ABSTRACT

Title of Dissertation: COORDINATING DEMAND FULFILLMENT WITH SUPPLY ACROSS A DYNAMIC SUPPLY CHAIN

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Today, technology enables companies to extend their reach in managing the supply chain and operating it in a coordinated fashion from raw materials to end consumers. Order promising and order fulfillment have become key supply chain capabilities which help companies win repeat business by promising orders competitively and reliably. In this dissertation, we study two issues related to moving a company from an Available to Promise (ATP) philosophy to a Profitable to Promise (PTP) philosophy: pseudo order promising and coordinating demand fulfillment with supply.

To address the first issue, a single time period analytical ATP model for *n* confirmed customer orders and *m* pseudo orders is presented by considering both material constraints and production capacity constraints. At the outset, some analytical properties of the optimal policies are derived and then a particular customer promising scheme that depends on the ratio between customer service level and profit changes is presented. To tackle the second issue, we create a mathematical programming model and explore two cases: a deterministic demand curve or stochastic demand. A simple, yet generic optimal solution structure is derived and a

series of numerical studies and sensitivity analyses are carried out to investigate the impact of different factors on profit and fulfilled demand quantity. Further, the firm's optimal response to a one-time-period discount offered by the supplier of a key component is studied. Unlike most models of this type in the literature, which define variables in terms of single arc flows, we employ path variables to directly identify and manipulate profitable and non-profitable products. Numerical experiments based on Toshiba's global notebook supply chain are conducted. In addition, we present an analytical model to explore balanced supply. Implementation of these policies can reduce response time and improve demand fulfillment; further, the structure of the policies and our related analysis can give managers broad insight into this general decision-making environment.

COORDINATING DEMAND FULFILLMENT WITH SUPPLY ACROSS A GLOBAL SUPPLY CHAIN

by

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Dedicated to

Yanmei Zhao, my lovely wife

Alina M. Chen, my daughter

Bangying Chen and Yunju Yan, my parents

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Chapter 1

Introduction

Today's technology enables companies to extend their reach in managing the supply chain and operating it in a coordinated fashion from purchasing raw material to fulfilling end consumers' demands. Traditional cost and profit based supply chain strategies are no longer sufficient in the present competitive business environment. Leading companies are creating synchronized supply chains that are driven by market needs and, in essence, are moving the supply chain closer to the customer. As a result, demand fulfillment capabilities have become the key to the competitive strategies of many companies. Available to Promise (ATP) directly links customer orders with enterprise resources to achieve supply chain optimization. ATP had its origins in the late 1980's with Manufacturing Resource Planning (MRP II). Traditionally, the ATP function provides a response to customer order requests based on resource availability by checking the uncommitted portion of a company's inventory and planned production, maintained in the master schedule to support customer order promising (Ball, Chen, 2002). Supply Chain Management (SCM) introduces processes and systems to generate an ATP that is feasible and optimal with respect to resource constraints (Ervolina, 2001). Since this ATP strategy has the ability to optimize resource utilization through complicated material and process constraints, it is also referred as "advanced" ATP (Chen and Ball, 2000). Due to the complexity of ATP, only a very limited number of papers present quantitative models to support ATP

(Ball and Chen, 2001). One objective of this dissertation is to introduce the analytical model to deduce the generic rules that can provide managers with useful insights into the optimal policies for improving demand fulfillment.

1.1 Research Motivation

Our research is motivated both by business needs and gaps prior ATP research.

1.1.1 Business Driving Forces

Global competition and widely adopted e-commerce business models have imposed tremendous pressure on product and service providers to get closer to their customers. At the same time end consumers are increasingly knowledgeable and demanding. Supply chains are confronting the essential challenges in the current customer-centric business environment: real-time responsiveness, uncertain customer orders, globally dispersed locations and diminishing profit margin. As the front-end of a supply chain, order management must treat these challenges as the diving forces to gain the advantage.

Detailed business transaction information has become accessible in real–time mode or near real-time modes throughout the supply chain, providing the possibility of real–time order management and optimization. Meanwhile, broad application of e-commence technology has challenged demand fulfillment manner and created the needs for new order promising styles. Customer order response time has become critical to customer satisfaction, especially when a real-time customer response is required. If a company does not meet customer expectation in “real-time”, customers may look towards their competitors while waiting for their order promise. In addition, by responding in real-time to the customer, manufacturers and suppliers could also better collaborate to present jointly constructed campaigns to end-customers and therefore provide both the manufacturers and suppliers with unparalleled means of promising and earning new business (Zweben,1996). As the number of customer orders increase, batch ATP becomes inefficient; and rule-based decision mechanism becomes a requirement for achieving real time response. The solutions from the *analytical model* in this dissertation provide such a mechanism.

Uncertainty is another challenge for ATP. Uncertainties across a supply chain generally come from inaccurate forecasting. Under severe competition, companies have to offer customers more flexibility canceling orders. It has become common for some customer orders to not show up or for customers to make changes that require "what-if" problem solving around cancellations, substitutions or reshuffling of orders. According to Fisher (1997), customer uncertainty is inherent in order promising and has a considerable impact on the supply chain structure. Similarly, uncertainty can be from supply side such as, e.g., delayed delivery of raw material or factory shutdowns. The company can hedge against uncertainty with excess inventory or excess capacity but this results in high inventory cost and capacity waste. The effective exploitation of uncertainty allows the supply chain to better calibrate service levels to meet the needs of various customer segments, as well as to reduce costs. Therefore: considering uncertainty in the ATP decision-making process is necessary to provide a greater degree of stability, continuity and predictability in the customer base in order to earn the more business. We are going to study the uncertainty from the customers' perspective by introducing the *pseudo order.*

Today's supply chains are continually increasing in complexity. Low level, even negative profit margins for a single product may make sense in the context of a large, complex supply chain. With improved communications and increased competition, consumers have been provided with more choice, while most competitors are similar in product performance, quality and price. As consumers expect new products, better quality, and shorter lead times at a reasonable price, strategic use of non-profitable products is not unusual. (Bhattacharjee, 2000). In facing this predicament, companies are showing enthusiasm in discovering how they can better use profitable products to serve their customers. From the supply chain's perspective, this means products have to reach the customers from the right supplier. Supply chain diversity ranging from globally dispersed manufacturers, distribution centers and sales subsidiaries with different production cost, capacities, capabilities and lead-times for different products demands identifying the profitable path for effective order promising, capacity utilization and production smoothness. Overcoming the diminishing profit margin and achieving resource allocation efficiency stimulates us to perform path analyses for the global supply chain.

1.1.2 ATP Trend

As the name suggests, the ATP function provides information regarding resource availability to promise delivery, in response to a customer order request. Conventional ATP quantity is a row under the Master Production Schedule (MPS), and is responsible for keeping track of the uncommitted portion of current and future available finished goods. Unlike conventional ATP, which assigns existing inventory or pre–planned production capacity, advanced ATP refers to a systematic process for making best use of available resources including raw materials, work–in–process, and production and distribution capacity, in addition to finished goods.

An increasingly dynamic and customer-centric environment is heightening the requirements in which companies perform order promising and fulfillment. However, even the most expensive and complex commercial ATP currently available typically promises orders on an incremental, first-come, first-served basis, and as such, have some obvious drawbacks:

● They do not consider the opportunity cost associated with committing supply to a particular order; for example, promising supply to a lower-margin order may preclude that supply from going to a higher-margin order that has yet to come in;

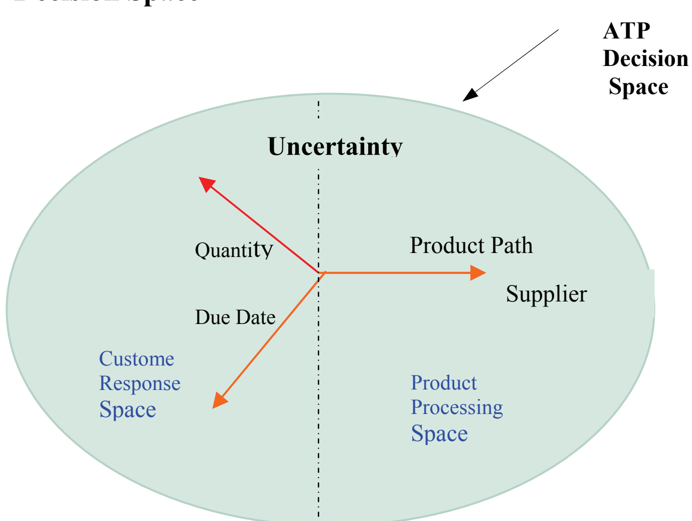
● They do not attempt to maximize the potential revenue of each order;

● They do not distinguish the products from different paths.

In addition, the current advanced-ATP research generally focuses on two elements: quantity and due-date. Quantity quoting gives the customer flexibility often seen in the supply contract. Due-date quoting gives the customer a time buffer in which the order has to be delivered. We know that the primary purpose of the ATP function is to provide a response to customer orders. There are two levels of response: *customer response space* -- one involves the direct response to the customer and the most fundamental decision related to an order is whether or not to accept the order and if accepted, its committed delivery date and quantity; *product processing space* --

this involves the underlying activities required of the production and distribution systems to carry out the customer commitment. We can see that the decision based on quantity and delivery date elements is only in customer response space. Therefore, we present an additional element of ATP: product path (see the Figure 1.1). In addition, as we described before, uncertainty resulting from order cancellations is a critical factor in ATP decisions regardless of the decision space. However, only a few stochastic push-based ATP models have been built, and little work related to stochastic pull-based ATP has been completed. Here, push-based ATP models are designed to allocate available resources for promising future customer demands, and pull-based ATP performs dynamic resource allocation in direct response to actual customer orders (Chen, Zhao and Ball, 2001). According to this definition, our research falls within the pull-based ATP domain. Incorporating path analysis and uncertainty into our model allows us to set and manage customer expectations with accurate supply availability and build a responsive, agile and truly customer-centric supply chain.

Figure 1.1: ATP Decision Space



1.2 Research Questions

Available-to-Promise (ATP) applications originated as a means for controlling the allocation of finished goods inventory and improving the quality of delivery promises to customers. It has since developed into a major operational tool that supports the management of customer demands, safety stocks, production efficiency and the available resource. ATP demonstrates the tremendous synergistic opportunities available within integrated manufacturing planning and control systems. Unfortunately, though easily understood by most users and characterized by some researchers, many companies defer the development and/or implementation of ATP due to the shortage of the efficient ATP systems to clearly and visibly link the external commitments to the supporting manufacturing plan. These are the questions we would address at the strategic level:

Question 1: *Under what conditions do some of the commonly used ATP rules perform well? Are there any other appropriate rules? How can the model parameters be effectively set?*

We believe that our rule-based ATP solution is the answer to this question. Rule-based results can be obtained from analytical models and their solution can be easily implemented and deployed in Decision Support Systems (DSS).

Our previous discussion has clearly shown the value of explicitly including product path analyses in addressing ATP decision. It also shows the importance of including model of uncertainty in such analysis. Accomplishing these objectives is not simple and is our major focus.

Profitability and customer service are two fundamental drivers in determining a company’s performance. Of course, without profitable orders, a business cannot survive. By putting customers at the center of the supply chain, and using information about customer needs to drive it, companies can lower costs, boost revenues and greatly increase customer satisfaction. A clear understanding of customer profitability is critical, because it enables the organization to differentiate the various level of service it provides to various customer segments according to their needs and value to the company. It is important for different customers to get the service that is most appropriate for their needs and the company's profitability. A comprehensive view of customer profitability and customer service lets companies focus resources where they will do the most good in terms of strengthening key customer relationships and bolstering top-line growth. This leads us to the question below:

Question 2: *How can uncertainty be incorporated into ATP optimization models?*

We are interested in answering Question 2 by developing an analytical ATP model, which generates a set of business rules to guide ATP execution. We incorporate both profit and customer service in the objective and include consideration of the pseudo orders. Here, we use a simple pseudo order to aggregate all potential future customer orders and the uncertainties surrounding them. The simple solution structure as well as the empirical result is provided. We devote Chapter 3 to this study.

It has been generally recognized that the coordination of demand fulfillment and purchasing is of critical importance to marketing and product managers, because this translates into increased customer satisfaction and cost. To counteract price erosion and the accompanying reduction in profit margin, manufacturers need to align production and logistics planning with end-sales to choose the right path. In addition, to avoid vulnerability in market competition and reduce the risk, the company also needs the right path. When we say "product path", we refer to the path from the supplier where the raw materials are purchased to manufacturer where products are produced, through the distribution center, to the sales locations where the products are ultimately sold to the customers.

Obviously, as the cost varies from every path, we need to ask:

Question 3: *What kind of strategies and models produce effective demand fulfillment through the right supplier/supply chain paths?*

We build constrained non-linear integer programming models in Chapter 4 to decide optimal supplying quantity of the products from each supplier to meet end customer's demand so that the profit of the company is maximized.

We also investigate the dynamic management of the available resource in accordance with the order promising, for example, if the supplier offers price discount on the component /raw material, how should the company respond to such situation? We think the resource and order management (sales) interact with each other, and should be managed in such a way.

Question 4: *How should companies coordinate the raw material and end product discounts?*

We use the Toshiba global supply chain as a case study, and introduce "path variables" to build a MIP (Mixed Integer Programming) model. Numerical results are presented in Chapter 4.2. We introduce "path variables" to directly determine the unit cost of all products, and identify, for example, the percentage of demand that is met by those cost effective paths. It is necessary to employ "path variables" to obtain such information and produce the appropriate decisions. This analysis leads to a question raised above: when can a discount on raw materials generate more profit, and when should a complementary discount on sales prices be offered to stimulate demand?

1.3 Contributions

1.3.1 Research Contributions

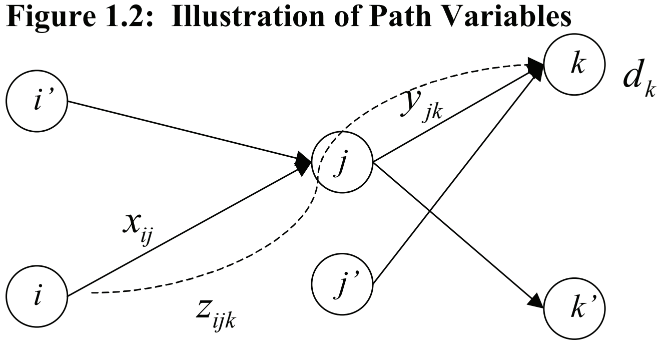
This dissertation provides several contributions to ATP research:

Production pooling considerations in ATP models: Production cost is particularly significant in the ATP research as it is always a major factor in affecting resource allocation. The shapes of the production cost functions depend on many issues (Ghali, 2003), such as industry, the length of the time horizon, capacity, and even the product life cycle. It is recognized that production cost curves, with current technological change, become concave. We build ATP models that evaluate the impact of such changes.

Supply chain collaboration to achieve better demand fulfillment: We build our models to reflect global supply chain goals, while addressing demand fulfillment so as to enable sellers, distributors, manufacturers and suppliers to easily satisfy the end-customer and to help them collaborate in sales, marketing and service initiatives. We should clarify, however, that our current research is related to classical revenue management but has certain differences as well. Revenue management encompasses all practices of discriminatory pricing used to enhance delivery reliability and maximize the profit generated from the resources. The key question facing us is how to allocate the resources shared by the various products, which some RM models address. In addition, demand fulfillment involves issues such as inventory and the resulting holding cost that are not covered in revenue management.

Employing path variables to directly identify and manipulate profitable and non-profitable products: Unlike most models in the global supply chain literature, which define variables in terms of flows along a single arc in the network and use flow balancing constraints at nodes, we employ *path variables*, which provide location–specific cost information and directly identify profitable and non–profitable products (see Figure 1.2). In Figure 1.2, x_{ij} and y_{jk} are traditional flow variables for the product in the network. Demand is represented by d_k at node *k*. Note that this choice of variables does not capture the unit costs for products sold at node *k* (for example, it is not possible to determine whether a particular product at node *k* comes from node *i* or *i'*.) By defining a path variable z_{ijk}, one can directly determine the unit cost of all products, and identify, for example, the percentage of demand that is met by profitable products. In our two–year research project with Toshiba, we found this to be very important managerial information, as it serves as an aid for other strategic decisions such as product line offering at different locations. Although this modeling approach certainly increases the number of decision variables (for example, a typical model we study here has over a million variables and a similar number of constraints), we find that the resulting models, even with real–world data, are manageable with solution times being around 5 minutes using typical computing environments.

Figure 1.2: Illustration of Path Variables



1.3.2 Managerial Implications

We would like to show the practitioners that our research can also serve as a strategic weapon along several dimensions:

ePromise Capability: Through our easy-to-implement analytical ATP solutions, a company can provide a so-called ePromise capability, which supports real-time resource allocation based on actual availability and dynamic order requests, in accordance with the company's business objectives. Such capabilities enable a company to automate order-promising decisions based on electronic information, which is essential for an e-commerce environment.

Enhanced Revenue and Service: Embedding the pseudo (future) order into confirmed orders to deal with uncertainty and finding the “right supplier” to serve the customer based on varying production status allow companies to identify customers’ real demand and provide a clear understanding of their internal capabilities, thus enabling managers to enhance service and profit simultaneously.

Buy Smart – How to Benefit from Recession: Our model also identifies the best quantity of the raw materials/components to buy from suppliers when they offer price discounts. This helps understand how to coordinate raw material discounts with the end sales discounts to achieve the best resource utilization. Such a proactive approach to managing procurement can make a substantial difference during a recession, and can help managers capitalize on future opportunities. As a result, the proactive purchaser is able to take advantage of the recession and shape the supply chain to their long-term advantage.

1.4 Overview of Chapters

The remaining chapters in this dissertation are organized as follows. Chapter 2 gives the literature review. In Chapter 3, we introduce an analytical model that considers multiple confirmed orders, multiple pseudo orders and production pooling. This model can evaluate the usefulness of responding to customer enquiries in real time. In Chapter 4, we create mathematical programming models for two scenarios: a company facing a certain demand curve or uncertain demand. These are strategic level models that provide insight into what quantity levels to purchase from multiple suppliers based on several cost factors. We also develop a mixed integer programming model that addresses how to coordinate raw material discounts offered by s supplier with end-consumers policies. In Chapter 5, we conclude the dissertation by summarizing the results and the contributions.

Chapter 2

Literature Review

2.1 Current ATP Research

Traditional ATP systems are based on the Master Production Schedule, which is derived from the aggregate production plan, detailed end item forecasts, and existing inventory and orders (Vollman, 1992). Thus, raw materials and production capacity constraints are taken into account in the MPS to the extent that they were previously considered in the firm's aggregate production plan—an infeasible MPS is only detected after a more detailed resource planning, later in the planning process. In a differentiated product portfolio, detailed item forecasts can be highly inaccurate, unexpected demand events are more frequent, and developing a feasible MPS is more challenging thus compromising the availability of reliable ATP information. It is not surprising to find several papers (eB2x 2000) , (Fordyce and Sullivan, 1999), (Lee and Billingtion, 1995), (Robinson and Dilts, 1999), and (Zweben, 1996) that discuss the need for advanced ATP systems, which provide order promising capabilities based on current capacity and inventory conditions within the firm's supply chain.

Interestingly, there are relatively few papers that address quantitative models for order promising. Taylor (1999) introduces a heuristic that keeps track of traditional ATP quantities to generate feasible due dates for order promising. Kilger (2000) also proposes a search heuristic to promise orders, motivated by yield management algorithms used in the airline industry (see below). Ervolina (2001) presents models developed at IBM for resource allocation in a CTO production environment. Moses (2002) investigates highly scalable methods for real–time ATP that are applicable to discrete BTO environments facing dynamic order arrivals, focusing on production scheduling. At an operational level, Yongjin (2002) discusses the relationship between the performance of dynamic vehicle routing algorithms and online ordering in conditions of demand saturation—where demand exceeds service capacity. Chen and Ball (2001) provide mixed–integer programming (MIP) formulations for order promising and due–date quoting, taking into account existing inventories for raw materials, components and finished goods, production capacities, and a flexible bill of materials (BoM) environment (where customers can select different suppliers for the same raw material). Their models address a static situation, computing ATP quantities for orders in a *batching interval*—a batching interval is the time window over which customer orders are collected before the ATP function is executed to schedule their production —which is an input to their models. These models maximize profit for a batching interval only, without consideration of future demands. We propose to address the stochastic and dynamic nature of the problem. More importantly, we address the ATP decision not just from customer response space, but also from product processing space that includes the product path analysis and uncertainty. In the following sections, we are going to review these topics.

2.2 Stochastic ATP - Uncertainty

In our analytical stochastic model we analyze how to allocate a resource to optimally promise customer orders when the future orders are uncertain. The problem of allocating scarce capacity to customer orders to promise for future deliveries can be viewed from another perspective when customers' sensitivity to lead times (or their due dates) is significantly different. In this environment, the firm may design a menu of price and lead–time combinations to segment the market (F.Chen, 2001). Once the menu is designed, the firm may use operational policies based on resource rationing to maximize profit. But ATP models and systems are essentially different from inventory models and systems. Inventory focused systems with different demand classes, different margins, and with different stock–rationing policies, for example Kaplan (1969) and Topkis (1968), have dynamic replenishment. There are *n* demand classes, and the penalty for not satisfying demand depends on the demand class (for example, one demand class pays a higher price, or can be met at a lower cost, thus having a higher priority for the firm). A significant stream of research exists on inventory rationing, depending on how assumptions of review period, demand distribution, and unsatisfied demand are handled (Cattani 2002), (Deshpande and Donhoue, 2001), and (Ha,1997). These models, however, do not consider capacity limitations in the replenishment decision, and therefore are fundamentally different from our dynamic ATP models.

A body of literature closely related to this topic is newsvendor-like problems, which build inventory model to trade off order placing cost and holding cost. The work of Agrawal and Seshadri (1999) considers a single-period inventory model in which a risk-averse retailer faces uncertain customer demand and makes a purchasing-order-quantity and a selling-price decision with the objective of maximizing expected utility. They analyze how price affects the distribution of the demand and in turn how the quantity to be committed should be determined. The paper by Cattani (2000) examines a single-period stochastic inventory problem where N distinct kinds of demand can be satisfied with a single kind of product. But it assumes that priority is hierarchical -- all demand for a higher priority product is met before meeting demand for the next lower priority product. Also, we address the demand pooling effect from economies of the scale, but in Cattani's paper (2000) the author studies pooling effect from negatively correlated demands.

2.3 Coordinating Demand Fulfillment with Supply

In order to coordinate demand fulfillment with supply, we build models that identify profitable paths from different suppliers and to different sales subsidiaries. We provide both an analytical model and a global supply chain MIP model to investigate that issue, since they involve interactions among several individual factors as well as thousands of products and product locations. Of the three levels of planning in a supply chain—strategic, tactical, and operational (see, e.g., Vidal and Goetschalckx, 1997) —our models address primarily tactical decisions, that is, production and distribution decisions that span a maximum of four months, such as production and transportation choices that maximize profits, subject to capacity and other constraints. Thus, our models neither address strategic decisions such as facility location nor operational decisions such as daily production scheduling at plants. There are several streams of literature that are relevant to our research: global supply chains, supply chain coordination mechanisms, inventory models with pricing, and product line offering; we review each stream separately and discuss our study accordingly.

Past literature addresses issues such as supplier–buyer coordination, for example, an ordering policy that minimizes supply chain costs for both parties (for a review see Thomas and Griffin, 1996)). Most of these models, however, are stylized extensions of the basic EOQ model or the Wagner–Within algorithm, focusing on a *single* product under independent and deterministic demand. Closely related is the literature on supply chain contracts (e.g. Bassok and Anupindi 1997, Urban 2000, Chen and Krass 2001, Serel, Dada and Moskowitz 2001), where, under uncertain demand for a single product, the buyer commits to a minimum cumulative procurement quantity *over a long–term planning horizon* in exchange for price discounts. Our model differs from the past literature in various aspects: in the analytical model, we consider demand curves that vary by sales locations; in the resource analysis of Toshiba global supply chain, we consider a situation where a firm orders a *component* periodically (there are no fixed ordering costs), however, the supplier offers a one–time price discount.

The literature on inventory and pricing is also relevant and extensive—where the firm decides, in addition to order quantity, on the price of the product (which influences demand). For a review of single–period models, readers can see Petruzzi and Dada (1999), and for multi–period models, see Federgruen and Heching (1999), Chen, Federgruen and Zheng (2001) and Zhao and Wang (2002). Unlike our work, again, this stream of research assumes a single product with unlimited capacity.

Cohen and Lee (1989) argue that differences between supply–chain planning within a single country and for a global network include the existence of duties, tariffs, tax rates across countries, currency exchange rates, multiple transportation modes, and local content rules, among others. There is a considerable body of research in strategic production–distribution models for global supply chains, and the reader could refer to Thomas and Griffin (1996), Vidal and Goetschalckx (1997), Goetschalckx, Vidal and Dogan (2002) for detailed literature reviews. In addition to production and distribution decisions, this body of literature addresses the more strategic problem of network design, which is usually formulated as a mixed–integer programming (MIP) or a non–linear programming (NLP) model. The global nature of the problem may require careful modeling of transfer pricing (e.g. Vidal and Goetschalckx 2001), and exchange rates (e.g. Huchzermeier and Cohen 1996).

Our research is also related to the product line offering question—which products are profitable and should be offered at each location. The literature on the design of a product line that maximizes profitability focuses primarily on marketing issues, such as the interactions of a set of products, given their relative utilities and prices, in the market place (for a review see Yano and Dobson, 1998). A few papers consider the manufacturing and/or inventory implications of product line breadth (e.g., Van Ryzin and Mahajan 1999, Smith and Agrawal 2000, Morgan, Daniels and Kouvelis 2001), such as variable production cost, holding and setup costs, but these papers do not consider the complex interactions of sourcing, manufacturing, and distribution in global, capacitated, supply chains, where the profitability of a product can be different, depending on its supplier or path in the supply chain and the location where it is sold. Summarily, our analysis bridges the gap between sales' product profitability and supplier's variety.

2.4 Revenue Management

Finally, we discuss the revenue management literature, which clearly has strong relevance to our work. Most revenue management models assume fixed (or "almost fixed") resource availability, (e.g., airline seats) and balance resource allocation among multiple demand classes (e.g., fair segment of price-sensitive customers). A common way to model the airline booking process is to model it as a sequential decision problem over a fixed time period, in which one decides whether each request for a ticket should be accepted or rejected. The classical example is that of customers traveling for leisure and those traveling on business. The former group typically books in advance and is more price-sensitive, whereas the latter behaves in the opposite way. Airline companies attempt to sell as many seats as possible to high-fare paying customers and at the same time avoid the potential loss resulting from unsold seats. In most cases, rejecting an early (and lower-fare) request saves the seat for a later (and higher-fare) booking, but at the same time that creates the risk of flying with empty seats. On the other hand, accepting early requests raises the percentage of occupation but creates the risk of rejecting a future high-fare request because of the constraints on capacity. The airline booking problem was first addressed by Littlewood in 1972, when he proposed what is now known as the "Littlewood Rule". Roughly speaking, the rule — proposed for a two class model — says that low-fare bookings should be accepted as long as their revenue value exceeds the expected revenue of future full fare bookings. This basic idea was subsequently extended to multiple classes (Belaboba, 1990). Later, it was shown that, under certain conditions, it is optimal to accept a request only if its fare level is higher or equal to the difference between the expected total revenues from the current time to the end when respectively rejecting and accepting the request. This rule immediately leads to the question "How to evaluate or approximate the expected total revenue from the current time until the end of booking?" However, the drawback of solving it as a sequential decision problem is also clear in that the booking policy is only locally optimized and it cannot guarantee global optimality.

