The Strange Quark Contribution to the Proton’s Magnetic Moment


1Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801
2Department of Physics, College of William and Mary, Williamsburg, VA 23187
3Bates Linear Accelerator Center, Laboratory for Nuclear Science, Massachusetts Institute of Technology, Middleton, MA 01949
4Department of Physics, University of Maryland, College Park, MD 20742
5W.K.Kellogg Radiation Laboratory, California Institute of Technology Pasadena, CA 91125
6Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506
7Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, MA 02139
8Physics Division, Argonne National Laboratory, Argonne, IL 60439
9Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061
10Department of Physics, University of Connecticut, Storrs, CT 06269
11Department of Physics, Louisiana Tech University, Ruston, LA 71272
(Dated: February 4, 2008)

We report a new determination of the strange quark contribution to the proton’s magnetic form factor at a four-momentum transfer $Q^2 = 0.1(\text{GeV}/c)^2$ from parity-violating $e-p$ elastic scattering. The results use a revised analysis of data from the SAMPLE experiment which was carried out at the MIT-Bates Laboratory. The data are combined with a calculation of the proton’s axial form factor from theoretical expectation as computed in [11]. These data have been further analyzed, and a new experiment was carried out at lower momentum transfer, and both measurements are now in good agreement with the calculation. The deuterium data are discussed in a separate report [12]. Here we re-evaluate the implications of the hydrogen data in light of this new conclusion, using the calculation of [11] for the axial form factor. We also assess its implication for the contribution of strange quarks to the proton’s magnetic moment using a theoretical extrapolation of $G_M^s$ to its static limit. In parity-violating elastic $e-p$ scattering, the asymmetry in the scattering cross section with respect to the incident electron’s helicity is

$$A_{PV} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = -\frac{G_E Q^2}{4\pi \alpha Z^2} \times \left[ G_M^2 \frac{1}{(G_E^2)^{1+\tau}(G_M^2)^{1-\tau}} \right] 	imes 
\left[ \varepsilon G_E^Z G_M^Z + \tau G_M^Z G_M^Z - \varepsilon' (1 - 4\sin^2 \theta_W) G_M^z G_M^z \right]$$

where $\varepsilon = \left[ 1 + 2(1 + \tau) \tan^2 \theta_W \right]^{-1}$, $\tau = Q^2/4M^2$, and $\varepsilon' = \sqrt{\tau(1 + \tau)(1 - \varepsilon^2)}$. At the backward-angle kinematics of the SAMPLE experiment, the $e-p$ asymmetry is dominated by the contribution from the magnetic neutral weak form factor, $G_M^z$. When combined with measurements of the proton and neutron electromagnetic form factors, would allow the identification of possible strange quark components of these neutral weak matrix elements as seen by an electron from the parity-violating elastic scattering. Experiments have since been carried out [4, 5, 6, 7, 8] and proposed [9, 10] at several electron scattering facilities, and many theoretical calculations of the strange quark components of these neutral weak matrix elements have appeared in the literature.

In this paper we present a new determination of the strange quark contribution to the proton’s magnetic form factor, $G_M^s$, from the parity-violating $e-p$ elastic scattering data of the SAMPLE experiment, which was carried out at the MIT-Bates Laboratory in 1998. Preliminary results were presented in [1]: in the intervening period two measurements with a deuterium target were performed along with a more detailed analysis of the hydrogen data. The deuterium data are relatively insensitive to the strange vector matrix elements, but provide the first experimental information about the nucleon’s neutral weak axial form factor as seen by an electron probe, $G_A^z$, which is necessary for a reliable extraction of $G_M^s$. Preliminary results using the first of the two deuterium measurements [2] indicated a relatively small contribution from strange quarks to the proton’s magnetic moment but an unexpectedly large deviation of the axial form factor from theoretical expectation as computed in [11]. These data have been further analyzed, and a new experiment was carried out at lower momentum transfer, and both measurements are now in good agreement with the calculation. The deuterium data are discussed in a separate report [12].
factors, the strange quark component can be extracted through the relation

$$G_M^s = (1 - 4 \sin^2 \theta_W) (1 + R_P^s) G_M^p - (1 + R_P^s) G_M^n - G_M^Z.$$  

(2)

