. 2.2). The total capacitance $C_\Sigma = C_J + C_B + C_g$ shunting the junction reduces the charging energy of the transmon:

$$E_C = \frac{e^2}{2C_\Sigma} = \frac{e^2}{2(C_J + C_B + C_g)}$$

so that $E_J \approx 50E_C$ to $100E_C$.

The transmon Hamiltonian can be written as [Koc07]:

$$\mathcal{H}_T = 4E_C(n - n_g)^2 - E_J \cos(\phi)$$

**Fig. 2.2**: Schematic of the transmon, consisting of a Josephson junction shunted by a large capacitance $C_B$, coupled via gate capacitance $C_g$ to the drive voltage $V$.  

16
where $E_C$ is the charging energy of the transmon, $n_g = C_g V_g / 2e$ is the effective offset charge on the device, $n$ is the number of Cooper pairs transferred through the junction, and $\phi$ is the gauge-invariant phase-difference across the junction. The first term in Eq. (2.5) accounts for the electrostatic energy of Cooper pairs on the effective capacitance $C_\Sigma$, while the second term describes the Josephson tunneling energy. By expressing $n$ in terms of the phase operator,

$$n = i \frac{\partial}{\partial \phi}, \quad (2.6)$$

Eq. (2.5) can be re-written as [Sch07]:

$$\mathcal{H}_T = \int_0^{2\pi} \left[ 4E_C \left( i \frac{\partial}{\partial \phi} - \frac{n_g}{2} \right)^2 |\phi\rangle \langle \phi| - E_J \cos(\phi) |\phi\rangle \langle \phi| \right] d\phi, \quad (2.7)$$

where

$$|\phi\rangle = \sum_{n=-\infty}^{\infty} e^{i\phi n} |n\rangle \quad (2.8)$$

are the phase eigenkets and $|n\rangle$ are the charge eigenkets. For an energy eigenket $|k\rangle$, the Schrödinger equation $\mathcal{H}_T |k\rangle = E_k |k\rangle$ becomes a Mathieu equation [Arf05] for the phase wave function $\psi_k(\phi) \equiv \langle \phi |k\rangle$:

$$4E_C \left( i \frac{\partial}{\partial \phi} - \frac{n_g}{2} \right)^2 \psi_k(\phi) - E_J \psi_k(\phi) \cos(\phi) = E_k \psi_k(\phi). \quad (2.9)$$
2.1 Transmon theory

Equation (2.9) can be solved analytically [Cot02] to obtain the eigenenergies $E_k$ of the transmon:

$$E_k(n_g) = E_C A_{2[n_g + m(k,n_g)]}(-E_J/2E_C)$$  \hspace{1cm} (2.10)

where $A_r(q)$ is the characteristic value for the even Mathieu functions with characteristic exponent $r$ and parameter $q$, and $m(k,n_g)$ is function that sorts the eigenvalues (see [Koc07]).

Using the eigenenergies $E_k(n_g)$, I define the level spacing between the adjacent states at $n_g = 0$ as:

$$E_{ij} \equiv E_j(n_g = 0) - E_i(n_g = 0)$$  \hspace{1cm} (2.11)

To measure how well the transmon levels can be isolated to form a qubit, I define the anharmonicity as

$$\alpha \equiv E_{12} - E_{01}$$  \hspace{1cm} (2.12)

and the relative anharmonicity $\alpha_r$ as

$$\alpha_r \equiv \frac{\alpha}{E_{01}}$$  \hspace{1cm} (2.13)

In general, $E_k(n_g)$ depends on the offset charge $n_g$. Because the resulting eigenvalues
2.1 Transmon theory

have a periodicity in $n_g$ of unity (one Cooper pair), I define the energy dispersion $\varepsilon_k$ of the $k$-th level due to charge offsets as

$$\varepsilon_k \equiv E_k(n_g = 1/2) - E_k(n_g = 0).$$  \hspace{1cm} (2.14)$$

The charge dispersion is important because any noise in the offset charge induces broadening of the $i \leftrightarrow j$ transmon transition by up to $|\varepsilon_j - \varepsilon_i|$, resulting in dephasing of the qubit. However, in the transmon regime, $E_J \gg E_C$, it was shown [Koc07] that the dispersion is suppressed exponentially with $\sqrt{E_J/E_C}$ ratio:

$$\lim_{E_J/E_C \to \infty} \varepsilon_k = (-1)^k E_C \frac{2^{4k+5}}{k!} \sqrt{\frac{2}{\pi}} \left(\frac{E_J}{2E_C}\right)^{\frac{k+3}{2}} e^{-\sqrt{8E_J/E_C}}$$

due to flattening of the energy bands $E_k$ with $n_g$ as $E_J/E_C \to \infty$. Remarkably, the relative anharmonicity is suppressed only polynomially:

$$\lim_{E_J/E_C \to \infty} \alpha_r = -\frac{1}{\sqrt{8E_J/E_C}}.$$  \hspace{1cm} (2.16)$$

In practice, this means one can find parameters where the device is insensitive to charge noise while retaining sufficient anharmonicity to operate as a qubit.

I used Mathematica to evaluate Eqs. (2.10) to (2.14) numerically Figure 2.3 shows the resulting dependence of transmon bands $E_k$ on $E_J/E_C$, and Fig. 2.4 shows the charge dispersion and anharmonicity. As will be seen in Section 3.2.3, my transmons had $E_J/E_C = 38 - 164$. 

19
2.1 Transmon theory

Fig. 2.3: Three lowest energy levels \( k = 0,1,2 \) (black, red, and blue) of the Hamiltonian Eq. (2.5) for (a) \( E_J/E_C = 1 \), (b) \( E_J/E_C = 5 \), (c) \( E_J/E_C = 10 \), and (d) \( E_J/E_C = 50 \).

In the limit \( E_J/E_C \gg 1 \), the Hamiltonian of Eq. (2.5) can be approximated by that of an anharmonic oscillator [Koc07]:

\[
\mathcal{H}_{AHO} = \sqrt{8E_JE_C} \left( b^\dagger b + \frac{1}{2} \right) - E_J - \frac{E_C}{12} \left( b + b^\dagger \right)^4 ,
\]  

(2.17)

where \( b^\dagger \) (\( b \)) is the creation (annihilation) operator for the transmon excitations. Diagonalizing this Hamiltonian, as was done in [Koc07], leads to a model that describes the transmon well (as long as \( E_J/E_C \gg 1 \) is valid), and allows one to develop intuition about the qubit. To zeroth order, the transmon is a harmonic oscillator with the \( j \leftrightarrow j + 1 \)
2.1 Transmon theory

Fig. 2.4: (a) Relative energy dispersion $\epsilon_k/E_{01}$ for the three lowest levels $k = 0, 1, 2$ (black, red, and blue) of the transmon. (b) Relative anharmonicity $\alpha_r$ of the transmon.
2.1 Transmon theory

transition energy given by

\[ E_{j,j+1}^{(0)} = \sqrt{8E_J E_C} \]  \quad (2.18)

The first-order correction due to the \((b + b^\dagger)^4\) term can be perturbatively calculated:

\[ E_{j,j+1}^{(1)} = -E_C(j + 1) \]  \quad (2.19)

The Cooper pair number operator \(n\) can be written in terms of the creation and annihilation operators:

\[ n = -i \left( \frac{E_J}{8E_C} \right)^{1/4} \frac{1}{\sqrt{2}} (b - b^\dagger) \]  \quad (2.20)

Transitions between states \(|i\rangle\) and \(|j\rangle\) are governed by the \((j|n|i)\) matrix elements. As with the harmonic oscillator, transitions between non-adjacent energy levels are forbidden in the limit \(E_J/E_C \to \infty\):

\[ \lim_{E_J/E_C \to \infty} \langle j|n|i \rangle = 0 \quad \forall j \neq i \pm 1 \]  \quad (2.21)

Between the adjacent energy levels \(|j\rangle\) and \(|j + 1\rangle\), the matrix element is

\[ \lim_{E_J/E_C \to \infty} \langle j + 1|n|j \rangle = \sqrt{\frac{j + 1}{2}} \left( \frac{E_J}{8E_C} \right)^{1/4} \]  \quad (2.22)

I note the harmonic oscillator-like \(\sqrt{j}\) dependence of the transition matrix element on
2.2 Coupling a transmon to a cavity – the Jaynes-Cummings Hamiltonian

the energy level. This implies the decay $\Gamma_j$ rate of $|j\rangle$ into $|j-1\rangle$ is linear, according to Fermi’s golden rule [Sak94]:

$$\Gamma_j \propto \frac{2\pi}{\hbar} |(j-1|n|j)\rangle^2 \rho \sim j \ .$$

(2.23)

This is a critical fact that will be revisited in Section 2.3, where I discuss the possibility of using only transmon levels to observe quantum interference.

2.2 Coupling a transmon to a cavity – the

Jaynes-Cummings Hamiltonian

As I described in Section 1.3, there are many reasons to couple a superconducting qubit to a resonant cavity: protection from dissipation, dispersive readout, rich physics that atom-cavity systems offer. This thesis is a direct proof of the last statement. Similar to regular atomic cavity QED, a transmon qubit coupled to a resonant cavity can be described by the Jaynes-Cummings Hamiltonian [Jay63; Sho93]:

$$\mathcal{H}_{JC} = \frac{1}{2} \hbar \omega_q \sigma_z + \hbar \omega_c (a^\dagger a + \frac{1}{2}) + \hbar g (a^\dagger \sigma_- + a \sigma_+) \ ,$$

(2.24)

where $\omega_q$ is the qubit transition frequency, $\sigma_z$ is the Pauli matrix for the qubit subspace, $\omega_c$ is the cavity resonance frequency, $a^\dagger$ ($a$) is the cavity photon creation (annihilation) operator, $\sigma_+$ ($\sigma_-$) is the qubit excitation creation (annihilation) operator, and $g$ is the qubit-cavity coupling, also known as the vacuum Rabi splitting.
2.2 Coupling a transmon to a cavity – the Jaynes-Cummings Hamiltonian

A conceptual diagram of a qubit coupled to a cavity is presented in Fig. 2.5, showing the coupling $g$ along with the decay rates $\Gamma$ of the qubit and $\kappa$ of the cavity. The case when $g \gg \Gamma$ and $\kappa$ is called the strong coupling regime of cavity QED [Sch07]. In this limit, coherent qubit-cavity dynamics are not obscured by the qubit or cavity decoherence. Large $g$ are easily attainable with superconducting qubits [Fin08; Wal04], and all of my experiments were done in the strong coupling regime.

The interplay between $g$ and the qubit-cavity detuning $\Delta \equiv \omega_q - \omega_c$ can lead to either a resonant ($\Delta = 0 \ll g$) or a dispersive ($\Delta \gg g$) limit. The experiments I performed were always in the dispersive limit of detuning, where the system can be approximated by the dispersive Jaynes-Cummings Hamiltonian [Sch07]:

$$H_{\text{DJC}} = \frac{1}{2}\hbar \omega_q \sigma_z + \hbar \left( \omega_c + \chi \sigma_z \right) a^\dagger a.$$  \hspace{1cm} (2.25)

Here the dispersive shift $\chi \equiv g^2/\Delta$ results in a frequency shift of the cavity resonance depending on the state of the qubit [see Fig. 2.6 (a)]. This allows the dispersive shift of

Fig. 2.5: Conceptual diagram of a Jaynes-Cummings system illustrating the vacuum Rabi coupling $g$ between the qubit and the cavity, qubit decay rate $\Gamma$, and cavity decay rate $\kappa$. 

24
2.2 Coupling a transmon to a cavity – the Jaynes-Cummings Hamiltonian

The cavity to be used to perform readout of the qubit state [Bia09b] by probing cavity transmission or reflection at $\omega_c - \chi$ or $\omega_c + \chi$.

The Hamiltonian in Eq. (2.25) is a perturbative expansion of Eq. (2.24) to second order in the small parameter $g/\Delta$ [Boi09]. The eigenstates of the dispersive Hamiltonian can be written in the form [Sch07]:

$$
|+,n\rangle = |e,n-1\rangle + \sqrt{n} \frac{g}{\Delta} |g,n\rangle,
$$

$$
|-,n\rangle = |g,n\rangle + \sqrt{n} \frac{g}{\Delta} |e,n-1\rangle,
$$

where $|g\rangle$ and $|e\rangle$ are the qubit ground or excited state\(^1\), respectively, and $|n\rangle$ is the state of the cavity when it has $n$ photons (i.e. a Fock state [Man95]). Thus, in the dispersive approximation, each eigenstate consists of an uncoupled qubit-cavity state, as well as a small contribution from another uncoupled qubit-cavity state with the same total

---

\(^1\) To avoid confusion with photon number states, I refer to transmon states $|0\rangle$, $|1\rangle$, $|2\rangle$, and $|3\rangle$ as $|g\rangle$, $|e\rangle$, $|f\rangle$, and $|h\rangle$ for the rest of the thesis, unless noted otherwise.
2.3 Cascade and \( \Lambda \) systems

number of excitations.

Because my experiments were well into the dispersive regime, I could also sometimes ignore the \( g/\Delta \) contribution and use the approximation:

\[
|+,n\rangle \approx |e,n - 1\rangle, \\
|-,n\rangle \approx |g,n\rangle,
\]

\( i.e. \) simply writing the hybridized states of the Hamiltonian as \( |g,n\rangle \) or \( |e,n\rangle \). Figure 2.6 (b) shows the energy level diagram in this case.

The energy levels \( |g,1\rangle, |e,1\rangle, \) and \( |e,0\rangle \) of this dispersively coupled system were used by B. Suri \textit{et al.} [Sur13] to observe the Autler-Townes doublet. As will be seen in Section 2.4 and Chapter 7, I used a different configuration of the Jaynes-Cummings dressed states to create a quasi-\( \Lambda \) system for observing CPT and a new phenomenon of electromagnetically suppressed transmission (EST).

2.3 Cascade and \( \Lambda \) systems

In this section, I model a cascade system consisting of three levels (\( |0\rangle, |1\rangle, \) and \( |2\rangle \)) and two microwave drives (probe and coupler). Figure 2.7 (a) shows the level diagram for this configuration. I used this system to study ATD (see Chapter 6), with the levels as transmon states: \( |0\rangle \equiv |g\rangle, |1\rangle \equiv |e\rangle, \) and \( |2\rangle \equiv |f\rangle \).

Without decoherence, the cascade system is equivalent to a \( \Lambda \) system [see Fig. 2.7 (b)]. With decays present, however, the dynamics become different as the decay process \( |2\rangle \rightarrow |1\rangle \) in the cascade is replaced by the \( |1\rangle \rightarrow |2\rangle \) decay process in the \( \Lambda \). The
2.3 Cascade and $\Lambda$ systems

equations I derive apply to both with a proper choice of decay rates. In Section 2.3.2 I show the calculation results for both cascade and $\Lambda$ configurations for completeness. To reduce clutter, I assume for $\hbar = 1$ for this section and Section 2.4.

2.3.1 Driven Hamiltonian

I write the Hamiltonian for three level-system without drives as

$$H = \omega_{10} \langle 1 | + \omega_{20} | 2 \rangle \langle 2 | . \tag{2.28}$$

In order to add a drive, I assume a monochromatic field $\vec{E} \cos(\omega t)$ coupling to the dipole moment $\vec{\mu} \equiv \vec{\mu}_{ab} (|b \rangle \langle a | + |a \rangle \langle b |)$ of an $|a \rangle \leftrightarrow |b \rangle$ transition in the system. Defining the

![Energy level diagram](image)

**Fig. 2.7:** (a) Energy level diagram for a cascade system, showing the probe (blue) and coupler (red). The three lowest transmon states can be used to implement this: $|0 \rangle \equiv |g \rangle$, $|1 \rangle \equiv |e \rangle$, $|2 \rangle \equiv |f \rangle$. (b) An equivalent $\Lambda$ system whose dynamics become different once relaxation is included.
2.3 Cascade and Λ systems

Rabi frequency as $\Omega_{ab} \equiv -\vec{\mu}_{ab} \cdot \vec{E}$, I write the semi-classical interaction Hamiltonian

$$\mathcal{H}_{\text{int}} = -\vec{\mu} \cdot \vec{E} \cos(\omega t) = \Omega_{ab} \left[ |b\rangle \langle a| \cos(\omega t) + |a\rangle \langle b| \cos(\omega t) \right]$$

$$= \frac{1}{2} \Omega_{ab} \left[ |b\rangle \langle a| e^{-i\omega t} + |a\rangle \langle b| e^{i\omega t} + |b\rangle \langle b| e^{i\omega t} + |a\rangle \langle a| e^{-i\omega t} \right].$$

(2.29)

The last two terms in the square brackets represent excitation with a simultaneous photon emission and decay with a photon absorption. These non-resonant processes can be neglected (the rotating-wave approximation, see [CT98]). Thus, the single monochromatic drive Hamiltonian is

$$\mathcal{H}_{\text{int}} = \frac{1}{2} \Omega_{ab} \left[ |b\rangle \langle a| e^{-i\omega t} + |a\rangle \langle b| e^{i\omega t} \right].$$

(2.30)

Let us define probe frequency $\omega_p$ and amplitude $\Omega_p$, as well as coupler frequency $\omega_c$ and amplitude $\Omega_c$. Adding both probe and coupler drives to $\mathcal{H}$ using Eq. (2.30), and re-writing the result in terms of $\omega_p, \omega_c, \Delta_p \equiv \omega_p - \omega_{10}$, and $\Delta_c \equiv \omega_c - \omega_{21}$ yields

$$\mathcal{H}_{\text{tot}} = \omega_p |1\rangle \langle 1| + (\omega_p + \omega_c) |2\rangle \langle 2| - (\omega_p - \omega_{10}) |1\rangle \langle 1| - (\omega_p - \omega_{10} + \omega_c - \omega_{21}) |2\rangle \langle 2|
$$

$$+ \frac{1}{2} \Omega_p \left[ |1\rangle \langle 0| e^{-i\omega_p t} + \text{H.c.} \right] + \frac{1}{2} \Omega_c \left[ |2\rangle \langle 1| e^{-i\omega_c t} + \text{H.c.} \right]$$

$$= \omega_p |1\rangle \langle 1| + (\omega_p + \omega_c) |2\rangle \langle 2| - \Delta_p |1\rangle \langle 1| - (\Delta_p + \Delta_c) |2\rangle \langle 2|
$$

$$+ \frac{1}{2} \Omega_p \left[ |1\rangle \langle 0| e^{-i\omega_p t} + \text{H.c.} \right] + \frac{1}{2} \Omega_c \left[ |2\rangle \langle 1| e^{-i\omega_c t} + \text{H.c.} \right]$$

$$= \mathcal{H}_0 + \mathcal{V},$$

(2.31)
where

\[ \mathcal{H}_0 \equiv \omega_p \ket{1} \bra{1} + (\omega_p + \omega_c) \ket{2} \bra{2} \]  

(2.32)

and

\[
\mathcal{V} = -\Delta_p \ket{1} \bra{1} - (\Delta_p + \Delta_c) \ket{2} \bra{2} + \frac{1}{2} \Omega_p \left[ \ket{1} \bra{0} e^{-i\omega_p t} + \text{H.c.} \right] + \frac{1}{2} \Omega_c \left[ \ket{2} \bra{1} e^{-i\omega_c t} + \text{H.c.} \right]
\]  

(2.33)

I remove the time dependence of \( \mathcal{V} \) by transforming to a frame co-rotating with the two drives via the following unitary:

\[ \mathcal{U} = e^{iH_0 t} \]  

(2.34)

In this interaction picture, the Hamiltonian becomes

\[
\mathcal{V}^t = \mathcal{U} \mathcal{V} \mathcal{U}^\dagger
\]

\[
= -\Delta_p \ket{1} \bra{1} - (\Delta_p + \Delta_c) \ket{2} \bra{2} + \frac{1}{2} \Omega_p \left[ \ket{1} \bra{0} + \text{H.c.} \right] + \frac{1}{2} \Omega_c \left[ \ket{2} \bra{1} + \text{H.c.} \right]
\]  

(2.35)

Diagonalizing Eq. (2.35) for the special case of \( \Delta_p = \Delta_c = 0 \) sheds some light on the
2.3 Cascade and Λ systems

eigenstates of the system:

\[
|D\rangle = \cos \Theta |0\rangle - \sin \Theta |2\rangle, \\
|+\rangle = \frac{1}{\sqrt{2}} (\sin \Theta |0\rangle + |1\rangle + \cos \Theta |2\rangle), \\
|-\rangle = \frac{1}{\sqrt{2}} (\sin \Theta |0\rangle - |1\rangle + \cos \Theta |2\rangle),
\]  

(2.36)

where the mixing angle \( \Theta = \tan^{-1}(\Omega_p/\Omega_c) \). State \(|D\rangle\) corresponds to the zero eigenvalue of \( V^I \), while \(|\pm\rangle\) correspond to \( \pm \frac{1}{2} \sqrt{\Omega_p^2 + \Omega_c^2} \), respectively. Note that \(|D\rangle\) has no contribution from intermediate state \(|1\rangle\). It turns out that this leads to neither the probe (driving \(|0\rangle \leftrightarrow |1\rangle\)) nor the coupler (driving \(|1\rangle \leftrightarrow |2\rangle\)) being absorbed when the system is in \(|D\rangle\). Consequently, state \(|D\rangle\) earned the name *dark state* [Ste14].

In the limit of \( \Omega_c \ll \Omega_p \), the existence of the dark state is a purely quantum-mechanical phenomenon that occurs due to Fano interference of different excitation pathways in the system [Fle05; Mar98]. Two particular cases are of importance. First, \( \Omega_c/\Omega_p = 0 \) results in population inversion, with \(|D\rangle = |2\rangle\). Second, when \( \Omega_c = \Omega_p \), quantum interference results in a maximally coherent dark state \(|D\rangle = (|0\rangle - |2\rangle)/\sqrt{2}\). Experimental observation of these two cases is essential for demonstrating Raman coherence.

It should be noted that as \( \Omega_c/\Omega_p \to \infty \), \(|D\rangle \to |0\rangle\) – the dark state becomes the trivial ground state of the system, and \(|2\rangle\) does not contribute to the superposition. This is the ATD regime of the system, where the two non-dark eigenstates \(|+\rangle\) and \(|-\rangle\) are spectroscopically observed at \( \pm \frac{1}{2} \sqrt{\Omega_p^2 + \Omega_c^2} \) and \( -\frac{1}{2} \sqrt{\Omega_p^2 + \Omega_c^2} \) in energy, respectively. In contrast, CPT and EIT rely on the superposition of \(|0\rangle\) and \(|2\rangle\) in the dark state, and are impossible to observe in the limit \( \Omega_c \gg \Omega_p \) [Ani11].
2.3 Cascade and $\Lambda$ systems

2.3.2 Adding decoherence

While $\Omega_c \gg \Omega_p$ can suppress the contribution of $|2\rangle$ to the dark state, decoherence can destroy the superposition. For example, fast decay from $|2\rangle$ to $|1\rangle$ can poison $|D\rangle$ by introducing $|1\rangle$ into the dark state. In fact, as I show later in this chapter, this particular mode of decay is the bane of any EIT experiment that utilizes the cascade system.

To model decoherence I construct a Lindblad-Kossakowski \cite{Kos72, Lin76} master equation for the density matrix $\rho^I$ in the interaction picture:

\begin{equation}
\frac{d\rho^I}{dt} = i[\rho^I, V^I] + \sum_j \Gamma_j D(A^I_j)\rho^I ,
\end{equation}

where the summation goes over all possible decoherence channels, $\Gamma_j$ is the decoherence (dissipation or dephasing) rate for $j$-th channel (see Eq. (2.23)), and the operator $D$ is defined by

\begin{equation}
D(A)\rho \equiv A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\} .
\end{equation}

Each jump operator $A_i$ represents a particular way dissipation or dephasing can occur. For example, spontaneous decay or excitation from $|a\rangle$ to $|b\rangle$ can be written in the lab frame as $A_{\text{diss}} = |b\rangle \langle a|$. Similarly, the dephasing operator for state $|a\rangle$ is $A_{\text{deph}} = |a\rangle \langle a|$ in the lab frame. Note that I transform the operators into the interaction picture via

\begin{equation}
A^I = UAU^\dagger .
\end{equation}
2.3 Cascade and $\Lambda$ systems

With these definitions, the off-diagonal density matrix elements decay at rate which I can define as

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi} = \frac{\Gamma}{2} + \gamma_\phi \ .$$  

(2.40)

The transmon decay rate $\Gamma = 1/T_1$ and dephasing rate $\gamma_\phi = 1/T_\phi$ can be found from the experimentally measurable quantities $T_1$ and $T_\phi$.

Summing over all of the decay channels $|i\rangle \rightarrow |j\rangle$ with $i > j$, I obtain the following dissipator

$$\mathcal{D}(A_{\text{diss}})\rho = \begin{pmatrix}
\Gamma_{10}\rho_{11} + \Gamma_{20}\rho_{22} & -\frac{1}{2}\Gamma_{10}\rho_{01} & -\frac{1}{2}(\Gamma_{20} + \Gamma_{21})\rho_{02} \\
-\frac{1}{2}\Gamma_{10}\rho_{10} & -\Gamma_{10}\rho_{11} + \Gamma_{21}\rho_{22} & -\frac{1}{2}(\Gamma_{10} + \Gamma_{20} + \Gamma_{21})\rho_{12} \\
-\frac{1}{2}(\Gamma_{20} + \Gamma_{21})\rho_{20} & -\frac{1}{2}(\Gamma_{10} + \Gamma_{20} + \Gamma_{21})\rho_{21} & -\frac{1}{2}(\Gamma_{20} + \Gamma_{21})\rho_{22}
\end{pmatrix}$$

(2.41)

Summing over all of the excitation channels $|i\rangle \rightarrow |j\rangle$ with $i < j$, I obtain the following

$$\mathcal{D}(A_{\text{exc}})\rho = \begin{pmatrix}
-(\Gamma_{01} + \Gamma_{02})\rho_{00} & -\frac{1}{2}(\Gamma_{01} + \Gamma_{02} + \Gamma_{12})\rho_{01} & -\frac{1}{2}(\Gamma_{01} + \Gamma_{02})\rho_{02} \\
-\frac{1}{2}(\Gamma_{01} + \Gamma_{02})\rho_{10} & \Gamma_{01}\rho_{00} - \Gamma_{12}\rho_{11} & -\frac{1}{2}\Gamma_{12}\rho_{12} \\
-\frac{1}{2}(\Gamma_{01} + \Gamma_{02})\rho_{20} & -\frac{1}{2}\Gamma_{12}\rho_{21} & \Gamma_{12}\rho_{11}
\end{pmatrix}$$

(2.42)
Finally, for the dephasing channels I find

\[
\mathcal{D}(A_{\text{deph}}) \rho = \begin{pmatrix}
0 & -\gamma_1 \rho_{01} & -\gamma_2 \rho_{02} \\
-\gamma_1 \rho_{10} & 0 & - (\gamma_1 + \gamma_2) \rho_{12} \\
-\gamma_2 \rho_{20} & - (\gamma_1 + \gamma_2) \rho_{21} & 0
\end{pmatrix}
\]  

(2.43)

After transforming the above dissipators into the rotating frame by means of Eq. (2.39), I set \(d \rho / dt = 0\) and used symbolic manipulation features of Mathematica to solve Eq. (2.37) in the steady state. I arrive at the following set of equations

\[
\rho_{11} = \frac{2(\Gamma_{01} + \Gamma_{02}) \rho_{00} - 2 \Gamma_{20} \rho_{22} - i \Omega_p (\rho_{01} - \rho_{10})}{2 \Gamma_{10}}
\]

\[
\rho_{22} = \frac{2(\Gamma_{02} \rho_{00} + \Gamma_{12} \rho_{11}) - i \Omega_c (\rho_{12} - \rho_{21})}{2(\Gamma_{20} + \Gamma_{21})}
\]

\[
\rho_{10} = - \frac{i [\Omega_c \rho_{20} + \Omega_p (\rho_{00} - \rho_{11})]}{\Gamma_{01} + \Gamma_{02} + \Gamma_{10} + \Gamma_{12} + 2 \gamma_1 - 2 i \Delta_p}
\]

\[
\rho_{21} = - \frac{i [\Omega_c (\rho_{11} - \rho_{22}) - \Omega_p \rho_{20}]}{\Gamma_{10} + \Gamma_{12} + \Gamma_{20} + \Gamma_{21} + 2 \gamma_1 + 2 \gamma_2 - 2 i \Delta_c}
\]

\[
\rho_{20} = - \frac{i [\Omega_c \rho_{10} - \Omega_p \rho_{21}]}{\Gamma_{01} + \Gamma_{02} + \Gamma_{20} + \Gamma_{21} + 2 \gamma_2 - 2 i (\Delta_p + \Delta_c)}
\]  

(2.44)

The set of Eq. (2.44) describes the steady state of the driven cascade system with decoherence. With the additional constraint of completeness \(\rho_{00} + \rho_{11} + \rho_{22} = 1\), I can solve these equations numerically using independently determined system parameters \(\Gamma_{ij}, \gamma_i, \Omega_p, \Omega_c, \Delta_p, \Delta_c\), and obtain \(\rho_{11} + \rho_{22}\) which I can then compare directly to my experimental data (see Chapter 6).

