Title of dissertation: ESSAYS ON THE DISTRIBUTIONAL AND WELFARE CONSEQUENCES OF DISINFLATION IN EMERGING ECONOMIES

Enes Sunel, Doctor of Philosophy, 2011

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This study undertakes a quantitative investigation of the distributional and welfare consequences of a sharp reduction in inflation in a small open economy. In the first chapter, a monetary model of a small open economy with uninsured idiosyncratic earnings risk is analyzed. In this model, consumers hold non-interest bearing real balances (demand deposits) that economize transactions costs of consumption and internationally-traded risk-free bonds (term deposits) that are useful for consumption smoothing. Bonds are modeled as inflation-indexed to incorporate financial dollarization. The model is calibrated to Turkish data and is used to compare stationary equilibria with quarterly inflation rates of 14.25% (for 1987:1-2002:4) and 2.25% (for 2003:1-2010:3) under alternative fiscal arrangements. The results show that (i) when uniform transfers are endogenous, reducing inflation lowers aggregate welfare by 1.25% in terms of compensating consumption variation. This is because the reduction in the costs of inflation for the poor is less than the reduction in their...
transfers income. This also tightens natural debt limits and increases precautionary savings motive. (ii) When endogenous transfers depend on individual-specific inflation tax payments, aggregate welfare increases by 0.45%. This is because proportional transfers do not drive redistributive effects. (iii) Welfare gains increase further (1.62%) if wasteful spending is endogenous. The model also generates a cross sectional portfolio consistent with the disaggregated deposits data and the literature.

The second chapter examines quantitative properties of the transitional dynamics produced by gradual disinflation (as opposed to the stationary equilibria analysis conducted in the first chapter). The main exercise is to feed the empirically observed declining path for inflation into the calibrated model and account for its macroeconomic, distributional and welfare effects under alternative fiscal arrangements. The results show that (i) when uniform transfers are endogenous, gradual decline in the inflation rate from 14.25% to 2.25% increases aggregate welfare by 0.28%. (ii) When wasteful spending is endogenous, aggregate welfare increases by 0.53%. These welfare effects are substantially different from those implied by steady state comparisons. This is because when transition is accounted for, fiscal variables do not jump to their low inflation steady state levels immediately.
ESSAYS ON THE DISTRIBUTIONAL AND WELFARE
CONSEQUENCES OF DISINFLATION
IN EMERGING ECONOMIES

by

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Dedication

To my darling, Betül.
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List of Abbreviations

CBRT Central Bank of the Republic of Turkey
CRRA Constant Relative Risk Aversion
SPO State Planning Organization
TURKSTAT Turkish Statistical Institute
Chapter 1

On the Distributional and Welfare Consequences of Disinflation in Stationary Small Open Economies

1.1 Introduction

This paper undertakes a quantitative investigation of the distributional and welfare consequences of a sharp decline in inflation in a small open economy. The study derives its motivation mainly from the observation that globally observed disinflation in the last two decades has been more predominant in emerging economies. In Table 1.1 below, I report the time series average of annual CPI inflation rate for a number industrialized and emerging countries. For each country, two periods, for which inflation has been high and low respectively, are pointed out. It appears that structural change in inflation has been more predominant in emerging economies (who have a record of high inflation) compared to industrialized countries.¹

Disinflation of magnitudes observed in emerging economies might derive non-trivial welfare effects because inflation (i) reduces the purchasing power of individuals; (ii) distorts consumption; and (iii) governments’ response to reduction in inflation tax revenues might create redistributive wealth effects. In addition to these

¹See Appendix D for the methodology of determining structural break dates and a complete list of countries.
Table 1.1: Disinflation as a Structural Change

<table>
<thead>
<tr>
<th>Advanced</th>
<th>High (Per.)</th>
<th>Low (Per.)</th>
<th>Emerging</th>
<th>High (Per.)</th>
<th>Low (Per.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>15&lt;sup&gt;a&lt;/sup&gt; (73-85)</td>
<td>4 (86-08)</td>
<td>Brazil</td>
<td>135 (60-94)</td>
<td>11 (95-08)</td>
</tr>
<tr>
<td>UK</td>
<td>10 (70-91)</td>
<td>3 (92-08)</td>
<td>Argentina</td>
<td>115 (75-94)</td>
<td>6 (95-08)</td>
</tr>
<tr>
<td>France</td>
<td>9 (68-85)</td>
<td>2 (86-08)</td>
<td>Peru</td>
<td>71 (74-91)</td>
<td>9 (92-08)</td>
</tr>
<tr>
<td>Japan</td>
<td>7 (60-81)</td>
<td>1 (82-08)</td>
<td>Turkey</td>
<td>60 (77-02)</td>
<td>10 (03-08)</td>
</tr>
<tr>
<td>U.S.</td>
<td>7 (70-85)</td>
<td>3 (86-08)</td>
<td>Mexico</td>
<td>53 (74-88)</td>
<td>14 (89-08)</td>
</tr>
<tr>
<td>Canada</td>
<td>7 (71-91)</td>
<td>2 (92-08)</td>
<td>Bolivia</td>
<td>35 (73-83)</td>
<td>9 (84-08)</td>
</tr>
<tr>
<td>Advanced&lt;sup&gt;b&lt;/sup&gt;</td>
<td>7 (60-90)</td>
<td>2 (91-08)</td>
<td>Emerg.&amp;Dev.&lt;sup&gt;b&lt;/sup&gt;</td>
<td>49 (79-95)</td>
<td>10 (96-08)</td>
</tr>
</tbody>
</table>

<sup>a</sup> Period average of annual CPI inflation rate, %.
<sup>a</sup> Data points that correspond to annual inflation rate of more than 200% are omitted.
<sup>b</sup> These classifications reflect aggregations in the International Financial Statistics database of the IMF.
effects, the walkway to the research question of this paper is paved by financial
system characteristics of emerging economies in relation to disinflation. Particularly,
in emerging economies, (i) the distribution of monetary assets displays substantial
inequality and financial assets portfolio is not uniform across people; (ii) financial
system exhibits a high degree of dollarization that affects vulnerability of monetary
assets to inflation in a particular way; and (iii) financial dollarization (FD, hereafter)
is systemically more predominant in countries that have an inflationary past and
that exhibit strong exchange rate pass through.2

This paper develops a monetary model of a small open economy with idiosyn-
cratic earnings risk and incomplete markets. The model economy is populated by
a continuum of consumers and a government. There is no aggregate uncertainty.
Infinitely-lived consumers face idiosyncratic earnings shocks and consume a tradable
consumption good. They hold, (i) non-interest bearing real balances that economize
transactions costs of consumption; and (ii) internationally-traded risk-free bonds
that are useful for consumption smoothing. Furthermore, consumers face ad-hoc
borrowing constraints, which dampen their ability to smooth consumption.

I assume perfect mobility in capital and goods markets so that domestic nom-
inal interest rate is determined by a parity condition and domestic price level is
determined by the law of one price. Because of the latter, domestic inflation rate
is equal to the depreciation rate of currency. These assumptions cause bonds to
be fully dollarized (inflation-indexed) so that their real return does not depend on
domestic inflation. I assume that the de facto exchange rate regime is practically

2See Section 1.2 for a detailed documentation of these facts.
a managed float. That is, monetary authority is able to manipulate the level of the devaluation (depreciation) rate exogenously and shall print as much money as private sector demands at this given rate.

Empirical literature has documented a positive and strong relationship between fiscal deficits and inflation in emerging (high inflation) economies (e.g. see, Fischer et al. (2002), Catão and Terrones (2005)). To that end, I assume that the government uses seigniorage revenues to finance lump-sum transfers and wasteful spending. In order to explore the mediating role of fiscal policy on the distributional and welfare consequences of disinflation, I study fiscal arrangements with (i) endogenous uniform transfers; (ii) endogenous government spending; and (iii) endogenous transfers that depend on individual specific inflation tax payments.

In this model, welfare of consumers are affected by inflation through the following channels: (i) Inflation leads to wealth eroding due to inflation taxation. (ii) Inflation creates a distortion in consumption since it makes real balances (that economize transactions costs) less desirable. (iii) The necessity of a balanced government budget (in equilibrium) creates redistributive wealth effects driven by particular fiscal arrangement.

The theory developed in this paper is consistent with the findings of the empirical literature that the poor hold a larger fraction of their assets in cash. This is due to the proportional relationship between consumption and money holdings of consumers who are not borrowing constrained and the typical property of incomplete markets models that consumption-to-wealth ratio decreases with wealth.

I calibrate the model to the low inflation period (2003:1-2010:2) of the Turk-
ish economy, which is representative of the disinflation phenomenon and the afore-
mentioned financial system characteristics of emerging economies. The main quan-
titative exercise is to compare stationary equilibria with quarterly inflation rates of
14.25% (for 1987:1-2002:4) and 2.25% under alternative fiscal arrangements.

I find that (i) when uniform transfers are endogenous, reducing (the quarterly)
inflation rate from 14.25% to 2.25% lowers aggregate welfare by 1.25% in terms of
compensating consumption variation. This is because reduction in the costs of
inflation to the poor is less than reduction in their transfers income that creates
positive wealth effects in an inflationary environment. (ii) When wasteful spending
is endogenous, aggregate welfare increases by 1.62%. This is because wealth effects
created by inflationary finance (that favor the poor) are muted when transfers are
constant. (iii) Finally, when endogenous transfers are proportional to individual-
specific inflation tax payments, aggregate welfare increases by 0.45%. Welfare gains
in this case are lower than the endogenous spending case because wasteful spending
does not decrease when inflation is lower.

The impact of disinflation on portfolio choice manifests itself through substi-
tution and wealth effects, where the latter crucially depends on the fiscal response to
reduction in inflationary finance. Specifically, when endogenous lump-sum transfers
are uniform, reduction in transfers (driven by disinflation) tightens natural debt
limits of the poor and increases their precautionary savings motive. This causes
the distribution of bonds to be more equitable at the left tail. In contrast, when
transfers are proportional, such wealth effects are partially neutralized and due to
substitution effects, consumers demand less bonds. This reduces the interest in-
come unambiguously, since the real interest-rate is exogenous and constant. In fact, if wealth effects are eliminated completely, disinflation reduces welfare (by 0.31% on aggregate) through this channel.

I abstract from the redistributive role of inflation among debtors and creditors of local currency denominated nominal contracts. The motivation of doing so derives from the phenomenon that (i) high inflation economies have developed particular methods (such as financial dollarization) to cope with inflation and (ii) a methodological point that, steady state comparisons do not allow to keep track of the portfolio evolution of particular agents across high and low inflation economies. As a result, portfolio revaluation effects are not traceable.

The most related work to this paper are the studies of Algan and Ragot (2010) and Berriel and Zilberman (2011). The former study explores the impact of inflation and borrowing constraints on aggregate capital accumulation in a heterogeneous agents environment. Yet, their study is contained within a closed economy framework and they do not explore welfare consequences of inflation. The latter study explores the distributional and welfare consequences of cash transfers in Brazil and find that cash transfers increase welfare and reduce the precautionary savings motive of the poor. As a result, while income inequality does not change much, wealth inequality increases with transfers. An important difference of the current paper from their work is that the redistributive nature of “cash transfers” implied by inflationary finance depend on fiscal and monetary interactions and there is no threshold level for poverty that is exogenously determined by the government. In this paper, redistributive effects of inflationary finance are purely driven by variation in
endogenous portfolio composition and wealth level of consumers.

Other closely related studies are the work of Erosa and Ventura (2002) and Albanesi (2007). Both studies incorporate a costly transaction technology that displays economies of scale so that the poor choose to consume “cash” goods. The current paper differs from the former study by showing that if the redistribution effect is predominant, the poor benefit from inflationary finance while holding a portfolio which is more vulnerable to inflation. Moreover, I analyze the effect of inflation on financial wealth inequality, changing the direction of causality emphasized by the latter study.

This paper contributes to the monetary economics literature that incorporates imperfectly insured, idiosyncratic risk. Imrohoroglu (1992) and Molico (2006) study the precautionary demand for money but abstract from portfolio composition. Imrohoroglu and Prescott (1991), Chatterjee and Corbae (1992), Akyol (2004), Ragot (2009) and Wen (2010) include interest bearing assets but do not model money as an asset that economizes transactions costs. Therefore, inflation acts as a savings tax on households (not as an indirect consumption tax), and most of welfare effects originate from increased consumption volatility. The recent work by Doepke and Schneider (2006) and Meh et al. (2008) study welfare effects of an inflation shock that is modeled as a zero sum redistribution of real wealth. Chiu and Molico (2007) explore the welfare cost of inflation in developed economies in a search-theoretic environment with costly liquidity management and find that welfare costs of inflation are smaller than those estimated by representative agent models. Kehoe et al. (1992) analytically find that optimal inflation rate might be positive if lump-sum
transfers are considered. The theoretical contribution of the current paper is to reconcile the monetary model of small open economy (which has been commonly used to study exchange-rate-based stabilizations) with incomplete markets, uninsured idiosyncratic risk framework. On empirical grounds, this paper contributes to the literature by documenting (i) the structural change in inflation as a worldwide phenomenon; and (ii) the distributional aspects of the financial system in emerging economies by using disaggregated deposits data.

The rest of the paper is organized as follows. Section 1.2 reviews key facts regarding financial dollarization and the distribution of financial assets in emerging economies. Section 1.3 describes the theoretical model. Section 1.4 shows the workings of the model and defines the stationary recursive equilibrium. Section 1.5 presents the parameterization of the model and reports findings. Section 1.6 performs sensitivity analysis, and finally, Section 1.7 concludes the paper.

1.2 Key Facts

In this section, I document some financial system characteristics of emerging economies, which are among the building blocks of the question asked in this paper. In particular, I document (i) financial dollarization and (ii) properties of the distribution of deposits in emerging economies, from which the motivation of modeling bonds as inflation-indexed and using a heterogeneous agents framework derives, respectively.
1.2.1 Financial Dollarization in Emerging Economies

Dollarization in emerging economies has been understood as a currency substitution phenomenon. However, as Ize and Levy Yeyati (2003) argue, what is analyzed as “currency substitution” is actually “asset substitution”, since dollarization of interest-bearing financial assets is more predominant. Following this argument, I list key observations from the dollarization literature:

1. Cross-country average of the share of dollarized deposits at the end of 2000 was 35% in all developing economies (Levy Yeyati (2006)). Furthermore, dollarization of deposits is coupled with dollarization of loans. The elasticity of dollarized loans with respect to dollarized deposits is 0.73 for 100 emerging, developing and transition economies in the period 1990-2001 (De Nicoló et al. (2003)).

2. Only countries that have managed to keep inflation below 35% per annum between 1990 and 2005, do not exhibit a high degree of dollarization (i.e., a FX deposit share of more than 50%) between 2000 and 2004 (Honohan (2007)). There is a positive relationship between the likelihood of having an inflationary past and the degree of dollarization (Reinhart et al. (2003)) as well. Levy Yeyati (2006) finds that the correlation between average deposits dollarization and inflation rates is 0.50 and FD is stronger in economies with more inflation elastic monetary shocks.

\[^3\text{For example, in the period 2005:4-2008:4, the average share of foreign currency denominated demand and term deposits (with a maturity more than 6 months) in Turkey are 44\% and 72\% respectively. Source: Banking Regulation and Supervision Agency.}\]
3. Theoretically, under perfect pass-through, real value of dollar assets becomes fixed and interest-bearing segment of the financial system fully dollarizes (Ize and Levy Yeyati (2003) and Levy Yeyati (2006)). The data show that, a 10% increase in dollarization is associated with an 8% increase in pass-through (Honohan and Shi (2001)).

The main findings of the dollarization literature are: (i) FD is commonly observed in emerging economies; (ii) the degree of FD is positively related to inflation; (iii) data support the prediction of theory that the stronger pass-through, the stronger FD is. I now present properties of the distribution of deposits in emerging economies.

1.2.2 Distribution of Demand and Term Deposits

In this section, I document inequality in deposits positions and portfolio heterogeneity for a selected group of emerging economies. The selection criterion is availability of the data. The top panels of Figure 1.1 illustrate Lorenz curves of demand and term deposits for Turkey, Peru, Bolivia and Thailand. As figure clearly shows, demand and term deposits distributions display substantial inequality. Gini coefficients implied by the top panels of Figure 1.1 vary between 65% and 95%. For Bulgaria, Chile, Georgia and Lithuania, disaggregation into demand and term deposits is not available. Therefore, those countries cannot be included in Figure

---

4 They use quarterly data from over 50 countries for the period 1980-2000. The implied t-statistic from the estimation is equal to 4.5.

5 Deposits represent an important fraction of the financial system in emerging economies. For example, average share of deposits in total financial assets for the period 1970-2006 is 61% in the Turkish economy. Source: State Planning Organization (SPO, hereafter).
Figure 1.1: Deposits Distributions and Portfolio Share in Emerging Economies
1.1. Gini coefficients for total deposits in those countries vary between 80% and 95%.\(^6\)

Finally, the bottom panels of Figure 1.1 represent the share of term deposits (which bear more interest than demand deposits) for increasing account sizes. Both for Turkey and Bolivia, this share increases with account size.\(^7\) This suggests that portfolio of heterogeneous consumers is not uniform across the wealth distribution. This observation is in line with findings of Mulligan and Sala-i-Martin (2000), Avery et al. (1987) and Easterly and Fischer (2001) that the poor hold a financial portfolio that is more vulnerable to inflation. Consequently, I study a heterogeneous agents framework, which is a natural laboratory to analyze portfolio heterogeneity across the wealth distribution. This completes the presentation of key facts and I now proceed to the next section in which I describe the theoretical framework used to analyze the distributional and welfare implications of disinflation in emerging economies.

1.3 The Model Economy

I study a monetary model of a small open economy with uninsurable idiosyncratic earnings risk. There is no production. The economy is inhabited by two agents: A continuum of infinitely lived households of total mass 1 and a govern-

\(^6\)In Appendix D, I report data sources, describe how Figure 1.1 is plotted and include Table D.2 that reports the data for Turkey. Data for other countries are available from the author upon request.

\(^7\)It is puzzling to see that share of term deposits is falling for the largest size accounts in the case of Bolivia. This could be because Bolivia experiences “currency substitution” so that even cash is dollarized and is less vulnerable to the depreciation of domestic currency. This feature reduces the asymmetric advantage of the rich in say Bolivia, relative to the advantage of the rich in a less dollarized economy. Yet, possession of zero return, dollarized cash still provides better insurance than negative return, domestic currency.
ment. To focus on the implications of a reduction in inflation, I abstract from aggregate uncertainty. Time is discrete. The consolidated government determines fiscal and monetary policy.

1.3.1 Households

The stochastic process of earnings is independently and identically distributed across consumers and follows a finite state Markov chain with conditional probabilities $p_{\varepsilon'|\varepsilon} = Pr(\varepsilon_{t+1} = \varepsilon' | \varepsilon_t = \varepsilon)$ for $\varepsilon'$ and $\varepsilon \in E$ where $E$ is a finite dimensional vector. The invariant distribution of this Markov process is denoted by $P$.

Households derive utility from consumption. Preferences over flows of a single, tradable consumption good are given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

where $0 < \beta < 1$ is the subjective discount factor (which is same across individuals) and $u(c)$ is a continuous and strictly concave function defined over the flow of consumption. Utility function satisfies the Inada condition, $\lim_{c \to 0^+} u'(c) = \infty$. $E$ is the mathematical expectation operator.

Households have access to two financial assets: Real balances (demand deposits), $m$, issued by the monetary authority, and one-period, risk free bonds (term deposits), $b$, that are internationally traded. From now on, I use real balances (bonds) and demand (term) deposits interchangeably.

The decision of real balances position is made at the beginning of the period. Consumers use real balances during the period to economize transactions costs of consumption and once consumption takes
place, they carry over their position in this asset to the next period. Small letters denote real values of individual specific variables. Capital letters denote aggregate real variables. If inflation from date \( t - 1 \) to date \( t \) is \( \pi_t \), then real deposits, \( a_t \), at time \( t \) are defined as \( a_t = Rb_t + \frac{m_t}{1 + \pi_t} \) where \( R \) is the gross real interest rate and \( b_t \), \( m_t \) are the beginning of period \( t \) positions in bonds and real balances respectively.

Consumers face the flow budget constraint,

\[
c_t \left[ 1 + S \left( \frac{c_t}{m_{t+1}} \right) \right] + b_{t+1} + m_{t+1} = \varepsilon_t + a_t + \tau_t \tag{1.2}
\]

The left-hand-side of (1.2) represents total consumption expenditures and asset demands. Following Kimbrough (1986) and Mendoza and Uribe (2000), transactions costs are assumed to be an increasing function \( S \) of the consumption velocity of money, \( \kappa_t = \frac{c_t}{m_{t+1}} \). The unit transactions costs function is assumed to take the form \( S = \phi \kappa^\gamma \), where \( \phi > 0 \) and \( \gamma > 1 \). \( \tau_t \) is a lump-sum transfer made by the government. Since utility function satisfies the Inada condition, consumption has to be strictly positive (\( c_t > 0 \ \forall t \)). Moreover, for the convex function \( S \) to be defined, real balances should be strictly positive as well (\( m_t > 0 \ \forall t \)). I assume that financial markets are underdeveloped in this economy. Therefore consumers face a borrowing constraint so that \( b_{t+1} \geq \Omega \) with \( \Omega \leq 0 \).

There is perfect mobility in capital and goods markets. Therefore, small open economy assumption ensures that \( R \) is taken as given from the international capital market. As is shown on the left hand side of (1.2), real balances chosen in period \( t \), \( m_{t+1} \), economize current transactions and are carried over to the next period.

Even without ad-hoc borrowing constraints, consumers will never borrow more than a “natural debt limit” to ensure nonnegative consumption in each period. This debt limit implies the lower bound \( \Psi = \left( \frac{c_{\text{min}} + \varepsilon_t - \pi_t}{1 - R} \right) \) for \( b_t \) and is a variation of the one studied by Aiyagari (1994).
markets.\textsuperscript{11} Under the law of one price and the assumption of zero foreign inflation rate, domestic inflation rate, $\pi_t$, becomes identical to the depreciation rate of the currency, $e_t$.\textsuperscript{12}

At any period $t$, a household is characterized by a double $(a_t, \varepsilon_t) \in A \times E$, where the terms in parentheses denote the real deposits position and earnings level of an individual. If $\mathcal{A}$ denote the Borel sets that are subsets of $A$ and $\mathcal{E}$ denote the set of all subsets of $E$, then $(X, \mathcal{X}) = (A \times E, \mathcal{A} \times \mathcal{E})$ denotes the product space whereas $X$ denotes the state space of this economy. Let $\Gamma(a, \varepsilon)$ be the measure of agents who are in the idiosyncratic state $(a, \varepsilon)$. I discretize the state space. Real deposits holdings is a member of the grid $A = [a_1 < a_2 < \ldots < a_n]$. The choices of real balances and bonds that govern the evolution of total deposits are also restricted to be members of the grids $M = [m_1 < m_2 < \ldots < m_{nm}]$ and $B = [b_1 < b_2 < \ldots < b_{nb}]$, respectively. Real deposits and earnings are used to indicate the state of an individual both for expositional simplicity and numerical tractability. However, portfolio choice between real balances and bonds is still explicit in the model, as I describe below.\textsuperscript{13}

1.3.2 Government and Alternative Fiscal Arrangements

Equation (1.3) describes the budget constraint of the government. As part of the monetary policy, the government issues the currency and announces the de-

\textsuperscript{11}For a given $R$, I restrict $\beta$ to satisfy $\beta R < 1$ in order to guarantee the existence of an ergodic distribution of total deposits. For a discussion of this property of incomplete markets models, see Huggett (1993).

\textsuperscript{12}Motivated by Section 1.2.1, bonds are thought to be fully dollarized so that real interest rate earned on them, $R$, is independent of the depreciation rate of currency by the interest parity condition.