The radiative corrections $R_P^{s,n}$ represent (small) contributions from higher order processes. While the SAMPLE measurement is dominated by this term, it is also sensitive to the proton’s neutral weak axial form factor $G_A^p$. At tree level, isospin symmetry relates $G_A^p$ to the axial vector coupling that enters neutron decay. Small corrections of order $-10\%$ are generated by strange quarks $[14]$. Electroweak radiative corrections introduce a more substantial, $O(50\%)$ effect. While electroweak corrections that renormalize the individual quark axial vector currents can be reliably computed, other effects that involve strong and weak $qq'$ correlations, such as $Z-\gamma$ box graphs and the nucleon “anapole moment” $[15]$, present a theoretical challenge. Predictions for these corrections, including estimates of the theoretical uncertainties obtained with chiral perturbation theory ($\chi$PT), were obtained in $[16]$ and updated in $[17]$. The results of the SAMPLE deuteron measurements are consistent with these predictions, and in what follows, we use the value for $G_A^p$ obtained in $[17]$ to interpret the results of the SAMPLE $e-p$ measurement.

The SAMPLE $e-p$ measurement was carried out at the MIT-Bates Linear Accelerator Center in 1998. A beam of 200 MeV circularly polarized electrons was incident on a 40 cm liquid hydrogen target, and Čerenkov light from backward-scattered electrons were detected in an array of 16 photomultiplier tubes after reflection from ellipsoidal mirrors. The yield in each photomultiplier tube was integrated over the 25 $\mu$sec long beam pulse and sorted by beam helicity state. The measured asymmetry was computed as the difference between the yields in the two helicity states over the sum, with corrections coming from helicity correlations in the beam, and from dilution factors associated with the beam polarization ($36.2\pm0.1\%$), electromagnetic radiative corrections, and electromagnetic background ($\sim30\%$ of the yield). Preliminary results were published in $[18]$, in which additional details of the experimental method can be found.

Subsequent refinement of the data analysis, along with development of a GEANT-based Monte Carlo simulation $[19]$ of the full experimental geometry, has revealed three corrections, all of which act to increase the magnitude of the experimental asymmetry. First, the electromagnetic radiative corrections were recomputed within the context of the simulation, whereas in $[18]$ they were computed at the central kinematics of each detector, resulting in a 4% increase in the dilution factor. In both cases a spin-dependent $[18]$ modification to the formalism of Mo and Tsai $[19]$ was used to compute the radiative effects. In the simulation, scattered electron events were generated uniformly in energy, angle and along the length of the 40 cm target. Energy loss due to ionization and collisions in the aluminum entrance window to the target, and in the thickness of liquid hydrogen upstream of the randomly chosen interaction point, was accounted for before computation of the scattered electron kinematics. Each scattered electron was assigned a cross section and a parity-violating asymmetry, and propagated through the target exit windows and the scattering chamber. A detection efficiency based on the velocity of the outgoing electron and the path length of the event’s track in the Čerenkov medium was combined with the computed cross section as an event weight. The radiative correction factor of approximately 1.13 was evaluated separately for each detector module, and was computed as the ratio of the (weighted) asymmetry without and with the radiative effects included.

Secondly, a background associated with threshold photo-pion production, which had been neglected in $[18]$, was evaluated using the GEANT simulation. Such processes contribute to the detector yield through their decay products, but have a negligible parity-violating asymmetry $[20]$. The $\pi^0$ ($\pi^+\pi^-$) channel was modeled based on data from $[21]$ ($[22]$). The $\pi^+$ production yield was found to be consistent with experimental observation of an exponential tail in the detector signal corresponding to the arrival of decay products of secondary muons. The net additional dilution factor coming from the pion background was 1.04.

