ABSTRACT

Title of dissertation: ESSAYS ON THE OPTIMAL LONG-RUN INFLATION RATE.

Salem Mosa Abo Zaid, Doctor of Philosophy, 2011

Dissertation directed by: Professor Sanjay Chugh
Department of Economics

Chapter 1: Optimal Long-Run Inflation with Occasionally-Binding Financial Constraints.
This paper studies the optimal inflation rate in a simple New Keynesian model with occasionally-binding collateral constraints that intermediate-good firms face on hiring labor. For empirically-relevant degrees of price rigidity, the optimal long-run annual inflation rate is in the range of half a percent to 2 percent, depending on whether it is TFP risk or markup risk or both that is the source of uncertainty in the economy. The shadow value on the collateral constraint is akin to an endogenous cost-push shock. Differently from usual cost-push shocks, however, this shock is asymmetric as it takes non-negative values only. Inflation is positive when the collateral constraint is binding and it is zero when it does not. Since the mean of this asymmetric endogenous cost-push shock is positive, inflation is also positive on average. In addition, a binding collateral constraint resembles a time-varying tax on labor, which the monetary authority can smooth by setting a positive inflation rate. More generally, the basic result is related to standard Ramsey theory in that optimal policy smoothes distortions over time.
Chapter 2: Optimal Monetary Policy and Downward Nominal Wage Rigidity in Frictional Labor Markets.

Empirical evidence suggests that nominal wages in the U.S. are downwardly rigid. This paper studies the optimal long-run inflation rate in a labor search and matching framework under the presence of Downward Nominal Wage Rigidity (DNWR). In this environment, optimal monetary policy targets a positive inflation rate; the annual long-run inflation rate for the U.S. is around 2 percent. Positive inflation “greases the wheels” of the labor market by facilitating real wage adjustments, and hence it eases job creation and prevents excessive increase in unemployment following recessionary shocks. These findings are related to standard Ramsey theory of “wedge smoothing”; by following a positive-inflation policy under sticky prices, the monetary authority manages to reduce the volatility and the size of the intertemporal distortion significantly. The intertemporal wedge is completely smoothed when prices are fully flexible. Since the optimal long-run inflation rate predicted by this study is considerably higher than in otherwise neoclassical labor markets, the nature of the labor market in which DNWR is studied can be relevant for policy recommendations.

Chapter 3: Sticky Wages, Incomplete Pass-Through and Inflation Targeting: What is the Right Index to Target?

This paper studies strict monetary policy rules in a small open economy with Inflation Targeting, incomplete pass-through and rigid nominal wages. The paper shows that, when nominal wages are fully flexible and pass-through is low to moderate, the monetary authority should target the Consumer Price Index (CPI) rather than the Domestic Price Index (DPI). When pass-through is high, an economy with high degrees of nominal wage rigidity and wage indexation should either target the CPI or fully stabilize nominal wages. These results suggest that, by committing to a common monetary policy in a common-currency area, some countries may not be following the right monetary policy rules.
ESSAYS ON THE OPTIMAL LONG-RUN INFLATION RATE

by

Salem Mosa Abo Zaid

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Advisory Committee: Professor Sanjay Chugh, Chair Professor John Shea Professor Enrique Mendoza Professor Irwin Morris Professor Pablo D’Erasmo
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CHAPTER 1

Optimal Long-Run Inflation with Occasionally-Binding Financial Constraints

1.1 INTRODUCTION
Recent economic events have revived interest in the optimal long-run inflation rate. This paper studies the optimal long-run inflation rate in a simple calibrated New Keynesian (NK) framework with occasionally-binding financial constraints. For empirically-plausible sizes of exogenous shocks, optimal monetary policy entails a strictly positive inflation rate in the long-run. In particular, the optimal annual long-run inflation rate in the benchmark calibration of the model is about 0.5 percent when the economy is only subject to TFP shocks and slightly above 1 percent when the economy is hit by only markup shocks. When the economy is subject to both shocks simultaneously, the optimal long-run inflation rate is about 2 percent annually. The main result of the paper, namely the optimality of a positive inflation rate, is robust to introducing a motive for holding money.

The baseline setup assumes three types of agents in the economy: households, entrepreneurs (or intermediate-good firms), and sticky-price firms that produce final goods. Financial frictions arise because hiring labor services by an entrepreneur is constrained by the level of her net worth. The collateral constraint is motivated by a type of the hold-up problem. Prior to supplying their labor services, households require the entrepreneur to show collateral that can be seized if needed.\footnote{The accumulation of net worth is via purchases of shares that are claims on the profits of final-good firms. These shares pay out the profits of final-good firms as dividends to shareholders.} The accumulation of net worth is via purchases of shares that are claims on the profits of final-good firms. This setup is similar to a model in which the entrepreneur borrows at the beginning of each period to pay wages ahead of production, and borrowing is constrained by collateral.

There are two main differences between this paper and typical papers that study optimal monetary policy within a New Keynesian framework featuring financial frictions. First, this paper assumes an occasionally-binding collateral constraint rather than always-binding collateral constraints as usually assumed in this literature. Second, this paper focuses on the optimal long-run inflation rate (i.e. the mean of the inflation rate in the “stochastic steady state” of the model), whereas the focus of most existing literature on monetary policy and...
financial frictions is on mainly on the short-run dynamics of inflation around the deterministic steady state.

The assumption of an occasionally-binding collateral constraint not only renders the environment more realistic, but it generates asymmetry in the behavior of the economy in response to favorable vs. adverse shocks. The computational approach that I use to deal with occasionally-binding constraints is a penalty-function algorithm within a second-order approximation. This approach has been extensively used recently (e.g. Kim, Kollmann and Kim, 2010; Den Haan and Ocaktan, 2009; De Wind, 2008 and Preston and Roca, 2007). A detailed description of this methodology can be found in Judd (1998).

When the collateral constraint binds, the shadow value of relaxing the collateral constraint is akin to a cost-push shock that generates inflation. The reason for that is straightforward: other things equal, a binding collateral constraint implies increases in the marginal costs of final-good firms which they accommodate by increasing prices. The inflation rate is positive on average due to the nature of this endogenous cost-push shock; it is asymmetric as it takes only non-negative values. In periods with a binding collateral constraint, inflation is positive. In periods with a non-binding collateral constraint, inflation is zero. Hence, inflation is positive on average since the shadow value on the collateral constraint is positive on average.

The results of this paper also highlight the role of inflation in mitigating the impact of adverse shocks on the economy. A binding collateral constraint distorts the choice of labor by entrepreneurs, and thus it magnifies the wedge between the marginal rate of substitution between labor and consumption and the marginal product of labor (which exists due to the monopolistic power of final-good firms). This implies a deviation from the first-best level of output. The wedge (to which we refer as the “labor wedge”) resembles a labor-income tax, and it increases with the shadow value of relaxing the collateral constraint. The analyses show that, under optimal policy, the monetary authority counteracts the effects of a binding collateral constraint, and it thus smoothes the “tax rate” on labor. Since the collateral constraint is more likely to bind during downturns, monetary policy makers aim for, at least, avoiding excessive increase in the “tax rate” during such episodes.

The ability of the monetary authority to smooth the “labor-income tax” (and more generally, the “labor wedge”) is limited due to the monopolistic power of final-good firms and the price rigidity. Put differently, the monetary authority does not have enough instruments to completely and simultaneously close the three distortions in the economy- the nominal distortion due to price rigidity, the monopolistic power of final-good firms, and the financial
distortion. Policy makers choose to spread the distortions across margins. Spreading distortions across all margins is well-known in the literature (Dupor, 2002).

Recent work has suggested other factors that justify a positive inflation rate. Related to the current study, Antinolfi, Azariadis and Bullard (2010) point to the role of positive inflation in deepening asset markets and loosening debt contracts. Kim and Ruge-Murcia (2009) show, assuming a neoclassical labor environment, that the optimal long-run inflation rate is positive (around 0.4 percent annually) if nominal wages are downwardly rigid. Abo-Zaid (Chapter 2 of this document) reports a significantly higher optimal long-run inflation rate (around 2 percent annually) in a labor search and matching framework in the presence of downward nominal wage rigidity. Fagan and Messina (2009) suggest that the optimal inflation rate for the U.S. ranges between 2 percent and 5 percent when nominal wages are downwardly rigid. This paper contributes to the growing literature that study motives for setting positive long-run inflation rates.

The remainder of the paper proceeds as follows. Section 2 outlines the model economy with the collateral constraint and defines the private-sector equilibrium and the optimal monetary policy problem. Section 3 discusses the labor wedge and the role of inflation in smoothing this wedge. Section 4 describes the calibration and the solution methodology of the model. Section 5 presents the optimal long-run inflation rate suggested by this paper. Impulse responses and the frequency of hitting the collateral constraint limit are also presented in this section. Section 6 presents the results of robustness analyses and section 7 concludes.

1.2 THE MODEL ECONOMY

The model is a variation of the standard New Keynesian model, with the basic structure by which financial frictions are modeled similar to the recent work of Carlstrom, Fuerst and Paustian (2010, CFP hereafter). The economy is populated by households, entrepreneurs that produce intermediate goods (in what follows, I refer to this sector as entrepreneurs and intermediate-good firms interchangeably), and final-good firms. Households consume differentiated final goods and supply labor on spot markets. Entrepreneurs hire labor services to produce homogenous intermediate goods. Entrepreneurs’ labor demand is constrained by the accumulated value of their net worth. This constraint is the source of the financial friction. Final-good firms are monopolistic competitors that purchase intermediate goods from
entrepreneurs and costlessly produce final goods. The pricing of a final-good firm is subject to a direct resource cost, which is the source of price rigidity in this model.

1.2.1 Households

The representative household purchases the differentiated final goods and enjoy utility from a composite consumption index \( c_t \) and supplies labor \( l_t \) in each period \( t \). Households have access to two financial instruments. The first is a standard one-period bond that pays a riskless nominal gross interest rate of \( R_t \). These bonds are in zero net supply, and, as in CFP, they make explicit pricing the nominal interest rate. In period \( t \), households also purchase \( s_t \) shares of final-good firms at a nominal per-share price of \( Q_t \). Total shares pay nominal dividends of \( D_t \), and their market supply is normalized to unity.

Households maximize their expected discounted lifetime utility given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(l_t)],
\]

where \( \beta < 1 \) is the standard subjective discount factor, \( E_0 \) is the expectation operator, \( u(c_t) \) is the period utility function from consumption and \( v(l_t) \) is the period disutility function from supplying labor. These functions satisfy the Inada conditions and the usual properties:

\[
\frac{\partial u(\cdot)}{\partial c} > 0, \quad \frac{\partial^2 u(\cdot)}{\partial c^2} < 0, \quad \frac{\partial v(\cdot)}{\partial l} > 0 \quad \text{and} \quad \frac{\partial^2 v(\cdot)}{\partial l^2} > 0.
\]

As standard in NK models, consumption \( (c_t) \) is a Dixit-Stiglitz aggregator of final goods \( (c_{jt}) \) produced by monopolistically-competitive firms:

\[
c_t = \left( \int_0^1 \frac{c_{jt}}{\varepsilon_t} \, dj \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}},
\]

where \( \varepsilon_t > 1 \) measures the elasticity of substitution between two varieties of final goods. The elasticity of substitution is allowed to be time-varying in order to allow for shocks to the desired markup, or, put differently, cost-push shocks. Other things equal, an increase in \( \varepsilon_t \) leads to a fall in the desired markup (the optimal ratio of price to the marginal cost), and
hence to less inflationary pressures in equilibrium. I allow for markup shocks both due to their familiarity in New Keynesian models and because they generate a tradeoff for the monetary authority between stabilizing inflation and stabilizing output. In some of the experiments in section 5, I consider constant elasticity of substitution, and the main results are not, qualitatively, sensitive to whether markups are stochastic or not.

Following standard derivations in Dixit-Stiglitz based NK models, the optimal allocation of expenditures on each variety is given by

\[ c_{jt} = \left( \frac{P_j}{P_t} \right)^{-\epsilon_j} c_t, \tag{3} \]

where \( P_t = \left( \int_0^t P_j^{1-\epsilon_j} \, dj \right)^{\frac{1}{1-\epsilon_j}} \) is the Dixit-Stiglitz price index that results from cost minimization.

Maximization is subject to the sequence of nominal budget constraints of the form:

\[ P_t c_t + Q_t s_t + B_t = R_{t-1} B_{t-1} + P_t (1 + \tau) w_t l_t + s_{t-1} (Q_t + D_t) + P_t \Pi_t + P_t T_t, \tag{4} \]

with \( c_t \) denoting consumption of the final good, \( P_t \) is the nominal price of the final good, \( w_t \) is the real wage, \( \tau \) is a labor market subsidy that is introduced to ensure the efficiency of the deterministic steady state (i.e. to achieve the first-best level of output; see Appendix 1E for details). Finally, \( T_t \) are real lump-sum transfers by the government, and \( \Pi_t \) are real profits from the ownership of firms.

The households’ budget constraint may be expressed in real terms as follows:

\[ c_t + q_s s_t + b_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + (1 + \tau) w_t l_t + s_{t-1} (q_t + d_t) + \Pi_t + T_t \tag{5} \]

where \( q_t = \frac{Q_t}{P_t} \) denotes the real price of shares, \( b_t = \frac{B_t}{P_t} \) is real bond holdings at the end of period \( t \), and \( d_t = \frac{D_t}{P_t} \) stands for real dividends.

The optimal choices of consumption, bonds, labor supply and shares of final-good firms yield the following optimization conditions (see Appendix 1A for derivations):

\[ \frac{v_{l,t}}{u_{c,t}} = (1 + \tau) w_t, \tag{6} \]
\[ u_{c,t} = \beta R E_t \left( \frac{u_{c,t+1}}{\pi_{t+1}} \right), \tag{7} \]

\[ u_{c,t} = \beta E_t \left[ u_{c,t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \right], \tag{8} \]

where \( u_{c,t} \) is the marginal utility of consumption in period \( t \), \( u_{l,t} \) is the marginal disutility of supplying labor in period \( t \), and \( \pi_t = \frac{P_t}{P_{t-1}} \) denotes the gross price inflation rate. Equation (6) is the standard labor-supply condition, and equation (7) is the standard consumption Euler equation. Equation (8) prices shares of final-good firms; it equates the period-\( t \) marginal utility of consumption to the expected utility of expanding future consumption through the gross one-period return on holding shares, given by \( \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \).

### 1.2.2 Entrepreneurs/Intermediate-Good Firms

There is a continuum of long-lived entrepreneurs, each of whom produces intermediate goods. An entrepreneur hires labor services on sport markets in order to produce a homogenous good using the linear production function,

\[ x_t = A_t l_t, \tag{9} \]

with \( A_t \) denoting total factor productivity, which is identical across all entrepreneurs.

Prior to supplying labor to an intermediate-good firm, households require that a part \( \alpha \) of their wages be backed up by collateral. This is the source of the financial friction in the model, about which more is discussed below. Given that a share of wage payments is collateralized, the intermediate-good firm then hires labor and starts production. Realized operating profits (revenues net of wage costs) and the beginning-of-the-period net worth can then be used to buy shares (\( e_t \)) for the next period. Positive operating profits are possible if the collateral constraint binds (see Appendix 1D for a proof).

The collateral constraint can be motivated by the hold-up problem, as in Kiyotaki and Moore (1997). Prior to supplying their labor services, households require some “guarantee” from the entrepreneur so that she does not force their wages down ex-post. In other words, the entrepreneur is required to back up the promised wage by some collateral that can be seized if
needed. Introducing the financial friction follows CFP and allows me to obtain the main results in a simple way.

Formally, hiring labor is constrained by the beginning-of-period net worth, as follows

$$e_{t-1} \leq \kappa n_t + d_t = \kappa n_t,$$

where, $e_{t-1}$ stands for the share-holdings by the entrepreneur at the beginning of period $t$, and $n_t$ is the real value of net worth. The maximum share of net worth that can be used as collateral is $\kappa$ (which is equivalent to the loan-to-value ratio in models with borrowing constraints). The parameter $\alpha$ measures the “significance” of the financial friction: the higher this parameter is, the more “significant” (or “severe”) the financial friction. Clearly, if $\alpha = 0$ then the model collapses to a standard new Keynesian model with no financial frictions.

As shown in Appendix 1J, this setup is isomorphic to a model in which part of wages is required to be paid in advance (“working capital”), the entrepreneur obtains intra-period loans to finance this part of wages, and borrowing is constrained by collateral. The parameters $\alpha$ and $\kappa$ come from two different constraints: $\alpha$ comes from the constraint that requires the collateralized wage payment to be lower than borrowing, and the parameter $\kappa$ comes from the constraint that limits borrowing. Therefore, I use two separate parameters in condition (10) rather than only one parameter that is equal to their ratio.

The most realistic setup, which is the main focus of this paper, is one in which the collateral constraint may only occasionally bind. For example, the constraint may not bind after a long series of positive shocks (Iacoviello, 2005). Assuming this constraint is always binding, as in CFP and other New Keynesian models with financial frictions do, imposes a restriction on the model’s dynamics. Also, even if the constraint always binds at the deterministic steady state and for small (positive) shocks, it does not necessarily bind for large shocks. Because large shocks are of course sometimes observed in reality, it is important to understand the model’s dynamics when constraints need not always bind.

To my knowledge, allowing the collateral constraint to only occasionally bind is an innovation compared to studies of monetary policy in the presence of financial frictions. Recent studies assume always-binding collateral constraints (e.g. Iacoviello, 2005; Monacelli, 2009 and Carlstrom, Fuerst and Paustian, 2010). Studying optimal monetary policy with occasionally-binding financial constraints can be viewed as another contribution of the paper. The way I computationally handle the occasionally-binding constraint is discussed in section 4.
I assume that any remaining resources (or “profits”) will be remitted to households in a lump-sum fashion, and that in the process of accumulating shares, entrepreneurs are more impatient than households. For this reason, they discount the future using a discount factor of \( \delta \), where \( \delta = \frac{\frac{1}{1 + \mu_{t+1}}}{u_{t+1}} \) and \( \delta < 1 \). The parameter \( \delta \) is needed to ensure that an entrepreneur will not accumulate enough assets so that the collateral constraint never binds.

Finally, as will be discussed in subsection 2.6, the assumption that entrepreneurs remit their “profits” to households simplifies the objective function of the monetary policy maker; the goal is only maximizing the lifetime utility of households. An entrepreneur thus chooses labor demand and shares to maximize expected present discounted value of profit payouts to households,

\[
E_0 \sum_{t=0}^{\infty} \delta^t \Xi_{0,t} \left[ p_t \lambda_t - w_t l_t + e_{t-1} (q_t + d_t) - e_t q_t \right],
\]

subject to the sequence of collateral constraints (10). The variable \( p_t \) denotes the relative price of the intermediate good in terms of the final good (and, in equilibrium, equals the marginal cost of final-good firms). The term in the square brackets is what I refer to as “profits,” and it corresponds, in equilibrium, to part of \( \Pi_t \) in the budget constraint of households.

Denoting the Lagrange multiplier on (10) by \( \lambda_t \), the optimal choices of labor and shares by an entrepreneur are characterized by (see Appendix 1B for details):

\[
A_t p_t = w_t (1 + \alpha \mu_t),
\]

\[
1 = \delta E_t \left[ \Xi_{t+1,t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) (1 + \kappa \mu_{t+1}) \right],
\]

where, as in CFP, the variable \( \alpha \mu_t \) can be interpreted as a “real interest rate” on a loan required for paying the wage bill of \( l \) in advance. Equation (12) states that, at the optimum, the marginal product of labor is equated to the real wage adjusted by a “financial markup” (i.e. the effective real wage from the viewpoint of the firm in the beginning of the period). Hence, if \( \alpha > 0 \), then labor demand will be distorted by the existence of the collateral constraint if it binds. Ex ante, the cost of hiring a unit of labor is higher the tighter the collateral constraint. \(^2\)

\(^2\) Condition (12) makes clear that profits are positive when the collateral constraint binds: under the optimal choice of the firm, the marginal product of labor exceeds the real wage.
Moreover, if $\alpha = 0$, this condition reads $p_t = \frac{w_t}{A_t}$, as is standard in NK models. Finally, equation (13) is a typical asset-pricing condition, but expanded to account for the imposition of the collateral constraint.\textsuperscript{3}

1.2.3 Final-Good Firms

Firms in this market are monopolistically competitive. A final-good firm $j$ purchases the homogenous intermediate goods from entrepreneurs at a relative price $p_t$ and transforms each unit of the intermediate good into a final good $y_j$ using a one-to-one technology.\textsuperscript{4} Each firm chooses its own price ($P_{jt}$) to maximize profits subject to the downward-sloping demand for its product (see Appendix 1G for more details). The pricing of a final-good firm is subject to a quadratic adjustment cost as in Rotemberg (1982), expressed in units of the final good:

$$\varphi \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 y_t,$$

where $\varphi$ is a parameter that governs the degree of rigidity. In a symmetric equilibrium, in which all firms set the same price, Rotemberg pricing leads to the following forward-looking Phillips curve:

$$1 - \varphi(\pi_t - 1)\pi_t + \beta E_t \left[ \left( \frac{u_{t+1}}{u_{t+1}} \right)(\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right] = \epsilon_t(1 - mc_t),$$

where, $mc_t$ is the marginal cost of the final-good firm, which equals $p_t$. As usual, because of the assumptions of one-to-one technology and zero fixed costs, the real marginal cost equals the real average cost. In the case of fully flexible prices ($\varphi = 0$) or fully stable prices ($\pi_t = 1$

\textsuperscript{3} Condition (13) is consistent with condition (8) because of the variations of the Lagrange multiplier on the collateral constraint and the additional discount factor ($\delta$). To fix ideas, consider the deterministic steady state versions of the two conditions. In this case, from (13) we get $1 = \delta(1 + \kappa t)$, which makes the two conditions consistent.

\textsuperscript{4} I assume two types of firms in the production sector since the “asset” in this model is shares of final-good firms. To avoid adding an asset (e.g. capital), and hence deviate from the linear-in-labor technology that is typically assumed in NK models, I assume two types of firms and introduce each friction in one sector.
for all \( t \), equation (15) collapses to the familiar condition, \( mc_t = \frac{e_t - 1}{e_t} \). Hence, in the absence of price adjustment costs, the real marginal cost equals the inverse of the optimal price markup.

By combining conditions (6) and (12) and using the fact that \( (mc_t = p_t) \), the Phillips curve can be written as

\[
(\pi_t - 1)\pi_t = \frac{1 - e_t}{\varphi} + \beta E_t \left[ \left( \frac{u_{t+1}}{\varepsilon_{t+1}} \right) (\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right] + \left[ \frac{\varepsilon_{y_{1,t}}}{\varphi(1+\tau)A_{u_{1,t}}} \right] + \left[ \frac{\alpha e_t v_{l,t}}{\varphi(1+\tau)A_{u_{1,t}}} \right] \mu_t, \tag{16}
\]

which explicitly shows the relationship between inflation and the financial friction (as measured by the multiplier \( \mu_t \)). This is a key equation since it directly links inflation and the (binding) collateral constraint. The left hand side of condition (16) is increasing in \( \mu_t \), which implies that, other things equal, an increase in \( \mu_t \) leads to an increase in inflation. In this regard, the Lagrange multiplier on the collateral constraint acts as an endogenous cost-push shock that generates inflation. Differently from typical cost-push shocks, however, the shock in this model is asymmetric as it may not be negative. This fact has implications for the average inflation rate: since the mean of the endogenous asymmetric cost-push shock is positive (positive when the collateral binds and zero when it does not), the average inflation rate is, accordingly, positive.

It is also worth noting that, other things equal, the impact of \( \mu_t \) on inflation is decreasing in the degree of price rigidity and increasing in the “degree” of the financial friction. With very high degrees of price rigidity, the channel introduced through the collateral constraint is expected to be dominated by the cost of deviating from zero inflation. Also, the elasticity of labor supply is another factor that determines the impact of the collateral constraint on inflation. In the limiting case when labor is inelastically supplied, the collateral constraint has no effects on inflation. This can be easily seen by setting \( v_{l,t} = 0 \) in condition (16). The intuition behind this result is straightforward: when the equilibrium quantity of labor is independent of the financial friction, there is no inefficiency to correct for. Therefore, zero inflation is optimal for each period \( t \) under TFP shocks: when the quantity of labor is efficient, output is efficient as well. Therefore, setting a zero inflation rate does not lead to inefficiencies in production and since zero inflation minimizes the resource costs of adjusting price, it is the optimal policy.
Finally, due to monopolistic competition, firms in this sector earn positive profits in equilibrium. These profits are paid in the form of dividends to shareholders. Real dividends are thus given by:

\[ d_t = y_t - mc_t y_t - \frac{\varphi}{2} (\pi_t - 1)^2 y_t. \]  

(17)

1.2.4 Market Clearing

In equilibrium, the resource constraint of the economy reads as follows:

\[ y_t = c_t + \frac{\varphi}{2} (\pi_t - 1)^2 y_t. \]  

(18)

Finally, market clearing for shares implies:

\[ e_t + s_t = 1. \]  

(19)

1.2.5 The Private Sector Equilibrium

**Definition 1**: Given the exogenous processes \( \{ R_t, A_t \} \), the private sector equilibrium is a state-contingent sequence of allocations \( \{ c_t, l_t, w_t, mc_t, \pi_t, q_t, d_t, e_t, \mu_t \} \) that satisfy the equilibrium conditions (6)-(8), (12)-(13), (15) and (17)-(18), and the complementary slackness condition \( \mu_t (\kappa e_{t-1} (q_t + d_t) - \alpha w_t l_t) = 0. \)

1.2.6 The Optimal Monetary Policy Problem

I use a Ramsey-type approach to study optimal monetary policy. The monetary authority chooses allocations to maximize the lifetime utility of households subject to the resource constraint and the private-sector equilibrium conditions.\(^5\) The monetary authority is also assumed to solve a commitment problem.