Equation (2.44) applies to both cascade and \(\Lambda\) systems. First, consider a cascade system [see Fig. 2.7 (a)] with \(\Gamma_{10} = 100\) kHz and \(\Gamma_{21} = 1\) kHz. Such system has a
Fig. 2.8: Metastable case simulation of the density matrix $\rho$ for a cascade system with $\Gamma_{10} = 100$ kHz, $\Gamma_{21} = 1$ kHz, $\Omega_p/2\pi = 100$ kHz. (a) State populations $\rho_{jj}$ and (b) coherences $\rho_{jk}$ for the EIT regime $\Omega_c/2\pi = 100$ kHz. (c) State populations $\rho_{jj}$ and (d) coherences $\rho_{jk}$ for the ATD regime $\Omega_c/2\pi = 2000$ kHz.

metastable state $|2\rangle$ in which population can be coherently trapped. I set $\Delta_c = 0$, $\Omega_p/2\pi = 100$ kHz, $\Omega_c/2\pi = 100$ kHz for the EIT regime and $\Omega_c/2\pi = 2000$ kHz for the ATD regime. Simulation results for $\rho$ vs $\Delta_p$ are plotted in Fig. 2.8. At this relatively low coupler power of $\Omega_c/2\pi = 100$ kHz and zero probe detuning $\Delta_p = 0$, the system is in a coherent dark state with $\rho_{00} = \rho_{22} \approx 0.5$ and no population in the intermediate state $|1\rangle$ [see Fig. 2.8 (a)]. Dark state coherence is evident from the off-diagonal $\text{Re}\rho_{02} \approx -0.5$ [see Fig. 2.8 (b)]. In the ATD regime of relatively high power $\Omega_c/2\pi = 1000$ kHz, the dark state consists of mostly $|0\rangle$ [see Fig. 2.8 (c)] and has very little coherence left
2.3 Cascade and Λ systems

![Graphs showing state populations and coherences for different regimes.](image)

Fig. 2.9: Fast-decaying case simulation of the density matrix \( \rho \) for a transmon-like cascade system with \( \Gamma_{10} = 100 \text{ kHz} \), \( \Gamma_{21} = 200 \text{ kHz} \), \( \Omega_p/2\pi = 100 \text{ kHz} \). (a) State populations \( \rho_{jj} \) and (b) coherences \( \rho_{jk} \) for the EIT regime \( \Omega_c/2\pi = 100 \text{ kHz} \). (c) State populations \( \rho_{jj} \) and (d) coherences \( \rho_{jk} \) for the ATD regime \( \Omega_c/2\pi = 2000 \text{ kHz} \).

As I discussed above [see Eq. (2.23)], the transmon’s second excited state decays twice as fast as the first. Accordingly, to model the case of the transmon, I set \( \Gamma_{21} = 2\Gamma_{10} = 200 \text{ kHz} \). Figure 2.9 shows my simulation results for the density matrix. One can see that in the EIT regime of \( \Omega_c/2\pi = 100 \text{ kHz} \), the “dark state” has a large contribution from \( |1\rangle \) [see Fig. 2.9 (a)]. This happens because state \( |2\rangle \) decays twice as quickly as \( |1\rangle \) in this case. Also, the dark state has low coherence, reflected in \( \text{Re} \rho_{02} \approx 0.05 \) [see Fig. 2.9 (b)], ten times smaller than in the previous case.
2.3 Cascade and $\Lambda$ systems

Fig. 2.10: Metastable case simulation of the density matrix $\rho$ for a $\Lambda$ system with $\Gamma_{10} = \Gamma_{12} = 100\,\text{kHz}$, $\Gamma_{20} = 1\,\text{kHz}$, $\Omega_p/2\pi = 100\,\text{kHz}$. (a) State populations $\rho_{jj}$ and (b) coherences $\rho_{jk}$ for the EIT regime $\Omega_c/2\pi = 100\,\text{kHz}$. (c) State populations $\rho_{jj}$ and (d) coherences $\rho_{jk}$ for the ATD regime $\Omega_c/2\pi = 2000\,\text{kHz}$.

To simulate the same effects in a $\Lambda$ system [see Fig. 2.7 (b)], I set $\Gamma_{21} = 0$ but $\Gamma_{12} = 100\,\text{kHz}$. The metastable case (see Fig. 2.10) has $\Gamma_{20} = 1\,\text{kHz}$, while the fast-decaying case (see Fig. 2.11) has $\Gamma_{20} = 200\,\text{kHz}$. As one can see, the results for the $\Lambda$ system are very similar to the corresponding results for the cascade system (compare Fig. 2.10 with Fig. 2.8 and Fig. 2.11 with Fig. 2.9).

2.3.3 Dark state fidelity

The coherence of the dark state in a system with decoherence can be quantified by finding the overlap with the ideal dark state $|D\rangle$ given in Eq. (2.36). This overlap is
Fig. 2.11: Fast-decaying case simulation of the density matrix $\rho$ for a Λ system with $\Gamma_{10} = \Gamma_{12} = 100$ kHz, $\Gamma_{20} = 200$ kHz, $\Omega_p/2\pi = 100$ kHz. (a) State populations $\rho_{jj}$ and (b) coherences $\rho_{jk}$ for the EIT regime $\Omega_c/2\pi = 100$ kHz. (c) State populations $\rho_{jj}$ and (d) coherences $\rho_{jk}$ for the ATD regime $\Omega_c/2\pi = 2000$ kHz.

called the dark state fidelity [Li11; Nov13] and can be computed using:

$$
F_D \equiv \sqrt{\langle D | \rho | D \rangle} \\
= \frac{\cos 2\Theta}{2} (\rho_{00} - \rho_{22}) - \frac{\sin 2\Theta}{2} (\rho_{20} + \rho_{02}) + \frac{1}{2} (1 - \rho_{11}) ,
$$

(2.45)

where $\Theta = \tan^{-1}(\Omega_p/\Omega_c)$. Using this definition, and Eq. (2.44), I calculated $F_D$ as a function of $\Omega_c/\Omega_p$ for the four cases presented in Figs. 2.8 to 2.11. The results are shown in Fig. 2.12. In all cases, $F_D \to 1$ as $\Omega_c/\Omega_p \to \infty$. To see why, first note that in this limit $\Theta \to 0$ and $|D\rangle \to |0\rangle$. Next note that in this limit, regardless of whether $|2\rangle$ decays
2.3 Cascade and $\Lambda$ systems

quickly or slowly, the system is never pumped strongly by the probe, and stays in $|0\rangle$ independent of the decay rates of $|2\rangle$. This is the ATD limit.

The picture changes drastically as $\Omega_c/\Omega_p \rightarrow 0$. The fidelities for the fast-decaying cases of both the cascade and $\Lambda$ systems approach zero, as population cannot be coherently trapped in state $|2\rangle$. The metastable cases start out with nonzero fidelity but there is a fundamental difference: the $\Lambda$ can trap the population much better than the cascade even at low powers. This can be understood by recalling that state $|2\rangle$ in the cascade system decays to $|1\rangle$, which, in turn, decays to $|0\rangle$. The contribution from $|1\rangle$ poisons the dark state, reducing its fidelity. In contrast, state $|2\rangle$ in the $\Lambda$ decays directly to $|0\rangle$, which then can be quickly pumped back. The $\Lambda$ system is therefore intrinsically much better for studying quantum interference effects that are associated with CPT and EIT, due to the high fidelity of its dark state even at low powers. The results in Fig. 2.12 suggest one possible application of such a $\Lambda$ system is creating a maximum superposition state $(|0\rangle - |2\rangle)/\sqrt{2}$ with fidelities greater than 99%.
From the previous section, I conclude that a cascade system comprised of transmon levels cannot exhibit EIT because state $|2\rangle = |f\rangle$ decays fast compared to $|1\rangle = |e\rangle$ (to lowest order the transmon is a harmonic oscillator). Combining a transmon with a cavity, as in Section 2.2, allowed me to select the transmon decay $\Gamma$ and cavity decay $\kappa$ independently. As I show in Chapter 7, the quasi-$\Lambda$ system made out of $|0\rangle \equiv |g,0\rangle$, $|1\rangle \equiv |e,1\rangle$, and $|2\rangle \equiv |e,0\rangle$ can exhibit CPT and EST (a counterpart of EIT) due to quantum interference. This section describes how I model that system.
2.4 Quasi-Λ system

2.4.1 Model assumptions

Although I would like to isolate states $|0\rangle \equiv |g,0\rangle$, $|1\rangle \equiv |e,1\rangle$, and $|2\rangle \equiv |e,0\rangle$, there are many more levels present in the Jaynes-Cummings ladder (see Fig. 2.13). The coupler tone is resonant with the $|e,n\rangle \leftrightarrow |e,n+1\rangle$ transitions and can drive those transitions. Restricting the system to the three levels of the Λ is impossible because of this. Instead, I worked numerically in a truncated Hilbert space with $n = 9$ photon levels. This turned out to be acceptable because the cavity-like decay $\kappa_n$ through the $|e,n\rangle \rightarrow |e,n-1\rangle$ is

$$\kappa_n = n\kappa$$  \hspace{1cm} (2.46)

Fig. 2.13: Level diagram for the quasi-Λ system, showing the probe (blue) and the coupler (red), qubit-like decays $\Gamma$ and cavity-like decays $n\kappa$. The three energy levels comprising the Λ are $|0\rangle \equiv |g,0\rangle$, $|1\rangle \equiv |e,1\rangle$, and $|2\rangle \equiv |e,0\rangle$. 
This means a sufficiently weak coupler will not drive the system too far up the ladder. An estimate can be made of how many states to include in the simulation by setting \( \Omega_c = n\kappa \), which gives

\[
N_{\text{max}} \sim \frac{\Omega_c}{\kappa}.
\]  (2.47)

Although I used \( \Omega_c < \kappa \) in most of my experiments, I included nine \( |g,n\rangle \) and nine \( |e,n\rangle \) states in my simulations. This resulted in good modeling accuracy as well as reasonable computation times.

Another important factor in using the quasi-\( \Lambda \) system shown in Fig. 2.13 is that a direct single-photon \( |g,0\rangle \leftrightarrow |e,1\rangle \) transition is parity-forbidden, as it involves a simultaneous excitation of both the cavity and the qubit. In practice, I used a two-photon drive at half of the transition frequency. Modeling a two-photon drive is more difficult than modeling a one-photon drive because one has to couple the probe to states such as \( |g,0\rangle \) and \( |e,0\rangle \) that serve as virtual levels supporting the two-photon transition. These intermediate states create multiple pathways for the probe to excite the system. Multiply connected systems [Rab79] are difficult to solve analytically, and the steady-state equations analogous to Eq. (2.44) would be very messy. In this case, the time-dependent master differential equation needs to be solved numerically, which takes orders of magnitude longer than solving a coupled system of linear equations. At the same time, the two-photon nature of the probe is not essential to the CPT and EIT effects I want to focus on, and modeling the probe as a single-photon drive is acceptable at low powers because excitations in
2.4 Quasi-Λ system

the system will not propagate far up the harmonic ladder due to the linear nature of the decay rate scaling with energy. For these reasons, I simply modeled the probe as a single-photon drive.

I note that the states used in the quasi-Λ system are not exactly the bare qubit-cavity states but, according to Eq. (2.26), have small contributions to them of \( O(\frac{g}{\Delta}) \) from states in the same Jaynes-Cummings excitation manifold. Including these contributions would once again make the equations very difficult to solve. On the other hand, in the dispersive limit of my experiments \( \frac{g}{\Delta} < 0.1 \), it was reasonable to treat the states as bare qubit-cavity states.

Even with these simplifications, the model captured the relevant physics of the system, and for low coupler powers the model produced excellent agreement with the data (see Chapter 7). At higher drives, ac-Stark shifts affect energy levels of the system \cite{Sch05}. These shifts may be accounted by introducing \textit{ad hoc} terms into the Hamiltonian. Fully capturing the high-power behavior would require solving the multiply-connected system with true Jaynes-Cummings eigenstates – a computationally intensive task for any reasonable truncation of the Hilbert space.
2.4 Quasi-Λ system

2.4.2 Driven Hamiltonian

Given the assumptions I stated above, the Hamiltonian for the quasi-Λ system in the truncated Hilbert space with up to eight photon excitations can be written as

\[
\mathcal{H}_\Lambda = \sum_{n=0}^{8} \left[ -n(\Delta_c + 2\chi)|g,n\rangle \langle g,n| - (\Delta_p + (n-1)\Delta_c)|e,n\rangle \langle e,n| \\
+ \frac{\Omega_c}{2}\sqrt{n+1}|e,n+1\rangle \langle e,n| + \frac{\Omega_p}{2}|e,1\rangle \langle g,0| + \text{H.c.} \right].
\]  

(2.48)

2.4.3 Adding decoherence

I took decoherence into account in the quasi-Λ system using the same approach described in Section 2.3.2. The cavity-like \(|g,n\rangle \rightarrow |g,n-1\rangle\) and \(|e,n\rangle \rightarrow |e,n-1\rangle\) decays are described by jump operators \(A_{\text{cav}}^{\text{diss}} = |g,n-1\rangle \langle g,n|\) or \(|e,n-1\rangle \langle e,n|\) with decay rates \(n\kappa\). Similarly, qubit-like \(|e,n\rangle \rightarrow |g,n\rangle\) decays happen at rate \(\Gamma\), and are included via \(A_{\text{qubit}}^{\text{diss}} = |g,n\rangle \langle e,n|\).

In steady state, the Lindblad-Kossakowski master equation [Kos72; Lin76] becomes a system of coupled linear equations. I solve these equations numerically to obtain the density matrix \(\rho\) for the system. I consider two cases – metastable \(\Gamma \ll \kappa\) (see Fig. 2.14) and fast-decaying \(\Gamma \gg \kappa\) (see Fig. 2.15). In both cases, I set \(\chi/2\pi = -10\) MHz.

At low coupler powers (the EIT regime), the metastable case behaves like a three-level \(\Lambda\) system, with only a small residual population \(\rho_{\text{res}} = 1 - (\rho_{00} + \rho_{11} + \rho_{22})\) outside of the \(\Lambda\) manifold. In fact, the residual population is zero for \(\Delta_p = 0\), when the system is in the dark state. At large coupler powers (the ATD regime), the transitions do not
2.4 Quasi-Λ system

\[ \text{Fig. 2.14: Simulation of the density matrix } \rho \text{ for a quasi-Λ system with a metastable state } |2\rangle: \Gamma = 10 \text{kHz, } \kappa = 1000 \text{kHz, } \Omega_p/2\pi = 100 \text{kHz.} \]  

(a) State populations \( \rho_{jj} \) and (b) coherences \( \rho_{jk} \) for the EIT regime \( \Omega_c/2\pi = 100 \text{kHz} \). (c) State populations \( \rho_{jj} \) and (d) coherences \( \rho_{jk} \) for the ATD regime \( \Omega_c/2\pi = 500 \text{kHz} \). Residual population in non-Λ states \( \rho_{\text{res}} = 1 - (\rho_{00} + \rho_{11} + \rho_{22}) \) is shown as a dashed magenta line.

saturate because of the harmonicity of the system, and instead of a typical ATD, I observe a drastic increase in \( \rho_{\text{res}} \) [see Fig. 2.14 (c)]. At the same time, coherence of the dark state is diminished, as expected for this regime.

The fast-decaying case at low coupler power presents similar behavior as the metastable case without population trapping in \( |2\rangle \) and coherence of the dark state. At large coupler powers, the drives again transfer population into the higher states, rather than saturating the three levels of the Λ.

Based on these results from my theoretical model, I concluded that it was possible to
2.4 Quasi-Λ system

Fig. 2.15: Simulation of the density matrix $\rho$ for a quasi-Λ system with a fast-decaying state $|2\rangle$: $\Gamma = 2000 \text{ kHz}$, $\kappa = 1000 \text{ kHz}$, $\Omega_p/2\pi = 100 \text{ kHz}$. (a) State populations $\rho_{jj}$ and (b) coherences $\rho_{jk}$ for the EIT regime $\Omega_c/2\pi = 100 \text{ kHz}$. (c) State populations $\rho_{jj}$ and (d) coherences $\rho_{jk}$ for the ATD regime $\Omega_c/2\pi = 500 \text{ kHz}$. Residual population in non-Λ states $\rho_{\text{res}} = 1 - (\rho_{00} + \rho_{11} + \rho_{22})$ is shown as a dashed magenta line.

build a quasi-Λ system out of bare Jaynes-Cummings eigenstates, where EIT and CPT effects should be possible to observe.

2.4.4 Dark state fidelity

To quantify how different the quasi-Λ system is from a true Λ, I calculated the fidelity of the dark state for both systems (see Fig. 2.16). From these results, I concluded that the quasi-Λ system possesses similar advantages over the cascade system in terms its ability to produce high-fidelity dark states at low coupler powers.
2.4 Quasi-Λ system

Fig. 2.16: Simulated dark state fidelity $\mathcal{F}_D$ versus $\Omega_c/\Omega_p$. Solid curves show $\mathcal{F}_D$ for a Λ system with $\Gamma_{10} = 10$ kHz, $\Gamma_{12} = 1000$ kHz, $\Omega_p/2\pi = 100$ kHz. The metastable case with $\Gamma_{20} = 10$ kHz is shown in solid red and fast-decaying case with $\Gamma_{20} = 2000$ kHz is shown in solid black. Dashed curves show $\mathcal{F}_D$ versus $\Omega_c/\Omega_p$ for a quasi-Λ system with $\kappa = 1000$ kHz and $\Omega_p/2\pi = 100$ kHz. The metastable case with $\Gamma = 10$ kHz is shown in dashed red (coincides with the solid red curve) and the fast-decaying case $\Gamma = 2000$ kHz is shown in dashed black.

The utility of the quasi-Λ system for superposition generation can be deduced from the fidelity of the maximum superposition dark state $|D\rangle = (|0\rangle - |2\rangle)/\sqrt{2}$ that is achievable in the system. I used my model to simulate this fidelity while varying the ratios $\Omega_p/(2\pi \times \Gamma)$ and $\kappa/\Gamma$ while keeping $\Omega_c = \Omega_p$ and $\Gamma = 10$ kHz (see Fig. 2.17). Consistent with an intuitive picture, the simulations show that $\kappa/\Gamma$ ratio limits the maximum possible fidelity by setting the “meta-stability” of $|e,0\rangle$. In order to achieve those fidelities, a simultaneous increase in $\Omega_p/\Gamma$ is required. Physically, larger probe powers allow to invert the population into $|e,0\rangle$ faster, before significant decoherence can occur. Figure 2.17 can be used as a “map” to aid in designing a system for high-fidelity dark state generation.
2.4 Quasi-Λ system

Fig. 2.17: False-color plot of the simulated dark state fidelity $F_D$ for the maximum superposition state ($\Omega_p = \Omega_c$ or $\Theta = \pi/4$) in the quasi-Λ system versus $\Omega_p/(2\pi \times \Gamma)$ and $\kappa/\Gamma$. The black curves represent equal-fidelity contours, and are labeled with the corresponding fidelities.
CHAPTER 3

Device design and fabrication

The goal of this chapter is to explain how I made devices and why I made them that way. The main design parameters for the devices were set by the Hamiltonian and the decay rates I wished to achieve. There were other, much more practical, constraints which came from a desire to have robust devices and repeatable fabrication and experimental outcomes. Here I describe the reasoning behind these constraints, to the best of my abilities, with sufficient detail that the procedures can be used as a starting point for someone who is new to the fabrication of superconducting qubits.

This chapter is divided into sections on the main components of the device – the cavity and the qubit – as well as a section on how I used simulations to predict the behavior of the devices in the refrigerator.

3.1 Cavity

The superconducting or normal metal resonant microwave three-dimensional cavity comprises half of our quantum system. It is the harmonic oscillator part of the Jaynes-
C.1.1 Design considerations

Superconducting qubits are typically operated at dilution refrigerator temperatures (10 mK) to avoid thermal excitation. In order to have the cavity in the ground state at this temperature, we need the Boltzmann factor $e^{-\hbar \omega / k_B T} \ll 1$, where $T$ is the temperature and $\omega / 2 \pi = f_c$ is the resonance frequency. Since $T = 100 \text{ mK}$ corresponds to approximately $f = k_B T / \hbar \approx 2 \text{ GHz}$, a cavity with a resonant frequency of the fundamental mode $f_c = 8 \text{ GHz}$ would have a Boltzmann factor of about $2 \times 10^{-17}$ for $T = 10 \text{ mK}$. This frequency is well within the range of commercially available microwave components, but lies towards the upper end of the range of some of the components in our laboratory [the high electron mobility transistor (HEMT) amplifier and the circulators]. Placing the fundamental mode fairly high in frequency is preferable for two reasons. First, it permits a large frequency space below $f_c$ for a qubit. Second, having the qubit below the cavity also reduces loss from the multi-mode Purcell effect [Hou08], which can adversely affect $T_1$ of the qubit.

A conventional transmon couples to its surroundings mostly via the electric field. Thus, to achieve strong coupling between the cavity and the transmon, the cavity should be designed such that the fundamental mode has an anti-node of the electric field in a
3.1 Cavity

Convenient place. A rectangular waveguide cavity (see Fig. 3.1) lets us do just that with the TE$_{101}$ (transverse electric) mode [Poz98]. In fact, placing the qubit at the anti-node of the TE$_{101}$ mode means it will be at the node of the TE$_{102}$ mode (and, in general, at the node of the TE$_{10(2n)}$ mode $\forall n \geq 1$), see Fig. 3.2. This effectively decouples the qubit from many other modes of the cavity, protecting the qubit from spontaneous decay into those modes. For a standard box-shaped cavity, the frequency of the TE$_{101}$ mode can be calculated from a simple rectangular waveguide equation [Poz98]:

$$f_{101} = \frac{c}{2\epsilon_r\mu_r} \sqrt{\frac{1}{x_0^2} + \frac{1}{z_0^2}}, \quad (3.1)$$

where $\epsilon_r$ is the relative permittivity, $\mu_r$ is the relative permeability, $x_0$ and $z_0$ are the two larger dimensions of the cavity.
3.1 Cavity

Fig. 3.2: HFSS simulation of the electric field inside the cavity for the (a) $\text{TE}_{101}$ mode, and (b) $\text{TE}_{102}$ mode. Field direction is indicated by the arrows, and its strength is proportional to the size of the arrows.

The smallest dimension $y_0$ of the cavity does not have an effect on $f_c$ but is rather determined by the physical size of the transmon chip substrate ($\approx 7\text{ mm by } 5\text{ mm}$), as the chip needs to fit inside and across the cavity to have the transmon parallel to the electric field. Setting $y_0 = 0.198\text{ inch} = 5.0\text{ mm}$, $x_0 = 0.85\text{ inch} = 21.6\text{ mm}$, and $z_0 = 1.378\text{ inch} = 35.0\text{ mm}$ in Eq. (3.1) yields $f_{101} = 8.163\text{ GHz}$. This is a good starting point, as the dielectric constant of the transmon substrate will lower the final frequency.

An early version of the cavity C2 that I used is shown in Fig. 3.1. It consisted of two halves that were bolted together. Machined to the dimensions given in the preceding
3.1 Cavity

paragraph, the resulting resonant frequency was $f_c = 7.98937 \text{ GHz}$ at $T = 20 \text{ mK}$.

From Fig. 3.1, one can see that the as-built cavity had rounded ends, so it was not exactly a rectangular box. To get a more accurate understanding of the modes, the physical design of the cavity was exported from Autodesk AutoCAD into Ansys HFSS [Cor15] to simulate the mode structure via finite element analysis. The resulting field distribution for the TE$_{101}$ and TE$_{102}$ modes is shown in Fig. 3.2. In my initial cavity designs, the cavity featured a horizontal notch or shelf [Fig. 3.3 (a)] in which the qubit chip would sit, with small pieces of indium holding it in place. I soon found this to be a bad design, because the qubit placement and stability were intimately coupled with how tight the two cavity halves were bolted together. Tightening the bolts too much sometimes resulted in a shattered qubit substrate. Figure 3.3 (b) shows a better design I created that decoupled the action of closing the cavity and supporting the qubit chip. It allowed secure mounting of the qubit chip independently of having the cavity tightly closed.

![Fig. 3.3: Copper cavity (a) C3D showing the old way and (b) C6A showing the new way (b) of mounting a transmon in the cavity.](image)

52
3.1 Cavity

The next step in designing the cavity was to determine how much to couple it to the outside world. Our microwave setup was wired for transmission measurements, with the cavity operating as a narrow bandpass filter. I used SubMiniature version A (SMA) straight-solder jack connectors with flange mounting (such as Amphenol 132142) to connect the coaxial input and output microwave lines to the cavity. The quality factor $Q$ characterizes how much energy is lost from the cavity through a particular decay channel, be it an external or internal one. I coupled to the input port of the cavity lightly, with a quality factor $Q^{in} = Q_{i1} = 10^6$, because I could control how hard I drove the cavity by simply changing the amplitude on the signal generators. At the same time, I coupled much more strongly to the output of the cavity ($Q^{out} = Q_{e2} = 3 \times 10^3$) to ensure that the transmission through the cavity was directional, and that I did not lose much information about the system via the input port (which I could not monitor).

The internal quality factor $Q_i$ was typically in the range of tens of thousands for my Cu cavities, and a few million for my Al cavities. This means the loaded quality factor (or the total quality factor) was dominated by the output coupling, i.e.:

$$Q_L = \frac{1}{\frac{1}{Q_i} + \frac{1}{Q_{e1}} + \frac{1}{Q_{e2}}} \approx Q_{e2}^{out} = Q_{e2} . \quad (3.2)$$

The loaded quality factor $Q_L$ is related to the relaxation rate of the cavity

$$\kappa = \frac{\omega_c}{Q_L} \approx \frac{\omega_c}{Q_{e2}^{out}} . \quad (3.3)$$

Spectroscopically, $\kappa$ is the full-width at half-maximum (FWHM) of the power resonance.
3.1 Cavity

peak, assuming there is no dephasing. The corresponding characteristic time \( \tau_\kappa = 2\pi/\kappa \)
determines how quickly photons leak out of the cavity. If the photons contain information
about the qubit, \( \tau_\kappa \) determines how much time we have to integrate the signal in order
to readout the qubit.

Some of my experiments required additional cavity design considerations. While the
ATD experiment (see Chapter 6) did not place specific constraints on \( \kappa \), creating a good
\( \Lambda \) system (see Chapter 7) required \( \kappa \gg \Gamma \), with \( \Gamma \) being the qubit relaxation rate. Using
a longer or shorter pin for the output connector allowed us to respectively increase or
decrease \( \kappa \). The dependence of \( Q_e \) on the pin length is very rapid. I simulated in HFSS
that a coupling pin that ends flush with the inner cavity surface results in \( Q_e = 418k \),
and shortening the pin by 1 mm yields \( Q_e = 46M \).

The internal quality factor \( Q_i \), pertaining to losses within the cavity, was an important
parameter that I could not completely control. The best I could do was to make it as
large as possible, so that any loss channels would be determined by \( Q_{e\text{out}} \). This was
done by mechanical and chemical polishing of the cavity walls, as discussed below in
Section 3.1.2.

