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(First draft)

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Abstract

In This paper we investigate the design of controllers, for discrete-time Markovian jump linear systems, that achieve optimal reference tracking in the presence of preview. In particular, given a reference sequence, we obtain the optimal control law for the fully observed case, while the output feedback case is also briefly discussed. We provide the optimal control law for the infinite and finite optimization-horizon cases. The optimal control policy consists of the additive contribution of two terms: a feedforward term and a feedback term which is identical to the standard LQR solution. We provide explicit formulas for computing the feedforward term, while establishing a comparison with the internal model principle.

I. Introduction

This paper deals with the problem of designing control systems that achieve optimal reference tracking in discrete-time. More specifically, we consider the *servomechanism* problem, i.e., given an output reference, the objective is to design feedback and feedforward strategies so that pre-selected maeasured variables of the plant track the reference optimally, according to a quadratic cost. In contrast with existing work in optimal reference tracking, we consider a plant that is linear but varies in time according to a Markovian process that takes values in a finite alphabet, such systems are denoted as *Markovian jump linear systems*.

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Definition 1.1: (**Fully-observed Markovian jump linear system**) Let $\bar{m}$, n and q be given positive integers along with a matrix of conditional probabilities $M \in [0,1]^{m \times m}$ satisfying $\sum_{i=1}^{\bar{m}} M_{ij} = 1$, for each j in the set $\{1, \dots, \bar{m}\}$. Consider also a given collection of matrices $\{A_i\}_{i=1}^{\bar{m}}$ and $\{B_i\}_{i=1}^{\bar{m}}$, where for each integer i in the set $\{1, \dots, \bar{m}\}$ it holds that $A_i \in \mathbb{R}^{n \times n}$ and $B_i \in \mathbb{R}^{n \times q}$. In addition, consider two independent random variables $\mathbf{x}(0)$ and $\mathbf{m}(0)$ taking values in $\mathbb{R}^n$ and $\{1, \dots, \bar{m}\}$, respectively. The following specifies a discrete-time fully-observed Markovian jump linear system:

$$\mathbf{x}(k+1) = A_{\mathbf{m}(k)}\mathbf{x}(k) + B_{\mathbf{m}(k)}\mathbf{u}(k), \;\; k \geq 0 \tag{1}$$

where $\mathbf{m}(k)$ is an autonomous Markovian process taking values in the set $\{1, \dots, \bar{m}\}$ and whose statistical behavior is governed by $Pr(\mathbf{m}(k+1) = i | \mathbf{m}(k) = j) = M_{ij}$. In this description, $\mathbf{u}(k)$ takes values in $\mathbb{R}^q$ and it represents the plant's input.

Notice that the Markovian jump linear system defined by (1) has a hybrid state composed by $\mathbf{x}(k)$, the *continuous* component, and by $\mathbf{m}(k)$, the *discrete* part of the state. The system features $\bar{m}$ modes of operation which are specified by (A_1, B_1) through (A_m, B_m). The mode process $\mathbf{m}(k)$ determines which mode of operation is active at each instant of time.

A. Brief survey of related results and summary of the technical contributions of this paper

The problem of designing controllers for Markovian jump linear systems that achieve optimal reference tracking, also referred to as the *servomechanism problem*, has not been investigated. This section starts with a short survey, of the state of the art in the design of optimal controllers of Markovian jump linear systems. This is followed by a discussion of existing results in optimal reference tracking for deterministic systems.

Results on Optimal Control of Markovian Jump Linear Systems Motivated by a wide spectrum of applications, for the last thirty years, there has been active research in the analysis [32], [31] and in the design of controllers [30] for Markovian jump linear systems. More specifically, in the last fifteen years, the classical paradigms of optimal control have been solved for Markovian jump linear systems, such as the ones defined by H_2 and mixed H_2/H_∞ measures of performance [29], [28], [27] (see [2] for a more detailed survey of existing work). Other approaches aiming at the design of robust controllers can be found in [26], [24]. Not only optimal solutions were fully characterized but also the optimal cost and its associated control law can be computed by means of solving *linear matrix inequalities* (LMIs) [25], which are convex programs that can be solved very efficiently by a variety of widely available mathematical software tools.

