TECHNICAL RESEARCH REPORT

Using Iterative Strategies for Aligning Attributed Points in Space

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TR 2005-2

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Abstract

Signatures based on shape features are expected to be useful for several similarity assessment applications. Such signatures can extract the relevant shape information needed for similarity assessment and ignore the shape information not needed for a particular application. Since different feature-based models are created in different coordinate systems, in order to compute distance between two sets of features, one needs to include geometric transformations for aligning features. Optimal alignment is the alignment that results in the minimum distance between two sets of features. In general, features can be viewed as attributed points in space. Hence, we need to develop techniques for aligning attributed points in space.

This report introduces iterative strategies for optimally aligning attributed points in space. Iterative strategies presented in this report involve successively applying algorithms that perform alignment under restricted rigid body transformations (e.g., rotations only or translations only) to solve point alignment problems that require higher dimension transformations (e.g., combinations rotations and translations). Our preliminary experimental results show that the idea of using of iterative strategies to solve higher dimension attributed point alignment problems is promising and can be used to perform feature-based shape similarity assessment.

1. Introduction

Popularity of 3D CAD systems is resulting in a large number of CAD models being generated. Availability of these CAD models is opening up new ways in which information can be archived, analyzed, and reused. 3D geometric information is one of the main components of CAD models. Therefore shape similarity assessment is a fundamental geometric reasoning problem that finds application in several different applications. The following two examples illustrate how shape similarity assessment can be used:

- **Cost Estimation For Machined Parts:** Nowadays, many job shops allow designers to submit a 3D model of the part to be machined over the Internet and provide a cost estimate based on the 3D part model. For some manufacturing domains such as rapid prototyping, reasonably accurate estimates of cost can be achieved by estimating volume or weight of the part. However, for some manufacturing domains such as machining, cost estimate depends on the geometric details of the object and automated procedures are not available for doing accurate cost estimation. Currently in such cases, humans perform cost estimation. In the Internet era, where designers solicit many quotes to make a decision, manual cost estimation is not economical. Cost of manufacturing a new part can be quickly estimated by finding previously manufactured parts that are similar in shape to the new part. If a sufficiently similar part can be found in the database of previously manufactured objects, then the cost of the new part can be estimated by suitably modifying the actual cost of the previously manufactured similar part.

- **Redesign Suggestion Generation For Multi Material Molding:** Converting a traditional molded assembly to a multi-material molded assembly often requires redesigning it. We believe that shape similarity assessment can be used in identifying the appropriate examples from the past that can be used as a guide to carry out the redesign task. Let us assume that we have constructed a repository of previously completed redesign projects. This repository includes both the initial and redesigned assemblies. Now let us assume that we need to redesign an assembly to make it manufacturable by multi-material molding. In this case we can find the initial designs in the repository that are closest to the assembly being redesigned. Now we can consult the repository to determine how those initial designs were modified. Hopefully, these examples can provide meaningful suggestions on how to carry out the redesign in the new project.

In many design and manufacturing applications the gross shape of the 3D parts does not play an important role in similarity assessment. Instead certain attributes of part features play a dominant role in determining the similarity between two parts. The attributes of part features used for similarity assessment need to be chosen depending on the application. For instance in cost estimation of machined parts the cost depends on attributes of machining features in the parts. Hence it is necessary to identify an appropriate feature-based representation for each application.

Feature-based similarity measures are defined using feature-based representations of the 3D parts. Hence a 3D part is represented by sets of features. In order to assess similarity between two sets of features it is necessary to compute the distance between them. The distance depends on the relative position of the two sets of features vectors. Applying a rigid transformation to a set of feature vectors in general yields different distance values. So in order to assess similarity between two parts, the rigid transformation that yields the minimum distance between the two sets of features needs to be found.

In general, geometric objects A and B would be modeled using their own local coordinate systems. Therefore, attributes of feature sets $F(A)$ and $F(B)$ are not invariant with respect to transformation due to presence of feature location and orientation attributes. When computing distance from $F(A)$ to $F(B)$, we need to first transform either $F(B)$ or $F(A)$ such that the distance between them is minimized. In this report, we will assume that $F(B)$ will be transformed by applying transformation $\mathbf{T}$. Transformed $F(B)$ will be denoted as $\mathbf{T}F(B)$. Now the asymmetric distance from $F(A)$ to $\mathbf{T}F(B)$ can be defined as:

$$\vec{d}(F(A), \mathbf{T}F(B)) = \frac{\sum_{i=1}^{n} \min_{f' \in F(B)} d(f_i, \mathbf{T}f')}{n}$$

Where, f and f' are features of sets $F(A)$ and $F(B)$ and n is the number of features of set $F(B)$. A symmetric distance function can simply be defined by taking average of asymmetric distance from $F(A)$ to $\mathbf{T}F(B)$ and $\mathbf{T}F(B)$ to $F(A)$.

