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Abstract

In this paper, we study a distributed opportunistic scheduling problem to exploit the channel fluctuations in wireless ad-hoc networks. In this problem, channel probing is followed by a transmission scheduling procedure executed independently within each link in the network. We study this problem for the popular block-fading channel model, where channel dependencies are inevitable between different time instances during the channel probing phase. Different from existing works, we explicitly consider this type of channel dependencies and its impact on the transmission scheduling and hence the system performance. We use optimal stopping theory to formulate this problem, but at carefully chosen time instances at which effective decisions are made. The problem can then be solved by a new stopping rule problem where the observations are independent between different time instances. Since the stopping rule problem has an implicit horizon determined by the network size, we first characterize the system performance using backward induction. We develop one recursive approach to solve the problem and show that the computational complexity is linear with respect to network size. Due to its computational complexity, we present an approximated approach for performance analysis and develop a metric to check how good the approximation is. We characterize the achievable system performance if we ignore the finite horizon constraint and design stopping rules based on the infinite horizon analysis nevertheless. We present an improved protocol to reduce the probing costs which requires no additional cost. We

characterize the performance improvement and the energy savings in terms of the probing signals. We show numerical results based on our mathematical analysis with various settings of parameters.

Index Terms

Opportunistic scheduling, media access control, ad-hoc networks, channel probing, block fading, optimal stopping, backward induction.

I. INTRODUCTION

There have been many works on opportunistic scheduling to exploit the channel fluctuations in the past decade. Instead of treating fading as a source of unreliability and trying to mitigate such channel fluctuations, fading can be exploited by transmitting information *opportunistically* when and where the channel is strong [1], [2]. On the other hand, opportunistic spectrum access and spectrum sharing has been widely studied for cognitive radio networks [3]. Hence it is important to understand the trade-off between the costs spent for channel sensing and the opportunities (e.g. system throughputs) obtained for such systems. Most existing works on opportunistic scheduling assume a cellular like system where a central scheduler tries to optimize the overall system performance by selecting the *on-peak* user for data transmission [1], [4]–[8]. In contrast, in ad-hoc networks it is necessary to access the wireless medium and schedule data transmission in a distributed fashion. So far few existing works have studied this problem. Such examples include rate adaptation with MAC design based on the RTS/CTS handshaking for IEEE 802.11 networks [9]–[11] and channel-aware ALOHA for uplink communications [12]–[14]. However, rate adaptation focuses on exploiting temporal opportunities while leaving the media access issue to the RTS/CTS mechanism. On the other hand, channel-aware ALOHA associates the probability to access the uplink with the channel state information (CSI) assuming that each user knows its own CSI. These schemes ignore the overhead due to the distributed nature of ad-hoc networks when considering the joint media access and scheduling problem. In fact, these costs should be counted into the protocol design in order to fully exploit the channel fluctuations in the network.

In [15], the authors proposed to study a distributed opportunistic scheduling (DOS) problem for ad-hoc networks, where M links contend the wireless medium and schedule data transmissions in a distributed fashion. In such networks, the transmitter has no knowledge of other links'

channel conditions, and even its own channel condition is not available before a successful channel probing. The channel quality corresponding to one successful probing can either be good or poor due to channel fluctuations. In each round of channel probing, the winner makes a decision on whether or not to send data over the channel. If the winner gives up the current opportunity, all links re-contend again, hoping that some link with better channel condition can utilize the channel after re-contention. The goal is to optimize the overall system performance. The authors show that the decision on further channel probing or data transmission is based only on local channel conditions, and the optimal strategy is a threshold policy.

One key issue in the design and analysis of opportunistic scheduling protocols for wireless ad-hoc networks is to seek an optimal trade-off between the costs to obtain the CSIs and the opportunities that can be exploited based on these information. When channel probing is adopted for this purpose, the problem reduces to the tradeoff between the durations elapsed for channel probing and those remaining for data transmissions. The authors in [15] consider the constant data time (CDT) model, where a fixed duration of T is available for data transmission regardless of the time consumed for channel probing. To further understand this tradeoff and its impact on the system performance, we consider the constant access time (CAT) model, where the total time duration available is a fixed amount T and the protocol needs to decide how to split T between channel probing and data transmissions in order to improve the system performance. On the other hand, in [15] the winners' channel rates were explicitly assumed to be independent during the channel probing phase, which is an ideal assumption. As we will explain in Section III, there are inevitable dependencies between the winners' rates at different time instances during the channel probing phase. In our previous conference paper [16], we analyzed the distributed opportunistic scheduling problem for the CAT problem under the ideal assumption that the winners' channel rates are independent during the channel probing phase. In this paper, we further investigate this problem under the popular block fading channel model. We explicitly consider how such dependencies could impact the transmission scheduling and hence the system performance. We use optimal stopping theory [17]–[19] to describe this problem, where we only choose the time instances when an effective decision is taken to make our mathematical analysis tractable. The new contributions of this paper include:

1) We study a distributed opportunistic scheduling problem under the popular block fading channel model where there are inevitable dependencies between the winners' channel rates

during the channel probing phase. To the best of our knowledge, this problem has not been studied in the literature.

2) We present a concept named "effective observation points", where we only take observations at time instances when effective decisions are made. In this approach, repeated decisions by the same link are properly treated as a single decision. This approach makes our mathematical analysis tractable, where winners' channel rates in the probing phase are not independent in the first place.

3) We characterize the optimal stopping rules and network throughputs for networks at different scales. We show that the finite horizon analysis is necessary for networks whose sizes are not large enough, otherwise the actual achievable network throughputs may deviate a lot from the infinite horizon analysis results.

4) We propose a modified protocol to reduce the probing costs, which requires no additional overhead for protocol design. By analytical and numerical results, we show that the new protocol improves the system performance, in particular for scenarios when the network size is not large or the network is "over-probed". Furthermore, we show that the new protocol can reduce the energy consumed in the channel probing phase considerably. This makes the improved protocol of particular interest for networks whose nodes have limited battery life.

This paper is organized as follows. In Section II we describe our system model for the distributed opportunistic scheduling problem. In Section III we formulate the problem as an optimal stopping problem and present our concept of effective observation points for analyzing the problem. We first present a rigorous analysis for the CAT problem based on the finite horizon approach in Section IV. Due to its computational complexity, in Section V we introduce an approximate approach to characterize the system performance. In Section VI we present a modified protocol to reduce the probing costs, and analyze the performance improvement in network throughputs and energy savings in the channel probing phase. In Section VII we introduce the results for the CDT problem and a performance comparison to the CAT problem. We show our numerical results in Section VIII and finally conclude the paper in Section IX.

II. System Model and Motivation

In this section, we introduce our system model for the distributed opportunistic scheduling problem. Similar to the problem discussed in [15], we assume M links share the wireless medium without any centralized coordinator in an ad-hoc network. To access the wireless medium, all links have to probe first. Suppose the links adopt a fixed probing duration τ. A collision channel model is assumed, where a link wins the channel if and only if no other links are probing simultaneously. If link m probes the channel with probability $p^{(m)}$, the duration of the n-th round of channel probing is $T_n = \tau K_n$, where K_n is the number of probings before the channel is won by some link. Hence K_n has a geometric distribution $\text{Geom}(p_s)$ with parameter p_s, where

$$p_s = \sum_{m=1}^{M} p^{(m)} \prod_{j \neq m} \left(1 - p^{(j)}\right) \tag{1}$$

is the successful probing probability. Throughout this paper, we use superscript (m) to denote variables related to the m-th link, and subscript n to denote variables related to the winner in the n-th round of channel probing. We also use the terms "n-th round" of channel probing and "time n" interchangeably. At the end of the n-th round, winner s_n has an option to send data through the channel at the current available rate R_n or to give up this opportunity. Based on the current rate R_n, s_n makes a decision on whether or not to utilize the channel for data transmission in order to optimize the overall network throughput. If s_n gives up the opportunity, all links re-contend again. This procedure repeats until some link finally utilizes the channel. The goal is that all links cooperate indirectly to make the channel accessible by some link with a good enough channel quality.

The performance analysis in [15] relies on an important assumption: the winners' channel rates R_n are independent with respect to time n in the channel probing phase but can be locked for a constant duration T in the data transmission phase. It should be noted that the independence of $R^{(m)}$ within one block does not necessarily imply the independence of the winners' rates R_n. In fact, possible dependencies do exist between the winners' channel rates R_n, since some link $\tilde{m}$ might win the channel for multiple times within one block. This assumption can generally hold when the network size (i.e. the number of links in the network) is infinitely large. It is not necessarily true for a network with a finite size M. On the other hand, although opportunistic scheduling has been shown to improve the system performance dramatically for large networks [2], [8], [15], there are other factors we need to consider in the design of such systems. For

example, we could take a look at the average waiting time for any link to access the medium [20]. Suppose the channel fading are *i.i.d.* for all M links in the network. Then based on the distributed opportunistic scheduling scheme [15], [16], any link m is able to access the current block with a probability $\frac{1}{M}$. Hence it takes roughly M blocks before link m is able to send data over the wireless channel. This will lead to a long delay for large networks. Hence for such kind of systems, one practice approach is to consider multi-cell or multi-channel scheme [21]–[23] to trade-off several design goals (e.g. throughput, delays). In line with that, we argue that it is important to consider this problem for a network with a finite size M, which is the basis for a more complex multi-cell or multi-channel system.

To investigate how the dependencies of the winners' channel rates in the channel probing phase affect the system performance, we study this distributed opportunistic scheduling problem for the popular block fading channel model. We assume the channel rates are flat fading within one block. Hence the channel rate $R^{(m)}$ for any link m does not change within one block. The total block length T_s is separated into two parts as $T_s = T_p + T_d$, where T_p is for channel probing and T_d is for data transmission. At the end of the n-th round of channel probing, the total time duration for channel probing is $T_p = \sum_{i=1}^n T_i$. We consider the CAT model [16], [21], [22], where the transmitter has a fixed duration $T_s = T$ in total, leaving the available duration for data transmission as $T_d = T - \sum_{i=1}^n T_i$. If we decide to send data at the end of the n-th round, the normalized network throughput is

$$Y_n = \frac{R_n \cdot (T - \sum_{i=1}^n T_i)}{T}. \tag{2}$$

III. THE OPTIMAL STOPPING PROBLEM FORMULATION

In this section, we formulate the distributed opportunistic scheduling problem as an optimal stopping problem. In particular, we present a concept named *effective observation points* to facilitate the mathematical treatment of our problem.