\textsuperscript{13}The other option is to consider the triple of real balances holdings, bond holdings and earnings as the idiosyncratic state of the consumer.
preciation rate of the nominal exchange rate, \( e_t \).\(^{14}\) Since the focus is on stationary equilibria, \( e_t = e \forall t \). Aggregate real seigniorage revenues are denoted by \( M_{s,t+1} - \frac{M_{s,t}}{1+e_t} \), where \( M_{s,t} \) is the aggregate real money supply at the beginning of period \( t \).\(^{15}\) I abstract from international reserves for simplicity.

\[
G_t + \tau_t = M_{s,t+1} - \frac{M_{s,t}}{1+e_t} \tag{1.3}
\]

Fiscal policy is conducted by making unproductive expenditures, \( G_t \), and remitting transfers, \( \tau_t \) to households. To explore the distributional role of a reduction in inflation, I study alternative fiscal arrangements in response to monetary policy described above. In Economy 1 (Economy 2), I assume that the government spending (uniform transfers) is (are) constant, \( G_t = G \forall t \ (\tau_t = \tau \forall t) \), which leaves uniform transfers (spending) as responsive to changes in seigniorage revenues. These two arrangements are meant to capture the redistributive role of uniform transfers. I consider Economy 1 as the benchmark case, since a well-known practice in the literature is to couple monetary creation by lump-sum transfers. Lastly, in Economy 3, I assume again that spending is constant but now model transfers as proportional to individual specific inflation tax payments.\(^{16}\) In this case, transfers are meant to partially neutralize wealth effects caused by changes in inflation.\(^{17}\)

\(^{14}\)It is assumed that the government can perfectly manipulate the depreciation rate of currency, although the de jure exchange rate regime is not necessarily pre-determined. To that end, I take the disinflation phenomenon as given.

\(^{15}\)Money is demand determined, i.e., for a pre-determined depreciation rate, the central bank prints as much money as the economy demands on aggregate.

\(^{16}\)I assume that the government is not capable of identifying the money holdings of heterogeneous agents in Economies 1 and 2 whereas in Economy 3, it can perfectly track the inflation tax paid by each consumer without consumers having the chance to internalize this transfers policy.

\(^{17}\)Wealth effects would be fully neutralized if inflation tax payments and transactions costs are completely rebated in an individual specific manner which also requires \( G = 0 \).
1.4 Analytical Framework

In this section, I formulate the optimization problem solved by the consumer in the benchmark economy, analyze the workings of the model on the portfolio heterogeneity and welfare, and define the stationary recursive equilibrium.

1.4.1 The Household’s Decision Problem

Dynamic programming problem solved by a household who is in state \((a, \varepsilon)\) is:

\[
v(a, \varepsilon) = \max_{c, m', b'} \left[ u(c) + \beta E \left\{ v \left( Rb' + \frac{m'}{1 + \varepsilon'}, \varepsilon' \right) \, \varepsilon' \right\} \right] \tag{1.4}
\]

subject to

\[
c \left[ 1 + S \left( \frac{c}{m'} \right) \right] + b' + m' = \varepsilon + a + \tau \tag{1.5}
\]

\[
c, m' \geq 0 \text{ and } b' \geq \Omega \tag{1.6}
\]

where \(a = Rb + \frac{m}{1 + \varepsilon} \) and \(-\Omega\) is an ad-hoc debt limit.

The decision rules of an individual that govern the demand for real money balances, bonds and consumption are time invariant functions \(m' = m'(a, \varepsilon), b' = b'(a, \varepsilon)\) and \(c = c(a, \varepsilon)\). The optimality conditions that come out of combining the first order conditions of this problem are:

\[
\lambda [1 - S'(\kappa)\kappa^2] = \frac{\beta}{1 + \varepsilon} E \{ \lambda \} \tag{1.7}
\]
\[ \lambda - \varphi = \beta RE\{\lambda^t\} \]  
\hspace{10cm} (1.8)

\[ c[1 + S(\kappa)] + b' + m' = \varepsilon + a + \tau \]  
\hspace{10cm} (1.9)

where \( \kappa = \frac{\varepsilon}{m} \) and \( a = Rb + \frac{m}{1+\varepsilon} \).

Lagrange multipliers of the budget constraint and the borrowing constraint (\( \lambda \) and \( \varphi \)) are shadow prices of total (real) deposits and relaxing the borrowing constraint by one unit respectively.\(^{18}\) Equation (1.7) is the Euler equation related to real balances decision. The left hand side is the marginal cost of saving in real balances (i.e., foregone marginal utility of consumption net of economized unit transactions costs) whereas the right hand side is the marginal benefit of saving in real balances (i.e., the expected discounted marginal utility of consuming the gross return in the next period). The real return from holding real balances is negative if \( e > 0 \). Equation (1.8) is the Euler equation for bonds, which equates the marginal cost of saving in interest-bearing bonds (net of the shadow price of relaxing the borrowing constraint by one unit) to the expected discounted marginal utility of consuming the gross return in the next period. Equation (1.9) is the flow budget constraint of the household.

\(^{18}\)Both Lagrange multipliers are functions of idiosyncratic states due to the history dependence implied by incomplete markets.
1.4.2 Heterogeneity in Opportunity Cost of Holding Real Balances and Portfolio Composition

**Proposition 1.1** For a given constant depreciation rate, $e$, and real interest rate, $R$, consumption velocity of individuals who do not face a binding borrowing constraint is identical, i.e. $\kappa(a, \varepsilon) = \kappa \forall (a, \varepsilon)$ if $b'(a, \varepsilon) > \Omega$. Moreover, consumption velocity of borrowing constrained individuals, $\kappa^c$ is strictly greater than $\kappa$ and is increasing in $\varphi(a, \varepsilon)$. For a proof, see Appendix A.

Proposition 1.1 elaborates how the effective opportunity cost of holding real balances is determined across different agents. I point out that it is higher for constrained individuals and the more constrained an individual (i.e. the larger $\varphi(a, \varepsilon)$) is, the larger the discrepancy. The intuition here is as follows: Consider a borrowing constrained individual who is hit by a negative earnings shock. The only way to dissave for such a consumer is to reduce real balances holdings, $m'$, which results in a higher consumption velocity, $\kappa^c = \frac{c}{m'}$ for a given consumption level. This is costly for such an individual because higher consumption velocity means higher effective price of consumption\(^{19}\).

The second important implication of Proposition 1.1 is on portfolio heterogeneity. In a standard incomplete markets economy with uninsurable idiosyncratic risk, the consumption-to-wealth ratio (i.e. $\frac{c(a, \varepsilon)}{a}$ in the current model) typically falls with wealth, since the marginal utility of consumption is higher for the poor.\(^{20}\) Now,

\(^{19}\)See the budget constraint in equation (1.9)

\(^{20}\)This is especially the case when the precautionary savings motive is less predominant, that is, when consumers are sufficiently far away from the natural debt limit.
the first part of Proposition 1.1 (i.e., \( \frac{c(a,\varepsilon)}{m'(a,\varepsilon)} = \kappa \forall (a,\varepsilon) \) with \( b'(a,\varepsilon) > \Omega \)) coupled with the afore-mentioned property causes \( \frac{d(\frac{m'(a,\varepsilon)}{a})}{da} < 0 \). As a result, the poor hold a larger fraction of their total deposits in demand deposits, consistent with the bottom panel of Figure 1.1 and the empirical literature on financial asset portfolio across the wealth distribution.\(^{21}\)

1.4.3 Distributional and Welfare Implications of Inflation in Relation to Fiscal and Monetary Interactions

In the current paper, inflation has the following two adverse effects: (i) a wealth eroding effect through inflation taxation and (ii) distortion in the consumption decision led by changes in the real transactions costs per unit of consumption. These effects can be listed among the classical adverse effects of inflation. However, the particular way that fiscal authority responds to monetary authority (which I call fiscal and monetary interactions) might create substantial wealth effects on the private sector. Moreover, these wealth effects can be asymmetric due to the heterogeneous agents nature of the model economy studied in this paper. In order to gain intuition on the implications of alternative fiscal policy arrangements, I make a detour here and analyze the deterministic version of the model economy described in Section 1.3.

\(^{21}\)In generating this result, I do not resort to any economies of scale assumption on the transactions costs function (i.e., average transactions costs \( \phi_n \) do not depend on consumption) in contrast with Erosa and Ventura (2002). Note also that the focus is on the “portfolio share” of real balances. Otherwise, it follows again from Proposition 1.1 that the poor hold less real balances in “absolute terms”, because they consume less.
1.4.3.1 A Deterministic Economy with Heterogeneous Households

In this section, I simplify the economy studied in Section 1.3 by assuming that the economy is now inhabited by a finite number of household types \( i \in \{1, \ldots, I\} \) who are endowed with a time-invariant flow of earnings \( \varepsilon_i \). Each cohort \( i \) includes a large number of identical households. If the total population is normalized to 1, the measure of each cohort becomes \( \mu_i > 0 \) with \( \sum_i \mu_i = 1 \). Here I focus on the welfare implications of the alternative fiscal arrangements within no-earnings risk framework. The solutions to these models including portfolio effects of inflation are available in Appendix B.

Endogenous Uniform Transfers

From an optimal inflation point of view, the Friedman rule establishes an important theoretical benchmark. In general, the inflation rate that follows the Friedman rule is the one that implies zero non-pecuniary returns from holding real balances.\(^{23}\)

Assuming CRRA utility and using the closed form solutions for \( c_i^* \) and \( \tau^* \), the long-run welfare of a type \( i \) consumer \( (W_i) \) becomes

\[
W_i = \left[ \frac{\varepsilon_i + \tau^* - (1 - R) \Omega}{(1 + \phi \kappa)^{1 - \sigma}} \right]^{1 - \sigma} - 1
\]

\[\text{(1.10)}\]

\(^{22}\)The problem of a type \( i \) consumer looks similar to the problem formulated by equations (1.4), (1.5) and (1.6) with the only difference that the deterministic \( \varepsilon_i \) is no more a state variable. For the following, I denote economic variables related to type \( i \) consumers by an \( i \) subscript.

\(^{23}\)In the current paper, it is the inflation rate that implies a zero consumption velocity, which would eliminate the inefficiency caused by real transactions costs of consumption. Hence, \( e^{Fr} = \beta - 1 < \frac{1}{\rho} - 1 \) by the solution for consumption-velocity, \( \kappa = \frac{c_i^*}{\mu_i} = \left[ \frac{1}{1 + \phi \kappa} \right]^{1 - \sigma} \) and \( \beta R < 1 \). In this case, opportunity cost of cash is higher than the interest rate, therefore \( e^{Fr} \) becomes smaller than the zero nominal interest rate rule, \( \frac{1}{\rho} - 1 \).
It is crucial to see that long-run welfare of consumers is affected by (i) their “disposable income”, \( \varepsilon_i + \tau^* - (1 - R)\Omega \); (ii) the inefficiency brought by transaction costs of consumption \( \phi\kappa \gamma \); and (iii) inflation tax paid-per consumption, \( \frac{\epsilon}{1 + e^{Fr}\kappa} \). The second and the third effects denote the classical adverse effects of inflation, i.e., consumption distortion and wealth eroding whereas the first effect denotes the “re-distribution” effect introduced by the current paper. Indeed, equation (1.10) shows that the poor would benefit from inflationary finance provided that seigniorage revenues (and therefore lump-sum transfers) increase with inflation. In particular, although the effects (ii) and (iii) worsen with higher inflation, incidence of inflation tax and transactions costs would fall short of the aggregate transfers earned by the poor, in “absolute terms”. This is because they consume less (and therefore hold less real balances by constant consumption velocity). This creates an increase in the disposable income of the poor at the expense of the rich. Consequently, (assuming that the measure of the poor is larger than the rich) the inflation rate that maximizes aggregate welfare (\( \sum \mu_i W_i \)) would be positive and high. It is also interesting to observe that the third term breaks down the optimality of the Friedman rule, since \( \kappa \to 0 \) implies that \( \frac{e^{Fr}}{1 + e^{Fr}\kappa} \frac{1}{\kappa} \to -\infty \), deteriorating consumption. Indeed, as inflation gets closer to the Friedman rule, although the distortions in the economy are eliminated, aggregate welfare would keep falling since it would be inefficient to redistribute resources from the poor to the rich by means of decreasing lump-sum transfers.
Endogenous Government Spending

Long-run welfare of type $i$ individuals when spending is endogenous can be written as:

$$W_i = \frac{\left[\frac{\varepsilon_i + \tau - (1 - R)\Omega}{(1 + \phi \kappa \gamma) + \frac{\varepsilon_i + \tau - (1 - R)\Omega}{(1 + \phi \kappa \gamma)}}\right]^{1-\sigma} - 1}{(1 - \beta)(1 - \sigma)}.$$  \hfill (1.11)

This expression is identical with equation (1.10) except for the crucial difference that redistribution effect is now muted since lump-sum transfers (and disposable income) do not respond to inflation.\textsuperscript{24} Therefore, optimality requires the inefficiencies to be eliminated. Yet, the Friedman rule is still suboptimal because it would imply negative government spending, which is not feasible. Consequently, we have a constrained efficiency problem and aggregate welfare ($\sum_i \mu_i W_i$) is maximized when $G^* = 0$. The closed form solution for $G^*$ enables us to solve for this inflation rate analytically:\textsuperscript{25}

$$e^{CE} = \frac{1}{1 - \frac{\tau \kappa (1 + \phi \kappa \gamma)}{Y - (1 - R)\Omega}} - 1.$$  \hfill (1.12)

Endogenous Proportional Transfers

Lastly, when transfers depend on idiosyncratic inflation tax payments, long-run welfare of type $i$ consumers becomes

\textsuperscript{24}Strictly speaking, redistribution is not absent as long as transfers are positive. But changes in the degree of redistribution are mostly eliminated.

\textsuperscript{25}Notice that $e^{CE}$ depends on $\tau$, which in turn is the equilibrium transfers level in the previous economy for a benchmark inflation rate.
\[ W_i = \left[ \frac{\varepsilon_i - G - (1 - R) \Omega}{(1 + \phi \kappa \gamma)} \right]^{1-\sigma} - 1 \]

This expression (and consequently aggregate welfare, \( \sum_i \mu_i W_i \)) can be maximized only if \( \kappa \to 0 \), which is achieved when inflation is equal to the Friedman rule level. Therefore, the redistribution effect in this economy is completely shut down and changes in inflation taxation are exactly compensated by changes in proportionate transfers.

A Deterministic Economy with a Representative Household

The deterministic version of Economy 1 would be identical to a representative agent economy if each cohort \( i \) possesses the same deterministic earnings profile \( \varepsilon_i = Y \), where \( Y \) is the GDP per-capita of the economy. Long-run welfare of the representative household becomes:

\[ W = \left[ \frac{Y - G - (1 - R) \Omega}{(1 + \phi \kappa \gamma)} \right]^{1-\sigma} - 1. \]

It is clear that the Friedman rule is optimal in this case given that the disposable income of the aggregate economy does not depend on inflation. Furthermore, wealth eroding term does not even show up in the welfare expression since lump-sum transfers exactly match the inflation tax net of government spending.\(^{26}\)

In summary, introducing heterogeneity to the small open economy model might drive non-trivial departures from the representative agent framework depending on fiscal and monetary interactions. Moving toward the economy with idiosyncratic

\(^{26}\)As a result, the first-best in this economy would require \( G = 0 \) and \( c = c^{Fr} \).
uncertainty and incomplete markets brings an additional channel that would affect the disposable income of households. Changes in inflation (which is a relative price between bonds and real balances) would create wealth and substitution effects regarding the portfolio decision. Depending on the relative dominance of these effects, welfare impacts might be strengthened or weakened. Notice also that small open economy takes the interest rate as given. Therefore, the celebrated precautionary savings (PS, hereafter) outcome of reduction in the equilibrium real interest rate (see Huggett (1993)) is absent in this model. This will result in large magnitudes of movements in the equilibrium quantity of interest-bearing assets, which in turn affects the disposable income and welfare. Figure 1.2 illustrates this idea by considering an increase in the PS motive. In closed-economy setup, an increase in the PS motive shifts the demand for assets, $S_0$, (supply of funds, $B_0$) to the right (left). This results in higher bond price, $q_1 > q_0 = q^*$ (lower interest rate) and an increase in the PS of $PS_{closed}$. However, since the supply of funds, $B_{open}$ is flat in the small open economy, the adjustment in the PS happens much larger, i.e. $PS_{open} > PS_{closed}$. Therefore, individuals might experience sharp variations in their interest income when the PS motive changes.

This completes the description of the framework and I now proceed to the definition of the stationary recursive equilibrium in the benchmark economy with idiosyncratic uncertainty.

1.4.4 Stationary Recursive Equilibrium

I assume that conditions that guarantee the existence of unique invariant meas-
Figure 1.2: Precautionary Savings in Small Open Economy
ure \( \Gamma^* \) are satisfied (see Hugget (1993)). Below is a formal definition of the stationary recursive equilibrium:

**Definition 1.1** Given a constant level of government expenditures \( G \), the international gross real interest rate \( R \) and a constant depreciation rate \( e \), a stationary recursive equilibrium is a time invariant value function \( v \), time invariant policy functions \( m' = m'(a, \varepsilon; e) \), \( b' = b'(a, \varepsilon; e) \), \( c = c(a, \varepsilon; e) \), constant lump-sum transfers \( \tau^* \) and a stationary distribution \( \Gamma \), such that: (i) Given \( \tau^* \), \( R \), and \( e \); \( v \), \( m' = m'(a, \varepsilon; e) \), \( b' = b'(a, \varepsilon; e) \) and \( c = c(a, \varepsilon; e) \) solve the household’s problem (1.4.1); (ii) Given \( G \), \( \Gamma \), \( e \) and the policy functions of households, \( \tau^* \) is consistent with the balanced budget of the government; \( G + \tau^* = \left( \frac{e}{1 + e} \right) M^* \); (iii) Given \( \Gamma \) and policy functions of households, aggregate goods market clears (i.e. the national income identity holds), \( C + G + (1 - R)B + Tr = Y \) with \( C = \sum_{a, \varepsilon} \Gamma(a, \varepsilon)c, \) \( B = \sum_{a, \varepsilon} \Gamma(a, \varepsilon)b', \) \( Y = \sum_{a, \varepsilon} \Gamma(a, \varepsilon)e, \) and \( Tr = \sum_{a, \varepsilon} \Gamma(a, \varepsilon)cS(\frac{1}{m'}). \) Money market equilibrium, \( M^* = \sum_{a, \varepsilon} \Gamma(a, \varepsilon)m' \) follows from the de facto exchange rate regime; (iv) Given policy rules for assets and the Markov transition of earnings, \( [b'(a, \varepsilon), m'(a, \varepsilon), p_{\varepsilon} | \varepsilon] \), the distribution of total deposits and earnings satisfies the following fixed point equation: \( \Gamma(a', \varepsilon') = \sum_{\varepsilon} \sum_{a:a'=Ra'(a, \varepsilon)+m'(a, \varepsilon)} \sum_{\varepsilon} \frac{\varepsilon}{m'} \Gamma(a, \varepsilon)p_{\varepsilon} | \varepsilon. \)

1.5 Quantitative Analysis

In this section I study the model’s quantitative predictions using a version calibrated to the Turkish economy. From a parameterization and calibration per-
pective, the focus is on the low inflation period 2003:1-2010:2, for which data on aggregates, inequality measures and government transfers are available. The main experiment is to make long-run equilibria comparisons between high \( e = 14.25\% \) for the period 1987:1-2002:4 and low \( e = 2.25\% \) inflation economies. Throughout the analysis, I carry this parameterization and calibration into Economies 2 and 3. I now describe the parameterization of Economy 1.

1.5.1 The Parameterization of the Benchmark Economy

The parameter values that are used in Economy 1 are reported in Table 1.2 below. Following the literature, \( \sigma = 2 \) is chosen as the risk aversion parameter of the CRRA utility. The model period is a quarter. \( R \) is set to 1.0276 in line with the exercise of Uribe and Yue (2006) to reflect an emerging economy country risk premium of 7% above the average US Treasury bill rate of 4% per year. A risk premium of 7% interestingly proves consistent with using Turkish data and the calibration strategy adopted in this paper as I explain below.

1.5.1.1 Idiosyncratic Earnings Process

I assume that the natural logarithm of the idiosyncratic earnings, \( \varepsilon_t \), follows an AR(1) process subject to normally distributed disturbances \( u_t \), with zero mean and constant variance, \( \sigma_u^2 \). Therefore I have

\[
\log \varepsilon_{t+1} = (1 - \rho)\mu + \rho \log \varepsilon_t + u_t
\]

(1.15)

where \( \mu \) is the mean of the logarithm of the earnings process and \( \rho \) is the persistence parameter. Due to lack of longitudinal panel studies on micro level earnings dyna-
Table 1.2: Benchmark Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Target</th>
<th>Moment</th>
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<tr>
<td><strong>Fixed</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.0000</td>
<td>Risk aversion</td>
<td>Literature</td>
<td>N/A</td>
</tr>
<tr>
<td>$R$</td>
<td>1.0276</td>
<td>Gross real interest rate</td>
<td>US Treasury + 7% spread</td>
<td>N/A</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9625</td>
<td>Persis. of earnings shocks</td>
<td>Literature</td>
<td>N/A</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.1400</td>
<td>Volat. of shocks to log-earnings</td>
<td>Literature</td>
<td>N/A</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.2175</td>
<td>Curv. of the trans. costs function</td>
<td>Int. elas. of M1 demand = -0.4510</td>
<td>N/A</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>0.1611</td>
<td>Real gov. spending-to-GDP</td>
<td>Average of 2003:1-2010:2</td>
<td>N/A</td>
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<tr>
<td><strong>Jointly Calibrated</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9215</td>
<td>Discount factor</td>
<td>$NX/GDP = -0.0334$</td>
<td>-0.0333</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.00175</td>
<td>Multip. trans. costs parameter</td>
<td>$C/M1 = 4.1925$</td>
<td>4.1920</td>
</tr>
<tr>
<td>$\Omega$</td>
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<td>Lower bound for bonds</td>
<td>$(M2Y - M1)/M2Y = 0.8493$</td>
<td>0.8543</td>
</tr>
</tbody>
</table>
nics for the Turkish economy, I choose values for earnings process parameters that are in an acceptable range studied by the literature. As mentioned in Algan and Ragot (2010), parameters of persistence of about 0.90 and standard deviation of innovations in the range of 0.12 and 0.25 are used in the literature (see also Hubbard et al. (1995) and Heathcote et al. (2010). The former study even considers values for persistence around 0.95). Therefore, I fix $\rho = 0.9625$ and $\sigma_u = 0.14$. I then approximate a normally distributed log-earnings process with a Markov chain, using the double $(\sigma_u, \rho)$, in Tauchen’s (1986) procedure. To capture most of the domain of normal density, the spread parameter is set to 3. 21 nodes that are located symmetrically around zero are used on the shocks to log-earnings grid. I normalize grid points to ensure that GDP in the model is equal to 1.

Note that in contrast with the case of Turkey, the literature is not silent on the evolution of inequality in earnings as far as some other emerging economies are concerned. For instance, Binelli and Attaniso (2010) document that earnings inequality in Mexico has been substantially high (with variance of log-earnings (VLE) around 2.2) and steady in the decade of 1990s, whereas Gorodnichenko et al. (2010) point out that inequality and volatility in earnings has been reducing in Russia (VLE, from about 0.8 in the mid-1990s to 0.6) in the last decade. The double $(\sigma_u, \rho)$ used in the current paper imply a low value for VLE, 0.3 implying a coefficient of variation of 0.55. The latter number is also lower than the value reported (0.97 for male regular wage earners) in the study of Dayioglu and Baslevent (2007) which uses cross sectional data from Household Budget Survey (2003) conducted by TURK-STAT (Turkish Statistical Institute). Nevertheless, it is necessary to have an upper
bound for the volatility of earnings process in this framework because with more volatile shocks, it is considerably difficult to generate substantial inequality in financial wealth due to the increased precautionary savings motive. The reader might judge the performance of the model on these grounds in section 1.5.2 below, in which I compare the benchmark model against data on distributional measures of assets and disposable income for these fixed values of earnings parameters. Additionally in Section 1.6, I conduct sensitivity analysis to explore the impact of tweaking these parameter values.