Good thermalization of the cavity and the transmon within it were key factors in
achieving long coherence times. Although I started out by building Al cavities, other
researchers found that their Al cavities did not thermalize well [Sea12], while long
dehasing times could be achieved with copper cavities [Rig12]. Even though these
copper cavities only reached \( Q_i \sim 20000 \), the losses were still dominated by external
coupling \( Q_{e\text{out}} \sim 1000 \). Furthermore, the qubit relaxation through the cavity mode
(internally or to the outside) was suppressed by a Purcell factor of \((g/\Delta)^2 \approx 1/400\),
where \(g/2\pi \sim 100\text{MHz}\) is the qubit-cavity coupling, and \(\Delta/2\pi \sim 2\text{GHz}\) is the qubit-cavity detuning. After testing qubit devices Q1-Q6 with Al cavities C3A and C3B, I switched to Cu cavities C3C, C3D, C6A, and C6B for most of the remaining experiments I performed.

3.1.2 Improving the internal quality factor of cavities

As I noted earlier, it is important for cavity-qubit experiments to use a cavity with low internal loss or a large \(Q_i\). I used a few approaches to increase \(Q_i\) in both aluminum and copper cavities. Low-loss microwave cavities are very important in particle accelerators, and there have been many studies on how to reduce loss [Pro98]. A key factor is the cleanliness and roughness of the cavity surface. To reduce surface loss, I mechanically polished the inside of my cavities with various grit sizes of ScotchBrite attached to a Dremel. The effects of polishing on \(Q_i\) could be seen even at room temperature (see Table 3.1) – an encouraging sign. In order to test that the improvement at room temperature correlate positively with the behavior at low temperatures, I also measured the cavities when they were cooled to 350 mK in a \(^3\text{He}\) fridge. The results for various preparation procedures (and with substrates inserted into the cavity) are shown in Table 3.2. The table also shows results for aluminum alloy 6061-T6 (cavity C2A) and alloy 6063 (cavity C3A). The 6063 cavity had generally higher \(Q_i\) due to fewer Mn magnetic impurities in the alloy [Boa66].

I also built cavities from oxygen-free high-conductivity (OFHC) copper and subjected
them to several different chemical etch procedures (see Appendix A.1 for recipes). Etching removed work-hardened surface material that could contribute to loss. Table 3.4 shows the effect of the etches on the room-temperature quality factor of the cavities. Because copper is not a superconductor, we did not expect the low-temperature $Q_i$ to reach values as high as those of the aluminum cavities, and in the interest of saving time I only measured the losses at low temperature after polishing.

3.1.3 Summary of cavity parameters

Tables 3.1 to 3.3 provide a summary of my measurements of the internal quality factor $Q_i$ of the fundamental mode of aluminum cavities C2A and C3A at 300 K (see Table 3.1), 350 mK (see Table 3.2), and 20 mK (see Table 3.3) for various preparation conditions. Results of various etching preparations of copper cavity C3C are presented in Table 3.4.

Table 3.1: TE$_{101}$ internal quality factor for aluminum cavities C2A and C3A under various preparation conditions at 300 K. Both cavities had dimensions $y_0 = 0.198$ inch $= 5.0$ mm, $x_0 = 0.85$ inch $= 21.6$ mm, and $z_0 = 1.378$ inch $= 35.0$ mm with resonance frequencies $f_c = 7.98937$ GHz for C2A and $f_c = 7.96884$ GHz for C3A.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Cavity name</th>
<th>Al alloy</th>
<th>$Q_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unpolished (baseline)</td>
<td>C2A</td>
<td>6061-T6</td>
<td>2500</td>
</tr>
<tr>
<td>Unpolished (baseline)</td>
<td>C3A</td>
<td>6063</td>
<td>2971</td>
</tr>
<tr>
<td>Mechanically polished</td>
<td>C2A</td>
<td>6061-T6</td>
<td>2754</td>
</tr>
<tr>
<td>Sapphire mounted with Ag paste</td>
<td>C2A</td>
<td>6061-T6</td>
<td>2356</td>
</tr>
<tr>
<td>Cleaned after the Ag paste</td>
<td>C2A</td>
<td>6061-T6</td>
<td>2841</td>
</tr>
</tbody>
</table>
3.1 Cavity

Table 3.2: TE$_{101}$ internal quality factor for aluminum cavities C2A and C3A under various preparation conditions at 350 mK. Both cavities had dimensions $y_0 = 0.198$ inch = 5.0 mm, $x_0 = 0.85$ inch = 21.6 mm, and $z_0 = 1.378$ inch = 35.0 mm with resonance frequencies $f_c = 7.98937$ GHz for C2A and $f_c = 7.96884$ GHz for C3A.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Cavity name</th>
<th>Al alloy</th>
<th>$Q_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unpolished (baseline)</td>
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<td>6061-T6</td>
<td>$1.3 \times 10^6$</td>
</tr>
<tr>
<td>Unpolished (baseline)</td>
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<td>6063</td>
<td>$4.1 \times 10^6$</td>
</tr>
<tr>
<td>Mechanically polished</td>
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<td>6061-T6</td>
<td>$1.7 \times 10^6$</td>
</tr>
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<td>Sapphire mounted alone</td>
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<td>6061-T6</td>
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<tr>
<td>Sapphire mounted with Ag paste</td>
<td>C2A</td>
<td>6061-T6</td>
<td>$0.14 \times 10^6$</td>
</tr>
<tr>
<td>Sapphire mounted with In bits</td>
<td>C2A</td>
<td>6061-T6</td>
<td>$0.9 \times 10^6$</td>
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</table>

Table 3.3: TE$_{101}$ internal quality factor for aluminum cavities C2A and C3A at 20 mK. Both cavities had dimensions $y_0 = 0.198$ inch = 5.0 mm, $x_0 = 0.85$ inch = 21.6 mm, and $z_0 = 1.378$ inch = 35.0 mm with resonance frequencies $f_c = 7.98937$ GHz for C2A and $f_c = 7.96884$ GHz for C3A.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Cavity name</th>
<th>Al alloy</th>
<th>$Q_i$</th>
</tr>
</thead>
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<td>Unpolished (baseline)</td>
<td>C3A</td>
<td>6063</td>
<td>$10^7$</td>
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Table 3.4: TE$_{101}$ internal quality factor for OFHC copper cavity C3C under various chemical etches. See Appendix A.1 for the etch recipes. The cavity had dimensions $y_0 = 0.198$ inch = 5.0 mm, $x_0 = 0.85$ inch = 21.6 mm, and $z_0 = 1.378$ inch = 35.0 mm with resonance frequency $f_c = 7.958$ GHz.

<table>
<thead>
<tr>
<th>Etch process</th>
<th>$T$ (K)</th>
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<tr>
<td>Baseline (no etch)</td>
<td>300 K</td>
<td>3770</td>
</tr>
<tr>
<td>HNO$_3$ : H$_2$O</td>
<td>300 K</td>
<td>4700</td>
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<tr>
<td>HNO$_3$ : H$_2$O</td>
<td>70 K</td>
<td>12000</td>
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<tr>
<td>HNO$_3$ : H$_2$O</td>
<td>4 K</td>
<td>18700</td>
</tr>
<tr>
<td>H$_2$SO$_4$ : HNO$_3$ : H$_2$O</td>
<td>300 K</td>
<td>4600</td>
</tr>
<tr>
<td>H$_2$O : H$_2$SO$_4$ : H$_2$O$_2$</td>
<td>300 K</td>
<td>4350</td>
</tr>
</tbody>
</table>
3.2 Qubit

The transmon is the qubit in our system. The design and fabrication are much more involved than those of the cavity. Also, there are many more unknowns and random variables that were beyond my control. Even a perfectly fabricated device could fail to work well because of microscopic fluctuators within it. Nevertheless, the goal of this section to provide a guide to the design and fabrication process I used and explain the reasoning behind the steps.

3.2.1 Design considerations

As with the cavity, the transition frequency $f_q$ from the ground to the first excited state of the qubit is a key design parameter. Ideally, $f_q$ should be below the cavity mode $f_c$, and sufficiently detuned from $f_c$ to receive Purcell protection from spontaneous emission into the cavity mode \cite{Purcell46}. At the same time, $f_q$ must be high enough to not be subject to thermal excitation. With these factors in mind, I chose $f_q \sim 4 - 6 \text{ GHz}$.

The frequency $f_q$ is determined by [see Eq. (2.18)]:

$$f_q \approx \sqrt{\frac{8E_J}{E_C}} h,$$

(3.4)

where $E_C$ is the charging energy of the transmon and $E_J$ is the tunneling energy of the junction. To avoid sensitivity to charge noise, we need to have $E_J/E_C \gg 1$ (see Section 2.1). In this limit, the anharmonicity of the transmon is approximately equal to $E_C$. While we need to have $E_C$ sufficiently low to avoid charge noise, we also need
it to be sufficiently high to be able to address the lowest transition of the transmon exclusively. Typical values of $E_C/h \sim 150 - 250\,\text{MHz}$ and $E_J/E_C \sim 100$ do the trick. For $f_q \sim 4 - 6\,\text{GHz}$, Eq. (3.4) then implies $E_J/h \sim 20 - 30\,\text{GHz}$. $E_C$ is determined by the physical layout of the transmon and does not change for nominally identical devices. On the other hand, $E_J$ depends inversely on the thickness of dielectric in the transmon’s tunnel junction. Usually one expects a relatively large variation in $E_J$ from device to device even for identical fabrication procedures. Due to this, one tends to adjust $E_J$, rather than $E_C$, in order to achieve the desired $f_q$.

The next key design parameter is the qubit-cavity coupling strength. The coupling of the transmon to the cavity can be quantified by the vacuum Rabi splitting $g$. The interplay between the coupling and the transmon-cavity detuning determines whether the system will be in the strong or weak dispersive limit (see Section 2.2). In order to maximize coherence times in the presence of cavity photons, one would want to be in the weak dispersive limit, where individual photon number peaks overlap within the qubit linewidth, and the dephasing from photon shot noise is minimal. On the other hand, for the quasi-$\Lambda$ system I discuss in Chapter 7, I needed to have very distinct cavity frequencies depending on whether the qubit was in the ground or excited state. Although I went through several iterations of the transmon design, I kept the dipole approximately the same size, always resulting in a relatively large $g$, and pushed the system into the strong- or weak-coupling regime by changing the detuning.

The geometry of the transmon not only determines its coupling to the cavity but also the shunt capacitance $C_\Sigma$, which, in turn determines $E_C$. The electric field generated by
the shunt capacitance permeates the vacuum surrounding the plates on one side and the lossy substrate dielectric on the other. The participation ratio of AlO$_x$ dielectric on the shunt capacitor surfaces can be large when the shunt pads are close together, resulting in low $T_1$, as will be seen in Section 3.2.3 for some of my early transmon designs. I note that the transmon is shunted by its own pads and by the stray capacitance to the cavity. When the pads are spread too far, a significant portion of $C_{\Sigma}$ can be due to the cavity. This is especially important if the cavity is made of lossy materials, such as copper. As will be seen in Section 3.2.3, device C6AQ042314A had a rather low $T_1$, possibly for this reason. The evolution of my qubit designs is presented in Fig. 3.4.

Each of my transmons had a single Al/AlO$_x$/Al Josephson junction made by double-angle evaporation [Dol77]. The e-beam resist pattern was designed to provide a suspended resist bridge that produced a small fully overlapping area between the first and second evaporated layers. That way, the area of the junction did not depend on the evaporation angles but only on the widths of the lines defining the junction leads, nominally 100 nm.

I tuned the tunneling energy from device to device by changing the oxidation time, rather than the junction area or the oxidation pressure. To get a handle on the oxidation parameters, I made test devices at the same time nominally identical junctions. From room-temperature measurements of the junction resistance $R_J$ of the test devices, the critical current of the junction $I_c$ could be estimated using the Ambegaokar-Baratoff
Fig. 3.4: Evolution of my transmon designs. Images taken in the Keyence confocal microscope. (a) Device Q6. (b) Devices Q15 and Q16. (c) Device Q29. (d) Device Q35. (e) Devices Q42 and Q58. (f) Device Q042314A.

3.2 Qubit

relation [Amb63]:

\[ I_c = \frac{\pi \Delta_{\text{gap}}}{2eR_J} , \tag{3.5} \]

where \( \Delta_{\text{gap}} \approx 170 \mu \text{eV} \) is the superconducting energy gap of Aluminum. Expressing the tunneling energy in terms of the critical current \( I_c \) or the resistance quantum \( R_Q = \hbar/(2e)^2 \approx 6.5 \text{ k}\Omega \), we have

\[ E_J = \frac{\hbar I_c}{2e} = \frac{1}{2} \frac{R_Q}{R_J} \Delta_{\text{gap}} . \tag{3.6} \]
3.2 Qubit

Typically, I fabricated devices with $R_j = 5 - 10 \, \text{k}\Omega$ to achieve the desired $E_j$.

3.2.2 Fabrication

The pattern for both the large shunt capacitor pads, and the transmon junction are fabricated in the same e-beam step (see Appendix A.3 for step-by-step instructions). The tunnel junction is made via the Dolan suspended bridge technique [Dol77]. To create the suspended bridge lithographically, I uses a bi-layer resist stack. The bottom layer consists of MMA(8.5)MAA EL11 co-polymer [Cor01], which is much more sensitive to e-beam exposure than the top layer of ZEP520A [Cor10]. After development in Amyl Acetate and water/IPA mixture, this results in the ZEP being undercut so much that the MMA is completely absent under the junction bridge, as required for the suspended bridge technique. Figure 3.5 gives an overview of the procedure.

![Fig. 3.5: Junction fabrication steps. (a) E-beam exposure of resist stack in the SEM. (b) Development of the resists. (c) First evaporation of Al (black) at angle $\theta_1$. (d) Oxidation of the first Al layer to create the $\text{AlO}_x$ junction dielectric (red). (e) Second Al evaporation (black) at angle $\theta_2$. (f) The junction after lift-off.](image-url)
To prepare device-sized chips for e-beam writing in the scanning electron microscope (SEM), the following steps are done. First, I spun MMA(8.5)MAA EL11 resist at 1000 RPM for 60 s on a clean 3” sapphire wafer, resulting in nominal resist thickness of 800 nm. The wafer was then baked on a hotplate at 180 °C for 5 minutes to evaporate the solvent from the resist. The wafer was then placed on the surface of a metal table to cool to room temperature before the next step. I then spun on ZEP520A resist with a dilution ratio (DR) of 2.3 at 5000 RPM for 60 s, resulting in nominal resist thickness of 100 nm. The wafer was then baked on a hotplate at 180 °C for 5 minutes, followed by a hard bake in an oven at 180 °C for 30 minutes to further harden the resist.

Because the sapphire substrate is a very good insulator at room temperature, electrons accumulate on the substrate during the SEM writing step, resulting in charging, deflection of the e-beam, and ultimately causing a distortion in the pattern being written. To prevent charging, I deposited 10 nm of Al in a thermal evaporator on the top of the resist stack on the wafer. This anti-charging layer was thin enough to allow electrons through during writing, yet thick enough to conduct scattered electrons away from the wafer once their work exposing the resist was done. At this point, both the resist stack and the anti-charging layer are fragile and need to be protected from dust and debris. In order to protect the wafer during the dicing step, I spun FSC-M photoresist at 2000 RPM for 60 s, resulting in a few microns of protective layer on top of the anti-charging layer. The wafer was then baked on a hotplate at 120 °C for 3 min and 30 s. Since FSC-M is sensitive to UV light, I wrapped the wafer carrier in aluminum foil to keep light out when not in use. I then diced the wafer into 5 mm × 7 mm chips using a CX-010-325-080-H hub resinoid
3.2 Qubit

diamond blade on a Disco-DAD 321 dicing saw. For a 3” wafer, this typically gives about 70 usable chips.

Before e-beam writing, I cleaned the FSC-M off by immersing a chip in beakers with Acetone, Methanol, and IPA for 30 s each, followed by N₂ blow drying. I then put Ag nanoparticles mixed with IPA on one corner of each chip to provide a good starting point for focusing. The chips were then mounted in the JEOL JSM-6500F SEM, and I used the following procedure to write the transmon pattern. With the working distance (WD) set to 7 mm (required by the NPGS writing software), I found a rough focus by moving the stage with the devices up or down. After a rough focus was achieved, the beam itself was focused finely without significantly perturbing the WD. This was done by going to the maximum magnification of the SEM (500000×) and parking the beam in one spot for a few seconds. If the focus is good, one will see afterwards a contamination spot of less than 10 nm in diameter.

After a good focus was achieved, I next performed stigmation correction. Good stigmation will make contamination spots look round, and will also improve the focus. Because the chip might be slightly tilted in the holder, it was important to check the focus with contamination spots as one moves in closer to the center of the chip, where the junction is to be written. The last such check was typically done 150 µm away from the center of the chip. The beam was then blanked, and the stage moved to the writing area blindly. The junction was then written at 500× magnification using an area dose of 175 µC/cm² at a typical beam current of about 30 pA. I arrived at this particular area dose after fabricating test devices with a range exposures (see Appendix C). The rest of
the pattern, containing the shunt capacitor pads, was then written at 75× magnification and 100µC/cm² with a typical beam current of about 800 pA. After removing the chip from the SEM, the anti-charging layer was stripped in OPD4262 for 60 s, followed by a DI rinse for 60 s, and a 3 s IPA dip, before being blown dry. To develop the top layer (ZEP), I used Amyl Acetate, which is a generic alternative to the commercial ZED-N50 and works just as well. The chips were agitated in the developer for 2 minutes, then rinsed with IPA for 60 s, and blown dry. Figure 3.6 (a) shows an image of the junction pattern at this step, taken in a Keyence confocal microscope. The bottom layer (MMA) was next developed in 5:1 solution of IPA : H₂O for 4 min and 20 s, then rinsed with DI water for 60 s, and blown dry [see Fig. 3.6 (b)]. I found that this particular development time gave me a good undercut without compromising the structural integrity of the suspended bridge.

![Image](image-url)  
**Fig. 3.6:** (a) Optical image of the junction after developing the ZEP and (b) the MMA layers.
3.2 Qubit

Fig. 3.7: SEM images showing (a) junction pattern, (b) close-up of the bridge, and (c) tilted view showing junction leads and the resist undercut after the double-angle evaporation.

After development, chips were mounted in a thermal evaporator, and the chamber was pumped down to the base pressure of $10^{-7}$ Torr by a turbomolecular pump. The first evaporation of Al was done at angle $\theta_1 = 12.5^\circ$ from the normal, at rates of $4 - 7 \text{Å s}^{-1}$, resulting in a 30 nm bottom layer. The devices were then oxidized in 120 – 150 mTorr of research-grade O$_2$ for 3 – 5 minutes, depending on the desired $R_J$. Finally, the second evaporation was done at $\theta_2 = -12.5^\circ$ at rates of $4 - 7 \text{Å s}^{-1}$, resulting in a 50 nm Al top layer. The top layer was made thicker intentionally, so that the continuity of the metal would be preserved over the step where the junction leads overlap.
3.2 Qubit

Before lift-off, some devices were imaged in the Zeiss LEO 1550VP SEM (see Fig. 3.7) to assess the quality of the junction bridge. Overall quality of fabrication can be gleaned from Fig. 3.7 (a). Problems with the junction bridge, such as a collapse or break in it, are apparent in the close-up [see Fig. 3.7 (b)]. A tilted view of the junction leads [see Fig. 3.7 (c)] let me assess the amount and quality of the resist undercut.

Following the deposition, a lift-off was done in N-Methyl-2-pyrrolidone (NMP) heated to $70 - 80^\circ$C. The devices were then rinsed in DI water for 60 s, IPA for 60 s, and blown dry. After lift-off, I imaged devices in the confocal microscope [see Fig. 3.8 (a)] to see if the overall transmon pattern looked correct. A thin gold layer was sputtered in a

![Fig. 3.8: Junction after lift-off. (a) Confocal optical micrograph. (b) Image of the whole junction pattern measured in the SEM. Close-up of the junction measured in the SEM taken looking (c) head-on and (d) tilted.](image)
Denton Vacuum Desk II on some of the test devices (including ones that failed) to form an anti-charging layer, and they were later imaged in the LEO SEM [Fig. 3.8 (b)-(d)] to determine the junction areas and examine the quality of the resulting devices.

3.2.3 Summary of transmon devices built

For my thesis, I ended up building several transmon devices. Characterizing these devices helped me learn how to perform quantum control, improve coherence times, and debug the experimental system. A few of these devices stand out.

Transmon Q15 was cooled down in both our Oxford-100 and our Leiden CF-450 dilution refrigerators. I was surprised to see the coherence times increase by an order of magnitude in the Leiden. Apparently, the Leiden provided better thermal anchoring for the device. It also had Cryoperm magnetic shielding, and additional filtering on the microwave lines (K&L 12 GHz low-pass filters). This result furthered our belief that proper low-temperature microwave setup and shielding were extremely important in protecting the qubits from extrinsic noise and achieving long coherence times. The Leiden setup will be discussed in detail in Section 4.1.

I also found that cycling devices to room temperature seemed to help with the coherence times in some cases. For example, when Q29 was re-cooled, I noticed disappearance of a fluctuator that was plaguing the measurements during the first cooldown, accompanied by an almost 30% increase in the coherence times (from $T_1 = 31 \mu s$ to $T_1 = 39 \mu s$). It is possible that thermal cycling of devices helps relax metal or junction stress, or just randomly rearranges two-level systems.
3.2 Qubit

Finally, I note that Q58 had the longest $T_1$ for any transmon in our lab, up to $T_1 = 171 \mu s$, but unfortunately suffered from large charge dispersion (since $E_J/E_C = 38$), resulting in large inhomogeneous broadening and thus relatively low $T_2^* \approx 29 \mu s$.

Measurements of my transmons are discussed in detail in Chapter 5, and key parameters are summarized in Table 3.5.
Table 3.5: Summary of measured transmon and cavity device parameters.

<table>
<thead>
<tr>
<th></th>
<th>Q6</th>
<th>Q15</th>
<th>Q15</th>
<th>Q16</th>
<th>Q29</th>
<th>Q29</th>
<th>Q35</th>
<th>Q42</th>
<th>Q58</th>
<th>Q042314A</th>
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<tr>
<td>Cavity</td>
<td>C3A</td>
<td>C3D</td>
<td>C3D</td>
<td>C3C</td>
<td>C3D</td>
<td>C3D</td>
<td>C3D</td>
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<td>C6B</td>
<td>C6A</td>
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<td>$\omega_c/2\pi$ (GHz)</td>
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<td>7.567</td>
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<td>$\omega_{ge}/2\pi$ (GHz)</td>
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<td>130</td>
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<td>5.2</td>
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<td>$T_2$ (µs)</td>
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<td>70</td>
<td>11.2</td>
<td>42.5</td>
<td>50.8</td>
<td>27.7</td>
<td>8.6</td>
<td>24</td>
<td>7.4</td>
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<tr>
<td>$T_2$ (µs)</td>
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<td>–</td>
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<tr>
<td>Notes</td>
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<td>$T_1$ limited by pads surface participation</td>
<td>$T_1 = 83$–171 µs</td>
<td>$T_1$ limited by transmon shunting through cavity</td>
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</tr>
</tbody>
</table>
3.3 Full-system simulation with black-box quantization

Recently, a new method called black-box quantization (BBQ) was developed [Nig12] for modeling the low-energy spectrum of a transmon-cavity system. The crux of this technique lies in treating the transmon as a harmonic oscillator to zeroth order, and simulating the whole system as two coupled harmonic oscillators, with transmon-induced non-linearities added later. The simulation of two coupled cavities automatically takes care of the hybridized transmon-cavity modes. This approach is useful in the design of transmon-cavity systems, which is why I briefly review it here.

In practice, this method works as follows. The cavity, coupling pins, vacuum, and the substrate are drawn in AutoCAD and exported to Ansys HFSS [Cor15]. The transmon is then defined as a 2D metallization in HFSS, with a lumped port instead of the junction. The system is simulated in HFSS to find the admittance $Y_{\text{sim}}(\omega)$ that the junction port sees due to both the cavity, and the shunt capacitor pads of the transmon. This way, both the shunt capacitance and the transmon-cavity coupling are automatically included in the simulation. The admittance $Y_{\text{sim}}(\omega)$ is interpolated in Mathematica, and the junction admittance (due to $L_J$ and $C_J$) is then added analytically to obtain the total admittance:

$$Y(\omega) = i\omega C_J + \frac{1}{i\omega L_J} + Y_{\text{sim}}(\omega) \quad .$$

(3.7)

From the total admittance $Y(\omega)$, the low-energy spectrum of the system (transmon and cavity modes), as well as transmon-cavity coupling, can be calculated. Zeroes of the impedance are the modes of the coupled linear system, with the lowest mode with
3.3 Full-system simulation with black-box quantization

the largest slope typically being the qubit mode. Let $\omega_q$ and $\omega_c$ be the zeroes of the admittance corresponding to the qubit and the cavity modes, respectively. Let subscripts $q$ and $c$ denote the qubit and cavity modes, respectively. For any mode $p = \{q, c\}$, let

$$\text{Im} Y'_p = \frac{d\text{Im} Y(\omega)}{d\omega} \bigg|_{\omega=\omega_p}$$

be the derivative of the admittance at the zero of the admittance $\omega_p$. Then, the capacitance due to the environment (of either the qubit or the cavity mode) is

$$C_p = \frac{1}{2} \text{Im} Y'_p.$$  

(3.9)

The lifetime of the qubit due to losses (both due to Purcell effect and any lossy materials included in the simulation) is then:

$$T_1 = \frac{\text{Im} Y'_q}{2\text{Re} Y(\omega_q)}.$$ 

(3.10)

The mode impedance seen by the port is

$$Z_p = \frac{2}{\omega_p \text{Im} Y'_p}$$  

(3.11)

and the mode inductance is

$$L_p = \frac{1}{\omega_p^2 C_p}.$$ 

(3.12)
BBQ also allows me to calculate corrections due to the junction’s non-linearity. The anharmonicity (i.e. self-Kerr) of either qubit or cavity modes is:

\[
\alpha_p = -\frac{L_p e^2}{2 L_J C_p}.
\]  

(3.13)

The qubit-cavity dispersive shift (i.e. cross-Kerr) is given by

\[
\chi = -2 \sqrt{\alpha_q \alpha_c}.
\]

(3.14)

The Lamb shift of the qubit frequency is

\[
\Delta f_q = -\frac{e^2}{2 L_J} \left[ Z_q (Z_q + Z_c) - \frac{Z_q^2}{2} \right].
\]

(3.15)

and the Lamb shift of the cavity frequency is

\[
\Delta f_c = -\frac{e^2}{2 L_J} \left[ Z_c (Z_c + Z_q) - \frac{Z_c^2}{2} \right].
\]

(3.16)

I employed the BBQ technique to estimate parameters of several devices, as shown in Table 3.6. The main difficulty was my inability to measure \(R_J\) (and, hence, estimate \(L_J\)) on the exact devices that were cooled down, because I did not want to risk damaging the junctions through electrostatic discharge (ESD). Instead, I determined \(E_J\) from qubit spectroscopy. A well-setup probe station and strict ESD precautions should be able to alleviate this difficulty. Furthermore, the junction capacitance \(C_J\) was not accurately
3.3 Full-system simulation with black-box quantization

Table 3.6: Summary of simulated transmon devices, with a comparison to experimentally extracted values.

