

ABSTRACT

Title of Document: UNDERSTANDING THE EQUAL SIGN AS KEY TO ALGEBRAIC SUCCESS: THE EFFECTS OF BLENDED INSTRUCTION ON SOLVING ONE- AND TWO-STEP EQUATIONS AND CONCEPTIONS OF THE EQUAL SIGN FOR SEVENTH GRADE STUDENTS WITH MATHEMATICS LEARNING DIFFICULTIES

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The current study examined the effect of an instructional package on the ability of students with mathematics difficulties to solve one- and two-step linear equations and understand the equal sign as a relational symbol. The instructional package included a blend of elements including explicit/systematic instruction, concrete-semi-concrete-abstract instruction, and graphic organizers while also developing students' capacity to meet the Common Core Standards for Mathematical Practice. A concurrent multiple probe design across three groups replicated across three other groups was utilized where the researcher instructed one section of three groups while other classroom teachers instructed the second section of three groups. The participants were 17 seventh grade students identified as having a learning disability or difficulty in mathematics (MD). Results of the study indicated that all groups significantly improved their performance when solving one- and two-step equations and significantly improved their understanding of the equal sign as a relational symbol. The study supports the use of blended instruction

with visual representations and graphic organizers to improve mathematical performance of students with MD.

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Chapter 1: Introduction

Algebra is a “gatekeeper” course for many students who would otherwise wish to further their education and employment opportunities (Usiskin, 2004; Kortering, deBettencourt, & Braziel, 2005). In the United States, students seeking entrance into competitive universities or skilled professions are required to pass algebra as a pre-qualifying requirement. Despite algebra’s importance, many students enter their first year algebra course with an inadequate understanding of fundamental topics necessary to develop a coherent, conceptual understanding of the course (Asquith et al., 2007; McNeil et al., 2006; Booth, 1988; Schoenfeld, & Arcavi, 1988). This is especially true for students with disabilities who consistently perform lower than their non-disabled peers in mathematics according to the National Assessment of Educational Progress (NAEP, 2005; 2007; 2009; 2011). As students with disabilities progress through grade levels, their mathematical achievement decreases with 41%, 64%, and 74% scoring below basic in grades 4, 8, and 12 respectively (NAEP, 2009). Consequently, secondary students with learning disabilities (LD) may have limited understanding of the skills and concepts needed to be successful in algebra and other secondary mathematics courses, and therefore their overall development and proficiency in mathematics is hindered. In this chapter, mathematical proficiency will be defined along with the status of mathematics proficiency in the U.S. and policies that are in place which impact mathematics instruction for students with disabilities. Following this is a discussion of factors impacting learning for students with LD, and successful mathematics interventions for students with LD. The section concludes with the statement of purpose, proposed research questions, hypotheses, and definition of terms.

Mathematical Proficiency

To successfully learn mathematics and attain mathematical proficiency, an individual must master five interwoven and interdependent strands of knowledge as outlined by the National Research Council (NRC). These NRC strands of proficiency include addressing (a) *conceptual understanding* (i.e., mathematical concepts, operations, and relations); (b) *procedural fluency* (i.e., applying procedures flexibly, accurately, efficiently, and appropriately); (c) *strategic competence* (i.e., formulating, representing, and solving mathematical problems); (d) *adaptive reasoning* (i.e., capacity for logical thought, reflection, explanation, and justification); and (e) *productive disposition* (i.e., see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's efficacy) (NRC, 2005, p. 116).

The National Council of Teachers of Mathematics (NCTM) process standards (2000) are closely linked to the NRC strands. The NCTM (2000) process standards offer ways that students should acquire and use mathematical content knowledge and include: problem solving, reasoning and proof, communication, connections, and representation. Both NCTM and the NRC recommend that mathematics instruction include the strands or standards to help all learners become mathematically proficient. Despite these standards being in existence for over 20 years, evidence from international, national, and state assessments suggests that across grade levels, many students in the U.S., particularly secondary students with LD, do not adequately demonstrate mathematics proficiency (Blackorby et al., 2003; National Center for Education Statistics, 2009a; Fleischman et al., 2010; Gonzales et al., 2008). Until the passage of IDEA in 2004, most special educators working with students with LD focused on instructional strategies to help

students meet specific educational goals and rather than concepts linked to standards in the general education mathematics curriculum (Maccini & Gagnon, 2002). Practices such as these are not conducive to developing a conceptual understanding of mathematics and may be one contributing factor to the status of mathematical proficiency in the U.S.

Mathematical Proficiency in the United States

Internationally, the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) have tracked the mathematical performance of students for over 10 years. The TIMSS broadly assesses students' mastery of specific knowledge, concepts, and skills reflecting the curricula of participating countries while the PISA assesses the ability of students to apply knowledge to problems in real-life contexts (Fleischman et al., 2010). In 2007, TIMSS reported on the overall performance of eighth grade students across 48 countries. The results showed that U.S. students performed better than 37 countries in mathematics, but fell behind 5 countries including: Chinese Taipei, Republic of Korea, Singapore, Hong Kong, and Japan (Gonzales et al., 2008). The results from the PISA 2009 study revealed that U.S. students fell towards the middle of the 33 participating Organization for Economic Co-operation and Development (OECD) countries with 17 countries having higher average scores, 11 countries having comparable averages scores, and 5 countries having lower average scores (Fleischman et al., 2010).

While neither the PISA nor TIMSS explicitly examined algebra performance, some insight can be gained from a report linking the 2000 NAEP achievement levels to the 1999 TIMSS (Philips, 2007). In the report, 8th grade achievement levels are defined as basic (i.e. having and understanding of arithmetic operations), proficient (i.e. applying

concepts and procedures consistently to complex problems), and advanced (i.e. generalizing and synthesizing concepts and principles). When the 2000 NAEP was projected onto the 1999 TIMSS scale, 65% of the students scored at or below basic, 27% scored at or above proficient, and 6% scored at or above advanced. These results suggest that the majority of 8th grade U.S. students are unable to engage in the skills necessary to be successful in algebra and other higher-level mathematics classes. Although there is no disaggregated data regarding students with disabilities in these assessments, there is no reason to suspect that students with disabilities are adequately prepared for advanced mathematics classes given that many students with disabilities perform consistently lower than their peers (Blackorby et al, 2003).

The NAEP has been administered periodically since 1969 in subjects including reading, writing, mathematics, science, history, and geography, to assess and evaluate the condition and progress of education. With regard to mathematics, 8th and 12th grade students continually perform poorly on these assessments. Although the percent of students who perform at or above the proficient level has increased with subsequent administrations of the NAEP since 2000, the NAEP 2011 reported that only 35% of students scored at this level (NAEP, 2011). Further, 8th grade students with disabilities have consistently scored far below their non-disabled peers over the last 7 administrations of the NAEP for which there is comparison data. Specifically, the most recent report found that only 9% of students with disabilities scored at or above proficiency level while 64% scored below basic (NAEP, 2011). A basic level of achievement entails using basic algebraic and geometric concepts along with structural aids such as diagrams, charts and graphs to arrive at the correct solution for a problem (NCES, 2009a). Scoring below basic

suggests that the majority of students with disabilities do not have the foundational skills needed at the 8th grade level and are therefore unlikely to be successful in Algebra and beyond.

The NAEP (2011) results are consistent with those found by the National Longitudinal Transition Study (NLTS-2), which also determined that students with disabilities demonstrate poor mathematical achievement. Specifically, the NLTS-2 data revealed that only 13.6% of students with LD are above, at, or less than a year below grade level in mathematics while the average discrepancy between tested and actual student level is 3.2 years (Blackorby et al, 2003).

In Maryland, between the 2005 and 2009 NAEP, the percent of students scoring below basic decreased nine percentage points for all students and 22 percentage points for students with disabilities (NCES, 2009a). On the 2011 administration, however, the percent of students scoring below basic increased by 1 percentage point for all students and 8 percentage points for students with disabilities (NAEP, 2011). Over half of students with disabilities (54%) scored below basic with only 12% scoring proficient, compared to 26% and 40% for all students, respectively. Despite progress towards closing these gaps as shown by NAEP 2007 and 2009, the 2011 data showed that the progress was not maintained and the gap widened to almost erase gains reported from the previous two administrations of the test (NAEP, 2011). This increase in the gap by almost 7 percentage points suggests that more work is needed to prevent students with disabilities from falling further behind their non-disabled peers.

The gap is even more pronounced between students with and without disabilities as shown on the Maryland State Assessment (MSA), a standardized measure

administered to students in grades 3 to 8 in mathematics and reading. The 2011 Mathematics Grade 8 MSA reported 73% of students without disabilities scored proficient or higher, as compared to only 33% of students with disabilities (MSDE, 2012). When comparing the Grade 8 data to those same student's Grade 5 MSA scores, there was an 18.3% and 11.5% decrease for students with and without disabilities, respectively, scoring proficient or higher (MSDE, 2012). The decrease for students without disabilities is mostly consistent across the grades while more than half of the decrease for students with disabilities occurred between the 7th and 8th grade. Students in Maryland also take high school assessments (HSA) in English, government, algebra/data analysis, and biology. Data from the 2010 algebra assessment found that only 48.3% of students with disabilities scored proficient or higher, as compared to 83.2% of students without disabilities. This is an area of critical concern given that students are required to pass the algebra HSA in order to graduate. Nationally these discrepancies have been noticed and policies and guidelines have been implemented to help students with disabilities obtain the support and opportunities they need to be as successful as their non-disabled peers in school and after.

Education Policies

Increasing the academic achievement and improving the quality of education for U.S. students to be competitive in a global market has been the focus of education policy over the last several decades. The Elementary and Secondary Education Act (ESEA, 1965) was the first federal policy that addressed the education for disadvantaged children in poor areas. The law focused more on assisting specific groups of children (i.e. students from low income families) rather than addressing general education programs in local

schools (Department of Education, 1996). In 1994 the ESEA was amended to the Improving America's Schools Act (IASA) which emphasized the need for *all* students to meet higher learning outcomes while continuing to focus on equity (IASA, 1994). The Goals 2000: Educate America Act (GOALS, 1994) was also passed in 1994 to further support states in developing world-class academic standards, annual progress monitoring techniques, and systems to judge student progress towards attainment of the standards. These laws were the first federal efforts to mandate the use of standards in education.

In 2002, the ESEA was reauthorized as the No Child Left Behind Act (NCLB), with a stronger focus on increasing the quality and effectiveness of education for all students in U.S. schools through high standards and accountability (NCLB, 2002). While the IASA and Goals 2000 included provisions for reporting on student progress towards the standards, few states set clear goals or disaggregated the data for students in at-risk groups. NCLB, however, held schools accountable by mandating that all subgroups of students within the student population show adequate yearly progress (AYP) on grade level state reading, mathematics, and science standards with the requirement that all students, including those with disabilities, be proficient by the 2013 – 2014 school year. Additional policies were also enacted that pertained specifically to students with disabilities.

Educational policies for students with disabilities. Although the ESEA of 1965 was intended to improve educational opportunities for all students, until the passage of the Education of All Handicapped Children Act (EAHCA) in 1975, there were no federal laws that entitled students with disabilities to an education. The EAHCA (1975) stated that children with disabilities had to be provided with a “free appropriate public

education” designed to meet an individual’s unique needs. Like the ESEA, the EAHCA underwent several amendments and in 1990 the name was changed to the Individuals with Disabilities Education Act (IDEA), reflecting the use of person first language (IDEA, 1990).

A major change to IDEA occurred in the 1997 amendments which, for the first time, included language that changed the focus from providing students with disabilities *access* to education to *improving the educational results* of students in schools (IDEA, 1997). This focus on educational results brought IDEA more in line with the goals of the general education policy of IASA and Goals 2000. The latest reauthorization occurred in 2004, and IDEA became the Individuals with Disabilities Education Improvement Act (IDEIA), to reflect changes in education and align the law with the new standards set in NCLB (IDEIA 2004; Yell, Shriener, & Katsiyannis, 2006). Among the primary changes included that: a) all students with disabilities must participate in state assessments; b) special education teachers must be highly qualified (i.e. hold a minimum of a bachelor’s degree, have full state certification, and have subject matter competency in all core subjects they teach (IDEIA, 2004); and c) any services, aides or accommodations provided to a student must be based on peer-reviewed research (Yell et al., 2006). These three educational laws had major implications for school systems to deliver high quality education to all students. The EAHCA (1975) and subsequent reauthorizations ensured that students with disabilities would be afforded the same educational opportunities as their non-disabled peers.

Reforms in Mathematics Education

While Congress passed laws requiring that all students receive a high quality education, organizations such as the National Council of Teachers of Mathematics (NCTM), the National Research Council (NCR), and the American Diploma Project (ADP) provided guidelines for establishing a high quality mathematics program. In 1989, the NCTM formally adopted and published the *Curriculum and Evaluation Standards* to promote quality instruction in mathematics (NCTM, 2000). These were updated and published in 2000 as *Principals and Standards for School Mathematics* to continue the effort to improve students' mathematics instruction by providing a guide to educators. The NCTM guidelines include both content standards (number and operations, algebra, geometry, measurement, data analysis and probability) and process standards (problem solving, reasoning and proof, communication, connections, and representation). Rather than focus on procedures and the memorization of algorithms, the purpose of the standards was to develop student's conceptual understanding. The standards are based on a constructivist view of learning in which learning and the cognitive structures associated with knowledge are built from experiences and interactions with the environment (Noddings, 1990). With this view of learning, students will gain a greater conceptual understanding of the topics and form an interconnected schema for learning mathematics rather than memorizing and applying formulas in a procedural fashion.

Teaching using constructivist approach requires considerable content knowledge and pedagogical skill (Noddings, 1990). In particular, algebra builds upon and formalizes many concepts learned in arithmetic, and teachers must be able to identify and provide learning experiences to ensure that students hold appropriate conceptions of these key

underlying ideas before introducing new material (i.e. understanding the meaning of equal sign before learning to solve equations).

While the NCTM standards focus on developing conceptual understandings of mathematics through constructivist means, the NCR guidelines focus more broadly on how students learn mathematics as a whole which includes conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition (NCTM, 2000; NCR, 2005). The NRC's *How Students Learn: Mathematics in the Classroom* (2005) focuses on three principles of learning that educators should understand and be aware of while planning and teaching. These principles include: 1) engaging students' prior understandings (i.e., using the resources students already have as a building block for instruction of new content); 2) knowing the role of factual knowledge and conceptual frameworks in student understanding (i.e., knowing how concepts and procedures previously learned must be integrated to support new understandings); and 3) developing student self-monitoring (i.e. using metacognitive skills to actively assess progress and adjust mathematical processes during problem solving). The goal is to explain how students learn mathematics and provide applications and examples for teachers to develop a deep understanding of how the principles of learning could be incorporated in the classroom.

Unlike the NCTM standards and the NCR guidelines that focused on what and how to teach mathematics, the American Diploma Project (ADP) focused more broadly on policies that should be in place to ensure high school graduates are adequately prepared for college or careers in high-performance, high-growth jobs (ADP, 2004). The ADP was initiated to establish a stronger link between secondary schooling and post-

secondary institutions and employers by providing college and workplace readiness benchmarks in English and mathematics that are reflective of the skills and content that students need to be successful after graduating high school (ADP, 2004). To comply with the requirements of NCLB, every state had to develop content and achievement standards and assessments in English, mathematics, and science in grades 3-8 and once during high school. According to ADP (2004) most state standards reflected what was *desirable*, not necessarily *essential*, for students to learn and exams generally tested material at the 8th and 9th grade level while rarely reflecting the real-world demands of postsecondary education and work. To address these shortcomings, the report offered suggestions to states regarding standards and assessments at the secondary level, as well as suggestions for postsecondary institutions. With regard to standards, ADP advised states to: 1) align high school standards with the knowledge and skills required for college and the workplace; 2) create a coherent, focused, grade-by-grade progression of standards from kindergarten through graduation; and 3) require students to take specific courses in English and mathematics with specified core content rather than “three years” of mathematics. To assess students on the standards, ADP suggested that high school graduation exams be required for all students, assess a significant portion of the standards, and be validated as accurate predictors of postsecondary performance.

Common Core State Standards. Findings from research studies of mathematics education programs in high-performing countries and suggestions from organizations such as NCTM, NCR, and ADP, pointed to the need for more focused and coherent mathematics standards in the U.S. Together the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) collaborated to create a national

set of standards in both language arts and mathematics called the Common Core States Standards (CCSS) (CCSS, 2010). The CCSS in mathematics focus on understanding key concepts by reviewing and building on the organizing principles of mathematics and how the properties of operations lead up to more advanced concepts (CCSS, 2010). The CCSS provide a clear and consistent framework of the knowledge and skills students should learn from kindergarten to graduation in mathematics and language arts and which could be adopted across the country.

The mathematics standards outline grade specific content standards derived from NCTM's standards, NCR's strands of mathematical proficiency, and ADP's college and career readiness standards (CCSS, 2010). The CCSS are divided into standards for mathematical practice and mathematical content similar to the content and process standards presented in NCTM's *Principles and Standards*. The CCSS do not, however, define or provide suggestions for how the standards should be taught, meaning that states and districts are free to choose their own curricular materials and teachers must determine the best methods for teaching individual standards (CCSS, 2010). As of September 2012, the CCSS have been adopted by 45 states and the District of Columbia (Kober & Rentner, 2012). These states are developing comprehensive state implementation plans, revising curriculum materials, and conducting statewide professional development to help teachers master the standards which most states find to be more rigorous than their current standards (Kober & Rentner, 2012).

Assessments. Standards such as the CCSS provide the guidance or framework for the subject matter knowledge and skills that students should master at the conclusion of each grade with the ultimate goal of successfully preparing students with the skills and

knowledge necessary to enter college or the workforce. Assessments however, play a large role in how those standards are operationalized. Although NCLB requires states to test students in mathematics in grades 10–12, states are able to create their own assessments that align to their particular standards. While NCLB does not require that students pass these assessments to obtain a diploma, the Center on Education Policy (2011) found that 25 states required, or planned to require, students to pass an exit exam in order to receive a high school diploma. The most common purpose of these exit exams however, was to assess student mastery of the state curriculum largely using standardized tests that focused on content at the 8th or 9th grade level. Most of the exams failed to assess advanced high school content and did not reflect the demands students would encounter in college and careers (Achieve, 2011).

Rather than using assessments primarily to measure student mastery of state curriculum, Achieve (2011) suggested that states administer assessments that could be used by postsecondary institutions to make decisions about students' readiness for college. According to a report by Achieve (2011), 20 states and the District of Columbia require all students to complete a college and career ready curriculum that includes mathematics content up to that typically taught in Algebra II although students are not necessarily assessed at this same level. The report, also suggested that the assessments could be used as tools to improve instruction and strengthen student preparation for post secondary work or education. At the time of publication, Georgia was the only state with a statewide policy that references postsecondary uses for its state exit exams although 16 other states administer or offer all students the opportunity to take an assessment (i.e., SAT or ACT) intended to determine college or career readiness (CEP, 2011).

Currently 45 states and the District of Columbia are part of two multistate consortia developing new assessments aligned to the CCSS. The two consortia, Smarter Balanced and the Partnership for Assessment of Readiness for College and Careers (PARCC), are in the process of creating the new assessments that will first be administered in 2014 – 2015 school year (Achieve, 2011). Both consortia are developing assessments that will collect data at multiple points throughout the year, which can be used to inform instruction, along with an end of year measure for accountability purposes.

Implications for students with disabilities. Although it is desirable to have assessments based on the CCSS that are consistent across multiple states, there is cause for some concern regarding how the new assessments could impact students with disabilities. Of the states participating in the two new assessment consortia, 16 reported that the assessments under development would be more rigorous than their current high school exit exams (CEP, 2011). This is concerning considering that students with disabilities already perform significantly lower than their general education peers on currently administered exit exams (CEP, 2007). Therefore, interventions are needed that are aligned to the CCSS to help prepare students to be successful. Since the passage of NCLB (2001) and IDEA (2004), both general education and special education teachers have been responsible for providing instruction that enable all students to access an age-appropriate general education math curriculum. While effective mathematics instruction focusing on conceptual understanding is needed for all students (NCTM, 2000; NCR, 2005), additional instructional supports may be necessary for students with special needs

to help them access the general education mathematics curriculum based on their learning characteristics.

Characteristics of Students with Learning Disabilities and Learning Difficulties

Between 5% and 8% of the student population experience some form of mathematical learning disability (Geary, 2004) which is defined as a “disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations” (IDEIA, 2004). Additionally, between 5% and 10% of students have mathematical difficulties (MD) (Kroesberg & Van Luit, 2003). Students with MD do not have a disability, but perform below average in mathematics for a variety of non-biological reasons (Mazzocco, 2007). Students with MD are often included in studies alongside students with LD as these two groups present similar characteristics, and there is currently no distinct, measureable boundary separating a child with LD from a child with MD (Mazzocco, 2007).

Within mathematics, students with MD or LD present a wide and varying range of characteristics and often struggle in a range of areas including: a) basic computational skills (Little, 2009; Montague & Applegate, 2000; Maccini, McNaughton, Ruhl, 1999); b) retrieving mathematics facts due to long-term memory deficits (Geary, 2004; Maccini, Mulcahy & Wilson, 2007; Garnett, 1998); c) judging the difficulty of problems (Montague & Applegate, 2000); d) organizing information (Maccini et al., 2007); e) selecting appropriate strategies (Montague, 2008; Maccini et al., 1999); f) monitoring performance (Montague, Bos, & Doucette; 1991); g) difficulty with abstract symbols (Garnett, 1998; Geary, 2004); h) limited conceptual understanding of procedures (Geary,

2004); and i) evaluating solutions for accuracy and reasonableness (Miller & Mercer, 1997). Given the overlapping similarities in characteristics for purposes of this proposal students with mathematics difficulties and/or LD will be referred to as MD.

When considering algebraic concepts, such as one-variable equations, students with MD may struggle because of the abstract or symbolic reasoning involved with variables and symbols (Miles & Forcht, 1995) and have difficulty understanding procedural and/or conceptual processes represented with symbols and signs such as ‘=’ (Bryant, Hartman, & Kim, 2003; Bryant, Bryant, & Hammill, 2000). Other factors contribute to low mathematics performance such as poor attention control (Geary, 2004) and low academic self-perceptions and confidence (Montague & Applegate, 2000). These factors may result in students giving up rather than persevering through a tough problem which may eventually lead to students giving up on mathematics instead of continuing to higher-level courses.

Course Taking and Students with Learning Disabilities

For students with LD, poor performance in elementary and middle school mathematics classes often limits their enrollment in mathematics classes in subsequent years. For instance, while 98% of 9th graders with LD enroll in a mathematics class, that number decreases to 85% by 11th grade (Wagner, 2003). Further, the courses students with LD enroll in tend to have less rigorous curriculum and are not based on age appropriate standards (Maccini & Gagnon, 2002). In fact, only 62% of secondary students with LD are enrolled in general education mathematics classes (Newman, 2006) despite research that shows students with disabilities who are in a general education setting are closer to grade level in mathematics than their peers in special education

classes (Blackorby et al., 2003). Strategies are needed to help all students access an age-appropriate mathematics curriculum.

Existing Research

In a literature review that focused on mathematics interventions for secondary students with LD, Maccini et al. (2007) found only 6 of 23 identified studies from 1995 - 2006 focused on mathematical domains required in the high school curriculum and of these studies, only 2 studies focused on concepts related to Algebra. Maccini and colleagues identified the following teaching practices as promising: graduated instructional sequencing, schema-based instruction, peer-mediated instruction, and contextualized instruction through the use of videodisks. However, despite showing promise, there are several limitations including studies that did not include enough detail about an intervention for replication or generalization of results, interventions that led to statistically significant gains while having little practical significance, and instruction on mathematics topics that were not based on age/grade appropriate standards. Future research should address interventions and strategies that focus on grade appropriate standards, can be easily implemented into the general education classroom, and be applicable across a range of content in the course (Foegen, 2008; Maccini et al., 1999). Although there are a few interventions on the topic of algebraic expressions and equations for students with LD, interventions that focus on the nature of the equal sign are absent from the literature. This is particularly worrisome as CCSS (2010) specifically notes that using the equal sign consistently and appropriately is a critical skill for proficient mathematics students to master.

Research on the topic of equality and the nature of the equal sign is critically

needed for several reasons: 1) it is a topic important in all years of secondary school mathematics, 2) involves symbols and abstract reasoning - a process that many students with LD struggle with – and, 3) is an area where many students (both general and special education) and teachers, do not have a complete understanding. Additionally, although several studies (Essien & Setati, 2006; Godfrey & Thomas, 2008; Alibali et al., 2007; McNeil et al., 2006; Knuth et al., 2006; Knuth et al., 2008) have indicated that students, in general, have incomplete or incorrect conceptions of the equal sign as they progress into the secondary grades, no intervention studies have been conducted with this age population.

Statement of Purpose

This study addressed an existing need in the literature on effective grade-level interventions and teaching strategies to prepare middle school students with MD for Algebra. It examined the effects of a researcher created instructional package around expressions and equations with a focus on the property of equality and the nature of the equal sign. The intervention was designed by incorporating empirically validated instructional practices from both the mathematics education and special education literature including explicit/systematic instruction, visual aids, and technology.

Research Questions

The overarching quantitative research question posed by the research reported here was to determine if instructional practices from both the mathematics education and special education literature on the relational nature of the equal sign and solving equations lead to improved performance on algebraic tasks requiring these skills for students with MD. The following specific questions will guide the study.

1. To what extent do students with mathematics difficulties who receive instructional intervention on the relational nature of the equal sign and solving one- and two-step equations have increased accuracy when completing algebraic tasks involving the equal sign?
2. To what extent do students with mathematics difficulties maintain performance on algebraic tasks involving the equal sign four-to-six weeks after the conclusion of the intervention?
3. How do students conceive of the equal sign prior to intervention and are there changes in those conceptions post intervention?
4. To what extent do middle school students with mathematics difficulties consider blended instruction with visual representations and graphic organizers beneficial (i.e., social validity)?
5. To what extent do middle school teachers consider blended instruction with visual representations and graphic organizers a viable intervention strategy?

Definition of Terms

Abstract Phase: the final stage of the graduated instructional sequence where students are able to manipulate traditional mathematical symbols

Blended Instruction: Incorporates elements of instructional practices found to be effective from both the special education and general education literature (i.e. explicit/systematic instruction, concrete-semiconcrete-abstract instruction and graphic organizers).

