

## ABSTRACT

Title of dissertation: THE DILATON, THE RADION AND DUALITY

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In this dissertation, scenarios where strong conformal dynamics constitutes the ultraviolet completion of the physics that drives electroweak symmetry breaking are considered. It is shown that in theories where the operator responsible for the breaking of conformal symmetry is close to marginal at the breaking scale, the dilaton mass can naturally lie below the scale of the strong dynamics. However, in general this condition is not satisfied in the scenarios of interest for electroweak symmetry breaking, and so the presence of a light dilaton in these theories is associated with mild tuning. The effective theory of the light dilaton is constructed in this framework, and the form of its couplings to Standard Model states are determined. It is shown that corrections to the form of the dilaton interactions arising from conformal symmetry violating effects are suppressed by the square of the ratio of the dilaton mass to the strong coupling scale, and are under good theoretical control. These corrections are generally subleading, except in the case of dilaton couplings to marginal operators, when symmetry violating effects can sometimes

dominate. Phenomenological implications of these results are investigated for models of technicolor, and for models of the Higgs as a pseudo-Nambu-Goldstone boson, that involve strong conformal dynamics in the ultraviolet.

Using AdS/CFT correspondence, a holographic realization of this scenario is obtained by constructing the effective theory of the graviscalar radion in the Randall-Sundrum models, taking stabilization into account. The conditions under which the radion can remain light are explored, and the corrections to its couplings to Standard Model (SM) states localized on the visible brane are determined. It is shown that in the theories of interest for electroweak symmetry breaking that have a holographic dual, the presence of a light radion requires mild tuning. Corrections to the form of the radion coupling to SM states arising from effects associated with brane stabilization are also calculated. These corrections scale as the square of the ratio of the radion mass to the Kaluza-Klein scale, and are generally subleading, except in the case of gluons and photon, when they can sometimes dominate. These results are in agreement with and lend robustness to the conclusions for the dilaton.

# THE DILATON, THE RADION AND DUALITY

by

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## Dedication

To my grandparents – Ramchandra Mishra, Ronawati Devi, Narendra Prasad Dixit and Rajkumari Devi, for their care, guidance and sweet scoldings.

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# Table of Contents

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List of Tables	v
List of Figures	v
List of Abbreviations	vi
1 Introduction	1
1.1 Standard Model, Hierarchy problem and Flavor constraints . . . . .	5
1.2 A CFT primer . . . . .	13
1.3 AdS/CFT correspondence . . . . .	18
1.4 Beyond Standard Model scenarios involving Strong Conformal Dy- namics . . . . .	20
1.5 Beyond Standard Model scenarios involving Warped Extra-dimensions	27
1.6 Summary and Plan of the Dissertation . . . . .	37
2 Effective Theory of the Dilaton	40
2.1 Dilaton Lagrangian in the Limit of Exact Conformal Invariance . . .	43
2.1.1 One Loop Analysis . . . . .	44
2.1.2 General Analysis . . . . .	47
2.2 Dilaton Lagrangian in the Presence of Conformal Symmetry Violating Effects . . . . .	49
2.2.1 One Loop Analysis . . . . .	50
2.2.2 General Analysis . . . . .	55
2.2.2.1 Limit When the Corrections Are Small . . . . .	58
2.2.2.2 Limit When the Corrections Are Large . . . . .	65
2.3 Dilaton Interactions to SM fields . . . . .	68
2.3.1 Dilaton Interactions in a Conformal SM . . . . .	69
2.3.2 Technicolor . . . . .	77
2.3.2.1 Couplings to Gauge Bosons . . . . .	79
2.3.2.2 Couplings to Fermions: Elementary Fermions . . . . .	84
2.3.2.3 Couplings to Fermions: Partially Composite Fermions	87

2.3.3	Higgs as a pNGB . . . . .	93
2.3.3.1	Couplings to Gauge Bosons . . . . .	95
2.3.3.2	Couplings to Fermions: Elementary Fermions . . . . .	96
2.3.3.3	Couplings to Fermions: Partially Composite Fermions . . . . .	98
2.3.3.4	Coupling to the Higgs . . . . .	100
3	Effective Theory of the Radion . . . . .	104
3.1	Radion Lagrangian without a Stabilization Mechanism . . . . .	105
3.2	Radion Lagrangian in the Presence of a Stabilization Mechanism . . . . .	108
3.2.1	Massive GW Scalar with no Bulk Interactions . . . . .	113
3.2.2	Massive GW Scalar with a Cubic Interaction in Bulk . . . . .	118
3.3	Mixing with Other Scalars . . . . .	120
3.4	Radion Interactions to SM fields . . . . .	121
3.4.1	Massive Gauge Bosons . . . . .	124
3.4.2	Massless Gauge Bosons . . . . .	127
3.4.3	Fermions . . . . .	129
4	Dilaton vs Radion: the Duality . . . . .	131
4.1	Comparison of the Lagrangian for the Dilaton and the Radion . . . . .	134
4.1.1	Dynamics in the Absence of a Stabilization Mechanism . . . . .	134
4.1.2	Dynamics in the Presence of a Stabilization Mechanism . . . . .	135
4.2	Comparison of Dilaton and Radion Couplings . . . . .	142
5	Conclusion . . . . .	145
A	Radion Potential . . . . .	148
A.1	In the absence of a GW Scalar . . . . .	148
A.2	In the presence of a GW Scalar . . . . .	150
A.2.1	GW Scalar without Bulk Interactions . . . . .	151
A.2.2	GW Scalar with Bulk Cubic Self Interaction . . . . .	153
B	NDA Estimation of Parameters . . . . .	156
B.1	5D Gravity . . . . .	157
B.2	GW Scalar . . . . .	158
B.3	Radion Potential . . . . .	159
B.4	Radion Couplings to SM . . . . .	160
	Bibliography . . . . .	162

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## List of Tables

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- 1.1 The quantum numbers of the fields in the SM under the gauge groups. The entries in the  $SU(3)_c$  and  $SU(2)_L$  columns denote the representation under which the fields transform, while the entries in the  $U(1)_Y$  column represent the charge under this gauge group. . . . . 6

---

## List of Figures

---

- 1.1 The top loop contribution to the self energy of the Higgs in the Standard Model. . . . . 12
- 1.2 The geometry of the two brane RS model. The location of the hidden and the visible branes are shown by the shaded planes and the warp factor of the geometry is shown by the solid curve. The space between the branes is referred as the bulk region. . . . . 30
- 3.1 **1a:** The approximate solution (solid line) matches well with the BR solution (large dotted) and the OR solution (small dotted) for  $\epsilon = -0.1$ ,  $k\pi r_c = 10$ . We have also taken  $v = 0.05$  and  $\alpha = -0.5$ . The shaded region separates the boundary region on its right from the outer region on its left. Asymptotic matching is done in the shaded region. **1b:** The approximate solution (dotted) agrees well with the exact solution (solid) for the same parameter values, and we show the agreement near the  $\theta = \pi$  boundary. . 115



## List of Abbreviations

SM	Standard Model
QFT	Quantum Field Theory
BSM	Beyond Standard Model
CFT	Conformal Field Theory
NGB	Nambu-Goldstone Boson
AdS	Anti-de Sitter
RS	Randall Sundrum
EW	Electroweak
EM	Electromagnetism
VEV	Vacuum Expectation Value
CKM	Cabibbo-Kobayashi-Maskawa
FCNC	Flavor Changing Neutral Current
GIM	Glashow Iliopoulos Maiani
QCD	Quantum Chromodynamics
TC	Technicolor
ETC	Extended Technicolor
WTC	Walking Technicolor
OPE	Operator Product Expansion
pNGB	Pseudo-Nambu-Goldstone Boson
PC	Partial Composite
UV	Ultraviolet
IR	Infrared
KK	Kaluza-Klein
GW	Goldberger-Wise
MS	Minimal Subtraction
EFT	Effective Field Theory
OR	Outer Region
BR	Boundary Region
RGE	Renormalization Group Equation
NDA	Naive Dimensional Analysis
LHC	Large Hadron Collider

# CHAPTER 1

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## Introduction

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Since time immemorial, mankind has pursued the existence of simple unifying descriptions capable of explaining a wide range of observed phenomena. There are several examples of such descriptions, spanning across scientific and non-scientific endeavors, in fields as varying as the natural and life sciences to political science, economics, sociology, psychology and even mythology. To the question, “What are all the objects around us fundamentally composed of, and how do they interact with each other?”, modern theoretical physics attempts an answer in the form of the Standard Model (SM) [1, 2]. This model provides a concrete mathematical description for the physics of the fundamental particles and the forces through which they interact with each other. There are presently four well established fundamental forces: the electromagnetic force, the weak force, the strong force and the gravita-

tional force<sup>1</sup>. These forces are at work all around us. While electromagnetic forces keep the electrons in an atom stable in their orbit around nucleus, weak forces are responsible for radioactive decays. The strong force holds nuclei together, while the ubiquitous force of gravity keeps planets in orbit.

The language of the SM is that of a relativistic quantum field theory (QFT), which is a consistent framework that incorporates both special relativity and quantum mechanics. After being developed as a collaborative effort, the SM has been held to scrutiny time and again, and has experienced more than 30 years of experimental success. However, we have reasons to question its validity to arbitrarily high energies, because the description of fundamental physics by the SM fails to address several issues satisfactorily. For example, in its original form, the SM contains three copies of massless particles called as neutrinos. However, recent experiments have shown that the neutrinos cannot be massless. As another shortcoming, there is compelling astrophysical evidence for the existence of dark matter, while there are no candidate particles in the SM with the right properties. Furthermore, taking the force of gravity into account, the SM is not capable of describing physical processes at energies above the Planck scale  $M_{pl}$ . These and other limitations of the SM motivate Beyond the Standard Model (BSM) theories that augment the SM with additional structure to address its shortcomings.

One of the biggest puzzles in the SM is that of the extreme weakness of the gravitational force compared to the other forces. Consider natural units, in which

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<sup>1</sup>In its earliest form, SM did not include gravity, but the modern understanding in the context of effective theories allows the inclusion of gravity in a consistent way [3].

the fundamental constants  $c$  (speed of light in vacuum) and  $\hbar$  ( $= h/2\pi$ ,  $h$  being the Planck constant) are set to 1. In these units, all dimensionful quantities can be expressed in units of energy, which is taken to be electron-volts (eV). In natural units, the value of the coupling constant of the gravitational force, Newton’s constant of gravitation  $G$ , is given by  $G \sim 10^{-56}eV^{-2}$ . In the same units, the coupling constant of the weak force, the Fermi constant  $G_F$  is given by  $G_F \sim 10^{-22}eV^{-2}$ , which is  $10^{34}$  times larger than the corresponding gravitational coupling constant! This unnatural difference in scales is not addressed in the SM, and constitutes the “hierarchy problem” [4, 5] of the SM.

A class of theories with an enhanced spacetime symmetry structure, called *conformal symmetry*, can play a role in addressing the hierarchy problem of SM. QFTs that are based on this spacetime symmetry are called as conformal field theories (CFTs). In the context of BSM theories, the focus is on strongly coupled theories that have approximate conformal symmetry at high energies. The motivation for this class of theories comes from the fact that apart from providing a way to solve the hierarchy problem, they can remain consistent with experimental constraints [6]. It then becomes important to understand what are the robust implications of this framework. In realistic BSM scenarios, conformal symmetry is broken spontaneously at some scale. This results in the presence of a scalar particle in the theory, the dilaton. The dilaton is the Nambu-Goldstone boson (NGB) of spontaneously broken conformal symmetry [7–10], and may be accessible in modern colliders. It then becomes important to construct the effective theory of the dilaton, in order to understand the conditions under which it can remain light, and its

couplings to the SM particles [11–14].

Apart from these features that make conformal dynamics attractive in the context of BSM models, there are other independently motivated theoretical reasons to study them. In the late 90s, a lot of excitement was generated by the AdS/CFT conjecture [15–18], which relates theories in 4 spacetime dimensions with conformal symmetry to the theory of gravity in 5 dimensional anti-de Sitter (AdS) space. If the  $4D$  CFT is strongly coupled, the corresponding theory on the  $5D$  side is weakly coupled. This has led to enormous calculational and conceptual progress in understanding strongly coupled theories. In the context of BSM scenarios involving strong conformal dynamics, the correspondence relates these theories to Randall-Sundrum (RS) models [19]. RS models are a specific class of BSM theories that offer a geometrical solution to the hierarchy problem in the SM. Using the duality, RS models provide a weakly coupled description that can be used to calculate quantities of interest on the strongly coupled side indirectly. Apart from that, RS models can also provide a way to verify results obtained on the strongly coupled side. The radion field, which is a scalar gravitational mode in RS models, is dual to the dilaton field in spontaneously broken CFTs [20]. In this way, the construction of effective theory of a radion on the RS side provides a way to verify the results concerning the mass and the couplings of the dilaton in BSM scenarios based on strong conformal dynamics [13, 14, 21].

A quick outline of this chapter and the dissertation is in order. In chapter 2 of this dissertation, we construct the effective theory of the dilaton in realistic BSM scenarios, focusing on its mass and couplings. In chapter 3, we construct the ef-

fective theory of the radion in the dual RS scenario, and compare the results for the mass and the couplings of the radion and the dilaton in chapter 4. A good agreement provides a verification of the results. In the rest of this chapter, we first present the relevant details of the SM, and then focus on the hierarchy problem in the SM. We also discuss the flavor processes in the SM, which provide stringent experimental constraints on the BSM scenarios involving strong dynamics. We then focus on relevant details of CFTs. Next, we discuss the AdS/CFT correspondence which relates CFTs to the theory of gravity in AdS space. Once equipped with a knowledge of CFTs and the AdS/CFT duality, we motivate the BSM models that are based on strong conformal dynamics and the RS models based on warped extra dimensions and outline several of their interesting features, particularly elaborating on how they address the hierarchy problem, and how they are related by the correspondence. Finally, we summarize and present a more detailed outline of the rest of the dissertation.

## 1.1 Standard Model, Hierarchy problem and Flavor constraints

The SM is a relativistic QFT with gauge groups  $SU(3)_c \times SU(2)_L \times U(1)_Y$ , and contains additional fields that are charged under these gauge symmetries. These fields include spin 1/2 particles, the fermions, and a spin 0 particle, the Higgs. In addition to these, the SM contains vector bosons associated with the corresponding gauge symmetries. The fermions in the SM can be arranged into three broad groups, each group being called a generation. The fermions in each generation have

Fields	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_L$	3	2	1/6
$u_R$	3	1	2/3
$d_R$	3	1	-1/3
$l_L$	1	2	-1/2
$e_R$	1	1	-1
$H$	1	2	1/2

Table 1.1: The quantum numbers of the fields in the SM under the gauge groups. The entries in the  $SU(3)_c$  and  $SU(2)_L$  columns denote the representation under which the fields transform, while the entries in the  $U(1)_Y$  column represent the charge under this gauge group.

identical quantum numbers under the three gauge groups. Each generation contains the fermions  $q_L, u_R, d_R, l_L, e_R$ . These fermions transform in an irreducible spin 1/2 representation of the Lorentz group. The subscripts  $L, R$  stand for left handed and right handed respectively, and specify the transformation properties of the corresponding fermion under the Lorentz group. The scalar Higgs  $H$  in the SM is a singlet under the Lorentz group. This field transforms as a doublet under  $SU(2)_L$  and is called the Higgs doublet in the SM. The quantum numbers for the Higgs doublet and for one generation of fermions under the three gauge groups are shown in table 1.1. The vector bosons of the SM transform in the adjoint representation of their respective gauge groups.

The gauge symmetries of the SM are partly realized non-linearly below the electroweak (EW) scale  $M_{EW}$ . Below this scale, the  $SU(3)_c$  and a subset  $U(1)_{EM}$  of the  $SU(2)_L \times U(1)_Y$  are realized linearly. These correspond to the gauge groups associated with the strong interactions and the electromagnetic (EM) interactions respectively. Due to the ‘‘Higgs mechanism’’ in the SM, the three generators corresponding to the gauge symmetries realized non-linearly give rise to three massive gauge bosons. The gauge bosons associated with the weak interactions, the  $W^\pm$  and  $Z$ , acquire a mass due to this effect. In the unitary gauge, where the Higgs mechanism is most transparent, there is another scalar particle  $h$  in the spectrum, called as the Higgs particle. The mass and couplings of this particle to all the other SM fields are fixed. Let us elaborate some of these features of the SM now.

In the SM, the Higgs mechanism is realized by a Higgs doublet  $H$  [22,23]. Let us focus on the part of the SM that involves the Higgs doublet. The Lagrangian for this sector, consistent with the symmetries, is given by

$$\mathcal{L}_H = (\mathcal{D}_\mu H)^\dagger (\mathcal{D}^\mu H) - V(H^\dagger H) - Y_l \bar{l}_L H e_R - Y_d \bar{q}_L H d_R - Y_u \bar{q}_L \tilde{H} u_R + h.c. \quad (1.1)$$

Let us explain various terms in (1.1). The quantity  $\mathcal{D}_\mu$  is the gauge covariant derivative, and its action on the Higgs doublet  $H$  is given as

$$\mathcal{D}_\mu H = \left( \partial_\mu - \frac{ig}{2} W_\mu^a \sigma^a - \frac{ig'}{2} B_\mu \right) H, \quad (1.2)$$

where  $g, g'$  are the gauge couplings of the  $SU(2)_L, U(1)_Y$  gauge groups respectively and  $W_\mu^a, B_\mu$  are the corresponding gauge bosons. Here,  $\sigma^a, a = 1, 2, 3$  are the Pauli matrices. These gauge bosons  $W_\mu^a, B_\mu$  are related by a linear combination to the



observed massive gauge bosons  $W_\mu^\pm, Z_\mu$  and the massless photon  $A_\mu$ .

The quantity  $V(H^\dagger H)$  is the potential for the Higgs doublet, and the terms involving  $Y_l, Y_d, Y_u$  are the Yukawa couplings. We have suppressed the generation indices in the Yukawa couplings, but they are to be understood as matrices in that space. The quantity  $\tilde{H}$  is related to  $H$  as  $\tilde{H} = \epsilon \cdot H^\dagger$ ,  $\epsilon$  being the completely antisymmetric two index tensor.

The potential for the Higgs doublet is given by

$$V(H^\dagger H) = -\mu^2(H^\dagger H) + \frac{\lambda}{2}(H^\dagger H)^2. \quad (1.3)$$

The quartic coupling  $\lambda$  in (1.3) is required to be positive, so that the potential is bounded from below. However, the parameter  $\mu^2$  can be of either sign. In the SM,  $\mu^2$  is chosen to be *positive*. This is done to ensure that minimum of the Higgs potential occurs at  $\langle H \rangle \neq 0$ . This is necessary to realize some of the gauge symmetries non-linearly, as stated earlier, and to let the Higgs mechanism play out in the SM. For convenience, a gauge transformation can be performed to put  $\langle H \rangle$  in the form  $\langle H \rangle = (0, v/\sqrt{2})^T$ . In this parametrization, the potential is minimized at  $v = \sqrt{2\mu^2/\lambda}$ . The Higgs doublet is said to have acquired a Vacuum Expectation Value (VEV)  $v$ . Replacing  $H$  by  $\langle H \rangle$ , three linear combinations of the gauge bosons  $W^a, B$  acquire a mass, and are identified with the  $W^\pm, Z$  gauge bosons. The linear combination that remains massless is identified as the photon  $A$ . The masses of the  $W^\pm, Z$  gauge bosons are proportional to the gauge couplings  $g, g'$  and  $v$ . Experimental results on the gauge couplings and the masses of the bosons fix  $v$  to be approximately 246 GeV.

Once the Higgs doublet acquires a VEV, the SM fermions also get a mass due to the Yukawa couplings. The mass of the fermions is proportional to the VEV  $v$  and the dimensionless Yukawa coupling. The numerical values of the Yukawa couplings are chosen to achieve the observed fermion masses. However, the fermions masses in SM vary by six orders of magnitude, from the lightest, the electron ( $\sim 0.5$  MeV), to the heaviest ( $\sim 10^5$  MeV), the top. Within the SM, there is no understanding of this large hierarchy. This is known as the *fermion mass hierarchy* puzzle in the SM.

The structure of the fermion masses is made more complicated in the presence of three generations of fermions. To understand that, let us put back the generation indices in the Yukawa couplings. The Yukawa couplings are now matrices in this space. In the absence of the Yukawa matrices, the SM Lagrangian is classically invariant under global unitary transformations of the form  $u_L \rightarrow V_L^u u_L$ ,  $u_R \rightarrow V_R^u u_R$ ,  $d_L \rightarrow V_L^d d_L$ ,  $d_R \rightarrow V_R^d d_R$ . Once  $H$  acquires a VEV, the Yukawa matrices lead to mass matrices for the fermions, which can be diagonalized using these transformations. The three fermion mass eigenstates, called as three flavors, are a linear combination of the three generations of the fermions. This diagonalization however has observational consequences. Once the fermion masses are diagonalized, the fermion couplings to the  $W$  gauge bosons, which come from the fermion kinetic term written in terms of the corresponding gauge covariant derivative, become non-diagonal in flavor space,

$$\mathcal{L}_W \supset \frac{g}{2} W_\mu^+ \bar{u}_L \gamma^\mu V_L^{u\dagger} V_L^d d_L. \quad (1.4)$$

The combination  $V_L^{u\dagger} V_L^d \equiv V_{CKM}$  is called Cabibbo-Kobayashi-Maskawa (CKM)

matrix. The fermion couplings to the  $Z$  boson remains diagonal in the flavor space. However, non-diagonal flavor couplings are generated at the level of loops. These couplings lead to Flavor Changing Neutral Current (FCNC) processes in the SM. In addition to being loop suppressed, these processes involving the light fermions are additionally suppressed in the SM. This is because of the common origin of the masses and the FCNCs from the Yukawa couplings, which are small for the light fermions. In the SM, the Yukawa couplings are the only source of flavor violation, and there is an approximate flavor symmetry that is explicitly broken by the Yukawa terms. Therefore, the FCNCs must be additionally suppressed by the Yukawa couplings, which are small for lighter fermions. This is the Glashow-Iliopoulos-Maiani (GIM) mechanism [24] in the SM. Experimental tests so far have shown that the predictions of the SM are in good agreement with data.

In the unitary gauge, where the Higgs mechanism is most transparent, there is another scalar mode  $h$  in the spectrum, known as the Higgs particle. The mass and the couplings of  $h$  can be obtained by substituting

$$H = \begin{pmatrix} 0 & (v + h)/\sqrt{2} \end{pmatrix}^T \quad (1.5)$$

in the relevant parts of the SM action. This results in a mass for  $h$ , given as  $m_h = \sqrt{\lambda} v$ . Recall that  $\lambda$  here is the quartic in the Higgs potential in (1.3). For perturbative control, we need the quartic  $\lambda \ll 16\pi^2$ . In that case, the mass  $m_h$  must be close to the scale  $v$ . A particle with mass in this range and properties similar to  $h$  has been reported recently [25, 26].

The couplings of  $h$  to the SM fermions are obtained by substituting (1.5) in the

Yukawa couplings involving the doublet  $H$  in (1.1). This leads to Yukawa couplings between the Higgs particle and the SM fermions. The same substitution in the Higgs potential in (1.3) leads to self couplings of the Higgs particle. The strength of the Yukawa coupling of the Higgs particle to a given fermion is directly proportional to the mass of the fermion, due to the common origin of the mass and the coupling. For example, the largest Yukawa coupling of  $h$  is to the top:

$$\mathcal{L}_Y \supset \lambda_t \bar{t} t h. \quad (1.6)$$

The dimensionless number  $\lambda_t$  is called the top Yukawa coupling, and is an order 1 number.

Having briefly reviewed the SM, we now discuss one of the problems in the SM, the hierarchy problem. In the SM, the mass squared  $m_h^2$  for the Higgs particle  $h$  is sensitive quadratically to the cut-off of the theory. The one loop correction to  $m_h^2$  gets its biggest contribution from the top loop (see figure 1.1), which scales as

$$\delta m_h^2 \sim \frac{\lambda_t^2}{16\pi^2} \Lambda_{UV}^2, \quad (1.7)$$

where  $\lambda_t \sim 1$  is the top Yukawa coupling and  $\Lambda_{UV}$  is the UV cutoff. In the absence of any new dynamics beyond the SM involving the Higgs particle, this cut-off scale is the Planck scale  $M_{pl} \sim 10^{19}$  GeV. In that case, the loop corrections are of the order  $10^{36}(GeV)^2$ . To get a mass close to the EW scale ( $\sim 100$  GeV), the tree level contribution to the mass squared of  $h$  must be fantastically tuned against the loop contribution, by more than 30 orders of magnitude! This is the source of the hierarchy problem. Since the mass of  $h$  is tied to  $v$  and therefore to  $M_{EW}$  where the EW symmetry is broken to EM, this translates the hierarchy problem to the

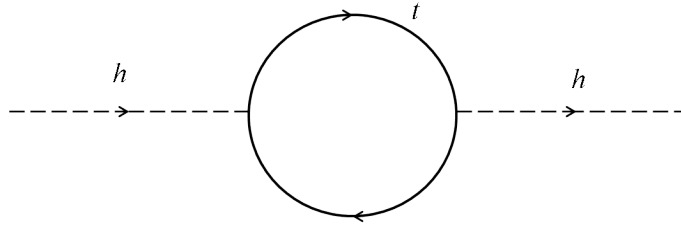


Figure 1.1: The top loop contribution to the self energy of the Higgs in the Standard Model.

unnatural separation between the Planck scale and the EW scale, or to the extreme weakness of gravity compared to the weak force. BSM models that address the hierarchy problem posit new dynamics at a scale  $M_{new}$ . This effectively means that SM is to be understood as an effective theory below this scale. If this new scale is not too far from the EW scale, the hierarchy problem of SM is addressed.

BSM models that address the hierarchy problem however generically generate additional contributions to FCNC effects from physics near the scale  $M_{new}$ . The experimental bounds on such processes are in close agreement with the theoretical results obtained within the SM. Therefore, the additional contribution to the FCNCs must be suppressed in realistic scenarios. This can be done by pushing  $M_{new}$  up, but this then starts to “un-solve” the hierarchy problem that motivated the BSM model at the first place. This tension must be addressed by a realistic model. In this sense, flavor constraints provide a useful guide to the BSM models that address the hierarchy problem. The class of BSM models that are considered in this dissertation present a way to address the hierarchy problem and the flavor constraints at the same time. The ability to deal with the apparent tension between these two effects

is an attractive feature of these BSM models that are based on conformal dynamics.

## 1.2 A CFT primer

In this section, we review some relevant aspects of conformal symmetry in the context of quantum field theories. The aim is to explain some of the terminology used in the later parts of this work.

Conformal symmetry is a spacetime symmetry defined by its action on  $4D$  Minkowski spacetime. Consider general infinitesimal spacetime transformations of the form  $x^\mu \rightarrow x'^\mu = x^\mu + \delta x^\mu$ . Then, conformal transformations are defined as those transformations under which the Minkowski metric  $\eta_{\mu\nu} \equiv \text{diag}\{1, -1, -1, -1\}$  transforms as  $\eta_{\mu\nu} \rightarrow f(x)\eta_{\mu\nu}$  where  $f(x)$  is a general function of spacetime. In 4 spacetime dimensions, this requirement fixes the number of generators of infinitesimal conformal transformation to be 15. These generators are the spacetime translations  $P_\mu$ , Lorentz rotations  $M_{\mu\nu}$ , dilatations  $S$ , and special conformal transformations  $K_\mu$ . More specifically, these transformations act on spacetime as

$$\begin{aligned}
 \delta_{P_\mu} x^\nu &= \delta_\mu^\nu \\
 \delta_{J_{\mu\nu}} x^\rho &= \delta_\mu^\rho x_\nu - \delta_\nu^\rho x_\mu \\
 \delta_S x^\nu &= x^\nu \\
 \delta_{K_\mu} x^\nu &= x^2 \delta_\mu^\nu - 2x_\mu x^\nu,
 \end{aligned} \tag{1.8}$$

and form a closed algebra. Quantum field theories that are invariant under conformal symmetry are given the name of conformal field theories (CFTs). The action of the transformations on the Hilbert space of the CFT is realized by Hermitian operators

in the usual way. In addition to the Poincare algebra,

$$\begin{aligned}
[J_{\mu\nu}, P_\rho] &= -i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu) \\
[J_{\mu\nu}, J_{\rho\sigma}] &= -i\eta_{\mu\rho}J_{\nu\sigma} \pm \text{permutations} \\
[P_\mu, P_\nu] &= 0,
\end{aligned} \tag{1.9}$$

we have the additional commutation relations

$$\begin{aligned}
[S, K_\mu] &= iK_\mu \\
[S, P_\mu] &= -iP_\mu \\
[J_{\mu\nu}, K_\rho] &= -i(\eta_{\mu\rho}K_\nu - \eta_{\nu\rho}K_\mu) \\
[J_{\mu\nu}, S] &= 0 \\
[P_\mu, K_\nu] &= 2iJ_{\mu\nu} - 2i\eta_{\mu\nu}S.
\end{aligned} \tag{1.10}$$

The algebra of conformal transformations acts linearly on the local operators of the CFT, by commutation. The irreducible representations are labeled by *primary* operators  $\mathcal{O}_n(x)$ , which are also in irreducible representations of the Lorentz algebra. The subscript  $n$  here represents a possible Lorentz index under which the primary operator transforms, depending on whether it is a scalar or has non-zero spin.

A primary local operator  $\mathcal{O}_n(x)$  transforms under conformal symmetry by the

action of the generators as

$$\begin{aligned}
[P_\mu, \mathcal{O}_n(x)] &= i\partial_\mu \mathcal{O}_n(x) \\
[J_{\mu\nu}, \mathcal{O}_n^\alpha(x)] &= [i(x_\mu\partial_\nu - x_\nu\partial_\mu)\delta_\beta^\alpha + (\Sigma_{\mu\nu}^n)_\beta^\alpha] \mathcal{O}_n^\beta(x) \\
[S, \mathcal{O}_n(x)] &= -i(\Delta_n + x \cdot \partial) \mathcal{O}_n(x) \\
[K_\mu, \mathcal{O}_n^\alpha(x)] &= -i(x^2\partial_\mu - 2x_\mu x \cdot \partial - 2x_\mu\Delta_n) \mathcal{O}_n^\alpha(x) - 2x^\nu (\Sigma_{\mu\nu}^n)_\beta^\alpha \mathcal{O}_n^\beta(x), \quad (1.11)
\end{aligned}$$

where  $\alpha, \beta$  are the indices of the Lorentz representation of  $\mathcal{O}_n$  and  $\Sigma_{\mu\nu}^n$  are the corresponding matrices (in  $\alpha, \beta$ ). The quantity  $\Delta_n$  is called the *primary scaling dimension* of the operator  $\mathcal{O}$ . The last result in the commutation relations in (1.11) defines a primary operator – at the origin  $x = 0$ , the primary operator is annihilated by the special conformal generator  $K_\mu$ . The irreducible representation labeled by the primary operator contains other operators called *descendants*, which are given by derivatives of the primary operator. The set of primary operators, labeled by its scaling dimension and spin, along with all its descendants span the space of all local operators of the CFT.

The scaling dimension of a primary operator is a real number bounded below by unitarity [27]. For example, for a scalar primary,  $\Delta \geq 1$ . Notice that the scaling dimension is independent of the canonical (engineering) dimension of the operator  $\mathcal{O}$ . However, for convenience, all operators are multiplied with a suitable power of the renormalization scale to make their scaling dimension equal to their canonical dimension. We will therefore assume in the rest of the discussions that the scaling dimension of the operator is same as its canonical dimension.

In theories where an exact conformal symmetry is spontaneously broken to



Poincare symmetry, the low energy effective theory below the breaking scale contains a massless scalar, the dilaton  $\sigma(x)$ . This scalar is the NGB associated with the breaking of conformal symmetry [7–10] to Poincare symmetry. Even though the number of broken generators are 5, only one Goldstone boson results in the spectrum. This is because the rules of Goldstone boson counting are different in the case of spacetime symmetries.

To be more precise, consider the case of dilatations  $S$ , under which  $x \rightarrow e^{-\omega}x$ . Denoting the current associated with dilatations as  $S^\mu$ , it is possible to define an energy-momentum tensor  $\Theta^{\mu\nu}$  such that

$$S^\mu = x_\nu \Theta^{\mu\nu}. \tag{1.12}$$

This should not be a surprise, as the currents associated with Lorentz transformations and translations can also be written in terms of the energy-momentum tensor. The tensor  $\Theta^{\mu\nu}$  is a symmetric tensor constructed out of the regular energy-momentum tensor, and is called the improved energy-momentum tensor [28].

Given the dilatation current, it is straightforward to see that  $\partial_\mu S^\mu = \Theta_\lambda^\lambda$ . If a theory possesses scale symmetry, the dilatation current must have a zero divergence. Therefore, for scale invariant theories, the energy-momentum tensor must be traceless. If the symmetry is realized non linearly, the low energy theorems that result from Ward identities can be used to obtain the matrix element of operators with one insertion of the trace of the improved energy momentum tensor at zero momentum transfer. This trace  $\Theta_\lambda^\lambda$  is proportional to the dilaton field  $\sigma$ . In this way, matrix elements with one insertion of the dilaton field can be calculated.

It turns out that the currents associated with special conformal transformations  $K^\mu$ , which we will denote by  $K^{\lambda\mu}$  can also be obtained in terms of  $\Theta^{\mu\nu}$  as

$$K^{\lambda\mu} = x^2\Theta^{\lambda\mu} - 2x^\lambda x_\rho\Theta^{\rho\mu}. \quad (1.13)$$

It is again straightforward to see that the divergence of the currents  $K^{\lambda\mu}$  vanish,

$$\partial_\mu K^{\lambda\mu} = 2x_\mu\Theta^{\lambda\mu} - 2x_\rho\Theta^{\rho\lambda} - 2x^\lambda\Theta_\rho^\rho = 0, \quad (1.14)$$

by the symmetric nature of  $\Theta$  and its tracelessness due to dilatation symmetry.

In the case when the symmetry is broken, the divergence of the currents associated with the special conformal transformation is proportional to the trace of the improved energy-momentum tensor,  $\partial_\mu K^{\lambda\mu} = -2x^\lambda\Theta_\rho^\rho$ . This is where the crucial difference appears that changes the Goldstone boson counting. The low energy theorems that are obtained by Fourier transforming the Ward identities, now give information about the derivative of the momentum space matrix elements of operators with one insertion of  $\Theta_\mu^\mu$ . (This is because multiplication by  $x^\lambda$  in position space translates to derivative w.r.t  $p^\lambda$  in the momentum space). Because  $\Theta_\lambda^\lambda$  is proportional to the dilaton field  $\sigma$ , the low energy theorems give information about the derivative of the matrix elements involving one  $\sigma$  field. This is completely unlike the case of internal symmetries. We therefore see that only one field  $\sigma$  suffices to parametrize the Goldstone modes in the this case, even though there are five sets of relations concerning its matrix elements, in accordance with five broken generators. For a more elaborate and pedagogical discussion of this topic, see [29, 30].

The effective theory of the dilaton below the conformal symmetry breaking scale can be constructed by the requirement that the symmetry be realized non-

linearly. For convenience, a quantity  $\chi(x)$  can be constructed which is defined in terms of the dilaton field  $\sigma$ . Even though the dilaton field itself transforms non linearly,  $\chi$  transforms linearly. This makes the construction of the effective theory much more straightforward. We will go into more details of this process in chapter 2, and therefore postpone the discussions till that point.

### 1.3 AdS/CFT correspondence

The AdS/CFT correspondence relates the theory of gravity in  $d+1$  dimensional Anti-de Sitter space to a conformal field theory in  $d$  spacetime dimensions [15–18]. We will focus here on the case of  $d = 4$ . More specifically, for a given operator  $\mathcal{O}$  in a  $4D$  CFT, there is a corresponding  $5D$  AdS field  $\Phi$  such that for a boundary value  $\Phi_0(x)$  of  $\Phi$  at the  $4D$  boundary of AdS, there is a unique solution to the equations of motion in the bulk. Representing by  $\Gamma[\Phi_0]$  the effective action associated with this solution, the correspondence relates the partition function on the CFT side and the effective action on the AdS side as

$$\left\langle \exp \left( - \int d^4x \Phi_0 \mathcal{O} \right) \right\rangle_{CFT} = \exp (-\Gamma[\Phi_0]) . \quad (1.15)$$

If the CFT is a strongly coupled  $SU(N)$  gauge theory and admits a large  $N$  expansion, the corresponding AdS fields become weakly coupled. A given  $5D$  AdS field in this limit is in one to one correspondence with a gauge invariant color singlet primary CFT operator. In addition, if there exists a gap in scaling dimension of these primary operators, the description of the conformal dynamics involving the low scaling dimension operators corresponds to an effective theory on the AdS side

with corresponding finite number of AdS particles. In this way, a dictionary can be built which relates quantities on one side to those on the other side. A particularly important class of such entries in the dictionary relates the scaling dimension of CFT operators to the  $5D$  mass of the corresponding AdS fields. For a modern review of this topic, see [31].

The correspondence goes beyond the vanilla case of an exact CFT being dual to an exact AdS gravitational theory, thereby making it much more powerful. For example, if the conformal symmetry is only approximate, or the vacuum is not invariant under conformal symmetry in a given CFT, the corresponding AdS theory can still be constructed [20, 32]. The correspondence also respects additional structure on either side of the duality, e.g., CFTs with additional global symmetries are dual to AdS theories with spin 1 gauge fields propagating in the fifth dimension. These robust features make the correspondence much more useful both theoretically as well as practically.

Using the duality, the AdS construction can provide a holographic realization of the corresponding CFT dynamics. Further more, because the correspondence relates a strongly coupled theory on one side to a weakly coupled theory on the other, a quantity of interest can be calculated on the side which offers more calculational simplicity, and then using the dictionary, can be translated to the result on the other side. In this way, the duality can be used to provide non-trivial checks on the results obtained on either side. In the wake of the general excitement in BSM models that involve strong conformal dynamics, the duality assumes more interest, because the duality can be used to obtain weakly coupled realizations of these scenarios, allowing

quantities of interest to be calculated.

## 1.4 Beyond Standard Model scenarios involving Strong Conformal Dynamics

In this section, we outline several BSM scenarios based on strong dynamics that address the hierarchy problem of the SM. We elaborate on the shortcomings of the earlier versions of these scenarios, in the process motivating the variants. Finally alluding to the flavor constraints, we elaborate how strong dynamics with conformal symmetry can address both the hierarchy problem and the flavor constraints.

As explained earlier, the hierarchy problem in the SM is related to the large separation between the EW scale and the Planck scale. The SM also contains another scale called  $\Lambda_{QCD} \sim 200 MeV$ , which is the scale where the  $SU(3)_c$  interactions become strong. A natural question to ask then is: why is there no hierarchy problem associated with this scale  $\Lambda_{QCD}$ ? The answer to this puzzle lies in the fact that this scale emerges from strong dynamics. The QCD coupling  $g$  is small and perturbative at very high energies, near the UV cutoff  $\Lambda_{UV}$  of the theory. At lower energies, this coupling grows, till it becomes close to the critical value  $g_c \sim 4\pi$ , where QCD becomes strongly coupled. In such a scenario, the UV scale and the QCD scale are related as

$$\Lambda_{QCD} \sim \Lambda_{UV} e^{-g_c^2/g_{UV}^2} . \quad (1.16)$$

Since this is an exponential relation, a large hierarchy can be achieved if  $g_{UV} \ll g_c \sim 4\pi$ .