**Definition 2**: Given the exogenous process for technology \( A_t \), the monetary authority chooses a sequence of allocations \( \{ c_t, l_t, w_t, mc_t, \pi_t, q_t, d_t, e_t, \mu_t \} \) to maximize (1) subject to the conditions (6), (8), (12)-(13), (15) and (17)-(18).

\(^5\) The fact that “profits” of entrepreneurs are transferred to households simplifies the problem of the monetary policy maker; instead of maximizing some weighted average of the lifetime objective functions of households and entrepreneurs, the objective function of the monetary policy maker is only the lifetime utility of households.
1.3 OPTIMAL MONETARY POLICY AND THE LABOR WEDGE

This section presents an alternative way, related to basic Ramsey theory, to view the implications of the collateral constraint for optimal monetary policy. In the basic Ramsey theory, the aim of the planner is to smooth distortions (or “wedges”) over time. In this paper, a binding collateral constraint distorts the choice of labor by entrepreneurs and hence leads to suboptimal choice of labor. To see this, consider first the problem of the social planner who maximizes the expected present discounted utility of households subject to the goods-market resource constraint (see Appendix 1E for details). The condition characterizing the social planner’s problem is given by

\[
\frac{v_{l,t}}{u_{c,t}} = A_t, \tag{20}
\]

which states that the marginal rate of substitution between labor and consumption should be equal to the marginal product of labor. In the decentralized economy, the equivalent condition is given by:

\[
\frac{v_{l,t}}{u_{c,t}} = A_t \left[ \frac{(1 + \tau)mc_t}{1 + \alpha\mu_t} \right]. \tag{21}
\]

The “labor wedge” is given by the term in the brackets (more precisely, the wedge is the difference between 1 and this term). In this paper, the labor wedge is a function of the Lagrange multiplier on the collateral constraint and the monopoly power of monopolistically-competitive firms.

The role of positive inflation in smoothing the wedge can be seen by substituting for \(mc_t\) using the Phillips curve (condition 15) as follows:

\[
\frac{v_{l,t}}{u_{c,t}} = A_t \left[ \frac{(1 + \tau)\left( \frac{\varepsilon_t - 1}{\varepsilon_t} + \phi (\pi_t - 1)\pi_t - \beta qE_t \left( \frac{u_{x+1}}{u_{x+1}} (\pi_{x+1} - 1)\pi_{x+1} \frac{y_{x+1}}{y_t} \right) \right)}{1 + \alpha\mu_t} \right]. \tag{22}
\]

To fix ideas, assume that the economy is subject only to a TFP shock (i.e., \(\varepsilon_t\) is constant) and let the difference between 1 and the term in brackets be defined as “labor-income tax”. Under zero-inflation policy, the numerator of (22) is constant, but the denominator varies \(\mu_t\). If the monetary authority implements zero-inflation policy, then a negative shock that leads to
an increase in $\mu$ will also result in a higher “tax rate”. This increase in the “tax rate” can be alleviated by appropriate setting of the inflation rate. In this case, by setting a positive inflation rate, the monetary authority can decrease the “tax rate” and smooth its variation.

If $\varepsilon_t$ is allowed to be exogenously time-varying, setting a positive inflation rate has a similar role. Suppose that $\varepsilon_t$ falls (which implies a decrease in the degree of competitiveness in the final-good sector). Under zero-inflation policy, the numerator $\left(\frac{\varepsilon_t - 1}{\varepsilon_t}\right)$ decreases, but the denominator increases. Both effects lead to increases in the “labor-income tax” and thus require greater response by the monetary authority.

More generally, the aim of setting a positive inflation rate is to reduce and smooth the labor wedge, and thus to position the economy as close as possible to the efficient state. In this regard, optimal monetary policy in this paper is in line with basic Ramsey policy of smoothing distortions over the business cycle.

### 1.4 Computational Strategy and Calibration

The first subsection presents some discussion about the solution methodology applied in this study. Subsection 4.2 then discusses the parameterization of the model.

#### 1.4.1 Computational Strategy

Ideally, occasionally-binding constraints should be handled using global computational methods, but this comes at the expense of tractability. Hence, I resort to local methods in order to approximate the solution of the model. However, standard perturbation methods, as they

\[ \frac{V_{t+1}}{u_{t+1}} = A_t(1 - \gamma_t), \]

with $\gamma_t$ being the labor-income tax rate. In our case, the “tax rate” is defined as $\gamma_t = 1 - \frac{(1 + \tau)\left[\frac{e_t - 1}{e_t} + \frac{\phi}{e_t}(\pi_t - 1)\pi_t - \beta\phi\psi_t\left(\frac{u_{t+1}}{u_t}(\pi_{t+1} - 1)\pi_{t+1}\frac{y_{t+1}}{y_t}\right)\right]}{1 + \alpha e_t}$. Hence, if $\pi_t = 1$, then any increase in the shadow value of relaxing the collateral constraint will lead to an increase in $\gamma_t$. 

---

6 In general, we can write $\frac{V_{t+1}}{u_{t+1}} = A_t(1 - \gamma_t)$, with $\gamma_t$ being the labor-income tax rate. In our case, the”tax rate” is defined as $\gamma_t = 1 - \frac{(1 + \tau)\left[\frac{e_t - 1}{e_t} + \frac{\phi}{e_t}(\pi_t - 1)\pi_t - \beta\phi\psi_t\left(\frac{u_{t+1}}{u_t}(\pi_{t+1} - 1)\pi_{t+1}\frac{y_{t+1}}{y_t}\right)\right]}{1 + \alpha e_t}$. Hence, if $\pi_t = 1$, then any increase in the shadow value of relaxing the collateral constraint will lead to an increase in $\gamma_t$. 

13
stand, cannot deal with occasionally-binding constraints. Therefore, I modify the problem by using the penalty function approach; this approach allows for any value of \(\alpha w_i l_i\) to be possible in principle, but it imposes penalty once the collateral constraint is violated. Since the constraint is imposed on the labor choice of an entrepreneur, her objective function is modified so that it explicitly includes the penalty on violating the collateral constraint. Once the objective function of an entrepreneur is enlarged with the penalty function, the collateral constraint is removed. Thus, the computational problem that I solve, in place of the problem described in subsection (2.2), is 7

\[
\max_{i} \sum_{t=0}^{\infty} \delta^t \Xi_{0,t} \left\{ p_t A_i l_i - w_i l_t + e_{t-1} (q_t + d_t) - e_t q_t - \frac{1}{\psi^2} \exp\left[ \psi (\alpha w_i l_i - \kappa e_{t-1} (q_t + d_t)) \right] \right\}. \quad (23)
\]

The parameter \(\psi\) governs the curvature of the penalty function and it will be a key parameter in the analyses below. Also, the penalty approaches zero when the collateral constraint is not violated (see Figure 1.1).8

Computationally, the optimality conditions (12) and (13) are replaced by

\[
A_i p_t = w_i (1 + \alpha \Omega_i), \quad (24)
\]

\[
1 = \delta E_i \left[ \Xi_{t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) (1 + \kappa \Omega_{t+1}) \right], \quad (25)
\]

where, \(\Omega_i = \frac{1}{\psi} \exp[\psi (\alpha w_i l_i - \kappa e_{t-1} (q_t + d_t))].\)

Comparing (24) and (25) with (12) and (13), it is apparent that the approximation method replaces the economic variable \(\mu_i\) by \(\Omega_i\). This variable satisfies the requirement of being nonnegative and it approaches zero when the collateral constraint does not bind.

The decision rules that solve this approximation to the equilibrium are obtained through a second-order approximation to the optimality conditions of the monetary authority. Using a second order approximation, rather than linearization, is necessary in order to capture the asymmetry inherent in the occasionally-binding collateral constraint. A second-order approximation also allows for the long-run mean of a variable to be different from its respective deterministic steady state value. The second-order approximation procedure I apply is the one developed by Schmitt-Grohe and Uribe (2004).

7 This function is similar to the one used by Den Haan and Ocaktan (2009).
8 The horizontal axis shows \(\alpha w_i l_i - \kappa e_{t-1} (q_t + d_t).\)
1.4.2 Parameterization

In what follows, I assume a time unit of a quarter and hence the discount factor $\beta$ is set to 0.99. Following CFP, I set the parameter $\alpha$ to 0.5 in the benchmark calibration of the model. Following Iacoviello (2005) the maximum loan-to-value ratio $\kappa$ is set 0.89 in the benchmark calibration. In addition, I assume the following period utility function for households:

$$u(c_t) - v(l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{l_t^{1+\theta}}{1+\theta},$$

with the benchmark value of $\sigma$ being set to 1.5. The parameter $\theta$ is set to 0.5, implying a labor supply elasticity of 2. This relatively high labor supply elasticity is needed to capture the volatility of total hours in a model with no extensive margin, as is the case in this paper. The parameter $\chi$ is then calibrated so that the steady state value of $l$ is 0.3.

Productivity is governed by the following AR(1) process:

$$\ln(A_t) = (1 - \rho_A)\ln(A) + \rho_A\ln(A_{t-1}) + u_t,$$

with $\rho_A$ being 0.95 in line with standard calibration of the TFP process. The innovation term $u_t$ is normally distributed with zero mean and a standard deviation of $\sigma_u = 0.007$. The mean of $A$ is 1.

Similarly, the elasticity of substitution $\varepsilon_t$ evolves according to the following process:

$$\ln(\varepsilon_t) = (1 - \rho_\varepsilon)\ln(\varepsilon) + \rho_\varepsilon\ln(\varepsilon_{t-1}) + v_t,$$

where $\rho_\varepsilon$ is set to 0.9672 and the innovation term $v_t$ is normally distributed with zero mean and a standard deviation of $\sigma_v = 0.0729$, in line with Ireland (2002). The deterministic steady state value of $\varepsilon_t$ is set to 6, implying a deterministic steady state markup of 20 percent.

The benchmark value of $\psi$ is chosen so that the collateral constraint holds with equality in the deterministic steady state of the model. This condition on the deterministic steady state implies $\Omega = \frac{1}{\psi}$. In addition, combining the steady state versions of (8) and (25) gives $\Omega = \frac{1-\delta}{\delta\kappa}$. The combination of these two relationships gives $\psi = \frac{\delta\kappa}{1-\delta}$. The implied value of this parameter is 889.11. There are two reasons for the assumption that the collateral constraint binds in the deterministic steady state. First, for the constraint not to bind in the deterministic steady state, the additional discount factor ($\delta$) must be 1. In this case, however,
entrepreneurs accumulate enough assets so that the collateral constraint never binds. Second, starting from a deterministic steady state in which the collateral constraint binds enables good comparison with the case in which the constraint always binds (because in both cases the deterministic steady state is the same). Hence, any differences regarding the optimal long-run inflation rate in the “stochastic” steady state can be attributed to the assumption of occasionally-binding collateral constraint.

The value of $\psi$ is obtained for $\delta = 0.999$. This value of $\delta$ is larger than the values typically assumed in the literature (which usually lie between 0.95 and 0.99). There are two reasons for choosing a higher than usual value for $\delta$. First, in models with always-binding constraints, this parameter is chosen so that the collateral constraint always binds. Other things equal, a lower value of $\delta$ increases the chance for the constraint to bind. This fact is also apparent in the relationship $\Omega = \frac{1-\delta}{\delta \kappa}$; the value of $\Omega$ in the deterministic steady state is decreasing in $\delta$. In particular, the constraint does not bind in the deterministic steady state of the model if $\delta = 1$. Second, the accuracy of the approximation using the penalty function depends on the value of $\psi$; the higher $\psi$ is, the closer the penalty function to obtain the L-shape, which clearly improves the approximation (put differently, a higher $\psi$ reduces the probability of violating the collateral constraint since any violation entails a higher penalty).

But the equation ($\psi = \frac{\delta \kappa}{1-\delta}$) suggests that $\psi$ depends positively on $\delta$. Hence, setting a higher value of $\delta$ is equivalent to setting a higher value of $\psi$. In the robustness checks section, I also show the results for $\delta = 0.99$.

Finally, the parameter governing the adjustment cost of prices $\varphi$ is set to 18.47 in my benchmark calibration. This value is based on the recent evidence regarding the duration of price contracts: Bils and Klenow (2004) show that the average duration of prices is between 4.5 and 5.5 months; Ravenna and Walsh (2006) suggest price duration of between 2 and 3 quarters, and Christiano, Eichenbaum and Evans (2005) use price duration of 2.5 quarters. I follow Christiano, Eichenbaum and Evans (2005) and set my benchmark price duration to 2.5 quarters, but I also show the results for various price durations between 2 and 4 quarters.
I map the price duration to the adjustment cost parameter $\varphi$ using the relationship

$$\varphi = \frac{\lambda(\lambda - 1)(\varepsilon - 1)}{\lambda - \beta(\lambda - 1)},$$

with $\lambda$ denoting the price duration. This approach follows Faia and Monacelli (2007). In short, the price rigidity parameter $\varphi$ is pinned down when the slope of the Philips curve in a linearized model with Calvo (1983)’s parameterization is equalized to the slope of the Philips curve in a linearized model with a Rotemberg (1982)’s parameterization. For more details, refer to Appendix 1H.

1.5 THE OPTIMAL LONG-RUN INFLATION RATE

This section presents the main findings regarding the optimal long-run inflation rate in the presence of financial frictions.

1.5.1 The Optimal Inflation Rate in the Deterministic Steady State

Before turning to present the optimal long-run inflation rate, a note on the deterministic steady state (i.e. the state with constant technology) is in order. Given the parameter $\psi$, the deterministic steady state of the model is invariant to the degree of price stickiness. The main result is that the optimal deterministic steady state of inflation is exactly zero (see Appendix 1K for a proof). This result is as expected: in the absence of shocks, inflation is not beneficial, and due to the resource cost of deviations from zero inflation, the monetary authority completely stabilizes prices in the deterministic steady state. This is true regardless of the degree of price rigidity assumed (since there is no benefit from non-zero inflation but there is a cost of non-zero inflation for any positive value of price rigidity) and regardless of whether there is a labor market subsidy or not. Also, given that the deterministic steady state value of inflation is zero regardless of the degree of price rigidity, the deterministic steady state values of other variables will not vary with the degree of price rigidity.

1.5.2 The Optimal Long-Run Inflation Rate

This subsection presents the main results regarding the optimal long-run inflation rate. Before presenting the results allowing for occasionally-binding constraint, I comment on the optimal inflation rate with in the absence of financial frictions. In this case, the optimal long-run
inflation rate is zero regardless of the type of the underlying shock. Furthermore, inflation does not respond to TFP shocks in the short run (“divine coincidence”).

The results with an occasionally-binding collateral constraint are considerably different (Table 1.1). In this case, optimal monetary policy deviates from full price stability in the long run. When the economy is subject to both shocks simultaneously, the optimal long-run inflation rate is around 2 percent annually in the benchmark calibration of the model. This is an important result, since in the real world the economy is subject to ongoing TFP and markup shocks, among others. The optimal inflation rate is also positive and around 1 percent for other empirically-plausible price durations. Hence, regardless of what the actual price duration in the U.S. is, the optimal inflation rate should, generally speaking, lie between 1 percent and 2 percent annually.

<table>
<thead>
<tr>
<th>Price Duration (Quarters)</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP and Markup shocks</td>
<td>2.08</td>
<td>1.82</td>
<td>1.44</td>
<td>1.06</td>
<td>0.81</td>
</tr>
<tr>
<td>TFP shocks only</td>
<td>0.63</td>
<td>0.54</td>
<td>0.43</td>
<td>0.32</td>
<td>0.22</td>
</tr>
<tr>
<td>Markup Shocks only</td>
<td>1.61</td>
<td>1.16</td>
<td>0.91</td>
<td>0.73</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Table 1.1: The optimal long-run inflation rate (in annualized percentages terms) for various price durations.

Also, for plausible price durations, the optimal inflation target is falling in the degree of price rigidity. This result is due to the higher resource cost associated with a higher inflation rate, which in turn negatively affects the mean value of consumption. The nominal distortion seems to be less dominant for relatively low degrees of price stickiness, but becomes more dominant as the degree of price rigidity increases. Indeed, as the cost of price adjustment increases, the optimal inflation rate decreases. The optimal inflation rate reaches zero for very high degrees of price rigidity, but this happens outside the empirically plausible range of \( \phi \) considered in this paper.

It is also interesting to consider each shock separately in order to learn about the contribution of each shock in driving the results in Table 1.1. When the economy is only subject to TFP risk, the optimal annual long-run inflation rate is around 0.5 percent in the

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9 The results here are, conceptually, in line with the findings of Schmitt-Grohe and Uribe (2007); they show that the optimal inflation rate is highly sensitive to the degree of price rigidity. In their study, there is a tension between the monetary distortion, which calls for a negative inflation rate, and the nominal distortion, which calls for full price stability. In the current study, the tension is between the financial friction, which calls for a positive inflation rate, and the nominal distortion.
baseline calibration of the model, and it is also strictly positive for other plausible price durations. This result suggests that the presence of occasionally-binding constraints not only leads to variations in inflation following productivity shocks (as will be shown in the impulse-response subsection below), but also implies positive inflation on average.

Under only markup shocks, the optimal long-run inflation rate is almost twice as large as under TFP shocks only and decreasing in the degree of price rigidity. Hence, most of the positive inflation rate found above is due to markup shocks. This result is in line with the fact that markup shocks have stronger impact on inflation, through the Phillips curve, and they account for higher portion of the variability in inflation compared to TFP shocks (Ireland, 2002). Furthermore, the existence of the collateral constraint magnifies this effect since, as discussed in subsection (2.3), the variable \( \Omega \), acts as an endogenous markup shifter.

1.5.3 Discussion
When the collateral constraint binds, the Lagrange multiplier is akin to a cost-push shock that leads to positive inflation. When the collateral constraint does not bind, the optimal inflation rate is zero. Since this cost-push shock is asymmetric, in the sense that it may not be negative, it has a positive mean. The positive mean of the cost-push shock leads to positive mean of inflation. This case differs from the standard cost-push shock; the latter has a zero mean and introduced in a long-linearized model, which is symmetric by construction. The endogenous cost-push shock in this paper not only allows us to study the dynamics of inflation but also its long-run mean.

1.5.4 The Labor Wedge
As discussed in section 3, a binding collateral constraint generates a wedge between the marginal rate of substitution between labor and consumption and the marginal product of labor, thus leading to a rise in the “labor-income tax”, as defined above. These effects can be counteracted by setting positive inflation.

Figure 1.2 shows the volatility of the labor wedge for various optimal inflation rates as the price duration is varied between 2 and 4 quarters. For the sake of exposition, I consider the case with simultaneous markup and TFP shocks, but the results hold under each shock individually. Clearly, the volatility of the wedge at the optimum is decreasing in the optimal inflation rate (put differently, a lower degree of price rigidity is associated with higher optimal
inflation and lower volatility of the labor wedge). The Ramsey planner cannot completely close and/or smooth the wedge due to the lack of a sufficient set of policy instruments to completely and simultaneously offset all distortions along the business cycle. Since prices are not fully flexible, the Ramsey planner must trade off between stabilizing the wedge and stabilizing inflation.

More generally, the main results of the paper can be related to the basic Ramsey theory of smoothing distortions over time. In this case, smoothing the labor wedge requires smoothing of the “labor-income tax”, which by itself can be achieved through appropriate setting of inflation.

1.5.5 Impulse Responses

It is useful to observe the behavior of some key variables following TFP and markup shocks in order to gain some insight about the mechanics of the model. Figure 1.3 displays the responses of some key economic aggregates under the optimal policy following negative and positive markup shocks of the same magnitude. Figure 1.4 shows the behavior of these variables following TFP shocks. The figures plot the percentage deviation of each variable from its deterministic steady state value. The main observation is the asymmetry in the response of these variables to negative and positive shocks of either type. The asymmetry is more apparent for the case of markup shocks, which, as we have seen above, generate a higher inflation rate.

Following a negative one standard deviation markup shock, the fall in nominal share prices and the increase in good prices lead to a drop in the real price of shares ($q$) of about 5 percent below their steady state value. The asymmetry in the response of net worth is mainly driven by the asymmetry in the real price of shares (notice the similarity of their movements) and, to a lesser extent, the asymmetry in the behavior of dividends. Shares ($e$) display little asymmetry (and their overall response is relatively small). Output, consumption, labor and the financial friction variable ($\Omega$) all display clear asymmetry under both types of shocks. The fall in output following a negative markup shock (i.e. an increase in the price markup) suggests negative co-movement between output and the desired price markup.

Inflation behaves as expected; a negative TFP shocks leads to an increase in the marginal cost and consequently to an increase in inflation. This is apparent from examining condition (16). In this paper, the existence of the collateral constraint is the reason for inflation to respond to TFP shocks (i.e. the collateral constraint breaks down the “divine coincidence”). A negative markup shock (which is modeled here as a fall in the elasticity of substitution
between different types of final goods) is akin to a cost-push shock that generates inflation. Clearly, the response of inflation to markup shocks is considerably larger than the response of inflation to TFP shocks.

1.6 ROBUSTNESS ANALYSES

1.6.1 Introducing Money Demand

Friedman (1969) suggested that a negative inflation rate is optimal in order to eliminate monetary distortions. In this subsection, I consider the implications of adding a money demand motive for the optimal inflation rate. I assume that households derive utility from holding money (i.e. “Money in the Utility”). Households’ optimization, which is presented in Appendix 1L, gives the following money demand condition:

\[ m_t = \beta \phi c_t^\sigma \frac{R_t}{(R_t - 1)} \]  \hspace{1cm} (26)

Real money holdings is positively related to consumption and negatively related to the nominal interest rate. As the interest rate approaches 1, real money holding approaches infinity (i.e. the economy is satiated with money balances). In addition, the motive to holding money is affected by the parameter \( \phi \); when this parameter is set to zero, the model collapses to the standard cashless New-Keynesian model.\(^{10}\) I set this parameter to 0.0128, implying a money-consumption ratio of 0.7, which equals the ratio of M1 to consumption in the US (Walsh, 2003).

The main result of the paper, that the optimal inflation rate is positive, is robust to the introduction of money demand. Interestingly, adding money demand only moderately affects the optimal long-run inflation rate. This is particularly true with relatively high price rigidity; in this case, price rigidity is very dominant and introducing another factor that imply non-zero inflation rate does not affect the optimal inflation rate significantly. More generally, the results

\(^{10}\) The rate of time preference appears in the demand function for money because of the timing assumption: households derive utility from the real money balances that are available at the beginning of time \( t \) (which are giving by \( \frac{M_{t-1}}{P_t} \)). The more patient households are, the more money they are willing to hold for the next period (thus leading to higher \( m_t = \frac{M}{P_t} \)).
of this subsection suggest that the motive for a positive inflation rate introduced in this paper outweighs the motive for a negative inflation rate that arises due to money demand.\textsuperscript{11}

<table>
<thead>
<tr>
<th>Price Duration (Quarters)</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP and Markup shocks</td>
<td>1.86</td>
<td>1.51</td>
<td>1.18</td>
<td>0.87</td>
<td>0.61</td>
</tr>
<tr>
<td>TFP shocks only</td>
<td>0.52</td>
<td>0.44</td>
<td>0.33</td>
<td>0.25</td>
<td>0.17</td>
</tr>
<tr>
<td>Markup Shocks only</td>
<td>1.25</td>
<td>0.98</td>
<td>0.77</td>
<td>0.61</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 1.2: The optimal long-run inflation rate (in annualized percentages terms) for various price durations with money demand (\( \phi =0.0128 \)).

1.6.2 Changing the Impatience Rate of Entrepreneurs

I start by changing the value of the parameter \( \delta \) and, consequently, the value of \( \psi \). I set \( \delta =0.99 \), which implies a subjective discount factor of entrepreneurs (\( \beta \delta \)) of about 0.98, in line with Iacoviello (2005). The implied value of \( \psi \) is 88.11.

<table>
<thead>
<tr>
<th>Price Duration (Quarters)</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP and Markup shocks</td>
<td>1.38</td>
<td>1.17</td>
<td>0.98</td>
<td>0.86</td>
<td>0.80</td>
</tr>
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<td>0.35</td>
<td>0.29</td>
<td>0.25</td>
<td>0.20</td>
</tr>
<tr>
<td>Markup Shocks only</td>
<td>0.95</td>
<td>0.81</td>
<td>0.62</td>
<td>0.47</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 1.3: The optimal long-run inflation rate (in annualized percentages terms) for various price durations with \( \delta =0.99 \) and \( \psi =88.11 \).

