# TECHNICAL RESEARCH REPORT

## Admission Control Schemes for Spot-Beam Satellite Networks

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## Abstract

In this report, we consider communication networks with a satellite with multiple spot beams. We describe the structure and features of these networks, and we consider admission control schemes which optimize the network revenue if several services types with different revenues are present. We show that in some cases the blocking of some calls even if capacity is available can considerably increase the network revenue. We will point out, however, that *complete sharing*, i.e., accepting calls on a first–come, first–served basis, is optimum for systems with similar traffic types.

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# Contents

# Chapter 1

# Introduction

In satellite systems, real–time capacity is often allocated on a first–come, first–served basis, as long as capacity is available. This form of admission control, which only takes into account the order of arrival and the capacity, is known as *complete sharing.* Although it is very simple to implement, it may not be optimum from a utilization point of view. Alternative admission control schemes might block a call even when capacity is available to improve overall blocking probability or to increase revenue generated by the network.

Imagine a situation in which the traffic through a given satellite approaches full capacity, and a new call which is known to occupy a large bandwidth and/or to have a long duration requests admission to the system. Assume that if this call is accepted, no more satellite capacity is available; then all other calls arriving before the termination of another on-going call will be blocked. On the other hand, if that first call was blocked, some more calls might have been accepted, therefore possibly reducing the overall blocking probability, increasing revenues, or both.

If the traffic over a satellite system can be separated into different types with known statistics, such as call arrival rate, departure rate and revenue per time unit, it may be advantageous to apply some admission control which depends on the state of the network. Such an admission control scheme is essentially defined by a set of allowed network states. If a new call arrives, the admission control algorithm simply computes the network state that would result if that call were accepted. If this new network state is not in the set of allowed states, that call is blocked, otherwise it is accepted.

Such an admission control scheme was suggested, among others, by Foschini and Gopinath [Fos81,Fos83] for systems where multiple service types are competing for a single resource. This work was extended by Jordan and Varaiya [Jor91b] to the more general case of Multiple Service, Multiple Resource (MSMR) networks. In our work, we try to apply the MSMR–theory to communication networks with a spot–beam satellite, and we investigate admission control strategies that maximize the system revenue.

In Chapter 2, we describe the general theory of admission control in multiple service, multiple resource networks, and we introduce the traffic model and the performance measures used in the subsequent optimization. In Chapter 3, we analyze the features of satellites with a single hopping beam and with multiple spot beams. The investigation of admission control schemes for networks with such satellites is performed in Chapter 4, and the report closes with some conclusions in Chapter 5. Mathematical details and proofs are given in the Appendix.

# Chapter 2

# Admission Control in Multiple Service, Multiple Resource Networks

In this chapter, we introduce the model of our satellite communication system, and we give a brief outline of the theory of Multiple Service, Multiple Resource (MSMR) networks, which will be useful for the remainder of this report.

## 2.1 System Model

An MSMR network is a system that offers $J$ types of services or calls (we will be using both terms). Each service requires a set of resources (dependent on the service type) to process. The state of our system at a given time is the number of active services of each type, which we denote by the vector $x = [x_1, x_2, \ldots, x_J]$, where $x_j$ is the number of active services of type $j$. The set of possible vectors $x$ forms a $J$-dimensional state space $\Omega$. We demand that service completion is never blocked, which implies that the state space $\Omega$ is *coordinate-convex*, i.e., if $x \in \Omega$ and $x_j > 0$, then for all $j = 1, \ldots, J$, we require $[x_1, \ldots, x_j - 1, \ldots, x_J] \in \Omega$.

### 2.1.1 Traffic Model

Some of the traffic models used in the literature are reviewed in Appendix A.1. For the remainder of this report, we assume that there is an infinite source population, that for service type $j$ call arrivals are Poisson with arrival rate $\lambda_j$, and that call duration is exponential with mean $1/\mu_j$; the corresponding load is $\rho_j = \lambda_j/\mu_j$. If a call is accepted, it uses all resources immediately, and releases them simultaneously upon service completion. Otherwise the blocked call is cleared (not queued), which corresponds to a "blocked-calls-cleared" mode of operation. We further assume that there are shared resources (as opposed to dedicated resources), such that all services of a given type can be served by any of the resources of that type. With these assumptions, the system can be represented as a Markov chain (*Erlang* model) with transition probabilities

$$P_{xy} = \begin{cases} \lambda_j & \text{if } y = [x_1, \ldots, x_j + 1, \ldots, x_J] \text{ and } y \in \Omega \\ \mu_j x_j & \text{if } y = [x_1, \ldots, x_j - 1, \ldots, x_J] \text{ and } y \in \Omega \\ 0 & \text{otherwise.} \end{cases} \tag{2.1}$$

Note that in this model, calls arrive at a rate $\lambda_j$ and depart at a rate $\mu_j x_j$, as in an $M/M/c$ or an $M/M/\infty$ system. For the above assumptions on traffic characteristics, the Markov chain is time reversible, and the stationary distribution of being in state $\pi(x)$ can be expressed in the following product form [Aei78]:

$$\pi(x) = \pi(0) \prod_{j=1}^{J} \frac{\rho_j^{x_j}}{x_j!}, \tag{2.2}$$

where $\rho_j = \lambda_j/\mu_j$, and where $\pi_0$ is the normalization constant given by

$$\pi(0) = \left\{ \sum_{x\in\Omega} \prod_{j=1}^{J} \frac{\rho_j^{x_j}}{x_j!} \right\}^{-1}. \tag{2.3}$$

Because $\pi(0)$ depends on $\Omega$, it is convenient to introduce the quantity

$$G(\Omega') = \sum_{x\in\Omega'} \prod_{j=1}^{J} \frac{\rho_j^{x_j}}{x_j!}, \tag{2.4}$$

where $\Omega'$ is any subset of the state space $\Omega$. Note that $\pi_0 = G(\Omega)^{-1}$.

### 2.1.2 Performance Measures

There are two classes of optimization criteria that are commonly used in the context of Multiple Service/Single Resource (MSSR) and Multiple Service/Multiple Resource (MSMR) systems: the reward-type and the blocking-type performance measures. Although they are inherently different in nature, it can be shown that maximizing some type of reward is equal to minimizing the blocking probability. They are discussed to some detail in Appendix A.1.

For the optimization of our satellite communication system, we will use as our performance measure the average revenue per time unit $R(\Omega)$ generated by a system with state space $\Omega$. We assume that the revenue of a call of type $j$ is $r_j$. The average system revenue $R(\Omega)$ is then defined as

$$R(\Omega) = \mathrm{E}\left[r(x) | x \in \Omega\right] = \sum_{x\in\Omega} r(x)\pi(x), \tag{2.5}$$

where $\pi(x)$ is defined by Eq. 2.2, and where $r(x)$ is the rate of revenue generated while in state $x$,

$$r(x) = \sum_{j=1}^{J} r_j x_j. \tag{2.6}$$

Note that the revenues $r_j$ depend on the type of service $j$ only, and not on the control policy.

## 2.2 Admission Control

In this section, we discuss admission control schemes for MSMR networks in general. We will use the system model described in the previous section, and our goal is to find a policy which maximizes the revenue of the network. To ensure that service completion is never blocked, we consider only *coordinate convex (c.c.)* policies, which base admission-control decisions on the state of the network. These policies can be expressed as a c.c. subsets $\Omega'$ of the state space $\Omega$. If a new call of type $j$ arrives when the system is in state $x = [x_1, \ldots, x_j, \ldots, x_J]$, this call is only admitted if the state $x = [x_1, \ldots, x_j + 1, \ldots, x_J]$ is in $\Omega'$.

Well known policies are *complete sharing (CS)* and *complete partitioning (CP)*. In CS, new calls are admitted to the system on a first-come, first-served basis as long as resources are available, such that the corresponding state space of the system can be expressed by $\Omega_{CS} = \Omega$. The CP policy partitions the capacity $C$ into $J$ sets $C_j$ corresponding to the $J$ call types, and a call of type $j$ is accepted as long as there is enough capacity $C_j$ available.

In our work, we are interested in the c.c. policy $\Omega^* \subseteq \Omega$ which maximizes the average system revenue $R(\Omega^*)$, cf. Eq. 2.5. Note that if $\Omega^* \neq \Omega$, i.e., for all policies other than complete sharing, calls are blocked even if resources are available. Such a policy can increase the system revenue if eg. a call which generates a small revenue is blocked, leaving capacity for a call which generates a higher revenue. Note that the optimum policy may not be fair, if for example all calls of some type are blocked. Hence, for a practical system, some fairness rule will have to be added.

We will now introduce some notation which will be helpful in the sequel. A set of states $\Gamma$ is *annexable* to $\Omega'$ if no element of $\Gamma$ is in $\Omega'$, and if the union of $\Gamma$ and $\Omega'$ is coordinate convex [Jor91b]. Accordingly, $\Gamma$ is *removable* from $\Omega'$ if all elements of $\Gamma$ are in $\Omega'$, and if the difference of $\Omega'$ and $\Gamma$ is coordinate convex:

$$\begin{array}{lll} \Gamma \text{ is } \textit{annexable} \text{ to } \Omega' & \text{iff} \quad \Gamma \cap \Omega' = \emptyset & \wedge\ \Gamma \bigcup \Omega' \ \text{ is c.c.} \\ \Gamma \text{ is } \textit{removable} \text{ from } \Omega' & \text{iff} \quad \Gamma \subseteq \Omega' & \wedge\ \Omega' - \Gamma \ \text{ is c.c.} \end{array}$$

We now show that the network revenue can be increased by restricting the state space ($\Omega \to \Omega - \Gamma$) if and only if the revenue generated by the removed states is smaller than the average revenue. This is intuitively obvious, but it has to be taken into account that the probability of being in state $x \in \Omega - \Gamma$ differs from the probability of being in state $x \in \Omega$ because the normalization constant is different. The two quantities to be compared are the average revenue $R(\Omega)$, and the revenue $R(\Gamma)$ generated in states $\Gamma$. We now state the following lemma:

**Lemma 1** *A subset of states $\Omega - \Gamma$ generates a higher revenue than the state space $\Omega$ if and only if the average revenue generated in $\Gamma$ is smaller than the average in $\Omega$:*

$$R(\Omega - \Gamma) \geq R(\Omega) \quad \text{iff} \quad R(\Gamma) \leq R(\Omega). \tag{2.7}$$

The proof is given in Appendix A.2. Similarly, it can be shown the revenue can be increased by adding states if and only if the added states generate above average revenue. With this result we can characterize an optimal c.c. policy as a subset of $\Omega$ to which nothing above average can be added and nothing below average removed [Fos81,Jor91b], i.e.,

A c.c. set $\Omega^* \subseteq \Omega$ is **optimal** iff:

$$\begin{array}{lll} \nexists \Gamma \subseteq \Omega & \text{s.t. } \Gamma \text{ is annexable to } \Omega^* & \wedge\ R(\Gamma) > R(\Omega^*) \\ \nexists \Gamma & \text{s.t. } \Gamma \text{ is removable from } \Omega^* & \wedge\ R(\Gamma) < R(\Omega^*) \end{array}$$

We will use this optimality criterion for the design of an admission control scheme in Chapter 4.

