

CONCEPTUALIZING TEACHERS' KNOWLEDGE OF STUDENTS'  
MATHEMATICS IDENTITY FORMATION AND DEVELOPMENT

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## Introduction

*In my third year of teaching mathematics at Edgewood<sup>1</sup> Middle School, I taught four preparations each day – a remedial class for eighth graders titled “Math 8”, a Pre-Algebra class, a “general” Algebra class, and an “advanced” Algebra class for students identified as the most mathematically able and capable. The school was located in an affluent, mostly White community on the outskirts of Atlanta, yet, due to a long-running school district desegregation effort, Edgewood’s racial composition was highly diverse, with approximately 50 percent of its students identifying as African American or Latino. This racial composition, however, came with a hidden price - most of Edgewood’s African American students lived more than twenty miles south of the school and rode busses for upwards of an hour and a half every morning and afternoon. The school day started at 7:50 a.m., meaning many of Edgewood’s African American students stood at their bus stops before six in the morning. As the busses unloaded each morning, it was not uncommon to witness African American students emerge wrapped in blankets and holding pillows – evidence of their need to finish off their morning sleep during the bus ride.*

*Although Edgewood’s demographics would suggest that students from different racial and class backgrounds learned together, this was not necessarily the case. Edgewood students were strictly grouped by ability (as reflected in my responsibility to teach four different eighth grade mathematics courses) and, due to racial gaps in mathematics performance, students were consequently grouped by race. As a result, throughout my day, the racial mix of my mathematics classes shifted from overwhelmingly White in my advanced Algebra class to exclusively African American and Latino in my Math 8 class.*

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<sup>1</sup> Edgewood is a pseudonym

*Mya<sup>2</sup>, a bright-eyed, always-smiling African American girl in my general Algebra class took the long ride from the southern part of the county to Edgewood's doors every morning. A fourth of the general Algebra class was from the southern part of the county, and Mya was clearly the social and intellectual leader of the group. She was unquestionably the most engaged, inquisitive, and mathematically confident student in the entire class, and she consistently outperformed her classmates on tasks and assessments. She thrived in her position as one who I could call on to assist struggling students. Her ability to communicate her mathematical thinking and problem solving approaches in front of the entire class was unmatched. Mya fared less well in other academic areas, but it was evident to everyone, including herself, that she was comfortable and in her element when interacting in the general Algebra class.*

*Yet, despite her love for mathematics and her outstanding performance, I could not help but feel as though something was wrong. How did a child this mathematically talented get scheduled into my general Algebra class? Why had she not been enrolled in the advanced Algebra class, a class more aligned to her mathematical ability? From my perspective, a placement error had occurred. Somehow, somewhere, a mistake had been made. It was my belief that an injustice was unfolding before my eyes, that, if left unchecked, could result in an extremely mathematically confident and able child tracked into a mathematics course taking path that may limit her educational and professional opportunities in the near and distant future. As an African American mathematics learner and teacher, I was well aware of the critical role knowledgeable adults play in positioning students - particularly students of color - for academic*

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<sup>2</sup> Mya is a pseudonym

*success, and the ways students with little advocacy in their lives miss seemingly minor opportunities that have a major impact down their educational road. I felt that it was my professional and personal duty to redirect Mya's path by reorganizing her schedule so that she could enroll in my advanced Algebra class. The semester was still young and it was my belief that she could quickly get caught up on any material she had missed. I contacted Mya's mother and she was supportive of the move, although she confided that she could provide little help for her daughter; there were no other adults in the home and she could not think of anyone off hand that might be able to support Mya with homework. I assured her that I would provide Mya additional support if she needed it. I prepared her schedule change and held Mya after class to tell her of our decision.*

*On hearing the news, Mya's face froze in a shocked, silent stare. In that moment, I knew that her mind's eyes were viewing images of herself in the advanced Algebra class, images that did not make sense to her. Tears formed in her eyes and she buried her face in her folded arms. She did not want to do it. She did not want to go. She did not know those people. She pleaded to stay in the general Algebra class. Although it crossed my mind that she might be a bit apprehensive, I did not anticipate the depth of emotion associated with her resistance. I recall feeling that I did not have a sense or understanding of from where her reaction emerged – she was an excellent student and clearly capable of managing the mathematical work in the advanced Algebra class.*

*Despite her objections, I felt the move was in her best interest and went forward with her schedule change. For the next several weeks, Mya sat in the advanced Algebra class with little resemblance to the energetic child I knew in the general Algebra class. She displayed little of her former confidence and pride in her mathematical ability. Despite my firm belief that she was*

*more than capable of performing at a high level in the class and my conviction that she would eventually “come around”, her struggles persisted. The quality of her work was inconsistent with her talents and capacity. Her effort was sporadic. She became a silent member of the class and began to fade into the margins. The mathematics class had become something else to her and she had become something else within its walls. I could not help but think that placing her in the advanced Algebra class with peers that she felt disconnected to and in an environment where she felt disempowered had somehow shifted her perception of herself as a competent mathematics learner.*

I have memories of many students, yet Mya stands out for two reasons. Primarily, to this day, I am intrigued with what Mya experienced when the learning context changed from one in which she felt competent and known by her peers, to one that she perceived herself as a foreigner. In her mind, who was she in the general Algebra class and who did she become in the advanced Algebra class? Secondly, I often wonder what I, as her mathematics teacher, should have noticed or done differently. As the primary facilitator of mathematics learning in her life at that time, a critical time in her emotional, social, and academic development, shouldn't I have possessed some professional knowledge of her experience (and the experiences of all mathematics learners) that could prepare me to better surmise the situation and facilitate the transition? From a professional practice standpoint, did I do the right thing? If so, did I do it right?

