

**Optimized Redundancy Allocation
For Electronic Equipment**

by

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OPTIMIZED REDUNDANCY ALLOCATION FOR ELECTRONIC EQUIPMENT

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Abstract - This paper describes a methodology for optimizing the temperature dependent reliability of electronic equipment using active redundancy, while satisfying an area constraint. The process consists of transforming the objective unreliability function by adding to it a suitable penalty term so as to force the optimum solution to an "almost integer" value, and minimizing the transformed objective function by a sequential quadratic programming technique. A sensitivity analysis, which avoids "perturb-and-reoptimize" methods, is then conducted to show how an incremental change in an input parameter affects the optimum solution.

Keywords - Redundancy allocation, system reliability, nonlinear programming, sensitivity analysis

1. INTRODUCTION

Even with the availability of electronic components with low failure rates, achieving high reliability in electronic equipment is still an important issue. This is due in part to the complexities of modern electronics. As a consequence, techniques must be utilized for reliability improvement.

The reliability of electronic equipment can be increased through improved design, improved production quality, use of screened components, through a decreased time of continuous operation which can be prescribed at the planning stage, and by adding redundancies. Implementing redundancy in the design of electronic equipment involves designing for alternate signal paths to connect the redundant components while conforming to acceptable power requirements and the available space for components. Thus, it is necessary to trade off the "cost" of the improvement in reliability with the "cost" of the constraints. Such a trade-off can be achieved through an optimization procedure, whereby the system unreliability is minimized by adding redundant components which are constrained by the available space.

The increase in stage reliability which can be gained by adding redundant components is a function of the individual component reliabilities and the number of redundant components per stage, as shown in Figure 1. It is apparent that if the component reliability is high, the rate of improvement in stage reliability decreases as the components are added. Moreover, each redundant component adds to the heat which must be dissipated by the system. This in turn will increase the temperature and thus the failure rate of each component on the board. Figure 2 shows the relationship between the stage reliability and the number of redundant components for a microelectronic com-

ponent on a printed wiring board. The decrease in reliability, which occurs when the optimal redundancy level is exceeded, is due to the increased failure rate resulting from the temperature increase. Hence, the optimum number of redundant components at a particular stage must satisfy the constraint of allowable area and should not deteriorate the stage reliability.

A great deal of progress has been made in the area of the optimized redundancy allocation. An excellent review of the literature in this field was given by Tillman et al.¹ up to 1977. In general, various heuristic and/or nonlinear programming methods have been developed to improve the system reliability. The heuristic methods though sometimes "conceptually and computationally simple"² can not guarantee the optimality of the final solution. On the other hand, the nonlinear programming methods including integer and dynamic programming methods,³⁻⁶ etc., though guarantee an optimum solution, may require excessive computational time.

In this paper, a methodology for optimizing the number of redundant components for electronic equipment is presented. The optimization procedure is based on a sequential quadratic programming formulated by Powell^{7,8}. An area constraint which includes the extra area required for interconnections is taken into account through the application of Tillman et al.'s⁹ formulation. In addition, the reliability of the individual components is taken to be a function of the failure rate, which in turn is a function of component temperature. In order to generate (almost) integer rather than real-valued solutions, the objective function is modified by appending to it a "penalty" term. The optimization procedure is then extended to calculate the sensitivity of the design variables, i.e., the changes in the number of redundant

components at different stages with respect to incremental changes in the parameters. This sensitivity or post-optimality analysis is different from the parametric programming procedure suggested by Chern and Jan¹⁰. Furthermore, it guides the designer towards a desirable reliability improvement.

2. PROBLEM FORMULATION

In the formulation of the problem, the following assumptions are made:

1. The system consists of a series of k stages, where each stage i ($i=1,2,\dots,k$) is a parallel combination of m_i components such that $m_i > 1$.
2. All the components in the stage are active. Thus, for a stage to fail, all the elements in that stage must fail.
3. Component failures can be represented as random events, such that component failure rates are assumed to be constant and exponentially distributed.
4. All the redundant components at a given stage are identical and hence, equally reliable.
5. Components are not repaired or replaced during their operation.

For active redundancy with m_i-1 redundant components, all the m_i components operate simultaneously. The system reliability, R_S , for a serial system with k stages is

$$R_S = \prod_{i=1}^k (1 - q_i^{m_i}) \quad (2.1)$$

where q_i is the i th stage component unreliability given by

$$q_i = 1 - e^{-\lambda_i t} \quad (2.2)$$

λ_i is the failure rate of i th stage components, t is the allocated mission time, λ_i is an exponential (Arrhenius type) function of temperature.

$$\lambda_i = K_i + M_i e^{-\left(\frac{N_i}{T_i}\right)} \quad (2.3)$$

where K_i , M_i , and N_i are constants, and T_i is the internal component temperature (i.e. junction temperature), expressed as

$$T_i = T_S + P_i \theta_i \quad (2.4)$$

where T_S is the system temperature, and P_i and θ_i are the heat dissipation and thermal resistance of the i th component, respectively. The system temperature, T_S , is a function of the type of heat transfer.

$$T_S = T_{ref} + \frac{P_{total}}{K_{cooling}} \quad (2.5)$$

where P_{total} is the total power dissipated, $K_{cooling}$ is a cooling parameter dependent on the type of heat transfer and T_{ref} is a reference or ambient temperature. For forced convection cooling, T_S is dependent on the air temperature. In such a case, T_S could be taken as the air temperature at the outlet of the system in order to profile to a worst case approximation.

