This dissertation explores the role of strategic behavior in financial markets and highlights the effects of such behavior on portfolio choice, trading behavior and asset prices.

In Chapter 1 I study portfolio choice of strategic fund managers in the presence of a peer-based underperformance penalty. While the penalty generates herding behavior, correlated trading among managers is exacerbated when a strategic setting is considered. The equilibrium portfolios are driven by the least restricted manager, who may vary according to the realization of returns. I compare model predictions to evidence from the Colombian pension fund management industry, where six asset managers are in charge of portfolio allocation for the mandatory contributions of the working population. These managers are subject to a peer based underperformance penalty, known as the Minimum Return Guarantee (MRG). I study trading behavior by managers before and after a change in the strictness of the MRG in June 2007. The evidence suggests that a tighter MRG results in more trading in the direction of peers, a behavior that is more pronounced for underperforming managers. I show
that these findings are consistent with the qualitative and quantitative predictions of the theoretical model.

In Chapter 2 I explore the limits to the allocational role of stock prices in a strategic setting. Stock prices are thought to help firms’ managers make more efficient real investment decisions, because they aggregate information about fundamentals that is not otherwise known to managers. In this chapter I identify a limitation to this view. I show that if informed traders internalize that firms use prices as signal, stock price informativeness depends on the quality of managers’ prior information. In particular, managers with low quality information would like to learn about their own fundamental by relying on the information aggregated in the stock price. However, in this case, the profitability of trading falls for informed speculators, who therefore reduce their trading volume, reducing the informativeness of prices. As a result, stock prices are not as useful to guide capital towards the most productive use, leading to inefficient investment decisions. Using a sample of U.S. publicly traded companies between 1990-2010, I document a positive correlation between the quality of managerial information and stock price informativeness. Contrary to the conventional view that less informed managers should rely more on stock prices when making investment decisions, I find no differences in the sensitivity of investment to stock price for different levels of managerial information. The evidence suggests that while firms do learn from prices, the learning channel and its effects on real investment are limited.
STRATEGIC BEHAVIOR IN FINANCIAL MARKETS

by

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Dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park in partial fulfillment of the requirements for the degree of Doctor of Philosophy 2014

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Dedication

To my family
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Chapter 1: Strategic Interactions and Portfolio Choice in Money Management: Theory and Evidence

1.1 Introduction

In financial markets, institutional investors manage a significant portion of the total assets and comprise an even greater portion of the trading volume. Given the size of the portfolio management industry, models addressing the agency issues of delegated portfolio management and their effects on asset pricing have become popular over the last few years.

In this paper, I study portfolio choice of strategic fund managers in the presence of a peer-based underperformance penalty. While the penalty might generate crowd effects among managers even under competitive behavior, correlated trading is potentially exacerbated when strategic behavior is considered.

Relative performance concerns among managers may be present for several reasons. The most common explicit compensation schemes in the asset management industry depend linearly on the volume of assets under management and non-linearly on excess performance relative to a benchmark (as exemplified by success fees or performance bonuses). Another implicit source for relative performance concerns is
the potential increase in funds flowing towards the best performing managers. Such empirical regularities have been documented by Chevalier and Ellison (1997) for mutual funds and Agarwal et al. (2004) for the hedge fund industry. The evidence suggests that a manager will get additional money flows and thus a higher future compensation if her relative return is above a threshold.

A less studied source of relative performance concerns comes from regulation. In particular, in countries that have moved from pay-as-you-go (PAYGO) pension systems to Defined Contributions (DC) systems based on individual accounts, regulation typically includes a Minimum Return Guarantee or an underperformance penalty levied on portfolio managers. The rationale for such regulations is to discourage excessive risk taking by the managers of these accounts. In most cases, the formula to determine the MRG is calculated based on peer performance. Hence, managers have an explicit reason to care about the returns of their peers.

Relative performance concerns generated by excess performance fees or the performance-flow relationship typically imply a convex payoff based on relative performance, which gives rise to more risk taking among managers. In contrast, an underperformance penalty represents the opposite kind of performance incentive—a serious penalty for being the loser, as opposed to a big price for being the winner—and therefore one would expect to find the opposite sort of behavior, namely herding. More specifically, an underperformance penalty based on peer returns introduces an explicit reason for managers to track each others’ portfolios, possibly generating

\[1\] See for instance Turner and Rajnes (2001) for a review on these systems. Castaneda and Rudolph (2010) present a theoretical analysis of portfolio choice under peer-based and index-based MRG.
crowd effects as managers minimize the risk from behaving differently from others. Of course managers might also herd into (or out of) the same securities over some period of time for other reasons not related to underperformance penalties. First, managers may receive correlated private information, perhaps from analyzing the same indicator (Hirshleifer et al. (1994)). Second, a manager might infer private information from the prior trades of better-informed managers and trade in the same direction (Bikhchandani et al. (1992), Sias (2004)). Third, managers might disregard private information and trade with the crowd due to the reputational risk of acting differently from other managers (Scharfstein and Stein (1990)). Finally, managers might simply have correlated specific preferences over certain types of securities. An underperformance penalty, such as the MRG, resembles a reputational risk, in that the manager might be penalized for having lower returns than her peers. With the MRG, the risk is explicit as the manager will be penalized financially if returns are below the maximum allowed shortfall relative to the peer benchmark.

When the number of competing money managers is small, relative performance concerns might lead to strategic behavior. In this environment, strategic interactions imply that a manager’s optimal portfolio choice needs to take into account the impact of his trades on other managers’ decisions. In such setting, the effects from a peer performance penalty on portfolio strategies and trading dynamics might be more pronounced.

The DC pension industries in several Latin American and Eastern European countries are natural candidates to display strategic interaction, because they consist of a small number of competing Pension Fund Administrators (PFAs), who act as
asset managers and make portfolio choices on behalf of the working population.

I use detailed data on the security allocations chosen by Colombian PFAs between 2004 and 2010 to study the strategic interaction between managers under relative performance concerns. The Colombian institutional set up satisfies several key conditions necessary for testing for strategic behavior of managers. PFAs manage the savings of a captive market, namely individual retirement accounts, so their set of competitors is restricted to other PFAs, and excludes other asset managers. Each PFA must comply with a MRG that is calculated based on peer performance, creating an explicit incentive to care about the portfolio choice and performance of other managers. Since the number of PFAs is small (six), strategic behavior might be more pronounced than with a large number of competitors. Previous empirical work on strategic behavior has examined data on managers’ broad asset allocation or overall portfolio returns. By using monthly detailed portfolio holdings, I am able to test richer implications of models with strategic behavior. Finally, the Colombian government changed the MRG formula in June 2007, increasing the maximum allowed shortfall and thereby loosening the MRG. This policy experiment allows me to measure the change in behavior associated with the change in the underperformance penalty, arguably holding constant other possible explanations for correlated trading.

The evidence suggests that a tighter MRG results in more trading in the direction of peers and a smaller cross-section dispersion of returns between pension funds. Moreover, the ranking among managers in terms of performance seems to play a role in portfolio balancing decisions. With a tighter MRG, underperforming
managers are more likely than their competitors to trade in the direction of their peers. This is done by buying stocks in which the manager has smaller weights in her portfolio relative to her peers, as opposed to selling stocks with larger weights relative to her peers.

I next present a partial equilibrium model of portfolio choice in which a small number of managers behave strategically in the presence of relative performance concerns. Relative performance concerns arise from a peer-based underperformance penalty similar to the MRG. Fund managers are endowed with the wealth of a group of fund investors and charge a management fee for carrying out the investment strategy. Managers take prices/returns as given and are strategic in the sense that they internalize that their choices affect the strategy of their peers and vice-versa due to the underperformance penalty. I calculate the equilibrium policies and trading strategies. I calibrate the model to the data before the change in regulation and simulate the quantitative effects of a change in regulation comparable to the one carried out by the Colombian government. The model captures the observed change in behavior after the change in policy. In particular it can account both qualitatively and quantitatively for the observed changes in the extent of correlated trading and the observed increase in dispersion of returns across managers.

The model shows that the presence of relative performance concerns through a peer based underperformance penalty affects the asset allocation in two distinct ways. First, the MRG regulation gives rise to time varying investment policies, with the managers making procyclical trades, buying more of the asset that performed well in the previous period. Second, given the strategic nature of the managers, the
model suggests that the equilibrium portfolios are driven by the choices of the least restricted manager (i.e. the manager that has superior accumulated returns at any period in time), while the more restricted manager is likely to end up selecting a portfolio that is similar to her competitor. The strategic nature of the managers exacerbates the extent of procyclical trading, as both the more and less restricted manager recognize their relative position and the action of their competitor. For example, the overperforming manager moves heavily towards her normal portfolio (the optimal portfolio without relative performance concerns) as she recognizes that the underperforming manager, who is more exposed to the penalty, will rebalance her portfolio in the same direction. The overall extent of this correlated trading is more pronounced than in a setting with no strategic interactions among managers.

The rest of the document is organized as follows: In section 1.1.1, I review the leading literature on strategic behavior by fund managers. The empirical evidence is presented in section 1.2, where I conduct two empirical exercises that describe the overall trading behavior of Colombian PFAs. In section 1.3, I introduce a model of strategic fund managers in the presence of an underperformance penalty. I calculate the optimal portfolio choice and trading strategies. Finally, in section 1.4, I present the conclusions and discuss future work.

1.1.1 Related Literature

This paper is related to several strands of the literature. The empirical literature on strategic behavior of money managers has focused on the trading strategies of asset managers competing for leadership to gain status, higher compensation
or increased future flows of funds. The game is similar to a typical tournament, where winners get a large prize and the losers end up with much less. In such tournaments, managers optimally increase their risk taking to maximize the probability of reaching a top position at some target date (usually at year end). Using U.S. data, Chevalier and Ellison (1997) document strong gambling incentives among top-performing mutual funds. Examining strategic behavior in the context of fund families, Kempf and Ruenzi (2008) document that mutual fund managers belonging to families with a small number of funds behave differently from managers belonging to large families. They argue that this result is driven by strategic interactions that might be more pronounced in small fund families. For UK funds, Jans and Otten (2008) present evidence of strategic behavior, finding that fund managers recognize the impact of their own decisions on the actions of their peers, rather than treating competing managers as exogenous benchmarks. A possible explanation for the findings of Kempf and Ruenzi (2008) and Jans and Otten (2008) is that, with strategic managers, the interim leader expects the laggard to increase risk, and therefore the leader also increases risk to maintain his lead (Taylor 2003).

I complement this literature by presenting empirical evidence on the trading behavior of Pension Fund Administrators in Colombia, where a small number of managers compete and set their strategies to avoid a peer-based underperformance penalty. With only six PFAs, it is highly likely that managers are strategic, as they recognize that the other managers will react to their own portfolio choice as it will affect each manager’s future compensation. In contrast to the previous literature, where risk taking behavior arises as managers try to outperform their peers, I study
the effects of the opposite kind of performance incentive, a serious penalty for being
the loser. In this setting one would expect to find the opposite outcome, meaning
herding among managers.

Despite strong theoretical foundations and a common perception that profes-
sional investors herd, earlier studies found little evidence of herding behavior, and
in most cases herding was mostly associated with only particular types of assets,
like small stocks (Wermers (1999) for US mutual funds and Lakonishok and Vishny
(1992) for US pension funds). In a more recent study, however, Sias (2004) shows
that changes in security positions of institutional asset managers over a quarter
are strongly correlated with the trades of other institutions over the previous
quarter. The author also finds that changes in positions on particular stocks are
weakly but positively related to returns over the following year. The results favor
the hypothesis that herding is a result of institutions inferring information from each
other’s trades. Raddatz and Schmukler (2012) find that Chilean pension funds, also
subject to a peer-based MRG, tend to herd, buying and selling the same assets at the
same time. The authors also find differences in the extent of herding across assets.
By comparing the trading behavior of PFAs before and after the MRG change, I am
able to identify the effects of the underperformance penalty holding other factors
constant.

On the theoretical side, papers on portfolio choice with underperformance
constraints include Deelstra et al. (2003) and Tepla (2001). In a general equi-
librium framework Cuoco and Kaniel (2009) study asset price effects of different
performance-based fees for money managers, including both excess performance
bonuses and underperformance penalties. However, these authors abstract from the potential role of strategic interactions in trading behavior. In a partial equilibrium framework, Basak and Makarov (2012) model strategic interactions between two managers competing for additional flows. My model is related to theirs in that I assume a discrete and small (two) number of strategic managers, but I focus on the effects of an underperformance penalty based on peer returns, as opposed to the effects of payoffs that are convex in portfolio returns due to a flow-performance relationship. In my setting herding is the optimal strategy, in contrast to gambling which can be present when managers face a convex payoff structure as in Basak and Makarov (2012). In my model, the overperforming manager moves heavily towards her preferred portfolio as she recognizes that the underperforming manager has to rebalance her portfolio in the same direction. The extent of this correlated trading is more pronounced given the strategic nature of the managers, and the combined portfolio follows the preferences of the least restricted manager.

1.2 Empirical Evidence

1.2.1 The Colombian Private Pension Industry

In 1993 the Colombian Congress approved Law 100, which among other reforms introduced major changes in the pension system. The country adopted a dual pension scheme, in which a defined contribution (DC) system of individual accounts was created in addition to the already existing defined benefit system. Under the new system, pensions were financed by compulsory contributions made by both
the employer and the employee. The law also provided guiding principles for the establishment, operation and supervision of Pension Fund Administrators (PFAs). Under the new scheme, all workers who chose the DC system were required to select a PFA to manage their retirement accounts. The worker’s investment decision was restricted to the choice of the PFA, while the government regulated PFAs’ portfolio strategies by imposing limits on specific asset classes and individual securities\(^2\) and through other provisions such as banning short selling. Workers were allowed to switch PFAs every six months.

The law also determined the compensation structure of the PFAs and the Minimum Return Guarantee (MRG). The PFAs were allowed to charge fees for collecting contributions, managing the fund and giving benefits. In particular, PFAs charge a front end load fee of 5.5\% on new contributions. On average, the fee on new contributions represents close to 90\% of the annual compensation of the PFAs. If a worker makes consistent contributions he does not face any additional charges\(^3\).

The MRG is a lower threshold of returns that each individual PFA needs to guarantee for its investors. If a PFA fails to provide at least this return, the PFA must transfer part of its own net worth to the fund to make up the shortfall. The MRG is assessed monthly by comparing the fund’s average annual return over the previous three years to the average of the six PFAs\(^4\). Between January 2004 and

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\(^2\) In June 2008 some of the limits were: (i) Maximum 50\% in domestic government debt. (ii) Maximum 40\% in equity securities. (iii) Maximum 40\% in foreign securities.

\(^3\) Other smaller fees apply in special cases (e.g. there is a fee when the worker changes PFA, as well as a proportional fee on the value of the account when the worker has not made a contribution for six consecutive months).

\(^4\) A similar provision is in place in other countries, including Chile, Peru, the Dominican Republic and Uruguay.
June 2007, the minimum return guarantee was calculated as the average across PFAs of the average annual return over the previous three years ($\Pi_t$), minus 30%, so that $MRG_t = 70\% \Pi_t$. After June 2007 the government changed the formula to $MRG_t = \min\{70\% \Pi_t, \Pi_t - 2.6\%\}$. For average industry returns below 8.66%, the new formula implies a MRG equal to $\Pi_t - 2.6\%$, as $70\% \Pi_t > \Pi_t - 2.6\%$. Effectively, for this set of returns, the new formula yielded a lower MRG (equivalently, a larger allowed shortfall) than what would have been calculated before June 2007.\footnote{As an example of how this new formula loosened the MRG constraint (increased the maximum allowed shortfall) consider the date December 31, 2009. Between December 31, 2006 and December 31, 2009 the industry annual average returns were 6.01%. With the new formula in place, the MRG was 3.41%, instead of the 4.20% that would have occurred under the older formula.}

Within this institutional setting, the MRG creates an explicit reason for each PFA to track peer portfolios and performance. The penalty for falling too far behind the industry average returns may lead the PFA to bankruptcy. Given the size of each PFA, and the total value of assets under management, a typical Colombian PFA falling 50bps below the MRG threshold would use up its entire net worth compensating its investors.\footnote{In the 15 year history of the private pension system (between 1996 and 2010), no PFA ever yielded returns below the MRG. Even in the turmoil of October 2008, the PFA with the lowest returns managed to have returns 118bps above the MRG (this is the closest any PFA was to the MRG in the sample period).}

With such a severe penalty, one should expect that the MRG is of first order importance when PFAs set their strategies.

Data on Colombian pension funds was provided by ASOFONDOS (Colombian Association of Pension Fund Administrators). The database includes the detailed security allocations for the funds managed by each of the six PFAs, on a monthly basis for the period 2004:1 to 2010:12. Summary statistics for this data set are presented in Table A.1 at two-year intervals. As of June 2010, total assets under
management were US$44.1 billion (equal to 17% of Colombian GDP). At that time, 32% of these funds were invested in Colombian stocks, which amounted to 7.1% of the total domestic market capitalization. Throughout the sample period, net flows to these funds were positive, which reflects the fact that most of the workers contributing to these funds were still young (more than 70% were younger than 40 years old).

In addition to the pension funds, PFAs manage voluntary retirement funds in separate accounts. These voluntary accounts supplement the compulsory retirement savings in the pension funds. Contrary to pension funds, these accounts are subject to very few regulations. In particular, they are not subject to the MRG and do not have limits on individual securities or asset classes. Moreover, workers are typically directly involved in the asset allocation of their voluntary portfolios. Panels D and E in Table A.1 present summary statistics of the voluntary funds.

In the following sections I present two empirical exercises suggesting that relative performance concerns are important for the portfolio dynamics of PFAs. For this, I introduce two measures that describe trading activity by PFAs. The first is an aggregate fund measure that describes how each manager rebalances her portfolio relative to the peer portfolio. The second focuses on fund trades of individual stocks. For both empirical exercises, I focus on the trading behavior across domestic stocks. While these represent only a fraction of the total portfolio, correlated behavior among managers is likely to be more pronounced for these securities, which display higher dispersion of returns than other assets in PFAs portfolios.

\footnote{The only major restriction that these funds share is the short selling ban.}
1.2.2 Trading Strategies and Relative Performance

In this section I introduce a measure of the direction of a PFA’s trades relative to its peers. The objective is to summarize the trading behavior and strategies of Colombian pension fund managers in a parsimonious way.

At the end of each month, each fund’s location is defined by its portfolio weights. The vector of portfolio weights for a fund $i$ in month $t$ is denoted $w^i_t \in \mathbb{R}^{S+1}$, where each element $s = \{1, 2, \ldots, S\}$ represents a domestic stock in the fund’s portfolio, $w^i_{st} = \frac{\text{shares}^i_{st} \times p^i_{st}}{V^i_t}$. Here shares is the number of shares of stock $s$ held by the fund, $p$ is the stock price and $V^i_t$ is the total value of the fund. The element $w^i_{S+1,t}$ in the vector of portfolio weights represents the fund’s participation in assets other than domestic stocks (i.e. domestic corporate debt and government debt).

For each PFA $i = 1, 2, \ldots, 6$, the average peer fund portfolio has weights denoted by the vector $\pi^i_t = \frac{1}{5} \sum_{-i} w^i_t$, where $\sum_{-i}$ is the sum of all funds excluding fund $i$.

To measure a fund’s trading strategy, or its change in portfolio weights, I first adjust for passive portfolio evolution due to changes in prices. Including changes in weights due to price changes may overstate the degree of coordination among funds. If the gross return of stock $s$ between period $t$ and $t+1$ is defined as $ret_{st}$, the adjusted vector of weight changes for fund $i$ from $t$ to $t+1$ can be denoted $\Delta w^i_{t}$, where each element $s$ is defined by $\Delta w^i_{st} = w^i_{st+1} - \frac{w^i_{st} \times ret_{st}}{\sum_s w^i_{st} \times ret_{st}}$. The last term accounts for the change in the weights due to differences in returns among stocks in the portfolio. To measure the position of fund $i$’s portfolio relative to its peers at period $t$, I calculate a vector of differences between the fund and its
competitors, \( d_i^t = \pi_i^t - w_i^t \). To capture the direction of portfolio weight changes, I measure the angle between the change of a PFA’s weights and the distance from its peers’ portfolio, as follows:

\[
\text{direction}_i^t = \cos(\theta) = \frac{\Delta w_i^t \cdot d_i^t}{||\Delta w_i^t|| \ ||d_i^t||}
\]  

(1.1)

In this specification, \( \text{direction} \) measures the correlation across securities between portfolio weight changes for fund \( i \) and the initial distance between \( i \) and its peers.\(^8\) If fund \( i \) is moving exactly towards its peers, the angle is zero and direction is equal to 1. If the manager is rebalancing the portfolio in exactly the opposite direction of its peers, the angle is 180 degrees and the direction measure equals -1.

Figure A.1 displays two examples of the angle between the vector of weight changes for fund manager \( i \) and the vector of initial distances between \( i \) and the other managers. This figure assumes that there are three securities; given that the portfolio weights add up to one, the third dimension is redundant. Initially, the peer portfolio \( \pi_i^t \) has a larger share of stock A than manager \( i \). In panel (a) the manager increases her participation in stock A, moving towards peers. In panel (b) the manager increases her participation in stock B, moving away from the peer portfolio.

