ABSTRACT

Title of dissertation:	Utilization of Channel State Information in Transmission Control for Wireless Communication Networks
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Dissertation directed by:	Professor Anthony Ephremides Department of Electrical and Computer Engineering

This dissertation deals with the utilization of channel knowledge in improving the performance of wireless communication systems. The first part is about energy harvesting networks. The transmission policies in energy harvesting wireless systems need to adapt to the harvested energy availability and the channel characteristics. We start by considering the scheduling policy for a single energy harvesting source node that operates over a time varying channel. The goal of the source is to maximize the average number of successfully delivered packets per time slot. The transmission decisions depend on the available channel information and the length of the energy queue. Then, we investigate the case in which the source is helped by a relay through a network-level cooperation protocol. We investigate the case of a single relay node in which we optimize the transmission control based on channel measurements. Then, we assess the benefits of using partial relaying. We provide an exact characterization of the stability region of a network which consists of a source, a relay and a destination with random data arrivals to both

the source and the relay. We derive the optimal value of the relaying parameter to maximize the stable throughput of the source for a given data arrival rate to the relay. Finally, we introduce the problem of general relaying cost minimization for cooperative energy harvesting networks with multiple relays. Then, we introduce the energy consumption as a cost criterion for the optimization problem to find an energy-efficient partial relaying protocol.

In the second part, we investigate the techniques to optimally exploit channel information in transmission control for interfering sources. We discuss the scheduling problem for different levels of channel knowledge because learning instantaneous channels states may be costly or infeasible. We consider a network that consists of two transmitter-receiver pairs which operate over time varying channels. We derive the optimal scheduling policies which maximize the expected weighted sum-rate of the network per time slot. The decision depends on the information about the channels between nodes.

In the third part, we investigate the effect of channel estimation on the performance of a secondary network in a cognitive radio system. We focus on estimating the sensing-channel from the primary source to the secondary source which helps in assessing the reliability of the sensing decision. The channel is estimated opportunistically when the secondary source senses the primary source to be active. We consider the performance criterion to be the energy consumed by the secondary system constrained by a required average data transmission rate for the secondary system and an allowable average failure probability for the primary system.

Utilization of Channel State Information in Transmission Control for Wireless Communication Networks

by

Mohamed Tawfeek Kashef Hany

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Advisory Committee:
Professor Anthony Ephremides, Chair/Advisor
Professor Sennur Ulukus
Professor Richard J. La
Professor Prakash Narayan
Professor Jeffrey Herrmann

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Chapter 1: Introduction

Channel variation is a source of randomness in data transmission; therefore, system design should exploit such randomness. The knowledge of channel state information plays a fundamental role in exploiting channel variation. This dissertation focuses on obtaining efficient techniques for wireless communication systems which exploit channel knowledge in various ways depending on different levels of channel state information availability in newly emerging topics in wireless communications and networking. These topics include energy harvesting, cooperative communication and cognitive radio.

1.1 Utilization of Channel Information

The time varying nature of the wireless channels leads to decrease in the reliability of transmission over these channels. The availability of instantaneous channel state information (CSI) of links plays an important role in enhancing the performance of wireless networks [1, 2].

There are two main challenges that face reliable wireless communications. The first is the multipath fading in addition to the classical additive white Gaussian noise. The other is the multi-user interference which results due to the fact that the wireless channel is a shared medium and hence simultaneous transmissions interfere with each other. The knowledge of channel state information, either accurately or partially, plays a key role in achieving reliable communication over unreliable wireless channels. Thus, understanding the impact of the knowledge of different levels of channel state information becomes indispensable to the overall system design. Recent research has demonstrated that deeper understanding of CSI can lead to new views on fading channels and new communication techniques such as multi-user diversity [3] and interference alignment [4].

In this dissertation, we discuss techniques to efficiently utilize the available knowledge about the wireless channels of a system in enhancing some performance measure. We consider different performance measures such as the throughput, the stability and the consumed energy by the system. We also consider different types of communication systems such as energy harvesting networks, cooperative communication networks and cognitive radio networks.

1.2 Energy Harvesting

Energy harvesting enables wireless nodes to be recharged by the surrounding environment. Thus, wireless communication networks with energy harvesting capability have extended lifetime and are self-sufficient. Recent advances in energy harvesting materials and ultra-low-power communications will soon enable the realization of energy harvesting networks [5, 6]. Nodes can harvest energy from nature through various different sources, such as solar cells, vibration absorption devices, water mills, thermoelectric generators and microbial fuel cells. Examples of the techniques of energy harvesting from nature can be found in [7, 8]. The energy harvesting nodes are used in different types of networks such as rechargeable sensor networks [9], and Energy Harvesting Active Networked Tags (EnHANTs) [10]. Such networks have applications in various areas which motivates studying different aspects related to energy harvesting networks.

In the systems where nodes harvest energy from nature, energy can be modeled as an exogenous recharge process. Therefore, unlike traditional battery-powered systems, energy is not a deterministic quantity in these systems, but is a random process which varies stochastically in time. In our work, we deal with the harvested energy as a stochastic process without considering the energy harvesting technique. When dealing with nodes powered by non-rechargeable batteries, the common objectives are short term such as maximizing the lifetime of the network [11, 12]. The harvesting capability enables considering different performance measures such as the throughput and the stability of the network [13].

There has been recent research effort on understanding data transmission in energy harvesting networks [14]- [24]. In [14], an optimal admission control policy is obtained for data transmission with energy harvesting sensors. In [15], energy management policies which stabilize the data queue are proposed for single-user communication under a linearity assumption for the power-rate relation. In [16], the problem of throughput optimal energy allocation is studied for energy harvesting systems in a time constrained slotted setting. In [17, 18], minimization of the transmission completion time is considered in an energy harvesting single-user system. In [19], the problem of minimization of the transmission completion time for energy harvesting transmitters with batteries of finite energy storage is considered. In [20, 21], optimal transmission policies are obtained for a single energy harvesting transmitter operating over a time varying channel. In [22–24], optimal transmission policies are developed for broadcast channel with an energy harvesting transmitter.

Channel knowledge in energy harvesting networks helps in efficiently consuming the limited renewable available energy in the transmission process. Energy harvesting nodes should forward data over wireless channels when they have good conditions.

1.3 Cooperative Communication

Cooperative diversity enables single antenna users to benefit from the spatial diversity by delivering data with the help of relay nodes. Numerous works have been done to analyze cooperative diversity at the physical layer based on information theoretic considerations [25, 26]. It has also been shown that cooperation can be applied at the network layer. In [27], a network-level cooperation protocol has been used to increase the stable throughput region for the uplink of a wireless network. Also in [28], a network-level cooperation protocol has been exploited to enhance the performance in a multicasting scenario. A network-level partial relaying protocol has been considered before in [29] where the stability region of a system with a source, a relay and a destination has been characterized. The nodes are non-energy harvesting and they access the channel through a random access technique. In [29], the effect of relaying control on the system performance has been investigated.

Channel knowledge in cooperative networks allows smarter cooperation between nodes. Channel characteristics help in determining the suitable situations for cooperation and channel state information availability helps in selecting the suitable transmission decisions.

1.4 Cognitive Radio

Cognitive radio [30] is a paradigm in which unlicensed secondary users may access licensed frequency bands in order to efficiently exploit the available radio spectrum. A huge amount of research has been carried out in recent years on cognitive radio techniques, since there is widespread interest in this technology. Classical cognitive radio is based on the use of temporarily unused frequency bands, and so its implementation requires that proper spectrum sensing procedures must be deployed so that white spaces are detected, and, mostly important, secondary users interrupt their communications as soon as a white space becomes no longer white that it is again used by the primary users. It is apparent that this is a quite difficult task, especially when it is to be implemented in a simple device with limited hardware capabilities and computational power. An alternative approach, instead, is based on the idea that secondary users are allowed to transmit in the same frequency band licensed to an active primary network, but subject to the constraint that they must not be too much disturbing for the primary users.

Channel knowledge is an essential part in the cognitive nature of the secondary networks. It helps in taking more reliable decisions either in the sensing process or in the transmission process. Thus, it helps in reaching the main goals for the secondary systems in cognitive radio networks. The goals are to opportunistically access the unused frequency bands and not disturb the primary users.

1.5 Outline of the Dissertation

This dissertation is organized as follows. Chapter 2 introduces the problem of scheduling of energy harvesting sources which operate over time varying wireless channels. Specifically, in section 2.3, we obtain the structure of the optimal transmission policy for an energy harvesting source. In section 2.4, we obtain an upper bound on the performance of the source node in the proposed scenario. Chapter 3 is about the stability analysis of an energy harvesting source which is helped by an energy harvesting relay while both operate over time varying wireless channels. We start by the case of perfect channel measurements. Then in section 3.4, we consider the case of imperfect channel measurements. In section 3.5, we obtain the optimal transmission strategy for the source node to maximize its stable throughput. Chapter 4 introduces a partial relaying cooperation protocol for energy harvesting networks. We characterize the stability region of a system which contains a source and a relay with energy harvesting capability that exploits partial relaying. In section 4.6, we show the improvement in the system performance because of using partial relaying compared to simple relaying strategies. Chapter 5 extends the analysis to the case of multiple relays. In this case, we consider a general cost minimization problem over the partial relaying parameters. Chapter 6 is about the problem of the scheduling of two sources over time varying channels. Different levels of channel state information availability are considered in different sections in the chapter. In section 6.8, we consider the case of distributed scheduling for the two sources. Chapter 7 is about the transmission control in cognitive radio networks with the availability of the sensing-channel information. The performance is compared for the cases of no channel estimation, accurate channel estimation and opportunistic channel estimation. Finally, chapter 8 summarizes the contributions of this dissertation and points out possible future research directions.

Chapter 2: Optimal Scheduling for Energy Harvesting Sources

2.1 Introduction

Energy harvesting is naturally a stochastic process. One important problem is to decide whether to use the available energy for transmission or continue storing it for future transmissions. Efficient scheduling techniques should be able to take the full advantage of energy harvesting. There have been many previous works that consider scheduling techniques in energy harvesting networks. In our work, we consider scheduling of transmissions based on the energy queue state, energy harvesting statistics and the channel state as we consider the case of time varying channels. In [14, 15, 31], scheduling for source nodes with energy harvesting capability is considered under a fixed channel assumption. In [15], an energy management policy to maintain the stability of the data queue is considered for single-user communication under a linear approximation of the rate-power relation. In [31], power adaptation is considered to maximize a general rate utility function for a single user where the decision depends on the energy queue state without considering the energy arrival process statistics. In [14], an optimal threshold on the data queue as a function of the energy queue state is found where the source node takes the decision to transmit data if the data queue level is above this threshold.

On the other hand, off-line scheduling is also considered in [18], [32]. In off-line scheduling, it is assumed that the time instants of energy arrival and data arrival events are known prior to the scheduling. In [18], optimal power allocation for each transmission is considered over a fixed channel where the power allocation is done off-line. The goal is to maximize the transmitted data over a fixed period of time. In [32], the same problem as in [18] is considered but for a time varying channel where also the states of the channel and the time instants for the states change are known prior to the scheduling.

Online scheduling for a source node with energy harvesting capability that transmits over a time varying channel is considered in [33]. The scheduling is done based on the energy queue state and the energy arrival process statistics. It was assumed that the energy arrivals and channel state variations can happen at any time instant and hence the change of the power used in transmission. The optimal policy is stated as a continuous time stochastic dynamic program which requires excessive computation. Then, suboptimal techniques are considered. Similar problem was considered in [16] where the problem of energy allocation of a single source node with energy harvesting capability was considered. The goal is the maximization of the finite horizon throughput. Both off-line and online policies are discussed. Structural results for the optimal energy allocation policy were obtained via the use of dynamic programming and convex optimization techniques. In case of online energy allocation, the policy calculations may be done off-line and implemented via a lookup table. In our work, we consider a time slotted system and we prove that the optimal online policy is a simple threshold type policy based on a Markov decision process model.

We consider a communication system which operates over a Gilbert-Elliot channel. The source node has an energy harvesting capability. Also, it takes the decision to transmit a data packet or defer the transmission for the future depending on the channel measurements and the energy queue length. At the beginning of each time slot, the source performs channel measurement to know whether the channel is in the good state or not, and it checks the length of the energy queue. Depending on this information, the source decides either to transmit a data packet over the channel or to defer the transmission for later time slots. The objective for the source is the maximization of the average number of packets that are received correctly by the destination per time slot.

We formulate this problem as a Markovian Decision Problem (MDP). The objective of the problem is the maximization of the expected infinite horizon discounted number of packets transmitted by the source node. The limit of this problem, when the discount factor tends to be 1, is equivalent to the problem of maximization of the expected average number of packets successfully delivered per time slot. We deal with the discounted reward problem for mathematical convenience. We determine the optimal policy for decision making via the use of value iteration. We derive structural results regarding the optimal policy and show that the optimal policy is a threshold-type policy in the energy queue length. Also, we calculate an upper bound for the performance of the system. The case of no channel measurements available at the source is also considered. Numerical results show the difference in the performance between the optimal policy and simpler policies that are the greedy and the conservative policies. Also, we compare the optimal performance in both cases where channel measurements are either available or not at the source node. Thus, we can assess the impact of the time varying nature of the channel as well as that of CSI availability. This work was presented in [21, 34].

2.2 System Model and Problem Formulation

We consider a source node that has a data queue and an infinite energy queue as shown in figure 2.1. The system is time slotted. During each time slot, the source can transmit a single data packet. Transmitting a data packet requires using a single energy unit from the energy queue. The energy queue length is denoted by E. The source node can acquire, at most, a single energy unit at each time slot with probability q. We assume that the source has a saturated data queue such that there is always data to be sent at every time slot. The saturated data queue models the case when the source has a large volume of data.

The channel is modeled by a two-state Markov chain (Gilbert Elliot model). Each state corresponds to a degree of channel quality. State 1 corresponds to good connectivity, while state 0 corresponds to poor connectivity. The success probability of a transmitted packet, when the channel is in state $i = \{0, 1\}$, is denoted by f_i. From this definition, we find that f_1 is larger than f_0. Time is slotted and the channel remains fixed within each slot and moves into another state in the next slot following the state transition probability of the underlying Markov chain. The transition probability from state 0 to state 1 is λ_0 and the transition probability

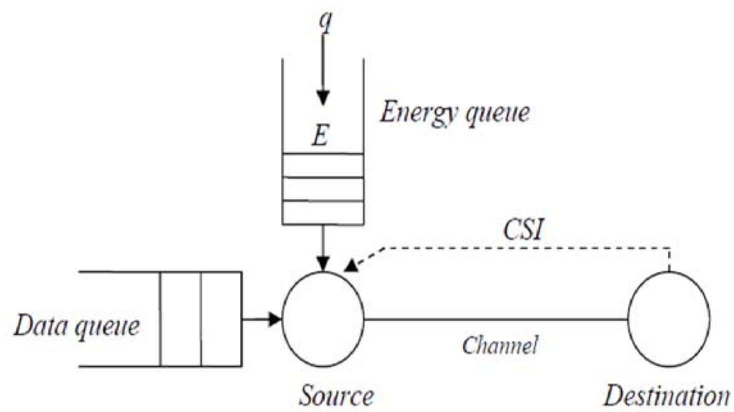

Figure 2.1: System Model

from state 1 to state 1 is λ_1 as shown in figure 2.2. We consider the case that λ_1 is larger than λ_0 which is noted as a positive memory channel as described in [35].

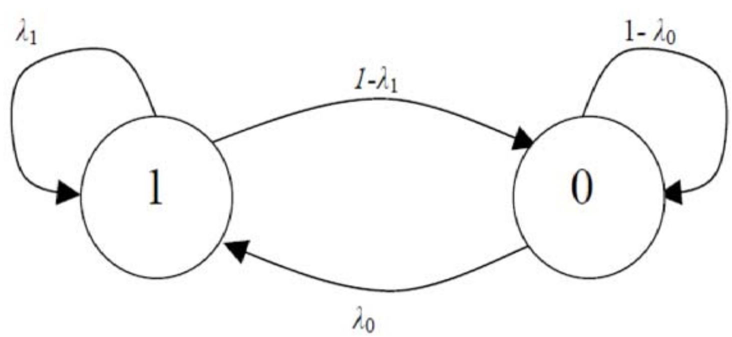

Figure 2.2: Gilbert-Elliot Channel Model

At the beginning of each slot, the source node chooses between two actions: transmit a packet or defer transmission. The action is taken based on the available CSI and the energy queue length. We denote the channel state by C. We assume that the source gets a feedback from the destination with the CSI at the beginning of the time slot. Based on the CSI and the value of E, the decision of the source node is taken. The action to transmit is denoted by T and the action to defer transmission is denoted by D.

MDP formulation-Because of the Markovian property of the channel and the Markovian property of the energy queue which depends on the decision chosen by the source, the decision problem at the source node is an MDP. We define u as the scheduling policy used by the source node and it is a mapping from the state space to the action space. Let $V^u(E, C)$ be the expected discounted reward with initial state $X_0 = (E, C)$, u be the policy followed, and $\beta \in [0, 1)$ be the discount factor. The expected discounted reward has the following expression

$$V^u(E, C) = \mathbb{E}^u \left[\sum_{t=0}^{\infty} \beta^t R(X_t, A_t) \,|X_0 = (E, C) \right] \tag{2.1}$$

$$R(X_t, A_t) = \begin{cases} f_C & \text{if } A_t = T \\ 0 & \text{if } A_t = D \end{cases} \tag{2.2}$$

The expected reward represents the expected number of packets delivered to the destination given that a certain action was chosen. First, if the action D is chosen, the source does not attempt to transmit any packets. As a result, the instantaneous expected reward has the value 0. When the action T is chosen, the reward will be 1 if the packet is delivered correctly to the destination. The probability that a packet is delivered depends on the channel state and equals f_C. The expected number of packets delivered at a time slot, when the action T is taken, is f_C.

Define now the value function $V(E,C)$ as

$$V(E,C) = \max_u V^u(E,C) \quad \text{for all } E \in \{0,1,2,......\} \text{ and } C \in \{0,1\} \tag{2.3}$$

From [36], there exists a stationary policy u^*such that $V(E,C)$= $V^{u*}(E,C)$. This value function satisfies Bellman's equation, namely,

$$V(E,C) = \max_{A\in\{T,D\}} \{V_A(E,C)\} \tag{2.4}$$

where $V_A(E,C)$ is the value achieved by taking the action A when the state is (E,C). The expression of $V_A(E,C)$ can be written as follows:

$$V_A(E,C) = R((E,C),A) + \beta\mathbb{E}_{(a,b)}\left[V(a,b)\,|X_0 = (E,C), A_0 = A\right] \tag{2.5}$$

where (a,b) is the next state when the action A is taken and the initial state is (E,C).

When the action T is chosen, the expected reward is f_C. Also, the energy queue will lose one energy unit. On the other hand, the energy queue can acquire a new energy unit with probability q. When the channel is in state C at the current time slot, the channel state at the next time slot is 1 with probability λ_C and 0 with probability (1-λ_C). The expression for $V_T(E,C)$ is written as follows:

$$\begin{aligned} V_T(E,C) = f_C + \beta\,[q\lambda_C V(E,1) + q(1-\lambda_C)V(E,0) \\ + (1-q)\lambda_C V(E-1,1) + (1-q)(1-\lambda_C)V(E-1,0)] \end{aligned} \tag{2.6}$$

When the action D is taken, the same explanation holds except that no energy units will be consumed and there is no instantaneous expected reward. As a result, the expression of $V_D(E,C)$ will be given by

$$V_D(E,C) = 0 + \beta\left[q\lambda_C V(E+1,1) + q(1-\lambda_C)V(E+1,0)\right.$$
$$\left. + (1-q)\lambda_C V(E,1) + (1-q)(1-\lambda_C)V(E,0)\right] \quad (2.7)$$

Finally, the Bellman's equation for the problem is written as follows

$$V(E,C) = \max\left\{V_T(E,C), V_D(E,C)\right\} \quad (2.8)$$

2.3 Structure of the Optimal Policy

In this section, we will prove some properties for the value function and prove the optimality of a threshold type policy.

Lemma 2.1. $V(E,1)$ is larger than or equal to $V(E,0)$

Proof. We are going to use mathematical induction in this proof. Define $V(E,C,n)$ as the optimal value function when the decision horizon spans n stages. The value function recursion is written as follows:

$$V(E,C,n) = \max\left\{f_C + \beta\left[q\lambda_C V(E,1,n-1) + q(1-\lambda_C)V(E,0,n-1)\right.\right.$$
$$\left. + (1-q)\lambda_C V(E-1,1,n-1) + (1-q)(1-\lambda_C)V(E-1,0,n-1)\right],$$
$$\beta\left[q\lambda_C V(E+1,1,n-1) + q(1-\lambda_C)V(E+1,0,n-1)\right.$$
$$\left.\left. + (1-q)\lambda_C V(E,1,n-1) + (1-q)(1-\lambda_C)V(E,0,n-1)\right]\right\} \quad (2.9)$$

We start by showing that the hypothesis is true at n=1. For $E \geq 1$, $V(E,1,1)=f_1$ and $V(E,0,1)=f_0$. Also for E=0, both are equal to zero, that $V(0,1,1)= V(0,0,1)=0$. Thus, the hypothesis is true for n=1 for all E. Assume that the lemma is true for n-1, then we start by calculating

$$\begin{aligned} V_T(E,1,n) - V_T(E,0,n) = \\ f_1 - f_0 + \beta(\lambda_1 - \lambda_0)\left[q\left(V(E,1,n-1) - V(E,0,n-1)\right)\right. \\ \left. + (1-q)\left(V(E-1,1,n-1) - V(E-1,0,n-1)\right)\right] \end{aligned} \quad (2.10)$$

where $V_T(E,C,n)$ is the value function when the action T is chosen, the channel at state C and the decision horizon spans n stages. We have that f_1 is larger than f_0. Also from the hypothesis at n-1 and $\lambda_1 > \lambda_0$, the quantity in (2.10) is larger than 0. Then, we consider the difference when the action D is chosen, that is,

$$\begin{aligned} V_D(E,1,n) - V_D(E,0,n) = \\ \beta\left[q\left(V(E+1,1,n-1) - V(E+1,0,n-1)\right)(\lambda_1 - \lambda_0)\right. \\ \left. + (1-q)\left(V(E,1,n-1) - V(E,0,n-1)\right)(\lambda_1 - \lambda_0)\right] \end{aligned} \quad (2.11)$$

where $V_D(E,C,n)$ is the value function when the action D is chosen, the channel is at state C and the decision horizon spans n stages. This quantity is also larger than or equal to 0. From the definition of the value function at (2.8), we conclude that $V(E,1,n) \geq V(E,0,n)$ for all n. Then, $V(E,1) \geq V(E,0)$ is true by considering the limit as n goes to infinity. □

Lemma 2.2. $V(E,C)$ is non decreasing function in E.

Proof. We are going to use mathematical induction in the proof with similar steps as in the proof of the previous lemma. We start by showing the validity of the hypothesis at n=1. For E=0, we found $V(0,C,1)$=0. Also for $E \geq 1$, we have found $V(E,C,1)=f_C$. Then, the hypothesis is true for n=1 as the value function at $E \geq 1$ is larger than the value function at E=0.

Assume that the hypothesis is true for n-1. Then, we have the value function expression

$$\begin{aligned} V(E,C,n) = \max \{ & f_C + \beta [q\lambda_C V(E,1,n-1) + q(1-\lambda_C) V(E,0,n-1) \\ & + (1-q)\lambda_C V(E-1,1,n-1) + (1-q)(1-\lambda_C) V(E-1,0,n-1)], \\ & \beta [q\lambda_C V(E+1,1,n-1) + q(1-\lambda_C) V(E+1,0,n-1) + \\ & + (1-q)\lambda_C V(E,1,n-1) + (1-q)(1-\lambda_C) V(E,0,n-1)] \} \end{aligned} \quad (2.12)$$

Each argument in the max function is the summation of positive weighted non-decreasing functions. Then, both arguments of the max function are non-decreasing functions. The maximum of two non-decreasing function is also non-decreasing.

Then, we consider the limit as n goes to infinity to prove that the value function $V(E,C)$ is a non-decreasing function of E for a fixed C. □

Lemma 2.3. For $E \geq 1$, $V(E+1,C)$-$V(E,C) \leq f_1$

Proof. We are going to use mathematical induction in the proof. We start by the validity of the hypothesis at n=1. We have $V(E+1,C,1)$-$V(E,C,1)$=0. Then, the hypothesis is true for n=1. Assume that the hypothesis is true for n-1, namely that $V(E+1,C,n\text{-}1)$-$V(E,C,n\text{-}1) \leq f_1$. Then by using the hypothesis at C=1, we have the following

$$V_T(E,1,n) - V_D\left(E,1,n\right)$$

$$\geq f_1 - \beta\left[q\lambda_C f_1 + q\left(1-\lambda_C\right)f_1 + \left(1-q\right)\lambda_C f_1 + \left(1-q\right)\left(1-\lambda_C\right)f_1\right]$$

$$= f_1 - \beta f_1 = f_1\left(1-\beta\right) > 0 \quad (2.13)$$

Then, the action to be chosen is T. As a result, $V(E,1,n)=V_T(E,1,n)$. This leads to the following difference between the value functions

$$V\left(E+1,1,n\right) - V\left(E,1,n\right) = V_T\left(E+1,1,n\right) - V_T\left(E,1,n\right) =$$

$$\beta\left[q\lambda_C\left[V\left(E+1,1,n-1\right) - V\left(E,1,n-1\right)\right]\right.$$

$$+ q\left(1-\lambda_C\right)\left[V\left(E+1,0,n-1\right) - V\left(E,0,n-1\right)\right]$$

$$+ \left(1-q\right)\lambda_C\left[V\left(E,1,n-1\right) - V\left(E-1,1,n-1\right)\right]$$

$$\left.+ \left(1-q\right)\left(1-\lambda_C\right)\left[V\left(E,0,n-1\right) - V\left(E-1,0,n-1\right)\right]\right] \leq \beta f_1 < f_1 \quad (2.14)$$

To explain the above result, note that the summation of the quantities $q\lambda_s$, (1-q)λ_s, q(1-λ_s) and (1-q)(1-λ_s) equals 1. Every term in the difference at equation (2.14) is multiplied by β and one of the quantities $q\lambda_s$, (1-q)λ_s, q(1-λ_s) and (1-q)(1-λ_s). Also, Every one of the terms $V(E$+1,1,n-1)-$V(E$,1,n-1), $V(E$+1,0,n-1)-$V(E$,0,n-1), $V(E$,1,n-1)-$V(E$-1,1,n-1) and $V(E$,1,n-1)-$V(E$-1,1,n-1) is less than or equal to f_1 by assumption. Then, the summation will be less than or equal to βf_1 which is less than $f_1$from the definition of β.

Then, we consider the case when C=0. There exist four cases for the actions to be selected when the energy queue has the lengths E and E+1. The first case is that the action T is chosen when the energy queue length is E+1 and the action D

is chosen when the energy queue length is E. The difference in this case is given by

$$V_T(E+1,0,n) - V_D(E,0,n) =$$

$$f_0 + \beta\left[q\lambda_0 V(E+1,1,n-1) + q(1-\lambda_0)V(E+1,0,n-1)\right.$$

$$\left. + (1-q)\lambda_0 V(E,1,n-1) + (1-q)(1-\lambda_0)V(E,0,n-1)\right]$$

$$- \beta\left[q\lambda_0 V(E+1,1,n-1) + q(1-\lambda_0)V(E+1,0,n-1)\right.$$

$$\left. + (1-q)\lambda_0 V(E,1,n-1) + (1-q)(1-\lambda_0)V(E,0,n-1)\right] = f_0 \quad (2.15)$$

The second case is that the action T is chosen in both cases when the energy queue length is E+1 or E. The difference in this case is given by

$$V_T(E+1,0,n) - V_T(E,0,n) =$$

$$\beta\left[q\lambda_0 V(E+1,1,n-1) + q(1-\lambda_0)V(E+1,0,n-1)\right.$$

$$\left. + (1-q)\lambda_0 V(E,1,n-1) + (1-q)(1-\lambda_0)V(E,0,n-1)\right]$$

$$- \beta\left[q\lambda_0 V(E,1,n-1) + q(1-\lambda_0)V(E,0,n-1)\right.$$

$$\left. + (1-q)\lambda_0 V(E-1,1,n-1) + (1-q)(1-\lambda_0)V(E-1,0,n-1)\right] \leq \beta f_1 < f_1$$

$$(2.16)$$

The third case is that the action D is chosen in both cases when the energy queue length is E+1 or E. The difference in this case is given by

$$\begin{aligned}
&V_D(E+1,0,n) - V_D(E,0,n) = \\
&\qquad \beta\left[q\lambda_0 V(E+2,1,n-1) + q(1-\lambda_0)V(E+2,0,n-1)\right. \\
&\qquad \left. + (1-q)\lambda_0 V(E+1,1,n-1) + (1-q)(1-\lambda_0)V(E+1,0,n-1)\right] \\
&\qquad - \beta\left[q\lambda_0 V(E+1,1,n-1) + q(1-\lambda_0)V(E+1,0,n-1)\right. \\
&\qquad \left. + (1-q)\lambda_0 V(E,1,n-1) + (1-q)(1-\lambda_0)V(E,0,n-1)\right] \leq \beta f_1 < f_1
\end{aligned} \tag{2.17}$$

The fourth case, finally, is that the action D is chosen when the energy queue length is E+1 and the action T is chosen when the energy queue length is E. The difference in this case is given by

$$\begin{aligned}
&V_D(E+1,0,n) - V_T(E,0,n) = \\
&\qquad \beta\left[q\lambda_0 V(E+2,1,n-1) + q(1-\lambda_0)V(E+2,0,n-1)\right. \\
&\qquad \left. + (1-q)\lambda_0 V(E+1,1,n-1) + (1-q)(1-\lambda_0)V(E+1,0,n-1)\right] \\
&\qquad - f_0 - \beta\left[q\lambda_0 V(E,1,n-1) + q(1-\lambda_0)V(E,0,n-1)\right. \\
&\qquad \left. + (1-q)\lambda_0 V(E-1,1,n-1) + (1-q)(1-\lambda_0)V(E-1,0,n-1)\right]
\end{aligned} \tag{2.18}$$

As we consider the case when the optimal action to be chosen is T for energy queue value E and channel state 0. Then, $V_T(E,0,n)$ is larger than $V_D(E,0,n)$ and

$$\begin{aligned}
&V_D(E,0,n) - V_T(E,0,n) = \\
&\qquad \beta\left[q\lambda_0 V(E+1,1,n-1) + q(1-\lambda_0)V(E+1,1,n-1)\right. \\
&\qquad + (1-q)\lambda_0 V(E,1,n-1) + (1-q)(1-\lambda_0)V(E,1,n-1)] \\
&\qquad - f_0 - \beta\left[q\lambda_0 V(E,1,n-1) + q(1-\lambda_0)V(E,1,n-1)\right. \\
&+ (1-q)\lambda_0 V(E-1,1,n-1) + (1-q)(1-\lambda_0)V(E-1,1,n-1)] \leq 0 \qquad (2.19)
\end{aligned}$$

Adding and subtracting $V_D(E,0,n)$ in (2.18) leads to

$$V_D(E+1,0,n) - V_D(E,0,n) + V_D(E,0,n) - V_T(E,0,n) < f_1 + 0 = f_1 \qquad (2.20)$$

After considering the four cases for C=0, we found that the hypothesis is true for C=0.

The hypothesis is true for every C and every n. By considering the limit as n goes to infinity, we have proved the lemma. □

Proposition 2.1. The optimal action for the source when the channel is at state 1 and the length of the energy queue is larger than 0 is to transmit, i.e. $V_T(E,1) \geq V_D(E,1)$.

Proof. We start by subtracting $V_D(E,1)$ from $V_T(E,1)$ and after rearranging terms, we get

$$V_T(E,1) - V_D(E,1) =$$

$$f_1 - \beta\left[q\lambda_1 \left(V(E+1,1) - V(E,1)\right) + q(1-\lambda_1)\left(V(E+1,0) - V(E,0)\right)\right.$$

$$\left. + (1-q)\lambda_1 \left(V(E,1) - V(E-1,1)\right) + (1-q)(1-\lambda_1)\left(V(E,0) - V(E-1,0)\right)\right]$$

$$\geq f_1 - \beta f_1 > 0 \quad (2.21)$$

The inequality is a result of applying lemma 2.2 to the terms in the equality above. □

Proposition 2.2. At C=0 and E≥1, the state action reward function $V_A(E,0)$ is *supermodular* in (E,A), that is, $V_T(E{+}1{,}0){+}V_D(E{,}0){\geq}V_D(E{+}1{,}0){+}V_T(E{,}0)$. Then the optimal policy is a threshold-type policy in the available energy in the queue. Consequently, there is an η such that

$$A^*(E,0) = \begin{cases} D & \text{if} \quad 0 \leq E \leq \eta \\ T & \text{if} \quad E > \eta \end{cases}$$

Proof. We want to prove that the difference between $V_T(.,0)$ and $V_D(.,0)$ is non-decreasing in E, that is,

$$V_T(E+1,0) - V_D(E+1,0) \geq V_T(E,0) - V_D(E,0) \quad (2.22)$$

We start by calculating the difference between $V_T(E,0)$ and $V_D(E,0)$. We have

$$V_T(E,0) - V_D(E,0) =$$

$$f_0 - \beta \left[q\lambda_0 \left(V\left(E+1,1\right) - V\left(E,1\right) \right) + q\left(1-\lambda_0\right) \left(V\left(E+1,0\right) - V\left(E,0\right) \right) \right.$$

$$\left. + \left(1-q\right) \lambda_0 \left(V\left(E,1\right) - V\left(E-1,1\right) \right) + \left(1-q\right) \left(1-\lambda_0\right) \left(V\left(E,0\right) - V\left(E-1,0\right) \right) \right] \tag{2.23}$$

Then, we subtract $V_D(E{+}1{,}0)$ from $V_T(E{+}1{,}0)$ which leads to

$$V_T(E+1,0) - V_D(E+1,0) =$$

$$f_0 - \beta \left[q\lambda_0 \left(V\left(E+2,1\right) - V\left(E+1,1\right) \right) + q\left(1-\lambda_0\right) \left(V\left(E+2,0\right) - V\left(E+1,0\right) \right) \right.$$

$$\left. + \left(1-q\right) \lambda_0 \left(V\left(E+1,1\right) - V\left(E,1\right) \right) + \left(1-q\right) \left(1-\lambda_0\right) \left(V\left(E+1,0\right) - V\left(E,0\right) \right) \right] \tag{2.24}$$

By subtracting equation (2.23) from equation (2.24) and for the difference to be larger than or equal 0, a sufficient condition is

$$V(E,C) - V(E-1,C) \geq V(E+1,C) - V(E,C) \tag{2.25}$$

This condition is that the difference in the value function is non-increasing function of E.