The system unreliability can be determined from the relationship

$$Q_S = 1 - R_S \quad (2.6)$$

Substituting equation (2.1) into (2.6) gives

$$Q_S = 1 - \prod_{i=1}^k (1 - q_i^{m_i}) \quad (2.7)$$

If the individual component unreliability is sufficiently small ($q_i < 1$) such that

$$q_i^{m_i} \ll 1 \quad (2.8)$$

then, the system unreliability, Q_S , in equation (2.7) can be approximated by

$$Q_S \approx \sum_{i=1}^k q_i^{m_i} \quad (2.9)$$

To show the validity of this approximation, both the exact, equation (2.7), and the approximate, equation (2.9), have been considered in the analysis. The minimization of unreliability is preferred to the maximization of reliability, since it is more convenient to compute equation (2.9) compared to equation (2.1). Thus, the redundancy allocation problem reduces to the selection of redundant components at each of the k stages so as to minimize system unreliability.

In this model, the available or free space has been considered as a constraint. Besides the area occupied by the physical dimensions of the redundant components, an extra area is required to layout the redundant interconnections while maintaining the routing track tolerances. A model which describes the increased area due to interconnections has been formulated by Tillman et al.⁹ and is of the form

$$g(m): \sum_{i=1}^k A_i m_i e^{m_i \beta} - A_{av} \leq 0 \quad (2.10)$$

where A_i is the area of an individual component at the i th stage, $e^{m_i \beta}$ is due

to the area of interconnections and the space required for their layout, and A_{av} is the allowable area of the system. The interconnections area factor (β) for a given component is determined by the designer depending on the geometrical tolerances (distance) between interconnections. A closer distance between components will require a lower value of β .

3. OPTIMIZATION PROCEDURE

In the optimization procedure, the objective function of system unreliability and the area constraint are considered to have the form

$$\begin{aligned} &\text{minimize } F(m) \\ &\text{subject to} \\ &g(m) \leq 0 \end{aligned} \tag{3.1}$$

where m represents the real-valued (almost integer) vector of redundant components at stage i , m_i s, with i ranging from 1 to k , F is the objective function given by equation (2.9), and g is the area constraint given by the equation (2.10). It is assumed that F and g are continuous and differentiable. To solve equation (3.1), a sequential quadratic programming (SQP) procedure^{7,8} was used. The algorithm for the SQP defines a Quadratic Programming Subproblem (QPS) in each iteration in order to minimize a quadratic approximation of the Lagrangian subject to the linearized constraint. The solution to the QPS estimates the Lagrange multiplier and the direction of search used in a subsequent one-dimensional search. The one-dimensional search decreases the objective function and the constraint infeasibility.

Since the SQP method results in solutions of real values, an integer solution (an integer number of components) must be developed. Conventionally, branch and bound methods such as those proposed by Fletcher¹¹ and Gupta and Ravindran¹² have been used to obtain integer solutions. Such methods usually require the investigation of all the integer solutions which exist around the continuous solution. However, these methods are extremely time consuming when the system consists of a large number of stages.

In this paper, the objective function is modified using a penalty term and the transformed objective is then minimized by a SQP method to generate an almost integer solution. The modified objective function for the problem is

$$\text{Minimize } F(m) + r_k Q_k(m) \quad (3.2)$$

where r_k is a weighting factor or the penalty parameter. The function $Q_k(m)$ is constructed such that a penalty is suffered when the variables take values other than integers. Gisvold and Moe¹³ suggested a candidate function, $Q_k(m)$, of the form

$$Q_k(m) = \sum_{i=1}^k \left\{ 4 \left(\frac{m_i - y_i}{z_i - y_i} \right) \left(1 - \frac{m_i - y_i}{z_i - y_i} \right) \right\}^{\beta_k} \quad (3.3)$$

where

$$y_i \leq m_i, \quad i=1,2,\dots,k \quad (3.4)$$

$$z_i > m_i \quad i=1,2,\dots,k \quad (3.5)$$

and $\beta_k > 1$ is a constant. Here, y_i and z_i are the two neighboring upper and lower integer values for m_i . The function $Q_k(m)$ is a normalized and symmetric

function. The variation of each of the terms under the summation sign in equation (3.3) for different values of β_k is shown in Figure 3. Continuous values of m_j placed in the middle of two neighboring integer points are penalized severely, while values symmetrically placed between the middle and y_j or z_j are penalized depending on their distance from y_j and z_j . For example, a continuous value of 2.5 will be penalized more since it may result in either 2 or 3, whereas real values of 2.2 and 2.8 will be penalized equally to reach 2 and 3, respectively. As shown in Figure 3, the value of β_k must be greater than or equal to unity in order to make the function $Q_k(m)$ continuous in its first derivative within the range of integer points. The use of the penalty term makes it possible to change the shape of the transformed objective function by changing β_k , while the amplitude is controlled by the weighting factor r_k .