The theory of optimal stopping [17]–[19] is about the problem of choosing a time to take a given action based on sequentially observed random variables in order to maximize an expected payoff. The stopping rule problem is defined by a sequence of random variables $\mathbf{X}_1, \mathbf{X}_2, \ldots$ whose joint distribution is known and a sequence of real-valued reward functions $Y_0, Y_1(\mathbf{x}_1), \ldots$. Let $(\Omega, \mathcal{B}, P)$ be the probability space, and $\mathcal{F}_n$ be the sub-σ-field of $\mathcal{B}$ generated by $\mathbf{X}_1, \ldots, \mathbf{X}_n$. We have a sequence of σ-fields as $\mathcal{F}_0 \subset \mathcal{F}_1 \subset \ldots \subset \mathcal{F}_n \subset \ldots \subset \mathcal{B}$. A stopping rule is defined

as a random variable $N \in \{0, 1, \ldots, \infty\}$ such that the event $\{N = n\}$ is in $\mathcal{F}_n$. Our goal is to choose a stopping rule N^* to maximize the expected reward $E[Y_N]$. If there is no bound on the number of stages at which one has to stop, this is an infinite horizon problem and the optimal return can be computed via the optimality equation. When there is a known upper bound on the number of stages, it is a finite horizon problem and the optimal return can be solved by backward induction. Details on this topic can be found in [17]–[19].

At the end of the n-th round, winner s_n observes the probing duration T_n and the available channel rate R_n. Recalling that $T_n = \tau K_n$ and the fact that τ is a constant, we denote the observations at time n as a random vector $\mathbf{X}_n = (R_n, K_n)$ and one realization of $\mathbf{X}_n$ as $\mathbf{x}_n = (r_n, k_n)$. The σ-fields can be denoted as

$$\mathcal{F}_n = \{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n\} = \{R_1, K_1; R_2, K_2; \ldots; R_n, K_n\}. \tag{3}$$

Then s_n makes a decision on whether or not to stop based on $\mathcal{F}_n$, to maximize the overall network throughput (2). Here a decision to "stop" means that s_n decides to utilize the remaining time duration for data transmissions. A decision to "continue" means that s_n decides to give up the current opportunity. Another round of channel probing and decision making then begins. This probing and decision behavior continues within this block until winner s_N finally utilizes the channel for data transmissions, where N is the stopping time. It could be easily sensed and detected by all other links at this point. Hence all links don't send probing signals anymore until the beginning of the next block. If this procedure is repeated for I blocks independently, the decision making process can be described as

$$Y^* = \max_{N \in \mathcal{F}_n} \frac{E\left[R_N \cdot (T - \tau \sum_{i=1}^{N} K_i)\right]}{T}, \tag{4}$$

where N is the stopping time. This procedure can be described as in Fig. 1.

Now the problem is to find an optimal rule N^* to maximize the overall network throughput. To do this, we need to characterize the joint distribution of R_n and K_n. We notice that R_n and K_n are independent of each other, and K_n are also independent with respect to time n. However, the winners' channel rates R_n are not independent due to the block fading assumption. The dependencies of R_n makes the mathematical analysis of this problem intractable. In this paper, we tackle this problem by using effective observation points instead of the original observation points in (3). The whole idea is motivated from the following fact: at time n, if the winner s_n

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1:  for each link m do
2:      m probes the channel with a fixed probability p^(m);
3:      if m wins the channel then
4:          m makes a decision on whether or not to send data over the channel;
5:          if m decides to utilize the channel then
6:              m sends data through the channel for a duration of T − Σ_{i=1}^{n} T_i (CAT) or T
                (CDT), where n is the current index of channel probing;
7:          end if
8:      end if
9:  end for
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Fig. 1.   The distributed opportunistic scheduling protocol

decides to give up the opportunity, the same decision will be repeated for all future $\tilde{n} > n$ in this block when the channel is won by s_n again at time $\tilde{n}$. This is because utilizing the channel at time $\tilde{n}$ will only yield a smaller reward, i.e.

$$Y_{\tilde{n}} = \frac{R_{\tilde{n}}(T - \tau \sum_{i=1}^{\tilde{n}} K_i)}{T} < \frac{R_n(T - \tau \sum_{i=1}^{n} K_i)}{T} = Y_n, \tag{5}$$

where we used the fact that $R_{\tilde{n}} = R^{(s_{\tilde{n}})} = R^{(s_n)} = R_n$. It implies that an *effective* decision is always made at the time instances when a link wins the channel *for the first time*. If we only take observations at these time instances, the channel rates $\tilde{R}_n$ are independent. We denote the σ-fields at these time instances as

$$\tilde{\mathcal{F}}_n = \left\{ \tilde{R}_1, \ \tilde{K}_1; \ \tilde{R}_2, \ \tilde{K}_2; \ \ldots; \ \tilde{R}_n, \ \tilde{K}_n \right\}. \tag{6}$$

Lemma 1: The solutions to the optimal stopping problem based on $\tilde{\mathcal{F}}_n$ and $\mathcal{F}_n$ have different distributions for the stopping time N. However, both solutions have the same network throughputs and distributions for the elapsed probing durations $L = \sum_{i=1}^{N} K_i$.

Hence if we do not care how many times a given link m has given up its opportunity upon winning the wireless medium, the problem is equivalent to analyzing the problem using $\tilde{\mathcal{F}}_n$ instead of $\mathcal{F}_n$. For the rest of this paper, we *always refer to the σ-fields at the effective observation points unless noted otherwise*. Hence we use the notations $\mathcal{F}_n$, T_n, K_n instead of $\tilde{\mathcal{F}}_n$, $\tilde{T}_n$, $\tilde{K}_n$ for short for the rest of the paper.

IV. A RIGOROUS PERFORMANCE ANALYSIS: THE FINITE HORIZON APPROACH

In this section, we characterize the optimal stopping rules and the network throughputs. We analyze the protocol using σ-fields (6) recorded at those effective observation points. By this notation, the number of effective probing links is *monotonically* decreasing as time n moves on, even though physically all links are still probing the wireless medium as in Fig. 1. On the other hand, since no recall is allowed, if link m gives up its opportunity at some point, link m cannot reclaim it at a later time. As a result, the "last" winner must utilize the wireless medium for data transmission, otherwise the channel will be completely wasted for this block. Hence the stopping rule problem always has an implicit horizon at M, where M is the network size. The problem should be treated as a finite horizon problem and be solved by the backward induction approach [17]–[19].

We denote the optimal expected reward based on observations until the n-th round of channel probing as $\lambda_n^* = \lambda_n^*(\mathbf{x}_1, \ldots, \mathbf{x}_n)$. We will use the term the n-th round of channel probing, "time n" or "stage n" interchangeably in this section. The backward induction procedure can be described as

$$\lambda_M^*(\mathbf{x}_1, \ldots, \mathbf{x}_M) = Y_M(\mathbf{x}_1, \ldots, \mathbf{x}_M), \tag{7}$$

$$\lambda_n^*(\mathbf{x}_1, \ldots, \mathbf{x}_n) = \max \left\{ Y_n(\mathbf{x}_1, \ldots, \mathbf{x}_n), E\left[\lambda_{n+1}^*(\mathbf{X}_1, \ldots, \mathbf{X}_n, \mathbf{X}_{n+1}) | \mathbf{X}_1 = \mathbf{x}_1, \ldots, \mathbf{X}_n = \mathbf{x}_n \right] \right\}, \tag{8}$$

where $n = 0, 1, \ldots, M-1$, and $Y_n(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ is the instant reward based on $\mathcal{F}_n$. At stage n, it is optimal to stop if $Y_n(\mathbf{x}_1, \ldots, \mathbf{x}_n) \geq \lambda_n^*(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ and to continue otherwise. The optimal return at stage n is the instant payoff if the decision is to stop and the expected payoff if the decision is to continue. The optimal network throughput is λ_0^*, i.e. the optimal expected reward before taking any observations.

However, it is not practical to directly solve this problem using (7) and (8) for two reasons. First, the channel rates r_n are generally continuous variables. We have to discretize r_n to use (7) and (8). Second, the instant observation $\mathbf{x}_n$ at time n is a two dimensional vector. To directly apply the backward induction procedure on $\mathbf{x}_n$, there will be too many states in the state space. The overwhelming computational complexity will restrict us to solve problems only with a small M. In this paper, we develop one approach to reduce the computational complexity for this procedure. First we note that the last item in (8) only depends on $\mathbf{x}_1, \ldots, \mathbf{x}_n$ since the

expectation is taken with respect to $\mathbf{X}_{n+1}$. Hence we can denote it as

$$w_n(\mathbf{x}_1, \ldots, \mathbf{x}_n) = E\left[\lambda_{n+1}^*(\mathbf{X}_1, \ldots, \mathbf{X}_n, \mathbf{X}_{n+1})|\mathbf{X}_1 = \mathbf{x}_1, \ldots, \mathbf{X}_n = \mathbf{x}_n\right] \tag{9}$$

for short. Now the problem in (7) and (8) reduces to the calculation of $w_n(\mathbf{x}_1, \ldots, \mathbf{x}_n)$. Next, we show that the calculation of $w_n(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ does not need all of these observations $\mathbf{x}_1, \ldots, \mathbf{x}_n$. To show this, we define the total number of probings up to time n as $L_n = \sum_{i=1}^n K_i$. Note that L_n is a random variable. We denote one realization of L_n as l_n.

Lemma 2: Suppose the network size is $M \geq 2$, the expected reward at time n can be characterized as

$$w_n(\mathbf{x}_1, \ldots, \mathbf{x}_n) = \begin{cases} w_M(r_M, l_M) & \text{for } n = M, \\ w_n(l_n) & \text{for } n = M-1, \ldots, 1, \\ w_0 = \lambda_0^* & \text{for } n = 0. \end{cases} \tag{10}$$

Proof: Since the network size is M, the backward induction procedure has a horizon at stage M. The reward at stage M is $w_M(\mathbf{x}_1, \ldots, \mathbf{x}_M) = \max\left\{0, \frac{r_M(T - \tau l_M)}{T}\right\}$. Hence $w_M(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ only depends on r_M and l_M, and it can be denoted as $w_M(r_M, l_M)$ for short.

Now we let $n = M - 1$ in (9). We can see that the expectation in (9) is taken with respect to $\mathbf{X}_M$, i.e. R_M and K_M. We have showed that $w_M(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ only depends on r_M and l_M, but is independent of r_{M-1}. Hence after taking the expectation, $w_{M-1}(\mathbf{x}_1, \ldots, \mathbf{x}_{M-1})$ is still independent of r_{M-1}. On the other hand, $w_{M-1}(\mathbf{x}_1, \ldots, \mathbf{x}_{M-1})$ does depend on l_{M-1}, since l_{M-1} remains in the expression after taking expectations with respect to K_M, where $L_M = L_{M-1} + K_M$. Hence w_{M-1} only depends on l_{M-1}. We can iterate this procedure from $n = M - 2$ to $n = 1$. As a result, for $n = M - 1, \ldots, 1$, $w_n(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ can be denoted as $w_n(l_n)$ for short.

