This is supplemental material pertaining to the qualitative data presented in the manuscript: Kuo. E., Hull, M. M., Gupta, A., & Elby, A., *How Students Blend Conceptual and Formal Mathematical Reasoning in Solving Physics Problems*, accepted for publication in *Science Education* on 08/08/2012.

The following material consists of 4 parts:

A. The context of the study (p. 2)
B. The question protocol that students were asked to answer. (pp. 3-4)
C. The transcript of Alex's interview (pp. 5-17)
D. The transcript of Pat's interview (pp. 18-31)

A. Context of Study

From fall 2008 through spring 2011, we interviewed 13 engineering majors taking the University of Maryland's first-semester, calculus-based, introductory physics course for the NSF funded project *"Improving students' mathematical sense-making in engineering: research and development*." The course, geared toward engineering majors, covers mechanics. Students were recruited by email and through an in-class announcement. All the students who expressed an interest were interviewed (except for 2-3 students who could not be interviewed due to scheduling issues). Interviews were done at least one month into the semester, so students were familiar with kinematics, but not necessarily as familiar with forces and Newton's laws, and not at all familiar with pressure. Interviews lasted about one hour.

The semi-structured interviews were designed to probe engineering students' approaches to using equations while solving quantitative physics problems, specifically what mathematical and conceptual tools they bring to bear and which epistemological stances they take toward the nature of the knowledge they invoked (e.g., is it integrated or piecemeal?). To that end, we had students think aloud while solving specific problems. We also asked them to explain the meaning of both familiar and unfamiliar equations and to discuss more generally how they know when they "understand" an equation.

Not all questions were asked of all students. All students were asked the first two questions ("explain the velocity equation" and the "two balls problem") and most were asked the next three questions ("explain the pressure equation," "pressure at 5 and 7 meters," and "pressure in a lake on Mars"). The interviews also contained many verbal-only prompts, which are not presented here. For these prompts, see the interview transcript.

Further details on the methodology can be found in the journal manuscript.

B. Interview Protocol

1. Velocity Equation (V)

(V1) Here's an equation you've probably seen in physics class: $v = v_0 + at$. How would you explain this equation to a friend from class?
(V2) How would you explain this on an exam? or to a 12-year old?

2. The Two Balls Problem (B)

(B1) Suppose you are standing with two tennis balls in the balcony of a tall building. You throw one ball down with an initial speed of 2 m/s; You just let go of the other ball, i.e., just let it fall. I would like you to think aloud while figuring out what is the *difference in the speed of the two balls* after 5 seconds – is it less than, more than, or equal to 2 m/s? (Acceleration due to gravity is 10m/s^2)
(B2) Could you have answered this without doing the calculations?

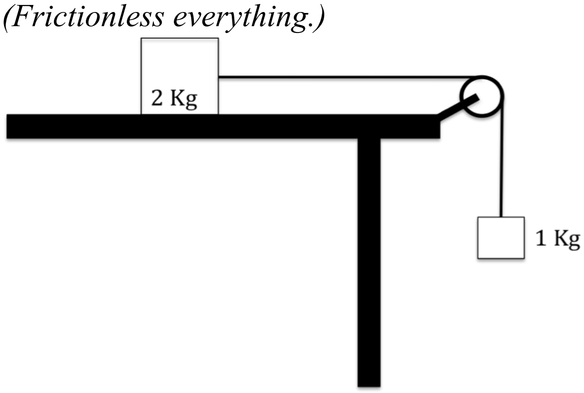
3. Hydrostatic Pressure Equation (P)

(P1) Here's an equation you perhaps haven't yet learned. It's a formula for the pressure at a given depth under the surface of a lake, ocean, or whatever: $\mathrm{p} = \mathrm{p}_{\text{at top}} + \rho_{\text{water}} gh$, where $\mathrm{p}_{\text{at top}}$ is the pressure at the surface of the water, ρ_{water} is the density of water, and h is the distance below the surface. How would you explain that equation to yourself?
(P2) Is the pressure at h=5 meters under water greater than, less than or equal to the pressure at h=7 meters under water?
(P3) Consider a lake on the surface of Mars that has weaker gravity compared to earth. What that means is that "g" for Mars is lower than "g" for earth, which is 10 m/s^2. Is the pressure at a depth in the earth-lake greater than, less than, or equal to the pressure at the same depth for the mars-lake?

4. Atwood's Machine Problem

4.1 Here's a problem you may have encountered in physics. A block of mass *M* sits on a table, with no friction, and a block of mass *m* is suspended by a cord of negligible mass over a frictionless pulley. At $t = 0s$ the blocks are at rest, and the hanging mass starts descend.

(Frictionless everything.)



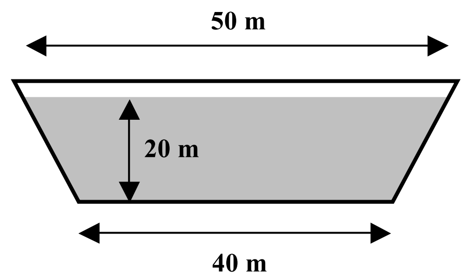
Solve for the acceleration of the 1 kg block.

I would like you to think aloud as you are working on this. We're not interested in the answer you get but in how you think about it.

4.2 Student X says: "The hanging mass is pulling down on the rope, so the tension in the rope is mg. That's the force on the block on the table, so that block's acceleration is $a = F/M = mg/M$." How would you respond to Student X.

5. Trapezoid Dam Problem

This trapezoidal dam is 20 meters high from the bottom to the water's surface. (The shaded section represents the part of the dam in contact with the water it traps.) The dam is 50 meters wide at the water's surface and 40 meters wide at the bottom, and it's 2.5 meters thick. Recall that the pressure at depth h is given by $p = p_0 + \rho gh$.



(a) A student, asked to find the overall force exerted by the trapped water on the dam, reasons as follows: "The force is the pressure times the dam's area, $A = 900\ \mathrm{m}^2$. And the pressure is just $p = p_{\mathrm{air}} + \rho_{\mathrm{water}}gh$, with $h = 20$ meters. So, I'll calculate that p and plug it into $F = pA$." Does that approach overestimate, underestimate, or correctly estimate the force? Explain.

(b) Another student suggests modifying the strategy from part (b). "To make that strategy work, you just need to use the average pressure on the dam, not the pressure at the bottom. So, plug $h = 10$ meters instead of $h = 20$ meters into $p = p_{\mathrm{air}} + \rho_{\mathrm{water}}gh$, and then use that average pressure in $F = pA$." Does this modified approach work? If not, does it overestimate or underestimate the actual force on the dam? Explain.

(c) Now use calculus to find the force exerted by the trapped water on the dam. Explain why you set up the integral in the way you did.

(d) If the dam were rectangular rather than trapezoidal, would the approach of part (b) or (c) work? If so, which h would you use, and why? Would an integral also work? Explain.

6. Questions probing epistemology of equations in the physics context (E)

(E1) How do you know when you really understand an equation?

(E2) What's hard about learning or using the math in this physics course?

(E3) Suppose you had photographic memory for equations. Would that improve your performance? Why? *Follow-ups would try to tease apart whether the advantage is course specific or a more general failure of what it means to know math in a physics context.*

(E4) Suppose a student is taking the course for fun, and not getting graded, with the goal of understanding physics more deeply. She's not interested in learning to solve the quantitative problems, but she's willing to study outside of class to learn the concepts better. What role if any should equations play in her studying?

(E5) Suppose you were given a list of equations on the exam – would that help you?

C. Transcript of Interview with Alex

Date of Alex's Interview – 10/15/2008
Transcript (numbers on the left indicate the turn number for utterances; The speaker marked I is the interviewer and the speaker marked A is Alex; numbers within square brackets [] indicate the time of the utterance in [min min: sec sec] format with the 00:00 time stamp being when the camera is turned on at the start of the interview):

0. I: So, that's the consent form. It just basically just tells a little about what the study is about and, uh, we're just interested in, you know, how students think about mathematics in the context of physics. And it also explains that, yeah, that we might be using, that we will be videotaping the interview and that we could use clips of that in education conferences and if you're OK with that.
1. A: Cool. So that's cool. What's the date?
2. I: It's 15th. Ok so, one more thing I wanted to say is that, so you know, I'll give you a bunch of questions on different situations and things like that, and we're not really interested in whether you get to an answer or not, more about how it is that you're thinking about it, ok? If you get to an answer that's fine. If not, that should not be a major concern. If at the end of the interview you wanted to talk about correct answers and wanted to, then I will be happy to discuss that with you. OK?
3. A: Oh Ok, ok, so this is like Math questions?
4. I: Yeah, you'll get to see what's going on. It'll just, you know. But don't feel concerned, just tell us what your thoughts are, and if you wanted to discuss that, as I said, we can talk about that at the end.
5. A: OK.
6. I: So, here's an equation, ok? And you've probably seen it.
7. A: Yeah.
8. I: Right? So suppose you had to explain this equation to a friend from class. How would you go about doing that?
9. A: Umm…OK… Well… umm… I guess, first of all, well, it's the equation for velocity. Umm…umm, well, I would… I would tell them that it's uh… I mean, it's the integral of acceleration, the derivative of [furrows brow] position, right? So, that's how they could figure it out… uh… I don't know. I don't really [laugh], I'm not too sure what else I would say about it. You can find the velocity, like, I guess it's interesting 'cause you can find the velocity at any time if you have the initial velocity, the acceleration, time and…
10. [2:43] I: OK. And, uh, is that what you would have said on an exam?
11. A: Umm, no.
12. I: What would you say on an exam?
13. A: Um…well, it depends on what it was asking…um…'cause I feel like your question's kind of vague, but, I mean, I would probably just say 'it's the velocity equation'[nods and laughs]. I mean, if it was a more specific question, I could probably like, elaborate, I guess.
14. I: So here's another situation: suppose there was a 12-year old who knew some amount of math but doesn't really know physics. Umm, how would you explain that equation to a 12-year old?
15. A: Um, well, these two sums will tell you how fast something is going. If you know how fast it's going when it first starts, an object first starts moving, and you know its speed when it first starts moving, and you know a certain point in time, like, you're looking at a certain point in time at which that object is moving, and you know how fast it's changing its speed,

you can find how fast it's moving at that time, or you can find out the acceleration from it if you know how fast it's going at that time.

