This dissertation explores the connection between the structure of labor markets and business cycle dynamics, with a focus on informality. The first chapter summarizes the main contributions of the dissertation. Institutional quality is one of the most important determinants of cross-country differences in informality.

The second chapter analyzes the link between institutions, the size of the informal sector, and aggregate volatility. I build a business cycle search and matching model with informal labor markets that captures the positive connection between informal sector size and consumption and investment volatility in the data. In addition, I show that the root cause of changes in the size of the informal sector matters for establishing the relationship between (1) informality and long-run macroeconomic outcomes and (2) informality and aggregate volatility. For the same change in informal sector size, changes in different parameters of institutional quality in the model have contrasting quantitative implications for the steady state and the volatility of unemployment in the economy. These results highlight the importance
of identifying the specific source behind changes in the size of the informal sector to characterize the link between informality and business cycle dynamics.

The third chapter explores the connection between the share of self-employment in the economy and the pace of economic recoveries. Self-employment comprises an important share of employment in many countries. Recent studies document that self-employment expands during downturns, a fact that arises from higher transition rates out of unemployment and into self-employment in recessions. Furthermore, countries with higher self-employment shares exhibit lower output persistence over the business cycle. I build a novel business cycle model with frictional labor markets where individuals can be self-employed or employed in salaried firms. I show that economies with larger self-employment shares exhibit faster recoveries following a negative economy-wide productivity shock. Differences in the ease of entry into self-employment as the economy recovers play a key role in explaining contrasting labor market and output dynamics. The model successfully captures some of the key cyclical patterns of self-employment absent in existing models, as well as the quantitative relationship between self-employment and cyclical output persistence in the data.
THE BUSINESS CYCLE CONSEQUENCES
OF INFORMAL LABOR MARKETS

by

Alan Finkelstein Shapiro

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<tr>
<td>DSGE</td>
<td>Dynamic Stochastic General Equilibrium</td>
</tr>
<tr>
<td>ENEU</td>
<td>Encuesta Nacional de Empleo Urbano</td>
</tr>
<tr>
<td>ENAMIN</td>
<td>Encuesta Nacional de Micronegocios</td>
</tr>
<tr>
<td>GDP</td>
<td>Gross Domestic Product</td>
</tr>
<tr>
<td>IDB</td>
<td>Inter-American Development Bank</td>
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<tr>
<td>ILO</td>
<td>International Labor Organization</td>
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<tr>
<td>OECD</td>
<td>Organization for Economic Cooperation and Development</td>
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<tr>
<td>SE</td>
<td>Self-Employment</td>
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<tr>
<td>TFP</td>
<td>Total Factor Productivity</td>
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<td>WB</td>
<td>World Bank</td>
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Chapter 1

Introduction

Labor markets play a fundamental role in the economic restructuring process over the business cycle. A comparison of labor market structures across economies unveils a high degree of cross-country heterogeneity in employment arrangements. Developed economies tend to have large salaried employment shares (often protected by social security and labor laws) as well as small self-employment shares in total employment. Conversely, developing countries exhibit the opposite pattern: a substantial portion of salaried workers have no access to social security and coverage by employment laws, and a large proportion of individuals run owner-only firms as self-employed workers. Understanding how labor market structures differ and how they can alter the dynamic behavior of the economy can yield important insights into the observed disparities in macroeconomic performance across economies. Also, delving deeper into the structural differences of labor markets and their implications for short-run and long-run economic activity can help us devise the best policy measures to respond adequately to adverse shocks to the economy.

This dissertation analyzes the link between two related themes in the business cycle literature – aggregate volatility and cyclical persistence – and the structure of labor markets, with an emphasis on informality. I follow the International Labor Organization (ILO) and consider informal employment as embodying two main
groups: employed workers who lack a formal employment contract (not registered with social security by their employer); and the self-employed. I also label firms that do not pay payroll taxes or comply with labor regulations (and hence do not register their workers) as informal (Bacchetta et al., 2009). Both informal salaried workers and the self-employed are more vulnerable to increased turnover and to higher separation rates, unemployment risk, and earnings volatility, which in turn affect labor flows. From a macroeconomic perspective, informal labor flows can play an important role in driving the resource reallocation process over the business cycle and hence determining how economies respond to shocks. By circumventing burdensome regulations and contributing to a flexible sector, firms and workers in the informal sector may lower the costs of adjustment that would otherwise prevent a swift adjustment of the economy in response to shocks. However, cyclical labor flows between the different employment states also have the potential to exacerbate short-run fluctuations in economic activity by modifying the incentives that affect firms’ and individuals’ decisions. Surprisingly few studies have explored the consequences of informality for short-run economic performance. One of the main objectives of this dissertation is to shed light on how differences in the structure of labor markets can influence short-run economic activity using quantitative business cycle models with frictional labor markets.

A large part of the empirical literature has focused on the determinants of informality as well as the relationship between the informal sector and long-run macroeconomic outcomes. For example, a series of cross-country studies have docu-
mented a positive relationship between employment regulations and the size of the informal sector and a negative one between institutional quality and informality (Djankov et al., 2002; Botero et al., 2004; Loayza, Oviedo, and Servén, 2005; Johnson, Kaufman, and Zoido-Lobatón, 1999; and Perry et al., 2007). Others have found a positive link between the size of the informal sector and unemployment, and a negative link between informality and the level of development (Djankov and Ramalho, 2009; Loayza and Rigolini, 2011). In general, the literature has highlighted a number of plausible determinants of informality, ranging from the regulatory framework and the tax structure to the quality of governance and institutions in the economy, with the latter being particularly relevant for explaining cross-country differences in the size of the informal sector. The empirical literature on informality and business cycles is much less developed, with Ferreira-Tiryaki (2008) being one of the first to document a positive causal relationship between the size of the informal sector and the volatility of consumption, investment, and output.  

In “Institutions, Informal Labor Markets, and Business Cycle Volatility,” I explore whether the underlying source of cross-country differences in the size of the informal sector is important for characterizing the link between informality and business cycle volatility. I build a parsimonious business cycle model with labor market frictions and informal salaried employment that can shed light on the connection between the economy’s institutional foundations, the structure of labor markets, and volatilities.  

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Loayza, Oviedo, and Servén (2004) find that more rigid labor regulations lead to higher variability in the cyclical component of real GDP per capita. Barseghyan and DiCecio (2009) show that higher entry costs lead to greater output volatility in a sample of developed and developing countries, while property rights do not seem to have a significant impact on volatility. These papers suggest an interesting link between the determinants of informality and aggregate volatility without explicitly mentioning the informal sector.
and the business cycle. The main idea behind this paper is simple: the empirical literature on informality has highlighted institutional quality as one of the most important determinants of informal sector size. However, institutional quality can manifest itself and operate through different channels and hence alter agents’ decisions in distinct ways. While institutions themselves may have similar consequences for the size of the informal sector, the specific margin through which these institutions affect informality may not lead to a uniform relationship between the informal sector and macroeconomic aggregates such as consumption, unemployment, investment, and output.\(^2\) Addressing whether the root source of differences in informality across economies matters for the behavior of short-run and long-run economic activity is particularly challenging from an empirical standpoint since most studies make use of aggregate measures of institutions. Thus, relying on a model to address these issues is particularly useful since we can identify the channels through which changes in the economy’s underlying institutional structure operate, how they determine the composition of employment, and how economic activity is affected. I use a standard business cycle model and expand it to have search frictions in the labor market and informal salaried employment. Search frictions allow me to address the implications of the size of the informal sector for unemployment volatility, which have received much less attention in the business cycle literature on informality.

The main results of the paper are as follows. I show that the model can successfully capture the positive connection between informal sector size and consumption

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\(^2\)This idea is related to Acemoglu, Johnson, Robinson, and Thaicharoen (2003), who link the quality of institutions to the economy’s aggregate volatility and crisis incidence. The authors claim that institutional quality is an important contributor to volatility by altering the economy’s micro and macro linkages.
and investment volatility documented in the empirical literature. In addition, I find that the root cause of changes in the size of the informal sector matters for establishing the relationship between (1) informality and particular long-run macroeconomic outcomes, and (2) informality and the volatility of unemployment. In particular, for the same change in informal sector size, changes in different proxies for institutional quality in the model have contrasting quantitative implications for the economy’s steady state, the volatility of the labor market, and the volatility of output. I consider two main experiments. The first exercise considers changes in institutional quality that are purely reflected in differences in enforcement of regulations in the informal sector. In this case, stricter enforcement reduces informality and unemployment volatility, but generates a counterfactual relationship between informality and the level of output in the economy. The second experiment consists of changes in overall or economy-wide institutional quality that affect productivity in the formal sector and the likelihood of detection in the informal sector simultaneously. In this case, a general improvement in institutions reduces informality and increases output levels, in line with the data, but leads to higher unemployment volatility. In both cases, more informality leads to higher volatility in consumption and investment, but to drastically different outcomes for labor market dynamics and long-run macroeconomic aggregates across economies with the same informal sector size. The reason behind these results can be traced back to the contrasting response of the average level of unemployment to institutional quality changes. While stricter en-

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Some of the results regarding the link between informality, enforcement, and unemployment in my paper confirm the findings in Ulyssea (2010), who focuses on regulations and enforcement, informality, and welfare.
forcement in the informal sector leads to a sharp increase in unemployment, better overall institutional quality reduces unemployment in the economy. The level of unemployment determines how fluctuations in formal and informal employment affect the variability of unemployment over the business cycle.

A limited number of papers have recently explored the link between informality and aggregate volatility in a business cycle framework. Restrepo-Echavarría (2011) uses a two-sector real business cycle (RBC) model where the informal sector is modeled as home production and studies the connection between informality and consumption volatility. She stresses that having a sector whose activity is imperfectly measured leads to higher observed volatility in consumption; as measurement improves, consumption volatility should decrease. Granda-Carvajal (2012) builds a two-sector business cycle model with a multitude of shocks and explores the link between informal activity and business cycle fluctuations. She generates changes in informal sector size through differences in the probability of detection and through changes in tax rates. Neither of these papers allow for unemployment. Castillo and Montoro (2010) study how technology and demand shocks are transmitted through the informal sector in a New-Keynesian model with labor search frictions. In related work, Bridji and Charpe (2011) use an RBC model with search frictions to analyze how a dual labor market affects aggregate volatility. Both papers suggest that the presence of informal labor generate more volatile aggregate fluctuations.

A subset of the results in my work are in line with some of the particular findings in these papers. Similar to Restrepo-Echavarría (2011) and Granda-Carvajal (2012), I capture a positive link between informality and consumption volatility, as
in the data. In my model, the source of volatility in consumption and investment arises mainly from the impact of informality on formal employment, which in turn influences the steady-state marginal product of capital in the formal sector, and the sensitivity of the hiring behavior of formal firms to changes in aggregate economic conditions. The fact that the steady-state marginal productivity of capital in the formal sector and the value of employing formal workers change with the size of the informal sector affects the sensitivity of formal vacancy postings and investment in response to aggregate shocks. Since households own all firms in the economy, changes in the volatility of investment across economies translate into similar qualitative changes in the volatility of consumption. Also, in contrast to most of the literature, I explicitly address the volatility of unemployment, which turns out to depend critically on both the root cause of changes in informality and on the substitutability between formal and informal production. More importantly, I show that only focusing on the size of the informal sector and aggregate volatility without taking into account the root cause of differences in the size of the informal sector across economies will yield an incomplete picture of the aggregate consequences of changes in informality. This point is critical for several reasons. First, informality itself should not be immediately associated with higher volatility or lower levels of economic activity. Second, we must be careful not to make inferences about changes in economic activity from observed differences in informality without identifying the factors that might explain the size of the informal sector. Third, the fact that informality and particular indicators of business cycle volatility are positively related does not mean that changes in informality will have a uniform impact on
unemployment and labor market volatility. In other words, a comprehensive view of each market is warranted to make the correct inferences about the business cycle consequences of informality. These results are particularly relevant for analyzing cross-country differences in business cycle dynamics, but also for policymakers in economies that aim to implement reforms that explicitly address the size of the informal sector, or other reforms whose implementation may result in changes in the composition of employment.

While there is a rapidly expanding theoretical literature on informality and labor market policy using search frictions, the majority of existing business cycle models have ignored the role of self-employment.\footnote{Rissman (2003, 2007), Kumar and Schuetze (2007), and more recently Narita (2011) and Margolis, Navarro, and Robalino (2012) incorporate self-employment as an additional employment state in labor search models. Rasteletti (2009) and Astebro, Chen and Thompson (2010) merge occupational choice with a search and matching framework. None of these papers focus on business cycle dynamics.} In the previous chapter, I abstracted from self-employment to have a tractable environment and to convey the message from the model in a clear manner. There are two main reasons why moving beyond salaried employment and addressing the presence of self-employment is important. As previously pointed out, a large segment of total employment is composed of self-employed workers, who generally operate owner-only businesses but still account for an important share of economic activity (Perry et al., 2007; Kucera and Roncolato, 2008). Indeed, self-employment can range from 10 or 20 percent of employment in developed countries up to 70 or 80 percent in certain developing countries. More relevant to the study of business cycles, recent studies have shown that self-employment exhibits different cyclical dynamics relative to salaried
employment: self-employment tends to be countercyclical, and more surprisingly, entry into self-employment from non-employment increases during recessions (Bosch and Maloney, 2007; Loayza and Rigolini, 2011). In “Self-Employment and Business Cycle Persistence: Does the Composition of Employment Matter for Economic Recoveries?” I explore whether differences in the composition of employment between salaried work and self-employment (as opposed to within salaried employment) affect the speed of economic recoveries in the aftermath of adverse aggregate shocks.

To motivate my focus on business cycle persistence, I document a new stylized fact that shows a negative connection between the share of self-employment in the economy and the persistence of output over the business cycle for a large sample of countries. The existing literature on small firm financing documents that interfirm input credit is one of the most important sources of financing for small firms. I use this fact as a basis to introduce search-based entry frictions in the self-employment sector in a two-sector business cycle model with standard search frictions in salaried employment. Self-employment in the model is meant to capture own-account (or independent) workers, which make up the majority of the self-employed in both developed and developing countries.

Whether self-employment can have an impact on recoveries from recessions is an area that has not been explored in recent theoretical models of the labor market and the business cycle. In particular, existing business cycle models with self-employment and liquidity constraints would predict that self-employment expands during booms, contrary to what we see on average across countries. Moreover, recent search models with self-employment would also struggle to reconcile the fact
that self-employment increases during downturns and individuals require external resources to produce. From a broader perspective, the specific channels through which self-employment and cyclical persistence are connected are ex-ante unclear. Self-employment may represent a large share of the employed labor force in certain countries, but does this difference in employment composition by itself necessarily have a non-negligible impact on how aggregate production responds to shocks? In other words, while there may be a large universe of small, owner-only firms in the economy, their weight in total production may not be large enough to affect macroeconomic aggregates and their behavior over the business cycle. If this is the case, what are the mechanisms that generate a negative relationship between self-employment rates and the cyclical persistence of output across countries? A natural starting point to address this issue is to determine how self-employment affects the static and dynamic structure of the labor market, and hence the decisions of salaried firms that generally account for a large share of total output in the economy. Understanding how self-employment affects the allocation of resources in the economy can therefore help us understand how the labor market structure may influence the speed of recoveries from recessions.

The model can successfully replicate the countercyclicality of self-employment and self-employment entry in the data, even if unemployed individuals require capital to transition into self-employment. Furthermore, the model highlights three key characteristics of self-employment that generate differences in the recovery path in the model economy: (1) the role of self-employment as an alternative outside option to salaried work; (2) the frictional nature of entry into self-employment, which is al-
allowed to vary with aggregate economic conditions; and (3) the impact of fluctuations in this outside option on salaried wage dynamics, which in turn modify the incentives of firms to hire workers and invest in the salaried sector. I show that differences in the ease of entry into self-employment in the aftermath of an adverse economy-wide productivity shock play a key role in driving employment, investment, and output dynamics during the recovery phase. In particular, I find that after a negative aggregate productivity shock, economies with larger self-employment shares exhibit long-lasting salaried wage contractions. The path of wages during the recovery phase pushes firms in the salaried sector to increase investment and hiring at a faster pace, thereby creating a faster recovery in salaried employment, investment, and output in the economy. Differences in the contraction of salaried wages are driven by the cyclical response of the ease of entering self-employment and the influence of the self-employment sector on aggregate labor market conditions. Thus, even if self-employment and output in the self-employment sector expand by more during downturns in economies with lower self-employment shares (consistent with the findings in Loayza and Rigolini, 2011), it is the salaried sector that determines the recovery speed of the economy.

When taken at face value, the main results from the paper may initially suggest that more self-employment in the economy is beneficial as it leads to faster recoveries. However, taking a more comprehensive view of the model’s results makes clear that there is an important tradeoff involved: economies with more self-employment may exhibit faster recoveries, but they also experience sharper fluctuations in investment, capital usage, unemployment, output, as well as lower levels of consumption
investment, employment, and output. These results may be important to consider when devising policy interventions that may generate changes in the composition of employment in the economy.

To summarize, this dissertation makes two relevant contributions to the literature on informality and business cycles, and to the literature on search frictions in the labor market more generally. The chapter on institutions and business cycle volatility draws attention to a subtle but crucial point about the link between informality and the business cycle. It stresses that simply observing changes in the size of the informal sector is not sufficient to draw appropriate conclusions about the impact of informality on labor market volatility, aggregate fluctuations, and long-run macroeconomic outcomes. Even though my analysis is purely positive, the main conclusions of the paper have important policy implications: if policymakers aim to reduce informality, the specific way in which informality is reduced is critical for what will happen to the volatility of the labor market, and to unemployment, consumption, investment, and output. The chapter on self-employment adds to the literature on cross-country differences in labor market structures by offering a tractable way to introduce frictional and endogenous entry into self-employment in a business cycle model with labor search frictions. To the best of my knowledge, this is the first business cycle search model that explicitly accounts for self-employment in the form of own-account work as an additional employment state while simultaneously capturing the cyclical dynamics of self-employment and self-employment entry in the data. This chapter also highlights an important yet scarcely explored channel through which the composition of employment in an economy can have important
consequences for economic recoveries.
Chapter 2
Institutions, Informal Labor Markets, and Business Cycle Volatility

2.1 Introduction

The informal sector, where firms and workers produce legal, market-based goods and services but explicitly circumvent government regulations, accounts for an important share of economic activity, ranging from 8 percent of GDP in some developed countries to almost 70 percent of GDP in some developing countries. The empirical literature on informality has found that stringent regulations and weak institutional quality are relevant in explaining cross-country differences in the size of the informal sector, with institutions being particularly important.\(^1\) The existence of a large proportion of individuals and firms operating outside the official regulatory and institutional framework has important implications for worker flows and the speed of factor reallocation in response to fluctuations in aggregate economic conditions. On the one hand, the informal sector can act as a shock absorber during downturns (Bosch and Maloney, 2006; OECD, 2009b). On the other hand, informal firms can swiftly adjust their inputs in response to aggregate shocks (Boeri et al., \(^1\)Throughout the paper, I consider institutions, institutional quality, or governance broadly as embodying the effectiveness of the rules and norms, established by a country’s governing body, that underlie and support economic transactions and interactions, including but not restricted to the effectiveness and quality of the legal system, of contract enforcement, and of property rights (Worldwide Governance Indicators, 2012). See Djankov et al. (2002) and Botero et al. (2004) on the regulation of labor markets and firm entry, and Loayza, Oviedo, and Servén (2005), Johnson, Kaufman, and Zoido-Lobatón (1999) and Perry et al. (2007) for more on informality and institutions.)
2008), thereby making aggregate economic activity more volatile. This suggests that the presence of a large informal sector can have important consequences at the aggregate level (Boeri, Helmppie, and Macis, 2008). Ferreira-Tiryaki (2008) is one of the first to provide evidence suggesting that a larger informal sector is associated with higher volatility in consumption, investment, and output using a sample of developed and developing countries. The theoretical literature has followed suit by analyzing how the informal sector and business cycle volatility are related, but most papers have ignored whether the determinants of informality play a role in this relationship. In fact, while the quality of institutions appears to have a uniform effect on the size of the informal sector in the data, institutional quality can manifest itself in different ways, work through different channels and hence lead to contrasting effects on long-run and short-run aggregate economic activity. This implies that the relationship between informality and macroeconomic performance is not clear-cut and may depend on the root cause of informality in the economy.

In this paper, I explore whether the cross-country relationship between informality and aggregate volatility depends on the underlying cause of differences in the size of the informal sector. I focus on different manifestations of institutions (as opposed to differences in regulations or the tax structure) as the main source of differences in the size of the informal sector across economies. The aggregate effects of particular institutional differences may be difficult to disentangle in the data since

\[\text{For example, institutional quality can be reflected in the effectiveness of debt enforcement or}
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\[\text{the enforcement of regulations, in the quality of public services that benefits the private sector, or}
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\[\text{in the productivity of the economy as a whole.}
\]

\[\text{See Acemoglu and Johnson (2005) as well as Mott (2010) on the relevance of exploring the}
\]

\[\text{impact of institutions on economic outcomes at a more disaggregated level.}\]
most institutional quality measures in the empirical literature are highly correlated with each other. I use a tractable two-sector business cycle model with labor search frictions and informal salaried employment that allows me to examine the impact of different institutional proxies on informality, and in turn the impact of the informal sector on economic activity. Using the model, I find that both labor market volatility and the levels of particular macroeconomic aggregates such as output, investment, and unemployment can differ widely across economies with the same informal sector size depending on the root cause of the differences in the level of informality. My results suggest that identifying the specific causes of differences in informal sector size (and understanding how these factors affect the allocation of resources) is crucial for correctly characterizing the short-run and long-run macroeconomic consequences of informality. While a similar result concerning the link between informality and long-run outcomes was documented in Ulyssea (2010), I delve deeper by considering the relationship between the determinants of informality, the size of the informal sector, and aggregate short-run fluctuations.

The model I use is a dynamic stochastic general equilibrium (DSGE) version of the search and matching framework in Ulyssea (2010), with capital accumulation in the formal sector and an endogenous detection probability in the informal sector. There are two sectors, formal and informal, that produce intermediate goods. Formal firms have higher hiring costs and must pay payroll taxes. They accumulate capital and face search frictions to find formal workers. Informal firms only use labor, but must still spend resources to find workers. They avoid payroll taxes and have lower hiring costs, but face an endogenous detection probability that depends
on the measure of informal workers and on an exogenous enforcement quality parameter. I analyze the impact of changing two parameters that proxy for institutional quality in the model. The first parameter affects the productivity of the economy as a whole, while the second reflects the quality of enforcement of regulations in the informal sector, either through higher informal worker separation rates or a higher detection probability.

The quantitative experiments show that the model can successfully capture the negative relationship between institutional quality and informality as well as the positive link between informality and consumption and investment volatility in the data. These results hold regardless of whether variation in informality is driven by parameters affecting institutional quality in the overall economy or just in the informal sector. However, the model predicts the observed negative relationship between informality and the level of total output only if the variation in informality is driven by economy-wide institutional quality. Better enforcement of regulations in the informal sector (either through a higher separation rate for informal workers or a higher detection probability) reduces informality but generates a fall in the level of total output.\(^4\) Despite this fact, all experiments yield a negative relationship between the capital-output ratio and the size of the informal sector, although this link is driven by the counterfactual response of output when variation in the size of the informal sector is driven by the enforcement parameter. Finally, I find that the root cause of differences in the size of the informal sector plays a key role in

\(^4\)See Elgin and Öztunah (2013) for recent evidence suggesting that informality and output levels are positively related in economies with weaker institutional quality.
the behavior of unemployment volatility: a decline in informality is driven by an increase in overall institutional quality generates higher unemployment volatility, whereas a similar-sized decline in informality driven by an increase in the quality of enforcement in the informal sector leads to lower unemployment volatility.

The response of consumption and investment volatility to changes in the size of the informal sector can be attributed to the effect of institutional quality on the marginal product of capital in the formal sector, and to the value of hiring workers in the formal sector. Regardless of the root cause of informality, economies with a larger informal sector have smaller formal employment shares and a lower marginal product of capital. As a result, these economies exhibit higher volatility in formal vacancy postings and in the marginal product of capital. In response to a positive aggregate productivity shock, formal firms in economies with a larger informal sector will post vacancies and accumulate capital more aggressively to take advantage of the increase in productivity. Thus, a larger expansion in formal vacancies will be accompanied by a sharper increase in investment in response to the aggregate shock, thereby generating higher investment volatility. Finally, the increase in investment volatility leads to higher consumption volatility since the evolution of capital affects the resources available to households for consumption (Andrés, Doménech, and Fatás, 2008). Ferreira-Tiryaki (2008) suggests that a larger informal sector could lead to higher volatility since informal firms are credit-constrained and do not have access to financial markets to smooth shocks. My work shows that one can explain the link between informality and aggregate volatility without resorting to credit constraints.
The link between unemployment volatility and informal sector size depends on how the steady-state level of unemployment is affected by the underlying factors driving the level of informality. In particular, a higher informal worker separation rate (which proxies for better enforcement) generates a sharp rise in steady-state unemployment and creates a negative relationship between steady-state informality and unemployment. The higher average level of unemployment dampens the effect of fluctuations in formal and informal employment on unemployment volatility. As a result, variation in informal worker separation rates create a positive link between informality and unemployment volatility. Conversely, a rise in overall institutional quality reduces unemployment in the economy, and yields a positive connection between steady-state informality and unemployment. The fall in unemployment in turn magnifies the impact of cyclical movements in formal and informal employment on the variability of unemployment. Thus, informality and unemployment volatility are negatively related. Furthermore, the less substitutable formal and informal output are in total production, the starker the differences in unemployment volatility between these two cases. One of the contributions of the paper is to show that making inferences about the business cycle consequences of informality solely by looking at the size of the sector can lead to an incomplete assessment of the effects of informality on both short-run and long-run economic activity: changes in different parameters reflecting institutional quality that have similar impacts on informal sector size can generate drastically different outcomes for particular macroeconomic aggregates and labor market volatility. These results can have important policy implications since policies to reduce the size of the informal sector can have different
consequences for aggregate fluctuations and long-run outcomes depending on the specific approach taken to address the size of the informal sector.

The paper is organized as follows: the next section offers an overview of related literature. Section 2.3 provides a brief empirical motivation. Section 2.4 describes the model and Section 2.5 discusses the calibration strategy. Section 2.6 presents the key results that emerge from the model’s simulations and provides some discussion. Section 2.7 concludes.

2.2 Related Literature

Djankov and Ramalho (2009) provide a summary of several papers on the link between employment laws, the informal sector, and unemployment in developing countries.\(^5\) In general, more stringent employment regulations tend to be associated with a larger informal sector and higher unemployment. In related work, Loayza, Oviedo, and Servén (2005) find a positive relationship between labor market regulations and informality using a sample of developed and developing countries. They suggest that the quality of governance is important in understanding the link between regulations and informal sector size. Indeed, better institutions may allow stringent regulations and low levels of informality to coexist. In complementary research, Loayza, Oviedo, and Servén (2004) consider the connection between labor market regulations and the volatility of per capita output and find that more rigid

labor regulations lead to higher variability in the cyclical component of real GDP per capita. Acemoglu, Johnson, Robinson, and Thaicharoen (2003) provide an extensive empirical analysis that links the quality of institutions to the economy’s macroeconomic volatility and crisis incidence. They conclude that, by affecting the economy’s micro and macro linkages, the institutional foundations of the economy are one of the leading determinants of macroeconomic volatility. Similarly, Barseghyan and DiCecio (2009) take a complementary approach to Loayza et al. (2004) and document that higher entry costs lead to greater output volatility using a large sample of developed and developing countries, while property rights do not seem to have a significant impact on volatility.

The surge in empirical studies on informality and the role of employment regulations and institutional quality has been complemented by an expanding theoretical literature.\textsuperscript{6} Ulyssea (2010) proposes a two-sector search and matching model based on Acemoglu (2001) that captures various features of labor market institutions in Latin America. In particular, he studies the connection between labor regulations and informality with a focus on unemployment and welfare. His findings suggest that reducing the cost of entry into the formal sector causes a reduction in informality without having a detrimental impact on unemployment. Conversely, reducing informality through better enforcement increases unemployment and is welfare-detrimental. My model extends his framework to a dynamic stochastic general equilibrium environment, and confirms some of his findings. In particular, he

\textsuperscript{6}For some examples of search models of informality in partial equilibrium, see Zenou (2008) and Bosch and Esteban-Pretel (2009, 2012). In a DSGE framework, see Cook and Nosaka (2005) and Castillo and Montoro (2010). For the growth implications of informality in labor markets, see Satchi and Temple (2008), among many others.
documents that increasing enforcement leads to a reduction in informality, to a rise in unemployment, and a fall in welfare in his model. The two papers can be seen as complements in the sense that they address the links between regulations and institutions and informality. However, I take a broader approach and consider the impact of different manifestations of institutional quality on informality, and on the behavior of macroeconomic aggregates over the business cycle. In contrast to Ulyssea’s work, I abstract from the normative implications of informality and offer a purely positive analysis.7

Restrepo-Echavarría (2011) studies the connection between informality and consumption volatility by proposing a home production two-sector RBC closed-economy framework with an informal sector. She finds that having a large informal sector that is not well measured leads to higher observed consumption volatility, and argues that this volatility should decrease with the level of development of the economy, which is accompanied by better measurement of informal activity. One of the key elements driving changes in relative consumption volatility in her model is the substitutability between formal and informal consumption goods. My approach assumes that all data are equally (mis)measured and focuses on the root causes of informality and the behavior of consumption, investment, employment, and output in the short run. I also show that the observed relationship between

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7If I were to take a normative stand, the paper suggests that more better enforcement leads to higher consumption, investment, and output levels only if higher enforcement has a positive effect on the formal sector as well. In this case, welfare would presumably be higher, which stands in contrast to Ulyssea’s results. This discrepancy depends critically on whether changes in institutions only affect the informal sector, or whether they affect the formal sector as well. Also, note that I take the view that enforcement only affects the informal sector. If enforcement also affected the formal sector, it is plausible that better enforcement may generate an increase in informality.
informal sector size and consumption volatility can be explained without assuming mismeasurement of informal activity in the data, or differences between formal and informal consumption goods.

Granda-Carvajal (2012) presents a real business cycle model with an underground economy to explore the role of the informal sector in driving short-run aggregate fluctuations in the economy. My model is similar to hers in that I assume that labor is the only input in production in the informal sector. However, I assume that the only source of fluctuations in the economy is an aggregate productivity shock that affects both sectors. Furthermore, my emphasis is on whether different root causes of informality have different implications for long-run and short-run macroeconomic outcomes, and I also address the implications of informality for unemployment fluctuations. Granda-Carvajal (2012) shows that a higher exogenous detection probability in the informal sector reduces output and consumption volatility, and increases investment and formal labor volatility marginally. In contrast to her results, I find that higher enforcement leads to a large reduction in investment volatility.

Castillo and Montoro (2010) analyze the role of informality in the transmission of demand and technology shocks in a two-sector sticky-price DSGE model with search frictions and focus on the effects of informality on inflation dynamics. Using the structure presented in Blanchard and Gal (2006), they proxy for the cost of regulations through time-varying hiring costs, but do not explicitly analyze the economy under different regulatory regimes that have similar consequences for informality. Their findings suggest that the informal sector acts as a shock absorber
for inflation and wages in response to demand shocks. In addition, their work offers an explanation for the positive link between informality and output volatility documented in Ferreira-Tiryaki (2008). In contrast to my work, the paper focuses solely on regulations as the source of differences in informality, focuses on the behavior of inflation, and abstracts from the behavior of investment and unemployment dynamics.

Bridji and Charpe (2011) explore how a dual labor market affects the cyclical behavior of output and unemployment using a business cycle model with search frictions in the formal sector and a frictionless market for informal labor. They find that informal employment tends to expand during downturns, in line with the conventional wisdom on the cyclical behavior of informal labor markets, and that informality tends to lead to higher output volatility. They stress that search costs and the barriers to entry in the two sectors play an important role in generating differences in the cyclical behavior of employment and output. In contrast to their setup, I introduce search frictions that afflict both formal and informal employment. Furthermore, my main focus is on whether different determinants of informality that have similar implications for the size of the informal sector can have different consequences for business cycle dynamics and long-run macro outcomes.

