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Noninteracting Control with Stability for a Class of Nonlinear Systems

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NONINTERACTING CONTROL WITH STABILITY FOR A CLASS OF NONLINEAR SYSTEMS

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Abstract

In this paper we address the problem of noninteracting control with stability for the class of nonlinear square systems for which noninteraction can be achieved (without stability) by means of invertible static state-feedback. The use of both static state-feedback and dynamic state-feedback is investigated. We prove that in both cases the asymptotic stabilizability of certain subsystems is necessary to achieve noninteraction and stability. We use this and some recent results to state a complete set of necessary and sufficient conditions in order to solve the problem.

Keywords : noninteracting control, stability, dynamic feedback

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I. Notations, basic assumptions and preliminary results

We consider a square nonlinear system Σ of the form

$$\begin{aligned} \dot{x} &= f(x) + \sum_{j=1}^m g_j(x)u_j, \\ y_i &= h_i(x) \quad i = 1, \dots, m, \end{aligned} \tag{1}$$

where f and g_i are smooth vector fields, and h_i are smooth scalar output functions defined on some open subset of $\mathbb{R}^n$. Moreover, $x_0 = 0$ is an equilibrium point of $f(x)$, i.e. $f(0) = 0$.

We say that (1) is noninteractive with respect to the partition $\{u_1, \dots, u_m\}$ if the i -th input does not influence the j -th output for $j \neq i$. If (1) is also asymptotically stable, we say that (1) is noninteractive and stable.

We denote the m -tuple $\{g_1(x), \dots, g_m(x)\}$ by $g(x)$ and set $G = \text{span}\{g(x)\}$. Moreover, for any static state-feedback,

$$u = \alpha(x) + \beta(x)v, \tag{2}$$

let $\tilde{f} = f + g\alpha$, $\tilde{g}_j = g\beta_j$, $j = 1, \dots, m$, where β_j is the j -th column of β , and for any dynamic state-feedback,

$$\begin{aligned} u &= \alpha(x, w) + \beta(x, w)v, \\ \dot{w} &= \delta(x, w) + \gamma(x, w)v, \end{aligned} \tag{3}$$

let, $\tilde{f}^e(x, w) = ((f(x) + g(x)\alpha(x, w))^T, (\delta(x, w))^T)$, $\tilde{g}_j^e(x, w) = ((g(x)\beta_j(x, w))^T, (\gamma_j(x, w))^T)$, where γ_j and β_j are respectively the j -th columns of γ and β . Similarly, we denote the closed-loop system resulting from applying (2) to (1) by $\tilde{\Sigma}$ and the closed-loop system resulting from applying (3) to (1) by $\tilde{\Sigma}^e$.

In what follows, we implicitly restrict our analysis to a neighborhood of the equilibrium point and suppose that every feedback law we consider preserves the equilibrium point.

Both linear and nonlinear noninteracting control problems have been widely discussed in the literature ([1,3,4,5,6,7,8,9,11,12,13,14,15,16]) and necessary and sufficient conditions have been given to solve this problem. On the other hand, the problem of stability was not addressed until recently in the enlighting papers [17], [18] and [19], where some necessary conditions are given. In [23], the necessary condition given in [19], plus some rank conditions, has been shown to be also sufficient to solve the problem. In [22], these results are carried over to a global setting and sufficient conditions are given in terms of stability (resp. stabilizability) of certain dynamics (resp. subsystems). The purpose of our paper is to give a complete set of necessary and sufficient conditions to solve the problem of local noninteraction and stability for the class of systems considered in [17], [19] and [23].

The paper is organized in the following way. In Section II necessary and sufficient conditions are given to solve the problem of noninteracting control with stability by means of *regular (i.e. β is nonsingular at x_0) static state-feedback*. In Section III the dynamic case is addressed.

First we review some basic definitions and algorithms (the reader is referred to [20]). A smooth distribution Δ is said to be invariant under f and g_j for $j = 1, \dots, m$ if $[f, \tau] \subset \Delta$ and $[g_j, \tau] \subset \Delta$ for $j = 1, \dots, m$ and for all $\tau \in \Delta$. A smooth distribution Δ is said to be controlled invariant if there exists a feedback law (2) such that $[\tilde{f}, \tau] \subset \Delta$ and $[\tilde{g}_j, \tau] \subset \Delta$ for $j = 1, \dots, m$ and for all $\tau \in \Delta$. The smallest distribution which is invariant under f and g_j for $j = 1, \dots, m$ and contains a given smooth distribution D , denoted in what follows by $\langle f, g_1, \dots, g_m | D \rangle$, always exists and it is smooth. A smooth distribution Δ is said to be a controllability distribution if it is involutive and there exists a regular feedback (2) and a subset $I \subset \{1, \dots, m\}$ such that

$$\Delta = \langle \tilde{f}, \tilde{g}_1, \dots, \tilde{g}_m | \text{span}\{\tilde{g}_j : j \in I\} \rangle.$$

Let $g_0 = f$. The following algorithm,

$$\begin{aligned} I_0 &= D, \\ I_k &= \sum_{j=0}^m [g_j, I_{k-1}] + I_{k-1}, \end{aligned}$$

can be used to compute $\langle f, g_1, \dots, g_m | D \rangle$ on a dense and open set $U^* \subset \mathbb{R}^n$, i.e. on U^* there exists $k^* < n$ such that $\langle f, g_1, \dots, g_m | D \rangle = I_{k^*} = I_{k^*+1}$ and each I_k is of constant dimension for $k = 0, \dots, k^*$. Let $I(D) = I_{k^*}$. If $x_0 \in U^*$, then we say that $I(D)$ is regularly computable at x_0 .

Finally, let Γ be a smooth involutive distribution. Let us consider the sequence,

$$\begin{aligned} S_0 &= V^*(\Gamma) \cap G, \\ S_k &= \left(\sum_{j=0}^m [g_j, S_{k-1}] \right) \cap G + S_{k-1}, \end{aligned}$$

where $V^*(\Gamma)$ is the maximal controlled invariant distribution contained in Γ [20]. If there exists $k^* < n$ such that $S_{k^*} = S_{k^*+1}$ and S_{k^*} has constant dimension, then S_{k^*} is the maximal controllability distribution contained in Γ . In this case, we denote this distribution by $R^*(\Gamma)$ and we say that $R^*(\Gamma)$ is regularly computable at x_0 .

