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Optimally Allocating MedKits to Defend Urban Areas from Anthrax Attacks

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Abstract

The deliberate release of aerosolized anthrax spores in a large city will expose thousands to this deadly disease. Although state and local health departments have developed contingency plans for promptly opening points of dispensing (PODs) and distributing antibiotics to those exposed after an attack is detected, other risk mitigation strategies have been proposed. This study focuses on the pre-event placement of pharmaceuticals in individual households for use only as directed by public health authorities. The pre-deployed medications are commonly known as “MedKits.” This paper considers the problem of a defender who wishes to minimize the expected fatalities of an anthrax attack by allocating a limited number of MedKits to various urban areas. Under the condition that the attacker wishes to maximize the expected fatalities, the defender’s optimal policy is to keep all of the potential targets equally attractive. The paper presents a methodology for finding this optimal policy. The paper considers a specific example using ten urban areas in the United States and compares the optimal policies with those in which the MedKit allocations are proportional to population. The approach can be adapted to consider a wide range of scenarios and local factors to help public health officials manage the risk of an anthrax attack. Having good solutions to this problem should be valuable to public health officials who are considering how to pre-deploy MedKits.

Keywords: bioterrorism, resource allocation
The deliberate release of aerosolized anthrax spores in a large city will expose many thousands of residents to this deadly disease. Promptly distributing antibiotics to those exposed is a key step in preventing illness and deaths. Avoiding delays in this distribution is critical, but such a response will require enormous resources. State and local health departments have developed contingency plans for points of dispensing (PODs), the primary distribution channel (CDC, 2011a). These health departments realize, however, that they may not have the staff required to operate enough PODs to distribute medication to a large number of people sufficiently quickly. Therefore, other strategies have been proposed and tested (CDC, 2011b). These include employing the U.S. Postal Service to deliver antibiotics directly to residences, pre-deploying pharmaceuticals to hospitals, pre-event dispensing of pharmaceuticals to first responders, and the pre-event placement of pharmaceuticals in individual households for use only as directed by public health authorities. This paper focuses on using this last option along with PODs. The pre-deployed medications are commonly known as “MedKits.”

The use of MedKits has been advocated because they can reduce the time needed to distribute medication (Bicknell, 2003). Concerns about safety and the inappropriate use of MedKits have slowed the development of this option (Troy, 2010). In order to show how pre-deploying MedKits would reduce the risk of an anthrax attack, Houck and Herrmann (2011a, b) presented the MedKits model, which predicts the deaths and hospitalizations from an anthrax attack when MedKits are pre-deployed and PODs are opened after an attack (the model extends one described by Zaric et al., 2008). Their results showed that, as more MedKits are pre-deployed, the expected number of deaths and the mortality rate decrease. The reduction in mortality rate is greater when the number of potential exposures is large. Essentially, distributing MedKits counteracts the problems caused by the large number of potential exposures delaying the prophylaxis of those who were truly exposed. Thus, MedKits help both those who have them and those who don’t.

This paper considers the problem of allocating a store of MedKits to multiple urban areas. Predeploying
MedKits in a city reduces the expected fatalities of an anthrax attack in that city. The defender allocates MedKits before knowing which city the terrorist will attack. The terrorist (attacker) wishes to maximize expected fatalities and will exploit any weaknesses in the defender’s strategy. Thus, the defender, to minimize expected fatalities, must consider the attacker’s decision. The approach presented here finds the optimal allocation. Having good solutions to this problem should be valuable to public health officials who are considering how to pre-deploy MedKits.

**Model and Assumptions**

The model presented here uses the following notation. There is a set of $n$ potential targets (urban areas, or cities). The defender has a total budget of $B$ MedKits available and will allocate these to the cities. Let $L_i(c_i)$ be the expected fatalities in city $i$ when $c_i$ MedKits are predeployed in that city. We assume that $L_i(c_i)$ is a continuous, monotonically decreasing function. Let $P_i$ be the population of city $i$. This is the upper limit on $c_i$, and $L_i(c_i)$ reaches its minimum at this value. Because the $L_i(c_i)$ are monotonically decreasing, they can be inverted: $c_i = L_i^{-1}(y)$.

After observing the defender’s allocation, the attacker wishes to maximize his expected utility, so he will attack the target that has the greatest value of $L_i(c_i)$. Let $h_i(c_1, \ldots, c_n) = 1$ if the terrorist will attack target $i$ (that is, $L_i(c_i)$ is the maximum) and 0 otherwise.

