

ABSTRACT

Title of dissertation: UNDERSTANDING AND TEACHING RATIONAL NUMBERS: A CRITICAL CASE STUDY OF MIDDLE SCHOOL PROFESSIONAL DEVELOPMENT

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A lot of money is spent each year on teacher professional development, but researchers and policymakers are still trying to determine what that investment yields in terms of improvements in teacher knowledge and practice. This study focuses on the extent to which middle school mathematics teachers comprehended and made use of the core content, pedagogical content and pedagogical components of a well designed professional development model. At the time of data collection, the teachers were participating in a large, federally funded randomized field trial on professional development that focused on rational numbers. Compared with many other teachers participating in the randomized study, these three teachers were highly receptive to the intensive, content-focused model and thus represent a critical case study of professional development.

Using interview and classroom observation data from the 2007-08 school year, the study indicates that teachers understood and implemented many of the pedagogical components emphasized in the model, but they had difficulty comprehending and articulating the core rational number content. Within the domain of rational numbers, the study shows that teachers had more difficulty understanding

ratio and proportion concepts as compared with fraction and decimal concepts. The study also describes sources of variation in teachers' understanding of the professional development material and the extent to which they utilized the professional development material while teaching.

Teachers' understanding of math content is a critical link in the theory of action driving current educational policies that call for increased rigor and coherence in K-12 mathematics. This case study illustrates that even well designed and well implemented professional development models may be incapable of improving teachers' content knowledge to levels that positively affect their instructional practices.

UNDERSTANDING AND TEACHING RATIONAL NUMBERS:
A CRITICAL CASE STUDY OF MIDDLE SCHOOL PROFESSIONAL
DEVELOPMENT

By

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Dedicated to my former middle and high school math students, who introduced me to the joys (mostly) and challenges of math teaching

And to my family, Randi, Maia and Ellie, who have supported me tirelessly throughout this endeavor

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CHAPTER 1: INTRODUCTION AND OVERVIEW

Problem Statement

Increasing student achievement in mathematics is a prominent federal education policy aim. Most policymakers acknowledge that improving students' mathematics achievement is highly dependent on the quality of instruction they receive. Researchers and policymakers also tend to agree that instructional quality is highly dependent on teachers' knowledge of the subject matter (Ball, 1990; Wu, 1996; Ma, 1999; Kilpatrick et al., 2001; Milgram 2005) and teachers' knowledge of how to utilize and integrate the subject matter into the classroom (Shulman, 1986; Ma, 1999; Kilpatrick et al., 2001). Given the shortage of well qualified mathematics teachers in the U.S. (Ingersoll, 1999 & 2001; Darling-Hammond & Sykes, 2003), policymakers increasingly are looking to professional development as a critical means to help existing teachers boost their knowledge and thereby improve their instructional practices.

Though professional development is viewed as a viable means to boost teachers' content and pedagogical knowledge and improve instruction, much of the professional development teachers typically receive is unfocused, sporadic and therefore unlikely to influence substantively what teachers know and do (Wilson & Berne, 1999). Professional development models that have certain characteristics, however, are considered more likely to impact what teachers know and how they teach (Garet et al., 2001). Among these characteristics are activities that are (a) focused on student outcomes, (b) sustained and intensive, (c) aligned with district standards, (d) connected to teachers' daily work, and (e) linked to a theoretical

rationale (Hawley & Valli, 1999; Garet et al., 2001; Wilson & Berne, 1999). A few studies have even shown a link between professional development and student achievement (e.g., Carpenter et al., 1989), but these studies are extremely rare (Yoon, et al. 2007).

Though certain types of professional development models are presumed to be more effective than others, very little is known about what teachers actually learn from professional development and the extent to which that knowledge influences instructional practices. In their review of research on teacher professional development, Wilson and Berne (1999) note this deficiency: “the ‘what’ of teacher learning [in professional development] needs to be identified, conceptualized, and assessed” (p.203). This study seeks to fill this void, by describing what a select group of teachers learned from participating in a high-quality professional development model.

Study Context and Rationale

This case study is set within the context of a larger study, a five-year, federally funded evaluation of mathematics professional development (hereafter, PD Math Study) The goal of this large-scale, randomized field trial is to determine whether sustained, content-focused, classroom-embedded professional development influences what 7th grade math teachers know, how they teach, and what their students learn. The key PD Math Study outcomes are student achievement in the domain of rational numbers, teacher content and pedagogical content knowledge in the domain of rational numbers, and teacher instructional practices in keeping with the pedagogical emphases of the professional development. Findings will be

available in late 2009 and that report will include contrasts between treatment and control groups on all these dimensions.

The primary purpose of this large, randomized study is to provide data that allow broad conclusions to be made about the impact of professional development on teacher and student outcomes. However, the larger study's analysis plan does not include in-depth analyses of the extent to which particular teachers comprehended and made use of the core content in the model. Such in-depth studies are important because they provide descriptive data about the complexities of teacher learning. According to Yin (2003), a case study is an appropriate research method when the purpose of the study is to examine a contemporary phenomenon in its natural context and pay attention to the technical complexities and potential sources of variation that are due to context. This case study examines a phenomenon – in this study, professional development – in relation to its contextual factors, while randomized control trials, like the larger study, seek to control contextual variables between the treatment and “business as usual” conditions.

Given this information, the data from this case study could be used to inform the results of the larger study when they are available. If the larger study finds an impact on teacher knowledge, teacher practice or student achievement, the data from this critical case could be used to describe particular aspects of teacher knowledge or teacher practice that might have contributed to the positive effect. If the larger study does not find an impact on key study outcomes, the descriptive case study data could be used to build or support hypotheses for why the intervention failed to have an impact or to illustrate exceptions to broad patterns.

This study focuses on the extent to which three middle school mathematics teachers comprehended and made use of the core content, pedagogical content and pedagogical components of a professional development model. The teachers represent all the targeted teachers from one of the 38 schools (in 12 districts) that participated in the study. Though the larger study relied on randomization in its design, this case study selected these three teachers purposefully. Unlike some of the other school-level teams of teachers participating in the study, these teachers were highly receptive to the professional development intervention. They actively participated in the study workshops, quickly established a strong rapport with the study-appointed professional development facilitator, and worked in a district and school that promoted a vision for teaching and learning consistent with the professional development model used in the larger study. By selecting a data collection site in which participants were hospitable to the intervention, this inquiry represents a critical case study of professional development. Critical case studies employ strategic sampling methods to allow generalizations of the sort, “if not here, then where” to be made (Flyvberg, 2001; Yin, 2003).

This study also draws upon my experience and expertise as the primary investigator. I previously taught middle and high school mathematics, developed and delivered mathematics professional development, and oversaw the development and evaluation of the professional development intervention implemented in the larger study. This relevant experience allowed me to play the role of expert observer, which means that the analyses in this study contain specific information about the

components of this model and the teaching and learning of rational number concepts in middle school.

This introductory chapter provides a brief preview of the dissertation. In addition to the previous discussions of the problems driving this inquiry and the study context, the chapter includes an overview of the primary research questions and conceptual approach, and a discussion about the study's significance. The chapter concludes with a description of how the dissertation is organized and the main focus of each chapter.

Research Questions

This case study seeks to answer two sets of questions: one that focuses on how much teachers learned about rational number content and pedagogical content through the professional development and one that focuses on how visible these core elements were during instruction. The inquiry also explores potential sources of variation among the study's three teachers related to these questions. More formally, the primary research questions guiding this inquiry are:

1. How deeply do the case study teachers understand the core content and pedagogical content components of the professional development model? What might explain any variation among teachers?
2. How effectively do the case study teachers integrate into their instructional routines new content, pedagogical content and pedagogical knowledge? What might explain any variation among teachers?

The potential sources of variation among teachers are based on a review of the literature and from theorizing based on practical experience.

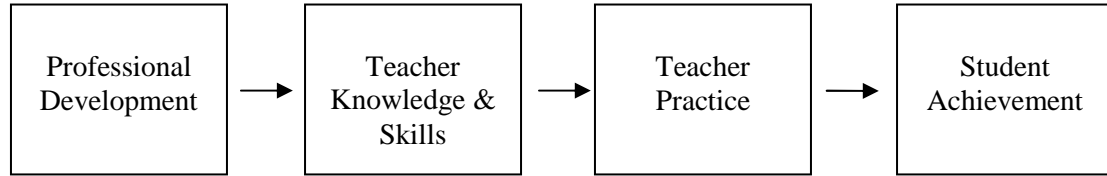
The potential sources of variation for the first question, which focus on teachers' understanding of the content, include the complexity of the different

components of the model; teachers' prior knowledge related to the professional development topics; teachers' prior experience with comparable curricula; extra self study or collaboration with the study's professional development facilitator; and teachers' beliefs and attitudes about the teaching strategies promoted in the professional development. The potential sources of variation for the second question, which focuses on the extent to which the core emphases of the professional development are visible during instruction, include teachers' understanding of the core math content and pedagogical content; whether or not teachers were working directly with an instructional coach; the complexity of the different components of the professional development model; and the frequency with which different components of the model occur.

Conceptual Approach

The PD Math Study's theory of action identifies teachers' knowledge and teachers' instructional practices as critical links to achieving the study's ultimate outcome, improving student achievement in the domain of rational numbers (see Figure 1). The logic assumes that if teachers participate in high-quality professional development, they will boost their knowledge and skills, which, in turn, will influence the quality of their instruction. As instructional quality improves, so does student achievement. Each intermediate outcome simultaneously represents a critical link to improving student achievement and a point at which the model can break down. For example, if teachers don't learn key aspects of the content being emphasized in the professional development, they won't be able to integrate that knowledge into their instructional routines. Or, if teachers learn the core content, they might not know

Figure 1. PD Math Study Theory of Action



how or choose not to implement that knowledge during instruction. This case study utilizes the larger study's theory of action by providing detailed descriptions of how a group of teachers, working in an environment hospitable to the professional development intervention, perform on the intermediate study outcomes of teacher knowledge and practice. Thus, the conceptual approach draws upon two streams of literature, the literature base addressing the types of content and pedagogical knowledge required for effective teaching and the base addressing most effective types of professional development.

The literature on the content and pedagogical content that math teachers need to know is substantial. Over the past two decades, policymakers and national organizations, such as the National Council of Teachers of Mathematics (NCTM), have been calling for improvements to the teaching and learning of mathematics. They argue that traditional, procedural-based approaches to mathematics teaching do not prepare enough students for higher level mathematics. Success in higher level mathematics is dependent on students' understanding of concepts as well as procedures. In order for students to receive mathematics instruction that attends to concepts as well as procedures, teachers must receive additional training and support to deliver instruction in a more conceptually-based way (National Council of Teachers of Mathematics [NCTM], 1989; 1991; 2000; Kilpatrick et al., 2001).

Though mathematicians and math education experts sometimes differ over the relative emphasis of particular mathematics concepts, they agree that conceptual understanding is a necessary condition for effective teaching. Teachers need to know why, not just how, so that they can deliver meaningful instruction to students. And they need to be able to make connections among mathematical concepts so that students get a coherent rather than a fragmented view of mathematics (National Math Panel Advisory Panel [NMAP], 2008).

Related to strong conceptual understanding of the topics they teach, experts also agree that teachers need pedagogical content knowledge, which is knowledge about how the concepts relate to student understanding (Shulman, 1986). For example, a teacher with conceptual understanding about the meaning of fraction might use that knowledge to improve the precision or coherence of an explanation about fractions. If the same teacher possessed strong pedagogical content knowledge in this area, the teacher also would be able to identify the most common student misconceptions and present the most appropriate representations of the concepts. Both types of knowledge, conceptual knowledge and pedagogical content knowledge, are considered to be critical for effective teaching (NRC, 2003; NCTM, 1989; 1991; 2000), and the professional development model used in the larger study attends to both types of knowledge in the domain of rational numbers.

The second literature base focuses on promising structures and mechanisms for triggering improvements in teacher knowledge. Most experts argue that pre-service training only goes so far in preparing math teachers to deliver high quality lessons (Kilpatrick et al., 2001; Ma, 1999; NCTM, 2000; NMAP, 2008). Teachers

learn a great deal about the content when they teach it, especially when working with students who struggle to understand the material. Professional development allows teachers to expand their knowledge based on what they learn from their teaching experience. However, teachers appear to learn more from certain types of professional development models than others. More effective professional development models tend to be: (a) focused on student performance outcomes, (b) aligned with district standards, (c) connected to teachers' daily work, (d) sustained and intensive, (e) collaborative or collective, and (f) linked to a theoretical rationale (Hawley & Valli, 1999; Garet et al., 2001; Wilson & Berne, 1999).

The design of the larger study's professional development model includes the key ideas from both the professional development and teacher knowledge literature streams. The model is sustained and intensive and focuses on material that matters to teachers' daily work. The model also builds teachers' conceptual understanding and pedagogical content knowledge in a narrow range of topics. Thus, the study's overall theory of action is well grounded in the literature on improving teachers' knowledge through professional development.

Significance of Study

This study is important for several reasons. First, the data in this study describe teachers' understanding of important mathematics content. Rational numbers is considered to be among the most important topics that students need to know in order to be successful in Algebra I, which is considered to be a critical "gatekeeper" course to advanced math and science tracks in high school (Kilpatrick et al., 2001; NMAP, 2008). Yet, assessments such as the National Assessment of

Education Progress (NAEP) have consistently shown that many middle school students have a limited understanding of basic rational number concepts. The study's focus on helping teachers deepen their understanding of and improve their instruction around rational numbers is therefore targeted on critical mathematics content – content that students have few opportunities to revisit once they leave middle school.

Second, the study provides information on the complex nexus between knowledge and practice. The research suggests that teacher knowledge is important to effective instruction (Hill, Rowan & Ball, 2005), but how that knowledge manifests itself during instruction is not well understood. Though classrooms are inherently messy and extremely complex from a data collection standpoint, they represent the point at which teacher behaviors influence student understanding. This study provides descriptive data on the visibility of content, pedagogical content, and pedagogical knowledge in this complex yet critical arena.

Third, this study provides information on how much teachers learn from participating in what is considered to be a high-quality professional development model. A lot of money is spent each year on teacher professional development, yet little is known about what that investment actually yields in terms of improvements in teacher knowledge and practice. By focusing on a hospitable setting for teacher learning, this study describes the type of return that can be expected on an investment in this type of professional development model.

Finally, this case study provides data on the multiple factors that influence what teachers learn through professional development activities and what they take back to the classroom. Among these factors, teachers' beliefs about teaching and

learning mathematics and the complexity of the components of the model are discussed in great detail. These data could be used to help fill the void in the teacher learning literature that calls for more “systematic theorizing about the mechanisms by which teachers learn” (Wilson, 1999, p.204).

Dissertation Organization

The dissertation has seven chapters, including this introductory chapter. The next chapter, Chapter 2, presents the two literature streams supporting the design of this study. These streams represent the literature about (1) the types of mathematics knowledge required for effective teaching, and (2) the structure and characteristics of effective professional development models. Chapter 2 also connects the major ideas from both literature streams to the structure of the mathematics professional development intervention used in the PD Math Study, the focus of this inquiry.

Chapter 3 expands upon the description of the professional development model of the PD Math Study. The chapter includes a brief overview of the development, timing and structure of the model, and a catalogue of the topics and strategies of the summer institute, follow-up seminars and coaching activities. Each of the core components of the model – the math content, pedagogical content and pedagogical elements – is described in detail, to provide a framework from which subsequent descriptive analyses are conducted.

Chapter 4 includes an overview of the study’s research and evaluation methods and procedures. The chapter includes a rationale for a qualitative research strategy, an elaboration of the study’s primary research questions, a description of

data collection procedures and analytic techniques, and a discussion of steps used to foster validity and reliability of findings.

Chapters 5 and 6 contain the analyses of each of the two primary research questions. Chapter 5 summarizes teachers' understanding of the core rational number content and pedagogical content emphasized in the study. The chapter includes a rubric that depicts teachers' level of understanding. The rubric is supplemented by relevant teacher responses to the questions. Chapter 6 summarizes the level of visibility of the core content, pedagogical content and pedagogical knowledge during instruction. Like Chapter 5, this chapter includes a rubric, but instead of focusing on the level of teachers' understanding it focuses on the level of teachers' use of the professional development content during instruction. In addition to providing rubric scores on the classroom observation protocol, the chapter contains classroom dialogue that illustrates various components of the observation protocol.

Chapter 7, the final chapter of the dissertation, discusses the findings from Chapters 5 and 6 in light of the literature on teachers' knowledge and pedagogical knowledge in mathematics and the literature on effective forms of professional development. The chapter concludes with a discussion of the limitations of the study and implications for future research.

CHAPTER 2: LITERATURE BASE

This case study draws from two distinct literature bases. The first literature base addresses the types of knowledge required for effective mathematics teaching. This topic is hotly debated among scholars in the mathematics and mathematics education communities. Consequently, this literature review addresses competing notions about what constitutes sufficient knowledge for teaching. Though certain aspects are contested, other aspects of teacher knowledge are considered essential to quality teaching and learning. This review identifies these domains of knowledge and describes such knowledge in relation to the core content and pedagogical content emphasized in the professional development intervention.

The second literature base examines what is currently known about professional development as it relates to improvements in teaching and learning. More specifically, this review describes several key features of effective professional development and explains why these features are thought to be promising, especially in terms of teacher learning and instructional decision making. The review of effective professional development models includes data from experimental, survey-based and qualitative studies as well as syntheses of the literature and recommendations from professional organizations. This section of the chapter concludes with a discussion about the extent to which the professional development delivered in the PD Math Study can be characterized along these key, research-based features of effective professional development presented in the literature.

What Types of Mathematical Knowledge Are Required for Effective Teaching?

It would be extremely difficult today to find a mathematician or mathematics education expert who would describe the quality of K-12 mathematics education in the U.S. as adequate (Kilpatrick et al., 2001; Wu, 2005; Milgram, 2005; NCTM, 2007). Student achievement data from state, national and international tests over the past several decades have shown consistently that many U.S. schoolchildren, at best, have a very limited understanding of basic mathematical concepts (U.S. Department of Education [USDE], 2000; Kilpatrick et al., 2001; Perie, Grigg & Dion, 2005). A well known mathematics problem taken from the 1978 National Assessment of Education Progress (NAEP) test illustrates this point: Among the 13-year olds asked to estimate the sum of $\frac{7}{8}$ and $\frac{12}{13}$, a mere 24% percent chose the correct answer, “2.” Despite modest achievement gains on the grades 4 and 8 mathematics portions of the NAEP during the 1990s, the overwhelming majority of U.S. students still have not reached the proficient level. These results are consistent with findings from large international studies, such as the 1995 Third International Math and Science Study (TIMSS) and the 1999 TIMSS-Repeat (TIMSS-R). In both studies, U.S. 4th and 8th graders scored significantly lower than the top-scoring countries, with the 8th grade cohort performing worse than the 4th grade cohort (USDE, 2000).

These data paint a similar picture. Many U.S. schoolchildren have a shallow understanding of basic concepts and struggle greatly when they are asked to do much more than perform routine computations (NCTM 2000; NRC 2001; NCTM 2007). Since computers and calculators (rather than pencil and paper) are now the most efficient media for performing most computations, students’ understanding of the

concepts behind the procedures has become increasingly critical (NCTM 2000; NRC 2001; NCTM 2007). These technological advances do not suggest that students' ability to perform accurate computations is unimportant or unrelated to the development of conceptual knowledge (Rittle-Johnson, Siegler & Alibali, 2001; Milgram, 1999). However, in order for the U.S. to keep afloat internationally in an increasingly competitive technical marketplace, students must be able to do much more than compute accurately; they should be given ample opportunities to learn high-level, conceptually rich mathematics (Milgram, 1999).

While policymakers agree that U.S. achievement levels in mathematics are far too low, many debate how the country has gotten to this point and what should be done to fix the problem. Explanations range from faulty curricula (Wu, 1994; NCTM 2007), an under-supply of qualified teachers (Ingersoll, 2001; Darling-Hammond & Sykes, 2003), lack of quality professional development opportunities (Stigler & Hiebert, 1999; Garet, Porter, Desimone, Birman & Yoon, 2001), low expectations of teachers (USDE, 2002), lack of parental support, and so on. It is difficult to know for sure which of these explanations or combinations of explanations has the most merit. One source of the problem, however, is hardly disputed: teachers' lack of knowledge of the subject matter (Ball, 1990; NCTM, 1991; Wu, 1996; Ma, 1999; Kilpatrick et al., 2001; Milgram 2005). Adages such as "you can't teach what you don't know" are commonplace, and though there is a limited amount of direct evidence supporting this claim, some research does exist (Hill, Rowan & Ball, 2005). Most people who have spent time in schools know that a teacher's grasp of the subject matter is a critical component of sound instruction.

But what does it mean exactly for a teacher to have a solid grasp of the subject matter? This review discusses competing notions and commonalities to this question from various stakeholders in the mathematics education research community. In particular, the review discusses the formal and informal aspects of mathematics and how these conceptions of knowledge relate to the mathematical knowledge required for teaching.

What is Mathematics?

Before delving into a discussion about the types of mathematical knowledge required for teaching, it is important to clarify the term “mathematics.” If one were to ask a typical adult to define the term, most of the answers would probably have something to do with numbers or calculations. Common responses might include “it’s about numbers,” “it’s a way of quantifying things,” or “it’s about calculations and problem solving.” These informal definitions, which focus on number and number calculations, though not incorrect, are incomplete. The more formal definitions of mathematics, though far from uniform, extend well beyond the idea of number and number calculations:

- *Mathematics is the science of pattern and order.* (Kilpatrick et al., 2001)
- *Mathematics is a collection of abstract structures.* (Parsons, 1983)
- *Mathematics is the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions and of space configurations and their structure, measurement, transformations, and generalizations.* (Merriam-Webster, 2007)
- *Mathematics is [almost impossible to define]; so, realistically, the best we can do is discuss the most important characteristics: precision (precise definitions of all terms, operations, and the properties of these operations) and stating well-posed problems and solving them (well-posed problems are*

problems where all the terms are precisely defined and refer to a single universe where mathematics can be done). (Milgram, 2005)

Formal definitions of mathematics tend to focus on the *theoretical* or abstract aspects of mathematics, such as a clearly defined structure of logic, the study of pattern and change, or a problem solving arena governed by precisely defined terms and interconnected properties. Informal definitions tend to emphasize the *applied* aspects of mathematics, such as calculations with numbers or approaches to solving problems.¹

In truth, mathematics includes both theoretical and applied ideas, even though it is the applied aspects with which most people – including many math educators – are most familiar. This viewpoint is not surprising, since abstract ideas by nature are generally harder to grasp and less likely to be emphasized in U.S. schools. For example, many adults probably could recall that in order to add two fractions, the denominators must be the same. Some might even remember the exact procedure, but it is highly unlikely that many could explain why common denominators are needed for addition or how the rules for adding fractions are consistent with the rules for adding whole numbers. Explaining why a procedure works or how one concept is embedded in another involves a level of theoretical knowledge rarely emphasized in the U.S. curriculum (Stigler and Hiebert, 1999; NRC, 2001). In the previously mentioned NAEP item, if students knew that $\frac{7}{8}$ and $\frac{2}{3}$ were both “a little less than 1” they easily would have been able to identify the correct answer “2.” Instead, they chose other answers that could be found by misapplying fraction procedures, such as adding the denominators instead of finding a common denominator.

¹ Even Plato, who despised the empirical world, pointed out that mathematics was the most practical of the abstract sciences (Reeve, 2003).

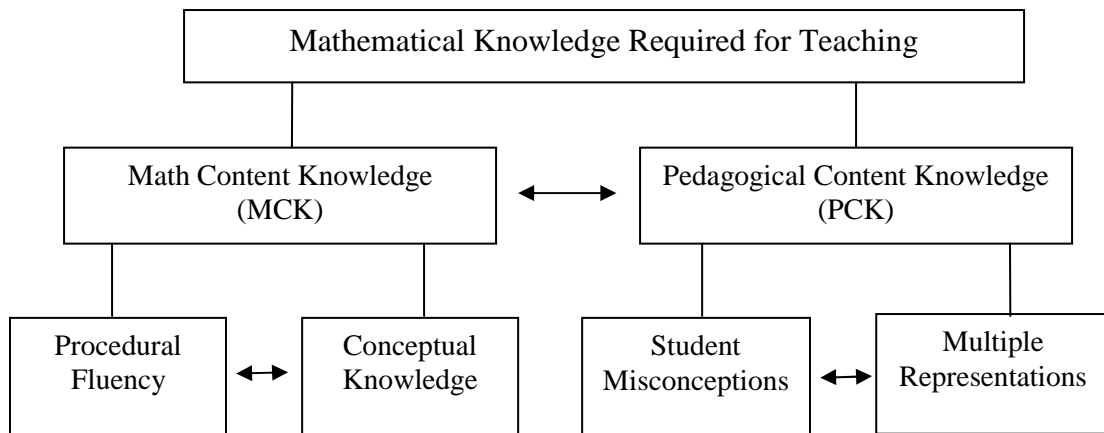
The distinction between the theoretical and applied aspects of mathematics is important because it frames the types of knowledge required for teaching. In the same way that a complete definition of mathematics includes both theoretical and applied ideas, a complete knowledge base for teaching includes both conceptual and procedural understanding. Researchers and practitioners tend to agree that teachers must know not only how a procedure works, but they also must understand and be able to represent the concepts underlying procedures as well as how such concepts are related to one another (NCTM, 2000; Kilpatrick et al., 2001; Wu, 2005; Milgram 2005). Additionally, many experts think that teachers need to be able to apply their conceptual and procedural understanding to the classroom to make instruction more meaningful for students (Shulman, 1986; Ball, 1990; Ma, 1999; NCTM, 2000).

Two Broad Types of Knowledge Required for Teaching

Conceptual and procedural knowledge fall under the umbrella of math content knowledge (MCK). Teachers typically acquire MCK in formal educational settings, through activities such as reading textbooks, taking notes, observing teacher demonstrations, listening to teacher explanations, and completing practice problems. This type of knowledge is an important foundation for teachers. In recent decades, however, a growing body of research suggests that math content knowledge is a necessary but perhaps insufficient knowledge base for effective teaching (Kilpatrick et al., 2001). Teachers also need targeted mathematical knowledge about (1) how students conceptualize and operationalize mathematics, (2) how to identify the most common student approaches and errors associated with a given topic or concept, and (3) how and when to intervene with students productively. These types of

understandings fall under the umbrella of pedagogical content knowledge (PCK), a term coined by Shulman (1986) two decades ago. Figure 2 illustrates these two general types of knowledge required for effective teaching. Given that both types of knowledge are associated with math content, they influence each other, as indicated by the double arrow. MCK consists of procedural and conceptual dimensions, and the arrow indicates connections between procedures and concepts. PCK consists of the knowledge about student misconceptions and multiple representations of concepts. Like procedural and conceptual understanding, these dimensions of knowledge interact with each others as indicated by the double arrow. The arrow from MCK to PCK indicates that these types of knowledge can inform the other. The strength of teachers' PCK is often dependent upon the strength of their MCK. For example, it is difficult for a teacher to know how to handle a particular misconception if he or she doesn't understand the math concepts underlying the misconception. Each of these components is examined more closely in the following sections.

Figure 2. Two Types of Mathematical Knowledge Required for Teaching



Mathematical Content Knowledge (MCK)

Although researchers do not always agree about which type of knowledge develops first and how both forms of knowledge interact with each other (Rittle-Johnson, Siegler, & Alibali, 2001), procedural and conceptual knowledge are widely used to categorize the knowledge base for teaching (NCTM 2000; Kilpatrick et al., 2001). Procedural knowledge is the most common type of content knowledge exhibited by U.S. teachers (Stigler & Hiebert, 1999) and is elaborated upon in the next section.

Procedural knowledge. A procedure is essentially a recipe for finding the answer to a problem. Individuals with strong procedural skills are good at following and executing steps. Although some conceptual understanding might be required to perform certain procedures, many procedures can be followed correctly with little to no understanding of the concepts underlying them. To illustrate this point, a common procedure for adding two fractions with unlike denominators is to (1) find a common denominator by multiplying the denominators, (2) multiply each numerator by the other denominator, (3) add the numerators, (4) keep the common denominator, and (5) simplify the result.² Little to no conceptual understanding is required to carry out this procedure. For example, it is not necessary to understand that the numerator and denominator of a fraction represent the number and relative size of the parts into which the unit has been divided. In the previously mentioned NAEP problem, many of the distracter answer choices were aberrations of these procedures.

² For example, $\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$ because 1) common denominator is 15 (3x5), 2) $1 \times 5 = 5$ & $2 \times 3 = 6$, (3) $5+6=11$, 4) keep 15 as the denominator, and 5) $\frac{11}{15}$ is already simplified, so there is no step 4 in this example.

Opinions differ on the importance of procedural knowledge to effective teaching. These differences generally fall into two camps.³ In one camp are the “reformers,” who argue that the U.S. curriculum and teacher practices are too procedurally focused. The National Council of Teachers of Mathematics (NCTM) is generally credited for initiating the math education reform movement in the 1990’s with the publication of two influential documents, *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Professional Standards for Teaching Mathematics* (1991). NCTM argued in these two publications that new technologies and new demands in the global marketplace required major changes to what and how mathematics should be taught. Business leaders “no longer seek workers with strong backs, clever hands, and shopkeeper arithmetic skills;” instead they seek employees that understand complex technologies, ask good questions, assimilate novel information and work cooperatively to solve challenging problems (NCTM, 1989, p.3). To meet these new goals the U.S. would have to make dramatic changes to its math curricula and its approaches to teacher preparation and professional development. Teachers would need new curricula and supporting professional development opportunities that focused more on problem solving and critical thinking and less on procedures and algorithms. According to the NCTM authors, when procedures and algorithms were presented to students, they needed to be connected

³ In this paper, I will use “reformers” to describe the mathematicians and mathematics educators who generally support the principles of the NCTM standards and/or the National Science Foundation (NSF) sponsored curricula and professional development in the 1990s. Some of the reformers are formally trained mathematicians, while others have degrees in mathematics education or other fields. Most teach in university education departments rather than the math department. When I refer to the “mathematicians,” I refer to the subset of formally trained mathematicians who have taken an interest in K-12 education but still teach primarily in university math departments. There are several prominent university mathematicians – Hung-His Wu, James Milgram, Jim Lewis, Richard Askey, Wilfren Schmid – who are very involved in K-12 education, but Wu and Milgram have written the most on the subject, which is why I reference their work most often.

explicitly to underlying concepts and represented in multiple ways that were meaningful to students (NCTM, 1991). These calls for change represented huge shifts in how math teachers were trained and how instruction was typically delivered.

In the other camp is a subset of university mathematicians with a strong interest in K-12 education. These mathematicians reacted very strongly against the NCTM standards because they thought the standards deemphasized the use of precise mathematical language and terminology, overemphasized open-ended problems and problem solving approaches, and discounted the importance of learning established procedures and algorithms. James Milgram, one of the most vocal mathematician critics, argued that many of the standard algorithms in basic arithmetic – which many of the math reformers were trying to deemphasize or alter to make them more student-friendly – were critical to success in subsequent mathematics courses. For example, Milgram (1999) argued that the long division algorithm – which proponents of the NCTM standards said should be deemphasized because it did not promote students' conceptual understanding of division – was critical to success in secondary mathematics. He argued that instead of throwing out or deemphasizing the procedure because it could be done with a calculator, teachers should teach it and explain why students would need it in subsequent math courses.⁴

Despite differing with mathematicians such as Milgram regarding the importance of procedural understanding to sound instruction, supporters of the NCTM standards were in no way suggesting that procedures be thrown out completely. Perhaps teaching had become too procedurally driven, but it was still

⁴ Milgram argued that they would need the long division algorithm in order to (1) understand terminating and repeating decimals in middle school and (2) manipulate and factor polynomials in high school (Algebra I and beyond).

fundamental for teachers to have strong basic skills and to become fluent with procedures (NCTM, 1989; Ball, 1990; NCTM 1991).

Studies conducted in the late 1980s and 1990s showed that U.S. math teachers generally had strong procedural skills (Graeber, Tirosh & Glover, 1989; Ball, 1990; Ma, 1999), although differences across math topics and differences between teachers in other countries were evident. Hardly any of the 250 preservice elementary teachers in Ball's (1990) study had difficulty carrying out a variety of procedures for operations with fractions, but they struggled significantly when asked to explain why a procedure worked or create a story problem that accurately reflected the procedure. Liping Ma (1999) also found that most of the U.S. elementary teachers in her study could execute procedures fairly well, except for some of the more difficult procedures, such as dividing mixed numbers. About half of the U.S. teachers – teachers who said math was their strength and who were assumed to be “above average” based on experience and reputation – could not accurately compute a fraction division problem. In contrast, none of the 72 Chinese math teachers in her comparative study – teachers who on average had much less formal schooling than the U.S. cohort – incorrectly answered the same fraction division problem. This finding suggests that teachers' level of procedural knowledge varies with the complexity of the procedure. The division algorithm is more complicated than other procedures because it involves multiple steps. The problem Ma used, $1\frac{3}{4} \div \frac{1}{2}$, involves (1) converting the mixed number to an improper fraction, (2) changing the \div to a \times , (3) finding the reciprocal of $\frac{1}{2}$, and (4) simplifying the result.

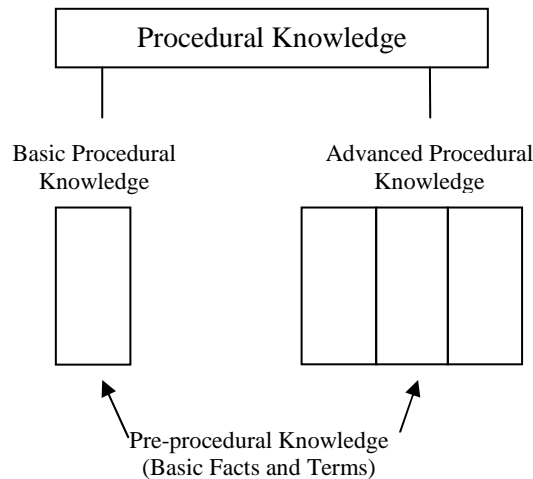
The 1999 TIMSS video study, which included large national probability samples of 8th grade teachers in the U.S., Japan and Germany, also captured varying levels of teachers' procedural understanding, but through the lens of instructional practice. The TIMSS researchers found distinct patterns of instruction across the three countries and came up with a motto to describe each teaching style. The motto they used to describe U.S. teachers in the sample was "learning terms and practicing procedures" (Stigler & Hiebert, 1999, p. 41). In contrast to Japanese and German teachers, U.S. teachers tended to present definitions and demonstrate procedures for specific problems or topics. The German teachers, who also placed an emphasis on following established procedures, tended to provide the rationale for procedures and often explained how procedures could be extended to solve more general classes of problems. The motto they used to describe the German teachers was "developing advanced procedures," since they tended to use multi-step procedures and provide rationales (Stigler & Hiebert, 1999, p. 27).

The Japanese teachers tended to place much less emphasis on established procedures and even encouraged students to generate their own procedures and algorithms. They tended to assign fewer problems, but the problems they assigned tended to be very demanding both procedurally and conceptually. The motto the TIMSS researchers assigned to Japanese teachers was "structured problem solving," which many of the NCTM reformers argued was actually similar to the instructional model espoused by the NCTM standards (Stigler & Hiebert, 1999, p. 36).

Teachers' procedural fluency, then, is not a monolithic construct. Figure 3 depicts two types of procedural fluency, basic and advanced procedural knowledge.

Basic procedural knowledge is represented as a rectangle; advanced procedural knowledge is depicted as a series of narrower rectangles, since this type of knowledge often involves extending and combining basic procedures. The multi-step algorithm for dividing mixed numbers in Ma's (1999) book is an example of advanced procedural knowledge.

Figure 3. Two Types of Procedural Knowledge



The basic facts and definitions that undergird both types of procedural knowledge include basic terminology that is necessary to carry out procedures. For example, prior to knowing how to carry out the steps involved in dividing mixed numbers, teachers need to know terms like numerator, denominator, mixed number and reciprocal. Other examples include basic addition, subtraction, multiplication and division facts. When a teacher can explain what a term means – e.g., the denominator of a fraction represents the relative size of the parts of the whole – the teacher is exhibiting conceptual understanding.

In summary, procedural knowledge is the most basic and typical type of content knowledge U.S. teachers exhibit and, consequently, the type of knowledge

that is most likely to influence their instructional routines. While t policymakers are concerned about U.S. teachers' level of procedural understanding, they are more concerned about teachers' conceptual understanding, which is elaborated upon in the next section.

Conceptual knowledge. Behind every mathematical procedure is a concept or set of concepts. Returning to our previous example, the procedure for adding fractions with like denominators rests on the concept of addition. In primary grades, children learn about the concept of addition by combining concrete, like objects (e.g., two cookies plus two cookies equals four cookies). When students encounter fraction addition problems in upper elementary school, however, many lose site of the notion of adding “like things” ($\frac{1}{4}$ of a cookie plus $\frac{3}{4}$ of a cookie can be combined because fourths are like units). They lose site of the concept because teachers tend to focus more on the series of steps required to generate the correct answer than the conceptual rationale behind the steps. The findings from the TIMSS study suggest that few U.S. teachers provide students with a rationale for why procedures work or make other instructional decisions that promote conceptual understanding. Probably the primary reason U.S. teachers avoid mathematical concepts during instruction is because they lack a strong conceptual understanding of the topics they teach (Ma, 1999; Ball, Phelps, & Thames, 2006). This deficit is why both the reformers and mathematicians (and essentially everyone in between) place a strong emphasis on improving teachers' understanding of the concepts and connections between concepts and procedures.

On the reform side, NCTM's (1991) *Professional Standards for Teaching Mathematics* outlined a broad set of knowledge objectives for teachers, including

knowledge of (1) mathematical concepts and procedures and the connections among them, (2) multiple representations of concepts and procedures, and (3) ways to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality. NCTM's (2000) *Principles for School Mathematics* called for teachers to “know and understand deeply the mathematics they are teaching” and to “understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise” (p. 373). The NCTM authors argued that the traditional teaching approaches and curricula portrayed mathematics as a narrow set of facts to be memorized (and soon forgotten) rather than a coherent and potentially powerful system of logic. Students needed teachers who had a deep understanding of the mathematics they taught and were able to represent concepts in multiple, meaningful ways. The previous example about adding fractions could be extended to illustrate this point. A teacher with a solid conceptual understanding of this topic would be able to explain why like denominators were necessary for addition – the concept of adding “like things” – and also be able to use a number line, pie chart or other representations to illustrate the concept of “likeness.”