The third modification to the previous analysis was in the treatment of the background coming from charged particles that were not blocked by shutters placed in front of the photomultiplier tubes. As discussed in $[18]$, the net measured “shutter closed” background asymmetry was consistent with zero, but the detector-by-detector distribution appeared to have a nonstatistical component, and a systematic error accounting for the nonstatistical behavior was added to the experimental uncertainty for each individual detector before combining them. Subsequent analysis revealed that this shutter closed distribution had a $\phi$-dependence which fit the function $f(\phi) = A_0 + A_1 \cos(2\phi + \phi_0)$ with a significantly better $\chi^2$/d.o.f (8.0/7) than the presumed flat distribution (33.3/9). The OPEN shutter data did not show such behavior with statistical significance. The $A_0$ coefficient, $-0.06\pm0.71$ ppm after all dilution corrections, was then subtracted from the OPEN asymmetry and its uncertainty added in quadrature. This method produced a 5% larger result than the method used in $[18]$. The three effects combined result in an experimental $e-p$ parity-violating asymmetry of $[24]$

$$A(Q^2 = 0.1) = -5.61 \pm 0.67 \pm 0.88 \text{ ppm}.$$  

(3)

The Monte Carlo simulation was also used to determine the appropriate theoretical asymmetry to which the data should be compared. Averaging over detector and target length acceptance effects results in an approximately 3%
smaller theoretical asymmetry,
\[ A(Q^2 = 0.1) = -5.56 + 3.37G_M^s + 1.54G_A^{(T=1)} \text{ ppm}. \]  
(4)

The small isoscalar component of \( G_A^s \) has been absorbed into the first term. In the model, dipole form factors were used for \( G_{E,M}^p \) and \( G_M^A \), and the Galster parameterization \cite{24} for \( G_E^A \).

The two SAMPLE deuterium measurements were also analyzed using the GEANT simulation, as described in \cite{12}. Both measurements are in agreement with the theoretical prediction for the asymmetry using the electroweak radiative corrections of \cite{11} in the computation of \( G_A^{(T=1)}(Q^2) \). The results for the two 200 MeV data sets, along with the computation of \cite{11}, are shown in Figure 1 as 1-\( \sigma \) bands in the space of \( G_M^s \) vs. \( G_A^{(T=1)} \). Both the overlap of the hydrogen data and the calculation, and the overlap of the two data sets, are shown as ellipses (1-\( \sigma \)), demonstrating the good agreement between the deuterium data and the theoretical expectation of \( G_A^{(T=1)} = -0.83 \pm 0.26 \). Using this value results in

\[ G_M^s(Q^2 = 0.1) = 0.37 \pm 0.20 \pm 0.26 \pm 0.07 \]  
(5)

where the uncertainties are statistical, experimental systematic and the uncertainty due to electroweak radiative corrections, respectively. Inclusion of the SAMPLE deuterium data in the extraction of \( G_M^s \) produces essentially the same result. Our new result is slightly shifted in the positive direction relative to, but consistent with, the analysis in Ref. \cite{4}.

The SAMPLE results for \( G_M^s \), together with the considerable body of theoretical work on this topic undertaken over the past decade, has substantially sharpened our picture of strange quark contributions to the nucleon’s electromagnetic structure. Here, we summarize the insights provided by the various theoretical studies of strange magnetism (for a more extensive review, see \cite{25}).

The most direct, first principles approach in QCD relies on lattice simulations. Such calculations are particularly challenging for operators such as \( s\gamma_\mu s \) that do not connect with nucleon sources, since one must sum disconnected quark loops over all lattice sites. Although a complete calculation is prohibitively time consuming and expensive, numerically tractable estimates can be obtained by approximating the sum with so-called \( Z_2 \) noise methods \cite{26}. A few calculations have been carried out, but with little consensus on the value of \( G_M^s(Q^2) \). The calculation in \cite{27} that relied on a relatively small sample of gauge field configurations and large number of \( Z_2 \) noise vectors yielded a statistically significant, non-zero, negative value for \( G_M^s \) over a range of momentum transfer, with \( \mu_s = G_M^s(0) = -0.36 \pm 0.20 \). In contrast, the computation reported in \cite{28}, which drew upon a much larger gauge configuration sample and smaller sample of noise vectors, found \( \mu_s = 0.05 \pm 0.06 \). A lattice-based computation by the Adelaide group is similarly consistent with zero \cite{29}, and our experimental result tends to favor these more recent calculations. One anticipates that further refinements in lattice methods, such as the use of chiral fermions and partially-quenched or unquenched computations, will eventually lead to a consensus for the precise value of \( G_M^s \).

