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**Mathematical Modeling of the Uncertainty for
Improving Quality in Machining Operations**

by G.M. Zhang, T.W. Hwang, and R. Ratnakar

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Mathematical Modeling of the Uncertainty for Improving Quality in Machining Operations

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Abstract The difficulty in quality improvement of machining performance comes from the uncertainty about the cutting force generated during the material removal process. This paper presents the results from the research aimed at developing a new approach to capture the uncertainty through mathematically modeling the physical machining system. A case study is used to demonstrate the procedure to interpret the cutting force variation through a three stage process. By integrating deterministic and stochastic approaches, an observed cutting force variation, which was recorded from an experiment, can be explained satisfactorily. The reduction of uncertainty allows an accurate prediction of the cutting force variation and forms a basis for developing a control strategy for improving the machining performance.

1. Introduction

Mathematical modeling of physical systems and simulating the performance of physical systems on computer based on the developed model have been widely used in engineering applications. For example, aerospace industry solely relies on simulation models to study the dynamics of spacecrafts. Vehicle simulation models are effectively used in automobile industry to evaluate safe operations during the vehicle design. In the manufacturing industry, due to the increasing emphasis on precision and cost effective machining, there is a pressing need for modeling and simulating machining systems [1-3].

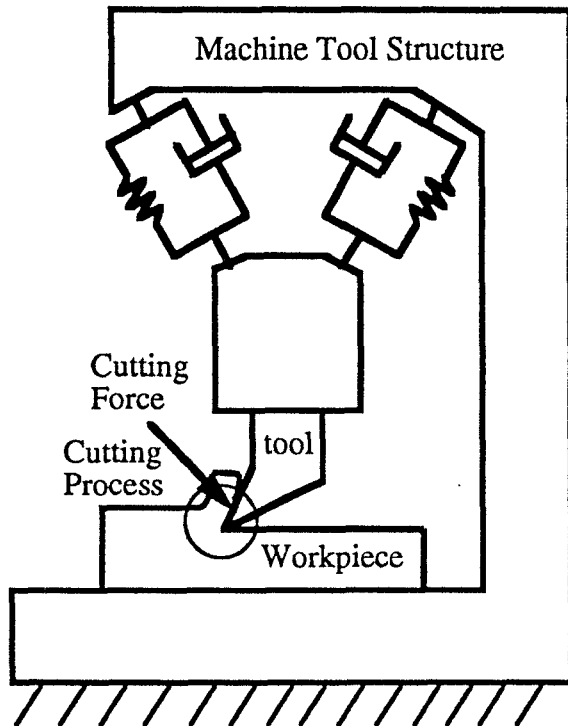
Research into the understanding of machining systems has been an active field of study for several decades with focus on predicting the cutting force generated during machining

The difficulty in modeling a machining system mainly comes from the uncertainty about the material removal process and dynamics of the machining system. Significant efforts have been made in attempt to explain the uncertainty through mathematical modeling approaches. For example, lumped-parameter analysis was developed to describe dynamics of the machine tool structure where the tool motion during machining was assumed to be purely periodic [4-5]. Finite element method has been used to model the material removal process as a continuous system [6-7]. However, limited success has been achieved due to the uncertainty caused by complexity of the machining system.

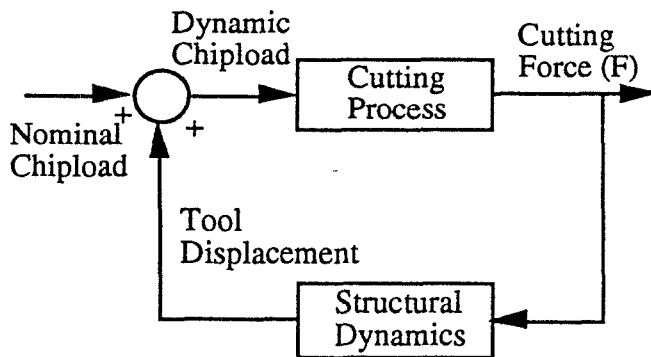
In this paper, we present a rational approach to improve the predictability of the cutting force generated during machining. The approach is analytically based and consists of three stages. In the first stage, a deterministic model, which assumes the periodic nature of the cutting force signal, is developed. In the second stage, a stochastic model is introduced to interpret part of the random variation in the cutting force signal. In the third stage, a stationary Markov chain model is used to add the model flexibility for describing possible state transition in the process of cutting force generation. A case study is presented to demonstrate the procedure how the cutting force variation observed during machining can be effectively interpreted by this new approach.