<table>
<thead>
<tr>
<th>Device</th>
<th>C3AQ6</th>
<th>C3DQ15</th>
<th>C3DQ42</th>
<th>C6BQ58</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>exp</td>
<td>sim</td>
<td>exp</td>
<td>sim</td>
</tr>
<tr>
<td>$\omega_c/2\pi$ (GHz)</td>
<td>7.594</td>
<td>7.51</td>
<td>7.567</td>
<td>7.53</td>
</tr>
<tr>
<td>$\omega_{ge}/2\pi$ (GHz)</td>
<td>8.767</td>
<td>8.63</td>
<td>4.265</td>
<td>5.14</td>
</tr>
<tr>
<td>$E_C/h$ (MHz)</td>
<td>400</td>
<td>340</td>
<td>176</td>
<td>198</td>
</tr>
<tr>
<td>$g_{tot}/2\pi$ (MHz)</td>
<td>188</td>
<td>86</td>
<td>97</td>
<td>65</td>
</tr>
<tr>
<td>$T_1$ (µs)</td>
<td>0.43</td>
<td>416</td>
<td>91</td>
<td>1000</td>
</tr>
</tbody>
</table>

known, and was set to a nominal value of 5 fF, which is typical for junctions. Because the environment shunt capacitance $C_q \sim 100 \text{fF} \gg C_J$, it is not as crucial to know $C_J$ precisely. The simulation results show that BBQ was reasonably good (10% accuracy) at predicting the low-energy spectrum of the transmon-cavity system. However, it usually underestimated the transmon-cavity coupling $g$ by a factor of about 2. Because perfect electrical conductor material was used to simulate the cavities, any losses associated with cavity walls participating in the transmon shunt capacitance were not included. Furthermore, as seen most prominently with device C3AQ6, the simulations did not predict $T_1$ accurately when it was low due to large participation of lossy dielectrics. Nevertheless, combined with room-temperature junction resistance measurements, it was still a very useful tool for designing the qubits.
CHAPTER 4

Experimental setup

This chapter describes in detail various aspects of the experimental setup I used to acquire the results I describe in Chapters 5 to 7. I first discuss cryogenics and low-temperature microwave engineering that are relevant to the apparatus. The systems I studied consisted of resonant microwave cavities with embedded transmon qubits. I probed the system via microwave transmission, and this led to a clear separation between the input and the output parts of the setup. On the input side, pulse shaping of the input microwave signals was done using either arbitrary waveform generator (AWG) or a specialized digital-to-analog converter (DAC) board. The transmitted output microwave signal from the dilution refrigerator was mixed down and demodulated using the setup I describe in Section 4.7. Based on the type of input signals that needed to be sent to the device, the measurements I performed could be sorted into four broad categories: continuous microwave transmission measurements through the device, pulsed spectroscopic measurements of the steady state of the qubit-cavity system, pulsed time-domain coherence measurements of the system (Rabi, Ramsey, spin-echo,
tomography), and pulsed transmission measurements through the device. In Sections 4.2 to 4.5 I provide details on how I performed each type of measurement.

4.1 Leiden dilution refrigerator cryogenic setup

Most of the measurements I made were done in the Leiden Cryogenics CF-450 dilution refrigerator. The Oxford-100 refrigerator, used in a few of my experiments, is described in detail by B. Suri [Sur15].

The Leiden refrigerator was placed in an RF-tight shielded room, with microwave and vacuum lines fed through the room walls. The gas handling system, the compressor, and the pumps of CF-450 were positioned in an adjacent pump room. The walls of the pump room were lined with thick sheets of rubber to reduce vibrations and acoustic noise from the compressor and the pumps.

A photograph of the cold parts of the CF-450 setup is shown in Fig. 4.1. The Leiden refrigerator has multiple stages that operate at successively lower temperatures, ultimately reaching the base temperature of about 15 mK at the mixing chamber. The CF-450 is cryogen-free, with the 3K stage cooled by a Cryomech compressor system, and the mixing chamber has a cooling power of 450 µW at 100 mK.

After mounting the device (typically a copper cavity with an embedded transmon) on an OFHC copper tail bolted to the mixing chamber plate, I attached two Cryoperm magnetic shields around the tail. I then bolted on the gold-plated copper radiation shields (one on the cold plate, one on the still plate). The stainless steel inner-vacuum can (IVC) was then attached, providing a vacuum tight environment as well as shielding
4.1 Leiden dilution refrigerator cryogenic setup

Fig. 4.1: Photographs of the Leiden CF-450 dilution refrigerator setup, showing (a) the overall setup, (b) the 3K and still plates, (c) the cold plate and the top of the mixing chamber plate, (d) the bottom of the mixing chamber plate where the devices are mounted.

at 3 K. I then attached the 50K shield, and, finally, the outer-vacuum can (OVC). The OVC provided the final vacuum seal from the outside world.

I next pumped out the IVC and the OVC, turned on the pulse-tube compressor, and flowed liquid nitrogen through the system to pre-cool it to 77 K. The flow of N\textsubscript{2} was then stopped, and the pulse-tube cooled the rest of the way to 3 K. Then, I used the
4.1 Leiden dilution refrigerator cryogenic setup

Leiden’s software control to condense and circulate the $^3\text{He} - ^4\text{He}$ mixture, resulting in mixing chamber plate being cooled down to the base temperature in about 4 hours. Detailed steps for the cooldown and warmup procedures can be found in Appendix B.

4.1.1 Filtering and isolation

The refrigerator was mounted in an RF-shielded room, and all lines were filtered to prevent RF from reaching the device. At temperatures above $\sim 100\text{ mK}$ and microwave frequencies $\sim 5\text{ GHz}$, the Nyquist-Johnson thermal noise is fairly large. That is, if $100\text{ mK}$ microwave radiation reaches the cavity or the transmon, it can cause decoherence due

Fig. 4.2: Schematic of the microwave lines in the Leiden refrigerator.
to increased thermal excitation of the cavity and the qubit. Because I needed large
bandwidths to operate my devices (to be able to access both $\omega_q$ and $\omega_r$), and the noise
was in the same band as the signal, it was not possible to selectively filter out thermal
noise. Instead, I heavily attenuated both the signal and the noise. This meant that I
needed to increase the amplitude of the applied signal sent to the device. I was able to
attenuate the noise so that it would not affect the device while keeping the signal as
large as I needed. A schematic of the microwave lines showing attenuation at different
stages in the refrigerator is presented in Fig. 4.2. Stray radiation above 12 GHz was
filtered at the mixing chamber by K&L 11L250-12000/T20000 low-pass filters on the
input and output lines.

After passing through the qubit-cavity system, the transmitted microwave signal
carried useful information about the state of the system. Therefore, it was imperative to
have as little attenuation as possible on the output side of the device. However, I still
needed to protect the device against the noise coming down from room temperature and
the refrigerator’s higher-temperature stages on the output side. This was done by means
of isolators – passive non-reciprocal components that allow propagation of microwaves
in one direction. Two Pamtech isolators were connected in series with the device output
port at the mixing chamber stage, providing $\sim 40$ dB of isolation in the $4-12$ GHz range.

From the output of the isolators, the signal was passed through a low-loss Nb super-
conducting cable to a Caltech HEMT amplifier attached to the 3 K stage. The amplifier
had a noise temperature $T_N < 5$ K and gain $G \approx 35$ dB within its operating range of
$4-12$ GHz.
4.2 Continuous-wave transmission measurements

The most basic measurement I made on the experimental system was to find the microwave transmission (see Fig. 4.3). To do this, I used an Agilent E5071C vector network analyzer (VNA) to measure the scattering parameter $S_{21}$:

$$S_{21} \equiv \frac{V_{\text{out}}}{V_{\text{in}}},$$

where $V_{\text{in}}$ is the voltage amplitude that the VNA sends from port 1, and $V_{\text{out}}$ is the voltage amplitude that the VNA receives at port 2.

The cavity was connected in a bandpass configuration. This leads to a Lorentzian

![Diagram](image.png)

**Fig. 4.3:** (a) Simple $S_{21}$ transmission measurement setup with a VNA. (b) $S_{21}$ transmission measurement in the presence of a second continuous tone ($f_2$). I used this arrangement to obtain EST data in Section 7.8.
4.3 Spectroscopic measurements

peak when the magnitude $|S_{21}|$ of the transmission is plotted as a function of frequency. The center frequency of this peak is due to the cavity resonance, which shifts depending on the state of the qubit (see Section 2.2). This behavior means that by measuring $S_{21}$ I could perform a dispersive readout of the qubit state [Sch07]. The location of the peak also depends non-linearly on the amplitude of the input signal sent to the cavity [Ree10]. I note that I used the presence of this non-linearity to verify that a working transmon was coupled to the cavity. The amount of frequency shift in the cavity resonance peak, and its dependence on power, was used to approximately determine the qubit-cavity coupling $g$, as I discuss in Chapter 5. Furthermore, as I describe in Section 5.1, I also used this nonlinear power-dependent behavior to perform qubit state readout [Ree10] with a high signal-to-noise ratio (SNR).

4.3 Spectroscopic measurements

Measurements of the qubit-cavity spectrum were done by pulsing spectroscopy tones as well as the readout tone. Figure 4.4 shows the experimental setup, while parts (a) and (b) of Fig. 4.5 show the pulse timing. I used an Agilent 33250A AWG to trigger two readout microwave pulses, as well as a second 33250A AWG. I set up this second AWG to trigger spectroscopy pulses of the probe (and coupler, if it was used) microwave source. A small fraction of the readout signal was split off and sent to the demodulation box to provide a reference (see Section 4.7 for demodulation and acquisition details). The spectroscopy and readout pulses were combined using a Pasternack 2215-10 directional coupler, and sent to the refrigerator input line. The first readout pulse measured the ground-state
4.3 Spectroscopic measurements

Fig. 4.4: Schematic of the spectroscopy and typical time domain measurements setup. Double dashed lines indicate trigger pulses. Grey components are not necessary for single-tone spectroscopy, Rabi, and Ramsey experiments. Single-dashed purple lines denote RF signal paths inside the demodulation box. The details of the box are discussed in Section 4.7.

qubit population $\rho_{\text{tot}}$ before any spectroscopy pulses. Then, the spectroscopy pulses were turned on for a time (typically $> 30 \mu s$) much longer than any coherence times or relaxation times of the system. This allowed the system to reach a driven steady state. Immediately after the spectroscopy pulses, a second readout pulse was sent find $\rho_{\text{tot}}$. The readout pulses typically lasted $5 \mu s$, and optimization of the readout is discussed in Section 5.1.

With proper choice for the frequency and amplitude of the applied readout pulse, the
transmitted readout pulse amplitude was proportional to the total qubit excited state population

\[ \rho_{\text{tot}} \equiv \sum_{j=e,f,\ldots} |\langle \psi | j \rangle|^2 = 1 - |\langle \psi | g \rangle|^2, \]

(4.2)

where \( |\psi \rangle \) is the state of the qubit, \( |g \rangle \) is the ground state of the qubit, and \( |e \rangle, |f \rangle, \ldots \) are the qubit excited states.

After leaving the refrigerator, the transmitted readout pulse was further amplified by a Miteq AMF-4F-04000800-12-10P low-noise amplifier (LNA), and then mixed down to an intermediate frequency (IF) \( f_{\text{IF}} = 0.5 - 10 \text{ MHz} \) inside the demodulation box (described in detail in Section 4.7). The readout reference and signal pulse waveforms were then acquired at 1 ns time steps, averaged by an Agilent U1082A data acquisition (DAQ) card, and then demodulated in LabVIEW to give the in-phase and quadrature of the

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**Fig. 4.5:** Typical pulse sequences sent to the device. (a) Single-tone spectroscopy. (b) Two-tone spectroscopy. (c) Rabi oscillations for a Rabi pulse of length \( t \). (d) Ramsey fringes for free evolution time \( t \) between the \( X_\pi/2 \) rotations. (e) Spin-echo with delay \( t/2 \) between pulses. (d) State tomography, consisting of a preparation rotation \( R_\theta \) followed by a tomography rotation \( R_{\text{tomo}} \). Labels \( \rho_{\text{tot}} \) denote the readout pulses. The wait time \( t_w \) was variable so that the total repetition time of the whole pulse sequence remained constant.
4.3 Spectroscopic measurements

transmitted signal.
4.4 Time-domain measurements

The time-domain measurements I made used a setup that was similar to the one used for spectroscopy, except the system was not necessarily driven into the steady state by long pulses. Instead, I used short pulses (from $\sim 1$ ns to $\sim 100\mu$s) to perform Rabi, Ramsey, spin-echo or tomography experiments. Typical pulse sequences are shown in Fig. 4.5 (c)-(f). The typical experimental setup was the same as the one I used for spectroscopy measurements (see Fig. 4.4), with AWGs triggering the microwave sources to shape the pulses as needed. The exceptions were spin-echo and tomography experiments, in which I used a DAC board to create the pulses, and an IQ mixer to achieve control of their amplitudes and phases. Section 4.5 describes the details of the spin-echo and tomography setup.

4.5 Nanosecond vector pulse control

To perform spin-echo and tomography, vector control (amplitude and phase) of the microwave pulses was required. In this section, I describe the pulse shaping system that I build which uses a DAC board to modulate in-phase and quadrature components of a continuous microwave signal via an IQ mixer (see Fig. 4.7). The DAC board was designed by Prof. John Martinis [Mar11] for controlling superconducting phase qubits, and was built by Steve Waltman of High Speed Circuit Consultants [Wal]. The aim of the design was to achieve low noise and high scalability. The IQ mixer is a readily available commercial component [Mar].
4.5 Nanosecond vector pulse control

Fig. 4.6: Schematic of the DAC setup for spin-echo and tomography measurements.

4.5.1 DAC specifications

A simplified schematic of the DAC board is presented in Fig. 4.7. The board operates at 1 GSa/s with 14-bit voltage resolution. A loss of 2 bits of voltage resolution is typical for high-speed DACs; 14-bit output ensures sufficient accuracy of the generated waveforms. An Altera Stratix II EP2S15 field-programmable gate array (FPGA) is used to store, manage and generate the digital data. The waveform data is loaded into the FPGA’s static random-access memory (SRAM). The size of the SRAM (∼32 kB) limits the waveforms to approximately 8 µs at the maximum resolution of 1 ns per word (1 word = 4 bytes). The board is driven with a 10 MHz reference, which is then used to generate
4.5 Nanosecond vector pulse control

Fig. 4.7: Schematic of main DAC components. On-board parts are shown inside the dashed box.

the main 1 GHz clock via a voltage-controlled oscillator (VCO) and a phase-locked loop (PLL).

Control of the FPGA and transfer of data is achieved through a 100BASE-T Ethernet port (re-programming of the FPGA can be done via a serial port). The low-level IEEE 802.3 standard used requires a dedicated network interface card (NIC) when interfacing with a computer. Multiple boards can be connected via a low-voltage differential signaling (LVDS) daisy chain. Thus, only one NIC is needed to address multiple boards. Each board can be programmed to have a unique internet protocol (IP) address, with the lowest 6 bits defined by an on-board dual in-line package (DIP) switch.
4.5 Nanosecond vector pulse control

Two Analog Devices 9736 DACs (DACA and DACB in Fig. 4.7) generate the output, thereby creating two independent channels for separate I and Q control. Under a 50 Ω load, the differential output voltages have a maximum range of ±500 mV. Although this is too low to drive an IQ mixer directly, these outputs can be amplified as required. In addition to the two differential analog DAC outputs, the board also has four differential digital outputs that use positive emitter-coupled logic (PECL). These outputs allow simple timing, triggering, and on-off commands to other instrumentation at the maximum sampling rate.

4.5.2 Basic commands

Ethernet commands to the DAC board are of three types: memory-write, SRAM-write and register-write. The board recognizes each type of command from the length of the IP packet it receives. Memory-write commands are 769 bytes in length, SRAM-write are 1026, and register-write are 56.

Memory-write packets are written to FPGA's internal memory (not SRAM). They carry a set of 256 operations to be executed by the board. The first byte in the packet sets the memory page where the following 256 commands are written. At present, only two pages are available, so the address is either 0 or 1. The next 768 bytes are a set of 256 operations to be executed by the board. Each operation is encoded in a 3-byte word, as shown in Table 4.1. Other commands are possible but are not relevant for this dissertation. As an example, a hexadecimal command to output 0 through 1024 ns of SRAM is written as
4.5 Nanosecond vector pulse control

Table 4.1: Hexadecimal commands for DAC memory operations.

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0x000000</td>
<td>No operation</td>
</tr>
<tr>
<td>0x3abcde</td>
<td>Delay extra abcde+1 clock cycles before next command</td>
</tr>
<tr>
<td>0x8abcde</td>
<td>Set SRAM start address to abcde</td>
</tr>
<tr>
<td>0xAabcde</td>
<td>Set SRAM end address to abcde</td>
</tr>
<tr>
<td>0xC00000</td>
<td>Call SRAM sequence</td>
</tr>
<tr>
<td>0xF00000</td>
<td>End of memory command sequence</td>
</tr>
</tbody>
</table>

The first byte (00) is the page address, followed by SRAM start address (000080), SRAM end address (0004A0), a call command (0000C0), and an end-of-sequence command (0000F0). Note that the commands are in little-endian format [Wik], i.e. the least-significant byte is written first. In order to ensure proper packet length, 756 bytes of zeroes should be appended to this command before sending it to the board. The memory is clocked at 25 MHz, so the minimum delay command is 40 ns. Programmable delays allow low-duty-cycle pulse sequences of lengths $\gg 8 \mu s$ where short pulses are encoded at maximum resolution, and long on-off pulses of $\sim 100 \mu s$ are present in between. By encoding various delays and calling specific parts of SRAM it is possible to create complicated pulse sequences and even trigger external instrumentation.

SRAM-write commands consist of 2 address bytes followed by 256 4-byte words. Each word corresponds to 1 ns of output and encodes amplitudes of both analog and the PECL digital outputs. The address is any number from 0 to 8192 but for simplicity I use $n \times 256$, $n = 0, 1, ..., 31$. This ensures writing to SRAM in 256-word blocks. Similar to memory commands, these are written in little-endian format. Bits 0-13 of each word
4.5 Nanosecond vector pulse control

after the address bytes encode DACA output, bits 14-27 encode DACB output, and bits 28-31 switch on and off the four serial PECL outputs. Therefore, each 32-bit SRAM word can be written as

\[ W = DAC_A + 2^{14}DAC_B + 2^{28}S_0 + 2^{29}S_1 + 2^{30}S_2 + 2^{31}S_3, \]

(4.3)

where \(-8192 \leq DAC_i \leq 8191\) are the desired analog output values written in 2’s complement form and \(S_i = 0, 1\) determine if \(i^{th}\) PECL serial channel is low or high, respectively.

Register-write commands provide essential control over the board. The first byte determines whether to stop (0), start executing memory commands (1), perform continuous output from registers (2), perform continuous output from the SRAM (3) or perform single cycle of SRAM output (4). Most of the data in each register-write packet are for advanced features such as multiple board synchronization. Bytes 14-19 are of particular importance. When performing execution of memory commands (start byte = 1), bytes 14-15 represent the number of times the memory sequence is called. During continuous SRAM output (start byte = 3), bytes 14-16 and 17-19 represent the starting and ending points of the SRAM block being output, respectively.

The procedure for programming the DAC board can be summarized in the following steps:

1. Connect to the Ethernet adapter.

2. Blank DAC memory.
4.5 Nanosecond vector pulse control

3. Blank DAC SRAM.

4. Generate and write combined DAC and PECL serial pulses to SRAM.

5. Generate and write commands that control execution to FPGAs internal memory.

6. Use register-write command to synchronize multiple boards and/or start execution of memory commands.

7. Repeat steps 2-6 as needed.

4.5.3 LabVIEW implementation

In order to access low-level network layers on Windows OS, I used the WinPcap library [Win]. While other research groups implement their code to control the DAC board in Python, I found it easier to use LabVIEW bindings for WinPcap [Lab]. This allowed quick integration with the existing LabVIEW code used to control our experiments. In order to facilitate rapid application development and encourage modularity, many LabVIEW subVIs have been created. Each subVI performs a very specific function, similar to a sub-routine in other programming languages. All raw hexadecimal code is hidden inside the subVIs, so in the spirit of object-oriented programming one can build up complete LabVIEW programs from these subVIs without needing to know all the exact details of how to program the board. An example of such a subVI is presented in Fig. 4.8. Because LabVIEW is a graphical language, it is difficult to present the code in its entirety here. The code can be requested from the author by email.
4.5.4 IQ mixer

A mixer uses nonlinear devices such as diodes to generate an output that is the product of two input signals. The resulting output contains the sum and difference frequencies of two input signals [Poz98]. In my case, one of the input signals was the local oscillator (LO), while the other was the IF signal. The resulting radio frequency (RF) output has frequencies $f_{RF} = f_{LO} \pm f_{IF}$. The process of mixing can be viewed in simple terms as modulating LO with the IF signal. When $f_{IF} \ll f_{LO}$, we have $f_{RF} \approx f_{LO}$ and the IF signal can be used for amplitude modulation (as opposed to side-band mixing). A quadrature IF mixer (or “IQ mixer”) has two input IF ports. The in-phase (I) port signal is mixed with the LO while the quadrature (Q) port is mixed with a $\pi/2$ phase-shifted LO. Hence,
4.5 Nanosecond vector pulse control

the output in the time domain is

\[ y_{RF}(t) = I(t) \cos(\omega_{LO} t) + Q(t) \sin(\omega_{LO} t) \]  \hspace{1cm} (4.4)

with \( \omega_{LO} \equiv 2\pi f_{LO} \). Here I will assume that the IF is a DC signal with very slow time-dependence such that \( f_{IF} \ll f_{LO} \), so sideband mixing can be neglected. By applying DC voltages to both the I and Q ports one can obtain arbitrary phase and amplitude modulation of the LO signal.

In our system we use a Marki IQ-4509 mixer with a nominal LO/RF working band of 4.5 – 9 GHz, 30 dB LO-RF isolation, 0.3 dB dB amplitude and 4° phase deviations [Mar]. While these figures might be good enough for many commercial applications, in my qubit experiments I needed to achieve more precise control of microwave pulses. The two most significant imperfections that plague IQ mixers are the LO leakage and the arm imbalance [Bal02].

For quantum information processing applications, a high on/off ratio of the drive microwave pulses ensures the qubit is not being unintentionally excited. In an ideal IQ mixer, when voltages on the I and Q ports are set to zero, the output RF signal will also be zero, providing an infinite on/off ratio. Due to imperfect diodes, interference and other effects, a real mixer allows some LO power to leak through even when \( I = Q = 0 \). However, it is generally possible to find minimum transmission through the mixer somewhere else in the I-Q plane. In order to find the offset coordinates \((I_0, Q_0)\) required to give zero RF output for the various LO frequencies of interest, I used the setup shown
in Fig. 4.9.

The LO port of the mixer is driven with an Agilent E5071C VNA. The maximum output of the VNA is 10 dBm while the mixer requires $10 - 13$ dBm of drive power. To compensate for loss in the SMA cables, a MITEQ AFS-4-02001800-24-10P-4 LNA was used to boost the LO power. The LNA has a gain of approximately $34.6 \pm 0.1$ dB in the band of interest, and the VNA output from port 1 was adjusted to have $P_{\text{VNA}} \approx -23$ dBm, providing $P_{\text{LO}} \approx 11$ dBm at the LO port of the mixer. The IQ ports of the mixer were driven by DC voltages from the DAC board. The raw differential outputs from the board were passed through two differential amplifiers to remove arbitrary voltage offsets and obtain levels high enough to drive the mixer. I used two 6 dB attenuators in front of the

![Fig. 4.9: Mixer calibration setup.](image-url)
4.5 Nanosecond vector pulse control

I and Q ports to reduce any standing waves that might affect the system’s performance and effectively provide 50Ω coupling. Mixer calibration was performed by using the VNA to measure the LO-to-RF transmission $S_{21}$ through the mixer as a function of $(I, Q)$. Previous work has shown that the minimum transmission is highly localized in the I-Q plane [Bal02], and my data confirms this (see Fig. 4.10).

Obtaining the data shown in Fig. 4.10 took considerable time (∼ 10 hours) and was a very inefficient way of finding the precise minimum. In locating the exact minimum, a linear search needs to have a use small step size. To speed up the process, I instead implemented a gradient search algorithm with variable step size in LabVIEW. The use of gradient search is further justified by smoothness of $S_{21}$ in I-Q plane. The pseudo-code outlining the algorithm is as follows:

1. Evaluate $S_{21}$ on a 3x3 grid of IQ points with distance $d$ between points.

![Figure 4.10](image)

**Fig. 4.10:** False-color map of $|S_{21}|$ versus $I$ and $Q$ showing localization of the $(I_0, Q_0)$ offset point for the IQ mixer.
2. Move the center of the 3x3 grid to the point with minimum $S_{21}$.

3. Repeat 1-2 until the minimum stays in the center.

4. Let $d \rightarrow d/\sqrt{10}$.

5. Repeat 1-4 until desired $S_{21}$ or maximum number of steps ($\sim 100$) are reached.

I found that the transmission minimum had a strong dependence on frequency, and this necessitated calibrating the offsets as a function of $f_{LO}$. The gradient search calibration of the offsets takes $\sim 30$ sec per frequency, resulting in total calibration time $\sim 10$-20 hours per GHz for small frequency step sizes (0.5 MHz). This makes frequent re-calibration unfeasible, and it is important to investigate the stability of the minimum against day-to-day temperature and LO power variations. I found temperature variations of the leakage to be negligible; a thorough analysis has been done by others [Bal02]. LO power, on the other hand, greatly affected the $S_{21}$ minimum. When driven even slightly below the recommended LO power, the mixer showed very poor performance. Moreover, variations of $\sim 0.1$ dB in LO power resulted in $\sim 10$ dB increase in $S_{21}$, so supplying consistent power to the mixer was crucial.

In order to provide a reference point for the power, I use a 20 dB MAC C320520 directional coupler between the LNA and the LO port. The coupler split off approximately $-9$ dBm and sent it to a Herotek DHM 124AA power meter diode. The resulting voltage on the diode was then digitized via an Agilent 34401A multimeter. The diode was first calibrated with a separate microwave source and a spectrum analyzer to record the output voltage versus microwave frequency while keeping the power at 11 dBm. In the
setup of Fig. 4.9, the voltage from the diode was fed back to the LabVIEW code and VNA power is adjusted until the diode voltage matches its previous calibration. One can thus achieve the same power at the mixer regardless of the instrument used to generate it and independent of the cables used to deliver it. This became important when the setup was used in some of my qubit experiments (see Chapter 5), where I had to drive the mixer with a microwave source and had to use the calibration obtained with the VNA.

To benchmark the performance of the offset calibration, I found the \((I_0, Q_0)\) that gave minimum \(S_{21}\) using the setup and algorithm described above. With the DAC outputting the offsets, \(S_{21}\) was then recorded several times at various LO frequencies (see Fig. 4.11).

It was possible to achieve a 60 dB on-off ratio consistently. The discrepancy between

![Fig. 4.11: LO leakage through an un-calibrated mixer with \(I_0 = Q_0 = 0\) (red), after \((I_0, Q_0)\) calibration (black circles), and using the calibration at later times (blue and green). Solid black line at \(-90\) dB was the calibration goal.](image)
the calibration values and the ones obtained later was most likely due to the limited ability to achieve the same LO power. This might prove to be the fundamental limitation of the above correction method, since the power resolution of most current microwave sources is $> 0.01$ dB, and the mixer sensitivity is of the same order.

Differences in the mixer diodes were responsible for different responses to I and Q signals, as well as interference between the two. Even with the LO leakage offsets accounted for, this so-called arm imbalance results in an output that is no longer described exactly by Eq. (4.4). Representing the in-phase and quadrature parts of the IQ drives by a column vector, we can write the most general transformation that the mixer performs on the IQ drives as

$$
\begin{pmatrix}
I_{\text{out}} \\
Q_{\text{out}}
\end{pmatrix} =
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}
\begin{pmatrix}
I_{\text{in}} \\
Q_{\text{in}}
\end{pmatrix} = M
\begin{pmatrix}
I_{\text{in}} \\
Q_{\text{in}}
\end{pmatrix}.
$$

(4.5)

Applying $M^{-1}$, the inverse of the transformation matrix $M$, to the input IQ drive signal before sending it to the mixer will therefore produce the desired output. The matrix elements of $M$ can be obtained by sending basis vectors (1,0) and (0,1) to the (I,Q) ports of the mixer and recording the resulting I and Q output RF signal with the VNA, effectively projecting matrix the onto a basis. To account for an ambiguity in the overall multiplicative factor, I divided the resulting matrix by its largest eigenvalue.