16. I: OK. [mumbles a little to himself as he checks to see if the camera is recording correctly] Ok, and is that an answer that you could feel comfortable giving on an exam?
17. [4:50] A: [Shakes head] No. I would probably feel like that would be too uh… like, conceptual, maybe? Not, I'm not sure, I mean, I've honestly never been in the situation where I'd have to say, like, answer that kind of question, so I'm not sure.
18. [5:14] I: OK. So you've not really faced any questions that ask you to explain an equation, is that what you're saying?
19. A: Well, I've never taken physics before until now. Yeah, so, um…like, the test, we've had one test, and there just wouldn't be a question on there that asks, like, to explain the equation. You know, it would ask you to assume that you understand what it is, and then use it to figure out something more complicated, so I guess that's what I mean. If it was on there, sure, I'd feel comfortable with that answer, but I've just have never really seen it on a test.
20. I: Ok. Here's a problem. You're standing on a 4th floor apartment balcony and you're throwing down a ball with an initial speed of 2 meters per second down, throwing it down, and the second one you're just letting it go. And the question is asking you that after 5 seconds, what's the difference in the speeds of the two balls.
21. A: OK. Um…[mumbles inaudibly]
22. I: How would you figure that out?
23. A: OK. Do you want me to write it down? And like…
24. I: Sure. Go ahead. Feel free to do whatever.
25. A: OK, so first I would draw a diagram, I guess, of the two balls, and their speed. So I know that one's going 2 meters per second, and if I just drop this one, it should be going um… 9.8 meters per second… um, so to figure out the difference in the speeds after 5 seconds, uh, I would just, I would use the [pause] velocity equation? Looking at this equation. [points to equation]
26. I: OK
27. A: So I know their initial velocity and, ok, but I… uh…don't, oh wait, then…I mean, a is…
28. I: You can use 10 if you wanted. You can, feel free to use 9.8 if you, if that makes you feel comfortable. Whichever.
29. A: No, I think 10 is probably easier.
30. [8:02] A: Ok. So, first I figure out one is going [writing] 2 plus 10 times 5. Um…[finishes writing] ok, so after I plug this into the velocity equation, I use the acceleration and the initial velocity that's given, multiply the acceleration by the time that we're looking at, 5 seconds, and then, once I know the velocities after 5 seconds of each of them, I subtract one from the other and get 2. So the question asks is it less than, more than, or equal to 2, so I guess I would say equal to.
31. I: Equal to 2, OK. Alex, could somebody have, could somebody who knows this equation, could they have answered this question without really working out the numbers, without going through, calculating explicitly the velocities?
32. A: Um, yeah.
33. I: OK. Could you tell me about that?
34. [9:30] A: Well, I'd have to think about it, since you're dropping one and throwing one. If you're, I mean I guess if you think you're throwing one 2 meters per second and the other has zero velocity since you're just dropping it, it's not, it's only accelerating due to gravity. You can just say that since you know one of them is going at 2 meters per second, it's going to get there 2 meters per second faster, so 5 seconds faster, it would get there 2 seconds… er… it's going 2 meters per second faster, I guess.

35. I: OK. So, they would say that you threw one, so this was getting 2 meters per second faster. So what happens 5 seconds later?
36. [10:21] A: Uh… it's going…uh…I don't know [laugh].
37. I: So, ok. So, you concluded that 5 seconds later, it's still going 2 meters per second faster, the ball that started out 2 meters per second faster?
38. A: Uh, maybe.
39. I: Ok.
40. A: Uh… I'm not sure.
41. I: Ok.
42. A: Umm… yeah, I guess so.
43. I: Can you tell me, what [points to her work] what is it?
44. A: Umm…
45. I: Say this is ball #1 and ball #2 [points to her drawing of the two balls].
46. A: Ok, so it would be going 2 meters faster because this one's initial velocity is zero, I guess?
47. I: Ok.
48. A: Yeah.
49. [11:37] I: So you're saying that they need not have actually plugged in the numbers? Is that what I'm hearing?
50. [11:55] A: No, I think you'd have to plug in the numbers because…I mean you just would to be sure. I guess you, I don't think you could just guess about it.
51. I: Ok. So, now look at another equation. Have you seen that equation before?
52. A: No
53. I: Ok. Um, the equation is about pressure, and it just tells you the pressure at certain depths in a lake or whatever and P top is the pressure at the surface of the water, this is the density, the rho water is the density, and the acceleration due to gravity, g, and h, which is the depth below the surface, ok? So suppose you were trying to understand the equation. How would you explain it to yourself?
54. A: Um…[pause] [mumbles inaudibly] Um…well, I would think that, um, if like a particle of water is just sitting on the top, it doesn't have any acceleration, and the only depth that it has is at the top, so you don't need to think about it in terms of how far up or down it is, it's just on the top, like if that was, if you were thinking that like, well I guess if you were thinking about a math problem, so that that was the origin, like the top of the water, like if the particle of water was at the top, then that would be like X equals zero. So it would just be the pressure, right, if you know that? Um, and then a particle of water, if it's like somewhere in between, you know, like just like floating around, it needs acceleration due to gravity, because it's like wherever, you know it's not just sitting on the top. And you also need, you also need to know how far down it is below the water. So, if you know that and then you know the pressure at the top… umm… it seems that if you add them together you get the formula for the depth at any given depth. So I guess it kind of makes sense if you, if you, I understand like how this would make sense, but I don't really understand why if you add them together… umm… you'd get, like… [pause] so I'm not too sure about the first, the pressure at the top. I mean, I guess it kind of makes sense.
55. I: Can you tell me…?
56. [15:19] A: Hmm?
57. I: So, you said that you didn't understand about the top and then you said you, kind of made sense, so is there something else that you could do?
58. A: Oh, I was just, um… not really. I'm still not really sure why you would add it, the, at the top. Oh! These are two different variables? Are those? Oh, ok.

59. I: Yeah, this is, which two are you talking about?
60. A: I didn't, sorry, I guess I wasn't looking close enough. I didn't notice that this is a P and this is a, I'm not sure what…so those are basically just different values.
61. I: Yeah, so does that…?
62. A: Yeah, I guess that makes things a little bit different. Oh, 'cause it, I'm sorry, I guess I misread the question, kind of. Well then yeah, then it would make more sense if this was the density and this was the pressure at the top, because if you know the pressure at the top and then you know the density, and the density at any given height…um…because the change is due to density, it would change due to density, so if this was zero, and you just wanted to know the pressure at the top and then you added the particle's density at a different height, then you would get whatever height it's at, I guess. I guess it's kind of like this formula, too. [points to velocity equation]
63. I: Ok. Tell me how?
64. A: Because you're looking for something, you have the variable, and you're looking for, initially, like, if there's no acceleration or time or, in this case, I guess, height. If it's just at a certain point and you know what its density is, kind of like this one, if you know what its initial velocity is and then you add, I would think of like acceleration times time or density of water times gravity times height as, you know, how, just like where it is [left-right gesture with her hands] on like a graph, like graph like where it is. So if you just, I guess they are really similar.
65. I: Ok, what kind of graph are you thinking about? Do you want to…?
66. A: I guess like a position versus time graph.
67. I: Do you want to draw something so that I have an idea?
68. A: I was thinking, um, more like this, like if this is the water, this is the top of the water and this is like X equals zero, and this could be, it goes pressure at the top and then if you have, you can move up or down on the y-axis if you are trying to find out, trying to find out the pressure at a different height.
69. I: Ok, and how is that similar to that equation?
70. A: Oh, ok. Well, I guess I can, this is the water… [draws graph] I guess they're… [Interviewer shows velocity equation and Alex smiles] oh, I got it. I need to know that formula anyway [laughs]. So if this was position, yeah, just like position, this could be um, like this is where it initially starts, and then when you know its acceleration and time, you can move wherever. The same thing as this, I guess. Except, maybe you'd only be able to move up and down on this one, but, or I guess these would be two different, this would be a, since this wouldn't be position vs time, this would be velocity graph, so then this would have to be a position vs time graph. Do you understand what I mean [laughing]?
71. I: Mhm.
72. A: OK
73. [20:00] I: So, umm… suppose we asked this question to you in an exam, that explain this equation that P is equal to P at top plus density of water times G times H. How would you explain that on an exam?
74. A: Well, the pressure at the top is like an initial value and then the density of water times the gravity and height is a variable that will let you find the density at another point. Umm, so, if you add it, you…can figure out the pressure, I guess.
75. I: Um, Alex, you said that it will help you find the density at another point. What did you mean by that?
76. A: Or, the pressure.
77. I: Ok, so you mean, if you add this and that, you can find the pressure at a different point, is that what you were saying?