We also say that (1) has relative degree $\{r_1, \dots, r_m\}$ at x_0 if the following properties hold:

- a) $L_{g_j} L_f^k h_i(x) = 0$ for $0 \leq k < r_i - 1, j = 1, \dots, m$ and all x in a neighborhood of x_0 and $L_{g_j} L_f^{r_i-1} h_i(x_0) \neq 0$ for some j ;

- b) the decoupling matrix,

$$\begin{pmatrix} L_{g_1} L_f^{r_1-1} h_1 & \cdots & L_{g_m} L_f^{r_1-1} h_1 \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m & \cdots & L_{g_m} L_f^{r_m-1} h_m \end{pmatrix},$$

is nonsingular at x_0 .

Set $R_i^* = R^*(\cap_{j \neq i} \ker dh_j)$, $i = 1, \dots, m$, (whenever these distributions exist) and $R_0 = I(G)$. Moreover, set $R^* = \cap_{i=1}^m \sum_{j \neq i} R_j^*$. We assume the following in the remainder of the paper.

(H1) $\dim R_0 = n$;

(H2) R_i^* , $i = 1, \dots, m$, and R_0 are regularly computable at x_0 ;

(H3) the distributions $R_0, R^*, R_i^*, \sum_{j \neq i} R_j^*$ and $\sum_{j=1}^m R_j^*$, $i = 1, \dots, m$,

have constant dimension;

(H4) Σ has some relative degree at x_0 .

(H1) means that (1) is strongly accessible at x_0 [2]. Should (H1) not hold, there would be a subsystem of (1) which is not influenced by the input at all, so that if the dynamics of such subsystem is unstable our problem becomes unsolvable (see [23] for the case that this dynamics is asymptotically stable).

(H4) is a well-known sufficient condition to find a regular feedback (2) such that $\tilde{\Sigma}$ is noninteractive (if we require that the relative degree is defined for each output, then (H4) is also necessary) [13, 20]. The next lemma tells us that, no matter how we choose this feedback law, the distributions R_i^* , $i = 1, \dots, m$, are invariant under $\tilde{f}$ and $\tilde{g}_j$ for $j = 1, \dots, m$.

The proof of this lemma is rather easy and we will leave it to the reader. The arguments needed can be found in the proofs of lemmas I.1.8 and VI.4.2 of [20], lemma III.1 of [23] and proposition 4.14 in [25].

Lemma II.1. Suppose that (H2)–(H4) hold. Consider any regular feedback (2) such that $\tilde{\Sigma}$ is noninteractive and suppose that $\langle \tilde{f}, \tilde{g}_1, \dots, \tilde{g}_m | \text{span}\{\tilde{g}_i\} \rangle$, $i = 1, \dots, m$, has constant dimension. Then,

$$\begin{aligned} R_i^* &= \langle \tilde{f}, \tilde{g}_1, \dots, \tilde{g}_m | \text{span}\{\tilde{g}_i\} \rangle, \quad i = 1, \dots, m, \\ \sum_{j \neq i} R_j^* &= \langle \tilde{f}, \tilde{g}_1, \dots, \tilde{g}_m | \text{span}\{\tilde{g}_j : j \neq i\} \rangle, \quad i = 1, \dots, m, \\ \sum_{j=1}^m R_j^* &= \langle \tilde{f}, \tilde{g}_1, \dots, \tilde{g}_m | \text{span}\{\tilde{g}_j : j = 1, \dots, m\} \rangle. \end{aligned}$$

Thus, $\sum_{j \neq i} R_j^*$, $i = 1, \dots, m$, and $\sum_{i=1}^m R_i^*$ are controllability distributions (in particular, involutive). $\diamond$

Let $\tilde{f}, \tilde{g}_1, \dots, \tilde{g}_m$ be as in lemma II.1. Note that,

$$R_i^* + \bigcap_{j \neq i} \sum_{k \neq j} R_k^* = \sum_{i=1}^m R_i^*.$$

From this and lemma II.1, using essentially the same arguments as in [17, lemma 4.1], it is easy to see that there is a coordinate system $(x_1, \dots, x_{m+1})$ such that (1) takes the form

$$\begin{aligned} \dot{x}_i &= \tilde{f}_i(x_i) + \tilde{g}_{ii}(x_i)u_i, & i = 1, \dots, m, \\ \dot{x}_{m+1} &= \tilde{f}_{m+1}(x) + \sum_{j=1}^m \tilde{g}_{m+1,j}(x)u_j, \\ y_i &= h_i(x_i), & i = 1, \dots, m, \end{aligned} \tag{4}$$

where $\sum_{j \neq i} R_j^* = \text{span}\{\partial/\partial x_j : j \neq i\}$, $i = 1, \dots, m$, and $R^* = \text{span}\{\partial/\partial x_{m+1}\}$. In what follows, we suppose that (1) has already been put in the form (4) and for simplicity we omit the tildas.

We now define the following distribution. Let $\mathcal{I}$ be the Lie ideal generated by the vector fields $\{[g_i, \text{ad}_f^k g_j] : j \neq i ; j, i = 1, \dots, m ; k \geq 0\}$ in the Lie algebra generated by $\{f, g_1, \dots, g_m\}$. Let,

$$\Delta_{MIX} = \text{span}\{\tau : \tau \in \mathcal{I}\}.$$

We state the additional regularity assumption,

(H5) Δ_{MIX} and $R_i^* + \Delta_{MIX}$, $i = 1, \dots, m$, have constant dimension in a neighborhood of x_0 .

Note that $\Delta_{MIX} \subset R^*$, since $\Delta_{MIX} \subset \sum_{j \neq i} R_j^*$ for $i = 1, \dots, m$.