Using this mechanism, a class of theories was put forth that required that the EW scale  $M_{EW}$  emerge in a similar way, thereby addressing the hierarchy problem. This class of theories is referred as Technicolor (TC) theories [4, 33, 34]. These theories are based on  $SU(N)_{TC}$  gauge theories at high energies, with fundamental fermions (called technifermions), that get strongly coupled at some low energy scale  $\Lambda_{TC} \sim M_{EW}$ . The technifermions acquire a mass due to this effect, in analogy with QCD. Continuing the analogy further, the technifermions have a chiral symmetry, which is realized non-linearly below this scale. A  $SU(2) \times U(1)$  subgroup of this chiral symmetry group is weakly gauged, and is identified with the SM gauge groups  $SU(2)_L \times U(1)_Y$ . This results in three linear combinations of Goldstone bosons being eaten by the gauge bosons  $W^\pm$  and  $Z$ , giving them a mass in the process. In this way, the necessity of a Higgs boson is eliminated.

However, the Higgs boson in the SM performs another task – of transmitting EW symmetry breaking to the fermions, giving them their observed mass. To incorporate this, TC theories are generalized to Extended technicolor (ETC) theories [35, 36], where the technifermions and the SM fermions transform in the same ETC gauge group, which contains the TC gauge group as a subgroup. The ETC gauge group is broken at the scale  $\Lambda_{ETC}$  to the TC gauge group, below which, the spectrum contains the technifermions charged under the TC gauge group, and the SM fermions. In this scenario, the SM fermions are outside the TC strong sector, and get mass by an operator of the form

$$\frac{(\bar{\psi}\psi)_{SM} (\bar{\psi}\psi)_{TC}}{\Lambda_{ETC}^2} . \tag{1.17}$$

At the scale  $\Lambda_{TC}$ , the operator  $(\bar{\psi}\psi)_{TC}$  acquires a VEV and gives the SM fermion a mass. If this VEV is related to the EW scale, it must scale as  $M_{EW}^3 \sim (4\pi)^3(246)^3 GeV^3$  from dimensional analysis. To get a realistic mass for the heaviest fermion (top), we therefore require that  $\Lambda_{ETC} \lesssim 10$  TeV.

The dynamics that generates the operator in (1.17) will in general also generate operators of the form

$$\frac{(\bar{\psi}\psi)_{SM} (\bar{\psi}\psi)_{SM}}{\Lambda_{ETC}^2}. \quad (1.18)$$

However, these operators contribute to FCNCs at tree level. The most stringent bound on these operators come from  $K - \bar{K}$  mixing, which require that  $\Lambda_{ETC} \geq 10^5 TeV$ . Therefore the simplest form of ETC models are ruled out by FCNC considerations. The large scaling dimension (in this case 3) of the operator that acquires a VEV to break the EW symmetry is the root behind the cause.

To address these issues, a variant of ETC theories was proposed, which goes by the name of walking technicolor (WTC) [37–40]. In these theories, the dimension of the operator  $(\bar{\psi}\psi)_{TC}$  receives large negative anomalous contribution to its scaling dimension, which effectively reduce its strength at the EW symmetry breaking scale. To achieve this, it is assumed that the theory is near a non-trivial fixed point. In spite of initial promise, progress in this direction has been limited by the difficulty in dealing with a strongly coupled theory. A truncation of Schwinger-Dyson equations [41] for the self energy of the operator  $(\bar{\psi}\psi)_{TC}$ , called as gap equations, have led to some progress in this direction. However, the approximations leading to gap equations are not fully defensible. Further, for asymptotically free gauge theories,

gap equations suggest that the operator  $(\bar{\psi}\psi)_{TC}$  must have at least dimension 2 at the quasi-fixed point [42]. In that case, some tension still remains between the requirements to generate a realistic mass for the heavy fermions and to address the flavor constraints. A precise way to handle this scenario therefore remains to be arrived at. For a review of these and related topics, see [43].

Conformal dynamics can play an important role in addressing the flavor constraints in BSM scenarios that involve strong dynamics. In theories of TC, if the dynamics that breaks EW symmetry is part of a conformal sector, the hierarchy problem and the flavor constraints can be addressed simultaneously. Such theories are studied under conformal technicolor [6]. In these models, the SM fermion mass is generated by an operator of the form

$$\frac{1}{\Lambda^{d-1}} (\bar{\psi}\psi)_{SM} \mathcal{H}, \quad (1.19)$$

where  $d$  is the scaling dimension of the operator  $\mathcal{H}$  and  $\Lambda$  is the scale of new physics. The operator  $\mathcal{H}$  is part of a conformal sector, and carries the right quantum numbers to break the EW symmetry. The situation in the SM corresponds to the case of  $d = 1$ , when  $\mathcal{H}$  is elementary. In the class of technicolor models,  $\mathcal{H} = (\bar{\psi}\psi)_{TC}$  can have  $d = 3$  (Technicolor) or  $d < 3$  (Walking Technicolor). When  $\mathcal{H}$  acquires a VEV, the SM fermions get a mass. In the absence of any symmetry reasons, the same dynamics also generates operators of the form

$$\frac{1}{\Lambda^2} (\bar{\psi}\psi)_{SM}^2, \quad (1.20)$$

that can contribute to the FCNCs. To satisfy the flavor constraints, even if  $\Lambda$  is made as high as  $10^5 TeV$ , a value of  $d$  close to 1.3 in (1.19) can still give the fermions a



realistic mass [6]. Such values of  $d$  are allowed because  $\mathcal{H}$  is an operator of a strongly coupled CFT. At the same time, the hierarchy problem can be addressed if the dimension of the operator  $\mathcal{H}^\dagger\mathcal{H}$  is close to 4. These possibilities have been studied in a model independent way using general properties of CFTs such as unitarity and Operator Product Expansion (OPE) in [44–49]. In such scenarios, the SM fermions do not constitute part of the strong conformal sector. However, in some variants, the heavier fermions such as the top are made part of the strong sector.

As we saw earlier, in the simplest TC models involving strong conformal dynamics, the conformal symmetry breaking also breaks the EW symmetry. In these models, an operator from the strong conformal sector accomplishes the task of the Higgs doublet in the SM. The low energy spectrum does not contain any Higgs like scalar resonances. In a closely related variant, called the *Composite Higgs* scenario, the EW symmetry is still broken by a scalar Higgs doublet at scale  $v$ , but the Higgs is a composite resulting from the breaking of the strong dynamics, at a higher scale  $f$ . In this scenario, the hierarchy problem can still be addressed. Since the Higgs is a composite, the loop corrections to the self energy of the Higgs are cutoff at the scale  $4\pi f$ . A small amount of tuning is still needed generically to get a light Higgs in the spectrum, because the mass of the Higgs is generically of the order of  $4\pi f$ .

To get a light Higgs naturally, a further variant of this scenario exists. In the scenario called as *Higgs as a pNGB*, the Higgs field emerges as the pseudo-Nambu-Goldstone boson (pNGB) of a global symmetry that is broken by strong dynamics [50–52]. Making the global symmetry approximate results in a potential for the Higgs field. In this way, a light SM like Higgs in the low energy theory

can be arranged. This potential of the Higgs below the symmetry breaking scale is such that the Higgs acquires a VEV, like in the SM. Fluctuations about this VEV lead to a light Higgs particle in the spectrum. This consideration attains more plausibility in light of the recent discovery of a Higgs like particle at mass close to 125 GeV. This class of theories includes little Higgs models [53–55] and twin Higgs models [56, 57]. If the dynamics above the scale at which the global symmetry is broken involves strong conformal dynamics, the flavor scale can again be separated from the electroweak scale, allowing new contributions to flavor violating processes to be small enough to satisfy the existing constraints.

BSM models with strong conformal dynamics in the UV can also provide a way to explain the fermion mass hierarchy of the SM. In the scenario called as Partial Composite (PC) [58], the SM fermions do not constitute part of the conformal sector. However, they acquire a mass by mixing to the composites of the conformal sector. Depending on the dimension of the composite operator with which they mix, the resulting mass can be made to vary by several orders of magnitude [58]. This allows a simple explanation of the observed hierarchy in the fermion masses, making these class of models even more appealing.

We therefore see that conformal dynamics can play an important role in BSM models that try to address the hierarchy problem while remaining consistent with flavor constraints. We explained earlier that if the conformal symmetry is broken spontaneously, the spectrum below the breaking scale contains the NGB, the dilaton. The form of the dilaton couplings is fixed by the requirement that conformal symmetry be realized nonlinearly. In the context of realistic BSM models, the theory

at high energies has only have an approximate conformal symmetry, which is spontaneously broken at low energies. This results in a mass for the dilaton in the low energy theory, and affects its couplings to other light states. It becomes important to understand the conditions under which the dilaton can be light, and its couplings to the SM fields in these scenarios. This is because if the dilaton is light, it may be accessible at the present colliders, and would signal involvement of conformal dynamics in the real world. However, in these BSM models, conformal symmetry is expected to be *explicitly* broken by operators that grow in the infrared to become strong at the breaking scale. Therefore, at least naively, there is no reason to expect a light dilaton in the low energy effective theory. This explicit breaking could also significantly affect the predictions for the couplings of the dilaton in a spontaneously broken CFT. This motivates a theoretical study of the mass and couplings of the dilaton in realistic scenarios.

The couplings of a light dilaton in the context of theories of electroweak symmetry breaking have been studied before [20, 59, 60]. Remarkably, the interactions of a dilaton with the SM fields are very similar to those of the SM Higgs [59]. This can be traced to the fact that at the classical level the SM has an approximate conformal symmetry which is spontaneously broken by the VEV of the Higgs, so that the Higgs can be understood as a dilaton in this limit. A theoretical study of the form of the couplings of the dilaton assumes further significance now, because only a precise understanding can allow a clear distinction between a dilaton and a Higgs at particle colliders. One possibility is that the new particle that has been observed [25, 26] close to 125 GeV is not the SM Higgs, but instead a dilaton that

emerges from a strongly interacting conformal sector that breaks electroweak symmetry dynamically [61–65]. In such a scenario, an understanding of the general form of the dilaton couplings, including conformal symmetry violating effects, is crucial to distinguishing it from the SM Higgs [65]. Another possibility is that the new particle which has been observed is indeed the SM Higgs, which arises as the pNGB of an approximate global symmetry that is broken by strong conformal dynamics. In such a scenario, there may be an additional light scalar in the low energy effective theory beyond the SM Higgs whose couplings to the SM fields can be predicted. These considerations form the motivation of our work [11, 21], on which this dissertation is based.

## 1.5 Beyond Standard Model scenarios involving Warped Extra-dimensions

Around the time when AdS/CFT correspondence was conjectured, Randall and Sundrum proposed a mechanism to address the hierarchy problem using extra dimensions [19]. The class of BSM theories that incorporate this mechanism are studied under the name of RS models. Via the AdS/CFT correspondence, RS models are closely related to BSM theories based on conformal dynamics. In this section, we develop the basic setup of the RS models and elaborate some of its important features. In particular, we outline how they address the hierarchy problem in the SM. We then use the duality to relate to the corresponding features of BSM scenarios involving conformal dynamics that are dual to the RS models.

In the RS models, a slice of AdS space in  $4 + 1$  dimensions is considered. The

metric in this space can be written as

$$ds^2 = e^{-2kr_c|\theta|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\theta^2, \quad (1.21)$$

where the extra-dimensional coordinate  $\theta$  goes from  $-\pi$  to  $\pi$ . The range  $(-\pi, 0]$  is identified with the range  $(0, \pi]$  using  $\mathcal{Z}_2$  orbifolding. In the set-up, the location  $\theta = 0$  and  $\theta = \pi$  are the location of the hidden/ultraviolet (UV) and the visible/infrared (IR) branes respectively. The exponential factor  $\exp(-2kr_c|\theta|)$  in (1.21) is called the warp factor of this geometry. Such a geometry can be obtained as a static solution of Einstein equations in the following way. Consider the 5D gravity action with a cosmological constant in the bulk and brane tensions on the two branes,

$$\mathcal{S} = \int d^4x \int_{-\pi}^{\pi} d\theta \left[ \sqrt{G} (-2M_5^3 \mathcal{R}[G] - \Lambda_b) - \sqrt{-G_h} \delta(\theta) T_h - \sqrt{-G_v} \delta(\theta - \pi) T_v \right]. \quad (1.22)$$

Here  $M_5$  is the 5D Planck scale,  $\Lambda_b$  is the bulk cosmological constant and  $T_h, T_v$  are the brane tensions on the hidden and the visible branes respectively.  $G_h$  and  $G_v$  are the induced metrics, obtained from the full background metric  $G$ . Using an ansatz for the metric  $G$  that respects 4 dimensional Poincare invariance in the  $x^\mu$  coordinates, such that

$$ds^2 = e^{-2\sigma(\theta)} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\theta^2, \quad (1.23)$$

the Einstein equations reduce to

$$\begin{aligned} \frac{6\sigma'^2}{r_c^2} &= \frac{-\Lambda_b}{4M_5^3} \\ \frac{3\sigma''}{r_c^2} &= \frac{T_h \delta(\theta)}{4M_5^2 r_c} + \frac{T_v \delta(\theta - \pi)}{4M_5^2 r_c}. \end{aligned} \quad (1.24)$$

The solution of these equations, consistent with the  $\mathcal{Z}_2$  orbifold symmetry, under which  $\theta \rightarrow -\theta$ , is given by

$$\sigma = r_c |\theta| \sqrt{\frac{-\Lambda_b}{24M_5^3}}. \quad (1.25)$$

The additive integration constant in (1.25) is absorbed by a constant rescaling of the  $x^\mu$  coordinates. The delta functions in the Einstein equation in (1.24) force the value of  $T_h, T_v$  to be

$$T_h = -T_v = 24M_5^3 \sqrt{\frac{-\Lambda_b}{24M_5^3}}. \quad (1.26)$$

For convenience, we define

$$k = \sqrt{\frac{-\Lambda_b}{24M_5^3}}, \quad (1.27)$$

which is identified as the curvature of AdS geometry. The quantities  $\Lambda_b$  and  $T_h, T_v$  are then related to  $M_5$  and  $k$  as  $T_h = -T_v = -\Lambda_b/k = 24M_5^3 k$ . Figure 1.2 illustrates the spacetime structure of the geometry.

Given this static background geometry, the natural step is to consider the gravitational fluctuations about this background. To be able to use this geometrical construction for a real world scenario, we must be able to recover  $4D$  gravity in the low energy limit. Consider small fluctuations about the full RS metric in (1.21). Without going into explicit details of the calculation, we outline the steps involved in getting the spectrum in the low energy. By expanding the  $5D$  graviton as a linear combination of  $4D$  fields, we can obtain all the fields present in the  $4D$  theory. This expansion is generically called as Kaluza-Klein (KK) decomposition of a  $5D$  field into  $4D$  fields. Under this expansion, all the  $4D$  fields get a profile in the extra dimension.

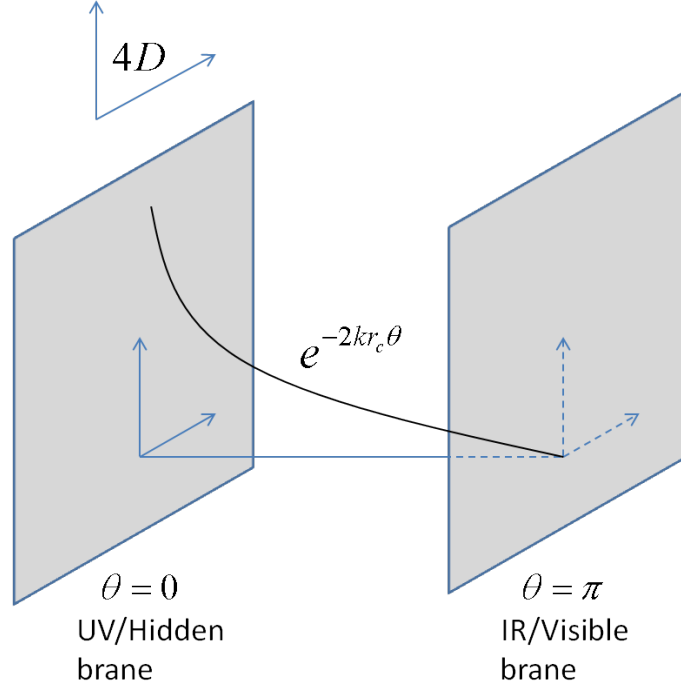


Figure 1.2: The geometry of the two brane RS model. The location of the hidden and the visible branes are shown by the shaded planes and the warp factor of the geometry is shown by the solid curve. The space between the branes is referred as the bulk region.

The profile is a function of the extra dimension, and is the coefficient multiplying the corresponding  $4D$  field in the KK decomposition. For the case of  $5D$  graviton, these  $4D$  fields include the massless mode  $h_{\mu\nu}$  corresponding to  $4D$  graviton, and other heavier modes referred to as KK gravitons. The  $4D$  graviton field  $h_{\mu\nu}$  has a profile given by  $e^{-2kr_c|\theta|}$ , and therefore corresponds to replacing  $\eta_{\mu\nu} \rightarrow g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$  in the  $5D$  action. Substituting the metric with this replacement into the action and focusing on the curvature term, we have

$$\mathcal{S}_{eff} \supset \int d^4x \int_{-\pi}^{\pi} d\theta 2M_5^3 r_c e^{-2kr_c|\theta|} \sqrt{-g} \mathcal{R}[g], \quad (1.28)$$

where  $\mathcal{R}[g]$  is the Ricci scalar constructed from the metric  $g$ . This part of action

has the same form as of  $4D$  gravity, with  $h_{\mu\nu}$  being the corresponding  $4D$  graviton. Performing the  $\theta$  integral, we can therefore relate the  $4D$  Planck scale  $M_4$  to the parameters of the  $5D$  geometry,

$$M_4^2 = \frac{M_5^3}{k} (1 - e^{-2kr_c\pi}) . \quad (1.29)$$

In the limit of  $k\pi r_c \ll 1$ , the relation simplifies to  $M_4^2 = M_5^3/k$ .

The massive  $4D$  fields that are obtained as a result of the KK decomposition, the KK gravitons, have a mass of the order

$$m_n \sim n \pi k e^{-kr_c\pi} \quad n = 1, 2, \dots \quad (1.30)$$

The form of interactions between these KK gravitons is fixed by  $5D$  gravity action, when written in terms of these  $4D$  fields. The overlap integral involving the profiles sets the coefficients of the interaction terms involving the KK gravitons. The existence of these new spin 2 resonances is a robust prediction of the RS scenario, and may be the first signal of these models in the present colliders [66].

Once the SM fields are included in a QFT based on this background geometry, this construction can serve as a BSM scenario that addresses the hierarchy problem. Such models are called as the RS models. In its earliest version, all the SM fields including the Higgs were considered localized on the IR brane. The later variants kept the Higgs localized on or towards the IR brane, but allowed the fermion and the gauge fields to propagate in the extra-dimension.

The warp factor in the geometry can be used to solve the hierarchy problem for a scalar field such as a Higgs. Consider the case of a brane localized Higgs  $H$ .



The relevant part of the action is given by

$$\mathcal{S} = \int d^4x \sqrt{-G_v} [D_\mu H^\dagger D_\nu H G_v^{\mu\nu} - \lambda(|H|^2 - v^2)^2] . \quad (1.31)$$

The action has one dimensionful parameter  $v$ . Using the explicit form of the induced metric,  $G_v^{\mu\nu} = \exp(-2kr_c\pi)\eta^{\mu\nu}$ , the action takes the form

$$\int d^4x e^{-4kr_c\pi} [e^{2kr_c\pi} D_\mu H^\dagger D_\nu H \eta^{\mu\nu} - \lambda(|H|^2 - v^2)^2] . \quad (1.32)$$

After wavefunction renormalization  $H \rightarrow e^{kr_c\pi} H$ , which ensures a canonical  $4D$  kinetic term for the Higgs, we are left with

$$\int d^4x [D_\mu H^\dagger D_\nu H \eta^{\mu\nu} - \lambda(|H|^2 - (ve^{-kr_c\pi})^2)^2] . \quad (1.33)$$

We see that the mass parameter  $v$  is warped down. This is a rather robust feature of this model, where any mass parameter in the fundamental higher dimensional theory appears warped down to an observer on the IR brane. Since the mass parameter appearing in the Higgs potential sets the EW scale, an exponential hierarchy can be achieved between the EW scale and the fundamental cutoff of the theory, say Planck scale. Even if  $v$  is of the order of  $M_{pl}$ , it is possible to have  $ve^{-kr_c\pi} \sim M_{EW}$  if  $kr_c \sim 10$ . Since there is no parametric hierarchy between the parameters  $k, r_c$ , such values of  $kr_c$  are not unnatural. In this way, an order one tuning of the parameters in the RS geometry can generate the large hierarchy between the Planck scale and the weak scale. This is the celebrated solution to the hierarchy problem in RS models.

The original RS proposal, did not explain how to get such a value of  $k\pi r_c$ . The quantity  $r_c$  is a free parameter in the RS model, and is associated with a modulus field, the radion. In the absence of a mechanism that gives a VEV to the radion,

thereby fixing a value for  $r_c$ , the radion is a massless  $4D$  field in the low energy theory and corresponds to the freedom to move the distance between the two branes. Such a situation is not acceptable phenomenologically, because experimental results severely restrict the presence of a massless scalar. However, it was shown later by Goldberger and Wise [67] that using a form of potential for a  $5D$  scalar field  $\Phi$ , a potential for the radion can be generated. This allows the VEV  $r_c$  to be non-zero, and gives the radion a mass. In this way, a realistic value of  $kr_c\pi$  can be obtained which addresses the hierarchy problem. We will not outline the mechanism here, but we will go into it in more detail in chapter 3 of this dissertation.

This geometric approach to solve the hierarchy problem can be cast into a more familiar language. In the geometry given by (1.21), the cutoff of a  $4D$  QFT localized on a brane depends on the location of the brane in the extra dimension. In particular, the cut-off at the UV and the IR branes are related by

$$\Lambda_{IR} = \Lambda_{UV} e^{-kr_c\pi} . \quad (1.34)$$

If the Higgs field is localized to the IR brane, the loops that contribute to the self energy of the Higgs particle are cutoff at the scale  $\Lambda_{IR}$ . For  $\Lambda_{IR}$  of order the weak scale  $M_{EW}$ , the hierarchy problem can be solved. The low energy effective theory below the IR cutoff contains the low lying KK gravitons, that get strongly coupled at the cutoff. In this way, the cutoff on the IR brane can be estimated.

RS models with the SM localized on the IR brane however quickly runs into severe problems. Because the cutoff on the IR brane is lowered all the way down, close to the EW scale, the higher dimensional operators that one can construct from

SM fields are also suppressed by the same scale. Such operators are those associated with flavor changing processes, which get enhanced tremendously by this lowering of the cutoff. This leads to a conflict with the flavor constraints.

To maintain this elegant solution to hierarchy problem while still suppressing the unwanted higher dimensional operators, the Higgs field is kept localized on the IR brane, while the other SM fields are allowed to propagate in the bulk of the space. This is because unlike the fermion and the gauge boson masses, only the Higgs mass suffers from the hierarchy problem. Once the fields are allowed to propagate in the bulk, the unwanted flavor violating operators are suppressed. In addition, RS models in this scenario can also address the fermion mass hierarchy puzzle [68–70] in a natural way. Let us briefly outline these two features now.

In the case when the  $5D$  fermion fields propagate in the bulk, they are KK decomposed into a set of  $4D$  mass eigenstates. The zero modes are made massless by  $Z_2$  orbifolding and are identified with the SM fermions before EW symmetry breaking. The KK modes have masses of the order of the warped down curvature. The zero mode and the KK modes get a profile in the extra dimension after the KK decomposition. The profile of the zero mode can be made to be exponentially localized to the UV or the IR brane by a small dialing of the  $5D$  mass parameter of the corresponding fermion. The profiles of the KK modes are all localized towards the IR brane. This universal localization of the profile is closely related to the fact that their masses are of the order of the cut-off on the IR brane.

The hierarchy problem continues to be solved if the Higgs profile remains localized on the IR brane. This scenario can now address the fermion mass hierarchy

in a natural way. The mass of the 4D SM fermion is proportional to the overlap between the Higgs profile and the corresponding zero mode profile. Therefore by a small dialing of the 5D parameters, one can move the zero mode profile towards (away) from the IR brane and generate a heavy (light) fermion. In this way, the desired fermion mass hierarchy of several orders of magnitude can be obtained by an order 1 tuning of the 5D mass parameters. This approach also keeps the contribution to the FCNC processes suppressed. This is because, the diagrams contributing to flavor violating processes involve the KK modes. However, the KK modes are all universally localized towards the IR brane. This results in a suppression in the coupling of the KK modes to the light fermions, due to a small overlap in their profiles. The lightness of the fermion therefore guarantees small flavor violating couplings. This is reminiscent of the GIM mechanism in the SM. Therefore, this feature of RS models is known as the RS-GIM mechanism [71, 72].

RS models are closely related to BSM scenarios based on conformal dynamics, through the AdS/CFT duality. In this way, several important features of RS models can be understood from a dual point of view. Consider concretely the case of two brane RS models. In the AdS/CFT dictionary, the coordinate corresponding to the fifth dimension of AdS space is associated with the renormalization scale  $\mu$  in the dual theory. Making a change of coordinates in AdS space from  $\theta$  to  $z$ , where  $z$  is defined as

$$z = \frac{e^{kr_c\theta}}{k}, \quad (1.35)$$

the renormalization scale  $\mu$  in the dual CFT is related to  $z$  as  $\mu \sim 1/z$  [20]. RS

models with two branes are therefore dual to a strongly coupled theory that is well approximated by a CFT in the energy regime between the two branes. The hidden brane corresponds to the UV cut-off of the theory. The visible brane corresponds to the scale where the CFT is spontaneously broken [20]. The boundary conditions on the bulk fields at the UV brane determine the coefficients of the deformation (in the UV) of the CFT in the dual picture. The RS geometry contains a massless graviscalar mode in the low energy, the radion. This mode corresponds to the freedom to move the distance between the two branes, and is identified with the NGB of spontaneously broken conformal invariance, the dilaton, on the  $4D$  side of the correspondence. The AdS geometry is stabilized by adding a Goldberger-Wise (GW) scalar  $\Phi$  to the theory. In the dual picture, this corresponds to deforming the CFT by a primary scalar operator  $\mathcal{O}$ . The boundary condition for  $\Phi$  on the UV brane is related to the strength of this deformation. Presence of  $\Phi$  that stabilizes the RS geometry generates a mass for the radion. Correspondingly, the deformation of CFT by  $\mathcal{O}$  generates a mass for the dilaton.

We saw that in the RS scenario, the SM fields could be localized to the IR brane, or could propagate in the bulk. In the dual  $4D$  theory, the brane localized fields correspond to composites of the conformal dynamics. The bulk fermionic fields in RS models correspond to elementary fields that mix with CFT operators of a given scaling dimension. The scaling dimension is related to the  $5D$  mass parameter of the fermion in the AdS space. The gauge fields that propagate in the bulk correspond to global symmetries of the conformal dynamics that are weakly gauged.

We see that AdS/CFT duality allows us to match BSM scenarios in RS models to those involving conformal dynamics. In this way, extra dimensional realizations of technicolor [73] and of the Higgs as a pNGB [74] have been obtained. We use this approach in this dissertation, based on our results in [21]. In particular, we use this correspondence to construct an explicit holographic realization of the setup for the dilaton, thereby providing a verification of the results concerning its mass and couplings.

## 1.6 Summary and Plan of the Dissertation

In this chapter, I have motivated the role of conformal dynamics in a class of BSM models that address the hierarchy problem and satisfy the flavor constraints naturally. Using AdS/CFT duality, conformal dynamics can be indirectly studied using the geometry of warped extra dimension. In this way, BSM models involving conformal dynamics can be related to RS models. Spontaneous breaking of conformal invariance in these scenarios leads to a pNGB in the spectrum, the dilaton. In the limit of exact conformal invariance, the dilaton is massless and its couplings are fixed by the requirement of nonlinear realization of conformal symmetry. In realistic scenarios, the conformal symmetry is only approximate, which results in the dilaton acquiring a mass, and modifies its couplings. If conformal dynamics is directly responsible for EW symmetry breaking, the dilaton may be the only new light particle in the spectrum. In a scenario where the Higgs is a composite pNGB, the dilaton can be another state in the low energy theory apart from the Higgs.

The discovery of a dilaton would be indicative of the involvement of conformal dynamics in the real world. It is therefore important to arrive at the conditions under which the dilaton can be light, and to understand the modifications to its couplings in such a scenario. Using the AdS/CFT duality, the dilaton is identified with the radion mode in the RS models. In this way, RS models can provide a holographic realization of this scenario, thereby providing a way to verify the results obtained for the mass and couplings of the dilaton.

In the wake of the recent discovery of a SM like Higgs particle at mass close to 125 GeV, studying the mass and the couplings of the dilaton becomes even more important. The dilaton couplings in the absence of conformal symmetry violating effects are of the same form as the SM Higgs in the classical limit. It therefore becomes important to study the dilaton couplings in realistic scenarios, and arrive at ways to distinguish it from a SM like Higgs. There are several possibilities – the observed particle is a dilaton and there may or may not be other light states (like a SM like Higgs) in the spectrum, the observed particle is a SM like Higgs and there may or may not be other light states (like a dilaton) in the spectrum, or the dilaton and the Higgs mix with each other, and the observed particle is an admixture. Phenomenological studies in this direction have already been reported.

With this motivation, we focus in this dissertation on the mass and the couplings of the dilaton in a spontaneously broken CFT, and those of the radion in the corresponding holographic duals. In chapter 2, we will focus on the theory of the dilaton. We will construct the effective theory of the dilaton, focusing on the mass and the couplings to light states. We will consider the scenarios when the correc-

tion to the scaling dimension of the symmetry breaking operator are small near the breaking scale, and when they are large. In both cases, we will construct the low energy effective theory. In chapter 3 we will present the effective theory of the radion in RS models. We will calculate the mass and the couplings of the radion to the light fields. This will be done closely keeping the CFT construction in mind, which we will elaborate when needed. In chapter 4 we will use the idea of duality to show that the results for the radion and the dilaton agree with each other, which confirms that the extra dimensional setup provides an explicit holographic realization of the conformal dynamics. Finally in chapter 5 we will conclude and summarize.



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### Effective Theory of the Dilaton

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In this chapter we construct the effective theory of the dilaton, which is the NGB of spontaneously broken conformal symmetry. Guided by the requirement to realize the symmetry non-linearly in the broken phase, we construct the effective potential for the dilaton. We show that the obtained potential in this case requires severe fine-tuning to allow for a realistic breaking scale and fails to give the dilaton a mass. This motivates incorporating conformal symmetry violating effects for the theory in the high energy. We construct the effective potential for the dilaton in the low energy in the presence of such effects and show that if the operator that breaks conformal symmetry at high energies is marginal at the breaking scale, the dilaton can naturally be light.

After establishing the results about the mass of the dilaton, we proceed to calculate the couplings of the dilaton to other light fields in the low energy, focusing on various scenarios. In particular, we consider the following cases:

1. **Conformal SM:** All the SM fields are composites of the CFT. EW symmetry is broken by the strong dynamics. There is no light Higgs.
2. **Technicolor:** The SM gauge fields are elementary, whereas the matter fields can be elementary, composites, or partially composites of the CFT. EW symmetry is broken by the strong dynamics. There is no light Higgs.
3. **Higgs as pNGB:** The SM gauge fields are elementary, whereas the matter fields can be elementary, composites, or partially composites of the CFT. Breaking of a global symmetry of the CFT gives a Higgs doublet, which breaks EW symmetry. There is a light Higgs.

In all these cases, we calculate the form of the couplings to the gauge bosons and the fermions. We obtain the leading corrections to the couplings that come from incorporating conformal symmetry violating effects.

The fifteen parameter conformal group extends the ten parameter Poincare group to include scale transformations and special conformal transformations. While it has long been conjectured that any Poincare invariant, unitary theory that realizes scale invariance linearly will also respect conformal symmetry [75], there exists no complete proof. The validity of this conjecture has been the subject of considerable interest in the recent literature [76–78]. In the rest of this dissertation, we will not concern ourselves with this yet unsettled interesting question, and will use scale invariance and conformal invariance interchangeably.

Consider a theory where conformal invariance is spontaneously broken. Then the low energy effective theory contains a dilaton field  $\sigma(x)$ , which can be thought

of as the NGB associated with the breaking of scale invariance [7, 8, 10, 79]. The additional four NGBs associated with the breaking of the special conformal symmetry can be identified with the derivatives of the dilaton, rather than as independent propagating fields. Below the breaking scale the symmetry is realized non-linearly, with the dilaton undergoing a shift  $\sigma(x) \rightarrow \sigma'(x') = \sigma(x) + \omega f$  under the scale transformation  $x^\mu \rightarrow x'^\mu = e^{-\omega} x^\mu$ . Here  $f$  is the scale associated with the breaking of conformal symmetry. For the purpose of writing interactions of the dilaton it is convenient to define the object

$$\chi(x) = f e^{\sigma(x)/f} \quad (2.1)$$

which transforms linearly under scale transformations. Specifically, under the scale transformation  $x^\mu \rightarrow x'^\mu = e^{-\omega} x^\mu$ ,  $\chi(x)$  transforms as a conformal compensator

$$\chi(x) \rightarrow \chi'(x') = e^\omega \chi(x) . \quad (2.2)$$

The low energy effective theory for the dilaton will in general include all terms consistent with this transformation, but with some additional restrictions and relations among their coefficients from the requirement that the theory be invariant not just under scale transformations, but under the full conformal group. These restrictions will not affect our discussion in any significant way, and so operationally we shall only require that the action for  $\chi$  be scale invariant.

In writing down the Lagrangian for the dilaton, it is necessary to take into account the implicit breaking of conformal invariance associated with the regulator. Since the theory in the ultraviolet possesses exact conformal invariance, this effect is of course completely spurious. However, it has the consequence that the Lagrangian

for the dilaton is not manifestly scale invariant. It is only at the quantum level, when effects of the regulator are incorporated, that conformal invariance is realized. This complicates the problem of finding the form of the effective theory.

Perhaps the simplest way to incorporate the effect of the regulator is to begin in a framework where the renormalization scale  $\mu_\chi$  is itself a function of  $\chi$ ,  $\mu_\chi = \mu\hat{\chi}$ , where  $\hat{\chi} = \chi/f$ . In such a framework, correlation functions can be obtained from the effective action, which has exactly the same form as in a conventional renormalization scheme, but with  $\mu$  replaced by  $\mu_\chi$  [80]. Such a choice of renormalization scheme has the advantage that the action for  $\chi$  is then manifestly scale invariant and therefore easy to write down. Starting from this action, the form of the Lagrangian in a more conventional scheme where the renormalization scale is independent of  $\chi$  can be determined. This is the approach we shall follow.

## 2.1 Dilaton Lagrangian in the Limit of Exact Conformal Invariance

We begin by constructing the effective theory for the dilaton in the case when conformal invariance is exact, and effects that explicitly violate conformal symmetry are absent. In a framework where the renormalization scale  $\mu_\chi$  is proportional to  $\chi$ , the low energy effective action for the dilaton will be manifestly scale invariant. This symmetry allows derivative terms in the Lagrangian of the form

$$\frac{1}{2}Z\partial_\mu\chi\partial^\mu\chi + \frac{c}{\chi^4}(\partial_\mu\chi\partial^\mu\chi)^2 + \dots \quad (2.3)$$

For reasons that will become clear, we postpone rescaling  $Z$  to one. Crucially, however, in contrast to the effective theory of the NGB of spontaneously broken

global symmetry, a non-derivative term in the potential is also allowed,

$$V(\chi) = \frac{Z^2 \kappa_0}{4!} \chi^4 . \quad (2.4)$$

The existence of this non-derivative term indicates that even in the absence of effects that explicitly violate conformal symmetry, there is a preferred value of  $f = \langle \chi \rangle$ . This is in sharp contrast to the case of a spontaneously broken global symmetry, where all points on the vacuum manifold are identical. In order to determine the location of the minimum, the effective potential must be obtained and minimized.

In order to bring the theory into a standard form, we now go over to a scheme in which the renormalization scale  $\mu$  is independent of  $\chi$ . In order to clarify the discussion, we first illustrate the procedure at one loop. We will obtain the Lagrangian for the low energy effective theory to this order, and use it to determine the effective potential and dilaton mass. We will then show how the result generalizes to arbitrary numbers of loops. It will be convenient to work in a mass-independent scheme, such as  $\overline{\text{MS}}$ . We label  $Z$  and the coupling constants  $c$ ,  $Z^2 \kappa_0$  etc. by  $g_i$ , where  $i$  is an index. The  $g_i$  are all dimensionless.

### 2.1.1 One Loop Analysis

At one loop, going over to a scheme where the renormalization scale  $\mu$  is independent of  $\chi$  is equivalent to evolving the parameters  $g_i$  etc. from  $\mu_\chi$  to  $\mu$  using the renormalization group. Running the renormalization group leads to  $g_i$  evolving into  $g'_i$ , where

$$g'_i = g_i - \frac{dg_i}{d \log \mu} \log \left( \frac{\chi}{f} \right) . \quad (2.5)$$

To keep the analysis simple we focus on the case when all the  $g_i$  are zero, except  $Z$  and  $Z^2\kappa_0$ . Then to this order, the potential for the dilaton takes the form

$$V(\chi) = \left\{ Z^2\kappa_0 - \frac{d(Z^2\kappa_0)}{d \log\mu} \log\left(\frac{\chi}{f}\right) \right\} \frac{\chi^4}{4!}. \quad (2.6)$$

Note that the potential is no longer manifestly scale invariant. In this theory at one loop order there is no wave function renormalization,

$$\frac{d \log Z}{d \log\mu} = -2\gamma = 0. \quad (2.7)$$

The derivative of  $\kappa_0$  can be evaluated in perturbation theory, leading to

$$\frac{d\kappa_0}{d \log\mu} = \frac{3\kappa_0^2}{16\pi^2}. \quad (2.8)$$

After using these expressions to replace the terms involving derivatives in the Lagrangian, we may choose to rescale  $Z$  to one.

The conformal invariance of this Lagrangian can be made more transparent in a basis where all the mass scales are expressed as powers of the renormalization scale  $\mu$ , and all coupling constants are dimensionless. In such a basis, the dilaton kinetic term can be written as

$$\frac{1}{2} \bar{Z} \partial_\mu \chi \partial^\mu \chi, \quad (2.9)$$

where  $\bar{Z}$  is given to one loop order by

$$\bar{Z} = Z - \frac{dZ}{d \log\mu} \log\left(\frac{\mu}{f}\right). \quad (2.10)$$

$\bar{Z}$ , which is equal to  $Z$  since wave function renormalization vanishes to this order, is a renormalization group invariant and does not change with  $\mu$ . At this point we choose to rescale  $\bar{Z}$  to one.

In this basis the potential for the dilaton takes the form

$$V(\chi) = \left\{ \bar{\kappa}_0 - \frac{d(Z^2 \kappa_0)}{d \log \mu} \log \left( \frac{\chi}{\mu} \right) \right\} \frac{\chi^4}{4!} . \quad (2.11)$$

where  $\bar{\kappa}_0$  is given by

$$\bar{\kappa}_0 = Z^2 \kappa_0 - \frac{d(Z^2 \kappa_0)}{d \log \mu} \log \left( \frac{\mu}{f} \right) . \quad (2.12)$$

Note that  $\bar{\kappa}_0$ , like  $\bar{Z}$ , is independent of the renormalization scale  $\mu$  to this loop order, as dictated by conformal invariance.