Leading mathematicians Milgram and Wu also argued that traditional teachers' understanding of the content and approaches to instruction fell well short of what students deserved. Wu (1996) points to the abysmal student test scores and high drop out rates in K-12 math classes during the 1970s and 80s – the two decades before the NCTM reform movement which was characterized by traditional curricula and teaching approaches – as evidence that a superficial approach to mathematics content has damaging effects. What is needed, he says, is a mathematics curriculum

that deals with “the basic questions of why something is true and why something is important” (Wu, 1996, p. 4). For this type of curriculum to be delivered effectively, teachers must have a solid grasp on these basic why questions, which often are avoided or underemphasized in typical teacher preparation programs and professional development opportunities.

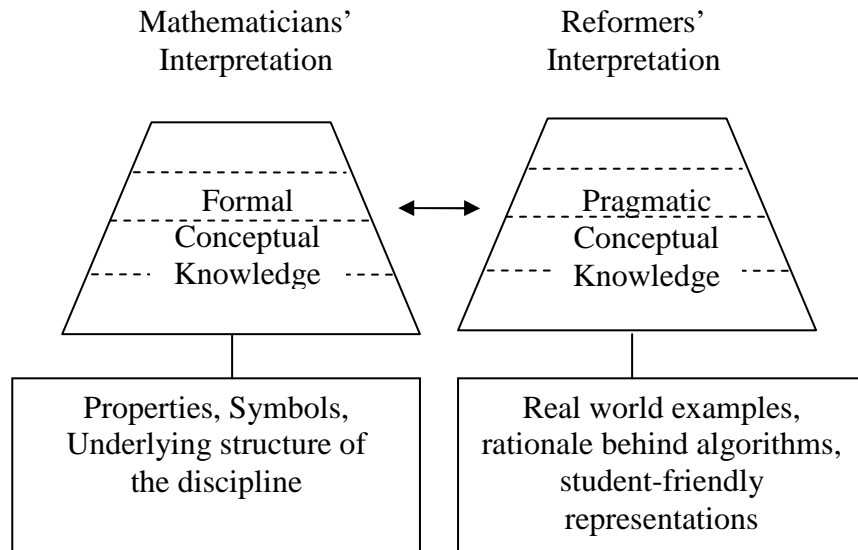
While both the mathematicians and the reformers advocate improving teachers’ conceptual knowledge, they do not agree on all aspects of this domain. The reformers emphasize teachers’ ability to represent concepts in student-friendly ways and take a more *pragmatic* view of teachers’ conceptual understanding. They contend that teachers need to understand how mathematical concepts are related and how they can be represented in order to help more students become successful in school mathematics. The mathematicians focus on teachers’ understanding of the underlying structure of mathematics, which includes precise definitions and properties and formal proofs, and, therefore, have a more *formal* view of teachers’ conceptual understanding. According to Wu, precise definitions, symbolic computations and exact answers are defining characteristics of mathematics as a discipline and yet characteristics that he believes the reformers downplay or dismiss entirely. He describes the pragmatic approach of reformers as wrongly favoring “process over product,” and relying too heavily on heuristic arguments rather than formal proofs and proper technique (Wu, 1996, p. 7).

Wu points to the following problem in NCTM’s (1991) *Professional Standards for Teaching Mathematics* to illustrate how differently the two camps address conceptual understanding: “If 30 points were scored in a basketball game

without a single foul shot, how were the 30 points scored?” (Basketball has 2 and 3 point shots.) He argues that although this problem requires students to find the number of possible combinations without resorting to a teacher imposed algorithm, it is imprecisely and therefore improperly posed. Instead of having students generate a list of outcomes that likely will include some students listing all the 2-point combinations, others listing all the 3-points combinations, and others listing both the 2 and 3-point combinations, it would have been better (and truer to the discipline) to pose the question precisely as “List all the possible ways the 30 points were scored,” or to use the imprecisely posed question as an opportunity to illustrate how such a problem can be translated into a mathematically precise statement. While the reformers see an imprecisely posed question as an opportunity to gauge student understanding of and approaches to a particular concept, the mathematicians view such as prompt as blatantly contradicting the foundational principle of precision.

Despite these distinctions between formal and pragmatic conceptual understanding, both the mathematicians and the reformers depict conceptual understanding as a hierarchical process rather than knowledge teachers either simply have or don't have. Teachers possess varying degrees of understanding of concepts, since mathematical ideas rest on a series of supporting and interconnected concepts (Figure 4). Teachers with deeper levels of conceptual understanding of a topic are located at progressively lower bands of each trapezoid. For example, a teacher with an extremely deep formal conceptual understanding of a topic would be located in the bottom band of the trapezoid to indicate that the teacher grasps the foundational principles of the concept as well as the relevant bands above.

Figure 4. Two Interpretations of Conceptual Knowledge⁵



The National Research Council's *Adding It Up* (2001) outlined a knowledge base encompassing both the pragmatic and formal aspects of conceptual understanding. According to its authors, which included math educators and mathematicians, K-8 mathematics teachers should possess a knowledge base that includes:

Knowledge of mathematical facts, concepts, procedures, and the relationships among them; knowledge of the ways that mathematical ideas can be represented; and knowledge of mathematics as a discipline – in particular, how mathematical knowledge is produced, the nature of discourse in mathematics, and the norms and standards of evidence and proof... Teachers certainly need to be able to understand concepts correctly and perform procedures accurately, but they must also be able to understand the conceptual foundations of that knowledge. (Kilpatrick et al., 2001, p.371)

This description includes pragmatic aspects of conceptual knowledge, such as representing mathematical ideas and connecting concepts and procedures. It also

⁵ I borrowed part of this conceptual model from Liping Ma's (1999) description of Profound Understanding of Fundamental Mathematics.

addresses formal aspects of conceptual knowledge, such as knowing how mathematical knowledge is produced and the role of proofs in verifying such knowledge.

Both the mathematicians and the reformers advocate improving teacher conceptual understanding because they believe it will improve the quality of instruction. Without a deep understanding of the concepts they teach – whether that understanding is more formal or pragmatic – teachers will be ill-equipped to help students overcome the various misconceptions they bring to the classroom. How teachers’ conceptual knowledge connects to the instruction is part of the other broad category of teachers’ content knowledge, pedagogical content knowledge.

Pedagogical Content Knowledge

The second broad category of teacher content knowledge is pedagogical content knowledge (PCK), a construct introduced by AERA president Lee Shulman during his 1985 inaugural address. Shulman argued that teacher preparation programs and teacher evaluation systems placed too little emphasis on teachers’ content knowledge. Instead, these systems were based on non-content aims such as lesson planning, classroom management, cultural awareness and evaluation. Although Shulman did not discount the importance of these aspects in teacher preparation, he believed that the lack of attention to content knowledge represented a significant “blind spot” in the academy. Shulman and his colleagues – as well as other groups of researchers from the University of Pittsburgh and Michigan State University – believed that a new theoretical framework was needed to capture what teachers needed to know to teach their students effectively.

Shulman's (1986) research focused on classifying the domains of content knowledge, understanding how content and general pedagogical knowledge are related, and identifying the most promising ways of enhancing the acquisition and development of the types of teacher content knowledge most relevant to teaching. One of the key outcomes of Shulman's (1986) research was a new theoretical framework that included the domain of pedagogical content knowledge (PCK),⁶ which he described broadly as the type of content knowledge required to teach a topic well. More specifically, PCK encompassed the most commonly taught topics within a given subject area, the most intuitive representations of those concepts, and the most powerful explanations and demonstrations – “in a word, the ways of representing and formulating the subject that make it comprehensible to others”(Shulman, 1986, p. 9). PCK also contained knowledge of what made particular topics easier or harder for students to understand, including students' most common conceptions and most persistent misconceptions.

The construct of PCK resonated deeply in the mathematics education research community and spurred a number of studies in the late 1980s and 1990s. Post, Harel, Behr and Lesh (1988) found that in addition to lacking basic content knowledge, the majority of the grade 4-6 teachers in their sample could not explain the problems they solved correctly in a “pedagogically acceptable manner,” i.e., a manner that included a clear and correct explanation and an awareness of how and when to assist students when they are confused. In a study of prospective elementary and secondary

⁶ Shulman presented three types of teacher content knowledge: content knowledge, pedagogical content knowledge and curricular knowledge. Content knowledge refers to what is described as MCK in this paper. Curricular knowledge did not have the same impact as pedagogical content knowledge, which quickly resonated and became a central component of many subsequent research studies, and therefore did not warrant a third category of teacher content knowledge.

teachers' understanding of division, Ball (1990) showed that although most of the teachers in her sample could calculate $1\frac{3}{4} \div \frac{1}{2}$ correctly, few could explain the underlying meaning or generate a student-intuitive representation of the problem, both of which are key aspects of PCK. She used the phrase "rule bound and thin" to describe pre-service teachers' understanding of division. Graeber, Tirosh and Glover (1989) used similar language – "rigid and segmented" – to describe prospective elementary teachers' knowledge of rational numbers topics and argued that it was essential to increase teachers' familiarity with common representations of rational number topics and increase their understanding of the rationales behind common procedures.

Student misconceptions. Being able to identify, anticipate and debug the most common student misconceptions for a given math topic or concept is an important component of PCK (NCTM, 1989; 1991; 2000; Kilpatrick et al., 2001). Though the level of this type of knowledge is often commensurate with experience – i.e., the more exposure a teacher has to students' work and approaches, the more specific misconceptions the teacher will know about and the more ways the teacher will have tried to resolve the misconception – teachers also can learn about student misconceptions through written activities in coursework or professional development activities. For example, examining student work is a common activity in teacher professional development in many subject areas. Being able to identify and debug student misconceptions requires strong conceptual understanding of the material students are studying, as indicated in the arrow from MCK to PCK (see Figure 2).

Multiple representations of concepts. Another core aspect of PCK is the teacher's ability to represent a single concept in multiple ways. For example, suppose a group of students is struggling with the meaning of denominator during a lesson on fractions. The teacher with strong PCK is able to represent denominator in a number of ways, such as a number line or an area model in addition to the classic pie or circle model. Like identifying and debugging student misconceptions, being able to represent concepts in multiple ways requires a strong understanding of the relevant mathematical concepts and how mathematical concepts relate to each other. The research and professional organizations describe multiple representations of concepts as a core aspect of mathematics PCK (Kilpatrick et al., 2001; NCTM, 1989; 1991; 2000).

Further clarification of PCK. Although PCK has become a household term in the education research community and spurred a variety of studies, some researchers believe that the term needs to be further refined in order for its impact to be maximized. Ball, Phelps and Thames (2006) suggest that PCK – although a useful construct – could be better utilized if it were broken into two sub-domains, knowledge of students and knowledge of teaching. Knowledge of students refers to knowledge of the most common approaches, conceptions and misconceptions that students bring to particular topics or concepts. Knowledge of teaching refers to the most intuitive representations and explanations to be used with students to make particular concepts meaningful to students. Although these two types of knowledge can interact with each other – e.g., a teacher identifies a student misconception in a particular topic and then introduces specific teaching strategies to clarify the

misconception – such interactions do not always occur. For example, a teacher might be able to recognize a particular misconception but not know the representation that could be used to help the student clarify the misconception.

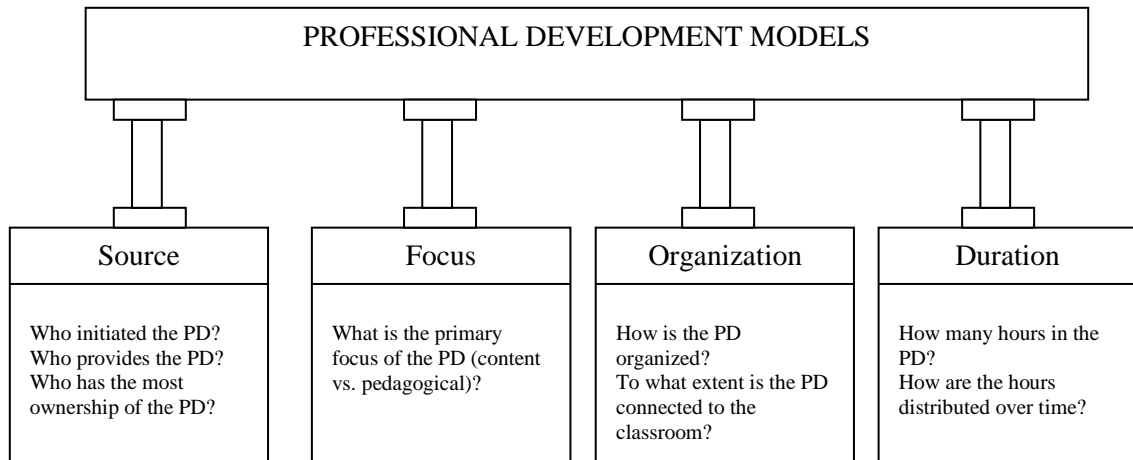
As the Ball et al. (2006) paper suggests, PCK and pragmatic conceptual knowledge overlap. For example, both deal with student friendly representations and teachers' flexible understanding of math concepts. They differ in that PCK is driven more heavily by experiences in the classroom. The teacher with highly developed PCK in a particular topic most likely has developed that knowledge by extensive analyses of student work and interactions with students. Although the border between PCK and Specialized Content Knowledge might still seem blurred, both categories point to a similar problem: until teachers' knowledge of the subjects they teach becomes more conceptually deep and pedagogically flexible, student achievement will continue to lag.

Mathematics content focus of the larger study. The PD Math study had a dual emphasis on building teachers' conceptual knowledge and pedagogical content knowledge. The model addressed teachers' conceptual knowledge of core rational number content by including workshop activities and resource materials that focused on improving the precision of definitions and math language of rational number concepts, making connections among rational number topics and concepts, and providing the rationales behind common rational number procedures. The model addressed teachers' pedagogical content knowledge by including activities and resources that focused on looking at student work in light of related misconceptions and presenting concepts in multiple, student-friendly ways.

What Are the Characteristics of Promising Professional Development Models?

Given the documented deficits in teachers' content and pedagogical content knowledge and given that a replacement pool of well qualified math teachers does not exist (Ingersoll, 1999 & 2001; Darling-Hammond & Sykes, 2003), policymakers increasingly are looking toward professional development as a critical means to help existing teachers boost their knowledge and improve their instructional practices. Professional development has been identified as an important policy tool or "policy pathway" to professionalize teaching and ultimately improve the quality of teaching and learning (Knapp, 2003). Yet, research rarely links professional development with gains in student achievement (Yoon et al., 2007). The professional development literature, however, does identify specific features of professional development models that are deemed to be more promising or effective (Hawley & Valli, 1999; Garet et al., 2001). Since professional development is a broad term that encompasses a variety of structures and delivery types, it is useful to frame a discussion about the more effective types of professional development around a few, core structural dimensions that apply to most, if not all, professional development models. These dimensions include the *source*, *focus*, *organization* and *duration* of the professional development model or activity. Figure 5 illustrates each of these four dimensions as pillars upon which professional development models rest. Each is described more fully in the following sections.

Figure 5. Structural Components of Professional Development Models



Source

The *source* or impetus for a professional development activity can arise from a number of places. A teacher might seek out additional support for a particular aspect of teaching, such as in-depth study of particular subject matter content, and then attend a relevant workshop at a conference. In this case, the teacher initiated the professional development based on a self-assessment of teaching practice. On the other hand, a school or district might decide that teachers need targeted professional development in some area, and then mandate that teachers participate in a relevant district-sponsored professional development activity. Thus, professional development can either be voluntary or involuntary, although sometimes the distinction can be blurred. In general, though, professional development activities can be described as more or less teacher-initiated, which is an important distinction to make when categorizing the level of teacher ownership of learning opportunities.

In their review of the literature on effective types of professional development, Hawley and Valli (1999) suggest that effective professional development programs, to the greatest extent possible, involve teachers in the identification and development of what they need to learn and how they can learn it. Involving teachers directly in this way increases their sense of ownership and the amount of motivation and effort they bring to each learning situation. Further, according to the authors, when teachers take more ownership of what they are learning, the learning is more likely to be linked to instruction and more likely to promote collaboration with colleagues who struggle with the same issues. Teachers are likely to dismiss professional development that is imposed from outside experts who do not pay careful attention to their daily work. In these instances, professional development has little chance of influencing what teachers know and do.

Hawley and Valli (1999) also indicate that Alexander and Murphy's (1998) five learner-centered principles have implications related to the source of professional development programs. Alexander and Murphy's (1998) motivation principal says that "intrinsic motivation, attributes for learning, along with personal goals, along with the motivational characteristics of the learning tasks, play a significant role in the learning process" (as quoted in Hawley and Valli, 1999, p.133). Teachers' beliefs play a lesser role in professional development activities that they initiate; but, when the source of the professional development activity is external, teachers' beliefs about the program can influence dramatically the degree of teacher learning. Reform mathematics programs, which emphasize conceptual understanding over rote memorization, are almost impossible to deliver well if the teacher believes that this

approach to teaching is invalid. Thus, professional development as a policy for change can encounter stiff resistance during implementation. Cohen, Moffitt and Goldin (2007) argue that the “further policy departs from extant practice, the more likely is conflict” (p.80). Teachers’ beliefs and attitudes about the nature of the professional development, particularly when it is imposed from the outside, figure heavily into how much the intervention influences practice.

In her discussion of professional development in the context of reform initiatives, which, like the core aspects of the PD Math professional development model, require significant shifts in what teachers know and how they normally operate, Little (1993) identifies six principles of professional development that “stand up to the complexity” (p.138) of educational reforms, and aspects of these principles address the *source* of the professional development activity. Her first principle suggests that professional development should be meaningful to teachers both intellectually and socially. In contrast to the shallow, fragmented learning opportunities teachers often receive, learning opportunities should be available that require teachers to be actively involved in deepening their understanding of the content through access to and collaboration with experts in the field.

Little (1993) also says that professional development should encourage and provide opportunities for “informed dissent.” Given the difficulty of achieving consensus or only achieving it superficially when reform initiatives are being implemented, she suggests incorporating into professional development models time and support for developing “well-informed dissent.” Such dissent ultimately can bolster both group and individual decisions. She builds on this aspect of teacher-

centered learning by stating that professional development should prepare teachers to “employ the techniques and perspectives of inquiry” (Little, 1993, p.139). She argues that professional learning opportunities should promote teachers’ generating and assessing their own knowledge rather than rely on consuming external knowledge.

Though the importance of teachers initiating and engaging in professional learning opportunities seems clear, studies also show that teacher learning can be enhanced when teachers collaborate with and draw upon the knowledge of outside experts. A good example of such a professional development model, one that utilizes the expertise of researchers and outside content experts but also incorporates teachers’ interests and daily work, is the widely studied Cognitively Guided Instruction (CGI) program. CGI is one of the few professional development programs that met stringent What Works Clearinghouse (WWC) standards of evidence and showed an impact on student achievement (Yoon et al., 2007).⁷

The CGI researchers developed a knowledge framework around students’ thinking in several different mathematical content areas. The framework included key problems and common student approaches to the problems for each content area. Instead of presenting a rigid framework that teachers were expected to follow, though, the CGI researchers developed and implemented a framework that teachers

⁷ The What Works Clearinghouse (WWC) standards of evidence include: (1) Topic – The study had to deal with the effects of in-service teacher professional development on student achievement; (2) Population – The sample had to include teachers of English, mathematics, or science and their K-12 students; (3) Study Design – The review of evidence was limited to final manuscripts that were based on empirical studies using randomized controlled trials or quasi-experimental designs, as defined by the WWC study design classification; (4) Outcome – The study had to measure student achievement outcomes; (5) Outcome – The study had to use measures demonstrated to be valid and reliable; (6) Time – The study had to be conducted between 1986 and 2006; (7) Country – The study had to take place in the U.S., Canada, Australia, or the United Kingdom, due to concerns about the external validity of the findings.

could mold to meet the specific instructional challenges of their students. As Franke and Kazemi (2001) put it:

The frameworks provided teachers the opportunity to understand how this knowledge about the development of children's thinking fits together so that the teachers could make it their own. The teachers discuss CGI as a philosophy, a way of thinking about the teaching and learning of mathematics, not as a recipe, a prescription or a limited set of knowledge (p.102).

The model was designed carefully by outside researchers who identified the most salient student misconceptions for each content area. At the same time, the CGI model was structured flexibly so that teachers could “make it their own.” Thus, the source of the CGI model was part outside expert-driven and part teacher-driven.

These studies suggest that effective professional development models provide opportunities for teachers to take ownership of their learning and to engage in sustained, active inquiry. Outside experts are certainly important to facilitating teacher learning (Hawley & Valli, 1999). But a key factor shaping the success of professional development is the extent to which teachers buy into and build upon the core aspects of the model.

Focus

The *focus* of a professional development activity varies across activities and models. One might focus on deepening teachers' understanding of the subject matter; another might emphasize pedagogical skills; and another might focus on aligning teaching practices with district standards and assessments. For example, in response to a district mandate requiring that all 8th grade students take Algebra I, a middle

school math department might decide to offer content focused professional development in foundational Algebra I content. In contrast to content-focused professional development, professional development activities focused on classroom management techniques might be assigned to a group of new teachers. The focus of the activity, then, is another distinguishing characteristic of any professional development activity or model.

Professional development activities that focus on deepening teachers' content and/or pedagogical content knowledge are thought to be more effective than other models. For example, Cohen and Hill (1998) conducted a study of mathematics teachers in California and found that student achievement was higher in schools where teachers had participated in extensive, content-focused professional development. Garet et al. (2001), utilizing a national probability sample of over 1000 math and science teachers, came to a similar conclusion. They found that content-focused professional development had a significant positive direct effect on teacher self-reported knowledge and skills and a significant positive indirect effect on changes in teacher practice. These findings are consistent with Kennedy's (1998) review of studies that linked various types of professional development to student achievement. She found that content and pedagogical content-focused professional development had larger positive effects on student achievement – particularly on students' conceptual understanding – than more general types of professional development.

Garet et al. (2001) also found that professional development activities that were coherent with other aspects of teachers' work – e.g., closely aligned to district

curricula or standards – had a direct positive effect on changes in teacher skills and practice. The researchers created an index that included whether the professional development was connected to information that teachers learned previously, aligned with state and district standards and assessments, and involved teachers in professional communication with other teachers and administrators. Hawley and Valli (1999) make a similar claim about coherence by stating that professional development should be driven by analyses of student achievement in relation to curriculum standards and benchmark assessments.

Publications by the National Council of Teachers of Mathematics (NCTM 1989; 1991; 2000) and the National Research Council (2001) also highlight the importance of content and pedagogical content focused professional development activities. Both organizations recommend sustained, intensive professional learning opportunities for teachers in these areas. In her case study comparing U.S. and Chinese elementary math teachers' knowledge, Ma (1999) documents how differences in teachers' subject area and pedagogical content knowledge impact the quality of teaching and learning. Her comparative case study showed that Chinese teachers were able to identify and articulate core math concepts and connect the concepts to student work, while U.S. teachers could not.

These studies suggest that professional development models that focus on building teachers' content or pedagogical content knowledge are promising in terms of promoting teacher learning that can improve the quality of instruction. However, we know from a recent report that included a nationally representative sample of teachers that only 16 percent of secondary math teachers report spending more than

24 hours per year on professional development focused in math (Birman et al., 2007). The literature also suggests that when the professional development is coherent with district standards and assessments, it is likely to impact the quality of teaching and learning.

Organization

The *organization* of the professional development can take many forms. The activity or model might be formal or informal, individual or group-based, classroom-based or a traditional workshop. For example, many teachers in Japan participate in lesson study groups that meet regularly over the course of a school year to focus on improving a lesson or series of interconnected lessons (Stigler & Hiebert, 1999). Since teachers meet together regularly with a clear set of goals, the organization is formal and group-based. Since they bring examples of student work and sometimes observe each other teaching, lesson study is also classroom-based. This type of professional development activity is in contrast to a one-time workshop where teachers, at most, bring back to the classroom a few, isolated activities. Attending a workshop tends to be a formal, individualized learning opportunity for teachers that may or may not be classroom based. An example of an informal learning situation would be if a group of teachers decided to meet to discuss some aspect teaching. These examples illustrate that the organization of the professional development activity can take many forms.

The literature contains considerable support for professional development models that promote collective and collegial participation among teachers. Garet et al. (2001) found that collective participation had modest positive effects on the core

features of coherence and active learning. They defined collective participation as group participation in professional development, such as participation by a department or grade-level group of teachers. Talbert and McLaughlin (1994) found that teacher participation in active learning communities enhanced professional knowledge and overall professionalism. Little (1993) argued that teachers should have regular opportunities to engage intellectually with colleagues both inside and outside of teaching. Thus, teacher learning communities should include not only teachers within a department or within a school, but also content experts and university researchers who are capable of infusing the learning communities with relevant professional knowledge.

Cochran-Smith and Lytle (1999), in their literature review on teachers' acquisition of professional knowledge in learning communities, argue that collegial learning opportunities are more productive when teachers participate as active inquirers in the learning community. This finding incorporates the promise of collegial learning with the earlier recommendation regarding the *source* of the professional development: that teacher-initiated learning activities and opportunities for active inquiry are thought to be productive.

Despite the promise of collegial learning opportunities, Elmore and colleagues (1996) caution that changing the structure of teachers work – such as creating small learning communities – won't automatically promote improvements in teacher learning, particularly in mathematics. In their case study research, they found instances where creating opportunities for cooperative learning had only modest effects on classroom teaching practice. The main barriers to changing teachers'

practice in these instances included “teachers’ deep-seated ideas about content and pedagogy and their limited access to experiences and external contacts that would help them develop alternative conceptions of knowledge and pedagogy” (Elmore et al., 1996, p.137). Thus, teacher beliefs about the underlying philosophy of the professional development coupled with access to outside experts can impact the extent to which the professional learning community impacts teacher learning. Nevertheless, collegial learning opportunities are thought to be more promising than isolated ones.

Duration

Like the previously discussed dimensions, the *duration* of professional development can vary widely across activities or models. A district might offer a series of connected workshops that span one or two academic years rather than a one-time workshop. A teacher might take an intensive, three-week university course during the summer while another teacher takes the semester-long version of the course. Thus, duration and intensity are important characteristics to consider when describing professional development.

Garet et al. (2001) defined duration as including both the total number of teacher contact hours and the time span of the professional development. The researchers found that both dimensions of duration had substantial positive direct effects on active learning and coherence and modest positive effects on content focus. This finding suggests that both “how much” and “how long” are important characteristics of professional development. In Yoon et al.’s (2007) meta-analysis of professional development and student achievement, they found that the professional

development models that showed gains in student achievement averaged about 70 hours of professional development for one year.

Wu (2005) argues that content-focused professional development for mathematics teachers requires a great deal of time and commitment. His content-focused courses include a 3-week summer institute and 5 follow up seminars spread throughout the school year. This structure highlights the importance of intensity and duration to Wu's approach to enhancing teachers' acquisition of professional knowledge. Professional organizations, such as the National Council of Teachers of Mathematics and the National Research Council also point to the importance of intensive, sustained teacher learning opportunities. For example, NCTM (1991; 2000) says that teachers should be provided with regular, ongoing opportunities to reflect on student learning with colleagues, participate in professional organizations and even design and evaluate professional development opportunities. Thus, the evidence is overwhelmingly against one-shot or short-term professional development opportunities and overwhelmingly in favor of intensive, sustained professional learning opportunities.

Recent Syntheses of the Literature and Professional Recommendations

Given the emerging consensus regarding the importance of professional development to improving the quality of teaching and learning and given the perception that much of the professional development currently available to teachers is ineffective or misguided, a number of scholars have synthesized the literature on the effectiveness of professional development. These syntheses have identified several key principles or attributes of effective professional development models,

even though very few rigorous impact studies have been conducted that show which attributes matter most. Hawley and Valli (1999), drawing upon the previous decade of research, outlined eight design principles for effective professional development. Principle one refers to student performance relative to standards for student learning. They suggest that effective professional development programs focus on the differences between the goals or standards for student learning and students' actual performance. This approach is especially important when professional development is targeted to students with historically lower levels of achievement. Such a "student centered" focus, though seemingly obvious, was not the norm in professional development at that time. NCLB's requirement that all groups of students show adequately yearly progress on standards-based assessments indicates that this recommendation continues to be, or perhaps is even more, relevant today.

Hawley and Valli's (1999) second principle refers to the level of teacher involvement in the development of professional development learning opportunities. Effective professional development programs, to the greatest extent possible, involve teachers in the identification and development of what they need to learn and how to learn it. Involving teachers directly in this way increases their sense of ownership and the amount of motivation and effort they bring to each learning situation. Further, according to the authors, when teachers take more ownership for what they are learning, the learning is more likely to be linked to instruction and more likely to promote collaboration with colleagues who struggle with the same issues. Professional development that is imposed from outside experts who do not pay

careful attention to teachers' daily work is likely to be dismissed by teachers and has little chance of influencing what teachers know and do.

Principle three is that effective professional development should be mainly school based and linked to teachers' daily work. The logic behind this principle is that teachers are more likely to be motivated to solve "job embedded" problems, which are more pressing and authentic than other type of problems. Ideally, school based professional development includes groups of teachers continuously collaborating to improve the quality of teaching and learning.

Principle four builds on the idea of teacher collaboration and refers to the extent to which professional development is organized around collaborative problem solving. Collaborative problem solving includes activities such as interdisciplinary planning and study groups, where teachers bring their individual expertise to solve a joint problem. When teachers collaborate in this way, a culture of professional respect replaces the teachers' sense of isolation.

Principle five is that professional learning experiences should be continuous and supported, as opposed to episodic and isolated. Continuous and supported professional development is necessary, in part, because many of the reform efforts are complex and require teachers to understand the subject matter deeply so they can teach in ways that promote student understanding (see NCTM 1989, 1991 & 2000; Kilpatrick et al., 2001). This sort of professional learning takes time – sometimes several years – so professional development models should allow adequate time and provide support, such as structured opportunities to learn from more experienced or expert colleagues, to solidify teacher understanding.

Principle six suggests that professional development should be evaluated and refined through multiple sources of rich information. Both practitioner and outside expert knowledge should be considered throughout the various stages of professional development, so that it can be adjusted to maximize teacher and student learning.

Principle seven refers to the importance of providing teachers with a clear theoretical base that underlying the professional development. Teachers are less likely to engage in learning new material or implementing new teaching strategies if they are not given a clear rationale for why the new material or strategy is thought to be promising or worthwhile. The theoretical background, though important by itself, is much more likely to be internalized if the theory is accompanied by attention to teachers' beliefs and experiences. For example, if teachers believe that instruction should be more teacher-centered than student-centered, they aren't likely to change their beliefs suddenly. Rather, they must be given gradual opportunities to try out new techniques and apply new forms of knowledge.

Principle eight is that effective professional development should be part of a larger, more comprehensive change process. Professional development activities that are consistent with state, district or school initiatives are more likely to be supported by district and school instructional leaders and aligned with teachers' daily work.

In his description of how professional development can be used as a policy tool to improve teaching and learning, Knapp (2003) synthesized the literature on "high quality" professional development and concluded that powerful professional learning experiences tend to 1) focus on teaching practices that support students achieving high learning standards; 2) build teachers' pedagogical content knowledge;

3) employ engaging and proven instructional practices for adult learners; 4) promote collegial, collaborate learning; 5) provide rigorous, sustained learning opportunities; and 6) align with district and state standards and assessments or reform initiatives.

Although Knapp cautions that these six attributes have not been linked definitively to gains in student achievement, he thinks professional development models characterized by one or more of these features are likely to benefit teachers and students.

In her examination of research on effective professional development in the 1990s, Wilson (1999) identified similar features of promising professional development. She includes findings from several sources of literature, including a synthesis by Little (1988), who claimed that effective professional development should 1) ensure collaboration and promote shared understanding among teachers; 2) require collective or group participation; 3) emphasize the most critical problems in curriculum and instruction; 4) occur with regularity to promote progressive levels of understanding and skill; and 5) be consistent with established professional habits and norms regarding collegial learning and cooperation.

Wilson also included a set of characteristics of high quality professional development that Abdal-Haqq (1995) proposed. According to Abdal-Haqq, effective professional development:

- 1) is continuous or ongoing;
- 2) includes a feedback loop grounded in teacher practice;
- 3) is school-based and closely connected to teachers' daily work;
- 4) is collaborative and includes opportunities for collegial interaction;
- 5) should focus on and be driven by student learning;
- 6) promotes school-based and teacher-based initiative to expand learning;
- 7) is firmly grounded in the knowledge base for teaching;
- 8) includes constructivist theories of teaching and learning;

- 9) acknowledges teachers as professional learners;
- 10) includes ample time for teachers to receive follow-up on what they are learning;
- 11) is pitched at a level that is accessible to all types of teachers (Wilson, 1999).

Wilson notes that these lists are not mutually exclusive and address a few essential aspects or “mantras,” which Putnam and Borko (1997) describe as

1. Teachers should be treated as active learners who construct their own understanding.
2. Teachers should be empowered and treated as professionals.
3. Teacher education must be situated in classroom practice.
4. Teacher educators should treat teachers as they expect teachers to treat students (Wilson, 1999, p.176).

Snow-Renner’s and Lauer’s (2005) review of the literature identified the following dimensions of professional development models that are most likely to improve teacher practice and student achievement. Such models are (1) of considerable duration; (2) focused on particular rather than general content or pedagogical strategies; (3) typified by collegial or collective participation; (4) coherent with district standards and curricula; and (5) infused with active learning opportunities. The authors indicated that despite the promise of these dimensions, a minority of current professional development models possess one or more of these characteristics.

Taken together, these literature reviews address several principles related to the source, focus, organization and duration of professional development. The reviews indicate that professional development activities should provide opportunities for teachers to be actively involved in their learning, to collaborate with other teachers, to make connections to the classroom, and to participate in sustained and intensive learning activities.

Professional Recommendations

In addition to recent syntheses of the literature, I reviewed the recommendations of many different professional organizations related to professional development. I then assessed the extent to which the professional recommendations matched the attributes of effective professional development models identified in the literature. Since the professional recommendations are based to varying degrees on syntheses of the literature and individual studies, the two data sources agree considerably.

The National Staff Development Council (NSDC) recently issued a plan for improving staff development, which included the development of a federal clearinghouse that would store information on effective professional development. Sparks and Hirsh (2006) reviewed the literature and provided a preliminary list of key features of effective professional development. According to NSDC, high-quality professional development is (1) results-driven; (2) job-embedded; (3) subject-matter, pedagogical, or pedagogical content-focused; (4) curriculum and/or standards-based; and (5) sustained and cumulative (Sparks and Hirsh, 2006).

The National Board for Professional Teaching Standards (NBPTS) originated in the mid-1980s, in response to the Carnegie publication, *A Nation Prepared: Teachers for the 21st Century*. The NBPTS features a voluntary national certification program, which teachers are increasingly seeking, in part because states are more and more offering financial and other type of incentives for National Board-certified teachers (Birman et al., 2007). Although the NBPTS does not outline particular

features of effective professional development, it does identify five, related core propositions regarding professional teaching.

Proposition 1 is that teachers are committed to all students and learning. This proposition assumes that teachers are committed to making knowledge accessible to all types and levels of students. The second, related proposition assumes that teachers know the content and pedagogical content subject matter of the courses they teach so the material can be presented solidly and meaningfully to all types of students. The third proposition is that professional teachers know how to manage and monitor student learning, which requires a thorough understanding of state and district standards and assessments and students' relative performance. The fourth proposition is that teachers should engage in habits that help them think systematically about their practice. Professional teachers read, question, learn and experiment. They regularly examine their practice as well as the theory behind practice. The final proposition is that the development of teachers' professional knowledge is strengthened and solidified through participation in professional learning communities. Professional teachers collaborate with other teachers primarily, but also with parents and business leaders to refine instructional practices and curriculum development.

Professional organizations in various content areas also have recommendations for how professional development should be organized and delivered. The National Council of Teachers of Mathematics (NCTM), one of the pioneers of the standards movement of the 1990s, has produced a number of reports that describe the characteristics of effective professional development in mathematics. NCTM's (1991) *Professional Standards for Teaching Mathematics* included six

standards to guide professional development. According to this report, high-quality professional development should include opportunities for teachers to (1) experience good teaching and be given opportunities to pose mathematical tasks and engage in meaningful mathematical discourse; (2) deepen their understanding of mathematics and school mathematics, including mathematical concepts, procedures and representations; (3) understand students as learners of mathematics and be given opportunities to learn about multiple student approaches, conceptions and misconceptions around core concepts; (4) understand mathematics pedagogy, such as facility with representations, assessment strategies and discourse strategies; and (5) develop continually as a mathematics teacher, such as collaborative opportunities to examine and revise assumptions about the nature of mathematics and mathematics teaching. The sixth and final standard addresses the teacher's role and responsibility in professional development. This standard implores teachers to take an active role in their learning and contends that learning should include:

- Participating actively in a professional learning community of mathematics teachers;
- Participating in the design and evaluation of professional development activities;
- Discussing and reflecting individually and with colleagues about issues in mathematics and mathematics teaching;
- Reading and discussing ideas presented in professional publications and professional meetings;

Another professional organization, the Association for Supervision and Curriculum Development (ASCD), published a report based on the work of Joyce and Showers (2002), who identified a list of key findings relating professional development to student achievement. The authors, whose research is targeted primarily at instructional coaching, suggested that professional training should (1)

provide opportunities for teachers to develop knowledge – including the underlying theory of what they are learning, observe demonstration lessons, and practice the skill and receive peer coaching; (2) help participants learn how to acquire new knowledge and become more effective and persistent learners; and (3) feature collaborative learning opportunities, such as joint planning of lessons; and (4) be supported by strong school leadership. The authors argue that instructional coaching is especially critical in this list. They say that teachers need 8-10 weeks to practice the new skill they are acquiring. Coached teachers, in contrast to non-coached teachers, are more likely to practice the new skill or strategy more often and more accurately, adapt the new strategy to meet their own goals, retain and increase the level of skill over time, and explain the new instructional models to their students.

The syntheses of the literature and the recommendations from professional organizations overlap considerably. Using the four dimensions of professional development previously discussed and Hawley and Valli's (1999) eight principles for designing effective professional development, Table 1 displays the consistency with which these attributes have been described across recent syntheses of the literature and from reports issued by professional organizations.

Professional Development Structure of the Larger Study

The professional development model in the larger study addressed the most critical aspects of teachers' knowledge, conceptual understanding and pedagogical content knowledge, and also adhered to these principles of high quality professional development. In terms of the *source* of the professional development, the PD Math

model is a voluntary program. The role of the professional development providers is a non-evaluative, supportive one.