Lemma II.2. Suppose that **(H2)**–**(H5)** hold. Let Δ_{MIX}^e be the distribution defined in the same way as Δ_{MIX} but with f and g_j replaced by $\tilde{f}^e$ and $\tilde{g}_j^e$ for $j = 1, \dots, m$ and set $R_i^e = \langle \tilde{f}^e, \tilde{g}_1^e, \dots, \tilde{g}_m^e | \text{span}\{\tilde{g}_i^e\} \rangle$, $i = 1, \dots, m$. Suppose also that R_i^e is regularly computable at $x_0^e = (x_0^T \ w_0^T)^T$, Δ_{MIX}^e and $R_i^e + \Delta_{MIX}^e$ have constant dimension and $\tilde{\Sigma}^e$ has some relative degree at x_0^e . Then, setting $\alpha = (\alpha_1^T, \dots, \alpha_m^T)^T$, the functions α_j are constant along leaves of $R_i^e + \Delta_{MIX}^e$ for $j \neq i$ and $i, j = 1, \dots, m$. Moreover, for $\pi : U \times V \rightarrow U$, $\pi(x, w) = x$, where U and V are open neighborhoods respectively of x_0 and w_0 , we have

$$(\pi_*)_x^e(\Delta_{MIX}^e + R_i^e)(x^e) = (\Delta_{MIX} + R_i^*)(x), \quad i = 1, \dots, m. \tag{5}$$

◇

Proof. Our assumptions meet those stated in [19, Prop. 2.4]. Thus we have

$$L_{\tilde{g}_i^e} D^e \alpha_j = 0, \quad j, i = 1, \dots, m ; j \neq i \tag{6}$$

and D^e is any product of factors $L_{\tilde{f}^e}$ and $L_{\tilde{g}_h^e}$ for $h = 1, \dots, m$. Since R_i^* , $i = 1, \dots, m$, is regularly computable at x_0 , as in [20, lemma 1.8.6], it can be shown that it is spanned by vector fields in the set

$$\{[g_{j_h}, \dots, [g_{j_1}, g_i], \dots] : n-1 \geq h \geq 1 ; j_k = 0, \dots, m ; 0 \leq k \leq h\} \tag{7}$$

with $g_0 = f$. Since Δ_{MIX}^e and $R_i^e + \Delta_{MIX}^e$, $i = 1, \dots, m$, have constant dimension, using the definition of Δ_{MIX}^e and the Jacobi identity, it is easy to see that the distributions $R_i^e + \Delta_{MIX}^e$, $i = 1, \dots, m$, are spanned by vector fields in the set

$$\{[ad_{f^e}^{l_h} \tilde{g}_{j_h}^e, [\dots, [ad_{f^e}^{l_2} \tilde{g}_{j_2}^e, ad_{f^e}^{l_1} \tilde{g}_{j_1}^e] \dots]] : (j_2 \neq j_1 \text{ and } h \geq 2) \text{ or } (j_1 = i \text{ and } h \geq 1); \\ j_k = 1, \dots, m ; l_k \geq 0 ; 1 \leq k \leq h\} . \quad (8.1)$$

Similarly, the distributions $R_i^* + \Delta_{MIX}$, $i = 1, \dots, m$, are spanned by vector fields in the set

$$\{[ad_f^{l_h} g_{j_h}, [\dots, [ad_f^{l_2} g_{j_2}, ad_f^{l_1} g_{j_1}] \dots]] : (j_2 \neq j_1 \text{ and } h \geq 2) \text{ or } (j_1 = i \text{ and } h \geq 1); \\ j_k = 1, \dots, m ; l_k \geq 0 ; 1 \leq k \leq h\} . \quad (8.2)$$

Thus, the first part of the lemma follows from (6) and (8.1). The equality (5) can be proven by using (6), (8.1), (8.2) and induction as in [19].•

Under assumption **(H5)**, Δ_{MIX} is nonsingular and thus by definition invariant under f and g_j for $j = 1, \dots, m$. As a consequence, (4) can be further decomposed in the following way

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_{ii}(x_i)u_i \quad i = 1, \dots, m \\ \dot{x}_{m+1,1} &= f_{m+1,1}(x_1, \dots, x_m, x_{m+1,1}) + \sum_{j=1}^m g_{m+1,1j}(x_1, \dots, x_m, x_{m+1,1})u_j \\ \dot{x}_{m+1,2} &= f_{m+2}(x_1, \dots, x_m, x_{m+1,1}, x_{m+1,2}) + \sum_{j=1}^m g_{m+2,j}(x_1, \dots, x_m, x_{m+1,1}, x_{m+1,2})u_j \\ y_i &= h_i(x_i) \quad i = 1, \dots, m, \end{aligned} \quad (9)$$

where $R^* = \text{span}\{\partial/\partial x_{m+1,1}, \partial/\partial x_{m+1,2}\}$ and $\Delta_{MIX} = \text{span}\{\partial/\partial x_{m+1,2}\}$.

We introduce now another coordinate system for (4), which will be useful in Section III. The distributions $R_i^* + \Delta_{MIX}$, $i = 1, \dots, m$, are constant dimensional (assumption **(H5)**) and involutive. Moreover, they are also invariant under f and g_j for $j = 1, \dots, m$, since they are constant dimensional and R_i^* , $i = 1, \dots, m$, and Δ_{MIX} are invariant under f and g_j for $j = 1, \dots, m$. As a consequence, the distributions $(R_i^* + \Delta_{MIX}) \cap R^*$ are constant dimensional (from **(H5)**), involutive and invariant under f and g_j for $j = 1, \dots, m$. Since $\Delta_{MIX} \subset R^*$, for each $i = 1, \dots, m$ there exist a coordinate system $(z_1^T, \dots, z_6^T)^T$ (which depends on i) such that

$$\begin{aligned} R_i^* + \Delta_{MIX} &= \text{span}\{\partial/\partial z_1, \partial/\partial z_3, \partial/\partial z_4, \partial/\partial z_5\}, \\ R^* &= \text{span}\{\partial/\partial z_2, \partial/\partial z_3, \partial/\partial z_4, \partial/\partial z_5\}, \\ R^* \cap (R_i^* + \Delta_{MIX}) &= \text{span}\{\partial/\partial z_3, \partial/\partial z_4, \partial/\partial z_5\}, \\ \Delta_{MIX} &= \text{span}\{\partial/\partial z_4, \partial/\partial z_5\} \\ \Delta_{MIX} \cap R_i^* &= \text{span}\{\partial/\partial z_5\} . \end{aligned} \quad (10)$$