Finally, consumption and income inequality in Turkish economy for the period 2004-2008 have been relatively stable. Gini coefficients that I compute from the approximate Lorenz curves for income and consumption are 39% and 33% respectively.28 Surprisingly, consumption inequality is much less than inequality in deposits positions and income. I attribute this phenomenon partly to the existence of informal insurance mechanisms such as worker remittances that are denominated in foreign currency and intra-family insurance.29

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28The data come from Household Budget Survey (2004-2005), Income and Living Conditions Survey (2006-2007) and Consumption Expenditures Survey conducted by TURKSTAT.
29Recall that I only model earnings, interest and transfers as potential sources of income. Unfortunately, disaggregated data on each potential source of income are not available for the Turkish economy. However, Income and Living Conditions Survey (2006-2007) shows that these three sources of income indeed captures total income inequality quite well. This is because (i) earnings, transfers and interest income that I model add up to 67.57% of the aggregate disposable income; (ii) the major type of income which I do not model (i.e., entrepreneurial income, making 24% of total income) is distributed among quintiles similar to the way that earnings and social transfers are distributed. See Tables D.3 and D.4 in Appendix D on income and consumption inequality in Turkey.
1.5.1.2 Interest Elasticity of Money Demand

I use a standard form for the transactions costs function, \( S(\kappa_t) = \phi \kappa_t^\gamma \), which is introduced by Kimbrough (1986), and used by Mendoza and Uribe (2000) to analyze exchange-rate-based stabilizations in emerging economies. I choose to calibrate the curvature parameter of the transactions costs function, \( \gamma \), by estimating an aggregate money demand equation since the data on aggregates such as M1, nominal interest rates and consumption are available. The estimated equation is a log-linear relationship between real balances demand, consumption and the opportunity cost of holding money implied by the functional form of \( S \), the optimality condition regarding real balances (i.e. \( S'(\kappa_t)\kappa_t^2 = \frac{i_t}{1 + i_t} \)) and the definition of consumption velocity \( \kappa_t = \frac{c_t}{m_{t+1}} = \left[ \frac{1}{\gamma \phi} \left( \frac{i_t}{1 + i_t} \right) \right]^{\frac{1}{1+\gamma}} \). I also consider the cointegrating relationship between real money demand, opportunity cost of holding money and consumption and estimate

\[
\log(m_{t+1}) = \alpha_1 \log(c_t) + \alpha_2 \log \left( \frac{i_t}{1 + i_t} \right) + d_c(L)\Delta c_t + d_i(L)\Delta \left( \frac{i_t}{1 + i_t} \right) + \epsilon_t \quad (1.16)
\]

following Stock and Watson (1993) using dynamic ordinary least squares (DOLS) where \( d_c(L) \) and \( d_i(L) \) are polynomial operators with 2 leads and 2 lags.\(^{30}\) Note that equations (A.2) and (A.4) in Appendix A imply that both constrained and unconstrained individuals are subject to this functional relationship. Since the opportunity cost of holding money is higher for constrained individuals, the second explanatory variable on the right hand side of equation (1.16) should ideally incorporate this feature. However, since I do not have a measure of the “effective

\(^{30}\) Sequential modified LR test statistic, final prediction error and Akaike information criteria pointed out 2 as the optimal lag number in the VAR. \( m_{t+1} \) is determined at date \( t \).
opportunity cost of holding money for constrained individuals” in the data, I use the quarterly nominal deposit interest rates as an explanatory variable for all individuals and estimate a single equation.\footnote{This issue would be problematic had $\gamma$ been a parameter of consumer characteristics, such as a taste parameter for real balances in the MIU specification. However, in this framework, consumers have no preference over the way that the transactions are carried on. Therefore, I argue that it is appropriate not to model heterogeneity over $\gamma$ and use nominal interest rates as the common variable to capture the opportunity cost of holding money. For this matter, I used 1, 3, 6 and 12-month maturity deposit rates. The quarterly time series for nominal interest rates are constructed by adjusting maturities properly and computing weighted average rates by using the share of various (maturity) type-deposits in the whole system.} I use the time series for aggregate consumption and real M1 (deflated by the GDP deflator) in the estimation of equation (1.16) for the period 1992:2-2010:1 (after the lag adjustments), which roughly captures the weekly monotonic decline of the opportunity cost of holding money in the Turkish economy. The data source for all these aggregates is the Central Bank of Republic of Turkey, CBRT.

I estimate the interest elasticity of real money demand to be equal to $\alpha_2 = -0.4510$ (Lucas (2000) argues that interest elasticity of money demand for the US economy is -0.50 for the period 1900-1994). The standard error of this coefficient is 0.0362, implying a p-value of zero. The estimation also implies close to unitary elasticity of money demand with respect to consumption: I estimate $\alpha_1 = 0.8371$, with a standard error of 0.0054 and a p-value of again zero. The adjusted $R^2$ of the regression is 88.31\%. Since $\alpha_2 = -\frac{1}{1+\gamma}$, I solve for $\gamma$ to be equal to 1.2175.

1.5.1.3 Simultaneously Chosen Parameters

I choose values for the discount factor ($\beta$), multiplicative parameter of the transactions costs function ($\phi$) and negative of the debt limit ($\Omega$) simultaneously.
to match moments from the data. The corresponding moments in order are (i) net exports-to-GDP ratio, (ii) aggregate consumption velocity of M1 and (iii) aggregate portfolio composition.\(^{32}\) Long-run net exports-to-GDP ratio is denoted by \(\frac{(1-R)B}{Y}\) in the model and therefore naturally depends on the economy’s aggregate stock of interest bearing asset. I measure total stock of deposits \((B + M)\) with M2Y, which includes currency in circulation, checkable deposits, term deposits and foreign currency denominated deposits.\(^{33}\) Aggregate portfolio composition is captured by \(\frac{B}{B+M}\) in the model.

Calibration of above parameters requires the solution of the model. In particular, I use the Simulated Annealing method to find the set of parameters that minimizes the absolute values of deviations of model generated moments from their empirical counterparts. The empirical targets and the model generated moments are reported in Table 1.2. The solution to these parameters remain in 0.75% error level.

I pin down the exact value of \(R\) by imposing the long-run relationship between aggregate bonds position of the small open economy and the trade balance, i.e.

\[
(1 - R) \left( \frac{M2Y-M1}{GDP} \right)_{Avg} = \left( \frac{X-M}{GDP} \right)_{Avg},
\]

to the data and then search for \(\beta\) to match the empirical trade balance-to-GDP ratio. Interestingly, the implied real interest rate from this calculation coincides with the value that Uribe and Yue (2006) use.\(^{34}\)

\(^{32}\)Time series averages over the period 2003:1-2010:2 are used as aggregate targets.

\(^{33}\)The interest parity condition \((1 + i = (1 + i^*)(1 + e))\) and the law of one price \((1 + \pi = (1 + \pi^*)(1 + e))\) imply that a local currency denominated interest-bearing deposit is equivalent to a dollarized asset in rate of return. Consequently, the aggregate portfolio composition \(\frac{M2Y-M1}{M2Y}\), can be thought of as an effective dollarization ratio.

\(^{34}\)The high real interest earned on the internationally traded bonds by savers in the emerging economy should be thought of as facing a no arbitrage condition between saving either in domestic term deposits or re-saving in euro-bonds issued by their own country.
Recall that in Bewley models, $\beta R < 1$ has to hold in order to obtain a well defined ergodic distribution of wealth. Otherwise, consumers accumulate an unbounded magnitude of assets to avoid negative consumption in any state of nature.\textsuperscript{35} This feature of incomplete-markets models, the fixed value of the real interest rate, $R = 1.0276$, and the objective of matching the empirical ratio of $\frac{NX}{GDP} = -0.0334$ (Source: CBRT) necessitate using a lower $\beta$ (0.9215) than the value used in standard quarterly models. The implied $\beta R$ value in the model is equal to 0.9469. As a result, model generated $(1-R)\beta$ ratio is -0.0333.

The multiplicative parameter of the transactions costs function $\phi$ is calibrated to match the quarterly aggregate consumption velocity (measured by the time series average of aggregate consumption-to-M1 ratio) of 4.1925 (Source: CBRT). Model generated value, $C/M$, is 4.1920.\textsuperscript{36}

I use $\Omega = -0.0329$ (implying a debt-to-lowest earnings ratio of 17.85%) to target the aggregate portfolio composition (i.e. $\frac{M2Y-M1}{M2Y} = 0.8493$, source: CBRT). The model generated aggregate share of interest-bearing deposits, $B/(B + M)$, is 0.8543.

1.5.1.4 Public Sector

Finally, I close the section on the calibration of the baseline economy by mentioning parameters related to the government. I consider the depreciation rate $e = 0.0225$ in the baseline economy. This is the average quarterly inflation rate

\textsuperscript{35}For examples, see Aiyagari (1994) and Ljungqvist and Sargent (2004).

\textsuperscript{36}Aggregate consumption series that I use includes private investment expenditures added to private consumption. I lump private investment in $C$ as well, since I do not model physical capital accumulation. Following a similar reasoning, I lump public investment in the term $G$ while calibrating it (Source: State Planning Organization, SPO).
in the period 2003:1-2010:2. I computed the inflation rate from the GDP deflator (Source: CBRT). The time series average of aggregate government spending (final goods consumption plus investment expenditures of the central government) was about 16% of GDP for the same period (Source: CBRT). Therefore I set $G = 0.1611 Y$.

Note that the government budget constraint in the model is very simplistic. For example, I abstract from public finance elements such as government debt and conventional taxes (such as capital income tax) other than the inflation tax. Therefore, I need to take an empirical stance on what the endogenous variable $\tau$ represents. Equation (1.17) below illustrates the budget constraint of the consolidated government in general:

$$G_t + \text{Transfers}_t + (R - 1)B^G_t = \text{Revenues}_t + B^G_{t+1} - B^G_t$$  \hspace{1cm} (1.17)

where $G_t$ is government spending, $(R - 1)B^G_t$ is debt service, $\text{Transfers}$ are pure transfers to households and $\text{Revenues}$ are any kind of government revenues (mainly taxes). Now if I rewrite the government budget constraint in the model, I have,

$$G_t + \tau_t = M^s_{t+1} - \frac{M^s_t}{1 + \epsilon_t}$$  \hspace{1cm} (1.18)

where the right hand side are seigniorage revenues, which do not explicitly show up in the public finance data and $\tau_t$ is a lump-sum variable that tends to capture what I do not model in the public side. Since leaving $\tau_t$ as a residual is not informative for the matter of accounting pure transfers to households, I decompose it into two parts
τ₁ᵗ and τ₀ᵗ, where the former represents pure transfers to households (such as social security transfers, direct transfers and transfers to the health and education sectors) and the latter represents any component of equation (1.17) that is not modeled. The crucial feature of the baseline economy here is that I fix \( G \) and \( τ₀ᵗ \) so that they are independent of inflation and let \( τ₁ᵗ \) and \( \frac{M_{t+1} - M_{t}}{1+e₁} \) respond to inflation. Since pure transfers to households are about 4.5%, government spending is about 16% and seigniorage revenues are about 1% of GDP in the period 2003:1-2010:2, I set \( τ₀ᵗ = (M_{t+1} - \frac{M_t}{1+e₁})^{Data} - G^{Data} - τ₁^{Data} = 0.0111Y - 0.1611Y - 0.0463Y = -0.1963Y \) (Source: CBRT).

This closes the discussion of the parameterization and calibration of the benchmark model. Numerical solution method for the computation of the stationary recursive equilibrium is described in Appendix C. I now proceed to the analysis of the benchmark economy.

1.5.2 Benchmark Model vs. Data

In this section, I compare the aggregate and distributional variables implied by the calibrated model (Economy 1) with their empirical counterparts. The aggregate statistics that I report in Table 1.3 are the ratios of the aggregate stock of real balances plus bonds \((B+M)/Y\), consumption \((C/Y)\), trade balance \((1-R)B/Y\), transactions costs \((Tr/Y)\), lump-sum transfers (the part that respond to inflation, i.e. \((τ₁/Y)\), see Section 1.5.1.3) and real seigniorage revenues-to-GDP \(((eM/(1 + e)))/Y\), the aggregate consumption velocity of money \((C/M)\) and the dollarization

\footnote{Economies 2 and 3 obviously deviate from this setup.}
Table 1.3: Benchmark Model vs. Data, $e = 2.25\%$

<table>
<thead>
<tr>
<th>Aggregates</th>
<th>DATA</th>
<th>$E1$</th>
<th>Distributional Variables$^a$</th>
<th>DATA</th>
<th>$E1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(B + M)/Y^b$</td>
<td>1.423</td>
<td>1.414</td>
<td>$Gini_y$</td>
<td>0.390</td>
<td>0.338</td>
</tr>
<tr>
<td>$C/Y$</td>
<td>0.871</td>
<td>0.864</td>
<td>$Gini_b$</td>
<td>0.781</td>
<td>0.637</td>
</tr>
<tr>
<td>$TB/Y$</td>
<td>-0.033</td>
<td>-0.033</td>
<td>$Gini_m$</td>
<td>0.775</td>
<td>0.345</td>
</tr>
<tr>
<td>$Tr/Y$</td>
<td>$N/A$</td>
<td>0.009</td>
<td>$Gini_c$</td>
<td>0.330</td>
<td>0.335</td>
</tr>
<tr>
<td>$C/M$</td>
<td>4.193</td>
<td>4.192</td>
<td>$(Top_{20}/Bottom_{20})_y$</td>
<td>9.146</td>
<td>6.089</td>
</tr>
<tr>
<td>$SE/Y$</td>
<td>0.011</td>
<td>0.005</td>
<td>$(Mean/Median)_y$</td>
<td>1.350</td>
<td>1.209</td>
</tr>
<tr>
<td>$\tau_1/Y$</td>
<td>0.046</td>
<td>0.040</td>
<td>$(Top_{20}/Bottom_{20})_c$</td>
<td>5.984</td>
<td>6.192</td>
</tr>
<tr>
<td>$B/(B + M)$</td>
<td>0.849</td>
<td>0.847</td>
<td>Frac. of Constrained</td>
<td>$N/A$</td>
<td>0.057</td>
</tr>
</tbody>
</table>


$^b$Y denotes the GDP of the economy.
ratio, $(B/(B+M))$. The distributional variables are the Gini coefficients of disposable income, bonds, real balances and consumption, top quintile-to-bottom quintile ratios of income and consumption, mean-to-median ratio of income and the measure of borrowing constrained individuals. The model performs considerably well in terms of matching the ratios of aggregate stock of deposits, consumption and transfers-to-GDP although they are not targeted. Aggregate transactions costs are estimated to be 0.9% of GDP.

In Figure 1.3, I compare model generated (straight plots) Lorenz curves of disposable income (which is defined as the summation of earnings, interest income and transfers, i.e., $y = \varepsilon + (R-1)b' + \tau$), bonds, real balances and consumption and portfolio share of bonds with their empirical counterparts (dashed plots). Consistent with the right panel of Table 1.3, the model does well in replicating the inequality patterns of income and consumption, partially underestimates inequality in bond positions and considerably fails in capturing inequality in real balances holdings. The Gini coefficient of real balances is larger than that of consumption because of the existence of borrowing constraints, nevertheless, since only 5.7% of the population is borrowing constrained, the distribution of real balances is slightly decoupled from that of consumption. This finding proves that the distribution of consumption and cash holdings might be decoupled even in an environment in which there is a strong relationship between cash holdings and consumption. Lastly, in the bottom-right panel of Figure 1.3, I show the portfolio (plotted in a comparable way to the bottom-left panel of Figure 1.1) as a function of total deposits position. The model is consistent with the concavely increasing share of bonds as individuals become richer.
Figure 1.3: Lorenz Curves and Portfolio in the Benchmark Economy, e=2.25%
This completes the assessment of benchmark model’s performance against data and I now proceed to the discussion of the stationary equilibria consequences of disinflation under alternative fiscal arrangements.

1.5.3 Aggregate Implications of Disinflation

Table 1.4 shows that under all fiscal arrangements, the ratios $Tr/Y$, $M_{t+1}/Y$ and $C/M$ decrease when inflation declines. Reduction in inflation dominates the increase in aggregate real balances demand causing seigniorage revenues (inflationary finance) to diminish. Transfers decrease in Economies 1 and 3, whereas government spending decreases in Economy 2. The model is consistent with the observations of the dollarization literature that a reduction in inflation lowers the degree of dollarization, $B/(B + M)$. In Economy 3, aggregate asset position decreases (resulting in trade deficit to fall) in contrast with Economies 1 and 2. I discuss this difference below, in regards to distributional implications.

1.5.4 Distributional Implications of Disinflation

In the top panels of Figure 1.4 below, I plot the model generated cumulative distribution functions (c.d.f.) of term and demand deposits for permanent depreciation rates of 2.25 (dashed plot) and 14.25 (straight plot) percent.\textsuperscript{38} The top-right panel shows that c.d.f. of demand deposits in the low inflation economy first order stochastically dominates that in high inflation economy. This is because wealth and substitution effects work in the same direction for real balances demand: A reduction in inflation creates (i) a positive \textit{wealth} effect that induces consumers to

\textsuperscript{38}For visual clarity, I restrict the plot on the top-left panel to display a subset of the range of the c.d.f.
Table 1.4: Aggregate Implications of Disinflation

<table>
<thead>
<tr>
<th>Aggregates</th>
<th>Uniform $\tau$</th>
<th>Endogenous $G$</th>
<th>Proportionate $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e = 14.25%$</td>
<td>$e = 2.25%$</td>
<td>$e = 14.25%$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(B + M)/Y$</td>
<td>1.319</td>
<td>1.414</td>
<td>1.406</td>
</tr>
<tr>
<td>$C/Y$</td>
<td>0.856</td>
<td>0.864</td>
<td>0.848</td>
</tr>
<tr>
<td>$TB/Y$</td>
<td>-0.033</td>
<td>-0.033</td>
<td>-0.035</td>
</tr>
<tr>
<td>$Tr/Y$</td>
<td>0.016</td>
<td>0.009</td>
<td>0.015</td>
</tr>
<tr>
<td>$SE/Y$</td>
<td>0.015</td>
<td>0.005</td>
<td>0.015</td>
</tr>
<tr>
<td>$\tau_1/Y$</td>
<td>0.050</td>
<td>0.040</td>
<td>0.040</td>
</tr>
<tr>
<td>$B/(B + M)$</td>
<td>0.907</td>
<td>0.854</td>
<td>0.913</td>
</tr>
</tbody>
</table>

*G/Y = 17.2% in Economy 2 when $e = 14.25\%$. 
Figure 1.4: Disinflation, Portfolio and Asset Distributions (Economy 1)
demand more of both assets and (ii) a substitution effect driven by a reduction in the relative price of real balances in terms bonds that induces an individual to demand more (less) real balances (bonds). Therefore, whether rich or poor, consumers increase their real balances demand when inflation is lower whereas this is not the case for bonds since the dominance of wealth and substitution effects displays heterogeneity across the wealth distribution.39

The bottom panels of Figure 1.4 illustrate the asset portfolio of households as functions of total deposits-earnings \((a, \varepsilon)\) and total deposits-depreciation rate \((a, e)\) doubles. Portfolio is defined as \(\frac{b'}{b' + m'}\) at the individual level. The increasing, concave shape is in line with the empirical facts documented by the literature that the poor hold a larger fraction of their portfolio in non-interest bearing assets and the facts documented in this paper.40 The bottom-left panel shows that earnings-poor individuals hold a portfolio which is more biased towards bonds because of the increased PS motive of the poor. The bottom-right panel illustrates that the share of bonds shifts down when inflation is reduced. Yet, a lower portfolio share of bonds does not mean that the ‘absolute value’ of the bond position of a particular consumer is decreasing. Indeed, the bond position of an individual may rise if the wealth effect dominates the substitution effect. Since the PS motive will be strong for consumers who are affected by wealth effects the most, at the heart of the analysis is the relative dominance of these effects.

Table 1.5 presents the distributional implications of reduction in inflation un-

---

39Hence the absence of first order stochastic dominance between c.d.f.s of bonds across high and low inflation economies.
40See section 1.4.2 for the discussion of the mechanism that generates this phenomenon in the model.
der alternative fiscal arrangements. Rows 1-4 report the Gini coefficients of income, consumption, bonds and real balances and rows 5-8 show the top quintile-to-bottom quintile and mean-to-median ratios of income and consumption. These statistics establish that the distribution of income, real balances and consumption are almost intact when inflation is reduced irrespective of the particular fiscal arrangement. On the other hand, when inflation is reduced, Gini coefficient of bonds stays intact in Economy 1, increases by about 1.5% in Economy 2 and 2% in Economy 3.

The difference between alternative fiscal arrangements can be explained by variations in the equilibrium precautionary savings (PS) motive across Economies 1, 2 and 3. Rows 9 and 10 in Table 1.5 report the fraction of population that hit the debt limit, and portfolio share of the poorest total deposits percentile respectively. In Economy 1, there is an inverse relationship between comovements in the fraction of borrowing constrained and portfolio share of bonds for the poorest 1st percentile with inflation. In particular, disinflation causes substantial decline in the former (from 10.5% to 5.7%), suggesting a surge in the precautionary savings motive. For a given stochastic process of idiosyncratic earnings, a decrease in the measure of borrowing constrained represents an increase in the equilibrium PS motive in the economy, since it points out the desire to avoid hitting debt limits. On the other

\[ \sum_{a,\varepsilon} \Gamma(a,\varepsilon;e) b'(a,\varepsilon;e) \]
\[ \sum_{a,\varepsilon} \Gamma(a,\varepsilon;e) b'(a,\varepsilon;e) + \sum_{a,\varepsilon} \Gamma(a,\varepsilon;e) m'(a,\varepsilon;e) \]

where \( a : \Phi(a;e) = 0.01 \) with \( \Phi(.) \) being the c.d.f. of \( a \).

\[ \text{Observing the natural debt limit, } -\Psi = \left( \frac{\tau - \tau_{\min}}{R-1} \right), \] is useful here. Since uniform transfers decline with disinflation (see Table 1.4), natural debt limits become tighter.
### Table 1.5: Distributional Implications of Disinflation

<table>
<thead>
<tr>
<th>Aggregates</th>
<th>Uniform $\tau$</th>
<th>Endogenous $G$</th>
<th>Proportionate $\tau$</th>
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<tbody>
<tr>
<td></td>
<td>$e = 14.25%$</td>
<td>$e = 2.25%$</td>
<td>$e = 14.25%$</td>
</tr>
<tr>
<td>$Gini_y$</td>
<td>0.335</td>
<td>0.338</td>
<td>0.338</td>
</tr>
<tr>
<td>$Gini_b$</td>
<td>0.637</td>
<td>0.637</td>
<td>0.623</td>
</tr>
<tr>
<td>$Gini_m$</td>
<td>0.341</td>
<td>0.345</td>
<td>0.341</td>
</tr>
<tr>
<td>$Gini_c$</td>
<td>0.332</td>
<td>0.335</td>
<td>0.335</td>
</tr>
<tr>
<td>$(\text{Top20/Bottom20})_y$</td>
<td>5.941</td>
<td>6.088</td>
<td>6.093</td>
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<tr>
<td>$(\text{Median/Median})_y$</td>
<td>1.204</td>
<td>1.209</td>
<td>1.210</td>
</tr>
<tr>
<td>$(\text{Mean/Median})_c$</td>
<td>1.149</td>
<td>1.172</td>
<td>1.168</td>
</tr>
<tr>
<td>Frac. of Constrained Portf. of the 1st Percentile</td>
<td>0.105</td>
<td>0.057</td>
<td>0.084</td>
</tr>
<tr>
<td>Portf. of the 1st Percentile</td>
<td>0.471</td>
<td>0.527</td>
<td>0.581</td>
</tr>
</tbody>
</table>
hand, the portfolio share of bonds for the poor rises from 47% to 53%. This is in contrast with the portfolio effect of disinflation in the aggregate economy because of the dominance of wealth effects on the poor.

In Economy 2, disinflation causes less increase in PS incentive because transfers do not respond to inflation (8.4% of the population hits the debt limit when inflation is high). Lastly in Economy 3, since transfers are much smaller than Economies 1 and 2, natural debt limits are much tighter so that the measure of borrowing constraint is lower. Since wealth effects are partially neutralized in this economy, the poor buffer more assets to make up for the insurance not provided by transfers. Moreover, \( \frac{d}{de} \left( \frac{\tau(a,e)-m'(a,e)e}{1+e} \right) = 0 \), so that PS incentive is less sensitive to disinflation. In Economies 2 and 3, similar to the aggregate economy, the poor reduces the share of bonds in the portfolio when inflation is lower due to the dominance of substitution effects. Nevertheless, since transfers are still large in Economy 2, wealth effects in Economy 3 are substantially weaker than in Economy 2. Higher precautionary savings motive in Economy 3 also explains why aggregate asset position of this economy is much larger than those in Economies 1 and 2.