Glover et al. were perhaps the first to describe a network revenue management problem in airlines. By assuming that passenger demands are deterministic, they focus on the network aspects of the model (e.g., using network flow theory) rather than on the stochastic aspect of customer arrivals. Dror, Trudeau, and Ladany propose a similar network model, again with deterministic demand. The proposed improvements allow for cancellations, which often happens in the real booking process. Booking methods based on linear programming were thoroughly investigated by Williamson (Williamson 1992). The basic models take stochastic demand into account only through expected values, thus yielding a deterministic program that can be easily solved. The major drawback of the approach above is that it ignores any distributional information about the demand.

Later many other industries also applied these techniques to control their perishable or even non-perishable assets. Weatherford and Bodily (1992) not only propose the new name, Perishable-Asset Revenue Management (PARM), but also provide a comprehensive taxonomy and research overview of the field. They identify fourteen important elements for defining revenue management problems. Although most of these elements are airline-orientated, many ATP problems share the similar characteristics: particularly the last three modeling-related elements: bumping procedure (for handling "overbooking"), asset control mechanism (for resource reservation), and decision rule (for resource allocation). More recently, McGill and Van Ryzin (1999) classify over 190 research papers into four groups: 1) forecasting, 2) overbooking research, 3) seat inventory control, and 4) pricing. The papers in the third group are more relevant to ATP models discussed here. For example, Weatherford and Bodily (1995) present a generic multiple-class PARM allocation problem. They first study a simplified two-class problem without diversion. The problem assumes that there are two demand classes, full-price and discount, share the fixed available capacity of q_0 units and that no full-price customer would pay less than their willingness to pay (i.e., the full price). The purpose is to determine the number of units that should be allocated to discount-price customers before the number of full-price customer is realized. The authors further extended the problem to allow diversion in the multiple-class setting. Sen and Zhang (1999) worked on a similar but more complicated problem by treating the initial availability as the decision variable and model the problem as a newsboy problem with multiple demand classes. To some degree, our work can be seen as the extension of Littlewood two-class model under the special business setting. Yet due to the characteristics of ATP, the holding cost and customer service level, which normally is not in the scope of revenue management, are taken into consideration and plays a critical role in our model (see Chapter 3.2).

Chapter 3

Pseudo Order Consideration in Available to Promise (ATP)

3.1 ATP Framework

The fundamental decisions ATP models must address are: 1) which orders to accept 2) the committed quantity for accepted orders. A sophisticated approach to carry out ATP functionality, introduced by Chen et al. (Chen et. al., 2002), is to employ large-scale mixed–integer-programming (MIP) models. Other researchers have also developed ATP models like allocated ATP (a-ATP) and capacity ATP (c-ATP) to support ATP decision-making process. This model-based approach for ATP execution can make effective use of resource flexibility and generate reliable order-promising results. It is indeed efficient in some complicated business environments to support resource allocation and rescheduling. However, it usually takes more execution time to solve these models compared to conventional simple finished-product level ATP search results.

As described in Chapter 1, real-time response is becoming a requirement based on customer service. Moreover, large numbers of customer orders with both accurate information and inaccurate configurations may come simultaneously in the e-business environment and/or large number of customer service channels. With this in mind, the MIP-based ATP mechanism may not be suitable because of its heavy computation. In contrast, analytical ATP models, which are based on simple rules and principles, can provide effective mechanisms for order promising solutions by trading off multiple business objectives under resource constraints. Another benefit of using analytical models to solve ATP problems is their capability to tackle uncertainty. Unconfirmed customer orders can capture real-life customer's inquiry and order cancellation, which is difficult for MIP types of models to handle. Undoubtedly, a further advantage of analytical models is that they offer generic solutions that don't require extensive experiments.

In this chapter, an analytical ATP model will be introduced for order-promising and fulfillment decisions based on consideration of both profit and customer service. Instead of putting the customer service level in the constraints, we include it in the objective function. This reflects the trend toward pushing the service levels as high as possible. Other feature of this analytical ATP model is: A pseudo order with stochastic characteristics is considered with other confirmed orders to represent uncertain customer inquires and order cancellations. Since one pseudo order may have a higher profit margin but also uncertainty, it will have an impact on the commitment of the confirmed orders. It's worth mentioning here that the uncertainties in ATP are mainly caused by three factors: demand, lead-time and raw material purchasing price. According to Weber's survey (Weber and Current 2000), the effect of lead time is only 10-20% of effect of demand on a company's total profit. On the other hand, the uncertainty from purchase price can be compromised by supply contracts as analyzed in our model. Thus we believe that using a pseudo order to incorporate the current orders and future orders together not only reflects price uncertainty in rapidly changing environment, but also captures the Achilles' heel.

We shall see that our objective function is neither convex nor concave, but a *d.c. function*, i.e. a function that can be represented as the difference of two convex (concave) functions (Horst, 1993). The problem of maximizing a *d.c. function* under linear constraints is a nonconvex global optimization problem, which may have multiple local minima with substantially different values. Such multiextremal problems cannot be solved by standard methods of nonlinear programming which can at best locate a local minimum. Outer approximation methods along with branch and bound methods for finding a global minimum have been suggested in (Tuy, 1987). However, most of these methods are able to solve limited size problem instances. This should not be surprising, since the problem is known to be NP-hard, see e.g. (Pardalos, 1984). Therefore, simultaneous consideration of the uncertain demand makes the problem more general, and also more difficult. Fortunately, we have derived some rule-based solutions which are presented later in this section.

3.2 Problem Formulation & Model

The problem under consideration is a single period, single product, multi-order ATP model. This model consists of N confirmed customer orders, which are assumed to be deterministic, and one pseudo order, which is stochastic. The fundamental decision in the model is to determine promised order quantities for the confirmed customer orders and a reserved quantity for the pseudo order by considering both production impact and material limitations. The objective of including the pseudo order is to anticipate near-term future customer orders based on customer inquiry information. The resultant model is a constrained non-linear stochastic programming problem.

In the model, we use "*total benefit*" as the objective function, which is defined as the sum of weighted expected profit and customer service level. This reflects common practice in most order fulfillment and optimization processes. The relative weights of expected profit and customer service level can be adjusted in the model to reflect their importance in specific business settings. Meanwhile, we assume the committed quantity of the confirmed order can never exceed the requested quantity. For the stochastic pseudo order, holding costs may be incurred if the committed quantity is greater than the specified quantity. Below is the notation that will be used in the formulation.

3.2.1 Notation and Remarks on Function Properties

Let $I = \{1,2,\cdots,N\}$ be the index of a set of the confirmed customer orders. For all $i \in I$, let q_i and r_i be the requested quantity and sales price of the confirmed order i, respectively; β_i is a weighted constant of the customer service level for the confirmed order i. For the pseudo order, let u be the pseudo order quantity, which is a random variable with known probability density function (PDF) as $f(\cdot)$, and p, h the unit sales price and unit holding cost, respectively.

For the order promising decision, we consider two kinds of resources: material availability and production capacity. Here, we assume the production cost is a convex function of the quantity produced. A convex production cost exhibits non-increasing returns to scale. Basically, two factors can lead to that. One is overload or overtime. As an example, consider a factory with a regular workforce. If demand is beyond the capacity of the regular workforce, management has to employ overtime at a higher cost, and, if needed, it may subcontract production at an even higher cost. Convex production costs therefore are incurred and reflect *diseconomies* of scale (Galeotti and Maccini 2004). The other factor that leads to convex production cost is quality. Consumers have heterogeneous willingness to pay for quality, and the unit cost as a function of product quality is technology specific. Based on a distinct engineering principle, for a given production technology, the unit production cost tends to rise more rapidly as quality increases, and an increasing, convex cost function effectively captures such decreasing returns (Rochet and Chone 1998). We let $m(\cdot)$ be the convex production cost function. Another kind of resource for order promising is material availability. Let $g(\cdot)$ be the material cost function, which is a concave function of material quantity used to reflect the economies of scale.

Since ATP is used to serve customers, both total profit and customer service level should be considered vital performance criteria. In this paper, we employ order fill-rate to model customer service level. The order fill-rate is defined as the quantity committed over the quantity requested for a customer order. Thus, the "*customer service benefit*" is a function of the order fill rate. Let $s(\cdot)$ be the customer service benefit function. It should be a concave function since most companies only pursue a certain higher level of customer service and penalize seriously very lower fill rates.

To formulate the committed quantity of the customer orders, we define the following decision variables. For all $i \in I$, let:

X_i equals the committed quantity of the confirmed order i;

Y equals the committed quantity of the pseudo order.

3.2.2 Model Formulation

Based on the previous notation, the total revenue, which comes from both confirmed customer orders and the pseudo order, can be formulated as

$$\sum_{i \in I} (r_i X_i) + pMin(Y,u), \tag{3.1}$$

Where the first term is the confirmed order's revenue, and the second term is the pseudo order's revenue. We assume that the production cost and material cost are both measured in terms of a unit of product. Then, the material used is proportioned to the number of products committed to customers. The material cost and production cost is given as equation (3.2) and (3.3), respectively.

$$g(\sum_{i \in I} X_i + Y) \tag{3.2}$$

$$m(\sum_{i \in I} X_i + Y) \tag{3.3}$$

The holding cost incurred by the pseudo order will be:

$$h \times Max(Y - u, 0). \tag{3.4}$$

Hence, the total expected profit is:

$$EP = \sum_{i \in I} r_i X_i + E(pMin(Y,u) - g(\sum_{i \in I} X_i + Y)$$

$$- m(\sum_{i \in I} X_i + Y) - h \times E(Max(Y - u, 0)) \tag{3.5}$$

Substituting the expected revenue and holding cost of the pseudo order into the PDF function in the above equation yields:

$$EP = \sum_{i \in I} r_i X_i + p(\int_0^Y uf(u)du + \int_Y^\infty Y f(u)du) - g(\sum_{i \in I} X_i + Y)$$

$$- m(\sum_{i \in I} X_i + Y) - h\int_0^Y (Y-u) f(u)du \tag{3.6}$$

We note that the unit profit margin of an individual customer order may be negative like most discount sales. The reason to promise such orders is due to customer service level considerations. For the *i*th customer order, the customer service level, specifically, order fill rate, is X_i/q_i. Hence, the total customer service benefit from all customer orders can be represented as below:

$$\sum_{i \in I} \beta_i s(\frac{X_i}{q_i}) \tag{3.7}$$

in which β_i is the customer service benefit weight. The value of β_i indicates the importance of customer service in comparison to profit for the *i*th customer order. Therefore, the objective function, which is defined as the "*benefit function*" since it includes both profit and customer service level, can be written as:

$$TB = \sum_{i \in I} r_i X_i + p(\int_0^Y uf(u)du + \int_Y^\infty Y f(u)du) - g(\sum_{i \in I} X_i + Y)$$

$$- m(\sum_{i \in I} X_i + Y) - h\int_0^Y (Y-u) f(u)du + \sum_{i \in I} \beta_i s(\tfrac{X_i}{q_i}) \tag{3.8}$$

subject to constraints described as following:

$$q_i \text{ - } X_i \geq 0 \tag{3.9}$$

$$X_i \geq 0 \tag{3.10}$$

$$Y \geq 0 \tag{3.11}$$

Constraints (3.9), (3.10) and (3.11) are order-promising limitations and non-negativity. They are similar to the widely accepted concept of "booking limits" or "protection level" in the revenue management area.

Using the Lagrange method, the first order condition of the problem (3.8)-(3.11) can be given as follows for all $i \in I$.

$$r_i - g'(\sum_{i \in I} X_i + Y) - m'(\sum_{i \in I} X_i + Y) + \tfrac{\beta_i}{q_i} s'(\tfrac{X_i}{q_i}) - \lambda_i + \mu_i = 0 \tag{3.12}$$

$$p(1 - F(Y)) - g'(\sum_{i \in I} X_i + Y) - m'(\sum_{i \in I} X_i + Y) - hF(Y) + w = 0 \tag{3.13}$$

$$\lambda_i (q_i \text{ - } X_i) = 0 \tag{3.14}$$

$$\mu \times X_i = 0 \tag{3.15}$$

$$\lambda_i \geq 0 \tag{3.16}$$

$$\mu_i \geq 0 \tag{3.17}$$

$$w \times Y = 0 \tag{3.18}$$

$$w \geq 0 \tag{3.19}$$

Where λ_i, μ_i and w are Lagrange multipliers for constraints (3.9), (3.10) and (3.11), respectively.

3.2.3 Model Analysis

As stated earlier the objective function is neither convex nor concave, but a *d.c. function*. We present the following Theorem to assure the existence of optimal solution for the problem (3.8)-(3.11). The proof is shown in Appendix 1.

Theorem 3.1: The objective function defined by (3.8) will be strictly concave on the feasible region defined by problem (3.9)-(3.11) if,

$$M > \frac{Max(LU, -(h+p)K)}{n+1} \tag{3.20}$$

Where $U = \underset{0 \le x \le 1}{Max}\, s''(x)$; $L = \underset{1 \le i \le n}{Min} \frac{\beta_i}{q_i^2}$; $K = \underset{0 \le y \le y_0}{Min} f(y)$ with $y_0 = F^{-1}(\frac{p}{p+h})$; and

$M = \underset{0 \le z \le z_0}{Min}[g''(z) + m''(z)]$ with $z_0 = y_0 + \sum_{i=1}^{n} q_i$.

Remark 3.1: There exists one and only one optimal solution for the nonlinear stochastic problem (3.8)-(3.11) if $g(\cdot)$ is a concave function, $s(\cdot)$ is a strictly concave function, and

$$g''(\cdot) + m''(\cdot) > 0 \tag{3.21}$$

Proof: see Appendix 2**.**

One can observe that condition (3.21) is the special case of condition (3.20). Theorem 3.1 gives conditions for the existence of an optimal solution for the problem (3.8)–(3.11). Condition (3.20) basically states that the total cost (production cost plus material cost) function should be "convex enough" to compensate the concaveness of projected customer service level functions.

Obviously, the problem (3.8)–(3.11) is a multiple constrained newsvendor problem, which has been shown by Lau (1995) to be very difficult to solve, and not much work has been done on capacitated systems (Tayur, 1998). From an order promising point of view, we are more interested in the solution structure, rather than the solution itself, of the problem since the solution structure can provide insights and guidance for optimal order promising. We present the following Theorem to illustrate the structure of the solutions for the problem (3.8)-(3.11).

Theorem 3.2: For $\forall i, j \in I$, if the following condition holds

$$[Z_{ij} - s'(0)][Z_{ij} - s'(1)] \geq 0, \tag{3.22}$$

where Z_{ij} is defined by Equation (3.23) with $\alpha_{ij} = \frac{\beta_i}{q_i} - \frac{\beta_j}{q_j}$

$$Z_{ij} = \begin{cases} -1, & if \alpha_{ij} = 0 \\ \frac{r_i - r_j}{\alpha_{ij}}, & \text{Otherwise} \end{cases} \tag{3.23}$$

then there exist two points $k, k' \in I$ $(k' \le k)$, the optimal solutions of problem (3.8)-(3.11) have the following structure:

$X_i = 0$ for all $i < k'$,

$X_i = q_i$ for all $i > k$,

X_i^* for all $k' \le i \le k$ is solved from the following equations:

$$s'(\frac{X_i^*}{q_k}) = \frac{q_i}{\beta_i}[g'(\sum_{l=1}^{n} X_l + Y) + m'(\sum_{l=1}^{n} X_l + Y) - r_i] \tag{3.24}$$

$$F(Y) = \frac{p - g'(\sum_{i=1}^{n} X_i + Y) - m'(\sum_{i=1}^{n} X_i + Y)}{p+h} \tag{3.25}$$

Proof: see Appendix 3.

Theorem 3.2 provides the structure of the optimal solution. Based on this Theorem we can see that some customer orders should be one hundred percent committed, and some others should not committed at all when the conditions above are hold. The Z_{ij} variable can be interpreted as the ratio between profit and customer service level, and Theorem 3.2 just states how a particular customer order promising scheme should be adopted depending on that ratio.

Remark 3.2: If the following condition holds, $\forall i \in I$,

$$\frac{\beta_i}{q_i} s'(0) + r_i < \frac{\beta_{i+1}}{q_{i+1}} s'(1) + r_{i+1}, \tag{3.26}$$

then the optimal solution for problem (3.8)–(3.11) is as given in Theorem 3.2 with $k = k'$.

Proof: see Appendix 4.

From this remark, one can observe that the optimal order commitment policy is characterized by a single key order, which represents a threshold between orders whose quantities are either 0 or q_i. That quantity can be found by simply searching the linear solution space *I*. This gives us a simplified policy to make the optimal order promising for customers.

In real life situations, one company may only concentrate on a single criteria, choosing between customer service level and profit in order promising practices. In such case, condition (3.22)-(3.23) in Theorem 3.2 can be further simplified as the following:

- In the case when product sales price plays a dominant role in order promising practice, condition (3.22)-(3.23) becomes (3.27) in deriving optimal solution to problem (3.8)-(3.11) in Theorem 3.2.

  $$r_{i+1} - r_i > \frac{c}{l}, \quad \text{where } c = s'(0) Max(\tfrac{\beta_i}{q_i}) \quad l \in I \tag{3.27}$$

  This solution structure can be easily checked and allows sales personnel to determine how the orders should be committed. When $l = 1$, then there exists one and only one solution.

- In the case when customer service level plays a dominant role in order promising practice, condition (3.22)-(3.23) would be simplified to (3.28) in Theorem 3.2.

  $$\frac{\beta_i}{q_i} \sqrt[l]{\varepsilon} < \frac{\beta_{i+1}}{q_{i+1}} \text{ where } \varepsilon = \frac{s'(0)}{s'(1)} \tag{3.28}$$

This is symmetric to condition (3.27).

Remark: Extension of Littlewood Model

We can also explain the work above from the perspective of revenue management. If we aggregate all the confirmed orders into one class (since they are all deterministic and order promising can be easily carried out by ordering the price from high to low), and the problem represented by (3.8-3.11) becomes tradeoff of pseudo order class and confirmed order class. This is a two-class revenue management problem except we take customer service level into consideration. Thus, it can be viewed as an extension of the Littlewood two-class model under a special business setting. For example, if we manipulate this equation and treat Y as the optimal booking of (3.25)

$$F(Y) = \frac{p - g'(\sum_{i=1}^{n} X_i + Y) - m'(\sum_{i=1}^{n} X_i + Y)}{p + h}$$

Then we have

$$F(Y) = 1 - \frac{h + g'(\sum_{i=1}^{n} X_i + Y) + m'(\sum_{i=1}^{n} X_i + Y)}{p + h}$$

$$F(Y) = 1 - \frac{p_2}{p_1} \quad \text{(the Littlewood two-class model)}$$

where $p_2 = h + g'(\sum_{i=1}^{n} X_i + Y) + m'(\sum_{i=1}^{n} X_i + Y)$ and $p_1 = p + h$

This result is in the form of Littlewood model. Belobaba (1987a) heuristically extends Littlewood's rule to multiple fare classes and introduces the term Extended Marginal Seat Revenue (EMSR) for the general approach. The EMSR method does not produce optimal booking limits except in the two-fare case, and Robinson (1995) shows that, for more general demand distributions, the EMSR method can produce arbitrarily poor results. There has been very little published research on joint capacity allocation/pricing decisions in the revenue management context. Weatherford (1994) presents a formulation of the simultaneous pricing/allocation decision that assumes normally distributed demands, and models mean demand as a linear function of price. The corresponding expressions for total revenue as a function of both price and allocation are extremely complex, but no structural results are obtained. However, our study not only obtains the structural results but also shows the impact of holding costs on both pseudo order and confirmed order quantities.

More importantly, our model takes service level into consideration and allows not only 100% or 0% commitment but also portion commitment for each single order, unlike most discussions in the field of Airline or Yield Management. Formulas (3.26) (3.27) and (3.28) describe the rule of the accepting or rejecting order, which is determined by different order parameters such as price and SL. We can see the order's "marginal benefit", characterized in Remark 3.2, includes both price and SL; the "marginal revenue" as defined in the Littlewood rule and extended EMSR (Expected Marginal Seat Revenue) model in other Revenue Management research, includes only one parameter (price).). Thus, our work can be viewed as extending prior revenue management research.

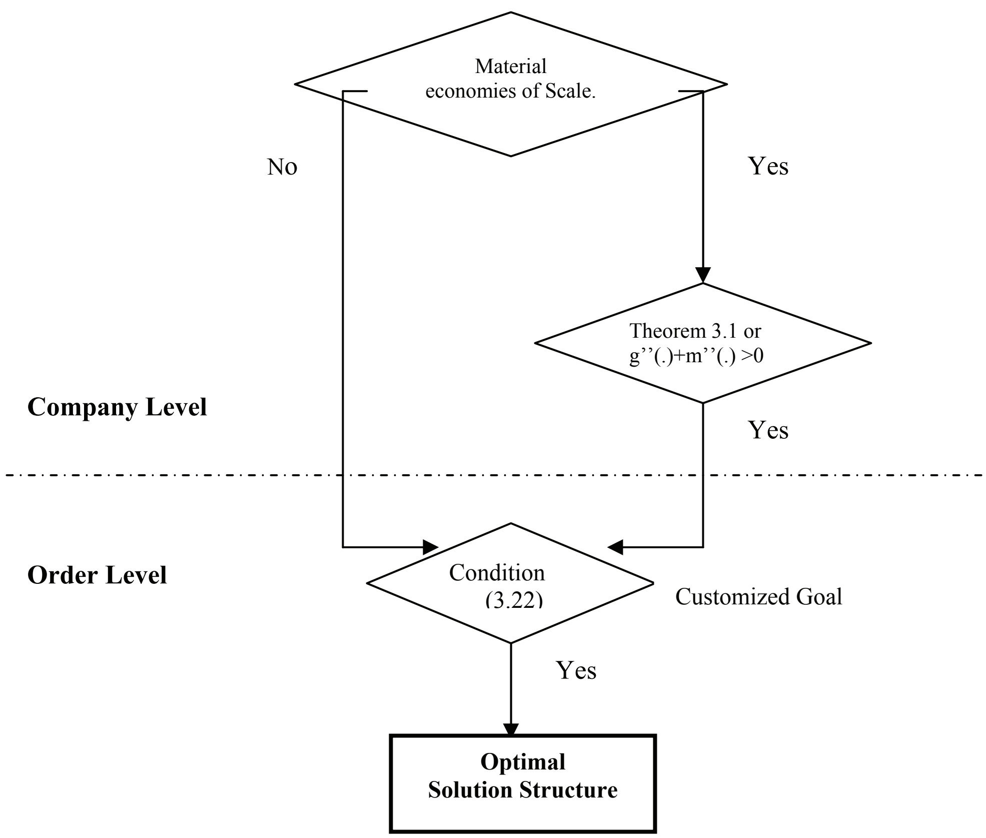
3.3 Implementation Rule

The analysis above provides decision makers the effective rule-based mechanism to implement and deploy critical decisions in real-time ATP systems. We summarize and illustrate it in Figure 3.1.

3.4 Experimental Study and Results

Our goal in developing analytical ATP model is to develop real-time strategies to support order-promising process. In order to gain strategic insight from the model numerically, we design and carry out the following experiments.

Figure 3.1 Pseudo order decision flow chart



The cost and service level functions are assumed to be as following:

$$s(X_i/q_i) = (X_i/q_i + \tau)^\alpha \qquad (0 < \alpha < 1)$$

$$m(\sum_{i \in I} X_i + Y) = k_1(\sum_{i \in I} X_i + Y)^\gamma \qquad (0 < \gamma < 1)$$

$$g((\sum_{i \in I} X_i + Y)) = k_2(\sum_{i \in I} X_i + Y)^\lambda \qquad (\lambda > 1)$$

$$f(Y) = N(\mu, \delta)$$

where the coefficients are given as below:

$$\beta_i = 800 \quad h = 3$$

$$\alpha = 0.9 \quad l = 1,$$

$$k_1 = 43 \quad \gamma = 0.8$$

$$k_2 = 0.002 \quad \lambda = 2$$

Four confirmed orders and one pseudo order are considered in this experiment with specifications as below:

$$r_1 = 5 \quad r_2 = 8 \quad r_3 = 12 \quad r_4 = 18 \qquad p = 20$$

$$q_1 = q_2 = q_3 = q_4 = 400, \ \mu = 500, \ \delta = 70$$

By using MathCAD software, we have the solution:

$$(X_1, X_2, X_3, X_4, Y) = (0, 0, 383.5, 400, 619.2)$$

We can observe that the solution has the exact same structure as Theorem 3.2 states.

Now let's see some interesting findings from the experiments.

