ABSTRACT

Title of Thesis: ANALYSIS AND BEHAVIOR INVESTIGATIONS OF BOX GIRDER BRIDGES

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Due to efficient dissemination of congested traffic, economic considerations, and aesthetic desirability horizontally curved steel box girder bridges have become increasingly popular nowadays in modern highway systems, including urban interchanges. Although significant research has been underway on advanced analysis for many years to better understand the behavior of all types of box-girder bridges, however, the results of these various research works are scattered and unevaluated. Hence, a clear understanding of more recent work on straight and curved box-girder bridges is highly desired. The non-composite steel section must support both the fresh concrete and the entire construction loads hence steel box girders are at their critical stage during construction. In the current study, non composite straight and curved steel boxes are analyzed with beam and shell elements using the three dimensional finite element analysis and their behavior is investigated.

The present research addresses comparison using beam and shell element models of the straight and curved box girder bridge. This task involves examining the stress patterns obtained using static three-dimensional finite element modeling. Comparisons are made between stresses obtained for the straight and curved box girder bridges, from the beam element model and shell element model for each. Further, the finite element results are compared to the BEST center program DESCUS-II results. Finally, the parametric investigations are performed on the curved steel box model to evaluate the effects of several important parameters on the behavior of the girder.

ANALYSIS AND BEHAVIOR INVESTIGATIONS OF BOX GIRDER BRIDGES

By

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Chapter 1

INTRODUCTION

1.1 General

Due to efficient dissemination of congested traffic, economic considerations, and aesthetic desirability horizontally curved steel box girder bridges have become increasingly popular nowadays in modern highway systems, including urban interchanges. Currently curved girders have replaced straight segments because in urban areas where elevated highways and multi-level structures are necessary, modern highway bridges are often subjected to severe geometric restrictions; therefore they must be built in curved alignment. Even though the cost of the super-structure for the curved girder is higher, the total cost of the curved girder system is reduced considerably since the number of intermediate supports, expansion joints and bearing details is reduced. The continuous curved girder also provides more aesthetically pleasing structures. Despite all the advantages mentioned above, horizontally curved girders are generally more complex than straight girders. Curved girders are subjected to vertical bending plus torsion caused by the girder curvature. To deal with such complexities, several approximate analysis methods were developed in the sixties. In the past, curved girders were generally composed of a series of straight segments that were used as chords in forming a curved alignment.

The first work on the static analysis of horizontally curved beams was published by De Saint Venant during the first half of the 19th century (Zurieck et al. 1994). In 1969, Federal Highway Administration (FHWA) and participating States Departments of Transportation formed The Consortium of University Research team (CURT) to conduct a research project on horizontally curved beams. This comprehensive and systematic research effort was undertaken between 1969 and 1976 and produced an extensive knowledge on the static and dynamic behavior of curved bridges.

The CURT research efforts resulted in the publication of the first edition of the Guide Specification, a working stress design guide for Horizontally Curved Bridges by the American Association of State Highway and Transportation Officials (AASHTO) in 1976 (Hall et al. 1999). Since then, there was a long period of inactivity during the 1980's and early 1990's that led to relatively antiquated design methodologies. The load factor design criteria were adopted by AASHTO in 1980 was based on the working stress results. In 1993 AASHTO Guide Specifications for Horizontally Curved Highway Bridges was primarily based upon research work conducted prior to 1978 and pertains only to multiple-spine composite type of box girders. Since then, a significant amount of work has been conducted to enhance the specifications and to better understand the behavior of all types of box-girder bridges. The latest edition of the guide specification was published by AASHTO (2003).

After the CURT project, AASHTO sponsored The National Cooperative Highway Research Program (NCHRP Project 12-26) in the mid 1980s to develop comprehensive specification provisions for distribution of wheel loads in straight highway bridges. The results of this program were reported elsewhere (Nutt et al. 1988; Zokaie et al. 1991) in the case of reinforced concrete and prestressed concrete multicell bridge cross sections.

In 1998, the National Cooperative Highway Research Program (NCHRP) indicated that, many provisions in the Guide Specification are overly conservative, and many may be difficult to implement (Hall et al. 1999). Other provisions lend themselves to misinterpretation, which may lead to uneconomical designs or designs with a lower factor of safety than intended. Many highway administrations and organizations have sponsored research projects to improve design methods for curved highway bridges. The Federal Highway Administration (FHWA) sponsored other projects to increase knowledge of the behavior of curved steel bridges.

The current design methods, which are primarily based on the research conducted prior to 1978, have a number of deficiencies. Although significant research has been underway on advanced analysis for many years to better understand the behavior of all types of box-girder bridges, however, the results of these various research works are scattered and unevaluated. Hence, a clear understanding of more recent work on straight and curved box-girder bridges is highly desired.

1.2 Box girder and I-beam girder

There are basically two types of steel cross sections currently being in used for curved alignment: an open section consisting of a number of I-shaped cross sections braced with a heavy transverse bracing system and the other type of section is a closed section consisting of few box girders. Compared to I-beam girders, box girders have a number of key advantages and disadvantages.

Steel box girders that serve to transfer loads directly from the concrete deck to the abutments and piers have emerged as the most common application in North America today (Canadian Institute of Steel Construction (CISC) workshop. 2001). Box girder cross sections may take the form of single cell (one box), multi-spline (separate boxes) or multi-cell with a common bottom flange. Box girders offer better resistance to torsion, which is particularly of benefit if the bridge deck is curved in plan.

Due to the high torsional stiffness of the closed cross section of the box girders, which often ranges from 100 to 1000 times larger than the torsional stiffness of comparable I-shaped sections, the torsional moment induced by the curvature of the girder can be resisted by the box girder with much less transverse bracing than the I-shaped girders. The fabrication of the box girder is more expensive compared to the I-shaped girder, but this additional cost is usually balanced by the reduction in sub-structuring for the box girder.

In addition to the large torsional stiffness, box girders provide higher corrosion resistance because a high percentage of the steel surface including the top of the bottom flange is not subjected to the environmental attack. The box girder also has a smooth shape that leads to better bridge aesthetics.

The trapezoidal shape, which is more popular nowadays, offers several advantages over rectangular shaped cross section. The trapezoidal box girder (bath-tub girder) provides a narrow bottom flange. Near the abutments where the bending moment is low, narrow flanges allow for steel savings. In addition, bath-tub girders are more aesthetically pleasing.

Additionally, larger girders can be constructed, because the presence of two webs allows wider and hence stronger flanges to be used. This in turn allows longer spans. A recently study by Hunley and Harik (2007) recognized comparable level of bridge redundancy for twin steel box girder bridges. On the other hand, box girders are more expensive to fabricate, and they are more difficult to maintain, because of the need for access to a confined space inside the box.

1.3 Construction of Steel Box girders

The steel box girder may be defined as a longitudinal structural member with four steel plates, two webs and two flanges, arranged to form a closed box section as shown in Figure 1(a). However, in modem highway structures, a more common arrangement is the box girder with open top which is usually referred to as the tub girder. In this case, two steel webs with narrow top flanges similar to those of the plate girders are joined at their bottoms by a full-width bottom flange as shown in Figure 1(b).

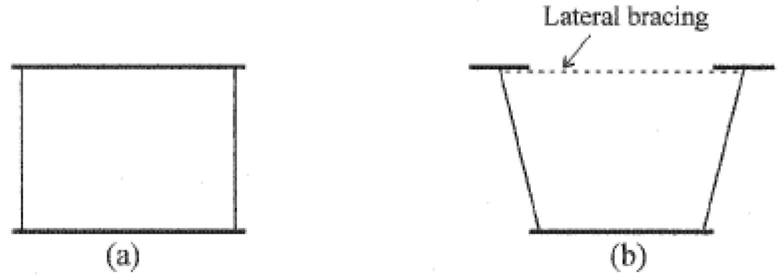

Figure 1 Steel box girders: (a) Enclosed box girder; (b) Tub girder with lateral bracing

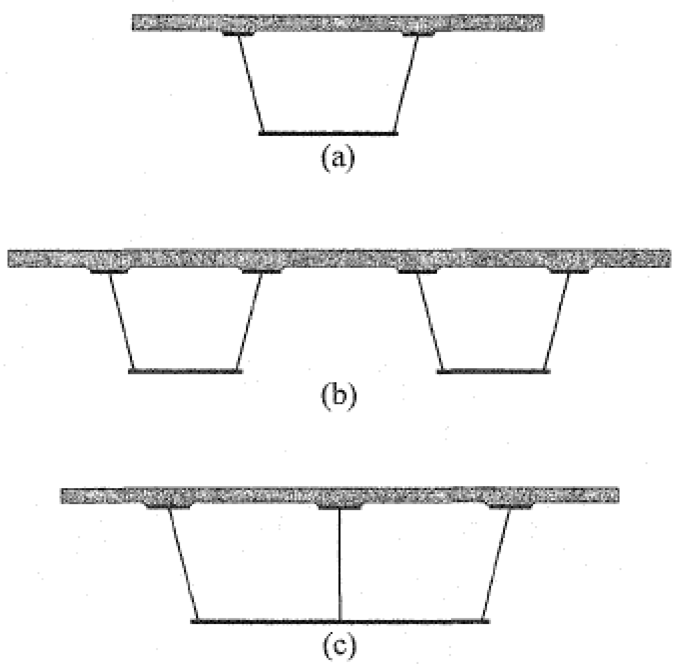

Figure 2 Steel/concrete composite box girders: (a) Single box; (b) Multi-box (twin-box); (c) Multi-cellular box

At fabrication and erection stages, the section may be completely open at the top or it may be braced by a top lateral bracing system to the top flanges. Composite box girder bridge systems may take the form of single box, multi-box also called twin-box or multi cellular box (Figure 2). To close the top opening and complete the box, a reinforced concrete deck slab is added which acts compositely with the steel section by a means of shear connectors, to ensure full interaction between them. Therefore, during construction, the steel girders are subjected to the wet concrete load in addition to other construction loads without the composite action that results from the hardened concrete deck.

During the construction stage, however, the behavior is not well understood. The usual practice of assuming the system to be non-composite during construction requires substantial top flange bracing to form a quasi-closed box section. The non composite steel section must support both the fresh concrete and the entire construction loads hence steel box girders are at their critical stage during construction. The open section of the bath-tub girder is a major concern because of its relatively low torsional stiffness. A lateral bracing system is usually installed at the top flange level in the open-top box girder to form a quasi-closed box, thereby increasing the torsional stiffness. Bracing systems commonly consist of a horizontal truss attached to the girder near its top flange to increase its torsional stiffness. The distortion of the cross section is reduced by using internal cross frames and diaphragms. External bracing between the interior and exterior box girders may be necessary in the case of very sharply curved bridges in order to control the deflections and rotations of the girders, thereby facilitating the placement of the concrete roadway deck. The box girder cross section possesses a high torsional stiffness after the concrete deck gains its full strength since the cross section is considered as a fully closed section.

1.4 Objectives and Scope

The current study is about the behavior and analysis investigation of the steel box girder bridges. The objectives and scope for the study are:

1. Literature review of the analytical methods, previous experimental and theoretical research work, and general behavior of box girder bridges.
2. Develop three-dimensional finite element beam and shell models of straight and curved box girders using the commercially available finite element computer program "ANSYS".

3. Study the behavior of straight and curved box girders and compare the analytical model results with the BEST Center program DESCUS-II.
4. Perform the parametric investigation utilizing the FEM model of the curved box to determine the effect of spacing of bracings on the stresses and effect of longitudinal stiffeners on the bottom plate stresses.

1.5 Outline of the Thesis

This thesis is organized into six chapters. The Chapter 1 is an introduction to the topic followed by the objectives and scope of the study. A literature review of the earlier analytical and experimental work on box girder bridges is presented in Chapter 2. The finite element analysis is described in Chapter 3 to study the behavior of the straight and curved box girders. In Chapter 4, the comparison between the Best Center program DESCUS-II results and those obtained by the finite element modeling is undertaken. Chapter 5 explains the parametric studies conducted on the FEM model of curved bridge. The summary and conclusions are presented in Chapter 6.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

The development of the curved beam theory by Saint-Venant (1843) and later the thin-walled beam theory by Vlasov (1965) marked the birth of all research efforts published to date on the analysis and design of straight and curved box-girder bridges. Many technical papers, reports, and books have been published in the literature concerning various applications of, and even modifications to, the two theories. Recent literature on straight and curved box girder bridges has dealt with analytical formulations to better understand their complex behavior of and few authors have undertaken experimental studies to investigate the accuracy of the existing methods. This chapter summarizes the static behavior of curved steel girder bridges and covers a wide range of topics that can be itemized as follows:

1. Literature pertaining to the elastic analysis methods
2. Experimental studies on elastic response of box girder bridges
3. Behavior of curved girder bridges

2.2 Analytical Methods for Box Girder Bridges

There are several methods available for the analysis of box girder bridges. In each analysis method, the three-dimensional bridge structure is usually simplified by means of assumptions in the geometry, materials and the relationship between its components. The accuracy of the structural analysis is dependent upon the choice of a particular method and its assumptions. The highlights of the references pertaining to elastic analysis methods of straight and curved box girder bridges is published by Sennah and Kennedy (2002). A review of different analytical methods for box girder bridges has been presented by Samaan (2004). Aldoori (2004) has discussed the theoretical aspects of some of the methods. A brief review of the analytical methods of box girder bridges is presented in the following sections.

2.2.1 Grillage Analogy Method

Grillage analysis has been applied to multiple cell boxes with vertical and sloping webs and voided slabs. In this method, the bridge deck was idealized as a grid assembly. Hambly and Pennells (1975) applied this idealization to the multicellular superstructure and Kissane and Beal (1975) to curved multispine box-girder bridges. The continuous curved bridge is modeled as a system of discrete curved longitudinal members intersecting orthogonally with transverse grillage members. As a result of the fall-off in stress at points remote from webs due to shear lag, the slab width is replaced by a reduced effective width over which the stress is assumed to be uniform. The equivalent stiffness of the continuum are lumped orthogonally along the grillage members. Cheung (1982) dealt with the calculation of the longitudinal bending moment and transverse shear in multispine box-girder bridges using the grillage-analogy method. The results from this method were compared favorably to the results obtained from 3D analysis using the finite-strip method. One problem which arises by using the grillage analogy method is in determining the effective width of the slab to include the shear lag effects. Another difficulty of this method lies in estimating the torsional stiffness of closed cells. Satisfactory, but approximate results can be achieved in modeling the torsional stiffness of a single closed cell by an equivalent I-beam torsional stiffness (Evans and Shanmugam 1984). Canadian Highway Bridge Design Code (CHBDC 2000) has limited the use of this method in the analysis of voided slab and box-girder bridges in which the number of cells or boxes is greater than two.

2.2.2 Orthotropic Plate Theory Method

The orthotropic plate theory method considers the interaction between the concrete deck and the curved girder of a box girder bridge. In this method the stiffness of the diaphragms is distributed over the girder length and the stiffness of the flanges and girders are lumped into an orthotropic plate of equivalent stiffness. However, the estimation of the flexural and torsional stiffness is considered to be one major problem in this method. Also, the evaluation of the stresses in the slab and girder presents another difficulty in adopting this approach. Bakht (1981) presented the various methods of calculating the equivalent plate parameters, which are necessary for 2D analysis of straight cellular and voided slab bridges. Cheung (1982) used the orthotropic plate method to calculate the longitudinal moments and transverse shear in multispine box-girder bridges. To establish the limits of validity of the orthotropic plate method, the results were compared to those obtained from 3D analysis using the finite-strip method. It was concluded that the orthotropic plate method gives accurate results provided that the number of spines is not less than three. This method is suggested mainly for multiple-girder straight bridges and curved bridges with high torsional rigidity. However, the Canadian Highway Bridge Design Code (CHBDC 2000) has recommended using this method only for the analysis of straight box-girder bridges of multispine cross section but not multicell cross section.

2.2.3 Folded Plate Method

This method produces solutions for linear elastic analysis of a box girder bridge, within the scope of the assumptions of the elasticity theory. In this method a box girder bridge can be modeled as a folded system which consists of an assembly of longitudinal plate elements interconnected at joints along their longitudinal edges and simply-supported at both ends by diaphragms. These diaphragms are infinitely stiff in their own planes but perfectly flexible perpendicular to their own plane. This method produces solution of simply supported straight or curved box-girder bridges for any arbitrary longitudinal load function by using direct stiffness harmonic analysis where any arbitrary longitudinal joint loading can be resolved into harmonic component of the loading using Fourier series and then, a direct stiffness analysis can be performed for each component. Scordelis (1960) developed an analytical procedure for determining longitudinal stresses, transverse moments and vertical deflections in folded plate structures by utilizing matrix algebra. The procedure can be easily programmed for digital computers. The method has been applied to analyze cellular structures by Meyer and Scordelis (1971), Al-Rifaie and Evans (1979), and Evans (1984). Marsh and Taylor (1990) developed a method that incorporates a classical folded plate analysis of an assemblage of orthotropic or isotropic plates to form box girders. One of the major drawbacks of the folded plate method is that it is tedious and complicated. According to the Canadian Highway Bridge Design Code (CHBDC 2000), the applicability of this method is restricted to bridges with support conditions that are closely equivalent to line supports at both ends and also line intermediate supports in the case of multi-span bridges.

2.2.4 Finite Strip Method

The finite strip method can be regarded as a special form of displacement formulation of finite element. Using a strain-displacement relationship, the strain energy of the structure and the potential energy of external loads can be expressed by displacement parameters. It employs the minimum potential energy theorem where at equilibrium; the values of the displacement parameters should make the total potential energy of the structure become minimal. In this method, the box girders section is discretized into annular finite strips running from one end support to the other. The strips are connected transversely along their edges by longitudinal nodal lines. Then the stiffness matrix is calculated for each strip based upon a displacement function in terms of Fourier series. Similar to the folded plate method, in the finite strip method the direct stiffness harmonic analysis is performed. The finite strip method is considered as a transition between the folded plate method and the finite element method. The finite element method is basically different from the strip method in terms of the assumed displacement interpolation functions. Unlike the finite element method, the displacement functions for the corresponding finite strip are assumed as combination of harmonics varying longitudinally and polynomials varying in the transverse direction.

The finite strip method was introduced by Cheung (1968) and then Cheung and Cheung (1971) applied this method to analyze curved box girders. They programmed this method and used the program to solve the numerical examples of curved bridges as well as straight bridges by making the radius of curvature very large and the subtended angle very small. Kabir and Scordelis (1974) developed a finite strip computer program to analyze curved continuous span cellular bridges, with interior radial diaphragms, on supporting planar frame bents. At the same time Cusens and Loo (1974) presented a general finite-strip technique to single and multispan box bridges with an extension to the analysis of prestressing forces. Cheung and Chan (1978) used this method to determine the effective width of the compression flange of straight multi-spine and multi-cell box-girder bridges. Cheung (1984) used a numerical technique based on the finite-strip method and the force method for the analysis of continuous curved multi cell box-girder bridges. Branco and Green (1984) used this method to investigate the effect of the cross bracing system, as well as the transverse web stiffeners, in resisting distortion and twist of straight composite twin-spine box girder bridges during service. Ho et al. (1989) used the finite strip to analyze three different types of simply supported highway bridges, slab-on-girder, two-cell box girder, and rectangular voided slab bridges. Gambhir and Singla (1988) presented an optimization study, using the finite-strip method of prismatic multi-cellular bridge decks for minimum cost. Cheung and Li (1989) extended the applicability of finite-strip method to analyze continuous haunched box-girder bridges (with variable depth web strip). Maleki (1991) further expanded the compound strip method for plates to analyze box girders. Shimizu and Yoshida (1991) used the finite-strip method to evaluate the reaction forces to be used in the design of load-bearing diaphragms at the intermediate support of two-span continuous curved box-girder bridges. Bradford and Wong (1992) used the finite strip method with one harmonic to study the local buckling of the straight composite concrete deck-steel box section in negative bending. Design graphs of the elastic buckling coefficients were produced. These graphs can be used to obtain accurate values of the web depth-thickness ratio that separates the boundary between slender and semi-compact sections. Using the finite strip method, Cheung and Foo (1995) presented the results of a parametric study on the relative behavior of curved and straight box girder bridges using the finite strip method. The parameters considered in the study included types of cross section, type/location and magnitude of loads, span length and radius of curvature. Empirical equations were developed for longitudinal moment ratios between curved and straight box girder bridges.

The finite strip method has an advantage over the finite element method that it requires shorter computer time and smaller computer storage because the amount of data input required in the analysis is reduced drastically because of strip idealization. Although the finite strip method has broader applicability as compared to folded plate method, however the drawback is that the Canadian Highway Bridge Design Code (CHBDC 2000) restricts its applicability to simply supported prismatic structures with simple line support.

2.2.5 Finite Element Method

The finite element method of analysis is generally the most powerful, versatile and accurate analytical method of all the available methods and has rapidly become a very popular technique for the computer solution of complex problems in engineering. It is very effective in the analysis of complicated structures such as that of a box girder bridge with complex geometry, material properties and support conditions and subjected to a variety of loading conditions. This method can be regarded as an extension of analysis techniques mentioned earlier in which a structure is represented as an assemblage of discrete elements interconnected at a finite number of nodal points. Canadian Highway Bridge Design Code has recommended the finite element method for all type of bridges.

A large number of elements have been developed for use in the finite element technique that includes one-dimensional beam-type elements, two dimensional plate or shell elements or even three-dimensional solid elements. Since the structure is composed of several finite elements interconnected at nodal points, the individual element stiffness matrix, which approximates the behavior in the continuum, is assembled based on assumed displacement or stress patterns. Then, the nodal displacements and hence the internal stresses in the finite element are obtained by the overall equilibrium equations. By using adequate mesh refinement, results obtained from finite element model usually satisfy compatibility and equilibrium (Zienkeiwicz, 1977).

Sisodiya, Cheung and Ghali (1970) presented finite element analyses of single box girder skew bridges that were curved in plan. The bridge that could be analyzed by this method may be of varying width, curved in any shape, not just a circular shape and with any support conditions. They used rectangular elements for the webs and parallelogram or triangular elements for top and bottom flanges. This approximation would require a large number of elements to achieve a satisfactory solution. Such an approach is impractical, especially for highly curved box bridges. Chapman et al (1971) conducted a finite element analysis on steel and concrete box girder bridges with different cross section shapes to investigate the effect of intermediate diaphragms on the warping and distortional stresses. They showed that curved steel boxes even with symmetrical load components gave rise to distortional stresses, and showed that the use of sloping webs resulted in an increase in distortional stresses. Lim et al. (1971) developed an element that has a beam-like-in-plane displacement field which is trapezoidal in shape, and hence, can be used to analyze right, skew, or curved box-girder bridges with constant depth and width.

Chu and Pinjarkar (1971) developed a finite element approach for analyzing curved box girder bridges. The top and bottom flanges were modeled as horizontal sector plates while the web was idealized as vertical cylindrical shell elements. The sloped web elements (Bath-Tub girders) were not considered as they require conical shell elements. Membrane and bending actions were both considered for the plate and shell elements, but no interaction between them was assumed. The method can be applied only to simply supported bridges without intermediate diaphragms.

William and Scordelis (1972) analyzed cellular structures of constant depth with arbitrary geometry in plan using quadrilateral elements in the finite element analysis. Bazant and El Nimeiri (1974) attributed the problems associated with the neglect of curvilinear boundaries in the elements used to model curved box beams by the loss of continuity at the end cross-section of two adjacent elements meeting at an angle. Instead of developing curvilinear element boundaries, they developed the skew-ended finite element with shear deformation using straight elements and adopted a more accurate theory that allows for transverse shear deformations.

Fam and Turkstra (1975) developed a finite element scheme for static and free vibration analysis of box girders with orthogonal boundaries and arbitrary combination of straight and horizontally curved sections. Four-node plate bending annular elements with two straight radial boundaries, for the top and bottom flanges, and conical elements for the inclined web members were used. The importance of warping and distortional stresses in single-cell curved bridges was established in relation to the longitudinal normal bending stresses, using the finite element method, by Truksta and Fam (1978).

Moffat and Lim (1976) demonstrated a finite-element technique to analyze straight composite box-girder bridges will complete or incomplete interaction with respect to the distribution of the shear connectors. Sargious et al. (1979) investigated the effect of providing end diaphragm with opening in single-cell concrete box-girder bridges supported by a central pier. At the same time, Daniels et al. (1979) studied the effect of spacing of the rigid interior diaphragms on the fatigue strength of curved steel box girders. The results showed that reducing the interior diaphragms spacing effectively controls the distortional normal and bending stresses and increases the fatigue strength of curved steel box girders.

Dezi (1985) examined the influence of some parameters including transverse and longitudinal locations of external loads, span-to-radius ratio, width-to-depth of the cell, and number of cross diaphragms on the deformation of the cross section in curved single-cell box beams over those in straight single-cell box beams. Ishac and Smith (1985) presented approximations for determining the transverse moments in single-span single-cell concrete box-girder bridges. Dilger et al. (1988) studied the effect of presence and orientation of diaphragms on the reaction, internal forces, and the behavior of skew, single cell, concrete box-girder bridges.

