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Carrier Frequency Estimation of MPSK Modulated Signals

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Abstract

In this paper we concentrate on MPSK carrier frequency estimation based on random data modulation. We present a fast, open-loop frequency estimation and tracking technique, which combines a feedforward estimator structure and a recursive least square (RLS) predictor. It is suitable for the frequency estimation and large frequency acquisition and tracking required of burst mode satellite modems operating under the condition of low SNR and large burst-to-burst frequency offset. The performance of the estimator is analyzed in detail and simulation results are shown. Finally, the non-linear impact of data modulation removal methods is discussed. The estimator we derived is easily implemented with digital hardware.

1 Introduction

Carrier frequency recovery is very important to MPSK modems. Fast frequency estimation and tracking is necessary for burst mode satellite modems operating in the presence of large frequency offset. An additional burden of low signal noise ratio (SNR) can make the task of frequency estimation quite difficult. Traditional methods such as phase locked loop (PLL, e.g. Costas loop) and Decision Directed Methods [13][14][15] are widely used in MPSK modems. A combination of PLL and frequency sweeping is commonly used to deal with large frequency offsets in continuous modems. For burst modems, some form of estimation is usually employed to speed up the acquisition process. The paper [15] shows that the PLL has a small frequency capture range and a long acquisition time[1][14]. The capture range of the PLL is around $2B_L$, where B_L is the loop bandwidth. A rough approximation of B_L is given as R_s/n . R_s is symbol rate, n is typically on the order of few hundred, depending on the SNR. The lower SNR, the larger n . Hence, we have a smaller capture range and a longer acquisition time at low SNR. Decision-Directed and Data-aided methods are more suitable for systems with a training sequence or operation at high SNR. Unfortunately, training sequences are not available for many burst modems. Continuous mode modems can also benefit from the faster acquisition time proposed. For these cases, open-loop frequency estimation methods, which have larger estimation range than PLL and require a smaller number of symbols and operate on random data modulation, are considered in this paper as a method to achieve fast frequency acquisition in the presence of large frequency offsets. An estimation module combined with a traditional PLL can achieve much faster synchronization. The technique presented can also be used for frequency tracking of burst mode modems that utilize a preamble for carrier

recovery, such as TDMA. The added benefit of this technique is the robustness of frequency estimation, and subsequent carrier recovery acquisition once phase is resolved, when frequency offsets are large compared to the symbol rate. This could permit less stringent and costly frequency control of TDMA networks.

Focusing on the carrier frequency recovery problem, a number of fast-converging methods are proposed. The paper[6] gives a survey of those methods operating on random data modulation which are easy to implement. A frequency estimator, based on power spectral density estimation, was first proposed by Fitz [5] for an unmodulated carrier. For an MPSK signal, the non-linear method in [1] can be used to remove data modulation. A variant of this algorithm was proposed by Luise[7]. The performance of these methods, at low SNR, is close to the *Cramér – Rao* lower bound (CRLB) [14] for a carrier with unknown frequency and phase. The maximum frequency error that can be estimated by the Fitz algorithm is $R_s/(2ML)$, where L is the maximum autocorrelation lag and M is the number of phase states in MPSK. Under the assumption that the carrier phase has a constant slope equal to the angular frequency offset, Tretter [2] and Bellini [3][4][6] proposed a frequency estimator by means of linear regression or line fit on the received signal phase. The maximum frequency error that can be digested is $R_s/(2M)$. The performance of this algorithm is good at high SNR (close to the CRLB for data modulated carrier) with low hardware complexity. Phase change over symbols is proportional to the frequency offset. Chuang and Sollenberger[8][9] use this idea and present algorithms based on differential symbol estimates. In this paper, we present a carrier recovery algorithm based on [8][9]. We propose a new data modulation removal method which performs better than [8] at small frequency offset. We also introduce an adaptive

filter to improve performance at low SNR($E_b/N_o \leq 5\text{dB}$).

In the second part, we revisit Viterbi's [1] feedforward phase estimator which is closely related to our algorithm. We then derive a simple version of the estimation and tracking algorithm and follow with the development of a more complex version. The complex estimator uses the idea of Viterbi's feedforward structure. An adaptive filtering technique is used for tracking and noise removal. A simple Recursive Least Square (RLS) one-step predictor is proposed. The performance of the estimation and tracking algorithm is analyzed in detail. An approximation for the variance of the estimate is derived for the Chuang algorithm[8]. In the third part, simulation results are shown and the non-linear effect of data modulation removal is discussed.