This essay seeks to explore these questions through considering current and potential conceptualizations of the knowledge base teachers of mathematics draw on to teach. This exploration has practical implications due to a host of simultaneous, interrelated narratives that are framing current mathematics and science education policy and teacher education initiatives.

The first narrative is the current perception and concern that U.S. students are consistently outperformed by numerous countries in international comparisons of mathematical knowledge (Glod, 2007; Organisation for Economic Co-operation and Development, 2009; National Center for Education Statistics, 2008), and that a future decline in U.S. students' performance has implications for U.S. innovation and the capacity of the U.S. to build a technologically-literate, globally-competitive workforce (National Research Council, 2009). The second narrative suggests that, if a technologically-literate workforce is to flourish in the U.S., considerable effort must be devoted to reversing historical trends in race and class achievement gaps in mathematics and science student performance, to increasing U.S. minority students' access to and performance in advanced mathematics and science courses, and to increasing access of U.S. minority citizens in mathematics, science, technology-based careers (National Academies of Sciences, 2010). Lastly, the third narrative suggests that, if any of this is to happen, the majority of the work rests on the skills and knowledge of current and future mathematics and science teachers. In his 2010 State of the Union Address, President Obama stated that the quality of U.S. K-12 mathematics and science education is a national priority, and, consequently, the U.S. must produce more knowledgeable, committed, and well-prepared mathematics and science teachers, and should do so quickly (The White House, 2010). The convergence of these narratives suggests that current and future teachers of mathematics will need to possess a knowledge base beyond what is typically considered the professional knowledge of mathematics teaching, namely a sophisticated understanding of the mathematics teachers are expected to teach and a firm understanding of how to teach it. If a large part of the work of mathematics teachers will be to support more students, particularly previously marginalized students, in seeing themselves as members of the mathematics learning community, this knowledge base will undoubtedly include

an understanding of the complexities of how and why some students participate in mathematical activity and thrive in mathematical contexts, while others do not.

In this essay, I explore the possibility that effective mathematics teachers in contemporary and future schooling contexts must tap into a distinctly different domain of teacher knowledge than previously identified in the research literature. This effort is accomplished through analysis and synthesis of two research foci that have garnered considerable interest in the mathematics education research community: 1) conceptualizing the distinct knowledge teachers of mathematics possess and use in their practice (Ball, Hill, & Bass, 2005; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005), and 2) conceptualizing and exploring students' mathematics identities (Anderson, 2007; Boaler, 1999; Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Martin, 2000, 2006; Nasir, 2007, Nasir & Hand, 2008). Work within each focus has produced important frameworks and constructs that highlight the ways students interact with mathematical content.

New directions of the first focus, the knowledge base that teachers of mathematics draw upon, acknowledges that the solution paths students choose or are drawn to, regardless of where any particular path might lead, are in many instances predictable. The extent to which a mathematics teacher can diagnose misconceptions and predict solution paths requires teachers to possess a sophisticated, multidimensional, integrated mathematical and pedagogical knowledge base. The second focus, students' mathematics identity formation and development, is concerned with how students come to orient themselves toward or away from engagement in mathematical activity due to a combination of perceptions of self as a mathematics learner and perceptions of how others (teachers, peers, parents) see the student in mathematical contexts. In this essay, I draw on these two bodies of literature in an effort to support the following claim: a focus on the

discussion and development of the *mathematical* knowledge teachers use in their practice (regardless of the extent to which this knowledge base is a reflection of teachers' understanding of students' mathematical cognition and problem solving patterns) in the absence of simultaneously considering knowledge of students' mathematics identity formation and development may limit our capacity to fully understand and influence mathematics teachers' "mathematical-pedagogical reasoning" (Hill, Ball, & Schilling, 2008) and identify the full set of intellectual resources teachers draw on in their efforts to engage students in meaningful mathematical activity. The claim has been stated in various forms (see Martin, 2007), however there has been little conceptual or empirical work focused on exploring, supporting, or refuting the claim. It is my hope that this essay will serve as a contribution to seminal efforts to better articulate and conceptualize a domain of teacher knowledge - teachers' knowledge of students' mathematics identity formation and development - that may serve as an indispensable and powerful resource future teachers will need to draw on to teach mathematics effectively to all students.