Concrete Phase: the first stage of the graduated instructional sequence where students physically manipulate concrete objects in order to solve problems and promote conceptual understanding of abstract mathematical ideas

Concrete/Representational/Abstract (CRA) Instruction: See graduated instructional sequence

Concrete/Semi-concrete/Abstract (CSA) Instruction: See graduated instructional sequence

Explicit Instruction: learning situations where the teacher leads students through a pre-determined instructional sequence (Steadly et al., 2008)

Explicit-Systematic Instruction: instruction that teaches students to become efficient learners by modeling the learning process and providing strategies and tools to use while problem solving (Steadly et al., 2008)

Graduated Instructional Sequence: instructional approach used to promote conceptual understanding of mathematical topics by moving students from using concrete manipulatives to using semi-concrete or representative drawings and finally to traditional abstract mathematical symbols

Instructional Package: Method of delivering instruction that combines effective practices from both the general education and special education literature bases

Learning Disability: “a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations” (IDEA 2004).

Manipulatives: concrete objects (i.e. counters, beans, algebra tiles) that can be physically manipulated by students while working on mathematical problems.

Math Difficulties (MD): includes students receiving special education services in mathematics and students at risk for mathematics failure

Representation/Semi-concrete Phase: the second stage of the graduated instructional sequence where students draw pictures representing concrete objects or mathematical ideas to assist with problem solving and conceptual understanding of abstract mathematical ideas

Self-Instruction: strategies that students can use to manage their learning, behavior, and attention

Chapter 2: Review of the Literature

Despite efforts to increase student achievement in mathematics and the increasing importance of mathematics for post-secondary education and working in a global economy, students in the U.S. continue to score below peers on international assessments. In 2007, the TIMSS reported that 8th grade students in the United States were ranked 6th out of 38 participating countries while the results from the 2009 PISA showed that students in the United States fell in the middle of the 33 participating Organization for Economic Co-operation and Development (OECD) countries (Fleischman et al., 2010). Nationally, the 2011 NAEP showed that only 35% of all 8th grade students scored proficient and 22% scored below basic (NAEP, 2011). The results are even poorer when considering students with disabilities as only 9% scored at or above the proficiency level while 64% scored below basic. The basic level of achievement entails using basic algebraic and geometric concepts (NCES, 2009a) and scoring below basic suggests that students do not have the foundational skills needed at the 8th grade level and are therefore unlikely to be successful in Algebra and beyond.

Within Maryland, 8th grade results from the 2005 to 2009 NAEP administrations showed promising movement towards closing the gap between students with disabilities and their non-disabled peers however the gains made were not maintained as reported on the NAEP 2011 administration suggesting more work is needed. On the assessments given by the state of Maryland the results were similar to those reported by the NAEP. The 2011 Grade 8 Mathematics MSA revealed a gap of 40 percentage points when comparing students with and without disabilities who scored proficient or higher. Analysis of past MSA results for this cohort of students, revealed that the percent of

students scoring proficient or higher decreased at each subsequent administration from grade 5 to grade 8 and while the decrease was evenly spread across grades for students without disabilities, 64% of the decline for students with disabilities occurred between the 7th and 8th grade administrations. This significant gap in achievement between 7th and 8th grade suggests a critical need for instructional change during those years and research on effective strategies for instruction is needed to assist teachers in making the necessary changes.

Organization of Literature Review

In this chapter, I present a comprehensive overview of the current intervention research involving solving equations and understanding the equal sign which are critical for success in algebra. This review serves to: 1) determine the current status of and additional areas of need for effective interventions on teaching equations and understanding the meaning of the equal sign; and 2) inform the proposed study by determining promising interventions and extending previous research for students with mathematics difficulties. Studies included in this review met the following criteria: 1) examined the effects of an instructional intervention on student performance on concepts related to solving equations or understanding the equal sign; 2) used experimental, quasi-experimental, or single-subject design; and 3) published between 1989 and 2011 in a peer-reviewed journal. The date range was chosen to reflect current mathematics education standards originating in NCTM's (1989) *Curriculum and Evaluation Standards for School Mathematics*. An electronic search was conducted using Google Scholar and the following databases: ERIC, EBSCO, JSTOR, and PsycINFO. Comprehensive searches were conducted using a combination of descriptors including: equations, equal

sign, algebra, math, mathematics, elementary, secondary, middle, learning disabilities, disabilities, intervention, struggl*, and at-risk. The search resulted in 12 articles meeting the criteria for inclusion (Araya et al., 2010; Hattikudur & Alibali, 2010; Hutchinson 1993; Ives, 2007; Mayfield & Glenn, 2008; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Star, 1999, Scheuermann, Deshler, & Schumaker, 2009; Witzel, 2005; Witzel, Mercer, & Miller, 2003). Upon closer examination it was determined that two articles (Witzel, 2005; Witzel et al., 2003) reported data on the same study, therefore, the first article (Witzel et al., 2003) was included in this review.

Overview of Studies

The current review includes 11 studies that met the criteria for inclusion with data on 882 participants. Of those participants, 189 (21.4%) were diagnosed as having LD and 27 (3%) were classified as at risk or having a mathematics difficulty (MD). Five studies (Araya et al. 2010; Hattikudur & Alibali, 2010; McNeil & Alibali, 2005b; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Star, 2009) did not disaggregate data based on disability, three studies (Mayfield & Glenn, 2008; Scheuermann et al., 2009; Witzel et al., 2003) included both students with and without MD, three studies (Hutchinson, 1993; Ives, 2007; Powell & Fuchs, 2010) included students with LD or MD, and one study (Witzel et al., 2003) included students who were considered at-risk for mathematics failure. Two studies (Mayfield & Glenn, 2008; Scheuermann et al., 2009) utilized single-subject design and nine studies (Araya et al., 2010; Hattikudur & Alibali, 2010; Hutchinson 1993; Ives, 2007; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Star, 1999; Witzel, 2005) utilized a group

design. The following literature review is divided into four major sections: a) nature of the sample, b) instructional content and focus, c) nature of mathematical practices and d) instructional approach.

Table 1:

Instructional Interventions on Concepts Related to Solving Equations or Understanding the Equal Sign

Author (year)	Purpose & Focus	Sample	Research Design/ Intervention	Dependent Variable	Results
Araya et al., (2010)	Assess impact that analogies has on learning to solve algebraic equations Equations: Variables left and right	N=236; M = 128 F = 108 Race: Participants in Chile Grade: 7 Age: 12 – 14	Posttest group design (a) Analogies for solving an equation (b) Traditional symbolic strategy	Researcher created equation solving test	Solving algebraic equations questions (a) > (b) Reasoning questions (a) > (b) Higher GPA = higher scores
Hattikudur & Alibali (2010)	Assess how learning about equality in conjunction with other comparison symbols impacts understanding Equal-sign	N=106 M=52 F=54 Race: NR Grade: 3 (n = 80) 4 (n = 26) Age: NR	Pretest/posttest (a) Comparison signs (b) equal sign (c) control	Researcher created: Problem solving Reconstruction Conceptual Definition Symbol Sort Statement Sort	Problem solving NS (a)=(b)=(c) Reconstruction NS (a)=(b)=(c) Conceptual (a)>(b)>(c) Definition (a)NS>(b)>(c) Symbol Sort (a)>(b)NS>(c) Statement Sort (a)>(b)>(c) Defining [(a)>(b)]>(c) Solving inequalities [(a)>(b)]>(c)

Hutchinson (1993)	Assess the impact that a cognitive strategy had on algebraic problem solving Equations: two variables	N = 20 LD = 20 M=9 F=11 Race: NR Grade: 8 (n=7) 9 (n=8) 10 (n=5) Age: 13 –15	Single subject multiple baseline across students; pretest/posttest group design (a) strategy instruction (b) traditional instruction	B.C. Mathematics Achievement test Metacognitive interviews Think-alouds, Problem classification task	(a) > (b) on all measures
Ives (2007)	Assess the impact that graphic organizer have while learning to solve systems of linear equations Equations: systems of equations	N = 40 LD = 33 M=31 F=9 Race: C=38 A=2 Grade: 6 – 12 Age: 13.6 – 19.3	Pretest/posttest (a) graphic organizer (b) traditional instruction	Researcher developed to conceptually justify procedures for systems of equations in two variables	(a) > (b) on posttest
Mayfield & Glenn (2008)	Assess the impact of 5 instructional interventions targeting 6 algebra skills on performance of 5 novel problem solving tasks Equations: two variables	N = 3 LD = 1 SPED = 1 Gender: M=1 F = 2 Race: NR Grade: 4 (n = 1) 7 (n = 1) 8 (n = 1) Age: 9, 13, 14	Single subject across skills and replicated across 3 students (a) cumulative practice (b) Tiered Feedback (c) Feedback + Solution Sequence Instruction (d) Review Practice (e) Transfer Training	Researcher developed target skills and problem-solving tests	(a) limited positive effects (b) no effect (c) positive effects (d) no effect (e) positive effects

McNeil & Alibali (2005)	Assess impact of a brief lesson on structure of equations or conception of equal sign has on understanding of equality Equal-sign	N=67 M=29 F=38 Race: AA=6 C=61 Grade: NR Age: 7.10 – 11.2	Pretest/posttest 2x2 factorial design (a) problem structure lesson (b) no problem structure lesson (c) equal concept lesson (d) no equal concept lesson	Researcher Adapted from prior studies Equation solving Problem Structure Equal sign definition	Problem Structure (a) > (b) main effect Equal sign definition (c) > (d) main effect
Powell & Fuchs (2010)	Assess the impact that explicit instruction on meaning of equal sign has on problem solving performance Equal-sign	N=80 LD=80 M=45 F=35 Race: AA=47 C=8 H=17 O=8 Grade: 3 Age: NR	Pretest/posttest design (a) Word Problem Tutoring (b) Word Problem plus equal-sign tutoring	Researcher Adapted from prior studies Equal sign tasks Open Equations Story Problems	Equal sign tasks (b) > (a) Open Equations (b) = (a) NS Story Problems (b) = (a) NS
Rittle-Johnson & Alibali (1999)	Assess the impact of procedural vs. conceptual instruction of equality Equal-sign	N=86 M= 46 F=40 Race: NR Grade: 4 (n= 58) 5 (n = 28) Age: mean = 10.2	Pretest/posttest counterbalanced design (a) conceptual (b) procedural (c) control	Researcher created Question task Evaluation task Standard equivalence problems Transfer problems	Question task (a) , (b) >(c) Evaluation task (a) , (b) >(c) Standard equivalence problems (a) = (b) Transfer problems (a) > (c) > (b)

Rittle-Johnson & Star (2009)	Assess impact of learning to solve algebraic equation solving through use of comparisons Equations: Variables left and right	N = 162 M=81 F=81 Race: AA=8 C=144 A=8 H= 2 Grade: 7 (n= 114) 8 (n= 18) Age: 11.9 – 15.1	Pretest/posttest design (a) compare solution methods (b) compare problem types (c) Compare equivalent equations	Researcher created on conceptual and procedural knowledge, equivalence, like terms, composite variables, use of shortcuts, flexibility knowledge	Conceptual knowledge (a) > (b),(c) Procedural knowledge: all improved (a) = (b) = (c) Flexible solution methods [(a) = (b)] > (c) Flexibility knowledge (a) > (c)
Scheuermann, Deshler, & Schumaker (2009)	Assess the effects of an Explicit Inquiry Routine on one-variable equations Equations: Variable on one side	N = 14; LD = 14 M=10 F=4 Race: AA=2 C=12 Grade: 6 (n=4) 7 (n = 4) 8 (n = 6) Age: 11 – 14	Single subject multiple-probe design across students Explicit Inquiry Routine (Explicit sequencing, Scaffolded inquiry, CRA illustrations	Researcher created test on Word problem, and Concrete manipulation; KeyMath revised	Word Problem: Mean of 95% on final instructional probe with mean difference of 74% from baseline; Manipulation: Mean of 88.6% on posttest with mean difference of 51% from baseline; KeyMath: showed significant improvement (a) > (b)
Witzel, Mercer, & Miller (2003)	Assess impact of an explicit instruction model involving CRA sequence has on solving algebraic equations Equations: Variables left and right	N = 68 LD = 41 AR = 27 Sex: NR Race: NR Grade: 6 (n =26) 7 (n= 42) Age: 11 – 14	Pretest/posttest Matched Pairs Design with random assignment across clusters (a) Explicit CRA instruction (b) Explicit Abstract instruction	Researcher Developed Assessment	(a) > (b)

Nature of Sample

This section provides an overview of the participant characteristics. Identified studies were reviewed for participant descriptions including demographics, gender, age, grade level, intervention setting, and identification criteria for students considered LD or MD.

Demographic Data. Of the studies that reported participant race/ethnicity (Ives, 2007; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Star, 2009; Scheuermann et al. 2009) 263 (29.8%) were Caucasian, 19 (2.1%) Hispanic, 63 (7.1%) African American, 10 (1.1%) Asian, and 8 (0.9%) were labeled as other. One study (Araya, 2010) was conducted in Chile and included 236 (26.8%) participants. Six studies (Araya et al., 2010; Ives, 2007; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Star, 1999; Rittle-Johnson & Star, 2009) reported SES data.

Gender, Age, Grade level. All but one (Witzel, 2003) study reported gender information with 432 (49%) male, 382 (43.3%) female, and 68 (7.7%) unknown. The age of participants was reported in nine studies and ranged between 7.10 years and 19.3 years of age. Nine studies (Araya et al., 2010; Hattikudur & Alibali, 2010; Hutchinson, 1993; Mayfield & Glenn, 2008; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Star, 2009; Scheuermann et al., 2009; Witzel et al., 2003) reported specific grade levels which included a total of 160 (18.1%) participants in grade 3, 85 (9.6%) participants in grade 4, 28 (3.1%) participants in grade 5, 30 (3.4%) participants in grade 6, 397 (45%) participants in grade 7, 32 (3.6%) participants in grade 8, 8 (0.9%) participants in grade 9, and 5 (0.5%) participants in grade 10. One study (Ives, 2007) reported that participants ranged in grades between 6 and 12 (n=40, 4.5%). The

remaining study (McNeil & Alibali, 2005b) did not report grade ($n=67$, 7.6%); however, based on participant age (7.10 -11.2), it can be assumed that participants were in the elementary grades.

Instructional Setting. Eight studies included participants who attended standard schools with instructional settings for the intervention including: one-on-one instruction in a quiet room (McNeil & Alibali, 2005b), small group instruction (Araya et al., 2010; Rittle-Johnson & Alibali, 1999), individualized instruction in resource room (Hutchinson, 1993), tutoring settings (Powell & Fuchs, 2010), and general education/inclusive classrooms (Hattikudur & Alibali, 2010; Rittle-Johnson & Star, 2009; Witzel et al., 2003). Two studies (Ives, 2007; Scheuermann et al., 2009) included participants attending special schools for students with LD with the intervention occurring in special education classrooms. The one remaining study (Mayfield & Glenn, 2008) was conducted in a residential setting using individualized instruction.

Identification Criteria for LD/MD. Three (Hutchinson, 1993; Scheuermann et al., 2009; Witzel et al., 2003) of the four studies that included students with LD reported the criteria used for the diagnosis as a discrepancy between intellectual ability and academic achievement. The remaining study (Mayfield & Glenn, 2008) did not include the criteria for diagnosis of LD. Only one study (Powell & Fuchs, 2010) noted the inclusion of students with MD who were defined by achieving below pre-determined scores on several different standardized tests.

Summary: Nature of sample. Seven of the 11 interventions included in this review occurred in settings outside of the general education classroom. While at times it may be necessary to pull some students for an intervention, many schools strive for

inclusion, the idea that students with disabilities spend the majority of their time in general education classrooms (Rozalski, Miller & Stewart, 2011), and pulling students out runs counter to this aim. Four studies (Ives, 2007, Rittle-Johnson & Star, 2009; Scheuermann et al., 2009; Witzel et al., 2003), in the current review were implemented with an entire classroom although only one study (Witzel et al., 2003) included both students with and without disabilities in the intervention. Interventions conducted in general education settings that demonstrate positive effects for both students with LD and their non-disabled peers are important as more students with LD are being included in these settings (Wagner, 2003). It is critical that future research find effective teaching practices and interventions that can be implemented in the general education setting that will be effective for all students.

Only half of the studies that were conducted in the U.S. or Canada reported information about participant race and/or SES status. Research in inclusive settings should also report student demographics including race and SES in addition to disability status, as data from the most recent NAEP (National Center for Educational Statistics, 2011) show that students from minority groups and low SES backgrounds are also under performing. It is possible that interventions, while targeted for students with MD, may also have a positive impact on the success of students from minority and low SES backgrounds in the setting as well (Freeman & Crawford, 2008; Garrison & Mora, 1999). Additionally, reporting information on demographics of the participants and the larger student body is needed in order to replicate and generalize results (Shadish, Cook & Campbell, 2002).

Instructional Content and Instructional Focus

This section includes a summary of the studies targeted in the current review based on their instructional content, instructional focus, and mathematical practices, three areas of classification which have been used in previous reviews of literature on mathematics interventions for students with disabilities (Maccini & Hughes, 1997; Mulcahy, 2007; Strickland, 2011). Implications for future research are discussed based on the review.

Instructional content. Instructional content refers to the skills or learning objectives taught during the intervention. The research in this literature review addressed concepts related to solving equations ($n=7$) or understanding the equal sign ($n=4$). Only one study (Scheuermann et al., 2009) focused on solving equations with the variable on one side of the equation, while four studies (Araya et al., 2010; Mayfield & Glenn, 2008; Rittle-Johnson & Star, 1999; Witzel et al., 2003) addressed solving equations with variables on both sides of the equation. Hutchinson (1993) included two-variable, two-equation problems in addition to single variable equations while the remaining study (Ives, 2007) focused on solving systems of equations including two-variable/two-equation and three-variable/three-equation problems (Ives, 2007).

The remaining four studies (Hattikudur & Alibali, 2010; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999) focused on how students understood and/or used the equal sign after receiving an intervention based on particular aspects of the equal sign. One study (Hattikudur & Alibali, 2010) taught students that the equal sign, along with the greater-than and less-than signs, are used to show a relationship between two quantities. In addition, three studies (McNeil & Alibali, 2005b;

Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999) taught students who were placed in the “conceptual understanding groups” that the equal sign means “the same as” (i.e. the left has to be the same as the right; or one side has to be the same amount as the other). Two of these studies also included comparison groups. For example, McNeil and Alibali (2005b) had students in the comparison group simply “notice where the equal sign is in the problem” whereas Rittle-Johnson and Alibali (1999) taught students in the comparison group a procedure for solving a problem without conceptual instruction on the equal sign.

Instructional focus. Whereas instructional content focuses on the standards or content objectives, the instructional focus includes the type of mathematical understanding that is being developed (i.e., conceptual knowledge, procedural knowledge, and problem solving). For students to be proficient in mathematics, they must develop and use multiple types of understanding including conceptual understanding and procedural fluency (Bransford, Brown, and Cocking, 1999). Students who only have a procedural understanding of a topic may be unsure when to use what they know and need a conceptual understanding to apply procedures appropriately to novel situations (NCTM, 2000). Although students need to develop both a conceptual and procedural understanding of mathematics, traditionally, most mathematics interventions for students in special education have focused primarily on teaching procedures (Maccini & Hughes, 1997; Maccini et al., 2007).

Conceptual knowledge. Conceptual knowledge refers to the ability to comprehend mathematical concepts, operations and relations, and understand where they fit in the scheme of mathematics (NRC, 2005). Knowing about the relationships and

foundational ideas of topics helps students form a network of associations (Van de Walle, Karp & Bay-Williams, 2010) and makes mathematics easier to remember and apply (NCTM, 2000). Interventions are needed that focus on conceptual understanding of topics as students with difficulties in mathematics may have trouble organizing information (Maccini et al., 2007) which is a critical skill necessary to form the networks and associations that underpin conceptual understanding.

Authors of six studies (Araya et al. 2010; Hattikudur & Alibali, 2010; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Star, 2009) focused on developing conceptual understanding. Four of these studies (Hattikudur & Alibali, 2010; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999) focused on conceptual understanding of the equal sign (i.e. as a relational sign or comparison sign). The remaining two studies (Araya et al. 2010; Rittle-Johnson & Star, 2009) focused on developing a conceptual understanding of solving algebraic equations by using analogies (Araya et al., 2010) and through comparison of multiple solution methods (Rittle-Johnson & Star, 2009).

Procedural knowledge. Procedural knowledge refers to the ability to carry out procedures and computations accurately and effectively and is important to master considering many everyday tasks require performing computations and/or using simple algorithms (NRC, 2005). Authors of three studies (Hutchinson, 1993; Ives, 2007; Rittle-Johnson & Alibali, 1999) developed interventions targeting procedural knowledge which involved using a strategy, procedure, or organizer to help students remember or organize specific steps. For instance, Hutchinson (1993) taught students a strategy to represent and solve three types of word problems involving equations. Ives (2007) used graphic

organizers as a way to organize the steps involved in solving systems of linear equations. Rittle-Johnson and Alibali (1999) taught students a specific grouping procedure to use to solve arithmetic equations (i.e., for the equation $3 + 4 + 5 = 3 + _$ students were told to add only the 4 and 5 because “there was a 3 here (point) and here (point)”).

Procedural to conceptual knowledge. Authors of two studies (Scheuermann et al., 2009; Witzel et al., 2003) incorporated the CRA sequence (concrete–representational–abstract) for representing problems in the interventions by having students first use manipulatives to model procedures for solving equations and then progressing to pictorial and symbolic methods. Scheuermann et al. (2009) used an explicit inquiry routine which involved inquiry (i.e., discovering or experiencing a concept), dialogue (i.e., Tell your neighbor how you know...), and explicit instruction (i.e., breaking a concept down into smaller/simpler components taught in a sequential manner) across multiple representations to develop understanding of one-variable equations. For instance, before solving equations that involved multiplying by a constant and subtracting a constant (i.e. $5x - 3 = 2$), students learned how to solve simpler equations involving subtracting a constant (i.e. $x - 3$) while concretely modeling the equations with cups and beans and discussing their ideas for problem representation and solution with their peers and teacher. Witzel et al. (2003) used the CRA sequence of instruction to teach students how to solve algebraic equations using manipulative objects at the concrete level, pictures at the representational level, and traditional symbolic notation at the abstract level.

Problem solving. Lastly, problem solving involves applying both conceptual knowledge and procedural knowledge to formulate, and represent problems in novel

situations (NCR, 2005). Only one study (Mayfield & Glenn; 2008) focused exclusively on problem solving although three studies (Araya et al., 2010; Hutchinson, 1993; Scheuermann et al., 2009) included measures which assessed students on problems that were not directly addressed in the intervention. Mayfield and Glenn (2008) used explicit instruction to teach six target algebra skills (i.e., multiplying/dividing/raising to a power/finding the roots of variables with coefficients and exponents, solving two-step linear equations and substituting into and simplifying two-step linear equations) and assessed the impact of five instructional interventions (i.e. cumulative practice, tiered feedback, feedback plus solution sequence instruction, review practice, and transfer training) on problem solving tasks that required novel use of 2 or more of the algebra skills that were taught. For example students were taught how the target skills of multiplying and dividing variables with coefficients independently (i.e., $5^6 f^3 \cdot 5^8 f^8$ and $\frac{49s^{12}}{7s^8}$) and tested on a problem that combined the two (i.e. $\frac{3x^6 \cdot 6x^8}{9x^7}$ or $\frac{2^{12}t^{13}}{2^3t^7} \cdot 2^5t^6$).

Summary of instructional content/instructional focus. The instructional content of the studies in this review focused on one variable equations, two variable equations, solving systems of equations and equal sign understanding (Table 2). Studies focusing on equal sign understanding (n=4) accounted for 38.4% of the total population of students included in this review and 42% of the students with LD. All of the studies were conducted with students the in 3rd – 5th grades. The remaining studies (n=7) focused on solving equations or systems of equations. None of these studies included instruction on the relational nature of the equal sign although Araya et al, (2010) used an analogy that implied the relational aspect of the equal sign without calling explicit attention to it.

Table 2:
Instructional Content and CCSS Mathematical Practice Standards

Author (Year)	Instructional Content	CCSS Mathematical Practice Standards								
		1	2	3	4	5	6	7	8	Total
Araya et al., (2010)	One-variable equations (both sides)							✓		1
Hattikudur & Alibali (2010)	Equal sign understanding	✓								1
Hutchinson (1993)	Two-variable equations	✓			✓			✓		3
Ives (2007)	Systems of Linear equations					✓		✓		2
Mayfield & Glenn (2008)	Two-variable equations							✓		1
McNeil & Alibali (2005)	Equal sign understanding						✓	✓		2
Powell and Fuchs (2010)	Equal sign understanding	✓						✓		2
Rittle-Johnson & Alibali (1999)	Equal sign understanding			✓				✓		2
Rittle-Johnson & Star (2009)	One-variable equations (both sides)	✓	✓	✓						3
Scheuermann et al., (2009)	One-variable equations (one side)		✓		✓	✓				3
Witzel et al., (2003)	One-variable equations (one side)		✓			✓				2
	Total	4	3	2	2	3	3	5	0	

Research has shown that incomplete understanding of equality can impede students' ability to conceptually understand expressions and equations (Essien & Setati, 2006; Falkner, Levi, & Carpenter, 1999; Stacey & Macgregor, 1997) and many secondary students continue to have a poor understanding (Essien & Setati, 2006; Godfrey & Thomas, 2008; Alibali, Knuth, Hattikudur, McNeil, Stephens, 2007; McNeil et al., 2006; Knuth, Stephens, McNeil, Alibali, 2006; Knuth, Alibali, Hattikudur, McNeil, Stephens, 2008; Capraro et al., 2010). Thus it is critical that future research address interventions or strategies for teaching equal sign understanding in the middle grades prior to or concurrently with learning to solve algebraic expressions.

Eight studies in this review focused on developing a conceptual understanding of the equal sign or solving equations. While procedural knowledge and problem solving are equally important skills to be developed, it is important that these abilities be developed in a conceptually meaningful way so that mathematics does not become just a series of

steps to memorize. Future research should include interventions designed to develop all three abilities in a logical and cohesive progression. For instance, developing a conceptual understanding of the equal sign as a relational symbol will provide students the foundational understanding needed to apply procedures and problem solving skills to equations in a meaningful way. This conceptual foundation for procedures is especially important for students with MD who have difficulties understanding procedural and/or conceptual processes represented with symbols and signs (Bryant, Hartman, & Kim, 2003) and who may impulsively attempt to solve problems by randomly combining numbers rather than using a logical process (Fuchs et al., 2003).