In the following part of the proof, we are going to prove that the condition (2.25) is true. Using mathematical induction, both sides of the inequality equal 0 for n=1. Assume that the hypothesis is true for n-1, then we will now prove that the difference in $V(E{,}C{,}n)$ has a non-increasing difference in E that is:

$$V(E+1,C,n) - V(E,C,n) \leq V(E,C,n) - V(E-1,C,n) \tag{2.26}$$

$$V(E+1,C,n) - V(E,C,n) - (V(E,C,n) - V(E-1,C,n)) \leq 0 \tag{2.27}$$

We assume that the actions $A1$, $A2$ and $A3$ from the action set are the optimal actions to be chosen when the energy queue contains E+1, E and E-1 units of energy, respectively. Then, we can write

$$V\,(E+1,C,n) = V_{A1}\,(E+1,C,n)$$

$$V\,(E,C,n) = V_{A2}\,(E,C,n)$$

$$V\,(E-1,C,n) = V_{A3}\,(E-1,C,n)$$

We substitute these values in (2.27). Hence, we have

$$V_{A1}(E+1,C,n) - V_{A2}(E,C,n) - (V_{A2}(E,C,n) - V_{A3}(E-1,C,n)) \leq 0 \tag{2.28}$$

or

$$\begin{aligned} &V_{A1}(E+1,C,n) - V_{A1}(E,C,n) + V_{A1}(E,C,n) - V_{A2}(E,C,n) \\ &\quad - V_{A2}(E,C,n) + V_{A3}(E,C,n) - (V_{A3}(E,C,n) - V_{A3}(E-1,C,n)) \leq 0 \end{aligned} \tag{2.29}$$

We know that $V_{A1}(E,C,n)$-$V_{A2}(E,C,n) \leq 0$ from the optimality of the action $A2$. The value function has its maximum value for energy E when the action $A2$ is taken.

Also, we know -$V_{A2}(E,C,n)$+$V_{A3}(E,C,n) \leq 0$ for the same reason, namely the optimality of the action $A2$.

Then, the remaining four terms are going to be considered together. We are going to consider the different four combinations for the actions $A1$ and $A3$. We want to show that the value of the quantity represented by the remaining four terms is less than or equal 0.

For the case of $A1{=}T$ and $A3{=}D$, we have

$$V_T(E+1,C,n)-V_T(E,C,n)-(V_D(E,C,n)-V_D(E-1,C,n))=$$

$$\beta\left[q\lambda_0\left(V\left(E+1,1,n-1\right)-V\left(E,1,n-1\right)\ -V\left(E+1,1,n-1\right)\right.\right.$$

$$\left.+V\left(E,1,n-1\right)\right)+q\left(1-\lambda_0\right)\left(V\left(E+1,0,n-1\right)-V\left(E,0,n-1\right)\right.$$

$$\left.-V\left(E+1,0,n-1\right)+V\left(E,0,n-1\right)\right)+\left(1-q\right)\lambda_0\left(V\left(E,1,n-1\right)\right.$$

$$\left.-\ V\left(E-1,1,n-1\right)-V\left(E,1,n-1\right)+V\left(E-1,1,n-1\right)\right)$$

$$+\left(1-q\right)\left(1-\lambda_0\right)\left(V\left(E,0,n-1\right)-V\left(E-1,0,n-1\right)\right.$$

$$\left.\left.-\ V\left(E,0,n-1\right)+V\left(E-1,0,n-1\right)\right)\right]=0 \quad (2.30)$$

Then, for the case of $A1{=}T$ and $A3{=}T$, we have

$$V_T(E+1,C,n)-V_T(E,C,n)-(V_T(E,C,n)-V_T(E-1,C,n))=$$

$$\beta\left[q\lambda_0\left(V\left(E+1,1,n-1\right)-V\left(E,1,n-1\right)\ -V\left(E,1,n-1\right)\right.\right.$$

$$\left.+V\left(E-1,1,n-1\right)\right)+q\left(1-\lambda_0\right)\left(V\left(E+1,0,n-1\right)-V\left(E,0,n-1\right)\right.$$

$$\left.V\left(E,0,n-1\right)+V\left(E-1,0,n-1\right)\right)+\left(1-q\right)\lambda_0\left(V\left(E,1,n-1\right)\right.$$

$$\left.-V\left(E-1,1,n-1\right)-V\left(E-1,1,n-1\right)+V\left(E-2,1,n-1\right)\right)+$$

$$\left(1-q\right)\left(1-\lambda_0\right)\left(V\left(E,0,n-1\right)-V\left(E-1,0,n-1\right)\right.$$

$$\left.\left.-V\left(E-1,0,n-1\right)+V\left(E-2,0,n-1\right)\right)\right]\leq 0 \quad (2.31)$$

The above is true since the differences are non-increasing at n-1 and the differences are multiplied by non-negative terms and summed together.

Then, using the hypothesis for $A1$=D and $A3$=T, we have

$$V_D(E+1,C,n) - V_D(E,C,n) - (V_T(E,C,n) - V_T(E-1,C,n)) =$$

$$\beta\left[q\lambda_0\left(V\left(E+2,1,n-1\right) - V\left(E+1,1,n-1\right) - V\left(E,1,n-1\right)\right.\right.$$

$$\left.+V\left(E-1,1,n-1\right)\right) + q\left(1-\lambda_0\right)\left(V\left(E+2,0,n-1\right) - V\left(E+1,0,n-1\right)\right.$$

$$\left.- V\left(E,0,n-1\right) + V\left(E-1,0,n-1\right)\right) + \left(1-q\right)\lambda_0\left(V\left(E+1,1,n-1\right)\right.$$

$$\left.- V\left(E,1,n-1\right) - V\left(E-1,1,n-1\right) + V\left(E-2,1,n-1\right)\right)$$

$$+ \left(1-q\right)\left(1-\lambda_0\right)\left(V\left(E+1,0,n-1\right) - V\left(E,0,n-1\right)\right.$$

$$\left.\left.- V\left(E-1,0,n-1\right) + V\left(E-2,0,n-1\right)\right)\right] \leq 0 \quad (2.32)$$

Finally, for the case where $A1$=D and $A3$=D, we have

$$V_D(E+1,C,n) - V_D(E,C,n) - (V_D(E,C,n) - V_D(E-1,C,n)) =$$

$$\beta\left[q\lambda_0\left(V\left(E+2,1,n-1\right) - V\left(E+1,1,n-1\right) - V\left(E+1,1,n-1\right)\right.\right.$$

$$\left.+V\left(E,1,n-1\right)\right) + q\left(1-\lambda_0\right)\left(V\left(E+2,0,n-1\right) - V\left(E+1,0,n-1\right)\right.$$

$$\left.- V\left(E+1,0,n-1\right) + V\left(E,0,n-1\right)\right) + \left(1-q\right)\lambda_0\left(V\left(E+1,1,n-1\right)\right.$$

$$\left.- V\left(E,1,n-1\right) - V\left(E,1,n-1\right) + V\left(E-1,1,n-1\right)\right)$$

$$+ \left(1-q\right)\left(1-\lambda_0\right)\left(V\left(E+1,0,n-1\right) - V\left(E,0,n-1\right)\right.$$

$$\left.\left.- V\left(E,0,n-1\right) + V\left(E-1,0,n-1\right)\right)\right] \leq 0 \quad (2.33)$$

Therefore, $V(E,C,n)$ has a non-increasing difference in E for all n, which implies that $V_A(E,C)$ is *supermodular* in (E,A).

From [37], if a function $F(x,y)$ is super-modular in (x,y), it follows that the function $y(x)$=argmax_y $F(x,y)$ is monotonically non-decreasing in the variable x.

Thus, the action A to be chosen is monotonically non-decreasing in the energy when the channel is at state 0. Therefore, the optimal policy is a threshold type policy. □

Note that determining the threshold value η is not simple and is not addressed here. It could be calculated numerically by exhaustive search for the threshold that gives the optimal throughput.

2.4 An Upper Bound on the Performance

Let γ be the average number of packets that are received successfully by the destination per time slot. Also, let x_{ij} be the indicator of taking the decision to transmit a packet in time slot j when the channel is in state i during this time slot. It is obvious that $x_{1j}+x_{0j}\leq 1$. If the source decides to transmit a packet while the channel is in state 0 or 1, then $x_{1j}+x_{0j}=1$. Also, if the source selects to defer the packet transmission then, $x_{1j}+x_{0j}=0$. Using the values of the packet success probabilities at different channel states, we can write the expression of γ as

$$\gamma = \lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} (f_1 x_{1j} + f_0 x_{0j}) \tag{2.34}$$

Due to the energy harvesting process characteristics, the average number of transmission attempts is limited. Thus, the maximum allowable transmission rate equals the energy acquiring rate at the source node. Therefore:

$$\lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} (x_{1j} + x_{0j}) \leq q \tag{2.35}$$

where $\lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} (x_{1j} + x_{0j})$ represents the average rate of transmissions attempted by the source under a certain scheduling policy.

We rewrite the inequality as

$$\lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} x_{0j} \le q - \lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} x_{1j} \tag{2.36}$$

We now calculate a bound for the average number of transmission attempts while the channel is at state 1. First, the proportion of time where the channel is at state 1 equals the steady state probability of the channel to be at state 1, which is denoted by π_1. The value of $\pi_1$is readily calculated as

$$\pi_1 = \frac{\lambda_0}{1 + \lambda_0 - \lambda_1} \tag{2.37}$$

Thus, the average number of the transmission attempts with the channel in state 1 satisfies

$$\lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} x_{1j} \le \pi_1 \tag{2.38}$$

Then from (2.35), the average number of transmission attempts from the source with the channel in state 1 is also bounded as

$$\lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} x_{1j} \le q \tag{2.39}$$

Combining the two bounds at (2.38) and (2.39), we obtain

$$\lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} x_{1j} \le \min\{q, \pi_1\} \tag{2.40}$$

By the linearity of the limit and the summation in (2.34), we can rewrite the expression of the average number of packets per time slot that are received successfully

$$\gamma = f_1 \lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} x_{1j} + f_0 \lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} x_{0j} \tag{2.41}$$

Substituting in (2.36), we obtain

$$\gamma \leq f_0 q + (f_1 - f_0) \lim_{T\to\infty} \frac{1}{T} \sum_{j=1}^{T} x_{1j} \tag{2.42}$$

The value of f_1 is larger than f_0 so that the value of the difference (f_1-f_0) is positive. Then, replacing the average number of transmission attempts from the source with channel in state 1 by its upper bound leads to an upper bound for γ, namely,

$$\gamma \leq f_0 q + (f_1 - f_0) \min\{q, \pi_1\} \tag{2.43}$$

The value of the minimum function in the bound leads to two different values for the bound. First, we consider the case when q is smaller than π_1. In this case, the upper bound is calculated as

$$\gamma \leq f_1 q \tag{2.44}$$

In this case, this upper bound could be reached if there is a policy that can force the source to transmit when the channel in state 1 only. Also, these transmissions are going to use all the energy acquired by the source.

On the other hand, we consider the case when $\pi_1$is smaller than q. In this case, the upper bound is calculated as

$$\gamma \leq f_1 \pi_1 + f_0 (q - \pi_1) \tag{2.45}$$

In this case, this upper bound could be reached if there is a policy that can force the source to transmit in every time slot in which the channel is in state 1.

The remaining energy is to be used for transmission attempts when the channel is in state 0. This policy needs to make sure that whenever the channel is in state 1, the source must have energy at its queue to be used for transmission. That can not be guaranteed due to the stochastic nature of the energy harvesting process.

2.5 Optimal Policy with No CSI Feedback

Let us denote the expected discounted number of packets successfully delivered to the destination when the source has E units of energy at its energy queue by $W(E)$. The source node does not have any channel measurements. The source knows that the channel is a Gilbert-Elliot and it knows its transition probabilities. We define π_1 and π_0 as the steady state probabilities for the channel to be at state 1 and state 0. The value of π_1 is given in (2.37) and the value of π_0 is found to be $(1\text{-}\lambda_1)/(1+\lambda_0\text{-}\lambda_1)$. Thus, we can write the value of $W(E)$ using Bellman's equation as

$$W(E) = \max\{W_T(E), W_D(E)\} \tag{2.46}$$

where $W_T(E)$ and $W_D(E)$ are the expected values of the discounted reward when the energy queue of the source has E units of energy and the source chooses to take the action T and the action D, respectively.

When the action T is chosen, the expected reward in the current time slot is calculated as $\pi_1 f_{1+}\ \pi_0 f_0$. The first term represents the probability of the channel being at state 1 multiplied by the probability of successful delivery when the channel is in state 1. The second term is the same but for the channel in state 0. We denote this instantaneous expected reward by f_{av}. Then, we can write the expression of $W_T(E)$ as

$$W_T(E) = \pi_1 f_1 + \pi_0 f_0 + \beta\left[qW(E) + (1-q)W(E-1)\right] \tag{2.47}$$

Also, we can write the expression of $W_D(E)$ as

$$W_D(E) = 0 + \beta\left[qW(E+1) + (1-q)W(E)\right] \tag{2.48}$$

Lemma 2.4. For $E \geq 1$, $W(E+1)$-$W(E) \leq f_{av}$

Proof. We are going to use mathematical induction in the proof. Define $W(E,n)$ as the optimal value function when the decision horizon spans n stages. Also, $W_A(E,n)$ is the value function when the action A is chosen and the decision horizon spans n stages where A belongs to $\{T,D\}$. We start by the validity of the hypothesis at n=1. We have that the difference $W(E+1,1)$-$W(E,1)$=0. Then, the hypothesis is true for n=1. The next step is to assume that the hypothesis is true for n-1, that is, $W(E+1,n\text{-}1)$-$W(E, n\text{-}1) \leq f_{av}$. Then, we have

$$\begin{aligned} W_T(E,n) - W_D(E,n) &\geq f_{av} - \beta\left[qf_{av} + (1-q)f_{av}\right] \\ &= f_{av} - \beta f_{av} = f_{av}(1-\beta) > 0 \end{aligned} \tag{2.49}$$

Then, the action to be chosen is T. As a result, $W(E,n)$=$W_T(E,n)$. This leads to have the following difference

$$\begin{aligned} W(E+1,n) - W(E,n) &= W_T(E+1,n) - W_T(E,n) \\ &= \beta\left[q\left[W(E+1,n-1) - W(E,n-1)\right]\right. \\ &\left.+ (1-q)\left[W(E,n-1) - W(E-1,n-1)\right]\right] \leq \beta f_{av} < f_{av} \end{aligned} \tag{2.50}$$

By considering the limit as n goes to infinity, we complete the proof. □

Proposition 2.3. The optimal action for the source when there is no CSI feedback and the length of the energy queue is larger than 0 is to transmit. i.e. $W_T(E) \geq W_D(E)$.

Proof. The proposition is going to be proved by directly applying the previous lemma. We start by subtracting $W_D(E)$ from $W_T(E)$, that is,

$$\begin{aligned} & W_T(E) - W_D(E) \\ & = f_{av} + \beta \left[qW(E) + (1-q) W(E-1) \right] - \beta \left[qW(E+1) + (1-q) W(E) \right] \end{aligned} \tag{2.51}$$

We rearrange the terms in the previous equation and obtain

$$\begin{aligned} & W_T(E) - W_D(E) = \\ & \qquad f_{av} - \beta \left[q \left(W(E+1) - W(E) \right) + (1-q) \left(W(E) - W(E-1) \right) \right] \end{aligned} \tag{2.52}$$

From the last lemma, we have

$$W_T(E) - W_D(E) \geq f_{av} - \beta f_{av} > 0 \tag{2.53}$$

□

Now, we calculate the expected number of packets successfully delivered to the destination per time slot. As stated before, the average number of transmission attempts by the source is limited by the average rate of the energy acquiring process. In the case of no CSI feedback to the source, the source is going to transmit whenever it has energy in its energy queue. Then, the probability of attempting transmission in any time slot is q. The probability of successful delivery of a transmitted packet is $\pi_1 f_{1+} \pi_0 f_0$. Then, the expected average number of successfully delivered packets equals $q(\pi_1 f_{1+} \pi_0 f_0)$.

From the discussion about the optimal policy in the case of no CSI availability, we found that the transmission policy is equivalent to the optimal policy of the case of fixed channel with packet successful delivery probability of f_{av}. The problem of finding the optimal policy of the fixed channel is the same as (2.46). The only difference is that the expected reward at the current time slot equals f_{av} which is the packet successful delivery probability of the channel.

Finally, note that the derived optimal policy for the case of no CSI availability is a greedy policy that always requires the source to transmit whenever there is energy which is available at the energy queue.

2.6 Numerical Results

In this section, we present numerical results to illustrate the previous analysis. We focus on comparing the performance of the different transmission strategies in terms of the throughput of the source node which is the average successfully delivered packets per time slot when the data queue is saturated. We also show the enhancement because of using the availability of CSI at the source node by comparing the optimal policy performance with no CSI at the source to the optimal policy of case at which the CSI is available at the source. Obviously, the values of f_C have a major impact on the results.

We compare three transmission strategies which are the optimal policy, the greedy policy and the conservative policy. The greedy policy is the policy in which the source node transmits a packet to the destination when there is energy which is available at the energy queue without considering the channel state. The conservative policy is the policy in which the source node transmits a packet only when there is energy which is available at the energy queue and the channel is in state 1.

We also compare the upper bound to these three strategies. The optimal policy performance for the case of no CSI is also shown. As mentioned in section 2.5, the optimal policy in that case is the greedy policy. Thus, a single curve on the figures is used to represent the performance of optimal policy with no CSI and the performance of the greedy policy. This Curve is noted as "Optimal No CSI (Greedy)".

The parameters considered for the system are λ_0=0.4, λ_1=0.8, f_0=0.2, f_1=0.5 and q=0.8. In the following figures, we compare the transmission strategies performance with varying the values of q and f_1. The threshold selection is done by exhaustive search for each system parameters set. We calculate the objective function for different threshold values. Then, we select the threshold value at which the objective function starts to decrease for the threshold values larger than this value.

In figure 2.3, we show the performance of the three compared transmission strategies against f_1. The figure shows the enhancement in the throughput because of using the optimal policy. Also, the greedy policy has better performance than the conservative one for low success probability values. The bound on the performance coincides with the performance of the optimal policy.

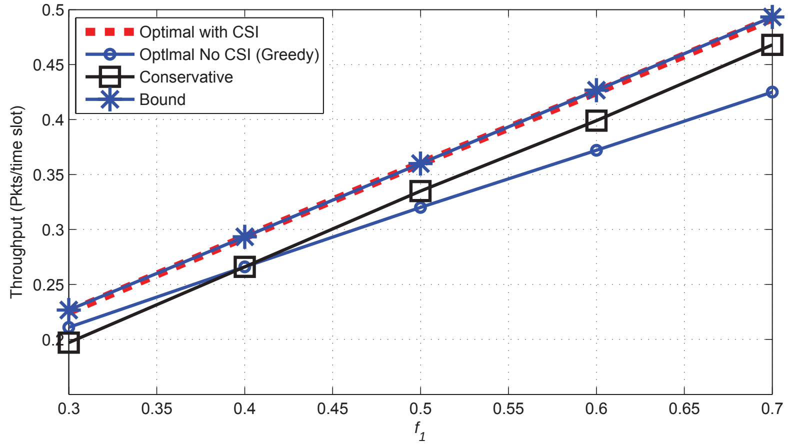

Figure 2.3: Effect of f_1 on the value function

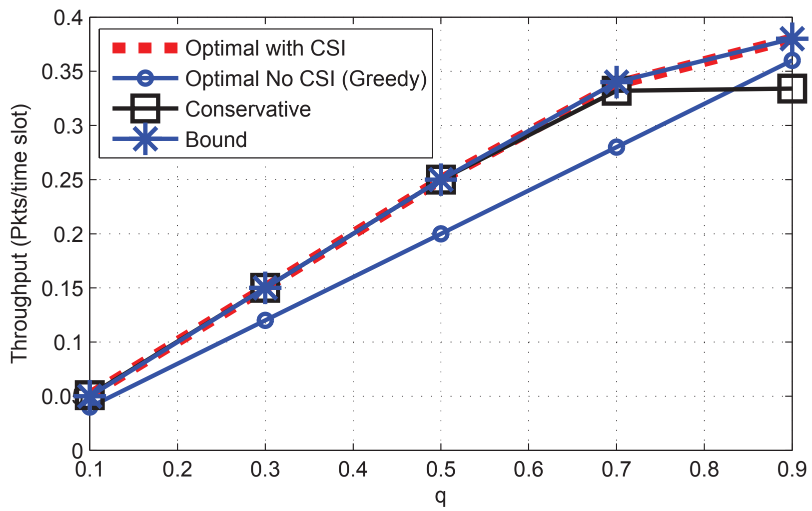

Figure 2.4: Effect of q on the value function

In figure 2.4, the performance of the three compared transmission strategies is shown against q. For this selection of system parameters, the performance of the optimal policy coincides with the performance of the conservative policy for a large range of the values of q. For small q values, the energy is scarce and the source node tends to store the available energy units to be used when the channel has good connectivity. This is the reason for the optimality of the conservative policy for the small values of q. Also in this figure, the bound on the performance coincides with the performance of the optimal policy. Thus, the calculated bound is tight.

2.7 Discussion

In this chapter, we have studied a communication link that operates over a Gilbert-Elliot channel. The source node has energy harvesting capability. In order to maximize the number of successfully delivered packets per time slot, the source decides in each time slot whether to transmit or defer the transmission. The problem has been formulated as a Markov decision problem and we have characterized the optimal policy. We have proved that it is a threshold-type policy, depending on the channel state and the energy queue length. Different properties of the optimal policy have been derived. An upper bound on the average number of packets per time slot that are successfully received by the destination has been derived. This bound has been shown to be tight on the performance of the optimal policy. The optimal policy for the case of no CSI availability has also been derived. Numerical results have been obtained to illustrate the analysis. We observe that the value of CSI can be significant. We also see that the channel fluctuation affects performance significantly as well.

Chapter 3: Energy Harvesting Sources over Time Varying Channels with Relays

3.1 Introduction

Cooperative diversity in energy harvesting networks at the physical layer has been considered before in a number of works as in [38,39]. Also, the problem of power optimization for energy harvesting networks with network-level cooperation has been discussed in [40]. The authors have derived the maximum stable throughput rate for a network consisting of a source, a relay and a destination. The relaying strategy is Time Division Multiple Access (TDMA). In this strategy, the odd time slots are assigned to the source transmissions and the even time slots are assigned to the relay transmissions. This strategy has low channel utilization because of the fixed assignment of the time slots. As a result, it has been shown in [40] that the direct transmission has higher stable throughput than this relaying scheme for high energy arrival rates. In our work, we propose a relaying scheme which has higher channel utilization than the relaying scheme in [40].

In this chapter, we investigate the impact of energy harvesting capability on the stable throughput rate of a source node. We start by calculating the stable throughput of the source while transmitting to the destination directly over a time varying channel. The channel is modeled by a two-state discrete-time process. The packets and energy arrivals into the source are modeled by discrete-time stochastic processes. Also, we derive the maximum stable throughput rate of a source node which is helped by a relay node through a network-level cooperation protocol. The relay also has energy harvesting capability. Due to the stochastic nature of the data arrivals to the source, we propose a strategy in which the relay transmits during the idle periods of the source to efficiently utilize the channel. The proposed transmission strategies exploit the knowledge of the CSI of the channel between the source and the destination. The source transmits with probability 1 when the channel is in the good state if its energy queue is not empty, but it randomly transmits with a certain probability if the channel is in the poor state. We calculate the optimal value of this probability. Also, we derive the stable throughput rate of the source when its decision depends on imperfect channel measurements. This work was presented in [41].

The study of a simple model consisting of only a source, a relay and a destination is both instructive and necessary. It reveals insights at the conceptual level about the effects of cooperative relaying and exploiting channel information on the stability of energy harvesting networks. More work needs to be done to exploit the results of this work in more realistic systems. Also, energy harvesting capability and channel knowledge can much affect the dynamic behavior of the proposed system but it is out of the scope of our work.

3.2 System Model

We consider a network which consists of a source node, a relay node, and a destination node as shown in figure 3.1. Each of the source and the relay has an infinite data queue for storing fixed length packets. These queues are denoted by Q_S and Q_R respectively. We assume that the source has its own traffic while the relay does not have its own traffic and is used only for cooperation. The data arrival to the source data queue is modeled by a Bernoulli process. Also, each of the source and the relay has an infinite energy queue. These queues are denoted by E_S and E_R respectively. The usage of infinite queues is a reasonable approximation when the data queues are large enough compared to the packet size and the energy queues are large enough compared to the energy unit [15]. All nodes are half-duplex and thus they can not transmit and receive simultaneously. Time is assumed to be slotted such that each packet transmission takes one time slot. Transmission of a data packet from a node requires using a single unit of energy from the corresponding energy queue. For simplicity, we assume that the energy consumption in a node is due to transmission only and therefore the processing and reception energy are considered to be negligible. Each of the source and the relay can acquire a single unit of energy at each time slot with probabilities q_S and q_R respectively that the energy arrival processes are modeled by Bernoulli processes.

All the channels, which are denoted by SD, SR and RD, are modeled by independent two-state discrete-time processes. The channels are also independent of the packet generation process and the energy harvesting at the source and the

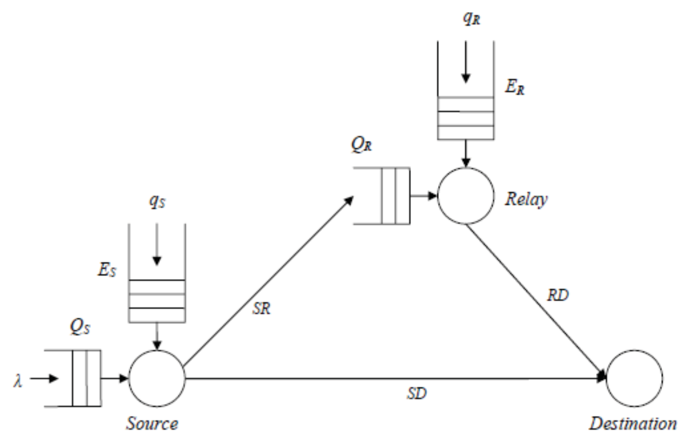

Figure 3.1: System Model

relay. Each channel state corresponds to a degree of channel connectivity. State 1 corresponds to good connectivity while state 0 corresponds to poor connectivity. The quality of the channels is represented by the success probability of a packet. The packet success probabilities are denoted by $f_{SD,i}$, $f_{SR,i}$ and $f_{RD,i}$ when the corresponding channels are in state $i = 0, 1$. These success probabilities are determined by the system physical parameters such as the transmission power, the modulation scheme, the coding scheme and the targeted bit-error rate. We assume that each channel remains fixed for a time slot and is able to move into another state in the next slot. The steady state probabilities for the channels to be in state $i = 0, 1$ are $\pi_{SD,i}$, $\pi_{SR,i}$ and $\pi_{RD,i}$ respectively.

In [42], Loynes' theorem states that if the arrival and service processes at a queue are jointly stationary, then the queue is stable if the average arrival rate is less than the average service rate. Throughout the chapter, we denote the average arrival rate at the source data queue by λ. The average arrival rate to the relay data queue is denoted by λ_R. The average service rate of the source data queue in the case of no relaying is denoted by μ_S^{NR}. The average service rate of the source data queue in the case of cooperative relaying is denoted by μ_S^{CR}. Also, the average service rate of the relay data queue is denoted by μ_R.

3.3 Network Protocols

In this section, we present two transmission protocols for delivering the packets from the source to the destination either directly with no relaying or by allowing the relay to help.

3.3.1 No Relaying

In this case, the system consists only of the source and the destination. The packets can reach the destination through the channel SD. The source can transmit only when both its energy queue and its data queue are not empty. The channel SD state is known at the source at the time of transmission and it is throughput-optimal for the source to transmit with probability 1 when the channel is in state 1. Thus, the transmission strategy when the data queue is not empty is described as follows: if the source energy queue is not empty and the channel SD is in state 1, the source is going to transmit. Also, if the source energy queue is not empty and the channel SD is in state 0, the source is going to transmit with some probability p_0. The packet is released from the source data queue if it is successfully received by the destination; otherwise it remains at the source data queue for retransmission. The feedback to the source is in the form of Acknowledgment or Negative-Acknowledgment. In this mechanism, a short-length error-free packets are broadcasted by the destination over a separate channel to inform the network users about the reception status.

The probability p_0 controls the utilization of the channel when the channel is in state 0. Increasing p_0 leads to one of the following two effects. First, it may increase the energy used when the channel is in state 0 by decreasing the energy used when the channel is in state 1. This leads to increase of the joint probability of the channel to be in state 1 and the source energy queue to be empty which affects the performance negatively. Second, increasing p_0 may increase the energy used when the channel is in state 0 by exploiting unused harvested energy without affecting the amount of energy used when the channel is in state 1. This effect improves the system performance.

3.3.2 Cooperation with the Relay

The source transmits its traffic with the help of the relay. At a time slot, the source is able to transmit if both its energy queue and its data queue are not empty. It transmits with probability 1 when the channel SD is in state 1 and with probability p_0 when the channel is in state 0. If the packet is successfully received by the destination or by the relay, it is released from the source data queue; otherwise it is kept in the source data queue for retransmission. The retransmission scheme is the same as mentioned in the last subsection. At the beginning of every time slot, the relay senses the channel. We assume perfect sensing by the relay for the source transmissions. If the source is not transmitting, the relay uses these idle time slots to transmit the packets in its data queue to the destination when its energy queue is not empty. Hence, no explicit channel resources are assigned to the relay. A packet is released from the relay data queue if it is successfully received by the destination; otherwise it is kept for retransmission.

In this protocol, we let the source transmission decisions depend only on the state of the channel SD that the source transmission control protocol is the same as the protocol in the case of no relaying. That allows us to illustrate the effect of relaying on the stability condition of the source. The proposed system can have better performance by allowing a different transmission protocol at the source in which the source considers both the channels SD and SR. Also, the relay can consider the channel RD while transmitting to the destination. Including this transmission control protocol in the analysis is straightforward but is not included for brevity.

3.4 Stable Throughput Analysis

In this section, we derive the maximum stable throughput rate of the source for the proposed transmission protocols.

3.4.1 No Relaying

In order to calculate the maximum stable throughput rate for the source data queue, we have to consider the maximum service rate for the source energy queue which is the rate of which the source node attempts to transmit. Each transmission attempt uses a single unit energy. As a result, the energy departure process is modeled by a Bernoulli process. Therefore, the source energy queue forms a discrete-time M/M/1 system. The transmission attempt rate equals $\pi_{SD,1} + \pi_{SD,0}p_0$. The arrival rate of the energy to the source is q_S. If the energy arrival rate to the source is larger than the transmission attempting rate, the number of energy units in the energy queue approaches infinity almost surely. Therefore, the probability of the energy queue to be empty is zero. On the other hand, if the energy arrival rate to the source node is smaller than or equal to the transmission attempting rate, it follows from [43] for discrete-time M/M/1 system that the probability of energy queue to be not empty is the ratio between the energy arrival rate and the transmission attempting rate. As a result, the probability of the energy queue to be not empty is written as follows

$$Pr[E_S \neq 0] = \frac{\min(q_S, \pi_{SD,1} + \pi_{SD,0}p_0)}{\pi_{SD,1} + \pi_{SD,0}p_0} \tag{3.1}$$

The probability of a packet to be delivered, given that the source is able to transmit, is $\pi_{SD,1}f_{SD,1} + \pi_{SD,0}p_0f_{SD,0}$. The source data queue service rate is the product of the success probability given that the source is able to transmit by the probability that the energy queue is not empty. The stability condition for the source data queue, when relaying is not used, is $\lambda < \mu_S^{NR}$ which can be written as

$$\lambda < Pr[E_S \neq 0](\pi_{SD,1}f_{SD,1} + \pi_{SD,0}p_0f_{SD,0}) \tag{3.2}$$

In the case of no availability of CSI at the source, it transmits with probability 1 when the energy queue is not empty. The expression of the stability condition can be evaluated by setting p_0 to be 1. The stability condition can be stated as follows

$$\lambda < q_S(\pi_{SD,1} f_{SD,1} + \pi_{SD,0} f_{SD,0}) \tag{3.3}$$

3.4.2 Cooperation with the Relay

In this protocol, the system is stable if both the source data queue and the relay data queue are stable. In the following subsections, we derive the stability conditions for each queue separately.