2 Frequency Estimation and Tracking Algorithm

In order to simplify our presentation, the following assumptions are made for the development of the algorithm:

1. The symbol timing is known
2. Discrete time samples are taken from the output of a pulse shape matched filter, one sample per symbol
3. The pulse shape satisfies the Nyquist criterion for zero intersymbol interference

The last assumption is reasonable for relatively small frequency offsets. The i th complex sample derived from matched filter can be expressed as

$$r_i = d_i \exp(j(2\pi\Delta f i T_s + \phi_0)) + n_i, \quad |d_i| = 1. \quad (1)$$

where d_i represents the i th complex symbol modulating the MPSK carrier, Δf is the frequency offset, T_s is the symbol interval, ϕ_0 is the carrier phase, n_i represents complex additive Gaussian noise. The channel noise has two-sided power spectral density $N_o/2$. The variance of the two quadrature components of n_i is $N_o/(2mE_b)$, where E_b is the energy per information bit and $m = \log_2 M$.

2.1 The Feedforward Phase Estimator

In their classical paper[1], Viterbi and Viterbi proposed a feedforward structure to estimate the phase ϕ_0 of data modulated MPSK signal. This estimator operates on a block of N symbols. It first removes the modulation from the complex sample r_i , obtaining,

$$R_i = I_i + jQ_i = F(|r_i|) \exp(j \text{Marg}(r_i)), \quad F(|r_i|) = |r_i|^k, \quad k \leq M \text{ even}. \quad (2)$$

Then it averages the N in-phase and quadrature components and finally generates the estimated carrier phase $\hat{\phi}$ for the block of symbols:

$$\hat{\phi} = \frac{1}{M} \tan^{-1} \left(\frac{\sum Q_i}{\sum I_i} \right). \quad (3)$$

This estimate is affected by a $(2\pi/M)$ -fold ambiguity, which can be resolved by differential encoding of channel symbols.

$F(|r_i|) = 1$ is best at $E_b/N_o \geq 6dB$, $F(|r_i|) = |r_i|^2$ is best at low $E_b/N_o \leq 0dB$ [1]. Ordinarily we pick up the zeroth power function because of the SNR we work with. The estimator is unbiased and has the variance,

$$\sigma^2 = \frac{1}{N2mE_b/N_o} \Gamma(M, \Delta f), \text{ for } F(|r_i|) = |r_i|^k, \quad (4)$$

$$\Gamma(M, 0) = 1 + \frac{(k-1)^2}{2mE_b/N_o} + O\left(\frac{1}{2mE_b/N_o}\right). \quad (5)$$

2.2 Frequency Estimation and Tracking Algorithm

The frequency offset causes the phase of unmodulated carrier to change by $2\pi\Delta fT_s$ every symbol, so if we differentiate the phase of adjacent symbols, we can get an estimate of the carrier frequency.

That's the basic idea of our algorithm.

The selection of the proper nonlinearity for data modulation removal is a difficult topic. Most frequency estimation methods suffer dramatic performance loss after going through data modulation removal. There are two common methods:

1. mod $2\pi/M$
2. M-th power.

We will discuss them separately. In the following discussion, we will focus on QPSK, the method also applies to all MPSK.

According to the work done by Tretter[2], we can absorb the noise term n_i in the received signal,

r_i , into phase noise at high SNR, i.e.

$$r_i = A \exp(j(2\pi\Delta f i T_s + \theta_i + \phi_0 + V_{Q_i})). \quad (6)$$

where $A=1$, θ_i is data modulation, V_{Q_i} is equivalent phase noise. Therefore, the phase ϕ_i of r_i can be modeled as

$$\phi_i = 2\pi\Delta f i T_s + \theta_i + \phi_0 + V_{Q_i}, \quad \theta_i = \frac{2\pi k}{M}, k = 0, \dots, M - 1. \quad (7)$$

If we differentiate ϕ_i , we can get

$$\delta_i = 2\pi\Delta f T_s + \theta_i - \theta_{i-1} + V_{Q_i} - V_{Q_{(i-1)}}. \quad (8)$$

Because data modulations θ_i and θ_{i-1} are multiples of $2\pi/M$ ($\pi/2$ for QPSK), if we keep only the remainder of $\delta_i/(\pi/2)$, (i.e. modulo operation) or select an l , such that $\gamma_i = \delta_i - l\frac{\pi}{2}$ is within the range $(-\pi/4, \pi/4)$, we can get

$$\gamma_i = 2\pi\Delta f T_s + N_i, \quad N_i = (V_{Q_i} - V_{Q_{(i-1)}}) \bmod \frac{\pi}{2}. \quad (9)$$

In order to prevent frequency aliasing, the frequency offset must satisfy $|\Delta f| < 1/(2MT_s)$. For QPSK, $|\Delta f| < \frac{1}{8}R_s$, is the bound of maximum frequency offset which can be estimated.

Equation(9) is a simple estimation of Δf based on adjacent symbols. γ_i is corrupted by phase noise N_i .

The other method for modulation removal is M-th (4 for QPSK) power, $4\cdot\delta_i$, i.e.

$$\gamma'_i = 4\delta_i = 4 \cdot 2\pi\Delta f T_s + 4(\theta_i - \theta_{i-1}) + 4(V_{Q_i} - V_{Q_{(i-1)}}). \quad (10)$$

γ'_i is passed through an exponential function $\exp(j(\cdot))$. It is similar to the algorithms in [1][8]. The same restriction on Δf applies as the $\text{mod}\pi/2$ method.