Correspondingly, we have

$$f(z) = \begin{pmatrix} f_{z_1}(z_1, z_6) \\ f_{z_2}(z_2, z_6), \\ f_{z_3}(z_1, z_2, z_3, z_6) \\ f_{z_4}(z_1, z_2, z_3, z_4, z_6), \\ f_{z_5}(z_1, z_2, z_3, z_4, z_5, z_6) \\ f_{z_6}(z_6), \end{pmatrix}, \quad g_i(z) = \begin{pmatrix} g_{z_1, i}(z_1, z_6) \\ 0 \\ g_{z_3, i}(z_1, z_2, z_3, z_6), \\ g_{z_4, i}(z_1, z_2, z_3, z_4, z_6) \\ g_{z_5, i}(z_1, z_2, z_3, z_4, z_5, z_6), \\ 0 \end{pmatrix}. \quad (11)$$

In what follows, we use also the following notation. Let Δ be a constant dimensional involutive distribution. Let also $\mathcal{L}_0^\Delta$ be the leaf of Δ passing through x_0 and $\mathcal{F}^\Delta$ the foliation induced by Δ . In what follows, we consider the restriction (when it is well-defined) of a smooth vector field X to $\mathcal{L}_0^\Delta$ and we denote it by $X|_{\mathcal{L}_0^\Delta}$.

II. Noninteracting control with stability via regular static state-feedback.

In this section we consider regular (i.e. β is nonsingular at x_0) feedback laws (2). Consider the system (4). Since $R_i^* + R^* = \text{span}\{\partial/\partial x_i, \partial/\partial x_{m+1}\}$ is invariant under f , $f(0) = 0$ and $g_i \in R_i^*$ for $i = 1, \dots, m$, the restrictions $f|_{\mathcal{L}_0^{R_i^* + R^*}}$ and $g_i|_{\mathcal{L}_0^{R_i^* + R^*}}$, $i = 1, \dots, m$, are well-defined. For each $i = 1, \dots, m$ we denote by

$$\Sigma_i^{R^*} : (f|_{(\mathcal{L}_0^{R_i^* + R^*} / \mathcal{F}^{R^*})}, g_i|_{(\mathcal{L}_0^{R_i^* + R^*} / \mathcal{F}^{R^*})}),$$

the system obtained from (4) by setting $u_j = 0$ for $j \neq i$ and taking the restrictions $f|_{\mathcal{L}_0^{R_i^* + R^*}}$ and $g_i|_{\mathcal{L}_0^{R_i^* + R^*}}$ with the x_{m+1} component omitted. Moreover, in what follows we refer to the dynamics

$$\dot{x}_{m+1} = f_{m+1}(0, \dots, 0, x_{m+1}) = f|_{\mathcal{L}_0^{R^*}}$$

as R^* dynamics.

The main result of this section is the following

Theorem 1. Under assumptions **(H1)**–**(H4)**, the problem of noninteracting control with stability is solvable by means of regular static state-feedback if

(S1) the R^* dynamics is asymptotically stable;

(S2) the subsystems $\Sigma_i^{R^*}$, $i = 1, \dots, m$, are asymptotically stabilizable via static state-feedback.

If in addition the distributions $\langle \tilde{f}, \tilde{g}_1, \dots, \tilde{g}_m | \text{span}\{\tilde{g}_i\} \rangle$, $i = 1, \dots, m$, have constant dimension, then **(S1)** and **(S2)** are also necessary. $\diamond$

Remark 1. Theorem 1 states that, modulo singularities and for the class of noninteractive (but not necessarily asymptotically stable) systems, the conditions **(S1)** and **(S2)** are

necessary and sufficient to stabilize these systems via regular static state–feedback without destroying the noninteraction property. The necessity of **(S1)** has been essentially proven in [17]. Moreover, in [20] it is proved that the exponential stability of the R^* dynamics and the controllability of the linear approximation of (4) are necessary and sufficient conditions to solve the problem of noninteracting control with exponential stability via regular static state–feedback. On the other hand, in [17] the asymptotic stabilizability of (4) and **(S1)** are shown to be sufficient to achieve noninteraction and asymptotic stability via dynamic state–feedback.

Proof. We show first the necessity of **(S1)** and **(S2)**. Suppose that $\tilde{\Sigma}$ is noninteractive and asymptotically stable. From lemma II.1 it follows that the distributions R_i^* , $i = 1, \dots, m$, are invariant under $\tilde{f}$ and $\tilde{g}_j$ for $j = 1, \dots, m$. Then, we must have

$$R_i^* \ni [\tilde{f}, \tau] = [f + \sum_{j=1}^m \alpha_j g_j, \tau] = [f, \tau] + \sum_{j=1}^m [g_j, \tau] \alpha_j + \sum_{j=1}^m (-L_\tau \alpha_j) g_j \quad i = 1, \dots, m ; \tau \in R_i^*. \quad (12)$$

Moreover, $\text{span}\{g_j : j \neq i\} \cap R_i^* = 0$. As a matter of fact, if $\text{span}\{g_j : j \neq i\} \cap R_i^* \neq 0$, then it would follow that $\text{span}\{g_j : j \neq i\} \cap V_i^* = \text{span}\{g_j : j \neq i\} \cap (\text{span}\{dh_j, \dots, dL_f^{r_i-1} h_j : j \neq i\})^\perp \neq 0$, where $\{r_1, \dots, r_m\}$ is the vector relative degree of (4) and “ $\perp$ ” means “annihilator of” (see [20]). This now implies that the decoupling matrix does not have full rank, which is a contradiction. Since R_i^* is invariant under f and g_j for $j = 1, \dots, m$, from (12) it easily follows that