In order to assess similarity between $F(A)$ and $F(B)$, the first step is to find transformation $\mathbf{T}$ that minimizes the distance function value. We will refer to such transformation as *optimal alignment* of feature-based models. Finding the optimal alignment is a task that involves a certain number of degrees of freedom, which depends on the transformation used and on the characteristics of the features being considered. In general finding the optimal alignment is harder if a higher dimension rigid transformation is involved. For lower dimension transformations it is possible to design algorithms that can directly find the optimal alignment.

The distance function makes use of the nearest neighbors. Existing point alignment techniques assume that the nearest neighbor information is available. Unfortunately, nearest neighbors change throughout the transformation space and hence it is difficult to apply classical optimization techniques to compute the transformation that leads to the minimum distance. Given two sets of features, there are exponentially many nearest neighbor combinations. Therefore solving this problem by enumeration is not likely to work. In practice, it turns out that actually a significantly lower number of combinations are geometrically feasible. One can imagine partitioning the transformation space into a cellular arrangement such that within each cell the nearest neighbors do not change. Now, within each cell one can solve the problem by using the analytical techniques and take the minimum over all the cells.

It appears to be very difficult to design algorithms to directly obtain the optimal alignment of feature-based models for higher dimension transformation spaces. However, many applications involve finding the optimal alignment for higher dimension transformations. We believe that in these cases solutions can be found by iteratively solving many different alignment problems in lower dimension transformation spaces. However, iterative schemes can get stuck in local minima and they may take a long time to converge. Hence it is necessary to identify the classes of alignment problems for which iterative schemes can be used in a computationally efficient manner.

Features can be viewed as attributed points in space. Hence aligning features is similar to aligning attributed points in space. In this report we investigate iterative strategies that use the lower dimension alignment algorithms to solve higher dimension alignment problems. Iterative strategies presented in this report involve successively applying algorithms that perform alignment under restricted rigid body transformations (e.g., rotations only or translations only) to solve point alignment problems that require higher dimension transformations (e.g., combinations rotations and translations). In Section 2, the problem and proposed strategies are described. In Section 3, experimental results are presented and in Section 4 the conclusions are drawn and the future research directions are established.

2. Problem Formulation

Consider two sets of attributed points P and Q. P and Q are compared using the following distance function.

$$\vec{d}(P,Q) = \frac{\sum_{i=1}^{n} \min_{q \in Q} d(p_i, q)}{n} \qquad (1)$$

The distance function between the attributed points $p \in P$ and $q \in Q$ is defined as follows.

$$d(p,q)=\sum_{i=1}^{d}(x_i^p-x_i^q)^2+(w^p-w^q)^2 \qquad (2)$$

In equation (2) d is the dimension of the points $p \in P$ and $q \in Q$, x_i^p and x_i^q are the i-th coordinates of points $p \in P$ and $q \in Q$ and finally w^p and w^q are the transformation-invariant attributed associated to points $p \in P$ and $q \in Q$. Consider the rigid transformation T applied to one set with respect to the other such that distance between the two sets is minimized. We refer to the problem of finding the above-defined transformation T as the attributed point alignment problem in this report. The distance function defined in Equation (1) can then be written as follows.

$$\vec{d}(TP,Q)=\frac{\sum_{i=1}^{n}\min_{q\in Q} d(Tp_i,q)}{n} \qquad (3)$$

The following two attributed point alignment problems have been analytically solved: alignment under (a) one DOF rotation in R^2 and (b) two DOF translation in R^2. The solutions are obtained by discretizing the transformation space into cells such that within each cell nearest neighbors are known and remain invariant. Hence, the distance function can be defined within each cell in a closed analytical form, and hence it is possible to minimize it by using standard techniques.

To solve problems involving higher degrees of transformations, we propose an iterative strategy. A formal definition is given below.

Consider again the two sets of points P and Q. P needs to be aligned with respect to Q using a transformation $T = (t_1, t_2, .., t_m)$ belonging to a transformation space Γ. Now let us assume that there is no direct algorithm available for this task.

Assume that a set of algorithms that can perform alignment between P and Q in a lower dimension transformation space Γ^s is available. Every $T^s \in \Gamma^s$ is of the form such that one or more of its components is zero (e.g., $T^s = (t_1, t_2, t_3, 0, 0, 0)$). Therefore, $\Gamma^s \subset \Gamma$.