In order to evaluate the effectiveness of the arm imbalance correction and the control over the phase of the microwaves, it was useful to work with the amplitude $R$ and phase
4.5 Nanosecond vector pulse control

$\phi$ of the signal:

\[
R \equiv \sqrt{I^2 + Q^2} , \quad (4.6)
\]
\[
\phi \equiv \tan^{-1}\left(\frac{Q}{I}\right) .
\]

Keeping the input amplitude $R_{in}$ constant, one can sweep the phase $\phi_{in}$ of the input signal. This can be viewed as traversing a circle $R(\phi)$ plotted in polar coordinates or a circle $Q(I)$ in the Cartesian plane. If the inverse transformation is done correctly, we expect to see the same circle on the RF output. To quantify discrepancies, I define the maximum normalized deviation from mean circle radius $\eta_R$ and the maximum phase deviation $\eta_\phi$:

\[
\eta_R \equiv \max_{0 \leq \phi_{in} < 2\pi} \left| 1 - \frac{R_{out}(\phi_{in})}{R_{in}(\phi_{in})} \right| \times 100\% ,
\]
\[
\eta_\phi \equiv \max_{0 \leq \phi_{in} < 2\pi} \left| \phi_{out}(\phi_{in}) - \phi_{in} - \phi_{out}(0) \right| .
\]

The lower $\eta_R (\eta_\phi)$, the better the magnitude (phase) precision. I aimed to achieve $\eta_R < 3\%$ and $\eta_\phi < 5^\circ$ in the region of interest. These error targets should be low enough for other parts of our system, such as the device and readout, to dominate the overall gate errors. In the setup shown in Fig. 4.9, measuring $S_{21}$ yields the output magnitude and phase with respect to the phase of the LO. A sample output radius $R_{out}$ is shown below as a function of the input phase $\phi_{in}$ before and after the matrix correction (see Fig. 4.12). Figure 4.13 shows a plot of $\eta_R$ and $\eta_\phi$ for several LO frequencies. It can be seen from the plot that in the 4.5 – 7.5 GHz band where most of my qubits were
4.5 Nanosecond vector pulse control

Fig. 4.12: Polar plot of $R_{out}(\phi_{in})$ in arb. units before (black, $\eta = 14.5\%$) and after (red, $\eta = 0.4\%$) correction. $f_{LO} = 6$ GHz.

operated, I was able to calibrate the mixer to achieve the $\eta_R < 3\%$ and $\eta_\phi < 5^\circ$ targets.

Fig. 4.13: Deviation from perfect circle in percent (left scale, black squares) or degrees (right scale, red circles) for the calibrated mixer in the band of interest.
If one increases the magnitude of the input IQ voltages with respect to the zero throughput, one can drive beyond the mixer’s compression point, and non-linear effects will result in $R_{out}$ no longer being a linear function of $R_{in}$. In this compression region, the output amplitude deviates from a circle and this has detrimental effect on the accuracy of the phase control.

The data plotted in Fig. 4.14 shows several “circles” with their respective $\eta_R$; the effect produced by non-linearity can be readily observed for the largest circle. With $\eta_R < 1\%$ and linearity as the two criteria, the maximum output $R_{out}^{max}$ can be determined. Since for the same IQ input the output varies with frequency, it is important to calibrate the mixer to achieve the same $R_{out}^{max}$ everywhere. This ensures that the qubit is excited with the same power at 5 GHz as it is at 8 GHz. Furthermore, pulse shaping can be done as a

**Fig. 4.14:** Polar plot of $R_{out}(\phi_{in})$ for input radii $R_{in} = 1, 1.5, 1.8, 2 \times R_{in}^{max}$ as black, red, blue and green, respectively. $f_{LO} = 6$ GHz. Calibration was done in the linear region of the mixer, and the same calibration parameters were then used for all circles.
4.5 Nanosecond vector pulse control

fraction of this maximum, to ensure one never crosses into the non-linear region. In practice, I needed to find $R_{in}^{max}(f)$, the input magnitude that yields output $R_{out}^{max}$. To do this, I sent two (I,Q) pulses proportional to each other, for example, (1,0) and (2,0). I then computed

$$R_{in}^{max}(f) = \left( \frac{dR_{out}(f)}{dR_{in}} \right)^{-1} R_{out}^{max}.$$  \hspace{1cm} (4.8)

The actual code was implemented in LabVIEW and the pseudo-code is as follows:

- Connect to DAC.
- Load $(I_0, Q_0)$ offsets from calibration file.
- For desired range of frequencies $f$:
  1. Send $(I + I_0, Q_0)$ to DAC and obtain $(I_{out}, Q_{out})$.
  2. Send $(I_0, Q + Q_0)$ to DAC and obtain $(I_{out}', Q_{out}')$.
  3. Invert the resulting transform matrix and normalize by the largest eigenvalue.
  4. Send offset- and matrix-corrected $(I_{in}^{1}, Q_{in}^{1})$ and $(I_{in}^{2}, Q_{in}^{2})$ to DAC and obtain $R_{out}^{1,2} = \sqrt{(I_{1,2}^{out})^2 + (Q_{1,2}^{out})^2}$ for the two inputs.
  5. Calculate slope $s \equiv (R_{2}^{out} - R_{1}^{out})/(R_{2}^{in} - R_{1}^{in})$ and maximum DAC amplitude $R_{max}$ to remain in linear region of the mixer.
  6. Re-take transform matrix data using offset- and matrix-corrected $(R_{max}, 0)$ and $(0, R_{max})$.
  7. Obtain a circle $R_{out}(\phi_{in})$, calculate $\eta$. 

102
8. Record $f$, matrix elements, $R_{\text{max}}$, $\eta$ etc. into a calibration file.

4.5.5 Experimental integration

In order to achieve synchronization between different instruments (microwave sources, DAC and DAQ cards), a triggering scheme must be used. I chose the DAC board's serial outputs S0-S3 as channels for triggering all other instrumentation. These channels make it possible to use complicated pulse sequences with gated external instrumentation. For example, I used one of the channels to turn the readout microwave signal on and perform a measurement immediately after the qubit was manipulated.

Most of the laboratory equipment used transistor-transistor logic (TTL) voltage levels for control, whereas the DAC serial outputs used differential PECL. I designed and implemented a custom PECL-to-TTL converter (see Fig. 4.15) to allow the DAC to trigger the other instruments. The converter uses a SY100ELT23ZG IC translator for the conversion and has SMA input/output connectors. All traces are microstrips with 50Ω nominal impedance.

While unnoticeable during DC calibration, when generating pulses from the DAC, I observed a 2 GHz feedthrough, most likely from the clock. The problem was solved by inserting low-pass filters between the output of the DAC and the input of the differential amplifiers; the results are presented in Fig. 4.16. I had to be careful and use non-reflective filters such as Gaussian dissipative filters [Tek15]. Brick-wall type filters (Chebyshev, Butterworth, etc.) add unwanted ring-up and ring-down. In contrast, Gaussian filters provide smoothing of square pulses, thereby reducing spurious sidebands in the frequency domain.
4.5 Nanosecond vector pulse control

Fig. 4.15: PCB drawing of the PECL-to-TTL converter. Metallization for the top layer is shown in red.
4.5 Nanosecond vector pulse control

Fig. 4.16: Measured voltage $V$ versus time $t$ of the output pulse edge from the DAC board with the high-frequency feedthrough (black), filtered with a 200 MHz reflective low-pass (red) and 35 MHz absorptive low-pass (blue).

With the above challenges solved, the final setup for the pulse-shaping part of experiment is presented in Fig. 4.17. The low-pass Gaussian filters were Picosecond Pulse Labs’ 5915 with a 3 dB point of 100 MHz, giving a risetime of $\sim 3.5$ ns. The band-pass filter on the RF output was MiniCircuits VBFZ-6260-S+. It ensured removal of higher-order LO harmonics and reduced the possibility of exciting unwanted qubit transitions.
4.5 Nanosecond vector pulse control

Fig. 4.17: Photograph of the vector pulse shaping setup. The DAC with attached LP filters can be seen at the top; PECL-to-TTL converter is in the middle; IQ mixer with attached differential amplifiers (DA) is at the bottom, connected to one of several SMA feed-throughs.
4.6 Pulsed transmission measurements

In Section 7.8, I describe my observation of the propagation of Gaussian-modulated microwave pulses through a cavity-transmon system while pumping the system with a continuous microwave probe tone. The setup I used for this experiment is shown in Fig. 4.18. A Gaussian envelope was generated by a Tektronix 3102 arbitrary function generator (AFG) and sent to the I-port of the IQ mixer driven by the coupler microwave probe.

![Fig. 4.18: Schematic of the setup used to measure propagation of Gaussian-modulated coupler pulses in presence of a probe tone.](image)

In Section 7.8, I describe my observation of the propagation of Gaussian-modulated microwave pulses through a cavity-transmon system while pumping the system with a continuous microwave probe tone. The setup I used for this experiment is shown in Fig. 4.18. A Gaussian envelope was generated by a Tektronix 3102 arbitrary function generator (AFG) and sent to the I-port of the IQ mixer driven by the coupler microwave probe.
source. The Q-port was terminated. A small part of this pulsed signal was split off, demodulated, and acquired to form the reference pulse. The rest of the signal was combined with the continuous probe tone, and sent to the fridge. Upon transmission through the cavity-transmon system, the amplified signal was demodulated and acquired. The comparison between the reference pulse and the signal pulse allowed me to calculate the time of flight for the signal pulse, and thus infer the group delay the signal pulse experienced when it traversed the device.

4.7 Signal demodulation and acquisition

I performed state readout in the spectroscopy and time-domain experiments using the setup illustrated in Fig. 4.19. A small fraction of the applied readout pulse was split off from the readout source and sent to the RF arm of the mixer to form a reference

![Fig. 4.19: Schematic of the demodulation setup used to mix down the measurement signal.](image-url)
measurement. The LO arm of the mixer was driven by another microwave source. The LO microwaves were sent through high-pass filters to reduce DC and low-frequency noise that could form an unwanted offset in the final signal. After being mixed down to a typical IF of $0.5 - 10$ MHz, both signal and reference were sent through low-pass filters to remove high-frequency noise and any LO or RF feedthrough. The signal was then amplified by an IF amplifier.

I then used an Agilent U1082A DAQ card to acquire both the transmitted signal pulse and the applied reference pulse. The signal was sent to channel 1 of the card, while the reference was sent to channel 2. I used on-board averaging capabilities of the DAQ to average of up to $65536$ times. The averaged pulses were then demodulated in LabVIEW software by convoluting them with sine and cosine functions. I then subtracted the reference from the signal, and calculated the in-phase and quadrature components, which were proportional to the transmon total excited state population $\rho_{\text{tot}}$ [see Eq. (4.2)]. The system bandwidth was $35$ MHz, allowing me to time-resolve the demodulated pulses down to about $30$ ns.
CHAPTER 5

Characterization of transmons

In this chapter, I briefly overview the techniques I used to characterize my devices. First, I show how the readout of the qubit state was accomplished. Next, I describe my technique for making both single- and two-tone spectroscopic measurements. I then discuss how I adapted the spectroscopic methods to acquire time-domain measurements and characterize coherence times of transmons. Finally, I describe full vector control of a transmon and show how I prepared qubit superpositions and then measured them using state tomography.

5.1 High-power readout

I used the high-power Jaynes-Cummings non-linearity [Bis10a; Boi10; Ree10] to measure the qubit state. In Section 2.2, I described how the interaction between a transmon and a cavity leads to a shift in the cavity frequency \( \omega_c \) that depends on the transmon state. The same interaction causes the shift in the cavity resonance to depend on the microwave field driving the cavity (see Fig. 5.1). At a sufficiently large driving power,
the cavity resonance reverts to the bare frequency $\omega_{\text{bare}}$ as if no transmon were present [Sur15]. The onset of this bare resonance occurs at a driving power that depends on the transmon state, and hence this behavior can be used to measure the state. Furthermore, because this effect happens at large driving powers (thousands to millions of photons in the cavity), the SNR of this type of readout can be large.

For the high-power readout, I used the setup described in Section 4.3 to obtain a pulsed measurement of the transmission through the cavity $S_{21}$. To find the optimal operating conditions, I measured $S_{21}$ while varying the cavity drive power and detuning. A false-color plot of $S_{21}$ versus frequency and power for device C3DQ29 is shown in Fig. 5.1 (a). Although the transmon was in the ground state for this measurement, its mere presence altered the cavity behavior. In particular, the cavity resonance shifted from $\omega_c/2\pi \approx 7.167 \text{GHz}$ at low drive power to $\omega_{\text{bare}}/2\pi \approx 7.1585 \text{GHz}$ at high drive power. A linecut at $\omega_{\text{bare}}$ shows nonlinear power dependence of the transmission magnitude $|S_{21}|$ around $P_0 = -8 \text{dBm}$ of drive power at the source [see Fig. 5.2 (a)]. Since the onset of this non-linearity depends on the transmon state, I set the readout tone to $P_0$ and $\omega_{\text{bare}}$ to bootstrap the spectroscopy measurement. I then swept the spectroscopy tone in a wide range of frequencies ($2 - 10 \text{GHz}$) while measuring $|S_{21}|$ with a cavity pulse of power $P_0$. When the spectroscopy tone excited the qubit, cavity transmission changed producing a peak in the transmission of the readout pulse.

From this crude spectroscopy, I obtained transition frequencies $\omega_{ge}$ and $\omega_{gf}/2$ of the $|g\rangle \leftrightarrow |e\rangle$ single-photon and the $|g\rangle \leftrightarrow |f\rangle$ two-photon transitions of the transmon, respectively. Next, I used a $\pi$-pulse to invert the transmon population into either $|e\rangle$ or
5.1 High-power readout

Fig. 5.1: Cavity transmission magnitude $|S_{21}|$ versus cavity drive frequency $f$ and microwave source power $P$ for transmon C3DQ29 in the (a) ground state $|g\rangle$, (b) first excited state $|e\rangle$, and (c) second excited state $|f\rangle$. The inset in (a) illustrates the pulse sequence.
5.1 High-power readout

Fig. 5.2: Linecuts from Fig. 5.1 showing cavity transmission magnitude $|S_{21}|$ versus (a) microwave frequency $f$ at $P = -43$ dBm of microwave source power and (b) power $P$ at $\omega_{\text{bare}}/2\pi = 7.1585$ GHz. Data for the transmon ground state $|g\rangle$ is in black, for the first excited state $|e\rangle$ is in red, and for second excited $|f\rangle$ state is in blue. Optimal readout powers $P_a$ for $\rho_{\text{tot}}$ and $P_b$ for $\rho_{ff}$ are indicated by the dashed lines. The data was measured in device C3DQ29.

$|f\rangle$, and then measured the cavity transmission as a function of cavity drive frequency and power [see Fig. 5.1 (b), (c)]. One can immediately see that at low powers, the cavity resonance shifts depending on the prepared transmon state, consistent with a dispersively coupled Jaynes-Cummings system [see Fig. 5.2 (a)]. As non-linearities beyond the dispersive approximation become significant, the resonance shifts towards lower frequencies as the drive power is increased. In the intermediate power region,
around $-32$ to $-14 \text{ dBm}$, the resonance disappears completely [Boi10]. At high powers, the resonance is recovered with its frequency $\omega_{\text{bare}}$ being power-independent but its onset point in drive power determined by the transmon state.

These measurements provided me with a map of the readout for the lowest three transmon states. Using this map, I was able to find optimal the readout frequency and power that maximized readout contrast. I note when I biased the readout at the optimal power $P_a \approx -10 \text{ dBm}$ to be sensitive to state $|e\rangle$ of the transmon results in the same cavity transmission even if the transmon were in state $|f\rangle$ [see Fig. 5.2 (b)]. This is consistent with the response being proportional to $\rho_{ee} + \rho_{ff}$, i.e. the total population of the two lowest transmon excited states. In fact, the readout of transmon state $|\psi\rangle$ at this bias point should be proportional to $\rho_{\text{tot}} = \sum_{j=e,f,...} \rho_{jj}$, the total excited state population of the transmon [see Eq. (4.2)]. However, at a lower readout power $P_b \approx -16 \text{ dBm}$, the cavity is only sensitive to the transmon being in $|f\rangle$. I used this fact to preferentially measure the second excited state population $\rho_{ff}$ of the transmon for some of the data I present in Chapter 6.

5.2 Spectroscopy

After I set up the high-power readout, I performed detailed single-tone spectroscopy by applying a long probe pulse of varying frequency before measuring the transmon state. To illustrate this technique, I present data from device C3DQ29. At low probe powers, only $|g\rangle \leftrightarrow |e\rangle$ transitions at $f_{ge}$ were seen [shown in Fig. 5.3 (a)]. As I increased the probe power, I was able to drive $|g\rangle \leftrightarrow |f\rangle$ via a two-photon transition at $f_{gf}/2$ [see
5.2 Spectroscopy

Fig. 5.3: Plot of $\rho_{\text{tot}} = 1 - \rho_{gg}$ versus frequency $f$ showing transmon transitions measured in device C3DQ29 for (a) $-12$ dBm, (b) 0 dBm, and (c) 12 dBm nominal spectroscopy tone power. The inset in (a) illustrates the pulse sequence.

Fig. 5.3 (b)] and $|g\rangle \leftrightarrow |h\rangle$ as a three-photon transition at $f_{gh}/3$ [see Fig. 5.3 (c)]. Using the measured values of these transition frequencies and equations from Section 2.1, I determined approximate transmon parameters such as the Josephson energy $E_J$, the charging energy $E_C$, and the anharmonicity $\alpha$. I also obtained more accurate estimates of device parameters by fitting the spectroscopic data to a numerically diagonalized Jaynes-Cummings Hamiltonian. The Mathematica code for that procedure was provided...
In the strong dispersive coupling regime, not only does the transmon state affect the cavity resonance, but the converse is also true – the qubit transition frequency $f_{ge}$ changes depending on the number of photons in the cavity [Sch07]. This effect, called photon number splitting, can be used to accurately calibrate the cavity occupation as well as determine the qubit-cavity coupling and the total attenuation of the input lines at the cavity frequency. To coherently populate the cavity, I added a second microwave tone to the spectroscopy measurement, resulting in a Poisson distribution of photon number (Fock) states [CT98]. Each photon number state corresponds to a different qubit transition frequency, and for the coherently driven cavity, I observed a Poisson-distributed comb of transitions (see Fig. 5.4 for data measured in device C3AQ6). For the n-th Fock state in the cavity, in the dispersive limit and for small occupancy the frequency $f_n$ of the $|g,n⟩ \leftrightarrow |e,n⟩$ transition is

$$f_n = f_{ge} + \frac{2n\chi}{(2\pi)} ,$$

(5.1)

where $\chi = g^2/\Delta$ is the dispersive shift. Therefore, the measurement of photon number splitting allowed me to determine $\chi$ and $g$ directly. At higher occupancy, the frequency dependence of $\chi$ becomes significant [Sur15]. I also fit a Poisson distribution of Lorentzian peaks to the data (similar to [Sch07]) to obtain the average cavity population $\bar{n}$ for a given cavity drive, which provided an accurate calibration of cavity drive power. I note that only in transmons strongly coupled to cavities (i.e. $\chi \gg 1/T_1, 1/T_2^*$) could photon
Spectroscopy

number splitting be resolved.

Fig. 5.4: Photon number splitting of the qubit frequency $f$ for different cavity occupations measured in device C3AQ6. The data is shown in red, and a fit of Poisson-distributed Lorentzian peaks is in black. The average cavity occupation $\bar{n}$ is determined from each fit. In (c) the dispersive shift $\chi$ is defined as half the difference between 0 and 1-photon peaks. The photon number peaks are numbered in (d) for reference. The inset in (d) illustrates the pulse sequence.
5.3 Relaxation, Rabi, Ramsey, and Spin-echo

Qubit decoherence can be modeled by coupling noise to the device (see [Sch02; Zar13b]). Transmon relaxation is attributed to high-frequency noise at the qubit transition frequency (\( \sim \) GHz), and is typically characterized by relaxation time \( T_1 \). Rabi oscillations are sensitive to the noise at the Rabi frequency (\( \sim 0.01 - 100 \) MHz), and have a corresponding Rabi decay time \( T' \). The low-frequency noise can be measured in a Ramsey experiment, producing the characteristic spectroscopic coherence time or Ramsey decay time \( T_2^* \). Finally, a spin-echo experiment modifies the Ramsey pulse sequence to be insensitive to noise at zero frequency, and measures the echo time or coherence time \( T_2 \).

Several important relations connect some of the different relaxation, decay, and coherence times. First, the Ramsey (free-evolution) decay rate is:

\[
\frac{1}{T_2^*} = \frac{1}{2T_1} + \frac{1}{T_\phi} + \frac{1}{T'}
\]

(5.2)

where \( T_\phi \) is the pure dephasing time of the qubit, and \( T' \) is the inhomogeneous decay time constant. In the absence of dephasing and inhomogeneous broadening the Ramsey decay is \( T_1 \)-limited, and \( T_2^* = 2T_1 \). Thus, if a device displays a Ramsey decay time that is twice its relaxation time, its suffers from negligible dephasing on that timescale and negligible inhomogeneous broadening on the measurement timescale.

The coherence time

\[
\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}
\]

(5.3)
is insensitive to inhomogeneous (shot-to-shot) variations.

The Rabi (driven) decay rate is

\[
\frac{1}{T'} = \frac{1}{2T_1} + \frac{1}{2T_2},
\]

(5.4)

which reduces to [Byl11]:

\[
\frac{1}{T'} = \frac{3}{4T_1} + \frac{1}{2T_\Omega},
\]

(5.5)

where \(T_\Omega\) includes dephasing and decoherence due to additional noise at the Rabi frequency \(\Omega\). If that noise and dephasing are negligible, the \(T_1\)-limited Rabi decay time is \(T' = \frac{4}{3} T_1\).

5.3.1 Relaxation measurements

To perform relaxation measurements, I excited the qubit using either a very long spectroscopy-like pulse or a \(\pi\)-pulse at \(f_{ge}\). After a variable amount of delay \(t\), I measured the qubit excited state population \(\rho_{tot}\). Typical data sets for three representative devices are shown in Fig. 5.5. I fit an exponential decay function:

\[
\rho_{tot}(t) = Ae^{-t/T_1} + B
\]

(5.6)

to the data, where \(A\) is the initial excited state population, \(B \approx 0\) is an offset parameter, and \(T_1\) is the relaxation time.
5.3 Relaxation, Rabi, Ramsey, and Spin-echo

Fig. 5.5: Transmon relaxation measurement showing $T_1$ data in red and fits in black for device (a) C3AQ6, (b) C3DQ15, and (c) C6BQ58. The inset in (a) illustrates the pulse sequence.

The qubit relaxation time is characteristic of the loss the qubit experiences. For my devices, dielectric loss [Gao08; Mar05] was believed to be the dominant relaxation mechanism. In my early devices, the dielectric participation ratio was large due to closely placed transmon shunt capacitor pads, resulting in a relatively low $T_1$. By separating the pads in my later designs, I was able to increase $T_1$ from 430 ns (device Q6) to 120 $\mu$s (device Q58). Some of this trend can be seen in Table 3.5. I note that my last transmon devices, Q042314A and Q042314B, had a low $T_1 \approx 4 \mu$s despite widely separated pads.
5.3 Relaxation, Rabi, Ramsey, and Spin-echo

Based on BBQ simulations [Nig12], I concluded that due to large pad separation, the shunt capacitance coupled the transmons to the lossy Cu cavity walls.

5.3.2 Rabi measurements

I performed Rabi measurements by driving the qubit on-resonance at $f_{ge}$ for some time $t$, and then reading out the excited state population. This produced exponentially decaying oscillations [CT98] that I fit to the following functional form:

$$\rho_{\text{tot}}(t) = A e^{-t/T'} \sin(\Omega t + \phi) + B,$$  \hspace{1cm} (5.7)

where $A \sim 0.5$ is the oscillation amplitude, $B \sim 0.5$ is the steady-state population value, $\Omega$ is the Rabi frequency, and $\phi \approx -\pi/2$ is a phase shift. Typical data for C3DQ29 is shown in Fig. 5.6 (a) and (b). For $\Omega \gg 1/\sqrt{T_1 T_2}$, the Rabi frequency $\Omega$ is proportional to the applied qubit drive amplitude $V$ (see [Sak94]):

$$\Omega = \zeta V,$$  \hspace{1cm} (5.8)

where I define the proportionality constant $\zeta$ as the Rabi coupling. The Rabi coupling not only describes how strongly the drive couples to the qubit, but also, reciprocally, how the qubit couples to the microwave environment. In particular, it was shown by Kim, et al. [Kim11] that $\zeta$ can be positively correlated with the relaxation rate $\Gamma_1 = 1/T_1$, due to coupling of the qubit to its microwave environment.

I obtained $\Omega$ by fitting Eq. (5.7) to my Rabi data. By varying $V$, and fitting a line to
5.3 Relaxation, Rabi, Ramsey, and Spin-echo

Fig. 5.6: Rabi oscillations for device C3DQ29 showing data in red and fits in black of (a) slow exponentially decaying oscillations and (b) fast oscillations before significant decay occurs. (c) Extracted Rabi frequency $\Omega$ versus drive amplitude $V$ (black squares), with a linear fit (red line) to obtain the slope, which is the Rabi coupling $\zeta$. The inset in (a) illustrates the pulse sequence.
5.3 Relaxation, Rabi, Ramsey, and Spin-echo

the Ω(V) data, I was able to measure the Rabi coupling [see Fig. 5.6 (c)]. For my devices
the Rabi coupling was small (ζ/2π ~ 0.2 − 0.4 MHz/µV) and contributed little to T₁. In
contrast, the effects of Rabi coupling on the loss were significant in B. Suri’s transmons
coupled to on-chip resonators [Sur15].

5.3.3 Ramsey measurements

Ramsey experiments quantify decoherence due to all sources, including low-frequency
noise coupled to the qubit, and shot-to-shot variations in the system. In order to do
a Ramsey experiment, a Xπ/2 pulse is sent at ω_R = ω_ge + Δ_R to create a superposition
state (|g⟩ + |e⟩)/√2 that precesses at the Ramsey detuning frequency Δ_R. The state is
left to evolve for some time t, resulting in rotation by angle φ = Δ_Rt around the z-axis
of the Bloch sphere. If dephasing occurs and changes the qubit transition frequency
(and thus Δ_R) during this free evolution, in different instances of the experiment the
superposition state will rotate by different amounts. A second Xπ/2 pulse is applied to
map φ onto ρ_{tot}. When averaging over many experimental instances, dephasing results
in the exponential decay of ρ_{tot} on the characteristic time scale T₂*. Figure 5.7 (a) shows
typical data for device C3DQ29.

I fit an exponentially decaying sine function, similar to the one used for Rabi oscillations, to the data:

\[ ρ_{tot}(t) = Ae^{-t/T_{2}^*} \sin(Δ_R t + φ) + B \]  \hspace{1cm} (5.9)

Ramsey oscillations can also be used to accurately find the qubit transition frequency
ω_{ge} by varying ω_R and measuring the Ramsey precession frequency Δ_R, as shown, for example, for device C3DQ29 in Fig. 5.7 (b).

**Fig. 5.7:** (a) Ramsey oscillations for device C3DQ29 showing data in red and fit in black. (b) Extracted Ramsey detuning Δ_R versus Ramsey drive frequency ω_R (black open circles), with a linear fit (red line). The intercept gives an accurate estimate of the qubit frequency ω_{ge}. The inset in (a) illustrates the pulse sequence.
5.3 Relaxation, Rabi, Ramsey, and Spin-echo

5.3.4 Spin-echo measurements

A spin-echo measurement is similar to a Ramsey measurement except an additional $Y_\pi$ pulse is applied in the middle of the free evolution. This pulse re-focuses the superposition ensemble by switching the rotation direction and effectively reversing the free evolution. Unlike the Ramsey experiment, spin-echo is insensitive to noise at low frequency and small levels of inhomogeneous broadening (shot-to-shot variations). As I discussed in Section 4.5, a good spin-echo experiment requires multiple pulses with good control over the qubit axis of rotation. Sample data I measured for device C3DQ15 is presented in Fig. 5.8. I fit the following exponentially decaying function to the data:

$$\rho_{\text{tot}}(t) = Ae^{-t/T_2} + B,$$  \hspace{1cm} (5.10)

where $A \sim 0.5$ is the maximum amplitude, which decays to the steady-state value $B \sim 0.5$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{spin_echo.png}
\caption{Fig. 5.8: Plot of $\rho_{\text{tot}}$ versus time $t$ for spin-echo measurement for device C3DQ15. The inset illustrates the pulse sequence.}
\end{figure}
5.4 State tomography

State tomography requires an even higher degree of control over a qubit than a spin-echo. Typically, it involves preparing an arbitrary state on the Bloch sphere, then rotating and measuring it to obtain projections of the x-, y-, and z-axes. Using the measured ensemble of projections, the initial state can be reconstructed. In my experiments, I prepared a few superposition states via pre-defined rotations. I then demonstrated arbitrary rotations of the prepared states around equatorial axes, before projecting them on the z-axis to readout $\rho_{\text{tot}}$ [see Fig. 5.9 (a)].