Elgin and Öztunah (2013) study the interaction between informality, institutions, and GDP per capita. They find evidence suggesting that the size of the informal sector and the level of output are positively related in economies with poorer institutions, while countries with better institutional quality exhibit a negative link between informality and output levels. They introduce a simple general
equilibrium model that can capture this relationship. My work is similar to theirs since I also consider the effect of institutions on informal activity and on long-run macroeconomic performance. Furthermore, my model is able to capture Elgin and Öztunali’s results, though I find that weaker institutions must be reflected in weaker enforcement alone to deliver a positive link between informality and output levels. Also, my work goes a step further by exploring the consequences of informality and its determinants for aggregate volatility (including labor market volatility) as well.

Wesselbaum (2010a, 2010b) studies the importance of labor market regulations in developed OECD countries in a one-sector DSGE search and matching model with firing costs and severance payments as proxies for labor institutions. He shows that the two types of costs have different implications for labor market volatility and overall welfare, which is a result that complements the findings in Garibaldi and Violante (2002). These results are similar to mine in that two seemingly similar regulatory features can have contrasting effects on volatility. Finally, Sala, Silva, and Toledo (2008) focus on the hiring margin and study the link between fixed-term contracts and the volatility of employment in a partial-equilibrium labor search environment. They conclude that fixed-term contracts represent an important adjustment mechanism for firms, and their use leads to higher labor market volatility.
2.3 Empirical Motivation

The literature has not reached a consensus on a general definition of informality (Maloney, 2004). In this paper, I use two related definitions. The first one, based on Schneider et al. (2010), expresses informal sector size as a percent of GDP. The second definition centers on informality in the labor market, which allows me to have a clear distinction between the two sectors in the model. One key characteristic of informal labor markets is non-compliance with the regulatory and tax structures that govern firm-worker relationships in the economy.

**Definition (Informal Sector)** The informal sector refers to the sector where legal, market-based production is explicitly hidden from the government, and therefore is not subject to government regulations. The size of the informal sector is expressed as a share of official GDP (Schneider et al., 2010).

**Definition (Informal Employment)** Employed workers who lack a formal employment contract (not registered by their employer) and are therefore unprotected by their country’s employment laws are informal. Likewise, otherwise legal firms that do not pay payroll taxes or comply with labor regulations, do not register their workers, and circumvent employment laws are considered informal (Bacchetta et al., 2009).

Since part of my focus is on the enforcement labor regulations, I explicitly abstract from self-employed workers, who are generally included in the definition of informal employment. Labor market regulations, employment laws that are supported by labor market institutions refer to the rules, terms and conditions, respon-
sibilities, and rights established by the country’s governing body that determine the relationship between firms and workers in the formal labor market. For the purposes of this paper, institutional quality encompasses the degree of law and regulatory enforcement as well as the strength of institutions that support economic activity.\textsuperscript{8}

2.3.1 Determinants of Informality

The literature on informality has documented that institutional quality, enforcement, and other types of regulations play an important role in determining the size of the informal sector (see Loayza, Oviedo, and Servén, 2005, and others). In this subsection, I briefly revisit the determinants of informality. I use these results as a basis for the quantitative experiments performed with the model.\textsuperscript{9}

I use a large sample of developed and developing countries. The dependent variable is the size of the informal sector as a percent of official GDP in 2002, obtained from Schneider et al. (2010). Since informality in the model is defined in terms of employment, I focus on labor market regulations when estimating the impact of regulations on informal sector size. I use data from 2002 since the variables I use to measure labor regulations are available starting in 2002. In particular, I use two specific components of labor regulations from the Freedom of the World report, regulations on hiring and firing, instead of an aggregate measure in order to isolate

\textsuperscript{8}For example, Rule of Law captures, among other things, the extent to which individuals follow the rules and norms of society that dictate economic transactions and interactions. See the World Bank’s Worldwide Governance Indicators (WGI) for the definitions of different proxies of institutional quality.

\textsuperscript{9}Some of the stylized facts in this section have already been documented in earlier literature (see Oviedo, 2009).
some of the particular elements of the institutional and regulatory environment that may have a direct impact on the informal sector. More stringent hiring regulations and dismissal costs are reflected in lower values for these indices.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Dep. Var.:</th>
<th>IS Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>French Legal Origin</td>
<td>5.859**</td>
<td>6.973**</td>
</tr>
<tr>
<td></td>
<td>(2.650)</td>
<td>(3.089)</td>
</tr>
<tr>
<td>Law and Order (LO)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hiring Reg. (HR)</td>
<td>-1.140**</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.459)</td>
<td></td>
</tr>
<tr>
<td>Dismissal Costs (DC)</td>
<td>-0.947***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.347)</td>
<td></td>
</tr>
<tr>
<td>Labor Regulations</td>
<td>-</td>
<td>-1.189</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.919)</td>
</tr>
<tr>
<td>Log RGDP 1999</td>
<td>-1.318***</td>
<td>-1.235***</td>
</tr>
<tr>
<td></td>
<td>(0.502)</td>
<td>(0.558)</td>
</tr>
</tbody>
</table>

Note: The constant term is omitted for expositional purposes. Robust standard errors are shown in parentheses. *** denotes significance at the 1 percent level, ** denotes significance at the 5 percent level, * denotes significance at the 10 percent.

I also consider a widely used measure of overall institutional quality in the literature, mainly Law and Order from the Political Risk Services (PRS) Group’s International Country Risk Guide (ICRG). I use this variable for year 1999. I use French legal origin as a proxy for enforcement (following Botero et al., 2004) when I include law and order in the specification. Finally, I use the log of real GDP.

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10 Another common measure is Rule of Law from the World Bank’s Worldwide Governance Indicators. The results are qualitatively the same if I use Rule of Law to control for institutional quality, though the significance of hiring regulations is sensitive to the proxy of institutions used.

11 The World Bank’s Worldwide Governance Indicators include a measure of regulatory quality,
in 1999, obtained from the World Bank’s World Development Indicators (WDI) as a measure of the level of development. The main details of the variables are presented in the Appendix for expositional purposes. Table 2.1 presents the main results, summarized as follows: more stringent hiring and firing regulations, worse enforcement (proxied by legal origin), and a lower level of development are associated with more informality. Once we include the measure of overall institutional quality (Law and Order), a deterioration in institutions, more stringent hiring regulations, and a lower a level of development are associated with a larger informal sector.\textsuperscript{12}

The proxy for enforcement and dismissal costs are no longer significant in explaining informality. The results for law and order and the level of GDP have been previously documented in the literature (see Loayza, Oviedo, and Servén, 2005, and Oviedo, 2009). One difference relative to existing studies is that the overall measure of labor regulations is insignificant, while individual measures of labor regulations that capture separately the ease of hiring and the cost of firing are still important even after controlling for enforcement and the level of development.

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\textsuperscript{12}The significance of hiring regulations is not robust to alternative measures of institutional quality and hence focus on institutions in the rest of the paper.
2.3.2 Informal Sector Size and Aggregate Volatility

In this subsection, I briefly revisit the analysis of aggregate volatility and informality presented in Ferreira-Tiryaki (2008). For the purpose of comparison, I use her dataset on annual consumption, investment, and output volatility, which includes a combination of developed and developing countries, for a total of 44 countries (see Appendix A for details). I use the same measure of informality (expressed as a percent of official GDP) from Schneider et al. (2010).\textsuperscript{13} Given that informal sector size is only available for a limited number of years, I use year 2002 for the data on informal sector size, which is the same year I use for the determinants of informality.\textsuperscript{14} To determine whether the relationship between volatility and informality is robust to other likely factors that could affect aggregate volatility, I use the government-to-GDP ratio, the share of credit to the private sector as a percent of GDP, and a measure of openness (imports plus exports as a percent of GDP), which are all obtained from the World Development Indicators.

While the general results in Table 2.2 confirm Ferreira-Tiryaki’s (2008) empirical analysis in a more parsimonious regression, I find no significant relationship between informality and output volatility once I control for other factors that might affect the volatility of output. The results for output volatility are particularly sensitive to the country sample and to the set of regressors included in the analysis. By contrast, the significance of informal sector size in the investment and consumption

\textsuperscript{13}The results for consumption volatility are sensitive to the inclusion of Argentina (which is part of the original country sample and has one of the highest levels of consumption volatility in the sample). The main results exclude Argentina to give a more general picture of the connection between informality and volatility.

\textsuperscript{14}Using the average for years 1999 through 2002 does not change the main conclusions.
Table 2.2: Informal Sector Size and Aggregate Volatility

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>$\sigma_{\text{consumption}}$</th>
<th>$\sigma_{\text{investment}}$</th>
<th>$\sigma_{\text{output}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS</td>
<td>0.035**</td>
<td>0.128***</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.041)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Gov. Spending</td>
<td>-0.041</td>
<td>-0.101</td>
<td>-0.066**</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.082)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Credit Private Sector</td>
<td>-0.003</td>
<td>0.007</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.014)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Openness</td>
<td>0.004</td>
<td>0.006</td>
<td>0.002*</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.007)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.25</td>
<td>0.38</td>
<td>0.31</td>
</tr>
<tr>
<td>Observations</td>
<td>43</td>
<td>43</td>
<td>43</td>
</tr>
</tbody>
</table>

Note: The constant term is omitted for expositional purposes. Robust standard errors are shown in parentheses. *** denotes significance at the 1 percent level, ** denotes significance at the 5 percent level, * denotes significance at the 10 percent.

Volatility regressions is robust to alternative specifications that include a richer set of regressors (such as a measure of inflation to proxy for monetary policy, trend GDP growth, and exchange rate volatility). In the rest of the paper, I focus on the relationship between aggregate volatility and informal sector size, and on the link between informality and labor market volatility. In the next section, I present a model that can capture the stylized facts about the determinants of informal sector size and the connection between informality and volatility. I then use the model to analyze whether the underlying source behind the differences in the size of the informal sector matters for characterizing the link between informality, macroeconomic

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15Following Ferreira-Tiryaki (2008), I also experiment with specifications where I instrument for informal sector size to address the potential endogeneity between informality and volatility. Note that using explicit measures of institutional quality as instruments, as in Ferreira-Tiryaki (2008), gives a mixed picture for the strength and validity of the instruments under richer specifications. For the specification in Table 2.2, the qualitative results when instrumenting on informal sector size remain unchanged. The most important result to take out of the evidence in Table 2.2 is that the positive relationship between informal sector size and consumption and investment volatility is significant even after controlling for other factors that affect aggregate volatility.
aggregates, and business cycle fluctuations.

2.4 The Model

I use a two-sector business cycle model that includes search and matching frictions in formal and informal salaried employment. The structure of the labor market is similar to the partial-equilibrium search and matching framework in Ulyssea (2010) and Acemoglu (2001). In contrast to Ulyssea (2010), I introduce capital accumulation in the formal sector to analyze the behavior of investment volatility.\textsuperscript{16} Formal firms post formal vacancies and accumulate capital. They face payroll taxes and higher hiring costs relative to informal firms. Formal workers also have higher bargaining power relative to informal workers. These last three elements reflect the presence of hiring regulations in the formal sector. Informal firms post vacancies to hire workers. They do not pay payroll taxes and have lower hiring costs, but they face an endogenous probability of detection that proxies for enforcement. There is no explicit segmentation in the labor market. Following the existing theoretical literature, I combine informal self-employment and informal salaried employment.\textsuperscript{17} I also abstract from firm entry and exit as well as job-to-job flows for simplicity.

The formal and informal sectors produce intermediate inputs used in the production of an aggregate consumption good. A representative final goods firm aggregates output from the two sectors with some degree of substitutability. I initially

\textsuperscript{16}In a modified version of the model, I introduce capital accumulation in the informal sector as well. The main conclusions of the paper remain unchanged.

\textsuperscript{17}For a setup with frictional entry into self-employment in a business cycle environment, see Finkelstein Shapiro (2012).
assume that the two inputs are imperfect substitutes, in line with the evidence (Chen, 2007) and the assumptions in Ulyssea (2010) and Acemoglu (2001). Output from a frictionless final goods firm represents aggregate output in the economy.\textsuperscript{18}

This setup captures the fact that formal firms often purchase inputs and contract services from other (formal and informal) firms in order to produce output that is later sold in formal output markets.

Improvements in institutional quality in the economy can manifest themselves in different ways, ranging from higher productivity in the formal sector (which can also be coupled with better enforcement in the informal sector and hence represent an economy-wide improvement in institutions) to better enforcement that only affects the informal sector. I consider two ways of varying the quality of enforcement. First, I change the separation rate of informal workers, similar to the experiment in Ulyssea (2010), such that a higher informal separation rate reflects better enforcement of regulations. Second, I introduce an endogenous probability of detection in the informal sector that depends on the measure of informal workers and a parameter that captures exogenous changes in enforcement. In the benchmark model, I assume that this parameter can also be affected by institutional quality in the formal sector, so that changes in the latter affect both sectors and hence represent changes in overall institutional quality.\textsuperscript{19}

\textsuperscript{18}See Krause and Lubik (2007) for a similar assumption in general equilibrium. While in practice there may exist informal sector output sold directly to consumers in informal output markets, I abstract from modeling informal final goods markets separately from formal ones for simplicity. This also avoids the problem of making assumptions about the degree of substitutability between formal and informal consumption goods. Also, most of the literature either assumes that a representative firm can hire both types of workers, i.e. formal and informal, or separates workers completely by sector. The latter assumption allows me to make a clearer distinction between sectors.

\textsuperscript{19}As I show in the firms’ problems below, the way I introduce this parameter allows me to
The size of the employed labor force in each of the two sectors is determined endogenously, but there is no firm entry or exit in either sector. The size of the informal sector can be described in two equally valid ways: by the measure of workers employed in informal firms or by the ratio of informal output to final output.

The timing of the model is as follows: at the beginning of time $t$, the aggregate productivity shock hits the economy. Once firms observe aggregate productivity, exogenous separations in the two sectors occur simultaneously. After separations, the total measure of workers in the economy (and hence unemployment) is determined. Informal workers are considered to be part of total employment. Firms post vacancies in their respective sectors and formal firms make investment decisions, and both previously unemployed and recently separated individuals look for jobs randomly. Once matching takes place, real wages are determined through Nash bargaining. Production occurs and workers receive their wage payments.

2.4.1 Households

The household’s problem is standard. Utility depends on an aggregate consumption good whose price is normalized to one (see Krause and Lubik, 2007). As in Andolfatto (1996), I assume an infinitely-lived representative household of measure one, consisting of a large number of family members who supply their labor inelastically. There is perfect risk-pooling in the economy. For simplicity, there analyze the behavior of the economy when changes in institutional quality affect only the formal sector, only the informal sector, or both sectors simultaneously. If institutions only affect the formal sector, the results are similar to the case where institutions affect both sectors simultaneously.

The informal sector is often characterized as being riskier. However, empirical evidence suggests that there are (formal and informal) insurance mechanisms that partially insulate households whose members work in the informal sector from idiosyncratic shocks (see, for example, Levy, 2008).
is no labor force participation margin, so the members of the household are either employed in one of the two sectors or unemployed. The household’s problem is to choose a stream of aggregate consumption and assets \( \{c_t,a_t\}_{t=0}^{\infty} \) to

\[
\text{max}_{\{c_t,a_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} \right\} 
\]

Subject to

\[
c_t + a_t = w_{I,t}n_{I,t} + w_{F,t}n_{F,t} + \Pi_{F,t} + \Pi_{I,t} + \Pi_t + bu_t + T_t + R_t a_{t-1} \quad (2.2)
\]

The choice over assets simply makes the pricing of interest rates explicit. As is typical in these models, the household does not choose the measure of individuals that work. \( \Pi_t, \Pi_{F,t} \) and \( \Pi_{I,t} \) are profits in the final goods sector, the formal intermediate goods sector, and the informal intermediate goods sector, respectively. \( w_{F,t} \) and \( n_{F,t} \) denote the real wage and the measure of individuals working in the formal sector, respectively, and \( w_{I,t} \) and \( n_{I,t} \) represent the same variables in the informal sector.\(^{21}\)

There is no intensive margin for labor. Both wages are measured in terms of the price of the aggregate consumption good. \( b \) denotes unemployment benefits. \( T_t \) are real lump-sum transfers from the government. At any point in time, a household member can be employed in either of the two sectors or unemployed. The total

\(^{21}\)Adding a labor income tax for workers in the formal sector does not change the main conclusions of the paper.
employed labor force $n_t$ is defined as

$$n_t = n_{F,t} + n_{I,t} \quad (2.3)$$

I normalize the total labor force to one and define unemployment as $u_t = 1 - n_t$. Let $U(c_t)$ be the utility function over the aggregate consumption good. The first-order conditions yield a standard consumption-savings decision

$$1 = R_tE_t \left\{ \beta \frac{U_{c_{t+1}}}{U_{c_t}} \right\} \quad (2.4)$$

where the stochastic discount factor is defined as $\Xi_{t+1|t} \equiv E_t \left\{ \beta \frac{U_{c_{t+1}}}{U_{c_t}} \right\}$.

### 2.4.2 Production

#### 2.4.2.1 Intermediate Formal Sector Firms

Production in the formal sector depends on formal labor $n_F$, formal capital $k_F$, and aggregate productivity $z$. In addition, institutional quality affects formal sector production and enters the production function as a productivity-enhancing parameter $a_F$.\textsuperscript{22} The firm’s production function is assumed to be constant-returns-to-scale and is given by

$$y_{F,t} = z_t a_F f(n_{F,t}, k_{F,t}) \quad (2.5)$$

\textsuperscript{22}For a recent empirical investigation on the impact of institutions on productivity and efficiency, see Bhaumik, Dimova, Kumbhakar, and Sun (2012).
Introducing institutional quality in this fashion is one reduced-form way of capturing the fact that better property rights, enforcement of contracts, and compliance with the rules of society are productivity-enhancing in the formal sector.

Firms choose a sequence of formal vacancies, formal labor and capital next period \(\{v_{F,t}, n_{F,t+1}, k_{F,t+1}\}_{t=0}^{\infty}\) to

\[
\max_{\{v_{F,t}, n_{F,t+1}, k_{F,t+1}\}_{t=0}^{\infty}} \!\! E_0 \sum_{t=0}^{\infty} \xi_t \left\{ p_{F,t} y_{F,t} - w_{F,t} n_{F,t}(1 + \tau^p) - \psi_F v_{F,t} - i_{F,t} \right\} \tag{2.6}
\]

subject to

\[
n_{F,t+1} = (1 - \delta^F)(n_{F,t} + v_{F,t} q(\theta_{F,t})) \tag{2.7}
\]

and

\[
k_{F,t+1} = (1 - \delta)k_{F,t} + i_{F,t} \tag{2.8}
\]

where the price of intermediate formal sector output relative to final output is given by \(p_{F,t}\), the separation rate for formal workers is \(\delta^F\), and the depreciation rate of capital is \(\delta\). Firms in the formal sector pay payroll taxes \(\tau^p\). \(q(\theta_F)\) stands for the job-filling rate, where \(\theta_F\) is labor market tightness in the formal sector. \(\psi_F\) is the formal hiring cost, which I assume reflects the cost of hiring regulations.\(^{23}\) Taking first-order conditions yields a job creation condition along with a standard Euler equation for capital:

\[
\frac{\psi^F}{q(\theta_{F,t})} = (1 - \delta^F)E_t \sum_{t=1}^{\infty} \left\{ p_{F,t+1} z_{t+1} a_F f_{n_F} (n_{F,t+1}, k_{F,t+1}) \right\} \tag{2.9}
\]

\(^{23}\)Introducing firing costs as well does not change the general conclusions of the paper.
and

\[ 1 = E_t \Xi_{t+1|t} \{ p_{F,t+1} a_F z_{t+1} f_k (n_{F,t+1}, k_{F,t+1}) + 1 - \delta \} \tag{2.10} \]

The job creation condition simply equates the expected marginal cost of posting a formal vacancy to the expected marginal benefit.

### 2.4.2.2 Intermediate Informal Sector Firms

Informal firms also face search frictions, but the cost of posting vacancies is lower than in the formal sector, \( \psi_F > \psi_I \) (Bosch and Esteban-Pretel, 2006). The production function for informal firms is linear in informal labor and is given by

\[ y_{I,t} = z_t f(n_{I,t}) = z_t n_{I,t}. \]

The detection probability is given by \( \rho(n_{I,t}, a_F, \iota) \), where \( \iota \) represents the exogenous level of enforcement in the informal sector, \( a_F \) represents institutional quality in the economy as a whole, and \( n_{I,t} \) is the amount of informal labor in the informal sector. I follow Choi and Thum (2005) in assuming that the detection probability is endogenous. The function \( \rho(n_{I,t}, a_F, \iota) \) is increasing in \( \iota, a_F, \) and \( n_{I,t} \), so that better enforcement in the informal sector, proxied by a higher \( \iota \), leads to an increase in the probability of getting caught for given levels of informal employment and institutional quality in the formal sector. Note that a higher \( a_F \) proxies for better overall institutional quality in the economy since it makes formal firms more productive and also affects informal firms by increasing the detection probability \( \rho(n_{I,t}, a_F, \iota) \). The representative informal firm chooses a sequence of

\[ ^{24}\text{Adding an exogenous productivity differential between formal and informal firms does not change the conclusions of the paper.} \]
vacancies and informal labor next period \( \{v_{I,t}, n_{I,t+1}\}_{t=0}^{\infty} \) to

\[
\max_{\{v_{I,t}, n_{I,t+1}\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \Xi_{t=0} \left( p_{I,t} (1 - \rho(n_{I,t}, a_F, \iota)) z_t f(n_{I,t}) - w_{I,t} n_{I,t} - \psi_I v_{I,t} \right) \right\}
\]

(2.11)

Subject to

\[
n_{I,t+1} = (1 - \delta^I)(n_{I,t} + v_{I,t} q(\theta_{I,t}))
\]

(2.12)

where the detection probability \( \rho(n_{I,t}, a_F, \iota) \) acts as a tax on informal output, \( q(\theta_I) \) is the informal sector job-filling probability as described above, and \( p_{I,t} \) is the relative price of informal output relative to final output. \( \delta^I \) is the separation rate in the informal sector. Bosch and Maloney (2008, 2010) find that separation rates are higher in the informal sector, which implies that \( \delta^F < \delta^I \). Taking first-order conditions, we obtain a job creation condition for the informal sector:

\[
\frac{\psi^I}{q(\theta_{I,t})} = (1 - \delta^I) E_t \Xi_{t+1|t} \left\{ \frac{p_{I,t+1} (1 - \rho(n_{I,t+1}, a_F, \iota)) z_{t+1}}{-p_{I,t+1} \rho_{n_I}(n_{I,t+1}, a_F, \iota) z_{t+1} - w_{I,t+1} + \frac{\psi^I}{q(\theta_{I,t+1})}} \right\}
\]

(2.13)

Note that the informal sector firm internalizes the probability of being discovered by the government. Indeed, the firm explicitly takes into consideration that its decision to use labor will have an impact on its visibility to the public authorities and thus will affect the likelihood of losing output due to government detection.

\[25\text{They also document that the job separation rate in the informal sector increases during recessions and is highly volatile. In addition, the volatility of unemployment is largely due to the volatility of the informal separation rate and the flows from informality. I assume exogenous separation rates to have a tractable environment.}\]
2.4.3 Labor Market Search and Nash Bargaining

Following the literature, I assume a constant-returns-to-scale matching function $m_j(u_t, v_{jt})$ for each sector $j = F, I$, where $v_{jt}$ denotes sectoral vacancies and $u_t$ is aggregate unemployment. The job-finding rate in each sector is $p(\theta_{jt}) = \frac{m_j(u_t, v_{jt})}{u_t}$, and the job-filling rate is given by $q(\theta_{jt}) = \frac{m_j(u_t, v_{jt})}{v_{jt}}$, so that sectoral market tightness is given by $\frac{p(\theta_{jt})}{q(\theta_{jt})} = \theta_{jt}$ for $j = F, I$.

2.4.3.1 Worker Value Functions

The value to a worker of being employed in the formal sector, $W_{F,t}$, is given by

$$W_{F,t} = w_{F,t} + E_t \Xi_{t+1|t} \{(1 - \delta^F)W_{F,t+1} + \delta^F W_{U,t+1}\} \quad (2.14)$$

The value function for a worker in the informal sector, $W_{I,t}$, is

$$W_{I,t} = w_{I,t} + E_t \Xi_{t+1|t} \{(1 - \delta^I)W_{I,t+1} + \delta^I W_{U,t+1}\} \quad (2.15)$$

The value function for an unemployed agent, $W_{U,t}$, can be written as

$$W_{U,t} = b + E_t \Xi_{t+1|t} \left\{ \begin{array}{l} p(\theta_{F,t})(1 - \delta^F)W_{F,t+1} + p(\theta_{I,t})(1 - \delta^I)W_{I,t+1} + \\ (1 - p(\theta_{F,t})(1 - \delta^F) - p(\theta_{I,t})(1 - \delta^I))W_{U,t+1} \end{array} \right\} \quad (2.16)$$

where I interpret $b$ as the value of unemployment benefits measured in consumption units.\footnote{The interpretation of this parameter is not clear-cut for developing countries. Indeed, if we take the interpretation of $b$ as pure unemployment benefits, then we should set the parameter to}

40
2.4.3.2 Firm Value Functions

Denoting the value function of having a worker in place during production for a formal sector firm by $J_{F,t}$, we have

$$J_{F,t} = p_{F,t} a_{F,t} f_{n_{F,t}}(n_{F,t}, k_{F,t}) - w_{F,t}(1 + \tau^p) + E_t \Xi_{t+1|t} \{ (1 - \delta^F) J_{F,t+1} \}$$

(2.17)

The value of having a worker for a firm in the informal sector, $J_{I,t}$, is

$$J_{I,t} = (1 - \rho(n_{I,t}, a_{F,t})) p_{I,t} z_{I,t} - \rho n_{I,t} a_{F,t} p_{I,t} z_{I,t} - w_{I,t} + E_t \Xi_{t+1|t} \{ (1 - \delta^I) J_{I,t+1} \}$$

(2.18)

where, once again, the probability that the firm’s output is confiscated by the government authorities is $\rho(n_{I,t}, a_{F,t})$. I assume free entry in vacancy posting in both sectors.

2.4.3.3 Nash Bargaining and Wage Determination

Formal sector firms and workers bargain over the real wage, so that $w_{F,t}$ solves

$$\max_{w_{F,t}} \left\{ (W_{F,t} - W_{U,t})^{\nu_F} (J_{F,t})^{1-\nu_F} \right\}$$

(2.19)

zero, as only a small number of developing countries have adequate safety nets. An alternative interpretation is to assume that $b$ includes the value of home production. For developing countries, some papers consider home production as a form of self-employment, even though the self-employed are counted as active labor market participants (Maloney, 2004; Fiess et al., 2010). This does not seem to be an adequate label since informal work, including self-employment, is tied to market activities. In this paper, I explicitly interpret $b$ as unemployment benefits.
where \((W_{F,t} - W_{U,t})\) is the worker’s surplus, \(J_{F,t}\) is the firm’s surplus, and \(\nu_F\) is the worker’s bargaining power. In a similar fashion, the Nash bargaining problem in the informal sector is

\[
\max_{w_{I,t}} \left\{ (W_{I,t} - W_{U,t})^{\nu_I} (J_{I,t})^{1-\nu_I} \right\}
\]  

(2.20)

where \((W_{I,t} - W_{U,t})\) is the worker’s surplus, \(J_{I,t}\) is the firm’s surplus, and the bargaining power for informal sector workers is \(\nu_I \leq \nu_F\). We can consider the difference in worker bargaining powers as an additional reduced-form way of capturing labor regulations tied to worker rights in the formal sector. Formal and informal wages are given by\(^{27}\)

\[
w_{F,t} = \xi^F \left[ p_{F,t} z_t a_{F,t} f_{n_{F,t},k_{F,t}} + \psi_F \theta_{F,t} \right] + \left(1 - \xi^F\right) \left[ b + \frac{\nu_I}{1 - \nu_I} \psi_I \theta_{I,t} \right]
\]  

(2.21)

and

\[
w_{I,t} = \nu_I \left[ p_{I,t} (1 - \rho(n_{I,t},a_{F,t},t)) z_t - \rho_{n_{I,t}} (n_{I,t},a_{F,t},t) p_{I,t} z_t n_{I,t} + \psi_I \theta_{I,t} \right]
\]  

(2.22)

\[
+ (1 - \nu_I) \left[ b + \frac{\xi^F}{1 - \xi^F} \psi_F \theta_{F,t} \right]
\]

where \(\xi^F = \frac{\nu_F}{\nu_F + (1 - \nu_F)(1 + \tau_p)}\) is the effective bargaining power of formal workers, which takes into account the impact of payroll taxes, \(\tau_p\). Intuitively, in a two-sector model with search frictions and interaction across sectors, the wage should not only depend on the marginal product of labor, but also on the value of a potential job in the

\(^{27}\)The Appendix presents a detailed derivation of the wage equations.
other sector as it represents an additional outside option. For example, consider the formal wage equation. Higher informal labor market tightness implies that the outside option in informal salaried work is more readily accessible if the individual were unemployed, which puts upward pressure on formal wages. This effect is similar to what we would obtain with higher unemployment benefits, with the exception that the informal salaried employment option responds to changes in aggregate economic conditions.

### 2.4.4 Final Goods Firm

A representative final goods firm uses output from both intermediate formal and informal firms to produce final consumption goods. For simplicity, these are the only inputs used in the production process. As in Ulyssea (2010), the representative final goods firm chooses output from the two intermediate sectors, \( \{ y_{F,t}, y_{I,t} \}_{t=0}^{\infty} \), to

\[
\max_{\{ y_{F,t}, y_{I,t} \}_{t=0}^{\infty}} \left\{ \left( \alpha_y y_{F,t}^\gamma + (1 - \alpha_y) y_{I,t}^\gamma \right)^\frac{1}{\gamma} - p_{F,t} y_{F,t} - p_{I,t} y_{I,t} \right\} \tag{2.23}
\]

where total output in the economy is \( y_t = \left( \alpha_y y_{F,t}^\gamma + (1 - \alpha_y) y_{I,t}^\gamma \right)^{\frac{1}{\gamma}} \), \( 0 \leq \alpha_y \leq 1 \) and \( \gamma < 1 \). Recall that \( p_{F,t} \) and \( p_{I,t} \) are the relative prices of intermediate formal and informal output, respectively. The first-order conditions yield the prices of formal and informal intermediate goods

\[
p_{F,t} = \alpha_y y_{F,t}^{\gamma-1} y_t^{1-\gamma} \tag{2.24}
\]
and
\[ p_{I,t} = (1 - \alpha_y) y_{I,t}^{\gamma-1} y_t^{1-\gamma} \quad (2.25) \]

2.4.5 Government

The government budget constraint is given by

\[ T_t + bu_t = w_{F,t} n_{F,t} r_p + \rho(n_{I,t}, a_F, \iota)p_{I,t} y_{I,t} \quad (2.26) \]

where recall that
\[ u_t = 1 - n_{F,t} - n_{I,t} \quad (2.27) \]

For simplicity, output confiscated from informal sector firms that are detected becomes government revenue.

2.4.6 Market Clearing

As in most models with labor search frictions, the costs of posting vacancies are a resource cost and thus enter explicitly into the economy’s resource constraint. Hence, the aggregate resource constraint is given by

\[ y_t = c_t + \psi_F v_{F,t} + i_{F,t} + \psi_I v_{I,t} \quad (2.28) \]

I assume that \( y \) in the model represents aggregate output in the data.
2.4.7 Competitive Equilibrium

**Definition** (Competitive Equilibrium) Taking the exogenous process \( \{z_t\} \) as given, a competitive equilibrium consists of prices \( \{w_{F,t}, w_{I,t}, p_{F,t}, p_{I,t}\}_{t=0}^{\infty} \) and allocations \( \{c_t, n_{F,t}, n_{I,t}, \theta_{F,t}, \theta_{I,t}, k_{F,t}, u_t, T_t\}_{t=0}^{\infty} \) that satisfy the formal capital Euler equation (2.10), the two laws of motion for formal and informal employment (2.7) and (2.12), the two job creation conditions (2.9) and (2.13), the formal and informal wages (2.21) and (2.22), the final goods firm’s optimality conditions for sectoral intermediate inputs (2.24) and (2.25), the government budget constraint (2.26), the expression for unemployment (2.27), and the aggregate resource constraint (2.28).