78. A: Um, yeah.
79. I: Ok, ok. So, have a look at a problem. Have a look at a problem. Suppose we asked you to compare the pressure at H equal to 5 vs pressure at H equal to 7. Which one is more, which one is less, how would you figure that out?
80. A: Um, well, you could…
81. I: Pull the chair up if you want to.
82. A: Well, you can just… well, you would have to know these, I guess.
83. I: I can tell you any quantity that you want to know.
84. A: Um, the one with, that's 7 meters under water would have a greater pressure unless these are negative. But if this is the same value, and these are both the same value, then all you're really changing is the H, so, um, the…would H have to be a negative value or would it be positive? Like, would you say it's negative 7 meters or would you say it's 7 meters.
85. I: Which one do you think it should be?
86. A: I think either could work, if you did the absolute value.
87. I: Ok.
88. A: But, um, well, it seems, I think it should be positive because, it's the distance below the surface, so you already know it's below, so you're just looking for, like a height, I guess.
89. I: Ok.
90. A: So, I would say this one would be, have more pressure.
91. [23:05] I: Ok, um, what about the next one? Have a look at the next one. There's a lake on the surface of Mars and a lake on the surface of Earth, and you're looking at the same depth below the surface, both on Earth and on Mars, and we know that the gravity on Mars is weaker, so G would be less than 10 meters per second squared. So how would you compare the pressures of the two?
92. A: Um, well I would say the, um, pressure at the lake that's on Mars would be weaker because, uh, the gravity is weaker, so the value would be, the value of g has to be less and then when you multiply that by however, um, how far it's under, it's going to be a smaller value. Um…just because the gravity is weaker. Yeah.
93. I: OK, so, so which one would be lesser, which one would be more?
94. A: Umm, well, I think the one on Mars would be less than the, and the one on Earth would be more.
95. I: OK. And you said that that's because the G value is less on Mars.
96. A: Yeah.
97. I: Ok. Ok.
98. I: Alex, one more thing, is that, so let's look back at the H equal to 5, H equal to 7. So, somebody who doesn't really know physics, per se, could they have figured that out?
99. A: Um, I guess if you have kind of uh, like a, understanding of, do you mean, like actually physically, like if you went and like, went underwater, you know that there's more pressure, just, just I guess, just by common sense, you don't necessarily have to use math. But if you think about it, like, in terms of this equation, it would make sense. Just because as the H value gets larger, which is, you know, most people would understand when you go lower underwater, there's more pressure. So I think someone, um, well do you mean, if someone understood math though, right? Like, could do math? Or just couldn't do physics?
100. [25:42] I: Mhm.
101. A: Yeah. I mean, I think so. Because this is almost like, it's an equation, but it's almost sort of like, more of a conceptual equation. You don't necessarily have any, you know, some people look at it and say like "Ah! There's too many like X's and Y's!" but this is just saying straightforward, like what it is, and so it's not very confusing.
102. I: So, could you explain that conceptually.

103. A: Sure, yeah!
104. I: And how would you do that?
105. A: Umm…ok well, um, ok because if you know the density of water and how deep under the water you are, if you multiply those two values, so that you know umm, how much the dens…er…how much the density is increasing as you go deeper, and then you add it to the actual pressure of the water when there's really like, very little, um, density, if you add those two together, then you'll know the pressure.
106. [27:06] I: Ok, ok. So um, are you saying that as, as you're going down, density is increasing and the density is less on top?
107. A: Um, no, because, I don't think so.
108. I: So, ok so, no, but that's what I heard, but that's not what you were saying.
109. A: 'Cause the density, shouldn't the density always be the same?
110. I: Ok, so when you, so you multiply this and what does that tell you? Because I heard you say that if you multiply the density with the H it tells you how much density is increasing. Did you mean density?
111. A: Oh, No. I don't think I meant density.
112. I: What did you mean?
113. A: Uh, the depth, I guess is more [laughs]…'Cause if the density is always the same, then it's really just, like, how far under the water you are, I, I guess. So it should matter less.
114. I: Ok. So, um, could you explain that equation to a 12-year old? Could you explain what's going on there?
115. [28:25] A: Um, I mean, it depends on the 12-year old. If they were willing to understand, 'cause, I mean there's a difference between a 12-year old that's, you know, they're like, above school, they're kind of at the stage where they don't really think they need to learn or, like, want to learn, you know, but some little kids are, like younger kids are, until a certain point, they're really interested, I think, in school. So, I mean, if this person was, like, interested and wanted to know what you're talking about, I think maybe…
116. I: What would you tell them?
117. A: Well, I would tell them, I don't know [laughs]. I would tell them…that however far under the water you are, um, determines the pressure. 'Cause that's really the only, I mean I guess that would really be the only, um, variable that's not constant.
118. I: Ok.
119. [29:41] I: So how far you are under determines the pressure.
120. A: Yeah. And the, also the gravity, because if you were on Earth, I guess if you wanted to go there, you could figure out on other planets with their gravity, so…
121. I: So, how do you know when you really understand an equation? When you feel comfortable, that "yeah, I really understand this equation."
122. A: I mean, I still don't even, I mean, I can know this: V equals initial velocity plus acceleration, time, like, I can know that equation, but sometimes it's hard, even when you think you know an equation, sometimes it's hard to know when to use it or how to use it or, um, what it really means. Because these two are very, they're really similar, but it's almost like if you just substitute in different kinds of variables, you still basically have the same equation.
123. I: So are you thinking that they have the same kind of a structure? Is that what you're saying?
124. A: Yeah, yeah, I think so. Definitely. Yeah.
125. I: Ok. And, um, so what kind of a structure do you think they have?

126. A: The structure where, that, if you have an initial value that's not, that's only dependent on itself plus a set of other values that are being multiplied together, I guess in this case, and those two variables are changing, they're usually changing, which gives you a value that's at any given, a different time or a different height or depth, can change the whole equation. But then that first one, it's still going to stay the same, it's still going to be constant. So, I guess it's an equation that's mostly dependent on the second, these two, these two sets of variables at the end.
127. I: Ok, ok. So you said something that "you know, I know the equation but I still might have a problem applying it." Did you mean, like, using it in problems, or what did you mean there?
128. A: Um, because just sometimes when you're doing a physics problem, uh, you know something can seem really complicated, but really it would be really simple and you wouldn't know, you would, there's just so much to know, I guess, that you might not think to realize that maybe I can use this basic kinematic equation rather than some really complicated equation. And it's just, like, knowing when to use which equation, I guess. That can make it hard sometimes.
129. I: Ok. Is it just a physics thing, or is it also a mathematics course thing?
130. A: I'd say it's more of a physics thing. I think math is more straightforward and usually it's stated in the problem, um, like in my calculus class usually I'll just, you know, I'll get all the formulas in my head, and then I'll read the question, and just instantly know, like ok, I have to use this formula. Like if I know the formula and I know what kind of problems to use it for I just, like, will instantly know "Ok, you use this formula." But with physics, it's harder to know sometimes. I mean, you can say like "I have to use these formulas," but you might not know which to use first or how to you use it, if that makes sense.
131. I: Ok. So, um, what's difficult about understanding the math in this course?
132. A: Um…I'm not, I don't think I really have trouble understanding the math. It's usually just, I mean, I can understand the basic equation and you know, like algebra-wise what's going on, you know, like how they're moving everything around or whatever they're doing, but it's usually just that, um…it's, it's easy to see the teacher do it and watch him use all the equations, and like, you know, whatever order he uses them in, but then when you have to do it on your own, it's sometimes harder to, um, you know, remember, because there's just so many different types of problems. So you might think you understand it, but then you actually try to do it, like, in a different type of problem, and it might not work. [laughs]
133. 34:46 I: Ok, ok. So if you had a photographic memory for equations, would that help you?
134. A: No.
135. I: Why not?
136. A: Because, um, you mean like if I could remember, like, every equation perfectly?
137. I: Mhm.
138. A: I still might not know how to use it. I still might not know, I mean, you know a lot of physics is thinking, like it's, anybody can memorize the formulas and, um, you know spend so much time memorizing the formulas, but it's all about, uh, what you do, like, how you think about what you're going to do and what order you're going to do things in and, um, you know, how to approach the problem, I guess.
139. I: Ok. So, so I heard, you know, you want to use the equations and use them properly, and so once, suppose you can use the equation properly. Do you feel that you have understood the equation?
140. [35:43] A: Um, yeah. I'd say, um, maybe when I understand an equation is when I know that when I feel comfortable putting in values and knowing that I can get them equal to each other. Do you know what I mean?
141. I: What do you mean?

142. A: Like, I guess when I can prove it, kind of, and just see that, like, with the, the values that the problem gives, will make both sides of the equation equal, then I kind of understand like, oh, you have to do that, and that, and then, like, now it's equal, I guess.
143. I: So do you mean like if you know V and if you know V NOT, A T, then you could calculate the right-hand side and you know the left-hand side, is that…?
144. A: Yeah, that's what I mean. Yeah, so then it kind of proves to you that it's actually what you think it is.
145. I: Ok. In most cases, you wouldn't really have one of the, like in this case, you didn't have the V.
146. A: Right, yeah, you usually wouldn't actually. I guess I kind of meant that more with regards to doing, like, math, like if you're trying to prove something and you had, you're just plugging in values kind of. But with this, you know, even after you, you're just solving the equation, so you might not know that, um, like back here, I can solve the equation, but I'm not going to necessarily going to know it's right unless I know that, unless I do it right. Unless I'm 100% sure that I'm doing everything right, I'm not, I can't really prove, I don't think, I can really prove that my answer's right just using that equation.
147. I: So how do you prove, how do you make sure, how do you make yourself comfortable that my answer is right?
148. A: [raises hands and shrugs shoulders] Look in the back of the book. Look at the answer in the back of the book.
149. I: Ok, ok right, ok, that's what you meant, that you do a problem and if you get an answer and you check back to see whether it matches or not. Suppose there were no answers in the back of the book. How would you figure out if you did it right or not?
150. A: Um, I'd probably think about it for a while, but if I didn't, that's, I think that's just personally that's how I learn, like I try to, I think about something and then try to see if I can get it on my own, and then, even if I know the answer before I started, I just know that what I'm doing is right because I got, I ended up with the same answer.
151. I: Ok. You said if you knew the answer before you started. how would you know the answer before you started?
152. A: Um, like if you're doing a problem in class. Say that the teacher does a problem in class and then he, you know, he gives you the answer, and then you might go and, like, try to do it on your own, and then you know the answer beforehand, because it's like, he told you the answer in class. But if you just try going through it without looking to see what he did to see if you can do it again on your own, then you have the answer.
153. I: Ok. Have a look at another problem, ok? You have a 2 kg block resting on a frictionless surface tied to a rope with a 1 kg block hanging down and we're asking you to solve for the acceleration of the 1 kg block. Once again, you know, it's not crucial that you reach a solution, but we want to see how you go about solving it, so keep talking as you think about it or work on this problem.
154. [39:50] A: For this one I would use, I'm used to, you have to use Newton's laws. And you know from Newton's 2nd law that, or would you have to use…? I'm not sure if you could, if using the 3rd law would really have, help you solve it, but if you used Newton's 2nd law and you say, like…well, actually, first off, I'd draw a free body diagram.
155. I: Go ahead.
156. A: So you'd have the 2 kg, and you know that there's tension, and then there's gravity and the normal force, and there's no friction, right?
157. I: No friction.
158. A: Ok, so then you know that the sum of the forces in the X direction is just T and no Y…and then you would do the second block too. Oh, I didn't draw the diagram. And then