The results are reported in Table 1.3 and they can be summarized as follows. First, the optimal long-run inflation rate with the benchmark price duration (of 2.5 quarters) is positive, ranging between about one third percent when TFP risk is the driving force and 1.2 percent when both TFP and markup shocks hit the economy. Second, the optimal inflation rate when both shocks are allowed is around 1 percent for most of the price durations considered.

1.6.3 Changing the Financial Friction Parameter

Table 1.4 presents the results for various values of the financial friction parameter \( \alpha \) under the assumption that the price duration is 2.5 quarters. The optimal inflation rate is increasing

\textsuperscript{11} Recently, Kim and Ruge-Murcia (2011) show that, in a model with downward nominal wage rigidity, sticky prices and money demand, the optimal inflation rate is positive, implying that the motive for positive inflation is stronger than the motive for negative inflation.
in $\alpha$ for all types of shocks, suggesting that the more “severe” the financial friction is, the higher the optimal inflation rate. Needless to say, the optimal long-run inflation rate for $\alpha = 0$ is zero regardless of the source of uncertainty, and hence it is not presented below.

<table>
<thead>
<tr>
<th>Financial Friction Parameter</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP and Markup shocks</td>
<td>0.92</td>
<td>1.82</td>
<td>2.81</td>
<td>3.85</td>
</tr>
<tr>
<td>TFP shocks only</td>
<td>0.27</td>
<td>0.54</td>
<td>0.87</td>
<td>1.17</td>
</tr>
<tr>
<td>Markup Shocks only</td>
<td>0.58</td>
<td>1.16</td>
<td>1.77</td>
<td>2.54</td>
</tr>
</tbody>
</table>

Table 1.4: The optimal long-run inflation rate (in annualized percentages terms) for various values of $\alpha$.

### 1.6.4 Increasing the Sizes of the Underlying Shocks

Blanchard, Dell’Ariccia and Mauro (2010) suggest that “the crisis has shown that large shocks to the system can and do happen,” and that “maybe policymakers should therefore aim for a higher target inflation rate in normal times, in order to allow for more room for monetary policy to react to such shocks.” Motivated by this statement, in what follows, I show the optimal long-run inflation rate when the shocks are larger than in the baseline case. This is not necessarily the only way to interpret the ideas of Blanchard, Dell’Ariccia and Mauro (2010), but it perhaps the simplest way to capture their suggestions. I consider the case when the shocks are 10 percent bigger than in the baseline calibration (i.e. $\sigma_u = 0.0077$ and $\sigma_v = 0.0802$). The results are summarized in Table 1.5.

<table>
<thead>
<tr>
<th>Price Duration (Quarters)</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP and Markup shocks</td>
<td>2.56</td>
<td>2.25</td>
<td>1.79</td>
<td>1.28</td>
<td>1.02</td>
</tr>
<tr>
<td>TFP shocks only</td>
<td>0.78</td>
<td>0.67</td>
<td>0.59</td>
<td>0.38</td>
<td>0.26</td>
</tr>
<tr>
<td>Markup Shocks only</td>
<td>1.61</td>
<td>1.48</td>
<td>1.16</td>
<td>0.82</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Table 1.5: The optimal long-run inflation rate (in annualized percentages terms) for various price durations, with larger risk.

With bigger shocks, the optimal inflation rate under simultaneous TFP and markup shocks is roughly 2.5 percent, about half a percent higher than in the benchmark calibration. Overall, the optimal inflation rate is around 2 percent for price durations between 2 and 3 quarters, which is the most empirically-relevant range. Hence, even a relatively small increase in the size of shocks leads to a considerable increase in the optimal inflation rate.
1.7 CONCLUSIONS

The main purpose of this paper is to study the optimal long-run inflation rate in the presence of financial frictions. The model is a variation of the standard New Keynesian framework in which the hiring of labor by entrepreneurs is constrained by their collateral. This study modifies the assumption of always-binding collateral constraints by assuming that the constraint may only occasionally bind. The main result is that optimal monetary policy sets a strictly positive inflation rate in the long-run (i.e. in the stochastic steady state of the model).

The optimal annual long-run inflation rate is about 2 percent when the economy faces both TFP and markup risks of empirically-relevant magnitudes.

When the collateral constraint binds, the shadow value of relaxing the constraint is equivalent to an endogenous asymmetric cost-push shock that generates inflation. Final-good firms set higher prices when they observe increases in their marginal cost as a result from a binding collateral constraint. Since the constraint binds on average and the shadow values takes non-negative values only, the effects of positive Lagrange multipliers on inflation are not offset in periods of non-binding collateral constraints. The positive average of the endogenous cost-push shock leads to a positive inflation rate on average.

Furthermore, a binding collateral constraint distorts labor demand and thus leads to suboptimal level of output. Basically, a binding collateral constraint is akin to a “tax” on labor which can be both reduced and smoothed by setting positive inflation. More generally, a positive inflation rate helps in smoothing the labor wedge that arise due to the existence of the collateral constraint and the monopolistic power of firms.

The current study also contributes to recent literature that attempts to justify the fact that central banks around the world target positive inflation rates. To my knowledge, the current study is the first to motivate a positive long-run inflation rate in an environment featuring occasionally-binding financial constraints. The recent debate about the optimal inflation rate makes this study particularly timely and significant.
CHAPTER 2

Optimal Monetary Policy and Downward Nominal Wage Rigidity in Frictional Labor Markets

2.1 INTRODUCTION

This paper studies optimal monetary policy in the presence of Downward Nominal Wage Rigidity (DNWR) within a labor search and matching model. When nominal wages are downwardly rigid, optimal monetary policy sets a strictly positive inflation rate, of about 2 percent annually, in the long run. A strictly positive long-run inflation rate is driven by precautionary considerations in the expectations of adverse shocks. Positive inflation allows for downward real wage adjustments (thus “greasing the wheels” of the labor market) which eases job creation and limit the increase in unemployment following adverse shocks.

The results of the paper are related to standard Ramsey theory of smoothing distortions (or “wedges”) over time. A virtually constant distortion across periods is the main insight of Barro (1979), in a partial equilibrium framework, and Chari, Christiano and Kehoe (1991) in a quantitative general equilibrium model, among others. Recently, Arseneau and Chugh (2010) have developed intertemporal and static notions of efficiency in general equilibrium models with labor search and matching frictions. They show that the intertemporal wedge should indeed be smoothed, but, contrarily to the cornerstone result of tax smoothing in the Ramsey literature, that occurs through volatility in labor income taxes. In this paper, optimal monetary policy, which calls for a positive inflation rate due to DNWR, reduces the size and the volatility of the intertemporal wedge when prices are sticky. This fact suggests that with both intertemporal and nominal distortions in place, the monetary authority cannot completely smooth both distortions simultaneously. When prices are fully flexible, however, monetary policy keeps the intertemporal wedge virtually constant over the business cycle by varying the

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12 DNWR means not only that wage increases are more likely than wage cuts, but also that the distribution of wage changes is not symmetric. Nominal wages tend to increase in good times but they do not tend to fall proportionally in bad times, thus generating an asymmetric distribution of wage changes. Note that the fact that wage increases are more common than wage cuts by itself is insufficient evidence for the presence of DNWR; a preponderance wage increases may reflect long-term productivity growth or steady state (positive) inflation.
inflation rate. Therefore, the volatility and the size of the intertemporal wedge are both falling in the inflation rate as the degree of price rigidity varies.

The results under zero-inflation policy at all dates and states are significantly different. In this case, the volatility of the intertemporal wedge is substantially higher; the wedge absorbs the shock. Similar results are obtained for labor market variables; the combination of DNWR and fully stable prices limit the decline in real wages considerably, thus generating unemployment increases and job creation declines far beyond their responses under a positive inflation target.

The paper is motivated by several recent empirical studies indicating DNWR. Some of the most notable recent evidence on DNWR is the comprehensive work of the International Wage Flexibility Project (IWFP), reviewed in Dickens et al. (2007a, 2007b). Their findings indicate asymmetry in the distribution of nominal wage changes in 16 OECD countries, with the U.S. being among the countries with very high degrees of DNWR. Gottschalk (2005) shows that after correcting for measurement errors that typically appear in wages reported in surveys, only about 5% of workers experienced wage cuts during a course of a year while working for the same employer. Card and Hyslop (1997) show a spike at zero in the distribution of nominal wage changes, indicating DNWR. The size of this spike is highly correlated with inflation; it significantly increased in the mid 1980’s when inflation rates fell relative to the 1970’s. In addition, their analysis reveals that, on average, real wages would have been lower by around 1% per year in the mid 1980’s had nominal wages not been downwardly rigid. Using large financial corporation wage and salary data, Altonji and Devereux (2000) find that only 0.5% of salaried workers had salary cuts and 2.5% of hourly workers had wage reductions.

The idea that positive inflation may be needed to “grease the wheels” of the labor market dates back at least to Tobin (1972). Following negative shocks that call for a fall in the real wage, Tobin (1972) suggests that setting a positive inflation rate, on one hand, and stabilizing nominal wages, on the other, would facilitate real wage adjustment in the presence of DNWR. Tobin’s idea has gained more attention in recent years for two reasons. First, inflation rates have become very low in the last two decades. Clearly, DNWR is more relevant in low inflation environments and during recessions. Second, central banks around the world do in fact target positive inflation rates, either explicitly or implicitly. DNWR may create a precautionary motive for positive inflation: since the timing of (negative) shocks is not fully
predictable, the monetary authority keeps the inflation rate positive on average in order to “ensure” against negative shocks once they materialize.

This study allows for staggered price setting, downwardly rigid nominal wages, and search and matching frictions in the labor market, the latter being consistent with positive unemployment in equilibrium. To model DNWR, I follow Kim and Ruge-Murcia (2009) and Fahr and Smets (2008) by using the Linex wage adjustment cost function. This function delivers higher costs in case of wage cuts relative to wage increases. To see the significance of this setup, consider the response of an economy to an adverse productivity shock. If inflation is high, then downward rigidity in nominal wages cannot prevent real wage drops, and hence inflation mitigates the potential increase in unemployment. In case of low inflation rates, however, DNWR may translate into Downward Real Wage Rigidity (DRWR). In this case, if the monetary authority seeks to keep prices stable (due to a direct cost of adjusting prices), downward rigidity in real wages implies higher unemployment than in the absence of DNWR. If the monetary authority instead chooses to stabilize employment, it inflates in order to achieve the desired cut in real wages. That is, the inflation rate needed ‘to grease the wheels’ of the economy is higher than it would be if nominal wages were not downwardly rigid. In short, the presence of labor market frictions may magnify the need for grease inflation if policy makers are trying to keep unemployment low, or it may create excessive unemployment when attempting to keep prices close to full stability.

The current study contributes to some recent literature that studies the optimal inflation rate in the presence of DNWR. In a frictionless labor market environment, Kim and Ruge-Murcia (2009) show that the optimal annual grease inflation in the U.S. is positive (around 0.4 percent). Unlike the current study, they estimate the model’s parameters based on some Taylor-type rule. In an earlier version of their paper (Kim and Ruge-Murcia, 2007), the monetary authority chooses allocations to maximize households’ welfare, but without assuming any Taylor-type rule. In that case, the optimal annual grease inflation is found to be 1.2 percent. Fagan and Messina (2009) introduce asymmetric menu costs for wage setting and show that the optimal inflation rate for the U.S. ranges between 2 percent and 5 percent when nominal wages are downwardly rigid. The optimal inflation rate in their model depends on the dataset used to measure the degree of DNWR. The optimal long run inflation rate found in the current paper is thus more in line with the results of Fagan and Messina (2009).

The fact that the inflation rate suggested by the current study is significantly higher than in Kim and Ruge-Murcia (2007, 2009), despite the use of a similar proxy for DNWR, suggests
that structure of the labor market in which DNWR is studied may matter for policy recommendations. Since the discussion is over the long-run inflation rate, these differences are economically significant. In addition, the average inflation rate in the United States has been around 2.5 percent in the last two decades. Therefore, the current study may also be seen as one that suggests a theoretical ground for targeting an inflation rate of around 2 percent.

The remainder of this paper proceeds as follows. Section 2 outlines the search and matching model economy with DNWR. Section 3 discusses the search-based efficient allocations and the intertemporal wedge. Section 4 describes the calibration methodology and the parameterization of the model. Section 5 presents the results regarding the optimal inflation rate. Impulse Response functions following productivity shocks are presented in section 6. Section 7 examines the performance of two extreme policies, full price stability and full employment stability, relative to optimal policy. Section 8 concludes.

2.2 THE MODEL ECONOMY

Apart from the monetary authority, the economy is populated by households and monopolistically-competitive firms that produce differentiated products. Hiring labor by firms is subject to search and matching frictions. Following literature, the model embeds the search and matching framework of Pissarides (2000), which has become the main framework within which optimal monetary policy is studied in the presence of labor market frictions. Each firm faces asymmetric adjustment cost for nominal wages that implies higher costs of cutting nominal wages relative to increasing nominal wages by the same magnitude. Changing prices by each firm is subject to a direct resource cost. The model allows for variations in total hours along both the extensive and the intensive margin.

2.2.1 Households

The economy is populated by a representative household which consists of family members of measure one. At each date $t$ a household member can be in either of two states: employed or unemployed and searching for a job. Employed individuals are of measure $n_t$, and the unemployed are of measure $u_t$, where $u_t = 1 - n_t$, as conventional in the literature.

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13 Note that my model assumes no idiosyncratic shocks, unlike, for example, Mortensen and Pissarides (1994).
Following the assumptions of consumption insurance in Merz (1995) and Andolfatto (1996), all family members in this household have the same consumption. The disutility of work is assumed to be the same for all employed individuals and the value of non-work is the same for all unemployed individuals. Given these assumptions, the household’s problem is to maximize

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - n_t v(h_t) \right], \]  

where \( \beta < 1 \) is the standard subjective discount factor, \( E_0 \) is the expectation operator, \( c_t \) is the composite consumption index, \( h_t \) denotes hours per worker, \( u(\cdot) \) is the period utility function from consumption and \( v(h_t) \) is the period disutility function from supplying labor. These functions satisfy the Inada conditions and the usual properties: 

\[ \frac{\partial u(\cdot)}{\partial c} > 0, \quad \frac{\partial^2 u(\cdot)}{\partial c^2} < 0, \]

\[ \frac{\partial v(\cdot)}{\partial h} > 0 \quad \text{and} \quad \frac{\partial^2 v(\cdot)}{\partial h^2} > 0. \]

As standard in New Keynesian models, consumption \( (c_t) \) is a Dixit-Stiglitz aggregator of differentiated products \( (c_{jt}) \) produced by monopolistically-competitive firms,

\[ c_t = \left( \int_0^1 c_{jt} \frac{1}{\varepsilon} \, dj \right)^{-\varepsilon}, \]  

where \( \varepsilon > 1 \) is the elasticity of substitution between two varieties of final goods. In line with standard Dixit-Stiglitz based NK models, the optimal allocation of expenditures on each variety is given by

\[ c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon} c_t, \]  

where \( P_t = \left( \int_0^1 P_{jt}^{-1/\varepsilon} \, dj \right)^{1/\varepsilon} \) is the Dixit-Stiglitz price index that results from cost minimization.

Maximization is subject to the sequence of budget constraints of the form

\[ c_t + \frac{B_t}{P_t} = \frac{n_t h_t W_t}{P_t} + (1-n_t) b + \frac{R_t}{P_t} \frac{T_t}{P_t} + \Theta_t, \]  

where

29
where $b$ stands for unemployment benefits, $B_t$ denotes nominal bonds, $W_t$ is the nominal wage, $R_t$ is the nominal gross interest rate on bonds, $P_t$ is the aggregate price level, $T_t$ are net transfers and $\Theta_t$ stands for profits from the ownership of firms.

Household’s choices of consumption and bond holdings yield the following optimality condition:

$$u_{ct} = \beta R_t E_t \left( \frac{u_{ct+1}}{\pi_{t+1}} \right),$$

(5)

in which $(\pi_t = P_t/P_{t-1})$ denotes the gross price inflation rate.

### 2.2.2 Firms in the Labor Market

There is a continuum of measure one of monopolistically-competitive firms. Each firm $j$ hires labor as the only input and produces differentiated products using the following linear technology

$$y_{jt} = z_t n_{jt} f(h_{jt}),$$

(6)

with $z_t$ denoting aggregate productivity, which is common to all firms, $n_{jt}$ is employment at time $t$ in firm $j$, and $h_{jt}$ is hours per worker supplied by each worker at the firm. The pricing of a firm is subject to a quadratic adjustment cost as in Rotemberg (1982), expressed in units of the final output.

Hiring workers by each firm is subject to search and matching costs. Each period firms post vacancies and they meet unemployed workers searching for jobs. Nominal wages and hours per worker are determined in a Nash bargaining process between workers and firms as will be outlined later. As noted by Krause and Lubik (2007), the assumption of quadratic adjustment cost and symmetry among firms allows for integrating price decisions and employment decisions in the same firm.

Each firm faces an asymmetric wage adjustment cost function that involves a higher cost in case of a nominal wage cut compared to a nominal wage increase. In particular, following

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14 The assumption that firms pay the adjustment costs of wages is without loss of generality. In this model, the wage rate is determined through bargaining between firms and workers. Therefore, it is not very clear who should pay the cost of adjusting wages. I assume that firms entail these costs without loss of generality. Note that this assumption has no effect on the economy-wide resource constraint.
Kim and Ruge-Murcia (2009), the real wage adjustment cost per employed individual is given by the following Linex function:

\[
\Phi_{W}^{jt} = \frac{\phi}{\psi^2} \left[ \exp[-\psi(\frac{W_{jt}}{W_{jt-1}} - 1)] + \psi(\frac{W_{jt}}{W_{jt-1}} - 1) - 1 \right].
\] (7)

For any positive value of \(\psi\), the cost of cutting nominal wages by a specific amount is higher than the cost of increasing wages by the same amount. Also, as \(\psi\) approaches zero, this function approaches the quadratic adjustment cost and hence enables comparison with the symmetric adjustment function. In the other extreme, as \(\psi\) approaches infinity, this function becomes L-shaped implying that nominal wages cannot fall. Naturally, this parameter will have special significance in my analyses, particularly regarding the optimal long-run inflation rate.

Posting a vacancy \(v\) entails a cost of \(\gamma\) for a firm. Matches between vacant jobs and unemployed individuals are governed by a constant return-to-scale-matching function of the form

\[
m(v_t, u_t) = \sigma_m u_t \theta_t^{1-\zeta},
\] (8)

where \(\sigma_m\) is a scaling parameter that reflects the efficiency of the matching process. Labor market tightness is measured as

\[
\theta_t = \frac{v_t}{u_t}.
\] (9)

The probability of the firm to fill a job (i.e. the job filling rate) is given by \(q(\theta_t) = \frac{m(v_t, u_t)}{v_t}\). Using the properties of the matching function it can be written as

\[
q(\theta_t) = \sigma_m \theta_t^{-\zeta},
\] (10)

which is decreasing in labor market tightness. Intuitively, the higher the ratio of vacancies to unemployment, the lower the probability for a specific vacancy to be filled. Similarly, the job finding rate (i.e. \(p(\theta_t) = \frac{m(v_t, u_t)}{u_t}\)) can be written as

\[\text{The variable } u \text{ measures the number of unemployed individuals at time } t. \text{ The corresponding unemployment rate is given by } ur_t = 1 - (1 - u_t)(1 - \rho)^{-1}.\]
\[ p(\theta_t) = \sigma_m \theta_t^{\epsilon - \gamma}, \quad (11) \]

and hence it increases in tighter labor markets. Finally, employment in each firm evolves according the following law of motion:

\[ n_{jt+1} = (1 - \rho)(n_{jt} + m(v_{jt}, u_t)), \quad (12) \]

with \( \rho \) denoting the separation rate from a match. Using the expression for the job-filling rate and the law if motion of employment can also be written as

\[ n_{jt+1} = (1 - \rho)(n_{jt} + v_{jt}q(\theta_t)). \quad (13) \]

In this formulation, I assume that a match formed at time \( t \) starts to produce at time \( t+1 \) given that it survived exogenous separations.

Each firm chooses its price vacancies and employment for next period to maximize the expected presented discounted stream of profits given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[ \frac{P_{jt}}{P_t} y_{jt} - n_{jt}W_{jt} h_{jt} - v_{jt} - \phi \psi \left( \exp\left[-\psi(W_{jt}/W_{jt-1})\right] + \psi(W_{jt}/W_{jt-1}) - 1 \right) n_{jt} - \phi \psi \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right) y_t \right],
\]

subject to the sequence of laws of motion of employment, and the:

\[ n_{jt+1} = (1 - \rho)(n_{jt} + v_{jt}q(\theta_t)), \quad (15) \]

and the downward-sloping demand function for its product

\[ z_t n_{jt} f(h_{jt}) = \left[ \frac{P_{jt}}{P_t} \right]^{-\epsilon} y_t. \quad (16) \]

Households are assumed to own the firms, and hence firms discount next period’s profits by the stochastic discount factor of households (i.e. \( \beta^\frac{\lambda_{t+1}}{\lambda_t} \)), where \( \lambda_t \) is the Lagrange multiplier on the households budget constraint.

Let \( \mu_{jt} \) be the Lagrange multiplier associated with the employment law of motion (equation 15), and \( \varphi_{jt} \) be the Lagrange multiplier associated with the output constraint (equation 16). The multiplier \( \varphi_{jt} \) measures the contribution of one additional unit of output to the revenue of the firm, and it equals, in equilibrium, the real marginal cost of the firm. Imposing symmetry across firms, and assuming that all workers supply the same number of hours (\( h \)), the first-order conditions with respect to \( n_{jt+1} \) and \( v_{jt} \) read, respectively, as follows:
\[ \mu_i \lambda_i = \beta E_i \left[ \lambda_{i+1} \left( \phi z_{i+1} f(h_{i+1}) - W_{i+1} h_{i+1} - \frac{\phi}{\psi^2} \left( \exp[-\psi(W_{i+1} - 1)] + \psi(W_{i+1} - 1) - 1 \right) \right) \right] \], \quad (17) \]

and,

\[ -\gamma + (1 - \rho) q(\theta_i) \mu_i = 0. \quad (18) \]

Combining conditions (15) and (16) and the fact that \( \lambda_i = u_{it} \) give the Job Creation (JC) condition:

\[ \frac{\gamma}{q(\theta_i)} = \beta(1 - \rho) E_i \left[ \frac{u_{it+1}}{u_{it}} \left( \phi z_{it+1} f(h_{it}) - W_{it+1} h_{it+1} - \frac{\phi}{\psi^2} \left( \exp[-\psi(\pi_{it+1}^w - 1)] + \psi(\pi_{it+1}^w - 1) - 1 \right) \right) \right], \quad (19) \]

where \( W_i = \frac{W}{P_i} \) is the real wage. Thus, in equilibrium, the firm equates the vacancy-creation cost to the discounted expected value of profits from the match. As the term in brackets makes clear, the flow profit to a firm from a match equals output net of wage payments and costs of adjusting wages.\(^{16}\) This condition is also referred to as the free-entry condition for posting vacancies.

By taking first order condition with respect to the price \( P_i \) and assuming symmetry among firms (since they all set the same price in equilibrium), we get the following price Philips curve (see Appendix 2A):

\[ 1 - \phi^p (\pi_t - 1) \pi_t + \beta \phi^p E_i \left[ \frac{u_{it+1}}{u_{it}} \right] (\pi_{it+1} - 1) \pi_{it+1} \frac{y_{it+1}}{y_t} = \varepsilon (1 - \rho_i). \quad (20) \]

This equation shows that the current inflation rate is an implicit function of the expected inflation rate and the real marginal cost. This equation collapses in the case of fully flexible entry conditions.

---

\(^{16}\) To see this clearly, one may write this condition in the following way:

\[ \gamma = q(\theta_i)(1 - \rho) E_i \left[ \beta u_{it+1} \left( \phi z_{it+1} f(h_{it+1}) - W_{it+1} h_{it+1} - \frac{\phi}{\psi^2} \left( \exp[-\psi(\pi_{it+1}^w - 1)] + \psi(\pi_{it+1}^w - 1) - 1 \right) \right) \right] \]

The LHS is the cost of posting a vacancy. The RHS shows the discounted expected value of profits from a given match. The firm enjoys profits from this match in case of being filled (which occurs with probability \( q(\theta_i) \)) and surviving exogenous separation (which occurs with probability \( (1 - \rho_i) \)). We can use the last term in the RHS to iterate forward and hence get the expected PDV of profits. In short, this equation equates the cost of posting a vacancy (the LHS) to the (expected) benefit of posting that vacancy.
prices ($\phi^p = 0$) or fully stable prices ($\pi_t = 1$ for all $t$) to the familiar condition, $\varphi_t = \frac{\varepsilon - 1}{\varepsilon}$, the inverse of the gross price markup.