1) The pseudo order's price sensitivity analysis:

Table 3.1: Effect of Pseudo Order's Price Change

Y's Price	X1	X2	X3	X4	**Y**	**SUM(X)**	**TB**
18	0	0	387.2	400	614	787.2	6391
19	0	0	385.8	400	617	785.8	6860
20	0	0	383.5	400	619	783.5	7330
21	0	0	381	400	621	781	7802
22	0	0	380	400	623	780	8276

Figure 3.2 Committed Order as a function of pseudo order's Price

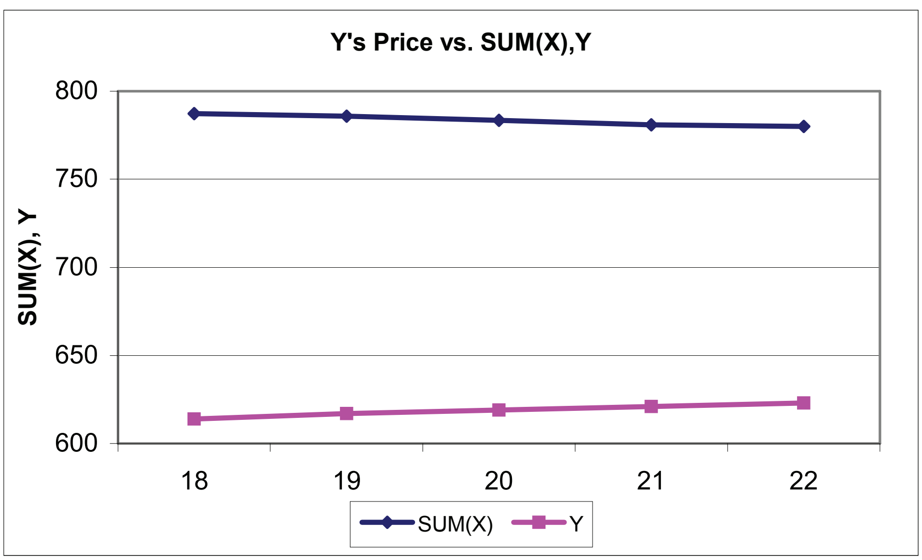


Figure 3.3 Total Benefit as a Function of Pseudo Order's Price

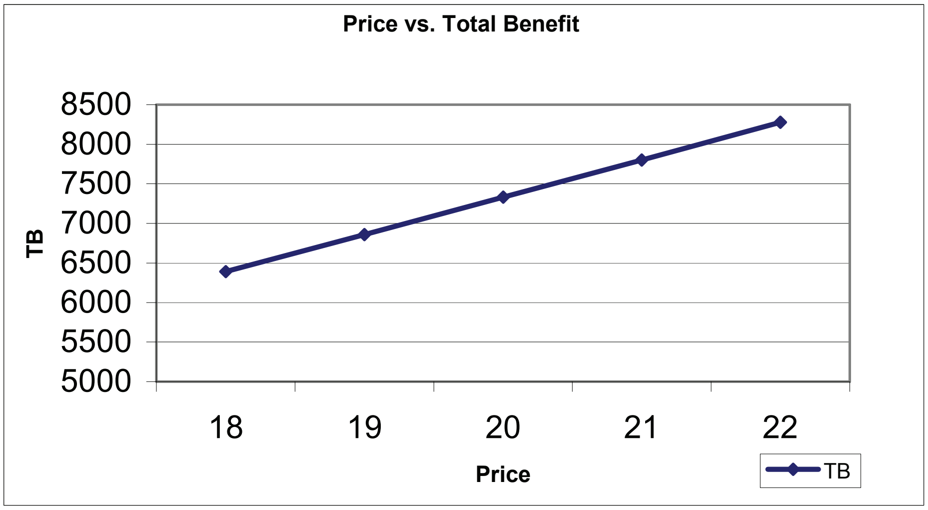


Figure 3.2 and Figure 3.3 shows the quantity of committed orders and total benefit as a function of pseudo order's price respectively. We observe that as the pseudo order's price increases, both the commitment quantity of the pseudo order and total benefit increases. These results imply that even with embedded uncertainty, the pseudo order's price is still a decisive factor in determining the commitment quantity and total benefit. The commitment quantity of the total confirmed orders decreases as the pseudo order's price increases. This is because they are sharing the same materials whose supply is tight. As in our case, the material cost function $m(\cdot)$ is a increasing convex function. So as the pseudo order's price increases, the confirmed orders become less economically attractive.

2) Pseudo order's variance sensitivity analysis:

Table 3.2 Effect of Pseudo Order's Variance

Variance of Y	X1	X2	X3	X4	**Y**	**SUM(X)**	**TB**
60	0	0	394.5	400	606.7	794.5	7639
70	0	0	383.5	400	619.2	783.5	7330
80	0	0	373	400	630	773	7031
90	0	0	360.6	400	641.5	760.6	6741
100	0	0	352.5	400	651.5	752.5	6458

Figure 3.4 Committed Order Quantity as a Function of Pseudo Order's Variance

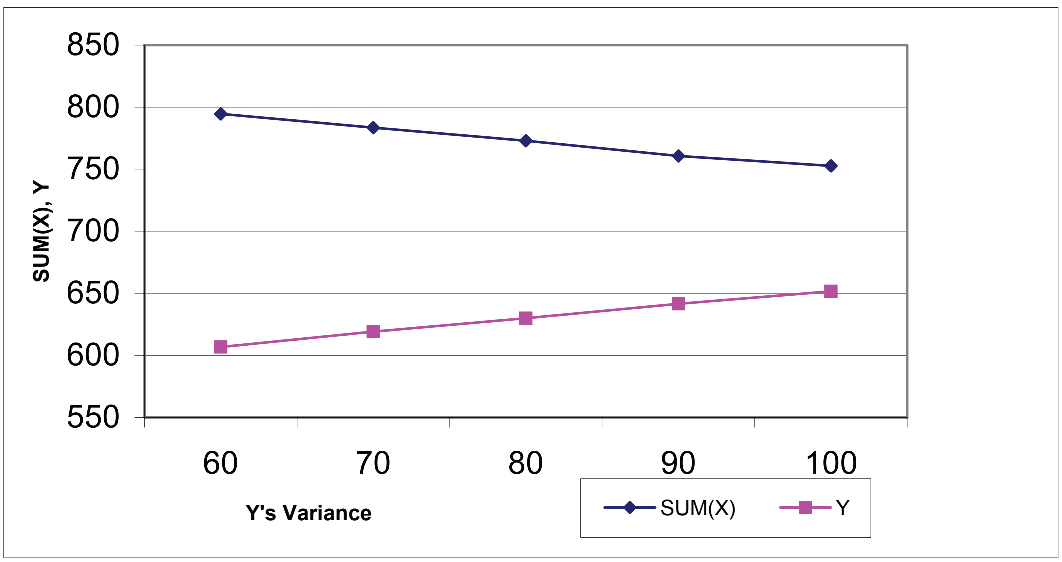


Figure 3.5 Total Benefit as a Function of Pseudo Order's Variance

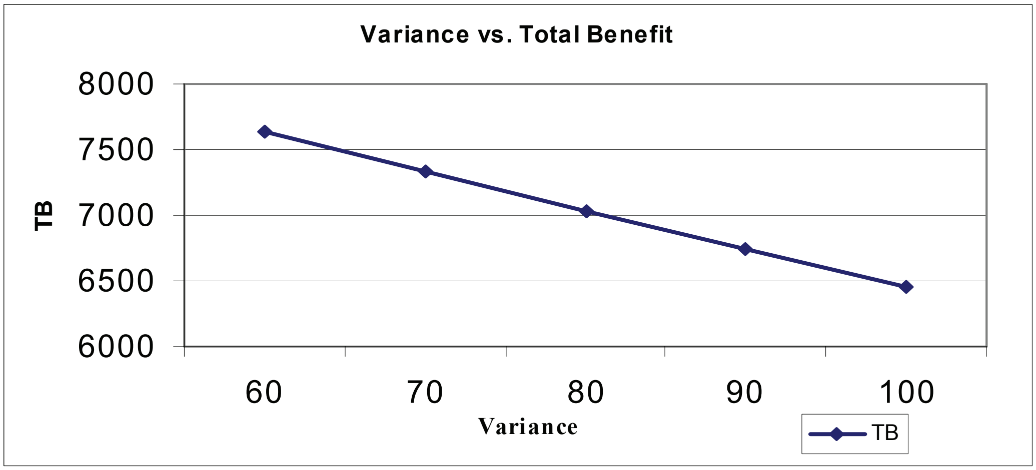


Figure 3.4 shows the committed order quantity as a function of the pseudo order's variance. As variance increases, the total commitment quantity of confirmed order decreases, and the commitment quantity of the pseudo order increases. Note that the reason that more pseudo orders are committed is to offset the larger uncertainty, not to generate more profit, as evidenced by Figure 3.5, the total benefit decreases when variance and commitment quantity of the pseudo order increase. This is an interesting finding and reveals that uncertainty of the forecasted order will not only lead to the bullwhip effect via information distortion, but ultimately will hurt the efficiency of the supply chain in the form of excess raw material inventory, misguided production schedules, missed target orders, and poor customer service levels. Predictably, more and more companies are adopting the various cutting-edge technologies like Electronic Data Interchange (EDI) to improve their forecasting ability and mitigate such uncertainty.

3) The service level sensitivity analysis:

Table 3.3 **Effects of Service Level on Order Promising**

Beta	X1	X2	X3	X4	Y	SUM(X)	TB
750	0	0	352.4	400	614.8	752.4	6108
760	0	0	359	400	614.7	759	6164
770	0	0	366	400	614.6	766	6221
780	0	0	373	400	614.4	773	6278
790	0	0	380	400	614.3	780	6334
800	0	0	387	400	614.2	787	6391

Figure 3.6 Total Committed Quantity of Confirmed Order as a Function of Service Level

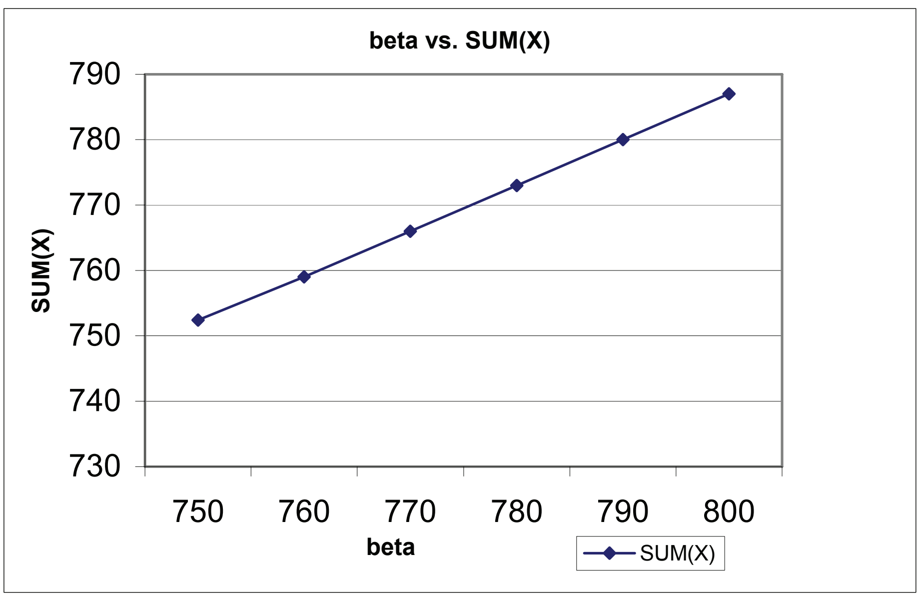


Figure 3.6 shows that the total commitment quantity of confirmed order increases with service level's weight. This reflects the significance of service level, which, like price, can be a driving force of commitment quantity. This observation allows us to integrate the parameter beta, the service level's weight, into a company's decision support system, providing a way to differentiate customers, e.g. in customer

relationship management (CRM) system. This provides an approach to offer superb service levels to the most valuable customers, develop strategies for unprofitable customers, and differentiate proactively the handling of different needs-based customer categories. Simply put, look for individual solutions rather than mass solutions.

3.5 Multiple Pseudo Orders in ATP

We further consider the case that captures more facts in real-life: a company not only has multiple confirmed orders, but also has multiple pseudo orders to fulfill. The stochastic characteristics associated with pseudo orders represent uncertain customer inquires and order cancellations. Particularly, we are interested to see what commitment decisions should be made if those pseudo orders' profit function is concave of the order quantity. This is very true in most of the mass-production environments such as lot-by-lot manufacturing process, or pallet-by-pallet transportation among semi-final and final assembly factories.

3.5.1 Notation

The problem under consideration is a single period, single product, multi-order ATP model. The model consists of N confirmed customer orders, which are assumed to be deterministic; and M pseudo orders, which are stochastic. The fundamental decision in the model is to determine promised order quantities for the confirmed customer orders and the reserved quantity for the pseudo orders by considering both production impact and material limitations. The objective of including the pseudo order is to anticipate near-term future customer orders based on customer inquiry information. The resultant model is a constrained non-linear stochastic programming problem.

In the model, we use "*total benefit*" as the objective function, which is defined as the sum of weighted expected profit and customer service level. This reflects common practice in most order fulfillment and optimization processes. The relative weights of expected profit and customer service level can be adjusted in the model to reflect their importance in specific business settings. Meanwhile, we assume the committed quantity of the confirmed order can never exceed the requested quantity. For the stochastic pseudo orders, holding costs may be incurred if the committed quantities are greater than the specified quantity.

Let $I = \{1,2,\cdots,N\}$ be the index of a set of the confirmed customer orders. For all $i \in I$, let q_i and r_i be the requested quantity and sales price of the confirmed order i, respectively; β_i a weighted constant of the customer service level for the confirmed order i. Let $J = \{1,2,\cdots,M\}$ be the index of a set of the pseudo orders, p_j and h_j the unit sales price and unit holding cost of the pseudo order *j*, respectively. u is the pseudo order quantity, which is a random variable with known probability density function (PDF) as $f(\cdot)$.

As described in section 3.2.1, let $g(\cdot)$ be the concave material cost function and $m(\cdot)$ be the convex production cost function. Also, let $s(\cdot)$ be the customer service benefit function. We model this function as a concave function, which would reflect a natural business perspective of increasing the per unit penalty for customer service deviations as the level of deviation increases (lower fill-rate are associated with higher deviations).

To formulate the committed quantity of the customer orders, we define the following decision variables. For all $i \in I$, let:

X_i equals the committed quantity of the confirmed order i;

Y_j equals the committed quantity of the pseudo order j;

3.5.2 Model Formulation

Based on the previous notation, the total revenue, which comes from both confirmed customer orders and the pseudo orders, can be formulated as

$$\sum_{i \in I}(r_i X_i) + \sum_{j \in J} p_j E\min(Yj, u), \tag{3.29}$$

where the first term is the confirmed order's revenue and the second term is the pseudo order's revenue. We assume that the production cost and material cost are both measured in terms of a unit of product. Then, the material used is proportional to the number of products committed to customers. The material cost and production cost are given by equations (3.30) and (3.31), respectively.

$$g(\sum_{i \in I} X_i + \sum_{j \in J} Y_j) \tag{3.30}$$

$$m(\sum_{i \in I} X_i + \sum_{j \in J} Y_j) \tag{3.31}$$

The holding cost incurred by the pseudo orders will be:

$$\sum_{j\in J} h_j E\max(Y_j - u, 0)\,. \tag{3.32}$$

Hence, the total expected profit is:

$$TP = \sum_{i\in I} r_i X_i + \sum_{j\in J} p_j E\min(Y_j, u) - g(\sum_{i\in I} X_i + \sum_{j\in J} Y_j)$$

$$-m(\sum_{i\in I} X_i + \sum_{j\in J} Y_j) - \sum_{j\in J} h_j E\max(Y_j - u, 0) \tag{3.33}$$

We note that the unit profit margin of an individual customer order could be negative like most discount sales. The reason to promise such orders is due to customer service level considerations. For the *i*th customer order, the customer service level, specifically, order fill rate, is X_i/q. Hence, the total customer service benefit from all customer orders can be represented as below:

$$\sum_{i\in I} \beta_i s(\frac{X_i}{q_i}) \tag{3.34}$$

in which β_i is the customer service benefit weight. The value of β_i indicates the importance of customer service in comparison to profit for the *i*th customer order.

Therefore, the objective function, which is defined as the "*reward function*" since it includes both profit and customer service level, can be written as:

$$TB = \sum_{i\in I} r_i X_i + \sum_{j\in J} p_j E\min(Y_j, u) - g(\sum_{i\in I} X_i + \sum_{j\in J} Y_j)$$

$$-m(\sum_{i\in I} X_i + \sum_{j\in J} Y_j) - \sum_{j\in J} h_j E\max(Y_j - u, 0) + \sum_{i\in I} \beta_i s(\tfrac{X_i}{q_i}) \tag{3.35}$$

subject to constraints described as following:

$$q_i - X_i \geq 0 \tag{3.36}$$

$$X_i \geq 0 \tag{3.37}$$

$$Y_j \geq 0 \tag{3.38}$$

Constraints (3.36), (3.37) and (3.38) are order-promising limitations and non-negativity.

Using the Lagrange method, the first order condition of the problem (3.35)-(3.38) can be given as follows for all $i \in I$.

$$r_i - g'(\sum_{i \in I} X_i + \sum_{j \in J} Y_j) - m'(\sum_{i \in I} X_i + \sum_{j \in J} Y_j) + \tfrac{\beta_i}{q_i} s'(\tfrac{X_i}{q_i}) - \lambda_i + \mu_i = 0 \tag{3.39}$$

$$p_j(1 - F(Y_j)) - g'(\sum_{i \in I} X_i + \sum_{j \in J} Y_j) - m'(\sum_{i \in I} X_i + \sum_{j \in J} Y_j) - h_j F(Y_j) + w_j = 0 \tag{3.40}$$

$$\lambda_i(q_i - X_i) = 0 \tag{3.41}$$

$$\mu \times X_i = 0 \tag{3.42}$$

$$\lambda_i \geq 0 \tag{3.43}$$

$$\mu_i \geq 0 \tag{3.44}$$

$$w_j \times Y_j = 0 \tag{3.45}$$

$$w_j \geq 0 \tag{3.46}$$

Where λ_i, μ_i and w are Lagrange multipliers for constraints (3.36), (3.37) and (3.38), respectively.

3.5.3 Model Analysis

Lemma 3.5.1 : *The profit function of pseudo orders is concave in terms of order quantity.*

Proof: See Appendix 5.

As stated earlier the objective function (3.35) is neither convex nor concave, but a *d.c. function*. We present the following Theorem to assure the existence of optimal solution for the problem (3.35)-(3.38). The proof is shown in Appendix 6.

Theorem 3.3: The objective function defined by (3.35) will be strictly concave on the feasible region defined by problem (3.36)-(3.38) if,

$$M > \frac{Max(LU, -K)}{n + m} \tag{3.47}$$

Where $U = \underset{0 \le x \le 1}{Max}\, s''(x)$; $L = \underset{1 \le i \le n}{Min} \frac{\beta_i}{q_i^2}$; $K = \underset{1 \le j \le m}{Min}(p_j + h_j) K_j$; $\text{Min} K_j = \underset{0 \le y \le y_j}{Min} f(y)$

with $y_j = F^{-1}(\frac{p_j}{p_j + h_j})$; and $M = \underset{0 \le z \le z_0}{Min}[g''(z) + m''(z)]$ with $z_0 = y_0 + \sum_{i=1}^{n} q_i$.

Corollary 3.5.2: There exists one and only one optimal solution for the nonlinear stochastic problem (3.35)-(3.38) if $g(\cdot)$ is a concave function, $s(\cdot)$ is a strictly concave function, and

$$g''(\cdot) - m''(\cdot) > 0 \tag{3.48}$$

Proof: Obvious from Theorem 3.3.

One can observe that condition (3.48) is the special case of condition (3.47). Theorem 3.3 gives conditions for the existence of an optimal solution. Condition (3.48) basically states that the total cost (production cost plus material cost) function should be convex enough to compensate for the concavity of projected customer service level functions. This is very much true when material supply is tight.

From an order promising point of view, we are more interested in the solution structure rather than the solution itself, since the solution structure can provide insights and guidance for optimal order promising. We present the following Theorem to illustrate the structure of the solutions for the problem (3.35)-(3.38).

Theorem 3.4:

For confirmed orders, we have:

Let $Z_{ij} = r_i - r_j$ and $\alpha_{ij} = \dfrac{\beta_i}{q_i} - \dfrac{\beta_j}{q_j}$. If we assume, $\forall i, j \in I$,

$[Z_{ij} + \alpha_{ij} s'(0)][Z_{ij} + \alpha_{ij} s'(1)] \geq 0$.

Then there exist two points $k, k' \in I$ $(k' \leq k)$ such that the optimal solution

$$X_i = \begin{cases} 0; & \forall i < k' \\ X_i^*; & \forall k' \leq i \leq k, \quad \text{Where } X_i^* \text{ solves equation} \\ q_i; & \forall i > k \end{cases}$$

$$s'\left(\frac{X_i^*}{q_k}\right) = \frac{q_i}{\beta_i}\left[g'\left(\sum_{l=1}^{n} X_l + \sum_{j=1}^{m} Y_j\right) + m'\left(\sum_{l=1}^{n} X_l + \sum_{j=1}^{m} Y_j\right) - r_i\right] \tag{3.49}$$

For pseudo orders: we have:

If we assume $p_1 \leq p_2 \leq \cdots \leq p_m$, then there exists $k'' \in I$ such that

$$Y_j = \begin{cases} 0; & \forall j < k'' \\ Y_j^*; & \forall j \geq k'' \end{cases}, \text{ where } Y_j^* \text{ solves equation}$$

$$F(Y_j^*) = \frac{p_j - g'(\sum_{i=1}^{n} X_i + \sum_{l=1}^{m} Y_l) - m'(\sum_{i=1}^{n} X_i + \sum_{l=1}^{m} Y_l)}{p_j + h_j} \tag{3.50}$$

Proof: see Appendix 7.

Theorem 3.4 provides a simple structure for the optimal solution for both confirmed orders and pseudo orders: some customer orders should be one hundred percent committed, and some others should not be committed at all. More specifically, confirmed orders' commitment quantities are determined by *K* and *K*' and pseudo orders' commitment quantities are determined by K. Therefore we only need to compute *K*, *K*' for confirmed orders and *K"* for pseudo orders.

The interpretation for Z_{ij} and β_i are the ratio between profit and customer service level, and Theorem 3.4 just states how a particular customer order promising scheme should be adopted depending on that ratio.

Corollary 3.5.3: For $\forall i \in I$, if the following condition holds,

$$\frac{\beta_i}{q_i} s'(0) + r_i < \frac{\beta_{i+1}}{q_{i+1}} s'(1) + r_{i+1}, \tag{3.51}$$

then the optimal solution for problem (3.35)–(3.38) would be as given in Theorem 3.4 with $k = k'$. Moreover, we have $k'' \geq \arg\min\{j \mid p_j > g'(q^*) + m'(q^*), j \in J\}$, where

$q^* = \sum_{i=k+1}^{n} q_i$.

Proof: see Appendix 8.

From this Corollary, one can observe that the optimal order commitment policy is decided by only one order commitment, and the others becomes either 0 or q_i if the condition (3.51) is satisfied. That quantity can be found by simply searching the linear solution space *I+J.* It gives us a simplified policy to make the optimal order promising for customers.

3.6 Remarks

It is clear to see that demand uncertainty will not only lead to the bullwhip effect via information distortion, but ultimately will hurt the efficiency of a supply chain in the form of excess raw material inventory, misguided production schedules, missed target orders, and poor customer service level. Our analytical ATP model, as the core of customer-driven order management, indeed provides the manager with useful insights to conquer the resulting problems. Our models can provide guidance in both customer-service dominant and profit dominant business environments.

There are two other types of approaches to pseudo orders promising: a traditional simulation-based approach and the more recent "scenario generation" approach. The simulation-based approach for promising the delivery of future orders is based on dynamic buffer adjustment coupled with forecasting the amount of buffer required. The primary objective of that is to frame the problem and suggest methods of analysis. Since this approach is simulation-based, optimal policies or solutions are not obtained; simulation run times can also grow very large, e.g. from tens of minutes to several days to run one data set. Grant and Moses (2002) applied this approach but have to deal with an issue that inherently exists in this methodology: multiple yet conflicting performance measures, such as average lateness to promise, variance of the lateness to promise, and constraining the probability of a failed promise. We can see, all of these ask for more rather than less judgment calls from decision-makers. We should also note that systems based on simulation are not only extremely sensitive to the distribution of the process times but also the congestion (arrivals of orders) of the system. Meixell and Chen develop a "scenario generation" approach for efficiently incorporating uncertainty about future demand into an ATP system, so that demand scenarios with their associated probability distributions may be initially established and then continually revised. They first define demand as a random variable and then estimate the parameters of its distribution to define the uncertainty associated with demand for future order. From this distribution they derive scenario probabilities based on expected values and variances associated with the demand model for each order type, and finally to assign values for each of the branches of the scenario tree. They use "scenario generation" methods to convert *stochastic* pseudo orders into a *pre-defined* order types so that a dynamic linear model, instead of stochastic programming model, can be built. Further, in this approach, only a procedure for generating demand scenarios that is compatible with a stochastic mixed-integer-programming (MIP) model is developed, neither optimal solution structures nor optimal order promising policies are presented.

In contrast to these two approaches, we derive a new stochastic programming-based customer order promising scheme along with efficient algorithms to implement it. These rule-based algorithms can be solved in real-time, and provide decision makers a clear answer: accept or reject certain orders based on a pre-set ratio of profit and customer service level without having to provide estimates of order arrivals.

Chapter 4

Coordinating Demand Fulfillment with Supply

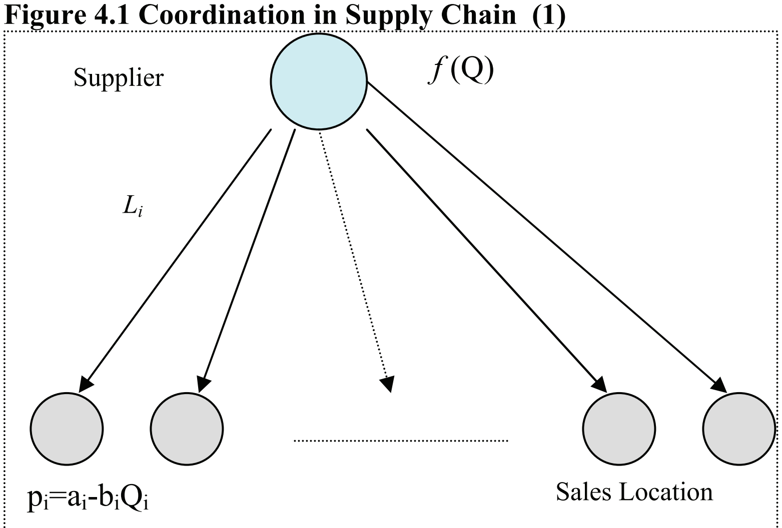
In the current supply chains, suppliers, manufacturers and retailers are globally dispersed. Material cost differs among suppliers; production cost varies across manufacturers, and transportation cost depends on locations. On the other hand, as customers' demand for the end products varies across the sales locations and is highly correlated to the sales price, the revenues are also different among paths. Consequently, the same product might be profitable along one path but unprofitable along another path. The choice of supplier or manufacturer can provide location–specific cost information and directly identify profitable and non–profitable products. In this section, we will answer two related questions: 1) How to coordinate demand fulfillment with supplies through the right path? 2) How to manage the available resource through the coordination of the raw material discount and end sales based on path analysis? First, we build and solve the analytical models, then provide the results of numerical experiments.

4.1 Analytical Model with Deterministic Demand Curve

4.1.1 Model & Formulation

The problem under consideration below is a single time period, single product, multiple-tier supply chain model. To simplify the deriving process, we assume the model consists of one manufacturer and n sales locations. The manufacturer buys raw materials from the supplier, and after assembling, sells finished products to end-customers across all sales locations. In most situations, the supplier will offer price discounts to buyers if their purchase quantity increases and therefore total purchase cost is concave. For every sales location we assume the demand curve is different. A supply chain path is from supplier to sales location. (See Figure 4.1). The fundamental decision in the model is to derive optimal purchase quantity of the raw materials and sales quantity of the end products in every location to maximize the profit of the company. This is a constrained non-linear programming problem.

