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ABSTRACT

In this paper, we present a new loading algorithm for subband coded image transmission on multicarrier modulation system. The image subbands are transmitted simultaneously, each occupying a number of subchannels. Different modulation rate and power are assigned to the subchannels transmitting different subbands. Unlike the traditional loading algorithms which flat the error performance of all the subchannels, the proposed loading algorithm assigns different error performance to the subchannels in order to provide unequal error protection for the subbands data. Numerical examples show that the proposed algorithm yield significant improvement over the traditional loading algorithms especially for spectral shaped channels.

1. INTRODUCTION

In recent year, multicarrier modulation (MCM) [1] also referred to as orthogonal frequency division multiplexing (OFDM) or discrete multition (DMT) has generated a tremendous amount of interest. It is currently considered as a standard channel coding scheme for asymmetric digital subscriber lines(ADSL), high rate digital high definition television (HDTV) and wireless personal communication systems[2, 3]. By applying the discrete Fourier transform (DFT) or fast Fourier transform (FFT) and their inverse, the available channel bandwidth is subdivided into a number of subchannels that achieves bandwidth efficiency. The cyclic prefix is added in between two consecutive symbols to avoid intersymbol interference(ISI) and to preserve the orthogonality between subchannels. At the receiver, a zero forcing equalization is used to compensate for the channel distortion. Multicarrier modulation yields many important advantages over single carrier modulation, such as canceling the noise interference caused by linear equalization[1].

A crucial aspect in the design of MCM system is the need to optimize the system transmission bandwidth and power through an optimal loading algorithm. At present three algorithms achieves practical importance[1, 6, 7]. By assigning a high modulation rate to carrier with high signal-to-noise ratio (SNR) and low modulation rate to carrier with low SNR or even no transmission on the carriers with very low SNR, these algorithms share the same criterion: the optimal solution is that all the usable subcarriers perform with the same error rate. Recently, [8] presented a combined source-channel coding using multicarrier modulation which assigns different power to different bits of the binary codeword to achieve unequal error protection according to their importance. They also applied this idea to image transmission over noisy channels[9].

For image transmission, the channel bit error need not be very low to achieve preferable quality. Thus, the noisy channels considered here may have the bit error rate (BER) from 10^{-5} to 10^{-1}. To transmit subband coded image [4] over noisy channel, if applying the traditional loading algorithm[1, 6, 7], the subbands are transmitted one by one in consecutive order with same error performance. Or if applying [8, 9] algorithm, the same power is assigned to every subbands although the power allocated to each bit of the codeword varies according to the importance of the bit. However, for subband coded data, these algorithm will not yield the best performance.

It is well known that subband coding distributes energy unequally among the subbands. The importance of the subbands can be classified as the energy amount they contains. For a octave-band decomposi-

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tion, the lowest subband contains most of the information and therefore is the most important. The bit error happening to the subbands with higher importance will cause larger amplitude change and thus more distortion. Unequal error protection for each subband was adopted by many schemes and yielded tremendous improvement over equal error protection. Thus, for subband transmission using multicarrier modulation, different error performance must be assigned to each subband. As such, a new loading algorithm is needed to optimally distribute the available power and bandwidth to each subchannel to minimize the total distortion, and yet no solution has been obtained in this area. This can also be viewed as a combined source and channel coding approach which is well known as a powerful solution to noisy channel transmission.

2. SUBBAND MCM

The MCM system can be modeled as C_T parallel, independent subchannels[7], where C_T is total number of subchannels. Thus the power, modulation rate and even the channel encoder of each subchannel can be changed flexibly without and effect on other subchannels. For transmitting subband coded image, this property shows more promising advantage since it can be used to provide unequal error protection for each subband. More important subbands are transmitted with better channel protection in order to minimize the channel error effect.

There can be two kind of transmission method for subband coded image. Type one transmits one subband at a time with different power allocated to the whole channel. The loading algorithm assigns the same error performance to each subchannel every time,similar to that of [1, 6, 7]. The portion of power assigned to each subband then is optimally allocated to achieve the best result. This method require two level of optimization and thus heavy computation. Also the power sum of all the subchannels varies in time which is not practical for real applications. In contrast, the second method tries to maintain the property of different noise effect at each subchannels and use it to achieve unequal error protection. As shown in Figure 1, all the subbands are transmitted at the same time, each occupying a number of subchannels according to their total number of bits. Instead of making the error performance of each subchannel equal, the loading algorithm tries to assign the power and modulation rate to each subchannel to achieve the best error performance for each subband which in general is not equal for noisy channel transmission. This method utilizes the flexibility of MCM and the total transmission power stays the same in time. The only disadvantage is that the receiver must wait until the transmission end to get every subband. But as we will show later, this fact becomes minor comparing to the performance improvement.