Galuta and Cheung (1995) developed a hybrid analytical solution that combines the boundary element method with the finite-element method to analyze box-girder bridges. The finite-element method was used to model the webs and bottom flange of the bridge, while the boundary element method was employed to model the deck. The bending moments and vertical deflection were found to be in good agreement when compared with the finite strip solution.

Abdelfattah (1997) used the 3D finite-element method to study the efficiency of different systems for stiffening steel box girders against shear lag. Davidson et al. (1996) utilized the finite element method to develop a detailed model for horizontally curved steel I-girder bridges. Sennah and Kennedy (1998) conducted an extensive parametric study on composite multi-cell box girder bridges using the finite element analysis. The results obtained from the finite element method were in good agreement with the experimental findings.

2.2.6 Thin-Walled Beam Theory Method

Saint-Venant (1843) established the curved beam theory for the case of a solid curved bar loaded in a direction normal to the plane of curvature. In general, curved beam theory cannot be applied to curved box girders bridges, because it can not account for warping, distortion, and bending deformations of the individual wall elements of the box. Curved beam theory can only provide the designer with an accurate distribution of the resultant bending moments, torque, and shear at any section of a curved beam if the axial, torsional and bending rigidities of the section are accurately known. The thin-walled beam theory was established by Vlasov (1965) for axisymmetric sections, and then extended by Dabrowski (1968) for asymmetric section who derived the fundamental equations that account for warping deformations caused by the gradient of normal stresses in individual box element. The theory assumes non-distortional cross section and, hence, does not account for all warping or bending stresses. The predication of shear lag or the response of deck slabs to local wheel load cannot be obtained using the theory.

Oleinik and Heins (1975), and Heins and Oleinik (1976) analyzed the curved box girders in two parts. In the first part of the analysis, the box sections were assumed to retain their shape under the load. The load-deformation response of such a curved box that considers bending, torsion and warping deformations was developed by Vlasov. Vlasov's differential equations were solved using a finite difference approach to calculate the normal bending and normal warping stresses. In the second part of the analysis, the effect of cross sectional deformations was considered. These cross sectional deformations were calculated using a differential equation developed by Dabrowski. This equation was also solved using the finite difference approach and the normal stresses that resulting from cross-sectional deformations were calculated. The effects of both parts were summed to give the total normal stress distribution. The above-mentioned formulations and the final solutions of these basic differential equations were programmed by Heins and Sheu (1982). A single straight or curved box girder with prismatic or nonprismatic section can be analyzed using this program. The box girder may have internal transverse diaphragms spaced along the box and top lateral bracing. A parametric study was conducted using this program to investigate the effect of internal diaphragms on the induced normal stresses in curved box girder bridges due to dead and live loads.

Maisel (1982, 1985, 1986) Extended Vlasov's thin-walled beam theory to account for torsional, distortional, and shear lag effects of straight, thin-walled cellular box beams. Mavaddat and Mirza (1989) utilized Maisel's formulations to develop computer programs to analyze straight concrete box beams with one, two, or three cells and side cantilevers over a simple span or two spans with symmetric midspan loadings. The structure was idealized as a beam, and the normal and shear stresses were calculated using the simple bending theory and Saint- Venant's (1843) theory of torsion. Then, the secondary stresses arising from torsional and distortional warping and shear lag were calculated.

Li (1992) and Razaqpur and Li (1990, 1994, 1997) developed a box girder finite element, which includes extension, torsion, distortion, and shear lag analysis of straight, skew, and curved multi-cell box girders using thin-walled finite element based on Vlasov's theory. Hsu, Fu and Schelling (1990), Fu and Hsu (1994) have published their development of curved steel box girder bridges using an improved curvilinear thin-walled Vlasov's element. Exact shape functions were used to eliminate the need for dividing the box into many elements in the longitudinal direction. The results of the proposed element agreed well with those results obtained from full three-dimensional shell finite element analysis. For both static and dynamic analyses of multi-cell box girder bridges, Vlasov's thin-walled beam theory was cast in a finite element formulation and exact shape function was used by EI-Azab (1999) to derive the stiffness matrix.

Hsu et al. (1995) presented a practical approach for the distortional analysis of steel box-girder bridges using an equivalent beam-on-elastic-foundation method that accounts for the deformation of the cross section due to the presence of rigid or flexible interior diaphragms and continuity over the supports. Fu and Hsu (1995) generated a new finite element based on Vlasov's theory of curved thin walled beams. The horizontally curved thin walled beam element stiffness was developed directly in the cylindrical coordinate system. The element stiffnesses of a curved thin-walled beam with four degrees of freedom (torsion, bending, vertical shear and bimoment) per node were presented in this study. The results produced using this element was in good agreement with Dabrowski's closed form solution. Hsu, Fu and Schelling (1996) and Hsu and Fu (2002) further studied the application of EBEF Method for the distortional analysis of steel box girder bridge superstructures during construction and under operating conditions. Kim, Fu and Kim (2007 & 2007) published two papers on forming the dynamic stiffness matrix of thin-walled curved beam on elastic foundation and buckling/vibration evaluation.

2.3 Experimental Studies on Elastic Response of Box Girder Bridges

The experimental studies have been conducted to examine the validity of the available methods of analysis and computer programs to investigate the behavior of box-girder bridges. The results of field testing of existing box-girder bridges have been reported by few authors to validate the existing methods of elastic analysis. This section presents the experimental studies on box girder bridges and includes both field testing and model testing.

Aneja and Roll (1971) tested a plastic model of a horizontally curved box beam highway bridge and compared the results to a finite-element analysis. The comparison showed a close agreement between the shapes but not the magnitude of the stress plots. In 1973, Buchanan, Yoo and Heins (1973) conducted a field test on a composite twin-box girder bridge located near Baltimore, Maryland. Two sets of tests were reported where the first one measured the response of the steel section to the placement of the concrete deck and the second test measured strains, deflections and rotations induced during live loading of the completed structure. The finite difference approach was used to analyze the bridge for both test measurements. The experimental results obtained from the test programs were then compared with the analytical solutions, and it showed good agreement while a comparison with design estimates showed that the design calculations were generally conservative.

Heins, Bonakdarpour and Bell (1972) presented an investigation of the behavior of a small plexiglass curved box beam model bridge. They tested a three-box girder bridge model as both a single span and two-span structure. Strain and dial gages were used to measure strains, deflections and rotations under the effect of a series of concentrated loads. The models were analyzed using the slope deflection theory. This theory assumes no cross-sectional distortions and thus adequate internal cross frames should be provided to minimize cross sectional distortion. The cross-sectional properties like moment of inertia I_x, St.Venant torsional constant K_t and warping constant I_w required for the analytical solution were computed for the bridge section. It was expected that the warping stresses would be negligible. Thus, the warping constant I_w was computed for both the individual girder section and multi-girder section. Correlation with the experimental values provided good results. For this reason, it was concluded that the warping phenomenon was negligible for these closed box girder sections and the warping constant, I_w need not be considered in the analysis.

Aslam and Godden (1975) tested elastically a series of small-scale aluminum, straight, skew, and curved four-cell box-girder bridge models. The main purpose of this study was to develop an accurate experimental data for checking the validity of computer analyses of such structures. The models were tested elastically, with and without a mid-span radial diaphragm for different locations of a single point load. It was concluded that the folded plate analysis is sufficiently accurate for the elastic analysis of box-girder bridges.

Kissane and Beal (1975) presented an experimentally determined behavior a horizontally curved, two-span, continuous composite concrete, deck-steel, three-spine, box bridge located in New York. Strains, deflections, rotations and cross-sectional deformations in one span of the structure were recorded. The structure was analyzed using a curved girder analysis program that idealized the structure as a planar grid and used the stiffness method to solve for the unknown joint displacements and subsequently the moment, shear and torque at each joint. Test results showed that the experimental in-plane bending moments for dead load and static live loads were about 86 percent of their respective theoretical values. However, comparisons between experimental and theoretical results concerning the proportion of the total load carried by the individual girder were within 6 percent. The experimental deflection compared well with the theoretical deflection. The authors believed that the high torsional stiffness resulting from heavy and closely spaced internal diaphragms was the reason behind getting the low live load distribution factor compared to the factor used in the design of straight box girder structures. Similar testing was performed by Yoo et al. (1976) on a three-span, continuous curved, composite concrete, deck-steel, twin-spine, box-girder bridge located in Baltimore.

Evans and AI-Rifaie (1975) presented experimental and theoretical studies on the behavior of curved box girder models. They tested eighteen simply supported, single-cell models with rigid end diaphragms of different curvatures and different cross-sectional dimensions subjected to different loading conditions. The models were built either from sand/araldite material or from steel. The models were instrumented with strain gages. Deflections, longitudinal stresses and transverse moments were measured. The models were analyzed using the finite element method. It was concluded that the finite element method was capable of accurately predicting the behavior of curved box girders. After that, the finite element method was used successfully as a tool to conduct a parametric study to show the curvature effect on the deflections, longitudinal stresses and transverse moments of the models.

Fam and Turkstra (1976) reported on studies of two, single-cell, plexiglass models with high curvature to investigate the effects of intermediate diaphragms and the adequacy of the 3D finite-element modeling of curved, single-cell structures. The models were tested under the effect of concentrated and line loadings. Strain gages and vertical dial gages were used to measure strains and vertical deflections. The results obtained from the finite element program were found to be in good agreement with experimental results for deflections, radial stresses and tangential stresses. It was concluded that the introduction of diaphragms had an effect on the magnitude and distribution of both longitudinal and radial stresses in the flanges of the girders with severe curvature. Nakai et al. (1980) tested four plexiglass, box-girder models of the same span length, radius of curvature, and central angle but with different numbers of intermediate diaphragms to verify the results obtained from thin-walled beam theory.

Shanmugam and Balendra (1985) conducted experimental studies on the elastic behavior of two, prespex, multicell bridge models—one with solid webs and the other with web openings. Results from analytical solution, using the grillage technique and the finite-element method, compared favorably with the experimental values for deflections and flange-edge stresses.

Siddiqui and Ng (1988) tested elastically two, straight plexiglass, single cell, box girder bridge models to determine the effect of transverse diaphragms in reducing the warping and distortional stresses that usually developed in box girders from deformation of the cross section. One of the models had a rectangular cross section while the other one was trapezoidal. Experimental results were compared with analytical results obtained from the beam on elastic foundation (BEF) analogy. Both models were tested as a simply supported beam. The testing program was divided into three major parts: case I, the models were tested using end diaphragms only; Case II, two interior diaphragms were inserted in the models and case III; six interior diaphragms were inserted in the models. The models were subjected to a concentrated load at different positions along the span and with different eccentricities. Both models were instrumented with dial gages and strain gages to measure vertical and rotational displacements as well as longitudinal strains. The experimental results indicated 1) that the deformation of a box girder cross section eccentrically loaded may cause substantial warping and distortional stresses and 2) that these stresses can be effectively controlled by the installation of rigid diaphragms along the span of the girder. Warping and distortional stresses that were calculated by the BEF analogy compared reasonably well with the experimental stresses.

Yasunori, Hamada and Oshiro (1988) tested three curved composite box girders with end diaphragms to investigate their distortional response and slip behavior. The test specimens had different radii of curvature, cross sections and placements of shear connectors. The test results were compared with analytical results based on the curved beam theory and the distortional theory proposed by Dabrowski, as well as with analytical results from the finite strip method. The authors concluded that the cross-sectional deformations of curved composite box girders with only end diaphragms were considerably significant and produced large additional longitudinal stresses. The major conclusion from the investigation was that if a sufficient number of diaphragms were provided, the effects of cross-sectional deformations might be disregarded in the design of curved composite box girders.

Yabuki and Arizumi (1989) investigated experimentally the normal stress distribution and the distortional deformation in the cross section of two, steel box girders models of the same curvature and span length but with different number of intermediate diaphragms. The experimental results were used to verify the analytical results from the beam on- elastic-foundation method, as well as the empirical equations proposed by Nakai and Murayama (1981) for predicting the distortional stresses in simply supported, curved, box-girder bridges.

Ng, Heung and Hachem (1993) conducted a model study to evaluate the elastic response of a curved composite bridge. The specimen was a 1/24 linear scale model of the Cyrville road bridge over the Queensway, east of Ottawa, Ontario. The prototype bridge was a two-lane, two-span concrete curved box girder structure, while the model bridge was a composite construction of concrete and aluminum. The model bridge was instrumented by strain gages and dial gages to measure strain and vertical deflections. ADINA (finite element program) was used to examine the bridge analytically. Analytical results of both vertical displacements and normal stresses at critical sections compared fairly well with those observed experimentally; the authors concluded that ADINA could be successfully applied in modeling an accurate elastic response of composite curved box girder bridges.

2.4 General Behavior of Curved Box Girders

Horizontally curved box girders applicable for both simple and continuous spans are used for grade-separation and elevated bridges where the structure must coincide with the curved roadway alignment. This condition occurs frequently at urban crossings and interchanges and also at rural intersections where the structure must conform to the geometric requirements of the highway. The objective of this section of the material is to present the overview of the general behavior of the box girder bridges.

Horizontally curved bridges will undergo bending and associated shear stresses as well as torsional stresses because of the horizontal curvature even if they are only subjected to their own gravitational load. Figure 3 shows the general behavior of an open box section under gravity load showing separate effect. An arbitrary line load on a simple span box girder (Figure 3(a)) contains bending and torsional load components that have corresponding bending and torsional effects.

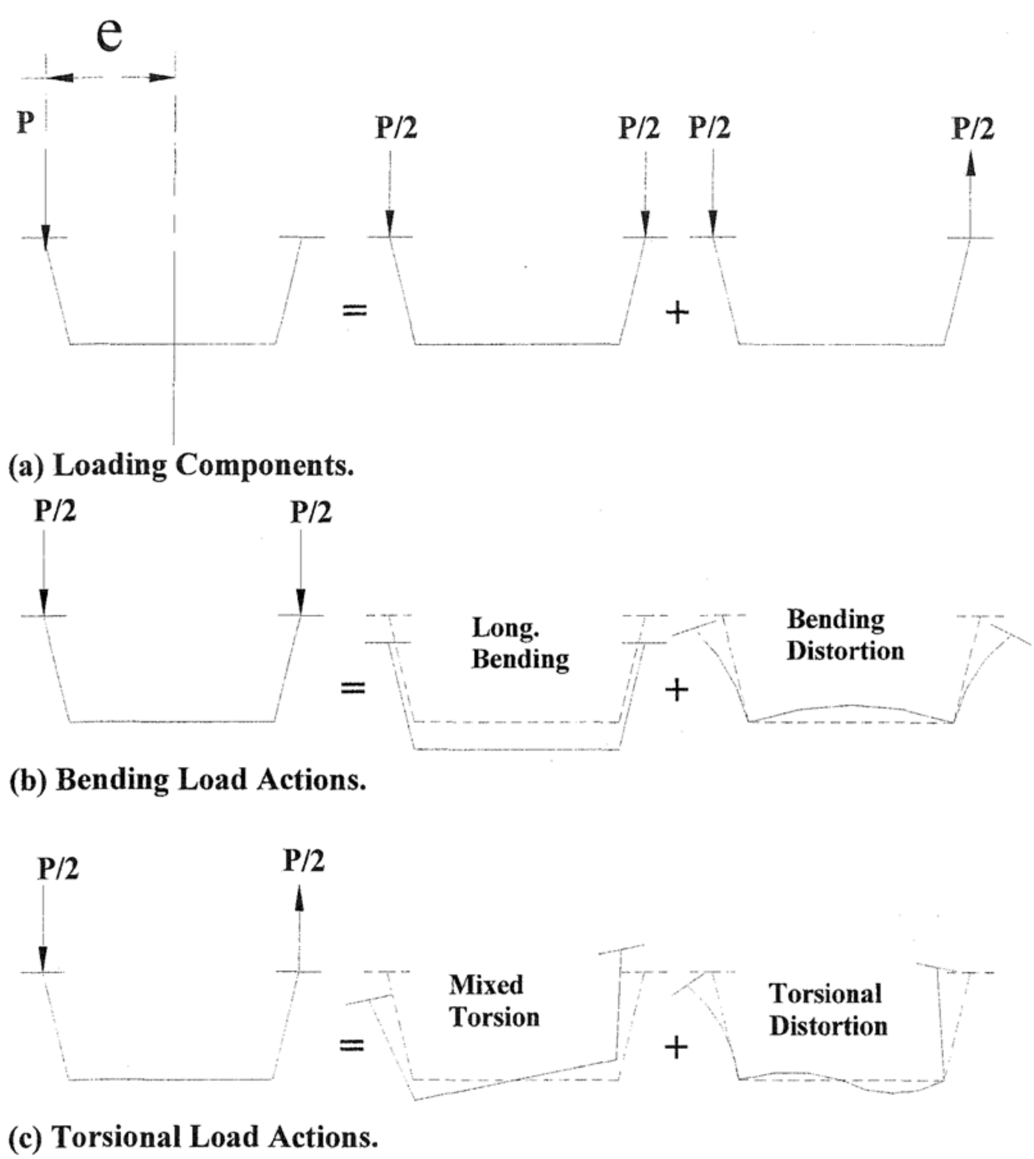

Figure 3 General behavior of an open box section under gravity load showing separate effect

2.4.1 Bending Effects

The bending load (Figure 3 (b)), causes the section to:

1. Deflect rigidly (longitudinal bending), and

2. Deform (bending distortion).

Longitudinal Bending

The box girders have large span/depth ratio and due to that transverse load causes significant bending stresses in the girder. Heins (1978) described that according to a survey conducted by ASCE task committee on horizontally curved steel box girder bridges; box girders typically have an average span to depth ratio of 23 for single spans and 25 for continuous girder spans.

Assuming elastic behavior and that plane sections remain plane under bending, bending normal stresses, f, arising from the equilibrium of the cross section (Figure 4 (a)) are given by:

$f = M/S$ …………………………………………………………………. 2.1

Where,
M = bending moment,
S = section modulus.

Shear stresses associated with the moment gradient also occur (Figure 4(b)) and are calculated by:

$f_v = VQ/It$………………………………………………………………….. 2.2

Where,
V = shear force,
I = moment of inertia of the section,
Q = first moment of area under consideration,
t = thickness of the segment.

Bending Distortion

Bending distortion occurs when transverse loads are applied to the open box. The guide specifications states that if the box girder does not have a full width steel top flange, the girder must be treated as an open section. In open box girders, this distortion causes outward bending of the webs, upward bending of the bottom flange and in-plane bending of the top flange (Figure 3(b). The out of plane bending of the plates forming the girder also causes the cross section to change shape. Therefore, to prevent bending distortion the top bracing (ties and struts) as shown in Figure 7 are usually placed between top flanges.

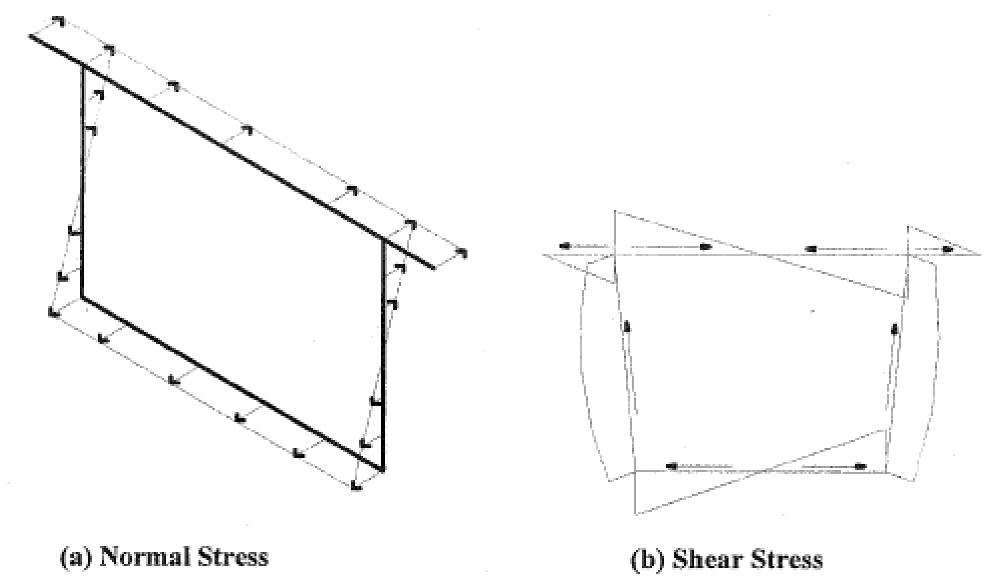

Figure 4 Normal and shear components of longitudinal bending stress.

2.4.2 Torsional Effects

The torsional load (Figure 3 (c)) causes the section to:

1. Rotate rigidly (mixed torsion) and
2. Deform the section (torsional distortion).

Mixed Torsion

In curved box girder bridges, the transverse loads acting on the girder causes twisting about its longitudinal axis because of the bridge curvature. Uniform torsion occurs if the rate of change of the angle of twist is constant along the girder and warping is constant and unrestrained. St. Venant analyzed this problem and found that the St. Venant shear stresses occur in the cross section (Figure 5). If there is a variation of torque or if warping is prevented or altered along the girder, longitudinal torsional warping stresses develop.

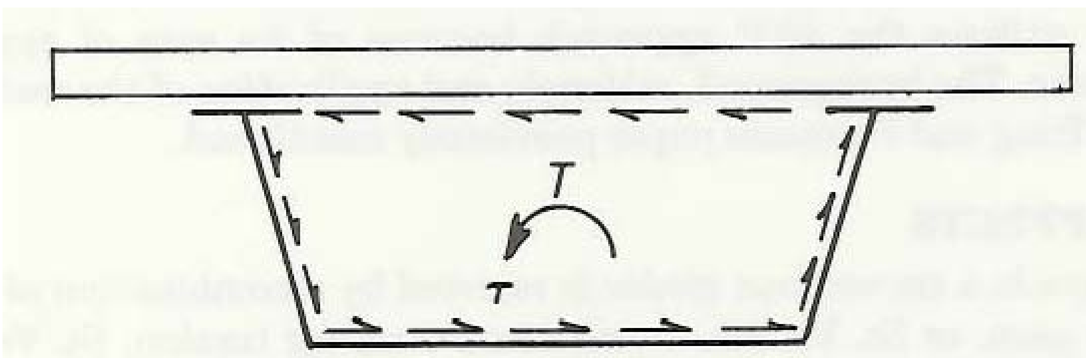

Figure 5 Saint-Venant Torsion in a closed section

In general, both St.Venant torsion and the warping torsion are developed when thin-walled members are twisted. Box girders are usually dominated by St.Venant torsion because the closed cross section has a high torsional stiffness. Box girders have large St.Venant stiffness, which may be 100-1000 times larger than that of a comparable I-section. Kollbrunner and Basler (1969) indicated that the longitudinal normal stresses resulting from the non-uniform warping torsion are usually negligible.

St.Venant stiffness of the box section is a function of the shear modulus of the steel, G, and the torsional constant K_t, which related to the cross section geometry. In curved box girders bridges St.Venant torsion provides most of the resistance that is given by:

$$T = GK_t \, d\theta / dz \qquad 2.3$$

Where,

T = torque on the cross section of the member,

θ = twist angle of the cross section,

z = longitudinal axis of the member.

The torsional constant for a single cell box girder is given by:

$$K_t = (4A^2) / \sqrt{(b/t)} \qquad 2.4$$

Where,

A = enclosed area of the box section,

b = width of the individual plate element in the box,

t = thickness of the plate element in the box.

A closed box-girder section is usually several thousand times stiffer than an open section. For this reason, if a curved box girder does not have a permanent, solid, top flange plate, the girder is braced by a lateral system at or near the top flange, to "quasi –close" the box during the wet-concrete stage of construction.

For analysis purposes, top lateral bracing may be transformed to an equivalent thickness of plate t_{eq}, in. by

$$t_{eq} = (E/G)(2A_d/b)\,(\cos^2\alpha \sin\alpha) \qquad 2.5$$

Where;

E = steel modulus of elasticity, ksi

G = steel shearing modulus of elasticity, ksi

A_d = area of the lateral-bracing diagonal, sq in.

b = clear box width in., between top flanges

α = angle of lateral- bracing diagonal with respect to transverse direction

To properly close the section and minimize warping stresses, the cross-sectional area of the lateral- bracing diagonal should be at least

$A_d = 0.03\ b$

The internal stresses produced by St. Venant torsion in a closed section are shearing stresses around the perimeter, as shown in the following sketch and defined by

$\tau = T/\ (2At)$……………………………………………………………. 2.6

Where,

τ = St. Venant shear stress in any plate, ksi

T = internal torgue, in-kips

A = enclosed area within the box girder, sq in.

t = thickness of plate, in.

Torsional Distortion

Torsional load causes the cross section to deform through bending of the walls (Figure 3(c)). Normal stresses as shown in Figure 6 result from warping torsion restraint and from distortion of the cross-section. If the box girder has no cross frames or diaphragms, the distortion is restrained only by the transverse stiffness of the plate elements. In the open box girder cross section, due to the lack of distortional stiffness the torsional distortion of the cross section can be prevented through the use of internal cross frames (Figure 7) connecting top and bottom flanges. Horizontal bracing can be placed at a small distance below the top flanges to increase the torsional stiffness of the open box cross section and reduce the twist.