### 2.2.3 Nash Bargaining

As is typical in the literature, wage payments and hours per employed individual are determined by Nash bargaining between firms and individuals. I follow Thomas (2008) and Arseneau and Chugh (2008) among others by assuming that bargaining is over nominal wages $W_t$ rather than real wages $w_t$ (as typically has been the assumption). This assumption allows focusing on nominal wages, which are the subject of this study. To have a good notion for downward wage rigidity one should focus on the determination of nominal wages, since if bargaining is over real wages, downward real wage rigidity will have no implications for monetary policy. As discussed in Fahr and Smets (2008), Downward Real Wage Rigidity means than nominal wages are indexed to inflation, which in case of full indexation, implies a zero greasing inflation rate. Put differently, the fact that real wages cannot fall following negative shocks regardless of the inflation rate makes grease inflation irrelevant. Given that deviation from price stability is costly, optimal policy will fully stabilize prices. This renders the discussion here less relevant.

Firms and workers split the surplus of a match according to their bargaining power. The asset value for an employed worker from a job is given by

$$V_t^W = W_t h_t - \frac{P_t v(h_t)}{u_{ct}} + \beta E \left[ \frac{u_{ct+1}}{u_{ct}} \left( \frac{P_t}{P_{t+1}} \right) (1 - \rho) V_{t+1}^W + \rho V_t^U \right],$$

where the disutility from work is expressed in terms of the marginal utility of consumption (and hence is equal to the marginal rate of substitution between consumption and labor). Therefore, the asset value for an employed individual is the difference between his current wage payment and the disutility from labor together with the discounted continuation value of staying employed or becoming unemployed next period, with the two events taking place with probabilities $(1 - \rho)$ and $\rho$, respectively.

Similarly, the asset value for an unemployed worker can be expressed as

$$V_t^U = b + \beta E \left[ \frac{u_{ct+1}}{u_{ct}} \left( \frac{P_t}{P_{t+1}} \right) \theta q(\theta)(1 - \rho) V_{t+1}^W + (1 - \theta) q(\theta)(1 - \rho) W_{t+1}^U \right],$$

(21)
which equals unemployment benefits plus the continuation value. The latter is the weighted sum of the values of staying unemployed next period (which occurs with probability \(1 - \theta_t q(\theta_t)(1 - \rho)\)) and becoming employed (which occurs with probability \(\theta_t q(\theta_t)(1 - \rho)\)).

Finally, the value of a filled job for a firm (after suppressing the index \(j\)) is

\[
V_t^V = \phi_t \pi_t f(h_t) - W_t h_t - \frac{\phi}{\psi t} \left( \exp[-\psi_t(\frac{W_t}{W_t^t} - 1)] + \psi_t(\frac{W_t}{W_t^t} - 1) \right) P_t + \beta E_t \left[ \left( u_{r+1} \frac{P_{r+1}}{u_{r}} \right) (1 - \rho) V_{r+1}^V \right].
\] (23)

Therefore, the value of each match equals the flow value of its product net of wage payments and wage adjustment costs plus the continuation value of that match in case of surviving separation.

The Nash bargaining problem is to choose \(W_t\) and \(h_t\) to maximize

\[
(V_t^W - V_t^U)^\eta V_t^{1-\eta}
\] (24)

where \(\eta\) denotes the bargaining power of workers (and their share in the match surplus). In equilibrium, the value of posting a vacancy is zero and hence the threat point of firms is set to zero in the above formulation. The first-order condition with respect to \(W_t\) reads

\[
\eta(V_t^W - V_t^U)^{\eta-1} \left( \frac{\partial V_t^W}{\partial W_t} - \frac{\partial V_t^U}{\partial W_t} \right) V_t^{1-\eta} + (1 - \eta) V_t^{\eta-\eta} \frac{\partial V_t^V}{\partial W_t} (V_t^W - V_t^U)^{\eta} = 0.
\] (25)

Denoting the effective bargaining power of workers by \(\omega_t\), the first-order condition with respect to \(W_t\) can be re-written as

\[
(V_t^W - V_t^U) = \frac{\omega_t}{(1 - \omega_t)} V_t^V,
\] (26)

with \(\omega_t = \frac{\eta}{\eta + (1 - \eta) \Delta_t^{\eta}}\), \(\Delta_t^{W} = -\left( \frac{\partial V_t^W}{\partial W_t} - \frac{\partial V_t^U}{\partial W_t} \right)\) and \(\Delta_t^{F} = \frac{\partial V_t^V}{\partial W_t}\).

If nominal wages are costless to adjust, \(\omega_t\) will be exactly equal to \(\eta\). The wage adjustment cost drives a wedge between the effective and the ex-ante bargaining powers. Also,

\[\text{To see this, notice that if } \phi = 0, \text{ then } \frac{\partial V_t^W}{\partial W_t} = H_t \text{ and } \frac{\partial V_t^V}{\partial W_t} = -H_t. \text{ Hence, } \frac{\Delta_t^{F}}{\Delta_t^{W}} = 1, \text{ and } \omega_t = \eta.\]
since the parameter $\psi$ appears in the expression for $\Delta F$, the presence of DNWR plays here a role in determining the effective bargaining power of workers. As $\psi$ increases, the cost of increasing wages becomes very low and hence the effective bargaining power approaches its ex-ante value, $\eta$.

Combining the job creation condition (19) with the asset value for the firm from a match (23) gives

$$V_t^V = \varphi, z, f(h_t) - W_t h_t - \frac{\phi}{\psi^t} \left[ \exp[-\psi (\pi^*_t - 1)] + \psi (\pi^*_t - 1) - 1 \right] P_t + \frac{\gamma P_t}{q(\theta)} .$$

(27)

It is evident that the more downwardly rigid nominal wages, the lower the value to a firm from a given match. Also, substituting the expression for $V_t^V$ yields the equation characterizing the real wage setting:

$$\frac{\omega}{1 - \omega} \left[ \varphi, z, f(h_t) - w, h_t - \frac{\phi}{\psi^t} \left[ \exp[-\psi (\pi^*_t - 1)] + \psi (\pi^*_t - 1) - 1 \right] + \frac{\gamma}{q(\theta)} \right] = w, h_t - \frac{v(h_t)}{u_t} - b + E \left[ \frac{\omega_{t+1}}{1 - \omega_{t+1}} \left( \frac{\gamma}{q(\theta)} - \gamma \theta \right) \right].$$

(28)

The current wage is affected by the cost of adjusting nominal wages, the outside option ($b$), the disutility from labor and the continuation value of the worker being employed.\(^{18}\)

Finally, the equation characterizing the determination of hours per employed individual is given by

$$\frac{\Gamma_t}{1 - \Gamma_t} \left[ \varphi, z, f(h_t) - w, h_t - \frac{\phi}{\psi^t} \left[ \exp[-\psi (\pi^*_t - 1)] + \psi (\pi^*_t - 1) - 1 \right] + \frac{\gamma}{q(\theta)} \right] = v(h_t) - \frac{v(h_t)}{u_t} - b + E \left[ \frac{\Gamma_{t+1}}{1 - \Gamma_{t+1}} \left( \frac{\gamma}{q(\theta)} - \gamma \theta \right) \right].$$

(29)

where $\Gamma_t = \frac{\eta}{\eta + (1 - \eta) \Delta \delta^F}$, $\delta^W = \frac{\partial V_t^W}{\partial h_t}$ and $\delta^V = \frac{\partial V_t^V}{\partial h_t}$.

\(^{18}\) Condition (29) can also be written in the following way:

$$w, h_t = \omega \left[ \varphi, z, f(h_t) - \frac{\phi}{\psi^t} \left[ \exp[-\psi (\pi^*_t - 1)] + \psi (\pi^*_t - 1) - 1 \right] + \frac{\gamma}{q(\theta)} \right] + (1 - \omega) \left[ \frac{v(h_t)}{u_t} + b \right] + E \left[ \frac{\omega_{t+1}(1 - \omega_{t+1})}{1 - \omega_{t+1}} \left( \gamma \theta - \frac{\gamma}{q(\theta)} \right) \right].$$

Therefore, the wage paid to a worker is a weighted average of the value of his output (net of wage adjustment costs), the value of his outside options, the disutility of work, and the present discounted value of his expected gain from search. In the absence of wage adjustment costs, this expression collapses to the more familiar equation

$$w, h_t = \eta \left[ \varphi, z, f(h_t) + \gamma \theta \right] + (1 - \eta) \left[ \frac{v(h_t)}{u_t} + b \right].$$

Hence, the real wage of a worker is equal to the share $\eta$ of the revenue and saving of hiring costs, and he is compensated by the share $(1 - \eta)$ of the disutility from supplying work and the foregone unemployment benefits.
To find expression (29), the FOC with respect to $h$ was expressed as

$$(V_t^w - V_t^u) = \frac{\Gamma_t}{(1 - \Gamma_t)} V_t^y.$$ 

### 2.2.4 The Private Sector Equilibrium

The equilibrium conditions of the private sector are the consumption Euler equation (5) describing intertemporal choices, the law of motion for employment (13), the job creation condition (19), the price Philips curve (20), the wage setting equation (28), the hours determination equation (29), the resource constraint of the economy given by

$$n_t z_t h_t^\alpha - c_t - \gamma \theta_t u_t - \frac{\phi}{\psi^2} (\exp[-\psi(\pi_t^w - 1)] + \psi(\pi_t^w - 1) - 1) h_t - \frac{\phi^\nu}{2} (\pi_t - 1)^2 n_t z_t h_t^\alpha = 0, \quad (30)$$

the constraint on unemployment

$$u_t = 1 - n_t, \quad (31)$$

and finally, the identity describing real wage growth

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}, \quad (32)$$

which is typically introduced in sticky price and sticky nominal wage models. As explained in Chugh (2006) and Arseneau and Chugh (2008), this identity does not hold trivially in the case of rigid nominal wages and hence it should be added to the equilibrium conditions of the private sector.19

Note that in condition (30), I substitute for $v_t$ using the expression for labor market tightness ($v_t = \theta_t u_t$).

**Definition 1:** *Given the exogenous processes $\{R_t, z_t\}$, the private sector equilibrium is a sequence of allocations $\{c_t, h_t, n_t, u_t, \theta_t, \nu_t, w_t, \pi_t, \pi_t^w\}$ that satisfy the equilibrium conditions (5), (13), (19), (20) and (28)-(32).*

### 2.2.5 The Optimal Monetary Policy Problem

The monetary authority in this economy seeks to maximize the household’s welfare subject to the resource constraint and the first order conditions of individuals and firms (see Appendix

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19 This constraint also appears in the study of Erceg, Henderson and Levin (2000).
2B for the full optimal monetary policy problem). Formally, given the exogenous process for technology $z_t$, the monetary authority chooses \( \{c_t, h_t, n_t, u_t, \theta_t, \varphi_t, w_t, \pi_t, \pi_t^n \} \) in order to maximize (1) subject to (13), (19), (20) and (28)-(32).

### 2.3 SEARCH EFFICIENCY AND THE INTERTEMPORAL WEDGE

In the basic Ramsey theory of public finance, the planner aims at smoothing distortions (or “wedges”) over the business cycle. This is the main insight of the partial-equilibrium “tax-smoothing” result of Barro (1979). Chari, Christiano and Kehoe (1991) show that this result is carried over to a general equilibrium framework. Judd (1985) and Chamley (1986) established that the optimal capital income tax in the steady state is zero and that there are no intertemporal distortions. Albanesi and Armenter (2007) generalize this idea and show that, in the deterministic steady state of a general class of optimal policy problems, it is optimal to achieve zero intertemporal distortion. More recently, Arseneau and Chugh (2010) have shown, within a labor search and matching model, that the Ramsey planner aims indeed at smoothing intertemporal wedges, but that is not mapped into tax smoothing. In the current paper, the only “tax” available to the Ramsey planner is the inflation rate. Hence, I also examine whether the notion of “wedge smoothing” applies to the current setup, and if does, how that is mapped into smoothing the inflation rate.

The derivations, presented in Appendix 2C, give to the following definitions

\[
IMRS = \frac{u_{ct}}{\beta u_{ct+1}},
\]

and,

\[
IMRT = \frac{\gamma}{m_t (1 - n_t, v_t)} \frac{(1 - \rho) \left[ z_{t+1} h_{t+1}^n - \frac{v(h_{t+1})}{u_{ct+1}} + \gamma \left[ 1 - m_t (1 - n_{t+1}, v_{t+1}) \right] \right]}{m_t (1 - n_{t+1}, v_{t+1})}.
\]

These definitions are borrowed from the recent work of Arseneau and Chugh (2010). \( IMRS \) is the intertemporal marginal rate of substitution between consumption choices across periods (put differently, the ratio of marginal utilities at time $t$ and time $t+1$). \( IMRT \) stands for the intertemporal marginal rate of transformation, and it measures the increase in consumption at time $t+1$ as a result of a forgone one unit of consumption at time $t$. 

38
As shown in Appendix 2C, efficiency requires IMRT=IMRS for all t. The efficiency condition can also be written as

\[
1 = (1 - \rho)E_t \left\{ \frac{\beta u_{zt+1}}{u_{zt}} \left[ \frac{z_{zt+1}f(h_{zt+1}) - \psi(h_{zt+1})}{u_{zt+1}} - \Phi_{zt+1}^{w} - \frac{\gamma(1 - \zeta)}{m_{s}(1 - n_{zt+1}, v_{zt+1})} \right] \right\},
\]

(35)

In the decentralized economy, the equivalent condition to (36) is given by

\[
1 = (1 - \rho)E_t \left\{ \frac{\beta u_{zt+1}}{u_{zt}} \left[ \frac{\varphi_{zt+1}z_{zt+1}f(h_{zt+1}) - \psi(h_{zt+1})}{u_{zt+1}} - \Phi_{zt+1}^{w} - \frac{\gamma(1 - \zeta)}{m_{s}(1 - n_{zt+1}, v_{zt+1})} \right] \right\}.
\]

(36)

Comparing the square brackets in condition (35) with the square brackets in condition (36) implicitly defines the “intertemporal wedge”. By comparing (35) with (36), sufficient conditions for efficiency are: nominal wages are fully flexible or fully stabilized (i.e. \( \Phi_{zt+1}^{w} = 0 \)), and hence \( \omega_{zt+1} = \eta \), the Hosios condition holds (\( \zeta = \eta \)), the unemployment benefits are zero (\( b = 0 \)), and no monopolistic power in the final-good sector (which implies \( \varphi_{zt+1} = 1 \)). See Appendix 2D for further details.

The fact that the adjustment cost of nominal wages appears in the term defining the intertemporal wedge is of special significance. When nominal wages are not fully flexible (or not fully stabilized), they induce a direct effect on the intertemporal wedge. But, nominal wage rigidity has also an indirect effect on the intertemporal wedge, which happens through the deviation of the effective bargaining power of workers (\( \omega_{t} \)) from its ex-ante value (\( \eta \)). This is well reflected in the last term of the numerator of condition (36). The fact that nominal wages are downwardly rigid only magnifies the two effects in periods of downturn. Hence, “smoothing” nominal wages is one way to smooth the wedge over time. Setting a positive inflation rate lead, at least partially, to smoothing nominal wages, and hence the intertemporal wedge. This can be easily seen by substituting for the real marginal cost (\( \varphi_{t} \)) using the Phillips curve.
2.4 CALIBRATION

The first subsection discusses the parameterization of the model, and the second subsection presents some discussion about the calibration methodology applied in this study.

2.4.1 Parameterization:

There are two groups of parameters. The values of the first group will be set to conventional values in the existing literature. The second group of parameters, namely the adjustment cost parameters of prices and nominal wages and the measure for DNWR, are obtained to match certain data moments. I follow this approach since most parameters of the model are commonly used in existing literature and hence there is no necessity to estimate them. The adjustment costs parameters will thus be consistent with commonly-used values for other parameters.

I assume the following period utility function:

\[ u(c_t, h_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - n_t \chi \frac{h_t^{1+\vartheta}}{1+\vartheta} \]

I set the parameter \( \sigma \) to 2. The parameter \( \chi \) is set at 2, implying a labor supply elasticity of 0.5. I then calibrate \( \chi \) such that the SS level of hours is 0.3, as is conventional in literature. I assume a time unit of a quarter and hence the discount factor \( \beta \) is set to 0.99.

Output per worker has diminishing returns in hours per worker, as follows:

\[ f(h_t) = h_t^\alpha, \]

where \( \alpha \) is set to 2/3 implying a labor share of output of about 67%, in line with literature.

The matching process between vacancies and unemployed individuals is governed by the following constant return-to-scale function:

\[ m(v_t, u_t) = \sigma_m u_t^{\zeta} v_t^{1-\zeta} \]

The parameter \( \zeta \) measures the elasticity of matches with respect to unemployment and is set here to 0.40 in line with several studies (e.g. Arseneau and Chugh, 2008 and Faia, 2008). The parameter \( \sigma_m \) measures the efficiency of the matching process and is calibrated in my benchmark case to be 0.658. This value has been calibrated assuming that the probability to

\[ \text{Previous studies considered elasticity between 0.1 and 1, corresponding to values of 10 and 1 for } \vartheta, \text{ respectively. I choose here an intermediate value for the elasticity of hours.} \]
fill a vacancy is 0.7 and the probability to find a job is 0.6, as conventional in the literature. The implied steady state value of labor market tightness is 6/7. I therefore calibrate the value of posting vacancies \( \gamma \) to match this SS level of \( \theta \); the value obtained for \( \gamma \) is 0.413. Also, following Shimer (2005) and Arseneau and Chugh (2008), among others, I set the quarterly separation rate \( \rho \) at 0.10.

As is standard in literature, I assume that the Hosios (1990) condition holds and hence that the Nash bargaining power of workers is equal to the contribution of an unemployed individual to the match (i.e. \( \eta = \zeta \)). As shown in Hosios (1990), this condition guarantees the efficiency of the matching process.

Productivity is governed by the following AR(1) process:

\[
\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t
\]

\( \rho \) is set to 0.95 in line with previous literature. The innovation term \( \varepsilon_t \) is normally distributed with zero mean and a standard deviation of \( \sigma_{\varepsilon} = 0.007 \), as typically assumed in the literature.

The parameters governing the adjustment cost functions of prices and nominal wages (i.e. \( \phi^p \), \( \phi \) and \( \psi \)) are estimated using the Simulated Method of Moments (SMM). In my baseline calibration, I choose parameters to match the standard deviations of consumption, wage inflation, price inflation, real wages, hours per employed individual and employment. However, as a robustness check, I also redo my work using other groups of moments to match. The resulting parameter estimates in my baseline calibration are \( \phi^p = 26.9 \), \( \phi = 87.3 \) and \( \psi = 2567.3 \).

### 2.4.2 Computational Solution

The main purpose of this paper is to address optimal monetary policy in the presence of asymmetries in the adjustment of nominal wages. Linearization cannot account for this asymmetry since, by construction, it eliminates the asymmetries of the model. Therefore, I need to apply second-order approximations for the monetary authority’s equilibrium conditions. A second-order approximation allows for the unconditional mean of the variable in the “stochastic steady state” to be different from its deterministic steady state value and to be affected by the size of the underlying shock. Finally, I apply the second order approximation procedure of Schmitt-Grohe and Uribe (2004).
2.5 THE OPTIMAL INFLATION RATE

This section presents the main findings regarding optimal monetary policy under search and matching frictions in the presence of downward rigidity in nominal wages. I first discuss the deterministic steady state and then turn to the dynamics of the model.

2.5.1 The Optimal Inflation Rate in the Deterministic Steady State

The deterministic steady state of the model is invariant to the degrees of price stickiness, wage stickiness and asymmetry in the adjustment of nominal wages. In the absence of shocks, inflation is not beneficial, and due to the direct cost of deviation from complete price stability, the monetary authority completely stabilizes prices (and nominal wages) in the deterministic steady state. This fact is unrelated to whether wages are flexible or rigid.

2.5.2 The Optimal Long-Run Inflation Rate

In this subsection I discuss the dynamics of the model. As a benchmark, I first examine the case with fully flexible wages (i.e. $\phi = 0$). When wages are costless to adjust, and prices are rigid, optimal monetary policy fully stabilizes prices (Panel I, Table 2.1). In this case, all the adjustment of real wages occurs through instantaneous adjustment of nominal wages. When nominal wages are rigid, but the adjustment cost function is symmetric ($\phi > 0, \psi = 0$), the optimal grease inflation rate is zero (Panel I, Table 2.1): in the latter case, the symmetry of the wage adjustment cost eliminates the precautionary motive for inflation.21

This is the main result of the paper is reported in panel III. When nominal wages are downwardly rigid ($\psi > 0$), optimal monetary policy deviates from full price stability in the long run; the optimal annual long-run inflation rate is slightly above 2 percents. This is an important result for, at least, two reasons. First, the optimal inflation rate is significantly higher than in a model with neoclassical labor markets (as for example in the work of Kim and Ruge-Murcia, 2007). This fact suggests that the nature of the labor market in which DNWR is studied can be important for policy recommendations. Second, the optimal inflation rate suggested by this paper is only about half a percent lower than the average annual inflation

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21 In both cases, the 95 percent confidence intervals include the zero. The confidence interval for the case with fully flexible wages is (-0.0057, 0.0020), and the confidence interval for the case with a symmetric adjustment cost of wages is (-0.0193, 0.0221). Hence, the hypothesis that the optimal inflation rate is zero cannot be rejected in either of the two cases.
rate in the U.S. during the last two decades. Hence, the current environment can be seen as one that justifies an inflation rate of more than 2 percents as observed in U.S. data.

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Table 2.1: Simulated moments- Second order approximation. $\phi = 87.3$ and $\phi^p = 26.9$.

$\mu_x$ - the mean of the variable. $\sigma_x$ - the standard deviation of the variable (in percents).

Price inflation and wage inflation are presented in annualized terms.

The model does well in accounting for the volatilities of key macroeconomic aggregates (e.g. output, consumption and hours per worker). The standard deviations of labor market variables (i.e. vacancies, unemployment, unemployment rate, and labor market tightness) are well below their values in actual U.S. data. The failure of the labor search and matching model to account for the true volatilities of labor market aggregates is well known in the literature since the seminal paper of Shimer (2005). The model, however, manages to capture the volatilities of $v$ and $u$ relative to the volatility of $\theta$ as shown in data. The model also captures the relative volatilities of $v$ and $ur$, and the level of the unemployment rate (around 6.25 percent, in line with the literature). Finally, the model with DNWR better accounts for the volatilities of these three variables compared to the model with fully flexible nominal wages and the model with symmetric adjustment cost of nominal wages.
2.5.3 Discussion- The Optimal Inflation Rate and Intertemporal Wedge Smoothing

The strictly positive long-run inflation rate is due to precautionary behavior by the monetary authority. Since the timing (and magnitude) of adverse shocks is not fully predictable, a positive inflation rate over time allows for real wage adjustments when nominal wages are downwardly rigid, and hence it eases job creation and limits the increase in unemployment.

This result can also be rationalized by considering basic Ramsey theory of “wedge smoothing”. As discussed in section (3), DNWR acts, directly and indirectly, towards generating a wedge between the intertemporal marginal rate of substitution and the intertemporal marginal rate of transformation, thus positioning the economy away from the efficient state. Therefore, the Ramsey planner acts in a manner that aims at closing this source of distortion, at least partially. This happens through stabilizing nominal wages to the maximum possible extent. Holding a positive inflation rate over time serves in achieving the real wage adjustments with less nominal wage adjustments and hence lower distortion.

The role of inflation in smoothing the intertemporal wedge is well documented in Figure 2.1. The figure shows the standard deviation of the intertemporal wedge for various optimal inflation rates obtained under different values of the price rigidity parameter ($\phi^P$). Clearly, the volatility of the intertemporal wedge is falling with the optimal annual long-run inflation rate. This finding supports the earlier expectations that positive inflation helps in reducing the volatility of the intertemporal wedge.

In the standard Ramsey theory, the intertemporal wedge should be completely smoothed over time. This is not the result here for the following reasons. First, the monetary authority does not have enough set of instruments to completely and simultaneously stabilize all distortions, including the intertemporal distortion, over the business cycle. Hence, the monetary authority chooses to spread the distortions across all margins. This is a well-known result in the literature (Dupor, 2002). Second, deviations from zero inflation rate are costly, and hence the monetary authority trades-off between stabilizing the inflation rate and stabilizing the intertemporal wedge through stabilization of nominal wages. The convexity of the adjustment costs of nominal wages and prices makes it not optimal to fully stabilize one of the two variables and allow for the other variable to vary.

It is interesting to contrast these findings with the results under a policy that commits to a zero-inflation rate at all dates. Figure 2.3 shows that standard deviation of the intertemporal
wedge for various degrees of DNWR ($\psi$) under both optimal policy and a zero-inflation policy. The standard deviation of the wedge is considerably smaller under a positive inflation rate policy than under a zero-inflation policy, particularly for the relevant values of $\psi$ (which are around 2600 given a benchmark calibration value of 2567.3). Moreover, as the degree of DNWR increases, the difference between the standard deviations becomes bigger. Hence, inflation has a more significant role in smoothing the intertemporal wedge as nominal wages become more downwardly rigid.