$$L_\tau \alpha_j = 0 \quad j = 1, \dots, m, \tau \in \sum_{i \neq j} R_i^*.$$

Since $\sum_{i \neq j} R_i^* = \text{span}\{\partial/\partial x_i : i \neq j\}$, α_j depends only on x_j , hence α_j is constant along leaves of $R_i^* + R^* = \text{span}\{\partial/\partial x_i, \partial/\partial x_{m+1}\}$ for $j \neq i$. Since $\alpha(0) = 0$, it follows that $\alpha_j|_{\mathcal{L}_0^{R_i^*+R^*}} = 0$ for $j \neq i$. Since $R_i^* + R^*$ is also invariant under $\tilde{f}$, $\tilde{g}_i \in R_i^* + R^*$ and the class of feedback laws we consider preserves the equilibrium point, the restrictions $\tilde{f}|_{\mathcal{L}_0^{R_i^*+R^*}}$ and $\tilde{g}_i|_{\mathcal{L}_0^{R_i^*+R^*}}$ are well-defined. Thus we have,

$$\begin{aligned} \tilde{f}|_{\mathcal{L}_0^{R_i^*+R^*}} &= (f + g_i \alpha_i)|_{\mathcal{L}_0^{R_i^*+R^*}} \\ &= (0, \dots, (f_i(x_i))^T, 0, \dots, (f_{m+1}(0, \dots, x_i, 0, \dots, x_{m+1}))^T) + \\ &\quad (0, \dots, (g_i(x_i))^T, 0, \dots, (g_{m+1,i}(0, \dots, x_i, 0, \dots, x_{m+1}))^T) \alpha_i(x_i) \end{aligned}$$

Since $\tilde{f}|_{\mathcal{L}_0^{R_i^*+R^*}}$ must be asymptotically stable and $\tilde{f}$ is tangent to $\mathcal{L}_0^{R^*}$, then the R^* dynamics must be asymptotically stable also and the subsystems $\Sigma_i^{R^*}$, $i = 1, \dots, m$, must be asymptotically stabilizable by means of static feedback $u_i = \alpha_i(x_i)$.

The sufficiency of **(S1)** and **(S2)** easily follows from the triangular structure of (4) and standard stability properties of triangular systems ([20, Appendix B]).

III. Noninteracting control with stability via dynamic state–feedback

In this section we consider the class of feedback laws (3) such that $\tilde{\Sigma}^e$ is noninteractive and has still some relative degree at x_0^e . In analogy with [19], we call them *regular dynamic noninteraction feedback* laws.

Fix $i = 1, \dots, m$. Since $R_i^* + \Delta_{MIX}$ is invariant under f , $f(0) = 0$ and $g_i \in R_i^* + \Delta_{MIX}$, it follows that the restrictions $f|_{\mathcal{L}_0^{R_i^* + \Delta_{MIX}}}$ and $g_i|_{\mathcal{L}_0^{R_i^* + \Delta_{MIX}}}$ are well-defined. Thus, we can define,

$$\Sigma_i^{\Delta_{MIX}} : (f|_{(\mathcal{L}_0^{R_i^* + \Delta_{MIX}} / \mathcal{F}^{\Delta_{MIX}})}, g_i|_{(\mathcal{L}_0^{R_i^* + \Delta_{MIX}} / \mathcal{F}^{\Delta_{MIX}})}) ,$$

where $f|_{(\mathcal{L}_0^{R_i^* + \Delta_{MIX}} / \mathcal{F}^{\Delta_{MIX}})}$ and $g_i|_{(\mathcal{L}_0^{R_i^* + \Delta_{MIX}} / \mathcal{F}^{\Delta_{MIX}})}$ are given respectively by $f|_{\mathcal{L}_0^{R_i^* + \Delta_{MIX}}}$ and $f|_{\mathcal{L}_0^{R_i^* + \Delta_{MIX}}}$ with the z_4, z_5 and z_6 components omitted (see (11)). Moreover, in what follows we call Δ_{MIX} *dynamics* the dynamics

$$\begin{pmatrix} \dot{z}_4 \\ \dot{z}_5 \end{pmatrix} = \begin{pmatrix} f_{z_4}(0, 0, 0, z_4, 0) \\ f_{z_5}(0, 0, 0, z_4, z_5, 0) \end{pmatrix} = f|_{\mathcal{L}_0^{\Delta_{MIX}}}$$

Let be Δ_{MIX}^e the distribution defined as Δ_{MIX} but with f and g_j replaced by $\tilde{f}^e$ and $\tilde{g}_j^e$ for $j = 1, \dots, m$. The main result of this section is the following.

Theorem 2. Under assumptions **(H1)**–**(H5)**, the problem of noninteracting control with stability is solvable via regular dynamic state–feedback if

- (D1)** the Δ_{MIX} dynamics is asymptotically stable;
- (D2)** the subsystems $\Sigma_i^{\Delta_{MIX}}$, $i = 1, \dots, m$, are asymptotically stabilizable via dynamic state–feedback.

If in addition the distributions $\langle \tilde{f}^e, \tilde{g}_1^e, \dots, \tilde{g}_m^e | \text{span}\{\tilde{g}_i^e\} \rangle$, $i = 1, \dots, m$, are regularly computable at x_0^e and $\langle \tilde{f}^e, \tilde{g}_1^e, \dots, \tilde{g}_m^e | \text{span}\{\tilde{g}_i^e\} \rangle$, Δ_{MIX}^e and $\Delta_{MIX}^e + \langle \tilde{f}^e, \tilde{g}_1^e, \dots, \tilde{g}_m^e | \text{span}\{\tilde{g}_i^e\} \rangle$, $i = 1, \dots, m$, have constant dimension, then **(D1)** and **(D2)** are also necessary. $\diamond$

Remark 2. Theorem 2 states that, modulo singularities and for the class of noninteractive systems (not necessarily asymptotically stable), the conditions (D1) and (D2) are necessary and sufficient to stabilize these systems via regular dynamic state–feedback without destroying the noninteraction property. Note that (D2) replaces the stronger condition given in [23], which requires that the systems $\Sigma_i : (f|_{\mathcal{L}_0^{R_i^*}}, g_i|_{\mathcal{L}_0^{R_i^*}})$, $i = 1, \dots, m$, be exponentially stabilizable by static state–feedback.