If there is a constraint on short selling, the space becomes a Simplex of portfolio weights and the measure would be naturally biased towards higher values of

\(^8\) A similar measure of direction was first introduced by [Koch (2012)]. Here I define the angle between the active change in weights and the initial distance to the peer benchmark, as opposed to the angle between the active change in weights and the peer benchmark active change in weights as in [Koch (2012)].
direction. For example, if fund $i$ is currently invested only in stock B, it is located along the vertical axis, and the only way to continue to move away from its competitors would be to move along the axis, in which case the angle would be smaller than 180 degrees and direction would be greater than -1. In this example, moving away from one’s peers would mean buying more of what you already own, as opposed to short selling securities in which your peers have larger weights.

Figure $A.2$ depicts the time series behavior of the measure of direction for both pension funds and voluntary funds. For each month in the sample, I calculate the direction of weight changes for each PFA over the next quarter, and take the average across PFAs. A high value indicates that PFAs on average are moving towards their peers. Evidently, for the pension funds, PFAs on average traded more in the direction of their peers prior to the loosening of the MRG formula in June 2007 than after this date. For the voluntary funds, the behavior of direction seems to be same before and after the policy change.

Table $A.2$ presents summary statistics on direction. The statistics are split for the period before and after the loosening of the MRG. For the pension funds, mean direction fell from 0.32 in the early period to 0.14 after the change in the MRG, suggesting that the policy change may have affected managers’ behavior. Furthermore, a Chow test in the direction series indicates a structural break with 99% confidence in June 2007.

Table $A.2$ also reports statistics on the relative performance between pension funds before and after the MRG change. Relative performance with respect to the peer portfolio is defined as $rel_t^i = R_t^i - R_t^{-i}$, where $R_t$ are 36 month returns.
prior to \( t \) (consistent with the measurement period of the MRG). The relative performance variable \( rel_i^t \) measures whether fund \( i \) is over-performing (\( rel_i^t > 0 \)) or under-performing (\( rel_i^t < 0 \)) at time \( t \) relative to the other managers. After June 2007, there seems to be some increase in the cross-section dispersion of PFA returns. If portfolios are less alike, returns are likely to vary more cross-sectionally.

A separate question is whether managers’ strategies depend on relative performance. Panel C in Table A.2 presents the correlation between \( direction_{it} \) and \( rel_i^t \). The negative correlation between relative performance and \( direction \) indicates that before June 2007, PFAs with poor relative performance tended to move more strongly towards peers. After June 2007 there is no evidence that relative performance is correlated with the direction of trades.

To summarize, the loosening in the MRG in June 2007 is associated with three important changes in the data for pension funds: (F1) Less trading in the direction of peers; (F2) An increase in cross-section dispersion of returns between funds; (F3) A disappearance of the negative correlation between relative performance and trading in the direction of peers.

1.2.3 Individual Stocks and Trading Strategies

In this section I further investigate herding behavior using data on individual stock trades. For each stock, the fund’s distance to the peer benchmark is measured as \( d_{st}^i = \pi_{st}^i - w_{st}^i \), where the fund can be overexposed (\( d_{st}^i < 0 \)), underexposed (\( d_{st}^i > 0 \)) or have the same weight (\( d_{st}^i = 0 \)) as its peers. I estimate the following model of a fund’s changes in individual stock weights:
\[ \Delta w_{st}^i = \beta_0 + \sum_{m=1}^{M} \beta_m x_{st}^i + \gamma_0 MRG_t + \sum_{m=1}^{M} \gamma_m MRG_t \cdot x_{st}^i \] (1.2)

where \( \Delta w_{st}^i \) is adjusted for stock returns as in the previous section \[^9\] \( x_{st}^i \) are fund and stock specific characteristics and \( MRG_t \) is a time dummy equal to one for dates before July 2007 and zero thereafter, representing the policy change. The objective here is twofold; first, to determine what fund based characteristics determine PFA trading on individual stocks, and second to measure whether there was any change in the impact of these characteristics after the MRG formula was modified.

More specifically, I set \( x_{st}^i = (d_{st}^i, rel_{st}^i, d_{st}^i \times rel_{st}^i, size_{st}^i, Controls_{st}, Market_{st}) \). Here \( size_{st}^i \) is the share of assets under management of fund \( i \) relative to the industry. The vector of \( Controls_{st} \) contains stock specific variables. I introduce lagged returns at one, three, six and twelve months to account for momentum trading, defined as purchasing (selling) assets with positive (negative) past returns \[^{10}\] This popular investment strategy has been widely documented for institutional investors \[^{11}\] Chan et al. (1996) suggest that momentum trading may be caused by a delayed reaction of investors to the information in past returns and past earnings. I also control for firm size and liquidity, as institutional investors may share an aversion to stocks with certain characteristics, as documented by Wermers (1999), who found evidence that US mutual funds tend to herd in small stocks.

[^9]: Alternatively, a PFA might opt for a “passive” rebalancing of its portfolio by accounting for changes in security prices. For this reason, I also estimate equation (1.2) using the unadjusted change in weights, i.e. \( \Delta w_{st}^i = w_{st+1}^i - w_{st}^i \).

[^10]: Selling past losers can also be explained by window dressing. For US pension funds see Lakonishok et al. (1991).\n
Finally, to verify that the results are driven by managers trading relative to their peers and not by trading relative to a broad market benchmark, I calculate Market Distance as the difference between the IGBC index weight on stock $s$ and fund $i$’s weight in stock $s$ for each period, $Market_{ist}^i = \Pi_{ist}^{IGBC} - w_{ist}^i$. The IGBC is a widely used value and liquidity based index for the Colombian stock market. I also interact this measure with relative performance.

This specification is motivated by Basak et al. (2007), who find different behavior in U.S. equity mutual funds depending on whether managers are ahead or behind the S&P 500 index. In their specification, the authors define risk shifting as an increase in the absolute difference between a fund’s returns and the S&P 500 returns. They regress this variable on an interaction between current relative returns and the market returns. Their question is whether underperforming funds move towards or away from the market index, thereby increasing or decreasing the size of deviations from market returns. My specification is analogous to theirs, in that one of my objectives is to measure whether underperforming funds move towards or away from a reference portfolio, the peers’ portfolio (F3), by increasing or decreasing their holdings of stocks in which they are underexposed or overexposed. Note that while Basak et al. (2007) only observe return outcomes, I observe portfolio weights and thus the actual strategy of each manager. In a setting with a small number of managers it might be hard to distinguish if changes in the cross-section dispersion of returns are due to managers’ strategies or to the realization of stock returns.

Table A.3 documents the results of linear regressions for weight changes $\Delta w_{ist}^i$. The results suggest that regardless of relative performance, managers were more
likely to increase their holdings of stocks in which they were already overexposed after the MRG was loosened in June 2007 than before this date. That is, there was less trading towards peers once the MRG was loosened. This change in behavior associated with the change in regulation is consistent with the average behavior of trading direction presented in Figure A.2 and Table A.2. This result holds for both adjusted and unadjusted measures of weight changes. However, the coefficient of the interaction between the MRG dummy and peer distance seems smaller for the specification with unadjusted weights. This difference is potentially explained by the fact that changes in weights that do not correct for price changes overestimates the coordination between the funds, possibly underestimating the change in behavior after the loosening of the MRG.

Figure A.3 presents differences in marginal effects of distance on adjustments in portfolio weights before and after the policy change \( \left( \frac{\partial \Delta w(MRG=1)}{\partial d} - \frac{\partial \Delta w(MRG=0)}{\partial d} \right) \) as a function of a fund’s relative performance, along with corresponding confidence intervals. Underperforming managers \( (rel < 0) \) were more likely to increase their holdings of stocks in which they were underexposed \( (d > 0) \) prior to June 2007 than after this date. This result for individual stocks is consistent with the decrease after June 2007 in the correlation between direction and relative performance documented in Panel C of Table A.2. To give a sense of the quantitative importance of these estimates, the results indicate that a fund lagging in returns by 200bps relative to its peers, and with 5% underexposure in an individual stock \( s \), would increase the weight on \( s \) by 0.89% more prior to June 2007 than after the MRG was loosened.

For high performing managers (with relative returns above 182bps in Figure
A.3), the estimated marginal effects on distance are negative but not statistically significant. That is, there is no evidence that the change in the strictness of the penalty affected the way top-performing managers traded stocks in which they were overexposed, perhaps because the MRG impacts more strongly the average and worst-performing managers. Top-performing managers, unconstrained by the MRG, might deviate from the peer portfolio in hopes of attracting more funds. However, as the results indicate, the MRG policy change seems to have little effect on the incentives for managers in the high end of the return spectrum.

Column (2) and (4) in Table A.3 add variables including distance from the market portfolio to the benchmark specification and the interactions with the MRG dummy variable. The coefficients for the interactions between MRG, the market portfolio and relative performance are statistically insignificant which suggests that the loosening of the MRG affected managers’ trading strategies relative to their PFA peers in particular, rather than to their position relative to the market portfolio.

**Buy and Sell Strategies**

In a final empirical exercise, I complement the above results by distinguishing buys and sells of individual stocks. This is a discrete version of the previous specification. Here, fund i’s trading strategy for a particular stock s is measured by whether the fund buys or sells the stock prior to the following period:
\[
(buy^i_{st}, sell^i_{st}) = \begin{cases} 
(1, 0) & \text{if } shares^i_{st+1} > shares^i_{st} \\
(0, 1) & \text{if } shares^i_{st+1} < shares^i_{st} \\
(0, 0) & \text{if } shares^i_{st+1} = shares^i_{st} 
\end{cases}
\]

corrected for stock splits at period \(t\). In this setting, the analog to the direction measure introduced before is as follows: When a fund buys shares in stocks in which it is already overexposed (underexposed), it moves away from (towards) the peer benchmark. When a fund sells shares in stocks in which it is already overexposed (underexposed) it moves towards (away from) the peer benchmark.

Panel B in Table A.1 shows some trading statistics for different months within the data set. For example, in June 2008, PFAs collectively held 44 different stocks and each fund on average had 26.3 stocks in its portfolio. That month, each PFA traded on average 8.33 stocks, with 6.66 of those trades as buys. In this setting a trading strategy is measured by the probability at time \(t\) that fund \(i\) buys or sells stock \(s\) within the set of stocks owned by all PFAs (i.e. the probability of fund \(i\) buys stock \(s\) from among the 44 total stocks is the likelihood that \(s\) was among the 6.6 stocks that fund \(i\) bought that period).

**Note on Short Selling:** As was the case for the direction measure discussed in the previous section, the short selling ban for these funds introduces a bias against using sales to move away from peers. Consider the previous example. The average PFA sold 1.67 stocks during June 2008. Given the short selling constraint, those sells must come from the set of stocks owned in the previous month (25.8 as of May
31, 2007) not from the total set of stocks held by the PFA industry (44 as of May
31, 2007). Hence, a measure of the probability of selling a stock that considers the
total set of securities is naturally biased towards smaller values, which is not the
case for the probability of buying a stock, since a fund can buy any stock in the
peer portfolio, whether owned at the beginning of the period or not. For this reason
I examine buying and selling strategies separately in what follows. Moreover, when
estimating the probability of selling a stock, I condition on stock ownership at the
beginning of the previous period.

I estimate a Probit specification of the probability of buying \((y = buy)\) or
selling \((y = sell)\) a stock as
\[
Pr(y^i_{st} = 1) = \Phi(\beta_0 + \sum_m \beta_m x^i_{st} + \sum_m \gamma_m MRG_t \cdot x^i_{st}),
\]
where \(\Phi\) is the cumulative distribution function of the standard normal, and the
vector of independent variables \(x\) is the same as in equation (1.2).

Columns one and two of Table A.4 document the results of the Probit regres-
sion for the probability of buying a stock \((buy^i_{st})\). Consistent with the results in the
continuous regression, managers were more likely to buy stocks in which they were
already overexposed after the MRG was loosened in June 2007 than before. Mean-
while, an underperforming manager \((rel < 0)\) was more likely to buy stocks in which
she was underexposed \((d > 0)\) prior to June 2007 than after this date. Columns
three and four of Table A.4 present the results from the Probit regression for the
probability of selling a stock \((sell^i_{st})\) conditional on stock ownership. The coeffi-
cients on the interactions between MRG, Peer Distance and Relative Performance
are all indistinguishable from zero, suggesting that the policy change in June 2007
had no impact on how PFAs sold stocks in which they were overexposed, regardless
of relative performance. Given that between 2004 and 2010 the yearly net flows to these funds were about 8.5% of the value of the fund, underperforming managers had the option of reducing their relative participation in any given stock by holding their number of shares constant, as opposed to selling shares.

To summarize the main empirical findings, the evidence suggests that the more strict MRG prior to June 2007 is associated with more trading in the direction of peers, and in particular more buying of stocks in which managers were underexposed. Meanwhile, underperforming managers traded more heavily towards the peer portfolio prior to June 2007, by buying stocks in which they were underexposed, as opposed to selling stocks in which they were overexposed. This asymmetric behavior between buys and sells could be explained by the fact that these funds were growing within the sample period. As I will show in section 2.2, these results are largely in line with the predictions of a model in which money managers behave strategically due to relative performance concerns.

1.2.4 Alternative Explanations

The specification strategy above assumes that the policy change is exogenous to the domestic stocks’ return process. In the estimation I control for stock-specific attributes such as past returns and trading volume. However, one cannot control for all stock characteristics that might have changed after July 2007 and that might have induced the funds to adjust their trading behavior. For example, PFAs might have received more good signals about the fundamentals of stocks in which they were underexposed prior to July 2007 than after, inducing them to buy more of
those stocks before the policy change than after. The shortcoming of this argument is that if a PFA is underexposed in a particular stock relative to the peer portfolio, by construction there must at least one PFA overexposed in the same stock. As favorable new information arrives about a stock, both underexposed and overexposed PFAs should increase their holdings. Hence, one would need some sort of argument for why PFAs with underexposure were the only ones receiving good signals.

Another possible explanation for the results is that PFAs altered their trading strategies due to managerial changes around the time of the policy change. For example, trading strategies might result from changes in management within the firms or shifts in preferences among the top investment officials. A closer look at PFAs’ CEO replacement indicates that, while there were some changes in management over the sample period, there is no evidence of an industry wide event before or after the MRG adjustment.\footnote{12} In terms of preference shocks, interactions between PFAs dummies and the MRG dummy should account for individual PFA changes before and after the policy experiment. However, an industry-wide taste shock occurring in mid 2007 would be indistinguishable from the policy experiment. While such event is unlikely, it cannot be ruled out under the current empirical specification.

\footnote{12} According to ASOFONDOS, four of the six PFAs had only one CEO replacement each during the sample period, occurring on the following dates: October 2006, February 2008, October 2008 and May 2010. The other two PFAs changed their CEO four times each between January 2004 and December 2010.
1.3 The General Model

Motivated by the above empirical findings, I consider a model in which the presence of relative performance concerns among a small number of money managers leads to strategic behavior. In particular, I focus on portfolio choice when there is an underperformance penalty, such as the MRG described in the previous section. I show that the behavior of institutional asset managers in the presence of the MRG is consistent qualitatively and quantitatively with the observed data.

1.3.1 Model Assumptions

I consider a finite horizon economy \( t = 0, 1, \ldots, T \), modeled as follows:

**Securities:** The investment opportunities are represented by a riskless bond and a risky stock. The bond is a claim to a riskless payoff \( B > 0 \). Without loss of generality the net interest rate is normalized to zero (B price is normalized to \( B = 1 \)).

The gross stock return follows a random process with states \( r^s = \{r^H, r^L\} \) and probabilities \( \{p, 1-p\} \). This is a partial equilibrium model as the return process is exogenous. The return between \( t \) and \( t+1 \) on a portfolio with fraction \( \phi_t \) of wealth invested in the stock and \( 1 - \phi_t \) in the risk free bond is denoted by

\[
R_{t+1}^s(\phi_t) = \phi_t (r_{t+1}^s - B) + B
\]  
(1.3)

**Fund Managers:** I consider two fund managers i and j. Each manager
chooses an investment policy \( \phi_t \). For this, they are compensated at time \( T \) with a management fee \( F_{iT} \), which is a function of the terminal value of their portfolio \( W_{iT} \) and that of their competitor \( W_{jT} \). Specifically I assume that

\[
F_{iT} = F(W_{iT}, W_{jT}) = \beta W_{iT} + \gamma W_{i0} \min \left\{ 0, \frac{W_{iT}}{W_{i0}} - \frac{W_{jT}}{W_{j0}} + x \right\}
\]  

(1.4)

In this specification, the fund managers’ compensation at time \( T \) consists of two components: a proportional fee, which depends on the final value of the portfolio \( \beta W_{iT} \), and the underperformance penalty \( \gamma W_{i0} \min \left\{ 0, \frac{W_{iT}}{W_{i0}} - \frac{W_{jT}}{W_{j0}} + x \right\} \) which depends on the manager’s performance relative to the other manager \( j \). Here \( x \geq 0 \) is the maximum allowed shortfall, i.e., \( x = \infty \) implies that the manager is unrestricted, while \( x = 2\% \) means that the maximum return shortfall allowed is 2\% relative to the peer returns. Any cumulative returns below this threshold result in a penalty that reduces the manager’s net fee, possibly to compensate the investors for the lack of returns. The size of the penalty is modeled by \( \gamma \). In the Colombian setting, the PFAs face an underperformance penalty with \( \gamma = 1 \), since under the Colombian MRG the manager pays a penalty that guarantees that the investors’ net returns are exactly the peer benchmark \( \frac{W_{jT}}{W_{j0}} - x \).

Throughout this document I refer to the manager with the lowest returns in any given state as the loser in that state. In this setting strategic interactions arise as each manager needs to consider each other’s policy reaction so as to avoid the underperformance penalty.

Fund managers are assumed not to have any private wealth. They therefore
act so as to maximize the expected utility $E_0[u_i(F(W_{iT}, W_{jT}))]$ given initial wealth $W_{i0}$, subject to the period by period budget constraint $W_{i,t+1} = R^s_{i,t+1}(\phi_{it})W_{it}$.

**The Normal Policy:** In the rest of the document I refer to the normal policy $\phi_{it}^{NP}$ of fund manager $i$ as the optimal portfolio allocation when no relative performance concerns are present. In this case $\gamma = 0$ and each fund manager solves a standard portfolio choice problem. The optimal share in stocks is given by the first order condition

$$p(r^H - B)u_i'(R^H_{i,t+1}(\phi_{it})W_{it}) + (1 - p)(r^L - B)u_i'(R^L_{i,t+1}(\phi_{it})W_{it}) = 0 \quad (1.5)$$

for $t = 0, 1, \ldots, T - 1$.

Fund managers $(i, j)$ are assumed to have CRRA preferences defined over their final wealth, $u_m(W) = \frac{1}{1-\sigma_m}W^{1-\sigma_m}$ with $m = i, j$. To generate differences in the normal policy, I assume that manager $i$ is less risk averse than manager $j$, $\sigma_i < \sigma_j$. Using data from U.S. mutual funds, Koijen (2008) documents substantial heterogeneity in estimates of fund managers’ risk aversion. Portfolios may also differ between managers because of ability, information, or pay for performance incentives. Here, my objective is to study how the introduction of an underperformance penalty such as the MRG causes fund managers to deviate from their normal strategies.

In this paper, I appeal to the Nash equilibrium concept to characterize managers’ strategic interactions in the presence of relative performance concerns. Below, I define the structure of the game between managers at time 0, where each manager
draws up a plan of how she is going to invest throughout the whole time period 
\{0, 1, \ldots, T\}. This definition of the game eases exposition, and is without loss of 
generality, since neither manager would want to deviate from the initial policy choice 
at any subsequent date \( t \), so that the equilibrium policies are time-consistent.

**Information sets:** I consider a complete information game at time 0, in 
which each manager knows all the primitives and parameters of the model described 
above, namely the stock return process, own initial wealth and risk aversion and 
those of the other manager. Finally, each manager knows the functional form of the 
underperformance penalty (\( \gamma \) and \( x \)).

**Strategy sets:** A strategy of manager \( i \) is a function \( \phi_i(t, W_{it}, W_{jt}) \) defined 
over the space \( \{0, 1, \ldots, T\} \times (0, +\infty) \times (0, +\infty) \), where \( \phi_i(t, W_{it}, W_{jt}) \) is manager i’s 
investment policy at time \( t \) for given values of wealth under management, \( W_{it} \), and 
that of her opponent, \( W_{jt} \). For convenience, I will use \( \phi_{it} \) as a shorthand notation 
for manager \( i \)’s time \( t \) investment strategy and drop its arguments.

**Manager’s payoffs:** The manager’s payoffs for policy vectors \( \{\phi_{it}, \phi_{jt}\}_{t=0}^{T-1} \) 
are given as follows. First, period T wealth is obtained by substituting \( \phi_{it} \) and \( \phi_{jt} \) 
into the dynamic wealth process of each manager. Given terminal wealth \( W_{iT} \) and 
\( W_{jT} \), fees are computed according to (1.4), yielding the final payoff.

### 1.3.2 Two Period Model (\( T = 1 \)): Portfolio Choice

I start by solving the model in a two period version of the above economy. The 
objective here is to show how best response and equilibrium policies are calculated 
and to address in the simplest environment the effects of the underperformance
penalty on managers’ equilibrium portfolios.

In a two period and two state economy \((s = \{H, L\})\), for a given initial level of wealth under management \(W_{i0}\), a portfolio allocation \(\phi_{i0}\) by manager \(i\) at period 0 determines two possible values of final wealth \(W_{i1}^H\) and \(W_{i1}^L\) according to equation (1.3). For each value of final wealth there is an associated management fee, calculated using (1.4). With no underperformance penalty \(\gamma = 0\), the manager’s fee is proportional to the final wealth under management \((\beta W_{i1}^s)\). With the underperformance penalty, the net fee depends on whether manager \(i\)’s returns are above or below the peer benchmark \(W_{j1} - x\) for each of the two states. Hence a portfolio choice \(\phi_{i0}\), for a given choice \(\phi_{j0}\), is effectively a choice of a pair of management fees received in the high and low states of the economy.