Brief Survey of Results on the Theory of Optimal Reference Tracking for Deterministic Systems (Optimal servomechanism design) A classical approach in servomechanism design is to guarantee asymptotic reference tracking via the internal model principle [23]. Simple applications of this idea are practical rules that date back to the early twentieth century, such as achieving asymptotic tracking of step references by making sure that the open loop gain of a linear, time-invariant feedback system has pole at 1, or at 0 for continuous time systems. Asymptotic tracking of many other periodic references can be achieved using the internal model principle, at the expense of *state augmentation techniques*. In the late eighties, techniques based on operator theory were used to derive control laws for linear and time-invariant systems that guarantee optimal reference tracking, under the assumption of finite horizon and infinite horizon preview [22], [21]. The papers [20], [19], [18], [34], [33] are also relevant contributions for the particular case of no reference preview. Examples of application can be found in [17], [35], [36], [16], [15]. More recently, since the nineties, the theory of control leading to optimal reference tracking, for deterministic systems, achieved a level of completion. In particular, more general performance metrics, such as H_∞, were considered [14], [13], [12]. There is also a substantial collection of results on fundamental limits of optimal reference tracking [37], [38] for a variety of metrics [11], constraints [10], [9], [8]and plant classes [7], [6], [5], [4], [3]. All of these results, in one way or another, conclude that reference preview may lead to a substantial increase in the tracking performance.

B. Paper Organization

This paper has three sections, besides the introduction: Section II gives preliminary definitions and a review of the linear quadratic optimal control of Markovian jump linear systems, while Sections III and IV focus on the problem formulation of the optimal preview control problem and its solution for the infinite horizon case, respectively.

II. Preliminary definitions and Review of the optimal linear quadratic regulator (LQR) for Markovian jump linear systems

Definition 2.1: (**Regulator**) Let $\mathbf{s}(k) = (\mathbf{x}(k), \mathbf{m}(k))$ be the state of a n dimensional and fully-observed Markovian jump linear system (MJLS) with input $\mathbf{u}(k)$ taking values in $\mathbb{R}^q$. The class of regulators $\mathbb{U}^{Reg}$ consists of all feedback policies $\mathcal{U}^{Reg}$ with the following structure:

$$\mathbf{u}(k) = \mathcal{U}^{Reg}(k, \{\mathbf{s}(l)\}_{l=0}^{k}) \tag{2}$$

Definition 2.2: (**Linear quadratic regulator (LQR): problem formulation**) Consider a fully-observed Markovian jump linear system, as in Definition 1.1, and denote by n and q its order and dimension of the input, respectively. Given a regulator $\mathcal{U} \in \mathbb{U}^{Reg}$, time horizon $T \in \mathbb{N} \bigcup \{\infty\}$, and symmetric matrices $R \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{q\times q}$, which are semi-definite and positive definite, respectively, we adopt the following cost function:

$$\mathcal{J}^{LQR}(\mathcal{U}, T) = E_{\mathbf{x}(0),\{\mathbf{m}(l)\}_{l=0}^{T}} \left[\sum_{l=0}^{T} \mathbf{x}(l)' R \mathbf{x}(l) + \mathbf{u}(l)' Q \mathbf{u}(l) \right],\ T < \infty \tag{3}$$

where $\mathbf{u}(k) = \mathcal{U}(k, \{\mathbf{s}(l)\}_{l=0}^{k})$. The *linear quadratic regulator* paradigm is defined by the following optimization problem:

$$\mathcal{U}^{*,LQR,T} = arg \min_{\mathcal{U}\in\mathbb{U}^{Reg}} \mathcal{J}^{LQR}(\mathcal{U}, T) \tag{4}$$