Nature of the Mathematical Practices

In addition to considering the mathematics content and the instructional focus, the interventions were also classified by the mathematical practices that are being developed alongside the content (Table 2). Mathematical practices refer to the *processes* proficient students use while learning and practicing mathematics as outlined in the Common Core State Standards for Mathematics (CCSS, 2010). As the CCSS (2010) have been adopted for use in 45 states and DC it is critical for studies to include strategies and methods that support the development of these practices. There are eight Common Core Standards for Mathematical Practice (CCSS, 2010) in total, which were derived from the NCTM (2000) process standards (i.e. problem solving, reasoning and proof, communication, representation, and connections) and the strands of mathematical proficiency described above (conceptual understanding, procedural fluency, and productive disposition) as outlined by the NRC (2005). The sections below describe these eight standards for

mathematical practice and highlight interventions from the current review that would promote the development of student's proficiency with each standard.

Make sense of problems and persevere in solving them. This practice standard involves students performing actions such as explaining the meaning of a problem to themselves, looking for entry points, making conjectures and a plan to solve a problem, considering analogous problems, monitoring and evaluating progress, checking answers with alternative methods and generally insuring if steps taken or answers arrived at make sense in the context of the problem (CCSS, 2010). Interventions in four studies (Hattikudur & Alibali, 2010; Hutchinson, 1993; Powell & Fuchs, 2010; Rittle-Johnson & Star, 2009) included elements to develop students' abilities to make sense of problems and persevere in solving them. For instance, Hattikudur and Alibali (2010) focused on developing students understanding of comparing symbols (i.e. greater-than and less-than) simultaneously with their understanding of the equal sign. During the intervention the experimenter phrased questions to students in a manner that would promote self-reflection while working independently (i.e. "would make sense to put a greater-than sign here to make this math sentence correct," p. 20). Further, Hutchinson (1993) investigated the use of a cognitive strategy to help students with algebra problem solving. The intervention involved the use of self-question prompt cards which reminded students questions to ask themselves while solving problems such as "have I understood each sentence...do I have the whole picture...what should I look for in a new problem to see if it is the same kind of problem" (p. 39).

Powel and Fuchs (2010) examined how instruction focused on the equal sign contributed to students' problem solving skills. One tutoring session focused on checking

written work to ensure the correct math, operations, and labels were used during the problem solving process thus ensuring students understood what they were doing at each point of the problem solving process. Rittle-Johnson and Star (2009) examined the advantages and disadvantages of teaching students to use different approaches to solve similar equations to support their learning and transfer. The researchers demonstrated that multiple possible methods could be used, and included questions that asked students to identify all possible “next steps” that could be taken while solving an equation. In doing so, students were reminded to focus on making sense of the problem to mind multiple solution methods. While using a cognitive strategy was the primary focus in only one of the studies (Hutchinson, 1993), all of the studies that included verbal prompts or cue cards to assist students with sense-making and perseverance were determined to have positive results on student learning.

Reason abstractly and quantitatively. This practice standard involves students demonstrating the ability to create coherent representations of problems, attend to the meaning of quantities, use properties of operations and objects, decontextualize situations by manipulating mathematical symbols abstractly without necessarily attending to the original referents, and to pause and contextualize the symbols during the manipulation to make sense of the symbols concretely in terms of the original context (CCSS, 2010). Three studies (Rittle-Johnson & Star, 2009; Scheuermann et al., 2009; Witzel et al., 2003) included activities that would develop students’ abilities to reason abstractly and quantitatively. For instance, Scheuermann et al., (2009) investigated the effects of an explicit inquiry routine – involving inquiry, dialogue, explicit instruction and the use of concrete manipulatives, representations and abstract symbols (CRA) – on student’s

understanding of one-variable equations. Researchers used multiple modes of representation including objects (e.g., beans, buttons) to illustrate abstract concepts and help students understand and perform abstract processes. Problems required students to represent a unit defined in the problem with concrete objects, manipulate the objects to solve a particular problem and provide an accurate answer. Witzel et al. (2003) also investigated the impact of the CRA instructional sequence on students' ability to solve algebraic equations. Students used physical models to concretely represent quantities and operations in a problem, manipulated the objects to arrive at a solution, and then interpreted the solution using the original abstract context of the problem. Rittle-Johnson and Star (2009) included items that asked students to decide if two equations were equivalent without solving the problem in order for students to attend to properties the object rather than resorting to computations. All three studies included methods to assist students with abstract reasoning. Two studies (Scheuermann et al., 2009; Witzel et al., 2003) included the CRA sequence, which has been shown to be effective at promoting conceptual understanding of abstract mathematical ideas in all students including those with MD (Gersten et al., 2009). The remaining study (Rittle-Johnson & Star, 2009) started with an abstract concept and used explicit questioning to guide students to use abstract reasoning rather than computation. While explicit instruction has been shown to be effective for teaching a specific strategy or skill (Kroesbergen & Van Luit, 2003), it is unknown if this type of questioning would develop students' ability to reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others. Students who demonstrate this standard are able understand stated assumptions/definitions, make

conjectures, build progressions of statements to explore the truth of conjectures, justify conclusions, compare the effectiveness of two plausible arguments, and distinguish and use correct logic and reasoning. They are also able to communicate findings, respond to the arguments of others, decide if an argument makes sense, and ask clarifying questions (CCSS, 2010). Interventions in two studies (Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Star, 2009) included elements that would develop students' abilities to construct viable argument and critique the reasoning of others. For example, Rittle-Johnson and Alibali (2010) included tasks that required participants to evaluate procedures they believed were done by other students as, *very smart*, *kind of smart*, or *not so smart* and explain their reasoning for making the choice. Rittle-Johnson and Star (2009) had participants compare solutions of equations completed by other students, evaluate nonconventional solution methods, and explain their reasoning for the evaluation provided. While neither of the studies measured students abilities to construct arguments or critique the reasoning of others directly as a result of the intervention, overall the interventions showed promising results and including elements that allowed students to rate or compare other students work which, at a minimum, exposed students to this mathematical practice. Future research is needed that explicitly measures changes in student's abilities to construct viable arguments and critique the reasoning of others as a result of an intervention. Additionally, neither of these studies included participants with MD. It is critical that future research design interventions for students with MD as many of these students have processing problems associated with interpreting information visually or auditorily and/or oral language problems associated with

understanding or providing oral arguments (Steele, 2010), which may impact their ability to construct viable arguments and critique the reasoning of others.

Model with mathematics. This practice standard involves students applying mathematics to solve everyday problems, using multiple representations such as graphs, tables and equations and interpreting/reflecting on the mathematical results in the context of the situation (CCSS, 2010). Only two studies (Hutchinson, 1993; Scheuermann et al., 2009) included elements of modeling with mathematics. Hutchinson (1993) described a task for solving two-variable two-equation problems that had participants create a chart to record values and related outcomes to help facilitate systematic selection when using trial and error during problem solving. Further, the CRA method used by Scheuermann et al. (2009) had students model a scenario with a graphic representation to help facilitate understanding of the abstract/symbolic mathematical representation of the problem. Given that many students with MD have difficulty with abstract symbols (Garnett, 1998; Geary, 2004) and limited conceptual understanding of procedures (Geary, 2004), alternative models that do not rely as heavily on these skills such as graphics and charts may be helpful for students understanding of topics as evidenced by the positive impacts on student learning found by the above studies (Hutchinson, 1993; Scheuermann et al., 2009). Future research is needed to replicate previous findings and expand the use of models to additional topics.

Use appropriate tools strategically. This practice standard involves students considering and using available tools (e.g., concrete model, ruler, compass, calculator, spreadsheets, dynamic geometry software) and evaluating their effectiveness for a given situation to help students explore and deepen their understanding of mathematical

concepts (CCSS, 2010). Interventions in three studies (Ives, 2007; Scheuermann et al., 2009; Witzel et al., 2003) included elements to help students use appropriate tools strategically. Ives (2007), for example, examined the use of graphic organizers during instruction on student's understanding with solving systems of equations. The graphic organizer served as a tool to help students organize their work and support their understanding when solving systems of two and three equations. Further, Scheuermann et al. (2009) and Witzel et al. (2003) incorporated the use of objects (e.g., buttons, beans) to represent abstract quantities to help students understand the abstract mathematical concepts presented. While graphic organizers were used as a tool in one study (Ives, 2007) and concrete objects were used as tools in the remaining two (Scheuermann et al., 2009; Witzel et al., (2003), all of the studies that included tools had a positive impact on student learning. Further research is needed however, that promotes 'strategic' tool use as all of the studies required use of tools and it is unknown whether students would know to use them on their own in novel situations.

Attend to precision. This practice standard involves students using clear and precise definitions in discussions and explanations of their own work, stating the meaning of and/or correctly using symbols specifying units of measure, calculating with precision that is appropriate for a problem context, and giving carefully formulated explanations to each other (CCSS, 2010). Interventions in three studies (McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999) included instruction that had students attend to precision in their work. For instance, McNeil and Alibali (2005b) examined if arithmetic ideas (e.g. 'operations = answer,' equal means "the total") contributed to student difficulties with equations. During instruction on the concept of the

equal sign, students were told the relational definition of the equal sign, which is a more accurate interpretation than the operational view. Powell and Fuchs (2010) and Rittle-Johnson and Alibali (1999) also taught students the relational definition of the equal sign and included additional activities that required students to provide explanations on problems they had solved. All three of the studies focused on teaching students a more precise definition of the equal sign (i.e. the same as) and determined that having a more precise definition resulted in positive student gains in problems that had the equal sign in them. While one study (Powell & Fuchs, 2010) included students with MD, all of the studies were conducted with elementary students and further research is needed to determine if similar results can be found with middle school students.

Look for and make use of structure. This practice standard involves students analyzing objects and examples to discern patterns and structures, extending lines in geometric figures to assist in problem solving, and “seeing” complicated things such as an algebraic expression as either a single whole or composed of several objects (CCSS, 2010). Interventions in five studies (Araya et al., 2010; Hutchinson, 1993; Ives, 2007; Mayfield & Glenn, 2008; McNeil & Alibali, 1999) included activities or tasks that promoted students ability to make use of structure. For example, Araya et al. (2010) investigated the effects that analogies (e.g. two-pan balance) had on learning to solve algebraic equations. Students were given novel tasks after receiving the instruction with analogies to see if they were able to generalize known procedures to equations with variables in novel positions. Mayfield and Glenn (2008) examined the effect of five instructional interventions on problem solving tasks that required students to apply six algebra skills that were focused on during the intervention in novel combinations. To

successfully complete the problem solving tasks, students had to recognize that the complex exponent expressions were composed of smaller parts that were included in the six algebra skills targeted in the intervention. McNeil and Alibali (1999) had students point out where the equal sign was located in a particular problem in the problem structure instructional group. Hutchinson (1993) engaged students in conversations about the importance of mathematical structure for determining problem type, how to represent a problem while Ives (2007) included questions for participants that had to be solved by recognizing and applying properties of equation structure. Four of the studies (Araya et al., 2010; Mayfield & Glenn, 2008; McNeil & Alibali, 1999; Hutchinson 1993) directly focused on student's attention to the structure of mathematical objects and making use of those structures to solve problems during the intervention while the remaining study (Ives, 2007) simply included questions meant to assess students ability to attend to structure. All of the studies that assisted students to look for and make use of structure had positive impacts on student learning. Therefore, future interventions should include components that focus student attention on the structure of mathematical objects to improve understanding of the content. Additionally, research is needed to find teaching methods and scenarios that will allow students to develop the practice of looking for and using structure for generalization across all mathematical domains.

Look for and express regularity in repeated reasoning. This practice standard involves students noticing repeated calculations and looking for general methods or shortcuts, maintaining oversight of problem solving process, and continually evaluating reasonableness of approach and results (CCSS, 2010). None of the studies described instruction or activities in the interventions that appeared to foster this mathematical

practice with regards to regularity and repeated reasoning. Future studies are needed that included evidence of this practice standard.

Summary of the mathematical practices. Of the studies in this review, three included evidence of incorporating three practice standards, five included evidence of two practice standards and the remaining three only had evidence of one practice standard. All of the practice standards were represented with the exception of ‘look for and express regularity in repeated reasoning.’ The practice of ‘attend to precision’ was only represented in studies focusing on the equal sign in the elementary grades. The practices of ‘construct viable arguments and critique reasoning of others,’ ‘model with mathematics,’ and ‘use appropriate tools strategically’ were only included in studies in the secondary grades. While not all interventions and topics lend themselves to include all eight of the mathematical practice standards, it is critical that future studies be designed to include as many mathematical practices as possible and reported with enough detail to let readers know how they were included (Maccini, Miller, & Toronto, 2013). This is critical as proficiency with the mathematical practice standards can assist students with problem solving skills and conceptual understanding of, both of which are areas where students with MD struggle (Greary, 2004; Montague & Applegate, 2000). Including and reporting the mathematical practices standards can be done by not only stating the targeted skills and content of an intervention but also by explicitly stating how the intervention addresses the need for students to gain proficiency on the mathematical practice standards.

Instructional Approach

This section organizes the studies in the current review based on the nature of the instructional approach. The instructional approaches for students with MD include the following: explicit instruction, explicit/systematic instruction, self-instruction, and visual representations (Steadly et al., 2008).

Explicit instruction. Explicit instruction refers to learning situations where the teacher leads students through a pre-determined instructional sequence (Steadly et al., 2008). Lessons that rely on explicit instruction begin with an advance organizer to prepare students for the upcoming lesson, a demonstration of the content by the teacher, guided practice with a gradual release of teacher support, and finally independent student practice (Hudson, Miller & Butler, 2006). This approach to teaching has been found to be especially successful for teaching a specific procedural strategy or isolated skill (Kroesbergen & Van Luit, 2003) and as a general approach for teaching mathematics to students with MD (Gersten et al., 2009). Authors of three studies (Hattikudur & Alibali, 2010; McNeil & Alibali, 2005b; Rittle-Johnson & Alibali, 1999) incorporated components of explicit instruction in their studies.

All three of the studies utilized a pre/post test comparison design with the intervention group focusing on conceptual understanding of the equal sign. For example Hattikudur and Alibali (2010) examined the effects of the following three treatment conditions: a) instruction that involved comparing the equal sign to other relational symbols (i.e. greater-than and less-than); b) instruction that only taught the relational interpretation of the equal sign; and c) a control group that participated in a filler tasks where students identified which numbers were bigger but no comparison signs were used. In the comparison group students were explicitly told that the three signs were used for

showing relationships between quantities and then were taught how each was used. Participants were then provided examples of the symbols in use and were asked to consider statements and if they made sense, followed by independent practice via a worksheet. Instruction in the equal sign group proceeded in the same manner; however, students were told three variations of relational definitions of the equal sign instead of being introduced to the three symbols. In the control group, students were presented cards with three numbers with no symbols between them and simply asked to choose the biggest number. The results showed that the comparison group demonstrated significantly higher conceptual understanding on the posttest than students in the equal sign group, although both treatment groups scored significantly higher than the control group.

McNeil and Alibali (2005b) investigated the relationship between elementary students operational patterns and likelihood of learning from a brief lesson on arithmetic equations. Participants were randomly assigned to one of four conditions and presented with an equation with a correct solution in the blank ($6+4+7 = 6 + \underline{11}$) followed by a timed 1 minute lesson delivered via explicit instruction on what to notice or think about. The instruction focused on a) problem-structure (i.e. where the equal sign was located), b) equal sign (i.e. relational definition of equal sign), c) problem structure + equal sign or d) control condition (i.e. told to think about the problem). The results showed that students in the problem-structure conditions improved their performance on the problem structure measure more than those in the other conditions while students in the equal sign conditions were better able to judge relational definitions of the equal sign as 'smarter' or

more sophisticated than other definitions such as “the end of the problem” compared to those students in the non-equal sign conditions.

Rittle-Johnson and Alibali (1999) examined the relation between 4th and 5th grade students’ conceptual understanding of equivalence and the procedures used for solving equivalence problems. The students were randomly assigned to three instructional groups (conceptual instruction, procedural instruction, control) and received 15-25 minute lessons. Students in the conceptual instruction group were presented with an equation ($3 + 4 + 5 = 3 + \underline{\quad}$) and taught that the amount before and after the equal sign is the same and the numbers on each side had of the equation add up to the same amount. The students were not given a procedure to solve the problem or provided a solution. Students in the procedural instruction group were told there were multiple ways to solve the problem ($3 + 4 + 5 = 3 + \underline{\quad}$) and were instructed on how to apply one specific procedure. Following initial instruction, students in both groups solved a novel problem independently, and were then provided with repeated instruction with another problem. Students in the control group received no instruction. The results showed that students in both instructional groups performed significantly better on the four post-test problems than students in the control group, although there was no significant difference between the two treatment groups. All three of these used explicit instruction as the main instructional delivery and students in intervention conditions showed improvement on problems pertaining to the problem types that they were explicitly taught. Future research is needed to extend the findings to students with MD at the middle school level, and to topic areas beyond the equal sign.

Explicit/systematic instruction. Systematic instruction focuses on teaching students to become efficient learners by modeling the learning process and providing strategies and tools to use while problem solving (Steadly et al., 2008). The combination of explicit and systematic instruction has been found to be especially effective for remediating students with LD (Swanson, 2001) and as a general approach for teaching mathematics to students with MD (Gersten et al., 2009). Authors of three studies (Mayfield & Glenn, 2008; Powell & Fuchs, 2010; Scheuermann et al., 2009) used explicit/systematic instruction as the primary instructional approach in their interventions and authors in two other studies (Hutchinson, 1993; Witzel et al., 2003) incorporated aspects of explicit/systematic instruction as well.

Mayfield and Glenn (2008) used a single subject design across interventions and participants to examine the effect of five instructional interventions on problem solving tasks that required students to apply six algebra skills in novel combinations. Participants were explicitly taught the six target skills related to solving linear equations and multiplying and dividing variables with exponents and coefficients using scripted instruction, guided and independent practice, and corrective feedback. The intervention phases (cumulative practice, tiered feedback, feedback with solution sequence instruction, review, and individualized transfer training) were implemented to determine the effect on students' problem solving performance. While the first four components are characteristic of explicit instruction the final component, individualized transfer training helped students apply skills to new situations. Specifically transfer training involved breaking a complex task into component target skills, which the students then solved in order to build back up to the original complex task. Prompting was provided throughout

the process as needed. Researchers noted some improvements for cumulative practice and feedback with solution sequence instruction and consistent improvement for individualized transfer training.

Similar to Mayfield and Glenn, Scheuermann et al. (2009) used a multiple-probe-across-students design to evaluate the effects of an Explicit Inquiry Routine (EIR) on the ability to solve one variable equation word problems for middle school students with LD. The EIR process consisted of three steps: (1) *explicit sequencing* which involved the process of solving equations by first “breaking down” the equation into small instructional pieces and teaching in sequence from simplest (e.g. $x + 3 = 10$) to complex (e.g. $3x + 2x - 4 = 51$) (2) *scaffolded inquiry* which involved students illustrating and manipulating the problems while explaining their thinking aloud; and (3) *modes of illustration* which involved the graduated sequence instruction in which students first solved equations by manipulating concrete objects then by drawing and manipulating representations, and finally by using mathematical symbols and notation. Following instruction, all but one student met or surpassed 80% accuracy on the word problem test final probe. The mean score was 78% accuracy across all probes in the instructional phase. Students scored significantly better on the post-test measure with a moderate effect size ($\Delta = .54$) on the KeyMath-Revised standardized test

Powell and Fuchs (2010) assessed the efficacy of embedding equal sign instruction within word problem tutoring. The authors used a pre/post-test design and randomly assigned 3rd grade students diagnosed with LD to two groups, a word-problem tutoring and control (no tutoring) groups, while blocking for LD subtype to insure equal proportions. A third cohort of students was recruited from a similar population as the first

two groups and assigned to the combined group (word problem and equal sign). The control group received conventional classroom instruction with no tutoring. Both tutoring groups received schema-broadening instruction to help broaden student's schemas for 'total' problems (e.g. Fred ate 3 pieces of cheese pizza and 2 pieces of mushroom pizza. How many pieces of pizza did Fred eat?) to include those with novel features such as information presented using multiple representations (i.e. With charts, graphs or pictures). In addition, the combined group received explicit instruction on the relational meaning of the equal sign. The results showed that the combined tutoring group performed significantly better on equal sign tasks over the word-problem tutoring and control students. Of particular note was that the combined group was significantly better at solving non-standard equations, which were not taught, while there was no difference between the word-problem and control groups. Based on the results the authors suggested that a relational understanding of the equal sign may have transfer effects to nonstandard equations.

The three studies described above all incorporated systematic instruction in conjunction with explicit instruction and led to improved student outcomes not only on what was taught during the intervention, but also in novel situations as well. This outcome suggests that the systematic component of the instructional delivery may assist transfer of student learning. The results are especially promising given that all of the studies included students with MD. Future research should include explicit/systematic instruction as an instructional approach when teaching middle school students with MD.

Self instruction. Self-instruction refers to strategies that students can use to manage their learning, behavior, and attention. Students work on developing the skills

employed by efficient learners, such as setting goals, planning a solution path, checking work as they go, remembering strategies, monitoring their progress, and evaluating the reasonableness and accuracy of their solutions (Stedly et al., 2008). These strategies help students to regulate their strategy use and performance, take control of their actions, and move toward independence as they learn which is critical to academic success (Montegue, 2007). Only one study (Hutchinson, 1993) incorporated self-instruction as the instructional approach used during the intervention. Specifically, Hutchinson (1993) used a modified multiple baseline design across 12 students in conjunction with a two group design to investigate the effect of a cognitive strategy to help students with solving specific types of algebra problems (e.g. relational problems, proportion problems and two-variable two-equation problems). The intervention involved the use of a self-question prompt card that included questions students should ask themselves while solving algebra word problems (e.g. “Have I read and understood each sentence? Have I got the whole picture, a representation, for this problem?) and a structured worksheet to help guide students while solving problems involving equations (e.g. Have I written an equation? Have I expanded the terms?). Students remained in an instructional phase until they reached 80% accuracy on three consecutive assessments. It was determined that all students met the criteria although they progressed at individual rates. When compared to the control group, students in the intervention performed significantly better at the posttest assessment. As with explicit/systematic instruction the use of self-instruction techniques showed evidence of assisting in the transfer of learning and therefore future studies should consider including self-instruction techniques when teaching Algebra concepts to middle school students with MD.

Visual representations. Visual Representations involve making the abstract concepts of mathematics more concrete through the use of manipulatives, pictures, charts, graphs, graphic organizers and other instructional aids (Steedly et al., 2008). Visual representations are frequently used in mathematics through the graduated instruction sequence, which involves first modeling a concept with concrete objects (i.e., beans, two-color chips, algebra tiles, etc.) then progressing to representations (i.e. drawn pictures) and finally to the abstract symbolic notations. This instructional sequence has been shown to be effective for both general education and special education students (Gersten et al., 2009) and is in line with the common core mathematical practice standards encouraging students to model with mathematics and use tools strategically (CCSS, 2010). Authors of three studies (Ives, 2007; Scheuermann et al., 2009; Witzel et al., 2003) used visual representations in their interventions and all of these studies focused on solving equations.

Ives (2007) examined how the use of graphic organizers when solving systems of linear equations affected secondary students performance in two similar studies using a pretest/posttest design. In both studies, graphic organizers were used as a way to organize student work and thoughts as they completed a procedure to solve two-variable (study 1) and three-variable (study 2) systems of equations. Results from the first study indicated statistically significant differences between the graphic organizer and comparison group on the posttest measure, although neither group demonstrated practically significant results (i.e. students only performed with 40% accuracy). Results of study 2 also indicated significant differences between the graphic organizer and comparison groups although both study's conclusions were based on an alpha level of 0.10. The effect sizes

for both studies were in small range (.101 and .12). The results suggest that more research is needed to determine if graphic organizers are a viable instructional tool for teaching systems of equations to students with LD, and can lead to practically significant gains and achieve statistical significances at the conventional alpha level of 0.05.

Two of the studies (Scheuermann et al., 2009; Witzel et al., 2003) incorporated the CRA sequence into their interventions. For example Witzel and colleagues (2003) investigated the impact that the use of a CRA instructional sequence on students' ability to solve algebraic equations using a pretest/posttest/follow-up design with random assignment of clusters. Students in the treatment and comparison groups were taught with explicit instruction but students in the treatment group also received CRA instruction; whereas students in the comparison groups received repeated abstract instruction. The CRA instruction included an introduction to the lesson, modeling of a new procedure, guided practice with the procedure, and independent practice at each level of the CRA sequence (i.e., concrete manipulatives, representational drawings, abstract symbolic notation). While both groups showed improvement from pre- to posttest, students in the CRA group demonstrated significantly higher gains. Although both studies included a graduated instructional sequence, the interventions also included elements of explicit instruction and further research is needed to determine the unique impact that visual representations may have on student learning outcomes.

Summary of instructional approaches. Authors of the studies included in this review included a range of instructional approaches that have been shown to be effective for teaching mathematics. The three studies that used explicit instruction focused on developing a conceptual understanding of the equal sign with elementary students and

found positive results in moving students towards understanding the equal sign as relational. Although these studies did not disaggregate based on student disability, the results are promising as they are in line with results of a meta-analysis of mathematics interventions for students with disabilities (Kroesbergen & Van Luit, 2003) which found that direct/explicit instruction is an effective teaching method for students with special needs especially for teaching a specific procedural strategy or isolated skill.

Although categorized by the main instructional approach used, several studies (Scheuermann et al., Powell & Fuchs, 2010, Witzel et al., 2003) used a combination of explicit/systematic instruction and visual aids (i.e. manipulatives, CRA sequence) during the intervention and found positive results. This is consistent with results of a meta-analysis of instructional components that enhance proficiency of students with LD (Gersten et al., 2009) which found that while visual representation are effective on their own, better effects are observed when used in conjunction with other instructional components such as explicit/systematic instruction. Two of these studies (Powell & Fuchs, 2010; Witzel et al., 2003) included both students with and without MD in the intervention as well which suggests that CRA and explicit/systematic instruction may be an effective approach for teaching in an inclusive general education classroom.

The five studies that included explicit/systematic instruction as either the main instructional focus (Mayfield & Glenn, 2008; Powell & Fuchs, 2010; Scheuermann et al., 2009) or that had components of it as part of a larger intervention including self instruction (Hutchinson, 1993) or graduated sequence instruction (Witzel et al., 2003) demonstrated evidence that students were able to transfer knowledge to novel situations not covered in the intervention. These results are particularly promising as they provide

evidence that the explicit/systematic support not only improvement in students content knowledge but also improvement in their mathematical practices, making sense of problems and persevering in solving them and looking for and making use of structure.

Summary

The current review of the literature identified instructional content, instructional focus, mathematical practices, and instructional approaches used in interventions for teaching equations and understanding the meaning of the equal sign. These results were analyzed in relation to how they were or might be used for students with MD. The following section summarizes limitations of the current literature and suggestions for future research.