2.5 Calibration Strategy

Bosch and Maloney (2008) are the first to have characterized the labor market dynamics of informal labor markets in two developing countries, Brazil and Mexico, so they provide a good source of estimates for some of the model’s parameters. I choose Mexico as the benchmark economy because Mexico actively tracks informal activity through household and firm surveys and incorporates estimates of informal production into its national income accounts (Quintin and Pratap, 2006; Oviedo, 2009; United Nations, 2008). The first subsection below specifies the functional forms and the second subsection addresses the benchmark parametrization. The main purpose of the simulations is not to determine whether the model can capture particular stylized facts about Mexico, but to make a specific point through a series of numerical experiments. However, I use particular moments for the Mexican econ-
omy to calibrate those parameters for which there are no values in the literature. This allows me to have a more disciplined parametrization of the model.

2.5.1 Functional Forms and Stochastic Processes

The functional forms are standard in the business cycle literature. Utility is given by

\[ U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \]  

(2.29)

The production function in the formal sector is Cobb-Douglas while the production function in the informal sector is linear in informal labor:

\[ f(n_{F,t}, k_{F,t}) = (n_{F,t})^{1-\alpha_F} (k_{F,t})^{\alpha_F} \]  

(2.30)

and

\[ f(n_{I,t}) = n_{I,t} \]  

(2.31)

The matching function in each sector is constant-returns-to-scale, so \( m_j(u_t, v_{j,t}) = M_j u_t^\xi v_{j,t}^{1-\xi} \), \( 0 < \xi < 1 \), where \( M_j \) denotes the sectoral matching efficiency parameter, and \( v_{j,t} \) denotes sectoral vacancies for \( j = F, I \). Then, the job-finding rate in each sector is \( p(\theta_{j,t}) = \frac{m_j(u_t, v_{j,t})}{u_t} = M_j u_t^{\xi-1} v_{j,t}^{1-\xi} \), and the job-filling rate is given by \( q(\theta_{j,t}) = \frac{m_j(u_t, v_{j,t})}{v_{j,t}} = M_j u_t^{\xi} v_{j,t}^{-\xi} \) for \( j = F, I \). The aggregate technology shock \( z_t \) follows an AR(1) process

\[ \ln z_t = \rho_z \ln z_{t-1} + \varepsilon_t^z \]  

(2.32)
With \( \varepsilon_i \overset{iid}{\sim} N(0, \sigma_z) \) and \( \rho_z < 1 \). Finally, the enforcement probability is assumed to be \( \rho(n_{I,t}, a_F, \iota) = a_F n_{I,t} \), similar to Choi and Thum (2005).

2.5.2 Parametrization and Calibration Targets

To parameterize the model, I borrow a set of parameter values directly from related studies, and calibrate the parameters for which there are no known estimates using specific targets for Mexico. Table 2.3 presents the parameters commonly used in the literature, while Table 2.4 specifies the parameters whose values are simultaneously obtained by imposing particular calibration targets and solving for the model’s steady state.

**Parameters from Literature** The time period in the model is set to one quarter, so \( \beta = 0.98 \). The capital share in formal sector production is set to \( \alpha_F = 0.30 \), which is a common value in one-sector DSGE models for developing countries. The depreciation rate for capital is set to 0.02. The quarterly separation rates in the two sectors are taken from Bosch and Maloney (2008), who report quarterly transition probabilities between employment states using quarterly data from 1987 to 2004 from Mexico’s National Survey on Urban Employment (\textit{Encuesta Nacional de Empleo Urbano}, or \textit{ENEU}). The elasticity of substitution between formal and informal intermediate goods in final output is taken to be somewhat higher than the value assumed by Ulyssea (2010) for Brazil. There are no existing estimates for this parameter for Mexico. I initially assume that \( \gamma = 0.5 \), which implies some degree of
Table 2.3: Parameters from Literature

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_F$</td>
<td>0.30</td>
<td>Capital Share, Formal Prod.</td>
<td>DSGE Literature</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.98</td>
<td>Discount Factor</td>
<td>DSGE Literature</td>
</tr>
<tr>
<td>$\delta$</td>
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<td>Capital Depreciation Rate</td>
<td>DSGE Lit.</td>
</tr>
<tr>
<td>$\delta^F_{\ell}$</td>
<td>0.030</td>
<td>Sep. Rate, Formal</td>
<td>Bosch, Maloney (2008)</td>
</tr>
<tr>
<td>$\delta^I_{\ell}$</td>
<td>0.060</td>
<td>Sep. Rate, Informal</td>
<td>Bosch, Maloney (2008)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.50</td>
<td>Elasticity of Substit., Final Goods</td>
<td>Benchmark Assumption</td>
</tr>
<tr>
<td>$\nu_F$</td>
<td>0.50</td>
<td>Worker Bargaining Power, Formal</td>
<td>Benchmark Assumption</td>
</tr>
<tr>
<td>$\nu_I$</td>
<td>$\frac{\nu_F}{2}$</td>
<td>Worker Bargaining Power, Informal</td>
<td>Benchmark Assumption</td>
</tr>
<tr>
<td>$\rho_{\pi}$</td>
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<td>Autocorrelation of TFP</td>
<td>DSGE Lit.</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>0.12</td>
<td>Payroll Tax Rate</td>
<td>OECD Taxing Wages</td>
</tr>
<tr>
<td>$b$</td>
<td>0</td>
<td>Unempl. Insurance</td>
<td>-</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.50</td>
<td>Matching Function Elast.</td>
<td>Search, Matching Lit.</td>
</tr>
</tbody>
</table>

substitution between the two inputs.\textsuperscript{28} The autocorrelation of TFP is set to 0.90. The payroll tax is set to its average for years 2001 and 2002 (OECD, 2009a), and is at the lower bound of existing estimates (see, for example, Levy, 2007). The elasticity of the matching functions and the bargaining power of workers in the formal sector take standard values in the literature, so that $\xi = \nu_F = 0.50$. For the bargaining power of workers in the informal sector, I follow Ulyssea (2010) and set a lower bargaining power for informal workers. As a benchmark, I set $\nu_I = \frac{\nu_F}{2}$.\textsuperscript{29}

Calibrated Parameters We are left with nine parameters that are jointly calibrated using targets from Mexican data: $\alpha_y$, $M_F$, $M_I$, $\psi_F$, $\psi_I$, and $\iota$. I choose $\psi_F$ to

\textsuperscript{28} As I discuss further below, the degree of substitutability plays an important role for the response of unemployment volatility to changes in $\alpha_F$ and to $\delta^I$, but the main message of the paper holds for alternative values of $\gamma$.

\textsuperscript{29} This keeps the job-finding probability in the formal sector bounded below one when the size of the informal sector is very small in the quantitative experiments. Assuming the same effective bargaining power for both types of workers does not change the main conclusions of the paper.
Table 2.4: Calibrated Parameters in Benchmark Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Target</th>
<th>Target Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_y$</td>
<td>0.523</td>
<td>Formal Output Share</td>
<td>$\frac{y_I y}{y} = 0.40$</td>
<td>Benchmark</td>
</tr>
<tr>
<td>$M_F$</td>
<td>0.208</td>
<td>For. Matching Eff.</td>
<td>$n_F = 0.50$</td>
<td>ENEU, ILO</td>
</tr>
<tr>
<td>$M_I$</td>
<td>0.084</td>
<td>Infor. Matching Eff.</td>
<td>$n_I = 0.45$</td>
<td>ENEU, ILO</td>
</tr>
<tr>
<td>$\psi_F$</td>
<td>0.131</td>
<td>Formal Hiring Cost</td>
<td>$\psi_F = 0.30w_F$</td>
<td>Levy (2007)</td>
</tr>
<tr>
<td>$\psi_I$</td>
<td>0.014</td>
<td>Informal Hiring Cost</td>
<td>$\psi_I = 0.035w_I$</td>
<td>Benchmark</td>
</tr>
<tr>
<td>$\iota$</td>
<td>0.111</td>
<td>Instit. Quality</td>
<td>$\rho(.) = 0.05$</td>
<td>OECD (2009a)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0174</td>
<td>Std. Dev. TFP</td>
<td>$\sigma_y = 2.17$</td>
<td>Lama, Urrutia (2012)</td>
</tr>
</tbody>
</table>

be 30 percent of steady-state formal wages, as in Levy (2007). Since there are no data on vacancy posting costs in the informal sector, I choose $\psi_I$ to be 3.5 percent of steady-state informal wages, which is the estimate of the scale of hiring costs, excluding regulations, in Levy (2007). What matters is that hiring workers with formal contracts is more expensive than hiring informal workers due to the legal and regulatory requirements to formalize employment contracts. The weighting factor for aggregate output, $\alpha_y$, is chosen to match a steady-state informal-sector-to-total-output ratio of 40 percent. While this value is higher than the estimates for Mexico in Perry et al. (2007) and Schneider et al. (2010), I assume a higher steady-state share to be able to obtain a longer range for changes in the size of the informal sector in the numerical experiments. The conclusions of the paper remain unchanged with alternative calibration targets. Note that in the model, the measure of informal sector size as a share of final output is given by $p_Iy_I/y$.

I calibrate the matching scale parameters $M_j$, for $j = F, I$ to match the average shares of formal and informal employment as a percent of the labor force in Mexico.
from 1987 to 2004, where informal employment is the sum of informal salaried and self-employment.\textsuperscript{30} I use Mexico’s ratio of the amount of tax-debt collected to the total amount eligible for collection, available from the OECD (2009a), as a proxy for the detection probability $\rho(n_I, a_F, \iota)$ and choose $\iota$ to set the detection probability in the benchmark calibration to 5 percent.\textsuperscript{31} Finally, I set the standard deviation of TFP to match the volatility of output reported in Lama and Urrutia (2012).

2.6 Numerical Experiments

This section characterizes the static and dynamic behavior of the model economy under different institutional environments. I log-linearize the model around the non-stochastic steady state and compute a first-order approximation of the equilibrium conditions.\textsuperscript{32} I simulate the economy for 2100 periods, drop the first 100 periods, use the Hodrick-Prescott (HP) filter with smoothing parameter 1600 on the simulated data, extract the cyclical component and compute the standard deviations of the model’s variables.

\textsuperscript{30}Depending on the definition of informal labor and the dataset used (urban or national), the values for the share of informal labor in Mexico range from 27 percent to 60 percent of the labor force.

\textsuperscript{31}In 2007, which is the latest year available, the ratio for Mexico was reported to be 7.2 percent, whereas in previous years it was closer to 1 percent. Bosch and Esteban Pretel (2009) use a monitoring probability of 0.5 percent in their calibration for Brazil, which is closer to the 1.2 percent probability observed for 2005 and 2006 in Mexico.

\textsuperscript{32}I use Dynare for all dynamic simulations. Using a second-order approximations does not change the results of the paper.
2.6.1 Informal Sector Size and Institutional Quality

Figures 2.1 through 2.3 below show the relationship between the size of the informal sector and various macroeconomic aggregates of interest.

Figure 2.1: Informal Sector Size and Institutional Quality

<table>
<thead>
<tr>
<th>(1) $a_F$</th>
<th>(2) $\delta^I$</th>
<th>(3) $\rho(n_I, a_F, \iota)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
<td><img src="image2.png" alt="Graph 2" /></td>
<td><img src="image3.png" alt="Graph 3" /></td>
</tr>
</tbody>
</table>

The columns of Figure 2.1 show the relationship between the steady-state size of the informal sector (as a percent of total output) and one of the three parameters proxying for institutional quality, holding all other parameters fixed. Column 1 varies overall institutional quality, $a_F$; column 2 varies enforcement as proxied by the informal worker separation rate, $\delta^I$ (see Ulyssea, 2010); and column 3 varies enforcement as proxied by changes in $\iota$ that directly affect the detection probability, $\rho(n_I, a_F, \iota)$.\textsuperscript{33} As shown in Figure 2.1, the model can successfully capture one of the main stylized facts documented in Section 2.3: the negative link between insti-

\textsuperscript{33} A similar experiment can be performed by changing the hiring cost in the formal sector as a proxy for changes in hiring regulations. Since the empirical motivation above suggests that institutions play a more prominent role, I analyze the case of hiring regulations as an extension. The results are in general similar to those obtained by changing $a_F$. Also, the results in columns 1 and 2 are similar if we assume an exogenous detection probability.
tutional quality and informal sector size, regardless of the specific interpretation of institutional quality in the model. Indeed, both higher informal worker separation rates and a higher detection probability proxy for stronger enforcement, and lead to a smaller informal sector size. Higher overall institutional quality, reflected in a higher $a_F$, has a similar effect. However, Figures 2.2 and 2.3 below will show that the underlying source of differences in the size of the informal sector is crucial for characterizing the relationship between informality and some of the steady-state variables of the model.

**Higher overall institutional quality**  Column 1 of Figures 2.2 and 2.3 shows the relationship between informal sector size and various steady-state moments traced out by variations in overall institutional quality, $a_F$. Higher overall institutional quality increases the value of hiring formal workers and accumulating capital in the formal sector, leading to higher formal vacancies, a higher formal job-finding rate, and an increase in formal employment as formal firms expand production. Since formal firms increase vacancy postings and both capital and labor are used in the production process, formal sector labor and capital rise, and formal intermediate output increases. Formal market tightness and the marginal product of formal labor are now higher, putting upward pressure on formal wages. The higher formal job-finding rate puts upward pressure on informal wages as well, but since the informal job-finding rate is decreasing in $a_F$, a higher $a_F$ generates a negative link between informal wages and informal sector size.
Note that the increase in formal vacancies dominates the fall in informal vacancies as institutional quality increases, so that the unemployment rate and the size of the informal sector are positively related. In other words, the informal sector cannot fully absorb the fall in formal employment when institutions deteriorate (Heckman
and Pagés, 2000). Finally, since capital and formal output respond strongly to the increase in $a_F$, these economies have higher output and consumption levels, as well as a higher capital-to-output ratio.
Higher enforcement of regulations in the informal sector  A very different picture emerges in economies where higher institutional quality manifests itself in higher (or better) enforcement of regulations (i.e., a higher $\iota$ which raises the detection probability, or a higher informal separation rate).\textsuperscript{34} Column 2 of Figures 2.2 and 2.3 shows the relationship between steady-state aggregates and informal sector size traced out by changes in enforcement proxied by changes in the separation rate of informal workers.\textsuperscript{35} In this case, an improvement in institutions through stricter enforcement in the informal sector leads to a drastic rise in unemployment. Furthermore, output and consumption fall with higher enforcement, where the first result goes against the cross-country evidence on the link between informality and the level of development. Note that informal wages rise with the size of the informal sector. This takes place since changes in $\delta^{\iota}$ have a larger impact on informal employment and unemployment relative to changes in $\iota$ for the same change in informal sector size, which causes the price of informal intermediate output to rise sharply at the same time as market tightness in both sectors is falling. Both elements affect informal wages but the effect of the price of intermediate output dominates and causes informal wages to rise.

As previously discussed, an alternative way for enforcement to affect informality is through a higher $\iota$, which increases the detection probability, reduces informal

\textsuperscript{34}Changes in $\iota$ can generate a range of informality from 40 percent to 28 percent of total output before running into convergence problems. Conversely, changes in $\delta^{\iota}$ can generate an informal sector size from 40 percent of output to 10 percent, similar to changes in $a_F$.

\textsuperscript{35}One of the main experiments in Ulyssea (2010) consists in increasing the separation rate for informal workers, which he interprets as stricter institutional enforcement. This leads to a larger formal sector, higher unemployment, and to higher average productivity but lower welfare. As I discuss below, my results are in line with Ulyssea’s.
vacancy postings and leads to a fall in informal labor market tightness, a decline in informal sector wages, and a rise in unemployment (see column 3 in Figures 2.2 and 2.3). Since enforcement under this scenario affects mainly the informal sector, the rise in vacancies and capital in the formal sector is not large enough to compensate for the fall in informal output, so that total output falls. Consumption follows the response of output. The fall in total output is much larger than the rise in formal capital, which causes the capital-output ratio to rise with enforcement. However, this occurs for a very different reason than in the case of changes in \( a_F \). Finally, note that both formal and informal wages are increasing in the size of the informal sector, mainly because of the impact of \( \iota \) on unemployment and sectoral labor market tightness. Before concluding this section, note that formal job-finding rates exhibit important differences even if changes in the informal separation rate and in the detection probability are both interpreted as changes in the quality of enforcement. In the case of changes in the separation rate, the sharp rise in unemployment as informal separations rise leads to a fall in formal market tightness. This occurs even though formal vacancies rise as the informal sector becomes smaller. While unemployment also rises with a higher detection probability, the rise in formal vacancies relative to the rise in unemployment is large enough to generate a higher formal job-finding rate when informality falls.

The comparative statics results are in line with a point previously articulated in the literature: improving the economic environment for firms in the formal sector is more effective than making participation in the informal sector more costly.\(^{36}\)

\(^{36}\)Ulyssea (2010) makes this point somewhat differently by showing that lowering the burden of
However, the results in Figure 2.2 go beyond this point and highlight the fact that the cross-country link between informality and long-run macroeconomic outcomes in general will depend on the underlying reason for variation in informal sector size. While increasing informal separation rates can be interpreted as an improvement in institutional quality through better enforcement, which in turn reduces the size of the informal sector, the resulting link between informality and output levels goes against the cross-country evidence. A similar contradiction holds for improvements in institutions that work through a higher detection probability. The intuition is simple: a deterioration in institutions that only affects the informal sector allows informal firms to expand output without necessarily having a large impact on formal sector firms. This generates a counterfactual positive relationship between output and informality (measured as a share of total output) in the model since the elasticity of informal output to changes in $\rho(n, a_F, \iota)$ or $\delta I$ is larger than that for formal output.

2.6.2 Informal Sector Size and Aggregate Volatility

Business cycles in the model are driven by aggregate productivity shocks. Figures 2.4 and 2.5 show the relationship between the size of the informal sector and the volatility of several variables in the model. Changes in the size of the informal sector are engineered in three different ways, as was the case for the steady-state experiments above. Once again, each column in Figure 2.3 corresponds to a different regulations in the formal sector can lead to positive aggregate outcomes that would not be present when we focus on making informality more costly.
determinant of the size of the informal sector: (1) overall institutional quality, $a_F$; (2) enforcement reflected through the informal worker separation rate, $\delta^I$; and (3) enforcement reflected through changes in $\iota$ that directly affect the detection probability, $\rho(n_I, a_F, \iota)$. For expositional purposes, I focus on analyzing the differences between changes in $a_F$ and changes in $\delta^I$ in the discussion below.

For all three cases, there is a positive relationship between the size of the informal sector and the volatility of consumption and investment. To understand this result, note that economies with a larger informal sector have smaller steady-state formal employment shares, which in turn lower the steady-state marginal product of capital. This takes place despite the smaller capital stock in these economies, and affects formal firms’ decisions to post vacancies in response to fluctuations in aggregate productivity. Indeed, these differences in the marginal product of capital make formal vacancy postings more volatile in economies with a larger informal sector by influencing the sensitivity of the value of employing formal workers to aggregate shocks. The rise in the volatility of formal vacancies translates into higher variability in capital accumulation. For example, when a positive productivity shock hits the

---

37 The impulse response functions for other relevant variables are included in Appendix A (Figures A.1 and A.2) for expositional purposes. Consistent with the results in this section, the impulse responses suggest that the volatilities of consumption and investment are lower in economies with a smaller informal sector (even though the steady state exhibits some differences across economies), which is consistent with the evidence.

38 In the case of economies with a larger informal sector due to either worse overall institutions or a smaller detection probability, the value of employing a formal worker is generally lower, which also affects how formal firms respond to economy-wide shocks. The value of employing a worker is lower since formal firms in these economies tend to accumulate less capital, which (ceteris paribus) lowers the marginal product of labor. Also, depending on the institutional proxy that generates changes in informality, formal wages may rise with the size of the informal sector, and higher wages put downward pressure on the value of employing a formal worker from the firm’s perspective. Both mechanisms lower the steady-state value of hiring formal workers relative to an economy with a smaller informal sector, and this lower value makes formal firms’ decisions to hire workers more sensitive to productivity shocks. Comparing the three different institutional scenarios, the fact that the marginal product of capital is lower in economies with a larger informal sector plays a
economy, the value of hiring formal workers rises more sharply in these economies as firms want to take advantage of the increase in productivity (much more so relative to firms in economies with lower informality due to the difference in the steady-state marginal product of capital), which in turn leads to sharper adjustments in formal vacancy postings on impact.\footnote{To see more clearly the connection between the value of employing a formal worker and the sensitivity of vacancies to productivity shocks, consider the contemporaneous term in the value of having a worker for a formal firm, $\hat{J}_{F,t}$, expressed in log-linear form:}

$$
\hat{J}_{F,t} = \left( \frac{p_F a_F m_p}{J_F} \right) (\hat{p}_{F,t} + \hat{m}_p) - \left( \frac{w_F}{J_F} \right) \hat{w}_{F,t} + \ldots
$$

(2.33)

The increase in vacancy postings affects employment in subsequent periods and is accompanied by sharper adjustments in capital accumulation since capital and formal labor are used jointly in production. The more aggressive rise in vacancy postings translates into more volatile investment. Finally, since this volatility is eventually reflected in firm profits, and firm profits are part of the household’s resources, higher consumption volatility follows from higher investment volatility.\footnote{This last argument is similar to Andrés, Doménech, and Futás (2008), who find that an increase in government size leads to higher investment and consumption volatility.}

**Changes in overall institutional quality** Consider a change in $a_F$ that affects both sectors simultaneously. A higher $a_F$ raises the value of hiring workers and accumulating capital in the formal sector. It also lowers the incentive to hire workers in the informal sector by raising the detection probability. Even though higher
formal output puts downward pressure on the price of intermediate formal output, the rise in capital accumulation and the improvement in sectoral productivity due to a stronger institutional environment increases the steady-state value of hiring formal workers and the marginal product of capital over and above the direct influence of a higher $a_F$ on the marginal product of labor. As argued above, this makes vacancy postings and capital accumulation less sensitive to productivity shocks, and reduces the volatility of formal employment, investment and consumption. Despite the reduction in informal output, the rise in capital accumulation and output in the formal sector dominate and lead to a higher capital-output ratio where both steady-state capital and output are higher.

Recall that total output is a combination of formal and informal output. The volatility of formal output increases marginally with the size of the informal sector, whereas the volatility of informal output falls (marginally as well). If informality falls due to improvements in overall institutional quality, we obtain a very mild positive relationship between total output volatility and the size of the informal sector. Given the quantitative change in total output volatility is negligible, the link between informality and output volatility predicted by the model is in line with the evidence presented in the empirical motivation, where there is no significant relationship between output volatility and informal sector size.
Figure 2.4: Informal Sector Size and Aggregate Volatility

<table>
<thead>
<tr>
<th>(1) $a_F$</th>
<th>(2) $\delta^I$</th>
<th>(3) $\rho(n_{1,t}, a_F, t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="chart1.png" alt="" /></td>
<td><img src="chart2.png" alt="" /></td>
<td><img src="chart3.png" alt="" /></td>
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<td><img src="chart6.png" alt="" /></td>
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<td><img src="chart8.png" alt="" /></td>
<td><img src="chart9.png" alt="" /></td>
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<tr>
<td><img src="chart10.png" alt="" /></td>
<td><img src="chart11.png" alt="" /></td>
<td><img src="chart12.png" alt="" /></td>
</tr>
</tbody>
</table>

Note: The volatility of a variable is defined as the standard deviation of the cyclical component of the log of the variable of interest, where the cyclical component is obtained using an HP-filter with smoothing parameter 1600.

Going back to the resource constraint, note that the volatility of consumption, investment, and vacancy postings (as well as the comovement between these variables) have an impact on output volatility. As of the size of the informal sector
Note: The volatility of a variable is defined as the standard deviation of the cyclical component of the log of the variable of interest, where the cyclical component is obtained using an HP-filter with smoothing parameter 1600.

rises, the volatility of consumption, investment, and formal vacancies rise. This puts upward pressure on the volatility of output. Conversely, the volatility of informal vacancies falls. Note that the rise informality is also accompanied by a rise
in informal vacancies. Furthermore, steady-state total output falls with the size of the informal sector. Combining the last two results implies that the volatility of informal vacancies exerts more influence on the volatility of output, and counteracts the rise in the volatility of consumption, investment, and formal vacancies.\(^{41}\) The end result is a very small rise in the volatility of output as the size of the informal sector increases.

**Changes in enforcement** Now consider an economy with stricter enforcement in the informal sector through either a higher informal separation rate $\delta^I$ or a higher enforcement parameter $\iota$ in the detection probability. The positive relationship between consumption and investment volatility and informal sector size is similar to the case where informality falls due to $a_F$. The intuition for this result remains the same: a rise in $\iota$ (or a rise in $\delta^I$) lowers informal labor market tightness, which puts downward pressure on formal wages. The fall in formal wages increases the value of posting formal vacancies and the marginal product of capital (as formal employment rises) and this makes the response of formal firms less sensitive to aggregate productivity shocks. The cyclical behavior of formal vacancies reduces the volatility of investment, which in turn reduces the volatility of consumption through a reduction in the volatility of formal profits. In contrast to an economy with a higher $a_F$, a smaller informal sector due to a higher $\delta^I$ is associated with higher total output volatility. Since the informal sector is more sensitive to changes in $\delta^I$ relative

---

\(^{41}\) This can be seen more explicitly in a log-linearized version of the resource constraint. Note that if we were to subtract vacancy costs from the resource constraint and compute a measure of output net of these costs, the positive relationship between output volatility and informal sector size would be stronger.
to changes in $a_F$, higher enforcement increases the volatility of informal output by more than it decreases the volatility of formal output. From the perspective of the resource constraint of the economy, the counterfactual relationship between steady-state output and informal sector size diminishes the impact that the rise in consumption and investment volatility would have on total output volatility. While the changes in total output volatility are quantitatively small, their direction is contrary to the economy where informality changes due to $a_F$ (and also to the evidence in Ferreira-Tiryaki, 2008).

**Differences in labor market volatility** The steady-state results showed that the underlying source of changes in the size of the informal sector matters for the relationship between informality and particular long-run macro outcomes. Figure 2.3 suggests that the same result holds for labor market volatility.

The *qualitative* steady-state response of formal and informal employment to changes in either overall institutional quality or enforcement is similar regardless of the underlying source of changes in informality: formal employment rises and informal employment falls with better institutions. However, the quantitative response of steady-state unemployment is different depending on what makes informality change, which causes the volatility of employment and unemployment to respond in different ways to the same productivity shocks and to the same change in informal sector size (as a share of output).

In the case of changes in $\delta^I$, the relationship between formal employment volatility and the size of the informal sector is non-monotonic. As a starting point,
consider an economy with a very high level of informality (i.e., a very low $\delta^I$). As enforcement increases, the fall in informality puts downward pressure on formal wages, which raises the value of hiring formal workers. At the same time, the rise in steady-state unemployment implies a higher job-filling probability for formal firms. These elements lead to a reduction in the volatility of formal vacancies, and in turn to a reduction in the volatility of formal employment. However, there is an enforcement threshold beyond which the steady-state unemployment level becomes less sensitive to a higher $\delta^I$ (since the role of informal employment becomes less important. This attenuation is evident in column 2 of Figure 2.3). This affects the change in the formal job-filling probability as $\delta^I$ changes, as well as the incentive to post formal vacancies in such a way that further increases in $\delta^I$ increase the response of formal hires to shocks. This explains the U-shaped relationship between formal employment volatility and the size of the informal sector.

Now consider changes in $a_F$. The fact that the steady-state value of hiring formal workers rises with $a_F$ makes formal employment less volatile in the presence of aggregate shocks. The opposite holds true for informal employment, since a higher $a_F$ reduces the steady-state value of hiring informal workers, so that informal firms react more aggressively through vacancies when aggregate productivity fluctuates. Similarly, an increase in $\delta^I$ or $\iota$ reduces the future value of hiring informal workers (i.e. the right-hand side of the job creation condition is now smaller), which makes informal employment more volatile. For example, when a positive productivity shock hits the economy, informal firms will want to take advantage of the rise in productivity by hiring at a more rapid pace, thereby causing more volatility in
informal employment. Overall, however, unemployment volatility increases with a higher $a_F$, thereby leading to a negative relationship between informality and the variability of unemployment.

For all three economies, a smaller informal sector increases the volatility of informal employment. Note that a fall in informal sector size from 40 to 10 percent due to an increase in $a_F$ doubles the volatility of informal employment, whereas a comparable decline in informality due to a higher $\delta^I$ more than quadruples the volatility of informal employment. Furthermore, while a higher $a_F$ leads to a reduction in steady-state unemployment, stricter enforcement through a higher $\delta^I$ generates a sharp rise in steady-state unemployment since higher enforcement has a larger impact on the informal sector relative to the formal sector, and informal separation rates only affect formal job creation indirectly through the value of the unemployment option. The positive impact of a higher $\delta^I$ on steady-state unemployment outweighs its positive impact on formal employment such that cyclical fluctuations in formal employment have a smaller impact on the cyclical behavior of unemployment. Similarly, the influence of the response of informal employment to a positive productivity shock is also weakened by the higher level of steady-state unemployment, and the end result is a positive relationship between informality and unemployment volatility in response to changes in $\delta^I$. Combining these effects, we can begin to see how the dynamic behavior of unemployment may differ starkly across economies with the same informal sector size.

Figure 2.6 illustrates this point by showing the impulse response function of unemployment to a positive aggregate productivity shock under three scenarios: the
benchmark economy with a share of informality of 40 percent of output; an economy with a share of informality of 20 percent due to better overall institutional quality (a higher $a_F$); and an economy with a share of informality of 20 percent due to a higher informal separation rate $\delta^I$.

Figure 2.6: Impulse Response Function of Unemployment to a Positive Aggregate Shock: Benchmark Economy

Recall that an increase in $a_F$ reduces the size of the informal sector and lowers steady-state unemployment. Conversely, an increase in $\delta^I$ lowers informality but increases unemployment. Writing the definition of unemployment in log-linear terms yields

$$\hat{u}_t = - \left( n_F \frac{u}{n} \right) \hat{n}_{F,t} - \left( n_I \frac{u}{n} \right) \hat{n}_{FI,t}$$  \hspace{1cm} (2.34)$$

where hatted variables correspond to log deviations from steady state (i.e. $\hat{x}_t = \log x_t - \log x$). Since higher overall institutional quality increases formal employment and decreases both informal employment and unemployment in steady state, the
response of formal employment to a productivity shock has a larger impact on unemployment. This causes a larger fall in unemployment in response to a positive aggregate shock when the steady-state size of the informal sector is reduced through a higher $a_F$. Furthermore, since the variability of informal employment increases with a smaller informal sector, fluctuations in informal employment will also have a larger impact on unemployment dynamics.42

Now consider an increase in $\delta^I$. Despite the larger change in informal employment volatility under $\delta^I$, the sharp rise in unemployment as $\delta^I$ rises curtails the impact of this volatility on unemployment volatility. In fact, the rise in steady-state unemployment is large enough to more than offset the rise in informal employment volatility such that a smaller informal sector leads to lower unemployment volatility. The stark differences in the response of unemployment volatility for the same change in informal sector size echo the message outlined in the analysis of the steady state of the model for different shares of informality: the underlying source of changes in the size of the informal sector matters for characterizing the consequences of informality for macroeconomic outcomes, and for labor market volatility in particular.