Finally, the network throughput is the optimal expected reward before taking any observations. That is to say $n = 0$. In this case, l_n can only be 0. Hence we can write it as $w_0 = \lambda_0^*$ for short. ∎

Following Lemma 2, we can use l_n as the only state for the backward induction procedure. The problem is reduced to a one-dimensional problem. To calculate w_0, we need to calculate $w_1(l_1)$ for all possible l_1, and then $w_2(l_2)$ for all possible l_2, and so on until stage M. Hence the problem is to compute $w_n(l_n)$ for $n = 1, \ldots, M - 1$ and $w_M(r_M, l_M)$.

Theorem 1: The optimal stopping rule for the distributed opportunistic scheduling problem is

$$N^* = \min \left\{ n \geq 1 : R_n \geq \lambda_n^* \cdot \frac{T}{T - \tau \sum_{i=1}^n K_i} \right\}. \tag{11}$$

The optimal network throughput is $w_0 = \lambda_0^*$. Suppose the network size is M. The finite horizon analysis reduces to the calculation of w_0, which eventually iterates all $w_n(l_n)$ for $n = 1, \ldots, M-1$ and $w_M(r_M, l_M)$. The expected reward can be calculated recursively as

$$w_{n-1}(l_{n-1}) = \sum_{k \in \Omega_n(l_{n-1})} (1 - p_{s,n})^{k-1} p_{s,n} \cdot q_n(l_{n-1}, k) \tag{12}$$

$$q_n(l_{n-1}, k) = P_n(k) \cdot E_n(k) + [1 - P_n(k)] \cdot w_n(l_{n-1} + k), \tag{13}$$

where $q_n(l_{n-1}, k)$ is the conditional expected reward given $K_n = k$. K_n can take values in

$$\Omega_n(l_{n-1}) = \{k \mid \tau \cdot (l_{n-1} + k) < T, \, k \in \mathbb{N}\},$$

and $P_n(k)$ and $E_n(k)$ can be calculated as

$$P_n(k) = P \left[R_n > \frac{w_n(l_{n-1} + k) \cdot T}{T - \tau(l_{n-1} + k)} \right]$$

$$E_n(k) = E \left[R_n \Big| R_n > \frac{w_n(l_{n-1} + k) \cdot T}{T - \tau(l_{n-1} + k)} \right] \cdot \frac{T - \tau(l_{n-1} + k)}{T}.$$

Proof: To calculate $w_{n-1}(l_{n-1})$, we use n to substitute $n-1$ in (9) and take expectation on both sides of (8) as

$$w_{n-1}(l_{n-1}) = E \left[\max\{Y_n(\mathbf{x}_1, \ldots, \mathbf{x}_{n-1}, \mathbf{X}_n), w_n(\mathbf{x}_1, \ldots, \mathbf{x}_{n-1}, \mathbf{X}_n)\} \right], \tag{14}$$

where the expectation is taken with respect to $\mathbf{X}_n$. We further take its conditional expectation with respect to K_n and write it as

$$w_{n-1}(l_{n-1}) = \sum_{k \in \Omega_n(l_{n-1})} P[K_n = k] \cdot q_n(l_{n-1}, k),$$

where $q_n(l_{n-1}, k)$ is the conditional expectation of (14) given $K_n = k$. As we showed in Section V, K_n has a geometric distribution with parameter $p_{s,n}$. Hence we have $P[K_n = k] = (1 - p_{s,n})^{k-1} p_{s,n}$. On the other hand, combing (2) and Lemma 2, we have

$$q_{n-1}(l_{n-1}, k) = E \left[\max \left\{ \frac{R_n \left(T - \tau(l_{n-1} + k) \right)}{T}, w_n(l_{n-1} + k) \right\} \right].$$

Now if we take its conditional expectation with respect to the following event

$$\left\{ \frac{R_n \left(T - \tau(l_{n-1} + k) \right)}{T} > w_n(l_{n-1} + k) \right\},$$

we can immediately have (13). This proves the theorem. ∎

We can also bound the computational complexity of the procedure described in Theorem 1.

Proposition 1: To calculate the optimal network throughput w_0 and the expected reward based on a set of given observations $\{r_1, k_1; \ldots; r_M, k_M\}$ with a relative error less than ϵ where $0 < \epsilon \ll 1$, the computational complexity of the procedure in Theorem 1 is

$$\min \left\{ M \lceil T/\tau \rceil, \sum_{n=1}^{M} n \left\lceil \frac{\log \frac{\epsilon}{1+\epsilon}}{\log(1-p_{s,n})} \right\rceil \right\}. \tag{15}$$

Proof: For a network with size M, the backward induction procedure in Theorem 1 has M stages. In the n-th stage, the procedure involves calculation of all possible $w_n(l_n)$. For the CAT problem, l_n can simply be bounded as $1 \leq l_n \leq \lceil T/\tau \rceil$. Hence the computational complexity in the n-th stage is at most $\lceil T/\tau \rceil$, and the total computational complexity of the backward induction procedure is at most $M \lceil T/\tau \rceil$.

On the other hand, since $q_n(l_{n-1}, k)$ is the conditional expected reward if the probing duration is $K_n = k$ at time n, $q_n(l_{n-1}, k)$ is a decreasing function of k. For a given integer k_ϵ, we have

$$\frac{\sum_{k>k_\epsilon} P[K_n = k] \cdot q_n(l_{n-1}, k)}{\sum_{k \leq k_\epsilon} P[K_n = k] \cdot q_n(l_{n-1}, k)} < \frac{\sum_{k>k_\epsilon} P[K_n = k] \cdot q_n(l_{n-1}, k_\epsilon)}{\sum_{k \leq k_\epsilon} P[K_n = k] \cdot q_n(l_{n-1}, k_\epsilon)} = \frac{1}{1 - (1 - p_{s,n})^{k_\epsilon}} - 1, \tag{16}$$

where we used the fact that K_n has a geometric distribution $\mathrm{Geom}(p_{s,n})$ with parameter $p_{s,n}$. To ensure the relative error in the calculation of $w_{n-1}(l_{n-1})$ is less than ϵ, we let the right hand of (16) be less than ϵ. After some manipulation, we have $k_\epsilon \geq \frac{\log \frac{\epsilon}{1+\epsilon}}{\log(1-p_{s,n})}$. Hence we only need to iterate $\left\lceil \frac{\log \frac{\epsilon}{1+\epsilon}}{\log(1-p_{s,n})} \right\rceil$ items in the n-th stage. Iterating this procedure from the top level $n = 0$ to $n = M$ and noticing that $l_0 = 0$, we immediately have our conclusion. ∎

V. An Approximation for Performance Analysis: the Infinite Horizon Approach

As we can see in Section IV, the computational complexity of backward induction can quickly become overwhelming as M increases. In contrast, the infinite horizon analysis based on the optimality equation [17]–[19] has a much smaller computational complexity. Hence we would like to see if the performance analysis in Section IV can be approximated using the infinite horizon approach. In this section, we analyze the protocol using the infinite horizon horizon approach and develop a metric as a guideline to choose the appropriate approach for a given network.

Lemma 3: For the same stopping rule problem described in Section III, the infinite horizon analysis yields an optimal network throughput slightly larger than that from the finite horizon analysis. The gap decreases to 0 as the network size $M \to \infty$.

If the network size M is large enough, this problem does not have a finite horizon and can be analyzed using the optimality equation [17]–[19]. We make the following assumptions:

[A1] The total number of links M in the network is large enough;

[A2] The channel rates only take values in $(0, +\infty)$;

[A3] Each link m probes the wireless medium with probability $p^{(m)} = p$;

[A4] The channel rates for all links have the same cumulative distribution function (CDF) $F_R(r)$.

Here [A1] ensures the problem does not have a finite horizon, and [A2]-[A4] make our mathematical analysis tractable. To analyze this problem, we first characterize the distribution of K_n. By the n-th round, $n - 1$ links in total have given up their opportunities in previous rounds. Hence only the rest of the $M - n + 1$ links can contribute to an effective channel probing. If we ignore the events when the channel is won by any of these $n - 1$ links, K_n has a geometric distribution $\mathrm{Geom}(p_{s,n})$ with parameter $p_{s,n}$, where

$$p_{s,n} = (M - n + 1) \cdot p(1 - p)^{M-1} \tag{17}$$

is the successful probing probability in the n-th round. To better explain our results, we introduce some notations that will be used frequently in our proof. We define a sequence of parameters $f_n \triangleq \frac{p_{s,n}}{p_{s,1}} = \frac{M-n+1}{M}$ and a sequence of random variables $\tilde{K}_n = f_n K_n$. Since f_n is a constant, $\tilde{K}_n$ also has a geometric distribution with mean $E[\tilde{K}_n] = f_n E[K_n] = E[K_1]$. Hence $\tilde{K}_n$ and K_1 can be considered *equal in distribution* [24], [25].

Theorem 2: The average network throughput λ_O^* of Fig. 1 is the solution of

$$E\left[1 + \frac{M(M+1)}{(M+0.5)^2} \cdot \frac{\tau \tilde{K}_1}{T} - \frac{\lambda}{R_0}\right]^+ = \frac{M(M+1)}{(M+0.5)^2} \cdot \frac{\tau}{T p_{s,1}}, \tag{18}$$

where $E[\cdot]^+$ is defined as $E[X]^+ = E[\max(X, 0)]$. The optimal stopping rule is

$$N^* = \min\left\{n \geq 1 : R_n \geq \lambda_n^* \cdot \frac{T}{T - \tau \sum_{i=1}^n K_i}\right\}, \tag{19}$$

where λ_n^* is the solution of

$$E\left[1 - \frac{\tau}{T}\left\{\sum_{i=1}^n K_i - \frac{M(M+n+1)}{(M+0.5)^2}\tilde{K}_1\right\} - \frac{\lambda}{R_n}\right]^+ = \frac{M(M+n+1)}{(M+0.5)^2} \cdot \frac{\tau}{T p_{s,1}}. \tag{20}$$

Proof: By [A2], we can rewrite the network throughput (2) at time n as $Y_n = \frac{T - \tau \sum_{i=1}^{n} K_i}{T/R_n}$. This problem can be solved as a maximal rate of return problem. For a fixed rate λ, we define a new reward function at time n as

$$V_n(\lambda) = T - \tau \sum_{i=1}^{n} K_i - \frac{\lambda T}{R_n}. \tag{21}$$

The problem is then to characterize the optimal rate λ_n^* and the stopping rule to achieve λ_n^*. First, we need to show the existence of the optimal stopping rule. We notice that $E\{\sup_n V_n(\lambda)\} < T < \infty$. On the other hand, we can easily see that $\limsup_{n \to \infty} V_n(\lambda) \to -\infty$ and $V_n(\lambda) \to -\infty$ *a.s.*. Putting them together leads to $\limsup_{n \to \infty} V_n(\lambda) \to V_\infty(\lambda)$ *a.s.*. Hence an optimal stopping rule exists and can be described by the optimality equation. By the definition of $\tilde{K}_n$, we notice that $K_i = \frac{M}{M - i + 1} \tilde{K}_i$. Substituting it into (21) and using the *i.i.d.* property of $\tilde{K}_i$, we have

$$V_n(\lambda) = T - \tau \sum_{i=1}^{n} \frac{M}{M - i + 1} \tilde{K}_i - \frac{\lambda T}{R_n} = T - \tau \tilde{K}_1 \sum_{i=1}^{n} \frac{M}{M - i + 1} - \frac{\lambda T}{R_n}.$$