The intertemporal wedge is almost entirely smoothed when prices are fully flexible (Figure 2.4). In this case, the monetary authority has more room for policy actions that stabilize the intertemporal wedge. Put differently, the monetary authority faces less trade-offs in conducting policy. Indeed, with fully flexible prices, the intertemporal wedge is essentially smoothed over time, regardless of the degree of asymmetry in the adjustment cost of nominal wages.

Figure 2.5 also helps to shed light on this result. The figure is drawn under the assumption that the monetary authority commits to a certain inflation target at all dates and states. The figure shows the mean levels and the standard deviations of unemployment rate and vacancies as a function of the inflation target (ranging between zero and 3 percents). The unemployment rate drops significantly in the range between zero inflation rate and about 2 percents. On the other hand, vacancy posting increases significantly in this range. The standard deviations of both variables fall considerably as the target increases in this range. Basically, two important conclusions can be obtained from this figure. First, inflation is more important in “greasing” the wheels of the labor market for low inflation rates, as expected. In addition, the trade-off between inflation and unemployment is more significant for lower inflation rates. Second, the marginal “benefit” from increasing the inflation rate approaches zero as we move beyond, approximately, 2 percents. This observation just supports an optimal inflation rate of about 2 percents.

### 2.5.4 The Optimal Inflation Rate - Sensitivity Analyses

The goal of this subsection is to check whether the main result of this paper, an inflation rate of about 2 percents holds once other empirical moments to match are chosen. The results are

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22 In fact, for inflation to be determinate, prices in the current setup cannot be fully flexible. Hence, “fully flexible” prices refer to the case where the adjustment cost of price is set to $10^5$. 

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shown in Table 2.2. In general, I choose here two different cases: in the first, I allow for only 3 moments to match (and so the number of moments equals the number of parameters). In the second, the number of moments exceeds the number of parameters.

<table>
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</table>

Table 2.2: Simulated moments- Second order approximation. Each entry shows the annualized mean of the variable.

The results show that the long run inflation rate is usually around 2 percents, ranging from about 1.80 percents to about 2.44 percents. The higher inflation rate in the latter case (where the standard deviation of labor market tightness is targeted) is due to the fact that in order to account for the high standard deviation of labor market tightness, a higher $\psi$ is needed. This in turn leads to a higher inflation rate.

In general, the analyses in this subsection confirm my earlier result; the optimal long run inflation rate in a model with DNWR and labor market frictions is about 2 percent. In addition, all cases considered the optimal grease inflation rate is considerably higher than the optimal grease inflation rate in Kim and Ruge-Murcia (2007, 2009). Hence, the nature of the labor market in which DNWR is imbedded seems to matter for policy analysis.

### 2.6 IMPULSE RESPONSES

This section describes, under optimal policy, the behavior of the economy following negative and positive productivity shocks of the same magnitude (the size of the shock is one standard deviation of the shock to TFP). Figure 2.6 shows the behavior of the main variables of interest under the presence of DNWR. These variables display asymmetry in their responses to negative and positive shocks, particularly those of big sizes (a big shock is defined here as of two standard deviation size).

Nominal wages do not almost fall following a negative shock, but they increase considerably following a positive shock. The asymmetry in the response of nominal wages is more significant the bigger the size of the shock. Price inflation increases considerably following a negative shock, but it falls only slightly below its SS level following a positive
shock of the same magnitude. This finding confirms the role of inflation in greasing the wheels of the labor market following negative shocks under DNWR.

Unemployment displays asymmetry in the response to productivity shocks and it is more persistent following big negative shocks. The asymmetry in unemployment is less than the asymmetry in price inflation and wage inflation. This fact is due to less asymmetry in the behavior of real wages: since prices and nominal wages almost complement each other following shocks, the fall in real wages following a negative shock (which almost entirely occurs through the increase in prices) is roughly the same, in absolute terms, as the increase in real wages following a positive shock (in which case the adjustment is through both prices and nominal wages). Hence, the asymmetry in unemployment is smaller.

Other variables also display asymmetry in their response to negative and positive shocks. The asymmetry in the behaviors of vacancies and unemployment explain the asymmetry in the behavior of labor market tightness. Finally, there seems to be little asymmetry in the behavior of output and consumption. This is perhaps due to the symmetry in the productivity shock and the relatively small degree of asymmetry in unemployment.

The behavior of the economy following shocks under the presence of DNWR and its behavior under symmetric adjustment cost function are also compared. Figure 2.7 displays the response of the economy to positive and negative shocks in the absence of DNWR. As expected, all variables display symmetry in their response to positive and negative shocks.

Figure 2.8 shows the response of the economy to a negative shock in the three possible cases: fully flexible nominal wages, symmetric nominal wage adjustment cost and DNWR. When nominal wages are fully flexible, not only that inflation is set at zero on average (recall Table 2.1), but inflation is also irresponsible on impact. Given that prices are costly to adjust and nominal wages are fully flexible, nominal wages fall instantaneously to allow for the drop in real wages. The role of DNWR is clearly revealed in this case: the response of inflation to a negative shock under downwardly nominal wages is significantly larger than its response to a negative shock when the adjustment cost is symmetric.

The fall in real wages under fully flexible nominal wages is considerably larger than under wage rigidity (of either type). Unemployment increase only slightly under fully flexible wages and it displays less persistence. In addition, the fall on vacancies and labor market tightness is considerably smaller under fully flexible nominal wages than under rigid wages. The larger falls in unemployment and hours under rigid wages than under fully flexible nominal wages lead to larger drops in output and consumption. In overall, and as expected, the case of
symmetrically rigid wages is an intermediate case between the cases of fully flexible nominal wages and downwardly rigid nominal wages.

The discussion on optimal monetary policy can be summarized as follows. Under fully flexible nominal wages, inflation does not respond to negative shocks and it is always set at zero. When nominal wages are rigid and the adjustment cost function is symmetric, inflation responds to shocks but it is kept at zero on average. Finally, when nominal wages are downwardly rigid, the response of inflation is stronger on impact and monetary policy deviates, on average, from complete price stability.

2.7 PRICE STABILIZATION VS. UNEMPLOYMENT STABILIZATION

This section considers the performance of two extreme policies relative to optimal policy. In the first case, the monetary authority commits to full stabilization of prices at all dates and states (to which I refer as strict inflation targeting or zero-inflation policy). In the second, the monetary authority commits to stabilize unemployment at its steady state level.

When the monetary authority strictly targets zero inflation, unemployment responds more strongly than under the optimal policy (Figure 2.9). The initial response of unemployment for a shock of one standard deviation size is significantly larger than under optimal policy. The peak on the response, after about 3 quarters, is approximately three times as large under strict inflation targeting as under optimal policy. In addition, unemployment displays considerably more persistence under strict inflation targeting. In either case, unemployment displays the typical humped-shaped pattern that has been observed in previous studies with labor market frictions (e.g. Blanchard and Gali, 2008; Krause and Lubik, 2007).

Wage inflation falls more under strict inflation targeting than under constant unemployment and optimal policy (although in either case, the fall in wage inflation is relatively small due to the presence of DNWR). Since prices are fully stabilized, nominal wages must fall by more than under the optimal policy in order to allow for real wage adjustments. Note that the behavior of nominal wages under the constant unemployment regime is almost the same as under the optimal policy.

When the monetary authority fully stabilizes unemployment, inflation displays a stronger response than under the optimal policy. Intuitively, if unemployment cannot increase, prices must increase by more in order to generate the larger required decline in real wages. However,
it is interesting to notice that after about 4 quarters, the behavior of inflation under the constant unemployment regime is similar to its behavior under optimal policy.

The most muted decline in real wages is under strict inflation targeting (which is a result of both DNWR and zero inflation), while the strongest decline is under the constant unemployment regime (because nominal wages remain almost unchanged, while inflation displays a larger increase than under optimal policy). This result is as expected since, when the monetary authority commits to fully stable employment, real wages must fall significantly so that the economy can adjust to the negative productivity shock.

Following the behavior of unemployment, the largest drop in output occurs under strict inflation targeting. Output reaches its trough after about 3 quarters, when unemployment peaks (and hours per worker reach their lowest point). The behavior of consumption is similar to the behavior of output, while the largest decline in labor market tightness occurs under strict inflation targeting, as vacancies fall strongly and unemployment increases considerably.

These responses suggest that strict inflation targeting is far from being optimal under DNWR. Full stabilization of prices in the presence of DNWR limits the ability of the economy to adjust to adverse shocks. Stabilizing unemployment, however, delivers responses similar to those under optimal policy. Welfare analyses show that welfare under a zero-inflation policy is lower by roughly 8.48 percent than welfare under the optimal policy. Welfare under full stabilization of unemployment is lower by only 0.30 percent than welfare under optimal policy.

The results of this section are in line with recent studies that suggest the need for deviations from full price stability. Blanchard and Gali (2008) show that strict inflation targeting delivers a welfare loss which is more than twice as large as under full stabilization of unemployment, and more than four times as large as under the optimal policy. My results here suggest higher welfare loss under strict inflation targeting than in Blanchard and Gali (2008), which can possibly be attributed to DNWR that makes zero-inflation policy even more undesirable. Faia (2008) suggests that, in the presence of real wage rigidities, the optimal Taylor-type rule should respond to unemployment alongside with inflation. Thomas (2008) also argues for incomplete stabilization of prices following shocks when nominal wages are rigid. Although these studies and the current one may differ in their focus (i.e. the type of wage rigidity), they all suggest that optimal policy should deviate from price stability following shocks. This study shows that, due to precautionary behavior, DNWR also leads to a significant deviation from full price stability on average.
This paper studies the optimal long-run inflation rate within a labor search and matching framework in the presence of downward nominal wage rigidity. When nominal wages are downwardly rigid, the optimal long-run inflation rate is around 2.0 percent. Optimal monetary policy deviates from full price stability to allow for real wage adjustments, particularly following adverse shocks, which promotes job creation and prevents an excessive increase in unemployment.

The results of this paper are related to Ramsey theory of smoothing wedges over time. In this study, the concern is over the “intertemporal wedge”, which is defined here, generally speaking, as the deviation of the intertemporal marginal rate of substitution from the intertemporal marginal rate of transformation. Importantly, the asymmetric adjustment cost of nominal wages is part of this wedge. By setting a positive inflation rate, the Ramsey planner acts towards smoothing the intertemporal wedge and hence taking the economy closer to the efficient allocation. Indeed, the results suggest that the size and volatility of the wedge are both falling in the inflation rate as the degree of price rigidity is varied. The intertemporal wedge is virtually constant over time when prices are fully flexible.

Committing to a zero inflation rate over the business cycle has been found to perform the worst among other alternatives. Under a zero-inflation policy, the volatility of the intertemporal wedge is significantly higher (about 4 times as large as under optimal policy with sticky prices). The findings regarding labor market aggregates are similar; if the monetary authority strictly targets a zero inflation rate, the increases in unemployment is significantly larger than under the optimal policy. In addition, unemployment displays significantly higher volatility and reaches a higher level on average under full price stability, while vacancies are lower on average and far more volatile than under optimal policy.

The current paper can be further extended. One natural extension is to evaluate the performance of different Taylor-type rules compared to the optimal policy. Another extension is to allow for endogenous participation in the labor force. Finally, future work may consider the optimal long-run inflation rate in an economy with labor market frictions, price rigidity, DNWR and monetary distortions. It will be interesting to study the optimal inflation rate in this environment giving that each distortion calls for a different inflation rate.
CHAPTER 3

Sticky Wages, Incomplete Pass-Through and Inflation Targeting: What is the Right Index to Target?

3.1 INTRODUCTION

During the last two decades, Inflation Targeting has emerged as the main monetary policy regime of several countries. These countries differ in many aspects, like their development levels, sizes, openness degrees, labor markets and foreign exchange markets. These differences are perhaps the reason behind the debate on monetary policy in the era of Inflation Targeting. They also raise the question, whether countries with different characteristics should follow the same monetary policy once they commit to a region-wide policy.

This paper attempts to characterize monetary policy rules for various structures of economies. The main focus of the study is on the flexibility of labor markets, reflected on wage rigidity, and on the degrees of sensitivity of consumer prices to movements in exchange rates- the degree of pass-through. A country adopting the inflation targeting (IT) regime can either target the Consumer Price Index (CPI), which embodies the prices of imported goods, or targets a measure of Domestic Price Index (DPI) and allows for exchange rates adjustments. Based on the contribution of Gali and Monacelli (2005), hereafter GM, the current study revisits this topic by discussing the implications of allowing for both rigid import prices (i.e. incomplete pass-through) and rigid nominal wages on monetary policy making. In particular, the study considers three alternative strict monetary policy rules: full stabilization of the CPI, full stabilization of the DPI and full stabilization of nominal wages. Stable nominal wages can be seen as an intermediate goal for monetary policy. Previous literature had the focus on CPI vs. DPI targeting. Study the desirability of nominal wage targeting is one contribution of the current study.

In general, the paper shows that the right index to target depends on the structure of the individual economy; some countries may find targeting CPI better than targeting the alternative, while other economies may better choose to target their DPIs. Targeting nominal

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23 The rigidities in both nominal wages and imports prices have been reported in several studies of recent years. Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005) among others show evidence for wage rigidity. Campa and Goldberg (2005) among others show evidence for incomplete pass-through from exchange rate movements into import prices.
wages is favorable for countries with relatively high degrees of nominal wage rigidity and wage indexation. These findings may imply that adopting the same regime for countries that differ in their structures, as is this case in common-currency areas, may not be desirable for some nations.

The choice of the right index has been discussed in several recent theoretical studies. In a relatively similar model to GM, Clarida, Gali and Gertler (2001) found DPI to be better to target giving complete pass-through and fully flexible wages. Assuming complete pass-through, but rigid wages in the GM framework, Campolmi (2006) showed that, given positive indexation levels, the best Taylor Rule to follow is the one targeting wage inflation and CPI inflation. In a model with fully flexible wages, Devereux, Lane and Xu (2006) also recommend targeting DPI in high pass-through environments. They however show that targeting the CPI is more desirable when pass-through is low. Based on typical interest rate feedback rules, Huang and Liu (2005) suggest that the Monetary Authority (MA) should respond to a weighted average of CPI and DPI. Recently, Flamini (2007) suggest that targeting DPI is better even when pass-through is low. In general, the rationale behind the favorability of DPI targeting is that targeting the CPI requires responding to exchange rates movements which makes interest rates, and hence real activity, more volatile.

The model assumed here is a standard New Keynesian (NK) framework calibrated for a prototype small open economy. I assume that domestic prices, import prices and nominal wages are rigid. Wages that are not reset during a given period are indexed to past CPI inflation. The original work of GM abstracted from rigidity in nominal wages and imports prices and it supported targeting the DPI.

Wage stickiness and wage indexation to the CPI might be crucial for the choice of the right index to target. The indexation scheme gives a rise for CPI stabilization, since a variable CPI leads to variable aggregate wages and hence to more volatile marginal costs of domestic firms. This renders full stabilization of the DPI harder and costly to achieve. Indeed, in a closed economy framework, Erceg, Henderson and Levine (2000) show that strict Inflation Targeting is suboptimal when both prices and nominal wages are rigid.

Relaxing the assumption of complete pass-through (CPT) is another empirically-plausible modification. One advantage of a floating exchange rate is that it adjusts in response to external shocks and thus helps stabilizing the real economy (Devereux, Lane and Xu 2006). But, when pass-through is high, this boosts inflation. Therefore, if the Monetary Authority targets the CPI, any movement in the exchange rate requires stronger response, which leads to
higher variability in both the interest rate and the output gap. This renders CPI targeting less desirable. When pass-through is low, however, the cost of the variability of exchange rate is relatively low and hence the MA can target CPI inflation and still have the exchange rate responding to external shocks. Thus, the desirability of CPI targeting rises in this case. 24

This paper also considers the possibility of targeting nominal wages (i.e. targeting zero wage inflation rate). In this sense, nominal wages can be seen as an intermediate goal for monetary policy, since the stabilization of nominal wages helps stabilizing the marginal cost and hence domestic prices. The degree of indexation to CPI inflation is an important factor in determining the desirability of targeting nominal wages.

The key findings of this paper are as follows. When wages are fully flexible, CPI targeting is favorable in low to moderate pass-through degrees (around 0.40 or lower). On the other hand, when pass-through is complete, CPI is found to be favorable when both wage rigidity and indexation levels are high (around 0.75 or more). When both of the two frictions are incorporated, CPI is better to target for relatively high levels of pass-through, wage rigidity and indexation. The relevant degrees of pass-through, wage rigidity and indexation are, in general, in line with some empirical findings. Also, for high degrees of wage rigidity, indexation and pass-through, it might even be better to target nominal wages rather than the DPI. Finally, in other cases, the study shows that targeting the DPI is favorable.

Given these findings, one may wonder whether adopting a similar monetary policy for a group of countries that differ in their labor markets, pass through and domestic products markets, is desirable. Once committing to a region-wide policy, some countries may indeed be conducting monetary policy correctly. Others, however, may not be doing so. Although region-wide policy may has its advantages over time, it renders some countries committing to a policy rule that would not otherwise been chosen. A research that study differences between countries empirically can be helpful to assess these conjectures.

The remainder of the paper proceeds as follows. Section 2 outlines the open economy macro model with the proposed modifications. Section 3 describes the calibration methodology and the parameterization of the model. Section 4 presents the main results of the study. Some sensitivity analyses are presented in section 5. Section 6 concludes.

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24 Note that I abstract here from imported inputs. Allowing for imported inputs can significantly enrich the model since in this case marginal costs of domestic firms will be directly affected by movements in the exchange rate and therefore stabilizing domestic inflation will be harder to achieve.
3.2 THE MODEL

This section describes the model economy, a modification of the GM framework. The paper relaxes both the assumptions of complete pass-through and fully flexible wage setting. Since the model is based on GM and to keep the focus on the main modifications, in what follows I only outline the main setup of the model. Therefore, in several occasions the reader may refer to their work, as well as Monacelli (2005), for further details.

3.2.1 Households
The representative household in our Small Open Economy has an access to complete foreign asset markets and seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(C_t) - V(N_t)],$$

where $N$ denotes labor and $C$ stands for composite consumption. The maximization is subject to the following sequence of budget constraints

$$\int_0^1 [P_{H,t}(i)C_{H,t}(i)]di + \int_0^1 [P_{F,t}(i)C_{F,t}(i)]di + E_tQ_{t,t+1}D_{t+1} \leq D_t + W_tN_t + T_t,$$

with $P_{H,t}(i), P_{F,t}(i), C_{H,t}(i)$ and $C_{F,t}(i)$ denoting the price of domestic good $i$, the price of foreign good $i$ and their quantities respectively. $W_t$ is the nominal wage, $Q_{t,t+1}$ is a stochastic factor in time $t+1$ of the portfolio hold at the end of time $t$, $D_{t+1}$. $T_t$ denotes the net lump sum transfers.

Composite consumption is given by

$$C_t = \left[ (1-\alpha)^{\frac{\eta-1}{\eta}} C_{H,t}^{\frac{\eta}{\eta-1}} + \alpha^{\frac{\eta-1}{\eta}} C_{F,t}^{\frac{\eta}{\eta-1}} \right]^\frac{1}{\eta},$$

with $C_{H,t}$ and $C_{F,t}$ denoting aggregate consumption of domestic (home) goods and foreign goods, respectively. The parameter $\alpha$ represents the degree of openness of the economy while $\eta$ measures the elasticity of substitution between home and foreign goods. Both domestic and foreign consumption are given by the following CES aggregators.
In the above setup, the general price level, i.e. the Consumer Price Index (CPI), is given by

\[ P_t = \left[ (1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \]

Assuming the following functional forms

\[ U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \quad \text{and} \quad V(N_t) = \frac{N_t^{1+\varphi}}{1+\varphi} \]

maximizing preferences subject to the sequence of budget constraints give the following (Log-Linearized) optimality conditions

\[ w_t - p_t = \alpha \pi_t + \varphi \eta_t, \quad \text{(5)} \]

\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} \left( r_t - E_t \pi_{t+1} - \rho \right), \quad \text{(6)} \]

where a lower case letter denotes the log of the respective upper case variable, \( w_t \) is the wage rate, \( \pi_t \) is the CPI inflation rate, \( \sigma \) is the risk aversion parameter, \( \varphi \) is the inverse of the labor supply elasticity and \( \rho \) represents time preference. Condition (5) states that, in equilibrium, households equate the marginal rate of substitution between consumption and leisure to the real wage. Condition (6) is the typical Euler Equation in consumption, to which we also refer as the New Keynesian IS curve.

### 3.2.2 Firms

As is typical in NK models, each firm \( j \) is monopolistically competitive and a produces a differentiated good with a linear technology in labor of the form

\[ Y_t(j) = A_t N_t(j), \quad \text{(7)} \]

with \( A_t \) denoting technology. The aggregate production in this economy can be written (in a Log-Linearized form) as \( y_t = a_t + n_t. \)

Cost minimization by domestic firms gives the following expression for the real marginal cost
\[ mc_t = w_t - p_{H,t} - a_t - \nu, \]  

where \( \nu \) is an employment subsidy that offsets the market power of firms. Prices set by domestic firms are assumed to be staggering (as in Calvo, 1983), with only a fraction \( 1 - \theta_H \) allowed to reoptimize each period. Other firms simply keep their prices at time \( t \) similar to time \( t-1 \) prices. The Domestic Price Index (DPI) can thus be written as

\[ p_{H,t} = (1 - \theta_H) \tilde{p}_{H,t} + \theta_H p_{H,t-1}, \]

where \( \tilde{p}_{H,t} \) stands for the price set by firms who are allowed to change prices. Finally, the last result can be combined with the expression for the marginal cost to obtain the following forward-looking Phillips Curve (or the AS Curve) for domestic prices:

\[ \pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda_H m \hat{c}_t, \]

where \( \lambda_H = \frac{(1 - \theta_H)(1 - \beta \theta_H)}{\theta_H} \).

### 3.2.3 The Real Exchange Rate and Pass-Through

In this subsection I discuss how does the introduction of incomplete pass-through (IPT) affect the setup of GM (2005). Note first that Log-Linearization for CPI inflation is given by

\[ \pi_t = \alpha \pi_{F,t} + (1 - \alpha) \pi_{H,t}, \]

Also, the real exchange rate can be written as

\[ q_t = e_t + p_{t}^* - p_t, \]

with \( p_{t}^* \) and \( e_t \) being foreign prices and the nominal exchange rate, respectively. In terms of our model, complete pass-through (or the Law of One Price) implies that \( p_{F,t} = e_t + p_{F,t}^* \).

Assuming that the Rest of the World is big and hence the prices of the SOE are negligible in determining foreign prices, we have \( p_{F,t}^* = p_t^* \). Therefore

\[ p_{F,t} = e_t + p_{F,t}^*. \]

To allow for incomplete pass-through, I follow Monacelli (2005) and assume that each period only a fraction \( 1 - \theta_F \) of the local import retailers are allowed to change their prices. Also, retailers import foreign goods at a price (cost) of \( e_t \cdot P_{F,t}^* \) and charge a price of \( P_{F,t} \) for these goods. The above setup leads to an analog expression of (9) given by

\[ p_{F,t} = (1 - \theta_F) \tilde{p}_{F,t} + \theta_F p_{F,t-1}. \]
Having IPT in place, the deviation from the Law of One Price (LOP) is measured by
\[ \psi_{t,F} = (e_t + p_t^*) - p_{t,F}. \]  

In this setup, we can think about the deviation from the LOP as a marginal cost for the importers: they import foreign goods with a price of \((e_t + p_t^*)\) but charge only \(p_{t,F}\).

Given IPT, the real exchange rate can be written now as (with \(s_t\) denoting the terms of trade)
\[ q_t = (1 - \alpha)s_t + \psi_{t,F}. \]  

Also, one can obtain an analog for (10) given by
\[ \pi_{t,F} = \beta E_t \pi_{t,F+1} + \lambda_t \psi_{t,F}, \]  
where \(\lambda_t = \frac{(1 - \theta_F)(1 - \beta \theta_F)}{\theta_F}\). Therefore, import price inflation is higher the higher \(\psi_{t,F}\) is.

Also, the parameter \(\theta_F\) plays a major role in determining import price inflation. Other things equal, a lower \(\theta_F\) (implying higher degree of pass-through) leads to higher import price inflation. As discussed in Monacelli (2005), there are two sources for fluctuations in the real exchange rate. The first, due to terms of trade fluctuations, is captured by the first term in (16). The second arises because of deviations from the LOP.

### 3.2.4 Wage Setting

Motivated by the empirical evidence of wage rigidities reported in several papers in recent years (e.g. Christiano, Eichenbaum and Evans 2005; Smets and Wouters 2003), this paper relaxes the assumption of fully flexible wages. In particular, the aggregate labor input of each firm is given by a CES function of the different types of labor inputs hired. Formally,
\[ N_{i,t} = \left( \int_0^1 N_{i,t-j}(j) \right)^{\varepsilon_u - 1} \varepsilon_u, \]
where \(\varepsilon_u\) denotes the elasticity of substitution between different labor types. In addition, only a fraction \((1 - \theta_w)\) of households can reset their wages (to \(\tilde{w}_t\)) each period, while other households (partially) index their wages to past CPI inflation.\(^{25}\) Such an indexation scheme

\(^{25}\) Smets and Wouters (2003) reported a degree of indexation of about 0.75 for the EURO area.
appears in both Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005). Under these assumptions, the aggregate wage level each period is given by

\[ w_t = (1 - \theta_w)\tilde{w}_t + \theta_w w_{t-1} + \gamma w \theta w \pi_{t-1} w_{t-1}, \]

with \( \gamma \) capturing the degree of indexation (e.g. \( \gamma = 1 \) corresponds to full indexation).

