Supplementary Material: "Because math": Epistemological stance or defusing social tension in QM?

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1. Introduction

This document provides supporting materials for a paper submitted for review to the Physics Education Research Conference proceedings in 2015 titled, "'Because math': Epistemological stance or defusing social tension in QM?"

The Physics Education Research Group at the University of Maryland, College Park has been working to develop curriculum materials for engineering students learning quantum mechanics. The materials are designed to provoke more conscious decision making between quantum, classical, or hybrid reasoning. Many of the materials are integrated with computer simulations, which allow students to more easily visualize quantum phenomena.

In the paper, we discuss episodes from video data of students working through a tutorial developed by us on the Particle in a Box. The full tutorial is included at the end of this document. Students were recruited by email to participate in the clinical focus group and were offered $20 compensation for their time. The focus group took place in a small conference room where the students were seated in a semi-circular fashion at the end of a conference table. The interviewer sat near the camera, typically outside the camera view. The students were briefly asked for some background information before being handed the tutorial and asked to begin. The five students participating in the focus group, whose pseudonyms are Al, Bob, Chad, Dan and Ed, are all physics majors who were, at the time of the focus group, enrolled in the first semester of a two-semester, upper-level quantum mechanics course for physics majors.

The paper submitted to the PERC Proceedings documents outlets that students use as a means of relieving social tension, which we refer to as "escape hatches." In the analysis, we discuss three episodes from the video data. In the first episode, we see the students using the "because math" line as a verbal pivot which serves both the purpose of relieving the tension around the discussion of linear algebra as well as allowing the group to move on from their discussion of linear algebra, providing the group the opportunity to investigate resources other than math. We then contrast the students' statements in the first episode with those in the second episode. Although we cannot make any generalizations from these two episodes, the data indicate that the epistemic stances students take can be context-dependent as well as transactional in nature. The third episode shows the students finding relief from a tense discussion about speed in the wording of the tutorial.

In the transcript, two statements in the same line separated by a slash indicate that the students were speaking simultaneously. Comments followed by an ellipsis indicate that the student trailed off and comments followed by dashes indicate that the student was interrupted by another speaker. Laughter, gestures, and intonation were not noted on the transcript.

Section 2 below provides the transcript data and Section 3 presents the entire Particle in a Box tutorial relevant to this data.

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2. Transcript Data

Episode 1: Why can't a particle have zero energy?

In this episode, the students are discussing the question "Why isn't the ground state n=0? That is, why isn't it possible for the particle to have zero energy?" (question 3 in the tutorial).

Chad: Why isn't the ground state n=0?
Al: Uncertainty principle. I guess mathematically, I don't know why. But…
Bob: Well… Is this a definition we don't...
Al: No, but if you...
Dan: Well we talked about the difference between that and the harmonic oscillator. 'Cus the harmonic oscillator starts at n=0.
Al: Right but when you have like the state n=0 for a harmonic oscillator, you still like, in your equation for the energy levels, you still use n=1. You just call it n=0.
Chad: Well, no. It's n=0 but it's n plus one half. /**Dan**: Well your energy would be one half $\hbar$ omega.
Al: Yeah, yeah, yeah. You're right but for like these, like the square well or whatever, it's— there's an n multiplied by it so like, if you had n=0, the energy would be zero.
Dan: Right.
Ed: Which would be no wavefunction. It would just be a flat bar.
Chad: Well if the energy was zero, wouldn't it be like there was no particle in the box anyway? So it's not the same problem?
Bob: That's right, we're talking about particle in a box here. So that means we're talking about um, I imagine in this case, an infinite square well. I think that's what it usually refers to?
Al: Yeah.
Chad: Walls that you can't pass.
Bob: So, if they're saying the particle has zero energy...
Chad: It's not a particle.
Bob: A particle always has energy. It always has some sort of intrinsic--
Chad: You can't have a particle with no energy. That's like saying I have a whole bushel of no apples.
Al: No. What I think they're saying is like the difference... Like you could imagine in a classical sense, like a ball in a well, it could just be sitting there.
Bob: Well classically, it can just be sitting there.
Al: It could have no kinetic energy whatsoever.

