Capacity and Variability Analysis of the IEEE 802.11 MAC Protocol

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Abstract—Packet error in the IEEE 802.11 network is one source of performance degradation and its variability. Most of the previous works study how collision avoidance and hidden terminals affect 802.11 performance metrics, such as probability of a collision and saturation throughput. In this paper we focus on the effect of packet errors on capacity and variability of the 802.11 MAC protocol. We develop a new analytical model, called p_e-Model, by extending the existing model (Tay and Chua’s model) to incorporate packet error probability p_e. With p_e-Model, we successfully analyze capacity and variability of the 802.11 MAC protocol. The variability analysis shows that increasing packet error probability by Δp_e has more effect on saturation throughput, than adding 0.5WΔp_e stations, where W is the minimum contention window size. We also show the numerical validation of p_e-Model with 802.11 MAC-level simulator.

I. INTRODUCTION

With the popularity of the IEEE 802.11 [1] based wireless network, it has become increasingly important to analyze and predict the performance of the IEEE 802.11 protocol and its variability accurately. Carrier Sensing Multiple Access/Collision Avoidance (CSMA/CA) and hidden terminals [3], [4] have been considered to be important in performance analysis of the IEEE 802.11 PHY/MAC protocol.

CSMA/CA exploits binary exponential backoff mechanism to avoid collisions among the wireless stations contending for the medium as follows. Before sending a packet, a wireless station first senses the medium for the duration T_{DIFS} (DIFS is Distributed Interframe Space). If the medium is free for the duration, the wireless station starts sending the packet immediately. Otherwise, if the wireless station detects the medium was busy for the duration, the wireless station back off for a multiple of time slots (T_{slot}). The multiple is randomly chosen between \( [0, 2^W] \) (\( i = 0, 1, 2, \ldots, m \)). W is called minimum contention window (CW) size, which is set to the same value for all the wireless stations. If the wireless station transmitted a packet and received ACK frame correctly, then \( i \) is set to 0. If the wireless station failed to receive ACK frame, \( i \) is incremented by 1.

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A. Causes and Effects of Packet Errors

Packet errors usually occur due to non-ideal channel condition [7]. Partition loss in the building and multipath fading, combined with ambient noise, decrease SNR (Signal-to-Noise Ratio), therefore cause packet errors. Co-channel and adjacent channel interferences also cause packet errors.

Wireless device variability is another source of packet errors. Different devices have different output power, receive sensitivity and firmware, which may incur packet errors. Such errors would not occur if different pair of cards were used. In the experimental study on wireless monitoring [9], they observe that some firmwares send data at multirates (1, 2, 5.5 and 11 Mbps), but other firmwares receive them correctly only at 11 Mbps, generate packet errors at other rates. This is an example of the packet errors due to wireless device variability.

The impact of packet errors on performance metrics of the IEEE 802.11 protocol is different from that of hidden terminals. Hidden terminal problem focuses on unreliable carrier sensing ability, while packet error analysis concerns unreliable packet transmission ability. Another difference is that packet errors cause packet retransmissions at sender station and reception errors at receiver station, which is not the effect of hidden terminals. Packet retransmissions incur delay and packet loss at sender station. Reception errors incur additional delay at receiver station. When a reception error occurs, the receiver station waits for T_{EIFS} (Extended Interframe Space, usually more than 7 × T_{DIFS}) instead of T_{DIFS} so that the sender can have enough time to know that a reception error may have occurred.
one wireless station, say station A, has the corresponding variability. In this section we show the variability of the 802.11 MAC protocol. The variability analysis is described in detail in Section III. Capacity and variability analysis based on $p_e$-Model are followed in Section IV and V respectively. With $p_e$-Model, we successfully analyze capacity and variability of the 802.11 MAC protocol. The variability analysis shows that increasing packet error probability by $\Delta p_e$ has more effect on saturation throughput, than adding 0.5W $\Delta p_e$ stations to the current network. In Section VI we show the numerical results on validation and analysis of $p_e$-Model with 802.11 MAC-level simulator.

II. PERFORMANCE MODELS OF THE IEEE 802.11 PROTOCOL

In this section we overview the existing models and analysis techniques. For our model is based on Tay and Chua’s mathematical model [8]. We explain it in detail followingly.

A. Overview of the Performance Models

One of the issues in the analysis of the IEEE 802.11 protocol has been to devise an analytical model which can predict the collision probability and its effect on the performance metrics in consideration of CSMA/CD and hidden terminals. Bianchi [2] conducts Markov chain analysis for calculating collision probability and saturation throughput. Similarly Ho and Chen [4] derive the throughput and average delay by use of 2-dimensional Markovian analysis. Hidden terminals have been considered in the literatures [3], [4], [5].