4. SENSITIVITY ANALYSIS

A sensitivity or post-optimality analysis is performed to determine how small changes in the parameters affects the optimum solution. The purpose is to aid the designer in conducting a trade-off study and see whether the reliability can be increased by changing some design specifications.

Sensitivity information with regard to a particular input parameter is derived from the Karush-Kuhn-Tucker (KKT) conditions. Assuming that the solution \mathbf{m}^* to the optimization problem is known and based on the assumption that the objective function $F(\mathbf{m})$, the penalty term $Q_k(\mathbf{m})$ and the constraint $g(\mathbf{m})$ are continuous and differentiable in the neighborhood of the solution \mathbf{m}^* , the KKT optimality condition will require

$$\nabla (F(m) + r_k Q_k(m)) + u \nabla g(m) = 0 \quad (4.1)$$

and

$$u g(m) = 0 \quad (4.2)$$

where m and u are the vector of redundant components and Langrange multiplier ($u \geq 0$) at the optimum, respectively.

Assuming that after a small change in the input parameters, the KKT optimality conditions still remain valid. If equation (4.1) is differentiated with respect to a specified parameter, p , the resulting derivative must be equal to zero. It is to be noted that equation (4.1) actually contains k separate equations of the form

$$\frac{\partial [F(m) + r_k Q_k(m)]}{\partial m_i} + u \frac{\partial g(m)}{\partial m_i} = 0, \quad i=1,2,\dots,k \quad (4.3)$$

Differentiating equation (4.3) with respect to the parameter p , gives

$$\begin{aligned} \sum_{i=1}^k \left\{ \frac{\partial^2 [F(m) + r_k Q_k(m)]}{\partial m_i \partial m_k} + u \frac{\partial^2 g(m)}{\partial m_i \partial m_k} \right\} \frac{\partial m_k}{\partial p} + \frac{\partial g(m)}{\partial m_i} \frac{\partial u}{\partial p} \\ + \frac{\partial^2 [F(m) + r_k Q_k(m)]}{\partial m_i \partial p} + u \frac{\partial^2 g(m)}{\partial m_i \partial p} = 0, \quad i = 1,2,\dots,k \end{aligned} \quad (4.4)$$

Also, differentiating equation (4.2) with respect to p gives

$$u \left\{ \sum_{i=1}^k \left[\frac{\partial g(m)}{\partial m_i} \frac{\partial m_i}{\partial p} \right] + \frac{\partial g(m)}{\partial p} \right\} + g \frac{\partial u}{\partial p} = 0 \quad (4.5)$$

To determine the values of the terms $\partial m_i / \partial p$ and $\partial u / \partial p$, equation

(4.4) and equation (4.5) can be expressed in the matrix form

$$\begin{bmatrix} \left(\frac{\partial^2 L}{\partial m_i^2} \right)_{k \times k} & \left(\frac{\partial g}{\partial m_i} \right)_{k \times 1} \\ u \left(\frac{\partial g}{\partial m_i} \right)^T_{1 \times k} & (g)_{1 \times 1} \end{bmatrix} \begin{bmatrix} \delta m \\ \delta u \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial^2 L}{\partial m_i \partial p} \right)_{k \times 1} \\ \left(u \frac{\partial g}{\partial p} \right)_{1 \times 1} \end{bmatrix} = 0 \quad (4.6)$$

where the Langrangian, L, is

$$L = F(m) + r_k Q_k(m) + u g(m) \quad (4.7)$$

and

$$\delta m = \begin{bmatrix} \partial m_1 / \partial p \\ \partial m_2 / \partial p \\ \vdots \\ \partial m_k / \partial p \end{bmatrix} \quad \text{and} \quad \delta u = \partial u / \partial p \quad (4.8)$$

Equation (4.6) may be expressed in short as

$$M^* \delta + N^* = 0 \quad (4.9)$$

where M^* , δ and N^* are the contributing matrices of equation (4.6). Matrix M^* given by equation (4.9) remains unchanged for all the optimization parameters. Matrix N^* can be evaluated using equation (4.6) and it would differ for each of the input parameters.

Substituiting the objective function F(m) from equation (2.9) and g(m) from equation (2.10) into equation (4.7), the Lagrangian, L, is formulated as

$$L = \sum_{i=1}^k q_i^{m_i} + r_k Q_k(m) + u \left[\sum_{i=1}^k A_i m_i e^{(m_i \beta)} - A_{av} \right] \quad (4.10)$$

where $Q_k(m)$ is given by equation (3.3). Differentiating the Langrangian with respect to m_i , gives

$$\begin{aligned} \frac{\partial L}{\partial m_i} &= q_i^{m_i} \ln(q_i) + 4r_k \beta_k (Y_i - Y_i^2)^{\beta_k - 1} (1 - 2Y_i) \\ &\quad + u A_i (1 + m_i \beta) e^{(m_i \beta)} \end{aligned} \quad (4.11)$$

and

$$\frac{\partial^2 L}{\partial m_i^2} = H_i \quad (4.12)$$

where

$$\begin{aligned} H_i &= q_i^{m_i} \{ \ln(q_i) \}^2 + 4r_k \beta_k (\beta_k - 1) (Y_i - Y_i^2)^{\beta_k - 2} (1 - 2Y_i)^2 \\ &\quad + u A_i \beta e^{(m_i \beta)} (2 + m_i \beta), \quad i=1, 2, \dots, k \end{aligned} \quad (4.13)$$

and

$$Y_i = \frac{m_i - y_i}{z_i - y_i} \quad (4.14)$$

likewise, considering the area constraint

$$g = \sum_{i=1}^k A_i m_i e^{(m_i \beta)} - A_{av} \quad (4.15)$$

and differentiating with respect to m_i , gives

$$Q_i = \frac{\partial g}{\partial m_i} = A_i (1 + m_i \beta) e^{(m_i \beta)} \quad (4.16)$$