Sequence $\{\gamma_i\}$ or $\{\gamma'_i\}$ is composed of frequency information and noise. Processing them in the phase domain is numerically error prone. We apply the idea of Viterbi's feedforward structure, project γ_i or γ'_i onto in-phase and quadrature components, then average both in-phase and quadrature components. We can get the frequency estimate as follows, for $\gamma_i(9)$, we get

$$2\pi\hat{\Delta f}T_s = \tan^{-1}\left(\frac{\sum_{i=1}^N \sin(\gamma_i)}{\sum_{i=1}^N \cos(\gamma_i)}\right), \quad (11)$$

$$\hat{\Delta f} = \frac{R_s}{2\pi} \tan^{-1}\left(\frac{\sum_{i=1}^N \sin(\gamma_i)}{\sum_{i=1}^N \cos(\gamma_i)}\right). \quad (12)$$

For $\gamma'_i(10)$, we get

$$2\pi\hat{\Delta f}T_s = \frac{1}{4} \tan^{-1}\left(\frac{\sum_{i=1}^N \sin(\gamma'_i)}{\sum_{i=1}^N \cos(\gamma'_i)}\right), \quad (13)$$

$$\hat{\Delta f} = \frac{R_s}{4 \cdot 2\pi} \tan^{-1}\left(\frac{\sum_{i=1}^N \sin(\gamma'_i)}{\sum_{i=1}^N \cos(\gamma'_i)}\right). \quad (14)$$

N is the number of symbols. We call this estimator the differential feedforward estimator(DFE).

Equation (14) is the algorithm presented in [8].

Simulation shows that the estimation result of (12) and (14) can be modeled as

$$\hat{\Delta f} = \Delta f + N_p. \quad (15)$$

N_p is additive noise with zero mean and autocorrelation $\{r_{N_p}(k)\}$, $k=0,1,\dots$

In order to remove noise and track the frequency change, an adaptive algorithm [10][11][12] can be used. There are three criteria for our algorithm selection:

1. unbiased prediction
2. good compromise between fast convergence and small variance
3. low hardware complexity.

Therefore, according to our model in (9) and (15), a recursive least square (RLS)[10] one-step predictor is a good choice. We would like to minimize the following performance index:

$$\Lambda_n = \sum_{i=1}^n \lambda^{n-i} (\omega_n - \gamma_i)^2. \quad (16)$$

where ω_n is the carrier frequency offset estimate from the predictor at time n , γ_i is the same as γ_i of (9) or $\hat{\Delta}f$ of (15). λ is an exponential forgetting factor, satisfying $0 < \lambda < 1$. We minimize Λ_n based on ω_n ,

$$\frac{\partial \Lambda_n}{\partial \omega_n} = 2 \sum_{i=1}^n \lambda^{n-i} (\omega_n - \gamma_i) = 0, \quad (17)$$

$$\omega_n = \frac{\sum_{i=1}^n \lambda^{n-i} \gamma_i}{\sum_{i=1}^n \lambda^{n-i}}. \quad (18)$$

After simple arithmetic, we can find the relationship between ω_n and ω_{n-1} . If we define $F_n = \sum_{i=1}^n \lambda^{n-i}$, $K_n = 1 - 1/F_n$ and $M_n = 1/F_n$, we can get the following RLS one step predictor:

1. Initialization: Select a proper λ which controls convergence speed and variance. If λ is small(close to 0), the predictor converges faster but has large variance; if λ is large(close to 1), the predictor converges slower but has small variance. Let $F_0 = 0, \omega_0 = 0$.
2. Processing: for $n=1,2,\dots$

$$F_n = \lambda F_{n-1} + 1, \quad (19)$$

$$K_n = 1 - \frac{1}{F_n}, M_n = \frac{1}{F_n} \quad (20)$$

$$\omega_n = \omega_{n-1}K_n + \gamma_n M_n. \quad (21)$$

The structure of the RLS predictor is similar to that of the extended Kalman filter in [10]. In real hardware implementation, we can fix $K_n = K$ and $M_n = M$ by letting $M = 1 - K$, $0 < M < 1$.

The RLS predictor is a solution for the requirement of small variance and tracking capability. It is intuitive to see that the larger the N , the smaller the variance. But, if we increase the number of symbols in frequency estimation, we will cover small but non-negligible frequency change. The following performance analysis shows that the RLS predictor can remove noise (reduce variance) and keep the tracking capability of the DFE with small N . If the frequency changes dramatically, a higher order predictor should be considered to achieve faster convergence with smaller variance.