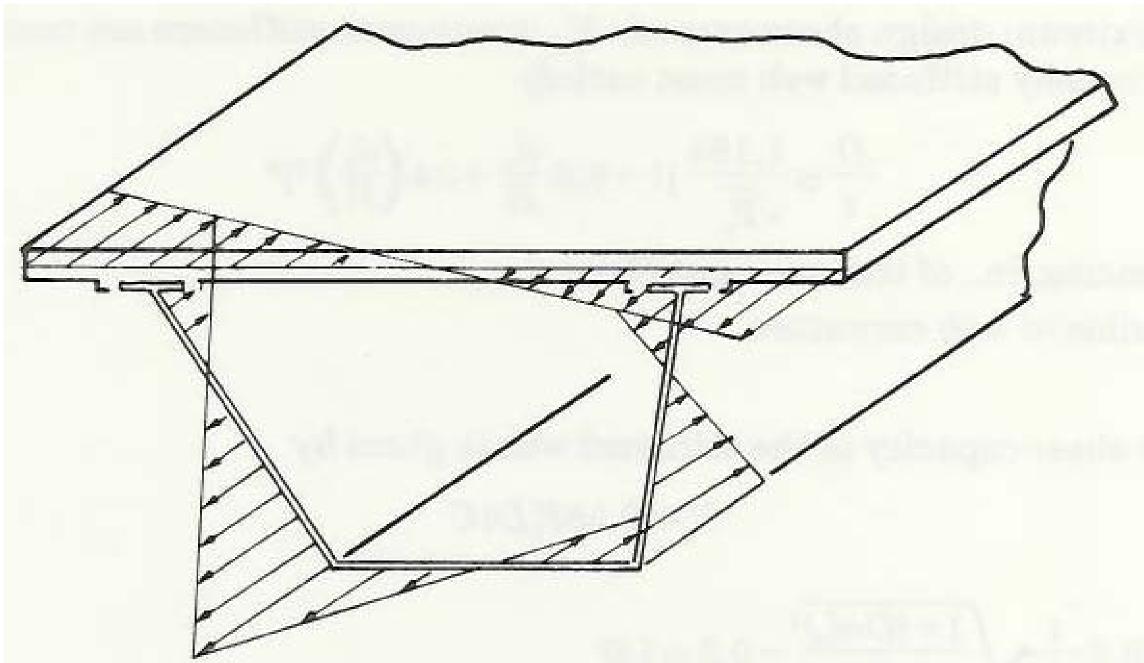

Figure 6 Warping stresses in a box girder

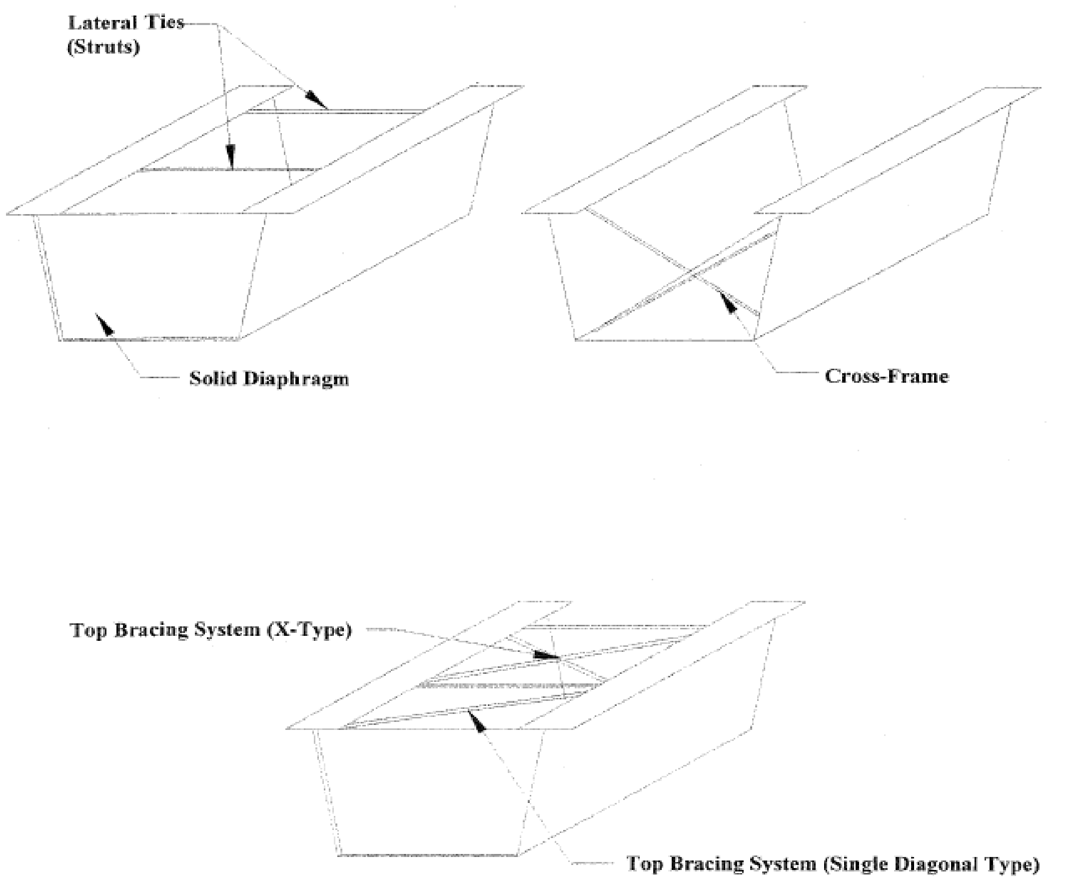

Figure 7 Bracing system terminology

Chapter 3

MATHEMATICAL MODELING

3.1 General

There are many methods available for analyzing curved bridges, as mentioned earlier in Chapter 2. However, of all the available analysis methods, the finite element method is considered to be the most powerful, versatile and flexible method. Among the refined methods allowed by AASHTO LRFD Specifications (1994), although the 3-D finite-element method is probably the most involved and time consuming, it is still the most general and comprehensive technique for static and dynamic analyses, capturing all aspects affecting the structural response. The other methods proved to be adequate but limited in scope and applicability. Due to recent development in computer technology, the method has become an important part of engineering analysis and design because nowadays finite element computer programs are used practically in all branches of engineering. A complex geometry, such as that of continuous curved steel box girder bridges, can be readily modeled using the finite element technique. The method is also capable of dealing with different material properties, relationships between structural components, boundary conditions, as well as statically or dynamically applied loads. The linear and nonlinear structural response of such bridges can be predicted with good accuracy using this method.

In the current research, various structural elements are modeled using finite element method. In this chapter, a general description of the finite element approach is presented next followed by background information pertaining to the finite element program ANSYS that was utilized throughout this study for the structural modeling and analysis and finally the description of the models of the straight and curved box bridges is presented.

3.2 The Finite Element Method

The finite-element method is a numerical procedure for solving problems in engineering and mathematical physics. In structural problems, the solution is typically concerned with determining stresses and displacements. Finite element model gives approximate values of the unknowns at discrete number of points in a continuum. This numerical method of analysis starts by discretizing a model. Discretization is the process where a body is divided into an equivalent system of smaller bodies or units called elements. These elements are interconnected with each other by means of certain points called nodes. An equation is then formulated combining all the elements to obtain a solution for one whole body.

In the case of small displacements and linear material response, using a displacement formulation, the stiffness matrix of each element is derived and the global stiffness matrix of the entire structure can be formulated by assembling the stiffness matrices of all elements using direct stiffness method. This global stiffness matrix, along with the given displacement boundary conditions and applied loads is then solved, thus that the displacements and stresses for the entire system are determined. The global stiffness matrix represents the nodal force-displacement relationships and can be expressed by the following equilibrium equation in matrix form:

$$[F] = [K]\,[U] \qquad (3.1)$$

Where,

[K] = global stiffness matrix assembled from the element stiffness matrices,

[U] = nodal displacement vector and

[F] = nodal load vector.

The above equation can be derived from the following basic relationships:

i. $v(x, y) = [\Phi(x, y)][\alpha]$ .... (3.2)

Where:

v = the internal displacement vector of the element.

Φ = the displacement function or shape function.

α = the generalized coordinates.

ii. $[U] = [A][\alpha]$

Then, $[\alpha] = [A]^{-1}[U]$ .... (3.3)

Where [A] is the transformation matrix from local to global coordinates

iii. $[\xi(x,y)] = [B(x,y)][\alpha] = [B(x,y)]\ [A]^{-1}[U]$ .... (3.4)

Where:

[ξ]= the strain matrix.

[B]= the strain-displacement matrix.

iv. $[(\sigma(x,y)j = [D]\ [\xi(x,y)] = [D][B(x,y)]\ [A]^{-1}[U]$ ………………………… (3.5)

Where; [D] is the stress-strain constitutive matrix or the elasticity matrix.

To obtain the finite element stiffness equations, the principle of minimization of the local potential energy for the total external work equal to 1/2 $[U]^T$ [F],

v. then

$W_E = [u']^T [F]$

$W_I = \int_{vol} [\xi]^T [\sigma] = [u']^T [A]^{-1}[k'] [A]^{-1}[U]$ .... (3.6)

Where:

W_E = the external virtual work;

W_I = the internal virtual work;

[u']] = the vector of virtual displacement;

[k'] = the element stiffness matrix.

Where $[k'] = \int_{vol} [B(x,y)]^T [D][B(x,y)]$ ………………………………………………(3.7)

vi. From the principle of virtual work, $W_E = W_I$. By taking one element of virtual nodal displacement vector [u'] equal to unity successfully, the solution becomes:

$$[F] = [K]\,[U] \quad \ldots\ldots\ldots\ldots \quad (3.8)$$

Where [K] = ∑[k'], so the global structural stiffness matrix is an assemblage of the element stiffness matrix [k'].

vii. The solution of the resulting system of equations yields the values of nodal displacement [U] and the internal forces for each element can be obtained from equation (3.4).

In the case of a linear elastic analysis, loads are first applied on a model and the response of the structure is obtained directly. In a non-linear case, the analysis follows a numerical method to obtain a solution. However, such analysis is beyond the scope of this thesis and is not discussed.

3.3 The Finite Element Program: ANSYS

The finite element modeling and analysis performed in this study were done using a general purpose, multi-discipline finite element program, ANSYS. ANSYS is a commercial finite element program developed by Swanson Analysis Systems, Inc.

(SAS IP Inc 10th edition). The program is available for both PC and UNIX based systems. The analyses presented in this thesis were performed using ANSYS version 10.0. ANSYS has an extensive library of truss, beam, shell and solid elements. The brief description of the elements used in the model is presented below:

1. **Shell 63 (elastic shell)**: A four noded element that has both bending and membrane capabilities. The element has six degree of freedom at each node, translations in the nodal X, Y, and Z directions and rotations about the nodal X, Y, and Z axes. Large deflection capabilities are included in the element. It is stated in ANSYS manual that an assemblage of this flat shell element can produce good results for a curved shell surface provided that each flat element does not extend over more than a 15° arc.

2. **Link 8 (3-D Spar)**: is a two-node, three-dimensional truss element. It is a uni-axial tension-compression element with three degrees of freedom at each node; translations in the nodal X, Y and Z directions. The element is a pin-jointed structure with no bending capabilities. Plasticity and large deflection capabilities are included. The required inputs for this element are the material properties and cross-sectional area.

3. **Beam 188 (3-D Linear Finite Strain Beam):** is a linear (2-node) or quadratic beam element in 3-D. Beam 188 has six or seven degrees of freedom at each node. These include translations in the X, Y, and Z directions and rotations about the X, Y and Z directions. A seventh degree of freedom (warping magnitude) can also be considered. This element is well-suited for linear, large rotation, and/or large strain nonlinear applications. The beam elements are one-dimensional line elements in space. The cross-section details are provided separately using the SECTYPE and SECDATA and commands.

4. **Beam 4 (3-D Elastic Beam):** is a uni-axial element with tension, compression, torsion and bending capabilities. The element has six degrees of

freedom at each node, translations in the nodal X, Y and Z directions and rotations about the nodal X, Y and Z axes. Stress stiffening and large deflection capability is included. The required inputs for this element are the cross-sectional properties such as, the moment of inertia, the cross-sectional area and the torsional properties.

Obtaining stresses from the finite element, ANSYS models can be utilized in understanding the box bridge behavior. In addition, it can also be used to compare the stress profiles. Therefore, creating the same general construction of straight and curved bridge models with same boundary conditions is required. In modeling the bridges using ANSYS, a FEA model was created using command prompt line input other than the Graphical User Interface (GUI). Details of the command code for building the entire model is given in the Appendix.

3.4 Design Considerations

Steel box girders are at their critical stage during construction because the non-composite steel section must support both the fresh concrete and the entire construction loads. In the current study, non-composite straight and curved steel boxes are analyzed with beam and shell elements using the three-dimensional finite element analysis and their behavior is investigated.

Curved box girders and straight box girders used in the current study consist of two span bridges that are of same general construction, consisting of a bottom flange, two webs, which are sloped, and top flanges. In negative–bending region,\ where the bottom flange is in compression, it is stiffened by longitudinal stiffener. They have internal diaphragms or cross frames at regular intervals along the span and lateral bracing at top flange. The cross frames maintain the shape of the cross section and are spaced at regular intervals to keep the transverse distorsional stresses and lateral bending stresses in flanges at acceptable levels. The effect of cross frames is discussed in more detail in parametric study in the next chapter.

3.5 Loading and boundary condition

The non-composite finite element beam and shell box bridge models are subjected to the deck load in addition to the self-weight of the box since the noncomposite steel section must support both the fresh concrete and the entire construction loads. The beam element ANSYS models of the straight and curved box bridge is analyzed with two cases of boundary conditions and compared with DESCUS-II results. In the first case twin bearings are provided at all supports and in the second case twin bearings are provided only in the interior support (pier location).

3.6 Description of the Non-composite Bridge Models

The non-composite steel box bridge models that are used in this chapter to study the behavior of the straight and curved box girder are two-span single box girder bridges of total span length 320 ft. There are four types of bridges that are modeled in ANSYS for the current study.

1. Straight box shell model (M1)
2. Curved box shell model (M2)
3. Straight box beam model (M3)
4. Curved box beam model (M4)

A lateral bracing system is installed at the top flange level in the open-top box girder to form a quasi-closed box, thereby increasing the torsional stiffness. Crossed diagonal bracing systems are considered part of lateral bracing systems. Internal transverse bracing or internal cross frames are provided at regular intervals in the box. In negative–bending region, longitudinal stiffener is provided in the bottom flange. The cross section dimensions are as shown in Figure 8.

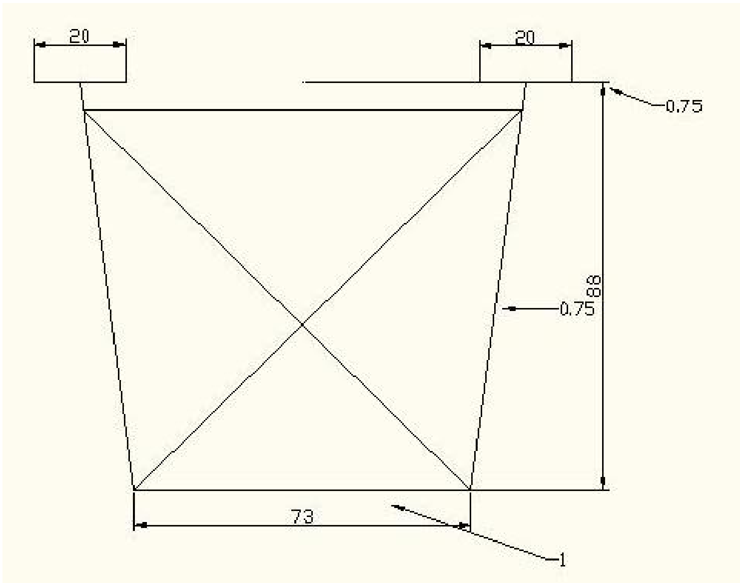

Figure 8 Cross Section Dimensions (in.) of the box

3.6.1 Straight box shell model (M1):

Straight box bridge model is made using shell 63 elements for webs, top flange, and the bottom flange. Shell63 elements are used as well to model longitudinal and transverse stiffeners and solid diaphragms at the supports location. The plate thicknesses and the material properties are required input for shell 63. Link 8 elements were used to model the top bracing truss and the cross frames (Figure 9) shows the completed straight box shell model. The stress contour of the straight box is as shown in the Figure 10.

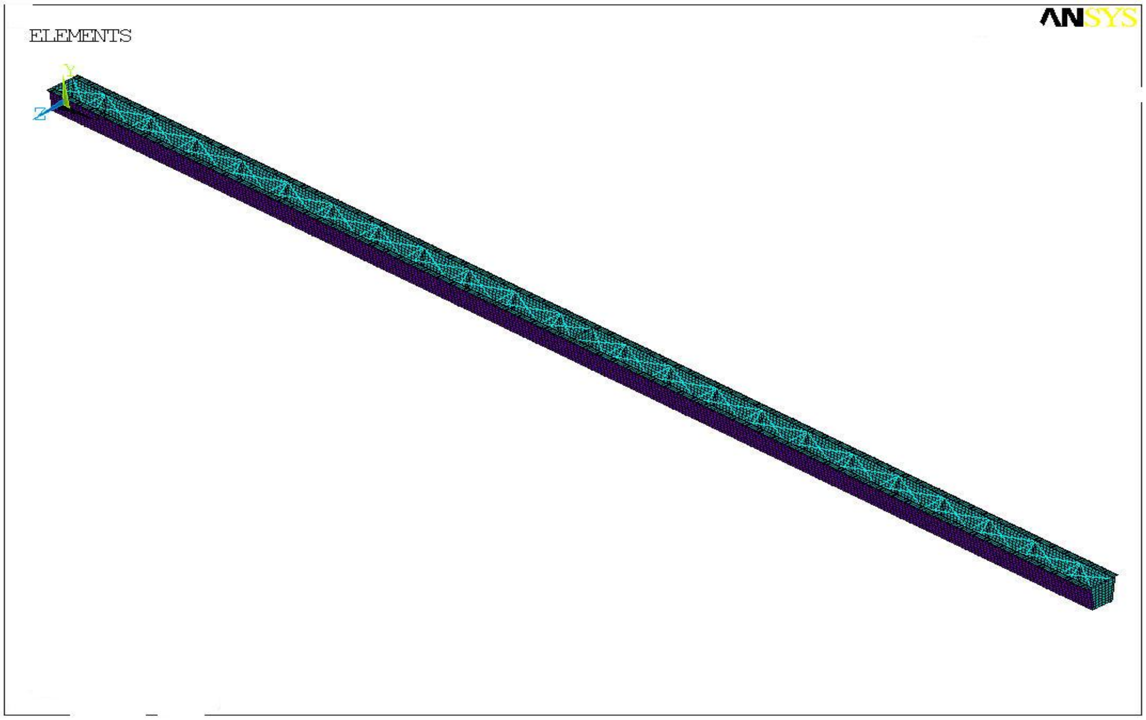

Figure 9 Isometric view of the straight box shell model

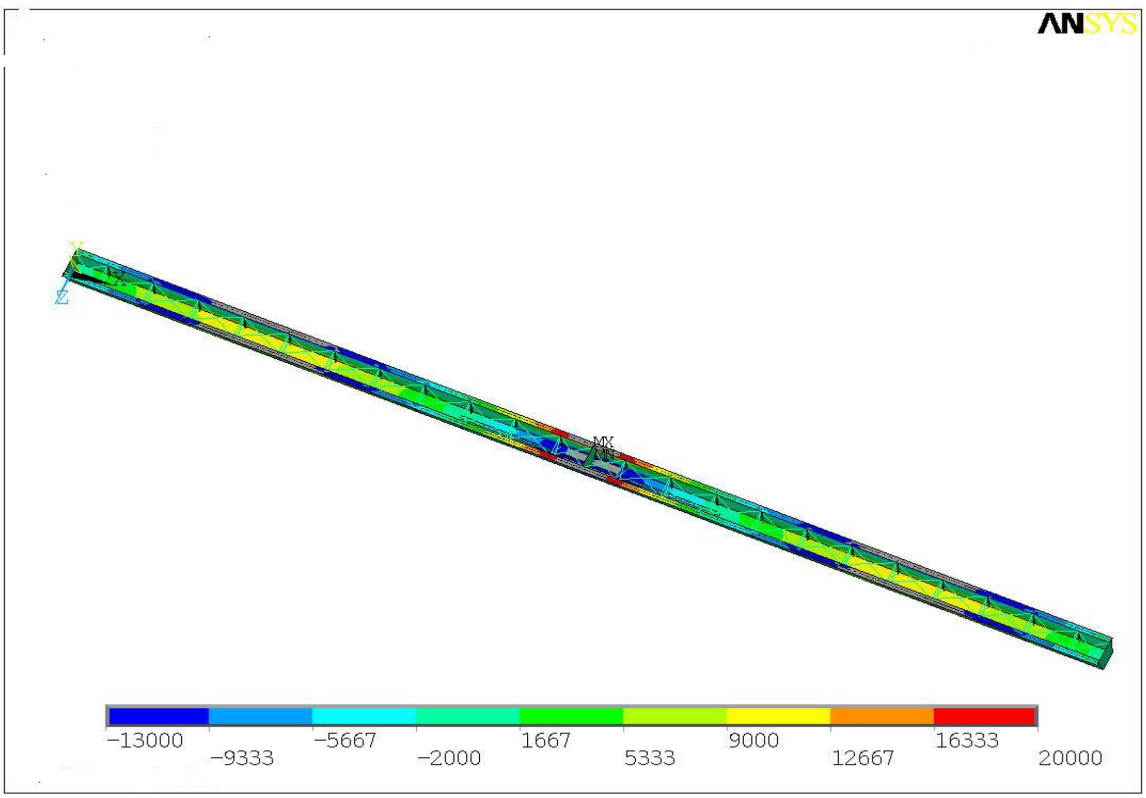

Figure 10 Stress contour of straight box shell model

3.6.2 Curved box shell model (M2):

Curved box having a radius of curvature of 300 ft is modeled using shell 63 similar to that of the straight box shell model. Just like the straight shell models, this one spanned 2-160 ft from support to support at center, this model was the same as those for model M1 with the exception that it is curved in plan. Since the models are used to study the comparisons, both the models are of the same general construction as mentioned in the description of the non-composite bridge model. Figure 11 shows the completed curved box shell model. The stress contour of the curved box is as shown in the Figure 12.