you'd know that the sum of the forces in the X direction for this one would be zero because it's not really moving, it's not moving left or right. It's just moving up or down or the forces are pushing it that, you know the tension, it has the tension going up and then the, um, gravity and mass pulling it down. So then you know the sum of forces in the Y direction would be T minus MG and, um, then once you have that, you, um, know that F equals MA, uh, so we're looking at this, right? Um, so since the, it's weight is just 1 kilogram, you can just say the force is equal to the acceleration, um, and the force should just be uh…[pause] gravity? I guess. Because, um…you have your forces in the Y direction and it's T minus MG, um, so that means the force going down is just MG, but they have to be, um, because of Newton's 3rd law, these have to be equal, uh, but one is negative. Um, so then I guess you can just say that the force is negative MG, and since we know the mass is one, you can just say it's negative G, which is just the acceleration.

159. I: Ok.
160. A: I don't know, I guess that works.
161. [43:48] I: Um, think about somebody, so this is all physics, right? So think about somebody who does not really know all these equations, right, who doesn't know that.
162. A: Ok.
163. I: And I give this situation to them, that there's a block resting on a really smooth table and another block hanging down, and there's a string in between two, ok? And the only thing that they know is that a freely falling block has an acceleration of, you know, G, or 10 meters per second squared. So, given that, just that much physics, they don't really know all the rest of it, could they have, you know, kind of made some kind of an inference about whether the acceleration of this block would be equal to G, less than G, or the same as G? What would they have thought?
164. A: Well, if they know that, I think some people, if you explain that to some people, they might get kind of confused, because it's kind of hard to understand acceleration if someone's not moving, I guess, so if this block is just hanging there and people are thinking, like, "how is this block accelerating, you know, if it's just hanging?"
165. I: Oh, so this is exactly the situation: we tell them that we have the block, so we have these two blocks connected with a string and you just let them go.
166. A: Oh! Oh, Ok.
167. [45:29] A: Yeah, I think people would understand, because if the block is, if they know that it's just falling due to gravity, it would make sense that the rate at which it's going to fall or the rate at, of the rate of the speed at which it's going to fall, it's just going to be gravity. If this only is 1 pound or 1 kilogram, and if this falls, it's not going to, I don't necessarily think it's going to speed up this acceleration.
168. I: Oh, so block 2 will not speed up at the same acceleration?
169. A: Oh, no, I don't think it's going to have anything to do with this block's acceleration.
170. I: Oh, so block 2's acceleration will be completely different. Is that what you're saying?
171. A: Yeah, because this, if this weighs 2 kilograms, it should be twice, uh…
172. I: Can you tell me why?
173. [46:29] A: Or… no, it would be half? I think, but this one would still just be G.
174. I: OK, so why should that be half?
175. A: Uh, actually, I'm not really sure. But, if you know, you know the equation F equals MA and then you divide by the mass to get the A, the force divided by the mass would be the acceleration, so since this weight is 2 kilograms, you make that 2, so you can say F, A is F divided by 2, and then…uh…then that's, yeah, that's really as much as I can go, I'm not really sure, what, or maybe it would be the same.

176. I: Could you, so, for a second, forget, once again that this is a physics problem. Suppose you were just thinking about the situation, just think about the situation. One block is resting like this on the table, and it's connected with a string and then another, the other block, second block is hanging down. And you're holding it, right? So nothing is moving and then you just let go, right? So if you were just seeing that in real life, what would you think about how will the blocks move?
177. A: Because of gravity, [shrugs shoulders] I guess, 'cause um…
178. I: So how, how do they move?
179. A: Because gravity pushes everything down.
180. I: OK
181. A: Yeah, and then you know, like, um… if everything, if gravity pushes everything down at a constant rate, then, um, it should, uh, its acceleration should be based on how much it weighs, too, so you can't just say its acceleration is equal to G unless it has no mass.
182. I: Ok. [pause]
183. I: What about the two blocks? Suppose you are seeing this movie, you know? Think about this movie, ready? Here we show (you?) this movie, and I'm holding these two blocks and I let them go. How, what will you see in the movie? Can you describe the scene?
184. A: I guess the heavier one would go first, right? So, they're like, this, and this is, this is like the heavier one and this is the lighter one [gestures with her hands], and the heavier one would start falling faster than the lighter one because it weighs more.
185. I: How will it fall?
186. A: Straight down? Oh, if it's on the table?
187. I: It's on the table, yeah.
188. A: Well, if it's on the table, it's going to go like [gestures from left to right, makes a "chhh" sound].
189. I: So start moving.
190. A: Like, say this is the, this one is going to start pulling it down, so this block is going to go horizontally, but then it's just going to, gravity is still pushing it down, so it has the string and the mass pulling it to the, uh, horizontally, but then after that, it's probably going to go like [gestures from her left to right, and then going straight down; makes a "shooom" sound].
191. I: Right, so before it falls, before it falls off the edge, we're not worrying about that. Will they be moving together, or will one be moving faster than the other, or…?
192. [50:15] A: Before it falls?
193. I: Before it falls off the edge. Right now this one is moving on the table, that one is falling down, right? And they're connected with the string, so will they be moving together or will one be moving faster than the other?
194. A: Well, this, this only stays put if you hold it, right?
195. I: Right.
196. A: Ok, so it can't stay, it can't, like, the 2 kilogram isn't holding up the…
197. I: Mm mm [meaning "no"], would you expect that?
198. A: No [laughs], err, I'm not sure. But, um, uh, if they both, let's see, no, I think this one will hit the ground first, because it weighs more, and if they both weigh the same thing, and it was this scenario and they both weighed the same thing, uh, wouldn't they hit the ground at the same time?
199. I: So tell me, tell me how you are thinking about this whole situation? What are you visualizing?
200. A: I'm visualizing, this, like, experiment that my teacher did where he had two little blocks, two little lead blocks, and one was, say they're sitting on a ledge kind of like this, and one block is here, uh, one block is here, and one block has a spring behind it, so it's going to get

acceler…err, accelerated. It hits the block and it goes [gestures block shooting to her left, makes "chhhh" sound] off, and then the other one just gets dropped straight down? They weigh the same, but you're trying to think of like what, uh, acceleration to gravity, and with the spring, which one is going to hit first. So I'm kind of thinking about that, like, in relation to this, and how they would be different or the same. Because I guess if this block could be accelerating is kind of like the spring, you know it's pulling down and it's making it move horizontally, but whereas this one is just going straight down, but it starts, they don't start at the same height, so it would be different I guess, then.

201. [52:37] I: So you are thinking of this block just falling straight down and this block being pulled from the table and then falling down. Is that what you're thinking? That the second block, like, going off the edge of the table and then moving to the ground, falling to the ground, is that…?
202. A: Mhm. Yeah. Uh, yeah. Yeah, I guess so. [laughs]
203. I: Hmm?
204. A: I'm just thinking about whether or not it would go farther horiz…I mean, I think it is going to go, like, it is going to have some distance horizontally because it's already got that horizontal acceleration, so it's going like this [gestures with her pen left to right, makes a "chuuu" sound], but this one is just going to land straight down, because all it has is gravity and its mass, so it's just going to go straight down because it has no X movement. This one is still probably going to be going a little bit horizontally, so it'll be like [gestures from left to right, makes "shooom" sound, gestures straight down], you know, it will probably land farther than that one.
205. [53:55] I: Ok. Suppose you have a friend who doesn't, who's in your physics class but is not taking it for credit. And she is only really interested in understanding physics. She's not interested in doing quantitative problems and solving these velocities, um, so she studies outside of class to understand physics better. What role should equations play in her studying?
206. A: Umm, that's a good question. I guess to…she just wants some basic understanding of why, because you can, you can watch something happen and then like have a theory about why it does that, but chances are you're not going to be able to put that into mathematical terms to say why it's doing this. So if you look at an equation that you think you understand and, like, um… it will help you understand what's going on and also how you can apply it to a different situation and how, like, what the difference is between different kinds of, like, situations with different mass or, um, different objects would change the outcome, I guess. And, uh…yeah.
207. [55:34] I: Ok. So equations should still be important for her, uh…
208. A: Yeah, definitely. Um, I mean it helps you get a better understanding of it.
209. I: Understanding of what?
210. A: Understanding of what's going on. Or what, I mean, I guess if she's not interested in it, she could, sure you can still understand physics, but it's a different kind of physics, I guess.
211. I: Can you tell me what's the difference?
212. A: Well, it's more of like an observational, like a conceptual understanding of what's going on, that you can, you explain it like in different terms and different words than if you're just, you know, seeing a math explanation: this is what [gestures at 2 points on the table]. It's totally different; some people think, some people can understand physics, but they think of it, uh, more like a book, maybe. You could write a book on physics and have there be no math in it, but you could still give someone an understanding of what's going on. But if someone wanted to learn why exactly, and like, or no, not why exactly, just, was interested in more specific things and examples, then you could explain to them in terms of math.
213. I: What do you mean by specific?