Table 1
Attributes of Effective Professional Development Models in Recent Literature Reviews & Professional Reports

	Source	----- Focus -----			----- Organization -----		Duration	
	Teacher Initiated or Empowered (2) ^a	Aligned with District Standards (8)	Linked to Theoretical Rationale (7)	Focused on Student Outcomes (1)	Part of Larger System of Change (8)	Collaborative and Collegial (4)	Connected to Teachers' Daily Work (3)	Sustained and/or Intensive (5)
Research Syntheses								
Hawley & Valli (1999) ^b	X	X	X	X	X	X	X	X
Knapp (2003)		X		X	X	X		X
Wilson & Berne (1999)	X	X	X	X	X	X	X	X
Snow-Renner & Lauer (2005)	X	X		X		X		X
Professional Reports								
NSCD (2002)		X		X		X	X	X
NCTM (1991;2000)	X		X	X	X	X	X	X
NBPTS (2007)	X	X	X	X		X	X	X
ASCD (2003)	X	X		X	X	X	X	X

- a. The numbers in this row refer to Hawley and Valli's eight principles of designing effective professional development. Only principle six, that professional development should be continuously evaluated and refined, is not included as a column heading. Principle eight is used twice because it can affect both the focus and the organization of the professional development.
- b. Hawley and Valli address all eight attributes because the column headings come from their work.

However, districts strongly encouraged school-level participation in the study, so the study was less voluntary for teachers than a purely voluntary professional

development offering. But teachers could still opt out of the program at any time. The source also includes highly skilled outside providers, which the research suggests enhances teacher learning, assuming the experts empower teachers rather than impose judgment.

The *focus* of the professional development in the PD Math Study is on deepening teachers' content and pedagogical content knowledge and expanding their instructional techniques that promote student understanding. The literature supports professional development models that seek to deepen teachers' content and pedagogical content knowledge, particularly when the content is aligned with district standards and teachers' daily work. The focus of this professional development model specifically addresses each of these criteria. The model also provides opportunities for teachers to solidify both their formal and pragmatic understanding of key rational number concepts.

The *organization* of the PD Math Study professional development model is consistent with what the research says is most promising in terms of promoting teacher learning. Professional development activities that include opportunities for collective participation and collaboration, that are linked to teachers' daily work, and that promote active learning are more likely to be perceived by teachers as beneficial. This professional development model was designed with these organizational aspects in mind in that teachers attend and participate in workshops as grade level teams, work with each other and the instructional coach in the classroom, and participate in active learning opportunities such as problem solving, group discussions and public presentations of their work.

Though the literature does not provide specific guidance as to what constitutes sufficient *duration* and *intensity*, the literature is clear that one-time workshops or other short-lived professional development opportunities rarely promote improvements in teacher learning or instruction. The PD Math Study model includes 18 days and roughly 70 hours of targeted learning opportunities spread over several months, which is substantial both in terms of duration and intensity. The content of each workshop and associated follow-up coaching activities is aligned to each district's pacing guide, so teachers are able to connect to the classroom the material presented in the professional development throughout the year at appropriate and potentially fruitful time points. This model was designed to be substantive in terms of duration and intensity, but it was also designed to be "policy relevant" in terms of what districts and schools could afford and implement if the model eventually shows an impact on teacher learning, teacher instructional practice and/or student achievement. Taken together, the PD Math Study has incorporated many of the most widely accepted elements of effective professional development.

The next chapter provides a detailed description of the key components of the PD Math professional development intervention. Such detail is important because the critical case focuses on the extent to which teachers comprehended and implemented specific components of the intervention.

CHAPTER 3: DESCRIPTION OF THE PROFESSIONAL DEVELOPMENT INTERVENTION USED IN THE PD MATH STUDY

While the previous chapter linked the attributes of the PD Math Study professional development to the core features of effective professional development identified in the literature, this chapter describes the specific components of the PD Math Study intervention. A detailed description of the model is important because this critical case study focuses on the extent to which teachers comprehended and integrated into their instructional routines the core components of the model. The descriptive analyses in Chapters 5 and 6 are linked to the core components of the model, which is why they are first delineated in this chapter. The chapter includes a brief description of the development of the intervention, an overview of the timing and structure of the model, and a catalogue of the topics and strategies of the summer institute, follow-up seminars and coaching activities.

Development of the Intervention

When the PD Math Study began in the fall of 2005, the professional development intervention had been generally, but not fully specified. The federal agency had indicated to the American Institutes for Research (AIR), its lead evaluator and my employer, that the intervention should focus on rational numbers, contain both workshops and in-class coaching, and occur in districts using both “reform” and traditional textbooks. The agency wanted AIR to work with the study’s external advisors and the two professional development providers, who were hired in early 2006, to finalize the design. The design went through a number of iterations during the 2006-07 school year, when the intervention was piloted in three districts. This

critical case study focuses on the intervention of one of the two study professional development providers. Both providers focused on the same topics, but they structured their learning activities differently. Most of the feedback to both providers during the pilot focused on establishing the appropriate difficulty of the math problems, the clarity and coherence of the professional development facilitator materials, the structure and focus of the coaching activities, and the concreteness of the instructional guidance. The pilot also uncovered one staffing issue, which resulted in a facilitator being dismissed from the project because she lacked sufficient math content knowledge to deliver the professional development activities coherently. By late spring of 2007, both providers had revised their materials based on feedback from the pilot and hired additional staff for the 2007-08 full study. The full study began during the summer of 2007 in 12 districts and 38 treatment schools and included 84 treatment teachers. Each provider worked in six districts, with roughly an equal number of schools and teachers.

Timing and Structure of the Model

The structure of the intervention delivered during the 2007-08 school year included a three-day summer institute, five days of follow-up seminars and 10 days of in-class coaching. The three-day summer institute focused primarily on building teachers' content and pedagogical content knowledge of specific rational number topics. Each seminar had a dual focus of math content and pedagogical strategies, such as question and answer routines and lesson planning. The coaching activities focused on connecting the seminar material to the classroom. Some of the coaching activities involved pairs or groups of teachers working with the coach while other

activities paired the coach with a single teacher. The focus within the domain of rational numbers included roughly half the workshop time devoted to fraction and decimal content and half the time devoted to ratio, proportion and percent content.

Table 2 depicts the number of days and hours offered and the content focus of each type of professional development activity. Each summer institute and seminar day contained six hours of professional development activities, beyond lunch and scheduled breaks. The coaching included approximately two hours of activities for each of the ten days of coaching. The coaching visits occurred in pairs of days, with each coaching visit taking place immediately after each seminar. Two facilitators led each summer institute and seminar workshop, which included all schools in the district. The two facilitators then divided the schools for the coaching. Each teacher spent 18 hours in the summer institute, 30 hours in the follow-up seminars and 20 hours in coaching activities, for a total of 68 hours over 18 days. In terms of rational number content, the summer institute focused on fraction and decimal topics. The seminars focused on ratio, proportion and percent.

Table 2
Hours of Professional Development, by Activity Type

	Summer Institute (18 hrs)			Seminar (30 hrs)					Coaching (20 hrs – per teacher)										Total
	Coach Visit 1	Coach Visit 2	Coach Visit 3	Coach Visit 4	Coach Visit 5	Coach Visit 1	Coach Visit 2	Coach Visit 3	Coach Visit 4	Coach Visit 5	Coach Visit 1	Coach Visit 2	Coach Visit 3	Coach Visit 4	Coach Visit 5				
Day	1	2	3	1	2	3	4	5	1	2	3	4	5	6	7	8	9	10	18
Hours	6	6	6	6	6	6	6	6	2	2	2	2	2	2	2	2	2	2	68
Content Focus ^a	F	F/D	F	R	R	R	R/P	F	R		R/P		R/P		M		M		

a. F = fractions; F/D = fractions and decimals; R = ratio and proportion; R/P = ratio, proportion and percent; M = Mix of rational number and non-rational number content.

Summer Institute Topics

The summer institute focused on increasing teachers' understanding of fraction and decimal concepts. The professional development provider that is the focus of this case study introduced participants to definitions of fraction and decimal that utilized the number line. Both definitions were new to most if not all of the participating teachers. During the review of the professional development materials, study experts pointed to the number line as an underutilized representation for these concepts. They argued that many of students' misconceptions with fractions were related to not understanding that fractions are numbers (the NAEP problem described in Chapter 1 illustrates this point). When students plot fractions on a number line, they are more likely to see fractions as numbers and less likely to treat the numerator and denominator as distinct entities that are unrelated to each other. In response to these concerns, the professional development provider created several summer institute activities in which teachers plotted, compared and ordered fractions and decimals on the number line and discussed how the study definitions related to these activities. In fact, on the last day of the summer institute, the provider gave teachers time to create their own number line posters to use with students. Table 3 summarizes the topics and key emphases of each day of the summer institute.

The summer institute also had a strong emphasis on connecting the study definitions of fraction and decimal to related student misconceptions. Teachers examined work samples that included common student mistakes and linked the misconceptions to the underlying concepts. They also discussed various types of representations that could be used to help students understand particular concepts

Table 3
Summer Institute Topics and Emphases, by Day

Day	Topic	Key Emphases
1	Fraction Representations	<ul style="list-style-type: none"> • Define fraction • Represent and order fractions on number line • Multiple representations of fractions • Equivalent fractions and identity property of multiplication • Student misconceptions with simplifying fractions
2	Compare and Order Decimals and Fractions	<ul style="list-style-type: none"> • Define decimal • Represent and order decimals on number line • Compare and order fractions and decimals on number line • Student misconceptions with decimals • Rationales for multiplying and dividing decimals
3	Multiply and Divide Fractions	<ul style="list-style-type: none"> • Representing multiplication of fractions • Understanding meaning of division of fractions • Two types of division • Rationale for “invert and multiply” procedure

and discussed why certain representations were more appropriate than others, depending on the concept or operation being presented.

Finally, the summer institute provided teachers with multiple opportunities to think about why common procedures with fractions and decimals were true. One of the criticisms of K-12 mathematics teaching in the U.S. is that teachers introduce rules or procedures that produce the correct answer but do not connect to the concepts underlying the procedures (Kilpatrick et al., 2001; NCTM 1989; 1991; 2000; USDE, 2003). This professional development model included activities in which teachers examined the rationale for common procedures with rational numbers, such as the

“invert and multiply” rule for dividing fractions and the “moving the decimal point” rules for multiplying and dividing fractions. In sum, the summer institute provided teachers with extensive opportunities to think deeply about the meaning of fractions and decimals. The study also provided teachers with a comprehensive set of reference materials so they could revisit the content and activities of the summer institute throughout the school year. Teachers received hard and electronic copies of the study definitions, key mathematics “take away points” for each topic, problem sets and other resources related to summer institute activities. They also received the book *Teaching Elementary and Middle School Mathematics*, by John Van de Walle (2007), which contains explanations of the topics covered in the summer institute.

Seminar Topics

The five follow-up seminars focused on improving teachers’ content and pedagogical knowledge and connecting that knowledge to instruction. Like the summer institute, each seminar day had a content focus, such as fractions or ratios. However, each seminar also had a pedagogical focus, such as lesson planning or a questioning technique. Roughly half of each seminar day focused on improving teachers’ understanding of the content associated with the given topic and the other half of the day focused on an associated pedagogical component. Table 4 summarizes the topics and key emphases of each seminar day.

Seminar Day 1 focused on building teachers’ understanding of the meaning of ratio, particularly through the use of the ratio table. The study definition of ratio focused on the multiplicative relationship between the two quantities being compared.

For example, the ratio of 4 to 5 means that 4 is $\frac{4}{5}$ of 5 and 5 is $\frac{5}{4}$ of 4. This definition illustrates that every ratio has two multiplicative comparisons, each of

Table 4
Seminar Topics and Emphases, by Day

Day	Topic	Key Emphases
1	Ratio Tables	<ul style="list-style-type: none"> • Define ratio • Multiple representations of ratios (special focus on ratio tables) • Connecting ratios to fractions • Connecting ratios to algebra • Lesson planning
2	Strip Diagrams and Scale Factor	<ul style="list-style-type: none"> • Represent ratios with strip diagrams • Use strip diagrams to understand scale factor • Pedagogical strategy: students restate each other's reasoning
3	Rate	<ul style="list-style-type: none"> • Defining rate and unit rate • Applications of rate • Student misconceptions associated with rate • Pedagogical strategy: "Say More" • Lesson Planning
4	Percents	<ul style="list-style-type: none"> • Define percent • Double number lines and percents • Solve percent application problems • Student misconceptions with percent • Lesson planning
5	Adding and Subtracting Fractions	<ul style="list-style-type: none"> • Rationale for common denominator • Multiple representations for adding and subtracting fractions • Mathematical justification for fraction operations • Summarizing the entire professional development program

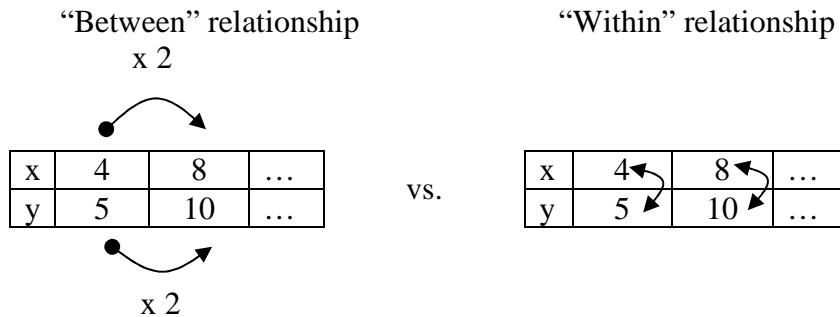
which represents the relative magnitude of the other. The reason that multiplicative relationships were emphasized is that many teachers and students see a ratio simply as a comparison of two quantities, but not necessarily a multiplicative comparison.

In the previous example, the comparison 4 to 5 could be an additive comparison: 4 is

one less than 5 and 5 is one more than 4. However, such additive comparisons are not ratios. Thus, the professional development focused on helping teachers understand the distinction between multiplicative and additive comparisons.

The professional development provider used the ratio table to illustrate the multiplicative relationships that occur within a single ratio and the relationships that occur between or among equivalent ratios. For example, the ratio of 4 to 5 is equal to the ratio of 8 to 10. Why is this true? One way to see why this statement is true is to compare the “within” ratios: 4 is $\frac{4}{5}$ of 5 and 8 is also $\frac{4}{5}$ (or $\frac{8}{10}$) of 10. Another way to see it is to look across the pair of ratios and notice that multiplying $\frac{4}{5}$ by a form of 1 ($\frac{2}{2}$), yields the same ratio. The sample ratio tables in Figure 6 illustrate these relationships.

Figure 6. Sample Ratio Table

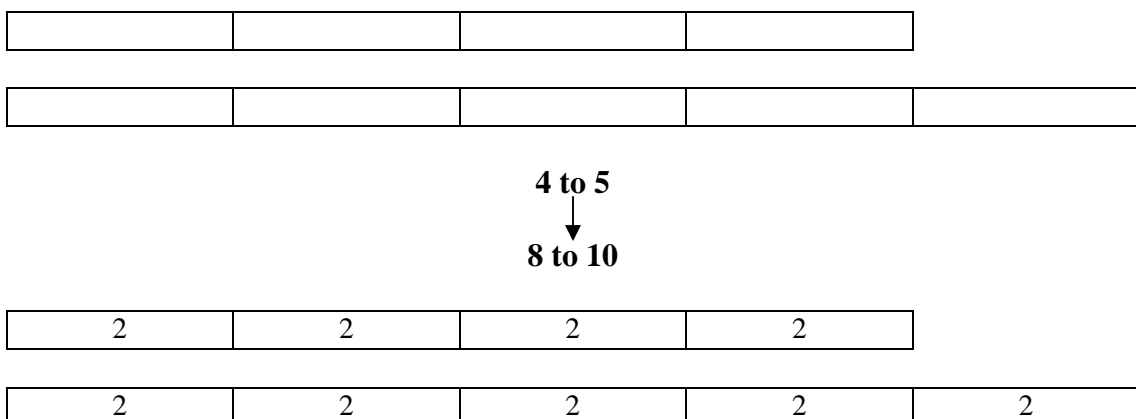


The first seminar also included a lesson planning segment in which participants planned a lesson based on some aspect of the seminar. The provider gave teachers a lesson planning template that matched the lesson structure of the teachers’ text, *Connected Mathematics* (hereafter, *CMP*). Teachers worked with the facilitators and other teachers to identify the student learning objective, list potential student

misconceptions and possible appropriate representation, and recognize the key math points to make during the summarize portion of the lessons.

Seminar 2 also focused on ratio and proportion concepts. The first seminar focused on the ratio table as a representation that could be used to promote students' understanding of the multiplicative relationships within and between ratios. This seminar introduced a second representation, the strip diagram, to help build students' understanding of ratios as multiplicative comparisons. The strip diagram is widely used in Singapore math curricula, which have been favorably reviewed in studies comparing the curricula of different countries (Ginsberg et al., 2005). Figure 7 shows how the ratios of 4 to 5 and 8 to 10 can be represented by a strip diagram. The strip diagram shows 5 rectangles for every 4 rectangles. If each rectangle is equal to 2, the 5 to 4 relationship is maintained visually even though the values change to 8 and 10. Teachers worked a variety of problems using strip diagrams and connected this representation to the core concepts of ratio and proportion.

Figure 7. Sample Strip Diagram



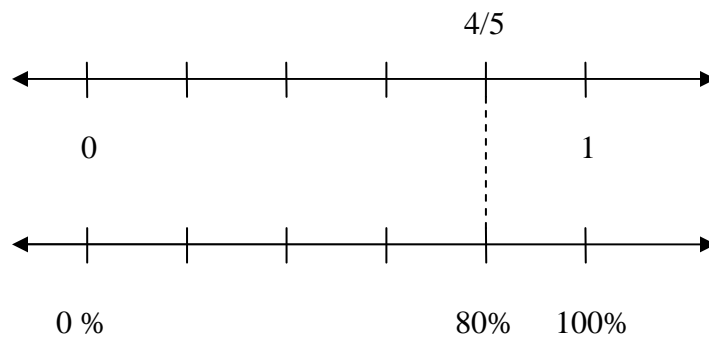
The second seminar also had a pedagogical focus of improving the quality of mathematical discussions. They introduced several prompts targeted to individual students, such as “Why did you do that?” or “Explain your thinking” as well as prompts targeted to stimulate discussion among students, e.g., “Johnny, can you restate what Sally said in your own words?” The facilitators referred to written descriptions of these strategies in the teacher materials and modeled each strategy throughout the seminar day.

Seminar 3 also focused on ratio and proportion content, the concept of rate. Teachers learned definitions of rate and unit rate and solved a variety of problems associated with the topics. They discussed how to identify and debug potential student misconceptions associated with rate. This seminar did not feature a specific representation like Seminars 1 (ratio table) and 2 (strip diagram). In terms of pedagogy, teachers built upon pedagogical strategies introduced in earlier seminars. They added the “Say More” discussion strategy to the list they started in Seminar 2 and continued to practice the other discussion techniques. They discussed the lesson plans they had created as a homework assignment between Seminars 2 and 3. They used the same lesson planning template that was introduced in Seminar 1.

Seminar 4 was the final workshop that focused on ratio, proportion and percent content (see Table 2 for math content focus of each day). Most of the focus of this seminar was on percent, though teachers participated in some activities that linked percent to other rational number topics. Teachers learned that a percent was a special type of ratio and they worked to solve percent application problems. They used the double number line (see Figure 8) to show how percents could be connected

with other rational numbers, such as fractions and decimals. Like the other seminars, this seminar provided opportunities for teachers to discuss common student misconceptions associated with percents and to link the misconceptions to the underlying mathematical concepts. In terms of a pedagogical focus, teachers examined two components of lesson planning: how to determine and articulate the core math of a lesson and how to anticipate student responses to the content.

Figure 8. Sample Double Number Line



The final seminar focused on adding and subtracting fractions. Teachers participated in activities that addressed the rationale for finding common denominators when adding or subtracting fractions. These activities incorporated multiple representations, including the number line and area models. Teachers also participated in other activities that focused on using mathematical justifications in their explanations. For example, teachers had to explain whether and why certain mathematical situations associated with fractions were always, sometimes or never true. The final seminar didn't have a specific pedagogical focus, but the closing activity summarized the lesson planning and discussion techniques used throughout the professional development sessions.

Coaching Topics

Each two-day coaching visit, which followed each seminar, had a particular pedagogical focus. Table 5 includes the coaching topics and activities associated with each coaching visit. The first coaching visit, which occurred after Seminar 1, focused on helping teachers plan, deliver and reflect upon a lesson that used a ratio table. The coach met individually with each teacher to establish the mathematical objective of the lesson and to review how ratio tables could be incorporated into the lesson.

The second coaching visit focused on the discussion techniques used during the seminars. These techniques encouraged students to explain their thinking to the teacher and to others students. The coach first modeled the discussion techniques with a small group of students during a segment of a lesson, while a substitute supervised the rest of the class. Then, the teacher modeled the strategies with a different group of students from the same class. During the next lesson, the teacher modeled the discussion strategies with a whole class. The coach and teacher debriefed the lesson that the teacher taught to the whole class and noted ways in which the questioning techniques could be strengthened.

The third coaching visit focused on anticipating student responses as well as continuing to practice the discussion strategies from the previous seminar. The coach and teacher used a tool for monitoring student understanding, which included space for teachers to record student responses and associated misconceptions. Like the first two coaching visits, this visit required the coach and teacher to meet before and after the lesson to clarify the focus of the observation and discuss ways in which future

Table 5
Coaching Topics and Activities, by Day

Visit ^a	Topic	Key Emphases
1	Lesson Planning	<ul style="list-style-type: none"> • Help teachers identify mathematical focus of lesson • Help teachers incorporate ratio table into lesson • Debrief lesson with each teacher • Co-teach lesson (coach and teacher) after ratio table lesson
2	Using Discussion Strategies	<ul style="list-style-type: none"> • Plan, deliver and modify lesson using discussion strategies • Coach and teacher work with small groups of students to practice discussion strategies • Debrief teacher's lesson using discussion strategies
3	Anticipating Student Responses	<ul style="list-style-type: none"> • Coach models discussion strategies • Teacher practices discussion strategies and receives feedback from coach • Teachers use monitoring tool to track student progress • Teacher and coach debrief use of discussion techniques.
4	Peer Observations	<ul style="list-style-type: none"> • Teachers plan, deliver and observe each other's lessons using a peer observation tool • Focus of peer observations sequencing of student questions
5	Co-teaching	<ul style="list-style-type: none"> • Teachers plan and co-teach lesson • Coach facilitates planning and debrief of lesson with co-teachers

a. Each coaching visit consisted of two days, with both days immediately following a seminar workshop.

lessons might be strengthened in terms of anticipating student misconceptions.

The fourth coaching visit required teachers to observe each other's lessons and to focus on pre-identified student behaviors. The coach and two teachers planned the pair of lessons in which each teacher would observe the other. Then, the teachers taught each lesson with the other teacher and coach completing the peer observation tool. The coach and both teachers then met after both teachers had taught their

lessons. They used the peer observation tool as the basis for the discussion. For example, if a teacher wanted the coach and the other teacher to focus on questioning techniques, the debrief focused on student questioning. To increase the comfort level of the teacher being observed, the coach allowed the teacher to pick the aspect of instruction that was the focus of the joint observation.

The final coaching visit had teachers plan, deliver and reflect upon a lesson together. Since the teachers had a double period, each teacher took turns leading segments of the lesson. For example, the launch, explore and summarize sections allowed teachers to take turns for a minimum of 10-15 minutes each. The coach observed these co-taught lessons and debriefed with teachers about what they learned and what they might do differently next time.

In sum, the professional development model included a variety of activities for teachers to build their content and pedagogical content knowledge in rational numbers and to practice integrating that knowledge into instruction. All three case study teachers participated in all these study professional development opportunities. The next chapter describes the research design and procedures I developed and followed to answer the two research questions guiding this case study.

CHAPTER 4: RESEARCH AND EVALUATION METHODS

This chapter provides an overview of the research and evaluation methods and procedures I used to conduct this case study. I begin by explaining why a case study is an appropriate research strategy to answer the study questions. Then, I describe the study research questions as they relate to the various components of the research design. Next, I explain the data collection procedures and analytic techniques that I adhered to in this study, and then conclude with a discussion of the steps I took to ensure the validity and reliability of findings.

Qualitative Research Strategy

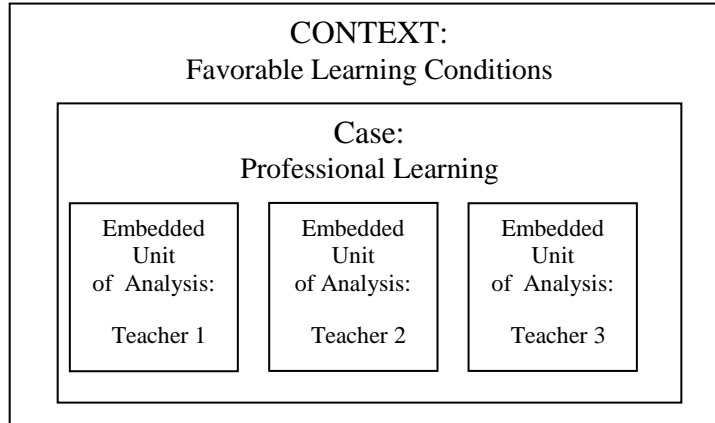
Qualitative studies, such as case studies, have been “stereotyped as a weak sibling among social science methods” (Yin, 2003, p.xiii). In contrast to randomized control trials and quasi-experiments that employ sophisticated quantitative analytic techniques that seek to control contextual variables, case studies are context-specific and lack quantification. Researchers such as Yin (2003) and Flyvberg (2001) dispute the “weak sibling” stereotype. Flyvberg argues that case study findings can contribute to the development and testing of broad theories and that critical cases, in particular, allow logical deductions of the type “if not here, then where?” to be made. In describing social science research more broadly, he adds, “a discipline without exemplars is an ineffective one” (Flyvberg, 2001, p.87). Yin contends that case studies provide a context in which specific aspects of theories can be tested. Theoretical generalizations are possible when case studies are rigorously designed. These arguments seemed particularly relevant as I considered conducting a study about teacher professional development. If a set of highly motivated, capable

teachers did not improve their knowledge and instructional routines from participating in high quality professional development, then the model was not likely to have an impact on teachers working in less favorable settings.

Yin (2003) also notes the importance of exemplars and contends that the appropriateness of any research method is dependent on the nature of the research questions. He says that when a researcher seeks to answer a question about “how” or “why” a phenomenon occurs in a contemporary setting in which the researcher has no control over behavioral events, the case study is a favorable research design. Since these conditions applied to my research questions, which are outlined in the next section, I determined that a case study design was an appropriate approach.

The case study method encompasses a number of different designs, including single- and multiple-case study designs with one or more embedded units of analysis. The study’s research questions determine whether a single- or multiple- case design should be utilized and whether a single or multiple units of analysis are appropriate (Yin, 2003). I employ a single-case study design because I am interested in studying a single phenomena – professional learning. Yin (2003) argues that it is virtually impossible to conduct a high quality case study without a clearly defined case. Because I am studying the professional learning of three different teachers, my single case design includes three embedded units of analysis, one unit for each teacher. Because the case study involves teachers who are receptive to the professional development intervention and who work in a district that is supportive of the intervention, the case study context is favorable. Figure 9, adapted from Yin (2003), depicts the single-case, embedded unit of analysis design.

Figure 9. Case Study Design



Research Questions

As mentioned in Chapter 1, this case study is organized around two sets of primary research questions, one that focuses on teachers' knowledge and one that focuses on teachers' instructional practices. The study also pays attention to potential sources of variation among teachers in relation to the two primary research questions. The primary research questions are:

1. How deeply do teachers understand the core content and pedagogical content components of the professional development model? What might explain any variation among teachers?
2. How effectively do teachers integrate into their instructional routines new content, pedagogical content and pedagogical knowledge? What might explain any variation among teachers?

The first question focuses on teachers' understanding of the content, and possible sources of variation include a) the complexity of the different components of the model – e.g., Do teachers have more difficulty with some rational number concepts than others?; b) teachers' prior knowledge in the professional development topics – e.g., Do teachers with stronger backgrounds in mathematics perform better

on the content interview than teachers with weaker content backgrounds?; c) teachers' prior experience with comparable curricula – e.g., Do teachers with more experience using *CMP*, which is aligned with the professional development model, perform better than teachers with less experience using *CMP*?; d) extra self study or collaboration with the study professional development facilitator – e.g., Do some teachers utilize the coach more than others and benefit from the extra time and attention?; and e) teachers' beliefs and attitudes about the teaching strategies promoted in the professional development and the district – e.g., Do teachers with stronger convictions about the importance of promoting conceptual understanding perform better than teachers with weaker convictions?

The second question focuses on the extent to which the core emphases of the professional development are visible during instruction and the potential sources of variation in visibility among teachers. These sources of variation include a) teachers' understanding of the core math content and pedagogical content – e.g. Do teachers who demonstrate stronger levels of knowledge on the content interview make the content more visible during instruction?; b) whether or not teachers were working directly with an instructional coach – e.g., Are certain aspects of the professional development more visible during instruction when teachers are working with an instructional coach than when they are not?; c) the complexity of the different components of the professional development model – e.g., Are certain aspects of the professional development more visible during instruction because they are simpler than other components?; and d) differences in the frequency with which various components of the model occur during typical lessons– e.g., Are the pedagogical

emphases of the model more visible than the content emphases because they occur more frequently within the *CMP* lesson structure? The case study design reflects these two primary research questions and investigates potential sources of variation associated with each question.

For both research questions, the potential sources of variation were generated from a review of the literature, specific characteristics of the professional development model, and hypotheses based on prior experience working with teachers in their classrooms. The literature suggests that ratio and proportion concepts are more difficult for teachers to understand than fraction and decimal concepts, which is why I included a question related to this issue. The professional development model includes an extensive coaching component, which is why I included a question about the visibility of the professional development when teachers were working directly with an instructional coach and when they were not. From observing teachers in prior projects using the same curriculum, I hypothesized that the pedagogical components of the mode would be more visible during instruction than the math content components. I identified as many potential sources as possible and incorporated them into my research questions.

Design Components

In this section, I first describe the criteria used to select the critical case from among the other study districts, teachers and schools. Then, I describe the characteristics of the district, school and teachers that are the focus of this inquiry.

Case Selection Criteria

Since the two research questions focus on the extent to which the professional development might impact what teachers know and how they teach, I wanted to study the intervention in a context in which teachers were receptive to and interacting with the material. As part of my supervisory role in the larger study, I attended numerous professional development events during the 2006-07 school year, when the intervention was being piloted, and during the summer of the 2007-08 school year, when the finalized intervention was being implemented in the full study. During these events, in which teachers actively participated by asking questions, solving problems and sharing their solutions publicly, I had no trouble distinguishing teachers who were engaged in the learning activities from those who were not. One pattern I noticed was that teachers in the districts using *CMP*, a conceptual and problem-solving based curriculum funded by the National Science Foundation, tended to be more interested in the professional development activities than the teachers in districts using more traditional textbooks.⁸ This pattern was not surprising, since the *CMP* curriculum was more compatible with the core math content and pedagogical goals of the study than the traditional texts. I noted that among the study's six study *CMP* districts, some districts appeared to have stronger curricular infrastructures than others. For example, some districts had detailed pacing guides that were tightly linked to the curriculum, while other districts provided teachers much less structure.

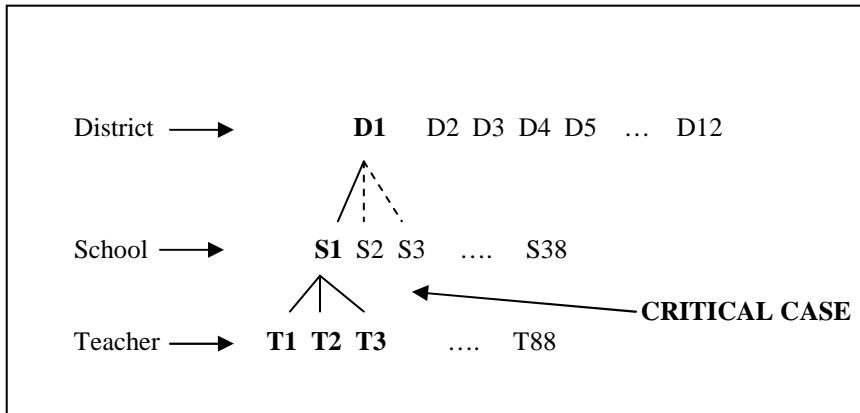
The teachers in this case study represent all the eligible teachers from one school in a *CMP* district that had one of the strongest curricular infrastructures in the

⁸ The study sample included six districts that used *CMP* and six districts that used one of two more traditional textbooks. The traditional textbooks focus more on learning discrete skills than building conceptual understanding through problem solving.

study. For example, the district math coordinator played an extremely active role in the recruitment of teachers to the study and as a participant in the professional development activities. The district also deployed math coaches and teacher leaders in every middle school to help teachers improve their instructional practices. Many of the schools arranged their schedules so that teachers had a common planning period in order to facilitate joint lesson planning and reflection time. Schools also adopted 85 minute double periods for math, which provided extra time for students to engage in the extended learning activities that are central to *CMP*.

The teachers in the school selected for this case study worked in a school with a strong teacher leader, a part-time district math coach and shared common planning time. During the summer institute, I took note of the thoughtful responses of two of the three teachers at this school and had interesting side conversations with them during breaks and lunch. All of these factors, particularly in comparison to my experiences in other districts that were less hospitable to the intervention, led me to think that this school was a site in which the professional development had a reasonably good chance of influencing what teachers knew and how they taught. I confirmed this hunch when I visited the teachers' classrooms during the fall, and I made the final decision to use this school as the site for an in-depth case study of professional development. Figure 10 illustrates how the teachers, school and district are situated within the sample of the larger study.

Figure 10. Critical Case in Relation to Larger Study Sample



District and School and Characteristics

As depicted in Figure 10, Adams⁹ Public Schools was one of the 12 districts that participated in the PD Math study during the 2007-08 school year. The district is located in a large metropolitan area in the central U.S. with more than 50 schools and 30,000 students. More than 70 percent of Adams’ students represent ethnic minority groups; 60 percent are eligible for free or reduced meals service; and 40 percent are second language students. In recent years, the proportion of students from ethnically diverse backgrounds and low socio-economic levels has steadily increased. Adams, like many other large urban districts throughout the country, is characterized by low levels of student achievement, high levels of student mobility, high proportions of second language students, and increasing levels of federal accountability under for underperforming schools.

Princeton Middle School, the site of this case study, is one of several middle schools in Adams and is representative of the district in terms of socio-demographic characteristics. More than 50 percent of Princeton students represent ethnic minority

⁹ Adams Public Schools, and Princeton Middle School, which is mentioned in the following paragraph, are pseudonyms.

groups and qualify for free or reduced meals service, and more than 30 percent are second language students. In 2007, approximately 60 percent of 7th grade students at Princeton scored below the proficient level on the state test.

Teacher Characteristics

The three teachers who are the focus of this inquiry represent all the 7th grade teachers from Princeton Middle School who were eligible to participate in the larger study. Eligible study teachers had to teach at least one section of middle-level 7th grade math. Special education teachers and the 7th grade teacher leader attended the workshops, but they did not participate in the coaching or other study data collection activities. Table 6 includes background characteristics for each case study teacher.

Table 6
Case Study Teacher Characteristics

	Gender	Age	Teaching Experience (CMP Experience)	Degree Type	Number of Math Courses Taken ^a	Number of Math Ed Courses Taken ^b
Hamlin	Female	40-45	2 nd year (2 nd year)	Elementary Ed (Math Emphasis)	6	5
Smith	Male	50-55	8 th year (3 rd year)	Elementary Ed	2	15 ^c
Wiggins	Female	30-35	9 th year (6 th year)	Secondary Math Ed	9	4

- The number of math courses represents teacher estimates of the number of mathematics courses taken from the mathematics department. These courses all represent undergraduate math credits; none of the teachers had taken graduate courses in “pure” mathematics.
- The number of math education courses represents teacher estimates of the number of mathematics or mathematics education courses taken outside the mathematics department (e.g., courses offered in the school of education). They represent both undergraduate and graduate courses in mathematics education.
- This teacher estimate, which seems high, represents a combination of courses offered from universities and continuing education credits. It should not be assumed that all 15 courses represented 3-credit university-based courses; however, the teacher reported taking approximately 45 credits in math education courses.

The sample includes two female teachers and one male teacher. These teachers have different levels of teaching experience and different types of educational backgrounds. Both Hamlin and Smith completed undergraduate programs in Elementary Education. Wiggins completed an undergraduate degree in Secondary Mathematics Education. Hamlin said that her undergraduate degree was unique, however, because her program required teachers to take a series of math classes in the mathematics department. Hamlin was in her 2nd year of teaching, while Smith and Wiggins were in their 8th and 9th years, respectively. Wiggins had the most experience teaching CMP (6 years); Smith had 3 years of experience; and Hamlin had 2 years of experience. The teachers also provided estimates of the number of math courses they took inside and outside a university mathematics department. Since Wiggins had a degree in secondary education, she reported taking the most math courses from the math department (9), but Wiggins also took several math courses (6) as part of the unique elementary program in which she was enrolled. Smith, who was enrolled in a more traditional Elementary Education program, estimated that he took only a few classes from a university math department. However, Smith has taken a number of math education courses after completing his undergraduate degree. He estimated that he has taken 15 such courses, compared to 4 and 5 for Wiggins and Hamlin, respectively. The teachers did not provide transcripts, so these numbers represent teacher estimates.

In addition to these background characteristics, I collected information from each teacher and from the study's instructional coach to try and establish a pedagogical baseline. Ideally, I would have been able to observe each teacher before

the professional development began, but I was not able to do this because districts were recruited too late in the previous spring semester to conduct such observations. To address this shortcoming, I asked the instructional coach to describe each teacher's practice at the beginning of the school year, since the coach spent time with teachers earlier in the school year than I did. I also asked the teachers to describe, compared to the previous year, the extent to which the professional development had altered specific instructional practices and their overall approach.

Hamlin's pedagogical baseline. Hamlin indicated that the professional development influenced her teaching primarily in two ways. First, she said that used more representations – models, diagrams, charts, etc. – in her teaching than she did previously. She credited the professional development activities, many of which featured multiple representations of rational number concepts, with providing her new ways to present concepts to students. Second, Hamlin said that she found the study's talk or discussion strategies – e.g., “Say more about that,” “Explain Johnny's approach in your own words,” – very useful. She said that she has always asked students to explain their thinking, but she thought that the study prompts were more succinct and effective. The coach also indicated that Hamlin's use of the discussion strategies increased noticeably over the course of the year. She said that at the beginning of the year, Hamlin would ask students questions, but the resulting discussions often fell flat. The coach said that the number of students participating in class discussions and the quality of student responses had improved as a result of the professional development.