The infinite-horizon LQR controller is defined as:

$$\mathcal{U}^{*,LQR,\infty} = \lim_{T\to\infty} \mathcal{U}^{*,LQR,T} \tag{5}$$

The solution to the infinite horizon ($T = \infty$) LQR for MJLS has the following form:

$$\mathcal{U}^{*,LQR,\infty}(k, \{\mathbf{s}(l)\}_{l=0}^{k}) = -K_{\mathbf{m}(k)}\mathbf{x}(k), k \geq 0 \tag{6}$$

where K_1 through $K_{\bar{m}}$ are matrices in $\mathbb{R}^{q\times n}$ given by:

$$K_i = \left(Q + B_i'\bar{P}_i B_i\right)^{-1} B_i'\bar{P}_i A_i \tag{7}$$

The characterization of the optimal LQR feedback law is completed by the following collection of coupled Riccati equations:

$$P_i = R + A_i'\bar{P}_i A_i - A_i'\bar{P}_i B_i \left(Q + B_i'\bar{P}_i B_i\right)^{-1} B_i'\bar{P}_i A_i,\ P_i = P_i' > 0,\ i \in \{1, \ldots, \bar{m}\} \tag{8}$$

$$\bar{P}_i = \sum_{j\in\{1,\ldots,\bar{m}\}} [M]_{j,i} P_j\ i \in \{1, \ldots, \bar{m}\} \tag{9}$$

Remark 2.1: The following are basic properties of the optimal solution to the infinite horizon LQR paradigm for MJLS:

- The optimal solution to (6) exists and the optimal cost is finite if and only if the coupled Riccati equations (8)-(9) have a solution.
- From (6) it follows that the optimal feedback policy is a memoryless function of the state $\mathbf{s}(k)$.
- The solution to (8)-(9), or a certificate of infeasibility, can be obtained via linear matrix inequalities (LMI) methods.

III. Optimal preview full-state feedback control: Problem formulation

In this section we formulate the optimal preview control paradigm, under full-state feedback. We start by defining the following class of allowable preview controllers:

Definition 3.1: (**Preview controller**) Let $\mathbf{s}(k) = (\mathbf{x}(k), \mathbf{m}(k))$ be the state of a n dimensional and fully-observed Markovian jump linear system (MJLS) with input $\mathbf{u}(k)$ taking values in $\mathbb{R}^q$. Given a reference sequence $\{r(l)\}_{l=0}^{\infty}$ taking values in $\mathbb{R}^n$, the class of preview controllers $\mathbb{U}^{Prev}$ consists of all feedback policies $\mathcal{U}^{Prev}$ with the following structure:

$$\mathbf{u}(k) = \mathcal{U}^{Prev}(k, \{\mathbf{s}(l)\}_{l=0}^{k}, \{r(l)\}_{l=0}^{\infty}) \tag{10}$$

Definition 3.2: (**Optimal preview control**) Consider a fully-observed Markovian jump linear system, as in Definition 1.1, and denote by n and q its order and dimension of the input, respectively. Given a sequence $\{r(l)\}_{l=0}^{\infty}$ taking values in $\mathbb{R}^n$, a preview controller $\mathcal{U} \in \mathbb{U}^{Prev}$, time horizon $T \in \mathbb{N} \bigcup \{\infty\}$, and symmetric matrices $R \in \mathbb{R}^{n \times n}$ and $Q \in \mathbb{R}^{q \times q}$, which are semi-definite and positive definite, respectively, we adopt the following cost function:

$$\mathcal{J}^{Prev}\left(\mathcal{U}, \{r(l)\}_{l=0}^{\infty}, T\right) = E_{\mathbf{x}(0), \{\mathbf{m}(l)\}_{l=0}^{T}}\left[\sum_{l=0}^{T} (\mathbf{x}(l) - r(l))' R (\mathbf{x}(l) - r(l)) + \mathbf{u}(l)' Q \mathbf{u}(l)\right], T < \infty \tag{11}$$