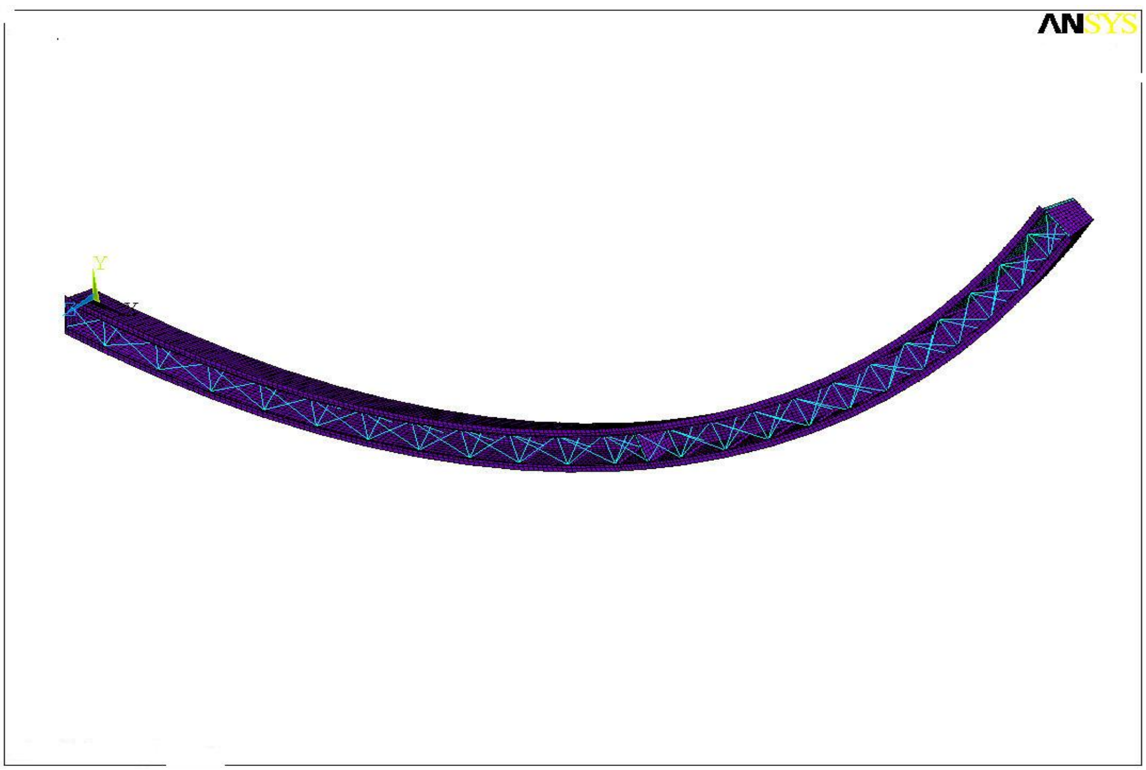

Figure 11 Isometric view of the curved box shell model

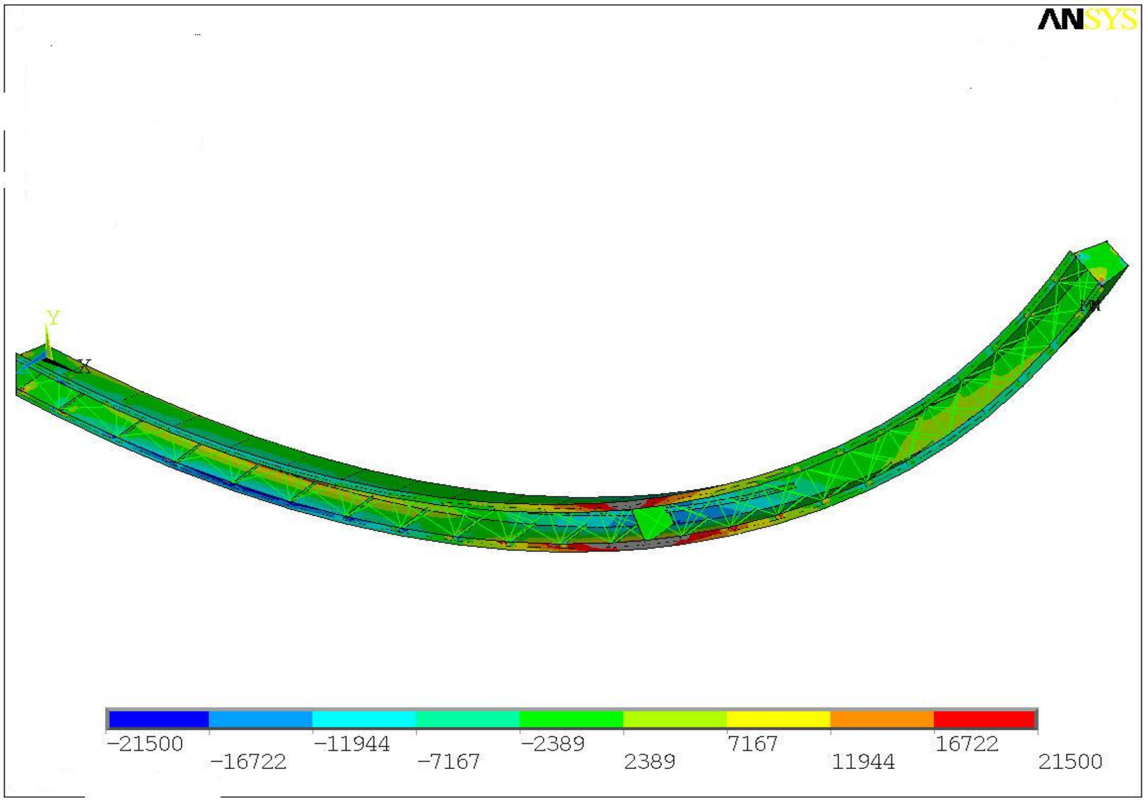

Figure 12 Stress contour of curved box shell model

3.6.3 Straight box beam model (M3)

The straight box beam model is made using beam 188 in the ANSYS using command prompt input line (refer Appendix for details). Beam 4 is used at supports to provide bearing support and apply boundary conditions. In the beam element model in ANSYS, the bracing and stiffener effects are not considered. Two cases are considered where in the first case two bearings are provided at all supports and in the second case two bearings are provided in the middle support (at pier) and single bearing is provided in end supports. Figure 13 shows the beam element model of the straight box showing boundary conditions and loading and Figure 14 shows the bending moment diagram throughout the span.

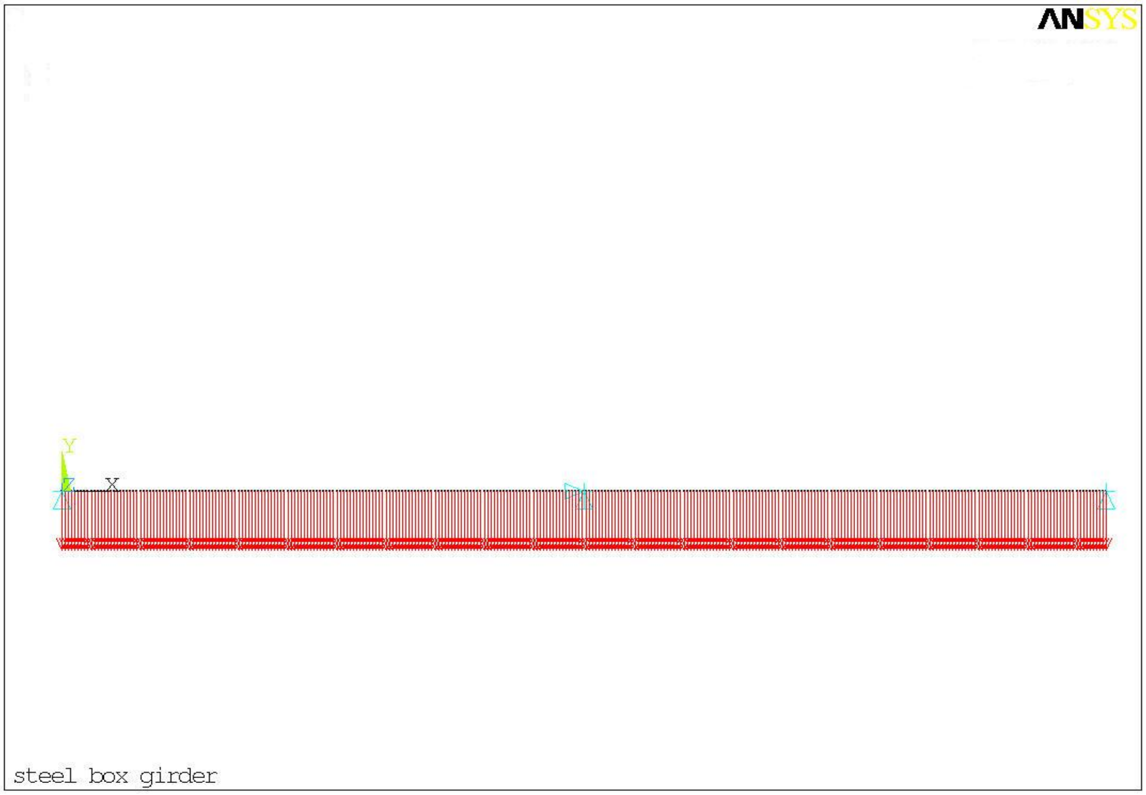

Figure 13Beam element model of straight box showing boundary conditions and loading

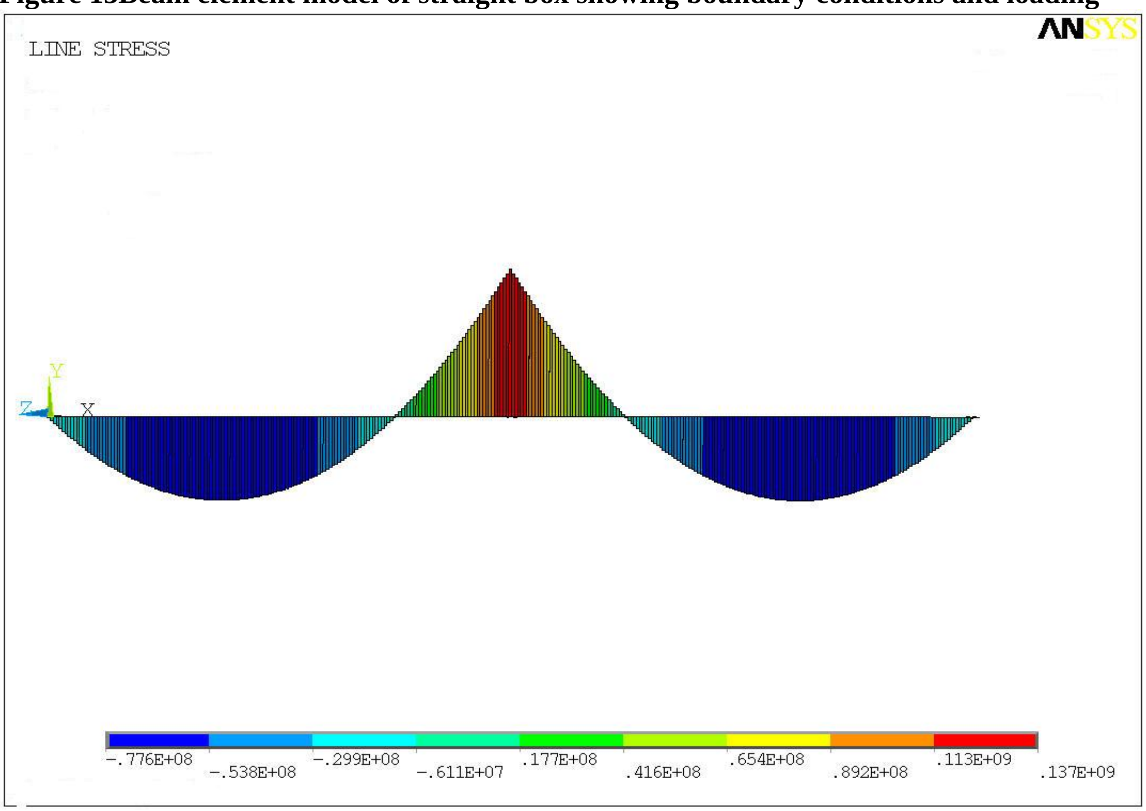

Figure 14 Bending moment diagram for M3

3.6.4 Curved box beam model (M4)

In this case also beam 188 is used to model the curved box. The model is similar to that of the straight box except that it is curved. The command prompt input line a box section is prepared first to obtain the properties of the box and then the section is deleted. Then changing the coordinate system to cylindrical coordinated the curved section is obtained (refer Appendix for details). In a study by C. C. Fu and Y. T Hsu (1993), they modified Vlasov's theory on curved thin-walled beams (originally developed for open sections such as I-girders) to represent the behavior of both open and closed sections, in order to develop a more exact horizontally curved beam finite element in a cylindrical coordinate system.

According to their study, by combining the effect of vertical deflection, the longitudinal torsion M_x, can be expressed as

$$M_x = M_{sv} + M_w = GK_t\,(\Phi`(x) - [\,\eta`(x)/R]) \\ - (EI_w/\mu)(\,\Phi```(x) - [\,\eta```(x)/R]) \text{-----------------------} \quad (3.9)$$

The longitudinal bending moment M_y is given by

$$M_y = EI_y\,(-\eta``(x) - \Phi(x)/R) \text{---} \quad (3.10)$$

Where,

M_{sv} = St Venant (or pure) torsion

M_w = warping torsion

GK_t = St Venant's torsional rigidity (central moment of inertia; sectional property associated with warping shear; warping shear decay coefficient)

Φ = DOF associated with torsion

η = DOF associated with longitudinal vertical shear

$\eta`$ = DOF associated with longitudinal bending

R = radius of the curved beam

EI_w = warping rigidity

EI_y = bending rigidity

Similar to the straight box beam model here beam 4 is also used at supports to provide bearings and apply boundary conditions and two cases are considered where in the first case two bearings are provided at all supports and in the second case two bearings are provided in the middle support (at pier) and single bearing is provided in end supports. Figure 15 shows the beam element model of the straight box and Figure 16 shows the bending moment diagram throughout the span.

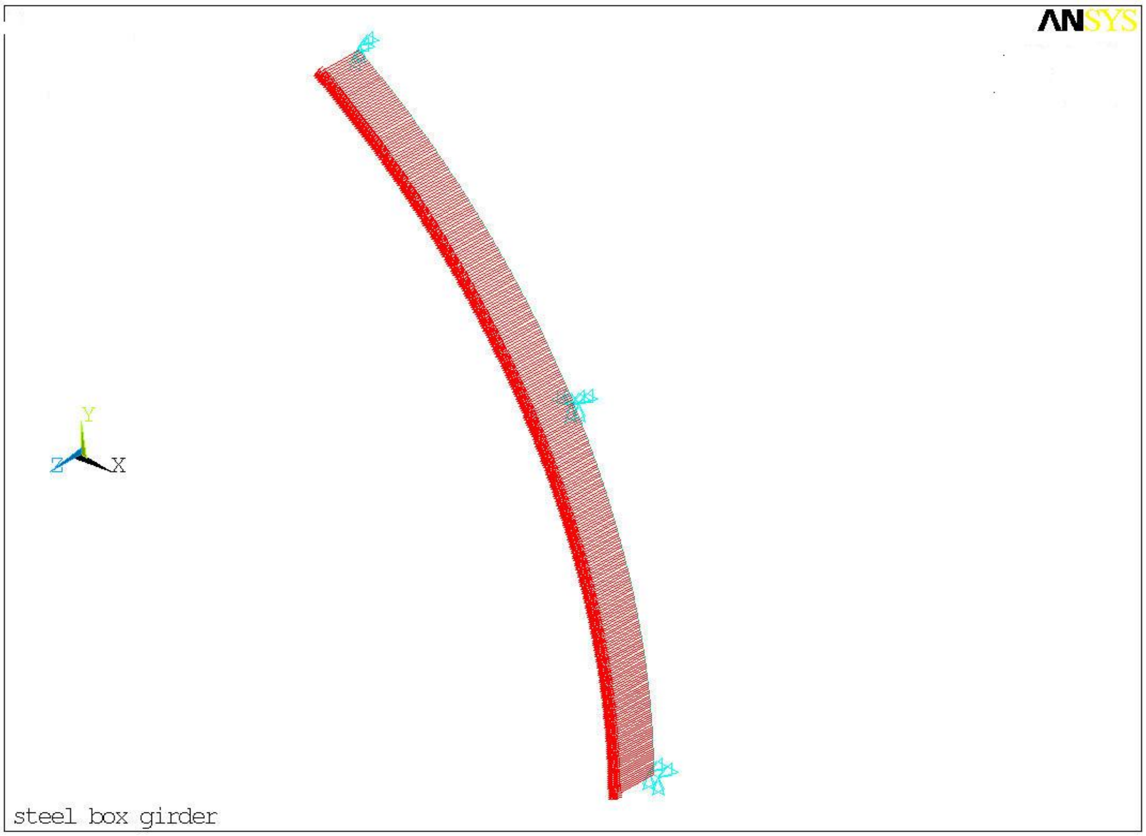

Figure 15 Beam element model of curved box showing boundary conditions and loading

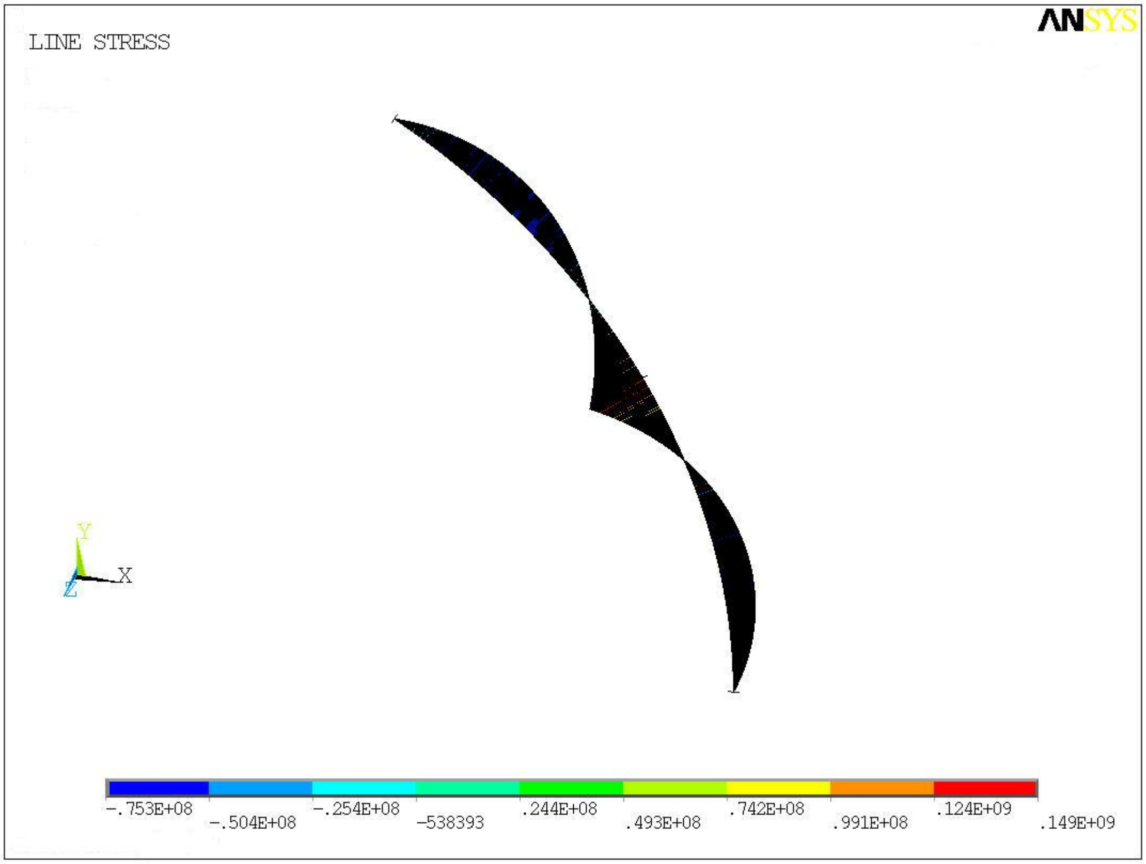

Figure 16 Bending moment diagram for M4

Chapter 4

RESULTS FROM THE BOX BRIDGE MODELS

4.1 Introduction

The design process of curved composite bridges involves tracking the stresses in the girders during different stages of construction and loading. Because curved bridges are typically less stable during erection than straight bridges, the construction of curved steel bridges is more complex than the construction of straight steel bridges. Therefore, to provide the more insight into the behavior of such structures comparisons were made on four non-composite box girder bridge models from Chapter 3, where the first two were shell element models (one straight and one curved box shell models), and the last two were beam element models of straight and curved box girders, respectively. This chapter summarizes and compares the results of the finite element analyses for the four different bridge models and BEST (Bridge Engineering Software Technology) Center program DESCUS-II, which is based on curved beam elements, models. It consists of the following comparisons:

- Comparison of curved box girder and straight box girder
- Comparison of beam models of ANSYS and DESCUS II
- Comparison of shell and beam ANSYS models

4.2 Results from the shell models

In order to compare the stress profiles of the straight and curved box bridges (models M1 and M2), from each model, stresses from the two sections are considered, the first section being the maximum positive moment section and the second one being the maximum negative moment section. Figures 17 and 23 show the position of the node numbers for in the maximum positive and maximum negative moment section of the straight box bridge and figures. 18 to 22 and figures 24 to 28 show the corresponding stress profiles. Similarly, figures 29 and 35 show the position of the node numbers in the maximum positive and maximum negative moment section respectively of the curved box bridge and figures 30 to 34 and figures 36 to 40 show the corresponding stress profiles.

4.2.1 Stresses in members of straight box girder shell model on the maximum positive moment region:

In this section, stresses in members of straight box girder shell model on the maximum positive moment region are shown in figures 17 to 22.

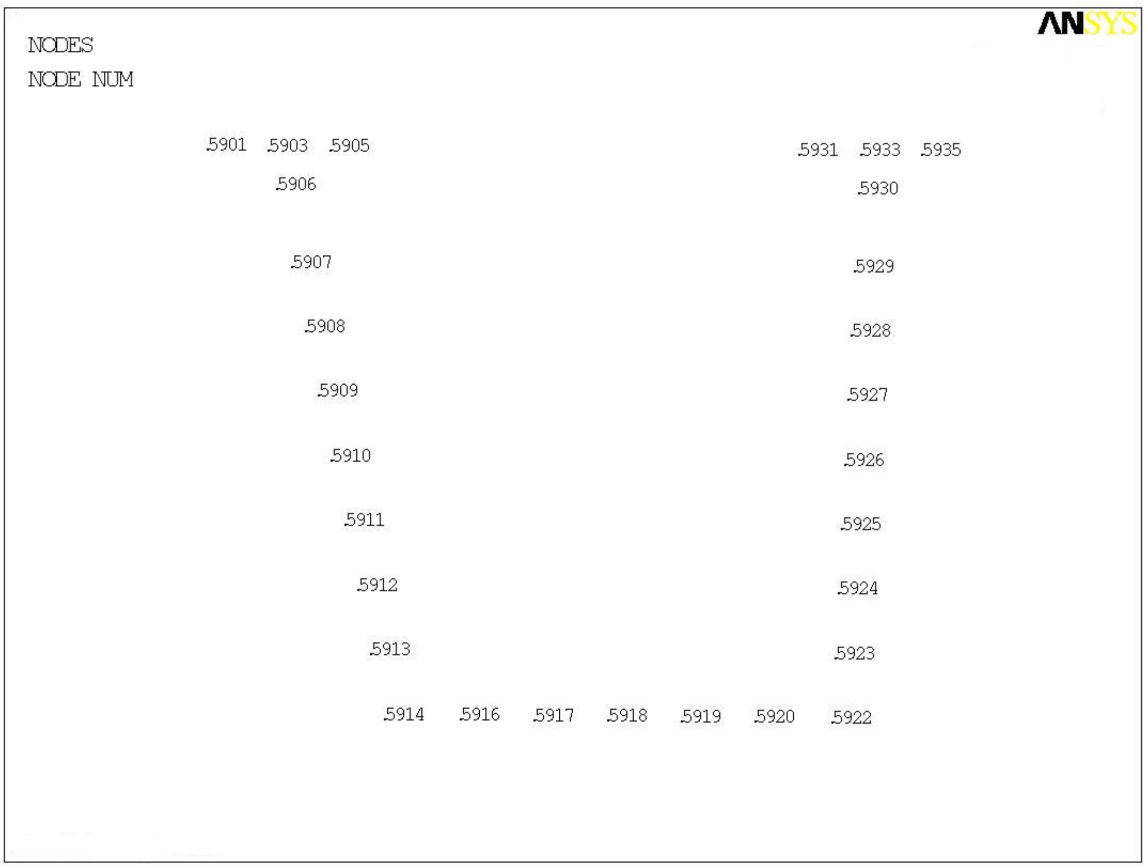

Figure 17 Cross section of model M1 showing the node numbers at maximum positive moment section

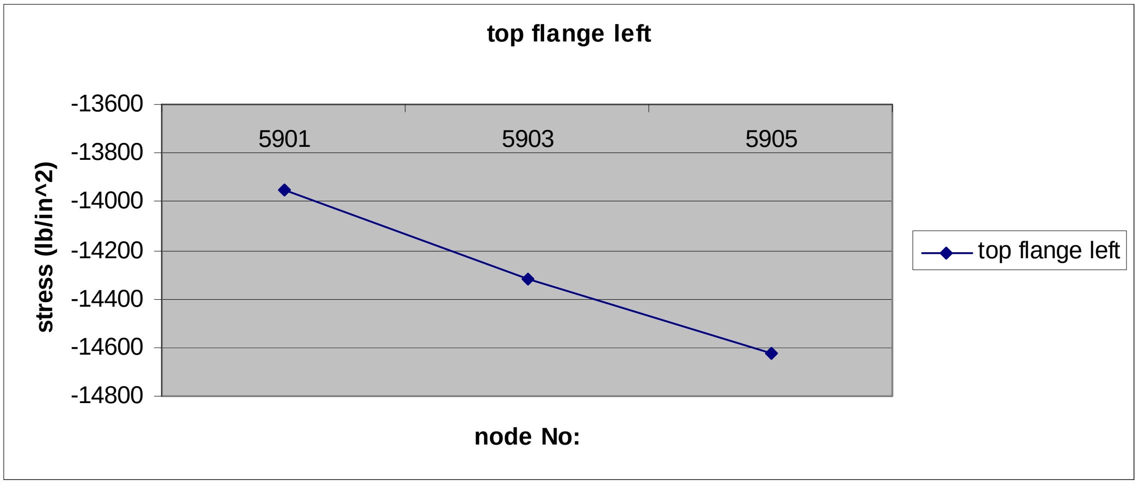

Figure 18 Stresses in top flange left of model M1 at max positive moment section

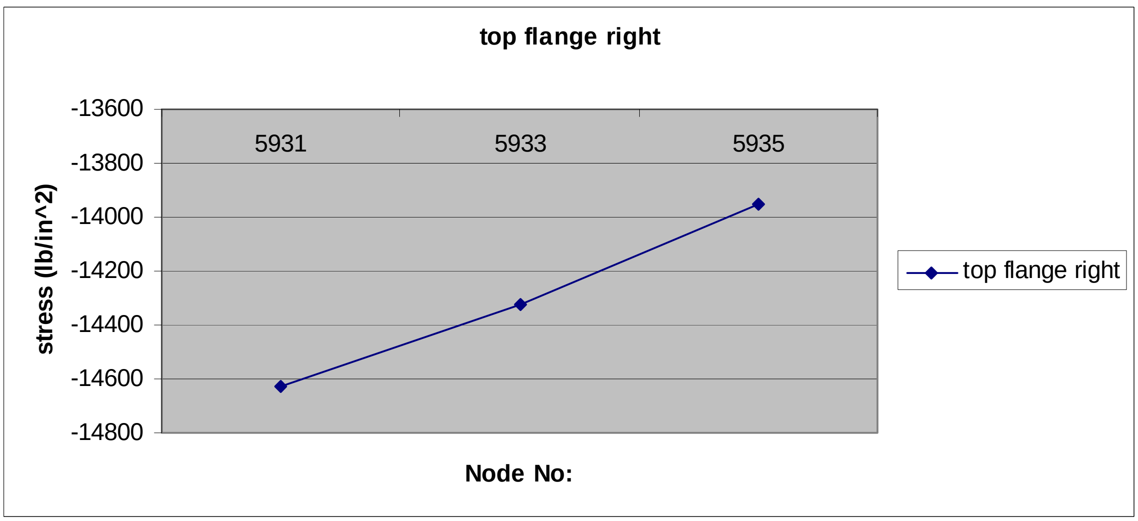

Figure 19 Stresses in top flange right of model M1 at max positive moment section