The next step is to determine the one loop effective potential. This can be computed from Eq. (2.6), after rescaling  $Z$  to one, using the Coleman-Weinberg formula,

$$V_{\text{eff}} = V \pm \frac{1}{64\pi^2} \sum_i M_i^4 \left( \log \frac{M_i^2}{\mu^2} - \frac{1}{2} \right) . \quad (2.13)$$

Here the sum is over the field dependent masses of all the states in the theory, the sign being positive for bosons and negative for fermions. This leads to

$$V_{\text{eff}}(\chi_{cl}) = \left\{ \kappa_0 - \frac{3\kappa_0^2}{32\pi^2} \left[ \log \left( \frac{\mu^2}{\frac{1}{2}\kappa_0 f^2} \right) - \frac{1}{2} \right] \right\} \frac{\chi_{cl}^4}{4!} \quad (2.14)$$

The conformal invariance of the theory can be made clear by rewriting this in terms of  $\bar{\kappa}_0$ . We obtain

$$V_{\text{eff}}(\chi_{cl}) = \frac{\hat{\kappa}_0}{4!} \chi_{cl}^4 . \quad (2.15)$$

where  $\hat{\kappa}_0$ , given to one loop order by

$$\hat{\kappa}_0 = \bar{\kappa}_0 + \frac{3\bar{\kappa}_0^2}{32\pi^2} \left[ \log \left( \frac{\bar{\kappa}_0}{2} \right) - \frac{1}{2} \right] , \quad (2.16)$$

is independent of the renormalization scale  $\mu$ , as required by conformal invariance.

Minimizing this effective potential, we find that the conformal symmetry breaking scale  $\langle \chi \rangle = f$  is driven to zero, corresponding to unbroken conformal

symmetry, if the sign of  $\hat{\kappa}_0$  is positive. Alternatively, if  $\hat{\kappa}_0$  is negative,  $f$  is driven to infinite values, and conformal symmetry is never realized. Only if the value of  $\hat{\kappa}_0$  is identically zero does the low energy effective theory possess a stable minimum, and a massless dilaton. In general setting  $\hat{\kappa}_0 = 0$  is associated with severe tuning, since there is no symmetry reason to expect it to vanish.

### 2.1.2 General Analysis

Although this result was obtained based on a one loop analysis, we now show that the same conclusion holds at arbitrary loop order. It can be verified that by replacing  $g_i$  in the theory renormalized at  $\mu_\chi$  by  $g'_i$ , where  $g'_i$  is given by

$$g'_i = g_i + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{d^n g_i}{d \log \mu^n} \left[ \log \left( \frac{\chi}{f} \right) \right]^n, \quad (2.17)$$

we obtain a Lagrangian which is conformally invariant when renormalized at  $\mu$ . The higher terms in this series are to be determined self-consistently order by order in perturbation theory. The potential for the dilaton now takes the form

$$V(\chi) = \left\{ \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{d^n (Z^2 \kappa_0)}{d \log \mu^n} \left[ \log \left( \frac{\chi}{f} \right) \right]^n \right\} \frac{\chi^4}{4!}. \quad (2.18)$$

As expected, the Lagrangian does not possess a manifestly scale invariant form. We can choose to rescale  $Z$  to one after the derivatives have been evaluated, but not before.

The conformal invariance of the theory can be made more transparent by going over to a basis where all the mass scales are expressed as powers of the renormalization scale  $\mu$ , and all coupling constants are dimensionless. In such a basis,  $\bar{Z}$ , the



coefficient of the dilaton kinetic term, is given by

$$\bar{Z} = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{d^m Z}{d \log \mu^m} \left[ \log \left( \frac{\mu}{f} \right) \right]^m. \quad (2.19)$$

$\bar{Z}$  does not change with the renormalization scale  $\mu$ , and we rescale it to one without loss of generality. The potential for the dilaton takes the form

$$V(\chi) = \left\{ \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \bar{\kappa}_{0,n} \left[ \log \left( \frac{\chi}{\mu} \right) \right]^n \right\} \frac{\chi^4}{4!}, \quad (2.20)$$

where  $\bar{\kappa}_{0,n}$  is given by

$$\bar{\kappa}_{0,n} = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{d^{m+n}(Z^2 \kappa_0)}{d \log \mu^{m+n}} \left[ \log \left( \frac{\mu}{f} \right) \right]^m. \quad (2.21)$$

The beta functions of all the  $\bar{\kappa}_{0,n}$  vanish by construction. This is a reflection of the conformal invariance of this theory. Going forward, we denote the  $\bar{\kappa}_{0,n}$  and all the other coupling constants in this basis by  $\bar{g}_i$ , where  $i$  is an index. The beta functions of all the  $\bar{g}_i$  vanish as a consequence of conformal invariance.

The next step is to obtain the effective potential  $V_{\text{eff}}(\chi_{cl})$  for this theory, and to minimize it. How is  $V_{\text{eff}}(\chi_{cl})$  to be determined? This time, rather than work directly from the Lagrangian, we employ the Callan-Symanzik equation for the effective potential,

$$\left\{ \mu \frac{\partial}{\partial \mu} + \beta_i \frac{\partial}{\partial \bar{g}_i} - \gamma \chi_{cl} \frac{\partial}{\partial \chi_{cl}} \right\} V_{\text{eff}}(\chi_{cl}, \bar{g}_i, \mu) = 0. \quad (2.22)$$

For a conformal theory, the beta functions  $\beta_i(\bar{g}_i)$  vanish. The anomalous dimension  $\gamma$  of  $\chi$ , which represents the difference between its mass and scaling dimensions, is also zero. Then the Callan-Symanzik equation reduces to

$$\mu \frac{\partial}{\partial \mu} V_{\text{eff}}(\chi_{cl}, \bar{g}_i, \mu) = 0. \quad (2.23)$$

The effective potential is then constrained by dimensional analysis to be of the especially simple form

$$V_{\text{eff}}(\chi_{cl}) = \frac{\hat{\kappa}_0}{4!} \chi_{cl}^4, \quad (2.24)$$

where  $\hat{\kappa}_0$  is a constant that depends on the  $\bar{g}_i$ , but is independent of  $\mu$ . We see that the theory does not have a stable minimum unless  $\hat{\kappa}_0 = 0$ , when the potential vanishes identically. The results of our one loop analysis are therefore confirmed.

## 2.2 Dilaton Lagrangian in the Presence of Conformal Symmetry Violating Effects

The situation changes if effects that explicitly break conformal symmetry are present in the theory. Consider an operator  $\mathcal{O}(x)$  of scaling dimension  $\Delta$  added to the Lagrangian,

$$\mathcal{L} = \mathcal{L}_{\text{CFT}} + \lambda_{\mathcal{O}} \mathcal{O}(x). \quad (2.25)$$

Under  $x \rightarrow x' = e^{-\omega}x$ , the operator  $\mathcal{O}(x) \rightarrow \mathcal{O}'(x') = e^{\omega\Delta} \mathcal{O}(x)$ . It is convenient to define the dimensionless coupling constant  $\hat{\lambda}_{\mathcal{O}} = \lambda_{\mathcal{O}} \mu^{\Delta-4}$ . We choose to normalize the operator  $\mathcal{O}(x)$  such that  $\hat{\lambda}_{\mathcal{O}}$  of order one corresponds to conformal symmetry violation becoming strong, so that it can no longer be treated as a perturbation on the conformal dynamics. This implies that if  $\hat{\lambda}_{\mathcal{O}} \ll 1$ , it satisfies the renormalization group equation (RGE)

$$\frac{d \log \hat{\lambda}_{\mathcal{O}}}{d \log \mu} = -(4 - \Delta). \quad (2.26)$$

We wish to determine the effect of this deformation on the form of the low energy effective theory. In order to do this, note that for small  $\hat{\lambda}_{\mathcal{O}}$  the action remains

formally invariant under  $x \rightarrow x' = e^{-\omega}x$  provided  $\lambda_{\mathcal{O}}$  is taken to be a spurion that transforms as

$$\lambda_{\mathcal{O}} \rightarrow \lambda'_{\mathcal{O}} = e^{(4-\Delta)\omega} \lambda_{\mathcal{O}}. \quad (2.27)$$

This implies that the effective theory for  $\chi$  will also respect conformal symmetry if  $\lambda_{\mathcal{O}}$  is treated as a spurion that transforms in this way. The combination invariant under the transformation is  $\lambda_{\mathcal{O}}\chi^{\Delta-4}$ . All low energy effects will be a power series in  $\lambda_{\mathcal{O}}$  as long as the deformation remains perturbative so that  $\lambda_{\mathcal{O}}$  is below its strong coupling value. By the spurion analysis, the specific form of  $\chi$  appearing in the low energy is then fixed.

In determining the low energy effective theory for the dilaton it is again simplest to begin in a framework where the renormalization scale depends on the conformal compensator as  $\mu_{\chi} = \mu\hat{\chi}$ , since the Lagrangian is then manifestly scale invariant. The potential for the dilaton is then

$$V(\chi) = \frac{Z^2\kappa_0}{4!}\chi^4 - \sum_{n=1}^{\infty} \frac{Z^{2-n\epsilon/2}\kappa_n}{4!}\lambda_{\mathcal{O}}^n \chi^{(4-n\epsilon)}, \quad (2.28)$$

where  $\epsilon$  is defined as  $4 - \Delta$ . The next step is to go over to a more conventional scheme where the renormalization scale  $\mu$  is independent of  $\chi$ .

### 2.2.1 One Loop Analysis

In order to clarify the discussion we will first work in the limit that  $\hat{\lambda}_{\mathcal{O}} \ll 1$  at scales  $\mu$  of order  $f$ , and determine the vacuum structure and the dilaton mass to one loop order. We will then relax the assumption on  $\hat{\lambda}_{\mathcal{O}}$  and also generalize the result to an arbitrary number of loops.

Keeping only the leading order term in  $\hat{\lambda}_{\mathcal{O}}$ , the potential for the dilaton Eq. (2.28) simplifies to

$$V(\chi) = \frac{Z^2 \kappa_0}{4!} \chi^4 - \frac{Z^{\Delta/2} \kappa_1}{4!} \lambda_{\mathcal{O}} \chi^{\Delta}, \quad (2.29)$$

where  $\kappa_0$  and  $\kappa_1$  are coupling constants. We can go over to a scheme where the renormalization scale is independent of  $\chi$  by using the renormalization group. The potential then becomes, to one loop order,

$$V(\chi) = \left\{ Z^2 \kappa_0 - \frac{d(Z^2 \kappa_0)}{d \log \mu} \log \left( \frac{\chi}{f} \right) \right\} \frac{\chi^4}{4!} - \left\{ Z^{\Delta/2} \kappa_1 - \frac{d(Z^{\Delta/2} \kappa_1)}{d \log \mu} \log \left( \frac{\chi}{f} \right) \right\} \frac{\lambda_{\mathcal{O}} \chi^{\Delta}}{4!}. \quad (2.30)$$

To keep the analysis simple we focus on the case when all the  $g_i$  are zero, except  $Z$ ,  $Z^2 \kappa_0$  and  $Z^{\Delta/2} \kappa_1$ . This theory does not experience wave function renormalization at one loop, and therefore the derivatives of  $Z$  in the expression above vanish. The derivatives of  $\kappa_0$  and  $\kappa_1$  can be evaluated in perturbation theory, leading to

$$\begin{aligned} \frac{d\kappa_0}{d \log \mu} &= \frac{3\kappa_0^2}{16\pi^2} \\ \frac{d\kappa_1}{d \log \mu} &= \frac{\Delta(\Delta-1)\kappa_1\kappa_0}{32\pi^2}. \end{aligned} \quad (2.31)$$

In order to understand how conformal symmetry is realized in this framework it is useful to go over to a basis where all mass scales are expressed in terms of the renormalization scale  $\mu$  and all coupling constants are dimensionless. In this basis,  $\bar{Z}$ , the coefficient of the dilaton kinetic term is given by

$$\bar{Z} = Z - \frac{dZ}{d \log \mu} \log \left( \frac{\mu}{f} \right). \quad (2.32)$$

$\bar{Z}$  is a renormalization group invariant. The absence of wave function renormalization in this theory at one loop means that  $Z = \bar{Z}$  to this order. We choose to rescale  $\bar{Z}$  to one.

The potential for the dilaton takes the form

$$V(\chi) = \left\{ \bar{\kappa}_0 - \frac{d(Z^2 \kappa_0)}{d \log \mu} \log \left( \frac{\chi}{\mu} \right) \right\} \frac{\chi^4}{4!} - \left\{ \bar{\kappa}_1 - \frac{d(Z^{\Delta/2} \kappa_1)}{d \log \mu} \log \left( \frac{\chi}{\mu} \right) \right\} \frac{\lambda_{\mathcal{O}} \chi^{\Delta}}{4!}. \quad (2.33)$$

Here  $\bar{\kappa}_0$  and  $\bar{\kappa}_1$  are related to  $Z$ ,  $\kappa_0$  and  $\kappa_1$  as

$$\begin{aligned} \bar{\kappa}_0 &= Z^2 \kappa_0 - \frac{d(Z^2 \kappa_0)}{d \log \mu} \log \left( \frac{\mu}{f} \right) \\ \bar{\kappa}_1 &= Z^{\Delta/2} \kappa_1 - \frac{d(Z^{\Delta/2} \kappa_1)}{d \log \mu} \log \left( \frac{\mu}{f} \right). \end{aligned} \quad (2.34)$$

Note that  $\lambda_{\mathcal{O}}$  is being treated as a spurion, not as a coupling constant, and therefore continues to carry mass dimension  $4 - \Delta$ , equal to its spurious scaling dimension. The beta functions of  $\bar{\kappa}_0$  and  $\bar{\kappa}_1$  can be seen to vanish to one loop order by construction. This is a consequence of the (spurious) conformal symmetry.

The next step is to obtain the effective potential for the theory at one loop order. We again use Eq. (2.13), after rescaling  $Z$  to one, leading to

$$\begin{aligned} V_{\text{eff}}(\chi_{cl}) &= \frac{\bar{\kappa}_0}{4!} \chi_{cl}^4 - \frac{\bar{\kappa}_1}{4!} \lambda_{\mathcal{O}} \chi_{cl}^{\Delta} + \\ &\frac{\bar{\kappa}_0}{4!} \left[ \frac{3\bar{\kappa}_0}{32\pi^2} \chi_{cl}^4 - \frac{\lambda_{\mathcal{O}} \Delta (\Delta - 1) \bar{\kappa}_1}{64\pi^2} \chi_{cl}^{\Delta} \right] \left[ \Sigma - \frac{1}{2} \right], \end{aligned} \quad (2.35)$$

where  $\Sigma$  is defined as

$$\Sigma = \log \left[ \frac{\bar{\kappa}_0}{2} - \frac{\bar{\kappa}_1}{4!} \Delta (\Delta - 1) \lambda_{\mathcal{O}} \chi_{cl}^{\Delta-4} \right]. \quad (2.36)$$

The next step is to find the minimum of this potential, and to obtain the dilaton mass. For simplicity, we neglect the loop suppressed terms on the second line of Eq. (2.35). We will later verify that including them does not alter our conclusions. The tree level potential admits a minimum when

$$f^{(\Delta-4)} = \frac{4\bar{\kappa}_0}{\bar{\kappa}_1\lambda_{\mathcal{O}}\Delta}. \quad (2.37)$$

The dilaton mass squared at the minimum, to this order, is given by

$$m_\sigma^2 = \frac{\bar{\kappa}_1}{4!}\lambda_{\mathcal{O}}\Delta(4-\Delta)f^{\Delta-2} = 4\frac{\bar{\kappa}_0}{4!}(4-\Delta)f^2. \quad (2.38)$$

If the conformal field theory is weakly coupled, the parameters  $\bar{\kappa}_0, \bar{\kappa}_1 \ll (4\pi)^2$ ,  $\hat{\lambda}_{\mathcal{O}} \ll 1 \Rightarrow \lambda_{\mathcal{O}}f^{(\Delta-4)} \ll 1$ , and the effective theory of the dilaton we have obtained is valid. Corrections to Eqs. (2.37) and (2.38) from the loop suppressed terms in Eq. (2.35) can be seen to be small in this limit, and we are justified in neglecting them.

However, if the conformal field theory under consideration is strongly coupled, as in the theories of interest for electroweak symmetry breaking, the effective theory of the dilaton is also expected to be strongly coupled at the scale  $\Lambda \sim 4\pi f$ . Then, in the absence of tuning, the parameters  $\bar{\kappa}_0$  and  $\bar{\kappa}_1$  are in general of order  $(4\pi)^2$  and, as is clear from Eq. (2.37), the assumption that  $\hat{\lambda}_{\mathcal{O}}$  is small at the scale  $f$  is no longer self consistent. Furthermore, it follows from Eq. (2.38) that the mass of the dilaton is of order the cutoff  $\Lambda$  and so it is no longer a light state. The loop suppressed terms we have neglected in obtaining Eq. (2.37) cannot alter this result. The conclusion to be drawn from this is that if a strongly coupled conformal field theory is explicitly

broken by a relevant operator that becomes strong in the infrared, in general there is no reason to expect a light dilaton.

However, a closer study of Eq. (2.38) reveals a very interesting feature. If the operator  $\mathcal{O}$  is very close to marginal so that  $(4 - \Delta) \ll 1$ , then even for  $\bar{\kappa}_0 \sim (4\pi)^2$  the dilaton mass is parametrically smaller than the strong coupling scale  $\Lambda$ . It is straightforward to verify that this conclusion remains true even when the loop suppressed terms in Eq. (2.35) are included in the analysis. This would suggest that in a scenario where the operator that breaks conformal symmetry is close to marginal, there is indeed a light dilaton in the effective theory. The dilaton mass depends on how close the dimension of  $\mathcal{O}$  is to the exactly marginal value of 4, scaling as  $m_\sigma \sim \sqrt{4 - \Delta}$ .

This is potentially a very important result. In a large class of theories of interest for electroweak symmetry breaking, the operator that breaks conformal symmetry is close to marginal in order to ensure that there is a large hierarchy between the flavor scale (or Planck scale) and the electroweak scale. This result would imply that in all such theories the low energy spectrum includes a light dilaton! Unfortunately, the steps leading up to Eq. (2.38) assumed that  $\hat{\lambda}_{\mathcal{O}} \ll 1$ . As is clear from Eq. (2.37), this assumption is not valid in the strong coupling limit. In order to validate this conclusion, we must show that the result continues to hold when this assumption is relaxed, and is valid beyond one loop.

## 2.2.2 General Analysis

Extending the analysis beyond small  $\lambda_{\mathcal{O}}$  involves incorporating two distinct effects. Firstly, if the coupling constant  $\lambda_{\mathcal{O}}$  is not small, the scaling behavior of the operator  $\mathcal{O}(x)$  is expected to receive corrections, and Eq. (2.26) is in general no longer valid. Instead, the RGE takes on the more general form

$$\frac{d \log \hat{\lambda}_{\mathcal{O}}}{d \log \mu} = -g(\hat{\lambda}_{\mathcal{O}}), \quad (2.39)$$

where  $g(\hat{\lambda}_{\mathcal{O}})$  is in general a polynomial in  $\hat{\lambda}_{\mathcal{O}}$ ,

$$g(\hat{\lambda}_{\mathcal{O}}) = \sum_{n=0}^{\infty} c_n \hat{\lambda}_{\mathcal{O}}^n, \quad (2.40)$$

that can be approximated by the lowest order term

$$g(\hat{\lambda}_{\mathcal{O}}) = c_0 = (4 - \Delta) \quad (2.41)$$

only in the limit when  $\hat{\lambda}_{\mathcal{O}}$  is small. In general, in a strongly coupled conformal field theory, the coefficients  $c_n$ ,  $n \geq 1$  are expected to be of order one. (This is consistent with the expectation that all the terms in the series should become comparable when  $\hat{\lambda}_{\mathcal{O}}$  is of order one.) This effect must be taken into account. Secondly, if  $\hat{\lambda}_{\mathcal{O}}$  is not small, the higher order terms in Eq. (2.28) are significant and must be included in our analysis.

While both these effects are important, the first has a particularly striking impact on the form of the low energy effective theory. The reason is that in this case, the leading order effect which is of order  $(4 - \Delta)$  receives corrections that begin at order  $\hat{\lambda}_{\mathcal{O}}$ . Since in the theories of interest  $(4 - \Delta) \ll 1$ , these corrections can potentially become large even before  $\hat{\lambda}_{\mathcal{O}}$  reaches its strong coupling value, significantly



impacting the final result. This is most easily seen by going beyond the leading term in Eq. (2.40), while neglecting the higher order corrections in  $\hat{\lambda}_\mathcal{O}$  that arise from other sources. Such an approximation is valid provided  $\hat{\lambda}_\mathcal{O}$  at the breaking scale is significantly below its strong coupling value,  $\hat{\lambda}_\mathcal{O} \ll 1$ . We are interested in the region of parameter space where  $(4 - \Delta) < c_1 \hat{\lambda}_\mathcal{O}$ , so that the corrections to the leading order term in Eq. (2.40) dominate. We postpone a more complete discussion that includes all the higher order effects in  $\hat{\lambda}_\mathcal{O}$  till later in this section.

Integrating Eq (2.39), it follows that  $\mathcal{G}(\hat{\lambda}_\mathcal{O})\mu^{-1}$  is a renormalization group invariant, where

$$\mathcal{G}(\hat{\lambda}_\mathcal{O}) = \exp \left( - \int \frac{d\hat{\lambda}_\mathcal{O}}{\hat{\lambda}_\mathcal{O}} \frac{1}{g(\hat{\lambda}_\mathcal{O})} \right). \quad (2.42)$$

We can make the theory defined by Eq. (2.25) formally invariant under scale transformations by promoting  $\hat{\lambda}_\mathcal{O}$  to a spurion that transforms as

$$\hat{\lambda}_\mathcal{O}(\mu) \rightarrow \hat{\lambda}'_\mathcal{O}(\mu) = \hat{\lambda}_\mathcal{O}(\mu e^{-\omega}) \quad (2.43)$$

under  $x \rightarrow x' = e^{-\omega}x$ . Under this transformation,

$$\mathcal{G}(\hat{\lambda}_\mathcal{O})\mu^{-1} \rightarrow \mathcal{G}(\hat{\lambda}'_\mathcal{O})\mu^{-1} = e^{-\omega} \mathcal{G}(\hat{\lambda}_\mathcal{O})\mu^{-1}. \quad (2.44)$$

The Lagrangian for the low energy effective theory must be invariant under this spurious scale transformation. Furthermore, it is restricted to terms involving positive integer powers of the spurion  $\hat{\lambda}_\mathcal{O}$ . Using Eq. (2.42), it follows that the combination  $\bar{\lambda}_\mathcal{O}$ , defined as

$$\bar{\lambda}_\mathcal{O} = \hat{\lambda}_\mathcal{O} \left[ 1 + g(\hat{\lambda}_\mathcal{O}) \log \mu \right], \quad (2.45)$$

is invariant under infinitesimal changes in the renormalization scale  $\mu$ . It then follows

from Eq. (2.44) that the object  $\bar{\Omega}(\hat{\lambda}_{\mathcal{O}}, \chi/\mu)$ , defined as

$$\bar{\Omega}(\hat{\lambda}_{\mathcal{O}}, \chi/\mu) = \hat{\lambda}_{\mathcal{O}} \left[ 1 - g(\hat{\lambda}_{\mathcal{O}}) \log \left( \frac{\chi}{\mu} \right) \right], \quad (2.46)$$

is invariant under infinitesimal (spurious) scale transformations. Furthermore,  $\bar{\Omega}$  is a polynomial in  $\hat{\lambda}_{\mathcal{O}}$ . Lagrangians that are invariant under infinitesimal (spurious) scale transformations can be constructed using  $\bar{\Omega}$ .

For values of  $\mu$  close to the symmetry breaking scale  $f$  and  $g(\hat{\lambda}_{\mathcal{O}}) \ll 1$ , we can approximate  $\bar{\Omega}$  as

$$\bar{\Omega}(\hat{\lambda}_{\mathcal{O}}, \chi/\mu) = \hat{\lambda}_{\mathcal{O}} \left( \frac{\chi}{\mu} \right)^{-g(\hat{\lambda}_{\mathcal{O}})}. \quad (2.47)$$

To leading order in  $\bar{\Omega}$  the potential for  $\chi$  takes the form

$$V(\chi) = \frac{\chi^4}{4!} (\kappa_0 - \kappa_1 \bar{\Omega}). \quad (2.48)$$

From this potential the dilaton mass at the minimum can be obtained as

$$m_{\sigma}^2 = 4 \frac{\kappa_0}{4!} g(\hat{\lambda}_{\mathcal{O}}) f^2 \quad (2.49)$$

This expression for the dilaton mass is very similar to that in Eq. (2.38), except in one important respect. We now see that it is the scaling behavior of the operator  $\mathcal{O}$  at the breaking scale that determines the dilaton mass, rather than the scaling dimension of  $\mathcal{O}$  in the far ultraviolet. In particular, this implies that for the dilaton of a spontaneously broken approximate conformal symmetry to be naturally light, it is not sufficient that  $(4 - \Delta) \ll 1$ , so that the operator that breaks the symmetry is close to marginal in the far ultraviolet. Instead, the requirement is that this operator be close to marginal at the symmetry breaking scale, so that  $g(\hat{\lambda}_{\mathcal{O}}) \ll 1$  at  $\mu = f$ .

Since in a general strongly coupled theory,  $\hat{\lambda}_{\mathcal{O}}$ , and therefore  $g(\hat{\lambda}_{\mathcal{O}})$ , are expected to be of order one at the breaking scale, this condition is not expected to be satisfied in the scenarios of interest for electroweak symmetry breaking (for which  $4 - \Delta \ll 1$  suffices to address the flavor problem). This suggests that the existence of a light dilaton in these theories is associated with tuning (or more precisely, a coincidence problem). However, since the consistency of this analysis requires that  $\hat{\lambda}_{\mathcal{O}} \ll 1$ , it remains to show that including the higher order corrections in  $\hat{\lambda}_{\mathcal{O}}$  that we have so far neglected, and which may be significant, does not affect this conclusion.

The next step is obtain the effective theory for the dilaton, consistently including all the higher order effects in  $\hat{\lambda}_{\mathcal{O}}$ . At this point it is convenient to separate out the corrections to the scaling behavior of  $\mathcal{O}$  from these effects. Recalling that  $\epsilon$  is defined as  $(4 - \Delta)$ , we write

$$g(\hat{\lambda}_{\mathcal{O}}) = \epsilon + \delta g(\hat{\lambda}_{\mathcal{O}}) , \quad (2.50)$$

where  $\delta g(\hat{\lambda}_{\mathcal{O}})$  represents the higher order corrections. In order to simplify our analysis we will consider the two cases  $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$  and  $|\delta g(\hat{\lambda}_{\mathcal{O}})| > \epsilon$ , corresponding to the corrections to the scaling dimension of the operator  $\mathcal{O}$  being smaller or larger than  $\epsilon$  at the breaking scale, separately.

### 2.2.2.1 Limit When the Corrections Are Small

We first consider the case when  $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$  at the breaking scale. In this limit we can simplify Eq. (2.42) by performing a binomial expansion,

$$\int d\hat{\lambda}_{\mathcal{O}} \frac{1}{\hat{\lambda}_{\mathcal{O}} g(\hat{\lambda}_{\mathcal{O}})} = \int \frac{d\hat{\lambda}_{\mathcal{O}}}{\epsilon \hat{\lambda}_{\mathcal{O}}} \left[ 1 - \frac{\delta g(\hat{\lambda}_{\mathcal{O}})}{\epsilon} + \dots \right] . \quad (2.51)$$

Then

$$\mathcal{G}(\hat{\lambda}_{\mathcal{O}}) = \hat{\lambda}_{\mathcal{O}}^{-1/\epsilon} \exp \left[ \int d\hat{\lambda}_{\mathcal{O}} \frac{\delta g(\hat{\lambda}_{\mathcal{O}})}{\epsilon^2 \hat{\lambda}_{\mathcal{O}}} + \dots \right]. \quad (2.52)$$

It follows from this that  $\bar{\mathcal{G}}(\hat{\lambda}_{\mathcal{O}})$ , defined as

$$\bar{\mathcal{G}}(\hat{\lambda}_{\mathcal{O}}) = \left[ \mathcal{G}(\hat{\lambda}_{\mathcal{O}}) \right]^{-\epsilon}, \quad (2.53)$$

can be expanded as a polynomial in  $\hat{\lambda}_{\mathcal{O}}$ ,

$$\bar{\mathcal{G}}(\hat{\lambda}_{\mathcal{O}}) = \hat{\lambda}_{\mathcal{O}} \left[ 1 - \int d\hat{\lambda}_{\mathcal{O}} \frac{\delta g(\hat{\lambda}_{\mathcal{O}})}{\epsilon \hat{\lambda}_{\mathcal{O}}} + \dots \right]. \quad (2.54)$$

Then the object  $\bar{\mathcal{G}}(\hat{\lambda}_{\mathcal{O}})\mu^\epsilon$ , which we denote by  $\bar{\lambda}_{\mathcal{O}}$ , is a renormalization group invariant that can be expanded as a polynomial in  $\hat{\lambda}_{\mathcal{O}}$ . It follows from Eq. (2.44) that the theory above the breaking scale is formally invariant under scale transformations,  $x \rightarrow x' = e^{-\omega}x$ , provided  $\bar{\lambda}_{\mathcal{O}}$  is taken to be a spurion that transforms as

$$\bar{\lambda}_{\mathcal{O}} \rightarrow \bar{\lambda}'_{\mathcal{O}} = e^{\epsilon\omega} \bar{\lambda}_{\mathcal{O}}. \quad (2.55)$$

The effective theory for  $\chi$  will then respect conformal symmetry if  $\bar{\lambda}_{\mathcal{O}}$  is treated as a spurion that transforms in this way. Note that this spurious transformation is identical to that of  $\lambda_{\mathcal{O}}$ , Eq. (2.27), in the case of small  $\hat{\lambda}_{\mathcal{O}}$ . Consider the object  $\Omega(\bar{\lambda}_{\mathcal{O}}, \chi)$ , defined as

$$\Omega(\bar{\lambda}_{\mathcal{O}}, \chi) = \bar{\lambda}_{\mathcal{O}} \chi^{-\epsilon}. \quad (2.56)$$

By construction,  $\Omega$  is invariant under (spurious) scale transformations. Furthermore, in the regime  $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$ , it can be expanded as a polynomial in  $\hat{\lambda}_{\mathcal{O}}$ .  $\Omega$  is useful in constructing the general Lagrangian for the low energy theory.

In a framework where the renormalization scale depends on the conformal compensator as  $\mu_\chi = \mu\hat{\chi}$ , the potential for  $\chi$  takes the form

$$V(\chi) = \frac{Z^2\chi^4}{4!} \left[ \kappa_0 - \sum_{n=1}^{\infty} \kappa_n \Omega^n(\bar{\lambda}_\mathcal{O}, \sqrt{Z}\chi) \right]. \quad (2.57)$$

This simplifies to the form of Eq (2.28), but with  $\lambda_\mathcal{O}$  replaced by  $\bar{\lambda}_\mathcal{O}$ ,

$$V(\chi) = \frac{Z^2\kappa_0}{4!} \chi^4 - \sum_{n=1}^{\infty} \frac{Z^{2-n\epsilon/2}\kappa_n}{4!} \bar{\lambda}_\mathcal{O}^n \chi^{(4-n\epsilon)}. \quad (2.58)$$

Going over to a more conventional scheme where the renormalization scale  $\mu$  is independent of  $\chi$ ,  $V(\chi)$  becomes

$$\begin{aligned} & \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{d^m(Z^2\kappa_0)}{d \log \mu^m} \left[ \log \left( \frac{\chi}{f} \right) \right]^m \frac{\chi^4}{4!} - \\ & \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{d^m(Z^{2-n\epsilon/2}\kappa_n)}{d \log \mu^m} \left[ \log \left( \frac{\chi}{f} \right) \right]^m \frac{\bar{\lambda}_\mathcal{O}^n \chi^{4-n\epsilon}}{4!}. \end{aligned} \quad (2.59)$$

We can choose to rescale  $Z$  to one, but only after the derivatives above have been evaluated.

The (spurious) conformal symmetry of the theory can be made more transparent in a basis where all mass scales are expressed in terms of the renormalization scale  $\mu$  and all coupling constants are dimensionless. In this basis,  $\bar{Z}$ , the coefficient of the dilaton kinetic term, is given by

$$\bar{Z} = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{d^m Z}{d \log \mu^m} \left[ \log \left( \frac{\mu}{f} \right) \right]^m. \quad (2.60)$$

$\bar{Z}$  is independent of the renormalization scale  $\mu$ . We again choose to set it to one.

The potential for the dilaton now takes the form

$$\begin{aligned} V(\chi) &= \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \bar{\kappa}_{0,m} \left[ \log \left( \frac{\chi}{\mu} \right) \right]^m \frac{\chi^4}{4!} \\ &- \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \bar{\kappa}_{n,m} \left[ \log \left( \frac{\chi}{\mu} \right) \right]^m \frac{\bar{\lambda}_\mathcal{O}^n \chi^{4-n\epsilon}}{4!} \end{aligned} \quad (2.61)$$

where the couplings constants  $\bar{\kappa}_{n,m}$  are given by

$$\sum_{r=0}^{\infty} \frac{(-1)^r}{r!} \frac{d^{m+r}(Z^{2-n\epsilon/2}\kappa_n)}{d \log \mu^{m+r}} \left[ \log \left( \frac{\mu}{f} \right) \right]^r . \quad (2.62)$$

The beta functions of all the  $\bar{\kappa}_{n,m}$  vanish by construction, reflecting the (spurious) conformal invariance of the theory.

The final step is to determine the form of the effective potential. We will again use the Callan-Symanzik equation for the effective potential,

$$\left\{ \mu \frac{\partial}{\partial \mu} + \beta_i \frac{\partial}{\partial \bar{g}_i} - \gamma_{\phi_\alpha} \phi_\alpha \frac{\partial}{\partial \phi_\alpha} \right\} V_{\text{eff}}(\phi_\alpha, \bar{g}_i, \mu) = 0 . \quad (2.63)$$

Here the index  $\alpha$  runs over the fields in the theory, namely  $\chi_{cl}$  and  $\hat{\lambda}_\mathcal{O}$ . The beta functions  $\beta_i(\bar{g}_i)$  vanish as a consequence of the (spurious) conformal symmetry, as does the anomalous dimension of  $\chi$ . The anomalous dimension of  $\hat{\lambda}_\mathcal{O}$  is  $g(\hat{\lambda}_\mathcal{O})$ , the difference between its scaling dimension and mass dimension. Then the Callan-Symanzik equation reduces to

$$\left\{ \mu \frac{\partial}{\partial \mu} - g(\hat{\lambda}_\mathcal{O}) \hat{\lambda}_\mathcal{O} \frac{\partial}{\partial \hat{\lambda}_\mathcal{O}} \right\} V_{\text{eff}}(\chi_{cl}, \hat{\lambda}_\mathcal{O}, \bar{g}_i, \mu) = 0 . \quad (2.64)$$

Making a change of variable from  $\hat{\lambda}_\mathcal{O}$  to  $\bar{\lambda}_\mathcal{O}$ , this becomes simply

$$\mu \frac{\partial}{\partial \mu} V_{\text{eff}}(\chi_{cl}, \bar{\lambda}_\mathcal{O}, \bar{g}_i, \mu) = 0 . \quad (2.65)$$

Dimensional analysis constrains the solution to be of the form

$$V_{\text{eff}}(\chi_{cl}) = \frac{1}{4!} \chi_{cl}^4 [\hat{\kappa}_0 - \mathcal{F}(\bar{\lambda}_\mathcal{O} \chi_{cl}^{-\epsilon})] , \quad (2.66)$$

where  $\hat{\kappa}_0$  is a constant that depends on the couplings  $\bar{g}_i$  but not on  $\bar{\lambda}_\mathcal{O}$ . The form of the function  $\mathcal{F}(\Omega)$  cannot be determined from symmetry considerations alone,

but depends on the dynamics of the conformal field theory under consideration, and on the operator  $\mathcal{O}$ . For values of  $\Omega$  less than one by a factor of at least a few, corresponding to  $\hat{\lambda}_{\mathcal{O}}$  being below its strong coupling value at the symmetry breaking scale  $f$ ,  $\mathcal{F}(\Omega)$  can be computed in perturbation theory. In general it is not a polynomial in  $\Omega$ , as can be seen from Eq. (2.35).

Minimizing the effective potential we find the condition that determines the symmetry breaking scale  $f$ ,

$$4\hat{\kappa}_0 - 4\mathcal{F}(\bar{\lambda}_{\mathcal{O}}f^{-\epsilon}) + \epsilon\bar{\lambda}_{\mathcal{O}}f^{-\epsilon}\mathcal{F}'(\bar{\lambda}_{\mathcal{O}}f^{-\epsilon}) = 0 . \quad (2.67)$$

The dilaton mass squared depends on the second derivative of the effective potential at the minimum, which is given by

$$\frac{\partial^2 V_{\text{eff}}}{\partial \sigma^2} = \frac{1}{4!} f^2 \{ 4\epsilon\bar{\lambda}_{\mathcal{O}}f^{-\epsilon}\mathcal{F}''(\bar{\lambda}_{\mathcal{O}}f^{-\epsilon}) \} . \quad (2.68)$$

Here we are neglecting effects of order  $\epsilon^2$ . We see from this that even at strong coupling, corresponding to  $\hat{\kappa}_0 \sim (4\pi)^2$ , if the function  $\mathcal{F}(\Omega)$  satisfies the condition  $\mathcal{F}(\Omega) \gtrsim \Omega\mathcal{F}'(\Omega)$  at the minimum, the dilaton mass is suppressed by a factor of  $\sqrt{\epsilon}$  relative to the strong coupling scale  $4\pi f$ , and therefore remains light. The question is whether this condition on the function  $\mathcal{F}(\Omega)$  is indeed satisfied in a general strongly coupled conformal field theory, for an arbitrary marginal operator  $\mathcal{O}$ . Unfortunately, in the absence of additional information about the function  $\mathcal{F}(\Omega)$ , we cannot establish such a conclusion. At the minimum, the value of  $\Omega$  is equal to that of  $\hat{\lambda}_{\mathcal{O}}$  evaluated at the symmetry breaking scale  $f$ . For  $\Omega$  of order one, corresponding to  $\hat{\lambda}_{\mathcal{O}}$  close to its strong coupling value, we expect that  $\mathcal{F}(\Omega)$  is of order  $(4\pi)^2$ , but its functional form is completely unknown.

However, there exists a class of strongly coupled theories where the condition  $\mathcal{F}(\Omega) \sim \Omega \mathcal{F}'(\Omega)$  is satisfied, and the dilaton is light. In the region of parameter space where  $\hat{\lambda}_{\mathcal{O}}$  and  $\hat{\kappa}_0$  are below their strong coupling values, the form of the function  $\mathcal{F}(\Omega)$  can be determined from perturbation theory. In this regime it is dominated by the term linear in  $\Omega$  in Eq. (2.57), since the other terms are loop suppressed or higher order in  $\hat{\lambda}_{\mathcal{O}}$ . Now, we expect that there exist strongly coupled conformal field theories where the parameter  $\hat{\kappa}_0$  is below its natural strong coupling value by a factor of order a few. This is quite natural, requiring at most mild tuning. From the minimization condition it follows that in such theories, symmetry breaking is realized for values of  $\mathcal{F}(\Omega)$  that correspond to values of  $\Omega$ , and therefore  $\hat{\lambda}_{\mathcal{O}}$ , that lie below their strong coupling values by roughly the same factor. Since  $\mathcal{F}(\Omega)$  is linear in  $\Omega$  in this regime, the condition  $\mathcal{F}(\Omega) \sim \Omega \mathcal{F}'(\Omega)$  is satisfied at the minimum. Therefore in this class of theories the conclusion  $m_\sigma \sim \sqrt{\epsilon}$  is valid, and the dilaton is light.