This closes the discussion of distributional implications of disinflation and I now proceed to the analysis of welfare consequences.

1.5.5 Welfare Implications of Disinflation

Welfare consequences of disinflation depend crucially on fiscal and monetary interactions as illustrated in section 1.4.3. Before analyzing models with uncertainty, \( \text{For the poor, } \frac{d}{de} \left( \frac{\tau(a,e)-m'(a,e)e}{1+e} \right) > 0 \) in Economy 1.
to warm up, first I develop a measure of aggregate welfare. Following Mendoza et al. (2007), welfare effects are computed as the proportional increase in consumption in the 14.25% inflation stationary equilibrium, $\eta$, that would make an individual consumer indifferent about remaining in that economy versus shifting to an economy with the inflation rate of 2.25%. Since the focus is on stationary equilibria analysis, I abstract from the effects of transitional dynamics on welfare. For each agent $i$ who is in state $(a, \varepsilon)$, $\eta(a, \varepsilon)$ solves

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}^{14.25\%}(1 + \eta(a, \varepsilon))) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}^{2.25\%})$$  \hspace{1cm} (1.19)$$

where $\{c_{i,t}^{14.25\%}\}_{t=0}^{\infty}$ is the infinite sequence of consumption of agent $i$ in state $(a, \varepsilon)$ in the high inflation economy and $\{c_{i,t}^{2.25\%}\}_{t=0}^{\infty}$ is the corresponding sequence of consumption in the low inflation economy.\(^{44}\)

Once I establish the consumption equivalent of welfare gains on the individual level, as a natural next step, I need to do an aggregation to achieve a normative assessment regarding the economy as a whole. The practice is to fix the wealth distribution of the high inflation economy, $\Gamma^{14.25\%}(a, \varepsilon)$ and use it to compute a weighted average of the welfare gains in terms of compensating consumption variation (CCV, hereafter). Hence, the consumption equivalent of the aggregate welfare gain from changing the inflation rate to 2.25% can be written as

$$\frac{1}{1 - \beta} \left[ (1 - \beta)(1 - \sigma) v^{14.25\%}(a, \varepsilon) + 1 \right] [1 + \eta(a, \varepsilon)]^{1-\sigma} = \left[ (1 - \beta)(1 - \sigma) v^{2.25\%}(a, \varepsilon) + 1 \right]$$  \hspace{1cm} (1.20)$$

where $v^{14.25\%}(a, \varepsilon)$ is the equilibrium value function in the high inflation economy and $v^{2.25\%}(a, \varepsilon)$ is the value function in the low inflation economy.

\(^{44}\)Given the particular functional form for the utility function and the notation so far, $\eta(a, \varepsilon)$ also solves

$$\Delta E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}^{14.25\%}(1 + \eta(a, \varepsilon))) = \Delta E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}^{2.25\%})$$  \hspace{1cm} (1.19)$$

where $\{c_{i,t}^{14.25\%}\}_{t=0}^{\infty}$ is the infinite sequence of consumption of agent $i$ in state $(a, \varepsilon)$ in the high inflation economy and $\{c_{i,t}^{2.25\%}\}_{t=0}^{\infty}$ is the corresponding sequence of consumption in the low inflation economy.\(^{44}\)
\[ W^{2.25\%} = \sum_{a, \varepsilon} \Gamma^{14.25\%}(a, \varepsilon) \eta(a, \varepsilon) \] (1.21)

Table 1.6 below presents the welfare implications of reducing inflation (from 14.25% to 2.25%) in model economies. The first row denotes the aggregate welfare gain (as defined above) of waking up in the low inflation stationary equilibrium. Rows 2, 3 and 4 include the disaggregation of this measure into the average gains of the poorest quintile, 50th percentile and the top percentile (ordered according to total deposit positions). Row 5 shows the welfare gains in the deterministic cases that I study in Section 1.4.

When transfers are uniform, (Economy 1), the rich benefit (welfare gain of 0.97% in terms of CCV from disinflation at the expense of the poor (welfare loss of 3.56%). This is because disinflation causes the poor to lose redistributive transfers that are mainly financed by inflation tax payments of the rich. Since welfare loss of the poor is disproportionately large, aggregate economy incurs a welfare loss of 1.25%.

If wasteful government spending responds to monetary policy, (Economy 2), welfare gains schedule observed in Economy 1 shifts up because transfers are not reduced following a contraction in the inflationary finance. Since the redistribution channel is absent in this economy, the poor do not incur a welfare loss anymore. Yet, their welfare gain (1.24%) is smaller than that of the rich (1.76%) because the reduction in inefficiencies caused by inflation taxation and transactions costs are mainly utilized by the latter, who consume and hold real balances more. Aggregate
Table 1.6: Welfare Consequences of Reducing Inflation from 14.25% to 2.25%

<table>
<thead>
<tr>
<th>Welfare Gains&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Uniform $\tau_1$</th>
<th>Endogenous $G$</th>
<th>Proportionate $\tau_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate</strong></td>
<td>-1.247</td>
<td>1.622</td>
<td>0.445</td>
</tr>
<tr>
<td><strong>Bottom 20%</strong></td>
<td>-3.564&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1.236</td>
<td>0.171</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>-1.226</td>
<td>1.655</td>
<td>0.471</td>
</tr>
<tr>
<td><strong>Top 1%</strong></td>
<td>0.967&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1.756</td>
<td>0.523</td>
</tr>
<tr>
<td><strong>Deterministic&lt;sup&gt;c&lt;/sup&gt;</strong></td>
<td>-0.126</td>
<td>1.816</td>
<td>0.625</td>
</tr>
</tbody>
</table>

<sup>a</sup>Welfare gains are computed as percentage change in terms of compensating consumption variation.

<sup>b</sup>Average welfare gains of percentiles ordered according to total deposits positions.

<sup>c</sup>Refers to the aggregate welfare effects in economies studied in section 1.4.3.
welfare increases by 1.62% when inflation is reduced to 2.25% in Economy 2.

When transfers are individual specific (i.e. $\tau(a, \varepsilon) = \frac{m'(a, \varepsilon)e}{1+\varepsilon} - G$), fiscal policy does not cause any redistribution among heterogeneous agents. As a result, reducing inflation in Economy 3 creates welfare gains mainly due to reduced transactions costs. Welfare gains are also much lower compared to Economy 2. This is mainly due to the absence of positive wealth affects created by a reduction in government spending in Economy 2. A second reason is that when wealth effects are not strong and real interest rate is constant, reduction in inflation causes consumers to earn unambiguously lower interest income (see the discussion in Section 1.4.3.).

Figure 1.5 brings yet another dimension on the pattern of disaggregated welfare gains from disinflation. The main message of the figure is that the afore-mentioned wealth effects led by fiscal arrangements have a substantial impact on the magnitudes of disaggregated welfare changes from disinflation. Additionally, consistent with Table 1.6, welfare gains increase as individuals get richer in terms of both earnings and total deposits. This finding is in contrast with the findings of Erosa and Ventura (2002) who find inflation as a regressive consumption tax. This is mainly because average transactions costs do not depend on consumption level in the current paper. Economies of scale assumption in their study ensures that distortions of inflation per consumption decreases as individuals get wealthier. In this paper, the ratio of distortions created by inflation has the same proportion to total consumption which causes the rich to incur substantive costs of inflation.

\footnote{In the Sensitivity Analysis section, I show that disinflation is welfare reducing when there are no wealth effects.}

\footnote{The ordering of welfare gains do not change if I rank people according to the summation of total deposits and earnings, $a + \varepsilon$.}
Figure 1.5: Disaggregated Welfare Gains from Disinflation
Coming to the role of uncertainty; I point out that in contrast with Economies 2 and 3, aggregate welfare loss (1.25%) is much larger in Economy 1 when there is idiosyncratic risk as opposed to the deterministic case (0.13%, see the last row of Table 1.6). This is because reduction in inflationary finance (and transfers) tightens natural debt limits substantially, which is only a feature of the stochastic economy. Economies 2 and 3 do not display this fundamental difference since transfers are fixed in the former and natural debt limits do not change with inflation at all in the latter.

In summary, I argue that welfare consequences of disinflation depend crucially on the particular fiscal arrangement. The main results are, (i) if the transfers system is redistributive, inflationary finance might be good for the poor at the expense of the rich. I also argue that this redistribution story might be a modest explanation to the chronic-inflation experience of some emerging economies. (ii) Inflationary finance causes substantial welfare losses if it is directed to government consumption and (iii) if agents’ financial wealth and their transfers income are positively related, then inflationary finance is again costly in terms of welfare.47

So far, the focus has been on comparing two inflation rates in the context of the empirically observed disinflation. But this experiment is silent on the distributional and welfare consequences of too-high or too-low inflation rates. In order to shed light on those questions, I analyze the steady state comovements of distributional and welfare measures with inflation in the next section.

47This is supported by the data for the Turkish economy as I discuss in Section 1.5.1.1.
1.5.6 Long-run Comovements

In this section, I analyze aggregate, distributional and welfare consequences of inflation in a broader context (i.e., one without limiting the focus on particular inflation rates). In particular, I lay out the stationary equilibrium implications of the model for a grid of inflation rates, \{-2\%, 0\%, 2.25\%, 5\%, 7\%, 14.25\%, 20\%, 30\%, 50\%\}, where these values are meant to capture some degree of variation in the opportunity cost of holding money, \(\frac{i}{1+i}\).

1.5.6.1 Aggregate Money Demand Curve

Arguably, the most important aggregate implication of the model is the predicted aggregate money demand curve in relation to the opportunity cost of holding money. Figure 1.6 plots model implied aggregate money per consumption (the dashed plot) as a function of the opportunity cost of holding money where the latter is computed by using Fisher equation, \((1 + i) = R(1 + e)\) under no aggregate uncertainty. \(R\) coincides with the calibrated value used in the model and inflation rates, \(e\), are the values used in this section. Circles (stars) represent the high (low) inflation period used in our aggregate money demand estimation in section 1.5.1.2.

As the figure lays out, the model performs well in capturing the convex, downward sloped pattern of the aggregate money demand curve.

1.5.6.2 Inflation and Inequality

The relationship between Gini coefficients of disposable income/assets and inflation (under alternative fiscal arrangements) is illustrated in Figure 1.7. Similar
Figure 1.6: Aggregate Money Demand Curve
Figure 1.7: Inflation and Inequality
to findings in section 1.5.4, inequality in income and real balances is not much responsive to inflation, whereas inequality in bonds is sensitive depending on the fiscal arrangement. In Economy 1, increase in inflation implies more inequality in distribution of bonds (when inflation is high enough). This finding mainly hinges on the feature of Economy 1 that endogenous transfers relax debt limits of individuals so that as inflation rises, more individuals start hitting debt limits. As expected, this feature is absent in Economies 2 and 3. In particular, inequality in bonds monotonically decreases with inflation in these economies.

To complete the argument, Figure 1.8 is included to sketch the relationship between fraction of borrowing constrained consumers and inflation. Now, when $e = 0\%$ is treated as a benchmark, the figure conveys three important pieces of information: First, wealth effects on the poor dominate substitution effects as inflation rises. The poor then demand less bonds and consequently, the measure of borrowing constrained increases. Second, this statistic is largest (smallest) in Economy 1 (3). This is because wealth effects are strongest in Economy 1 as manifested by aggressive increases in transfers, weaker in Economy 2 with constant transfers and weakest in Economy 3 with transfers that depend on individual money holdings. Third, for negative inflation rates, again more individuals start hitting debt limits. In addition, the ordering among three economies for this matter is now reversed. This is because the relaxation of debt limits are reversed when inflation tax paid by the poor is negative.
Figure 1.8: Inflation and Precautionary Savings
1.5.6.3 Inflation and Welfare

Finally, I analyze the long-run relationship between inflation and aggregate welfare as measured in section 1.5.5. Figure 1.9 strikingly suggests that welfare consequences of inflation crucially depend on fiscal arrangements. To be consistent with the analysis in section 1.5.5, changes in aggregate welfare in terms of ccv are plotted vis-à-vis the economy with $e = 14.25\%$. As per the analysis in section 1.4.3.1, aggregate welfare monotonically increases with inflation in Economy 1. This is because redistributive transfers are always increasing with inflation as highlighted by Lucas (2000) that seigniorage revenues are monotonically increasing with inflation with log-log money demand. On the other hand, in Economy 2, aggregate welfare is monotonically decreasing with inflation since changes in seigniorage revenues are directed to government spending. Finally, Economy 3 interestingly implies that the optimal inflation rate must be zero as opposed to the Friedman rule suggested by the deterministic version of the same economy. This finding can be explained again by referring to Figure 1.8: Despite the decrease in distortions created by inflation, negative inflation rates cause more of the poor to hit debt limits. Since real interest rate is fixed, this unambiguously causes them to earn less interest income, limiting their consumption opportunities.

This closes the analysis of long-run implications of inflation in general and I now proceed to the next section in which I explore the sensitivity of findings to the parameter values and fiscal arrangements.
Figure 1.9: Inflation and Welfare (Relative to $e = 14.25\%$)
1.6 Sensitivity Analysis

In this section I perform sensitivity analysis on two dimensions. First, I explore the role of changing calibrated parameters one at a time on main findings (rows 2-17 of Table 1.7) related to distributional and welfare consequences of reducing inflation from 14.25% to 2.25%. Second, I tweak the transfers policy of the fiscal authority by considering variations that might induce qualitatively and quantitatively different wealth effects (rows 18-21). I report equilibrium lump-sum transfers-to-GDP ratios in columns 1-2, Gini coefficients of asset holdings and fraction of borrowing constrained in columns 3-8 and disaggregated/aggregate welfare gains in columns 9-11.

*Discount Factor and Real Interest Rate ($\beta, R$):* In Bewley-style economies, a higher $\beta R$ implies stronger PS incentive of households. Second row of Table 1.7 shows that a higher (lower) $\beta$ reduces (increases) the measure of borrowing constrained individuals. As a result, a lower Gini coefficient for bonds is obtained when $\beta = 0.94$ and the distribution of real balances is more decoupled from that of consumption when $\beta = 0.90$. Increasing (decreasing) real return of bonds, $R$ has similar implications to that of $\beta$ because the higher $R$, the stronger the asset buffering motive of individuals is. Equilibrium transfers-to-GDP ratios and welfare implications are quite insensitive to these parameters.

*Risk Aversion ($\sigma$):* A higher $\sigma$ of 3 makes consumers more risk averse. Therefore, regardless of the inflation rate, almost no one hits the debt limit. In this case, the distribution of bonds become significantly more equitable and welfare implica-
Table 1.7: Sensitivity Analysis

<table>
<thead>
<tr>
<th></th>
<th>$\tau_1/Y$</th>
<th>Gini$_b$</th>
<th>Gini$_m$</th>
<th>Fraction of</th>
<th>Aggregate$^a$</th>
<th>Gain of</th>
<th>Gain of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Constrained</td>
<td>Welfare Gain</td>
<td>Bottom 20%</td>
<td>Top 1%</td>
</tr>
<tr>
<td>Benchmark$^b$</td>
<td>0.050</td>
<td>0.040</td>
<td>0.637</td>
<td>0.637</td>
<td>0.105</td>
<td>0.057</td>
<td>-1.247</td>
</tr>
<tr>
<td>$\beta = 0.94$</td>
<td>0.052</td>
<td>0.040</td>
<td>0.579</td>
<td>0.567</td>
<td>0.041</td>
<td>0.019</td>
<td>-1.008</td>
</tr>
<tr>
<td>$\beta = 0.90$</td>
<td>0.050</td>
<td>0.040</td>
<td>0.617</td>
<td>0.632</td>
<td>0.374</td>
<td>0.201</td>
<td>-1.290</td>
</tr>
<tr>
<td>$R = 1.040$</td>
<td>0.051</td>
<td>0.040</td>
<td>0.602</td>
<td>0.596</td>
<td>0.057</td>
<td>0.024</td>
<td>-1.118</td>
</tr>
<tr>
<td>$R = 1.010$</td>
<td>0.050</td>
<td>0.040</td>
<td>0.646</td>
<td>0.686</td>
<td>0.214</td>
<td>0.123</td>
<td>-1.215</td>
</tr>
<tr>
<td>$\sigma = 3$</td>
<td>0.052</td>
<td>0.040</td>
<td>0.468</td>
<td>0.452</td>
<td>0.004</td>
<td>0.001</td>
<td>-1.398</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>0.049</td>
<td>0.039</td>
<td>0.558</td>
<td>0.556</td>
<td>0.486</td>
<td>0.406</td>
<td>-0.509</td>
</tr>
<tr>
<td>$\gamma = 1.95$</td>
<td>0.065</td>
<td>0.043</td>
<td>0.654</td>
<td>0.670</td>
<td>0.051</td>
<td>0.014</td>
<td>-2.384</td>
</tr>
<tr>
<td>$\gamma = 1.05$</td>
<td>0.048</td>
<td>0.039</td>
<td>0.652</td>
<td>0.633</td>
<td>0.155</td>
<td>0.074</td>
<td>-1.035</td>
</tr>
<tr>
<td>$\phi = 0.003$</td>
<td>0.055</td>
<td>0.041</td>
<td>0.644</td>
<td>0.647</td>
<td>0.097</td>
<td>0.035</td>
<td>-1.437</td>
</tr>
<tr>
<td>$\phi = 0.0005$</td>
<td>0.044</td>
<td>0.038</td>
<td>0.652</td>
<td>0.635</td>
<td>0.176</td>
<td>0.108</td>
<td>-0.862</td>
</tr>
<tr>
<td>$\Omega = -0.01$</td>
<td>0.051</td>
<td>0.040</td>
<td>0.638</td>
<td>0.639</td>
<td>0.107</td>
<td>0.057</td>
<td>-1.237</td>
</tr>
<tr>
<td>$\Omega = -0.05$</td>
<td>0.050</td>
<td>0.040</td>
<td>0.637</td>
<td>0.634</td>
<td>0.105</td>
<td>0.054</td>
<td>-1.254</td>
</tr>
<tr>
<td>$\sigma_u = 0.16$</td>
<td>0.051</td>
<td>0.040</td>
<td>0.483</td>
<td>0.409</td>
<td>0.000</td>
<td>0.000</td>
<td>-8.584</td>
</tr>
<tr>
<td>$\sigma_u = 0.12$</td>
<td>0.050</td>
<td>0.040</td>
<td>0.640</td>
<td>0.655</td>
<td>0.200</td>
<td>0.120</td>
<td>-2.0442</td>
</tr>
<tr>
<td>$\rho = 0.97$</td>
<td>0.051</td>
<td>0.040</td>
<td>0.557</td>
<td>0.416</td>
<td>0.000</td>
<td>0.000</td>
<td>-14.063</td>
</tr>
<tr>
<td>$\rho = 0.95$</td>
<td>0.050</td>
<td>0.040</td>
<td>0.646</td>
<td>0.652</td>
<td>0.164</td>
<td>0.110</td>
<td>-0.590</td>
</tr>
</tbody>
</table>

$\tau^* = \frac{Me}{1+e} + Tr - G^c$

$\tau^* = \frac{Me}{1+e}$

$\tau^*(a, \epsilon) = \frac{w'(a, \epsilon)\epsilon}{1+e}$

$\tau^*(a, \epsilon) = \frac{w'(a, \epsilon)\epsilon}{1+e} + c(a, \epsilon)\phi \left( \frac{c(a, \epsilon)}{m'(a, \epsilon)} \right)^\gamma$

$^a$ Welfare gains of reducing inflation rate from 14.25% to 2.25% in terms of compensating consumption variation.

$^b$ Implications of disinflation for the benchmark parameterization of the model. See Table 2 for parameter values.

$^c$ This row and the following three rows consider alternative fiscal arrangements that are different than the ones considered in Economies 1,2 and 3.
tions are very similar to those in the benchmark economy. In the case of log utility, $\sigma = 1$, welfare losses of the poor reduce substantially because consumers continue to borrow (fraction of constrained is between 49% and 41% when inflation changes) since their risk aversion is lower compared to the benchmark case.

*Parameters of the transactions costs function ($\gamma, \phi$):* When the transactions function has more curvature, i.e., $\gamma = 1.95$, elasticity of money demand with respect to the opportunity cost of money becomes lower. This causes both transactions costs to be more sensitive to changes in consumption velocity and seigniorage revenues to change more in the direction of inflation. This increases welfare changes in absolute value. On the other hand when curvature is less, i.e., $\gamma = 1.05$, welfare changes are diminished in absolute value as expected. Similar to $\gamma$, when $\phi$ is increased (decreased), transactions costs become more (less) sensitive to changes in consumption velocity. Consequently welfare changes become more (less) emphasized. Distributional implications do not seem to be affected substantially from these parameters as shown by the implied Gini coefficients.

*Lower bound of bonds ($\Omega$):* The distributional and welfare results are almost the same as in the benchmark parameterization when I increase or decrease the lower bound for bonds.

*Parameters of the earnings process ($\rho, \sigma_u$):* The PS motive is lower and consumers are willing to borrow more when shocks are less volatile and persistent ($\rho, \sigma_u$, lower). Therefore, welfare changes are less emphasized when earnings shocks are less severe. As expected, when shocks are more volatile and persistent, nobody hits the debt limit, the distribution of bonds become substantially more equitable and wel-
fare changes are asymmetrically emphasized.

*Alternative fiscal arrangements:* Rows 18 and 19 keep the baseline calibration but additionally rebate transactions costs and set $G = 0$ respectively. Welfare results show that adding transactions costs amplifies redistributive effects (increasing (decreasing) welfare loss (gain) of the poor (rich) from disinflation). Setting $G = 0$ on the other hand (which is the case for rows 19-21) reduces welfare changes substantially. The reason is that when $G = 0$, transfers are already high. Therefore the effect of redistribution is lower when transfers increase with inflation. Row 20 shows the proportionate transfers case with $G = 0$. In comparison to Economy 3, changes in welfare are now lower due to the absence of wealth effects caused by $G > 0$. The last row of Table 1.7 is especially important because it corresponds to the case with no wealth effects from reducing inflation. In particular, all costs of inflation are rebated in an individual specific way and $G = 0$. Although welfare impacts are lower than those in Economy 1, disinflation is again welfare reducing. This is due to the presence of substitution effects that lead consumers to decrease their bond demand when inflation is lower. This unambiguously reduces their interest income, implying a reduction in their welfare. In the deterministic case with no wealth effects, inflation is welfare neutral because interest income in the deterministic model is exogenous and fixed. Consequently, the last row of Table 1.7 shows that introducing idiosyncratic uncertainty with incomplete markets is enough to find that disinflation to be welfare reducing in this framework. Finally, notice also that debt limits become substantially relaxed in these economies with reduced (and even absent) wealth effects. This causes both the measure of borrowing constrained and
inequality in the distribution of bonds to increase at the inflation rates analyzed in the paper.

1.7 Conclusion

This paper investigates the distributional and welfare implications of disinflation in a small open economy. The motivation of the study derives from the recently observed structural disinflation in emerging economies and financial system characteristics of these countries that highlight heterogeneity. The inflationary past of these countries caused their financial system to evolve in a particular way, which is reflected in the analytical framework adopted in this paper.

The analysis in this paper shows that apart from the classical adverse effects of inflation, the way that fiscal authority responds to monetary policy might create various wealth effects. The main policy conclusion is that unless the transfers system is of redistributive nature, inflationary finance reduces welfare. Another interpretation of this result is that if emerging economies are to experience inflationary episodes in the future, they are better direct the inflationary finance to social transfers of redistributive nature. The redistribution story may also be considered as a modest explanation as to why emerging economies endured chronic inflation experience for long periods.

For further research, there are several avenues to follow: Empirical literature has shown that fiscal deficits and inflation are positively related in (high inflation) developing economies. Normative findings of this paper suggest that it is important to identify whether the comovement between the two are driven by wasteful govern-
ment spending or transfers of uniform nature. Another important extension is to analyze the transitional dynamics implications of disinflation in a calibrated economy. Gradual adjustment in the aggregate money supply along the transition might cause volatility in transfers which would definitely affect welfare results. Lastly, it would be an important robustness check to incorporate additional costs of inflation into a production economy and explore whether the redistribution effect is still predominant or not.
Chapter 2

Transitional Dynamics of Disinflation in a Small Open Economy with Heterogeneous Agents

2.1 Introduction

The goal of this paper is to quantitatively investigate macroeconomic, distributional and welfare consequences of transitional dynamics produced by recent disinflation in the Turkish economy. The analysis derives its motivation from stylized macro implications caused by gradual decline in inflation observed in Turkey in the last decade.