While some of these studies use the stochastic analysis [2], [4], Tay and Chua [8] use the mathematical approximations by means of average values. This model provides closed-form expressions for the probability of a collision and the saturation throughput. Tay and Chua’s model is based on our $p_e$-Model, will be described in more detail in the following section.

The models mentioned so far assume ideal channel conditions, where packet error does not occur. Qiao and Choi [6] assume additive white Gaussian noise channel (AWGN) and calculate packet error probability, then derive the goodput performance of PHY/MAC protocol analytically. But their MAC model is oversimplified. They assume that there are only two stations (one sender and one receiver) therefore no collisions occur. In our model we consider both packet errors and the collisions among $n$ stations.

B. Capacity Metrics and Parameters

Before describing Tay and Chua’s model, we define important metrics and parameters used in capacity analysis of the IEEE 802.11 MAC protocol. Table I lists the all variables used in Tay and Chua’s analysis and our analysis, described in the following sections.

The capacity metrics include transmission failure probability $p_f$, collision probability $p_c$, total channel utilization $u_{total}$ and saturation throughput $S$. Each transmission has the probability of transmission failure $p_f$ and the probability of collision $p_c$. Total channel utilization $u_{total}$ is the fraction of non-idle period of the channel. Saturation throughput $S$ is the fraction of channel bandwidth that is used to successfully transmit payload bits if every station’s buffer is always occupied, i.e. if the network is under saturation condition.

The parameters used in the capacity analysis of the IEEE 802.11 protocol include minimum window size $W$, maximum

![Number of transmissions of distinct packet varies over time. Distinct packets are identified with Packet Number. Packets are sent at 200 packets/second. Number of retransmissions is limited to 3.](image)
C. Tay and Chua’s Model

Tay and Chua’s model [8] assumes that packet transmission fails only when collision occurs. Therefore each transmission failure occurs with the probability of \( p_c = p_f \). Average CW size \( W_{backoff} \) is calculated as the sum of possible average CW size \( 2^i W / 2^i (i = 0, 1, \ldots, m) \) times the corresponding probability:

\[
W_{backoff} = \sum_{i=0}^{m} \left( \frac{2^i W}{2^i} p_f (1 - p_c) + p_c^{m+1} (2^m W) / 2 \right)
= \frac{1 - p_c - p_c (2p_c)^m W}{1 - 2p_c}. \tag{1}
\]

Suppose station \( A \) sees the packet transmissions by station \( B \) under saturation condition. Every time \( A \) detects any \( B \)’s transmission, \( A \)’s backoff timer is suspended until \( B \)’s transmission completes [1], and resumes after the completion, as illustrated in Fig. 2. Therefore collision occurs only if \( A \) starts transmission at black slots, in Fig. 2. Because average interval between black slots equals to average CW size, \( W_{backoff} \), \( A \)’s probability of colliding with \( B \) is \( 1 / W_{backoff} \). Therefore \( p_c \), the probability of colliding with \( n - 1 \) stations is calculated as

\[
p_c = 1 - (1 - \frac{1}{W_{backoff}})^{n-1} = 1 - (1 - \frac{2 (1 - 2p_c)}{1 - p_c - p_c (2p_c)^m W})^{n-1}. \tag{2}
\]

Let \( T_{TX} = T_{physical} + T_{SIFS} + T_{ACK} + T_{DIFS} \), then saturation throughput \( S \) is derived based on \( p_c \) as

\[
S = \frac{2 (1 - p_c)}{2 - p_c} \times \frac{T_{payload}}{T_{TX} + W T_{slot} / (n + 1)}.
\]

The followings are the capacity analysis results in [8].

- Collision independence: the probability of a collision does not depend on packet length, the latency in crossing the MAC/PHY layers, the acknowledgement timeout, the interframe spaces and the slot size; it only depends on \( W \), \( m \), and \( n \).
- Inverse gap dependence: When \( n \) is large but smaller than \( 2^m - 1 W \), the protocol’s performance depends on \( W \) and \( n \) only through a new parameter \( q = \frac{n}{2^m} \), called an inverse gap.
- Maximum window size effect: suppose \( p_c < 0.5 \). The choice of maximum window size has minimal effect (namely \( O(p_c (2p_c)^m) \)) on the collision rate and saturation throughput.
- Approximation of \( p_c \) and \( S \): for large \( n \) the collision probability \( p_c \) and the saturation throughput \( S \) can be approximated by

\[
p_c = \frac{1}{2} \left( 1 + 4q - \sqrt{1 + 4q^2} \right),
S = \frac{2 (1 - p_c)}{2 - p_c} \times \frac{T_{payload}}{T_{TX} + W T_{slot} / q}.
\]

- Maximum throughput: suppose \( T_{TX} \gg 4 T_{slot} \). The saturation throughput is maximum when \( W = \sqrt{T_{TX} T_{slot} / (n - 1)} \).