The defender’s objective is to minimize the total expected loss from the terrorist attacks:

$$
\min_{c_1, \ldots, c_n} \sum_{i=1}^{n} h_i(c_1, \ldots, c_n) L_i(c_i)
$$

subject to the budget constraint

$$
\sum_{i=1}^{n} c_i \leq B
$$
Note that \( h_i(c_1, \ldots, c_n) L_i(c_i) = \max_j L_j(c_j) \) for the target \( i \) that will be attacked and is 0 otherwise. Thus,

\[
\sum_{i=1}^n h_i(c_1, \ldots, c_n) L_i(c_i) = \max_{j=1, \ldots, n} L_j(c_j)
\]

In an optimal solution, the defender should invest resources in (distribute MedKits to) the cities in such a way that equalizes the expected fatalities in the cities that receive MedKits (while the expected fatalities in any cities without MedKits is even lower).

The range of expected fatalities can be determined as follows:

\[
L_{\text{max}} = \max_{i=1, \ldots, n} L_i(0)
\]
\[
L_{\text{min}} = \max_{i=1, \ldots, n} L_i(P_i)
\]

It is not possible to reduce the number of expected fatalities beyond \( L_{\text{min}} \), so there is an upper limit on the number of MedKits that should be allocated and, in some cases, there is no benefit to distributing any MedKits to cities that will have a low number of expected fatalities (those with smaller populations and those that are otherwise well-prepared to respond to an anthrax attack).

For each city \( i \), if \( L_i(0) \geq L_{\text{min}} \), let \( c_i^{\text{max}} \) be the value of \( c_i \) such that \( L_i(c_i^{\text{max}}) = L_{\text{min}} \) (such a value must exist because \( L_i(0) \geq L_{\text{min}} \geq L_i(P_i) \)); otherwise, set \( c_i^{\text{max}} = 0 \). Then, the upper limit on the MedKit allocation equals

\[
B_{\text{max}} = \sum_{i=1}^n c_i^{\text{max}}.
\]

When \( B = B_{\text{max}} \), the optimal allocation to city \( i \) is \( c_i^{\text{max}} \).

If \( B \) (the total number of MedKits) is small, then the optimal allocation predeploys MedKits to only the
cities with the most expected fatalities. As $B$ (the total number of MedKits) increases, more cities will receive MedKits. Thus, it is valuable to determine the values of $B$ at which additional cities are added to the set of those that receive MedKits. Let $h$ be the number of cities with $c_i^{\text{max}} > 0$. Without loss of generality, renumber these cities so that

$$L_1(0) \geq L_2(0) \geq \cdots \geq L_h(0).$$

Then, let $c_{ij}^*$ be the value of $c_j$ such that $L_i(c_{ij}^*) = L_j(0)$ for $i < j \leq h$. Let $B_1 = 0$. Then, for $j = 2, \ldots, h$, define the breakpoints

$$B_j = \sum_{i=1}^{j-1} c_{ij}^*.$$

If $B = B_j$, the optimal allocation is $c_i = c_{ij}^*$ for $i < j$ and $c_i = 0$ for $i \geq j$. The number of expected fatalities equals $L_j(0)$. If $B \leq B_j$, no MedKits are allocated to cities $j$ to $h$. If $B > B_h$, all $h$ cities should receive some MedKits.

Given a value of $B$ in the range $[0, B_{\text{max}}]$, the optimal allocation can be found as follows: (1) let $j^*$ be the largest value of $j$ such that $B > B_j$; (2) find the value of $y$ such that $\sum_{i=1}^{j^*} L_i^{-1}(y) = B$ (because this sum is a monotonically decreasing function of $y$, a bisection search or other similar technique can be used) and then set $c_i = L_i^{-1}(y)$ for $i = 1, \ldots, j^*$ and $c_i = 0$ for all other cities.
Table 1. Ten urban areas and their populations.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Los Angeles-Long Beach</td>
<td>9,519,338</td>
</tr>
<tr>
<td>New York</td>
<td>9,314,235</td>
</tr>
<tr>
<td>Chicago</td>
<td>8,272,768</td>
</tr>
<tr>
<td>Philadelphia, PA-NJ</td>
<td>5,100,931</td>
</tr>
<tr>
<td>Washington, DC-MD-VA-WV</td>
<td>4,923,153</td>
</tr>
<tr>
<td>Houston</td>
<td>4,177,646</td>
</tr>
<tr>
<td>Boston, MA-NH</td>
<td>3,406,829</td>
</tr>
<tr>
<td>Seattle-Bellevue-Everett</td>
<td>2,414,616</td>
</tr>
<tr>
<td>Newark</td>
<td>2,032,989</td>
</tr>
<tr>
<td>San Francisco</td>
<td>1,731,183</td>
</tr>
</tbody>
</table>