# Chapter 3

# Spot–Beam Satellites

Here, we will discuss some of the features of satellites with multiple spot beams. We will consider satellites with a single hopping beam, satellites with multiple beams and an on-board switch, and satellites with hopping beams. We will focus on capacity requirements, and on the Multiple Service, Multiple Resource (MSMR) representation introduced in the previous chapter.

Early communication satellites, such as Early Bird (INTELSAT I), used simple dipole antennas, and they transmitted most of their power into deep space. Modern geostationary satellites with high-gain antennas can focus their signal on the desired area, be it the visible earth, a continent, or a footprint as small as 200 km in diameter (pencil beams with a beam-width of $1/3^o$). The use of spot beams greatly improves the link budget, because the (scarce) satellite power is not wasted by illuminating undesired areas. One of the results of the improved link budget are smaller and cheaper earth stations. The NASA Advanced Communication Technology Satellite (ACTS), for example, provides a data rate of about 1.5 Mbps with an earth station antenna diameter of only 1.2 meter. The use of spot-beams also allows frequency re-use, an important issue for the growing demand of bandwidth for satellite communications.

In the remainder of this chapter, we will describe three different types of spot-beam satellites, which differ in the number of beams and footprints. We denote $B$ the number of satellite beams (and transponders), $F$ the number of footprints, and $C$ the capacity of the satellite. We assume that the capacity of the satellite is time-shared among all users by applying some kind of TDMA scheme. For each user which is admitted to the system, one or several slots in a TDMA frame of length $C$ are allocated in a fashion not to be discussed here. The satellite can handle $B$ simultaneous basic calls per slot (one for each beam or transponder), which results in a total number of $B \cdot C$ simultaneous calls.

We will first describe satellites with a single hopping beam ($B = 1, F > 1$), we will then explain some features of satellites with an on-board switch and fixed beams ($B = F$), and we finally proceed to hopping-beam satellites ($F > B > 1$).

## 3.1 Satellite with a Single Hopping Beam

In this section, we consider a satellite with a single hopping beam (in fact two hopping beams, one for the uplink and one for the downlink). The uplink and the downlink beam can hop independently, thereby covering $F$ footprints. The transponder on board the satellite is transparent, and it can be shared among all calls. A link between footprints $F_1$ and $F_2$ can be established by switching the uplink beam to footprint $F_1$, and the downlink beam to footprint $F_2$. For a link from $F_2$ to $F_1$, the two beams have to be interchanged.

We assume that each call can be modeled as one of $J$ different traffic types, which might be characterized by the source and the destination, which may represent different priority classes, or which may require a different capacity (number of slots). Each traffic type $j$ is defined by the arrival rate $\lambda_j$, departure rate $\mu_j^{-1}$, the utilization $\rho_j = \lambda_j/\mu_j$, the required capacity $a_j$ and the revenue $r_j$.

For the service types described above, a hopping beam satellite with capacity $C$ can accept calls as long as the sum of all ongoing calls, weighted by their required capacity $a_j$, does not exceed $C$. Hence, the state space $\Omega$ of this system is given by

$$\Omega = \left\{ x \left| \sum_{j=1}^{J} x_j a_j \leq C \right. \right\}. \tag{3.1}$$

Recall that the vector $x$ has $J$ components, $x_j$ denoting the number of ongoing calls of type $j$.

This satellite network can be modeled as a Multiple Service Single Resource (MSSR) communication system [Jor91b], with the satellite being the single resource. MSSR systems have been investigated, among others, by Aein [Aei77,Aei78] and Foschini et. al. [Fos81].

It was conjectured by Foschini et. al. [Fos83] that the optimum policy which minimizes the blocking probability for an MSSR system with capacities $a_j = 1, \quad j = 1 \ldots J$ and with transition probabilities according to Eq. A.1 (memory sharing) is of the form

$$\Omega' = \{x | x \in \Omega \text{ and } \sum_{j \notin \mathcal{I}} x_j \leq c_{\mathcal{I}} \quad \forall \quad \mathcal{I} \subset \mathcal{J}\}, \tag{3.2}$$

where $\mathcal{J}$ is the set of all service types $\mathcal{J} = \{1, \ldots, J\}$, and $\mathcal{I}$ is a subset of $\mathcal{J}$. In other words, this conjecture states that the optimum policy is to limit sums of the number of each type in the system.

As an example, for $J = 2$, the optimum policy $\Omega'$ is of the form

$$\Omega' = \{x | x_1 \leq c_1, x_2 \leq c_2, x_1 + x_2 \leq c_{12}\}, \tag{3.3}$$

and for $J = 3$, the optimum policy is of the form

$$\Omega' = \{x | x_1 \leq c_1, x_2 \leq c_2, x_3 \leq c_3, x_1 + x_2 \leq c_{12}, x_1 + x_3 \leq c_{13}, x_2 + x_3 \leq c_{23}, x_1 + x_2 + x_3 \leq c_{123}\}. \tag{3.4}$$

Note that this result is very powerful, since it greatly reduces the number of policies to be investigated. This conjecture was proved by Jordan and Varaiya [Jor91b], however for the criterion of maximizing revenue.

Because of the limited practical interest in satellites with a single hopping-beam, we will not consider these systems in our upcoming analysis.

## 3.2 Satellite with Multiple Fixed Beams

We now consider a satellite with $B$ uplink- and downlink pairs of *fixed* beams. Each of these beam pairs covers one set of footprints, such that the number of footprints equals the number of beams ($F = B$). There are $B$ transparent transponders on board the satellite, and each transponder can handle traffic from one beam pair. A microwave switch matrix switches cyclically between each up- and downlink, thereby creating a fully connected network, as eg. used for satellite-switched TDMA (SS-TDMA) [Cam90].

For a single-hop link between two footprints, a service requires an uplink-beam from the source to the satellite, and a downlink-beam from the satellite to the destination simultaneously, i.e., there is no buffering on board the satellite. With $B$ pairs of beams, there are $B^2$ different links, including those where source and destination are in the same footprint. Taking into account that all up- and downlink beams are different, there are $B!$ possible switch patterns, as shown in Fig. 3.1 for $B = 3$.

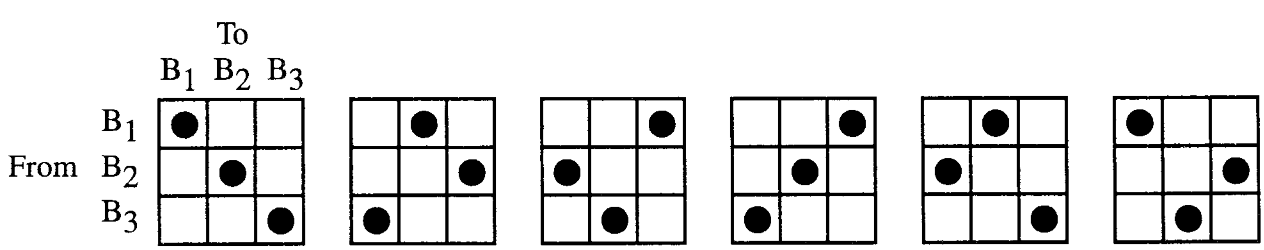


Figure 3.1: Switch States For $B = 3$ Beams Covering $F = 3$ Footprints

Since the satellite switch matrix can accommodate $B$ calls at a time, there are $B$ switching patterns which have to be executed cyclically to cover all $B^2$ possible call types. Hence, this model can be represented as a Multiple Service Multiple Resource (MSMR) network [Jor91b] with $J = B^2$ services and two sets of $B$ resources, each service of type $j$ requiring $a_j$ resources of each of the two sets.

In the sequel, we assume that each service requires only one resource of each of the two sets, i.e., $a_j = 1,\ j = 1, \dots, J$. Hence, because of the unit resource requirement and the $B$ transponders, the satellite can handle $B$ simultaneous calls in each time unit. We therefore define the capacity of the satellite to be $C \cdot B$ calls.

For the upcoming analysis, it is convenient to represent the system state by a *traffic matrix* $A$ with elements

$$a_{ij} = x_{(i-1)B+j}, \quad i, j = 1 \dots B. \tag{3.5}$$

Then, $a_{ij}$ represents the number of calls going from footprint $i$ to footprint $j$. We further define a *line* of a matrix to be a row or a column, and a *line sum* to be the sum of all elements in a line.

To illustrate this model, let us consider a satellite with $B = 2$ pairs of beams, $B_{1u}, B_{1d}, B_{2u}$, and $B_{2d}$, where the subscripts denote up- or downlink. There are two footprints, denoted $F_1$ and $F_2$, and four different service types, as shown in Table 3.1:

| Service Type | Source | Destination | Resources |
|---|---|---|---|
| 1 | $F_1$ | $F_1$ | $B_{1u} + B_{1d}$ |
| 2 | $F_1$ | $F_2$ | $B_{1u} + B_{2d}$ |
| 3 | $F_2$ | $F_1$ | $B_{2u} + B_{1d}$ |
| 4 | $F_2$ | $F_2$ | $B_{2u} + B_{2d}$ |

Table 3.1: Service Types And Resources For A Satellite With $B = 2$ Beams And $F = 2$ Footprints

For this simple example, there are only two states of the microwave switching matrix of the satellite. In the first state, $B_{1u}$ is connected to $B_{1d}$, and $B_{2u}$ is connected to $B_{2d}$, whereas in the second state $B_{1u}$ is connected to $B_{2d}$, and $B_{2u}$ is connected to $B_{1d}$. Hence, in state one, the satellite can provide services 1 and 4 simultaneously, whereas in state two, services 2 and 3 are available.

As already mentioned above, for $B$ beams there are $B!$ switching patterns and $B^2$ links. The optimum switch state time plan (SSTP) for a given number of calls of each type, denoted by the state vector or the traffic matrix $A$, can be computed by an extension of the greedy algorithm, as described by Inukai [Inu79]. Let us denote $L(x)$ the frame length of the SSTP which can accommodate a number of calls described by the vector $x$. With optimum algorithms, $L(x)$ is equal to the maximum line sum of the traffic matrix $A$, or, expressed with the state vector $x$,

$$L(x) = \max_{i=1\ldots B} \left\{ \max \left( \sum_{j=1}^{B} x_{j+(i-1)B}, \sum_{j=1}^{B} x_{i+(j-1)B} \right) \right\} \tag{3.6}$$

A given number of calls described by $x$ can be transmitted by a satellite with capacity $C \cdot B$ if $L(x) \leq C$, hence the state space can be expressed as

$$\Omega = \{x | L(x) \leq C\}. \tag{3.7}$$

Note that for a simple satellite with $B = 2$ beams, the above expression simplifies to

$$L(x) = \max(x_1, x_4) + \max(x_2, x_3), \tag{3.8}$$

such that the state space is described by

$$\Omega = \{x | \max(x_1, x_4) + \max(x_2, x_3) \leq C\}. \tag{3.9}$$

Let us illustrate this with an example for $B = 2$. Assume that there are two calls of type 1, three calls of type 2, four calls of type 3, and one call of type 4. Then, the state vector is $x = [2, 3, 4, 1]$, and the traffic matrix is

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \tag{3.10}$$

The maximum line sum of $A$ is six, i.e., we need a satellite with at least capacity $6 \times 2$ to handle this traffic.