III. \( p_c \)-Model: A Packet Error Extension of Tay and Chua Model

A. Assumptions

\( p_c \)-Model is an extension of Tay and Chua model by incorporating probability of packet error \( p_c \). The model assumes that transmission fails when either collisions or packet errors occurs. The two events are assumed to be independent. Therefore transmission failure probability \( p_f \) is given by

\[
p_f = p_c + p_c - p_c p_c \approx p_c + p_c. \tag{3}
\]

For simplicity we ignore the product of \( p_c \) and \( p_c \). We will show in Section VI the model does not lose much in the prediction accuracy due to this approximation. We also assume that packet errors always incur reception errors at the receiver station. Every time packet error occurs, therefore the sender experiences retransmission or loss of the packet, and the receiver suffers additional delay of \( (T_{EIFS} - T_{DIFS}) \). Another assumption is that \( n \) wireless stations all have the same packet error probability, i.e. \( p_c \) is assumed to be a global parameter for all the stations.
B. $p_c$-Model

We can calculate the collision probability $p_c$ in similar way as in (2), except that in $p_c$-Model $p_f$ is different from $p_c$, therefore $p_f$ is used to calculate $W_{backoff}$ in (1).

$$p_c = 1 - (1 - \frac{2(1 - 2p_f)}{1 - p_f - p_f(2p_f)^m} \frac{1}{W})^{n-1}$$

(4)

By (3), transmission failure probability $p_f$ is given by

$$p_f = p_c + p_e = 1 + p_e - (1 - \frac{2(1 - 2p_f)}{1 - p_f - p_f(2p_f)^m} \frac{1}{W})^{n-1}$$

(5)

$r_{xmit}$ and $r_{success}$ denote the rate of transmissions (including failures) and the rate of successful transmissions respectively. $(1 - p_f)$ is the probability of successful transmission, therefore we have

$$\frac{r_{success}}{r_{xmit}} = 1 - p_f.$$  

(6)

Let $r_{collision}$ and $r_{pkterr}$ to be the rate of collisions and the rate of packet errors respectively. We count multiple transmissions that collide as one collision. Approximately each collision is between just two transmissions, therefore $2r_{collision}$ contributes to the rate of transmission failures. Transmission failures are also due to packet errors, therefore we have

$$\frac{r_{xmit} - r_{success}}{r_{pkterr}} = 2r_{collision} + r_{pkterr}$$

(7)

$$\frac{2r_{collision}}{r_{pkterr}} = p_c.$$  

(8)

We define $T_{cycle}$ to be the average time between the start of two payload transmissions under saturation condition. Collided transmissions occur at the same time, therefore $r_{collision}$ (not $2r_{collision}$) contributes to $1/T_{cycle}$ as follows.

$$\frac{1}{T_{cycle}} = r_{success} + r_{collision} + r_{pkterr}.$$  

(9)

Solving the above equations (6), (7), (8) and (9), we can express $r_{xmit}$, $r_{success}$, $r_{pkterr}$ and $r_{collision}$ in terms of $p_f$ and $p_c$ as follows.

$$r_{collision} = \frac{p_f - p_e}{2 - p_f + p_e} \frac{1}{T_{cycle}}.$$  

(10)

$$r_{success} = \frac{2(1 - p_f)}{2 - p_f + p_e} \frac{1}{T_{cycle}}.$$  

(11)

$$r_{pkterr} = \frac{2p_e}{2 - p_f + p_e} \frac{1}{T_{cycle}}.$$  

(12)

$$r_{xmit} = \frac{2}{2 - p_f + p_e} \frac{1}{T_{cycle}}.$$  

(13)

Total utilization $u_{total}$, the fraction of non-idle period of the channel, is given by

$$u_{total} = r_{success}(T_{physical} + T_{ACK}) + (r_{collision} + r_{pkterr})T_{physical}.$$  

(14)