### The Knowledge Teachers Use to Teach Mathematics

An emerging view of teacher knowledge acknowledges its complexity and special character (Shulman, 1986). In recent years, considerable attention in the mathematics education community has focused on conceptualizing and measuring the unique knowledge base teachers of mathematics draw on to plan and facilitate mathematical tasks (Ball & Bass, 2003; Lampert, 2001). This view maintains the necessity that teachers know the mathematics they are teaching well, yet contends that the mastery of "common content knowledge" (Ball, Thames, & Phelps, 2008) is not sufficient for the work of effectively teaching mathematics to students. Although frameworks designed to organize and describe the knowledge that mathematics teachers use in



their practice vary (Campbell et al., 2011; Hill, Ball, & Schilling, 2008), there appears to be general consensus that the demands of mathematics teaching, particularly those related to articulating multiple representations of mathematical concepts (e.g., algebraic, graphical, tabular representations of the specific mathematical concepts) require that teachers possess an understanding of seemingly simple mathematics concepts and structures in sophisticated and flexible ways. A central component of this knowing revolves around a teacher's capacity to predict and interpret students' engagement in mathematical activity – the intersection of knowing specific mathematics concepts well while simultaneously knowing how students engage in tasks and activities involving those same mathematics concepts.

In an effort to bring structure and definition to the teacher knowledge domain conceptualized as the intersection of knowledge of mathematics and knowledge of the ways students interact with mathematics - titled “knowledge of content and students” (KCS) – Ball, Thames, and Phelps (2008) state:

KCS is knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing. When choosing an example, *teachers need to predict what students will find interesting and motivating*. When assigning a task, *teachers need to anticipate what students are likely to do with it and whether they will find it easy or hard*. Teachers must also be able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language. Each of these tasks requires an interaction between specific mathematical understanding and *familiarity with students* and their

mathematical thinking.<sup>3</sup> (p. 211)

In reading this statement, one can conclude that this knowledge domain has substantive mathematical entailments, yet departs from traditional notions of the mathematical knowledge teachers use to teach. However, although distinctly recognizable as a domain, particularly to those who have taught mathematics in K – 12 settings, an articulation of the structure of the domain itself remains incomplete. As teachers choose to facilitate a particular task or respond to a student’s solution strategy, it is not clear to what extent the resources they draw on are based on what teachers generally know about students’ misconceptions (knowledge that could be gained through reading an article in a professional journal, enrolling in a course related to students’ mathematical misconceptions, or recognizing patterns of students’ problem solving approaches after repeated interactions with students over years of teaching) or what they know through their ongoing interactions with their students in mathematical contexts, interactions that encourage particular pedagogical responses that are based on knowledge of social processes in the mathematics classroom and students’ perceptions of self as a mathematics learner.

For example, a teacher responsible for teaching a lesson on integer subtraction (and generally knowledgeable of students’ solution patterns when solving integer subtraction problems) may make distinctly different pedagogical decisions when faced with presenting the material to a large group of ninth graders identified as “low-tracked” than when faced with presenting the material to a group of “gifted” fourth graders. The important distinction between the two groups is that the older group most likely has been exposed to an integer subtraction unit or lesson every year for the past four years and are unlikely to be exuberant about repeating the lesson, regardless of their lack of success during the previous encounter. Furthermore, the older

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<sup>3</sup> Emphasis added

group, through reinforcements of perceptions of self as “slow mathematics learners” by tracking structures, peers, and previous teachers, may implicitly (or in some cases, explicitly) demand that their teacher engage in particular mathematics practices that meet their particular needs. In such instances, mathematics teachers must have a sense of what resonates with groups of students based on those students’ prior experiences in mathematical environments. In short, the knowledge base that teachers draw on to teach mathematics may include knowledge of contexts and students’ experiences that may encourage teachers to choose from a wide range of pedagogical approaches to teach a particular concept or facilitate a task. There is little debate that the ways teachers know and teach mathematics relies heavily on the ways students come in contact with mathematics, however it has been long established that student engagement in mathematical activity is not solely an unmediated, internal, cognitive process; a host of multilevel factors are at work as students perform (or underperform) in mathematical contexts (Martin, 2000).

Those involved in defining and characterizing KCS as a distinct knowledge domain, acknowledge challenges in establishing its definitive form and structure. Hill, Ball, & Schilling (2008) state:

The very notion of ‘knowledge of content and students’ as knowledge needs further development. Teachers ‘know’ that students often make certain errors in particular areas, or that some topics are likely to be difficult, or that some representations often work well. But teachers also reason about students’ mathematics: They see students work, hear students’ statements, and see students solving problems. Teachers must puzzle about what students are doing and thinking, using their own knowledge of the topic and *insights about students* [emphasis added]. Some teachers may do this with more skill and

precision than others, but we lack theory that would help us examine the nature of teachers' mathematical-pedagogical reasoning about students. Some teachers may 'know' things about students in particular domains, whereas others may be figuring out students' thinking as they teach. (p. 396)

In that current conceptualizations of the knowledge mathematics teachers use in practice has room for development, additional perspectives on the ways students interact with mathematics may be important referents. Namely, our current understandings of how students come to see themselves as mathematics learners in tandem with how others see them in mathematical contexts, often described as "students' mathematics identity formation and development", may provide insights and help us better understand and further extend conceptualizations of teacher knowledge domains. The following section summarizes current thinking on the role of mathematics identity in mathematics performance and learning.