Since this analysis is restricted to the region of parameter space where  $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$ , it is important to understand the circumstances under which this condition is satisfied. One possibility is that  $\mathcal{O}$  is a protected operator, so that all the coefficients  $c_n$  in the polynomial expansion of  $g(\hat{\lambda}_{\mathcal{O}})$  are of order  $\epsilon$ . The operator  $\mathcal{O}$  is then close to marginal for any value of  $\hat{\lambda}_{\mathcal{O}}$ . An example is a theory where the parameter  $\hat{\lambda}_{\mathcal{O}}$  corresponds to a fixed line, while the parameter  $\epsilon$  is associated with the coefficient of an operator that is very close to marginal (for all  $\hat{\lambda}_{\mathcal{O}}$ ) and which lifts the fixed line. However, theories that admit such protected operators are clearly rather special. There is no reason to expect the condition  $c_n \lesssim \epsilon$  to be satisfied by an arbitrary



marginal operator  $\mathcal{O}$  in a general conformal field theory.

Another possibility is that the parameter  $\hat{\kappa}_0$  lies significantly below its natural strong coupling value so that symmetry breaking is realized for values of  $\hat{\lambda}_{\mathcal{O}}$  less than  $\epsilon$ . The condition  $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$  can then be satisfied. Since in this regime  $\mathcal{F}(\Omega)$  is dominated by the term linear in  $\Omega$ ,  $\mathcal{F}(\Omega) \sim (4\pi)^2 \Omega \sim (4\pi)^2 \hat{\lambda}_{\mathcal{O}}$ , it follows from Eq. (2.67) that the condition  $\hat{\lambda}_{\mathcal{O}} < \epsilon$  translates into  $\hat{\kappa}_0/(4\pi)^2 \lesssim \epsilon$ . It follows from Eq. (2.68) and the minimization condition Eq. (2.67) that in this regime the dilaton mass scales as  $\sqrt{\hat{\kappa}_0 \epsilon}$ , and therefore receives additional suppression from the fact that  $\hat{\kappa}_0$  is small. However, small values of  $\hat{\kappa}_0$  are associated with tuning, and so this condition is not expected to be satisfied in a general conformal field theory. However, in the case of small hierarchies, such as between the flavor scale and the weak scale, values of  $\epsilon$  as large as  $1/5$  can still serve to address the problem. It follows from this that in such a theory, a dilaton mass a factor of 5 below the strong coupling scale can be realized for  $\hat{\kappa}_0$  a factor of 5 below its natural strong coupling value. Since the tuning scales with  $\hat{\kappa}_0$ , this theory need only be tuned at the level of 1 part in 5 (20%). This is to be contrasted with the case of a (non-pNGB) composite scalar of the same mass, which is tuned at the level of 1 part in 25 (4%). We see that although this scenario is tuned, the tuning is mild, scaling with the mass of the dilaton rather than the square of its mass.

### 2.2.2.2 Limit When the Corrections Are Large

We now turn our attention to the case when the corrections to the scaling behavior of  $\mathcal{O}$  are large in the neighborhood of the breaking scale, so that  $|\delta g(\hat{\lambda}_{\mathcal{O}})| > \epsilon$ . For simplicity, we will work in the limit that the renormalization group evolution of  $\log \hat{\lambda}_{\mathcal{O}}$  close to the breaking scale is dominated by the term linear in  $\hat{\lambda}_{\mathcal{O}}$  so that

$$\frac{d \log \hat{\lambda}_{\mathcal{O}}}{d \log \mu} = -c_1 \hat{\lambda}_{\mathcal{O}}, \quad (2.69)$$

Integrating this equation we find that  $\mathcal{G}(\hat{\lambda}_{\mathcal{O}})$  is now given by

$$\mathcal{G}(\hat{\lambda}_{\mathcal{O}}) = \exp\left(\frac{1}{c_1 \hat{\lambda}_{\mathcal{O}}}\right). \quad (2.70)$$

Since  $\mathcal{G}(\hat{\lambda}_{\mathcal{O}})\mu^{-1}$  is a renormalization group invariant, it follows that  $\bar{\lambda}_{\mathcal{O}}$ , now defined as

$$\bar{\lambda}_{\mathcal{O}} = \frac{\hat{\lambda}_{\mathcal{O}}}{1 - c_1 \hat{\lambda}_{\mathcal{O}} \log \mu} \quad (2.71)$$

is also a renormalization group invariant. Once  $\hat{\lambda}_{\mathcal{O}}$  is promoted to a spurion as in Eq (2.43), the object

$$\Omega(\bar{\lambda}_{\mathcal{O}}, \chi) = \frac{\bar{\lambda}_{\mathcal{O}}}{1 + c_1 \bar{\lambda}_{\mathcal{O}} \log \chi} \quad (2.72)$$

is invariant under (spurious) scale transformations. Furthermore, at scales  $\mu$  close to  $\langle \chi \rangle = f$ , it can be expanded as a polynomial in  $\hat{\lambda}_{\mathcal{O}}$ . The Lagrangian for the low energy effective theory can be constructed using  $\Omega$ .

In a scheme where the renormalization scale is proportional to  $\chi$ ,  $\mu_{\chi} = \mu \hat{\chi}$ , the potential takes the form

$$V(\chi) = \frac{Z^2 \chi^4}{4!} \left[ \kappa_0 - \sum_{n=1}^{\infty} \kappa_n \Omega^n(\bar{\lambda}_{\mathcal{O}}, \sqrt{Z} \chi) \right]. \quad (2.73)$$

It is straightforward to go over to a scheme where the renormalization scale  $\mu$  is independent of  $\chi$ , and where all mass parameters in the Lagrangian are expressed as powers of  $\mu$ . As before, the dimensionless coupling constants  $\bar{g}_i$  in such a scheme are independent of the renormalization scale  $\mu$ .

The effective potential for the low energy effective theory can once again be determined from the Callan-Symanzik equation, Eq (2.63). The anomalous dimension of  $\chi$  vanishes while that of the spurion  $\hat{\lambda}_{\mathcal{O}}$  is given by  $g(\hat{\lambda}_{\mathcal{O}}) = c_1 \hat{\lambda}_{\mathcal{O}}$ . As a consequence the Callan-Symanzik equation reduces to

$$\left\{ \mu \frac{\partial}{\partial \mu} - c_1 \hat{\lambda}_{\mathcal{O}}^2 \frac{\partial}{\partial \hat{\lambda}_{\mathcal{O}}} \right\} V_{\text{eff}}(\chi_{cl}, \hat{\lambda}_{\mathcal{O}}, \bar{g}_i, \mu) = 0 . \quad (2.74)$$

Making the change of variable from  $\hat{\lambda}_{\mathcal{O}}$  to  $\bar{\lambda}_{\mathcal{O}}$ , this simplifies to

$$\mu \frac{\partial}{\partial \mu} V_{\text{eff}}(\chi_{cl}, \bar{\lambda}_{\mathcal{O}}, \bar{g}_i, \mu) = 0 . \quad (2.75)$$

Dimensional analysis constrains the solution to be of the form

$$V_{\text{eff}}(\chi_{cl}) = \frac{1}{4!} \chi_{cl}^4 \{ \hat{\kappa}_0 - \mathcal{F}[\Omega(\bar{\lambda}_{\mathcal{O}}, \chi_{cl})] \} . \quad (2.76)$$

The form of the function  $\mathcal{F}[\Omega]$  cannot be determined from symmetry considerations alone, but depends on the dynamics of the conformal field theory under consideration, and on the operator  $\mathcal{O}$ . For values of  $\Omega$  less than one by at least a factor of a few, corresponding to  $\hat{\lambda}_{\mathcal{O}}$  being below its strong coupling value at the symmetry breaking scale,  $\mathcal{F}(\Omega)$  can be computed in perturbation theory.

Minimizing the effective potential we obtain the condition that determines  $f$ ,

$$4\hat{\kappa}_0 - 4\mathcal{F}[\Omega] + c_1 \Omega^2 \mathcal{F}'[\Omega] = 0 . \quad (2.77)$$

The dilaton mass squared depends on the second derivative of the effective potential at the minimum, which can be determined as

$$\frac{\partial^2 V_{\text{eff}}}{\partial \sigma^2} = \frac{c_1}{4!} f^2 \Omega^2 [(4 - 2c_1 \Omega) \mathcal{F}' - c_1 \Omega^2 \mathcal{F}''] . \quad (2.78)$$

Once again we focus on theories where the parameter  $\hat{\kappa}_0$  is below its natural strong coupling value by some factor, which could be as small as a few. In such theories, symmetry breaking is realized for values of  $\hat{\lambda}_{\mathcal{O}}$  that are below its strong coupling value. In this limit the form of the effective potential can be determined in perturbation theory, and  $\mathcal{F}(\Omega)$  is dominated by the term linear in  $\Omega$ . Then at the minimum the condition  $\mathcal{F}[\Omega] \sim \Omega \mathcal{F}'[\Omega]$  is satisfied. Noting that at the minimum the value of  $\Omega$  is equal to that of  $\hat{\lambda}_{\mathcal{O}}$  at the scale  $f$ , it follows from Eq. (2.77) and Eq. (2.78) that the dilaton mass squared scales as  $m_\sigma^2 \sim c_1 \hat{\lambda}_{\mathcal{O}} \hat{\kappa}_0$ .

We see from this that the dilaton mass depends on the scaling behavior of the operator  $\mathcal{O}$  at the symmetry breaking scale, and therefore on the value of  $c_1 \hat{\lambda}_{\mathcal{O}}$ . Since the minimization condition Eq. (2.77) relates  $\Omega$  (and therefore  $\hat{\lambda}_{\mathcal{O}}$  at the breaking scale) to  $\hat{\kappa}_0$ , for  $c_1$  of order its natural value of one we have that the dilaton mass squared scales as  $\hat{\kappa}_0^2$ . This suggests that for  $\hat{\kappa}_0$  of order its natural strong coupling value the dilaton mass lies near the cutoff of the theory, and it is not a light state. It follows from this that in general, the spectrum of a conformal field theory broken by an arbitrary marginal operator that grows strong in the infrared does not include a light dilaton.

The low energy effective theory will however contain a light dilaton if the parameter  $\hat{\kappa}_0$  lies significantly below its natural strong coupling value. In general,

this involves tuning, since this condition is not expected to be satisfied in an arbitrary conformal field theory. However, the tuning is mild, scaling with  $\hat{\kappa}_0$  and therefore as the mass of the dilaton, so that a dilaton that lies a factor of 5 below the strong coupling scale is only tuned at the level of 1 part in 5 (20 %).

It follows from this discussion that in strongly coupled theories where an approximate conformal symmetry is spontaneously broken, the low energy spectrum includes a light dilaton if the operator that breaks the symmetry is close to marginal at the breaking scale. This condition is in general not expected to be satisfied by the theories of interest for electroweak symmetry breaking, and so the presence of a light dilaton in these theories is associated with tuning. However, the tuning is mild, scaling as the mass of the dilaton rather than as the square of its mass.

### 2.3 Dilaton Interactions to SM fields

In this section, we calculate the form of the interaction of the dilaton to the SM gauge bosons and fermions. In particular we focus on the three scenarios mentioned earlier on page 40. In the limit that conformal invariance is exact, the form of the dilaton interactions with SM fields in the low energy effective theory is fixed by the requirement that the symmetry be realized nonlinearly. Since CFT is a spacetime symmetry, both elementary fields as well as composite operators transform non-trivially under the CFT. Therefore in both cases, the low energy effective theory is fixed by the requirement of non-linear realization of the symmetry. In the scenario of interest where the CFT is broken by the operator  $\mathcal{O}$ , we expect significant deviations

from the results for the case when there is exact conformal invariance. This is because effects associated with the operator  $\mathcal{O}$  that violate the symmetry are large at the breaking scale. It is crucial to understand the size of these effects, and the extent to which the predictions of the theory with exact conformal invariance are affected.

### 2.3.1 Dilaton Interactions in a Conformal SM

In this section, we consider a scenario where the SM gauge bosons and matter fields are all composites of a strongly interacting conformal sector that breaks electroweak symmetry dynamically, and there is no light Higgs. The AdS/CFT correspondence relates this scenario to Higgsless RS models where the SM matter and gauge fields are localized on the infrared brane. The couplings of the dilaton to the SM fields in such a framework have been determined [59], and agree with earlier results for the couplings of the radion to brane-localized fields in RS models [67,81–83]. Several authors have studied the question of distinguishing the dilaton from the Higgs at the Large Hadron Collider (LHC) in such a scenario [84–86], see also [87]. We will study the corrections to the dilaton couplings in this scenario when effects associated with the operator  $\mathcal{O}$  that explicitly violates conformal symmetry are incorporated.

We begin by considering the dilaton couplings to the  $W$  and  $Z$  gauge bosons. We choose to work in a basis where we write all gauge kinetic terms in the form

$$-\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} . \tag{2.79}$$

In the absence of conformal symmetry violating effects, the couplings of the dilaton to the  $W$  are such as to compensate for the breaking of conformal invariance by the gauge boson mass term. In unitary gauge these take the form

$$\left(\frac{\chi}{f}\right)^2 \frac{m_W^2}{g^2} W_\mu^+ W^{\mu-} \quad (2.80)$$

in the Lagrangian. Here  $m_W$  is the  $W$  gauge boson mass. Expanding the compensator  $\chi = f e^{\sigma/f}$  out in terms of  $\sigma$  to leading order in inverse powers of  $f$ , we find for the dilaton couplings

$$2 \frac{\sigma}{f} \frac{m_W^2}{g^2} W_\mu^+ W^{\mu-} . \quad (2.81)$$

Next we consider the corrections to the dilaton couplings when conformal symmetry violating effects are included. We will focus on the case when the corrections to the scaling behavior of  $\mathcal{O}$  are small, so that  $|\delta g(\hat{\lambda}_{\mathcal{O}})|$  is less than  $\epsilon$  at the breaking scale. We will later argue that the same conclusions are obtained in the limit when  $|\delta g(\hat{\lambda}_{\mathcal{O}})|$  is greater than  $\epsilon$ .

The presence of conformal symmetry violating effects allows additional two derivative terms in the dilaton action,

$$\frac{1}{2} \left[ 1 + \sum_{n=1}^{\infty} \alpha_{\chi,n} \bar{\lambda}_{\mathcal{O}}^n \chi^{(-n\epsilon)} \right] \partial_\mu \chi \partial^\mu \chi . \quad (2.82)$$

The dimensionless parameters  $\alpha_{\chi,n}$  depend both on the operator  $\mathcal{O}$  and the specific conformal field theory under consideration. They are expected to be of order one. These new terms contribute to the dilaton kinetic term, which now becomes

$$\frac{1}{2} \left[ 1 + \sum_{n=1}^{\infty} \alpha_{\chi,n} \bar{\lambda}_{\mathcal{O}}^n f^{(-n\epsilon)} \right] \partial_\mu \sigma \partial^\mu \sigma . \quad (2.83)$$

When  $\sigma$  is rescaled to make the dilaton kinetic term canonical, we see that the effective impact of these terms is to alter the effective value of  $f$  in Eq. (2.81), while leaving the form of the interaction unchanged. More generally, it follows that corrections to the dilaton kinetic term from conformal symmetry violating effects do not alter the form of the dilaton couplings to the SM fields. Instead, to leading order in  $\sigma/f$ , they lead to a universal rescaling in the effective value of  $f$ , leaving the relative strengths of the dilaton couplings to the various SM fields unchanged. Since to the order we are working this effect can be entirely absorbed into the parameter  $f$ , we will not consider it further.

The gauge kinetic term also receives corrections from conformal symmetry violating effects. It now takes the form

$$-\frac{1}{4\hat{g}^2} \left[ 1 + \sum_{n=1}^{\infty} \alpha_{W,n} \bar{\lambda}_{\mathcal{O}}^n \chi^{(-n\epsilon)} \right] F_{\mu\nu} F^{\mu\nu} , \quad (2.84)$$

where the parameters  $\alpha_{W,n}$  are dimensionless. They are expected to be of order one.

The physical gauge coupling is now given by

$$\frac{1}{g^2} = \frac{1}{\hat{g}^2} \left[ 1 + \sum_{n=1}^{\infty} \alpha_{W,n} \bar{\lambda}_{\mathcal{O}}^n f^{(-n\epsilon)} \right] . \quad (2.85)$$

Expanding Eq. (2.84) to leading order in  $\sigma$  we obtain

$$\epsilon \frac{\bar{c}_W}{4g^2} \frac{\sigma}{f} F_{\mu\nu} F^{\mu\nu} , \quad (2.86)$$

where the dimensionless parameter  $\bar{c}_W$  is given by

$$\bar{c}_W = \frac{\sum_{n=1}^{\infty} n \alpha_{W,n} \bar{\lambda}_{\mathcal{O}}^n f^{(-n\epsilon)}}{1 + \sum_{n=1}^{\infty} \alpha_{W,n} \bar{\lambda}_{\mathcal{O}}^n f^{(-n\epsilon)}} . \quad (2.87)$$

In a strongly coupled theory  $\bar{c}_W$  is expected to be of order  $\bar{\lambda}_{\mathcal{O}} f^{-\epsilon}$ , which is the value



of  $\hat{\lambda}_\mathcal{O}$  at the breaking scale  $f$ . It follows that this correction to the dilaton coupling is suppressed by  $\epsilon\hat{\lambda}_\mathcal{O}$ , which is of order  $m_\sigma^2/\Lambda^2$ .

Conformal symmetry violating effects also modify the gauge boson mass term, Eq. (2.80), which now becomes

$$\left(\frac{\chi}{f}\right)^2 \left[ 1 + \sum_{n=1}^{\infty} \beta_{W,n} \bar{\lambda}_\mathcal{O}^n \chi^{(-n\epsilon)} \right] \frac{\hat{m}_W^2}{\hat{g}^2} W_\mu^+ W^{\mu-} . \quad (2.88)$$

Here  $\hat{m}_W$  is the  $W$  boson mass in the unperturbed theory, and the dimensionless parameters  $\beta_{W,n}$  are of order one. Expanding this out in terms of  $\sigma(x)$ , we see that to leading order in inverse powers of  $f$ , the dilaton couples as

$$\frac{\sigma}{f} \frac{m_W^2}{g^2} [2 + c_W \epsilon] W_\mu^+ W^{\mu-} . \quad (2.89)$$

Here  $m_W^2$  is again the physical  $W$  boson mass,

$$m_W^2 = \hat{m}_W^2 \left[ \frac{1 + \sum_{n=1}^{\infty} \beta_{W,n} \bar{\lambda}_\mathcal{O}^n f^{(-n\epsilon)}}{1 + \sum_{n=1}^{\infty} \alpha_{W,n} \bar{\lambda}_\mathcal{O}^n f^{(-n\epsilon)}} \right] , \quad (2.90)$$

while the dimensionless parameter  $c_W$  is given by

$$c_W = - \frac{\sum_{n=1}^{\infty} n \beta_{W,n} \bar{\lambda}_\mathcal{O}^n f^{(-n\epsilon)}}{1 + \sum_{n=1}^{\infty} \beta_{W,n} \bar{\lambda}_\mathcal{O}^n f^{(-n\epsilon)}} . \quad (2.91)$$

In the strong coupling limit,  $c_W$  is expected to be of order  $\bar{\lambda}_\mathcal{O} f^{-\epsilon} \sim \hat{\lambda}_\mathcal{O}$ . We see that the effect of the conformal symmetry violating term is to correct the dilaton couplings by order  $\epsilon\hat{\lambda}_\mathcal{O} \sim m_\sigma^2/\Lambda^2$ .

If we instead consider the limit when  $|\delta g(\hat{\lambda}_\mathcal{O})|$  is greater than  $\epsilon$  at the symmetry breaking scale  $f$ , the analysis is very similar. The only significant difference is that  $\bar{\lambda}_\mathcal{O} \chi^{-\epsilon}$  in Eqs. (2.82), (2.84) and (2.88) is replaced by  $\Omega(\bar{\lambda}_\mathcal{O}, \chi)$ , which in this limit is given by

$$\Omega(\bar{\lambda}_\mathcal{O}, \chi) = \frac{\bar{\lambda}_\mathcal{O}}{1 + c_1 \bar{\lambda}_\mathcal{O} \log \chi} . \quad (2.92)$$

Following exactly the same sequence of steps we find that the corrections to the dilaton couplings have the same form, but are now suppressed by  $c_1\Omega^2(\bar{\lambda}_{\mathcal{O}}, f)$  rather than  $\epsilon\bar{\lambda}_{\mathcal{O}}f^{-\epsilon}$ . However, this new suppression factor is of order  $m_\sigma^2/\Lambda^2$ , exactly as before. We see that the corrections have the same form and are of the same size as in the case  $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$ . It is not difficult to verify that this result is quite general. Therefore, in the remainder of the paper we will limit our analysis to the case  $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$ , with the understanding that the same general conclusions apply to the case  $|\delta g(\hat{\lambda}_{\mathcal{O}})| > \epsilon$  as well.

Next we turn our attention to the dilaton couplings to the massless gauge bosons of the SM, the photon and the gluon. Unlike the  $W$  and  $Z$ , the Lagrangian for these particles does not break conformal invariance at the classical level, only at the quantum level. At one loop the RGEs for the corresponding gauge couplings are of the form

$$\frac{d}{d \log \mu} \frac{1}{g^2} = \frac{b_{<}}{8\pi^2} \quad (2.93)$$

where the constant  $b_{<} = -11/3$  for electromagnetism and  $+7$  for color, at scales above the mass of the top. This implies that under infinitesimal scale transformations  $x \rightarrow x' = e^{-\omega}x$ , the operator  $F_{\mu\nu}F^{\mu\nu}$  transforms as  $F_{\mu\nu}F^{\mu\nu}(x) \rightarrow F'_{\mu\nu}F'^{\mu\nu}(x')$ , where

$$F'_{\mu\nu}F'^{\mu\nu}(x') = e^{4\omega} \left( 1 + \frac{b_{<}}{8\pi^2} g^2 \omega \right) F_{\mu\nu}F^{\mu\nu}(x) \quad (2.94)$$

If conformal symmetry is to be realized nonlinearly, the couplings of the dilaton must be such as to compensate for this. It is then easy to see that the dilaton

couplings in the Lagrangian must take the form

$$\frac{b_{<}}{32\pi^2} \log\left(\frac{\chi}{f}\right) F_{\mu\nu} F^{\mu\nu} . \quad (2.95)$$

Expanding this out in terms of  $\sigma(x)$ , to leading order in inverse powers of  $f$ , we find for the dilaton coupling

$$\frac{b_{<}}{32\pi^2} \frac{\sigma}{f} F_{\mu\nu} F^{\mu\nu} . \quad (2.96)$$

It follows from this that the dilaton couples much more weakly to the massless gauge bosons than to the  $W$  or the  $Z$ . The reason is that the gauge interactions correspond to marginal operators in the low energy effective theory, while mass terms for the gauge bosons are relevant operators. Since the dilaton couples as a conformal compensator, it is to be expected that its couplings to massless gauge bosons are suppressed.

We now consider corrections to this interaction arising from conformal symmetry violating effects. These allow direct couplings of the compensator  $\chi$  to the gauge kinetic term of the form

$$-\frac{1}{4\hat{g}^2} \left[ 1 + \sum_{n=1}^{\infty} \alpha_{A,n} \bar{\lambda}_{\mathcal{O}}^n \chi^{(-n\epsilon)} \right] F_{\mu\nu} F^{\mu\nu} . \quad (2.97)$$

The physical gauge coupling is now given by

$$\frac{1}{g^2} = \frac{1}{\hat{g}^2} \left[ 1 + \sum_{n=1}^{\infty} \alpha_{A,n} \bar{\lambda}_{\mathcal{O}}^n f^{(-n\epsilon)} \right] . \quad (2.98)$$

Expanding Eq. (2.97) in terms of  $\sigma$ , and combining with Eq. (2.96) we find for the dilaton coupling to massless gauge bosons

$$\frac{\sigma}{f} \left[ \frac{b_{<}}{32\pi^2} + \frac{c_A}{4g^2} \epsilon \right] F_{\mu\nu} F^{\mu\nu} . \quad (2.99)$$

Here the dimensionless coupling  $c_A$  is given by

$$c_A = \frac{\sum_{n=1}^{\infty} n \alpha_{A,n} \bar{\lambda}_{\mathcal{O}}^n f^{(-n\epsilon)}}{1 + \sum_{n=1}^{\infty} \alpha_{A,n} \bar{\lambda}_{\mathcal{O}}^n f^{(-n\epsilon)}} . \quad (2.100)$$

In a strongly coupled theory the parameter  $c_A$  is expected to be of order  $\bar{\lambda}_{\mathcal{O}} f^{-\epsilon} \sim \hat{\lambda}_{\mathcal{O}}$  at the scale  $f$ . We see from this that the corrections to the dilaton coupling arising from symmetry breaking effects are suppressed by  $m_\sigma^2/\Lambda^2$ . Nevertheless, the fact that the leading order effect is loop suppressed and therefore small implies that the symmetry breaking contribution may dominate.

Finally we consider the couplings of the dilaton to the SM fermions. In the limit that conformal symmetry is exact, the coupling of the dilaton is such as to compensate for the spontaneous breaking of conformal invariance by the fermion mass terms. These interactions take the form

$$\frac{\chi}{f} m_\psi \bar{\psi} \psi \quad (2.101)$$

in the potential, where we have suppressed flavor indices. Expanding the compensator out in terms of  $\sigma$  we obtain

$$\sigma \frac{m_\psi}{f} \bar{\psi} \psi . \quad (2.102)$$

However, if a conformal symmetry breaking effect of the form considered in the previous section is present, Eq. (2.101) generalizes to

$$\frac{\chi}{f} \left[ 1 + \sum_{n=1}^{\infty} \beta_{\psi,n} \bar{\lambda}_{\mathcal{O}}^n \chi^{(-n\epsilon)} \right] \hat{m}_\psi \bar{\psi} \psi , \quad (2.103)$$

where  $\hat{m}_\psi$  is the fermion mass in the unperturbed theory, and the parameters  $\beta_{\psi,n}$  are dimensionless. In obtaining this we have assumed that the operator  $\mathcal{O}$  does not

violate the approximate  $U(3)^5$  flavor symmetry associated with the SM fermions in the chiral limit, which is broken by the fermion mass terms. This ensures that the dilaton couples diagonally in the mass basis.

In general the operator  $\mathcal{O}$  will also correct the fermion kinetic term, which generalizes to

$$\left[ 1 + \sum_{n=1}^{\infty} \alpha_{\psi,n} \bar{\lambda}_{\mathcal{O}}^n \chi^{(-n\epsilon)} \right] \bar{\psi} \gamma^\mu \partial_\mu \psi . \quad (2.104)$$

After expanding out Eqs. (2.103) and (2.104) in terms of  $\sigma$ , rescaling to make the fermion kinetic term canonical, and then using the equation of motion for  $\psi$ , we obtain a correction to the dilaton coupling of the form

$$\sigma \frac{m_\psi}{f} [1 + c_\psi \epsilon] \bar{\psi} \psi , \quad (2.105)$$

In a strongly coupled conformal field theory we expect that  $c_\psi$  is of order  $\bar{\lambda}_{\mathcal{O}} f^{-\epsilon}$ . We conclude from this that the effect of the conformal symmetry violating terms is to modify the dilaton couplings to the SM fermions by order  $m_\sigma^2/\Lambda^2$ .

From this discussion we see that conformal symmetry violating effects associated with the operator  $\mathcal{O}$  correct the parameters in the low energy effective theory at order  $\hat{\lambda}_{\mathcal{O}}$ . However, these effects can be absorbed into the masses and couplings of the light states, so that corrections to the form of the dilaton couplings to the SM only arise at order  $\epsilon \hat{\lambda}_{\mathcal{O}} \sim m_\sigma^2/\Lambda^2$ , and are therefore small if the dilaton is light. To understand why the corrections to the form of the dilaton couplings receive additional suppression, note that if  $g(\hat{\lambda}_{\mathcal{O}})$  were to vanish close to the breaking scale  $f$ , the effects of explicit conformal symmetry violation would disappear even though  $\hat{\lambda}_{\mathcal{O}}$  was non-zero. In this limit, the dilaton couplings must have exactly the same

form as in a theory without conformal symmetry violation, so that all the corrections of order  $\hat{\lambda}_{\mathcal{O}}$  must be able to be absorbed into the masses and couplings of the SM states. It follows that corrections to the form of the dilaton couplings must be suppressed by both  $\hat{\lambda}_{\mathcal{O}}$  and  $g(\hat{\lambda}_{\mathcal{O}})$  ( $\sim \epsilon$  in the limit we are working in).

In summary we see that corrections to the form of the dilaton couplings to SM states arising from conformal symmetry violating effects are suppressed by the square of the ratio of the dilaton mass to the strong coupling scale, and therefore under good theoretical control in the theories of interest. These contributions are generally subleading, except in the case of dilaton couplings to marginal operators, when symmetry violating effects can dominate.

### 2.3.2 Technicolor

In this section we determine the form of the couplings of a light dilaton to the SM fields in a scenario where electroweak symmetry is broken dynamically by a strongly interacting sector, and there is no light Higgs. The strongly interacting sector is assumed to be conformal in the far ultraviolet. However, conformal symmetry is explicitly broken by the operator  $\mathcal{O}$ , which grows large close to the TeV scale triggering electroweak symmetry breaking. The SM gauge fields do not constitute part of the strongly interacting sector. However, this sector transforms under the weak and electromagnetic gauge interactions. It may also transform under the SM color group. The SM gauge interactions constitute another small explicit breaking of the conformal symmetry. The SM fermions may be elementary, or may emerge

as composites or partial composites of the strong dynamics.

The AdS/CFT correspondence relates this class of theories to Higgsless RS models with the SM gauge fields propagating in the bulk. The couplings of the radion to SM fields in this framework have been determined, in the limit that effects associated with the dynamics that stabilizes the radion are neglected [88, 89]. We reproduce these results, and in doing so establish their validity beyond the large  $N$  limit. We also determine the corrections to the dilaton couplings that arise from conformal symmetry violating effects.

In order to avoid large corrections to precision electroweak observables, the strongly interacting sector must respect a custodial  $SU(2)$  symmetry. This symmetry is not exact, but is broken by the SM Yukawa couplings, and also by hypercharge. A simple way to realize custodial symmetry is to extend the  $SU(2)_L$  symmetry of the SM to  $SU(2)_L \times SU(2)_R$ . Only the diagonal generator of this new  $SU(2)_R$  is gauged, and is associated with hypercharge. The strong dynamics breaks this extended symmetry down to the diagonal  $SU(2)$ , which is identified with the custodial symmetry. Only a  $U(1)$  subgroup of the original  $SU(2) \times U(1)$  gauge symmetry survives, and is identified with electromagnetism.

The NGBs  $\pi(x)$  that arise from the breaking of  $SU(2) \times SU(2)$  gauge symmetry down to the custodial  $SU(2)$  can be parametrized in terms of a matrix  $\Sigma$ , defined as

$$\Sigma = e^{i\frac{\pi(x)}{v}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}. \quad (2.106)$$

Here  $v$  is the electroweak VEV.  $\Sigma$  transforms linearly under  $SU(2)_L \times SU(2)_R$ , and is therefore more convenient for writing interactions. In unitary gauge the NGBs

$\pi(x)$  are absorbed into the  $W$  and  $Z$  gauge bosons and  $\Sigma$  can be replaced by its VEV.

### 2.3.2.1 Couplings to Gauge Bosons

We begin by determining the dilaton couplings to the  $W$  and  $Z$  gauge bosons. In the ultraviolet, the SM gauge interactions do not violate the conformal symmetry of the theory at the classical level, only at the quantum level. Therefore, when these effects are included, the theory still respects conformal symmetry up to effects which are suppressed by loops involving the SM gauge bosons. Therefore, the dominant interactions of the dilaton to the  $W$  and  $Z$  bosons in the low energy effective theory arise from couplings which compensate for the breaking of conformal invariance by the gauge boson mass terms. These take exactly the same form in the Lagrangian as in the case of composite  $W$  and  $Z$  gauge bosons

$$\left(\frac{\chi}{f}\right)^2 \frac{m_W^2}{g^2} W_\mu^+ W^{\mu-} . \quad (2.107)$$

Expanding this out in terms of  $\sigma$  to leading order in inverse powers of  $f$ , we again find

$$2\frac{\sigma}{f} \frac{m_W^2}{g^2} W_\mu^+ W^{\mu-} . \quad (2.108)$$

This agrees with the known results for the coupling of the radion to bulk gauge bosons in RS models [88, 89]. Our analysis shows that this formula is valid beyond the large  $N$  limit.

When conformal symmetry violating effects associated with the operator  $\mathcal{O}$  are present, the gauge boson mass will in general depend on  $\hat{\lambda}_{\mathcal{O}}$ . Then Eq. 2.107)



generalizes to

$$\left(\frac{\chi}{f}\right)^2 \left[1 + \sum_{n=1}^{\infty} \beta_{W,n} \bar{\lambda}_{\mathcal{O}}^n \chi^{(-n\epsilon)}\right] \frac{\hat{m}_W^2}{\hat{g}^2} W_\mu^+ W^{\mu-} . \quad (2.109)$$

Here  $\hat{m}_W$  is the  $W$  boson mass in the unperturbed theory, and the dimensionless parameters  $\beta_{W,n}$  are of order one. Expanding this out in terms of  $\sigma(x)$ , we find that the dilaton couples as

$$\frac{\sigma}{f} \frac{m_W^2}{g^2} [2 + c_W \epsilon] W_\mu^+ W^{\mu-} , \quad (2.110)$$

where  $m_W^2$  is the physical  $W$  boson mass. The dimensionless parameter  $c_W$  is of order  $\bar{\lambda}_{\mathcal{O}} f^{-\epsilon} \sim \hat{\lambda}_{\mathcal{O}}$  at the scale  $f$ , so that the correction to the coupling is suppressed by  $m_\sigma^2/\Lambda^2$ .

We move on to consider the dilaton couplings to the massless gauge bosons of the SM, the gluon and the photon. We first determine the form of the couplings in the limit that effects arising from the operator  $\mathcal{O}$  are neglected. Above the breaking scale, the RGE for the corresponding gauge coupling takes the form

$$\frac{d}{d \log \mu} \frac{1}{g_{UV}^2} = \frac{b_>}{8\pi^2} , \quad (2.111)$$

where the constant  $b_>$  receives contributions from both elementary states and the strongly interacting sector. Similarly, below the breaking scale it takes the form

$$\frac{d}{d \log \mu} \frac{1}{g_{IR}^2} = \frac{b_<}{8\pi^2} \quad (2.112)$$

where  $b_<$  receives contributions from elementary states, and also from any additional light states that emerge from the strongly interacting sector after symmetry breaking.

Equation (2.111) indicates that above the symmetry breaking scale, under infinitesimal scale transformations  $x \rightarrow x' = e^{-\omega}x$ , the operator corresponding to the gauge kinetic term transforms as  $F_{\mu\nu}F^{\mu\nu}(x) \rightarrow F'_{\mu\nu}F'^{\mu\nu}(x')$ , where

$$F'_{\mu\nu}F'^{\mu\nu}(x') = e^{4\omega} \left( 1 + \frac{b_{>}}{8\pi^2} g_{UV}^2 \omega \right) F_{\mu\nu}F^{\mu\nu}(x) \quad (2.113)$$

Below the symmetry breaking scale the corresponding transformation is  $F_{\mu\nu}F^{\mu\nu}(x) \rightarrow F'_{\mu\nu}F'^{\mu\nu}(x')$ , where

$$F'_{\mu\nu}F'^{\mu\nu}(x') = e^{4\omega} \left( 1 + \frac{b_{<}}{8\pi^2} g_{IR}^2 \omega \right) F_{\mu\nu}F^{\mu\nu}(x) \quad (2.114)$$

Above the symmetry breaking scale we can make the gauge kinetic term formally invariant under infinitesimal scale transformations by promoting the gauge coupling constant  $g_{UV}$  to a spurion that under  $x \rightarrow x' = e^{-\omega}x$  transforms as

$$\frac{1}{g_{UV}^2} \rightarrow \frac{1}{g'^2_{UV}} = \frac{1}{g_{UV}^2} - \frac{b_{>}}{8\pi^2} \omega. \quad (2.115)$$

Now, matching at one loop across the symmetry breaking threshold we have

$$\frac{1}{g_{IR}^2} = \frac{1}{g_{UV}^2} + \frac{C}{8\pi^2} \quad (2.116)$$

where  $C$  is a dimensionless number that depends on the gauge quantum numbers of the states in the strongly interacting sector that have been integrated out at the threshold. While  $C$  cannot be calculated, since it depends on details of the strong dynamics, it is of order the number of states that have masses at the threshold. It is independent of  $g^2$  up to corrections which are additionally loop suppressed.

In the limit that conformal symmetry violating effects arising from the operator  $\mathcal{O}$  are neglected, it follows from Eqs. (2.114) and (2.115) that if the gauge kinetic

term in the low energy effective theory,

$$-\frac{1}{4}\frac{1}{g_{IR}^2}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}\left\{\frac{1}{g_{UV}^2} + \frac{C}{8\pi^2}\right\}F_{\mu\nu}F^{\mu\nu}, \quad (2.117)$$

is to be invariant under infinitesimal scale transformations, the conformal compensator must couple as

$$\frac{(b_{<} - b_{>})}{32\pi^2}\log\left(\frac{\chi}{f}\right)F_{\mu\nu}F^{\mu\nu} \quad (2.118)$$

in the Lagrangian. Expanding this out in terms of  $\sigma(x)$ , to leading order in inverse powers of  $f$ , we find for the dilaton coupling

$$\frac{(b_{<} - b_{>})}{32\pi^2}\frac{\sigma}{f}F_{\mu\nu}F^{\mu\nu}. \quad (2.119)$$

This agrees with the result in the literature for the coupling of the radion to massless bulk gauge bosons in RS models [88, 89]. Our analysis establishes that this result is valid beyond the large  $N$  limit. This formula is valid at scales slightly below the strong coupling scale  $4\pi f$ , and must be renormalization group evolved to the dilaton mass. If the conformal sector does not transform under the SM color group, as may be the case in theories where the top quark is not a composite of the strong dynamics, then  $b_{<} = b_{>}$  and the gluon does not couple to the dilaton at this order. In such a scenario, the leading interaction of the dilaton with the gluons is through a loop of top quarks, just as for the Higgs in the SM.

When conformal symmetry violating effects arising from the operator  $\mathcal{O}$  are included, this formula will receive corrections. The renormalization group evolution of the gauge coupling above the symmetry breaking scale is affected by the presence of the deformation, with the result that  $b_{>}$  is now a function of  $\hat{\lambda}_{\mathcal{O}}$ ,  $b_{>} = b_{>}(\hat{\lambda}_{\mathcal{O}})$ .