Smith's pedagogical baseline. Smith indicated that, for him, the most valuable component of the professional development was the opportunity to identify and discuss student approaches to different concepts. He said that he was more open to allowing students to share different approaches to problems because he was more familiar with what they might produce. He learned various student approaches from working with teachers in the seminars and from working one-on-one with the coach. Smith said that although he had learned a great deal about different student representations of concepts, it was an area that he needed additional support. The coach indicated that Smith's use of representations with students had been an area of focus in their coaching activities. She said that although she thought Smith was an extremely strong teacher at the beginning of the year, he was less likely to encourage students to represent multiple approaches to a single concept. She thinks he grew in his willingness to allow students to pursue multiple approaches and in his ability to know how to respond to specific student misconceptions.

Wiggins's pedagogical baseline. Wiggins said that her instructional practices had changed primarily in two ways. First, she said that she was more likely to encourage students to share multiple approaches to and representations of problems than she did previously. Second, she said that she was more likely to ask students to explain their thinking because of the study's questioning prompts. She said that the prompts were easy to remember and therefore easy to implement in the classroom. The coach indicated that she had observed an increase in Wiggins's willingness to ask students to explain and represent their thinking throughout the course of the professional development.

Data Collection Procedures

In this section, I first describe the procedures I followed to develop data collection instruments. Then, I provide the timeline for when these instruments were used in the field.

Instrument Development

I developed two instruments to answer the study's two primary research questions. The study's first question addresses whether teachers comprehended the core math content and pedagogical content emphasized in the model. Though all study teachers took a timed pre and post assessment in rational number content as part of the larger study, the assessment did not allow teachers to explain their thinking or elaborate upon their responses. After reading Liping Ma's (1999) book, *Teaching and Learning Mathematics*, I decided that one way to assess teachers' understanding of the math content would be through an extended interview. Ma used math scenarios, which were essentially open-ended prompts, as the basis of her interviews. I developed seven such prompts based on the core pedagogical and pedagogical content emphasized in the model, which became the structure for the extended interviews I conducted with teachers at the end of the 2007-08 school year. Chapter 5 contains the complete interview protocol, which I hereafter refer to as the structured content interview.

The study's second question addresses the extent to which the core components of the professional development were visible during instruction. Though I didn't have classroom observation data from before the beginning of the larger study, I decided that the model was specific enough that aspects of it could be

detected during instruction. In fact, during my first informal visit to the school, I observed teachers using specific questioning strategies that they had been introduced during a previous workshop. For instance, teachers used the prompts “Say More” and “Explain what [Johnny] said in your own words,” which were exactly how they were stated in the professional development. Such exact terminology helped compensate for the absence of baseline classroom observation data. Thus, the second instrument I developed was an observation protocol organized around three core components of the professional development model: mathematics content, pedagogical content and pedagogical strategies. Within each of these three broad categories, I included specific teaching behaviors emphasized in the professional development, which are described in Chapter 3. Appendix A contains the classroom observation protocol.

I also conducted interviews with teachers about their perceptions of the professional development and their general beliefs about math teaching and learning. To triangulate these data, I interviewed the district math coordinator and the professional development facilitator who worked with these three teachers in the larger study. I asked them about their experiences working in the district and with each of these teachers in particular. I developed protocols for these interviews, which are included in Appendices B-D.

Data Collection Timeline

I used collected data at several time points during the summer of 2007 and the 2007-08 school year, which I have organized into three study phases (see Table 7). The primary purpose of Phase 1 (July '07 – September' 07) was to collect data on teachers' perceptions of and interactions with the content of the summer institute.

The summer institute, which took place in July '07, focused primarily on boosting teachers' mathematical content knowledge in the domain of

Table 7
Study Timeline and Phases

	Case Study Phases	Primary Purpose(s)	
PD Math Intervention Calendar	July 2007 (summer institute)		
	August 2007 (seminar 1 and coaching)	Phase 1	
	Sept 2007 (seminar 2 and coaching)		
	Nov 2007 (seminar 3 and coaching)		
	Dec 2007 (seminar 4 and coaching)	Phase 2	
	Feb 2008 (seminar 5 and coaching)		
	May 2008 (all seminars and coaching complete)		
		Phase 3	<ul style="list-style-type: none"> • Collect and summarize data from classroom observations after coaching concluded • Administer teacher structured content interviews • Administer teacher background interviews

rational numbers. Teachers solved problems in small groups and took turns sharing out with the entire group on what they were learning. These observations and informal conversations with teachers led me to believe that Princeton Middle School might qualify as a critical case. The primary focus of Phase 2 was to collect data on the visibility of the professional development in teachers' instructional practices. The purpose of Phase 3 (May 2008) was two-fold: to conduct observations of lessons

after the professional development intervention had been withdrawn and to administer the structured content and teacher background interviews.

Analytic Approach

In order for the results of a study to be taken seriously, the study must be designed rigorously. Two hallmarks of rigorously designed case studies are validity – the extent to which research findings are consistent with what really happened – and reliability – the degree to which research findings are replicable (Merriam, 1998; Yin, 2003). Throughout data collection and analysis, I adhered to Yin’s (2003) three principles of establishing validity and reliability: use multiple sources of evidence, create a complete database, and maintain a detailed chain of evidence. Regarding multiple sources of evidence, I collected information on teachers’ knowledge through intensive interviews with the teachers, classroom observations and interviews with the instructional coach. I then triangulated these data during analysis. I also carefully created and maintained a study database, which included field notes from professional development workshops, classroom observations and interviews; documents and artifacts from professional development and classroom activities; and transcripts of the audio-recorded interview data. Finally, to improve reliability, I maintained a chain of evidence throughout data collection and analysis. For example, I described in great detail the specific components of the professional development model, so that teachers’ responses and behaviors could be linked to explicit criteria.

General Analytic Strategy

According to Yin (2003), establishing and adhering to a general analytic strategy is important because it restricts the focus of subsequent analyses and helps the research know which analytic tools will be most useful. Yin (2003) notes that, too often, researchers cling to the tools – e.g., a computer-assisted data indexing software, such as NUD:IST – before they have established a general analytic strategy. When researchers make this mistake, they often get so mired down in the details that they have trouble getting to the analytic phase and maintaining a clear analytic focus.

To avoid such confusion, I selected *case description* and *rival explanations* as my primary analytic approaches. I used case description because the answers to my primary research questions are descriptive in nature. I provide descriptions – often verbatim – of teachers’ understanding of mathematics and their teaching behaviors. These descriptions are anchored to specific information in the professional development program. I also used rival explanations because I am interested in studying the sources of variation in teachers’ responses to the professional development – in fact, rival explanations are embedded in my research questions (e.g., What might explain variation among teachers?). These rival explanations include characteristics of teachers and characteristics of various components of the professional development program that might influence teacher responses. Yin (2003) says that as the number of rival explanations to be tested and rejected increases, so does the amount of confidence that can be placed in study findings.

Throughout the design and analysis phases of this study, I made a systematic effort to account for and address potential rival explanations.

Specific Analytic Techniques

In addition to the two previously described general analytic strategies, I used pattern matching as a specific technique to analyze the observational and interview data. Pattern matching is process by which predicted and empirically-based patterns are linked. The specific components of the professional development model, as detailed in Chapter 3, comprise the predicted patterns in this case. For example, I used the study resources to inform the development of the structured content interview, so that the interview questions directly reflected the material presented in the professional development. I developed the classroom observation protocol in the same way, by incorporating into the instrument specific teaching behaviors emphasized in the professional development. With the predicted patterns outlined in advance, I was able to assess the extent to which teachers' empirical actions matched predicted behaviors.

Yin (2003) discusses several types of pattern matching techniques. The technique that was most appropriate for this study was pattern matching involving simple patterns. Unlike pattern matching techniques that focus on nonequivalent dependent variables or rival explanations, I relied on simple, straightforward patterns between predicted and actual behaviors. Chapters 5 and 6, which are linked to the criteria outlined in Chapter 3, describe the patterns that emerged from analyzing the data in this way.

Ensuring Validity and Reliability of Findings

Role of the Researcher

As mentioned previously, my background experience and role on the larger study in evaluating the professional development materials allowed me to play the role of expert observer. Like any research vantage point, the role of expert observer has advantages and disadvantages. One advantage of this role was that it allowed me to collect and analyze data more efficiently than an observer less familiar with the study and less familiar with mathematics teaching and learning, more generally. For example, I reviewed the professional development materials at multiple time points prior to delivery during the 2007-08 full study. I led the review of the materials during the 2006-07 pilot study, participated in the training of facilitators for the full study, and created the instruments for tracking the fidelity of implementation of the professional development during the full study. By the time these teachers in this participated in their first professional development activity, I had been through it several times. Such familiarity with the model allowed me to focus closely on teacher interactions with the materials, since I didn't have to acquaint myself with the purpose or details of the activity.

Another advantage is that I had access not only to the facilitators and coaches who were delivering the professional development but also to the senior staff member who wrote the materials. If I had questions about the purpose of an activity or a specific technique, I could get my question answered quickly – often an email or a phone call returned the same day. Someone less connected to the project would have had more difficulty getting clarification about the intended focus of an activity.

One potential disadvantage is that my role in the larger study as supervisor of the professional development could have caused the facilitator/coach to behave differently because of my presence – e.g., perhaps the facilitator would perform less well because of anxiety or perform better because she was trying to impress me. I was less worried about this aspect during the workshops because the professional development provider’s quality control person attended these events, and she provided much more critical feedback to the facilitator/coach than I did. Second, I established a strong rapport with the two facilitators working in the district, and they gradually came to see me as a supportive representative of the study. Further, the favorable conditions for delivering the professional development in Adams and Princeton made the facilitators comfortable with outside observers. In fact, the facilitator who also served as the coach at Princeton described the Princeton teachers as “a dream” compared with many of the other teachers she worked with in other districts. Even if my presence did influence the performance of the facilitators, I didn’t know in which direction, and I assumed that any effect would be small.

I was concerned that my role as a study representative might influence teachers’ behaviors. I was especially concerned that teachers might behave differently when I conducted classroom observations when the coach was no longer present. Would teachers see me as another coach or someone with a stake in seeing them implement particular aspects of the study? I addressed this potential drawback in a few ways. First, I took time to get to know the teachers individually during the workshops – during breaks, lunch, etc. I shared with them that I had traveled to many different districts around the country and thought that their district and school might

be a fruitful place to study the intervention in an in-depth way. I explained that their apparent eagerness and capacity to learn the material stood out from other districts and schools. Because teachers were used to having district coaches and other teachers in their classrooms, they said that they were happy to have me visit their classrooms. They also understood that the data I would be collecting were separate from the full study, and that I had no evaluative purpose and would protect their confidentiality throughout the study. It is possible that teachers behaved a little differently when I was present during the observations after the coach had left – during the observations when teachers were working with a coach, I do not think I altered what teachers were doing; their primary focus was on the coach – but I saw no evidence of such behavior. Chapter 6 includes analyses of classroom observations of both when the coach was present and not present. I found no dramatic differences between the two data collection time points, which suggests that my presence did not alter noticeably teachers' instructional practices.

My role can be described as *observer as participant*, which Glense and Peshkin (1992) explain as an observer who has contact with the participants but does not provide input, advice or any other information that would alter what participants would ordinarily do. I made this role clear to teachers, even though, on a few occasions, one of the teachers asked me for advice in handling a particular student's misconception. I did not intervene for research purposes, even though as a former teacher, I was eager to support the teacher in those instances.

I conducted all of the classroom observations, so I was not able to conduct inter-rater reliability checks with other observers. However, during the classroom

observations in which the teacher was working with the study coach, I did verify the information in my field notes with the coach. This process was not as straightforward as comparing my field notes with hers, since the coach played a variety of roles and often didn't take notes when she was modeling parts of a lesson or working with students. Yet, these informal conversations improved the quality of the data I collected.

I also took steps to reduce bias that I might have brought to the classroom observations in several ways. First, I tried to keep an objective frame of mind when I observed classrooms. Though I was looking for specific teaching behaviors, I made sure that I captured as much information during the classroom observations as possible. I recorded as many relevant direct quotes as I could, since direct quotations are extremely low inference. I did not score teachers on the classroom observations until I had typed and re-read my field notes several times. This process occurred within a few days of each observation, since it took roughly four hours per observation to type and expand upon my hand written field notes. I wanted to score teachers on the observation protocol soon after each observation, but I didn't want to do it until I had a complete set of notes to review. Chapter 6 includes excerpts from my field notes to justify teachers' scores on the observation protocol.

Since I was the only researcher in the study, I conducted all the interviews by myself. I took notes during each interview, mostly to help me keep focused on the interview protocol. I audio-recorded and later transcribed each interview as well. I assured participants that I wouldn't share any information from the interviews with anyone and that the audio-recordings and transcripts would be destroyed at the end of

the study. Though most of the information shared in the interviews was non-threatening, some participants did respond with personal information, which they asked me to keep confidential.

Inter-rater Reliability of the Structured Content Interview

I wasn't concerned about internal validity when participants were providing information about their background, perceptions of the study or beliefs about mathematics because they provided me this relatively straightforward information. However, I was concerned about ensuring internal validity during the analysis of the structured content interviews. Because these questions assessed teachers' understanding of the mathematics, I wanted to make sure that my assessments were valid. I dealt with validity by having the interviews scored independently by another math expert who was familiar with the study. He participated extensively in the pilot study and knew the intervention extremely well. He also is a leading math education expert and a colleague of mine at the American Institutes for Research. He provided input on the appropriateness of the questions in relation to the study design and later scored teachers' responses independently. In an attempt to be efficient, I provided the expert with key excerpts from the lengthy interviews rather than have him read the approximately 100 pages of transcribed interview data. These excerpts appear in Appendices I-O.

Fortunately, this approach seemed to work. Appendix E includes a table with both of our scores on each of the seven interview questions for each of the three teachers. As the table indicates, the inter-rater reliability was quite high. On 14 of the 21 questions, we had perfect agreement. Among the seven items on which we

disagreed, we only differed by 0.5 of a level on four questions and 1.0 on the remaining three items.¹⁰ Our overall averages for the three teachers were within 0.2 of a level – 1.9 vs 1.7 for Hamlin; 1.5 vs. 1.3 for Smith; and 1.4 vs. 1.3 for Wiggins. These small differences gave me confidence that my scoring criteria were reliable and valid. If this approach had not produced such consistent results, I would have shared all the transcripts with him and then held some follow up discussions about our differences.

Another way that I addressed internal validity in the structured content interviews was to provide as many direct quotes as possible when justifying individual teacher ratings. By including direct quotes and a brief rationale as to why these quotes indicated a particular level of understanding, I provide opportunities for other researchers to confirm or dispute the results, and thereby increase the transparency of my findings.

The next two chapters include findings and analyses related to the study's two primary research questions. Chapter 5 utilizes data from the structured content interviews and Chapter 6 utilizes data from the classroom observations to describe the extent to which the three teachers' understood the core math content emphasized in the professional development and integrated that content into their instructional practices.

¹⁰ 3 teachers x 7 items per structured content interview = 21 items total.

CHAPTER 5: TEACHERS' UNDERSTANDING OF MATHEMATICS CONTENT

The larger study's theory of action assumes that high quality mathematics instruction is dependent upon teachers' understanding of mathematics content. Thus, the professional development model emphasized both strengthening teachers' understanding of core mathematics content and improving teachers' ability to deliver that content meaningfully to students. Though one could make a strong argument that the impact of teachers' mathematical knowledge on student learning ultimately should be judged by how well that content is delivered in the classroom (e.g., the quality of an explanation, the ability to articulate student misconceptions), it is often difficult to make such assessments exclusively from classroom observation data. One limitation of using observation data to capture teachers' understanding of the content is that typical lessons provide only limited opportunities for teachers to demonstrate their content knowledge publicly. Another reason is that teachers sometimes choose to withhold particular aspects of knowledge because they don't think it is appropriate to share with students (Kennedy, 2005). For example, a teacher might decide that a particular definition or certain way of presenting the content is too complex for most students and introducing this information might introduce confusion or even frustration among students. Given these limitations of classroom observation data as a means to assess teachers' understanding of mathematics content, I conducted in-depth interviews with each teacher about the extent to which they understood the core mathematics content and pedagogical content emphasized in the professional development model.

Structured Content Interview

Unlike timed, close-ended assessments, which all teachers – including the three teachers who are the focus of this inquiry – completed as a requirement of the larger study, structured content interviews provide an opportunity for teachers to delve deeply into the content and to explain their reasoning over an extended period of time. Liping Ma (1999), in her case study comparing Chinese and U.S. elementary teachers' understanding of mathematics, used structured content interviews (she called them scenarios) to classify teachers' understanding of the subject matter. The data from these interviews illustrated sharp differences in understanding between the Chinese and U.S. teachers in her study. Other researchers, such as Ball (1990; 1991) and Prawat, Remillard, Putnam and Heaton (1992), have used these types of interviews to depict teachers' ability to articulate mathematical concepts and to identify student misconceptions associated with those concepts. Thus, the structured content interview was well suited for the design of this critical case study.

Components of the Structured Content Interview

As discussed in Chapter 4, the larger study focused on the domain of rational numbers, which includes fractions, decimals, ratio and proportion, and percent.¹¹ I weighted these rational number topics proportionally as I developed the interview protocol. The larger study also had a dual emphasis of strengthening teachers' mathematics content knowledge and pedagogical content knowledge, so I constructed interview prompts that addressed both of these dimensions as well. By mathematics

¹¹ The topic of percent received less attention in the professional development than the other rational number topics, so I excluded percent from the structured content interview. However, some of the teachers talked about percent in relation to some of the other interview prompts –e.g., when they were asked to connect fraction and decimal concepts.

content knowledge, I refer to teachers' understanding of key concepts and definitions, connections among rational number concepts, and rationales behind procedures or algorithms. By pedagogical content knowledge, I refer to teachers' ability to represent concepts in multiple, student-friendly ways and to identify and debug common student misconceptions. Appendix J indicates which of these rational number, math content knowledge and pedagogical content domains are addressed in each interview question. The final interview protocol included seven questions (see Appendix K for all the questions) that covered these domains. Each interview took about an hour to complete.

The interviews addressed the core content and pedagogical content topics emphasized in the professional development, but some topics were emphasized more than others. To capture the degree of emphasis on each topic, I report the number of hours spent on professional development activities related to each question. For most of the seven questions, teachers spent at least four hours in workshops (summer institutes and follow up seminars) on activities exclusively devoted to the core topic or topics in each question. Teachers spent time during coaching focusing on these topics, but the coaching activities were not as intensely focused on the content as were the workshop activities. Teachers also had a variety of study resources they could reference at any time. The study resources included a set of math content "take away points" for each topic, several problem sets, a math reference book, and a commercial college textbook. Sometimes they were assigned homework problems from these materials. Thus, it is difficult to know precisely how much time teachers spent on topics related to these questions. Teachers did report the approximate

amount of time they spent using the study resources, but they did not report how much time they spent on topics related to each question on the interview. A conservative way to describe the amount of time teachers spent on each topic is between four and six hours in workshops, a few hours in coaching activities where the time was less concentrated on the content than the workshop time, and a few hours of self-study of the supplemental resources. See Appendix L for a more detailed description of the number of hours devoted to each question by each professional development activity.

Criteria Used to Score the Content Interviews

The purpose of the structured content interview was to assess teachers' understanding of the mathematics content *emphasized in the professional development*, not to assess their global understanding of mathematics. However, given the extensive review process required by the federal agency during the development and pilot phases of the study, what teachers were expected to learn is consistent with what many experts would describe as general mathematical and pedagogical content knowledge. Leading mathematicians and experts in mathematics education carefully reviewed all study materials, so the content presented to teachers was conceptually clear and coherent.

For each of the seven questions, I applied a scoring rubric to capture the degree to which teachers accurately articulated the content emphasized in the professional development. The rubric ranged from no understanding to strong understanding. Table 8 contains full descriptions of the scoring criteria and includes the numeric values associated with each level of understanding. None of the teachers

scored at the level of no understanding on any question, so all of the teacher responses demonstrated weak, moderate, or strong levels of understanding.

Table 8
Structured Content Interview Scoring Criteria

Scoring Criteria	Level of Understanding	Score
No connections to study-emphasized definitions, approaches or explanations.	No Understanding	0
Superficial and/or limited connections to study-emphasized definitions, approaches or explanations.	Weak Understanding	1
Solid connections to many aspects of the study-emphasized definitions, approaches or explanations, but connections did not fully capture most essential elements of study-emphasized information and/or may not have been clearly stated.	Moderate Understanding	2
Strong connections to the core elements of the study-emphasized definitions, approaches or explanations. Connections capture most essential aspects of the study-emphasized information and are clearly stated.	Strong Understanding	3

Teachers' Scores on the Structured Content Interview

Table 9 displays the scores each of the three teachers received on each question and includes two average scores, as well. The values in the last column indicate each teacher's average score across the seven questions. The average scores in the bottom row indicate the average score for each question for the three study teachers. For example, Hamlin received a score of 2 on the first question of the interview, which indicates a moderate understanding of the study's definition of fraction. Her average across all 7 items was 1.9, or slightly below the moderate level of understanding. The average score for all teachers on this item was 1.7, slightly lower than Hamlin's score of 2.

Table 9
Teachers' Scores on Structured Content Interview Questions

Teacher	Question							Average
	1	2	3	4	5 ^a	6	7	
Hamlin	2	1	3	3	1	1	2	1.9
Smith	1	2	1.5	2	2	1	1	1.5
Wiggins	2	1	1.5	1	1	1	1.5	1.3
Average	1.7	1.3	2.0	2.0	1.3	1.0	1.5	1.6

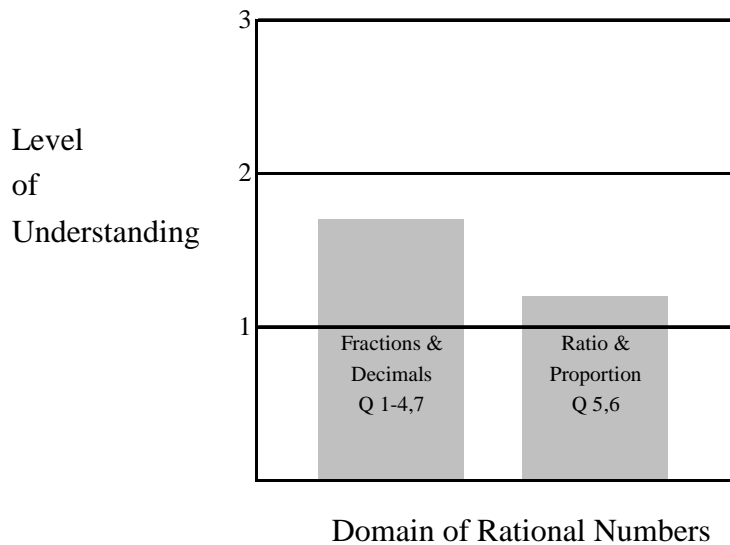
a. Columns 5 and 6 are shaded because they represent questions focused on ratio and proportion content; all other questions focus on fractions and decimals (fractions, primarily).

All three teachers' average across the seven items fell between the weak and moderate levels of understanding. The highest average was 1.9 (Hamlin) and the lowest average was 1.3 (Wiggins), for a range of 0.6. Hamlin was the only teacher to demonstrate a strong level of understanding for any question, which she reached on both questions 3 and 4. These two questions required teachers to explain the rationale for operations with fractions and decimals. Her high scores on these two items helped raise the averages on these two items to 2.0, the only questions on the interview that averaged a moderate level of understanding across all three teachers.

The two shaded columns in the table refer to the two questions from the interview that dealt explicitly with ratio and proportion content. Questions 1-4 and 7 dealt with fractions and decimals. Ratio and proportion generally are considered to be among the most complex rational number concepts. Perhaps that is why teachers in this study struggled more with these concepts than they did with fractions and decimals. When the averages for each item are grouped by fraction and decimal content and ratio and proportion content, teachers averaged 1.7, or close to the

moderate level of understanding, across the fraction and decimal questions. However, they averaged 1.2, or close to the weak level of understanding, for the two ratio and proportion questions. Figure 11 displays the difference in these average scores for these two domains of rational numbers content.

Figure 11: Teachers' Average Scores across Fraction/Decimal and Ratio/Proportion Items



Teachers' Understanding of Fraction and Decimal Content

Though Table 9 illustrates that teachers scored higher on questions with fractions and decimals, on average, they still displayed an understanding below the moderate level. However, teachers' level of understanding fluctuated across questions; and, it varied among teachers for a given question. To understand this variance more fully, I elaborate on each teacher's responses to each interview question, grouped by rational number topics. I begin by describing each teacher's responses to questions 1-4 and 7, the questions on the interview that addressed fractions and decimal concepts.

Define fraction and represent definition with students (Q1). The first question asked teachers to define fraction and explain how they would represent that definition to students.¹² It also asked teachers whether and how their definition of fraction had changed as a result of participating in the professional development. Students' difficulties understanding fraction concepts are well documented. Many experts believe that part of the problem is that students are presented with multiple definitions of fractions throughout upper elementary school and middle school (Wu, 1996; 2005; Milgram, 2005). For example, a fraction is often first defined as a part of a whole – e.g., what fraction of the circle is shaded? This definition resonates with many students (and probably with many adults as well). But later, a fraction also refers to parts of a set – e.g., what fraction of the chips is blue? – which is a different type of model. While both of these definitions are correct, students often have trouble understanding yet a third definition for fraction, which is the idea that a fraction is a single number that can be placed on a number line. This third definition is especially critical as students move into algebra, where fractions become subsumed under the domain of rational numbers and older notions of “part-whole” become less useful when manipulating expressions and equations with rational numbers (Wu, 2005).

Given this background information and the study's focus on 7th grade students who are transitioning into algebra, the study developed the following definition of fraction that supports students' understanding of fraction as a point on a number line:

For whole numbers a and b , with b not equal to zero, the *fraction* a/b is the number on the number line corresponding to a times the unit divided into b

¹² The actual question was: How would you define fraction and represent that definition with students? Has this definition changed because of what you learned in the professional development? If so, how and why?

equal parts. Using this definition, $\frac{3}{2}$ is a fraction. Since $\frac{3}{2} = 1\frac{1}{2}$, $1\frac{1}{2}$ is a fraction.

The professional development facilitators introduced this formal definition of fraction to teachers at the beginning of the summer institute and revisited it on several occasions during follow-up workshops. Teachers participated in several activities where they plotted, compared and ordered fractions on number lines and explained how their solutions connected to the study definition. The core ideas associated with this definition are that a) fractions are numbers, so every fraction has an exact location on a number line; b) the numerator of a fraction (a) represents the number of pieces being counted; and c) the denominator (b) represents the size of each piece relative to the unit. I expected teachers to touch on these aspects when they responded to the first part of the question, and, given that the importance and usefulness of the number line is embedded in this study definition, I expected teachers to explain why the number line is an appropriate representation to use with students when communicating these ideas.

Two of the three teachers – Hamlin and Wiggins – said that a fraction represented a point on the number line, which indicated that they had understood this key aspect of the definition. Hamlin went on to explain that “the top number being the number of pieces being counted, and the bottom number how many pieces make up the whole,” which was very similar to what the study emphasized. Wiggins also echoed the study definition by saying that “the denominator tells you how many pieces it’s split into and then the numerator is how many of those pieces you have.”

Both teachers scored at the moderate level of understanding because they

made solid connections to the study content. Had either teacher explained why the number line is a useful representation for showing that fractions are numbers, they would have scored at the strong level of understanding. Both teachers mentioned that a fraction was a point on a number line, but neither discussed what the numerator and denominator meant in terms of the number line. Neither teacher either emphasized that they would use the number line to explain the meaning of fraction to students, even though they participated in a number of activities focused on fractions and number lines – one of which even required teachers to make their own number lines for use in the classroom. Neither teacher referred to the more formal, algebraic aspects of the definition. For example, neither teacher referred to the algebraic aspects of the definition, such as using a or b to describe the number of pieces or the size of each piece with respect to the unit.

Smith responded to the prompt by stating that a fraction was a “representation of parts of a set compared by division,” and went on to say that a fraction could either be “parts of a whole or parts of a set.” He never mentioned that a fraction was a number that had an exact location on a number line; and, when prodded, he never fully explained what he meant by “part of a set compared by division.” Nor did he distinguish when it might make more sense to use the part of the whole or the parts of a set definition. In fact, two of the study emphases were to encourage teachers to be deliberate about using particular representations and to encourage teachers to use the number line because it was the most comprehensive representation. Since he missed the number line component completely and did not distinguish between parts of a whole and parts of a set, he scored at the weak level of understanding.

Connect fraction and decimal concepts (Q2). The second question addressing fraction and decimal content asked teachers what they would do to help students solidify their understanding of how fractions and decimals are related.¹³ The study research team worked with the professional development provider to craft a precise definition of decimal. To promote coherence between the two definitions, we agreed to use parallel language and similar symbolic notation. According to the study:

Decimals are numbers that can be written as $N.abc$, which means N plus $\frac{a}{10} + \frac{b}{100} + \frac{c}{1000}$ etc. Decimal notation is analogous to place value notation for whole numbers; positions to the right of the decimal point represent fractions with denominators that are powers of 10. Both fractions and decimals are numbers, and they can be ordered and compared. All fractions can be represented as decimals – some terminating and some repeating – but there are some decimals that cannot be represented as fractions. For example, the terminating decimal 1.23 is equal to $1\frac{23}{100}$ or $1 + \frac{2}{10} + \frac{3}{100}$. The repeating decimal, $.333\dots$ is equal to $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots$

The professional development facilitators introduced the definition of decimal early in the summer institute and revisited it at various time points during the follow-up seminars, though not quite as often as they referred to the study definition of fraction. Teachers participated in activities where they connected the meaning of fractions and decimals. They identified decimals as numbers, which could be compared and ordered on the number line. The core idea associated with this definition is that decimals, place value and fractions are closely linked and should be mutually reinforced. For example, the decimal 0.15 is equivalent to fifteen hundredths or $\frac{15}{100}$, with the denominator of 100 representing a power of 10. These connections can be reinforced when fractions and decimals are placed on the same number line. These

¹³ The actual question was: Suppose a group of students are having trouble connecting the meaning of fractions with the meaning of decimals. What might you do to help students solidify their understanding of how these two types of rational numbers are related?

emphases are what I expected teachers to address when they responded to this question.

Only one teacher, Smith, scored at the moderate level of understanding on this question. Both Hamlin and Wiggins, though they scored higher than Smith on the previous question, scored at the weak level of understanding. Smith used place value to connect fractions and decimals: “I would say the number one problem equating a fraction to a decimal is that idea that a decimal is based on the base 10 and a fraction can be based on whatever.” Though his point about decimals being based on base 10 is important, it could have been stronger if he had talked about how fractions with power of 10 denominators represent positions to the right of the decimal point. He actually didn’t refer to the study definition at all. Nevertheless, his initial, strong connection to place value was consistent with what was being promoted in the professional development. Later, Smith mentioned how he would connect fractions and decimals using a number line: “I’ve also used fractions that are 10^{th} s on the number line...then we start looking at those, including looking at little blow ups, like we did in the study.” This connection to the number line was consistent with what was being emphasized in the professional development – he even referred to a specific professional development activity from the summer institute. These responses put Smith safely into the category of moderate level of understanding.

Neither Hamlin nor Wiggins made solid connections to the study emphases around fractions and decimals. Instead, both used real world examples to promote student understanding. Hamlin used pizza to describe how the value of a number can take different forms:

We talk about how the value of a number and it being in a different form...Two out of five pieces might make more sense than 40% of something if you're talking about how much pizza you ate, for example...we talk about rational numbers in different forms having the same value.

Instead of connecting fractions and decimals using the study definitions or number line activities, Hamlin used an area model – pizza – to illustrate how rational numbers can come in different forms and yet maintain the same value. This idea is important, but it is not an idea that the study emphasized; and, her example connects percents, not decimals, with fractions.

Wiggins used money to connect fractions with decimals and focused on the particular numbers in the question, rather than on the concepts more broadly:

I would bring it back to common sense. If a student thinks .12 is equal to $\frac{1}{2}$, I would say 'you know what half looks like and .12 is like 12 cents. Do those match up?' ... Because, if you make anything from money for them, they'll get it.

Money is a common way to connect fractions with decimals, but the professional development actually warned against using money, since money only works for decimals carried out to the hundredth place. For example, money wouldn't necessarily help a student understand why 0.158 is equal to $\frac{158}{1000}$ or $\frac{1}{10} + \frac{5}{100} + \frac{8}{1000}$. Had Wiggins referred to either study definition or mentioned the importance of place value, she probably would have scored at moderate level of understanding. But since she and Hamlin relied on their own techniques and failed to mention these core emphases, they both scored at the "weak" level of understanding.

Rationale for decimal procedures (Q3). The third question focusing on fractions and decimals asked teachers to explain the rationale for operations with decimals, i.e., to explain why decimals must be "lined up" before they are added or

subtracted and why the decimal point “moves” when they are multiplied or divided.¹⁴ Teachers participated in several professional development activities that were designed to promote understanding of these rationales behind common procedures. The core idea behind adding and subtracting decimals is that only like quantities can be combined. When the decimal points are “lined up,” the associated place values are lined up as well. This distinction might seem subtle, but lining up place values is a much more conceptually rich idea than lining up the decimals points, which many students know how to do but can’t explain why it works. The “movement” of the decimal point when multiplying and dividing decimals is understood when decimals are converted to fractions with power of 10 denominators. The study included activities where teachers converted decimals to fractions and then discussed what was happening to the decimal point and why. For example, $0.23 \times 0.15 = \frac{23}{100} \times \frac{15}{100} = \frac{345}{10,000}$ or 0.0345. The product is carried out to the ten thousandths place because the two factors are in the hundredths place – hundredth x hundredth = ten thousandth.

Hamlin’s response to this question was one of only two responses across all the interviews that reached the strong level of understanding (Hamlin also scored the other strong response, on question 4). Her responses to both parts of the question – adding and subtracting and multiplying and dividing decimals – are strongly linked to what was emphasized in the professional development. When discussing the rationale for adding and subtracting decimals, Hamlin begins by referring to money:

¹⁴ The actual question was: Suppose a student asks you to explain why you “line up the decimals” when adding or subtracting but “move” the decimal place to the right or left when multiplying or dividing. What would you say?

We've done examples. That's come up actually pretty recently and we talked about money. First of all, I put up that I have 50 cents and I have \$5.00. So I [decided to] add them and so now I have 55 cents, right?

But she doesn't stop there. Instead, she connects this concrete example to the underlying concept of adding like place values and even gets the kids to think about why first:

Now does that make sense to you? ... Well, why? ... You've gotta add the same thing... the pieces that you're adding have to be the same size.

When Hamlin talked about the rationale for multiplying and dividing decimals, she referred to the study emphasis of converting the decimals to fractions with power of ten denominators:

When we multiply decimals we turn the decimals into fractions... 25 times .5...and they get, you know, 125/1000ths and then we talked about, um, you know, started looking at those. What do you notice?... You know, I'm ending up with thousandths here. Why am I ending up with thousandths? Well, 'cause I'm multiplying hundredths times tenths.

Hamlin's written work was consistent with this explanation: the "movement" of the decimal point is the result of multiplying the fraction equivalents with denominators that are powers of ten. The approach reinforces the decimal place value system. Because Hamlin succinctly and clearly captured the core study emphases related to this question, she received a rating of strong level of understanding.

Neither Smith nor Wiggins was able to answer this question as completely or succinctly as Hamlin. But Smith's rationale for lining up the decimals when adding or subtracting addressed several ideas emphasized in the professional development.

For example, he made an immediate connection to place value:

One thing, I think, you know, you start having a conversation about place value. You could start with whole numbers and we don't use a decimal point when we're just adding whole numbers but we could go -- you could have like

35 + 28, if you were to put a decimal point in there for each number and it would still be 35 and 28, where would you put the decimal point on each one? Okay. Does it does line up? Well, yeah because I've got the ones and I've got the tens.

Though he talked about lining up the ones and the tens, his explanation would have been stronger if he had explained that the ones and tens were part of a larger place value system. However, Smith returned to the importance of place value later in the interview, as part of a discussion about estimation:

The estimation is important. It's a way to keep place value so that we're adding 100ths to 100ths, 10ths to 10ths, 1s to 1s and so forth when we're doing a regrouping.

He continued to discuss the importance of estimation when explaining to students why the decimal point “moves” when multiplying or dividing: “I would tell the student, ‘Is the decimal place...in your answer based on your estimation [so that] it makes sense to you?’” The study discussed estimation as one strategy to promote number sense, but the professional development did not talk specifically about how estimation might be used to explain this question because estimation only makes sense when the numbers are manageable. Relying solely on estimation is similar to using money to explain the connection between decimals and fractions: even though they activate students' prior knowledge, both are limited in what they can explain. Given Smith's moderate-strong response to the part of the question focusing on decimal addition and subtraction and his weak response to the part of the question focusing on decimal multiplication and division, he scored at the weak-moderate level of understanding.

Wiggins also scored at the weak-moderate level of understanding on this question. Like Smith, she did better on the first part of the question and made a solid

connection to place value for why the decimal point must be lined up when adding or subtracting:

Well, as far as for the adding, I mean, we line 'em up when we're doing whole numbers 'cause each number has the same value... We've done a lot more with just place value and they're getting to being okay with the whole idea that they have a value.

Like Smith, who used estimation to explain why the decimal point “moves” when multiplying or dividing, Wiggins emphasized estimation and sense making when determining where the decimal point should be placed in any product or quotient. First, though, Wiggins went back to the definition of multiplication as the number of groups of a certain number:

Well, I'd probably bring them to like a simple question where it's like 3 times .2 ... 'cause like 3 times 2 is 3 groups of 2 or two groups of 3... So it's 3 groups of .2.

The professional development did not focus on the meaning of multiplication as a way to understand why the decimal point “moves,” but Wiggins concluded her explanation by saying that “[students] know that 3 groups of 2 is 6, so 3 groups of .2 has to be .6.” This latter part of her explanation reinforced estimation and number sense, which were general study foci but not specifically emphasized for this question. Like Smith, who focused on helping students understand where the decimal point should be placed in an answer, Wiggins did not answer directly the question of why the decimal point “moves.” Also like Smith, Wiggins only used simple numbers to reinforce students' understanding of decimal placement. Neither teacher mentioned the study focus of converting decimals to fractions, which explains the location of the decimal point in any product, not just simple questions where students'

number sense is sufficient. Given the similarities between Smith's and Wiggins's responses, Wiggins also scored at the weak-moderate level of understanding.