2.3 Hardware Implementation

Figure 1 shows the flow diagram of hardware implementation. (a) is the simple version that works well at $E_b/N_o \geq 10dB$. (b) is the slightly more complex DFE version in which two "arctan" modules can share one lookup table on a time division basis. We can also place $\text{mod}\pi/2$ (or 4 times phase) and NCO($\sin()$, $\cos()$) together into another lookup table. In order to simplify the RLS predictor, constant coefficients K and M can be used as an extended Kalman filter, however, it will suffer slower convergence. When we combine the RLS predictor (or extended Kalman filter) with the DFE we can track the frequency change without large performance loss. Simulation results show that this combination will improve the estimation variance, especially at low SNR.

The system(b) in Figure 1 operates as two stages: estimation and tracking. It works in the following manner:

1. Estimation: DFE estimates frequency offset over N symbols. During last L symbols, the RLS predictor is activated to remove noise.
2. Tracking: after estimation, the DFE tracks frequency offset over a rectangular sliding "window" of length N . The estimation result is passed through the RLS predictor.

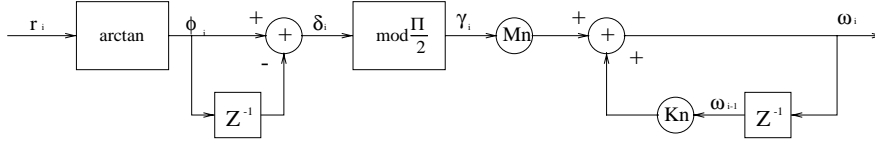
We activate the RLS predictor during last L symbols because as the number N increases, the estimation variance of DFE goes down at speed $O(1/N)$. The RLS predictor (or Kalman filter) removes noise, but it also accumulates noise from previous inaccurate (when time $\leq N$) estimation. There is an optimum point, $N - L$, at which we begin the filtering process to remove maximum noise. This point can be obtained by simulation.

2.4 Performance Analysis

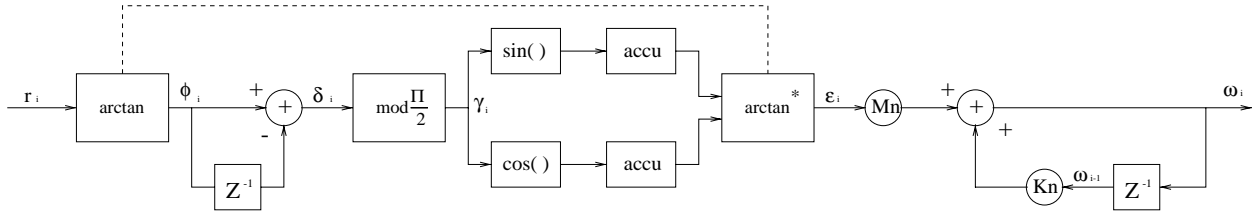
In this subsection, we first analyse the variance of DFE and then discuss the convergence of the RLS predictor. We follow with a discussion of some advantages of this technique.

The DFE algorithm is very similar to Viterbi's feedforward structure with the addition of differentiating input phase. It is intuitive to see that the relationship between the frequency estimation variance and N (the number of symbols) should be the same as (4) as shown in the simulation

The two nonlinear data removal methods discussed in this paper play different roles at low SNR ($E_b/N_o \leq$



(a). Simple frequency estimation and tracking module



(b) Differential Viterbi frequency estimation and tracking module

* In hardware implementation, this arctan module can share one lookup table with front.

Figure 1: Flow digram of frequency estimation and tracking module

5dB) and high SNR($E_b/N_o \geq 12dB$). At low SNR, $\text{mod}\pi/2$ has smaller variance but is biased when frequency offset is large. The 4-th power is unbiased but has a larger variance. At high SNR($E_b/N_o \geq 12dB$), $\text{mod}\pi/2$ is preferred. The $\text{mod}\pi/2$ hard limits the phase difference into $(-\pi/4, \pi/4)$, but this nonlinear operation will cause estimation error. The 4-th power method amplifies the noise 4 times when it removes data modulation, which introduces a (approximately)12dB noise penalty.

Because of the nonlinear operation above, it's difficult to get an analytical solution of the estimation variance. Fortunately, simulation shows that the variance of the estimator(14)(4-th power, Chuang algorithm [8]) exhibits some regularities. After some data processing, we get the following

approximation:

$$\text{var}\left(\frac{\Delta f}{R_s}\right) = \frac{C}{N(2\pi)^2(2m)(E_b/N_o)^{3.2}}, \text{ for } E_b/N_o > 2dB. \quad (22)$$

where C is a constant depending on the modulation scheme. We can use this formular(22) to predict the performance of the predictor.