$T_{cycle}$ consists of packet transmission time, $T_{SIFS}$, ACK transmission time, carrier sensing time ($T_{DIFS}$ or $T_{EIFS}$) and contention period before any station obtains the medium. If all $n$ stations experiences reception errors (due to packet errors), then $T_{EIFS}$ is used for carrier sensing time. Otherwise there exists at least one station that waits for $T_{DIFS}$ and the station obtains the medium. Therefore the average carrier sensing time is $(1 - p_e^m)T_{DIFS} + p_e^nT_{EIFS}$. There exist $(1 - p_e)n$ stations whose CW is $W$. When those stations uniformly choose a time in $W$, then the earliest slot will be $W/((1 - p_e)n + 1)$ slots, which is the average contention period. For simplicity, we approximate the average contention period to be $W/(n + 1)$ slots. We discuss the effect of this approximation in Section VI. $T_{cycle}$ therefore is given by

$$T_{cycle} = T_{physical} + T_{SIFS} + T_{ACK} + (1 - p_e^m)T_{DIFS} + p_e^nT_{EIFS} + \frac{W}{n + 1}T_{slot}.$$  

(15)

Saturation throughput $S$ is given by

$$S = r_{success} \times T_{payload} = \frac{2(1 - p_f)}{2 - p_f + p_e} \times \frac{T_{payload}}{T_{cycle}}.$$  

(16)

IV. Capacity Analysis

Claim 1: When $n$ is large (say, $n \geq 5$) but smaller than $2^n - 2$ and $p_e < 0.5$, the protocol’s performance metrics ($p_f$, $r_{success}$, $r_{xmit}$, $r_{collision}$, $r_{pkterr}$, $u_{total}$, $S$) depends on $W$ and $n$ only through the inverse gap $q = \frac{n - 1}{W}$.

Proof: From equation (5), taking first order approximation, we get

$$p_f = \frac{2(1 - 2p_f)}{1 - p_f - p_f(2p_f)^m} \frac{n - 1}{W} + p_e.$$  

(17)

Now let

$$f(x) = \frac{x - p_e}{2} \frac{1 - x - x(2x)^m}{2(1 - 2x)} - q$$

(18)

$$= \frac{x - p_e}{2} \left( x \sum_{k=0}^{m-1} (2x)^k + 1 \right) - q.$$  

(19)

where

$$0 \leq p_e \leq x \leq 1, p_e < \frac{1}{2}$$

$$0 \leq q = \frac{n - 1}{W} < 2^{m-2}$$

Then, if $p_f \neq \frac{1}{2}$, then $f(x)$ in (19) is increasing and continuous function. Also by substituting 0 and 1 for $x$ in (18) we get

$$f(0) = -\frac{1}{2}p_e - q < 0$$

$$f(1) = (1 - p_e)2^{m-1} - q \geq 2^{m-2} - \frac{n - 1}{W} > 0.$$  

Thus, \( f(x) = 0 \) has exactly one root in \((0,1)\), which is a valid and unique value for \( p_f \). \( p_f \) depends on \( n \) and \( W \) only through \( q \). Furthermore for large \( n \), we can approximate (15) by

\[
T_{cyd} = T_{physical} + T_{SIFS} + T_{ACK} + (1 - p_f^n)T_{DIFS} + p_f^nT_{EIFS} + \frac{W}{n-1}T_{std.}.
\]

Therefore \( T_{cyd} \) depends on \( q \). According to equations (10), (11), (12), (13), (14) and (16), \( r_{success}, r_{txmit}, r_{collision}, r_{pkterr}, u_{std} \) and \( S \) depend on \( q \) also.

Claim 2: Suppose \( p_f < 0.5 \). The choice of maximum window size has minimal effect (namely \( O(p_f^2) \)) on the transmission failure probability \( p_f \) and saturation throughput \( S \).

Proof: Suppose

\[
(p_m - p_c) \frac{1 - p_m - p_m(2p_m)^n}{1 - 2p_m} = 2q = (p_\infty - p_c) \frac{1 - p_\infty}{1 - 2p_\infty}
\]

\( p_m \) is the root of (17) for maximum window size \( 2^nW \) and \( p_\infty \) is the root for unbounded window size (using \( 2p_f < 1 \), so \( \lim_{m \to \infty}(2p_f)^m = 0 \)). Let \( \Delta_{p_f} = \frac{p_m - p_c}{p_m} \). Ignoring the term \( \Delta_{p_f}^2 \), this gives

\[
\Delta_{p_f} = \frac{(2p_m)^n(p_m - p_c)(1 - 2p_m)}{(p_m - p_c)(2p_m)^{n+1} + (1 - 2p_m)p_m - (1 - 2p_c)}
\]

Now let the denominator as \( g(x) \).

\[
g(x) = (x - p_c)(2x)^{n+1} + (1 - 2p_c)x - (1 - 2p_c).
\]

where \( 0 \leq p_c \leq x < \frac{1}{2} \).

\[
g'(x) = 2(x - p_c)(m + 1)(2x)^m > 0.
\]

\[
g(p_c) = (1 - 2p_c)(p_c - 1) \leq g(x) < (2p_c - 1) = g(\frac{1}{2}).
\]

\[
|\Delta_{p_f}| < \frac{(2p_m)^n(p_m - p_c)(1 - 2p_m)}{1 - 2p_c} < (p_m - p_c)(2p_m)^m
\]

Therefore the effect of \( m \) is bounded by \( O(p_f(2p_f)^m) \).