Student misconceptions with fraction procedures (Q4). The next question asked teachers to debug a student misconception related to simplifying fractions. Students are taught often to “cross cancel” as a quick way to simplify fractions, but they are rarely provided the underlying rationale for why it works. For example, $\frac{10}{150} = \frac{1}{15}$, but $\frac{8}{28} \neq \frac{1}{2}$. The reason the procedure works in the first equation but not the second is grounded in the identity property of multiplication. The identity property of multiplication states that the value of a number does not change when it is multiplied by one (any form of one). The study placed a strong emphasis on this property and included the more formal expression of it: $\frac{a}{b} \bullet \frac{n}{n} = \frac{a \bullet n}{b \bullet n} = \frac{a}{b}$. So, $\frac{10}{150} = \frac{1}{15}$ is true because $\frac{10}{150}$ can be expressed as $\frac{10 \bullet 1}{10 \bullet 15}$, where $\frac{10(n)}{10(n)} = 1$. And the value of anything (in this case $\frac{1}{15}$) multiplied by one (in this case $\frac{10}{10}$) does not change. The same cannot be said for the second inequality, since 28 does not have a factor of 8, which could be used to write the expression $\frac{8(n)}{8(n)} = 1$. To address the student misconception in the inequality, $\frac{8}{28} \neq \frac{1}{2}$, teachers would need to discuss the identity property of multiplication, either formally or informally.

Hamlin again demonstrated a solid grasp of the underlying student misconception and referred to the concepts emphasized in the professional development.

Well, if they're changing it into an equivalent fraction they have to divide by one -- a form of one -- and in this case they are not dividing by a form of one. They're just crossing off things.

When Hamlin talks about dividing by a form of one, she is referring to the $\frac{n}{n}$ component of $\frac{a}{b} \bullet \frac{n}{n} = \frac{a \bullet n}{b \bullet n} = \frac{a}{b}$. Though she says “dividing by a form of one” rather than multiplying by a form of one, the study emphasized that every multiplication problem can be rewritten as a division problem. Thus, her description addressed the core underlying concept of the identity property of multiplication. Hamlin elaborates further and demonstrates that she knows that the mistake stems from situations where it does work:

I think where [students] get this misconception is when they’re taught because of powers of ten, if you have 10/880, boom, you cross off the zeroes. Well, that works. You’re actually dividing the numerator and the denominator ... by ten tenths.

Though Hamlin did not refer to the formal property, she did address the key principles associated with the property. She also reinforced the study idea of providing rationales for procedures: “That’s my biggest thing with my kids ... if you don’t understand why that works, you can’t use it.” These responses placed Hamlin in the category of “strong” level of understanding.

Smith also addressed some of the core ideas emphasized in the study, but his responses were not as comprehensive or succinct as Hamlin’s. He begins by suggesting that the student think about benchmark fractions, which reinforce his earlier focus on building number sense and estimation:

Well, I think, you know one thing that we could instruct to have them do is come over here to $\frac{1}{2}$...Start asking ... why is this a half? Well, because it’s a one in the numerator and it’s a 2 in the denominator ... what does that really mean?...I might move to $\frac{2}{4}$ ths and $\frac{3}{6}$ ths and start looking for a pattern: it looks like the numerator has what relationship with the denominator? It’s always half...

Like other questions, the study did emphasize estimation and number sense generally, but the primary emphasis for this question – a question that was very similar to an activity the teachers completed in a workshop – was the identity property of multiplication. I prompted Smith to think about cases where “cross canceling” does work in an effort to trigger the identity property of multiplication:

Interviewer prompt: *If you have 80/160, [crossing out the zeros] actually would work in this case, wouldn't it? 80/160 -- 8/16ths would still equal a half.* Smith: They have no concept of dividing each number by 10, which is still dividing it by the value of one, but it's a tenth of the pieces. And, of course, they're ten times larger and so you'll need a tenth of them. They're using a procedure here.

Smith does touch on the concept of dividing each number by ten, which is the same as dividing by the value of one. Since Smith showed that he understood the basic concept behind the identity property of multiplication, I gave him a score of moderate understanding. Like Hamlin, he did not refer to the formal property. But unlike Hamlin, who extended the question to an example where the “cross canceling” worked, Smith only responded to my prompt. His response suggested that his initial response, which emphasized number sense more than the underlying property, is what he would most likely do to emphasize with students.

Like Smith, Wiggins responded to this prompt by referring to the benchmark fraction of $\frac{1}{2}$. She emphasized the visual representation of the common fraction and the relationship between the numerator and denominator:

I would probably go back and draw a picture of it. I mean, the odds are if you give them something to look at they can see that $\frac{8}{28}$ is definitely not $\frac{1}{2}$... I'd just say think about it. You know what $\frac{1}{2}$ is and you know that 1 is half of 2; is 8 half of 28?

While drawing a picture may be an effective way to help some students with this particular problem, the explanation does not address the identity property of multiplication, which can be used to explain why any fraction can be simplified. Wiggins focused on the incorrect student answer, which happened to be the benchmark fraction of $\frac{1}{2}$, and then worked backwards toward a meaningful answer. But what if the incorrect student answer had not been a benchmark fraction, then how would she have explained it?

Though Wiggins does hint at the identity property – she says that she would talk to students about why it works with tens – she doesn’t name it explicitly and has trouble articulating it succinctly:

As far as the division we’ve talked about just reducing a fraction and how you can divide top and bottom by the same thing and in the tens case it ... you can cancel out a zero... You have to reduce by the same number on both...it goes to that whole ratio and keeping it consistent.

Unlike Hamlin and Smith, who intentionally avoid using the term “cross cancel” and stress multiplying or dividing by a value of one, Wiggins still refers to “canceling out the zero” in her explanation and never mentions multiplying or dividing by a value of one. Given these responses, as well as her comment, “I don’t know how I’d say it to a seventh grader right now, though”, Hamlin scored at the “weak” level of understanding for this question.

Rationales and representations for fraction procedures (Q7). The final question addressing fraction and decimal content focused on the appropriateness of specific models or representations for particular operations with fractions. More specifically, I asked teachers to explain which representations would be most appropriate to help a student understand why $\frac{1}{3} + \frac{3}{4} = \frac{13}{12}$ and why $\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$.

The professional development model emphasized that linear models, such as the number line, were most appropriate for explaining additive situations, and area models, such as rectangular grids, were most appropriate for explaining multiplicative situations. For example, teachers participated in a number of workshop activities where they used number lines to show how fractions could be added or subtracted and why common denominators were needed. They participated in other activities and solved problems where area models were used to explain the multiplication of fractions. A major student misconception related to multiplying fractions is that the product of two fractions must be greater than the value of either factor. Teachers solved problems in the professional development that addressed this misconception through an area model – e.g., What fraction of a pan of brownies remains if $\frac{1}{2}$ the pan is eaten on one day and $\frac{1}{4}$ of what was left on the next day?

None of the teachers articulated the core distinction that linear models were generally more appropriate for additive situations and area models more appropriate for multiplicative situations. Hamlin said that she would use the same model to show addition and multiplication:

I would use the same model, so we could see the differences... Well, I'll go back to brownie problem [area model], okay? One-third, so we find out what one-third is. This would be one-third of three-fourths is the way I think of it so first I need to figure out what three-fourths is. So divide it into fourths or they would divide it into fourths for me.

Her description of the area model is consistent with what the study emphasized; in fact, she references the “brownie problem” in her explanation.¹⁵ She also says that she would use a set model to help students who didn't understand the area model. The

¹⁵ The “brownie problem” asked teachers to describe what fraction of a pan of brownies remained after consecutive days of eating. If half the pan is eaten on Monday and one fourth of what remained is eaten on Tuesday, what fraction of the pan remains? The brownie pan represents an area model.

professional development discussed set models as another appropriate representation for multiplying fractions, so her response reinforced this representation.

Though she says she is going to use the same model to show multiplication and division, she ends up using money to illustrate why like denominators are needed when adding or subtracting fractions:

You know, we'd go back to my money model [for addition]. When you're adding fractional pieces – pieces like pennies or pieces of a dollar – well, you need to be all adding the same thing. Well, if you're adding the same thing what's important about these denominators? They have to be the same size pieces.

This explanation, though conceptually clear and grounded in students' real world knowledge, does not include the number line or another linear model. Given Hamlin's strong response to the first part of the question, she scored at the moderate level of understanding overall.

Smith doesn't refer to an area model or linear model, but instead says that he will focus on building students' number sense. He starts by explaining that when $\frac{3}{4}$ is multiplied by a number great than one – in this case, two – the product is greater than $\frac{3}{4}$:

I think one place you might want to start is start using, you know, their number sense. Coming back to, all right, I've got $\frac{3}{4}$ of this bar, and I find out one group of that I have $\frac{3}{4}$, all right? If I have two of those, okay, then I have $\frac{3}{4}$ -- each one of these is $\frac{1}{4}$. Now I have another three so I have $\frac{6}{4}$. So $\frac{6}{4}$ is greater than $\frac{3}{4}$, okay?

Then, he explains that when $\frac{3}{4}$ is multiplied by a number less than one, the product is less than $\frac{3}{4}$:

So therefore, there must be something that we can do algorithmically or whatever that this is going to be something less than $\frac{3}{4}$. So I think that's where you start in a number sense understanding, [that] it's got to be less than $\frac{3}{4}$.

Unlike Hamlin, who referenced both the area and set models for the multiplication part of the prompt, Smith did not refer to a specific model or representation. Nor did Smith reference a model or representation for the addition part of the prompt.

Instead, he focuses again on estimation strategies with benchmark fractions:

And then with addition ... I started with $\frac{1}{3}$ and $\frac{3}{4}$ and now I'm dealing with 12ths in my answer and this numerator is bigger than my denominator... Tell me what part of a whole I have and, you know, start putting 'em together and they could estimate it.

Since Smith did not reference any representations, even when prodded, which was the primary point of this question, he scored at the weak level of understanding.

Like Smith, Wiggins emphasized students' number sense to address the part of the question dealing with multiplying fractions. She wanted students to see that the product of $\frac{1}{3}$ and $\frac{3}{4}$ had to be less than $\frac{3}{4}$:

A lot of times you can refresh them back to 2 times 3 is 6; it's 3 groups of 2 and this is $\frac{1}{3}$ of a group of $\frac{3}{4}$... And they can at least see it's gonna be a smaller answer

Unlike Smith, Wiggins said that she would also use a picture to illustrate why the product is $\frac{1}{4}$ in this problem. She didn't refer specifically to an area model, but her picture utilized an area model similar to the one featured in the "brownie problem." When Wiggins explained why common denominators were needed for addition, she said that she would use a combination of fraction strips and an area model. The professional development did not emphasize fraction strips, though this linear representation could be linked to the number line, but Wiggins did not make this connection:

I think I would go to like fraction strips. Where you take a strip and you cut it -- fold it into three and you take one of them, and you take another strip and

fold it into four equal parts and you have to take three of them and then you compare it to a couple of strips and say that yeah, you end up with more than what you began with...If you wanted to get to the common denominators then pull out the fraction circles and do that. Take the $\frac{1}{3}$ and the $\frac{1}{4}$ and then take other pieces to try and take another color and try and get the answer and you could put, okay, you have a $\frac{1}{3}$ piece, you'd have the $\frac{3}{4}$ piece and you have to put pieces on top of them if really wanted an exact answer.

Nor did the study professional development encourage teachers to use the area model for addition. Instead, the study focused on the number line as a more appropriate representation. Since Wiggins's response to the multiplication part of the question was relatively strong, she scored at the weak-moderate level of understanding for this question overall.

Teachers' Understanding of Ratio and Proportion Content

Compared with teachers' performance on questions dealing with fraction and decimal content, in which they on average scored between the weak and moderate levels of understanding, teachers scored lower on the ratio and proportion questions. Two of the three teachers scored at the weak level of understanding for both ratio and proportion items. The third teacher scored at the moderate level for one question and at the weak level of understanding for the other ratio and proportion question. On the one hand, this discrepancy between fractions/decimals and ratio/ proportion is not surprising, since ratio and proportion concepts are considered to be more complex than fraction and decimal concepts. On the other hand, given the strong emphasis on ratio and proportion in the professional development and the extensive amount of time devoted to ratio and proportion in this district's text, the discrepancy is puzzling.

Define and represent ratio (Q5). The first ratio and proportion question asked teachers to define ratio and explain how to represent that definition with students.

According to the study, a ratio is a comparison of two quantities by division, where the quotient represents the relative magnitude of the two quantities – i.e., the magnitude of one quantity as a multiple of the other. This definition emphasizes that ratios are not just comparisons between two quantities but *multiplicative* comparisons. This difference might seem subtle, but it's very important and often overlooked. For example, additive comparisons, such as “three more than a number,” are comparisons between two quantities but they are not multiplicative comparisons. The study also emphasized that ratios come in three forms – part to part, part to whole and whole to whole – distinguished “within” from “between” ratios. “Within” ratios consider comparisons from one similar figure, while “between” ratios compare corresponding dimensions from two similar figures. The study provided teachers with several ways to represent ratios with students, most notably strip diagrams and ratio tables.

Hamlin, who scored the highest on the fraction and decimal questions and highest overall, did not use a definition of ratio that highlighted the multiplicative relationship between the two quantities. She did mention that a ratio was a comparison, but then she connected ratios to fractions:

A ratio is a comparison between two things...put in fractional form, a lot of times, or with the little dot dots...We always talk to the kids, you know, the amount of girls to the amount of boys; the amount of left-hand people to the amount of right-hand people; how many are blue eyed and how many are brown eyed? We talk about what's a comparison...and sometimes we'll represent it as a fraction. It can be manipulated like a fraction...you can add ratios and subtract 'em and multiply and do all those fractional things with ratios.

To her, a ratio was a comparison between two quantities, but she did not distinguish multiplicative from additive comparisons. She used a real world example (boys to

girls) and prior knowledge (fractions) to explain what a ratio was, but these connections did not capture the core ideas of the study. When she mentioned that fractions and ratios were related, she didn't elaborate on how they were related or how they were different. Hamlin did not discuss how ratio tables and strip diagrams could be used to highlight what ratios are and to help students understand problems involving ratio and proportions. Given these responses, Hamlin scored at the weak level of understanding.

Smith was the only teacher who addressed the multiplicative aspect of ratios, though he didn't make this point immediately or succinctly. When asked to define ratio, he initially said:

It's a way to compare groups of numbers and I think that could be an introduction. Say we're gonna talk about ratios of boys to girls in the class, something that they can concretely see, [or] I have one cup of sugar to every two cups of flour.

This definition does not mention the multiplicative relationship between the two quantities, and like Hamlin's response, utilizes a real world, concrete example. But later in the interview, when I prompted Smith to explain the type of relationship between the flour and sugar, Smith said:

Well, it's a ratio but it's also a comparison of two values in a sense by division. Because if it's 1:2 – we'll say that one cup of the flour to two cups sugar – the flour is always half of the sugar. So no matter how much sugar I have, I need to divide that amount by two to get my flour or if I have my flour I could always multiply by two to get the amount of sugar.

In this explanation, Smith captures the multiplicative aspect of ratios: no matter how big the recipe, the amount of flour is always half the sugar (or the amount of sugar is always double the amount of flour). Smith didn't mention either ratio tables or strip

diagrams as a key representation for ratios, but because he captured the multiplicative aspect of the definition of ratio, he scored at the moderate level of understanding.

Like Hamlin but unlike Smith, Wiggins did not explain that ratios were multiplicative comparisons between two quantities. She said that a ratio is “just a comparison of two things,” and when prompted to explain that comparison, she said, “it’s got to keep that consistency between itself.” Though “consistency between itself” might have been a proxy for a multiplicative relationship, she was not able to articulate this comparison in the interview, nor did she distinguish additive from multiplicative comparisons. Hamlin did mention ratio tables – “I would use the ratio table; I love the ratio table” – but, when prompted, could not explain what mathematical concepts the ratio table might illustrate or illuminate. Given the lack of attention and clear articulation of the study emphases, she scored at the “weak” level of understanding.

Connect ratio and fraction concepts (Q6). The second and final question in the ratio and proportion domain asked teachers to explain how ratios and fractions are related and how they can be distinguished from one another. The study professional development emphasized that a fraction is a number with an exact location on the number line, but a ratio is a comparison between two numbers, and therefore does not have an exact location on the number line. Even though ratios can be expressed in fractional form – e.g., the ratio 3 to 4 can be expressed as 3:4 or $\frac{3}{4}$ – they are not exactly the same thing. The professional development also touched on how ratios and fractions give rise to each other – which is definitely true – but because this emphasis was not as great as the number vs. comparison distinction, I focused more on the

distinction between fractions and ratios for scoring purposes. However, if any teacher had discussed how fractions and ratios give rise to each other – none of them did – I would have incorporated those responses into the scoring.

Hamlin could not explain how fractions and ratios are related to one another: “That’s befuddling. I’ll have to think over that one. I haven’t really given much thought to it truthfully”. However, she did try to distinguish between a fraction as a “part of a whole” and a ratio as a comparison between two different things:

A fraction is a part-to-whole relationship. And a ratio can be between two different things – it can be between two different things. Where generally a fraction is a – you’ve got a whole and then the numerator is the part of that whole. [But] in boys to girls it’s not that way.

But this explanation was unclear and did not reference the underlying concept that fractions are numbers that can be placed on the number line but ratios are comparisons between numbers and cannot be placed on a number line. Hamlin, like Smith and Wiggins on this question, scored at the weak level of understanding.

Smith did not bring up the core way in which fractions and ratios are different – that one is a number and the other is a comparison between two numbers – but instead focused on how part-to-whole ratios and fractions are related:

If you talked about ratios as part-to-part and part-to-whole, and you’ve talked about fractions as part of a whole, whatever that whole is, then you can start talking about well, is a fraction always the same as a ratio? ... Then you can start having the discussion, okay, if a fraction basically means it’s the parts that I’m counting compared to the number of parts or pieces or parts that it takes to represent the whole... but I have two types of ratios over here -- a part-to-part and a part-to-whole -- which one does it sound like it’s more like?

This explanation does address the similarity between part-to-whole ratios and fractions – e.g., if the ratio of girls to students is 11 to 20, then $11/20$ describes the

fraction of the class that is girls.¹⁶ However, this connection was not an emphasis in the professional development. Rather, the strongest explanation for why fractions and ratios give rise to each other is rooted in the idea of rate.

Like Smith and Hamlin, Wiggins does not distinguish fractions from ratios as one being a number and the other being a comparison between two numbers. When prompted to think about whether the number line could be used to distinguish fractions from ratios, she said, “I probably wouldn’t go at it that way because I don’t use fractions on the number line. Like Smith, Wiggins talks about the relationship between part-to-whole ratios and fractions being a part of a whole:

When I think of ratios I think of the part, no, fractions I think of as being part-to-whole, so like you have three pieces out of five of a cake. But a ratio could be I’ve got 14 girls and 24 boys. You know ...a fraction is always part to whole, [but] a ratio could be part-to-part or a part-to-whole.

Since she did not address the core distinction and struggled to articulate the other connections between fractions and ratios, Wiggins also scored at the weak level of understanding.

Sources of Variation Among Teachers’ Responses

The previous discussion of teachers’ responses to the seven questions on the structured content interview depicted variation in understanding across the questions as well as variation across teachers for particular questions. One potential source of variation resides in the complexity of the content or components of the professional development model. For example, teachers on average scored lower on the ratio and

¹⁶For example, if the ratio of cups of lemon juice to cups of water in a lemonade recipe is 2 to 5, then there are $\frac{2}{5}$ cups of lemon juice for every cup of water (rate). Thus, the ratio 2:5 gives rise to the fraction $\frac{2}{5}$ in the rate of $\frac{2}{5}$ cups of lemon juice for every cup of water. Fractions give rise to ratios in the reverse fashion. For a complete discussion of this topic, see Beckman (2005).

proportion items (Questions 5 and 6) than they did on the fraction and decimal items (Questions 1-4; 7). Teachers also scored lower on questions that required them to make connections across multiple concepts (Questions 2 and 6) than on most of the other items.

A second potential source of variation resides in differences among teachers such as their knowledge and experience and their beliefs and attitudes toward learning. Teachers brought different levels of background knowledge and experience to the professional development. They also varied in the degree to which they were motivated to learn the content, as evidenced by the amount of time and effort they put into the various professional development activities. The next two sections address each of these broad sources of variation: the complexity of the content of different components of the model and individual differences among teachers.

Complexity of the Content

As previously discussed, teachers scored lower on the ratio and proportion items – just barely above the weak level of understanding – than they did on the fraction and decimal items – closer to the moderate level of understanding. What might explain this difference? According to Cramer and Lesh (1988), who worked on the Rational Number Project, a twenty year study focusing on the teaching and learning of rational numbers in the elementary and middle grades, elementary and middle school teachers’ understanding of ratio and proportion concepts is a major obstacle to promoting student understanding of these concepts. Other researchers, such as Lamon (1999), have documented similar challenges in teachers’ acquisition of knowledge related to teaching ratio and proportion concepts.

This research is consistent with how material is presented to students: students learn about fractions and decimals in elementary school, but don't delve deeply into ratio and proportion until middle school, when they are older and more able to understand more complicated concepts. Unlike fractions, which lend themselves to simple, concrete models like pie charts or sets and generally involve a single concept, a ratio is a multiplicative comparison between two quantities and involves more than one idea at a time. For example, if the ratio of lemons to cups of water in a lemonade recipe is 3 to 8, then the ratio of water to lemons is 8 to 3. Expressed as a unit rate, there is $\frac{3}{8}$ of a lemon per cup of water or $\frac{8}{3}$ of a cup of water per lemon. Notice that the description of this relationship requires an understanding of fractions and illustrates that ratios are more complex ideas than fractions.

To compound matters, the definition of ratio varies across different textbooks and resources available to teachers. According to the first edition of *CMP*, the textbook that all three teachers were using, a ratio is a comparison between two quantities, but it does not include the "by division" part of the definition. The text discusses the multiplicative aspect of ratios, but it is presented differently than in the professional development. Thus, the study's expansion of what "by division" means was likely new to teachers, which could partially explain why they scored lower on these items.

Another potential source of variation related to the complexity of the content is the distinction between pedagogical content knowledge and content knowledge. Though the range of averages across teachers for each question was 1.0 – from 2.0 to

1.0 – two of the lowest averages were on Questions 2 and 6, both of which dealt with making connections across rational number concepts. Question 2 asked teachers to make connections between fractions and decimals, and Question 6 asked teachers to make connections between fractions and ratios. Both averages, 1.3 for Question 2 and 1.0 for Question 6, were the lowest averages across the 7 questions. This pattern suggests that teachers struggle more with connecting concepts than they do articulating a single concept. Given the difficulty teachers had with individual concepts – e.g., they struggled to define both decimal and ratio – it is not surprising that they struggled more to make connections across concepts.

Individual Differences Among Teachers

In addition to the differences associated with the nature of the content, differences among teachers are another potential source of variation in their understanding of the core content emphasized in the professional development. Teachers' a) prior knowledge and experience, b) prior experiences with prior curricula compatible to the professional development, c) further self or group study on the professional development topics, d) extra collaboration with the coach, and e) beliefs and attitudes about the utility of such knowledge and about how students learn mathematics are all potential sources of variation in understanding. Among these potential sources, teachers' beliefs and attitudes about mathematics content and beliefs about how students learn mathematics appeared to be the most salient sources of variation.

Hamlin, who demonstrated the strongest overall level of understanding on the structured content interview, stood out even more in terms of her beliefs and attitudes

toward learning the content and promoting students understanding of the content. She was hungry to learn. She often vigorously pursued why something was true or false, and was always uncomfortable when students didn't understand something or when she was confused about the mathematics. For example, during one of the workshop activities, the teachers were solving a relatively difficult problem involving rates (several of the teachers were stuck):

It takes 4 people 10 hours to mow the grass in the city park. If they all work at this rate, how long will it take 6 people to mow the same park?

Hamlin managed to arrive at the correct answer of $6\frac{2}{3}$ hours, but she didn't know why this answer was correct. She asked the professional development facilitator during whole group discussion, "I know this is the right answer, but why does this work?" Unfortunately, the facilitator wasn't able to explain where this answer came from or how it was connected to what teachers had been doing that day.¹⁷ Hamlin then spent the next several minutes – including the entire afternoon break – trying to figure out the answer to the question on her own and discussing it with other teachers. This persistent attitude carried over to the classroom, as well. On several occasions, I observed Hamlin asking the coach (or me, when the coach was not present) for help in understanding the underlying mathematics in a lesson. One poignant example occurred at the end of a lesson, when several students were sharing different solutions to a particular problem. Hamlin was listening to the students talk about what they had done, but as she was trying to summarize the underlying mathematics in the students'

¹⁷ Teachers had been studying proportional relationships, but this problem featured an inversely proportional situation. Effort and time are inversely proportional, so if it takes 4 people 10 hours to complete the work, then 6 people would produce $\frac{6}{4}$ or 1.5 the amount of work in $\frac{4}{6}$ or $\frac{2}{3}$ the amount of time -- $\frac{2}{3}$ of 10 hours is 6 and $\frac{2}{3}$ hours.

thinking, she turned to the coach and said, “This is where I get stuck. I don’t know what to say. What should I say?”

Hamlin’s eagerness to understand the underlying mathematics stood out from Smith and Wiggins. Though both Smith and Wiggins actively participated in the professional development, neither teacher, at least publicly, was as determined to understand the mathematics (or as uncomfortable when confused) as Hamlin. Smith participated more publicly than Wiggins during the workshops – e.g., he asked more questions and made more presentations – but the difference between Smith’s and Wiggins’s apparent will to learn was much less pronounced than the difference between Hamlin’s and everyone else.

Among the other potential sources of variation, teachers’ prior knowledge and experience using similar instructional materials did not appear to affect teachers’ level of understanding of the core math content emphasized in the model. Using degree type and number of math courses taken as a proxy for prior knowledge, only Wiggins, who scored the lowest on the content interview, has a degree in secondary math education. Both Smith and Hamlin have degrees in elementary education, though Hamlin did specialize in mathematics in her elementary program. Hamlin said that her program was unique because it required students to take five courses in mathematics from the math department, as opposed to math courses in the education department. All three teachers have taken between 4 and 9 graduate courses in mathematics education or mathematics teaching, but none of the teachers has completed an advanced degree in mathematics or mathematics teaching. Using the number of math courses taken in an undergraduate math department as a proxy for

knowledge, the expected order would be Wiggins (12), Hamlin (5) and Smith (2); however, the order of the scores on the content interview was Hamlin (1.9), Smith (1.5) and Wiggins (1.3).

In addition to being the only teacher with a degree in secondary mathematics education, Wiggins also had the most middle school mathematics teaching experience (9 years) and the most experience teaching *CMP* (6 years). See Table 6 in Chapter 4 for a table of teachers' education and experience. Smith and Hamlin have been teaching middle school and the *CMP* text for three and two years, respectively. Thus, experience teaching middle school and experience teaching *CMP*, which more than traditional textbooks requires teachers to understand and be able to explain math concepts, do not appear to influence how much teachers were able to learn from the professional development. If length of experience mattered in this case, then Wiggins, who scored lowest, should have scored highest. However, the type of experience that each teacher had using the *CMP* materials is also likely a contributing factor. Though Wiggins has more years experience teaching *CMP*, she may not have made the most of opportunities to understand and explain the content.

None of the teachers collaborated with each other or with their instructional coach in between or after professional development activities. The absence of such contact eliminated potential sources of variation in understanding of the math content. Both Hamlin and Wiggins enrolled in math teaching courses during the year of professional development. Hamlin took a class on fractions, which might have supported her understanding of fraction concepts emphasized in the professional development. However, she reported that the class was not terribly informative and

that the main reason she enrolled in the course was to accrue credits toward a master's degree. Wiggins took a geometry class, which did not focus on rational number content and which Wiggins described as not very useful, so it is unlikely that this course contributed to Wiggins's understanding of content emphasized in the professional development.

Conclusion

In summary, all three teachers scored between the weak and moderate level of understanding on the structured content interview, with only one of the three teachers scoring close to the moderate level. The teachers, on average, scored higher on questions addressing fraction and decimal content than on questions addressing ratio and proportion content. Teachers' average scores on the ratio and proportion items were especially low, with teachers scoring weak on all but one of the ratio and proportion items. Teachers also struggled on questions where they were asked to make connections between rational number concepts – e.g., connections between fractions and decimals and ratios and fractions. Their average scores for these items were similar to their low averages on the ratio and proportion items.

Among the potential sources of variation in teachers' understanding, the complexity of the content – i.e., ratio and proportion concepts are considered to be more complex fraction and decimal concepts – and teachers' beliefs about learning mathematics stood out as salient factors. Teachers reported that the ratio and proportion content was more complex and newer than the fraction and decimal content. The teacher who was most eager to learn and most visibly troubled when she

didn't understand something scored noticeably higher than the other two teachers on the structured content interview.

The next chapter shifts from teachers' understanding of the content and pedagogical content through a structured interview to how that understanding manifests itself in the classroom, through teachers' instructional practice. In addition to capturing teachers' understanding of mathematics content and pedagogical content, the next chapter includes pedagogical knowledge, which the professional development model also emphasized.

CHAPTER SIX: TEACHERS' INTEGRATION OF THE PROFESSIONAL DEVELOPMENT INTO THEIR INSTRUCTIONAL ROUTINES

This chapter examines the extent to which teachers integrated into their instructional routines the content, pedagogical content and pedagogical components of the professional development model. Unlike the previous chapter, which utilized structured interviews to assess a single component of the model – teachers' understanding of math content and pedagogical content – this chapter utilizes data from classroom observations to assess all three main components of the model.

Classroom Observations

In order to assess the extent to which teachers integrated into their instructional routines various components of the professional development model, I developed an observation protocol based on the core content emphasized in the workshops, seminars and coaching activities. In addition to the math content and pedagogical content topics described in the structured content interview, the professional development emphasized general pedagogical strategies. General pedagogical strategies, as outlined in Chapter 3, are techniques that the professional development facilitators modeled in the workshops and coaching that can be applied to any lesson, regardless of whether the focus of the lesson is on rational numbers or not.

Components of the Classroom Observation Protocol

The classroom observation protocol is organized around these three general domains of knowledge: math content, pedagogical content and pedagogical knowledge (see Table 10). Math content knowledge encompasses several teacher behaviors, such as the degree to which teachers use mathematically precise language, make connections between or among rational number concepts, explain the rationale behind procedures or algorithms, and incorporate key ideas about fractions, decimals, ratio, proportion and percent. The content focus of most of the lessons I observed fell under the domain of rational numbers; however, parts of some of the lessons focused on other math topics. In the next section I describe how I scored teachers' understanding of non-rational number content.

Table 10
Core Components of Classroom Observation Protocol

Domain of Knowledge	Teacher Behaviors
Math Content Knowledge	Use mathematically precise language Make explicit connections between or among concepts Provide rationale for why procedure works Incorporate key ideas about ratio and proportion Incorporate key ideas about fractions and decimals
Pedagogical Content	Identify and debug student misconception(s) Use (or student use of) multiple representations of concepts Make connections among student errors or approaches
Pedagogical Knowledge	State the mathematical focus or objective of lesson Encourage students to pursue multiple strategies to solve problems Ask students to justify or extend their answers or explanations Ask students to engage in each other's reasoning Provide lesson summary or closure

Pedagogical content knowledge encompasses the extent to which teachers diagnose and debug student misconceptions, use or encourage student use of multiple representations of concepts, and make connections among student approaches, including student errors. The misconceptions and representations are rooted in specific math concepts, which is why the term is pedagogical *content* rather than straight pedagogical knowledge. Pedagogical knowledge refers to general actions that can be applied to any lesson, regardless of the mathematics content that is the focus of the lesson. Stating the lesson objective, encouraging students to pursue multiple solution strategies, asking students to justify or extend their answers, asking students to engage in each other's reasoning, and providing a lesson summary statement or closure are among the general pedagogical actions emphasized in the professional development.

Criteria Used to Score Classroom Observation Protocols

To capture the extent to which these three core aspects of teachers' knowledge manifested themselves during instruction, I developed scoring criteria similar to the criteria used to score the structured content interview. To score structured content interviews, I used a four-point scale, from 0 to 3, to capture teachers' level of understanding of the content. For the observation data, I also used a four-point scale, from 0 to 3, but to capture the visibility of the professional development in teachers' instructional practices. Thus, the four levels of "understanding" on the content rubric are replaced with levels of "evidence" on the classroom observation rubric. Table 11 includes complete descriptions of the four levels used to capture visibility: no, low, moderate and high visibility in teacher actions.

Table 11
Observation Protocol Scoring Criteria

Scoring Criteria	Level of Visibility	Score
Study-emphasized material is not visible in teacher instructional practices.	No Visibility	0
Study-emphasized material is occasionally or superficially visible in teacher instructional practices.	Low Visibility	1
Study-emphasized material is visible in teacher instructional practices.	Moderate Visibility	2
Study-emphasized material is highly visible in teacher instructional practices, including exact content or pedagogical techniques modeled in the professional development.	High Visibility	3

In contrast to conducting interviews, where teachers respond to a fixed set of questions in a fixed period of time with few distractions, collecting classroom data is much more difficult and unpredictable. For example, a teacher might have planned carefully for a particular lesson only to find out that the class period has been shortened considerably for a school-wide function. Or, more commonly, a teacher might launch a lesson only to find out that the students are confused because they don't have sufficient background knowledge to comprehend the new concept. As a result, the teacher must alter or even scrap the current lesson plan and focus on review material.

Despite the inherent messiness of the classroom, classroom data are potentially very powerful because they capture actual rather than intended teaching behaviors. A teacher may provide a correct definition for fraction when interviewed but decide to use an entirely different definition with students. Or a teacher might

plan to conduct a 10 minute lesson summary segment but decide to skip it. Because these differences are important to capture, I explain how I took these complexities into account when I scored the classroom observation data.

Scoring the Observation Data

One of the challenges in scoring the level of visibility of the math content, pedagogical content and pedagogical aspects of the professional development is that each phenomenon occurs at different frequencies. Pedagogical strategies, for example, were the most straightforward teacher actions to score. In every lesson, teachers either state the lesson objective or not, probe student thinking to some degree or not, and summarize the lesson or not. The professional development offered specific questioning techniques – e.g., teachers were encouraged to use probes like “Say more” and “Explain what Johnny did in your own words” – that teachers used often. The specificity and simplicity of the techniques made them easy to detect and quantify. However, the process was much less straightforward for capturing math content and pedagogical content teaching actions. Compared with the pedagogical strategies, teacher actions that addressed the content and pedagogical content emphases of the professional development were much less visible.

Given this difference, I weighted each content and pedagogical content occurrence more heavily than the pedagogical aspects of the model. In situations where items did not occur at all – e.g., fraction and decimal content was not a focus in ratio and proportion lessons – I excluded these items from the averages within and across the three general categories. These phenomenological differences are why I interviewed teachers on the content and pedagogical aspects of the model. Whenever

possible, I used the structured content interview data to cross check the associated, but less visible, math content and pedagogical content classroom data. The descriptive analyses in the next sections further outline how I analyzed the classroom observation data.

Teachers' Scores on the Observation Protocol

I observed each teacher five times over the course of the 2007-08 school year. The first three observations occurred when each teacher was working directly with the study's instructional coach. These lessons occurred immediately following a professional development seminar in which teachers planned and/or studied content relevant to the lessons they taught the following week. Each lesson was between 80 and 90 minutes in length and included a 20-30 minute numeracy block and the 50-60 minute lesson from *CMP*. In the numeracy block, teachers reviewed a variety of topics and skills that appeared on the state assessment. In the *CMP* block, teachers worked on an investigation or set of problems in the current unit.¹⁸ In terms of time, each observation was about twice the length of an average 40-45 minute middle school lesson.

One way to examine the level of evidence of the three types of knowledge during instruction is to look at average scores for teachers across the five lessons. Table 12 shows that, overall, all three teachers demonstrated between a low and moderate level of visibility of the core components of the model during instruction.

¹⁸ Unlike traditional textbooks, which are chapter and lesson based, *CMP* is unit and investigation based. Chapters typically take 2-3 weeks to complete and contain 7-10 lessons focusing on discrete topics. Units typically take 4-5 weeks to complete and include 4-5 big investigations or problems. The investigations are linked together by a common unit theme. Chapter lessons are linked as topics to the topic of the Chapter, but traditional textbooks typically don't have a theme that holds the lessons together.

These scores represent the weighted averages of the math content, pedagogical content and pedagogical knowledge across the five lessons. For instance, Hamlin’s overall score of 1.9 represents the average of her math content (1.2), pedagogical content (2.3) and pedagogical knowledge (2.2) scores, equally weighted. Each of the three knowledge domains represents the average scores across the five lessons. Hamlin’s and Smith’s averages of 1.9 and 1.7, respectively, were the highest and closer to moderate rather than low level of visibility. Wiggins’s overall average of 1.3 was closer to the low level of visibility. These scores are consistent with the ordering from the extended content interviews.

Table 12
Level of Evidence of Teachers’ Use of Math Content, Pedagogical Content and Pedagogical Knowledge in the Classroom

Teacher	Lesson					Average
	1	2	3	4 ^b	5	
Hamlin	1.9 ^a	2.1	1.7	1.9	1.8	1.9
Smith	2.0	2.1	1.6	1.8	1.2	1.7
Wiggins	1.4	1.9	0.9	0.7	1.6	1.3
Average	1.7	2.0	1.4	1.5	1.5	1.6

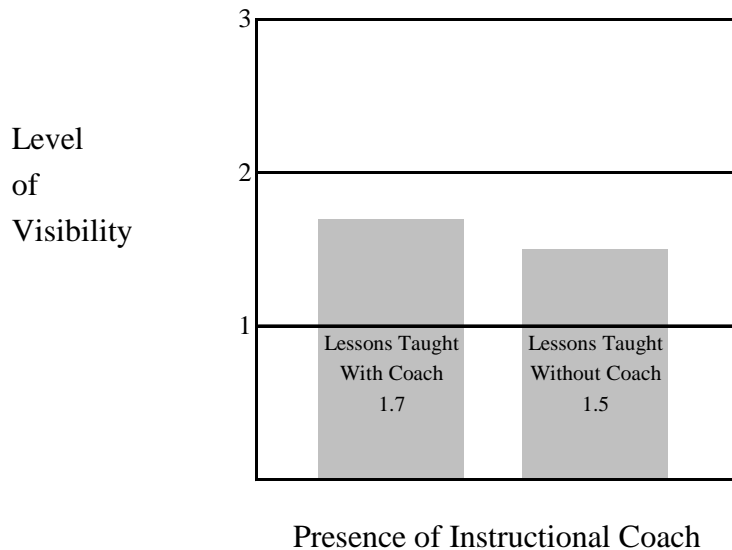
a. 1.9 represents the average level of evidence for all items in the math content, pedagogical content and pedagogical knowledge components in the protocol for the first lesson observation. This lesson was delivered in the presence of an instructional coach, where 0 = no evidence, 1 = weak level of evidence, 2 = moderate level of evidence, and 3 = strong level of evidence.

b. Columns 4 and 5 are shaded because they were observed after the professional development intervention had been withdrawn (i.e., the instructional coach was present in lessons 1-3 but not present in lessons 4 and 5).