However, the theory remains formally invariant under infinitesimal scale transformations if  $g_{UV}$  is promoted to a spurion as in Eq. (2.115). Hence this effect does not alter the form of Eq. (2.119). The operator  $\mathcal{O}$  also affects the low energy theory through the fact that the value of the gauge coupling at low energies depends on the detailed spectrum of states at the threshold, which in turn depends on  $\hat{\lambda}_{\mathcal{O}}$ . As a consequence the constant  $C$  in Eq. (2.116) is in general a function of  $\hat{\lambda}_{\mathcal{O}}$ . Now, conformal symmetry ensures that in the low energy effective theory  $C$  depends on  $\bar{\lambda}_{\mathcal{O}}$  in the specific combination  $C(\bar{\lambda}_{\mathcal{O}}\chi^{-\epsilon})$ . Since the Lagrangian is limited to terms with positive integer powers of  $\hat{\lambda}_{\mathcal{O}}$ , we can expand  $C$  as

$$C = \left[ C_0 + \sum_{n=1}^{\infty} C_n \bar{\lambda}_{\mathcal{O}}^n \chi^{(-n\epsilon)} \right]. \quad (2.120)$$

Inserting this into Eq. (2.117) and requiring invariance under (spurious) scale transformations, we find that the dilaton couplings must take the form

$$\frac{\sigma}{f} \left[ \frac{(b_{<} - b_{>})}{32\pi^2} + \frac{\bar{c}_A}{32\pi^2} \epsilon \bar{\lambda}_{\mathcal{O}} f^{-\epsilon} \right] F_{\mu\nu} F^{\mu\nu}. \quad (2.121)$$

Here the dimensionless constant  $\bar{c}_A$  is expected to be of order the number of states that transform under the gauge symmetry that have masses at the threshold, so that  $\bar{c}_A \sim (b_{<} - b_{>})$ . We can therefore rewrite Eq. (2.121) as

$$\frac{\sigma}{f} \frac{(b_{<} - b_{>})}{32\pi^2} [1 + c_A \epsilon] F_{\mu\nu} F^{\mu\nu}, \quad (2.122)$$

where the dimensionless constant  $c_A$  is of order  $\bar{\lambda}_{\mathcal{O}} f^{-\epsilon}$ , which is the value of  $\hat{\lambda}_{\mathcal{O}}$  at the symmetry breaking scale  $f$ . Here  $b_{>}(\hat{\lambda}_{\mathcal{O}})$  is to be evaluated close to the breaking scale. We see from this that in this scenario, corrections to the form of the dilaton couplings to massless gauge bosons arising from conformal symmetry

violating effects are subleading, being suppressed not just by  $m_\sigma^2/\Lambda^2$ , but also by a loop factor. This is in contrast to the case of composite gauge bosons considered in the previous section.

### 2.3.2.2 Couplings to Fermions: Elementary Fermions

Next we consider the dilaton couplings to the SM fermions, which we label by  $Q, U^c, D^c, L$  and  $E^c$ . These depend on how the fermion masses are generated. One possibility is that the fermion masses arise from a contact term that couples a scalar operator  $\mathcal{H}$  in the conformal field theory that carries the gauge quantum numbers of the SM Higgs to elementary fermions. For the up-type quarks, this takes the form

$$y^{ij}\mathcal{H}Q_iU_j^c + \text{h.c.} \quad (2.123)$$

in the Lagrangian. Here  $i$  and  $j$  are flavor indices. This leads to a mass term

$$m^{ij}Q_iU_j^c + \text{h.c.} \quad (2.124)$$

in the potential of the low energy effective theory. The generalization to the down-type quarks and leptons is straightforward. In the limit that  $y^{ij}$  is set to zero the ultraviolet theory has a  $U(3)_Q \times U(3)_U$  flavor symmetry, which can be restored by promoting  $y^{ij}$  to a spurion that transforms as an anti-fundamental under each of these symmetries. By requiring that the low energy effective theory be invariant under this spurious flavor symmetry, it follows that  $m^{ij}$  is proportional to  $y^{ij}$  to lowest order in the couplings  $y$ .

It has been shown that the flavor problem and the large mass of the top quark can both be addressed in this framework if the operator  $\mathcal{H}$  has dimension

$\Delta_{\mathcal{H}} \sim 1.3$ . However, if the hierarchy problem is to be solved, the dimension  $\Delta_{\mathcal{H}^\dagger\mathcal{H}}$  of the operator  $\mathcal{H}^\dagger\mathcal{H}$  must satisfy  $\Delta_{\mathcal{H}^\dagger\mathcal{H}} \gtrsim 4$  [6]. Determining whether scalar operators that satisfy these criteria can exist in unitary, causal conformal field theories is an open question that has attracted considerable recent interest [44,48,49], see also [46]. Note that this condition cannot be satisfied in the large  $N$  limit, and therefore realistic models of this type cannot be constructed within the RS framework.

In order to determine the coupling of the dilaton to the up-type quarks, we make the coupling in Eq. (2.123) formally invariant under scale transformations by promoting  $y^{ij}$  to a spurion that transforms as  $y^{ij} \rightarrow y'^{ij} = e^{\omega(1-\Delta_{\mathcal{H}})}y^{ij}$  under  $x \rightarrow x' = e^{-\omega}x$ . Then the coupling of the dilaton to the up-type quarks in the effective theory must respect this symmetry. Since the quark mass matrix  $m^{ij}$  is proportional to  $y^{ij}$  the conformal compensator couples as

$$m^{ij} \left(\frac{\chi}{f}\right)^{\Delta_{\mathcal{H}}} Q_i U_j^c + \text{h.c.} \quad (2.125)$$

Then to lowest order in inverse powers of  $f$ , the dilaton coupling to up-type quarks takes the form [60]

$$m^{ij} \Delta_{\mathcal{H}} \left(\frac{\sigma}{f}\right) Q_i U_j^c + \text{h.c.} \quad (2.126)$$

We see that to the extent that  $\Delta_{\mathcal{H}}$  differs from one, the dilaton couplings to fermions can differ significantly from those of a SM Higgs.

Once effects of the operator  $\mathcal{O}$  are included the scaling dimension of the operator  $\mathcal{H}$  receives corrections,  $\Delta_{\mathcal{H}} = \Delta_{\mathcal{H}}(\hat{\lambda}_{\mathcal{O}})$ . However, above the breaking scale the theory continues to remain invariant under the infinitesimal spurious scale transformation  $y^{ij} \rightarrow y'^{ij} = e^{\omega(1-\Delta_{\mathcal{H}})}y^{ij}$  when  $x \rightarrow x' = e^{-\omega}x$ , and so the form of Eq. (2.126)

is not affected by this. Instead, the leading correction arises from the fact that in addition to the term in Eq. (2.125), other terms involving the invariant  $\bar{\lambda}_\mathcal{O}\chi^{-\epsilon}$  can now also contribute. As a result, Eq. (2.126) is modified to

$$m^{ij} (\Delta_{\mathcal{H}} + c_q \epsilon) \frac{\sigma}{f} Q_i U_j^c + \text{h.c.} , \quad (2.127)$$

where the dimensionless parameter  $c_q$  is of order  $\bar{\lambda}_\mathcal{O} f^{-\epsilon}$ . Here  $\Delta_{\mathcal{H}}(\hat{\lambda}_\mathcal{O})$  is to be evaluated close to the symmetry breaking scale. We see that corrections to the form of Eq. (2.126) from conformal symmetry violating effects are of order  $m_\sigma^2/\Lambda^2$ , and under control.

More generally, there could be several scalar operators  $\mathcal{H}_\alpha$  in the conformal field theory that couple to the SM fermions. The coupling in Eq. (2.123) then generalizes to

$$y^{\alpha ij} \mathcal{H}_\alpha Q_i U_j^c + \text{h.c.} , \quad (2.128)$$

where the index  $\alpha$  runs over all the scalar operators in the theory with the quantum numbers of the SM Higgs. However, operators with dimension significantly larger than one are not expected to play a significant role.

It follows from the  $U(3)_Q \times U(3)_U$  flavor symmetry that the up-type fermion masses depend on the couplings  $y^{\alpha ij}$  as

$$m^{ij} = y^{\alpha ij} D_\alpha , \quad (2.129)$$

where the parameters  $D_\alpha$  depends on the details of the conformal field theory. In order to determine the couplings of the dilaton, we make the coupling in Eq. (2.128) formally invariant under scale transformations by promoting the  $y^{\alpha ij}$  to spurions

that transform as

$$y^{\alpha ij} \rightarrow y'^{\alpha ij} = e^{\omega(1-\Delta_{\mathcal{H}(\alpha)})} y^{(\alpha)ij} , \quad (2.130)$$

under  $x \rightarrow x' = e^{-\omega}x$ . There is no sum over  $\alpha$  on the right hand side of this equation. The various terms in the sum on the right hand side of Eq. (2.129) transform differently under this transformation. In order to account for this we define

$$m^{\alpha ij} = y^{(\alpha)ij} D_{(\alpha)} , \quad (2.131)$$

where again there is no sum over  $\alpha$  on the right hand side of this equation. Then the requirement that the fermion mass in the low energy effective theory be formally invariant under this symmetry constrains the conformal compensator to couple as

$$m^{\alpha ij} \left( \frac{\chi}{f} \right)^{\Delta_{\mathcal{H}\alpha}} Q_i U_j^c + \text{h.c.} \quad (2.132)$$

in the potential. This leads to the dilaton coupling

$$m^{\alpha ij} \Delta_{\mathcal{H}\alpha} \frac{\sigma}{f} Q_i U_j^c + \text{h.c.} \quad (2.133)$$

We see from this that if the  $\Delta_{\mathcal{H}\alpha}$  are not all equal, the couplings of the dilaton in the low energy effective theory violate flavor. However, in the absence of large cancellations among the contributions of different operators to the quark masses, the matrix  $m^{\alpha ij} \Delta_{\mathcal{H}\alpha}$  will be somewhat aligned with the quark mass matrix  $m^{ij}$ , leading to suppression of flavor violation.

### 2.3.2.3 Couplings to Fermions: Partially Composite Fermions

Another possible origin for the fermion masses is that the SM quarks and leptons are partial composites of the strongly interacting sector [58]. This scenario



can arise if the theory contains elementary fermions  $Q_i, U_i^c, D_i^c, L_i$  and  $E_i^c$  with the same gauge quantum numbers as the corresponding SM fermions that mix with operators in the conformal field theory. The physical SM fermions emerge as a linear combination of the corresponding elementary particles and states associated with the strongly interacting sector. Within the RS framework, this corresponds to putting the SM fermions in the bulk of the space [69].

To understand this in greater detail, let us consider the mass terms for the up-type quarks. These can be generated if the conformal field theory contains fermionic operators  $\mathcal{Q}_\alpha^c$  and  $\mathcal{U}_\alpha$ , with dimensions  $\Delta_{\mathcal{Q}}$  and  $\Delta_{\mathcal{U}}$  respectively, that couple to elementary fermions  $Q_i$  and  $U_i^c$  in the Lagrangian as

$$y_{\mathcal{Q}}^{\alpha i} \mathcal{Q}_\alpha^c Q_i + y_{\mathcal{U}}^{\beta j} \mathcal{U}_\beta U_j^c + \text{h.c.} \quad (2.134)$$

We assume that the indices  $\alpha$  and  $\beta$ , which run from 1 to 3, are associated with an internal  $U(3)$  symmetry of the conformal sector so that  $\Delta_{\mathcal{Q}}$  and  $\Delta_{\mathcal{U}}$  are independent of  $\alpha$  and  $\beta$ . We will relax this assumption later. If  $\Delta_{\mathcal{Q}}$  and  $\Delta_{\mathcal{U}}$  are close to  $5/2$ , these interactions correspond to marginal operators in the conformal field theory. These couplings will generate up-type quark masses in the potential of the form

$$m^{ij} Q_i U_j^c + \text{h.c.} \quad (2.135)$$

This framework can be extended to the down-type quarks and leptons in a straightforward way. The AdS/CFT correspondence relates the operator dimensions  $\Delta_{\mathcal{Q}}$  and  $\Delta_{\mathcal{U}}$  to the mass terms for bulk fermions in RS models.

In the limit that the couplings  $y_{\mathcal{Q}}$  and  $y_{\mathcal{U}}$  are set to zero the ultraviolet theory has a  $U(3)_Q \times U(3)_U$  flavor symmetry. This symmetry can be restored by promoting

$y_Q$  and  $y_U$  to spurions that transform as anti-fundamentals under  $U(3)_Q$  and  $U(3)_U$  respectively. Then, requiring the low energy effective theory to respect this spurious symmetry constrains the mass matrix to be proportional to the product of  $y_U$  and  $y_Q$ ,

$$m^{ij} \propto [y_Q^T y_U]^{ij}, \quad (2.136)$$

to lowest order in the couplings  $y$ . The kinetic terms of the quarks in the low energy effective theory also receive corrections from the couplings  $y_Q$  and  $y_U$  of the form

$$\Delta Z_Q \bar{Q} \gamma^\mu D_\mu Q + \Delta Z_U \bar{U}^c \gamma^\mu D_\mu U^c \quad (2.137)$$

where

$$\begin{aligned} \Delta Z_Q &\sim \frac{1}{16\pi^2} \frac{y_Q^\dagger y_Q}{f^{5-2\Delta_Q}} \\ \Delta Z_U &\sim \frac{1}{16\pi^2} \frac{y_U^\dagger y_U}{f^{5-2\Delta_U}}. \end{aligned} \quad (2.138)$$

The corrections to the kinetic terms are a consequence of the fact that the fermions in the low energy theory are partially composite.

In order to determine the coupling of the dilaton to the up-type quarks, we promote  $y_Q$  and  $y_U$  to spurions that transform as  $y_Q \rightarrow y'_Q = e^{\omega(5/2-\Delta_Q)} y_Q$  and  $y_U \rightarrow y'_U = e^{\omega(5/2-\Delta_U)} y_U$  under  $x \rightarrow x' = e^{-\omega} x$ . Then the couplings (2.134) are formally invariant under scale transformations, and the conformal compensator couples to quarks so as to make low energy effective theory consistent with this symmetry. To lowest order in powers of  $y_Q$  and  $y_U$ , and neglecting effects arising from  $\mathcal{O}$ , this coupling takes the form

$$m^{ij} Q_i U_j^c \left( \frac{\chi}{f} \right)^{(\Delta_U + \Delta_Q - 4)} + \text{h.c.} \quad (2.139)$$

in the potential. This leads to the dilaton couplings

$$m^{ij} (\Delta_{\mathcal{U}} + \Delta_{\mathcal{Q}} - 4) \frac{\sigma}{f} Q_i U_j^c + \text{h.c.} \quad (2.140)$$

This agrees with the results in the literature for the coupling of the dilaton to partially composite fermions in the large  $N$  limit [60], and for the coupling of the radion to bulk fermions in the RS model [88, 89]. Our analysis establishes that these results are valid beyond the large  $N$  limit.

When effects of the operator  $\mathcal{O}$  are included, Eq. (2.140) is modified to

$$m^{ij} [(\Delta_{\mathcal{U}} + \Delta_{\mathcal{Q}} - 4) + c_q \epsilon] \frac{\sigma}{f} Q_i U_j^c + \text{h.c.} , \quad (2.141)$$

where  $c_q$  is of order  $\bar{\lambda}_{\mathcal{O}} f^{-\epsilon}$ , which is the value of  $\hat{\lambda}_{\mathcal{O}}$  at the scale  $f$ . In this expression,  $\Delta_{\mathcal{U}}(\hat{\lambda}_{\mathcal{O}})$  and  $\Delta_{\mathcal{Q}}(\hat{\lambda}_{\mathcal{O}})$  are to be evaluated close to the symmetry breaking scale. In obtaining this result, we have assumed that the operator  $\mathcal{O}$  does not break the approximate SM flavor symmetries, or the internal  $U(3)$  symmetry of the conformal sector. It follows that corrections to the form of Eq. (2.140) from conformal symmetry violating effects are suppressed by  $m_{\sigma}^2/\Lambda^2$ , and are under good theoretical control.

There are additional contributions to the dilaton coupling to quarks associated with the corrections to the kinetic terms, Eq. (2.137). However, using the equations of motion, it can be shown these contributions are higher order in  $y_{\mathcal{Q}}$  and  $y_{\mathcal{U}}$  than the effects we have considered, and are therefore suppressed.

In the more general case the operators  $\mathcal{Q}_{\alpha}^c$  and  $\mathcal{U}_{\alpha}$ , could have dimensions  $\Delta_{\mathcal{Q}_{\alpha}}$  and  $\Delta_{\mathcal{U}_{\alpha}}$  that depend on the flavor index  $\alpha$ . Then it follows from the spurious flavor

symmetries that the SM fermion masses depend on the couplings  $y_Q$  and  $y_U$  as

$$m^{ij} = y_Q^{\alpha i} y_U^{\beta j} D_{\alpha\beta} , \quad (2.142)$$

where the parameter  $D_{\alpha\beta}$  depends on the details of the conformal field theory. We can make the theory formally invariant under scale transformations by promoting  $y_Q$  and  $y_U$  to spurions that transform as

$$\begin{aligned} y_Q^{\alpha i} &\rightarrow y_Q^{\prime\alpha i} = e^{\omega(5/2-\Delta_{Q(\alpha)})} y_Q^{(\alpha)i} \\ y_U^{\alpha i} &\rightarrow y_U^{\prime\alpha i} = e^{\omega(5/2-\Delta_{U(\alpha)})} y_U^{(\alpha)i} \end{aligned} \quad (2.143)$$

under  $x \rightarrow x' = e^{-\omega} x^\mu$ , where there is no sum over  $\alpha$  on the right hand side of the equations. The couplings of the dilaton in the low energy effective theory must respect this symmetry. We define

$$m^{\alpha\beta ij} = y_Q^{(\alpha)i} y_U^{(\beta)j} D_{(\alpha)(\beta)} , \quad (2.144)$$

where again there is no sum over  $\alpha$  and  $\beta$  on the right hand side of the equation.

We also define

$$\Delta_{\alpha\beta}^{QU} = \Delta_{Q\alpha} + \Delta_{U\beta} - 4 . \quad (2.145)$$

In terms of these new variables the coupling of the conformal compensator to the up-type quarks can be expressed as

$$m^{\alpha\beta ij} Q_i U_j^c \left( \frac{\chi}{f} \right)^{\Delta_{\alpha\beta}^{QU}} + \text{h.c.} \quad (2.146)$$

Expanding this out, we find for the dilaton couplings in the potential

$$m^{\alpha\beta ij} \Delta_{\alpha\beta}^{QU} \frac{\sigma}{f} Q_i U_j^c + \text{h.c.} \quad (2.147)$$

It follows that in this scenario, the couplings of the dilaton to the quarks in the low energy effective theory violate flavor. However, in the absence of large cancellations among the contributions of the different  $y_Q$  and  $y_U$  to  $m^{ij}$ , we expect some degree of alignment between the quark mass matrix and the dilaton coupling matrix, which may be sufficient to satisfy flavor bounds.

Since  $Q^c$  and  $U$  are part of the strongly interacting sector, they must arise from complete multiplets of  $SU(2)_L \times SU(2)_R$ . There are two distinct possibilities for the realization of this symmetry, which we consider in turn.

The first possibility is that  $Q^c$  transforms as  $(2, 1)$  under  $SU(2)_L \times SU(2)_R$  while  $U$  is partnered by another state  $\mathcal{D}$ , and together they transform as a  $(1, 2)$ . In the context of RS models, this realization of custodial symmetry was first proposed in [90]. The large mass of the top quark implies that the couplings  $y_Q$  and  $y_U$  must be sizable for the third generation quarks. This realization leads to mild tension with precision electroweak tests, since  $y_U$  distinguishes between  $U$  and  $\mathcal{D}$ , and therefore violates custodial  $SU(2)$  symmetry.

The alternative possibility [91] is that  $Q^c$  is partnered with a new state  $\hat{Q}^c$ , and together they transform as  $(2, 2)$  under  $SU(2)_L \times SU(2)_R$ . Meanwhile,  $U$  is now just a singlet. In this realization of the extended symmetry, it is  $y_Q$  that violates custodial  $SU(2)$  and leads to tension with precision tests. This difficulty can be avoided if the third generation  $SU(2)$  singlet up-type quark  $U_3^c$  is a composite of the strongly interacting sector. This allows  $y_Q$  to remain small enough to avoid conflict with the bound. In this scenario Eq. (2.147) remains valid, the only difference being that  $\Delta_{U_3}$  now takes the value  $5/2$ .

### 2.3.3 Higgs as a pNGB

Next we consider theories where the SM Higgs doublet emerges as the pNGB associated with the breaking of an approximate global symmetry by strong conformal dynamics. For concreteness, we will take the global symmetry to be  $SO(6)$ , which is broken to  $SO(5)$ . An  $SU(2) \times U(1)$  subgroup of the unbroken  $SO(5)$  is gauged, and identified with the electroweak gauge sector of the SM. Of the 5 pNGBs, 4 are identified with the SM Higgs doublet, while the remaining one is a SM singlet.

For the purpose of writing interactions, it is convenient to work in a framework where we keep only the symmetries associated with the  $SU(3) \times U(1)$  subgroup of the non-linearly realized  $SO(6)$  global symmetry manifest. As shown in [92], the 5 NGBs associated with the breaking of  $SO(6)$  to  $SO(5)$  can be identified with the 5 NGBs arising from the breaking of the  $SU(3) \times U(1)$  subgroup of  $SO(6)$  to  $SU(2) \times U(1)$ , since the corresponding coset spaces are identical. We parametrize the NGBs as  $h^a$ , where  $a$  runs from 1 to 5. Rather than work with the  $h^a$  directly it is more convenient to construct an object  $\phi$  which transforms linearly under  $SU(3) \times U(1)$ .

$$\phi = \hat{f} \exp(ih^a t^a) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \quad (2.148)$$

Note that we are employing a convention where the  $h^a$  carry no mass dimension. We expect that  $\hat{f}$  and  $f$  will be of the same order, since the same dynamics is responsible for the breaking of both conformal symmetry and the global symmetry.

The 5 matrices  $t^a$  span  $[\text{SU}(3) \times \text{U}(1)/\text{SU}(2) \times \text{U}(1)]$ , and are chosen as

$$\{t^a\} = \left\{ \begin{array}{l} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \end{array} \right\}. \quad (2.149)$$

This choice allows us to take  $h^a$ ,  $a = 1 \rightarrow 4$  to represent the SM Higgs doublet, which we denote by  $h$ , while  $h^5$  represents the additional singlet.

The low energy effective Lagrangian will in general contain all possible operators consistent with the  $\text{SU}(3) \times \text{U}(1)$  global symmetry, but with restrictions on the coefficients of various terms enforced by the larger  $\text{SO}(6)$  symmetry. In particular the dangerous custodial  $\text{SU}(2)$  violating operator

$$|\phi^\dagger D_\mu \phi - (D_\mu \phi)^\dagger \phi|^2, \quad (2.150)$$

while allowed by  $\text{SU}(3) \times \text{U}(1)$ , is forbidden by  $\text{SO}(6)$ . Here  $D_\mu$  is the gauge covariant derivative with respect to the SM  $\text{SU}(2) \times \text{U}(1)$  gauge symmetry.

The requirement of scale invariance implies that the non-linear sigma model condition  $|\phi|^2 = \hat{f}^2$  becomes  $|\phi|^2 = \hat{f}^2 \hat{\chi}^2$ . This means that we can make the low energy effective theory for the pNGBs invariant under scale transformations, up to terms arising from effects that explicitly violate the conformal and global symmetries, by making the replacement  $\hat{f} \rightarrow \hat{f} \hat{\chi}$  in Eq. (2.148). The net effect is that in the low energy effective theory the  $h^a$  transform as fields with scaling dimension equal to

zero, up to effects that violate conformal symmetry. This allows us to determine the form of the dilaton couplings to the SM fields. A major simplification is that since  $h$  has no scaling dimension, when replaced by its VEV the various operators have exactly the same scaling dimensions as in the technicolor models of the previous section, and many results can simply be carried over.

### 2.3.3.1 Couplings to Gauge Bosons

We begin by considering the dilaton couplings to the weak gauge bosons of the SM. These arise from the gauge covariant kinetic term for  $\phi$ ,

$$(D_\mu\phi)^\dagger D^\mu\phi. \quad (2.151)$$

Expanding out  $\phi$  to lowest order in  $h$ , we obtain the gauge covariant kinetic term for the SM Higgs doublet

$$\hat{\chi}^2 \hat{f}^2 (D_\mu h)^\dagger D^\mu h, \quad (2.152)$$

Working in unitary gauge, and replacing  $h$  by its VEV, we find the coupling of the conformal compensator to the  $W$  bosons in the Lagrangian

$$\frac{m_W^2}{g^2} \frac{\chi^2}{f^2} W_\mu^+ W^{\mu-}. \quad (2.153)$$

This leads to the dilaton coupling

$$2\frac{\sigma}{f} \frac{m_W^2}{g^2} W_\mu^+ W^{\mu-}. \quad (2.154)$$

As expected, this is identical to the corresponding formula in the technicolor case.

When conformal symmetry violating effects arising from  $\mathcal{O}$  are incorporated, the



non-linear sigma model condition is modified to

$$|\phi|^2 = \hat{f}^2 \hat{\chi}^2 \left[ 1 + \sum_{n=1}^{\infty} \alpha_{\phi,n} \bar{\lambda}_{\mathcal{O}}^n \chi^{(-n\epsilon)} \right], \quad (2.155)$$

where the dimensionless parameters  $\alpha_{\phi,n}$  are expected to be of order one. Then the dilaton coupling to  $W$  bosons is modified to

$$\frac{\sigma}{f} \frac{m_W^2}{g^2} (2 + c_W \epsilon) W_{\mu}^+ W^{\mu -}, \quad (2.156)$$

where  $c_W$  is of order  $\bar{\lambda}_{\mathcal{O}} f^{-\epsilon}$ . We see that the corrections are suppressed by  $m_{\sigma}^2/\Lambda^2$ , exactly as in the technicolor case.

Next we consider dilaton couplings to the massless gauge bosons of the SM, the gluons and the photon. The leading effect which breaks conformal invariance is again the running of the gauge couplings, just as in the previous section. The results can simply be carried over, and are given by Eq. (2.122),

$$\frac{\sigma}{f} \frac{(b_{<} - b_{>})}{32\pi^2} [1 + c_A \epsilon] F_{\mu\nu} F^{\mu\nu}. \quad (2.157)$$

Here  $b_{>}(\hat{\lambda}_{\mathcal{O}})$  is to be evaluated close to the breaking scale. As can be seen from this formula, corrections to the form of the dilaton couplings from conformal symmetry breaking effects are suppressed by  $m_{\sigma}^2/\Lambda^2$  and also by a loop factor, and are generally small.

### 2.3.3.2 Couplings to Fermions: Elementary Fermions

Next we consider dilaton couplings to the SM fermions. We begin with the case where the SM fermions are elementary, and their masses arise from direct contact

interactions with operators in the conformal field theory. The up-type fermion masses arise from terms in the Lagrangian of the form

$$\hat{y}^{ij} \mathcal{H} Q_i U_j^c + \text{h.c.} \quad (2.158)$$

that break the global symmetry. Here the operator  $\mathcal{H}$  has the quantum numbers of the SM Higgs doublet. This leads to Yukawa couplings for the up-type quarks in the potential of the low energy effective theory,

$$y^{ij} (\hat{f}h) Q_i U_j^c + \text{h.c.} , \quad (2.159)$$

where we are neglecting higher order terms in  $h$  which may also arise from the term in Eq. (2.158). It follows from the flavor symmetries that  $y^{ij}$  is proportional to  $\hat{y}^{ij}$  to lowest order in the couplings  $\hat{y}$ . We can find the dilaton couplings by promoting  $\hat{y}^{ij}$  to a spurion exactly as in the technicolor case. Noting that  $h$  has no scaling dimension, it follows that the conformal compensator couples to up-type quarks as

$$y^{ij} \left( \frac{\chi}{f} \right)^{\Delta_{\mathcal{H}}} (\hat{f}h) Q_i U_j^c + \text{h.c.} \quad (2.160)$$

in the potential. This leads to the dilaton coupling

$$y^{ij} \Delta_{\mathcal{H}} \frac{\sigma}{f} (\hat{f}h) Q_i U_j^c + \text{h.c.} . \quad (2.161)$$

Replacing  $h$  by its VEV we obtain

$$m^{ij} \Delta_{\mathcal{H}} \frac{\sigma}{f} Q_i U_j^c + \text{h.c.} , \quad (2.162)$$

exactly as in the technicolor case. When effects of the operator  $\mathcal{O}$  are included, this again becomes

$$m^{ij} (\Delta_{\mathcal{H}} + c_q \epsilon) \frac{\sigma}{f} Q_i U_j^c , \quad (2.163)$$

where  $c_q$  is of order  $\bar{\lambda}_\mathcal{O} f^{-\epsilon}$ . In this expression  $\Delta_{\mathcal{H}}(\hat{\lambda}_\mathcal{O})$  is to be evaluated close to the symmetry breaking scale.

In the case where there are multiple operators  $\mathcal{H}_\alpha$  that couple to the SM fermions,

$$\hat{y}_U^{\alpha ij} \mathcal{H}_\alpha Q_i U_j^c + \text{h.c.}, \quad (2.164)$$

Eq. (2.162) generalizes to the corresponding formula in the technicolor case, Eq. (2.133).

### 2.3.3.3 Couplings to Fermions: Partially Composite Fermions

We move on to the case where the SM quarks and leptons are partial composites of the strongly interacting sector. We introduce elementary fermions  $Q_i, U_i^c, D_i^c, L_i$  and  $E_i^c$  that have the same gauge quantum numbers as the corresponding SM fermions, and which mix with operators in the conformal field theory. The observed SM fermions are linear combinations of the corresponding elementary particles and states associated with the strongly interacting sector.

Mass terms for the up-type quarks arise from couplings of fermionic operators  $\mathcal{Q}_\alpha^c$  and  $\mathcal{U}_\alpha$ , with dimensions  $\Delta_\mathcal{Q}$  and  $\Delta_\mathcal{U}$  respectively, to the elementary fermions  $Q_i$  and  $U_i^c$  in the Lagrangian,

$$y_\mathcal{Q}^{\alpha i} \mathcal{Q}_\alpha^c Q_i + y_\mathcal{U}^{\beta i} \mathcal{U}_\beta U_i^c + \text{h.c.} \quad (2.165)$$

We assume that the indices  $\alpha$  and  $\beta$ , which run from 1 to 3, are associated with an internal U(3) symmetry of the conformal sector so that  $\Delta_\mathcal{Q}$  and  $\Delta_\mathcal{U}$  are independent of  $\alpha$  and  $\beta$ . We will relax this assumption later. We can determine the coupling of the dilaton to the up-type quarks by promoting  $y_\mathcal{Q}$  and  $y_\mathcal{U}$  to spurions, exactly as

in the technicolor case. Noting that  $h$  has scaling dimension zero, we find that the conformal compensator couples as

$$y^{ij} \left( \frac{\chi}{f} \right)^{(\Delta_{\mathcal{U}} + \Delta_{\mathcal{Q}} - 4)} (\hat{f}h) Q_i U_j^c + \text{h.c.} \quad (2.166)$$

Replacing  $h$  by its VEV and expanding  $\chi$  out in terms of  $\sigma$ , we obtain

$$m^{ij} (\Delta_{\mathcal{U}} + \Delta_{\mathcal{Q}} - 4) \frac{\sigma}{f} Q_i U_j^c + \text{h.c.} , \quad (2.167)$$

which is identical to the corresponding formula in the technicolor case, Eq. (2.140).

When effects of the operator  $\mathcal{O}$  are included, Eq. (2.167) receives corrections, and is again modified to

$$m_U^{ij} [(\Delta_{\mathcal{U}} + \Delta_{\mathcal{Q}} - 4) + c_q \epsilon] \frac{\sigma}{f} Q_i U_j^c + \text{h.c.} \quad (2.168)$$

In this expression,  $\Delta_{\mathcal{U}}(\hat{\lambda}_{\mathcal{O}})$  and  $\Delta_{\mathcal{Q}}(\hat{\lambda}_{\mathcal{O}})$  are to be evaluated close to the symmetry breaking scale. We see that corrections to the form of Eq. (2.167) from conformal symmetry violating effects are suppressed by  $m_\sigma^2/\Lambda^2$ , and are therefore small. In the more general case where the operators  $\mathcal{Q}_\alpha^c$  and  $\mathcal{U}_\alpha$  have dimensions  $\Delta_{\mathcal{Q}_\alpha}$  and  $\Delta_{\mathcal{U}_\alpha}$  that depend on the index  $\alpha$ , Eq. (2.167) generalizes to the corresponding formula in the technicolor case, Eq. (2.147).

Since  $\mathcal{Q}$  and  $\mathcal{U}$  are part of the strongly interacting sector, they must arise from complete multiplets of  $O(6)$ . Perhaps the simplest possibility is that  $\mathcal{Q}^c$  constitutes part of a multiplet that transforms as a fundamental of  $O(6)$ , while  $\mathcal{U}$  is just a singlet. In this realization of the extended symmetry  $y_{\mathcal{Q}}$  violates custodial  $SU(2)$ . The large mass of the top quark means that this coupling must be large for the third generation, leading to tension with precision tests. This difficulty can be avoided if

the third generation SU(2) singlet up-type quark  $U_3^c$  is a composite of the strongly interacting sector. This allows  $y_Q$  to remain small enough to avoid conflict with the bound. In this scenario Eq. (2.147) remains valid, but with  $\Delta_{u_3}$  taking the value  $5/2$ .

### 2.3.3.4 Coupling to the Higgs

Finally we consider the dilaton coupling to the SM Higgs. In general, this receives contributions from both the Higgs kinetic term and the Higgs potential. From the kinetic term for the Higgs doublet, Eq. (2.152), we obtain the coupling

$$\frac{\sigma}{f} \partial_\mu \rho \partial^\mu \rho \tag{2.169}$$

in the Lagrangian. Here  $\rho$  is the canonically normalized SM Higgs field, and we are working only to quadratic order in  $\rho$ . When corrections arising from the symmetry violating parameter  $\mathcal{O}$  are included, this is modified to

$$\frac{\sigma}{f} [1 + c_H \epsilon] \partial_\mu \rho \partial^\mu \rho . \tag{2.170}$$

where  $c_H$  is of order  $\bar{\lambda}_\mathcal{O} f^{-\epsilon}$ .

The kinetic term for  $\phi$ , Eq. (2.151), does not lead to mixing between between the dilaton and the SM Higgs field. Other two derivative terms, such as

$$\frac{\partial^\mu \chi}{\chi} [\phi^\dagger D_\mu \phi + (D_\mu \phi)^\dagger \phi] , \tag{2.171}$$

also do not generate such mixing. This conclusion remains true when conformal symmetry violating effects are included.

In this scenario, the potential for the Higgs doublet can only arise from effects that explicitly violate the global symmetry, such as the SM gauge and Yukawa interactions. If all such effects, however, respect conformal symmetry, then the potential for the Higgs doublet is of the very restrictive form

$$V = \chi^4 V_0(h) . \quad (2.172)$$

A potential of this form does not lead to mixing between the SM Higgs and the dilaton after minimization. The reason is that when the Higgs field is expanded about its VEV, there is no linear term in  $\rho$  at the minimum of the potential  $V(h)$ . However, expanding  $V(h)$  to quadratic order in  $\rho$ , we find a coupling of the dilaton to the Higgs of the form

$$2 \frac{\sigma}{f} m_\rho^2 \rho^2 \quad (2.173)$$

in the potential. This formula will receive corrections from any contribution to the Higgs potential that arises from an effect that violates conformal symmetry. Mixing between the Higgs and the dilaton may be generated by such effects.

In particular, when effects arising from the operator  $\mathcal{O}$  is taken into account, the Higgs potential takes the more general form

$$V = \chi^4 V_0(h) + \sum_{n=1}^{\infty} \bar{\lambda}_{\mathcal{O}}^n \chi^{(4-n\epsilon)} V_n(h) . \quad (2.174)$$

Eq. (2.173) is consequently modified to

$$\frac{\sigma}{f} (2 + c_\rho \epsilon) m_\rho^2 \rho^2 , \quad (2.175)$$

where, if the symmetry violating terms contribute significantly to the potential so that at the minimum  $V_1(h)$  is of order  $V_0(h)$ ,  $c_\rho$  is expected to be of order  $\bar{\lambda}_{\mathcal{O}} f^{-\epsilon}$ .

We see that the corrections are suppressed by  $m_\sigma^2/\Lambda^2$ . This effect also gives rise to mixing between the Higgs and the dilaton. However the mixing angle  $\theta$  is small,

$$\theta \lesssim \epsilon \bar{\lambda}_\mathcal{O} f^{-\epsilon} \left(\frac{v}{f}\right) \sim \frac{m_\sigma^2}{\Lambda^2} \left(\frac{v}{f}\right). \quad (2.176)$$

Here  $v$  is the electroweak VEV.

Since the SM gauge interactions also constitute an explicit breaking of conformal symmetry, there will be additional radiative corrections to the Higgs potential that are not of the simple form of Eq. (2.172). However, because the gauge interactions respect conformal symmetry at the classical level, and only break it through quantum effects, deviations away from this form are further loop suppressed, and generally small.

In theories where the top quarks are elementary or partially composite, the top Yukawa coupling also violates conformal symmetry. Then, if contributions to the Higgs potential from loops involving the top Yukawa coupling are sizable, there can be significant deviations away from the form of Eq. (2.172). We parametrize the coupling of the conformal compensator to the top quark as

$$\frac{m_t}{v} \left(\frac{\chi}{f}\right)^{(1+\bar{\Delta})} (\hat{f}h) \bar{t}t, \quad (2.177)$$

where  $\bar{\Delta}$  is equal to zero if the top quarks are composite, is equal to  $(\Delta_\mathcal{H} - 1)$  if the top quarks are elementary, and is equal to  $(\Delta_{\mathcal{U}_3} + \Delta_{\mathcal{Q}_3} - 5)$  if the top quarks are partially composite. Then one loop corrections to the Higgs potential from the top loop, which we label by  $\delta V_t$ , are of the form

$$\chi^4 \left[ \sum_{n=1}^2 \frac{\hat{\alpha}_{t,n}}{(16\pi^2)^{n-1}} \left(\frac{m_t}{v}\right)^{2n} \left(\frac{\chi}{f}\right)^{2n\bar{\Delta}} |h|^{2n} \right]. \quad (2.178)$$

Here the dimensionless parameters  $\hat{\alpha}_{t,n}$  are of order one. Then Eq. (2.173) is modified to

$$\frac{\sigma}{f} [2 + \bar{c}_\rho \bar{\Delta}] m_\rho^2 \rho^2. \quad (2.179)$$

If contributions to the Higgs potential arising from loops involving the top Yukawa coupling are significant, so that  $\delta V_t$  is comparable to  $V$  in Eq. (2.172) at the minimum, we expect  $\bar{c}_\rho$  to be of order one. This effect also gives rise to mixing between the dilaton and the Higgs, but the mixing angle  $\theta \lesssim \bar{\Delta} v/f$  is expected to be small in realistic models. Mixing will correct the dilaton couplings to other SM fields as well, and so a precise determination of these interactions requires this effect to be taken into account.



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## Effective Theory of the Radion

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In this chapter we construct the low energy effective theory of the radion incorporating the effects of the GW stabilization mechanism. Through out this chapter, we keep in mind the CFT construction from previous chapter and closely model the generalities developed there. While we stick to the extra-dimensional language in this chapter and an explicit comparison is made only in the next chapter 4, we appeal to the dual ideas when necessary to justify the non-minimal constructions in the extra-dimensional language. Our primary objective is to determine the dependence of the radion mass on the parameters of the extra dimensional theory and to calculate the coupling of the radion to SM fields in the presence of stabilization effects. We restrict to brane localized fields for calculating the radion couplings, as they suffice to make the general point.

Following Goldberger and Wise we consider a scenario where a  $5D$  scalar field  $\Phi$  acquires a VEV whose value depends on the location in the extra dimension [67].

However, we deviate from the original GW construction in that we allow the IR brane tension to be detuned away from its fine-tuned value in the original RS model, and also include non-linear self interaction terms for the GW field  $\Phi$  in the bulk. As we shall see, this alters the parametric dependence of the radion mass. We later elaborate on the significance of this by invoking the dual picture in the next chapter 4.