## 3.3 Satellite with Multiple Hopping Beams

For a satellite with $B$ pairs of hopping beams, we assume that each of these beam pairs covers one set of $F > B$ footprints $F_f$, $\bigcup_{f=1}^{F} F_f = F$, where the sets $F_f$ are non-overlapping, i.e., $F_f \cap F_g = \emptyset$ for $f \neq g$. As in the previous scenario, there are $B$ transparent transponders on board the satellite, and each transponder serves one beam pair. We can either assume that the up- and the downlink beams are "hard-wired", i.e., there is no switch on board the satellite, or that the satellite has $F$ fixed beams, $B$ transponders, and a switch which connects the beams to the transponders. The second model represents the technical implementation more closely.

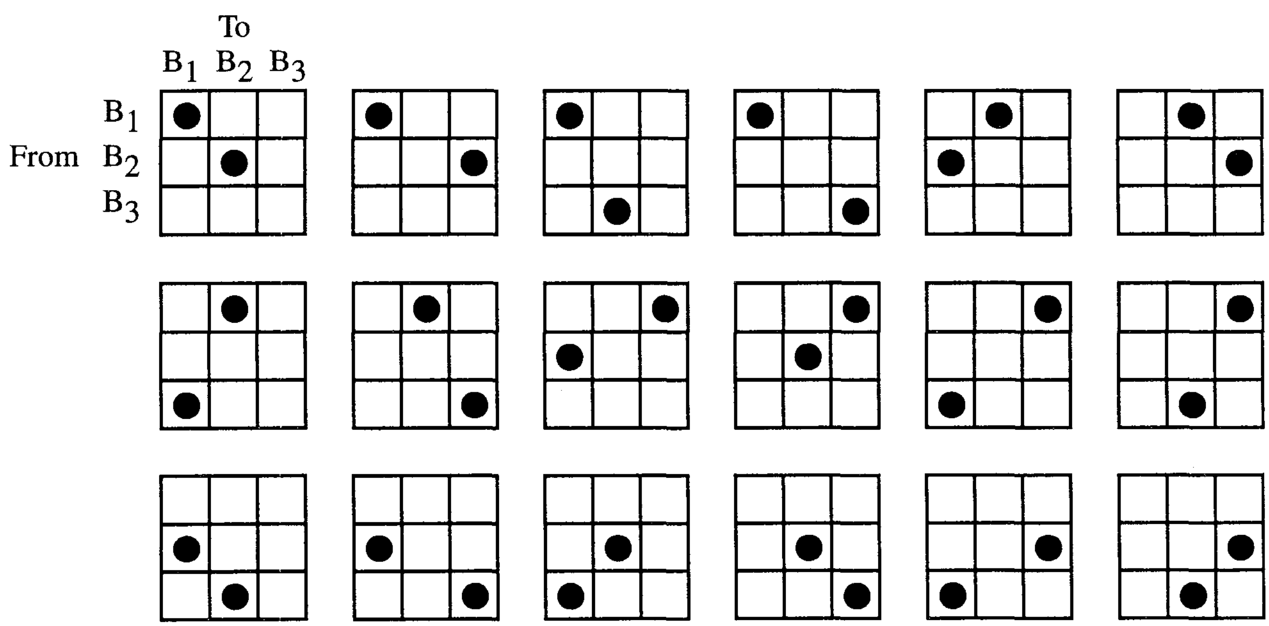


Figure 3.2: Switch States For $B = 2$ Beams Covering $F = 3$ Footprints

To allow for frequency re–use, we assume that all $B$ beam pairs operate in the same frequency band, such that no two up- or downlinks beams are allowed to point to the same footprint at the same time. Because the up- and the downlinks use different frequencies, it is possible, however, that *one* uplink and *one* downlink beam point to the same footprint.

The topology of such a network can also be represented as switch states, where we have to take into account that there are more footprints than transponders. For $B = 2$ beams and $F = 3$ footprints, there are 18 different switch states, as shown in Fig. 3.2.

We assume that the greedy algorithm used to optimize the satellite switch time plan for SS-TDMA can be adapted for these partly connected switching matrices. However, we did not attempt to prove this.

A given traffic pattern can be accommodated by a satellite with capacity $C$ if the maximum line sum is not greater than $C$ and if the total number of elements in the matrix is not greater than $C \cdot B$. The second restriction is due to the fact that in each slot only $B$ (and not $F$) calls can be active[1]. Hence, the state space of a system with $B$ beams, $F > B$ footprints and capacity $C$ is given by

$$\Omega = \left\{ x \;\middle|\; \max_{i=1\ldots F} \left\{ \max \left( \sum_{j=1}^{F} x_{j+(i-1)F}, \sum_{j=1}^{F} x_{i+(j-1)F} \right) \right\} \le C \;\bigwedge\; \sum_{j=1}^{J} x_j \le C \cdot B \right\}. \quad (3.11)$$

[1]Note that in the previous section with $B = F$ the restriction on the maximum line sum also implies the second one because there are $B = F$ lines in the matrix.

# Chapter 4

# Admission Control For Spot–Beam Satellites

Let us now investigate admission control schemes for spot-beam satellites. We will apply the methods introduced in Chapter 2 to the satellite networks described in Chapter 3. We will start with a simple system with two beams and two footprints, and then extend the investigation to larger systems.

## 4.1 Satellites with Two Beams and Two Footprints

In this section, we consider a satellite with $B = 2$ beams and $F = 2$ footprints. The satellite has a $2 \times 2$ switch on board, and we assume that it can handle at most $2 \cdot C$ calls. We recall from the discussion in Section 3.2 that for such a system there are four different service types that cover all possible connectivities of the two up- and downlink beams As shown in Fig. 4.1 and in Table 3.1, the source and destination of service type 1 both are footprint $F_1$, type 2 connects $F_1$ with $F_2$, type 3 connects $F_2$ with $F_1$, and, finally, type 4 services originate and end both in footprint $F_2$. Recall that the set of all possible states, the state space $\Omega$, is given by Eq. 3.9.

Note that the switch on board the satellite has two states, allowing either calls of types 1 and 4, or 2 and 3 to be served simultaneously. We denote a *group* to be the combination of simultaneous services, and it can be seen clearly from Eq. 3.9 that the sum of the maximum number of calls in each group determines the system behavior.

It is shown in Appendix A.3 that the number of states in $\Omega$, i.e., the cardinality card($\Omega$) is

$$\text{card}\,(\Omega) = \frac{1}{6}(C+1)(C+2)(C^2+3C+3). \tag{4.1}$$

We will consider optimum admission control schemes for this system in the next sections.

### 4.1.1 Two-Beam-Satellites With Capacity $C = 2$

We now investigate a system with a satellite that has $B = 2$ fixed beams that cover $F = 2$ footprints, which can handle up to $2 \cdot C = 4$ simultaneous calls. This is one of the most

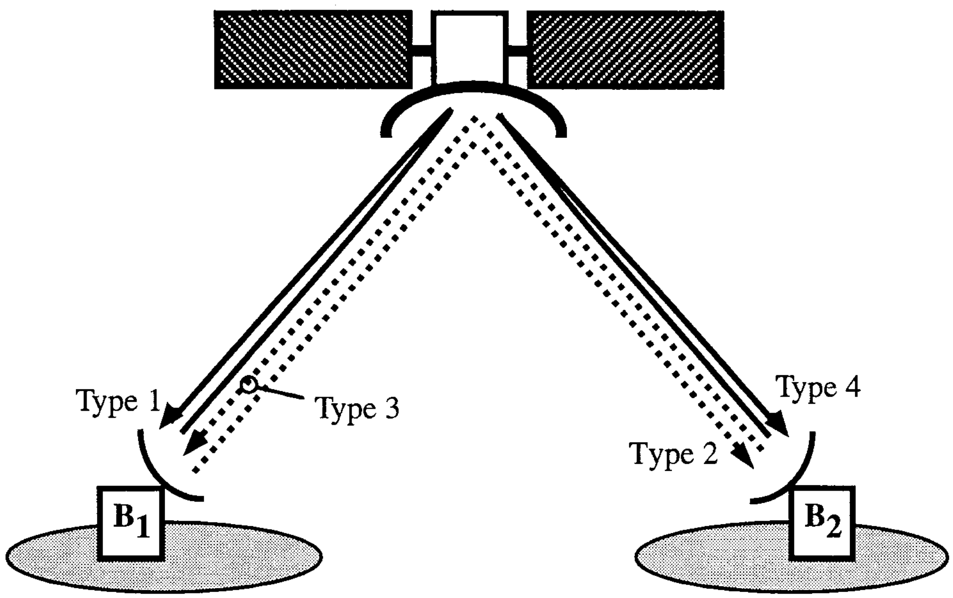


Figure 4.1: Satellite System With $B = 2$ Beams And $F = 2$ Footprints

simple systems one can think of, yet it is already fairly complex, and it provides some valuable insight into the nature of the optimum admission control scheme.

The state space of this system is described by

$$\Omega = \{x \,|\, \max(x_1, x_4) + \max(x_2, x_3) \leq 2\}, \tag{4.2}$$

and from Eq. 4.1 we know that there 26 states in $\Omega$.

Our goal is to find a coordinate convex (c.c.) admission policy which maximizes the revenue generated by our system. To estimate the complexity of this problem, we are interested in the number of c.c. subspaces of $\Omega$. There is a total number of $2^{26} \approx 67$ million possible subspaces, but only a few of them are c.c. We are not aware of an analytical method for computing the number of c.c. subspaces, so we did an exhaustive search by testing all $2^{26}$ possible subspaces for coordinate-convexity. This was done with an optimized computer program, which ran several minutes on a SUN workstation. We found 7653 c.c. subspaces, i.e., only about one subspace in 10000 is coordinate-convex.

The next step was to find the c.c. subspace which maximizes the system revenue for given revenues $r_j$ and loads $\rho_j$. We did this again with a computer program, by computing the revenue generated by each of the 7653 c.c. subspaces.

We first investigated systems with equal revenues for all four service types ($r_j = 1,\ j = 1 \ldots 4$) and randomly chosen loads $\rho_j$. We ran more than a million computations with $\rho_j$ in the range of $0.01 \ldots 0.99$, and the unrestricted state space $\Omega$ always generated the maximum system revenue. We therefore conjecture that *complete sharing*, i.e., an unrestricted state space $\Omega$ maximizes the system revenue for equal service revenues, and it is part of our ongoing work to elaborate a proof for this.

For unequal revenues, $r_i \neq r_j$ for some $i \neq j$, there is a different picture. Performing several hundred thousand computer runs with revenues varying from one to ten and loads in the range of 0.01 to 0.99, it was shown that restricting the state space can indeed increase the system revenue. Depending on the revenue and the load for each service type, the system revenue can be increased by up to 30%.

We found that 29 out of the 7653 c.c. subspaces were optimum for some values of the $r_j$ and $\rho_j$. These optimum admission control strategies are shown in Table 4.1.

### Optimum Policies

We will now discuss these 29 admission control policies to some detail. We expect to see some symmetry between traffic types 1 and 4, and types 2 and 3, because these two types belong to the same group and, hence, can each be accommodated by the same state of the satellite switch.