3.4.2.1 Source Data Queue

The maximum data arrival rate which maintains the stability of the source data queue is limited by its service rate. A packet at the source is served if it is successfully delivered to the relay or the destination. The service rate of the source data queue is calculated to be

$$\begin{aligned}\mu_S^{CR} = Pr[E_S \neq 0][\pi_{SD,1}(\pi_{SR,1}[1-(1-f_{SD,1})(1-f_{SR,1})]\\ + \pi_{SR,0}[1-(1-f_{SD,1})(1-f_{SR,0})]) + \pi_{SD,0} p_0(\pi_{SR,1}[1-(1-f_{SD,0})(1-f_{SR,1})]\\ + \pi_{SR,0}[1-(1-f_{SD,0})(1-f_{SR,0})])]\end{aligned} \tag{3.4}$$

3.4.2.2 Relay Data Queue

We start by calculating the probability that the channel is occupied by the source transmissions and this probability is denoted by ρ_S. As the source data queue forms a discrete-time M/M/1 system and assuming that the source data queue is stable, it follows from [43] that the probability ρ_S is calculated as follows

$$\rho_S = \frac{\lambda Pr[E_S \neq 0]}{\mu_S^{CR}} \tag{3.5}$$

The arrival rate for the relay data queue is the probability that a packet is received by the relay at any given time slot. It is calculated as follows

$$\lambda_R = \rho_S Pr[\text{Packet received by relay only}] \tag{3.6}$$

The Pr[Packet received by relay only] is denoted by P_R and its value is calculated as follows

$$\begin{aligned} P_R = {} & \pi_{SD,1}(\pi_{SR,1} f_{SR,1}(1 - f_{SD,1}) + \pi_{SR,0} f_{SR,0}(1 - f_{SD,1})) \\ & + \pi_{SD,0} p_0(\pi_{SR,1} f_{SR,1}(1 - f_{SD,0}) + \pi_{SR,0} f_{SR,0}(1 - f_{SD,0})) \end{aligned} \tag{3.7}$$

Also, we denote the probability that a packet is received by either the relay or the destination by P_E and we calculate its value as follows

$$\begin{aligned} P_E = {} & \pi_{SD,1}(\pi_{SR,1}[1 - (1 - f_{SD,1})(1 - f_{SR,1})] + \pi_{SR,0}[1 - (1 - f_{SD,1})(1 - f_{SR,0})]) \\ & + \pi_{SD,0} p_0(\pi_{SR,1}[1 - (1 - f_{SD,0})(1 - f_{SR,1})] + \pi_{SR,0}[1 - (1 - f_{SD,0})(1 - f_{SR,0})]) \end{aligned} \tag{3.8}$$

The expression of λ_R can be rewritten as follows

$$\lambda_R = \lambda \frac{P_R}{P_E} \tag{3.9}$$

Then, the service rate of the relay data queue equals

$$\mu_R = (\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) \min(q_R, 1 - \rho_S) \tag{3.10}$$

The complete derivation of the expression of the service rate of the relay data queue is found in the appendix at section 3.9.

3.4.2.3 Stability Conditions

To ensure that the system is stable, both source and relay data queues have to be stable. As a result, both the conditions $\lambda < \mu_S^{CR}$ and $\lambda_R < \mu_R$ should be satisfied. By substituting using equation (3.9) in the second condition, it is written as

$$\lambda < \frac{P_E}{P_R}(\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) \min(q_R, 1 - \rho_S) \tag{3.11}$$

Note that the right hand side of the inequality is still function of λ. By combining the conditions on λ, we get the general expression for the maximum stable throughput as follows

$$\lambda < \min\left(\mu_S^{CR}, \frac{P_E}{P_R}(\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) q_R, \frac{P_E(\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0})}{P_R + (\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0})}\right) \tag{3.12}$$

In the case of no availability of CSI at the source node, the expression of the stability condition is calculated by setting p_0 to be 1.

The same analysis is still valid when the energy arrival processes and the data arrival process are modeled by Poisson processes. In this case, the energy queues and the source data queue form M/G/1 systems.

3.5 Imperfect Channel Measurements

In this section, we study the effect of channel uncertainty on the stable throughput of the source for the proposed transmission strategies. The measured channel is the channel SD. We denote the probability of measuring the channel to be in state

1 given that the channel is in state 0 by $p_{1|0}$ and the probability of measuring the channel to be in state 0 given that the channel is in state 1 by $p_{0|1}$. Also, we denote the steady state probabilities of the channel SD to be measured in state 1 and 0 by $\hat{\pi}_{SD,1}$ and $\hat{\pi}_{SD,0}$ respectively. The expressions of the steady state probabilities are

$$\hat{\pi}_{SD,1} = \pi_{SD,1}(1 - p_{0|1}) + \pi_{SD,0}p_{1|0} \tag{3.13}$$

$$\hat{\pi}_{SD,0} = \pi_{SD,1}p_{0|1} + \pi_{SD,0}(1 - p_{1|0}) \tag{3.14}$$

3.5.1 No Relaying

In this case, the source transmits with probability 1 when the channel is measured to be in state 1. It transmits with probability p_0 when the channel is measured to be in state 0. As a result, the source energy queue service rate is $\hat{\pi}_{SD,1} + \hat{\pi}_{SD,0}p_0$. Thus, the probability of the source energy queue to be not empty is written as follows

$$Pr[E_S \neq 0] = \frac{\min(q_S, \hat{\pi}_{SD,1} + \hat{\pi}_{SD,0}p_0)}{\hat{\pi}_{SD,1} + \hat{\pi}_{SD,0}p_0} \tag{3.15}$$

The probability of a packet to be successfully received by the destination given that the source is able to transmit equals $(\pi_{SD,1}f_{SD,1}[(1 - p_{0|1}) + p_0p_{0|1}] + \pi_{SD,0}f_{SD,0}[p_{1|0} + (1 - p_{1|0})p_0])$. Hence, the stability condition for the source data queue is written as follows

$$\lambda < \frac{\min(q_S, \hat{\pi}_{SD,1} + \hat{\pi}_{SD,0}p_0)}{\hat{\pi}_{SD,1} + \hat{\pi}_{SD,0}p_0} \big(\pi_{SD,1}f_{SD,1}[(1 - p_{0|1}) + p_0p_{0|1}] + \pi_{SD,0}f_{SD,0}[p_{1|0} + (1 - p_{1|0})p_0]\big) \tag{3.16}$$

3.5.2 Cooperation with the relay

In this case, the service rate of the source data queue is affected by the erroneous channel measurements. The service rate can be written as follows

$$\mu_S^{CR} = Pr[E_S \neq 0] \left[\pi_{SD,1}[(1-p_{0|1}) + p_0 p_{0|1}]\right.$$
$$(\pi_{SR,1}[1-(1-f_{SD,1})(1-f_{SR,1})] + \pi_{SR,0}[1-(1-f_{SD,1})(1-f_{SR,0})])$$
$$+ \pi_{SD,0}[p_{1|0} + (1-p_{1|0})p_0] \left(\pi_{SR,1}[1-(1-f_{SD,0})(1-f_{SR,1})]\right.$$
$$\left.\left.+\pi_{SR,0}[1-(1-f_{SD,0})(1-f_{SR,0})]\right)\right] \quad (3.17)$$

As a result, the probability of the channel to be occupied by the source transmissions is updated by using the updated values of both μ_S^{CR} and $Pr[E_S \neq 0]$. Also, the values of P_R and P_E are updated because of the uncertainty of the channel measurements. The values are calculated as follows

$$P_E = \pi_{SD,1}[(1-p_{0|1}) + p_0 p_{0|1}](\pi_{SR,1}[1-(1-f_{SD,1})(1-f_{SR,1})]$$
$$+ \pi_{SR,0}[1-(1-f_{SD,1})(1-f_{SR,0})])$$
$$+ \pi_{SD,0}[p_{1|0} + (1-p_{1|0})p_0](\pi_{SR,1}[1-(1-f_{SD,0})(1-f_{SR,1})]$$
$$+ \pi_{SR,0}[1-(1-f_{SD,0})(1-f_{SR,0})]) \quad (3.18)$$

$$P_R = \pi_{SD,1}[(1-p_{0|1}) + p_0 p_{0|1}](\pi_{SR,1} f_{SR,1}(1-f_{SD,1}) + \pi_{SR,0} f_{SR,0}(1-f_{SD,1}))$$
$$+ \pi_{SD,0}[p_{1|0} + (1-p_{1|0})p_0](\pi_{SR,1} f_{SR,1}(1-f_{SD,0}) + \pi_{SR,0} f_{SR,0}(1-f_{SD,0})) \quad (3.19)$$

The expressions for λ_R and μ_R remain the same as in equations (3.9) and (3.10) but the values of ρ_S, P_E and P_R are updated as shown above. As a result, the stability condition is the same as in equation (3.12) using the updated values of the parameters.

3.6 Transmission Optimization

In this section, we evaluate the value of the parameter p_0 to maximize the maximum stable throughput rate for different protocols which is denoted by λ_{max}. The value of p_0 belongs to [0,1].

3.6.1 No Relaying

We have derived the stability condition in this case to have the expression in equation (3.2). We are going to consider two cases depending on the system parameters.

3.6.1.1 $\pi_{SD,1} > q_S$

The value of $\pi_{SD,0}p_0$ is always greater than or equal to 0. Then, we can rewrite the expression of λ_{max} as

$$\lambda_{max} = \frac{q_S(\pi_{SD,1}f_{SD,1} + \pi_{SD,0}p_0 f_{SD,0})}{\pi_{SD,1} + \pi_{SD,0}p_0} \tag{3.20}$$

This value as a function of p_0 is found to be a decreasing function of p_0 by calculating its first derivative. The first derivative is always negative for any value of p_0. As a result, the optimal value of p_0 is 0.

3.6.1.2 $\pi_{SD,1} \leq q_S$

In this case, we can rewrite the expression of λ_{max} as follows

$$\lambda_{max} = \begin{cases} \pi_{SD,1} f_{SD,1} + \pi_{SD,0} p_0 f_{SD,0}, & \text{if } p_0 \leq \frac{q_S - \pi_{SD,1}}{\pi_{SD,0}} \\ \frac{q_S(\pi_{SD,1} f_{SD,1} + \pi_{SD,0} p_0 f_{SD,0})}{\pi_{SD,1} + \pi_{SD,0} p_0}, & \text{if } p_0 > \frac{q_S - \pi_{SD,1}}{\pi_{SD,0}} \end{cases} \tag{3.21}$$

The first expression is an increasing function of p_0. The second one is a decreasing function of p_0. The optimal value of p_0 equals $(q_S - \pi_{SD,1})/\pi_{SD,0}$.

From these results, we can write the general expression for the optimal value of p_0 as follows

$$p_0^* = \max(0, \frac{q_S - \pi_{SD,1}}{\pi_{SD,0}}) \tag{3.22}$$

3.6.2 Transmission with Relaying

The optimal value of p_0 is the solution of the problem

$$p_0^* = \arg\max_{p_0}(\min\{f_1(p_0), f_2(p_0), f_3(p_0)\}) \tag{3.23}$$

where the values of $f_1(p_0)$, $f_2(p_0)$ and $f_3(p_0)$ are obtained from equation (3.12). It can be shown that $f_2(p_0)$ and $f_3(p_0)$ are decreasing functions by calculating the first derivative of each of the functions and showing that it is always negative. Also, if $\pi_{SD,1} > q_S$, we can show that $f_1(p_0)$ is a decreasing function in p_0. Then, the optimal value of p_0 should be 0.

On the other hand, we consider the case when $\pi_{SD,1} \leq q_S$ in which $f_1(p_0)$ is an increasing function in p_0 for p_0 belongs to $[0, (q_S - \pi_{SD,1})/\pi_{SD,0}]$ and a decreasing function in p_0 for p_0 belongs to $[(q_S - \pi_{SD,1})/\pi_{SD,0}, 1]$. We denote the increasing part of $f_1(p_0)$ by $f_{11}(p_0)$ which has the same expression as P_E.

We calculate the intersection points between $f_{11}(p_0)$ and both $f_2(p_0)$ and $f_3(p_0)$. We denote these points by PI_{12} and PI_{13} respectively. We calculate their values as follows

$$PI_{12} = \frac{(\pi_{RD,1}f_{RD,1} + \pi_{RD,0}f_{RD,0})q_R}{\pi_{SD,0}(\pi_{SR,1}f_{SR,1}(1-f_{SD,0}) + \pi_{SR,0}f_{SR,0}(1-f_{SD,0}))} - \frac{\pi_{SD,1}(\pi_{SR,1}f_{SR,1}(1-f_{SD,1}) + \pi_{SR,0}f_{SR,0}(1-f_{SD,1}))}{\pi_{SD,0}(\pi_{SR,1}f_{SR,1}(1-f_{SD,0}) + \pi_{SR,0}f_{SR,0}(1-f_{SD,0}))} \quad (3.24)$$

$$PI_{13} = \frac{1}{H}[(\pi_{RD,1}f_{RD,1} + \pi_{RD,0}f_{RD,0})(1-\pi_{SD,1}) - \pi_{SD,1}(\pi_{SR,1}f_{SR,1}(1-f_{SD,1}) + \pi_{SR,0}f_{SR,0}(1-f_{SD,1}))] \quad (3.25)$$

where $H = \pi_{SD,0}(\pi_{RD,1}f_{RD,1} + \pi_{RD,0}f_{RD,0}) + \pi_{SD,0}((\pi_{SR,1}f_{SR,1}(1-f_{SD,0}) + \pi_{SR,0}f_{SR,0}$ $(1-f_{SD,0})))$. We consider three cases for the values of these intersection points:

3.6.2.1 At least one point is less than 0

In this case the function $\min(f_1(p_0), f_2(p_0), f_3(p_0))$ is a decreasing function in p_0 for p_0 belongs to [0,1]. As a result, the optimal value of p_0 is 0.

3.6.2.2 At least one point belongs to $[0, (q_S - \pi_{SD,1})/\pi_{SD,0}]$ and no point less than 0

In this case, the function $\min(f_1(p_0), f_2(p_0), f_3(p_0))$ is increasing till the first intersection point and then it is decreasing. As a result, the optimal value of p_0 is $\min(PI_{12}, PI_{13})$.

3.6.2.3 Both points are larger than $(q_S - \pi_{SD,1})/\pi_{SD,0}$

In this case, the function $\min(f_1(p_0), f_2(p_0), f_3(p_0))$ is increasing till $(q_S - \pi_{SD,1})/\pi_{SD,0}$ and then it is decreasing. As a result, the optimal value of p_0 is $(q_S - \pi_{SD,1})/\pi_{SD,0}$.

Thus, we can generally write the optimal value of p_0 as follows

$$p_0^* = \max(0, \min(PI_{12}, PI_{13}, \frac{q_S - \pi_{SD,1}}{\pi_{SD,0}})) \tag{3.26}$$

3.7 Numerical Results

In this section, we present numerical results to illustrate the previous theoretical development. We illustrate the effects of different system parameters on the maximum stable throughput of the proposed transmission protocols. In the following results, we fix the channels success probabilities to be $f_{SD,1} = 0.4$, $f_{SD,0} = 0.1$, $f_{SR,1} = 0.8$, $f_{SR,0} = 0.2$, $f_{RD,1} = 0.8$ and $f_{RD,0} = 0.2$. Also, we let the channels distributions be identical such that $\pi_{SD,1} = \pi_{SR,1} = \pi_{RD,1} = \pi_1$ and $\pi_{SD,0} = \pi_{SR,0} = \pi_{RD,0} = \pi_0$. We denote the system with no relaying capability by "No Relaying". Also, we denote the system in which cooperative relaying is exploited by "With Relaying".

In figure 3.2, we show the maximum stable throughput of the two proposed network protocols against the probability of the channels to be in state 1. We fix the system parameters $q_S = 0.7$ and $q_R = 0.3$. The results are for p_0 with the values 0.25 and 0.75. For small values of π_1, the performance of the system is better for

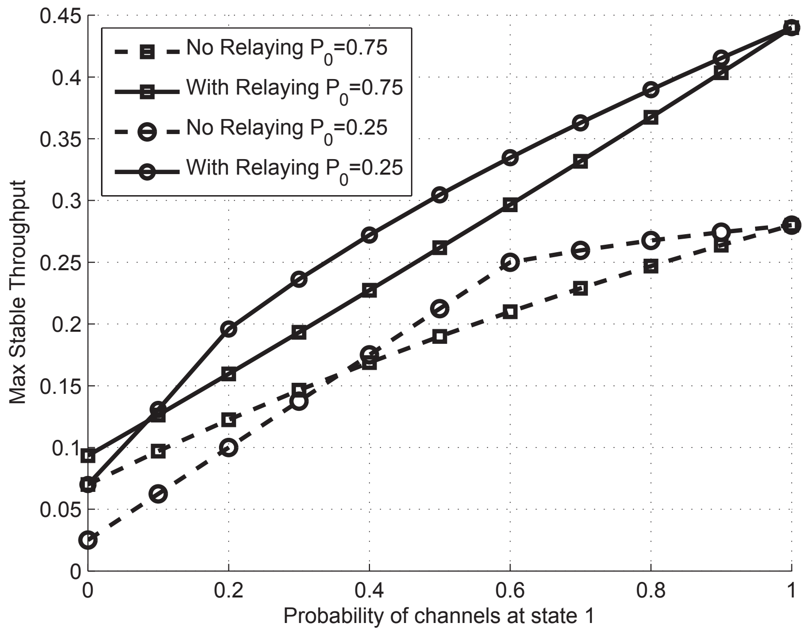

Figure 3.2: Maximum stable throughput against π_1

larger p_0 because it is better for the source to make more transmission attempts during the time slots in which the channel is in state 0. For large values of π_1, the performance of the system is better for smaller p_0 because the source should not waste much of its energy in transmission during the time slots in which the channel is in state 0.

In figure 3.3, we show the maximum stable throughput of the two proposed network protocols against the energy arrival rate to the source energy queue. We fix the system parameters $p_0 = 0.5$ and $\pi_1 = 0.6$. The results are for q_R with the values 0.1, 0.3 and 0.5. For the case $q_R = 0.1$, the maximum stable throughput of the cooperative relaying protocol becomes less than the throughput of the protocol with no relaying. That is because the channel SR has higher success probability than the channel SD. Then, most of the source packets are forwarded to the relay.

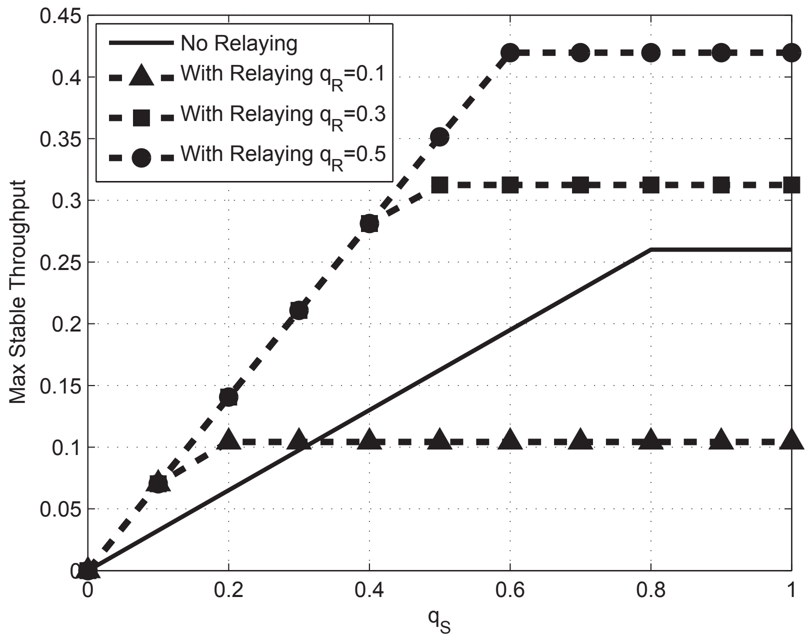

Figure 3.3: Maximum stable throughput against q_S

Also due to the limited energy at the relay and to maintain the stability of the relay data queue, the maximum stable throughput of the system is lowered.

In figure 3.4, we show the maximum stable throughput of the two proposed network protocols against the probability to attempt transmission while the channel in state 0. We fix the system parameters $q_S = 0.7$ and $q_R = 0.3$. The results are for π_1 with the values 0.6, 0.5 and 0.4. This figure shows the effect of exploiting the knowledge of the CSI of the channel between the source and destination. The performance when no CSI available is equivalent to the performance of the system with p_0 equals 1. For any value of π_1, the system is able to have higher stable throughput using the knowledge of the CSI than the system with no CSI at the source by selecting a suitable p_0.

In figure 3.5, we show the maximum stable throughput against the probability

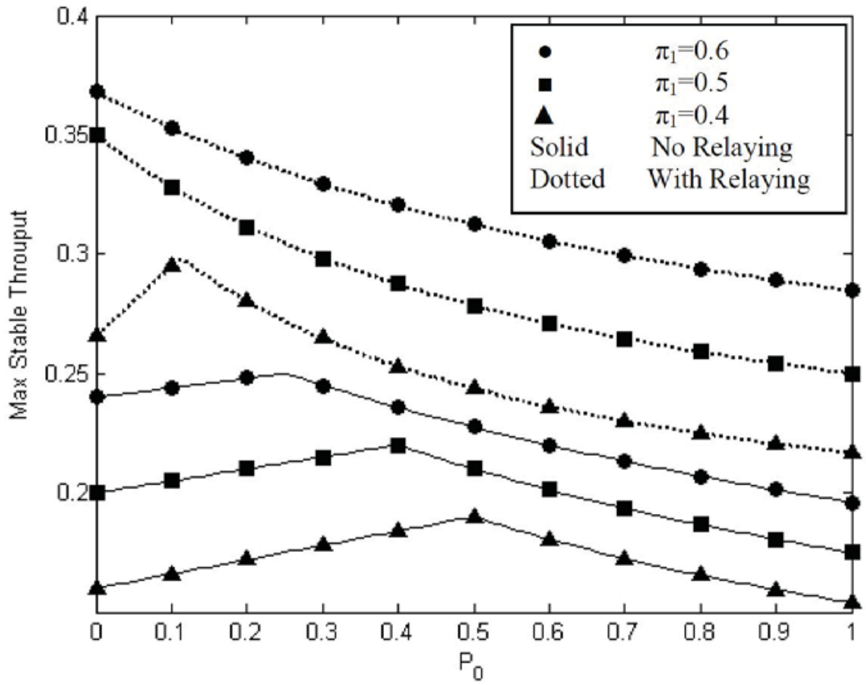

Figure 3.4: Maximum stable throughput against p_0

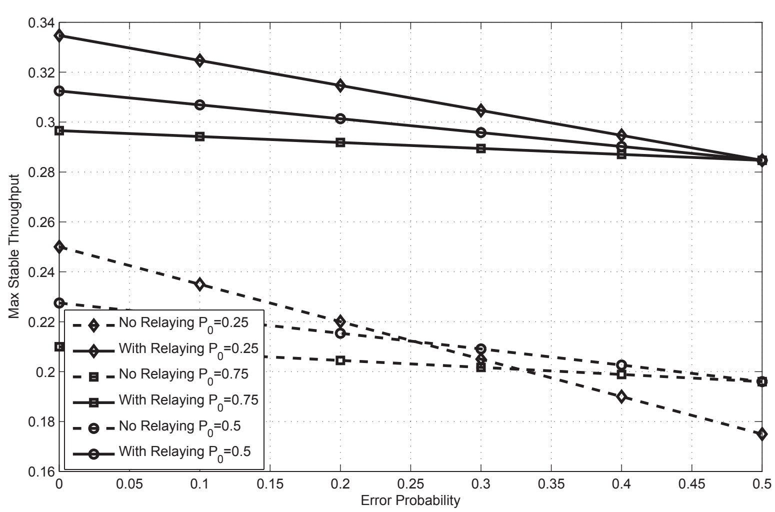

Figure 3.5: Maximum stable throughput against error probability

of error in channel measurement. We fix the system parameters $q_S = 0.7$, $q_R = 0.3$ and $\pi_1 = 0.6$. The results are for p_0 with the values 0.25, 0.5, and 0.75. The effect of error in channel measurement in case of transmission with relaying is less than the effect in case of direct transmission.

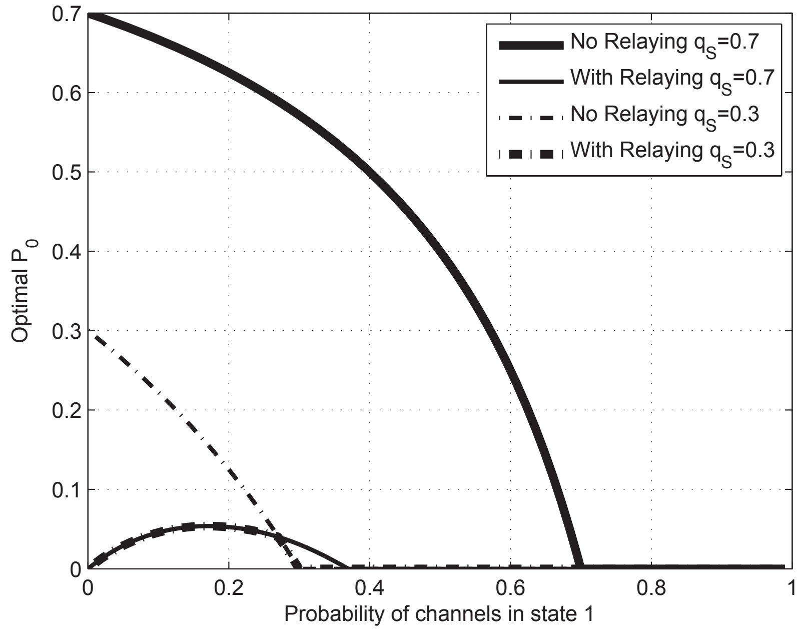

Figure 3.6: The value of p_0^* against π_1

In figure 3.6, we show the optimal transmission probability with the channel SD in state 0 against π_1. We fix $q_R = 0.3$. The results are for q_S with the values 0.3 and 0.7. The figure shows that the optimal p_0 takes small value when q_S is low as energy is better to be used when the channel in its good state. Also when the probability of the channel to be in state 1 is high, the optimal value of p_0 equals 0 as there will be no need to transmit while the channel is in state 0. In the case of no relaying, p_0 takes larger values than the case of cooperative relaying because there is no benefit for leaving the channel idle while there is unused energy at the source. In the case of cooperative relaying, keeping the channel idle allows the relay to transmit which can be more beneficial than allowing the source to transmit with the channel SD in state 0.

3.8 Discussion

In this chapter, we have proposed and analyzed protocols for transmission from a source that has energy harvesting capability. We have considered the case in which a relay is used to help the source transmissions. The relay also has energy harvesting capability. The proposed protocol allows the relay to use the idle time slots of the source and hence avoids allocating any explicit resources to the relay. Our analysis shows that cooperation increases the maximum stable throughput rate in most cases except when the energy harvesting rate of the relay is small. The proposed strategy exploits the knowledge of the CSI of the channel between the source and the destination such that the source transmits with probability 1 if the channel is in state 1 and transmits with a certain probability if the channel is in state 0. The optimal probability has also been calculated. The effect of imperfect channel measurements has been considered.

3.9 Appendix: Derivation of the Service Rate for the Relay Data Queue for Transmission Protocol with Relaying

We are going to calculate the service rate of the relay data queue. Let p_{RD} be the probability that a packet received by the destination due to a relay transmission. The packet is to be decoded successfully when the relay is able to transmit and the channel RD is not in outage. The relay is able to transmit when the relay energy queue is not empty. The value of P_{RD} is calculated as follows

$$P_{RD} = Pr[E_R \neq 0](\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) \tag{3.27}$$

The relay energy queue forms a discrete-time M/M/1 system for the same reasoning as the source energy queue. The service rate of the relay energy queue is the rate of attempting transmission of the relay node. The transmission attempting rate equals $(1 - \rho_S)$. The arrival rate of energy to the relay is q_R. Also, if the energy arrival rate of the relay node is larger than the transmission attempting rate, the number of energy units in the queue approaches infinity almost surely. Therefore, the probability of the energy queue to be empty is zero. On the other hand, if the energy arrival rate of the relay node is smaller than or equal to the transmission attempting rate, the probability of energy queue to be not empty is the ratio between the energy arrival rate and the transmission attempting rate. As a result, the probability of the energy queue to be not empty is written as follows

$$Pr[E_R \neq 0] = \frac{\min(q_R, 1 - \rho_S)}{1 - \rho_S} \tag{3.28}$$

Let T_R be the number of time slots needed for the relay to serve a packet in the relay data queue assuming that the relay continuously transmits. Then, T_R has a geometric probability distribution as follows

$$Pr[T_R = k] = P_{RD}(1 - P_{RD})^{k-1} \tag{3.29}$$

Then, the expected value of the number of time slots needed till the packet is decoded correctly by the destination, assuming that the relay continuously transmits, is shown to be

$$\mathbb{E}[T_R] = \frac{1}{P_{RD}} \tag{3.30}$$

Let v_1,v_2,.....be a sequence of random variables. The random variable v_i represents the number of successive time slots in which the source is going to be busy before the i^{th} relay retransmission. This sequence represents an i.i.d sequence. The probability of the source to be busy is ρ_S. Then, the number of successive time slots, in which the source is busy, follows a geometric distribution as follows

$$Pr[v = k] = \rho_S^k(1 - \rho_S) \tag{3.31}$$

The expected value of the number of successive time slots, in which the source is busy, is calculated as follows

$$\mathbb{E}[v] = \frac{\rho_S}{(1 - \rho_S)} \tag{3.32}$$

Let T be the number of time slots needed for the relay to get served including those in which the source will be transmitting, then we have

$$T = T_R + \sum_{i=1}^{T_R} v_i \tag{3.33}$$

This expression results from that the i^{th} transmission of the T_R relay transmissions is followed by busy period of length v_i. Then, the expected value of the number of time slots needed for the relay to get served, including those in which the source will be transmitting, is calculated as follows

$$\mathbb{E}[T] = \mathbb{E}[T_R](1 + \mathbb{E}[v]) = \frac{\mathbb{E}[T_R]}{(1 - \rho_S)} \tag{3.34}$$

Thus, the service rate of the relay data queue is shown as follows

$$\mu_R = \frac{1}{\mathbb{E}[T]} = (\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) \min(q_R, 1 - \rho_S) \tag{3.35}$$

Chapter 4: Partial Relaying for Energy Harvesting Networks

4.1 Introduction

In this chapter, we characterize the stability region of a system which contains a source, a relay and a destination. The source and the relay have energy harvesting capability. Each of the source and the relay has stochastic data arrivals. The relay regulates the relaying process by accepting only a proportion of the source successfully received packets. The relay transmits over the common medium only when the source is idle. We start by evaluating the stability conditions for the source and the relay data queues. Then, we combine the conditions to characterize the stability region as a function of the relaying parameter. Then, we solve the optimization problem of obtaining the relaying parameter which maximizes the stable throughput rate of the source for a given relay data arrival rate while maintaining the stability of the source and the relay data queues. Thus, we characterize the stability region of the system over the whole range of the relaying parameter. Then, we evaluate the stability region for simple transmission strategies such as no relaying strategy and fixed resource allocation strategy. We consider TDMA as an example for fixed resource allocation strategies. This work was presented in [44].

4.2 System Model

4.2.1 Network Model

We consider a network which consists of a source node, a relay node, and a destination node as shown in figure 4.1. Each of the source and the relay has an infinite data queue for storing fixed length packets. These queues are denoted by Q_S and Q_R respectively. We assume that the source generates its own traffic while the relay both generates its own traffic and relays the source traffic. The data arrival processes to the source and the relay data queues are modeled by Bernoulli processes. Also, each of the source and the relay has an infinite energy queue. These queues are denoted by E_S and E_R respectively. The usage of infinite queues is a reasonable approximation when the data queues are large enough compared to the packet size and the energy queues are large enough compared to the energy unit [15]. Each of the source and the relay can acquire a single unit of energy at each time slot with probabilities q_S and q_R respectively that the energy arrival processes are modeled by Bernoulli processes. All nodes are half-duplex and thus they can not transmit and receive simultaneously. Time is assumed to be slotted such that each packet transmission takes one time slot. Transmission of a data packet from a node requires using a single unit of energy from the corresponding energy queue. For simplicity, we assume that the energy consumption in a node is due to transmission only and therefore the processing and the reception energies are considered to be negligible.

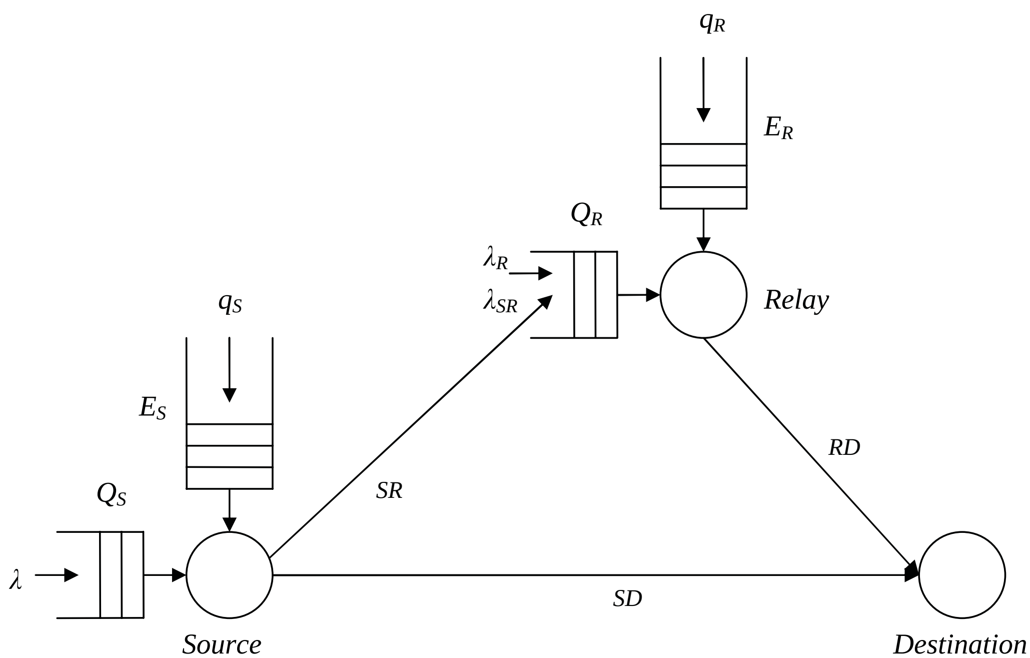

Figure 4.1: System Model

4.2.2 Channel Model

All the channels, which are denoted by SD, SR and RD, are modeled as independent erasure channels. The channels are independent of the packet arrival processes and the energy harvesting processes at the source and the relay. The quality of a channel is represented by the success probability of a packet. The average packet success probabilities are denoted by f_{SD}, f_{SR} and f_{RD}. These success probabilities are determined by the system physical parameters such as the transmission power, the modulation scheme, the coding scheme and the targeted bit-error rate.