For example, selecting the available area, A_{av} , as the parameter p , equations (4.11) and (4.15) can be differentiated with respect to A_{av} to give

$$\frac{\partial^2 L}{\partial m_i \partial p} = 0 \quad (4.18)$$

$$\frac{\partial g}{\partial p} = -1 \quad (4.19)$$

respectively.

With M^* and N^* known, the matrix equation (4.6) can be solved to give

$$H_i (\partial m_i / \partial p) + Q_i (\partial u / \partial p) = 0, \quad i=1,2,\dots,k \quad (4.20)$$

and

$$u \sum_{i=1}^k Q_i (\partial m_i / \partial p) + g (\partial u / \partial p) = u \quad (4.21)$$

The requirement of an almost integer solution can make the Lagrange multiplier zero and the constraint inactive. As a result, the stage variables may become insensitive to the change in the parameter, as can be seen from equations (4.20) and (4.21) if we select p to be A_{av} . In order to be able to determine the sensitivity factors for this condition, the used area, A_{used} , is taken as the right-hand term of equation (4.15) instead of the available area, A_{av} . This makes g always active ($g = 0, u \neq 0$). Substituting $g = 0$ in the equation (4.21) and cancelling u from both sides, gives

$$\sum_{i=1}^k Q_i (\partial m_i / \partial A_{used}) = 1 \quad (4.22)$$

From equation (4.20), $\partial m_i / \partial A_{used}$ can be expressed as

$$\partial m_i / \partial A_{used} = - \frac{Q_i (\partial u / \partial A_{used})}{H_i}, \quad i=1,2,\dots,k \quad (4.23)$$

Solving the two equations (4.22) and (4.23) gives

$$\partial u / \partial A_{used} = - \frac{1}{M} \quad (4.24)$$

where

$$M = \sum_{i=1}^k \frac{Q_i^2}{H_i} \quad (4.25)$$

$$\partial m_i / \partial A_{used} = - \frac{Q_i}{H_i M}, \quad i=1,2,\dots,k \quad (4.26)$$

Since equation (4.26) is a function of the i th stage parameters only, the analysis can be extended to systems with any number of stages.

Likewise, taking the area factor, β , as the parameter and following the same procedure as before will result in the sensitivity factor as

$$\partial m_i / \partial \beta = - \frac{u F_i}{H_i} + \frac{Q_i}{H_i M} \left(\sum_{i=1}^k \frac{Q_i F_i}{H_i} - T \right) \quad (4.27)$$

where

$$F_i = A_i e^{m_i \beta} (2 + m_i \beta) \quad (4.28)$$

and

$$T = \sum_{i=1}^k A_i m_i^2 e^{m_i \beta} \quad (4.29)$$

5. COMPUTATIONAL EXAMPLES

To demonstrate the methodology, two examples are given. The first example has five stages and the second example has ten stages. Each stage represents a microelectronic component, whose attributes were obtained from MIL-HDBK-217E¹⁴. The components are mounted on a printed wiring board which is cooled by forced convection. The worst case board temperature T_s , in equation (2.4), is calculated from the outlet air temperature based on the equation

$$P_{total} = hA(T_s - T_{outlet}) \quad (5.1)$$

where

$$T_{outlet} = T_{inlet} + \frac{P_{total}}{\dot{m}C_p} \quad (5.2)$$

T_{inlet} is the inlet air temperature, P_{total} is the total power dissipated, \dot{m} is the mass flow rate, C_p is the specific heat at constant pressure, h is the convective heat transfer coefficient, A is the area of the board and T_{outlet} is the outlet temperature. The failure rate of the individual components, λ_i , is calculated using equation (2.3).

The specifications of the heat dissipated, area occupied, the thermal resistances of each component, the available area A_{av} , area parameter β , $\dot{m}C_p$ value, and the convective heat transfer coefficient are given in Tables 1 and 2. A mission time of 10,000 hours was allocated for the board. Tables 1 and 2 also present the solutions in terms of the number of redundant components for each stage, and the sensitivity information. Sensitivity factors given in

the tables should serve as a tool to predict how an incremental change in a particular parameter will affect the optimum solution. The iteration history of the optimum solution, both for the exact and approximated objective is illustrated in Figures 4 and 5. To simplify the plotting procedure, $-\log(1-R_S)$ is taken as the ordinate instead of R_S .

Figures 4 and 5 also show that the solution initially converges smoothly and then oscillates before converging to the final solution. The oscillation is the result of the value of the penalty parameter used in the optimization procedure. Initially, the penalty parameter is kept at a low value, but it is gradually increased. This results in fluctuations of the objective values before the final solution is obtained.