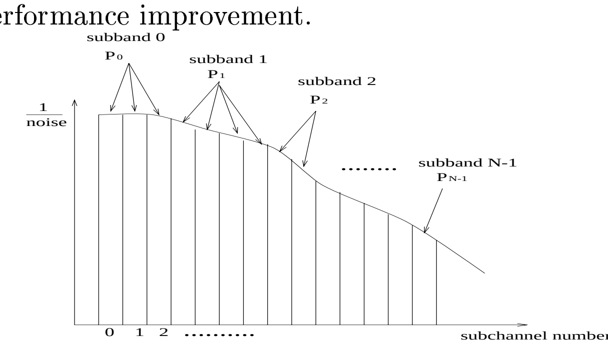

Figure 1: *Transmission Model: subband 0 to N-1 are transmitted through different subchannels with different error probability pi, i=0...N-1*

3. A MATHEMATIC MODEL

3.1. Channel Distortion

Assume the orthogonality of source and channel, the overall distortion can be written as the sum of source distortion and channel distortion. After the source rate is distributed according to bit allocation algorithm [5], the source distortion is then determined. The objective is to minimize the channel distortion by finding the optimal error performance for all the subbands under power constraint. For subband decomposed and vector quantized (VQ) image, the channel distortion is defined as

$$D_c = \sum_{m=1}^{M} \sum_{i=0}^{N_m-1} \sum_{j=0}^{N_m-1} P(i)P(j|i)(y_i - y_j)^2, \quad (1)$$

where M is total number of subbands, N_m is the cardinality of the subband m, y_i and i is the codeword for VQ and binary codeword for transmission. Assume only single bit error within one binary codeword with probability P_m, i.e. $P(j|i) = P_m$ is i, j differ in 1 bit, $1 - P_m$ if $i = j$ and otherwise 0. Thus, (1) becomes

$$D_c = \sum_{m=1}^{M} P_m \sum_{i=0}^{N_m-1} D_{i,m} = \sum_{m=1}^{M} P_m W_m, \quad (2)$$

where $D_{i,m}$ represents the distortion sum of j and i where j and i differ one bit, $W_m = \sum_{i=0}^{N_m-1} D_{i,m}$

represents the distortion caused by a single bit error for subband m and is deemed as the weighting factor of subband m. Subband with high energy has larger weighting factor and then more important. P_m is the bit error rate of subband m which is the function of power and noise variance of subband m.

3.2. Assignment of Subchannels to Subband

For transmitting all the subbands at the same time, the assignment of subchannels to each subband becomes an important procedure since the noise variance at each subchannel is quite different. W propose the following scheme:

1. More important subbands are assigned with better channels (smaller noise variance). First the subchannels are sorted in increasing noise variance order. Similarly, the subbands are sorted in decreasing importance (weighting factor) order.

2. Compute the number of subchannels that each subband needs. For the optimal design, all the subchannels belong to the same subband should have the same error performance. The algorithms of [1, 6, 7] can be used to accomplish this for each subband. For simplicity, we accomplish this by assigning the same modulation rate to the subchannels belong to the same subband and allocating power to these subchannels to achieve the same SNR value. Thus, for subband m, given modulation rate R_m, the number of subchannels occupied c_m is INT($\frac{Bit_m B}{R_m Bit_{total}}$), where Bit_m is the total number of bits of subband m and Bit_{total} is that of whole image, B is the number of bits that must be transmitted in every transmission. The Bit_m and Bit_{total} only depend on the source image while B depends on the MCM structure. $\{c_m\}_{m=0}^{N-1}$ are adjusted until $\sum_{m=1}^{N} c_m R_m = B$.