Note that the above equation holds *in distribution*. Since the network size M is large enough and the problem can be solved as an infinite-horizon problem, the number of rounds n is usually much smaller compared to M. To calculate the above summation, we appropriate $\frac{M}{M-i+1} + \frac{M}{M-(n+1-i)+1}$ as $\frac{2M}{M-n/2+0.5}$. By repeating this procedure for all $i \leq n/2$, we can appropriate $V_n(\lambda)$ as

$$V_n(\lambda) \approx T - \tau \tilde{K}_1 \cdot \frac{Mn}{M - n/2 + 0.5} - \frac{\lambda T}{R_n}.$$

Similarly, the payoff at time $n + 1$ can be written as

$$V_{n+1}(\lambda) \approx T - \tau \tilde{K}_1 \cdot \frac{M(n+1)}{M - (n+1)/2 + 0.5} - \frac{\lambda T}{R_{n+1}}.$$

Meanwhile, note that R_n are *i.i.d.* by [A4]. Hence in the sense of distribution the difference between $V_n(\lambda)$ and $V_{n+1}(\lambda)$ can be written as

$$\Delta V_n(\lambda) = V_{n+1}(\lambda) - V_n(\lambda) = -\tau \tilde{K}_1 \cdot \frac{M}{M + 0.5} \left[\frac{n+1}{1 - \frac{(n+1)/2}{M+0.5}} - \frac{n}{1 - \frac{n/2}{M+0.5}} \right].$$

By [A1], we can approximate the item in the above square bracket as

$$(n+1) \left\{ 1 + \frac{\frac{n+1}{2}}{M + 0.5} \right\} - n \left\{ 1 + \frac{\frac{n}{2}}{M + 0.5} \right\} = \frac{M + n + 1}{M + 0.5}. \tag{22}$$

Substituting it into the optimality equation $V_n^* = \max\{Y_n, E(V_{n+1}^* | \mathcal{F}_n)\}$ [17]–[19], we have

$$V_n^*(\lambda) = E \left[\max \left\{ T - \tau \sum_{i=1}^{n} K_i - \frac{\lambda T}{R_n}, V_n^*(\lambda) - \tau \tilde{K}_1 \cdot \frac{M(M + n + 1)}{(M + 0.5)^2} \right\} \right].$$

According to optimal stopping theory [17]–[19], the optimal rate λ_n^* that maximizes the rate of return should yield 0 for (21). If we substitute $V_n^*(\lambda_n^*) = 0$ into the above equation and note that $E[\tilde{K}_1] = 1/p_{s,1}$, we can rewrite the equation as (20). The uniqueness of λ_n^* can be easily verified. The optimal stopping rule can be written as

$$N^* = \min\left\{n \geq 1 : T - \tau \sum_{i=1}^{n} K_i - \frac{\lambda_n^* T}{R_n} \geq V_n^*(\lambda_n^*) = 0\right\},$$

which leads to (19) after some manipulation. The optimal network throughput is the expected rate of return before taking any observations. Hence we get the optimal network throughput λ_O^* if we let $n = 0$ in (20), which immediately yields (18). ∎

The optimal network throughput (18) can be further simplified under certain conditions.

Proposition 2: Assume $\tau \ll T$, the network throughput λ_O^* of Fig. 1 can be approximated as the solution of

$$E\left[1 - \frac{\lambda}{R_0}\right]^+ = \frac{M(M+1)}{(M+0.5)^2} \cdot \frac{\tau}{T p_{s,1}}. \tag{23}$$

Proof: By [A1], we have $\frac{M(M+1)}{(M+0.5)^2} \approx 1$. Since $\frac{\tau}{T} \ll 1$, the term $\frac{\tau}{T} \cdot \frac{M(M+1)}{(M+0.5)^2} \tilde{K}_1$ can be ignored compared to 1 on the left hand of (18). This completes the proof. ∎

An immediate question following Lemma 3 and Theorem 2 is: how good is the approximation compared to the rigorous analysis in Section IV, in particular for networks at a finite size M? In fact, we prefer to design stopping rule based on the analytical results from Theorem 2 due to its low computational complexity even for a finite network size M. What will the actual achievable network throughput be like?

To answer these questions, we present one metric which serves as a guideline when we decide whether or not we could use the infinite horizon analysis. For a given network, if the problem can be treated as in Section V, in a probabilistic sense the optimal stopping time N^* should be much smaller than the network size M. Hence one necessary condition is that the probability $P[N^* > M]$ should be small enough.

Theorem 3: For a network with size M, suppose the infinite horizon analysis in Theorem 2 yields a sequence of optimal expected network throughputs λ_n^* for Fig. 1. If $\tau \ll T$, the probability $P[N^* > M]$ can be approximated as

$$P[N^* > M] \approx \prod_{n=1}^{M} F_R(\lambda_n^*). \tag{24}$$

If this probability is not small enough, it is not recommended to design stopping rules based on the infinite horizon analysis.

Proof: For a given integer $k > 0$, we have

$$P\left[N^* > k\right] = P\left[\min\left\{n \geq 1 : R_n \geq \lambda_n^* \cdot \frac{T}{T - \tau \sum_{i=1}^n K_i}\right\} > k\right]. \tag{25}$$

Since $\tau \ll T$ and the optimal stopping time N^* is much smaller than M, we consider $\frac{\tau}{T} \cdot \sum_{i=1}^n K_i \ll 1$ for approximation. Substituting it into (25), we have

$$P\left[N^* > k\right] \approx P\left[\min\left\{n \geq 1 : R_n \geq \lambda_n^*\right\} > k\right]$$

$$= \prod_{n \leq k} P\left[R_n < \lambda_n^*\right], \tag{26}$$

where we used the fact that R_n are *i.i.d.*. To get (25), simply let $k = M$ in (26). ∎

On the other hand, if $P[N^* > M]$ is not small enough, it implies that the stopping rule problem cannot be treated as an infinite horizon problem. In this case, if we design a stopping rule based on Theorem 2 nevertheless, the procedure will quickly reach the last stage and be forced to stop then. In this case, the actually achieved network throughput is generally not optimal.

Theorem 4: Suppose the infinite horizon analysis yields a sequence of λ_n^* for a network with size M. Suppose we design a stopping rule $\hat{N}$ based on these rates and (19). If $\tau \ll T$, the achievable network throughput based on $\hat{N}$ is

$$\hat{\lambda}^* = \sum_{n=1}^M E\left[R_n | R_n \geq \lambda_n^*\right] \frac{T - \tau \sum_{i=1}^n 1/p_{s,i}}{T} \times \left[1 - F_R(\lambda_n^*)\right] \prod_{i=1}^{n-1} F_R(\lambda_i^*). \tag{27}$$

Proof: According to the stopping rule (19), the expected reward can be written as

$$\hat{\lambda}^* = \sum_{n=1}^M E\left[Y_n(\mathbf{X}_1, \ldots, \mathbf{X}_n) \cdot P(N = n | \mathbf{X}_1, \ldots, \mathbf{X}_n)\right]. \tag{28}$$

The condition to stop at time n is $R_n \geq \lambda_n^* \cdot \frac{T}{T - \tau \sum_{i=1}^n K_i}$. When $\tau \ll T$, this condition can be simplified as $R_n \geq \lambda_n^*$. Hence the expected reward at time n can be written as a conditional expectation, i.e.

$$E\left[R_n \cdot \frac{T - \tau \sum_{i=1}^n K_i}{T} \Bigg| R_n \geq \lambda_n^*\right] = E\left[R_n | R_n \geq \lambda_n^*\right] \cdot \frac{T - \tau \sum_{i=1}^n 1/p_{s,i}}{T},$$

where we used the fact that K_i has a geometric distribution $\text{Geom}(p_{s,i})$ and is independent from R_n. On the other hand, by (26) the probability to stop at time n can be approximated as

$$P(N = n | \mathbf{X}_1, \ldots, \mathbf{X}_n) = \prod_{i=1}^{n-1} F_R(\lambda_i^*) - \prod_{i=1}^n F_R(\lambda_i^*) = \left[1 - F_R(\lambda_n^*)\right] \prod_{i=1}^{n-1} F_R(\lambda_i^*). \tag{29}$$

Substituting it into (28) together with the expected reward at time n, we have (27). ∎

VI. An Energy Efficient Improvement of the Protocol

In this section, we present an improved distributed opportunistic scheduling protocol, which is directly motivated by the concept of effective observation points introduced in Section III.

According to (23), the network throughput λ_O^* decreases as the successful probing probability $p_{s,1}$ decreases. Hence to improve the network throughputs, we need to improve $p_{s,1}$. For a given network with size M, we can first tune the parameter p to maximize $p_{s,1}$. To do this, we let $n = 1$ in (17) and take the first-order derivative as $\frac{\partial p_{s,1}}{\partial p} = M(1-p)^{M-2}(1-Mp) = 0$. The non-trivial solution in $(0, 1)$ is $p^* = 1/M$. Hence to maximize $p_{s,1}$, on average there is exactly $Mp^* = 1$ link probing the channel. The maximal successful probing probability is $p_{s,1} = \frac{1}{1-\frac{1}{M}} \cdot \left(1 - \frac{1}{M}\right)^M$ for $M \geq 2$, which is a decreasing function of M. Hence the optimal throughput λ_O^* is a decreasing function as M increases. From the perspective of channel probing costs, a smaller M is preferred for better system performance. On the other hand, from (5) we can see that if link m ever gives up the current opportunity, m will always repeat the same decision in the current block. Hence if link m ever decides to send data in the current block, it should happen when m wins the channel for the first time. If after that m still contends the medium, it would not lead to an effective decision, and meanwhile it lowers the successful probability $p_{s,n}$. Based on this observation, we have an improved protocol as shown in Fig. 2 [16].