6. ADDITIONAL COMMENTS

The optimization procedure presented in this paper provides practical method of allocating active redundancies for heat dissipating electronic components whose failure rates are temperature dependent. The procedure is especially useful for space-based systems which employ nonrepairable elements. Although the presented examples pertained to the allocation of components on a printed wiring board (PWB), the analysis can also be applied to the allocation of PWBs within an assembly, or to electronic modules in an enclosed system.

In the optimization procedure developed here, an area constraint limits the number of components whose physical and interconnection areas are specified. The extension to a volume constraint requires modifying equation (2.10). Finally, this paper assumes a simplified convection cooling scheme for the determination of the component temperature. By an appropriate modifi-

cation of equations (5.1) and (5.2), the optimization procedure can be extended to conduction or other cooling schemes.

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TABLE 1

Component specifications					Results		
Stage Number	Part-Name [14]	Area (in ²)	Heat Dissipated (watts)	Thermal Resistance (°C/W)	Number of components (m_i)	Sensitivity Factor with respect to Used Area ($\partial m_i / \partial A$)	Sensitivity Factor with respect to Area factor ($\partial m_i / \partial \beta$)
1	STTL 20801-X	0.5	1.2	100.0	3 (3.09)	0.077	- 3.925
2	STTL 01801-F	0.4	0.8	125.0	3 (3.02)	0.117	- 5.939
3	TTL 00904-J	1.0	1.2	70.0	2 (2.08)	0.493	- 2.512
4	TTL 02306-A	0.5	1.0	90.0	2 (2.11)	0.022	- 1.117
5	HTTL 02305-E	0.7	1.2	70.0	2 (2.09)	0.035	- 1.767

Input parameters :

Available area (in²) = 20.00 β = 0.4

Mission time (hours) = 10,000.00

M_i [14] = 0.1 N_i [14] = 4,635.0

Cooling parameters :

Air inlet temperature (°C) = 20.0 Convective heat transfer
Coefficient h (W/m² °C) = 60.0

Mass flow rate (lbs/min) = 0.5

Output :

Series system reliability = 0.99531677

Redundant system reliability = 0.99999839

TABLE 2

Component specifications					Results		
Stage Number (i)	Part-Name [14]	Area (in ²)	Heat Dissipated (watts)	Thermal Resistance (°C/W)	Number of components (m _i)	Sensitivity Factor with respect to Used Area ($\partial m_i / \partial A$)	Sensitivity Factor with respect to Area factor ($\partial m_i / \partial \beta$)
1	STTL 50501-L	0.5	0.49	100.0	2 (2.03)	0.013	- 1.076
2	TTL 01801-F	0.4	0.77	68.0	2 (2.04)	0.009	- 0.760
3	HTTL 02305-D	1.0	0.24	125.0	2 (2.01)	0.059	- 4.764
4	TTL 02306-A	0.5	0.80	70.0	2 (2.04)	0.010	- 0.835
5	STTL 20904-J	0.7	0.50	125.0	2 (2.05)	0.011	- 0.942
6	TTL 01801-J	0.4	0.77	100.0	3 (3.01)	0.053	- 4.219
7	TTL 01701-F	0.5	0.44	70.0	2 (2.01)	0.029	- 2.301
8	HTTL 02305-D	0.4	1.20	70.0	3 (3.02)	0.037	- 2.977
9	TTL 00101-D	0.5	0.04	90.0	2 (1.99)	0.113	- 8.944
10	STTL 20801-C	0.6	1.00	100.0	3 (3.06)	0.280	- 2.312

Input parameters :

Available area (in²) = 33.00 β = 0.4

Mission time (hours) = 10,000.00

M_i [14] = 0.1 N_i [15] = 4635.0

Cooling parameters :

Air inlet temperature (°C) = 20.0 Convective heat transfer
Coefficient h (W/m² °C) = 60.0

Mass flow rate (lbs/min) = 0.5

Output :