Figure A.4 depicts the optimization problem in the space of returns. In the left panel, the normal policy is such that in both the high and low states, manager \(i\)’s returns are above her peer or below her peer by less than \(x\). In this case the underperformance penalty is not binding and the manager’s optimal portfolio is her normal policy. In the right panel, if the normal policy was played, the manager’s returns would be below the peer benchmark in the low state (loser in low). The manager optimally chooses a portfolio with less exposure to the risky asset \((\hat{\phi}_{i0} < \phi_{i}^{NP})\), generating smaller returns in the high state but greater returns in the low state than in her normal policy. Basically the manager is giving up a higher income in the high state to increase the income in the low state in order to reduce the penalty. In this example, manager \(i\) still pays the penalty in the low state, but less than what she would have paid had she played her normal policy.
In the rest of the document I will use $\hat{\phi}$ to denote best response functions and $\phi^*$ to denote equilibrium policies.

**Proposition 1.** For a given portfolio choice of manager $j$, $\phi_{j0}$, manager $i$’s best response $\hat{\phi}_{i0}$ is given by

$$\hat{\phi}_{i0} = \begin{cases} 
  a_i + b_i\phi_j - c_i x 
  & \phi_{iNP} \leq \phi_{j0} - \frac{x}{r_H-B} \text{ (loser in high)} \\
  \phi_{iNP} 
  & \phi_{j0} - \frac{x}{r_H-B} < \phi_{iNP} < \phi_{j0} + \frac{x}{B-r_L} \\
  \bar{a}_i + \bar{b}_i\phi_j + \bar{c}_i x 
  & \phi_{iNP} \geq \phi_{j0} + \frac{x}{B-r_L} \text{ (loser in low)}
\end{cases} \tag{1.6} \tag{1.7} \tag{1.8}$$

with $\bar{b}_i, \bar{b}_i, \bar{c}_i, \bar{c}_i \geq 0$. Switching subscripts $i$ and $j$ above yields manager $j$’s best response. The proof of the proposition and formulas for $a_i, \bar{a}_i, b_i, \bar{b}_i, c_i$ and $\bar{c}_i$ in terms of the underlying parameters of the model are presented in Appendix A.3.

**Definition 1.** The Shifting Region is the region in the parameter space such that a manager’s best response policy is to play a strategy different than her normal policy $\hat{\phi} \neq \phi_{NP}$.

In this two period economy, this region is defined by the conditions in (1.6) and (1.8). In other words, we are in the shifting region if $\phi_{iNP} \in (-\infty, \phi_{j0} - \frac{x}{r_H-B}] \cup [\phi_{j0} + \frac{x}{B-r_L}, \infty)$ which is the region to the left of $\frac{W_{j1}}{W_{j0}} - x$ and below $\frac{W_{j1}}{W_{j0}} - x$ in Figure A.4.

In the Shifting Region, the result that $\bar{b}_i, \bar{b}_i \geq 0$ indicates that the manager optimally chooses a portfolio that is shifted towards her peer, buying more or less of the risky asset depending on the other manager’s portfolio. If $x$ increases (e.g. the allowed shortfall is larger, implying a looser MRG) the shifting region becomes
smaller, which means that the manager will play her normal policy for a larger set of parameters.

Moreover, in the shifting region, a larger (smaller) $x$ implies a smaller (larger) shift. As an example, consider the best response when manager $i$ plays to lose in the low state. Here the manager selects an allocation with a greater share in the risky asset than her competitor ($\hat{\phi}_{i0} > \phi_{j0}$). More specifically, the allocation is such that the manager pays the underperformance penalty in the low state (loser in Low). A larger $x$ implies a greater share in the risky asset, or less shift from her normal policy. On the contrary, a smaller $x$ implies a smaller share in the risky asset or a larger absolute shift. A similar analysis can be made for the best response in (1.6). Note that when the manager is playing to pay the performance penalty in the high state (loser in high) she has a lower share of the risky asset than her competitor. As $x$ decreases, her participation in the risky asset increases, moving farther away from her normal policy and closer to her peer.

To summarize, the maximum allowed shortfall $x$ determines both the size of the shifting region and the size of the shift in the best response functions.

**Corollary 2.** In the shifting region, if the manager is playing to lose in the low state, the best response satisfies $\hat{\phi}_{i0} \in [\phi_{j0} + \frac{x}{B-r}, \phi_{NP}^i]$. If the manager is playing to lose in the high state, the best response satisfies $\hat{\phi}_{i0} \in [\phi_{NP}^i, \phi_{j0} - \frac{x}{r-B}]$.

Corollary 2 states that within the shifting region, the maximum shift is up to the point where the maximum allowed shortfall is met.

**Corollary 3.** If $\gamma >> \beta$, manager $i$’s best response in the shifting region is $\hat{\phi}_{i0} =$
\( \phi_{j0} + \frac{x}{B-r-g} \) when playing to lose in the low state, and \( \hat{\phi}_{j0} = \phi_{j0} - \frac{x}{r-n-B} \) when playing to lose in the high state.

Corollary 3 refers to the case when the size of the underperformance penalty \( \gamma \) is significantly larger than the proportional fee \( \beta \). When facing a large enough penalty the manager shifts her strategy to the point where she is never below the underperformance benchmark, and the optimal portfolio implies that the manager hits the maximum allowed shortfall exactly in either the high or the low state, such that \( \frac{W_{sT}^i}{W_{i0}} = \frac{W_{sT}^j}{W_{j0}} - x \), in which case the manager does not pay the penalty and receives the proportional fee \( \beta W_{sT}^j \). In figure A.4 this would look like a horizontal line in the left shifting region as \( \frac{\beta}{\beta+\gamma} \to 0 \). The manager is giving up a higher income in the high state, to guarantee that in the low state she doesn’t have to pay the underperformance penalty.

Given the definition of the MRG in the Colombian case, where the PFA is required to pay in full \( (\gamma = 1) \) any shortfall in returns below the benchmark, the penalty is significantly greater than the average proportional fee \( \beta = 0.008 \) on the assets under management (see Appendix A.5 for more details). In this case, corollary 3 suggests that the managers will chose a strategy to avoid the penalty in every state. As it turns out, as of December 2013, no PFA has ever fell below the MRG threshold. Even in the turmoil of October 2008, the PFA with the lowest returns managed to have returns 118bps above the MRG (this is the closest any PFA was to the MRG in the sample period).

Up to this point, I have characterized the best response functions. In this
setting with the competition restricted to a small number of managers, one should expect that the managers anticipate each others’ reactions to their strategies. In order to describe the behavior of these strategic managers, I appeal to Nash Equilibrium, in which strategies are mutual best responses.

**Definition 2. Nash Equilibrium** A pure-strategy Nash equilibrium is a pair of portfolio choices \((\hat{\phi}_i^*, \hat{\phi}_j^*)\) that solve the fixed point equations \(\hat{\phi}_i^* = \hat{\phi}_i \left( \hat{\phi}_j \left( \phi^*_i \right) \right)\) and \(\hat{\phi}_j^* = \hat{\phi}_j \left( \hat{\phi}_i \left( \phi^*_j \right) \right)\).

Proposition 4 describes these equilibrium policies for each manager and the conditions that determine when each one of these equilibria is played.

**Proposition 4.** The Nash equilibrium policies are given by:

\[
\left( \phi_{i0}, \phi_{j0} \right) = \begin{cases} 
(\phi^N_P, \phi^N_P) & \Phi_{i0} \geq \phi^N_P \text{ and } \Phi_{j0} \leq \phi^N_P \\
(\phi^N_P, a_j + b_j \phi^N_P - c_j x) & \Phi_{i0} \geq \phi^N_P \text{ and } \Phi_{j0} > \phi^N_P \\
(\bar{a}_i + \bar{b}_i \phi^N_P + \bar{c}_i x, \phi^N_P) & \Phi_{i0} < \phi^N_P \text{ and } \Phi_{j0} \leq \phi^N_P \\
(\Phi_{i0}, \Phi_{j0}) & \Phi_{i0} < \phi^N_P \text{ and } \Phi_{j0} > \phi^N_P 
\end{cases}
\]

where \(\Phi_{i0} = \frac{(a_i + b_i \alpha_i) + (c_i - \bar{b}_i \alpha_i) x}{1 - \bar{b}_i \beta_i}\) and \(\Phi_{j0} = \frac{(a_j + b_j \alpha_j) - (c_j - \bar{b}_j \alpha_j) x}{1 - \bar{b}_j \beta_j}\). Moreover, when \(x > (r^H - B) (\phi^N_P - \phi^N_P)\) and \(x > (B - r^L) (\phi^N_P - \phi^N_P)\) the conditions in (1.9) are always satisfied and managers play their normal policy. \(r^H - B > B - r^L\) is a necessary condition for (1.10) to be an equilibrium and \(r^H - B < B - r^L\) is a necessary condition for (1.11) to be an equilibrium.

The equilibrium portfolios take a simple form: (i) Both managers play their normal policy if (1.9) holds. (ii) One manager plays her normal policy and the other
shifts her portfolio if (1.10) or (1.11) hold. (iii) Both shift their portfolio towards their competitor if (1.12) holds.

As expected, for a large enough $x$, both managers play their normal policies, as they are guaranteed under their normal policies of not paying the penalty in either state. Within the shifting region, when condition (1.12) is satisfied, both managers shift their portfolios towards each other, but they do so taking into account that the other manager will also shift, resulting in a shift that is less than in a non-strategic case. In this equilibrium it is as if both managers agree to move from their normal portfolio towards the other manager. As a result, neither individual needs to shift as much. This case is illustrated in panel (a) of Figure A.5. The equilibrium portfolios in (1.10) and (1.11) are particularly interesting as they clearly illustrate the impact of strategic behavior on equilibria. In each case the equilibrium portfolios are highly shifted toward one of the two managers. In (1.10), the less risk averse manager (i throughout this document) plays her normal policy and the more risk averse manager (j) shifts her portfolio, buying a higher share in the risky asset than her normal policy. This case is illustrated in panel (b) of Figure A.5. Suppose that manager $i$ initially conjectures that manager $j$ will play her normal policy. In the shifting region, $i$ responds by playing a portfolio that is shifted towards $j$ ($\hat{\phi}^A_i$ in Figure A.5). For $\hat{\phi}^A_i$, manager $j$ is in the shifting region and responds by increasing her share in the risky asset, thus moving towards $i$. In response, manager $i$ plays a portfolio with less shift, as in $\hat{\phi}^B_{i0}$ in Figure A.5 for which $j$ responds with still more shifting towards $i$. In equilibrium, $i$ plays her normal policy and $j$ does all the shifting even though relative to their normal policies both managers
are in the shifting region. Note that $r^H - B > B - r^L$ is a necessary condition for this equilibrium to exist. With returns skewed to the right, the more risk averse manager $j$ is more exposed to the underperformance penalty, as manager $i$ can select a portfolio to avoid the penalty in the low state, at the same time that manager $j$ has to pay the penalty in the high state.

**The Non-Strategic Benchmark:** As a baseline for comparison, consider the equilibrium if managers were non-strategic. In this case each manager would take the peer portfolio as given and would not internalize that her own strategies affect the strategies of her peers. More specifically, suppose each manager assumes that her peer is playing her normal policy and sets her own strategy accordingly. The equilibrium in this benchmark is characterized according to Proposition 1 by replacing $\phi_j$ by $\phi_j^{NP}$ in manager $i$’s best response and replacing $\phi_i$ by $\phi_i^{NP}$ in manager $j$’s best response. When the two managers are in the shifting region, each individual portfolio is set closer to the peer’s normal policy, and the combined portfolio is close to the average of the normal policies of the two managers. Contrary to this result, in the strategic model, one could have a situation in which even though both managers are in the shifting region, the combined portfolio is heavily tilted towards the preferences of one manager. This arises as both managers internalize that one of them is more exposed to the penalty than the other, and despite both being in the shifting region, only one manager ends up doing all the shifting toward her peer while the other manager plays her normal policy. The risk of falling below the underperformance threshold is reflected by shifts in policy of one manager instead of both.
To summarize the results in the two period model, the shifting region and size of the shift depend among other things on the tightness of the MRG constraint \( x \). A prominent characteristic of the equilibria with strategic managers and an underperformance penalty based on peer performance is that managers might end up playing portfolios that are heavily shifted towards the normal policy of the other manager, more so than in a non-strategic environment.

1.3.3 Three Period Model \((T=2): Trading Strategies\)

To study how underperformance penalties affect manager’s trading strategies, and how this relationship depends on the manager’s current level of underperformance, I solve a three period version of the model with periods \( t=0,1 \) and \( 2 \) (details are presented in Appendix A.4).

The structure of the equilibrium policies in the intermediate period \( t = 1 \) is the same as in the initial period of the two period case. However the conditions under which each manager shifts from the normal policy and the size of the shift varies according to the accumulated returns. More specifically, Proposition A.1 states that in the shifting region, the outperforming manager is likely to play her normal policy, while the underperforming manager does all the shifting. Here the outperforming manager realizes not only that she is far away from the penalty threshold, but that her competitor is lagging in returns and is facing the risk of paying a higher penalty.\(^{13}\)

The main results of the three period model are summarized in Proposition...\(^{13}\)

\(^{13}\) One should note that if both managers have the same returns to start the period \( R_{s1} = R_{s11} \) the equilibrium strategies are exactly the same as in the two period model.
In period 0, either both managers select their normal policies, one shifts and the other plays her normal policy or both shift. After the first period, if high returns are realized in period 1, the manager with the greater initial share in the risky asset (manager i throughout this document) will either increase the share of the risky asset in her portfolio or continue to play her normal policy if that was her equilibrium strategy in the first period. Manager j, who has a smaller initial share in the risky asset, is now more vulnerable to the underperformance penalty, and will thus buy more shares of the risky asset, shifting more from her normal policy. Here both managers end up (weakly) increasing their shares in the risky asset, thus trading in the same direction. If instead low returns are realized in period 1, both managers (weakly) decrease their participation in the risky asset. Using the terminology from the previous section, the model states that the overperforming manager will move toward her normal policy while the underperforming manager moves toward her peer. The game is to follow the leader, where the interim winner moves away from the peer portfolio and the interim loser tries to catch up to minimize the risk of paying the underperformance penalty in the last period.

The size of these portfolio changes, depends among other things on the strictness of the MRG (x). This correlated trading may look as if both managers are chasing returns, but in fact they are simply chasing each other. When a manager is overperforming she is less exposed to the penalty, so she can now move toward her normal policy, which happens to be more of the asset that performed well. The underperforming manager is more constrained by the penalty and realizes that the leading manager will move toward her normal policy, so she will have to shift her
The model shows that the introduction of relative performance concerns through a peer based underperformance penalty affects the asset allocation in important ways. First, the MRG regulation gives rise to time varying investment policies, with the managers making procyclical trades, buying more of the asset that performed well in the previous period. Second, given the strategic interactions of the managers, the model suggests that the equilibrium portfolios and individual shifts are driven by the least restricted manager, and depending on the parameters, the more restricted manager may end up doing all the shifting. Hence the combined portfolio might be highly tilted towards the preferences of the best-performing manager.

As an additional exercise, I calibrate the three period model to the estimated data prior to June 2007 and calculate the change in the trading behavior predicted by the model in response to a change in regulation equivalent to the change in the Colombian MRG formula introduced in June 2007 (details are presented in Appendix A.5). The model does a reasonably good job explaining the observed magnitudes of the decline in correlated trading, the observed increase in dispersion of returns between managers and the observed reduction in the correlation between trading direction and relative performance.

1.4 Conclusions

In this paper I study portfolio choices of strategic fund managers in the presence of a peer-based underperformance penalty. The penalty generates herding be-
behavior, and the extent of correlated trading is exacerbated when a strategic setting is considered.

I document empirical evidence suggesting strategic behavior of asset managers when facing a peer-based underperformance penalty. The evidence is taken from the Colombian pension industry, where six Pension Fund Administrators compete to manage the saving accounts of the working population, and are subject to a Minimum Return Guarantee based on peer performance. The evidence suggests that a tighter MRG is associated with more trading in the direction of peers and a smaller cross-section dispersion of returns between pension funds. Moreover, the ranking among managers in terms of performance seems to affect how PFAs rebalance their portfolio. When the MRG is tight, underperforming managers are more likely than their competitors to trade in the direction of their peers; this is not true when the MRG is slack. Underperforming managers rebalance their portfolio by buying more heavily stocks in which the manager is underexposed relative to her peers, as opposed to selling stocks in which she is overexposed. Since these pension funds were in an accumulation stage during the sample period, with new flows accounting for an average of 8.5% the value of the fund each year, managers were presumably able to reduce their participation in stocks to which they were overexposed simply by maintaining a fixed number of shares.

There is an interesting time dimension that is not studied in this paper; namely managers’ behavior close to the MRG evaluation date. Unfortunately, since the data shows only monthly portfolio holdings, the same time frame as the MRG evaluation period, I cannot study whether PFAs alter their trading strategies within the month.
This is an important question but it requires access to portfolio data at higher frequencies.

Finally, I present a model of portfolio choice and strategic interactions among managers facing a peer-based underperformance penalty similar to the MRG. In the model, two fund managers select the trading strategy on behalf of their investors and are compensated based on their assets under management and their relative performance. The model shows that the introduction of a peer based underperformance penalty induces managers to make procyclical trades, buying more of the asset that performed well in the previous period. Such behavior is exacerbated when managers behave strategically, and the combined portfolio of the managers is highly tilted towards the preferences of the least restricted manager.

In the model analyzed in this paper asset prices and returns are exogenous. Given the size of the assets under management by the PFAs, their procyclical trading might have price effects; for example, procyclical trading might increase both volatility and correlation between stock prices. It would be interesting to study asset prices in a general equilibrium version of the above model where fund managers interact with other market participants. The challenge for such an extension is that since there is only a small number of managers, they might be strategic about the effect of their trading on asset prices, recognizing their market power. Previous work starting with [Lindenberg (1979)] suggests that these considerations might reduce the size of the trades but the direction of trading would still be the same. Finally, since I have data on individual security allocations for PFAs, any model prediction could be further tested with individual stocks. These extensions are left for future work.
Chapter 2: Strategic Information Revelation and Capital Allocation

2.1 Introduction

Do stock prices improve efficiency by directing capital towards more productive uses? A widely held view, dating back at least to Hayek (1945), is that stock prices are useful signals since they aggregate information about fundamentals that is not otherwise known to firms’ managers.\(^1\) In this sense, the stock price of a company might be informative to the manager when making a real investment decision.\(^2\) In a very practical way, informative prices enable superior decision-making (Fama and Miller (1972)).

In this paper I identify a limitation to this view. More precisely, in a model where informed traders in secondary markets internalize that stock prices are signals to firms’ managers, I show that trading volume and price informativeness depend on the quality of managers’ prior information. In other words, the amount of private information that is aggregated into the stock price through the trading process is a function of managers’ initial information. The learning channel is limited because prices are less informative for firms with low quality of managerial information,

\(^1\) Subrahmanyam and Titman (1999) argue that prices are useful to managers because they aggregate investors’ signals about future product demand.

\(^2\) This mechanism has received empirical support in the recent work of Durnev et al. (2004), Chen et al. (2007) and Bakke and Whited (2010).
precisely the case in which managers would like to learn more from the stock market. This happens despite the fact that some market participants are endowed with perfect information. As a result, stock prices are not as useful in guiding capital towards its most productive use.

Aside from identifying a fundamental limitation to the allocational role of stock prices theoretically, I make two novel empirical contributions. Using a sample of U.S. publicly traded companies, I document a positive correlation between the quality of managers’ prior information and stock price informativeness. I find a stronger correlation between prior information and stock price informativeness for firms with higher institutional ownership, which further suggests the presence of strategic behavior by informed traders. Also, contrary to the conventional view that less informed managers should rely more on stock prices when making investment decisions, I find no differences in the sensitivity of investment to stock prices for different levels of managerial information. The evidence suggests that while firms do learn from prices, the learning channel and its effects on real investment are limited.

I model learning from prices as follows. There is a continuum of publicly traded firms facing a real investment opportunity with uncertain net present value. Firms use stock prices to update their prior about their own fundamentals. Informed and noise traders submit demands for the firm’s shares in a secondary market. Informed traders are strategic in that they internalize the effect of their trades on both prices and on the firms’ inference problem.

The main result of the model can be summarized as follows: When firms’
managers are less informed a priori, informed traders realize that their expected trading profits are lower because the firm is less likely to undertake the project in the first place. When trading is costly, an informed speculator does not want to buy the stock of a firm with a potentially good investment project if the investment is likely to be cancelled. Similarly, a trader will not short sell a public firm with a negative NPV project if the firm is not likely to invest. Under these circumstances, traders with private information reduce their trading volume, and in turn prices are less informative about the fundamental. Investment sensitivity to price is lower in this case, as firms recognize that prices contain less information and are less inclined to rely on the stock price to update their prior. Overall, investment efficiency falls as the market signal is not as useful in helping managers distinguish between good and bad projects.

Stylized facts about speculative markets suggest that the best-informed traders are large. In the stock market, arbitrageurs with private information about merger prospects buy and sell significant percentages of the outstanding equity of publicly held companies. It is well recognized in existing literature that large traders take into account their effect on prices in choosing the quantities they trade (Grinblatt and Ross 1985 and Kyle 1985). If an investor has superior information, attempts to use it will “publicize” some of the information and instantly reduce its value. Price-taking behavior would be irrational in this case. This paper extends this logic to argue that large traders internalize that prices are also signals to firms’ managers when making real investment decisions.