### 3.1 Radion Lagrangian without a Stabilization Mechanism

We begin by considering the 5D gravity action in the absence of any stabilization mechanism. The action is given as

$$\mathcal{S}_{GR}^{5D} = \int d^4x \int_{-\pi}^{\pi} d\theta \left[ \sqrt{G} \left( -2M_5^3 \mathcal{R}[G] - \Lambda_b \right) - \sqrt{-G_h} \delta(\theta) T_h - \sqrt{-G_v} \delta(\theta - \pi) T_v \right], \quad (3.1)$$

where  $M_5$  is the 5D Planck mass,  $\Lambda_b$  is the bulk cosmological constant and  $T_h, T_v$  are the brane tensions at the hidden (UV) and visible (IR) brane respectively. We are using the  $(+, -, -, -, -)$  sign convention for the 5D metric. The classical solution for the geometry is a slice of AdS<sub>5</sub> compactified on a circle  $\mathcal{S}_1$  with  $\mathcal{Z}_2$  orbifolding.

The background metric  $G_{MN}$  is then given as

$$ds^2 = e^{-2kr_c|\theta|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\theta^2 \quad -\pi \leq \theta \leq \pi. \quad (3.2)$$

To obtain a static solution of this form we have set  $\Lambda_b/k = T_v = -T_h = -24M_5^3 k$ , while  $r_c$  is an arbitrary constant. To parametrize the light degrees of freedom, we promote  $r_c$  and  $\eta_{\mu\nu}$  to dynamical fields by letting  $r_c \rightarrow r(x)$  and  $\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x)$ ,

which are associated with the radion and the graviton fields respectively. After this replacement in the metric, we plug the metric back in the action and integrate over the extra dimension to obtain the effective theory for the 4D graviton  $g_{\mu\nu}(x)$  and the canonically normalized radion field  $\varphi(x)$ , see appendix A.1 for details. Up to exponentially small corrections, this is given by

$$\mathcal{S}_{GR}^{4D} = \frac{2M_5^3}{k} \int d^4x \sqrt{-g} \mathcal{R}[g_{\mu\nu}] + \int d^4x \sqrt{-g} \left( \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right). \quad (3.3)$$

From this we see that the 4D Planck mass is given by  $M_4^2 = M_5^3/k$ . The canonically normalized field  $\varphi(x)$  is related to  $r(x)$  as

$$\varphi(x) = F e^{-k\pi r(x)}, \quad (3.4)$$

where  $F$  is defined as  $F = \sqrt{24M_5^3/k} = \sqrt{24} M_4$ . The potential generated for the radion field from the 5D gravity sector is given by (see appendix A.1)

$$V_{GR}(\varphi) = \frac{\varphi^4}{F^4} \left( T_v - \frac{\Lambda_b}{k} \right), \quad (3.5)$$

and vanishes identically for the tuned values of  $\Lambda_b$  and  $T_v$  in the original RS solution.

This naive procedure of promoting  $\eta_{\mu\nu}$  and  $r_c$  to dynamical fields leads to a parametrization of the light modes that does not solve the linearized Einstein equations [83, 93]. One may therefore worry about the validity of the low energy effective field theory (EFT). However, this turns out not to be a concern, as shown in [94]. A general parametrization of the fluctuations that includes both heavy and light modes can be written as

$$ds^2 = e^{-2kr(x)|\theta|} \left[ g_{\mu\nu}(x) + H_{\mu\nu}(x, \theta) \right] dx^\mu dx^\nu + 2H_{\theta\mu}(x, \theta) dx^\mu d\theta - r^2(x) \left[ 1 + H_{\theta\theta}(x, \theta) \right] d\theta^2. \quad (3.6)$$

Here  $g_{\mu\nu}(x)$  and  $r(x)$  are again the light fields, while  $H_{\mu\nu}(x, \theta)$ ,  $H_{\theta\mu}(x, \theta)$  and  $H_{\theta\theta}(x, \theta)$  parameterize the heavy KK excitations of the 5D graviton after reducing to the four dimensional theory. This naive parametrization of the light degrees of freedom is consistent as long as there are no tadpole terms in the heavy fields with one derivative [94]. By 4D Lorentz invariance, such a tadpole must multiply  $H_{\mu\theta}$ , but an explicit computation shows that  $H_{\mu\theta}$  vanishes identically for the metric that solves the linearized Einstein equation [83, 93]. This ensures that our naive parametrization of light fields will indeed generate the correct low energy effective theory up to corrections that are suppressed by powers of the KK scale. This conclusion is not altered by the inclusion of the GW field.

In order to solve the hierarchy problem, the warp factor in the background geometry must be exponentially large. Defining  $f = \langle \varphi \rangle = F e^{-kr_c \pi}$ , we require a stable solution such that  $f \ll F$ . The radion potential in Eq. (3.5) generated from the gravitational part of the action does not allow for such a possibility dynamically, unless fine-tuned to vanish. The VEV of  $\varphi$  either vanishes or is driven to infinity depending on the sign of the coefficient of the quartic. In the original RS solution to the hierarchy problem, this issue is addressed by setting the quartic coefficient to zero by tuning the tension of the visible brane such that  $T_v = -24M_5^3 k = \Lambda_b/k$ . The cosmological constant must also be tuned to zero by adjusting the tension of the hidden brane,  $T_h = 24M_5^3 k = -\Lambda_b/k$ .

### 3.2 Radion Lagrangian in the Presence of a Stabilization Mechanism

In a realistic RS scenario, a mechanism is needed to stabilize the geometry and give the radion a mass. Such a mechanism was proposed by Goldberger and Wise [67]. In this construction, a massive 5D field  $\Phi$  is sourced at the boundaries and acquires a VEV whose value depends on the location in the extra dimension. After integrating over the extra dimension, this generates a potential for the radion field in the low energy effective theory. In the original GW construction, the quartic potential for the radion in Eq. (3.5) was tuned to zero, and only the dynamics of the scalar field  $\Phi$  contributed to the radion potential. Here we allow the quartic generated by the gravitational potential to be non-zero, and also include self-interaction terms in the potential for the GW scalar in the bulk. The effect of these additional terms, which are important in theories with a holographic dual, is to alter the parametric dependence of the radion mass.

We now construct the low energy effective theory for the radion that emerges after stabilization using the GW mechanism. The action for the GW scalar  $\Phi$  is given by

$$\mathcal{S}_{GW} = \int d^4x d\theta \left[ \sqrt{G} \left( \frac{1}{2} G^{AB} \partial_A \Phi \partial_B \Phi - V_b(\Phi) \right) - \sum_{i=v,h} \delta(\theta - \theta_i) \sqrt{-G_i} V_i(\Phi) \right]. \quad (3.7)$$

Here  $G_h$  and  $G_v$  are the induced metrics at the brane locations,  $\theta_h = 0$  and  $\theta_v = \pi$ . The corresponding brane potentials are given by  $V_h$  and  $V_v$  respectively, while  $V_b$  is the potential in the bulk. In our parametrization, the radion does not couple to

the hidden brane at  $\theta_h = 0$ . We therefore do not specify the form of  $V_h$ , but simply require that it fixes  $\Phi$  at  $\theta = 0$  to be  $k^{3/2}v$  and does not contribute to the hidden brane tension. Such a requirement can be easily arranged, and is in fact the one used in the original GW proposal.

On the visible brane, we consider a simple potential for  $\Phi$  of the form

$$V_v(\Phi) = 2k^{5/2}\alpha\Phi, \quad (3.8)$$

where  $\alpha$  is a dimensionless number. The choice of a linear potential for  $\Phi$  on the IR brane is not special and is expected to present if  $\Phi$  is not charged under any symmetries. Considering a more general brane potential does not alter our final conclusions.

The potential for the GW scalar  $\Phi$  in the bulk has the general form

$$V_b(\Phi) = \frac{1}{2}m^2\Phi^2 + \frac{1}{3!}\eta\Phi^3 + \frac{1}{4!}\zeta\Phi^4 + \dots \quad (3.9)$$

Since we are in AdS space, the mass squared parameter  $m^2$  for the GW scalar  $\Phi$  can be negative without giving rise to instabilities as long as the condition  $m^2/k^2 + 4 > 0$  is satisfied [95]. The theories of interest for electroweak symmetry breaking are characterized by a large hierarchy between the scales associated with the UV brane and the IR brane,  $\exp(-k\pi r_c) \ll 1$ . The size of the hierarchy is determined by the potential for the GW scalar, which fixes the brane spacing  $r_c$ . A large hierarchy can be obtained if the mass term in the bulk potential for the GW field is small in units of the inverse curvature  $k$ . As first observed by Goldberger and Wise [67], a value of  $|m^2/k^2|$  of order a tenth suffices to generate the hierarchy between the Planck and weak scales.

In the class of RS models that possess a holographic dual, the parameters in the  $5D$  gravity theory and in the potential for the GW scalar are related to parameters in the dual CFT. The AdS/CFT correspondence associates the GW field  $\Phi$  with a scalar operator  $\mathcal{O}$  that deforms the dual CFT. Our primary focus is on the extra-dimensional duals of  $4D$  theories where a nearly marginal deformation  $\mathcal{O}$ , although small in the UV, grows large in the IR, resulting in the breaking of the conformal symmetry. This dictates our choice of the values of the parameters in the  $5D$  theory. The mass of  $\Phi$  is related to the scaling dimension of  $\mathcal{O}$  at the fixed point, and  $|m^2/k^2| \ll 1$  corresponds to the operator  $\mathcal{O}$  being close to marginal. We choose  $m^2 < 0$  to obtain a solution for  $\Phi$  which grows in the IR. The dimensionless parameter  $v$ , which determines the value of  $\Phi$  at the UV brane is also chosen to be small, allowing us to work to linear order in  $v$ . This corresponds to the operator  $\mathcal{O}$  being a small deformation in the UV. The other parameters are taken to be of order their natural strong coupling values (see appendix B for a description of how to estimate the natural values of parameters in our limit of interest), but with an important caveat. In our analysis, we neglect the back-reaction of the scalar field dynamics on the metric. In order for this approximation to be consistent, the detuning of the IR brane tension away from that in the original RS model must be smaller, by a factor of order a few, than its natural strong coupling value. The value of the parameter  $\alpha$ , which controls the value of  $\Phi$  near the IR brane, must also be taken to be smaller than its natural value by a similar factor. We do not expect our general conclusions to be affected by the fact that we are working in a limit where the back reaction is small.

Given the action, we can solve for the classical field configuration  $\Phi(\theta)$  in the RS background. The equation satisfied by  $\Phi$  in the bulk with the boundary conditions resulting from brane potentials is given by

$$\begin{aligned}
& \partial_\theta^2 \Phi - 4kr_c \partial_\theta \Phi - r_c^2 V'_b(\Phi) = 0 \\
& \theta = 0 \quad : \quad \Phi = k^{3/2} v \\
& \theta = \pi \quad : \quad \partial_\theta \Phi = -\alpha k^{3/2} kr_c .
\end{aligned} \tag{3.10}$$

The classical equation is a second order differential equation and is not simple to solve analytically for a general bulk potential. If, however,  $4kr_c \gg 1$  and certain conditions are satisfied by the parameters in the bulk potential, then an approximate analytic solution may be obtained using the methods of singular perturbation theory [96]. The key observation is that in the limit  $4kr_c \gg 1$ , the bulk equation of motion in Eq. (3.10) possesses solutions in different regions of  $\theta$  that have the property that one of the terms in Eq. (3.10) is small compared to the other two. In particular, there are self-consistent solutions for  $\Phi$  in the bulk that satisfy the boundary condition on the UV brane such that the  $\partial_\theta \Phi$  term and  $V'_b$  term are parametrically larger than the  $\partial_\theta^2 \Phi$  term. Within this approximation, the equation effectively becomes first order and is readily solved. The full solution to the equation displays boundary layer formation very near the  $\theta = \pi$  boundary where the  $\partial_\theta^2 \Phi$  term cannot be self-consistently dropped. However, in this regime it is self-consistent to drop the potential term  $V'_b$ , in favor of the  $\partial_\theta \Phi$  and  $\partial_\theta^2 \Phi$  terms.

It follows that two independent approximate solutions to the second order differential equation can be obtained in the following way: once by balancing the



$\partial_\theta\Phi$  term against the potential term, and once by balancing the  $\partial_\theta\Phi$  term against the  $\partial_\theta^2\Phi$  term. We will refer to these two equations (solutions) as the outer region (OR) and boundary region (BR) equations (solutions) respectively. The OR solution holds in the bulk, while the BR solution holds close to  $\theta = \pi$  where a boundary layer is formed, as the name suggests. The thickness of the boundary layer is of the order  $\sim 1/4kr_c$ . More specifically, the equations and their domains of validity<sup>1</sup> are,

$$\begin{aligned} \text{OR :} \quad & \frac{d\Phi}{d\theta} = -\frac{r_c}{4k}V'_b(\Phi) & \left(0 \leq \theta \lesssim \pi - \frac{1}{4kr_c}\right) \\ \text{BR :} \quad & \frac{d^2\Phi}{d\theta^2} = 4kr_c\frac{d\Phi}{d\theta} & \left(\pi - \frac{1}{4kr_c} \lesssim \theta \leq \pi\right). \end{aligned} \quad (3.11)$$

Notice that the BR solution is independent of the choice of potential  $V_b$  and can be readily solved. After applying the boundary condition at  $\theta = \pi$ , it is given by

$$\Phi_{BR}(\theta) = -\frac{k^{3/2}\alpha}{4}e^{4kr_c(\theta-\pi)} + C. \quad (3.12)$$

The yet unspecified constant  $C$  is determined by requiring the BR solution to be consistent with the OR solution, using asymptotic matching to the OR solution [96]. This allows us to construct a smooth solution that is a good approximation to each of the two solutions in the appropriate region. It is clear that the BR solution is exponentially suppressed in the region  $0 \leq \theta \lesssim \pi - 1/4kr_c$ , but becomes important in the region  $\pi - 1/4kr_c \lesssim \theta \leq \pi$ , thereby justifying the approximations made. Different choices of  $V_b$  change the OR solution, and therefore change the constant  $C$  in the BR solution. The complete solution for  $\Phi$  exhibits the universal feature of

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<sup>1</sup>For the remainder of this work, we give solutions for  $0 \leq \theta \leq \pi$ . Because of the  $\mathcal{Z}_2$  orbifold symmetry, this is sufficient to reconstruct the solution for the entire space.

boundary layer formation close to  $\theta = \pi$ . As we shall see, this general characteristic is to be expected from the point of view of the holographic dual theory.

The potential for the GW scalar  $\Phi$  in the bulk takes the form

$$V_b(\Phi) = \frac{1}{2}m^2\Phi^2 + \frac{1}{3!}\eta\Phi^3 + \frac{1}{4!}\zeta\Phi^4 + \dots \quad . \quad (3.13)$$

For this potential, we can write the OR equation in a holographically suggestive form as:

$$\frac{d \log \Phi}{d(kr_c\theta)} = -\frac{m^2}{4k^2} - \frac{\eta}{8\sqrt{k}} \frac{\Phi}{k^{3/2}} - \frac{\zeta k}{24} \frac{\Phi^2}{k^3} + \dots \quad . \quad (3.14)$$

In what follows we restrict ourselves to specific forms of the bulk potential for the GW field. In particular, we will first study the case when  $V_b(\Phi)$  consists of just a mass term, and then consider the case of a cubic self-interaction term. For each theory, we will describe the conditions on the parameters of the bulk potential for our approximation to be valid. We will then obtain the OR solution and study the resulting radion potential. These special cases will suffice to determine the general features of the solution.

### 3.2.1 Massive GW Scalar with no Bulk Interactions

We begin by considering the case when the potential for  $\Phi$  is dominated by the mass term, and the higher powers of  $\Phi$  can be neglected,

$$V_b(\Phi) = \frac{1}{2}m^2\Phi^2 \quad . \quad (3.15)$$

The large hierarchy between the Planck and weak scales implies that  $e^{-kr_c\pi}$  is a very small number. To obtain this hierarchy we require the bulk mass squared

being small in units of the inverse curvature, namely  $\epsilon = m^2/4k^2$  is a small number,  $|\epsilon| \ll 1$ . We also take  $e^{-\epsilon kr_c \pi}$  to be an  $\mathcal{O}(1)$  number. These conditions are also sufficient for our boundary layer analysis to be valid. The OR equation in this case takes the form

$$\frac{d \log \Phi}{d(kr_c \theta)} = -\frac{m^2}{4k^2} = -\epsilon . \quad (3.16)$$

Applying the boundary condition at  $\theta = 0$ , the solution is

$$\Phi_{OR}(\theta) = k^{3/2} v e^{-\epsilon kr_c \theta} . \quad (3.17)$$

Given this OR solution, the constant  $C$  in the BR solution is fixed uniquely by requiring asymptotic matching to the OR solution, resulting in

$$\Phi_{BR}(\theta) = -\frac{k^{3/2} \alpha}{4} e^{4kr_c(\theta-\pi)} + k^{3/2} v e^{-\epsilon kr_c \pi} . \quad (3.18)$$

Using the form of the OR and BR solutions, a smooth approximate solution for  $\Phi$  that is valid in the entire region  $0 \leq \theta \leq \pi$  and that matches on to both  $\Phi_{BR}$  and  $\Phi_{OR}$  is given by

$$\Phi_{\text{approx}}(\theta) = -\frac{k^{3/2} \alpha}{4} e^{4kr_c(\theta-\pi)} + k^{3/2} v e^{-\epsilon kr_c \theta} . \quad (3.19)$$

For this choice of the bulk potential, an exact expression for  $\Phi$  can be obtained (see appendix A.2), which matches very well with the approximate solution. Figure 3.1 indicates the matching of the BR and IR solutions to the approximate solution (1a), and shows the agreement of the approximate solution with the exact solution (1b).

To calculate the resulting contribution to the radion potential, we promote  $r_c$  to a dynamical field and insert the complete solution for  $\Phi(\theta)$  with this replacement

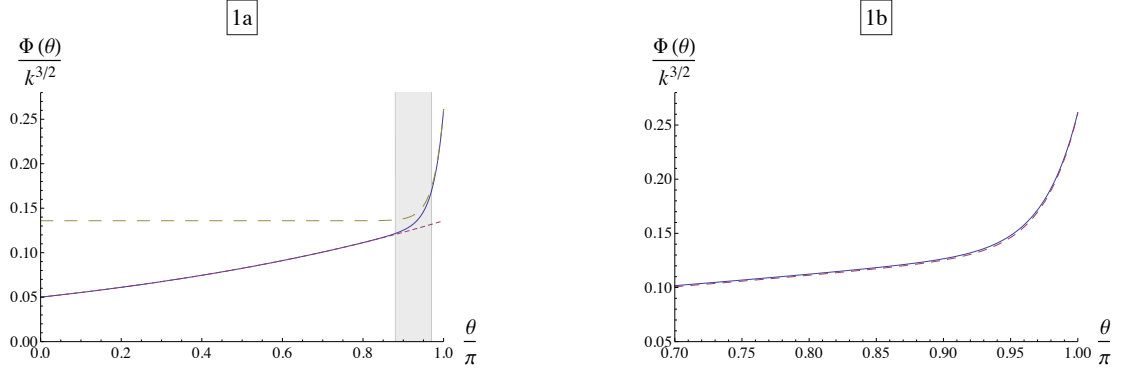


Figure 3.1: **1a**: The approximate solution (solid line) matches well with the BR solution (large dotted) and the OR solution (small dotted) for  $\epsilon = -0.1$ ,  $k\pi r_c = 10$ . We have also taken  $v = 0.05$  and  $\alpha = -0.5$ . The shaded region separates the boundary region on its right from the outer region on its left. Asymptotic matching is done in the shaded region. **1b**: The approximate solution (dotted) agrees well with the exact solution (solid) for the same parameter values, and we show the agreement near the  $\theta = \pi$  boundary.

back into the action. Integrating over the extra dimension then generates the contribution to the radion potential (see appendix A.2 for details). To leading order in  $v$  and  $\epsilon$ , the resulting potential takes the form

$$V_{GW}(\varphi) = k^4 \left(\frac{\varphi}{F}\right)^4 \left[ 2\alpha v \left(\frac{\varphi}{F}\right)^\epsilon - \frac{\alpha^2}{4} \right]. \quad (3.20)$$

Including the contribution from the gravity part of the action in Eq. (3.5), the full potential for the radion is given to leading order in  $v$  and  $\epsilon$  as

$$\begin{aligned} V(\varphi) &= V_{GR}(\varphi) + V_{GW}(\varphi) \\ &= 2k^4\alpha v \left(\frac{\varphi}{F}\right)^{4+\epsilon} + \left(T_v - \frac{\Lambda_b}{k} - \frac{k^4\alpha^2}{4}\right) \left(\frac{\varphi}{F}\right)^4. \end{aligned} \quad (3.21)$$

We can define  $\tau = (T_v - \Lambda_b/k - k^4\alpha^2/4)/k^4$  which represents the detuning of the brane tension away from the original RS solution. The first two terms in  $\tau$  are purely

gravitational, while the last term comes from the dynamics of the GW field on the visible brane and receives corrections higher order in  $\epsilon$ . With this, the minimum of the potential in Eq. (3.21) occurs at  $\langle\varphi\rangle = f$  where the condition

$$\left[\frac{f}{F}\right]^\epsilon \equiv e^{-\epsilon k r_c \pi} = -\frac{\tau}{2\alpha v} + \mathcal{O}(\epsilon) \quad (3.22)$$

is satisfied. For small  $\epsilon$ , an exponential hierarchy can be established between  $f$  and  $F$  provided the parameter  $v$  which controls the value of  $\Phi$  at the UV brane is also small. This is indeed the limit in which we are working. In particular, if  $\epsilon$  is of order a tenth and the other parameters are of order their natural strong coupling values, then  $v$  of order  $10^{-2}$  suffices to generate the hierarchy between the Planck and weak scales.

After  $\varphi$  acquires a VEV, we parametrize the physical radion  $\tilde{\varphi}$  that corresponds to the fluctuations over the minimum as  $\varphi = f \exp[\tilde{\varphi}/f] \approx f + \tilde{\varphi}$ . The mass of the radion  $\tilde{\varphi}$  is given to leading order in  $\epsilon$  as

$$m_\varphi^2 = -\frac{\epsilon\tau}{6} \frac{k^3}{M_5^3} \left( k e^{-k r_c \pi} \right)^2. \quad (3.23)$$

The warped down curvature scale  $k e^{-k r_c \pi} \equiv \tilde{k}$  appears naturally in this expression, and is identified with the KK scale  $m_{KK}$  in the  $4D$  theory. Including higher order terms in  $V(\varphi)$  does not alter the results. At the minimum, the value of  $T_h$  is tuned to set the  $4D$  cosmological constant to zero. Therefore, in the presence of a GW mechanism, only one fine-tuning is needed, which is the usual tuning to solve the cosmological constant problem.

We see from this analysis that  $m_\varphi^2 \sim |\epsilon|$ , which is consistent with previous studies [97, 98]. Either sign of  $\epsilon$  can give rise to a positive mass squared for the

radion. In the case of  $\epsilon > 0$  we require  $\tau < 0$  and  $\alpha v > 0$ , while for  $\epsilon < 0$  these conditions are reversed. (The condition on the sign of  $\tau$  and  $\alpha v$  for a given sign of  $\epsilon$  arises from requiring the consistency of the minimization condition and a positive mass squared for the radion.) From the holographic perspective, the case  $\epsilon < 0$  when the solution for  $\Phi$  grows in the IR is particularly interesting, and is our primary focus. The parametric dependence of the dilaton mass on  $\epsilon$  is robust and independent of the form of the potential for  $\Phi$  on the visible brane. Since  $\epsilon$  is small this would seem to suggest that the presence of a light dilaton is natural in this class of theories. However, this result assumed that the cubic self-interaction terms in the bulk potential for  $\Phi$  could be neglected in favor of the mass terms. For this assumption to be self-consistent we require that the condition

$$\frac{1}{3!} \eta \Phi^3 \ll \frac{1}{2} m^2 \Phi^2 \quad (3.24)$$

be satisfied at all points in the bulk. Using the solution for  $\Phi$  and noting that it has the largest magnitude at  $\theta = \pi$ , this translates to

$$\left| \left( \frac{\eta}{8\sqrt{k}} \right) \left( \tau + \frac{\alpha^2}{2} \right) \right| \ll 3 |\epsilon \alpha|. \quad (3.25)$$

For natural values of the parameters  $\alpha$ ,  $\tau$ , and  $\eta$ , this condition is not expected to be satisfied. Therefore, in determining whether a light radion is present in the spectrum, it is necessary to consider the bulk self-interaction terms.

### 3.2.2 Massive GW Scalar with a Cubic Interaction in Bulk

Next we consider the addition of a cubic self-interaction term in the bulk for  $\Phi$ ,

$$V_b(\Phi) = \frac{1}{2}m^2\Phi^2 + \frac{1}{3!}\eta\Phi^3. \quad (3.26)$$

The OR equation now becomes

$$\frac{d \log \Phi}{d(kr_c\theta)} = -\epsilon - \frac{\eta}{8\sqrt{k}} \left( \frac{\Phi}{k^{3/2}} \right). \quad (3.27)$$

We focus on the case when  $\eta/8\sqrt{k}$  is close to its natural strong coupling value. We further work in the limit that  $(\eta/8\sqrt{k})(\Phi/k^{3/2})$  is large enough that  $\epsilon$  can be neglected in Eq. (3.27). Although this latter condition is not necessarily satisfied at all points in the bulk, it is expected to be satisfied in the neighborhood of the IR brane. The reason is that in the theories of interest, while  $\epsilon$  is required to be small in order to generate a large hierarchy, there is no corresponding condition on the cubic self-coupling. Since the radion wave function is localized near the IR brane, neglecting  $\epsilon$  does not significantly affect the radion dynamics.

In the limit where the cubic term dominates the bulk potential for  $\Phi$ , the OR solution is given by

$$\Phi_{OR}(\theta) = \frac{k^{3/2}v}{1 + \xi kr_c\theta}. \quad (3.28)$$

Here  $\xi = \eta v/8\sqrt{k}$  and we have imposed the boundary condition  $\Phi(0) = k^{3/2}v$ . In order for our boundary layer analysis to be self-consistent, we require that both  $\alpha$  and  $v$  to lie below their natural values by a factor that could be as small as a few.

However, we had already chosen  $v$  to be small so that it corresponds to a small deformation of the dual CFT in the UV. We had also set  $\alpha$  somewhat smaller than its natural value so that the gravitational back reaction can be neglected. Hence our boundary layer analysis is valid without additional restrictions. Combining the OR and the BR solutions, we obtain the complete solution for  $\Phi$  as

$$\Phi(\theta) = -\frac{k^{3/2}\alpha}{4}e^{4kr_c(\theta-\pi)} + \frac{k^{3/2}v}{1 + \xi kr_c\theta}. \quad (3.29)$$

We can now compute the potential for the radion to leading order in  $v$ :

$$V(\varphi) = k^4 \left(\frac{\varphi}{F}\right)^4 \left[ \tau + \frac{1}{1 - \xi \log(\varphi/F)} \left( 2\alpha v + \frac{\alpha^2 \xi}{8} \right) \right]. \quad (3.30)$$

We can define  $w = 2\alpha v + \alpha^2 \xi/8$  and note that  $w$  is small because it is linear in  $v$ .

The potential is minimized when the condition

$$\tau + \frac{w}{1 - \xi \log(f/F)} = 0 \quad (3.31)$$

is satisfied. We can solve for  $\langle \varphi \rangle = f$ ,

$$\left[ \frac{f}{F} \right]^\xi \equiv e^{-\xi kr_c \pi} = e^{1+w/\tau} + \mathcal{O}(\xi). \quad (3.32)$$

This is more transparent when written in terms of the radius of the extra dimension,

$$k\pi r_c \approx -\frac{w + \tau}{\xi\tau} \approx -\frac{1}{\xi}. \quad (3.33)$$

To obtain a large warp factor  $k\pi r_c$  must be larger than 1. This condition is satisfied if  $v$  is of order  $10^{-2}$ , which justifies the last approximation in Eq. (3.33). From

Eqs. (3.30) and (3.31), we can compute the mass squared of the radion

$$m_\varphi^2 = \frac{\tau^2 \xi}{6w} \frac{k^3}{M_5^3} (k e^{-kr_c \pi})^2. \quad (3.34)$$



Details of the calculation are given in appendix A.2.

Putting in the natural values of the parameters given in appendix B, we find that the radion is parametrically of order the KK mass scale. If  $\eta$ , (as well as  $\epsilon$ ), was tuned to be small, then Eq. (3.34) would predict a light radion. In that case, however, the quartic and higher order terms in the  $\Phi$  bulk potential would give large contributions to the radion mass. If, however, all the terms in the  $\Phi$  bulk potential are small in units of curvature, the radion will be light. This could be realized naturally, if, for example, the GW field corresponds to a pseudo-Goldstone boson.

We also see from Eq. (3.34) that the mass of the radion scales as  $\tau$ . Therefore a light radion can arise if  $\tau$  lies below its natural strong coupling value. Small values of  $\tau$  are associated with tuning, since this condition is not expected to be satisfied in general. However, the tuning of the radion mass is mild, scaling as  $\tau$ . Therefore a radion that lies a factor of 5 below the KK scale is only tuned at the level of 1 in 5 (20%).

### 3.3 Mixing with Other Scalars

Along with the radion, the other scalars in the low energy spectrum are the first few KK modes  $\{\phi_n\}$  of the scalar  $\Phi$  which in general mix with the physical radion  $\tilde{\varphi}$ . Expanding the  $\Phi$  action to linear order in  $\tilde{\varphi}$  and  $\phi_n$ , all terms without a spacetime derivative vanish because of the classical  $\Phi$  equations of motion [20]. This implies there is only kinetic mixing between  $\tilde{\varphi}$  and  $\phi_n$ . Writing this kinetic

mixing term as

$$\kappa_n \int d^4x \partial_\mu \tilde{\varphi} \partial^\mu \phi_n, \quad (3.35)$$

the mixing coefficient  $\kappa_n$  is given as

$$\kappa_n = 2 \int_0^\pi d\theta \left( r_c e^{-2kr_c\theta} f_n(\theta) \frac{\partial \Phi(\theta, \varphi)}{\partial \varphi} \Big|_{\langle \varphi \rangle} \right), \quad (3.36)$$

where  $f_n(\theta)$  is the profile of the  $n$ th KK mode of  $\Phi$ . This affects the physical radion and KK modes as

$$\tilde{\varphi} \rightarrow \tilde{\varphi} - \kappa_n \frac{m_n^2}{m_n^2 - m_\varphi^2} \phi_n \quad \phi_n \rightarrow \phi_n - \kappa_n \frac{m_\varphi^2}{m_n^2 - m_\varphi^2} \tilde{\varphi}, \quad (3.37)$$

where  $m_n$  is the mass of the KK mode and  $\tilde{\varphi}$  is the radion field. Estimating the integral, we find that  $\kappa_n \sim m_\varphi^2/m_n^2$  for both types of bulk  $\Phi$  potentials considered in this section. As we will see in the later sections, this mixing induces a coupling of the radion to SM fields, but scales as  $m_\varphi^4/m_n^4$ , and is therefore subleading compared to a  $m_\varphi^2/m_n^2$  effect when the radion is light.

### 3.4 Radion Interactions to SM fields

In this section we calculate the leading order couplings to the SM fields and the corrections to the radion couplings that arise as a consequence of the GW stabilization mechanism. We restrict to brane localized SM fields which allows us to keep the discussion simple while at the same time makes the general point.

In the absence of the GW scalar, the radion being a massless scalar gravitational mode couples to the trace of the  $5D$  energy momentum tensor [99]. Its

couplings are therefore completely fixed. This has been used to determine the couplings of the radion, both in the case when the SM fields are confined to the IR brane [67, 81–83, 99] and the case when they reside in the bulk of the space [88, 89]. In this paper we will consider SM fields confined to a brane, and leave the case of bulk fields for future work.

Once the GW scalar  $\Phi$  is added to the theory, in general we expect interactions that couple  $\Phi$  to the SM fields. Since the VEV of  $\Phi$  depends on the brane spacing, and therefore on the background radion field, this effect contributes to the coupling of radion to SM fields. To understand this in detail, consider an arbitrary term in the Lagrangian that depends only on the SM fields,

$$\mathcal{L} \supseteq \sqrt{G} f(\Psi_i, \mathcal{A}_i^\mu, \mathcal{H}_i) . \quad (3.38)$$

Here  $f(\Psi_i, \mathcal{A}_i^\mu, \mathcal{H}_i)$  is a function of the SM fermions  $\Psi_i$ , the SM gauge fields  $\mathcal{A}_i^\mu$  and the Higgs field  $\mathcal{H}$ . Then, one can also write the following interaction term in the Lagrangian involving the GW field  $\Phi$ ,

$$\mathcal{L} \supseteq \alpha_{\text{int}} \sqrt{G} f(\Psi_i, \mathcal{A}_i^\mu, \mathcal{H}_i) k^{-3/2} \Phi(x, \theta) , \quad (3.39)$$

where  $\alpha_{\text{int}}$  is a dimensionless coupling constant. Terms containing higher powers of  $\Phi$  in the interaction do not change our conclusions as we work to leading order in  $v$ . In theories with a holographic dual, we expect  $\alpha_{\text{int}}$  to be of order its natural strong coupling value (see appendix B).

In order to determine the radion couplings, we allow the GW field to fluctuate

and KK expand the fluctuations over the classical value. This amounts to replacing

$$\Phi(\theta) \rightarrow \Phi(\theta) + \sum_n f_n(\theta) \phi_n(x) \quad (3.40)$$

in Eq. (3.39), where  $\phi_n$  are the KK modes of the GW field  $\Phi$ . The radion couplings to the light fields contained in  $f(\Psi_i, \mathcal{A}_i^\mu, \mathcal{H}_i)$  get contributions from two sources. Firstly, in the parametrization that we consider, the background value  $\Phi(\theta)$  is a function of the background radion field  $\varphi(x)$  after  $r_c$  is made dynamical. Expanding  $\Phi(\theta)$  about the radion VEV to linear order generates coupling to the physical radion  $\tilde{\varphi} \approx \varphi - f$  as

$$\Phi(\theta) \rightarrow \Phi(\varphi, \theta) = \Phi(f, \theta) + \tilde{\varphi} \partial_\varphi \Phi(f, \theta) + \dots \quad (3.41)$$

Secondly, the KK modes  $\phi_n$  in general have a kinetic mixing with the radion, and thereby generate a radion coupling to the SM fields. From section 3.3, it follows that this second effect scales as  $m_\varphi^4/m_{KK}^4$  for a light radion. As we will see below, this effect is subleading compared to the first effect, and can be neglected.

We can schematically understand how the corrections to the radion couplings scale. For simplicity, we focus on the case where the SM fields live on the visible brane. When the bulk potential for  $\Phi$  is dominated by the mass term, we find that the leading source of modification to the radion coupling, using Eqs. (3.41) and (3.19), scales as  $\partial_\varphi \Phi|_{\theta=\pi} \sim \epsilon v e^{-\epsilon k r_c \pi}$ . From the minimization condition Eq. (3.22) and the expression for the mass of the radion Eq. (3.23), we obtain

$$\epsilon v e^{-\epsilon k r_c \pi} \sim \epsilon \tau \sim \frac{m_\varphi^2}{m_{KK}^2} \quad (3.42)$$

We see that the correction scales as  $\sim m_\varphi^2/m_{KK}^2$  and is small for a light radion.

In the case when the the bulk potential for  $\Phi$  is dominated by the cubic self interaction term, using Eqs. (3.41) and (3.29), we find that the leading source of corrections to the radion coupling now scales as

$$\partial_\varphi \Phi|_{\theta=\pi} \sim v\xi/(1 - \xi \log f/F)^2 . \quad (3.43)$$

From the expressions for the minimization condition Eq. (3.31) and the mass of the radion Eq. (3.34), we find that

$$\frac{v\xi}{(1 - \xi \log(f/F))^2} \sim \tau^2 \sim \frac{m_\varphi^2}{m_{KK}^2} , \quad (3.44)$$

so that the corrections again scale as  $\sim m_\varphi^2/m_{KK}^2$ . It follows from this analysis that the form and magnitude of the leading corrections to the radion couplings does not depend on the details of the GW mechanism that generates the radion mass.

We now calculate the corrections to the radion couplings to SM fields in detail, focusing on the radion couplings to massive and massless gauge bosons, and to fermions. We will consider the case where the Higgs is the pNGB of an approximate global symmetry [50–52, 100, 101]. In this scenario, mixing between the Higgs and the radion, which can otherwise be significant, is suppressed [82], and therefore the Higgs can be replaced by its VEV in the action. This also allows the results from this section to be directly carried over to Higgsless models [73].

### 3.4.1 Massive Gauge Bosons

We begin by considering the radion couplings to the massive gauge bosons of the SM, the  $W^\pm$  and the  $Z$ . In our discussion, we shall focus exclusively on the  $W^\pm$ ,

the generalization to the  $Z$  being straightforward. In the limit when the effects of brane stabilization are neglected, the relevant terms in the action for the  $W$  bosons take the form

$$\mathcal{S} = \int d^4x d\theta \delta(\theta - \pi) \sqrt{-G_v} \left[ -\frac{1}{4g^2} G_v^{\mu\rho} G_v^{\nu\sigma} W_{\mu\nu} W_{\rho\sigma} + G_v^{\mu\nu} D_\mu H^\dagger D_\nu H \right], \quad (3.45)$$

where  $G_v$  is the induced metric on the visible brane and  $g$  is the gauge coupling. Here we use the convention where the gauge covariant derivative is given by  $D_\mu = \partial_\mu + A_\mu$ , and the gauge coupling appears in the kinetic term of the gauge fields. When  $H$  is replaced by its VEV in Eq. (3.45), the  $W$  bosons acquire a mass. Expanding out the components of the metric in terms of the radion and graviton, we find that the radion couples to the  $W^\pm$  as

$$\frac{2m_W^2}{g^2} \frac{\tilde{\varphi}}{f} W_\mu^+ W^{\mu-}, \quad (3.46)$$

where the index on  $W$  is now raised by the  $4D$  Minkowski metric of flat space. Just as for the SM Higgs, at tree level the coupling of the radion is proportional to the mass of the field it is coupling to.