### Identity

Identity literature often describes identity development as the process of becoming a member of a community of practice (Wenger, 1999). Communities come in many forms (ethnic, racial, familial, geographical, classroom), yet all communities of practice possess general characteristics, including mutual engagement of participants; a shared repertoire of actions, discourses, and stories; and a joint enterprise that is the "negotiated response to their situation" (Wenger, 1999, p.77). The relationships between identity development, learning, and schooling are obvious; students come to know themselves as members of particular subgroups in schooling situations (gifted, slow, challenged, brilliant) through interactions with teachers, peers, and parents. Furthermore, the ways students see themselves as academically inclined people are influenced by the norms of additional communities of which they are members, including those

defined by race, class, gender, and peer groupings. The link between identity development and learning is described by Nasir (2007):

The development of identity, or the process of identification, is linked to learning, in that learning is about becoming as well as knowing. It is my view that this issue of how learning settings afford ways of becoming or not becoming something or someone is central to understanding culture, race, and learning, particularly given the multiple ways that race (as well as social class) can influence both the kinds of practices within which one can “become,” as well as the trajectories available in those practices. (p. 135)

The growing interest in incorporating identity constructs in mathematics education literature is motivated by the opportunities these constructs provide for analyzing and explaining affective aspects of mathematics learning central to student mediation of the teaching and learning process; aspects such as students’ motivation, engagement, interest, anxiety, and participation (Cobb & Hodge, 2007). Furthermore, identity development constructs are useful tools when attempting to understand and articulate what coming to know mathematics is and means for marginalized groups of U.S. students (Gutierrez, 2008).

### *Mathematics Identity Frameworks*

It is theorized that successful mathematics students have well-developed “mathematical identities”<sup>4</sup> (Anderson, 2007; Martin, 2000, 2007). Anderson (2007) states:

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<sup>4</sup> It should be noted that there are multiple perspectives on the form, structure, and development of an individual’s identity; perspectives that range from defining identity as a collection of self-perceptions, to more complex frameworks that define identity as community membership, as practice, as performance, and as participation (Erikson, 1968; Gee, 2000; Wenger, 1999). There are that might argue that one cannot possess a *mathematics* identity; we all possess and experience identification, of which participation in mathematical contexts is a part. Despite these potential and possible objections, I choose to push based on the multidimensionality of the frameworks developed, and the considerations of students’ self-perceptions, perceptions by others, and practices that existing mathematics identity frameworks employ.

Our identity—who we are—is formed in relationships with others, extending from the past and stretching into the future...As students move through school, they come to learn who they are as mathematics learners through their experiences in mathematics classrooms; in interactions with teachers, parents, and peers; and in relation to their anticipated futures. (p. 31)

In the school setting, therefore, learning mathematics is far more complex than coming to know mathematics concepts that were once unknown. Mathematics learning is not the process of simply remembering and forgetting mathematics concepts and procedures (Lampert, 2001). Learning mathematics is doing what mathematics learners do (Lampert, 2001), being treated the way they are treated, and forming the community they form. Mathematics learning, therefore, is inextricably bound to their practices as mathematics learners (Boaler, 1999; Boaler & Greeno, 2000), the practices of other mathematics learners that surround them, their understanding of what it means to be a mathematics learner, and how this status is perceived by others.

From an identity development perspective, mathematics instruction consists of both socializing students into the norms and discourse practices of the mathematics classroom (Yackel & Cobb, 1996), and influencing students' perceptions of themselves as members of a community of mathematics learners (Boaler, 1999; Boaler & Greeno, 2000). This perspective suggests that the resources teachers draw on to teach mathematics may include an understanding of students' mathematics identity formation and development. Although considerable work has been done to document students' perceptions and beliefs related to mathematics, mathematics teaching, and mathematics learning (Op't Eynde, Corte, & Verschaffel, 2003), to date only three frameworks

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have been developed that organize mathematics identity into a cohesive set of dimensions or features. The three frameworks are Martin's four dimensions of mathematics identity (2000), Anderson's four 'faces' of mathematics identity (2007), and Cobb, Gresalfi, and Hodge's interpretive scheme (2009). A synthesis of these three frameworks suggests that a student's mathematics identity is, in part, perceptions based, consisting of a combination of perceptions by others and perceptions of self related to a set of interdependent features (Figure 1). These features include:

1. Students' perceptions of their mathematics ability and the ways these perceptions of students' mathematics ability influence students' mathematics experiences
2. Students' perceptions of the importance of mathematics inside and beyond their current experiences in the mathematics classroom
3. Students' perceptions of the engagement in and exposure to particular forms of mathematical activity, and the ways these engagements influence students seeing themselves as mathematics learners
4. Motivations a student possesses in his or her efforts to perform at a high level and attributions to their success or failure in mathematical contexts