This paper is related to a growing literature that studies feedback effects on
equilibrium asset prices. The basic idea of this literature is that if firms use market prices when deciding on their actions, traders should adjust their strategies to reflect this response. On the theoretical side, this paper is closely related to Bond et al. (2010). The authors suggest that when agents (e.g., directors, regulators, or managers) learn from stock prices, there is a complementarity between the agent’s direct sources of information and his use of market data. However, in their model, trading in the secondary market is not modeled explicitly. Instead, the stock price is set through a rational expectations condition, which is then used by the agent who is taking a corrective action. The main drawback in their model is that a rational expectations equilibrium does not always exist, which limits the model’s predictions.

In my model, trading by informed speculators and firms’ real investment decisions are the outcome of a strategic game. By assuming that informed speculators internalize the firms’ inference problem, I am able to show existence of equilibria for any level of managerial information, contrary to the non-existence result in Bond et al. (2010). In this sense, my model has a clear empirical prediction, namely that stock prices are less informative for managers with low prior quality of information. Furthermore, my model allows me to study informed trading behavior when there is learning from stock prices, a feature that is omitted in Bond et al. (2010).

Other papers studying feedback effects include Leland (1992), Dow et al. (2011), Goldstein and Guembel (2008), Edmans et al. (2011) and Goldstein et al. (2012). Leland (1992) shows that when insider trading is permitted, prices better

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3 See Bond et al. (2012) for an excellent survey of this literature.
4 The authors interpret non-existence as indicating a loss of information transmitted by prices.
5 This result holds independently of whether the investment decision is value-increasing or value-decreasing for the firm.
reflect information and expected real investment rises. Dow et al. (2011) find that information production in secondary markets is sensitive to the ex-ante likelihood of the firm undertaking the project and Edmans et al. (2011) study asymmetric trading behavior between good and bad information. Most of these papers assume a discrete space for firms’ fundamentals, typically specifying two possible valuations for the investment project (i.e. high and low). My model assumes a continuous space of firms’ fundamentals, which allows me to study the interaction between the quality of managerial information, trading behavior, stock price informativeness and investment efficiency, issues that to my knowledge have been overlooked by the existing literature.

This paper provides a novel explanation for why markets are limited in their ability to aggregate information and guide real decisions. Shleifer and Vishny (1997) provide an alternative explanation based on limits to arbitrage, in which the slow convergence of prices to fundamentals may deter speculators from trading on their information. Other explanations rely on market frictions such as short selling constraints. For example, Diamond and Verrechia (1987) show that short selling constraints affect the speed of price adjustment to private information. In my model, the ability of prices to fully reflect fundamentals and to coordinate investment is crucially related to the precision of firms’ prior beliefs about their fundamental value.

Empirical evidence that price informativeness is high in well-developed financial systems and low in emerging markets is presented by Morck et al. (2000). The

\footnote{Another example is Goldstein and Guembel (2008), who study price manipulation when traders are uninformed.}
authors argue that in countries with well-developed financial markets, traders are more motivated to gather information on individual firms. My model offers an alternative interpretation of their results. I argue that low price informativeness may result from the failure of stock prices to aggregate information when feedback effects are present and ex-ante fundamental uncertainty is high, which is potentially the case for emerging markets.

Finally, this paper is related to the empirical literature that studies learning from prices. Durnev et al. (2004), Chen et al. (2007) and Bakke and Whited (2010) show that investment sensitivity to stock prices is higher for firms for which the stock price is more informative about fundamentals. My empirical work addresses the determinants of stock price informativeness. The evidence suggests that stock prices are less informative for firms with low quality information ex-ante. I also estimate a standard investment equation as in Chen et al. (2007), and show that, contrary to the conventional view, less informed managers do not rely more on stock prices to make investment decisions. Collectively, the evidence suggests that while secondary market are a useful source of information, they are limited in their ability to guide real decisions.

The rest of this document is organized as follows. Section 2.2 introduces the model economy. Equilibrium results are derived in section 2.3. This section includes a model extension where firms can incur a cost to acquire information about the fundamental before observing the stock price. In section 2.4 I present the empirical exercise and I conclude in section 2.5.
2.2 Model

The model consists of three periods, \( t \in \{0, 1, 2\} \), with three types of continuum agents: firms, informed speculators (one for each firm) and noise traders. Stocks for each firm are traded in a secondary market. Each firm’s manager needs to decide whether to continue or abandon an investment project. The investment decision is taken to maximize firm value (there is no shareholder/manager agency problem).

2.2.1 Firms

The economy is populated by a continuum of firms. At period \( t = 0 \) each firm is uncertain about its own fundamental value \( \theta \), which determines its final profits (for instance, the firm may be uncertain about the viability of a project or future demand). \( \theta \) is unobservable and firms have a common prior \( \theta \sim N(\mu_\theta, \sigma_\theta^2) \).

At \( t = 1 \) firms observe their own stock price \( q \) and decide whether to invest \((d = i)\) or not \((d = n)\). If a firm decides to invest it pays a fixed investment cost \( c > 0 \). In period \( t = 2 \) payoffs are realized for each firm according to

\[
\Pi^d = \begin{cases} 
\theta - c & d = i \\
0 & d = n 
\end{cases}
\]  

(2.1)
2.2.2 Financial Markets

For each stock there is one risk neutral informed speculator. He learns the firm’s fundamental value $\theta$ at period $t = 0$. In this setting I am modeling the extreme case where the speculator is perfectly informed and the firm is not. This simplifying assumption allows tractability. I conjecture that similar results would hold if the speculator has some private information about the firm fundamental that is orthogonal to the firm’s information. This would generate some learning from prices. At date 1, conditional on their information, informed speculators submit a market order $X_I(\theta)$ to a Walrasian auctioneer. I assume that speculators do not observe the price when they trade, and hence submit a market order, as in Kyle (1985). This captures the idea that speculators, when they trade, do not have the market information that the firm will have when making the investment decision (recall that the firm bases its investment decision on the price of the security). I impose no any additional constraints on the demands by the informed speculators, such as short selling constraints. That is, the informed speculators either have deep pockets or have access to financing to buy or sell as many shares as they find profit maximizing.

The Walrasian auctioneer also observes a noisy supply curve from uninformed traders and sets a price to clear the market. The noisy supply for each stock is exogenously given by $X_N(\tilde{z}, q)$, a continuous function of an exogenous supply shock $\tilde{z}$ and a price $q$. The supply curve $X_N(\tilde{z}, q)$ is strictly decreasing in $\tilde{z}$ and increasing in $q$.

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7 This order is not observed by the firm.
$q$, so that supply is upward sloping in price. The supply shock $\tilde{z} \in \mathbb{R}$ is independent of other shocks in the economy, and $\tilde{z} \sim N(0, \sigma_{\tilde{z}}^2)$.

The usual interpretation of noisy supply is that there are agents who trade for exogenous reasons, such as liquidity or hedging needs. They are usually referred to as “noise traders”. In this setting, the presence of noise traders guarantees that prices will not be fully revealing, as there can be different prices for the same fundamental value.

To solve the model in closed form, I assume that $X_N(\tilde{z}, q)$ takes the following functional form: $X_N(\tilde{z}, q) = \epsilon q - \tilde{z}$. The parameter $\epsilon$ captures the elasticity of supply with respect to the price. It can be interpreted as the liquidity of the market: when $\epsilon$ is high, supply is very elastic with respect to the price, and large shifts in informed demand are easily absorbed in the price without having much of a price impact. This notion of liquidity is similar to that in Kyle (1985), where liquidity is considered high when the informed trader has a low price impact. These basic features, i.e., that supply is increasing in price and has a noisy component, are standard in the literature. It is also common in the literature to assume particular functional forms to obtain tractability. The specific functional form assumed here is close to Goldstein et al. (2012). The equilibrium price is given by the market clearing condition $\epsilon q - \tilde{z} = X_I(\theta)$.

In the last period $t = 2$, the informed speculators and noise traders earn a share of firms’ profits proportional to their stock ownership. The model timeline for each firm is depicted in Figure 3.1
2.2.3 Equilibrium

I now turn to the definition of equilibrium in this economy.

**Definition 3. Perfect Bayesian Nash Equilibrium** An equilibrium with imperfect competition among informed speculators and learning from prices is defined as follows: (i) Each informed speculator chooses a trading strategy \( X_I(\theta) \) that maximizes expected profits subject to the market clearing condition \( X_I(\theta) = X_N(\tilde{z}, q) \) and the investment strategy by the firm. (ii) Each firm chooses an investment rule to maximize expected payoffs given the observed stock price \( q \), prior beliefs about their own fundamental value and beliefs about the informed speculator trading strategy. (iii) Each player’s belief about the other players’ strategies is correct in equilibrium.

In other words, an equilibrium is a fixed point in strategies where each firm sets a best response (investment rule) to market prices, given prior beliefs and the informed speculator trading strategy, and speculators set their optimal demands recognizing the price impact of their trades and the firms’ reaction.

2.3 Solving the Model

In this section, I explain the main steps that are required to solve the model. Using the market clearing condition, I start by solving the optimal investment rule by the firm for a given stock price. I then characterize the optimization problem of the informed speculator for the given investment rule. Finally, given the investment rule by the firm and the trading rule by the informed speculator, I calculate the
fixed point.

2.3.1 Firms

After observing the stock price, the firm’s posterior distribution on its fundamental value is

$$\xi(\theta \mid q) = \frac{\varphi(eq - X_I(\theta))\xi(\theta)}{\int_{-\infty}^{\infty} \varphi(eq - X_I(\theta))\xi(\theta)d\theta}$$  \hspace{1cm} (2.2)

where $$\varphi()$$ is the density function of the normal distribution with mean 0 and variance $$\sigma_z^2$$ and $$\xi()$$ is the density function of a normal distribution with mean $$\mu_\theta$$ and variance $$\sigma_\theta^2$$.

Profit maximization implies that a firm with stock price $$q$$ will invest if the expected profit under the posterior is nonnegative, $$\int_{-\infty}^{\infty} \theta \xi(\theta \mid q)d\theta \geq c$$. In this setting, the firm’s decision is a cutoff rule, such that for any $$q \geq \bar{q}$$ the firm will invest ($$d = i$$), where $$\int_{-\infty}^{\infty} \theta \xi(\theta \mid \bar{q})d\theta = c$$, and will not invest ($$d = n$$) if $$q < \bar{q}$$.

**Lemma 5.** If firm managers conjecture a linear demand function by the informed speculators of the form $$X_I(\theta) = a + b\theta$$, then the cutoff price function $$\bar{q}_l$$ is given by:

$$\bar{q}_l = \frac{1}{\epsilon} \left[ a + cb - \frac{1}{b}(\mu_\theta - c)\frac{\sigma_z^2}{\sigma_\theta^2} \right]$$  \hspace{1cm} (2.3)

Proof in Appendix B.3.

**Lemma 5** refers to the functional form of the cutoff price when managers believe that informed speculators’ trades are linear in the fundamental. More precisely, $$\bar{q}_l$$

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8 **Lemma 5** is intended primarily to help build up intuition for the model mechanism. In section
in Lemma 5 is the firms’ best response to linear demands by the informed speculators. The cutoff price is set as an optimal weighting between the prior information of the manager and the price signal. The fraction $\sigma_z^2/\sigma_\theta^2$ represents the ratio of the precision of the stock price to the precision of the manager’s prior information. When the managers’ precision is large relative to the precision of the price signal, $\sigma_z^2/\sigma_\theta^2 \rightarrow \infty$, the cutoff price is $q \rightarrow -\infty$ (always invest) if $\mu_\theta > c$ and $\eta \rightarrow \infty$ (never invest) if $\mu_\theta < c$. In this case, the firm’s investment decision is independent of the stock price, as the manager’s decision is based exclusively on whether the ex-ante expected profits of the project are positive or negative. When the ratio of precisions between the signal and the prior is finite, managers set a finite $\bar{q}_t$, in which case the investment decision depends on the observed stock price.

The cutoff rule also depends on the conjectured trading strategy of the informed speculators, i.e. the parameters $a$ and $b$. For example, if firms believe that informed speculators set their demands independently of the fundamental, e.g. $b = 0$, managers understand that the price signal contains no idiosyncratic information that would be useful to infer the fundamental, and rely only on their prior to make the investment decision.

### 2.3.2 Informed Speculators

The risk neutral informed speculators maximize the expected profits of their trading strategies, $\max_{X_I(\theta)} E[X_I(\theta)(\Pi^d - q) \mid \theta]$, subject to the market clearing condition $X_I(\theta) = X_N(\bar{z}, q)$ and the firm’s investment rule described above. Since I solve the model numerically, in which case speculators demands are not linear and Lemma 5 does not hold.
each informed speculator internalizes his market power, the optimization problem is transformed to

$$\max_{X_I(\theta)} X_I(\theta) E[\Pi^d | \theta] - \frac{X_I(\theta)^2}{\epsilon} \tag{2.4}$$

The first term in (2.4) is expected total earnings given the investment profits. The trader is perfectly informed about the value of the firm’s project, so his expectation is taken with respect to whether the firm will invest or not. The second term in (2.4) is the cost of the trading strategy.

For a firm with fundamental value $\theta$, the probability that the stock price $q$ is above the threshold $\bar{q}$ is $Pr(q \geq \bar{q} | \theta) = \epsilon \int_{\bar{q}}^{\infty} \frac{1}{\sqrt{2\pi}} \varphi(\epsilon q - X_I(\theta)) dq = \Phi \left[ \frac{1}{\sigma_z} (X_I(\theta) - \epsilon \bar{q}) \right]$, where $\Phi$ is the cumulative distribution of the standard normal.

**Definition 4.** Let $\psi(\bar{q}, X_I(\theta))$ be defined as the probability that the stock price $q$ of a firm with fundamental $\theta$ is above the firm’s cutoff rule: $\psi(\bar{q}, X_I(\theta)) \equiv Pr(q \geq \bar{q} | \theta)$.

The informed speculator’s optimization problem becomes

$$\max_{X_I(\theta)} X_I(\theta) [\psi(\bar{q}, X_I(\theta))(\theta - c)] - \frac{X_I(\theta)^2}{\epsilon} \tag{2.5}$$

To summarize, expected trading profits depend on the probability that the firm undertakes the project. When the informed speculator trades, he always incurs the trading cost, which is quadratic the number of shares he demands, while the expected revenue is proportional to the likelihood that the firm undertakes the project. This maximization captures the idea that an informed trader does not
want to buy shares in a firm with a good investment project if the investment is likely to be cancelled. Similarly, is not optimal for a trader to short sell a firm with a negative NPV project if the firm is not likely to invest.

To learn more about the impact of strategic trading and firm learning, below I also consider the following alternatives to the benchmark model:

**Alternative #1 - Perfect information:** Firms learn their true fundamental before making the investment decision. In this case, only firms with profitable projects (good fundamentals) will invest, i.e. $\theta \geq c$. Since firms with bad projects $\theta < c$ don’t invest, speculators don’t trade these companies, while buying $X^1_I(\theta) = \frac{\epsilon}{2}(\theta - c)$ shares for firms with good fundamentals. At time zero, the expected price is zero for bad firms and $E_0q^1 = \frac{1}{2}(\theta - c)$ for firms with positive NPV. Here the expectation $E_0$ is taken over the noise shock. In this case the firm’s stock price is half its expected profit. The results follow from market power, as the informed speculator recognizes that every unit he demands of the stock will increase the price by a factor of $\frac{2}{\epsilon}$.

**Alternative #2 - No learning from prices:** Firms don’t use stock prices $q$ to update their beliefs about fundamentals. If the ex-ante expectation of the investment return is greater than the investment cost, i.e. $\mu_\theta > c$, all firms invest. In this case, the informed speculator demand is $X^2_I(\theta) = \frac{\epsilon}{2}(\theta - c)$. Here, informed speculators take a long position on firms with positive net present value and short positions on firms with negative present value.

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9 For instance, this would be the outcome if the firm is required to make its investment decision at the same time that the fundamental value is revealed to the informed speculator. In this case, the firm cannot use prices to update beliefs at the time of investment.
Alternative #3 - Speculators don’t internalize firms’ updating: In this alternative setting, informed speculators make their trading decision assuming that firms invest without updating their prior. If $\mu_\theta > c$, informed speculators’ demands are $X_3^I(\theta) = \frac{\epsilon}{2}(\theta - c)$. From Lemma 5, firms set their cutoff price as

$$q^3 = -\left(\mu_\theta - c\right) \frac{2\sigma_z^2}{\epsilon^2\sigma_\theta^2}$$

and the probability of investment is

$$\psi^3(\theta) = \Phi \left[ \frac{\epsilon(\theta - c)}{2\sigma_z} + \frac{2(\mu_\theta - c)\sigma_z}{\epsilon\sigma_\theta^2} \right]$$

which is monotonically increasing in $\theta$, indicating that firms with better fundamentals are more likely to invest than firms with bad fundamentals. In this alternative model, firms use the stock price as a signal and therefore make a better and more informed investment decisions. The result arises almost by construction, because some market participant (the informed speculator) is endowed with perfect information which makes the price a good signal to improve the firm’s decision. However, in what follows, I show that this learning channel is limited when the informed speculator internalizes the firm learning process.

I now turn to the solution of the benchmark model, when firms learn from stock prices and informed speculators are strategic in that they internalize both the price effect and the firm’s updating process. Proposition 6 presents the equilibrium.
Proposition 6. There exists an equilibrium with strategic informed traders and firms. The equilibrium in strategies can be approximated around $\theta = c$ as:

(i) If the expected NPV of the investment project is non-negative under the firm’s prior, $\mu_\theta \geq c$, then the equilibrium demands by speculators and firms’ cutoff rule are:

- Informed speculators’ demand: $X_I^*(\theta) = \frac{\epsilon(\theta - c)}{2} \Phi(\gamma)$
- Firms’ cutoff rule: $q^* = -(\mu_\theta - c) \frac{2\sigma^2}{\sigma^2_\theta} \frac{1}{\Phi(\gamma)}$

where $\Phi(\gamma)$ is the cumulative distribution of the standard normal and $\gamma = \frac{2(\mu_\theta - c)\sigma_z}{\epsilon \sigma^2_\theta}$. 

(ii) If the expected NPV of the investment project is negative under the firm’s prior, $\mu_\theta < c$, then informed speculators do not trade in equilibrium, $X_I^*(\theta) = 0$, and firms never invest, $q^* \to \infty$.

Proof in Appendix B.3.

Proposition 6 refers to the informed speculators demands’ and the firm strategy approximated around $\theta = c$. While a closed form solution for the entire space of $\theta$ is not available, the approximated solution provides the relevant economic intuition because it refers to the investment decision and the trading behavior for the marginal firm. More precisely, the approximated solution allows me to compare equilibrium strategies between firms with fundamentals slightly above and below the investment cost. While the comparative statics below are carried out with the
linear approximation, exact numerical solutions are presented later to establish the general validity of the results.\footnote{\textsuperscript{10}}

Case (i) in Proposition\footnote{\textsuperscript{6}} refers to the equilibrium when the ex-ante expectation of the firm’s profit is non-negative. Here, speculators’ demands and the cutoff price are scaled by a factor of $\Phi(\gamma)$ and $\frac{1}{\Phi(\gamma)}$ respectively, relative to Alternative \#3 above, in which speculators do not internalize the firm learning. From here onwards I will refer to $\gamma$ as the precision of managers’ information, which is inversely proportional to $\sigma^2_\theta$ (managers’ uncertainty about fundamentals).

Under Alternative \#3, the cutoff price is strictly decreasing in the managers’ precision, $\frac{d\tilde{q}^3}{d\gamma} = -\frac{\sigma_z}{\epsilon} < 0$. As discussed earlier, in this standard signal extraction problem, managers rely less on the stock price the more confident they are on their prior. In the benchmark model, when traders internalize the firm’s updating process, the cutoff price varies with the managers’ precision as follows:

$$\frac{d\tilde{q}^\ast}{d\gamma} = -\Phi(\gamma) + \gamma\Phi'(\gamma) \frac{\sigma_z}{\epsilon} \Phi(\gamma)^2 \tag{2.8}$$

In this case the cutoff price is also strictly decreasing in managers’ precision ($\frac{d\tilde{q}^\ast}{d\gamma} < 0$, proof in Appendix \textsuperscript{B.3}). However the second term in the numerator in (2.8) has the opposite sign compared to a standard signal extraction problem. In particular, if managers have a more precise prior, the are likely to rely less on the stock price (given by the factor $-\frac{1}{\Phi(\gamma)}$). However, this effect is dampened by the factor $\frac{\gamma\Phi'(\gamma)}{\Phi(\gamma)^2}$, since the manager understands that in this scenario stock prices

\footnote{From here onwards, all the analytical results in the benchmark model make use of this approximation unless stated otherwise.}
will have more private information, as informed speculators trade more in absolute terms \((X_t(\theta) \sim \Phi(\gamma))\) when \(\gamma\) is higher. On the contrary, when \(\gamma\) is low (less precise prior), the firm manager wants to rely more on the stock price to make the investment decision by increasing \(\bar{q}^*\), but he understands that when \(\gamma\) is low, informed speculators reduce their demands in absolute terms, which in turn makes the price signal less informative, dampening the learning channel. I expand on this discussion in the next section.