4.2.3 Transmission Strategy

At a time slot, the source is able to transmit if both its energy queue and its data queue are not empty. If the packet is accepted by the destination or by the relay, it is released from the source data queue; otherwise it is kept in the source data queue for retransmission. At the beginning of every time slot, the relay senses the channel. We assume perfect sensing by the relay for the source transmissions. If the source is not transmitting, the relay uses this idle time slot to transmit a packet from its data queue to the destination when its energy queue is not empty. Hence, no explicit channel resources are assigned to the relay. A packet is released from the relay data queue if it is successfully received by the destination; otherwise it is kept for retransmission.

We exploit partial relaying cooperation that the relay accepts only a certain proportion of the successfully received packets. This proportion of accepted packets should match the ability of the relay to forward the packets. This proportion is determined by the relaying parameter r which is the probability of accepting a packet at the relay data queue given that this packet has been successfully received.

In [42], Loynes' theorem states that if the arrival and service processes at a queue are jointly stationary, then the queue is stable if the average arrival rate is less than the average service rate. Throughout the chapter, we denote the average arrival rate at the source data queue by λ. The average arrival rate to the relay data queue due to the source transmissions is denoted by λ_{SR} and the average arrival rate to the relay data queue for the packets generated by the relay is denoted by λ_R. The average service rate of the source data queue is denoted by μ_S. Also, the average service rate of the relay data queue is denoted by μ_R.

4.3 Stability Analysis

In this section, we derive the stability region of the proposed transmission protocol. The system is stable if both the source data queue and the relay data queue are stable. In the following subsections, we derive the stability conditions for each queue separately. The following two probabilities are defined to be used in the stability analysis. First, the probability that a packet transmitted by the source is accepted by the relay is denoted by P_R and its value is calculated as follows

$$P_R = r f_{SR}(1 - f_{SD}) \tag{4.1}$$

Also, we denote the probability that a packet transmitted by the source is accepted by either the relay or the destination by P_E and we calculate its value as follows

$$P_E = 1 - (1 - f_{SD})(1 - r f_{SR}) \tag{4.2}$$

4.3.1 Source Data Queue

In order to calculate the maximum stable throughput rate for the source data queue, we have to start by considering the service rate for the source energy queue which is the rate of which the source node can transmit when its data queue is saturated [45]. Each transmission attempt uses a single energy unit. As a result, the energy departure process is modeled by a Bernoulli process. Therefore, the source energy queue forms a discrete-time M/M/1 system. The energy queue service rate equals 1. The arrival rate of energy to the source node is q_S. Then, it follows from [43] for discrete-time M/M/1 system that the probability of energy queue to be not empty is the ratio between the energy arrival rate and the energy queue service rate. As a result, the probability of the energy queue to be not empty is

$$Pr[E_S \neq 0] = q_S \tag{4.3}$$

The maximum data arrival rate which maintains the stability of the source data queue is limited by its service rate. A packet at the source is served if it is successfully delivered to the relay or the destination. The source data queue service rate is the product of the success probability given that the source is able to transmit by the probability that its energy queue is not empty. The service rate of the source data queue is

$$\mu_S = Pr[E_S \neq 0]P_E \tag{4.4}$$

The stability condition for the source data queue is $\lambda < \mu_S$.

4.3.2 Relay Data Queue

We start by calculating the probability that the channel is occupied by the source transmissions and this probability is denoted by ρ_S. As the source data queue forms a discrete-time M/M/1 system and assuming that the source data queue is stable, it follows from [43] that the probability ρ_S is calculated as follows

$$\rho_S = \frac{\lambda Pr[E_S \neq 0]}{\mu_S} = \frac{\lambda}{P_E} \tag{4.5}$$

The arrival rate for the relay data queue from the source transmissions is the probability that a packet is accepted by the relay at any given time slot. The relay has a successful arrival at a time slot if the source is transmitting and the channel SR is not in outage while the channel SD is in outage. The arrival rate to the relay λ_{SR} is calculated as follows

$$\lambda_{SR} = \rho_S P_R = \lambda \frac{P_R}{P_E} \tag{4.6}$$

Then, the service rate of the relay data queue equals

$$\mu_R = f_{RD} \min(q_R, 1 - \rho_S) \tag{4.7}$$

The derivation of the expression of the service rate of the relay data queue has been done using the same way in section 3.9. For the relay data queue to be stable, the summation of the relay own traffic arrival rate and the arrival rate of the packets due to the source transmissions has to be less than the relay data queue service rate. Thus, the stability condition for the relay data queue is $\lambda_R + \lambda_{SR} < \mu_R$.

4.3.3 Stability Conditions

To ensure that the system is stable, both the source and the relay data queues have to be stable. As a result, both the conditions $\lambda < \mu_S$ and $\lambda_R + \lambda_{SR} < \mu_R$ should be satisfied. By substituting using equation (4.6) in the second condition, it is written as

$$\lambda < \frac{P_E}{P_R} f_{RD} \min(q_R, 1 - \rho_S) - \frac{P_E}{P_R} \lambda_R \tag{4.8}$$

Note that the right hand side of the inequality is still a function of λ. By combining the conditions and moving λ to one side of the inequality, we get the general expression for the maximum stable throughput of the source as follows

$$\lambda < \min(q_S P_E, \frac{P_E}{P_R}(f_{RD}q_R - \lambda_R), \frac{P_E(f_{RD} - \lambda_R)}{P_R + f_{RD}}) \tag{4.9}$$

The same analysis is still valid when the energy arrival processes and the data arrival process are modeled by Poisson processes. In this case, the energy queues and the data queues form M/G/1 systems.

4.4 Partial Relaying Optimization

In this section, we evaluate the value of the parameter r to maximize the stable throughput of the source λ for a given value of λ_R. The value of the optimal r as a function in λ_R is substituted in equation (4.9) to get the bound of the stability region. The optimal value of r is the solution of the problem

$$r^* = \arg\max_r(\min\{f_1(r), f_2(r), f_3(r)\}) \tag{4.10}$$

where the values of $f_1(r)$, $f_2(r)$ and $f_3(r)$ are obtained from equation (4.9) as follows

$$f_1(r) = q_S[1 - (1 - f_{SD})(1 - rf_{SR})] \tag{4.11}$$

$$f_2(r) = \frac{1 - (1 - f_{SD})(1 - rf_{SR})}{rf_{SR}(1 - f_{SD})}(f_{RD}q_R - \lambda_R) \tag{4.12}$$

$$f_3(r) = \frac{(1 - (1 - f_{SD})(1 - rf_{SR}))(f_{RD} - \lambda_R)}{rf_{SR}(1 - f_{SD}) + f_{RD}} \tag{4.13}$$

The function $f_1(r)$ is a linear function of r with a non-negative slope. Thus, $f_1(r)$ is a non-decreasing function of r.

The function $f_2(r)$ is continuous over the interval $]0, 1]$. We calculate the first derivative of the function $f_2(r)$ to be

$$\frac{d}{dr}f_2(r) = \frac{-f_{SD}f_{SR}(1 - f_{SD})(f_{RD}q_R - \lambda_R)}{(rf_{SR}(1 - f_{SD}))^2} \tag{4.14}$$

Thus, the first derivative is always non-positive and the function $f_2(r)$ is a non-increasing function of r.

The function $f_3(r)$ is continuous over the interval $]0, 1]$. The function $f_3(r)$ can be either non-increasing or non-decreasing based on the system parameters. The value of the first derivative of $f_3(r)$ is calculated as follows

$$\frac{d}{dr} f_3(r) = \frac{(f_{RD} - \lambda_R) P'_R (P_R + f_{RD} - P_E)}{(P_R + f_{RD})^2} \tag{4.15}$$

where P'_R is the first derivative of P_R with respect to r. The value of P'_R is

$$P'_R = f_{SR}(1 - f_{SD}) \tag{4.16}$$

Also, the difference between P_R and P_E is

$$P_E - P_R = f_{SD} \tag{4.17}$$

Thus, the sign of the term $f_{RD} - f_{SD}$ determines the monotonicity of the function $f_3(r)$. If it is non-negative, the function is non-decreasing with respect to r and if the term $(f_{RD} - f_{SD})$ is non-positive, the function is non-increasing with respect to r. In the following subsections, we will consider the optimal value of r in both cases.

4.4.1 $f_{RD} > f_{SD}$

We consider the case in which the channel from the relay to the destination has better quality than the channel from the source to the destination. In this case, the function $f_3(r)$ is a non-decreasing function of r. We calculate the intersection points of $f_2(r)$ with each of $f_1(r)$ and $f_3(r)$. We denote these points by r_{12} and r_{23}

respectively. The value of r_{12} is evaluated to be

$$r_{12} = \frac{q_R f_{RD} - \lambda_R}{q_S f_{SR}(1 - f_{SD})} \tag{4.18}$$

Then, the value of the second intersection point r_{23} is calculated as follows

$$r_{23} = \frac{q_R f_{RD} - \lambda_R}{(1 - q_R) f_{SR}(1 - f_{SD})} \tag{4.19}$$

The values of r_{12} and r_{23} are always positive when the relay data queue is stable as the value of $q_R f_{RD} - \lambda_R$ is positive when the queue is stable.

The optimal value of r in this case is calculated as follows

$$r^*_{(f_{RD} > f_{SD})} = \min\left(1, \max\left(r_{12}, r_{23}\right)\right) \tag{4.20}$$

The optimal value of r is 1 if the maximum of the intersection points is larger than 1.

By substitution using the values of r_{12} and r_{23} and simplifying the resulted equation, we get

$$r^*_{(f_{RD} > f_{SD})} = \min\left(1, \frac{q_R f_{RD} - \lambda_R}{\min\left(q_S, 1 - q_R\right) f_{SR}(1 - f_{SD})}\right) \tag{4.21}$$

In the case when the quality of the channel RD is better than the quality of the channel SD, it is preferred to let the relay transmit with the maximum transmission attempt rate which is equal to q_R. The numerator $q_R f_{RD} - \lambda_R$ represents the rate with which the relay could forward the source transmissions. It is the difference between the relay service rate and the relay own traffic arrival rate. The denominator is the rate of the proportion of the source data that can be relayed. The term $\min\left(q_S, 1 - q_R\right)$ is the rate with which the source accesses the channel and it is multiplied by the probability that a packet transmitted by the source is received by the relay only.

4.4.2 $f_{RD} \leq f_{SD}$

In this subsection, we discuss the optimal relaying parameter when the channel from the source to the destination has better quality than the channel from the relay to the destination. In this case, the function $f_3(r)$ is a non-increasing function of r. We calculate the intersection points of $f_1(r)$ with each of $f_2(r)$ and $f_3(r)$. We denote these points by r_{12} and r_{13} respectively. The value of r_{12} is calculated in the previous subsection. The value of r_{13} is

$$r_{13} = \frac{(1 - q_S) f_{RD} - \lambda_R}{q_S f_{SR} (1 - f_{SD})} \tag{4.22}$$

The optimal value of r in this case is calculated as follows

$$r^*_{(f_{RD} \leq f_{SD})} = \max \left(0, \min \left(r_{12}, r_{13}, 1\right)\right) \tag{4.23}$$

The optimal value of r is 1 when both r_{12} and r_{13} are larger than 1 . On the other hand, the optimal value of r is 0 if r_{13} is non-positive considering that the value of r_{12} is always non-negative when the system is stable.

By substitution using the values of r_{12} and r_{23} and simplifying the resulted equation, we get

$$r^*_{(f_{RD} \leq f_{SD})} = \max \left(0, \min \left(1, \frac{\min \left(q_R, 1 - q_S\right) f_{RD} - \lambda_R}{q_S f_{SR} (1 - f_{SD})}\right)\right) \tag{4.24}$$

In the case when the quality of the channel SD is better than the quality of the channel RD, it is preferred to let the source transmit with the maximum transmission attempt rate which is equal to q_S. The denominator $q_S f_{SR}(1 - f_{SD})$ represents the rate with which the source data is relayed. The numerator is the rate with which the relay can forward the source transmissions. The term $\min(q_R, 1 - q_S)$ is the rate with which the relay accesses the channel and it is multiplied by the probability that a packet transmitted by the relay is received by the destination. Then, the relay data arrival rate is subtracted from the previous quantity to get the rate with which the relay can forward the source transmissions.

4.5 Special Cases

In this section, we consider the special cases when a node has a continuous source of energy for transmission.

4.5.1 The case of ($q_R = 1$)

This is the case in which the relay has a continuous source of energy. Thus, the service rate of the relay is limited only by the channel occupation due to the source transmissions. Then, the expression of the relay service rate can be rewritten as follows

$$\mu_R|_{q_R=1} = f_{RD}(1 - \rho_S) \tag{4.25}$$

As a result, the general expression for the maximum stable throughput of the source is stated as follows

$$\lambda|_{q_R=1} < \min\left(q_S P_E, \frac{P_E(f_{RD} - \lambda_R)}{P_R + f_{RD}}\right) \tag{4.26}$$

The expression of the maximum stable throughput of the source can be written as

$$\lambda|_{q_R=1} < \min\left(f_1(r)|_{q_R=1}, f_3(r)|_{q_R=1}\right) \tag{4.27}$$

As a result, the optimal value of r can be obtained following the same steps of the general case. When $f_{RD} > f_{SD}$, both $f_1(r)|_{q_R=1}$ and $f_3(r)|_{q_R=1}$ are non-decreasing functions in r. Then, the optimal value of r is

$$r^*_{(f_{RD}>f_{SD})}|_{q_R=1} = 1 \tag{4.28}$$

On the other hand when $f_{RD} \leq f_{SD}$, the optimal value of r is

$$r^*_{(f_{RD}\leq f_{SD})}|_{q_R=1} = \max\left(0, \min\left(r_{13}|_{q_R=1}, 1\right)\right) \tag{4.29}$$

$$r^*_{(f_{RD}\leq f_{SD})}|_{q_R=1} = \max\left(0, \min\left(1, \frac{(1-q_S)f_{RD} - \lambda_R}{q_S f_{SR}(1-f_{SD})}\right)\right) \tag{4.30}$$

It is intuitive that when $q_R = 1$ and $f_{RD} > f_{SD}$, the optimal value of r is 1. In this case, the channel from the relay to the destination has better quality than the channel from the source to the destination and there is no energy limitation at the relay. Then, there is no reason for the relay to reject a successfully received packet from the source. This explanation is true also when the value of q_R is large enough to forward all the successfully received packets from the source. To get the condition on q_R for the optimal r to be 1 when $f_{RD} > f_{SD}$, we get the value at which $\max(r_{12}, r_{23}) \geq 1$. Then, the condition is

$$q_R \geq \min\left(\frac{q_S f_{SR}(1-f_{SD}) + \lambda_R}{f_{RD}}, \frac{f_{SR}(1-f_{SD}) + \lambda_R}{f_{RD} + f_{SR}(1-f_{SD})}\right) \tag{4.31}$$

4.5.2 The case of $(q_S = 1)$

This is the case in which the source has a continuous source of energy. Thus, the service rate of the source is limited only by the channels success probabilities. Then, the expression of the source service rate can be rewritten as follows

$$\mu_S|_{q_S=1} = P_E \tag{4.32}$$

As a result, the general expression for the maximum stable throughput of the source is stated as follows

$$\lambda|_{q_S=1} < \min\,(P_E, \frac{P_E}{P_R}(f_{RD}q_R - \lambda_R), \frac{P_E(f_{RD} - \lambda_R)}{P_R + f_{RD}}) \tag{4.33}$$

The optimal value of r can be obtained following the same steps of the general case. When $f_{RD} > f_{SD}$, the value of $r_{23}|_{q_S=1}$ is greater than or equal to the value of $r_{12}|_{q_S=1}$ because $(1 - q_R) \leq 1$ that is

$$\frac{q_R f_{RD} - \lambda_R}{(1 - q_R) f_{SR}(1 - f_{SD})} \geq \frac{q_R f_{RD} - \lambda_R}{f_{SR}(1 - f_{SD})} \tag{4.34}$$

Then, the optimal r can be defined to be

$$r^*_{(f_{RD} > f_{SD})}|_{q_S=1} = \min\,(1, r_{23}|_{q_S=1}) \tag{4.35}$$

$$r^*_{(f_{RD} > f_{SD})}|_{q_S=1} = \min\,(1, \frac{q_R f_{RD} - \lambda_R}{(1 - q_R) f_{SR}(1 - f_{SD})}) \tag{4.36}$$

When $f_{RD} \leq f_{SD}$, we found that

$$r_{13}|_{q_S=1} = \frac{-\lambda_R}{f_{SR}(1 - f_{SD})} \tag{4.37}$$

This quantity is a non-positive quantity from the definition of λ_R. Thus, the optimal value of r is

$$r^*_{(f_{RD} \leq f_{SD})}|_{q_S=1} = 0 \tag{4.38}$$

It is intuitive that when $q_S = 1$ and $f_{RD} \leq f_{SD}$, the optimal value of r is 0. In this case, the channel from the source to the destination has better quality than the channel from the relay to the destination and there is no energy limitation at the source. Then, there is no reason for the source to be helped by the relay. This explanation is true also when the value of q_S is large enough to forward all the packets from the source. To get the condition on q_S for the optimal r to be 0 when $f_{RD} \leq f_{SD}$, we get the value at which $\min(r_{12}, r_{13}) \leq 0$. Then, the condition is

$$q_S \geq 1 - \frac{\lambda_R}{f_{RD}} \tag{4.39}$$

4.6 Stability Regions for Simple Strategies

4.6.1 No relaying

We consider the case in which the source and the relay do not cooperate. The source has higher priority than the relay that the relay transmits only when the source is idle. The results for this case are obtained by setting the parameter r to 0. The importance of this case is that it represents the case of resource allocation with no cooperation and no interference between energy harvesting nodes.

The value of the source data queue service rate with "No Relaying" which is denoted by $\mu_S^{(NR)}$ is calculated as follows

$$\mu_S^{(NR)} = q_S f_{SD} \tag{4.40}$$

The value of the probability that the channel is occupied by source transmissions is calculated as follows

$$\rho_S^{(NR)} = \frac{\lambda}{f_{SD}} \tag{4.41}$$

Then, the service rate of the relay data queue equals

$$\mu_R^{(NR)} = f_{RD} \min(q_R, 1 - \rho_S^{(NR)}) \tag{4.42}$$

Thus, the stability condition for the relay data queue is $\lambda_R < \mu_R^{(NR)}$. To ensure that the system is stable, both the conditions $\lambda < \mu_S^{(NR)}$ and $\lambda_R < \mu_R^{(NR)}$ should be satisfied. By combining the conditions on λ, we get the general expression for the stability region as follows

$$\lambda < f_{SD} \min(q_S, \frac{f_{RD} - \lambda_R}{f_{RD}}), \text{if} \lambda_R < q_R f_{RD} \tag{4.43}$$

4.6.2 Fixed Resource Allocation

In this section, we evaluate the stability region in the case of TDMA scheduling for the source and the relay. The same transmission strategy as in section 4.2 is exploited except of the time allocation. The odd time slots are assigned for the source and the even time slots are assigned for the relay which is the same technique used in [40].

The value of the source data queue service rate which is denoted by $\mu_S^{(TDMA)}$ is calculated using the same steps as the calculation of the relay service rate in section 3.9. The probability with which the source can access the channel is 1/2. Then, the probability of the source energy queue to be not empty is calculated as follows

$$Pr[E_S \neq 0]^{(TDMA)} = \frac{\min(q_S, 1/2)}{1/2} \tag{4.44}$$

Then, we obtain the expression for the source data queue service rate as follows

$$\mu_S^{(TDMA)} = \min(q_S, \frac{1}{2})P_E \tag{4.45}$$

The value of the probability that the channel is occupied by source transmissions is calculated as follows

$$\rho_S^{(TDMA)} = \frac{\lambda}{P_E} \tag{4.46}$$

The calculation of the service rate of the relay data queue follows the same steps. Also, the probability with which the relay can access the channel is 1/2. Then, the probability of the relay energy queue to be not empty is calculated as follows

$$Pr[E_R \neq 0]^{(TDMA)} = \frac{\min(q_R, 1/2)}{1/2} \tag{4.47}$$

Then, the service rate of the relay data queue equals

$$\mu_R^{(TDMA)} = \min(q_R, \frac{1}{2})f_{RD} \tag{4.48}$$

Thus, the stability condition for the relay data queue is $\lambda_{SR} + \lambda_R < \mu_R^{(TDMA)}$. To ensure that the system is stable, both the conditions $\lambda < \mu_S^{(TDMA)}$ and $\lambda_{SR}+\lambda_R < \mu_R^{(TDMA)}$ should be satisfied. By combining the conditions on λ, we get the general expression for the stability region as follows

$$\lambda < \min(\mu_S^{(TDMA)}, \frac{P_E}{P_R}(\mu_R^{(TDMA)} - \lambda_R)) \tag{4.49}$$

The maximum source stable throughput rate is the minimum of two functions. The first function is a non-decreasing function of r as shown in section IV. The second function is a non-increasing function of r. The optimal value of r is the intersection point of the two functions if this intersection point is less than 1; otherwise, the optimal value of r is 1. Thus, the expression for the optimal value of r is written as follows

$$r^{*(TDMA)} = \min(\frac{\min(q_R, \frac{1}{2}) f_{RD} - \lambda_R}{f_{SR}(1 - f_{SD}) \min(q_S, \frac{1}{2})}, 1) \tag{4.50}$$

4.7 Numerical Results

In this section, we present numerical results to illustrate the previous theoretical development. We illustrate the effects of system parameters on the performance of the system and the optimal value of relaying parameter. In the following results, we fix the channel success probabilities to be $f_{SD} = 0.25$, $f_{SR} = 0.5$ and $f_{RD} = 0.5$. We denote the system with optimal relaying parameter with the relay senses the channel by "Optimal" and the system with TDMA channel access techniques by "TDMA".

In figure 4.2, we show the stability regions of different relaying schemes. We set the energy arrival rates for the source and the relay to $q_S = 0.6$ and $q_R = 0.6$. We compare the optimal relaying to the cases of full relaying, no relaying and TDMA. We show that the stability region of the optimal partial relaying contains the stability region of other relaying schemes. For the selected parameters, it is optimal to use full relaying for $\lambda_R \leq 0.15$. When λ_R is larger, the relay does not have enough energy to forward all the successfully received packets by the source. Also, it is optimal not to relay source data when $\lambda \leq 0.1$. In this case, the source energy is enough to forward the source data through the channel SD and the relay uses its

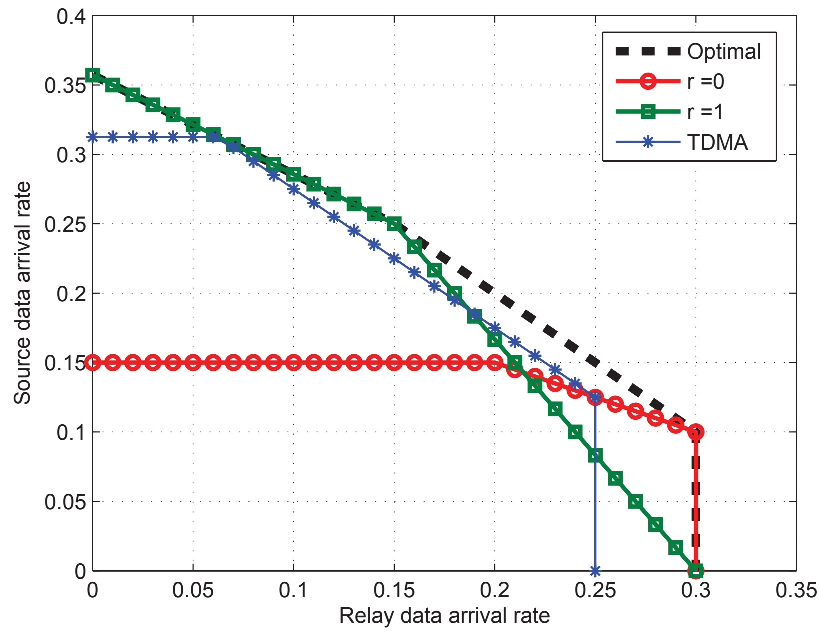

Figure 4.2: Stability Region

energy to forward its own data. On the other hand, the TDMA scheme is optimal at one point only when $\lambda_R = 0.06$.

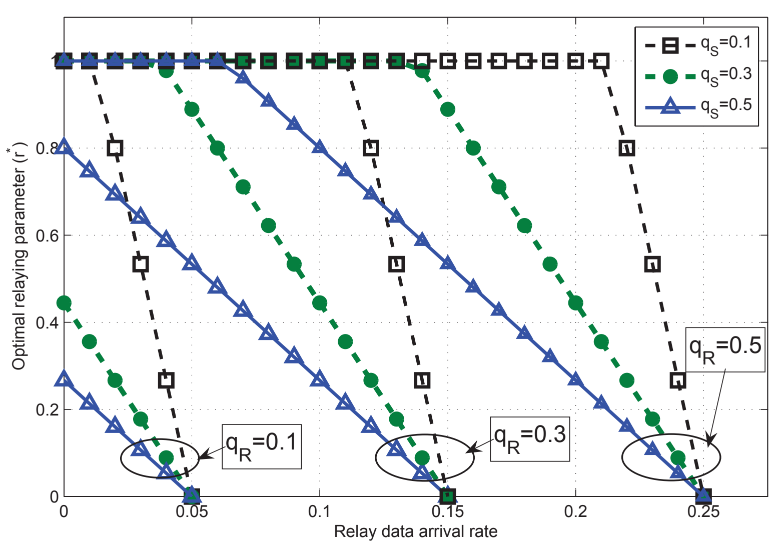

Figure 4.3: Optimal r against different system parameters

In figure 4.3, we show the value of the optimal relaying parameter against λ_R, q_S and q_R. The horizontal axis is λ_R and we show results for q_S and q_R with the values of 0.1, 0.3 and 0.5. The figure shows that full relaying is optimal for a wider range with the increase of relay energy arrival rate. Also, partial relaying is more important for enhancing the performance with the decrease of the source energy arrival rate. Finally, the value of the optimal relaying parameter decreases with the increase of the data arrival rate for the relay.

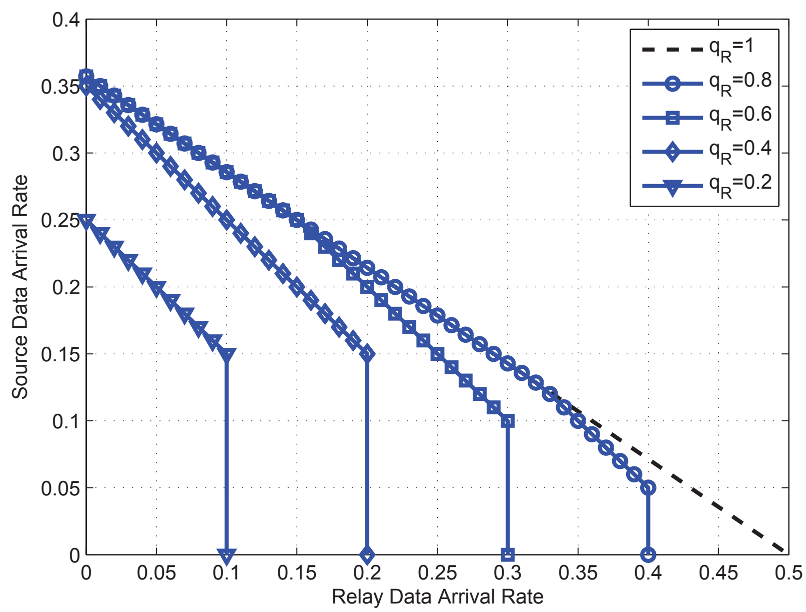

Figure 4.4: Stability Region against different values of q_R

In figure 4.4, we show the stability regions of the optimal partial relaying system for different values of q_R. We set the energy arrival rate for the source to $q_S = 0.6$. We show the increase in the stability region with the increase of q_R. This increase is much higher for low values of q_R while it has lower effect for large values of q_R. When q_R is large that the network can not exploit all the harvested energy, the enhancement in the stability region happens only when λ is small such that the source is not able to use all its harvested energy. The vertical lines in the figure represent the case when $r^* = 0$ such that the increase in λ does not affect the allowable λ_R in the system.

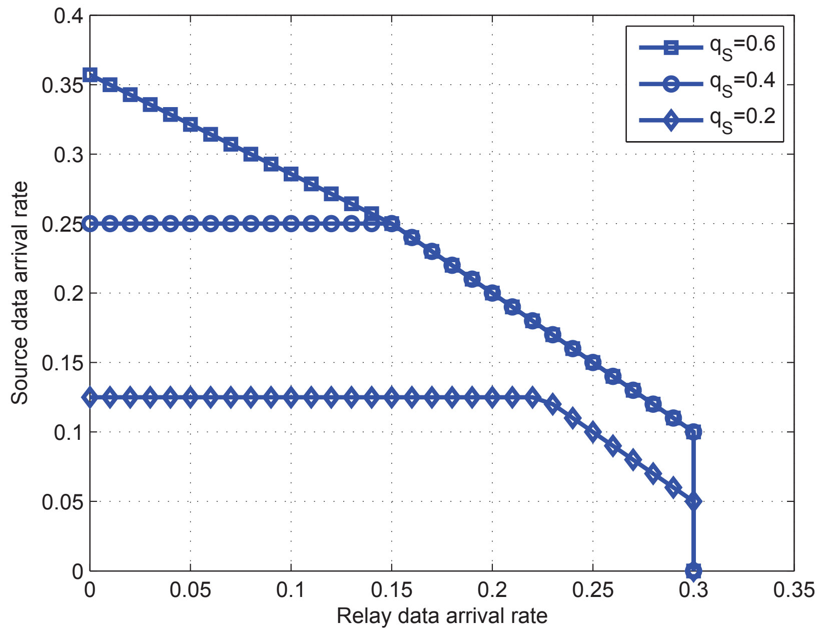

Figure 4.5: Stability Region against different values of q_S

In figure 4.5, we show the stability regions of the optimal partial relaying system for different values of q_S. We set the energy arrival rate for the relay to $q_R = 0.6$. We show the increase in the stability region with the increase of q_S. This increase is much higher for low values of q_S while it has lower effect for large values of q_S. The horizontal lines in the figure represent the case when $r^* = 1$ such that the increase in λ_R exploits unused harvested energy at the relay without affecting the source performance.

4.8 Discussion

In this chapter, we have introduced the notion of partial network-level cooperation for energy harvesting networks. The flow from the source through the relay is controlled. We provide an exact characterization of the stability region for the discussed system. We have shown that the performance of the system with optimal partial relaying is always better than or equals the performance of simple relaying schemes. Also, we have shown that it is optimal to use full relaying for a small data arrival rate at the relay while it is optimal to use no relaying when the source has a small data arrival rate.

Chapter 5: Relaying and Stability in Energy Harvesting Networks with Multiple Relays

5.1 Introduction

The use of multiple relays compared to a single relay leads to wider coverage and lower transmit power [46, 47]. Selecting a subset of multiple available relays according to a performance metric can further enhance the performance of cooperative networks. Relay selection schemes can be divided into two categories: single relay selection schemes and multiple relay selection schemes. The complexity of the multiple relay selection schemes increases exponentially with the number of available relays [48]. Thus, in our work, we consider the case of selecting a single relay from multiple available relays. Several relay selection schemes have been proposed in the literature. Examples of relay selection schemes can be found in [48]- [51]. In these works, the enhancement of the performance due to selecting a single relay from multiple available relays was shown. The main difference in our work is that the relays are energy harvesting nodes with random energy availability.

In this chapter, we consider a simple system which consists of a source, a destination and a number of relays. The source and the relays have energy harvesting capability. The nodes share the same band. The packets arrivals into the source and the energy arrivals into the source and the relays are modeled by discrete-time stochastic processes. We consider a two-hop network with the availability of the line of sight between the source and the destination that each packet can reach the destination by passing through a single relay at most. The study of a two-hop network is both instructive and necessary. It reveals insights at the conceptual levels about the effects of different system parameters in more practical scenarios such as the multi-hop networks. The importance of considering this simple model is to shed insights into the interaction between relaying, energy harvesting, and stability.

We consider a centralized transmission scheduling policy in the network. The studied centralized policy is analytically tractable and serves as a benchmark for the different distributed schemes that could be used. The centralized policies also can be applied for networks with small number of nodes and within the neighbor nodes in large networks. Due to the random nature of data arrivals, we introduce a transmission strategy in which relays transmit during the idle periods of the source. The transmission strategy allows partial relaying cooperation. The partial network-level cooperation between the source and the relays is achieved by adding a flow controller to each relay which controls the flow going through the relay. It controls the flow by setting a probability to accept packets at each relay. This partial cooperation was used before in [52] for non energy harvesting relays.