We counted the number of times each of the 29 subspaces was optimum ("probability"), and we tried to determine a typical set of $\rho_j$ and $r_j$ for which a subspace was optimum, as well as the average and maximum increase of the system revenue with respect to the unrestricted system. Note that the following statements about the probability of a state space being optimum depend on the chosen interval for $\rho_j$ and $r_j$. We will show, however, that there are some general conclusions which can be drawn from these observations.

The results are summarized in Table 4.2. Let us first consider the policies $\Omega_1^* \ldots \Omega_4^*$, which are of the form $\{x \mid x_j = 0\}$, i.e., all calls of type $j$ are blocked. Such a strategy is optimum if the revenue of type $j$ is considerably (about 4 times) smaller than that of all others, and if this type of call occurs very often ($\rho_j > 0.9$). Each of these four strategies are optimum with probability 1.3%, achieving average and maximum revenue increases of about 3% and 17%, respectively.

The state spaces $\Omega_5^* \ldots \Omega_8^*$ are similar, by not completely blocking calls of type $j$, but allowing not more than one call of this type. These strategies are also optimum for a small value of $r_j$ and a large $\rho_j$, with the addition that in this case the revenues of the services types in the other group are both very large. The probability for each of these admission control strategies to be optimum is about 7%, and average and maximum revenue increases are 0.6% and 7%, respectively.

The state spaces $\Omega_9^*$ and $\Omega_{10}^*$ block all calls of the same group, i.e., either all calls of type 1 and 4, or of type 2 and 3[1]. This is an optimum policy if the revenue of one group is much smaller than of the other group, and if the throughput of that group is rather high. This occurred with a probability of 1%, generating an average revenue increase of 7%, and a maximum revenue increase of more than 30%! As an example, a system with $r_1 = 8.9, r_2 = 1.1, r_3 = 1.0, r_4 = 7.8, \rho_1 = 0.92, \rho_2 = 0.99, \rho_3 = 0.99$, and $\rho_4 = 0.80$, which blocks a calls of type 2 and 3, generates a revenue increase of 30.7% with respect to the uncontrolled admission policy.

The control policies $\Omega_{11}^* \ldots \Omega_{22}^*$ block all calls of type $i$ and restrict the number of type $j$ calls to one, i.e., they are of the form $\{x \mid x_i = 0 \;\&\; x_j \leq 1\}$. Here, we expect to see a

[1] For symmetry reasons, we would also expect that policies that block two calls from different groups may be optimum. We assume, however, that the probability of these policies is so low that they were not found in the 100000 computer runs we performed.

$$
\begin{aligned}
\Omega_1^* &= \{x \mid x_1 = 0\} \\
\Omega_2^* &= \{x \mid x_2 = 0\} \\
\Omega_3^* &= \{x \mid x_3 = 0\} \\
\Omega_4^* &= \{x \mid x_4 = 0\} \\
\Omega_5^* &= \{x \mid x_1 \le 1\} \\
\Omega_6^* &= \{x \mid x_2 \le 1\} \\
\Omega_7^* &= \{x \mid x_3 \le 1\} \\
\Omega_8^* &= \{x \mid x_4 \le 1\} \\
\Omega_9^* &= \{x \mid x_1 = 0 \;\&\; x_4 = 0\} \\
\Omega_{10}^* &= \{x \mid x_2 = 0 \;\&\; x_3 = 0\} \\
\Omega_{11}^* &= \{x \mid x_1 = 0 \;\&\; x_2 \le 1\} \\
\Omega_{12}^* &= \{x \mid x_1 = 0 \;\&\; x_3 \le 1\} \\
\Omega_{13}^* &= \{x \mid x_1 = 0 \;\&\; x_4 \le 1\} \\
\Omega_{14}^* &= \{x \mid x_2 = 0 \;\&\; x_1 \le 1\} \\
\Omega_{15}^* &= \{x \mid x_2 = 0 \;\&\; x_3 \le 1\} \\
\Omega_{16}^* &= \{x \mid x_2 = 0 \;\&\; x_4 \le 1\} \\
\Omega_{17}^* &= \{x \mid x_3 = 0 \;\&\; x_1 \le 1\} \\
\Omega_{18}^* &= \{x \mid x_3 = 0 \;\&\; x_2 \le 1\} \\
\Omega_{19}^* &= \{x \mid x_3 = 0 \;\&\; x_4 \le 1\} \\
\Omega_{20}^* &= \{x \mid x_4 = 0 \;\&\; x_1 \le 1\} \\
\Omega_{21}^* &= \{x \mid x_4 = 0 \;\&\; x_2 \le 1\} \\
\Omega_{22}^* &= \{x \mid x_4 = 0 \;\&\; x_3 \le 1\} \\
\Omega_{23}^* &= \{x \mid x_1 \le 1 \;\&\; x_2 \le 1\} \\
\Omega_{24}^* &= \{x \mid x_1 \le 1 \;\&\; x_3 \le 1\} \\
\Omega_{25}^* &= \{x \mid x_1 \le 1 \;\&\; x_4 \le 1\} \\
\Omega_{26}^* &= \{x \mid x_2 \le 1 \;\&\; x_3 \le 1\} \\
\Omega_{27}^* &= \{x \mid x_2 \le 1 \;\&\; x_4 \le 1\} \\
\Omega_{28}^* &= \{x \mid x_3 \le 1 \;\&\; x_4 \le 1\} \\
\Omega_{29}^* &= \{x \mid \max(x_1, x_4) + \max(x_2, x_3) \le 2\}
\end{aligned}
$$

Table 4.1: Optimum Admission Control Policies For $B = M = C = 2$

| Policy | Condition | Probability | Max. Revenue Inc. | Avg. Revenue Inc. |
|---|---|---|---|---|
| $\Omega_1^* \ldots \Omega_4^*$ | $x_j = 0$ | 1.3% | 7.6% | 17.5% |
| $\Omega_5^* \ldots \Omega_8^*$ | $x_j \leq 1$ | 7.2% | 7.5% | 0.6% |
| $\Omega_9^*, \Omega_{10}^*$ | $x_i = 0$ & $x_j = 0$ | 1.1% | 30.7% | 6.6% |
| $\Omega_{11}^* \ldots \Omega_{22}^*$ | $x_i = 0$ & $x_j \leq 1$ | 0.5%/$10^{-6}$ | 17.3%/5.2% | 3.5%/1.6% |
| $\Omega_{23}^* \ldots \Omega_{28}^*$ | $x_i \leq 1$ & $x_j \leq 1$ | 2.3%/0.06% | 11.9%/3.0% | 1.4%/0.6% |
| $\Omega_{24}^*$ | unrestricted | 56% | 0% | 0% |

Table 4.2: Revenue Increase and Probability For Optimum Admission Control Policies, $B = M = 2$, $C = 2$. Columns with two values correspond to traffic types in the same and in different groups, respectively.

difference between the $i$ and $j$ which belong to the same group, and all others. Indeed, the cases where $i$ and $j$ are in different groups ($\Omega_{11}^*, \Omega_{12}^*, \Omega_{14}^*, \Omega_{16}^*, \Omega_{17}^*, \Omega_{19}^*, \Omega_{21}^*$, and $\Omega_{22}^*$), occur with a probability of less than $10^{-5}$, and generate an average revenue increase of less than 1%. The values for these policies are those on the right hand side in the columns of Table 4.2, and we will neglect these strategies in the sequel. For $i$ and $j$ belonging to the same group, however, a maximum revenue increase of up to 17% can be achieved, where the average increase is still about 4%. The probability of these events is about 0.5%, and they happen if the revenue $r_i$ is small, the throughput $\rho_i$ is high, and the throughput of type $j$, $\rho_j$, is low.

The state spaces $\Omega_{23}^* \ldots \Omega_{28}^*$ restrict the number of services of type $i$ and $j$ to one, i.e., $\{x \mid x_i \leq 1 \;\&\; x_j \leq 1\}$. Again, for $i$ and $j$ in different groups ($\Omega_{23}^*, \Omega_{24}^*, \Omega_{27}^*$, and $\Omega_{28}^*$), the probability is much below 1%, and the revenue increase is also less than 1%. Hence, we will discard these cases in the sequel. Similar to the case described previously, if $i$ and $j$ belong to the same group, a maximum revenue increase of up to 12% can be achieved; the average increase is about 1.5%. This occurs with a probability of about 2.3%, when both $r_i$ and $r_j$ are small, and when $\rho_i$ and $\rho_j$ are close to one.

Finally, strategy $\Omega_{29}^*$ is the unrestricted state space, which proved to be optimum in about 56% of all cases considered.

Computing the average system revenue increase for the system described above by adding the average revenue increase of each policy weighted by its probability yields the average revenue increase achieved by admission control, which is only about 0.6%.

### Non-Optimum Policies

To get an idea about the quality of the remaining non-optimum policies, we computed a histogram for the revenue generated by all policies relative to the unrestricted policy. The result is shown in Fig. 4.2, where the histogram ranges from a revenue of 0% (the all-blocking policy) to 127%. Note that the histogram is depicted in logarithmic scale to include the results for less probable revenues, and that it scaled such that the maximum value is 1.

Fig. 4.2 shows that if we randomly pick one of the 7653 admission control policies, the most likely system revenue is about 83% of the unrestricted system, which is surprisingly

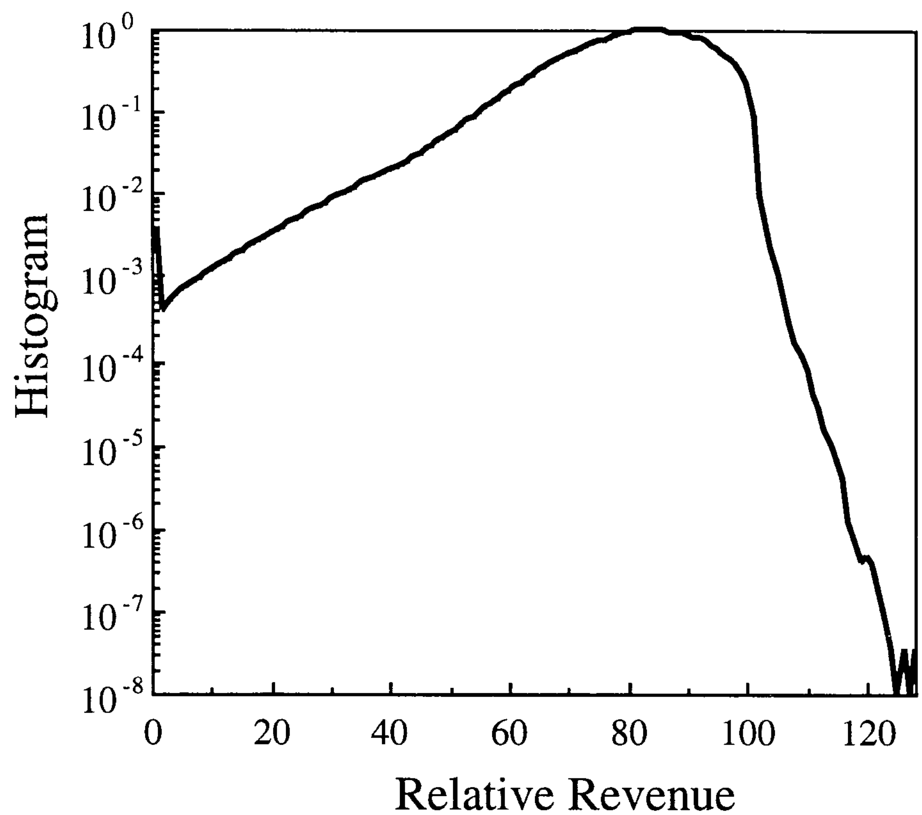


Figure 4.2: Histogram Of Relative Revenue Of All 7653 Admission Control Policies

high. The "3dB-interval" of the histogram, i.e., the interval in which the values are greater than 0.5, is between 68% and 96%, which is fairly broad, and which indicates that many of the control policies generate a similar revenue. Note, however, that the histogram drops sharply for values above 100%.