where $\mathbf{u}(k) = \mathcal{U}(k, \{\mathbf{s}(l)\}_{l=0}^{k}, \{r(l)\}_{l=0}^{\infty})$. The optimal *preview control* paradigm is defined by the following optimization problem:

$$\mathcal{U}^{*,Prev,T} = arg \min_{\mathcal{U} \in \mathbb{U}^{Prev}} \mathcal{J}^{Prev}\left(\mathcal{U}, \{r(l)\}_{l=0}^{\infty}, T\right) \tag{12}$$

The solution to the infinite-horizon optimal preview paradigm is defined by:

$$\mathcal{U}^{*,Prev,\infty} = \lim_{T \to \infty} \mathcal{U}^{*,Prev,T} \tag{13}$$

provided that the limit is well defined.

A. Preview control with respect to pre-selected performance variables

In most applications, there is no need to specify tracking objectives with respect to the entire state. For instance, consider a matrix $C \in \mathbb{R}^{\eta \times n}$, where η is an integer strictly smaller than n, the dimension of $\mathbf{x}(k)$. In this case, $\mathbf{z}(k) \stackrel{def}{=} C\mathbf{x}(k)$ may represent a vector of measured variables, such as the velocity vector of a moving vehicle. Given a sequence $\{\tilde{r}(l)\}_{l=0}^{\infty}$, taking values in $\mathbb{R}^{\eta}$, one might be interested in computing the optimal solution to the optimal control problem based on the following cost:

$$\mathcal{J}^{Prev,meas}\left(\mathcal{U}, \{\tilde{r}(l)\}_{l=0}^{\infty}, T\right) = E_{\mathbf{x}(0),\{\mathbf{m}(l)\}_{l=0}^{T}} \left[\sum_{l=0}^{T} (\mathbf{z}(l) - \tilde{r}(l))'(\mathbf{z}(l) - \tilde{r}(l)) + \mathbf{u}(l)'Q\mathbf{u}(l) \right] \tag{14}$$

However, notice that such a problem can be solved using our formulation of Definition 3.2 by selecting $Q = C'C$ and $\tilde{r}(l) = C^{\dagger} r(l)$, where $C^{\dagger}$ represents the Moore-Penrose pseudo-inverse of C.

IV. Optimal solution for the infinite-horizon case ($T = \infty$)

Theorem 4.1: (**Part-I:Existence of a solution**) Consider a the optimal preview control problem of Definition 3.2. The optimal solution to the infinite-horizon paradigm (see (12)) exists and the optimal cost is well defined if and only if the following two conditions hold:

- The optimal LQR Riccati equations (8)-(9) have a solution.
- The reference sequence $\{r(l)\}_{l=0}^{\infty}$ is such that the following limits are well defined for all k:

$$B_i' L_i(k, \infty) = B_i' \lim_{T \to \infty} L_i(k, T), \ \ i \in \{1, \ldots, \bar{m}\} \tag{15}$$

where

$$L_i(k,T) = \begin{cases} (A_i - B_i K_i)' \left[\bar{P}_i (A_i r(k) - r(k+1)) + \bar{L}_i(k,T) \right] & \text{if } k < T \\ 0 & \text{if } k = T \end{cases}, \ \ T < \infty \tag{16}$$

$$\bar{L}_i(k,T) = \sum_{j \in \{1,\ldots,\bar{m}\}} [M]_{j,i} L_j(k+1, T) \tag{17}$$