When examining the overall averages for teachers for lessons in which they worked with an instructional coach, the averages were slightly higher when teachers were working with the coach than when they were not. The average across the three teachers for the three lessons in which they worked with an instructional coach was 1.7 versus an average of 1.5 for the lessons in which the coach was not present (see Figure 12). These averages provide an overall portrait of the extent to which the

professional development penetrated the classroom. However, analyzing the observation data by each core emphasis – math content, pedagogical content and pedagogical knowledge – and by each teacher provides a more fulsome explanation of the visibility of each component of the professional development in teachers’ instructional practices.

Figure 12. Average Level of Visibility of Professional Development in Lessons Taught With and Without an Instructional Coach



Visibility of Mathematics Content in Teachers’ Lessons

The first category on the observation protocol was the visibility of teachers’ understanding of math content in the classroom. As previously mentioned, teachers’ understanding of the content included the extent to which they used precise language and explanations, made explicit connections between concepts, provided rationales for why procedures work, and incorporated key ideas about either ratios and proportion or fractions and decimals. The visibility of teachers’ implementation of

the study-emphasized math content was notably lower than the other two dimensions, pedagogical content knowledge and pedagogical knowledge (see Table 13).

Hamlin’s, Smith’s and Wiggins’s averages for math content knowledge were lower than their averages in the other two categories, and quite low overall. As with the overall averages in the structured content interview and in the classroom observation data overall, Hamlin, Smith and Wiggins, in descending order, exhibited the highest level of visibility of the math content during instruction.

Table 13
Visibility of Teachers’ Math Content Knowledge in the Classroom

Teacher	Lesson					Average
	1	2	3	4 ^b	5	
Hamlin	1.2 ^a	1.8	1.3	1.6	1.4	1.5
Smith	1.3	1.8	0.8	1.6	0.5	1.2
Wiggins	0.8	1.5	0.5	0.5	1.0	0.9
Average	1.1	1.7	0.9	1.2	1.0	1.2

a. 1.2 represents the average level of evidence for the math content items in the protocol for the first lesson observation. This lesson was delivered in the presence of an instructional coach, where 0 = no evidence, 1 = weak level of evidence, 2 = moderate level of evidence, and 3 = strong level of evidence.

b. Columns 4 and 5 are shaded because they were observed after the professional development intervention had been withdrawn (i.e., the instructional coach was present in lessons 1-3 but not present in lessons 4 and 5).

Visibility of the Math Content in Hamlin’s Lessons

The average level of visibility of Hamlin’s understanding of the core content emphasized in the professional development was 1.5 or the low-moderate level overall. The range across the five lessons with respect to math content was 0.6 – from 1.2 to 1.8.

The lesson where I rated Hamlin 1.2, or close to the low level of visibility, featured a problem about sharing pizza. The problem asked students to decide which

group of students would get more pizza, the students sitting at a table with four pizzas and 10 students or the students sitting a table with three pizzas and eight students (all pizzas on both tables were the same size).

The preceding three seminars, including one that occurred the day before this lesson, focused on ratio and proportion concepts. As described in Chapter 4, the first day focused on ratio tables; the second day focused on strip diagrams; the third day focused on rate. Since all three of these topics were relevant to the lesson, I was looking for any evidence of this content in the lesson. For example, if the teacher referenced the definition of ratio, which gets at the idea that ratios are multiplicative rather than additive comparisons, it would be counted as evidence that the professional development content had influenced instruction. Other examples include if the teacher 1) explained that comparison between pizzas and students was a rate, the type of ratio that compares different units, 2) made a connection to unit rate, which is a rate with a denominator of one (e.g., pizzas per one person or people per one pizza), or 3) used a strip diagram to illustrate how 3:8 and 4:10 compare when there was an equal number of students (e.g., 40). Even though this problem was in a unit devoted to ratio and proportion and the recent professional development activities focused on ratio and proportion, I also looked for evidence of the study's definition of fraction, since the pizza problem could be solved with fractions.

With these content possibilities in mind, I conducted the observation and ended up jotting down many more notes of "missed opportunities" than actual instances where the content manifested itself during instruction. For example, during an extended warm up problem, which was connected to the content of the pizza

lesson, students shared publicly how they determined the cost of 30 cans of soda if six cans cost \$2.40. One student said, “You’re trying to figure out what times six to get 30.” Since Hamlin often responded to students’ explanations with a comment or illustration, I expected her to explain or push the student to think about the definition of ratio, since this student was touching on the multiplicative relationship between the cans of soda and cost, or perhaps address the meaning of rate, since cans of soda and cost represent different units. Two other possibilities would have been for Hamlin to put the student’s work into a ratio table, a representation that received considerable attention during the professional development, or to make a connection to the identity property of multiplication. However, Hamlin made no mention of any of these ideas.

Initially, I attributed the avoidance of ratio content during this discussion to the nature of the activity. It was only a “warm up” after all, so perhaps Hamlin would weave more of the math content into the full lesson, when she had more time. But these connections rarely happened, or when they did, they were thin or under developed. For example, as students were beginning to work on the pizza problem, she asked students to re-state what the problem was asking them to do. One student raised his hand and said, “ratio table.” Since the professional development emphasized ratio tables, I wondered how Hamlin would respond to this answer. Instead of pushing the student to think harder about what the lesson was about – e.g., find out which table yields the most pizza per person or something similar – she simply nodded and said “Ok.” A ratio table is a representation that can be used to help students understand ratio and proportion concepts, including the content of this lesson, but it was not the focus of the lesson. If Hamlin had pointed out this

distinction, I would have coded the interaction as evidence of the content of the professional development impacting instruction. Throughout the rest of the lesson, as students worked in groups solving the problem and as students presented their solutions toward the end of the lesson, I noted similar “missed opportunities” to make connections to the underlying math content. Hamlin had planned to conclude the lesson with a brief summary statement, which would have been a final opportunity to make connections to the content, but she ran out of time.

Hamlin’s other four lessons generally were similar to this one in terms of the visibility of math content during instruction. The one slight exception was the lesson in which Hamlin’s composite content score was 1.8, or closer to the moderate level of visibility. In this lesson, Hamlin made more explicit connections to the underlying content during whole class discussions. For example, when students were sharing their solutions to a problem that involved the ratio of boys to girls, one student said that he multiplied the girls and boys by 40 to arrive at the correct answer. Instead of acknowledging the correct answer and moving on, Hamlin saw the student’s correct response as an opportunity to make sure students understood the identity property of multiplication. The problem asked students to determine the number of boys in the school if the ratio of girls to students was 3 to 7 and there were 280 students in the school. The student wrote the following equation on the board: $\frac{4}{7}x \frac{40}{40} = \frac{160}{280}$.

Hamlin said,

But why does this work? ... What do you notice about what I’m multiplying by? What do we know about the value of $\frac{40}{40}$? (Students, in unison, respond “One.”) When you multiply by one, you get the same value.

Because Hamlin made this link and some other connections to the math content during the lesson, she scored closer to the moderate level of visibility. However, like I did during the other lessons, I recorded in my field notes many missed opportunities or weak connections to the study content, which the lesson with the highest visibility of the content still fell below the moderate level.

Visibility of the Math Content in Smith's and Wiggins's Lessons

The visibility of the math content across Smith's and Wiggins's lessons was close to the "weak" level. Smith's overall average was 1.2 and Hamlin's 0.9. The range across Smith's five lessons was 1.1 – from 1.6 to 0.5 – while the range across Hamlin's lessons was 1.0 – from 1.5 to 0.5.

In the lessons where the visibility of the content was lowest, at the 0.5 level – Smith had one lesson at this level and Wiggins had two lessons at this level – connections to the core content were extremely rare. My field notes for all three of these lessons mostly describe missed opportunities for making connections to the study content. For example, in Smith's lesson at the 0.5 level, students solved an extended warm up problem in which they were asked a series of questions about the Exxon Valdez oil spill. Part of the problem required students to estimate the total number of gallons of oil that were lost from the tanker based on the total capacity of the tanker. Smith could have explained to students that the problem involved a rate, the amount of oil lost over a given amount of time, and reinforced students' understanding that rates are types of ratios that compare different units. Or, he could have made a connection to the meaning of fraction when students were comparing the

gallons of oil spilled with the total gallons of oil in the tanker. However, he made no such connections.

Both of Wiggins's lessons at this level contained similar missed opportunities. One of the lessons included an extended problem where students had to write and manipulate an equation related to a CD club. In order to join the club, students had to pay \$30 for a membership and then \$15 per month. Students solved several problems where they were given the number of months and had to find the total cost or where they were given total cost and had to find the number of months. Initially, I thought Wiggins was going to make a strong connection to rate, which is a core part of the problem: \$15 per month is an example of a unit rate. She asked the class, "Is there a constant rate of change in this problem?" One student raised his hand and said, "15m or 15 times m." While the student had given the correct answer, Wiggins didn't elaborate on why the student's correct response illustrated a constant rate. She might have questioned the student further or said something like the following:

Right. We know that a rate is a type of ratio that compares two different units. What are the two different units in this problem? The amount of money (dollars) and time (months). And what is constant about this situation? For every month, you have to pay \$15. It's a constant rate.

Instead of responding in this or a similar way, she quickly moved to the next part of the lesson. This lesson also illustrated Wiggins's use of informal rather than precise mathematics terminology. When Wiggins was helping students write an equation when they were given the total cost of the CD, she said, "if we mush all of this together, we can get an equation." By "mushing" she meant combining the constant (\$30) and the rate (\$15/month) and setting it equal to the total cost (\$195). This

description is an example of a missed opportunity to use more precise, and conceptually relevant mathematical language.

In the lessons where the visibility of the math content was higher – Smith’s highest score was 1.6 and Wiggins’s highest score was 1.5 – both teachers made some connections to the study content. One of the two lessons where Smith scored at the 1.6 level included the previously described pizza problem, where Wiggins scored at the 1.8 level. Smith had his students solve the problem in small groups. When Smith was circulating the classroom monitoring the small group work, he encountered a group that was completely stuck. To try and help them get unstuck, he said, “Let’s think about what a ratio is.” He then used an example like Hamlin had used in her warm up problem, where she asked students to estimate the number of girls in the school when the ratio of boys to girls and the total number of students were given. With this example, he was trying to get students to see that both situations had multiplicative relationships: pizzas per person (or people per pizza) and girls to boys. He asked the students, “What is the ratio here? What is remaining constant?” By asking students to think about what is constant, Smith was trying to get students to see that ratios were multiplicative rather than additive comparisons. If Smith had extended this conversation and explained to students how the constant comparison represented a multiplicative relationship, the visibility of the math content would have been higher. Instead, Smith moved students in the right direction but didn’t fully articulate the core content.

Wiggins’s lesson with the highest visibility of the content – 1.5 – included a problem similar to the previously discussed CD club problem. Though the visibility

of the content still fell between low and moderate, Wiggins did touch on two aspects of the content emphasized in the professional development: 1) distinguishing between the two types of division, i.e., measurement (how many groups?) versus partitive (how many in each group?) division, and 2) understanding that multiplication and division are inversely related, i.e., every multiplication problem can be rewritten as two, related division problems. For example, she used the equation $4 \times 5 = 20$ and the two related division problems $20 \div 5 = 4$ and $20 \div 4 = 5$ to show students how multiplication and division were inversely related. The teachers completed a professional development activity with this emphasis. She also discussed with students how division could be interpreted as “how many groups” of something, which was addressed in the professional development. These interactions elevated the visibility of the math content in Wiggins’s lessons to the low to moderate level.

Visibility of Pedagogical Content in Teachers’ Lessons

Compared with the math content emphasized in the professional development, the overall visibility of teachers’ pedagogical content knowledge was notably higher for all three teachers. Pedagogical content knowledge refers to the extent to which teachers are able to identify and debug student misconceptions, use multiple representations of concepts, and make connections among student errors or approaches. Table 14 displays the visibility of pedagogical content knowledge for all three teachers across the five lessons.

Unlike the visibility of the math content, all three teachers taught lessons that exceeded the moderate level of visibility. The overall average visibility of the pedagogical content during instruction was 1.9, or virtually the moderate level of

visibility. As with the math content, in order to get a clearer sense of what these averages mean. I analyze some examples from each teacher's lessons.

Table 14
Visibility of Teachers' Pedagogical Content Knowledge in the Classroom

Teacher	Lesson					Average
	1	2	3	4 ^b	5	
Hamlin	2.3 ^a	2.3	2.3	2.3	2.3	2.3
Smith	2.0	2.3	1.7	1.7	1.3	1.8
Wiggins	1.7	1.7	1.0	0.7	2.3	1.5
Average	1.9	2.1	1.7	1.6	2.0	1.9

a. 2.3 represents the average level of evidence for the pedagogical content items in the protocol for the first lesson observation. This lesson was delivered in the presence of an instructional coach, where 0 = no evidence, 1 = weak level of evidence, 2 = moderate level of evidence, and 3 = strong level of evidence.

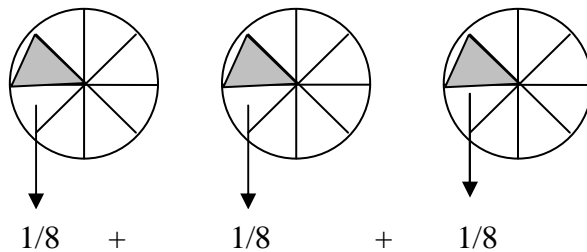
b. Columns 4 and 5 are shaded because they were observed after the professional development intervention had been withdrawn (i.e., the instructional coach was present in lessons 1-3 but not present in lessons 4 and 5).

Visibility of Pedagogical Content in Hamlin's Lessons

The visibility of the pedagogical content elements in Hamlin's lessons was remarkably consistent: she scored at the 2.3 level in each of the five lessons I observed.¹⁹ Unlike Hamlin's score on the math content, where I coded numerous missed and under-utilized opportunities to make connections to the study content, Hamlin consistently used multiple representations of concepts and often made connections among student approaches. She occasionally identified and debugged student misconceptions; if I had observed more instances in which she dealt with student misconceptions, the level of visibility would have been closer to strong than moderate.

¹⁹ The pedagogical content average of 2.3 is based on averaging the scores from the three sub-domains: teachers' (1) ability to identify and debug student misconceptions, (2) use of multiple representations, and (3) ability to make connections among student errors or approaches. Though Hamlin received a score of 2.3 on each lesson, the identical averages do not indicate identical scores on the sub-domains.

To illustrate Hamlin's consistent use of multiple representations, consider the previously described pizza problem, which Hamlin taught twice. In both lessons, she encouraged students to solve the problem using the approach that made the most sense and then required students to explain their thinking. Students used a variety of approaches and associated representations, which Hamlin had groups of students share with the rest of the class. For example, one group of students put the following representation on the board:



The student reporting for the group said that the picture showed that each of the eight people sitting at the table with three pizzas would get one piece from each pizza. Since each piece represented $\frac{1}{8}$ of one pizza, each student sitting at the table with three pizzas would get $\frac{3}{8}$ of one pizza. The student then said that the same illustration could be used for the table with four pizzas and ten students, which would show that each student would get one slice sized $\frac{1}{10}$ from each of the four pizzas, or $\frac{4}{10}$ of a pizza overall.

Another group used percents to solve the problem. They converted the fractions $\frac{3}{8}$ and $\frac{4}{10}$ into 37.5 and 40 percent, respectively. Though they didn't draw a picture or use a model to illustrate the concept of percent, they did explain why their numeric approach was valid. Another student wrote her two ratio tables on

her paper, which the teacher acknowledged but did not have the student present publicly. The first table contained the number of pizzas and people at the small table:

Pizzas	3	6	9	12	15
People	8	16	24	32	40

The second table contained the number of pizzas and people at the large table:

Pizzas	4	8	12	16
People	10	20	30	40

I assumed the student was trying to illustrate that when the number of pizzas is held constant – at 12 pizzas in this case – fewer people share the pizzas in the large table (30) compared to the small table (32). Thus, the students at the large table would each get a little bit more pizza. However, the student then created two ratios from the tables, $30/40$ and $32/40$. It wasn't clear to me (or the teacher) what these ratios were representing. Yet, since Hamlin encouraged the use of a table, I scored the episode as a case where the use of multiple representations was highly visible.

These sorts of episodes, where students were encouraged to solve problems in ways that made the most sense to them, were common in Hamlin's lessons. Hamlin also regularly encouraged students to make connections to each other's work. She made these connections happen in two ways: by organizing students into small groups and having students share their approaches with other group members and by requiring a sample of students to share their approaches publicly, often at the end of a

lesson. In both of these cases, I frequently coded Hamlin pushing students to think about their work and connect their approach to previous content or other student approaches. For example, one student said his group divided 4 by 10 and got 40% of one pizza. Hamlin encouraged the group to consider an alternative way to verify that their answer was correct. In another lesson, during an extended warm up problem where kids were matching an equation with a table and a graph, Hamlin asked different groups to share their answers. During the discussion, she said, “Can someone restate what he just said? Who can tell us which graph works?” These sorts of interchanges, where Hamlin called on a student or a group of students to respond to another group’s approach were quite common.

In contrast to lesson episodes in with a relatively high visibility of multiple representations and connections among student approaches, lesson episodes in which Hamlin both identified and debugged specific student misconceptions were uncommon. In all five of the lessons I observed, Hamlin rarely articulated specific student misconceptions and even more rarely explained the underlying source of the particular misconception. It is quite possible, perhaps even quite likely, that Hamlin identified more student misconceptions than I was able to code, since she spent a lot of time circulating among groups and encouraging them to think about their work. But since I coded few interchanges where she identified particular misconceptions to small groups of students or to the whole class, I rated these lessons at the no or low visibility level. These lower scores on this dimension of pedagogical content knowledge were why Hamlin’s overall pedagogical content score was closer to the moderate than high visibility level.

Visibility of Pedagogical Content in Smith's Lessons

Like Hamlin, Smith scored highest on the pedagogical content knowledge component for a single lesson was 2.3. Unlike Hamlin, who reached this level in all five lessons I observed, Smith reached this level only once. Smith's other lessons ranged from 1.3 to 2.0, yielding an overall average of 1.8, or slightly below the moderate level of visibility.

In the lesson where Smith reached the 2.3 level, he encouraged students to use multiple approaches to solve the previously described CD club problem and he made connections among various student approaches. Recall that the CD problem asked students several questions about the club if the total cost of membership included a \$30 enrollment fee and \$15 per month thereafter. In Smith's lesson, students used three primary approaches to solve the problem: guess and check, division, and a table. Smith either posted or had students post examples of each approach and then led a discussion about each one. The students who used the guess and check approach inserted various guesses for the number of months enrolled in the club into equation $\$195 - \$30 = 15n$, where n equaled the number of months enrolled in the club. They eventually arrived at the correct answer, 11, which was the point at which the total cost of the club – joining fee plus monthly membership – equaled \$165. Smith displayed this approach to the rest of the class.

The students who used division first subtracted the enrollment fee of \$30 and then divided \$165 by 15 to get the number of months enrolled in the club. In response to this approach, Smith wrote the following on the board:

$$11 \times 15 = 165$$

↓
“groups of”

He then responded to the class,

What does this represent? Does it mean how many 15s are in 165? [Then he turned to specific students, who had also used division] Christy, do you agree and why? Kim and David, does this make sense to you?

Though a complete representation of the measurement or “how many groups” interpretation of division might have included a picture,²⁰ Smith did illustrate with an arrow which number in the problem could be thought of as “the number of groups.” He also made an effort to connect several student approaches during this episode. Connecting multiple student approaches illustrates another aspect of pedagogical content knowledge.

The third group of students created a two-column table, with one column for the number of months enrolled in the club and the second column for the total cost of club membership. Mr. Smith had one of the students write the table on the board:

Months	0	1	2	3	4	5	6
Cost	30	45	60	75	90	105	120

The student then explained that with each additional month, they added \$15 until they got to \$165. Even though their table did not extend all the way to \$165, the student said that their group used the pattern to arrive at the correct answer of 11 months.

²⁰ In the professional development, the teachers discussed partitive and measurement division using pictures such as these ($6 \div 3 = 2$): $\begin{array}{c} \square \square \square \\ \square \square \square \end{array}$ versus $\begin{array}{c} \square | \square | \square \\ \square | \square | \square \end{array}$

The first picture shows measurement division or “how many groups of three?” The second picture shows partitive division or “how many in each of three equal groups?”

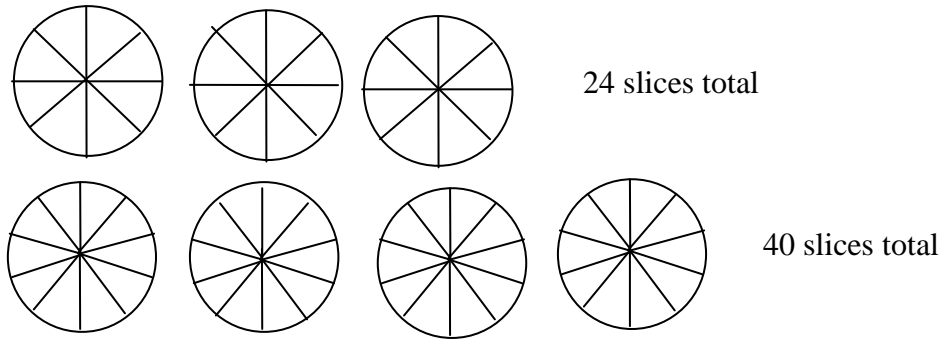
Smith asked this group where the down payment appeared in their table and one member of the group said that the first column represented the \$30 down payment.

Though the level of visibility of the use of multiple representations and connections to student approaches was high in this lesson, the visibility of identifying and debugging student misconceptions was low in this lesson, which is why Smith's overall average was closer to the moderate than high level. Smith's other lessons varied in the extent to which the pedagogical content emphases of the professional development were visible. In general, however, the use of multiple representations and connections among student approaches was more visible than the identification of particular student misconceptions.

Visibility of Pedagogical Content in Wiggins's Lessons

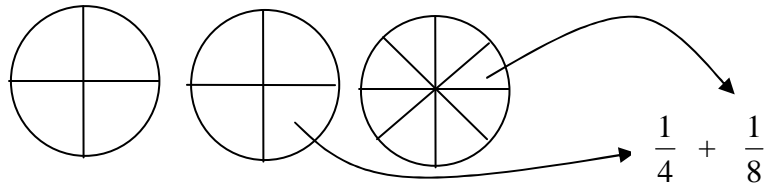
Like Hamlin and Smith, the highest level of visibility of the pedagogical content in Wiggins's lessons was 2.3. She reached this level once, in the last lesson I observed. Wiggins's other lessons ranged from 0.7 to 1.7. Her overall average was 1.5, or at the low moderate level of visibility.

The lesson where Wiggins scored at the 2.3 level was the previously described pizza problem. She included many of the same representations that Hamlin did when she taught the same problem. For example, one of the groups divided the pizzas into tenths and eighths:



They explained that a student sitting at the table with four pizzas would get more pizza than a student sitting at the table with three pizzas because of the large table had 40 slices and the small table only had 24 slices. This representation allowed Wiggins to address the group’s misconception with a question: “But what about the size of each piece?” She could have asked the group to think about the number of people per table, but the prompt got students to think about how they arrived at their answer.

Two other groups used the same representation but they divided the pizzas into different sized pieces. One of the groups divided all seven pizzas – the three at the small table and the four at the large table – into quarters. The student representing the group said that all students would get the same amount – one quarter of a pizza – even though both tables had leftover quarters of pizza. Wiggins used their picture in her follow up question, in which she asked students to address how the leftover pieces should be distributed. The other group divided two of the three pizzas at the small table into quarters and the third pizza into eighths. The group spokesperson said that every student at this table would get a quarter and an eighth:



The group divided each of the four pizzas at the large table into fifths and said that each person would get two-fifths or 40% of one pizza.

Another group converted the pizzas and people at each table into unit rates.

They found that the small table had $2.\bar{6}$ people per pizza and the large table 2.5 people per pizza. Yet another group used common denominators to create two fractions,

$\frac{30}{80}$ and $\frac{32}{80}$, in which the first fraction represented the small table and the second

fraction the large table. Since Wiggins required all groups to explain their thinking and show their work publicly, the use of multiple representations was highly visible in this lesson.

Like Hamlin and Smith, Wiggins was less likely to identify and debug specific student misconceptions than present multiple representations and make connections among student approaches. However, in the two lessons where the visibility of pedagogical content knowledge was very low, I recorded few instances in which multiple representations or connections among student errors or approaches occurred.

Visibility of Pedagogical Knowledge in Teachers' Lessons

Like the visibility of pedagogical content emphasized in the professional development, the visibility of the pedagogical strategies during instruction was

noticeably higher than content. Pedagogical knowledge refers to the extent to which teachers state the mathematics focus or objective of the lesson, encourage students to pursue multiple solution strategies, ask students to justify or extend their answers, ask students to engage in each other’s reasoning, and provide lesson summary or closure. Table 15 displays the visibility of pedagogical knowledge for all three teachers across the five lessons.

Table 15
Visibility of Teachers’ Pedagogical Knowledge in the Classroom

Teacher	Lesson					Average
	1	2	3	4 ^b	5	
Hamlin	2.2	2.2	1.4	1.8	1.6	2.0
Smith	2.6	2.2	2.3	2.0	1.8	2.2
Wiggins	1.8	2.4	1.2	1.0	1.4	1.6
Average	2.2	2.3	1.6	1.6	1.6	1.9

a. 2.3 represents the average level of evidence for the pedagogical content items in the protocol for the first lesson observation. This lesson was delivered in the presence of an instructional coach, where 0 = no evidence, 1 = weak level of evidence, 2 = moderate level of evidence, and 3 = strong level of evidence.

b. Columns 4 and 5 are shaded because they were observed after the professional development intervention had been withdrawn (i.e., the instructional coach was present in lessons 1-3 but not present in lessons 4 and 5).

The overall average visibility of the pedagogical strategies was 1.9, or close to the moderate level. This number is equal to the overall average level of visibility for the pedagogical content knowledge component. Smith’s and Wiggins’s averages of 2.2 and 1.6, respectively, were their individual highest levels of visibility for the math content, pedagogical content and pedagogical components of the observation protocol. Hamlin’s average of 2.0 was lower than her 2.3 pedagogical content average but higher than her 1.5 math content average. The pedagogical knowledge dimension is the first time in which Hamlin did not have the highest overall average; Smith’s average of 2.2 was the higher than Hamlin’s 2.0. As with the other two

dimensions, it is useful to look at specific examples from lessons to understand more about what these averages mean.

Visibility of Pedagogical Strategies in Hamlin's Lessons

Among the pedagogical strategies visible in Hamlin's lessons, encouraging students to pursue multiple solution strategies and justify or extend their reasoning were highly visible in all lessons. In each lesson I observed, Hamlin encouraged the use of multiple solution strategies and asked probing questions throughout each lesson. I routinely coded questions and probes such as "Say more about that," "Tell me why," and "What does your answer mean?" She was relentless, at times, with her questioning of individual students who were struggling with the material. During these episodes, it became impossible to write down all the questions she asked because of how quickly they were asked. A conservative estimate of the average number of probing questions Hamlin asked per lesson is 50.

Hamlin also encouraged students to engage in each other's reasoning, but these episodes were much rarer than instances when Hamlin directly engaged in students' reasoning. She stated the mathematical focus of the lesson regularly – she did this in four of the five lessons I observed – but the quality and clarity of the lesson objective varied somewhat. What is most striking, however, about Hamlin's use of the pedagogical strategies emphasized in the professional development is that she never provided a summary statement at the end of any lesson. Even when Hamlin and the instructional coach planned specifically for the lesson summary or closure piece of a lesson – they did this twice – Hamlin never reached that point of the lesson. The videotape component of the Third International Math and Science Study suggests

that Hamlin's avoidance of closure is common among U.S. teachers. Compared to their Japanese counterparts, U.S. 8th grade teachers rarely provided lesson closure.

One hypothesis for why Hamlin avoided lesson closure is that summarizing the lesson could be considered one of the more difficult tasks in teaching, particularly in lessons in which students use multiple approaches to solve a problem.

Summarizing these types of lessons is difficult because the teacher must be mindful of various student approaches and misconceptions as they relate to the core mathematics of the lesson. Though a lesson summary is a general pedagogical technique, it is also heavily dependent upon the content for these reasons. If lesson closure is treated as a content activity and removed from the pedagogical component, Hamlin's pedagogical score jumps from 2.0 to 2.4, or between the moderate to high level of visibility.

Visibility of Pedagogical Strategies in Smith's Lessons

Smith had the highest overall level of visibility of the pedagogical components, 2.2, or above the moderate level. Unlike Hamlin, who never reached closure in any of the lessons I observed, Smith summarized three of the five lessons I observed. He also consistently encouraged students to pursue multiple solution strategies and routinely asked students to justify their thinking.

Smith commonly used the questioning techniques and prompts emphasized in the professional development, such as "Say more about that," "Can you restate [Johnny's] answer in your own words," and "Say why." For at least part of all the lessons I observed, he circulated the room and pushed individual students and small groups of students to justify their thinking. He didn't ask students questions as

rapidly as Hamlin, but he did consistently ask students to justify their thinking. For example, when Smith taught the pizza problem, one of the groups of students got stuck because they divided all pizzas at both tables into tenths, instead of dividing the pizzas at the table with 10 people into tenths and dividing the pizzas at the table with 8 pizzas into eighths. They were able to show that students at the large table would each get $\frac{4}{10}$ of a pizza, but when they started numbering off the slices at the small table from 1 to 10, they realized that it didn't come out evenly. Smith responded with the following:

What if they bring out this other pizza and split it into 8ths? Would that be possible? What would you then do with $\frac{4}{10}$ and $\frac{3}{8}$? How do they compare?

These sorts of interchanges were common in Smith's lessons. He was comfortable asking students to explain their thinking, and the pedagogical techniques emphasized in the professional development seemed to complement the questioning techniques he ordinarily used with students.

Visibility of Pedagogical Strategies in Wiggins's Lessons

Compared with the visibility of the math content and pedagogical content aspects of the professional development model in Wiggins's lessons, the visibility of the pedagogical strategies were more visible. Her overall average was 1.6 or between the low and moderate levels of visibility, but she scored as high as 2.4 on a single lesson. Like Hamlin and Smith, she regularly asked students to use multiple approaches and somewhat consistently asked students to justify their thinking and explain their work.

The lesson where Wiggins scored a 2.4 featured the CD club problem. She managed to elicit seven different approaches to the problem from students and small groups of students. As she circulated the classroom monitoring student work, she routinely asked students to “Say why,” and show their work. When two students were confused about how one of the groups got 11 months from their table, she said, “How are your table and their approach similar? When you were in elementary school, didn’t they say that multiplication was repeated addition?” These sorts of interchanges were common in this lesson, where Wiggins asked students to engage in each other’s thinking. She also delivered a brief lesson summary statement, after students had presented the seven ways they had come up with to solve the problem.

In many of the other lessons, however, the pedagogical elements of the professional development were less visible. Part of the reason that the overall visibility was lower was due to the absence of closure or lesson summary statements. The only lesson that Wiggins closed was the previously mentioned CD problem. Another reason was that Wiggins only sporadically encouraged students to pursue multiple approaches to solutions. Another explanation could be the presence of the instructional coach. Both Smith and Wiggins had higher pedagogical scores during lessons in which they worked with the study coach.

Level of Student Engagement during Q and A Episodes

Though my primary focus during the classroom observations was teachers’ use of the information and strategies emphasized in the professional development, I also kept track of the level of student engagement during each lesson. At the end of each lesson, I indicated whether less than 25%, 25%, 50% or 75% or more of students

were engaged for most of the lesson (see Appendix A for complete description). During the question and answer episodes, in which all three teachers employed the pedagogical techniques emphasized in the professional development, I noticed that it was difficult for teachers to keep the whole class engaged. For example, Hamlin's use of questioning with an individual student or small group of students was relentless at times. She would push a student or small group of students to explain their thinking for several minutes on many occasions. However, while Hamlin was focused on getting these students to think harder about their approach, she was not able to keep track with what other students were doing. Many of the students were off task or waiting for Hamlin to make it around to their group. I coded the overwhelming majority of these episodes as only 25% or 50% of the class being engaged. I noticed a similar pattern in Smith's and Wiggins's classroom. Both used questioning strategies but were only able to focus on the student or small group of students being interrogated.

The professional development intervention focused primarily on teachers' use of questioning techniques that were targeted to an individual student or small group of students, not on techniques that could be used to keep the rest of the class engaged while they interrogated a few students. Thus, the primary emphasis of this section is on the extent to which teachers used those pedagogical strategies during instruction. However, I raise this issue because I think it is important to portray both the behaviors of the teacher and student(s) who are at the center of the question and answer episode as well as the behaviors of the rest of the students in the class, since they are affected by how and with whom teachers target instruction.

Sources of Variation among Teachers' Responses

The previous discussion included many examples in which teachers varied in the extent to which they integrated into their instructional routines the core content, pedagogical content, and pedagogical components of the professional development model. This section examines and is organized around two general types of variation: variation across the different components of the professional development model, and variation among teachers.

Variation Across Different Components of the Model

The preceding sections illustrated that the pedagogical content and pedagogical elements of the professional development were more visible than the math content elements in teachers' instructional practices. Three possible hypotheses for this variation include a) the complexity of the content – e.g., the math content is more difficult for teachers to articulate than a brief pedagogical technique, b) the nature of the phenomena – e.g., teachers have fewer opportunities to demonstrate their understanding of the content than a questioning technique, and c) whether or not the instructional coach was involved in the planning and delivery of the lesson.

Complexity of the content. The analyses in Chapter 5 showed that all three teachers struggled to comprehend aspects of the math content emphasized in the professional development. In particular, they had trouble articulating ratio and proportion content as well as making connections across rational number topics. Given that at least three of each teacher's five lessons focused on ratio and proportion content, it is not surprising that very few teaching episodes included a correct definition or a succinct explanation about these concepts. Many of the lessons also

included extended problems that could be solved a number of different ways and could use a number of different rational number concepts. For example, the pizza problem, which all the teachers taught at least once, could be solved with fractions, decimals, percents or ratios. None of the teachers made strong connections across these rational number topics when they taught this problem, perhaps because they lacked sufficient knowledge to articulate these connections.

Compared with the mathematics content emphasized in the professional development, the pedagogical strategies were less complex and therefore easier for teachers to digest. Chapter 5 included study definitions of fraction, decimal and ratio that incorporated algebraic notation and precise language. Most of these definitions were new to teachers and they had trouble articulating them. In contrast, the pedagogical strategies and discussion techniques were often short sayings, such as “Say more about that,” or “Tell me why,” or “Tell me what [Johnny] said in your own words.” All three teachers used these sayings frequently in all the lessons I observed, at least in part because they were easy to remember and they supported the type of instruction these teachers typically delivered.

The apparent higher complexity of the math content is also revealed when examining the individual teaching behaviors that comprise the pedagogical content and pedagogical components of the professional development. Some of the teaching behaviors in each component are more heavily dependent on the content than others. For example, in the pedagogical content strand, teachers generally encouraged students to take multiple approaches and discussed multiple representations of concepts. Though these teaching behaviors are connected to the math content of each

lesson, identifying and debugging specific student misconceptions require a stronger level of understanding of the content. The observation data support this assumption: the level of visibility of teachers debugging students' misconceptions was notably lower than the other teaching behaviors that comprised the pedagogical content component of the observation protocol.

Within the pedagogical component of the model, the lesson summary or closure piece is heavily dependent on teachers' understanding of the core math goal of the lesson and their ability to synthesize various student approaches and misconceptions related to the problem. Compared with the other pedagogical teaching behaviors, lesson summaries were extremely rare. Hamlin never summarized a lesson and Wiggins only summarized one lesson. Smith reached closure in three of the five lessons I observed. One possible explanation for why Smith reached closure is that he appeared to be the most structured and systematic of the three teachers in his lesson planning and delivery. In fact, the coach commented that Smith's lesson organization skills were advanced in comparison to the other two teachers. Running out of time is a plausible explanation for why teachers did not typically summarize lessons, but it is also possible that they avoided closure because it was difficult. Hamlin illustrated this point during the episode described in Chapter 5, in which she asked the coach for specific guidance in articulating student misconceptions toward the end of pizza problem lesson. Even though Hamlin had the strongest level of knowledge among the three teachers, she still found the summary part of the lesson difficult, at least for this problem.

CMP lesson structure. The district text, *CMP*, employs a three-part lesson structure: launch, explore and summarize. The launch is the brief lesson opening in which the teacher describes the context of the problem to be solved. The explore portion of the lesson is an extended period of time in which students work on an investigation or a series of problems independently or in small groups. The teacher's role during the explore period is to monitor student progress by asking questions and/or by clarifying parts of the task. The teacher is not supposed to do the work for the students; the program is based on giving students a substantial amount of time to work on intellectually challenging problems. The summarize portion of the lesson, like the launch, is much shorter than the explore time. Teachers are expected to make connections among student approaches and articulate the core content of the lesson.

These characteristics of the *CMP* lesson structure impact the extent to which the math content, pedagogical content and pedagogical components of the professional development are visible during instruction. The pedagogical techniques and questioning strategies, such as "Say more about that," or "Say why," are not only less complex than the math content, but also more applicable to the longest segment of each lesson, the explore period. Teachers were much more likely to use these pedagogical prompts as they circulated among groups of students than to provide succinct explanations or refer to study definitions. Thus, the frequent coding of these questions was partially due to the fact that students spent a lot of time in activities where these techniques were applicable.

Similar to the pedagogical techniques, the pedagogical content elements of the professional development model – using multiple representations, connecting student

approaches, etc. – also fit well with *CMP* because many of the tasks can be solved in multiple ways and students are expected to share their thinking. In contrast to the brief summarize portion of the lesson, in which teachers state the core mathematics content of the lesson as it relates to student approaches, other, longer portions of the lessons provided teachers regular opportunities to demonstrate aspects of pedagogical content knowledge. In addition, the summarize portion of the lesson was often skipped, so opportunities for teachers to demonstrate their understanding of the core math content during instruction were reduced. These aspects of the *CMP* lesson structure are another source of variation in the visibility of the math content, pedagogical content and pedagogical components of the professional development.

Presence of an instructional coach. Three of the five lessons I observed for each teacher occurred when the teacher was under the guidance of the study-appointed instructional coach. I conducted the last two observations for each teacher approximately 10 weeks after the coaching component was completed. In general, the presence of the coach did not appear to affect the visibility of the math content, pedagogical content and pedagogical elements of the professional development in teachers' lessons. Hamlin's and Wiggins's overall averages of 1.9 and 1.3, respectively, were essentially the same for the coach and non-coach lessons. Smith's coach and non-coach averages were somewhat different, however. Smith averaged 1.9 or essentially the moderate level of visibility in the lessons where the coach was present but averaged 1.5 or between the low and moderate levels of visibility in the two lessons without the coach. It is possible that the presence of the coach

contributed to the higher levels of visibility for Smith, though it is difficult to know for sure.