During the analysis of the RLS predictor, we must assume the models(9) and (15) hold, where N_p or N_i is additive noise with zero mean and autocorrelation $\{r_N(k)\}$. It is easy to verify the predictor (18)(21) is unbiased. If we assume the output of predictor is ω_n (shown in Figure 1), then we can get

$$\text{var}(\omega_n) = \frac{1}{(\sum_{i=1}^n \lambda^{n-i})^2} \left(\sum_{i=1}^n \sum_{k=1}^n \lambda^{2n-i-k} r_N(i-k) \right). \quad (23)$$

In order to simplify the problem, let us assume N_p or N_i is white (which is not quite accurate because the DFE estimation results are correlated), and let $r_N(0) = \sigma^2$,

$$\text{var}(\omega_n) = \frac{\sum_{i=1}^n \lambda^{2(n-i)}}{(\sum_{i=1}^n \lambda^{n-i})^2} \sigma^2 = \frac{(1 - \lambda^{-1})^2 (1 - \lambda^{-2(n+1)})}{(1 - \lambda^{-2})(1 - \lambda^{-(n+1)})^2} \sigma^2. \quad (24)$$

From (22)(24): $\text{var}(\Delta f/R_s)$ decreases at $O(1/N)$, but $\text{var}(\omega_n)$ decreases as (24). Further, our simulation shows some gain at low SNR from filtering.

We can summerize the advantages of the DFE & RLS predictor as follows:

1. At low SNR, the 4-th power estimation is unbiased (linear regression method exhibits a serious bias). RLS predictor reduces estimation noise during the "estimation" stage; during

the "tracking" stage, it reduces noise further and keeps tracking frequency changes. At high SNR, RLS plus $\text{mod}\pi/2$ provides a simpler solution. The DFE & RLS technique is simple and easy for hardware implementation when contrasted with the more complex spectrum estimation method.

2. The performance of this technique is scalable. We can get different performance (variance) by programming the length N of the accumulator by using (22).
3. If Viterbi's feedforward phase estimator is adopted, the hardware cost is even smaller in that two methods share most modules.

3 Simulation Results

The following simulation results are based on QPSK.

Figure 2 shows the normalized estimation variance(i.e., $\text{var}(\Delta f/R_s)$) of the 4-th power DFE (Chuang algorithm[8]) as a function of E_b/N_o and symbol length N . The "Δ" curve is equation (22) given $N=800$ and $C=30$ for QSPK. It shows that (22) is a good approximation of the performance of the Chuang algorithm. The most attractive advantage of 4-th power DFE is that it is unbiased and the performance is independent of frequency offset(less than $1/8 R_s$) at low SNR ($E_b/N_o \geq 1\text{dB}$), given the proper selection of N vs. SNR.

Figure 3 shows the normalized estimation variance of the $\text{mod}\pi/2$ DFE as a function of E_b/N_o and symbol length N . We can see that at low SNR, the variance decreases at approximately $O(1/N)$; at high SNR it decreases even faster. The variance is smaller than that of the Chuang

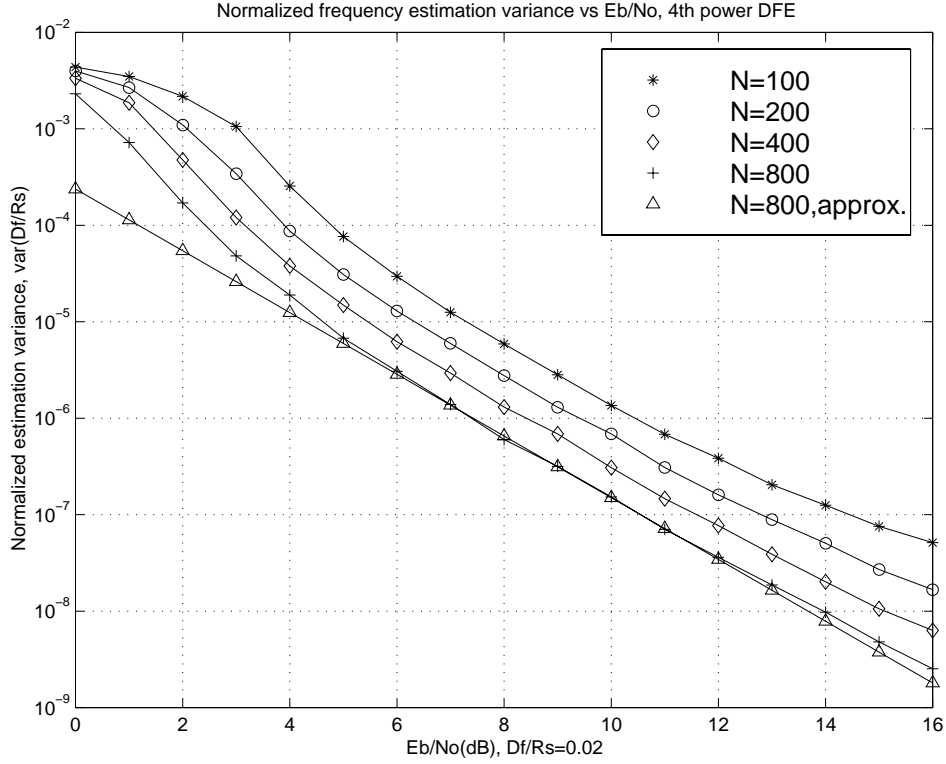


Figure 2: Normalized frequency estimation variance vs E_b/N_o and N , 4-th power DFE