214. [57:09] A: Like, if I was explaining to someone this problem but didn't really include the weights, or, um, height, uh, it would be more vague, like, I could just say, this block and this block, one's heavier and one's lighter, and then what happens? You can explain it, but you can only explain so much without, um, you can just only explain so much [laughs].
215. I: Right. Ok, Ok. I have one more question to ask you, it's that, suppose I cut this string. What would be the acceleration of the 1 kg block?
216. A: Um, it's mass and gravity, mass times gravity.
217. I: That's the force, Ok? But suppose that instead of cutting the string, I let this system go. Then what do you expect the acceleration to be?
218. A: Um, like, in terms of an equation?
219. I: No, ok, so what's the force on this thing, if we just, so if we cut it, then you said there's mass times gravity. If you don't cut the string, but just let it go, then how do the forces…
220. A: [inaudible] tension…
221. I: So there's tension on it.
222. A: Yeah.
223. I: Ok, so from there what can you tell about the acceleration?
224. A: Uh, it's based on tension and mass.
225. I: OK. Would it be less than G, more than G?
226. A: More.
227. I: OK, why?
228. A: Because it has more, well, I guess it depends, if the, if the tension was, um, if there was enough tension then it could decrease the acceleration, I guess. It would just, it would just depend.
229. I: Could you guess, could you guess numbers? You know the G. G is 10.
230. A: Ok, yeah.
231. I: So what do you think tension would be in this problem? If I asked you to pick a number for tension, you know that the downward is, you know, 10 times 1, right?
232. A: Mhm.
233. I: So that's the force down.
234. A: Ok, yeah.
235. I: So, if I asked you to pick a tension value, could you just, a random guess, what would you pick?
236. A: The same value?
237. I: 10?
238. A: yeah [laughs]
239. I: Ok…[pause] so…
240. A: Well, maybe not. I'm not sure, we just, I'm, I guess I'm just having a hard time with this one because we just learned the, how to do this kind of problem, but the way we learned it is, uh, the way, the problems we've been doing are more complicated, so I almost feel like I'm over-complicating it I think. [laughs]
241. I: Ok, so are you happy with picking 10 upwards?
242. A: No [laughs].
243. I: Why not?
244. A: Uh, because then it wouldn't be moving [laughs].
245. I: Right.
246. A: Um, I mean, wouldn't it be zero? I'm not really sure, like there would be slack in the rope, right? 'Cause if the, if there's tension and then all of a sudden it let's, you let go, the rope's not going to be, like if this is the rope, it's not going to be like [holds up piece of

paper vertically, makes "chhh" sound] it's going to start, like [makes the piece of paper wavy and not straight] twirling, or, you know, just, there would be slack in it. Uh…

247. I: So if you were seeing a movie of this thing happening, there's a block sitting here connected to a rope, say I connected it, connect that to a rope and there's something hanging there and I just let it go, that rope will have, uh, slack on it?
248. A: For a, a, for a point in time.
249. I: Ok. So then, then, there is no tension. If it is zero, then it falls down with gravity the same as a free fall.
250. A: Yeah [doesn't look confident], I think so.
251. I: Ok, that is great. We have reached the end. Awesome. This is just a receipt.
252. A: OK
253. I: It's just, you know, says that you participated in the interview and that you were…

D. Transcript of Interview with Pat

Date of Pat's Interview – 10/9/2008

Transcript (numbers on the left indicate the turn number for utterances; The speaker marked I is the interviewer and the speaker marked P is Pat; numbers within square brackets [] indicate the time of the utterance in [hour hour: min min: sec sec] format with the 00:00:00 time stamp being when the camera is turned on at the start of the interview):

1 I: [00:02:10] Ok. So here's probably an equation that you have seen before, right? And um, if you were to explain this equation to a friend from class, how would you go about explaining this?

2 P: [00:02:22] Well, I think that the first thing that you'd need to go over would be the definitions of each variable and what each one means, and I guess to get the intuition part, I'm not quite sure if I would start with dimensional analysis or try to explain each term before that. Because I mean if you look at it from the unit side, it's clear that acceleration times time is a velocity, but it might be easier if you think about, you start from an initial velocity and then the acceleration for a certain period of time increases that or decreases that velocity. And then of course you have signs, which doesn't make anything any easier.

3 I: Uh, signs?

4 P: Like positive or negative. Like I immediately thought increasing seeing the positive, and that doesn't help either. So I would think I would start from defining the variables and then I would probably write it out in terms of the dimensions, that sometimes makes things easier on exactly what cancels what.

5 I: Mhm.

6 P: [00:03:27] And then explain the significance of each term.

7 I: Ok, you mentioned something about intuitively. Um, did you mean the dimensional or did you mean something else?

8 P: I meant that, you mean from thinking that it was positive?

9 I: So right when you started you said something about "well, then from the intuitive side."

10 P: Yeah, the problem is, uh, dimensions are just numbers really, or units, and it doesn't really explain what's going on in the motion.

11 I: Mhm.

12 P: I mean it can help.

13 I: Ok.

14 P: But it's not really…I mean I don't even know how you can do a picture first for something like velocity and acceleration since, I mean, maybe a dot diagram but even then you can't show what's initial and what's the change very easily.

15 I: [00:04:15] Ok, so how would you explain it intuitively?

16 P: [00:04:18] I would say that an acceleration is the change in velocity, so you start from the velocity you have in the beginning, and you find out how the acceleration affects that velocity.

17 I: Ok.

18 P: And that would be the significance of each term.

19 I: [00:04:39] Ok. How, uh, how would you answer that on an exam? How would you explain this question on an exam?

20 P: I think I would write out the definition, then probably write out the dimensions 'cause it's the easiest thing to check against the textbook and then explain where both terms come from.

21 I: Ok. You mentioned something about the sign and, um, can you tell a little bit more about what you meant?

22 P: [00:05:15] I feel like when you see V oh plus A T and especially the word accelerating really doesn't help, and it's always been a problem of not being, not really, and of course the velocity could be negative, and you could be breaking in the negative direction, which doesn't make anything any easier, but I think that there's not really any way of getting around with that from just the equation. In that case you would need to have to have pictures of, I mean the easiest way to handle this is in one dimension, maybe have a little diagram of a particle's motion and exactly what each case means when both the initial velocity and the acceleration are positive or either is negative or when both are negative.

23 I: Ok.

24 P: So that may help if you were to put a sequence of 4 pictures that explain the differences between each of those 4 cases.

25 I: Could you, could you draw me a couple of examples?

26 P: I can try. [draws] Yeah, probably first and second differences is the easiest so…

27 P: [mumbles]

28 I: Ok.

29 P: [00:07:49] I mean there's a little bit of ambiguity here, but the idea that the velocity is decreasing and starting to go in the opposite direction is…helps.

30 I: Ok.

31 P: Like, some people are afraid of vectors, but I think they're really good for graphical descriptions, gets a little more complicated if you throw them in algebra, but…

32 I: Ok, so your thinking of these velocities as, as vectors helps you.

33 P: Yes.

34 I: Ok.

35 P: Because if you don't, then the whole concept of magnitudes increasing and then the idea of a velocity becoming positive or negative just isn't really congruous, like you have a increasing magnitude conceptio…possibly, and the decreasing velocity and since it's becoming more negative, and that's very awkward if you can't see the vector becoming larger, but in the negative direction.

36 I: Ok, ok, so would you write the same equation for these four cases or would you…

37 P: [00:08:45] I would write the same equation and in case of dealing with the signs, if for example the velocity was negative, I would use a parenthesis to kind of encapsulate it and show that each of these things is a representation of a single variable, and there may be a negative there, it doesn't really make sense there, but…

38 I: So V nought itself might be positive or negative or A itself might be positive or negative. That's…

39 P: Yeah, I would uh, my professor kind of warns me against this just because at that point you're not really thinking about vectors as much as you're thinking about components, but I feel that once you make the jump between an equation like this, vectors are much more difficult to deal with in their normal form and that it's really much easier to take all the horizontal or vertical or whatever components of everything you're dealing with, so that way you only have the algebra, and the signs will tell you which direction everything's going and hopefully everything works out if you've gotten it all right.

40 I: Ok, [turns page] so here's a problem.

41 P: [00:10:04] Oh wow, that's interesting. [exhale] Well the first thing I would think of is the equations. The velocity, I suppose, is the same equation as that other one and…I'm trying to think of calculus as well and what the differences do. So the acceleration is a constant and that means that velocity is linearly related to time and they're both at the same…so the first differences are the same. I *think* it's equal to two meters per second.