The bottom-right panel of Figure 2.1 shows the quarterly change in the GDP deflator (straight plot) in the period 1987:2-2010:3. This plot is consistent with the disinflation profile of emerging economies that generally starts around the mid-1990s. On the other hand, it shows that disinflation does not take place overnight. In particular, it takes about 6 years for the inflation rate to decline from 15.46\% to 1.86\% (the plot with asterisks) where the numbers are the time series average of inflation rates in the periods 1987:2-1999:2 and 2004:4-2010:3 respectively.

Panels 1-5 of Figure 2.1 plot the dynamics of main macroeconomic variables with a particular focus on the period 1999:2-2010:3, that encompasses the gradual decline in inflation (in the first 6 years). The straight plots represent actual data for aggregate consumption, M2Y-M1, M1 and trade balance-to-GDP ratios and aggre-
Figure 2.1: Macroeconomic Dynamics of Disinflation in Turkey
gate consumption velocity along disinflation.\textsuperscript{1} In all panels, the plots with asterisks denote the linear HP trend computed at quarterly frequency (for the bottom-right panel, HP trend is computed only for the period 1999:2-2004:4 that represent the gradual decline in inflation). First and foremost, the impact of the severe 2001 banking crisis is evident in the dynamics of consumption, trade balance and M2Y-M1-to-GDP ratios. Keeping this observation in mind, the most highlighted stylized facts that are present in Figure 2.1 can be listed as:

1. There is a secular rise in the aggregate money demand and a secular decline in consumption velocity along disinflation.

2. There is a consumption boom coupled with an increase in trade deficit.

3. Interest-bearing and dollarized deposits decline for two years right after the 2001 crisis and maintain a positive trend along disinflation.

Welfare dimension of the study finds its roots at the idea that disinflation of magnitudes observed in the Turkish economy might derive non-trivial wealth effects in an emerging economy. This is because, inflation \textit{(i)} reduces the purchasing power of individuals; \textit{(ii)} distorts consumption; and \textit{(iii)} government budget’s (dynamic) response to reduction in inflation tax revenues might matter from a redistributive aspect. Distributional dimension of the framework, on other hand, is thought to be important because the distribution of monetary assets in emerging economies displays substantial inequality. This suggests asymmetric vulnerability of individual

\textsuperscript{1}M2Y is the monetary aggregate that includes currency in circulation, checkable deposits, term deposits and foreign currency denominated deposits. In the plot, M2Y-M1 is meant to capture the interest-bearing segment of the deposits system that is less vulnerable to inflation.
portfolios to inflation, which creates heterogeneity in the afore-mentioned wealth effects.

This paper develops a monetary model of a small open economy with uninsured idiosyncratic risk and incomplete markets. The model economy is populated by a continuum of consumers and a government. Infinitely-lived consumers face idiosyncratic earnings shocks and consume a tradable consumption good. They hold (i) non-interest bearing real balances that economize transactions costs of consumption and (ii) internationally-traded risk-free bonds that are useful for consumption smoothing under the presence of idiosyncratic earnings shocks. Furthermore, financial system of this model economy is underdeveloped so that consumers face ad-hoc borrowing constraints.

I assume that there is perfect mobility in capital and goods markets so that domestic nominal interest rate is determined by a parity condition and domestic price level is determined by the law of one price. Because of the latter, domestic inflation rate is equal to the depreciation rate of the currency. These assumptions cause bonds to be fully dollarized (inflation-indexed) so that their real return does not depend on domestic inflation.

Turkish monetary authority in reality achieved disinflation by adopting a floating exchange rate regime coupled with inflation targeting after the 2001 crisis. However, since then, the Central Bank of Republic of Turkey has intervened the foreign exchange market many times with the discourse of “preventing excess volatility in the nominal exchange rate” and accordingly, accumulated substantial amount of international reserves. Consequently, I assume that the de facto exchange rate regime
is practically a managed float so that monetary authority is able to manipulate the level of the depreciation rate exogenously. Moreover, to focus on disinflation, I abstract from any kind of aggregate uncertainty, except for a one-time, unanticipated announcement of a disinflationary path, made by the government.

The assumption of using inflation-indexed bonds is motivated by the idea that high inflation economies have developed particular methods (such as financial dollarization) to cope with this phenomenon (see Section 1.2 in Chapter 1). Therefore, I abstract from nominal valuation effects created by surprise inflation that drive redistribution of wealth from creditors to debtors. Indeed, Iacoviello (2005) argues that debt-deflation effects are more important in low inflation (developed) economies. To explore this, Doepke and Schneider (2006) and Meh et al (2008) study welfare effects of an inflation shock that is modeled as a zero sum redistribution of real wealth in the context of industrialized economies. Berriel (2011), on the other hand, presents a provocative finding that endogenous portfolio decision (which is absent in the framework of Doepke and Schneider (2006)) offsets portfolio valuation effects in a general equilibrium setting even for the case of the U.S. economy. The current paper is immune to Lucas’ critique in this sense that portfolio decisions in this model endogenously change with inflation.

In order to focus on the mediating role of fiscal policy on the consequences of disinflation, I study alternative fiscal arrangements with (i) endogenous uniform transfers; and (ii) endogenous government spending. Adjustments in these fiscal

\footnote{Berument and Guner (1997) and Berument and Gunay (2003) find that nominal deposit and treasury auction rates have provided a good hedge against inflation and currency depreciation during the high inflation period of the Turkish economy.}
variables will naturally depend on the gradual decline in inflationary finance along the transition.

I calibrate the model to the low inflation period of the Turkish economy in the last decade. The main quantitative exercise is to feed the calibrated declining path for inflation rates (illustrated in Figure 2.1) into the model and to explore the macroeconomic and distributional dynamics and welfare consequences of disinflation under alternative fiscal arrangements. It is obvious that studying gradual disinflation rules out steady state comparisons. Therefore, I assume that the economy is initially at the 14.25% inflation equilibrium and the government makes an unanticipated, time-consistent and credible announcement at date 0 that inflation will follow a “declining path” for the next 6 years and will stay at 2.25% forever. These numbers are average inflation rates in the periods 1987:2-2002:4 and 2003:1-2010:3 that are separated by a structural change in inflation.

I find that accounting for the gradual disinflation is crucial for generating the secular downward pattern in consumption velocity and upward pattern in aggregate money demand. Strikingly, irrespective of the fiscal arrangement, model generated time profile of these macro variables are almost identical with their trends presented in Figure 2.1. The model is also qualitatively consistent with the dynamics of consumption and trade balance-to-GDP ratios.

Second, the evolution of Gini coefficients of bonds display non-trivial dynamics which are impossible to capture within a steady state comparison framework.

Third, welfare changes from disinflation are affected substantially from transitional dynamics. In particular, when transfers are endogenous, aggregate welfare
loss of 1.25% (with no transition) in terms of compensating consumption variation is transformed into a gain of 0.28% (with transitional dynamics). This is because the reduction in the costs of inflation to the poor is smaller than the reduction in their transfers income implied by disinflation. While transfers plummet immediately in the stationary world, in the transitional dynamics equilibrium, they follow a gradual path mainly dictated by the announced path of inflation. As a result, the reduction in the magnitude of redistribution from the rich to the poor is limited in the transitional dynamics economy. Due to a similar reason, when government spending is endogenous, aggregate welfare gain of 1.62% reduces to 0.53% with transitional dynamics.

Sensitivity analysis of this paper establishes that the secular pattern of aforementioned macro variables cannot be captured when disinflation is modeled as a one-time, unanticipated announcement of a sharp decline in inflation. In this case, welfare consequences are qualitatively similar to those implied by the steady state analysis when transfers are endogenous; and are exactly identical to those implied by the steady state analysis when government spending is endogenous. The reason for the discrepancy is that in the latter economy, sharp reversals in spending does not feed back to the utility maximization problem of consumers, whereas in the former economy, sharp reversals in transfers create excess consumption volatility.

This paper contributes to the monetary economics literature that incorporates imperfectly insured idiosyncratic risk framework. Among this vast literature, the work of Algan and Ragot (2010), Berriel and Zilberman (2011), Erosa and Ventura (2002) and Albanesi (2007) should be highlighted as Bewley-Huggett-Aiyagari type
stationary environments within monetary framework. However, none of these studies incorporate the transitional dynamics of disinflation by using a calibrated model of a small open economy.

The rest of the paper is organized as follows. Section 2.2 describes the theoretical model. Section 2.3 shows the workings of the model and defines the stationary (pre-disinflation) equilibrium and the transitional dynamics (disinflation) equilibrium. Section 2.4 describes the parameterization of the model and reports findings. Sensitivity analysis is performed in Section 2.5 and finally, Section 2.6 concludes the paper.

2.2 The Model Economy

The model in this chapter will essentially follow the one described in Chapter 1 with the only difference that the path of inflation is not static anymore. In particular, I study a monetary model of a small open economy with uninsured idiosyncratic earnings risk. There is no production. The economy is inhabited by two agents: a continuum of infinitely lived households of total mass 1 and a government. To highlight the unexpected decline in inflation at date 0, I abstract from any other type of aggregate uncertainty. Time is discrete. The consolidated government determines fiscal and monetary policy.

2.2.1 Households

The stochastic process of earnings is independently and identically distributed across consumers and follows a finite state Markov chain with conditional probabil-
ities \( p_{\varepsilon'|\varepsilon} = Pr(\varepsilon_{t+1} = \varepsilon'|\varepsilon_t = \varepsilon) \) for \( \varepsilon' \) and \( \varepsilon \in E \) where \( E \) is a finite dimensional vector. The invariant distribution of this Markov process (which does not depend on inflation) is denoted by \( P \).

Households derive utility from consumption. Preferences over flows of a single, tradable consumption good are given by

\[
E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]
\]  

(2.1)

where \( 0 < \beta < 1 \) is the subjective discount factor (which is the same across individuals) and \( u(c) \) is a continuous and strictly concave function defined over the flow of consumption. Utility function satisfies the Inada condition, \( \lim_{c \to 0^+} u'(c) = \infty \). \( E \) is the mathematical expectation operator.

Households have access to two financial assets: Real balances (demand deposits), \( m \), issued by monetary authority, and one-period, risk free bonds (term deposits), \( b \), that are internationally traded. If inflation from date \( t - 1 \) to date \( t \) is \( \pi_t \), then real deposits, \( a \), at time \( t \) are defined as \( a_t = Rb_t + \frac{m_t}{1+\pi_t} \) where \( R \) is the gross real interest rate and \( b_t, m_t \) are the beginning of period \( t \) positions in bonds and real balances respectively.

Consumers face the budget constraint,

\[
c_t \left[ 1 + S \left( \frac{c_t}{m_{t+1}} \right) \right] + b_{t+1} + m_{t+1} = \varepsilon_t + a_t + \tau_t
\]

(2.2)

The left-hand-side of (2.2) represents total consumption expenditures and asset demands. Transactions costs are assumed to be an increasing function \( S \) of
consumption velocity of money, \( \kappa_t = \frac{c_t}{m_{t+1}} \). The unit transactions costs function is assumed to take the form \( S = \phi \kappa^\gamma \), where \( \phi > 0 \) and \( \gamma > 1 \). \( \tau_t \) is a lump-sum transfer made by the government. I assume that financial markets are underdeveloped, therefore consumers face a borrowing constraint so that \( b_{t+1} \geq \Omega \) with \( \Omega \leq 0 \).

There is perfect mobility in capital and goods markets. Therefore, small open economy assumption ensures that \( R \) is taken as given from the international capital markets. Under the law of one price and the assumption of zero foreign inflation rate, domestic inflation rate, \( \pi_t \), becomes identical to the depreciation rate of currency, \( e_t \). Motivated by financial dollarization in emerging economies, I assume that the real interest rate earned on bonds stays constant even if there is a surprise change in inflation, i.e. nominal interest reflects the change in the depreciation rate of currency by the interest parity condition. Therefore, nominal portfolio valuation effects from unanticipated changes in inflation will be omitted in this framework.

At any period \( t \), a household is characterized by a double \( (a_t, \varepsilon_t) \in A \times E \), where the terms in parentheses denote real deposits position and earnings level of an individual. Let \( \Gamma_t(a_t, \varepsilon_t) \) be the measure of agents who are in the idiosyncratic state \( (a_t, \varepsilon_t) \) at date \( t \). I discretize the state and policy spaces. This omits one state variable and eases computation. However, portfolio choice between real balances and bonds is still explicit in the model, as I describe below.

2.2.2 Government and Alternative Fiscal Arrangements

Equation (2.3) describes the budget constraint of the government. As part of monetary policy, the government issues currency and announces the depreciation
rate of nominal exchange rate, \( \{\epsilon_t\}_{t=0}^{\infty} \). I do not bring any foundation to the disinflation phenomenon and model it as an unanticipated and credible policy announcement made by the monetary authority. Aggregate real seigniorage revenues are denoted by
\[
M_{s,t+1} - \frac{M_s^t}{1+\epsilon_t},
\]
where \( M_s^t \) is aggregate real money supply at the beginning of period \( t \). Money is demand determined, i.e., for a pre-determined depreciation rate, the central bank prints as much money as the economy demands on aggregate.

I abstract from international reserves for simplicity.

\[
G_t + \tau_t = M_{s,t+1}^t - \frac{M_s^t}{1+\epsilon_t} \quad (2.3)
\]

Fiscal policy is conducted by making unproductive expenditures, \( G_t \), and remitting transfers, \( \tau_t \) to households. To explore the distributional role of disinflation, I study alternative fiscal arrangements in response to monetary policy described above. In Economy 1, I assume that government spending is constant, \( G_t = G \forall t \), which leaves uniform transfers as responsive to changes in seigniorage revenues. In Economy 2, I assume that uniform transfers are constant, \( \tau_t = \tau \forall t \), so that spending responds to changes in seigniorage revenues. These two arrangements are meant to capture the redistributive role of uniform transfers. I consider Economy 1 as the benchmark case, since a well-known practice in the literature is to couple monetary creation by lump-sum transfers.

From a policy-making perspective, the idea is to study a credible stabilization plan which is in practice achieved by controlling the depreciation rate of currency. Indeed, Central Bank of the Republic of Turkey (CBRT) was able to reduce inflation
gradually by employing inflation targeting as the main policy rule since the 2001 crisis. Yet, during this period, Turkish residents have witnessed many occasions in which CBRT has intervened the foreign exchange market with the discourse of "preventing excess volatility in the exchange rate". This supports the perspective of modeling the exchange rate regime as a de facto managed float in this paper. Consequently, I assume that at $t = 0$, the government announces a declining time profile for the future sequence of inflation rates in an unanticipated way. That is

$$e_t = e^0 \text{ for } t = 0$$

$${e_t}^\infty_{t=1} = {e^1_t}^\infty_{t=1} \text{ for } t > 0$$

where $e^0$ is the pre-disinflation level of the depreciation rate of the currency and $\{e^1_t\}^\infty_{t=1}$ is the infinite sequence of future depreciation rates that satisfies, $e^0 > e_1^1 > e_2^1 ... > e_T^1$ and $\{e^1_t\}^\infty_{t=T+1} = e^1$ with $e^0 > e^1$ for a finite $T$.

2.3 Analytical Framework

In this section, I formulate the optimization problem solved by the consumer in the benchmark economy, and define the stationary and transitional dynamics recursive equilibria.

2.3.1 The Household’s Decision Problem

Dynamic programming problem solved by a household who is in state $(a_t, \varepsilon_t)$ at date $t$ is:
\[ u_t(a_t, \varepsilon_t; e_t) = \max_{c_t, m_{t+1}, b_{t+1}} \left[ u(c_t) + \beta E_t \{ v_{t+1} (a_{t+1}, \varepsilon_{t+1}; e_{t+1}) | \varepsilon_{t+1}, e_{t+1} \} \right] \]  

subject to

\[ c_t \left[ 1 + S \left( \frac{c_t}{m_{t+1}} \right) \right] + b_{t+1} + m_{t+1} = \varepsilon_t + a_t + \tau_t \]  

\[ c_t, m_{t+1} \geq 0 \text{ and } b_{t+1} \geq \Omega \]

where \( a_t = Rb_t + \frac{m_t}{1+\varepsilon_t} \) \( \forall \ t \) and \( -\Omega \) is an ad-hoc debt limit.

Decision rules of an individual that govern the demand for real money balances, bonds and consumption are functions \( m_{t+1} = m_{t+1}(a_t, \varepsilon_t), \ b_{t+1} = b_{t+1}(a_t, \varepsilon_t) \) and \( c_t = c_t(a_t, \varepsilon_t) \). Notice that the recursive problem of the household incorporates variations in inflation and transfers. Therefore, I use time subscripts for the value function and policy rules.

2.3.2 Equilibrium

I assume that conditions that guarantee the existence of unique invariant measure \( \Gamma^0 \) for the initial inflation rate and transfers are satisfied (see Huggett (1993)). Definitions 2.1 and 2.2 below describe the stationary recursive equilibrium (that represents pre-disinflation) and recursive transitional dynamics equilibrium (that represents disinflation) respectively:

**Definition 2.1 (Pre-disinflation)** Given a constant level of government expenditures \( G \), the international gross real interest rate \( R \) and a constant depreciation
rate \( e^0 \), a stationary recursive equilibrium is a time invariant value function \( v^0 \), time invariant policy functions \( m^{0} = m^{0}(a, \varepsilon; e) \), \( b^{0} = b^{0}(a, \varepsilon; e^0) \), \( c^0 = c^0(a, \varepsilon; e^0) \), constant lump-sum transfers \( \tau^{*0} \) and a stationary distribution \( \Gamma^{*0} \), such that: (i) Given \( \tau^{*0} \), \( R \), and \( e^0 \); \( m^{0} = m^{0}(a, \varepsilon; e^0) \), \( b^{0} = b^{0}(a, \varepsilon; e^0) \) and \( c^0 = c^0(a, \varepsilon; e^0) \) solve the household’s problem (2.3.1); (ii) Given \( G \), \( \Gamma^{*0} \), \( e^0 \) and the policy functions of households; \( \tau^{*0} \) is consistent with the balanced budget of the government; \( G + \tau^{*0} = \left( \frac{e^0}{1+e^0} \right) M^* \); (iii) Given \( \Gamma^{*0} \) and the policy functions of households, aggregate goods market clears (i.e. the national income identity holds), \( C + G + (1-R)B + Tr = Y \) with \( C = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)c^0 \), \( B = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)b^0 \), \( Y = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)\varepsilon \), and \( Tr = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)c^0 S \left( \frac{c^0}{m^0} \right) \). Money market equilibrium, \( M^* = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)m^{*0} \) follows from the de facto exchange rate regime; (iv) Given the policy rules for assets and the Markov transition of earnings, \( \{b^{*0}(a, \varepsilon), m^{*0}(a, \varepsilon), p_{\varepsilon'|\varepsilon}\} \), the distribution of total deposits and earnings satisfies the following fixed point equation:

\[
\Gamma^{*0}(a', \varepsilon') = \sum_{\varepsilon} \sum_{\{a, a^0 = Rb^0(a, \varepsilon) + m^0(a, \varepsilon)\}} \Gamma^{*0}(a, \varepsilon)p_{\varepsilon'|\varepsilon}.
\]

**Definition 2.2 (Disinflation)** Given a constant level of government expenditures \( G \), the international gross real interest rate \( R \) and the sequence of depreciation rates \( \{e_t\}_{t=0}^{\infty} \) which satisfies the disinflation profile described in (2.2.2), a recursive transitional dynamics equilibrium is a sequence of functions \( \{v_t, m_{t+1}, b_{t+1}, c_t\}_{t=0}^{\infty} \), lump-sum transfers \( \{\tau_t\}_{t=0}^{\infty} \) and distributions \( \{\Gamma_t^1\}_{t=0}^{\infty} \), such that: (i) Given \( \{\tau_t^1\}_{t=0}^{\infty} \), \( R \), and \( \{e_t\}_{t=0}^{\infty} \); \( \{v_t, m_{t+1}, b_{t+1}, c_t\}_{t=0}^{\infty} \) solve the household’s problem (2.3.1); (ii) Given \( G \), \( \Gamma_t^1(a, \varepsilon) \), \( \{e_t\}_{t=0}^{\infty} \) and the policy functions of households, \( \{\tau_t^1\}_{t=0}^{\infty} \) is consistent with the balanced budget of the government; \( G + \tau^{*0} = \left( \frac{e^0}{1+e^0} \right) M^* \); (iii) Given \( \Gamma^{*0} \) and the policy functions of households, aggregate goods market clears (i.e. the national income identity holds), \( C = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)c^0 \), \( B = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)b^0 \), \( Y = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)\varepsilon \), and \( Tr = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)c^0 S \left( \frac{c^0}{m^0} \right) \). Money market equilibrium, \( M^* = \sum_{a, \varepsilon} \Gamma^{*0}(a, \varepsilon)m^{*0} \) follows from the de facto exchange rate regime; (iv) Given the policy rules for assets and the Markov transition of earnings, \( \{b^{*0}(a, \varepsilon), m^{*0}(a, \varepsilon), p_{\varepsilon'|\varepsilon}\} \), the distribution of total deposits and earnings satisfies the following fixed point equation:

\[
\Gamma^{*0}(a', \varepsilon') = \sum_{\varepsilon} \sum_{\{a, a^0 = Rb^0(a, \varepsilon) + m^0(a, \varepsilon)\}} \Gamma^{*0}(a, \varepsilon)p_{\varepsilon'|\varepsilon}.
\]

\(^3\text{In Economy 2, } G^{*0} \text{ closes the equilibrium for a fixed } \tau.\)
sistent with the balanced budget of the government; $G + \tau_t^1 = M_{t+1}^s - \frac{M_t^s}{1+\varepsilon_t}$;  

(iii) Given $\{\Gamma_1^t\}_{t=0}^\infty$ and the policy functions of households, aggregate goods market clears (i.e. the national income identity holds), $C_t + G + B_{t+1} - RB_t + Tr_t = Y$ with $C_t = \sum_{a,\varepsilon} \Gamma_1^t(a_t, \varepsilon_t)c_t$, $B_{t+1} = \sum_{a,\varepsilon} \Gamma_1^t(a_t, \varepsilon_t)b_{t+1}$, $Y = \sum_{a,\varepsilon} \Gamma_1^t(a_t, \varepsilon_t)\varepsilon$, and $Tr_t = \sum_{a,\varepsilon} \Gamma_1^t(a_t, \varepsilon_t)c_t S(\frac{\alpha}{m_{t+1}}) \forall t$. Money market equilibrium, $M_{t+1}^s = \sum_{a,\varepsilon} \Gamma_1^t(a_t, \varepsilon_t)m_{t+1}$ $\forall t$, follows from the de facto exchange rate regime;  

(iv) Given the policy rules for assets and the Markov transition of earnings, $[b_{t+1}(a, \varepsilon), m_{t+1}(a, \varepsilon), p_{\varepsilon t+1|\varepsilon_t}]$; the distribution of total deposits and earnings follows the law of motion: $\Gamma_1^t(a_{t+1}, \varepsilon_{t+1}) = \sum_{a,\varepsilon} \sum_{\{\alpha: a_{t+1} = Rb_{t+1} + m_{t+1}\}} \Gamma_1^t(a_t, \varepsilon_t)p_{\varepsilon t+1|\varepsilon_t}$.

2.3.3 Characterization of Equilibrium

The optimality conditions that come out of combining the first order conditions of this problem are:

$$\lambda_t[1 - S'(\kappa_t)\kappa_t^2] = \frac{\beta}{1 + \varepsilon_{t+1}} E_t \{\lambda_{t+1}\} \tag{2.8}$$

$$\lambda_t - \varphi_t = \beta RE_t \{\lambda_{t+1}\} \tag{2.9}$$

$$c_t[1 + S(\kappa_t)] + b_{t+1} + m_{t+1} = \varepsilon_t + a_t + \tau_t \tag{2.10}$$

where $\kappa_t = \frac{\alpha}{m_{t+1}}$ and $a_t = Rb_t + \frac{m_t}{1+\varepsilon_t}$.