3. Assign the sorted subchannels to the sorted subbands consecutively. Assume the total power assign to subband m as E_m and subband m occupies c_m subchannels with noise variance $\{N_{m,k}\}_{k=1}^{c_m}$, the system model is equivalent to transmitting subband m over c_m same channel, each with power E_m/c_m and noise variance $N_m = \sum_{k=1}^{c_m} \frac{N_{m,k}}{c_m}$. After the optimal value of $E_m/(c_m N_m)$ is solved, the power allocate to subchannel k can be computed as $\frac{E_m N_{m,k}}{c_m N_m}$. Thus, the c_m subchannels belongs to subband m has the same channel model with SNR $\frac{E_m}{c_m N_m}$ and modulation rate R_m.

For some typical spectral shaped channels, the noise variances are quite different among all the subchannels. This assignment will ensure the most important subbands are transmitted over good channels without allocating much power to them. This shows more advantages for low power requirement. Also the total power needed at each transmission stays the same which makes our scheme more practical for the real applications. Under the above assumption, the optimization function is

$$\begin{aligned} \text{Min} \quad & \sum_{m=1}^{N} Pe(R_m, \frac{E_m}{c_m N_m}) W_m, \\ \text{subject to} \quad & \sum_{m=1}^{N} E_m \le E_T, \ \sum_{m=1}^{N} c_m \le C_T, \end{aligned} \quad (3)$$

where E_T is the total power constraint, C_T is the total number of subchannels and Pe is the bit error rate.

4. A NEW LOADING ALGORITHM

4.1. A Simple Example

Assume that all the subchannels have the same noise variance normalized to 1 and the same modulation type BPSK. The (3) is simplified to

$$\begin{aligned} \text{Min} \ & \sum_{m=1}^{N} Q(\sqrt{\frac{2E_m}{c_m}}) W_m, \\ \text{subject to} \ & \sum_{m=1}^{N} E_m \le E_T. \end{aligned} \quad (4)$$

By applying Lagrange multiplier, the optimal solution satisfies $W_m \sqrt{\frac{1}{E_m c_m}} \exp \frac{-E_m}{c_m} = \lambda$. Define $\Phi_{a,b}(x) = \sqrt{(\frac{1}{ax})} exp(-bx)$, the optimal λ can be found by solving

$$\sum_{m=1}^{N} \Phi^{-1}_{a_m,b_m}(\lambda/W_m)|_{a_m=E_m c_m, b_m=1/c_m} = E_T. \quad (5)$$

Since $\Phi_{a,b}(x)$ is a monotonic function of x for $a, b > 0$, Φ^{-1} can be solved simply by bisection method. Also since Φ^{-1} is also monotonic, the optimum λ can be solved by bisection method.

4.2. Loading Algorithm

- **Given Rate R_m determine Power E_m.** For square QAM constellation, the uncoded Pe

at the receiver is

$$Pe(R_m, \frac{E_m}{c_m N_m}) \approx 4Q(\sqrt{\frac{3E_m}{c_m N_m (2^{R_m}-1)}}). \tag{6}$$

Given $\{R_m\}$, the optimal power allocation is computed similarly as in the above section by applying Lagrange multiplier and finding Φ^{-1}, i.e. $E_m = \Phi^{-1}_{a_m,b_m}(\lambda/W_m)$ with $b_m = \frac{3}{2c_m N_m(2^{R_m}-1)}$ and $a_m = c_m N_m(2^{R_m}-1)$.

- **2. Determine Rate R_m.**
  It is equivalent to determine c_m. Since $\{c_m\}_{m=1}^{N-1}$ are discrete value, the derivation method can not be used. Usually in real applications the allowed modulation rates of all the subchannels are limited to the range $R_{min} \leq R_m \leq R_{max}$. R_{min}, R_{max} are the allowed upper bound and lower bound, respectively. Based on this assumption, we propose the following algorithm,

  1. Initially, let the modulation rate of all the subbands $\{R_m\}_{m=0}^{N-1}$ as R_{max}. Compute the corresponding $\{c_m\}_{m=0}^{N-1}$. If $C = \sum_{m=1}^{N} c_m \geq C_T$, exit. Compute $\{E_m\}_{m=0}^{N-1}$ and D_c as in (3).
  2. Find the subband k which satisfies $C \leq C_T$ and yields smallest D_c by subtracting one bit from R_k. Set $R_k = R_k - 1$;
  3. Continue step 2 until for all subbands, no subtraction of modulation rate satisfies the constraint $C \leq C_T$ or reduce D_c.

This algorithm is practical for real applications since the number of subbands are usually small. For l level subband decomposition, number of subbands is $3l+1$. If N is the number of total subbands with quantization rate > 0, the worst computation complexity is $N(R_{max} - R_{min}) \times$ (complexity of compute C and E).