2. Basic Methodology

Figure 1a illustrates a physical machining system, in which the material is being removed by the tool during the cutting process. The structural dynamics of the machine tool is



(a) A Physical Machining System



(b) System Block Diagram

Figure 1 Representation of a Machining System

represented by a two-degree-of-freedom model. As illustrated, the cutting area, or the nominal chip load, serves as the input of the machining system. The cutting force is generated when the material is removed to form chips. The generated cutting force excites the machine tool structure. The dynamic variation of tool displacement changes the chip load through a

feedback loop. Figure 1b is a block diagram of the machining system representation.

2.1 Stage 1 - Deterministic Model

Three plots in Figure 2 represent the cutting force signals recorded during three consecutive revolutions of the workpiece in a defined turning process. An FFT analysis identifies that the signal is mainly composed of two frequency components at 2500 Hz and 3750 Hz, respectively. Based on the initial data analysis and an assumption that the cutting force signal is purely periodic, a deterministic model can be formulated to predict the cutting force generated during machining. Using the deterministic model, we are able to simulate the machining process on computer. Results from the simulation can be used to predict the cutting force. Figure 3b is a plot of the data obtained from the simulation. Comparing it with the observed data shown in Fig. 3a, the deterministic model is capable of interpreting 30 - 40% of the total variation of the cutting force recorded during machining.

2.2 Stage 2 - Integration of a Deterministic Model and Stochastic Model

It is evident that prediction using a deterministic model is not satisfied. This is usually the case simply because the deterministic modeling approach fails to interpret the part of random variation. In reality, noise may come from phenomena, such as non-homogeneous distribution of microstructures in the material being removed. Consequently, the cutting process functions as a sampling process. It picks up the part of material that can be hard or soft, varying at a random fashion. By using the sampling variance theory [8-9], we add a stochastic model, which describes the random variation of the cutting force, to the deterministic model introduced in stage 1. Following a similar procedure, we implement the integrated modeling approach on computer and simulate the cutting force generation. Figures 4a and 4b present such a comparison. Figure 4b is a plot of the data obtained from the simulation. By examining these two plots, approximately, 70% of the total variation of the cutting force recorded during machining can be explained, an increase of almost 30% from stage 1.

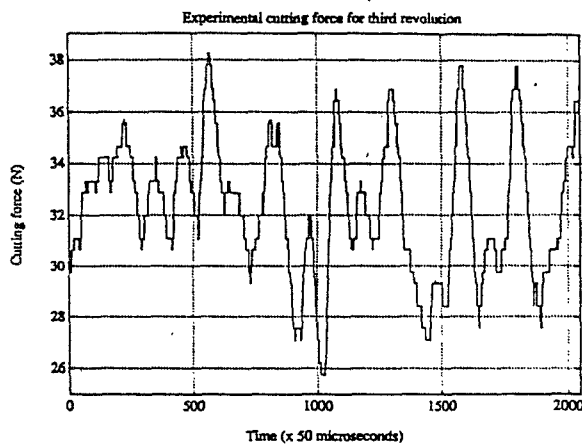
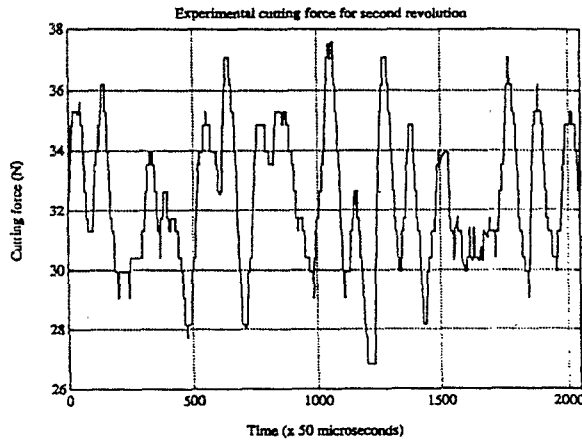
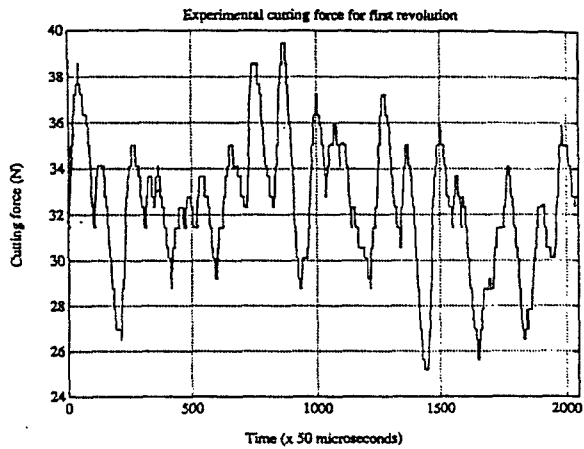