Finally, case (ii) in Proposition 6 refers to the equilibrium when the ex-ante expectation of the firm’s profit is negative. When \(\mu_\theta < c\), informed traders don’t trade and stock prices are determined solely by noise traders. Since firm managers understand this, they ignore the stock price, making the investment decision based exclusively on their prior information, which results in the investment project being canceled. While this result also holds when traders don’t internalize the firms’ updating process (Alternative #3), it is surprising that even when all agents understand the feedback from prices to investment, stock prices cannot promote better firm decisions by overcoming the information gap between traders and firms.\(^{11}\) Note that in this case, speculators cannot profit on their information despite having perfect knowledge of the fundamental. In this case, the informed trader would be better off taking over the firm as a private equity investor whenever the true \(\theta > c\), since he could then use his information to make efficient decisions for the firm.

The results indicate that the extent of information revelation through prices

\(^{11}\) The key assumption for this result is that the informed trader has no direct communication with the firm.
is sensitive to the ex-ante likelihood of the firm undertaking the project. Dow et al. (2011) has a similar result when studying information production in financial markets. In their model, a continuum of atomistic speculators pay a cost of acquiring information as long as others are also paying this cost. This in turn depends on whether the firm is likely to undertake the project in the first place. The less likely a firm is to invest, the less incentive traders have to produce information about the project. In my model, some speculators are endowed with perfect information and incur trading costs (i.e. the price effect of their own trades). While the model in Dow et al. (2011) studies complementarities between traders and information acquisition, I abstract from such concerns by assuming one informed speculator per firm. However, this simplification allows me to expand to a continuous space of firms’ fundamentals instead of the discrete setting in Dow et al. (2011) with only two possible valuations for the investment project (i.e. high and low). With this extension, my model is suitable for analyzing the interactions between the quality of ex-ante managerial information and trading behavior, stock price informativeness and investment efficiency. In what follows I present a detailed analysis of these interactions.

2.3.3 Informed Trading

Above, I showed that when informed traders do not trade \( X_I(\theta) = 0 \), the stock price depends on noise trading alone, which makes the stock price uninformative about the fundamental. Larger absolute informed speculator demands reflect the presence of informed trading in the stock, which leads to more informative prices.
In other words, price informativeness refers to the amount of information the speculator reveals through the stock price, which in turns allows managers to learn about the fundamental value. To be precise, price informativeness is proportional to informed speculators’ trading volume.

**Definition 5.** Let the informed trading volume $V_I(\theta)$ in a stock with fundamental $\theta$ be defined as the absolute value of informed speculators’ demands: $V_I(\theta) \equiv |X_I(\theta)|$.

**Corollary 7.** Informed trading volume decreases in managers’ uncertainty about the fundamental $\frac{\partial V_I(\theta)}{\partial \sigma_\theta} < 0$, for firms with fundamental value close to the investment cost ($\theta$ in the neighborhood of $c$).

Corollary 7 indicates that informed trading volume is lower, and hence price informativeness is lower, when firm managers are less informed ex-ante about the fundamental. This result is exclusive to the benchmark strategic model, as in all three of the alternative models, informed trading volume is independent of the precision of the firms’ prior. Figure B.2 presents equilibrium demands by informed speculators for different levels of managerial uncertainty $\sigma_\theta$, for the case $\mu_\theta = 1.05$, $c = \epsilon = \sigma_z = 1$. The equilibrium demands in Proposition 6 are a linear approximation around $\theta = c$. Figure B.2 displays exact numerical solutions for the given model parameters. Consistent with Corollary 7, informed speculator demands decrease in absolute value for larger values of managerial uncertainty.

There is one important distinction to be made. Price informativeness in the model is not the same as prices being unbiased. Prices are unbiased if they reflect the correct expected value of the firm. Take the case when $\mu_\theta < c$. According to
Proposition 6, firms never invest and speculators do not trade in equilibrium. The expected price at $t = 0$ is zero for any firm, independently of the fundamental. Hence, expected prices are unbiased as they reflect the fact that the firm is not investing. However, in this situation, prices are not informative, since they are not useful to the firm. In general, when the precision of managers’ prior information falls, prices informativeness falls, even though expected prices correctly reflect the real value of the firm (taking into account the investment decision).

According to Figure 3.2, equilibrium demands in the benchmark model are convex in $\theta$ for $\sigma_\theta > 0$. This result can be rationalized as follows: When speculators have positive information about a firm’s prospects, every share they buy of that firm increases the price of the stock, signaling to the firm that it should continue the project, which is the value-maximizing decision from the point of view of the speculators. Thus, in this case the incentives of the speculators and the firm are perfectly aligned. Meanwhile, when speculators have negative information about the firm’s fundamental ($\theta < c$), their inclination would be to short sell firm shares. However, speculators realize that every additional unit borrowed lowers the share price, making the firm more likely to cancel the investment project, which in turn reduces the payoff of the short position. As a result, the informed speculators reduce their short position when they have adverse information. This asymmetry in trading by informed speculators with positive or negative news about a firm’s investment outlook was first studied by Edmans et al. (2011) in a setting with a discrete distribution of firms’ payoff. While this is certainly an interesting result, asymmetric trading results from higher order terms in the solution. The first order
effect, namely the reason why informed speculators optimally reduce their trading volume for firms with low quality information ex-ante, is due to the fact that low quality information increases the likelihood that firms won’t invest and speculators lose money whenever they trade and the firm does not invest.

2.3.4 Investment

I now consider the real side of the economy. Corollary 8 presents the ex-ante probability that a firm with fundamental $\theta$ will invest.

Corollary 8. If $\mu_\theta > c$, for firms with fundamental value close to the investment cost ($\theta$ in the neighborhood of $c$), the probability of undertaking the project in the benchmark model is

$$\psi^*(\theta) = \Phi \left[ \frac{e(\theta - c)}{2\sigma_z} \Phi(\gamma) + \frac{2(\mu_\theta - c)\sigma_z}{e\sigma_\theta^2} \frac{1}{\Phi(\gamma)} \right]$$

(2.9)

where $\Phi()$ is the cumulative distribution of the standard normal and $\gamma = \frac{2(\mu_\theta - c)\sigma_z}{e\sigma_\theta^2}$. Proof in Appendix B.3.

The probability of investment is monotonically increasing in $\theta$. Similar to Alternative #3, firms with better projects are more likely to invest than firms with bad projects, suggesting that learning from prices improves the firms’ decision. However, in the benchmark model, the slope of the investment decision with respect to $\theta$ is scaled by a factor of $\Phi(\gamma)$. This indicates that the amount of information revealed through the stock price depends on $\gamma$. In particular, the slope around $\theta = c$ measures the efficiency of the investment decision, or how well firms distinguish between
good and bad investment projects. Figure B.3 depicts the probability of investment for the three alternative models and the benchmark strategic model. When the firm does not learn from prices (Alternative #2), the firm decides solely according to its prior. When \( \mu_\theta > c \), all firms invest and there is no distinction between different types of projects. The investment probability is one for all values of \( \theta \), and the slope around \( \theta = c \) is zero. Under perfect information (Alternative #1), the probability of investment is one for \( \theta \geq c \) and zero otherwise. In this case, firms perfectly differentiate between positive and negative NPV projects. The slope is undefined around \( \theta = c \), but one can think of it as infinity. For intermediate cases, a steeper slope around \( \theta = c \) indicates that managers are making better investment decisions. The main take away from this figure is that the slope around \( \theta = c \) rather than the level of the probability of investment (\( \psi(c) \)) measures investment efficiency in the model.

**Definition 6.** Investment efficiency is defined as the slope of the probability of investment with respect to \( \theta \) around the point \( \theta = c \): \( \frac{\partial \psi(\theta)}{\partial \theta} \big|_{\theta=c} \)

**Corollary 9.** If \( \mu_\theta > c \), the investment decision is less efficient with strategic traders than in the non-strategic alternative, i.e. \( \frac{\partial \psi^3(\theta)}{\partial \theta} \big|_{\theta=c} > \frac{\partial \psi^*(\theta)}{\partial \theta} \big|_{\theta=c} \). Proof in Appendix B.3.

Corollary 9 refers to the fact that the slope of \( \psi(\theta) \) around \( \theta = c \) is greater in Alternative #3 than in the model with strategic behavior, as shown in Figure B.3 for a particular set of parameters. More precisely, learning from prices improves investment efficiency in both models, but this improvement is smaller in the strategic
model.

In a standard signal extraction model, an uninformed manager is more likely to rely on the outside signal to learn about the fundamental. In such a case, one would expect that managers with less precise prior information rely more on the stock price to make their investment decision. In other words, the sensitivity of investment to the stock price should be higher for less informed managers. To study whether that intuition still holds in the benchmark model, I calculate the correlation between the expected stock price and the investment probability.

**Definition 7.** The correlation between the expected stock price and probability of investment is defined as follows:

\[
\text{Corr}(q, \psi) = \frac{1}{\epsilon} \int \left[ X_I(\theta) - \bar{X}_I \right] \left[ \psi(\theta) - \bar{\psi} \right] d\xi(\theta)
\]

where \( \frac{1}{\epsilon} \bar{X}_I \) and \( \bar{\psi} \) are averages of the expected price and investment probability respectively, taken with respect to the space of fundamentals \( \theta \).

**Corollary 10.** The correlation between the expected stock price and the probability of investment is:

1. In the benchmark model: \( \text{Corr}(q^*, \psi^*) = \frac{1}{2} \Phi(\gamma) \phi \left( \frac{\gamma \xi}{\Phi(\gamma)} \right) [\sigma^2 + (\mu - c)^2] \)
2. In the non-strategic model: \( \text{Corr}(q^*, \psi^*) = \frac{1}{2} \phi(\gamma) [\sigma^2 + (\mu - c)^2] \)

where \( \Phi() \) and \( \phi() \) are the cumulative and probability distribution functions of the standard normal respectively and \( \gamma = \frac{2(\mu - c)\xi}{\sigma^2} \). Proof in Appendix B.3
Corollary 10 implies that, all else equal, the correlation between stock prices and investment is increasing in $\sigma_\theta$ in both the benchmark model and the non-strategic alternative, as expected from the standard intuition discussed earlier. That is, managers with lower quality of information a priori are more likely to rely on their own stock price to make investment decisions. However, Corollary 10 implies that the correlation between investment and stock prices increases less with respect to managerial uncertainty ($\sigma_\theta$) in the strategic model than in the non-strategic alternative, i.e. \[
\frac{\partial \text{Corr}(q^*, \psi^*)}{\partial \sigma_\theta} < \frac{\partial \text{Corr}(q_3^*, \psi_3^*)}{\partial \sigma_\theta}. \]

Two salient features of the equilibrium drive this result. First, informed trading volume is lower when managers are less informed about the fundamental. Second, less informed firms increase the cutoff price, but by less in the strategic case than in the non-strategic alternative, because they internalize that prices are less informative. In summary, in the strategic model managers rely less on the stock price to make the investment decision than when informed speculators fail to internalize the learning channel. Figure B.4 presents the correlation between the stock price and the investment probability for different levels of managerial uncertainty, for the benchmark model and for alternative 3.\(^{13}\)

The correlation is increasing in managerial uncertainty for both cases, but less so when traders internalize the firms’ learning from prices.

To summarize, the model has three main implications. (i) For lower quality of managers’ information a priori, there is less trading volume by informed speculators.

\(^{12}\) Another implication of Corollary 10 is that the correlation between the expected stock price and the probability of investment is smaller in the benchmark model when traders are strategic than in the non-strategic model: \[
\text{Corr}(q^*, \psi^*) = \Phi(\gamma) \phi \left( \frac{2}{\Phi(\gamma)} \right) \text{Corr}(q_3^*, \psi_3^*) \text{ and } \Phi(\gamma) \phi \left( \frac{2}{\Phi(\gamma)} \right) \leq 1
\]

\(^{13}\) The figure presents exact numerical solutions.
and lower price informativeness (Corollary 7). (ii) The investment decision is less efficient when traders internalize the fact that firms learn from prices (Corollary 9). (iii) The correlation between expected stock price and investment is smaller when informed speculators behave strategically (Corollary 10).

2.4 Empirical Evidence

In this section I present empirical evidence on the connection between ex-ante managerial uncertainty about firms’ fundamentals and stock price informativeness. I also study how the correlation between investment and stock prices varies for different levels of managerial information. The empirical analysis that follows is based on a sample of U.S. public firms from 1990 to 2010. For each firm I construct two measures of managerial information and uncertainty, and one measure of stock price informativeness. These measures are described below.

2.4.1 Managerial Information and Uncertainty

When making corporate decisions, managers gather information about the outlook of their firms and the profitability of new products and projects. In the model outlined above, uncertainty about a firm’s fundamentals refers to the variance of the firm’s prior distribution of future profits. To measure firm uncertainty about fundamentals, one would need to know not only the firm’s point estimates of expected profits but the entire distribution. To my knowledge there are no surveys at the firm level with probability distributions on future earnings. For this reason,
I rely on two proxies to measure managerial uncertainty about the outlook of their firm.

The first measure is based upon analysts’ earnings forecasts. At the firm level, surveys of analysts’ forecasts typically report first moments, e.g. expected earnings, profits or sales. As a proxy for firm uncertainty I instead use dispersion of analysts’ earnings forecasts from the Institutional Brokers Estimate System (IBES). Empirical evidence suggests that a large fraction of the information used by analysts comes from discussions with firm managers, which also suggests that analysts’ information is not news to the firm (Bailey et al. (2003)). Analysts collect information from each firm and issue their own forecast. The assumption is that managers with more precise information are more likely to convey such information to analysts covering the firm, and thus, one would expect less disagreement in the analysts’ forecasts. On the contrary, more uncertainty about fundamentals is likely to be reflected in more disagreement in the forecasts issued by the analysts covering a firm.\(^{14}\) The caveat, of course, is that consensus among analysts need not imply a high degree of confidence in their point estimates. However, there is a large body of literature that has studied forecast dispersion, and on balance these studies confirm that forecast dispersion is a useful proxy for uncertainty.\(^{15}\)

\(^{14}\) For instance, analysts might be talking to different managers within a firm, and dispersion among analysts’ forecasts could thus reflect disagreement within the firm. Alternatively, disagreement among analysts could reflect precisely the fact that analysts don’t put a lot of weight on what they are hearing from the firm when firms provide noisy information.

\(^{15}\) Earlier papers using forecast dispersion to proxy for uncertainty include Bomberger and Frazer (1981), Lambros and Zarnowitz (1987) and Barron and Stuerke (1998). Using the Survey of Professional Forecasters (SPF), Lambros and Zarnowitz (1987) show a positive correlation between forecast dispersion and uncertainty, where uncertainty is proxied by the spread of the probability distribution of point forecasts. IBES distributes only point forecasts, but the SPF provides both point forecasts and the histogram of forecasts for GDP, unemployment, inflation, and other major macroeconomic variables. In recent papers, Avramov et al. (2009) and Guntay and Hackbarth
For each firm, I construct this proxy for uncertainty using all the forecasts issued by analysts within a fiscal year. Following Gilchrist et al. (2005), dispersion is defined as the logarithm of the fiscal year average of the monthly standard deviation of analysts’ forecasts of earnings per share, times the number of shares, scaled by the book value of total assets. That is,

\[
DIS_{i,t} = \log \left( \frac{\sum_{j=1}^{12} N_{tj}SD_{tj}/12}{ASSETS_{i,t}} \right)
\]

(2.10)

where \( t \) and \( j \) denote year and month respectively. \( N_{tj} \) is the number of shares outstanding, and \( SD_{tj} \) is the standard deviation of the per-share earnings forecasts for all analysts making forecasts for month \( j \).

The second measure of managerial information quality is based on insider trading activity (Chen et al. (2007) and Foucault and Fresard (2013)). Managers should be more likely to trade their own stock and make a profit on these trades if they are more confident in their information. Although managers don’t always trade on information, the premise is that on average, managers with better information will trade more. To build this proxy for managerial information, I obtain corporate insiders’ trades from the Thomson Financial Insider Trading database. I measure the quality of managers’ information with the intensity of a firm’s insider trading activity, \( INSIDER_{it} \), calculated as the ratio of the firm’s shares traded by insiders in a year to the total number of firm shares traded. As in other studies I only include open market stock transactions initiated by the top five executives (CEO, CFO, (2010) study forecast dispersion as a measure of uncertainty about firms’ future earnings. Guntay and Hackbardt (2010) find that dispersion is positively associated with credit spreads, and it appears to proxy largely for future cash flow uncertainty.
COO, President and Chairman of the Board). Since \( INSIDER_{it} \) is a measure of absolute insider trading activity, it captures managerial information but not the direction of such information, that is, whether the firm has a positive or a negative outlook. My second proxy for the firm’s uncertainty about fundamentals is \( 1 - INSIDER \). While insider trades may reveal managers’ firm-specific information not embodied in share prices, a potential drawback to this proxy for uncertainty is that the lack of insider trading might simply indicate that market prices are close to insider’s beliefs about the fundamentals of a firm, rather than indicating low precision of managerial information. Nonetheless, we should expect better informed managers who are more confident about the quality of their information to trade more.

2.4.2 Price Informativeness

To measure the amount of firm specific information contained in stock prices I use price nonsynchronicity. Specifically, I measure price informativeness for a firm as the share of its daily stock return variation that is firm-specific, defined as \( PI_{it} = 1 - R^2_{it} \), where \( R^2_{it} \) is the \( R^2 \) from the regression in year \( t \) of firm \( i \)’s daily returns on market and industry returns. The idea, first suggested by Roll (1988), is that trading on firm-specific information makes stock returns less correlated in the cross-section and thereby increases the fraction of total volatility due to idiosyncratic

\[ ^{16} \text{Foucault and Fresard (2013) and Peress (2010).} \]

\[ ^{17} \text{The market index and industry indices are value-weighted averages excluding the firm in question. This exclusion prevents spurious correlations between firm and industry returns in industries that contain few firms.} \]
This measure is related to price informativeness in the model, in that increased informed trading volume should increase the idiosyncratic volatility of a firm’s stock price.

Firms are matched to their specific three digit SIC industry. I exclude firm-year observations with less than $10 million book value of equity or with less than 30 days of trading activity in a year. I used CRSP data to measure stock returns and Compustat to measure book values. I exclude firms in financial industries (SIC code 6000-6999) and utility industries (SIC code 4000-4999). The final sample consists of an unbalanced panel with 5,607 firms and 33,610 firm-year observations of uncertainty and price informativeness between 1990 and 2010. I detail the construction of all the variables in Table B.1 and Table B.2 presents summary statistics. To reduce the effect of outliers all variables are winsorized at 1% in each tail. Finally, I scale all variables by their standard deviation so that the estimated coefficients are directly informative about the economic significance of the effects.

2.4.3 Empirical Methodology

To estimate the relationship between stock price informativeness and fundamental uncertainty, I consider the following baseline specification:

\[
PI_{it} = \alpha_i + \delta_t + \beta \text{UNCER}_{it} + \gamma X_{it} + \epsilon_{it}, \tag{2.11}
\]

where the subscripts \(i\) and \(t\) represent respectively firm \(i\) and year \(t\). The de-

\[18\] This measure has been used extensively in the literature studying feedback between prices and managerial decisions. See for example [Durnev et al. (2004), Chen et al. (2007) and Foucault and Presard (2013)].
ependent variable is price nonsynchronicity, my proxy for price informativeness. The explanatory variable \( UNCE R_{it} \) measures managers’ uncertainty about fundamentals as captured by one of the proxies discussed in subsection 2.4.1. The vector \( X \) includes control variables such as firm size, measured as the natural logarithm of the book value of assets, level of cash flows and the number of analysts issuing forecasts for each firm. In addition, I account for time-invariant firm heterogeneity by including firm fixed effects \( (\alpha_i) \) and time-specific effects by including year fixed effects \( (\delta_t) \). The coefficient \( \beta \) measures how managerial uncertainty about fundamentals is related to stock price informativeness over time and across firms.

Table B.3 reveals that the coefficients on the dispersion between analysts’ earnings forecasts and insider trading are significantly negative in all specifications. While this does not establish causality, it suggests a robust negative correlation between uncertainty and price nonsynchronicity even after controlling for year and firm fixed effects. Of course this interpretation depends on the assumption that analysts’ earnings forecast dispersion and insider trading capture the quality of managers’ information. That is, to the extent that forecast dispersion and insider trading capture managers’ uncertainty about the firms’ fundamentals, prices seem to be less informative about firm-specific information when the precision of managers’ information is low.

In the model, the negative relationship between uncertainty and price informativeness is derived from two main assumptions. The first is that there are feedback effects from prices to firms’ decisions. The second is that both traders and firms are strategic. This provides a potential test exploiting a priori cross-sectional differences
in the relationship between uncertainty and price informativeness. In particular, institutional investors are typically better informed than individual investors and their trades are more likely to have price effects. This suggests that stocks with larger institutional ownership should exhibit a stronger negative correlation between uncertainty and price informativeness. I test this hypothesis by adding to the baseling regression the interaction between the measure of uncertainty and the percentage of shares held by institutional investors in any given stock \((UNCER_{it} \times INST_{it})\)

\[
PI_{it} = \alpha_i + \delta_t + \beta_0 UNCER_{it} \times INST_{it} + \beta_1 UNCER_{it} + \beta_2 INST_{it} + \gamma X_{it} + \epsilon_{it}. \tag{2.12}
\]

Table B.4 shows that the magnitude of the correlation between uncertainty and price informativeness is indeed greater for firms with a larger share of institutional ownership. The results are similar for both proxies of firm fundamental uncertainty. Overall, the results suggest uncertainty and price informativeness are most strongly related for firms for which strategic behavior is most likely to be present. As shown in the model, such strategic behavior has important implications for how well markets reveal information, and for how firms use stock prices when making investment decisions.

\[19\] Economies of scale imply that institutional investors can acquire information at a lower cost per share traded than individual investors.
2.4.4 Investment sensitivity to price

The model above suggested that there are limits to firms’ ability to use stock prices as a guide in making real investment decisions. This is because stock price informativeness is not exogenous with respect to the precision of the managers’ prior information. When a manager is less informed a priori, strategic informed traders optimally reduce their trading volume, making stock prices less informative about the fundamental. In turn, managers themselves end up relying less on the price signal relative to the alternative case when informed traders are non-strategic.