Results

To illustrate this technique, we will consider the following scenario in which a store of MedKits will be predeployed to the ten urban areas (in the United States) that have the highest expected annual terrorism losses (Willis et al., 2005). Table 1 lists the urban areas and their populations from the 2000 U.S. Census.

We assume that the terrorist has enough anthrax to expose 500,000 individuals in any one of the cities. (We will also consider scenarios in which the number of exposures is 250,000 and 750,000.)

This scenario considered the following timeline. The attack occurs at $t = 0$ hours. The attack is detected at $t = 48$ hours. Local supplies of both intravenous antibiotics (IVs) for treatment and antibiotics for dispensing will become available 5 hours later at $t = 53$ hours. Intravenous antibiotics (IVs) for treatment, antibiotics for dispensing, and additional ventilators from the push pack will become available 16 hours after attack detection at $t = 64$ hours. (This is due to a 12 hour delay in receiving the push pack and another 4 hour delay in getting the material from the push pack ready.) Intravenous antibiotics (IVs) for treatment and antibiotics for dispensing from vendor-managed inventory (VMI) will become available 36 hours after attack detection at $t = 84$ hours. At $t = 96$ hours (48 hours after attack detection), complete POD capacity will be available.
The population consists of three large subpopulations: those who were exposed to the anthrax attack, those who were not exposed to the anthrax attack (or inhaled too few anthrax spores to become ill), and those who believe that they may have been exposed (because of their proximity to the attack or for other reasons). The persons in this last group, called “potential exposures,” will undergo prophylaxis (by going to PODs and taking their MedKits) but cannot become ill. In this scenario, the number of potential exposures equals 25% of the number not exposed.

Prophylaxis dispensing capacity is limited. It depends upon the facilities and staff available. In the scenarios considered in this paper, we assume that there is a fixed maximum prophylaxis dispensing capacity, which depends upon the city’s population $P_i$. In particular, the maximum equals $P_i/1000$ persons per hour. Prophylaxis dispensing is also limited by the availability of medication. Although a complete regimen has 60 days of medication, we assume that only 14-day abbreviated regimens are dispensed until the VMI becomes available. We assume that the local stockpile has a one day dose for every 100 persons, the push pack provides 2,718,000 doses (194,143 abbreviated regimens), and the VMI provides sufficient doses for everyone to receive a complete regimen and enough IV antibiotics for everyone who needs them.

Those who adhere to their prophylaxis will not become ill, but some who begin prophylaxis during the incubation stage will not adhere and may become ill. (We assume that those who are in the prodromal and fulminant stages will always adhere.) In this scenario the adherence rate was 90%.

Persons who become ill need treatment, which consists of three antibiotics administered intravenously in an intensive care unit (ICU). All who begin treatment adhere to it. Treatment capacity is limited by the availability of IV antibiotics, ventilators, respiratory technicians, and ICU beds. We assume that the local stockpile has one day of IV antibiotics for every 10,000 persons, the push pack provides 21,492 days of IV antibiotics, and the VMI provides sufficient IV antibiotics for everyone who is being treated. We assume that 100 ventilators are available when the attack occurs, and the push pack provides 100 more.
We assume that each respiratory technician can monitor 10 patients. There is one ICU bed available for every 2,500 persons and one respiratory technician available for every 25,000 persons.

The expected fatalities in city $i$ from an anthrax attack (when MedKits are predeployed and PODs are used after an attack) can be estimated using the MedKits model (Houck and Herrmann, 2011a, b). In this scenario, each urban area is considered as one population, the number of pre-deployed MedKits was set to 0, 1%, 2%, …, 100% of the urban area’s population, and the MedKits model is used to estimate $L_i(c_i)$, the expected number of fatalities in that urban area for the given scenario. For values of $c_i$ other than those 101 values evaluated, we use a linear interpolation to approximate $L_i(c_i)$.