Figure 2 Cutting Force Signal Recorded during Three Consecutive Revolutions

2.3 Stage 3 - Introduction of a Stationary Markov Chain Model

Based on experimental evidence, the cutting force variation displays chaotic dynamics. Traditionally, the variation due to the chaotic dynamics has to be interpreted through mathematical modeling that is non-linear. Very often, it leads to formidable equations with complex initial conditions and constraints. For mathematical modeling of machining systems, we propose to apply stationary Markov chain models to view a non-linear system as a linear system going through transition between a finite

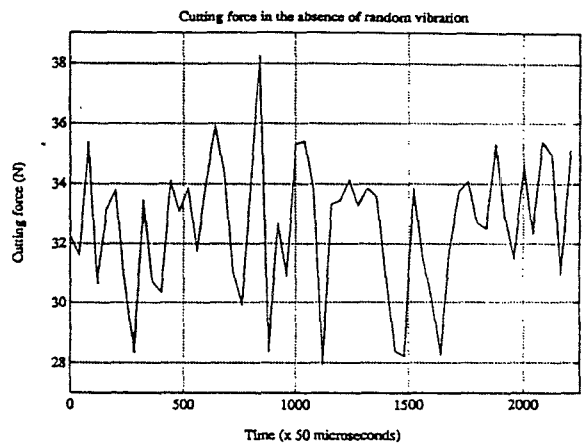
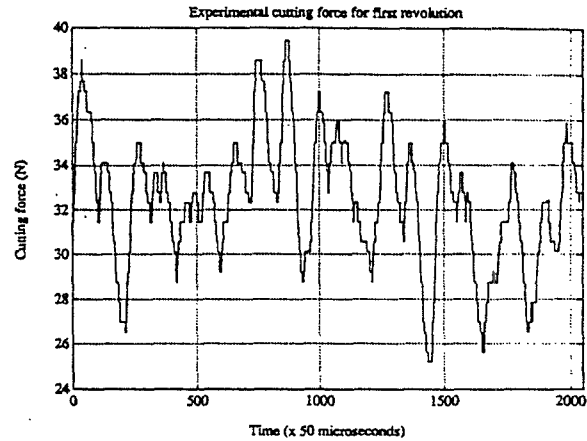


Figure 3 Comparison between the Experimental and Simulated Data (Stage 1)

number of states. As illustrated in Figs. 5a, 5b, and 5c, a N-state transition model is used to describe the distribution of microstructures from different cross-sections, a phenomenon often accounted in raw materials fabricated through a rolling process.