Suppose at time n the set of active probing links is $\mathcal{M}_n$. This is the set of links whose current state is TRUE in Fig. 2. Denoting its cardinality as $M_n \triangleq \|\mathcal{M}_n\|$, we have $M_n = M - n + 1$ following line 9 of Fig. 2. We can see that M_n is decreasing as time n moves on. The shrinking of $\mathcal{M}_n$ is an important feature of the improved protocol. It not only reduces the probing costs, but also ensures the winner s_n is different at each time n. Hence the winners' rates R_n are independent in Fig. 2. At time n, the successful probing probability can be written as

$$p_{s,n} = M_n p(1-p)^{M_n-1}. \tag{30}$$

We now characterize the performance of the improved protocol shown in Fig. 2. First of all, the finite horizon analyses described in Section IV can be applied in a similar way here. The computational complexity can also be estimated similarly. The only difference is that the successful probing probability $p_{s,n}$ in (12) should be calculated according to (30). Now we analyze this problem assuming that it can be treated as an infinite horizon problem. By [A1],

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1: Link m sets its state as TRUE, where $m = 1, \ldots, M$;

2: **for** each link m whose state is TRUE **do**

3:   m probes the channel with a fixed probability $p^{(m)}$;

4:   **if** m wins the channel **then**

5:     m makes a decision on whether or not to send data over the channel;

6:     **if** m decides to utilize the channel **then**

7:       m sends data through the channel for a duration of $T - \sum_{i=1}^{n} T_i$ (CAT) or T (CDT), where n is the current index of channel probing;

8:     **else**

9:       m sets its state as FALSE;

10:     **end if**

11:   **end if**

12: **end for**

Fig. 2. The improved distributed opportunistic scheduling protocol

we can approximate the successful probing probability (30) as

$$p_{s,n} \approx Mp(1-p)^{M_n - 1} = Mp(1-p)^{M-n}. \tag{31}$$

We can see that $p_{s,1} < p_{s,2} < \ldots < p_{s,n}$. Similar to Theorem 2, we introduce a sequence of parameters $g_n \triangleq \frac{p_{s,n}}{p_{s,1}} = (1-p)^{-(n-1)}$ and a sequence of random variables $\tilde{K}_n = g_n K_n$. It is easy to verify that $\tilde{K}_n$ and K_1 can be considered *equal in distribution* and thus $\{\tilde{K}_n\}$ are *i.i.d.*.

Theorem 5: The network throughput λ_P^* of Fig. 2 is the solution of

$$E\left[1 + \frac{\tau}{T} \cdot (1-p)^2 \tilde{K}_1 - \frac{\lambda}{R_0}\right]^+ = (1-p)^2 \frac{\tau}{Tp_{s,1}}. \tag{32}$$

The optimal stopping rule is

$$N^* = \min\left\{n \geq 1 : R_n \geq \lambda_n^* \cdot \frac{T}{T - \tau \sum_{i=1}^{n} K_i}\right\}, \tag{33}$$

where λ_n^* is the solution of

$$E\left[1 - \frac{\tau}{T}\left\{\sum_{i=1}^{n} K_i - (1-p)^{n+1} \tilde{K}_{n+1}\right\} - \frac{\lambda}{R_n}\right]^+ = (1-p)^{n+1} \frac{\tau}{Tp_{s,1}}. \tag{34}$$

Proof: We use V_n defined in (21) in our proof. The existence of the optimal stopping rule can be verified in the same way as in Theorem 2. To compute the optimal reward V_n^*, we take a look at the reward after l steps since time n. By the definition of g_n, we can write $K_n = (1-p)^{n-1} \tilde{K}_n$. Substituting it into (21), we have

$$V_{n+l}(\lambda) = T - \tau \sum_{i=1}^{n} (1-p)^{i-1} \tilde{K}_i - \left[\tau \sum_{i=n+1}^{n+l} (1-p)^{i-1} \tilde{K}_i + \frac{\lambda T}{R_{n+l}} \right].$$

If we start from time $n+1$, the reward after l rounds is

$$V_{n+l+1}(\lambda) = T - \tau \sum_{i=1}^{n} (1-p)^{i-1} \tilde{K}_i - \tau(1-p)^n \tilde{K}_{n+1} - \left[\tau \sum_{i=n+2}^{n+l+1} (1-p)^{i-1} \tilde{K}_i + \frac{\lambda T}{R_{n+l+1}} \right].$$

The item in the above square bracket is the recursive part for l rounds of observations since time $n+1$. We can rewrite it as

$$(1-p) \left\{ \tau \sum_{i=n+1}^{n+l} (1-p)^{i-1} \tilde{K}_{i+1} + \frac{\lambda T}{R_{n+l+1}} \right\} + p \cdot \frac{\lambda T}{R_{n+l+1}}.$$

By [A1], p should be reasonably small; otherwise the average number of probing links Mp will be much larger than 1, leading to increased probing costs. Hence we can ignore the last term and write the optimality equation as

$$V_n^*(\lambda) = E \left[\max \left\{ T - \tau \sum_{i=1}^{n} K_i - \frac{\lambda T}{R_n}, (1-p)\left(V_n^*(\lambda) - \tau K_{n+1} \right) \right\} \right].$$

Again, the optimal reward λ_n^* that maximizes the rate of return must satisfy $V_n^*(\lambda_n^*) = 0$. We substitute this into the optimality equation and rewrite it as

$$E \left[1 - \frac{\tau}{T} \left\{ \sum_{i=1}^{n} K_i - (1-p) K_{n+1} \right\} - \frac{\lambda_n^*}{R_n} \right]^{+} = (1-p) \cdot \frac{\tau}{T} E[K_{n+1}].$$

If we further notice that $K_{n+1} = 1/g_{n+1} \tilde{K}_{n+1} = (1-p)^n \tilde{K}_{n+1}$ and that $\tilde{K}_{n+1}$ and K_1 are *i.i.d.*, we can rewrite the above equation as (34). The optimal stopping rule N^* can be derived in the same way as in Theorem 2. To get the optimal system throughput λ_P^*, we let $n = 0$ in (34) and rewrite the equation as (32). ∎

Similar to Section V, we further simplify the network throughput as Proposition 3 if $\tau \ll T$. Based on this, we show that the modified protocol improves the network throughput as in Proposition 4. The proofs are straight forward and are skipped due to space limitations.

Proposition 3: If $\tau \ll T$, the network throughput λ_P^* can be approximated as the solution of

$$E \left[1 - \frac{\lambda}{R_0} \right]^{+} = (1-p)^2 \cdot \frac{\tau}{T p_{s,1}}. \tag{35}$$

Proposition 4: The improved protocol in Fig. 2 yields a higher network throughput compared to the protocol in Fig. 1, i.e. $\lambda_P^* > \lambda_O^*$.

In the improved protocol any link who decides to give up the current opportunity for data transmission will not probe the channel anymore until the beginning of the next block. Hence these links can temporarily switch to a sleep mode until the beginning of the next block and reduce the energy used for channel probing. This could be very useful for mobile ad-hoc or sensor networks where most of their mobile nodes have limited battery life.

Similar to the analyses for throughputs, we focus on the total energy savings for all links in the channel probing phase, not for a specific link. Suppose each probing signal consumes roughly a constant energy of c. Then the energy consumed during the channel probing phase can be written as $c \sum_{i=1}^{N} Z_i$, where Z_i is the number of probing signals sent during the i-th round of channel probing, and N is the stopping time associated with the stopping rule. Hence the average energy spent during the channel probing phase is $z = cE\left[\sum_{i=1}^{N} Z_i\right]$. Using law of total expectations, we can write

$$z = cE\left[E\left[\sum_{i=1}^{N} Z_i \,\middle|\, N\right]\right] = c\sum_{n} P[N = n]\sum_{i=1}^{n} E[Z_i]. \tag{36}$$

Theorem 6: The average energy consumed for channel probing of Fig. 1 can be written as

$$z_O = c\sum_{n} P[N_O^* = n] \cdot \frac{1}{(1-p)^{M-1}}\sum_{i=1}^{n} \frac{M}{M-i+1}, \tag{37}$$

where N_O^* is the optimal stopping rule for Fig. 1, and the average probing energy of Fig. 2 can be written as

$$z_P = c\sum_{n} P[N_P^* = n] \cdot \frac{1}{(1-p)^{M-1}}\frac{1-(1-p)^n}{1-(1-p)}, \tag{38}$$

where N_P^* is the optimal stopping rule for Fig. 2.

Proof: As we mentioned in Section III, we will use the notation of $\tilde{\mathcal{F}}_n$ in the proof. For the protocol in Fig. 1, in the i-th round there are a total of $\tilde{K}_i$ probings, each of which has a duration of τ and on average Mp links sending probing signals. Hence there are on average $E[Z_i] = Mp \cdot E[\tilde{K}_i]$ probing signals sent in the i-th round. Hence we can write

$$E[Z_i] = Mp \cdot \frac{1}{(M-i+1)p(1-p)^{M-1}} = \frac{1}{(1-p)^{M-1}} \cdot \frac{M}{M-i+1}.$$

Substitue the above equation into (36), we can immediately have (37).

On the other hand, for the improved protocol in Fig. 2, in the i-th round there are a total of $\tilde{K}_i$ probings, and each of them has on average $(M - i + 1)p$ links sending probing signals. This is because in the improved protocol once a link gives up its opportunity, he would not probe again until the beginning of the next block. Hence we can write

$$\sum_{i=1}^{n} E[Z_i] = \sum_{i=1}^{n} (M - i + 1)p \cdot \frac{1}{(M - i + 1)p(1 - p)^{M-i}} = \frac{1}{(1 - p)^{M-1}} \cdot \frac{1 - (1 - p)^n}{1 - (1 - p)}.$$

Combine the above equation with (36), we have (38). ∎

In Theorem 6, the probability of $P[N^* = n]$ can be approximated in the same way as (29).

VII. THE CONSTANT DATA TIME PROBLEM

Our analyses in Section IV, V and VI can be applied to the CDT problem in a similar way. In the CDT problem [15], [21], [22], the transmitter has a fixed duration $T_d = T$ for data transmission, regardless of the duration T_p elapsed for channel probing. The normalized network throughput to utilize the channel at the end of the n-th round is

$$Y_n = \frac{R_n \cdot T}{T + \sum_{i=1}^{n} T_i}. \tag{39}$$

We show the analytical results for the CDT problem in this section and compare its numerical results to that of the CAT problem in Section VIII.

First of all, due to the block fading assumption, the CDT problem also has an implicit horizon at M. Hence the CDT problem for the original protocol in Fig. 1 or the improved protocol in Fig. 2 should be treated as a finite horizon problem.