42 I: Ok, could you tell me how you figured it out?

43 P: Well I think of the acceleration being constant and using the other equation or just differentiating you would, well not differentiating that would be an integrate. But you uh find that velocity with constant acceleration is linear with respect to time and that means that the derivative, that is the acceleration is uh, it stays the same.
44 I: Ok.
45 P: So the first differences are the same.
46 I: Mhm.
47 P: And if the first differences are the same then the initial difference between the two speeds should not change.
48 I: What do you mean by first difference?
49 P: [00:11:37] When I was taking algebra one and we were, before we actually ended up in derivatives, but like for example taking a set of data points and figuring out the linear equation for them, we would consider the first difference to be two subsequent points, so basically delta y over delta x would be the first difference and then the second difference would be the difference between the first differences and you could use uh…
50 I: Ok. Ok.
51 P: Like depending on what you were doing, you could find out what kind of equation you were looking at from a set of data points.
52 I: Mhm.
53 P: Like if all the first differences are the same it's clearly linear, and if all the second differences are the same, it's quadratic which, and now I know through calculus why that's the case, but that was a analysis tool I used to use for looking at data.
54 I: Mhm.
55 P: So, to me, the question here is whether or not the velocity is, 'cause the initial difference in the velocity is two.
56 P: Ok.
57 P: So if you pick either of the balls as a reference frame, then basically one of them's initial velocity is 2 and the other one's initial velocity is 0, and I'm trying to figure out if their velocities are changing at different rates, and I decided that was not the case.
58 I: Ok.
59 P: There's a couple methods of attacking it and if I run into a kind of a mess, mess up in one of them, then I'll switch to another one.
60 I: Ok, so, uh, what would you have switched to?
61 P: So if I started from thinking about the equations and I'm not quite sure whether the velocities are changing at the same rate, then like sometimes I'll use several and see if they're consistent. Then I could switch to thinking about the derivatives of the velocity and I'll think, ok, so the initial conditions are off by 2 and then the velocities are changing at the same rate so that should mean they stay at 2. And then I could think about the reference frame because you're basically talking about the difference between 2 objects and, um, oh, uh oh, yeah that's a bit of a problem because they're accelerating, and you can't…and then the laws don't really work very well in accelerating reference frames, but, so that probably would have fallen through even though it seems to be consistent it's probably not the right way to go. But I think the easiest way especially when it's asking about values, even though it's a conceptual question.
62 I: Mhm.
63 P: It's good to look at the equations and see how they behave in relation to the motion.
64 I: Ok, so could you write down the equation that you were thinking about?
65 P: Yeah I would definitely write down the one we had before…plus A T.
66 I: Ok, so the equation that you talked about before.

67 P: Yeah, and then I would think about one of them is zero and the other one is two, and since their accelerations are the same, then, I mean if you wanted to attack it on paper clearly you can subtract these and find that everything cancels except this two.
68 I: Ok.
69 P: [00:14:58] And that would be another way to attack the problem, but thinking about it in my head I would realize that without actually writing out the subtraction of the equations that the A T term will be exactly the same for both of them since they say acceleration due to gravity is the same and they're both after 5 seconds.
70 I: Ok. Ok. [turn page] Here's another equation.
71 P: Oh wow.
72 I: Have you see this before?
73 P: A little, we looked at it, we looked at it for some kind of proof for some of Bernoulli's stuff in ENES 100, but we haven't used it extensively since…it's a hovercraft project and the differences in height between the inside of the plenum chamber are not significant enough for us to really care.
74 I: Ok.
75 P: So then.
76 I: So.
77 P: Yeah, it's definitely a little more complicated than I'm used to, at least I haven't seen it very often.
78 I: Ok, so suppose you were to explain, you were trying to understand this equation. How would you explain it to yourself?
79 P: Well, it's taking a familiar format in that the pressure at the top seems to be almost an initial condition, so I would start with an analogy to some equations that I'm already familiar with.
80 I: Mhm.
81 P: And I already know what G is. At least, I assume, it's not really noted, but…
82 I: Ok, yeah.
83 P: Is that, um, it's, uh, acceleration due to gravity?
84 I: Yes.
85 P: [00:16:36] Well, in that case I would question that and start with some kind of dimensional analysis, and I know that pressure is…so Newtons per meters squared. Density is kilograms per meters cubed. And height is in meters, so you end up with, so these terms are the same, so this one eventually has to come out in Newtons and then…not sure if I worked that one out or not…well, it's got to have a meters squared on the bottom eventually, so…ok. Ok, multiply a one there. So that would at least establish to me that all the terms were what I thought they were.
86 I: Ok.
87 P: That's kind of interesting.
88 I: Mhm. Can you tell me what you're thinking?
89 P: Well I'm having trouble squaring the dimensions because when I think of seconds squared, I think of time and clearly this doesn't have anything to do with time, or any elapsed time. It just has to do with that as an acceleration so seeing G…
90 I: Uh huh.
91 P: …in that equation is a little bit tough. So when I see a height, I think of potential energy.
92 I: Ok.
93 P: Which is kind of interesting, but I'm not sure if that would really help since there's not really any work being done here either. The density is the most obvious thing that makes a good bit of sense, because you're underwater. You can definitely feel that it's heavy.
94 I: Ok.

95 P: [19:29] Yeah, I'm starting to hit a wall. I would probably have to take a look at a textbook at this point to figure out how the equation was derived.
96 I: Ok. Ok, so um suppose you were doing this question on an exam. What would you do? How would you answer that?
97 P: Well, I'd probably start with what I knew with the dimensional analysis.
98 I: Ok.
99 P: And then explain a little bit and, uh, yeah that would be a tough one.
100 I: Ok.
101 P: I [might have to?] skip that one
102 I: Here's a different situation, ok? Now, think about, um suppose um, suppose there was a 12 year old and, who understands some rudimentary stuff but that hasn't really taken a science course, a physics course, so how, how would you go around explaining this equation to a 12 year old?
103 P: Well, I'd probably just have to start with what I knew and show how the units work out, I mean, I would think that for a 12 year old, it would be a little bit of a shock to be able to cancel out units more then anything else. It took me a little while to get that one, but after that I wouldn't really be able to explain how the, everything worked together, and I would probably have to end up, you know, here's how it works, use it when you need it.
104 I: Ok.
105 P: Not the best situation but…
106 I: So when do you think you would need this equation?
107 P: This one? Uh, possibly something to do with buoyancy or, well if you ever wanted to build something, say a submersible, you would need these values.
108 I: Mhm.
109 P: And probably has some, uh, applications in aerospace as well. Well not necessarily space, but certainly aero.
110 I: Uh, air. Did you say air?
111 P: [00:21:40] Yeah, I'm thinking about that. It might be, I mean, I know we're using at least a little bit in the hovercraft applications, not much with the height business but this is that small, conceivably. Good for pressures in fluids or at least in this situation.
112 I: Mhm. So you mentioned something about that this, you know you try to understand it. It looks analogous to some of the equations that you've seen before. Uh, could you tell a little more about that? Which equations it is?
113 P: Well pretty much any of the kinematic equations that start with an initial condition, well a lot of equations start with an initial condition. So I think of p at top and I see the other one and think of change and, the other one, like in my mind I'm kind of thinking about the area that this equation is describing, so you have a point underwater and you have a single line shooting up to it, and I guess that's probably where I fall short on the equation, because when I think of pressures, I think of areas and there's not really any area involved.
114 I: Ok.
115 P: But it reminds me of potential energy problems or any problem where you have a certain condition and then something else happens to it, say a force or energy transfer or something else and you, that's expressed as a change and an initial and final condition. 116 I: Ok, so you're seeing the P top as, as the initial condition and then the Rho G H as the change
117 P: Yes.
118 I: Giving the pressure P.
119 P: Yeah.

120 I: [00:23:18] Ok. So have a look at these 2 problems, ok? The first is, um if I ask you to compare the pressure at a depth of 5 meters versus at a depth of 7 meters, how would you go about doing that?
121 P: Well I would probably question whether comparing the division or subtraction would, usually subtraction's the best bet for this, especially with an equation like that where there's a lot more, well it's an addition thing mostly, the change I was talking about.
122 I: Ok.
123 P: So I would start by writing down the problem again and plugging in values and seeing what I had left.
124 I: Can you show me what you're doing?
125 P: Sure, so…I'm going to put in an initial symbol there. It's the thing that makes the most sense to me…equals...
126 P: So those are the two pressures.
127 I: Ok.
128 P: And if you were to subtract them from each other, the initial pressure would cancel.
129 I: Mhm.
130 P: And you would end up with the difference between these two values. Well, I suppose if you wanted to consider density as a variable.
131 I: Density is 1, suppose I told you.
132 P: Ok, well then it would just be 9.8 minus 9.8 times 5, and it's approximately 10.
133 I: Ok, so uh which of them is greater then?
134 P: The seven is greater.
135 I: The seven, seven meters is greater.
136 P: Well yeah, I would, uh, keep track of which value is which until the final calculations.
137 I: Ok. Ok, so um you said something about, if you looked at compare, you would either think about subtraction or division. So is this what you meant by subtraction, that you…?
138 P: Yeah, that I would subtract the two values instead of trying to find a ratio.
139 I: So how would the ratio help you?
140 P: Well, the ratio, not very much, because I don't very much like to see plus or minus signs in fractions, especially in the denominators, because it makes things very ugly.
141 I: Ok.
142 P: So without say, if it were, say Newton's gravitational law where there isn't anything like that and all of it is multiplication, then it would make more sense to do something like that. But in this case, not so much, because when you do the subtraction you can cancel the initial conditions very easily, where, which is theoretically interesting that the initial pressure has nothing to do with changes in pressure over distances, whereas the ratio wouldn't be as helpful because you would, I think that the initial conditions would not cancel, but I'm not quite sure.
143 I: So if you compute the ratio, what do you look for?
144 P: [00:27:11] Division. I would put them on top of each other, and I would start with the seven, I would prefer, uh greater-than-one ratios to less-than-one ratios and, for this one, prefer positive numbers over negative.
145 I: So you look for, uh, greater-than-one ratios, and...
146 P: Yeah.
147 I: So if the ratio was greater than one, what would that mean?
148 P: That would mean that the pressure at seven was greater.
149 I: Was greater, ok.
150 P: And you can establish a fraction for the five in relation to it.
151 I: Ok, so um could somebody have done this problem without actually working out the numbers? Or without, you know, solving the...