In summary, it can be said that although only 29 out of 7653 policies generate a revenue increase with respect to the complete sharing policy, the remaining policies are not much worse in the sense that their most probable relative revenue is still about 83%.

**Summary**

It was shown that a satellite with $B = 2$ hopping beams, $F = 2$ footprints a capacity $C = 2$ can accommodate $B \cdot C = 4$ simultaneous calls. The corresponding system has 26 states, whose state space was shown to have 7653 coordinate convex subspaces, which is only about 0.001% ($10^{-5}$) of all possible $2^{26} \approx 6.7 \cdot 10^{7}$ subspaces.

For systems which generate identical revenues for all four service types, the unrestricted state space seems to generate the maximum system revenue, i.e., *complete sharing* seems to be the optimum admission control policy for this case.

If the revenues for the service types vary in the range 1...10, and if the throughputs of each service type are chosen randomly in the interval 0.01...0.99, 29 out of the 7653 c.c. subspaces proved to be optimum, of which only 17 are of practical interest, however. It was shown that the revenue of a system with an optimum admission control policy can be increased by up to 30% with respect an unrestricted system with complete sharing. If the revenue increase is averaged over the entire range of throughputs and revenues that were considered, admission control can increase the system revenue by about 0.6%.

Note finally that the definition of the unrestricted state space is a non-linear function (cf. Eq. 4.2), such that one might have anticipated that complex, non-linear state spaces would correspond to optimum admission control policies. It was shown, however, that all optimum policies are simple threshold-type functions of the form

$$\Omega_k^* = \{x \mid x_i \leq c_1 \ \& \ x_j \leq c_2\}, \tag{4.3}$$

where $i$ and $j$ belong to the same group, and where $c_1$ and $c_2$ are in the interval $0..C$. This result is somewhat surprising, and we will make use of it in the next sections.

### 4.1.2 Two-Beam-Satellites With Capacity $C > 2$

In this section we investigate systems with a hopping beam satellite with $B = 2$ beams and $F = 2$ footprints, with a capacity $C > 2$. Recall that we were able to show that for $C = 2$ 7653 out of the 67 million subspaces are coordinate convex. For capacity $C = 3$, there are $2^{70} \approx 10^{21}$ subspaces, and this number explodes for $C > 3$. Even with a supercomputer it is no longer feasible to test all subspaces for coordinate-convexity. We therefore tried to use a constructive approach, which takes into account the structure of the problem. We found 857925 c.c. subspaces with 20 or less elements, and from our observations, we guess that the total number c.c. subspaces for $C = 3$ is somewhere in the range of $10^8 \ldots 10^{14}$. This is again considerably smaller than the total number of subspaces, but it is beyond size that is practically tractable.

However, it was shown that for $C = 2$ the optimum policies were of a simple threshold-type form, and we will assume that this observation can be extended for capacities $C > 2$. In the sequel, we will therefore only consider subspaces of the form shown in Eq. 4.3. It can be shown that for a system with capacity $C$ there are

$$N_{\Omega^*} = 2C^2 + 4C + 1 \tag{4.4}$$

subspaces to be considered, i.e., the size of the problem grows only with the square of the capacity.[2]

Recall that for a system with capacity $C = 2$, complete sharing was shown to be optimum when the revenues $r_j$ of all service types are equal. The same observation was

[2]Note that it was shown in a different context [Bar93a] that there are optimum policies that restrict the state space with hyperplanes of slope one, i.e., $x_i + x_j \leq c$, $c = 1 \ldots 2C$. We first included these policies in our investigation, but they were never optimum for the systems considered.

made for $C = 3\ldots6$, when the $\rho_j$ were chosen randomly in the interval $0.01\ldots0.99$. Hence, for equal revenues, complete sharing seems indeed to be optimum.

However, for unequal revenues, admission control can increase the system revenue. We will now show some results for $C = 3\ldots6$ in the next sections.

### Two-Beam-Satellites With Capacity $C = 3$

A system with capacity $C = 3$ has 70 states (cf. Eq. 4.1), and 31 c.c. subspaces need to be investigated (cf. Eq. 4.4), including the unrestricted state space

$$\Omega = \{x \,|\, \max(x_1, x_4) + \max(x_2, x_3) \leq 3\}. \tag{4.5}$$

As for capacity $C = 2$, we ran a computer program with ten million different revenues and throughputs in the range $r_j = 1\ldots10$, and $\rho_j = 0.01..0.99$, respectively. The results are summarized in Table 4.3. Note that the service types $i$ and $j$ belong to the same group, and that the figures are given for only one of the two or four policies of each type is given (eg. $x_j = 0$ stands for $x_1 = 0$ or $x_2 = 0$ or $x_3 = 0$ or $x_4 = 0$). Hence, the probabilities do not add up to one.

| Condition | Probability | Max. Revenue Inc. | Avg. Revenue Inc. |
|---|---|---|---|
| $x_j = 0$ | 0.03% | 7.6% | 2.0% |
| $x_j \leq 1$ | 1.5% | 4.5% | 0.7% |
| $x_j \leq 2$ | 6.1% | 1.7% | 0.1% |
| $x_i = 0$ & $x_j = 0$ | 0.01% | 10.4% | 3.4% |
| $x_i \leq 1$ & $x_j \leq 1$ | 0.3% | 7.4% | 1.5% |
| $x_i \leq 2$ & $x_j \leq 2$ | 1.1% | 2.5% | 0.2% |
| $x_i = 0$ & $x_j \leq 1$ | 0.01% | 7.7% | 2.2% |
| $x_i = 0$ & $x_j \leq 2$ | 0.01% | 8.0% | 2.1% |
| $x_i \leq 1$ & $x_j \leq 2$ | 0.4% | 4.5% | 0.8% |
| unrestricted | 65% | 0% | 0% |

Table 4.3: Revenue Increase and Probability For Optimum Admission Control Policies, $B = M = 2,\ C = 3$.

It can be seen from Table 4.3 that the unrestricted policy is optimum for 65% of the randomly chosen parameters $r_j$ and $\rho_j$. The next most probable policy is of type $x_j \leq 2$, which proved to be optimum in about 25% of the cases, yielding an average revenue increase of about 1.5%. The maximum achievable revenue increase is about 10%, and the average revenue increase is up to 3%. Policies that contain the term $x_j = 0$ occur rather seldom, but they result in a considerable revenue increase.

In conclusion, it can be said that for a system with capacity $C = 3$, a moderate revenue increase can be achieved by testing 31 admission control policies.

**Two-Beam-Satellites With Capacities $C = 4 \ldots 6$**

For a system with $C = 4$, which has 155 states, we had to consider 49 admission control policies. As for $C = 3$, we ran a computer program with a million different random revenues and throughputs in the range $r_j = 1 \ldots 10$, and $\rho_j = 0.01 \ldots 0.99$, respectively. The unrestricted policy proved to be optimum for 75% of the cases, and in all other cases the average revenue increase was below 1%. The maximum revenue increase is 2%, and it occurred with a probability of 0.1%. Policies which contain a term $x_j = 0$ did not occur at all, and the more probable policies were those of the form $x_j \leq 3$, which were optimum in 18% of the cases considered, however yielding a maximum revenue increase of less than 0.3%.

We repeated the same procedure for $C = 5$ (301 states, 71 admission control policies, 1 million runs with random revenues and throughputs, as above). For 84% of the cases considered, complete sharing was optimum. The policy $x_j \leq 4$ was chosen 14% of the time, yielding maximum and average revenue increases of only 0.04% and 0.002%, however. The maximum revenue increase found was 0.3%.

As expected, the results for $C = 6$ tend towards the same direction. Complete sharing is now optimum for more than 90% of the cases considered, and all revenue increases were well below 0.1%. Hence, we conclude that admission control for a system with $B = 2$ beams, $F = 2$ footprints, and a capacity of $C \geq 4$, the revenue increases achieved by admission control are probably too small justify the effort, and that complete sharing should be considered instead.

### 4.1.3 Summary For Two-Beam-Satellites

In the previous sections, we considered admission control schemes for a system with a satellite with $B = 2$ beams and $F = 2$ footprints. If the satellite has capacity $C = 2$, it was shown that the system state space contains 7653 coordinate convex subspaces, which is also the number of possible admission control schemes. We computed the optimum admission policies for such a system by randomly selecting some ten million different values for the revenues for each service type in the interval $r_j = 1 \ldots 10$ and throughputs in the range $\rho_j = 0.01 \ldots 0.99$. We showed that only 17 out of the 7653 admission policies are of practical interest, and that they are all of the form given in Eq. 4.3. This is a nice result, and it was not expected that a non-linearly defined optimization problem (cf. Eq. 3.9) admits linear admission control scheme solutions, which can be expressed as restricting the state space by one or two hyperplanes perpendicular to the axes of the coordinate system.

We then used this result to obtain the admission control policies for systems with capacity up to $C = 6$, which would not have been possible otherwise due to problems of complexity. It was shown that for all capacities considered, the unrestricted state space, i.e., the complete sharing policy, is the most probable to occur when the $r_j$ and $\rho_j$ are randomly chosen in the intervals described above; the corresponding probability increases from 56% for $C = 2$ to over 90% for $C = 6$. Maximum revenue increase for some policy was shown to be about 30% for $C = 2$, 10% for $C = 3$, 3% for $C = 4$, and below 1% for

$C \geq 5$. We then computed the average system revenue increase by adding the average revenue increase for each policy multiplied by its probability. For $C = 2$, this average revenue which can be achieved by admission control is about 0.6%, and even smaller for larger capacities. Note that these results depend on the range of the intervals chosen for $\rho_j$ and $r_j$.

In conclusion, we showed that the optimum admission control schemes for the systems considered are of a simple form, despite the non–linear definition of the optimization problem. We showed that for systems with capacities $C = 2$ and $C = 3$, considerable revenue increases can be achieved if the revenues generated by some service types are at least an order of magnitude different from others. For increasing capacities, the maximum increase gets smaller, such that admission control does not seem favorable in this case. If all service types generate the same revenue, complete sharing seems to be optimum.