Denoting the wage markup by \( \mu^w_t \) and the deviation of the markup from its frictionless level by \( \hat{\mu}^w_t \) give the following expression for wage inflation

\[ \pi_{w,t} = \beta E_t \pi_{w,t+1} - \lambda_w \hat{\mu}^w_t - \theta_w \gamma w \beta \pi_t + \gamma w w \pi_{t-1}, \]

where \( \lambda_w = \frac{(1 - \theta_w)(1 - \beta \theta_w)}{\theta_w (1 + \phi \theta_w)} \). \( \hat{\mu}^w_t = (w_t - p_t) - (\sigma \psi + \phi \pi) - \mu^w \) and \( \mu^w \) being the wage markup in a frictionless environment. Note that the first two terms of (19) are typical in wage inflation equations with no indexation (e.g. Gali 2002). Current wage inflation is higher the higher the expected future wage inflation. If the markup is higher than its frictionless level (i.e. \( \hat{\mu}^w > 0 \)), then wage inflation today tend to decrease in order to prevent a situation of losing competitiveness in the labor market. The indexation scheme assumed here introduces two more terms that will turn to be significant. Past inflation have a positive effect on wage inflation since workers who are not allowed to reset their wages at time \( t \) will have higher wages the higher past inflation is. On the other hand, because of indexation, households today know that even if they cannot reset their wages next period, the higher current inflation implies higher wages next period.26

Having sticky wages together with incomplete pass-through, it will prove useful to rewrite the above expressions for both DPI and CPI inflation rates in a more explicit way. Note first that the marginal cost of domestic firms can be rewritten as follows

\[ mc_t = (\varphi + \sigma) x_t + (1 - \frac{\omega}{\omega_s}) \psi_{f,t} + \hat{\mu}^w, \omega = 1 + \alpha(\sigma \eta - 1) \text{ and } \omega \psi = 1 + \alpha(2 - \alpha)(\sigma \eta - 1), \]

with \( x \) being output gap (the difference between output and its frictionless level). In GM, only the first term in the right hand side appears. The two modifications clearly affect the determination of the marginal cost of domestic firms: a fluctuating wage markup or LOP gap

\[ 26 \text{ In other words, households balance between low wages today in expectation of higher wages in future through the indexation channel. The reason is that setting too high wage today will render them losing some competitiveness in labor market.} \]
leads to a less stable marginal cost. Note also that the expression for $m\hat{c}$ can be substituted into (10) to obtain

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_\delta x_t + \kappa_\psi \psi_{f,t} + \lambda_H \hat{\pi}_t^{\psi}, \quad \kappa_\psi = \lambda_H (\varphi + \frac{\sigma}{\sigma_0}), \quad \kappa_{\varphi} = \lambda_H (1 - \frac{\sigma}{\sigma_0}). \quad (21)$$

The last two terms of (21) introduce more tradeoff for monetary policy makers and they endogenously justify the ad-hoc cost-push shock assumed in some NK studies. In particular, the Monetary Authority cannot stabilize DPI inflation, the output gap, the deviation from the LOP and the wage markup simultaneously. To see this consider for example a positive productivity shock. As a result, the output gap falls but there is a nominal depreciation that boosts the LOP gap (assuming $\kappa_{\varphi} > 0$). Increasing the interest rate to close the LOP gap will result in higher output gap. On the other hand, if the MA attempts to fully stabilize the output gap by lowering the interest rate, the LOP will rise thus boosting CPI inflation.

There is another reason for the inability of the monetary authority to fully stabilize all prices and wages when prices and wages are rigid. Since the path of the real wage is tied to the path of the marginal product of labor (i.e. technology), the real wage fluctuates with the fluctuations in technology. In this case, full stabilization of the wage inflation and price inflation is inconsistent with this path. Hence, the monetary authority should choose the best combination of price and wage stabilization that, on one hand allow for real wages to adjust, while on the other, leads to lower welfare losses.

Finally, by using the definition of CPI inflation (equation 11), a similar expression for CPI inflation is obtained

$$\pi_t = \beta E_t \pi_{t+1} + (1 - \alpha) \kappa_\delta x_t + ((1 - \alpha) \kappa_\psi + \alpha \lambda_H \hat{\pi}_t^{\psi} + \lambda_H \hat{\pi}_t^{\psi}. \quad (22)$$

As for the case of domestic price inflation, the presence of both rigid wages and import prices introduce more tradeoffs for policy making. Notice also the importance of the openness degree in this expression and in particular its role in the tradeoff between inflation stabilization and output gap stabilization. For this reason, the calibration part will devote special attention to the openness degree by presenting the effects of varying this parameter on the benchmark result.

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27 The lack of such tradeoff is called the Divine Coincidence in the terminology of Blanchard and Gali (2007) and it requires introducing more factors that the policy makers should account for.

28 Contrary to the case of DPI inflation, the tradeoff between stabilizing CPI inflation and other variables (i.e. output gap) exists also in the case of $\kappa_{\varphi} = 0$ (because of the term $\alpha \lambda_H \hat{\pi}_t^{\psi}$).
3.2.5 Optimal Monetary Policy and Inflation Targeting

As is typical in the NK literature, the optimal monetary authority is maximize the following welfare (loss) criterion subject to conditions A1-A7 in Appendix 3A:

\[
W = -\frac{(1-\alpha)}{2\omega} \sum_{i=0}^{\infty} \beta^i [(1 + \varphi)x_i^2 + \frac{\varepsilon_H}{\lambda_H} \pi_{H,i}^2 + \frac{\varepsilon_w}{\lambda_w} \pi_{w,i}^2 + \beta \gamma^2 \frac{\varepsilon_w}{\lambda_w} \pi_{w,i}^2 + \frac{\varepsilon_F}{\lambda_F} \pi_{F,i}^2] .
\]  

(23)

As shown in GM among others, this welfare function can also be written as

\[
L = -\frac{(1-\alpha)}{2\omega} [(1 + \varphi)Var(x_i) + \frac{\varepsilon_H}{\lambda_H} Var(\pi_{H,i}) + \frac{\varepsilon_w}{\lambda_w} Var(\pi_{w,i}) + \beta \gamma^2 \frac{\varepsilon_w}{\lambda_w} Var(\pi_{w,i}) + \frac{\varepsilon_F}{\lambda_F} Var(\pi_{F,i})] .
\]  

(24)

Here, \( \text{Var}(z) \) denotes the variance of the variable \( z \). In GM (2005), only the first two terms of (24) appear (i.e. the variances of both domestic inflation and the output gap). In this paper, however, the welfare criterion includes three more terms, namely the variabilities of wage inflation, CPI inflation and import price inflation. The Monetary Authority cannot now stabilize domestic inflation costlessly.

3.2.6 Policy Target in the Rest of the World

Following Gali and Monacelli (2005) and Monacelli (2005), I assume that the Monetary Authority in the Rest of the World simultaneously stabilizes foreign inflation and output gap and hence replicate the flexible price allocation. Note that under the assumption that the Rest of the World is too big, foreign CPI coincides with foreign DPI and hence the insignificance of the issue of pass-through. Also, for simplicity, I keep to assume no wage rigidity in the foreign economy. In short, the ROW is assumed to be as in study of GM (2005).

3.3 CALIBRATION

To allow for good comparisons to GM, I will use their parameter values in my benchmark calibration, although some sensitivity analysis will be presented later. As in their study I assume logarithmic utility in consumption (and hence \( \sigma \) is set to 1). This assumption makes the derivation of the welfare criterion simpler. Next, \( \varphi \) is set to be 3 implying a labor supply elasticity of 1/3. All gross markups (of domestic firms, importers and workers) will be set to 1.2 and hence all the elasticities are assumed to be 6 (i.e. \( \varepsilon_H = \varepsilon_F = \varepsilon_w = 6 \)). Domestic prices are assumed to be readjusted every one year, and therefore the parameter \( \theta_H \) takes the value of
0.75. The openness degree is set to 0.4 which implies ‘home bias’ in consumption. The degrees of wage rigidity, pass-through and indexation will be varied in the analysis below.

One major change will be in the elasticity of substitution between domestic and foreign goods (η) assumed to be unity in GM (2005). I follow Monacelli (2005) and set it to 1.5 in the baseline calibration although the effects of varying this parameter will be discussed in the sensitivity analysis section. The reasons for setting η different from one are twofold. First, setting η to 1 is a special and perhaps a restrictive assumption. Second, setting η to unity, together with σ =1, makes both ωϕ and ωζ being 1 and hence κϕ is zero. But, this renders the third term in the right hand side of (25) insignificant. Hence, in this case the whole idea of assuming incomplete pass-through is missed since there is no tradeoff stemming from this channel. Setting η =1.5 means that κϕ is positive and hence all the discussion becomes more relevant.

Note however that as discussed in GM (2005), when ση differs from one, some equations hold only up to first order approximation, while they hold exactly when ση is one. Hence, in choosing η, I trade-off between exact relationships on one hand and gaining some intuition from the other (which is more likely when η from one). Given the Linear-Quadratic (LQ) approach applied here, this assumption clearly adds to the relevance of the discussion with only a mild expense in terms of precision.

3.4 RESULTS

This section presents the main results of the study. The first subsection shows the results of the paper when only IPT is allowed for (i.e. wages are fully flexible). Subsection 4.2 presents my findings in an environment of rigid wages but complete pass-through. By so doing, we are able to see the effects of each modification on the choice of the monetary policy rule. Finally, subsection 4.3 presents the results of the study when both rigid wages and incomplete pass-through are allowed. The last step helps to assess monetary policy in a more realistic environment in which domestic prices, imported prices and nominal wages are not fully flexible.

29 In this regard, the parameter ωζ in the welfare function differs from one. Assuming η = 1.0 gives ωζ = 1 and hence delivers a loss function similar to that of GM (2005), but of course with more terms.
3.4.1 Incomplete Pass-Through and Fully Flexible Nominal Wages

Figure 3.1 shows the difference in welfare losses, the loss under CPI targeting minus the loss under DPI targeting for various degrees of pass-through (all losses are expressed in terms of steady state consumption). When PT is complete, targeting the DPI is highly favorable. DPI is better to target also in the case of intermediate to high degrees of PT, although the difference in losses shrinks. However, when PT is relatively low (around 0.43 or lower), it is better to target the CPI. This is the first important finding of the current study, and it suggests the significance of relaxing the assumption of complete pass-through.\(^{30}\)

To better understand these results, consider the behavior of our main variables under different degrees of PT (Figure 3.2). When PT is complete, targeting domestic prices delivers zero output gap’s variability, and hence the zero loss. On the other hand, when CPI is targeted, the variabilities of both DPI and output gap are relatively high. Reducing the degree of PT makes things different. As the degree of PT falls, the variabilities of both domestic inflation and the output gap under CPI targeting fall, thus implying lower welfare loss. In this case however, another factor comes into play- the variability of imported prices. Figure 3.2 shows that a country targeting its DPI allows for more fluctuations in imported prices. When the loss function is expanded to include the variability of these prices, the loss under DPI may turn to be higher. Our results above indicate that this is indeed the case.

Before closing this subsection I present the effects of varying the degree of openness on my main results (Table 3.1 and figures 3.3 and 3.4). Since my focus is mainly on the desirability of CPI targeting versus DPI targeting, I only show the losses under these two regimes. Also, the table presents the results for complete PT and then for PT of 0.35 and lower. I choose these values since complete PT is a useful benchmark, and for PT of 0.35 or lower, DPI may not be the right index to target. Contrarily, the favorability of DPI targeting seems to hold when the pass-through degrees is between 0.35 and 1.0 given that the economy is not completely open. Note that for scale reasons, I present the actual losses and the differences in losses between CPI targeting and DPI targeting and not the relative losses as I will do later.\(^{31}\) A positive difference in losses indicates that the DPI is better to target.

\(^{30}\) Notice also that in the limit (when PT is almost zero), imported prices are fully rigid and hence the two regimes coincide. In particular, if imported price are fully rigid, the only variability in CPI comes from domestic prices. Hence, setting domestic inflation to zero implies zero CPI inflation and vise versa.

\(^{31}\) When PT is complete, DPI targeting delivers zero loss while CPI targeting delivers positive losses. Therefore, dividing the loss under CPI by the loss under DPI creates scale problems.
Few observations are worth-noting. First, targeting CPI leads to lower welfare loss when PT is relatively low regardless of the openness degree. This is a significant finding since as shown in Campa and Goldberg (2005), few Inflation Targeting countries have degrees of PT around 0.40 or lower. Second, regardless of the openness degree, in the case of complete pass-through, targeting CPI cannot be favorable. Third, for a given openness degree, lowering the degree of pass-through gives higher loss under DPI targeting (with the exception of course of the case of zero PT). Contrarily, for a given openness degree, the loss under CPI targeting tends to decrease as PT falls. Finally, When PT is zero, import prices are fully rigid and therefore import price inflation is zero. In this case, the only source for fluctuations in the CPI is domestic price volatility. Hence, fully stabilizing domestic prices will fully stabilize consumer prices and vise versa (i.e. the two regimes coincide). In overall, the results here indicate that my earlier finding is robust to varying the degree of openness in the more plausible ranges.

3.4.2 Complete Pass-Through and Nominal Wage Rigidity

This subsection assumes perfect pass-through, but considers the case of rigid nominal wages. As discussed above, the degree of indexation ($\gamma_w$) is another important parameter to account for in this case. Hence, in what follows I will outline the results for some levels of wage rigidity as well as for specific degrees of indexation.

Figure 3.5 shows the losses under CPI and WPI targeting relative to the loss under DPI targeting. Hence, DPI serves here as a benchmark. I choose to compare the losses under CPI and WPI relative to DPI since the later has been typically suggested as the best to target. Also, Figure 3.5 assumes an indexation degree of 0.75 in line with empirical findings. It should be noted however that all results reported here holds qualitatively also for higher indexation degrees and in particular when indexation is full (i.e. $\gamma_w = 1$). Depending on the wage rigidity degree, some of the results hold also when the indexation degree is relatively low (around 0.65).

The main result is that the relative loss under both CPI targeting and nominal wage targeting is lower when the wage rigidity degree is around 0.80 (zero wage inflation is the best even for less than 0.80). The main explanation for this finding is as follows. When nominal wages are “fundamentally” highly rigid (i.e. $\theta_w$ is high), stabilizing nominal wages by policy means is relatively less costly than stabilizing the domestic price index. That is, the nature of
the labor market makes the costs of full stabilization of wages being relatively low: When nominal wages are “fundamentally” rigid, the monetary authority needs less manipulations of the interest rate in order to fully stabilize nominal wages. Although, of course, the rigidity in nominal wages affects the output gap considerably, it is still less costly than implementing a policy that aims at stabilizing domestic prices when nominal wage are highly indexed to CPI inflation or when nominal wage are highly flexible. Also, the stabilization of nominal wages helps stabilizing domestic prices and this offsets some of its negative effect on welfare through the output gap. In other words, targeting nominal wages delivers both zero wage inflation and lower variability in domestic inflation, hence lower welfare loss.

The result that targeting the CPI might be favorable for relatively high degrees of wage rigidity and wage indexation confirms our earlier expectations and it is the second important finding of the current study. Note that the required levels of wage rigidity and wage indexation to prefer WPI or CPI targeting over DPI targeting are in line with some empirical studies (e.g. Smets and Wouters 2003; Christiano, Eichenbaum and Evans 2005) and hence the importance of this result.

Two more important observations come out from Figure 3.5. First, the relative losses under either CPI targeting or WPI targeting are higher for low to moderate degrees of wage rigidity and they are actually increasing when $\theta_w$ varies between zero and 0.40. The main reason is the low loss under DPI targeting for relatively low degrees of wage rigidity. To see this, note that the losses under the three type of regimes are increasing in $\theta_w$, indicating higher nominal distortions. However, for $\theta_w$ less than 0.40, the loss under DPI increases by less compared to the other two regimes. This pattern changes for higher levels of $\theta_w$ since at some point the effect on the output gap under DPI targeting becomes very high and its targeting outweighs the losses under the two other indices.

Second, since the relative loss under CPI targeting is typically higher than relative loss under WPI targeting, we also infer that the loss under CPI targeting tends to be higher than under WPI targeting, especially for moderate levels of wage rigidity. It therefore seems that stabilizing nominal wages is the best policy to follow when the degrees of wage rigidity and indexation are around 75% or higher. Note however that when $\theta_w$ approaches 1, the two

---

32 Also, Bodart et al. (2006) and Bockerman, Laaksonen and Vainiomaki (2006) report similar estimates for wage rigidity in Belgium and Finland, respectively.
regimes deliver the same loss. This is result is as expected: when $\theta_w$ is one, the only variation in nominal wages comes from indexation to (past) CPI inflation. Hence if the CPI is completely stabilized at all dates, nominal wages will be stabilized as well. In fact, fully stabilizing nominal wages can occur only if CPI inflation is zero. In short, the two regimes coincide in the limit.

As in the previous case, I examine the effects of different openness degrees on my main results (Figure 3.6). The figure shows the loss under CPI relative to the loss under DPI targeting where the wage rigidity degree is 0.75 and the indexation rate is 0.50. CPI targeting is preferred when the openness degree is around 0.67. This is an interesting result since recall that for these degrees of wage rigidity and wage indexation, the benchmark case (which assumes openness degree of 0.40) indicates that DPI is favorable. Also, although not shown here, for higher degrees of wage rigidity and wage indexation, CPI becomes the right index to target for even lower degrees of openness. Finally, DPI is the is found as the right index to target given low levels of openness regardless of the wage indexation and wage rigidity. These results only suggest the intuitive idea that more open economies should try to stabilize the price index that embed the price of foreign goods since this is simply the more relevant one.

### 3.4.3 Incomplete Pass-Through and Nominal Wage Rigidity

I discuss here the ranking of the three indices when the two frictions are both introduced. In this case we look at three important parameters simultaneously: the degrees of PT, wage rigidity and wage indexation. To do so, I first choose some indexation level. Next I choose some PT degrees and finally the degrees of wage rigidity.

Figure 3.3 and Figure 3.4 show the results for $\gamma_w$ is 0.75 and 0.90, respectively. I choose these degrees of indexation both because they are in line with empirical evidence and since the results for lower indexation degrees generally indicate DPI as a better index to target, especially compared to CPI targeting. To focus on the main findings of the paper, on one hand, and in order to economize in presentation, on the other, I present only the results for these levels of indexations. Also, since I need to account for the degrees of pass-through, I show the results for two levels of pass-through (0.80 and 0.90 respectively).

33 Note that the former subsection can be seen as a particular case of the current with pass-through being complete.
As before, each figure presents the losses under CPI targeting and WPI targeting relative to DPI targeting. Figure 3.7 reveals that when nominal wage rigidity is relatively high ($\theta_w = 0.90$ or more) and the indexation degree is 0.75, targeting wage inflation is better than targeting the DPI. This result is particularly true when pass-through is 0.90. Also, targeting the wage inflation seems to be better than targeting the CPI inflation for almost all levels of wage rigidity (but note again that the two regimes coincide in the limit). Figure 3.8 supports these conclusions. In this case, targeting both the CPI and the WPI become favorable if wages are highly rigid (around 0.85 or more), although the degree of wage rigidity needed is a bit lower. This result holds for similar reasons as discussed in the last subsection.

Few more observations can be inferred from figures 3.3 and 3.4. The higher the degree of indexation is, the lower the relative losses under both CPI and WPI. It should be noted however that the loss under WPI increases with the degree of indexation since the higher the indexation rate, the more costly full stabilization of the wage inflation is. The loss under CPI targeting does not vary with the indexation degree since when CPI inflation is zero, the wage indexation degree is irrelevant. Hence, as the relative loss under CPI is considered, the difference between Figure 3.7 and Figure 3.8 comes from the fact that the losses under DPI targeting are higher for higher degrees of indexation.

I close this subsection by considering the effect of varying the openness degree on my main results. Since I need to control for few parameters, I choose here to show the results only when the degree of openness is 0.60, but with noting that some of the results hold qualitatively for other degrees of openness (e.g. 0.50). Moreover, I assume a relatively moderate degree of indexation (of 0.75) both because of its empirical plausibility and since around which the favorability of DPI targeting may cease to hold. As for PT, I assume two different levels, 0.50 and 0.80. I choose these levels of PT for two reasons. First, they are empirically plausible; the study of Campa and Goldberg (2005) indicates that the average PT is around 0.46 in the short run and 0.64 in the long run. The study of Campa, Goldberg and Gonzales-Minguez (2005), shows relatively higher averages of PT (0.66 and 0.80) in the Euro area. Second, the results above show that in the presence of wage rigidity, CPI is better to target only if PT is relatively high. Hence, it will be interesting to check whether the CPI is the right index to target for moderate levels of PT given higher openness degree. Needless to say, the main results reported below hold also in the case of higher PT and indexation degrees.
Figure 3.9 shows that, given an indexation degree of 0.75, targeting CPI is better if wage rigidity is high and PT is moderate to high. Notice that this result differs from the result above where, for the same indexation degree, DPI is always favorable. Although not shown here, CPI may be the best to target in the case of indexation degree of 0.65 given high degrees of wage rigidity. In short, the results found in my calibration regarding the desirability of CPI targeting are only supported and even strengthened for more open economies.

3.5 SENSITIVITY ANALYSES

This section presents some sensitivity analysis by changing some of the benchmark parameters. Note that I do not show the effects of different parameterization on the performance of wage inflation targeting relative to other rules, and hence keep the focus on the comparison between CPI and DPI targeting. The first parameter to change is the elasticity of substitution between home and foreign goods ($\eta$) assumed to be 1.5 in my benchmark calibration (and 1.0 in GM). Next, I will change the degree of domestic price rigidity (assumed above to be 0.75). Finally, the elasticities of substitution between domestic goods ($\varepsilon_H$), foreign goods ($\varepsilon_F$) as well as between labor inputs ($\varepsilon_w$) will be varied. This basically allows for different levels of markup in each of these markets.

3.5.1 Changing the Elasticity of Substitution between Home and Foreign Goods ($\eta$):

I assume that $\eta$ can take any level between 0.3 and 2.25. In addition, I assume the more relevant levels of indexation (set to be 0.75), wage rigidity (0.75 and 0.80) as well relatively high degree of PT (0.80). The results are presented in Figure 3.10.

When wage rigidity is 0.75, CPI targeting leads to lower loss given that $\eta$ falls below 0.70. for higher levels of $\eta$, DPI seems to be better to target. Increasing the wage rigidity degree only slightly (to 0.80) shows that CPI is favorable when $\eta$ is less than 1.0. Hence, as the degree of wage rigidity increases, CPI yields lower losses for a wider range of $\eta$. Also, given some wage rigidity degrees, he higher the indexation rate, the wider the range under which CPI targeting is favorable. Increasing the degree of PT in this case will also support CPI as the favourable index to target for more values of $\eta$. 
3.5.2 Changing the Degree of Domestic Price Rigidity ($\theta_H$):

So far, the study assumed a degree of domestic price rigidity of 0.75 for the case of CPI targeting. In this subsection I check whether the results can be altered once different levels of domestic price rigidities are assumed. Notice that since, by definition, DPI targeting corresponds to $\theta_H$ being 1, the only effects to consider are on the loss under CPI targeting. Moreover, I have chosen the more relevant degrees of wage indexation and pass-through (both set to 0.75).

I first change $\theta_H$ to 0.50. The results (not reported here) show that in this case the loss under CPI is even larger than under $\theta_H$ of 0.75, reflecting highly variable domestic prices. Next, I increase $\theta_H$ to 0.90 and found lower loss under CPI compared to the benchmark case. In all of these occasions however, the loss under CPI is higher than the loss under DPI. Hence, the benchmark calibration level of $\theta_H$ has no effect on the qualitative results.

3.5.3 Changing the Elasticities of Substitution between Domestic Goods ($\varepsilon_H$), Foreign Goods ($\varepsilon_F$) and Between Workers ($\varepsilon_W$):

This subsection conducts the last sensitivity analysis of the study. Since there are 3 different parameters to vary, I do not go into details here and only report the basic results qualitatively. The whole analysis is done assuming pass-through, wage rigidity and indexation degrees of 0.75. The main outcome of this exercise is that changing the three parameters in the more relevant range (between 4 and 11) do not change the basic results of the paper.