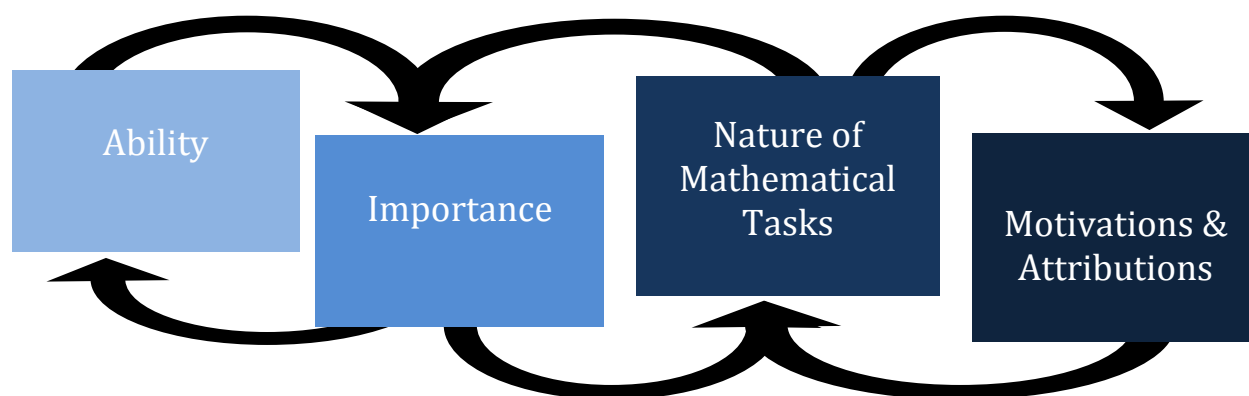


Figure 1. Four features of students' mathematics identity.

*Ability.* Of all academic disciplines, it could be argued that mathematics possesses the most socially-accepted, well-defined scale of ability on which most students have a very strong perception of where they sit. Adults in general, and students in particular, can articulate very clearly the extent to which they see themselves as either good or bad at math (Boaler, 2000). Students' perceptions of their mathematical ability have been shown to influence students' performance expectations in the mathematics classroom, and linkages between student perceptions of their ability, performance, and achievement are an important assumption in theories of learning and academic achievement (Meece, Wigfield, & Eccles, 1990). Furthermore, it has been shown that subgroups of students (e.g., low-performing vs. high-performing) often overestimate or underestimate their mathematics ability in comparison to their actual performance – a process identified as “calibration” (Chiu & Klassen, 2010; Pajares, 1996). The study of linkages between students' perceptions of their ability and performance, however, would be incomplete without consideration of the role of sociocultural factors, and the ways perceptions of ability influences students' engagement in mathematical contexts. Students receive explicit and implicit messages from parents, community members, peers, teachers that shape their perception of their ability to perform mathematically (Martin, 2007). Furthermore, students may also perceive their ability as genetically or biologically determined (e.g., possessing the “math gene”) (Anderson, 2007), and, in turn, a stable, immutable aspect of self. Cobb, Gresfaldi, and Hodge's (2009) interpretive scheme considers the ways mathematics students are constantly managing the distance between their perception of their own mathematical ability and the ability of the normative, mathematically-proficient student. The extent to which a student sees himself or herself as able and capable in the mathematics



classroom potentially has important implications for the design and facilitation of mathematics learning environments.

*Importance.* Students' hold perceptions of the importance of mathematics as an academic discipline and useful tool in their daily practices. In and out of school environments, the general perception of mathematics as important is often due to its practical utility, as an agent of access to a wide range of professions, and its cultural value (Gowers, 2002). Students, therefore, may orient themselves towards or away from mathematics depending on the extent to which they see mathematics as a useful, important tool in their current or future lives. Anderson (2007) contends that students engage in a process of "imagination" and "alignment" when forming perceptions of the importance of mathematics. Imagination refers to the extent to which students view mathematics as fitting into their broader lives. Students may see the completion of rigorous mathematics courses as necessary for performing well on the upcoming state assessment, as necessary for graduation, college, as relevant to their present or future workplace, or as important for the purposes of solving problems that occur in their present or future lives. Students that have a limited imagination related to the role mathematics plays in their broader and future lives may display limited effort in mathematics classes yet apply themselves at a high level and display considerable intellectual engagement in other courses and content.

Alignment refers to the extent to which students respond to their understanding of the ways mathematics fits into their broader lives; the extent to which students' respond to the imagination face. By following requirements and participating in the required activities that are aligned to the ways they see mathematics fitting into their broader lives, students come to see themselves as certain "types of people" (Gee, 2000). For example a "college-intending" student may take math classes required for admission to college or join a mathematics club because she

feels that her involvement will better prepare her for the SAT. Students that do not perceive themselves as college intending or are not members of a peer group that values participation in the math club may forego such activities. Acting on the ways they imagine mathematics as a part of their lives – alignment – communicates to self and others the importance of mathematics and the extent to which one sees self as a mathematics learner.