For this scenario (which we call Scenario 1), the expected number of fatalities in each city can vary within the ranges shown in Table 2, which also shows the $c_i^{\text{max}}$ for each city. Note that $B_{\text{max}} = 50,510,771$. San Francisco, with the greatest number of expected fatalities, will be the first city to receive MedKits, and Los Angeles will be the last.

Regardless of the number of MedKits distributed, some number of deaths is unavoidable. The unavoidable deaths result from the delays in detecting the attack and starting prophylaxis (during which time some exposed persons become very ill), the loss of MedKits among those who received them, and the imperfect adherence rate.

For any given value of $B$ between 0 and 50,510,771, we can determine the optimal allocation to minimize the expected number of fatalities using the procedure discussed earlier in this paper. We also evaluated a simple allocation rule in which the number of MedKits allocated to each city is proportional to that city’s population. Of course, when $B$ is very large (approaching the total population of all ten urban areas), the optimal and proportional allocations are nearly the same and yield the same expected number of fatalities. When $B$ is low, however, the allocations are very different, and the proportional allocation yields a higher
expected number of fatalities, as shown in Figure 1. Notice, in particular, how the expected number of fatalities drops quickly as $B$ increases when the optimal allocation is used.

We also considered the uncertainty in the attack scenario. This study investigated the uncertainty in the number of exposed, which could vary if the terrorist has more (or less) anthrax or if the conditions during the attack increase (or decrease) the number exposed. We let the number of exposed equal 750,000 (Scenario 2) and 250,000 (Scenario 3) and find, for various values of $B$, the optimal allocations for these new scenarios.

For any value of $B$, the allocation of MedKits that is optimal for Scenario 1 (500,000 exposed) is not optimal in Scenarios 2 and 3. We evaluate Scenario 1’s optimal allocation and the proportional allocation in these new scenarios and compare the expected number of fatalities to those that result from the optimal allocations for these scenarios, also shown in Figure 1.

We observe again that the proportional allocation yields an expected number of fatalities that is greater than the minimal expected number of fatalities. The difference between the expected number of fatalities with original optimal allocation and the scenario-specific optimal allocation is not as great. For instance, when $B = 20,000,000$ and the number exposed equals 750,000, the optimal policy allocates more MedKits to San Francisco than the proportional policy does (see Table 3). The expected number of fatalities is 106,809 if the optimal allocation is selected, 116,150 (which is 9% greater) if Scenario 1’s optimal allocation is selected, and 169,296 (58% greater) if the proportional allocation is selected. Thus, it appears that the original optimal allocation is robust with respect to the uncertainty in the number exposed.
Table 2. Range of expected fatalities for each urban area when 500,000 persons are exposed.

<table>
<thead>
<tr>
<th>Urban Area</th>
<th>( L_i (P_i) )</th>
<th>( L_i (0) )</th>
<th>( c_i^{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Los Angeles-Long Beach</td>
<td>41,396</td>
<td>130,375</td>
<td>9,427,066</td>
</tr>
<tr>
<td>New York</td>
<td>41,396</td>
<td>130,823</td>
<td>9,224,588</td>
</tr>
<tr>
<td>Chicago</td>
<td>41,397</td>
<td>133,423</td>
<td>8,196,345</td>
</tr>
<tr>
<td>Philadelphia, PA-NJ</td>
<td>41,400</td>
<td>147,310</td>
<td>5,063,803</td>
</tr>
<tr>
<td>Washington, DC-MD-VA-WV</td>
<td>41,401</td>
<td>148,567</td>
<td>4,888,162</td>
</tr>
<tr>
<td>Houston</td>
<td>41,402</td>
<td>154,859</td>
<td>4,151,522</td>
</tr>
<tr>
<td>Boston, MA-NH</td>
<td>41,405</td>
<td>163,825</td>
<td>3,865,652</td>
</tr>
<tr>
<td>Seattle-Bellevue-Everett</td>
<td>41,410</td>
<td>181,999</td>
<td>2,408,157</td>
</tr>
<tr>
<td>Newark</td>
<td>41,414</td>
<td>192,509</td>
<td>2,030,292</td>
</tr>
<tr>
<td>San Francisco</td>
<td>41,418</td>
<td>203,125</td>
<td>1,731,183</td>
</tr>
<tr>
<td>( B_{\text{max}} )</td>
<td></td>
<td></td>
<td>50,510,771</td>
</tr>
</tbody>
</table>

Table 3. Allocations of MedKits to each urban area under three different policies when \( B = 20,000,000 \).