In the studied model, we investigate the problem of constrained minimization of a linear cost objective function. Each packet has a cost associated with the path through which the packet reaches the destination. The cost which is associated with a certain path is generally determined by the channels characteristics and the energy harvesting rates at different nodes. The cost objective function may be selected to represent a network performance measure as the delay or the consumed energy. The minimization problem is constrained by the stability of the data queues of different nodes.

The results of this work quantify the enhancement in the performance due to the use of partial cooperation in the system. We compare the results when exploiting partial relaying to the case of no relaying and the case of full relay cooperation in which no flow control is applied at the relays.

We start the analysis by calculating the stability conditions of the data queues of the source and the relays which represent the constraints of the relaying cost minimization problem. Then, we get a closed-from expression for the maximum achievable rate of the source as a function of the relaying parameters which are the probabilities of accepting packets at the relays. Finally, we specify the cost minimization problem to the case of energy consumption minimization. We optimize the network energy consumption over the partial relaying parameters while maintaining the stability of the source and the relays data queues. This work was presented in [53].

5.2 System Model and Problem Formulation

5.2.1 Network Model

We consider a network which consists of a source node, a number of relay nodes, and a destination node as shown in figure 5.1. The number of relay nodes is N. We refer to each node by an index that each relay takes an index i which belongs to $\{1, 2, ...N\}$ and the source takes the index 0. Each of the source and the relays has an infinite data queue for storing fixed length packets. These queues are denoted by Q_i with i is the index of the node and belongs to $\{0, 1, 2....., N\}$. We assume that the source has its own traffic while the relays do not have their own traffic and are used only for cooperation with the source. The data arrival to the source data queue is modeled by a Bernoulli process. Also, each of the source and the relays has an infinite energy queue. These queues are denoted by E_i with i is the index of the node and belongs to $\{0, 1, 2....., N\}$. The usage of infinite queues is a reasonable approximation when the data queues are large enough compared to the packet size and the energy queues are large enough compared to the energy unit [15]. All nodes are half-duplex and thus they can not transmit and receive simultaneously. Time is assumed to be slotted such that each packet transmission takes one time slot. Transmission of a data packet from a node requires using a single unit of energy from the corresponding energy queue. The source and the relays can acquire a single unit of energy at each time slot with probabilities q_i that the energy arrival processes are modeled by Bernoulli processes. For simplicity, we assume that the energy consumption in a node is due to transmission only and therefore the energy for data processing and data reception does not affect our analysis.

5.2.2 Channel Model

All the channels are modeled as independent erasure channels. The channels are also independent of the packet generation process and the energy harvesting at the source and the relays. The channels from node i to node j and from node i to the destination are denoted by C_{ij} and C_{iD} respectively. The quality of a channel is represented by the average success probability of a packet. The average packet success probabilities over the channels from node i to node j and from node i to the destination are denoted by f_{ij} and f_{iD} respectively. These average success probabilities are determined by the system physical parameters such as the transmission power, the modulation scheme, the coding scheme and the targeted bit-error rate.

5.2.3 Transmission Strategy

The source transmits when both its data and energy queues are not empty. The transmitted packet is released from the source data queue if it is accepted by either the destination or any of the relay nodes; otherwise it is kept at the source data queue for retransmission. A packet is stored at the data queue of the relay i if the packet is accepted by the relay i and is not accepted by neither the destination nor the relays with indices belongs to $\{1, 2, ...i - 1\}$. When the source is idle, the

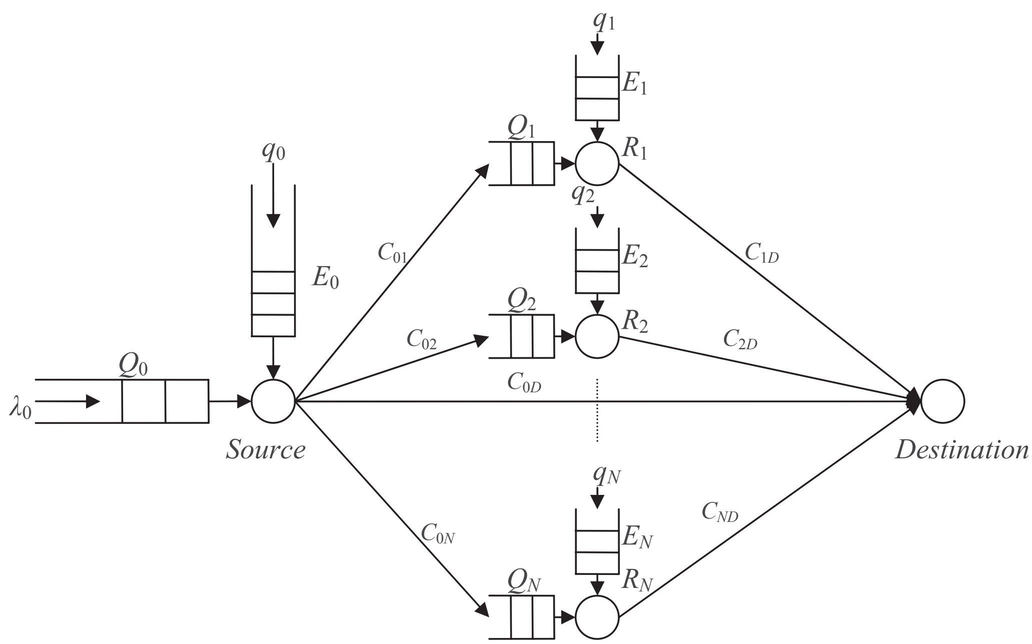

Figure 5.1: System Model

centralized controller allows the relay with the lowest index and both its energy and data queues are not empty to transmit a packet. The packet is released from a relay data queue when it is successfully received by the destination.

In the transmission strategy, we have considered the case in which the relays have fixed order. When a transmitted packet by the source is not received by the destination and is accepted by more than a single relay node, the packet is stored at the data queue of the relay with the lowest index. As a result, giving a lower index to a relay means that this node has a higher priority in storing received packets. A node with high energy harvesting rate is able to make more transmission attempts than a node with low energy harvesting rate. Also, a node with high average success probability for its channel to the destination is able to do less number of retransmissions than a node with low average success probability. Thus, we suggest in our work an ordering criterion based on the product of the energy harvesting rate by the success probability. The nodes are ordered such that lower index means higher value of the product to include both the effects of the energy arrival rate and the average channel success probability. The analysis is general for any ordering scheme.

We introduce partial relaying cooperation that each relay accepts only a certain proportion of the successfully received packets. This proportion of accepted packets should match the ability of the relay node to forward the packets. The proportion which is accepted by relay i is determined by the flow control parameter r_i, $i = 1, 2, ...N$. The parameter r_i is the probability of accepting a packet at the relay i data queue given that this packet has been successfully received. As a result, the packet accepting probability at relay i equals $r_i f_{0i}$. The vector that contains the values of r_i with $i = 1, 2, ...N$ is denoted by $\vec{r}$.

In [42], Loynes' theorem states that if the arrival and service processes at a queue are jointly stationary, then the queue is stable if the average arrival rate is less than the average service rate. Throughout the chapter, the average arrival rate to the node i data queue is denoted by λ_i. Also, the average service rate of the node i data queue is denoted by μ_i. The maximum achievable rate of the source for a certain relaying vector $\vec{r}$ is denoted by $\hat{\lambda}_0(\vec{r})$. Also, we denote the maximum achievable rate of the source over all the values of $\vec{r}$ by λ_0^*. Also, the proportion of the source data packets which arrives at the destination directly is denoted by $\tilde{\lambda}_0$ and it equals $\lambda_0 - \sum_{i=1}^{N} \lambda_i$.

5.2.4 Problem Formulation

The goal of the problem is minimizing the relaying cost while maintaining the stability of the source and the relays data queues. The problem objective function is denoted by J. The cost of a packet which is relayed by the relay i is denoted by c_i. The cost for forwarding a packet directly from the source to the destination is denoted by c_0. The cost could be selected to represent some network performance measure such as the average consumed energy or the average delay. The objective function is

$$J = c_0 \tilde{\lambda}_0 + \sum_{i=1}^{N} c_i \lambda_i \tag{5.1}$$

The relays do not generate their own traffic. As a result, the data arrival rates for different relays are functions in the source data arrival rate. For a certain partial relaying parameters vector $\vec{r}$, the stability of all queues is achieved by constraining the source data arrival rate to be less than the maximum achievable rate for this $\vec{r}$ that the problem is constrained by $\lambda_0 < \hat{\lambda}_0(\vec{r})$. Also, the relaying parameters r_i for all i have to belong to $[0, 1]$. Thus, the problem can be written as follows:

$$\min_{\vec{r}} \quad c_0 \tilde{\lambda}_0 + \sum_{i=1}^{N} c_i \lambda_i$$

$$\text{subject to} \quad \lambda_0 < \hat{\lambda}_0(\vec{r})$$

$$0 \le r_i \le 1, \quad \text{for } \ i = 1, 2, ...N$$

We start investigating the problem by calculating the value of $\hat{\lambda}_0(\vec{r})$ which is obtained by evaluating the stability conditions of the data queues of the source and the relays. Then, we discuss the optimization problem and specify the cost objective function to be the network energy consumption.

5.3 Stability Analysis

The system is stable if the source data queue and the relays data queues are stable. In the following subsections, we derive the stability conditions for all the system data queues.

5.3.1 Source Data Queue

The service rate of the source energy queue is the rate of which the source node attempts to transmit when its data queue is not empty. It equals the probability that the channel is not busy by other nodes transmissions. Each transmission attempt consumes a single energy unit. The transmission attempting rate equals 1 as the source node has the highest priority to transmit in the network. The arrival rate of energy to the source is q_0. The energy arrival rate to the source is smaller than or equal to the transmission attempting rate, then it follows from [43] that the probability of the energy queue to be not empty, when the data queue is saturated, is the ratio between the energy arrival rate and the transmission attempting rate. As a result, the probability of the source energy queue to be not empty is calculated as follows

$$\Pr[E_0 \neq 0] = q_0 \tag{5.2}$$

For more details on calculating the probability that an energy queue is not empty in an energy harvesting source, refer to [45].

The average probability, that a transmitted packet is released from the source data queue, is denoted by P_E and calculated as follows

$$P_E = (1 - (1 - f_{0D}) \prod_{i=1}^{N} (1 - r_i f_{0i})) \tag{5.3}$$

The source data queue service rate is the product of the success probability given that the source is able to transmit by the probability that the energy queue is not empty. The value of the source data queue service rate equals

$$\mu_0 = \Pr[E_0 \neq 0] P_E \tag{5.4}$$

To maintain the stability of the source data queue, the data arrival rate to the source data queue has to be less than the source data queue service rate that is $\lambda_0 < \mu_0$.

5.3.2 Relays Data Queues

To investigate the stability conditions for the relays data queues, we start by calculating the probability that the channel is occupied by the source transmissions which is denoted by ρ_0. When the source data queue is stable, the probability ρ_0 is calculated as follows

$$\rho_0 = \frac{\lambda_0 \Pr[E_0 \neq 0]}{\mu_0} = \lambda_0 \frac{1}{P_E} \tag{5.5}$$

It is the product of the average data rate λ_0 by the expected time for a packet to be accepted by either the destination or any of the relays which equals $1/P_E$.

The data arrival rate of the relay i is the probability that a packet is accepted by the relay at any given time slot. It is the product of the channel occupation probability due to the source transmissions by the probability that the packet is accepted by the relay i. Let P_{Ri} be the probability that a transmitted packet is accepted by the relay i and is not accepted by neither the destination nor the relays with indices belongs to $\{1, 2, ...i-1\}$. Then, the value of λ_i is calculated as follows

$$\lambda_i = \rho_0 P_{Ri} = \lambda_0 \frac{P_{Ri}}{P_E}, i = 1, 2, ...N \tag{5.6}$$

The value of P_{Ri} is calculated as follows

$$P_{Ri} = r_i f_{0i}(1 - f_{0D}) \prod_{j=1}^{i-1} (1 - r_j f_{0j}) \tag{5.7}$$

The relay i data queue service rate is derived using the same way as in section 3.9 and its value is calculated as follows

$$\mu_i = f_{iD}(1 - \sum_{m=0}^{i-1} \rho_m) \Pr[E_i \neq 0], i = 1, 2, ...N \tag{5.8}$$

where ρ_i is the probability that the channel is occupied by the transmissions of the relay i with i belongs to $\{1, 2, ...N\}$. The probability ρ_i is calculated as follows

$$\rho_i = \frac{\lambda_i}{f_{iD}} \tag{5.9}$$

Also, the probabilities that the relays energy queues are not empty are calculated using similar steps of deriving equation (5.2). The transmission attempting rate of the relay i is $1 - \sum_{m=0}^{i-1} \rho_m$ which is the probability that the channel is idle for this node to transmit. Then, the probabilities that the relays energy queues are not empty are calculated as follows

$$\Pr[E_i \neq 0] = \frac{\min\left(q_i, 1 - \sum_{m=0}^{i-1} \rho_m\right)}{1 - \sum_{m=0}^{i-1} \rho_m}, i = 1, 2, ...N \tag{5.10}$$

The stability condition for the data queue of the relay i is that $\lambda_i < \mu_i$ and can be written as follows

$$\lambda_0 < \frac{P_E}{P_{Ri}} f_{iD} \min{(q_i, 1 - \sum_{m=0}^{i-1} \rho_m)}, i = 1, ..N \tag{5.11}$$

Note that the right hand side of the inequality is still a function in λ_0 as the probabilities ρ_i are functions in λ_0.

In order to simplify the optimization problem, we find a closed-form expression for the maximum achievable rate of the source. We start by calculating the variable γ_i which represents the service rate of the corresponding relay when it operates alone over the channel. The value of this service rate for the relay i is calculated as follows

$$\gamma_i = q_i f_{iD}, i = 1, 2, ...N \tag{5.12}$$

The service rate is the product of two terms. First, the probability of the relay energy queue to be non-empty which equals q_i. The second term is the average success probability of a packet transmitted from the relay to the destination. Also in this case, the proportion of time in which the channel is occupied by the transmissions of node i while this node operates alone over the channel is still ρ_i.

The stability conditions for the system are written as follows $\lambda_0 < \mu_0$, $\lambda_i < \gamma_i$ and $\sum_{i=0}^{N} \rho_i < 1$. By substituting using equations (5.4), (5.6), (5.12), (5.5) and (5.9) for μ_0, λ_i, γ_i, ρ_0 and ρ_i respectively, and by combining the stability conditions, we get the general expression for the system stability condition that $\lambda_0 < \hat{\lambda}_0(\vec{r})$ where

$$\hat{\lambda}_0(\vec{r}) = \min\left(\mu_0, \frac{P_E}{P_{R1}}\gamma_1, ..., \frac{P_E}{P_{Ri}}\gamma_i, ...\frac{P_E}{P_{RN}}\gamma_N, (\frac{1}{P_E} + \sum_{i=1}^{N} \frac{P_{Ri}}{P_E f_{iD}})^{-1}\right) \tag{5.13}$$

5.3.3 Maximization of the achievable rate over all $\vec{r}$

In this section, we discuss the problem of finding the optimal $\vec{r}$ to maximize $\hat{\lambda}_0(\vec{r})$. we derive a number of properties of the solution. Some of these properties can help in evaluating some components of the vector $\vec{r}$ directly. In deriving these properties, we use equation (5.13) which has been derived to simplify the expression of the maximum achievable throughput.

Property 1: if $\mu_0|_{\vec{r}=\vec{1}} < \frac{P_E}{P_{Ri}}\gamma_i|_{\vec{r}=\vec{1}}$ for $i = 1, ...N$ and $\mu_0|_{\vec{r}=\vec{1}} < (\frac{1}{P_E} + \sum_{i=1}^{N} \frac{P_{Ri}}{P_E f_{iD}})^{-1}|_{\vec{r}=\vec{1}}$ then it is throughput optimal that $\vec{r} = \vec{1}$ where $\vec{1}$ is the vector of all ones.

Proof. In this proof, we show the monotonicity of P_E. Then, we use this result in proving the property using contradiction.

We start by showing that P_E is a non-decreasing function in r_i for all $i = 1, 2, ...N$. The function P_E is continuous and differentiable with respect to r_i with r_i belongs to $[0, 1]$. The first derivative of P_E is calculated as follows

$$\frac{dP_E}{dr_i} = f_{0i}(1 - f_{0D}) \prod_{\substack{n=1 \\ n \neq i}}^{N} (1 - r_n f_{0n}) \tag{5.14}$$

This value is larger than or equal to zero and as a result, the function P_E is a non-decreasing function in r_i.

The property is then proved by contradiction. The maximum stable throughput when $\vec{r} = \vec{1}$ is denoted by $\hat{\lambda}_0(\vec{1})$. Assume that there exists $\vec{\tilde{r}} \neq \vec{1}$ which gives a maximum stable throughput $\hat{\lambda}_0(\vec{\tilde{r}})$ that $\hat{\lambda}_0(\vec{\tilde{r}}) > \hat{\lambda}_0(\vec{1})$.

We have shown that P_E is a non-decreasing function in each r_i. As a result, $\mu_0|_{\vec{r}=\vec{1}} \geq \mu_0|_{\vec{r}=\vec{\tilde{r}}}$. Assuming that the conditions of the property are satisfied, if $\hat{\lambda}_0(\vec{\tilde{r}}) > \hat{\lambda}_0(\vec{1})$, there must be at least one of the terms $(\frac{P_E}{P_{R1}}\gamma_1, ...\frac{P_E}{P_{Ri}}\gamma_i, ...\frac{P_E}{P_{RN}}\gamma_N \;\; , (\frac{1}{P_E}+ \sum_{i=1}^{N} \frac{P_{Ri}}{P_E f_{iD}})^{-1})|_{\vec{r}=\vec{1}}$ which has a value less than $\mu_0|_{\vec{r}=\vec{1}}$. This contradicts the hypothesis that $\mu_0|_{\vec{r}=\vec{1}} < \frac{P_E}{P_{Ri}}\gamma_i|_{\vec{r}=\vec{1}}$ for $i = 1, ...N$ and $\mu_0|_{\vec{r}=\vec{1}} < (\frac{1}{P_E} + \sum_{i=1}^{N} \frac{P_{Ri}}{P_E f_{iD}})^{-1}|_{\vec{r}=\vec{1}}$. □

Property 2: if $\sum_{i=0}^{N} q_i \leq 1$ then the throughput optimal $\vec{r}$ satisfies $\hat{\lambda}_0(\vec{r}) = \min(\mu_0, \frac{P_E}{P_{R1}}\gamma_1, ..., \frac{P_E}{P_{Ri}}\gamma_i, ...\frac{P_E}{P_{RN}}\gamma_N)$ and $r_i > 0$ for $i = 1, 2, ...N$.

Proof. When the system is stable, the value of $\lambda_i/\mu_i < 1$. Also, $\Pr[E_0 \neq 0] \leq q_0$ and $\min(q_i, 1 - \sum_{m=0}^{i-1} \rho_m) \leq q_i$. Then, we can show that

$$\sum_{i=0}^{N} \rho_i = \Pr[E_0 \neq 0]\frac{\lambda_0}{\mu_0} + \sum_{i=1}^{N} \min(q_i, 1 - \sum_{m=0}^{i-1} \rho_m)\frac{\lambda_i}{\mu_i}$$
$$\leq \sum_{i=0}^{N} q_i \frac{\lambda_i}{\mu_i} \leq \max \frac{\lambda_i}{\mu_i} \sum_{i=0}^{N} q_i < \sum_{i=0}^{N} q_i \quad (5.15)$$

As a result, $\sum_{i=0}^{N} \rho_i < 1$ for all system parameters and the condition for channel occupation is always satisfied. We can write the expression of the maximum achievable rate as $\hat{\lambda}_0(\vec{r}) = \min(\mu_0, \frac{P_E}{P_{R1}}\gamma_1, ..., \frac{P_E}{P_{Ri}}\gamma_i, ...\frac{P_E}{P_{RN}}\gamma_N)$.

We now prove the second result that no element of the vector $\vec{r}$ which maximizes the service rate of the source data queue can equal zero. We prove this result by contradiction. Without loss of generality, we will prove the result for r_1. we assume that the optimal vector $\vec{\hat{r}}$ has the component $\hat{r}_1 = 0$ and the the remaining components are denoted by $\hat{r}_{-1}$. Let the value of $\tilde{r}_1 = \delta$ which is an arbitrary small value and the remaining components of the vector $\vec{\tilde{r}}$ are the same as $\hat{r}_{-1}$.

we show that the function P_{Ri} for $i = 2, 3, ..N$ is a non-increasing function in r_1. We get the first derivative of the function with respect to r_1. The derivative is calculated as follows

$$\frac{dP_{Ri}}{dr_1} = -f_{01}(1 - f_{0D}) \prod_{j=2}^{i-1} (1 - r_j f_{0j}) \tag{5.16}$$

The derivative is less than or equal zero. The function is continuous and differentiable over the range of r_1. Then, the function is non-increasing in r_1.

From the above result and knowing that P_E is non-decreasing in r_1. Then, $P_E|_{r_1=\delta} > P_E|_{r_1=0}$ and $P_{Ri}|_{r_1=\delta} < P_{Ri}|_{r_1=0}$ for $i = 2, 3, ...N$. As a result, $\frac{P_E}{P_{Ri}}|_{r_1=\delta} > \frac{P_E}{P_{Ri}}|_{r_1=0}$ for $i = 2, 3, ...N$. Also, $\mu_0|_{r_1=\delta} > \mu_0|_{r_1=0}$. All the terms of the maximum achievable rate of the source data queue have increased by setting $r_1 = \delta$ except the term $\frac{P_E}{P_{R1}}\gamma_1$. This term equals infinity when $r_1 = 0$ so we can select δ small enough such that the term is not the minimum term. Then, the vector $\vec{\tilde{r}} = [\tilde{r_1} \;\; \hat{r}_{-1}]$ can give higher stable throughput rate than the vector $\vec{\hat{r}}$ which contradicts that $\vec{\hat{r}}$ is optimal. □

Property 3: if $(1 + \sum_{i=1}^{N} \frac{P_{Ri}q_i}{\gamma_i})|_{max} \leq \frac{1}{\Pr[E_0 \neq 0]}$ then then the throughput optimal $\vec{r}$ satisfies $\hat{\lambda}_0(\vec{r}) = \min(\mu_0, \frac{P_E}{P_{R1}}\gamma_1, ..., \frac{P_E}{P_{Ri}}\gamma_i, ...\frac{P_E}{P_{RN}}\gamma_N)$ and $r_i > 0$ for $i = 1, 2, ...N$.

Proof. Starting by the condition $(1 + \sum_{i=1}^{N} \frac{P_{Ri}q_i}{\gamma_i})|_{max} \leq \frac{1}{\Pr[E_0 \neq 0]}$, we multiply both sides by $\frac{\lambda_0}{P_E}$ to obtain

$$\sum_{i=0}^{i=N} \rho_i|_{max} \leq \frac{\lambda_0}{\mu_0} < 1 \tag{5.17}$$

The last inequality is satisfied when the source data queue is stable. If the condition is satisfied for the maximum value of $\sum_{i=0}^{i=N} \rho_i$, then it is satisfied over all value of $\vec{r}$. Then, the condition of the channel occupation is always satisfied. As a result, the maximum stable throughput can be written as $\hat{\lambda}_0(\vec{r}) = \min\left(\mu_0, \frac{P_E}{P_{R1}}\gamma_1, ...\frac{P_E}{P_{RN}}\gamma_N\right)$. The remaining steps of the proof are exactly the same as the the proof of the property 2. □

Property 4: If the throughput optimal vector $\vec{r}$ contains an element which equals 0 then the maximum stable throughput equals $(1 + \sum_{i=1}^{N} \frac{P_{Ri}q_i}{\gamma_i})^{-1}$ that the channel occupation equals 1.

Proof. We are going to prove the result using contradiction. Assume that the optimal vector $\vec{r}$ contains an element r_1 which equals 0 and the maximum stable throughput does not equal $(1 + \sum_{i=1}^{N} \frac{P_{Ri}q_i}{\gamma_i})^{-1}$. We selected r_1 without loss of generality. As a result, the maximum stable throughput equals $\min\left(\mu_0, \frac{P_E}{P_{R2}}\gamma_2, ..., \frac{P_E}{P_{Ri}}\gamma_i, ...\frac{P_E}{P_{RN}}\gamma_N\right)$ as the term $\frac{P_E}{P_{R1}}\gamma_1$ goes to infinity. We have shown that all the terms in the maximum stable throughput in this case are increasing in r_1. As a result, there exist some value of r_1 larger than zero that gives higher maximum stable throughput than the one obtained. That contradicts the fact that r_1 equals 0 and the vector $\vec{r}$ is throughput optimal. □

5.4 Energy-Efficient Partial Relaying

In this section, we consider the case in which the cost is defined to be the energy consumed in the network. Thus, we solve the energy consumption minimization problem. We denote the average energy consumption for the network by J_E. Also, the average energy consumed by a packet delivered directly to the destination from the source is denoted by J_{E_0}. The average energy consumed by a packet delivered to the destination by the relay i is denoted by J_{E_i}. The value of J_{E_i} includes both the energy consumed by the source for the packet to reach the relay and the energy consumed by the relay for the packet to reach the destination. The total energy consumption can be written as follows

$$J_E = (\lambda_0 - \sum_{i=1}^{N} \lambda_i) J_{E_0} + \sum_{i=1}^{N} \lambda_i J_{E_i} \tag{5.18}$$

It also can be written as follows

$$J_E = \lambda_0 J_{E_0} + \sum_{i=1}^{N} \lambda_i (J_{E_i} - J_{E_0}) \tag{5.19}$$

The expression of J_E is equivalent to the relaying cost J when $c_i = J_{E_i}$ for $i = 0, 1, 2, ...N$.

To calculate the values of the average consumed energy per packet and knowing that each packet transmission attempt consumes a single unit of energy, we calculate the average number of time slots needed for a packet to be received by the destination. We start by calculating the value of J_{E_0}.

The probability P_E is the probability of a packet to be received by any of the relays or the destination at any time slot when the source transmits. The number of the time slots till the reception of a packet has a geometric distribution with probability P_E. Thus, the expected number of the time slots needed for a packet to reach the destination or any of the relays equals $1/P_E$. Then, the value of J_{E_0} is calculated as follows

$$J_{E_0} = \frac{1}{P_E} \tag{5.20}$$

On the other hand, the expected number of the time slots for a packet to reach the destination through the relay i is the sum of the expected number of the time slots for the packet to reach the relay i from the source and the expected number of the time slots to reach the destination from the relay i. Thus, it is calculated as follows

$$J_{E_i} = \frac{1}{P_E} + \frac{1}{f_{iD}} \tag{5.21}$$

Then, the problem is written as follows

$$\min_{\vec{r}} \quad \lambda_0 \frac{1}{P_E} + \sum_{i=1}^{N} \lambda_i \frac{1}{f_{iD}}$$

$$\text{subject to} \quad \lambda_0 < \hat{\lambda}_0(\vec{r})$$

$$0 \leq r_i \leq 1, \quad \text{for } \; i = 1, 2, ...N$$

Then by substituting using the optimal relaying parameters r_i^*, the optimal energy consumption is

$$J_E^* = \lambda_0 \frac{1}{(1-(1-f_{0D})\prod_{i=1}^{N}(1-r_i^* f_{0i}))} \cdot \\ (1 + \sum_{i=1}^{N} \frac{r_i^* f_{0i}(1-f_{0D})\prod_{j=1}^{i-1}(1-r_j^* f_{0j})}{f_{iD}}) \tag{5.22}$$

5.5 Numerical Results

In this section, we show numerical results to illustrate the theoretical development shown in the previous discussion. We illustrate the effects of different system parameters on the maximum stable throughput of the source and the minimum energy consumed in the network. In the following results, we fix the following system parameters except otherwise mentioned: $f_{0i} = 0.3$, $f_{iD} = 0.3$, $f_{0D} = 0.2$ and $q_i = 0.2$ for $i = 1, 2, ..N$.

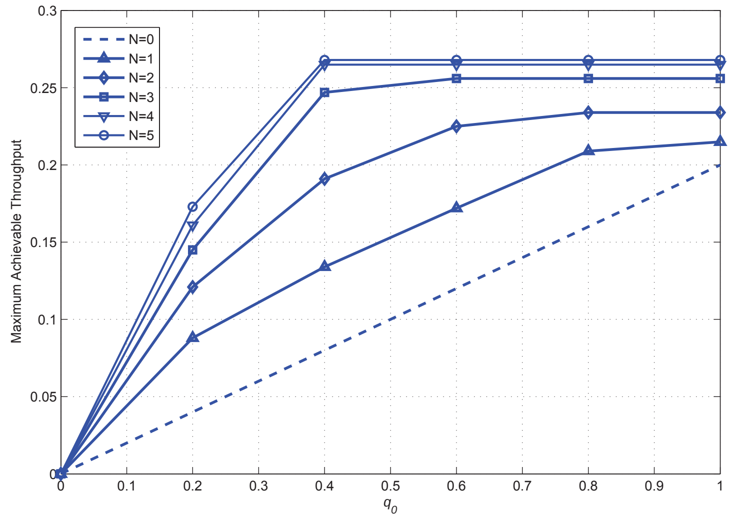

Figure 5.2: Maximum stable throughput against q_0 with different number of relays

In figure 5.2, we show the maximum achievable throughput against the energy harvesting rate at the source with different number of relay nodes. The figure shows the enhancement in the performance due to the use of cooperation in the network using optimal partial relaying. The improvement because of adding a single relay to the network is higher for lower number of relays. The throughput values are constant for large values of q_0 because of the fixed values of q_i that the relays can not accept more packets while the system remains stable. Hence, there is no enhancement in the performance with the increase of q_0.

In figure 5.3, we show the maximum achievable throughput against the number of relays with different values of energy harvesting rates at the relays. We set $q_0 = 0.1$. The figure shows the enhancement in the performance due to the use of

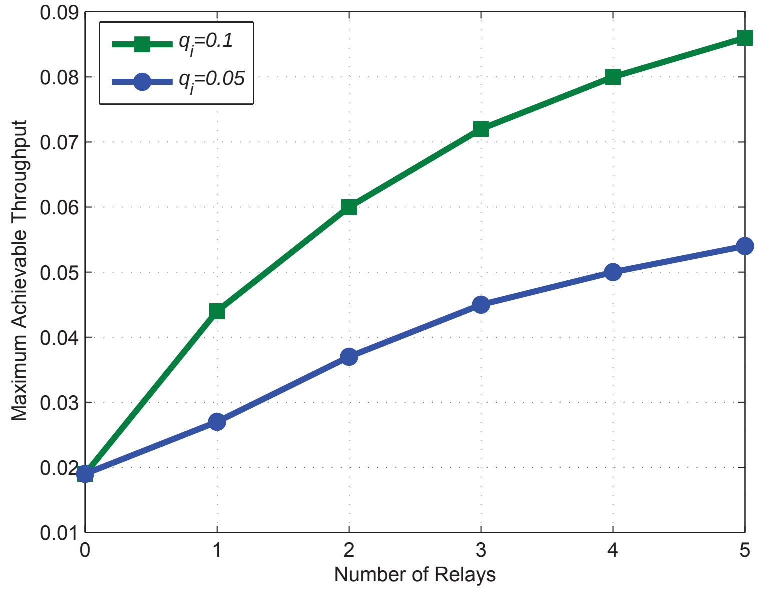

Figure 5.3: Maximum stable throughput against the number of relays with different values of q_i, $i = 1, 2, ...N$

cooperation in the network using optimal partial relaying. The slope of the curve with $q_i = 0.1$ is higher that the enhancement of the throughput is higher when using relays with higher energy harvesting rates.

In figure 5.4, we show the minimum consumed energy in the network against the average data arrival rate at the source with different number of relays. We set $q_0 = 0.3$. The curve for $N = 0$ is not complete as the system is not stable for $\lambda_0 \geq 0.06$. The figure shows the enhancement in the performance due to the use of cooperation in the network using optimal partial relaying. The enhancement due to the increase of a single relay is larger when the number of relays is small than the case of large number of relays.