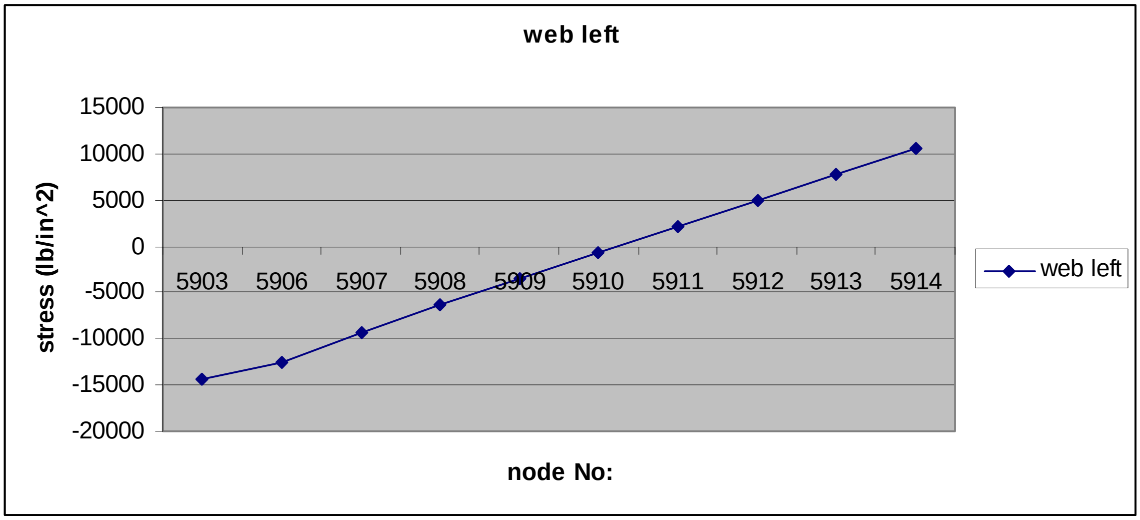

Figure 20 Stresses in web left of model M1 at max positive moment section

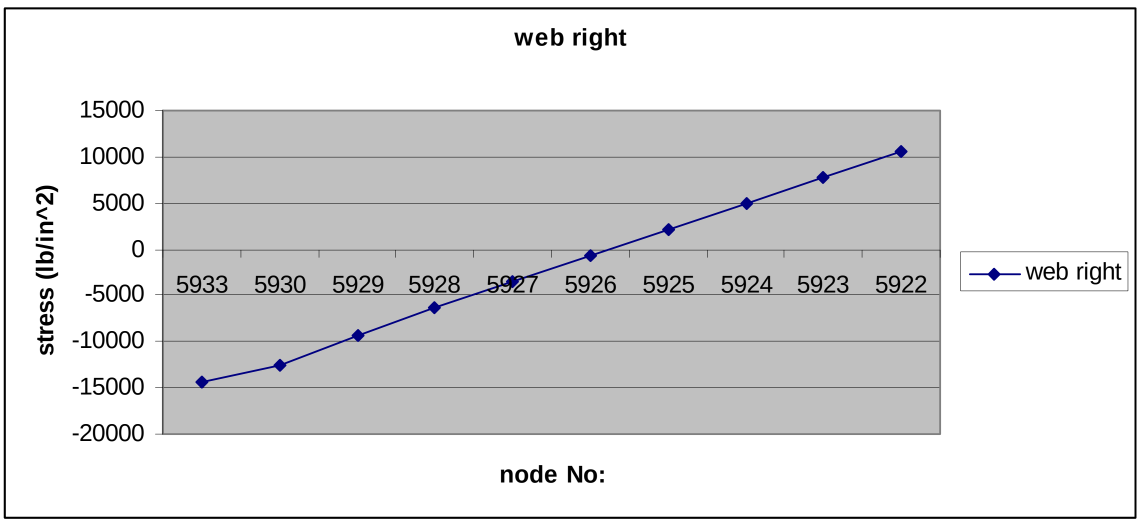

Figure 21 Stresses in web right of model M1 at max positive moment section

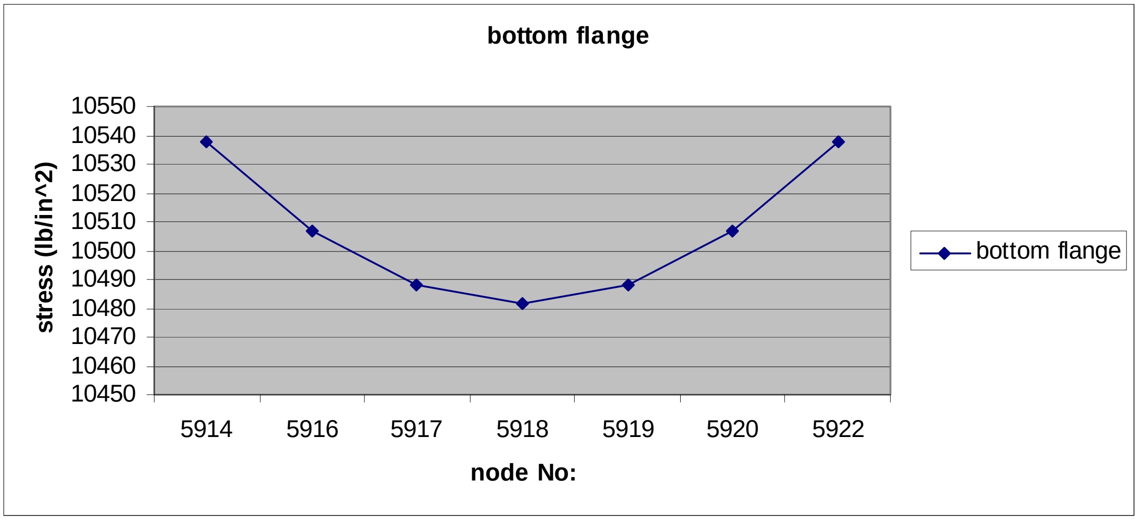

Figure 22 Stresses in bottom flange of model M1 at max positive moment section

4.2.2 Stresses in members of straight box girder shell model on the maximum negative moment region:

In this section, stresses in members of straight box girder shell model on the maximum negative moment region are shown in figures 23 to 28.

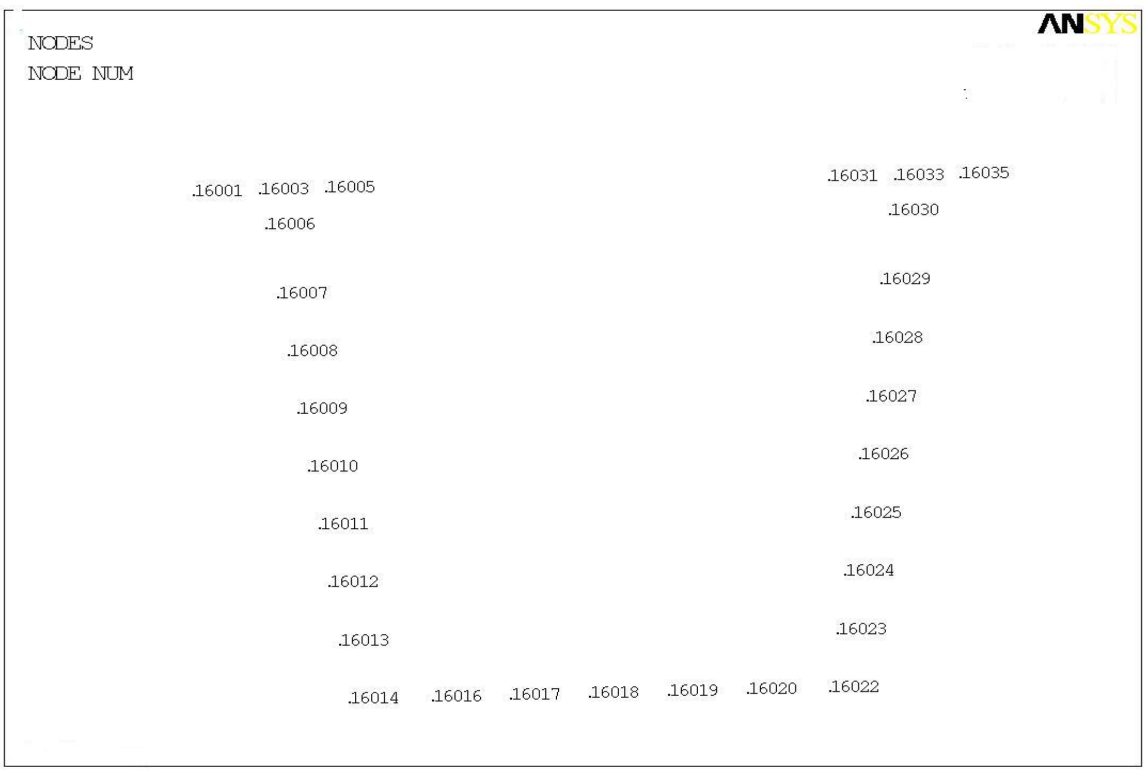

Figure 23 Cross section of box of model M1 showing the node numbers at max negative moment section

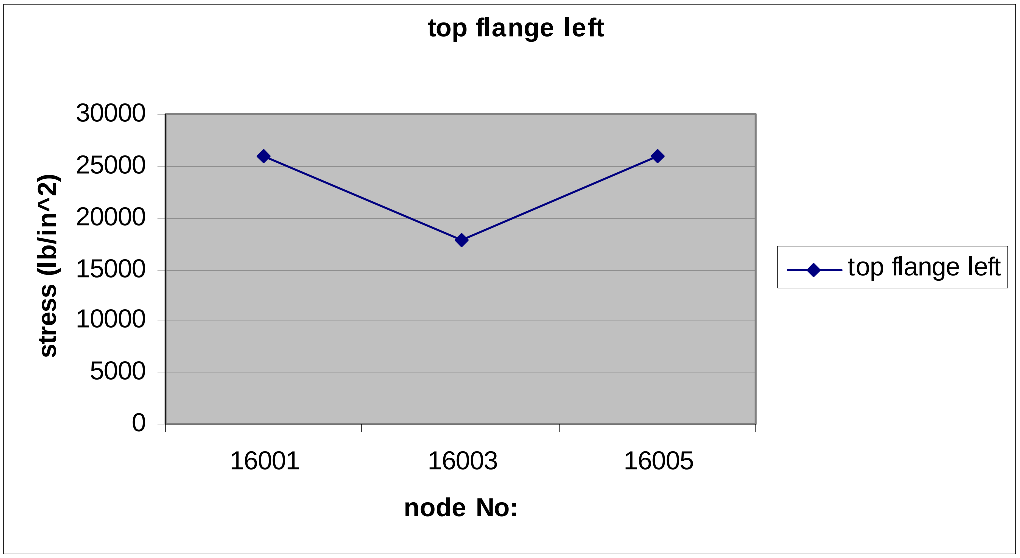

Figure 24 Stresses in top flange left of model M1 at max negative moment section

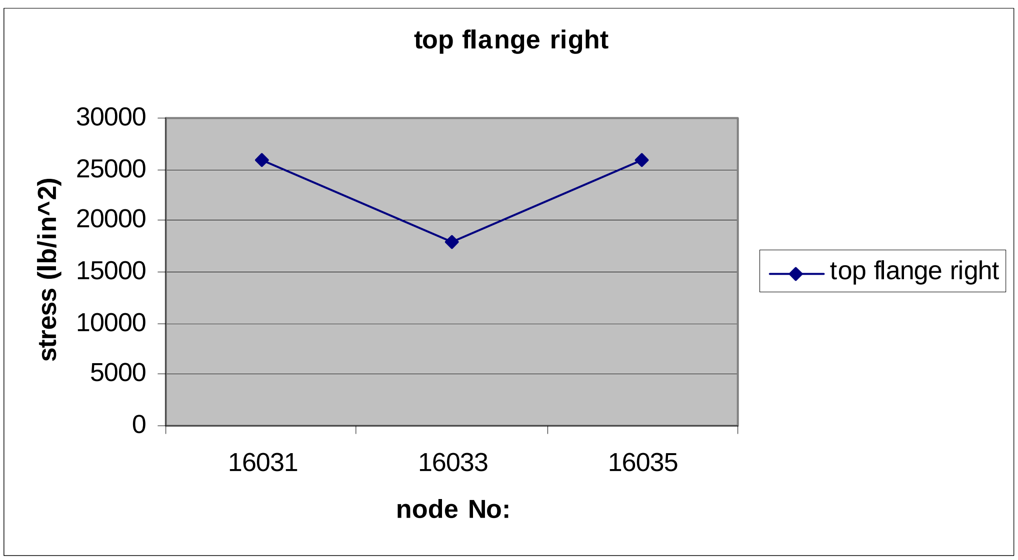

Figure 25 Stresses in top flange right of model M1 at max negative moment section

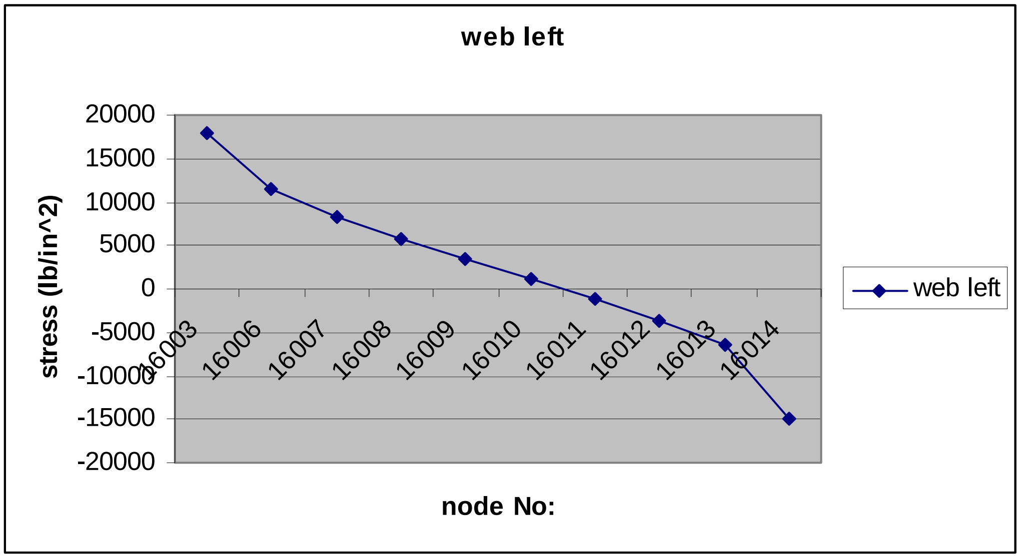

Figure 26 Stresses in web left of model M1 at max negative moment section

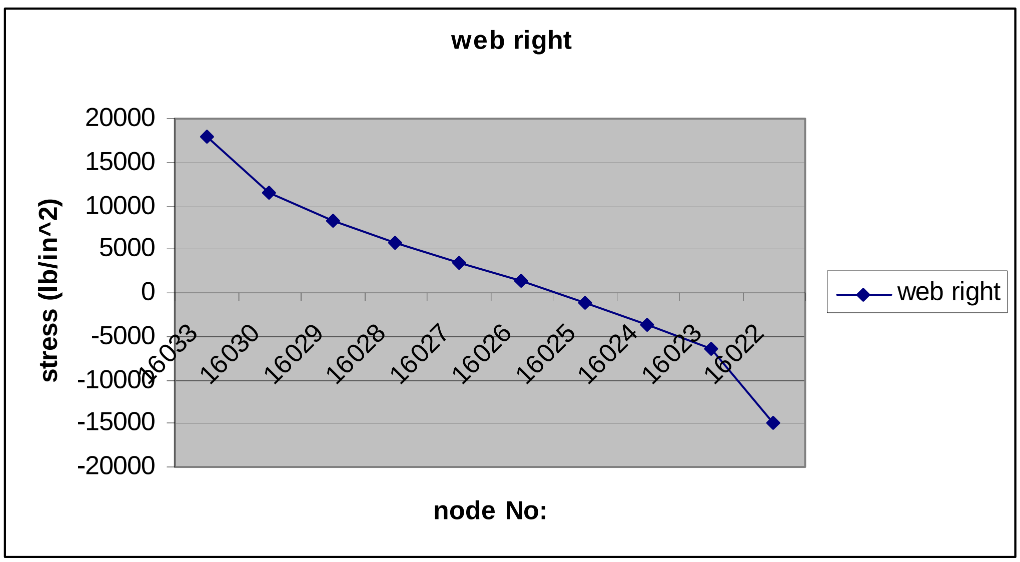

Figure 27 Stresses in web right of model M1 at max negative moment section

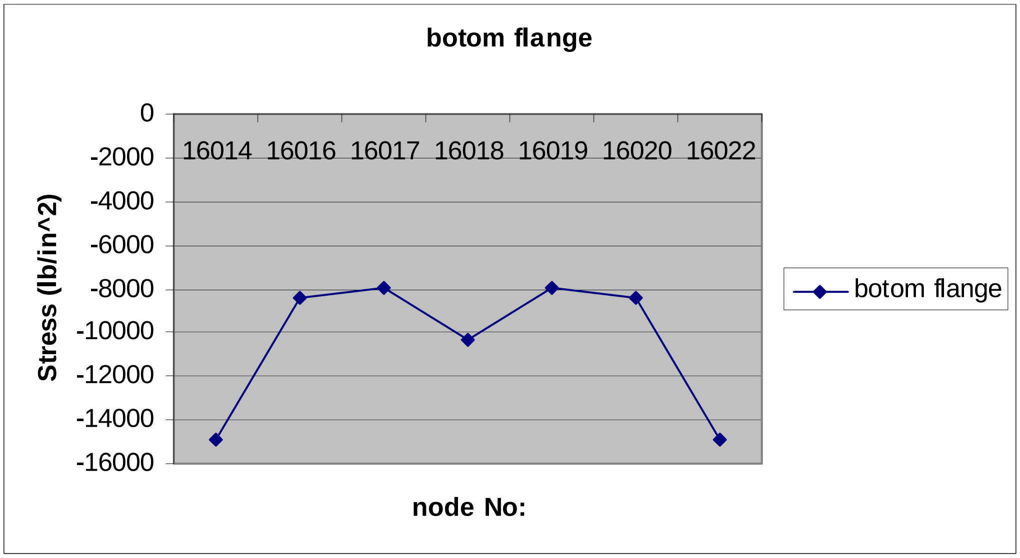

Figure 28 Stresses in bottom flange of model M1 at max negative moment section

4.2.3 Stresses in members of curved box girder shell model on the maximum positive moment region:

In this section, stresses in members of curved box girder shell model on the maximum positive moment region are shown in figures 29 to 34.

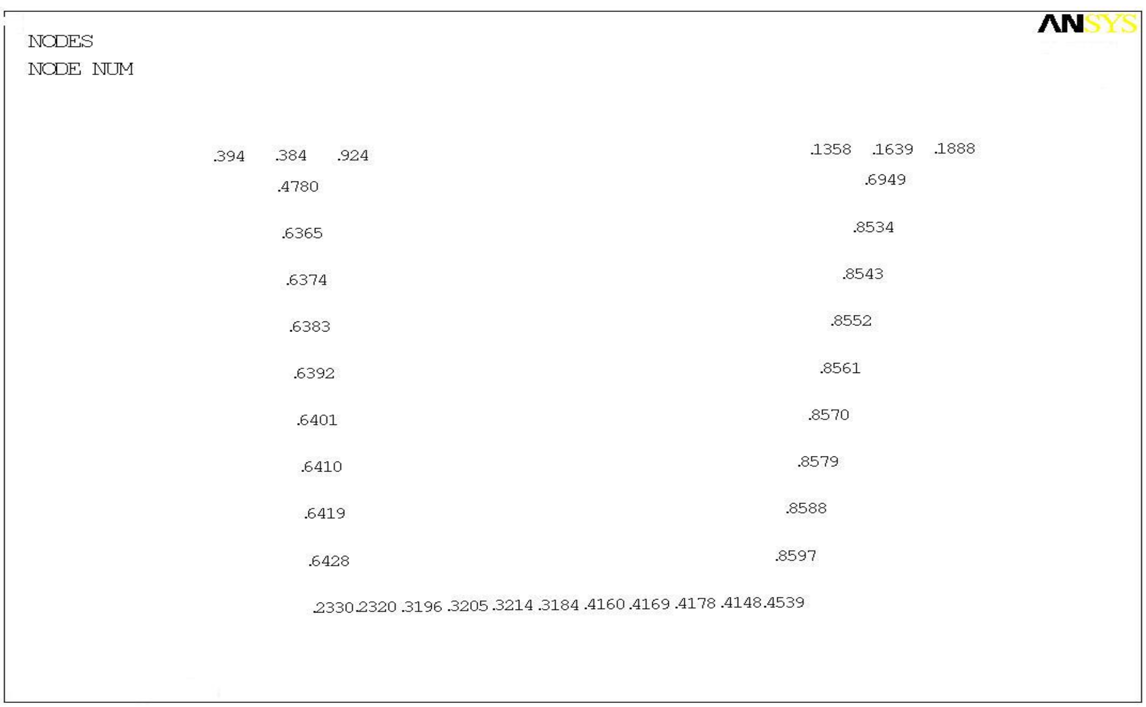

Figure 29 Cross section of model M2 showing the node numbers at max positive moment section

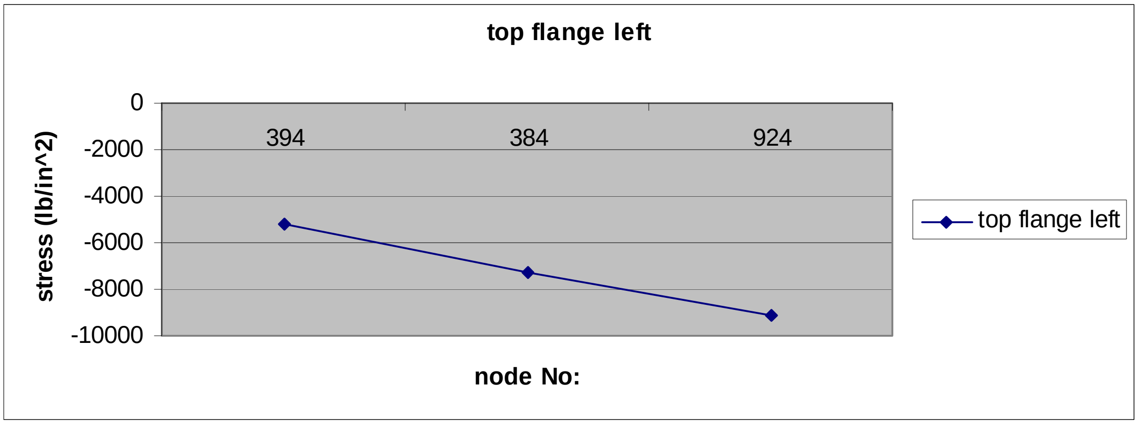

Figure 30 Stresses in top flange left of model M2 at max positive moment section

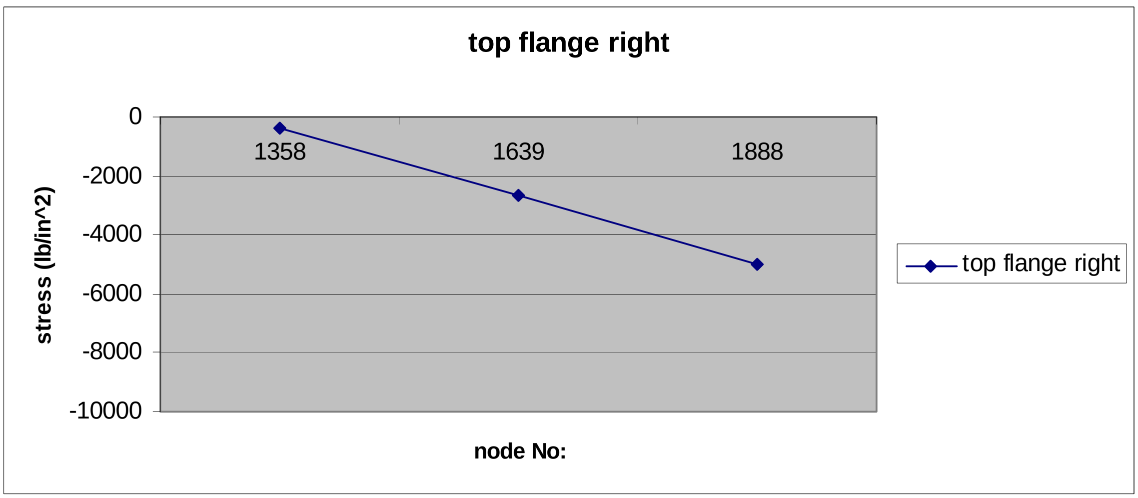

Figure 31 Stresses in top flange right of model M2 at max positive moment section