Series system reliability = 0.99828477

Redundant system reliability = 0.99999982

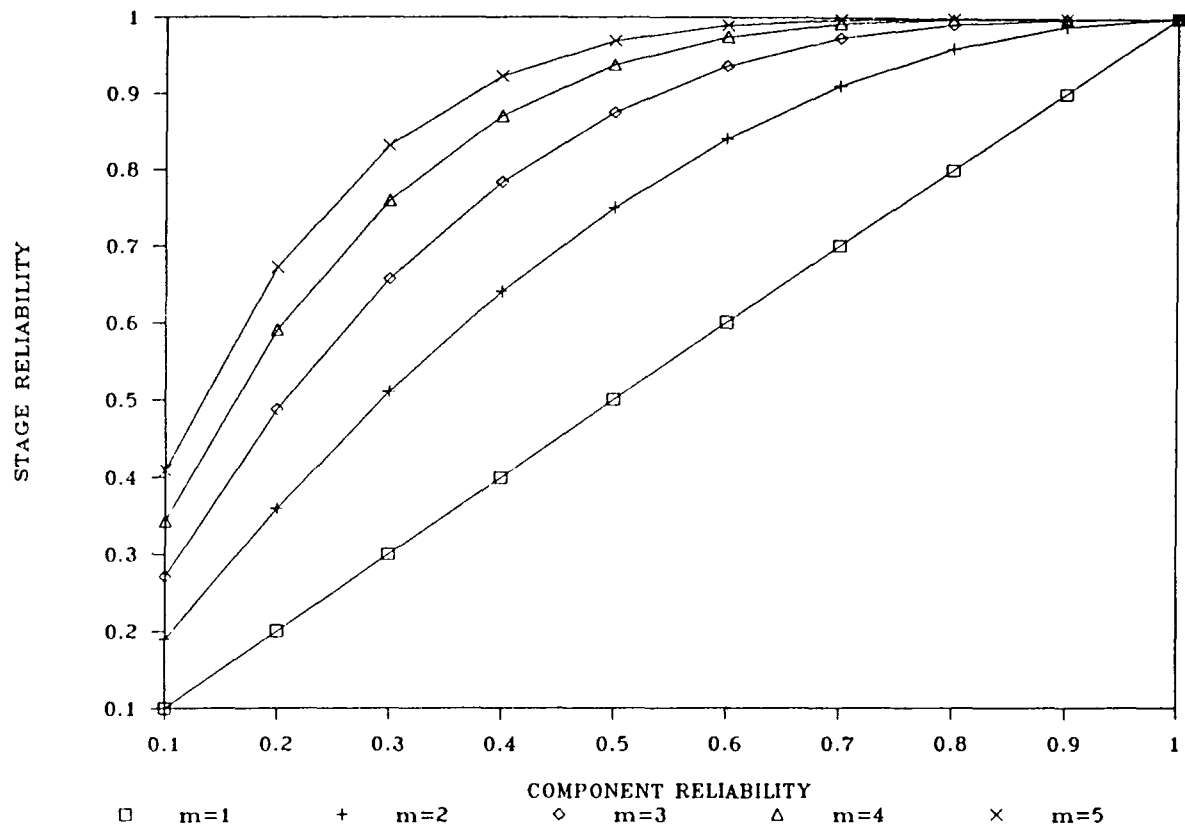


Figure (1) Graph showing the relationship between stage reliability and component reliability. As the component reliability increases, the rate of the improvement in stage reliability decreases.

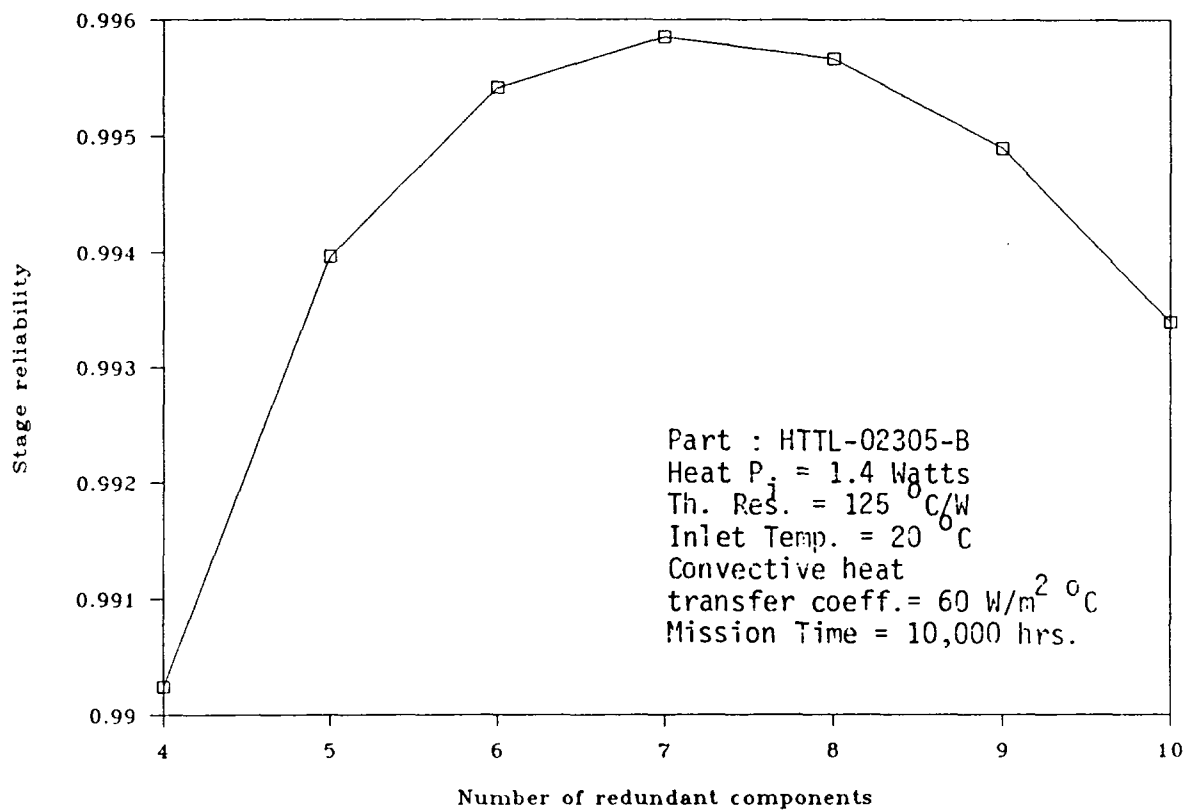


Figure (2) Graph illustrating stage reliability as a function of the number of redundant components when the thermal aspect of design is considered. As the number of redundant components increases, the failure rate of the components increase, consequently, decreasing the stage reliability.