To test whether the quality of managerial information affects the sensitivity of investment to the stock price, I estimate a variant of a standard empirical investment equation as follows:

\[ I_{it} = \alpha_i + \delta_t + \beta_1 Q_{it-1} + \beta_2 Q_{it-1} \cdot UNCER_{it-1} + \gamma X_{it} + \epsilon_{it} \]  

(2.13)

The dependent variable, \( I_{it} \), is the ratio of capital expenditures in that year to lagged fixed assets. Following other studies on the sensitivity of investment to stock price, I use Tobin’s average Q as a proxy for a firm’s market value. Average Q is defined as a firm’s stock price times the number of shares outstanding plus the book value of assets minus the book value of equity, divided by the book value of assets. The vector \( X \) includes control variables known to correlate with investment decisions such as cash flows and firm size. To test for the effect of the quality of managerial information on the relationship between investment and stock price, I interact lagged managerial uncertainty (\( UNCER_{it-1} \)) with Tobin’s Q. I also control
for the direct effect of the quality of managerial information on investment. Following the specification in the previous section, I include firm fixed effects and year fixed effects.

Results are presented in Table B.5. While the coefficient $\beta_2$ is positive for both proxies of managerial uncertainty, these coefficients are not statistically significant, suggesting that there are no differences in the sensitivity of investment to stock prices for different levels of managerial information. Recall that the model did predict a higher correlation between stock prices and investment for less informed managers (as stock prices do contain some private information not possessed by managers), but this interaction is limited in the benchmark model compared to the case when traders fail to internalize that firms learn from prices.

In columns (1) and (3) I control for stock price informativeness and its interaction with $Q$. As in previous studies (Chen et al. (2007), Bakke and Whited (2010) and Foucault and Fresard (2013)) I find that a firm’s investment is more sensitive to Tobin’s $Q$ when its stock price is more informative. These results suggest that firm managers learn from the private information aggregated into the stock price when making investment decisions, but market prices does not seem to provide more guidance to managers with low quality of information.

2.5 Conclusions

In this paper I find limits to the ability of secondary markets to inform and guide firms’ real investment decisions. The economy is modeled as a strategic game
between firms and informed speculators. Before making an investment choice, firms use stock prices to update their priors about their own fundamentals. Informed traders are strategic in that they internalize the firms’ inference problem. In this setting, I show that informed trading volume depends on the quality of managers’ prior information. In other words, the amount of private information that is aggregated into the stock price through the trading process is a function of managers’ initial information. Learning from prices is limited because prices are endogenously less informative for firms with low quality of managerial information, which are precisely those firms that would like to learn more from the stock market in the first place. In turn, real investment efficiency falls as the market signal is not as useful in helping managers differentiate between good and bad projects.

Using a sample of U.S. publicly traded companies, I document a positive correlation between the quality of managers’ information and stock price informativeness. I show that less informed managers do not rely more on the stock price to make investment decisions. Collectively, the evidence is suggestive of limits to the ability of firms to learn relevant information from stock prices.

The model presented here may have implications for firms’ decision about whether to be financed through public or private equity. More specifically, depending on the information gap between traders and the firm’s managers, the firm might benefit from an IPO or might be better off being held privately. In the same way, the quality of managers’ information should determine the choice of outside speculators to either become private or public equity investors. When managers have low quality information, speculators’ trading profits are limited, despite knowing the
true value of the fundamental. In this case, the speculator might be better off being a private equity investor in the firm, which would allow it to participate directly in the investment decision. On the contrary, when managers’ prior information is good, speculators’ trading profits are potentially large, and they can fully benefit from their information by trading on secondary markets. A further analysis of the links between managerial information, outsider information and the optimal form of equity finance is left for future work.
Chapter A: Appendix for Chapter 1
A.1 Appendix: Tables
Table A.1: Summary Statistics for Colombian Pension Funds Holdings

Key statistics are provided below (at two-year intervals) for the Colombian pension funds and voluntary funds. For each column, statistics are shown for the portfolios reported by June 30 of each year, except as noted. The database, made available by the Association of Pension Fund Administrators (ASOFONDOS), includes monthly portfolio holdings of each security in every pension fund and voluntary fund from January 31, 2004 to December 31, 2010. Panel A documents the total number of funds, the total assets under management and the share invested in stocks traded publicly in the domestic capital market. Panel B shows the average number of stocks held per fund at each date, the number of different stocks held by all six pension funds as a group and the number of stocks in the IGBC index, which is a major stock index for the Colombian stock market. Panel B also provides trading data, inferred from the difference in portfolio holding between May 31 and June 30 of each year. Panel C shows key statistics on relative performance between funds and portfolio differences between each fund and the peer portfolio and between each fund and the market portfolio. Relative performance is measured as the difference between the peer returns and individual fund returns, for annual returns measured over a three year rolling window. Distance measures a fund’s exposure to each stock relative to the benchmark: $d_{it} = \pi_{it} - w_{it}$. Panels D and E present key statistics for voluntary funds.

<table>
<thead>
<tr>
<th>Year</th>
<th>2004</th>
<th>2006</th>
<th>2008</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Pension Fund Count, Assets and Asset Allocation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of funds</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Total assets ($billions)</td>
<td>8.2</td>
<td>13.8</td>
<td>27.8</td>
<td>44.1</td>
</tr>
<tr>
<td>Net flows (contributions minus withdrawals $billions)</td>
<td>0.8</td>
<td>1.5</td>
<td>2.4</td>
<td>1.7</td>
</tr>
<tr>
<td>Percent invested in domestic stocks</td>
<td>5.0</td>
<td>12.6</td>
<td>22.4</td>
<td>32.1</td>
</tr>
<tr>
<td>Largest fund share (percentage of all PFA assets)</td>
<td>27.1</td>
<td>26.6</td>
<td>27.2</td>
<td>27.2</td>
</tr>
<tr>
<td>Smallest fund share (percentage of all PFA assets)</td>
<td>2.9</td>
<td>3.8</td>
<td>4.5</td>
<td>4.8</td>
</tr>
<tr>
<td><strong>Panel B. Pension Funds Domestic Stock Count and Trading Statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average number of stocks held per fund</td>
<td>16.2</td>
<td>21.2</td>
<td>26.3</td>
<td>30.0</td>
</tr>
<tr>
<td>Number of distinct stocks held by all pension funds</td>
<td>41</td>
<td>50</td>
<td>44</td>
<td>47</td>
</tr>
<tr>
<td>Number of stocks in the market index</td>
<td>26</td>
<td>33</td>
<td>27</td>
<td>32</td>
</tr>
<tr>
<td>Average stocks traded per fund</td>
<td>7.2</td>
<td>5.2</td>
<td>8.3</td>
<td>7.0</td>
</tr>
<tr>
<td>Proportion of trades that are buy (percent)</td>
<td>65.1</td>
<td>61.3</td>
<td>80.0</td>
<td>54.8</td>
</tr>
<tr>
<td>Total buys ($millions)</td>
<td>14.1</td>
<td>20.3</td>
<td>82.7</td>
<td>50.7</td>
</tr>
<tr>
<td>Total sells ($millions)</td>
<td>4.5</td>
<td>16.3</td>
<td>23.9</td>
<td>80.0</td>
</tr>
<tr>
<td>Average yearly sells (percentage of sell volume over total trades)</td>
<td>27.4</td>
<td>25.4</td>
<td>29.2</td>
<td>65.4</td>
</tr>
<tr>
<td><strong>Panel C. Pension Funds Performance and Portfolio Differences (standard deviation in parenthesis)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average relative returns (percent)</td>
<td>-0.10</td>
<td>0.13</td>
<td>0.27</td>
<td>0.29</td>
</tr>
<tr>
<td>Average peer distance (percent)</td>
<td>(0.49)</td>
<td>(0.79)</td>
<td>(1.30)</td>
<td>(1.30)</td>
</tr>
<tr>
<td>Average market distance (percent)</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Proportion of trades that are buy (percent)</td>
<td>(3.83)</td>
<td>(2.01)</td>
<td>(1.23)</td>
<td>(2.09)</td>
</tr>
<tr>
<td>Average market distance (percent)</td>
<td>0.06</td>
<td>0.00</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Proportion of trades that are sell (percent)</td>
<td>(5.39)</td>
<td>(3.10)</td>
<td>(2.94)</td>
<td>(3.08)</td>
</tr>
<tr>
<td><strong>Panel D. Voluntary Funds Count, Assets and Asset Allocation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total assets ($billions)</td>
<td>1.2</td>
<td>2.1</td>
<td>4.8</td>
<td>4.3</td>
</tr>
<tr>
<td>Percent invested in domestic stocks</td>
<td>1.8</td>
<td>4.5</td>
<td>9.2</td>
<td>14.5</td>
</tr>
<tr>
<td>Largest fund share (percentage of all voluntary funds)</td>
<td>38.1</td>
<td>28.0</td>
<td>47.0</td>
<td>36.1</td>
</tr>
<tr>
<td>Smallest fund share (percentage of all voluntary funds)</td>
<td>1.9</td>
<td>3.5</td>
<td>2.1</td>
<td>4.7</td>
</tr>
<tr>
<td><strong>Panel E. Voluntary Funds Domestic Stock Count and Trading Statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average number of stocks held per fund</td>
<td>5.2</td>
<td>13.8</td>
<td>17.8</td>
<td>20.3</td>
</tr>
<tr>
<td>Number of distinct stocks held by all pension funds</td>
<td>23</td>
<td>30</td>
<td>36</td>
<td>34</td>
</tr>
<tr>
<td>Average stocks traded per fund</td>
<td>0.8</td>
<td>6.5</td>
<td>10.2</td>
<td>12.8</td>
</tr>
<tr>
<td>Proportion of trades that are buy (percent)</td>
<td>79.5</td>
<td>38.5</td>
<td>39.4</td>
<td>59.2</td>
</tr>
<tr>
<td>Total buys ($millions)</td>
<td>1.9</td>
<td>10.9</td>
<td>12.8</td>
<td>34.3</td>
</tr>
<tr>
<td>Total sells ($millions)</td>
<td>0.5</td>
<td>7.75</td>
<td>32.9</td>
<td>77.8</td>
</tr>
<tr>
<td>Average yearly sells (percentage of sell volume over total trades)</td>
<td>19.8</td>
<td>41.5</td>
<td>71.9</td>
<td>69.4</td>
</tr>
</tbody>
</table>
Table A.2: Direction of portfolio weight changes

The direction measure, \( direction^i_t \), for a given fund \( i \) at some month \( t \) equals \( \frac{\Delta w^i_t d^i_t}{||\Delta w^i_t|| ||d^i_t||} \), where \( \Delta w^i_t \) is the active change in portfolio weights between \( t \) and \( t + 1 \) adjusted by individual stock returns. \( d^i_t \) is the distance in month \( t \) between fund \( i \)'s portfolio and the peer portfolio. Statistics are calculated for measures of direction across funds (\( direction^i_t \)). Direction captures whether each fund is moving towards or away from the peer benchmark.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>std dev</th>
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<td><strong>Panel A. Statistics for Direction</strong></td>
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<td></td>
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<tr>
<td>Pension Funds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before June 2007</td>
<td>0.32</td>
<td>0.32</td>
<td>-0.21</td>
<td>0.71</td>
<td>0.19</td>
</tr>
<tr>
<td>After June 2007</td>
<td>0.14</td>
<td>0.16</td>
<td>-0.68</td>
<td>0.50</td>
<td>0.18</td>
</tr>
<tr>
<td>Voluntary Funds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before June 2007</td>
<td>0.13</td>
<td>0.10</td>
<td>-0.73</td>
<td>0.99</td>
<td>0.38</td>
</tr>
<tr>
<td>After June 2007</td>
<td>0.13</td>
<td>0.16</td>
<td>-0.93</td>
<td>0.96</td>
<td>0.42</td>
</tr>
<tr>
<td><strong>Panel B. Statistics for Relative Performance (in bps)</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pension Funds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before June 2007</td>
<td>0.07</td>
<td>0.13</td>
<td>-3.63</td>
<td>3.71</td>
<td>1.29</td>
</tr>
<tr>
<td>After June 2007</td>
<td>-0.07</td>
<td>-0.35</td>
<td>-4.98</td>
<td>5.45</td>
<td>1.86</td>
</tr>
<tr>
<td>Voluntary Funds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before June 2007</td>
<td>0.01</td>
<td>0.10</td>
<td>-5.46</td>
<td>4.91</td>
<td>1.74</td>
</tr>
<tr>
<td>After June 2007</td>
<td>-0.02</td>
<td>-0.15</td>
<td>-6.77</td>
<td>6.12</td>
<td>1.91</td>
</tr>
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<td><strong>Panel C. Correlation between Direction and Relative Performance</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pension Funds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Before June 2007 = -0.31***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>After June 2007 = 0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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</tr>
<tr>
<td></td>
<td>Before June 2007 = 0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>After June 2007 = 0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table A.3: Linear Regressions for Adjusted and Unadjusted Weight Changes

The dependent variable is the change in weight $\Delta w_{ist}^{s+1}$ for stock $s$ between period $t$ and $t+1$ for fund $i$. The change in weights is adjusted by the stock returns for columns (1) and (2), i.e. $\Delta w_{ist}^{s+1} = w_{ist}^{s+1} - w_{ist}^{s} \times \text{ret}_{st}$, where $\text{ret}_{st}$ are the gross returns for stock $s$ between $t$ and $t+1$, and unadjusted for columns (3) and (4), i.e. $\Delta w_{ist}^{s+1} = w_{ist}^{s+1} - w_{ist}^{s}$. The unit of observation is a month. “MRG” is a dummy variable, equal to one for dates prior June 2007 and zero thereafter. “Peer (Market) Distance” is the difference between the weight of stock $s$ in the peer (market) portfolio and the weight of $s$ in fund $i$'s portfolio. The market portfolio is the IGBC, a major index in the Colombian stock market. “Relative Performance” is the difference in returns between manager $i$ and the overall pension industry, measured over the previous 36 months for each date. “Size” is the share of assets under management of pension fund $i$ as a percentage of the entire pension industry. Standard errors are in parenthesis. Note: ***/**/* indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Adjusted weight changes</th>
<th>Unadjusted weight changes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>MRG x Peer Distance</td>
<td>0.0599**</td>
<td>0.0867****</td>
</tr>
<tr>
<td></td>
<td>(0.0250)</td>
<td>(0.0257)</td>
</tr>
<tr>
<td>MRG x Relative Performance</td>
<td>0.0598</td>
<td>0.0547</td>
</tr>
<tr>
<td></td>
<td>(0.0479)</td>
<td>(0.0478)</td>
</tr>
<tr>
<td>MRG x Peer Distance x Relative Performance</td>
<td>-4.1280***</td>
<td>-4.6414***</td>
</tr>
<tr>
<td></td>
<td>(1.2137)</td>
<td>(1.4519)</td>
</tr>
<tr>
<td>MRG x Size</td>
<td>-0.0058</td>
<td>-0.0040</td>
</tr>
<tr>
<td></td>
<td>(0.0110)</td>
<td>(0.0110)</td>
</tr>
<tr>
<td>MRG x Size x Peer Distance</td>
<td>0.0774</td>
<td>0.1234</td>
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<tr>
<td></td>
<td>(0.1414)</td>
<td>(0.1412)</td>
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<tr>
<td>MRG</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0008)</td>
</tr>
<tr>
<td>Peer Distance</td>
<td>-0.0195</td>
<td>-0.0183</td>
</tr>
<tr>
<td></td>
<td>(0.0231)</td>
<td>(0.0235)</td>
</tr>
<tr>
<td>Relative Performance</td>
<td>-0.0270</td>
<td>-0.0255</td>
</tr>
<tr>
<td></td>
<td>(0.0377)</td>
<td>(0.0376)</td>
</tr>
<tr>
<td>Peer Distance x Relative Performance</td>
<td>4.2870***</td>
<td>4.8586***</td>
</tr>
<tr>
<td></td>
<td>(1.1573)</td>
<td>(1.3222)</td>
</tr>
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<td>Size</td>
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<td>0.0074</td>
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<td></td>
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<td>(0.0088)</td>
</tr>
<tr>
<td>Size x Peer Distance</td>
<td>0.0961</td>
<td>0.0989</td>
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<tr>
<td></td>
<td>(0.1306)</td>
<td>(0.1302)</td>
</tr>
<tr>
<td>MRG x Market Distance</td>
<td>0.0396</td>
<td>0.0396</td>
</tr>
<tr>
<td></td>
<td>(0.076)</td>
<td>(0.076)</td>
</tr>
<tr>
<td>MRG x Market Distance x Relative Performance</td>
<td>-1.0673</td>
<td>-0.0519</td>
</tr>
<tr>
<td></td>
<td>(0.7078)</td>
<td>(0.4802)</td>
</tr>
<tr>
<td>Market Distance</td>
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<td>0.0032*</td>
</tr>
<tr>
<td></td>
<td>(0.0048)</td>
<td>(0.0018)</td>
</tr>
<tr>
<td>Market Distance x Relative Performance</td>
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<td>0.4948</td>
</tr>
<tr>
<td></td>
<td>(0.5709)</td>
<td>(0.2445)</td>
</tr>
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<td>Constant</td>
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<td>-0.0014**</td>
</tr>
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<td>(0.0006)</td>
<td>(0.0006)</td>
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<tr>
<td>Controls</td>
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<td>yes</td>
</tr>
<tr>
<td>Pension fund fixed effects</td>
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<td>yes</td>
</tr>
<tr>
<td>Number of observations</td>
<td>18960</td>
<td>18960</td>
</tr>
</tbody>
</table>
Table A.4: Probit Regressions of Buying or Selling a Stock

The dependent variable is the dummy variable $buy_{it+1}^i$ or $sell_{it+1}^i$, which indicates whether a given fund $i$ in period $t+1$ increases or decreases the number of shares in stock $s$. The unit of observation is a month. “MRG” is a dummy variable equal to one for dates prior June 2007 and zero thereafter. “Peer (Market) Distance” is the difference between the weight of stock $s$ in the peer (market) portfolio and the weight of $s$ in fund $i$’s portfolio. The market portfolio is the IGBC, a major index in the Colombian stock market. “Relative Performance” is the difference in returns between manager $i$ and the overall pension industry, measured over the previous 36 months for each date. “Size” is the share of assets under management of pension fund $i$ as a percentage of the entire pension industry. Standard errors are in parenthesis. Note: ***/**/ indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Probability of buying a stock</th>
<th>Probability of selling a stock conditional on stock ownership</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>MRG x Peer Distance</td>
<td>54.52***</td>
<td>59.13***</td>
</tr>
<tr>
<td></td>
<td>(15.99)</td>
<td>(16.78)</td>
</tr>
<tr>
<td>MRG x Relative Performance</td>
<td>13.40***</td>
<td>13.97***</td>
</tr>
<tr>
<td></td>
<td>(4.52)</td>
<td>(4.53)</td>
</tr>
<tr>
<td>MRG x Peer Distance x Relative Performance</td>
<td>-2693.39***</td>
<td>-2330.73***</td>
</tr>
<tr>
<td></td>
<td>(605.10)</td>
<td>(889.02)</td>
</tr>
<tr>
<td>MRG x Size</td>
<td>2.23***</td>
<td>2.19***</td>
</tr>
<tr>
<td></td>
<td>(0.82)</td>
<td>(0.82)</td>
</tr>
<tr>
<td>MRG x Size x Peer Distance</td>
<td>-239.79***</td>
<td>-243.95***</td>
</tr>
<tr>
<td></td>
<td>(87.55)</td>
<td>(87.96)</td>
</tr>
<tr>
<td>MRG</td>
<td>-0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Peer Distance</td>
<td>-26.65*</td>
<td>-26.42*</td>
</tr>
<tr>
<td>Relative Performance</td>
<td>-12.35***</td>
<td>-12.20***</td>
</tr>
<tr>
<td></td>
<td>(3.28)</td>
<td>(3.28)</td>
</tr>
<tr>
<td>Peer Distance x Relative Performance</td>
<td>1444.90***</td>
<td>1077.75*</td>
</tr>
<tr>
<td></td>
<td>(502.71)</td>
<td>(557.29)</td>
</tr>
<tr>
<td>Size</td>
<td>-4.35***</td>
<td>-4.34***</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(0.50)</td>
</tr>
<tr>
<td>Size x Peer Distance</td>
<td>77.59*</td>
<td>76.15*</td>
</tr>
<tr>
<td></td>
<td>(43.62)</td>
<td>(43.58)</td>
</tr>
<tr>
<td></td>
<td>(8.35)</td>
<td>(8.35)</td>
</tr>
<tr>
<td>MRG x Market Distance x Relative Performance</td>
<td>503.32</td>
<td>823.02</td>
</tr>
<tr>
<td>Market Distance</td>
<td>0.10</td>
<td>2.12</td>
</tr>
<tr>
<td>Market Distance x Relative Performance</td>
<td>375.87</td>
<td>74.64</td>
</tr>
<tr>
<td>Constant</td>
<td>238.14***</td>
<td>238.25***</td>
</tr>
<tr>
<td></td>
<td>(29.27)</td>
<td>(29.28)</td>
</tr>
</tbody>
</table>

Pension fund fixed effects | yes                          | yes                          | yes                          | yes                          |
Number of observations     | 18960                        | 18960                        | 11299                        | 11299                        |
Table A.5: Calibration

The return process of the risky asset is calibrated according to the Colombian stock market historical returns (Data available after 1987). The risk aversion parameters $\sigma_i$ and $\sigma_j$ are calibrated to match the average direction of trading and the average cross-section dispersion of returns prior to June 2007.