Limitations. Overall, the authors of the studies utilized sound methodologies in the studies however there are several limitations to the current research.

- 1) While several studies (Hattikudur & Alibali, 2010; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999) examined the effects of an intervention on understanding the equal sign with elementary grade students, no studies were identified that included secondary students despite literature that reports that students have incomplete or incorrect conceptions about the equal sign in secondary grades (Essien & Setati, 2006; Godfrey & Thomas, 2008; Alibali et al., 2007; McNeil et al., 2006; Knuth et al., 2006; Knuth et al., 2008).
- 2) Only one study (Witzel et al., 2003) was conducted in an inclusive classroom with both students with and without MD therefore, the effectiveness of the remaining interventions is uncertain if implemented in inclusive classrooms.
- 3) Eight studies (Araya et al., 2010; Hattikudur & Alibali, 2010; Ives, 2007;

- Mayfield & Glenn, 2008; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Star, 1999, Scheuermann et al., 2009; Witzel et al., 2003), including all of the studies pertaining to equations, relied on researcher-created assessments whose reliability and validity were unknown or not reported. Without this information it is unknown how well the instruments tap what researchers intended and the validity of inferences drawn from the studies cannot be determined (Hill & Shih, 2009).
- 4) Eight of the studies (Hattikudur & Alibali, 2010; Hutchinson, 1993; Ives, 2007; Mayfield & Glenn, 2008; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Star, 2009; Scheuermann et al., 2009) were implemented by either the researcher or graduate assistants and it is not clear what the results would have been if they had been implemented by classroom teachers. This is particularly problematic for those studies (Ives, 2007; Powell & Fuchs, 2010) that focused on students with learning disabilities as effect size tends to be higher when interventions are implemented by the investigators as opposed to classroom teachers (Swanson, Hoskyn, & Lee, 1999).

Future research. Based on the studies included in this review, the following areas are in need of further research.

1. Several studies (Asquith et al., 2007; Booth, 1988; McNeil et al., 2006; Schoenfeld, & Arcavi, 1988) have identified that many students enter their first year algebra course with an inadequate understanding of the equal sign however no studies have disaggregated data to compare differences between students with and without MD.
2. No studies specifically examined an intervention or strategy for teaching the

relational meaning of the equal sign in the middle grades or concurrently with learning to solve algebraic expressions.

3. Only one study in the current review (Witzel et al., 2003) was conducted in an inclusive classroom containing both students with and without disabilities. Research is needed to find methods to support students with MD in these inclusive settings.
4. All of the studies analyzed, based on the CCSS for Mathematical Practice were published prior to the adoption of the standards. Future studies should be designed to promote student growth in both the content and practice standards.

Conclusion

This literature review synthesized research findings involving interventions for solving equations and understanding the equal sign. These instructional interventions emanated from a comprehensive review from the general educational and special education literature bases and address instructional practices to promote student understanding of these concepts fundamental to future success in algebra. This is a critical area of research as students with MD continue to score below their non-disabled peers on national and state assessments. Beliefs about how students learn impact teacher's instruction and curriculum choices and therefore instructional interventions need a theoretical grounding prior to development.

Within the special education research base, interventions have become increasingly more aligned with standards and practices outlined by the mathematics education literature base. Part of this shift in alignment could be attributed to laws such as the 1997 amendments to IDEA which included a focus improving the educational

achievement of students with disabilities and NCLB (2002) which placed a stronger focus on increasing the quality and effectiveness of education through high standards.

Consequently, more students with disabilities are being educated in general education classrooms with access to the same rigorous curriculum as their non-disabled peers and supports for students with MD in this environment are critical.

The CCSS (2010) explicitly states that *all* students must have the opportunity to reach the same high standards and recognizes that students with special needs may need support to reach the standards. The CCSS do not, however, define intervention methods or materials to support students with disabilities. However, the special education research base has shown three promising strategies and interventions to support students in accessing the rigorous mathematics standards, including the use scaffolded instruction (Gersten et al., 2009; Steedly et al., 2008), strategy instruction (Steedly et al., 2008; Montague, 2007), and visual representations (Gersten et al., 2009; Steedly et al., 2008). These three methods are grounded in both the special education and general education literature base and diverge from findings of earlier reviews of the literature in mathematics and special education (Maccini & Hughes, 1997; Woodward & Montague, 2000) that reported an emphasis on procedures, rote memorization, and didactic instruction. Despite this progress, more studies are needed that include supports for students with MD accessing age-appropriate standards, especially at the middle and secondary school level. The conclusion of this literature review addresses the equal sign and topic of equality, summarizes the constructivist theory of learning, and ends with the rationale for the proposed study.

Topic of equality. The topic of algebraic expressions and equations and specifically the nature of the equal sign is an area absent from the literature on interventions for students with MD. This topic is critical for several reasons: 1) it is a topic important in all years of secondary school mathematics, 2) involves symbols and abstract reasoning - a process that many students with LD struggle with – and, 3) is an area where many students (general and special education) and teachers, do not have a complete understanding of the topic. Without a complete understanding of the fundamental meanings of the symbols that are used in expressions and equations, particularly the equal sign, algebra simply becomes a series of procedures, and steps to memorize, rather than an interconnected subject of concepts and ideas for some students and teachers.

Research shows that most students believe the equal sign is an operational symbol that simply indicates, “answer goes here” or “find the total” (Asquith et al., 2008; Baroody & Ginsburg, 1983; Kieran, 1981; McNeil et. al., 2006;). Students holding an operational view may be at a disadvantage when progressing to algebra as they will not be able to interact with the mathematics as completely as those who hold a more complete understanding (Godfrey & Thomas, 2008). For example, expressions such as $17 \cdot 9 = 10 \cdot 9 + 7 \cdot 9$ or equations with quadratics require a broader understanding of the equal sign which includes the relational and symmetric properties (Prediger, 2010).

Four predominant reasons have been put forth as to why students develop and maintain a misconception of the equal sign, including limited cognitive ability during elementary school, frequent exposure to operational contexts, lack of explicit instruction, and knowledge depending on context. The first reason put forth by early researchers suggested that the children’s conception of the equal sign is limited by their cognitive

development in the elementary years. This theory is grounded in Piagetian research which suggests that students do not develop the logical cognition necessary to understand relations and equivalence until they are middle school aged (Inhelder & Piaget, 1958). The cognitive development view however, was called into question by Baroody and Ginsburg (1983) when they implemented a curriculum that carefully developed elementary students' relational understanding of the equal sign and helped them to understand and use both the operational and relational view. Based on the findings, Baroody and Ginsburg proposed that students arrive at the operational view through exposure to everyday experiences (i.e. putting together two counted sets and counting the total).

The idea of frequent exposure and reinforcement leads to the second possible reason for the misconception which is related to the curriculum students used and how the equal sign is presented to students by educators. Researchers that support this view of exposure and reinforcement (Baroody & Ginsburg, 1983; McNeil et al, 2005a) argue that the misconception develops as a consequence of working with the equal symbol in elementary schools where students do not need to understand the equal sign as a relational symbol in order to be successful in arithmetic. This is primarily due to the structure of arithmetic examples students are exposed to in standard curricula that almost exclusively emphasize the operational nature of the equal sign by presenting problems of the form " $a + b = \underline{\quad}$ " with the operations on the left and the "expected answer" being a single number on the right (Prediger, 2010).

In addition to students prior exposure, researchers report a lack of explicit attention on the concept of equality prior to and during middle school (Asquith et al,

2007; McNeil et al, 2006) as well as the use of the transitive, reflexive, and symmetric properties of equality by teachers in high school and university settings without explicitly exploring these properties with students (Godfrey & Thomas, 2008). The lack of focus as to the meaning of the equal sign may lead students to rote memorize procedures during later mathematics classes instead of having a conceptual understanding that would come with knowing the relational properties of the equal sign.

Finally, some researchers (Barsalou, 1982; Thelen & Smith, 1994) report that students' misconception persists because their conceptual knowledge may be "context dependent. McNeil et al. (2006) state that the relational properties of equal are not typically presented until middle school or later while the view of the equal sign as an operator has been firmly established throughout elementary school. Consequently, according to the idea of contextual dependency, the operational view of the equal sign would be activated in most contexts while the relational view of the equal sign would need more contextual support (McNeil & Alibali, 2005).

Several researchers spanning almost 30 years have shown that early focused instruction on the concept of equality can help students develop a relational view of the symbol (Baroody & Ginsburg, 1983; Powell and Fuchs, 2010) although no intervention research has been conducted at the middle school level that met the criteria for this review. Based on results obtained with 3rd grade students, Powell and Fuchs (2010), suggest that students who do have a relational understanding of the equal sign in middle school would be more likely able to solve non-standard arithmetic and algebraic problems (i.e., $8 + 3 = X + 2$, or $5 = 8 - x$) even without explicit instruction (i.e. shown how to solve these types of problems).

It has also been suggested that changes should be made to the curricula used in middle schools to provide more exposure to the types of questions and uses of the equal sign that promote the relational view (McNeil et al., 2006) although currently no studies have tested this hypothesis.

Constructivist theory of learning. The constructivist theory of learning is based on the idea that all knowledge is constructed and there exists cognitive structures that are used during the process of construction and that the cognitive structures themselves are all under continual development (Noddings, 1990). Within constructivism, students are viewed as active learners who construct their own conceptions of the material based on influences from previous knowledge, interactions with peers and teachers, and the environment (Goldin, 1996; Greeno, Collins & Resnick, 1996; Noddings, 1990; Smith, diSessa, & Roschelle, 1993). Some theorists view constructivism in a radical form believing that all 'knowledge' is necessarily constructed and that nothing truly exists except vague approximations of objects that come into being because of the constructs of the mind (Von Glasersveld, 1990). While an interesting thought experiment, putting radical constructivism into the context of teaching is a challenging if not impossible task. A more moderate view divides constructivism into a continuum with Endogenous constructivism (i.e. discovery learning with the teacher on the periphery) on one end to exogenous constructivism (i.e. teacher directed instruction) on the other (Harris & Graham, 1994; Moshman, 1982). Between these two is dialectical constructivism where the teacher acts more as a guide than a leader or observer on the periphery (Harris & Graham, 1994; Moshman, 1982). A teacher may rely on a range of instructional methods that could fall anywhere along the constructivist continuum based on the content being

taught, and the experience, pedagogical and content knowledge of the teacher. Regardless of how constructivism is viewed at the theoretical level, practically, methods that rely on constructivist ideas (i.e. CRA sequence) have been shown to be effective method for learning mathematical content for students with disabilities and their nondisabled peers (Gersten et al., 2009; Witzel, 2005).

Rational for Proposed Study

In order to attend a competitive university or enter into many skilled professions, students are required to pass algebra as a pre-qualifying requirement. Despite algebra's importance, many students enter their first year algebra course with an inadequate understanding of fundamental topics necessary to develop a coherent, conceptual understanding of the course (Asquith et al., 2007; McNeil et al., 2006; Booth, 1988; Schoenfeld, & Arcavi, 1988). To prepare students with MD for success in Algebra, instructional interventions addressing fundamental topics, such as equality and solving equations are needed. Currently, no studies at the middle school level examine interventions that address the topic of equality and only one of the three studies that addressed solving equations contained students with MD. To develop competency in solving equations, student need to develop a conceptual understanding of the equal sign, specifically that it means "the same as" along with both procedural and conceptual knowledge of solving equations. Additionally, these content skills should be developed using methods that support development of the mathematical practices standards outline by the CCSS (2010).

The proposed study will examine the effects of and instructional package on the performance of middle school students with MD when presented with tasks involving

equations. Explicit/systematic instruction will be used to teach the relational meaning of the equal sign while visual representations will be used to teach equations. Constructivist theory as outlined by Noddings (1990) will provide the theoretical framework. The intervention will include components of instruction found to be effective in this review and include methods to develop the standards for mathematical practice alongside the content knowledge.

Chapter 3: Methodology

Through the current inquiry, the researcher sought to develop a research-based method to help students with (MD) develop the foundational skills necessary for success in algebra and other higher mathematics courses. The researcher designed this study to expand the general education and special education literature on using blended instruction with representations and graphic organizers to teach students how to solve equations and understand the equal sign. The study focused on supports and methods shown to be effective for all students, including those with MD. These approaches included the use of a) explicit/systematic instruction, b) components of concrete, semi-concrete, and abstract strategies, and c) graphic organizers. The content addressed in the study aligned with the CCSS for content and mathematical practice. Specifically, the study addressed: a) the equal sign as a relational symbol and b) solving algebraic equations. Figure 1 depicts the conceptual model for the study and shows how the researcher adjusted instructional components based on participant characteristics to ensure students demonstrated improved mathematics performance.

The researcher utilized a single-case design, as it “provide[s] a rigorous experimental evaluation of intervention effects” (Kratochwill et al., 2010, p. 2) when there are not enough participants to utilize a traditional group design with adequate statistical power (Odom et al., 2005). Single-case design studies also examine specific learner characteristics that may contribute to their response or non-response to the intervention (Kratochwill et al., 2010). This chapter will provide a description of a) the research participants and setting, b) the instructional package and materials, and c) the experimental design (i.e. single-case design) and study procedures.

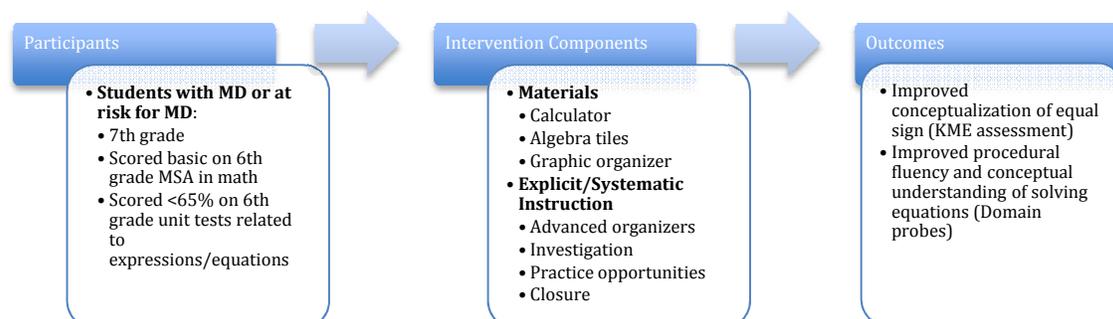


Figure 1: Conceptual model for the instructional package.

Participants and Setting

This section details the criteria used during the participant selection process and provides a description of the setting for the study. The section also includes the procedures for obtaining Institutional Review Board (IRB) approval and informed consent from participants and their guardians.

Participant eligibility. Participants were all seventh grade students at the target school who had learned about equations in the sixth grade. The timing of the study ensured that there was time to intervene before the students began to study equations in more depth as a part of the seventh grade curriculum. To take part in this study, participants had to demonstrate math difficulties (MD) in Grade 6 mathematics, specifically in the areas of algebraic concepts, expressions, and equations. The researcher selected participants based on the following criteria: 1) having a score of basic on the 2013 Grade 6 MSA exam, and 2) scoring below 65% on two sixth grade unit tests

administered during the 2012-2013 school year that related to equations/expressions and equations/inequalities that aligned with the CCSS. These participants also took an initial assessment related to the content of the intervention; however, the researcher did not exclude any students from the initial sample because none of them scored higher than a 65% on the assessment. Although 18 students met the criteria for inclusion in the study, the researcher removed one participant from the analysis because he missed five of the 10 lessons during the intervention. Table 3 provides demographic data for the student participants; including their gender, race/ethnicity, age, disability status, English for speakers of other languages (ESOL) status, and their eligibility for free and reduced meals (FARMS).

Table 3:
Demographic Information

Name*	Gender	Race / Ethnicity	Age (years)	Disability status	ESOL status	FARMS
Bryan	Male	White	12	-	-	Yes
Camila	Female	Hispanic	13	-	Released	Yes
Charissa	Female	Black	12	SLD	-	Yes
Daniel	Male	White	13	ADHD	-	Yes
Damion	Male	White	12	-	-	-
Justin	Male	White	12	SLD	-	Yes
Heather	Female	White	13	-	-	Yes
Harry	Male	White	12	-	-	-
Hyokwon	Male	Asian	13	SLD	Yes	Yes
Jessica	Female	Hispanic	12	-	-	Yes
Kevin	Male	Hispanic	13	-	Yes	Yes
Nick	Male	White	14	-	-	-
Jonah	Male	White	12	Autism	-	-
Pablo	Male	Hispanic	12	-	Yes	Yes
Ramona	Female	Hispanic	13	SLD	Yes	Yes
Sarah	Female	White	13	-	-	-
Sadie	Female	Hispanic	12	-	-	Yes

*All student names have been changed to maintain confidentiality

SLD = specific learning disability; ADHD = attention deficit with hyperactivity disorder; FARMS = Free and reduced meal student; ESOL = English for speakers of other languages

Instructor and setting. The researcher and other teachers at the target school site implemented the intervention. The researcher also preplanned all instructional lessons and assessments prior to the start of the intervention and facilitated the administration of the assessments used for data collection. The researcher and teachers had a pre-existing familiarity with the participants from occasionally assisting the students in their sixth grade math class during the 2012-2013 school year. The researcher also taught a majority of the participants in their regularly scheduled seventh grade co-taught math class for the 2013-2014 school year and had all of the participants during a homeroom period.

Teachers received copies of the lesson about a week prior to starting the intervention to review and had the opportunity to ask any questions they had prior to teaching the lesson. The two non-math teachers asked for a demonstration of how to use the instructional materials for one of the lessons. After seeing the example they felt ready to implement the lessons. The teachers did not have to meet any predetermined criteria in order to teach the intervention as they served as typical intervention agents who might implement the curriculum without extensive training.

The study took place at a public school in Maryland. The researcher collected baseline and post intervention data during homeroom; however, participants did not receive instruction related to mathematics at this time. During the intervention, the researcher removed participants from an elective class and provided instruction to a small group of 3 – 4 students. The intervention took place either in the room in which students received their regular mathematics instruction or in another room familiar to the students that had identical equipment and materials. The research team needed the two rooms

because they administered the intervention to two groups simultaneously. A smart board, document camera, and white board were available for use during instruction.

Internal Review Board (IRB). Prior to the beginning of the study, the researcher submitted a research proposal for approval by the IRB at the University of Maryland, College Park. Plans were also received approval from the Research Office of the target school district.

Informed consent. Participants and their guardians received a letter that detailed the purpose of the study, the content the study addressed, the data the researcher sought to collect, and the risks and benefits of the study. As a teacher in the school where the study took place, I had access to each participant's educational records; however, I sought permission from parents to include (anonymously) relevant data from these files in any papers or presentations stemming from the study. The letter also informed participants and their guardians that students could withdraw from the study at any time for any reason without ramifications, although none did. To participate in the study, the participants had to sign the Student Assent Form, and their guardians had to sign the Parent Consent Form (see Appendices A, B, & C).

Instructional Materials

This section provides an overview of the instructional materials used during the intervention to develop students' understanding of the equal sign and solving linear equations. The researcher developed lesson plans for the unit and utilized research-based supports that have shown promise for helping students with MD access mathematics.

Manipulative materials. The researcher used algebra tiles as a concrete manipulative during this study. These tiles included square and rectangular pieces that

represented units (1), a variable (typically x), and the variable squared (typically x^2). The tiles helped the researcher model the abstract symbolic manipulation of number and variable used when solving linear and quadratic equations. For the purpose of this intervention, the researcher only used the unit and variable tile.

Graphic organizer. Participants initially received an outline of a graphic organizer to complete before having to develop their own graphic organizer. The initial graphic organizer consisted of a linear flow map that modeled a sequential process. The flow map helped students organize their work and show that they were doing the same operations to both sides of the equation while trying to isolate the variable (see Appendix D for an example of the graphic organizer).

Calculator. The researcher allowed participants to use a graphing calculator throughout the intervention. The calculator served as a tool that students could use as necessary so they would not be penalized for poor computation skills.

Instructional Unit and Lesson Plans

The researcher developed lesson plans with objectives that aligned with the CCSS for mathematics content and practices (Common Core State Standards, 2010). The goal of the unit was to help students develop a relational understanding of equality and conceptual knowledge and procedural fluency when solving one and two-step linear equations (see Appendix E for alignment to CCSS content). In addition, the intervention addressed the following CCSS for mathematical practices: making sense of problems, persevering, reasoning abstractly and quantitatively, constructing viable arguments, and reasoning with others, strategically using tools, attending to precision, and making use of structure (see Appendix F for descriptions). The researcher incorporated practices shown

to promote mathematics achievement for students with MD into the instructional design and delivery. These instructional practices included components of explicit/systematic instruction, concrete manipulatives (algebra tiles), and graphic organizers. Although the study took place outside of the participants' regular Grade 7 math classes, the researcher designed the unit to allow for implementation in small groups or whole class instruction within an inclusive classroom.

Single-Case Design

As mentioned above, the researcher utilized a single case design for this study. A single case design involves the repeated, systematic measurement of the dependent variable through phases of active manipulation of an independent variable in an attempt to establish a functional relationship between the two components (Kratochwill et al., 2010). Researchers primarily use single case designs (SCD) when it is difficult to include the large numbers of participants necessary to achieve the statistical power required of traditional group designs (Odom et al., 2005). Although there are many variations of SCD, a common feature is the use of "cases" that consist of an individual or group of participants that serve as a unit of data analysis. Each case serves as its own control during data collection before, during, and/or after intervention (Horner & Odom, 2013; Kratochwill et al., 2010).

One benefit of using an SCD is that it focuses on the individual and allows for a detailed analysis of participants who respond or do not respond to the intervention (Horner et al., 2005; Kratochwill et al., 2010). Given that students with MD do not necessarily share the same defining characteristics (Geary, 2004; Little, 2009; Maccini, Mulcahy, & Wilson, 2007), group designs may miss important participant characteristics

that influence their response or non-response to the intervention (Kratochwill et al., 2010). Compared to group designs, SCDs are also more “practitioner friendly,” due to the lower number of participants required for teachers to conduct research in their own classrooms (Wolery & Gast, 2000). These benefits (e.g., small number of participants, focus on individual, easy replication by teachers) strongly influenced the researcher’s decision to utilize an SCD for this study. The following sections will describe the experimental design, independent variable, dependent variable, and data analysis procedures used in this study.

Experimental design and study procedures

The researcher used a concurrent multiple probe design across three groups replicated across three other groups. Each group consisted of three students (see Figure 2 for an example of the design). Given the academic nature of the intervention, continual use of probes for students during baseline would have been impractical (Kazdin, 2011), and it was unlikely that a change in performance would occur until the intervention was introduced (Horner & Baer, 1978; Richards, Taylor, & Ramasamy, 2014). Therefore, a multiple probe design was preferable over a multiple baseline design. Additionally, reducing the number of probes required of participants during baseline reduced the threat to internal validity caused by repeated testing (Kazdin, 2011).

The researcher taught the first section of three groups (A, B, C) while other teachers replicated this instruction for a second section of three groups (W, X, Y; see the *Fidelity of Treatment* section for details on how groups received instruction in a consistent manner). The second section provided an opportunity for replication across participants and instructors, which enhanced the external validity of the results (Horner et

al., 2005). Additionally, external validity increases with the random assignment of participants to groups, groups to instructors, and interventions to groups (Horner et al., 2005). As shown in Figure 2, the design consisted of two phases, baseline and intervention, with the independent variable having a staggered introduction across groups

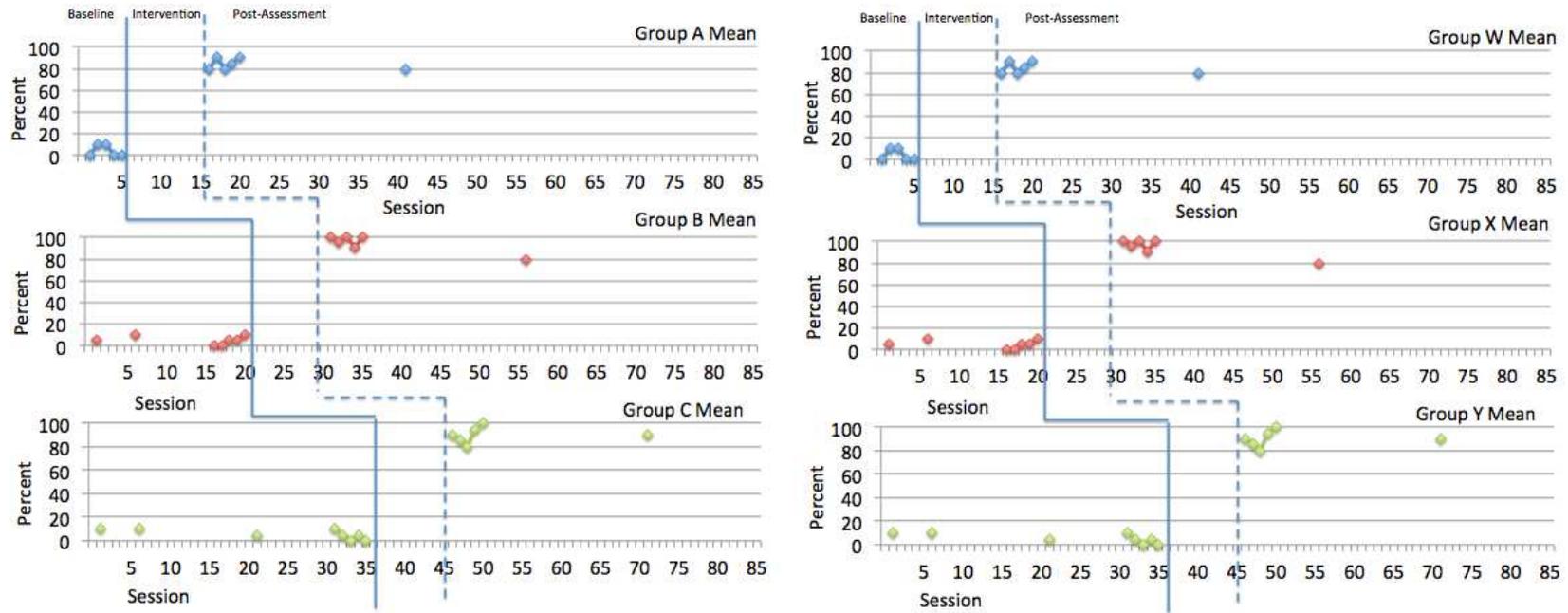


Figure 2: Example of design. Groups A, B and C represent section one and were instructed the researcher. Groups W, X, and Y represent section two and were instructed by teachers. Each point represents the group mean on the given domain probe.