3.6 CONCLUSIONS

This paper studies monetary policy rules in the era of Inflation Targeting in an economy with multiple nominal rigidities. Particularly, the paper assumes domestic price rigidity, import price rigidity (incomplete pass-through) and nominal wage rigidity. The study then contrasts welfare losses under two different inflation targeting regimes (of the domestic price index and the consumer price index) as well as the losses under fully stable nominal wages (to which we refer as wage inflation targeting). Wage inflation targeting basically examines the desirability of targeting an intermediate goal for monetary policy. The main focus however remains on comparing CPI targeting and DPI targeting.
Allowing for rigid import prices, but fully flexible wages, the study shows that targeting CPI is better when pass-through is relatively low to moderate (around 0.40 or lower). This degree of PT has been reported to be the case of few Inflation Targeting economies and hence the significance of the result. This finding is robust to changing the degree of openness.

When complete pass-through is restored and wage rigidity is assumed instead, CPI targeting turns to be better than DPI targeting for relatively high degrees of both wage rigidity and wage indexation to CPI inflation. Particularly, when the indexation degree and the wage rigidity degree are both around 0.75, the economy better target its CPI in order to avoid large fluctuations in marginal costs (through fluctuating nominal wages) and hence in both domestic prices and the output gap. Also, fully stabilizing nominal wages in such an environment (in which nominal wages are very rigid by nature) may even be the superior choice.

The key results for the case of both rigid wages and import prices are similar to the case of only rigid wages. Having high degrees of both wage rigidity and indexation, CPI tends to be a better index to target given high degrees of PT. This result is undoubtedly important since it suggests a different conclusion from GM’s even for high pass-through. When PT is low however, CPI ceases to be favorable even if wages are relatively rigid and highly indexed to CPI inflation. However, increasing the degree of openness reveals that CPI is better to target also in moderate PT environments (around 0.50). Moreover, the study points to the favorability of targeting nominal wages in this economic environment.

In overall, the paper suggests that the right index to target depends on the specific structure of the individual economy. Countries with low flexible nominal wages, high degrees of wage indexation and high pass-through should target their CPI. The same conclusion holds for countries with low degrees of pass-through and highly flexible wages. Relatively open countries with moderate to high indexation degrees and rigid wages should also target their CPIs. Economies with high degrees of wage rigidity may also consider the possibility of full stabilization of nominal wages. Other countries better target their Domestic Price Index. In this regard, some countries may not be following the best monetary policy rule once committing to a common policy.

This study can also be further extended. One possible extension is adding imported inputs and then considering the ranking of the different indices. Allowing for rigid export prices or incomplete pass-through in the foreign economy is another modification to consider. Finally, it would also be interesting to rank the indices according to some Taylor-Type Rules, which are believed to be the rules guiding monetary policy making in several countries.
Table 3.1: Welfare losses under CPI and DPI targeting for various values of openness and PT degrees (as percentage of steady state consumption)

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<th>PT=0.35</th>
<th>PT=0.25</th>
<th>PT=0.00</th>
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</table>
Figure 1.1: The Penalty Function ($\psi = 889.11$).

Figure 1.2: The standard deviation of the static wedge for various optimal annual inflation rate (in percents). The driving processes: Markup and TFP Shocks.
Figure 1.3: Response to negative and positive markup shocks with financial frictions (percentage deviations from SS levels). Inflation is shown in annualized terms.
Figure 1.4: Response to negative and positive TFP shocks with financial frictions (percentage deviations from SS levels). Inflation is shown in annualized terms.
Figure 2.1: The standard deviation of the intertemporal wedge and the optimal annual inflation rate with varying the degree of price rigidity.

Figure 2.2: The size of the intertemporal wedge and the optimal annual inflation rate with varying the degree of price rigidity.
Figure 2.3: The standard deviation of the intertemporal wedge as a function of the degree of DNWR with sticky prices under optimal policy and zero-inflation policy.

Figure 2.4: The standard deviation of the intertemporal wedge as a function of the degree of DNWR with flexible prices under optimal policy and zero-inflation policy.
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Figure 2.6: Response to a negative productivity shock with *asymmetric* wage adjustment cost function (percentage deviations from SS levels). $\sigma_1$: a positive 1 standard deviation shock. $-\sigma_1$: a negative 1 standard deviation shock. $2\sigma$: a positive 1 standard deviation shock. $-2\sigma$: a negative 1 standard deviation shock.
Figure 2.7: Responses to productivity shocks with symmetric wage adjustment cost function (percentage deviations from SS levels). $\sigma_1$: a positive 1 standard deviation shock. $-\sigma_1$: a negative 1 standard deviation shock. $2\sigma$: a positive 2 standard deviation shock. $-2\sigma$: a negative 2 standard deviation shock.
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Figure 2.9: Responses to a 1 standard deviation negative productivity shocks with asymmetric wage adjustment cost function under different policy rules (percentage deviations from SS levels).
Figure 3.1: Difference in welfare losses under CPI and DPI targeting for various values of PT.
Figure 3.2: Volatilities of Main Variables for various degrees of pass-through.
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Figure 3.5: The losses under CPI targeting and wage inflation targeting relative to the loss under DPI targeting with complete pass-through and rigid wages under different degrees of wage rigidity.

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Figure 3.7: The losses under CPI targeting and wage inflation targeting relative to the loss under DPI targeting with incomplete pass-through and rigid wages under different degrees of wage rigidity. Indexation degree=0.75.

Figure 3.8: The losses under CPI targeting and wage inflation targeting relative to the loss under DPI targeting with incomplete pass-through and rigid wages under different degrees of wage rigidity. Indexation degree=0.90.
Figure 3.9: The losses under CPI targeting relative to the loss under DPI targeting with incomplete pass-through and varying wage rigidity degree. Indexation degree=0.75; Openness degree=0.60.

Figure 3.10: The loss under CPI Targeting relative to the loss under DPI Targeting for various values of the elasticity of substitution between home and foreign goods. Pass-Through=0.80.
PROOFS AND DERIVATIONS

1A The Households’ Problem

Households maximize the following objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(l_t)],$$

subject to the sequence of budget constraints of the form:

$$c_t + q_t s_t + b_t = \frac{R_{t-1}b_{t-1}}{\pi_t} + (1 + \tau)w_t l_t + s_{t-1}(q_t + d_t) + \Pi_t + T_t$$

Denoting the Lagrange multiplier on the budget constraint by $\lambda_t$, the first order conditions with respect to $c_t$, $b_t$, $s_t$ and $l_t$ are, respectively:

$$\lambda_t = u_{c,t},$$

$$\lambda_t = \beta R_t E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} \right),$$

$$- q_t \lambda_t + \beta E_t [\lambda_{t+1}(q_{t+1} + d_{t+1})] = 0,$$

and,

$$- v_{t,l} + \lambda_t (1 + \tau)w_t = 0.$$  

Combining (A3) and (A6) gives equation (6) in the text. Combining (A3) and (A4) gives equation (7), and the combination of equations (A3) and (A5) yields equation (8) in the text.

1B The Entrepreneurs’ Problem

An entrepreneur chooses labor and shares to maximize:

$$E_0 \sum_{t=0}^{\infty} \delta^t \Xi_{0,t} \left[ p_t A_t l_t - w_t l_t + e_{t-1} (q_t + d_t) - e_t q_t \right],$$

subject to $\kappa e_{t-1} (q_t + d_t) - \alpha w_t l_t \geq 0.$

Denoting the Lagrange multiplier on constraint (B2) by $\mu_t$, the choice of labor yields:

$$A_t p_t - w_t - \alpha w_t \mu_t = 0,$$

or, after collecting terms,

$$A_t p_t = w_t (1 + \alpha \mu_t),$$
which is equation (12) in the text.

Similarly, the first-order condition with respect to \( e_t \) gives:

\[
\delta^t \Xi_{0,t}(-q_t) + \delta^{t+1} E_t \left[ \Xi_{0,t+1}(q_{t+1} + d_{t+1}) \right] + \delta^{t+1} E_t \left[ \Xi_{0,t+1}(-\kappa(q_{t+1} + d_{t+1}))\mu_{t+1} \right] = 0 , \tag{B5}
\]

By collecting terms and rearranging, condition (B5) can be written as:

\[
1 = \delta E_t \left[ \Xi_{t,t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) (1 + \kappa \mu_{t+1}) \right] . \tag{B6}
\]

This is condition (13) in the text.

1C The Approximated Entrepreneurs’ Problem

The problem of an entrepreneur in this case is to maximize:

\[
E_0 \sum_{t=0}^{\infty} \delta^t \Xi_{0,t} \left\{ p_t A_t l_t - w_t l_t + e_{t-1}(q_t + d_t) - e_q - \frac{1}{\psi^2} \exp \left[ \psi (\alpha w_t l_t - \kappa e_{t-1}(q_t + d_t)) \right]\right\} , \tag{C1}
\]

The first order condition with respect to \( l_t \) yields:

\[
A_t p_t - w_t - \frac{\psi \alpha w_t}{\psi^2} \exp \left[ \psi (\alpha w_t l_t - \kappa e_{t-1}(q_t + d_t)) \right] = 0 . \tag{C2}
\]

Letting \( \Omega_t = \frac{1}{\psi} \exp \left[ \psi (\alpha w_t l_t - \kappa e_{t-1}(q_t + d_t)) \right] \), condition (C2) can now be written as

\[
p_t A_t - w_t (1 + \alpha \Omega_t) = 0 , \tag{C3}
\]

which is equation (21) in the text.

Finally, the first order condition with respect to \( e_t \) yields:

\[
\delta^t \Xi_{0,t}(-q_t) + \delta^{t+1} E_t \left[ \Xi_{0,t+1}(q_{t+1} + d_{t+1}) \right] + \delta^{t+1} E_t \left[ \Xi_{0,t+1} \left( -\frac{1}{\psi} \exp \left[ \psi (\alpha w_t l_t - \kappa e_{t-1}(q_t + d_t)) \right] \right) \right] \left( -\kappa(q_{t+1} + d_{t+1}) \right) = 0 , \tag{C4}
\]

or, by using the definition of \( \Omega_t \),

\[
\Xi_{0,t}(-q_t) + \delta E_t \left[ \Xi_{0,t+1}(q_{t+1} + d_{t+1}) \right] + \delta E_t \left[ \Xi_{0,t+1}(-\Omega_{t+1})(-\kappa(q_{t+1} + d_{t+1})) \right] = 0 . \tag{C5}
\]

Rearranging equation (C5) gives equation (22) in the text.

1D Operating Profits of Entrepreneurs in the Approximated Model

Let us define the difference between revenues and wage costs by operating profits, as follows:
\[ \Pi_i^{op} = p_i A_i l_i - w_i l_i \]  

(D1)

Recall, from equation (B4) above, that \( p_i A_i = w_i (1 + \alpha \mu_i) \). Hence:

\[ p_i A_i l_i = w_i l_i (1 + \alpha \mu_i) , \]  

(D2)

Using the production function of entrepreneurs (\( x_i = A_i l_i \)), condition (D2) can be written as:

\[ p_i x_i = w_i l_i (1 + \alpha \mu_i) , \]  

(D3)

or,

\[ w_i l_i = \frac{p_i x_i}{1 + \alpha \mu_i} . \]  

(D4)

Substituting (D4) in (D1) and using the production function give:

\[ \Pi_i^{op} = p_i x_i - \frac{p_i x_i}{1 + \alpha \mu_i} , \]  

(D5)

which, after collecting terms, can be written as:

\[ \Pi_i^{op} = \frac{\alpha \mu_i}{1 + \alpha \mu_i} p_i x_i . \]  

(D6)

Condition (D6) states that operating profits are positive in an equilibrium with a binding collateral constraint. Clearly, if \( \alpha \) is zero (i.e. no part of wages is secured by net worth), then operating profits are zero in equilibrium (as one would expect in a perfectly competitive sector). Similarly, if the collateral constraint does not bind, then these profits will be zero as well, since in this case the economy is behaving as if there is no collateral constraint to begin with. Finally, in the approximated model discussed in section 4, the operating profits will be given by \( \Pi_i^{op} = \frac{\alpha \Omega_i}{1 + \alpha \Omega_i} p_i x_i \), with \( \Omega_i \) as defined in the text. The derivations are similar to the ones just shown, and therefore they are not presented here.

1E Efficient Allocations and the Labor Wedge

The social planner chooses consumption and labor to maximize

\[ E_\varnothing \sum_{i=0}^{\infty} \beta^i [u(c_i) - v(l_i)] , \]  

(E1)

subject to the sequence of resource constraints
\[ A_{t}I_{t} - c_{t} = 0. \quad (E2) \]

Let \( \eta_{t} \) be the Lagrange multiplier associated with (E2), then, the first-order conditions with respect to \( c_{t} \) and \( I_{t} \), respectively, read

\[ u_{c,t} = \eta_{t}, \quad (E3) \]

and

\[ v_{I,t} = \eta_{t}A_{t}. \quad (E4) \]

Combining (E3) and (E4) yields

\[ \frac{v_{I,t}}{u_{c,t}} = A_{t}, \quad (E5) \]

and hence efficiency requires the marginal rate of substitution (the left hand side of condition E5) to be equal to the marginal product of labor (given by the right-hand side of condition E5).

Given this result, one can derive the expression for the intratemporal (static) wedge. To do so, combine labor supply condition (6) and the labor demand condition (12) to get

\[ \frac{v_{I,t}}{u_{c,t}} = A_{t}\left(\frac{(1 + \tau)p_{t}}{1 + \alpha\mu_{t}}\right). \quad (E6) \]

Comparison of (E5) and (E6) reveals that the wedge is defined by the term in the parentheses. Clearly, this wedge is directly affected by the existence of the collateral constraint.

1F The Labor Market Subsidy

The labor market subsidy \( \tau \) is introduced to render the deterministic steady state of the model efficient. In particular, this subsidy is chosen so that, in the deterministic steady state, the marginal rate of substitution (MRS) between consumption and labor is equal to marginal product of labor (MPL). The derivations for labor market subsidy in the approximated model are similar to what follows, but with \( \Omega \) replacing \( \mu \) wherever it appears.

In what follows, undated variables denoted the deterministic steady state level of the corresponding variables. From equation (6), we have:

\[ \frac{v_{I}}{u_{c}} = (1 + \tau)w. \quad (F1) \]

The left-hand side of (F1) is the MRS between consumption and labor, hence:
\[ MRS = (1 + \tau)w \quad \text{(F2)} \]

Recalling that \( MPL = A \), equation (12) in the text implies:

\[ MPL = \frac{w(1 + \alpha \mu)}{p}. \quad \text{(F3)} \]

Setting \( MRS = MPL \), and canceling \( w \), yields:

\[ (1 + \tau) = \frac{(1 + \alpha \mu)}{p}, \quad \text{(F4)} \]

which, after rearranging terms, gives:

\[ \tau = \frac{(1 + \alpha \mu) - p}{p}. \quad \text{(F5)} \]

Finally, recall that \( p \) equals the marginal cost of final-good firms (\( mc \)), which, in the deterministic steady state, is given by the inverse of the gross markup (i.e. \( mc = \frac{\varepsilon - 1}{\varepsilon} \)). Substituting this result into equation (F5) yields:

\[ \tau = \frac{1 + \alpha \varepsilon \mu}{\varepsilon - 1}. \quad \text{(F6)} \]

Therefore, the labor market subsidy depends both on the level of the “real interest rate” and the degree of the monopolistic distortion (represented by \( \varepsilon \)). If no wage is required to be secured (i.e. \( \alpha = 0 \)), or if the collateral constraint does not bind (i.e. \( \mu = 0 \)), then the labor market subsidy should correct only the monopolistic distortion. On the other hand, when \( \varepsilon \) approaches infinity (which corresponds to perfect competition in the final-good sector), \( \tau \) approaches \( \alpha \mu \). This result is as expected since, with perfect competition, the only inefficiency in the allocation of \( l \) comes from existence of the financial friction. Clearly, if the choice of labor is unconstrained and the final-good sector is perfectly competitive, there is no distortion to correct for, and hence the labor market subsidy is zero.

1G Deriving the Philips Curve

The problem of a final-good firm \( j \) is to choose its price \( (P_{jt}) \) to maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\lambda_j}{\lambda_0} \left[ \frac{P_{jt}}{P_{jt-1}} y_{jt} - mc_{jt} y_{jt} - \frac{\phi \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right)^2}{2} y_{jt} \right] \right\},
\]

subject to the demand function for its product
\[ y_\beta = \left( \frac{P_{\beta}}{P_t} \right)^{-\epsilon_i} y_t. \]  \hfill (G2)

Rewrite (G1) as

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\lambda_\beta}{\lambda_0} \left[ \left( \frac{P_{\beta}}{P_t} \right)^{1-\epsilon_i} y_t - mc_{\beta} \left( \frac{P_{\beta}}{P_t} \right)^{-\epsilon_i} y_t - \phi \left( \frac{P_{\beta}}{P_{\beta-1}} - 1 \right) y_t \right] \right\}. \]  \hfill (G3)

The first-order condition with respect to the price \( P_{\beta} \) reads

\[ \beta' \frac{\lambda_\beta}{\lambda_0} \left\{ (1-\epsilon_i) \left( \frac{P_{\beta}}{P_t} \right)^{\epsilon_i} \frac{y_t}{y_t} - mc_{\beta} \left( \frac{P_{\beta}}{P_t} \right)^{-\epsilon_i} - \phi \left( \frac{P_{\beta}}{P_{\beta-1}} - 1 \right) \frac{y_t}{y_t} \right\} + \beta^\tau E_I \left\{ \left( \frac{\lambda_{\tau+1}}{\lambda_0} \right) \left[ - \phi \left( \frac{P_{\tau+1}}{P_{\tau}} - 1 \right) \left( \frac{P_{\tau+1}}{P_{\tau+1}} \right) \frac{y_{\tau+1}}{y_{\tau+1}} \right] \right\} = 0. \]  \hfill (G4)

In equilibrium, all firms set the same price (i.e. \( P_{\beta} = P_t \) for all \( j \)). Imposing symmetry on condition (G4) and canceling terms give

\[ \lambda_t \left\{ (1-\epsilon_i) \frac{y_t}{y_t} + mc_{t} - \phi \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{y_t}{y_t} \right\} + \beta E_I \left\{ \lambda_{\tau+1} \left[ - \phi \left( \frac{P_{\tau+1}}{P_{\tau}} - 1 \right) \frac{y_{\tau+1}}{y_{\tau+1}} \right] \right\} = 0. \]  \hfill (G5)

Multiplying by \( \frac{P_t}{y_t} \) yields

\[ \lambda_{\tau+1} \left\{ (1-\epsilon_i) + mc_{t-i} - \phi(\pi_t - 1) \right\} + \beta E_I \left\{ \lambda_{\tau+1} \left[ \phi(\pi_{\tau+1} - 1) \frac{y_{\tau+1}}{y_t} \right] \right\} = 0. \]  \hfill (G6)

Defining \( \pi_t = \frac{P_t}{P_{t-1}} \), we get

\[ \lambda_{\tau+1} \left\{ (1-\epsilon_i) + mc_{t-i} - \phi(\pi_t - 1) \right\} + \beta E_I \left\{ \lambda_{\tau+1} \left[ \phi(\pi_{\tau+1} - 1) \pi_{\tau+1} \frac{y_{\tau+1}}{y_t} \right] \right\} = 0, \]  \hfill (G7)

or, after rearranging and using the fact that \( \frac{\lambda_{\tau+1}}{\lambda_t} = \frac{u_{\tau+1}}{u_t} \), yields

\[ 1 - \phi(\pi_t - 1) \pi_t + \beta \phi E_I \left[ \left( \frac{u_{\tau+1}}{u_t} \right) (\pi_{\tau+1} - 1) \pi_{\tau+1} \frac{y_{\tau+1}}{y_t} \right] = \epsilon_t (1 - mc_t), \]  \hfill (G8)

which is equation (15) in the text.
1H Mapping Between the Price Duration and the Price Rigidity Parameter

I show here the way to map between the price duration and the price rigidity parameter. To do so, let us define the price duration by $\lambda$ and probability of not resetting the price during a given period by $\omega$. Hence,

$$\lambda = \frac{1}{(1-\omega)}. \tag{H1}$$

The slope of the Philips curve under the Rotemberg’s approach for price rigidity is given by

$$\frac{(\varepsilon-1)}{\varphi}. \tag{H2}$$

Similarly, the slope of the Philips curve when one follows Calvo’s approach for price rigidity is $\frac{(1-\omega)(1-\beta\omega)}{\omega}$. Substituting (H1) shows that the slope with the Calvo’s approach can be rewritten as

$$\frac{\lambda - \beta(\lambda - 1)}{\lambda(\lambda - 1)}. \tag{H3}$$

Setting equation (H2) equals to equation (H3) and rearranging yields $\varphi = \frac{\lambda(\lambda - 1)(\varepsilon - 1)}{\lambda - \beta(\lambda - 1)}$, which is the equation reported in the text.

II The link between Inflation and the Lagrange Multiplier on the Collateral constraint

Recall that the labor’s demand function is given by $A_i p_t = w_t (1 + \alpha \mu_t)$ and the real price of an intermediate-good firm equals the real marginal cost of a final-good firm ($p_t = mc_t$). Hence,

$$mc_t = \frac{w_t (1 + \alpha \mu_t)}{A_t}. \tag{I1}$$

Also, the labor-supply condition implies $w_t = \frac{v_{l,t}}{(1 + \tau) u_{c,t}}$. Substituting this result in (I1) gives:

$$mc_t = \frac{v_{l,t}}{u_{c,t} (1 + \tau) A_t} + \alpha \frac{v_{l,t}}{u_{c,t} (1 + \tau) A_t} \mu_t, \tag{I2}$$
and so, the real marginal cost is positively related to \( \mu_t \). This condition suggests the collateral constraint (represented by \( \mu_t \) in condition I2) affects inflation through the marginal cost. To see this more explicitly, rewrite the Philips Curve (equation 15 in the text) as

\[
(\pi_t - 1)\pi_t = \frac{1 - \varepsilon_t}{\varphi} + \beta E_t \left[ \frac{u_{\alpha t + 1}}{u_{\alpha t}} (\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right] + \varepsilon_t mc_t, \tag{I3}
\]

which shows inflation as an implicit function of the expected future inflation and the current marginal cost. Substituting (I2) in (I3) yields

\[
(\pi_t - 1)\pi_t = \frac{1 - \varepsilon_t}{\varphi} + \beta E_t \left[ \frac{u_{\alpha t + 1}}{u_{\alpha t}} (\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right] + \varepsilon_t \left[ \frac{v_{t, \tau}}{u_{e, \tau} (1 + \tau) A_t} \right] + \varphi \frac{\varepsilon_t}{u_{e, \tau} (1 + \tau) A_t} \mu_t. \tag{I4}
\]

Basically, \( \mu_t \) acts as cost-push shock (even when \( \varepsilon \) is constant), so that a rise in \( \mu_t \) is associated with an increase in inflation at time \( t \). This is similar to the idea in the log-linearized version of CFP, where \( \mu_t \) manifests itself as an endogenous mark-up shocks.

In the approximated model \( \Omega \), replaces \( \mu_t \), wherever it appears.

1J The Equivalence to a Model with Intra-Period Loans

I show here that there is equivalence between the main setup of the paper and a model where part of the wage bill needs to be paid ahead of production (the standard “working capital” requirement), entrepreneurs need to borrow in order pay this part of wages, and the borrowing is constrained by their net worth.

The model is modified in the following way. Households are assumed to lend to entrepreneurs (say through a perfectly-competitive intermediation sector). They deposit \( B^h_t \) in the beginning of period \( t \) and earn an interest rate of \( R^h_t \) in the end of the same period. Their problem will now be

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(l_t)], \tag{J1}
\]

subject to the sequence of budget constraints of the form:

\[
P_t c_t + Q_t s_t + B_t + B^h_t = R_{t-1} B_{t-1} + R^h_t B^h_t + P_t (1 + \tau) w_l l_t + s_{t-1} (Q_t + D_t) + P_t \Pi_t + P_T t, \tag{J2}
\]

where all variables are as in the main test. The households’ budget constraint in real terms reads:
\[ c_t + q_t s_t + b_t + b_t^h = \frac{R_{t-1} b_{t-1}}{\pi_t} + R^h_t b^h_t + (1 + \tau)w_t l_t + s_{t-1} (q_t + d_t) + \Pi_t + T_t \]  
\text{(J3)}

The choices of consumption, bonds, labor supply and shares of final-good firms yield the following optimization conditions:

\[ \frac{v_{t,t}}{u_{c,t}} = (1 + \tau)w_t , \]  
\text{(J4)}

\[ u_{c,t} = \beta R_t E_t \left( \frac{u_{c,t+1}}{\pi_t} \right) , \]  
\text{(J5)}

\[ u_{c,t} = \beta E_t \left[ u_{c,t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \right] , \]  
\text{(J6)}

and,

\[ R_t^h = 1 . \]  
\text{(J7)}

As for entrepreneurs, at the beginning of the period each entrepreneur obtains a loan \( B_t^e \) from households, which is to be paid in the end of the period at a nominal gross interest rate of \( R_t^e \). His borrowing, however, is constrained by the beginning-of-the-period net worth. Formally, an entrepreneur chooses labor, loans and shares to maximize:

\[ E_0 \sum_{t=0}^{\infty} \delta^t \Xi_{0,t} \left[ p_t A_t l_t + b_t^e - R_t^e b_t^e - w_t l_t + e_{t-1} (q_t + d_t) - e_t q_t \right] , \]  
\text{(J8)}

subject to

\[ \kappa e_{t-1} (q_t + d_t) - b_t^e \geq 0 , \]  
\text{(J9)}

and

\[ b_t^e - \alpha w_t l_t \geq 0 . \]  
\text{(J10)}

Letting \( \mu_t \) and \( \zeta_t \) denote the Lagrange multiplier on the constraints (J9) and (J10), respectively, the optimality condition with respect to \( b_t^e \) reads:

\[ \zeta_t = R_t^e + \mu_t - 1 . \]  
\text{(J11)}

Similarly, the first order condition with respect to \( l_t \) yields:

\[ A_t p_t - w_t (1 + \alpha \zeta_t) = 0 . \]  
\text{(J12)}

Finally, the first order condition with respect to \( e_t \) yields

\[ \Xi_{0,t} (-q_t) + \partial E_t \left[ \Xi_{0,t+1} (q_{t+1} + d_{t+1}) \right] + \partial E_t \left[ \Xi_{0,t+1} (-\mu_{t+1}) (-\kappa (q_{t+1} + d_{t+1})) \right] = 0 . \]  
\text{(J13)}
In equilibrium, the interest rate that households earn on their deposits will be equal to the interest rate that entrepreneurs pay, and hence $R_t^e = 1$. Using this fact, equation (J11) becomes:

$$\zeta_t = \mu_t,$$

which, by substituting in (J12) gives

$$A_t p_t - w_t (1 + \alpha \mu_t) = 0,$$

which is exactly as equation (12) in the text. Rearranging condition (J13) gives condition (13) in the text.