42If we consider the variance of unemployment in this expression, we can see that, even if the variance of formal employment falls with a smaller informal sector due to a higher $a_F$, the change in $\frac{n_F}{n_u}$ and $\frac{n_I}{n_u}$ may be large enough to increase the variance of unemployment, which is indeed the case in the benchmark simulations.
2.6.3 The Importance of the Elasticity of Substitution Between Formal and Informal Production

The benchmark model assumed an elasticity of substitution parameter between formal and informal intermediate goods in the final goods production function of 0.5, which makes formal and informal intermediate goods imperfect substitutes.\footnote{Recall that the elasticity of substitution between the two intermediate goods is given by \(\frac{1}{1-\gamma}\), where \(\gamma\) is the elasticity of substitution parameter.} A lower value for \(\gamma\) (as in Ulyssea, 2010) would strengthen the quantitative implications of changing the size of the informal sector, and would keep the relationship between output and informal sector size qualitatively intact in Figures 2.1, 2.2, and 2.3: an increase in \(a_F\) would deliver a higher level of output and a smaller informal sector, whereas the opposite would be the case under a higher \(\delta^I\) or a higher detection probability \(\rho(n_I, a_F, \iota)\) arising from a higher \(\iota\). A similar claim holds for the links between informal sector size and aggregate volatility. Moreover, the differences in the response of unemployment to productivity shocks when the size of the informal sector changes would become even starker. When formal and informal intermediate inputs are less substitutable, changes in enforcement in the informal sector have a larger impact on the steady-state level of unemployment. As stricter enforcement reduces the size of the informal sector and increases steady-state unemployment sharply, movements in formal and informal employment in response to productivity shocks have less influence on unemployment fluctuations. Following equation (2.34), as the steady-state ratios of sectoral employment to unemployment fall, the volatility of unemployment also falls. If a reduction in informality is due to an improvement
in overall institutional quality $a_F$, we observe a sharper reduction in unemployment after a positive aggregate shock mainly because of the beneficial impact that better institutions have on the steady-state level of unemployment in the economy. From (2.34), lower steady-state unemployment makes unemployment more sensitive to fluctuations in sectoral employment over the business cycle, i.e. the opposite result when compared to changes in $\delta^I$.

Results are different once formal and informal intermediate goods become better substitutes (i.e. if we assume a higher $\gamma$). Figure 2.5 shows how the responses of unemployment to a positive productivity shock change for values of $\gamma$ ranging from 0.4 to 0.7.\footnote{Acemoglu (2001) and Ulyssea (2010) suggest that the two intermediate inputs are unlikely to be perfect substitutes, and Ulyssea (2010) shows that in his model, there are multiple equilibria when $\gamma \in \{0.8, 0.9, 1\}$. Intuitively, when formal and informal intermediate production are very good substitutes, the fall in informal output resulting from a higher $\delta^I$ or a higher detection probability is easily offset by the rise in formal production. A smaller informal sector puts downward pressure on formal wages and pushes formal firms to increase formal vacancies, employment, capital, and finally output. However, the root cause of changes in the size of the informal sector still matters for the volatility of unemployment, even as the degree of substitutability between the two sectors increases. What ends up happening is that the results are reversed for high values of $\gamma$. This is due to a combination of the change in the sensitivity of steady-state unemployment to changes in $a_F$ and $\delta^I$, and the change in the volatility of each employment state as the substitutability between formal and informal intermediate goods increases.}

The impulse response functions in Figure 2.7 once again illustrate three different economies: the benchmark economy; an economy with an informal sector of 20 percent of output due to better overall institutional quality; and an economy with an informal sector of 20 percent of output due to stricter enforcement in the form of higher informal worker separation rates. As the exercise suggests, for low values of $\gamma$, unemployment is more responsive to productivity shocks in an economy with a smaller informal sector due to a higher $a_F$, while unemployment fluctuations are
more subdued when informality is lower due to more stringent enforcement through a higher $\delta^I$. Hence, the results from the benchmark experiments are magnified. However, for values of $\gamma$ above the benchmark value, the results are reversed and unemployment is now less volatile in economies with a smaller informal sector due
Figure 2.8: Changes in Steady-State Unemployment for Different Elasticities of Substitution: $\gamma = 0.4$ and $\gamma = 0.7$

Change in informal sector size due to $a_F$:

<table>
<thead>
<tr>
<th>$\gamma = 0.4$</th>
<th>$\gamma = 0.7$</th>
</tr>
</thead>
</table>

Change in informal sector size due to $\delta^I$:

<table>
<thead>
<tr>
<th>$\gamma = 0.4$</th>
<th>$\gamma = 0.7$</th>
</tr>
</thead>
</table>

to better overall institutional quality. Conversely, unemployment is more volatile in economies with a smaller informal sector due to stricter enforcement through a higher informal separation rate.
To understand this result, consider what happens to steady-state unemployment for different values of $\gamma$ when we change $a_F$ and $\delta^I$, respectively. This is shown in Figure 2.8. As the elasticity between intermediate goods in final output increases, the response of steady-state unemployment to exogenous changes in $a_F$ and $\delta^I$ falls. It is still the case that unemployment is decreasing (increasing) in $a_F$ ($\delta^I$), as in the benchmark model. However, the changes in unemployment in response to exogenous changes in institutional quality are less dramatic in both cases. To make this transparent, consider how the economy responds when enforcement $\delta^I$ increases and $\gamma$ is higher than its benchmark value: the changes in steady-state unemployment as informality falls are much smaller.\footnote{For a higher $\gamma$, unemployment rises 4 percentage points as opposed to almost 9 percentage points when $\delta^I$ generates a fall in informality from 40 to 10 percent of output.} As I discussed above, this ultimately affects how unemployment volatility responds to changes in the size of the informal sector. Furthermore, when $y_F$ and $y_I$ are more substitutable, the ease with which intermediate output in the two sectors can be substituted in the production of the final good increases, and this also affects how intermediate firms make hiring decisions in response to aggregate productivity shocks. Indeed, when $\gamma$ is higher, the absolute change in the volatility of formal employment and informal employment is larger as the size of the informal sector changes. For a higher $\delta^I$, the smaller change in steady-state unemployment combined with the larger absolute change in the volatility of formal employment and the negative link between informal employment volatility and informality will cause unemployment volatility to rise with a smaller informal sector.\footnote{The intuition for this result is as follows: since formal and informal output become more substitutable, a rise in enforcement reflected in a higher informal separation rate will tilt the}
ployment as the size of the informal sector falls is larger as well (relative to the same case under a lower $\gamma$). Furthermore, the fact that steady-state unemployment is less sensitive to changes in $a_F$ reduces the impact that changes in informal employment volatility would otherwise have on unemployment volatility (again, relative to the benchmark case). The larger fall in formal employment volatility as the size of the informal sector falls therefore leads to a fall in unemployment volatility.

At first sight, these results seem striking since in both cases the volatility of formal employment is increasing in the size of the informal sector. For a relatively high degree of substitutability between intermediate outputs, we should not expect substantial differences in unemployment dynamics. The reason for the discrepancy between an economy where $a_F$ rises and an economy where $\delta^I$ rises has to do with the influence of the volatility of informal employment on the variability of unemployment. In the case of a higher $\delta^I$ under a higher $\gamma$, the changes in informal employment volatility as the size of the informal sector changes are much larger than in an economy where $a_F$ is higher. As $\gamma$ increases, the volatility of informal employment plays a more important role in affecting the volatility of unemployment, even as the share of informal employment is decreasing. This is not the case when we change $a_F$ since the changes in informal employment volatility are much less.

Adjustment to aggregate shocks away from informal firms and into formal firms (relative to an economy with less substitutability between intermediate outputs). This is particularly the case as a rise in $\delta^I$ effectively reduces the value of long-term employment relationships in the informal sector. Thus, for a given productivity shock, formal employment will now be more sensitive to shocks under a higher $\gamma$. This, however, is not enough to lead to higher variability in unemployment. In addition to this effect, the lower sensitivity of steady-state unemployment to changes in $\delta^I$ increases the influence of the volatility of informal employment relative to the benchmark results. When the substitutability between the two types of intermediate output was lower, the volatility of informal employment played a small role in affecting unemployment. Under a higher $\gamma$, this is no longer the case, leading to higher unemployment volatility in economies where formal employment represents a larger share of total employment.
pronounced.

While the impact of informality on the volatility of unemployment across economies is sensitive to the elasticity of substitution between formal and informal output, the model still delivers a positive relationship between informal sector size and consumption and investment volatility, regardless of the main factor behind the change in informality. This takes place because the relationship between the value of having a formal worker and the size of the informal sector remains qualitatively intact to changes in $\gamma$. Also, despite the sensitivity of unemployment to changes in $\gamma$, the main message of the paper holds: even though the size of the informal sector may be the same across two economies, this does not imply that the labor market response of these economies to aggregate shocks (or the levels of consumption, output, and unemployment) will be the same.

2.6.4 Changes in Hiring Regulations

The results in Table 2.1 suggest that even after controlling for enforcement and overall institutional quality, hiring regulations could still play a role in explaining differences in the size of the informal sector. Changing the cost of posting vacancies in the formal sector in the model—a proxy for changes in hiring regulations—yields similar results to those obtained when overall institutional quality in the economy changes: the size of the informal sector and the level of output (the unemployment rate) are negatively (positively) correlated, while informality and investment and consumption volatility (unemployment volatility) are positively (negatively) re-
lated. This simple exercise suggests that policies that have a direct impact on the behavior of agents in the formal sector tend to yield results that are in line with the cross-country evidence on informality and its relationship to macroeconomic aggregates. Even though more stringent formal labor regulations imply a larger informal sector, aggregate output volatility changes marginally. This result is inconsistent with the empirical findings in Loayza, Oviedo and Servén (2004), who document a positive relationship between labor regulations and real GDP per capita volatility. Thus, the model suggests that informality itself may not be the channel through which stricter labor regulations may be associated with higher output variability in the data. However, the absence of a link between informality and output volatility in the model is consistent with some of the regression results in Table 2.2. Finally, note that if we were to increase hiring regulations and overall institutional quality simultaneously, we would obtain a smaller informal sector as well as lower consumption and investment volatility in the economy. This fact is consistent with some of the evidence for rich OECD countries with relatively strict labor market regulations.47

2.6.5 Fixed Detection Probability

Recall that the probability of detection in the benchmark model is an endogenous object that depends positively on the measure of informal workers. This means that even if the enforcement parameter, $\iota$, in the detection probability is held fixed in the benchmark model, the probability of detection in the informal sector can change

47Merkl and Schmitz (2010) and Rumler and Scharler (2009) document a negative relationship between labor regulations and the volatility of various macroeconomic aggregates in developed OECD countries.
with $\delta^I$ and $a_F$ when changes in these parameters affect informal vacancy postings and hence the measure of informal employment. Shutting down the endogenous detection probability in the model does not affect the main results from the benchmark model. In the case of changes in institutional quality that only directly affect the formal sector, the positive impact on total output is larger relative to the benchmark model since the endogenous detection probability has an additional negative impact on informal output when institutions improve. This effect would not be present in a model with exogenous detection.\(^{48}\) In this sense, the results from the benchmark model represent a lower bound for the impact of institutional quality on informal sector size (and in turn on aggregate volatility). These results also show that the behavior of the formal sector tends to dominate the response of the informal sector to changes in the institutional environment that directly affect formal firms.

2.6.6 Introducing Investment in the Informal Sector

In this modified version of the benchmark model, the production function for informal firms is given by $y_{I,t} = z_t f(n_{I,t}, k_{I,t})$, which is assumed to be constant-returns-to-scale. The representative informal firm therefore chooses a sequence of vacancies, informal labor and capital next period $\{v_{I,t}, n_{I,t+1}, k_{I,t+1}\}_{t=0}^{\infty}$ to

$$
\max_{\{v_{I,t}, n_{I,t+1}, k_{I,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \xi_t \left\{ \left((1 - \rho(n_{I,t}, a_F, t))p_{I,t}z_t f(n_{I,t}, k_{I,t}) \right) - w_{I,t}n_{I,t} - \psi_I v_{I,t} - i_{I,t} \right\}
$$

\(^{48}\)This is straightforward to see in the informal sector job creation condition, where detection would merely act as a constant tax on the marginal product of labor.
subject to
\[ n_{I,t+1} = (1 - \delta^I)(n_{I,t} + v_{I,t}q(\theta_{I,t})) \quad (2.36) \]
and
\[ k_{I,t+1} = (1 - \delta)k_{I,t} + i_{I,t} \quad (2.37) \]
The first-order conditions yield a standard job creation condition for informal employment
\[
\frac{\psi^I}{q(\theta_{I,t})} = (1 - \delta^I)E_t\Xi_{t+1} \left\{ p_{I,t+1}(1 - \rho(n_{I,t+1}, a_F, \iota))z_{t+1}f_{n_{I,t+1}}(n_{I,t+1}, k_{I,t+1}) \right. \\
\left. -p_{I,t+1}\rho n_{I,t+1}(n_{I,t+1}, a_F, \iota)z_{t+1}f(n_{I,t+1}, k_{I,t+1}) \right. \\
\left. -w_{I,t+1} + \frac{\psi^I}{q(\theta_{I,t+1})} \right\} \quad (2.38)
\]
and a standard Euler equation for capital used in informal firms
\[ 1 = E_t\Xi_{t+1} \left\{ p_{I,t+1}(1 - \rho(n_{I,t+1}, a_F, \iota))z_{t+1}f_{k_I}(n_{I,t+1}, k_{I,t+1}) + 1 - \delta \right\} \quad (2.39) \]
Now, total investment is determined by the sum of investment in the formal sector and investment in the informal sector, and the resource constraint of the economy is modified accordingly. An increase in overall institutional quality in the economy – an increase in \( a_F \) affecting both sectors – pushes formal firms to increase formal investment while informal firms decrease capital accumulation. The former dominates the latter such that investment and consumption volatility fall. At the same time, unemployment falls and unemployment volatility increases for the same reason as in the benchmark model. Conversely, a rise in \( \delta^I \) leads to a decrease in capital
accumulation in the informal sector, but to a larger fall in informal and total output. As was the case in the benchmark economy, investment and consumption volatility decrease with a rise in $\delta^I$. In summary, the main qualitative conclusions from the benchmark model do not change when I allow for informal sector investment.

2.7 Conclusion

The empirical evidence on informality shows that the quality of institutions is one of the most important factors behind cross-country differences in the size of the informal sector. Institutional quality can manifest itself in different ways, influencing the level of enforcement of regulations in specific sectors or having a broad impact on economy-wide productivity. While better institutional quality and better enforcement both lead to a lower level of informality in my model, I show that different parameters affecting institutions that have a similar impact on the size of the informal sector may have differing implications for both long-run macroeconomic outcomes and short-run aggregate volatility. I build a two-sector business cycle search and matching model with informal salaried employment that replicates several stylized facts in the data, including the link between institutions and informality, output levels and informality, and the relationship between informality and consumption and investment volatility.

Using the model, I consider two main experiments: the first one generates changes in the average size of the informal sector through stricter enforcement in the informal sector, proxied by a higher informal worker separation rate and a higher
probability of detection in the informal sector. The second focuses on changes in informality due to improvements in overall institutional quality that are reflected simultaneously in higher exogenous productivity in the formal sector and in a higher detection probability in the informal sector. Both cases recreate a negative link between informality and the quality of institutions. However, the first experiment generates a positive link between the size of the informal sector and the level of output, which goes against the existing cross-country evidence. The second experiment generates the correct link between output in the economy and informal sector size in the data. I also obtain a positive link between informality and consumption and investment volatility consistent with the empirical evidence, regardless of the root cause of the change in informality. Intuitively, this takes place because the hiring decisions of formal firms in economies with a larger informal sector are more sensitive to aggregate shocks, which leads to higher volatility in formal hiring and investment. This volatility ultimately translates into higher consumption volatility since firm profits are a component of household resources (similar to Andrés, Doménech, and Fatás, 2008).

While the results for consumption and investment volatility are independent of the the origin of differences in informality levels may not matter, the same is not true for labor market volatility. I find a negative relationship between the size of the informal sector and unemployment volatility when better overall institutional quality leads to lower informality. Conversely, I find a positive link between informality and unemployment volatility when informality falls due to stricter enforcement in the form of higher informal worker separation rates. The link between informal
sector size and unemployment volatility depends heavily on how the underlying source of changes in the size of the informal sector affects the steady-state level of unemployment in the economy. When overall institutional quality increases, both informality and unemployment fall in steady state. Conversely, with stricter enforcement in the informal sector, informality drops but steady-state unemployment rises sharply. This last result reduces the sensitivity of unemployment volatility to fluctuations in formal and informal employment in response to aggregate shocks, thereby generating a positive link between unemployment volatility and informal sector size.

In the model, formal and informal output are assumed to be imperfect substitutes in the production of final output. I find that the impact of informality on the volatility of unemployment is sensitive to the substitutability of output between sectors. For high levels of substitutability, improvements in overall institutional quality generate less unemployment volatility, whereas the opposite occurs with higher enforcement in the informal sector. Intuitively, the more substitutable formal and informal goods are in total output, the lower the impact of stricter enforcement on the level of unemployment in the economy, and the larger the impact of fluctuations in formal and informal employment on the variability of unemployment over the business cycle. Thus, a high level of substitutability can reverse the benchmark results for unemployment volatility, but the general message of the paper remains intact: the underlying reason for differences in the size of the informal sector matters critically for characterizing the connection between informality, long-run macroeconomic outcomes and the cyclical behavior of the labor market. Looking at the size
of the informal sector in isolation may yield misleading conclusions about the consequences of changes in the employment structure for labor market and business cycle fluctuations. These results carry an important message when thinking about policies to address informality.

The model abstracts from other important features that characterize informal economic activity, such as the rich heterogeneity in that exists across workers and firms. In light of the main conclusions of the paper, incorporating other types of rigidities (institutional or otherwise) that are likely to affect informality and investigating their impact on aggregate volatility may be worthwhile. Including the explicit (and endogenous) fiscal costs of enforcing regulations may be important as well. In addition, to preserve tractability, I ignored job-to-job flows between the two sectors. Bosch and Esteban-Pretel (2006, 2009) and Bosch and Maloney (2010) find these flows to be empirically relevant. These sectoral worker flows could have different implications for the behavior of employment, consumption, investment, and output in response to shocks. A similar comment applies to the exclusion of the self-employed in the model. Analyzing the root causes of informality and its implications for labor market and aggregate fluctuations in richer settings appears to be an important area of research. I plan to pursue this line of investigation in future work.
Chapter 3

Self-Employment and Business Cycle Persistence: Does the Composition of Employment Matter for Economic Recoveries?

3.1 Introduction

Countries are well-known to exhibit differences in employment arrangements. While developed economies have high salaried employment and low self-employment shares, developing economies exhibit the opposite pattern (see Table 3.1 and Gollin, 2008). A number of studies document that a majority of the self-employed are independent workers with no salaried employees, who nonetheless account for an important share of employment, firms, and economic activity (ILO, 2002; Perry et al., 2007; Sanandaji, 2010; Kucera and Roncolato, 2008). Despite their small scale and limited capital usage, these individuals are often capital-constrained and must rely on external financing from input suppliers and other informal sources (Beck et al., 2008).

Recent cross-country evidence highlights two more facts about self-employment. First, self-employment expands during downturns. Second, this expansion arises mainly from an increase in transitions from unemployment into self-employment in recessions (see Table 3.2 or Bosch and Maloney, 2008; and Loayza and Rigolini, 2011).\(^1\) The cyclical behavior of self-employment raises a number of questions: if

\(^1\)Transitions from salaried employment into self-employment are mildly procyclical, while tran-
self-employment is a feasible outside option to salaried work during recessions, how does it alter labor market dynamics? If owner-only firms are created during downturns, does this affect the pace of recoveries from adverse shocks? If so, through what channels? Figure 3.1 shows that there is a negative relationship between the cyclical persistence of output and the share of self-employment in non-agricultural employment for a sample of developed and developing countries. This fact, documented here for the first time, suggests that self-employment may play an important role in business cycle dynamics, and particularly, economic recoveries.

In this paper, I build a business cycle model with frictional labor markets where individuals can be self-employed or employed in salaried firms. I use the model to shed light on the channels through which self-employment influences the pace of economic recoveries, and show that economies with higher self-employment shares exhibit faster recoveries from negative aggregate productivity shocks. This result hinges critically on whether self-employment is a feasible outside option to salaried work in downturns, and whether the ease of entry into self-employment changes as the economy recovers. This last fact makes the cyclicality of transition rates into self-employment particularly important for understanding how differences from self-employment into unemployment are strongly countercyclical. Transitions from non-participation into self-employment are almost acyclical (see Bosch and Maloney, 2008). A decomposition by type of employment similar to the one in Table 3.2 is not possible for most countries due to limited data on labor flows.

This finding is in line with Calderón and Fuentes (2010) regarding the recovery speed in emerging market economies. I use real GDP for years 1985 through 2007 from the World Development Indicators to compute the cyclical persistence of output. Cyclical persistence is measured as the first-order autocorrelation of cyclical annual real GDP where the cyclical component of real GDP is obtained using an HP filter with smoothing parameter 100, consistent with annual data. The data on the share of self-employment as a percent of non-agricultural employment is obtained from the OECD for years 2000 through 2007. The relationship between output persistence and self-employment in Figure 3.1 holds even after controlling for the level of development and other factors that may influence the persistence of output. See Appendix B for details.
in the composition of employment affect business cycle dynamics. Rising entry into self-employment during downturns is difficult to explain with existing business cycle models of entrepreneurship and liquidity constraints: if individuals require external financing to start and run their businesses, they should be less likely to enter self-employment during bad times, when credit conditions deteriorate and the availability of credit from financial intermediaries falls. A series of empirical studies underscore the relevance of supplier or interfirm trade credit as a major source of external financing for small firms. Furthermore, a related line of research suggests that trade credit initially expands in downturns (see Section 3.3). This evidence offers a way to reconcile the fact that transitions into self-employment increase in recessions. It also hints at two relevant connections between larger salaried-sector firms that act as input suppliers and small-scale, self-employed enterprises: the first one rooted in trade credit relationships, and the second one embedded in the labor market.

Table 3.1: Self-Employment Rates Around the World

<table>
<thead>
<tr>
<th>Region</th>
<th>Self-Employment</th>
<th>Min.</th>
<th>Max.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>45.5</td>
<td>11.6</td>
<td>88.7</td>
<td>26</td>
</tr>
<tr>
<td>Asia</td>
<td>31.5</td>
<td>5</td>
<td>75.4</td>
<td>15</td>
</tr>
<tr>
<td>Latin America</td>
<td>37.9</td>
<td>18.9</td>
<td>64.7</td>
<td>19</td>
</tr>
<tr>
<td>Developed Countries</td>
<td>14.3</td>
<td>6.7</td>
<td>28.6</td>
<td>23</td>
</tr>
</tbody>
</table>

Source: OECD (2009).

Indeed, if self-employment is an attractive alternative to salaried work, fluctuations in the ease of entry into self-employment can affect salaried labor market dynamics in important ways. In economies with higher self-employment shares,
Table 3.2: Cyclical Correlations of Employment and Job-Finding Rates with Output

<table>
<thead>
<tr>
<th>Cyclical Correlation with Output:</th>
<th>Brazil</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-Employment</td>
<td>-0.219</td>
<td>-0.415</td>
</tr>
<tr>
<td>Formal Salaried Employment</td>
<td>0.616</td>
<td>0.780</td>
</tr>
</tbody>
</table>

Transition Rate from Unemployment to:

<table>
<thead>
<tr>
<th></th>
<th>Brazil</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-Employment</td>
<td>-0.600</td>
<td>-0.433</td>
</tr>
<tr>
<td>Formal Salaried Employment</td>
<td>0.652</td>
<td>0.798</td>
</tr>
</tbody>
</table>

Notes: Taken directly from Bosch and Maloney (2008). Self-employment corresponds to informal self-employment (own-account workers and business owners with less than 5 employees) as a share of the working-age population. The authors use quarterly data, and all variables are logged and HP-filtered with smoothing parameter 1600. Entry into informal (unprotected) salaried employment from unemployment is procyclical and therefore similar to formal salaried employment.

Figure 3.1: Self-Employment and Cyclical Output Persistence Across Countries

Notes: Author’s calculations. The self-employment share corresponds to the average of annual self-employment as a percent of non-agricultural employment from 2000 to 2007. The autocorrelation of output corresponds to the first-order autocorrelation of the cyclical component of log annual real GDP from 1985 to 2007, using an HP filter with smoothing parameter 100. See Appendix B for more details.
self-employment is likely to exert more influence on overall labor market conditions since the structure of labor markets is tilted towards low-scale, owner-only firms. The fact that self-employment has a larger presence in the labor market modifies the decisions of salaried firms and households in the wake of economic fluctuations, which in turn shape the economy’s response to shocks. This channel has not been explored in the existing literature.

In my framework, self-employment consists of own-account or independent workers who operate owner-only firms. I use these terms interchangeably throughout the paper to refer to individuals working on their own who do not hire workers. Furthermore, the self-employed rely on capital from input suppliers to produce. Since supplier credit is a relationship-based source of financing, capital matching frictions present a natural modeling choice to capture the difficulty in obtaining capital to finance self-employment ventures. I assume that unemployed individuals must match with a capital supplier to enter self-employment. This process is costly and time-consuming. Salaried firms act as trade credit suppliers by devoting unused capital in the salaried sector to the self-employed in frictional capital markets. Capital search frictions play three crucial roles in my model: they establish a link between the salaried firms that supply capital and the self-employed who demand capital;

---

3 It is well known that self-employment exhibits substantial heterogeneity, with a small share of individuals running businesses that start small but grow rapidly. In this paper, I explicitly abstract from these individuals (often labeled entrepreneurs) and instead focus on those that run their own businesses without hiring salaried workers (which represent the majority of the self-employed) to explore whether the presence of an alternative employment state matters for business cycle dynamics. I consider the model in this paper as a stepping stone that can easily be expanded to account for those self-employment who eventually expand their firms by hiring workers.

4 The International Labor Organization (ILO) considers own-account workers excluding (high-skilled) professionals; unpaid family workers; and business owners with less than five employees as “informal self-employed”.

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they endogenize self-employment entry, where the supply and demand for capital determine the availability of self-employment capital and therefore the likelihood of becoming self-employed; and finally, they allow me to capture the countercyclicality of both the stock of self-employment and the entry rate into self-employment from unemployment in the data.

Two key features are critical to explaining the relationship between the share of self-employment and the pace of recoveries in the model. First, flows into self-employment respond to aggregate conditions and depend on the availability of capital used in self-employment. So, entry into self-employment is determined by the explicit decisions of households and firms in the economy, and will therefore change depending on the phase of the business cycle. Second, self-employment may not be a readily accessible alternative to salaried employment, since capital is needed to set up shop and finance production. Thus, differences in the ease of entry into self-employment will have direct static and dynamic consequences for the returns to salaried work, and therefore on the hiring and investment decisions in the salaried sector. Economies with higher self-employment shares tend to have lower capital utilization rates in the salaried sector, and therefore a higher supply of capital to the self-employment sector. The higher the availability of these inputs to potential self-employed individuals, the more likely it is for an unemployed individual to enter self-employment, and the larger is the influence of the self-employment outside option on salaried wages and aggregate labor market conditions. This has important implications for the economy’s response to negative aggregate shocks.

In standard models with frictional labor markets, the probability of entering
salaried employment falls in downturns as firms cut back on vacancies. This result is present in my framework. However, inflows into self-employment increase in recessions as capital utilization in the salaried sector falls and hence the availability of unused capital for self-employment increases. The expansion in self-employment takes place even though demand for self-employment capital also increases during downturns. In economies with higher average self-employment shares, the self-employment outside option exerts more influence on labor market conditions, and its improvement at the onset of recessions lessens the initial fall in salaried wages when a negative aggregate shock hits the economy. Salaried firms in these economies respond by reducing vacancies and the share of capital used in production by more, causing a larger initial contraction in investment, salaried output, and total output when the shock hits the economy.

As aggregate productivity slowly returns to trend, salaried firms adjust their capital usage upwards more drastically relative to economies with less self-employment to take advantage of the recovery in aggregate productivity, which sharply curtails the initial increase in the availability of capital to the self-employed. This makes the increase in the entry rate into self-employment less persistent after the shock in economies with higher steady-state self-employment. Since the increase in the likelihood of moving into self-employment in these economies is more short-lived, and self-employment has a larger presence in these economies, the initial fall in wages becomes more persistent following the shock. This occurs since the deterioration in the self-employment outside option puts downward pressure on salaried sector wages, much more so in economies with higher average self-employment. A more
prolonged fall in wages coupled with a recovery in aggregate productivity creates a larger benefit from hiring workers, which in turn bolsters hiring and investment in the salaried sector. The faster recovery in salaried employment and investment boosts salaried sector output and translates into a faster recovery in total output in the economy. Thus, the model suggests that changes in the ease of entry into self-employment after a negative shock play a critical role in shaping the recovery path of wages in the salaried sector. This affects firms’ hiring and investment decisions in the salaried sector, and the recovery of output in the economy. This is an important mechanism that can explain why economies with more self-employment recover faster from downturns in the model. In particular, this last fact cannot be explained if the likelihood of becoming self-employed is fixed over the business cycle. Finally, I show that the model can successfully capture the empirical relationship between output persistence and self-employment shown in Figure 3.1. This gives additional validity to the framework used to capture self-employment that I propose in the paper.

My work expands on two different literatures. The recent theoretical literature on labor flows in developing countries, rooted in the Mortensen and Pissarides search and matching framework, has generally abstracted from modeling self-employment (see Albrecht, Navarro, and Vroman, 2006; Bosch and Esteban-Pretel, 2012, among others). Similarly, the business cycle labor search literature has placed little attention on the role of self-employment in business cycle dynamics. I offer a tractable way to introduce self-employment in a business cycle search environment that explicitly accounts for endogenous transition rates into self-employment. To my knowledge,
this is the first paper to capture important stylized facts about the cyclical behavior of self-employment in a business cycle framework with frictional labor markets.

The rest of the paper is organized as follows. Section 3.2 offers a summary of related literature and Section 3.3 presents four stylized facts on small firms and capital constraints. These facts motivate how I introduce endogenous and frictional entry into self-employment in the model, which I present in Section 3.4. Section 3.5 describes the calibration of the model, Section 3.6 discusses the simulation results, and Section 3.7 concludes.

3.2 Related Literature

Using data for 22 OECD countries, Koellinger and Thurik (2009) find that entrepreneurship is highly responsive to aggregate shocks, increasing during recessions when outside employment options become more scarce. These results are in line with Loayza and Rigolini (2011), who show that self-employment is on average countercyclical using a large sample of developed and developing countries.\footnote{The authors also show that the countercyclicality of self-employment is decreasing in the level of self-employment.} Using Argentinian data, Mandelman and Montes-Rojas (2009) find that transitions into self-employment are higher during downturns, and the probability of becoming self-employed is increasing in the length of the downturn. They find that most self-employed workers are own-account workers with no employees, in line with the evidence for other countries documented in ILO (2002) and Perry et al. (2007).

Kumar and Schuetze (2007) use a partial equilibrium labor search model where
transitions from unemployment into self-employment depend positively on the job-finding probability for salaried workers, and the self-employed do not require external resources to produce. They analyze the steady state impact of unemployment insurance, minimum wages, and taxation on self-employment, and their main focus is on business owners who hire workers.\footnote{One of their extensions addresses own-account work, but the endogenous transition rate from unemployment into self-employment in their benchmark model – which is driven by the job-finding rate in salaried work – does not change when they consider entry into own-account work.} While they do not analyze business cycle dynamics, the entry rate into self-employment would be procyclical in their model. Also, in contrast to my framework, the salaried wage in their model is independent of conditions in self-employment. This last fact eliminates the key channel through which self-employment affects aggregate dynamics in my model. Rissman (2007) presents a search model with salaried employment and own-account work (i.e., self-employed individuals with owner-only businesses) to analyze the influence of startup costs on employment transitions. She posits a probability of receiving a business idea linked to a stochastic profit that individuals observe before deciding to become self-employed given current conditions in the labor market.\footnote{Individuals entering self-employment must pay a fixed startup cost, but no resources are needed to operate in self-employment thereafter. Hobijn and Şahin (2007) have a similar framework with business idea shocks and occupational choice. Individuals with an idea above a given threshold become entrepreneurs subject to labor search frictions to hire workers. The authors abstract from owner-only firms and focus on steady state outcomes. For a related setup to Rissman (2007), see Rissman (2003), where self-employment is assumed to be a second-best alternative to salaried employment, where individuals can optimally search for salaried employment from self-employment. In this setup, entry into self-employment depends solely on paying a fixed cost. It still takes one period to become officially self-employed. Exit from self-employment depends on whether search by the self-employed yields an acceptable wage offer in salaried employment.} My modeling approach endogenizes the transition rate into self-employment by linking it to tightness in physical capital markets and hence the availability of resources needed to start production. Access to capital has been shown to be an important constraint for...
starting small business ventures. Endogenous transition rates into self-employment add a dynamic component to the self-employment outside option absent in other frameworks, and the latter ultimately affects salaried wages when the economy is subject to shocks. This channel is crucial in explaining the link between economic recoveries and self-employment in the model. Lastly, Narita (2010) introduces informal self-employment into a life-cycle partial-equilibrium labor search environment with formal and informal salaried labor. Transitions into self-employment require prior experience in salaried employment to learn about potential ability to be self-employed. I propose an alternative way of introducing self-employment in a search environment that is more suitable for the analysis of business cycles, where I can account for the empirical relevance of external resources required for production during self-employment.