*Nature of Mathematical Tasks.* Students construct a sense of who they are as mathematics learners in relation to the nature and type of mathematical activity they are engaged in. Students may see themselves as central or peripheral to the broader community of mathematics learners depending on what mathematical activities they are exposed to, or, more broadly, what types of mathematics courses they have historically or are currently enrolled in. For example, if a high school student associates engaging in particular mathematical tasks such as constructing proofs or enrolling in specific courses such as Advanced Placement Calculus with the practices of proficient mathematics learners, yet does not engage in these practices or enroll in these courses, he or she may, in turn, not see himself as a mathematics learner, regardless of the performance level on the tasks he engages in within their current or assigned mathematics course. Students hold perceptions of what is possible or challenging for them in mathematical contexts, and construct visions of self in relation to those opportunities and constraints (Martin, 2000). Mathematical contexts (and the specific activities within those contexts), therefore, suggest to students what types of mathematics learners they are or are perceived to be.

*Motivations and Attributions.* Students are motivated to engage in mathematical activity or disengage from mathematical activity due to a host of factors. Studies of student motivation in mathematical contexts have identified intrinsic (deep interest, joy, or pleasure in engaging in mathematics) and extrinsic (grades, approval by others, avoidance of punishment) motivators

that appear to influence student performance (Middleton & Spanias, 1999). Furthermore, studies have identified factors to which students attribute their mathematical success or failure, including ability, effort, task difficulty, luck, and classroom context (Yailagh, Lloyd, & Walsh, 2009). Students' motivations to engage and their attributions for their performance is a complicated system, however there is consistent evidence that students' motivations and attributions develop early, are stable over time, and are influenced by teacher actions and attitudes (Middleton & Spanias, 1999). Lastly, it is important to note that decades of "analyses of school achievement, course-taking patterns, and standardized-test data reveal prevalent patterns of inequity in students' access to significant mathematical ideas" (Nasir & Cobb, 2007, p.1). As a result, it is well evidenced that U.S. minority students, language-minority students, poor students, and, to some extent, girls underperform mathematically relative to U.S. middle class White students. Yet what is unclear are the ways patterns of performance, success, and failure between subgroups are shaped, in part, by subgroup collective motivations and attributions.

#### Teacher's Knowledge of Student's Mathematics Identity Formation and Development:

##### A Framework for Structure and Use

If then, student's mathematics identity has particular interdependent features as described above, how might we then attempt to describe a domain of teacher knowledge, of which student's mathematics identity formation and development is the primary content? In other words, how might teachers know about student's mathematics identity and how might this knowledge be organized? These questions are not trivial; as stated, identity is multidimensional, dynamic, and malleable. Therefore, it is important to ask, 'What can teachers know of student's mathematics identity, and for what purposes?' A first pass at articulating a comprehensive definition of teachers' knowledge of mathematics identity is as follows: knowledge and

understanding of the ways students' self perceptions and perceptions by others along the four features of mathematics identity influence students' mathematical participation, performance, and achievement. Admittedly, this definition demands considerable elaboration and specification before it can pass epistemological tests and enter as a construct into teacher knowledge discourse, however, it serves as a starting point. The elaboration and specification of this domain of teacher knowledge is a task of future work, however of immediate importance is identifying an approach to consider and organize students' individual mathematical experiences, students' collective or group experiences, and more general or universal experiences that may influence students' mathematics identity formation and development.

*Student's Mathematics Identity or Students' Mathematics Identity?*

Identity has consistently been defined as a unique, individual experience shaped by multitude of forces and factors. In that identity is often defined in terms of individuals, it would seem that the only way a teacher can know or understand their students' mathematics identities is at the individual level. It would seem plausible that if a teacher had a grasp of the ways students' perception of self and others' perceptions influenced students' mathematical performance, this could only be achieved through teacher's awareness of the influence of these perceptions for each and every individual student they come in contact with. Furthermore, identity is by definition idiosyncratic –individuals 'become' differently, even if the conditions within which they grow and exist appear similar. Yet, due primarily to school structures and student groupings, teachers often ascribe the characteristics of individuals to groups of students (i.e., "last year's class was more focused than this year's class"). In keeping with these observation, the ways in which teachers know and understand dimensions of students' mathematics identity may occur at three levels, or along a continuum that possesses three discernable spaces: the

individual, the collective, and the universal (Figure 2).

The “individual level” refers to the extent to which teachers are aware of their current students’ mathematical dispositions. New directions in mathematics education research suggest that a student’s productive disposition towards learning mathematics is an important component in his or her overall mathematical proficiency. A productive mathematical disposition is defined as, “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (Kilpatrick, Swafford, & Findell, 2001, p.31). There is growing evidence that suggests that teachers play an important role in influencing the development of their students’ dispositions towards learning mathematics (Walshaw & Anthony, 2008). It seems plausible, therefore, that this influence is in part based on the extent to which teachers possess a level of awareness of the mathematical dispositions of their individual students. Examples of this awareness are the extent to which a teacher has a good sense of whether or not his students (or the majority of his students) see themselves as “good” or “bad” at mathematics, or the extent to which a teacher has a good sense of why her students think math is important or unimportant in their current or future lives. There is emerging empirical evidence that suggests that teachers’ awareness of their students’ mathematical disposition varies and is positively associated with their students’ mathematics achievement (Campbell et al., 2001). It seems plausible, therefore, to consider the ways this awareness influences the mathematical tasks teachers choose for their class and the pedagogical practices teachers engage in.