<table>
<thead>
<tr>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Colombian Stock Market Historical Returns</strong></td>
<td></td>
</tr>
<tr>
<td>Risk Premium</td>
<td>12.5%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>19%</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.33</td>
</tr>
<tr>
<td><strong>Panel B. Model Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Probability of high state</td>
<td>$p=0.55$</td>
</tr>
<tr>
<td>High returns</td>
<td>$r^H = 25%$</td>
</tr>
<tr>
<td>Low returns</td>
<td>$r^L = -10%$</td>
</tr>
<tr>
<td>Minimum Return Guarantee</td>
<td>$x^0 = 3.3%$</td>
</tr>
<tr>
<td></td>
<td>$x^1 = 5.2%$</td>
</tr>
<tr>
<td></td>
<td>$\gamma = 1$</td>
</tr>
<tr>
<td>PFA management fee</td>
<td>$\beta = 0.8%$</td>
</tr>
<tr>
<td><strong>Panel C. Calibration</strong></td>
<td></td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>$\sigma_i = 2.75, \sigma_j = 2.21$</td>
</tr>
</tbody>
</table>

Table A.6: Effects from the change in the Minimum Return Guarantee formula

Empirical and model-implied moments before and after the change in the MRG formula in June 2007. The parameters are set according to the calibration in Table A.5.

<table>
<thead>
<tr>
<th></th>
<th>Before June 07</th>
<th>After June 07</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Mean Direction</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Std. dev. of relative returns</td>
<td>1.19%</td>
<td>1.08%</td>
</tr>
<tr>
<td>Corr(direction, rel)</td>
<td>-0.31</td>
<td>-0.90</td>
</tr>
</tbody>
</table>
A.2 Appendix: Figures

Figure A.1: Economy with three assets. This figure presents two examples of changes in the portfolio composition in the space of weights for stocks A and B. The portfolio of fund $i$ moves from $w_i^t$ to $w_i^{t+\Delta}$. The distance vector is $d_i^t = \Pi_i^t - w_i^t$, which represents the initial difference between fund $i$ and the peer portfolio at the beginning of the period $t$. $\theta$ is the angle formed between the change in the portfolio of manager $i$ and the distance vector. Direction is defined as $\cos \theta$. When manager $i$ moves towards the peer portfolio, $\theta$ is smaller and direction is closer to 1, as in panel (a). In panel (b) the manager moves away from the peer benchmark and direction takes smaller values as the angle increases.
Figure A.2: Average direction of portfolio change. The direction measure, $\text{direction}_t^i$, for a given fund $i$ at some month $t$ equals $\frac{\Delta w_t^i d_t^i}{||\Delta w_t^i|| ||d_t^i||}$, where $\Delta w_t^i$ is the active change in portfolio weights between $t$ and $t+1$ adjusted by individual stock returns. $d_t^i$ is the distance between fund $i$’s portfolio and the peer portfolio as of month $t$. The figure reports the monthly value of direction averaged across the six PFAs, for pension funds (solid line) and voluntary funds (dotted line).

Figure A.3: Marginal effects. Difference in marginal effects of distance on adjustments in portfolios weights before and after the policy change $\frac{\partial \Delta w(MRG=1)}{\partial d} - \frac{\partial \Delta w(MRG=0)}{\partial d}$ with 95% confidence intervals.
Figure A.4: Utility maximization for manager $i$ for a given portfolio choice by $j$, $\phi_j$. When the final wealth of manager $i$, $W_{iT}/W_{i0}$, is below the peer benchmark $W_{jT}/W_{j0} - x$ for either the high or low state, she pays the underperformance penalty, thus reducing the net compensation. In panel (a) the manager’s best response is to play her normal policy $\hat{\phi}_i = \phi_{iNP}$. Here, she does not pay the underperformance penalty in either state. In panel (b), the manager’s best response is a portfolio with a lower share in the risky asset than in her normal policy, $\hat{\phi}_i < \phi_{iNP}$, to reduce the underperformance penalty that is paid if the low state of returns is realized.

Figure A.5: Nash Equilibrium Portfolios $(\phi_i^*, \phi_j^*)$. Best responses by managers $i$ (solid line) and $j$ (dash-dotted line). In panel (a) both managers optimally shift their portfolio towards their peer. In panel (b) manager $j$ does all the shifting, while manager $i$ plays her normal policy.
Figure A.6: Nash Equilibria and trading behavior in a calibrated three period economy for different values of $x$. Panels (a) and (b) present the Nash equilibrium policies in period 0 and period 1 for managers i and j respectively. Direction is plotted in panel (c) using the formula $\frac{(\phi_{i0}^* - \phi_{i0}^{NP})(\phi_{j0}^{NP} - \phi_{i0}^{NP})}{(\phi_{i0}^{NP} - \phi_{i0}^{NP})^2}$ for period 0 and $\frac{(\phi_{i1}^* - \phi_{i0}^{NP})(\phi_{j0}^{NP} - \phi_{i0}^{NP})}{(\phi_{i1}^{NP} - \phi_{i0}^{NP})^2}$ for period 1. Direction is calculated for each manager and averaged across i and j for periods 0 and 1. The correlation between relative performance and direction presented in panel (d) is calculated for period 1 for each pair of direction and relative returns for both managers. Parameters are set according to the calibration in Table A.5.
A.3 Appendix: Proofs

Proof of Proposition 1

In the absence of an underperformance penalty, by definition the normal policy yields a higher expected utility than any other strategy. With the underperformance penalty in place, for a given portfolio choice by the other manager $\phi_j$, if the normal policy can be implemented without triggering the penalty in any state, the manager optimally chooses her normal policy as if there were no relative performance concerns. Note that if this is the case, the compensation for the manager in each state is $\beta W_{iT}$, which is the same as without the penalty. The maximum allowed shortfall for each state, $\frac{W^H_i}{W_{io}} \geq \frac{W^H_j}{W_{jo}} - x$ and $\frac{W^L_i}{W_{io}} \geq \frac{W^L_j}{W_{jo}} - x$, can be written in terms of the share in the risky asset $\phi$ using (1.3) as $\phi_j + \frac{x}{B - r} \geq \phi_i \geq \phi_j - \frac{x}{B - r}$. Hence if $\phi_{NP}^i$ is in this region the manager’s optimal policy is the normal policy.

If the manager cannot implement her normal policy without avoiding the underperformance penalty, the compensation (1.4) needs to be calculated accordingly with a penalty in that state. Also, noting that a manager cannot be a loser in both states, to solve the optimization problem I split the problem into two regions, one where the manager is a loser in the low state and another where the manager is a loser in the high state.

For the manager playing to lose in the low state the problem is transformed into a constrained maximization as follows (loser in the low state):
\[
\max_{\phi_{i0}} EU \left[ F^S_i (\phi_{i0}, \phi_{j0}) \right] + \mu \left[ \phi_i - \phi_j - \frac{x}{B - r^L} \right] \quad (A.1)
\]

The Kuhn-Tucker conditions are

\[
(1 - p)(\beta + \gamma)B - r^L U' \left[ F^L_i (\phi_{i0}, \phi_{j0}) \right] + \beta (r^H - B) p U' \left[ F^H_i (\phi_{i0}, \phi_{j0}) \right] + \mu = 0 \quad (A.2)
\]

\[
\mu \left( \phi_i - \phi_j - \frac{x}{B - r^L} \right) = 0, \quad \mu \geq 0 \quad (A.3)
\]

which imply that the best response in this region is:

\[
\hat{\phi}_{i0} = \begin{cases} 
\phi_{i0} & \phi_{i0} > \phi_{j0} + \frac{x}{B - r^L} \text{ (interior solution)} \\
\phi_{j0} + \frac{x}{B - r^L} & \text{otherwise (corner solution)}
\end{cases} \quad (A.4)
\]

where \( \phi_{i0} \) solves \( (A.2) \) with \( \mu = 0 \). Solving for \( \phi_{i0} \), the best response in this region can be written as a linear function of \( \phi_{j0} \) and \( x \) as

\[
\hat{\phi}_{i0} = a_i + b_i \phi_{j0} + c_i x,
\]

where the coefficients take the following values:

**Interior solution**

\[
a_i = (\beta + \xi \gamma) A_i B; \quad b_i = \xi \gamma (r^H - B) A_i; \quad c_i = \xi \gamma A_i
\]

where

\[
A_i = \left[ \xi (\beta + \gamma) (r^H - B) + \beta (B - r^L) \right]^{-1}
\]

and

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\( \xi_i = \left[ \frac{(\beta+\gamma)p(r^H - B)}{\beta(1-p)(B-r_L)} \right]^{-1/\sigma_i} \)

Corner solution

\( a_i = 0; \quad b_i = 1; \quad c_i = \frac{1}{r_H - B} + 1 \)

To solve the optimal portfolio in the region where manager i pays the underperformance penalty in the high state (loser in the high state), I proceed in similar fashion. The best response function of manager i in this region is again linear in \( x \) and \( \phi_{j0} \) as 
\( \hat{\phi}_{i0} = \overline{a}_i + \overline{b}_i \phi_{j0} - \overline{c}_i x \), with the coefficients defined as follows:

Interior solution

\[ \overline{a}_i = (1 - \overline{c}_i)\beta \overline{A}_i B; \quad \overline{b}_i = \gamma(B - r^L)\overline{A}_i; \quad \overline{c}_i = \gamma\overline{A}_i \]

where

\[ \overline{A}_i = \left[ \overline{c}_i \beta (r^H - B) + (\beta + \gamma)(B - r^L) \right]^{-1} \]

and

\[ \overline{c}_i = \left[ \frac{\beta p (r^H - B)}{\beta + \gamma(1-p)(B-r_L)} \right]^{-1/\sigma_i} \]

Corner solution

\( \overline{a}_i = 0; \quad \overline{b}_i = 1; \quad \overline{c}_i = \frac{1}{B - r^L} \)

Q.E.D
Proof of Corollary 2: The assertion follows directly from the corner solutions in Proposition 1 that effectively determine the boundaries of the shift.

Q.E.D

Proof of Corollary 3: The optimality condition for an interior solution for a manager playing to lose in the low state reads

\[
(1 - p)(B - r^L) \frac{u'(F^L)}{u'(F^H)} = \frac{\beta}{\beta + \gamma} \to 0
\]

(A.5)

In order for the marginal rate of substitution between the compensation in the low and high state to go to zero, the manager must end up with all the wealth in the low state, which implies \( \varphi \to -\infty \). Hence the boundary condition \( \varphi_{i0} \leq \varphi_{j0} + \frac{x}{B - r^L} \) in (A.4) is always satisfied and the manager’s best response is \( \hat{\varphi}_{i0} = \varphi_{j0} + \frac{x}{B - r^L} \).

The optimality condition for an interior solution for a manager playing to lose in the high state reads

\[
(1 - p)(B - r^L) \frac{u'(F^L)}{u'(F^H)} = \frac{\beta + \gamma}{\beta} \to \infty
\]

(A.6)

In order for the marginal rate of substitution between the compensation in the low and high state to go to infinity, the manager must end up with all the wealth in the high state, which implies \( \varphi \to \infty \). Hence the boundary condition \( \varphi_{i0} \geq \varphi_{j0} - \frac{x}{r^H - B} \) is always satisfied and the manager’s best response is \( \hat{\varphi}_{i0} = \varphi_{j0} - \frac{x}{r^H - B} \).

Q.E.D
A.4 Appendix: Three Period Model Solution

The timeline of events is as follows: In period \( t = 0 \), each manager chooses some \( \phi_{i0} \) and \( \phi_{j0} \).\(^1\) Returns for the risky asset \( r_i^1 \) are realized and managers enter period \( t = 1 \) with a new level of wealth \( R_i^1 W_0 \) (to simplify notation \( R_i^1 \equiv R_i^1 (\phi_0) \) where \( R_i^1 (\phi_0) \) is defined according to (1.3)). In this period each manager chooses a portfolio allocation \( \phi_{i1} \) and \( \phi_{j1} \). The last period returns \( (t=2) \) for the risky asset \( r_2 \) are realized, managers observe their relative wealth, and fees are calculated depending on their relative performance. Here I define a trading strategy as the change in portfolio allocations between period 0 and 1 \( (\phi_{i1} - \phi_{i0}) \). I solve this problem by backward induction.

**Period \( t=1 \) equilibrium policies:** Starting at period 1, each manager observes the realized returns \( R_{i1} \) and \( R_{j1} \) given portfolio allocations at time 0, \( \phi_{i0} \) and \( \phi_{j0} \), and the realization of the first period state. At this point, the best response functions and equilibrium policies are calculated as in the initial period of the two period economy studied in section 1.3.2. The only difference is that the underperformance penalty for manager i now satisfies

\[
\begin{align*}
W_{i0}^s &- W_{j0}^s + x = R_{i1}^s R_{21}^s (\phi_{i1}) - R_{j1}^s R_{21}^s (\phi_{j1}) + x
\end{align*}
\]

using the dynamic wealth process.

The shifting region, which was given by conditions in (1.6) and (1.8) in Proposition 1 for the two period case, is now

\(^1\) Note that these policies are not necessarily the same as in the initial period of the two-period model. I refer to these policies as \( \phi_{i0} \) and \( \phi_{j0} \) to conserve of notation.
The best response functions in period 1 for this case are similar to Proposition \[1\] and are omitted here. Instead in Proposition \[A.1\] I present the equilibrium policies for both managers at period \(t = 1\) for a given pair of accumulated returns \(R_{i1}^s\) and \(R_{j1}^s\).

**Proposition A.1.** The Nash equilibrium policies for managers \(i\) and \(j\) at period \(t = 1\) for a given pair of accumulated returns \(R_{i1}^s\) and \(R_{j1}^s\) by manager \(i\) and \(j\) respectively are given by:

\[
(\phi_{i1}^*, \phi_{j1}^*) = \begin{cases} 
(\phi_i^{NP}, \phi_j^{NP}) & \Phi_{i1} \geq \phi_i^{NP} \text{ and } \Phi_{j1} \leq \phi_j^{NP} \quad (A.8) \\
(\phi_i^{NP}, a_j + b_j \phi_i^{NP} - c_j x) & \Phi_{i1} \geq \phi_i^{NP} \text{ and } \Phi_{j1} > \phi_j^{NP} \quad (A.9) \\
(a_i + b_i \phi_j^{NP} + c_i x, \phi_j^{NP}) & \Phi_{i1} < \phi_i^{NP} \text{ and } \Phi_{j1} \leq \phi_j^{NP} \quad (A.10) \\
(\Phi_{i1}, \Phi_{j1}) & \Phi_{i1} < \phi_i^{NP} \text{ and } \Phi_{j1} > \phi_j^{NP} \quad (A.11)
\end{cases}
\]

where \(\Phi_{i0} = \frac{(a_i + b_i a_i) + (c_i - b_i c_i) x}{1 - b_i b_j}\), \(\Phi_{j0} = \frac{(a_j + b_j a_j) - (c_j - b_j c_j) x}{1 - b_i b_j}\) and the coefficients are defined as: \(a_i = a_i - \gamma \left(1 - \frac{R_{i1}^s}{R_{i1}^H}\right) / A_i\), \(b_i = \frac{R_{i1}^s}{R_{i1}^H} b_i\), \(c_i = \frac{c_i}{R_{i1}^H}\), \(a_j = \bar{a}_j - \gamma \left(1 - \frac{R_{j1}^s}{R_{j1}^L}\right) / \bar{A}_j\), \(b_j = \frac{R_{j1}^s}{R_{j1}^L} b_j\), \(c_j = \frac{c_j}{R_{j1}^L}\) where \(A_i, A_j, a_i, a_j, b_i, b_j, c_i, c_j\) are as defined in the proof of Proposition \[2\] in Appendix \[A.3\]. Moreover,

i. If \(x > (r^H - B) (R_{i1}^s \phi_i^{NP} - R_{j1}^s \phi_j^{NP}) + (R_{i1}^s - R_{j1}^s)B\) and \(x > (B - r^L)(R_{i1}^s \phi_i^{NP} - R_{j1}^s \phi_j^{NP})\)

\(-R_{j1}^s \phi_j^{NP} - (R_{i1}^s - R_{j1}^s) B\), the conditions in \(A.8\) are always satisfied and

\[93\]
managers play their normal policy at $t = 1$.

ii. $R^*_i \left( r^H - B \right) > R^*_j \left( B - r^L \right)$ is a necessary condition for (A.9) to be an equilibrium

iii. $R^*_i \left( r^H - B \right) < R^*_j \left( B - r^L \right)$ is a necessary condition for (A.10) to be an equilibrium

Intuitively, a manager that enters period 1 with smaller returns than her competitor $\left( \frac{R^*_i}{R^*_j} < 1 \right)$ is more likely to pay the underperformance penalty at $t=2$. From (A.7), the shifting region for i becomes larger as her accumulated relative returns with respect to her peer are smaller.

Nash equilibrium strategies at period 1 are functions of the portfolio choices by both managers at period 0 and the state of returns $s$ at period 1. Formally, $\phi^*_i = \phi^*_i (\phi_{i0}, \phi_{j0}, s)$.

**Period $t=0$ equilibrium policies:** With this setup one can write the maximization problem for manager i as a portfolio choice $\phi_{i0}$ at period 0 that maximizes expected terminal utility for a given policy by her peer $\phi_j$, with the equilibrium portfolios in period 1 for realized returns $r^*_i$ given by Proposition A.1.

Formally, the wealth process can be written as $W_{iT} = R^*_i(\phi_{i0})R^S_2(\phi^*_i(\phi_{i0}, \phi_{j0}, s))$, the manager’s compensation is calculated according to (1.4) and the expectation is calculated over the four possible states $\{sS\} = \{HH, HL, LH, LL\}$.

**Proposition A.2.** The unique Nash equilibrium policies are functions of $x$ and have the following properties:

i. At time 0, $\phi^*_{i0} \leq \phi^{NP}_i$ and $\phi^*_{j0} \geq \phi^{NP}_j$. 

ii. If the high returns are realized in period 1 \((r_1 = r^H)\) then \(\phi_{i1}^{NP} \geq \phi_{i1}^* \geq \phi_{i0}^*\) and 
\[\phi_{j1}^* \geq \phi_{j0}^*.\]

iii. If the low returns are realized in period 1 \((r_1 = r^L)\) then \(\phi_{i1}^* \leq \phi_{i0}^*\) and \(\phi_{j0}^* \geq \phi_{j1}^*\).
A.5 Appendix: Numerical Analysis

In this section I describe the calibration of the three period model and evaluate its quantitative implications for portfolio choice and trading strategies. I first calibrate the trading behavior of fund managers to the observed behavior in the Colombian pension industry before June 2007, and show that the change in behavior implied by the model following an exogenous change in the formula of the MRG is quantitatively similar to the changes in PFA behavior observed in the data after June 2007.

Before proceeding with the calibration it is important to keep in mind that I have constructed a simple model to highlight the potential impact of an MRG on portfolio choice with strategic managers. To obtain a parsimonious and tractable model I have made two important assumptions. First, managers differ only in their risk aversion. Second, there are only two securities, a risk-free and a risky asset. With two securities, $\Delta w$ and $d$ in section ?? are one dimensional objects, so direction as defined in equation (1.1) can only take three possible values $\{-1,0,1\}$, as the measure is normalized by the magnitude of both $\Delta w$ and $d$. To give a sense of the magnitude of the adjustment in portfolio weights I redefine direction in period 1 as follows:

$$direction_{i1} = \frac{(\phi_{11} - \phi_{i0}^*) (\phi_{j0}^* - \phi_{i0}^*)}{(\phi_{i1}^{NP} - \phi_{j1}^{NP})^2}$$

(A.12)

where the two terms in the numerator are the model analogues for $\Delta w$ and $d$. 
respectively. The denominator in (A.12) normalizes the measure by the maximum distance between the two portfolios when the managers are choosing their normal strategies. Finally, measuring direction in period 1 alone might underestimate important adjustments to the portfolio allocations carried out in period 0. For this period, I introduce an alternative measure of direction based on deviations from the normal policy, as follows:

\[
direction_{i0} = \frac{(\phi^{*}_{i0} - \phi_{i}^{NP})(\phi_{j}^{NP} - \phi_{i}^{NP})}{(\phi_{i}^{NP} - \phi_{j}^{NP})^2}
\] (A.13)

Direction for manager j is obtained by switching subscripts i for j in equations (A.12) and (A.13). Here \(\text{direction} \in [0, 1]\) is zero when both managers choose their normal policies in each period and takes positive values when the manager moves her portfolio towards the other manager.

A period in the model represents a quarter. The timing of events is as follows: Each manager chooses their equilibrium portfolio at \(t = 0\). After the first quarter, returns for the risky asset are realized and managers choose their optimal policy at \(t = 1\), changing their portfolio allocation depending on their relative performance. The risky asset returns for the second quarter are realized, and the MRG is enforced if a manager’s cumulative returns over the two quarters are below the benchmark.