(**Part-II:Optimal solution**) If the conditions above hold then the solution to the optimal preview control paradigm is given by:

$$\mathcal{U}^{*,Prev,\infty}(k,\{\mathbf{s}(l)\}_{l=0}^{k},\{r(l)\}_{l=0}^{\infty}) = \\ -\left(Q+B_i'\bar{P}_iB_i\right)^{-1}B_i'\left(\bar{L}_i(k,\infty)+\bar{P}_i(A_ir(k)-r(k+1))\right)-K_{\mathbf{m}(k)}(\mathbf{x}(k)-r(k)),\ k\geq 0 \tag{18}$$

where K_1 through $K_{\bar{m}}$ are matrices in $\mathbb{R}^{q\times n}$ given by the optimal LQR solution (7). The matrices $\bar{P}_i$ follow from the LQR coupled Riccati equations (8)-(9).

Proof: For any given optimization horizon T, we use a dynamic programming method, analogous to the one adopted in [2] for deriving the optimal LQR, to obtain the following optimal preview control:

$$\mathcal{U}^{*,Prev,T}\,(k,\{r(l)\}_{l=0}^{\infty},T) = \\ -\left(Q+B_{\mathbf{m}(k)}'\bar{P}_{\mathbf{m}(k)}(k,,T)B_{\mathbf{m}(k)}\right)^{-1}\left(\bar{L}_{\mathbf{m}(k)}(k,T)+\bar{P}_{\mathbf{m}(0)}(k,T)(\mathbf{x}(k)-r(k))\right)-K_{\mathbf{m}(k)}(\mathbf{x}(k)-r(k)) \tag{19}$$

where $\bar{L}_i(k,T)$ is computed from (16)-(17) and $\bar{P}_{\mathbf{m}(0)}(k,T)$ is given by the following backward iterations:

$$P_i(k,T)=\begin{cases} R & \text{if } k=T \\ R+A_i'\bar{P}_i(k,T)A_i-K_i(k,T)'\left(Q+B_i'\bar{P}_i(k,T)B_i\right)K_i(k,T) & \text{otherwise}\end{cases} \tag{20}$$

$$\bar{P}_i(k,T)=\sum_{j=1}^{\bar{m}}[M]_{j,i}P_j(k+1,T) \tag{21}$$

where

$$K_i(k,T)=-\left(Q+B_i'\bar{P}_i(k,T)B_i\right)^{-1}B_i'\bar{P}_i(k,T)A_i \tag{22}$$

The proof follows by taking the limit when T goes to infinity □.

A. Comparison between infinite-horizon ($T=\infty$) optimal preview control and the LQR

The following is a list of observations relating optimal preview control and the LQR:

- The optimal preview control law (18) results from the additive contribution of a feedback term and a feedforward component. Notice that if $r(l)\stackrel{\Delta}{=}0$ then the optimal preview control law reduces to the LQR.

- If $r(l)$ is such that $A_i r(l) = r(l+1)$ then (18) reduces to the solution we would obtain from the well known internal model principle. In fact, if $A_i r(l) = r(l+1)$ then the solution to the optimal preview control paradigm consists of a simple modification of the LQR where the gain matrices are multipltied by the tracking error $\mathbf{x}(k) - r(k)$.
- The LQR optimal solution is well defined and the minimum cost is bounded if and only if the coupled Riccati equations (8)-(9) have a solution (see Remark 2.1). In constrast, the optimal preview control framework requires extra condition related with the convergence of the limits defining the sequences $L_i(k, \infty)$, leading to the conclusion that the well posedness of such an optimization paradigm will depend on the reference $r(k)$. This motivates Section IV-B, where we study computable criteria for establishing the boundedness of $L_i(k, \infty)$.

B. *Computation of* $L_i(k, \infty)$

The solution to the optimal preview control paradigm for linear and time-invariant systems can be found in several papers and books, such as [1]. However, prior work on the computation of the feedforward term in the preview control for deterministic systems (time-invariant or time-varying) is not applicable to the paradigm addressed in this paper.

The following proposition gives an explicit formula for computing $L_i(k, \infty)$ in the presence of constant references. Before we state such a proposition, we first introduce the following notation.