Proof. The necessity of **(D1)** has been shown in [17]. We show now the necessity of **(D2)**. Suppose that $\tilde{\Sigma}^e$ is asymptotically stable and noninteractive. Set $R_i^e = \langle \tilde{f}^e, \tilde{g}_1^e, \dots, \tilde{g}_m^e | \text{span}\{\tilde{g}_i^e\} \rangle$, $i = 1, \dots, m$. Fix $i = 1, \dots, m$. Reasoning as for $R_i^* + \Delta_{MIX}$, it can be shown that the distribution $R_i^e + \Delta_{MIX}^e$ is constant dimensional, involutive and invariant under $\tilde{f}$ and $\tilde{g}_j^e$ for $j = 1, \dots, m$. Moreover, the restrictions $\tilde{f}^e|_{\mathcal{L}_0^{R_i^e + \Delta_{MIX}^e}}$ and $\tilde{g}_i^e|_{\mathcal{L}_0^{R_i^e + \Delta_{MIX}^e}}$ are well-defined. From lemma II.2, $\alpha_j = 0$ for $j \neq i$ is constant along the leaves of $R_i^e + \Delta_{MIX}^e$ and $(\pi_*)_{x^e}(R_i^e + \Delta_{MIX}^e)(x^e) = (R_i^* + \Delta_{MIX})(x)$. From (10) and (11), it follows that $z_2 = 0$ and

$z_6 = 0$ on $\mathcal{L}_0^{R_i^e + \Delta_{MIX}^e}$. Thus, z_1, z_3, z_4 and z_5 can be taken as part of a coordinate system on $\mathcal{L}_0^{R_i^e + \Delta_{MIX}^e}$ so that

$$\begin{aligned} \tilde{f}^e(z, w)|_{\mathcal{L}_0^{R_i^e + \Delta_{MIX}^e}} &= \begin{pmatrix} f_{z_1}(z_1, 0) \\ 0 \\ f_{z_3}(z_1, 0, z_3, 0) \\ f_{z_4}(z_1, 0, z_3, z_4, 0) \\ f_{z_5}(z_1, 0, z_3, z_4, z_5, 0) \\ 0 \\ \bar{f}(z_1, 0, z_3, z_4, z_5, 0, \bar{w}) \end{pmatrix} + \\ &+ \begin{pmatrix} g_{z_1, i}(z_1, 0) \\ 0 \\ g_{z_3, i}(z_1, 0, z_3, 0) \\ g_{z_4, i}(z_1, 0, z_3, z_4, 0) \\ g_{z_5, i}(z_1, 0, z_3, z_4, z_5, 0) \\ 0 \\ \bar{g}_i(z_1, 0, z_3, z_4, z_5, 0, \bar{w}) \end{pmatrix} \alpha_i(z_1, 0, z_3, z_4, z_5, 0, \bar{w}), \end{aligned}$$

with suitable $\bar{w}$, $\bar{f}$ and $\bar{g}_i$ (depending on i). Since $\tilde{f}^e|_{\mathcal{L}_0^{R_i^e + \Delta_{MIX}^e}}$ must be asymptotically stable, from (10) it follows that $\Sigma_i^{\Delta_{MIX}}$ must be asymptotically stabilizable by means of the dynamic feedback,

$$\begin{aligned} u_i &= \alpha_i(z_1, 0, z_3, z_4, z_5, 0, \bar{w}), \\ \dot{z}_4 &= f_{z_4}(z_1, 0, z_3, z_4, 0) + g_{z_4, i}(z_1, 0, z_3, z_4, 0)\alpha_i(z_1, 0, z_3, z_4, z_5, 0, \bar{w}), \\ \dot{z}_5 &= f_{z_5}(z_1, 0, z_3, z_4, z_5, 0) + g_{z_5, i}(z_1, 0, z_3, z_4, z_5, 0)\alpha_i(z_1, 0, z_3, z_4, z_5, 0, \bar{w}), \\ \dot{\bar{w}} &= \bar{f}(z_1, 0, z_3, z_4, z_5, 0, \bar{w}) + \bar{g}_i(z_1, 0, z_3, z_4, z_5, 0, \bar{w})\alpha_i(z_1, 0, z_3, z_4, z_5, 0, \bar{w}). \end{aligned}$$

The sufficiency of **(D1)** and **(D2)** can be essentially proved as in [1] and [2]. Since in this case the proof is simpler, we shortly recall it here.

Consider first the system (9) and the well-defined vector fields $\hat{f}$ and $\hat{g}_j$ for $j = 1, \dots, m$ obtained from f and g_j by omitting the $x_{m+1,2}$ component. Let also $\hat{x}$ denote the coordinate system $(x_1^T \dots x_{m+1,1}^T)^T$. It is easy to see that, denoting by $\hat{\Delta}_{MIX}$ the distribution defined in the same way as Δ_{MIX} but with f and g_j , $j = 1, \dots, m$, replaced by $\hat{f}$ and $\hat{g}_j$, we have $\hat{\Delta}_{MIX} = 0$. Consider the map $\sigma : U \rightarrow \hat{U}$, $\sigma(x) = \hat{x}$, where U and $\hat{U}$ are open neighborhoods respectively of x_0 and $\hat{x}_0$. Denote by $\hat{R}_i$ the distribution which assigns to each point $\sigma(x)$ the subspace $(\sigma_*)_x(R_i^* + \Delta_{MIX})(x)$ and set $\hat{s}_i = \dim \hat{R}_i$ (from the regularity assumption, this integer is well-defined). From (10) and (11), it follows that $\hat{R}_i$ is smooth and invariant under $\hat{f}$ and $\hat{g}_j$ for $j = 1, \dots, m$. From (10), (11) and the regular computability of R_i^* , it also follows that there exist vector fields $\{\hat{X}_{ik} : k = 1, \dots, \hat{s}_i\}$, $i = 1, \dots, m$, in the set

$$\{[\hat{g}_{j_h}, \dots, [\hat{g}_{j_0}, \hat{g}_i] \dots] : n-1 \geq h \geq 0 ; j_k = 0, \dots, m ; \text{ for } 0 \leq k \leq h\}$$

such that

$$\hat{R}_i = \text{span}\{\hat{X}_{ik} : 1 \leq k \leq \hat{s}_i\} \quad i = 1, \dots, m.$$