Lagrange multipliers of the budget constraint and the borrowing constraint ($\lambda_t$ and $\varphi_t$) are shadow prices of total (real) deposits and relaxing the borrowing constraint by one unit respectively. Equations (2.8) and (2.9) are Euler equations.
for real balances and bonds demand respectively. Equation (2.10) is the flow budget
constraint of the household.

Disinflation at $t = 0$ is unanticipated and agents re-optimize. Yet, since the
government announces a deterministic path (which is credible) for the sequence of
depreciation rates starting from period 1 and on, the one-period ahead depreciation
rate, $e_{t+1}$, does not enter into the expectation operator in equation (2.8). Given
this feature, for consumers who are not borrowing constrained (i.e. $\varphi_t(a_t, \varepsilon_t) = 0$),
equations (2.8) and (2.9) can be combined to obtain,

$$\left(\frac{1}{1 + e_{t+1}}\right)\left(\frac{1}{1 - S'(\kappa_t)\kappa_t^2}\right) = R. \quad (2.11)$$

which can also be rewritten as

$$S'(\kappa_t)\kappa_t^2 = \frac{i_{t+1}}{1 + i_{t+1}} \quad (2.12)$$

by using the definition of the nominal interest rate between periods $t$ and $t + 1,$

$$1 + i_{t+1} = (1 + e_{t+1})R \text{ under the absence of aggregate uncertainty.} \quad \text{Given that} \quad S(\kappa_t) = \phi\kappa_t \gamma$$

is a strictly convex and increasing function of $\kappa_t,$ equation (2.12)
implies a unique solution for the consumption velocity as,

$$\kappa_t = \left[\frac{1}{\gamma \phi} \left(\frac{i_{t+1}}{1 + i_{t+1}}\right)\right]^{\frac{1}{\gamma}}. \quad \text{Clearly, } \kappa_{t+1} \text{ does not depend on any idiosyncratic variable, therefore, consumption velocities of unconstrained individuals become identical.} \quad \text{On the other hand, for borrowing constrained individuals, we have } \varphi(a_t, \varepsilon_t) > 0. \quad \text{Now, equations (2.8) and (2.9) imply that} \quad \text{82}$$
\[
\frac{\beta E_t \{ \lambda_{t+1} \}}{\lambda_t} = (1 + e_{t+1}) [1 - S'(\kappa_t^c)\kappa_t^{c2}] = \frac{1}{R} \left[ 1 - \frac{\varphi_t}{\lambda_t} \right]. \tag{2.13}
\]

The first equality follows from equation (2.8) and the second equality follows from equation (2.9) after dividing the whole equation by \( R\lambda_t \). It is straightforward to show that the definition of nominal interest rate and rearranging terms yield

\[
S'(\kappa_t^c)\kappa_t^{c2} = \frac{i_{t+1} + \varphi_t}{1 + i_{t+1}} \tag{2.14}
\]

which implies the consumption velocity of constrained individuals to be, \( \kappa_t^c = \left[ \frac{1}{\gamma} \left( \frac{i_{t+1} + \varphi_t}{1 + i_{t+1}} \right) \right]^{\frac{1}{1+\gamma}} \). Since \( \varphi_t(a_t, \varepsilon_t) > 0 \) and \( \lambda_t(a_t, \varepsilon_t) > 0 \) \( \forall (a_t, \varepsilon_t) \), \( \kappa_t^c > \kappa_t \) \( \forall (a_t, \varepsilon_t) \). Furthermore, \( \gamma, \phi, \lambda_t, i_{t+1} > 0 \) implies that \( \kappa_t^c \) is increasing in \( \varphi_t \). This means that consumers who are borrowing constrained have a higher consumption velocity than those who are not. Moreover, the more constrained an individual (i.e. the larger \( \varphi_t(a_t, \varepsilon_t) \)) is, the larger the discrepancy.

### 2.3.4 Transitional Dynamics of Disinflation

In this section, I discuss the mechanics of transitional dynamics implied by gradual disinflation. First and foremost, from a methodological point of view, stationary equilibria analysis cannot incorporate gradual disinflation observed in the data. Therefore, one has to resort to the numerical solution of the transitional dynamics equilibrium. Coming to the expected implications, as equation (2.11) illustrates, consumption velocity of consumers depends on next period’s inflation rate. The calibrated disinflationary path will then imply a declining profile for consumption velocities along the transition. This will consequently cause aggregate

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consumption velocity to follow a secular decline as in data. Seigniorage revenues are also expected to decline over time as opposed to collapsing immediately. This point is rather crucial because as Chapter 1 illustrates, redistributive implications are closely tied to the time profile of seigniorage revenues. As a result, welfare consequences are expected to be less emphasized in the transitional dynamics experiment compared to the study of steady states.

Inflation does not cause any distortions in the production sector in this model. This would strengthen the transmission from the calibrated path of inflation to the variables in the government budget constraint (i.e., seigniorage revenues, government spending and transfers). However, in any case, the evolution of precautionary savings during disinflation will be non trivial. This is because there is a certain degree of persistence in the stochastic earnings process governed by, \[ p_{\epsilon'|\epsilon} = Pr(\epsilon_{t+1} = \epsilon'|\epsilon_t = \epsilon). \] Take the case of Economy 1. Now, persistence in earnings process would cause some poor consumers to be hit by adverse earnings shocks and stay borrowing constrained although the precautionary savings motive increases due to the lack of insurance provided by uniform transfers. As I establish above, the opportunity cost of holding real balances will be higher for those individuals and they will hold less real balances compared to unconstrained consumers. This might affect aggregate money demand together with consumption velocity and welfare consequences of disinflation.

I finish the discussion of analytical framework here and proceed to the quantitative assessment of disinflation in the next section.
2.4 Quantitative Analysis

In this section, I study the model’s quantitative predictions using a version calibrated to the Turkish economy. From a parameterization and calibration perspective, the focus is on the low inflation steady state represented by the period 2003:1-2010:2, for which data on aggregates, inequality measures and government transfers are available. The model period is a quarter. The main experiment is to assess the transitional dynamics of macroeconomic and distributional variables and welfare consequences of a gradual decline in inflation from $e^0 = 14.25\%$ to $e^1 = 2.25\%$. Following the exercise in plotting Figure 2.1, I HP filter the time series of inflation rates during disinflation (1999:2-2004:4) and feed the trend levels into the model to establish $\{e^1_t\}_{t=1}^T$ for $T = 23$.\(^4\) I use the same parameter values in the analysis of Economy 2. The major difference of Economy 2 is that (as opposed to uniform transfers in Economy 1) government spending is now an endogenous equilibrium object that responds to changes in seigniorage revenues to satisfy the government budget constraint.

2.4.1 The Parameterization of the Benchmark Economy

The parameters of Economy 1 are determined by treating the period 2003:1-2010:2 as a benchmark as in Chapter 1. Table 2.1 below includes a list and description parameter values used in the quantitative analysis. To avoid repetition, I skip the detailed description of the parameterization of Economy 1. The reader could

\(^4\)As mentioned in Introduction, average inflation in the periods 1987:2-1999:2 and 2004:4-2010:3 are 15.46\% and 1.86\% respectively. These inflation rates are not significantly different from the “high” and “low” inflation rates (14.25\% and 2.25\%) analyzed in Chapter 1. For comparability, I use those values as initial and terminal conditions of the transitional dynamics equilibrium.
Table 2.1: Benchmark Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Target</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.0000</td>
<td>Risk aversion</td>
<td>Literature</td>
<td>N/A</td>
</tr>
<tr>
<td>$R$</td>
<td>1.0276</td>
<td>Gross real interest rate</td>
<td>US Treasury + 7% spread</td>
<td>N/A</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9625</td>
<td>Persis. of earnings shocks</td>
<td>Literature</td>
<td>N/A</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.1400</td>
<td>Volat. of shocks to log-earnings</td>
<td>Literature</td>
<td>N/A</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.2175</td>
<td>Curv. of the trans. costs function</td>
<td>Int. elas. of M1 demand = -0.4510</td>
<td>N/A</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>0.1611</td>
<td>Real gov. spending-to-GDP</td>
<td>Average of 2003:1-2010:2</td>
<td>N/A</td>
</tr>
<tr>
<td>Jointly Calibrated</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9215</td>
<td>Discount factor</td>
<td>$NX/GDP = -0.0334$</td>
<td>-0.0333</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.00175</td>
<td>Multip. trans. costs parameter</td>
<td>$C/M1 = 4.1925$</td>
<td>4.1920</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>-0.0329</td>
<td>Lower bound for bonds</td>
<td>$(M2Y - M1)/M2Y = 0.8493$</td>
<td>0.8543</td>
</tr>
</tbody>
</table>
find such a detailed description in Section 1.5.1 of Chapter 1. The numerical solution algorithm of the transitional dynamics equilibrium is described in Appendix E.

2.4.2 Macroeconomic Consequences of Disinflation

In this section, I discuss the macroeconomic dynamics of gradual disinflation (calibrated to data) under alternative fiscal arrangements that govern how inflationary finance is used.

2.4.2.1 Uniform Transfers

Figure 2.2 below illustrates the time profile for main aggregate variables, the depreciation rate and the fraction of borrowing constrained when inflationary finance is directed to transfers. I plot ratios of aggregate transfers, consumption, net foreign assets position, money supply, transactions costs and trade balance-to-GDP in the top panels 1-6. Panels in the lower part of Figure 2.2 display the path of aggregate consumption velocity, measure of borrowing constrained and inflation. The dashed plots represent the dynamics observed in the data (see Figure 2.1).

The most striking observation is that the model is able to capture the secular decline in consumption velocity and rise in aggregate money supply. Following the “unanticipated decline” in inflation at date 0, money demand (and therefore supply) starts to increase. The upward slope of money supply causes seigniorage revenues to even increase slightly in early periods. The path of transfers is in turn driven by the path of seigniorage revenues, since government spending is constant. After reaching a maximum in about a year, transfers start decreasing during disinflation. However, since inflation is still relatively high compared to the terminal steady state, the red-
Figure 2.2: Macroeconomic Dynamics of Disinflation (Uniform Transfers)
uction in seigniorage revenues becomes limited. When the economy gets closer to the point at which agents know that inflation will be permanently lower in about 6 years, transfers sharply get closer to their terminal steady-state level.

This finding is rather interesting and needs to be elaborated: Inflation does not create any distortions in the “production” sector of this model economy since earnings are exogenous. It only creates (i) wealth effects transmitted by transactions costs of consumption and inflation taxation and (ii) substitution effects distorting the portfolio choice margin between money and bonds. Furthermore, as illustrated in Section 2.3.3, as long as “unconstrained” agents learn that opportunity cost of holding money will be lower permanently, they will immediately settle to the same consumption velocity which is pinned down by the terminal steady state.

While consumption velocity and transactions costs monotonically decrease along the transition, aggregate consumption and net foreign asset position follows a path guided by the evolution of transfers. A 1.25% increase in consumption is achieved in about 6 years. Notice that the sharp decline and then recovery around 2001 in Figure 2.1 is due to the most severe financial crisis of the Turkish economy in the last decade. Therefore, 2001 remains as an outlier within the disinflation period, around which most macroeconomic variables exhibit sharp movements (see Figure 2.1). Keeping this in mind, I argue that the model is qualitatively consistent with the upward trend in aggregate consumption during disinflation. On the other hand, bond position of the model economy exhibits a downward trend during the first 5 years. This is mainly due to relaxed debt limits facilitated by high transfers in that period. This also causes the fraction of borrowing constrained to follow a
smooth declining path along the transition. Since output and government spending are fixed in this economy, trade surplus follows an exactly opposite time profile to that of consumption. This is again qualitatively in line with the data since the trade balance starts deteriorating after 2001.

2.4.2.2 Endogenous Government Spending

Figure 2.3 below illustrates the time profile for same set of variables included in Figure 2.2 under the assumption that inflationary finance is now directed to wasteful spending. The transitional path of consumption velocity, transactions costs and the fraction of borrowing constrained are similar to the endogenous transfers case. Government spending follows a path very similar to that of transfers in the previous economy. This model is also able to explain the secular decline in consumption velocity and rise in money supply as does the previous model.

However, the endogeneity of government spending causes different dynamics for consumption and bonds position of the economy. In particular, the decline in spending and in the distortions created by inflation during 6 years imply an almost monotonically increasing path for consumption. The initial decline in consumption is due to the slight surge in government spending caused by the unanticipated change. But now, since the immediate fall in spending as the economy approaches to the terminal conditions does not feed back into consumers’ optimization problem described in 2.3.1, consumption does not plummet in year 6 in sharp contrast with Economy 1. Moreover, the path of trade balance reflects the combined effect of changes in consumption and government spending along the transition. Specifically,
Figure 2.3: Macroeconomic Dynamics of Disinflation (Endogenous Spending)
the increasing trade deficit along the transition is corrected by a discrete jump when spending collapses to its terminal equilibrium value. In line with these observations, net foreign asset position decreases for 5 years and settles down to its terminal value without showing a reversal as opposed to Economy 1. This is again due constant transfers in Economy 2, which shut down redistributive wealth effects. Since transfers do not respond to disinflation in this economy, natural debt limits do not tighten as much, causing a limited increase in the precautionary savings motive. Therefore, the measure of borrowing constrained falls less compared to Economy 1. Notice that natural debt limits depend on the inflation tax payments of the poorest as well. Therefore, even transfers are fixed, natural debt limits are affected by inflation in Economy 2.

Finally, in Table 2.2, I compare the time series averages of macroeconomic variables during disinflation, i.e., 1999:2-2004:4, to those implied by Economies 1 and 2. The model proves successful in predicting time series averages for consumption and money supply-to-GDP ratios and consumption-velocity that are very close to their empirical counterparts.

This completes the analysis of macroeconomic implications of disinflation in Economies 1 and 2 and I now proceed to comment on distributional consequences.

2.4.3 Distributional Consequences of Disinflation

In Figures 2.4 and 2.5, I present the time profile of Gini coefficients of bonds, money, consumption and income in Economies 1 and 2 respectively. Although the degree of inequality in bonds position is very similar at initial and terminal condi-
Table 2.2: Time Series Averages of Macroeconomic Variables Along Disinflation

<table>
<thead>
<tr>
<th></th>
<th>Data(^a)</th>
<th>Uniform (\tau_1)</th>
<th>Endogenous (G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C/GDP)</td>
<td>0.857</td>
<td>0.862</td>
<td>0.862</td>
</tr>
<tr>
<td>(NFA/GDP)</td>
<td>1.175</td>
<td>1.198</td>
<td>1.225</td>
</tr>
<tr>
<td>(M1/GDP)</td>
<td>0.190</td>
<td>0.194</td>
<td>0.194</td>
</tr>
<tr>
<td>(TB/GDP)</td>
<td>-0.021</td>
<td>-0.033</td>
<td>-0.035</td>
</tr>
<tr>
<td>(C/M1)</td>
<td>4.772</td>
<td>4.552</td>
<td>4.547</td>
</tr>
</tbody>
</table>

\(^a\)Disinflation period spans 1999:2-2004:4 as illustrated in Figure 2.1.
Figure 2.4: Disinflation and Inequality (Uniform Transfers)
Figure 2.5: Disinflation and Inequality (Endogenous Spending)
tions in Economy 1, accounting for the transitional dynamics shows that the Gini coefficient increases by about 1.5% in the first 6 years (top-left panel of Figure 2.4). This coincides with the reduction in precautionary savings motive, due to high level of endogenous transfers in this economy. This also prevents income inequality to increase (bottom-right panel) until transfers collapse to their low inflation steady state level. Money demand is a function of total deposits which is mainly composed of bonds. Consequently, more inequality in bonds imply more inequality in the distribution of money holdings. Specifically, the Gini coefficient of money holdings rise about 1% in 7 years and then settles down to its low inflation steady state value. Recall that as shown in Figure 2.2, the measure of borrowing constrained is declining along the transition. This requires inequality of consumption to get closer to that of real balances. Therefore, Gini coefficient of consumption surges by about 1 percentage point in a discrete manner when disinflation starts.

Evolution of money holdings and consumption inequality in Economy 2 is similar to those in Economy 1 (see Figure 2.5 below). From a quantitative point of view, income inequality is almost intact and does not display a discrete jump as the economy approaches to the terminal conditions in contrast with Economy 1. However, inequality in bonds secularly increases in the first 6 years and ultimately stays at a higher level compared to the high inflation steady state. Since transfers are fixed in Economy 2, reduction in precautionary savings motive is less compared to Economy 1. This prevents consumers from buffering bonds that leads to a more dispersed distribution.
2.4.4 Welfare Consequences of Disinflation

I develop a measure of aggregate welfare. Following Mendoza et al. (2007), welfare effects are computed as the proportional increase in consumption in the 14.25\% inflation stationary equilibrium, $\eta$, that would make an individual consumer indifferent about remaining in that state versus shifting to an economy that exhibits the disinflation profile described above. For each agent $i$ who is at the initial state $(a_0, \varepsilon_0)$, $\eta(a_0, \varepsilon_0)$ solves

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{i,14.25\%}(1 + \eta(a_0, \varepsilon_0))) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{i,Dis})$$

(2.15)

where $\{c_t^{i,14.25\%}\}_{t=0}^{\infty}$ is the infinite sequence of consumption of agent $i$ in state $(a_0, \varepsilon_0)$ in the high inflation economy and $\{c_t^{i,Dis}\}_{t=0}^{\infty}$ is the corresponding sequence of consumption in the disinflation economy.\footnote{Given the particular functional form for the utility function and the notation so far, $\eta(a_0, \varepsilon_0)$ also solves

$$[(1 - \beta)(1 - \sigma)v^{14.25\%}(a, \varepsilon) + 1](1 + \eta(a_0, \varepsilon_0))^{1-\sigma} = [(1 - \beta)(1 - \sigma)v_0^{Dis}(a_0, \varepsilon_0) + 1]$$

where $v^{14.25\%}(a, \varepsilon)$ is the equilibrium time invariant value function in the high inflation economy and $v_0^{Dis}(a_0, \varepsilon_0)$ is the $t = 0$ value of experiencing disinflation. Notice also that as per the recursive representation of households’ optimization problem, $v_0^{Dis}(a_0, \varepsilon_0)$ incorporates the value of experiencing the transitional dynamics of disinflation.}

Once I establish the consumption equivalent of welfare gains on the individual level, as a natural next step, I need to do an aggregation to achieve a normative assessment regarding the economy as a whole. The practice is to fix the deposits distribution of the high inflation economy as an initial condition, $\Gamma^{14.25\%}(a_0, \varepsilon_0)$ and use it to compute a weighted average of the welfare gains in terms of compensating consumption variation (CCV hereafter). Hence, the consumption equivalent of the...
aggregate welfare gain from disinflation becomes

\[
W^{Dis} = \sum_{a, \varepsilon} \Gamma^{14.25\%}(a_0, \varepsilon_0) \eta(a_0, \varepsilon_0) 
\] (2.16)

Table 2.3 below presents welfare consequences of reducing inflation from 14.25% to 2.25% under alternative fiscal arrangements. Furthermore, for each arrangement, I compare welfare consequences of immediately switching to the low inflation steady state versus experiencing the transitional dynamics. The first row denotes aggregate welfare gain of settling at the low inflation stationary equilibrium. Rows 2, 3 and 4 include the disaggregation of this measure into the average gains of the bottom quintile and the top percentile and the median gain (ordered according to total deposits positions). The results show that apart from capturing the secular downward (upward) pattern of consumption velocity (money demand), accounting for the gradual decline in inflation also has significantly different welfare consequences in comparison to stationary equilibria analysis. In particular, as illustrated in Chapter 1, instantaneous switch to the low inflation equilibrium in Economy 1 causes a sharp reduction in endogenous transfers. This causes the poor to incur substantial welfare losses (3.56% in terms of CCV) caused by a large reduction in redistributive transfers, which surpasses the reduction in distortions created by inflation (see the first column of Table 2.3). However, when transitional dynamics are taken into account, the evolution of transfers is gradual. This keeps redistribution alive for about 6 years and transforms welfare losses of the poor into gains (0.26%). It is straightforward at this point to see that welfare gains of the rich in the stationary
Table 2.3: Welfare Consequences of Reducing Inflation from 14.25% to 2.25%

<table>
<thead>
<tr>
<th>Welfare Gains&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Uniform &lt;sup&gt;τ_1&lt;/sup&gt;</th>
<th></th>
<th>Endogenous &lt;sup&gt;G&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steady States</td>
<td>Transition</td>
<td>Steady States</td>
</tr>
<tr>
<td>Aggregate</td>
<td>-1.247</td>
<td>0.284</td>
<td>1.622</td>
</tr>
<tr>
<td>Bottom 20%</td>
<td>-3.564&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.264</td>
<td>1.236</td>
</tr>
<tr>
<td>Median</td>
<td>-1.226</td>
<td>0.280</td>
<td>1.655</td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.967&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.326</td>
<td>1.756</td>
</tr>
</tbody>
</table>

<sup>a</sup>Welfare gains are computed as percentage change in terms of compensating consumption variation.

<sup>b</sup>Average welfare gains of percentiles ordered according to total deposits positions.
world (0.97%) are diminished in the transitional dynamics world (0.33%), since they are financing redistributive transfers by paying more inflation tax in comparison to the poor.

Moving to Economy 2 with endogenous spending, it is again observed that transitional dynamics dampen welfare consequences of steady state comparisons. This is mainly due to the persistently high level of government spending that distorts the production possibilities frontier of this economy. However, the magnitude of the change in welfare effects is not as strong as in Economy 1 so that disinflation is still welfare improving for all segments of the society (see the third and fourth columns of Table 2.3).

As per the steady state analysis in Chapter 1, instantaneous adjustments in fiscal variables financed by seigniorage create strong wealth effects. Although marginal utility of consumption is higher for the rich, immediate collapse of transfers/government spending in these economies increases the consumption of the rich substantially. Therefore, welfare gains are monotonic (and concave) in earnings and total deposits in Economies 1 and 2 but are larger in the latter, since spending has no value to consumers.

In Figure 2.6, I plot the disaggregated welfare gains implied by the transitional dynamics exercise. I let earnings to take 21 values in the numerical computation of the model and plot percentage changes in welfare as a function of earnings $\varepsilon_1 < \varepsilon_3 < \varepsilon_{11} < \varepsilon_{19} < \varepsilon_{21}$ (plots with dashes, diamonds, no dashes or shapes, circles and asterisks) and total deposits. Both panels suggest that disaggregated welfare changes are highly non-linear and non-monotonic in contrast with the steady state
Figure 2.6: Disaggregated Welfare Gains From Disinflation
analysis. The left panel illustrates the reversal of welfare changes in Economy 1. Consumers who are poor both in terms of earnings and total deposit positions enjoy substantial welfare gains thanks to persistently high redistributive transfers along the transition. Furthermore, welfare gains of the rich now start from a low level and are not increasing in total deposits position anymore. This is due to the limited increase in rich individuals’ consumption, because, gradual decline of transfers produce diminished wealth effects. On the other hand, in Economy 2, welfare gains of the earnings poor increases for low deposits levels. This explains why median gain is the largest in the fourth column of Table 2.3. Earnings rich individuals finance more of the fiscal spending with their inflation tax payments. Therefore, their welfare gains from disinflation are larger. But now, wealth effects of reducing spending is again spilled over time, which produces diminishing welfare gains in total deposits.

This completes the analysis of macroeconomic, distributional and welfare consequences of transitional dynamics of disinflation. It should be obvious at this point that accounting for gradual disinflation (which is observed in the data) improves this stylized model of small open economy with heterogeneous agents upon the steady state comparisons in three dimensions: First, stylized dynamics of consumption velocity and money demand during disinflation periods are captured better. Second, it shows that the path of financial assets inequality can be volatile along disinflation and third, welfare consequences are substantially different than those implied by steady state comparisons. Therefore, I argue that the most relevant sensitivity experiment within this framework would be to focus on the importance of calibrat-
ing the path of “gradual disinflation”. Consequently, in the next section, I perform sensitivity analysis by computing the transitional dynamics equilibrium, which now entails a stabilization policy of unanticipated “sudden” decline in inflation.