In figure 5.5, we show the maximum stable throughput against q_0 with different

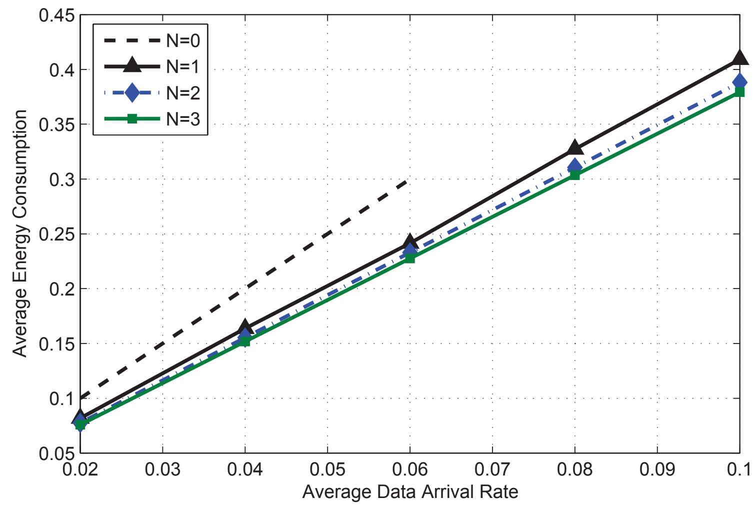

Figure 5.4: Minimum energy consumption against the source data arrival rate with different number of relays

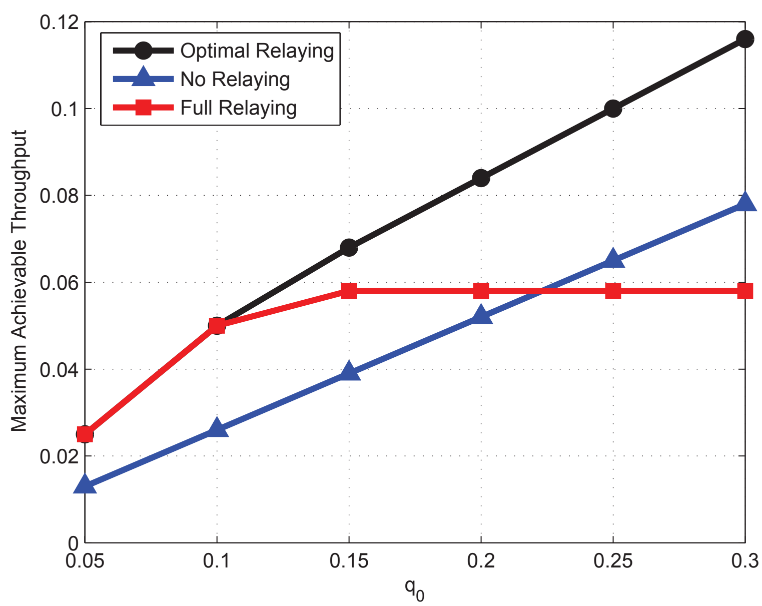

Figure 5.5: Maximum stable throughput against q_0 with partial relaying effect

techniques of relaying. We set $N = 2$, $f_{0D} = 0.2$, $f_{1D} = 0.3$, $f_{2D} = 0.2$, $f_{01} = 0.2$ and $f_{02} = 0.3$. The figure shows the enhancement of the performance because of using the optimal partial relaying in the network. At low values of q_0, it is throughput optimal to use full relaying for this parameters setting. This is true because the source at this case prefers to be helped by the relays as much as possible due to the limited availability of energy. Also at high values of q_0, the maximum achievable rate for the case of no relaying becomes higher than the maximum achievable rate for the case of full relaying. The case of full relaying is limited by the average harvesting rate for the source and the relays that increasing q_0 only can not enhance the performance over a certain limit while for the case of no relaying, the performance is enhanced directly by increasing the energy harvesting rate at the source.

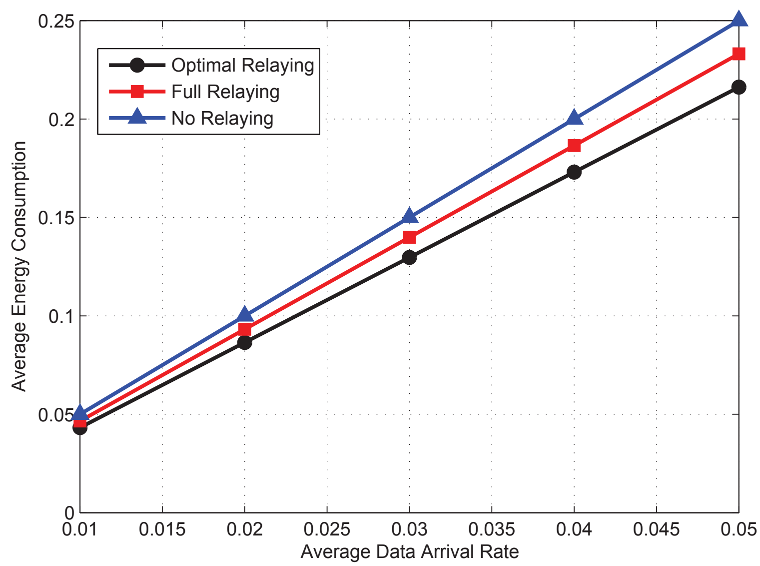

Figure 5.6: Minimum energy consumption against the source data arrival rate

In figure 5.6, we show the minimum consumed energy against the average data arrival rate at the source with different relaying techniques. In this figure, we use the same system parameters as in figure 5.5.

5.6 Discussion

In this chapter, we have investigated the problem of transmission control in a network with multiple energy harvesting relays. We have exploited partial relaying cooperation in the proposed network. We have derived the stability conditions for the source and the relays data queues. Our analysis shows that cooperation increases the maximum achievable rate of the source. We have discussed the problem of maximizing the achievable rate at the source data queue over the relaying parameters vector. Also, we have discussed the problem of relaying cost minimization. The problem is constrained by the stability of the system data queues. We have given an example for the cost to be the average energy consumed in the network. We have shown that partial relaying cooperation has equal or better performance than full relay cooperation.

Chapter 6: Transmission Scheduling of Two Sources over Time Varying Channels

6.1 Introduction

The problem of scheduling the transmissions over wireless channels that can cause interference to each other has been considered in a number of works as in [54]- [58]. In our work, we consider the effect of different levels of channel knowledge on the scheduling of source nodes transmissions.

The problem of transmission scheduling without perfect channel measurements was considered before in a number of works as in [59]- [62]. This problem is crucial because channel estimation usually uses a non-trivial amount of network resources that could otherwise be used for data transmission. In [59], the problem of opportunistic multiuser scheduling was considered in a downlink communication scenario. The scheduler estimates the channels by exploiting the memory of the channels by using the acknowledgment history of the network. In [60], the authors discussed a similar model to [59]. They obtained an inner and an outer bounds for the capacity region. In [61], a wireless downlink communication system was considered with limited sensing rate that the channels states can not be sensed every time slot.

The authors studied the trade-off between the throughput and the sensing rate in asymptotic sense. In [62], a downlink system was considered with the scheduler exploits the information about the channels from the acknowledgment history and the lengths of the queues corresponding to the destinations.

In this work, we show the optimal scheduling policies for the following cases: 1) Perfect channel measurements for all the channels; 2) Delayed channel information that is obtained from previous transmissions; 3) Infrequent channel measurements; 4) No channel measurements but only using the knowledge of steady state probabilities of the channels states; 5) Erroneous channel measurements with memory ; and 6) Distributed decisions where each source takes its decision depending on its knowledge about the channels . We consider the weighted sum-rate of the network as our performance criterion. Hence, we maximize the total amount of data transferred in the system with choosing a level of service for each user. A similar objective was considered before in several papers as in [63].

The belief vector is the vector of the probabilities of the channels being in certain states. When the channels are not measured perfectly, the belief vector is used by the scheduler to choose the optimal action with respect to the objective function. The belief vector value can be updated every time slot using both the channel characteristics and the new information obtained about the channels.

In the case of delayed channel information, the scheduler knows the states of the channels which have been used in previous transmissions. In this case, the information about the channels is the probabilities of the channels being in a certain state and thus the exact states of the channels are not known. Hence, we formulate the problem of finding the optimal policy as a POMDP [64] which is a controlling framework under which we deal with partially observable and stochastic environments. The optimal policy can be obtained using the value function iterations method which is computationally intensive even in a small problem with a small number of states and actions [65]. As a result, a suboptimal solution based on linear programming is studied. Authors of [66] describe linear programming approaches that can handle finite and infinite horizon problems for finite-state Markov decision problems. Also in [67], a grid based technique is introduced to approximate a POMDP to a finite-state Markovian decision problem. We can apply linear programming to the generated finite-state Markovian decision problem. We use the techniques in [66], [67] to find an approximate solution to the formulated POMDP.

In the case of infrequent channel measurements, the channels states are to be known at the scheduler periodically every fixed time interval. This technique could be used when the knowledge of all the system channels at all the users introduces a significant overhead. The effect of infrequent channel measurements was considered before in a number of works as [63], [68], [69] for different scenarios than the one considered in our work. In this work, we compare two decision making schemes when the channels are infrequently measured. First, we consider the case of infrequent decision making in which the action is taken directly after measuring the channels and this action remains fixed for the whole measurement interval. Also, we consider the case in which the belief of the channels is updated at each time slot during the measurement interval. Using the updated belief values, the action is taken every time slot.

In the case of imperfect channel knowledge, all the channels are measured with a certain error probability. The scheduler uses the erroneous channel measurements to update the belief vector about the channels. Thus, the belief vector takes into consideration all the history of the erroneous measurements. We derive the belief update function to update the value of the belief vector based on the current measurements.

In the case of distributed scheduling, each source takes its own decision depending on its own information about the channels. Distributed scheduling based on channel measurements was studied before in a number of works as in [70]- [72], [58]. In [70], a rate selection protocol is introduced in which the channel between the source and the destination is measured at the start of each transmission slot. Based on the measurements, a modulation technique is selected for transmission. In [71], a channel-aware transmission control protocol for a memoryless channel case is proposed such that random access probabilities vary based on the channels measurements. In [72], an opportunistic rate selection protocol is proposed in which the high quality channels are exploited via transmission of multiple back-to-back packets. In [58], the authors investigated channel-aware distributed scheduling, aiming to maximize the overall network throughput for a random access based ad hoc network under the physical interference model. In our work, each source exploits the information about both its direct channel to the corresponding destination and the interference channel to the other user's destination. The problem of finding the optimal transmission probabilities using the measured channels states, is formulated as a quadratic program [73]. The main advantage of formulating the problem as a quadratic program is the availability of computationally efficient algorithms for solving quadratic optimization problems [74].

The study of a simple model consisting of only two sources and two destinations is both instructive and necessary. It reveals insights at the conceptual level about exploiting channel characteristics on the performance of interfering sources. More work needs to be done to exploit the results of this work in more realistic systems. This work was presented in [75, 76].

6.2 System Model and Problem Formulation

We consider two transmit-receive pairs as shown in figure 6.1. We assume that time is slotted. During each time slot, each source can transmit a single data packet. We assume that each source has a saturated data queue such that there is always data to be transmitted at every time slot.

The channels are modeled by independent identically distributed two-state Markov chains (Gilbert Elliot model). State 1 corresponds to good connectivity, while state 0 corresponds to poor connectivity. The channel state between the source n and the destination m is denoted by c_{nm}. We denote the quadruple $(c_{11}, c_{12}, c_{22}, c_{21})$ by C. The transitions between states occur at the edges of the time slots. The transition probability from state 0 to state 1 is λ_0 and the transition probability from state 1 to state 1 is λ_1. Also, let $\pi_{nm}^{c_{nm}}$ denotes the steady state probability of the channel to be in state c_{nm}.

In the case of centralized scheduling and at the beginning of each time slot,

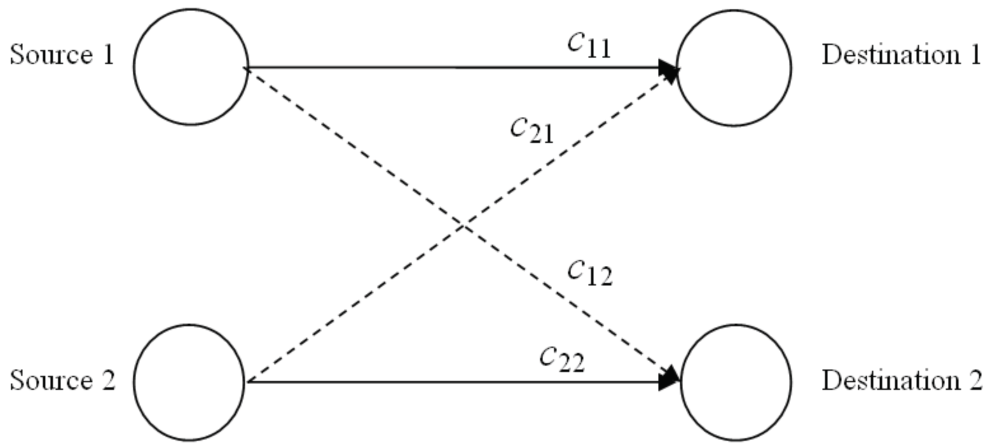

Figure 6.1: System Model

an action is chosen which is either the source 1 transmits a packet, the source 2 transmits a packet, or both sources transmit simultaneously. The action is chosen based on the channel state information (CSI). The action to let a single source n transmit is denoted by S_n and the action to let both sources transmit simultaneously is denoted by B.

The information about the channels is represented by the belief vector P which contains the quadruple $(p_{11}, p_{12}, p_{22}, p_{21})$. The element p_{nm} is the probability that the channel between the source n and the destination m is in state 1.

In the case of distributed scheduling and at the beginning of each time slot, each user selects either to transmit or not based on its own measurements. Each source estimates the channels over which this source transmits. In our work for the case of distributed scheduling, we consider only the case of perfect channel measurements.

If a single source n transmits and the channel c_{nn} is in state i, the probability that a packet is successfully decoded by the corresponding destination is denoted by $f_{n|n}^{(i)}$. Also, if both sources transmit simultaneously and the channels c_{nn} and c_{mn} $(m \neq n)$ are in states i and j respectively, the probability that a packet is successfully decoded by the destination n is denoted by $f_{n|1,2}^{(i,j)}$.

In the case of centralized scheduling, the policy u is the mapping from the belief vector P to an action A as follows

$$u : P \rightarrow A \in \{S_1, S_2, B\} \tag{6.1}$$

The objective is finding the optimal probabilities p_A^P of taking an action A while the belief vector is P to maximize the average weighted number of successful packets per time slot. Let $V^u(P)$ be the expected average reward with initial belief $P_0 = P$ and u be the policy followed. The expected average reward has the following expression

$$V^u(P) = \mathbb{E}^u \left[\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T} R(P_t, A_t) | P_0 = P \right] \tag{6.2}$$

where t is the time-slot index, A_t is the action taken at time t and P_t is the belief vector at time t. The term $R(P_t, A_t)$ denotes the expected reward when the belief is P_t and the action is A_t. At any time slot, the selected action is the one which maximizes this objective function with current belief equals P and the expectation over the policy is over all the following actions including the current action. The instantaneous expected reward is calculated as follows

$$R(P_t, A_t) = w_1 R_1(P_t, A_t) + w_2 R_2(P_t, A_t) \tag{6.3}$$

where

$$R_1(P, A) = \begin{cases} p_{11} f_{1|1}^{(1)} + (1 - p_{11}) f_{1|1}^{(0)} & \text{if} A = S_1 \\ 0 & \text{if} A = S_2 \\ p_{11} p_{21} f_{1|1,2}^{(1,1)} + p_{11}(1 - p_{21}) f_{1|1,2}^{(1,0)} & \\ +(1 - p_{11}) p_{21} f_{1|1,2}^{(0,1)} + (1 - p_{11})(1 - p_{21}) f_{1|1,2}^{(0,0)} & \text{if} A = B \end{cases}$$

$$R_2(P, A) = \begin{cases} 0 & \text{if} A = S_1 \\ p_{22} f_{2|2}^{(1)} + (1 - p_{22}) f_{2|2}^{(0)} & \text{if} A = S_2 \\ p_{22} p_{12} f_{2|1,2}^{(1,1)} + p_{22}(1 - p_{12}) f_{2|1,2}^{(1,0)} & \\ +(1 - p_{22}) p_{12} f_{2|1,2}^{(0,1)} + (1 - p_{22})(1 - p_{12}) f_{2|1,2}^{(0,0)} & \text{if} A = B \end{cases}$$

Then, the optimal objective function $V(P)$ is

$$V(P) = \max_u V^u(P) \text{for} P \in [0, 1]^4 \tag{6.4}$$

In the case of distributed scheduling, the objective is finding the optimal probabilities for each source to transmit based on its own channel measurements. The same objective function as the centralized case is considered.

6.3 Full Channel Knowledge

In this section, we consider the case in which all the channels are perfectly measured at each time slot. Then, the belief vector coincides with the true channel state vector. The belief vector in this case is $P = C = (c_{11}, c_{12}, c_{22}, c_{21})$. The expected reward under a policy u can be calculated as follows

$$
\begin{aligned}
V^u(P) = \mathbb{E}^u &\left[\lim_{T\to\infty} \frac{1}{T} \sum_{i=0}^{T} R(P_t, A_t) | P_0 = P \right] \\
&= \sum_C \sum_A \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T} R(C, A) I\{P_t = C, A_t = A\} \\
&= \sum_C \sum_A R(C, A) \Pr\{P_t = C, A_t = A\} = \sum_C \pi^C \sum_A R(C, A) p_A^C \quad (6.5)
\end{aligned}
$$

In these calculations, we use the independence between decisions in different slots. The outer summation is calculated over all the combinations the channels states vectors and the inner summation is calculated over all allowed actions. Also, π^C is the steady state probability of the channels states vector to be C. By the independence amongst the channels, we have

$$
\pi^C = \pi_{11}^{c_{11}} \pi_{12}^{c_{12}} \pi_{22}^{c_{22}} \pi_{21}^{c_{21}} \tag{6.6}
$$

By substituting equations (6.3) and (6.6) into equation (6.5),

$$
V^u(P) = \sum_C \pi_{11}^{c_{11}} \pi_{12}^{c_{12}} \pi_{22}^{c_{22}} \pi_{21}^{c_{21}} \sum_A p_A^C (w_1 R_1(C, A) + w_2 R_2(C, A)) \tag{6.7}
$$

Each term in the outer summation is positive. Hence, to get the optimal values of p_A^C, we need to maximize each term in the outer summation which is corresponding to a certain system state.

Let us define $J^C(A)$ to be the expected weighted reward when the system is in a certain state C and the action A is chosen i.e. it equals $w_1 R_1(C, A) + w_2 R_2(C, A)$. Then, we can write the objective function as follows

$$
V^u(P) = \sum_C \pi_{11}^{c_{11}} \pi_{12}^{c_{12}} \pi_{22}^{c_{22}} \pi_{21}^{c_{21}} (p_{S_1}^C J^C(S_1) + p_{S_2}^C J^C(S_2) + p_B^C J^C(B)) \tag{6.8}
$$

where $J^C(S_1) = w_1 q_{1|1}^{(c_{11})}$, $J^C(S_2) = w_2 q_{2|2}^{(c_{22})}$ and $J^C(B) = w_1 q_{1|1,2}^{(c_{11},c_{21})} + w_2 q_{2|1,2}^{(c_{22},c_{12})}$. Then, to maximize the term $(p_{S_1}^C J^C(S_1) + p_{S_2}^C J^C(S_2) + p_B^C J^C(B))$, we assign to the action with the highest reward a probability which equals 1. As a result, the optimal action is

$$A^*(C) = \underset{A \in \{S_1, S_2, B\}}{\text{argmax}} \; (J^C(A)) \tag{6.9}$$

6.4 Delayed Channel Knowledge

In this section, we consider the case in which partial information about the channels is available. The available information is the states of the channels which have been used in the previous transmissions. This case represents the situation in which the channels are estimated by the destinations and then fedback to the scheduler to be used in the following transmissions. This problem can be formulated as an infinite horizon average reward POMDP. We need to define the following components: 1) the states; 2) the observation; 3) the actions; 4) the transition probabilities; 5) the instantaneous rewards; and 6) the observation function indicating the relation between the actions and the states with the corresponding observation. Now, we start to define each component with respect to our problem:

1. The state space is the set that contains all the combinations of the channels states. Each state is represented by a channel vector C that is $(c_{11}, c_{12}, c_{22}, c_{21})$.

2. The observation is the feedback from the destinations about the channels that are used in the previous time slot. The channels which are not used in the previous time slot are not observed.

3. The actions are the same actions defined in section 6.2 which are S_1, S_2 and B.

4. The transition probability between two state vectors $C^{(1)}$ and $C^{(2)}$ is the product of the probabilities $\Pr[c_{ij}^{(2)}|c_{ij}^{(1)}]$ for all i and $j \in \{1, 2\}$.

5. The expected rewards are the weighted sum of the probabilities of successfully decoding packets by both destinations. The expression for the expected reward as a function of the belief vector P is found in equation (6.3).

6. The observation function is the updating function of the belief vector. The belief vector elements $p_{nm}(k)$ at time slot k are calculated as follows

$$p_{11}(k+1) = \begin{cases} c_{11}(k)\lambda_1 + (1 - c_{11}(k))\lambda_0 \ \text{if} A(k) = S_1 \text{or} B \\ p_{11}(k)\lambda_1 + (1 - p_{11}(k))\lambda_0 \ \text{if} A(k) = S_2 \end{cases} \tag{6.10}$$

$$p_{12}(k+1) = \begin{cases} c_{12}(k)\lambda_1 + (1 - c_{12}(k))\lambda_0 \ \text{if} A(k) = S_1 \text{or} B \\ p_{12}(k)\lambda_1 + (1 - p_{12}(k))\lambda_0 \ \text{if} A(k) = S_2 \end{cases} \tag{6.11}$$

$$p_{22}(k+1) = \begin{cases} p_{22}(k)\lambda_1 + (1 - p_{22}(k))\lambda_0 \ \text{if} A(k) = S_1 \\ c_{22}(k)\lambda_1 + (1 - c_{22}(k))\lambda_0 \ \text{if} A(k) = S_2 \text{or} B \end{cases} \tag{6.12}$$

$$p_{21}(k+1) = \begin{cases} p_{21}(k)\lambda_1 + (1 - p_{21}(k))\lambda_0 \ \text{if} A(k) = S_1 \\ c_{21}(k)\lambda_1 + (1 - c_{21}(k))\lambda_0 \ \text{if} A(k) = S_2 \text{or} B \end{cases} \tag{6.13}$$

In [77], the author shows that there exists a stationary policy which is optimal for solving POMDP with the average reward criterion under two conditions: 1) the immediate rewards $R(P, A)$ are non-negative; and 2) The corresponding Markov chain is irreducible and ergodic. From the definition of our problem, both conditions are satisfied. As a result, the optimal probabilities p_A^P are functions in the belief vector only and not in time. The problem can be solved using value function iterations but this technique is computationally intensive. The problem is then approximated by discretizing the belief vector components.

In order to approximate the problem of getting the optimal action with delayed channel information, each component of vector P is discretized to have one value from $(N+1)$ values. Each element of P takes a value between λ_0 and λ_1. Each of the discretized belief values for the channel from the source n to the destination m takes one of the following values

$$d_{nm} = \lambda_0 + (\lambda_1 - \lambda_0)(k/N), k = 0, 1, 2, ...N \tag{6.14}$$

Using the discrete states, the problem is a Markov decision problem. The transition probability for each channel from a state with the index k to a state with the index l by choosing the action A is denoted as $p_{nm}(l|k; A)$ where $k, l \in \{0, 1, 2, ...N\}$. This probability has the following expression

$$p_{nm}(l|k; A) = \begin{cases} \lambda_0 + (\lambda_1 - \lambda_0)\frac{k}{N} & \text{if} \quad l = N, A = S_n \text{or} B \\ 1 - \lambda_0 - (\lambda_1 - \lambda_0)\frac{k}{N} & \text{if} \quad l = 0, A = S_n \text{or} B \\ 1 & \text{if} \quad l = ||k\lambda_1 + (N-k)\lambda_0||, A = S_{3-n} \\ 0 & \text{otherwise} \end{cases} \tag{6.15}$$

The function $||\bullet||$ represents rounding to the nearest integer for the argument of the function. The state space of the discretized problem is denoted by Z. Any state D which belongs to Z is $(d_{11}^D, d_{12}^D, d_{22}^D, d_{21}^D)$. The transition probability from the state D to the state F by choosing the action A is denoted as $p_{DF}(A)$. Also, the probability $p_{DF}(A)$ equals the product of $p_{nm}(l|k;A)$ for all n,m where k is the discrete index corresponding to d_{nm}^D and l is corresponding to d_{nm}^F. Using the linear programming approach as in [66], we can write the linear program to find the optimal policy as follows

$$\max_{x_{DA}} \sum_{D \in Z} \sum_{A \in \{S_1, S_2, B\}} R(D, A) x_{DA}$$

subject to

$$\sum_{A \in \{S_1, S_2, B\}} x_{FA} - \sum_{D \in Z} \sum_{A \in \{S_1, S_2, B\}} p_{DF}(A) x_{DA} = 0, F \in Z$$
$$\sum_{D \in Z} \sum_{A \in \{S_1, S_2, B\}} x_{DA} = 1$$
$$x_{DA} \leq 0, D \in Z, A \in \{S_1, S_2, B\}$$

Let us denote the optimal values of x_{DA} by x_{DA}^*. It was shown in [66] that the problem has a randomized decision rule, therefore the optimal probability for a certain action A is

$$p_A^D = \frac{x_{DA}^*}{\sum_{A \in \{S_1, S_2, B\}} x_{DA}^*}, D \in Z \tag{6.16}$$

The optimal probabilities resulting from this linear programming approach are evaluated based on channel characteristics before the sources start transmitting. Then, these probabilities are used simply in the transmission process.

6.5 No Channel Knowledge

In this section, we consider the case in which there are no channel measurements. Then, the belief vector is fixed and equals the steady state probabilities of the channels to be in state 1. The belief vector in this case is $P = \Pi = (\pi^1_{11}, \pi^1_{12}, \pi^1_{22}, \pi^1_{21})$. The expected reward under a policy u can be calculated as follows

$$V^u(P) = \sum_A R(\Pi, A) p^\Pi_A \tag{6.17}$$

By substituting equation (6.3) into equation (6.17),

$$V^u(P) = \sum_A p^\Pi_A (w_1 R_1(\Pi, A) + w_2 R_2(\Pi, A)) \tag{6.18}$$

Let us define $J^\Pi(A)$ to be the expected weighted reward at the belief Π given that the action A is chosen i.e. it equals $w_1 R_1(\Pi, A) + w_2 R_2(\Pi, A)$. Then, we can write the objective function as follows

$$V^u(P) = p^\Pi_{S_1} J^\Pi(S_1) + p^\Pi_{S_2} J^\Pi(S_2) + p^\Pi_B J^\Pi(B) \tag{6.19}$$

where

$$J^\Pi(S_1) = w_1(\pi^1_{11} f^{(1)}_{1|1} + (1 - \pi^1_{11}) f^{(0)}_{1|1})$$

$$J^\Pi(S_2) = w_2(\pi^1_{22} f^{(1)}_{2|2} + (1 - \pi^1_{22}) f^{(0)}_{2|2})$$

$$J^\Pi(B) = w_1(\pi^1_{11}\pi^1_{21} f^{(1,1)}_{1|1,2} + \pi^1_{11}(1-\pi^1_{21}) f^{(1,0)}_{1|1,2} + (1-\pi^1_{11})\pi^1_{21} f^{(0,1)}_{1|1,2} + (1-\pi^1_{11})(1-\pi^1_{21}) f^{(0,0)}_{1|1,2})$$
$$+ w_2(\pi^1_{22}\pi^1_{12} f^{(1,1)}_{2|1,2} + \pi^1_{22}(1 - \pi^1_{12}) f^{(1,0)}_{2|1,2} + (1 - \pi^1_{22})\pi^1_{12} f^{(0,1)}_{2|1,2} + (1 - \pi^1_{22})(1 - \pi^1_{12}) f^{(0,0)}_{2|1,2})$$

Then, to maximize the objective function, we assign to the action with the highest reward a probability which equals 1. As a result, the optimal policy can be written as follows

$$A^* = \underset{A\in\{S_1,S_2,B\}}{\text{argmax}} \left(J^{\Pi}(A)\right) \tag{6.20}$$

From previous analysis, when there are no channel measurements at any time slot, it is optimal to maximize the steady state expected reward of the system.

6.6 Infrequent Channel Knowledge

In this section, we consider the case in which the channels are measured every τ time slots where $\tau \geq 1$. If the channels are measured every time slot, the existence of multiple channels may introduce significant overhead. To minimize the effect of this overhead, infrequent channel measurements technique is exploited. Two schemes are introduced which are: 1) action is selected every τ slots, and 2) action is selected every single slot depending on the characteristics of the channels and the belief values of the channels which are updated based on the measured values of the channels.

Let the time slots be grouped into intervals of length τ. Thus the $(k+1)$th interval consists of slots $k\tau, ..(k+1)\tau - 1$. Although the channels conditions may change every time slot, the channels are measured only at the beginning of each interval. Thus, the interval τ represents the duration between successive measuring instances of the channels. Each channel is modeled by a two-state Markov chain that has a transition matrix, Λ, and the transition matrix is written as follows

$$\Lambda = \begin{bmatrix} 1-\lambda_0 & \lambda_0 \\ 1-\lambda_1 & \lambda_1 \end{bmatrix} \tag{6.21}$$

The eigenvalues of the transition matrix are 1 and $\lambda_1 - \lambda_0$. The n-step transition matrix is calculated as follows

$$\Lambda^{(n)} = \frac{1}{1 + \lambda_0 - \lambda_1}(\Lambda - (\lambda_1 - \lambda_0)I - (\lambda_1 - \lambda_0)^n(\Lambda - I)) \tag{6.22}$$

where I is the 2x2 identity matrix. Substituting by the value of the matrix Λ, we calculate the values of the elements of the matrix $\Lambda^{(n)}$ which represent the n-step transition probabilities as follows

$$\Lambda^{(n)} = \frac{1}{1 + \lambda_0 - \lambda_1}\begin{bmatrix} 1 - \lambda_1 + \lambda_0(\lambda_1 - \lambda_0)^n & \lambda_0(1 - (\lambda_1 - \lambda_0)^n) \\ (1 - \lambda_1)(1 - (\lambda_1 - \lambda_0)^n) & \lambda_0 + (1 - \lambda_1)(\lambda_1 - \lambda_0)^n \end{bmatrix} \tag{6.23}$$

We denote the transition probability of a channel from state 0 to state 1 in n steps by $\lambda_0^{(n)}$ and the transition probability of a channel from state 1 to state 1 in n steps by $\lambda_1^{(n)}$.

6.6.1 Action is selected every τ slots

In this scheduling scheme, the channels are measured at the beginning of each measurement interval. An action which belongs to the set $\{S_1, S_2, B\}$ is selected at the beginning of the interval and continues for the whole interval. The decision for each interval depends only on the measurements at the beginning of this interval. Thus, the action which is selected for each interval is independent of time and hence the policy is stationary.

The belief vector at the time slot $k\tau + n$ is calculated using the measured channels states at the time slot $k\tau$ and the n-step transition probabilities where

$k = 0, 1, 2, ...$ and $n = 0, 1, ...\tau - 1$. The belief vector is calculated as follows

$$P(k\tau + n) = (\lambda^{(n)}_{c_{11}(k\tau)}, \lambda^{(n)}_{c_{12}(k\tau)}, \lambda^{(n)}_{c_{22}(k\tau)}, \lambda^{(n)}_{c_{21}(k\tau)}) \tag{6.24}$$

The expected reward under a policy u can be calculated as follows

$$\begin{aligned} V^u(P) &= \mathbb{E}^u[\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T} R(P_t, A_t)|P_0 = P] \\ &= \mathbb{E}^u[\lim_{K\to\infty} \frac{1}{K\tau} \sum_{k=0}^{K} \sum_{n=0}^{\tau-1} R(P_{k\tau+n}, A_{k\tau+n})|P_0 = P] \end{aligned} \tag{6.25}$$

$$V^u(P) = \mathbb{E}^A[\sum_{n=0}^{\tau-1} R(P_n, A)] + \mathbb{E}^u[\lim_{K\to\infty} \frac{1}{K\tau} \sum_{k=1}^{K} \sum_{n=0}^{\tau-1} R(P_{k\tau+n}, A_{k\tau+n})|P_0 = P] \tag{6.26}$$

All the actions and the beliefs at the following intervals are independent from the action A at the current time slot. Then, the maximization of the objective function is equivalent to the maximization of $\mathbb{E}^A[\sum_{n=0}^{\tau-1} R(P_t, A)]$. In this policy, the action is fixed in each interval of length τ. Also, the average reward of an interval of length τ is denoted by $\overline{R_\tau}(C, A)$. It is calculated as follows

$$\overline{R_\tau}(C(k\tau), A) = \frac{1}{\tau} \sum_{n=0}^{\tau-1} (w_1 R_1(P(k\tau + n), A) + w_2 R_2(P(k\tau + n), A)) \tag{6.27}$$

Let $p^C_{A,\tau}$ is the probability of taking the action A for an interval at which the channels states vector has the value C at the beginning of the interval. Hence, to get the optimal values of $p^C_{A,\tau}$, we need to maximize

$$\mathbb{E}^A[\sum_{n=0}^{\tau-1} R(P_n, A)] = p^C_{S_1,\tau} \overline{R_\tau}(C, S_1) + p^C_{S_2,\tau} \overline{R_\tau}(C, S_2) + p^C_{B,\tau} \overline{R_\tau}(C, B) \tag{6.28}$$

Then, we assign to the action with the highest reward a probability which equals 1. As a result, the optimal action for an interval of length τ is

$$A^*_\tau(C) = \underset{A\in\{S_1,S_2,B\}}{\operatorname{argmax}} (\overline{R_\tau}(C, A)) \tag{6.29}$$

6.6.2 Action is selected every time slot

In this scheduling scheme, the channels are measured at the beginning of each interval. Then, an action belonging to the set $\{S_1, S_2, B\}$ is selected at every time slot depending on the updated belief values. The belief updating process at each time slot may lead to different optimal actions at different time slots within the same measurement interval. Thus, the optimal value of the objective function in this case is always larger than or equal to the value of the objective function in the case of fixed action in the whole measurement interval.