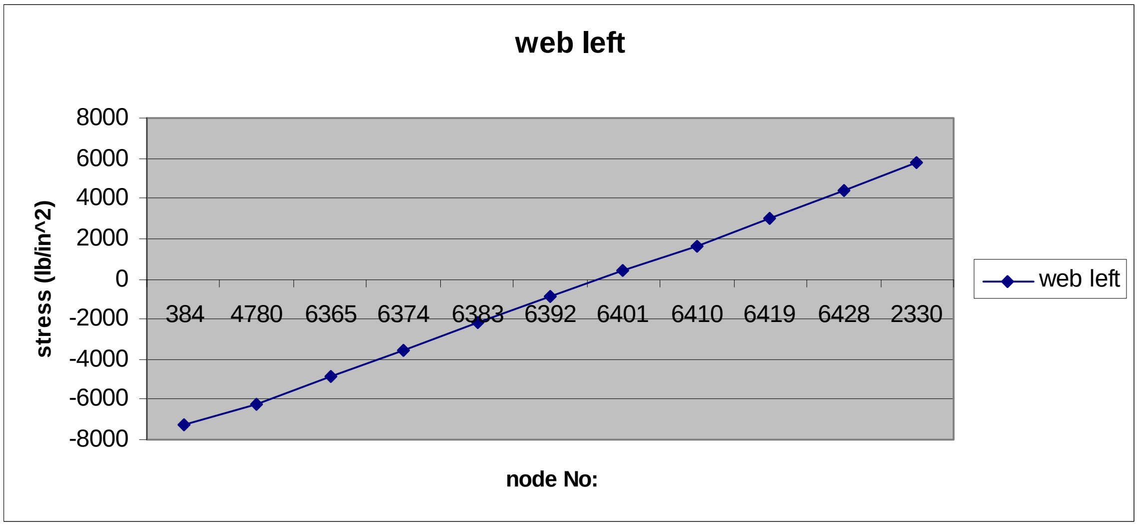

Figure 32 Stresses in web left of model M2 at max positive moment section

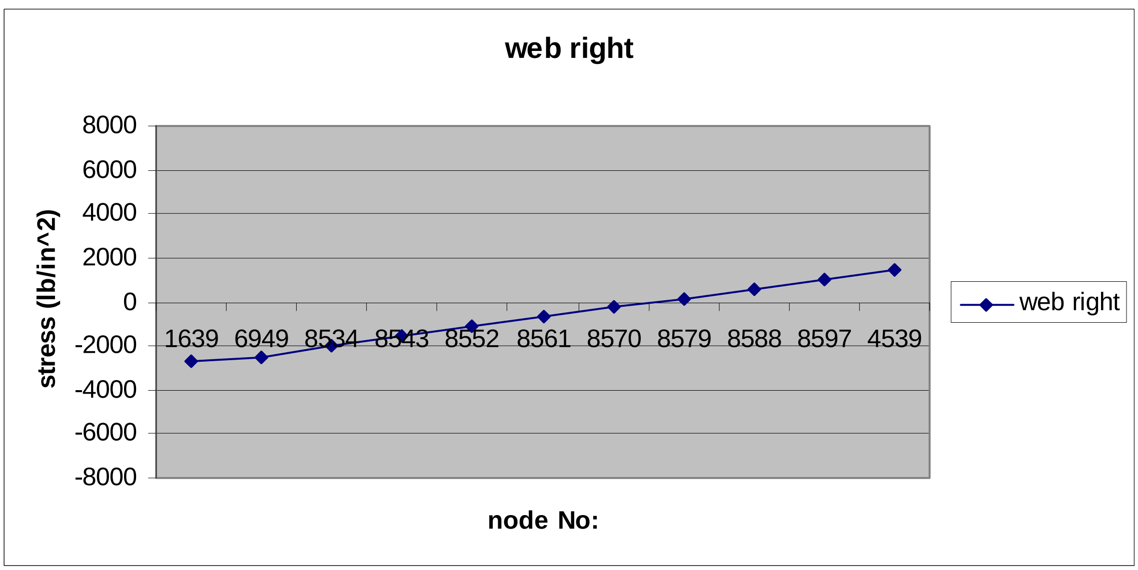

Figure 33 Stresses in web right of model M2 at max positive moment section

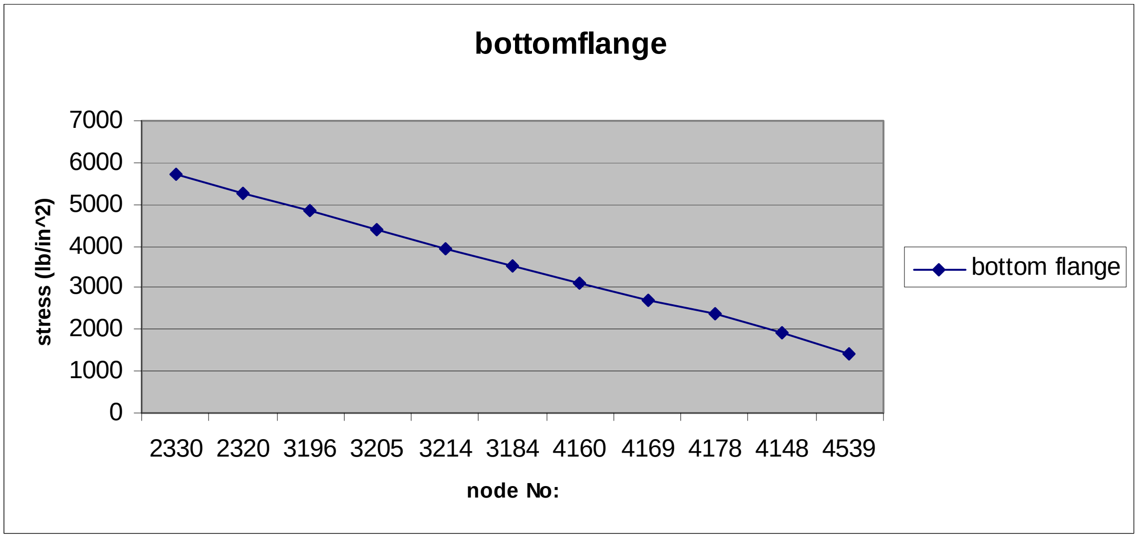

Figure 34 Stresses in bottom flange of model M2 at max positive moment section

4.2.4 Stresses in members of curved box girder shell model on the maximum negative moment region:

In this section, stresses in members of curved box girder shell model on the maximum negative moment region are shown in figures 35 to 40.

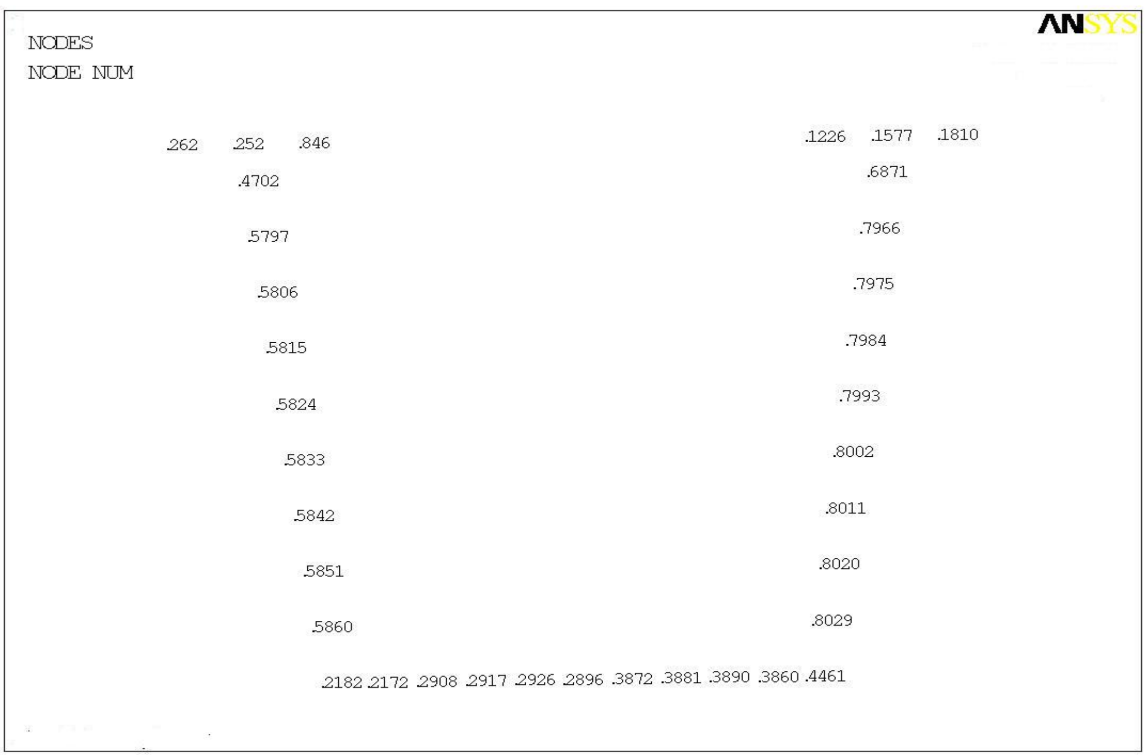

Figure 35 Cross section of model M2 showing the node numbers at maximum negative moment section

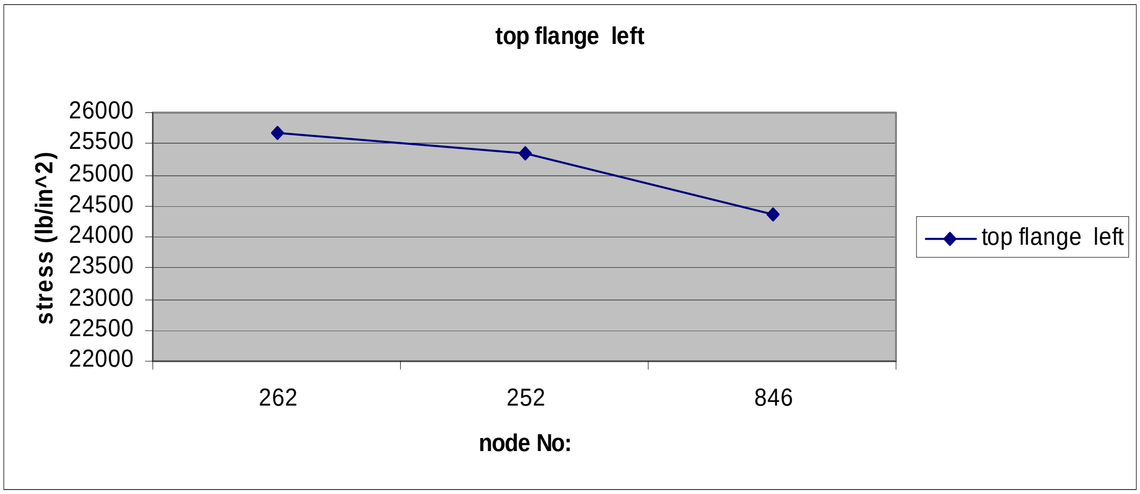

Figure 36 Stresses in top flange left of model M2 at max negative moment section

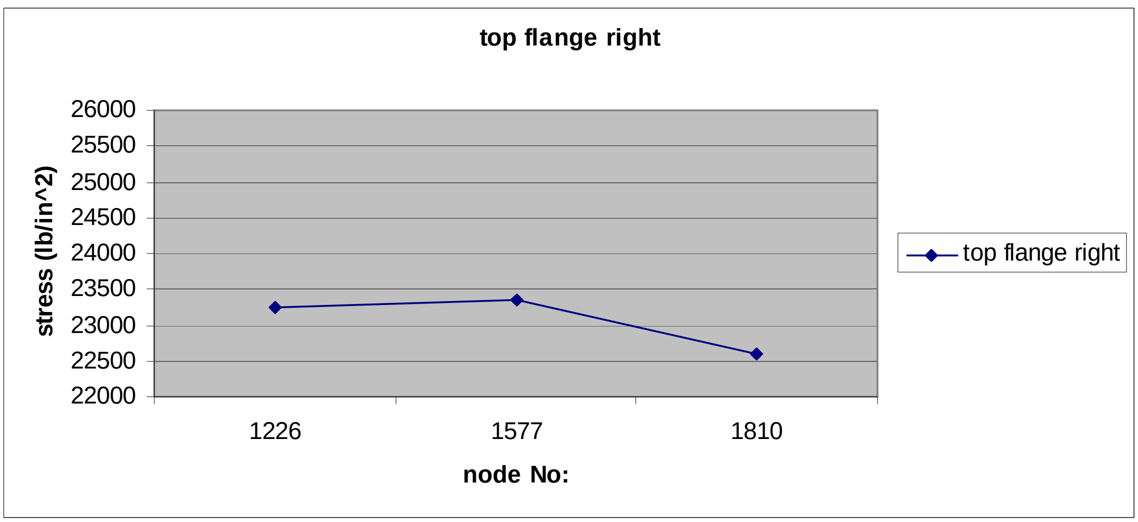

Figure 37 Stresses in top flange right of model M2 at max negative moment section

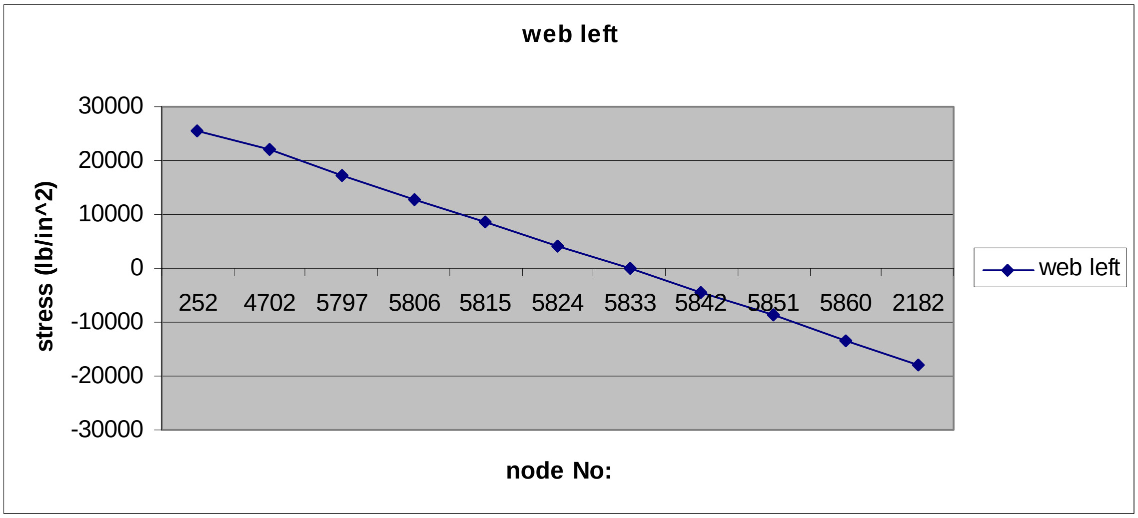

Figure 38 Stresses in web left of model M2 at max negative moment section

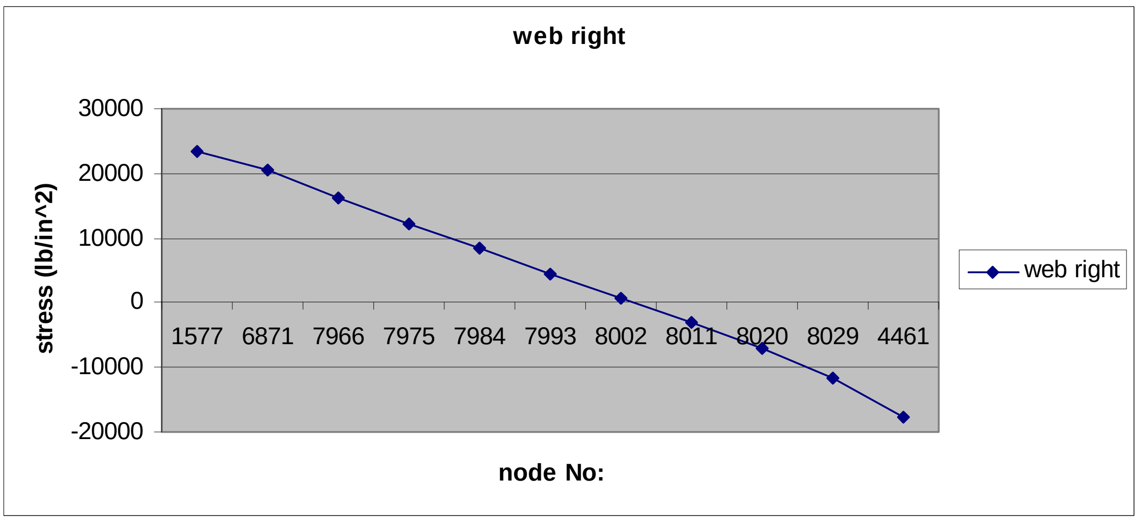

Figure 39 Stresses in web right of model M2 at max negative moment section

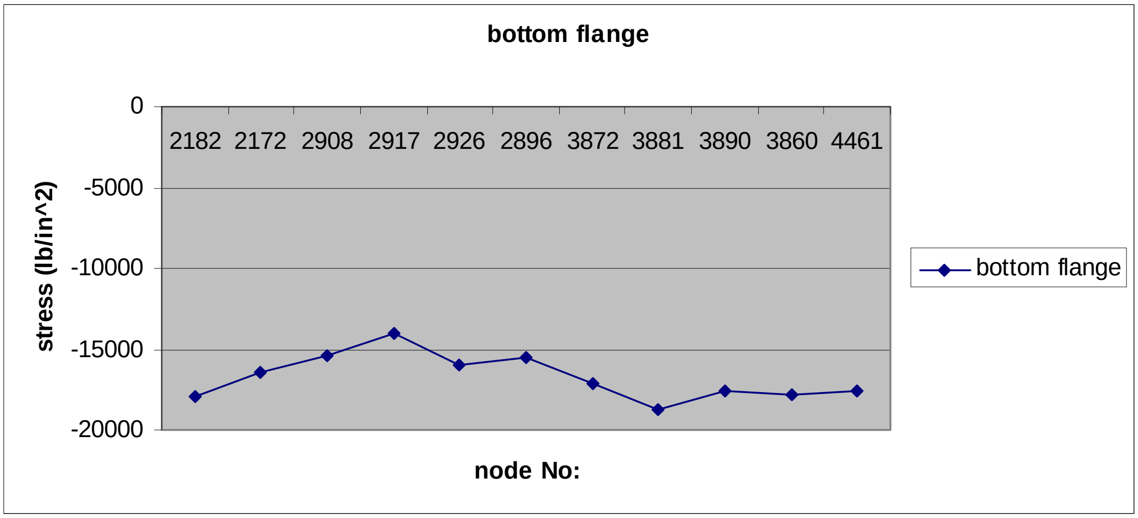

Figure 40 Stresses in bottom flange of model M2 at max negative moment section

4.3 Comparison tables

The behavior of the box girder bridge is investigated by comparing the different models. The comparison of the beam models of ANSYS and the BEST Center program DESCUS-II of the straight and curved box with different boundary conditions are presented in the following tables 1 to 4. Table 5 and 6 shows the comparison between beam and shell models of ANSYS for straight and curved box girder bridges respectively.

Table 1 Comparison between beam models of ANSYS and DESCUS of straight box girder: Boundary condition: Twin bearings at all supports

Support Reactions (kips)			Moment due to DL (lb-in)			Stress due to bending (kip/in^2)		
Location	DESCUS	ANSYS	Location	DESCUS	ANSYS	Location	DESCUS	ANSYS
at support 1	205.3	217.07	M at 4	7.4E+07	7.7E+07	Top DL Stress at 4	-14.24	-14.16
at support 2	684.4	715.71				Bottom DL Stress at 4	9.55	10.266
at support 3	205.3	217.06	M at 10	-1.3E+08	-1.37E+08	Top DL Stress at 10	25.43	25.07
						Bottom DL Stress at 10	-17.05	-18.17

Table 2 Comparison between beam models of ANSYS and DESCUS of straight box girder: Boundary condition: Twin bearings at support 2 only

Support Reactions (kips)			Moment due to DL (lb-in)			Stress due to bending (kip/in^2)		
location	DESCUS	ANSYS	Location	DESCUS	ANSYS	Location	DESCUS	ANSYS
at support 1	205.3	217.07	M at 4	7.4E+07	7.7E+07	Top DL Stress at 4	-14.24	-14.16
at support 2	684.4	715.71				Bottom DL Stress at 4	9.55	10.266
at support 3	205.3	217.06	M at 10	-1.3E+08	-1.37E+08	Top DL Stress at 10	25.43	25.07
						Bottom DL Stress at 10	-17.05	-18.17

Table 3 Comparison between beam models of ANSYS and DESCUS of curved box girder: Boundary condition: Twin bearings at all supports

Support Reactions (kips)			Moment due to DL (lb-in)			Stress due to bending (kip/in^2)		
location	DESCUS	ANSYS	Location	DESCUS	ANSYS	Location	DESCUS	ANSYS
at support 1	202.2	211.973	M at 4	7.2E+07	7.5E+07	Top DL Stress at 4	-13.98	-13.69
at support 2	680.8	729.61				Bottom DL Stress at 4	9.37	9.926
at support 3	211.9	208.255	M at 10	-1.4E+08	-1.51E+08	Top DL Stress at 10	26.67	27.616
						Bottom DL Stress at 10	-17.88	-20.018

Table 4 Comparison between beam models of ANSYS and DESCUS of curved box girder: Boundary condition: Twin bearings at support2 only

Support Reactions (kips)			Moment due to DL (lb-in)			Stress due to bending (kip/in^2)		
location	DESCUS	ANSYS	Location	DESCUS	ANSYS	Location	DESCUS	ANSYS
at support 1	202.2	206.09	M at 4	6.5E+07	6.8E+07	Top DL Stress at 4	-12.66	-12.43
at support 2	680.8	740.15				Bottom DL Stress at 4	8.49	9.011
at support 3	211.9	203.6	M at 10	-1.5E+08	- 1.67E+08	Top DL Stress at 10	29.84	30.643
						Bottom DL Stress at 10	-20	-22.21

Table 5 Comparison of stresses between shell and beam ANSYS models of straight box girder:

Stress due to bending (kip/in^2)		
Location	Shell model	Beam model
Top DL Stress at 4	-14.63	-14.162
Bottom DL Stress at 4	10.53	10.266
Top DL Stress at 10	25.9	25.07
Bottom DL Stress at 10	-14.9	-18.173

Table 6 Comparison of stresses between shell and beam ANSYS models of curved box girder:

Stress due to bending (kip/in^2)		
Location	Shell model	Beam model
Top DL Stress at 4	-9.124	-13.693
Bottom DL Stress at 4	5.73	9.926
Top DL Stress at 10	25.68	27.616
Bottom DL Stress at 10	-17.96	-20.018

4.4 Discussion of the results

Results at the two different cross sections are used to compare the stresses and moments one being the maximum positive moment section (location 4) and the other being maximum negative moment section (location 10). Since the main objective of the current study is to analyze and study the behavior of box girder bridges, the following comparisons of the different models are made:

4.4.1 Comparison of straight box girder and curved box girder

Details of the bridge models of curved and straight box is presented in the previous chapter. To compare the box bridges models, the same modeling techniques were employed for both the straight and curved bridge models. As expected, the maximum longitudinal tensile stress (maximum positive moment section) in the steel bottom flange was obtained at the middle of the first span, while the maximum longitudinal compression stress (maximum negative moment section) at the bottom steel flanges was obtained at the interior support line. Longitudinal stresses at cross sections of these two locations were obtained. It should be noted that the stresses are recorded at node points and the stress profiles were made to make the comparison of the stresses and thus understand their behavior. It can be noted that in the straight box girder the stress distribution is symmetric from one end to the other (in left and right top flanges and web) whereas in curved box girder the stress profile is not symmetric. The left top flange and left web side show much higher stress values than the right side.