algorithm within the estimation range of the $\text{mod}\pi/2$ DFE (e.g. when $E_b/N_o=4\text{dB}$, $N=400$, $\Delta f/R_s=2\%$, $\text{var}_{\text{mod}\pi/2}=8.2979\text{e-}6$, $\text{var}_{4th}=3.7916\text{e-}5$, when $E_b/N_o=12\text{dB}$, N and Δf remain the same, $\text{var}_{\text{mod}\pi/2}=1.5778\text{e-}8$, $\text{var}_{4th}=7.7097\text{e-}8$). The shortcoming of the $\text{mod}\pi/2$ DFE is that it is biased given large frequency offset at low SNR. The actual frequency estimation range of $\text{mod}\pi/2$ at low SNR is much smaller than that of 4-th power DFE. However, with increasing SNR, $\text{mod}\pi/2$ exhibits an increased frequency estimation range.

Figure 4 shows the normalized mean square (MS) estimation error ($E[(\hat{\Delta f}/R_s - \Delta f/R_s)^2]$) of 4-th power and $\text{mod}\pi/2$ DFE as a function of $\Delta f/R_s$, given $E_b/N_o=14\text{dB}$, $N=400$. We can see that

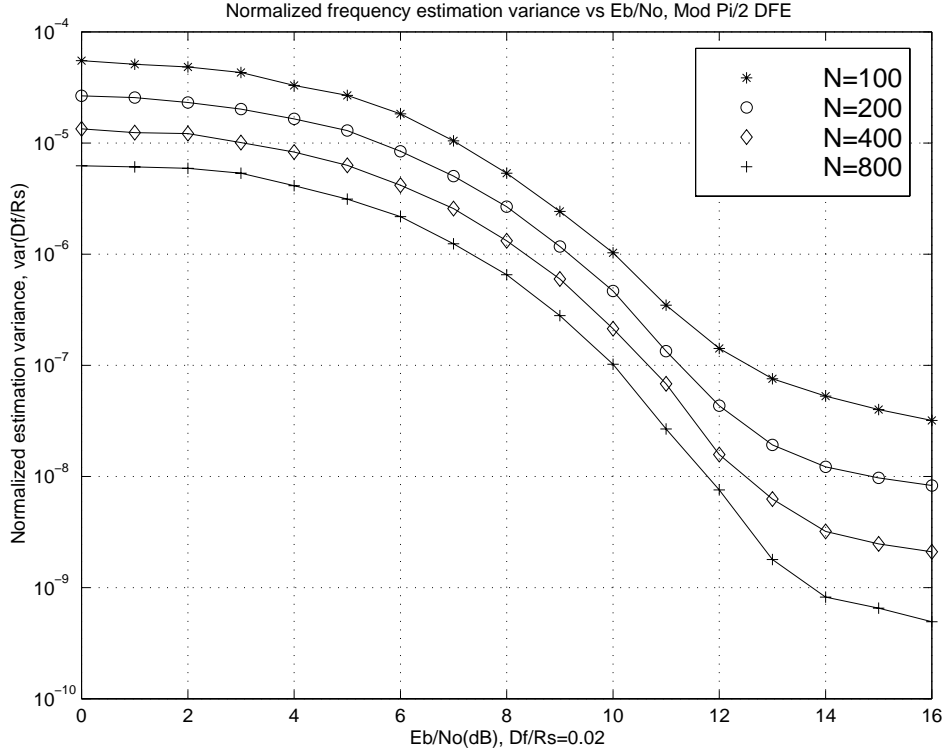


Figure 3: Normalized frequency estimation variance vs E_b/N_o and N , mod $\pi/2$ DFE

the maximum frequency offset Δf_{max} of the 4-th power DFE is very close to our theoretical value $0.125R_s$. The mod $\pi/2$ DFE has a small frequency estimation range (e.g., under $4\%R_s$, it performs better than 4-th power). It exhibits even smaller frequency estimation range (around $2\%R_s$) at lower SNR, but it has a smaller MS estimation error within its estimation range.

Figure 5 shows the performance comparison of the 4-th power DFE, the 4-th power DFE plus extended Kalman filter and line fit algorithm[3][4][6], given $N=200$. The coefficient M of the extended Kalman filter is $1/64$. The “*” curve shows the filtered result (after 100 symbols training). We can see that the RLS predictor (or simplified extended Kalman filter) removes more noise,