(Horner & Odom, 2013). The staggered introduction occurred by collecting a baseline series of data (consisting of at least five data points) for all groups, and then introducing the intervention to the first group. After the first group reached a predetermined criteria, the researcher introduced the next group to the intervention. This process continued until at least three groups completed the intervention and met the required criteria (Horner & Odom, 2013). The minimum number of replications (i.e., participants) needed for a multiple probe design is three (Kratowill et al., 2013).

During the probe sessions, students received a pencil, paper, algebra tiles, and a calculator. Each of the probes had written directions that the probe administrator read verbatim. This process helped improve the fidelity of the implementation and ensure the validity and reliability of the measures. All members of the researcher team responded to participants' questions with a standard response, "Just do the best you can." The researcher then collected the probes for scoring. The research team introduced the intervention to the first two groups (Groups A and W) simultaneously after baseline data for all members within that group reflected stability in both level (i.e., at or below 65% accuracy for at least three data points with little variability in scores) and trend (i.e., did not indicate an increase in scores over time).

The groups remaining in the baseline condition (B, C, X, Y) received an additional domain probe on the first day of intervention for Groups A and W (see Figure 2). The researcher gave probes at this point to insure that there were no external factors acting on the groups in baseline that might have affected the dependent variable. At the conclusion of the intervention, Groups A and W began post-assessment probes, while the next groups (B and X) began a series of five baseline probes. Once Groups B and X

reached a stable baseline, they entered intervention; and the remaining groups (C, Y) received a domain probe on the first day of intervention for Groups B and X. This cycle continued until all groups finished the intervention and post assessments.

Baseline phase. Throughout the duration of the study participants remained in their regular math classroom where they covered the topics of proportions, rational numbers, and integers. Students received no instruction related to solving equations or the relational nature of the equal sign until after the conclusion of the study. The researcher had participating students in his homeroom and for their regular mathematics instruction, as he is a teacher in the school where the intervention occurred. The researcher was not, however, responsible for planning instruction or grading during the study period. All participants were in the researcher's homeroom and completed the baseline assessment measures during that time. No feedback was provided to students after completing the probes. If a student asked how she did, she was told she could see her results at the end of the study. The assessment sessions were video recorded for use during fidelity checks.

Dependent variables and measurement procedures. The primary dependent measures consisted of domain probes developed by the researcher, along with curriculum-based measurements (CBM) pertaining to solving one- and two-step equations. Additionally an assessment of knowledge of mathematical equivalence (KME) (Rittle-Johnson et al., 2011) was used as a pre- and post-test to measure changes in how students performed on tasks involving the equal sign.

Domain probes. During the baseline condition, participants received a minimum of five randomly-chosen parallel versions of the domain probe administered across five sessions. Domain probes measured content sampled from all of the objectives across the

instructional unit. The researcher established content validity by having two experts in the field of mathematics special education review the probes to determine if they contained the same content and were of the same level of difficulty. The domain probes helped the researcher a) establish a baseline level of performance prior to the intervention that included at least five data points (Kratochwill et al., 2010), b) establish a level of performance after intervention, and c) determine maintenance of performance four to six weeks after the intervention. See Appendix G for an example domain probe.

Knowledge of mathematical equivalence (KME) probes. The researcher also administered the KME measure to all participants (Rittle-Johnson et al., 2011) to assess their understanding of the equal sign. The KME assessment detects systematic changes in students' knowledge of equivalence across elementary grades (see Appendix H; Rittle-Johnson et al., 2011). While the domain probes focuses more broadly on application and use of the equal sign in a procedural manner, the KME assessment measures how a student conceptually views the equal sign itself. The assessment draws from a construct map that has four continuous levels of understanding equivalences proceeding from operational to comparative relational understanding (see Appendix I). The assessment built upon prior research on mathematical equivalence, with each of the four levels incorporating three classes of research-supported items involving solving equations, evaluating the structure of equations, and defining the equal sign. The two versions of the assessment had high reliability and validity, as reported in Appendix J (Rittle-Johnson et al., 2011). The researcher administered this probe once to participants during baseline and once following completion of the intervention.

Intervention phase. During the intervention, the researcher removed students

from an encore class (e.g., gym, art, music, etc.) in groups of three to receive instruction (independent variable). The researcher provided all instruction to the three groups in the first section (Groups A, B, C). Three additional teachers provided the intervention materials and instruction for one group in the second section for Groups W, X, Y. The research team staggered the intervention across the groups, with each group in a section entering intervention after the conclusion of the post-test probes from the previous group and the demonstration of a stable baseline by the group entering intervention, as described above and in Figure 2.

During the intervention phase, participants received instruction based on the lesson objectives outlined in the unit plan (see Appendix K). The group of participants advanced through the sequence of one-step equations and did not advance to two-step equations until over half of the group scored at least 70% on the objective probes by the end of Lesson 6. These objective probes were end-of-lesson exit tickets used for formative purposes and not included in the graphs. The intervention concluded after the participants received instruction on all of the objectives. After the conclusion of the intervention, the researcher administered parallel versions of the domain probes over the next five consecutive school days during homeroom period. The participants that just concluded the intervention also received Form B of the equal sign measure; and all participants, including those in baseline, received parallel versions of the CBM probes during this five-day period. Four-to-six weeks after the conclusion of the intervention, participants also completed a long-term maintenance domain probe.

Independent variable. In a SCD, the researcher must actively manipulate the independent variable to document experimental control (Horner et al., 2005). In this

study, the independent variable (i.e. the instructional package) integrated features of instruction identified by the special education research community and aligned with the CCSS for mathematical practices. The independent variable incorporated components of explicit/systematic instruction, along with manipulatives and graphic organizers, to help students develop a relational understanding of the equal sign and conceptual knowledge and procedural fluency when solving one and two-step linear equations. The following sections describe components of the instructional package.

Instructional procedures. The instructional package incorporated critical practices identified by practitioners and researchers in the special education and math education fields. Specifically, the lesson plans included the following components:

- *Advanced organizers*, which included lesson objectives, reviews of pre-requisite skills, and links to current lesson topics;
- *Investigation*, which included a teacher demonstration or facilitation of a new task using instructional practices and materials from the special education and mathematics education literature;
- *Practice opportunities*, including guided and independent practice and collaboration with peers; and
- *Closure*, which consisted of a review of the objective through questioning and discussion and a brief formative assessment of the skills taught.

Throughout the lessons, the instructor acted as a facilitator by guiding participants towards concepts and procedures through questioning and group discussion. For example, when teaching the procedures for solving a one-step equation, the instructor showed participants a pan balance and demonstrated how one must add and subtract various

quantities to both sides to maintain balance. After a few examples, the instructor asked, “What will happen if you add or subtract from one side only?” and “What must happen for an equation to stay the same when isolating the variable?” Additionally, when students arrived at a solution, the instructor prompted them by stating, “Explain to your partner how you got your answer” and “How can you check to know your solution is correct?” The instructor provided these prompts for correct, partially correct, and incorrect responses. The research team also provided explicit instruction using think-alouds and teacher modeling to students who could not rectify errors through questioning and discussion.

The researcher embedded the Common Core Standards for Mathematical Practice (CCSMP) throughout the lessons, and in many cases, students engaged in multiple practices simultaneously. For example, students took part in discussions with their peers to defend and critique strategies (CCSMP #3) used while working with the manipulatives (CCSMP # 2) and graphic organizers (CCSMP #5). The lessons all targeted the appropriate use of the equal sign, which the CCSMP explicitly state are one of the skills that characterizes students who are proficient at attending to precision (CCSMP #6).

Blended instruction. Instructors taught lesson objectives using blended instruction, which incorporated elements of explicit/systematic instruction, concrete-semi-concrete-abstract instruction, and graphic organizers (see Appendix L for a sample lesson plan). The following paragraphs provide an overview of each lesson plan component used in the intervention.

First, an advanced organizer provided students with the objective and review of prerequisite skills as a warm-up and link to the current topic. For example, for the

introduction lesson on solving one-step equations, the teacher read the day's objective, and students reviewed the previous day's lesson on the comparison symbols as a warm-up to the new lesson. After reviewing the answers to the warm-up, the teacher presented students with an equation that was similar to the statements in the warm-up, but that replaced one of the numbers with a variable. The teacher asked students to discuss the differences in the equations and then explain how to decide what the variable should equal to make the equation true.

Second, the students engaged in a teacher-led investigation that included teacher modeling and think alouds, along with prompts, questioning, and peer discussions, to help maintain engagement. The lessons also incorporated explicit sequencing, where the instructor presented the content in concise instructional pieces that built in a systematic way towards the full concept (Scheuermann et al., 2009). For example, in the introduction to solving one-step equations, the instructor showed students a balance scale and explained that it's purpose (i.e. to compare different weights). The instructor then asked questions like, "What needs to happen in order for the scale to stay balanced?" and "What will happen if you add or subtract from one side only?" The lesson progressed as students learned to solve one-step equations that involved addition with manipulatives. Once they acquired that skill, the instructor moved to solving one-step equations involving subtraction with manipulatives.

Third, the students engaged in practice opportunities in lessons that were teacher-guided, independent, or cooperative. In each of the lessons, the instructor encouraged or required the use of manipulatives or graphic organizers. The instructor modeled how to solve an equation abstractly using algebra tiles while the students used their own algebra

tiles to follow along. During the guided practice section of the example lesson, the students and teacher worked through the example $4 + x = 9$ as the teacher asked probing questions like, “How can the variable be isolated?” and “How can you check to make sure your answer is correct?” Students then completed several examples independently while the teacher monitored their progress during the independent practice session.

Lastly, the students engaged in a closure activity where the teacher read the objective and asked students to share with a partner how the activities of the lesson helped move them towards meeting the objective. After this discussion, the students took a short formative assessment on the content covered. For instance, in the example lesson, students had to choose an equation to solve and provide a written description of the processes involved with determining a solution.

Visual representations/organizers. The instructional package included manipulatives (i.e. algebra tiles) and graphic organizers. Special education researchers and the CCSMP recommend using multiple representations of mathematical concepts. Researchers have identified the use of manipulatives, particularly in the concrete-semi-concrete-abstract instructional strategy, as an effective strategy for teaching algebraic skills and concepts like integers (Maccini & Hughes, 2000; Maccini & Ruhl, 2000), linear equations (Witzel, 2003), and quadratic equations (Strickland & Maccini, 2013). Additionally, graphic organizers have shown promise as an instructional aid and a tool for students to use while working with systems of equations (Ives, 2007) and quadratics (Strickland & Maccini, 2013). In the present study, students used manipulatives to model how to solve one- and two-step equations and transitioned to using a graphic organizer (see Appendix D).

Inter-assessor reliability. The researcher attained inter-assessor reliability for at least 20% of the data points from the domain probes in each of the phases to monitor the reliability in the measurement of the dependent variable (Kratochwill et al., 2010, 2013). A teacher in the target school independently scored the domain probes and equal sign measure. The teacher received mock probes and an answer sheet to practice scoring, and completed training until (HE/SHE) scored a minimum of three mock domain probes and three mock equal sign measures with at least 90% agreement with the researcher.

The researcher measured inter-assessor agreement by comparing the scores of the two assessors and calculating the percentage obtained by dividing the number of actual agreements by the number of possible agreements (O'Neill et al., 2011), with the goal of obtaining a minimum acceptable value of agreement of 80% (Hartman, Barrios, & Wood, 2004).

Fidelity of treatment. Fidelity of treatment refers to how well the instructors implemented the intervention in comparison to the original design of the intervention (O'Donnell, 2008; Swanson, Wanzek, Haring, Ciullo, & McCulley, 2012). Fidelity of treatment is a concern in a SCD study, because the independent variable is implemented over time, which increases the chance of variation in the implementation. As a result, the researcher must document and record fidelity of implementation during each phase of the study (Horner et al., 2005). For the current study, two independent observers (i.e., trained graduate students or teachers) reviewed the video-recorded lessons for treatment fidelity using a checklist (see Appendix M) containing critical features of the intervention (O'Donnell, 2008). The independent observers also reviewed video-recorded assessment sessions to evaluate fidelity using a checklist (see Appendix N). The researcher trained

the independent observers by explaining the checklists and reviewing the written lesson plans, assessments, and recorded sessions. The independent observers completed their training when they obtained at least 90% agreement with the researcher on three selected intervention sessions.

Two independent observers conducted fidelity observations on 33% of the instructional and assessment sessions using video recordings of the sessions (Kennedy, 2005). The researcher calculated fidelity by dividing the number of observed components by the number of total possible components and multiplying the result by 100 (O'Neill, et al., 2011). The researcher determined inter-observer agreement by comparing the results of the two observer's fidelity checks and calculating the percentage of actual agreements out of the number of possible agreements (O'Neill et al., 2011). The goal was to obtain a minimum mean interobserver agreement of 80% across all observations for both treatment and assessment conditions.

Data analysis procedures. For this study, the researcher collected and graphed data continually for each individual group. The researcher averaged the scores earned by the participants on each domain probe and plotted the mean for the group.

Data analysis in an SCD involves visual analysis of linear data graphs that compares the effects of the intervention to the performance during the baseline condition and/or other treatment phases (Horner et al., 2005; Horner & Odom, 2013; Kratochwill et al., 2013). Conducting a visual analysis involves looking at the graph as a whole and considering four steps and six features of the outcome measure (Kratochwill et al., 2013). The four steps include: 1) demonstrating a stable, predictable baseline pattern of data; 2) assessing within-phase data to decide if a predictable pattern of responding has occurred;

3) comparing data from adjacent phases to determine whether the independent variable can be associated with an effect; and 4) determining if there have been at least three demonstrations of effect at different points in time (Kratochwill et al., 2013). The researcher assessed each of the six features described in Table 4, individually and collectively, to decide if a causal relation existed between the independent and outcome variables (Kratochwill et al., 2013).

Table 4
Six Features of the Outcome Measure

Category	Feature	Description
Within-phase	Level	Mean of an outcome measure within a phase
	Trend	Slope of the best-fitting line within a phase
	Variability	Range, variance, or standard deviation of outcome measure around the trend line; degree of overall scatter
Between-phase	Immediacy of the effect	Change in level between last three points in one phase and first three points in following phase
	Overlap	Proportion of data that overlaps from one phase to the following phase
	Consistency of data patterns across similar phases	Reviewing data from all similar phases within the same condition

Source: Kratochwill et al., 2013

In addition to visual analysis, there are several statistical analyses that researchers can use to determine an effect size. Researchers have applied ordinary least squares regressions analysis to SCD data; however, because it is a parametric statistical test, certain necessary assumptions may prove difficult to achieve with the SCD data (Parker,

Vannest, Davis, & Sauber, 2011). Newer statistical tests like the Tau-U (Parker et al., 2011) and the improvement rate difference (IRD; Parker, Vannest, & Brown, 2009) were specifically designed for use with SCD data and have shown promising results in determining effect size (Parker, Vannest, & Davis, 2011). The current study used these two methods, Tau-U and IRD, to calculate effect size which allowed for comparisons over multiple estimators (Kratochwill et al., 2013).

Social validity. Social validity refers to the social acceptance of an intervention's importance, effectiveness, and appropriateness (Carter, 2010; Foster & Mash, 1999). Horner et al. (2005) suggested that researchers could enhance the social validity of their research goals by using design features that included (a) dependent variables with high social importance; (b) independent variables that are applied with fidelity by typical intervention agents; (c) procedures that are acceptable, feasible, and effective as reported by typical intervention agents; and (d) an intervention that meets a defined need.

The researcher received positive feedback on the proposed intervention from personnel at the target school, including the principal, special education department chair, mathematics department chair, and classroom teachers, which helped increase the social validity prior to implementation. At the conclusion of the study, participants completed an researcher-developed survey (see Appendix O) based on other social validity measures (Mulcahy, 2007; Strickland, 2011), which assessed students' perceptions of the intervention and its design features (i.e. manipulatives and graphic organizers). Additionally, teachers who implemented the intervention in section two completed a survey about their thoughts on the intervention (see Appendix P).

Chapter 4: Results

In this chapter, I present the results of the study. I have organized the chapter using the following five research questions that guided this inquiry:

1. To what extent do students with mathematics difficulties who receive instructional intervention on the relational nature of the equal sign and solving one- and two-step equations have increased accuracy when completing algebraic tasks involving the equal sign?
2. To what extent do students with mathematics difficulties maintain performance on algebraic tasks involving the equal sign four-to-six weeks after the conclusion of the intervention?
3. How do students conceive of the equal sign prior to intervention and are there changes in those conceptions post intervention?
4. To what extent do middle school students with mathematics difficulties consider blended instruction with visual representations and graphic organizers beneficial (i.e., social validity)?
5. To what extent do middle school teachers consider blended instruction with visual representations and graphic organizers a viable intervention strategy?

I will begin the chapter by addressing those questions directly related to the single subject design (Research Questions 1 and 2) and will follow with data relating to the pre-test and post-test equal sign measures (Research Question 3). I will conclude with the results of the social validity surveys (Research Questions 4 and 5).

Research Question 1 and 2: Increased Accuracy on Algebraic Tasks Involving the Equal Sign and Maintenance of Performance

The researcher measured students' increased accuracy on algebraic tasks involving the equal sign by monitoring their performance on domain probes. This section presents data from both the researcher-instructed and teacher-instructed groups, and then arrays the data for each individual participant.

Researcher-instructed groups. As shown in Figure 3, students in all researcher-instructed groups increased their overall accuracy on domain probes from an average of 9.8% during baseline to an average of 87.7% after the intervention. Specifically, the baseline scores ranged from 7% to 12%, and the post-intervention scores ranged from 76% to 96%, with an average increase of 77.9% points over baseline. Students completed a domain probe 4 – 6 weeks after the intervention, which measured their maintenance of performance, and all groups demonstrated a high degree of retention, with an average score of 81% (range = 72% - 91%).

The researcher analyzed graphs of the data using visual analysis to identify patterns within and between phases. All groups demonstrated a predictable stable baseline pattern prior to entering intervention, and a within-phase analysis showed a predictable pattern of responses for all groups with little variability. An analysis of between-phase patterns indicated an increase in level for all groups when assessed after the intervention. The aggregated Tau U effect size for the three groups was 1 (confidence interval .5964 <> 1.4036) meaning that 100% of the data from baseline to post-assessment did not overlap. This effect size was confirmed using the improvement rate difference (IRD) measure which also found an effect size of 1. Additionally, all groups maintained

the same level of performance when assessed 4 – 6 weeks after the intervention. Table 5 provides a summary of the data for each researcher instructed group.

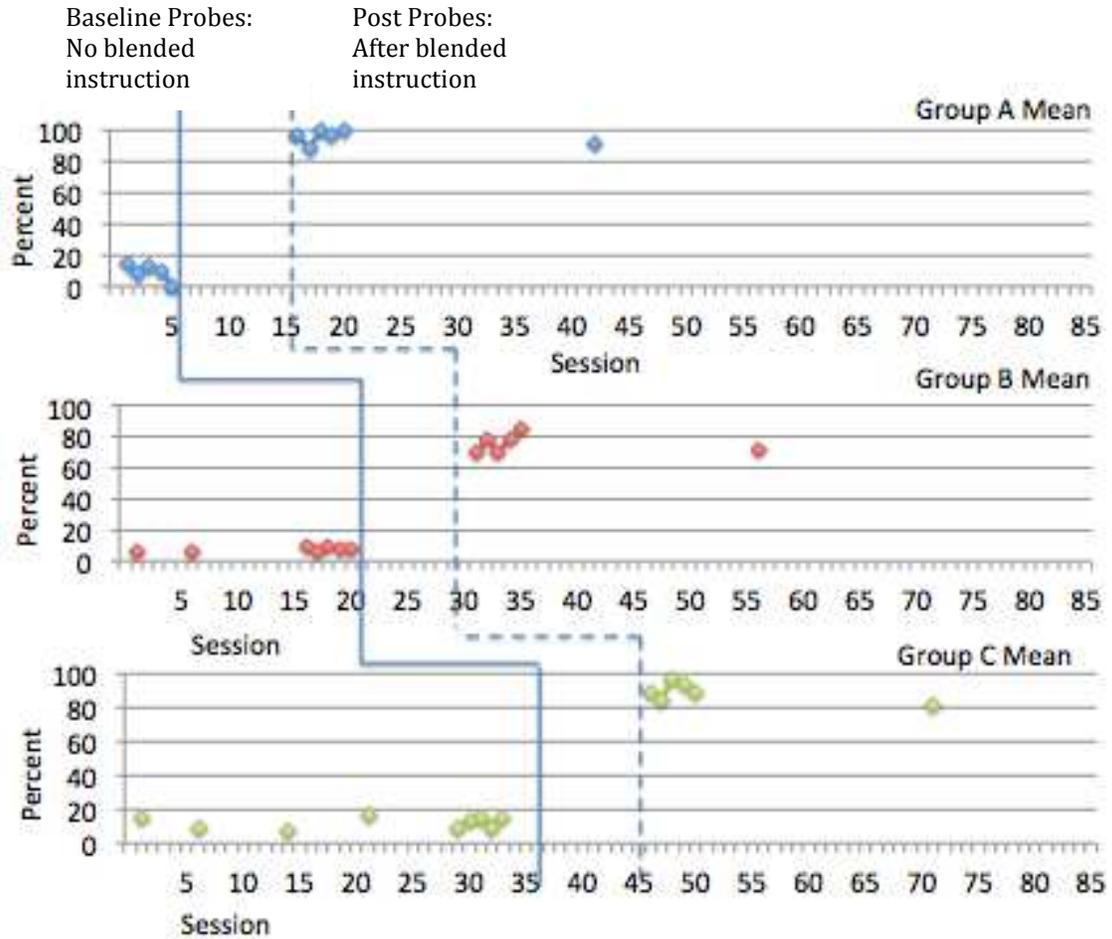


Figure 3: Domain probes for researcher-instructed groups with percent correct on the y-axis and the session number on the x-axis.

Table 5

Average Percentage of Accuracy & Increases in Percentages on Domain Probes for Researcher Instructed Groups

<u>Group</u>	<u>Baseline</u>	<u>Post-intervention</u>	<u>Increase</u>	<u>Maintenance</u>	<u>Tau U</u> <u>(90% CI)</u>
A	10% (r = 6% - 15%)	96% (r = 89% - 100%)	86% points	91%	100% (CI=0.37<>1.63)
B	7% (r = 6% - 9%)	76% (r = 70% - 85%)	69% points	72%	100% (CI=0.42<>1.58)
C	12% (r = 7% - 17%)	90% (r = 85% - 96%)	78% points	81%	100% (CI=0.45<>1.55)
Mean	9.6% (r = 6% - 17%)	87% (r = 70% - 100%)	77% points	81%	100% (CI=.60<>1.40)

Overall, an error analysis revealed that participants missed the most points for three reasons: a) they did not justify their solution when asked, b) they made a computational error, or c) they used the incorrect operation. Across the groups, students missed 47 of 90 possible points because they did not justify the accuracy of their solutions when asked to do so on two questions. Only one student justified his solutions correctly every time, and two students did not earn any points for justification. The remaining students missed an average of 5 points (range =1 – 9) because of this error.

The next most frequent mistake involved computational errors made by four students that resulted in a total of 12 missed points (e.g., $3 + 3 = 9$; $7 - 5 = 3$). Additionally, three students lost a total of 7 points because they used the wrong operation when determining the inverse of a number to solve for the variable. For example, when solving for a in Figure 4, the student correctly subtracted 7 from both sides of the equation $11 = a/4 + 7$ to get $4 = a/4$. The student then divided by 4 when he should have multiplied by 4 and arrived at the incorrect solution of $a = 1$.

$$\begin{array}{l}
 \boxed{11} \\
 \downarrow \\
 \boxed{-7} \\
 \downarrow \\
 \boxed{4} \\
 \downarrow \\
 \boxed{-} \\
 \downarrow \\
 \boxed{1}
 \end{array}
 =
 \begin{array}{l}
 \boxed{\frac{a}{4} + 7} \\
 \downarrow \\
 \boxed{-7} \\
 \downarrow \\
 \boxed{\frac{a}{4}} \\
 \downarrow \\
 \boxed{-4} \\
 \downarrow \\
 \boxed{a}
 \end{array}$$

Figure 4: Example of an error where the student did not use the correct inverse operation when trying to isolate the variable.

Teacher-instructed groups. As shown in Figure 5, all teacher-instructed groups increased their overall accuracy on domain probes from an average of 16.4% during baseline to an average of 68.0% after the intervention. Specifically, the baseline scores ranged from 16% to 17%, and the post-intervention scores ranged from 51% to 83%, with an average increase of 51.7% points over baseline. The aggregated Tau U effect size for the three teacher instructed groups was 1 (confidence interval .5964 <> 1.4036). This effect size was confirmed using the IRD measure which also found an effect size of 1. The students received a domain probe 4 – 6 weeks after the intervention to measure their maintenance of performance, and all groups demonstrated a high degree of retention, with an average score of 73% (range = 58% - 83%).

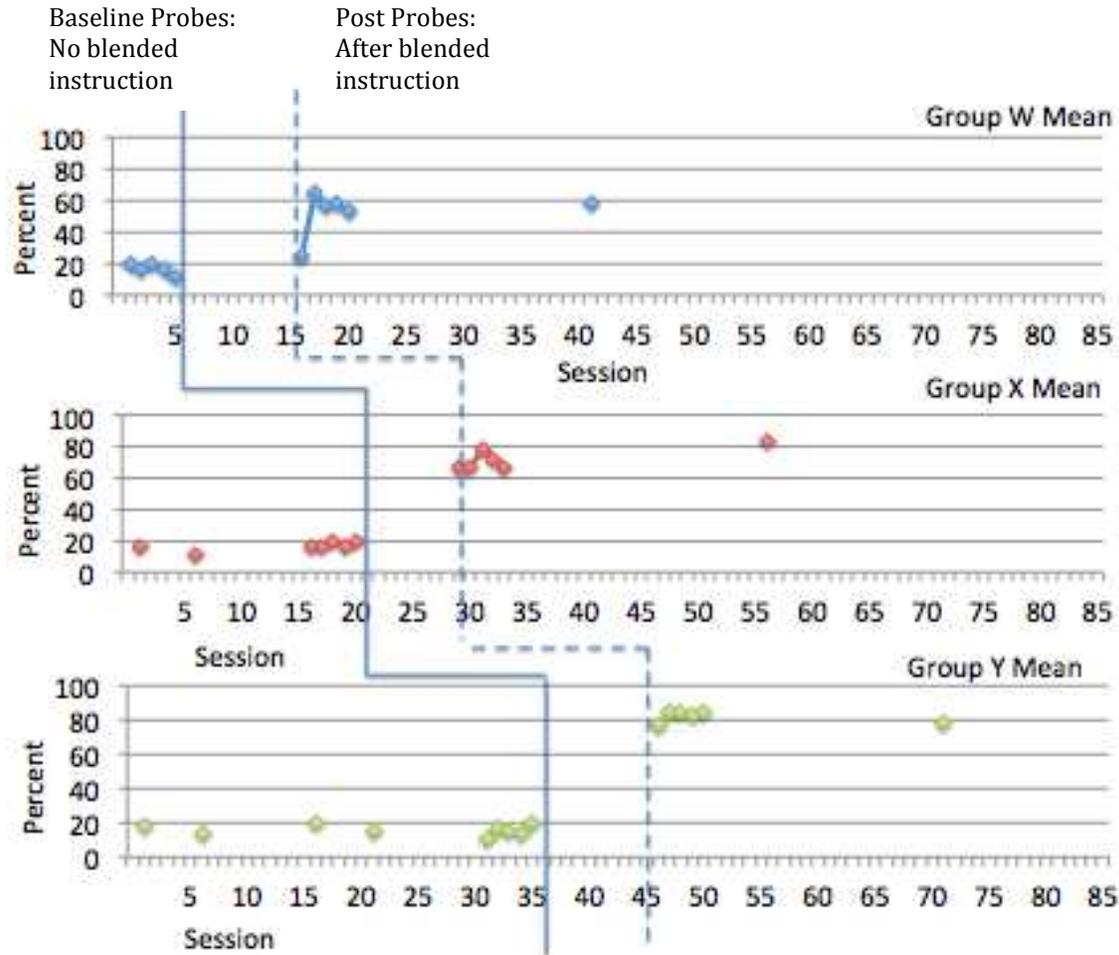


Figure 5: Domain probes for teacher-instructed groups (each group taught by a different teacher) with percent correct on the y-axis and the session number on the x-axis.