Let the given set of algorithms for alignment in the lower dimension transformation space be:

$$\{\text{ALIGN-T}^s_1, \text{ALIGN-T}^s_2, \ldots, \text{ALIGN-T}^s_n\} \qquad (4)$$

where ALIGN-T^s_i performs the best alignment of P with respect to Q using a T^s_i transformation. We use the following notation to describe the effect of alignment.

$$P' = \text{ALIGN-T}^s_i\,(P,Q) \qquad (5)$$

where P' is transformed P after applying the transformation that results in the optimal alignment of P with respect to Q.

Assume that the transformation set $\{T^s_1, T^s_2, \ldots, T^s_n\}$ that corresponds to alignment algorithms described above is such that for every component t_i of general transformation T, there exists a lower dimension transformation with a corresponding non-zero component. If this condition is met then we say that set $\{T^s_1, T^s_2, \ldots, T^s_n\}$ spans the dimension of T.

Now let us consider the following sequence of application of algorithms.

$$(P_1 = \text{ALIGN}_1\,(P,Q),\ P_2 = \text{ALIGN}_2\,(P_1,Q),\ \ldots,\ P_k = \text{ALIGN}_k\,(P_{k\text{-}1},Q)) \tag{6}$$

Where, $\text{ALIGN}_i \in \{\text{ALIGN-T}^s{}_1, \text{ALIGN-T}^s{}_2, \ldots, \text{ALIGN-T}^s{}_n\}$

This sequence terminates when the following condition is met.

$$\left|\vec{d}(P_k,Q) - \vec{d}(P',Q)\right| < \varepsilon \tag{7}$$

where P' = ALIGN (P_k,Q) and ALIGN $\in \{\text{ALIGN-T}^s{}_1, \text{ALIGN-T}^s{}_2, \ldots, \text{ALIGN-T}^s{}_n\}$.

We refer to the sequential application of lower dimension algorithms as an iterative strategy. This basically corresponds to searching for the best alignments in certain projections of the transformation space in an iterative manner.

The following property holds for 2D attributed point alignment problems. Let us consider the alignment of 2D attributed points under three degree of freedom transformations. The following lower dimension transformations are utilized: translation in the coordinate plane *xy* (T_{xy}) and rotation about *z* axis (R_z) with the center of mass of the point set as center of rotation. The center of mass is computed without considering the transformation-invariant attributes. As explained above, the two transformations described correspond to the two lower dimension alignment problems already solved. Consider the iterative strategy consisting of the translation T_{xy} followed by the rotation R_z. It is possible to prove that the described iterative strategy is guaranteed to lead to a local minimum of the distance function defined in equation (2). The above-mentioned property is proved by checking the positive definiteness of the Hessian of the distance function to minimize for point alignment.

We can make two observations on the class of iterative strategies described above. Different sequences sometimes lead to different local minima and they have significantly different computational performances. Moreover, the local minimum produced by this strategy depends on the initial condition. A way to circumvent this problem is to try many different initial conditions. For example one can apply a randomly generated (or a fixed) transformation to one of the sets to produce a different initial condition. In general it may be best to try many different initial conditions to make sure that one of them leads to the global minimum. A series of experiments has been run to assess the performance of the above-described iterative scheme. In the Section 3 these experiments are presented.

Iterative strategies can be applied to attributed point alignment problems of any dimension. A class of alignment problems whose solution is useful in many applications is the alignment of 3D attributed points under six DOF transformations. Te following known lower dimension transformations are utilized: translation in the coordinate plane *xy* (T_{xy}), translation in the coordinate plane *yz* (T_{yz}), translation in the coordinate plane *xz* (T_{xz}), rotation about *x* axis (R_x), rotation about *y* axis (R_y), and rotation about *z* axis (R_z). All the above-defined rotations are performed with the center of mass of the point set as center of rotation. Consider the iterative strategy consisting of all the possible sequences of the three coordinate planes (i.e. 3! = 6). Even in this case we have tried to prove whether this class of iterative strategies is guaranteed to lead to a local minimum or not. For this purpose the Hessian of the distance function has been studied. Its positive definiteness cannot be guaranteed. Experiments have been carried out in order to assess the performance of this class of iterative strategies on the 3D attributed point alignment problem in 3D.

3. Experimental Results

In this section the experiments performed on the iterative strategy are presented.

In the first two experiments, the alignment of 2D points under three degree of freedom transformations was considered. The lower dimension transformations consisting of translation and rotation in plane were utilized.