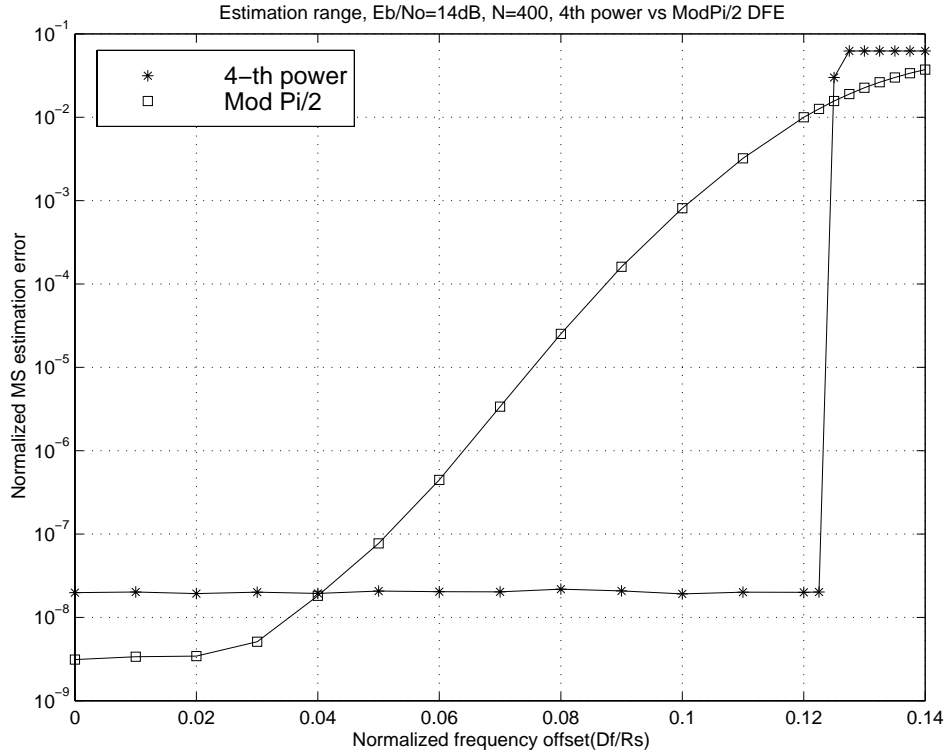


Figure 4: Frequency offset estimation range, 4-th power vs mod $\pi/2$ DFE

especially at low SNR, (e.g. at $E_b/N_o=1\text{dB}$, $var_{DFE}=2.6767\text{e-}3$, $var_{DFE+RLS}=1.5113\text{e-}3$). The filter improves the performance of the estimator by 0.5dB to 1dB. It also shows that the line fit algorithm (linear regression) is biased at low SNR.

The following table shows the performance comparison of the pure 4-th power DFE and the 4-th power DFE plus RLS predictor in the “estimation” stage at low SNR. The condition is: $N=250$, $L=50$ (i.e. RLS predictor starts filtering at $N-L=200$), $\lambda = 0.97$.

From the simulation result we can see that the combination of DFE and RLS predictor reduces the frequency uncertainty to a small range at low SNR, which could be used to speed up the subsequent