The researcher analyzed graphs of the data using visual analysis to identify patterns within and between phases. All groups demonstrated a predictable, stable baseline pattern prior to entering intervention. Within-phase analysis of both baseline and post-intervention phases showed a predictable pattern of responses with little variability for Groups X and Y; however, for Group W, the post-intervention Probe 1 was 33 percentage points lower than the average of the remaining four post-intervention probes. An analysis of between-phase patterns indicated an increase in level for all groups,

although the increase for Group W was not as large as the increase for the remaining five groups.

Additionally, all groups maintained the same level of performance when assessed 4 – 6 weeks after the intervention. Group X *appears* to have improved on the maintenance measure; however, the lowest achieving participant moved prior to taking the maintenance probe, which resulted in the *appearance* of a higher group score. Table 6 provides a summary of the data for each teacher instructed group.

Table 6

Average Percentage of Accuracy & Increases in Percentages on Domain Probes for Teacher Instructed Groups

<u>Group</u>	<u>Baseline</u>	<u>Post-intervention</u>	<u>Increase</u>	<u>Maintenance</u>	<u>Tau U (90% CI)</u>
W	17% (r= 11% - 20%)	51% (r= 25% - 65%)	34% points	58%	100% (CI=0.37<>1.63)
X	16% (r= 11% - 19%)	70% (r= 67% - 78%)	54% points	83%*	100% (CI=0.42<>1.58)
Y	16% (r= 11% - 20%)	83% (r= 76% - 85%)	67% points	78%	100% (CI=0.45<>1.55)
Mean	16% (r= 11% - 20%)	68% (r=25% - 85%)	52% points	73%	100% (CI=.60<>1.40)

Note. * *The lowest achieving participant moved prior to taking the maintenance probe, which resulted in a higher score.*

An error analysis revealed that, unlike the researcher groups, where errors were consistent across participants, the teacher groups had one participant in each group who made a unique and repeated error that inflated the number of errors for the groups. These participants' unique errors were not included in the teacher group error analysis and are discussed in the *Individual results* section. After the researcher removed these three unique individual errors, the data showed that participants missed the most points for two of the same reasons as the researcher-instructed groups—missing justification and

computational error. Across the groups, students missed a total of 54 of 80 possible points because they did not justify their solution when prompted on two questions. None of the students justified their solutions correctly every time, and three students did not earn any points for justification. The remaining students missed an average of 6 points (range = 4 – 9 points). Five students made computation errors, the next most frequent mistake, which resulted in a total of 27 missed points.

Individual results. The group data were replicated by all but one of the individual participants, as shown in Table 7 (researcher instructed groups) and Table 8 (teacher instructed groups). The outlying participant (Nick) was a part of Group X and did not demonstrate a meaningful change in level from baseline to post-assessment. Of the 17 participants, 15 (88%) demonstrated *clinically* significant gains. Clinical significance refers to the importance of the results, as interpreted by the individual, and the degree to which an intervention makes a meaningful difference in participants' everyday lives (Bothe & Richardson, 2011; Kazdin, 1999). In the present study, participants exhibited clinically significant gains when their scores progressed from failing to passing (i.e., above 60%).

Error analysis on the individual level showed that three individuals, one from each of the three teacher instructed groups, had systematic errors that were unique and not shown by other individuals. Jonah did not arrive at the correct solution for the one-step problems, although he correctly solved the two-step problems. When he used the

Table 7

Average Percentage of Accuracy & Increases in Percentages on Domain Probes by Individual Participant with Researcher as Instructor for all Groups

<u>Group</u>	<u>Participant</u>	<u>Baseline</u>	<u>Post Intervention</u>	<u>Increase</u>
A	Harry	11% (r = 0% - 22%)	99% (r = 94% - 100%)	88%
	Hyokwon	13% (r = 6% - 28%)	93% (r = 72% - 100%)	80%
	Kevin	6% (r = 6% - 6%)	97% (r = 89% - 100%)	91%
B	Charissa	10% (r = 6% - 17%)	68% (r = 56% - 78%)	57%
	Daniel	0% (r = 0% - 0%)	74% (r = 56% - 89%)	74%
	Pablo	11% (r = 6% - 11%)	87% (r = 78% - 89%)	76%
C	Justin	4% (r = 0% - 6%)	89% (r = 89% - 89%)	85%
	Ramona	10% (r = 6% - 11%)	93% (r = 89% - 100%)	83%
	Sadie	23% (r = 11% - 33%)	89% (r = 78% - 89%)	66%

Table 8

Average Percentage of Accuracy & Increases in Percentages on Domain Probes by Individual Participant with Different Teacher Instructor for Each Group

<u>Group</u>	<u>Participant</u>	<u>Baseline</u>	<u>Post Intervention</u>	<u>Increase</u>
W	Jonah	8% (r = 6% - 11%)	37% (r = 6% - 44%)	29%
	Sara	27% (r = 17% - 33%)	66% (r = 44% - 83%)	39%
X	Camila	32% (r = 11% - 33%)	87% (r = 83% - 89%)	54%
	Damion	8% (r = 0% - 11%)	80% (r = 67% - 89%)	72%
	Nick	21% (r = 17% - 28%)	43% (r = 33% - 56%)	14%
Y	Bryan	26% (r = 17% - 33%)	96% (r = 89% - 100%)	70%
	Heather	10% (r = 6% - 17%)	68% (r = 61% - 72%)	57%
	Jessica	11% (r = 6% - 22%)	86% (r = 67% - 100%)	74%

graphic organizers, Jonah solved the one-step problems as if they were two-step problems, which may have been due to the similarities between the organizers (i.e., the graphic organizers looked like the organizers used for two-step equations and not one-step equations). For example, given the equation $n - 4 = 2$, Jonah correctly added 4 to both sides and wrote $n = 6$. He then divided both sides by 4 and obtained $n = 1.5$ as his final answer.

Heather was able to solve approximately half of the two-step equations correctly. However, she made errors on the remaining problems by forgetting that the variable had a coefficient (8 problems) or incorrectly completing the process of inverse operations (9 problems). For example, given the equation $2y - 5 = 3$, Heather correctly added 5 to both sides and wrote $y = 8$ as her next and final step instead of $2y = 8$. In another problem, $8 + 5n = 23$, Heather wrote $\div 5$ as the step she was completing and then subtracted 5 and noted $n = 18$ as her answer. Her errors were not consistent, however, as she solved similar two-step equation problems accurately.

Lastly, Nick lost a point on 21 problems. On these problems he relied on facts or guess and check to arrive at the correct solution and showed no evidence of relying on inverse operations to solve the problem.

Research Question 3: Equal Sign Conception Pre and Post Intervention

The researcher administered the KME assessment to participant groups right before they began the intervention and approximately one week following the conclusion of the intervention. Although the participants completed the measure at different points in time corresponding to the multiple baseline design of the study, the researcher conducted the analysis as if the participants were a single group ($n=17$). The results for the total

score indicate there were significant differences between the pre-test ($M = 44.02$, $SD = 17.18$) and posttest ($M = 65.56$, $SD = 21.88$) scores on the KME measure; $t(16) = -5.37$, $p < .01$, $d = 1.09$. As shown in Table 9, there were statistically significant differences across all three subcategories of the KME measure with solving equations, recognizing structure, and defining the equal sign. With regards to the items associated with the level of understanding the equal sign, there was a significant difference from pre-test to post-test achievement scores on the items associated with Levels 3 and 4 only.

Table 9

Descriptive Statistics and t-test Results for KME Measure

Outcome	Pre-test		Post-test		n	95% CI for Mean Difference	r	t	Df	Cohen's <i>d</i>
	M	SD	M	SD						
Total Score	44.02	17.18	65.52	21.88	17	-29.99, -13.01	.66*	-5.37*	16	1.09
Solving	42.25	27.82	63.10	28.65	17	-33.52, -8.19	.62*	-3.49*	16	.74
Structure	38.46	19.23	62.35	22.23	17	0.55, 0.83	.19	-2.86*	16	1.15
Definition	62.35	22.23	89.41	18.86	17	-39.65, -15.03	.36	-4.77*	16	1.31
Level 1	94.12	13.10	92.16	14.57	17	-7.56, 11.48	.11	.436	16	-
Level 2	69.41	23.58	76.47	21.49	17	-21.11, 6.99	.27	-1.07	16	-
Level 3	44.92	31.90	72.19	29.19	17	-41.00, -13.55	.62*	-4.21*	16	.89
Level 4	17.65	14.20	42.48	27.84	17	-39.61, -10.06	.19	-3.56*	16	1.12

* $p < .01$

Research Question 4: Social Validity Students

The mean score from the participant social validity measure equaled 4.3 (range = 2 – 5; mode = 4.67; see Table 10). Most students agreed or strongly agreed that the intervention was worth their time (n= 15), and that they would recommend it to other students (n=16). Further, all students agreed that the intervention made them feel better about their math skills. The students mostly agreed (n=14) that the flow maps were helpful in solving two-step equations; however, one student strongly disagreed. Students' feelings about the use of manipulatives were mixed, with 12 students agreeing that algebra tiles were helpful and eight students noting that use of the balance scale helped them to remember that the equal sign means “the same.”

Overall, participants responded positively to the open ended questions from the social validity measure. For example, Hyun Woo said, “ I felt smarter,” and Jessica said, “It really helped me understand math with the problems I didn't get.” The majority of students (n=13) mentioned that the flow maps were what they liked most about the intervention. For example, Daniel said he liked “the flow maps because they help me set it up.” The most frequent component (n=8) that students reported that they liked *least* was the algebra tiles (e.g., “ confusing to use with all the moving of them,” “hard to understand”).

Table 10

Participants' Responses on Social Validity Measure

<u>Questions</u>	<u>Groups</u>						Mean
	A	B	C	D	E	F	
I am confident in my ability to solve one-step equations.	4.67	4.33	4.67	5.00	4.67	4.67	4.67
I am confident in my ability to solve two-step equations.	4.33	4.00	4.33	5.00	4.67	4.67	4.50
The use of the algebra tiles helped me to see what I was doing when I solved the equations.	3.67	4.00	4.33	4.00	2.00	4.00	3.67
The use of the flow map helped me to solve two-step equations.	4.67	4.33	4.00	4.50	3.67	5.00	4.36
The use of the balance helps me to remember that the equal sign means that both sides have to be the same.	4.33	4.00	3.33	3.50	2.00	3.33	3.42
Working with the teacher on this intervention was worth my time.	4.33	4.67	4.67	4.50	4.33	4.33	4.47
I would recommend this intervention to other students.	4.00	5.00	4.67	4.00	4.33	4.33	4.39
As a result of this intervention, I feel better about my math skills.	4.67	5.00	4.67	5.00	5.00	5.0	4.89

*Note: 1 – Strongly Disagree 2 – Disagree 3 – Neutral 4 – Agree 5 – Strongly Agree

Research Question 5: Social Validity Instructors

The mean score from the instructor social validity measure equaled 4.81 (range = 3 – 5; mode = 5; see Table 11). All of the instructors agreed that students should be proficient in solving equations and that the intervention was necessary to help students become successful in math. The instructors all agreed that the intervention was not difficult to implement and that participating was worth their time. Additionally, they agreed that they could teach the intervention within the regular classroom and within the normal constraints of the school day. One instructor provided a neutral response about whether typical special education teachers could teach the intervention.

Overall, the instructors responded positively to the open-ended questions from the social validity measure. All of the teachers also had positive responses to the variety of activities and methods, and all mentioned that the flow maps were especially helpful. For example, one teacher said, “I liked that there was a variety of activities throughout the intervention time span,” while another stated that “the flow maps were a great manipulative for the students.” One of the instructors commented that students did not like the algebra tiles, which “were difficult for the students to manipulate” and “caused a lot of frustration,” while another instructor stated that “the time span for the instruction and use of the models was quick.”

Table 11

Instructor Responses on Social Validity Measure

<u>Questions</u>	<u>Teacher 1</u>	<u>Teacher 2</u>	<u>Teacher 3</u>	<u>Mean</u>
The intervention was not difficult to implement.	4	4	5	4.3
The intervention would be able to be taught by typical math teachers.	5	5	5	5
The intervention would be able to be taught by typical special education teachers.	3	5	5	4.3
The intervention would be able to be taught by other intervention agents (i.e. PAM, etc.)	5	5	5	5
This intervention could be implemented within the regular classroom.	5	5	5	5
This intervention could occur within the normal constraints of school.	5	5	5	5
The intervention is needed to help students be successful in math.	5	5	4	4.6
Solving equations is an important topic for students to be proficient in.	5	5	5	5
I felt that it was worth my time to implement this intervention.	5	5	5	5

*Note: 1 – Strongly Disagree 2 – Disagree 3 – Neutral 4 – Agree 5 – Strongly Agree

Interrater reliability. The researcher attained interrater reliability for at least 20% of the data points from the domain probes in each of the phases (Kratochwill et al., 2010, 2013). As explained in Chapter 3, the researcher measured interrater agreement by comparing the scores of the two assessors and calculating the percentage obtained by dividing the number of actual agreements by the number of possible agreements (O’Neill et al., 2011). Initial reliability on domain and transfer probes was 98% (range = 94 – 100%), with a reliability of 100% following a discussion of differences. Disagreement occurred because of an error in scoring caused by confusion about when to assigning full or partial credit and an instance when the handwriting of the participant was difficult to read. After discussion, the assessors scored two additional probes in any set of domain probes where there was initial disagreement and achieved 100% reliability.

Initial reliability on the KME measure was 99.5% (range = 96.7% – 100%), but increased to 100% following a discussion of differences. Disagreement occurred on a question where one of the reviewers accidentally marked a solution correct when it was incorrect.

Treatment fidelity. According to an independent observer, the instructors and the researcher implemented the intervention as intended using 100% of the instructional components. A second independent observer viewed 18 lessons, three from each group, and obtained inter-observer agreement of 100%.

Assessment fidelity. According to an independent observer, the researcher implemented the assessments as intended and followed 100% of the assessment guidelines for both the domain probe and KME measures. A second independent observer viewed eight videos of the domain probe assessment sessions (35%) and two

videos of the KME assessment sessions (50%). The interobserver agreement was 100% for both assessment types.

Chapter 5: Discussion

The purpose of this study was to investigate how blended instruction with representations and graphic organizers affected the ability of students with MD to solve one- and two-step linear equations and understand the equal sign as a relational symbol. Overall, the intervention successfully helped participants learn both of these key mathematics skills. This chapter will begin with a summary of the research findings and a discussion of the importance of these findings in relation to the relevant literature. The chapter will then present an interpretation of the findings as they relate to the research questions, and will conclude by detailing the limitations of the study and the implications for research and practice.

Summary of the Results

Since 1989, when the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards for School Mathematics*, only 11 published research studies have examined the effects of an instructional intervention on students' ability to solve equations or understand the equal sign. Of these 11 studies, only four (Hattikudur & Alibali, 2010; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999) focused on developing students' understanding of the equal sign as a relational symbol, and all targeted students in the elementary grades. At the time of this study, no studies specifically examined an intervention or strategy for teaching the relational meaning of the equal sign concurrently with algebraic expressions in the middle grades. Additionally, in eight of the studies (Hattikudur & Alibali, 2010; Hutchinson, 1993; Ives, 2007; Mayfield & Glenn, 2008; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Star, 2009; Scheuermann et al., 2009), either the

researcher or graduate assistants conducted the intervention, and it is not clear how the results would have differed if classroom teachers had provided the instruction.

The current study addressed this gap in the literature by implementing a research-based instructional package that taught students the relational meaning of the equal sign while they learned to solve one- and two-step equations. This study was also the first to use a concurrent multiple-probe design, replicated across three groups, where the instructors for the replication groups were practicing classroom teachers.

The researcher developed the instructional package for this study after a review of the literature and incorporated strategies shown to be effective for both general education students and students with MD. These strategies included (a) explicit/systematic instruction (Gersten et al., 2009; Mayfield & Glenn, 2008; Powell & Fuchs, 2010; Scheuermann et al., 2009; Steedly et al., 2008; Swanson, 2001); (b) CSA instruction (Scheuermann et al., 2009; Witzel et al., 2003); and (c) graphic organizers (Ives, 2007). The instructional package blended these strategies while incorporating constructivist-based activities that promoted the following CCSMP: reasoning abstractly and quantitatively, using tools strategically, and looking for and making use of structure.

Emphasis on equations and equal sign. Several studies (Alibali et al., 2007; Essien & Setati, 2006; Godfrey & Thomas, 2008; Knuth et al., 2006; Knuth et al., 2008; McNeil et al., 2006) have indicated that, in general, middle school students have incomplete or incorrect conceptions of the equal sign as they progress into the secondary grades, yet no intervention studies have targeted this population. For some students and teachers, without a complete understanding of the fundamental meanings of the symbols used in expressions and equations, particularly the equal sign, algebra simply becomes a

series of procedures and steps to memorize, rather than an interconnected subject of concepts and ideas.

In the current study, students explicitly learned the relational meaning of the equal sign and then used that relational meaning in conjunction with the CSA sequence and graphic organizers to develop a conceptual understanding of the procedures for solving one- and two-step equations. This process aligns with recommendations that explicit instruction is beneficial for teaching a specific strategy or isolated skills (Kroesbergen & Van Luit, 2003) and for teaching mathematics generally to students with MD (Gersten et al., 2009).

Use of visual representations. The instructional package implemented in this study included the use of visual representations to model linear equations and the relational nature of the equal sign. The visual representations included a two-pan balance to model equations, concrete manipulatives (i.e. algebra tiles), a drawing of a balance, and a graphic organizer (abstract notation). The use of manipulatives when teaching mathematics has proven an effective strategy for both general education and special education students (Gersten et al., 2009), is in line with the CCSMP, and encourages students to model with mathematics and use tools strategically (CCSS, 2010). Additionally, two previous studies (Scheuermann et al., 2009; Witzel et al., 2003) identified the use of manipulatives and the CSA sequence as an effective strategy for teaching linear equations to middle school students with MD.

The current study replaced the semi-concrete step of the CSA sequence with a more integrative approach (Strickland & Maccini, 2013). Specifically, students first used the concrete manipulatives and then progressed to using the manipulatives while

simultaneously writing the abstract notation in a graphic organizer. The transition to abstract notation can be difficult for students with MD, as they may have trouble organizing information (Maccini et al., 2007) and typically have limited conceptual understanding of procedures (Geary, 2004). To assist with the transition, the researcher provided students with a graphic organizer in the form of a flow map, which helped students to organize the parts of an equation, provided structure as they solved equations abstractly while, and reinforced the idea of the equal sign as a relational symbol.

Use of blended instruction. As mentioned above, the researcher utilized blended instruction, which incorporated elements of explicit/systematic instruction, CSA, and graphic organizers, while supporting the CCSMP. Use of the CSA instructional sequence has proven effective for both general education and special education students (Gersten et al., 2009) and supports *modeling with mathematics* (CCSMP #4) and *using tools strategically* (CCSMP #5). Furthermore, the students justified the strategies they attempted and *critiqued their peers' strategies* (CCSMP #3) while working with the manipulatives. Additionally, while the students learned to solve equations, the lessons incorporated the appropriate use of the equal sign, which the CCSMP explicitly states is one of the skills that characterizes students who are proficient with *attending to precision* (CCSMP 6). The blending of instructional strategies that address the CCSMP from the special education and mathematics education literature is critical in helping students with MD gain access to and participate in general education curricula and settings (Maccini et al., 2013; Strickland & Maccini, 2012).

The current study investigated the effects of blended instruction and visual representations on participants' accuracy when solving one- and two-step equations.

Additionally the researcher examined the conceptions of the equal sign held by the students prior to intervention and determined whether these conceptions changed over the course of the study.

Interpretation of Findings

This study utilized a concurrent multiple probe design replicated across three other groups to answer Research Questions 1 and 2. For the third question, the researcher utilized a pre- and posttest analysis to determine how students conceived of the equal sign and to determine whether there were any changes in their understanding of the symbol post intervention. Finally, the researcher administered surveys to the participants and instructors to gather social validity data to answer Research Questions 4 and 5.

Research Question 1: To what extent do students with mathematics difficulties who receive instructional intervention on the relational nature of the equal sign and solving one- and two-step equations have increased accuracy when completing algebraic tasks involving the equal sign? This section presents a discussion of the data collected from the researcher-instructed groups and the teacher-instructed group. The section concludes with an exploration of select individual data.

Researcher-instructed groups. The effectiveness of this instructional package is evident by the change in level demonstrated by each group from baseline to post-assessment (see Figure 3). All researcher-instructed group scores showed marked improvements from baseline to post-assessment, and average increases on domain probes ranging from 69% - 86% percentage points (average 78%). Specifically, the groups' baseline scores ranged from 6% to 17% (average 10%), while the post-assessment scores ranged from 70 to 100% (average 87%).

The baseline condition for each group was stable and predictable prior to entering intervention. The large change in level demonstrated by each group from pre-intervention to post-intervention suggests that the instructional package positively affected students' performance as they solved one- and two-step equations. Additionally, this effect was evident across three different groups at different points in time. These conditions suggest causal relationship between the independent and outcome variables (Kratochwill et al., 2013).

The data on students' ability to solve equations aligned with previous research in which students with MD demonstrated mostly significant gains after explicit/systematic instruction (Mayfield & Glenn, 2008; Scheuermann et al., 2009) that included the CRA sequence (Scheuermann et al., 2009; Witzel et al., 2003) and graphic organizers (Ives, 2007). However, the significance of the results in this study differed from those in the Ives (2007) study, which reported a lower average performance of 40% accuracy post intervention. These findings were significantly lower than the results of the current study, which yielded an average accuracy of 87%. One possible explanation for this difference may be the difficulty of the mathematics addressed, as Ives examined systems of equations, while the current study focused on a prerequisite skill of solving one- and two-step equations.

An error analysis revealed that participants missed the most points because they did not justify their solution when asked, made a computation error, and/or used an incorrect operation. Only one student justified his solutions correctly every time, two students did not earn any points for justification, and the remaining students missed an average of 5 points (range = 1 – 9) out of the possible 10 they could earn for justifying

their solutions. Students rarely lost points for incorrect justification; however, several simply skipped the justification step entirely. This phenomenon was not altogether surprising for two reasons: a lack of desire to check their work and a lack of understanding about the requirement to do so. First, students did not like having to check their work during the intervention. Several students asked, “We found the answer why, do we have to check?” and only begrudgingly checked at the insistence of the instructor. This type of response is common among students with MD who may have difficulty evaluating solutions for accuracy and reasonableness (Miller & Mercer, 1997).

Second, the students may not have understood the directions on the probe due to a design mistake on the part of the researcher. For instance, the graphic organizers students used during the intervention had a space next to the flow map that said “Check” or “Check your work” as a reminder, whereas the domain probes simply stated “Solve and justify why your solution is correct.” It is possible that more students would have consistently justified their solution if the prompt used on the assessment was consistent with the one used during the intervention.

The next most common mistakes were computational errors (e.g., $3 + 3 = 9$; $7 - 5 = 3$), for which four students missed a total of 12 points. This finding is consistent with previous research, which indicates that students with MD struggle with basic computational skills (Little, 2009; Montague & Applegate, 2000; Maccini, McNaughton, & Ruhl, 1999).

Lastly, three students lost a total of 7 points because they used the wrong operation when finding the inverse of a number to solve for the variable. For example, when solving for a in Figure 4, the student correctly subtracted 7 from both sides of the

equation $11 = a/4 + 7$ to get $4 = a/4$. The student then divided by 4 when he should have multiplied by 4, and he arrived at the incorrect solution of $a = 1$. Again, these mistakes are consistent with previous research, which indicates that students with MD have difficulty selecting appropriate strategies (Montague, 2008; Maccini et al., 1999) and monitoring performance (Montague, Bos, & Doucette, 1991). The students might have also had a limited conceptual understanding of procedures (Geary, 2004); however, it is more likely that these errors were careless mistakes, as the students' performance on the KME measure showed a growth in conceptual understanding, as described in detail in the section addressing Research Question 3.

Teacher instructed groups. The teacher instructed groups all showed a change in level from baseline to post-assessment, although the change in level for Group W was not as dramatic as those demonstrated by the other groups included in the study (see Figure 3 and Figure 5). The average increase on domain probes ranged from 34% to 67%. Specifically, groups' baseline scores ranged from 11% to 20% (average 16%), while post-assessment scores ranged from 25% to 85% (average 68%). Although the overall post-assessment average of the teacher instructed groups was lower than the researcher instructed groups, closer examination of the individuals composing the groups partially explains these differences.

In each of the three teacher instructed groups, an individual made systematic errors unique to themselves that significantly lowered their performance when compared to the peers in their group (see the *Individual results* section for further detail on this phenomenon). When the researcher removed these individuals' scores, the teacher groups' overall average (79%) was closer to that of the researcher instructed groups

(87%). Although the average of the teacher instructed groups was lower than the researcher instructed groups, the difference was relatively small compared to the overall gains made by the students. Further, the students still made clinically significant gains by progressing from failing to passing on the domain probes. The visual analysis of the data from the teacher-instructed groups also met the same criteria as the data from the researcher instructed groups, which suggests a causal relationship between the independent and outcome variables (Kratochwill, et al., 2013).