Theorem 7: The network throughput of the CDT problem based on backward induction is $w_0 = \lambda_0^*$, and the optimal stopping rule is

$$N^* = \min \left\{ n \geq 1 : R_n \geq \lambda_n^* \cdot \left(1 + \frac{\tau}{T} \sum_{i=1}^{n} K_i \right) \right\}. \tag{40}$$

The finite horizon analysis reduces to the calculation of w_0, which eventually iterates all $w_n(l_n)$ for $n = 1, \ldots, M - 1$ and $w_M(r_M, l_M)$. The expected reward can be calculated recursively as

$$w_{n-1}(l_{n-1}) = \sum_{k \in \mathbb{N}} (1 - p_{s,n})^{k-1} p_{s,n} \cdot q_n(l_{n-1}, k) \tag{41}$$

$$q_n(l_{n-1}, k) = P_n(k) \cdot E_n(k) + [1 - P_n(k)] \cdot w_n(l_{n-1} + k), \tag{42}$$

where $q_n(l_{n-1}, k)$ is the conditional expected reward given $K_n = k$, and $P_n(k)$ and $E_n(k)$ can be calculated as

$$P_n(k) = P\left[R_n > w_n(l_{n-1} + k) \cdot \frac{T + \tau(l_{n-1} + k)}{T}\right]$$

$$E_n(k) = E\left[R_n \middle| R_n > w_n(l_{n-1} + k) \cdot \frac{T + \tau(l_{n-1} + k)}{T}\right] \times \frac{T}{T + \tau(l_{n-1} + k)}.$$

Proposition 5: For the CDT problem, to calculate the optimal network throughput w_0 and the expected reward for given observations $\{r_1, k_1; \ldots; r_M, k_M\}$ with a relative error less than ϵ where $0 < \epsilon \ll 1$, the computational complexity of the procedure in Theorem 7 is at most $\sum_{n=1}^{M} n \left\lceil \frac{\log \frac{\epsilon}{1+\epsilon}}{\log(1-p_{s,n})} \right\rceil$.

Similar to Section V, we prefer to analyze the CDT problem using infinite horizon approach when it can yield a good approximation to the finite horizon approach. The proof of Theorem 8 can be found in Appendix A.

Theorem 8: The average network throughput λ_O^* of the CDT problem is the solution of

$$E\left[\frac{R_0}{\lambda} + \frac{M(M+1)}{(M+0.5)^2} \cdot \frac{\tau \tilde{K}_1}{T} - 1\right]^+ = \frac{M(M+1)}{(M+0.5)^2} \cdot \frac{\tau}{Tp_{s,1}}. \tag{43}$$

The optimal stopping rule N^* is

$$N^* = \min\left\{n \geq 1 : R_n \geq \lambda_n^* \cdot \left(1 + \frac{\tau}{T}\sum_{i=1}^{n} K_i\right)\right\}, \tag{44}$$

and λ_n^* is the solution of

$$E\left[\frac{R_n}{\lambda} - \frac{\tau}{T}\left\{\sum_{i=1}^{n} K_i - \frac{M(M+n+1)}{(M+0.5)^2}\tilde{K}_1\right\} - 1\right]^+ = \frac{M(M+n+1)}{(M+0.5)^2} \cdot \frac{\tau}{Tp_{s,1}}. \tag{45}$$

Proposition 6: Assume $\tau \ll T$, the network throughput λ_O^* for the CDT problem can be approximated as the solution of

$$E\left[\frac{R_0}{\lambda} - 1\right]^+ = \frac{M(M+1)}{(M+0.5)^2} \cdot \frac{\tau}{Tp_{s,1}}. \tag{46}$$

In one block, the available duration for data transmission is $T - \tau\sum_{i=1}^{n} K_i$ for the CAT problem and T for the CDT problem respectively. Hence intuitively the protocol in Fig. 1 should yield a higher network throughput for the CDT model.

Proposition 7: Denote λ_{CAT}^* and λ_{CDT}^* as the optimal network throughput for the CAT and CDT problem respectively, we have $\lambda_{CAT}^* < \lambda_{CDT}^*$.

Proof: For any $r > \lambda^*_{CAT}$, we have $1 - \frac{\lambda*_{CAT}}{r} < \frac{r}{\lambda^*_{CAT}} - 1$. By taking integration on both sides of the inequality, we have

$$E\left[\frac{R_0}{\lambda^*_{CAT}} - 1\right]^+ > E\left[1 - \frac{\lambda^*_{CAT}}{R_0}\right]^+ = \frac{M(M+1)}{(M+0.5)^2} \cdot \frac{\tau}{Tp_{s,1}} = E\left[\frac{R_0}{\lambda^*_{CDT}} - 1\right]^+,$$

where the first and second equality is from Proposition 2 and Proposition 6 respectively. If we compare the first and last item in the above inequality, we have $\lambda^*_{CAT} < \lambda^*_{CDT}$. ∎

Theorem 9: Suppose the infinite horizon analysis yields a sequence of rates λ^*_n for a network of size M. If $\tau \ll T$, the probability $P[N^* > M]$ can be approximated as

$$P[N^* > M] \approx \prod_{n=1}^{M} F_R(\lambda^*_n). \tag{47}$$

If this probability is not small enough, it is not recommended to design stopping rules based on the infinite horizon analysis. Otherwise if we use the stopping rule based on these rates and (44), the achievable network throughput is

$$\hat{\lambda}^* = \sum_{n=1}^{M} E\left[R_n | R_n \geq \lambda^*_n\right] \frac{T}{T + \tau \sum_{i=1}^{n} 1/p_{s,i}} \times [1 - F_R(\lambda^*_n)] \prod_{i=1}^{n-1} F_R(\lambda^*_i). \tag{48}$$

For the improved protocol shown in Fig. 2, the performance for the CDT problem can be shown in Theorem 10. The proof of Theorem 10 can be found in Appendix B.

Theorem 10: The network throughput λ^*_P of Fig. 2 for the CDT problem is the solution of

$$E\left[\frac{R_0}{\lambda} + \frac{\tau}{T} \cdot (1-p)^2 \tilde{K}_1 - 1\right]^+ = (1-p)^2 \frac{\tau}{Tp_{s,1}}. \tag{49}$$

The optimal stopping rule N^* is

$$N^* = \min\left\{n \geq 1 : R_n \geq \lambda^*_n \cdot \left(1 + \frac{\tau}{T} \sum_{i=1}^{n} K_i\right)\right\}, \tag{50}$$

where λ^*_n is the solution of

$$E\left[\frac{R_n}{\lambda} - \frac{\tau}{T}\left\{\sum_{i=1}^{n} K_i - (1-p)^{n+1} \tilde{K}_{n+1}\right\} - 1\right]^+ = (1-p)^{n+1} \frac{\tau}{Tp_{s,1}}. \tag{51}$$

Proposition 8: If $\tau \ll T$, we can approximate the network throughput λ^*_P as the solution of

$$E\left[\frac{R_0}{\lambda} - 1\right]^+ = (1-p)^2 \frac{\tau}{Tp_{s,1}}. \tag{52}$$

VIII. NUMERICAL RESULTS

In this section, we show numerical results based on our discussions from Section IV to Section VII. We consider an ad-hoc network where the wireless medium is Rayleigh fading within each block. The channel rate can be written as

$$R(h) = \log_2(1 + \rho h) \qquad \text{bits/s/Hz},$$

where ρ is the average signal-to-noise ratio (SNR), and h is the channel gain corresponding to Rayleigh fading. Hence the probability density function (PDF) of h can be written as

$$f(h) = \frac{h}{\sigma^2} e^{-\frac{h^2}{2\sigma^2}}, \quad h \geq 0.$$

We assume $T = 1$ fixed throughout all simulations in this section. We compare numerical results from the finite horizon and the infinite horizon analysis with various settings of the parameters M, p, τ and ρ. For performance comparison purposes, we also show network throughputs from a pure random access approach, where the *first* winner of the wireless medium always utilizes the channel for data transmission, regardless of the available channel rates.

In Fig. 3, we show numerical results from both the infinite horizon and finite horizon analysis, where the network size is M and other parameters are $p = 1/M$, $\tau = 0.01$, $\rho = -10$dB and $\sigma = 1$. In Fig. 3(a), the dashed line shows the network throughputs for the pure random access scheme. Clearly we can see that the distributed opportunistic scheduling schemes show a considerable performance improvement, e.g. 57% improvement at a network size $M = 30$ in Fig. 3(a).

On the other hand, we notice that the finite horizon and infinite horizon analysis yield quite different network throughputs, especially when the network size M is not large enough. Fig. 3(a) shows the network throughputs for the distributed opportunistic scheduling protocol described in Fig. 1, where the line with "○" is from the finite horizon analysis and the line with "□" is from the infinite horizon analysis. The network throughputs show opposite trends as the network size M increases in Fig. 3(a). The network throughput from the infinite horizon analysis decreases while the network throughput from the finite horizon analysis increases. This is because in the infinite horizon analysis, there is enough multiuser diversity to be exploited. In the finite horizon analysis, there is not enough multiuser diversity to be exploited when the network size M is small, which is constrained by the finite horizon. Hence the infinite horizon analysis always
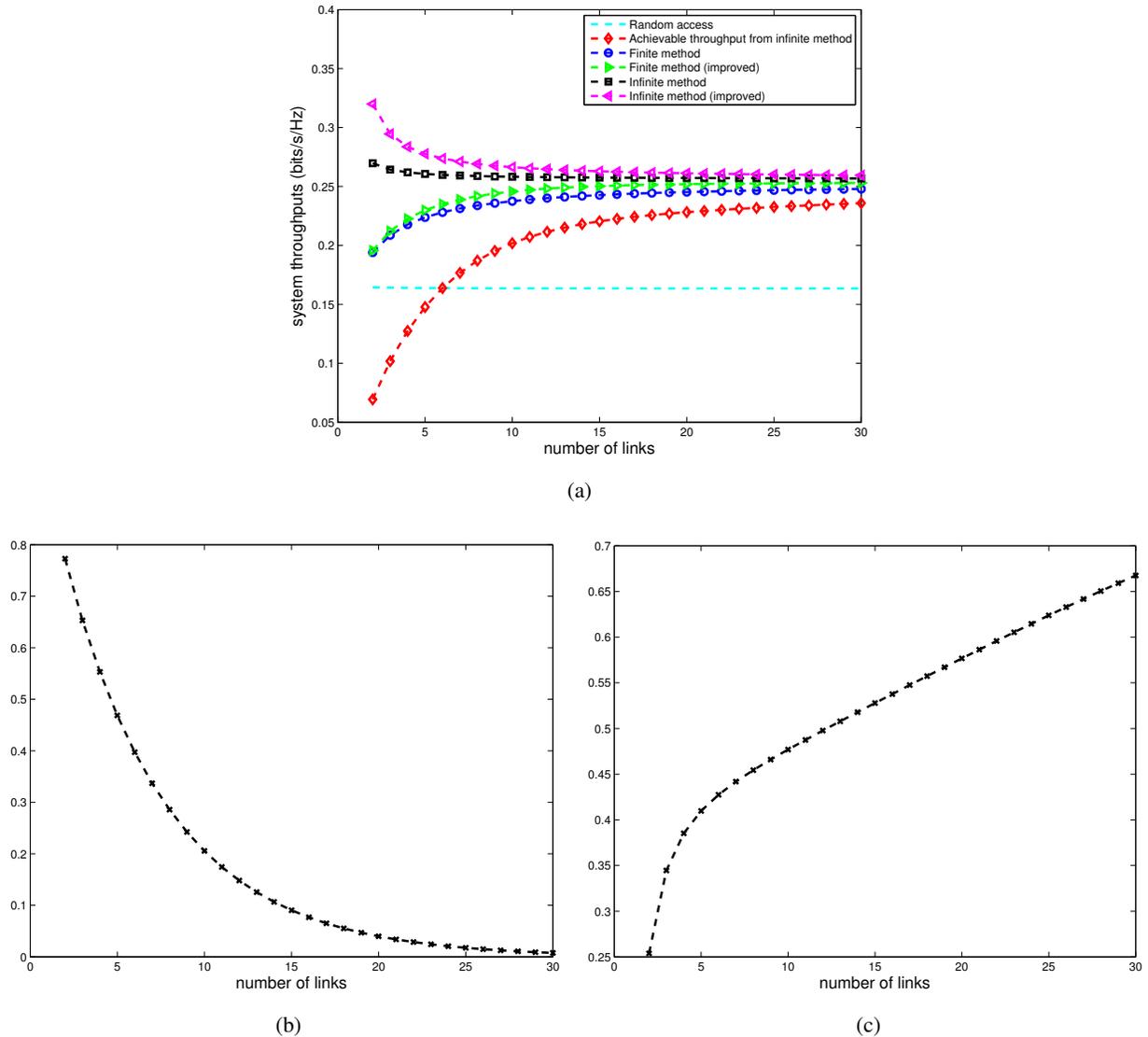
Fig. 3. Numerical results for ad-hoc networks with M links, where the parameters are $p = 1/M$, $\tau = 0.01$, $\rho = -10$dB and $\sigma = 1$: (a) network throughputs; (b) $P[N^* > M]$; (c) energy savings in probing signals.