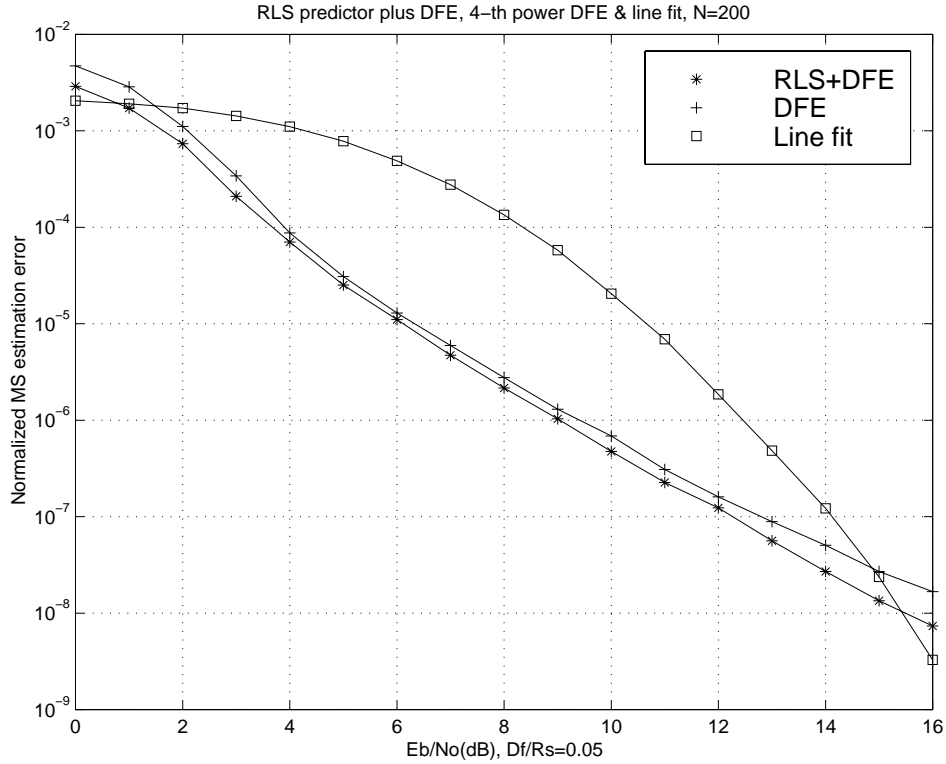


Figure 5: Performance comparison between DFE+RLS and DFE

PLL pull-in process. At high SNR, a PLL is not necessary for very short bursts in that an open loop frequency estimator plus feedforward phase estimator can recover the carrier. According to the analysis in [6], the maximum value of $\Delta f/R_s$ that Viterbi's estimator can tolerate is $1/2Mn$, where n is the block length for phase estimation[1]. Typical values of n are around 20 to 25 for QPSK [3]. Hence, the variance of $\Delta f/R_s$ which should be guaranteed is around 6×10^{-6} . If we use the $\text{mod}\pi/2$ DFE, $E_b/N_o=12\text{dB}$, $N=100$, $\text{var}(\Delta f/R_s)=1.4181\text{e-}7$.

E_b/N_o (dB)	0	1	2	3	4
$var(\Delta f/Rs)_{DFE}$	3.7135e-3	2.3406e-3	9.7756e-4	2.3223e-4	7.8873e-5
$var(\Delta f/Rs)_{DFE+RLS}$	3.1208e-3	2.0396e-3	8.7936e-4	2.2489e-4	6.8964e-5

Data modulation removal is a key point to carrier frequency offset estimation based on random data modulation. The 4-th power method reduces SNR by approximately 12 dB. At low SNR, the harmonics generated by the nonlinear operation of data removal are serious. These harmonics cause bias of the frequency estimates at low SNR, which could distort the results of other algorithms (e.g. line fit). The 4-th power DFE described above shows good performance at low SNR.

4 Conclusion

This paper presents a simple open loop frequency estimation and tracking algorithm based on random data modulation. Two data removal nonlinear methods are discussed. The 4-th power DFE is unbiased at low SNR. The $\text{mod}\pi/2$ DFE exhibits a smaller variance within its smaller estimation range. A formula for performance approximation is given. The combination of this algorithm and a PLL can be used to reduce carrier synchronization time. It is also suitable for the frequency acquisition and tracking of burst mode modems operating under the condition of large burst-to-burst frequency offset.

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6 References

1. A. J. Viterbi, A. M. Viterbi, "Nonlinear Estimation of PSK-Modulated Carrier Phase with Application to Burst Digital Transmission", IEEE Trans. Inform. Theory, vol. IT-29, pp. 543-551, July 1983.
2. S. Tretter, "Estimating the Frequency of a Noisy Sinusoid by Linear Regression", IEEE Trans. Inform. Theory, vol. IT-31, pp. 832-835, November 1985.
3. S. Bellini, C. Molinari, G. Tartara, "Digital Frequency Estimation in Burst Mode QPSK transmission", IEEE Trans. Commun. pp.959-961, July 1990.
4. S. Bellini, C. Molinari, G. Tartara " Digital Carrier Recovery With Frequency offset in TDMA transmission", International Conference on Communications (ICC '91), Denver, June 1991, pp. 789-793.
5. M. P. Fitz, "Planar Filtered Techniques for Burst Mode Carrier Synchronization", GLOBE-COM '91, pp. 365-369.
6. S. Bellini, "Frequency Estimators for M-PSK Operating At one Sample Per Symbol", Global Telecommunications Conference (GLOBECOM '94), Volume: 2 , pp. 962-966.

7. M. Luise, R. Reggiannini, "Carrier Frequency Recovery in All-Digital Modems for Burst-Mode Transmissions", *IEEE Trans. Commun.* pp. 1169-1178, Feb./Mar./Apr. 1995.
8. J. Chuang, N. Sollenberger, "Burst Coherent Demodulation with Combined Symbol Timing, Frequency Offset Estimation, and Diversity Selection", *IEEE Trans. Commun.* vol 39, pp.1157-1164, July 1991.
9. N. Sollenberger, J. Chuang, "Low-overhead Symbol Timing and Carrier Recovery for TDMA Portable Radio Systems", *IEEE Trans. Commun.*, vol. 38, pp.1886-1892, Oct. 1990.
10. S. Haykin, *Adaptive Filter Theory*, 3rd Ed., Upper Saddle River, NJ: Prentice Hall, 1996.
11. S. Haykin, *Adaptive Filter Theory*, 2nd Ed., Englewood Cliffs, NJ: Prentice Hall, 1991.
12. G. C. Goodwin, K. S. Sin, *Adaptive Filtering Prediction and Control*, Englewood Cliffs, NJ: Prentice Hall 1984.
13. J. Bingham, *The Theory and Practice of Modem Design*, New York: Wiley, 1988.
14. F. Gardner, *Phaselock Techniques*, New York: Wiley, 1979.
15. L. Kenney, "Signal Acquisition with a Digital PLL", *Communication Systems Design*, June 1997.
16. H. L. Van Trees, *Detection, Estimation and Modulation Theory, Part I*, New York: Wiley, 1968.