An error analysis revealed that, unlike the researcher instructed groups, where errors were consistent across participants, the teacher instructed groups had one participant in each group that made a particular error unique to the individual, which inflated the number of errors for the group and decreased the groups overall performance. These particular student errors were removed for the group error analysis and will be discussed in the following subsection, Individual Results. After the removal of the unique errors, the participants missed the most points for two of the same reasons as the researcher-instructed groups: missing justification and computation error. The most frequent error involved missing justification, which supports the idea that a flaw in the probe design may be the strongest contributing factor for the errors. The second most common mistake involved computational errors, which was consistent with the results of previous studies that showed that students with MD struggled with basic computational skills (Little, 2009; Maccini et al., 1999; Montague & Applegate, 2000).

Individual results. The group data were replicated by all but one of the participants. Nick, from teacher instructed Group X, did not demonstrate a meaningful change in level from baseline to post-assessment (i.e. increase between baseline and post

assessment was less than 15%). Additionally, Jonah did demonstrate a meaningful change in level however he did not increase his score to above passing (i.e., 60%), as defined by the school system in which the study took place.

Nick was the only student who did not demonstrate a meaningful change in level, as he only increased his score from an average of 21% on the pre-assessments to an average of 43% on the post-assessments. Nick lost a point on 21 problems across the five post-assessment domain probes because he did not show evidence of using inverse operations to solve the problem. Specially, he used facts and guess and check on the pre-assessments and did not show any evidence of using the strategies taught during the intervention to assist him in solving the problems on the post-assessment. His score was higher on the post-assessments for two reasons. First, he attempted more problems on the post-assessments and wrote IDK (i.e. abbreviation for I don't know) once on each of the post-assessments, as compared to writing IDK an average of four times (range = 3 - 5) on each of the pre-assessments. On the pre-assessments, he wrote IDK on most of the division problems. Across the five post-assessments, most of the problems for which he wrote IDK involved division, which suggests it was harder for him to guess and check the answer for those types of problems. Additionally, while Nick did not check his work on the pre-assessment, he did check his work correctly four times on the post-assessments.

Nick may have failed to show improvement when solving equations because of his resistance to using key components of the intervention like the algebra tiles and graphic organizers. The instructor of Nick's group reported that he was reluctant to use the algebra tiles and would try to figure out the answers in his head rather than using the tiles to facilitate the process. He also expressed a lack of desire to use the graphic

organizers and only did so correctly while the teacher was explicitly teaching a problem. During independent practice, he did not use the organizers or inverse operations to find his answers. Although his work from the lessons frequently showed the correct answer written in the graphic organizer, the steps leading to the answer were incorrect; and, in several instances, he failed to write the equation correctly.

The instructor reported that she insisted that Nick use the graphic organizers to show his work. He used his calculator and guess and check to obtain the correct solution and then went back to fill in the organizer. Nick was the only student to show a strong resistance to using the graphic organizers and algebra tiles during the intervention, which significantly impacted his performance on the post assessment. He was also the only student who did not demonstrate a meaningful change in level from the pre-assessment.

Jonah was the only other student who did not improve his post-assessment scores to above a 60%, although he did show a meaningful change in level (i.e. increase of more than 15%). Unlike Nick, who resisted the intervention, attributes associated with Jonah's disability area of autism, may have impacted his performance. Jonah was the only student who did not show any change in level from the last probe in baseline to the first post-assessment. The reason that Jonah did not show improvement on the first post-assessment probe was that he did not utilize a graphic organizer because he did not raise his hand to ask for one. Of the 17 students, 5 students did not ask to use the graphic organizers for any of the post assessment probes. Students were required to request the graphic organizer, rather than starting with one already available, to address the mathematical practice of using tools strategically and recognizing when a tool (i.e. graphic organizer) is appropriate for a situation.

After the first post-assessment period, when the other students had left the room, Jonah asked if he could use the graphic organizer when completing the next probe and if I could give him the probe when I handed out the assessment. Although he would have used a graphic organizer on the first post assessment probe, a characteristic of his disability, namely a struggle to initiate a social exchange in a group (Myles & Simpson, 2002), may have prevented him from doing so. On the remaining four probes, Jonah's scores did reflect a change in level, although he did not show as much improvement as the other students.

The error analysis revealed that Jonah answered every two-step equation correctly and every one-step equation incorrectly on the remaining four post-assessment domain probes, because he solved the one-step problems as if they were two-step problems when he used the graphic organizers. For example, given the equation $z - 2 = 9$, Jonah correctly added 2 to both sides of the equation and wrote $z = 11$. He then divided both sides by 2 and obtained $n = 5.5$ as his final answer (see Figure 6). Regardless of the type of one-step problem, Jonah added or subtracted first, then divided or multiplied, which are the correct procedures for solving two-step equations.

$$\begin{array}{l} \boxed{z - 2} = \boxed{9} \\ \downarrow \\ \boxed{+2} = \boxed{+2} \\ \downarrow \\ \boxed{z} = \boxed{11} \\ \downarrow \\ \boxed{\div 2} = \boxed{\div 2} \\ \downarrow \\ \boxed{z} = \boxed{5.5} \end{array}$$

$$\begin{array}{l} \boxed{\frac{a}{2}} = \boxed{10} \\ \downarrow \\ \boxed{+2} = \boxed{+2} \\ \downarrow \\ \boxed{\frac{a}{2}} = \boxed{12} \\ \downarrow \\ \boxed{\cdot 2} = \boxed{\cdot 2} \\ \downarrow \\ \boxed{a} = \boxed{24} \end{array}$$

Figure 6: Examples of Jonah's error of solving one-step equations using the procedures for two-step equations in order to fill all the boxes.

Jonah may have made these errors for two reasons. First, students with autism tend to focus on small parts of a topic or concept and subsequently may experience difficulty with synthesizing aspects of a situation into a complete picture (Meyer & Minshew, 2002). More specifically, individuals with autism may attend to details at the expense of organizing information, which can strongly impact reasoning and problem solving (Meyer & Minshew, 2002). In Jonah's case, he may have noticed that the graphic organizers provided were the same ones he used for solving two-step equations and then followed the rules for solving two-step equations, regardless of the equations given. Second, Jonah was absent during the last two days of the intervention, when he would have learned about solving one- and two-step equations during the same lesson while using the two-step graphic organizers as a tool. Had he attended those lessons, the instructor could have intervened and helped him focus on the equation structure and not the number of boxes in the graphic organizer as he determined how to solve the given equations. As a result, he might have realized that he did not have to use all of the boxes provided. Although this intervention targeted students with MD, and no other students made this error, an easy change could be made to the intervention that may have prevented Jonah's error. The intervention could only have used the same graphic organizer for both one- and two-step equations instead of using a simplified version first for the one-step equations. The instructors could just explain to the students that they do not always need to use all of the boxes provided.

Research Question 2: To what extent do students with mathematics difficulties maintain performance on algebraic tasks involving the equal sign four-to-six weeks after the conclusion of the intervention? All of the participants demonstrated a high degree of maintenance when solving one- and two-step equations four-to-six weeks after the end of the intervention. The mean score on the maintenance probe for the participants in the researcher-instructed group was 81%, with a range of 44% to 91%. The mean score of the participants in the teacher instructed groups was 73%, with a range of 44% to 89%. One student (Nick) from teacher instructed Group X moved prior to taking the maintenance measure. Nick was the lowest-achieving student in the group, and, as a result, there appeared to be a large increase from the post-assessment average to maintenance for Group X.

When excluding Nick's data, the average post-assessment and maintenance scores for Group X both equaled 83%. The remaining groups did not show a significant change from post-assessment to maintenance; however, they maintained a significant level change from the pre-assessment to the maintenance measure. These findings are consistent with research on solving equations in which students with MD maintained an increase in performance that resulted from explicit/systematic instruction (Scheuermann et al., 2009), the CRA sequence (Scheuermann et al., 2009; Witzel et al., 2003), and graphic organizers (Ives, 2007). The group data were replicated by all but one of the individual participants who self-reported that he forgot to take his ADHD medicine, which may have resulted in his earning of 56% accuracy score. This score equaled his lowest post-assessment measure and was 18 percentage points below the average of the five post-assessment measures (average = 74%, range = 56% – 89%).

Research Question 3: How do students conceive of the equal sign prior to intervention and are there changes in those conceptions post intervention? The KME assessment detects systematic changes in students' knowledge of equivalence across elementary grades (Rittle-Johnson et al., 2011). In the current study, the assessment identified changes that may have occurred because of the intervention. The domain probes focused more broadly on application and use of the equal sign in a procedural manner, whereas the KME assessment focused explicitly on measuring how a student conceptually viewed the equal sign itself (Rittle-Johnson et al., 2011). The assessment draws from a construct map that has four continuous levels of understanding the equal sign proceeding from operational to comparative relational understanding (see Appendix M). The assessment builds upon prior research on mathematical uses of the equal sign, and each of the four levels incorporates three classes of research-supported items involving solving equations, evaluating the structure of equations, and defining the equal sign. Overall, there were significant differences between the pre-test ($M = 44.02$, $SD = 17.18$) and post-test ($M = 65.56$, $SD = 21.88$) total scores on the KME measure; $t(16) = -5.37$, $p < .01$, $d = 1.09$. This suggests that there was overall improvement in tasks involving the equal sign as a result of the intervention. The following subsections detail student responses from pre-test to post-test for the levels within the KME measure.

Level 1 and Level 2. There were no significant differences found from pre-test to post-test on items identified at the lowest levels of the continuum. These items included equations in the operations-equals-answer structure ($a + b = c$) and atypical problems compatible with the operational definition ($c = a + b$ or $c = c$), and students had to define the equal sign operationally (Rittle-Johnson et al., 2011). On the pre-test, the students

scored high on items in Level 1 ($M = 94.12$, range = 67-100) and level 2 ($M = 69.41$, range = 20-100), with minimal room for significant improvement. The average scores on the Level 2 items increased slightly ($M = 76$). The high scores obtained by the students on the pre-test were in line with previous research that showed that students are exposed to the operational understanding throughout elementary school, and students at the second grade level have moderate success with these types of items (Rittle-Johnson et al., 2011).

Level 3 and Level 4. There were significant differences found from pre-test ($M = 44.92$, $SD = 31.90$) to post-test ($M = 72.19$, $SD = 29.19$) on Level 3 items. These items included equations with operations on both sides of the equal sign ($a + b = c + d$ or $a + b - c = d + e$), and students had to recognize and generate a relational definition of the equal sign. Additionally, there were significant differences found from pre-test ($M = 17.65$, $SD = 14.20$) to post-test ($M = 42.48$, $SD = 27.84$) on Level 4 items, although the increase was not as high as for the Level 3 items. These results align with those of previous studies, which determined that explicit instruction led to improvement in elementary school students' relational understanding of the equal sign (Hattikudur & Alibali, 2010; McNeil & Alibali, 2005b; Rittle-Johnson & Alibali, 1999). The most notable change in student responses from pre-test to post-test at these levels related to how the students defined the equal sign.

When asked "What does the equal sign (=) mean?" on the pre-test, 12 students responded with an operational answer like "It means it's giving you an answer" or "It means what the total is." Only three students responded with a relational definition like "two numbers are the same" or "It means a same amount of something." Lastly, two

students responded by saying, “It means equal,” and it was not possible to determine if they were thinking relationally or operationally. These results align with previous studies that reported that less than half of middle school students could provide a relational definition of the equal sign (Alibali, Knuth, & Hattikudur, 2007; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005; Knuth, Stephens, McNeil, & Alibali, 2006).

On the post-test, only one student provided an operational definition, and 12 students provided a relational definition, stating, “It means if they are the same” or “It means equal to or the same amount.” Additionally, four students responded by saying, “It is equal to” or “It means to be equal.” While these responses represent an improvement from saying, “It means the answer,” it is still not possible to determine if the students were thinking relationally or operationally when they provided the definition.

These results are similar to the findings reported by McNeil and Alibali (2005b), as the students could identify the relational definition as “smarter” or more sophisticated than other definitions, such as “the end of the problem,” after receiving explicit instruction on the meaning of the equal sign. In addition to explicitly teaching the relational definition of the equal sign, the current study exposed students to non-standard equation structures that also can help students understand the equal sign as a relational symbol (McNeil et al., 2009). Due to the pre post-test design of the study, and the lack of comparison groups, the researcher could not determine which of these two factors contributed most to students’ development of a relational definition of the equal sign.

Several studies have shown a positive correlation between students’ relational understanding of the equal sign and their performance when solving equations (Alibali et al., 2007; Knuth et al., 2006), and the findings from the current study support this view.

Only three students could provide a relational definition of the equal sign during the pre-assessment, and the students achieved an average accuracy score of 42% on the equation solving items. On the post-assessment, 12 students provided a relational definition, and the students achieved an average accuracy score of 63% on the equation solving items.

Although previous studies (Kieran, 1982; Rittle-Johnson & Alibali, 1999) found that the ability to solve equations with operations on both sides developed faster than the ability to provide a relational definition of the equal sign, findings in the current study determined that more students ($n=12$) correctly provided the relational definition than did the average number of students who accurately solved equations with operations on both sides of the equal sign ($M = 9.5$, range = 6 – 12). One possible explanation for this finding is that both of the previous studies included general education students, while the current study focused on students with MD. An examination of the responses to the equation solving items in the current study revealed that many students attempted to solve the equations using correct strategies that demonstrated a relational understanding; however, they arrived at an incorrect answer due to poor calculation skills, an area in which students with MD struggle (Little, 2009; Maccini et al., 1999; Montague & Applegate, 2000).

Research Question 4: To what extent do middle school students with mathematics difficulties consider blended instruction with visual representations and graphic organizers beneficial (i.e., social validity)? The researcher administered the social validity measure to each group of students after they completed the post-assessment domain probes. The assessment contained both 5-point Likert scale questions and open-ended questions. Both the researcher and teacher instructed groups reported that

(a) the intervention was worth their time, (b) they would recommend it to other students, and (c) they felt better about their math skills because of the intervention. Additionally, most students agreed ($n=14$) that the graphic organizers were helpful in solving two-step equations, although one student (Nick) strongly disagreed. The open-ended comments supported the Likert scale responses, as 13 students noted that the graphic organizers were their favorite component of the intervention. The previous study that utilized graphic organizers (Ives, 2007) did not collect social validity data on the use of graphic organizers and listed it as a limitation of the study. The results of the current study suggest (a) that graphic organizers lead to an increase in student performance when solving one- and two-step equations, and (b) that students enjoy learning with them.

There were small differences between the researcher-instructed groups and teacher instructed groups when using algebra tiles and the balance. The students in the researcher-instructed groups reported that they agreed (4.0) that use of the algebra tiles helped them see what they were doing when solving equations, and they were neutral/agreed (3.89) that use of the balance helped them to remember that the equal sign meant that both sides had to be the same.

The teacher instructed groups were about one point lower than the research instructed groups. Students reported that they were neutral (3.33) about using algebra tiles and were neutral/disagreed (2.94) that the balance helped them remember that the equal sign meant the same. Some of these differences may result from the background of the instructors. For instance, the results of the social validity survey were similar for the researcher and teacher instructor for Group W. Both instructors had mathematics backgrounds and had worked with the algebra tiles and balance model prior to the

intervention. The remaining two teachers (Groups X and Y) did not have mathematics backgrounds and had never used the algebra tiles or the balance model prior to teaching the intervention. As a result, they may not have been as comfortable with the manipulatives, which may have impacted the student perceptions. The students in the groups led by instructors with mathematics backgrounds agreed that the algebra manipulatives were helpful, which is consistent with a previous study (Maccini & Ruhl, 2000). No previous studies have collected social validity data on students who used manipulatives and received instruction from “non-math” teachers.

Research Question 5: To what extent do middle school teachers consider blended instruction with visual representations and graphic organizers a viable intervention strategy? Like the student social validity assessment, the teacher instructor version contained both 5-point Likert scale questions and open-ended questions. All of the teachers agreed that the intervention was not difficult to implement and that participating was worth their time. Although the design of the study made it impossible for the research to take place in a regular, intact classroom, all of the instructors agreed that the intervention could occur within the regular classroom setting and within the normal constraints of the school day. The teacher with the mathematics background responded with the only neutral response about whether typical special education teachers could deliver the intervention to students. The two other teachers, one with a special education background and one with a social studies background, reported that they strongly agreed that typical special education teachers could deliver the intervention to students.

Summary. These results suggest that blended instruction with representations and graphic organizers can (a) improve the performance of students with MD as they solve one- and two-step linear equations and (b) increase their understanding of the equal sign as a relational symbol. All but one participant across the groups demonstrated significant increases in their accuracy scores when solving one- and two-step equations; with 11 of the 17 students scoring at or above 80% accuracy on the post-intervention domain probes, and all but two students scoring above 60% accuracy. Additionally, all but one student maintained their performance on a domain probe administered four-to-six weeks after the intervention. Students also showed significant differences from pre- to post-test, with large effect sizes related to their ability to solve equations, utilize structure, and define the equal sign on the KME assessment. Students also showed significant gains with large effect sizes on problems that required them to demonstrate a relational understanding of the equal sign when solving the equations.

Limitations and Future Research

Although the results of this study are promising, there are limitations and suggestions for future research that researchers should consider. First, the researcher conducted the current study outside of the general education classroom setting, with a small teacher-to-student ratio (1:3). Using small groups allowed the teacher to allot more time to each individual student than would have been possible in an inclusive classroom setting. Future studies with larger groups or intact classrooms would allow researchers to generalize their findings to a typical classroom setting. In addition future research could compare the performance of students who received the intervention in small groups to

students who received the intervention as part of regular classroom instruction in order to judge the effectiveness of the intervention across settings.

Second, the study involved students who demonstrated a history of difficulty with solving equations and who earned low scores on baseline domain probes, which resulted in a heterogeneous group of students. Some participating students had been formally diagnosed with learning disabilities, including ADHD, Autism, and SLD; whereas other students did not have a formally diagnosed disability. All of these students fall under the umbrella of having MD and were in need of some type of specialized assistance in math. The type of assistance needed however could vary widely depending on the specific disability diagnosis of the student. An intervention that is successful for non-disabled students with MD is not guaranteed to be effective for students with MD and a diagnosed disability. Therefore, future research is needed to determine the effectiveness of the intervention for students with specific diagnosed mathematics disabilities.

Third, the researcher developed the domain probes used in the study, so the tools did not have an established reliability and validity prior to implementation. Additionally, the probes aligned with the content of the intervention and did not include tasks that would involve transfer of knowledge or application to unfamiliar situations. While the KME assessment did have established reliability and validity for students in Grades 2-6, researchers had not yet tested it for use in with seventh graders. Future studies should include dependent measures that have established validity and reliability, and that include questions that the research team has not directly taught as part of the intervention.

Fourth, the researcher administered the KME assessment as a pre- and post-test measure, with no comparison groups. Additionally, the researcher gave the pre- and post-

test versions at different points in time corresponding to the SCD of the study. After consulting with the author of the measure, I decided to conduct the analysis as if the participants formed an intact group due to the low number of participants. Because I used only one group, it is possible that there were historical and maturation effects that threatened the internal validity of the study. Future research is needed with larger groups and a control group to strengthen the internal validity of the design.

Fifth, the results from the social validity assessments by have been impacted by the researchers relationships with the participants. The researcher worked in the school where the study took place and the three teachers that participated in the study were colleagues who volunteered to assist the researcher. Additionally the researcher was well liked by the student participants in the study and had interactions with the students as part of normal school activities. Future studies should collect social validity data in settings where the researcher does not have pre-established familiarity with the participants.

Lastly, future research should include qualitative data to support the quantitative data. Specifically the inclusion of vignettes or case studies would help determine how and why the graphic organizers assist students in solving equations and provide more specifics on how and why the intervention package impacted students' conceptual understanding of the equal sign. Additionally, researchers might be able to determine if other intangible factors such as increased student confidence also impact achievement.

Implications for Practice

The current study contributes to the literature in several ways: 1) it addresses the need for an intervention or strategy to teach the relational meaning of the equal sign at the middle school level; 2) it incorporates research-based strategies for accessibility; 3) it

addresses social validity; and 4) it contributes to single case research design research. First, this study addressed solving equations and understanding the equal sign as a relational symbol. While several studies (Hattikudur & Alibali, 2010; McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999) examined the effects of an intervention on understanding the equal sign with elementary grade students, the researcher found no studies that included secondary students, despite literature that reports that students have incomplete or incorrect conceptions about the equal sign in secondary grades (Alibali et al., 2007; Essien & Setati, 2006; Godfrey & Thomas, 2008; Knuth et al., 2006; Knuth et al., 2008; McNeil et al., 2006). The current study extends the research conducted at the elementary levels to develop students' ability to see the equal sign as a relational symbol at the middle school level. This skill is critical, as many students enter their first-year algebra course with an inadequate understanding of the equal sign (Asquith et al., 2007; Booth, 1988; McNeil et al., 2006; Schoenfeld & Arcavi, 1988), and a relational understanding of the equal sign correlates to improved performance when solving equations (Alibali et al., 2007; Knuth et al., 2006), which is a major topic within the CCSS (2010).

Second, this study blended research-based strategies from both the general education and special education literature to provide access to the CCSS for students with MD. The study included the use of manipulatives that have proven effective for teaching math to students with MD (Gersten et al., 2009), while supporting *modeling with mathematics* (CCSMP #4) and *using tools strategically* (CCSMP #5). Additionally, while the students learned to solve equations, the lessons incorporated instruction on the

appropriate use of the equal sign, which is explicitly stated as one of the skills that characterizes students who are proficient with *attending to precision* (CCSMP #6).

Third, this study addressed the social validity of the intervention from both the student and instructor perspective. None of the previous studies included in the review that examined solving equations or understanding the equal sign collected social validity data from the participants to determine if they found the intervention valuable. Additionally, only three studies included interventions conducted by teachers, and none of the researchers surveyed the teachers to gain social validity information about whether the procedures were acceptable, feasible, and effective (Horner et al., 2005). This information is critical to obtain, especially when working with manipulatives, as many secondary teachers do not use manipulatives with students (Howard, Perry, & Conroy, 1995); and those that do use them may not see their value or may view manipulatives as fun for the students without having any connection to “real math” (Moyer, 2001). It is critical that teachers understand that manipulatives are an effective method for teaching concepts, as teachers attitudes can blunt the benefits of manipulatives and hinder learning (Moyer, 2001).

Lastly, this study was also the first to use a concurrent multiple-probe design replicated across three groups, where the instructors for the replication groups were typical intervention agents (classroom teachers). While three replications of a result over time is the minimum needed to establish a functional relationship in a single case design (Kratochwill et al., 2010), the replication in this study strengthens the internal validity by increasing the number of replications (Kratochwill & Levin, 2010; Horner & Odom, 2013). Additionally, external validity was increased by showing the intervention resulted

in the same effects when replicated (Horner et al., 2005) by typical intervention agents. The random assignment of participants to groups, groups to instructions, and groups to start order also added additional strength to the study's internal validity (Kratochwill & Leven, 2010). It is critical that studies employing the SCD methodology have strong internal validity with randomization to make casual connections between the intervention and outcome that are as valid as those obtained by group designs, which the research community tends to view as more scientifically credible (Kratochwill & Levin, 2010).

Conclusion

Federal legislation mandates that students with disabilities not only have access to the general education curriculum but also demonstrate proficiency with the standards alongside their non-disabled peers (IDEA, 2004; NCLB, 2002). Despite this legislation, students identified as having a disability continue to score poorly when compared to their non-disabled peers on national assessments (NAEP, 2013). The current study investigated the effects of blended instruction on students' ability to solve one- and two-step equations while developing their understanding of the equal sign as a relational symbol. Prior to this study, no research specifically examined an intervention or strategy for teaching the relational meaning of the equal sign in the middle grades or concurrently with learning to solve algebraic expressions. The results of this study show promising evidence that the instructional package helped students can improve their ability to understand the equal sign as a relational symbol while increasing their ability to accurately solve one- and two-step equations.

Continued research is necessary to identify practices that educators can implement in the general education setting that will increase access to the mathematics curriculum

and improve results for students with MD. Proficiency in algebra is one of the requirements most students must demonstrate to graduate from high school and pursue a postsecondary education or enter a skilled profession. Therefore, identifying research-supported practices that help students become proficient in mathematics may contribute to desirable outcomes like improved high school graduation rates, increased enrollment in college, and increased numbers of students who enter professions in science, technology, engineering, and math.

Appendix A: Letter to Parents

[Date]

Dear Parent/Guardian and Student:

We are conducting a study on the effectiveness of a pre-algebra instructional package for middle school students who may be struggling in math. The instructional package will target the idea of equality as it relates to solving equations, skills that are important for success in middle school math and beyond. The instructional package will be taught by certified teachers at Corkran Middle School.

We are looking for students to participate in this study. The study will last for the first semester of school. Participating students will be taught the instructional package for about ten days during their regular scheduled encore periods during normal school hours at some point the first semester. Students may also take some assessments related to the study during their homeroom period throughout the 9 weeks. Mr. Jason Miller will access confidential student education records to obtain pertinent data related to the study including IEP information, cognitive skills (i.e. IQ), and academic achievement (i.e. report cards). All data regarding your child will be kept confidential and only accessed by Mr. Miller. Data will be destroyed five years after the study ends.

Risks associated with this study include possible frustration with difficult tasks and the possibility of your child's image being viewed in research presentations, publications, and/or teacher trainings, if permission for video recording is granted. Participation will not affect your child's grades. You may request that your child be withdrawn from participating at any time without penalty. There are no direct benefits to participants, but possible benefits may include improvements in understanding and performance on grade level math objectives.