In the presence of the GW field, the gauge kinetic term on the brane is modified to

$$\mathcal{L} \supseteq \delta(\theta - \pi) \sqrt{-G_v} \left[ -\frac{1}{4\hat{g}^2} G_v^{\mu\rho} G_v^{\nu\sigma} W_{\mu\nu} W_{\rho\sigma} \left( 1 + \alpha_W \frac{\Phi}{k^{3/2}} \right) \right]. \quad (3.47)$$

Here  $\hat{g}$  represents the gauge coupling in the absence of the GW stabilization mechanism, and  $\alpha_W$  is a dimensionless number. We take  $\alpha_W$  to be its natural value but continue to work in a regime where  $v$  is small and work to leading order in  $v$ . This turns out to be equivalent to working to linear order in  $\alpha_W$ . The physical gauge

coupling is now given by

$$\frac{1}{4g^2} = \frac{1}{4\hat{g}^2} \left( 1 + \alpha_W \frac{\Phi(\pi)}{k^{3/2}} \right). \quad (3.48)$$

To incorporate radion fluctuations about the VEV of  $\Phi$ , we let

$$\Phi(\pi) \rightarrow \Phi(\pi) \left( 1 + \frac{\partial_\varphi \Phi(\pi)}{\Phi(\pi)} \tilde{\varphi} \right). \quad (3.49)$$

Using the classical solution for  $\Phi$ , the interaction term leads to a correction to the radion coupling. We consider first the case when the bulk potential for  $\Phi$  is dominated by the mass term and the self-interactions can be neglected. In this limit, the radion coupling is given by

$$\bar{c}_W (\epsilon v e^{-\epsilon k r_c \pi}) \frac{1}{4g^2} \frac{\tilde{\varphi}}{f} W_{\mu\nu} W^{\mu\nu}. \quad (3.50)$$

Here  $\bar{c}_W$  is an  $\mathcal{O}(1)$  number, and indices on  $W$  are raised by the Minkowski metric  $\eta^{\mu\nu}$ . Using the minimization condition and the formula for the mass of the radion, we have  $v \epsilon e^{-\epsilon k r_c \pi} \sim \epsilon \tau \sim m_\varphi^2 / m_{KK}^2$ . Therefore the correction scales as  $m_\varphi^2 / m_{KK}^2$ , and is small for a light radion.<sup>2</sup>

The GW scalar also couples to the gauge covariant kinetic term for the Higgs. These interactions affect the gauge boson masses, and lead to corrections to the radion couplings to these fields. We therefore consider the term

$$\mathcal{L} \supseteq \delta(\theta - \pi) \sqrt{-G_v} \left[ \left( 1 + \beta_W \frac{\Phi}{k^{3/2}} \right) G_v^{\mu\nu} D_\mu H^\dagger D_\nu H \right], \quad (3.51)$$

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<sup>2</sup>We note that the operators in Eq. (3.46) and Eq. (3.50) are different, and that it may be possible to tell their contributions apart using gauge boson polarizations, especially at high energy.

where  $\beta_W$  is a dimensionless number. Working in unitary gauge, we replace the Higgs field  $H$  by its VEV. The physical gauge boson mass is now modified to

$$m_W^2 = \hat{m}_W^2 \frac{\left(1 + \beta_W \Phi(\pi)/k^{3/2}\right)}{\left(1 + \alpha_W \Phi(\pi)/k^{3/2}\right)}, \quad (3.52)$$

where  $\hat{m}_W$  is defined as the gauge boson mass in the absence of the GW stabilization mechanism. Including the radion fluctuations about the VEV of  $\Phi$ , we obtain for the radion coupling

$$\frac{\tilde{\varphi}}{f} \left[2 + c_W (\epsilon v e^{-\epsilon k r_c \pi})\right] \frac{m_W^2}{g^2} W_\mu^+ W^{\mu-}. \quad (3.53)$$

Here  $c_W$  is an  $\mathcal{O}(1)$  number. We see that the correction is again suppressed by  $m_\varphi^2/m_{KK}^2$ , and is small for a light radion.

For the case when the bulk potential for  $\Phi$  has a cubic interaction and no mass term, we can repeat the steps above to obtain the corrections to the radion coupling. The end result however remains the same, with the corrections again scaling as  $m_\varphi^2/m_{KK}^2$ .

### 3.4.2 Massless Gauge Bosons

Next we consider the case of radion couplings to the massless gauge bosons of the SM, the photon and the gluons, on the IR brane. The kinetic term has the same form as Eq. (3.45), but now the coupling to the Higgs is absent. Expanding out the components of the metric in terms of the  $4D$  graviton and radion, we find that the radion does not couple to the massless gauge bosons. However this statement is true only at the classical level. In general, quantum effects generate a coupling



of the massless gauge bosons to the radion at one loop. This can be understood as arising from quantum corrections to the trace of the energy momentum tensor; the trace anomaly. These contributions have been calculated in [82], and the resulting coupling is given by

$$\frac{b_{<}}{32\pi^2} \frac{\tilde{\varphi}}{f} F_{\mu\nu} F^{\mu\nu} , \quad (3.54)$$

where again the indices are raised by  $\eta^{\mu\nu}$ . Here  $b_{<}$  is proportional to the one-loop  $\beta$ -function coefficient for the running of the gauge coupling,

$$\frac{d}{d \log \mu} \frac{1}{g^2} = \frac{b_{<}}{8\pi^2} . \quad (3.55)$$

This formula is valid at the KK scale and must be renormalization group (RG) evolved to the radion mass to determine the couplings to the photon and gluons for the on-shell radion.

In the presence of the GW stabilization mechanism, there is a coupling between  $\Phi$  and the gauge bosons given in Eq. (3.47), which modifies the gauge coupling as in Eq. (3.48). In the case where the GW scalar has a bulk mass term, a coupling between the radion and gauge bosons of the form of Eq. (3.50) is generated by the stabilization dynamics. This coupling is of order  $m_\varphi^2/m_{KK}^2$ , and, in the case where the bulk potential of  $\Phi$  has no mass but a cubic interaction, it is straightforward to verify that the correction is of the same form and again scales as  $m_\varphi^2/m_{KK}^2$ . Although the corrections arising from stabilization are small, the fact that the leading order effect is loop suppressed implies that when the radion is only moderately lighter than the KK scale, the effects of GW stabilization are significant, and may even dominate.

### 3.4.3 Fermions

We finally consider the case of the radion couplings to brane localized SM fermions. For concreteness we will focus on the radion couplings to up-type quarks,  $Q$  and  $U$ . The generalization to the other fermions is straightforward. In the absence of a stabilization mechanism, the relevant part of the action has the form

$$\mathcal{S} = \int d^4x d\theta \delta(\theta - \pi) \sqrt{-G_v} \left[ \frac{i}{2} e_a^\mu \left( \bar{Q} \Gamma^a \overleftrightarrow{\partial}_\mu Q + \bar{U} \Gamma^a \overleftrightarrow{\partial}_\mu U \right) - y \left( \bar{Q} H U + \bar{U} H^\dagger Q \right) \right], \quad (3.56)$$

where  $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$  and  $e_a^\mu$  is the vierbein. We replace  $H$  by its VEV and expand the components of the metric and vierbein out in terms of the  $4D$  graviton and radion. After expanding  $\varphi$  about its VEV, canonically normalizing the kinetic terms of the fields  $Q$  and  $U$ , and using the equations of motion for the fermions [102, 103], we obtain the coupling of the fermions to the radion as

$$-m_f \frac{\tilde{\varphi}}{f} (\bar{Q} U + h.c.) , \quad (3.57)$$

showing that the radion couples proportional to mass as expected.

The GW stabilization mechanism allows the following additional interaction terms involving  $\Phi$ ,

$$\mathcal{L}_{\text{int}} = \delta(\theta - \pi) \sqrt{-G_v} \frac{\Phi}{k^{3/2}} \left[ \frac{i}{2} e_a^\mu \left( \alpha_q \bar{Q} \Gamma^a \overleftrightarrow{\partial}_\mu Q + \alpha_u \bar{U} \Gamma^a \overleftrightarrow{\partial}_\mu U \right) - \beta y \left( \bar{Q} H U + \bar{U} H^\dagger Q \right) \right], \quad (3.58)$$

where  $\alpha_q, \alpha_u$  and  $\beta$  are dimensionless numbers. After  $\Phi$  gets a VEV, the kinetic and mass terms receive corrections. The fluctuations of  $\Phi$  about its VEV give

rise to corrections to the radion couplings. We first focus on the case when the bulk potential for  $\Phi$  consists only of a mass term. After making the kinetic terms canonical, and using the equations of motion for the fermions, the coupling to  $\tilde{\varphi}$  is determined to be of the form

$$-m_f \frac{\tilde{\varphi}}{f} (\bar{Q}U + h.c.) [1 + c_\psi \epsilon v e^{-\epsilon k r c \pi}] . \quad (3.59)$$

Here  $c_\psi$  is an  $\mathcal{O}(1)$  number. We see that the corrections are suppressed by  $\sim m_\varphi^2/m_{KK}^2$ , and are therefore small for a light radion. In the case of a cubic bulk potential for  $\Phi$ , the corrections are of the same form and also suppressed by  $\sim m_\varphi^2/m_{KK}^2$ .

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## Dilaton vs Radion: the Duality

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In this chapter, we apply the rules of the AdS/CFT dictionary to compare the results obtained for the mass and the couplings of the dilaton to those of the radion, and we find good agreement. This accomplishes the task of arriving at an extra-dimensional realization of the set-up developed in chapter 2. The comparison of the two sides also explains some of the choices made in the extra-dimensional construction, which are necessary to model the dilaton dynamics. In particular, we establish that the bulk self-interactions for  $\Phi$  are well motivated in the dual picture, and are in fact necessary to model large corrections to the scaling dimension of the operator that breaks the conformal symmetry, on the  $4D$  side of the correspondence.

In the AdS/CFT dictionary, the coordinate corresponding to the fifth dimension of AdS space is associated with the renormalization scale  $\mu$  in the dual theory. To make this more precise, consider making a change of coordinates in AdS space

from  $\theta$  to  $z$ , where  $z$  is defined as

$$z = \frac{e^{kr_c\theta}}{k} . \tag{4.1}$$

Then the renormalization scale  $\mu$  in the dual CFT is given by  $\mu \sim 1/z$ . Therefore, the RS set-up with two branes is dual to a strongly coupled theory that is well approximated by a CFT in the energy regime between the two branes. The hidden brane corresponds to the UV cut-off of the theory. The boundary conditions on the bulk fields at this boundary determine the coefficients of the deformation of the CFT in the UV, in the dual picture. The visible brane ends the AdS space in the IR, signaling the breakdown of the CFT. Various checks can be performed that are suggestive of a spontaneous breakdown. In particular, the trace of the energy momentum tensor is unchanged by the presence of the visible brane. Furthermore, the two point function of the dilatation current has a massless pole of the appropriate strength as dictated by Goldstone's theorem [20]. The scalar excitation associated with this massless pole, the radion, is identified with the NGB of broken scale invariance, the dilaton.

The AdS geometry is stabilized by adding a GW scalar  $\Phi$  to the theory. In the dual picture, this corresponds to deforming the CFT by a primary scalar operator  $\mathcal{O}$ . The boundary condition for  $\Phi$  on the UV brane is related to the strength of the deformation of the CFT at the UV scale. The mass of the scalar  $\Phi$  fixes the scaling dimension of the operator  $\mathcal{O}$  in the UV, while the self interactions of  $\Phi$  in the bulk correspond to the corrections to the scaling dimension of the operator  $\mathcal{O}$  due to the deformation. In what follows, we compare the potentials and the couplings

for the radion and the dilaton on the two sides of the duality, both in the presence of corrections to the scaling dimension of  $\mathcal{O}$  and in the absence of such effects. We first focus on the original model where there is no stabilization mechanism before considering the scenario when the GW scalar is present. We will make use of the ideas of holographic renormalization [104–110] when identifying parameters on the two sides of the correspondence.

To be able to compare the results on the two sides and thereby provide an extra-dimensional realization of the dilaton results, we will use Naive Dimensional Analysis (NDA) values of the  $5D$  parameters for the radion. To be in the semi-classical limit on the AdS side, AdS/CFT correspondence assumes the hierarchy  $M_5 \gg M_S \gg k$ , where  $M_5$  is the  $5D$  Planck scale,  $M_S$  is the scale at which string excitations enter the picture and  $k$  is the AdS curvature. This corresponds to requiring  $N_c \gg 1$  and  $g^2 N_c \gg 1$ , where  $N_c$  is the number of colors and  $g$  is the gauge coupling in the  $4D$  CFT. We will take the approach that in a realistic CFT for which our results from chapter 2 hold, these special conditions may not apply. We will estimate the NDA value of the  $5D$  parameters keeping this in mind. While this implies that the results we obtain using our NDA estimates can only be taken as a very rough estimate because the correspondence is being pushed outside its domain of validity, this will nevertheless provide a way to verify the results obtained for the dilaton explicitly.

## 4.1 Comparison of the Lagrangian for the Dilaton and the Radion

### 4.1.1 Dynamics in the Absence of a Stabilization Mechanism

As a warm up, we begin with the scenario in the absence of a deformation in the CFT. This means that in the extra-dimensional picture, there is no GW mechanism to fix the brane separation. From Eqn. (2.24), the effective potential of the dilaton in the absence of deformation is of the form

$$V_{eff}(\chi) = \kappa_0 \frac{\chi^4}{4!} . \quad (4.2)$$

Here,  $\kappa_0$  is a number that does not depend on  $\chi$ . We can see that unless  $\kappa_0$  is fine-tuned to zero, this potential does not admit a stable minimum away from the origin. Apart from that, the potential fails to give the dilaton a mass.

Now consider the radion potential, which is very similar. We showed in Eqn. (3.5) that before stabilization,  $\varphi$  is massless in AdS and corresponds to the freedom to alter the distance between the two branes. The potential for  $\varphi$  in the absence of stabilization is given by (see appendix A.1)

$$V(\varphi) = \frac{k^4}{F^4} \tau \varphi^4 . \quad (4.3)$$

We see that the forms of the potential for the dilaton and the radion are identical, and therefore suffer from the same problems. The parameter  $\tau$  on the AdS side is given as  $\tau = (T_v - \Lambda_b/k)/k^4$ . This allows us to identify

$$\tau \Leftrightarrow \kappa_0 , \quad (4.4)$$

where we use the double arrow to denote that these two quantities are related by the duality. In the absence of a stabilization mechanism, the original RS solution fine-tunes the IR brane tension  $T_v$  such as to set  $\tau = 0$ . In the dual theory, in the absence of explicit breaking of CFT, this is equivalent to fine-tuning  $\kappa_0$  to vanish.

#### 4.1.2 Dynamics in the Presence of a Stabilization Mechanism

Once the GW mechanism is switched on, the brane spacing is stabilized in the extra-dimensional picture, and the radion gets a mass. In the dual description, the addition of the GW scalar corresponds to deforming the CFT by a primary scalar operator  $\mathcal{O}$  that has scaling dimension  $\Delta$  in the far UV. The dimension  $\Delta$  is related to the mass of the GW scalar,  $m^2$  in units of the curvature as

$$\Delta(\Delta - 4) = m^2/k^2 \tag{4.5}$$

For  $m^2/k^2 \ll 1$ , this relation becomes  $\Delta = 4 + m^2/4k^2$  (The second root of the quadratic equation involving  $\Delta$  is discarded in this limit by unitarity bounds).

In the  $4D$  picture, the cases of interest for electroweak symmetry breaking correspond to those where there is a large hierarchy between the cutoff scale  $\Lambda_{UV}$  and the scale  $f$  where conformal symmetry is broken. A large hierarchy can be generated if  $\hat{\lambda}_{\mathcal{O}}$ , the dimensionless coefficient of deformation of the CFT, is exponentially small in the UV. That would ensure that it takes enough running to make it large enough to break the symmetry. This is technically natural if the operator  $\mathcal{O}$  breaks a symmetry of the CFT, and is therefore protected. The AdS/CFT correspondence relates this scenario to the case when the value of  $\Phi$  at the UV brane is exponentially small. A



second scenario exists where a large hierarchy can be generated if the operator  $\mathcal{O}$  is close to marginal in the UV, so that  $|4 - \Delta| \ll 1$ . This ensures that the deformation runs slowly enough to be able to grow significant only at an IR scale quite separated from the UV scale. Duality relates this scenario to the case when the bulk mass term for the GW scalar is small in units of the curvature and is negative. This discussion justifies why we take the route of  $m^2 < 0$  in the  $5D$  picture. This choice of the sign of the mass squared on the extra-dimensional side is needed to model the  $4D$  scenario developed in chapter 2 earlier. It is this latter scenario which we will focus on.

Below the breaking scale, the potential for the dilaton  $\chi$  in the presence of this explicit breaking of CFT can be determined from a spurion analysis. In the presence of the deformation, the dimensionless coefficient  $\hat{\lambda}_{\mathcal{O}}$  of the deformation satisfies a renormalization group (RG) equation above the symmetry breaking scale

$$\frac{d \log \hat{\lambda}_{\mathcal{O}}}{d \log \mu} = -g(\hat{\lambda}_{\mathcal{O}}), \quad (4.6)$$

where  $g(\hat{\lambda}_{\mathcal{O}})$  is a polynomial in  $\hat{\lambda}_{\mathcal{O}}$  that can be parametrized as

$$g(\hat{\lambda}_{\mathcal{O}}) = (4 - \Delta) + c_1 \hat{\lambda}_{\mathcal{O}} + c_2 \hat{\lambda}_{\mathcal{O}}^2 + \dots \quad (4.7)$$

With our normalization of the operator  $\mathcal{O}$ , the coefficients  $c_i$  are expected to be of order one.

We saw in chapter 2 that in the limit of  $g(\hat{\lambda}_{\mathcal{O}}) \ll 1$ , the potential for the dilaton to leading order in  $\hat{\lambda}_{\mathcal{O}}$  was given as

$$V(\chi) = \frac{\chi^4}{4!} \left[ \kappa_0 - \kappa_1 \hat{\lambda}_{\mathcal{O}} \left( \frac{\chi}{f} \right)^{-g(\hat{\lambda}_{\mathcal{O}})} \right]. \quad (4.8)$$

Here  $\kappa_0$  and  $\kappa_1$  are parameters in the low energy theory. The potential now admits a stable minimum at  $\langle\chi\rangle = f$  without fine-tuning  $\kappa_0$  to zero and results in dilaton acquiring a mass. In the limit in which we are working, the minimization condition takes the form  $\kappa_0 = \kappa_1 \hat{\lambda}_{\mathcal{O}}$  and the mass of the dilaton  $\sigma$  is given as  $m_\sigma^2/f^2 = 4\kappa_0 g(\hat{\lambda}_{\mathcal{O}})/4!$  It follows that the mass of the dilaton depends on  $\kappa_0$  and  $g(\hat{\lambda}_{\mathcal{O}})$ , which are related to each other by the minimization condition. We also saw in chapter 2 that depending on how much  $\kappa_0$  is tuned below its natural value, or alternatively how much  $\hat{\lambda}_{\mathcal{O}}$  is pushed below 1, the mass squared of the dilaton scales as  $m_\sigma^2 \sim \kappa_0(4-\Delta)$  or  $m_\sigma^2 \sim \kappa_0^2$ . These two different scalings for the mass squared of the dilaton result from whether the second and subsequent terms in the expansion for  $g(\hat{\lambda}_{\mathcal{O}})$  (4.7) can be dropped or not. Hence we see that, depending on the extent to which  $\kappa_0$  lies below its natural strong coupling value, the dilaton mass may scale either as  $\sqrt{\kappa_0(4-\Delta)}$  or as  $\kappa_0$ . Recall that duality relates  $\kappa_0$  to  $\tau$ , the detuning of the IR brane tension. We had arrived at same scaling results for the scaling of the radion mass in chapter 3, with the detuned IR brane tension  $\tau$  replaced by the quartic  $\kappa_0$ . Therefore the radion dynamics indeed captures the general story of the dilaton.

We can draw more detailed parallels to the AdS side of the duality in the presence of stabilization by studying the RG equation on the  $4D$  side and the classical equation satisfied by  $\Phi$  on the  $5D$  side more closely. Recall that the classical equation satisfied by  $\Phi$  has two independent solutions, which in the limit that  $4kr_c \gg 1$ , are well approximated by the OR and BR solutions defined in chapter 3. The OR differential equation is only first order, and is a good approximation to the classical equation that is satisfied by  $\Phi$  everywhere except in the boundary layer region

close to the IR brane. In the absence of an IR brane, the AdS/CFT dictionary relates the value of  $\Phi$  at the UV brane to the coefficient of the deformation of the CFT,  $\hat{\lambda}_{\mathcal{O}}$ , evaluated at the renormalization scale  $\mu = k$  [17, 18, 20]. We denote this correspondence by

$$\frac{\Phi(\theta = 0)}{k^{3/2}} \Leftrightarrow \hat{\lambda}_{\mathcal{O}}(\Lambda_{\text{UV}}) . \quad (4.9)$$

The ideas of holographic renormalization extend this identification further. In the AdS theory without an IR brane, the BR solution is replaced by the requirement of regularity at the AdS horizon. In that case, the value of  $\Phi$  at an arbitrary point with coordinates  $(x^\mu, \theta)$  in the bulk corresponds to  $\hat{\lambda}_{\mathcal{O}}$  evaluated at the renormalization scale  $\mu = k \exp(-kr_c\theta)$ . This then implies that the first order OR differential equation for  $\Phi$  corresponds to the RGE for  $\hat{\lambda}_{\mathcal{O}}$  at energies below the UV cutoff.

In our case, the AdS space has an IR brane and we cannot discard the BR solution, which captures the physics associated with the conformal symmetry breaking phase transition. However, given that the boundary layer has a thickness of just  $\sim 1/4kr_c \sim \epsilon$  in  $\theta$  coordinates, as long as we are at least this distance from the IR brane, the correspondence between the OR equation for  $\Phi$  and the RGE for  $\hat{\lambda}_{\mathcal{O}}$  holds. From Eq. (3.14), the OR equation satisfied by  $\Phi$  is

$$\frac{d \log \Phi_{OR}}{d(kr_c\theta)} = -\frac{m^2}{4k^2} - \frac{\eta}{8\sqrt{k}} \frac{\Phi_{OR}}{k^{3/2}} - \frac{\zeta k}{24} \frac{\Phi_{OR}^2}{k^3} + \dots . \quad (4.10)$$

The corresponding RGE satisfied by the coefficient of the deformation  $\hat{\lambda}_{\mathcal{O}}$  is given as

$$\frac{d \log \hat{\lambda}_{\mathcal{O}}}{d \log \mu} = (4 - \Delta) + c_1 \hat{\lambda}_{\mathcal{O}} + c_2 \hat{\lambda}_{\mathcal{O}}^2 + \dots . \quad (4.11)$$

From the AdS/CFT dictionary, we have that the coordinate in the fifth dimension  $\theta$  corresponds to the renormalization scale  $\mu$  in the  $4D$  theory,  $k \exp(-kr_c\theta) = \mu$ . While it is tempting to use this to relate the various terms in Eq. (4.10) and Eq. (4.11), we are prevented from doing so by the fact that RGEs are in general scheme dependent. However, because this restriction does not apply to the lowest order term in an RGE, we are able to reproduce the familiar AdS/CFT relation between the dimensions of operators in the  $4D$  theory and the masses of the corresponding scalar fields in  $5D$ ,  $\Delta = 4 + m^2/4k^2 = 4 + \epsilon$  for  $\epsilon \ll 1$ . In addition, in the case when the bulk mass term is small so that the potential for the GW scalar is dominated by the cubic self-interaction term, we can also equate

$$c_1 \hat{\lambda}_{\mathcal{O}} = -\frac{\eta}{8\sqrt{k}} \frac{\Phi_{OR}}{k^{3/2}}. \quad (4.12)$$

Comparing the RGE for  $\hat{\lambda}_{\mathcal{O}}$  and the OR differential equation for  $\Phi$ , it is clear that the limit when  $\hat{\lambda}_{\mathcal{O}}$  is small and  $g(\hat{\lambda}_{\mathcal{O}})$  is dominated by the constant term, it corresponds to the AdS potential for  $\Phi$  being dominated by the mass term. For larger values of  $\hat{\lambda}_{\mathcal{O}}$ , the linear and higher order terms in  $g(\hat{\lambda}_{\mathcal{O}})$  dominate. This corresponds to the cubic and higher order self-interaction terms becoming important in the bulk potential for  $\Phi$ .

The potential for the dilaton is to be compared to that for the radion in the two cases that we studied in chapter 3. Consider first the case when the self-interactions for  $\Phi$  in the bulk can be neglected. In this limit,  $g(\hat{\lambda}_{\mathcal{O}})$  in the RGE for  $\hat{\lambda}_{\mathcal{O}}$  is dominated by the constant term. The potential for the dilaton using Eq. (4.8) is

then given by

$$V(\chi) = \frac{\chi^4}{4!} \left[ \kappa_0 - \kappa_1 \hat{\lambda}_{\mathcal{O}} \left( \frac{\chi}{f} \right)^{\Delta-4} \right]. \quad (4.13)$$

The resulting potential for the radion from Eq. (3.21) is given by

$$V(\varphi) = \frac{k^4}{F^4} \varphi^4 \left[ \tau + \frac{2\alpha v}{F^\epsilon} \varphi^\epsilon \right] = \frac{k^4}{F^4} \varphi^4 \left[ \tau + \frac{2\alpha (ve^{-\epsilon kr_c \pi})}{(Fe^{-kr_c \pi})^\epsilon} \varphi^\epsilon \right], \quad (4.14)$$

where we have rewritten it in a form that is convenient for making the comparison.

Bearing in mind that the symmetry breaking scale  $f$  is of order  $ke^{-kr_c \pi}$ , the duality allows us to relate

$$\begin{aligned} \hat{\lambda}_{\mathcal{O}}(f) &\Leftrightarrow \frac{\Phi_{OR}(\theta \sim \pi)}{k^{3/2}} \approx ve^{-\epsilon kr_c \pi} \\ \Delta &= 4 + \epsilon. \end{aligned} \quad (4.15)$$

With this identification, the potentials for the radion and the dilaton are of the same form. The low energy parameter  $\kappa_1$  on the CFT side is related to the coefficient  $\alpha$  of the potential for  $\Phi$  on the visible brane. Our initial identification  $\kappa_0 \Leftrightarrow \tau$  continues to hold even though  $\tau$  now receives additional  $\mathcal{O}(1)$  contributions from visible brane dynamics. Since the potentials for  $\varphi$  and  $\chi$  are of the same form, the leading order expression for the masses of the fluctuations of  $\varphi$  and  $\chi$  have the same parametric dependence. Therefore the radion construction models the dilaton set-up honestly.

Consider next the scenario when the potential for the GW scalar  $\Phi$  is dominated by the cubic self-interaction term in the bulk. By duality this is related to the case when  $g(\hat{\lambda}_{\mathcal{O}})$  in the RGE for  $\hat{\lambda}_{\mathcal{O}}$  is dominated by the linear term  $c_1 \hat{\lambda}_{\mathcal{O}}$ . For this case, we can read off the potential for the dilaton from Eq. (4.8). In this limit,

$$V(\chi) = \frac{\chi^4}{4!} \left[ \kappa_0 - \kappa_1 \hat{\lambda}_{\mathcal{O}} \left( \frac{\chi}{f} \right)^{-c_1 \hat{\lambda}_{\mathcal{O}}} \right]. \quad (4.16)$$

In the corresponding limit, we obtain the potential for the radion from Eq. (3.30)

as

$$V(\varphi) = \frac{k^4}{F^4} \varphi^4 \left[ \tau + \frac{2\alpha v}{1 - \xi \log(\varphi/F)} \right] = \frac{k^4}{F^4} \varphi^4 \left[ \tau + \frac{2\alpha v}{1 + \xi k r_c \pi - \xi \log(\varphi/F e^{-k r_c \pi})} \right], \quad (4.17)$$

where  $\xi = \eta v/8\sqrt{k}$ . In the second equality we have added and subtracted  $\xi k r_c \pi$  in the denominator. For  $\xi/(1 + \xi k r_c \pi) \ll 1$ , we can approximate the potential as

$$V(\varphi) \approx \varphi^4 \left[ \tau + \frac{2\alpha v}{1 + \xi k r_c \pi} \left( \frac{\varphi}{F e^{-k r_c \pi}} \right)^{\frac{\xi}{1 + \xi k r_c \pi}} \right]. \quad (4.18)$$

This is a more convenient form to compare against the dilaton potential. In the limit in which we are working,  $f$  is of order  $k e^{-k r_c \pi}$ . The duality allows us to relate

$$\begin{aligned} \hat{\lambda}_{\mathcal{O}}(f) &\Leftrightarrow \frac{v}{1 + \xi k r_c \pi} \\ c_1 \hat{\lambda}_{\mathcal{O}}(f) &= -\frac{\xi}{1 + \xi k r_c \pi}. \end{aligned} \quad (4.19)$$

We once again see that the potentials for the radion and the dilaton are of the same form. The low energy parameters in the CFT,  $\kappa_0$  and  $\kappa_1$  are again related to the AdS parameters  $\tau$  and  $\alpha$  respectively. The parametric dependences of the masses of the radion and the dilaton then agree in a straightforward manner. Once again, the radion construction models the dilaton set-up honestly.

Finally, we can use the correspondence from Eq. (4.9) to justify the choice of parameters on the  $5D$  side that ensured a behavior of  $\Phi(\theta)$  shown in Fig. 3.1. Since  $\theta$  is dual to  $\log \mu$ , we see that in the outer region, which corresponds to most of the space, the behavior of  $\Phi(\theta)$  captures the logarithmic running of the coupling, as expected for a nearly marginal deformation of the CFT. Near the IR brane at  $\theta = \pi$ ,

the behavior of  $\Phi$  changes due to the formation of a boundary layer. This is the region where the behavior of  $\Phi(\theta)$  can no longer be described by the OR Eq. (3.14) and we need a different description. This is once again in accordance with the dual picture, where, a phase transition associated with the spontaneous breakdown of the CFT occurs.

## 4.2 Comparison of Dilaton and Radion Couplings

In this section, we compare the results for the dilaton couplings to those for the radion couplings. We restrict to the case of conformal SM on the  $4D$  side of the correspondence, which amounts to studying brane localized SM fields on the  $5D$  side of the duality. We find good agreement between the results on the two sides. This suffices to make the point that the radion dynamics studied in chapter 2, captures the important features of the physics on the  $4D$  side of the correspondence.

In the absence of explicit CFT breaking sources, the couplings of the fermion and the massive gauge bosons are generated by their mass terms in the action, and were obtained earlier as

$$-m_f \frac{\sigma}{f} \bar{\psi} \psi, \quad 2m_W^2 \frac{\sigma}{f} W_\mu^+ W^{\mu-}. \quad (4.20)$$

The corresponding results for the radion are of the same form, and are given here for the sake of comparison.

$$-m_f \frac{\tilde{\varphi}}{f} \bar{\psi} \psi, \quad 2m_W^2 \frac{\tilde{\varphi}}{f} W_\mu^+ W^{\mu-}. \quad (4.21)$$

For the case of massless gauge bosons, we found that the dilaton does not couple at

the tree level, but a coupling is generated by the trace anomaly as

$$\frac{b_{<}}{32\pi^2} \frac{\sigma}{f} F_{\mu\nu} F^{\mu\nu} . \quad (4.22)$$

These results are also true in the extra-dimensional scenario, where the radion does not couple to the massless gauge bosons at the classical level, but a coupling is generated at the loop level and has the exact same form as (4.22), with the  $\sigma$  field replaced by  $\tilde{\varphi}$ ,

$$\frac{b_{<}}{32\pi^2} \frac{\tilde{\varphi}}{f} F_{\mu\nu} F^{\mu\nu} . \quad (4.23)$$

In the presence of explicit source of CFT breaking, the corrections to the dilaton couplings were calculated for two cases: when the corrections to the scaling dimension of the CFT breaking operator  $\mathcal{O}$  are small and when they are significant. In both the cases, the corrections scaled as  $m_\sigma^2/\Lambda^2$ , where  $\Lambda = 4\pi f$ . More specifically, the corrections to the coupling of the dilaton to the fermions and massive gauge bosons were given as

$$-c_\psi \frac{m_\sigma^2}{\Lambda^2} m_f \frac{\sigma}{f} \bar{\psi} \psi , \quad c_W \frac{m_\sigma^2}{\Lambda^2} m_W^2 \frac{\sigma}{f} W_\mu^+ W^{\mu-} , \quad (4.24)$$

where  $c_\psi$  and  $c_W$  are  $\mathcal{O}(1)$  numbers. These corrections were calculated by a spurion analysis in the CFT, by promoting the coefficient of the deformation  $\hat{\lambda}_\mathcal{O}$  to be a spurion and considering insertions of  $\hat{\lambda}_\mathcal{O}$  in the low energy. The results were independent of whether the corrections to the scaling dimension of the operator  $\mathcal{O}$  were small or large. In the extra-dimensional setup, the corrections were calculated by coupling the SM fields to one power of the GW field  $\Phi$  on the IR brane. Since  $\Phi \sim \hat{\lambda}_\mathcal{O}$ , this amounted to considering one insertion of  $\hat{\lambda}_\mathcal{O}$  in the CFT. In the small



$\hat{\lambda}_{\mathcal{O}}$  limit, this therefore reproduces the CFT results. We considered two forms of the bulk  $\Phi$  potential to model the two CFT scenarios and found the final form of the coupling was again independent of this intermediate detail. In these limits, the corrections were of the form

$$-c_{\psi} \frac{m_{\varphi}^2}{m_{KK}^2} m_f \frac{\tilde{\varphi}}{f} \bar{\psi} \psi, \quad c_W \frac{m_{\varphi}^2}{m_{KK}^2} m_W^2 \frac{\tilde{\varphi}}{f} W_{\mu}^{+} W^{\mu-}, \quad (4.25)$$

These corrections are small compared to the leading coupling and have same form as that for the dilaton. As we can see, the results agree with each other after  $\Lambda$  is identified with  $m_{KK}$ . For the case of massless gauge bosons, the corrections on the dilaton and the radion side of the correspondence were

$$c_A \frac{m_{\sigma}^2}{\Lambda^2} \frac{\sigma}{f} F_{\mu\nu} F^{\mu\nu}, \quad c_A \frac{m_{\varphi}^2}{m_{KK}^2} \frac{\tilde{\varphi}}{f} F_{\mu\nu} F^{\mu\nu}. \quad (4.26)$$

These corrections can be important as they can be of the same order or even larger than the leading coupling generated by loops.

We therefore see that for the two scenarios considered in the CFT, the results for the leading coupling and the corrections are in good agreement on the two sides of the correspondence. Therefore, the radion scenario considered is a good extra-dimensional realization for the dilaton dynamics.

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## Conclusion

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In this dissertation, we considered scenarios where strong conformal dynamics constitutes the ultraviolet completion of the physics responsible for electroweak symmetry breaking. We have constructed the effective theory of a light dilaton in such a framework, taking into account the explicit conformal symmetry violating effects that are necessarily present in realistic models. We have considered two cases: when the corrections to the scaling behavior of the operator that breaks the conformal symmetry are small, and when they are large. In both these cases, the presence of a light dilaton is associated with mild tuning. This tuning scales with the mass of the dilaton rather than with the square of the mass. As part of our analysis we have obtained the couplings of the dilaton to gauge bosons and fermions in the technicolor and Higgs as a pNGB cases and also determined the size of the corrections to these couplings from conformal symmetry violating effects. We found that they are under good theoretical control in theories where the dilaton is light.

These corrections are subleading, but can be important for marginal operators.

Using the extra-dimensional realization of such scenarios, we have constructed the effective theory for a radion stabilized by the GW mechanism. Our analysis differs from the original GW construction in that we do not tune the gravitational potential for the radion to vanish, but allow an interplay between the dynamics of the GW scalar and gravity to stabilize the radion. We required that the bulk mass for the GW scalar  $\Phi$  is small compared to the inverse curvature scale in order to generate a large hierarchy between the scales of the hidden and visible branes. We have considered two different cases for the bulk potential of the GW scalar. In the first, the mass for  $\Phi$ , although small, is still the dominant term in the potential. This is the case most often studied in the literature, and we find that the radion is light. In particular, the smallness of the bulk mass in units of the inverse AdS curvature translates to radion being lighter than the KK scale. The second case we have studied is when the bulk potential for  $\Phi$  is dominated by a cubic term. This captures the features of a general interacting potential. Taking the coefficient of the cubic coupling to be around its strong coupling value, we showed that the radion is generically not light – its mass being controlled by the visible brane tension which can be tuned to allow the radion to lie below the KK scale. The tuning is mild, scaling as the mass of the radion rather than as the square of the mass. In the absence of symmetry reasons, the general form of the bulk potential for  $\Phi$  must contain interactions. Therefore, a light radion is associated with tuning. For both choices of the bulk potential, we have also analyzed radion couplings to SM fields living on the visible brane, focusing in particular on corrections due to the GW

mechanism. These corrections are proportional to the mass squared of the radion in units of the KK scale, and are small if the radion is light. We have shown that these corrections from stabilization are subleading for massive SM fields, but can be important and possibly dominant for the photon and the gluon.

We finally compared the results for the radion and the dilaton in light of the AdS/CFT correspondence, showing the agreement clearly and thereby establishing that the RS construction captures the essence of the dilaton results. We focused on CFTs which are deformed by marginal operators that are small in the UV, but grow large in the IR to trigger the breaking of conformal symmetry. In general, the scaling behavior of these operators near the breaking scale is very different from their scaling behavior in the UV. The AdS/CFT dictionary associates this change in scaling behavior with the presence of self-interaction terms for the GW scalar  $\Phi$  in the bulk. This necessitated the inclusion of bulk self-interactions for  $\Phi$  to capture this effect. Once incorporated, the results for the radion and the dilaton are in good agreement.

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## Radion Potential

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In this appendix we give intermediate steps involved in obtaining the radion potential in various scenarios considered in this dissertation. We also give the minimization condition and the mass of the radion in each case.

### A.1 In the absence of a GW Scalar

In the absence of a stabilization mechanism, the 5D action is

$$\mathcal{S}_{GR}^{5D} = \int d^4x \int_{-\pi}^{\pi} d\theta \sqrt{G} \left( -2M_5^3 \mathcal{R}[G] - \Lambda_b \right) - \sqrt{-G_h} \delta(\theta) T_h - \sqrt{-G_v} \delta(\theta - \pi) T_v , \quad (\text{A.1})$$

where the geometry is a slice of AdS<sub>5</sub> compactified on a circle  $\mathcal{S}_1$  with  $\mathcal{Z}_2$  orbifolding.