Bob: Although I do remember reading--
Al: What they're saying I think, is like why in a quantum realm, why it can't.
Bob: I do remember reading yesterday in another physics book I have that there's a minimum speed that a particle can have. And for a macroscopic object like a ball it's like 10^-36. Which is basically zero, but ummm...
Chad: Well, hen it would also be at zero Kelvin, wouldn't it? If it had no speed?
Bob: Well it's just its translational speed, not its molecular...
Chad: Oh okay.
Bob: For a particle though, it was much larger. I think for an electron it was like... what was the number? I don't remember but it was a lot larger. But anyway, for the particle in a box... isn't there some sort of theorem in linear algebra that says that zero can't be an eigenvalue or is it can't be an eigenvector?
Al: It's zero isn't an eigenvector.
Chad: 'Cus it's arbitrary.
Al: Right, 'cus then everything would be...
Bob: Oh so eigenvector? Ok never mind, so that doesn't work.
Al: Zero can't...
Chad: Can't be an eigenvalue, 'cus eigenvectors--
Al: 'Cus then any vector could be an eigenvector.
Bob: Oh, so it can't be an eigenvalue?
Al: No, no. Zero can be an eigenvalue. Right?
Chad: No it can't.
Dan: No, zero can't be an eigenvalue.
Al: Then zero can be an eigenvector.
Dan: Yes, I think.
Bob: So then if zero can't be an eigenvalue and if the way you--
Al: It's whatever one that makes it like trivial.
Dan: Trivial? I think it's the eigenvalue. 'Cus that would be like H psi equals zero psi. Right?
Al: So it would work for any wavefunction.
Bob: So, in other words, if I take H on psi and get zero as an eigenvalue, it doesn't work?
Al: So we can say linear algebra.
Dan: Because math.
Al: No, what I was arguing at the very beginning is that, I thought at least qualitatively, it boiled down to the uncertainty principle. Like there's always, you can't say it has zero energy.
Bob: Oh yeah... I think that's what it is...
Al: Well yeah. I really think so too, but...
Dan: How do you explain that?
Al: I don't know, like beyond that.
Bob: If it has no energy then it now has a definite position, and--
Dan: And a definite momentum
Al: Something along the lines of that.
Bob: Yeah! It has a definite position and a definite momentum, which is impossible. 'Cus you know its momentum is zero, and you know its position is right there, which is not possible.
Chad: Well, do you know where its position in the box is, if it has no momentum? Unless you know the initial state, you don't know. Which, I don't know how that plays into it.

Bob: But you know it's somewhere in a discrete position and you know it's, once you find it, it's going to be right there.
Chad: Well yeah, but it's still a probability distribution of where it's going to be.
Bob: Well, that's exactly the point. It's going to have a probability distribution and I think that gives rise to some sort of energy.
Chad: Well yeah, that would give it energy, if it had a distribution, 'cus then you could do the Hamiltonian and it won't be a zero eigenvector... eigenvalue.
Bob: Well yeah that makes sense. I have no idea how to answer this question.
Chad: I think it's just indistinguishable from no particle, so you can't have a particle with no energy.
Dan: Oh yeah, I'm going with that.
Bob: Yeah, I'm going with that 'cus I don't know.
Chad: Then it just becomes trivial.

Episode 2: Measurements of Energy

In this episode, the students are working through the question, "If you were to measure the energy of the particle at some point in time, what would you expect to measure? Why? Will you get the same measurement every time?" (question 5 in the tutorial).

Ed: Energy would be the same.
Chad: Is it still in the ground state? For question 5?
Dan: Yeah I think that's the assumption for the whole...
Ed: Like every measurement it would just go back to this.
Interviewer: This is energy now? Ok.
Dan: Isn't the energy just based on, isn't this the n squared pi squared...
Ed: Oh for an infinite square well? The energy is different than--
Dan: But it's just based, it's just based on what state you're in. So it's going to be constant throughout.
Ed: I just don't know what that actual term is.
Chad: Well, just say it's En, based on the number of states.
Al: It's like, pi squared n squared hbar squared, over 2 m-- /**Dan**: Over 2 m...**Al**: And then the length, it would be L here.
Bob: a squared?
Al/Dan: L squared.
Chad: L squared 'cus it's...
Bob: Sorry, yeah.
Al: I guess that's what they want, like an actual value? Or do they want something that's, like an eigenvalue for time-independent?
Al: Like you know how you can only measure the energy to be eigenvalues.
Chad: Yeah, so it has to be one of the states.
Al: Yeah.
Chad: Will it always give you the first state if it's in the ground state?
Bob: We know it's in the ground state, so it should give us pi squared...
Chad: Unless it evolves... I don't know if it could evolve between states.
Ed: I think you're overcomplicating this.

Chad: I could be.
Al: So we're saying you would get the energy of the ground state. And you would always get that.
Ed: Yeah.
Bob: Yeah, I'm not sure…
Ed: What's that energy, what's that energy equal to?
Chad: E0.
Dan: I'm just saying, I'm just saying it's equal to E1 for the ground state.
Chad: Oh right E1, not E0. That's a harmonic oscillator.
Chad: Pi squared.
Dan: Pi squared, hbar squared over..
Chad: 2m.
Dan: L squared.
Bob: Pi squared what?
Chad: Pi squared, hbar squared over 2m L squared. I think... Yeah 'cus there's n squared on the top, and we're at 1.
Bob: Yup. So...
Chad: Seems good.
Bob: And we will get the same measurement every time.
Dan: Yeah.
Chad: Yeah 'cus it can't just change energy.
Bob: Unless you add energy.
Chad: Yes.

Episode 3: Measurements of Speed

In this episode, the students work through the question "Can we define a 'speed' for the wave?" (question 10 in the tutorial). The students are asked to compare the wavefunction of the particle in a box to a standing wave on a string in this section of the tutorial.