4.4.2 Comparison of beam models of DESCUS II and ANSYS:

Results obtained from the beam model of DESCUS- II were compared with those obtained from beam element model of finite element software "ANSYS". The results from the non composite straight and curved box models presented in this chapter include support reactions, moments and longitudinal stresses for the two cases of boundary conditions, where in the first case twin bearings are provided at all three supports and in second case twin bearings are provided only for support 2 (interior support) whereas single bearing are provided at support 1 and 3 (exterior supports). Comparison of the results obtained from DESCUS –II and those obtained by the finite element model shows that there is good agreement between the two set of results.

Results for support reactions are shown in tables 1 to 4. The maximum support reaction occurred at the interior support, as expected. It can be observed that the values deviate by less than 5% for the support reactions. The moments and stresses are obtained from two different cross sections. Location 4 corresponds to maximum positive moment cross section where the moments deviate by 4% approx, the stresses at top deviate by 1.5% and the stresses at bottom deviate by 6 %. Location 10 corresponds to maximum negative moment cross section where the moments deviate by 7 % approx, the stresses at top deviate by 1.5% and the stresses at bottom deviate by 10%.

4.4.3 Comparison of shell and beam ANSYS models:

The stresses are obtained at the same two cross sectional locations mentioned earlier (locations 4 and 10) and their values are compared. In the beam model the stresses are obtained only at top and bottom whereas in shell model the stresses can be obtained at every node point (see stress profiles). The shell and beam element models comparison is shown in table 5 and 6. It is interesting to note that in the case of straight box girder there is very good agreement between the two set of results whereas in the case of curved box model stresses model stresses show some deviation.

Chapter 5

PARAMETRIC STUDY

5.1 Introduction

In this chapter the parametric study was conducted to obtain information about the curved box girder bridges that could aid in understanding the factors affecting the stress distribution. The parametric investigation is performed utilizing three dimensional FEM model of the curved steel box (model M2) from Chapter 3. The objectives of this parametric study were to:

1. Determine the effect of spacing of bracings on the stresses and
2. Determine the effect of longitudinal stiffeners on the bottom plate stresses.

5.2 Bracing Effect:

In terms of bracing effect, internal bracings improve the ability of the cross section to transfer loads from one girder to an adjacent one. Furthermore it is possible to reduce the adverse effects of distortional warping to acceptable or negligible levels by providing an adequate number of cross frames (Oleinik and Heins 1975). Figure 41, which is constructed using data in Oleinik and Heins , quantifies the statement about distortional warping. The ratio between normal stresses resulting from distortional warping and flexural stresses drops rapidly as the number of cross frames increases. However, the addition of external bracing (between boxes) did not have a significant effect on the stress distribution. Based on such findings, even though the presence of external bracing was insignificant, for consistency reasons, this parametric study used internal and external bracing spaced at regular intervals within the box girders.

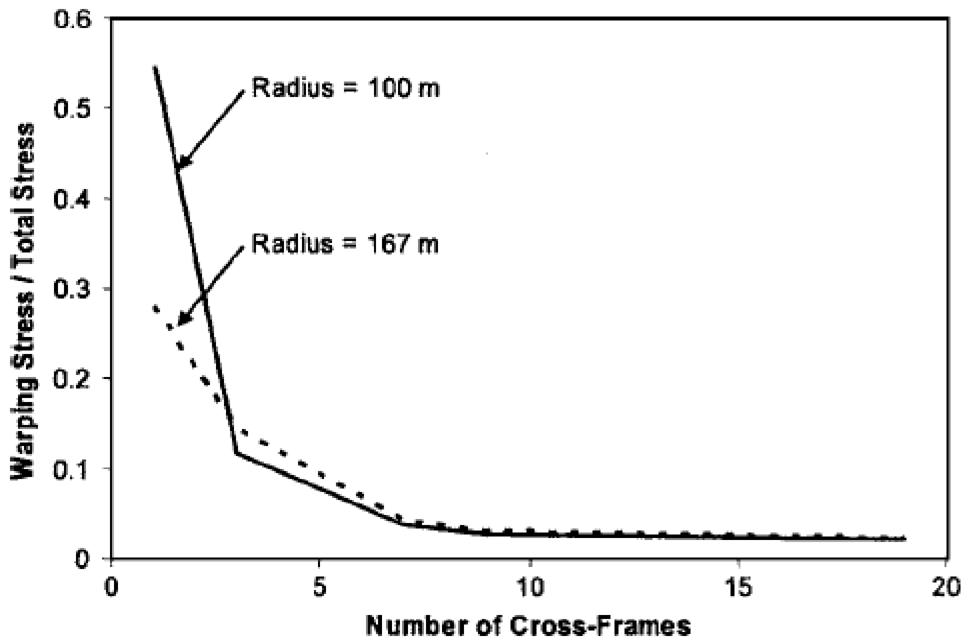

Figure 41 Ratio of warping normal stress to total stress as a function of number of cross frames

To study the effect of internal cross bracings on the stresses, the shell element FEM model of the two span curved box girder (M2) from the previous chapter is used. Three cases are considered in which the spacing of the cross frames is reduced in each case such that in case I there are 23 bracings, in case II there are 10 bracings and case III consists of 4 bracings. Figure 42 shows the position of the nodes along the cross section. Table 7 shows the variation of stresses against the number of bracings in the maximum positive moment region and Table 8 shows the variation of stresses against the number of bracings in the maximum negative moment region. It can be observed that the stresses decrease with increasing number of cross frames.

Figure 42 Position of the nodes

Table 7 Variation of stresses against the number of bracings in Max +M region

Nodes	Normal stress in long direction S_x (lb/in^2)			Normal stress in transverse direction S_y (lb/in^2)		
position No:	23 bracings	10 bracings	4 bracings	23 bracings	10 bracings	4 bracings
1	-10745.5	-13707	-21755.5	104.728	188.523	233.155
2	-10847	-11189.1	-12441.3	1234.557	1366.927	1566.175
3	-21790.5	-19547	-14745.5	-751.02	-729.635	-629.052
4	-320.3	-2339.05	-7771.05	5.485	-4.015	-73.99
5	-4569.34	-4206.98	-3315.65	4.538333	-82.6267	-132.334
6	-13676	-10604	-2433.9	-414.655	-421.195	-267.465
7	9377.098	10097.55	11912.65	-754.037	-705.357	-730.263
8	1530.989	767.7408	-884.989	-307.916	-387.555	-461.278

Table 8 Variation of stresses against the number of bracings in Max -M region

Nodes	Normal stress in long direction S_x (lb/in^2)			Normal stress in transverse direction S_y (lb/in^2)		
position No:	23 bracings	10 bracings	4 bracings	23 bracings	10 bracings	4 bracings
1	26451	29915	43787	8441.6	9429.65	13751
2	16784.13	17180.35	18856.97	3192.767	3098.467	3304.8
3	24886	22327	12689.2	8877.55	8069.5	4885.25
4	24509	26969	35559.5	8153.3	8904.3	11712
5	15747.4	15220.42	12818.8	4208.267	4234.567	3671.6
6	23006.5	19070	3201.5	8256.55	7064	1804.8
7	-13002.5	-14164.7	-17413.5	-3915.72	-4300.6	-5287.08
8	-13874.4	-12542.2	-8581.06	-4665.43	-4208.52	-2917.55

5.3 Longitudinal Stiffener Effect:

In the non-composite FEM model of the curved box (model M2), the longitudinal stiffener with (T) section in the bottom flange is also installed. Heins et.al. (1973) estimated that the longitudinal stiffener contribution to the total moment of inertia of the non-composite section was less than 1 %. However, the stiffeners contribution in the composite section should be considered because of the shifting of the centroidal axis.

Lateral bending stresses due to curvature also occurs in the flanges of the longitudinal stiffeners attached to the bottom flange. These stiffener flanges participate with the girder flanges in resisting bending moments and carry a stress f_s, ksi, as shown in the Figure 43 and is given by

$f_s = (y_b - y_s)\, f_b / y_b$ ……………………………….. (5.1)

where;

f_b = maximum bending stress, ksi ,in the girder bottom flange

y_b = distance, in., from neutral axis to bottom of girder

y_s = distance, in., from neutral axis to top of stiffener flange

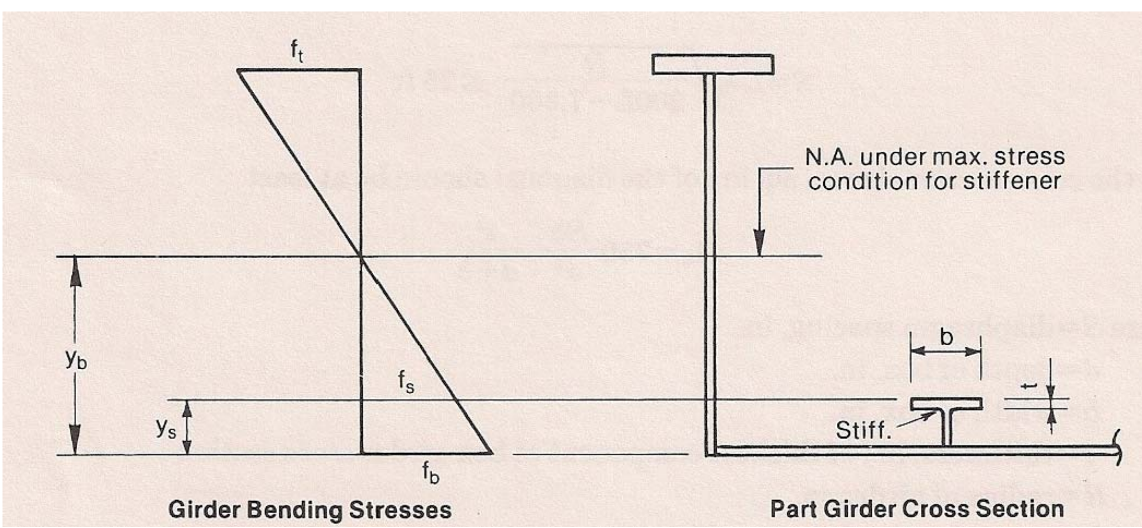

Figure 43 Bending stresses in longitudinal stiffener

Since the stiffener is curved, its flange is subjected to a lateral bending moment

$$M_{LC} = f_s btd^2/10R \text{………………………………} \quad (5.2)$$

Where;

d = unbraced length, in. of stiffener flange

t = thickness, in., of stiffener flange

b = width, in., of stiffener flange

R = radius of curvature, in., of stiffener

The corresponding lateral bending stress is

$$f_{wc} = 6 f_s d^2/10Rb \text{……………………………….} \quad (5.3)$$

With the direct stress and the lateral bending stress in the stiffener flange known, f_s may be checked against the allowable stresses for noncomposite I-girder flanges under construction loading.

In the current analysis using finite element program ANSYS, the longitudinal stiffeners were modeled using shell elements in non-composite model. To study the effect of longitudinal stiffeners on bottom plate stresses two models were used with the same geometric configuration, one with the longitudinal stiffener and one without the longitudinal stiffener. Table 9 and 10 shows the comparison of stresses with and without the longitudinal stiffener in the bottom flange in the max +M and max -M region respectively. Figures 44 and 45 shows the comparison curves thus explaining the longitudinal stiffener contribution to the bottom plate stresses.

Table 9 Stresses in bottom flange in Max +M region

Node No:	stresses in BF with longitudinal stiffener(lb/in^2)	stresses in BF without long stiffener (lb/in^2)
2330	5734.2	5770.025
2320	5267.5	5301.75
3196	4830.15	4860.85
3205	4394.8	4422
3214	3956	3740.05
3184	3524.1	3544.65
4160	3107.6	3125.15
4169	2721.35	2735.95
4178	2380.15	2392.35
4148	1917.8	1927
4539	1425.3	1431.45

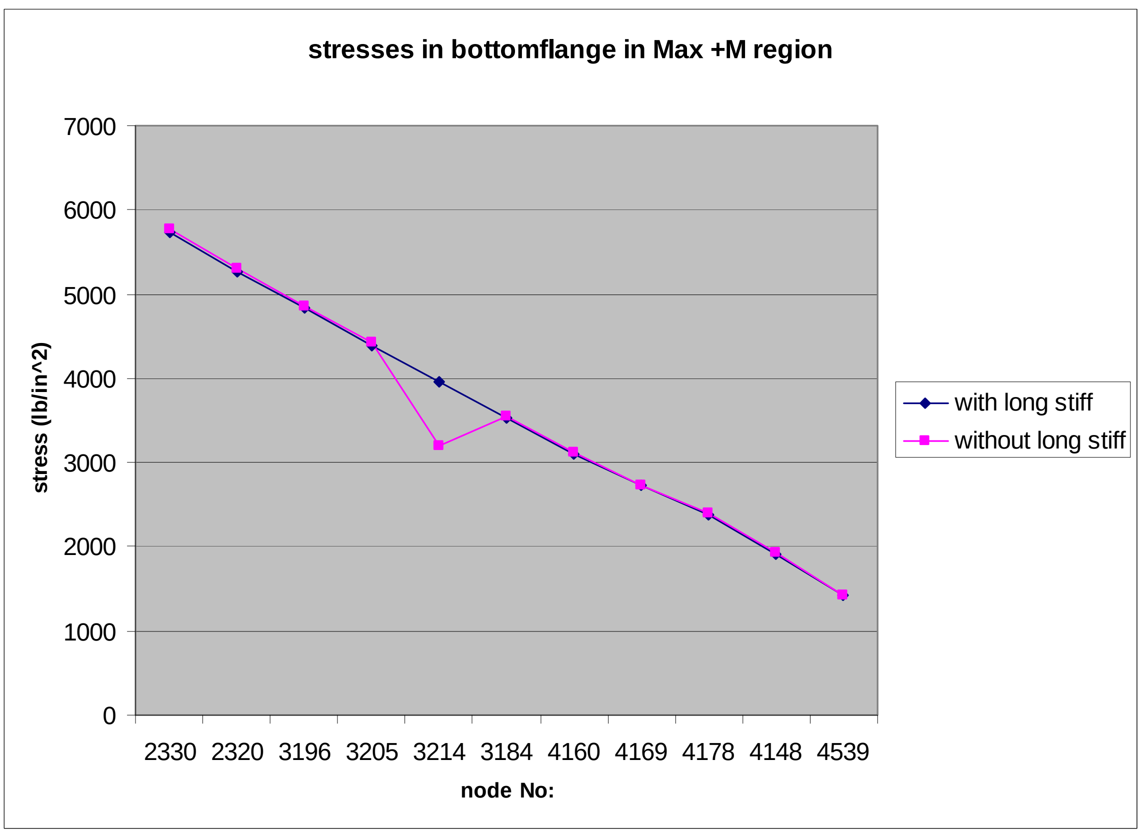

Figure 44 Comparison curves showing stresses in bottom flange in Max +M region

Table 10 Stresses in bottom flange in Max -M region

Node No:	stresses in BF with longitudinal stiffener(lb/in^2)	stresses in BF without long stiffener (lb/in^2)	% difference between stresses w/ & w/o long. stiff
2182	-17975.5	-18989.5	5.64
2172	-16453.5	-17395.5	5.73
2908	-15367.5	-16271	5.88
2917	-14073	-14939.5	6.16
2926	-15924.5	-16846	5.79
2896	-15462.75	-17201.5	11.2
3872	-17157	-17999	4.91
3881	-18739.5	-19636	4.78
3890	-17630.5	-18475.7	4.79
3860	-17786.5	-18611.5	4.64
4461	-17571.75	-18360.25	4.49

Note: Avg difference is 5.82%

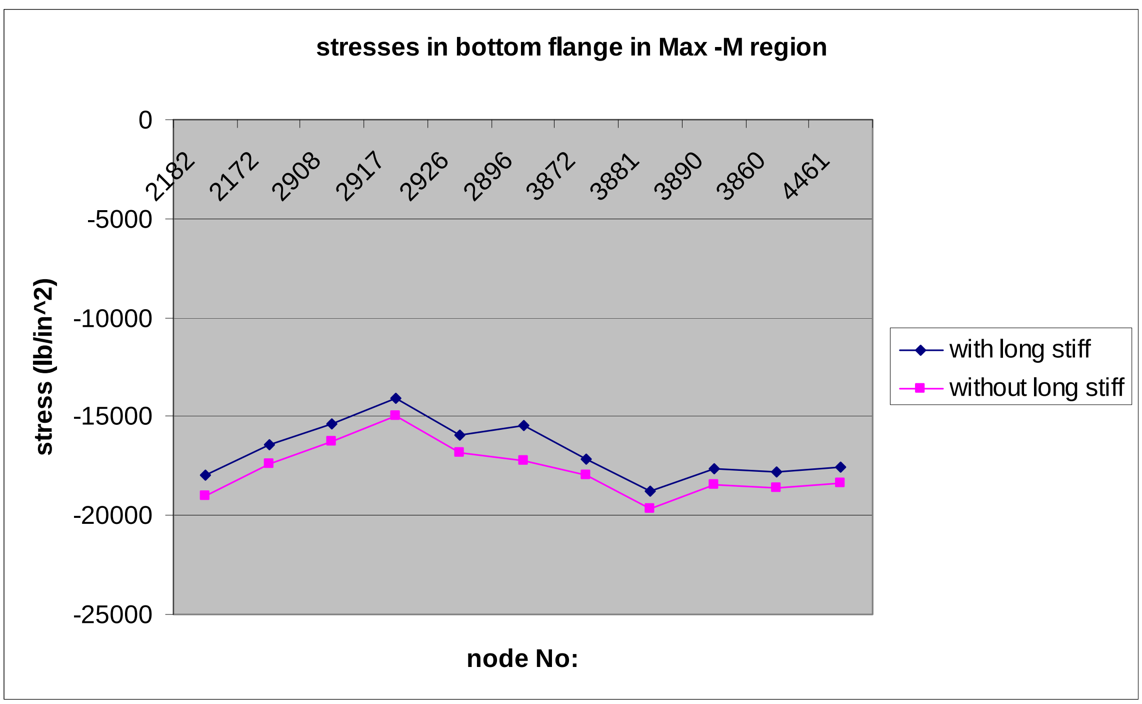

Figure 45 Comparison curves showing stresses in bottom flange in Max -M region

It is observed that the longitudinal stiffener has comparatively much significant effect on the bottom plate stresses of negative moment cross section than on the bottom plate stresses of positive moment cross section.

Chapter 6

SUMMARY AND CONCLUSION

6.1 Summary

In this study, the main objective is to investigate the static structural behavior of straight and curved non- composite box girder bridges. The behavior of the box girder system is more complex than that of the I-girder system. It was observed that the majority of the experimental and analytical research efforts have focused on I-girder systems, and very little on box girders. Therefore, for the box girder systems, more study to establish sound design criteria is needed.

A literature review was conducted in order to establish the foundation for this study in chapter 2.The review of the literature commenced with a review of different methods that are commonly used in the static analysis of box girder bridges. This was followed with a discussion about the experimental studies conducted to investigate the box girders that include both field test and model test. Finally a general description of the behavior of curved box girders is presented with the emphases on behavior during construction.

In chapter 3, the finite element program ANSYS was used to analyze the non-composite two span box girder models. The objective of the use of this ANSYS is to employ advanced analytical techniques such as the finite element method. Four models were developed, straight box shell model (M1), curved box shell model (M2), straight box beam model (M3) and curved box beam model (M4). These models were used to further study the box girder behavior in the next chapters.

The results obtained from the four different bridge models and the BEST (Bridge Engineering Software Technology) Center program DESCUS-II models, which use curved beam elements, was presented in chapter 4. It was followed by the discussion of the results which include comparison of curved box girder and straight box girder, comparison of beam models of ANSYS and DESCUS II and comparison of shell and beam ANSYS models.

The model M2 which is a curved box shell model from chapter 3 was further used in chapter 5 to carry out parametric study to evaluate the influence of several important parameters on the response of the girders stresses. The objective of this parametric study was to determine the effect of spacing of bracings on the stresses and determine the effect of longitudinal stiffeners on the bottom plate stresses.

6.2 Observations and Conclusions

Based on the results obtained from the different models of straight and curved box-girder bridges, the following observations and conclusions are made:

1. It is observed from the stress profiles of shell element models that in the straight box girder the longitudinal stress distribution is symmetric from one end to the other whereas in curved box girder the stress profile is not symmetric. The left (inside) top flange and left web side show much higher stress values than the right (outside) side.

2. Comparison of the results obtained from DESCUS –II and those obtained by the ANSYS finite element model shows that there is good agreement between the two set of results.

3. In the beam model the stresses are obtained only at top and bottom whereas in shell model the stresses can be obtained at every node point. It is interesting to note that in the case of straight box girder there is very good agreement

between beam model and shell model results whereas in the case of curved box model stresses show some deviation.

4. In the study of effect of spacing of bracings on the stresses it is observed that the stresses increase as the spacing of the internal cross frames is reduced (decreasing the number of internal cross frames).

5. The longitudinal stiffener has comparatively much significant effect on the bottom plate stresses of negative moment cross section than on the bottom plate stresses of positive moment cross section.

APPENDIX

Straight box beam model

```
fini
/clear
/filename,straight beam
/title,steel box girder

/prep7

!!BOX-SECTION
et,1,82

k,1,-58.2417,88.8750,0.0000
k,2,-38.2417,88.8750,0.0000
k,3,-58.2417,88.1250,0.0000
k,4,-48.6167,88.1250,0.0000
k,5,-47.8667,88.1250,0.0000
k,6,-38.2417,88.1250,0.0000
k,7,38.2417,88.8750,0.0000
k,8,58.2417,88.8750,0.0000
k,9,38.2417,88.1250,0.0000
k,10,47.8667,88.1250,0.0000
k,11,48.6167,88.1250,0.0000
k,12,58.2417,88.1250,0.0000
k,13,-35.7500,0.9722,0.0000
k,14,35.7500,0.9722,0.0000
k,15,-36.5000,0.0000,0.0000
k,16,36.5000,0.0000,0.0000
k,17,-47.5539,85.8750,0.0000
k,18,-47.5469,85.8250,0.0000
k,19,47.5539,85.8750,0.0000
k,20,47.5469,85.8250,0.0000

A,1,2,6,5,4,3
A,7,8,12,11,10,9
A,4,5,17,18,13,15
A,13,14,16,15
A,10,11,16,14,20,19
A,17,18,20,19

ALLSEL
AADD,ALL
```

```
SMRTSIZE,8
AMESH,ALL
SECWRITE,BOXSHAPE,SECT,,1
SECTYPE,2,BEAM,MESH,
SECOFFSET,USER,0.60
SECREAD,'BOXSHAPE','SECT',,MESH

!delete the section
ASEL,ALL
ACLEAR,ALL
ADELE,ALL,,,1

et,2,beam188
mp,ex,1,29e6
mp,prxy,1,0.3

n,1,0,0,0
n,2,12,0,0
n,320,3840,0,0
fill

n,500,0,5000,0

type,2
mat,1
secnum,2

*do,i,1,319
e,i,i+1,500
*enddo

et,3,beam4

r,3,730.44,7120400,7120400,6,6
mp,ex,2,29e6
mp,prxy,2,0.3

n,10001,1920,0,-30
n,10002,1920,0,30

type,3
mat,2
real,3

e,160,10001
e,160,10002
```

```
n,20001,0,0,-30
n,20002,0,0.0000,30

n,30001,3840,0,-30
n,30002,3840,0,30

e,1,20001
e,1,20002

e,320,30001
e,320,30002
```

Curved box beam model

```
fini
/clear
/filename,curved beam 320
/title,steel box girder

/prep7

!!BOX-SECTION
et,1,82

k,1,-58.2417,88.8750,0.0000
k,2,-38.2417,88.8750,0.0000
k,3,-58.2417,88.1250,0.0000
k,4,-48.6167,88.1250,0.0000
k,5,-47.8667,88.1250,0.0000
k,6,-38.2417,88.1250,0.0000
k,7,38.2417,88.8750,0.0000
k,8,58.2417,88.8750,0.0000
k,9,38.2417,88.1250,0.0000
k,10,47.8667,88.1250,0.0000
k,11,48.6167,88.1250,0.0000
k,12,58.2417,88.1250,0.0000
k,13,-35.7500,0.9722,0.0000
k,14,35.7500,0.9722,0.0000
k,15,-36.5000,0.0000,0.0000
k,16,36.5000,0.0000,0.0000
k,17,-47.5539,85.8750,0.0000
k,18,-47.5469,85.8250,0.0000
k,19,47.5539,85.8750,0.0000
```

```
k,20,47.5469,85.8250,0.0000

A,1,2,6,5,4,3
A,7,8,12,11,10,9
A,4,5,17,18,13,15
A,13,14,16,15
A,10,11,16,14,20,19
A,17,18,20,19

ALLSEL
AADD,ALL
SMRTSIZE,8
AMESH,ALL
SECWRITE,BOXSHAPE,SECT,,1
SECTYPE,2,BEAM,MESH,
SECOFFSET,USER,0.60
SECREAD,'BOXSHAPE','SECT',,MESH

!delete the section
ASEL,ALL
ACLEAR,ALL
ADELE,ALL,,,1

et,2,beam188
mp,ex,1,29e6
mp,prxy,1,0.3

!!!!!change the coordinate to cylindrical coordinate system

*do,i,0,319
n,i+1,3600,i*0.19098593171
*enddo

!!!!!change the coordinate system back
ngen,2,320,1,320,1,,,50
type,2

type,2
mat,1
secnum,2

*do,i,1,319
e,i,i+1,i+320
*enddo

et,3,beam4
```

```
r,3,730.44,8900500,8900500,12,12
mp,ex,2,29e6
mp,prxy,2,0.3

n,10000,3100,1830.3,-6
n,10001,3125.3523,1845.3844,-6
n,10002,3074.5475,1815.3864,-6

type,3
mat,2
real,3

e,160,10000
e,10000,10001
e,10000,10002

n,20001,3629.5000,0.0000,0.0000
n,20002,3570.5000,0.0000,0.0000

n,30001,1752.9641,3178.1106,0.0000
n,30002,1724.4685,3126.4482,0.0000

e,1,20001
e,1,20002

e,320,30001
e,320,30002
```