Variation Among Teachers

The previous section outlined ways in which differences among the components of the model might have contributed to the variation in the visibility of the professional development during instruction. This section examines potential sources of variation among the three teachers, such as teachers' knowledge and their beliefs about mathematics teaching and learning.

Teachers' knowledge. Though all of the teachers' average scores were below the moderate level of understanding on the structured content interview, Hamlin's, Smith's and Wiggins's scores on this instrument varied. Hamlin's overall average of 1.9 was notably higher than Smith's 1.5 and Wiggins's 1.3, but it is difficult to distinguish between Smith's and Wiggins's level of understanding. The visibility of the study-emphasized content during instruction followed the same pattern as the structured content interview: Hamlin scored highest (1.5), followed by Smith (1.2) and Wiggins (0.9). Thus, content knowledge is one potential source of variation among teachers' scores on the observation protocol. Hamlin demonstrated the highest level of understanding on the structured content interview and therefore probably had more information to draw upon during instruction. Hamlin also had the highest average level of pedagogical content knowledge (2.3) on the classroom observation protocol. Since the structured content interview contain parts of questions that asked teachers to describe the most appropriate representations of

concepts, it is possible that knowledge affected the level of visibility of pedagogical content during instruction.

Teachers' beliefs. Two other potential sources of variation are teachers' attitudes toward learning mathematics content and promoting student understanding and teachers' level of agreement with the instructional practices espoused by the professional development. The first potential source of variation, teachers' attitudes and beliefs about learning, might have contributed to Hamlin's higher scores on the observation protocol. As discussed in Chapter 5, Hamlin displayed an eagerness to learn that was far more intensive than the other two teachers. Not only did she display this fervor for learning during the workshops – recall the episode in which Hamlin had the right answer but was unsatisfied because neither she (nor the coach) knew why her answer was correct – she displayed it in the classroom. On more than a few occasions, Hamlin expressed frustration over how little her students appeared to be learning. During one lesson, she asked the coach for direct feedback – “What am I doing wrong?” After another lesson, she spent time doing extra research on how students learn. She read a journal article about Piaget that night and reported back to me the following morning about what she had learned.

It is possible that these attitudes and behaviors increased the visibility of the core content in Hamlin's lessons. Though all three teachers employed the pedagogical techniques, such as asking students to explain their thinking, Hamlin's level of questioning was more rapid and persistent than the other two teachers. This behavior is consistent with the tenacity she approached learning the content. She also was the only teacher of the three who said that she did extensive reading about

rational numbers outside the professional development. She said that she consistently read excerpts from John Van de Walle's (2007) book *Elementary and Middle School Mathematics: Teaching Developmentally*. The professional development provider gave all teachers a copy of the book and had teachers read short excerpts for homework assignments during the professional development. Hamlin said that she used the book as a resource that went well beyond the brief homework assignments. Thus, Hamlin's eagerness to learn may have contributed to the knowledge she obtained, which may have contributed to the increased visibility of certain elements of the professional development during instruction.

A second type of belief or attitude is teachers' appraisal of the district text (*CMP*) and its associated instructional practices. All three teachers expressed concerns about the *CMP* curriculum being right for their students. Hamlin said that *CMP* and the district's vision for teaching and learning focused too much on conceptual understanding and not enough on procedural fluency:

I think there needs to be a balance...between conceptual development and procedural fluency...the [balance] has tipped in favor of conceptual understanding...our kids don't know their multiplication tables; they don't know the meaning of division even though they've been in conceptual things forever.

However, she still thinks conceptual understanding is important and said that *CMP* contained "very rich problems that get kids to think about the mathematics." Smith, too, had concerns about the *CMP* approach. He said that *CMP* "is not a great fit," because students do not have the skills to solve the problems successfully. However, Smith said that the professional development was helpful because it described the most essential math content in the rational number *CMP* units and because the talk

strategies – “Say more,” “Say why,” etc. – supported the pedagogical techniques promoted in *CMP*. Wiggins echoed Hamlin’s and Smith’s concerns about the lack of skill practice in *CMP*:

I think there needs to be a mix of the two [concepts and skills]. I think [students] need that conceptual to fall back on but I think *CMP* is very much lacking in time to practice and perfect the skills...the kids don’t ever have time to put it in their brain.

But like Hamlin, Wiggins thought that the *CMP* problems were good and that a total skill-based problem would be bad for her students, even though the majority lacked basic skills.

The consistency of these beliefs – that *CMP* contained good conceptual problems but not enough attention to skills – is perhaps why none of the teachers delivered what I would refer to as a complete *CMP* lesson. By a complete lesson, I mean that the teacher knows what the core math goals of the lesson are, keeps track of the extent to which students understand the goals, and succinctly summarizes the key student approaches as they relate to the core math goals. All three of the teachers, though somewhat less in Hamlin’s case, supplemented *CMP* with skill practice during the first 20 minutes of each lesson. If they had determined that this practice was unnecessary, they perhaps could have used the time to ensure that each *CMP* lesson had an adequate summarize section. However, it is also quite possible that teachers would have used the extra time to extend the warm up or explore sections of the lesson.

Conclusion

This chapter indicated that the average visibility of the core professional development content fell between the low and moderate levels for all three teachers. Among the three primary components of the intervention, the visibility of the pedagogical content and pedagogical aspects were more visible during instruction than the math content. Within the pedagogical content component, teachers' use of multiple representations of concepts was highly visible compared with the other pedagogical content items. Within the pedagogical component, teachers' encouragement of students to pursue multiple solution strategies and to justify or extend their answers was highly visible, especially in comparison to teachers' summarizing or closing lessons.

Among the potential sources of variation in the visibility of the professional development during instruction, the complexity of the professional development component, the nature of the phenomena in the *CMP* curriculum, and teachers' knowledge and beliefs appeared to be the most significant factors. The next and final chapter synthesizes findings from Chapters 5 and 6 and discusses implications for future research.

CHAPTER SEVEN: DISCUSSION

This study provides insight into two important questions: How much do teachers learn from an intensive mathematics professional development intervention? And, how much of what teachers learn is visible during instruction? The study also examines possible sources of variation that are associated with the answers to these two questions. The data in this study provide insight into the complex and dynamic relationship between policy and practice, an important nexus in education policy studies. In this chapter, I first discuss the main issues that this inquiry uncovered related to the dynamic relationship between policy aims, policy instruments and practitioners' capacity. Then, I describe the primary limitations of the study design. I conclude with a discussion of how the study findings relate to potential avenues of future research in mathematics teacher professional development.

The Dynamic Relationship Between Policy and Practice

This critical case study provides a window into the dynamic, two-way relationship between policy and practice: policymakers seek to solve problems related to practice and yet they must rely on faulty practitioners to carry out the proposed solutions (Cohen, Moffitt & Goldin, 2007). One of the central challenges for policymakers is to figure out how to enable and support practitioners in making the improvements outlined in the policy. Knowing effective ways to build capacity for practitioners, however, is dependent upon what the policy seeks to accomplish. Policies that are ambitious and deviate from typical modes of practice are likely to fail unless such policies are coupled with well designed instruments that build and utilize practitioners' capacity.

The policy aims driving this study were somewhat ambitious: deepening teachers' understanding of the subject matter and improving teachers' instructional practices associated with that subject matter. The policy instrument – a well designed, intensive professional development model – and the capacity of the practitioners – a hand-picked group of receptive teachers – initially seemed commensurate with these ambitious policy aims. What do the results of this study, which are somewhat disappointing, say about the nature of policy aims, policy instruments and practitioners' capacity as related to mathematics teacher professional development?

Policy Aims

The data in this study illustrate the challenges associated with the ambitious policy aim of improving teachers' content knowledge. Even though these three, carefully selected teachers participated in a well designed, year-long professional development model with a restricted content focus, they struggled to articulate much of the core math content. Though they did employ many of the pedagogical techniques emphasized in the professional development, the teachers struggled to keep all students engaged during question and answer episodes and had difficulty summarizing student approaches at the end of lessons. Why did these teachers struggle so much? Were the policy aims too ambitious, even for these hand-picked teachers? Isn't it reasonable to expect math teachers to improve their content knowledge through this type of professional development model? Shouldn't teachers be able to question individual students and keep the rest of the class engaged?

Perhaps these aspects of the professional development model are more complex than they seem, as the following explanations suggest.

Complexity of rational number concepts. One theory for why teachers struggled to understand and articulate key ideas related to rational numbers is the complexity of the content itself. Throughout the development and review of the professional development materials, internal and external study experts routinely commented on the difficulty of the subject matter. Two of the experts said that rational number topics were more complex than most topics in Algebra I, even though Algebra I occurs after rational numbers in the K-12 curricular sequence. In fact, mastery of rational numbers – especially fractions – is considered critical to success in Algebra I (NMAP, 2008).

The relationship between ratios and fractions – how they give rise to the other – is one example of the complexity of rational number concepts. This relationship is very subtle, and it is even disputed among mathematicians. The prevailing view among mathematicians, which the study adopted, says that fractions are numbers that “live” on the number line, but ratios are quotients between two numbers and hence do not “live” on the number line. However, some mathematicians argue that fractions and ratios are actually equal to each other (Personal communication with study expert, 2008). If accomplished mathematicians dispute some of these ideas, it is no wonder that teachers, who have far less formal training in mathematics, lack clarity on these ideas. Other aspects of rational numbers are also complex, such as the study definitions of fraction and decimal, the relationships between fractions and decimals, and the rationales behind familiar algorithms. Thus, the complexity of the rational

number content might have contributed to how much teachers were able to learn through this professional development model.

Formal math concepts. Another issue related to the acquisition of math content is the distinction between formal and pragmatic content knowledge. Formal mathematical knowledge tends to be highly symbolic and theorem or proof-based. In contrast, pragmatic content knowledge focuses more on student-friendly terms and representations. Many of the definitions and key math take-away points in the professional development were formal. The definitions of fraction and decimal, for example, used algebraic notation that is common in higher-level math textbooks. All three teachers in this case study said that the study definitions seemed too advanced for their students, which is why none of them introduced the definitions to students. Had the study definitions been less formal, the teachers would have been more likely to use the definitions with students, which might have further solidified that information.

However, being able to articulate formal mathematical content is important, especially given the National Math Advisory Panel's (NMAP) recent report, which calls for all students to learn "authentic algebra" before they graduate from high school. By "authentic algebra," the authors refer to content that is highly symbolic and formal. The authors argue that students should be provided opportunities to learn formal content in the earlier grades if they hope to be successful in high school. They single out fractions as a critical topic that should be introduced more formally. The panel's description of formal fraction content is consistent with the content emphasized in this study's professional development model. Nonetheless, policies

that call for teachers and students to become more facile with formal, symbolic mathematics should not assume that teachers will acquire such knowledge quickly or easily, even when they participate in well designed and well delivered professional development activities.

Complexity of “reform” mathematics teaching. In her book, *Inside Teaching: How Classroom Life Undermines Reform*, Mary Kennedy (2005) argues that some educational reform ideals, such as teaching rigorous content in such a way that all students are intellectually engaged, are probably unrealistic and sometimes even impede rather than improve student understanding. She says that such reform efforts, which rely on extensive question and answer episodes between the teacher and students, underestimate the time and energy required to teach in this way and overlook the multiple demands that teachers must simultaneously manage. For example, teachers typically must juggle how to maintain lesson momentum, cover the required curriculum and monitor student behaviors when teaching in this way. When teachers also are asked to stimulate and maintain students’ intellectual engagement through questioning, they end up being forced to make tough choices, such as deciding to cover only a portion of a lesson or a portion of the curriculum. In fact, one of the teachers in Kennedy’s study only made it through one third of her textbook by the end of the year because of the extra time she spent teaching in an intellectually demanding way.

The teachers in this study followed this pattern: even though they had extended class periods, they had trouble finishing lessons because of the extensive amount of time they spent maintaining student engagement through frequent and

extended question and answer episodes. In addition, none of the three teachers knew how to maintain the intellectual engagement of students who were not directly involved in a particular question and answer episode. Kennedy would argue that this further illustrates the complexity of reform teaching. I agree with this assessment and would add that such instruction, at least in this country, is rare and only observed among exceptional teachers.

I happened to observe one such expert teacher at Adams Middle School. She was the teacher leader for the 7th grade teachers in this case study, but since she taught 6th grade, she wasn't part of the larger study. The district math coordinator encouraged me to stop by her class when I had a chance because the coordinator said she was one of the best teachers in the district. I had some extra time on one of my visits to Princeton, so I asked her if I could observe one of her classes on the spur of the moment. She welcomed me into her classroom, which was a 6th grade class containing mostly advanced students. Given that she taught a different grade level and that most of her students were high performing, it is difficult to make blanket comparisons between her class and the other 7th grade classes I observed. However, her classroom environment was dramatically different than any other lesson I observed at the school. In fact, it was one of the most well orchestrated math lessons I have ever observed in my 15 years as a teacher and researcher. The teacher had several distinct components of the lesson, which included whole group discussions, individual seat work and small group math "centers." The teacher leader rotated students in and out of centers that included games and hands on materials that were tailored to skill deficits of particular students. During the whole group discussions,

when the teacher pushed students to explain their thinking, she knew how to get the students to bat ideas back and forth so that she didn't have to focus on one student or group and ignore the rest of the class. The teacher leader directed these discussions efficiently, with virtually all students actively engaged.

Each of the three case study teachers, when they discussed working with the teacher leader, attributed much of what she is able to do to the intellectual capacity of her students. Their common sentiment was roughly, "I'd like to see her teach that way with my kids." While this observation might be true – that keeping all students intellectually engaged in high level content is dependent on the ability levels of students – the instructional routines and rituals that this teacher had in place seemingly would benefit students at all ability levels. However, what is possible instructionally should not be equated with what is feasible or likely. This teacher's instructional practices highlight the complexity of the type of teaching promoted by the study professional development and depicted in Kennedy's (2005) work.

These three aspects of the professional development intervention – the complexity of rational number concepts, the formality of the math content and the complexity of the instructional routines – illustrate how the twin policy aims of improving teacher knowledge and instructional practice are highly ambitious and extremely complex. And ambitious and complex policies require well designed and comprehensive policy instruments if they hope to succeed. In this study, the professional development model is the policy instrument. The next section describes the extent to which the model supported practitioners in meeting these complex policy aims.

Policy Instruments

Policy instruments are “the capability that policy brings to its relations with practice” (Cohen et al., 2007, p.536). In this study, the professional development model represents the policy instrument used to support the policy aims of improved teacher knowledge and improved instructional practice. According to Cohen and colleagues:

[Policy instruments] vary in strength, or their influence in practice, and in salience, or how closely they connect with what must happen in practice to achieve policy aims. The further those aims depart from conventional practice, the more acute the dilemma becomes and the more likely it is that strong and salient instruments will be required to enable practitioners to respond constructively. (p.536)

Though the professional development model was well designed and well delivered, it is reasonable to ask whether the model was “strong and salient” enough to enable the practitioners to meet the ambitious aims of the professional development. Two components of the professional development design, the intensity and duration of the model and the extent to which collaborative learning structures were utilized, are discussed in relation to the aims of increased knowledge and improved instructional practice.

Intensity and duration. In this study, teachers participated in 48 hours of professional development workshops that focused largely on content and pedagogical content, which was notably more than the 84% of secondary math teachers who recently reported spending 24 or fewer hours in similarly focused professional development activities (Birman et al., 2007). The teachers also participated in approximately 20 hours of school-based coaching activities. The coaching activities included one-on-one and grade level meetings with the coach and combined in-class

observations and out-of-class lesson planning and lesson debriefing sessions. The workshop and coaching activities spanned seven months of the school year so the professional development activities could be delivered when teachers were teaching relevant rational number topics and with time for the teachers and the coach to establish an ongoing relationship. These aspects of the model make it much more intensive and much more connected to teachers' daily work than typical professional development opportunities offered by external providers.

However, the model was for one year,²¹ which some researchers would argue is too short a timeframe to detect substantive changes in teacher knowledge and practice. Hawley and Valli's (1999) professional development design principle of "continuous and supported" suggests that a 3- to 5-year time frame might be more realistic. In fact, they refer to a finding from the Prichard Committee (1995), which is particularly relevant to the professional development aims associated with this study. The committee found that a group of teachers who were engaged in professional development activities that focused on pedagogical strategies consistent with the NCTM standards needed a stronger content knowledge base – and hence, more professional development time – to deliver the pedagogical techniques effectively. This finding is consistent with one of the key findings in this study: pedagogical techniques, such as asking students to explain their thinking and to summarize the key ideas in a lesson, depend on teachers' understanding of the content. Solidifying

²¹ The initial design of the study was for one-year of professional development. However, in the spring of 2008, the Federal agency decided to add a second year of professional development in 6 of the 12 original districts. The second year intervention, which is still currently being delivered, is a little less intensive than the first year intervention. It contains 30 hours of workshops and 16 hours of coaching for returning teachers and 42 hours of workshops and 16 hours of coaching for new teachers – i.e., teacher who did not participated in the first year intervention.

teachers' understanding of mathematics content may require an intensive model that spans multiple years.

Other researchers describe continuous professional development models used in other countries. The Japanese approach to mathematics teaching and learning is widely cited as an example of continuous, job-embedded, content-rich professional development. Unlike the U.S., in which educational reforms are relatively short-lived (Cuban, 1990), Japan takes a much longer-term view of improvements in teaching and considers teaching to be a very complex activity. In order to teach their subjects well, Japanese educators assume that teachers need time to participate in continuous, school-based professional development. Unlike the U.S., where teachers are considered to be competent once they have completed teacher-training programs, Japan assumes that competence can be improved over time and expects teachers to participate in on-going training throughout their careers. This continuous school-based professional development model is called *kounaikenshuu*. One of its core components is lesson study (Stigler and Hiebert, 1999). Lesson study is year-long process in which teachers define a problem, plan a lesson around the problem, teach the lesson, refine and re-teach the lesson, and then reflect upon and summarize the process, which often takes the form of a written report. At the end of the year-long process, teachers have a product that they can use again and share with other schools. Since Japan has a national curriculum, these lessons can be shared among teachers and schools nationwide.

The Japanese approach to teacher learning is based on normative expectations that are quite different than those governing the U.S. system. As Stigler and Hiebert

(1999) contend, teaching is a distinct cultural activity. Thus, any lessons that might be learned from Japan should be interpreted in light of the cultural context of teaching. However, as U.S. educators and policymakers are calling for increased content rigor and improved teaching practices in mathematics (NMAP, 2008), they might look at the Japanese system as one that has made strides in both areas. As the TIMSS video study illustrated, Japanese 8th grade teachers deliver advanced, formal mathematics content in methods that engage students – teaching methods that, according to Stigler and Hiebert, are closely aligned with the NCTM standards. Japanese students outperformed their U.S. counterparts, and this difference in achievement highlights the promise of their system.

These arguments suggest that the study's professional development model, though much more intensive and sustained than typical models, might not have been intensive or sustained enough to trigger noticeable improvements in teachers' knowledge. However, cost and teacher turnover issues make dramatically increasing the intensity of professional development a less than straightforward policy proposition. Providing high quality professional development is extremely expensive. Even this model, which probably wasn't intensive enough, would be difficult for districts and schools to afford. Further, professional development programs that require 3- to 5-years to affect teacher practices assume that the teacher workforce is relatively stable. In secondary mathematics, particularly in urban schools, teacher turnover is quite high, a condition that longer-term professional development programs must take into account. In this study, over one third of the teachers who taught 7th grade math in the 2007-08 school year no longer taught that

subject and grade level in the 2008-09 school year. These turnover rates illustrate the challenges associated with providing sustained, ongoing professional development to accomplish ambitious policy goals.

Collaborative learning structures. The study design incorporated many collective learning opportunities for teachers, such as co-teaching, co-planning and peer observations. The literature on effective professional development highlights the importance of providing teachers with opportunities to work and learn together (Garet et al., 2001; Hawley & Valli, 1999). In mathematics, international studies, such as the TIMSS Study and Ma's (1999) comparative case study, indicate that some of the countries whose students performed better than the U.S. have much more comprehensive collective communication structures in place. In the previous discussion about lesson study, Japanese teachers meet regularly to carry out the planning, revising and re-teaching of the targeted lesson. They are able to meet regularly because their teaching schedules include ample time for teachers to meet and collaborate. Ma indicated that the Chinese teachers also have more time to meet together and discuss mathematics than their U.S. counterparts. Thus, the push to provide teachers more time to spend with each other to improve mathematics teaching and learning is well founded.

In this case study, the teachers worked in a school that valued teacher collaboration. The principal created a schedule in which each grade-level team of math teachers shared a common planning period, which allowed teachers to plan and debrief lessons or meet and discuss other ideas related to teaching. The district appointed math teacher leaders in each school and supported an extra planning period

so the teacher leaders could work in a coaching capacity. The district also provided a part-time math coach, who, in addition to the teacher leader, visited teachers during their common planning time and while they were teaching to provide additional feedback. Teachers had a variety of ways to collaborate with other teachers and/or specialists to reflect upon and improve their instructional practices.

This substantial infrastructure for collective participation seemed underutilized, however. When I asked teachers how they used their joint planning period, they said it was rare for them to meet and plan lessons together outside of the PD Math Study. When I asked if they had ever used the time to discuss any aspects of the PD Math Study, such as the math content emphasized in the professional development or the problem sets, none of the teachers reported doing so. Why were these collective opportunities underutilized? One possible explanation is that the district and school did not have strong cultural norms about how teachers should use the time. They knew that it was important to provide teachers time to collaborate, but they let teachers decide how to use the common time. Without guidance, the teachers did very little collaborating on their own. If the district had provided expectations or incentives about how to use the time, teachers might have been more likely to take advantage of the common planning time and learn from each other. However, as the discussion on the complexity of the math content indicates, simply providing time for teachers to meet does not ensure that they will be able to and motivated to learn sophisticated content.

Practitioners' Capacity

The capacity or capability of practitioners also influences the likelihood that policy aims will be realized. Capacity is critical because it “consists of the resources practitioners bring to policy,” and yet capacity is relative, since “what is sufficient for a policy that departs only a bit from conventional practice is unlikely to be sufficient for a policy that departs much more dramatically” (Cohen et. al, 2007, p.537).

Capacity is often associated with individual practitioners, but it also can be associated with social sources in the policy environment. Practitioners' capacity is much more than the skills and knowledge that practitioners bring to the policy environment and through which policy instruments are activated. Though knowledge and skills are vital, practitioners' values, interests, and dispositions are also important attributes of capacity. This critical case study illuminates how these attributes of capacity interact with each other and influence the extent to which policy aims were accomplished.

Teachers' beliefs about mathematics teaching and learning. Reform-based instruction, which emphasizes problem solving and conceptual understanding over rote memorization and procedural fluency, represents a dramatic shift from the way in which most teachers experienced mathematics as K-12 students. This shift is important because, as Lortie (1975) points out,

Teaching is unusual in that those who decide to enter it have had exceptional opportunity to observe members of the occupation at work; unlike most occupations today, the activities of teachers are not shielded from youngsters (p.65).

Teachers are not “blank slates,” then, when it comes to learning instructional techniques or curricular programs; they have prior experiences in mathematics classrooms – experiences that are both good and bad – from which to compare current

programs and modes of instruction. Since none of the three teachers in this case study was old enough to experience the New Math curricular movement of the 1960s, it is safe to say that all three experienced traditional, procedural based mathematics instruction as K-12 students. In fact, over the course of the year I spent with these teachers, all three mentioned in some way that the *CMP* program represented a different mode of instruction than they had experienced as students.

All three teachers mentioned aspects of reform-based instruction that they liked and disliked. They all thought it was important to ask students to explain their thinking, and they found the questioning techniques emphasized in the professional development to be helpful in stimulating and sustaining student discourse. All three teachers thought that presenting students with interesting problems and encouraging them to explain their thinking were important components of quality instruction. They liked the extended problems and activities featured during the professional development, which they said supported the types of problem solving activities in the *CMP* program.

However, all three teachers thought that the skill deficits of their students interfered with their ability to deliver meaningful, conceptually-based lessons. Though the teachers provided students with skill practice during the 20-30 minute numeracy block – one of the teachers used this time for skill practice more so than the other two teachers – none of them said that the balance was quite right. They wished that they had more time to work on skills than the numeracy block provided. Wiggins had an especially strong opinion about this issue; she thought the *CMP* curriculum was too difficult for students and wished the district had a skill based program that

could be supplemented with CMP problems. Hamlin, the teacher with the strongest level of knowledge and the strongest will to learn, said that her husband had encouraged her to write her own textbook so she could achieve the proper balance of concepts and skills. Even though all three teachers taught CMP in the way that the district envisioned – i.e., they followed the district pacing guide and devoted most of the class time to CMP – they all wished CMP provided more opportunities for students to practice and build skills.

These mixed beliefs about teaching and learning manifested themselves during instruction. On one hand, all three teachers said that the questioning strategies emphasized in the professional development had helped them manage their classroom discussions. And in each of the 15 lessons I observed, teachers used the pedagogical techniques to probe student understanding. They demonstrated these behaviors both when they were working with the instructional coach and when they were not. In fact, these teaching behaviors are consistent with what the district promotes. The district math coordinator indicated that she encourages teachers to probe students' thinking and is especially aware when teachers are unwilling to let students take on sufficient intellectual burden. Hiebert and Grouws (2007), in their literature review on common characteristics of effective mathematics teaching, refer to such intellectual burden as “the engagement of students in struggling or wrestling with important mathematical ideas” (p.387). Unlike teachers who might have rejected this mode of instruction, these teachers allowed students to struggle with important mathematics.

On the other hand, the teachers did not fully endorse the CMP program, which might be partly why they did not pursue the math content more deeply. The pedagogical techniques were easy to comprehend and they reinforce the type of questioning promoted by the district. But understanding the core content, articulating the most common student misconceptions, and summarizing the most salient points of the lesson are much harder to grasp than the pedagogical techniques and require a stronger belief in the philosophy of the program. Thus, even though the majority of the pedagogical behaviors I observed were consistent with much of what the district was promoting, teachers' partial endorsement of the CMP approach might have contributed to challenges associated with acquiring content knowledge and integrating such knowledge into instruction.

Teachers' intrinsic motivation to learn. In addition to teachers' mixed beliefs about the underlying premises of reform-based instruction, teachers' beliefs about themselves as learners seemed to influence the extent to which they understood and made use of the professional development. In the paper, *Exploring Teachers' Will to Learn*, Van Eekeln and colleagues (2006) argue that a teacher's "will to learn" is an understudied phenomenon in the field of teacher professional development. They believe the topic should receive more attention because a teacher's "will to learn" is a precondition for acquiring additional knowledge and skills.

Among the three case study teachers, one was extremely eager to learn. Hamlin, the teacher most eager to learn the material, reported spending the most time outside of class thinking about why her students were not learning as much as she thought they needed to learn. She read journal articles, made up her own problems

and even took a math class at a university during the year I spent with her. She wasn't afraid to admit when she was confused or frustrated about some aspect of her teaching. In fact, she was more critical of her own teaching than the instructional coach. Even though she had a weaker background in mathematics than one of the other two teachers and fewer years teaching experience than both teachers, she demonstrated the highest level of understanding of the math content and the highest level of visibility of the math content during instruction.

The other two teachers – albeit to varying degrees – expressed a relatively strong “will to learn” when they were participating in the professional development activities. They asked questions and seemed engaged in understanding the material, which contributed to my initial assessment that the school would be a good site for a critical case study of professional development. However, neither teacher sustained that “will to learn” outside the professional development – at least not to the extent that Hamlin did. Unlike Hamlin, who routinely mulled over what she might have done differently after teaching a lesson, these teachers were less bothered and dismayed when their students failed to grasp an idea or when a lesson didn't go as well as they had planned. They didn't approach the problem with the same sense of urgency that Hamlin did. Wiggins, the only teacher with a degree in secondary mathematics education and the teacher with the most experience teaching *CMP*, had the weakest “will to learn” and demonstrated the lowest level of understanding of the math content. Smith, who scored in the middle in terms of understanding and implementation of the math content, was also in the middle in terms of motivation to learn the material. Though he didn't share Hamlin's intense thirst for knowledge, he

was thoughtful about the material and participated fully in the professional development.

The sample is much too small to make generalizations, but Hamlin's eagerness to learn, which included being visibly upset when her students were confused, at least partially contributed to her comprehending higher levels of the math content than the other two teachers. At minimum, these data confirm that identifying and stimulating teachers' "will to learn" should be a consideration when designing and implementing teacher professional development.

Summary

In sum, the ambitious policy aims of the study demanded a policy instrument that was strong enough to support participants' capacity to fulfill the aims. Had the aims been less ambitious or the instrument and/or practitioners' capacity been stronger, the results likely would have been different. When trying to understand the extent to which policy aims were or were not realized, policymakers should examine all three of these dimensions, rather than a single dimension. For example, focusing exclusively on participants' capacity or capability would be short sighted, since the level of capacity required for a policy to be successful is highly correlated with the level of complexity and ambition of the policy itself. As Cohen (2007) and his colleagues put it:

Capability is relational, like the other resources that we have discussed. It waxes and wanes in interaction with the aims that policies set, the instruments that they deploy, and the environments in which policy and practice subsist. One can speak accurately of capability only if one speaks in relational terms, and one can shape capability only by shaping it in relation to those other resources (p.540).

This case study suggests that, even in relatively favorable conditions, practitioners' capacity should be expanded in order to meet the ambitious aims of improved knowledge and practice. If expanding capacity is not possible, then the policy aims must be adjusted downward to match lower levels of capacity and/or the instrument must be extended. Otherwise, professional development models such as this one can, at best, show limited results. In terms of this particular policy, a more comprehensive package of policies would likely be necessary to accomplish these ambitious aims. For instance, policies that address teacher preparation programs and policies that address normative expectations for math teachers likely would need to be initiated in concert with initiatives to improve the quality of professional development.

Limitations

Like other empirical studies that focus on complex phenomena, this study has limitations that should be considered. A major limitation of this study design is the absence of baseline data for teacher knowledge and practice. Ideally, I would have been able to administer to teachers a pre-assessment that addressed the key constructs in the structured content interview and used that assessment to measure teachers' growth from the pre to the post assessment. Such a design would have allowed me to distinguish between what teachers learned from the professional development and what teachers knew before they participated in the professional development. This issue would be especially problematic if the teachers had demonstrated high levels of understanding of the math content and pedagogical content on the structured content interview. Since they did not – the teachers averaged between the low and moderate

levels of understanding – this issue is less of a problem. In fact, it probably means that teachers’ overall levels of understanding of the professional development are partially inflated by their prior knowledge. This hypothesis suggests that what teachers learned from the professional development is probably a little lower than indicated in this study – particularly in the domain of pedagogical knowledge.

Even though teachers’ overall low level of understanding made this issue less problematic, I tried to address this weakness in the design in a few ways. First, I focused on how the content was presented in the professional development and used that information to anchor my analyses. For example, when a teacher used language that was similar to or exactly the same as language featured in the professional development, I used that text as the basis to measure teachers’ level of understanding of the professional development content. I also included that text as much as possible in Chapters 5 and 6 to illustrate the link between the content presented in the professional development and teachers’ actual responses.

I also addressed the absence of baseline data by asking teachers to teach a lesson that they taught the previous year and then interviewed them afterward about any differences that they attributed to the professional development. I used the pizza problem, described in Chapter 6, as the core lesson. All three teachers said that the professional development didn’t impact how they thought about the math content of the lesson, but it did impact how they delivered the lesson. For instance, two of the three teachers mentioned that the coach gave them a novel way to introduce the problem, which they used and would continue to use in the future. All three teachers said that they used the questioning strategies introduced in the professional

development and would continue to use those in the future. These interview data supported findings from the classroom observations that indicated higher levels of visibility of the pedagogical aspects of the model compared with the visibility of math content. However, if I were going to conduct a similar study in the future, I would make a concerted effort to secure baseline data.

Another limitation of this study relates to the numeric values assigned to teachers' understanding of the math content on the structured content interview. Though I tried to standardize the scoring process as much as possible, the numeric values still represent estimates of what teachers' understood. The estimates are less problematic when comparing large differences between teachers. For example, Hamlin scored a "3" or at the strong level of understanding on two of the questions on the interview, and I am confident that her understanding was notably higher than the other two teachers on these items because her scores were 1 to 2 points higher on the 0 to 3 scale. That an independent math content expert reached the same conclusion on these items bolsters my confidence even further.

However, after teachers' scores are averaged, some of the overall differences are harder to understand. Smith's and Wiggins's overall averages were 1.5 and 1.3, respectively, but I have less confidence that Smith scored notably higher than Wiggins. Smith scored higher than Wiggins on only three of the seven items and yet had a higher overall average. If the content of the structured interview had varied even somewhat, it is possible that Wiggins and Smith would have scored the same or possibly switched places. The averages are still useful in describing overall patterns – e.g., none of the teachers came away with a strong understanding of the math content

and one of the three teachers appeared to understand more than the other two – but small differences between average scores should be interpreted cautiously

A third limitation of this study is that I chose not to audio or videotape the lessons. I decided against either approach primarily because I didn't want to interfere with the design of the larger study. I didn't want these three teachers to act differently because they were being examined twice – as a participant in the larger study and as a participant in my supplemental case study. I also took my cues from the instructional coach, who was present during the first three observations and indicated that she thought audio or videotaping a lesson would interfere with the rapport she was trying to build as a coach. Though I could have asked teachers for permission to audio or videotape lessons after they were no longer being coached, I decided against this practice because I wanted to follow the same procedures I used to score the observation data when the coach was present.

The procedure I used to record classroom observation data involved keeping detailed hand written field notes and then typing them up within a day or two of each observation. I determined that this approach was sufficient for completing the observation instrument, but I would have preferred to have a more complete transcript of classroom interactions from which to portray classroom life and evaluate the visibility of the professional development during instruction. I compensated for this deficit by recording exact quotes by hand whenever I could and whenever the exchanges seemed to capture something essential to the goals of the study. This approach worked for the most part, but the classroom story would have been much richer if I had more data about classroom interactions to use.

Directions for Future Research

This study has several implications for future research related to mathematics professional development. First, this study captured what three teachers comprehended and implemented from professional development, but it only speculated about *how* teachers learned the content. Wilson and Berne (1999) argue that the mechanisms by which teachers learn is an important yet understudied phenomenon. This study also suggests that this avenue is worth pursuing. For instance, it would be interesting to know more about how Hamlin developed and fed her curiosity and her thirst for knowledge; more generally, it would be useful to know which aspects of teachers' "will to learn" can be fostered and which aspects are more innate. However, the design of this case study did not allow for an in-depth analysis of this topic.

A second and related avenue of future research is to learn more about how teachers' beliefs and motivations are related to the mechanisms that trigger teacher learning. The research consistently points to the importance of recognizing teachers' values and beliefs when designing professional development models. Models that ignore or minimize teachers' values about the content or about the nature of teaching and learning are likely to fail. But more research is needed to classify and categorize teachers' values and beliefs so that professional development models might be tailored to maximize the involvement of teachers with various types of beliefs. Instead, at least in mathematics, two somewhat crude categories distinguish teachers' beliefs about teaching and learning: reform-based teachers assert that students should learn high level concepts through questioning and purposeful struggle with content

while traditional teachers believe that students should learn rules and procedures through practice and explicit instruction.

These distinctions are not fine grained enough, as this study indicated. Unlike Cohen's (1990) case study of Ms. Oublier, who saw herself as a reform-based teacher and yet often refused to probe student understanding, these teachers allowed students to grapple with the material – often with such persistence and to such a degree that they were not able to keep the rest of the class intellectually engaged while they were questioning a student or small group of students. Here is how Cohen depicts Ms. Oublier's interpretation of reform-based teaching:

Make no mistake: Mrs. O was teaching math for understanding. The work with number sentences certainly was calculated to help students see how addition worked, and to see that addition and subtraction were reversible. That mathematical idea is well worth understanding, and the students seemed to understand it at some level. They were, after all, producing the appropriate sorts of sentences. Yet it was difficult to understand how or how well they understood it, for the didactic form of the lesson inhibited explanation or exploration of students' ideas. Additionally, mathematical knowledge was treated in a traditional way: Correct answers were accepted, and wrong ones simply rejected. No answers were unpacked. There was teaching for mathematical understanding here, but it was blended with other elements of instruction that seemed likely to inhibit understanding (p.313).

Like Ms. Oublier, the teachers in this case study blended teaching for mathematical understanding with other elements of instruction that inhibited understanding. For example, the case study teachers persistently questioned students about their thinking, but they had trouble summarizing student approaches and articulating the core mathematics underlying the approaches. This deficiency was problematic because students often seemed confused at the conclusion of lessons, even though they had just spent a lot of time and energy pursuing various approaches and responding to the teacher's questions. Unlike Ms. Oublier, however, the case study teachers did not

simply reject wrong student answers. They questioned students vigorously and tried to unpack student approaches.

Both Ms. Oublier and the teachers in this case study thought they were teaching reform-based mathematics, or teaching math for understanding. But both fell short of comprehensive reform-based instruction, yet in different ways. Ms. Oublier was more didactic and less likely to engaged students in what Hiebert and Grouws (2007) refer to as “purposeful struggle.” The teachers in this case study were less didactic and very likely to engage students in such struggle. In fact, the case study teachers needed help bringing resolution to lessons in which students spent extensive amounts of time grappling with ideas. Future research should depict teachers’ actions along a *continuum*, rather than a dichotomous distinction, between reform-based and traditional teaching so that professional development activities can be developed accordingly. Such tailoring would maximize what teachers choose to learn and are able to learn through professional development.

A third area of future research relates to the specification of sustained and intensive professional development activities. Most experts think that one-shot professional development workshops or programs have little chance of influencing what teachers know and how they teach. But less is known about what constitutes professional development models that are sustained and intensive enough to impact teachers’ knowledge and practice and yet feasible enough that districts and schools can adopt them. The professional development model developed for this study sought to achieve the right balance of intensity and feasibility, which are inversely related. Other workshop-based models include a more extensive summer component,

often two to three weeks, which increase intensity but is less feasible in terms of ensuring teacher participation. Most of the more intensive summer workshops are voluntary (Wu, 1996). Future research could explore the tradeoffs associated with adjusting the intensity and feasibility of professional development models. The degree to which a professional development program is intensive affects the extent to which it is feasible and vice versa.

A fourth area of research relates to the teaching and learning of formal, rigorous mathematics, such as the mathematics outlined in the National Math Advisory Panel's (2008) recent report. Currently, few curricular materials exist that move students from a pragmatic to a formal understanding of the content. Available programs tend to favor one approach more heavily than the other. But both types of understanding are important. Students are unlikely to learn formal ideas unless the teacher presents the ideas in meaningful ways. Yet meaningful explorations of the content are ultimately unsatisfying if the teacher is not able to connect students' ideas to formal mathematics concepts. This case study illustrated that teachers may not be able to move between formal and pragmatic understanding of the content even when they are being supported through professional development. If states and districts follow the recommendations in the NMAP report, they must think carefully about how teachers and students will be supported to meet these challenging content standards. It is unreasonable and unfair to think that teachers and students will be capable and motivated to meet these demands without extensive support, such as intensive professional development activities focused on helping teachers understand and articulate challenging math content.