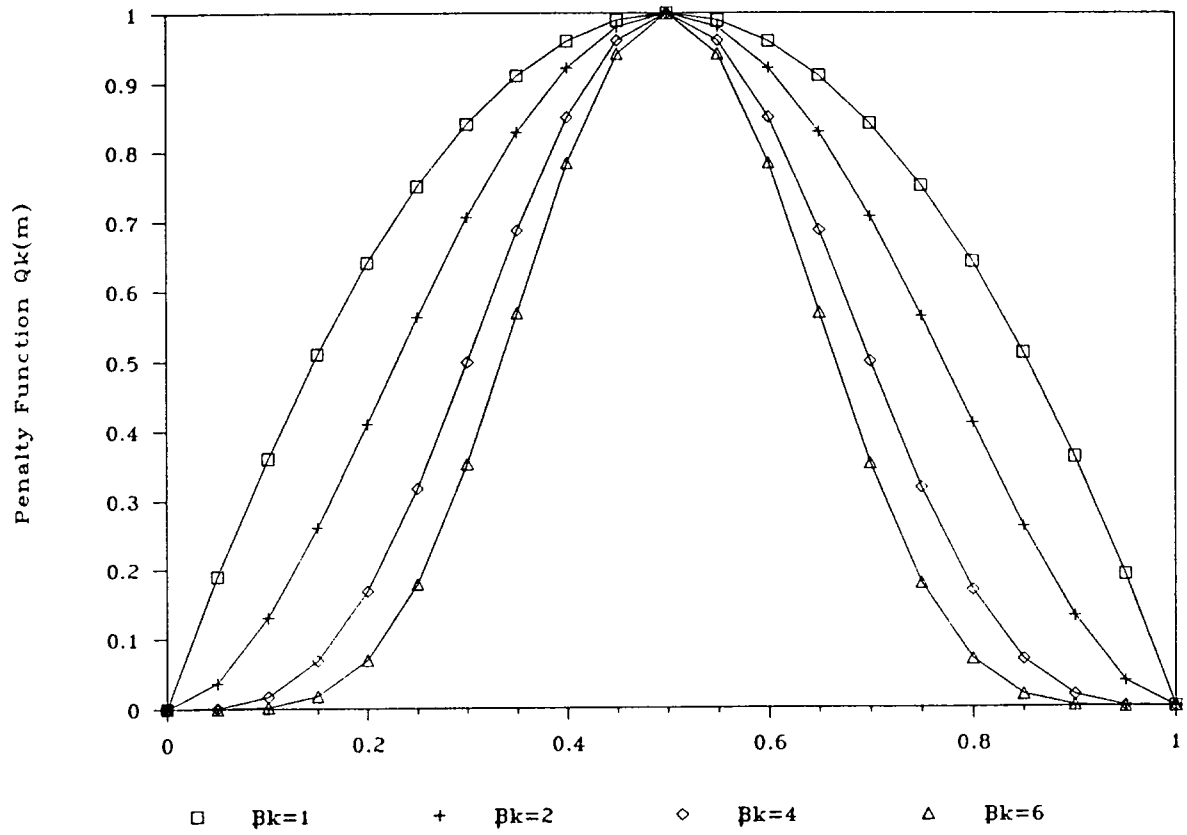


Figure (3) Graph showing the penalty function $Q_k(m)$ as a function of $\frac{m_i - y_i}{z_i - y_i}$ and parameter β_k . Continuous values symmetrically located around 0.5 are penalized equally. Increasing β_k introduces discontinuity in the penalty function [13].