shows a larger network throughput than the finite horizon analysis, and the gap between these two lines gradually decreases to 0 as the network size M increases. For example, the two lines show a gap of 8.7% at $M = 10$, and the gap drops to 4.9% at $M = 20$. In Fig. 3(b), we show the estimated probability $P[N^* > M]$ in Theorem 3. We can see that $P[N^* > M]$ is as high as 20% at $M = 10$, but drops quickly to 5% at $M = 20$. Hence for a given network, the estimated $P[N^* > M]$ serves as a measure of how well the problem can be treated as an infinite horizon problem. In line with this guideline, the line with "$\diamond$" in Fig. 3(a) shows the actual

achievable rewards based on Theorem 4 if the stopping rule is designed based on the results from the infinite horizon analysis. To our surprise, the actual reward is much smaller than the one from the infinite horizon analysis. This gap is pretty large when the network size M is not large enough, say $M \leq 20$ in Fig. 3(a). This observation agrees with the trend of $P[N^* > M]$ in Fig. 3(b). Hence if the problem is not suitable to be treated as an infinite horizon problem, it is not recommended to design stopping rules based on the infinite horizon analysis; otherwise the actual achievable rewards may deviate a lot from the infinite horizon analysis results for small and medium-size networks.

In addition, Fig. 3(a) shows the network throughputs for the improved protocol described in Fig. 2, where the line with "▷" is from the finite horizon analysis and the line with "◁" is from the infinite horizon analysis respectively. We can see that the improved protocol always yields a slightly better performance. For example, the line with "▷" steadily shows a 2% performance improvement over the line with "○" based on the finite horizon analysis. This coincides with our theoretical result in Proposition 4. Even though the performance improvement is not significant, it is still worth mentioning since there is no additional cost in the protocol design of Fig. 2. This performance improvement can be considered as a "free ride" based on the concept of effective observation points. On the other hand, in Fig. 3(c) we show the energy savings in probing signals that can be achieved by the improved protocol, where the y-axis is z_P/z_O for each M. We can see that the improved protocol can considerably reduce the total number of probing signals sent in the network. For example, at $M = 30$ the improved protocol only needs 67% of the probing signals sent in the original protocol in Fig. 1. This results in an saving of 33% in energy savings for probing signals. Hence with only a simple modification, the improved protocol can slightly improves the network throughputs while considerably saves energy used for probing signals. This is of particular interest for mobile ad-hoc networks or sensor networks where many nodes in the network have limited battery life.

In Fig. 4(a)-(d), we compare network throughputs with different parameters, where we vary one parameter at a time from the default parameter settings. We first show the network throughputs under two different scenarios for p in Fig. 4(a) and Fig. 4(b) respectively. Fig. 4(a) shows the network throughputs for $p = 0.01$, which represents an "under-probed" scenario since $Mp < 1$. We can see that the protocols yield smaller throughputs compared to Fig. 4(a). On the other hand, the improved protocol almost has the same performance as the original protocol. In this case, it
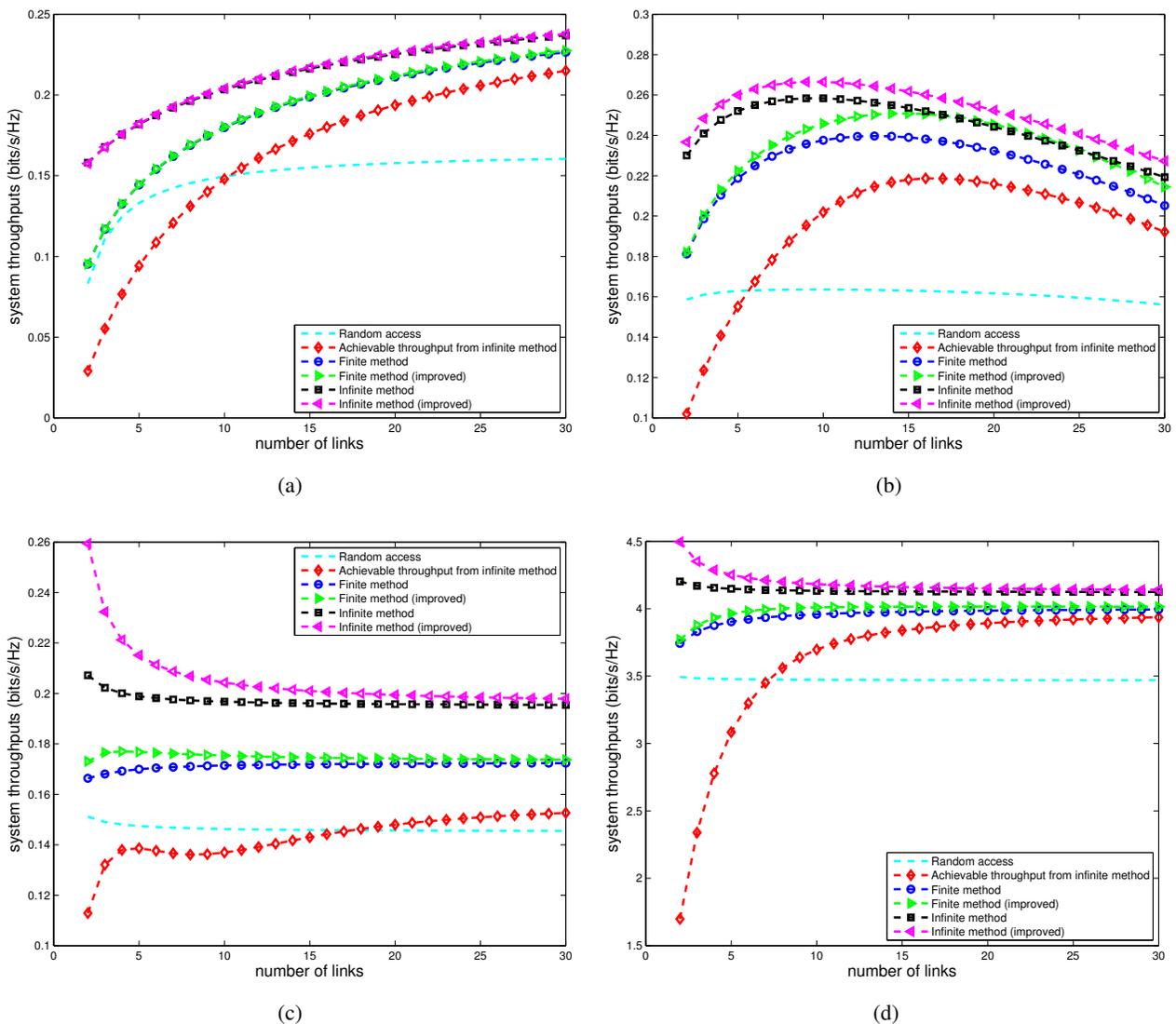
Fig. 4. Numerical results for ad-hoc networks with M links, where the default parameters are $p = 1/M$, $\tau = 0.01$, $\rho = -10$dB and $\sigma = 1$: (a) network throughputs with $p = 0.01$; (b) network throughputs with $p = 0.1$; (c) network throughputs with $\tau = 0.05$; (d) network throughputs with $\rho = 10$dB.

would not help to reduce the probing costs since the system is already under-probed. Fig. 4(b) shows the opposite scenario with $p = 0.1$ where the medium is "over-probed" since $Mp > 1$. The network throughputs are also smaller compared to Fig. 4(a). However, the improved protocol shows a 5% performance improvement compared to the original protocol. Recall that this quantity is 2% in Fig. 4(a). In this case, it helps to reduce the probing costs since the network is over-probed. In Fig. 4(c), we show the network throughputs with a larger probing cost $\tau = 0.05$.
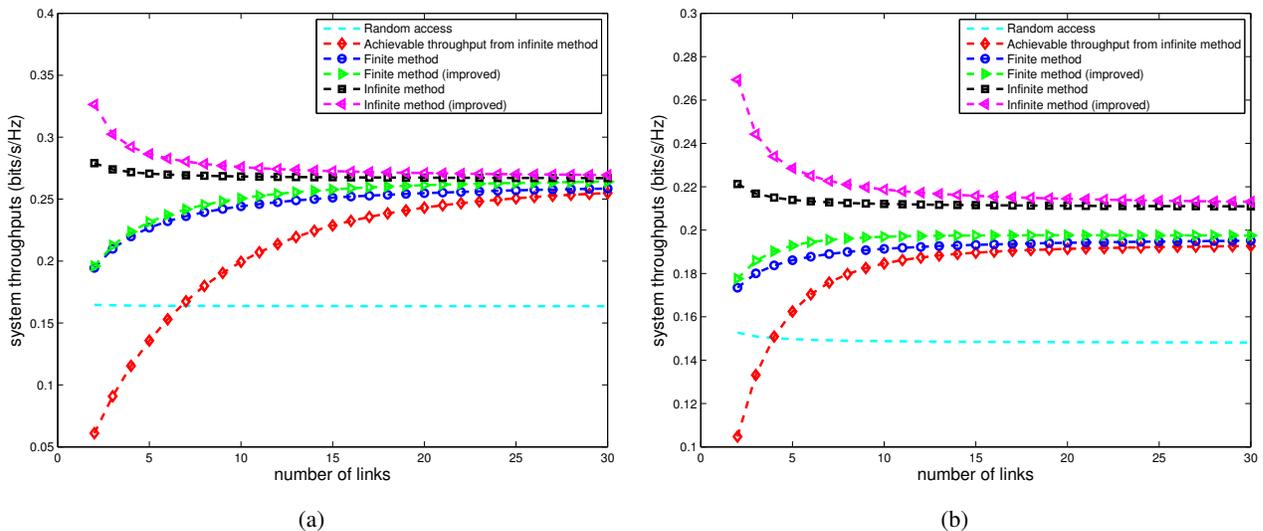
Fig. 5. Numerical results for the CDT problem for ad-hoc networks with M links, where the parameters are $p = 1/M$, $\rho = -10$dB and $\sigma = 1$: (a) network throughputs with $\tau = 0.01$; (b) network throughputs with $\tau = 0.05$.