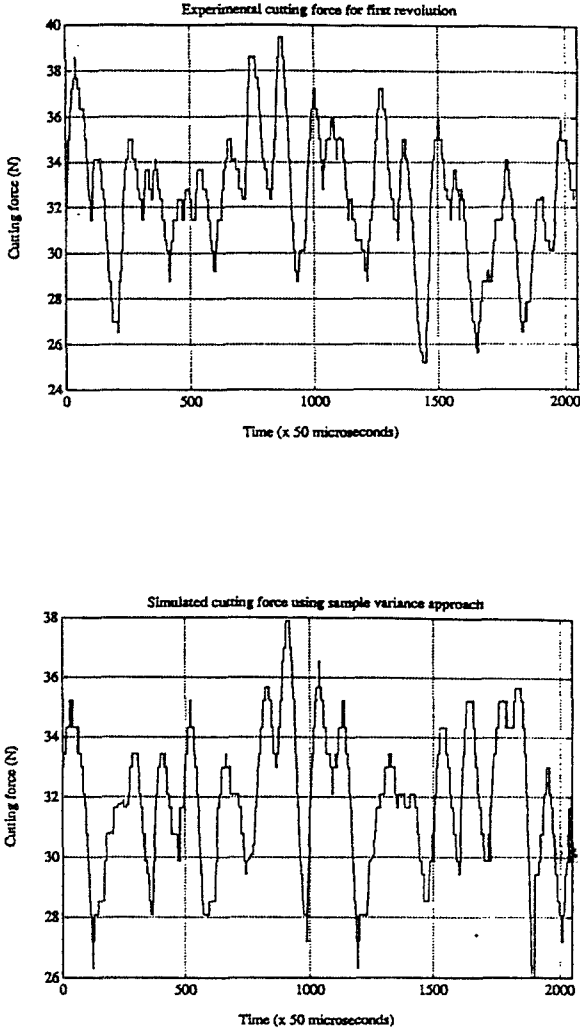


Figure 4 Comparison between the Experimental and Simulated Data (Stage 2)

3. Discussion of Results

Our approach to mathematically model machining systems through a three-stage

process has been proven effective. Not only does the case study presented in this paper demonstrate its effectiveness, other examples can also be found [10-12].

The uniqueness of our approach is the integration of deterministic and stochastic models. We use the deterministic model to characterize the system identity and use the stochastic model to capture effects of the system noise on the system response. Through a three-stage modeling process, the system uncertainty is gradually reduced to a low and acceptable level, thus allowing an effective evaluation of the performance measures of interest.

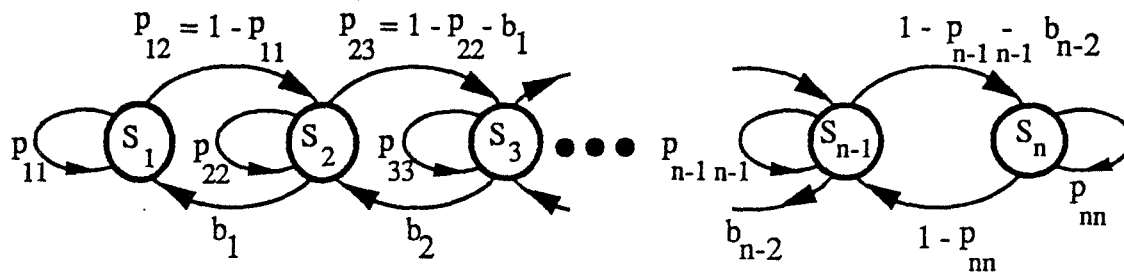
The introduction of a stochastic model allows the evaluation of the performance measures of interest statistically. For example, the results obtained from the uncertainty analysis can be used to construct a control chart to perform an on-line monitoring of a machining system. Assume that the topographical data of a machined surface is available through computer simulation. Assume that $R_{a1}, R_{a2}, \dots, R_{a99}, R_{a100}$ are the R_a values selected from the simulation data to represent the R_a at 100 different locations, two relations may be obtained.

$$R_{a\text{-mean}} = \frac{1}{100} [R_{a1} + R_{a2} + \dots + R_{a99} + R_{a100}] \quad (1)$$

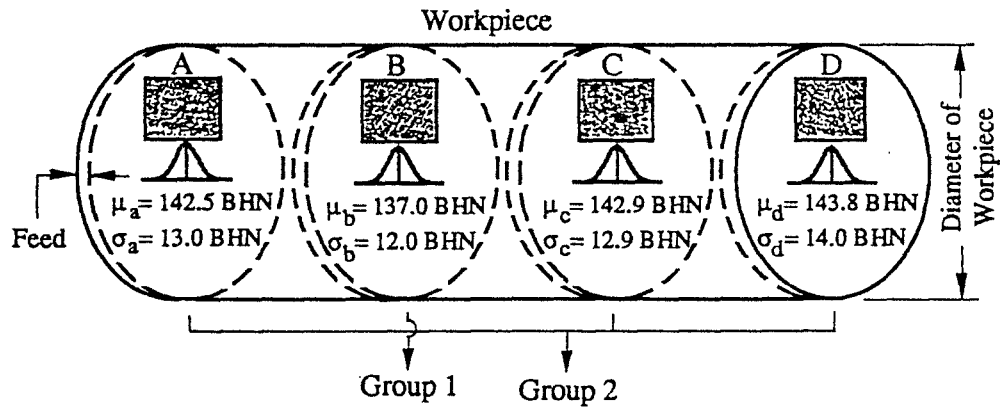
$$\sigma_{Ra}^2 = \frac{1}{100} \sum_{i=1}^{100} (R_{ai} - R_{a\text{-mean}})^2 \quad (2)$$