As stated previously, it is no secret that subgroups of students in U.S. mathematics classrooms experience mathematics differently. Researchers have documented the extent to which the quality and nature of students’ mathematical experiences are in part a function of

students' group membership and affiliation, such as gender (Catsambis, 1994; Gallagher & Kaufman, 2005; Nagy et al., 2010), socioeconomic status (McGraw, Lubienski, Strutchens, 2006), race/ethnicity (Ladson Billings, 1997; Martin, 2009, Tate, 1994), and ability group (Karsenty, Arcavi, & Hadas, 2007; Slavin & Karweit, 1985). The "collective level", therefore, refers to teachers' knowledge and understanding of the experiences of particular groups of students in mathematical contexts. The collective level also refers to teachers' awareness and understanding of developmental trajectories of students' mathematics identity and formation throughout students' school careers. A student may hold a perception of self as a competent, successful mathematics learner at one point in their developmental trajectory, yet hold a distinctly different self image, an image incongruent with that of an idealized mathematics learner, at another point in their trajectory. For example, there is evidence that students experience dilemmas when shifting from elementary mathematics to algebra (Kaput, Blanton, & Maren, 2008), and when shifting from algebra, geometry, and trigonometry courses to calculus (Ferrini-Mundy & Lauten, 1993). Both shifts demand that students engage in abstraction in ways they have not in prior mathematics courses. Upon contact with algebra, students must learn to reason through complex symbolization processes, whereas calculus demands that students have a firm understanding of the nature of functions, limits, and continuity. At both points, students' perceptions of self as mathematics learner may experience shifts ("In fourth grade I was good mathematics student, I loved math, but now that I'm in eighth grade I am not good anymore"). Students' perception of self as a learner of mathematics, therefore, may be on a developmental trajectory that has spikes and dips at mathematics content-specific points. A consequence of this possibility is students' perceptions of self as a mathematics learner is distinct from students' more general perceptions of self as academically inclined being. A teachers' knowledge of the

ways students hold congruent and incongruent images of themselves (personal identity) and that of the successful mathematics doer/learner (normative identity) (Cobb, Gresalfi, & Hodge, 2009) along their developmental trajectory could prove to be a useful resource to draw on in their practice. In my interactions with Mya, I believe it would have been helpful if I had known more about the collective perceptions students held – such as the perceptions the general Algebra students held of the advanced Algebra students, the perceptions Mya’s peer group held of the mathematics tracking and ability grouping structures, and Mya’s own mathematics learning trajectory prior to coming into my class.

The “universal level” consists of teachers’ broader understanding of the ways, theoretically, features of students’ mathematics identity are thought to influence students’ mathematical participation and performance. A considerable amount of theoretical work in this space, particularly theories related to perceptions of intelligence and ability (Mueller & Dweck, 1998), self-efficacy (Pajares & Graham, 1999) and motivation (Eccles & Wigfield, 2002; Stipek et al. 1998; Wigfield, 1994), are based on empirical work conducted in mathematical contexts. Yet these theories are rarely seriously considered in the training and preparation of mathematics teachers beyond brief treatments in adolescent development and student cognition and learning courses that typically group all teacher education candidates together, regardless of discipline. In that teachers develop or draw on lay versions of these theories in their perceptions of student engagement and participation, it is reasonable to argue that teachers’ understandings of these broader theories might currently or potentially influence their pedagogical choices and practices. For example, I am convinced that my decision to change Mya’s schedule was in part based on my personal, lay theory of what happens when capable students enter learning environments that are perceived to be both more challenging and unfamiliar to them – a trajectory that is closely

aligned to legitimate peripheral participation (Lave & Wenger, 1991). I anticipated that Mya would observe in the beginning, adapt to the norms of the advanced Algebra classroom, slowly become a member of the class community, and excel.

In viewing teachers' knowledge of students mathematics identity at multiple levels, it is plausible to claim that teachers engage in reasoning that reflects meaningful, valuable knowledge at these three levels, such as "in my experience students' math ability definitely influences the kinds of math we can do in my class (universal), but this year the high level of discourse in my class helps my special education students see themselves as contributors to the class, versus a drag on the whole class (collective), although I still need to work with Jennifer (individual), she still seems resistant to let the class know what she doesn't know." Figure 2 illustrates levels of teachers' understanding of students' mathematics identity.

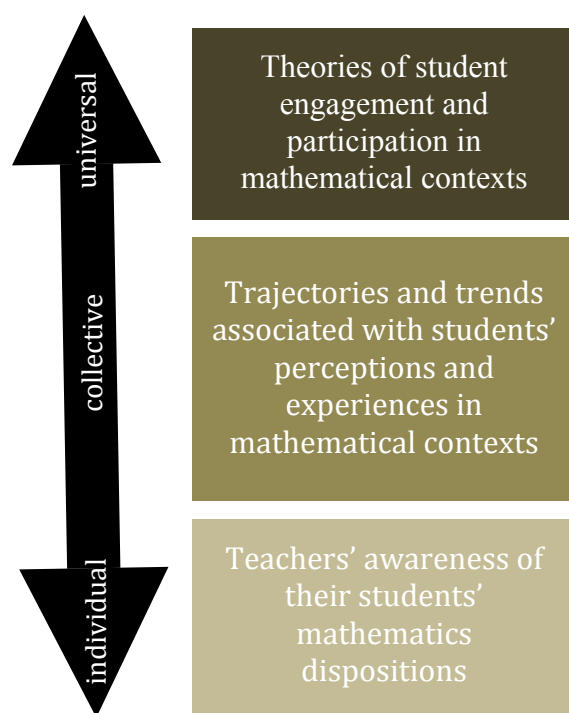


Figure 2. Levels of teachers' understanding of students' mathematics identity



*A Framework for Teachers' Knowledge of Students' Mathematics Identity Formation and Development*