152 P: Conceivably. I mean with 9.8? Not so much. But they would definitely have had to plug in something somewhere, either in their head or on paper.
153 I: Ok.
154 P: I would be very surprised if they had memorized the difference between five and seven meters in water.
155 I: Ok. Does this make sense to you?
156 P: Yeah, I mean, I'd still like to figure out how the equation's derived, probably go back and look at my textbook when I get back, but…
157 I: So how do you make sense of this, that, the answer that you got?
158 P: [00:28:34] But, right now the answer that I got makes sense, because I know from my own experience that the deeper you go in a pool of water, the higher pressure it is. So the value, I mean, first of all I came up with a value that made sense, 20 Newton-meters between 2 meters, and the values of 70 and 50 seem to be reasonable. I haven't, don't think I missed any orders of magnitude or anything.
159 I: Ok. Ok, have a look at the second problem?
160 P: [00:29:15] Ok, well the Mars lake pressure the grav…force of gravity or the gravitational acceleration is not given, so the first thing I would do is establish a variable that represents it. Probably subscript M is the best way to go.
161 I: Ok.
162 P: And then start to work out values. Now one of the annoying things about this problem is that, uh, the initial conditions on Mars aren't really given ,and they would probably be different than Earth as well, but from up here, if I don't use the ratio, which doesn't make much sense since it's dependent on the initial conditions.
163 I: Mhm.
164 P: If I use the subtraction, the subtraction instead, then I end up with values that I'm familiar with.
165 I: Ok.
166 P: Except for temperature. Temperature's also kind of awkward, but say it's noon on Mars. So probably the same thing up here but with uh H instead of the values given, and I'd throw the initial value in again just to be safe, because I'm not quite sure how everything's going to work out algebraically.
167 I: Ok.
168 P: And in this case, I may want to, just for convenience, establish a variable for G of Earth as well, so I know that I'm not really doing a full calculation.
169 I: Ok.
170 P: Because I think what's really going on here is looking for some kind of relationship, and if you don't use 9.8, then you can expand the relationship to, say there's a question about other planets…
171 I: Ok.
172 P: …that aren't 9.8, and it gives you a more useful answer. [writing] So subtracting these yields…and again I'm going for positive numbers as opposed to negative ones.
173 I: Ok.
174 P: So it says clearly that the Earth gravity is larger.
175 I: Mhm.
176 P: And at this point I might sub in the 10 at that point, depending on what the question exactly was asking for.
177 I: Ok, and G M is less than 10, right?
178 P: Yes.
179 I: Ok, so pressure at Earth is…
180 P: The pressure at Earth appears to be higher.
181 I: Appears to be higher, ok. You mentioned something about temperature.

182 P: Yes.
183 I: Tell me how you are thinking about that for this.
184 P: Well, the density is going to change depending on the temperature and that will affect the pressure according to the equation.
185 I: Ok.
186 P: [00:32:36] Though it doesn't look like the initial pressure at the surface matters in terms of distance, in terms of the difference between the pressures. Actually, I just realized that you can't cancel those, so it will matter. So now we need initial values for these as well.
187 I: Ok, so you're giving the initial values of the pressure on top for Earth and for Mars.
188 P: Yeah.
189 I: Ok.
190 P: And presumably the Earth pressure is greater, because as far as I can tell, Mars doesn't have much of an atmosphere.
191 I: Ok.
192 P: If it's not then, I suppose we'll find out. But I subtracted the equations anyways, so…
193 I: Right.
194 P: …they have to stay in the same order.
195 P: [00:33:42] And if the densities were different that would make things worse, and you would have to throw in a value for that as well.
196 I: Ok, suppose the densities were the same, so how would you figure out now, um whether the delta P that you have written out is, no, whether Earth pressure is greater or less than Mars pressure?
197 P: Mmm…well based on the assumption that the Mars pressure at the surface is less, then I would say that the earth pressure continues to be greater but…
198 I: Ok.
199 P: …if that was not the case, then it would change depending on exactly what the height was, um, which of these terms was more significant.
200 I: Ok, so you would compare this term and that term?
201 P: Yes, if the Mars initial pressure, pressure at the surface, is higher.
202 I: [00:34:40] Ok. Ok. Um, actually let me ask you a, a, another question right now. Um, how do you know when you really understand an equation?
203 P: I know I really understand an equation when I can tell where each of the values, where each of the terms is coming from. I mean, the first thing you need to know is what each of the values represents in a real world application of the motion or phenomenon that it's describing, but I usually know when I really understand an equation when I understand what each term means and can conceptualize in my head where all of the values are, what each term is doing I suppose. In the sense of, say you have a function or a variety of values over a graph or something like that. Let's say like, if this value is higher, what does that mean for the motion and what does it, err, what does that mean for the, what you're given? And what does it mean for what you get? And how do they interact with each other? So If I have all of those interactions going together in my head, say for the F equals M A problems.
204 I: Ok.
205 P: Then I would know that, it makes sense to me that, uh acceleration would be proportional to the force and inversely proportional to the mass just from my own experiences, and I would understand what each of those values meant in relation to the others and why they are in the positions they're in.
206 I: [00:36:20] Ok, ok, so you can understand each of those terms and how they interact with each other?
207 P: Yeah, whereas for something like this, I'm not sure what an acceleration has to do with pressure, and that's starting to confuse me.

208 I: Ok, right. So that's what you were trying to do here, is figure out…

209 P: Yeah, I started out with the dimensional analysis and found out that without anything to do with time, since it's really kind of a static thing, there's just the pressure and you're not moving, you're not changing any distances.

210 I: [00:36:57] Ok, um, here's another problem and, think I will [interviewer corrects typo on prompt], misprinting. Everything is frictionless. You have a 2 kg block resting on a frictionless, um surface with a rope around a frictionless pulley and a 1 kg block is hanging, uh from the rope and, um, the question asks you to solve for the acceleration of the 1 kg block. So once again just, you know, talk aloud as you're working on this problem, and we're just interested in how you're thinking about it.

211 P: I haven't dealt with something like this in a while. Well, I could do a free body diagram but I'm too lazy, so uh, it's clear that the normal force of this table or whatever is counteracting the gravitational force on the 2 kilogram block, so I'm going to disregard that one entirely, especially since it's frictionless. If it wasn't, then that would be a concern, but the 1 kg block being suspended in air, looking at this that's the only force that I can see, the gravitational force on this 1 kg block. Well also the tension here but as far as, from say a surroundings onto the system, what is happening here, clearly the 1 kg block is going to move down if it's frictionless and that's the only, gravity is the only force involved in moving that block. So I guess this is kind of complicated to explain because you have contact forces in here clearly that aren't gravity.

212 I: Mhm.

213 P: But I kind of think of those contact forces as almost dependent on the gravitational force, because if you had these things tied together on a table it wouldn't matter, even though they were connected, so when I see a problem like this and I think about motion, I start thinking about which forces are moving what, and then what the other objects do in response to that, instead of kind of writing out all the forces at once. So knowing that the only thing that gravity is affecting is this 1 kilogram block, I would start with the force for that, and since I can see the picture and I'm familiar with which direction everything's going, in this case I wouldn't be very careful with where the vectors are going, and I know that this represents 9.8 down.

214 I: You can use 10 if that helps you.

215 P: That works. So we have 10 Newtons.

216 I: Mhm.

217 P: So 10 Newtons of force here and no other, to me I would think of outside forces are really acting on this. But since it's tethered to this, it's not going to be just 10 Newtons acting on this, so these things are going to move as a unit since they're connected by a string. This thing isn't contributing at all to the gravitational force. This is the only one contributing to the gravitational force.

218 I: Ok.

219 P: So since the force is only being only derived from the 1 kilogram but the 1 kilogram is tied to the 2 kilograms, looking at this and thinking about how it moves I'm thinking that this is going to move more slowly than it would normally, so since there's no friction in here, the only way to slow that down is to limit the effectiveness of force and the force, so by effectiveness I mean how the force affects the acceleration.

220 I: Ok.

221 P: And that will happen when the mass increases, so I'm wondering what am I going to do with this value? And I think that even though this mass is 1 kg in thinking about gravity, for the acceleration of the 2 boxes as a unit 3 kg is the value I'm going to have to use.

222 I: Ok.

223 P: So solving for acceleration we have the force that we have and we have the effective mass of the system. We have force divided by mass, and if they were not attached it

would come up with the same value as before but, it would come up as 9.8 or 10, and that's about…

224 I: Ok.

225 P: 3.3 or whatever, if I'm not crazy. And now I think about that value, clearly 3.3 is less than 9.8 as expected.

226 I: You were expecting that the block would, uh move less, have an acceleration less than 9.8.

227 P: Yeah, and I mean if I knew that the professor was picky, which they usually are, um, I don't know all the fancy notation.

228 I: [00:42:12] Ok, ok um so why did you guess that the acceleration would be less than 9.8? What was that expectation?

229 P: Well I just put it in my mind of a block, a pulley, and another block sliding across the table, and, from my own understanding of the physical world, I realize that the 1 kg block is going to fall more slowly than it would if it were not tied to a string.

230 I: Ok, so…

231 P: And I don't expect it to go up, because the only force acting on this in here, none, there's no motor in here or anything this is not accelerating of it's own volition this is the only thing where a force is acting on it, and I guess if I could say an independent force, would probably be the best way to describe it.

232 I: Ok.

233 P: Where the tension in the cable would be more of a dependent force.

234 I: Ok, dependent on?

235 P: Dependent on gravity.

236 I: Ok.

237 P: Since if they were just lying flat, not doing anything, there wouldn't be any force there even though there are molecular bonds.

238 I: Ok. Ok. [turns page] [00:43:31] Have a look at this problem?

239 P: [reading] Oh, that's obnoxious. Oh, 50 meter wide at the water's surface, ok. Yeah, for a second, I thought that this was above the water's surface and that didn't make any sense at all.