Straight box shell model

```
fini
/clear
/filename,straight shell
/title,steel box girder

/prep7
/REP,FAST

n2=8
n1=3*n2
esteel=29e6
econ1=esteel/n1
econ2=esteel/n2
dsteel=0.28155
dcon=0.0868
```

```
et,1,shell63
et,2,link8

r,1,0.75   !!!!!!!flange
r,2,0.75   !!!!!!!web
r,3,1     !!!!!!!bot
r,4,9.5  !!!!!!!deck
r,5,0.75     !!!!!!!sti1
r,6,0.6  !!!!!!!sti2
r,7,0.75  !!!!dia

r,9,8.82   !!!!!!!bracing

mp,ex,1,esteel  !!!!!!!!!steel
mp,prxy,1,0.3
mp,dens,1,dsteel

mp,ex,2,econ2   !!!!!!!!!concrete
mp,prxy,2,0.2
mp,dens,2,dcon

n,1,,88,-58.2417
n,2,,88,-55.2417
n,3,,88,-48.2417
n,4,,88,-41.2417
n,5,,88,-38.2417

n,6,,82.0000,-47.4411
n,7,,70,-45.8400
n,14,,0,-36.5
fill

n,15,,0,-29.5
n,16,,0,-24
n,20,,0,24
fill
n,21,,0,29.5
n,22,,0,36.5
n,29,,70,45.84
fill

n,30,,82,47.4411
n,31,,88,38.2417
n,32,,88,41.2417
n,33,,88,48.2417
```

```
n,34,,88,55.2417
n,35,,88,58.2417

n,36,,88,-140.2417
n,42,,88,-68.2417
fill
n,43,,88,-58.2417
n,44,,88,-55.2417
n,45,,88,-48.2417
n,46,,88,-41.2417
n,47,,88,-38.2417

n,48,,88,-26.2417
n,49,,88,-14.2417
n,50,,88,
n,51,,88,14.2417
n,52,,88,26.2417
n,53,,88,38.2417
n,54,,88,41.2417
n,55,,88,48.2417
n,56,,88,55.2417
n,57,,88,58.2417

n,58,,88,68.2417
n,64,,88,140.2417
fill


ngen,321,100,1,64,1,12,

!!!!!!!!!!!
type,1       !!!!
real,1
mat,1
e,1,2,102,101

egen,4,1,1


real,2
mat,1
e,3,6,106,103
e,6,7,107,106
egen,8,1,6,6,1

real,3
```

```
mat,1
e,14,15,115,114
egen,8,1,14,14,1

real,2
mat,1
e,22,23,123,122
egen,8,1,22,22,1

e,30,33,133,130

real,1
mat,1
e,31,32,132,131
egen,4,1,31,31,1


egen,320,100,1,34,1

esel,s,real,,1
cm,flange,elem
allsel


esel,s,real,,2
cm,web,elem
allsel

esel,s,real,,3
cm,bot,elem
allsel


!!!!!!!!!!!!!!!!add diaphragm

*do,i,0,10
ngen,2,59,i*1400+1006,i*1400+1013,1,,,7
ngen,2,50,i*1400+1023,i*1400+1030,1,,,-7
*enddo


*do,i,0,10
ngen,2,59,i*1400+17006,i*1400+17013,1,,,7
ngen,2,50,i*1400+17023,i*1400+17030,1,,,-7
*enddo
```

```
!!!!!!!biuld the stiffeners
type,1
real,5
mat,1

e,1003,1004,1065,1006
e,1006,1065,1066,1007
egen,7,1,10882,10882,1
e,1013,1072,1015,1014

e,1021,1022,1023,1073
e,1023,1024,1074,1073
egen,7,1,10891
e,1030,1033,1032,1080

egen,11,1400,10881,10898,1
egen,2,2000,11061,11078,1
egen,11,1400,11079,11096,1

esel,s,real,,5
cm,stiffener,elem
allsel

!!!!!!!!node of stif2   start from 33000
ngen,2,21000,12018,20018,100,,7,-5.5

ngen,3,1,33018,41018,100,,,5.5

!!!!!!!!biuld the element of longitidunal stif
type,1
real,6
mat,1
e,33018,33019,33119,33118
egen,80,100,11277
egen,2,1,11277,11356,1

e,33019,12018,12118,33119
egen,80,100,11437

esel,s,real,,6
cm,stiffener2,elem
allsel

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!node of dia1
ngen,2,41995,6,13,1,,,7
n,42009,,88,-24
```

```
n,42013,,88,24
fill
ngen,2,5,42009,42013,1,,-6
ngen,2,5,42014,42018,1,,-12
ngen,7,5,42019,42023,1,,-10

ngen,2,42031,23,30,1,,,-7

!!!!!node of dia2
ngen,2,26056,16006,16013,1,,,7
n,42070,1920,88,-24
n,42074,1920,88,24
fill
ngen,2,5,42070,42074,1,,-6
ngen,2,5,42075,42079,1,,-12
ngen,7,5,42080,42084,1,,-10

ngen,2,26092,16023,16030,1,,,-7

!!!!!node of dia3
ngen,2,10117,32006,32013,1,,,7
n,42131,3840,88,-24
n,42135,3840,88,24
fill
ngen,2,5,42131,42135,1,,-6
ngen,2,5,42136,42140,1,,-12
ngen,7,5,42141,42145,1,,-10
ngen,2,10153,32023,32030,1,,,-7


!!!!!!!build the element of dia1
type,1
real,7
mat,1
e,3,4,42001,6
e,6,42001,42002,7
egen,7,1,11518

e,4,42009,42014,42001
e,42001,42014,42019,42002
e,42002,42019,42024,42003
e,42003,42024,42029,42004
e,42004,42029,42034,42005
e,42005,42034,42039,42006
e,42006,42039,42044,42007
e,42007,42044,42049,42008
```

```
e,42009,42010,42015,42014

egen,4,1,11533
egen,8,5,11533,11536,1

e,42013,32,42061,42018
e,42018,42061,42060,42023
e,42023,42060,42059,42028
e,42028,42059,42058,42033
e,42033,42058,42057,42038
e,42038,42057,42056,42043
e,42043,42056,42055,42048
e,42048,42055,42054,42053

e,32,33,30,42061

e,42054,23,24,42055
egen,7,1,11574

e,13,42008,15,14
e,42008,42049,16,15
e,42049,42050,17,16
egen,4,1,11583

e,42053,42054,21,20
e,42054,23,22,21

!!!!!!build the element of dia2
type,1
real,7
mat,1
e,16003,16004,42062,16006
e,16006,42062,42063,16007
egen,7,1,11590

e,16004,42070,42075,42062
e,42062,42075,42080,42063
e,42063,42080,42085,42064
e,42064,42085,42090,42065
e,42065,42090,42095,42066
e,42066,42095,42100,42067
e,42067,42100,42105,42068
e,42068,42105,42110,42069

e,42070,42071,42076,42075
```

```
egen,4,1,11605
egen,8,5,11605,11608,1

e,42074,16032,42122,42079
e,42079,42122,42121,42084
e,42084,42121,42120,42089
e,42089,42120,42119,42094
e,42094,42119,42118,42099
e,42099,42118,42117,42104
e,42104,42117,42116,42109
e,42109,42116,42115,42114

e,16032,16033,16030,42122

e,42115,16023,16024,42116
egen,7,1,11646

e,16013,42069,16015,16014
e,42069,42110,16016,16015
e,42110,42111,16017,16016
egen,4,1,11655

e,42114,42115,16021,16020
e,42115,16023,16022,16021

!!!!!!build the element of dia3
type,1
real,7
mat,1
e,32003,32004,42123,32006
e,32006,42123,42124,32007
egen,7,1,11662

e,32004,42131,42136,42123
e,42123,42136,42141,42124
e,42124,42141,42146,42125
e,42125,42146,42151,42126
e,42126,42151,42156,42127
e,42127,42156,42161,42128
e,42128,42161,42166,42129
e,42129,42166,42171,42130

e,42131,42132,42137,42136
egen,4,1,11677
egen,8,5,11677,11680,1
```

```
e,42135,32032,42183,42140
e,42140,42183,42182,42145
e,42145,42182,42181,42150
e,42150,42181,42180,42155
e,42155,42180,42179,42160
e,42160,42179,42178,42165
e,42165,42178,42177,42170
e,42170,42177,42176,42175

e,32032,32033,32030,42183

e,42176,32023,32024,42177
egen,7,1,11718

e,32013,42130,32015,32014
e,42130,42171,32016,32015
e,42171,42172,32017,32016
egen,4,1,11727

e,42175,42176,32021,32020
e,42176,32023,32022,32021

esel,s,real,,7
cm,dia,elem
allsel


type,2       !!!!
real,9
mat,1



*do,i,0,10
e,i*1400+1065,i*1400+1080
e,i*1400+1065,i*1400+1073
e,i*1400+1072,i*1400+1080
*enddo

*do,i,0,10
e,i*1400+17065,i*1400+17080
e,i*1400+17065,i*1400+17073
e,i*1400+17072,i*1400+17080
*enddo

*do,i,0,9
```

```
e,i*1400+1065,i*1400+2480
*enddo

*do,i,0,9
e,i*1400+1080,i*1400+2465
*enddo

*do,i,0,9
e,i*1400+17065,i*1400+18480
*enddo

*do,i,0,9
e,i*1400+17080,i*1400+18465
*enddo

e,42001,1080
e,42061,1065

e,15065,42122
e,15080,42062

e,42062,17080
e,42122,17065

e,42123,31080
e,42183,31065

esel,s,real,,9
cm,bracing,elem
allsel


nsel,s,loc,x,0
nsel,r,loc,z,-24
nsel,r,loc,y,0
d,all,uy
allsel

nsel,s,loc,x,0
nsel,r,loc,z,24
nsel,r,loc,y,0
d,all,uy
allsel

nsel,s,loc,x,1919.9,1920.1
nsel,r,loc,z,-24
```

```
nsel,r,loc,y,0
d,all,UX
d,all,UY
d,all,UZ
allsel

nsel,s,loc,x,1919.9,1920.1
nsel,r,loc,z,24
nsel,r,loc,y,0
d,all,UX
d,all,UY
d,all,UZ
allsel

nsel,s,loc,x,3839.9,3840.1
nsel,r,loc,z,-24
nsel,r,loc,y,0
d,all,uy
allsel

nsel,s,loc,x,3839.9,3840.1
nsel,r,loc,z,24
nsel,r,loc,y,0
d,all,uy
allsel

allsel
finish

/solu

ACEL,,1,                    !GRAVITY LOAD


!!!!!!!!!!!!weight of deck
*do,i,0,320
f,3+i*100,fy,-1387.7
f,33+i*100,fy,-1387.7
*enddo

allsel

solve
finish
```

Curved box shell model

```
fini
/clear
/filename,Curved box bridge
/title,steel box girder

/PREP7
/REP,FAST

n2=8
n1=3*n2
esteel=29e6
econ1=esteel/n1
econ2=esteel/n2
dsteel=0.28155
dcon=0.0868

et,1,shell63
et,2,link8

r,1,0.75   !!!!!!!flange
r,2,0.75   !!!!!!!web
r,3,1     !!!!!!!bot
r,4,9.5  !!!!!!!deck
r,5,0.75     !!!!!!!sti1
r,6,0.6  !!!!!!!sti2
r,7,0.75  !!!!dia

r,9,8.82   !!!!!!!bracing

mp,ex,1,esteel  !!!!!!!!!steel
mp,prxy,1,0.3
mp,dens,1,dsteel

mp,ex,2,econ2   !!!!!!!!!concrete
mp,prxy,2,0.2
mp,dens,2,dcon

!!!build node
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k,2,0.0000,-48.2417,88.0000
k,3,0.0000,-41.2417,88.0000
k,4,0.0000,-38.2417,88.0000
k,5,0.0000,0.0000,88.0000
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k,11,0.0000,-29.5000,0.0000
k,12,0.0000,0.0000,0.0000
k,13,0.0000,29.5000,0.0000
k,14,0.0000,36.5000,0.0000
k,15,0.0000,-140,88.0000
k,16,0.0000,140,88.0000

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k,103,121.3612,-39.2187,88.0000
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k,108,118.3787,50.2150,88.0000
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k,203,291.0101,-29.5942,88.0000
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k,206,284.6577,49.6349,88.0000
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k,211,290.0717,-17.8901,0.0000
k,212,287.7140,11.5156,0.0000
k,213,285.3564,40.9212,0.0000
k,214,284.7969,47.8988,0.0000
k,215,298.9029,-128.0366,88.0000
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k,2407,3116.1668,1881.2025,88.0000
k,2408,3110.0374,1884.5833,88.0000
k,2409,3101.2811,1889.4131,88.0000
k,2410,3184.2400,1843.6550,0.0000
k,2411,3178.1106,1847.0359,0.0000
k,2412,3152.2794,1861.2837,0.0000
k,2413,3126.4482,1875.5315,0.0000

```
k,2414,3120.3188,1878.9123,0.0000
k,2415,3274.8681,1793.6669,88.0000
k,2416,3029.6908,1928.9004,88.0000

k,917,1426.2549,288.5914,7.0000
k,918,1424.0792,293.6427,7.0000
k,919,1421.9035,298.6941,7.0000

k,1017,1579.1896,358.7363,7.0000
k,1018,1576.7806,363.6807,7.0000
k,1019,1574.3717,368.6250,7.0000

k,1117,1728.6853,435.9397,7.0000
k,1118,1726.0483,440.7663,7.0000
k,1119,1723.4113,445.5930,7.0000

k,1217,1833.1818,495.3141,7.0000
k,1218,1830.3854,500.0501,7.0000
k,1219,1827.5889,504.7861,7.0000

k,1317,1935.6413,558.1382,7.0000
k,1318,1932.6886,562.7784,7.0000
k,1319,1929.7359,567.4186,7.0000

k,1417,2075.4461,651.7542,7.0000
k,1418,2072.2802,656.2515,7.0000
k,1419,2069.1142,660.7489,7.0000

k,1517,2210.7313,751.7905,7.0000
k,1518,2207.3589,756.1353,7.0000
k,1519,2203.9866,760.4801,7.0000

!!!!! for bracing

k,20,0.0000,-47.4411,82.0000
k,21,0.0000,-40.4411,82.0000
k,22,0.0000,40.4411,82.0000
k,23,0.0000,47.4411,82.0000
k,24,0.0000,0.0000,82.0000

k,120,121.5678,-45.4147,82.0000
k,121,121.3345,-38.4186,82.0000
k,122,118.6387,42.4188,82.0000
k,123,118.4054,49.4149,82.0000
k,220,291.5056,-35.7738,82.0000
k,221,290.9461,-28.7962,82.0000
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k,222,284.4820,51.8273,82.0000
k,223,283.9225,58.8049,82.0000

k,320,460.8085,-18.2153,82.0000
k,321,459.9242,-11.2714,82.0000
k,322,449.7057,68.9627,82.0000
k,323,448.8213,75.9066,82.0000

k,420,629.1080,7.2225,82.0000
k,421,627.9006,14.1176,82.0000
k,422,613.9501,93.7877,82.0000
k,423,612.7427,100.6828,82.0000

k,520,796.0374,40.4844,82.0000
k,521,794.5097,47.3157,82.0000
k,522,776.8575,126.2482,82.0000
k,523,775.3298,133.0794,82.0000

k,620,961.2333,81.4979,82.0000
k,621,959.3885,88.2504,82.0000
k,622,938.0731,166.2734,82.0000
k,623,936.2284,173.0260,82.0000

k,720,1124.3359,130.1736,82.0000
k,721,1122.1781,136.8327,82.0000
k,722,1097.2459,213.7763,82.0000
k,723,1095.0881,220.4355,82.0000

k,820,1284.9900,186.4055,82.0000
k,821,1282.5240,192.9567,82.0000
k,822,1254.0292,268.6534,82.0000
k,823,1251.5631,275.2046,82.0000

k,920,1442.8459,250.0713,82.0000
k,921,1440.0768,256.5003,82.0000
k,922,1408.0816,330.7852,82.0000
k,923,1405.3126,337.2142,82.0000

k,1020,1597.5596,321.0322,82.0000
k,1021,1594.4936,327.3250,82.0000
k,1022,1559.0676,400.0363,82.0000
k,1023,1556.0017,406.3292,82.0000

k,1120,1748.7943,399.1337,82.0000
k,1121,1745.4381,405.2766,82.0000
k,1122,1706.6585,476.2561,82.0000
```

k,1123,1703.3023,482.3990,82.0000

k,1220,1854.5064,459.1987,82.0000
k,1221,1850.9473,465.2264,82.0000
k,1222,1809.8235,534.8738,82.0000
k,1223,1806.2644,540.9015,82.0000
k,1224,1830.3854,500.0501,82.0000

k,1320,1958.1577,522.7536,82.0000
k,1321,1954.3997,528.6594,82.0000
k,1322,1910.9774,596.8975,82.0000
k,1323,1907.2194,602.8032,82.0000

k,1420,2099.5889,617.4586,82.0000
k,1421,2095.5594,623.1825,82.0000
k,1422,2049.0009,689.3206,82.0000
k,1423,2044.9715,695.0445,82.0000

k,1520,2236.4477,718.6586,82.0000
k,1521,2232.1556,724.1883,82.0000
k,1522,2182.5622,788.0823,82.0000
k,1523,2178.2702,793.6120,82.0000

k,1620,2368.4362,826.1333,82.0000
k,1621,2363.8908,831.4568,82.0000
k,1622,2311.3706,892.9674,82.0000
k,1623,2306.8252,898.2909,82.0000

k,1720,2495.2670,939.6487,82.0000
k,1721,2490.4782,944.7543,82.0000
k,1722,2435.1455,1003.7478,82.0000
k,1723,2430.3567,1008.8534,82.0000

k,1820,2616.6639,1058.9575,82.0000
k,1821,2611.6421,1063.8341,82.0000
k,1822,2553.6174,1120.1819,82.0000
k,1823,2548.5956,1125.0586,82.0000

k,1920,2732.3624,1183.8000,82.0000
k,1921,2727.1185,1188.4370,82.0000
k,1922,2666.5282,1242.0164,82.0000
k,1923,2661.2844,1246.6535,82.0000

k,2020,2842.1106,1313.9042,82.0000
k,2021,2836.6562,1318.2916,82.0000

```
k,2022,2773.6322,1368.9859,82.0000
k,2023,2768.1777,1373.3733,82.0000

k,2120,2945.6696,1448.9869,82.0000
k,2121,2940.0164,1453.1150,82.0000
k,2122,2874.6960,1500.8138,82.0000
k,2123,2869.0428,1504.9420,82.0000

k,2220,3042.8137,1588.7538,82.0000
k,2221,3036.9741,1592.6137,82.0000
k,2222,2969.4995,1637.2132,82.0000
k,2223,2963.6599,1641.0731,82.0000

k,2320,3133.3316,1732.9006,82.0000
k,2321,3127.3182,1736.4839,82.0000
k,2322,3057.8364,1777.8869,82.0000
k,2323,3051.8230,1781.4702,82.0000

k,2420,3193.8204,1838.3707,82.0000
k,2421,3187.6910,1841.7515,82.0000
k,2422,3116.8678,1880.8158,82.0000
k,2423,3110.7384,1884.1966,82.0000
k,2424,3152.2794,1861.2837,82.0000

!!!!!web

*do,i,0,23,1
a,2+i*100,20+i*100,120+i*100,102+i*100
*enddo

*do,i,0,23,1
a,20+i*100,10+i*100,110+i*100,120+i*100
*enddo

*do,i,0,23,1
a,8+i*100,23+i*100,123+i*100,108+i*100
*enddo

*do,i,0,23,1
a,23+i*100,14+i*100,114+i*100,123+i*100
*enddo

!!!!! bottom flange

*do,i,0,23,1
a,10+i*100,11+i*100,111+i*100,110+i*100
```

```
*enddo

*do,i,0,23,1
a,11+i*100,12+i*100,112+i*100,111+i*100
*enddo

*do,i,0,23,1
a,12+i*100,13+i*100,113+i*100,112+i*100
*enddo

*do,i,0,23,1
a,13+i*100,14+i*100,114+i*100,113+i*100
*enddo

!!!!!Top flange

*do,i,0,23,1
a,1+i*100,2+i*100,102+i*100,101+i*100
*enddo

*do,i,0,23,1
a,2+i*100,3+i*100,103+i*100,102+i*100
*enddo

*do,i,0,23,1
a,3+i*100,4+i*100,104+i*100,103+i*100
*enddo

*do,i,0,23,1
a,6+i*100,7+i*100,107+i*100,106+i*100
*enddo

*do,i,0,23,1
a,7+i*100,8+i*100,108+i*100,107+i*100
*enddo

*do,i,0,23,1
a,8+i*100,9+i*100,109+i*100,108+i*100
*enddo

!!!!! stifferners

*do,i,0,10,1
a,102+i*100,103+i*100,121+i*100,120+i*100
*enddo
```

```
*do,i,0,10,1
a,120+i*100,121+i*100,111+i*100,110+i*100
*enddo

*do,i,0,10,1
a,107+i*100,108+i*100,123+i*100,122+i*100
*enddo

*do,i,0,10,1
a,122+i*100,123+i*100,114+i*100,113+i*100
*enddo

*do,i,0,10,1
a,1302+i*100,1303+i*100,1321+i*100,1320+i*100
*enddo

*do,i,0,10,1
a,1320+i*100,1321+i*100,1311+i*100,1310+i*100
*enddo

*do,i,0,10,1
a,1307+i*100,1308+i*100,1323+i*100,1322+i*100
*enddo

*do,i,0,10,1
a,1322+i*100,1323+i*100,1314+i*100,1313+i*100
*enddo



*do,i,0,5,1
a,912+i*100,918+i*100,1018+i*100,1012+i*100
*enddo

*do,i,0,5,1
a,917+i*100,918+i*100,1018+i*100,1017+i*100
*enddo

*do,i,0,5,1
a,918+i*100,919+i*100,1019+i*100,1018+i*100
*enddo



!!!!!!diaphrams
```

```
a,8,7,22,23
a,23,22,13,14
a,7,5,24,22
a,24,22,13,12
a,5,3,21,24
a,21,24,12,11
a,3,2,20,21
a,20,21,11,10

a,1208,1207,1222,1223
a,1223,1222,1213,1214
a,1207,1205,1224,1222
a,1224,1222,1213,1212
a,1205,1203,1221,1224
a,1221,1224,1212,1211
a,1203,1202,1220,1221
a,1220,1221,1211,1210

a,2408,2407,2422,2423
a,2423,2422,2413,2414
a,2407,2405,2424,2422
a,2424,2422,2413,2412
a,2405,2403,2421,2424
a,2421,2424,2412,2411
a,2403,2402,2420,2421
a,2420,2421,2411,2410

!!!!! Bracing

!!!Top bracing

*do,i,0,23,1
l,21+i*100,122+i*100
*enddo

*do,i,0,23,1
l,22+i*100,121+i*100
*enddo

*do,i,0,10,1
l,121+i*100,122+i*100
*enddo


*do,i,0,10,1
l,1321+i*100,1322+i*100
```

```
*enddo

!!!!!internal bracing

*do,i,0,10,1
l,122+i*100,111+i*100
*enddo

*do,i,0,10,1
l,121+i*100,113+i*100
*enddo

*do,i,0,10,1
l,1322+i*100,1311+i*100
*enddo

*do,i,0,10,1
l,1321+i*100,1313+i*100
*enddo

!!!!Meshing

type,1
mat,1
real,1
amesh,atf

type,1
mat,1
real,3
amesh,abf

type,1
mat,1
real,2
amesh,aw

type,1
mat,1
real,5
amesh,as1



type,1
mat,1
```

```
real,6
amesh,as2-1

type,1
mat,1
real,6
amesh,as2-2

type,1
mat,1
real,7
amesh,ad

type,2
mat,1
real,9
lmesh,IBL

type,2
mat,1
real,9
lmesh,ltb
```

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