Similarly, if \( S_m \) and \( S_\infty \) are the corresponding saturation throughputs and \( \Delta_S = (S_\infty - S_m)/S_m \), then we get from (16)

\[
\Delta_S = \frac{2(1 - p_m(1 + \Delta_{p_f}))}{2 + p_m(1 + \Delta_{p_f})} \frac{(2 + p_c - p_m)}{2(1 - p_m)} - 1 = \frac{(1 + p_c)p_m\Delta_{p_f}}{(1 - p_m)(2 + p_c - p_m)(1 + \Delta_{p_f})}
\]

Since \( \Delta_{p_f} < 0 \) and \( 0 \leq p_c \leq p_f < 0.5 \), we have

\[
|\Delta_S| < \frac{(1 + p_c)p_m|\Delta_{p_f}|}{(1 - p_m)(2 + p_c - p_m)} < \frac{(1 + p_c)|\Delta_{p_f}|}{1.5 + p_c} = O(p_f(2p_f)^m).
\]

Claim 3: For large \( n \) the saturation throughput can be approximated by

\[
S = \frac{2(1 - p_f)}{2 - p_f + p_c} \times \frac{T_{payload}}{T_{physical} + T_{SIFS} + T_{ACK} + T_{DIFS} \frac{T_{std.}}{T}} \tag{20}
\]

where

\[
p_f = \frac{1}{2}(1 + p_c + 4q - \sqrt{1 + (p_c + 4q)^2 - 2p_c}),
\]

\[
q = \frac{n - 1}{W}.
\]

Proof: Since the choice of \( m \) has minimal impact on \( p_f \), we can approximate (17) by

\[
\frac{(p_f - p_c)(1 - p_f)}{1 - 2p_f} = 2q. \tag{21}
\]

This has solution

\[
p_f = \frac{1}{2} \left(1 + p_c + 4q - \sqrt{(p_c + 4q)^2 - 2p_c}\right). \tag{22}
\]

(The positive square root gives \( p_f > 1 \), which is impossible.)

The claim follows from (16) and (20).

Claim 4: Suppose \( T_{physical} + T_{SIFS} + T_{ACK} + T_{DIFS} \gg 4T_{std} \). The saturation throughput is maximum when

\[
q = \frac{n - 1}{W} = \frac{(1 - p_f)^2}{(1 - 2p_f)\sqrt{1 + p_c} - 4(1 - p_c)},
\]

where

\[
c = \frac{T_{physical} + T_{SIFS} + T_{ACK} + T_{DIFS}}{T_{std}}/T_{std}
\]

and

\[
\frac{p_c + 2(1 - p_c)}{\sqrt{1 + p_c}} < 0.5.
\]

Proof: From (20),

\[
\frac{dS}{dq} = -\frac{2T_{payload}}{T_{std}(2 - p_f + p_c)^2} \left( q(1 + p_c)\frac{dp_f}{dq} - (1 - p_f)(2 - p_f + p_c) \right) c_q + 1,
\]

so the maximum occurs when

\[
\frac{dp_f}{dq} = \frac{1 - p_f}{1 + p_c}q(c_q + 1)
\]

By (21), we have

\[
\frac{dp_f}{dq} = \frac{2(1 - 2p_f)^2}{2p_f^2 - 2p_f + 1 - p_c}
\]

These two equations give

\[
e = \frac{4(1 - (3 + 2p_c + p_f^2)p_f + (4 + 2p_c)p_f^2 - p_f^3)}{(1 + p_c)(p_f - p_c)^2(1 - p_f)}
\]
By this equation and (21) we get
\[
p_f \approx \frac{p_f \sqrt{(1 + p_c) c + 2}}{(1 + p_c) c + 2}, \text{ since } \sqrt{c} \gg 2,
\]
\[
\approx p_c + \frac{2(1 - p_c)}{(1 + p_c) c} < 0.5.
\]