Figure 4.1 Coordination in Supply Chain (1)



Below is the notation which will be used in the model formulation.

Notation:

Let $I = \{1,2,\cdots,N\}$ be the index set of the sales locations. For all $i \in I$, let the path unit cost to be l_i and the demand curve be: $p_i = a_i - b_i Q_i$ (We use this format just to simplify the following deductive process. Of course, it is equivalent to the widely used format: $Q_i = \frac{a_i}{b_i} - \frac{1}{b_i} p_i$). p_i is the sales price. Let the purchase cost function be $f(\cdot)$. It is a concave function of the purchased quantity. So the decision variables are:

Q: the raw material purchase quantity.

Q_i: the quantity of finished product sold at location i.

We assume one raw material unit per finished product unit, so that:

$$\sum_{i=1}^{n} Q_i = Q$$

Model Formulation:

Based on the previous notation, the total revenue can be formulated as

$$\sum_{i \in I} p_i Q_i = \sum_{i \in I} (a_i - b_i Q_i) Q_i \tag{4.1}$$

The raw material purchase cost and the path cost is given by expressions (4.2) and (4.3), respectively.

$$f\left(\sum_{i \in I} Q_i\right) \tag{4.2}$$

$$\sum_{i \in I} l_i Q_i \tag{4.3}$$

Hence, the total profit is:

$$TP = \sum_{i \in I} (a_i - b_i Q_i) Q_i - f\left(\sum_{i \in I} Q_i\right) - \sum_{i \in I} l_i Q_i \tag{4.4}$$

We reorganize the formula and have the objective function:

$$TP = \sum_{i \in I} (a_i - l_i - b_i Q_i) Q_i \text{ - } f(\sum_{i \in I} Q_i) \tag{4.5}$$

Subject to constraints described as following:

$$Q_i \geq 0 \tag{4.6}$$

From the Lagrange method, the first order condition of problem (4.5)-(4.6) can be given as following: for all $i \in I$.

$$\lambda_i = a_i - l_i - 2b_i Q_i - f'(\sum_{i \in I} Q_i) \leq 0$$

$$\lambda_i Q_i = 0 \tag{4.7}$$

Where λ_i is Lagrange multiplier for constraint (4.6).

4.1.2 Model Analysis

Theorem 4.1: Let $(a_i \text{ - } l_i)$ be in increasing order of *i*. There exists a point $k \in I$, the optimal solutions of problem (4.5)–(4.6) have the following structure:

$$Q_i = 0 \qquad \text{for all } i \leq k,$$

$$Q_i = \frac{a_i - l_i - f'(\sum_{i \in I} Q_i)}{2b_i} \qquad \text{for all } i > k.$$

Proof: see the Appendix 9.

We have observed that $f'(\sum_{i \in I} Q_i)$ is the marginal purchase cost, and we can clearly see how the material price discounts would have an impact on the end sales Q_i based on the numerical experiments we conducted in Chapter 4.4. However, Theorem 4.1 reveals in close-form how the discount offered by a supplier can have an impact on profit and purchased quantity. It will not only change the Q_i ($i > k$), but it will also change the Q_i ($i \le k$). It gives us a formula, i.e. $a_i - l_i - f'(\sum_{i \in I} Q_i)$, to identify "profitable" and "non-profitable" demand, and only "profitable" demand will be fulfilled and "non-profitable" demand will not be fulfilled -- a conclusion which is different from the one shown in Chapter 4.4 because we don't have a "minimum fill rate" constraint in this case. Furthermore, since material cost function $f(\cdot)$ is a concave function, i. e. $f'(\cdot) < 0$, the following applies: when the supplier offers discount, $f'(\cdot)$ decreases and Q_i increases, which is in line with the one in Chapter 4.4. The other mechanism being investigated in Chapter 4.4 is offering of discount on profitable products, or the equivalent of the following: If we reduce b_i, then from Theorem 4.1 Q_i will increase, which is also in line with conclusion of Chapter 4.4: Offering discount for profitable products will increase total purchased component quantity.

From this Theorem we can have the following steps to derive the optimal Q_i.

Algorithm:

Let $\mu_1 = \sum_{i=k+1}^{n} \frac{a_i - l_i}{2b_i}$, $\mu_2 = \sum_{i=k+1}^{n} \frac{1}{2b_i}$.

Beginning

Step 1: Let $k = 1$

Step 2: Derive the MC by solving the equation: $f'(\mu_1 - \mu_2 \cdot MC) = MC$

Step 3: Check if the MC satisfy the assumption:

$$a_1 - l_1 \le \dots \le a_k - l_k \le MC < a_{k+1} - l_{k+1} \le \dots \le a_n - l_n \tag{4.8}$$

If Yes, go to step 4; otherwise go to step 5.

Step 4: Check if the Hessian matrix at this point is negative definitive. If so, store this solution, go to step 5; otherwise, go to step 5 without storing solution.

Step 5: If $k < n$, let $k = k + 1$ go to step 2. If $k = n$, go to step 6;

Step 6: Choose the solution with maximal objective value from the stored, this is the optimal solution.

End. (See Figure 4.2)

Since $f(\cdot)$ is a concave function, we have $f''(\cdot) \le 0$. Further, if the $f(\cdot)$ satisfy: $-\frac{2}{\sum_{i \in I} \frac{1}{b_i}} < f''(\cdot) < 0$, we can have the following conclusion:

Remark 4.1: Let the $(a_i - l_i)$ in increasing order of i. If the $f(\cdot)$ satisfy: $-\frac{2}{\sum_{i \in I} \frac{1}{b_i}} < f''(\cdot) < 0$, there exists one and only one point $k \in I$, the optimal solutions of problem (5)–(6) have the following structure:

$$Q_i = 0 \qquad \text{for all } i \le k,$$

$$Q_i = \frac{a_i - l_i - f'(\sum_{i \in I} Q_i)}{2b_i} \qquad \text{for all } i > k.$$

Proof: see the Appendix 10.

With Remark 4.1, we can simplify the solution process as following:

Beginning

Step 1: Let $k = 1$

Step 2: Derive the MC by solving the equation: $f'(\mu_1 - \mu_2 \cdot MC) = MC$,

Step 3: Check if the MC satisfies the assumption:

$$a_1 - l_1 \leq \ldots \leq a_k - l_k \leq MC < a_{k+1} - l_{k+1} \leq \ldots \leq a_n - l_n$$

If it does, stop, this is the optimal solution. Otherwise, let $k = k + 1$ go to step 2.

End. (See Figure 4.3)

4.1.3 Numerical Experiments

Based on the analysis above, the most important factors in our model that determine the *structure* of optimal solution to fulfill demand would be: maximal sales price a_i and unit transportation cost l_i. The price elasticity b_i and purchase cost function $f(\cdot)$ would only influence the optimal value of demand fulfillment. In the following experiments we show how the structure of optimal solution is in line with Theorem 4.1; then we run multiple scenarios for some sensitivity analysis. Assuming we have 3 sales locations, i.e. i =3. With demand curve $p_i = a_i - b_i Q_i$, the coefficients are shown in Table 4.1 below:

Table 4.1: Experiment Coefficients

Parameters/ Locations	**1**	**2**	**3**
Maximal Price: (a_i)	4	6	10
Price Elasticity (b_i)	1.8	2	1.9
Transportation Cost (l_i)	0.12	0.11	0.15

Thus $(a_i - l_i)$ is in increasing order of i. Further, let the purchase cost function $f(Q) = 6\sqrt{Q}$, which is an increasing concave function.

Figure 4.2 Coordination Algorithm (1)

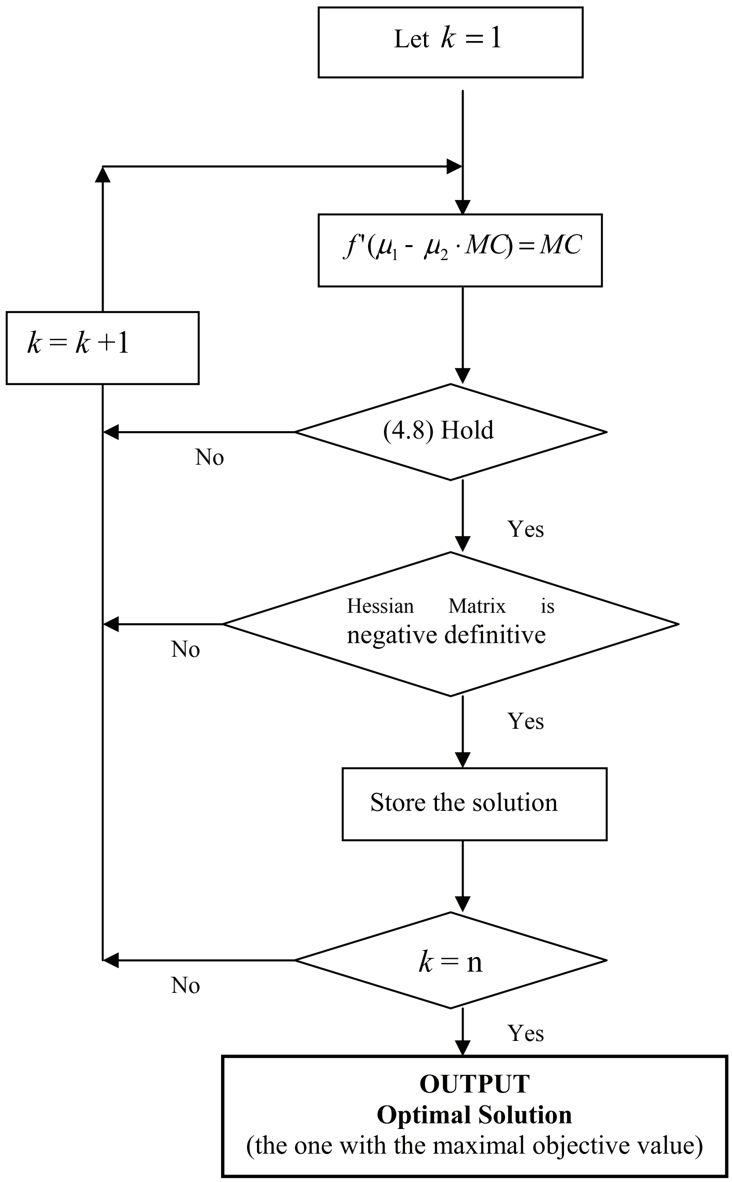
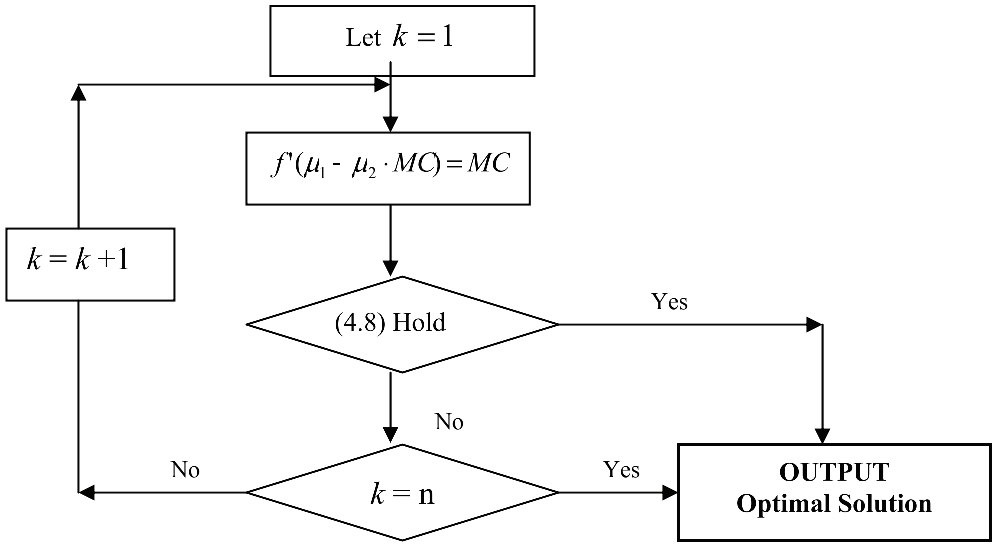


Figure 4.3 Coordination Algorithm (2)



First, we solve the problem analytically (4.5 - 4.6) by Theorem 4.1 and algorithm shown in Figure 4.2, the optimal solution is: (Q_1, Q_2, Q_3) = (0, 0, 2.05), the objective function value (*total profit)* is 3.62. Now, let us examine the numerical experiments to determine where our optimal solution stands in terms of the possible different solution structures: (0, 0, Q_3), (0, Q_2, Q_3), (Q_1, 0, Q_3), (Q_1, Q_2, Q_3), which are shown in Table 4.2 - 4.5 and the corresponding Figure 4.4 – 4.7 , respectively. Figure 4.4 clearly shows that as Q_3 increases, total profit increases, but it begins to decline beyond a certain value: 2.05. This occurs because, in order to have more demand, the charged price has to be at a lower level. Note that after sales quantity reaches 2.05, the additional profit can not offset the lost margin and increased transportation cost anymore. Figure 4.5 – 4.7 show that if we increase Q_1 or Q_2, or both from 0 to some value without following Theorem 4.1, the total profit would indeed decrease. The reason that the additional fulfilled demand quantity can not overcome the loss in revenues is because $(a_i - l_i)$ is too low to be profitable. In fact, under such situation the more Q_1 or Q_2 sold, the less profit generated. Overall, these experiments numerically demonstrate the implication of the solution structure in the Theorem and Remark 4.1. Price elasticity and purchase cost function only affect the optimal solution's value. However, more importantly, the factors that determine the fundamental structure of the optimal solution is the maximum price a_i and transportation unit cost l_i.

Now we examine some other interesting findings from further experiments. First, we

Table 4.2: Q_3 vs. Total Profit

Q_1	Q_2	Q_3	$SUM(Q)$	TP
0	0	1.9	1.9	3.59
0	0	1.95	1.95	3.6
0	0	2	2	3.61
	0	**2.05**	**2.05**	**3.62**
0	0	2.1	2.1	3.61
0	0	2.15	2.15	3.6
0	0	2.2	2.2	3.57

Figure 4.4: Q_3 vs. Total Profit

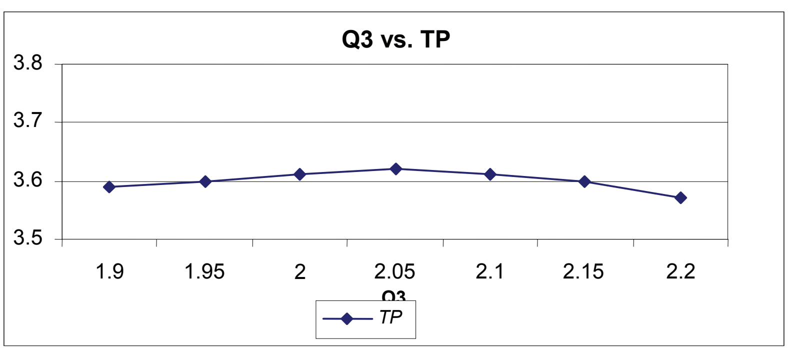


Table 4.3: Q_2, Q_3 vs. Total Profit:

Q_1	Q_2	Q_3	$SUM(Q)$	TP
0	**0**	**2.05**	**2.05**	**3.62**
0	1.05	2.05	3.1	3.52
0	1.1	2.05	3.15	3.42
0	1.15	2.05	3.2	3.3
0	1.2	2.05	3.25	3.18
0	1.25	2.05	3.3	1.58
0	1.3	2.05	3.35	1.45

Figure 4.5: Q_2, Q_3 vs. Total Profit

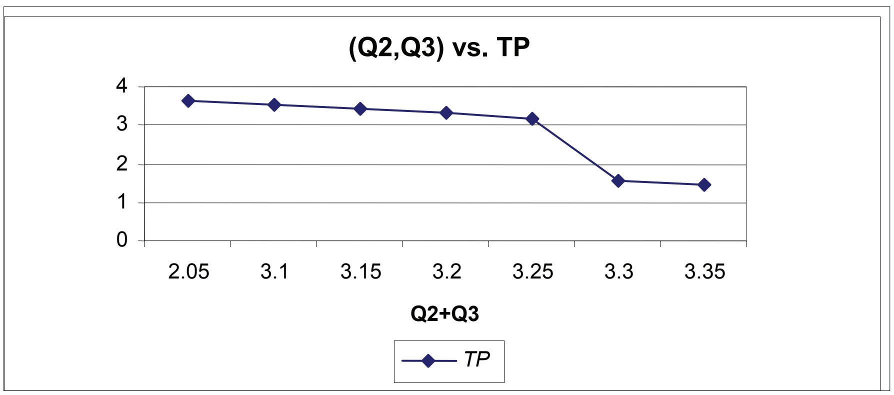


Table 4.4: Q_1, Q_3 vs. Total Profit:

Q_1	Q_2	Q_3	$SUM(Q)$	TP
0	**0**	**2.05**	**2.05**	**3.62**
1.05	0	2.05	3.1	1.63
1.1	0	2.05	3.15	1.45
1.15	0	2.05	3.2	1.26
1.2	0	2.05	3.25	1.06
1.25	0	2.05	3.3	0.85
1.3	0	2.05	3.35	0.63

Figure 4.6: Q_1, Q_3 vs. Total Profit

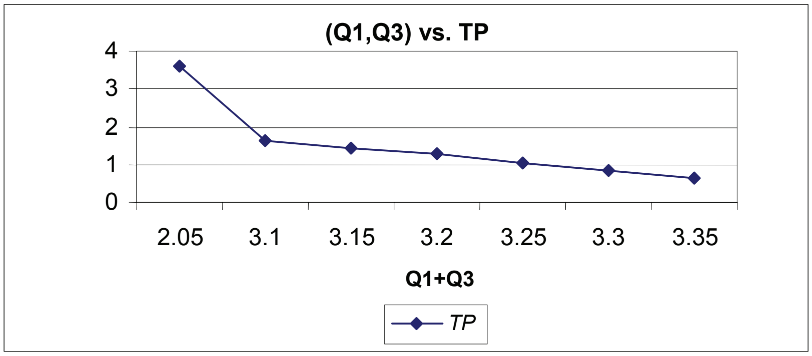
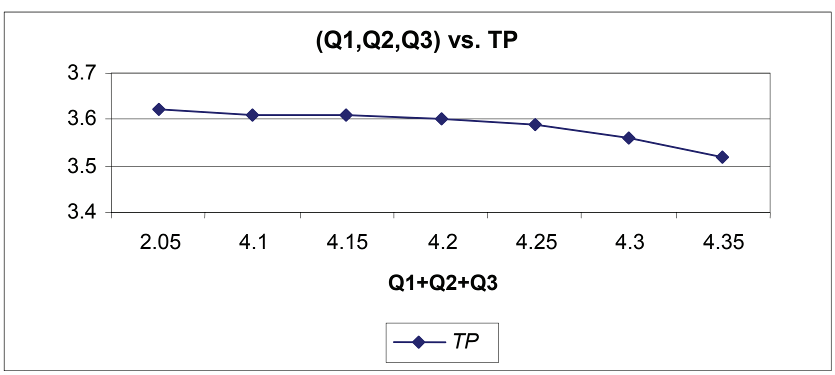


Table 4.5: Q_1, Q_2, Q_3 vs. Total Profit:

Q_1	Q_2	Q_3	$SUM(Q)$	TP
0	**0**	**2.05**	**2.05**	**3.62**
1.05	1	2.05	4.1	3.61
1.1	1	2.05	4.15	3.61
1.15	1	2.05	4.2	3.6
1.2	1	2.05	4.25	3.59
1.25	1	2.05	4.3	3.56
1.3	1	2.05	4.35	3.52

Figure 4.7: Q_1, Q_2, Q_3 vs. Total Profit

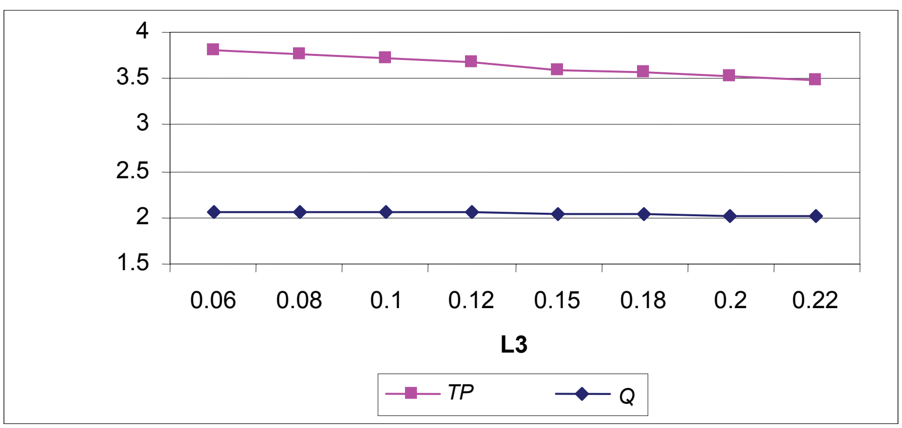


examine the effect of transportation cost l_3. Not surprisingly, with transportation cost increasing, both fulfilled demand quantity and total profit decrease, and its effect on profit is more significant since it results in both a decrease in fulfilled demand and an increase in cost, as shown in Figure 4.8. Note that as long as the $(a_i - l_i)$ is increasing in the order of i, and the condition in the Remark 4.1 holds, the solution still has Q_1 and Q_2 at 0.

Table 4.6: Effect of Transportation Cost l_3 $(l_1 = 0.12,\ l_2 = 0.11)$

l_3	Q_1	Q_2	Q_3	TP
0.06	0	0	2.068	3.8
0.08	0	0	2.062	3.76
0.1	0	0	2.058	3.72
0.12	0	0	2.053	3.69
0.15	0	0	2.05	3.62
0.18	0	0	2.03	3.57
0.2	0	0	2.028	3.51
0.22	0	0	2.025	3.44

Figure 4.8: Effect of Transportation Cost l_3



Next, we look at the impact of the maximal unit price that a firm can charge on demand quantity and profit by varying a_3 from 8 to 14. These are shown in Table 4.7 and Figure 4.9. They display both Q and total profit increase as maximal price a_3 increases. This is not surprising; as matter of the fact, maximal price reflects the customers' desire and affordability of the product. Total profit increases more significantly than Q due to the dual effects of raising both Q and price. Again, we should notice that $(a_3 - l_3)$ in this scenario is always larger than $(a_1 - l_1)$ and $(a_2 - l_2)$, and in optimal solutions Q_1 and Q_2 remain at 0, which is exactly as Theorem 4.1 points out.

Table 4.7: Effect of Maximal Price a_3 ($a_1 = 4,\ a_2 = 6$)

a_3	Q_1	Q_2	Q_3	TP
8	0	0	1.4	0.17
9	0	0	1.8	1.72
10	0	0	2.05	3.62
11	0	0	2.35	5.81
12	0	0	2.65	8.29
13	0	0	2.96	11.07
14	0	0	3.2	14.13

Figure 4.9: Effect of Maximal Price a_3

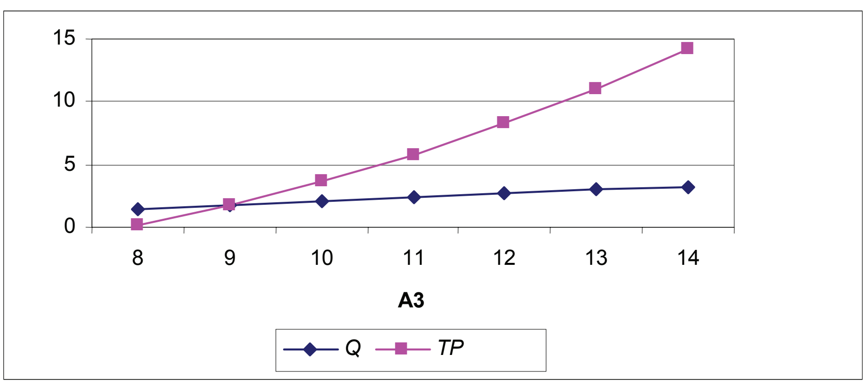


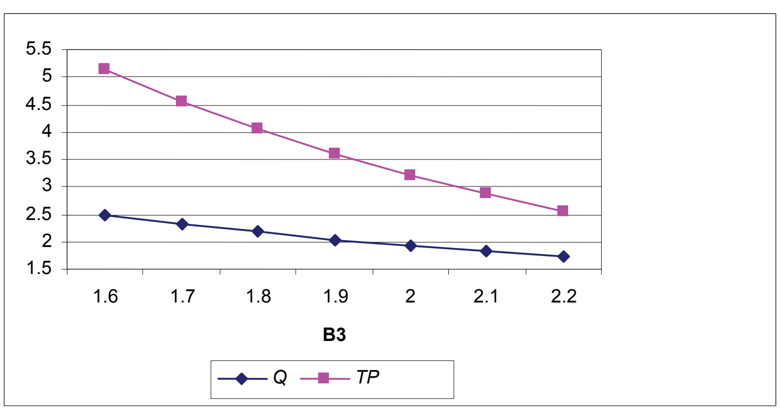
Table 4.8 and Figure 4.10 show the optimal solution of fulfilled demand quantity and total profit as a function of price elasticity. For various levels as price elasticity, b_3, increases, both Q and total profit decrease, and profit decreases more steeply due to the same dual effects as mentioned in maximal price discussion. Further, if we

compare Figure 4.8, 4.9 and 4.10, we can see that price-related coefficients, i. e., maximal price a_3 and price elasticity b_3, have a more significant effect on profit than transportation cost, does as evidenced by the “flat” curve in Figure 4.8 vs. “steep” curve in Figure 4.9 and 4.10. This occurs because when end customers are buying something, they are always more sensitive to visible price than transparent transportation cost. In addition, in most cases, price accounts for a larger portion of the cost than the transportation cost itself. Thus the firm should carefully determine the quantity of demand to fulfill as demonstrated in Theorem 4.1 and shown in the results above. Fulfilling more demand doesn’t necessarily mean more profit is secured. Sometimes it may even hurt the business.

Table 4.8: Effect of Price Elasticity’s b_3 (b_1=1.8, b_2 = 2)

b_3	Q_1	Q_2	Q_3	TP
1.6	0	0	2.49	5.14
1.7	0	0	2.32	4.56
1.8	0	0	2.18	4.06
1.9	0	0	2.05	3.62
2	0	0	1.92	3.22
2.1	0	0	1.82	2.88
2.2	0	0	1.72	2.56

Figure 4.10: Effect of Price Elasticity’s b_3



4.2 Analytical Model with Stochastic Demand

In Section 4.1, we assumed that the demand is price-dependent with different demand curves at each sales location. Therefore no inventory cost is considered. This scenario is true as stated in the theory of monopolistic competition (Chamberlain and Robinson 1954), where the demand curve of a monopoly firm is downward sloping and the inventory will not be an issue when price can be used as a decision tool.