I took the tomography data using device C3DQ15 for four initial qubit states. The data for the ground state $|g\rangle$ as the initial state is shown in Fig. 5.9 (b). In this case, I set $R_{\text{prep}} = 0$. Then, I applied $R(\phi, t)$ to rotate the prepared state around an equatorial axis with azimuthal angle $\phi \in [0, 2\pi]$ for duration $t$. The measured population $\rho_{\text{tot}}$ displayed Rabi oscillations in time, which I plotted in the radial direction at angle $\phi$. Because the starting state was $|g\rangle$, rotation around any equatorial axis resulted in Rabi oscillations, yielding a rotationally symmetric plot. A slight asymmetry can be observed, with the oscillation peaks at $\phi = 0$ occurring at earlier times than the peaks at $\phi = \pi/2$. This was due to a small mixer arm imbalance, as I discussed in Section 4.5.4, resulting in a slightly larger Rabi drive amplitude delivered to the qubit at $\phi = 0$.

The tomography data changed drastically when the superposition state $(|g\rangle + |e\rangle)/\sqrt{2}$ was prepared by $R_{\text{prep}} = X_{\pi/2}$ [see Fig. 5.9 (c)]. Rotation of this state around the y-axis by $R(\phi = \pi/2, t)$ resulted in the typical Rabi oscillations. However, rotation around
the x-axis (parallel to the prepared state vector) by \( R(\phi = 0, t) \) did not produce Rabi oscillations but instead a constant \( \rho_{\text{tot}} \) in the radial direction.

A similar situation occurred when \( (|g\rangle + i|e\rangle)/\sqrt{2} \) was prepared by \( R_{\text{prep}} = Y_{\pi/2} \) [see Fig. 5.9 (d)]. Because this state was oriented along the y-axis, tomographic pulses for \( R(\phi = \pi/2, t) \) produced constant \( \rho_{\text{tot}} \), i.e. no rotation. I note that the radial line of constant \( \rho_{\text{tot}} \) did not show up precisely at \( \phi = \pi/2 \) but at a slightly lower value. This was also due to mixer imperfections resulting in small phase errors (as I discussed in Section 4.5.4).

Although direct z-rotations were not possible in my experimental system, I achieved them by combining x- and y-rotations as follows:

\[
Z_{\theta} = X_{\pi/2}Y_{\theta}X_{-\pi/2}.
\] (5.11)

To demonstrate this rotation, I set up a state \( (|g\rangle + |e\rangle)/\sqrt{2} \) via \( R_{\text{prep}} = X_{\pi/2} \), and then rotated it using \( Z_{\pi/4} \), resulting in the prepared state \( [(|g\rangle + (1 + i)|e\rangle)/\sqrt{2}]/\sqrt{2} \). I then proceeded to perform the usual tomographic rotations on this state [see Fig. 5.9 (e)]. The data shows that the prepared state was indeed half-way between states \( (|g\rangle + |e\rangle)/\sqrt{2} \) and \( (|g\rangle + i|e\rangle)/\sqrt{2} \) in the equatorial plane.

The data I present in Fig. 5.9 demonstrates the ability to perform rotations around any equatorial axis and, by means of Eq. (5.11), any rotations around the z-axis. Furthermore, the data reveals mixer imperfections, and therefore can be used to evaluate the experimental setup. I note that one can think of the qubit as providing an unbiased
5.4 State tomography

A system to test one's ability to perform Bloch sphere rotations, with any imperfections coming from the measurement setup.
5.4 State tomography

Fig. 5.9: State tomography measurement for device C3DQ15. (a) The pulse sequence consisted of a preparation pulse $R_{\text{prep}}$ followed by an arbitrary tomography rotation $R(\phi, t)$ of duration $t$ around an axis in the equatorial plane making angle $\phi$ with the x-axis. (b) Ground state $|g\rangle$ prepared using $R_{\text{prep}} = 0$. (c) State $(|g\rangle + |e\rangle)/\sqrt{2}$ prepared using $R_{\text{prep}} = X_{\pi/2}$. (d) State $(|g\rangle + i|e\rangle)/\sqrt{2}$ prepared using $R_{\text{prep}} = Y_{\pi/2}$. (e) State $|(g\rangle + (1 + i)|e\rangle)/\sqrt{2}\sqrt{2}$ prepared using $R_{\text{prep}} = X_{\pi/2}Z_{\pi/4}$. 
5.5 Conclusions

The measurements presented in this chapter constituted the backbone of single qubit characterization in my work. Each technique required a certain degree of control in frequency or in the time domain. The tomography measurement placed the most stringent requirements on the control of the microwaves and provided sensitive tests for the imperfections of the control.

These characterizations provided fairly complete descriptions of my devices. Further measurements, such as randomized benchmarking [Cho09] and gate-set tomography [BK13], for evaluating both the devices and the experimental control, would be useful in the context of quantum computation. I did not pursue them as part of my thesis, and they are subject of future work.
CHAPTER 6

Observation of Autler-Townes doublet in a 3D transmon

6.1 Introduction

The Autler-Townes effect [Aut55; CT98] involves a three-level quantum system interacting with an applied coupling drive field that is nearly resonant with two of the levels. For a sufficiently strong coupling field, one of the transitions will split into a doublet, which can be probed by a weak second tone. The effect is an example of electromagnetic dressing of quantum states, and it has been proposed as a basis for fast, high ON/OFF ratio microwave routers for quantum computation [Hoi13; Li12]. As I outlined in Chapter 1, the Autler-Townes doublet (ATD) is closely related to EIT and quantum effects such as slow light [Tur01]. I showed in Section 2.3.2 that EIT poses more stringent requirements on the coherence of the system. Although EIT has been shown in atomic systems [Fie91], prior to my work it had not been conclusively demonstrated with superconducting qubits [Ani11]. EIT in superconducting systems had been proposed as a sensitive probe of decoherence [Mur04].
The ATD has been studied in atomic [Cah76; Del76; Pic76], molecular systems [Tam95], quantum dots [Xu07], and superconducting qubits. Groups studying the effect in superconducting qubits have employed transmon levels with continuous-tone cavity readout [Bau09], phase qubit levels with tunneling readout [Kel10; Sil09], and levels of flux [Abd10] and transmon [Hoi13] qubits coupled directly to a transmission line.

In this chapter, I present experimental measurements of the ATD in a 3D transmon [Pai11]. In contrast to previous studies involving transmons [Bau09], which had to include transmon-cavity effects of the Jaynes-Cummings Hamiltonian, I isolated the three lowest transmon levels by using pulsed spectroscopy. My method eliminated the need to re-tune microwave drives to account for power-dependent dispersive shifts [Bau09], and I achieved a large signal-to-noise ratio by using a qubit-induced-non-linearity readout [Bis10a; Boi10; Ree10]. I also note that previous experiments using superconducting qubits [Sil09] employed coupling drives that were large compared to the energy level anharmonicity to compensate for relatively short coherence times. At such strong drives, multi-photon transitions are possible, and accurate modeling of the system requires a Hilbert space of more than three levels. In contrast, my device possessed long enough coherence times to observe the ATD even at low drive amplitudes, and, as I show, my data is well-explained by a three-level density matrix with no free parameters.

6.2 Device and setup

My qubit and cavity device C3DQ29 consisted of a transmon [Koc07] embedded in a 3D microwave cavity [Pai11; Rig12]. I fabricated transmon Q29 [see Fig. 6.1 (a)] via
standard e-beam lithography, double-angle evaporation [Dol77], and lift-off procedures, which I described in Section 3.2. The transmon had a single Al/AlO$_x$/Al Josephson junction capacitively shunted by two 375 $\times$ 800 $\mu$m Al pads on a sapphire substrate. The tunneling energy of the junction was $E_J/h = 16.5$ GHz, and the pads lowered the charging energy to $E_C/h = 177$ MHz. The pads were fabricated as a mesh of 2.5 $\mu$m wide lines placed every 10 $\mu$m in both directions to enhance expulsion of external magnetic fields and prevent trapping of magnetic vortices in the films. The shunting pads also formed a dipole antenna which coupled the transmon to the cavity with strength $g/2\pi = 151$ MHz. Cavity C3D was a rectangular box made from oxygen-free-high-conductivity Cu, with the fundamental TE$_{101}$ mode at $\omega_{cav}/2\pi = 7.1585$ GHz (see Section 3.1). This mode was used for qubit readout, with loss limited by the internal quality factor $Q_i = 18,000$. The cavity was probed in transmission, with the output connector coupled much more strongly ($Q_e^{\text{out}} = 30,000$) than the input connector ($Q_e^{\text{in}} = 120,000$). The cavity was
mounted on the mixing chamber of a Leiden Cryogenics CF-450 dilution refrigerator at $T = 22$ mK. The microwave lines to the cavity were heavily attenuated, filtered and isolated to protect the device from extrinsic noise, as described in Section 4.1. The output signal from the cavity was passed to a HEMT amplifier at the 3 K stage, and then further amplified, mixed down, and digitized at room temperature.

Three microwave drives were used: cavity, probe, and coupler. The cavity drive was turned on at time $t = 0$ for $5 \mu s$ to record the initial (ground) state of the system, and then again at $t = 290 \mu s$ to read out the final state of the system. Within the $290 \mu s$ window between readout pulses, transmon control microwaves (either probe or coupler, or both) were applied. The whole sequence was repeated every $600 \mu s$.

In this arrangement, the cavity was used solely for readout of the qubit state, did not participate in the Autler-Townes manifold, and had zero photon occupation while probe and coupler were applied. The detailed description of the measurement setup can be found in Section 4.3.

6.3 Modeling

Based on Section 2.3, I modeled the system using a density matrix that included just the ground, first, and second excited states of the transmon: $|0\rangle \equiv |g\rangle$, $|1\rangle \equiv |e\rangle$, and $|2\rangle \equiv |f\rangle$, as shown in Fig. 6.1 (b). These states were separated by two transition frequencies: $\omega_{01}$ and $\omega_{12} \equiv \omega_{02} - \omega_{01} = \omega_{01} + \alpha$, where $\omega_{ij} \equiv \omega_j - \omega_i$, and $\alpha$ is the level anharmonicity. Two microwave drives, the probe and the coupler, were applied at $\omega_p = \omega_{01} + \Delta_p$ and $\omega_c = \omega_{12} + \Delta_c$. Their amplitudes $\Omega_p$ and $\Omega_c$ determined the
Rabi oscillation frequencies of the $\omega_{01}$ and $\omega_{12}$ transitions, respectively. In the frame co-rotating with the drives, the system Hamiltonian is

$$
\mathcal{H}_{\text{tot}} = \omega_p \left|1\right\rangle \langle 1\right| + (\omega_p + \omega_c) \left|2\right\rangle \langle 2\right| - (\omega_p - \omega_{10}) \left|1\right\rangle \langle 1\right| - (\omega_p - \omega_{10} + \omega_c - \omega_{21}) \left|2\right\rangle \langle 2\right|
+ \frac{1}{2} \Omega_p \left|1\right\rangle \langle 0| e^{-i\omega_{p}t} + \text{H.c.} \right| + \frac{1}{2} \Omega_c \left|2\right\rangle \langle 1| e^{-i\omega_{c}t} + \text{H.c.} \right|,
$$

where $\hbar = 1$ is assumed. I included dissipation and dephasing via the Kossakowski-Lindblad [Kos72; Lin76] master equation for the density matrix $\rho$

$$
\frac{d\rho^I}{dt} = i[\rho^I, \mathcal{V}^I] + \sum_j \Gamma_j \mathcal{D}(A_j)\rho^I,
$$

where $\mathcal{V}^I$ is the system Hamiltonian in the frame co-rotating with the probe and coupler drives, and $\mathcal{D}_j$ is the Lindblad operator describing decoherence at a rate $\Gamma_j$ of the system through a particular channel $j$ (see Section 2.3). I solved Eq. (6.2) numerically in steady-state to obtain the theoretical description of my data.

The Mathematica notebook I made for these simulations is available in an online repository [Nov15], or can be requested from the author by email (snovikov AT gmail DOT com).
6.4 High-power readout

I measured of the transmon state with a high signal-to-noise ratio by using the Jaynes-Cummings non-linearity readout [Bis10a; Boi10; Ree10]. The readout process is described in detail in Section 5.1. The cavity pulses were applied at the bare cavity frequency $\omega_{\text{cav}}/2\pi = 7.1585\,\text{GHz}$ with an amplitude that provided maximum contrast between the ground and excited states, as shown in Fig. 6.2 (a). At amplitude $P_a = -10\,\text{dBm}$ the cavity did not discriminate between the transmon being in $|1\rangle$ or $|2\rangle$, while at $P_b = -16\,\text{dBm}$ it was mostly sensitive to $|2\rangle$. That is, the signal from the readout at $P_a$ was proportional to the sum total of the first and second excited state probabilities $\rho_{11}$ and $\rho_{22}$, and was used to obtain the 0-to-1 and the ATD data. The signal at $P_b$ was used for the characterization of 1-to-2 transition only.
6.5 System characterization

I determined the parameters in Eq. (6.1) and Eq. (6.2) from a set of independent measurements as follows. I characterized the $\omega_{01}$ transition by applying the probe and cavity readout tones. At low probe powers, I observed a small shoulder on the left-hand
side of the $\omega_{01}$ peak [see 6.2 (b)], which I attribute to a fluctuator affecting the transmon. Similar fluctuators, possibly due to a microscopic defect in or near the junction, have been studied in other superconducting qubits [Con07; Mar05; Zar13a]. Apart from the slight background, the fluctuator did not appear to otherwise affect the system. I fit the data using the steady-state solution to Eq. (6.2):

$$\rho_{11} + \rho_{22} = \frac{\Omega_p T_1 T_2 / 2}{1 + T_z^2 (\omega - \omega_{01})^2 + \Omega_p^2 T_1 T_2},$$

(6.3)

with an additional Lorentzian to account for the fluctuator background. With decoherence rates determined independently via $T_1$ and Ramsey experiments (see Fig. 6.3), I extracted the best fit values $\Omega_p / 2\pi = 186$ kHz and $\omega_{01} / 2\pi = 4.294085$ GHz. This value for $\Omega_p$ corresponded to $-44$ dBm of applied probe power at the source, and this was the power I used for all of the ATD measurements. I also found the position, width and amplitude of the Lorentzian background to be $-1.46$ MHz, $1.57$ MHz, and $0.0526$, respectively.

By pulsing the probe, I found the transmon first-excited- to ground-state relaxation time $T_1 = 39 \mu$s, limited by internal loss. The measured Ramsey decay time $T_2^* = 51 \mu$s of the transmon was less than the relaxation-limited value of $2T_1$, indicating the presence of additional dephasing. From $T_1$ and $T_2^*$ I obtained relaxation (denoted by $\Gamma_{ij}$ for $|i\rangle \rightarrow |j\rangle$ process) and dephasing (denoted by $\gamma_i$ for state $|i\rangle$) rates of $\Gamma_{10} = 1/T_1 = 26 \times 10^3$ s$^{-1}$, $\gamma_2 = \gamma_1 = 1/T_2^* - 1/2T_1 = 6.6 \times 10^3$ s$^{-1}$. I set $\Gamma_{21} = 2 \times 26 \times 10^3$ s$^{-1}$ and $\Gamma_{20} = 0$ based on the ratio of transmon transition matrix elements [see Eq. (2.23)]. Using $\gamma_1$, I placed
6.5 System characterization

![Fig. 6.3: (a) Relaxation and (b) Ramsey decay measurements of device C3DQ29 showing data in black and fits in red.](image)

Fig. 6.3: (a) Relaxation and (b) Ramsey decay measurements of device C3DQ29 showing data in black and fits in red.

...a bound of less than $\bar{n} = 0.02$ thermal photons in the cavity [Sea12] by estimating the photon number as

$$\bar{n} \approx \frac{\tau}{T_\phi} = \frac{2\pi\gamma_1}{\kappa},$$  \hspace{1cm} (6.4)

where $\tau = 2\pi/\kappa$ is the cavity lifetime, $T_\phi = 1/\gamma_1$ is the pure dephasing time due to photons in the cavity, $\kappa$ is the cavity linewidth, and $\gamma_1$ is the dephasing rate. I assumed negligible upward rates in the system, and set $\Gamma_{ij} = 0$ for all $i < j$.

In order to characterize the $\omega_{12}$ transition, I applied a $\pi$-pulse at $\omega_{01}$ with the probe...
6.5 System characterization

tone, followed by a \( \pi \)-pulse near \( \omega_{12} \) with the coupler. I measured the population of \( |2\rangle \) alone at a cavity power \( P_b \approx P_a - 10 \) dB that provided contrast only when \( |2\rangle \) was excited [see Fig. 6.2 (a)]. I fit the spectroscopic peak shown in Fig. 6.2 (c) to the function

\[
\rho_{22} = A \text{sinc} \left( \frac{\omega - \omega_{12}}{\delta \omega} \right) + B
\]  

(6.5)

to obtain \( \omega_{12}/2\pi = 4.116 \, 609 \) GHz. To calibrate \( \Omega_c \) for the AT experiment, I measured the Rabi frequency of the \( |1\rangle \leftrightarrow |2\rangle \) transition as a function of coupler amplitude by replacing the \( \pi \)-pulse on the coupler with a variable-length pulse at \( \Delta_c = 0 \).

Finally, I calibrated the probability scale by performing Rabi oscillations on \( \omega_{01} \). I fit the data, and set the amplitude of the exponentially decaying sine function fit to unity. This calibrated \( \rho_{11} \), and, with the readout at cavity power \( P_a \) being equally sensitive to \( \rho_{11} \) and \( \rho_{22} \), also calibrated \( \rho_{11} + \rho_{22} \). Table 6.1 summarizes all of the parameters I used in the model. I emphasize that all of the parameters were obtained independently of the ATD data discussed in the next section.

**Table 6.1**: Parameters for the simulation of ATD data in Fig. 6.5. Coupler detuning was set to \( \Delta_c = 0 \), and dephasing rates were set to \( \gamma_1 = \gamma_2 = 6.6 \times 10^3 \) [s\(^{-1}\)] throughout.

<table>
<thead>
<tr>
<th>Data set</th>
<th>( \Omega_c/2\pi ) [MHz]</th>
<th>( \Omega_p/2\pi ) [kHz]</th>
<th>( \Gamma_{10} \times 10^3 ) [s(^{-1})]</th>
<th>( \Gamma_{21} \times 10^3 ) [s(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.3538</td>
<td>186</td>
<td>26</td>
<td>52</td>
</tr>
<tr>
<td>b</td>
<td>0.7071</td>
<td>186</td>
<td>26</td>
<td>52</td>
</tr>
<tr>
<td>c</td>
<td>1.412</td>
<td>186</td>
<td>26</td>
<td>52</td>
</tr>
<tr>
<td>d</td>
<td>2.818</td>
<td>186</td>
<td>26</td>
<td>52</td>
</tr>
<tr>
<td>e</td>
<td>5.625</td>
<td>186</td>
<td>26</td>
<td>52</td>
</tr>
<tr>
<td>f</td>
<td>11.224</td>
<td>186</td>
<td>26</td>
<td>52</td>
</tr>
</tbody>
</table>
6.6 ATD data

To observe the Autler-Townes doublet, I turned the probe and the coupler on for 280 μs between the two cavity readout pulses, and fixed the probe power at −44 dBm on the microwave source, corresponding to Ω_p/2π = 186 kHz. Being much longer than any coherence times in the system, this probe and coupler pulse length ensured the system had achieved a steady state before the transmon state was measured. Sweeping both Δ_p and Δ_c around zero, and measuring ρ_{11} + ρ_{22}, I observed emergence of the Autler-Townes doublet as Ω_c was increased (see Fig. 6.4). At a relatively low coupler drive of Ω_c/2π = 0.177 MHz, I saw a crossing of ω_{01} [vertical band in Fig. 6.4 (a)] with the two-photon sideband excitation of ω_{02} [diagonal streak in Fig. 6.4 (a)]. As the coupler strength was increased four-fold, ω_{01} became dressed by the coupler photons and showed the emergence of an anti-crossing at zero detuning [see Fig. 6.4 (b)]. Increasing Ω_c another four-fold resulted in a completely separated splitting [see Fig. 6.4 (c)]. Parts (d)-(e) of Fig. 6.4 show the corresponding simulations I performed by solving Eq. (6.2) with no fitting parameters. I found excellent agreement between the simulations and the data.

To observe a well-separated AT doublet, I needed to apply a sufficiently strong coupler tone while keeping excitations to a three-level manifold. The anharmonicity of the device, α ≡ ω_{01} − ω_{12} = E_C/h = 2π × 177 MHz, set an upper limit for the strengths of the drives that could be used. The proximity of the |0⟩ ↔ |2⟩ two-photon transition at ω_{02}/2 = ω_{01} − α/2 could also, at sufficiently strong drives, interfere with the AT signal.
Fig. 6.4: False-color plot of excited state population $\rho_{11} + \rho_{22}$ versus probe detuning $\Delta_p$ and coupler detuning $\Delta_c$ for (a)-(c) data and (d)-(f) simulations of the Autler-Townes splitting. Coupler strengths $\Omega_c/2\pi$ were: (a) and (d) 0.177 MHz, (b) and (e) 0.707 MHz, and (c) and (f) 2.82 MHz, respectively. To account for the larger peak separation, the scale is increased on the bottom row of plots.
6.6 ATD data

[Sil09]. Although the transitions were power-broadened to $\Gamma/2\pi \approx 350\,\text{kHz}$ at the probe amplitude of $\Omega_c/2\pi = 186\,\text{kHz}$, they remained much smaller than the anharmonicity $\alpha$. Therefore, I required $\Omega_c \gg \Gamma = 2\pi \times 350\,\text{kHz}$, $\Omega_c \gg \Omega_p = 2\pi \times 186\,\text{kHz}$ for a well-separated AT doublet, as well as $\Omega_c \ll \alpha = 2\pi \times 177\,\text{MHz}$ to restrict the Hilbert space to the three lowest levels.

From Fig. 6.5, one can see that for coupler detuning $\Delta_c = 0$, the splitting was symmetric around probe detuning $\Delta_p = 0$. Figure 6.5 also shows that I saw an excellent agreement between the data and the density matrix simulation with all parameters independently determined, and an additional Lorentzian added at $\pm \Omega_c/2$ to account for the small background due to the aforementioned fluctuator.

As I described in detail in Section 2.3, at $\Delta_c = \Delta_p = 0$ the eigenstates of the three-level system can be written in the form:

$$|D\rangle = \cos \Theta |0\rangle - \sin \Theta |2\rangle \quad ,$$

$$|+\rangle = \frac{1}{\sqrt{2}} (\sin \Theta |0\rangle + |1\rangle + \cos \Theta |2\rangle) \quad ,$$

$$|-\rangle = \frac{1}{\sqrt{2}} (\sin \Theta |0\rangle - |1\rangle + \cos \Theta |2\rangle) \quad ,$$

(6.6)

where the mixing angle $\Theta = \tan^{-1}(\Omega_p/\Omega_c)$. State $|D\rangle$ is the dark state with eigenvalue of zero, while states $|\pm\rangle$ correspond to eigenvalues $\pm \frac{1}{2} \sqrt{\Omega_p^2 + \Omega_c^2}$, i.e. separated from each other by the generalized Rabi frequency. For large peak separation, the dark state mostly consists of the ground state. In the ATD regime, we have $\Omega_c \gg \Omega_p$, and we find $\Theta \approx 0$. Thus, the dark state is not achieved by population inversion into $|2\rangle$ but, rather, by driving the population in $|0\rangle$. Nevertheless, it should be possible to use the ATD dark
Fig. 6.5: Data (black dots) and simulation (red curve) of the Autler-Townes doublet at $\Delta_c = 0$. Coupler strengths $\Omega_c/2\pi$ were: (a) 0.354 MHz, (b) 0.707 MHz, (c) 1.41 MHz, (d) 2.82 MHz, (e) 5.63 MHz, and (f) 11.2 MHz.
state as the OFF state in router applications [Hoi13] due to vanishing contributions of $|1\rangle$ at large peak separations.

In Section 2.3.3, I defined the dark state fidelity:

$$
\mathcal{F}_D \equiv \sqrt{\langle D | \rho | D \rangle} \\
= \frac{\cos 2\Theta}{2}(\rho_{00} - \rho_{22}) - \frac{\sin 2\Theta}{2}(\rho_{20} + \rho_{02}) + \frac{1}{2}(1 - \rho_{11}) .
$$

(6.7)

From the experimental values of $\Omega_p$ and $\Omega_c$, as well as the density matrix elements calculated in the simulations, I inferred dark state fidelities of the measured data, shown as black points in Fig. 6.6. At the two highest coupler powers [$\Omega_c/2\pi = 5.63$ MHz in Fig. 6.5 (e) and $\Omega_c/2\pi = 11.2$ MHz in Fig. 6.5 (f)] the data simulation differ slightly, as shown in Fig. 6.6.

**Fig. 6.6:** Dark state fidelity $\mathcal{F}_D$ inferred from simulations (black dots), and theoretical fidelity (colored lines) versus $\Omega_c/\Omega_p$ for a system with $\Gamma_{21}^n = \Gamma_{21}/2^n$, $n = 0,1,\ldots,9$ (red to violet). A crossover to the $\Gamma_{21} \ll \Gamma$ regime where EIT is possible is manifest as an increased fidelity even at small $\Omega_c/\Omega_p$. Inset shows detailed view of $\mathcal{F}_D$ results for $\Omega_c/\Omega_p > 10$. 


presumably due to increased proximity to the $\omega_{02}/2$ transition. This causes the ATD to be pushed to a higher frequency and not centered at $\Delta_p = 0$. The fluctuator parameters also changed slightly. To account for these discrepancies in my calculation of $\mathcal{F}_D$, I fit both the AT peaks and the background with Lorentzians to determine new probe detuning $\Delta'_p$, and new background parameters to feed into the simulation. At the maximum separation (29 linewidths), I calculated the dark state fidelity to be $99.6 - 99.9\%$ (see inset in Fig. 6.6). I note that even at the largest coupler powers used, the effects of the higher levels were dispersive, manifesting themselves not as additional excitations but as slight frequency shifts of the doublet.

Figure 6.6 also shows theoretical predictions for the fidelity as a function of $\Omega_c/\Omega_p$ for parameters that would put the system in the EIT regime (see colored curves), which would require $\Gamma_{21} \ll \Gamma_{10}$. I model the EIT regime by replacing $\Gamma_{21}$ by $\Gamma'_2 = \Gamma_{21}/2^n$ for $n = 0, 1, \ldots, 9$ while keeping all other simulation parameters the same. A metastable $|2\rangle$ makes population trapping in that state possible, opening a narrow EIT window, and resulting in high fidelities even for $\Omega_c/\Omega_p \ll 1$. Of course, these are unrealistic parameters for the decay rate of $|2\rangle$, but they illustrate the difference between ATD and EIT regimes.

6.7 Conclusions

In summary, I have observed an Autler-Townes splitting in a 3D transmon system by dressing the three lowest levels of the transmon with two drives (coupler and probe) and reading out the state of the system by pulsing an additional cavity tone. Even at the
highest coupler powers, the data stands in good agreement with a three-level density matrix simulation with no adjustable parameters. The simulations imply that I achieved $99.6 - 99.9\%$ maximum dark state fidelity at 29 linewidths of separation. Although I did not have a direct measurement of the dark state fidelity, my technique for determining $F_D$ as a function of $\Omega_c/\Omega_p$ using independently measured parameters provided a useful metric for assessing EIT, distinguishing EIT from ATD, characterizing the ON/OFF ratio, and estimating dark state coherence.
CHAPTER 7

Raman coherence in a cavity-transmon Λ system

7.1 Introduction

Interference is a fundamental feature of quantum mechanics arising from the addition of complex probability amplitudes. In a three-level system driven by two coherent light fields (probe and coupler), interference leads to a range of phenomena including CPT [Aga93; Ari96] and EIT [Bol91; Mar98]. These two effects are manifestations of Raman coherence that is generated when the system is driven.

CPT results in the system being coherently trapped in the dark state – a zero-eigenvalue eigenstate of the driven system. While in the dark state, the system becomes transparent to both the probe and coupler, giving rise to EIT. Complementary to EIT is EST, which happens when an otherwise transparent system becomes opaque when in the dark state. EST drastically modifies the dispersion properties of the system, and can yield superluminal or negative group velocities [Big03; Chu82; Wan00]. Both CPT and EST are manifestations of Raman coherence, with CPT referring to the “atomic” dynamics, and
EST referring to the dynamics of the propagating electromagnetic fields. Achieving these effects implies the ability to create atomic superpositions and control light propagation in the medium, and has applications to quantum state initialization, quantum computation and routing [Din13; Hei13].