In the first experiment, 200 initial sets of 20 attributed points inside a circle of a fixed size were randomly generated. The transformation-invariant attributes were also generated randomly. Then a random transformation was applied to each of the 200 sets, creating 200 more sets of attributed points. Hence finally 400 sets of points were obtained. Consider all the pairs of sets consisting of one initial set and one corresponding additional sets created as explained above. The iterative scheme was applied to each pair of point sets, evaluating a total of 200 instances. The expected minimum distance corresponding to the perfect alignment computed among the sets of each pair is 0. In the cases in which the perfect alignment was not found for some instances, the following procedure was used. A random transformation was applied to the initial set of the pair in order to create a different initial condition. Then the experiment was repeated with the different initial condition that had been obtained for those instances. This procedure was repeated until the perfect alignment was found or the limit of ten different initial conditions was reached. Hence out of the 200 instances, a perfect alignment (i.e. distance = 0) was found in all of them. The same experiment was then repeated generating 200 initial sets of 40 and 80 attributed points inside a circle. A total of 400 instances were evaluated for 40 and 80 attributed point sets. Out of the 400 instances, a perfect alignment (i.e., distance = 0) was found in all of them.

In the second experiment, 200 initial sets of 20 points were randomly generated and then again a random transformation was applied to each of them, creating 200 more sets of points. This time before applying the iterative scheme one, two, four and eight points were erased from one of the sets being compared. We expected the erasing of the points not to affect the alignment of each pair of sets, as the distance function used is defined also for sets of different cardinality. Out of the 800 instances (i.e. 200 instances for each distinct number of erased points), a perfect alignment was found in all of them.

In the third and fourth experiment, the alignment of 3D points under six degree of freedom transformations was considered. The lower dimension transformations defined in Section 2 were utilized.

In the third experiment, 4 initial sets of 5 points inside a cube of a fixed size were randomly generated. Then a random transformation was applied to each of the 4 sets, creating 12 more sets of points. Hence finally 16 sets of points were obtained. Consider all the 4-ples of sets consisting of one initial set and three corresponding additional sets created as explained above. The expected minimum distance corresponding to the perfect alignment computed among the sets of each 4-ple is 0. The iterative scheme utilized 6 different sequences of transformations. They consist of all the possible sequences of the three coordinate planes (i.e. 3!=6). Within each plane, first a rotation then a translation was applied iteratively. The iterative scheme was applied to each 4-ple of point, a total of 24 instances. Out of the 24 instances, a perfect alignment (i.e. distance = 0) was found in 23 instances (95.8%). Then the experiment was repeated by randomly generating 8 initial sets of 10 points and then again applying a random transformation to each of them, creating 24 more sets of points. The same iterative scheme was utilized, and was applied to 48 instances. Out of the 48 instances, a perfect alignment was found in 45 instances (93.7%).

In the fourth experiment 10 initial sets of 20 points were randomly generated and then again a random transformation was applied to each of them, creating 30 more sets of points. This time before applying the iterative scheme two points were erased from one of the sets being compared. Out of the 60 instances, a perfect alignment was found in 59 instances (98.3%).

4. Conclusions

In this report the use of iterative strategies to solve higher dimension alignment problems is proposed. Experimental results are presented in order to evaluate the performance of the iterative strategy and to identify future research directions.

As already mentioned, both the iteration sequences and the initial conditions affect the performance of the iterative strategy. In the first two experiments the perfect alignment was obtained in all instances with different numbers of points and under different conditions. Furthermore additional initial conditions were not needed to obtain the perfect alignment. Hence it can be inferred that that the number of local minima is very small, and the global minimum can be found by just using a few randomly generated initial conditions.

The third and fourth experiments focus on the alignment of 3D points under six degree of freedom transformations. The outcome suggests that the iterative strategy might be used to solve attributed point alignment problems in 3D as well. In this case both the iteration sequences and the initial conditions affect the performance of the iterative strategy. It is necessary to perform a larger number of experiments in order to confirm these impressions.

Our preliminary experimental results show that the idea of using of iterative strategies to solve higher dimension attributed point alignment problems is promising and can be used to perform feature-based shape similarity assessment.

For 3D attributed point alignment problems we will perform additional experiments with the same structure as the one described in Section 3 for 2D attributed point alignment problems. The lower dimension transformations and iterative strategy used will be the one used in the third and fourth experiments described in Section 3. We will evaluate the performance of the iterative strategy in 3D alignment problems. It will also be experimentally verified if this class of iterative strategies is guaranteed to lead to a local minimum in 3D point alignment problems. For this purpose the Hessian of the distance function will be evaluated when the iterative strategy does not lead to the perfect alignment expected. If cases in which the Hessian is not positive definite are found, then it will be finally proved that the class of iterative strategies used is not guaranteed to lead to a local minimum in 3D point alignment problems. Otherwise it will be inferred that the above-mentioned property may hold, and further experiments and theoretical studies will be needed.