Finally, all discussions surrounding the teaching and learning of mathematics depend upon whether one question is answered: As a society, how much do we really care about supporting teachers and students in learning high-level conceptual mathematics? If we are as serious as the experts say we should be, then achieving these aims will require nothing less than a comprehensive, sustained overhaul of the current system, which provides teachers few incentives to learn advanced mathematics content and to deliver that content meaningfully to students. Such an overhaul likely would move beyond professional development and would focus on restructuring teacher training programs, which are responsible for providing teachers with threshold levels of content knowledge related to teaching. Unless we address this challenge carefully and comprehensively, interventions, such as the professional development model that I examined, are unlikely to be successful.

APPENDIX A: CLASSROOM OBSERVATION PROTOCOL

Teacher _____ Class _____ Date _____

I. Lesson Focus/Objective:

II. Instructional Practices

PROFESSIONAL DEVELOPMENT EMPHASIS	EVIDENCE EXHIBITED IN LESSON			
	None/NA	Weak	Moderate	Strong
Mathematics Content Knowledge				
Use mathematically precise language (e.g., correct definitions, properties.)				
Make explicit connections between or among concepts				
Provide rationale for why procedure works (e.g., show property, justify with a proof.)				
Incorporate key ideas about ratio and proportion (e.g., stress multiplicative reasoning instead of use cross product algorithm)				
Incorporate key ideas about fractions, decimals or percents (e.g., connect decimals to place value and fractions)				
Pedagogical Content Knowledge				
Identify and debug student misconception(s) associated with focus of lesson				
Use (or encourages student use of) multiple representations of concepts (e.g., number line, area model, table)				
Make connections among student errors or approaches				
Pedagogical Knowledge				
State the mathematical focus or objective of lesson				
Encourage students to pursue multiple strategies to solve the problem				
Ask students to justify or extend their answers or explanations (e.g., “How did you get that?”, “Say more about your answer.”)				
Ask students to engage in each other’s reasoning (e.g., saying whether they agree or disagree with each other’s approaches and why)				
Provide lesson summary or closure (e.g., summary statement, exit card)				
Overall Rating				
Teacher exhibits behaviors that exemplify the core components of the professional development.				

III. Student Behavior

Less than 25% of students were engaged for most of the lesson	<input type="checkbox"/>
About 25% of students were engaged for most of the lesson	<input type="checkbox"/>
About 50% of students were engaged for most of the lesson	<input type="checkbox"/>
75% or more of students were engaged for most of the lesson	<input type="checkbox"/>

APPENDIX B: TEACHER PERCEPTION INTERVIEW PROTOCOL

Name _____ Position/Title _____ Date _____

Background Information

Type of undergraduate degree _____
Number of undergraduate math courses taken _____
Number of undergraduate math education courses taken _____
Type of graduate degree _____
Number of graduate math courses taken _____
Number of graduate math education courses taken _____

Approaches to Acquiring Professional Knowledge

1. The study professional development focuses on building conceptual understanding of core rational number concepts (content knowledge), understanding of student approaches and misconceptions (pedagogical content knowledge) and utilizing pedagogical strategies that promote student discourse (pedagogical knowledge). Which components of the professional development have you found most valuable and why? Which components have you found least useful and why? Please describe any specific strategies or aspects of the professional development that you have incorporated into your teaching and the frequency with which you have employed such strategies.
2. What (if anything) have you done after the professional development workshop or coaching activity to solidify your understanding of these components or aspects of the professional development? (e.g., utilized supplemental readings, met with other teachers to discuss, follow up emails or calls with coach, trial and error in own classroom)
3. How (if at all) has the coaching component of this study influenced how much of the information from the summer institutes and seminars you have (1) comprehended and (2) implemented in the classroom? How would you rate the overall quality of the coaching?
4. If you could spend more time on any of the topics or strategies included in the professional development model, what would they be and why?
5. Compared with the other professional development workshops you have participated in, how does this model stack up in terms of improving the quality of instruction and increasing levels of student understanding? Explain.
6. To what extent are the goals of this professional development model consistent with the instructional goals of the district? How relevant is such alignment (or misalignment) to the extent to which you embrace and alter your teaching practices?

APPENDIX C: PD FACILITATOR INTERVIEW PROTOCOL

Name _____ Date _____

Re: teacher _____

Background Information

Type of undergraduate degree _____
Number of undergraduate math courses taken _____
Number of undergraduate math education courses taken _____
Type of graduate degree _____
Number of graduate math courses taken _____
Number of graduate math education courses taken _____

Perceptions of Teacher Learning

1. What aspects of the professional development has this teacher been most and least receptive to? Why do you think this is the case?
2. What aspects of the professional development have been easiest and most difficult for this teacher to implement? Why do you think this is the case?
3. Compared with other districts, schools and teachers, where does this teacher fall on the continuum of (1) grasp of the subject matter and (2) teaching for understanding?
4. If you had more time to work with this teacher, what would you focus on and why?

APPENDIX D: DISTRICT MATH COORDINATOR INTERVIEW PROTOCOL

Name _____ Position/Title _____ Date _____

District Middle School Mathematics Philosophy

1. Describe or state APS's mission for middle school mathematics.
2. How long has this mission been in place?
3. How would you describe APS teachers' familiarity and commitment to the instructional habits and practices associated with the mission? Are the study schools and Columbia Middle School representative of the other middle schools in the district? Why or why not?
4. What do you see as the key barriers preventing teachers from embracing and implementing the key aspects of your mission?
5. What steps does the district office take to ensure that teachers understand and implement the core aspects of your mission and view of instructional practice? Has the district implemented structural support for teaching learning? If so, explain.
6. You are one of approximately 20% of districts across the country that uses a "reform" mathematics text (*Connected Mathematics*). Describe when and why you adopted this text and any challenges associated with teacher implementation of the program.

Evaluation of the PD Math Professional Development Model

7. How would you rate the overall quality and usefulness of the PD Math professional development model? What aspects have been especially helpful or useful for your teachers? What aspects have been less helpful or useful?
8. Describe any evidence of changed teacher beliefs, knowledge and/or practice related to the goals of the professional development.

Alignment between District Philosophy and PD Math Model

9. To what extent what are the key ideas and aims of the professional development consistent with (1) district standards, curriculum and assessments, (2) district conceptions of pedagogy, and (3) school or district professional development initiatives?
10. Is there anything else you would like to add about the district's middle school mathematics program or your perceptions of the professional development? Is there anything else you would like to add about Columbia Middle School in particular? For example, can you say more about how Columbia and its teachers compare to other schools and teachers in terms of (1) receptivity and adherence to the district's vision of quality teaching and learning and (2) fostering teacher learning?

APPENDIX E: INTER-RATER RELIABILITY OF STRUCTURED CONTENT INTERVIEW SCORING^a

Teacher	Question							Average
	1	2	3	4	5 ^a	6	7	
Hamlin	2/2	1/1	3/3	3/3	1/1	1/1	2/1	1.9/1.7
Smith	1/1	2/2	1.5/1	2/1	2/2	1/1	1/1	1.5/1.3
Wiggins	2/1.5	1/1	1.5/1	1/1.5	1/1	1/1	1.5/2.5	1.3/1.4
Average	1.7/1.5	1.3/1.3	2.0/1.6	2.0/1.5	1.3/1.3	1.0/1.0	1.5/1.5	1.6/1.5

a. Hamlin's two scores represent my score and the content experts independent score, i.e., 2/2 means that I and the expert, respectively, scored Hamlin's understanding of Question 1 at the moderate level. The bolded cells indicate differences between myself and the expert.

APPENDIX F: DOMAINS OF KNOWLEDGE IN STRUCTURED CONTENT INTERVIEW, BY QUESTION AND SHORT CODE

Question/Short Code	Domain of Rational Numbers			Math Content Knowledge			Pedagogical Content Knowledge	
	Fractions	Decimals	Ratio/ Proportion	Definitions	Connections between Concepts	Rationale for Procedures	Representations	Student Misconceptions
1. Define and represent fraction	X			X			X	
2. Connect fraction and decimal concepts	X	X				X		
3. Rationale for decimal procedures		X					X	
4. Rationale for misconceptions with fraction procedures	X						X	X
5. Define and represent ratio			X	X			X	
6. Connect ratio and fraction concepts	X		X			X		X
7. Rationale & representations of fraction procedures	X						X	X

APPENDIX G: STRUCTURED CONTENT INTERVIEW QUESTIONS AND SHORT CODES

Question	Short Code
1. How would you define fraction and represent that definition to students? Has this definition changed because of what you have learned in the professional development? If so, how and why?	Define and represent fraction
2. Suppose a group of students are having trouble connecting the meaning of fractions with the meaning of decimals. What might you do to help students solidify their understanding of how these two types of rational numbers are related?	Connect fraction and decimal concepts
3. Suppose a student asks you to explain why you “line up the decimals” when adding or subtracting but “move” the decimal place to the right or left when multiplying or dividing. What would you say?	Rationale for decimal procedures
4. Suppose a student is confused with the “cross canceling” shortcut to simplifying fractions. For example, the student thinks this is correct: $\frac{8}{28} = \frac{1}{2}$. What is the student’s underlying misconception? How would you help the student overcome this conceptual hurdle?	Rationale for and student misconceptions with fraction procedures
5. How would you define ratio and represent that definition to students? Has this definition changed because of what you have learned in the professional development? If so, how and why?	Define and represent ratio
6. Suppose a student thinks that all fractions are ratios and vice versa. Is this student correct? If not, what would you say to help him distinguish fractions from ratios?	Connect ratio and fraction concepts
7. Suppose a student is struggling to understand why $\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$ and why $\frac{1}{3} + \frac{3}{4} = \frac{13}{12}$. What representation or representations would be most useful to help this student understand why multiplying fractions can sometimes yield a product that is smaller than both the factors and why the sum of two fractions doesn’t always have the same denominators as the addends?	Rationale for and appropriate representations of fraction procedures

APPENDIX H: NUMBER OF HOURS OF PROFESSIONAL DEVELOPMENT DEVOTED TO EACH STRUCTURED CONTENT INTERVIEW QUESTION

Question/Short Code	Hours			Total (68) ^a
	Summer Institute (18)	Follow up Seminars (30)	Coaching (20)	
1. Define and represent fraction	4.5	1.0	1.0 ^b	6.5
2. Connect fraction and decimal concepts	2.5	1.5		4.0
3. Rationale for decimal procedures	1.5		1.0	2.5
4. Rationale for and student misconceptions with fraction procedures	2.5	4.0		6.5
5. Define and represent ratio		7.5	5.0	11.5
6. Connections between ratios and fractions		2.5	1.0	3.5
7. Rationale for and appropriate representations of fraction procedures	2.5 ^c	4.0 ^c	1.0	7.5

a. Teachers participated in 68 hours of professional development during the summer of 2007 and the 2007-08 school year; 18 hours in a three-day summer institute, 30 hours in five, one-day follow up seminars, and 20 hours in ten, one-day coaching sessions.

b. Calculating the time spent on each topic during the summer institute and seminars was relatively straightforward, since each workshop devoted a particular amount of time to each topic. Calculating the number of hours spent on each topic during coaching, however, was much more difficult because coaching activities included both individual and group meetings between the coach and teacher(s) that occurred *outside of the classroom* and other coaching activities, such as modeling, observing and co-teaching, that occurred *inside the classroom*. A further complication was that the amount of time each of the three teachers spent on various coaching activities was only roughly the same. To address these issues, the coaching hours reflect only those hours of coaching for that took place *outside of the classroom* and reflect the hours for the teacher with the lowest amount of time spent on each topic. Thus, the coaching hours are underestimates, since some teachers spent more time on these activities and since in-class learning also occurred.

c. Only emphasis from same PD segment in summer institute and seminars; all other summer institute and seminar emphases are mutually exclusive.

APPENDIX I: TEACHERS' SCORES ON STRUCTURED CONTENT INTERVIEW (Q1)

Study Definition: For whole numbers a and b , with b not equal to zero, the *fraction* a/b is the number on the number line corresponding to a times the unit divided into b equal parts. Using this definition, $3/2$ is a fraction. Since $3/2 = 1\frac{1}{2}$, $1\frac{1}{2}$ is a fraction.

Core Emphases: A fraction is a number, so every fraction has an exact location on a number line. The numerator of a fraction represents the number of pieces that are being counted, and the denominator represents the size of each piece, relative to the unit.

Teacher	Teacher Definition	Score	Level of Understanding
Hamlin	“Point that can be placed on the number line, the top number being the number of pieces being counted, and the bottom number how many pieces make up the whole.”	2	Moderate
Smith	“Representation of parts of a set compared by division...parts of a whole or parts of a set.”	1	Weak
Wiggins	“Part to the whole. You have a circle or a square and it’s put into the denominator, and it tells you how many pieces it’s split into and then the numerator is how many of those pieces you have...[as opposed to] it’s a number on a number line”	2	Moderate

APPENDIX J: TEACHERS' SCORES ON STRUCTURED CONTENT INTERVIEW (Q2)

Study Definition: Decimals are numbers that can be written as N.abc, which means N plus $\frac{a}{10} + \frac{b}{100} + \frac{c}{1000}$ etc. Decimal notation is analogous to place value notation for whole numbers; positions to the right of the decimal point represent fractions with denominators that are powers of 10. Both fractions and decimals are numbers, and they can be ordered and compared. All fractions can be represented as decimals – some terminating and some repeating – but there are some decimals that cannot be represented as fractions. For example, the terminating decimal 1.23 is equal to $1 + \frac{23}{100}$ or $1 + \frac{2}{10} + \frac{3}{100}$. The repeating decimal, .333... is equal to $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots$

Core Emphases: Decimals, place value and fractions with power of ten denominators are closely linked and should be mutually reinforced. 0.15 represents 15 hundredths, which can be expressed as the fraction $\frac{23}{100}$, which is equal to $\frac{2}{10} + \frac{3}{100}$.

Teacher	Teacher Definition	Score	Level of Understanding
Hamlin	“We talk about how the value of a number and it being in a different form...Two out of five pieces might make more sense than 40% of something if you’re talking about how much pizza you ate, for example...we talk about rational numbers in different forms having the same value.”	1	Weak
Smith	“Take ¼. We talk about it being 25%, which is .25 because it’s 25 out of 100. And 25 out of 100 is another form of 25%.” “I would say the number one problem equating a fraction to a decimal is that idea that a decimal is based on the base 10 and a fraction can be based on whatever.” [connection to representations] “1/4, we can also express it as 25/100. Or 5/1000. It’s relative size. I can write 1/1000 th and it looks – and they say, ‘wow, that looks really small’ when they see 1000 squares [area representation].” “I’ve also used fractions that are 10 th s on the number line...then we start looking at those, including looking at little blow ups, like we did in the study.”	2	Moderate
Wiggins	“I would bring it back to common sense. If a student thinks .12 is equal to ½, I would say ‘you know what half looks like and .12 is like 12 cents. Do those match up?’ ...Because, if you make anything from money for them, they’ll get it.”	1	Weak

APPENDIX K: TEACHERS' SCORES ON STRUCTURED CONTENT INTERVIEW (Q3)

Core Emphases: Decimals must be “lined up” when they are added or subtracted because only like quantities – e.g., tenths and tenths, thousands and thousands – can be combined. This is the same logic for finding a common denominator when adding or subtracting fractions. The most straightforward way to see why decimals “move” when multiplying or dividing is to convert the decimals into fractions with power of ten denominators (connection to study definition of decimal). For example, $0.23 \times 2.5 = \frac{23}{100} \times \frac{25}{10} = \frac{575}{1000}$ or 0.575.

Teacher	Teacher Definition	Score	Level of Understanding
Hamlin	<p>“We’ve done examples. That’s come up actually pretty recently and we talked about money, first of all, and I put up that I have 50 cents and I have \$5.00 so I put, you know, .50 or whatever and \$5.00 and I added them and so now I have 55 cents, right? Now does that make sense to you? ... Well, why? ... You’ve gotta add the same thing... the pieces that you’re adding have to be the same size.” ... “When we multiply decimals we turn the decimals into fractions... 25 times .5...and they get, you know, 125/1000ths and then we talked about, um, you know, started looking a those. What do you notice?... You know, I’m ending up with thousandths here. Why am I ending up with thousandths? Well, ‘cause I’m multiplying hundredths times tenths.”</p>	3	Strong
Smith	<p>“One thing -- I think, you know, you start having a conversation about place value. You could start with whole numbers and we don’t use a decimal point when we’re just adding whole numbers but we could go -- you could have like $35 + 28$, if you were to put a decimal point in there for each number and it would still be 35 and 28, where would you put the decimal point on each one? Okay. Does it does it line up? Well, yeah because I’ve got the ones and I’ve got the tens... The estimation is important. It’s a way to keep place value so that we’re adding 100ths to 100ths, 10ths to 10ths, 1s to 1s and so forth when we’re doing a regrouping.” ... “I would tell the student is the decimal place for right now is placed in your answer so that based on your estimation it makes sense to you.”</p>	1.5	Weak-Moderate
Wiggins	<p>“Well, as far as for the adding, I mean, we line ‘em up when we’re doing whole numbers ‘cause each number has the same value... We’ve done a lot more with just place value and they’re getting to being okay with the whole idea that they have a value.” ... “Well, I’d probably bring them to like a simple question where it’s like 3 times .2 ... ‘cause like 3 times 2 is 3 groups of 2 or two groups of 3... So it’s 3 groups of .2... They know [that] 3 groups of 2 is 6, so 3 groups of .2 has to be .6.”</p>	1.5	Weak-Moderate

APPENDIX L: TEACHERS' SCORES ON STRUCTURED CONTENT INTERVIEW (Q4)

Core Emphases: The underlying student misconception is associated with the identity property of multiplication, which says that the value of a number does not change when it is multiplied by one (any form of one). More formally, $\frac{a}{b} \bullet \frac{n}{n} = \frac{a \bullet n}{b \bullet n} = \frac{a}{b}$. This student thought $\frac{2}{2}$ was a form of one, but the represented parts of numbers, not the factors of numbers, as in $\frac{a \bullet n}{b \bullet n}$. Since every multiplication problem can be rewritten as a division problem, the property also holds for division.

Teacher	Teacher Definition	Score	Level of Understanding
Hamlin	<p>“Well, if they’re changing it into an equivalent fraction they have to divide by one -- a form of one -- and in this case they are not dividing by a form of one. They’re just crossing off things... I think where they get this misconception is when they’re taught because of powers of ten, if you have 10/880, boom, you cross off the zeroes. Well, that works. You’re actually dividing the numerator and the denominator ...by ten tenths... That’s my biggest thing with my kids: ... if you don’t understand why that works, you can’t use it.”</p>	3	Strong
Smith	<p>“Well, I think, you know one thing that we could instruct to have them do is come over here to $\frac{1}{2}$...Start asking ... why is this a half? Well, because it’s a one in the numerator and it’s a 2 in the denominator ... what does that really mean?...I might move to $\frac{2}{4}$ths and $\frac{3}{6}$ths and start looking for a pattern: it looks like the numerator has what relationship with the denominator? It’s always half... <u>Interviewer prompt:</u> <i>If you have 80/160, that [crossing out the zeros] actually would work in this case, wouldn’t it? 80/160 -- 8/16ths would still equal a half.</i> “They have no concept of dividing each number by 10, which is still dividing it by the value of one, but it’s a tenth of the pieces. And, of course, they’re ten times larger and so you’ll need a tenth of them. They’re using a procedure here.”</p>	2	Moderate
Wiggins	<p>“I would probably go back and draw a picture of it. I mean, the odds are if you give them something to look at they can see that $\frac{8}{28}$ is definitely not $\frac{1}{2}$...You’d have to have that whole conversation about why it works with 10s...they understand that when you multiply by 10 it’s 2 -- like 2×10 is just 20. They know that and so you can just build off of what they know. So 100×2 has got to be 200...as far as the division we’ve talked about just reducing a fraction and how you can divide top and bottom by the same thing and in the 10s case it just -- you can cancel out a zero...you have to reduce by the same number on both..it goes to that whole ratio and keeping it consistent...I don’t know how I’d say it to a seventh grader right now though...I mean, even some kids I wouldn’t even have to go -- I’d just say think about it. You know what $\frac{1}{2}$ is and you know that 1 is half of 2; is 8 half of 28?”</p>	2	Moderate

APPENDIX M: TEACHERS' SCORES ON STRUCTURED CONTENT INTERVIEW (Q5)

Study Definition: A ratio is a comparison of two quantities by *division*. Sometimes a ratio is defined simply as “a comparison of two quantities,” but this definition is incomplete since it is unclear whether the comparison is additive or multiplicative. An additive comparison is the absolute difference between two quantities (e.g., my brother is 3 years older than I am), where a multiplicative comparison is a relative relationship between two quantities (e.g., my brother is twice as old as my niece). Ratios are multiplicative comparisons, which is why the “by division” phrase is critical to the definition. For any ratio, the quotient represents the relative magnitude of the two quantities, that is, the magnitude of one quantity as a multiple of the other. For example, suppose there are 2 cookies for every 5 students in a math class. The ratio $\frac{2}{5}$ means that the number of cookies is always $\frac{2}{5}$ the number of students and the number of students will always be $\frac{5}{2}$ the number of cookies. The relative magnitude of the two quantities holds for equivalent ratios, such as $\frac{4}{10}$ or $\frac{20}{50}$; more generally the ratio $\frac{a}{b}$ refers to the comparison of na to nb for any n not 0 that is defined in the situation.

Teacher	Teacher Definition	Score	Level of Understanding
Hamlin	“A ratio is a comparison between two things...put in fractional form, a lot of times, or with the little dot dots... We always talk to the kids, you know, the amount of girls to the amount of boys; the amount of left-hand people to the amount of right-hand people; how many are blue eyed and how many are brown eyed? We talk about what’s a comparison...and sometimes we’ll represent it as a fraction. It can be manipulated like a fraction...you can add ratios and subtract ‘em and multiply and do all those fractional things with ratios.”	1	Weak
Smith	“It’s a way to compare groups of numbers and I think that could be an introduction. Say we could start with all right, we’re gonna talk about ratios of boys to girls in the class, something that they can concretely see...I have one cup of sugar to every two cups of flour so if I want to make a recipe and whatever that is is ‘gonna feed 12 people, I need to feed 60 people or 300 people. I can either do this and repeat my recipe enough times until I can feed 300 or I can just put it all together in one big bowl at one time. <u>Interviewer prompt:</u> <i>So that relationship between the flour and the sugar, what kind of relationship is that mathematically?</i> Well, it’s a ratio but it’s also -- it’s a comparison of two values a sense by division because if it’s 1:2 the flour -- we’ll say that one cup of the flour to two cups sugar, the flour is always half of the sugar. So no matter how much sugar I have, I need to divide that amount by two to get my flour or if I have my flour I could always multiply by two to get the amount of sugar.”	2	Moderate
Wiggins	“A ratio is just a comparison of two things and ...it’s got to keep that consistency between itself. I mean, just like I buy two shoes for two feet. I can’t buy three shoes for two feet... I would use the ratio table. I love the ratio tables.”	1	Weak

APPENDIX N: TEACHERS' SCORES ON STRUCTURED CONTENT INTERVIEW (Q6)

Core Emphases: A fraction is a number, while a ratio is a comparison between two numbers. A fraction has an exact location on a number line, but a ratio does not. The phrase “part of a whole” is commonly applied to fractions, though the study is trying to get teachers to get away from using this definition exclusively, while “part:whole” is a type of ratio (e.g., the ratio of the number of boys in a class to the total number of students). **Need to add more here – Beckman: how ratios give rise to fractions (and vice versa).**

Teacher	Teacher Definition	Score	Level of Understanding
Hamlin	<p><u>Interviewer:</u> <i>So [you said that] ratio is a comparison of two quantities.</i> “And a fraction is a part-to-whole relationship. And a ratio can be between two different things – it can be between two different things – where generally a fraction is a – you’ve got a whole and then the numerator is the part of that whole, where like in boys to girls it’s not that way. ...[Regarding the connection between ratios and fractions]: That’s befuddling. I’ll have to think over that one. I haven’t really given that much thought to it truthfully.”</p>	1	Weak
Smith	<p>“Well, if you talked about ratios part-to-part and part-to-whole and you’ve talked about fractions as part of a whole, whatever that whole is, then you can start talking about well, is a fraction always the same as a ratio? ...Then you start -- you can start having the discussion, okay, if a fraction basically means it’s the parts that I’m counting compared to the number of parts or pieces or parts that it takes to represent the whole, so that’s the number of parts and the whole, but if I have two types of ratios over here -- a part-to-part and a part-to-whole -- which one does it sound like it’s more like? ... A fraction can sometimes be a ratio but it is always going to be a ratio. And I think we -- I would talk about the part-to-part and the part-to-whole.”</p>	1	Weak
Wiggins	<p>“I when I think of ratios I think of the part, no, fractions I think of as being part-to-whole, so like you have three pieces out of five of a cake. But a ratio could be or it could be I’ve got 14 girls and 24 boys. You know ...a fraction is always part to whole, [but] a ratio could be part-to-part or a part-to-whole. <u>Interviewer prompt:</u> <i>Do you see the number line coming into play at all with fractions and ratios?</i> I probably wouldn’t go at it that way just because I don’t -- we don’t use the fractions on a number line.”</p>	1	Weak

APPENDIX O: TEACHERS' SCORES ON STRUCTURED CONTENT INTERVIEW (Q7)

Core Emphases: A linear model (e.g., number lines, fraction strips) is an appropriate way to show how fractions can be added, including the rationale for common denominators. An area model (e.g., rectangle, circle) is an appropriate way to what happens when fractions are multiplied. For example, an area model can be used to show how $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$. The professional development also presented set models as a good way to represent multiplication of fractions, though the area model was emphasized more.

Teacher	Teacher Definition	Score	Level of Understanding
Hamlin	<p>“I would use the same model, so we could see the differences... Well, I’ll go back to brownie problem [area model], okay? One-third, so we find out what one-third is. This would be one-third of three-fourths is the way I think of it so first I need to figure out what three-fourths is. So divide it into fourths or they would divide it into fourths for me... Why don’t you draw a picture – draw the cookies out [set model], now what does that one-third mean? How do I equally divide that? So then I told them – well you could read that fraction one-third as one out of every three. So, John ate one out of every three cookies, right? ... You know, we’d go back to my money model [for addition]. When you’re adding fractional pieces – pieces like pennies or pieces of a dollar – well, you need to be all adding the same thing. Well, if you’re adding the same thing what’s important about these denominators? They have to be the same size pieces.”</p>	2	Moderate
Smith	<p>“I think one place you might want to start is start using, you know, their number sense. Coming back to, all right, I’ve got $\frac{3}{4}$ of this bar, and I find out one group of that I have $\frac{3}{4}$, all right? If I have two of those, okay, then I have $\frac{3}{4}$ -- each one of these is $\frac{1}{4}$. Now I have another three so I have $\frac{6}{4}$. So $\frac{6}{4}$ is greater than $\frac{3}{4}$, okay? ... So using number sense, if this number stays the same and I multiply it by a smaller number just using yours what’s going to have to happen to my product if I’m multiplying simply by a smaller value? ... And then with addition ... I started with $\frac{1}{3}$ and $\frac{3}{4}$ and now I’m dealing with 12ths in my answer and this numerator is bigger than my denominator... Tell me what part of a whole I have and, you know, start putting ‘em together and they could estimate it.”</p>	1	Weak
Wiggins	<p>“I a lot of times you can refresh them back to like the 2 times 3 is 6, it’s 3 groups of 2 and this is $\frac{1}{3}$ of a group of $\frac{3}{4}$... And they can at least see it’s gonna be a smaller answer ... I think I would eventually have to go to a picture.[insert teacher diagram] ... [For addition]: I think I would go to like fraction strips. Where you take a strip and you cut it -- fold it into three and you take one of them, and you take another strip and fold it into four equal parts and you have to take three of them and then you compare it to a couple of strips and say that yeah, you end up with more than what you began with... if you wanted to get to the common denominators then pull out the fraction circles and do that. Take the $\frac{1}{3}$ and the $\frac{1}{4}$ and then take other pieces to try and take another color and try and get the answer and you could put, okay, you have a $\frac{1}{3}$ piece, you’d have the $\frac{3}{4}$ piece and you have to put pieces on top of them if really wanted an exact answer.”</p>	1.5	Moderate

REFERENCES

- Abdal-Haqq, I. (1995). *Making time for teacher professional development* (Digest 95-4). Washington, DC: ERIC Clearinghouse on Teaching and Teacher Education.
- Alexander, P.A., & Murpy, P.K. (1998). The research base for APA's learner-centered psychological principles. In Lambert, N.M. and McCombs, B.L. (eds), *Issues in school reform: A sampler of psychological perspectives on learner-centered schools*. Washington, D.C.: The American Psychological Association.
- Ball, D. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal*, 90(4), 449-466.
- Ball, D. L., Phelps, G., & Thames, M. H. (2006). *Content knowledge for teaching: What makes it special*. Unpublished manuscript, University of Michigan, Ann Arbor.
- Beckman, S. (2005). *Mathematics for elementary teachers*. New York, NY: Pearson-Addison Wesley
- Birman, B., Le Floch, K., Klekotka, A., Ludwig, M., Taylor, J., Walters, K., Wayne, A., Yoon, K., Vernez, G., Garet, M., & O'Day, J. (2007) *State and Local Implementation of the No Child Left Behind Act, Volume II – Teacher Quality Under NCLB: Interim Report*. Washington D.C.: U.S. Department of Education.
- Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, C., & Loeff, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26, 499-53.

- Cochran-Smith, M., & S. L. Lytle (1999). Relationships of knowledge and practice: Teacher learning in communities. *Review of Educational Research in Education* 24: 249-305.
- Cohen, D.C. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis*, 12(3), 311-329.
- Cohen, D.K., & Hill, H.C. (1998). Instructional policy and classroom performance: The mathematics reform in California. *Consortium for Policy Research in Education (CPRE) Research Report Series RR-39*. Philadelphia: CPRE.
- Cohen, D.K., Moffitt, S.L., & Goldin, S. (2007). Policy and practice: The dilemma. *American Journal of Education*, 113, 515-548.
- Cramer, K., & Lesh, R. (1988). [Rational number knowledge of preservice elementary education teachers](#). In M. Behr (ed.), *Proceedings of the 10th Annual Meeting of the North American Chapter of the International Group for Psychology of Mathematics Education* (pp. 425-431). DeKalb, IL.: PME.
- Cuban, L. (1990). Reforming again, again, and again. *Educational Researcher*, 19(1), 3-13.
- Darling-Hammond, L., & Sykes, G. (2003). Wanted: A national teacher supply policy for education: The right way to meet the "Highly Qualified Teacher" challenge. *Education Policy Analysis Archives*, 11(33). Retrieved May 10, 2007, from <http://epaa.asu.edu/epaa/v11n33/>.
- Elmore, R., Peterson, P., & McCarthey, S. (1996). *Restructuring in the classroom: Teaching, Learning & School Organization*. San Francisco, CA: Jossey-Bass.

- Flyvberg, B. (2001). *Making social science matter: Why social inquiry fails and how it can succeed again*. Cambridge, UK: Cambridge University Press.
- Franke, M.L., & Kazemi, E. (2001). Learning to teach mathematics: Developing a focus on students' mathematical thinking. *Theory into Practice*, 40, 102-109.
- Garet, M., Porter, A., Desimone, L., Birman, B., & Yoon, K. (2001). What makes professional development effective? Analysis of a national sample of teachers. *American Education Research Journal*, 38(3), 915-945.
- Graeber, A., Tirosh, D., & Glover, R. (1989) Preservice teachers' misconceptions in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, 20(1), 95-102.
- Hawley, W., & Valli, L. (1999). The essentials of effective professional development. In Darling-Hammond, L. & Sykes, G. (eds.) *Teaching as the learning profession: Handbook of policy and practice*. San Francisco: Jossey-Bass Publishers.
- Hiebert, J. & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Charlotte, NC: Information Age Publishing.
- Hiebert, J., Gallimore, R., & Stigler, J.W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one? *Educational Researcher*, 31(5), 3-15.

- Hill, H., Rowan, B., & Ball, D. (2005) Effects of teachers' mathematical knowledge on student achievement. *American Education Research Journal*, 42(2), 371-406.
- Ingersoll, R. (1999). The Problem of Underqualified Teachers in American Secondary Schools. *Educational Researcher*, 28(2), 26-37.
- Ingersoll, R. (2001). Teacher turnover and teacher shortages. *American Educational Research Journal* 38(2), 499-534
- Johnson, R., Siegler, R., & Alibali, M. (2001). Developing conceptual understanding and procedural skill: An iterative process. *Journal of Education Psychology*, 93(2), 346-362.
- Joyce, B., & Showers, B. (2002). *Student achievement through staff development*. Alexandria, VA: ASCD
- Kennedy, M. (1998) Form and Substance in Inservice Teacher Education. Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA.
- Kennedy, M. (2005). *Inside teaching: How classroom life undermines reform*. Cambridge, MA: Harvard University Press.
- Knapp, M. (2003). Professional development as a policy pathway. *Review of Research in Education*, 27, 109-157.
- Kilpatrick, J., Swafford, J., & Findell, B. (eds.) (2001) *Adding it up: Helping children learn mathematics*. Washington, D.C.: National Academy Press.

- Lamon, S. (1999). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Lortie, D.C. (1975). *Schoolteacher: A sociological study*. Chicago, IL: University of Chicago Press.
- Ginsberg, A., Leinwand, S., Anstrom, T., & Pollock, E. (2005). *What the world can learn from Singapore's world class mathematics system (and what Singapore can learn from the United States): An exploratory study*. Washington, D.C.: American Institutes for Research.
- Glesne, C., & Peshkin, A. (1992). *Becoming qualitative researchers: An introduction*. White Plains, NY: Longman.
- Little, J.W. (1993). Teachers' professional development in a climate of educational reform. *Education Evaluation and Policy Analysis*, 15(2), 129-151.
- Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Merriam, S.B. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Jossey-Bass.
- Merriam-Webster (2007). Retrieved January 4, 2007, from www.m-w.com/dictionary/mathematics.
- Milgram, R.J. (1999). Why long division? Paper presented at the conference on standards-based K-12 education. California State University, Northridge, CA.

Retrieved January 12, 2007, from

http://www.shearonforschools.com/why_long_division.htm

Milgram, R.J. (2005). *The mathematics pre-service teachers need to know*. Stanford, CA: Department of Mathematics.

National Board for Professional Teaching Standards (2007). *The five core propositions*. Retrieved November 12, 2007, from http://www.nbpts.org/the_standards/the_five_core_propositio

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (2000). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (2007). *Curricular focal points*. Reston, VA: NCTM.

National Mathematics Advisory Panel (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, U.S. Department of Education: Washington, DC.

Parsons, C. (1983). *Mathematics in philosophy: Selected essays*. Ithaca, N.Y.: Cornell University Press.

Perie, M., Grigg, W.S., & Dion, G.S. (2005). *The nation's report card: Mathematics 2005* (NCES 2006-453). U.S. Department of Education, Institute of Education

Sciences, National Center for Education Statistics. Washington, D.C.: U.S. Government Printing Office.

- Post, T., Harel, G., Behr, M., & Lesh, R. (1988). Intermediate teachers' knowledge of rational number concepts. In Fenneman, et al. (Eds.), *Papers from first Wisconsin symposium for research on teaching and learning mathematics*, pp.194-219. Madison, WI: Wisconsin Center for Education Research.
- Prawat, R.S., Remillard, J., Putnam, R. T., & Heaton, R. M. (1992). Teaching mathematics for understanding: Case studies of four fifth-grade teachers. *Elementary School Journal*, 93,(2), pp. 145-152.
- Prichard Committee for Academic Excellence and Partnership for Kentucky School Reform (1995). *Professional development in the service of of systemic reform: Policy action research and dissemination project*. Proposal submitted to Pew Charitalbe Trusts.
- Putnam, R.T., & Borko, H. (1997). Teacher learning: Implications of the new view of cognition. In B.J. Biddle, T.L. Good & I.F. Goodson (Eds.) *The international handbook of teachers and teaching*. Dordrecht, Netherlands: Kluwer.
- Reeve, C.D.C. (2003). *The Socratic movement*. In Curren, R. (Ed) *A companion to the philosophy of education*. Oxford, UK: Blackwell.
- Rittle-Johnson, B., Siegler, R.S., & Alibali, M.W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, 93(2), 346-62

- Sparks, D., & Hirsh, S. (2006). A national plan for improving professional development. Retrieved November 4, 2006, from <http://www.nsd.org/library/authors/NSDCPlan.cfm>
- Stigler, J.W., & Hiebert (1997). Understanding and improving classroom mathematics instruction: An overview of the TIMSS video study. *Phi Delta Kappan*, 79(1), 14-21.
- Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15, 4-14.
- Snow-Renner, R., & Lauer, P.A. (2005). Professional development insights. Retrieved October 22, 2007, from http://www.mcrel.org/PDF/ProfessionalDevelopment/5051IR_Prof_dvlpmt_analyses
- Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: The Free Press.
- Talbert, J., & McLaughlin, M. (1994). Teacher professionalism in local school contexts. *American Journal of Education*, 102, 123-153.
- U.S. Department of Education, National Center for Education Statistics (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study*, (NCES 2003-013 Revised). Washington, D.C.: U.S. Government Printing Office.

- Van Eekelen, I.M., Vermunt, J.D., & Boshuizen, H.P.A. (2006). Exploring teachers' will to learn. *Teaching and Teacher Education*, 22, 408-423.
- Wilson, S.M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iron-Nejad and P.D. Pearson (Eds.), *Review of Research in Education*, 24, 173-209.
- Wu, H. (1996) On the educating of mathematics teachers. Retrieve January 7, 2007, from <http://math.berkeley.edu/~wu/>.
- Wu, H. (1996) The mathematician and the mathematics education reform. Retrieved January 7, 2007, from <http://math.berkeley.edu/~wu/>.
- Wu, H. (2005) Professional development: The hard work of learning mathematics. Retrived January 7, 2007, from <http://math.berkeley.edu/~wu/>.
- Yin, R. (2003). *Case study research: Design and methods*. Thousand Oaks, CA: Sage.
- Yoon, K.S., Duncan, T., Lee, S., Scarloss, B., & Shapley, K. (2007). *Reviewing the evidence on how teacher professional development affects student achievement* (Issues & Answers Report, REL 2007-No.033). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Educational Evaluation and Regional Assistance, Regional Educational Laboratory Southwest. Retrieved October 12, 2007, from <http://ies.ed.gov/ncee/edlabs>.