By this equation and (21) we get
\[
2q = \frac{2 n - 1}{W},
\]
\[
= \frac{2(1 - p_c)(1 - p_c - \sqrt{p_c + 4q})}{1 - 2(p_c + \frac{2(1 - p_c)}{\sqrt{p_c + 4q}})} \text{, since } \sqrt{c} \gg 2,
\]
\[
\approx \frac{2(1 - p_c)^2}{(1 - 2p_c)(\sqrt{p_c + 4q} - 1)}.
\]

V. Variability Analysis

A. Variability of \(p_f\)

By (22),
\[
\frac{dp_f}{dq} = 2 \left(1 - \frac{p_c + 4q}{\sqrt{p_c + 4q}^2 + 1 - 2p_c}\right), \quad (23)
\]
\[
\frac{dp_f}{dp_c} = \frac{1}{2} \left(1 - \frac{p_c + 4q - 1}{\sqrt{(p_c + 4q)^2 + 1 - 2p_c}}\right). \quad (24)
\]

From (23) and (24), and let \(D = \sqrt{(p_c + 4q)^2 + 1 - 2p_c} > 0,\)
\[
\frac{d}{dq} \left(\frac{dp_f}{dq}\right) = \frac{2}{D} \left(\frac{D^2 - 1 - 2p_c}{D^2} - 1\right) < 0.
\]
\[
dp_f/dq \text{ is decreasing in terms of } q, \text{ and } 0 \leq p_c < 0.5, \text{ we have}
\[
\left[\frac{dp_f}{dq}\right]_{q=1} = 2(1 - \frac{p_c + 4q}{\sqrt{p_c + 4q}^2 + 1 - 2p_c}) > 0,
\]
\[
\left[\frac{dp_f}{dq}\right]_{q=0} = 2(1 - \frac{p_c + 4q - 1}{\sqrt{(p_c + 4q)^2 + 1 - 2p_c}}) < 2
\]
Therefore, \(0 < \frac{dp_f}{dq} < 2. \quad (25)\)

\[
\frac{d}{dp_c} \left(\frac{dp_f}{dp_c}\right) = \frac{1}{2D} \left(\frac{D^2 - 8q}{D^2} - 1\right) < 0.
\]
\(dp_f/dp_c \text{ is decreasing in terms of } p_c, \text{ and } 0 < q \leq 1, \text{ we have}
\]
\[
\left[\frac{dp_f}{dp_c}\right]_{p_c=0.5} = \frac{1}{2} \left(1 - \frac{8q - 1}{8q + 1}\right) > \frac{1}{9},
\]
\[
\left[\frac{dp_f}{dp_c}\right]_{p_c=0} = \frac{1}{2} \left(1 - \frac{4q - 1}{1 + 16q^2}\right) < 1
\]
Therefore, \(\frac{1}{9} < \frac{dp_f}{dp_c} < 1. \quad (26)\)

We compare the effects of \(q\)'s variability and \(p_c\)'s variability on transmission failure error \(p_f\).
Let \(D = \sqrt{(p_c + 4q)^2 + 1 - 2p_c} > 0,\)
\[
\frac{dp_f}{dq} = F(p_c, q) = \frac{4(1 - \frac{1}{D(p_c + 4q - 1)})}{D}
\]
\[
\frac{dF}{dq} = \frac{4}{(D - (p_f + 4q - 1))^2} \left(\frac{4(p_c + 4q)}{D} - 4\right)
\]
\[
\frac{16}{(D - (p_c + 4q - 1))^2} \left(\frac{p_c + 4q}{\sqrt{(p_c + 4q)^2 + 1 - 2p_c}} - 1\right) < 0.
\]

Therefore \(dF/dq\) is decreasing in terms of \(q, 0 < q \leq 1\) and \(0 \leq p_c < 0.5, \text{ so}
\]
\[
[F(p_c, q)]_{q=1} = 4(1 - \sqrt{p_c^2 + 6p_c + 17} + p_c + 3) > 0
\]
\[
[F(p_c, q)]_{q=0} = 4(1 - \frac{1}{2(1 - p_c)}) = 2
\]
Therefore, \(0 < \left|\frac{dp_f}{dq}\right| \leq 2. \quad (27)\)

Claim 5: With \(W\) fixed, increasing packet error probability by \(\Delta p_c\) causes at least the same effect as adding \(0.5W\Delta p_c\) stations, on transmission failure probability \(p_f\).