By signing the attached permission form, you are agreeing to allow your child to participate in this study

If you have questions about this study, please contact Jason Miller at:

(email) millerj@umd.edu

(phone) 410-787-6350

Jason Miller
Student Investigator
Special Education / Mathematics Teacher
Corkran Middle School

Dr. Paula Maccini
Faculty Advisor
Associate Professor
UMD: College Park

Jolyn Davis
Principal
Corkran Middle School

Appendix B: Parent Consent Form

Project Title
The Effects of Blended Instruction and Visual Representations on Understanding the Equal Sign and Solving Equations for Middle School Students with Math Difficulties.
Purpose of the Study
This is a research project being conducted by Mr. Jason Miller, a special education teacher at Corkran middle school as part of his doctoral studies at the University of Maryland: College Park, under the supervision of Dr. Paula Maccini. We are inviting your child to participate in this research because he or she has a history of difficulty in mathematics, particularly with solving equations. The purpose of this research project is to advance current knowledge on effective interventions for middle school students having difficulty with mathematics.
What will my child be asked to do?
<p>The procedures involve the collection of information from your child's confidential school file, including IQ scores, achievement scores, and grades from past and current mathematics courses to determine if your child is eligible for the intervention.</p> <p>Your child will complete a minimum of five pretests on solving one- and two-step equations before instruction is provided. After the pre-assessments your child will be asked to participate in daily instructional sessions for 10 days during a period when your child has an encore subject. During the instruction sessions your student will be taught how to solve one- and two-step equations. Additionally, your child will be asked to complete periodic assessments on solving equations during the first semester during homeroom. After completing all instructional sessions, your child will complete a minimum of five post-tests to determine any changes in his or her understanding of solving one- and two-step equations. Two to four weeks after the end of the intervention, your child will be asked to complete a short assessment to see if he or she remembers the content that was taught. Your child will also complete a short assessment during homeroom on algebra basic skills approximately every three weeks for a total of six to nine times. Your child will complete two assessments on their understanding of the equal sign, one before and one after receiving instruction. Additionally your child will be asked his or her opinion regarding instruction. For example, your child will be asked if the intervention helped him/her learn the targeted objectives and what he/she liked most and least about the intervention. While the assessments that your child will take will be given periodically over the course of the first semester, the intervention component of the study will only take place over a period of 10 days.</p> <p>During the study, we will be video recording the instructional and assessment sessions only. We would like your permission to use portions of these videos in four ways:</p> <ol style="list-style-type: none"> 1.) To determine your child's thinking about the math topics 2.) To determine if the intervention is being implemented as planned 3.) To determine if the assessments were delivered as planned. 4.) In research presentations, publications, and/or teacher trainings <p>If you choose not to have your child video recorded, he or she may still participate in the study. Students without permission to be recorded will be seated outside the view of the camera.</p>

Potential Risks
There may be some risks from participating in this research study. Risks include possible frustration with some of the tasks and the possibility of your child's likeness being viewed in research presentations, publications, and/or teacher trainings.
Potential Benefits
This research is not designed to help your child personally, but the results may help me learn more about mathematics instruction for students who have difficulty with math. We hope that, in the future, other people might benefit from this study through improved understanding of instructional practices in algebra. Although there are no direct benefits to participants, your child may benefit by participating because the study is designed to improve understanding of mathematics, specifically how to solve equations, which may lead to improved grades in math class.
Confidentiality
All information collected by this study will be kept confidential to the extent permitted by law. All data collected will be kept in a locked file cabinet in my room at Corkran Middle School or digitally on a password protected hard drive. Video recordings will be stored on a password protected hard drive. Access to these data will be provided to trained graduate students and/or higher education faculty members for fidelity checks.. If we write a report or article about this research project, your child's identity will be protected to the maximum extent possible and your child's name will not be used. Data will be identified using false names or an identification code. Your child's information may be shared with representatives of the University of Maryland: College Park, or governmental authorities if your child or someone else is in danger or if we are required to do so by law.
Does my child have to be in this research? / Can my child stop participating at any time?
Your child's participation in this research is completely voluntary. You may choose not to have your child take part at all. If you decide to have your child participate in this research, you may request that he/she stop participating at any time. If you decide not to have your child participate in this study or if you request that he/she stop participating at any time, your child will not be penalized or lose any benefits to which he/she otherwise qualifies. Your child's participation or nonparticipation in this study will not directly affect his or her grades because it is voluntary and the participating instructors do not have access to assigning grades.
What if I have Questions?
This is a research project being conducted by Mr. Jason Miller, a special education teacher at Corkran Middle School as part of his doctoral studies at the University of Maryland, College Park, under the supervision of Dr. Paula Maccini. If you have any questions about the research study itself, please contact Mr. Jason Miller at: 7600 Quarterfield Rd Glen Burnie, MD 21061 (telephone) 410-787-6350, (e-mail) millerj@umd.edu If you have any questions about the study's implementation at Corkran Middle School please contact Jolyn Davis at: 7600 Quarterfield Rd Glen Burnie, MD 21061, (telephone) 410-787-6350, (e-mail) jmdavis1@aacps.org If you have questions about your rights as a research participant or wish to report a research-related injury, please contact: University of Maryland College Park, Institutional Review Board Office, 1204 Marie Mount Hall College Park, Maryland, 20742 (Telephone) 301-405-0678

(E-mail) irb@umd.edu <i>This research has been reviewed according to the University of Maryland, College Park IRB procedures for research involving human subjects.</i>		
Statement of Consent		
Your signature indicates that you are at least 18 years of age and you hereby give permission for your child or legal ward to participate in an educational study; the research has been explained to you; your questions have been fully answered; and you freely and voluntarily choose to participate in this research project.		
PRINTED NAME OF CHILD		
I agree to:	<p>___ have my child video recorded for internal use to determine his or her thinking processes about the algebra topics and to insure the intervention is being implemented as planned.</p> <p>___ have my child video recorded for external use in research presentations, publications, and/or teacher trainings.</p>	
PRINTED NAME OF PARENT/GUARDIAN	SIGNATURE OF PARENT/GUARDIAN	DATE

Appendix C: Student Assent Form

The Effects of Blended Instruction and Visual Representations on Understanding the Equal Sign and Solving Equations for Middle School Students with Math Difficulties.

We are requesting your participation in an educational project conducted by Mr. Jason Miller a teacher a Corkran Middle School and doctoral student at the University of Maryland, College Park. You are under 18 years of age, and your parent or legal guardian has agreed that you can participate in this study.

The purpose of this study is to learn more about good pre-algebra instruction for middle school students with learning difficulties in mathematics. You will participate in instructional sessions lasting 1 period, for 8-10 consecutive days and participate in short assessments (<15 minutes) on solving equations, equal-sign knowledge, and algebra basic skills during homeroom periodically over the course of 10 weeks. Instruction will take place at school, during regular school hours and the instructional sessions will be video recorded. Video recordings may be used for three reasons: (1) to determine how you think about the questions; (2) to determine how the teacher is teaching the topics; and (3) to use your picture in research presentations, publications, and/or teacher trainings. If you do not want to be video recorded, you may still participate in the study. You will complete assessments before, during and after the study. You will also be asked your opinion about the study, such as what you like best and what you would change. Mr. Miller will also collect information from your confidential school records such as IQ scores, academic achievement scores and current math grades. Any information collected by Mr. Miller will be confidential, which means it will not be shared with anyone.

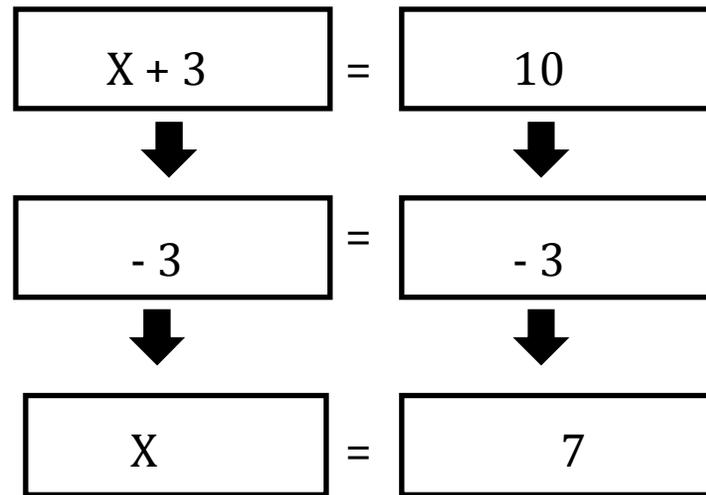
Participation in this study will not affect your math grade. You may feel frustrated with some of the math work. This study is not designed to help you personally but you may benefit from this study because the project is designed to improve your math skills. You are free to ask questions anytime and you may stop participating at any time. If you stop participating, your grades in class will not be affected.

Print Name

Date

Signature

Appendix D: Graphic Organizer (Researcher developed)



Appendix E: CCSS Content Standards Addressed in Intervention

6.EE.2.c Evaluate expressions at specific values of their variables.

6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q , and x are all non-negative rational numbers.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Appendix F: CCSS for Mathematical Practice

Standard	Description/Look For
Standard 1: Make sense of problems and persevere in solving them	Explain the meaning of the problem, make conjectures about form and meaning, use concrete objects to help conceptualize problem, check answers to problems using different methods, ask “Does this make sense”
Standard 2: Reason abstractly and quantitatively	Use real-world contexts and manipulatives (e.g., algebra tiles, colored chips) to make sense of abstract concepts; Consider units involved; Use different properties of operations and objects
Standard 3: Construct viable arguments and critique the reasoning of others	Analyze situations, make conjectures, justify reasoning, communicate with and respond to arguments of others.
Standard 4: Model with mathematics	Consider models such as diagrams, two-way tables, graphs, flowcharts, formulas, graphic organizers when problem solving.
Standard 5: Use appropriate tools strategically	Knowing how and when to use tools such as: manipulatives, calculators, rulers, statistical packages, graphic organizers, etc.
Standard 6: Attend to precision	Use clear definitions, label quantities, specifying units, using equal sign consistently and appropriately, communicate precisely with others.
Standard 7: Look for and make use of structure	Apply foundational skills and concepts to novel or more complex situations. Look for patterns or structure that may offer insight or assist with solving.
Standard 8: Look for and express regularity in repeated reasoning	Repeated reasoning, Look for general methods and shortcuts; attend to details of a problem, maintain oversight of the process.

Appendix G: Sample Domain Probe A

Solve: $14 = a + 4$

Solve: $3x - 1 = 8$

Solve: $\frac{m}{16} = 2$

Solve: $17 = 2x - 15$

Solve: **$9b = 72$**

Solve: **$7 = \frac{m}{8} + 4$**

Solve and justify why your solution is correct: **$11 = 3 + 4x$**

Solve and justify why your solution is correct: **$y - 3 = 4$**

Appendix H: Knowledge of Mathematical Equivalence Assessment

First Name _____

SECTION TIME - 10 minutes**1.** (ST1) Memory

Practice _____

a-b) _____

b-d) _____

c-f) _____

d-h) _____

2. (ST2) For each example, decide if the number sentence is true. In other words, does it make sense?

After each problem, circle True, False, or Don't Know.

Samples:

$3 + 4 = 7$

True

False

Don't Know

$3 + 4 = 12$

True

False

Don't Know

$a_{-b}) 8 = 8$

True

False

Don't Know

$b_{-d}) 7 + 6 = 0$
 Know

True

False

Don't

$c_{-e}) 5 + 3 = 3 + 5$

True

False

Don't Know

$d_{-g}) 8 = 5 + 10$

True

False

Don't Know

$e_{-i}) 3 + 1 = 1 + 1 + 2$

True

False

Don't Know

- 3.** (ST3) For each example, decide if the number sentence is true. Then, explain how you know.

A.) $8 = 5 + 3$ True False Don't Know

How do you know?

b) $4 + 1 = 2 + 3$ True False Don't Know

How do you know?

- 4.** (ST4b) This problem has two sides. Circle the choice that correctly breaks the problem into its two sides.

$$4 + 3 + 6 = 2 + \underline{\quad}$$

a.

Side A	Side B
$4 + 3 +$	$6 = 2 + \underline{\quad}$

b.

Side A	Side B
$4 + 3 + 6 + 2$	$= \underline{\quad}$

c. ?

d.

Side A	Side B
$4 + 3 + 6$	$2 + \underline{\quad}$

e.

Side A	Side B
$4 + 3 + 6 = 2 + \underline{\quad}$	$\underline{\quad} + 2 = 6 + 3 + 4$

5. (ST5@) *Without adding* $89 + 44$, can you tell if the statement below is true or false?

$$89 + 44 = 87 + 46$$

True

False

Can't tell without adding

How do you know?

6. (ST6) *Without subtracting* the 9, can you tell if the statement below is true or false?

$$76 + 45 = 121 \text{ is true.}$$

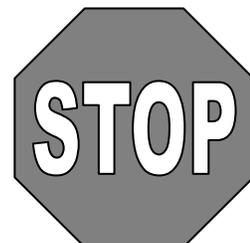
Is $76 + 45 - 9 = 121 - 9$ true or false?

True

False

Can't tell without subtracting

How do you know?



SECTION TIME - 5 minutes

7. (ES1) What does the equal sign (=) mean?

Can it mean anything else?

8. (ES3⑥) Which of these pairs of numbers is equal to $3 + 6$? Circle your answer.

a) $2 + 7$

b) $3 + 3$

c) $3 + 9$

d) none of the above

9. (ES4) Which answer choice below would you put in the empty box to show that two nickels are the same amount of money as one dime? Circle your answer.



- a) 5¢
- b) =
- c) +
- d) don't know

10. (ES5) Is this a good definition of the equal sign? Circle good or not good.

- a. The equal sign means two amounts are the same. Good Not good
- b. The equal sign means count higher. Good Not good
- c. The equal sign means what the answer is. Good Not good

11. (ES6) Which of the definitions above is the **best** definition of the equal sign? Write a, b, or c in the box below.

12. (ES7) Please circle your choice.

The equal sign (=) is more like:

- a) 8 and 4
- b) < and >
- c) + and -
- d) don't know



SECTION TIME - 10 minutes

DIRECTIONS: *Find the number that goes in each box.*

13. (OE2) $6 + \square$

14. (OE4) $\square + 5 = 9$

15. (OE6) $\square + 3$

DIRECTIONS: *On these problems, we really need you to show your work by writing down the numbers you add or subtract. Write your answer in the box.*

16. (OE8) $\square = 6 + 2$

17. (OE10) $3 + 6 = 8 + \square$

18. (OE11) $4 + 5 + 8 = \square + 8$

19. (OE13) $\square + 9 = 8 + 5 + 9$

20. (OE15) $8 + 5 - 3 = 8 + \square$

DIRECTIONS: Find the number that goes in each box. You can try to find a shortcut so you don't have to do all the adding. Show your work and write your answer in the box.

21. (OE21) $67 + 84 = \boxed{}83$

22. (OE23) $\boxed{}55 = 37 + 54$

23. (OE25) Find the value of c . Explain your answer.
 $c + c + 4 = 16$

Explain:



For each statement below, check (1) Very Rarely, (2) Rarely, (3) Often, or (4) Very Often

When I do my math work, I try to:

1. Explain to myself why each answer is correct or incorrect
 1 2 3 4
 Very Rarely Rarely Often Very Often
2. Skip and not do the problems that are confusing
 1 2 3 4
 Very Rarely Rarely Often Very Often
3. Connect the new things we are learning to the things that I already know
 1 2 3 4
 Very Rarely Rarely Often Very Often
4. Memorize the answers
 1 2 3 4
 Very Rarely Rarely Often Very Often
5. Double check my answers to make sure that they make sense
 1 2 3 4
 Very Rarely Rarely Often Very Often
6. Ask for help right away if I don't understand something
 1 2 3 4
 Very Rarely Rarely Often Very Often

For each statement below, check (1) Disagree, (2) Disagree a little, (3) Agree a little, or (4) Agree

7. In general, I find math to be very interesting:
 1 2 3 4
 Disagree Disagree a little Agree a little Agree
8. Understanding math is very important to me:
 1 2 3 4
 Disagree Disagree a little Agree a little Agree
9. I'm willing to work really hard to learn about math
 1 2 3 4
 Disagree Disagree a little Agree a little Agree

Appendix I: Construct Map for Mathematical Equivalence
(Rittle-Johnson, Matthews, Taylor and McEldoon, 2011)

Levels	Description	Example
Level 1: Basic Operational	-Operational view of equal sign -Can successfully solve equations with operations – equals – answer structure	$a + b = c$
Level 2: Flexible Operational	-Operational view of equal sign -Can successfully solve equations with answer – equals – operations structure	$c = a + b$
Level 3: Basic Relational	-Relational definition of equal sign -Can successfully solve equations with operations on both sides of equal sign -Involves single digit numbers only	$a + b = c + d$ $a + b + c = d + e$ $3 + 4 = 5 + 2$
Level 4: Comparative Relational	-Relational definition of equal sign -Compares expressions on both sides of equal sign -Recognizes performing the same operation on both sides maintains equivalence -Involves multi-digit number and or multiple instances of a variable	$28 + 32 = 27 + \square$ while stating 27 is 1 less than 28 so \square has to be 1 more than 32

Appendix J: Reliability and Validity of KME Measures

Knowledge of Mathematical Equivalence Assessment (Rittle-Johnson et al., 2011)		
Reliability	Internal consistency	Form 1 = .94 Form 2 = .95
	Test-Retest	Form 1, $r(26) = .94$ Form 2, $r(26) = .95$
Internal structure-dimensionality	Rasch Model	Accounted for 57.3% of variance in the data, second largest factor was 2.2%
	Confirmatory Factor Analysis	One-factor model = .980 Two-factor model = .980 Three-factor model = .981
Convergent validity	Correlation to Iowa Test of Basic Skills	Form 1, $r(26) = .79$ Form 2, $r(26) = .80$

Appendix K: Unit Objectives for Instructional Package

Lesson 1: Given number sentences, students will correctly identify if the comparison symbol used ($<$, $>$, $=$) makes the statement true or false.

Lesson 2: With the aid of algebra tiles and a balance mat, students will solve one-step equations involving addition and subtraction.

Lesson 3: With the aid of a flow map, students will be able to solve one-step equations involving addition and subtraction.

Lesson 4: With the aid of algebra tiles and a balance mat and/or flow map, students will solve one-step equations involving multiplication and division.

Lesson 5: With the aid of algebra tiles, balance mat and/or flow map, students will complete a scavenger hunt activity and complete exit ticket on one-step equations with at least 80% accuracy.

Lesson 6: With the aid of algebra tiles and a balance mat, students will solve two-step equations.

Lesson 7: With the aid of a flow map, students will be able to solve two-step equations.

Lesson 8: With the aid of algebra tiles, balance mat and/or flow map, students will be able to solve two-step equations involving decimals and complete exit ticket on integer two-step equations with 80% accuracy.

Lesson 9: (if needed) With the aid of a flow map, students will compete a review game activity (horse racing) and complete exit ticket on two-step equations with 80% accuracy on lesson probe.

Lesson 10 (if needed): reteach/review for groups not achieving 80% accuracy on exit ticket.

Appendix L: Sample Lesson Plan

Lesson 2 – Balance Addition & Subtraction

Content Standards: 6.EE.2.c, 6.EE.5, 6.EE.7

Practice Standards: 2. Reason abstractly and quantitatively, 3. Construct viable arguments and critique reasoning of others, 5. Use appropriate tools strategically, 6. Attend to precision, 7. Look for an make use of structure

Materials: Pan Balance, blocks, algebra tiles, balance mats, whiteboard and markers

Objective: With the aid of algebra tiles and a balance mat, students will solve one-step equations involving addition and subtraction.

Advance Organizer: Provide students with the following statements and ask them to determine whether or not each statement is true.

1.) $7 = 5 + 7$

2.) $12 - 5 > 4 + 6$

3.) $4(5) < 5 + 17$

Ask

- How do we know if an equation is true?
- Who can tell me what a variable is?

Show: $7 = 5 + x$

Ask: What is different between this and number 1 from the warm-up?

Say: In order to decide what number makes this statement true, we are going to use algebra tiles and a balance mat to help find a rule that will always give us the correct answer.

Investigation:

Part 1: Show students the pan balance scale. Explain that it is used to compare different weights. Ask them when comparison symbol they should use when the scale is balanced. Guide towards saying it is equal.

Place 3 blocks on one side of the scale and 3 blocks on the other. **Ask** students what they would be allowed to add or subtract to keep the scale balanced. After a few examples of adding and subtracting different quantities, ask students what would happen if they only added or subtracted from one side? **Ask** what must always happen for the balance to stay equal.

Part 2: Take out algebra tiles and balance mat.

- Place a little yellow square on the teacher mat and **ask** students what they think the little yellow square is worth? Guide them to saying it is worth 1.
- Flip over the tile so the red side is showing and ask what they think it is worth now. Guide to saying -1.

- Take out the x-bar and place green side up. Pass out a green bar to each student and a bag of 1's. **Ask** the students to decide how much the green bar is worth. Guide students to saying they can't decide because it is shorter than 6 and longer than 5. **Tell** students that they can't measure it with the tiles because it represents an unknown or a variable.
- **Ask** students what a yellow and a red make. If needed remind them that it makes a 'zero pair' so a positive and a negative make 0. Model $2 + 0$ and ask the value. Model $-1 + 0 + 0 + 0$ and ask value. Continue modeling until all students are able to articulate that it does not matter how many zero pairs there are, they can be taken away until only all reds or all yellows are left to determine the value.

Part 3: Now set up the example $7 = 5 + x$ on the balance mat and **ask** students when they think x should be in order to stay balanced. After they provide the answer of 2 show them how to get the answer of 2 on the scale by subtracting 5 from both sides to get the variable alone.

- **Ask** students how they think they can check that the answer is actually 2. Set up the equation again and tell students that since we know x is two, we can take the x out and substitute two tiles in its place. **Ask** students if the scale balances. **Explain** that is how we check our answer by substituting back in.
- Set up the next example. $4 = x - 2$. **Ask** students to predict what the x should be. **Model** adding 2 to both sides. **Say** that since we have two reds and two yellows they cancel to equal zero so we can slide them off the mat. Now we see that $x = 6$. **Ask** students how to check. Set up the equation again, then take out the x and put in 6 instead. Show how 2 zero pairs are made so that both sides equal 4 and the equation balanced.

Guided Practice: Set of the example $4 + x = 9$ and have students do the same on their mats. **Ask** what the equal sign means. **Ask** students how they can isolate (get by itself) the variable. Together model taking 4 from both sides. **Ask** what the answer is. **Ask** students how they can check to make sure their answer is correct.

With the students set up the equation again and substitute in 5, count to make sure both sides are balanced.

Set up example $2 = x - 1$. Complete the same procedures as above.

Independent practice.

Provide the students with the following examples and monitor as they complete each one.

$$x + 4 = 7 \quad 8 = 2 + 8 \quad x - 5 = 3 \quad 7 = x - 1$$

Closure.

Think-pair-share: Read the objective to the students and ask them to think how the activities of the lesson helped move them towards meeting the objective. What were the big ideas from the lesson? Then have students discuss with a partner and finally have partners share out as group.

Exit Ticket: Provide students with the equations $5 = x + 3$ and $7 = x - 2$ and ask them to select one of the equations to solve and provide a written description of the process involved to get the solution.

Appendix M: Fidelity of Treatment Checklist

Observer:		Date:	Time:
Directions:			
Indicate the observed behaviors by placing a check mark in the spaces below.			
Item	Description	Observed?	Notes
Advanced Organizer:			
1	Review of prerequisite skills		
2	Lesson objective stated at the beginning of instruction		
Investigation:			
4	Maximizing student engagement via questions and prompts		
5	Modeling the thinking and action for procedures needed to solve the problem		
6	Prompting questions to facilitate student exploration		
Multiple Practice Opportunities:			
7	Opportunities for students to practice tasks demonstrated or explored. Teacher acts as facilitator. May include guided practice and/or individual work.		
Visual Representations:			
8	Multiple opportunities to utilize a variety of visual representations (i.e. algebra		

	tiles, drawings, graphic organizer)		
CCSS Practice Standards: <i>(All practice standards do not need to be present in every lesson, each lesson should include a minimum of 4)</i>			
9	Standard 1: Make sense of problems and persevere in solving them <i>Look for: Persevering, trying alternative methods</i>		
10	Standard 2: Reason abstractly and quantitatively <i>Look for: Manipulatives</i>		
11	Standard 3: Construct viable arguments and critique the reasoning of others <i>Look for: explaining/justifying with peers/teachers</i>		
12	Standard 4: Model with mathematics <i>Look for: Graphic Organizers</i>		
13	Standard 5: Use appropriate tools strategically <i>Look for: Using manipulatives, graphic organizers if needed</i>		
14	Standard 6: Attend to precision <i>Look for: Equal sign as relational symbol</i>		
15	Standard 7: Look for and make use of structure <i>Look for: Applying skills learned to novel situations</i>		
16	Standard 8: Look for and express regularity in repeated reasoning <i>Look for: shortcuts, attending to details</i>		

Closure:			
17	Review the main ideas at the end of the lesson		
18	Assessment, which includes student completing an independent task or responding orally to teacher's questions.		

Appendix N: Fidelity of Assessment Checklist

Observer:		Assessment:	Session:
Directions: Indicate the observed behaviors by placing a check mark in the spaces below.			
		Observed?	Notes
1	Teacher read directions for assessment verbatim to the students.		
2	Teacher provided the correct amount of time for each measure.		
3	Teacher did not provide any assistance to students other than clarifying directions or saying "do the best you can."		
4	Teacher provided blank copies of graphic organizer to only students that requested it.		

Appendix O: Social Validity Measure (Students)

Part 1: Please indicate the degree to which you agree with the following statements.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
I am confident in my ability to solve one-step equations.	1	2	3	4	5
I am confident in my ability to solve two-step equations.	1	2	3	4	5
The use of the algebra tiles helped me to see what I was doing when I solved the equations.	1	2	3	4	5
The use of the flow map helped me to solve two-step equations.	1	2	3	4	5
The use of the balance helps me to remember that the equal sign means that both sides have to be the same.	1	2	3	4	5
Working with the teacher on this intervention was worth my time.	1	2	3	4	5
I would recommend this intervention to other students.	1	2	3	4	5
As a result of this intervention I feel better about my math skills.	1	2	3	4	5

Part 2: Open Response

How did the intervention help you to solve equations?

What did you like best about the intervention?

What did you like least about the intervention?

How do you think the intervention will help you outside of math?

Do you have any suggestions on how we can make it better?

Appendix P: Social Validity Measure (Instructors)

Part 1: Please indicate the degree to which you agree with the following statements.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
The intervention was not difficult to implement	1	2	3	4	5
The intervention would be able to be taught by typical math teachers.	1	2	3	4	5
The intervention would be able to be taught by typical special education teachers.	1	2	3	4	5
The intervention would be able to be taught by other intervention agents (i.e. PAM, etc.).	1	2	3	4	5
This intervention could be implemented within the regular classroom.	1	2	3	4	5
The intervention could occur within the normal constraints of the school.	1	2	3	4	5
The intervention is needed to help students be successful in math.	1	2	3	4	5
Solving equations is an important topic for students to be proficient in.	1	2	3	4	5
I felt that it was worth my time to implement this intervention.	1	2	3	4	5

Part 2: Open Response

- What did you like best about the intervention?
- What did you like least about the intervention?
- Did you have any concerns about the structure of the intervention?
- Were there any particular barriers to implementation in terms of time, training, resources, and or supports?
- Do you have any suggestions for improving the intervention?
- Any additional thoughts or comments?

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