However, if one company did not posses the monopolistic power, e.g., under fierce global competition, it has to face the uncertainty of customer demand. In such cases, inventory holding to support sales becomes significant. Effectively managing and minimizing the inventory holding cost while satisfying customer demands could provide a competitive advantage to any firm in the marketplace. Besides the inventory cost, we also take into account the lost sales (supply shortage) penalty that would be incurred when demand can not be met. We further assume that the uncertain demand on each path can have different distributions. The fundamental decision in this constrained stochastic model is still to derive an optimal purchase quantity for the raw materials and a sales quantity for the end products in every location to maximize the profit of the company.

A body of literature closely related to this topic is newsvendor-like problems, which are to find the order quantity which maximizes the expected profit in a single-period probabilistic demand framework. By assuming that the retailers' demand obeys a normal or lognormal distribution and that the retailers place orders according to the Newsvendor Rule, researchers derive the necessary and sufficient conditions for the optimal solution of production size (Khouja, 2000; Z.Weng 2003). Our model differs from theirs on: 1) It is more generalized and all demands are equally treated; (Unlike Cattni (2000)'s paper, we don't need hierarchical priority in the model.) 2) it can be applied to any unknown-but-bounded disturbances; 3) with both manufacturers and retailers in the picture, it is a constrained stochastic problem.

We also would like to point out that this model is different from a multi-inventory system. Most papers that have discussed multi-inventory systems have assumed that the retailer's ordering policy is only related to the demand and supplier but unrelated to the other retailers. Some supply-chain and inventory models use the following two-echelon symmetric-information and deterministic gaming structure: a ''manufacturer'' wholesales a product to a ''retailer,'' who in turn retails it to the consumer. The retail market demand varies either with the retail price according to a deterministic ''demand function'' that is known to both the manufacturer and the retailer or with the known distribution. The manufacturer is a Stackelberg leader and the retailer is a Stackelberg follower (from Franco Blanchini etc. 2004). Thus they completely separate the "market channel" from "manufacturing channel" to make decisions. Under the approach described, in the next section, we derive an optimal promising policy from a centralized supply chain's point, in which the company considers the "market channel" and "manufacturing channel" together as a *path*.

4.2.1 Model & Formulation

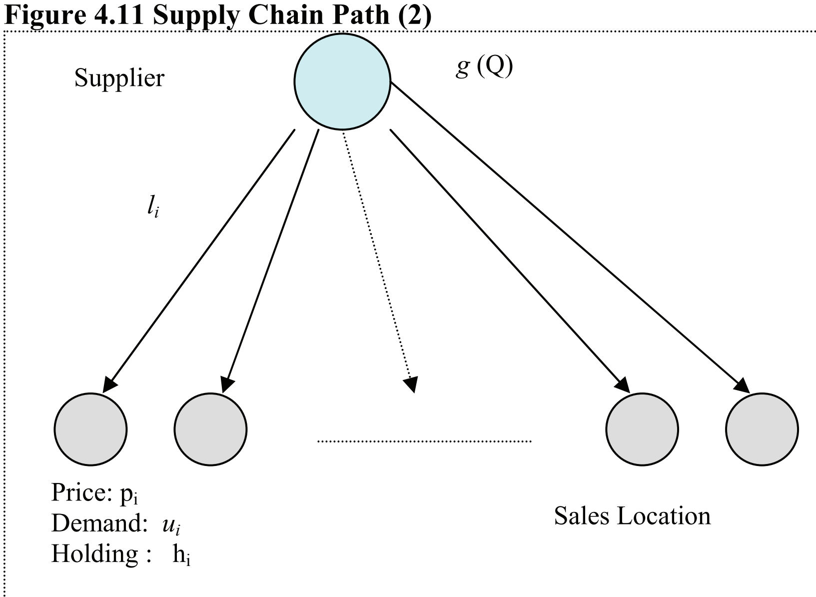
The problem under consideration is a single time period, single product, two-echelon supply chain system. As before, we assume there is one supplier and n sales locations. The manufacturer buys raw materials from the supplier, and after assembling, sells finished products to end-customers via sales locations. In practical situations, the supplier usually offers price discounts to manufactures if their purchase quantity increases and therefore total purchase cost is concave (the cost function is denoted by $g(\cdot)$). The supply chain *path* is the path from one manufacturer to sales location (Figure 4.11). Let $I = \{1,2,\cdots,N\}$ be the index set of the sales locations. For all $i \in I$, let the path unit cost be l_i and the demand of each market as u_i, which is a random variable with known probability density function (PDF) denoted by $f(\cdot)$ and *c. d. f.* denoted by $F(\cdot)$. p_i is the sales price in each market. We assume one raw material unit per finished product unit.

Our decision variables are:

Q: the raw material purchase quantity.

S_i: the quantity of finished product sold at location i.

Figure 4.11 Supply Chain Path (2)



Formulation:

Revenue: $\sum_i p_i \min(S_i, u_i) = \sum_i p_i (\int_0^{S_i} u_i f(u_i) du_i + \int_{S_i}^{\infty} S_i f(u_i) du_i)$

Purchase Cost: $g(Q)$, $g(\cdot)$ is a concave function

Transportation Cost: $\sum_i l_i S_i$

Holding Costs on sales location:

$$\sum_i h_i \times \max(S_i - u_i, 0) = \sum_i [h_i \int_0^{S_i} (S_i - u_i) f(u_i) du_i]$$

Shortage Cost: $\sum_i d_i \times \max(u_i - S_i, 0) = \sum_i [d_i \int_{S_i}^{\infty_i} (u_i - S_i) f(u_i) du_i]$

We would like to:

Maximize:

Profit = Revenue - Purchase Cost - Transportation Cost - Holding Costs – Shortage cost

So the problem now is:

$$\text{Max: } \sum_i p_i (\int_0^{S_i} u_i f(u_i) du_i + \int_{S_i}^{\infty} S_i f(u_i) du_i) - g(Q) - \sum_i l_i S_i - \sum_i [h_i \int_0^{S_i} (S_i - u_i) f(u_i) du_i] -$$

$$\sum_i [d_i \int_{S_i}^{\infty} (u_i - S_i) f(u_i) du_i]$$

s. t. $Q - \sum_i S_i \geq 0$

$S_i \geq 0$,

$Q \geq 0$

It is equivalent to:

$$\text{Min: } \sum_i (p_i + h_i + d_i) \int_0^{S_i} (S_i - u_i) f(u_i) du_i + g(\sum_i S_i) - \sum_i (p_i + d_i - l_i) S_i \tag{4.9}$$

$$\text{s. t.} \quad S_i \ge 0 \tag{4.10}$$

4.2.2 Model Analysis

From the Lagrange method, the first order condition of this problem can be given as the following: Kuhn-Tucker condition for all $i \in I$.

$$\lambda_i = (p_i + d_i + h_i) W_i^{'}(S_i) + g^{'}(\sum_i S_i) - (p_i + d_i - l_i) \ge 0 \tag{4.11}$$

$$\lambda_i S_i = 0$$

$$\lambda_i \ge 0$$

$W_i(S_i)$ is defined by $E_i[S_i - u_i]^+ = \int_0^{S_i} (S_i - u_i) f(u_i) du_i$. Assume $W_i(\cdot)$ is everywhere differentiable, we have: $W_i'(S_i) = F(S_i)$. It's easy to show: $W_i(0) = W_i^{'}(0) = 0$, and $W_i(\cdot)$ is convex (see Appendix 11).

Theorem 4.2: Let the $(p_i + d_i - l_i)$ in increasing order of *i*. There exists a point $k \in I$ such that the *optimal* solutions of problem (4.9)–(4.10) has the following structure:

$$S_i = 0 \qquad \text{for all } i \le k, \tag{4.12}$$

$$F(S_i) = \frac{(p_i + d_i - l_i) - g'(\sum_{i \in I} S_i)}{p_i + d_i + h_i} \qquad \text{for all } i > k. \tag{4.13}$$

Proof: see Appendix 12.

We can further show that an upper bound exists for Q (see Appendix 13), which reveals that, even with a discount on raw material, it would not be foolish to limit the total quantity purchased. Unsurprisingly, the transportation cost l_i, shortage penalty d_i and sales price p_i have an impact on the decision. Also, we notice that $g'(\sum_{i \in I} S_i)$ can be interpreted as the *marginal purchase cost*. From Theorem 4.2 we can easily see it is a decisive factor of the end sales S_i. Another interesting finding is that inventory cost h_i has no effect at the first stage where we determine whether the finished product should be sold at one location or not. It only plays a role at the second stage where we determine the sold quantity S_i (for $i > k$).

4.2.3 Algorithm:

BEGIN:

Step 1: Let $k = 1$

Step 2: Derive the S_i by solving the equations (4.12)-(4.13).

Step 3: Check if the $g'(\sum_i S_i)$ satisfy the assumption:

$$p_1 + d_1 - l_1 \leq \ldots \leq p_k + d_k - l_k \leq g'(\sum_i S_i) < p_{k+1} + d_{k=1} - l_{k+1} \leq \ldots \leq p_n + d_n - l_n \tag{4.14}$$

If so, go to step 4; otherwise go to step 5.

Step 4: Check if the Hessian matrix at this point is negative definitive. If so, store this solution, go to step 5; otherwise, go to step 5 without storing.

Step 5: If $k < n$, let $k = k + 1$ go to step 2. If $k = n$, go to step 6;

Step 6: Choose the solution with maximal objective value from the stored, this is the optimal solution.

END. (see the Figure 4.12)

Remark 4.2: Let the $(p_i + d_i - l_i)$ be increasing i. If

$$\sum_i \frac{1}{(p_i + h_i + d_i)} > UF \times UG$$

where UF and UG is the upper bound of $f(\cdot)$ and - $g''(\cdot)$ respectively, then there exists a unique $k \in I$ for which the optimal solutions of problem (4.9) – (4.11) have the following structure:

$$S_i = 0 \qquad \text{for all } i \leq k,$$

$$F(S_i) = \frac{(p_i + d_i - l_i) - g'(\sum_{i \in I} S_i)}{p_i + d_i + h_i} \qquad \text{for all } i > k.$$

Proof: see the Appendix 14.

Using Remark 4.2, we can state the following simplified algorithm:

Algorithm 2:

BEGIN:

Step 1: Let $k = 1$

Step 2: Derive the S_i by solving the equations (4.12)-(4.13).

Step 3: Check if the $g'(\sum_i S_i)$ satisfy the assumption:

$$p_1 + d_1 - l_1 \leq \ldots \leq p_k + d_k - l_k \leq g'(\sum_i S_i) < p_{k+1} + d_{k=1} - l_{k+1} \leq \ldots \leq p_n + d_n - l_n$$

If so, Stop; Otherwise, let $k = k + 1$ go to step 2.

END. (see the Figure 4.13)

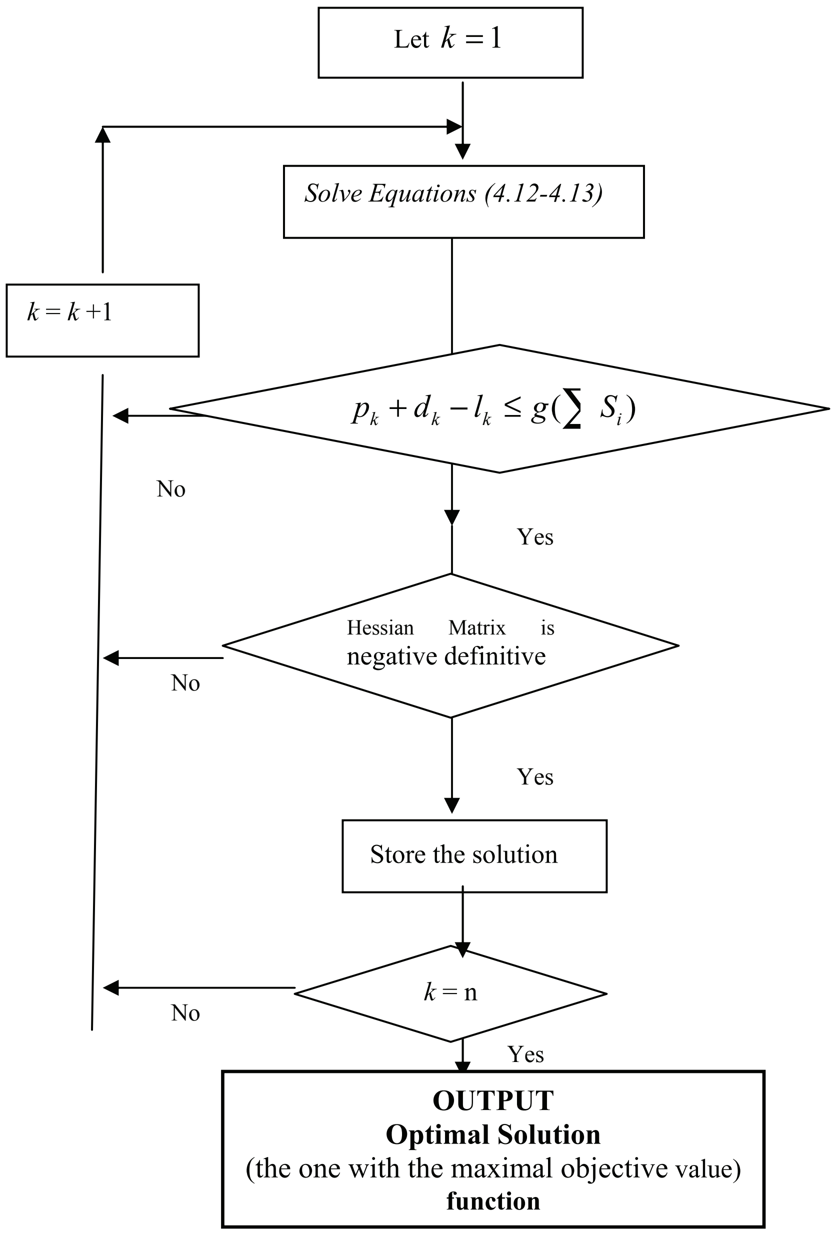


Figure 4.12. Coordination Algorithm (3)

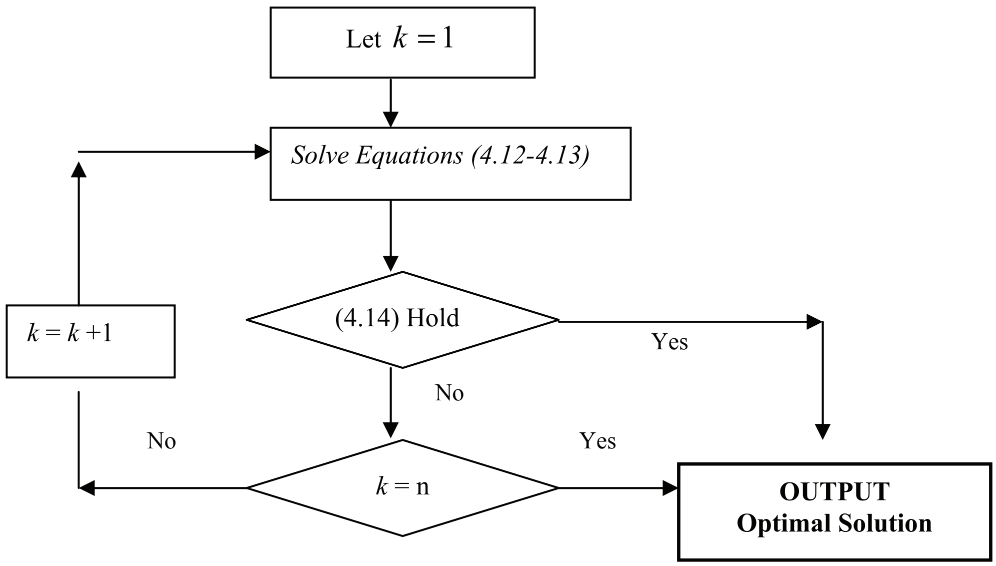


Figure 4.13. Coordination Algorithm (4)

4.2.4 Numerical Experiments

Based on the analysis above, the most important factors in our model that determine the *structure* of optimal solution to fulfill demand are: sales price p_i, unit transportation cost l_i, and unit shortage cost d_i. Holding cost h_i and purchase cost function $f(\cdot)$ would only influence the optimal value but not the structure. In the following experiments we show how the structure of optimal solution is in line with Theorem 4.2; then we run multiple scenarios for some sensitivity analysis. Assuming we have 3 sales locations, i.e. i = 3. The demand of each market u_i is a random variable with known probability density function (PDF), $f(\cdot)$, and *c. d. f.* , $F(\cdot)$. Assume $f(\cdot)$ is the widely-used lognormal distribution with *E(u)* = 300, and *std(u)* =

60. In addition, let the purchase cost function $f(Q) = 6\sqrt{Q}$, which is an increasing concave function. The other coefficients are shown in Table 4.9 below:

Table 4.9: Experiment Coefficients

Parameters/ Locations	**1**	**2**	**3**
Sales Price: (p_i)	4	8	10
Transportation Cost (l_i)	0.7	0.8	0.9
Holding Cost (h_i)	0.2	0.25	0.4
Shortage Cost (d_i)	0.5	0.8	1

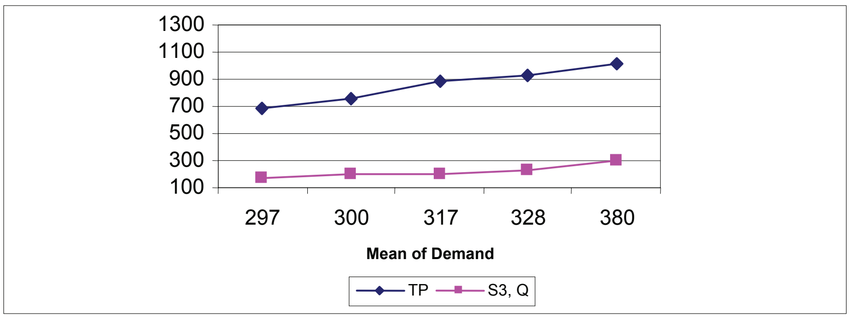
First, we solve the problem analytically (4.9 - 4.11) by Theorem 4.2 and using the algorithm shown in Figure 4.12 yields the optimal solution: $(S_1, S_{2,} S_3)$ = (0, 0, 198), with objective function value (*total profit)* 762. Like the experiments shown in Table 4.2 - 4.5, the solution structure has exactly the same form as Theorem 4.2 and Remark 4.2, since in our case $(p_i + d_i - l_i)$ is in increasing order of i. In addition, it is confirmed numerically that the purchase cost function and holding cost only affect the optimal solution's value. Most importantly, the sales price p_i, transportation cost l_i and shortage cost d_i determine the fundamental structure of the optimal solution.

Now let us examine some interesting findings from the following experiments. First, we investigate the effect of varying the demand mean. Unsurprisingly, as the mean of demand increases, both fulfilled demand quantity and total profit increases, its effect on profit is more significant since it results in both sales increase and purchase cost decrease due to economies of scale, as shown in Figure 4.14. Note that as long as $(p_i + d_i - l_i)$ is increasing in the order of i, and the condition of Remark 4.2 holds. S_1 and S_2 will remain at 0.

Table 4.10: Effect of Demand's Mean (STD = 60)

AVG	S_1	S_2	S_3	$Q = sum(S_i)$	TP
297	0	0	170	170	684
300	0	0	**198**	**198**	**762**
317	0	0	205	205	886
328	0	0	224	224	932
380	0	0	296	296	1013

Figure 4.14: Effect of Demand's Mean



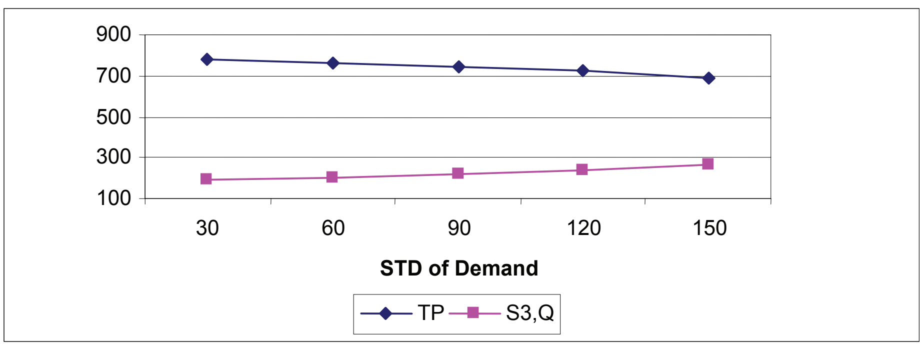
Next, we look at the impact of demand's standard deviation on fulfilled demand quantity and profit by varying std from 30 to 150. These are shown in Table 4.11 and Figure 4.15. They show that the fulfilled demand quantity increases, while the total profit decreases as the standard deviation of demand increases. This can be explained as follows: large variance in demand patterns create havoc for a company, which in turn must sell more quantity to offset it. Even so, the generated revenues are still not enough to overcome the loss due to overstocking (holding cost) and lost sales (shortage cost). Thus, accurate demand forecasts are a business imperative. After all, the better you can match supply with actual demand, the more streamlined your business operations will be. Thus, a company needs the capability to track forecast accuracy in real-time, allowing management to take immediate action to eliminate forecast errors and improve the overall demand forecasting process.

Again, we should notice that $(p_3 + d_3 - l_3)$ in this scenario is always larger than $(p_1 + d_1 - l_1)$ and $(p_2 + d_2 - l_2)$, and in optimal solutions S_1 and S_2 stay at 0, which is exactly what Theorem 4.1 points out.

Table 4.11: Effect of Demand's Stand Deviation (AVG = 300)

STD	S_1	S_2	S_3	$Q = sum(S_i)$	TP
30	0	0	191	191	778
60	0	0	**198**	**198**	**762**
90	0	0	216	216	746
120	0	0	237	237	722
150	0	0	263	263	688

Figure 4.15: Effect of Demand's Stand Deviation

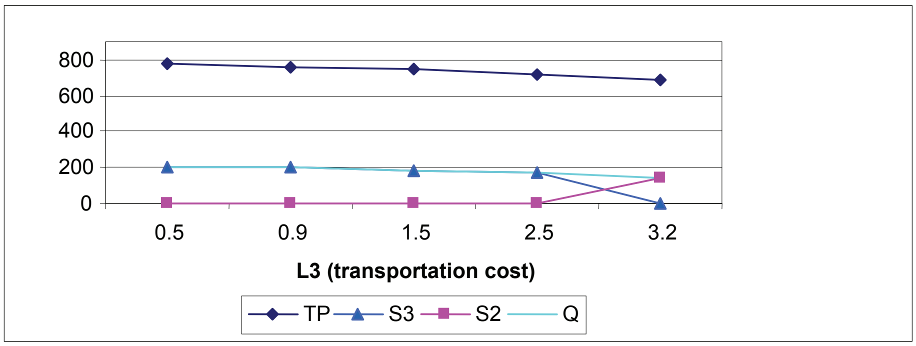


For various levels of transportation cost l_3, as l_3 increases (Table 4.12), both Q and total profit monotonically decrease. This result is obvious as the company is increasingly more costly now, profits decrease more steeply (Figure 4.16) due to the dual effects from increased cost and reduced fulfilled quantity. We also notice that S_2 changes from 0 to 142 when l_3 increases to 3.2. This should not surprise us as, according to Theorem 4.2, the break point for $(p_3 + d_3 - l_3)$ in this scenario is $l_3 =$ 3.0. In addition, even the optimal value of S_2 changes, the solution structure still observes the formula (4.12) and (4.13).

Table 4.12: Effect of Transportation Cost ($l_1 = 0.7,\ l_2 = 0.8$)

l_3	S_1	S_2	S_3	$Q = sum(S_i)$	TP
0.5	0	0	207	207	778
0.9	**0**	**0**	**198**	**198**	**762**
1.5	0	0	184	184	746
2.5	0	0	169	169	722
3.2	0	142	0	142	688

Figure 4.16: Effect of Transportation Cost



It's also interesting to study the effect of price (p_i) on sales price and total sales quantity, and we have:

Remark 4.3: If there exists optimal solution S_i^* for problem (4.9)-(4.10), then Q^* ($Q^* = \sum_i S_i^*$) increases in p_i.

Proof: See appendix 15.

This result can be intuitively explained as follows: sales locations are only tied together by a concave production function so if something induces more sales at one location then this will drive down unit costs and cause an increase in profitability at other locations.

4.3 Analytical Model for Balanced Supply

Competition becomes stiffer and margins get smaller. Globalization, more demanding customers and shortened product lifecycles are challenges in today's competition. Companies are continuously forced to improve their performance in order to create value-added (VA) to customers from day to day, if they want to remain profitable. As companies continuously seek to provide their products and services to customers faster, cheaper, and better than their competitors, they realized that they cannot do it without considering the "product path". From supply chain's perspective, this means that products have to reach the customers from the right supplier. Supply chain diversity ranges from globally dispersed manufacturers, distribution centers and sales subsidiaries with different production cost, capacities, capabilities and lead-times for different products. Therefore identifying the right supplier/path is crucial for effective order promising, capacity utilization and production smoothness. Overcoming the diminishing profit margin and achieving the resource allocation efficiency stimulates us to do *path analysis* for the global supply chain. Again, when we say *product path*, we refer to the path from the supplier where the raw materials are purchased to manufacturer where products are produced, through the distribution center (D.C.), and to the sales locations where the products are finally sold to the customers.

In previous section we stated that, to counteract price erosion and the resulting reduction in profit margin, manufacturers need to align the production and logistics planning with the end-sales to choose the right path. Especially, in such a risky and uncertainty environment, from the purchasing of raw material to manufacturing, to the D.C, and finally to the customer, companies have to use different suppliers in order to avoid vulnerability in market competition and reduce risk (L.Hunter 2004). This brings us our major concern: the "*balanced supply*". To meet customer demand, decision-makers on the supply side include suppliers, manufacturers and distributors. They are concerned not only with profit maximization but also with risk minimization (A. Nagruney 2004). At the same time, developing more responsive strategies requires multiple suppliers to respond to different supply chain drivers. One of the time-based competition strategies proposed by D. Kritchanchaj (1999). focuses on how to improve flexibility and responsiveness of business processes to meet customer requirements. Organizations need to ensure that they continually monitor the changing demands of customers and then attempt to meet their customer's expectations in order to defend their market position against competitors. The capability of responding quickly to customers' demand is a key business process. But the bottom line is the responsiveness of different suppliers varies. Hence, meeting customer demands with different suppliers instead of adopting only one supplier, thus has become a prerequisite for business survival in the face of market globalization where rapidly changing business environments and seemingly insatiable customer expectations have become the norm. This is especially true for companies that incur the burden of high logistics costs. For example, Amazon.com has at least 2 fulfillment centers to cover *any* customer zone in order to satisfy its trademark "shipping guarantee policy". Achieving this feat during Christmas season is paramount for any retailer. All these point us in one direction: reducing the risk through balanced supply.