Proof: From (27), for the same change of \(p_f\) (\(\Delta p_f\)), \(\Delta p_c\) and \(\Delta q\) have the following inequality.
\[
\frac{\Delta p_f}{\Delta q} \leq \frac{2\Delta p_c}{\Delta p_c},
\]
\[
\Delta q = \frac{\Delta(n - 1)}{W} \geq 0.5\Delta p_c, \text{ therefore},
\]
\[
\Delta n \geq 0.5W\Delta p_c
\]

B. Variability of \(S\)

Restating (16) for reading convenience,
\[
S = \frac{2(1 - p_f)}{2 - p_f + p_c} \times \frac{qT_{\text{payload}}}{q(b + p_c'(T_{EIFS} - T_{DIFS}) + T_{\text{slot}})}
\]

where
\[
q = \frac{n - 1}{W}, b = T_{\text{physical}} + T_{\text{IFS}} + T_{\text{ACK}} + T_{\text{DIFS}}.
\]

\(S\) can be restated as \(S = A \times B\) where
\[
A = \frac{2(1 - p_f)}{2 - p_f + p_c} \times \frac{qT_{\text{payload}}}{C}, \quad B = \frac{q(b + p_c'(T_{EIFS} - T_{DIFS}) + T_{\text{slot}})}{C}
\]
We now compare the effects of $q$'s variability and $p_e$'s variability on throughput $S$.

\[
\frac{dS/dq}{dS/dp_e} = \frac{\frac{dA}{dp} B + \frac{dA}{dq}}{\frac{dA}{dp} B + \frac{dA}{dq}}
\]

Let $\frac{dA}{dp} = A'$, 

\[
= \frac{\frac{dA}{dp} B + \frac{dA}{dq}}{\frac{qC^{dp}_{dp} + \frac{A}{A'} T_{slot}}{qC^{dp}_{dp} + \frac{A}{A'} T_{slot}}}
\]

To get the bounds of $A/A'$, $-G(p_e, q)$.

\[
\frac{A}{A'} = -G(p_e, q) = \frac{(1 - p_f)(2 - p_f + p_e)}{1 + p_e}
\]

\[
\frac{dG}{dq} = \frac{1}{1 + p_e} \frac{dp}{dq} \left( (p_e - 1/2 + 9/2) > 0 \right) < 0
\]

$G(p_e, q)$ is decreasing. With (28) and (22), we have

\[
[G(p_e, q)]_{q=1} = \frac{p_e + 15 - 3\sqrt{p_e^2 + 6p_e + 17}}{2(1 + p_e)} > 0
\]

\[
\frac{[G(p_e, q)]_{q=0}}{\frac{2(1 - p_e)}{1 + p_e}} < 2
\]

Therefore, 

\[
\frac{2}{3} < G \left( - \frac{A'}{A} \right) < 2
\]

For simplicity, we use $T_p$ for $T_{p_{agload}}$, $T_s$ for $T_s$, $T_E$ for $T_E$ and $T_D$ for $T_D$.

\[
\frac{dR}{dq} = \frac{1}{C_{max} (\frac{dp}{dq} + \frac{2}{3}q(T_E - T_D)n_p^{n-1})} \times \left( \frac{-16}{3}q(T_E - T_D)n_p^{n-1} \frac{(D - (p_e + 4q)^2)}{D^3} \right)
\]

All the four terms in the numerator are negative, so $dR/dq < 0$. $R(p_e, q)$ is decreasing in terms of $q$, therefore $R(p_e, q)$ has the maximum when $q = 0$, hence, $D = 1 - p_e$.

\[
\frac{dS/dq}{dS/dp_e} < R(p_e, q = 0)
\]

\[
\frac{C_{max} 2 - \frac{2}{3} T_s}{C_{max} 2 - \frac{2}{3} T_s} < \frac{2}{C_{max}}
\]

(28)

Claim 6: With $W$ fixed, increasing packet error probability by $\Delta p_e$ causes at least the same effect as adding $0.5W \Delta p_e$ stations, on throughput $S$.

Proof: From (28), for the same change of $S$ ($\Delta S$), $\Delta p_e$ and $\Delta q$ have the following inequality.

\[
\frac{\Delta S}{\Delta q} < 2 \frac{\Delta S}{\Delta p_e}
\]

\[
\Delta q = \frac{\Delta (n - 1)}{W} > 0.5\Delta p_e, \text{ therefore,}
\]

\[
\Delta n > 0.5W \Delta p_e
\]

VI. Numerical Results

We use DCF simulator [2] by Bianchi et al. for numerical validation and analysis. We modify the simulator to add the behaviors of packet errors and the delay due to the packet errors ($T_{EIFS}$). We obtain the results by varying the simulation factors, which are $m$, $W$, $n$ and $p_e$. Other simulation parameters are summarized in Table II.