Den Haan, Ramey and Watson (2003) use capital matching frictions to model long-term relationships between borrowers (entrepreneurs) and lenders (financial intermediaries) and analyze the propagation of aggregate shocks. In their framework, households channel savings through financial intermediaries while entrepreneurs match with lenders to obtain funds for production. In contrast, Kurmann and Petrosky-Nadeau (2007) explore the role of capital frictions as a propagation mecha-

---

8Margolis, Navarro, and Robalino (2012) and Bargain et al. (2012) introduce informal self-employment in a partial equilibrium search environment with formal and informal salaried employment. In these papers, as in Rissman (2003, 2007), self-employment opportunities arise randomly each period, after which individuals decide whether or not to enter self-employment, and transition rates into self-employment are exogenous. None of these papers address the business cycle implications of self-employment.

9In contrast to other papers modeling the search for capital, their model includes asymmetric information and other contracting rigidities that complement search frictions.
anism in an environment without financial intermediaries.\footnote{For an optimal policy approach, see Arseneau, Chugh, and Kurmann (2008).} In a similar vein, Nicoletti and Pierrard (2006) propose an RBC model with labor and capital search frictions in which small firms match with banks to obtain capital before hiring workers, while large firms face frictionless capital markets. Their setup yields a procyclical probability of finding a financial intermediary that supplies capital. In contrast to these papers, I focus on the behavior of own-account work and its influence on labor market and output dynamics. I also eliminate financial intermediaries and assume a direct lending relationship between salaried firms supplying capital and the self-employed. Finally, my framework yields a countercyclical probability of finding a capital supplier, which is a key component needed to generate important cyclical differences in wage dynamics, salaried employment, and output in the model. To the best of my knowledge, my model is the first to capture the cyclicality of the transition rate from unemployment into self-employment observed in the data. Furthermore, I show that cyclical fluctuations in this transition rate have important implications for labor market and business cycle dynamics.

Fiess, Fugazza, and Maloney (2010) propose a small open economy general equilibrium model where the informal self-employed are subject to entry barriers in the form of liquidity constraints. Their work highlights the importance of identifying the sources of shocks to understand the cyclical movements in informal self-employment, since the latter depend on the type of shock affecting the economy. I emphasize search for productive inputs as an entry barrier to self-employment, focus on the implications of self-employment for the cyclical persistence of output in a
closed economy setting, and investigate the model’s implications quantitatively. My model can capture the main argument in Fiess et al. (2010) by including a sectoral productivity shock in self-employment.

Finally, Bergoeing, Loayza, and Repetto (2004) argue that regulatory impediments distort the resource reallocation process among firms in the aftermath of adverse aggregate shocks, thereby leading to slower recoveries. They suggest that policies and regulations that push otherwise inefficient firms to stay in the market during downturns can lead to slower recoveries by distorting the restructuring process that takes place during recessions. My paper is related to Bergoeing et al.’s in that countries with high self-employment have many small inefficient firms and a weaker institutional environment that adversely affects productivity in the salaried sector. I show that, despite an expansion of the low-productivity sector in recessions, economies with more self-employment exhibit faster recoveries from aggregate shocks.

3.3 Capital Frictions and Self-Employment: The Role of Input Suppliers

In this section, I outline four facts established by previous literature that motivate the use of capital search frictions to model entry into self-employment.

Frictions in obtaining capital affect firm creation and financing

Blanchflower (2004) documents that frictions in obtaining capital are an important obstacle
for small-firm creation in the United Kingdom and other developed countries. Other studies show that micro and small firms – a large majority of which are owner-only – tend to be more credit constrained and are more likely to consider access to credit as a major obstacle relative to medium and large firms (Kantis et al., 2002; IDB, 2005a; Cull, McKenzie, and Woodruff, 2008).

**Trade and supplier credit is a major source of financing for small firms**

Chavis, Klapper, and Love (2011) use a sample of more than 100 countries and document that young firms in developing and developed countries tend to rely more on trade credit from input suppliers, friends and family, and informal lenders.\(^{11}\) Input suppliers and customers act as important sources of working capital for both young and older firms (Kuntchev et al., 2012), and are one of the most relevant sources of financing among small nascent firms in Latin America and Asia (IDB, 2005b).\(^{12}\) These are two regions where interfirm trade credit is particularly prevalent as a source of external finance and working capital for small firms (Demirgüç-Kunt and Maksimovic, 2001; Beck, Demirgüç-Kunt, and Maksimovic, 2008).

**Supplier credit often involves goods and not cash loans** Poor institutional quality in developing countries worsens access to formal credit and pushes young firms to rely on trade credit as a source of finance.\(^{11}\) For a comprehensive review of trade credit in the U.S., see Petersen and Rajan (1997). For evidence on the extent of trade credit across countries, see Brown, Chavis, and Klapper (2011) and Kuntchev et al. (2012). Klapper, Laeven, and Rajan (2011) cover in detail the characteristics of trade credit contracts. McMillan and Woodruff (1999) analyze the use of trade credit in Vietnam. Fabbri and Klapper (2009) have a related study on China. Hall and Monge-Naranjo (2003) present evidence on trade-credit use by Costa Rican manufacturing firms.

\(^{12}\) Chavis, Klapper, and Love (2010) find that only 15 percent of young firms in low-income countries use formal banking as a source of financing. Credit through input suppliers is also used to avoid the regulatory burden that often characterizes formal credit relations (IDB, 2005b; Safavian and Wimpey, 2007).
firms to tap informal financing sources (McMillan and Woodruff, 1999). A deficient institutional environment in these economies, combined with the asymmetric information problems intrinsic to lending relationships, makes credit based on goods—in-kind or input credit—more prevalent than cash-based credit since goods are easier to monitor than cash loans (Burkart and Ellingsen, 2004). Relatedly, Eisfeldt and Rampini (2009) show that in the U.S., many small firms lease a substantial amount of capital to finance their operations, confirming that input credit and input leasing through suppliers and customers are important sources of external finance in developed countries as well.\textsuperscript{13}

**Trade credit tends to be countercyclical** Ramey (1992) suggests that trade credit is countercyclical in the U.S. Using evidence on Mexico and East Asia, Love, Preve, and Sarria-Allende (2007) find that trade credit provision to firms by suppliers increases right after a financial crisis, and then falls in the aftermath, suggesting that trade credit is countercyclical. Using the World Bank’s Financial Crisis Survey, Klapper and Randall (2011) document that a non-negligible fraction of firms in Eastern European countries extended supplier credit during the 2008-2009 financial crisis. The countercyclicality of trade credit may offer one way to rationalize the countercyclicality of self-employment, since the self-employed rely on trade credit to operate their firms.

I take the facts above as supporting evidence for using capital search frictions

\textsuperscript{13}As Chavis, Klapper, and Love (2011) point out, leasing is different from trade credit since the former is typically backed by assets, and hence more prevalent in developed economies. However, both leasing and trade credit seem to involve establishing a (long-term) relationship of some sort with input suppliers.
to model self-employment: entry into the sector requires external finance through in-kind trade credit. Since trade credit relationships take time to materialize and are often long-lived, capital search frictions are a natural way to model frictional and endogenous transition rates into self-employment.

3.4 The Model

The economy is comprised of households and salaried firms. Salaried firms hire wage workers in frictional labor markets and accumulate capital. In contrast to the standard RBC labor search model, these firms also make a capital allocation decision: each period, they decide on the fraction of capital used inside the firm. Any unused capital is devoted to matching with potential self-employed individuals outside the firm, who use matched capital to produce in self-employment. All goods in the economy are identical. A representative household with many family members posts self-employment projects to attract capital in frictional capital markets, and sends its members to find salaried employment. There is perfect risk-pooling in the economy. There is no on-the-job search and search for salaried employment is undirected. Separations from salaried employment and self-employment are exogenous. Lump-sum taxes are used to finance unemployment benefits. I focus on urban labor markets and therefore abstract from self-employment in the agricultural sector.

The search process for the self-employed works in the following way: each period, after stochastic productivity is realized and separations from each employment state take place, the household chooses the number of self-employment projects
posted today as well as the measure of self-employed individuals it would like to have producing next period. At the same time, salaried firms accumulate capital in a frictionless environment and choose the fraction of capital to be used for production inside the firm, which determines the fraction of capital available for matching with self-employment projects. The capital supplied by salaried firms can be considered as input trade credit to the self-employed.\textsuperscript{14} A matching function brings together unused salaried firm capital and potential projects and determines the creation of productive self-employment ventures. Capital market tightness is defined as the ratio of self-employment projects (capital demand) to the salaried firms’ supply of unused capital for matching (capital supply). A tighter capital market implies that households find it more difficult to match with capital suppliers, which in turn lowers the probability of entering self-employment. Each self-employed individual requires one unit of capital to produce, so the measure of self-employed individuals in the current period is given by last period’s amount of newly matched capital plus the stock of last period’s self-employed individuals remaining after exogenous separations have taken place.

A successful match allows the self-employed to access a stochastic production technology that depends on an aggregate technology shock and a sectoral shock that reflects the additional volatility of the sector.\textsuperscript{15} For each unit of matched capital, households must supply one inelastic unit of self-employed labor for production to

\textsuperscript{14}An equally valid way to interpret this is to assume that firms offer funds to the self-employed, where the funds are considered a “productive input” (den Haan, Ramey and Watson, 2003).

\textsuperscript{15}This sectoral shock plays an important role when comparing the prediction of the model against the data as it allows me to introduce additional volatility in the self-employment sector. The shock also generates differences in the volatility of self-employment earnings relative to wage earnings, which is supported by empirical evidence.
take place. To focus on the behavior of the self-employed, I abstract from the reasons why the self-employed do not expand their projects by hiring salaried workers since most self-employed run owner-only businesses. Salaried firms rent the matched capital to the self-employed while retaining full ownership during the length of the match. The rental rate paid by self-employed household members is determined by Nash bargaining between the self-employed individual and the firm. This setup captures the fact that establishing a trade credit relationship is costly and time-consuming and characterizes the prevalence of external financing barriers for new firms.\footnote{Informational asymmetries may distort trade credit relationships as well, but these can be overcome with time. I abstract from information frictions to keep the model tractable.}

The timing of events is as follows: at the beginning of the period, the productivity shocks are realized. Separations in all employment states take place and unemployment is determined. Salaried firms recover the capital separated at the beginning of the period and cover the depreciation of capital in surviving self-employment relationships. Firms also post salaried vacancies, choose the fraction of the capital stock used inside the firm, and choose investment. They also choose next period’s salaried employment and self-employment capital. At the same time, households post self-employment projects and decide on next period’s self-employment capital. Matching in labor and physical capital markets takes place. Firms and salaried workers bargain over wages while firms and the self-employed bargain over the rental rate on capital. Finally, production takes place, workers receive wage payments, and the self-employed pay the rental rate to firms.
3.4.1 Households

As in Andolfatto (1996), I assume an infinitely-lived representative household of measure one, consisting of a large number of family members with perfect risk-pooling across household members. Within the household, individuals can be in salaried employment, self-employment (i.e., own-account or independent work), or unemployment. There is no labor force participation margin and labor supply is perfectly inelastic. Utility only depends on an aggregate consumption good whose price is normalized to one. Households cannot accumulate capital but they are the final owners of salaried firms in the economy.\footnote{This is similar to den Haan, Ramey, and Watson (2003), who assume that entrepreneurs use external funds from financial intermediaries and their own effort to produce. In other words, entrepreneurs cannot accumulate their own capital and must rely solely on external financing. An alternative would be to assume that the household accumulates capital but only uses part of it to finance the projects of the self-employed. The remaining amount would come from external sources such as trade credit. To keep the model tractable, I assume the potential self-employed rely solely on external finance. If the model included on-the-job search where individuals can move from wage employment into self-employment, then capital accumulation while on-the-job may be an alternative assumption to model transitions into self-employment. However, assuming some dependence on external financing or inputs suppliers is in line with the evidence on the constraints faced by small firms.} To move into self-employment, potential self-employed household members must be matched to salaried firms to rent capital for their projects. This is a key distinction from Arseneau, Chugh, and Kurmann (2008) and Kurmann and Petrosky-Nadeau (2007), where firms are the ones posting projects to attract capital in a frictional capital market. The household’s problem is to choose paths of consumption $c_t$, total capital demand next period $k_{SE,t+1}$ (where each self-employed household member uses one unit of
capital to produce), and potential self-employment projects $v_{SE,t}$ to

$$\max \{c_t, k_{SE,t+1}, v_{SE,t}\}_{t=0}^{\infty} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} \right\}$$ (3.1)

subject to the budget constraint

$$c_t + \psi_{SE} \kappa (v_{SE,t}) = (z_t a_{SE,t} - r_{SE,t}) k_{SE,t}^h + w_{S,t} n_{S,t} + \Pi_{S,t} - T_t + bu_t$$ (3.2)

and the law of motion for capital used by the self-employed

$$k_{SE,t+1}^h = (1 - \delta_{SE})(k_{SE,t}^h + v_{SE,t} \rho(\theta_{SE,t}))$$ (3.3)

where $\psi_{SE}$ is the resource cost of posting projects and $\kappa (v_{SE,t})$ is a convex function of self-employment projects such that $\kappa' (v_{SE,t}) > 0$ and $\kappa'' (v_{SE,t}) > 0$. The term $\psi_{SE} \kappa (v_{SE,t})$ can be considered as a start-up cost that the household pays to attract capital suppliers. This cost is required to move into self-employment but does not guarantee that the projects will become productive (i.e., regardless of the payment, there is a positive probability that some projects may not match with capital suppliers and hence may not become active).\(^{18}\) Total production in the self-employment sector is $y_{SE,t} = z_t a_{SE,t} k_{SE,t}^h$, which depends on matched capital, $k_{SE,t}^h$, aggregate productivity in the economy, $z_t$, and self-employment sectoral pro-

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\(^{18}\)This idea is similar to the cost that entrepreneurs have to pay during the initial phase of a project in den Haan and Kaltenbrunner (2009). Note that once a match with an input supplier occurs, $\psi_{SE}$ no longer has to be paid to maintain an existing relationship. Instead, matched self-employed individuals pay a rental rate on capital each period for as long as the relationship with the capital supplier lasts.
ductivity, $a_{SE,t}$. Both $z_t$ and $a_{SE,t}$ follow independent stochastic processes. The shock $a_{SE,t}$ captures the additional volatility inherent to the self-employment sector (Maloney, Cunningham, and Bosch, 2004). Each self-employed individual uses a single unit of matched capital for production. Thus, $h_{SE,t}^h$ is also the measure of active self-employed individuals. Individual earnings for each self-employed member of the household are given by $\pi_{SE,t} = (z_t a_{SE,t} - r_{SE,t})$. $w_{S,t}$ and $\Pi_{S,t}$ stand for the salaried sector wage and salaried firm profits, respectively. $b$ represents unemployment benefits. $\delta^{SE}$ is the exogenous separation rate for matched capital and 
\[
p(\theta_{SE,t}) = \frac{m_{SE}(v_{SE,t}, (1 - \omega_t)k_{S,t})}{v_{SE,t}}
\] is the probability of finding a capital supplier, where $m(v_{SE,t}, (1 - \omega_t)k_{S,t})$ is a constant-returns-to-scale matching function that takes self-employment projects from households $v_{SE,t}$ and capital supply from salaried firms $(1 - \omega_t)k_{S,t}$ as its inputs. Capital market tightness is defined as $\theta_{SE,t} \equiv \frac{v_{SE,t}}{(1 - \omega_t)k_{S,t}}$. Note that a higher $\theta_{SE,t}$ implies a tighter capital market and corresponds to households finding it more difficult to match with a capital supplier. This reduces the unemployed’s probability of entering self-employment, so that $p(\theta_{SE,t})$ is a decreasing function of capital market tightness.

Since a household member who is matched to a firm in the capital market has access to the capital necessary to start production, self-employment $n_{SE,t}$ evolves in exactly the same way as $h_{SE,t}^h$, so that

\[
n_{SE,t+1} = (1 - \delta^{SE})(n_{SE,t} + m(v_{SE,t}, (1 - \omega_t)k_{S,t}))
\] (3.4)

\[\text{In a setting with idiosyncratic shocks, this shock would capture project risk (see Akyol and Athreya, 2009, or Buera and Shin, 2011).}\]
The total employed labor force is given by

\[ n_t = n_{S,t} + n_{SE,t} \]  

(3.5)

where \( n_{j,t} \) represents the labor measure in employment state \( j = S, SE \). I normalize the total labor force to one and define unemployment as

\[ u_t = 1 - n_{S,t} - n_{SE,t} \]  

(3.6)

Denote the representative household’s instantaneous utility function by \( U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \) and the marginal utility of consumption as \( U_c(c_t) \). Combining first-order conditions yields the following self-employment project posting condition:

\[
\frac{\psi_{SE}}{p(\theta_{SE,t})} \left( v_{SE,t} \right) = (1 - \delta_{SE}) E_t \left\{ \beta \frac{U_c(c_{t+1})}{U_c(c_t)} \left( z_{t+1} a_{SE,t+1} - r_{SE,t+1} + \frac{\psi_{SE}}{p(\theta_{SE,t+1})} \left( v_{SE,t+1} \right) \right) \right\}
\]  

(3.7)

The left-hand side represents the expected marginal cost of looking for capital to start a project while the right-hand side captures the expected marginal benefit of matching with a salaried firm that supplies capital. The expected benefit includes both the benefit from producing with that unit of capital net of the cost of renting the unit of capital, \( za_{SE} - r_{SE} \), and the benefit of having the lending relationship continue into the future, i.e. the continuation value. Since households are the ultimate owners of the firms and for future reference, define \( \Xi_t = \beta^t \left( \frac{U_c(c_t)}{U_c(c_0)} \right) \) as the salaried firms’ stochastic discount factor.
3.4.2 Production

The production sector is comprised of a salaried sector with firms that hire salaried workers in a frictional labor market and a self-employment sector with owner-only firms operated by the self-employed who use rented capital and their fixed effort to produce. This section focuses on the salaried firms’ problem since the self-employed’s problem was described above. Output from salaried firms is a function of salaried labor $n_S$, physical capital $k_S$, the share of capital used in production $\omega$, stochastic aggregate productivity $z$, and a time-invariant parameter $a_S$ meant to capture the quality of the institutional environment in the economy. Varying this parameter will affect the steady-state composition of employment in the economy.\(^{20}\) The firm’s production function is assumed to be constant-returns-to-scale. Salaried sector production is given by

$$y_{S,t} = z_t a_S f(n_{S,t}, \omega_t k_{S,t})$$  \hspace{1cm} (3.8)

Since $\omega_t$ is the share of the firm’s capital stock $k_{S,t}$ used in salaried production in period $t$, unused capital $(1-\omega_t)k_{S,t}$ represents the supply of capital for matching with the self-employed. Firms choose a sequence of vacancies for salaried employment $v_{S,t}$, salaried employment next period $n_{S,t+1}$, total capital next period $k_{S,t+1}$, the fraction of the capital stock used in production today $\omega_t$, and capital to be lent out

\(^{20}\)This is a natural parameter to vary since the institutional setting of a country, particularly as it influences employment arrangements, affects mainly the salaried sector (Akyol and Athreya, 2009; Fiess et al., 2010). See Pietrobelli, Rabello, and Aquilina (2004) and Ardagna and Lusardi (2008) for an overview of some of the main determinants of self-employment and entrepreneurship.
to the self-employed next period \( k_{SE,t+1} \) to

\[
\max_{\{v_{S,t},n_{S,t+1},k_{S,t+1},\omega_t,k_{SE,t+1}\}} \sum_{t=0}^{\infty} E_0 \Xi_t \begin{cases} 
z_t a_S f(n_{S,t},\omega_t k_{S,t}) - w_{S,t} n_{S,t} \\
-\psi_S v_{S,t} - i_t + r_{SE,t} k_{SE,t}^f \end{cases} \tag{3.9}
\]

subject to the law of motion for salaried employment

\[ n_{S,t+1} = (1 - \delta^S)(n_{S,t} + v_{S,t} q(\theta_{S,t})) \tag{3.10} \]

the evolution of capital used in the self-employment sector

\[ k_{SE,t+1}^f = (1 - \delta^{SE}) (k_{SE,t}^f + (1 - \omega_t) k_{S,t} q(\theta_{SE,t})) \tag{3.11} \]

and the evolution of the firm’s total capital stock\(^{21}\)

\[ k_{S,t+1} = i_t + (1 - \delta) \omega_t k_{S,t} + (\delta^{SE} - \delta) k_{SE,t}^f + [(1 - \delta)(1 - \omega_t) k_{S,t} - (1 - \delta^{SE})(1 - \omega_t) k_{S,t} q(\theta_{SE,t})] \tag{3.12} \]

The term \( \psi_S v_{S,t} \) captures the total cost of posting salaried vacancies.\(^{22}\) The salaried employment job-filling probability is \( q(\theta_{S,t}) = \frac{m_S (u_t, v_{S,t})}{v_{S,t}} \), where \( m_S (v_{S,t}, u_t) \) is a constant-returns-to-scale matching function for salaried employment, \( \delta^S \) is the separation rate for salaried workers, and \( \delta^{SE} \) is the self-employment separation rate.

\(^{21}\)Note that combining the law of motion for capital held by the firm and the law of motion for capital in self-employment yields a standard law of motion for total capital in the economy.

\(^{22}\)One can introduce a resource cost of supplying capital to proxy for the administrative and monitoring costs associated with the supply of input credit. Introducing such a cost does not change the main results of the paper. A similar comment applies to investment adjustment costs.
The probability of finding a self-employment project is \( q(\theta_{SE,t}) = \frac{m_{SE}(v_{SE,t} (1-\omega_t) k_{S,t})}{(1-\omega_t) k_{S,t}} \), where once again \( \theta_{SE,t} = \frac{v_{SE,t}}{(1-\omega_t) k_{S,t}} \) embodies tightness in the market for physical capital. Note that higher capital market tightness from the salaried firms’ perspective means that firms find it easier to match their unused capital with self-employment projects. Thus, \( q(\theta_{SE,t}) \) is increasing in \( \theta_{SE,t} \). The expression \([ (1-\delta) - (1-\delta^{SE}) q(\theta_{SE,t}) ] (1-\omega_t) k_{S,t} \) captures the fact that firms subtract the capital matched in \( t \) after taking into account separations, which is given by \((1-\delta^{SE}) q(\theta_{SE,t})(1-\omega_t) k_{S,t} \), from the capital that was devoted to matching in \( t \), net of depreciation, which is given by \((1-\delta)(1-\omega_t) k_{S,t} \). As in Kurmann and Petrosky-Nadeau (2007) and Arseneau et al. (2008), the firm considers unmatched capital as part of capital accumulation, net of depreciation. The term \( \delta k_{SE,t}^f \) is an expense for the firm since capital in self-employment depreciates each period, so the term must be subtracted from the capital available for allocation at the beginning of period \( t+1 \) after accounting for the capital that is returned to the firm due to separations, given by \( \delta^{SE} k_{SE,t}^f \). For future reference, define \( u_{k,t} \equiv (1- q(\theta_{SE,t}))(1-\omega_t) k_{S,t} \) as unmatched idle capital before depreciation. Combining first-order conditions yields a standard job creation condition:

\[
\frac{\psi_S}{q(\theta_{S,t})} = (1-\delta^S) E_t \Xi_{t+1} | t \left\{ z_{t+1} a_S f_{n_2} (n_{S,t+1}, \omega_{t+1} k_{S,t+1}) - w_{S,t+1} + \frac{\psi_S}{q(\theta_{S,t+1})} \right\} \tag{3.13}
\]

a standard Euler equation for capital \( k_S \):

\[
1 = E_t \Xi_{t+1} | t \left\{ z_{t+1} a_S f_{\omega k_S} (n_{F,t+1}, n_{I,t+1}, \omega_{t+1} k_{S,t+1}) + (1-\delta) \right\} \tag{3.14}
\]
and a capital supply condition for capital allocated to matching with self-employment projects:

\[
(1 - \delta^{SE})E_t z_{t+1} a_S f_\omega k_S(n_{S,t}, \omega_t k_{S,t} + 1) + (1 - \delta^{SE})q(\theta_{SE,t} + 1)
\]

\[
= \left( \frac{z_t a_S f_\omega k_S(n_{S,t}, \omega_t k_{S,t}) + (1 - \delta^{SE})q(\theta_{SE,t})}{q(\theta_{SE,t})} \right) \left( r_{SE,t+1} + (\delta^{SE} - \delta) \right)
\]

The job creation condition states that the expected marginal cost of a vacancy is equal to the expected marginal benefit of having a worker in the firm, which takes into account the continuation value since employment relationships are long-lived. The capital Euler equation balances the cost and benefit of obtaining an additional unit of capital, regardless of whether it is used within the firm or lent out to a self-employed individual.

The capital supply condition is similar but not identical to the one in Arsenneau et al. (2008). The left-hand side gives the expected marginal cost of lending an additional unit of capital to self-employed workers. This cost includes the opportunity cost of devoting a unit of capital to matching, \( z_t a_S f_\omega k_S(n_{S,t}, \omega_t k_{S,t}) \), which is the marginal product of capital used inside salaried firms. The second term captures the fact that if capital is matched today with probability \( q(\theta_{SE,t}) \) and survives with probability \( 1 - \delta^{SE} \), then the firm must set aside that capital and hence cannot count it as part of idle capital (after matching) within the firm. The right-hand side gives the expected benefit of a matched unit of capital. The benefit for the firm is the rental rate obtained if the relationship survives next period, net of the depreci-
ation of capital $\delta$, which the firm must cover for surviving relationships. Also, the firm takes into account that the match ends with probability $\delta^{SE}$, in which case the firm gets back a unit of capital. The last term on the right-hand side reflects the continuation value of a long-term capital relationship. The intertemporal nature of the optimal capital supply decision arises due to the existence of long-term relationships between the self-employed and their input suppliers. Note that the capital supply condition is a decision made by the firm and not by the household, since salaried firms are the ones making capital accumulation decisions. Furthermore, the expected cost of investing capital in self-employment ventures will depend on the marginal product of capital within the firm since the firm faces a tradeoff between trying to match capital and using it in-house. These are two key distinctions relative to the setting in Arseneau et al. (2008).

### 3.4.3 Labor Market Search, Capital Search, and Nash Bargaining

The job-finding and job-filling rates are given by $p(\theta_{S,t}) = \frac{m_s(u_{t},v_{S,t})}{u_{t}}$ and $q(\theta_{S,t}) = \frac{m_s(u_{t},v_{S,t})}{v_{S,t}}$, respectively. Thus, salaried labor market tightness is $\theta_{S,t} = \frac{p(\theta_{S,t})}{q(\theta_{S,t})}$. The probability of finding a capital supplier is $p(\theta_{SE,t}) = \frac{m_{SE}(v_{SE,t},(1-\omega_t)k_{S,t})}{v_{SE,t}}$ and the probability of finding a self-employment project is $q(\theta_{SE,t}) = \frac{m_{SE}(v_{SE,t},(1-\omega_t)k_{S,t})}{(1-\omega_t)k_{S,t}}$. Then, capital market tightness is $\theta_{SE,t} = \frac{q(\theta_{SE,t})}{p(\theta_{SE,t})}$. 

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3.4.3.1 Household Value Function

The value function of a salaried worker currently employed at a firm, $W_{S,t}$, is given by

$$W_{S,t} = \lambda^h w_{S,t} + E_t \beta \left\{ (1 - \delta^S)W_{S,t+1} + \delta^S W_{U,t+1} \right\}$$  \hspace{1cm} (3.16)

where labor income is weighted by the marginal utility of consumption, $\lambda^h_t = U_c(c_t)$.

The value function of a self-employed individual currently matched with a capital supplier, $W_{SE,t}$, is given by

$$W_{SE,t} = \lambda^h_t (z_t a_{SE,t} - r_{SE,t}) + E_t \beta \left\{ (1 - \delta^{SE})W_{SE,t+1} + \delta^{SE} W_{U,t+1} \right\}$$  \hspace{1cm} (3.17)

To incorporate the probability of entry into self-employment in the value function of an unemployed individual, define $v^u_{SE,t} \equiv \frac{v_{SE,t}}{u}$, which gives the number of project postings per unemployed household member. Hence, the effective probability of finding a capital supplier per unemployed individual, which I denote by $p^u(\theta_{SE,t})$, is given by $v^u_{SE,t} p(\theta_{SE,t})$. The value function of an unemployed individual, $W_{U,t}$, can then be written as

$$W_{U,t} = \lambda^h_t b + E_t \beta \left\{ \begin{array}{l}
(1 - \delta^S)p(\theta_{S,t}) W_{S,t+1} \\
+(1 - \delta^{SE}) v^u_{SE,t} p(\theta_{SE,t}) W_{SE,t+1} \\
+[1 - (1 - \delta^S)p(\theta_{S,t})] W_{U,t+1} \\
-(1 - \delta^{SE}) v^u_{SE,t} p(\theta_{SE,t}) \end{array} \right\}$$  \hspace{1cm} (3.18)
where \( b \) denotes unemployment benefits measured in consumption units.\(^{23}\)

### 3.4.3.2 Firm Value Functions

The marginal value of a salaried firm from having a salaried worker, \( J_{S,t} \), is given by

\[
J_{S,t} = z_t a_S f_{n_S} (n_{S,t}, \omega_t k_{S,t}) - w_{S,t} + E_{t} \Xi_{t+1|t} \left\{ (1 - \delta^S) J_{S,t+1} \right\}
\]

(3.19)

The marginal value of a salaried firm from renting a unit of capital to a self-employed household member is

\[
J_{SE,t} = r_{SE,t} + (\delta^{SE} - \delta) + E_{t} \Xi_{t+1|t} \left\{ (1 - \delta^{SE}) J_{SE,t+1} \right\}
\]

(3.20)

where the firm takes into account that the credit relationship will survive into the next period with probability \((1 - \delta^{SE})\). Also, recall that firms must cover the depreciation of the matched unit of capital, \( \delta \), but gain a unit of previously matched capital if it is separated with probability \( \delta^{SE} \). This yields the term \((\delta^{SE} - \delta)\) in the expression for \( J_{SE,t} \) above. Finally, the value of having unused, idle capital is given

\(^{23}\)In principle, the expression for \( W_{U,t} \) suggests that it is possible for an unemployed individual to be matched simultaneously to more than one type of employment. Assuming otherwise would rule out closed-form solutions for the wage and the rental rate. Since these price expressions provide intuition for the influence of self-employment on wages (and therefore hiring), I assume that individuals can only be in a single type of employment at any given time after being matched. An alternative would be to fully account for the fact that, if there is a match in a given employment state, the same individual cannot be matched in the other employment state. This would imply that the wage in period \( t \) can only be expressed as a function of each of the value functions in \( t + 1 \) and no closed-form solutions could be obtained. The downside of this approach is that we would not be able to explicitly observe how the possibility of self-employment affects the wage and the capital rental rate.
by $J_{u_k,t} = (1 - \delta)$ since capital depreciates every period at rate $\delta$.

3.4.3.3 Nash Bargaining: Wage and Rental Rate Determination

Real wages for salaried workers are determined by Nash bargaining with salaried firms. The real wage $w_{S,t}$ solves

$$
\max_{w_{S,t}} \left\{ \left( \frac{W_{S,t} - W_{U,t}}{\lambda_t^h} \right)^{\nu_S} (J_{S,t})^{1-\nu_S} \right\}
$$

(3.21)

where $\left( \frac{W_{S,t} - W_{U,t}}{\lambda_t^h} \right)$ is the worker’s surplus, $J_{S,t}$ is the salaried firm’s surplus, and $\nu_S$ is the worker’s bargaining power. In a similar fashion, the Nash bargaining problem in the capital market is

$$
\max_{r_{SE,t}} \left\{ \left( \frac{W_{SE,t} - W_{U,t}}{\lambda_t^h} \right)^{\nu_{SE}} (J_{SE,t} - J_{u_k,t})^{1-\nu_{SE}} \right\}
$$

(3.22)

where the self-employed household member’s surplus is $\left( \frac{W_{SE,t} - W_{U,t}}{\lambda_t^h} \right)$ and the salaried firm’s surplus is $(J_{SE,t} - J_{u_k,t})$, where the threat point is the value of unmatched, idle capital.