## 4.2 Satellites with Three Beams and Three Footprints

We now extend our investigation to satellites with three beams and three footprints. Such a satellite uses an SS–TDMA access scheme and a $3 \times 3$ switch. There are $3^2 = 9$ different source–destination pairs, as shown in Table 4.4, and 3!=6 different states of the switch matrix. These switch states are depicted in Fig. 4.3, together with the services types that can be accommodated by each state. Note that each service type is contained in two switch states.

| Service Type | Source | Destination |
|---|---|---|
| 1 | $F_1$ | $F_1$ |
| 2 | $F_1$ | $F_2$ |
| 3 | $F_1$ | $F_3$ |
| 4 | $F_2$ | $F_1$ |
| 5 | $F_2$ | $F_2$ |
| 6 | $F_2$ | $F_3$ |
| 7 | $F_3$ | $F_1$ |
| 8 | $F_3$ | $F_2$ |
| 9 | $F_3$ | $F_3$ |

Table 4.4: Service Types For A Satellite With $B = 3$ Beams And $F = 3$ Footprints

Recall from Eqs. 3.6 and 3.7 that the state space is defined by

$$\begin{aligned} \Omega \;=\; & \{x \,|\, \max(x_1 + x_2 + x_3, x_4 + x_5 + x_6, x_7 + x_8 + x_9, \\ & x_1 + x_4 + x_7, x_2 + x_5 + x_8, x_3 + x_6 + x_9) \leq C\}. \end{aligned} \tag{4.6}$$

This expression is much more complex than for $B = F = 2$, since it involves a maximization over six sums of three terms each. It can seen from the above equation that in order to accept at least one call of each type, a capacity of $C \geq 3$ is required.

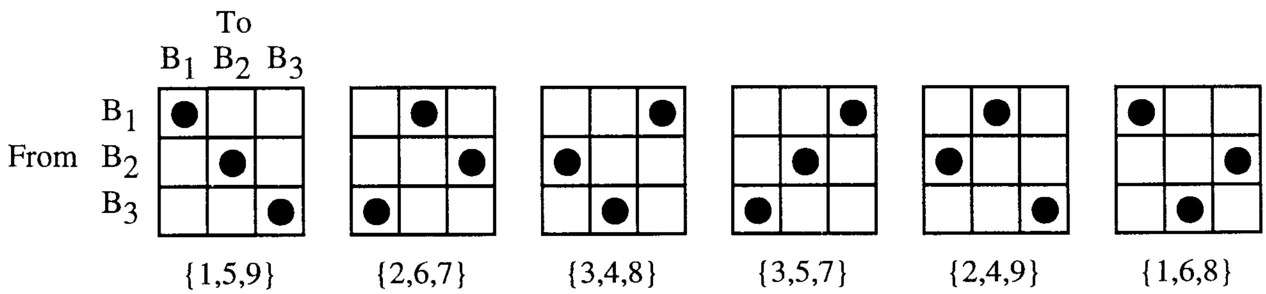


Figure 4.3: Switch States And Service Types For A Satellite With $B = 3$ Beams Covering $F = 3$ Footprints

We have not been able to find either an analytical expression or a bound on cardinality of the state space. However, we were able to transform this question into an equivalent problem in combinatorial matrix theory, which seems to be unsolved (cf. Appendix A.3). We therefore counted the number of states of $\Omega$ with a computer program for capacities $C$ up to 6. The result is shown in Table A.3 in Appendix A.3. Note that the states space grows very fast, from 3380 states for $C = 3$ to more that 200000 states for $C = 6$.

Based on the experience from the system with $B = F = 2$, we did not attempt to consider all possible coordinate-convex subspaces of $\Omega$ for the optimization of the admission control scheme, but we restricted our investigations to policies of the threshold–type and of the slope–one–type:

$$\begin{aligned} \Omega^*_{th} &= \{x \mid x_i \le c_1 \;\&\; x_j \le c_2 \;\&\; x_k \le c_3\} \\ \Omega^*_{sl} &= \{x \mid x_i + x_j \le c_4 \;\vee\; x_i + x_j + x_k \le c_5\}, \end{aligned} \tag{4.7}$$

where $c_1 \ldots c_3 = 0 \ldots C$, $c_4 = 1 \ldots 2C$, $c_5 = 1 \ldots 3C$, and where $x_i, x_j$, and $x_k$ are chosen such that the corresponding state lies in $\Omega$. Note that $\Omega^*_{th}$ describes a set of three hyperplanes which are perpendicular to the axes $x_i, x_j$, and $x_k$, and that $\Omega^*_{sl}$ describes hyperplanes with slope 'one' in the subspace spanned by $\{x_i, x_j\}$ and $\{x_i, x_j, x_k\}$, respectively. It might seem arbitrary that we consider only policies that involve up to three of the nine possible service types, but we think that this well justified by the results obtained for the two–beam satellite system.

For a system with capacity $C = 3$, we found 2620 policies $\Omega^*_{th}$ (including the unrestricted policy), and 834 policies $\Omega^*_{sl}$ which are not contained in $\Omega^*_{th}$. Hence, we have to consider a fairly complex optimization problem which involves 3454 policies for a system with 9 service types and 3380 states. To find the optimum policies, we ran a computer program similar to the one for $B = F = 2$ that randomly selected values for the throughputs for each service type in the range $\rho_i = 0.01..0.99$ and revenues in the range $1 \ldots 10$.

Results of 59000 optimization runs are shown in Table 4.5 for the unrestricted policy, for the 19 policies of type $\Omega^*_{th}$, and for the 13 policies of type $\Omega^*_{sl}$. We listed the policy, its probability of being optimum, and the average revenue increase it generates. Note that, as opposed to the previous case with two beams, policies of type $\Omega^*_{sl}$ are now sometimes optimum, but with a very low probability. They will therefore be discarded in the se-

| Policy | Probability | Revenue Increase (%) |
|---|---|---|
| unrestricted | 0.082 | 0.000 |
| $x_i = 0$ | 0.024 | 1.305 |
| $x_i \leq 1$ | 0.092 | 0.355 |
| $x_i \leq 2$ | 0.151 | 0.039 |
| $x_i = 0 \wedge x_j = 0$ | 0.006 | 2.533 |
| $x_i \leq 1 \wedge x_j \leq 1$ | 0.039 | 0.667 |
| $x_i \leq 2 \wedge x_j \leq 2$ | 0.092 | 0.069 |
| $x_i = 0 \wedge x_j = 0 \wedge x_k = 0$ | 0.001 | 3.991 |
| $x_i \leq 1 \wedge x_j \leq 1 \wedge x_k \leq 1$ | 0.018 | 1.073 |
| $x_i \leq 2 \wedge x_j \leq 2 \wedge x_k \leq 2$ | 0.033 | 0.097 |
| $x_i = 0 \wedge x_j \leq 1$ | 0.027 | 1.727 |
| $x_i = 0 \wedge x_j \leq 2$ | 0.038 | 1.385 |
| $x_i \leq 1 \wedge x_j \leq 2$ | 0.110 | 0.378 |
| $x_i = 0 \wedge x_j = 0 \wedge x_k \leq 1$ | 0.009 | 3.090 |
| $x_i = 0 \wedge x_j = 0 \wedge x_k \leq 2$ | 0.012 | 2.622 |
| $x_i = 0 \wedge x_j \leq 1 \wedge x_k \leq 1$ | 0.019 | 2.149 |
| $x_i = 0 \wedge x_j \leq 1 \wedge x_k \leq 2$ | 0.040 | 1.774 |
| $x_i = 0 \wedge x_j \leq 2 \wedge x_k \leq 2$ | 0.027 | 1.468 |
| $x_i \leq 1 \wedge x_j \leq 1 \wedge x_k \leq 2$ | 0.052 | 0.690 |
| $x_i \leq 1 \wedge x_j \leq 2 \wedge x_k \leq 2$ | 0.060 | 0.379 |
| $x_i + x_j \leq 1$ | 0.006 | 1.236 |
| $x_i + x_j \leq 2$ | 0.027 | 0.218 |
| $x_i + x_j \leq 3$ | $<< 10^{-4}$ | |
| $x_i + x_j \leq 4$ | $<< 10^{-4}$ | |
| $x_i + x_j \leq 5$ | $< 10^{-4}$ | $< 10^{-4}$ |
| $x_i + x_j + x_k \leq 1$ | $< 10^{-4}$ | 1.895 |
| $x_i + x_j + x_k \leq 2$ | 0.001 | 1.123 |
| $x_i + x_j + x_k \leq 3$ | 0.028 | 0.599 |
| $x_i + x_j + x_k \leq 4$ | 0.007 | 0.101 |
| $x_i + x_j + x_k \leq 5$ | $<< 10^{-4}$ | |
| $x_i + x_j + x_k \leq 6$ | $<< 10^{-4}$ | |
| $x_i + x_j + x_k \leq 7$ | $<< 10^{-4}$ | |
| $x_i + x_j + x_k \leq 8$ | $< 10^{-4}$ | $< 10^{-4}$ |

Table 4.5: Optimum admission control policies for $F = B = C = 3$

quel. With threshold-policies, the revenue can be increased by up to 4% with respect to complete sharing. However, if we compute the average revenue increase by multiplying the probability of each policy by its revenue increase, and summing over all policies, we get an average increase of 0.6%. This figure is again disappointingly small, and it hardly justifies the effort of implying admission control at all.

We increased the capacity to $C = 4$ and performed the same optimization, where in this case 7225 policies and a state space with 11290 elements had to be considered. As expected, the average revenue increase got even smaller, namely 0.13%. We assume that this trend continues for larger capacities, and we recommend in that case to use the complete sharing admission policy for its excellent performance and its simple implementation.

## 4.3 Satellites with Two Beams and Three Footprints

We also investigated hopping–beam satellite systems, which are characterized by the fact that they cover more footprints than there are beams. Such systems were discussed in general in Section 3.3, and here we investigate optimum admission control schemes. The most simple system is the one with two beams and three footprints. Recall from Fig. 3.2 that such a small system offers 9 different services and has 20 different switch states. For a capacity $C = 3$, there are 2299 states in the state space to be considered. Due to problems of complexity, we only investigated the 3454 admission control schemes of the threshold– and the slope–one–type (cf. Eq. 4.7).

As before, we computed the optimum policy for revenues ranging from one to ten, and for utilizations $\rho_j = 0.01 \ldots 0.99$. The results are shown in Table 4.6. Although networks with hopping beams are inherently different in nature from those with fixed beams, we get similar results for both systems. In this case, the average revenue is about 1%, which is slightly more than that for a corresponding fixed–beam system. Again, policies of the slope–one–type occur very seldom, and the highest increase in revenue can be achieved by restricting the number of one or more call types which generate a small revenue.