It can be easily checked that the vector fields $\widehat{X}_{ik}(\widehat{x})$ are of the form

$$\widehat{X}_{ik}(\widehat{x}) = (\partial/\partial x_i)\widehat{Y}_{ik}(x_i) + (\partial/\partial x_{m+1,1})\widehat{Z}_{ik}(\widehat{x}) \quad i = 1, \dots, m; \quad k = 1, \dots, \widehat{s}_i, \quad (13)$$

where $\widehat{Y}_{ik}$ and $\widehat{Z}_{ik}$ are respectively the x_i and $x_{m+1,1}$ components of $\widehat{X}_{ik}$ (see also [23]). In what follows, for simplicity we omit the hats. Set

$$\begin{aligned} n_i &= \dim x_i & i &= 1, \dots, m \\ n_0 &= \dim x_{m+1} \\ n_{wi} &= n_i + n_0 & i &= 1, \dots, m \\ n_w &= \sum_{i=1}^m n_{wi} \\ n^e &= n + n_w. \end{aligned}$$

Consider the extended system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ \dot{w} &= u_w \\ y &= h(x), \end{aligned} \quad (14)$$

with

$$w = \begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix}, \quad \dim w = n_w, \quad w_i = \begin{pmatrix} \lambda_i \\ \mu_i \end{pmatrix}, \quad \dim \lambda_i = n_i, \quad \dim \mu_i = n_0 \quad i = 1, \dots, m.$$

Set

$$x^e = \begin{pmatrix} x \\ w \end{pmatrix}, \quad u^e = \begin{pmatrix} u \\ u_w \end{pmatrix}, \quad x_0^e = \begin{pmatrix} x_0 \\ w_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

and

$$\begin{aligned} f^e(x^e) &= \begin{pmatrix} f(x) \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad g_i^e(x^e) = \begin{pmatrix} g_i(x) \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad g_{m+i}^e(x^e) = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ I_{n_{wi} \times n_{wi}} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad i = 1, \dots, m, \\ g^e(x^e) &= (g_1^e(x^e) \quad \dots \quad g_{2m}^e(x^e)) \quad \text{and} \quad h_i^e(x^e) = h_i(x_i) \quad i = 1, \dots, m. \end{aligned}$$

Let us now construct the following extended vector fields

$$X_{ik}^e(x^e) = \begin{pmatrix} X_{ik}(x) \\ 0 \\ \vdots \\ X_{ik}^*(x_i, \mu_i) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad i = 1, \dots, m,$$

where from (13) we define

$$X_{ik}^*(x_i, \mu_i) = \left(\begin{array}{c} Y_{ik}(x_i) \\ Z_{ik}(x) \end{array} \right) \Big|_{x_j=0 \text{ for } j \neq i; x_{m+1,1}=\mu_i}$$

and set

$$R_i^e = \text{span}\{X_{ik}^e : 1 \leq k \leq s_i\} \quad i = 1, \dots, m.$$

Since $\Delta_{MIX} = 0$, the distributions R_i^e , $i = 1, \dots, m$, are independent, involutive and have the same dimension as R_i^* in a neighborhood of x_0^e [23, lemma II.2].

It can be also shown [1, 23] that there exists a smooth state-feedback

$$u^e = \alpha^e(x^e) + \beta^e(x^e)v^e \quad (15)$$

such that

- a) $\alpha^e(0) = 0$ and $\beta^e(x^e)$ is nonsingular at x_0^e ;
- b) R_i^e is invariant under $\tilde{f}^e = f^e + g^e \alpha^e$ and $\tilde{g}_j^e = g^e \beta_j^e$ for $i = 1, \dots, 2m; j = 1, \dots, 2m$;
- c) $\tilde{g}_i^e \in R_i^e \quad i = 1, \dots, m$.

Moreover, it can be shown that for each $i = 1, \dots, m$ the two well-defined subsystems $(f|_{\mathcal{L}_0^{R_i}}, g_i|_{\mathcal{L}_0^{R_i}})$ and $(\tilde{f}^e|_{\mathcal{L}_0^{R_i^e}}, \tilde{g}_i|_{\mathcal{L}_0^{R_i^e}})$ are diffeomorphic. For, denoting by π the canonical projection onto the first n coordinates, we prove first that π maps $\mathcal{L}_0^{R_i^e}$ diffeomorphically onto $\mathcal{L}_0^{R_i^*}$. Fix $i = 1, \dots, m$. By Chow's Theorem [24], any point in $\mathcal{L}_0^{R_i^e}$ can be joined to the origin by a concatenation of integral curves of X_{ik}^e , which satisfy the differential equations,

$$\begin{aligned} \dot{x}_i &= Y_{ik}(x_i), \\ \dot{x}_j &= 0 \quad j \neq \{i, (m+1, 1), (m+1, 2)\}, \\ \dot{x}_{m+1,1} &= Z_{ik}(x_1, \dots, x_{m+1,1}), \\ \dot{\mu}_i &= Z_{ik}(0, \dots, x_i, 0, \dots, \mu_i), \\ \dot{\lambda}_i &= Y_{ik}(x_i). \end{aligned}$$

This means that on $\mathcal{L}_0^{R_i^e}$,

$$\begin{aligned} x_j &= 0 \quad j \neq \{i, (m+1, 1), (m+1, 2)\}, \\ x_i &= \lambda_i, \\ x_{m+1} &= \mu_i, \end{aligned}$$

and our claim follows at once.

Since $\pi_*(\tilde{f}^e) = f$ and $\pi_*(\tilde{g}_i^e) = g_i$, $i = 1, \dots, m$, it follows that for each $i = 1, \dots, m$ the two subsystems $(f|_{\mathcal{L}_0^{R_i}}, g_i|_{\mathcal{L}_0^{R_i}})$ and $(\tilde{f}^e|_{\mathcal{L}_0^{R_i^e}}, \tilde{g}_i^e|_{\mathcal{L}_0^{R_i^e}})$ are diffeomorphic.