2.5 Sensitivity Analysis

In this section, I repeat the transitional dynamics exercise with the only difference that the calibrated disinflation path now involves a one time, unanticipated decline in inflation. That is,

\[ e_t = e^0 \quad \text{for} \quad t = 0 \quad (2.17) \]

\[ \{e_t\}_{t=1}^{\infty} = e^1 \quad \text{for} \quad t > 0 \]

with \( e^0 = 14.25\% \) and \( e^1 = 2.25\% \).

2.5.1 Macroeconomic Consequences of Sudden Disinflation

Macroeconomic dynamics implied by sudden disinflation (illustrated in Figures 2.7 and 2.8) are strikingly different from those implied by gradual disinflation (illustrated in Figures 2.2 and 2.3). When transfers are endogenous, unanticipated disinflation causes discrete jumps in money demand and transfers-to-GDP ratios on impact (a huge 10% for the latter, see Figure 2.7). The surge in money (which economizes transactions costs) and transfers, make people richer, cause consumption-to-GDP ratio to increase by about 5 percentage points and create a trade deficit on impact. The surge in consumption causes a slight increase in consumption velocity on impact and transactions costs immediately collapse to 50% of their original lev-
Figure 2.7: Macroeconomic Dynamics of Sudden Disinflation (Uniform Transfers)
Figure 2.8: Macroeconomic Dynamics of Sudden Disinflation (Endogenous Spending)
el. The most striking observation is that the model is now unable to capture the endured secular decline in consumption velocity and increase in money demand along the transition. This is because transfers quickly adjust by plummeting to very low levels as per consumers’ perfect information on disinflation that it will be implemented credibly. Consequently, debt limits are tightened, precautionary savings motive increases substantially and a discrete fall in the fraction of borrowing constraint of almost 10% takes place. This coincides with an increase in bonds position-to-GDP ratio that follows a one time decline created by the surge in transfers. Along the transition, the measure of borrowing constrained and net foreign assets converge to their terminal values gradually. Since government spending is fixed, trade balance again follows the opposite of the path of consumption.

Figure 2.8 illustrates macroeconomic dynamics of sudden disinflation when spending is endogenous. The plots have a very stark message. Except for the measure of borrowing constrained, all other variables settle down to their terminal values almost instantaneously. This is again because there is no feedback from the adjusting variable, spending, to consumers’ optimization problem. The surge in spending on impact is the result of unanticipated disinflation because money demand shoots up on impact. Since transfers are fixed in this economy, fraction of borrowing constrained does not display any movement on impact but converges to its terminal value along transition. The gradual convergence of this measure (as opposed to steady state comparisons) is purely attributable to the persistence in idiosyncratic earnings process (see the discussion in Section 2.3.4). The increase in aggregate consumption on impact is now very limited because government spending increases
a lot. Substantial rise in domestic absorption deteriorates the trade balance about 10% and causes a decline in the net foreign asset position. Most notably, similar to the endogenous transfers case, this model also cannot resemble the persistent decline in consumption velocity and increase in aggregate money demand.

2.5.2 Distributional Consequences of Sudden Disinflation

Figure 2.9 suggests that reducing inflation overnight with endogenous transfers is a short-term remedy for inequality in financial assets and consumption. The reduction in the measure of borrowing constrained causes the Gini coefficient of bonds to decline by about 7%. The initial surge in redistributive transfers causes the distribution of money holdings and consumption to be almost perfectly equitable on impact. The distribution of the two are also very similar as per the collapse in the fraction of constrained. Similar to gradual disinflation (see Figure 2.4), income inequality resembles the opposite of the path of redistributive transfers.

Distributional implications of sudden disinflation with endogenous spending are displayed in Figure 2.10 below. The paths of Gini coefficients clearly establish that sudden disinflation with endogenous spending would have undesired distributional consequences in the short run. This is because the rise in spending on impact creates adverse wealth effects that dominate the typical increase in the precautionary savings motive caused by disinflation in this model. In particular, Gini coefficients of bond holdings, money holdings and consumption increase by about 2%, 1.5% and 1% on impact. Income inequality does not exhibit a substantial change.

2.5.3 Welfare Consequences of Sudden Disinflation
Figure 2.9: Sudden Disinflation and Inequality (Uniform Transfers)
Figure 2.10: Sudden Disinflation and Inequality (Endogenous Spending)
Table 2.4 reports the comparison of welfare consequences of disinflation within steady state and transitional dynamics frameworks where the latter now incorporates the unanticipated announcement of reduction in inflation from 14.25% to 2.25%. The main observation is that unlike gradual disinflation, accounting for transitional dynamics within sudden disinflation produces qualitatively similar welfare consequences in Economy 1 and exactly identical consequences in Economy 2. In comparison with the steady state analysis, welfare changes in Economy 1 are magnified due to the volatility in transfers. In Economy 2, since there is no feedback from spending to households’ utility maximization problem, consumers immediately adjust their portfolio decisions according to the low inflation steady state. This is a very interesting finding that if one limits focus on sudden disinflation in Economy 2, steady state comparisons perform (in welfare dimension) almost as good as explicitly accounting for transitional dynamics.

Finally, for completeness, I illustrate disaggregated welfare gains from sudden disinflation with transitional dynamics in Figure 2.11. As in Figure 2.7, welfare gains are plotted as function of earnings and total deposits. The left panel (representing Economy 1) shows that welfare losses of the poorest are magnified by the sharp decline in redistributive transfers that follows the initial hike as per the unanticipated announcement. The reader should be urged at this point that in this economy, most of the population lie in the range of \([\Omega - 5]\) in the total deposits dimension. Therefore, only the rich enjoy welfare gains from disinflation. Finally, as the right panel of Figure 2.11 illustrates, disaggregated welfare changes in Economy 2 are strikingly identical to those implied by the steady state analysis in Chapter 1, consistent with
Table 2.4: Welfare Consequences of Reducing Inflation from 14.25% to 2.25%

<table>
<thead>
<tr>
<th>Welfare Gains&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Uniform $\tau_1$ Steady States</th>
<th>Transition&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Endogenous $G$ Steady States</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate</td>
<td>-1.247</td>
<td>-8.317</td>
<td>1.622</td>
<td>1.622</td>
</tr>
<tr>
<td>Bottom 20%</td>
<td>-3.564&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-35.939</td>
<td>1.236</td>
<td>1.237</td>
</tr>
<tr>
<td>Median</td>
<td>-1.226</td>
<td>-6.535</td>
<td>1.655</td>
<td>1.655</td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.967&lt;sup&gt;c&lt;/sup&gt;</td>
<td>4.268</td>
<td>1.756</td>
<td>1.763</td>
</tr>
</tbody>
</table>

<sup>a</sup>Welfare gains are computed percentage change in terms of compensating consumption variation.

<sup>b</sup>Transition columns represent unanticipated, $t = 0$ announcement of sudden disinflation.

<sup>c</sup>Average welfare gains of percentiles ordered according to total deposits positions.
Figure 2.11: Disaggregated Welfare Gains from Sudden Disinflation
In summary, the main message of the sensitivity analysis is that accounting for “the gradual decline” in inflation which is modeled as an unanticipated, time consistent and credible announcement (i) is necessary to produce the stylized macroeconomic consequences of disinflation; and (ii) produces qualitative and quantitative differences from the steady state framework. The transitional dynamics exercise fails in improving upon steady state analysis in these dimensions if disinflation is modeled as an unanticipated announcement of a sudden collapse in the inflation rate.

2.6 Conclusion

This paper explores the unanswered question of “what might be the macroeconomic, distributional and welfare consequences of recent disinflation in emerging economies?”. The analysis starts with documenting stylized macroeconomic facts for the case of Turkey’s disinflation which has endured since the last two decades. Apart from disinflation, Turkey constitutes a good example of important financial system characteristics of emerging economies, such as financial dollarization and substantial inequality in the distribution of monetary assets.

To that end, I calibrated the disinflation profile of the Turkish economy and fed it into a monetary model of a small open economy with uninsured idiosyncratic earnings risk and incomplete markets. The policy experiment is to have the monetary authority make an unanticipated, time-consistent and credible announcement of a disinflation profile at date 0.
As established in Chapter 1, fiscal and monetary interactions play a decisive role on part of distributional and welfare consequences of disinflation. Guided by those findings, I incorporate alternative adjustment rules in the government budget constraint in response to reduction in inflationary finance and find that, explicitly accounting for transitional dynamics is a necessary condition to capture the stylized dynamics of macroeconomic variables. Moreover, analyzing gradual disinflation produces interesting welfare consequences that are different from those implied by the steady state analysis. Another important finding is that due to the absence of disruptive effects of inflation in the production sector, the first order impact of disinflation is observed on macro variables related to public finance. This implied that accounting for transitional dynamics does not matter much if policy experiment is switched to one that involves a sharp, one-time decline in inflation.

The most relevant research avenue for further work would be to analyze an environment in which idiosyncratic and aggregate uncertainty coexists. In such an environment, volatility of inflation (which appears to be reducing in the disinflation era), beside total factor productivity shocks should be modeled as the source of aggregate uncertainty. That environment would enrich the distortions created by inflation (specifically in the production sector) and provide an enhanced metric of macroeconomic, distributional and welfare consequences of both the level and the volatility of inflation.
Appendix A

A.1 Proof of Proposition 1.1

The Lagrange multiplier on the borrowing constraint ($\varphi$) will be equal to zero for unconstrained individuals. Therefore one can combine equations (1.7) and (1.8) to obtain the following:

\[
\left(\frac{1}{1+e}\right) \left(\frac{1}{1 - S'(\kappa)\kappa^2}\right) = R \tag{A.1}
\]

which can also be rewritten as

\[
S'(\kappa)\kappa^2 = \frac{i}{1 + i} \tag{A.2}
\]

by using the definition of the nominal interest rate, $1+i = (1+e)R$ under the absence of aggregate uncertainty. Given that $S(\kappa) = \varphi \kappa^\gamma$ is a strictly convex and increasing function of $\kappa$, equation (A.2) implies a unique solution for the consumption velocity as, $\kappa = \left[\frac{1}{\gamma \varphi (1+e)^{\gamma^{-1}}}\right]^{1/\gamma}$. Clearly, $\kappa$ does not depend on any idiosyncratic variable. On the other hand, for borrowing constrained individuals, we have $\varphi(a, \varepsilon) > 0$. Now, equations (1.7) and (1.8) imply that

\[
\frac{\beta E \{X\}}{\lambda} = (1 + e)[1 - S'(\kappa^c)\kappa^c] \frac{1}{R} = \frac{1}{\lambda} \left[1 - \frac{\varphi}{\lambda}\right]. \tag{A.3}
\]

The first equality follows from equation (1.7) and the second equality follows from equation (1.8) after dividing the whole equation by $R \lambda$. It is straightforward to
show that the definition of nominal interest rate and rearranging terms imply

\[ S'(\kappa^c)\kappa^c^2 = \frac{i + \frac{\phi}{\lambda}}{1 + i} \quad \text{(A.4)} \]

The proof can be completed by imposing the functional form of \( S(.) \) again and solving for the consumption velocity of a constrained individual as,

\[ \kappa^c = \left[ \frac{1}{\gamma \phi} \left( \frac{i + \frac{\phi}{\lambda}}{1 + i} \right) \right]^{\frac{1}{1+\gamma}}. \]

Since \( \varphi > 0 \) and \( \lambda > 0 \) \( \forall (a, \varepsilon) \), \( \kappa^c > \kappa \) \( \forall (a, \varepsilon) \). Furthermore, \( \gamma, \phi, \lambda, i > 0 \) implies that \( \kappa^c \) is increasing in \( \varphi \).\(^1\)

---

\(^1\)Equations (A.2) and (A.4) are the consumption-money optimality conditions that illustrate the marginal benefit-opportunity cost trade off regarding the real balances holding decision for unconstrained and constrained individuals respectively.
Appendix B

Solutions of Deterministic Economies

B.1 Economy with a Representative Household

In this example, I assume that the economy is inhabited by a large number of identical households with given initial asset positions of $M_0$ and $B_0$. I also assume that the economy is deterministic in the sense that the representative consumer faces a time-invariant profile of earnings $Y$, which can be thought of as the aggregate GDP of this economy (RH, henceforth). The monetary arrangement is the same as in the benchmark model, i.e. central bank can set the depreciation rate of the currency (price inflation of the single tradable good). Hence, aggregate money supply is demand determined. The recursive representation of the households’ problem then is:

$$v(A) = \max_{C, M', B'} \left[ u(C) + \beta v \left( RB' + \frac{M'}{1 + e} \right) \right]$$

subject to

$$C \left[ 1 + \phi \left( \frac{C}{M'} \right) \right] + B' + M' = Y + A + \tau$$

$$(B_0, M_0) \text{ given, } C, M' \geq 0, B' \geq \Omega \text{ and } A = RB + \frac{M}{1 + e}$$
Note that since there is no uncertainty on earnings, \( Y \) is not an argument of the value function. Assuming the same role for the government described by equation (1.3), the equilibrium conditions will read:

\[
\begin{align*}
    u'(C_t) &= \lambda_t \left[ 1 + \phi (1 + \gamma) \left( \frac{C_t}{M_{t+1}} \right)^{1+\gamma} \right] \quad (B.4) \\
    \lambda_t \left[ 1 - \phi \gamma \left( \frac{C_t}{M_{t+1}} \right)^{1+\gamma} \right] &= \frac{\beta}{1 + e} \lambda_{t+1} \quad (B.5) \\
    \lambda_t - \varphi_t &= \beta R \lambda_{t+1} \quad (B.6) \\
    C_t \left[ 1 + \phi \left( \frac{C_t}{M_{t+1}} \right)^{\gamma} \right] + B_{t+1} + M_{t+1} &= Y + RB_t + \frac{M_t}{1 + e} + \tau \quad (B.7) \\
    B_{t+1} &\geq \Omega \quad (B.8) \\
    G_t + \tau &= M^*_{t+1} - \frac{M^*_t}{1 + e} \quad (B.9) \\
    M^*_t &= M_t \quad \forall t \quad (B.10)
\end{align*}
\]

where \( \lambda_t \) and \( \varphi_t \) are the Lagrange multipliers of the budget constraint and borrowing constraint respectively.\(^1\)

The steady state equilibrium is characterized by a time-invariant profile for endogenous real variables, \( \lambda_t = \lambda^* \), \( \varphi_t = \varphi^* \), \( C_t = C^* \), \( M^*_t = M_t = M^* \), \( B_t = B^* \),

\(^1\)Note that condition (B.10) holds due to the exchange rate regime.
\[ \tau_t = \tau^* \ \forall \ t \] and the system of equations (B.4)-(B.9) evaluated at these constant values:

\[ u'(C^*) = \lambda^* \left[ 1 + \phi (1 + \gamma) \left( \frac{C^*}{M^*} \right)^{1+\gamma} \right] \] (B.11)

\[ \left[ 1 - \phi \gamma \left( \frac{C^*}{M^*} \right)^{1+\gamma} \right] = \frac{\beta}{1 + e} \] (B.12)

\[ \lambda^* (1 - \beta R) = \varphi^* \] (B.13)

\[ C^* \left[ 1 + \phi \left( \frac{C^*}{M^*} \right)^{\gamma} \right] + B^* + M^* = Y + RB^* + \frac{M^*}{1 + e} + \tau^* \] (B.14)

\[ B^* \geq \Omega \] (B.15)

\[ G + \tau^* = \frac{M^* e}{1 + e} \] (B.16)

which is a system of 6 conditions and 6 unknowns: \((C^*, M^*, B^*, \tau^*, \varphi^*, \lambda^*)\).\(^2\) It is possible to find a closed-form solution to this system. For now, assume that \(\beta R < 1\).

Then, since \(\lambda^* > 0\), equation (B.13) implies that \(\varphi^* > 0\), i.e., the borrowing constraint is binding. Therefore, by equation (B.15), \(B^* = \Omega\) so that the representative consumer roles over a constant interest payment of \((R - 1)\Omega\). On the other hand, it is straightforward to show that equation (B.12) implies a constant consumption velocity, which can be denoted by

\(^2\)Since \(Y\) is the exogenous aggregate GDP of this economy, \(G\) is known and proportional to \(Y\).
Now by (i) substituting equation (B.16) for $\tau^*$, (ii) using $B^* = \Omega$ and (iii) substituting $M^*$ for $C^*$ by the help of equation (B.17), I rewrite equation (B.14) as:

$$M^*\kappa(1 + \phi\kappa) + (1 - R)\Omega = Y - G$$  \hspace{1cm} (B.18)

which implies that $M^* = \frac{Y - G - (1 - R)\Omega}{\kappa(1 + \phi\kappa)}$. Now, constant consumption velocity pins down the closed-form solution for consumption as, $C^* = \frac{Y - G - (1 - R)\Omega}{(1 + \phi\kappa)}$. Finally, substituting $M^*$ in equation (B.16) yields the equilibrium level of government transfers as, $\tau^* = \frac{\kappa(1 + \phi\kappa)\Omega}{Y - G - [1 - R - \kappa(1 + \phi\kappa)]\Omega} - \bar{G}$.\(^3\)

Alternatively, if $\beta R = 1$, equation (B.13) implies that $\varphi^* = 0$ and therefore, equilibrium bond position is determined by the initial conditions, $B^* = b_0$ which is given.\(^4\) Therefore, all closed form solutions hold except for the difference that $\Omega$ is replaced by $b_0$.

**Portfolio Composition**

Using closed form solutions, I can pin down the share of bonds in total stock of assets as:

$$\frac{B^*}{B^* + M^*} = \frac{\kappa(1 + \phi\kappa)\Omega}{Y - G - [1 - R - \kappa(1 + \phi\kappa)]\Omega}. \hspace{1cm} (B.19)$$
How the portfolio composition is affected by earnings, the lower bound for bonds position and inflation are illustrated in equations (B.20) to (B.22) below.

\[
\frac{d \left( \frac{B^*/B^*+M^*}{\Omega} \right)}{dY} = -\frac{\kappa(1+\phi\kappa^\gamma)\Omega}{(Y-G-[1-R-\kappa(1+\phi\kappa^\gamma)]\Omega)^2} \tag{B.20}
\]

\[
\frac{d \left( \frac{B^*/B^*+M^*}{\Omega} \right)}{d\Omega} = \frac{\kappa(1+\phi\kappa^\gamma)(Y-G)}{(Y-G-\Omega[1-R-\kappa(1+\phi\kappa^\gamma)])^2} \tag{B.21}
\]

\[
\frac{d \left( \frac{B^*/B^*+M^*}{\Delta e} \right)}{d\kappa} = \frac{dk}{d\kappa} \cdot \left( \frac{B^*/B^*+M^*}{\Omega} \right)^{-2} \cdot \frac{[Y-G-(1-R)\Omega]}{\Omega[1+\phi(1+\gamma)\kappa^\gamma]} > 0. \tag{B.22}
\]

where

\[
\frac{dk}{de} = \frac{\beta}{(1+\gamma)\kappa^\frac{1}{1+\gamma} \phi\gamma(1+e)^2} > 0. \tag{B.23}
\]

Given parameter signs, restrictions on endogenous variables and inequality (B.23), equation (B.20) implies that, share of bonds increases with output if the lower bound for bond position is negative (i.e., \( \Omega < 0 \)). Since \( B^* = \Omega \), this implies that if the economy is indebted, it is more desirable to accumulate foreign assets rather than increasing real balances (consumption).\(^5\) Equation (B.21) on the other hand, reveals that share of bonds increases with total asset position. This is in line with a decrease in the consumption-to-total asset position ratio due to reduced marginal utility of consumption with larger \( M^*+\Omega \). Finally, equation (B.22) shows that, if the economy is saving, portfolio becomes biased towards bonds with higher

\(^5\)Recall that consumption and real balances are proportional.
inflation, that is, consumers do not refrain from reducing real balances demand substantially.\footnote{Note that }6

\textbf{B.2 Economies with Heterogeneous Households}

\textbf{B.2.1 Endogenous Uniform Transfers}

The steady state equilibrium is characterized by a time-invariant profile for endogenous real variables, $\lambda_{it} = \lambda_i^*$, $\varphi_{it} = \varphi_i^*$, $c_{it} = c_i^*$, $M_t^* = \sum_i \mu_i m_{it} = \sum_i \mu_i m_i^* = m^*$, $b_{it} = b_i^*$, $\tau_t = \tau^* \forall i, t$ and the system of equations (B.24)-(B.29) evaluated at these constant values:

\begin{align*}
\frac{u'(c_i^*)}{\lambda_i^*} &= \left[1 + \phi (1 + \gamma) \left(\frac{c_i^*}{m_i^*}\right)^{1+\gamma} \right] \tag{B.24} \\
\frac{1 - \phi \gamma \left(\frac{c_i^*}{m_i^*}\right)^{1+\gamma}}{1 + e} &= \frac{\beta}{1 + e} \tag{B.25} \\
\lambda_i^* (1 - \beta R) &= \varphi_i^* \tag{B.26} \\
\frac{c_i^* \left[1 + \phi \left(\frac{c_i^*}{m_i^*}\right)^{\gamma}\right] + b_i^* + m_i^* = \varepsilon_i + R b_i^* + \frac{m_i^*}{1 + e} + \tau^* \tag{B.27} \\
\beta_i^* &\geq \Omega \tag{B.28} \\
G + \tau^* &= \sum_i \mu_i m_i^* \frac{e}{1 + e} \tag{B.29}
\end{align*}

\footnote{Note that }$Y - G - (1 - R)\Omega = C^*(1 + \phi \kappa^*) > 0$ by the resource constraint.
which is a system of \((5 \times I) + 1\) conditions and \((5 \times I) + 1\) unknowns: \((c_i^*, m_i^*, b_i^*, \tau^*, \varphi_i^*, \lambda_i^*)\). It is possible to find a closed-form solution to this system. If I assume that \(\beta R < 1\), then, since \(\lambda_i^* > 0\), equation (B.26) implies that the borrowing constraint is binding (i.e., \(\varphi_i^* > 0\)). Therefore, by equation (B.28), \(b_i^* = \Omega\) that is, consumers role over a constant interest payment of \((1 - R)\Omega\). It is straightforward to show that equation (B.25) implies a consumer type independent-consumption velocity, which can be denoted by:\(^8\)

\[
\kappa = \frac{c_i^*}{m_i^*} = \left[ \frac{1}{\gamma \phi} \left( 1 - \frac{\beta}{1 + e} \right) \right]^{1/\gamma}. \tag{B.30}
\]

The budget constraint, (B.27), \(b_i^* = \Omega\) and equation (B.30) yields,

\[
m_i^* = \frac{\varepsilon_i + \tau^* - (1 - R)\Omega}{\kappa(1 + \phi \kappa^\gamma) + \frac{e}{1 + e}} \tag{B.31}
\]

Aggregating this equation and using the government budget constraint implies,

\[
(G + \tau^*) \frac{1 + e}{e} = m^* = \sum_i \mu_i \left[ \frac{\varepsilon_i + \tau^* - (1 - R)\Omega}{\kappa(1 + \phi \kappa^\gamma) + \frac{e}{1 + e}} \right] = \frac{Y + \tau^* - (1 - R)\Omega}{\kappa(1 + \phi \kappa^\gamma) + \frac{e}{1 + e}} \tag{B.32}
\]

which delivers equilibrium transfers as \(\tau^* = \left(\frac{e}{1 + e}\right)\frac{Y - G - (1 - R)\Omega}{\kappa(1 + \phi \kappa^\gamma)} - G\). Finally, plugging this solution in equation (B.31) yields,

\[
m_i^* = \frac{\varepsilon_i - G - (1 - R)\Omega + \left(\frac{e}{1 + e}\right)\frac{Y - G - (1 - R)\Omega}{\kappa(1 + \phi \kappa^\gamma)}}{\kappa(1 + \phi \kappa^\gamma) + \frac{e}{1 + e}}. \tag{B.33}
\]

\(^7\)Note that if equation (B.27) is aggregated with \(\sum_i \mu_i c_i^* = C^*\), \(\sum_i \mu_i b_i^* = B^*\), \(\sum_i \mu_i \varepsilon_i = Y^*\) and substituted in equation (B.29), the resource constraint, \(C + TR + G + (1 - R) = Y\), where \(TR = \sum_i \mu_i c_i \phi(\frac{e}{m})\gamma\) obtains.

\(^8\)Consumption velocity is increasing in the inflation rate by \(\frac{\partial \kappa}{\partial \phi} = \frac{\beta}{(1 + \gamma) \phi(1 + \phi \gamma(1 + e)^2) > 0\), given that \(\beta, \gamma, \kappa, \phi > 0\).
as the closed-form solution for real balances. Now by equation (B.30), $c_i^* = m_i^* \kappa$.  