In this Chapter, I report on my observation of Raman coherence with a superconducting qubit acting as an artificial atom. I demonstrated CPT and EST using a single superconducting qubit in a resonant microwave cavity. Through dissipation engineering, I created a Λ system with a metastable state in which population could be coherently trapped. Driving the system via a three-photon Raman process, I obtained a spectroscopic signature of CPT and demonstrated the coherence of the CPT dark state directly in the time domain. By measuring transmission through the cavity, EST was observed at a single-photon level. EST drastically modified the dispersion properties of the cavity, and Gaussian pulses sent within the EST window propagated at large negative group velocities. The corresponding maximum refractive index $n_g = -(1.64 \pm 0.13) \times 10^5$ was the largest negative group index reported to date that we were aware of (cf. [Kea12]). My results suggest that quantum superconducting circuits provide a viable platform for studying quantum optics of multi-level systems and discovering new phenomena such as EST.

### 7.2 Device and setup

I used device C6AQ042314A, which consisted of a transmon superconducting qubit [Koc07] embedded in a three-dimensional microwave cavity [Pai11; Rig12], as shown in
7.2 Device and setup

Fig. 7.1: (a) Experimental arrangement for CPT measurements showing the transmon inside the cavity, as well as the probe (blue), coupler (red) and readout (green) tones. (b) Level diagram for the transmon-cavity system, indicating the probe (blue) and the coupler (red) of frequencies $\omega_p$ and $\omega_c$, and amplitudes $\Omega_p$ and $\Omega_c$, respectively. $\kappa$ is the cavity decay rate (fast), and $\Gamma$ is the qubit decay rate (slow). (c) Simplified model of the energy levels comprising the $\Lambda$. 
7.2 Device and setup

Fig. 7.1 (a). I fabricated transmon Q042314A via standard e-beam lithography, double-angle evaporation [Dol77], and lift-off, which I described in Section 3.2. The transmon had a single Al/$\text{AlO}_x$/Al Josephson junction with tunnelling energy $E_J/h = 25.8$ GHz. Two 500 $\mu$m $\times$ 675 $\mu$m Al pads capacitively shunted the junction reducing the charging energy of the transmon to $E_C/h = 0.198$ GHz. The pads also acted as a dipole antenna, and coupled the qubit to the fundamental $\text{TE}_{101}$ mode of the cavity with strength $g/2\pi = 79$ MHz. Cavity C6A was machined from oxygen-free high-conductivity copper, and had the fundamental mode at $\omega_{\text{cav}}/2\pi = 7.9271$ GHz with an intrinsic quality factor $Q_i = 2 \times 10^4$. I probed this mode in transmission, with the output microwave port coupled much stronger ($Q_{\text{out}} = 3 \times 10^3$) than the input port ($Q_{\text{in}} = 10^6$).

The device was mounted on the mixing chamber of a Leiden Cryogenics CF-450 cryogen-free dilution refrigerator and cooled to 22 mK. To protect the device from Johnson-Nyquist noise, the input microwave line had 64 dB of cold attenuation, while the output microwave line had 40 dB of directional isolation. The cavity output signal was amplified at 3 K by a high-electron mobility transistor amplifier, and then further amplified at room temperature, before being mixed down, digitized, and recorded as described in Chapter 4.

I measured the qubit ground- to excited-state transition frequency to be $\omega_q/2\pi = 6.395194$ GHz, and the cavity resonance with the qubit in the ground state to be $(\omega_{\text{cav}} - \chi)/2\pi = 7.9271$ GHz. Because $g/(\omega_{\text{cav}} - \omega_q) = 0.05 \ll 1$, the system was well-described
by the dispersive Jaynes-Cummings Hamiltonian (see Section 2.2 for details):

\[ H_{JC} = \frac{1}{2} \hbar \omega q \sigma_z + \hbar \left( \omega_{\text{cav}} + \chi \sigma_z \right) a^\dagger a, \tag{7.1} \]

where \( \sigma_z \) is the Pauli z-matrix, \( a^\dagger (a) \) is the cavity creation (annihilation) operator, and the qubit-cavity coupling results in a dispersive shift of \( \chi / 2\pi = -4.1 \text{ MHz} \). The interaction between the qubit and the cavity shifted the resonance of the cavity from \( \omega_{\text{cav}} - \chi \) when the qubit was in the ground state to \( \omega_{\text{cav}} + \chi \) when the qubit was in the excited state. Because \( \chi \) was much larger than the qubit transition linewidth \( (1/T_2^* \sim 150 \text{ kHz}) \), the system was in the dispersive strong coupling limit of the Jaynes-Cummings interaction.

In the dispersive limit, I approximated (to order \( g/\Delta \)) the eigenstates of the Hamiltonian as uncoupled states of the form \( |\text{qubit, cavity}\rangle \), allowing a simplified theoretical treatment of the system that still captured the essential physics. Details of the model assumptions were presented in Section 2.4. I denote the qubit ground and excited states by \( |g\rangle \) and \( |e\rangle \), respectively, and cavity photon number states by \( |n\rangle \). In the dispersive limit, the system has two harmonic ladders: one consisting of states \( |g,n\rangle \), and the other – of states \( |e,n\rangle \) [see Fig. 7.1 (b)].

Diagonal transitions \( |g,n\rangle \leftrightarrow |e,n\rangle \) were qubit-like, with a decay rate \( \Gamma = 1/T_1 = 0.25 \times 10^6 \text{ s}^{-1} \) and Ramsey decay time \( T_2^* = 7.4 \mu\text{s} \). Similarly, ladder transitions \( |g,n\rangle \leftrightarrow |g,n \pm 1\rangle \) and \( |e,n\rangle \leftrightarrow |e,n \pm 1\rangle \) were cavity-like, with decay rate \( n\kappa \) where \( \kappa = 18 \times 10^6 \text{ s}^{-1} \). I engineered the cavity decay rates to be much larger than the qubit decay rate by strongly coupling the output port of the cavity to the microwave line. This guaranteed
preferential downward, rather than diagonal, decay of states in my system.

I formed the Λ system with states \(|g,0\rangle, |e,0\rangle, \text{ and } |e,1\rangle\), and drove it with two microwave tones: probe and coupler as shown in Fig. 7.1 (c). The coupler, of amplitude \(\Omega_c\) and frequency \(\omega_c\), was detuned by \(\Delta_c = \omega_c - \omega_{\text{cav}} - \chi\) from the \(|e,0\rangle \leftrightarrow |e,1\rangle\) transition, driving the fast-decaying arm of the Λ. The probe tone had amplitude \(\Omega_p\) and frequency \(\omega_p\), which was detuned by \(\Delta_p = 2\omega_p - \bar{\omega}_q - \omega_{\text{cav}} - \chi\) from the \(|g,0\rangle \leftrightarrow |e,1\rangle\) transition, driving the slow-decaying arm of the Λ. Here, the qubit frequency \(\omega_q\) is ac-Stark shifted by \(\delta_{ac}\) to \(\bar{\omega}_q = \omega_q + \delta_{ac}\) off-resonantly due to the presence of probe and coupler \([\text{Sch05; Wal07}]\). I note that the direct single-photon \(|g,0\rangle \leftrightarrow |e,1\rangle\) transition was parity-forbidden as it involved a simultaneous excitation of a cavity and a qubit state. Instead, the probe drove a two-photon transition at half the transition frequency.

Henceforth, I refer to the effective single-photon probe amplitude \(\tilde{\Omega}_p \propto \Omega_p^2\) as the probe amplitude, as shown in Fig. 7.1 (d).

For \(\kappa, \tilde{\Omega}_p \gg 2\pi \times \Gamma\), the state can be transferred from \(|g,0\rangle\) into \(|e,0\rangle\) by continuously pumping into \(|e,1\rangle\) and letting it decay to \(|e,0\rangle\). Since \(|e,0\rangle\) is a metastable qubit-like state, when both probe and coupler are present I expected the CPT dark state to be a coherent superposition of \(|g,0\rangle\) and \(|e,0\rangle\). As I discussed in Section 2.3, in the absence of decoherence the dark state \(|D\rangle\) can be written as:

\[
|D\rangle = \cos \Theta \, |g,0\rangle - \sin \Theta \, |e,0\rangle , \tag{7.2}
\]

where \(\Theta = \tan^{-1}(\tilde{\Omega}_p/\Omega_c)\) is the mixing angle.
7.3 Modeling

Based on the discussion in Section 2.4, I modeled the system Hamiltonian by considering 18 states (9 for the \(|g,n\rangle\) ladder, and 9 for the \(|e,n\rangle\) ladder). The probe was modeled as a single-photon drive connecting states \(|g,0\rangle\) and \(|e,1\rangle\), while the coupler connected \(|e,n\rangle\) to \(|e,n \pm 1\rangle\). In the frame co-rotating with the drives, the system Hamiltonian then becomes

\[
\mathcal{H}_\Lambda = \sum_{n=0}^{8} \left[ -n(\Delta_c + 2\chi) |g,n\rangle \langle g,n| - (\Delta_p + (n-1)\Delta_c) |e,n\rangle \langle e,n| \\
+ \frac{\Omega_p}{2} \sqrt{n+1} |e,n+1\rangle \langle e,n| + \frac{\tilde{\Omega}_p}{2} |e,1\rangle \langle g,0| + \text{H.c.} \right].
\] (7.3)

As with the model in Chapter 6, I added decoherence via the Lindblad master equation formalism [Kos72; Lin76] for the density matrix \(\rho\):

\[
\frac{d\rho}{dt} = i \left[ \rho, \mathcal{H}_\Lambda \right] + \sum_j \left( \mathcal{L}_j \rho \mathcal{L}_j^\dagger - \frac{1}{2} \{ \rho, \mathcal{L}_j^\dagger \mathcal{L}_j \} \right),
\] (7.4)

where \(\hbar = 1\) is assumed, Lindblad operators \(\mathcal{L}_j\) included decay channels \(|e,n+1\rangle \rightarrow |e,n\rangle\), \(|g,n+1\rangle \rightarrow |g,n\rangle\), and \(|e,n\rangle \rightarrow |g,n\rangle\), as well as a qubit dephasing term due to the cavity being off-resonantly driven by the coupler [Boi09]. I solved Eq. (7.4) in Mathematica by numerical diagonalization to find the steady state of the system.

Modeling the probe as a single-photon drive simplified the numerics and is acceptable at low drive powers because excitations in the system will not propagate far up the harmonic ladder due to the linear nature of the decay rate scaling with energy. Further-
more, any two states in the model are connected by the probe and coupler via at most a single pathway. Although states in the system may in reality be multiply connected [Rab79], the presence of multiple pathways will only become apparent at large coupler and probe powers. Removing time dependence of the drives from a multiply-connected system is a difficult endeavor, while numerically solving a full time-dependent master equation in a Hilbert space with eighteen levels can be very computationally intensive.

The Mathematica notebook I made for these simulations is available in an online repository [Nov15], or can be requested from the author by email (snovikov AT gmail DOT com).

7.4 Coherent population trapping

In order to obtain the CPT data, I set the coupler on resonance ($\Delta_c = 0$), fixed the probe amplitude $\tilde{\Omega}_p/2\pi = 0.56\text{MHz}$, and swept the probe detuning $\Delta_p$ and the coupler power $\Omega_c$. The experimental details of this two-tone spectroscopy are described in Section 4.3. I pulsed the probe and coupler microwave tones for $30\mu s$, which was much greater than $T_1 = 4\mu s$ and $T_2^* = 7.4\mu s$, to bring the system into steady state before measuring the total excited state population of the qubit $\rho_{\text{tot}} = \sum_{n=0}^{\infty} |\langle \psi | e, n \rangle|^2$ (see Fig. 7.2). For $\Omega_c = 0$ the transition spectral width of $2.3\text{MHz}$ was determined primarily by the natural linewidth of this process, i.e. $\kappa/2\pi = 2.9\text{MHz}$. Given the slow qubit decay rate $\Gamma = 0.25 \times 10^6\text{s}^{-1} \ll \kappa = 18 \times 10^6\text{s}^{-1}$, the maximum excited state population $\rho_{\text{tot}}^{\text{max}} \approx 0.8 > 0.5$ at $\Delta_p = 0$ implies significant population inversion into the metastable state $|e,0\rangle$ of the $\Lambda$ system, rather than two-level saturation. $\rho_{\text{tot}}^{\text{max}}$ was
7.4 Coherent population trapping

Fig. 7.2: (a) CPT spectroscopy data (left) and simulation (right) of the two-photon sideband $|g, 0\rangle \leftrightarrow |e, 1\rangle$ showing excited state population $\rho_{\text{tot}}$ versus probe detuning $\Delta_p$ and coupler amplitude $\Omega_c$ at $\Delta_c = 0$ and $\Omega_p/2\pi = 0.56$ MHz. Individual linecuts of data (red) and theory (black) for $\Omega_c/2\pi$ of (b) 0.32 MHz, (c) 0.64 MHz, (d) 1.28 MHz, and (e) 2.56 MHz.
Coherent population trapping

limited because $\tilde{\Omega}_p/2\pi \sim 2\Gamma$, and even more population inversion could be achieved if $\tilde{\Omega}_p/2\pi \gg \Gamma$. Unfortunately, the output power limitation of the probe microwave source placed an upper bound on the maximum possible $\tilde{\Omega}_p$.

As the coupler amplitude $\Omega_c$ was increased from zero, a narrow CPT window centered at $\Delta_p = 0$ appeared [see Fig. 7.2 (b)-(e)]. My simulations of the system’s steady-state density matrix can be used to understand the relevant physics of the system in this limit. The simulation parameters used to obtain the theory curves in Fig. 7.2 are shown in Table 7.1. For low coupler powers I found excellent agreement between the simulations (black curve) and the data (red points), as shown in Fig. 7.2 (b)-(e). One expects in the steady state at $\Delta_p = 0$ for the system to be in the dark state, a coherent superposition of $|e,0\rangle$ and $|g,0\rangle$. The simulated populations and coherences of the dark state density matrix $\rho$ as a function of $\Omega_c$ are shown in Fig. 7.3. Despite serving as an intermediary through which the tones couple $|g,0\rangle$ to $|e,0\rangle$, state $|e,1\rangle$ did not accrue population due to destructive quantum interference from different excitation pathways. This is consistent with the ideal case of Eq. (7.2), where $|e,1\rangle$ does not contribute to the dark state superposition. Due to this interference, the width of the CPT window (0.15 MHz for $\Omega_c/2\pi = 0.64$ MHz) was much smaller than the width of the overall three-photon Raman transition (2.3 MHz).

For $\Omega_c/2\pi = 2.56$ MHz, my simulations indicated that $> 99.5\%$ of the population remained within the $\Lambda$ system comprised of $|g,0\rangle$, $|e,0\rangle$, and $|e,1\rangle$. The excitation did not propagate to higher states of the $|e,n\rangle$ harmonic ladder because of the increasing cavity decay rates: state $|e,n\rangle$ decays at a rate $n\kappa$ to $|e,n-1\rangle$ and $\Gamma$ to $|g,n\rangle$. This ensured that
7.4 Coherent population trapping

Fig. 7.3: Simulated dark state (a) populations $\langle g,0|\rho|g,0 \rangle$ in black, $\langle e,0|\rho|e,0 \rangle$ in red, $\langle e,1|\rho|e,1 \rangle$ in blue, and (b) coherences $\langle g,0|\rho|e,0 \rangle$ in black, $\langle g,0|\rho|e,1 \rangle$ in red, and $\langle e,0|\rho|e,1 \rangle$ in blue versus $\Omega_c$ for $\Delta_p = 0$. Vertical grey line indicates the location of the maximum-coherence state.

Table 7.1: Parameters for the simulation of CPT data in Fig. 7.2 (b)-(e). Here, $\gamma$ is a dephasing rate of the qubit due to coupler photons in the cavity. Coupler detuning was set to $\Delta_c = 0$ throughout.

<table>
<thead>
<tr>
<th>Data set</th>
<th>$\Omega_c/2\pi$ [MHz]</th>
<th>$\Omega_p/2\pi$ [MHz]</th>
<th>$\chi/2\pi$ [MHz]</th>
<th>$\Gamma \times 10^6$ [s$^{-1}$]</th>
<th>$\gamma \times 10^6$ [s$^{-1}$]</th>
</tr>
</thead>
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<tr>
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<td>0.56</td>
<td>4.05</td>
<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>c</td>
<td>0.64</td>
<td>0.56</td>
<td>4.05</td>
<td>0.25</td>
<td>0.12</td>
</tr>
<tr>
<td>d</td>
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<td>0.56</td>
<td>4.05</td>
<td>0.25</td>
<td>0.24</td>
</tr>
<tr>
<td>e</td>
<td>2.56</td>
<td>0.56</td>
<td>4.05</td>
<td>0.25</td>
<td>0.48</td>
</tr>
</tbody>
</table>
7.5 Dark state coherence

excitations fell down the ladder and stayed in the low energy manifold, at least in the regime of \( \Omega_c \ll n \kappa = n \times 18 \times 10^6 \text{ s}^{-1} \).

According to Eq. (7.2), when there is no decoherence and \( \Omega_c = \tilde{\Omega}_p \) the dark state is an equal superposition of \( |g,0\rangle \) and \( |e,0\rangle \). When decoherence is included, my simulations show that for \( \tilde{\Omega}_p/2\pi = 0.56 \text{ MHz} \), the maximum superposition state occurs at a coupler amplitude of \( \Omega_c/2\pi \approx 0.64 \text{ MHz} \) [Fig. 7.2 (c)]. For this case, the simulation yielded \( \langle g,0|\rho|g,0\rangle = 0.60 \), \( \langle e,0|\rho|e,0\rangle = 0.39 \approx \rho_{\text{tot}}^{\text{max}}/2 \) for the diagonal elements and \( \langle g,0|\rho|e,0\rangle = -0.41 \) for the off-diagonal coherence term.

7.5 Dark state coherence

I used the simulations to find the dark states corresponding to the four coupler amplitudes shown in Fig. 7.2 (b)-(d). These states can be represented as vectors on the Bloch sphere, as I show in Fig. 7.4 (a). Here, the azimuthal angle \( 2\Theta \) is measured from the south pole of the Bloch sphere, so the population in \( |g,0\rangle \) is \( \cos(\Theta) \) while the coherence is \( \sin(2\Theta)/2 \). The length of each vector is less than unity because of decoherence, i.e. the system is not a pure superposition of states but is in a mixed state with significant coherence remaining. A pure superposition state would lie on the surface of the sphere, while a mixed state with no coherence would be represented as a vector of zero length. The color of each vector on the plot depicts the relative contributions of \( |g,0\rangle \) (red) and \( |e,0\rangle \) (blue) to the superposition.

Although the narrow linewidth of the CPT window and good agreement with simulations are indicative of quantum interference, spectroscopy cannot directly show that
7.5 Dark state coherence

Fig. 7.4: Dark state coherence in time domain. (a) Energy level diagram and a Bloch sphere illustration of the dark state versus the mixing angle $\Theta$. Four double arrows represent the prepared dark states. Dot-dashed line shows the population inversion and coherence limitations imposed on the dark state by $T_1$ decay of $|e,0\rangle$. (b) Pulse sequence used to demonstrate dark state coherence. (c) Ramsey oscillations of dark states prepared via CPT. Data (dots) and fits (lines) corresponding, top to bottom, to $\Omega_c/2\pi$ of 0.32 MHz (offset by 0.145), 0.64 MHz (offset by 0.075), 1.28 MHz (offset by 0.025), and 2.56 MHz.
the superposition of $|g,0\rangle$ and $|e,0\rangle$ comprising the dark state is coherent. In order to demonstrate coherence, I carried out a Ramsey-like free evolution experiment with the pulse sequence shown in Fig. 7.4 (b). Although the probe and coupler detunings were $\Delta_p = \Delta_c = 0$, the ac-Stark shift of $\omega_q$ to $\tilde{\omega}_q = \omega_q + \delta_{ac}$ due to the presence of probe and coupler [Sch05; Wal07] resulted in $2\omega_p - \omega_c - \omega_q = \delta_{ac} = -2.85\,\text{MHz}$. The system was initialized to a dark state by turning the two tones on for $30\,\mu\text{s} \gg \{T_1, T_2^*, 1/\kappa\}$. Then, with the tones turned off, the state was allowed to evolve freely for time $t$. During this time, the state precessed around the $z$-axis of the Bloch sphere. Next, the state was rotated around the $x$-axis by $\sim 5^\circ$ with a $1\,\mu\text{s}$-long pulse of the same probe and coupler tones, after which $\rho_{\text{tot}}$ was measured. Figure 7.4 (c) shows $\rho_{\text{tot}}$ data for different coupler amplitudes. One sees a decaying Ramsey oscillation with different amplitudes on top of an exponential decay. I note that all of the measured oscillations are in phase with each other since the initialization pulse is in phase with the rotation pulse. Furthermore, at the probe power used, $\bar{\Omega}_p/2\pi = 0.56\,\text{MHz}$, the amplitude of oscillations is largest for $\Omega_c/2\pi = 0.64\,\text{MHz}$, which is the coupler amplitude where the CPT dark state is expected to have nearly maximum coherence [see Fig. 7.3 (b)].

The oscillations were observable because the system was prepared in a superposition state with a well-defined phase, and the free-evolution frequency of the state $\omega_q$ when the drives were turned off was different from the drive frequency $2\omega_p - \omega_c = \tilde{\omega}_q = \omega_q + \delta_{ac}$, which was the ac-Stark-shifted frequency. The dark state precessed with respect to the drive at $\delta_{ac}$, while slowly losing coherence. The starting point of the oscillations corresponded to the steady-state population for the dark states observed in
the spectroscopy [$\Delta p = 0$ on Fig. 7.2 (b)-(e)].

I fit the phenomenological function

$$\rho_{tot} = Ae^{-t/\tau_2} \sin(\delta_R t + \phi) + Be^{-t/\tau_1} + C$$ (7.5)

to the data using the values given in Table 7.2. The first term in Eq. (7.5) describes the free evolution of the dark state, and the fits confirmed that it decayed on a Ramsey-like timescale with $\tau_2 = (7.1 - 7.8) \mu s \approx T_2^\star$. Also, $\delta_R = \delta_{ac}$ within 0.07%, confirming that I was indeed performing an off-resonant Ramsey precession. The second term describes the overall population decay, with the fits confirming a characteristic decay time $\tau_1 = (3.9 - 4.3) \mu s \approx T_1$.

Table 7.2: Parameters for fits in Fig. 7.4. Note that $\tau_1 \approx T_1 = 4.0 \mu s$, $\tau_2 \approx T_2^\star = 7.4 \mu s$, and $\delta_R \approx \delta_{ac} = -2\pi \times 2.85 \text{MHz}$.

<table>
<thead>
<tr>
<th>$\Omega_c/2\pi \text{[MHz]}$</th>
<th>$\tau_1 \text{[\mu s]}$</th>
<th>$\tau_2 \text{[\mu s]}$</th>
<th>$\delta_R/2\pi \text{[MHz]}$</th>
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<td>4.0</td>
<td>7.6</td>
<td>2.845</td>
</tr>
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</table>
7.6 Dressed state blockade

To investigate how the coupler dressed the $|e,n\rangle \leftrightarrow |e,n+1\rangle$ harmonic ladder, I set the coupler detuning $\Delta_c/2\pi = -30\,\text{MHz}$ and measured $\rho_{\text{tot}}$ while sweeping coupler source voltage $V_c \propto \Omega_c$ and probe detuning $\Delta_p$ (see Fig. 7.5). I observed the main $|g,0\rangle \leftrightarrow |e,1\rangle$ two-photon transition requiring $2\omega_p$ of energy, $|g,0\rangle \leftrightarrow |e,0\rangle$ three-photon transition requiring $2\omega_p - \omega_c$ of energy, $|g,0\rangle \leftrightarrow |e,2\rangle$ three-photon transition requiring $2\omega_p + \omega_c$ of energy, and, at large coupler powers, $|g,0\rangle \leftrightarrow |e,3\rangle$ four-photon transition requiring $2\omega_p + 2\omega_c$ of energy and $|g,0\rangle \leftrightarrow |e,4\rangle$ five-photon transition requiring $2\omega_p + 3\omega_c$ of energy (see Fig. 7.5 inset).

I modeled the system by considering the following Hamiltonian of a harmonic ladder dressed by the coupler ($\hbar = 1$ here):

$$
\mathcal{H}_d = \sum_{n=0}^{99} \left[ -n\Delta_c \langle e,n|e,n\rangle + \alpha \frac{\Omega_c}{2} \sqrt{n+1} \langle e,n+1|e,n+1 \rangle + \text{H.c.} \right]
$$

(7.6)

where $\alpha$ is a proportionality constant between $V_c$ and $\Omega_c$, and a truncated Hilbert space of 100 states is used. For fit parameter $\alpha = 0.599$, the eigenvalues of $\mathcal{H}_d$ are in good agreement with the data (see dashed curves in Fig. 7.5). Because the system is harmonic (i.e. not saturable), the coupler ac-Stark shifts the whole ladder without separating the individual ladder states further from each other.

For the most part, each eigenstate $|\tilde{n}\rangle$ (with $\tilde{n} = 0,1,2,\ldots$) of $\mathcal{H}_d$ contains some contribution from $|e,1\rangle$, making it possible for the probe to excite the $|g,0\rangle \leftrightarrow |\tilde{n}\rangle$ transition. However, when the condition $\alpha \sqrt{\tilde{n} + 1}\Omega_c/2 = \tilde{n}\Delta_c$ is met, the eigenvalue
7.6 Dressed state blockade

corresponding to $|\tilde{n}\rangle$ becomes zero, and the eigenstate loses all contribution from $|e, 1\rangle$.

At coupler amplitudes meeting this condition, the probe transition $|g, 0\rangle \leftrightarrow |\tilde{n}\rangle$ becomes forbidden, as seen in the reduction of $\rho_{\text{tot}}$ at $V_c \approx 0, 100, 120, 150$ mV in Fig. 7.5. This feature could potentially be used to accurately calibrate $\Omega_c$.

Fig. 7.5: False-color plot of excited state population $\rho_{\text{tot}}$ versus probe detuning $\Delta_p$ and coupler source amplitude $V_c \propto \Omega_c$ for an off-resonant coupler at $\Delta_c/2\pi = -30$ MHz. Dashed black curves are a theoretical fit calculated by diagonalizing a drive truncated harmonic ladder. Inset shows a level diagram with three of the transitions labeled.
7.7 3-photon Rabi oscillations

The presence of an intermediate state $|e,1\rangle$ can be used to transfer population in the system off-resonantly. In order to demonstrate coherent off-resonant transfer of population in the time-domain, I detuned the coupler to $\omega_c/2\pi \approx -96.5$ MHz. Besides the main $|g,0\rangle \leftrightarrow |e,1\rangle$ peak at $\Delta_p = 0$, probe spectroscopy showed an additional sideband at $\Delta_p = \Delta_c$, as evident in Fig. 7.6 (a). Pulsing both probe and coupler for a short period of time with $\Delta_p \approx \Delta_c$, I observed Rabi oscillations [see Fig. 7.6 (b)]. These oscillations imply an off-resonant transfer of population between $|g,0\rangle$ and $|e,0\rangle$ mediated by the detuned intermediate state $|e,1\rangle$. Two photons of the probe, and one photon of the coupler were combined to make the transition. I note that this technique could be used to perform off-resonant quantum gates when excitation of other states in the system is undesired.
Fig. 7.6: Off-resonant three-photon Rabi oscillations in a quasi-$\Lambda$ system. (a) With the coupler detuned, an additional sideband peak appears in the data (red) when $\Delta_p = \Delta_c \approx -96.5$ MHz. (b) False-color plot of excited state population versus probe detuning $\Delta_p$ and Rabi pulse time $t$ of the three-photon drive tones, showing three-photon Rabi oscillations and off-resonant population transfer from $|g,0\rangle$ to $|e,0\rangle$.