The closed-loop system resulting from applying (15) to (14) can be expressed in local coordinates $\psi = (\psi_1^T \ \cdots \ \psi_{m+1}^T)^T$ by,

$$\begin{aligned}\dot{\psi}_i &= \tilde{f}_{\psi_i}(\psi_i, \psi_{m+1}) + \tilde{g}_{\psi_i, i}(\psi_i, \psi_{m+1})v_i + \tilde{g}_{\psi_i, m+1}(\psi_i, \psi_{m+1})v_{m+1}, \quad i = 1, \dots, m, \\ \dot{\psi}_{m+1} &= \tilde{f}_{\psi_{m+1}}(\psi_{m+1}) + \tilde{g}_{\psi_{m+1}, m+1}(\psi_{m+1})v_{m+1}, \\ y_i &= h_i(\psi_i, \psi_{m+1}), \quad i = 1, \dots, m,\end{aligned}\tag{16}$$

where $\dim v_i = 1$ and $\dim v_{m+1} = n_w$. It is worth noting that, if we set $v_{m+1} = 0$, (16) is noninteractive. We show now that (16) can be stabilized without destroying the above noninteraction property. First, we prove that the matrix $\tilde{g}_{\psi_{m+1}, m+1}(\psi_{m+1})$ has full rank. It is sufficient to prove this fact pointwise, since in this case the regularity assumptions ensure that it holds also in a neighborhood of x_0^e . For, note that $\tilde{g}_{\psi_{m+1}, m+1}$ has n_w columns and $n + n_w - \sum_{i=1}^m s_i$ rows. If we show that $\dim((\sum_{i=1}^m R_i^e) \cap G_w) = \sum_{i=1}^m s_i - n$, then the rows of $\tilde{g}_{\psi_{m+1}, m+1}$ are independent, since otherwise $\dim((\sum_{i=1}^m R_i^e) \cap G_w) > \sum_{i=1}^m s_i - n$, which is a contradiction. For, $(\sum_{i=1}^m R_i^e) \cap G_w$ is given by the kernel of the matrix,

$$X = (X_{1i} \ \cdots \ X_{1s_1} \ \cdots \ X_{m1} \ \cdots \ X_{ms_m}).$$

But $\dim(\text{span}\{X_{ik} : i = 1, \dots, m ; k = 1, \dots, s_i\}) = n$, since from lemma II.1.c) $\sum_{i=1}^m R_i^* = \text{span}\{\partial/\partial x\}$. Our claim follows from well-known results of linear algebra.

Since, as it has been shown above, the two systems $(f|_{\mathcal{L}_0^{R_i}}, g_i|_{\mathcal{L}_0^{R_i}})$ and $(\tilde{f}^e|_{\mathcal{L}_0^{R_i^e}}, \tilde{g}_i^e|_{\mathcal{L}_0^{R_i^e}})$ are diffeomorphic, from (D2) it follows that $(\tilde{f}^e|_{\mathcal{L}_0^{R_i^e}}, \tilde{g}_i^e|_{\mathcal{L}_0^{R_i^e}})$ can be stabilized by means of a suitable smooth dynamic feedback,

$$\begin{aligned}v_i &= F_{i1}(\psi_i, \zeta_i) + \bar{v}_i \\ \dot{\zeta}_i &= F_{2i}(\psi_i, \zeta_i),\end{aligned}\tag{17}$$

with $F_{i1}(0, 0) = 0$ and $F_{i2}(0, 0) = 0$. It is worth noting that each feedback law (17) possibly enlarges the dimension of the dynamic extension. Since $\tilde{g}_{\psi_{m+1}, m+1}(\psi_{m+1})$ has full rank, in order to stabilize the ψ_{m+1} dynamics, we can apply the feedback

$$v_{m+1} = g_{\psi_{m+1}, m+1}^S(\psi_{m+1})[-f_{\psi_{m+1}}(\psi_{m+1}) - \psi_{m+1}],\tag{18}$$

where $g_{\psi_{m+1}, m+1}^S(\psi_{m+1})$ is a suitable permutation matrix. Our claim follows from the noninteracting structure of the stabilizing feedback (17)–(18) and from standard stability results on triangular systems [20, Appendix B].

Using again standard stability results on triangular systems [20, Appendix B], assumption (D1) and the fact that the $x_{m+1,2}$ dynamics does not influence the outputs imply that the closed-loop system, resulting from applying to (9) the composite feedback given by (15), (17) and (18), is asymptotically stable and noninteractive. •

Remark 3. It is worth noting that the composite feedback given by (15), (17) and (18) is a regular dynamic feedback, since Σ and $\tilde{\Sigma}^e$ have the same relative degree respectively at x_0 and x_0^e (see [23]).

Remark 4. It is important to note that **(S1)** and **(S2)** imply **(D1)** and **(D2)**. First, (10), (11) and **(S1)** imply **(D1)** since $\Delta_{MIX} \subset R^*$. Moreover, (10), (11) and **(S2)** imply that,

$$z_1 = f_{z_1}(z_1, 0) + g_{z_1, i}(z_1, 0)u_i,$$

must be stabilizable via static state-feedback. On the other hand, **(S1)** implies that,

$$\dot{z}_3 = f_{z_3}(0, 0, z_3, 0),$$

is asymptotically stable. It follows that $\Sigma_i^{\Delta_{MIX}}$ is asymptotically stabilizable via static state-feedback, which implies **(D2)**.

IV. Conclusions

In this paper we have given a complete set of necessary and sufficient conditions to solve the problem of local noninteracting control with stability for a class of nonlinear systems. Besides some geometric obstructions recently pointed out in the literature (see **(S1)** and **(D1)**), the stabilizability of certain subsystems has been shown to be crucial to achieve noninteraction and stability. It is now clear that the R^* and Δ_{MIX} play analogous roles in this problem. In particular, in Remark 3 it is shown how **(D1)** and **(D2)** weaken **(S1)** and **(S2)**. For linear systems, while **(S2)** is trivially satisfied since Σ is controllable (assumption **(H1)**), **(S1)** still survives [1] ; this obstruction can be always overcome by means of dynamic feedback, since **(D1)** and **(D2)** are always satisfied [3]. Note that in a nonlinear setting **(D2)** is not satisfied in general, even if **(H1)** holds.

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