5. SIMULATION AND DISCUSSION

The image is four level subband decomposed using Daubechies 16 wavelet filter and then vector quantized using full search LBG algorithm. The weighting factors are computed. The noise in the subchannels are White Gaussian Noise. Although for low SNR value, channel coding can be applied to reduce the noise effect, we assume no channel coding to maintain simplicity.

To demonstrate the advantage of our loading algorithm, we compare our result to the results of [9] and single carrier modulation on Gaussian channels. Assume that the noise variance for all the subchannels are the same and only the BPSK modulation is allowed, Figure 2,3 and 4 illustrate the PSNR as a function of the average SNR of the channel. The "Lena" image is 4 level subband coded to 0.1bpp, 0.398bpp and 0.96bpp. For channel SNR value less than 8dB, the noise effect becomes observable for the BPSK modulation system and causing large degradation on the received image. As can be seen, our algorithm performs better than that of [9] and single carrier modulation. At 4dB channel SNR for different source rate at 0.398bpp and 0.96bpp, our system is 1.97dB and 2.83dB better than that of [9], 4.00dB and 5.80dB better than single carrier modulation.

For single carrier modulation, no unequal error protection is involved and bit error rate of the most important subband is as same as the least important subband. Our results outperform that of [9] since for subband coded image and noisy channel, the difference of the subbands importance appears as a dominate factor on the system design compared to the difference of the bits importance inside a codeword. Different error performance for different subbands become a more powerful solution, especially for large source rate and low SNR value. Larger source rate will increase the number of subbands, which leads to the importance of the subbands being classified more precisely. More tradeoff can be achieved among the subbands in the error performance assignment. On the other hand, decreasing SNR results in larger noise effect. And assigning most of the power to the lowest subband will reduce the degradation due to the noise effect. From the implementation point of view, our algorithm is more practical than that of [8, 9] since the total number of subchannels and total power for each transmission is fixed to the same. After the loading procedure at the initial transmission, the whole system stays to the same parameters.

We also compare our loading algorithm to that of [6, 7] by transmitting the subband coded image through a spectrally shaped channel. The subchannel noise variance is shown in Figure 5. For a total of 256 subchannels, each MCM symbol carries 512 bits, which means $C_T = 256, B = 512$. Set $R_{max} = 6$ and $R_{min} = 2$. Figure 6 sketches the received image PSNR value versus $E_{avg} = \frac{\text{total power}}{B}$ for "Lena" image coded from 0.1bpp to 0.49bpp. The new loading algorithm shows 5-10dB PSNR improvement for E_{avg} ranging from 1dB to 20dB. The image results for $E_{avg} = 15$dB are shown in Figure 7 and 8. For spectrally shaped channels, the noise variance differs a lot among the subchannels causing good channels and bad channels. The traditional loading algorithm flats the BER of the subchannels, turning both the good channels and the bad channels into the average performance channels. The new loading algorithm maintains good/bad channel situation and uses good channel to transmit important information. According to the importance measure, the power and rate allocation adjusts the good/bad channel by a small amount in order to achieve the best error performance distribution among the subbands. As E_{avg} increases, the noise effect becomes less significant and the performance converges. As can be seen, our algorithm achieves more improvement over the other algorithms at lower SNR and larger source rate which can be explained similarly as that in BPSK example.

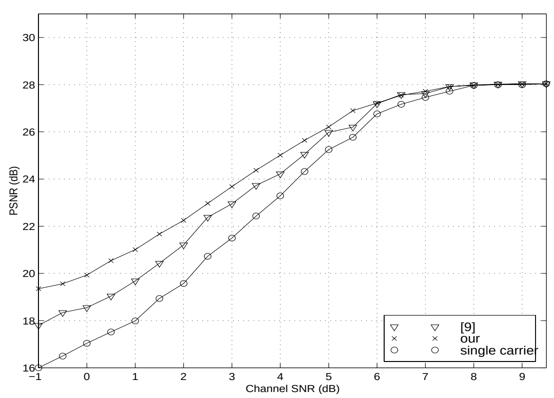

Figure 2: *Comparison of our algorithm to [9] and single carrier modulation algorithm using "Lena" coded at 0.1bpp*