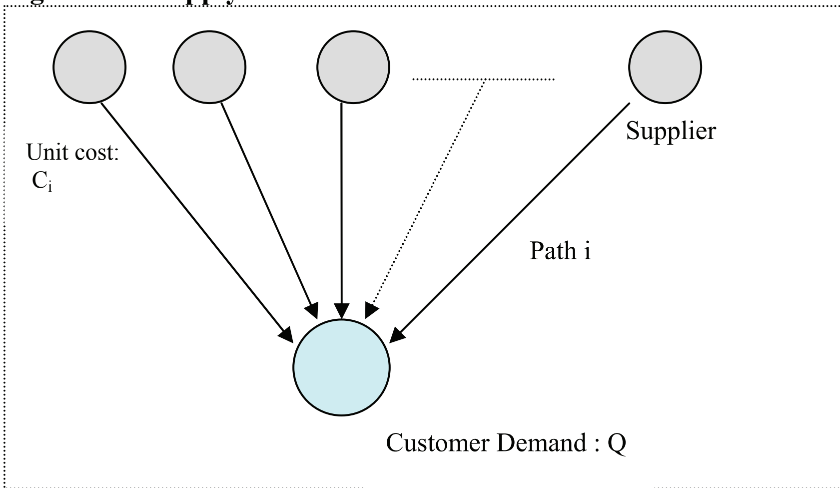
4.3.1 Model & Formulation

The problem under consideration below is a single time period, single product, multiple-tier supply chain model. In particular, we consider one focal company with multiple suppliers, which provide the material to manufacturing sites / distribution sites. The manufacturing sites are involved in the production of a homogeneous product which is then shipped to distributors, who, in turn, ship the product to end customers. To simplify the deriving process, the supply chain path is illustrated to be from one supplier to one sales location. (See Figure 4.17). The fundamental decision in the model is to decide the optimal supply of products from each supplier to meet the end customer's demand so that the profit of the company is maximized, while *supply balance* is also achieved. This problem is modeled as a constrained non-linear integer programming problem.

4.3.1.1 Inputs

- Set F of Suppliers, $|I| = n$. In general elements of I will be identified by i.
- Customer demand Q: a given quantity.
- Unit cost C_i: This represents linear cost such as materials needed

Figure 4.17 Supply Balance



- Cost function *f*: This cost function represents nonlinear costs such as production costs and transportation costs, which include shipping and handling fees. It is concave function to reflect economies of the scale.
- Supply balance reward B_i: With more suppliers to commit the customer demand, the company has more flexibility to quickly respond to changes and more alternatives to overcome the uncertainties, and thus reduce the risk. We use B_i to represent such benefits in the model.
- Supply Lower bound l_i: the minimum quantity that each supplier should provide in one time period, so that path shutdown can be avoided and meanwhile, production smoothness, TL transportation and the economies of the scale can be achieved.

4.3.1.2 Decision Variables

x_i: the quantity committed by path *i*.

y_j = 1 if supplier *i* is chosen to fulfill demand , and y_j = 0 otherwise.

In other words: $y_i = 1$ if $x_i > 0$,

$y_i = 0$ if $x_i = 0$.

4.3.1.3 Model

Minimize $$\sum_i (f(x_i) + C_i x_i) - \sum_i B_i y_i \tag{4.15}$$

Subject to:

$$\sum_i x_i = Q \tag{4.16}$$

$y_i = 1$ if $x_i > 0$,

$$y_i = 0 \text{ if } x_i = 0. \tag{4.17}$$

$$x_i \geq l_i \text{ if } x_i > 0 \tag{4.18}$$

We can see, (4.17) and (4.18) are equivalent to: if $y_i = 1$, then $x_i \geq l_i$; if $y_i = 0$, then $x_i = 0$, which can be expressed as:

$$x_i \leq M \cdot y_i$$

$$x_i \geq l_i \cdot y_i$$

$$x_i \geq 0$$

$$y_i = 0,1$$

and the model can be simplified as:

Min $$\sum_i (f(x_i) + C_i x_i) - \sum_i B_i y_i \tag{4.19}$$

subject to:

$$\sum_i x_i = Q \tag{4.20}$$

$$x_i \,:\, M \cdot y_i \tag{4.21}$$

$$x_i \geq l_i \cdot y_i \tag{4.22}$$

$$x_i \geq 0 \tag{4.23}$$

$$y_i = 0,1 \tag{4.24}$$

We will always assume that $Q > \sum_i l_i$, all parameters are positive, and $f(\cdot)$ is concave.

4.3.2 Model Analysis

Without loss of generality, we assume: $C_1 < C_2 < ..... < C_n$, the following results are based on assumption that: $C_2 - C_1 > f'(0) - f'(Q)$.

Theorem 4.3: For any optimal solution $X^*, x_1 \geq l_1$.

Proof: see Appendix 16.

This Theorem tells us 1) that the supplier with the lowest unit cost will definitely be allocated at least the lower-bound quantity 2) it only depends on the unit cost, the nonlinear shipping and handling cost doesn't play any role here if $C_2 - C_1 > f'(0) - f'(Q)$.

From Theorem 4.3, we can further have:

Theorem 4.4: For any optimal solution X^*, we have: $x_i : l_i$. (Since l_i is lower bound, this is equivalent to say: for all $i \geq 2$: either $x_i = 0$ or $x_i = l_i$)

Proof: see Appendix 17.

These Theorems give us a clear and simple solution. It shows us that the first supplier is used at l_1 and for all the other suppliers, the solution space is not infinite but a very limited number of possibilities.

From Theorem 4.3 and Theorem 4.4, we can see that, without any complicated assumptions and constraints involved, this model provides a simple structure relative to demand fulfillment and balanced utilization of the supply chain. The model's conclusion can also be broadly applied to other similar situations.

4.4 Available Resource Analysis

Identifying profitable paths is also a powerful weapon coordinating an available resource such as component inventory or capacity, with order management (sales subsidiaries) policies because it enables a company to know which path is profitable and therefore can improve the customer service at least cost. The path in the real-life global supply chain is difficult to handle analytically since they involve interactions among several individual factors as well as thousands of products and product locations. To reduce complexities and improve insight, we create an aggregate level model, which means sales subsidiaries, rather than the individual orders, are the unit of attention.

In the research below we study the firm's optimal response to a price discount on a component (raw material) under two different strategies. In the first strategy, the firm does not pass along the discount to its customers (sales subsidiaries); the firm simply coordinates sales among the different products and subsidiaries to minimize the financial impact of non–profitable products. In the second strategy, the firm offers price discounts in different sales subsidiaries to increase the demand for profitable products. We carried out experiments for the two strategies based on a mathematical programming model, built around Toshiba's global notebook supply chain. Model constraints include, among others, material constraints, bill–of–materials, capacity and transportation constraints, and a constraint on minimum fill rate (service level constraint). Unlike most models of this type in the literature, which define variables in terms of single arc flows, we employ path variables, which allow for direct direction identification and manipulation of profitable and non–profitable products.

4.4.1 Model background

Facing fierce competition, many companies have to cope with two conflicting goals: one is to maximize total profit; the other one is to maintain a high level of service, usually measured by *fill–rate*. For strategic reasons, a firm may set a lower bound on its fill rate, be it global (across products and locations), or local for a given location. This constraint usually forces a company to sell *non–profitable* products. Meantime, due to the different path involved, similar products might be profitable in one path (location) but non-profitable in the other. In this study, we consider n

products, which are variations of a generic product, and can be thought of as configurations for a notebook PC. The products are manufactured in different factories at different costs, and sold in different sales subsidiaries, also called sales locations, at different prices.

Temporary price discounts by suppliers are a relatively common phenomenon—these occur as a result of supply–demand imbalances (e.g., production overruns, poor forecasting, etc.), competition among suppliers, retooling, etc. (Tersine and Barman 1995). In this research we consider the issue of coordinating available resource with sales at different locations in a global, capacitated, sourcing–manufacturing–distribution supply chain under such temporary price discount. This research is based on a two–year project conducted with Toshiba Corporation. All products have a common and critical component, for which the supplier offers a price discount in one time period.

Customers are price–sensitive and demand for a product is assumed to decrease linearly with its price. We define a sales subsidiary's *fill rate* as total committed quantity divided by its total demand. There are different minimum fill rate requirements across different sales subsidiaries. This minimum fill rate requirement may induce sales products that are non–profitable in that location. We propose two coordinating mechanisms to improve the resource utilization and reduce the financial impact of non–profitable products. The first mechanism simply buys a large amount of components at the discounted price to use them in non–profitable products for as long as possible—the length of time is related to inventory holding costs, and capacity considerations along the chain (transportation, production, etc.). An optimal procurement quantity trades off gains from the discounted price against additional holding costs, taking into account the various capacity constraints. The second mechanism reduces the quantity of non–profitable products needed to meet the minimum service level constraint by increasing sales of profitable products via a price discount. Note that offering price discounts for profitable products has the potential of increasing profits due to two effects: increased demand, and decreased sales of non–profitable products, however, too large a price discount can ultimately hurt profits by decreasing total revenue. We should point out here, without the product path to identify profitable product, such coordinating mechanism is impossible to be carried out.

4.4.2 Toshiba Supply Chain

In this section we model Toshiba's notebook (PC) global supply chain (Figure 4.18), which comprises four final assembly and testing (FAT) factories and six sales subsidiaries, including locations in Japan, The Philippines, United States and Germany. Toshiba buys PC components, including CPU, hard disk drive, keyboard, LCD, CD–ROM, and DVD–ROM, directly from suppliers, and transports the motherboard subassembly from subassembly factories; these are shipped to the FAT factories for assembly, and finished PCs are finally shipped to the different sales subsidiaries. The motherboard components are bought from suppliers, and all subassembly factories can produce all types of motherboards. There are over 3,500 different product models offered across the different sales subsidiaries. Although our model is based on Toshiba Corporation, it is clear that the structure of its supply chain is similar to many other firms operating in similar industries.

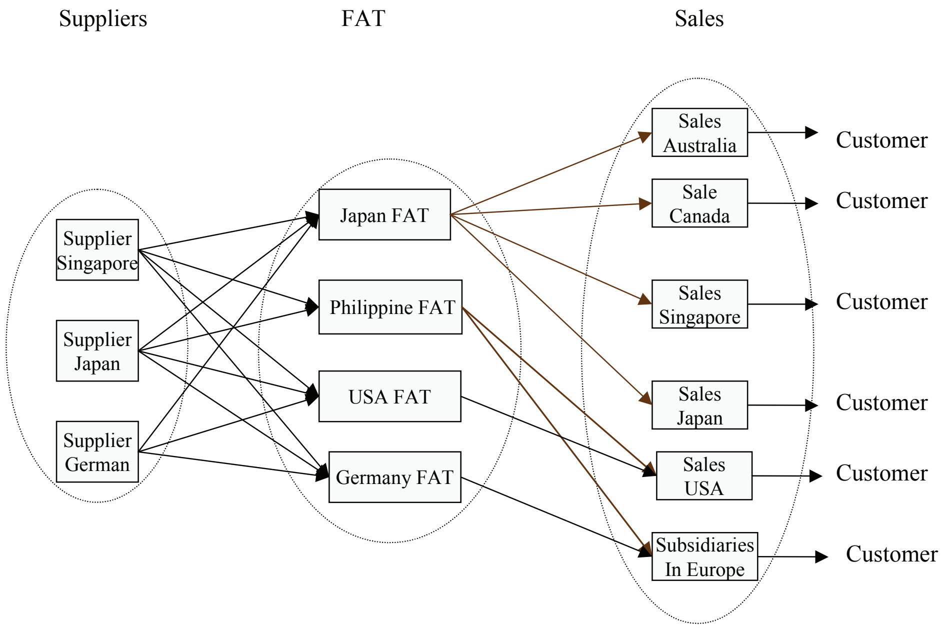


Figure 4.18 Toshiba Global Supply Chain

Our sourcing–production–distribution planning model spans a time horizon of 13 weeks. Unmet demand for a sales subsidiary at any period cannot be backordered; consequently committed quantity is equal or lower than demand. The supplier network is not considered in this model and thus we ignore holding costs at the suppliers. The integrated supply chain model is a multi–period, multi–echelon and multi–product MIP model (integer variables are necessary because of minimum lot size requirements), which is presented in §4.4.3

From a modeling perspective, our global supply chain model is rather straightforward, except for a choice in the decision variables, which allows us to answer the research questions posed in §4.4.1. As explained in the Chapter 1, unlike most models in the global supply chain literature, which define variables in terms of flows along a single arc in the network and use flow balancing constraints at nodes, we employ *path variables*, which provide location–specific cost information and directly identify profitable and non–profitable products In our two–year research project with Toshiba, we found this to be very important managerial information, as it serves as an aid for other strategic decisions such as product line offering at different locations. Although this modeling approach certainly increases the number of decision variables (for example, a typical model we study here has over a million variables and a similar number of constraints), we find that the resulting models, even with real–world data, are manageable with solution times being around 5 minutes using typical computing environments.

4.4.3 Model Formulation

Notation

Index Use (Indices and Index Sets)

$s \in S$ Sales subsidiaries

$f \in F$ Final assembly and testing (FAT) factories

$l \in L$ Subassembly factories.

$i \in I$ Product (notebook PC models)

$j \in J$ Components sourced directly from suppliers

$\bar{j} \in \bar{J}$ Motherboard (sourced from subassembly factories)

$k \in K$ Motherboard components.

Data (Lower Case Letters)

δ Tangible profit weight (objective function tradeoff parameter)

$d_i^s(t)$ Demand for product $i \in I$ at sales subsidiary $s \in S$ at time $t = 1,2,\cdots,T$

$p_i^s(t)$ Sales price for product $i \in I$ at sales subsidiary $s \in S$ at time $t = 1,2,\cdots,T$

$a_j^f(t)$ Quantity of component $j \in J$ available at factory $f \in F$ at time $t = 1,2,\cdots,T$

b_{ij} Quantity of component $j \in J$ needed to produce a unit of product $i \in I$; similarly for $b_{i\bar{j}}$ and $b_{\bar{j}k}$

γ^s Minimum fill rate for sales subsidiary $s \in S$ (total quantity committed divided by total demand)

$q^{fs}(t)$ Transportation capacity between factory $f \in F$ and sales subsidiary $s \in S$ at time $t = 1,2,\cdots,T$; similarly for $q^{lf}(t)$

$c^f(t)$ Maximum production rate at factory $f \in F$, notebook PC units, at time $t = 1,2,\cdots,T$; $c^l(t)$ is similarly defined for motherboard units

z_i^f Minimum lot–size for product $i \in I$ at factory $f \in F$; $z_{\bar{j}}^l$ is similarly defined

τ^{fs} Transportation lead–time between factory $f \in F$ and sales subsidiary $s \in S$; τ^{lf} is similarly defined

ω_i^f Production lead–time for product $i \in I$ at factory $f \in F$; $\omega_{\bar{j}}^l$ is similarly defined

r_j^f Unit purchasing cost of component $j \in J$ at factory $f \in F$ at time $t = 1,2,\cdots,T$; r_k^l is similarly defined

h_i^s Unit holding cost for product $i \in I$ at sales subsidiary $s \in S$; h_k^l, h_j^f are similarly defined

v_i^f Production cost per unit for product $i \in I$ at factory $f \in F$; $v_{\bar{j}}^l$ is similarly defined

w^{fs} Transportation cost per unit between FAT factory $f \in F$ and sales subsidiary $s \in S$, including taxes or duties; w^{lf} is similarly defined.

Path Decision Variables (capital letters)

$D_i^{lfs}(t)$ Quantity of product $i \in I$ produced at factory $f \in F$ by using motherboard from sub–assembly factory $l \in L$ and shipped to sales subsidiary $s \in S$ at time $t = 1,2,\cdots,T$

Auxiliary Variables (Capital Letters)

$M_i^s(t)$ Demand commitment for product $i \in I$ at sales subsidiary $s \in S$ at time $t = 1,2,\cdots,T$

$Q_i^f(t)$ Quantity of product $i \in I$ produced at factory $f \in F$ at time $t = 1,2,\cdots,T$; $Q_{\bar{j}}^l(t)$ is similarly defined

$Z_i^f(t)$ Binary variable (= 1, if product $i \in I$ is produced at factory $f \in F$ at time $t = 1,2,\cdots,T$; 0 otherwise); $Z_{\bar{j}}^l(t)$ is similarly defined

$H_i^s(t)$ Inventory on hand for product $i \in I$ at sales subsidiary $s \in S$ at time $t = 1,2,\cdots,T$ ($H_i^s(0)$ is known and given); $H_j^f(t)$ and $H_k^l(t)$ are similarly defined

$C_i^s(t)$ Total cost for product $i \in I$ sold at sales subsidiary $s \in S$ at time $t = 1,2,\cdots,T$

$P_i^s(t)$ Total profit for product $i \in I$ sold at sales subsidiary $s \in S$ at time $t = 1,2,\cdots,T$

$\tilde{C}(t)$ Total PC component and motherboard component holding cost at time $t = 1,2,\cdots,T$

The objective function maximizes profit—revenue from promised orders minus transportation, production, duty, component, and inventory costs. Constraints include: demand commitment and fill rate constraints, inventory balance constraints at factories and sales subsidiaries, minimum lot size, and production and transportation capacity constraints.

Objective function: maximize profit

$$\max \quad \sum_{t=1}^{T} \sum_{i}^{s} P_i^s(t) \tag{4.25}$$

Subject to:

Demand commitment and minimum fill rate constraints

$$M_i^s(t) \le d_i^s(t) \quad i \in I,\ 1 \le t \le T, \tag{4.26}$$

$$\sum_{i\in I} M_i^s(t) \ge \gamma^s \sum_{i\in I} d_i^s(t) \quad s\in S,\ 1\le t\le T, \tag{4.27}$$

Inventory balance constraints at sales subsidiaries

$$H_i^s(t) = H_i^s(t-1) + \sum_{l\in L, f\in F} D_i^{lfs}(t-\tau^{fs}) - M_i^s(t) \quad i\in I, s\in S, 1\le t\le T \tag{4.28}$$

Flow conservation constraints at factories

$$Q_i^f(t-\omega_i^f) = \sum_{l\in L, s\in S} D_i^{lfs}(t) \quad i\in I, f\in F, 1\le t\le T \tag{4.29}$$

Inventory balance constraints for components at factories

$$H_j^f(t) = H_j^f(t-1) + a_j^f(t) - \sum_{i\in I} Q_i^f(t)\cdot b_{ij} \quad j\in J, f\in F, 1\le t\le T \tag{4.30}$$

$$\sum_{i\in I} Q_i^f(t)\cdot b_{i\bar{j}} = \sum_{i\in I, s\in S, l\in L} D_i^{lfs}(t-\tau^{lf})\cdot b_{i\bar{j}} \quad \bar{j}\in \bar{J}, f\in F, 1\le t\le T \tag{4.31}$$

Flow conservation constraints at subassembly factories

$$Q_{\bar{j}}^l(t-\omega_{\bar{j}}^l) = \sum_{i\in I, s\in S, f\in F} D_i^{lfs}(t)\cdot b_{i\bar{j}} \quad \bar{j}\in \bar{J}, l\in L, 1\le t\le T \tag{4.32}$$

Inventory balance constraints for raw material at sub–assembly factories

$$H_k^l(t) = H_k^l(t-1) + a_k^l(t) - \sum_{\bar{j}\in \bar{J}} Q_{\bar{j}}^l(t)\cdot b_{\bar{j}k} \quad k\in K, l\in L, 1\le t\le T \tag{4.33}$$

Minimum lot size constraints at factories

$$Q_i^f(t) \ge Z_i^f(t)\cdot z_i^f \quad i\in I,\ f\in F, \tag{4.34}$$

$$Q_i^f(t) \le Z_i^f(t)\cdot N \quad i\in I, f\in F, \tag{4.35}$$

Minimum lot size constraints at subassembly factories

$$Q_{\bar{j}}^l(t) \ge Z_{\bar{j}}^l(t)\cdot z_{\bar{j}}^l \quad \bar{j}\in \bar{J},\ l\in L, \tag{4.36}$$

$$Q_{\bar{j}}^l(t) \le Z_{\bar{j}}^l(t)\cdot N \quad \bar{j}\in \bar{J}, l\in L, \tag{4.37}$$

Production and transportation capacity constraints

$$\sum_{i \in I} Q_i^f(t) \le c^f(t) \quad f \in F, 1 \le t \le T \tag{4.38}$$

$$\sum_{\bar{j} \in J} Q_{\bar{j}}^l(t) \le c^l(t) \quad l \in L, 1 \le t \le T \tag{4.39}$$

$$\sum_{i \in I, l \in L} D_i^{lfs}(t) \le q^{fs}(t) \quad f \in F, s \in S, 1 \le t \le T \tag{4.40}$$

$$\sum_{i \in I, s \in L, \bar{j} \in \bar{J}} D_i^{lfs}(t) \cdot b_{i\bar{j}} \le q^{lf}(t) \quad l \in L, f \in F, 1 \le t \le T \tag{4.41}$$

Cost and profit calculation constraints

$$C_i^s(t) = h_i^s \cdot H_i^s(t) + \sum_{f \in F, l \in L} D_i^{lfs}(t) \begin{bmatrix} v_i^f + w^{fs} + \sum_{j \in J} b_{ij} \cdot r_j^f + \\ \sum_{\bar{j} \in \bar{J}} b_{i\bar{j}} \cdot v_{\bar{j}}^l + \sum_{\bar{j} \in \bar{J}, k \in K} b_{i\bar{j}} \cdot b_{\bar{j}k} \cdot r_k^l \end{bmatrix} \\ i \in I, s \in S, 1 \le t \le T, \tag{4.42}$$

$$\tilde{C}(t) = \sum_{j \in J, f \in F} h_j^f \cdot H_j^f(t) + \sum_{k \in K, l \in L} h_k^l(t) \cdot H_k^l(t) \quad 1 \le t \le T, \tag{4.43}$$

$$P_i^s(t) = p_i^s(t) \cdot M_i^s(t) - C_i^s(t) - \tilde{C}(t) / d_i^s(t) \quad i \in I, s \in S, 1 \le t \le T, \tag{4.44}$$

Integrality and Non–negativity

$$Z_i^f(t) \in \{0,1\}, Z_{\bar{j}}^l(t) \in \{0,1\}, \tag{4.45}$$

$$\begin{gathered} C_i^s(t) \ge 0,\ Q_i^f(t) \ge 0, Q_j^l(t) \ge 0, D_i^{lfs}(t) \ge 0, H_i^f(t) \ge 0, \\ H_j^f(t), H_k^l(t) \ge 0, P_i^s(t) \ge 0 \end{gathered} \tag{4.46}$$

We now elaborate on how (purchasing) component costs are handled in the model. Materials availability, given by $a_j^f(t)$'s, are parameters (inputs) to the model. Consequently, one would be inclined to treat purchasing costs as “sunk” costs; in this manner they would not influence the optimal solution—given by the path decision variables $D_i^{lfs}(t)$. In the computation of profits (4.42)– (4.44), however, we explicitly incorporate these purchasing costs as part of the unit cost for the path decision variables, and thus they *affect* the firm's optimal solution. We justify this approach by noting that the problem has a long (13–period) planning horizon, and therefore the firm may hold components in inventory for future use in any given period.

4.4.4 Result Analysis

In the fist coordinating mechanism, we don't differentiate the profitable or non-profitable product and we shall see how the available resource would affect the profit and interact each other under such scenario. Then in the second strategy, we will treat the profitable product differently to get optimal response.

The model is solved with the MPL 4.11, with a Cplex 7.0 solver (Cplex 1998) operating on an IBM server with a Pentium III processor, under Windows NT (4.0). The computer RAM is 1,047,960. A typical model has 1.16 million variables and 810,000 constraints; a typical running time is 7 – 10 minutes.

4.4.4.1

To address the first research question, we design a numerical study based on data collected at Toshiba (Figure 4.18), however, we disguise the actual magnitude of profits and demands for obvious reasons. Based on the literature review in Chapter 2, the most important factors in our model that influence the firm's optimal response to a one–time discount by a critical supplier would be: the magnitude of the discount rate, the minimum fill rate (service level), production and transportation capacity, and the holding cost. Accordingly, we design a full–factorial experimental design with three levels for each of these factors, where one of the levels represent the current value used at Toshiba; the other two levels represent alternative scenarios that are considered plausible by the company. We thus have a 3^4 factorial design, which is shown in Table 4.13 and explained below.

Table 4.13: Experimental Design for First Research Question

Parameter	Levels		
	Low (LO)	Medium (MID)	High (HI)
Discount	0%	30%	50%
Fill Rate	0.65	0.75	0.85
Capacity	0.65	0.85	1.00
Holding Cost	10%	30%	60%

Discount is defined as a % reduction in the nominal price of a critical component from a major supplier (in this case, CPU), from no reduction to a 50% reduction. The firm has a contractual obligation to purchase "normal" amounts of components from its suppliers throughout the planning horizon; these "normal" amounts are the parameters $a_j^f(t)$. The discount mentioned above is given *only* to the *marginal* amount above the "normal" amount. We set three values for a common minimum fill rate γ^s across all sales subsidiaries *s*: 0.65, 0.75 and 0.85. We consider three scenarios for capacity as follows: we take existing values for the capacity parameters $c^f(t)$ and $q^{fs}(t)$ at Toshiba, and consider three multipliers; the relative proportions among these three multipliers are the capacity ratios shown in Table 4.13. (we disguise which one is the current capacity at Toshiba). Finally, we consider three values for annual holding cost, expressed as a percentage of the product's price at the sales subsidiary (h_i^s) or as a percentage of the component's price from the supplier (h_j^f and h_k^l): 10%, 30% and 60%; these simulate various levels of obsolescence in this high–velocity environment.

We report key summary measures of the model's optimal solution for this research question: total profit, and total quantity of components purchased (the component for which the supplier offers a price discount). Given the full factorial study design, an effective way to measure the sensitivity of profit and purchased quantity (dependent variables) to the factors (Table 4.13, independent variables) is to compute t–statistics for the respective coefficients in multiple regressions for each dependent variable as a function of all independent variables (Wagner 1995). These statistics are reported in Table 4.14.

Table 4.14: T–Statistic for Multiple Regressions Where Dependent Variables are Profit and Purchased Quantity and Independent Variables are Factors (n = 81)

Factor	**Profit (R^2 = 0.78)**	**Purchased Quantity (R^2 = 0.95)**
Discount	0.4	2.6
Fill Rate	–11.4	39.1
Capacity	12.3	10.8
Holding Cost	1.1	–4.2

Not surprisingly, both fill rate and capacity have a strong effect on both profit and purchased quantity, as evidenced by the high values of the t–statistic. The effect of the discount rate on profit is non–significant (t–value = 0.4) since purchasing cost for the component represents only a small fraction of total profit; the same is true for holding cost. Note that the discount is offered only in one period, and total profit is defined as the sum over the entire planning horizon of 13 periods. The effect of the discount rate on purchased quantity is relatively mild compared to the effects of the minimum fill rate and capacity. There are several reasons for this surprising mild effect. There are production and transportation capacities in the supply chain, as well as constraints for other components. In addition, the discount is offered only for units above the minimum purchased quantity specified in the contract with the supplier. Thus, the discounted components, without a change in the final product's price, does not result in a demand increase for the final product, and is therefore mostly used to increase the fraction of demand that is met with profitable products.