The selected action is the action that maximizes the expected instantaneous reward for the system as a function of the belief vector. The belief values are updated as shown in equation (6.24). The optimal action, in the time slot at which the belief vector is P, is obtained as follows

$$A^*(P) = \underset{A \in \{S_1, S_2, B\}}{\text{argmax}} \left(J^P(A)\right) \tag{6.30}$$

where

$$J^P(S_1) = w_1(p_{11} f_{1|1}^{(1)} + (1 - p_{11}) f_{1|1}^{(0)})$$

$$J^P(S_2) = w_2(p_{22} f_{2|2}^{(1)} + (1 - p_{22}) f_{2|2}^{(0)})$$

$$\begin{aligned} J^P(B) = w_1(&p_{11}p_{21}f_{1|1,2}^{(1,1)} + p_{11}(1-p_{21})f_{1|1,2}^{(1,0)} \\ &+ (1-p_{11})p_{21}f_{1|1,2}^{(0,1)} + (1-p_{11})(1-p_{21})f_{1|1,2}^{(0,0)}) + w_2(p_{22}p_{12}f_{2|1,2}^{(1,1)} + p_{22}(1-p_{12})f_{2|1,2}^{(1,0)} \\ &+ (1-p_{22})p_{12}f_{2|1,2}^{(0,1)} + (1-p_{22})(1-p_{12})f_{2|1,2}^{(0,0)}) \end{aligned}$$

6.7 Erroneous Channel Knowledge

In this section, we consider the case in which all the channels are measured at each time slot and the measurements of the channels are imperfect. The action is selected based on the whole history of the previous measurements. We use the previous erroneous channel measurements in forming the belief vector. This vector contains the probabilities of channels to be in state 1. The current measurements are then used to update the belief values. Let us denote the probability of error in measuring the channel between the source n and the destination m by $p_{nm}^{(\epsilon)}$ and the measured state of the channel by $\hat{c}_{nm}$. The quadruple of all the measured channels states is denoted by $\hat{C}$.

6.7.1 The Belief Vector

In this subsection, we derive the update equation of the belief vector components. We start by using basic probability formulas that set the relation between the belief vector value at a certain time slot with its value at the previous one. Then, we show that the pair of the true state of each channel and the measured state of the same channel follows a Markov chain. The transition probabilities of the Markov chain of the channels are used in the update equation of the belief vector.

The belief vector elements are the probabilities of the channels being in state 1 given that the measured channel state is $\hat{c}_{nm}$ and the probability of the channel being in state 1 in the previous time slot is known which is denoted by $\Pr[c_{nm}^{(-1)} = 1|\hat{c}_{nm}^{(-1)}, H]$ where H is the history of all previous measurements. The belief is the probability

$\Pr[c_{nm} = 1|\hat{c}_{nm}, \hat{c}_{nm}^{(-1)}, H]$ and it is calculated using bayes' rule as follows

$$\Pr[c_{nm} = 1|\hat{c}_{nm}, \hat{c}_{nm}^{(-1)}, H] = \frac{\Pr[c_{nm} = 1, \hat{c}_{nm}|\hat{c}_{nm}^{(-1)}, H]}{\Pr[\hat{c}_{nm}|\hat{c}_{nm}^{(-1)}, H]} \tag{6.31}$$

The numerator of the equation can be calculated as follows

$$\begin{aligned}
&\Pr[c_{nm} = 1, \hat{c}_{nm}|\hat{c}_{nm}^{(-1)}, H] = \\
&\qquad \Pr[c_{nm}^{(-1)} = 1|\hat{c}_{nm}^{(-1)}, H]\Pr[c_{nm} = 1, \hat{c}_{nm}|c_{nm}^{(-1)} = 1, \hat{c}_{nm}^{(-1)}, H] \\
&\qquad + (1 - \Pr[c_{nm}^{(-1)} = 1|\hat{c}_{nm}^{(-1)}, H])\Pr[c_{nm} = 1, \hat{c}_{nm}|c_{nm}^{(-1)} = 0, \hat{c}_{nm}^{(-1)}, H]
\end{aligned} \tag{6.32}$$

where $\Pr[c_{nm}^{(-1)} = 1|\hat{c}_{nm}^{(-1)}, H]$ is the previous belief value which is to be updated.

On the other hand, we get the value of the denominator of equation (6.31) as follows

$$\begin{aligned}
\Pr[\hat{c}_{nm}|\hat{c}_{nm}^{(-1)}, H] &= \Pr[c_{nm}^{(-1)} = 1|\hat{c}_{nm}^{(-1)}, H]\Pr[\hat{c}_{nm}|c_{nm}^{(-1)} = 1, \hat{c}_{nm}^{(-1)}, H] \\
&\quad + (1 - \Pr[c_{nm}^{(-1)} = 1|\hat{c}_{nm}^{(-1)}, H])\Pr[\hat{c}_{nm}|c_{nm}^{(-1)} = 0, \hat{c}_{nm}^{(-1)}, H]
\end{aligned} \tag{6.33}$$

To calculate the value of $\Pr[\hat{c}_{nm}|c_{nm}^{(-1)}, \hat{c}_{nm}^{(-1)}, H]$ for both the cases when $c_{nm}^{(-1)} = 0, 1$, we use the following formula

$$\begin{aligned}
&\Pr[\hat{c}_{nm}|c_{nm}^{(-1)}, \hat{c}_{nm}^{(-1)}, H] = \\
&\qquad \Pr[\hat{c}_{nm}, c_{nm} = 1|c_{nm}^{(-1)}, \hat{c}_{nm}^{(-1)}, H] + \Pr[\hat{c}_{nm}, c_{nm} = 0|c_{nm}^{(-1)}, \hat{c}_{nm}^{(-1)}, H]
\end{aligned} \tag{6.34}$$

To calculate $\Pr[\hat{c}_{nm}, c_{nm}|c_{nm}^{(-1)}, \hat{c}_{nm}^{(-1)}, H]$ in both equations (6.32), (6.34), we show that the pair $(c_{nm}, \hat{c}_{nm})$ follows a first order Markov chain. The probability of the current state given all the previous states is calculated as follows

$$\begin{aligned}
\Pr[c_{nm}, \hat{c}_{nm}|c_{nm}^{(-1)}, \hat{c}_{nm}^{(-1)}, c_{nm}^{(-2)}, \hat{c}_{nm}^{(-2)}, ...] &= \Pr[c_{nm}, \hat{c}_{nm}|c_{nm}^{(-1)}] \\
&= \Pr[\hat{c}_{nm}|c_{nm}, c_{nm}^{(-1)}]\Pr[c_{nm}|c_{nm}^{(-1)}]
\end{aligned} \tag{6.35}$$

where $c_{nm}^{(-2)}, \hat{c}_{nm}^{(-2)}$ are the channel state and the measured channel state at two time slots before the current time slot. The first equality comes from the fact that the channels are first order Markov chains and the error in measurement depends only on the current channel state. Also, $\Pr[c_{nm}, \hat{c}_{nm}|c_{nm}^{(-1)}]$ in the first equality is equivalent to $\Pr[c_{nm}, \hat{c}_{nm}|c_{nm}^{(-1)}, \hat{c}_{nm}^{(-1)}]$ that we prove that the pair $(c_{nm}, \hat{c}_{nm})$ follows a Markov chain. The transition probabilities of the chain are shown in figure 6.2.

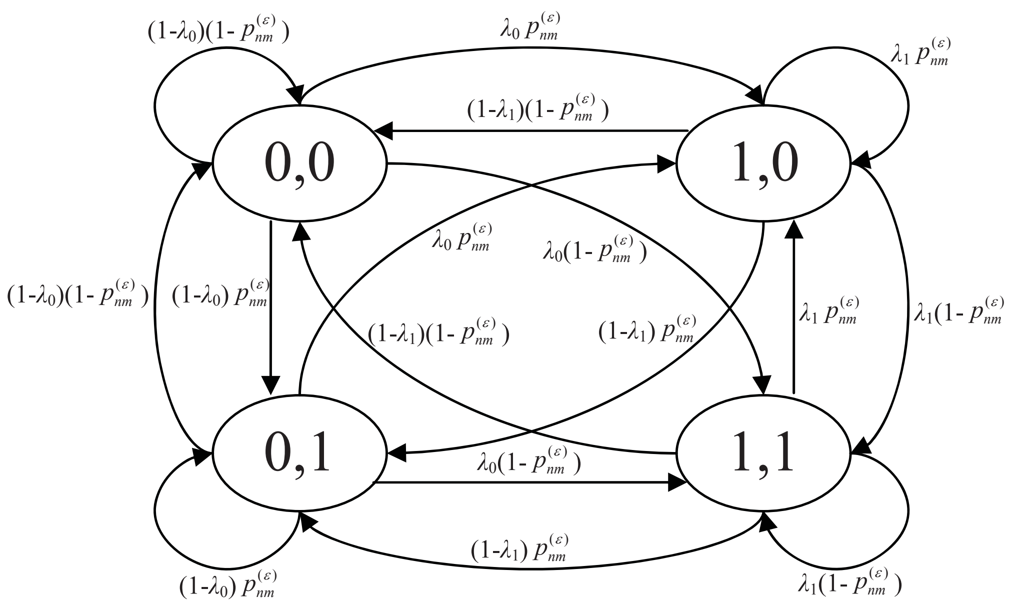

Figure 6.2: The Markov Chain

Then, the belief vector in this case is $P = \hat{P} = (\hat{p}_{11}, \hat{p}_{12}, \hat{p}_{22}, \hat{p}_{21})$. The updating function of the belief vector with the belief vector elements $p_{nm}(k)$ at time slot k is

$$\hat{p}_{nm}(k+1) = \frac{N(\hat{p}_{nm}(k))}{D(\hat{p}_{nm}(k))} \tag{6.36}$$

where

$$N(\hat{p}_{nm}(k)) = \hat{p}_{nm}(k)\Pr[c_{nm} = 1, \hat{c}_{nm}|c_{nm}^{(-1)} = 1, \hat{c}_{nm}^{(-1)}]$$
$$+ (1 - \hat{p}_{nm}(k))\Pr[c_{nm} = 1, \hat{c}_{nm}|c_{nm}^{(-1)} = 0, \hat{c}_{nm}^{(-1)}] \quad (6.37)$$

$$D(\hat{p}_{nm}(k)) = \hat{p}_{nm}(k)(\Pr[c_{nm} = 1, \hat{c}_{nm}|c_{nm}^{(-1)} = 1, \hat{c}_{nm}^{(-1)}]$$
$$+ \Pr[c_{nm} = 0, \hat{c}_{nm}|c_{nm}^{(-1)} = 1, \hat{c}_{nm}^{(-1)}]) + (1 - \hat{p}_{nm}(k))(\Pr[c_{nm} = 1, \hat{c}_{nm}|c_{nm}^{(-1)} = 0, \hat{c}_{nm}^{(-1)}]$$
$$+ \Pr[c_{nm} = 0, \hat{c}_{nm}|c_{nm}^{(-1)} = 0, \hat{c}_{nm}^{(-1)}]) \quad (6.38)$$

6.7.2 The Problem Formulation

The actions in different time slots are independent. Thus, the maximization of the objective function is equivalent to the maximization of $\mathbb{E}^A[R(\hat{P}, A)]$.

Let us define $J^{\hat{P}}(A)$ to be the expected weighted reward when the belief is $\hat{P}$ given the action A is chosen i.e. it equals $w_1 R_1(\hat{P}, A) + w_2 R_2(\hat{P}, A)$. Then, we can write

$$\mathbb{E}^A[R(\hat{P}, A)] = p_{S_1}^{\hat{P}} J^{\hat{P}}(S_1) + p_{S_2}^{\hat{P}} J^{\hat{P}}(S_2) + p_B^{\hat{P}} J^{\hat{P}}(B) \quad (6.39)$$

where

$$J^{\hat{P}}(S_1) = w_1(\hat{p}_{11}^1 f_{1|1}^{(1)} + (1 - \hat{p}_{11}^1) f_{1|1}^{(0)})$$

$$J^{\hat{P}}(S_2) = w_2(\hat{p}_{22}^1 f_{2|2}^{(1)} + (1 - \hat{p}_{22}^1) f_{2|2}^{(0)})$$

$$J^{\hat{P}}(B) = w_1(\hat{p}_{11}^1 \hat{p}_{21}^1 f_{1|1,2}^{(1,1)} + \hat{p}_{11}^1 (1 - \hat{p}_{21}^1) f_{1|1,2}^{(1,0)}$$
$$+ (1 - \hat{p}_{11}^1)\hat{p}_{21}^1 f_{1|1,2}^{(0,1)} + (1 - \hat{p}_{11}^1)(1 - \hat{p}_{21}^1) f_{1|1,2}^{(0,0)}) + w_2(\hat{p}_{22}^1 \hat{p}_{12}^1 f_{2|1,2}^{(1,1)} + \hat{p}_{22}^1 (1 - \hat{p}_{12}^1) f_{2|1,2}^{(1,0)}$$
$$+ (1 - \hat{p}_{22}^1)\hat{p}_{12}^1 f_{2|1,2}^{(0,1)} + (1 - \hat{p}_{22}^1)(1 - \hat{p}_{12}^1) f_{2|1,2}^{(0,0)})$$

Then, to maximize each term in the summation, we maximize the following term $(p_{S_1}^{\hat{P}} J^{\hat{P}}(S_1) + p_{S_2}^{\hat{P}} J^{\hat{P}}(S_2) + p_B^{\hat{P}} J^{\hat{P}}(B))$ by giving to the action with the highest reward a probability which equals 1. As a result, the optimal policy can be written as follows

$$A^*(\hat{P}) = \underset{A \in \{S_1, S_2, B\}}{\text{argmax}} \; (J^{\hat{P}}(A)) \tag{6.40}$$

6.8 Distributed Scheduling

In this section, we consider the case of distributed scheduling in which each source takes its own decision. Each source takes its decision based on its observations about the channels over which this source transmits. This case represents the scenario where there is no information passing between the sources. Information passing can be exploited to enhance the system performance but the complexity increases with the increase of the number of sources in the network.

At the beginning of each time slot, each source chooses either to transmit or not based on the available CSI. Each source has the knowledge of the exact states of the channels over which it can transmit. The source i transmits with probability $m^{(i)}_{c_{ii},c_{ij}}$, $i, j = 1, 2$ and $i \neq j$ depending on the measurements of the channels c_{ii} and c_{ij}.

The expected average reward under a policy u can be written as follows

$$V^u(P) = \sum_C \pi^C [m^{(1)}_{c_{11},c_{12}}(1 - m^{(2)}_{c_{22},c_{21}}) R(C, S_1) + \\ (1 - m^{(1)}_{c_{11},c_{12}}) m^{(2)}_{c_{22},c_{21}} R(C, S_2) + m^{(1)}_{c_{11},c_{12}} m^{(2)}_{c_{22},c_{21}} R(C, B)] \tag{6.41}$$

We denote the vector of the transmission probabilities by M and it is defined as follows

$$M = \begin{bmatrix} m_{00}^{(1)} & m_{01}^{(1)} & m_{10}^{(1)} & m_{11}^{(1)} & m_{00}^{(2)} & m_{01}^{(2)} & m_{10}^{(2)} & m_{11}^{(2)} \end{bmatrix}^T \tag{6.42}$$

where $(.)^T$ denotes the transpose of the vector.

We can write the expected average reward in the matrix form as follows

$$V^u(P) = M^T G M + D^T M \tag{6.43}$$

where the matrix G and the vector D are to be defined. For simplicity of terms, we define

$$H(C) = \pi^C (R(C, B) - R(C, S_1) - R(C, S_2)) \tag{6.44}$$

The matrix G is defined as follows

$$G = \begin{bmatrix} 0_4 & G_1 \\ G_1^T & 0_4 \end{bmatrix} \tag{6.45}$$

where 0_4 is the 4x4 matrix of zeros and G_1 is a 4x4 matrix defined as follows

$$G_1 = \begin{bmatrix} H(0,0,0,0) & H(0,0,0,1) & H(0,0,1,0) & H(0,0,1,1) \\ H(0,1,0,0) & H(0,1,0,1) & H(0,1,1,0) & H(0,1,1,1) \\ H(1,0,0,0) & H(1,0,0,1) & H(1,0,1,0) & H(1,0,1,1) \\ H(1,1,0,0) & H(1,1,0,1) & H(1,1,1,0) & H(1,1,1,1) \end{bmatrix} \tag{6.46}$$

Also, the vector D is defined as follows

$$D = \begin{bmatrix} \sum_{c_{22},c_{21}} \pi_{22}^{c_{22}} \pi_{21}^{c_{21}} R((0,0,c_{22},c_{21}),S_1) \\ \sum_{c_{22},c_{21}} \pi_{22}^{c_{22}} \pi_{21}^{c_{21}} R((0,1,c_{22},c_{21}),S_1) \\ \sum_{c_{22},c_{21}} \pi_{22}^{c_{22}} \pi_{21}^{c_{21}} R((1,0,c_{22},c_{21}),S_1) \\ \sum_{c_{22},c_{21}} \pi_{22}^{c_{22}} \pi_{21}^{c_{21}} R((1,1,c_{22},c_{21}),S_1) \\ \sum_{c_{11},c_{12}} \pi_{11}^{c_{11}} \pi_{12}^{c_{12}} R((c_{11},c_{12},0,0),S_2) \\ \sum_{c_{11},c_{12}} \pi_{11}^{c_{11}} \pi_{12}^{c_{12}} R((c_{11},c_{12},0,1),S_2) \\ \sum_{c_{11},c_{12}} \pi_{11}^{c_{11}} \pi_{12}^{c_{12}} R((c_{11},c_{12},1,0),S_2) \\ \sum_{c_{11},c_{12}} \pi_{11}^{c_{11}} \pi_{12}^{c_{12}} R((c_{11},c_{12},1,1),S_2) \end{bmatrix} \tag{6.47}$$

To calculate the optimal values of transmission probabilities, we solve the following quadratic programming problem

$$\begin{aligned} &\max_{M} M^T G M + D^T M \\ &\text{s.t. } 0 \leq M_{ij} \leq 1, 1 \leq i,j \leq 8 \end{aligned} \tag{6.48}$$

6.9 Numerical Results

In this section, we present some numerical examples to illustrate the previous development. We focus on comparing the performance of the different scheduling strategies in terms of the weighted sum rate of the two sources per time slot. We compare the optimal policy in which the channel measurements are perfectly available, the optimal policy using partial channels information, the optimal policy when there are no channel measurements, the optimal policy with erroneous channel measurements, the optimal policy with infrequent measurements and decisions, the optimal policy with infrequent measurements only but the decisions are taken every time slot and the optimal policy in distributed manner. We denote these policies by "Full CSI", "Partial CSI", "No CSI", "Erroneous CSI", "Infrequent Decision", "Infrequent Measurement" and "Distributed" repectively.

We set the system parameters as follows: $\lambda_0 = 0.4$, $\lambda_1 = 0.7$, $f^{(1)}_{n|n} = 0.4$, $f^{(0)}_{n|n} = 0.25$, $f^{(1,0)}_{n|1,2} = 0.35$, $f^{(1,1)}_{n|1,2} = 0.2$, $f^{(0,1)}_{n|1,2} = 0.1$, $f^{(0,0)}_{n|1,2} = 0.2$ with $(n = 1, 2)$, $w_1 = w_2 = 0.5$ and $p^{(\epsilon)}_{11} = p^{(\epsilon)}_{12} = p^{(\epsilon)}_{21} = p^{(\epsilon)}_{22} = p^{(\epsilon)} = 0.1$. Then, we start to change these system parameters to study their effects on the system performance. All the figures show the enhancement in the throughput because of exploiting different levels of CSI knowledge in the scheduling process.

In figure 6.3, the performance is shown against the channel transition probability λ_1. Measuring all system channels, even with small errors in measurement, leads to higher throughput than the case of no CSI available at the scheduler. Also, in case of partial CSI, the enhancement due to partial channel knowledge is larger as λ_1 becomes larger. That is because the channel correlation becomes larger which leads to that the channel measurements are more effective in the channel prediction and the control strategy.

In figure 6.4, the performance against the change in the weighting factors is shown. The x-axis represents w_1 and we set $w_2 = 1 - w_1$. For very small values of w_1, the optimal action for all the scheduling policies is S_2 in all time slots so all policies have the same throughput. Then for larger values of w_1, the enhancement in performance due to the CSI knowledge is shown.

In figure 6.5, we show the system performance against the change in the error

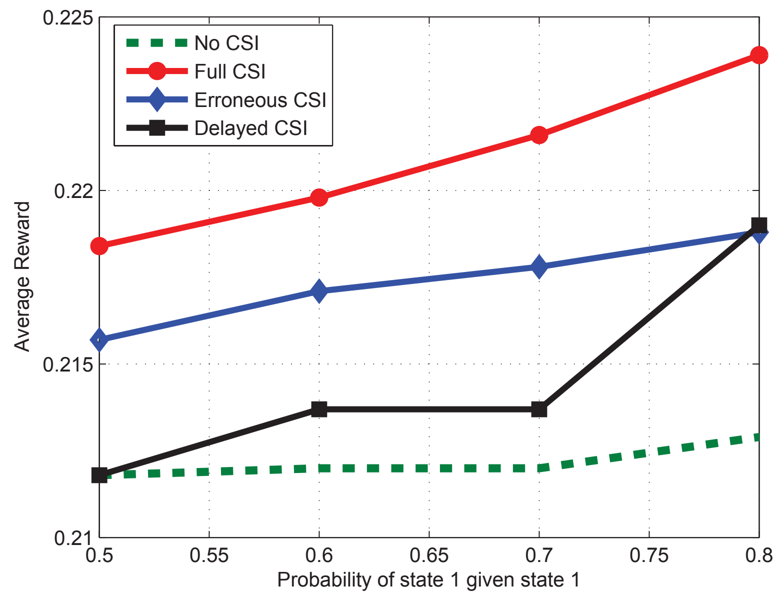

Figure 6.3: Throughput against λ_1

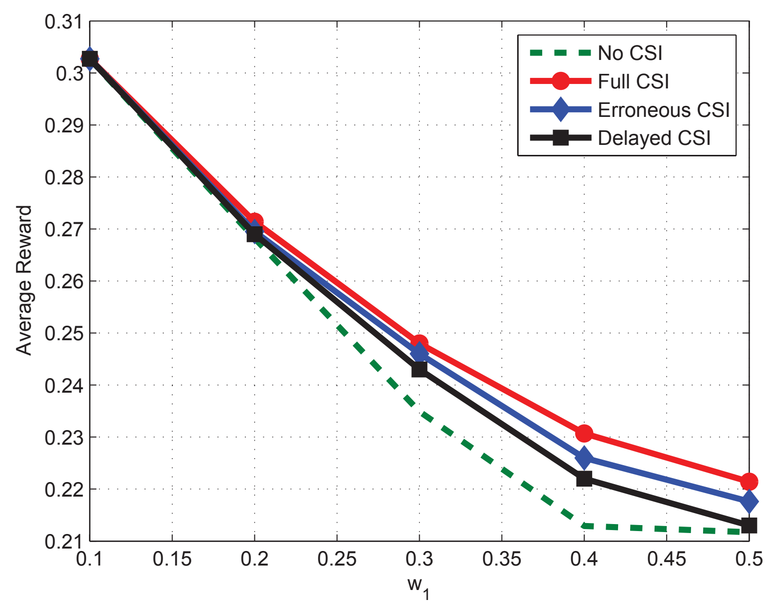

Figure 6.4: Throughput against w_1

probability of the channel measurements. It shows that measuring the channels with errors can have better performance than not measuring the channels for a certain range of error probability. In this system setting, as the probability of error is less than 0.3, it is better to measure the channel than to use the steady state probabilities of the channels.

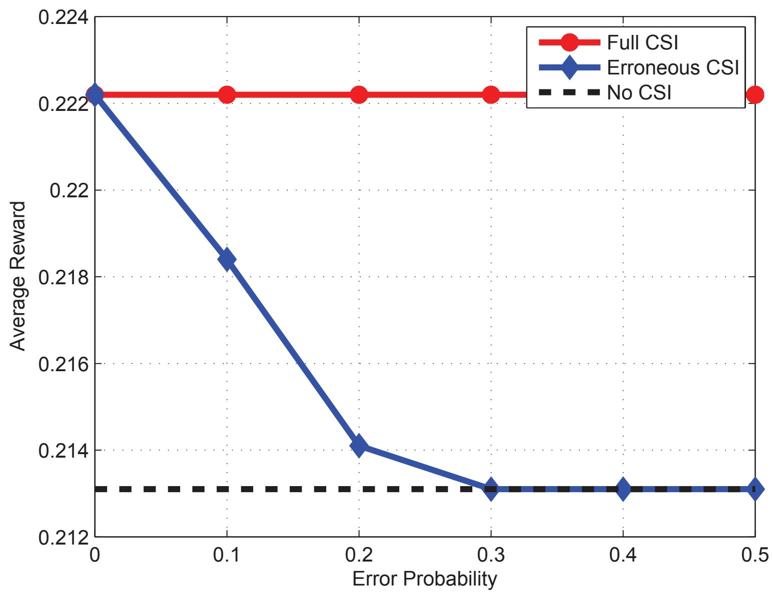

Figure 6.5: Throughput against $p^{(\epsilon)}$

In figure 6.6, the performance against the measurement interval τ is shown. The performance for the case of making decision every time slot is better than the case of infrequent decision. The difference between the two cases becomes larger with the increase of τ.

In figure 6.7, we mainly illustrate the performance of the distributed scheduling against the centralized scheduling with full CSI and with no CSI. The amount of channel information in the distributed scheduling case is less than the amount of information in the centralized scheduling case with full CSI that the performance of the distributed case is worse than the performance of the centralized case with full

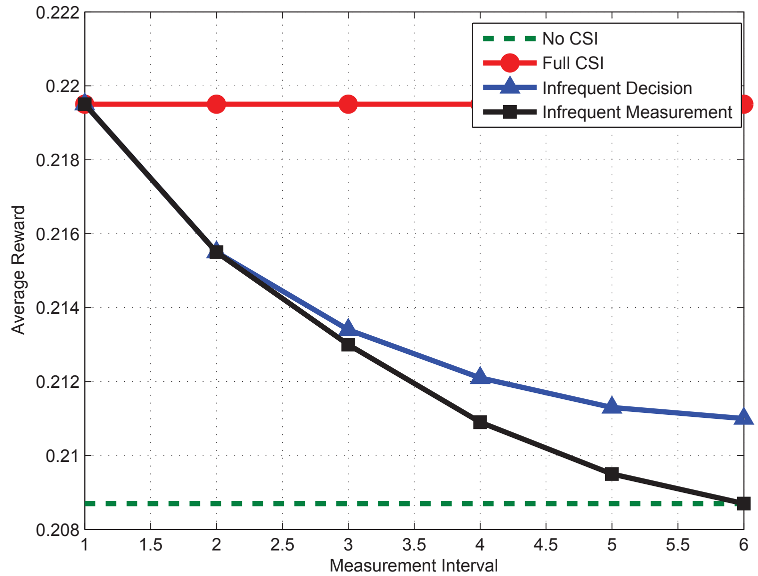

Figure 6.6: Throughput against measurement interval (τ)

CSI.

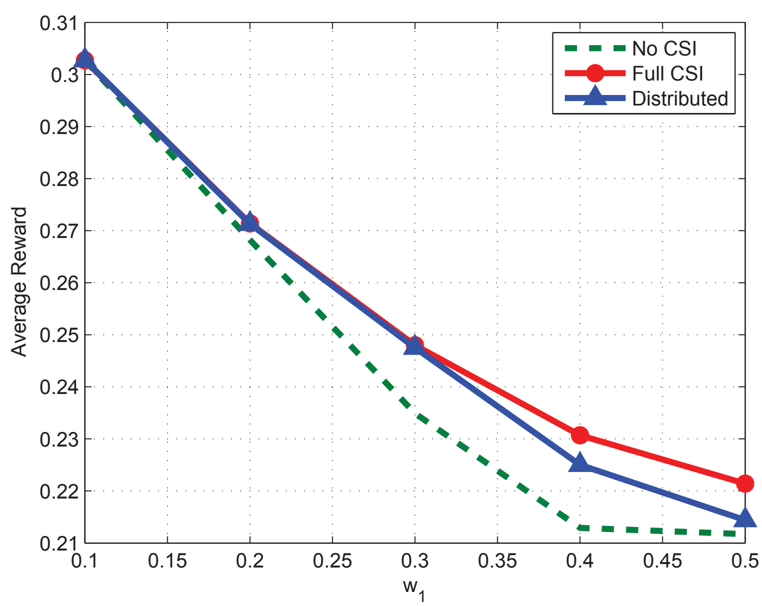

Figure 6.7: Throughput against w_1

6.10 Discussion

In this chapter, we have derived the optimal scheduling policies for a communication system that contains two transmitter-receiver pairs which operate over Gilbert-Elliot channels. We have considered exploiting CSI of the system channels in the scheduling policies. Due to the difficulty of the analysis of the problem, we traced the solutions in the case of two pairs only. In the case of full channel knowledge, we have shown that it is optimal to maximize the instantaneous expected reward of the system. Then, the problem of delayed channel information has been formulated as a Partially Observable Markovian Decision Problem for which we have found an approximate solution using linear programming. Also, for the case in which no channel measurements are available, it is optimal to select a fixed action that maximizes the steady state expected reward of the system. Then, we calculated the system expected reward as a function of the error probability when the channel measurements are inaccurate. In this case, it is also optimal to maximize the instantaneous expected reward of the system. In the case of infrequent channel measurements, we have shown the effects of changing the measurement interval length on the performance and we have shown that taking a decision every time slot can lead to better performance than taking a decision every measurement interval. We have also considered the scheduling in a distributed manner. We have formulated this scheduling problem as a quadratic program. We compared the throughput performance for all these cases and assessed the value of different levels of channel state information knowledge.

Chapter 7: Transmission Control in Cognitive Radio Networks

7.1 Introduction

Rapidly rising energy costs have led to an emerging trend of addressing energy efficiency aspect of wireless communication technologies [78]. In a typical wireless cellular network, the radio access part accounts for most of the total energy consumption [79]. Therefore, increasing the energy efficiency of radio networks is very important to meet the challenges raised by the high demands of traffic and energy consumption. As a result, energy efficient communications have recently attracted more research effort [80]. Reducing energy consumption is very important in order to reduce the impact from wireless networks on the environment. It is also important because mobile terminals have batteries with limited energy supply.

Cognitive radio technology can play an important role in improving energy efficiency in wireless networks [81]. The cognitive abilities have a wide range of properties, including spectrum sensing [82], spectrum sharing [83] and adaptive transmission [84], which are beneficial to improve the trade-off among energy efficiency, spectrum efficiency, bandwidth, and deployment efficiency in wireless networks [79]. Also, some works have been done to consider energy efficiency in cognitive radio networks. In [85], the authors have studied the hierarchy in energy games for cognitive radio networks. Authors of [86] have studied the distributed power control game to maximize the transmission energy-efficiency for secondary users in cognitive radio networks. Energy-efficient power control and receiver design in cognitive radio networks have been studied in [87].

In this work, we consider a system which contains one primary and one secondary source-destination communications pairs. The secondary source senses the primary activity with certain missed detection and false alarm probabilities and it has also knowledge about the steady state probability of the channel being busy by the primary source transmissions. The secondary source estimates the channel from the primary source to have knowledge about the reliability of the sensing decision. The channel is estimated opportunistically when the channel is sensed to be busy. The enhancement in the performance due to the channel knowledge is studied. We consider the consumed energy by the secondary system as the performance criterion and the system is constrained by the a maximum allowable probability of failure for the primary system and a minimum required average throughput for the secondary system. We compare the performance of the system with opportunistic channel estimation to the benchmarks of the system with no channel estimation and with accurate channel estimation at every time slot.

7.2 System Model and Problem Formulation

7.2.1 System Model

We consider a simple cognitive radio network with a single primary source-destination pair and a single secondary source-destination pair as shown in figure 7.1. We assume that time is slotted. During each time slot, each source can transmit a single data packet. We assume that the steady state probability of the channel to be busy by the primary source transmissions is denoted by π_v.

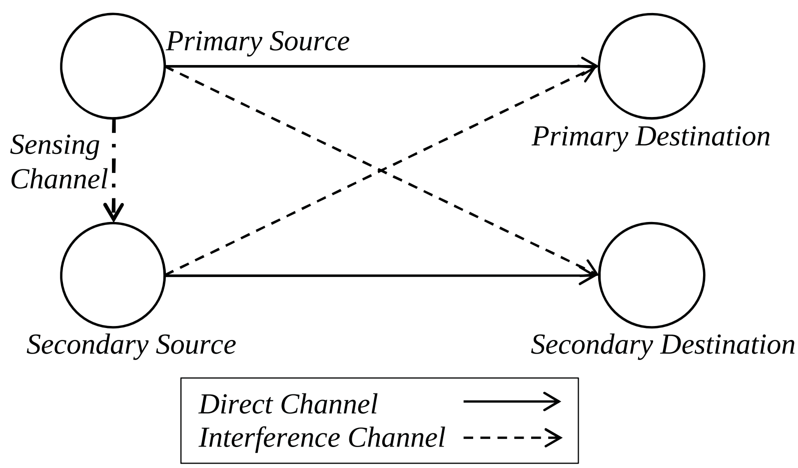

Figure 7.1: System Model

The sensing-channel is modeled by a two-state Markov chain, "Gilbert-Elliot" Model. The state of the channel is denoted by C and it belongs to $\{0, 1\}$. The transition probability from state 1 to state 1 is denoted by λ_1 and the transition probability from state 0 to state 1 is denoted by λ_0. The probability of missed detection when the channel state is C is denoted by $p_m(C)$. The probability of false alarm when the channel state is C is denoted by $p_f(C)$. The steady state probability of the channel to be in state 1 is denoted by π_C. The belief about the channel state is denoted by p and it defined to be the probability that the channel is in state 1.

All channels are assumed to be independent erasure channels. Also, each of the primary and the secondary destinations can decode the transmitted packet from the corresponding source when both sources transmit with some probability. The success probability of a transmitted packet from the secondary source when the secondary source transmits alone is denoted by $f_{S|S}$ and when both sources transmit is denoted by $f_{S|P,S}$. The success probability of a transmitted packet from the primary source when the primary source transmits alone is denoted by $f_{P|P}$ and when both sources transmit is denoted by $f_{P|P,S}$.