Increasing the capacity led again to an even smaller increase of revenue (0.2% for $C = 4$), and we assume that for networks with a larger number of beams and footprints, optimum admission control would not be much better than the simple complete sharing policy.

| Policy | Probability | Revenue Increase (%) |
|---|---|---|
| unrestricted | 0.055 | 0.000 |
| $x_i = 0$ | 0.038 | 1.434 |
| $x_i \leq 1$ | 0.074 | 0.348 |
| $x_i \leq 2$ | 0.099 | 0.037 |
| $x_i = 0 \wedge x_j = 0$ | 0.015 | 3.002 |
| $x_i \leq 1 \wedge x_j \leq 1$ | 0.036 | 0.682 |
| $x_i \leq 2 \wedge x_j \leq 2$ | 0.061 | 0.066 |
| $x_i = 0 \wedge x_j = 0 \wedge x_k = 0$ | 0.006 | 5.414 |
| $x_i \leq 1 \wedge x_j \leq 1 \wedge x_k \leq 1$ | 0.020 | 1.036 |
| $x_i \leq 2 \wedge x_j \leq 2 \wedge x_k \leq 2$ | 0.020 | 0.092 |
| $x_i = 0 \wedge x_j \leq 1$ | 0.046 | 1.868 |
| $x_i = 0 \wedge x_j \leq 2$ | 0.059 | 1.420 |
| $x_i \leq 1 \wedge x_j \leq 2$ | 0.083 | 0.365 |
| $x_i = 0 \wedge x_j = 0 \wedge x_k \leq 1$ | 0.027 | 3.701 |
| $x_i = 0 \wedge x_j = 0 \wedge x_k \leq 2$ | 0.027 | 3.107 |
| $x_i = 0 \wedge x_j \leq 1 \wedge x_k \leq 1$ | 0.039 | 2.283 |
| $x_i = 0 \wedge x_j \leq 1 \wedge x_k \leq 2$ | 0.069 | 1.822 |
| $x_i = 0 \wedge x_j \leq 2 \wedge x_k \leq 2$ | 0.042 | 1.479 |
| $x_i \leq 1 \wedge x_j \leq 1 \wedge x_k \leq 2$ | 0.047 | 0.688 |
| $x_i \leq 1 \wedge x_j \leq 2 \wedge x_k \leq 2$ | 0.045 | 0.375 |
| $x_i + x_j \leq 1$ | 0.016 | 1.131 |
| $x_i + x_j \leq 2$ | 0.027 | 0.201 |
| $x_i + x_j \leq 3$ | $< 10^{-4}$ | 0.011 |
| $x_i + x_j \leq 4$ | $< 10^{-4}$ | $< 10^{-4}$ |
| $x_i + x_j \leq 5$ | 0.001 | $< 10^{-4}$ |
| $x_i + x_j + x_k \leq 1$ | 0.002 | 1.976 |
| $x_i + x_j + x_k \leq 2$ | 0.005 | 0.610 |
| $x_i + x_j + x_k \leq 3$ | 0.033 | 0.551 |
| $x_i + x_j + x_k \leq 4$ | 0.007 | 0.086 |
| $x_i + x_j + x_k \leq 5$ | 0.002 | 0.077 |
| $x_i + x_j + x_k \leq 6$ | $< 10^{-4}$ | 0.001 |
| $x_i + x_j + x_k \leq 7$ | $< 10^{-4}$ | $< 10^{-4}$ |
| $x_i + x_j + x_k \leq 8$ | $< 10^{-4}$ | $<< 10^{-4}$ |

Table 4.6: Optimum admission control policies for $F = 3,\ B = 2,\ C = 3$

# Chapter 5

# Outlook and Conclusions

In this report, we investigated admission control schemes for satellites with multiple spot beams and multiple service types. As a reference policy, we used the *complete sharing (CS)* strategy, which accepts calls on a first–come, first–served basis as long as capacity is available. We first considered a small system with two beams, two footprints and a capacity to handle four simultaneous calls, and we showed that in this case blocking some calls, even if capacity is available, can actually increase the network revenue, or, accordingly, decrease the overall call blocking probability. It was shown that all optimum policies are limiting the state space by one or two hyperplanes perpendicular to the axes of the coordinate system. This is somewhat surprising, in that a non–linearly defined optimization problem admits linear admission control scheme solutions.

The amount of improvement with respect to the CS policy strongly depends on the difference between the traffic types. If the revenue generated by some traffic type is considerably smaller than that of the other ones, blocking calls of this type results in an increase of the network revenue. The difference between the optimum policy and CS gets smaller if the service types become more similar, and eventually, if all traffic types generate the same revenue, CS is the optimum strategy.

For systems with larger capacity and several spot beams, the revenue increase due to optimum admission control gets smaller even if there is a considerable difference between traffic types. A possible explanation is offered by means of a geometrical reasoning: For small systems, the state space consists only of a small number of states, and the surface that defines the boundaries of the state space is relatively rugged compared to the size of the state space. Furthermore, the surface that defines states that generate average revenue lies relatively close to the space boundary. Hence, it is easily possible to remove chunks of the state space that generate below–average revenue (recall from Lemma 1 that this increases the average revenue), such that the remaining state space is still coordinate convex. For larger systems, however, the boundary of the state space tends to get smoother, and the surface of average revenue moves away from that boundary, so that it becomes more difficult to remove states which generate below–average revenue and still leave a c.c. state space.

Based on the results of our work, we propose to use an *optimum c.c. admission control* policy for communication networks with a spot beam satellite

- if the satellite has a small number of beams,
- if the satellite has a small capacity, and
- if there is a considerable variation of revenues generated by the various traffic types.

In all other cases, the *complete sharing* policy, which is very simple to implement, is either optimum or so close to optimum that the effort of implementing an optimum admission control scheme may not be justified.

In our on–going work, we try to analytically prove that the complete sharing policy is indeed optimum if all traffic types generate the same revenue. We are further investigating new issues which are specific to spot–beam and hopping–beam satellites, such as scheduling and dynamic adjustments of the satellite switch plan or the hopping pattern. By taking an integrated design look at these systems, we hope to come up with new, optimum solutions for dynamic beam allocation.

# Appendix A

# Mathematical Details and Proofs

## A.1 Traffic Models and Performance Measures

In this section, we review traffic models and performance measures used in previous publications on admission control schemes, and we describe those selected in Chapter 2 to some detail.

We assume that for service type $j$ call arrivals are Poisson with arrival rate $\lambda_j$, and call duration is exponential with mean $1/\mu_j$; the corresponding load is $\rho_j = \lambda_j/\mu_j$. The corresponding system can be modeled as a $J$-dimensional Markov chain on the state space $\Omega$. If we consider an infinite source population, and we assume further that only one call of each type can be served at a time (dedicated resources), the transition rates of the Markov process are

$$P_{xy} = \begin{cases} \lambda_j & \text{if } y = [x_1, \ldots, x_j + 1, \ldots, x_J] \text{ and } y \in \Omega \\ \mu_j & \text{if } y = [x_1, \ldots, x_j - 1, \ldots, x_J] \text{ and } y \in \Omega \\ 0 & \text{otherwise.} \end{cases} \tag{A.1}$$

This model was used by Foschini et. al. [Fos83] to describe memory sharing in a computer with multiple processors. In this case, $J$ dedicated processors, one for each service type, share a common waiting room (memory), where jobs destined for each of the processors wait to be served in a first-come first-served fashion. Similar to an $M/M/1$ queue, jobs arrive at rate $\lambda_j$ and leave the system (i.e., are being processed) at a rate $\mu_j^{-1}$.

If we allow all services to be served by any of the resources (shared resources), and if we assume a finite source population $N_j$, we get the following transition rates:

$$P_{xy} = \begin{cases} \lambda_j(N_j - x_j) & \text{if } x_j < N_j \text{ and } y = [x_1, \ldots, x_j + 1, \ldots, x_J] \text{ and } y \in \Omega \\ \mu_j x_j & \text{if } y = [x_1, \ldots, x_j - 1, \ldots, x_J] \text{ and } y \in \Omega \\ 0 & \text{otherwise.} \end{cases} \tag{A.2}$$

This model, which is referred to as the *Engset* model was used by Aein [Aei77,Aei78] and Foschini [Fos81] to represent systems with a limited number of users $N_j$ generating requests of type $j$ with rate $\lambda_j$. In such a system, calls of type $j$ arrive at a rate $\lambda_j(N_j - x_j)^+$, where $x_j$ is the number of ongoing calls of type $j$, and where $(\alpha)^+$ denotes $\max(0, \alpha)$. Note that this is different from an infinite source population, where calls are generated at a rate $\lambda_j$ (see below). Departure rate is $\mu_j x_j$, as in an $M/M/c$ or an $M/M/\infty$ queue.

If we assume an infinite source population, we get the *Erlang* model, which is obtained from the Engset model by letting $N_j \to \infty$ and $N_j\lambda_j \to \lambda_j$. In this case, the transition probabilities of the Markov chain are

$$P_{xy} = \begin{cases} \lambda_j & \text{if } y = [x_1, \ldots, x_j + 1, \ldots, x_J] \text{ and } y \in \Omega \\ \mu_j x_j & \text{if } y = [x_1, \ldots, x_j - 1, \ldots, x_J] \text{ and } y \in \Omega \\ 0 & \text{otherwise.} \end{cases} \tag{A.3}$$

This model was used by Aein [Aei78], Jordan and Varaiya [Jor91b] and Barnhart et. al. [Bar93a,Bar93b]. Calls arrive at a rate $\lambda_j$ and depart at a rate $\mu_j x_j$, as in an $M/M/c$ or an $M/M/\infty$ system.

In this report, we will restrict ourselves to the case of infinite source population and shared resources, where the transition probabilities are described in Eq. A.3, see Section 2.1.

### A.1.1 Performance Measures

There are two classes of optimization criteria that are commonly used in the context of Multiple Service/Single Resource (MSSR) and (Multiple Service/Multiple Resource (MSMR) systems: the reward-type and the blocking-type performance measures. Although they are inherently different in nature, it can be shown that maximizing some type of reward is equal to minimizing the blocking probability.

#### The Reward-Type Performance Measures

This class of performance measures considers some type of average revenue generated per time unit, and it is used by Aein [Aei78], Foschini et. al. [Fos81], Jordan and Varaiya [Jor91b] and Barnhart et. al. [Bar93a,Bar93b].

The average revenue per time unit $R(\Omega)$ generated by a system with state space $\Omega$ is defined as

$$R(\Omega) = \mathrm{E}\left[r(x)|x \in \Omega\right] = \sum_{x\in\Omega} r(x)\pi(x), \tag{A.4}$$

where $r(x)$ is the rate of revenue generated while in state $x$. If we assume that the revenue of a call of type $j$ is $r_j$, we get

$$r(x) = \sum_{j=1}^{J} r_j x_j. \tag{A.5}$$

The throughput $\Gamma(\Omega)$, defined as the expected number of active calls averaged over system state, is a special case of revenue, where the revenue generated in a state equals the number of active calls. Setting $r_j = 1, j = 1, \ldots, J$, yields

$$\Gamma(\Omega) = \sum_{x\in\Omega} \gamma(x)\pi(x), \tag{A.6}$$

where

$$\gamma(x) = \sum_{j=1}^{J} x_j. \tag{A.7}$$

The throughput as defined by Foschini et. al. in [Fos81] as the average number of completed jobs (calls) per unit time, can be obtained by computing $R(\Omega)$ with $r_j = \mu_j$, where $\mu_j^{-1}$ is the average holding time of a call of type $j$.

Note that the revenue $r_j$ and the throughput $\gamma_j$ depend on the type of service $j$ only, and not on the control policy.

### The Blocking-Type Performance Measures

Another class of performance measures, which is considered by Foschini et. al. [Fos83], Jordan and Varaiya [Jor91b] and Barnhart et. al. [Bar93a,Bar93b], is related to the blocking probability, which, obviously, depends on the control policy.