The effect of unit holding cost on profit is also non–significant (Table 4.14); this occurs because holding costs comprise only a fraction of total profit over the 13–period planning horizon. The effect of holding cost on the purchased quantity is mild compared to the effect of the minimum fill rate and the capacity, which is a surprising result, considering the lot–sizing literature. For example, if the unit holding cost increases six–fold, the EOQ formula posits that the optimal lot size would decrease by 60% (= $1 - \sqrt{1/6}$); our model's optimal solution recommends instead that the total quantity of purchased components decreases by an average of only 7.3% (Figure 4.19). Overall, these are interesting results because they point out weaknesses of simple models in the literature—where assumptions of a single product, uncapacitated supply chains, and no service level constraints are frequently used—in dealing with the complexities of global supply chains, where these assumptions do not hold, and which significantly influence a firm's optimal sourcing–production–distribution decision.

Next, we look at the impact of each factor on profit and purchased quantity, by taking an average across experiments, for each factor level. These are shown in Figure 4.19 and Figure 4.20 for profit and purchased quantity, respectively. The relationship between (minimum) fill rate and profit (Figure 4.19) displays the usual shape of profit–service trade–off curves—that is, increasing minimum fill rates decreases profits at an increasing rate. Also, increasing capacity beyond a certain level (here, mid level) does not result in significant profit increases (Figure 4.19) or in additional purchased components (Figure 4.20) and this can be explained as follows. Additional capacity beyond the low level is used to produce profitable products, however, additional capacity beyond the middle level *would* be used to produce non–profitable products beyond the required service level, which is not economically attractive and thus does not happen (Figure 4.21); the purchased quantity for non–profitable products remains unchanged with the capacity level at 267). As discussed before, holding cost and discount do not significantly affect profits, as evidenced by the respective "flat" curves.

Figure 4.19: Effect of Holding Cost, Capacity, Fill Rate and Discount on Profit

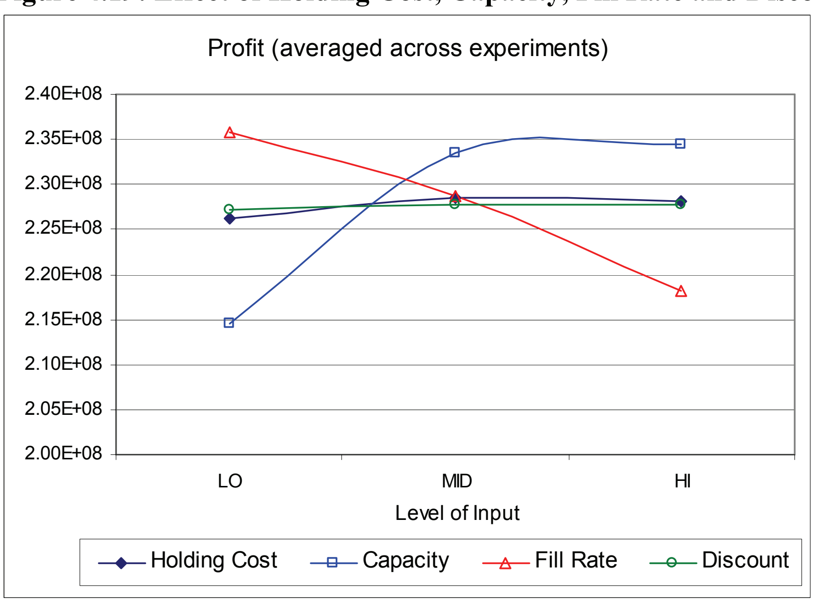


Figure 4.20: Effect of Holding Cost, Capacity, Fill Rate, and Discount on the Purchased Quantity

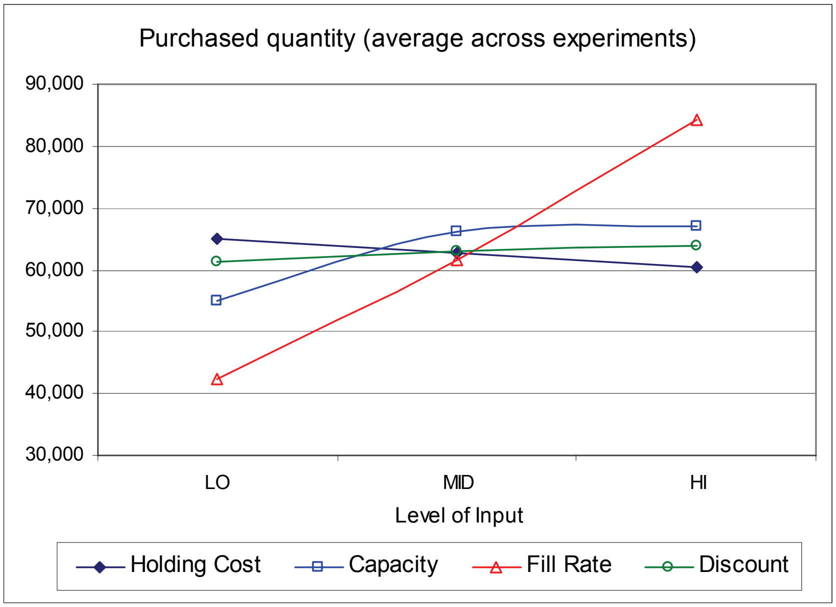
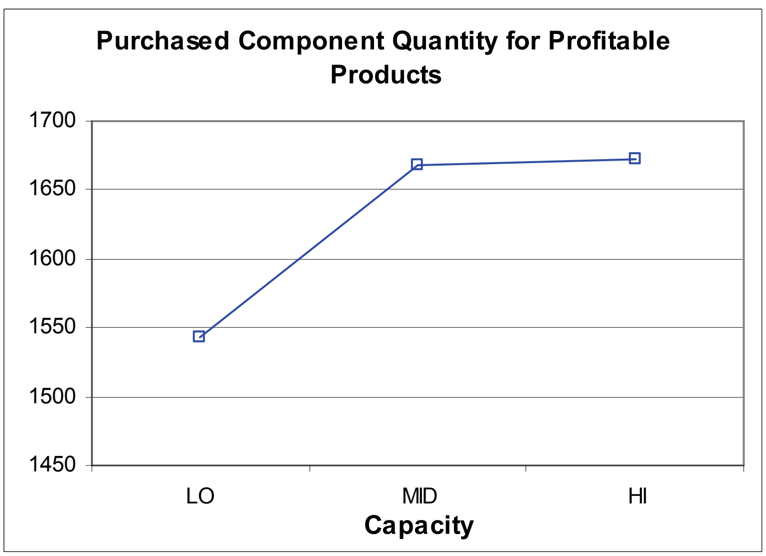


Figure 4.21: Effect of Capacity Increase in the Production of Profitable Products (Average Across Experiments Where Minimum Fill Rate = 0.85)



4.4.4.2

We now investigate the possibility of passing on part of the component savings to the customers by offering price discounts for profitable products at sales subsidiaries, which increases their demand. As demand for profitable products increases, the amount of non–profitable products needed to meet the minimum fill rate requirement decreases (we note that the total demand at a sales subsidiary s is composed of several individual demand quantities, $d_i^s(t)$.). For ease of presentation, we assume that all *profitable* products at all locations are offered the same price discount β, and for simplicity we assume the same (linear) demand curve across products and locations: $\tilde{d}_i^s(t) = d_i^s(t) \cdot (1 + \theta \cdot \beta)$, where θ is a price elasticity parameter, and we use the "~" to differentiate between the "discounted" demand and the base demand data $d_i^s(t)$. Offering discounts for profitable products potentially increases profit due to two effects: increased overall demand, and decreased sales of non–profitable products, however, too large a price discount may hurt profits by decreasing total revenue.

We use the same base data from §4.4.4.1; in addition, we fix the capacity ratio at its high level 1.0, the holding cost at its low level 10%, and the minimum fill rate at a high level of 0.9; we vary the price discount β from 5% to 20%. The price discount is initially offered in period 2 (of our 13–period model), and we study four scenarios, where we discount the product for 1, 2, 3 and 4 consecutive periods. We consider three levels for the demand elasticity parameter θ: 0.5, 1 and 10 (these values indicate that a 10% price discount results in a demand increase of 5%, 10% and 100%, respectively.) Across all numerical examples, the CPU supplier offers a discount of 25% on the price of the *component*, where, as described in §4.4.4.1, the discount is offered only to the *marginal* amount *above* the "normal" amount, or contractual obligation, $a_j^f(t)$. In contrast, the price discount β offered to the final product customers is *applied* to *all* products sold in that period and location (that is, there is no price discrimination across customers in a location.)

The curves showing profit as a function of price discount can be seen in Figure 4.22, for various levels of demand elasticity θ, when the price discount is offered 4 periods. Note that for θ = 0.5—an inelastic demand curve—profit decreases monotonically with the discount rate, and thus the firm should not offer a price discount. Note that profits decrease steeply for discount rates above 10% when θ = 0.5; this is because the additional demand generated is not large enough to overcome the loss in revenues, and several products may become non–profitable. The optimal discount rate is around 10% for θ = 1 and 10. To meet the additional demand generated by the price discount, the firm increases the quantity of components purchased at the discounted rate, as shown in Figure 4.23 (note that Figure 4.23 shows the total quantity of the component purchased at the discounted price not the total quantity of the component used, which is higher).

Figure 4.22: Profit vs. Price Discount

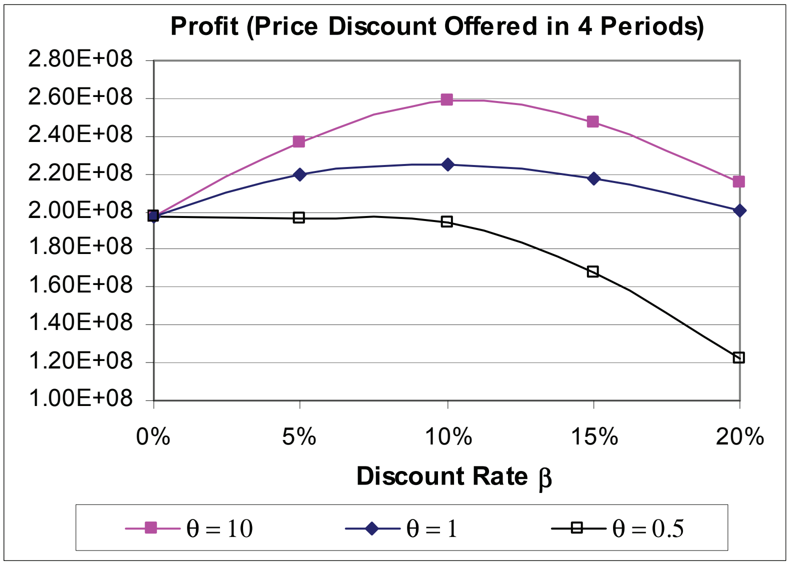


Figure 4.23: Purchased Component Quantity vs. Price Discount

4.4.5 Remarks

We have studied the problem of responding to a supplier's one–time price discount for a critical component that is common to several products on a global supply chain, using real data from Toshiba Corporation. We use an MIP–type formulation for a 13–week planning horizon, but for implementation purposes we simplify it to a linear program. Our model is a tactical planning model; we neither consider strategic issues such as facility location nor tactical issues such as production scheduling. Our model's primary decision variables are flows of materials in the supply chain along *paths* (as opposed to flows along single arcs), which isolates critical information regarding profitable and non–profitable products across locations—products that are profitable in one location may not be profitable in another location. This information serves as an input to the more strategic decision of product line offering. Our model maximizes profit subject to a critical service level constraint—a minimum fill rate, which may be different across locations. Because of the minimum fill rate, the firm may need to sell non–profitable products. Thus, our research studies one of many weapons a firm has to mitigate the effect of non–profitable products on the firm's profitability.

Temporary price discounts by suppliers are a relatively common phenomenon. We consider the issue of coordinating a temporary price discount for a critical component with pricing and sales at different locations in a global, capacitated, sourcing–manufacturing–distribution supply chain. Based on data from Toshiba, we perform a numerical study where we vary critical parameters on the supply chain—capacity, discount rate, holding cost, number of discounted periods, and minimum fill rate—and analyze their impact on the firm's profitability. We find that capacity critically impact profits, however, adding capacity beyond a certain level, without changes in the firm's pricing strategy, only impacts the production of non–profitable products, which does not result in profit improvement if the firm is above the minimum fill rate threshold. The relationship between minimum fill rate and profit has the usual shape of a profit–service trade–off curve. We also point out the weaknesses of analytical models in dealing with the complexities of decision–making in global supply chains. For example, we show how a six–fold increase in holding cost for a component only results in a 7% increase in the optimal lot size, whereas the EOQ model predicts a 60% increase. This occurs because the assumptions of uncapacitated supply chains—including production and transportation—as well as a single product type, typical of analytical models, do not hold in complex supply chains.

We find that passing on temporary component price discounts to final product customers is attractive if the demand curve is "elastic" enough such that a price discount significantly stimulates demand. Also, a price discount needs to be offered for several periods (weeks in this case) for it to be effective and profitable. For example, it is not optimal to offer price discount on the final product during only one period under any scenario analyzed in this paper. We find that the relationship between price discount and the number of periods when the discount is offered is non monotonic, requiring careful optimization.

Although our study was conducted with real data from a global company, we caution on some of its limitations. The primary limitation regards modeling the demand curve. Essentially, we have assumed demands to be independent across products—discounts offered in one product would not impact demand for *other* products. Also, our analysis considers competitive issues only on a limited basis, through the estimated demand curve—for example, we assume that if the firm offers no discounts on its final product, even after receiving a discount on a critical component by a key supplier, then its demand does not change. In reality, it is likely that the supplier offers discounts to several manufacturers; thus the demand curve should include price dynamics by all other competitive products in the marketplace; this, however, significantly complicates modeling and parameter estimation for the demand curves.

Chapter 5

Conclusions

Global competition and e-commerce have imposed tremendous pressure on product and service providers to get closer to the customers. At the same time end consumers are becoming increasingly knowledgeable and demanding. Consequently, in the current customer-centric business environment, supply chains must be designed to accommodate for real-time responsiveness, uncertain customer orders, globally dispersed locations and diminishing profit margin. Therefore, managing supply chains in today's distributed manufacturing environment has become more complex. To remain competitive in today's global marketplace, organizations must streamline their supply chains. The practice of coordinating design, procurement, flow of goods, services, information and finances, beginning from raw material flows, parts supplier, manufacturer, distributor, retailer, and finally to consumer requires synchronized planning and execution. It is of critical importance to understand how an efficient and effective supply chain is affected by order promising and order fulfillment strategies. In this dissertation, we study two issues related to moving a company from an Available to Promise (ATP) philosophy to a Profitable to Promise (PTP) philosophy: pseudo order promising and coordinating demand fulfillment with supply.

To address the first issue, a single time period analytical ATP model for n confirmed customer orders and m pseudo orders is presented by considering both material constraints and production capacity constraints. At the outset, some analytical properties of the optimal policies are derived and then a particular customer order promising scheme is derived based on the ratio between customer service level and profit. Algorithms presented to solve this problem provide decision makers the effective rule-based mechanism to implement and deploy critical decisions in real-time ATP systems.

To tackle the second issue, we explore two cases: a deterministic demand curve or stochastic demand. In the first case, a constrained non-linear programming model is developed. For the second case a stochastic constrained model is formulated to determine both the quantity of raw materials to purchase from suppliers and demand fulfillment levels for each end product in every location. A simple, yet generic optimal solution structure is derived and a series of numerical studies and sensitivity analysis are carried out to investigate the impact of different factors on profit and fulfilled demand quantity. The objective of this analysis is to understand the implication of factors like price elasticity, path cost, shortage cost and holding cost, along with their interaction effects within the firm's supply chain. Further, we present an analytical model to explore *balanced supply.* Implementation of the resulting generic rules reduces response time and provides managers with insight into the optimal policies that improve demand fulfillment and simultaneously reduce risk through balanced supply.

Further, the firm's optimal response to a one-time-period discount offered by the supplier of a key component is studied under two different strategies: a) Not passing along the discount to its customers b) Offering price discounts to increase the demand for profitable products. Unlike most models of this type in the literature, which define variables in terms of single arc flow, this model employs path variables to directly identify and manipulate profitable and non-profitable products. Numerical experiments based on Toshiba's global notebook supply chain are conducted. Based on the results, an interesting relationship among capacity, fill rate and profit is observed. Furthermore, it is found that passing on temporary component price discounts to end product customers is attractive if the demand curve is "elastic" enough so that a price discount significantly stimulates demand.

Appendices

Appendix 1:

Proof of Theorem 3.1:

It suffices to show that the Hessian matrix of the objective function, $H_{n+1\times n+1}$, is negative definite. Let $E_{n+1\times n+1}$ be the matrix with each entry of 1. Then we have

$$H=\text{diag}\left(\frac{\beta_1}{q_1^2}s''\left(\frac{X_1}{q_1^2}\right), \frac{\beta_2}{q_2^2}s''\left(\frac{X_2}{q_2^2}\right), \ldots, \frac{\beta_n}{q_n^2}s''\left(\frac{X_n}{q_n^2}\right), -(p+h)f(Y)\right)-(g''(z)+m''(z))E$$

where $z=\sum_{i=1}^{n} X_i + Y$.

Let $\theta=(\theta_1,\ldots,\theta_{n+1})^T$,

$$\begin{aligned}
&\theta^T H\theta \\
&= \sum_{i=1}^{n} \frac{\beta_i}{q_i} s''\left(\frac{X_i}{q_i}\right)\theta_i^{\,2} - (p+h)f(Y)\theta_{n+1}^{\,2} - [g''(z)+m''(z)]\left(\sum_{i=1}^{n+1}\theta_i\right)^2 \\
&\le -M\left(\sum_{i=1}^{n+1}\theta_i\right)^2 + LU\sum_{i=1}^{n}\theta_i^{\,2} - (p+h)K\theta_{n+1}^{\,2}
\end{aligned}$$

If $M > 0$, RHS ≤ 0 and RHS $= 0$ only if $\theta = 0$. Hence H will be negative definite.

If $M \le 0$, we have,

$$\begin{aligned}
&\theta^T H\theta \\
&\le -M(n+1)\sum_{i=1}^{n+1}\theta_i^{\,2} + LU\sum_{i=1}^{n}\theta_i^{\,2} - (p+h)K\theta_{n+1}^{\,2} \\
&= [LU - M(n+1)]\sum_{i=1}^{n}\theta_i^{\,2} - [(p+h)K + M(n+1)]\theta_{n+1}^{\,2}
\end{aligned}$$

then, by the assumption of the theorem, H is still negative definitive. Q.E.D.

Appendix 2:

Proof of Remark 3.1:

The second derivative of TB at X_i and Y:

$$\frac{\partial^2 TB}{\partial^2 X_i} = -g''(\sum_{i=1}^{n} X_i + Y) - m''(\sum_{i=1}^{n} X_i + Y) + \frac{\beta_i}{q_i^{\ 2}} s''(\frac{X_i}{q_i})$$

$$\frac{\partial^2 TB}{\partial^2 Y} = -pf(Y) - g''(\sum_{i=1}^{n} X_i + Y) - m''(\sum_{i=1}^{n} X_i + Y) - hf(Y)$$

$$\frac{\partial^2 TB}{\partial X_i \partial X_j} = -g''(\sum_{i=1}^{n} X_i + Y) - m''(\sum_{i=1}^{n} X_i + Y)$$

$$\frac{\partial^2 TB}{\partial X_i \partial Y} = -g''(\sum_{i=1}^{n} X_i + Y) - m''(\sum_{i=1}^{n} X_i + Y)$$

$$\frac{\partial^2 TB}{\partial^2 X_i} = -g''(\sum_{i=1}^{n} X_i + Y) - m''(\sum_{i=1}^{n} X_i + Y) + \frac{\beta_i}{q_i^{\ 2}} s''(\frac{X_i}{q_i}) \le 0$$

$$\frac{\partial^2 TB}{\partial^2 Y} = -pf(Y) - g''(\sum_{i=1}^{n} X_i + Y) - m''(\sum_{i=1}^{n} X_i + Y) - hf(Y) \le 0$$

Now we need to prove the Hessian matrix to be *negative definitive*.

Hessian Matrix:

$$
\begin{pmatrix}
\frac{\partial^2 TB}{\partial^2 X_1} & \frac{\partial^2 TB}{\partial X_1 \partial X_2} & .. & \frac{\partial^2 TB}{\partial X_1 \partial X_n} & \frac{\partial^2 TB}{\partial X_1 \partial Y} \\
\frac{\partial^2 TB}{\partial X_2 \partial X_1} & \frac{\partial^2 TB}{\partial^2 X_2} & .. & \frac{\partial^2 TB}{\partial X_2 \partial X_n} & \frac{\partial^2 TB}{\partial X_2 \partial Y} \\
. & . & . & . & . \\
\frac{\partial^2 TB}{\partial X_n \partial X_1} & \frac{\partial^2 TB}{\partial X_n \partial X_2} & . & \frac{\partial^2 TB}{\partial X_n \partial X_n} & \frac{\partial^2 TB}{\partial X_n \partial Y} \\
\frac{\partial^2 TB}{\partial Y \partial X_1} & \frac{\partial^2 TB}{\partial Y \partial X_2} & . & \frac{\partial^2 TB}{\partial Y \partial X_n} & \frac{\partial^2 TB}{\partial^2 Y}
\end{pmatrix} = HM^{(n+1)\times(n+1)}
$$

For simplicity, let $g(\cdot) = g(\sum_{i=1}^{n} X_i + Y)$, and $m(\cdot) = m(\sum_{i=1}^{n} X_i + Y)$

We can decompose the $HM^{(n+1)\times(n+1)}$ to be:

$HM^{(n+1)\times(n+1)} = H_1 + H_2 + H_3$

where : $H_1 = \begin{pmatrix}
\frac{\beta_1}{q_1^2} s''(\frac{X_1}{q_1^2}) & 0 & 0 & . & 0 \\
0 & \frac{\beta_2}{q_2^2} s''(\frac{X_2}{q_2^2}) & 0 & . & 0 \\
. & . & . & . & . \\
0 & 0 & . & \frac{\beta_n}{q_n^2} s''(\frac{X_n}{q_n^2}) & . \\
0 & 0 & 0 & . & -(p+h)f(Y)
\end{pmatrix}$,which is a *semi-negative definitive.*

$\because \quad g''() + m''() > 0$

$H_2 = - g''(\cdot)(I)$

$H_3 = - m''(\cdot)(I)$,

. $H_2 + H_3 = - [g''(\cdot) + m''(\cdot)](I)$, which is a *negative definitive*.

$HM^{(n+1)\times(n+1)} = H_1 + H_2 + H_3$,

. $HM^{(n+1)\times(n+1)}$ is *negative definitive*. Q.E.D.

Appendix 3:

Proof of Theorem 3.2:

i) Let the sequences $\{ r_i + \frac{\beta_i}{q_i} s'(1) \}$ and $\{ r_i + \frac{\beta_i}{q_i} s'(0) \}$ be non-decreasingly ordered.

By $[Z_{ij} - s'(0)][Z_{ij} - s'(1)] \geq 0$, we can prove for any i, j

$$r_i + \frac{\beta_i}{q_i} s'(1) < r_j + \frac{\beta_j}{q_j} s'(1) \qquad r_i + \frac{\beta_i}{q_i} s'(0) \leq r_j + \frac{\beta_j}{q_j} s'(0) \text{ and}$$

$$r_i + \frac{\beta_i}{q_i} s'(1) > r_j + \frac{\beta_j}{q_j} s'(1) \qquad r_i + \frac{\beta_i}{q_i} s'(0) \geq r_j + \frac{\beta_j}{q_j} s'(0)$$

Then, if necessary, we can adjust the order of elements in the above two sequences to make sure we have simultaneously for any i,

$$r_i + \frac{\beta_i}{q_i} s'(1) \leq r_{i+1} + \frac{\beta_{i+1}}{q_{i+1}} s'(1) \text{ and } r_i + \frac{\beta_i}{q_i} s'(0) \leq r_{i+1} + \frac{\beta_{i+1}}{q_{i+1}} s'(0)$$

ii) For simplicity, let $f^* = g'(\sum_{i=1}^{n} X_i^* + Y^*) + m'(\sum_{i=1}^{n} X_i^* + Y^*)$.

Then there exist unique k and k' such that

$$r_k + \frac{\beta_k}{q_k} s'(1) \leq f^* < r_{k+1} + \frac{\beta_{k+1}}{q_{k+1}} s'(1) \text{ and } r_{k'} + \frac{\beta_{k'}}{q_{k'}} s'(0) < f^* \leq r_{k'+1} + \frac{\beta_{k'+1}}{q_{k'+1}} s'(0)$$

By s'(0)>s'(1)>0, it is trivial to prove k ≥ k'

iii)

For all $i \geq k+1$: $r_i - f(\cdot) + \frac{\beta}{q_i} s'(\frac{X_i}{q_i}) = \lambda_i - \mu_i > 0$,

i.e.: $\lambda_i > 0$, from $\lambda_i (q_i - X_i) = 0$, we have: $X_i = q_i$, $(i \geq k+1)$

For all $i \geq k'$: $r_i - f(\cdot) + \frac{\beta_i}{q_i} s'(\frac{X_i}{q_i}) < r_i - f(\cdot) + \frac{\beta_i}{q_i} s'(0) < 0$

$$\therefore \quad \lambda_i - \mu_i = r_i - f(\cdot) + \frac{\beta}{q_i} s'(\frac{X_i}{q_i}) < 0,$$

i.e. $\mu_i > 0$, from: $\mu \times X_i = 0$, we have: $X_i = 0, (i \geq k')$.

Q.E.D.

Appendix 4:

Proof of Remark 3.2.

Let $\{X_i^*\}$ and Y^* be an arbitrary solution to (3.24)-(3.25).