Using the first-order conditions from (21) and (22), the Nash bargaining solu-
tion for the wage and the rental rate can be shown to be

\[ w_{S,t} = \nu_S [z_t a_S f_{n_S}(n_{S,t}, \omega_t k_{S,t}) + \psi_S \theta_{S,t}] + (1 - \nu_S)b \]  

(3.23)

\[ + \frac{(1 - \nu_S)\nu_{SE}}{1 - \nu_{SE}} v_{SE,t}^u p(\theta_{SE,t}) \left[ \frac{z_t a_S f_{\omega k_S}(n_{S,t}, \omega_t k_{S,t})}{q(\theta_{SE,t})} \right] \]

\[ + \frac{(1 - \nu_S)\nu_{SE}}{1 - \nu_{SE}} v_{SE,t}^u p(\theta_{SE,t})(1 - \delta^{SE}) \left[ 1 - E_t \Xi_{t+1|t}(1 - \delta) \right] \]

Similarly, the rental rate for matched capital is

\[ r_{SE,t} = (1 - \nu_{SE}) \left[ z_t a_{SE,t} - \frac{\nu_S}{1 - \nu_S} \psi_S \theta_{S,t} - b \right] \]  

(3.24)

\[ - \nu_{SE} v_{SE,t}^u p(\theta_{SE,t}) \left[ \frac{z_t a_S f_{\omega k_S}(n_{S,t}, \omega_t k_{S,t})}{q(\theta_{SE,t})} \right] \]

\[ + \nu_{SE} (1 - v_{SE,t}^u p(\theta_{SE,t}))(1 - \delta^{SE}) \left[ 1 - E_t \Xi_{t+1|t}(1 - \delta) \right] \]

The wage and the rental rate depend, among other things, on market tightness in the other employment state. Intuitively, in a model with search frictions and two types of employment, the prices of factors of production should not only depend on the marginal product of each of the factors but also on the potential opportunities in other employment states, which are embodied in market tightness in each of these states. Thus, salaried wages and the rental rate on capital used in self-employment

\[ ^{24}\text{Appendix B describes the derivation of the wage and rental rate equations.} \]
are affected by the outside options available in the economy.

3.4.3.4 Intuition Behind the Wage and Rental Rate Equations

To understand better how conditions in one labor market spill over into other markets, consider the wage in the salaried sector. The wage equation in a standard one-sector search model without self-employment would be given by

\[ w_{s,t} = \nu_s \left( z_t s m p_{s,t} + \psi_s \theta_{s,t} \right) + (1 - \nu_s) b \]

where \( m p_{s,t} \) is the marginal product of labor and \( \nu_s \) is the bargaining weight for workers. Both higher labor market tightness \( \theta_{s,t} \) and a higher outside option in unemployment \( b \) lead to a higher wage.

Now, consider the expression for the wage in the economy with self-employment. To see how market tightness in the capital market – and hence the self-employment outside option – affects the wage, rewrite the last two lines of the wage equation as

\[ v_{SE,t} \left( 1 - \nu_{SE} \right) \nu_{SE} \left( 1 - \nu_{SE} \right) \left( z_t s f_k (n_{s,t} \omega_t k_{s,t}) + p(\theta_{SE,t}) \right) \left( 1 - \delta^{SE} \right) \left[ 1 - E_t \Xi_{t+1,t} (1 - \delta) \right] \]

where recall that capital market tightness is defined as \( \theta_{SE,t} = \frac{v_{SE,t}}{(1 - \omega_t) k_{s,t}} \) and given constant-returns in matching, \( \theta_{SE,t} = \frac{a(\theta_{SE,t})}{p(\theta_{SE,t})} \). The first term inside the brackets represents the foregone marginal product of capital if the capital devoted to matching were instead used inside the firm. The second element captures the fact that firms participating in the capital matching process must set aside \( (1 - \delta^{SE}) \) units of capital.
in the current period for those new matches that become productive next period, and must cover depreciation costs for matched capital next period. This second term is also part of the opportunity cost of successfully matching capital. Thus, it is weighted by the probability of a successful match from the household’s perspective. This cost is \((1 - \delta SE) \left[ 1 - E_t \Xi_{t+1:t}(1 - \delta) \right] \) since firms net out the depreciation of capital from the total opportunity cost of storing the capital today until it becomes productive next period. In summary, the term in brackets is akin to the vacancy posting cost for salaried employment, except that the cost of matching capital depends on the opportunity cost of capital and the cost of covering depreciation for capital relationships.

Having described each component of the wage, note that an increase in capital market tightness \(\theta SE,t\) ceteris paribus decreases the expected cost to the firm of devoting capital to matching. Given that the household’s probability of finding a capital supplier \(p(\theta SE,t)\) is decreasing in \(\theta SE,t\), an increase in capital market tightness reduces an unemployed individual’s probability of entering self-employment.\(^{25}\) This implies that an increase in \(\theta SE,t\) reduces the salaried sector wage since the worker’s outside option of going into self-employment is now less valuable. In other words, while an increase in salaried labor market tightness \(\theta S,t\) tends to push the wage up, an increase in capital market tightness \(\theta SE,t\) goes in the opposite direction since the latter implies a lower probability of moving into self-employment – and therefore a

\(^{25}\)Clearly, an increase in \(\theta SE,t\) has implications for the marginal product of capital used inside the salaried firm if \(\theta SE,t\) changes due to \((1 - \omega_t)k_{S,t}\). Thus, the impact of a change in \(\theta SE,t\) is more complex than what my description suggests. However, for the purposes of developing intuition about the wage and the rental rate, it proves useful to see how changes in \(\theta SE,t\) due to \(v_{SE,t}\), holding everything else constant, would affect each of the prices.
weaker outside option – from the point of view of household members.

Now consider the determinants of the capital rental rate for the self-employed. First, from equation (3.24), the rental rate depends positively on output of a given self-employed individual, \( z_t a_{SE,t} \). This is similar to the wage equation, in which the wage is increasing in the marginal product of labor. Also, an increase in the value of the other two outside options for the household (unemployment or salaried employment) through higher salaried market tightness \( \theta_{S,t} \) or a higher outside option \( b \) puts downward pressure on the rental rate and increases self-employment earnings. Meanwhile, an increase in capital market tightness \( \theta_{SE,t} \) puts upward pressure on the rental rate, since a tighter capital market reduces the matching probability for households \( p(\theta_{SE,t}) \) and raises the capital matching probability for salaried firms \( q(\theta_{SE,t}) \). Finally, note that a higher marginal product of capital for salaried firms increases the opportunity cost of devoting capital to matching, which pushes firms to decrease the supply of capital for matching. Since this increases \( \theta_{SE,t} \) for a given number of self-employment projects, the impact on the rental rate is in principle ambiguous. However, note that a higher marginal product of capital also pushes firms to post more salaried vacancies, thereby increasing salaried labor market tightness. Higher labor market tightness, in turn, puts downward pressure on the rental rate. Then, we should expect the rental rate to be lower if the change in labor market tightness dominates the influence exerted by capital market tightness and the opportunity cost of capital on the rental rate. This discussion can be summarized
as:

\[ w_{S,t} = \Theta_{S,t}(mpl_S, \theta_S, \theta_{SE}, mpk_S), \quad r_{SE,t} = \Theta_{SE,t}(z_S, \theta_S, \theta_{SE}, mpk_S) \]  \hspace{1cm} (3.25) \\

Lower capital market tightness \( \theta_{SE,t} \) puts upward pressure on \( w_{S,t} \) since the self-employment option becomes a more accessible employment alternative. Also, as I discuss in detail in the results section below, \( w_{S,t} \) and \( r_{SE,t} \) are more sensitive to fluctuations in \( \theta_{SE,t} \) in economies with higher steady-state self-employment (or equivalently, lower \( \theta_{SE} \)).

3.4.4 Government, Total Output, and Resource Constraint

The government levies lump-sum taxes to finance unemployment benefits, so the government budget constraint is

\[ T_t = bu_t \]  \hspace{1cm} (3.26) \\

Total output \( y_t \) is given by the sum of output from salaried firms, \( y_{S,t} \), and output from the self-employed, \( y_{SE,t} \):

\[ y_t = y_{S,t} + y_{SE,t} \]  \hspace{1cm} (3.27) \\

In the model, \( y_t \) represents observed output in the data. Most countries follow United Nations guidelines and incorporate estimates of output from the informal sec-
tor, which most of the self-employed belong to, into their national income accounts (Quintin and Pratap, 2006; United Nations, 2008).²⁶

As in most models with labor search frictions, the costs of posting vacancies and posting projects are resource costs and enter explicitly into the economy’s resource constraint. Thus, the aggregate resource constraint is given by

\[ y_t = c_t + i_t + \psi_S v_{S,t} + \psi_{SE}(v_{SE,t}) \]  (3.28)

where total output \( y_t \) and investment \( i_t \) were defined above.

### 3.4.5 Competitive Equilibrium

**Definition** (Competitive Equilibrium) Taking the set of exogenous processes \( \{z_t, a_{SE,t}\} \) as given, the allocations \( \{c_t, n_{S,t}, n_{SE,t}, \theta_{S,t}, \theta_{SE,t}, k_{S,t}, k_{SE,t}, \omega_t, u_t, T_t, y_t\}_{t=0}^{\infty} \) as well as prices \( \{w_{S,t}, r_{SE,t}\}_{t=0}^{\infty} \) satisfy the law of motion for capital used in self-employment (3.3), the law of motion for self-employment (3.4), the definition of unemployment (3.6), the self-employed’s demand for self-employed capital (3.7), the law of motion for salaried employment (3.10), the salaried firms’ salaried job creation condition (3.13), the Euler equation for capital (3.14), the salaried firms’ capital supply of self-employed capital (3.15), the Nash wage and the Nash rental rate equations (3.23) and (3.24), the government budget constraint (3.26), the definition of total output (3.27), and the economy’s resource constraint (3.28).

²⁶I focus on urban employment and therefore abstract from economic activity in agriculture, even though developing countries have large self-employment shares in the sector. Strictly speaking, total output should include an additional term that captures the contribution of the agricultural sector to total production. In my framework, this could be done by including a constant term in the definition of total output.
3.5 Calibration and Solution Method

Given my interest in economies with sizeable self-employment shares, a natural choice for the calibration of the model is a developing country. I choose Mexico as a benchmark because its household and firm surveys give a detailed overview of self-employment in the labor market. Moreover, Bosch and Maloney (2008) have documented the cyclical dynamics of salaried employment and self-employment in Mexico, so their work is an excellent source for some of the parameter values and calibration targets used to simulate the model.

To analyze the dynamic behavior of the economy, I log-linearize the model around the non-stochastic steady state and compute a first-order approximation of the equilibrium conditions. I simulate the economy for 2100 periods, drop the first 100 periods, and use the Hodrick-Prescott (HP) filter with smoothing parameter 1600 to filter the simulated data, extract the cyclical component and compute the moments of interest.\footnote{I use Dynare for all dynamic simulations. Using higher-order approximations does not change the main conclusions of the paper.}

3.5.1 Functional Forms and Stochastic Processes

Aggregate productivity $z_t$ and self-employment productivity $a_{SE,t}$ follow independent AR(1) processes:

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon^z_t$$  \hspace{1cm} (3.29)
\[
\ln a_{SE,t} = \rho_{SE} \ln a_{SE,t-1} + \varepsilon_{t}^{SE} \tag{3.30}
\]

where \( \varepsilon_{t}^{z} \sim \text{iid } N(0, \sigma_{z}) \), \( \rho_{z} < 1 \), \( \varepsilon_{t}^{SE} \sim \text{iid } N(0, \sigma_{SE}) \), and \( \rho_{SE} < 1 \). Salaried output is Cobb-Douglas and given by \( y_{S,t} = z_{t} a_{S}(n_{S,t})^{1-\alpha_{S}} (\omega_{t} k_{S,t})^{\alpha_{S}} \), \( 0 < \alpha_{S} < 1 \). Recall that \( a_{S} \) is a time-invariant parameter in the salaried sector and not a shock. The cost of posting projects is given by \( \kappa(v_{SE,t}) = (v_{SE,t})^{\eta_{SE}} \) with \( \eta_{SE} > 1 \). The matching function for salaried employment is constant-returns-to-scale, so \( m_{S}(u_{t}, v_{S,t}) = M_{S} u_{t}^{1-\xi} \), \( 0 < \xi < 1 \), where \( M_{S} \) is the salaried matching efficiency parameter, \( u_{t} \) is unemployment, and \( v_{S,t} \) denotes salaried vacancies.

Then the salaried job-finding rate is \( p(\theta_{S,t}) = \frac{m_{S}(u_{t}, v_{S,t})}{u_{t}} = M_{S} u_{t}^{1-\xi} u_{t}^{\xi-1} = M_{S} \theta_{S,t}^{1-\xi} \), and the salaried job-filling rate is given by \( q(\theta_{S,t}) = \frac{m_{S}(u_{t}, v_{S,t})}{v_{S,t}} = M_{S} u_{t}^{1-\xi} v_{S,t}^{\xi} = M_{S} \theta_{S,t}^{-\xi} \). The matching function in the capital market is also Cobb-Douglas, so that \( m_{SE}((1-\omega_{t}) k_{S,t}, v_{SE,t}) = M_{SE}((1-\omega_{t}) k_{S,t})^{\xi_{SE}} (v_{SE,t})^{1-\xi_{SE}} \), \( 0 < \xi_{SE} < 1 \). The probability of finding a self-employed individual from the point of view of the firm is \( q(\theta_{SE,t}) = \frac{m_{SE}((1-\omega_{t}) k_{S,t}, v_{SE,t})}{(1-\omega_{t}) k_{S,t}} = M_{SE}((1-\omega_{t}) k_{S,t})^{\xi_{SE}-1} (v_{SE,t})^{1-\xi_{SE}} = M_{SE} \theta_{SE,t}^{\xi_{SE}} \), while the probability of finding a capital supplier is \( p(\theta_{SE,t}) = \frac{m_{SE}((1-\omega_{t}) k_{S,t}, v_{SE,t})}{v_{SE,t}} = M_{SE}((1-\omega_{t}) k_{S,t})^{\xi_{SE}} (v_{SE,t})^{-\xi_{SE}} = M_{SE} \theta_{SE,t}^{-\xi_{SE}} \).

### 3.5.2 Parametrization and Calibration Targets

I borrow a number of parameter values that have been used in related studies and calibrate the remaining parameters using specific targets for Mexico. Table 3.3 below presents the parameter values adopted from existing literature. Table 3.4
shows the parameters whose values are obtained by imposing particular calibration targets from the data and solving for the model’s non-stochastic steady state.

**Parameters Taken from Existing Literature**  The time period in the model is one quarter, so I set the discount factor $\beta$ to 0.98, consistent with common values assumed for developing countries. The capital share in the firm’s production function is set to $\alpha_S = 0.30$, a common value in DSGE models. The depreciation rate of capital is set to 0.02. Bosch and Maloney (2007) compute transition probabilities between employment states for Mexico using quarterly data from the National Survey on Urban Employment (*Encuesta Nacional de Empleo Urbano*, or ENEU) for years 1987 through 2002. They find that separation rates for informal salaried workers are at least twice as high as those for formal salaried workers. Self-employment separation rates are closer to those of formal workers.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter Description</th>
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<td>Salaried Separation Rate</td>
<td>Bosch, Maloney (2007)</td>
</tr>
<tr>
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<td>SE Separation Rate</td>
<td>Bosch, Maloney (2007)</td>
</tr>
<tr>
<td>$\nu_S$</td>
<td>0.50</td>
<td>Salaried Bargaining Power</td>
<td>Search Lit.</td>
</tr>
<tr>
<td>$\nu_{SE}$</td>
<td>0.50</td>
<td>SE Bargaining Power</td>
<td>Search Lit.</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.90</td>
<td>Autocorrelation of TFP</td>
<td>DSGE Lit.</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.50</td>
<td>Matching Elasticity, Labor</td>
<td>Search Lit.</td>
</tr>
<tr>
<td>$\xi_{SE}$</td>
<td>0.50</td>
<td>Matching Elasticity, Capital</td>
<td>Search Lit.</td>
</tr>
</tbody>
</table>
Since I have a single type of salaried employment, I set the salaried separation rate to be a weighted average of the two salaried separation rates, where the weights are given by each salaried type’s contribution to total salaried employment. This implies that $\delta^{SE} < \delta^S$. The conclusions in the paper remain intact if I make a distinction between formal and informal salaried employment in the model. I set the persistence parameter for $z_t$ to 0.90, in line with the DSGE literature. The elasticities of the matching functions and the bargaining power parameters are set to $\xi = \xi_{SE} = 0.50$ and $\nu_S = \nu_{SE} = 0.50$, respectively, consistent with the labor search literature (and to maintain as much symmetry as possible). $a_S$ is a free parameter that I vary to generate changes in steady-state self-employment in the model. I fix its value to 0.60 in the benchmark economy. Variations in $a_S$ allow me to generate sizeable changes in steady-state self-employment, ranging from 6 percent to 79 percent of the labor force. The benchmark value for $a_S$ allows me to obtain these changes in steady-state self-employment while keeping the share of capital used in salaried production bounded below one. 28

As I discuss below, I treat this parameter as a proxy for institutional quality.

**Calibrated Parameters** The remaining parameters are jointly calibrated by solving for the non-stochastic steady state of the model and imposing specific data targets. The calibrated parameters are: $\eta_{SE}, M_S, M_{SE}, \psi_S, \psi_{SE}, \rho_{SE}, \sigma_{SE},$ and $\sigma_z$.

Recall that $\eta_{SE}$ governs the curvature of the cost of posting self-employment projects
and therefore the probability of entry into self-employment through the household’s project posting condition. I calibrate $\eta_{SE}$ to match the volatility of the transition probability from unemployment to self-employment reported in Bosch and Maloney (2008). Given my benchmark assumption for $a_S$, I choose the matching scale parameters $M_j$, for $j = S, SE$ to match the average shares of salaried employment and self-employment in the total labor force for Mexico from 1987 to 2002, which are obtained from the ENEU and the ILO. The sectoral productivity shock to self-employment $a_{SE,t}$ aims to capture the added volatility and riskiness of the self-employment sector. Given the structure of self-employment entry in the model, I calibrate the persistence of the self-employment productivity shock to match the cyclical correlation between output and the transition rate from unemployment to self-employment documented in Bosch and Maloney (2008). I set the vacancy posting cost for salaried positions $\psi_S$ to be 3.5 percent of wages, as documented in Levy (2007). The target for the posting project cost parameter $\psi_{SE}$ is taken from McKenzie and Woodruff (2006), who document evidence on startup costs among microenterprises for different sectors using Mexico’s microenterprise survey (Encuesta Nacional de Micronegocios, or ENAMIN). The target for $\psi_{SE}$ represents three months of wages, which is a lower-bound estimate for micro firms in construction and personal services.

I calibrate $\sigma_{SE}$ to match the cyclical correlation between the transition rate from unemployment into salaried employment and output in Bosch and Maloney (2008). Note that this also generates a volatility differential between self-employment...

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29While I use the cyclical correlation between the transition rate from unemployment to self-employment and output in the data to calibrate the persistence of the shock to self-employment...
Table 3.4: Calibrated Parameters and Targets: Benchmark Economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter Description</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{SE}$</td>
<td>1.11</td>
<td>Curvature, Project Postings</td>
<td>$\sigma_{\psi(\theta_{SE,t})} = 9.4$</td>
</tr>
<tr>
<td>$M_S$</td>
<td>0.198</td>
<td>Formal Matching Efficiency</td>
<td>$n_S = 0.72$</td>
</tr>
<tr>
<td>$M_{SE}$</td>
<td>0.028</td>
<td>SE Matching Efficiency</td>
<td>$n_{SE} = 0.23$</td>
</tr>
<tr>
<td>$\psi_S$</td>
<td>0.027</td>
<td>Formal Vacancy Cost</td>
<td>3.5 percent of wages</td>
</tr>
<tr>
<td>$\psi_{SE}$</td>
<td>0.759</td>
<td>Project Posting Cost</td>
<td>3 months of wages</td>
</tr>
<tr>
<td>$\rho_{SE}$</td>
<td>0.67</td>
<td>Autocorr. SE Prod.</td>
<td>$\rho_{p(\theta_{SE,t}),yt} = -0.43$</td>
</tr>
<tr>
<td>$\sigma_{SE}$</td>
<td>0.025</td>
<td>SD SE Productivity Shock</td>
<td>$\rho_{p(\theta_{S,t}),yt} = 0.66$</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0155</td>
<td>SD Aggregate TFP Shock</td>
<td>$\sigma_y = 2.17$</td>
</tr>
</tbody>
</table>

earnings and salaried employment earnings, which is in line with the evidence on
the higher earnings volatility in self-employment (see Hamilton, 2000, for the U.S.,
and Bargain and Kwenda, 2010, for Brazil and Mexico). Finally, the standard
deviation of TFP is chosen to match the standard deviation of output for Mexico
as reported in Lama and Urrutia (2012).

Table 3.5 shows the moments generated by the benchmark calibration of the
model. It also shows three additional moments that are not targeted, mainly the
cyclical correlation between self-employment and output, the cyclical correlation be-
tween salaried employment and output, and the correlation between unemployment
and output for Mexico. The third column shows the same moments generated by
an alternative version of the benchmark model where there are no capital search
productivity, the model still generates the countercyclical transition rate from unemployment into
self-employment (and the countercyclicity of self-employment) when I shut down the sectoral
shock. The only reason I use this moment as a target is to avoid assigning arbitrary parameter
values to the self-employment productivity process. Calibrating the process also allows me to have
a more disciplined quantitative experiment.

30Since I make no distinction between formal and informal salaried workers in the model, the
target for the cyclical correlation of entry into salaried employment and output is a weighted
average of the cyclical correlations for the two employment states, where the weights are given by
the respective shares of formal and informal salaried workers in total salaried employment.
frictions, and transitions from unemployment into self-employment depend solely on a fixed probability of becoming self-employed.\textsuperscript{31}

Table 3.5: Benchmark Economy vs. Model with Exogenous Transition Rate Into Self-Employment

<table>
<thead>
<tr>
<th>Targeted Moments</th>
<th>Benchmark Model</th>
<th>Model with Exogenous SE Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_S$</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>$n_{SE}$</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>$\rho_{p^u(\theta_{SE,t}),y_t}$</td>
<td>$-0.42$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\rho_{p(\theta_{S,t}),y_t}$</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>$\sigma_{p^u(\theta_{SE,t})}$</td>
<td>9.4</td>
<td>$-$</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>2.18</td>
<td>2.18</td>
</tr>
</tbody>
</table>

| Other Moments                                 |                 |                               |
| $\rho_{n_{SE,t},y_t}$                         | $-0.56$         | $0.12$                        |
| $\rho_{n_{S,t},y_t}$                          | 0.69            | 0.57                          |
| $\rho_{u_t, y_t}$                             | $-0.65$         | $-0.49$                       |

The benchmark model is able to explain the negative correlations between output and self-employment and unemployment observed in the data. In contrast, a model with an exogenous transition rate into self-employment and no capital search frictions cannot replicate the contemporaneous cyclical correlation between self-employment and output observed in the data.\textsuperscript{32} This model also yields weaker cyclical correlations between salaried employment and output and unemployment and output, respectively. The results in Table 3.5 highlight the importance of endogenizing the transition rate into self-employment to capture the cyclical facts about

\textsuperscript{31}The latter can be interpreted as the probability of receiving an idea for a self-employment venture. This alternative version of the model assumes that (fixed) effort is the only input in self-employment.

\textsuperscript{32}There are no plausible parameter values for the self-employment productivity process that yield the cyclicality of entry into self-employment (or the cyclicality of self-employment) in the model with exogenous entry into self-employment.
self-employment as well as other cyclical features of the labor market.

Changes in the steady-state share of self-employment across simulations are engineered by varying the salaried sector productivity parameter, $a_S$. I interpret changes in this parameter as differences in the quality of institutions, which have been shown to be important for cross-country differences in self-employment rates (see, for example, Loayza and Rigolini, 2011).  

3.6 Simulation Results

3.6.1 Steady State

Figure 3.2 shows the steady-state relationship between self-employment and other model variables traced out by variations in the institutional quality parameter $a_S$, holding other model parameters fixed. The parameter $a_S$ is centered at 0.60 for the benchmark economy and varies between 0.54 and 0.63 to generate steady-state self-employment shares ranging from 6.6 percent of the labor force to 79 percent of the labor force. First, note that an increase in $a_S$ causes a fall in steady-state self-employment by raising the marginal product of salaried labor and the marginal product of capital used in the salaried sector.

These changes in marginal products increase the value of posting salaried

---

33The correlation between $a_S$ in the model and the index of Law and Order from Political Risk Services is 0.55 and significant at the 1 percent level, while the correlation between $a_S$ and Rule of Law from the World Bank’s Worldwide Governance Indicators is 0.63 and significant at the 1 percent level as well. The correlation between the self-employment share in the data and Law and Order (Rule of Law) is -0.62 (-0.68) and significant at the 1 percent level.

34Given the response of steady-state unemployment to changes in $a_S$, this implies a range for self-employment of 7 percent to 87 percent of total employment (which is the variable plotted in Figure 3.1).
Figure 3.2: Model Steady State and Self-Employment (Share of Labor Force)

Thus, the marginal product of salaried employment would be lower even if increases in steady-state self-employment originated from changes in capital matching efficiency or other parameters that affect the share of self-employment in the economy. Intuitively, regardless of the parameter I use to change the share of self-employment in the model, economies with more self-employment use less capital in the salaried sector, which decreases the marginal product of labor for a given level of employment. Thus, the general conclusions I present do not depend on using $a_S$ to change the share of self-employment.
vacancies in salaried employment fall as the steady-state share of self-employment rises in the economy, as do steady-state salaried-firm capital and investment. The fall in vacancies and the increase in unemployment explain the fall in salaried labor market tightness for higher steady-state self-employment levels. Naturally, given the behavior of salaried vacancies, salaried employment and self-employment are negatively related. Since both physical capital and the share of the capital stock used in salaried production are higher in economies with less self-employment, salaried output and self-employment are negatively related.

The positive relationship between steady-state self-employment and unemployment suggests that self-employment cannot fully absorb the individuals who move into unemployment when vacancies in the salaried sector fall. While the household has an incentive to post more self-employment projects when salaried labor market tightness is lower, the increase in project postings does not cause an increase in capital market tightness, because a lower $a_S$ reduces the share of capital devoted to production in the salaried sector, which implies an increase in the supply of capital to matching. The increase in capital supply is larger than the increase in capital demand, causing a decrease in capital market tightness. Output in the self-employment sector is positively related to self-employment, since production in the sector in the steady state depends solely on the share of self-employment in the economy. The share of self-employment output in total output is therefore increasing in self-employment.\(^{36}\)

\(^{36}\)The ratio for the benchmark economy is close to 0.20. We can consider this as the share of output from informal enterprises, which excludes output from informal workers in formal firms but includes most of the output from self-employment, in total output. In Mexico, this ratio was 0.173 for years 2003 through 2006 (see http://wiego.org/informal-economy/statistical-picture).
The model yields a negative relationship between total output and the share of self-employment, which is in line with the cross-country empirical evidence on the link between GDP and self-employment. Market conditions in each of the employment states are important determinants of the wage and the rental rate. In particular, recall that salaried sector wages are increasing in salaried labor market tightness and decreasing in capital market tightness. Plotting steady-state self-employment against the wage and the rental rate reveals that wages decline as self-employment rises, suggesting that the impact on wages from lower labor market tightness outweighs the impact from lower capital market tightness. If we consider the rental rate of capital, an increase in labor market tightness puts downward pressure on the rental rate because the probability of transitioning into salaried employment is higher. The opposite holds true for capital market tightness. However, similar to the case of wages, the effect of labor market tightness is stronger in steady state, which makes the rental rate rise with self-employment. Thus, wages and self-employment are negatively related while self-employment and the rental rate are positively related. This implies that the cost of renting capital is lower in economies with higher institutional quality $a_S$. In terms of understanding the cyclical dynamics reported below, the most relevant steady-state results are the negative relationship between self-employment and capital market tightness, the positive links between self-employment, project postings, and the supply of capital, and the negative connection between self-employment and salaried labor market tightness.
3.6.2 Dynamic Response to a Negative Aggregate Productivity Shock

I first describe the dynamic response of the model’s variables to a negative aggregate productivity shock $z_t$ for the benchmark economy. I then discuss how cyclical dynamics differ across three economies with different steady-state self-employment levels, obtained by varying the salaried sector productivity parameter $a_S$. The shocks are the same across economies. I examine an economy with steady-state self-employment of 7.5 percent of total employment, which corresponds to the level in the United States; an economy with 23 percent self-employment, which corresponds to the level in the benchmark economy, Mexico; and an economy with 50 percent self-employment, which corresponds to the level in Colombia. These three choices are only meant to be illustrative. The following discussion refers to the results in Figures 3.3 through 3.5.

**Benchmark Economy** On impact, the fall in aggregate TFP immediately lowers the marginal product of labor in salaried employment and the marginal product of capital used by salaried firms. Firms cut back on vacancies and investment. Since unemployment is predetermined in period $t$, salaried labor market tightness responds solely to changes in vacancies and falls, causing matching in the salaried sector to fall. This implies an upward jump in the probability of filling a salaried vacancy from the point of view of salaried firms.

---

37 The only parameter that varies across economies is $a_S$. I do not claim that the model economies with 7.5 and 50 percent self-employment capture the U.S. and the Colombian economies, respectively.
The fall in aggregate productivity pushes firms to decrease the share of capital used in salaried sector production, $\omega$. Since the capital stock is predetermined in
Figure 3.4: Impulse Response Functions to a Negative Aggregate Productivity Shock (Continued)
Figure 3.5: Impulse Response Functions to a Negative Aggregate Productivity Shock (Continued 2)

In period \( t \), a fall in the share of capital used by salaried firms implies that the supply of capital for matching, \((1 - \omega)k_S\), increases on impact. As I describe below, the
rental rate on self-employment capital falls on impact. Households respond to the decline in both the probability of finding a salaried job and in the capital rental rate by posting more self-employment projects relative to the previous period. The increase in the supply of capital by salaried firms dominates the increase in project postings, so that capital market tightness falls on impact.38 Recalling that $p(\theta_{SE})$ is decreasing in capital market tightness, the fall in $\theta_{SE}$ translates into an increase in the probability of finding a capital supplier from the point of view of the household, and hence an increase in the probability of entering self-employment.39 Therefore, transition rates from unemployment into self-employment are countercyclical. Even though self-employment expands, output in the self-employment sector falls due to the fall in aggregate TFP. Since salaried output is also lower, this implies that total output in the economy falls on impact as well.

The initial responses of the wage and the rental rate are more subtle to analyze. First, note that a fall in capital market tightness raises the probability of becoming self-employed and puts upward pressure on wages. Second, a fall in salaried labor market tightness puts upward pressure on the rental rate, whereas the fall in capital market tightness has the opposite effect. In the end, the effect of salaried labor market tightness drives the behavior of wages, while the effect of capital market tightness dominates in explaining the behavior of the rental rate. Hence, both the wage and the rental rate fall on impact. This result is intuitive: for a given level

---

38 In principle, the impact response of capital market tightness is ambiguous since both the supply and the demand for self-employment capital have increased. For a reasonable calibration, the supply of capital always dominates.

39 This holds whether we look at the total probability of entering self-employment, $p(\theta_{SE})$, or the probability per unemployed individual, $v_{SE}p(\theta_{SE})$. The latter is the appropriate variable to compare against the salaried job-finding probability.
of self-employment, conditions in salaried employment should exert more influence on wages than conditions in self-employment, even though the latter still affects the behavior of wages. As I discuss below, important differences emerge once we analyze economies with different steady-state self-employment levels since capital market tightness plays an increasingly important role in shaping the response of wages as the steady-state level of self-employment increases.