7.8 Electromagnetically suppressed transmission

The quantum interference responsible for dark state coherence also affects propagation properties of the electromagnetic fields driving the system. I investigated these properties by directly measuring the microwave transmission through the cavity (see Fig. 7.7). In an optically dense medium with atomic absorption, creation of the dark state can lead to
a substantial increase in the transmission – EIT. For my system the opposite happened: ordinarily there was a large transmission through the cavity at $|g,n\rangle \leftrightarrow |g,n+1\rangle$ or $|e,n\rangle \leftrightarrow |e,n+1\rangle$ resonant transitions. However, driving the system into the dark state by applying a probe tone at $\Delta_p = 0$ resulted in a drastic decrease in transmission through the cavity – a phenomenon I call electromagnetically suppressed transmission (EST).

I observed EST by measuring the ratio of the coupler output amplitude to the coupler input amplitude, $S_{21} \equiv V_{out}/V_{in}$ for fixed probe amplitude $\tilde{\Omega}_p/2\pi = 0.56$ MHz, as illustrated in Fig. 7.7. The magnitude $|S_{21}|$ and the phase $\phi = \arg(S_{21})$ of coupler transmission, measured with Agilent E5071C VNA as a function of the probe and coupler detunings, are shown in Fig. 7.8. When $|\Delta_p| \gg \kappa \approx 2.9$ MHz, transmission through the cavity
occurred via the $|g,n\rangle \leftrightarrow |g,n+1\rangle$ cavity-like transitions. As the two-photon probe frequency approached that of the $|g,0\rangle \leftrightarrow |e,1\rangle$ transition (i.e. $\Delta_p \sim \kappa$), an excitation appeared in the $|e,n\rangle$ ladder, resulting in increased transmission via the $|e,n\rangle \leftrightarrow |e,n+1\rangle$ channel. At $\Delta_p = \Delta_c = 0$, however, the system went into the dark state, shutting down the $|e,n\rangle \leftrightarrow |e,n+1\rangle$ cavity transmission by about 10 dB. This resulted in a reflected signal with very little transmission through the cavity, i.e. EST. Although the magnitude of the transmission $|S_{21}|$ at the bottom of the narrow EST window was the same as
that of an unperturbed cavity, the phase $\phi$ experienced large changes [see Fig. 7.8 (b)] consistent with the Kramers-Kronig relations.

I examined the non-linear nature of EST by varying the average cavity occupation $\bar{n}$ by changing the coupler power $\Omega_c$ while keeping the probe off [see Fig. 7.9 (a)] or on with $\tilde{\Omega}_p/2\pi = 0.56$ MHz [see Fig. 7.9 (b)]. At low coupler photon numbers ($\bar{n} \lesssim 3$), the EST window appears in the transmission when the probe is on. The feature persists for a decade and a half in photon numbers, down to $\bar{n} \approx 0.1$. The coupler power $\Omega_c/2\pi = 0.64$ MHz that produced the maximum-superposition dark state in Section 7.4 corresponded to $\bar{n} \approx 0.8$, implying that both CPT and EST happened in the same region of drive amplitudes. The occurrence of both effects at low photon numbers is consistent with a $\Lambda$ system from a single qubit-cavity system.

The change in the transmission $\Delta|S_{21}|$ shows that inside the EST window ($\Delta_c \approx 0$) the magnitude $|S_{21}|$ did not change relative to the probe-off case, and is equal to the default off-resonant transmission through the cavity when the qubit is in the ground state [see Fig. 7.10 (a)]. Because the system had one strongly coupled “artificial atom”, it became nonlinear as soon as the average number of photons approached one. For $\bar{n} > 3$, the EST feature disappeared completely, and the transmission reverted to the probe-off case. This corresponded to the shifting of the original $|g,n\rangle \leftrightarrow |g,n + 1\rangle$ cavity resonance at $\Delta_c/2\pi = 8.2$ MHz due to the Jaynes-Cummings non-linearity [Bis10a; Boi10; Ree10].

Since EST arises due to interference between two different pathways, $\Delta|S_{21}|$ is described by a difference between two Lorentzian resonances. I fit my $\Delta|S_{21}|$ data at
7.8 Electromagnetically suppressed transmission

Fig. 7.9: EST measurements versus coupler power. Magnitude of transmission when probe is (a) off and (b) on at probe amplitude $\tilde{\Omega}_p/2\pi = 0.56$ MHz versus the coupler detuning $\Delta_c$ the average photon number $\bar{n}$ in the cavity due to the coupler tone.

$\bar{n} = 0.26$ to the following function [see Fig. 7.10 (b)]:

$$\Delta |S_{21}|(f_c) = \frac{2A_1 w_1}{4(f_c - f_1)^2 + w_1^2} + \frac{2A_2 w_2}{4(f_c - f_2)^2 + w_2^2} + S_0, \quad (7.7)$$

where $f_c = \omega_c/2\pi$ is the coupler frequency, $S_0$ is the background transmission, and $f_j, A_j,$
7.8 Electromagnetically suppressed transmission

$w_j$ are the peak center, amplitude, and width for the $j$-th Lorentzian, respectively. I note that the EST window linewidth (0.18 MHz) is much smaller than the cavity linewidth (2.9 MHz), consistent with quantum interference.

![Figure 7.10](image)

**Fig. 7.10:** (a) Difference in the magnitude of transmission between probe on and off versus the average photon number $\bar{n}$ in the cavity due to the coupler tone. The transmission is unchanged in the EST window at $\Delta_c = 0$, as well as at large $\bar{n}$. (b) Transmission at $\bar{n} = 0.26$ showing a narrow EST window in the data (red) and fit (black). Inset details the narrow EST interference region.
7.9 Superluminal pulse propagation

A rapid variation of phase with coupler detuning $\Delta_c$ near $\Delta_c = 0$ implies a large dispersion of coupler microwaves propagating through the system. The phase $\phi$ of a coupler pulse propagating forward in time can be written as $\phi = k x - \omega_c t$, where $k = 2\pi/\lambda$ and the group delay $\tau_g$ of a narrow-band pulse propagating through the system is:

$$\tau_g = -\frac{d\phi}{d\omega_c}.$$  \hspace{1cm} (7.8)

I investigated the group delay by sending coupler pulses with a Gaussian envelope through the system and measuring the arrival time of the transmitted pulses [see Fig. 7.11 (a)]. Transmitted pulses were fit to a sinusoidally-modulated Gaussian function, from which I extracted the arrival time $t_c$, as defined by the center of the pulse. The group delay $\tau_g$ was then defined as the difference between the pulse centers of the signal (when probe is on with $\tilde{\Omega}_p/2\pi = 0.56$ MHz) and the reference (when probe is off): $\tau_g \equiv t^{\text{sig}}_c - t^{\text{ref}}_c$. Figure 7.11 (c) shows the delay versus the coupler frequency. As expected for negative dispersion, the group delays were negative, implying that a Gaussian pulse exited the cavity before it had fully entered it. Negative group velocities do not violate causality [Boy09; Gar98; Geh06; Ste03] because information contained in the pulse is encoded in discontinuities. The high-frequency Fourier components comprising a discontinuity are not affected by the EST dispersion and travel at most with the speed of light.

At $\Delta_c = 0$, I found the maximum negative group delay of $\tau_g = -9.40 \pm 0.79 \, \mu s$.  

172
Fig. 7.11: (a) Demodulated coupler pulse (black, offset by 3 mV) with $\Delta_c = 0$ after traversing the cavity when the probe is turned off and when the probe is present (red). Gaussian envelopes of the least-squares fits to the data are shown as dashed lines. Arrows indicate pulse centers $t_c$ extracted from fits. (b) Fit (black) of Eq. (7.8) to the measured phase shift in transmission (red). (c) Red points show group delay $\tau_g$ data extracted from pulse propagation data versus coupler frequency, and black curve shows $\tau_g$ from Eq. (7.9) using the measured phase shift in transmission shown in (b).
7.9 Superluminal pulse propagation

Given the distance \( l = 17 \text{ mm} \) between the input and output ports of the cavity, the corresponding group velocity of the pulse was \( v_g = l/\tau_g = -1.82 \pm 0.14 \text{ km/sec} \), and the group refractive index \( n_g = -(1.64 \pm 0.13) \times 10^5 \). I emphasize that my data was obtained with a single superconducting qubit in a microwave cavity at single photon numbers, whereas previous experiments measured large changes in \( n_g \) from an ensemble of atoms.

The measured phase \( \phi(f) \) dependence of the continuous-wave coupler transmission, such as in Fig. 7.8 (b), can also be used to theoretically predict the group delay. The phase response of the coupler is obtained via the Kramers-Kronig relations by integrating Eq. (7.7) with respect to \( f_c \):

\[
\phi(f_c) = \frac{2A_1w_1(f_c-f_1)/\pi}{4(f_c-f_1)^2+w_1^2} + \frac{2A_2w_2(f_c-f_2)/\pi}{4(f_c-f_2)^2+w_2^2} + \phi_0, \tag{7.9}
\]

where \( f_c = \omega_c/2\pi \) is the coupler frequency, \( \phi_0 \) is the background transmission, and \( f_j, A_j, w_j \) are the peak center, amplitude, and width for the \( j \)-th Lorentzian, respectively. Figure 7.9 (c) shows a fit of Eq. (7.9) to the phase data. From this fit and Eq. (7.8), I obtained a prediction for group delay. The theory showed good agreement with the directly measured group delay except for a frequency offset of \(-25 \text{ kHz} \). I believe this small offset is a result of distortion of pulses because their frequency width was comparable to that of the EST window.
7.10 Conclusions

In conclusion, I have engineered and measured a transmon-cavity Λ system based on the $|g,0\rangle$, $|e,0\rangle$, and $|e,1\rangle$ states of the dispersive Jaynes-Cummings Hamiltonian. By designing the system with a qubit decay rate $\Gamma$ much less than the cavity decay rate $\kappa$, I was able to invert population and achieve CPT due to quantum interference. Although the system had many harmonic cavity levels, the dark state was effectively confined to three levels of the desired Λ. I showed the dark state coherence to be preserved while the drives were present, and with the drives off the coherence decayed on the timescale of the metastable state $|e,0\rangle$. By measuring microwave transmission through the cavity, I demonstrated that in my system, EST accompanies CPT. Although the magnitude of the transmission within the EST window was not modified, the dispersion changed drastically, leading to superluminal propagation of Gaussian pulses, and a record negative refractive index.

I note that my results are applicable to any qubit-cavity system that can achieve the strong dispersive regime of cavity QED, and this approach can be used to generate superposition and entanglement with continuous-wave tones. It should be possible to generate a high-fidelity (> 99.5%) dark state by increasing the probe power $\tilde{\Omega}_p$, as well as the coherence time of the metastable state to $T_1$, $T_2^* \sim 100 \, \mu s$ (as was shown on the fidelity “map” in Fig. 2.17). Large probe power can be obtained with an addition of a high-power amplifier. The long coherence times are achievable with 3D transmons [Pai11; Rig12], and I observed long coherence times in a different transmon
(see Chapter 5). Moreover, by using a notch resonator design, a complementary system that exhibits EIT can be made. Such a system would allow for slow and stopped light, and would have direct applications to routing and storage of single microwave photons.
CHAPTER 8

Conclusions

8.1 Summary of main findings

In this thesis I have detailed my investigations of effective three-level systems based on superconducting transmon qubits driven with two microwave tones and cooled to millikelvin temperatures. In Chapter 6, I described how I used the three lowest energy levels of a transmon in a cascade configuration to observe the Autler-Townes doublet. The long coherence times of my device, described in Section 6.5, resulted in excitations being confined to the three levels of the transmon, allowing me to theoretically model the system with almost no approximations (see Section 6.3). At the same time, I used a state-of-the-art Jaynes-Cummings non-linearity readout (see Section 6.4) to characterize the spectrum and coherence properties of the qubit-cavity system, and this allowed me to independently determine all the free parameters. I defined the dark state fidelity and showed that my cascade system could not have a coherent dark state in the low-coupler limit (see Section 6.6). My model of the cascade system, which I presented in Section 2.3,
8.1 Summary of main findings

confirmed that without a metastable state CPT and EIT could not be achieved. These results lead me to recognize the need for a three-level system with a metastable state in order to observe CPT and EIT effects.

In Chapter 7, I described how I created a cQED Λ-system from the \(|g,0\rangle, |e,0\rangle, \) and \(|e,1\rangle\) combined levels of a transmon and a cavity. By designing the cavity decay \(\kappa\) to be much larger than the qubit decay \(\Gamma\), I made the qubit-like state \(|e,0\rangle\) metastable relative to other states of the Λ. In agreement with my model, I was able to observe CPT spectroscopically, as I showed in Section 7.4. I demonstrated that the dark state was indeed coherent by performing a Ramsey-like experiment in which the dark state evolved freely before being measured (see Section 7.5). Although I expected EIT to accompany CPT, microwave transmission measurements showed a related but new phenomenon – electromagnetically suppressed transmission, or EST (see Section 7.8). The negative dispersion of EST led to superluminal propagation of pulses through the system, which I measured in a time-of-flight experiment, described in Section 7.9. The corresponding negative refractive group index \(n_g = -(1.64 \pm 0.13) \times 10^5\) was the largest reported to date.

The ability to establish Raman coherence in the Λ system presented in this thesis opens several potential avenues for quantum control and quantum optics experiments with microwave photons. I present a few of the potential studies of interest in the next sections. Because of the large photon-matter coupling achievable in cQED, these experiments can be done at a single-microwave-photon and single-qubit level. As shown with EST, the superconducting architecture has the potential for revealing new phenomena, previously
unobserved with other systems.

8.2 State stabilization

An immediate application of CPT is as a simple and robust method for preparing high-fidelity superposition states. The typical method to generate superposition states in superconducting qubits has used short microwave pulses (e.g. $\pi/2$ pulses), which can lead to infidelity in the final state due to timing errors and mixer imperfections (see, for example, Section 4.5.4). Preparing an arbitrary superposition with CPT relies on continuous tones, and it is the relative amplitudes and phases of the probe and coupler which determine the superposition state that is prepared, rather than pulse timing. These parameters are much easier to control, and the resulting state will be stabilized in the superposition as long as the drives are present. This means the state can wait in the required superposition until it needs to be used, helping align gate operations in time. The qubit can be pumped into the ground state via CPT, and this would allow resetting the quantum computation faster than the typical initialization by relaxation. Given the typical coherence times $T_1, T_2^* \sim 100\,\mu s$ of 3D transmons, my simulations show that state preparation fidelities $> 99\%$ can be achieved.

8.3 Off-resonant control

By using off-resonant Raman drives, it is possible to perform without exciting unwanted states. The three-photon Rabi oscillations I presented in Section 7.7 are an example of such procedure. Of course, in that particular device, a single-photon drive on the
8.3 Off-resonant control

$|g,0\rangle \leftrightarrow |e,0\rangle$ transition would lead to the same result. However, in systems with many qubits or where direct dipole transitions are forbidden (or states are too close in energy), off-resonant control can prove useful.

A particular version of off-resonant control called stimulated Raman adiabatic passage (STIRAP) [Cub05; Kuk89], has been used by the atomic physics community to perform population inversion. In this procedure, shown in Fig. 8.1, probe and coupler tones can transfer population between two states, $|0\rangle$ and $|2\rangle$, via an intermediate third state $|1\rangle$. By the virtue of being detuned from the $|1\rangle$, the drives do not populate the intermediate state. The timing of the pulses is counter-intuitive, as the coupler needs to be turned on first, before the probe. After the coupling between $|1\rangle$ and $|2\rangle$ is created this way, the probe is turned on to transfer the population.

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**Fig. 8.1:** (a) Transfer of population from $|0\rangle$ to $|2\rangle$ via two-tone Raman drive detuned from $|1\rangle$. (b) Counter-intuitive timing of the pulses to achieve optimal transfer.
8.4 EIT with a notch-style cavity

In Section 7.8, I showed that my qubit-cavity system displayed CPT, and this was accompanied by EST and negative dispersion. It should be possible to construct a system where the typical EIT with large positive dispersion occurs. For example, instead of measuring the cavity in a bandpass configuration, the cavity could be coupled to a common input-output transmission line, i.e. the notch-style configuration that is commonly used in transmons coupled to lumped-element resonators. The coupler microwave transmission through such a system would normally be diminished on resonance. Turning on an appropriate probe tone would result in dark state creation, restoring full transmission within the EIT window. At the same time, because of Kramers-Kronig relations, the dispersion would be positive, and large, inside the window. This would effectively create a very large refractive index, slowing down coupler or probe pulses propagating through the system.

A superconducting system exhibiting EIT would allow for arbitrary on-chip delays of microwave signals at single-photon level. By pulsing the coupler, information carried by the probe could also be coherently stored and retrieved (“stopped light”), as was done in other systems [Hei13; Liu01; Phi01]. An illustration of the pulse sequence that can be used to store microwave photons in a $\Lambda$ system is shown in Fig. 8.2. Not only can this system be used for information storage but also as a potential deterministic single-photon source, releasing a photon when the coupler is turned on the second time.
8.5 Raman coherence in the ultra-strong coupling limit

Recently, vacuum fluctuations were shown to enhance EIT [Müc10; TS11]. These fluctuations can be made large by using a high-impedance cavity with the help of, for example, superinductance [Man12], resulting in ultra-strong light-matter coupling. The interaction between the superconducting qubits and light in such systems would be highly non-trivial, and could provide insight into this relatively inaccessible area of physics. Raman coherence in these systems can potentially result in very interesting effects and new phenomena, both expanding our understanding of nature and enabling applications to the fields of quantum computation and quantum optics.

Fig. 8.2: (a) Level diagram and (b) the pulse sequence for writing, storing, and retrieving a probe photon in a Λ system.
A. Fabrication recipes

A.1 Cavity cleaning, etching, and polishing

A.1.1 Recipe 1

This recipe produced the best results with the fewest number of ingredients.

1. Rinse with Acetone, Methanol, IPA.

2. Dip and agitate in a 5:1 solution of $\text{HNO}_3 : \text{H}_2\text{O}$ for several minutes to etch (in a fume hood).

3. Dip and agitate in $\text{NH}_4\text{OH}$ to stop the etching.

4. Rinse thoroughly with DI water.

5. Rinse with IPA.

A.1 Cavity cleaning, etching, and polishing

A.1.2 Recipe 2

This recipe has extra steps and uses a different etch formula.

1. Rinse in hot TCE and Methanol to degrease.
2. Rinse with Acetone, Methanol, IPA.
3. Dip and agitate in NH₄OH to strip the oxide.
4. Quench with DI, rinse with IPA, blow dry.
5. Dip and agitate in a 3:1:3 solution of H₂SO₄ : HNO₃ : H₂O for several minutes to etch (in a fume hood).
6. Rinse thoroughly with DI water.
7. Rinse with IPA.
8. Blow dry.

A.1.3 Recipe 3

The original recipe which showed some improvement in the quality factor.

1. Rinse with Acetone, Methanol, IPA.
2. Dip and agitate in 0.97:0.02:0.01 solution of H₂O : H₂SO₄ : H₂O₂ for several minutes to etch (in a fume hood).
3. Rinse thoroughly with DI water.
4. Rinse with IPA.
5. Blow dry.
A.2 Optical lithography

A.2.1 Spinning resist

1. Spin LOR-5A at 3000 RPM for 60 sec.

2. Bake on a hotplate at 190 °C for 60 sec. Allow to cool down.

3. Spin OIR 906-10 at 3500 RPM for 60 sec.

4. Bake on a hotplate at 90 °C for 60 sec. Allow to cool down.

5. After UV exposure (4 sec), bake on a hotplate at 120 °C for 60 sec.

A.2.2 Development

1. Agitate in OPD4262 for 80 sec.

2. Rinse in running DI water for at least 60 sec.

3. (optional) Dip in IPA for several seconds.


A.2.3 Lift-off

1. Place in a bath of Remover-PG (or NMP) at 70 °C for 1 hour, while agitating with a magnetic stirrer. Allow to cool down.

2. Move to a second bath of Remover-PG (or NMP) at 70 °C for 15 minutes, while agitating with a magnetic stirrer.

3. Place the beaker with the wafer in an ultrasonic tank for 15 minutes.
A.3 Electron-beam lithography

4. Rinse with running DI water for 60 sec.

5. Rinse with IPA for 60 sec.


A.3 Electron-beam lithography

A.3.1 Spinning resist

1. Spin MMA(8.5)MAA EL11 at 1000 RPM for 60 sec.

2. Bake on a hotplate at 180 °C for 5 min. Allow to cool down.

3. Spin ZEP520A DR2.3 at 5000 RPM for 60 sec.

4. Bake on a hotplate at 180 °C for 5 min.

5. Hard bake in an oven at 180 °C for 30 min.

6. After depositing the anti-charging Al layer, spin FSC-M at 2000 RPM for 60 sec.

7. Bake on a hotplate at 120 °C for 3 min 30 sec.

A.3.2 Development

1. Agitate in OPD4262 for 60 sec.

2. Agitate in DI water for 60 sec.

3. Dip in IPA for 3 sec.


5. Agitate in Amyl Acetate for 2 min.
A.3 Electron-beam lithography

6. Agitate in IPA for 60 sec.


8. (optional) Image in an optical microscope.


10. Agitate in DI water for 60 sec.


A.3.3 Lift-off

1. Place in a bath of NMP at 70-80 °C (195 °C on hotplate) for 40 minutes, agitating every 10 mins. Allow to cool down.

2. Move to a second bath of NMP at 70-80 °C (195 °C on hotplate) for 20 minutes, agitating every 5 mins. Allow to cool down.

3. Rinse with DI water for 60 sec.

4. Rinse with IPA for 60 sec.

5. Blow dry.
APPENDIX B

Leiden CF-450 operation

B.1 Cooldown

B.1.1 Pumping out the dilution unit and the traps

Connect a portable turbo pump unit to Aux. Port 1, and rough out the lines by opening M3, then A9, 8, 2, 0, Gate Valve, 5, 16, Compressor Bypass, 6, 7. Turn on the portable turbo unit to further pump on the dilution unit, and turn on turbo S1 to pump on the still.

B.1.2 Pumping out the OVC and the IVC

Once the still pressure reaches $< 5 \times 10^{-4}$ mbar, close the Gate Valve, turn off S1, close 0, 2, 8, 6, 7, 5, 16, A9, Compressor Bypass, M3. Stop and disconnect the portable turbo unit from Aux. Port 1. Turn on S4 and open A8, A2, A0 to pump the IVC below 1 mbar. Close A0, connect a He-4 bottle to Aux. Port 1, open M3, and flush a little He-4 gas by pumping with S4. Close A8 and fill the space between A8 and A0 with $\sim 50$ mbar of
B.1 Cooldown

He-4, then open A0 to transfer it into the IVC. Repeat until the IVC pressure reaches 10 mbar. Close A0, A2, M3, A8, and open A7, A1 to pump on the OVC.

B.1.3 Pre-cooling to 77 K

Once the OVC pressure is below 1 mbar, switch on the pulse tube compressor. Connect a liquid $N_2$ dewar to the pre-cooling system, and start flowing the $N_2$. Open the pressurizing coils valve on the dewar to pressurize the $N_2$ to $\sim 20$ PSI. It takes about 12 hours to reach 77 K with a full dewar. At that point, the OVC pressure should be $< 5 \times 10^{-4}$ mbar. Close A1 and A7, and turn off S4 to stop pumping on the OVC.

B.1.4 Cooling down to 3 K

Close the pre-cooling valves, and disconnect the $N_2$ dewar. Turn on the IVC sorb heater current to 25 – 30 mA. After the mixing chamber temperature reaches $\sim 3$ K, turn off the sorb heater current and wait 20 min for the temperature to stabilize.

B.1.5 Condensing the mixture

Open manual dump valves M1 and M2. Press "Condensing 3He", then "AUTO" buttons on either the gas handling system or in the LabVIEW program. The system will take 3 – 4 hours to reach the base temperature of 20 mK.
B.2 Warmup

B.2.1 Recovering the mixture

First, make sure the manual dump valves M1 and M2 are open. Press "Recovery", then "AUTO" buttons on either the gas handling system or in the LabVIEW program. The mixing chamber current can be set to 6—8 mA to speed-up the He-3 recovery. Once all of He-3 is recovered, close M1 to the He-3 dump. At this point, the pulse tube compressor can be turned off. The system will stop after finishing the recovery of the remaining mixture. Close M2 after the recovery is complete.

B.2.2 Warming up to 300 K

After the mixture has been recovered, set the 3 K and 50 K heaters to 80 — 100 V for 6—8 hours. After the mixing chamber reaches > 77 K, N₂ can be let into the OVC and IVC to act as an exchange gas and speed up the process. If quick warm-up is needed, He-4 can be added instead as soon as the mixing chamber is above 4 K.

B.2.3 Cleaning the nitrogen traps

Rough out the line by turning on S4, opening A8, and waiting for P3 to display ~ −2. Close A8 and open A9, 5, and 16. Take the traps out of the liquid N₂ dewar and let them warm up. Record the pressure P3. Open A8 to pump out the traps. When P3 reaches ~ −2, close 5, 16, A9, A8, and turn off S4.
To calibrate optimal e-beam exposure, I wrote several patterns in the SEM at different area doses. I then processed and imaged the patterns after MMA development and Al lift-off. For large 10µm × 10µm squares, 100µC/cm² resulted in optimal patterns after the lift-off (see Fig. C.1). Lower exposures led to under-development and poor lift-off, while large exposures resulted in rounding of the corners. I tested single junction patterns using the same method (see Fig. C.2 and Fig. C.3). The junction lines were nominally 100 nm wide, and the bridge width was nominally 320 nm. Exposures below ∼ 570µC/cm² resulted in poor undercuts, as evidenced in the absence of a lighter “shadow” around the pattern after MMA development [Fig. C.2 (a) and Fig. C.3 (a)]. I also tested two-junction patterns (see Fig. C.4, Fig. C.5, and Fig. C.6). The junction lines were nominally 100 nm wide, and the bridge width was nominally 320 nm. The large rectangle connecting two of the junction leads was written at the previously determined optimal exposure for large areas, 100µC/cm². Exposures below ∼ 630µC/cm² very narrow horizontal junction leads [Fig. C.4 (b) and Fig. C.5 (b)]. Good-looking patterns
occurred at $\sim 790 \mu C/cm^2$. Higher exposures resulted in complete undercutting of the resist between the two vertical junction leads, with the pattern failing during the development of MMA (see Fig. C.6).
Fig. C.1: Optical images of $10\mu m \times 10\mu m$ squares written with e-beam at different exposures, showing (a) the resist after MMA development (b) Al (light) on sapphire (dark) after lift-off. The numbers next to each square are the e-beam area doses used, in $\mu C/cm^2$. 
Fig. C.2: Optical images of 1-junction test patterns written with e-beam at different exposures, showing (a) the resist after MMA development (b) Al (light) on sapphire (dark) after lift-off. The numbers next to each pattern are the junction e-beam area doses used, in $\mu$C/cm$^2$. The large rectangular areas were written at 100$\mu$C/cm$^2$. The nominal junction bridge width was 320 nm.
Fig. C.3: Optical images of 1-junction test patterns written with e-beam at different exposures, showing (a) the resist after MMA development (b) Al (light) on sapphire (dark) after lift-off. The numbers next to each pattern are the junction e-beam area doses used, in µC/cm². The large rectangular areas were written at 100µC/cm². The nominal junction bridge width was 320 nm.
Fig. C.4: Optical images of 2-junction test patterns written with e-beam at different exposures, showing (a) the resist after MMA development (b) Al (light) on sapphire (dark) after lift-off. The numbers next to each pattern are the junction e-beam area doses used, in $\mu$C/cm$^2$. The large rectangular areas were written at 100 $\mu$C/cm$^2$. The nominal junction bridge width was 320 nm.
Fig. C.5: Optical images of 2-junction test patterns written with e-beam at different exposures, showing (a) the resist after MMA development (b) Al (light) on sapphire (dark) after lift-off. The numbers next to each pattern are the junction e-beam area doses used, in \(\mu\text{C/cm}^2\). The large rectangular areas were written at 100\(\mu\text{C/cm}^2\). The nominal junction bridge width was 320 nm.
Fig. C.6: Optical images of 2-junction test patterns written with e-beam at different exposures, showing (a) the resist after MMA development (b) Al (light) on sapphire (dark) after lift-off. The numbers next to each pattern are the junction e-beam area doses used, in $\mu$C/cm$^2$. The large rectangular areas were written at $100\mu$C/cm$^2$. The nominal junction bridge width was 320 nm.
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