The *weighted* blocking probability $P_{bw}(\Omega)$ is the weighted sum of blocking probabilities of type $j$:

$$P_{bw}(\Omega) = \sum_{j=1}^{J} w_j P_{bj}(\Omega) = \sum_{x\in\Omega} \sum_{j=1}^{J} 1\left((x_j+1) \notin \Omega\right) w_j \pi(x), \tag{A.8}$$

where $w_j$, $\sum_{j=1}^{J} w_j = 1$ are the weights associated with service type $j$, where $P_{bj}$ is the fraction of calls of type $j$ that are blocked, and where $1(.)$ is the indicator function, which is 1 if the argument is true and 0 otherwise. Defining $e_j$ as the vector of length $J$, whose elements are all zero with the exception of the $j$-th element, which is one, we get

$$(x + e_j) = [x_1, x_2, \ldots, x_j + 1, \ldots, x_J], \tag{A.9}$$

and we can rewrite $P_{bw}(\Omega)$ as

$$P_{bw}(\Omega) = \sum_{j=1}^{J} \sum_{\substack{x\in\Omega \\ (x+e_j)\notin\Omega}} w_j \pi(x). \tag{A.10}$$

The *average* blocking probability $P_b(\Omega)$ is the ratio of the expected number of blocked calls per unit time to the expected total number of call arrivals per unit time. It can be computed from $P_{bw}(\Omega)$ by setting the weights $w_j = \rho_j / \sum_{j=1}^{J} \rho_j$:

$$P_b(\Omega) = \frac{\sum_{j=1}^{J} \sum_{\substack{x\in\Omega \\ (x+e_j)\notin\Omega}} \rho_j \pi(x)}{\sum_{j=1}^{J} \rho_j}. \tag{A.11}$$

In [Fos83,Jor91b], throughput is defined as

$$T = \sum_{j=1}^{J} \lambda_j w_j (1 - P_{bj}), \tag{A.12}$$

which is the weighted average rate of accepted service requests. Note that $T$ differs from the throughput $\Gamma$ of Eq. A.6. However, as already noted in the introduction to this section, it can be shown that minimizing $T$ is equivalent to maximizing the revenue.

In our report, we will focus on the *average revenue* $R$, as defined in Eq. A.4, as our primary performance measure (see Section 2.1).

## A.2 Proof of Lemma 1

Here, we prove Lemma 1 (cf. p. 6), which states that the network revenue can be increased by restricting the state space ($\Omega \rightarrow \Omega'$) if and only if the revenue generated by the removed states is smaller than the average revenue.

**Proof:** We start by writing the revenue generated in $\Omega' = \Omega - \Gamma$ as

$$R(\Omega') = \sum_{x \in \Omega'} r(x)\pi(x) = \sum_{x \in \Omega'} \frac{r(x)}{G(\Omega')} \prod_{j=1}^{J} \frac{\rho^{x_j}}{x_j!}, \tag{A.13}$$

which, using simple algebraic transformations, can be written as

$$R(\Omega') = \frac{G(\Omega)}{G(\Omega) - G(\Omega - \Omega')} \left[ R(\Omega) - R(\Omega - \Omega') \frac{G(\Omega - \Omega')}{G(\Omega')} \right]. \tag{A.14}$$

We now express $R(\Gamma) = R(\Omega - \Omega')$ as a multiple of $R(\Omega)$:

$$R(\Gamma) =: R(\Omega)(1 + \epsilon), \quad \epsilon \geq -1, \tag{A.15}$$

Combining Eqs. A.14 and A.15 yields

$$R(\Omega') = R(\Omega) \left[ 1 - \epsilon \cdot \frac{G(\Omega)}{G(\Omega) - G(\Gamma)} \right]. \tag{A.16}$$

It is obvious that $R(\Omega') > R(\Omega)$ if and only if $\epsilon < 0$, i.e., the average revenue of the restricted state space $\Omega'$ is bigger than that of the unrestricted state space $\Omega$ iff the average revenue generated in $\Gamma$ is smaller than $R(\Omega)$. □

## A.3 Cardinality of the State Space $\Omega$

In this section, we try to find the cardinality of the state space $\Omega$, i.e., the number of possible states for a network with a hopping-beam satellite with $B$ beams, $F \geq B$ footprints, and capacity $C \cdot B$. Note that for $F = B$, this corresponds to an SS-TDMA satellite with fixed beams.

We consider a $F \times F$ traffic matrix $A$, where the element $a_{ij}$ represents the number of calls from footprint $i$ to $j$. Clearly, the elements $a_{ij}$ are nonzero integers. We know that the capacity of the satellite is $C \cdot B$, hence, the sum of all elements of $A$ may not exceed $C \cdot B$. We also know that the state space is limited by $L(x) \leq C$, where $L(x)$ is the maximum line sum of $A$. With this, we can find the following mathematical formulation:

*Find the number of $F \times F$ matrices with non-negative integer elements such that the sum of all elements does not exceed $C \cdot B$, and each line sum does not exceed $C$.*

For $C = 1$, a solution can be found in a straight-forward way, using a geometrical argument: We want to find the number of $F \times F$ matrices with elements '0' or '1', such that the sum of all elements does not exceed $B$, and each line sum is either zero or one.

This is equivalent to finding the number matrices that contain at most $B$ '1's, and at most one '1' in each line.

We first find the number of matrices that contain *exactly* $b$ '1's, and at most one '1' in each line. In other words: How many ways are there to put $b$ '1's into a $F \times F$ matrix, such that is no more than one '1' in each row and in each column. All possible matrices can be constructed by selecting $b$ rows and $b$ columns containing the '1's, and permuting over all possible row-column pairs. There are $\binom{F}{b}$ ways to pick the rows or the columns, and there are $b!$ permutations of the row-column pairs, such that we end up with $\binom{F}{b}^2 \cdot b!$ different matrices. Summing over all values of $b$ from 0 to $B$, we finally get the following result:

For a satellite with $B$ beams, $F$ footprints, and capacity $B$ (i.e., $C = 1$), there are

$$\mathcal{T}(C = 1) = \sum_{b=0}^{B} \binom{F}{b}^2 \cdot b! \tag{A.17}$$

different traffic matrices, where we defined $\binom{F}{0} = 1, F \geq 1$, and $0! = 1$.

Next, we compute the cardinality of the state space for a satellite with $B = 2$ beams, $F = 2$ footprints, and capacity $C$. As in the example in the last section, we denote number of ongoing calls of each type by the state vector $x = [x_1, x_2, x_3, x_4]$. We want to find the number of possible states that can be accommodated by a satellite with capacity $C$:

$$\max(x_1, x_3) + \max(x_2, x_4) \leq C. \tag{A.18}$$

As above, we first determine the number of possibilities such that

$$\max(x_1, x_3) + \max(x_2, x_4) = c, \tag{A.19}$$

and then we sum over all values of $c$ up to $C$. Note that there are $2l + 1$ pairs $x_1, x_3$ such that $\max(x_1, x_3) = l$, and there are $c + 1$ pairs of $l_1, l_2$ such that $l_1 + l_2 = c$, namely $0 + c, 1 + (c - 1), \ldots, (c - 1) + 1, c + 0$. Combining these two results, we get the following number of $x$ that satisfy Eq. A.19:

$$\sum_{l=0}^{c} (2l + 1)(2(c - l) + 1) = \frac{1}{3}(c + 1)(2c^2 + 4c + 3) \tag{A.20}$$

Summing the above expression over $c = 0 \ldots C$, we finally get the cardinality of the state space for $F = 2$ and $B = 2$:

$$\mathcal{T}(F = 2, B = 2) = \frac{1}{6}(C + 1)(C + 2)(C^2 + 3C + 3) \tag{A.21}$$

Hence, the cardinality of the state space increases with the fourth power of $C$. The values of $\mathcal{T}(F = 2, B = 2)$ up to capacity $C = 10$ are listed in Table A.1.

We assume that for general values of $F$, $B$, and $C$ no solution is known to the mathematical problem given above. We therefore counted all possible states with a computer program; the results are listed in the following tables (bold numbers denoting analytical results).

| $C$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{T}(F=2, B=2)$ | 1 | 7 | 26 | 70 | 155 | 301 | 532 | 876 | 1365 | 2035 | 2926 |

Table A.1: Cardinality of the state space $\Omega$ for a satellite with $B = 2$ beams, $F = 2$ footprints, and capacity up to $2C = 20$ calls.

| $F \setminus C$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 2 | **1** | **7** | **26** | **70** | **155** | **301** | **532** |
| 3 | **1** | **28** | 331 | 2299 | 11290 | 43538 | 140546 |
| 4 | **1** | **89** | 2901 | 47677 | 496679 | | |
| 5 | **1** | **226** | | | | | |

Table A.2: Cardinality of the state space $\Omega$ for a satellite with $B = 2$ beams, $\mathcal{T}(B = 2)$

| $F \setminus C$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 3 | **1** | **34** | 451 | 3380 | 17531 | 70466 | 235014 |
| 4 | **1** | **185** | 10749 | 273767 | 4011527 | | |
| 5 | **1** | **826** | | | | | |

Table A.3: Cardinality of the state space $\Omega$ for a satellite with $B = 3$ beams, $\mathcal{T}(B = 3)$

| $F \setminus C$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 4 | **1** | **209** | 12951 | 344279 | 5206930 | | |
| 5 | **1** | **1426** | | | | | |

Table A.4: Cardinality of the state space $\Omega$ for a satellite with $B = 4$ beams, $\mathcal{T}(B = 4)$

# Bibliography


[Aei78] J.M. Aein, "A Multi-User-Class, Blocked-Calls-Cleared, Demand Access Model," *IEEE Transactions on Communications*, Vol. 26, No. 3, p. 378–385, March 1978

[Aei77] J.M. Aein and O.P Kosovych, "Satellite Capacity Allocation," *Proceedings of the IEEE*, Vol. 65, No. 3, p. 332–342, March 1977

[Bar93a] C.M. Barnhart, J.E. Wieselthier, A. Ephremides, " Admission Control in Integrated Voice/Data Multihop Radio Networks," *NRL Report*, NRL/MR/5521–93-7196, January 18, 1993

[Bar93b] C.M. Barnhart, J.E. Wieselthier, A. Ephremides, " An Approach to Voice Admission Control in Multihop Wireless Networks," *Proc. of IEEE INFOCOM'93*, Paper 3a.1, March 1993

[Cam90] S.J. Campanella, G-P. Forcina, B.A. Pontano, and J.L. Dicks, "SS-TDMA system considerations," *COMSAT Technical Review*, Vol. 20, No. 2, pp. 335–369, Fall 1990

[Fos83] G.J. Foschini and B. Gopinath, "Sharing Memory Optimally," *IEEE Transactions on Communications*, Vol. 31, No. 3, p. 352–360, March 1983

[Fos81] G.J. Foschini, B. Gopinath, and J.F. Hayes, "Optimum Allocation of Servers to Two Types of Competing Customers," *IEEE Transactions on Communications*, Vol. 29, No. 7, p. 1051–1055, July 1981

[Inu79] T. Inukai, "An Efficient SS/TDMA Time Slot Assignment Algorithm," *IEEE Transactions on Communications*, Vol. 27, No. 10, p. 1449–1455, October 1979

[Jor91a] S. Jordan and P. Varaiya, "Throughput in Multiple Service, Multiple Resource Communication Networks," *IEEE Transactions on Communications*, Vol. 39, No. 8, pp. 1216–22, Aug. 1991

[Jor91b] S. Jordan and P. Varaiya, "Control of Multiple Service, Multiple Resource Communication Network," submitted to *IEEE Transactions on Communications*, Jan. 1991