Alternative treatments have been pursued using dispersion relations and chiral perturbation theory (\( \chi \)PT). In principle, these methods are model-independent since they rely on general properties of QCD, such as analyticity and causality (dispersion relations) or the approximate chiral symmetry of QCD (\( \chi \)PT) to organize in a hadronic basis various contributions to \( G_M^s \). In either case, one must rely upon experimental input to determine these contributions. In practice, dispersion relations and \( \chi \)PT have yet to yield entirely model-independent predictions, though considerable insight has been derived from their application.

The benchmark dispersion relation analysis was performed by Jaffe \cite{30}, who employed the narrow resonance approximation for the hadronic spectrum that is consis-
tent with the large $N_c$ limit of QCD and an ansatz for the high-$Q^2$ behavior of $G_M^s$ consistent with known quark counting rules. Use of such an ansatz, which introduces a degree of model-dependence [31], was needed to account in a physically realistic way for in calculable contributions from the higher-mass part of the QCD spectrum. The results, which drew upon the measured isoscalar electromagnetic form factors and the flavor content of the lowest lying $1^{-+}$ mesons, suggested that $G_M^s$ would be both sizable and negative. An updated version of this analysis was carried out by the authors of [32], who included logarithmic corrections to the power-law asymptotic behavior used in [30] but found no substantial modification of the original Jaffe prediction. The results in Eq. (4), however, tend to disfavor such a substantially negative value, thereby suggesting a re-examination of the assumptions used in [30].

Subsequent dispersion theoretic work in [33, 34, 35] avoided a priori reliance on the narrow resonance approximation and high-$Q^2$ ansatz and instead constructed the isoscalar and strange magnetic spectral functions from measured $K$-$N$ scattering amplitudes and $e^+e^-$ partial widths. This analysis, which entailed a truncation of the spectrum at $\sim 1$ GeV due to the lack of experimental input, produced results consistent with the Jaffe predictions for the low-mass spectral content of $G_M^s$ and demonstrate that the nucleon’s “kaon cloud” is dominated by the $\phi(1020)$ resonance. Thus, the narrow resonance approximation for the low-mass spectral function, as assumed in [30], appears justified. By itself, however, the low-mass spectral content would imply $\mu_s \sim -0.3$, so inclusion of higher-mass contributions to the strangeness vector spectral functions appears necessary in order to account for the SAMPLE value for $G_M^s$. At present, no model-independent, first principles treatment of this higher-mass region has yet been achieved, though a number of one-loop model calculations [36, 37, 38, 39] have indicated the importance of the higher-mass states in moving the value $\mu_s$ in the right direction to better agree with the SAMPLE result.

More generally, one-loop computations using a hadronic basis represent a perturbative approximation to the full dispersion theoretic treatment. Such loop calculations entail truncation of an expansion in the strong hadronic coupling $g$ at second order and, therefore, tend to be subject to a substantial degree of model-dependent ambiguities [37, 40]. In particular, the one-loop amplitudes rely on a unitarity violating approximation to pseudoscalar meson-nucleon scattering amplitudes, the omission of higher-order (in $g$) rescattering contributions that restore unitarity and generate the physically important resonant behavior of the spectral functions, and use of unphysical, point-like pseudoscalar vector current form factors [40, 41]. In principle, these deficiencies are remedied in a systematic way using $\chi$PT, wherein the a priori unknown low-energy constants (LEC’s) determined by fits to experimental data embody the physics omitted from the one-loop graphs. In the case of $G_M^s$, however, this program encounters an intrinsic limitation due to the symmetry properties of the strangeness vector current (see below), so any one-loop predictions for $G_M^s$ necessarily entail unquantifiable model-dependent uncertainties.

The earliest hadron loop calculations [42, 44, 45] suggested that the non-resonant part of the nucleon’s kaon cloud should make a fairly small contribution to $G_M^s$, an insight confirmed by the subsequent dispersion theory analyses discussed above. A variation on this theme was carried out by Geiger and Isgur [38], who summed up a complete tower of meson-baryon one loop graphs, using the quark model to determine the relevant hadronic vertices, and found a pattern of cancellations among successively higher-mass intermediate states. Although this computation suffers from the same ambiguities as the earlier one-loop calculations, it is nevertheless suggestive that higher mass contributions may, as indicated by Eq. (5) and the dispersion theoretic studies, play an important role in the dynamics of $G_M^s$.