The secondary source senses the primary activity every time slot. The state of the sensed primary activity is denoted by $\hat{v}$ and it belongs to $\{0, 1\}$. If the sensing result is that the channel is busy, the channel from the primary source to the secondary source is estimated. We assume that the channel state C is estimated accurately if the primary activity is correctly sensed. Otherwise; it is estimated accurately with probability 1/2. If the sensing result is that the primary source is idle, the channel state is not estimated. At every time slot, the secondary source transmits with a probability which depends on the sensing decision and the belief about the channel. This probability is denoted by $\rho(\hat{v}, p)$.

We compare the results to the cases of no channel estimation. In this case, the average probability of missed detection is denoted by $\overline{p_m}$ and the average probability of false alarm is denoted by $\overline{p_f}$. Also, we compare to the case of estimating the channel accurately at every time slot.

7.2.2 Problem Formulation

The goal of the problem is to design an energy efficient transmission strategy by selecting the transmission probabilities $\rho(\hat{v}, p)$ to minimize the average energy consumed subject an allowable failure probability for the primary system and a required average success rate for the secondary system. The average energy consumed by the secondary system is calculated as follows

$$\overline{E} = \int_0^1 \sum_{\hat{v}=0}^{1} f(\hat{v}, p)\rho(\hat{v}, p)dp \tag{7.1}$$

where $f(\hat{v}, p)$ is the joint probability distribution of $\hat{v}$ and p.

The average probability of failure for the primary system is calculated as

$$\overline{Q} = \int_0^1 \sum_{\hat{v}=0}^{1} f(\hat{v}, p)\Pr(v|\hat{v})(\rho(\hat{v}, p)(1 - f_{P|P,S}) + (1 - \rho(\hat{v}, p))(1 - f_{P|P}))dp \tag{7.2}$$

where $\Pr(v|\hat{v})$ is the conditional probability that the source is transmitting given that its activity was sensed to be at state $\hat{v}$.

The average probability of success for the secondary system is calculated as

$$\overline{R} = \int_0^1 \sum_{\hat{v}=0}^{1} f(\hat{v}, p)\rho(\hat{v}, p)(\Pr(v|\hat{v})f_{S|P,S} + (1 - \Pr(v|\hat{v}))f_{S|S})dp \tag{7.3}$$

Thus, the problem is formulated as follows

$$\min_{\rho(\hat{v},p)} \overline{E}$$

$$\text{s.t.} \ \ \overline{R} \geq \lambda$$

$$\overline{Q} \leq \delta$$

$$0 \leq \rho(\hat{v}, p) \leq 1, \forall p, \hat{v}$$

where λ is the required average success rate for the secondary system and δ is the allowable average failure probability for the primary system.

7.3 No Channel Estimation

In this case, the channel from the primary source to the secondary source is not estimated at all. The belief about the channel is fixed and equal to π_C. Then, the problem is simplified such that the transmission probabilities are $\rho^{(NC)}(\hat{v})$ which depends only on the sensing output. The average energy consumed can be then written as follows

$$\overline{E} = \pi_v[(1-\overline{p_m})\rho^{(NC)}(1) + \overline{p_m}\rho^{(NC)}(0)] + (1-\pi_v)[\overline{p_f}\rho^{(NC)}(1) + (1-\overline{p_f})\rho^{(NC)}(0)] \quad (7.4)$$

The average probability of failure for the primary system can be rewritten as follows

$$\overline{Q} = \pi_v(1-\overline{p_m})[\rho^{(NC)}(1)(1-f_{P|P,S}) + (1-\rho^{(NC)}(1))(1-f_{P|P})] + \pi_v\overline{p_m}[\rho^{(NC)}(0)(1-f_{P|P,S}) + (1-\rho^{(NC)}(0))(1-f_{P|P})] \quad (7.5)$$

The average probability of success for the secondary system can be rewritten as follows

$$\overline{R} = \pi_v[(1-\overline{p_m})\rho^{(NC)}(1) + \overline{p_m}\rho^{(NC)}(0)]f_{S|P,S} + (1-\pi_v)[\overline{p_f}\rho^{(NC)}(1) + (1-\overline{p_f})\rho^{(NC)}(0)]f_{S|S} \quad (7.6)$$

We start by discussing the feasibility conditions of the problem. The problem is feasible when there exist transmission probabilities for which the average success probability of the secondary system is achieved with the failure probability of the primary system is less than or equal to the maximum allowable failure probability. In order to get the maximum allowable data arrival rate for the feasibility of the problem, we solve the following problem

$$\max_{\rho^{(NC)}(1),\rho^{(NC)}(0)} \overline{R}$$

$$\text{s.t.} \qquad \overline{Q} \le \delta$$

$$0 \le \rho^{(NC)}(1), \rho^{(NC)}(0) \le 1$$

This problem is a linear knapsack problem for the two variables. The optimal probabilities for this problem are denoted by $\hat{\rho}^{(NC)}(1)$ and $\hat{\rho}^{(NC)}(0)$. These probabilities are calculated as follows based on the following conditions [88].

If $\overline{p_m} + \overline{p_f} \ge 1$, then

$$\hat{\rho}^{(NC)}(1) = \min\left(1, \frac{\delta - \pi_v(1-\overline{p_m})(1-f_{P|P})}{\pi_v(1-\overline{p_m})(f_{P|P} - f_{P|P,S})}\right)$$

$$\hat{\rho}^{(NC)}(0) = \min\left(1, \max\left(0, \frac{\delta - \pi_v((1-\overline{p_m})(1-f_{P|P,S}) + \overline{p_m}(1-f_{P|P}))}{\pi_v\overline{p_m}(f_{P|P} - f_{P|P,S})}\right)\right)$$

If $\overline{p_m} + \overline{p_f} \le 1$, then

$$\hat{\rho}^{(NC)}(0) = \min\left(1, \frac{\delta - \pi_v\overline{p_m}(1-f_{P|P})}{\pi_v\overline{p_m}(f_{P|P} - f_{P|P,S})}\right)$$

$$\hat{\rho}^{(NC)}(1) = \min\left(1, \max\left(0, \frac{\delta - \pi_v(\overline{p_m}(1-f_{P|P,S}) + (1-\overline{p_m})(1-f_{P|P}))}{\pi_v(1-\overline{p_m})(f_{P|P} - f_{P|P,S})}\right)\right)$$

In both cases, the maximum achievable average success probability for the secondary system is calculated as follows

$$\hat{\overline{R}} = \pi_v[(1-\overline{p_m})\hat{\rho}^{(NC)}(1)f_{S|P,S} + \overline{p_m}\hat{\rho}^{(NC)}(0)f_{S|P,S}] + (1-\pi_v)[\overline{p_f}\hat{\rho}^{(NC)}(1)f_{S|S} + (1-\overline{p_f})\hat{\rho}^{(NC)}(0)f_{S|S}] \quad (7.7)$$

From the previous result, if $\lambda > \hat{\overline{R}}$, then the problem is not feasible and has no solution. On the other hand if $\lambda \leq \hat{\overline{R}}$, there exist a solution for the energy minimization problem.

Then, we start discussing the problem of finding the optimal transmission probabilities to minimize the average energy which is consumed by the secondary system. We start by setting two assumptions under which we will be able to find a closed-form solution for the problem.

Assumption 7.1: $\overline{p_m} \leq \frac{1}{2}$ and $\pi_v(1-2\overline{p_m}) \geq (1-\pi_v)(1-2\overline{p_f})$.

Assumption 7.2: $\overline{p_m} \geq \frac{1}{2}$ and $\pi_v(1-2\overline{p_m}) \leq (1-\pi_v)(1-2\overline{p_f})$.

proposition 7.1: If either assumption 7.1 or assumption 7.2 is satisfied, then the energy minimization problem can be written as follows

$$\min_{\rho^{(NC)}(1),\rho^{(NC)}(0)} \quad \overline{E}$$

$$\text{s.t.} \qquad \overline{R} \geq \lambda$$

$$0 \leq \rho^{(NC)}(1), \rho^{(NC)}(0) \leq 1$$

Thus, this is also a linear knapsack minimization problem that can be solved as follows: If $\overline{p_m} + \overline{p_f} \geq 1$, then

$$\rho^{*(NC)}(1) = \min\left(1, \frac{\lambda}{\pi_v(1-\overline{p_m})f_{S|P,S} + (1-\pi_v)\overline{p_f}f_{S|S}}\right)$$

$$\rho^{*(NC)}(0) = \min\left(1, \max\left(0, \frac{\lambda - (\pi_v(1-\overline{p_m})f_{S|P,S} + (1-\pi_v)\overline{p_f}f_{S|S})}{\pi_v\overline{p_m}f_{S|P,S} + (1-\pi_v)(1-\overline{p_f})f_{S|S}}\right)\right)$$

If $\overline{p_m} + \overline{p_f} \leq 1$, then

$$\rho^{*(NC)}(0) = \min\left(1, \frac{\lambda}{\pi_v\overline{p_m}f_{S|P,S} + (1-\pi_v)(1-\overline{p_f})f_{S|S}}\right)$$

$$\rho^{*(NC)}(1) = \min\left(1, \max\left(0, \frac{\lambda - (\pi_v \overline{p_m} f_{S|P,S} + (1-\pi_v)(1-\overline{p_f}) f_{S|S})}{\pi_v (1-\overline{p_m}) f_{S|P,S} + (1-\pi_v)\overline{p_f} f_{S|S}}\right)\right)$$

In both cases, the optimal consumed energy for the secondary system is calculated as follows

$$\overline{E}^* = \pi_v[(1-\overline{p_m})\rho^{*(NC)}(1) + \overline{p_m}\rho^{*(NC)}(0)]+ \\ (1-\pi_v)[\overline{p_f}\rho^{*(NC)}(1) + (1-\overline{p_f})\rho^{*(NC)}(0)] \quad (7.8)$$

On the other hand, when neither of the assumptions is satisfied, the problem is a simple linear program with two unknowns that can be solved by any one of a variety of algorithms for linear programming.

7.4 Accurate Channel Estimation

In this section, we consider the case in which the channel from the primary source to the secondary source is accurately estimated at every time slot. In this case, the belief of the channel equals the channel state that $p = C$ and it takes only two values of 0 and 1. Then , the problem is simplified such that the transmission probabilities are $\rho^{(AC)}(\hat{v}, C)$ which depend on the sensing output and the exact channel state. The average consumed energy can be rewritten as follows

$$\overline{E} = \pi_v\pi_C[(1-p_m(1))\rho^{(AC)}(1,1) + p_m(1)\rho^{(AC)}(0,1)]+ \\ \pi_v(1-\pi_C)[(1-p_m(0))\rho^{(AC)}(1,0) + p_m(0)\rho^{(AC)}(0,0)]+ \\ (1-\pi_v)\pi_C[p_f(1)\rho^{(AC)}(1,1) + (1-p_f(1))\rho^{(AC)}(0,1)]+ \\ (1-\pi_v)(1-\pi_C)[p_f(0)\rho^{(AC)}(1,0) + (1-p_f(0))\rho^{(AC)}(0,0)] \quad (7.9)$$

The average probability of failure for the primary system can be written as follows

$$\begin{aligned}\overline{Q} = \pi_v - \pi_v\pi_C(1-p_m(1))[\rho^{(AC)}(1,1)f_{P|P,S} + (1-\rho^{(AC)}(1,1))f_{P|P}] \\ - \pi_v\pi_C p_m(1)[\rho^{(AC)}(0,1)f_{P|P,S} + (1-\rho^{(AC)}(0,1))f_{P|P}] \\ - \pi_v(1-\pi_C)(1-p_m(0))[\rho^{(AC)}(1,0)f_{P|P,S} + (1-\rho^{(AC)}(1,0))f_{P|P}] \\ - \pi_v(1-\pi_C)p_m(0)[\rho^{(AC)}(0,0)f_{P|P,S} + (1-\rho^{(AC)}(0,0))f_{P|P}] \quad (7.10)\end{aligned}$$

The average probability of success for the secondary system can be written as follows

$$\begin{aligned}\overline{R} = \pi_v\pi_C[(1-p_m(1))\rho^{(AC)}(1,1) + p_m(1)\rho^{(AC)}(0,1)]f_{S|P,S} + \\ \pi_v(1-\pi_C)[(1-p_m(0))\rho^{(AC)}(1,0) + p_m(0)\rho^{(AC)}(0,0)]f_{S|P,S} + \\ (1-\pi_v)\pi_C[(1-p_f(1))\rho^{(AC)}(0,1) + p_f(1)\rho^{(AC)}(1,1)]f_{S|S} + \\ (1-\pi_v)(1-\pi_C)[(1-p_f(0))\rho^{(AC)}(0,0) + p_f(0)\rho^{(AC)}(1,0)]f_{S|S} \quad (7.11)\end{aligned}$$

The problem is a simple linear program with four unknowns that can be solved by any of linear programming solving algorithms.

7.5 Opportunistic Channel Estimation

In this section, we consider the case in which the channel is estimated when the channel is sensed to be busy by the primary transmissions. The estimated channel state when the channel is sensed to be busy is denoted by $\hat{C}$. To obtain the expressions of different system quantities, we need to obtain the probability distribution of p. It is difficult to obtain this expression. Thus, we formulate the system evolution as a Markov chain. The state of the Markov chain is denoted by S and represented by the pair $(\hat{C}, n)$ where $\hat{C}$ is the last estimated value of the channel and n is the number of the time slots since the channel has been estimated.

We start by calculating the transition probabilities of the chain. The probability $Pr(\hat{C}, n|\hat{C}, n-1)$ is the probability that the sensing result is negative while the last measured channel was $\hat{C}$. It is calculated as follows

$$Pr(\hat{C}, n|\hat{C}, n-1) = Pr(\hat{v} = 0|\hat{C}, n-1) =$$
$$\pi_v[p_{\hat{C}}^{(n)} p_m(1) + (1 - p_{\hat{C}}^{(n)}) p_m(0)] + (1 - \pi_v)[p_{\hat{C}}^{(n)}(1 - p_f(1)) + (1 - p_{\hat{C}}^{(n)})(1 - p_f(0))] \tag{7.12}$$

where $p_{\hat{C}}^{(n)}$ is the belief about the channel when the last estimated channel is $\hat{C}$ and the channel has been estimated from n time slots.

The probability $Pr(1, 0|\hat{C}, n-1)$ is the probability that the sensing result is positive and the estimated channel is 1. It is calculated as follows

$$Pr(1, 0|\hat{C}, n-1) = Pr(\hat{v} = 1, \hat{C} = 1|\hat{C}, n-1) =$$
$$\pi_v p_{\hat{C}}^{(n)}(1 - p_m(1)) + \frac{1}{2}(1 - \pi_v)[p_{\hat{C}}^{(n)} p_f(1) + (1 - p_{\hat{C}}^{(n)}) p_f(0)] \tag{7.13}$$

Similarly, the probability for the channel state to be measured 0 is calculated as follows

$$Pr(0, 0|\hat{C}, n-1) = Pr(\hat{v} = 1, \hat{C} = 0|\hat{C}, n-1) =$$
$$\pi_v(1 - p_{\hat{C}}^{(n)})(1 - p_m(0)) + \frac{1}{2}(1 - \pi_v)[p_{\hat{C}}^{(n)} p_f(1) + (1 - p_{\hat{C}}^{(n)}) p_f(0)] \tag{7.14}$$

The probability $p_{\hat{C}}^{(n)}$ is the n step transition probability from the measured channel state being $\hat{C}$ to the channel state being 1.

Then, we calculate the steady state probabilities of all the states. We denote the steady state probability of the state $S = (\hat{C}, n)$ by π_S which equals $\pi_{\hat{C},n}$. We write the balance equations of the Markov chain as follows

$$\pi_{0,0}(Pr(1,0|0,0) + Pr(0,1|0,0)) = \sum_{i=1}^{\infty} \pi_{0,i} + \sum_{i=0}^{\infty} \pi_{1,i}$$

$$\pi_{1,0}(Pr(0,0|1,0) + Pr(1,1|1,0)) = \sum_{i=1}^{\infty} \pi_{1,i} + \sum_{i=0}^{\infty} \pi_{0,i}$$

$$\pi_{0,n}(Pr(0,n+1|0,n) + Pr(0,0|0,n) + Pr(1,0|0,n)) = \pi_{0,n-1}Pr(0,n|0,n+1)$$

$$\pi_{1,n}(Pr(1,n+1|1,n) + Pr(0,0|1,n) + Pr(1,0|1,n)) = \pi_{1,n-1}Pr(0,n|1,n+1)$$

Note that in the last two equations, both the terms $Pr(0,n+1|0,n)+Pr(0,0|0,n)+Pr(1,0|0,n)$ and $Pr(1,n+1|1,n) + Pr(0,0|1,n) + Pr(1,0|1,n)$ equal 1. Then, the equations can be rewritten as follows

$$\pi_{0,n} = \pi_{0,n-1}Pr(0,n|0,n+1)$$

$$\pi_{1,n} = \pi_{1,n-1}Pr(0,n|1,n+1)$$

The general relations of the steady state probabilities are

$$\pi_{0,n} = \prod_{i=1}^{n} Pr(0,i|0,i-1)\pi_{0,0}$$

$$\pi_{1,n} = \prod_{i=1}^{n} Pr(1,i|1,i-1)\pi_{1,0}$$

We denote the term $\prod_{i=1}^{n} Pr(0,i|0,i-1)$ by A_n with A_0 defined to be 1 and the term $\prod_{i=1}^{n} Pr(1,i|1,i-1)$ by B_n with B_0 defined to be 1. Then by solving the first two equations after substituting using the last two, we get

$$\pi_{1,0} = \frac{Pr(1,0|0,0) + Pr(0,1|0,0) - \sum_{i=1}^{\infty} A_i Pr(0,0|0,i)}{\sum_{i=0}^{\infty} B_i Pr(0,0|0,i)} \pi_{0,0} \tag{7.15}$$

we denote the constant $\frac{Pr(1,0|0,0)+Pr(0,1|0,0)-\sum_{i=1}^{\infty} A_i Pr(0,0|0,i)}{\sum_{i=0}^{\infty} B_i Pr(0,0|0,i)}$ by K. Then, the value of $\pi_{0,0}$ is obtained to be

$$\pi_{0,0} = \frac{1}{\sum_{i=0}^{\infty} A_i + K \sum_{i=0}^{\infty} B_i} \tag{7.16}$$

Using the Markov chain states, we write the expressions of the system quantities as functions in the obtained steady state probabilities.

$$\overline{E} = \sum_{S} \pi_S \rho(S) \tag{7.17}$$

$$\overline{R} = \sum_{S} \pi_S \rho(S)(Pr(B|S) f_{S|P,S} + (1 - Pr(B|S)) f_{S|S}) \tag{7.18}$$

$$\overline{Q} = \sum_{S} \pi_S Pr(B|S)(\rho(S)(1 - f_{P|P,S}) + (1 - \rho(S))(1 - f_{P|P})) \tag{7.19}$$

where B is the event that the channel is busy by the primary source transmissions. The probability $Pr(B|S)$ remains to be calculated and it is the probability that the primary source is transmitting given a certain measured state. Using basic probability Bayes' rule, we can calculate this probability as follows

$$Pr(B|S) = \frac{Pr(S|B)\pi_v}{\pi_S} \tag{7.20}$$

Thus, we calculate the probability $Pr(S|B)$ for all the states of the Markov chain as follows

$$Pr(0,0|B) = \sum_{S} \pi_S \left(p_{\hat{C}}^{(n)}(1-\lambda_1)(1-p_m(0)) + (1-p_{\hat{C}}^{(n)})(1-\lambda_0)(1-p_m(0)) \right)$$

$$Pr(1,0|B) = \sum_{S} \pi_S (1-p_m(1))(p_{\hat{C}}^{(n)}\lambda_1 + (1-p_{\hat{C}}^{(n)})\lambda_0)$$

$$Pr(\hat{C}, n|B) = \pi_{\hat{C},n-1} \Big[p_m(1)(p_{\hat{C}}^{(n-1)}\lambda_1 + (1 - p_{\hat{C}}^{(n-1)})\lambda_0) +$$
$$p_m(0)(p_{\hat{C}}^{(n-1)}(1-\lambda_1) + (1 - p_{\hat{C}}^{(n-1)})(1-\lambda_0)) \Big]$$

The optimization problem is an infinite linear program that could be solved using shadow simplex method in [89] or it can be approximated using the truncation method in [90].

7.6 Numerical Results

In this section, we present some numerical examples to illustrate the previous analysis. We focus on comparing the performance of the different channel estimation strategies in terms of the constrained average consumed energy by the secondary system and the maximum reachable throughput at the secondary system. We choose the following system parameters values: $\pi_v = 0.4$, $\lambda_1 = 0.6$, $\lambda_0 = 0.3$, $p_m(0) = 0.4$, $p_m(1) = 0.2$, $p_f(0) = 0.2$, $p_f(1) = 0.1$, $\delta = 0.05$, $\lambda = 0.05$, $f_{P|P,S} = 0.1$, $f_{P|P} = 0.5$, $f_{S|P,S} = 0.1$ and $f_{S|S} = 0.5$. Then, we vary these system parameters to study their effects on the system performance. All the figures show the enhancement in the throughput as a result of different levels of CSI knowledge.

In figure 7.2, we show the average throughput of the secondary source constrained by the allowable primary probability of failure against the steady state probability of the channel to be busy by the primary transmissions.

In figure 7.3, we show the average constrained energy consumed by the secondary source against the steady state probability of the channel to be busy by the primary transmissions. When the primary system is more active, the enhancement

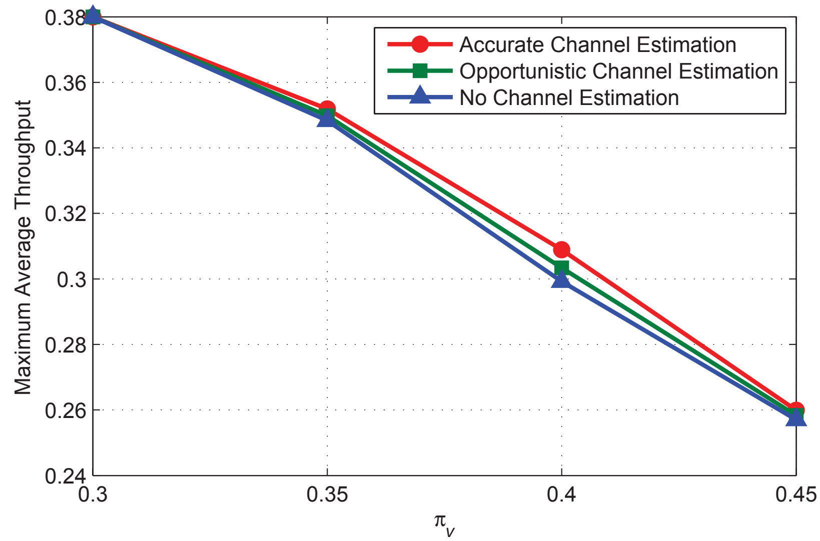

Figure 7.2: Maximum achievable throughput against π_v

due to the channel knowledge increases. That is because the importance of the reliability of the sensing decision is higher when π_v increases.

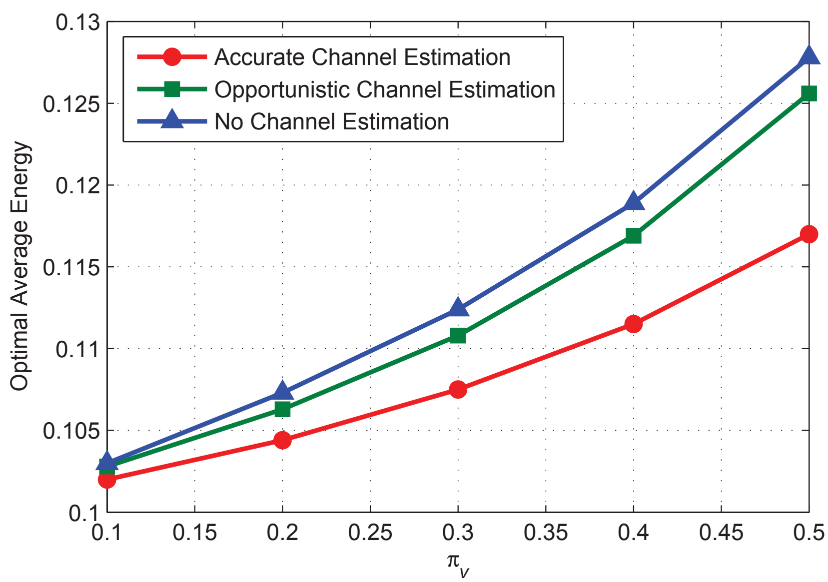

Figure 7.3: Optimal average energy against π_v

In figure 7.4, we show the average constrained throughput of the secondary source against the maximum allowable probability of failure for the primary system.

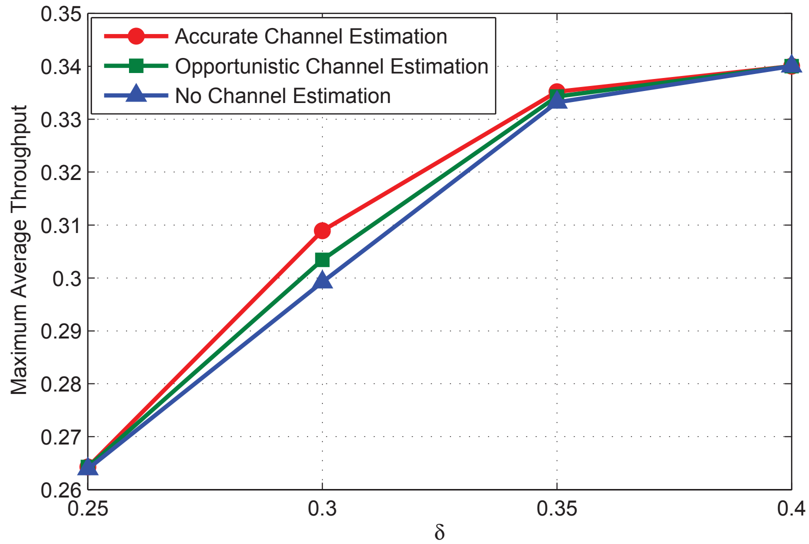

Figure 7.4: Maximum achievable throughput against δ

Finally in figure 7.5, we show the average constrained energy consumed by the secondary source against λ_1. When the channel correlation increases, the enhancement due to the channel knowledge also increases.

7.7 Discussion

In this chapter, we investigated the effect of estimating the sensing-channel by the secondary source in a cognitive radio system. We have shown that obtaining the optimal transmission probabilities can be done through linear programming in the cases of no channel estimation and accurate channel estimation. In the case of opportunistic channel estimation, the system is modeled by a Markov chain and then the problem of finding the optimal transmission probabilities is formulated as an infinite linear program. We quantify the enhancement in the performance and

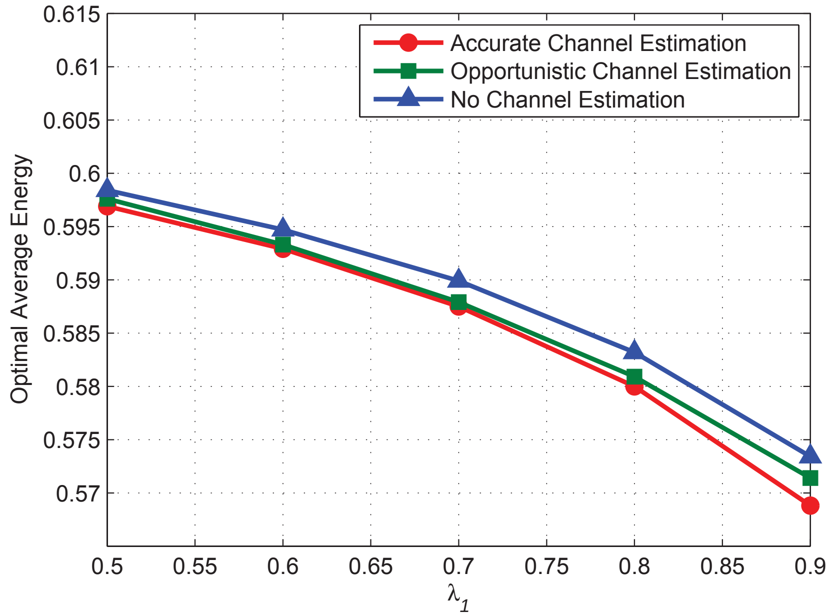

Figure 7.5: Optimal average energy against λ_1

show that the enhancement due to channel knowledge increases when the primary activity increases.

Chapter 8: Conclusion

8.1 Summary of Contributions

We have studied in chapter 2 a communication link that operates over a Gilbert-Elliot channel. The source node has energy harvesting capability. In order to maximize the number of successfully delivered packets per time slot, the source decides in each time slot whether to transmit or defer the transmission. The problem has been formulated as a Markov decision problem and we have characterized the optimal policy. We have proved that it is a threshold-type policy, depending on the channel state and the energy queue length. Different properties of the optimal policy have been derived. An upper bound on the average number of packets per time slot that are successfully received by the destination has been derived. This bound has been shown to be tight on the performance of the optimal policy. The optimal policy for the case of no CSI availability has also been derived. Numerical results have been obtained to illustrate the analysis. We observe that the value of CSI can be significant. We also see that the channel fluctuations affect performance significantly as well.

In chapter 3, we have proposed and analyzed protocols for transmission from a source that has energy harvesting capability. We have considered the case in which a relay is used to help the source transmissions. The relay also has energy harvesting capability. The proposed protocol allows the relay to use the idle time slots of the source and hence avoids allocating any explicit resources to the relay. Our analysis shows that cooperation increases the maximum stable throughput rate in most cases except when the energy harvesting rate of the relay is small. The proposed strategy exploits the knowledge of the CSI of the channel between the source and the destination such that the source transmits with probability 1 if the channel is in state 1 and transmits with a certain probability if the channel is in state 0. The optimal probability has also been calculated. The effect of imperfect channel measurements has been considered.

In chapter 4, we have introduced the notion of partial network-level cooperation for energy harvesting networks. The flow from the source through the relay is controlled. We provide an exact characterization of the stability region for the discussed system. We have shown that the performance of the system with optimal partial relaying is always better than or equals the performance of simple relaying schemes. Also, we have shown that it is optimal to use full relaying for a small data arrival rate at the relay while it is optimal to use no relaying when the source has a small data arrival rate.

In chapter 5, we have investigated the problem of transmission control in a network with multiple energy harvesting relays. We have exploited partial relaying cooperation in the proposed network. We have derived the stability conditions for the source and the relays data queues. Our analysis shows that cooperation increases the maximum achievable rate of the source. We have discussed the problem of maximizing the achievable rate at the source data queue over the relaying parameters vector. Also, we have discussed the problem of relaying cost minimization. The problem is constrained by the stability of the system data queues. We have given an example for the cost to be the average consumed energy in the network. We have shown that optimal partial relaying cooperation has equal or better performance than full relay cooperation.

In chapter 6, we have derived the optimal scheduling policies for a communication system that contains two transmitter-receiver pairs which operate over Gilbert-Elliot channels. We have considered exploiting CSI of the system channels in the scheduling policies. Due to the difficulty of the analysis of the problem, we traced the solutions in the case of two pairs only. In the case of full channel knowledge, we have shown that it is optimal to maximize the instantaneous expected reward of the system. Then, the problem of delayed channel information has been formulated as a Partially Observable Markovian Decision Problem for which we have found an approximate solution using linear programming. Also, for the case in which no channel measurements are available, it is optimal to select a fixed action that maximizes the steady state expected reward of the system. Then, we calculated the system expected reward as a function of the error probability when the channel measurements are inaccurate. In this case, it is also optimal to maximize the instantaneous expected reward of the system. In the case of infrequent channel measurements, we have shown the effects of changing the measurement interval length on the performance and we have shown that taking a decision every time slot can lead to better performance than taking a decision every measurement interval. We have also considered the scheduling in a distributed manner. We have formulated this scheduling problem as a quadratic program. We compared the throughput performance for all these cases and assessed the value of different levels of channel state information knowledge.

In chapter 7, we investigated the effect of estimating the sensing-channel by the secondary source in a cognitive radio system. We have shown that obtaining the optimal transmission probabilities can be done through linear programming in the cases of no channel estimation and accurate channel estimation. In the case of opportunistic channel estimation, the system is modeled by a Markov chain and then the problem of finding the optimal transmission probabilities is formulated as an infinite linear program. Then, we quantify the enhancement in the performance and show that the enhancement due to channel knowledge increases when the primary activity increases.

8.2 Future Directions

The solid theoretical analysis in this dissertation provides useful insights for better understanding of the communication architecture in wireless networks and its ultimate performance limits. There remains a number of questions for future investigation.

A fundamental issue that naturally arises is the need for a distributed cooperative communication protocol. The proposed cooperation strategy for the multiple access system with multiple relays, implicitly assumes that there exists a centralized controller which activates at most one relay in a time slot, such that all other users can overhear the transmission and possibly relay the received packet. However, such centralized controller may not exist, or too costly to implement in a real wireless network. Further, the strategy requires all the users that capture the transmission to send back acknowledgements, upon which the best of them can be selected as the relay. This can result in the feedback implosion problem. Thus, a distributed cooperation policy with feedback suppression mechanism will be of both theoretical and practical interest. The performance in our centralized policy can serve as an upper bound to evaluate the effectiveness of the distributed policy.

Also, the issue of rate and power control in energy harvesting networks needs to be investigated for improving the performance. Transmission control allows the nodes to decide whether to transmit or not while rate and power control gives more degrees of freedom for the transmission action. Thus, the nodes are allowed to select more tailored transmission parameters for the network conditions.

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