1K The Deterministic Steady State

In this appendix, I present some analytical solutions for the deterministic steady state. The starting point is the assumption that households devote 30 percent of their time for work, and hence $\ell$ is set to 0.3 in the SS. In addition, in the absence of shocks, the optimal inflation rate is zero, and hence $\pi = 1$. This result can be shown by considering the first-order condition of the optimal Ramsey planner with respect to inflation ($\pi_t$) in the deterministic steady state. In this case, this condition reads

$$\varphi(\lambda_7 + \lambda_8)(\pi - 1)y = 0.$$  \hspace{1cm} (K1)

$\lambda_7$ and $\lambda_8$ are the Lagrange multipliers on the resource constraint (condition 18) and dividends (equation 19), respectively. Both of these condition holds with equality in the deterministic steady state and hence $\lambda_7$ and $\lambda_8$ are both positive. Hence, the solution is $\pi = 1$.

Imposing deterministic steady state on equation (15) in the text, the deterministic steady state value of $mc$ equals the inverse of the gross markup (i.e. $mc = \frac{\varepsilon - 1}{\varepsilon}$). The deterministic steady state value of technology ($A$) is set to 1.

Under the assumption that the collateral constraint holds with equality in the deterministic steady state, we have $\Omega = \frac{1}{\psi}$. By setting $mc = p$, equation (8) in the text yields $w = \frac{mc}{1 + \alpha \Omega}$.

Substituting for $mc$ and $\Omega$ gives

$$w = \frac{(\varepsilon - 1)\psi}{\varepsilon (\alpha + \psi)}.$$  \hspace{1cm} (K2)

Imposing SS on equation (17) gives the SS value of dividends $d = Al(1 - mc)$, which, after substituting for $A$ and $mc$, can be written as $d = \frac{1}{\varepsilon}$.  \hspace{1cm} (K3)
Equation (8) in the text yields $q = \frac{\beta}{1 - \beta} d$, and hence

$$q = \frac{\beta \cdot 1}{1 - \beta} \varepsilon . \quad (K4)$$

Since the collateral constraint holds with equality in the SS, shares of entrepreneurs can be written as $e = \frac{\alpha w l}{\kappa (q + d)}$.

After substituting for $q$, $d$, and $w$, we get,

$$e = \frac{\alpha \varphi (1 - \beta) (\varepsilon - 1)}{\kappa (\alpha + \psi)} , \quad (K5)$$

which is zero when $\alpha = 0$. Intuitively, if no wage is required to be backed by collateral, then the entrepreneur has no reason to accumulate assets. Also, the SS value of $e$ is increasing in $\psi$, as expected. The higher the curvature parameter in the penalty function the higher the penalty for any violation of the collateral constraint. Hence, in order to avoid occasions where the constraint is violated, the entrepreneur tends to acquire more assets.

Recalling that $\Omega = \frac{1}{\psi}$, equation (K5) can also be written as

$$e = \frac{\alpha (1 - \beta) (\varepsilon - 1)}{\kappa (1 + \alpha \Omega)} , \quad (K6)$$

which implies a negative relationship between $e$ and $\Omega$. Intuitively, the more shares entrepreneurs have the more collateral they will have which reduces the value of the $\Omega$, the equivalent of the Lagrange multiplier.

11. The Households’ Problem with Money in the Utility

Households maximize the following objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1 - \sigma}}{1 - \sigma} - \frac{l_{t+1}^{1 + \theta}}{1 + \theta} + \phi \log \left( \frac{M_{t-1}}{P_t} \right) \right] , \quad (L1)$$

subject to the sequence of budget constraints of the form:

$$P_t c_t + Q_t s_t + B_t + M_t = R_{t-1} B_{t-1} + M_{t-1} + P_t (1 + \tau) w_t l_t + s_{t-1} (Q_t + D_t) + P_t \Pi_t + P_t T_t \ , \quad (L2)$$

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with \( M_{t-1} \) denoting nominal money holdings at the beginning of period \( t \). Real money balances at the beginning of period \( t \) is denoted by \( m_{t-1} \). The optimality conditions read:

\[
\frac{\lambda_i^{1+\theta}}{c_i^{1-\sigma}} = (1 + \tau)w_i, \quad (L3)
\]

\[
c_i^{-\sigma} = \beta R_i E_t \left( \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} \right), \quad (L4)
\]

\[
c_i^{-\sigma} = \beta E_t \left[ c_i^{-\sigma} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \right], \quad (L5)
\]

and,

\[
m_t = \beta \frac{\phi c_i^{1-\sigma} R_i}{(R_i - 1)} \quad (L6)
\]

The households’ equilibrium conditions include now the money demand function (condition L6). Other conditions are not affected by introducing the money demand motive.
2A Deriving the Philips Curve:

I show here the derivation of the Phillips curve which is the outcome of the first-order condition with respect to the price $P_{jt}$. The firm $j$ chooses the price $P_{jt}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \frac{P_{jt}}{P_t} y_t - n_{jt} W_t h_t - \frac{\phi}{\psi} \left( \exp[\psi(W_t) - 1] + \psi(W_t) - 1 \right) n_{jt} - \frac{\phi}{2} \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right) y_t \right],$$

subject to the sequence of laws of motion of employment, and the : 

$$n_{jt+1} = (1 - \rho)(n_{jt} + v_{jt} q(\theta_j)), \quad \text{(A2)}$$

and the downward-sloping demand function for its product

$$z_t n_{jt} f(h_{jt}) = \left[ \frac{P_{jt}}{P_t} \right]^{-\varepsilon} y_t. \quad \text{(A3)}$$

Associating a Lagrange multiplier $\varphi_{jt}$ with (A3), the first-order condition with respect to the price $P_{jt}$ reads

$$\beta^t \frac{\lambda_t}{\lambda_0} \left[ (1 - \varepsilon) \left( \frac{P_{jt}}{P_t} \right)^{\varepsilon} y_t + \varepsilon \varphi_{jt} \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon-1} y_t - \frac{\phi}{\psi} \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right) y_t \right] +$$

$$\beta^{t+1} E_t \left[ \frac{\lambda_{t+1}}{\lambda_0} \right] - \frac{\phi}{\psi} \left( \frac{P_{jt+1}}{P_t} - 1 \right) \left( \frac{P_{jt+1}^2}{P_{jt}^2} \right) y_{t+1} \right] = 0 \quad \text{(A4)}$$

In equilibrium, all firms set the same price (i.e. $P_{jt} = P_t$ for all $j$). Imposing symmetry on condition (A4) and canceling terms give

$$\lambda_t \left[ (1 - \varepsilon) \frac{y_t}{P_t} + \varepsilon \varphi_t \frac{y_t}{P_t} - \frac{\phi}{\psi} \left( \frac{P_{jt+1}}{P_{jt}} - 1 \right) \frac{y_t}{P_t} \right] + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left[ - \frac{\phi}{\psi} \left( \frac{P_{jt+1}}{P_t} - 1 \right) \left( \frac{P_{jt+1}^2}{P_{jt}^2} \right) y_{t+1} \right] \right] = 0 \quad \text{(A5)}$$

Multiplying by $\frac{P_t}{y_t}$ yields

$$\lambda_t \left[ (1 - \varepsilon) + \varepsilon \varphi_t - \frac{\phi}{\psi} \left( \frac{P_{jt+1}}{P_{jt}} - 1 \right) \frac{P_t}{P_{jt}} \right] + \beta E_t \left[ \lambda_{t+1} \left[ \frac{\phi}{\psi} \left( \frac{P_{jt+1}}{P_t} - 1 \right) \frac{P_{jt+1}^2}{P_{jt}^2} \right] \frac{y_{t+1}}{y_t} \right] = 0. \quad \text{(A6)}$$

Defining $\pi_t = \frac{P_t}{P_{jt}}$, rearranging and using the fact that $\frac{\lambda_{t+1}}{\lambda_t} = \frac{u_{ct+1}}{u_{ct}}$, we get

$$1 - \phi^\psi (\pi_t - 1) \pi_t + \beta \phi^\psi E_t \left[ \frac{u_{ct+1}}{u_{ct}} \left( \pi_{t+1} - 1 \right) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] = \varepsilon (1 - \varphi_t), \quad \text{(A7)}$$

which is equation (20) in the text.
2B Optimal Monetary Policy Problem:

The optimal monetary policy problem is to choose \( \{c_t, h_t, n_t, u_t, \theta_t, \varphi_t, w_t, \pi_t, \pi_w^t \} \) to maximize household’s expected discounted lifetime utility subject to the resource constraint of the economy and the equilibrium conditions of firms and individuals. Formally,

\[
Max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - n_t \chi \frac{h_t^{1+\beta}}{1+\beta} \right],
\]

subject to,

\[
\frac{y}{q(\theta_t)} = \beta(1-\rho)E_0 \left[ \left( \frac{c_{t+1}^{1-\sigma}}{c_t^{1-\sigma}} \right) \left( \frac{\varphi_{t+1} z_{t+1} f(h_{t+1}) - w_{t+1} h_{t+1} - \frac{\phi}{\psi} \left[ \exp[-\psi(\pi_w^t - 1)] + \psi(\pi_w^t - 1) - 1 \right]}{h_t} \right) \right],
\]

\[
\frac{\omega_t}{1-\omega_t} \left[ \varphi_t z_t f(h_t) - w_t h_t - \frac{\phi}{\psi} \left[ \exp[-\psi(\pi_w^t - 1)] + \psi(\pi_w^t - 1) - 1 \right] + \frac{\gamma}{q(\theta_t)} \right] = w_t h_t - \frac{v(h_t)}{u_t} - b + E_t \left[ \frac{\omega_{t+1}}{1-\omega_{t+1}} \left( \frac{\gamma}{q(\theta_t)} - \gamma \theta_t \right) \right],
\]

\[
\frac{\Gamma_t}{1-\Gamma_t} \left[ \varphi_t z_t f(h_t) - w_t h_t - \frac{\phi}{\psi} \left[ \exp[-\psi(\pi_w^t - 1)] + \psi(\pi_w^t - 1) - 1 \right] + \frac{\gamma}{q(\theta_t)} \right] = w_t h_t - \frac{v(h_t)}{u_t} - b + E_t \left[ \frac{\Gamma_{t+1}}{1-\Gamma_{t+1}} \left( \frac{\gamma}{q(\theta_t)} - \gamma \theta_t \right) \right],
\]

\[
1 - \phi^t (\pi_t - 1) \pi_t + \beta \phi^t E_t \left[ \frac{u_{t+1}}{u_t} (\pi_{t+1} - 1) \pi_{t+1} y_{t+1} \right] = \varepsilon(1 - m c_t),
\]

\[
\frac{w_t}{w_{t-1}} - \pi_w^t \pi_t = 0,
\]

\[
1 - n_t - u_t = 0,
\]

\[
n_{t+1} - (1-\rho)(n_t + \sigma_m u_t \theta_t^{1-\gamma}) = 0,
\]

and,

\[
n_t z_t f(h_t) - c_t - \gamma \theta_t u_t - \frac{\phi}{\psi} \left[ \exp[-\psi(\pi_w^t - 1)] + \psi(\pi_w^t - 1) - 1 \right] n_t - \frac{\psi^t}{2} (\pi_t - 1)^2 n_t z_t f(h_t) = 0.
\]
2C Efficient Allocations:

The problem of the social planner is to maximize

\[ E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - n_tv(h_t)], \]  

subject to the sequence of the economy-wide resource constraints

\[ n_tz_t f(h_t) - c_t - \gamma v_t = 0, \]

and the sequence laws of motion of employment

\[ (1-\rho)[n_t + m(1-n_t, v_t)] - n_{t+1} = 0 \]

Let, \( \lambda_{it} \) and \( \lambda_{it} \) denote the Lagrange multipliers on constraints (C2) and (C3), respectively.

Then, the first-order condition with respect to \( c_t \), \( v_t \) and \( n_{t+1} \), respectively, are

\[ u_{ct} = \lambda_{it}, \]

\[ -\gamma \lambda_{it} + \lambda_{2t}[(1-\rho)m_t(1-n_t, v_t)] = 0, \]

and,

\[ -\lambda_{2t} - \beta v(h_{t+1}) + \beta E_t \lambda_{2t+1}[z_{t+1}f(h_{t+1})] + \beta(1-\rho)E_t \lambda_{2t+1}[1-m_u(1-n_t, v_t)] = 0. \]

From (C5) we have

\[ \lambda_{2t} = \frac{\gamma}{(1-\rho)m_t(1-n_t, v_t)} \lambda_{it}, \]

which, after substituting (C4), yields

\[ \lambda_{2t} = \frac{\gamma}{(1-\rho)m_t(1-n_t, v_t)} u_{ct}. \]

By substituting (C8) in (C6) and rearranging, we get

\[ \frac{\gamma}{m_t(1-n_t, v_t)} = \beta(1-\rho)E_t \left[ \frac{u_{ct+1}}{u_{ct}} \left( \frac{z_{t+1}f(h_{t+1})}{u_{ct+1}} - \frac{v(h_{t+1})}{u_{ct+1}} + \frac{\gamma[1-m_u(1-n_{t+1}, v_{t+1})]}{m_t(1-n_{t+1}, v_{t+1})} \right) \right], \]

Finally, condition (C9) can also be written as
\[ \frac{u_{ct}}{\beta u_{ct+1}} = \frac{(1 - \rho) \left[ z_{t+1} f(h_{t+1}) - \frac{v(h_{t+1})}{u_{ct+1}} + \gamma \left[ 1 - m_u (1 - n_{t+1}, v_{t+1}) \right] \right]}{m_i (1 - n_t, v_i)} \]. \tag{C10} 

The left hand side is the Intertemporal Marginal Rate of Substitution (IMRS), while the right hand side is Intertemporal Marginal Rate of Transformation (IMRT). Efficiency, thus, requires the IMRT being equal to the IMRS for all \( t \).

Finally, this condition can also be written as,

\[ 1 = (1 - \rho) E_t \left\{ \frac{\beta u_{ct+1}}{u_{ct}} \left[ z_{t+1} f(h_{t+1}) - \frac{v(h_{t+1})}{u_{ct+1}} + \gamma \left[ 1 - m_u (1 - n_{t+1}, v_{t+1}) \right] \right] \right\}. \tag{C11} \]

**2D The Intertemporal Wedge**

In order to derive the intertemporal wedge, I make use of the vacancy-posting condition

\[ \frac{\gamma}{q(\theta_i)} = \beta (1 - \rho) E_t \left\{ \frac{u_{ct+1}}{u_{ct}} \left[ \phi_{r,t+1} z_{r,t+1} f(h_{r,t+1}) - w_{r,t+1} h_{r,t+1} - \Phi^w_{r,t+1} + \frac{\gamma}{q(\theta_{r,t+1})} \right] \right\}, \tag{D1} \]

and the wage bargaining condition

\[ \frac{\omega_i}{1 - \omega_i} \left[ \phi_{r,t} z_{r,t} f(h_{r,t}) - w_t h_t - \Phi^w_t + \frac{\gamma}{q(\theta_t)} \right] = w_t h_t - \frac{v(h_t)}{u_{ct}} - b + E_t \left[ \frac{\omega_{t+1}}{1 - \omega_{t+1}} \left( \frac{\gamma}{q(\theta_{t+1})} - \gamma \theta_{t+1} \right) \right], \tag{D2} \]

where \( \Phi^w_t = \frac{\phi}{\psi^2} \left( \exp[-\psi(\pi^w_t - 1)] + \psi(\pi^w_t - 1) - 1 \right) \). Using the properties of the matching function, we have \( q(\theta_t) = \frac{m_i (1 - n_t, v_i)}{(1 - \zeta)} \). Then, conditions (D1) and (D2) can, respectively, be written as
\[
\frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} = \beta (1 - \rho) E_i \left[ \frac{u_{a+t}}{u_a} \left( \frac{\varphi_{r+1} z_{r+1} f (h_{r+1}) - w_{r+1} h_{r+1} - \Phi_{r+1}^w}{m_i (1 - n_i, v_i)} + \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} \right) \right],
\]
(D3)

and,
\[
\frac{\omega_t}{1 - \omega_t} \left[ \varphi_{r+1} z_{r+1} f (h_{r+1}) - w_{r+1} h_{r+1} - \Phi_{r+1}^w + \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} \right] = w_t h_t - \frac{\nu (h_t)}{u_a} - b + E_t \left[ \frac{\omega_{a+t}}{1 - \omega_{a+t}} \left( \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} \right) + \frac{m_i (1 - n_i, v_i)}{\zeta} \right].
\]
(D4)

Rearranging condition (D4) yields:
\[
w_t h_t = \omega_t \left[ \varphi_{r+1} z_{r+1} f (h_{r+1}) - \Phi_{r+1}^w + \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} \right] + (1 - \omega_t) \left( \frac{\nu (h_t)}{u_a} - b + E_t \left[ \frac{\omega_{a+t}}{1 - \omega_{a+t}} \left( \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} \right) + \frac{m_i (1 - n_i, v_i)}{\zeta} \right] \right).
\]
(D5)

After iterating one period ahead and collecting terms, equation (D5) can now be substituted into (D3) to yield:
\[
\frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} = \beta (1 - \rho) E_i \left[ \frac{u_{a+t}}{u_a} \left( \frac{1 - \omega_{a+t}}{1 - \omega_t} \left( \frac{\varphi_{r+1} z_{r+1} f (h_{r+1}) - \frac{\nu (h_t)}{u_a} - b + \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)}}{1 + E_t \left[ \frac{\omega_{a+t}}{1 - \omega_{a+t}} \left( \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} \right) + \frac{m_i (1 - n_i, v_i)}{\zeta} \right]} \right) \right] \right].
\]
(D6)

Then, dividing by \((1 - \zeta)\) gives
\[
\gamma = \beta (1 - \rho) E_i \left[ \frac{u_{a+t}}{u_a} \left( \frac{1 - \omega_{a+t}}{1 - \omega_t} \left( \frac{\varphi_{r+1} z_{r+1} f (h_{r+1}) - \frac{\nu (h_t)}{u_a} - b + \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)}}{1 + E_t \left[ \frac{\omega_{a+t}}{1 - \omega_{a+t}} \left( \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} \right) + \frac{m_i (1 - n_i, v_i)}{\zeta} \right]} \right) \right] \right].
\]
(D7)

By comparing (D7) with (C9), efficiency is restored if nominal wages are fully flexible or fully stabilized (i.e. \(\Phi_{r+1}^w = 0\), and hence \(\omega_{a+1} = \eta\), the Hosios condition holds (\(\zeta = \eta\)), the unemployment benefits are zero (\(b = 0\)), and no monopolistic power in the final-good sector (which implies \(\varphi_{r+1} = 1\)). Finally, the wedge is implicitly defined by the comparing equation (D7) and (C9).

An analogous to condition (C10) can be obtained by rearranging terms in (D7), as follows
\[
\frac{u_a}{\beta u_{a+t}} = \left( 1 - \rho \right) \left[ \frac{1 - \omega_{a+1}}{1 - \zeta} \left( \frac{\varphi_{r+1} z_{r+1} f (h_{r+1}) - \frac{\nu (h_{r+1})}{u_{a+1}} - b + \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} \left( 1 + E_{a+1} \left[ \frac{\omega_{a+1}}{1 - \omega_{a+1}} \left( \frac{\gamma (1 - \zeta)}{m_i (1 - n_i, v_i)} \right) + \frac{m_i (1 - n_i, v_i)}{\zeta} \right] \right) \right) \right] \right].
\]
(D8)

where the left-hand side is the intertemporal rate of substitution (IMRS) and the right-hand side is the intertemporal rate of transformation (IMRT). Finally, rewrite condition (D8) as

\[
\frac{u_a}{\beta u_{a+t}} = \frac{\gamma}{m_i (1 - n_i, v_i)}
\]
\[ 1 = (1 - \rho)E_t \left( \frac{\beta_{t+1}}{u_{t+1}} \right) \left( \frac{1 - \omega_{t+1}}{(1 - \zeta)} \right) \left( \frac{\phi_{t+1}z_{t+1}f(h_{t+1}) - \psi(h_{t+1}) - \Phi_{t+1} - b + \gamma(1 - \zeta)}{u_{t+1} - \phi_{t+1} - \psi(h_{t+1}) - b} \right) \left( 1 + E_{t+1} \left[ \frac{\omega_{t+2}}{1 - \omega_{t+2}} \left( 1 - \frac{m_s(1 - n_{t+1}, y_{t+1})}{\zeta} \right) \right] \right) \]. \quad (D9)

Comparison of the square brackets in (D9) with the square brackets of (C11) implicitly defines the intertemporal wedge.
3A. The Optimal Monetary Policy Problem

In the general case where the economy features rigidities in domestic prices, import prices and nominal wages, the problem of the Monetary Authority is to choose allocations \( \{x_t, \pi_t, \pi_{H,t}, \pi_{F,t}, \pi_{w,t}, \psi_{F,t}, W_t, r_t\} \) to maximize

\[
W = -\frac{(1-\alpha)}{2\omega} \sum_{t=0}^{\infty} \beta^t \left[ \left(1 + \phi\right)x_t^2 + \frac{\varepsilon_H}{\lambda_H} \pi_{H,t}^2 + \frac{\varepsilon_w}{\lambda_w} \pi_{w,t}^2 + \beta \frac{\varepsilon_{F}}{\lambda_{F}} \pi_{F,t}^2 \right]
\]

Subject to,

\[
\pi_{H,t} = \beta E_{t} \pi_{H,t+1} + \kappa x_t + \lambda \left[ \frac{1-\alpha - \omega_s}{\omega_s} y_t + \frac{(1-\alpha - \omega_s)}{\omega_s} + \phi(x_t + \log(1-\alpha) - \frac{(1-\alpha)}{1+\phi} a_t) \right] - \frac{\lambda \alpha \eta}{\omega_s} \psi_{F,t} + (1-\alpha) \pi_{H,t}, \quad (A1)
\]

\[
\pi_{F,t} = \beta E_{t} \pi_{F,t+1} + \lambda \psi_{F,t}, \quad (A2)
\]

\[
\psi_{F,t+1} - \psi_{F,t} = r_t - r^*_t + \pi_{F,t+1} - \pi_{F,t}, \quad (A3)
\]

\[
\pi_t - \pi_t = w_t - w_{t-1}, \quad (A4)
\]

\[
\pi_{w,t} = \beta E_{t} \pi_{w,t+1} - \lambda \left[ \frac{1-\alpha - \omega_s}{\omega_s} y_t + \frac{(1-\alpha - \omega_s)}{\omega_s} + \phi(x_t + \log(1-\alpha) - \frac{(1-\alpha)}{1+\phi} a_t) \right] + \frac{\lambda \alpha \eta}{\omega_s} \psi_{F,t} - \beta t \phi \pi_t + \gamma \pi_{t+1}, \quad (A6)
\]

\[
x_t = E_{t} x_{t+1} - \frac{\alpha}{\sigma} \left( r_t - \pi_{H,t+1} - rr_t \right) + \Gamma x_t \left( \psi_{F,t+1} - \psi_{F,t} \right), \quad (A7)
\]

where,

\[
rr_t = \frac{\sigma \varphi (\omega_s - 1)}{\sigma + \varphi \omega_s} \Delta x_{t+1}^* - \frac{\sigma (1-\rho_s)(1+\phi)}{\sigma + \varphi \omega_s} a_t + \rho
\]

\[
\Gamma x = \frac{\alpha (1-\alpha)(\sigma \eta - 1)}{\sigma}
\]
REFERENCES