Table A.5 presents details on how I calibrate model parameters. Using historic returns for the Colombian stock market calculated with the IGBC index, I estimate the mean, standard deviation and skewness of these overall returns, and then solve for \(p, r^H\) and \(r^L\) to match these three moments. In a two manager setting, the MRG
between January 2004 and June 2007 can be written as \( MRG = 70\% \left( \frac{R_i + R_j}{2} \right) \) where \( R \) are individual fund 3-year annual returns. The equivalent MRG measure for fund \( i \) in terms of the compensation formula (1.4) defined in the model is calculated as

\[ 70\% \left( \frac{R_i + R_j}{2} \right) = R_j - x. \]

In the data, the yearly average PFA return over a three year window is 11%. This would imply a yearly \( x = 1.65\% \). In the model economy, the MRG is applied to the cumulative returns of two quarters, so the actual measure of MRG for a semester is adjusted by a factor of two, so that \( x^0 = 3.3\% \).

Finally I calibrate the risk aversion parameters \( \sigma_i \) and \( \sigma_j \) to match the average direction of trading and the average cross-section dispersion of portfolio returns across managers prior to June 2007. Average direction in the model is calculated as the average of \( \text{direction} \) for both managers and across all states. The cross-section dispersion of returns is calculated for the observed returns of both managers in both periods 1 and 2. The model moments implied by this calibration are presented in Table A.6. Compared to the data, the model generates too strong of a negative correlation between relative performance and trading towards ones peers prior to June 2007. Table A.6 also presents the implied model moments generated by the change in the MRG formula to an annual \( x = 2.6\% \) starting in July 2007 (in terms of the model analogue this represents a change in the MRG formula to a semi-annual \( x = 5.2\% \)). After the change in regulation, model managers trade less towards their peers and exhibit a less negative correlation between relative performance and the direction of trading. In addition, the cross section dispersion of returns increases. All of these moment changes are consistent with the data.
Figures A.6a and A.6b present the equilibrium strategies for each manager for the calibrated model. At period 0, if the MRG is strict enough, manager j optimally shifts her portfolio towards i, while i plays her normal policy. If the stock yields high returns in period 1, manager j finds herself behind the other manager and then moves her portfolio closer to i ($\phi_{j1}^* \geq \phi_{j0}^*$), buying more shares in the stock, while manager i plays her normal strategy again and doesn’t rebalance her portfolio. If the low returns are realized, manager j is overperforming and can play her normal policy given that she is now not constrained by the MRG, potentially rebalancing her portfolio by increasing her participation in the risk-free asset ($\phi_{i1}^* \leq \phi_{i0}^*$), moving away from the other manager. Manager i, underperforming in this state, finds it optimal to shift her strategy towards j to avoid the underperformance penalty, and does so by increasing her participation in the risk-free asset as well ($\phi_{i1}^* \leq \phi_{i0}^*$). Aggregate trading behavior is procyclical in this partial equilibrium model, increasing the risky asset share after good returns and decreasing it after bad returns. However, managers are not chasing returns, but are instead chasing each other, setting their portfolio to avoid being below the MRG.

Figures A.6c and A.6d display the average of direction across funds for both periods 0 and 1 and the correlation between direction and relative performance for different values of $x$. As expected, a tighter MRG constraint (small $x$) results in more shifting towards one’s peers. The U-shape of the correlation measure can be explained as follows. With a tight MRG, both managers set their portfolio close to each other. Hence, the cross-section dispersion of returns is small, and subsequent portfolio adjustments are small, even for underperforming managers. As the MRG
is loosened, the cross-section dispersion of returns increases, but the MRG might still bind, and portfolio adjustments also increase after returns are realized to avoid the penalty. In the limit, with a loose MRG (large $x$), managers can simply play their normal policies, and the cross-section dispersion of returns is thus larger, but no portfolio adjustment is required.
Chapter B: Appendix for Chapter 2
## B.1 Appendix: Tables

### Table B.1: Variable definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - R^2$</td>
<td>Price informativeness, defined as one minus $R^2$ from regressing a firm’s daily returns on market and industry indices over year $t$</td>
<td>CRSP</td>
</tr>
<tr>
<td>SIZE</td>
<td>Logarithm of the book value of assets</td>
<td>Compustat</td>
</tr>
<tr>
<td>INSIDER</td>
<td>Ratio of firm’s shares traded by insiders in a given year to the total number of shares traded (in percentage terms). Insider traders refers to open market stock transactions by the top five executives (CEO, CFO, COO, President and Chairman of the board)</td>
<td>Thompson Financial Insider Trading Database and CRSP</td>
</tr>
<tr>
<td>INST</td>
<td>Percentage of shares held by institutional investors</td>
<td>Thompson Financial</td>
</tr>
<tr>
<td>ANALYST</td>
<td>Number of analysts issuing forecasts for each stock</td>
<td>IBES</td>
</tr>
<tr>
<td>DIS</td>
<td>Analyst forecast dispersion, defined as the natural logarithm of the standard deviation of analysts’ forecasts of earnings per share, times the number of shares, scaled by the book value of total assets as in <a href="#">Gilchrist et al. (2005)</a></td>
<td>IBES and Compustat</td>
</tr>
<tr>
<td>CF</td>
<td>Cash flow, defined as net income before extraordinary items + depreciation and amortization expenses + R&amp;D expenses scaled by assets</td>
<td>Compustat</td>
</tr>
<tr>
<td>Q</td>
<td>Average Q, defined as $[\text{Book value of assets} - \text{book value of equity} + \text{market value of equity}]/\text{book value of assets}$</td>
<td>Compustat</td>
</tr>
<tr>
<td>I</td>
<td>Investment rate, defined as the ratio of capital expenditures to lagged fixed assets</td>
<td>Compustat</td>
</tr>
</tbody>
</table>

### Table B.2: Summary statistics

This table reports the summary statistics of the main variables used in the analysis. For each variable I present its mean, standard deviation, 5th, 25th, 50th, 75th and 95th percentile. All variables are defined in Table B.1. The sample period is from 1990 to 2010.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Number of observations</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - R^2$</td>
<td>34819</td>
<td>0.82</td>
<td>0.18</td>
<td>0.43</td>
<td>0.72</td>
<td>0.89</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>SIZE</td>
<td>34819</td>
<td>7.67</td>
<td>2.61</td>
<td>3.70</td>
<td>5.80</td>
<td>7.30</td>
<td>9.37</td>
<td>12.04</td>
</tr>
<tr>
<td>INSIDER</td>
<td>34819</td>
<td>6.89</td>
<td>12.69</td>
<td>0.01</td>
<td>0.08</td>
<td>0.43</td>
<td>3.16</td>
<td>34.04</td>
</tr>
<tr>
<td>INST</td>
<td>34819</td>
<td>0.52</td>
<td>0.26</td>
<td>0.09</td>
<td>0.31</td>
<td>0.53</td>
<td>0.73</td>
<td>0.93</td>
</tr>
<tr>
<td>DIS</td>
<td>34819</td>
<td>-3.05</td>
<td>1.12</td>
<td>-4.89</td>
<td>-3.78</td>
<td>-3.07</td>
<td>-2.34</td>
<td>-1.15</td>
</tr>
<tr>
<td>CF</td>
<td>34819</td>
<td>16.27</td>
<td>2502</td>
<td>-5.37</td>
<td>17.02</td>
<td>141</td>
<td>5706</td>
<td>7805</td>
</tr>
<tr>
<td>Q</td>
<td>35051</td>
<td>2.81</td>
<td>2.16</td>
<td>0.93</td>
<td>1.14</td>
<td>1.63</td>
<td>6.31</td>
<td>8.31</td>
</tr>
<tr>
<td>I</td>
<td>24155</td>
<td>0.29</td>
<td>0.42</td>
<td>0.49</td>
<td>0.11</td>
<td>0.19</td>
<td>0.34</td>
<td>0.79</td>
</tr>
</tbody>
</table>
Table B.3: Price informativeness and managerial information

Definitions of all variables are listed in Table B.1. The dependent variable is Price Informativeness. Managers’ uncertainty is proxied by the dispersion of analysts’ earnings forecasts (DIS) and insider trading activity 1 − INSIDER. T-statistics are in parentheses. ***/***/ indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Measure of Uncertainty</th>
<th>DIS</th>
<th>1 − INSIDER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3)</td>
<td>(4) (5) (6)</td>
</tr>
<tr>
<td>Dependent variable: PI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(UNCER_{it})</td>
<td>-0.10***</td>
<td>-0.18***</td>
</tr>
<tr>
<td></td>
<td>(17.90)</td>
<td>(26.30)</td>
</tr>
<tr>
<td>(SIZE_{it})</td>
<td>-0.41***</td>
<td>-0.45***</td>
</tr>
<tr>
<td></td>
<td>(55.13)</td>
<td>(27.87)</td>
</tr>
<tr>
<td>(CF_{it})</td>
<td>0.25***</td>
<td>0.34***</td>
</tr>
<tr>
<td></td>
<td>(32.97)</td>
<td>(18.91)</td>
</tr>
<tr>
<td>Firm fixed effects</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.14</td>
<td>0.52</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>24753</td>
<td>24753</td>
</tr>
</tbody>
</table>

Table B.4: Price informativeness and managerial information: Interaction with institutional ownership

Definitions of all variables are listed in Table B.1. The dependent variable is Price Informativeness. Managers’ uncertainty is proxied by the dispersion of analysts’ earnings forecasts (DIS) and insider trading activity 1 − INSIDER. \(SIZE\) and \(CF\) coefficients are omitted. T-statistics are in parentheses. ***/***/ indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Measure of Uncertainty</th>
<th>DIS</th>
<th>1 − INSIDER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2)</td>
<td></td>
</tr>
<tr>
<td>Dependent variable PI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(UNCER_{it})</td>
<td>-0.02***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.40)</td>
<td></td>
</tr>
<tr>
<td>(UNCER_{it} \times INST_{it})</td>
<td>-0.05***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(11.95)</td>
<td></td>
</tr>
<tr>
<td>(INST_{it})</td>
<td>-0.22***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(28.30)</td>
<td></td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.73</td>
<td>0.74</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>24753</td>
<td>24921</td>
</tr>
</tbody>
</table>
Table B.5: Managerial information and investment sensitivity to price

Definitions of all variables are listed in Table B.1. The dependent variable is investment. Managers’ uncertainty is proxied by the dispersion of analysts’ earnings forecasts (DIS) and insider trading activity $1 - INSIDER$. T-statistics are in parentheses. ***/**/ indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Dependent variable: $I$</th>
<th>DIS</th>
<th>1 – INSIDER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.215***</td>
<td>0.225***</td>
</tr>
<tr>
<td></td>
<td>(8.11)</td>
<td>(8.45)</td>
</tr>
<tr>
<td>$Q \times PI$</td>
<td>0.038***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.88)</td>
<td></td>
</tr>
<tr>
<td>$Q \times UNCER$</td>
<td>0.012</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>$PI$</td>
<td>-0.002</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td></td>
</tr>
<tr>
<td>$UNCER$</td>
<td>-0.023*</td>
<td>-0.025**</td>
</tr>
<tr>
<td></td>
<td>(1.84)</td>
<td>(2.03)</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.39</td>
<td>0.38</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>19009</td>
<td>19009</td>
</tr>
</tbody>
</table>
B.2 Appendix: Figures

Figure B.1: Timeline for each firm. Informed speculator (IS), Noise traders (NS).

(i) IS and NT submit demands
(ii) Firm observes stock price and decides whether to invest or not
Firm’s output is realized

Figure B.2: Equilibrium demands by informed speculators. The figure shows informed speculators’ demands in the model with strategic behavior and learning from prices for different levels of managerial uncertainty (i.e. different $\sigma_\theta$). The figure presents numerical solutions with model parameters as follows: $c = 1$, $\mu_\theta = 1.05$, $\sigma_z = 1$ and $\epsilon = 1$. 

Informed Speculators’ Optimal Demands: $X^*_I(\theta)$

<table>
<thead>
<tr>
<th>$\sigma_\theta$</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend:
- $\sigma_\theta=0$
- $\sigma_\theta=0.5$
- $\sigma_\theta=1$
- $\sigma_\theta=4$
Figure B.3: Probability of investment. Probability of the firm undertaking the project under perfect information (alternative #1), no learning from prices (alternative #2), no strategic behavior (alternative #3) and strategic behavior. The parameters are $c = 1$, $\mu_\theta = 1.05$, $\sigma_\theta = 1$, $\sigma_z = 1$, $\epsilon = 1$, $c = 1$ and $\epsilon = 1$. 
Figure B.4: Correlation between the stock price and the probability of investment. Correlation between price and investment for different levels of managerial uncertainty ($\sigma_\theta$) for the strategic and non-strategic (alternative #3) models. The parameters are $c = 1$, $\mu_\theta = 1.05$, $\sigma_z = 1$ and $\epsilon = 1$. 

![Correlation diagram between stock price and investment probability](image-url)
B.3 Appendix: Proofs

Proof of Lemma 5: If the firm manager conjectures a linear demand by the informed speculator of the form $X_I(\theta) = a + b\theta$, the posterior on the fundamental is:

$$\xi(\theta | q) = \frac{\varphi(\epsilon q - a - b\theta)\xi(\theta)}{\int_{-\infty}^{\infty} \varphi(\epsilon q - a - b\theta)\xi(\theta)d\theta}. \quad (B.1)$$

where $\varphi()$ is density function of the normal distribution with mean 0 and variance $\sigma_z^2$ and $\xi()$ is the density function of a normal distribution with mean $\mu_{\theta}$ and variance $\sigma_{\theta}^2$. Under the posterior, the cutoff rule is given by

$$\int_{-\infty}^{\infty} \theta \varphi(\epsilon \bar{q}_l - a - b\theta)\xi(\theta)\int_{-\infty}^{\infty} \varphi(\epsilon \bar{q}_l - a - b\theta)\xi(\theta)d\theta = c \quad (B.2)$$

Using the properties of the normal distribution I solve for the cutoff price, $\bar{q}_l = \frac{1}{\epsilon} \left[ a + cb - \frac{1}{b}(\mu_{\theta} - c)\frac{\sigma_{\theta}^2}{\sigma_z^2} \right]$.

Proof of Proposition 6: The method of this proof is to iterate among best responses to find the fixed point in strategies.

Step 1a. Assume a linear function for the informed speculators’ demand $X_I^0(\theta) = \frac{\epsilon(\theta - c)}{2}$. From Lemma 5, the firm’s best response (cutoff price) is

$$\bar{q}^1 = - (\mu_{\theta} - c) \frac{2\sigma_{\theta}^2}{\epsilon^2\sigma_z^2}$$

Step 1b. Using $\bar{q}^1$, I find the optimal decision rule of the informed speculator.
This is the solution to the first order condition from profit maximization \([2.5]\):

\[
(\theta - c) \left[ \Phi \left( \frac{X_I(\theta) - \epsilon q^1}{\sigma_z} \right) + \frac{1}{\sigma_z} X_I(\theta) \Phi' \left( \frac{X_I(\theta) - \epsilon q^1}{\sigma_z} \right) \right] - \frac{2}{\epsilon} X_I(\theta) = 0
\]

For the reasons described in the main text, in this paper I am interested in the solution around \(\theta = c\). Linearizing the FOC around \(\theta = c\), I can guess and verify that the demand function of the informed speculators is of the form \(X_I(\theta) = k(\theta - c)\).

The first order approximation of the FOC is:

\[
(\theta - c) \Phi \left( -\frac{\epsilon q^1}{\sigma_z} \right) - \frac{2k(\theta - c)}{\epsilon} = 0
\]

Solving for \(k\), \(k = \frac{\epsilon \Phi(\gamma)}{2}\), where \(\gamma = \frac{2(\mu_\theta - c)}{\epsilon \sigma_\theta^2}\). Informed speculators’ demands are:

\[
X^1_I(\theta) = \frac{\epsilon(\theta - c)}{2} \Phi(\gamma)
\]

**Step 2a.** Using the linear demands above \(X^1(\theta)\), I solve for the firm’s cutoff rule.

\[
\bar{q}^2 = -(\mu_\theta - c) \frac{2\sigma_z^2}{\epsilon^2 \sigma_\theta^4} \frac{1}{\Phi(\gamma)}
\]

**Step 2b.** The iteration continues by assuming \(\bar{q}^2\) to find the optimal decision rule of the informed speculator. Following the linearization in step 1b, the informed
speculators' demands are:

\[ X^1_I(\theta) = \frac{\epsilon(\theta - c)}{2} \Phi \left( \gamma \Phi(\gamma) \right) \]

Continuing the iteration procedure, in the \( n \)th iteration the firm’s cutoff rule and the speculators’ demands are:

\[ q_n = -\left( \mu_\theta - c \right) \frac{2\sigma^2}{\epsilon^2\sigma^2_\theta} \frac{1}{f^{n-1}(\gamma)} \]

\[ X^*_n(\theta) = \frac{\epsilon(\theta - c)}{2} f^n(\gamma) \]

where \( f^n(\gamma) \) is a continued fraction of cumulative normal distributions of the form:

\[
f^n(\gamma) = \Phi \left( \frac{\gamma}{\Phi \left( \frac{\gamma}{\Phi \left( \frac{\gamma}{\Phi \left( \cdots \right)} \right)} \right)} \right) \tag{B.3}
\]

**Lemma B.1.** If \( \gamma \geq 0 \), the infinite continued fraction \( f^{(\infty)}(\gamma) \) converges to \( \Phi(\gamma) \). That is, \( \lim_{n \to \infty} f^n(\gamma) = \Phi(\gamma) \). If \( \gamma < 0 \), \( \lim_{n \to \infty} f^n(\gamma) = 0 \).

Using Lemma [B.1], the fixed point in strategies has the following form:

- If the expected NPV of the investment project is non-negative under the firm’s prior, \( \mu_\theta \geq c \) then the equilibrium demands by speculators and firms’ cutoff
rule are:

\[ X_t^*(\theta) = \frac{\epsilon(\theta - c)}{2} \Phi(\gamma) \]

\[ \bar{q}^* = -(\mu_\theta - c) \frac{2\sigma_z^2}{\epsilon^2 \sigma^2_\theta} \frac{1}{\Phi(\gamma)} \]

- If the expected NPV of the investment project is negative under the firm’s prior, \( \mu_\theta < c \) then, informed speculators don’t trade in equilibrium \( X_t^*(\theta) = 0 \) and firms never invest, \( \bar{q}^* \rightarrow \infty \).

**Proof that** \( \frac{d\bar{q}^*}{d\gamma} < 0 \): From equation (2.8) it is sufficient to prove that \( g(\gamma) \equiv -\Phi(\gamma) + \gamma \Phi'(\gamma) < 0 \) for \( \gamma \geq 0 \). First, note that \( g() \) is monotonically decreasing in \( \gamma \), \( g'(\gamma) = \Phi''(\gamma) < 0 \). Also \( g(0) = -\frac{1}{2} \). Hence \( g(\gamma) < 0 \).

**Proof of Corollary 8**: Replace the equilibrium demands by the informed speculators and firms’ cutoff rule in the probability of investment: \( \Pr(q \geq \bar{q} \mid \theta) = \Phi \left[ \frac{1}{\sigma_z} (X_t^*(\theta) - c\bar{q}^*) \right] \).

**Proof of Corollary 9**: Investment efficiency when informed speculators internalize the firm’s updating process is:

\[ \left. \frac{\partial \psi^*(\theta)}{\partial \theta} \right|_{\theta=c} = \frac{\epsilon}{2\sigma_z} \Phi(\gamma) \Phi' \left( \frac{\gamma}{\Phi(\gamma)} \right) \] (B.4)

Investment efficiency when informed speculators don’t internalize the firm’s updating process is:

\[ \left. \frac{\partial \hat{\psi}(\theta)}{\partial \theta} \right|_{\theta=c} = \frac{\epsilon}{2\sigma_z} \Phi' (\gamma) \] (B.5)
The $g(\gamma)$ can be defined as the ratio between the investment efficiency measure in the non-strategic case and the strategic case:

$$
g(\gamma) \equiv \frac{\partial \psi^b(\theta)}{\partial \theta} \bigg|_{\theta=c} \frac{\partial \psi^*(\theta)}{\partial \theta} \bigg|_{\theta=c}
$$

(B.6)

This ratio is strictly decreasing in $\gamma$ if $\gamma > 0$, that is

$$
g'(\gamma) = -\frac{\gamma \Phi'(\gamma) \Phi(\gamma)^3 + \gamma^2 \Phi'(\gamma)^2}{\Phi(\gamma)^4 \Phi'\left(\frac{c}{\Phi(\gamma)}\right)} < 0, \text{ if } \gamma > 0
$$

Also, $\lim_{\gamma \to \infty} g(\gamma) = 1$. Combining these two results, for any finite level of precision of managerial information (i.e. $\gamma$ finite) and $\mu_\theta > c$, the investment decision is less efficient with strategic traders than in the non-strategic benchmark.

**Proof of Corollary 10**: The first order approximation of the probability of investment for a firm with fundamental $\theta$ around $\theta = c$ can be written as: $\psi^3(\theta) = \Phi(\gamma) + \phi(\gamma)(\theta-c)$ for the alternative case #3 and $\psi^*(\theta) = \Phi\left(\frac{\gamma}{\Phi(\gamma)}\right) + \phi\left(\frac{\gamma}{\Phi(\gamma)}\right)(\theta-c)$ for the benchmark model.

In the non-strategic model (Alternative #3), the correlation between the probability of investment and stock prices, following Definition is:

$$
Corr(q^3, \psi^3) = \int_{-\infty}^{\infty} \frac{\phi(\gamma)}{2} (\theta - c)^2 d\xi(\theta)
$$

Integrating above I obtain $Corr(q^3, \psi^3) = \frac{\phi(\gamma)}{2} [\sigma^2_\theta + (\mu_\theta - c)^2]$. Similarly, for the strategic model I use the linear approximation of the investment probability and
integrate to calculate the correlation between prices and investment.
Bibliography


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