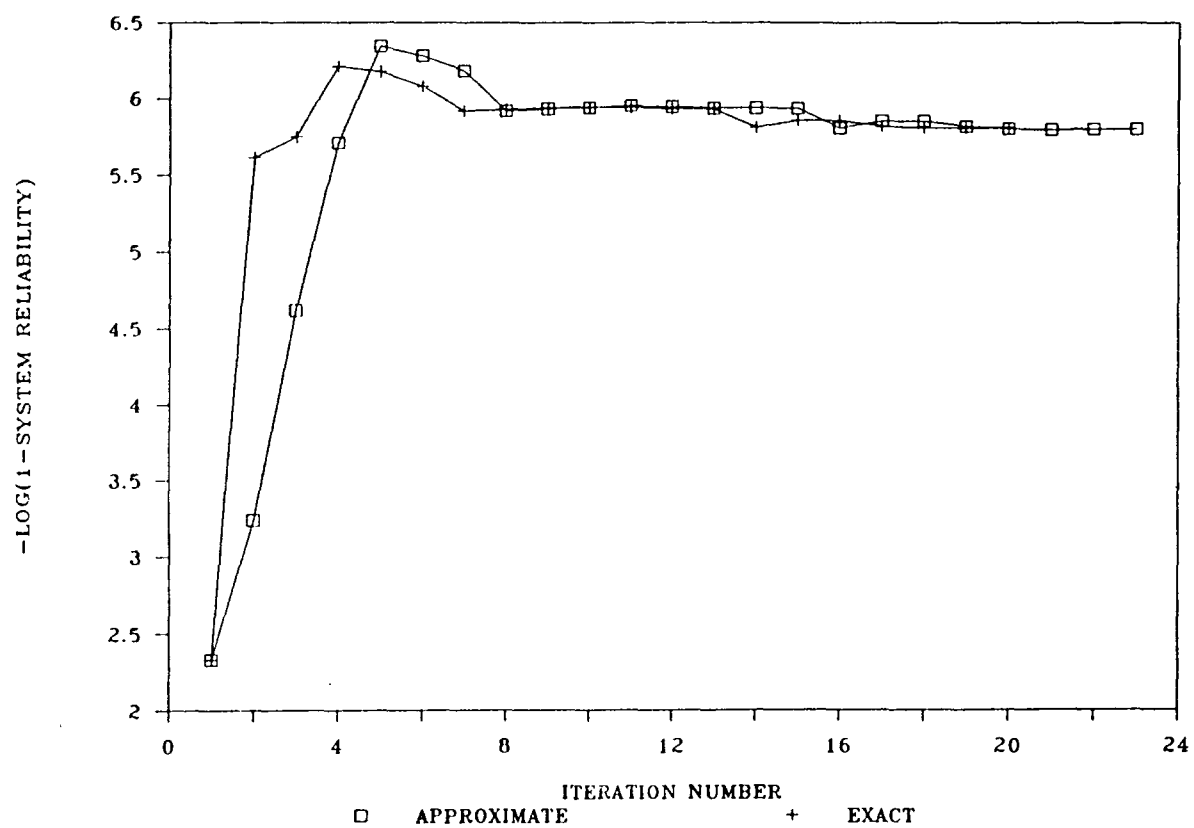


Figure (4) Iteration history of Example 1 for the approximate
 objective, $\sum_{i=1}^5 q_i^m i$, and the exact objective, $1 - \prod_{i=1}^5 (1 - q_i^m i)$, functions.

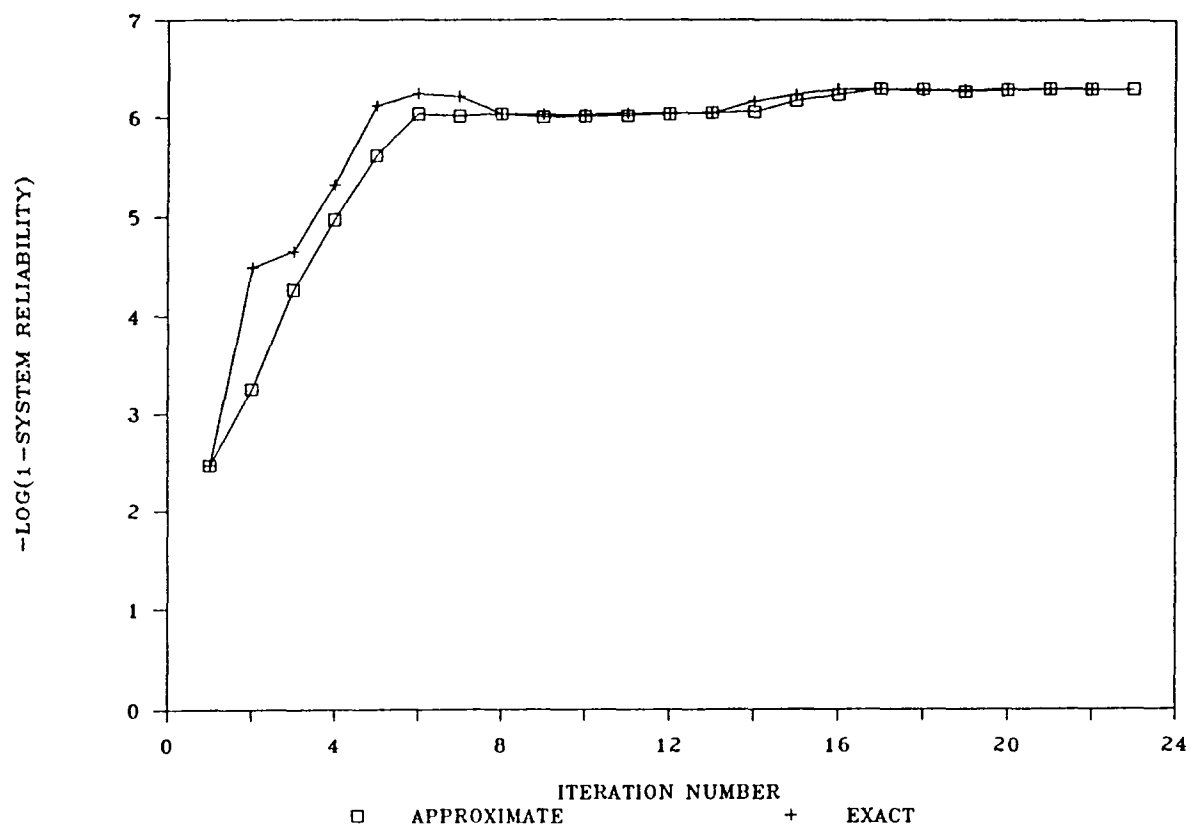


Figure (5) Iteration history of Example 2 for the approximate and the exact objective functions.