Notation: Given a collection of matrices (or vectors) W_1 through $W_{\bar{m}}$, we denote the corresponding block diagonal matrix as:

$$\mathfrak{D}\left(\{W_i\}_{i=1}^{\bar{m}}\right) \overset{def}{=} \begin{pmatrix} W_1 & 0 & \cdots & 0 \\ 0 & W_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & W_{\bar{m}} \end{pmatrix} \tag{23}$$

Given vectors v_1 through $v_{\bar{m}}$, we use the following notation to denote vectorization:

$$\mathfrak{V}(\{v_i\}_{i=1}^{\bar{m}}) \overset{def}{=} \begin{pmatrix} v_1 \\ \vdots \\ v_{\bar{m}} \end{pmatrix} \tag{24}$$

The Kronecker product between two matrices X and $Y \in \mathbb{R}^{n_1\times n_2}$ is denoted as:

$$X \otimes Y \stackrel{def}{=} \begin{pmatrix} X\ [Y]_{1,1} & X\ [Y]_{1,2} & \cdots & X\ [Y]_{1,n_2} \\ X\ [Y]_{2,1} & \cdots & \cdots & X\ [Y]_{2,n_2} \\ \vdots & \vdots & \vdots & \vdots \\ X\ [Y]_{n_1,1} & X\ [Y]_{n_1,2} & \cdots & X\ [Y]_{n_1,n_2} \end{pmatrix} \tag{25}$$

Proposition 4.2: Let integers $\bar{m}$, n, q, a vector $r \in \mathbb{R}^n$ and a stochastic matrix $M \in \mathbb{R}^{\bar{m}\times\bar{m}}$ be given. Consider matrices A_1 through $A_{\bar{m}}$ taking values in $\mathbb{R}^{n\times n}$ and symmetric positive semi-definite matrices $\bar{P}_1$ through $\bar{P}_{\bar{m}}$ also taking values in $\mathbb{R}^{n\times n}$. The sequences $L_1(k,\infty)$ through $L_{\bar{m}}(k,\infty)$, given in (15)-(17), are constant with respect to k and they can be computed as:

$$\mathfrak{V}(\{L_i(k,\infty)\}_{i=1}^{\bar{m}}) = \\ \Big(I - \mathfrak{D}\left(\{(A_i - B_iK_i)'\}_{i=1}^{\bar{m}}\right)(M'\otimes I_{n\times n})\Big)^{-1}\ \mathfrak{V}\left(\{(A_i - B_iK_i)'\bar{P}_i(A_i - I)r\}_{i=1}^{\bar{m}}\right),\ k\geq 0 \tag{26}$$

Proof: We start by representing the backward equations (16)-(17) in the following equivalent form:

$$\mathfrak{V}(\{L_i(k,T)\}_{i=1}^{\bar{m}}) = \mathfrak{D}\left(\{(A_i - B_iK_i)'\}_{i=1}^{\bar{m}}\right)(M'\otimes I_{n\times n})\mathfrak{V}(\{L_i(k+1,T)\}_{i=1}^{\bar{m}}) + \\ \mathfrak{V}\left(\{(A_i - B_iK_i)'\bar{P}_i(A_ir(k) - r(k+1))\}_{i=1}^{\bar{m}}\right),\ k < T \tag{27}$$

The proposition follows by noticing that if $r(l)$ is constant and equal to r then:

$$\mathfrak{V}(\{L_i(k,\infty)\}_{i=1}^{\bar{m}}) = \\ \sum_{l=0}^{\infty}\left[\mathfrak{D}\left(\{(A_i - B_iK_i)'\}_{i=1}^{\bar{m}}\right)(M'\otimes I_{n\times n})\right]^l \mathfrak{V}\left(\{(A_i - B_iK_i)'\bar{P}_i(A_i - I)r\}_{i=1}^{\bar{m}}\right) \tag{28}$$

□

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