**Portfolio Composition**

What does heterogeneity bring to the representative agent economy in terms of portfolio composition? Given the closed form solutions, I can solve for the share of bonds in the portfolio of type $i$ consumers as,

\[
\frac{b_i^*}{b_i^* + m_i^*} = \frac{\Omega}{\Omega + \frac{\varepsilon_i - G - (1 - R)\Omega + \frac{(c/1+\phi)(Y_G - G - (1 - R)\Omega)}{\kappa(1+\phi_\kappa\gamma)}}{\kappa(1+\phi_\kappa\gamma)}} \tag{B.34}
\]

How the portfolio composition is affected by earnings, lower bound for bonds position and inflation are illustrated in equations (B.35) to (B.37) below.

\[
d\left(\frac{b_i^*}{b_i^* + m_i^*}\right) = -\frac{(b_i^*/b_i^* + m_i^*)^2}{\Omega} \cdot \frac{1}{\kappa(1+\phi_\kappa\gamma) + \frac{\varepsilon_i}{1+\varepsilon}} \tag{B.35}
\]

\[
d\left(\frac{b_i^*}{b_i^* + m_i^*}\right) = \left(\frac{b_i^*/b_i^* + m_i^*}{\Omega}\right)^2 \cdot \frac{\varepsilon_i - G + \frac{(c/1+\phi)(Y_G - G)}{\kappa(1+\phi_\kappa\gamma)}}{\kappa(1+\phi_\kappa\gamma) + \frac{\varepsilon_i}{1+\varepsilon}} \tag{B.36}
\]

\[
d\left(\frac{b_i^*}{b_i^* + m_i^*}\right) = \left(\frac{b_i^*/b_i^* + m_i^*}{\Omega}\right)^2 \cdot \frac{(\varepsilon_i - G - (1 - R)\Omega)}{F^2} \cdot \frac{dF}{de} + \frac{(Y_G - (1 - R)\Omega)\left[\frac{1}{(1+\varepsilon)^2} + \frac{\varepsilon_i}{1+\varepsilon}(\frac{dT}{de} + \frac{dT}{de} + \frac{dF}{de} + \frac{dF}{de})\right]}{TF} \tag{B.37}
\]

where $T = \kappa(1 + \phi_\kappa\gamma)$ and $F = \kappa(1 + \phi_\kappa\gamma) + \frac{\varepsilon_i}{1+\varepsilon}$. Equation (B.35) shows that if consumers are not indebted, then portfolio share of bonds declines with earnings.

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9If $\beta R = 1$, equation (B.26) implies that $\phi_i^* = 0$ and therefore, equilibrium bond position is determined by the initial conditions, $b_i^* = b_0$ which is given. Note that $b_0 \geq \Omega$ should hold in this case. Therefore, all closed form solutions hold except for the difference that $\Omega$ is replaced by $b_0$. 

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This brings the idea that the poor have a stronger tendency to build up bonds position to make up for their low earnings. The role of heterogeneity is even starker when I consider the effect of a change in the lower bound of bond position on the portfolio. While share of bonds is always increasing with higher $\Omega$ in the representative agent economy, it depends on the sign of the term $\varepsilon_i - G + \frac{\varepsilon'(\psi)}{\psi} \left( \frac{\psi}{1+\phi\psi} \right) \left( \frac{\psi}{1+\phi\psi} \right) \left( Y - G \right) \kappa (1+\phi \kappa \gamma)$ in the heterogeneous agents economy.\footnote{See equation B.36.} As individuals get poorer, this term would tend to be negative and bonds share in the portfolio would decrease with higher $\Omega$. Consequently, tighter borrowing constraints imply lower bonds share for the earnings-poor since they choose to allocate more of resources to real balances to increase consumption, while the earnings-rich choose to increase savings. Finally, as equation (B.37) points out, the response of the portfolio to inflation again depends on the degree of heterogeneity in earnings across individuals. The term in the parenthesis can only be negative for the earnings-poor. Therefore, when consumers are not indebted, share of bonds increases with inflation with the only exception of the earnings-poor. The intuition is that the poor find it more difficult to reduce real balances demand even when inflation is high, since marginal utility of consumption (which is supported by real balances) to them is too high.

B.2.2 Endogenous Government Expenditures

In this economy, the stationary equilibrium conditions (B.24)-(B.29) would again follow with the only difference that the endogenous unknowns are now $(c^*_i, m^*_i, b^*_i, G^*, \varphi^*_i, \lambda^*_i)$ whereas $\tau$ is just a parameter. Straightforward calculations deliver that
equation (B.31) is replaced by

\[ m^*_i = \frac{\varepsilon_i + \tau - (1 - R)\Omega}{\kappa(1 + \phi\kappa^\gamma) + \frac{e}{1+e}} \]  \hspace{1cm} (B.38)

which is now a closed-form solution since \( \tau \) is a known parameter. Equilibrium conditions and aggregation implies that \( G^* = \frac{(\varepsilon_i + \tau - (1 - R)\Omega)}{\kappa(1 + \phi\kappa^\gamma) + \frac{e}{1+e}} - \tau \). The rest of the system can be solved in a straightforward way.

**Portfolio Composition**

Given closed-form solutions, the portfolio composition can be written as:

\[
\frac{b^*_i}{b^*_i + m^*_i} = \frac{\Omega}{\Omega + \frac{\varepsilon_i + \tau - (1 - R)\Omega}{\kappa(1 + \phi\kappa^\gamma) + \frac{e}{1+e}}} \]  \hspace{1cm} (B.39)

How the portfolio composition is affected by earnings, lower bound for bonds position and inflation are illustrated in equations (B.40) to (B.42) below.

\[
\frac{d \left( \frac{b^*_i}{b^*_i + m^*_i} \right)}{d\varepsilon_i} = -\frac{(b^*_i/b^*_i + m^*_i)^2}{\Omega} \cdot \frac{1}{\kappa(1 + \phi\kappa^\gamma) + \frac{e}{1+e}} \]  \hspace{1cm} (B.40)

\[
\frac{d \left( \frac{b^*_i}{b^*_i + m^*_i} \right)}{d\Omega} = \left( \frac{b^*_i/b^*_i + m^*_i}{\Omega} \right)^2 \cdot \frac{\varepsilon_i + \tau}{\kappa(1 + \phi\kappa^\gamma) + \frac{e}{1+e}} \]  \hspace{1cm} (B.41)

\[
\frac{d \left( \frac{b^*_i}{b^*_i + m^*_i} \right)}{de} = \frac{\left( b^*_i/b^*_i + m^*_i \right)^2}{\Omega} \cdot \left( \frac{\varepsilon_i + \tau - (1 - R)\Omega}{F^2} \right) \frac{dF}{de} \]  \hspace{1cm} (B.42)

where \( F = \kappa(1 + \phi\kappa^\gamma) + \frac{e}{1+e} \) as in above and \( \frac{dF}{de} = [1 + (1 + \gamma)\phi\kappa^\gamma] \frac{de}{de} + \frac{1}{(1+e)^2} > 0 \).

While the response of the portfolio to changes in earnings is identical to that of uniform transfers economy, the response of bonds share to the lower bound of bonds
depends on the sign of the term $\varepsilon_i + \tau$. In particular, if consumers are rich enough and transfers are positive, then portfolio share of bonds unambiguously increases when borrowing constraints are tighter. However, $\varepsilon_i + \tau$ could be negative when transfers are negative and individuals are poor. Those individuals increase the share of real balances in the portfolio when borrowing constraints are tighter. Finally, assuming that $\Omega > 0$, the portfolio response to inflation is positive if $\varepsilon_i + \tau - (1 - R)\Omega > 0$, that is when the consumers are rich enough. This observation clearly follows from the dominance of a substitution effect, since inflation increases the relative cost of saving in real balances vis-à-vis bonds.

### B.2.3 Endogenous Proportional Transfers

In this economy, transfers received by type $i$ consumers equal $\tau_i(a_i) = \frac{me}{1+e} - G$ so that any change in the inflation tax paid by consumer $i$ is reflected to transfers. Stationary equilibrium conditions (B.24)-(B.29) of the benchmark economy follow with the modification that the budget constraint, (B.27), includes type-specific transfers $\tau_i$ and instead of the government budget constraint, (B.29), I write $\tau_i = \frac{me}{1+e} - G$ as $I$ additional equilibrium conditions.\(^{11}\) Therefore, the equation system is composed of $6 \times I$ unknowns, $(c^*_i, m^*_i, b^*_i, \tau^*_i, \varphi^*_i, \lambda^*_i)$ and $6 \times I$ equations. These conditions would yield,\(^{12}\)

\[
m^*_i = \frac{\varepsilon_i - G - (1 - R)\Omega}{\kappa(1 + \phi\kappa^\gamma)}.
\]  

\(^{11}\)The aggregation of these transfers imply the government budget constraint.

\(^{12}\varepsilon_i - G - (1 - R)\Omega \geq 0 \forall i \) should hold for the real balances and consumption to be non-negative.
Portfolio Composition

Given closed-form solutions, the portfolio composition can be written as:

\[
\frac{b_i^*}{b_i^* + m_i^*} = \frac{\Omega}{\Omega + \frac{\varepsilon_i - G - (1-R)\Omega}{\kappa(1+\phi\kappa^\gamma)}}. \tag{B.44}
\]

How the portfolio composition is affected by earnings, lower bound off bonds position and inflation are illustrated in equations (B.45) to (B.47) below.

\[
\frac{d \left( \frac{b_i^*}{b_i^* + m_i^*} \right)}{d\varepsilon_i} = -\left( \frac{b_i^*}{b_i^* + m_i^*} \right)^2 \cdot \frac{1}{\Omega} \cdot \frac{1}{\kappa(1+\phi\kappa^\gamma)} \tag{B.45}
\]

\[
\frac{d \left( \frac{b_i^*}{b_i^* + m_i^*} \right)}{d\Omega} = \left( \frac{b_i^*}{b_i^* + m_i^*} \right)^2 \cdot \frac{\varepsilon_i - G}{\Omega} \cdot \frac{1}{\kappa(1+\phi\kappa^\gamma)} \tag{B.46}
\]

\[
\frac{d \left( \frac{b_i^*}{b_i^* + m_i^*} \right)}{de} = \left( \frac{b_i^*}{b_i^* + m_i^*} \right)^2 \cdot \frac{\varepsilon_i - G - (1-R)\Omega}{\Omega} \cdot \frac{1}{\kappa(1+\phi\kappa^\gamma)} \cdot \frac{dK}{de} \tag{B.47}
\]

Portfolio response to earnings changes is similar to the economies explored above. Moving to the portfolio response to changes in the lower bound of bonds, if \( \Omega > 0 \), then there is the chance that \( \varepsilon_i - G < 0 \) for the poor so that an increase in the lower bound of bonds decreases bonds share. On the other hand, \( \Omega \leq 0 \) causes share of bonds to increase with \( \Omega \). Finally equation (B.47) shows that bonds share unambiguously increases with inflation if \( \Omega > 0 \). This last observation is intuitive since wealth effects created by changes in inflation are partly neutralized when transfers are proportional to individual specific inflation tax payments.
Appendix C

Numerical Solution Algorithm of Stationary Economies

C.1 Economy 1

I solve the household optimization problem formulated in section 1.4.1 by value function iteration on a discretized space for total deposits and idiosyncratic earnings. I use separate grids for real balances and bonds choices. The grids that I use for total deposits, earnings, bonds (in \([\Omega,30]\)) and real balances (in \([0.001,5]\)) have 100, 21, 3200 and 400 nodes respectively. When \(b' > \Omega\), by Proposition 1.1, consumption can be computed for given states and the bond choice. On the other hand when, \(b' = \Omega\), the consumer budget constraint becomes non-linear in consumption and real balances choice needs to be handled separately.\(^1\) For each pair of real balances and bonds choice, I keep track of the law of motion of real deposits and linearly interpolate the next iteration’s value by using this law of motion. Once I find the decision rules, I solve for the stationary distribution of total deposits by employing standard methods, aggregate over heterogeneous agents by the help of the stationary distribution and compute public surplus \(\left(\frac{M_e}{1+e} - G - \tau\right)\) from the government budget constraint.

\(^1\)I exploit this property of the model by solving the non-linear budget constraint only when the borrowing constraint binds. I achieve this by implementing Newton’s univariate method for solving the roots of non-linear equations.
equilibrium is as follows:

- Pin down two lump-sum transfers levels ($\tau^1$ and $\tau^2$) for which the above-mentioned solution of the private sector implies public surplus ($\frac{Me}{1+\varepsilon} - G - \tau^1 > 0$) and public deficit ($\frac{Me}{1+\varepsilon} - G - \tau^2 < 0$) respectively,

- Initialize lump-sum transfers by setting $\tau_0 = \frac{(\tau^1 + \tau^2)}{2}$. If there is public surplus, update lump-sum transfers in order to bring them closer to the public deficit-generating transfers level (i.e. $\tau_1 = \frac{(\tau_0 + \tau^2)}{2}$) and set $\tau^1 = \tau_0$. If there is public deficit, update lump-sum transfers in order to bring them closer to the public surplus-generating transfers level (i.e. $\tau_1 = \frac{(\tau_0 + \tau^1)}{2}$) and set $\tau^2 = \tau_0$).

- Repeat step 2 until the absolute value of the public surplus is smaller than a tolerance level. ²

C.2 Economy 2

The numerical solution algorithm of Economy 2 involves fixing $\tau$ and iterating on $G$ by using an algorithm in the spirit of the above-mentioned steps.

C.3 Economy 3

The solution of Economy 3 involves initiating a state-dependent matrix of lump-sum transfers $\tau^0(a, \varepsilon) = \tau^0_1(a, \varepsilon) + \tau_0$ (instead of a uniform value) and solving the problem of the private sector by respecting this transfers schedule. ³ Once the

²I use $10^{-6}$ as the tolerance value.
³Notice that total transfers still have the lump-sum component $\tau_0$ which tends to capture the taxes that are not modeled.
private sector problem is solved and aggregation is done, the transfers schedule is
updated following the rule,

$$\tau^1_1(a, \varepsilon) = \omega \tau^0_1(a, \varepsilon) + (1 - \omega)\left(\frac{m'(a, \varepsilon)e}{1 + e} - G - \tau_0\right)$$

(C.1)

where $\omega$ is a number between 0 and 1 and $m'(a, \varepsilon)$ is the policy for real balances
of an agent who is in state $(a, \varepsilon)$. Once I find $\tau^1_1(a, \varepsilon) = \tau^1_1(a, \varepsilon) + \tau_0$, I use it as
the new candidate transfers schedule and repeat the above steps until the whole
transfers schedule converges (i.e. $\sup|\tau^0_0(a, \varepsilon) - \tau^1_1(a, \varepsilon)|| < 10^{-4}$) and the implied
public surplus is less than a tolerance level.

\footnote{I set $\omega = 0.75$. This parameter might change depending on the inflation rate. The second term
on the right hand side of equation (C.1) might change in accordance with what the government
rebates back to households.}
Appendix D

Data

D.1 Structural Change in Inflation

In general, inflation has followed a low-high-low time profile in the periods (1960-1975), (1975-1995) and (1995-2008) around the world. In order not to bias structural change results, I omit data points that correspond to annual inflation rates of more than 200%. In Table D.1, I report a complete list of developing and industrialized countries for which the annual CPI inflation data for the period 1989-2008 are available from the International Financial Statistics, published by the IMF. Observing this general pattern, I regress the time series of inflation for each country on a constant and perform the Chow test that incorporates two structural break dates, one around the mid-1970s and another around the mid-1990s. If a country displays a high-low profile, I use only one structural break point. If for a country, there is not a pattern at all, I just compute averages for the aforementioned periods. For each country, I search over alternative break dates and choose the ones that imply the highest F-statistic in the Chow test. Since I focus on disinflation, I only include the period averages for which inflation has been high and low historically. Countries are listed in descending order according to their average inflation rates in the first period. Among 134 countries listed in Table D.1, 104 pass the structural break test (at 99% significance level). Countries that did not pass the test are
marked by an asterisk.

D.2 Deposits Distributions

Data in Table D.2 are used to plot the parts of Figure 1.1 that are related to the Turkish economy. Columns denote account groups that are classified by the sizes of accounts. Rows on the other hand (from 1 to 4), report shares of account balances and shares of number of accounts for each account group. The last row of the table reports share of term deposits within each account group.

The data sources for deposits are: Autoridad de Supervision del Sistema Financiero (Bolivia), Bulgarian National Bank, Superintendencia de Bancos e Instituciones Financieras (Chile), National Bank of Georgia, Bank of Lithuania, Central Reserve Bank of Peru, Bank of Thailand and Banking Regulation and Supervision Agency (Turkey). It should be noted that the data are on the number of accounts, not depositors. Therefore, if an individual possesses multiple accounts with small balances, then inequality in the distribution of these deposits would be understated. Second, depending on the country specific institutional arrangements, demand deposits might be dollarized or effectively pay interest that is closely related to the inflation rate, missing the vulnerability of cash to inflation. Considering that the existing Gini coefficients are already too high, I believe that the first caveat is not that important. The second issue is difficult to address since the currency composition data are not available in the disaggregated level.
D.3 Income and Consumption Inequality

Tables 10 and 11 include data on income-consumption inequality and the distribution of income earned by various sources among quintiles that are ordered according to the disposable income.
Table D.1: Disinflation as a Worldwide Phenomenon

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<tr>
<th>Country</th>
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<th>Low (Per.)</th>
<th>Country</th>
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<th>Low (Per.)</th>
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Table D.2: Summary Statistics on the Distribution of Deposits in Turkey

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<td>up to 10K&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>Share of DD&lt;sup&gt;b&lt;/sup&gt; Bal.</td>
<td>21.11&lt;sup&gt;c&lt;/sup&gt;</td>
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<td>Share of DD # of Acc.</td>
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<tr>
<td>Share of TD&lt;sup&gt;d&lt;/sup&gt; Bal.</td>
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<td>Share of TD in group</td>
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<sup>a</sup>In Turkish Liras.
<sup>b</sup>Demand deposits.
<sup>c</sup>In percentage terms, the average over the period.
<sup>d</sup>Term deposits.
Table D.3: Income and Consumption Inequality in Turkey, 2004-2008

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<th></th>
<th>1&lt;sup&gt;st&lt;/sup&gt; Quintile</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; Quintile</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt; Quintile</th>
<th>4&lt;sup&gt;th&lt;/sup&gt; Quintile</th>
<th>5&lt;sup&gt;th&lt;/sup&gt; Quintile</th>
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<td>Household Income Avg. of 2004-2007</td>
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<td>10.03</td>
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<td>21.75</td>
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<td>Consumption    Avg. of 2004-2008</td>
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<td>11.95</td>
<td>16.28</td>
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<sup>a</sup>Percentage share of quintiles in total household income and consumption.

Data sources are Household Budget Survey (2004-2005), Income and Living Conditions Survey (2006-2007) and Consumption Expenditures Survey conducted by TURKSTAT.
Table D.4: Inequality and Income Types, 2006-2007

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<th>Avg. of 2006-2007 Types of Income</th>
<th>Aggregate Share of Types</th>
<th>Share of Quintiles Within Type</th>
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<th>2&lt;sup&gt;nd&lt;/sup&gt;</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt;</th>
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<th>5&lt;sup&gt;th&lt;/sup&gt;</th>
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<td>Total</td>
<td>100</td>
<td>4.47&lt;sup&gt;a&lt;/sup&gt;</td>
<td>9.17</td>
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<td>21.26</td>
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<td>Wage and Salary</td>
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<td>43.06</td>
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<sup>a</sup>Percentage share of the relevant income quintile. Quintiles are always ordered according to total income.

Appendix E

Numerical Solution Algorithm of Transitional Dynamics Equilibrium

E.1 Economy 1

The numerical solution algorithm of the transitional dynamics exercise involves the following steps:

1. Solve for the stationary equilibria that correspond to high (14.25%) and low (2.25%) inflation rates by following the algorithm presented in Appendix C. Store $v^{14.25\%}(a, \varepsilon), \Gamma^{14.25\%}(a, \varepsilon), B = \sum_{a, \varepsilon} \Gamma^{14.25\%}(a, \varepsilon)v^{14.25\%}$, $M = \sum_{a, \varepsilon} \Gamma^{14.25\%}(a, \varepsilon)m^{14.25\%}$ as initial conditions and $v^{2.25\%}(a, \varepsilon), \Gamma^{2.25\%}(a, \varepsilon), \tau^{2.25\%}$ as terminal conditions.

2. Feed the calibrated time profile for depreciation rates,

$$e_t = 14.25\% \text{ for } t = 0$$

$$\{e_t\}_{t=1}^{T_1} = \{e_t^{2.25\%}\}_{t=1}^{T_1} \text{ for } t > 0$$

where $\{e_t^{2.25\%}\}_{t=1}^{T_1}$ is the finite sequence of depreciation rates that satisfies,

$$e_0 > e_1^{2.25\%} > e_2^{2.25\%} > e_3^{2.25\%} \ldots > e_T^{2.25\%} \text{ and } \{e_t^{2.25\%}\}_{t=T+1}^{T_1} = 2.25\% \text{ for finite } T \text{ and } T_1.$$

3. Set $\tau^{T_1} = \tau^{2.25\%}$. Guess a sequence of uniform transfers $\{\tau_t\}_{t=0}^{T_1-1}$. Set $v_{T_1}(a, \varepsilon) =$
\[ v^{2.25\%}(a, \varepsilon). \] Solve for the sequence of functions \( \{v_t, b_{t+1}, m_{t+1}, c_t\}_{t=0}^{T_1-1} \) by backward recursion. The solution takes as given the guessed sequence for transfers.

4. Compute the sequence of distributions over total deposits and earnings, \( \{\Gamma_t\}_{t=0}^{T_1-1} \) by using the Markov transition probabilities of the earnings process and the policy functions for assets.

\[
\Gamma_{t+1}(a_{t+1}, \varepsilon_{t+1}) = \sum_{\varepsilon} \sum_{a:a_t+1=R b_t+1} \Gamma_t(a_t, \varepsilon_t) p_{\varepsilon_{t+1}|\varepsilon_t} \tag{E.2}
\]

5. Use the obtained decision rules and distributions to do aggregation. Since the economy is at the 14.25% inflation rate equilibrium at \( t = 0 \), set \( B_0 = B_0^{14.25\%} \) and \( M_0 = M_0^{14.25\%} \). Compute \( \{B_t+1, M_t+1, C_t, T r_t\}_{t=0}^{T_1-1} \).

6. Compute the sequence of public surpluses, \( \{M_{t+1} - \frac{M_t}{1+e_t} - G - \tau_t^0\}_{t=0}^{T_1-1} \) and update the guess for equilibrium sequence of transfers;

\[
\{\tau_t^1\}_{t=0}^{T_1-1} = \left\{ \chi t_t^0 + (1 - \chi) \left( M_{t+1} - \frac{M_t}{1+e_t} - G \right) \right\}_{t=0}^{T_1-1} \tag{E.3}
\]

for \( 0 < \chi < 1 \). Set \( \chi = 0.75 \).

7. If \( \max \left\{ \{\tau_t^0 - \tau^{2.25\%}\}, \sup ||\Gamma_t - \Gamma^{2.25\%}||\right\} < 10^{-4} \), go to the next step.

Otherwise, increase \( T_1 \) and go back to step 2.

8. If \( \max \left\{ \{\tau_t^1 - \tau_t^0\}_{t=0}^{T_1-1}, \{M_{t+1} - \frac{M_t}{1+e_t} - G - \tau_t^0\}_{t=0}^{T_1-1} \right\} < 10^{-4} \), the transition is solved for. Otherwise, set \( \{\tau_t^0\}_{t=0}^{T_1-1} = \{\tau_t^1\}_{t=0}^{T_1-1} \) and go back to step 3.
Economy 2: The solution algorithm of Economy 2 is similar to the one described above. The main difference is that now I search for an equilibrium sequence of government spending, \( \{G^1_t - G^0_t\}_{t=0}^{T_1-1} \), for a given constant stream of transfers, \( \tau \). Naturally, steps 6 and 8 have to be modified to update the candidate spending vector and perform the convergence test.
Bibliography