<table>
<thead>
<tr>
<th>Urban Area</th>
<th>Optimal for 750,000 exposed</th>
<th>Optimal for 500,000 exposed</th>
<th>Proportional allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Los Angeles-Long Beach</td>
<td>3,177,574</td>
<td>3,307,654</td>
<td>3,740,872</td>
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<tr>
<td>New York</td>
<td>3,128,173</td>
<td>3,249,777</td>
<td>3,660,271</td>
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<tr>
<td>Chicago</td>
<td>2,874,805</td>
<td>2,955,753</td>
<td>3,251,000</td>
</tr>
<tr>
<td>Philadelphia, PA-NJ</td>
<td>2,062,797</td>
<td>2,038,265</td>
<td>2,004,554</td>
</tr>
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<td>Washington, DC-MD-VA-WV</td>
<td>2,014,199</td>
<td>1,985,396</td>
<td>1,934,681</td>
</tr>
<tr>
<td>Houston</td>
<td>1,805,264</td>
<td>1,759,649</td>
<td>1,641,715</td>
</tr>
<tr>
<td>Boston, MA-NH</td>
<td>1,576,456</td>
<td>1,518,868</td>
<td>1,338,802</td>
</tr>
<tr>
<td>Seattle-Bellevue-Everett</td>
<td>1,251,882</td>
<td>1,189,857</td>
<td>948,886</td>
</tr>
<tr>
<td>Newark</td>
<td>1,113,225</td>
<td>1,053,957</td>
<td>798,916</td>
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<tr>
<td>San Francisco</td>
<td>995,625</td>
<td>940,824</td>
<td>680,313</td>
</tr>
</tbody>
</table>
Figure 1. Expected fatalities over the range of $B$, optimal and proportional allocations, and three scenarios: (1) 500,000 exposed, (2) 750,000 exposed, and (3) 250,000 exposed.

**Discussion**

Clearly, if the defender has enough MedKits for everyone in every city, the allocation decision is trivial. When the number of MedKits is low, however, the allocation decision has a significant impact on the expected number of fatalities. Optimally allocating the MedKits is much better than a proportional allocation. Moreover, the optimal allocation for one scenario can be a very good allocation even if the scenario changes, which indicates that it is a robust solution.

These results also show that hedging (allocating resources to targets that are initially less attractive to the attacker) is optimal when there are sufficient resources. Previous work has shown that, in the optimal resource allocation, the most valuable target receives most of the resources when cost effectiveness (the rate at which investments reduce the probability of a successful attack) is low (Bier, 2008). As cost effectiveness increases, hedging becomes optimal, and more targets receive some resources for defense.
In the context of MedKits allocation, cost-effectiveness is not directly relevant, but the total number of MedKits available for allocation does affect how many cities receive MedKits.

This study does not address the question of how many MedKits should be obtained, but the results seem to indicate that the marginal benefit of additional MedKits is large if they are allocated optimally. As the number of MedKits available increases, the marginal benefit of additional MedKits decreases (cf. Houck and Herrmann, 2011a, b).

This study considered only the allocation of MedKits that will be predeployed in the general population. The predeployment of MedKits to first responders and other personnel who are essential to continuity of operations will reduce the number of MedKits available to the general population. In general, the allocation of scarce resources to different urban areas and to groups within an urban area must be considered within a framework of ethical guidelines that emphasize the relevant moral principles.  

**Summary and Conclusions**

This paper discussed the problem of allocating a store of MedKits to multiple cities. A game theory-based approach is adopted, and the attacker’s objective is used to define the objective function that the defender needs to optimize. In particular, the objective is to minimize the maximum expected fatalities.

When the total number of MedKits is low, the optimal solution allocates MedKits to a small number of cities that have the highest expected number of fatalities. When more MedKits are available, all of the cities receive some, but the optimal allocation is not proportional to the cities’ populations. Based on this analysis, finding the optimal solution is not difficult. An illustrative example was used to demonstrate the essential characteristics of the problem.

Solving this problem requires having a useful model that can estimate the expected number of fatalities in a city in a given scenario when that city has been allocated a number of MedKits. The example presented
in this paper used the MedKits model, which uses various approximations, but other models could be used.

This study does not address the question of how many MedKits should be obtained. Answering this question would require weighing the cost of procuring and predeploying MedKits against the resulting risk reduction.

ACKNOWLEDGEMENTS

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REFERENCES


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