Due to labor matching frictions, the initial fall in vacancies leads to a persistent fall in salaried employment. Also, since investment falls on impact, the lower capital stock next period slows down the recovery of the marginal product of salaried labor, which negatively affects salaried employment in subsequent periods. This has a direct impact on the recovery of salaried employment and output in the aftermath of the shock. Despite the effect of capital on salaried employment, there is a large rebound in salaried vacancies (and salaried labor market tightness) in the period after the shock. To understand this, note that in the benchmark economy wages fall further in the period after the shock, and the self-employment outside option plays a key role in this result. The recovery in aggregate productivity (and hence the marginal product of capital) pushes salaried firms to use more capital in-house. This reduces the supply of capital to the self-employed, which exerts upward pressure on the self-employment capital rental rate. In response to the rise in the rental rate, households post fewer self-employment projects. The lower demand for capital in

\footnote{Note the tension between the rise in the marginal product of capital – which pushes firms to devote less capital to matching and hence increases $\omega$ – and the rise in the rental rate – which pushes firms to devote more capital to matching and hence decreases $\omega$. The first effect eventually dominates the second effect, which brings the share of capital used in salaried production back to steady state.}
self-employment lowers the probability of entering self-employment and effectively reduces the self-employment outside option by enough that wages fall further after the period of the shock. Since the marginal product of labor also starts to recover as aggregate productivity slowly rises back to steady state, the salaried firm’s surplus from hiring workers increases, which further feeds the rise in salaried vacancies. Eventually, the downward pressure from the value of the self-employment option on wages winds down and is offset by rising labor market tightness, and wages begin their upward path back to steady state.

Absent the effect of endogenous movements in capital market tightness on salaried wages after the shock, real wages in the economy would simply fall on impact and start recovering in the subsequent period. This is a natural consequence of the fact that vacancies, labor market tightness, and the marginal product of labor all rise with the recovery of aggregate productivity, as in the standard labor search model. Hence, the dynamic response of the ease of entry into self-employment is critical to the speed of recovery of salaried employment and output, as the probability of becoming self-employed affects the recovery path of salaried wages. I show below that this mechanism becomes more relevant as self-employment in the economy increases.

**Differences Across Economies on Impact**  The impulse response function for total output at the bottom of Figure 5 shows that economies with more steady-state self-employment recover faster from an adverse aggregate productivity shock. This is one of the main results of the paper. To understand this result, it proves useful
to first see how the impact responses differ across economies.

Economies with more self-employment have a lower steady-state value of vacancies, which reflects the lower level of labor productivity in the salaried sector.\footnote{Once again, note that this is not an artifact of the way I obtain different levels of steady-state self-employment through changes in the salaried sector productivity parameter $a_S$. For example, if I change steady-state self-employment by varying the capital matching efficiency parameter $M_{SE}$, the results are qualitatively the same: the value of salaried vacancies is lower in economies with more self-employment because labor productivity is lower. This last result is due to the allocation of capital in the economy: more capital allocated to the self-employed implies a lower marginal product of labor in the salaried sector because the amount of capital used by salaried firms is lower for a given level of salaried employment.} Thus, when salaried firms in these economies face an adverse aggregate shock, the proportional downward adjustment in salaried vacancies will be larger, causing salaried labor market tightness to fall by more as well, as shown in Figure 3.3. The larger decrease in vacancies is accompanied by a larger fall in the share of capital used in production, $\omega$. The fall in investment is larger in economies with more steady-state self-employment, which is due both to the larger drop in vacancies – a larger fall in vacancies lowers the incentive to accumulate capital since salaried employment will be lower next period – and to the lower steady-state capital-output ratio in these economies. Even though $\omega$ falls by more in economies with more self-employment, the lower steady-state capital stock in these economies implies that capital supply to the self-employed, given by $(1 - \omega)k_S$, expands by less in the period of the shock. Hence, the increase in transitions into self-employment is smaller in economies with more steady-state self-employment due to the weaker surge in capital supply for matching. Lastly, note that the larger fall in total output in economies with more steady-state self-employment is driven by the response of salaried sector output on impact, which drops by more due to the behavior of the
share of capital used in the salaried sector, $\omega$.

Since the fall in salaried labor market tightness is larger in economies with higher self-employment shares, we would expect wages to fall by more as well, but this is not the case in the model. To understand this result, recall that steady-state salaried labor market tightness is decreasing in self-employment. This implies that, for a given deviation from steady-state, a fall in labor market tightness has less of an effect on wages in economies with more self-employment. Intuitively, relative to economies with less self-employment, the self-employment outside option plays a larger role in stabilizing wages since the steady-state supply of capital for self-employment is larger, implying a higher likelihood of entering the sector. Even if conditions in salaried employment change, they will have less of an effect on wages since self-employment plays a more prominent role in shaping labor market conditions in these economies. This unique feature of the model arises from the fact that capital market tightness, and therefore the probability of becoming self-employed, is endogenous and affects how wages respond to aggregate productivity shocks.

To make this argument more transparent, consider the terms involving self-employment from a log-linearized version of the salaried sector wage equation:

$$
\left[ \frac{\omega_{SE}^{\theta SE}}{w_{SE}^\theta} \left( \hat{v}_{SE,t} + \hat{z}_t + \hat{m}_{\omega k_{SE}} - \hat{\theta}_{SE,t} \right) \right] + 
\left[ \frac{(1-\delta^{SE})[1-\beta(1-\delta)]M_{SE}v_{SE}^\theta \xi_{SE}}{w_{SE}^{\theta SE}} \left( \hat{v}_{SE,t} - \hat{\theta}_{SE,t} + \hat{\lambda}_{t+1} - \hat{\lambda}_t \right) \right]
$$

(3.31)

where hats refer to variables are variables expressed in log-deviations from steady
state, $mp_{\omega k, t}$ is the marginal product of capital for salaried firms, and $\lambda^h_t$ is the marginal utility of consumption. There are several things to note. First, the combined impact of the deviation in capital market tightness $\hat{\theta}_{SE, t}$ can be written as

$$\left[ \frac{v^u_{SE} z_{SE} f_{\omega k}}{w_t \theta_{SE}} + \frac{(1 - \delta^{SE}) [1 - \beta(1 - \delta)] M_{SE} v^u_{SE} \xi_{SE}}{w_t \theta_{SE}^{kSE}} \right]$$

which is larger in absolute value in the calibrated economies with more steady-state self-employment, mainly due to the lower steady-state value of salaried wages and capital market tightness. This makes intuitive sense: given that the self-employment sector is larger in these economies, any movements in capital market tightness will have a larger impact on salaried wages through variations in the self-employment outside option, and conditions in self-employment more generally. Since $\hat{\theta}_{SE, t}$ has a negative impact on wages, this implies that a fall in capital market tightness will limit the fall in wages resulting from lower labor market tightness and a lower marginal product of salaried labor in the period of the shock. Put another way, the counteracting effect on wages due to the self-employment outside option is greater in economies with more steady-state self-employment, which implies that wages fall by less on impact. A similar claim holds for increases in project postings per unemployed individual $v^u_{SE, t}$ in response to the negative shock, since the weight on $v^u_{SE, t}$ in absolute value is the same as for $\hat{\theta}_{SE, t}$. Again, this is intuitive since the larger presence of self-employment in the economy implies that changes in market conditions in self-employment exert more influence on salaried labor market condi-

\[42\footnote{Note that this is the case even if capital market tightness falls by less on impact in economies with more self-employment.} \]
tions. Hence, a given change in individual project postings in economies with more steady-state self-employment will have a larger stabilizing influence on the response of salaried wages in the period of the shock, and will make wages more rigid on impact.

Analyzing numerically each of the components of the coefficient on $\hat{\theta}_{SE,t}$ in (32) shows that the larger steady-state supply of capital to self-employment is key to explaining the greater influence of $\hat{\theta}_{SE,t}$ in economies with more self-employment: the supply of capital ultimately determines how accessible the self-employment outside option is, even though the availability of self-employment capital also depends on project postings.\(^{43}\) The upshot is that wages will fall by less on impact in economies with more self-employment: the larger steady-state supply of capital for matching effectively bolsters the self-employment outside option in the period of the downturn, which in turn reduces the impact response of wages to changes in aggregate economic conditions. This relative rigidity in wages causes vacancies to take a larger hit in the period of the shock in economies with more steady-state self-employment.

In contrast to the behavior of wages, the rental rate is more sensitive to changes in capital market conditions in economies with more steady-state self-employment. Going back to the rental rate equation, recall that a fall in salaried labor market tightness puts upward pressure on the rental rate since the probability of finding salaried employment is lower. This effect is weaker in economies with more

\(^{43}\)Using the definition of $\theta_{SE}$ and $v_{SE}$, we can write $\left(\frac{v_{SE}aSf_{ks}}{(\theta_{SE}wS)}\right)$ as $\left(\frac{(1 - \omega)kSzaSf_{ks}}{(uwS)}\right)$. While $aS$ and the supply of capital $(1 - \omega)kS$ move in the opposite direction, the change in $(1 - \omega)kS$ dominates the change in $aS$ such that $\left(\frac{(1 - \omega)kSzaSf_{ks}}{(uwS)}\right)$ increases with the share of self-employment in the economy. What is key here is that the numerator rises by more than the denominator.
self-employment since conditions in salaried employment have less influence on self-
employment, and on labor market conditions in general. Conversely, a fall in cap-
ital market tightness puts downward pressure on the rental rate, and this effect is
stronger in economies with more self-employment due to the higher steady-state
supply of capital devoted to self-employment.\footnote{The coefficient on \( \theta_{SE,t} \) in the log-linearized rental rate equation is the same as the one in the log-linearized wage equation, but with a positive sign. Thus, a fall in \( \theta_{SE,t} \) in response to a negative aggregate productivity shock tends to lower the rental rate, ceteris paribus.} As it turns out, the second effect
dominate, so that the rental rate will fall by more in economies with more self-
employment.

As I argue below, the response of salaried wages plays a key role in the recovery
process, so a brief summary of the impact response of wages is in order. First,
recall that steady-state salaried labor market tightness is lower in economies with
more self-employment, which makes wages fall by less when vacancies (and therefore
salaried labor market tightness) fall in response to the shock. Second, the steady-
state supply of capital to the self-employed in these economies is higher, implying
a higher steady-state entry probability into self-employment. Thus, a fall in capital
market tightness arising from an adverse aggregate shock will put more upward
pressure on wages in economies with more self-employment. Both effects make
wages initially less responsive to shocks in economies with more self-employment,
while the opposite is true for the rental rate. The intuition is clear: economies
with more self-employment are associated with a more accessible outside option in
self-employment, which in turn has more influence on salaried-sector wages in the
economy and makes wages initially less responsive when an aggregate shock hits the
Differences Across Economies After the Shock  To understand how the economy’s steady-state level of self-employment affects the recovery path of total output, consider how the hiring pattern in the salaried sector evolves differently across economies in the aftermath of the shock. We know that the marginal product of labor starts to recover after the shock since TFP slowly returns back to steady state. In fact, the recovery path of this variable is virtually identical across economies since all economies face the same shock. What differs, however, is the behavior of wages. Recall that wages fall less on impact in economies with more self-employment. By itself, this causes a larger initial drop in salaried employment and output, and would tend to slow down their recovery. However, as shown in Figure 3.4, wages in these economies also have a more pronounced U-shaped recovery and keep falling several periods after the shock. This last fact is crucial to explaining why these economies recover faster in the model. As I describe below, the self-employment outside option plays a central role in shaping the path of wages after the shock.

Intuitively, changes in the probability of becoming self-employed as aggregate productivity recovers effectively determine how accessible the self-employment outside option is in the periods following the shock. In particular, the likelihood of becoming self-employed falls back to steady state more quickly in economies with more self-employment, thereby making self-employment a less accessible employment option. This puts downward pressure on wages, so that wages fall even further in the aftermath of the shock. In turn, the more persistent fall in wages promotes a
more rapid recovery of salaried vacancies, which leads to a faster recovery of salaried employment, investment and output.

The mechanism works as follows: first, steady-state capital market tightness is lower in economies with more steady-state self-employment, which implies a smaller rise in the probability of self-employment entry on impact. Second, since the pace of recovery in the marginal product of capital due to TFP is similar across economies, the share of capital used in salaried production recovers at a faster pace in economies with more self-employment since the former experienced a larger initial fall in these economies. This reduces the amount of capital devoted to matching with the self-employed by more, which in turn causes a faster recovery in the rental rate in the periods subsequent to the shock since the rental rate is more sensitive to changes in the supply of capital in these economies. The faster recovery in the rental rate in economies with more self-employment after the period of the shock sharply reduces the benefit from posting self-employment projects. Moreover, the larger collapse in salaried vacancy postings from the period of the shock translates into a larger increase in unemployment. Combining these last two facts, we see that self-employment project postings per unemployed individual fall back to steady state earlier in economies with more self-employment as the shock subsides. The larger contraction in capital supply reduces the availability of self-employment capital, and hence the probability of entry into self-employment falls back more quickly after the shock in economies with more self-employment. In simple terms, the self-employment outside option deteriorates faster in the aftermath of the shock in these economies.
As the possibility of becoming self-employed becomes less and less likely – which happens much earlier in economies with more self-employment as the availability of capital for self-employment falls more rapidly – the drop in the self-employment outside option puts continued downward pressure on wages in these economies. Thus, wages continue to fall for several periods after the shock, which creates a larger surplus from posting salaried vacancies. Salaried labor market tightness also starts to return to steady state in the period after the shock, which tends to raise wages. Eventually, the downward pressure arising from the lower availability of self-employment capital winds down, and the rise in salaried vacancies allows wages to begin their recovery back to steady state. What is important is that wages fall further after the shock in economies with more steady-state self-employment, even though wages are initially more rigid. Since the marginal product of labor begins its recovery immediately after the period of the shock as TFP recovers, this creates a larger difference between the marginal product of salaried labor and the wage – a larger contemporaneous value from hiring workers – which bolsters a faster recovery in vacancy postings in subsequent periods. In turn, the faster recovery of vacancies and employment in the salaried sector leads to a faster surge in investment in economies with higher steady-state self-employment shares. Finally, a faster recovery in salaried employment and investment bolster the recovery in salaried output, which in turn leads to a faster recovery in total output.

\[45\] To make this argument clear, consider the ratio of the wage to the marginal product of salaried employment, shown in Figure 3.4. On impact, this ratio is greater in economies with more self-employment exactly because wages fail to fall as much as in economies with less self-employment. The persistent fall in wages induced by the evolution of the self-employment outside option pushes the ratio of the wage to the marginal product of labor down by a larger proportion in subsequent periods. This drives the larger surge in salaried vacancies.
The Importance of Endogenous Transition Rates into Self-Employment

The existence of self-employment by itself is not sufficient to explain faster output recoveries in economies with higher steady-state self-employment. The evolution of the value of the self-employment outside option in response to shocks is critical to explaining why total output recovers faster in economies with more self-employment. A model with an exogenous transition rate into self-employment and no capital search frictions would not be able to explain the observed stylized fact in the data.\textsuperscript{46} Furthermore, the wage channel – whereby the dynamic behavior of the ease of entry into self-employment affects the self-employment outside option and hence the recovery path of wages after a negative shock – would not be present in such a model. Hence, an alternative model with a fixed transition probability into self-employment cannot explain why output would recover faster in economies with more self-employment. The contributions of this paper are therefore twofold: I offer a tractable framework that replicates the cyclicality of self-employment and the transition rate into self-employment from unemployment in the data, and I propose a plausible channel through which differences in the composition of employment across economies may have important consequences for the pace of economic recoveries.

3.6.3 Model and Data Comparison

To determine whether the model can yield reasonable predictions about the relationship between self-employment and business cycle persistence, I check whether

\textsuperscript{46}In this alternative model, salaried employment, investment, salaried output, and total output recover more slowly in economies with higher steady-state self-employment.
it can capture the empirical relationship initially shown in Figure 3.1.

Figure 3.6: Output Persistence and Self-Employment Rates: Data and Model Comparison

To compare the prediction of the model against the relationship in the data, I take the simulated series for quarterly output and create an unfiltered annual output series in levels. I then log the series and extract the cyclical component of the annual series using the HP filter with smoothing parameter 100, as I did with the data. Finally, I compute the first-order autocorrelation of the cyclical output series for different economies by varying the parameter $a_S$ to change the steady-state level of self-employment. I then plot a second-order polynomial fit for the data, the data points for all the countries in the sample, and the relationship generated by the model. Figure 3.6 shows that the model can capture the change
in the autocorrelation of output for countries with high shares of self-employment reasonably well.

As a simple exercise, I regress the autocorrelation of output in the data, $AC(y)_{data}$ on the autocorrelation of output computed from the simulated series, $AC(y)_{simulated}$. This yields:

$$AC(y)_{data} = -0.273(0.199) + 1.602(0.376) AC(y)_{simulated}$$

(3.33)

where standard errors are shown in parentheses. The R-squared of the regression is 0.17, implying that the model can explain 17 percent of the variation in the autocorrelation of cyclical output.

The model's ability to replicate the negative relationship between output persistence and self-employment rates in the data depends on the response of the variability of self-employment output to changes in the share of self-employment in the economy. As I briefly discuss below, the presence of the self-employment productivity shock plays an important role in capturing the quantitative relationship between self-employment and output persistence.

In a version of the model without a self-employment productivity shock, the increase in the volatility of salaried output when self-employment increases would be offset by the decrease in the volatility of self-employment output as the sectoral output weights shift and raise the share of self-employment output in total output.\footnote{The idea for this experiment is borrowed directly from Moscoso Boedo and Mukoyama (2012). Using the model, I target the share of self-employment (as a percent of employment) for each country in Figure 3.1 (for a total of 83 countries). Then, for each self-employment share, I compute the autocorrelation of cyclical output using the simulated series, which yields $AC(y)_{simulated}$.}

\footnote{The covariance term between salaried output and self-employment output plays a minor role}
The change in employment composition would merely change the composition of the volatility of total output without changing its level. This, in turn, would result in virtually no change in the persistence of output in the economy as the share of self-employment increases. Having a sectoral productivity shock in self-employment effectively reduces the sensitivity of self-employment output volatility to steady-state changes in the composition of employment. Intuitively, this is the case since the added variability from the sectoral shock dilutes the effect of changes in the volatility of self-employment on the variability of output in self-employment. Including a sectoral productivity shock to self-employment output substantially improves the model’s ability to reproduce the empirical link between output persistence and self-employment, and also allows the model to capture additional qualitative stylized facts about the volatility of self-employment earnings, and the volatility of output across economies.

The inclusion of a sectoral shock in the self-employment sector can be justified on three grounds. First, this sector is well-known to face higher risk and higher volatility (Maloney, Cunningham, and Bosch, 2004). One way to capture this is to include a sectoral shock. Second, shutting down the sectoral shock generates similar volatilities in self-employment earnings and salaried labor earnings. This fact is inconsistent with the empirical evidence for various countries, which documents that self-employment earnings are more volatile than wage earnings (Hamilton, 2000; in determining changes in the variance of total output.

\(^{49}\)This holds even though the volatility of self-employment changes with the variance of the sectoral shock. Note that increasing the volatility of the sectoral shock beyond the benchmark value would generate a stronger negative relationship between output persistence and the share of self-employment, even though the volatility of self-employment output is decreasing in the share of self-employment.
Bargain and Kwenda, 2010; Narita, 2010). In the simulations, I can capture this last stylized fact with the self-employment productivity shock $a_{SE,t}$. Finally, as I discuss below, if the shock hitting the economy originates in the self-employment sector, the model can deliver episodes of procyclical self-employment, which Fiess et al. (2010) have documented for particular time periods in certain Latin American countries. The inclusion of this shock allows me to test this fact in the data.

Even though I highlighted the model’s quantitative success in replicating Figure 3.1, the main conclusion to draw from this exercise is that the model can capture the negative relationship between self-employment and output persistence in a qualitative fashion.\footnote{Clearly, the polynomial fit (or a simple linear fit of the data) will change slightly if the countries with the highest shares of self-employment from the sample (70 percent or more) or some of the apparent outliers are excluded. However, the negative link between self-employment and cyclical output persistence would still hold despite changes in the country sample.} I take this to be an important indicator that the model of self-employment I propose is heading in the right direction.

3.6.4 Additional Comments

**Total Factor Productivity Differences** Recall that production in the salaried sector is given by

$$y_{S,t} = z_t a_S f(n_{S,t}, \omega_t k_{S,t}) = z_t a_S (n_{S,t})^{1-a_S} (\omega_t k_{S,t})^{a_S}, 0 < a_S < 1,$$

where recall that $a_S$ is a proxy for institutional quality and $z_t$ is exogenous aggregate productivity. Naturally, economies can differ in total factor productivity (TFP) because of differences in $a_S$, which I consider exogenous. Following Lama and Urrutia (2012), we can rewrite $y_{S,t}$ as
\[ y_{S,t} = \underbrace{\text{Measured } TFP_t}_{\text{Exog. TFP}} - \underbrace{\omega_t^{\alpha S}}_{\text{Endog. TFP}} (n_{S,t})^{1-\alpha S} (k_{S,t})^{\alpha S} \quad (3.34) \]

and hence decompose measured \( TFP_t \) into an exogenous component, \( z_t a_S \), and an endogenous component, \( \omega_t^{\alpha S} \). I can then compute steady-state differences in measured and endogenous TFP across economies.\(^{51}\)

The simulations under the benchmark calibration show that measured TFP in an economy with 7 percent self-employment – equivalent to a country like the United States – is only 5 percent higher than in an economy with 23 percent self-employment, 16 percent higher than in an economy with 50 percent self-employment, and 57 percent higher than in an economy with 87 percent self-employment (the upper bound for the country sample in Figure 1). Similarly, endogenous TFP in an economy with 7 percent self-employment is only 1.5 percent higher than in an economy with 23 percent self-employment, and 6 percent higher than in an economy with 50 percent self-employment. These results suggest that the allocation of capital across sectors in each economy accounts for a small share of the differences in measured TFP across economies.

The Response to a Positive Shock to Self-Employment Productivity

The model can capture the countercyclicality of self-employment and self-employment

\(^{51}\) For some related literature on cross-country differences in productivity, see Hsieh and Klenow (2009), Bartelsman, Haltiwanger, and Scarpetta (2013), Buera, Kaboski, and Shin (2011), among others. For a recent study on capital misallocation and institutions in Africa, see Kalemli-Ozcan and Sorensen (2012). Also, see Hopenhayn (2012), who proposes a simple mapping between distortions and aggregate productivity and finds much smaller effects on TFP from these distortions, in contrast to the existing literature.
entry in the data. Using data from four Latin American countries, Fiess et al. (2010) document particular episodes where self-employment is procyclical in these countries. They argue that the source of shocks may be important in determining whether self-employment expands or contracts in response to shocks to the economy. The inclusion of a sectoral productivity shock in self-employment allows me test this claim in the model. Both total output and self-employment expand in response to a positive productivity shock in the self-employment sector. Intuitively, the shock increases the value of devoting resources to the self-employment sector, thereby causing a reallocation of accumulated capital in the salaried sector towards the self-employment sector and generating increased entry into self-employment. The contraction in capital used by salaried firms reduces vacancy postings in the salaried sector, but the increase in the supply of self-employment capital causes wages in the salaried sector to expand. Despite the higher value of supplying capital to the sector, investment falls. The expansion in self-employment output due to higher entry into the sector more than offsets the contraction in salaried output, leading to an expansion in total output. Thus, the model is able to both capture the evidence in Fiess et al. (2010) and generate the countercyclical behavior of self-employment documented in Loayza and Rigolini (2011).

**Endogenous Separations in Self-Employment** Bosch and Maloney (2008) document that separations from self-employment are strongly countercyclical. Would the model be able to generate the cyclicality of the separation rate in self-employment if separations are endogenous? Relatedly, would the main results in the paper be ro-
bust to the inclusion of endogenous separations in self-employment? The benchmark model shows that salaried firms will respond to a negative TFP shock by using less capital in production and devoting more capital to the self-employment sector. This takes place despite the fact that the shock affects both sectors. With endogenous separations, salaried firms have an additional margin of adjustment and can decide to destroy less capital relationships since having capital in the self-employment sector acts as an additional source of income during bad times. If we assume that both old and new matches in self-employment start with the same idiosyncratic productivity each period, the threshold idiosyncratic productivity value that determines destruction in the sector can either rise or fall in response to a negative aggregate productivity shock. Thus, it is not clear whether separations would be countercyclical.

Assuming that new self-employment matches always start with higher productivity than existing matches would push salaried firms to destroy old matches in response to an adverse aggregate shock, and reallocate that capital to new matches that start with the highest productivity. This would not only yield countercyclical separations in self-employment, as in the data, but would also allow for a net expansion of self-employment during recessions. More importantly, the mechanism through which differences in the share of self-employment affect the speed of recoveries – mainly, how the ease of self-employment entry affects the dynamic path of salaried wages in the aftermath of an aggregate shock – would not change relative

\textsuperscript{52}For a setup that uses this assumption in a model with two types of salaried employment, see Bosch and Esteban-Pretel (2006).
to the benchmark economy. For example, as aggregate productivity slowly recovers after a negative shock, the separation rate from self-employment would begin to fall back to steady-state, and salaried firms would start to use more capital in-house. The fall in the amount of capital supplied to the self-employed would reduce the self-employment outside option, thereby putting downward pressure on salaried wages. This would generate similar qualitative dynamics relative to the benchmark model.

3.7 Conclusion

In this paper, I explore the role of self-employment in the recovery process from recessions. Four key facts motivate my focus on self-employment and business cycle dynamics: self-employment is a pervasive feature of labor markets around the world; most self-employed run owner-only businesses with no salaried workers; and self-employment generally expands in recessions. I also document a robust negative relationship between the share of self-employment and the persistence of output over the business cycle in a sample of developed and developing countries. This link suggests that self-employment may be important for understanding differences in economic recoveries.

I build a business cycle model with frictional labor markets where individuals can be self-employed or work in salaried firms, and explore the channels through which self-employment influences the pace of recoveries. I find that economies with higher self-employment shares recover faster from negative aggregate productivity shocks. In the model, unemployed individuals who want to enter self-
employment must match with capital suppliers in order to finance their business ventures. Salaried firms supply unused capital in the salaried sector to the self-employed in frictional capital markets. Capital search frictions play three key roles. First, they determine the availability of capital to the self-employed, thereby making transition rates into self-employment endogenous and dependent on aggregate economic conditions. Second, the supply and demand for self-employment capital create a direct link between conditions in self-employment and salaried wages. Third, access to self-employment, and therefore the self-employment outside option, changes in response to fluctuations in aggregate economic conditions. In particular, transition rates from unemployment into self-employment in the model expand in downturns, as in the data. This last fact is difficult to obtain in existing business cycle models of entrepreneurship and liquidity constraints. One contribution of my paper is to reconcile the fact that the self-employed require external resources to produce with the empirical evidence on the countercyclicality of self-employment. Furthermore, the model I propose is consistent with other stylized facts about self-employment and labor market dynamics.

Since self-employment is an alternative to salaried employment, cyclical movements in the ease of entry into self-employment affect salaried wages directly. Furthermore, the average level of self-employment in the economy determines how much the self-employment outside option influences the response of wages at the onset of a downturn, as well as the path of wages in the aftermath of a negative aggregate shock. When a negative aggregate shock hits the economy, wages in the salaried sector are initially more rigid in economies with higher average self-employment. These
economies exhibit larger drops in vacancies, investment, and output, thereby generating sharper contractions. As aggregate productivity begins to recover, the availability of self-employment capital falls, which weakens the self-employment outside option much faster in these economies. This causes more persistent, or long-lasting, salaried wage contractions in economies with more self-employment, which creates a faster recovery in salaried employment and investment as salaried firms post vacancies and invest at a faster rate. Salaried output recovers faster, and this drives the faster recovery in total output in economies with higher average self-employment rates. The model helps us identify a subtle channel through which the long-run level of self-employment has implications for the economy’s speed of recovery from downturns. In addition, the dynamic behavior of unemployment, investment, and output in response to shocks is consistent with the empirical evidence on business cycles in developed and developing economies.

The model goes a long way in capturing the relationship between the autocorrelation of output and the share of self-employment observed in the data. I find that allowing for a sectoral shock to self-employment productivity is important to capturing this relationship quantitatively. Furthermore, the inclusion of the shock allows the model to capture two additional stylized facts in the data: the higher output volatility in economies with more self-employment, and the higher volatility in self-employment earnings relative to wage earnings, which is consistent with empirical evidence on the higher riskiness and volatility in the self-employment sector. More broadly, this paper suggest that it is critical to account for the fact that the likelihood of entering into self-employment – and hence the self-employment outside

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option – is not fixed but rather fluctuates with the aggregate state of the economy. Existing models have ignored this relevant cyclical feature of self-employment. Movements in the attractiveness of, and ease of entry into, self-employment over the business cycle – which are ultimately affected by the structure of the labor market – feed into the decisions of agents in the salaried sector, with important consequences for economic recoveries and business cycle dynamics.

The model does not differentiate between high and low-ability self-employed individuals, which prevents me from capturing other very relevant features of self-employment highlighted in the empirical literature, such as the fact that a proportion of the self-employed enter the sector during economic expansions and expand their firms by creating salaried positions. It would be particularly interesting to incorporate this heterogeneity in the current version of the model and address business cycle dynamics and firm growth simultaneously. For example, we would be able to explore the policy implications of promoting small firm creation during downturns. It would also allow us to investigate whether targeting small firm creation based on observables such as the human capital of firm owners matters for the policy’s success. This framework would be relevant even for countries like the U.S. since a large majority of the self-employed tend to be own-account workers. Relatedly, the fact that agents are perfectly insured within the household – a common assumption in labor search models – prevents me from appropriately addressing the relevance of higher riskiness in self-employment. In principle, the higher volatility of self-employment earnings should arise endogenously due to occupational choice risk, production risk, or other characteristics that are determined within the economic
environment, and should not rely on an exogenous process as is currently the case in the model. Indeed, some of the benefits from self-employment may be reduced once we take into account the risk involved in entering the sector. Also, to the extent that risk and uncertainty may increase in recessions, the model might yield an incomplete picture of the actual behavior of the self-employed. Finally, while the existing evidence on trade credit supported the use of capital search frictions as a reduced-form modeling device, there is little empirical work done in this area. The increasing availability of surveys that capture the obstacles faced by micro-firms in developing countries provides an exciting avenue that can take us deeper into the frictions that affect firm creation over the business cycle. Much remains to be understood about the cyclical implications of differences in the structure of labor markets. Taking these shortcomings into account, this paper offers a framework that incorporates self-employment in a tractable way to help us deepen our understanding of the link between business cycle dynamics and the composition of employment. I plan to explore the limitations and extensions outlined above in future work.
Appendix A

Appendix for Chapter 2

A.1 Data Sources and Empirical Specification

A.1.1 Data Sources

For the determinants of informal sector size, the largest sample includes 94 countries (in Africa, Asia, Latin America, as well as rich OECD countries). For the analysis of aggregate volatility, I take the same country sample as Ferreira-Tiryaki (2008) to have easily comparable results, but exclude Argentina from the sample since it represents an outlier in the sample, leaving 43 countries. Most of the series for the explanatory variables I use are taken from Catini, Panizza and Saade’s (2010) compilation of the World Bank’s World Development Indicators (WDI). The list below includes a brief description of each of the variables I use in the empirical analysis, along with its corresponding source. More detailed information can be obtained by visiting the primary sources.

**Informal sector size**  Measured as a share of official GDP and borrowed from Schneider et al. (2010). Their work provides comparable yearly estimates of the size of the informal sector for 162 countries from 1999 to 2007. I use the estimates from year 2002 to have a single point estimate per country.
**Distance from the Equator**  Measured as the distance (in degrees) from the Equator. Taken from Rodrik et al. (2004).

**French legal origin**  Dummy variable equal to one if the legal system is based on the French legal system. Taken from Djankov et al. (2004).

**Law and Order**  Measure of institutional quality from the Political Risk Services (PRS) Group’s International Country Risk Guide (ICRG) for years 1998 through 2007. It takes values from 1 to 6, where higher values denote better institutions. I use year 1999 to have a point estimate for each country, and also experiment with years 2000 and 2001 as a robustness check.

**Rule of Law**  Measure of institutional quality. It takes values from -2.5 to 2.5, where higher values denote better institutions. I use year 1998 to obtain a point estimate for each country since year 1999 is not available, and experiment with year 2000 as a robustness check. Source: World Bank’s Worldwide Governance Indicators.

**Engfrac and Eurofrac**  Engfrac and Eurofrac refer to the fraction of the population that speaks English, and the fraction of the population that speaks a Western European language, respectively. Source: Rodrik et al. (2004).