Chad: Can you define a speed for the wave?
Al: Well, you could talk about how fast, like, it moves, the points on it move vertically up and down. Then if it's...
Bob: Well that's a good question. Which speed is it talking about? /**Dan**: Translational versus...
Al: Well it says it's a standing wave.
Bob: A standing wave, right, right, ok so... To define the speed of... Why do they say speed of the wave?
Chad: I would just say it's--
Bob: Translational speed wouldn't be the speed of the wave, it'd be the speed of the particles.
Al: Yeah, when they usually say speed of the wave they mean like how fast it's propagating.
Dan: So there's no translational speed.
Chad: Are we assuming that--
Bob: A standing wave is equivalent of two waves moving in opposite directions, superimposed on each other.
Chad: That's true. So if we just picked out the one part of it that's going to the right, that would be the speed of it. 'Cus it would be the same in the other direction.

Bob: Yeah, they're both going the same speed.
Chad: It'd have to be the same speed.
Al: I don't know about that because let's say if it was actually a traveling wave, then it could be one wave going this way and one going a little slower this way.
Bob: But the whole point is it's a standing wave. /**Dan**: But they're saying it's a standing wave so it's going this way and this way and they're the same speed.
Al: No I get that.
Chad: It must be the same speed or else it wouldn't be a standing wave, it'd be beating patterns.
Al: I know what you're saying, but like if you did like.... Remember when we did the free particle? How there's like the group velocity and the...
Bob: The phase velocity.
Al: Phase velocity. It's like... when we solve the Schrodinger Equation we pictured one going this way and one this way. What I'm saying if you're doing what you're saying, then a standing wave is just they're going in opposite directions at the same speed, and you just pick one and say that's the speed, then how do you determine which one?
Chad: Well, speed doesn't have a direction. /**Bob**: Wouldn't you just say speed...
Al: No what I'm saying is if it's a traveling, well.... I guess it's a standing wave so--
Chad: If you were to release the edges of the standing wave and put it to a string, there would just have two separate waves that go off at the same speed, right?
Bob: If you had an infinitely long string, but we have these boundary conditions that literally cause those two traveling waves going in opposite directions to be reflecting off the boundaries.
Chad: They have to be imposed on each other.
Bob: Yeah...
Chad: Yeah, ‘cus the nodes don't move on a standing wave. So total speed....
Ed: So how would you say we can define the speed?
Chad: Uhhh, phase velocity in one direction?
Bob: I don't know.
Al: I just don't think it's good to define the speed of the wave in like the speed of one of the things that's superimposed.
Bob: Okay so, what are, how are we defining it? How should we define it?
Chad: Well what would be a speed then?
Al: I don't think there is one. For it's a standing wave, I don't think it has like...
Chad: Oh! It's can we define!
Al: Yeah.
Chad: No, we cannot.
Bob: No.

3. Particle in a box Tutorial

Consider the “particle in a box”: a quantum system where one particle (for example, let’s say an electron) is confined between positions 0 and L. This can be in 1, 2, or 3 dimensions, but for now, let’s focus on the 1-dimensional system.

1. Let’s say the system is in the ground state (n = 1). Sketch a graph of the wavefunction.

2. How would you explain the physical meaning of the wavefunction?

3. Why isn’t the ground state n = 0? That is, why isn’t it possible for the particle to have zero energy?

4. If you were to measure the position of the particle at some point in time, what position(s) would you expect to measure? Why? Will you get the same measurement every time?

5. If you were to measure the energy of the particle at some point in time, what would you expect to measure? Why? Will you get the same measurement every time?

6. If you were to measure the speed of the particle at some point in time, what would you expect to measure? Why? Will you get the same measurement every time?

7. Is your answer to #6 (speed) consistent with your answers to #4 (position) and #5 (energy)? If not, how do you reconcile them?

8. Can you come up with an example of a real-world system for which the particle in a box is a useful model?

Let's compare the wavefunction of the "particle in a box" to a (classical) standing wave on a string.

9. Does it make sense to talk about the energy of this system? What physical properties does it correspond to?

10. Can we define a "speed" for the wave?

11. What is the relationship between the energy (in #9) and the speed (in #10)?

Now let's consider a classical particle, moving in 1 dimension in an actual box.

12. What would the "ground state" look like for the classical particle? (I.e. what is the least possible energy that the particle could have, and what would the particle be doing in that case?)

13. Suppose the particle has some energy, and is bouncing back and forth. If you were to measure the position of the particle at some point in time, what position(s) would you expect to measure? Why? Will you get the same measurement every time?

14. Graph the probability of measuring the particle at each position, as a function of position.

15. If you were to measure the energy of the particle at some point in time, what would you expect to measure? Why? Will you get the same measurement every time?

16. If you were to measure the speed of the particle at some point in time, what would you expect to measure? Why? Will you get the same measurement every time?

17. Is your answer to #16 (speed) consistent with your answers to #13 (position) and #15 (energy)? If not, how do you reconcile them?

Returning to the quantum “particle in a box”...

18. Is it more like a classical particle, a classical wave, both, or neither? Why?