The metric is then given as

$$ds^2 = e^{-2kr(x)|\theta|} g_{\mu\nu} dx^\mu dx^\nu - r^2(x) d\theta^2 , \quad (\text{A.2})$$

where  $g_{\mu\nu}$  is the 4D graviton and  $r(x)$  is the (non-canonical) radion. The 5D Ricci scalar for this metric is

$$\begin{aligned} \mathcal{R}[G] = \frac{2}{r(x)} \left[ e^{2kr(x)|\theta|} \left( -\frac{r}{2} \mathcal{R}[g] - \partial^2 r + 3kr|\theta| \partial^2 r + 2k|\theta| \partial r \partial r - 3k^2 r |\theta|^2 \partial r \partial r \right) \right. \\ \left. + 10k^2 r - 8k\delta(\theta) + 8k\delta(\theta - \pi) \right]. \end{aligned} \quad (\text{A.3})$$

Using  $\sqrt{G} = r(x)e^{-4kr(x)|\theta|}\sqrt{-g}$ , the integrand in the action is

$$\begin{aligned} \sqrt{-g} e^{-2kr(x)|\theta|} \left[ 2M_5^3 \left( r \mathcal{R}[g] + 2\partial^2 r - 4k|\theta| \partial r \partial r - 6kr|\theta| \partial^2 r + 6k^2 r |\theta|^2 \partial r \partial r \right) \right] \\ + \sqrt{-g} e^{-4kr(x)|\theta|} \left[ -r\Lambda_b + 2M_5^3 \left( -20k^2 r + 16k\delta(\theta) - 16k\delta(\theta - \pi) \right) - T_h \delta(\theta) - T_v \delta(\theta - \pi) \right]. \end{aligned} \quad (\text{A.4})$$

Using integration by parts, the integrand further reduces to

$$\begin{aligned} \sqrt{-g} e^{-2kr(x)|\theta|} \left[ 2M_5^3 \left( r \mathcal{R}[g] + 6k|\theta| \partial r \partial r - 6k^2 |\theta|^2 r \partial r \partial r \right) \right] \\ + \sqrt{-g} e^{-4kr(x)|\theta|} \left[ -r\Lambda_b + 2M_5^3 \left( -20k^2 r + 16k\delta(\theta) - 16k\delta(\theta - \pi) \right) - T_h \delta(\theta) - T_v \delta(\theta - \pi) \right]. \end{aligned} \quad (\text{A.5})$$

Integrating over  $\theta$  leads to a cancellation between the  $|\theta|$  and the  $|\theta|^2$  terms as

$$\int_{-\pi}^{\pi} d\theta e^{-2kr|\theta|} (6k|\theta| - 6k^2 r |\theta|^2) = 6k\pi^2 e^{-2kr\pi} \quad (\text{A.6})$$

and leads to the effective action for the 4D graviton  $g_{\mu\nu}(x)$  and the modulus field  $r(x)$  as

$$\begin{aligned} \mathcal{S}_{GR}^{4D} = \frac{2M_5^3}{k} \int d^4x \sqrt{-g} \left( 1 - e^{-2k\pi r(x)} \right) \mathcal{R}[g] + \frac{12M_5^3}{k} \int d^4x \sqrt{-g} \partial_\mu \left( e^{-k\pi r(x)} \right) \partial^\mu \left( e^{-k\pi r(x)} \right) \\ - \int d^4x \sqrt{-g} V(r). \end{aligned} \quad (\text{A.7})$$

The potential  $V(r)$  for the modulus field has contributions both from bulk and brane, and is given by

$$\begin{aligned}
V(r) &= - \int_{-\pi}^{\pi} d\theta e^{-4kr(x)|\theta|} \left( -r\Lambda_b - 40M_5^3 k^2 r \right) + \left( T_h - 32M_5^3 k \right) + \left( T_v + 32M_5^3 k \right) e^{-4kr(x)\pi} \\
&= e^{-4kr(x)\pi} \left( T_v + 32M_5^3 k - 2 \frac{r\Lambda_b + 40M_5^3 k^2 r}{4kr} \right) + \left( T_h - 32M_5^3 k + 2 \frac{r\Lambda_b + 40M_5^3 k^2 r}{4kr} \right).
\end{aligned} \tag{A.8}$$

In terms of the canonically normalized 4D radion field  $\varphi \equiv F e^{-k\pi r(x)}$ , where  $F = \sqrt{24M_5^3/k}$ , the 4D action looks like

$$\begin{aligned}
\mathcal{S}_{GR}^{4D} &= 2M_5^3/k \int d^4x \sqrt{-g} \left( 1 - (\varphi/F)^2 \right) \mathcal{R} + \frac{1}{2} \int d^4x \sqrt{-g} \partial_\mu \varphi \partial^\mu \varphi \\
&\quad - \int d^4x \sqrt{-g} \left[ (\varphi/F)^4 \left( T_v - \frac{\Lambda_b}{2k} + 12M_5^3 k \right) + \left( T_h + \frac{\Lambda_b}{2k} - 12M_5^3 k \right) \right].
\end{aligned} \tag{A.9}$$

We ignore the interactions between the Ricci scalar and radion because they are small and irrelevant for phenomenology. Using  $k^2 = -\Lambda_b/24M_5^3$ , the radion potential is given as

$$V(\varphi) = \left[ (\varphi/F)^4 \left( T_v - \frac{\Lambda_b}{k} \right) + \left( T_h + \frac{\Lambda_b}{k} \right) \right] \equiv \frac{k^4}{F^4} (\tau \varphi^4 + \varrho), \tag{A.10}$$

where  $\varrho = \left( T_h + \frac{\Lambda_b}{k} \right)/k^4$  is the 4D cosmological constant which we tune to be small, and  $\tau = \left( T_v - \frac{\Lambda_b}{k} \right)/k^4$ . When the GW scalar is included,  $\tau$  will receive additional corrections.

## A.2 In the presence of a GW Scalar

In the presence of a GW scalar, we consider two cases for the bulk potential  $V_b(\Phi)$ : (i) the bulk potential dominated by a mass term and (ii) the bulk potential

dominated by a cubic term. In the case where the bulk potential only has a mass term, the equation for  $\Phi$  admits exact solution. We compare the approximate OR and BR solutions for  $\Phi$  obtained earlier to the exact solution of  $\Phi$  in this case.

### A.2.1 GW Scalar without Bulk Interactions

Consider first the case of

$$V_b(\Phi) = \frac{1}{2}m^2\Phi^2 . \quad (\text{A.11})$$

The equation satisfied by  $\Phi$  in the bulk becomes

$$\partial_\theta^2\Phi - 4kr_c\partial_\theta\Phi - m^2r_c^2\Phi = 0 . \quad (\text{A.12})$$

The equation being a homogeneous second order differential equation, admits exact solutions given by

$$\Phi(\theta, r) = Ae^{\nu_1kr_c|\theta|} + Be^{\nu_2kr_c|\theta|} , \quad (\text{A.13})$$

where  $\nu_{1,2} = 2 \pm \sqrt{4 + m^2/k^2}$  and  $A, B$  are constants, determined by the brane potentials as

$$\begin{aligned} A\nu_1 + B\nu_2 &= \frac{1}{2k}V'_h(\Phi) \\ \nu_1Ae^{\nu_1kr_c\pi} + \nu_2Be^{\nu_2kr_c\pi} &= -\frac{1}{2k}V'_v(\Phi) \end{aligned} \quad (\text{A.14})$$

Without specifying the form of  $V_h$ , we require that the value of  $\Phi(\theta = 0) = k^{3/2}v$ .

For the visible brane, we assume a simple form  $V_v(\Phi) = 2k^{5/2}\alpha\Phi$ . This gives

$$\begin{aligned} A + B &= k^{3/2}v \\ \nu_1Ae^{\nu_1kr_c\pi} + \nu_2Be^{\nu_2kr_c\pi} &= -k^{3/2}\alpha . \end{aligned} \quad (\text{A.15})$$



We take  $\epsilon = m^2/4k^2 \ll 1$  and  $e^{-kr_c\pi} \ll 1$  but  $e^{\pm\epsilon kr_c\pi}$  unhierarchical. The two conditions on  $A, B$  fix them to be

$$\begin{aligned} A &= k^{3/2} \frac{v \nu_2 e^{\nu_2 kr_c\pi} + \alpha}{\nu_2 e^{\nu_2 kr_c\pi} - \nu_1 e^{\nu_1 kr_c\pi}} \approx -\frac{k^{3/2}\alpha}{4} e^{-(4+\epsilon)kr_c\pi} \\ B &= k^{3/2} \frac{v \nu_1 e^{\nu_1 kr_c\pi} + \alpha}{\nu_1 e^{\nu_1 kr_c\pi} - \nu_2 e^{\nu_2 kr_c\pi}} \approx k^{3/2} v . \end{aligned} \quad (\text{A.16})$$

Therefore, the leading order solution of  $\Phi$  is given as

$$\Phi(\theta) = -\frac{k^{3/2}\alpha}{4} e^{(4+\epsilon)kr_c(\theta-\pi)} + k^{3/2} v e^{-\epsilon kr_c\theta} . \quad (\text{A.17})$$

Using the arguments from chapter 3, an approximate solution for  $\Phi$  can be obtained by a boundary layer analysis:

$$\Phi_{\text{approx}}(\theta) = -\frac{k^{3/2}\alpha}{4} e^{4kr_c(\theta-\pi)} + k^{3/2} v e^{-\epsilon kr_c\theta} . \quad (\text{A.18})$$

Figure 3.1 in chapter 3 shows that the approximate solution agrees very well with the exact solution.

Substituting either solution of  $\Phi$  in the action, integrating over the extra dimension and letting  $r_c \rightarrow r(x)$  generates a potential for the radion. The potential gets contribution both from the bulk and the brane terms in the GW action, and is given in general by

$$V_{GW}(r) = \int_0^\pi d\theta \frac{1}{r} e^{-4kr\theta} \left( \partial_\theta \Phi \partial_\theta \Phi + r^2 m^2 \Phi^2 \right) + e^{-4kr\pi} 2\alpha k^{5/2} \Phi(\pi) . \quad (\text{A.19})$$

Working in the limit where  $|\epsilon| \ll 1$ , the bulk contribution to the potential goes as

$$\begin{aligned} V_{GW}^{\text{bulk}}(r) &= k \left[ (4+\epsilon)A^2 \left( e^{(2\nu_1-4)kr\pi} - 1 \right) - \epsilon B^2 \left( e^{(2\nu_2-4)kr\pi} - 1 \right) \right] \\ &= k^4 \frac{e^{-4kr\pi}}{4+\epsilon} \left( \alpha - \epsilon v e^{-\epsilon kr_c\pi} \right)^2 - \epsilon k^4 v^2 e^{-(4+2\epsilon)kr_c\pi} + \epsilon k^4 v^2 . \end{aligned} \quad (\text{A.20})$$

The brane contribution to the potential is

$$\begin{aligned} V_{GW}^{\text{brane}}(r) &= 2k^{5/2} \alpha e^{-4kr\pi} \Phi(\pi) \\ &= k^4 e^{-4kr\pi} 2\alpha \left[ -\frac{\alpha - \epsilon v e^{-\epsilon kr_c \pi}}{4 + \epsilon} + v e^{-\epsilon kr_c \pi} \right]. \end{aligned} \quad (\text{A.21})$$

Keeping to linear order in  $v$  and  $\epsilon$  we find that the bulk contribution to the potential is subleading. We can now write the leading radion potential, including the gravitational contributions from Eq. (A.10), in terms of the canonical field  $\varphi$ :

$$V(\varphi) = k^4 \left( \frac{\varphi}{F} \right)^4 \left[ \tau + 2\alpha v \left( \frac{\varphi}{F} \right)^\epsilon \right]. \quad (\text{A.22})$$

Here,  $\tau$  receives contributions from both the gravity sector and the visible brane dynamics, and, to leading order in  $\epsilon$ , it is given by

$$\tau = \frac{T_v - \Lambda_b/k - k^4 \alpha^2/4}{k^4}. \quad (\text{A.23})$$

The minimization condition gives  $\langle \varphi \rangle = f$  as

$$\tau + 2\alpha v \left( \frac{f}{F} \right)^\epsilon = 0. \quad (\text{A.24})$$

At the minimum, the mass squared is given as

$$m_\varphi^2 = -\frac{\epsilon \tau}{6} \frac{k^3}{M_5^3} (k e^{-kr_c \pi})^2. \quad (\text{A.25})$$

Including higher order terms does not change the parametric dependences.

## A.2.2 GW Scalar with Bulk Cubic Self Interaction

We next consider the case of

$$V_b(\Phi) = \frac{1}{3!} \eta \Phi^3. \quad (\text{A.26})$$

In this case, the classical equation satisfied by  $\Phi$  is given as

$$\partial_\theta^2 \Phi - 4kr_c \partial_\theta \Phi - \frac{\eta}{2} r_c^2 \Phi^2 = 0. \quad (\text{A.27})$$

In the absence of an exact solution, we use the boundary layer theory described in section 3.1 to get the approximate solution for  $\Phi$  as

$$\Phi_{\text{approx}}(\theta) = -\frac{k^{3/2}\alpha}{4} e^{4kr_c(\theta-\pi)} + \frac{k^{3/2}v}{1 + \xi kr\theta}, \quad (\text{A.28})$$

where  $\xi = \eta v / 8\sqrt{k}$ .

The radion potential coming from the GW action with a cubic interaction in the bulk is given by

$$V_{GW}(r) = \int_0^\pi d\theta \frac{1}{r} e^{-4kr\theta} \left( \partial_\theta \Phi \partial_\theta \Phi + \frac{r^2 \eta}{3} \Phi^3 \right) + e^{-4kr\pi} 2\alpha k^{5/2} \Phi(\pi). \quad (\text{A.29})$$

We now plug the solution from Eq. (A.28) into Eq. (A.29) to get an explicit form of the radion potential. Including the gravitational contribution and working to leading order in  $v$  we get

$$V(\varphi) = k^4 \left( \frac{\varphi}{F} \right)^4 \left[ \tau + \frac{1}{1 - \xi \log(\varphi/F)} \left( 2\alpha v + \frac{\alpha^2 \xi}{8} \right) \right]. \quad (\text{A.30})$$

We define  $w = 2\alpha v + \alpha^2 \xi / 8$ . The first term in  $w$  comes from the brane potential while the second term comes from the bulk, and we note that both are small because they are proportional to  $v$ .

The potential is minimized at  $\langle \varphi \rangle = f$  as

$$\tau + \frac{w}{1 - \xi \log(f/F)} = 0. \quad (\text{A.31})$$

The mass squared at the minimum is given by

$$m_\varphi^2 = \frac{\tau^2 \xi}{6w} \frac{k^3}{M_5^3} (k e^{-kr_c \pi})^2. \quad (\text{A.32})$$

At the NDA values of the parameters (see the next section), we find that  $m_\varphi^2/m_{KK}^2 \sim \tau^2$ .

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## NDA Estimation of Parameters

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In this appendix we give a brief review of estimates using NDA [111, 112] and its application in five dimensions [113] as in this dissertation. The general idea of NDA is to estimate the size of Lagrangian parameters by assuming that quantum corrections are the same size at every loop order.

The loop factor  $\ell_D$  that comes from integrating over  $D$ -dimensional phase space is given by

$$\ell_D = 2^D \pi^{D/2} \Gamma(D/2) , \tag{B.1}$$

giving the familiar  $\ell_4 = 16\pi^2$ . The relevant number for five dimensions is  $\ell_5 = 24\pi^3$ . If there are  $N$  states in the theory which go around in loops, then each loop contribution gets multiplied by  $N$ . For our case  $N$  is small, of order a few. We can then write the  $D$  dimensional Lagrangian as follows

$$\mathcal{L}_D \sim \frac{N\Lambda^D}{\ell_D} \hat{\mathcal{L}}(\hat{\Phi}, \partial/\Lambda) , \tag{B.2}$$

where  $\hat{\Phi}$  is a field whose kinetic term is not canonically normalized,  $\Lambda$  is the cutoff of the theory, and all parameters in  $\hat{\mathcal{L}}$  are dimensionless and  $O(1)$ .

## B.1 5D Gravity

We can begin with the gravity Lagrangian of Eq. (A.1) and use the fact that the kinetic term has two derivatives to estimate the cutoff:

$$\Lambda \sim \left(\frac{\ell_5}{N}\right)^{1/3} M_5 = \left(\frac{3}{N}\right)^{1/3} 2\pi M_5, \quad (\text{B.3})$$

where  $M_5$  is the five dimensional Planck mass. From this we see that there is a regime of the effective field theory before 5D gravity becomes strongly coupled if  $N$  is not too large. Next we estimate the natural value of the cosmological constant  $\Lambda_b$

$$\Lambda_b \sim \frac{N\Lambda^5}{\ell_5} \sim \left(\frac{\ell_5}{N}\right)^{2/3} M_5^5, \quad (\text{B.4})$$

from which we can estimate the inverse of the AdS curvature  $k$ ,

$$k = \sqrt{\frac{-\Lambda_b}{24M_5^3}} \sim \frac{1}{2\sqrt{6}} \left(\frac{\ell_5}{N}\right)^{1/3} M_5 \sim \frac{2}{N^{1/3}} M_5 \sim \frac{\Lambda}{\sqrt{24}}. \quad (\text{B.5})$$

From this we see that the AdS curvature scale is only separated by a factor of a few from the cutoff  $\Lambda$  and the higher dimensional Planck scale  $M_5$ . This implies that if the bulk cosmological constant is of order its natural value, only a handful of KK states are present below the cutoff.

The AdS/CFT correspondence assumes the hierarchy  $M_5 \gg M_S \gg k$ . Here  $M_S$  represents the string scale, the energy scale at which string excitations enter the picture. From the CFT perspective, this corresponds to requiring that  $N_c \gg 1$  and

$g^2 N_c \gg 1$ , where  $N_c$  is the number of colors and  $g$  the coupling constant in the dual gauge theory. The fact that  $M_5$  and  $k$  differ only by a factor of a few implies that we are not actually in the regime where  $N_c$  and  $g^2 N_c$  are large. This implies that the results we obtain using our NDA estimates can only be taken as a very rough guide, since the correspondence is being pushed to the edge of its domain of validity.

We can also use NDA to estimate the natural values of the four dimensional cosmological constants, which in this case are the brane tensions:

$$T_h \sim T_v \sim \frac{N\Lambda^4}{\ell_4} \sim \frac{\ell_5^{4/3}}{\ell_4 N^{1/3}} M_5^4 . \quad (\text{B.6})$$

These parameters are restricted to a four dimensional brane, so it is the four dimensional loop factor which goes into the estimate.

## B.2 GW Scalar

The next step is to estimate the values of the parameters in the potential of the GW field,  $\Phi$ . We begin with the bulk parameters defined in Eq. (3.13). In order to get from the Lagrangian in Eq. (B.2), to one with field that have canonical kinetic terms, we have to rescale  $\Phi$  by  $\sqrt{\ell_5/N\Lambda^3}$ . Therefore, for a mass term  $m^2\Phi^2/2$ , the natural value of the mass is given by

$$m^2 \sim \Lambda^2 \sim 24k^2 . \quad (\text{B.7})$$

In order to generate a large hierarchy, the mass parameter is taken to lie significantly below its NDA value. The natural value of the cubic interaction is also easily

obtained as

$$\eta \sim \sqrt{\frac{\ell_5 \Lambda}{N}} \sim 24^{1/4} \sqrt{\frac{\ell_5 k}{N}} \sim 60 \sqrt{\frac{k}{N}}. \quad (\text{B.8})$$

We also want to compute the natural values of the parameters  $\alpha$  and  $v$ . The visible brane potential for the GW field is given by

$$V_v = \delta(\theta - \pi) 2k^{5/2} \alpha \Phi. \quad (\text{B.9})$$

Using the NDA prescription, we estimate the size of this term to be

$$2\alpha k^{5/2} \sim \frac{\sqrt{N} \ell_5}{\ell_4} \Lambda^{5/2} \sim \frac{\ell_5^{4/3}}{\ell_4 N^{1/3}} M_5^{5/2} \quad (\text{B.10})$$

$$\alpha \sim 2^{11/4} 3^{5/4} \frac{\sqrt{N} \ell_5}{\ell_4} \sim 3 \left( \frac{54}{\pi^2} \right)^{1/4} \sqrt{N} \sim 5\sqrt{N}. \quad (\text{B.11})$$

As discussed in section 3.1, we only require that  $\Phi(\theta = 0) = k^{3/2}v$ , but leave the potential unspecified. One potential that can generate this boundary condition is

$$V_h = \delta(\theta) \lambda (\Phi^2 - k^3 v^2)^2, \quad (\text{B.12})$$

from which we estimate the natural value of  $v$  as

$$v \sim 24^{3/4} \left( \frac{N}{\ell_5} \right)^{1/2} = \frac{24^{1/4}}{\pi^{3/2}} \sqrt{N} \sim 0.4\sqrt{N}. \quad (\text{B.13})$$

Using other possible potentials such as  $\lambda(\Phi - k^{3/2}v)^2$  give the same estimate for  $v$ .

In order to generate a hierarchy we take  $v$  to lie below its NDA value.

### B.3 Radion Potential

The radion parameter  $\tau$  associated with the quartic is defined below Eq. (3.21)

and can be estimated as follows:

$$\tau = \frac{1}{k^4} \left( T_v - \frac{\Lambda_b}{k} - \frac{k^4 \alpha^2}{4} \right) \sim 72 N \left( \frac{8}{\ell_4} - \frac{16\sqrt{6}}{\ell_5} - \frac{\sqrt{6} \ell_5}{\ell_4^2} \right) \sim 5 N. \quad (\text{B.14})$$



The three contributions we show are roughly the same size. Since this is just an estimate, there are  $O(1)$  coefficients on each term, so we assume there is no cancellation and that total size of  $\tau$  is the size of each individual term.

We have computed the mass of the radion in the regime where the OR Eq. (3.14) is dominated by  $m^2$  and by  $\eta$  in Eqs. (3.23) and (3.34) respectively:

$$\frac{m_\varphi^2}{(ke^{-k\pi r_c})^2} = \begin{cases} -\frac{\epsilon\tau}{6} \frac{k^3}{M_5^3} & \text{if } m^2 \text{ dominates} \\ \frac{\tau^2\xi}{6w} \frac{k^3}{M_5^3} \sim 10 & \text{if } \eta \text{ dominates.} \end{cases} \quad (\text{B.15})$$

These formulae require that the mass term be well below its natural value, so we only give an NDA estimate for the second case in which the radion mass does not depend on  $\epsilon$ . As the mass of the KK gravitons is typically  $\sim 3$  times larger than  $ke^{-k\pi r_c}$ , we see that in the cubic case NDA gives us the expected result that the radion mass is roughly equal to the KK scale.

## B.4 Radion Couplings to SM

Finally we estimate the coupling of the GW field to other SM fields on the visible brane. These couplings take the form

$$(1 + \alpha_{\text{int}}\Phi/k^{3/2}) \cdot \mathcal{O}_{\text{SM}} . \quad (\text{B.16})$$

We can estimate the size of  $\alpha_{\text{int}}$  using NDA and assuming that the operator  $\mathcal{O}_{\text{SM}}$  has already been normalized, so we just need to rescale  $\Phi$  to get a canonical kinetic term:

$$\alpha_{\text{int}}/k^{3/2} \sim \ell_5^{1/2}/N^{1/2}\Lambda^{3/2} \quad (\text{B.17})$$

$$\alpha_{\text{int}} \sim \frac{1}{24^{3/4}} \sqrt{\frac{\ell_5}{N}} \sim \frac{1}{\sqrt{N}} \left( \frac{\pi^6}{24} \right)^{1/4} \sim \frac{2.5}{\sqrt{N}}. \quad (\text{B.18})$$

We will take these parameters to be their NDA sizes, but since they will always multiply  $\Phi/k^{3/2}$ , we will end up only working to first order in them.

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## Bibliography

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- [1] S.L. Glashow. Partial Symmetries of Weak Interactions. *Nucl.Phys.*, 22:579–588, 1961.
- [2] Steven Weinberg. A Model of Leptons. *Phys.Rev.Lett.*, 19:1264–1266, 1967.
- [3] John F. Donoghue. The effective field theory treatment of quantum gravity. *AIP Conf.Proc.*, 1483:73–94, 2012.
- [4] Leonard Susskind. Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory. *Phys.Rev.*, D20:2619–2625, 1979.
- [5] (ed.) 't Hooft, Gerard, (ed.) Itzykson, C., (ed.) Jaffe, A., (ed.) Lehmann, H., (ed.) Mitter, P.K., et al. Recent Developments in Gauge Theories. Proceedings, Nato Advanced Study Institute, Cargese, France, August 26 - September 8, 1979. *NATO Adv.Study Inst.Ser.B Phys.*, 59:pp.1–438, 1980.
- [6] Markus A. Luty and Takemichi Okui. Conformal technicolor. *JHEP*, 0609:070, 2006.
- [7] Abdus Salam and J.A. Strathdee. Nonlinear realizations. 2. Conformal symmetry. *Phys.Rev.*, 184:1760–1768, 1969.
- [8] C.J. Isham, Abdus Salam, and J.A. Strathdee. Nonlinear realizations of space-time symmetries. Scalar and tensor gravity. *Annals Phys.*, 62:98–119, 1971.
- [9] B. Zumino. Effective Lagrangians and broken symmetries. 1970.
- [10] John R. Ellis. Aspects of conformal symmetry and chirality. *Nucl.Phys.*, B22:478–492, 1970.
- [11] Zackaria Chacko and Rashmish K. Mishra. Effective Theory of a Light Dilaton. 2012.

- [12] Brando Bellazzini, Csaba Csaki, Jay Hubisz, Javi Serra, and John Terning. A Higgslike Dilaton. 2012.
- [13] B. Bellazzini, C. Csaki, J. Hubisz, J. Serra, and J. Terning. A Naturally Light Dilaton and a Small Cosmological Constant. 2013.
- [14] Francesco Coradeschi, Paolo Lodone, Duccio Pappadopulo, Riccardo Rattazzi, and Lorenzo Vitale. A naturally light dilaton. 2013.
- [15] Juan Martin Maldacena. The Large N limit of superconformal field theories and supergravity. *Adv.Theor.Math.Phys.*, 2:231–252, 1998.
- [16] Edward Witten. Anti-de Sitter space and holography. *Adv.Theor.Math.Phys.*, 2:253–291, 1998.
- [17] S.S. Gubser, Igor R. Klebanov, and Alexander M. Polyakov. Gauge theory correlators from noncritical string theory. *Phys.Lett.*, B428:105–114, 1998.
- [18] Igor R. Klebanov and Edward Witten. AdS / CFT correspondence and symmetry breaking. *Nucl.Phys.*, B556:89–114, 1999.
- [19] Lisa Randall and Raman Sundrum. A Large mass hierarchy from a small extra dimension. *Phys.Rev.Lett.*, 83:3370–3373, 1999.
- [20] R. Rattazzi and A. Zaffaroni. Comments on the holographic picture of the Randall-Sundrum model. *JHEP*, 0104:021, 2001.
- [21] Zackaria Chacko, Rashmish K. Mishra, and Daniel Stolarski. Dynamics of a Stabilized Radion and Duality. 2013.
- [22] Peter W. Higgs. Broken Symmetries and the Masses of Gauge Bosons. *Phys.Rev.Lett.*, 13:508–509, 1964.
- [23] F. Englert and R. Brout. Broken Symmetry and the Mass of Gauge Vector Mesons. *Phys.Rev.Lett.*, 13:321–323, 1964.
- [24] S.L. Glashow, J. Iliopoulos, and L. Maiani. Weak Interactions with Lepton-Hadron Symmetry. *Phys.Rev.*, D2:1285–1292, 1970.
- [25] Serguei Chatrchyan et al. Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. *Phys.Lett.*, B716:30–61, 2012.
- [26] Georges Aad et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys.Lett.*, B716:1–29, 2012.
- [27] G. Mack. All Unitary Ray Representations of the Conformal Group SU(2,2) with Positive Energy. *Commun.Math.Phys.*, 55:1, 1977.

- [28] Jr. Callan, Curtis G., Sidney R. Coleman, and Roman Jackiw. A New improved energy - momentum tensor. *Annals Phys.*, 59:42–73, 1970.
- [29] Sidney Coleman. *Aspects of Symmetry*. Cambridge University Press, 1985.
- [30] Ian Low and Aneesh V. Manohar. Spontaneously broken space-time symmetries and Goldstone’s theorem. *Phys.Rev.Lett.*, 88:101602, 2002.
- [31] Raman Sundrum. *From Fixed Points to the Fifth Dimension*. 2011.
- [32] Nima Arkani-Hamed, Massimo Porrati, and Lisa Randall. Holography and phenomenology. *JHEP*, 0108:017, 2001.
- [33] Steven Weinberg. Implications of Dynamical Symmetry Breaking. *Phys.Rev.*, D13:974–996, 1976.
- [34] Steven Weinberg. Implications of Dynamical Symmetry Breaking: An Addendum. *Phys.Rev.*, D19:1277–1280, 1979.
- [35] Estia Eichten and Kenneth D. Lane. Dynamical Breaking of Weak Interaction Symmetries. *Phys.Lett.*, B90:125–130, 1980.
- [36] Savas Dimopoulos and Leonard Susskind. Mass Without Scalars. *Nucl.Phys.*, B155:237–252, 1979.
- [37] Bob Holdom. Technicolor. *Phys.Lett.*, B150:301, 1985.
- [38] Thomas W. Appelquist, Dimitra Karabali, and L.C.R. Wijewardhana. Chiral Hierarchies and the Flavor Changing Neutral Current Problem in Technicolor. *Phys.Rev.Lett.*, 57:957, 1986.
- [39] Koichi Yamawaki, Masako Bando, and Ken-iti Matumoto. Scale Invariant Technicolor Model and a Technidilaton. *Phys.Rev.Lett.*, 56:1335, 1986.
- [40] Thomas Appelquist and L.C.R. Wijewardhana. Chiral Hierarchies and Chiral Perturbations in Technicolor. *Phys.Rev.*, D35:774, 1987.
- [41] F.J. Dyson. The S matrix in quantum electrodynamics. *Phys.Rev.*, 75:1736–1755, 1949.
- [42] Andrew G. Cohen and Howard Georgi. WALKING BEYOND THE RAINBOW. *Nucl.Phys.*, B314:7, 1989.
- [43] Christopher T. Hill and Elizabeth H. Simmons. Strong dynamics and electroweak symmetry breaking. *Phys.Rept.*, 381:235–402, 2003.
- [44] Riccardo Rattazzi, Vyacheslav S. Rychkov, Erik Tonni, and Alessandro Vichi. Bounding scalar operator dimensions in 4D CFT. *JHEP*, 0812:031, 2008.
- [45] Vyacheslav S. Rychkov and Alessandro Vichi. Universal Constraints on Conformal Operator Dimensions. *Phys.Rev.*, D80:045006, 2009.

- [46] David Poland and David Simmons-Duffin. Bounds on 4D Conformal and Superconformal Field Theories. *JHEP*, 1105:017, 2011.
- [47] Riccardo Rattazzi, Slava Rychkov, and Alessandro Vichi. Bounds in 4D Conformal Field Theories with Global Symmetry. *J.Phys.*, A44:035402, 2011.
- [48] David Poland, David Simmons-Duffin, and Alessandro Vichi. Carving Out the Space of 4D CFTs. *JHEP*, 1205:110, 2012.
- [49] Alessandro Vichi. Improved bounds for CFT's with global symmetries. *JHEP*, 1201:162, 2012.
- [50] Howard Georgi and A. Pais. Calculability and Naturalness in Gauge Theories. *Phys.Rev.*, D10:539, 1974.
- [51] David B. Kaplan and Howard Georgi.  $SU(2) \times U(1)$  Breaking by Vacuum Misalignment. *Phys.Lett.*, B136:183, 1984.
- [52] David B. Kaplan, Howard Georgi, and Savas Dimopoulos. Composite Higgs Scalars. *Phys.Lett.*, B136:187, 1984.
- [53] Nima Arkani-Hamed, Andrew G. Cohen, and Howard Georgi. Electroweak symmetry breaking from dimensional deconstruction. *Phys.Lett.*, B513:232–240, 2001.
- [54] N. Arkani-Hamed, A.G. Cohen, E. Katz, and A.E. Nelson. The Littlest Higgs. *JHEP*, 0207:034, 2002.
- [55] Thomas Gregoire and Jay G. Wacker. Mooses, topology and Higgs. *JHEP*, 0208:019, 2002.
- [56] Z. Chacko, Hock-Seng Goh, and Roni Harnik. The Twin Higgs: Natural electroweak breaking from mirror symmetry. *Phys.Rev.Lett.*, 96:231802, 2006.
- [57] Z. Chacko, Hock-Seng Goh, and Roni Harnik. A Twin Higgs model from left-right symmetry. *JHEP*, 0601:108, 2006.
- [58] David B. Kaplan. Flavor at SSC energies: A New mechanism for dynamically generated fermion masses. *Nucl.Phys.*, B365:259–278, 1991.
- [59] Walter D. Goldberger, Benjamin Grinstein, and Witold Skiba. Distinguishing the Higgs boson from the dilaton at the Large Hadron Collider. *Phys.Rev.Lett.*, 100:111802, 2008.
- [60] Luca Vecchi. Phenomenology of a light scalar: the dilaton. *Phys.Rev.*, D82:076009, 2010.
- [61] Kingman Cheung and Tzu-Chiang Yuan. Could the excess seen at 124-126 GeV be due to the Randall-Sundrum Radion? *Phys.Rev.Lett.*, 108:141602, 2012.

- [62] Shinya Matsuzaki and Koichi Yamawaki. Techni-dilaton at 125 GeV. *Phys.Rev.*, D85:095020, 2012.
- [63] Bohdan Grzadkowski, John F. Gunion, and Manuel Toharia. Higgs-Radion interpretation of the LHC data? *Phys.Lett.*, B712:70–80, 2012.
- [64] Daniel Elander and Maurizio Piai. The decay constant of the holographic techni-dilaton and the 125 GeV boson. *Nucl.Phys.*, B867:779809, 2013.
- [65] Zackaria Chacko, Roberto Franceschini, and Rashmish K. Mishra. Resonance at 125 GeV: Higgs or Dilaton/Radion? 2012.
- [66] Kaustubh Agashe, Hooman Davoudiasl, Gilad Perez, and Amarjit Soni. Warped Gravitons at the LHC and Beyond. *Phys.Rev.*, D76:036006, 2007.
- [67] Walter D. Goldberger and Mark B. Wise. Modulus stabilization with bulk fields. *Phys.Rev.Lett.*, 83:4922–4925, 1999.
- [68] Tony Gherghetta and Alex Pomarol. Bulk fields and supersymmetry in a slice of AdS. *Nucl.Phys.*, B586:141–162, 2000.
- [69] Yuval Grossman and Matthias Neubert. Neutrino masses and mixings in nonfactorizable geometry. *Phys.Lett.*, B474:361–371, 2000.
- [70] Kaustubh Agashe, Takemichi Okui, and Raman Sundrum. A Common Origin for Neutrino Anarchy and Charged Hierarchies. *Phys.Rev.Lett.*, 102:101801, 2009.
- [71] Kaustubh Agashe, Gilad Perez, and Amarjit Soni. B-factory signals for a warped extra dimension. *Phys.Rev.Lett.*, 93:201804, 2004.
- [72] Kaustubh Agashe, Gilad Perez, and Amarjit Soni. Flavor structure of warped extra dimension models. *Phys.Rev.*, D71:016002, 2005.
- [73] Csaba Csaki, Christophe Grojean, Luigi Pilo, and John Terning. Towards a realistic model of Higgsless electroweak symmetry breaking. *Phys.Rev.Lett.*, 92:101802, 2004.
- [74] Roberto Contino, Yasunori Nomura, and Alex Pomarol. Higgs as a holographic pseudoGoldstone boson. *Nucl.Phys.*, B671:148–174, 2003.
- [75] Joseph Polchinski. SCALE AND CONFORMAL INVARIANCE IN QUANTUM FIELD THEORY. *Nucl.Phys.*, B303:226, 1988.
- [76] Jean-Francois Fortin, Benjamin Grinstein, and Andreas Stergiou. Scale without Conformal Invariance: An Example. *Phys.Lett.*, B704:74–80, 2011.
- [77] Markus A. Luty, Joseph Polchinski, and Riccardo Rattazzi. The  $a$ -theorem and the Asymptotics of 4D Quantum Field Theory. 2012.

- [78] Yu Nakayama. Supercurrent, Supervirial and Superimprovement. 2012.
- [79] B. Zumino. Lectures on elementary particles and quantum field theory. In S. Deser, editor, *1970 Brandeis Summer School*. MIT Press, 1970.
- [80] Raman Sundrum. Gravity’s scalar cousin. 2003.
- [81] Csaba Csaki, Michael Graesser, Lisa Randall, and John Terning. Cosmology of brane models with radion stabilization. *Phys.Rev.*, D62:045015, 2000.
- [82] Gian F. Giudice, Riccardo Rattazzi, and James D. Wells. Graviscalars from higher dimensional metrics and curvature Higgs mixing. *Nucl.Phys.*, B595:250–276, 2001.
- [83] Csaba Csaki, Michael L. Graesser, and Graham D. Kribs. Radion dynamics and electroweak physics. *Phys.Rev.*, D63:065002, 2001.
- [84] JiJi Fan, Walter D. Goldberger, Andreas Ross, and Witold Skiba. Standard Model couplings and collider signatures of a light scalar. *Phys.Rev.*, D79:035017, 2009.
- [85] Vernon Barger, Muneyuki Ishida, and Wai-Yee Keung. Dilaton at the LHC. *Phys.Rev.*, D85:015024, 2012.
- [86] Baradhwaj Coleppa, Thomas Gregoire, and Heather E. Logan. Dilaton constraints and LHC prospects. *Phys.Rev.*, D85:055001, 2012.
- [87] Bruce A. Campbell, John Ellis, and Keith A. Olive. Phenomenology and Cosmology of an Electroweak Pseudo-Dilaton and Electroweak Baryons. *JHEP*, 1203:026, 2012.
- [88] Thomas G. Rizzo. Radion couplings to bulk fields in the Randall-Sundrum model. *JHEP*, 0206:056, 2002.
- [89] Csaba Csaki, Jay Hubisz, and Seung J. Lee. Radion phenomenology in realistic warped space models. *Phys.Rev.*, D76:125015, 2007.
- [90] Kaustubh Agashe, Antonio Delgado, Michael J. May, and Raman Sundrum. RS1, custodial isospin and precision tests. *JHEP*, 0308:050, 2003.
- [91] Kaustubh Agashe, Roberto Contino, Leandro Da Rold, and Alex Pomarol. A Custodial symmetry for Zb anti-b. *Phys.Lett.*, B641:62–66, 2006.
- [92] Puneet Batra and Z. Chacko. Symmetry Breaking Patterns for the Little Higgs from Strong Dynamics. *Phys.Rev.*, D77:055015, 2008.
- [93] Christos Charmousis, Ruth Gregory, and V.A. Rubakov. Wave function of the radion in a brane world. *Phys.Rev.*, D62:067505, 2000.



- [94] Markus A. Luty and Raman Sundrum. Hierarchy stabilization in warped supersymmetry. *Phys.Rev.*, D64:065012, 2001.
- [95] Peter Breitenlohner and Daniel Z. Freedman. Stability in Gauged Extended Supergravity. *Annals Phys.*, 144:249, 1982.
- [96] Carl M. Bender and Steven A. Orszag. *Advanced Mathematical methods for Scientists and Engineers*. McGraw-Hill Publishing Company, 1978.
- [97] Thomas Konstandin, Germano Nardini, and Mariano Quiros. Gravitational Backreaction Effects on the Holographic Phase Transition. *Phys.Rev.*, D82:083513, 2010.
- [98] Yochay Eshel, Seung J. Lee, Gilad Perez, and Yotam Soreq. Shining Flavor and Radion Phenomenology in Warped Extra Dimension. *JHEP*, 1110:015, 2011.
- [99] Walter D. Goldberger and Mark B. Wise. Phenomenology of a stabilized modulus. *Phys.Lett.*, B475:275–279, 2000.
- [100] Howard Georgi and A. Pais. Vacuum Symmetry and the PseudoGoldstone Phenomenon. *Phys.Rev.*, D12:508, 1975.
- [101] Howard Georgi and David B. Kaplan. Composite Higgs and Custodial SU(2). *Phys.Lett.*, B145:216, 1984.
- [102] Christopher Arzt. Reduced effective Lagrangians. *Phys.Lett.*, B342:189–195, 1995.
- [103] Howard Georgi. On-shell effective field theory. *Nucl.Phys.*, B361:339–350, 1991.
- [104] M. Porrati and A. Starinets. RG fixed points in supergravity duals of 4-D field theory and asymptotically AdS spaces. *Phys.Lett.*, B454:77–83, 1999.
- [105] Vijay Balasubramanian and Per Kraus. Space-time and the holographic renormalization group. *Phys.Rev.Lett.*, 83:3605–3608, 1999.
- [106] Herman L. Verlinde. Holography and compactification. *Nucl.Phys.*, B580:264–274, 2000.
- [107] Jan de Boer, Erik P. Verlinde, and Herman L. Verlinde. On the holographic renormalization group. *JHEP*, 0008:003, 2000.
- [108] Erik P. Verlinde and Herman L. Verlinde. RG flow, gravity and the cosmological constant. *JHEP*, 0005:034, 2000.
- [109] Vijay Balasubramanian, Eric G. Gimon, and Djordje Minic. Consistency conditions for holographic duality. *JHEP*, 0005:014, 2000.

- [110] Sebastian de Haro, Sergey N. Solodukhin, and Kostas Skenderis. Holographic reconstruction of space-time and renormalization in the AdS / CFT correspondence. *Commun.Math.Phys.*, 217:595–622, 2001.
- [111] Aneesh Manohar and Howard Georgi. Chiral Quarks and the Nonrelativistic Quark Model. *Nucl.Phys.*, B234:189, 1984.
- [112] Howard Georgi and Lisa Randall. Flavor Conserving CP Violation in Invisible Axion Models. *Nucl.Phys.*, B276:241, 1986.
- [113] Z. Chacko, Markus A. Luty, and Eduardo Ponton. Massive higher dimensional gauge fields as messengers of supersymmetry breaking. *JHEP*, 0007:036, 2000.