240 I: Ok.

241 P: [reading]

242 I: [00:44:58] So just think about part A for now.

243 P: Part a?

244 I: Uh huh.

245 P: Yeah that's a complicated one, because thinking instinctually about what the equation does, it doesn't point sideways. It seems to me that pressure means down in general.

246 I: Ok, sort of like pushing down?

247 P: Yeah.

248 I: Ok.

249 P: But from what I know about air pressure that's probably not the case. The pressure probably just means pressure. Like if you think about a submersible, except for the distances, there's really not significantly more pressure at the top than there is at the bottom or on the sides, actually there's probably less because there's less water there, so that's probably the case, but I would say that the first student underest…er he overestimates the pressure, because he's only using the bottom area. The bottom…the pressure at the bottom of the, clearly the pressure at the bottom is not the same as the pressure at the top.

250 I: Ok so why do you think it's an overestimation?

251 P: Because the pressure at the bottom is greater than the pressure at the top

252 I: Ok.

253 P: And since the, he's using the bottom pressure, thinking about…the first thing that comes to my mind is, uh Riemann sums and the estimations there and I, because the function of the pressure slowly increases until it reaches the maximum of 20 meters, if you use the 20 meter value for the entire pressure, you're going to get an incorrect answer.

254 I: Ok.

255 P: And it's going to be larger than the correct answer.

256 I: Ok, so you think the correct answer will be less than this person just using, uh 20 meter depth?

257 P: Yeah.

258 I: [00:47:05] So have a look at part b then.

259 P: That one I think would work, but I'm not quite sure. Actually no. My geometry teacher's tricking me right now by flipping this over and making it into a rectangle, but the pressure in these upper triangles is not the same as it would be if they were down here, so that doesn't really make any sense either. So there's less of the trapezoid in the higher pressure zones and more of it in the lower pressure zones, so that doesn't work either and it uh it again overestimates the force because it takes low pressure zones and considers them to be high pressure, so that would, uh, I would think about the geometry and how you would transform them into a rectangle and make it make sense.

260 I: Ok.

261 P: Which I guess is the answer to d as well, but…

262 I: [00:48:07] Ok, so you're thinking that essentially just taking the mid pressure will not work because that makes…

263 P: Yeah.

264 I: …trapezoid geometry.

265 P: Yeah.

266 I: [00:48:24] Ok um, Pat, what do you find difficult or, uh you know, difficult to understand or use about math in this course?

267 P: About math?

268 I: In this course.

269 P: In a physics course?

270 I: Mhm.

271 P: Not much. The most confusing bit for me, like the one thing that doesn't quite make sense, is angular problems where angular accelerations, the actual vector, is perpendicular to the rotation. That's a little bit unusual, but it works and since vectors can't be curved, when I think about it I realize it's the only sensible way to describe curved motion. Well, uh circular motion, because the way planes are defined by normal lines.

272 I: Ok.

273 P: And the magnitude clearly corresponds to whatever the magnitude of the circular phenomenon is.

274 I: Ok.

275 P: But that kind of thing, when you get into more abstract representations, like vectors aren't bad because they pretty clearly represent normal concepts like position or speed or acceleration, so so far, well most of my physics stuff right now is review, so I'm not really having a lot of struggle with the mathematics.

276 I: It's review and…

277 P: Well I've already, I'm a freshman and when I was in high school, we had non-calculus based physics but taking calculus then as well, meshing the two is very intuitive, since, I don't know for sure, but I assume that Newton's contributions to both fields weren't an accident, and they're very interconnected.

278 I: Ok.

279 P: But I think the hardest thing actually is part of the arithmetic. The sign changes are just a huge mess, always very difficult to deal with, easy to make a mistake.
280 I: [00:50:27] Can you tell me what are you thinking about the connection? You said that the two fields are connected?
281 P: Yeah, in that physical phenomena doesn't really follow linear equations very often. When they do you're lucky, but even if it does, there are, you think about rates a lot and integration is also very helpful when you have a rate and you'd like to get a position, so tools like that have obvious applications like the first things you learn, or at least I learned in calculus when talking about derivatives and integrals were position, velocity, and acceleration graphs and seeing how they relate, how the graphs relate to each other, and how the equations relate to each other, the idea of the area under the curve or tangential slope.
282 I: Ok.
283 P: Just, that's the most, I think people use that example so often, because it's very intuitive and because you do have a little bit of background in that, in that when you're explaining it obviously velocity is the change in distance over time and acceleration is the change of velocity over time, so relating that concept of slope is pretty easy. The integral is a little bit more of a pain, but it also starts to make a little more sense like when you do it over time, well suppose, I guess there's an inverse operation to that as well.
284 I: Mhm.
285 P: [00:51:57] So the connection between calculus and physics is very clear to me.
286 I: Ok. So, Pat, if you had a photographic memory for equations, you know, you see an equation and it's just imprinted, would that help you in your performance?
287 P: Kind of. I mean, you would know you would have a set of equations that you were able to use so that you wouldn't…like sometimes I'll have an idea of what I want to do, but not quite sure exactly what the relationship is and usually when that happens I'll think of a few values and fall back on dimensional analysis. [?] If I'm not quite sure if I want to square something or not, I'll figure that out, how it works with the units.
288 I: Mhm.
289 P: I think it would help in the sense that you have more information, but it wouldn't be very useful for comprehension. It would be useful for calculations and assuming you knew what the equations were supposed to do as well. Just the equation won't help you very much.
290 I: [00:53:02] Ok, why?
291 P: Because they're just letters, and if you don't know where the numbers are supposed to go or what the numbers represent, like at least if you know that where the numbers are supposed to go, you might get a correct value, but if you're not sure what they represent, then you're probably going to make some sign mistakes. Especially if there are angles in it, and you don't know where those angles, what those angles are for. Like I remember one slightly confusing incident in my physics class. We were discussing fields and planes and a lot of the equations didn't measure the angle between the plane and the field. They measured the angle between the normal vector for the plane and the field because it's much more, a much less ambiguous definition of a plane.
292 I: Ok.
293 P: And if you don't know that you're very likely to be off by 90 minus theta degrees.
294 I: [00:53:56] Ok. Ok. Um, here is a situation. Suppose there's a friend of yours who, who was in the course with you but not getting graded. Um, she was just interested in learning physics in a very deep way but not really interested in solving quantitative problems.
295 P: Huh.

296 I: And she's happy to study outside the course to build this deep understanding of physics. Um, what role do you think should equations play in her studying?
297 P: Well, I guess she's not very interested in constants, which is fine, but I think you would have to both read about what happens, and I think the equations give you a pretty clear picture of how it happens. Like pictures and text explanations can only get you so far, because they really just give you a framework to understand the equation and the equation is really, like I have a very dry calculus textbook, but it reads almost like a legal document, because the, it's basically documentation for the universe, and it has to be extremely precise in order for you to understand it fully.
298 I: Ok.
299 P: So I feel like things like pictures in texts and conceptual understanding, well it doesn't perfectly lead to conceptual understand because it's really just a framework in which to view certain kinds of phenomena, and you have to have the equation to tie it to exactly what happens, like if you say that higher mass means lower forces, that's great and it makes sense in your head, but you don't actually know what that means until you take a look at the equation, and you realize this is exactly how all of the components interact with each other.
300 I: Ok
301 P: Until you do that, you can really only look at, say one interaction at a time and an equation really gives you a holistic view of whatever's going on. I mean if you're not interested in the algebra or solving techniques then that's fine, but you definitely need to at least look at how the different values interact.
302 I: [00:56:14] Ok, so for F equal to M A what is the, what would you call the holistic view?
303 P: Well that you can see that, say for example A is usually what people consider to be the dependent variable there because you usually have a force given and a mass given and you're trying to figure out what the mass is going to do.
304 I: Ok.
305 P: Physics tends to be, I guess, a lot about prediction given a set of parameters.
306 I: Ok.
307 P: So you would be able to easily tell the acceleration increases as the force increases and decreases as the mass increases, ehh yeah.
308 I: Ok.
309 P: [00:56:57] So that would be the easiest way to do this, split it up into, I think that's actually the way my textbook presented it at first, as A equals F over M, because that's the, that's a slightly more intuitive way of presenting it. And you can also see that the force represents a mass being accelerated a certain amount, and if you think about that in your head, like most people's conception of force is rather vague, like, ok here's a force. What does that mean? And then if you think about it from the equation, then you don't really need the dimensions per se for solving purposes but you realize, ok so the force doubles say if the mass doubles. Ok, that makes sense, and say you, the force, uh, that the mass accelerates twice as fast. Then, that means twice as much force. And that gives you a more precise understanding of, like not necessarily values to 10 decimal points but basic ratios.
310 I: [00:57:57] Ok, Pat could somebody have, without seeing the equation, figured out that mass uh, if the mass is more then acceleration will be less or the force is more then the acceleration would be more.
311 P: I suppose they could memorize it, but, I mean, without knowing it beforehand, you mean like an empirical thing?
312 I: Right, without the equation could they have gotten this understanding?
313 P: Well, someone did.
314 I: Ok.

315 P: [00:58:28] You'd have to use experiment. You would have to uh start with a set of masses and, well it would be a little awkward because you wouldn't be, we wouldn't be quite sure how force is measured. Like to me when I first, uh got into electronics, I mean, what's a volt? I don't know what that is, so if someone was trying to measure force maybe they would mistake it for energy or some similar concept, and you would have to know that a spring scale is the easiest way to do that…

316 I: Ok.

317 P: …or a similar device, so I suppose you would try a few things and come up with relationships, have large numbers of data points, so say you have a mass and you apply a certain amount of force to it find it how much it accelerates, and you double the mass or double the force and see what that does to the acceleration, and you come up with a set of values and a set of relationships and work them together into a single relationship.

318 I: Ok. Ok. Do you have any questions for me?