These two parameters characterize the mean and variation of the R_a measurements. They provide quantitative information about the natural performance variation and form a basis for the construction of $\overline{R_a}$ - and σ_{Ra} - charts, as shown in Fig. 6, where the sample size = 5

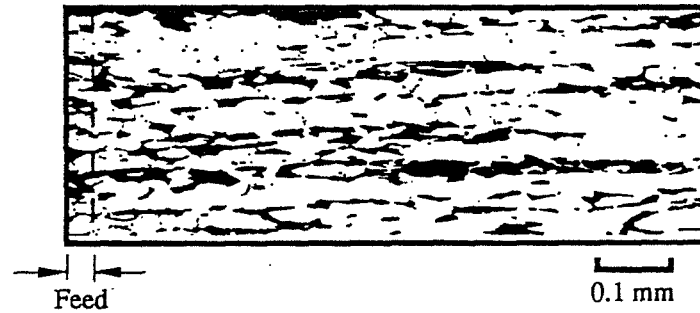
and $3\sigma/\sqrt{5}$ principle are assumed. During machining, on-line detected R_a values can be plotted on the chart against the upper and lower limits to ensure statistical control of the machining process. Consequently, the on-line monitoring system is capable of indirectly tracking the cutting force variation. As long as the monitored values of $\overline{R_{ai}}$ do not exceed the



a. A stationary Markov Chain Model



b. Microstructures from Different Cross-Sections



c. Microstructures from Longitudinal Direction

Figure 5 Introduction of A Markov Chain Model (stage 3)

control limits and display a normal variation pattern about the center line of the control chart, no human intervention is needed.

It is important to note that difficulties arise when using control charts to perform in-process monitoring, namely, false alarming and low sensitivity to detect process abnormalities. The application of Markov chain models offers flexibility that significantly enhances the sensitivity of detection and reduces the risk of false alarming. By choosing a state associated

with the narrowest control chart, sensitivity of detection can be improved significantly. By choosing a state associated with the widest control chart, we are able of avoiding false alarming effectively. More important is the fact that using probabilities of the state transition matrix allows the utilization of Bayes' theorem for decision making. When an abnormal situation occurs on the control chart, for example one \bar{R}_{ai} value exceeds the upper limit, the risk of making a false alarming would be greatly

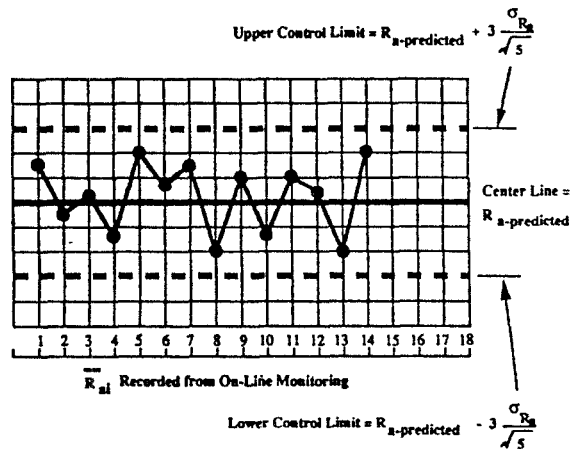


Figure 6 Control Chart for On-Line Monitoring

reduced if the probabilities for associated state transition are taken into consideration.

4. Conclusions

This paper presents a new approach to investigate the uncertainty accounted in the process of improving machining operations. It advocates an integration of deterministic and stochastic models for data/signal interpretation. The deterministic model characterizes the system identity while the stochastic model captures effects of the system noise on the system response. Through a three-stage modeling process, the system uncertainty gradually reduces to a low and acceptable level in the evaluation of the performance measures of interest. The effectiveness of employing Markov chain models is demonstrated in the case study for providing a quantitative measure to balance the need to increase sensitivity meanwhile keeping false alarming at a minimum level for on-line monitoring.

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