**Legal enforcement of contracts**  Fraser Institute’s Economic Freedom of the World Report 2010. Based on the Doing Business report, it provides a summary of the time and monetary cost of collecting a debt. More information about the
construction of this index is provided in the Explanatory Notes and Data Sources from the Economic Freedom of the World: 2010 Annual Report.

**Log Real Gross Domestic Product in 1999** The logarithm of real GDP in 1999 expressed in 2000 dollars. Source: WDI.

**Hiring regulations (HR)** Fraser Institute’s Economic Freedom of the World Report 2010. This index is available starting in 2002. Hiring regulations (HR) captures whether fixed-term contracts can be offered for tasks or jobs that generally do not expire, the maximum duration and number of renewals of fixed-term contracts, and the ratio between the minimum wage and the average value added by the worker. This index is based on the Difficulty of Hiring Index (DHI) in the World Bank’s Doing Business Report. HR takes values from 0 to 10, where higher values of HR denote less stringent hiring regulations (Gwartney, Hall, and Lawson, 2010). I use HR from 2002. More information about the construction of HR is provided in the Explanatory Notes and Data Sources from the Economic Freedom of the World: 2010 Annual Report. http://www.freetheworld.com/2010/reports/world/EFW2010-appendix.pdf.

**Dismissal costs (DC)** Fraser Institute’s Economic Freedom of the World Report 2010. This index is available starting in 2002. Dismissal costs (DC) are expressed in weeks of salary and capture any severance payments, explicit costs of notifying workers and third parties in advance of termination, and penalties that the firm faces when the firm formalizes a worker separation. We can think of this measure as an approximation of the explicit resource costs that a firm has to incur when a worker


**Consumption volatility** Standard deviation of the cyclical component of annual private consumption using the Baxter-King band-pass filter from 1985 to 2002. Source: Ferreira-Tiryaki (2008). The original data is obtained from the International Monetary Fund’s (IMF) International Financial Statistics (IFS).

**Investment volatility** Standard deviation of the cyclical component of annual gross capital formation using the Baxter-King band-pass filter from 1985 to 2002.
Source: Ferreira-Tiryaki (2008). The original data is obtained from the International Monetary Fund’s (IMF) International Financial Statistics (IFS).


**Government spending-GDP ratio**  General government final consumption expenditure as a percent of GDP for years 1985 through 2002. Source: WDI.

**Openness**  Constructed as the sum of imports and exports divided by GDP from 1985 to 2002. Averaged over the sample period to obtain one point estimate per country. Constructed with data from the WDI.

**Domestic credit**  Domestic credit to the private sector as a share of GDP from 1985 to 2002. Averaged over the sample period to obtain one point estimate per country. Source: WDI.

Table A1 presents descriptive statistics of the main variables used in the empirical motivation.

**A.1.2 Estimation: Determinants of Informal Sector Size**

Table A2 presents the results of an OLS regression where the dependent variable is a measure of institutional quality. I present the regression results for a
wide range of institutions measures to determine whether there is a particular variable that is common across institutional quality measures. The results show that Distance from the Equator is strongly significant for all measures of institutions, whereas the fraction of the population that speaks English (engfrac) is strongly significant for all measures except Law and Order (from PRS). More importantly, French legal origin is significant only for Legal Enforcement of Contracts.

The estimation equation for the determinants of informality has the form

\[ IS_i = \beta_0 + \beta_1 Enforcement_i + \beta_2 Institutions_i + \beta_3 HR_i + \beta_4 DC_i + \sum_{k=5}^{n} \beta_k X_{ki} + \varepsilon_i \]

Where \( n \geq 5 \) and \( IS_i \) denotes the size of the informal sector as a percent of official GDP in country \( i \), \( Enforcement_i \) is captured by French legal origin, 

\(^1\)I do so because there is no a priori reason to think that one measure of institutional quality is better than another. The choice of rule of law is simply guided by its use in other empirical papers.
Table A.2: Institutional Quality and Instruments

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<th>Rule of Law</th>
<th>Control of Corruption</th>
<th>Regulatory Quality</th>
<th>Legal Enforc. Contracts</th>
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<td>Eurofrac</td>
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<td>(0.200)</td>
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<td></td>
<td>(0.317)</td>
<td>(0.208)</td>
<td>(0.221)</td>
<td>(0.203)</td>
<td>(0.521)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.56</td>
<td>0.61</td>
<td>0.60</td>
<td>0.53</td>
<td>0.45</td>
</tr>
<tr>
<td>Obs.</td>
<td>101</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>93</td>
</tr>
</tbody>
</table>

Note: The constant term is omitted for expositional purposes. Robust standard errors in parentheses. Law and Order is from the PRS Group’s ICRG. Rule of Law, Control of Corruption, Government Effectiveness, and Regulatory Quality are from the World Bank’s Worldwide Governance Indicators (WGI). Legal Enforcement of Contracts is from the Fraser Institute’s Economic Freedom of the World Report 2010. Engfrac and Eurofrac refer to the fraction of the population that speaks English, and the fraction of the population that speaks a Western European language, respectively (see Rodrik et al., 2004). *** denotes significance at the 1 percent level, ** denotes significance at the 5 percent level, * denotes significance at the 10 percent. Robust standard errors are shown in parentheses.
Institutions\textsubscript{i} denotes the measure of institutions (Law and Order), HR\textsubscript{i}, DC\textsubscript{i} are hiring regulations and dismissal costs, respectively, from the Economic Freedom of the World dataset, and ε\textsubscript{i} is an error term. Higher values for hiring regulations and dismissal costs denote less stringent regulations. X\textsubscript{k\textsubscript{i}} includes the log of real GDP in 1999.\textsuperscript{2}

A.1.3 Estimation: Informal Sector Size and Aggregate Volatility

The estimation equation has the form

\[ \sigma_{j,i} = \gamma_0 + \gamma_1 IS_i + \sum_{k=2}^{n} \gamma_k X_k + \mu_i \]

Where \( \sigma_{j,i} \) denotes the volatility of variable \( j \) in country \( i \), where \( j \) can be aggregate consumption, investment, or output. IS\textsubscript{i} is informal sector size, and \( \mu_i \) is an error term. As before, \( X_k \) encompasses mean government spending as a share of GDP, a measure of openness to trade, and the mean of domestic credit to the private sector as a share of GDP. Additional controls to test for robustness (not shown in the main regression results) include the median inflation rate, a measure of exchange rate fluctuations, and the growth rate in trend real GDP, similar to the

\textsuperscript{2}The log of real GDP per capita is highly correlated with the institutional quality measure. As a robustness test, \( X_k \) includes other variables that could potentially affect the size of the informal sector, including the inflation rate, a measure that captures the mean tax rate, and business regulations. Based on Loayza, Servén, and Sugawara (2009), it may be important to control for business regulations. However, when I do so, the measure of business regulations turns out to be very highly correlated with institutional quality. Also, a natural extension would be to include interaction terms between institutions and the two measures for labor regulations to test whether the impact of regulations falls with better institutions. I tested this by introducing interaction terms but found that institutional quality becomes insignificant since there is a high degree of multicollinearity between the interaction terms and the measure of institutional quality. Thus, the main specifications do not include any interaction terms.
regressors used in Ferreira-Tiryaki (2008).³

A.2 Solution to the Nash Bargaining Problem and Wage Equations

To derive the wage equations, I use the following equations:

\[ W_{F,t} = w_{F,t} + E_t \Xi_{t+1} \{ (1 - \delta^F)W_{F,t+1} + \delta^F W_{U,t+1} \} \]  (A.1)

\[ W_{I,t} = w_{I,t} + E_t \Xi_{t+1} \{ (1 - \delta^I)W_{I,t+1} + \delta^I W_{U,t+1} \} \]  (A.2)

\[ W_{U,t} = b + E_t \Xi_{t+1} \left\{ p(\theta_{F,t})(1 - \delta^F)W_{F,t+1} + p(\theta_{I,t})(1 - \delta^I)W_{I,t+1} + \right\} \]  (A.3)

\[ J_{F,t} = p_{F,t} z_t a_F f_{n_{F,t}}(n_{F,t}, k_{F,t}) - w_{F,t}(1 + \tau^p) + E_t \Xi_{t+1} \{ (1 - \delta^F)J_{F,t+1} \} \]  (A.4)

And

\[ J_{I,t} = p_{I,t}(1 - \rho(n_{I,t}, a_F, t))z_t - \rho n_t(n_{I,t}, a_F, t) p_{I,t} z_t n_{I,t} - w_{I,t} + \right\} \]  (A.5)

The Nash bargaining solutions in the formal and informal sectors can be expressed as

\[ w_{F,t} = \arg \max \{ (W_{F,t} - W_{U,t})^{\nu_F}(J_{F,t})^{1-\nu_F} \} \]  (A.6)

and

\[ w_{I,t} = \arg \max \{ (W_{I,t} - W_{U,t})^{\nu_I}(J_{I,t})^{1-\nu_I} \} \]  (A.7)

³One difference relative to Ferreira-Tiryaki’s (2008) work is that I do not include the standard deviation of the cyclical component of the Solow residual as an explanatory variable in the volatility regressions.
Solving each one of these problems, we obtain

\[ \nu_F (W_{F,t} - W_{U,t})^{\nu_F - 1} (J_{F,t})^{1 - \nu_F} \left( \frac{\partial W_{F,t}}{\partial w_{F,t}} \right) = \]  

\[ -(1 - \nu_F) (W_{F,t} - W_{U,t})^{\nu_F} (J_{F,t})^{-\nu_F} \left( \frac{\partial J_{F,t}}{\partial w_{F,t}} \right) \]  

(A.8)

and

\[ \nu_I (W_{I,t} - W_{U,t})^{\nu_I - 1} (J_{I,t})^{1 - \nu_I} \left( \frac{\partial W_{I,t}}{\partial w_{I,t}} \right) = -(1 - \nu_I) (W_{I,t} - W_{U,t})^{\nu_I} (J_{I,t})^{-\nu_I} \left( \frac{\partial J_{I,t}}{\partial w_{I,t}} \right) \]  

(A.9)

Now, following Arseneau and Chugh (2008), define

\[ \xi_F = \frac{\nu_F}{\nu_F + (1 - \nu_F) \Delta_1} \]  

(A.10)

where

\[ \Delta_1 = \frac{\left( \frac{\partial J_{I,t}}{\partial w_{I,t}} \right)}{\left( \frac{\partial W_{I,t}}{\partial w_{I,t}} \right)} \]  

(A.11)

Hence, we can rewrite the first-order conditions for the Nash bargaining problems as

\[ \left( \frac{\xi_F}{1 - \xi_F} \right) (J_{F,t}) = (W_{F,t} - W_{U,t}) \]  

and

\[ \left( \frac{\nu_I}{(1 - \nu_I)} \right) J_{I,t} = (W_{I,t} - W_{U,t}) \]  

(A.12)
Now, using the asset values for the workers, we have

\[
W_{F,t} - W_{U,t} = w_{F,t} - b + E_t \Xi_{t+1|t} \left\{ (1 - \delta^F) W_{F,t+1} + \delta^F W_{U,t+1} \right\} \tag{A.13}
\]

\[
- E_t \Xi_{t+1|t} \left\{ p(\theta_{F,t})(1 - \delta^F) W_{F,t+1} + p(\theta_{I,t})(1 - \delta^I) W_{I,t+1} + \\
(1 - p(\theta_{F,t})(1 - \delta^F) - p(\theta_{I,t})(1 - \delta^I)) W_{U,t+1} \right\}
\]

\[
W_{F,t} - W_{U,t} = w_{F,t} - b \tag{A.14}
\]

\[
+ E_t \Xi_{t+1|t} \left\{ (1 - p(\theta_{F,t})(1 - \delta^F))(W_{F,t+1} - W_{U,t+1}) \right\} \]

\[
- p(\theta_{I,t})(1 - \delta^I)(W_{I,t+1} - W_{U,t+1}) \right\}
\]

Using the modified outcome for the Nash bargaining problem, we have

\[
\left( \frac{\xi^F}{1 - \xi^F} \right) (J_{F,t}) = w_{F,t} - b \tag{A.15}
\]

\[
+ E_t \Xi_{t+1|t} \left\{ \frac{(1 - \delta^F)(1 - p(\theta_{F,t})) \xi^F}{1 - \xi^F} J_{F,t+1} \right\} \]

\[
- \frac{p(\theta_{I,t})(1 - \delta^I) \nu_I}{1 - \nu_I} J_{I,t+1} \right\}
\]

Writing \((W_{I,t} - W_{U,t})\) in a similar way, we have

\[
\frac{\nu_I}{1 - \nu_I} J_{I,t} = w_{I,t} - b + E_t \Xi_{t+1|t} \left\{ \frac{p(\theta_{I,t})(1 - \delta^I) \nu_I}{1 - \nu_I} J_{I,t+1} \right\} \tag{A.16}
\]

\[
- E_t \Xi_{t+1|t} \left\{ \frac{(1 - \delta^F)(1 - p(\theta_{F,t})) \xi^F}{1 - \xi^F} (J_{F,t+1}) \right\}
\]

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Now, using the job creation conditions we derived in the text,

\[
\frac{\psi_F}{q(\theta_{F,t})} = E_t \Xi_{t+1|t} \left\{ (1 - \delta^F) \left( p_{F,t+1} z_{t+1} a_F f_{n_{F,t+1}, k_{F,t+1}} (n_{F,t+1} k_{F,t+1}) - w_{F,t+1} (1 + \tau^p) + \frac{\psi_F}{q(\theta_{F,t+1})} \right) \right\} \tag{A.17}
\]

\[
\frac{\psi_I}{q(\theta_{I,t})} = E_t \Xi_{t+1|t} \left\{ (1 - \delta^I) \left( p_{I,t+1} z_{t+1} a_F f_{n_{I,t+1}, k_{I,t+1}} (n_{I,t+1} k_{I,t+1}) - w_{I,t+1} + \frac{\psi_I}{q(\theta_{I,t+1})} \right) \right\} \tag{A.18}
\]

Along with the asset values for the firms, we can write

\[
\frac{\psi_F}{q(\theta_{F,t})} = E_t \Xi_{t+1|t} \left\{ (1 - \delta^F) J_{F,t+1} \right\} \tag{A.19}
\]

\[
\frac{\psi_I}{q(\theta_{I,t})} = E_t \Xi_{t+1|t} \left\{ (1 - \delta^I) J_{I,t+1} \right\} \tag{A.20}
\]

which, using the firms’ asset values once again, implies that

\[
J_{F,t} = p_{F,t} z_{t} a_F f_{n_{F,t}, k_{F,t}} (n_{F,t}, k_{F,t}) - w_{F,t} (1 + \tau^p) + \frac{\psi_F}{q(\theta_{F,t})} \tag{A.21}
\]

and

\[
J_{I,t} = p_{I,t} (1 - \rho(n_{I,t}, a_F, \ell)) z_{t} - \rho(n_{I,t}, a_F, \ell) p_{I,t} z_{t} n_{I,t} - w_{I,t} + \frac{\psi_I}{q(\theta_{I,t})} \tag{A.22}
\]

Taking these last two expressions forward, we can then write the wage equations as

\[
w_{F,t} = \xi^F \left[ p_{F,t} z_{t} a_F f_{n_{F,t}, k_{F,t}} + \psi_F \theta_{F,t} \right] + (1 - \xi^F) \left[ b + \frac{\nu_I}{1 - \nu_I} \psi_I \theta_{I,t} \right] \tag{A.23}
\]
and

\[
    w_{I,t} = \nu_I \left[ \rho_I (1 - \rho(n_{I,t}, a_F, \iota)) z_t - \rho_{n_I} (n_{I,t}, a_F, \iota) p_{I,t} z_t n_{I,t} + \psi_I \theta_{I,t} \right] ~ (A.24)
\]

\[
    + (1 - \nu_I) \left[ b + \frac{\xi_F}{1 - \xi_F \psi_F \theta_{F,t}} \right]
\]
A.3 Additional Impulse Response Functions

Figure A.1: Impulse Response Functions to a Positive Aggregate Shock: Benchmark Economy
Figure A.2: Impulse Response Functions to a Positive Aggregate Shock: Benchmark Economy (Continued)
Figure A.3: Impulse Response Functions to a Positive Aggregate Shock: Economy with $\gamma = 0.7$
Appendix B

Appendix for Chapter 3

B.1 Summary Statistics and Relationship Between Self-Employment and Output Persistence

Table B.1: Cross-Country Descriptive Statistics: Full Sample

<table>
<thead>
<tr>
<th>Variable</th>
<th>Years</th>
<th>Obs.</th>
<th>Mean</th>
<th>SD</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE Rate</td>
<td>2000-2007</td>
<td>86</td>
<td>31.8</td>
<td>20.72</td>
<td>5.00</td>
<td>88.7</td>
</tr>
<tr>
<td>AC of Output</td>
<td>1985-2007</td>
<td>115</td>
<td>0.574</td>
<td>0.20</td>
<td>-0.251</td>
<td>0.840</td>
</tr>
<tr>
<td>Log Real GDP 1985</td>
<td>1985</td>
<td>115</td>
<td>23.4</td>
<td>2.24</td>
<td>18.9</td>
<td>29.4</td>
</tr>
</tbody>
</table>

AC of Output refers to the autocorrelation of the cyclical component of output. SE refers to Self-Employment. I use Catini, Panizza, and Saade’s (2010) compilation of the World Bank’s World Development Indicators (WDI) to obtain the annual series for real GDP for each country from 1985 to 2007. I HP-filter the series with a smoothing parameter of 100 to extract the cyclical component of the series and compute the first-order autocorrelation, which I take to be the measure of cyclical persistence in the paper. Self-employment is obtained from the OECD (2009) and corresponds to the average share of self-employed individuals in non-agricultural employment for years 2000 through 2007. Self-employment includes own-account workers, employers, and contributing family members. Figure 3.1 in the text uses
83 observations and excludes countries with an autocorrelation of output lower than 0.1.

The estimation equation for the persistence of the cyclical component of real GDP can be written as

\[ AC(GDP_i) = \gamma_0 + \gamma_1 SE_i + \sum_{k=2}^{n} \gamma_k X_{ki} + \mu_i \]

Where \( AC(GDP_i) \) denotes the first-order autocorrelation of the cyclical component of the log of real GDP in country \( i \). \( SE_i \) is the average self-employment rate as share of non-agricultural employment for years 2000 through 2007, \( \mu_i \) is an error term, and \( X_{ki} \) encompasses other regressors of interest including a measure of openness over the sample period, the log of real GDP per capita in 1985 as a proxy for the country’s level of development, the government spending-to-GDP ratio over the sample period, and a measure of the quality of institutions (Law and Order from Political Risk Services, PRS).\(^1\) Openness is defined as the sum of imports and exports divided by GDP. All the regressors except for the self-employment rate and Law and Order are taken from the World Development Indicators (WDI) for years 1985 through 2007. The results in Table B2 omit the constant term in the regression for expositional purposes. As the table shows, the coefficient on self-employment is significant at conventional levels and fairly stable across specifications.\(^2\)

\(^1\)All these variables except for log real GDP per capita in 1985 and Law and Order are averages from 1985 to 2007. I also experiment with log real GDP in 1985, which has a lower correlation with self-employment, when excluding institutional quality and obtain qualitatively similar results.

\(^2\)For now, I ignore the possibility that self-employment might be influenced by the persistence of output, though it is not strictly correct to claim causation from self-employment to output persistence without taking care of the potential endogeneity between the two variables. One piece of evidence regarding entrepreneurship that acts in favor of the approach I present here is that,
Table B.2: Self-Employment and Cyclical Persistence: Cross-Country Evidence

<table>
<thead>
<tr>
<th>Specification</th>
<th>Dep. Variable:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-Employment</td>
<td>Autocorrelation of Output</td>
<td>-0.003***</td>
<td>-0.003**</td>
<td>-0.003**</td>
<td>-0.003**</td>
<td>-0.004***</td>
</tr>
<tr>
<td>Log RGDP 1985</td>
<td></td>
<td>-</td>
<td>0.015</td>
<td>0.015</td>
<td>0.013</td>
<td>-</td>
</tr>
<tr>
<td>Openness</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-0.000**</td>
<td>-0.000**</td>
<td>-0.000*</td>
</tr>
<tr>
<td>Gov. Spending</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td>Law and Order</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.001</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.14</td>
<td>0.17</td>
<td>0.20</td>
<td>0.20</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>83</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>78</td>
<td></td>
</tr>
</tbody>
</table>

Note: The results above use the sample for self-employment with 83 observations, which excludes countries with an autocorrelation of output lower than 0.1.

Restricting potential outliers (countries with autocorrelations less than 0.2 and countries with a share of self-employment higher than 85 percent) reduces the coefficient on self-employment only slightly. However, the results are still significant at conventional levels and remain essentially the same. Restricting the sample to developing countries does not change the main conclusions either.

While the level of income and the quality of institutions are considered important determinants of self-employment in the literature, it is important to assess how controlling for these two factors affects the results. After all, institutional quality might influence the persistence of cyclical output in other ways unrelated to this while the business cycle might affect entry into self-employment over the life-cycle, this effect seems to be temporary and only delays entry into entrepreneurship (see, for example, Yu, Orazem, and Jolly, 2009). Thus, the deep determinants of self-employment are likely to be less dependent on short-run economic activity and more dependent on other structural features of the economy.
variable's impact on self-employment. Table B2 shows that the significance of self-employment holds for all specifications, with little variation in the coefficient value. Using a smoothing parameter of 6.25 to extract the cyclical component of output, which is another common value used in the literature, does not alter the qualitative results or the strength of the relationship between the cyclical persistence of output and self-employment. The measure of self-employment I use only considers self-employment in non-agricultural employment. Since developing countries have large self-employment shares in agriculture as well, one may wonder whether the link between self-employment and output persistence over the business cycle still holds when we control for the share of employment in agriculture or the share of agricultural production in GDP. The sign and significance of the relationship remain virtually unchanged when I control for either one of these measures.\textsuperscript{3} Finally, the results remain qualitatively the same if I use non-agricultural real GDP as opposed to total real GDP to compute the measure of cyclical persistence.\textsuperscript{4}

\textsuperscript{3}The share of employment in agriculture and the share of agricultural production in GDP are highly correlated with initial GDP per capita. A similar comment applies to the proxy for institutional quality, Law and Order. Thus, when I regress output persistence on the share of non-agricultural self-employment, I exclude initial GDP per capita as well as Law and Order.

\textsuperscript{4}In fact, the relationship between self-employment and the cyclical persistence of non-agricultural output is still negative, but becomes stronger.
Table B.3: List of Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Country</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algeria</td>
<td>Argentina</td>
<td>Australia</td>
</tr>
<tr>
<td>Austria</td>
<td>Bahrain</td>
<td>Bangladesh</td>
</tr>
<tr>
<td>Belgium</td>
<td>Benin</td>
<td>Bolivia</td>
</tr>
<tr>
<td>Botswana</td>
<td>Brazil</td>
<td>Bulgaria</td>
</tr>
<tr>
<td>Burkina Faso</td>
<td>Burundi</td>
<td>Cameroon</td>
</tr>
<tr>
<td>Canada</td>
<td>Central African Republic</td>
<td>Chile</td>
</tr>
<tr>
<td>China</td>
<td>Colombia</td>
<td>Comoros</td>
</tr>
<tr>
<td>Denmark</td>
<td>Dominican Republic</td>
<td>Ecuador</td>
</tr>
<tr>
<td>Egypt</td>
<td>El Salvador</td>
<td>Finland</td>
</tr>
<tr>
<td>France</td>
<td>Gambia</td>
<td>Germany</td>
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<tr>
<td>Greece</td>
<td>Guatemala</td>
<td>Guinea</td>
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<tr>
<td>Haiti</td>
<td>Honduras</td>
<td>Hong Kong</td>
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<tr>
<td>Hungary</td>
<td>India</td>
<td>Indonesia</td>
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<tr>
<td>Ireland</td>
<td>Italy</td>
<td>Japan</td>
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<td>Jordan</td>
<td>Kenya</td>
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<td>Mali</td>
<td>Mauritius</td>
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<td>Mexico</td>
<td>Morocco</td>
<td>Mozambique</td>
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<td>Netherlands</td>
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<td>Singapore</td>
<td>South Africa</td>
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<td>Spain</td>
<td>Sri Lanka</td>
<td>Sweden</td>
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<td>Syrian Arab Republic</td>
<td>Thailand</td>
<td>Togo</td>
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<td>Turkey</td>
<td>United Arab Emirates</td>
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<td>United Kingdom</td>
<td>United States</td>
<td>Uruguay</td>
</tr>
<tr>
<td>Venezuela</td>
<td>Zambia</td>
<td></td>
</tr>
</tbody>
</table>

B.2 Derivation of Nash Wage and Rental Rate Equations

Recall that the value functions for the household and the salaried firm are given by:

\[ W_{S,t} = \lambda_t^h w_{S,t} + E_t \beta \left\{ (1 - \delta^S) W_{S,t+1} + \delta^S W_{U,t+1} \right\} \]  
(B.1)

\[ W_{SE,t} = \lambda_t^h (z_t a_{SE,t} - r_{SE,t}) + E_t \beta \left\{ (1 - \delta^{SE}) W_{SE,t+1} + \delta^{SE} W_{U,t+1} \right\} \]  
(B.2)
\begin{align*}
W_{U,t} &= \lambda_t^h b + E_t \beta \begin{pmatrix}
(1 - \delta^S)p(\theta_{S,t})W_{S,t+1} \\
+ (1 - \delta^{SE})p(\theta_{SE,t})W_{SE,t+1} \\
+ [1 - (1 - \delta^S)p(\theta_{S,t})]W_{U,t+1} \\
- (1 - \delta^{SE})u_{SE,t}^n p(\theta_{SE,t})W_{U,t+1}
\end{pmatrix} \tag{B.3}
\end{align*}

\begin{align*}
J_{S,t} &= \frac{z_t a_{Sf}(n_{S,t}, \omega_t k_{S,t}) - w_{S,t}}{1 - (1 - \delta^S)} \{1 - \delta^S\}J_{S,t+1} \tag{B.4}
\end{align*}

\begin{align*}
J_{SE,t} &= \frac{r_{SE,t}}{1 - \delta^{SE}} \{\delta^{SE} - \delta\} + E_t \Xi_{t+1} \{1 - \delta^{SE}\}J_{SE,t+1} \tag{B.5}
\end{align*}

Where \(\Xi_{t+1} = \beta \left(\frac{U_t(c_{t+1})}{U_t(c_t)}\right)\) and \(\lambda_t^h = U_t(c_t)\). The Nash bargaining first-order condition for the wage is given by

\begin{align*}
\left(\frac{W_{S,t} - W_{U,t}}{\lambda_t^h}\right) &= \frac{\nu_S}{1 - \nu_S} (J_{S,t}) \tag{B.6}
\end{align*}

For the rental rate on capital used in self-employment, we have

\begin{align*}
\left(\frac{W_{SE,t} - W_{U,t}}{\lambda_t^h}\right) &= \frac{\nu_{SE}}{1 - \nu_{SE}} (J_{SE,t} - J_{u_k,t}) \tag{B.7}
\end{align*}

First, write

\begin{align*}
(W_{SE,t} - W_{U,t}) &= (z_t a_{SE,t} - r_{SE,t}) - b \\
&= \left(1 - \nu_{SE}^n p(\theta_{SE,t})\right)(1 - \delta^{SE}) + E_t \Xi_{t+1} \left(\frac{1 - \delta^S}{1 - \delta^{SE}}(W_{SE,t+1} - W_{U,t+1}) - p(\theta_{S,t})(1 - \delta^S)(W_{S,t+1} - W_{U,t+1})\right) \tag{B.8}
\end{align*}
Similarly, we can write

\[ (W_{S,t} - W_{U,t}) = w_{S,t} - b \quad \text{(B.9)} \]

\[ + E_t \Xi_{t+1|t} \left\{ (1 - p(\theta_{S,t})) (1 - \delta^S) (W_{S,t+1} - W_{U,t+1}) \right\} \]

\[ - v_{SE,t}^u p(\theta_{SE,t}) (1 - \delta^{SE}) (W_{SE,t+1} - W_{U,t+1}) \]

Using the first-order conditions from Nash bargaining, we have:

\[
\frac{\nu_{SE}}{1 - \nu_{SE}} (J_{SE,t} - J_{u_k,t}) = (z_t a_{SE,t} - r_{SE,t}) - b + \]

\[ E_t \Xi_{t+1|t} \left\{ \frac{\nu_{SE}(1 - v_{SE}^u p(\theta_{SE,t}))}{1 - \nu_{SE}} (1 - \delta^{SE}) \right\} \]

\[ \times (J_{SE,t+1} - J_{u_k,t+1}) \]

\[ - \frac{p(\theta_{SE,t}) \nu_{SE}}{1 - \nu_{SE}} (1 - \delta^{SE}) J_{S,t+1} \]

and

\[
\frac{\nu_{S}}{1 - \nu_{S}} (J_{S,t}) = w_{S,t} - b + E_t \Xi_{t+1|t} \left\{ \frac{(1 - p(\theta_{S,t})) \nu_{S}}{1 - \nu_{S}} (1 - \delta^S) J_{S,t+1} \right\} \]

\[ - \frac{\nu_{SE} v_{SE}^u p(\theta_{SE,t})}{1 - \nu_{SE}} (1 - \delta^{SE}) \]

\[ \times (J_{SE,t+1} - J_{u_k,t+1}) \]

\[ \right\} \quad \text{(B.11)} \]

Using the optimality conditions from the firm’s and household’s problems, we can write:

\[
\frac{\psi_{S}}{q(\theta_{S,t})} = E_t \Xi_{t+1|t} \left\{ (1 - \delta^S) J_{S,t+1} \right\} \quad \text{(B.12)}
\]
and

\[
\frac{[z_t a S_f \omega_k (n_{S,t}, \omega_t k_{S,t}) + (1 - \delta^{SE}) q(\theta_{SE,t})]}{q(\theta_{SE,t})} = E_t \Xi_{t+1|t} \left\{ (1 - \delta^{SE}) J_{SE,t+1} \right\} \quad \text{(B.13)}
\]

Recall that \( \Xi_{t+1|t} = \frac{\beta h_t}{\lambda_t} = \frac{z_t a S_f n_{S,t} \omega_t k_{S,t}}{v_{SE}(\theta_t)} \cdot \frac{p(\theta_{S,t})}{q(\theta_{S,t})} = \frac{\theta_{S,t}}{v_{SE,t}} \cdot \frac{p(\theta_{SE,t})}{q(\theta_{SE,t})} = \frac{1}{v_{SE,t}} \), and \( J_{u_{k,t}} = (1 - \delta) \).

After some algebra and using the three facts above along with \( \lambda_t^h = U_c(c_t) \), we obtain expressions for \( w_{S,t} \) and \( r_{SE,t} \):

\[
w_{S,t} = \nu_s [z_t a S_f n_{S,t} \omega_t k_{S,t} + \psi S \theta_{S,t}] + (1 - \nu_s) b \quad \text{(B.14)}
\]

\[
+ \frac{(1 - \nu_s) \nu_{SE}}{1 - \nu_{SE}} v_{SE,t} p(\theta_{SE,t}) \left[ \frac{z_t a S_f \omega_k (n_{S,t}, \omega_t k_{S,t})}{q(\theta_{SE,t})} \right]
\]

\[
+ \frac{(1 - \nu_s) \nu_{SE}}{1 - \nu_{SE}} v_{SE,t} p(\theta_{SE,t}) (1 - \delta^{SE}) \left[ 1 - E_t \Xi_{t+1|t} (1 - \delta) \right]
\]

\[
r_{SE,t} = (1 - \nu_{SE}) \left[ z_t a S_f \theta_{SE,t} - \frac{\nu_s}{1 - \nu_s} \psi S \theta_{S,t} - b \right] \quad \text{(B.15)}
\]

\[
- \nu_{SE} v_{SE,t} p(\theta_{SE,t}) \left[ \frac{z_t a S_f \omega_k (n_{S,t}, \omega_t k_{S,t})}{q(\theta_{SE,t})} \right]
\]

\[
+ \nu_{SE} (1 - v_{SE,t} p(\theta_{SE,t})) (1 - \delta^{SE}) \left[ 1 - E_t \Xi_{t+1|t} (1 - \delta) \right]
\]
Bibliography


[71] Hobijn, Bart, and Ayşegül Şahin. 2007. “Firms and Flexibility;” *mimeo*.