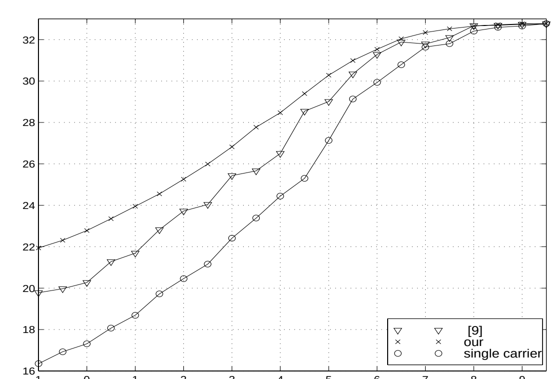

Figure 3: *Comparison of our algorithm to [9] and single carrier modulation using "Lena" coded at 0.398bpp*

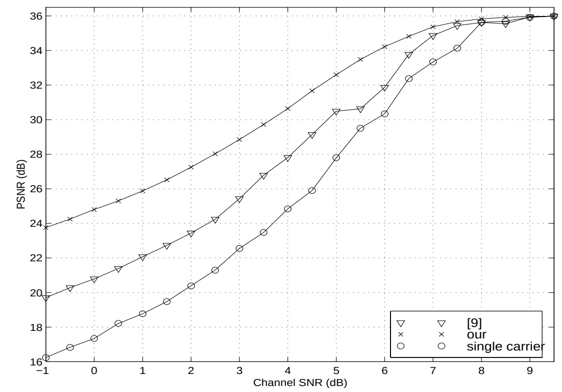

Figure 4: *Comparison of our algorithm to [9] and single carrier modulation using "Lena" coded at 0.96bpp*

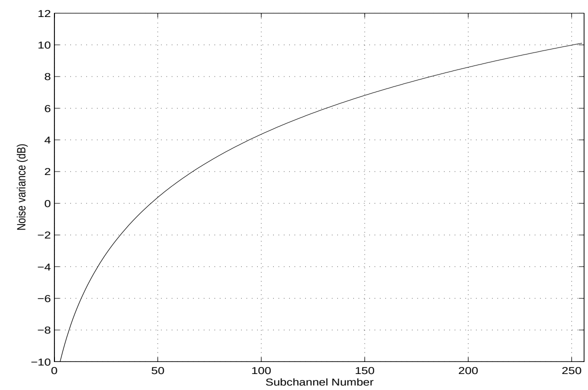

Figure 5: *Noise variance distribution for multicarrier modulation, total 256 subchannels.*

6. CONCLUSION

We have proposed a power and modulation rate loading algorithm for subband coded image transmitting over noisy channel using MCM. Unlike the traditional method, we propose to transmit all the subbands simultaneously but using different subchannels. Our loading algorithm efficiently allocates the subchannel's power and modulation rate according to the importance of the information they transmitted. It achieves significant PSNR improvement for spectrally shaped channels based on the idea of good channel transmitting important information and bad channel transmit nonimportant information. Numerical examples show that on very noisy AGWN channel or CSNR ranging from -1dB to 6dB, our result achieves 0.5-4dB and 1-8dB PSNR improvement over result in [9] and single carrier modulation, respectively. On spectrally shaped channel, our result yield 10 to 15dB PSNR improvement over that of [7].

The implementation of our algorithm requires only small amount of computation at the initial transmission. After initialization, the power and modulation

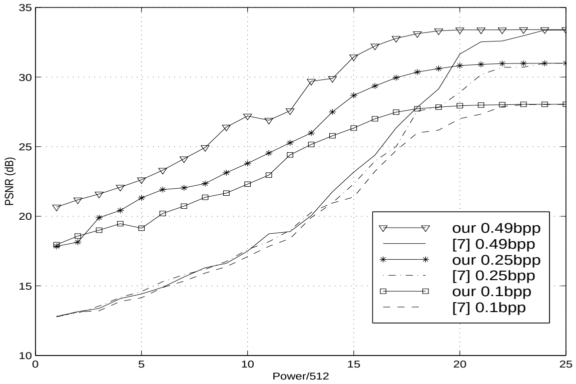

Figure 6: *Comparison of our algorithm to [7] using "Lena" coded at various rates*

Figure 7: *Received "Lena" at 0.25bpp, our result*

remains the same for all the subbands. The total power and rate at each transmission remains the same also. Those factors make our algorithm practical in real applications.

7. REFERENCES

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Figure 8: *Received "Lena" at 0.25bpp, [7] result*

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