Teacher knowledge of dimensions of students' mathematics identity, therefore, may be considered and organized at these levels. Furthermore, these levels may be crossed with features of students' mathematics identity, resulting in a comprehensive framework for teachers' knowledge of students' mathematics identity development and formation (Table 1). Admittedly, for some cells of this framework, there is sparse data and information currently available that can be used to adequately build this knowledge base in mathematics teachers. This framework, however can serve as a tool to organize what is currently known and to engage in research related to what is not known.

		<b>Features of students' mathematics identity</b>			
		<i>Ability</i>	<i>Importance</i>	<i>Nature of mathematical tasks</i>	<i>Motivation &amp; attributions</i>
<b>Levels of teachers' understanding of students' math identity</b>	<b>Theories of students' engagement and participation in mathematical contexts</b>	Theoretical role students' perception of their ability plays in their capacity to participate and perform in mathematical contexts	Theoretical role students' perception of the importance of mathematics plays in their capacity to participate, and perform in mathematical contexts	Theoretical role students' perception of the nature of the mathematical tasks at hand plays in their capacity to participate and perform in mathematical contexts	Motivation theory in mathematical contexts  Attribution theory in mathematical contexts
	<b>Trajectories and trends associated with students' perceptions and experiences in mathematical contexts</b>	Developmental trajectories of students' perceptions of their mathematics ability  Subgroup trends associated with students' perceptions of their mathematics ability	Developmental trajectories of students' perceptions of the importance of mathematics in their current and future lives  Subgroup trends associated with students' perceptions of the importance of mathematics in their current and future lives	Developmental trajectories of students' perceptions of the mathematical tasks at hand  Subgroup trends associated with students' perceptions of the nature of the mathematical tasks at hand	Developmental trajectories of students' motivations to participate and perform in mathematical contexts  Subgroup trends associated with students' motivations to engage or disengage in mathematics  Subgroup attribution trends
	<b>Teacher's awareness of their students' mathematics dispositions</b>	Teachers' awareness their students' perceptions of their mathematics ability	Teachers' awareness of their students' perceptions of the importance of mathematics in their current and future lives	Teachers' awareness of their students' perceptions of what the mathematics tasks they are engaged in communicate to themselves and others about who they are as mathematics learners	Teachers' awareness of what motivates their students to engage or disengage in the mathematical activity at hand  Teachers' awareness of their students' attributions

Table 1. Teachers' knowledge of students' mathematics identity development and formation framework

## Conclusion

*Mya's engagement with mathematics and her peers in the advanced Algebra class did not improve. Math class became something that she no longer looked forward to. Perhaps it was due to my inability to create an environment in which she felt empowered, confident, and included. Perhaps with deeper knowledge and further training, I could have done more or something different. In November of that year, two months after moving her into the advanced Algebra class, I moved Mya back to the general Algebra class.*

It is not uncommon for mathematics teacher candidates to feel that something is missing in their professional preparation and that their university experiences do not fully prepare them for their classroom duties. This lack of preparation is often located in their inability to manage complex social processes in the classroom, engage the disengaged student, and facilitate mathematical tasks in ways that promote high levels of participation, excitement, and interest. I conjecture that this sense of under preparation is partially due to the focus in mathematics teacher preparation on mathematics content courses, lesson planning, curriculum exploration, and assessment design. These foci are indisputably critical; teachers need to have a deep understanding and knowledge base in these areas. Sound, appropriate pedagogical decisions, however, are not always based on knowledge in these areas. At times, and perhaps more often than we collectively acknowledge, teachers' mathematical pedagogical reasoning is based on managing students' historical and current relationship with mathematics and relationships with other mathematics learners in the classroom.

There appears to be very little focus in mathematics teacher preparation on discussing trends in how students come to see themselves as mathematics learners through social and historical processes, and how these perceptions shape performance. It is timely that we

incorporate our understanding of how students come to see themselves and be seen as mathematics learners in mathematics teacher knowledge discourses, and through that, mathematics teacher education. If we focus solely on understanding and developing teachers' mathematics content or pedagogical content knowledge (and its underlying focus on students' mathematics cognition and solution strategies), we are not acknowledging a stubborn reality – for more students to more fully engage in mathematical activity they must have some sense of themselves as mathematics learners, have more confidence in their mathematics ability, view mathematics as important, and imagine themselves in future contexts where advanced mathematical knowledge is indispensable.

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