A. Numerical Validation of $p_e$-Model

Comparing our approximation of $p_f$ in (22) (lines in Fig. 3) with the simulation results (points in Fig. 3), we observe that $p_e$-Model makes substantially accurate predictions of $p_f$. 
Table II

<table>
<thead>
<tr>
<th>Packet Format and Timing Parameters Used in the Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>Packet payload</td>
</tr>
<tr>
<td>MAC header</td>
</tr>
<tr>
<td>PHY header</td>
</tr>
<tr>
<td>ACK length</td>
</tr>
<tr>
<td>Channel Bit Rate</td>
</tr>
<tr>
<td>Propagation Delay</td>
</tr>
<tr>
<td>RX-TX Trunaround Time Delay</td>
</tr>
<tr>
<td>Busy Detect Time</td>
</tr>
<tr>
<td>SIFS</td>
</tr>
<tr>
<td>DIFS</td>
</tr>
<tr>
<td>EIFS</td>
</tr>
<tr>
<td>ACK Timeout</td>
</tr>
<tr>
<td>Slot Time</td>
</tr>
<tr>
<td>Maximum Cycle (DATA-ACK) Duration</td>
</tr>
<tr>
<td>Maximum Packet Rate for Single Station</td>
</tr>
<tr>
<td>Packet Rate for Single Station</td>
</tr>
</tbody>
</table>

In $p_e$-Model we have used many approximations, which might affect the prediction accuracy of $p_f$. We made an approximation by ignoring $p_e \times p_e$ in (3), which we call $P$-APPROX. $P$-APPROX can make $p_f$ in the model greater than the simulation results, i.e. incur positive errors. P-APPROX's positive errors become higher as $p_e$ and $n$ increase. First-order approximation used in (17), called $F$-APPROX, can introduce positive errors. The $F$-APPROX's positive errors increase as $n$ increases. Ignoring $(2p_f)^m$ term in (21) (called $M$-APPROX) can cause negative errors on $p_f$. As $p_e$ increases and $m$ decreases, $M$-APPROX causes more negative errors.

In Fig. 4 $M$-APPROX's negative errors are higher for $p_e = 0.4$ than for $p_e = 0.1$. As $m$ changes from 2 to 10, $M$-APPROX’s negative errors are reduced significantly. For high $p_e$ (e.g. $0.4$) negative errors are dominant due to $M$-APPROX, as shown in Fig. 3. $p_f$ is not much affected by P-APPROX and $F$-APPROX, even for high $p_e$ and $n$.

For low $p_e$ (e.g. $= 0.1$) errors change from positive to negative as $n$ increases. For small $n$ $F$-APPROX’s positive errors become dominant.

Fig. 5 shows that $p_e$-Model accurately predicts $S$ also, comparing $S$ in (20) (lines in Fig. 5) with the simulation results (points in Fig. 5). In (20) errors on $p_f$ introduce the errors on $S$ in the opposite sign, i.e. positive $p_f$ errors incur negative $S$ errors. Fig. 3 and Fig. 5 show that the $p_f$ errors are negatively reflected in the $S$ errors.

B. Numerical Results for Variability Analysis

To validate Claim 6 we run the simulator with $W = 32$ and $m = 5$, which are the typical setup specified in the standard [1]. As shown in Fig. 7, change of $p_e$ from 0.1 to 0.3 and that from 0.3 to 0.5 have the same effect on $S$ as adding 38 and 66 stations.
Transmission Failure Probability $p_f$

Number of stations $n$

For the same $\Delta p$, $\Delta n > 0.5 W \Delta p_f$

Fig. 6. Variability results for transmission failure probability: increasing $p_e$ by $\Delta p_e$ has more effect on $p_f$, than adding $0.5W \Delta p_e$ stations.

For the same $\Delta S$, $\Delta n > 0.5 W \Delta p_e$

Fig. 7. Variability results for saturation throughput: increasing $p_e$ by $\Delta p_e$ has more effect on $S$, than adding $0.5W \Delta p_e$ stations.

respectively. 38 and 66 both are greater than 4 ($\approx 0.5W \Delta p_e = 0.5 \times 32 \times 0.2$), thus Claim 6 is validated.

VII. CONCLUSION

In this work we have extensively studied the effect of packet errors on capacity and variability of the 802.11 MAC protocol. We develop $p_e$-Model and successfully model transmission failure probability $p_f$ and saturation throughput $S$ in terms of packet error probability $p_e$ and $q = (n-1)/W$.

Furthermore, introduction of $p_e$ in the model enables us to make variability analysis on the effect of $p_e$ on the performance metrics, such as $p_f$ and $S$.

Numerical results show that our model can accurately predict capacity and variability of the real-world wireless LAN, where packet errors are common due to non-ideal channel condition and device variability.

REFERENCES