In general, the systematic, model-independent treatment of hadronic loop effects in $\chi$PT does not yield predictions that are independent of the $G_M^s$ measurements since the operator $\bar{s}\gamma_\mu s$ contains an SU(3)-singlet component [41]. Apart from the SAMPLE result for $G_M^s$ itself, there exists no other experimental information on the SU(3)-singlet component of the nucleon vector current that would allow one to determine the LEC’s relevant to strange magnetism. An exception occurs for the strange magnetic radius, $r_s^2$, that governs the slope of $G_M^s$ at the origin, for which a parameter-free prediction can be made at $O(p^3)$ [11]. However, inclusion of $O(p^4)$ loop contributions, nominally suppressed by one power of $m_K/\Lambda_{\chi}$ where $\Lambda_{\chi} = 4\pi F_\pi$ is the chiral scale, nearly cancels most of the $O(p^3)$ term, leaving a residual dependence on an a priori unknown strange magnetic radius LEC, $b_s^\pi$ [42]. Rigorously speaking, the latter must be determined from future measurements of the $Q^2$-dependence of $G_M^s$. However, a reasonable range may be estimated by comparing the model-independent dispersion relation and lattice QCD calculations as indicated above, leading to $-1 \leq b_s^\pi \leq 1$. In this case, $\chi$PT provides reasonable guidance for an extrapolation of $G_M^s(Q^2)$ to the photon point. The static moment can be written as [42]

$$\mu_s = G_M^s(Q^2 = 0.1) - 0.13b_s^\pi$$  \hspace{1cm} (6)

resulting in $\mu_s = 0.37 \pm 0.20 \pm 0.26 \pm 0.15$, where now the two sources of theoretical uncertainty (electroweak radiative corrections and strange magnetic radius) have been combined.

Nucleon model calculations, though less transparently connected to QCD, have also provided some insights into the dynamics of strange quarks. One model that has received considerable attention recently is the chiral quark
soliton model (χQSM), in which constituent quarks interact with the Goldstone bosons of the spontaneously-broken, approximate chiral symmetry of QCD in a self-consistent way. A recent χQSM computation of $G_M^s$ reported in [36] yields a small positive result, $0.05 \lesssim G_M^s(Q^2 = 0.1) \lesssim 0.1$ that is consistent with the SAMPLE result. The same model, however, underpredicts the proton and neutron magnetic moments by $\sim 40\%$, and the prediction for $G_M^s$ is strongly dependent on ad hoc assumptions about the long-distance behavior of the Goldstone boson field. The computation is nevertheless suggestive that, at the microscopic level, the topology of the QCD vacuum plays a non-trivial role in the dynamics of the $s\bar{s}$ sea.

Completion of the SAMPLE program and the corresponding insights derived from a comparison of the experimental results to the last decade of theoretical efforts represents a milestone in the quest to understand the quark substructure of the nucleon. Nonetheless, work is on-going, and new experimental data will become available from Jefferson Laboratory and from the Mainz PVA4 program [35]. An independent determination of $G_M^s$ at low momentum transfer is planned by the HAPPEX collaboration using combined forward angle parity-violation measurements on hydrogen and helium targets [18, 19]. Precise determination of the $Q^2$ dependence of both $G_E^s$ and $G_M^s$ over the range $0.3 < Q^2 < 1.0$ (GeV/c)$^2$ will be available from the planned program of measurements by the G0 collaboration [40], for which data taking will soon begin. Theoretically, new efforts to carry out unquenched lattice computations and to resolve the numerical sampling challenges are underway. One expects these efforts to flesh out the framework that has now emerged after more than a decade of experimental and theoretical work. Of course, future surprises are always a possibility, and one may ultimately find that some deeper principle governs the dynamics of sea quarks than is apparent to our current understanding.

This work was supported by NSF grants PHY-9420470 (Caltech), PHY-9420787 (Illinois), PHY-9457906/9971819 (Maryland), PHY-9733772 (VPI), DOE Cooperative agreement DE-FC02-94-ER40818 (MIT-Bates), and DOR contract W-31-109-ENG-38 (ANL).

---

* Electronic address: beise@physics.umd.edu

[17] GEANT, CERN program library.