With larger probing costs the protocols yield smaller network throughputs. Meanwhile, there is a larger gap between the finite horizon and infinite horizon analysis results. This is because with larger τ/T, a smaller horizon is imposed for the CAT problem, which makes it less likely to be treated as an infinite horizon problem. Fig. 4(d) shows the network throughputs with $\rho = 10$dB. With higher SNR, the protocols have much better network throughputs. However, compared to the random access scheme, the performance gain from opportunistic scheduling is only 13%. This shows that the opportunistic scheduling scheme is particularly useful at lower SNR regions, where the random access scheme does not perform well in the first place.

In comparison, Fig. 5 shows numerical results for the CDT problem with the same default parameters. Similar to the CAT problem, in Fig. 5(a) we can see the infinite horizon analysis always yields larger network throughputs than the finite horizon analysis. The gap of the network throughputs between them is more than 30%, but eventually decreases to 0 as the network size M is large enough. On the other hand, with the same parameters the CDT problem in Fig. 5(a) yields slightly larger network throughputs than the CAT problem in Fig. 3(a). This coincides with our theoretical result in Proposition 7. On the other hand, we can see that the line with "$\diamond$" in Fig. 5(a) approaches the finite horizon analysis faster than that of Fig. 3(a). It implies that the CDT problem requires a smaller network size M than the CAT problem for using the

infinite horizon analysis. In addition, Fig. 5(a) shows the network throughputs from the improved protocol. Similar to Fig. 3(a), the improved protocol always yields a slightly better performance from both analyses. Finally Fig. 5(b) shows the network throughputs for a larger probing cost $\tau = 0.05$. We can see that the gap in the network throughputs between the two analyses is 10.6%, while this gap for the CAT problem is 14.7% in Fig. 3(c). It implies that for the same network the CAT problem shows a smaller horizon compared to the CDT problem. This coincides with our earlier observation: to safely use infinite horizon analysis, the CAT problem generally requires a larger network size M. Furthermore, comparing both lines with "$\diamond$" in Fig. 3(c) and Fig. 5(b), we can see that the real rewards that can be achieved by the stopping rules from the infinite horizon analysis are very different. For the CAT problem, the expected real reward has a huge gap from the result based on the finite horizon analysis. For the CDT problem, the expected real reward approximates the result based on finite horizon analysis pretty well when the network size M is large enough, say $M = 15$. This implies that when the probing cost is high, it is particularly not recommended to design stopping rules based on the infinite horizon analysis for the CAT problem. The source of this difference lies in that there is always a constant duration of T for data transmission in the CDT problem.

IX. CONCLUSIONS

In this paper, we studied a distributed opportunistic scheduling problem for wireless ad-hoc networks under the popular block fading model. In this problem, we considered the inevitable dependencies between winners' channel rates at different time instances during the channel probing phase and their impact on the transmission scheduling. We formulated this problem using optimal stopping theory, but at carefully chosen time instances when effective decisions are made by merging repeated decisions. We mainly introduced our results using the CAT model. Since the problem has an implicit finite horizon constraint, we first characterized its performance using backward induction. We presented one recursive approach to reduce its computational overhead and derived an upper bound for its computational complexity. Due to the computational complexity, we proposed an approximated approach based on the infinite horizon analysis and developed a measure to check how well the problem can be treated as an infinite horizon problem. We estimated the achievable network throughput if we ignore the finite horizon constraint and use the stopping rule based on the infinite horizon analysis nevertheless. We then presented

an improved protocol to reduce the probing costs which requires no additional design cost. We showed that the modified protocol can slightly improve the network throughputs and considerably save energy for channel probing.

APPENDIX A

Similar to the CAT problem, we solve this problem as a maximal rate of return problem. For a fixed rate $\lambda > 0$, we define a new payoff at time n as

$$V_n(\lambda) = R_n T - \lambda \left(T + \tau \sum_{i=1}^{n} K_i \right). \tag{53}$$

To show the existence of the optimal rule, we first notice that $E\{\sup_n V_n\} < \infty$. On the other hand, we can see that $\limsup_{n \to \infty} V_n \to -\infty$ and $V_n \to -\infty$ *a.s.*. Putting them together leads to $\limsup_{n \to \infty} V_n \to V_\infty$ *a.s.*. Hence an optimal stopping rule exists and can be given by the optimality equation. Note that we used the equation $K_i = \frac{M}{M-i+1} \tilde{K}_i$ in the proof of Theorem 2. If we substitute it into (53) and notice the *i.i.d.* property of $\tilde{K}_i$, we can rewrite (53) as

$$V_n(\lambda) = R_n T - \lambda T - \lambda \tau \tilde{K}_1 \sum_{i=1}^{n} \frac{M}{M-i+1}.$$

The above equation should be understood *in distribution*. By taking the average of M and $M - n + 1$, we approximate $V_n(\lambda)$ as

$$V_n(\lambda) \approx R_n T - \lambda T - \lambda \tau \tilde{K}_1 \cdot \frac{Mn}{M - n/2 + 0.5}.$$

Similarly, the payoff at time $n + 1$ can be written as

$$V_{n+1}(\lambda) \approx R_{n+1} T - \lambda T - \lambda \tau \tilde{K}_1 \cdot \frac{M(n+1)}{M - (n+1)/2 + 0.5}.$$

Meanwhile, note that R_n are *i.i.d.* according to [A4]. Hence in the sense of distribution the difference between $V_n(\lambda)$ and $V_{n+1}(\lambda)$ can be written as

$$\lambda \tau \tilde{K}_1 \cdot \frac{M}{M + 0.5} \left[\frac{n+1}{1 - \frac{(n+1)/2}{M+0.5}} - \frac{n}{1 - \frac{n/2}{M+0.5}} \right].$$

The item in the above square bracket has been calculated as (22) in the proof of Theorem 2. If we substitute it into the optimality equation, we have

$$V_n^*(\lambda) = E \left[\max \left\{ R_n T - \lambda T - \lambda \tau \sum_{i=1}^{n} K_i, V_n^*(\lambda) - \frac{M(M + n + 1)}{(M + 0.5)^2} \cdot \lambda \tau \tilde{K}_1 \right\} \right].$$

The optimal rate λ_n^* that maximizes the rate of return should yield $V_n^*(\lambda_n^*) = 0$. If we substitute it into the optimality equation and notice $E[\tilde{K}_1] = 1/p_{s,1}$, we immediately have (45). The uniqueness of λ_n^* can be verified easily. The optimal stopping rule can be written as

$$N^* = \min \left\{ n \geq 1 : R_n T - \lambda_n^* T - \lambda_n^* \tau \sum_{i=1}^{n} K_i \geq V_n^*(\lambda_n^*) = 0 \right\},$$

which immediately leads to (44). If we let $n = 0$ in (45), we get (43). The solution of (43) is the optimal system throughput λ_O^*.

APPENDIX B

We use V_n defined in (53) in our proof. The existence of the optimal stopping rule can be verified in the same way as Theorem 8. To compute the optimal payoff V_n^*, we take a look at the payoff after l steps since time n. Note that we have used the equation $K_n = (1 - p)^{n-1} \tilde{K}_n$ in the proof of Theorem 5. If we substitute it into (53), we have

$$V_{n+l}(\lambda) = -\lambda T - \lambda \tau \sum_{i=1}^{n} (1 - p)^{i-1} \tilde{K}_i + \left[R_{n+l} T - \lambda \tau \sum_{i=n+1}^{n+l} (1 - p)^{i-1} \tilde{K}_i \right].$$

If we start from time $n + 1$, the payoff after l rounds is

$$V_{n+l+1}(\lambda) = -\lambda T - \lambda \tau \sum_{i=1}^{n} (1 - p)^{i-1} \tilde{K}_i - \lambda \tau (1-p)^n \tilde{K}_{n+1} + \left[R_{n+l+1} T - \lambda \tau \sum_{i=n+2}^{n+l+1} (1 - p)^{i-1} \tilde{K}_i \right].$$

The item in the above square bracket is the recursive part for l rounds of observations since time $n + 1$. We can rewrite it as

$$(1 - p) \left\{ R_{n+l+1} T - \lambda \tau \sum_{i=n+1}^{n+l} (1 - p)^{i-1} \tilde{K}_{i+1} \right\} + p \cdot R_{n+l+1} T.$$

By [A1], p should be reasonably small; otherwise the average number of probing links Mp are much larger than 1, leading to increased probing costs. Hence we can ignore the last term and write the optimality equation as

$$V_n^*(\lambda) = E \left[\max \left\{ R_n T - \lambda T - \lambda \tau \sum_{i=1}^{n} K_i, (1 - p) \left(V_n^*(\lambda) - \tau K_{n+1} \right) \right\} \right].$$

Again, the optimal payoff λ_n^* that maximizes the rate of return must satisfy $V_n^*(\lambda_n^*) = 0$. We substitute it into the optimality equation and rewrite it as

$$E \left[\frac{R_n}{\lambda_n^*} - \frac{\tau}{T} \left\{ \sum_{i=1}^{n} K_i - (1 - p) K_{n+1} \right\} - 1 \right]^+ = (1 - p) \cdot \frac{\tau}{T} E[K_{n+1}].$$

If we further notice that $K_{n+1} = 1/g_{n+1}\tilde{K}_{n+1} = (1-p)^n\tilde{K}_{n+1}$ and $\tilde{K}_{n+1}$ and K_1 are *i.i.d.*, we can rewrite the above equation as (51). The optimal stopping rule N^* can be derived in the same way as in Theorem 8. To get the overall optimal system throughput λ_P^*, we let $n = 0$ in (51) and rewrite the equation as (49).

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