From $\frac{\beta_i}{q_i}s'(0)+r_i < \frac{\beta_{i+1}}{q_{i+1}}s'(1)+r_{i+1}$, we can have:

$$r_1+\frac{\beta_1}{q_1}s'(1)<r_1+\frac{\beta_1}{q_1}s'(0)<r_2+\frac{\beta_2}{q_2}s'(1)<r_2+\frac{\beta_2}{q_2}s'(0)<....<r_n+\frac{\beta_n}{q_n}s'(1)<r_n+\frac{\beta_n}{q_n}s'(0)$$

For simplicity, let $f(\cdot)=g'(\sum_{i=1}^{n}X_i^*+Y)+m'(\sum_{i=1}^{n}X_i^*+Y)$

Without the loss of the generality, there are two cases:

(A): if $r_k+\frac{\beta_k}{q_k}s'(0)\le f(\cdot)\le r_{k+1}+\frac{\beta_{k+1}}{q_{k+1}}s'(1)$

then: 1)for $i\ge k+1$: $r_i-f(\cdot)+\frac{\beta}{q_i}s'(\frac{X_i}{q_i})=\lambda_i-\mu_i>0$,

i.e.: $\lambda_i>0$, from $\lambda_i(q_i - X_i)=0$, we have: $X_i=q_i$, $(i\ge k+1)$

2) For all $i < k$:

$$r_i-f(\cdot)+\frac{\beta_i}{q_i}s'(\frac{X_i}{q_i})<r_i-f(\cdot)+\frac{\beta_i}{q_i}s'(0)<0$$

. $$\lambda_i-\mu_i=r_i-f(\cdot)+\frac{\beta}{q_i}s'(\frac{X_i}{q_i})<0,$$

i.e. $\mu_i>0$, from: $\mu \times X_i=0$, we have: $X_i=0,(i<k)$

(B) if: $r_k+\frac{\beta_k}{q_k}s'(1)<g'(\cdot)+m'(\cdot)<r_k+\frac{\beta_k}{q_k}s'(0)$

then: 1) for $i\ge k+1$: $r_i-f(\cdot)+\frac{\beta}{q_i}s'(\frac{X_i}{q_i})=\lambda_i-\mu_i>0$, i.e.: $\lambda_i > 0$, from $\lambda_i(q_i - X_i) = 0$, we have: $X_i = q_i$, $(i \geq k + 1)$

2) For all $i \leq k - 1$:

$$r_i - f(\cdot) + \frac{\beta_i}{q_i} s^{'}(\frac{X_i}{q_i}) < r_i - g'(\sum_{i=1}^{n} X_i + Y) - m'(\sum_{i=1}^{n} X_i + Y) + \frac{\beta_k}{q_k} s^{'}(1) < 0 \text{ .}$$

$$\lambda_i - \mu_i = r_i - f(\cdot) + \frac{\beta_i}{q_i} s^{'}(\frac{X_i}{q_i}) < 0 \text{ ,}$$

i.e. $\mu_i > 0$, from: $\mu \times X_i = 0$, we have: $X_i = 0, (i < k)$

for i =k, need to be determined by (3.24) and (3.25).

Combining both (A) and (B), the conclusion is proven. Q.E.D.

Appendix 5:

Proof of Corollary 3.5.1.

After separating the confirmed orders and pseudo orders, the profit function of pseudo orders is:

$$\text{PP} = \sum_{j \in J} p_j E \min(Y_j, u) - \sum_{j \in J} h_j E \max(Y_j - u, 0)$$

Its first order derivative would be:

$$\text{PP'} = \sum_{j \in J} (p_j (1 - F(Y_j) - h_j F(Y_j))$$

$$\text{PP'} = \sum_{j \in J} (p_j - (p_j + h_j) F(Y_j)) \geq 0$$

And its second order derivative is:

$$\text{PP''} = \sum_{j \in J} (-p_j f(Y_j) - h_j f(Y_j)) \text{, which is:}$$

$$\text{PP''} = \sum_{j \in J} -(p_j + h_j) f(Y_j)) \leq 0$$

So we can see, the first order of that function is greater than or equal to 0; and its second order is smaller than or equal to 0; thus the Profit function of Pseudo orders is concave in terms of order quantity. Q. E. D.

Appendix 6:

Proof of Theorem 3.3:

It suffices to show that the Hessian matrix of the objective function, $H_{n+m \times n+m}$, is negative definite. Let $E_{n+m \times n+m}$ be the matrix with each entry of 1. Then we have

$$H=\mathrm{diag}\left(\frac{\beta_1}{q_1^2}s''\left(\frac{X_1}{q_1^2}\right), \frac{\beta_2}{q_2^2}s''\left(\frac{X_2}{q_2^2}\right), \ldots, \frac{\beta_n}{q_n^2}s''\left(\frac{X_n}{q_n^2}\right), -(p_j+h_j)f(Y_j)\right)-(g''(z)+m''(z))E$$

where $z=\sum_{i=1}^{n} X_i + \sum_{j\in J} Y_j$.

Let $\theta=(\theta_1,\ldots,\theta_{n+m})^T$,

$$\theta^T H\theta$$
$$=\sum_{i=1}^{n}\frac{\beta_i}{q_i}s''\left(\frac{X_i}{q_i}\right)\theta_i^{\,2}-\sum_{j\in J}(p_j+h_j)f(Y_j)\theta_{n+j}^{\,2}-[g''(z)+m''(z)]\left(\sum_{i=1}^{n+m}\theta_i\right)^2$$
$$\le -M\left(\sum_{i=1}^{n+m}\theta_i\right)^2+LU\sum_{i=1}^{n}\theta_i^{\,2}-\sum_{j\in J}(p_j+h_j)K_j\theta_{n+j}^{\,2}$$

If $M > 0$, RHS ≤ 0 and RHS $= 0$ only if $\theta = 0$. Hence H will be negative definite.

If $M \le 0$, we have,

$$\theta^T H\theta$$
$$\le -M(n+m)\sum_{i=1}^{n+m}\theta_i^{\,2}+LU\sum_{i=1}^{n}\theta_i^{\,2}-\sum_{j\in J}(p_j+h_j)K_j\theta_{n+j}^{\,2}$$
$$\le [LU-M(n+m)]\sum_{i=1}^{n}\theta_i^{\,2}-\sum_{j\in J}[K+M(n+m)]\theta_{n+j}^{\,2}$$

then, by the assumption of the Theorem, H is still negative definitive. Q.E.D.

Appendix 7:

Proof of Theorem 3.4: We first prove part i)

Let the sequences $\{ r_i + \frac{\beta_i}{q_i} s'(1) \}$ and $\{ r_i + \frac{\beta_i}{q_i} s'(0) \}$ be non-decreasingly ordered.

By $[Z_{ij} + \alpha_{ij} s'(0)][Z_{ij} + \alpha_{ij} s'(1)] \geq 0$, we can prove for any i, j

$$r_i + \frac{\beta_i}{q_i} s'(1) < r_j + \frac{\beta_j}{q_j} s'(1) \qquad r_i + \frac{\beta_i}{q_i} s'(0) \leq r_j + \frac{\beta_j}{q_j} s'(0) \text{ and}$$

$$r_i + \frac{\beta_i}{q_i} s'(1) > r_j + \frac{\beta_j}{q_j} s'(1) \qquad r_i + \frac{\beta_i}{q_i} s'(0) \geq r_j + \frac{\beta_j}{q_j} s'(0)$$

Then, if necessary, we can adjust the order of elements in the above two sequences to make sure we have simultaneously for any i,

$$r_i + \frac{\beta_i}{q_i} s'(1) \leq r_{i+1} + \frac{\beta_{i+1}}{q_{i+1}} s'(1) \text{ and } r_i + \frac{\beta_i}{q_i} s'(0) \leq r_{i+1} + \frac{\beta_{i+1}}{q_{i+1}} s'(0)$$

For simplicity, let $f^* = g'(\sum_{i=1}^{n} X_i^* + \sum_{j=1}^{m} Y_j^*) + m'(\sum_{i=1}^{n} X_i^* + \sum_{j=1}^{m} Y_j^*)$.

Then there exist unique k and k' such that

$$r_k + \frac{\beta_k}{q_k} s'(1) \leq f^* < r_{k+1} + \frac{\beta_{k+1}}{q_{k+1}} s'(1) \text{ and } r_{k'} + \frac{\beta_{k'}}{q_{k'}} s'(0) < f^* \leq r_{k'+1} + \frac{\beta_{k'+1}}{q_{k'+1}} s'(0)$$

By s'(0)>s'(1)>0, it is trivial to prove $k \geq k'$

For all $i \geq k+1$: $r_i - f^* + \frac{\beta}{q_i} s'(\frac{X_i}{q_i}) = \lambda_i - \mu_i > 0$,

i.e.: $\lambda_i > 0$, from $\lambda_i (q_i - X_i) = 0$, we have: $X_i = q_i$, $(i \geq k+1)$

For all $i \leq k'$: $r_i - f^* + \frac{\beta_i}{q_i} s'(\frac{X_i}{q_i}) < r_i - f^* + \frac{\beta_i}{q_i} s'(0) < 0$

. $$\lambda_i - \mu_i = r_i - f^* + \frac{\beta}{q_i} s' (\frac{X_i}{q_i}) < 0 ,$$

i.e. $\mu_i > 0$, from: $\mu \ \times X_i = 0$, we have: $X_i = 0, (i \geq k')$.

Now we will show ii)

From (3-47), we have $F(Y_j) = \frac{p_j + w_j - f^*}{p_j + h_j}$, where w_j>=0. Obviously when $p_j < f^*$, w_j has to be positive which then implies Y_j=0 by (3-47). On the other hand, if $p_j < f^*$, we have Y_j>0 and solves equation (3-50). Apply similar arguments to the case where $p_j = f^*$, we still get Y_j=0. We complete part (ii) by rephrasing our arguments above.

Q.E.D.

Appendix 8:

Proof of Corollary 3.5.3.

Let $\{X_i^*\}$ and $\{Y_j\}^*$ be an arbitrary solution to (3-49)-(3-50).

From $\frac{\beta_i}{q_i}s'(0)+r_i < \frac{\beta_{i+1}}{q_{i+1}}s'(1)+r_{i+1}$, we can have:

$$r_1+\frac{\beta_1}{q_1}s'(1) < r_1+\frac{\beta_1}{q_1}s'(0) < r_2+\frac{\beta_2}{q_2}s'(1) < r_2+\frac{\beta_2}{q_2}s'(0) < .... < r_n+\frac{\beta_n}{q_n}s'(1) < r_n+\frac{\beta_n}{q_n}s'(0)$$

For simplicity, let $f(\cdot)=g'(\sum_{i=1}^{n}X_i^*+\sum_{j\in}Y_j)+m'(\sum_{i=1}^{n}X_i^*+\sum_{j\in}Y_j)$

Without the loss of the generality, there are two cases:

(A): if $r_k+\frac{\beta_k}{q_k}s'(0) \le f(\cdot) \le r_{k+1}+\frac{\beta_{k+1}}{q_{k+1}}s'(1)$

then: 1)for $i \ge k+1$: $r_i - f(\cdot)+\frac{\beta}{q_i}s'(\frac{X_i}{q_i}) = \lambda_i - \mu_i > 0$,

i.e.: $\lambda_i > 0$, from $\lambda_i(q_i - X_i)=0$, we have: $X_i = q_i$, $(i \ge k+1)$

2) For all $i < k$:

$$r_i - f(\cdot)+\frac{\beta_i}{q_i}s'(\frac{X_i}{q_i}) < r_i - f(\cdot)+\frac{\beta_i}{q_i}s'(0) < 0$$

.
$$\lambda_i - \mu_i = r_i - f(\cdot)+\frac{\beta}{q_i}s'(\frac{X_i}{q_i}) < 0,$$

i.e. $\mu_i > 0$, from: $\mu \times X_i = 0$, we have: $X_i = 0, (i < k)$

(B) if: $r_k+\frac{\beta_k}{q_k}s'(1) < g'(\cdot)+m'(\cdot) < r_k+\frac{\beta_k}{q_k}s'(0)$

then: 1) for $i \ge k+1$: $r_i - f(\cdot)+\frac{\beta}{q_i}s'(\frac{X_i}{q_i}) = \lambda_i - \mu_i > 0$, i.e.: $\lambda_i > 0$, from $\lambda_i(q_i - X_i) = 0$, we have: $X_i = q_i$, $(i \geq k+1)$

2) For all $i \leq k-1$:

$$r_i - f(\cdot) + \frac{\beta_i}{q_i} s'(\frac{X_i}{q_i}) < r_i - g'(\sum_{i=1}^{n} X_i^* + \sum_{j\in} Y_j) - m'(\sum_{i=1}^{n} X_i^* + \sum_{j\in} Y_j) + \frac{\beta_k}{q_k} s'(1) < 0 .$$

$$\lambda_i - \mu_i = r_i - f(\cdot) + \frac{\beta_i}{q_i} s'(\frac{X_i}{q_i}) < 0 ,$$

i.e. $\mu_i > 0$, from: $\mu \times X_i = 0$, we have: $X_i = 0, (i < k)$

for i =k, need to be determined by (3-49) and (3-50).

Combining both (A) and (B), the first part is proven.

Now we know that $\sum_{i\in I} X_i \geq q^*$ From part ii) of Theorem 3.4, Y_j>0 only if

$$p_j > g'(\sum_{i\in I} X_i + \sum_{j\in J} Y_j) + m'(\sum_{i\in I} X_i + \sum_{j\in J} Y_j)$$

Since $g''(\sum_{i\in I} X_i + \sum_{j\in J} Y_j) + m''(\sum_{i\in I} X_i + \sum_{j\in J} Y_j) > 0$, we know

$g'(\sum_{i\in I} X_i + \sum_{j\in J} Y_j) + m'(\sum_{i\in I} X_i + \sum_{j\in J} Y_j)$ is increasing.

Thus we have

$$p_j > g'(\sum_{i\in I} X_i + \sum_{j\in J} Y_j) + m'(\sum_{i\in I} X_i + \sum_{j\in J} Y_j) > g'(q^*) + m'(q^*)$$

when $Y_j > 0$. This is equivalent to the second part of the corollary.

Q.E.D.

Appendix 9:

Proof of Theorem 4.1.

Let the sequences $(a_i - l_i)$ non-decreasingly ordered. Without loss of generality, assume we have:

$$a_1 - l_1 \leq \ldots \leq a_k - l_k \leq f'(\sum_{i \in I} Q_i) < a_{k+1} - l_{k+1} \leq \ldots \leq a_n - l_n$$

then for all $i \leq k$, the formula (4.7) is strict negative, so (4.8) must hold, and we have $Q_i = 0$, for all $i \leq k$; for all $i > k$, formula (4.7) = 0, so we have:

$$Q_i = \frac{a_i - l_i - f'(\sum_{i \in I} Q_i)}{2b_i} \quad \text{for all } i > k.$$

Q.E.D.

Appendix 10:

Proof of Remark 4.1.

It suffices to show that the Hessian matrix of the objective function (4.5) is negative definitive.

$$H = -2\times\begin{bmatrix} b_1 & 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & b_{n-1} & 0 \\ 0 & 0 & 0 & 0 & b_n \end{bmatrix} - f''(\sum_{i\in I} Q_i) I \times I^T$$

We need to show: if $-\dfrac{2}{\sum_{i\in I}\dfrac{1}{b_i}} < f''(\cdot) < 0$, then: $X^T HX < 0$.

$$X^T HX = -2\sum_{i\in I} b_i x_i^{\,2} - f''(\sum_{i\in I} Q_i)(\sum_{i\in I} x_i^{\,2})$$

According to Cauchy formula, we have:

$(\sum_{i\in I} x_i^{\,2}) = (\sum_{i\in I}\frac{1}{\sqrt{b_i}}\cdot\sqrt{b_i}\cdot x_i)^2 \le (\sum_{i\in I}\frac{1}{\sqrt{b_i}})(\sum_{i\in I} b_i x_i^{\,2})$, then we easily have:

$$-2\sum_{i\in I} b_i x_i^{\,2} - f''(\sum_{i\in I} Q_i)(\sum_{i\in I} x_i^{\,2}) \le 0$$

Q.E.D.

Appendix 11:

Proof of Remark 4.1

$$W_i(0) = \int_0^0 (S_i - u_i) f(u_i) du_i \ = 0$$

$$W_i'(0) = F(0) = 0$$

and: $\because \ W_i'(S_i) = F(S_i)$

$$\therefore \ W_i''(S_i) = f(S_i) > 0$$

So the $W_i(\cdot)$ is a convex function.

Q.E.D.

Appendix 12 :

Proof of Theorem 4.2.

Let the sequences $(p_i + d_i - l_i)$ non-decreasingly ordered. Without loss of generality, assume we have:

$$p_1 + d_1 - l_1 \le \ldots \le p_k + d_k - l_k \le g'(\sum_{i \in I} S_i) < p_{k+1} + d_{k+1} - l_{k+1} \le \ldots \le p_n + d_n - l_n$$

then for all $i \le k$, the formula (4.12) is strict positive; to let (4.13) hold, we have $S_i = 0$ for all $i \le k$;

On the other hand, for all $i > k$, when $g'(\sum_{i \in I} S_i) < p_{k+1} + d_{k+1} - l_{k+1} \le \ldots \le p_n + d_n - l_n$,

we have: $W_i^{'}(S_i) > 0$. We also know: $W_i(\cdot)$ is convex and $W_i^{'}(0) = 0$,

$$\therefore\ S_i > 0.$$

$$\because\ S_i > 0 \ \text{ and } \ \lambda_i S_i = 0$$

$$\therefore\ \lambda_i = (p_i + d_i + h_i) W_i^{'}(S_i) + g^{'}(\sum_i S_i) - (p_i + d_i - l_i) = 0$$

so we have: $W_i'(S_i) = F(S_i) = \dfrac{(p_i + d_i - l_i) - g'(\sum_{i \in I} S_i)}{p_i + d_i + h_i}$ for all $i > k$.

Q.E.D.

Appendix 13.

$\because$ $W_i(\cdot)$ is convex function, . $W_i'(\cdot)$ is a increasing function which goes from 0 when S_i=0 to 1 when S_i= infinity

$\because W_i'(0) = 0$, and

$$W_i'(S_i) < \frac{(p_i + d_i - l_i)}{p_i + d_i + h_i} < 1$$

. S_i has upper bound.

. $Q = \sum_i S_i$ has upper bound.

Q.E.D.

Appendix 14:

Proof of Remark 4.2.

It suffices to show that the Hessian matrix of the objective function

$$\sum_i (p_i + h_i + d_i)\int_0^{S_i}(S_i - u_i) f(u_i) du_i + g(\sum_i S_i) - \sum_i (p_i + d_i - l_i) S_i$$

is positive definitive. We have:

$$H = \begin{bmatrix} (p_1+h_1+d_1)f(S_1) & 0 & 0 & 0 & 0 \\ 0 & (p_2+h_2+d_2)f(S_2) & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & (p_{n-1}+h_{n-1}+d_{n-1})f(S_{n-1}) & 0 \\ 0 & 0 & 0 & 0 & (p_n+h_n+d_n)f(S_n) \end{bmatrix}$$
$$+ g''(\sum_{i\in I} S_i) I \times I^T$$

Now we need to show: if $\sum_i \frac{1}{(p_i + h_i + d_i)} > UF \times UG$, then: $X^T HX > 0$; where UF

and UG is the upper bound of $f(\cdot)$ and - $g''(\cdot)$ respectively.

$$X^T HX = \sum_{i\in I} (p_i + h_i + d_i) f(S_i) x_i^{\,2} + g''(\sum_{i\in I} S_i)(\sum_{i\in I} x_i^{\,2})$$

According to Cauchy formula, we have:

$$\begin{aligned}(\sum_{i\in I} x_i^{\,2}) &= (\sum_{i\in I} \frac{1}{\sqrt{(p_i + h_i + d_i) f(S_i)}} \cdot \sqrt{(p_i + h_i + d_i) f(S_i)} \cdot x_i)^2 \\ &\le (\sum_{i\in I} \frac{1}{\sqrt{(p_i + h_i + d_i) f(S_i)}})(\sum_{i\in I} \sqrt{(p_i + h_i + d_i) f(S_i)} x_i^{\,2})\end{aligned},$$

then we easily have:

$$\sum_{i\in I} (p_i + h_i + d_i) f(S_i) x_i^{\,2} > -g''(\sum_{i\in I} S_i)(\sum_{i\in I} x_i^{\,2})$$

$$. \sum_{i\in I} (p_i + h_i + d_i) f(S_i) x_i^{\,2} + g''(\sum_{i\in I} S_i)(\sum_{i\in I} x_i^{\,2}) > 0 \qquad \text{Q.E.D.}$$

Appendix 15:

Proof of Remark 4.3.

Proof: If there exists optimal solution for problem (4.9)-(4.10), then Hessian Matrix of the objective function

$$\sum_i (p_i + h_i + d_i) \int_0^{S_i} (S_i - u) f(u) du + g(\sum_i S_i) - \sum_i (p_i + d_i - l_i) S_i$$

is positive definite, and its Hessian matrix can be written as:

$$H = \begin{pmatrix} (p_1 + h_1 + d_1) f(S_1) & 0 & 0 & . & 0 \\ 0 & (p_2 + h_2 + d_2) f(S_2) & 0 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & . & . & . \\ 0 & 0 & 0 & . & (p_n + h_n + d_n) f(S_n) \end{pmatrix} +$$

$$g''(\sum_i S_i) \begin{pmatrix} 1 & 1 & 1 & . & 1 \\ 1 & 1 & 1 & . & 1 \\ . & . & . & . & . \\ 1 & 1 & . & . & 1. \\ 1 & 1 & 1 & . & 1 \end{pmatrix}$$

H being positive definite means, for any $X = (X_1, X_2, ... X_n) \neq 0$, $XHX^T > 0$.

We have:

$$XHX^T = \sum_i (p_i + h_i + d_i) f(S_i) X_i^2 + g''(\sum_i S_i)(\sum_i X_i)^2$$

Using Cauchy inequality, we have:

$$(\sum_i \frac{1}{(p_i + h_i + d_i) f(S_i)})(\sum_i (p_i + h_i + d_i) f(S_i) X_i^2) \geq (\sum_i X_i)^2$$

So:

$$XHX^T \geq (\frac{1}{\sum_i \frac{1}{(p_i + h_i + d_i) f(S_i)}} + g''(\sum_i S_i))(\sum_i X_i)^2$$

And Positive Definite implies:

$$\frac{1}{\sum_i \frac{1}{(p_i + h_i + d_i) f(S_i)}} + g''(\sum_i S_i) > 0$$

In Theorem 4.2, we have optimal solution:

$$S_i = 0 \qquad \text{for all } i \le k,$$

(4.12)

$$F(S_i) = \frac{(p_i + d_i - l_i) - g'(\sum_{i \in I} S_i)}{p_i + d_i + h_i} \qquad \text{for all } i > k.$$

(4.13)

So from (4.13), we have:

$$S_i = F^{-1}(1 - \frac{h_i + l_i + g'(\sum_{i \in I} S_i)}{p_i + d_i + h_i})$$

$$Q = \sum_i S_i = \sum_{i \in K^*} F^{-1}\left(1 - \frac{h_i + l_i + g'(Q)}{p_i + d_i + h_i}\right)$$

K^* is the set of i such that (4.13) holds.

Without loss of generality, we assume $1 \in K^*$, Now we look at $\frac{\partial Q}{\partial p_1}$:

Define function $R(Q)$ as: $R(Q) = \sum_{i \in K^*} F^{-1}\left(1 - \frac{h_i + l_i + g'(Q)}{p_i + d_i + h_i}\right) - Q$

$\frac{\partial Q}{\partial p_1} = -\frac{\frac{\partial R}{\partial p_1}}{\frac{\partial R}{\partial Q}}$, and let's look as signs of $\frac{\partial R}{\partial p_1}$ and $\frac{\partial R}{\partial Q}$:

$$\frac{\partial R}{\partial p_1} = \frac{\frac{h_1 + l_1 + g'(Q)}{(p_1 + d_1 + h_1)^2}}{f\left(F^{-1}\left(1 - \frac{h_i + l_i + g'(Q)}{p_i + d_i + h_i}\right)\right)} \geq 0$$

$$\frac{\partial R}{\partial Q} = \sum_{i \in K^*} \frac{-\frac{g''(Q)}{p_i + d_i + h_i}}{f\left(F^{-1}\left(1 - \frac{h_i + l_i + g'(Q)}{p_i + d_i + h_i}\right)\right)} - 1$$

$$= -\sum_{i \in K^*} \frac{g''(Q)}{f(S_i)(p_i + d_i + h_i)} - 1,$$

$$\leq -\sum_{1 \leq i \leq n} \frac{g''(Q)}{f(S_i)(p_i + d_i + h_i)} - 1$$

$$< 0$$

Therefore, $\frac{\partial Q}{\partial p_1} = -\frac{\frac{\partial R}{\partial p_1}}{\frac{\partial R}{\partial Q}} \geq 0$.

so $Q = \sum_i S_i$ is increasing of p_i.

Q.E.D.

Appendix 16.

Proof of Theorem 4.3.

Proof: suppose the optimal solution is $(x_1, x_2, \ldots. x_n)$, where $x_1 = 0$. We will construct a new solution ($x_1', x_2', \ldots. x_n'$). If we can show this new solution is better than the old one, we complete our proof by contraposition. The construction of new solution is as follows:

For $2 \le i \le n$,

we have: $x_i' = 0$ if $x_i = 0$; $x_i' = l_i$ if $x_i \ge l_i$,

For $i = 1$,

we have: $x_1' = Q - \sum_{1 < i \le n} x_i'$.

Since we assume $Q > \sum_i l_i$, it is trivial to show that $x_1' \ge l_1$.

Now we compare the objective value of the two solutions. Let z and z' as the objective values associated with old solution and new solution respectively. We only need to show that: $z' < z$.

We have: $z - z' = \sum_i (f(x_i) - f(x_i') + C_i(x_i - x_i')) + B_1$, (note the reason we have time B_1 is because $x_1 = 0$ and $x_1' \ge l_1$)

We denote $\delta_i = x_i - x_i'$, and we have:

$\sum_i \delta_i = 0$,

$\delta_1 < 0$,)

$\delta_i \ge 0$ for $1 \le i \le n$

Applying Mean-Value Theorem to the term $f(x_i)$ - $f(x_i')$ yields:

$$z - z' = \sum_i (\xi_i \delta_i + C_i \delta_i) + B_1$$

where $\xi_i = \dfrac{f(x_i) - f(x_i')}{\delta_i}$ and $0 < f'(Q) \le \xi_i \le f'(0)$.

Plugging $\sum_i \delta_i = 0$ into the above formula, we have:

$$z - z' = \sum_{1<i\le n} (C_i - C_1 + \xi_i - \xi_1)\delta_i + B_1 \ge \sum_{1<i\le n} (C_2 - C_1 + f'(Q) - f'(0))\delta_i + B_1 \ge B_1 > 0$$

Q.E.D.

Appendix 17

Proof of Theorem 4.4.

suppose some x_i, say $x_2 > l_2$, we will construct a new solution which turns out to be better than the old one. Now we know $x_1 \geq l_1$ from Theorem 1. To construct a new solution, we keep all x_i except x_1 and x_2 unchanged in the new solution. However, $x_2' = l_2$, and $x_1' = x_1 + x_2 - l_2$.

It is trivial to show the new solution is feasible. So we have:

$$z - z' = f(x_1) + f(x_2) - f(x_1') - f(x_2') + C_1\delta_1 + C_2\delta_2$$

where $0 < \delta_2 = -\delta_1$, still applying Mean-Value Theorem, we have:

$$z - z' = (C_2 - C_1 + \xi_2 - \xi_1)\delta_2 > 0$$

Q.E.D.

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